



Pre-Leaving Certificate Examination 2026

Mathematics

(Option B)

Paper 1

Higher Level

2 hours 30 mins

300 marks

Name:
School:
Address:
Class:
Teacher:

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

Running total	
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Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer questions as follows:

- **any five** questions from Section A – Concepts and Skills
- **any three** questions from Section B – Contexts and Applications

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet.

If you need to use it, label any extra work clearly with the question number and part.

The superintendent may give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination.

In general, diagrams are not to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

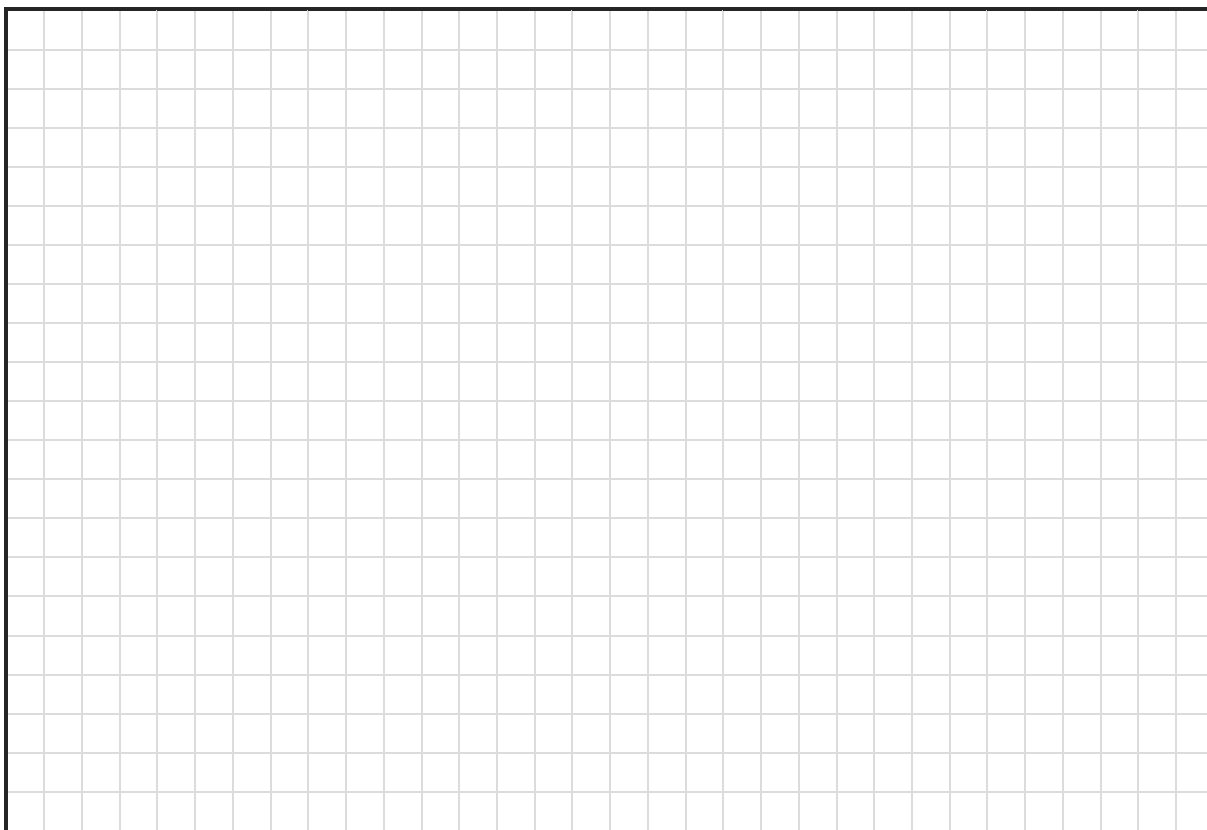
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Answer **any five questions** from this section.

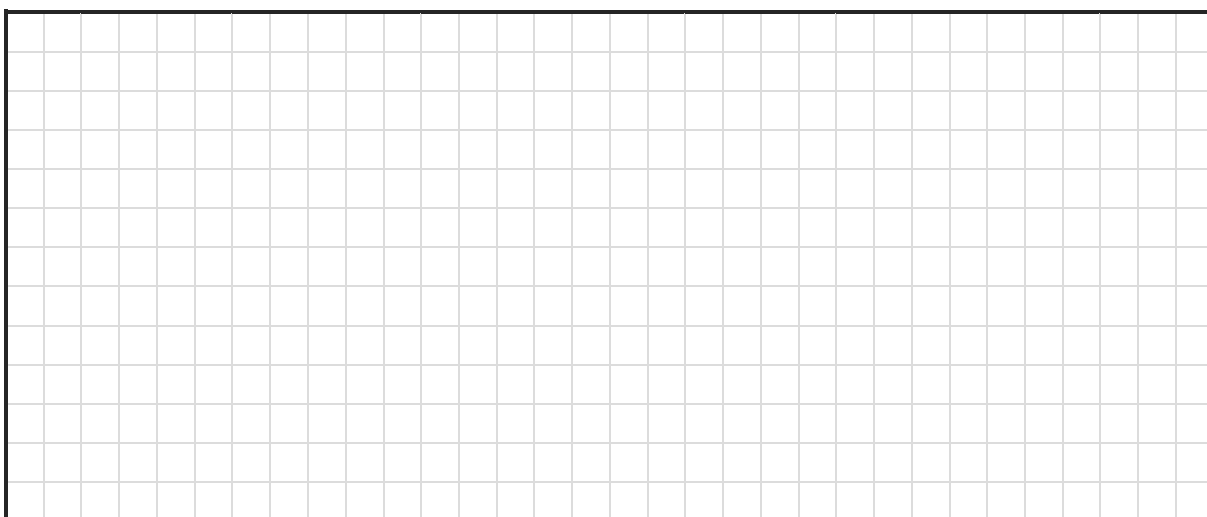
Question 1**(30 marks)**

- (a) Write the following as a single fraction, in its simplest form.

$$\frac{6}{x^2 - 2x - 8} + \frac{1}{x^2 + 5x + 6}$$



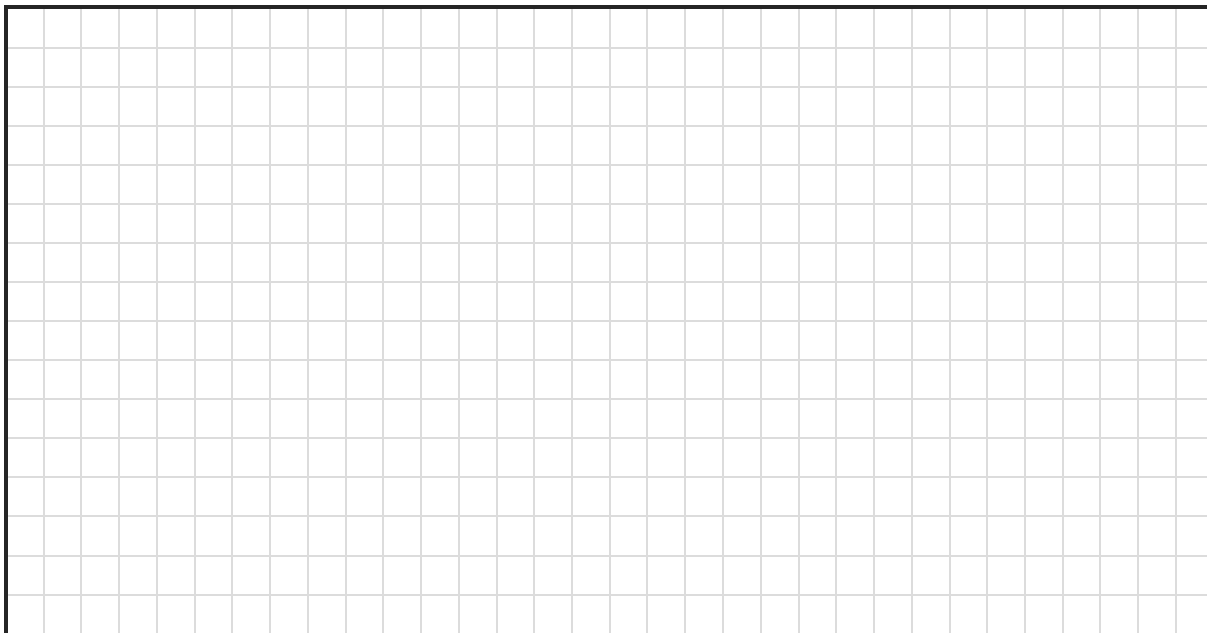
- (b) Fully factorise $18x^2 - 8y^2$.



- (c) The terms of the binomial expansion of

$$\left(3x + \frac{1}{x}\right)^6$$

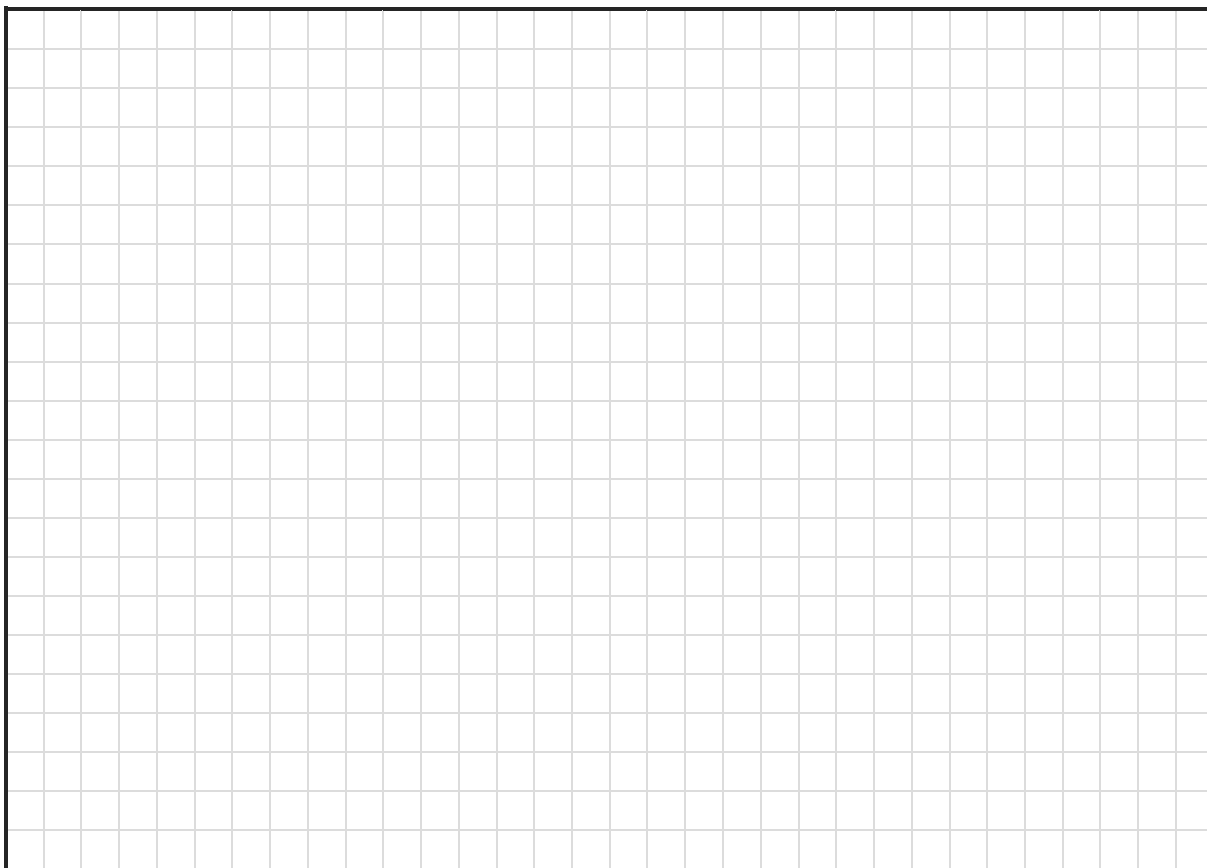
are written in **descending** powers of x , where $x \in \mathbb{R}$.
Find the value of the term independent of x .



- (d) Solve the following simultaneous equations:

$$2x + y = 3$$

$$2x^2 - xy = 10$$

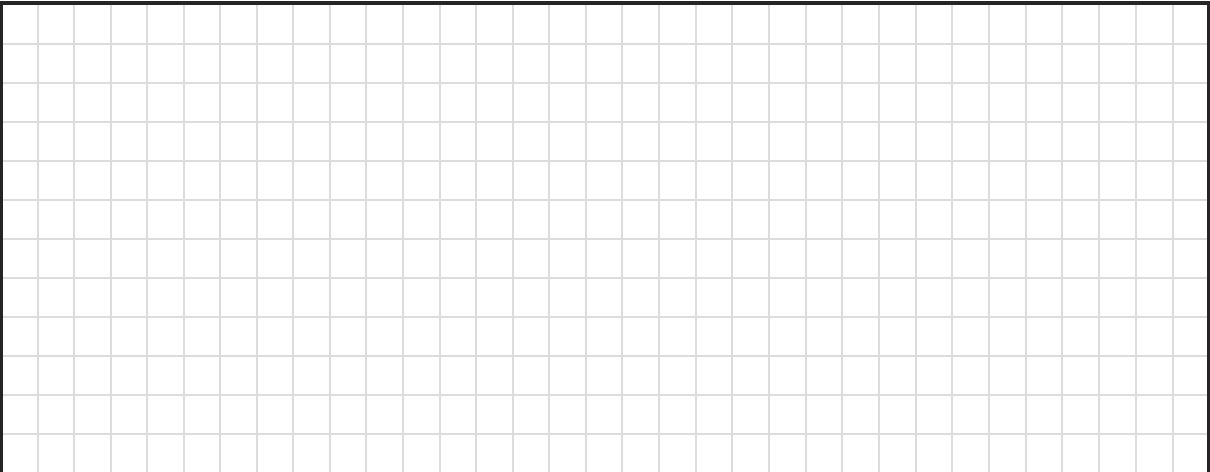


Question 2**(30 marks)**

- (a) The n th term of a sequence is defined by

$$T_n = \frac{1}{2}n(n+1)$$

- (i) Find the value of $(T_1 + T_2)$ and $(T_2 + T_3)$.



$(T_1 + T_2) =$ $(T_2 + T_3) =$

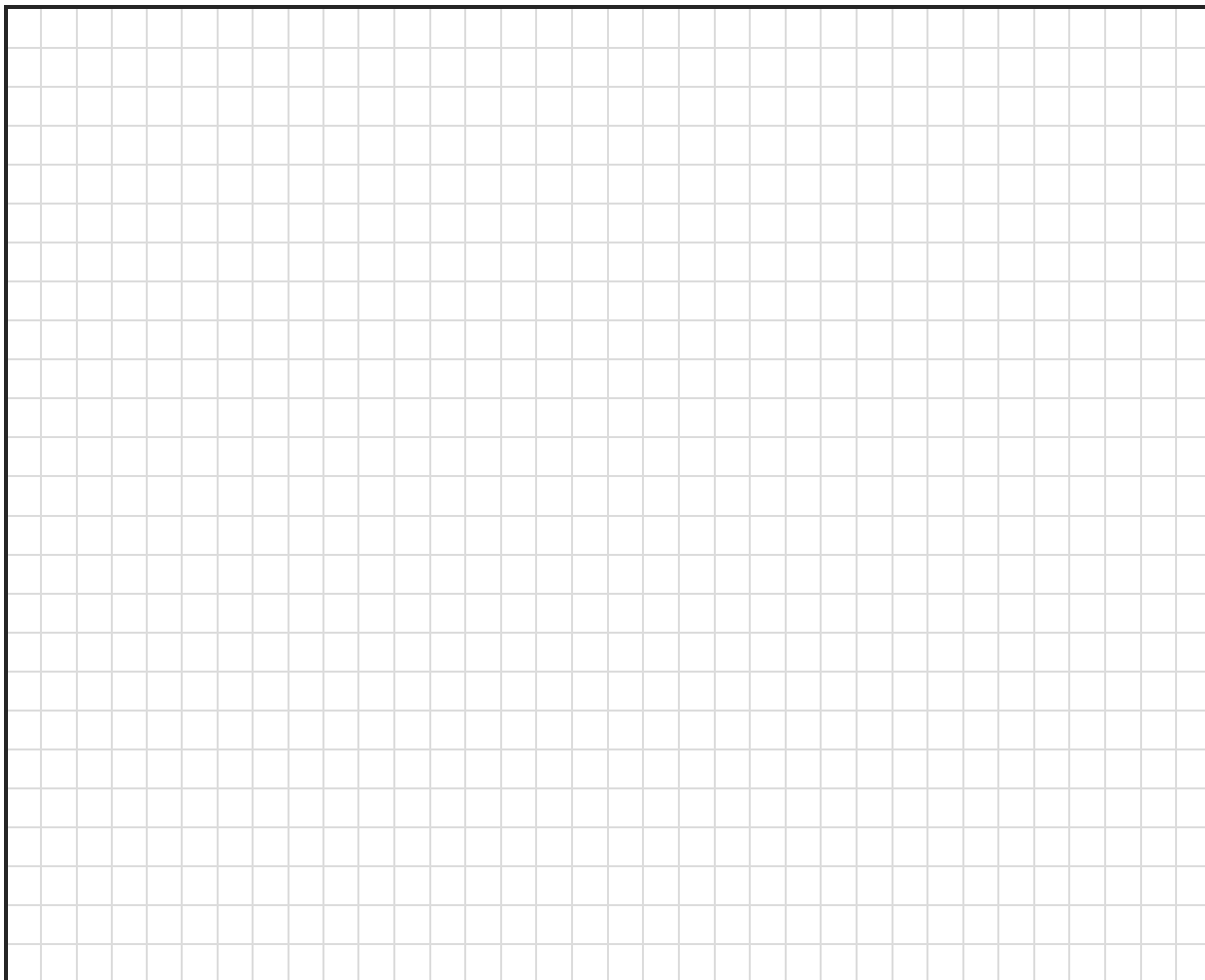
- (ii) By simplifying the expression $T_n + T_{n+1}$, show that the sum of any two consecutive terms is a perfect square.



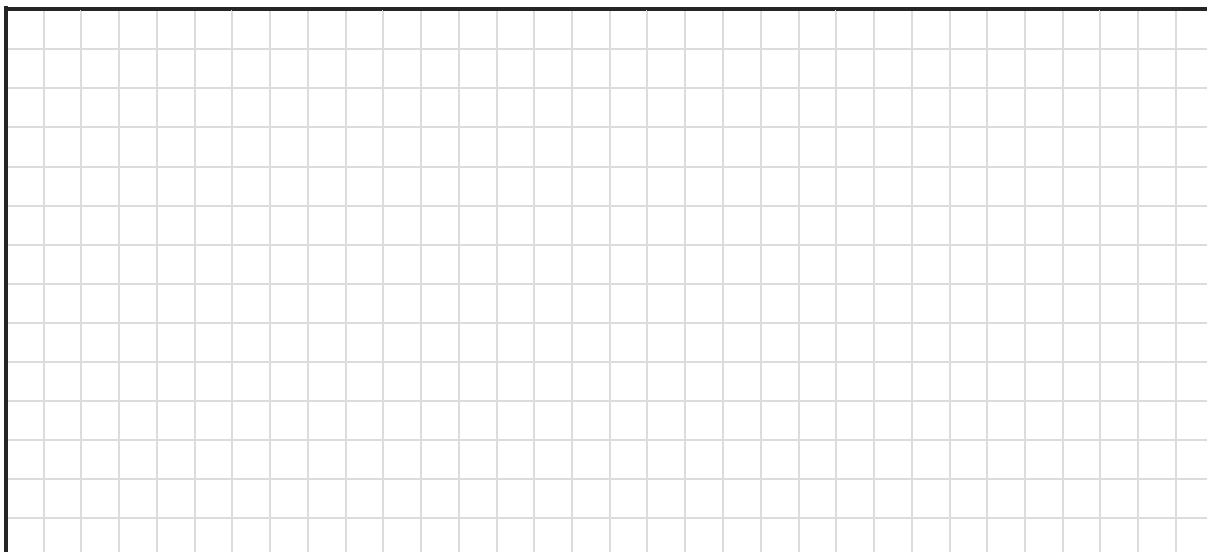
(b) A quadratic sequence has the following properties:

- The second difference is 10
- The first two terms are equal, i.e. $T_1 = T_2$
- $T_1 + T_2 + T_3 = 28$

(i) Show that the general term of the sequence is $T_n = 5n^2 - 15n + 16$.



(ii) Is 216 a term of the sequence? Justify your answer showing necessary calculations.



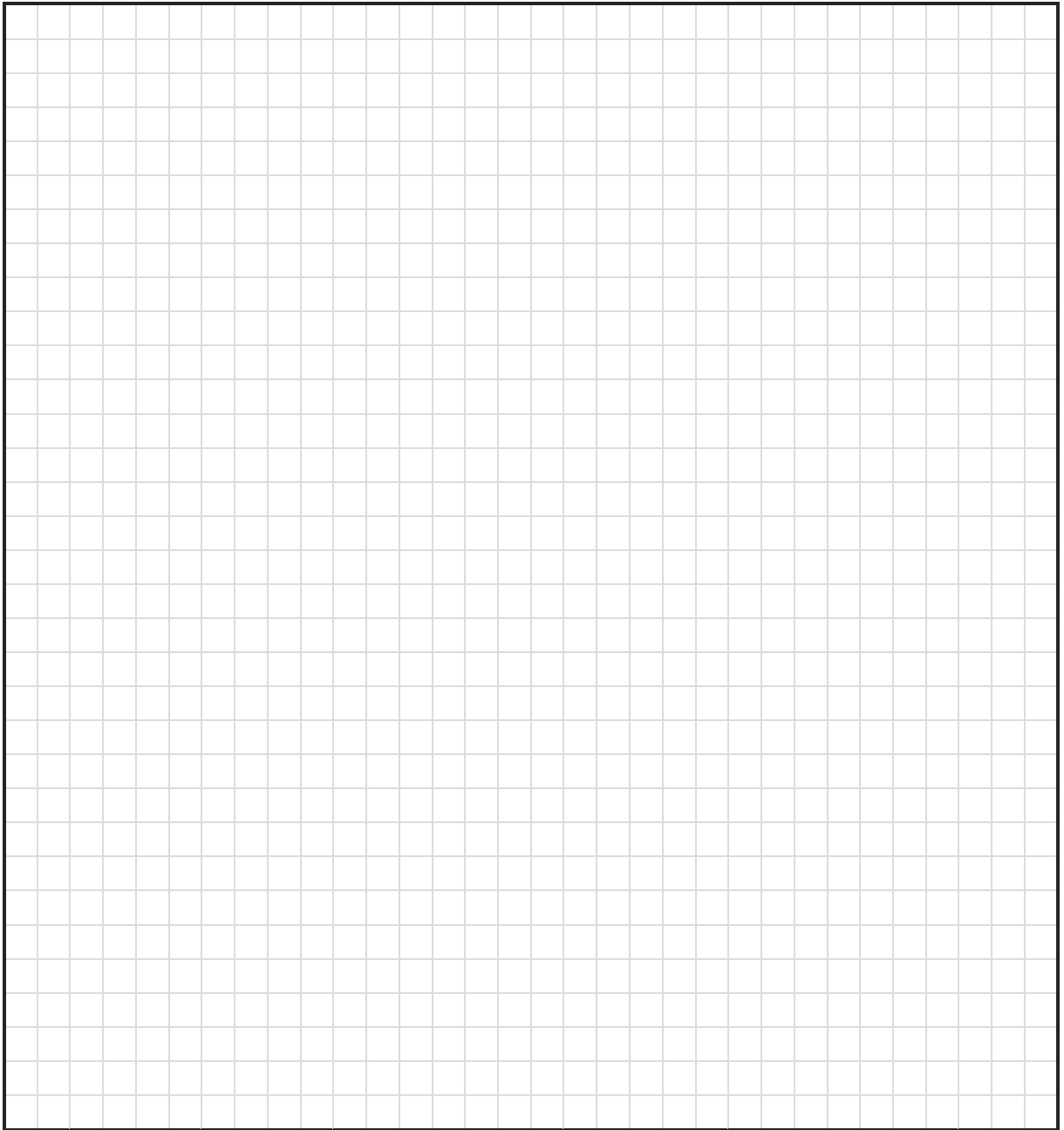
Question 3**(30 marks)**

In this question, $i^2 = -1$.

- (a) Find the two possible values of $\sqrt{-5 - 12i}$ in the form $a + bi$, where $a, b \in \mathbb{R}$.

- (b) Show that $2 - 3i$ is a root of the equation $z^3 - 2z^2 + 5z + 26 = 0$, and hence, find the other two roots of the equation.

- (c) Express $\left(\frac{1 + \sqrt{3}i}{1 - \sqrt{3}i}\right)^{10}$ in the form $a + bi$, where $a, b \in \mathbb{R}$.



Question 4**(30 marks)**

- (a) Find the integral

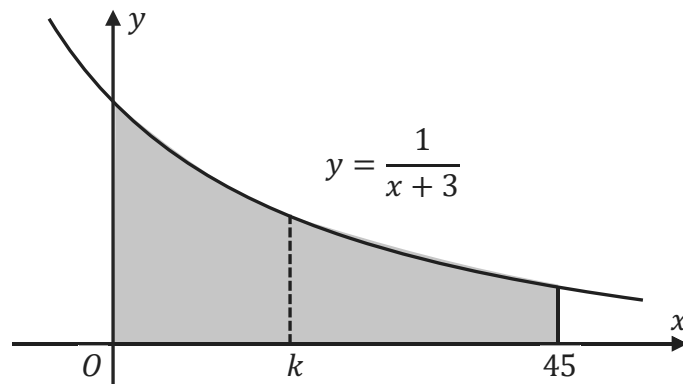
$$\int \sin 8x \, dx$$

- (b) The function $f(x)$ is defined for $x \in \mathbb{R}$ by:

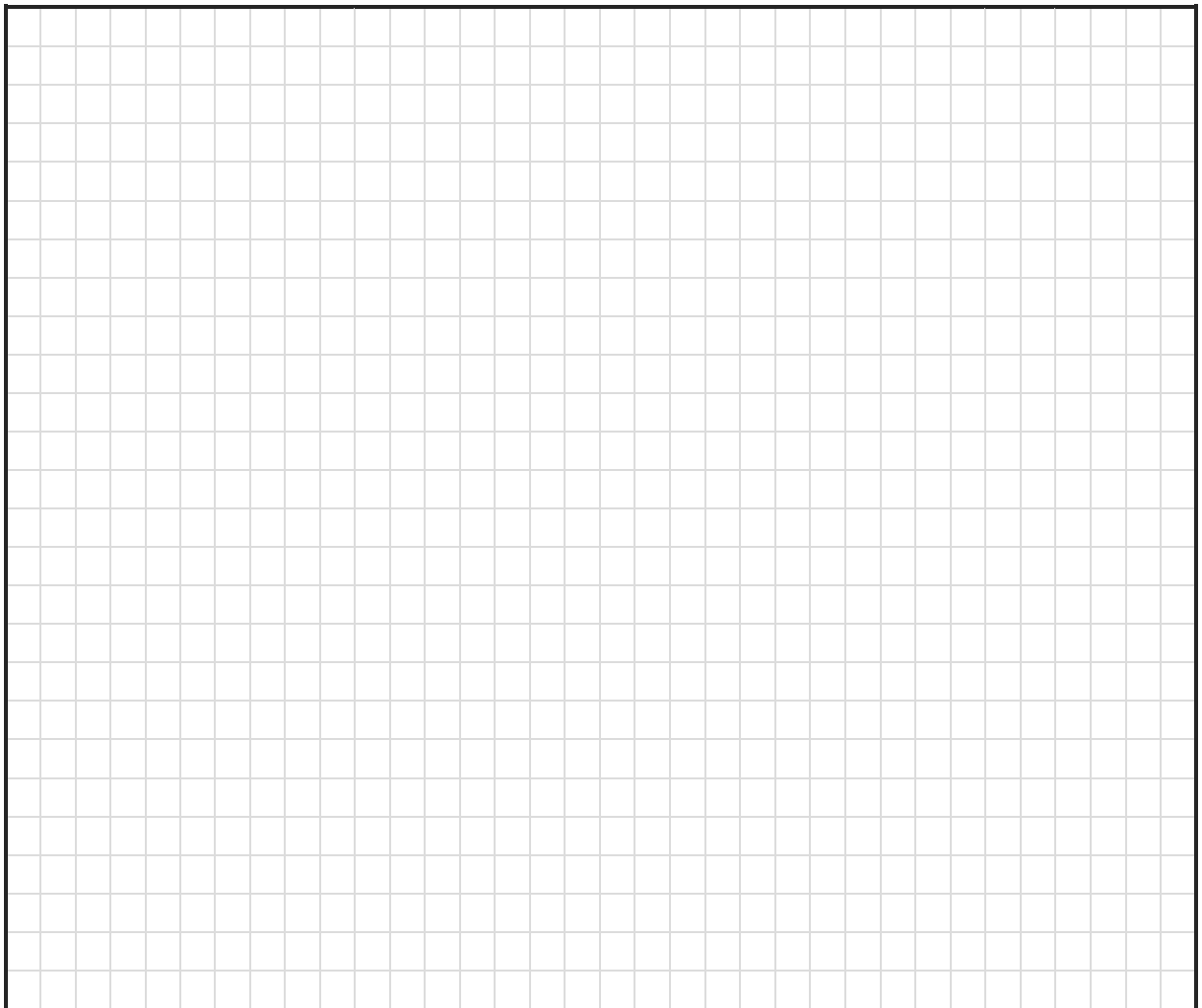
$$f(x) = \frac{e^x}{1 + e^x}$$

Using the Quotient Rule, find $f'(x)$, the differential of $f(x)$.
Give your answer in the simplest form.

- (c) The diagram below shows the region bounded by $y = \frac{1}{x+3}$, the x -axis, the y -axis and $x = 45$. The region is divided into two parts of equal area by the line $x = k$, where $k \in \mathbb{Z}$.



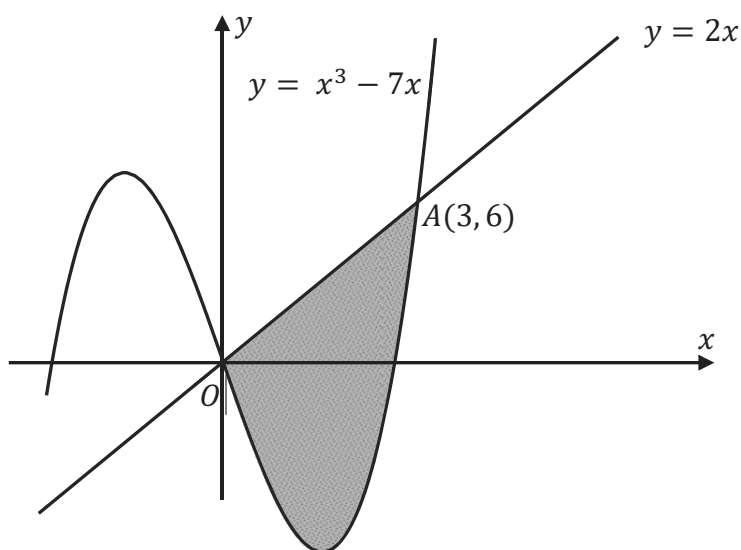
What is the value of the integer k , given that the two parts have equal areas?



Question 5**(30 marks)**

The shaded region is enclosed by the curve $y = x^3 - 7x$ and the line $y = 2x$, as shown in the diagram.

The line $y = 2x$ intersects the curve $y = x^3 - 7x$ at $O(0, 0)$ and $A(3, 6)$.

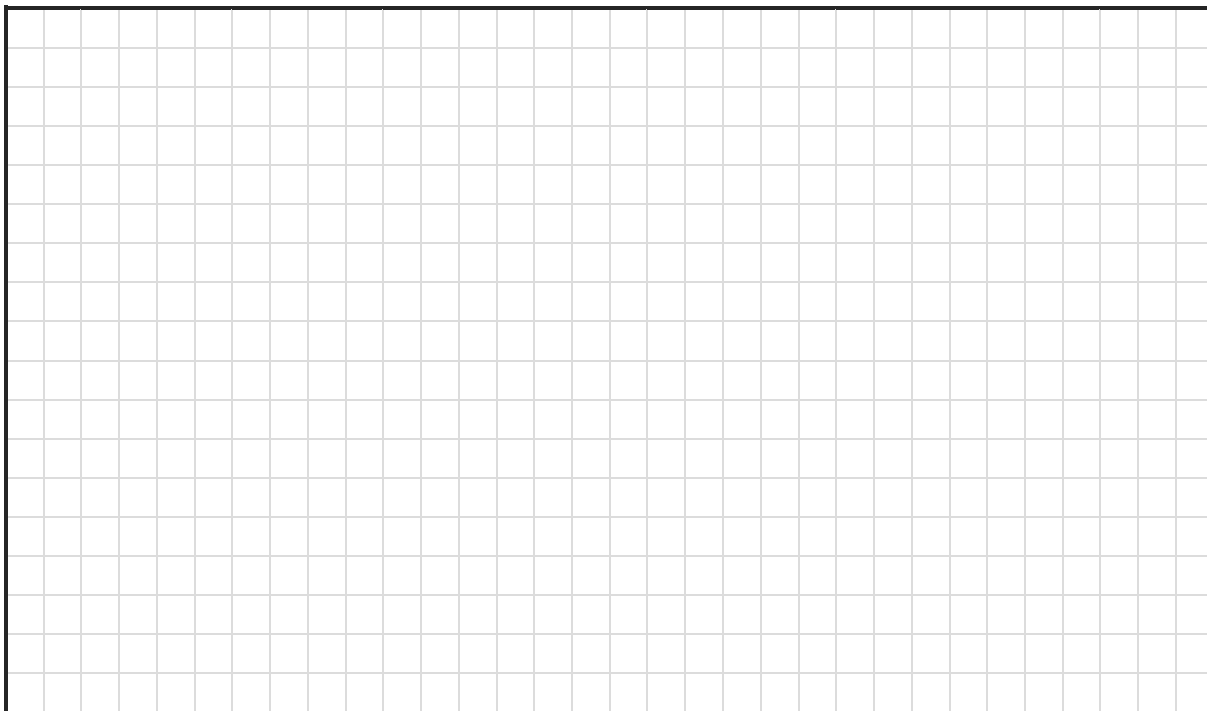


- (a) Find the area of the shaded region in square units.

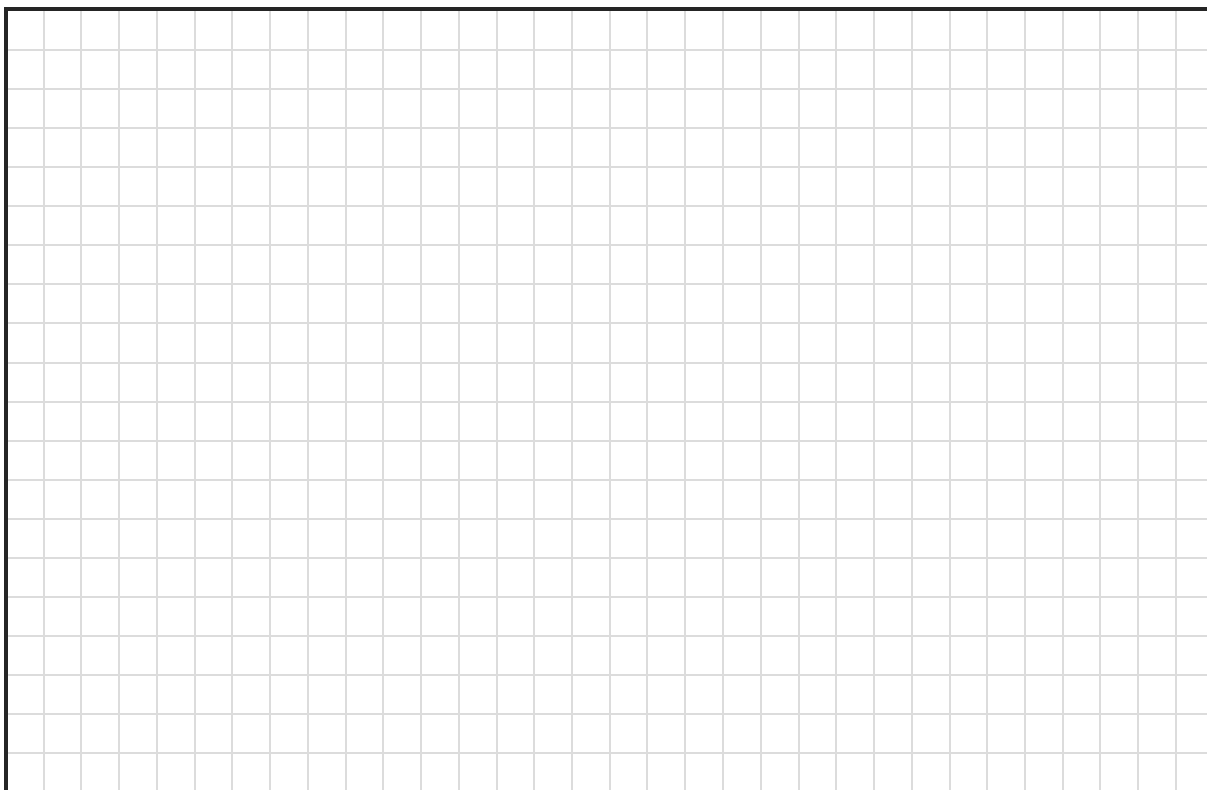


- (b) The point B is chosen on the curve $y = x^3 - 7x$ so that the tangent at B is parallel to the line $y = 2x$ and the x -coordinate of B is positive.

(i) Show that the coordinates of B are $(\sqrt{3}, -4\sqrt{3})$.



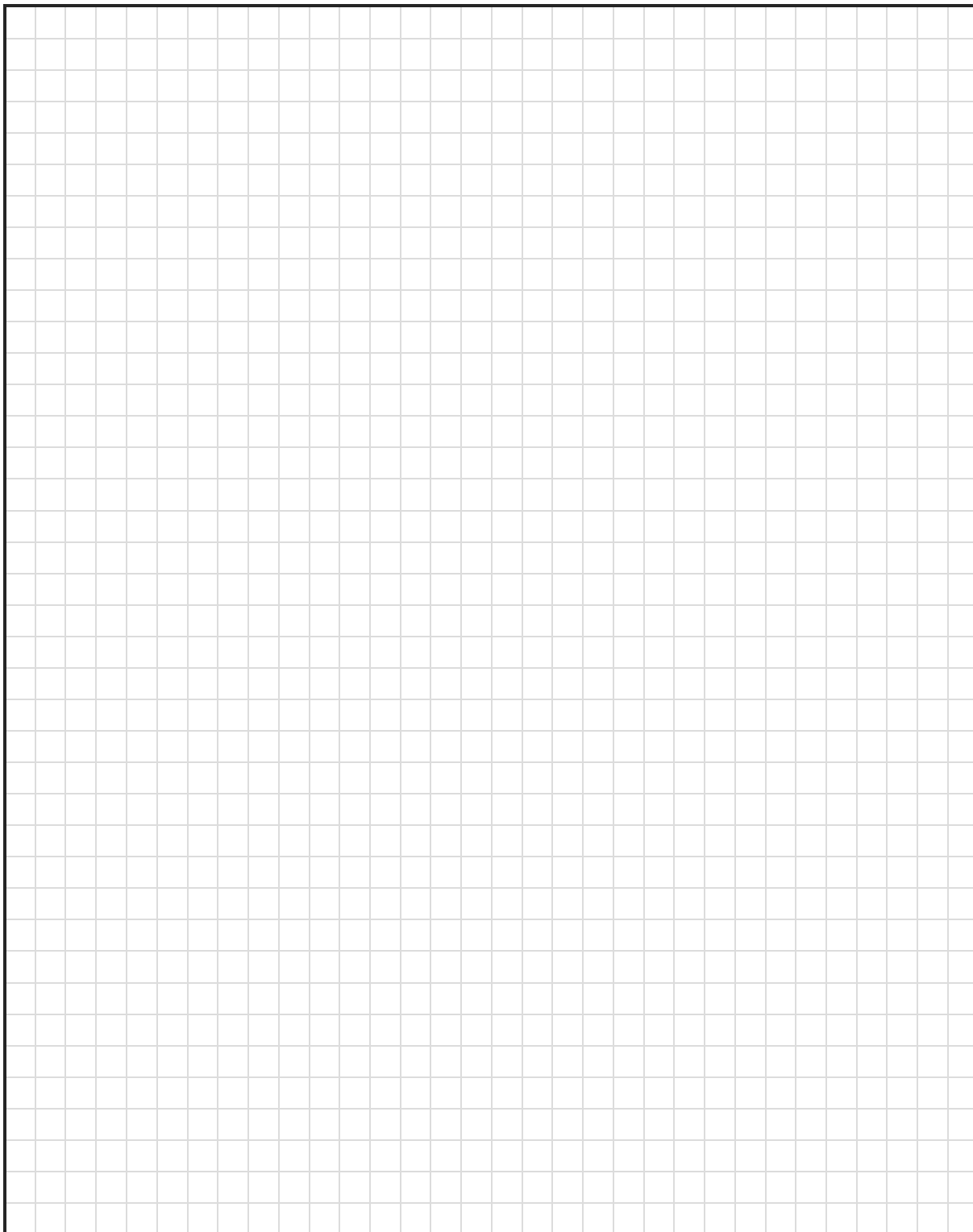
(ii) Find the **exact** area of triangle AOB , in square units.



Question 6**(30 marks)**

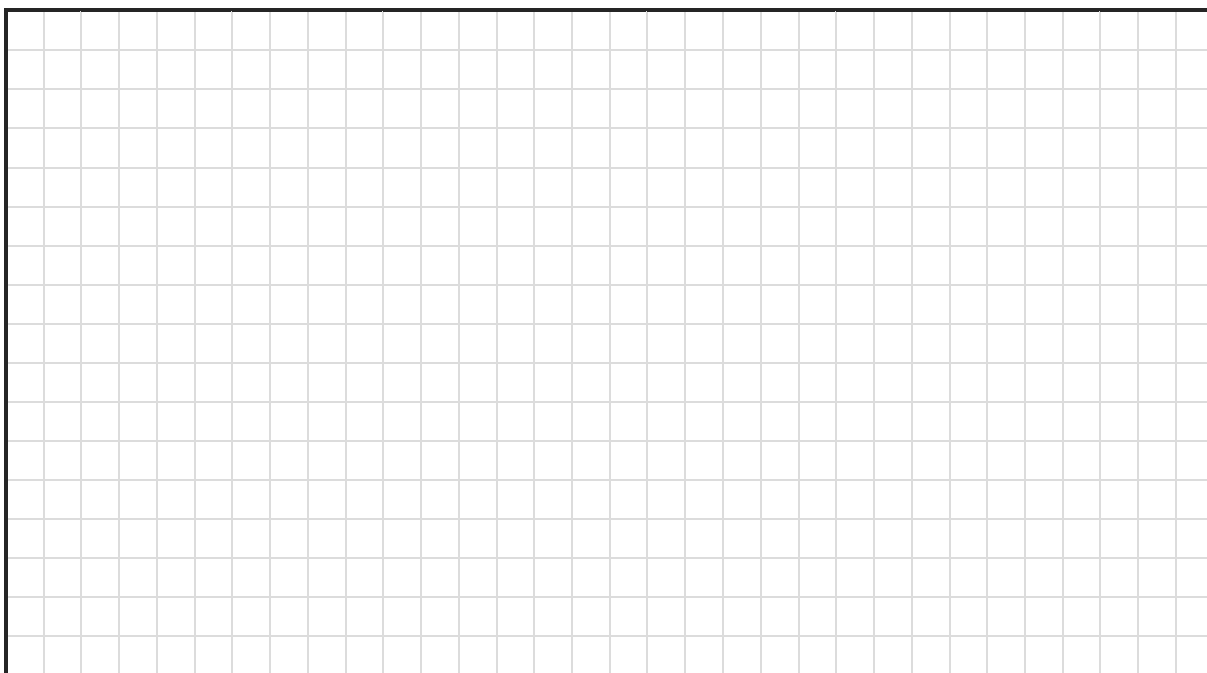
- (a) Use **induction** to prove that the sum of the first n natural numbers can be found using the formula:

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$



- (b)** Hence, or otherwise, prove that the sum of the first n even natural numbers

$$2 + 4 + 6 + \cdots + 2n = n^2 + n$$



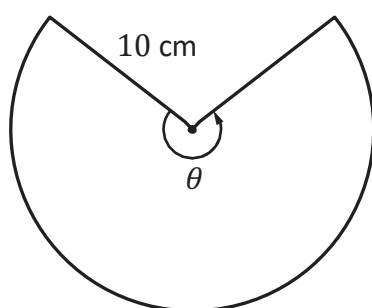
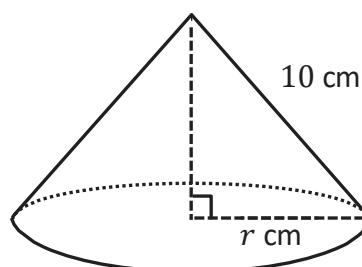
- (c)** Using the results from **(a)** and **(b)** above, find an expression for the sum of the first n odd natural numbers in its simplest form.



Answer **any three questions** from this section.

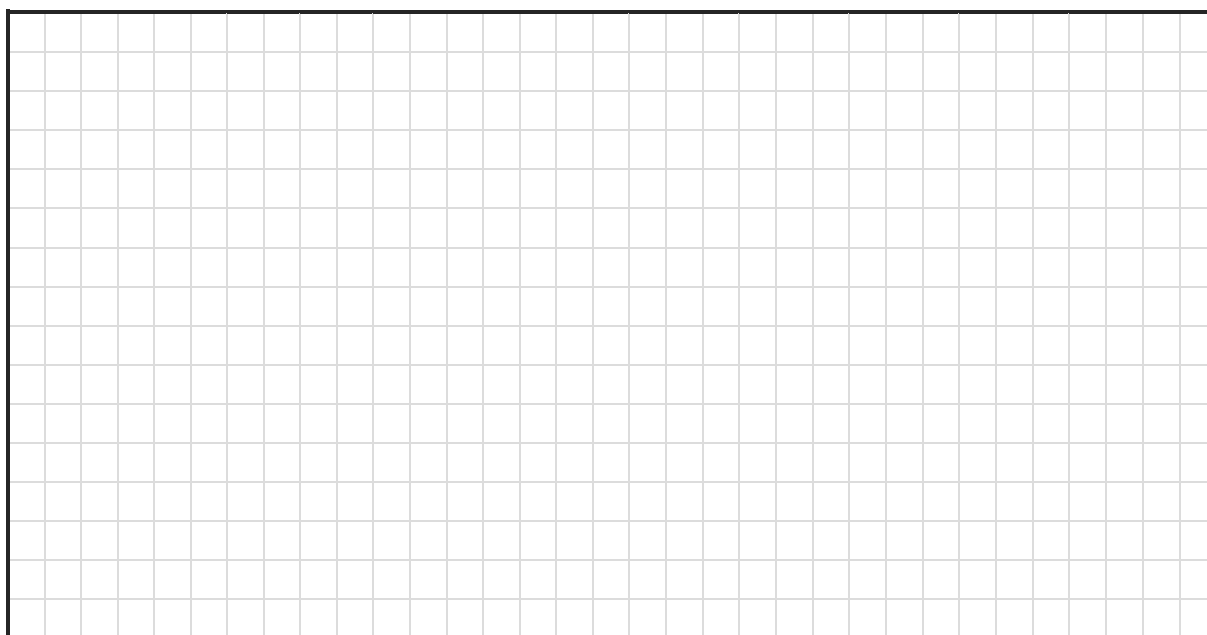
Question 7**(50 marks)**

A sector with radius 10 cm and angle θ is used to form the curved surface of a cone with base radius r cm, as shown in the diagram.



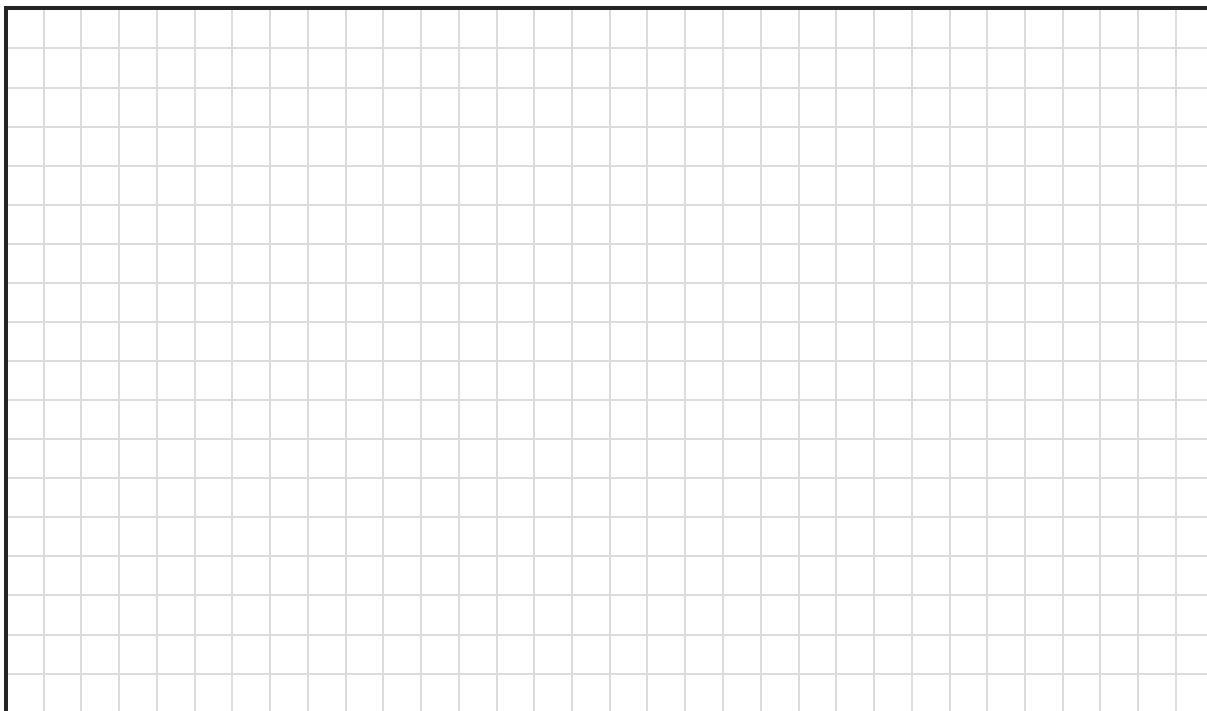
- (a) By expressing r^2 in terms of h , show that the volume of the cone, $V \text{ cm}^3$, can be written as

$$V = \frac{\pi}{3} h(100 - h^2)$$

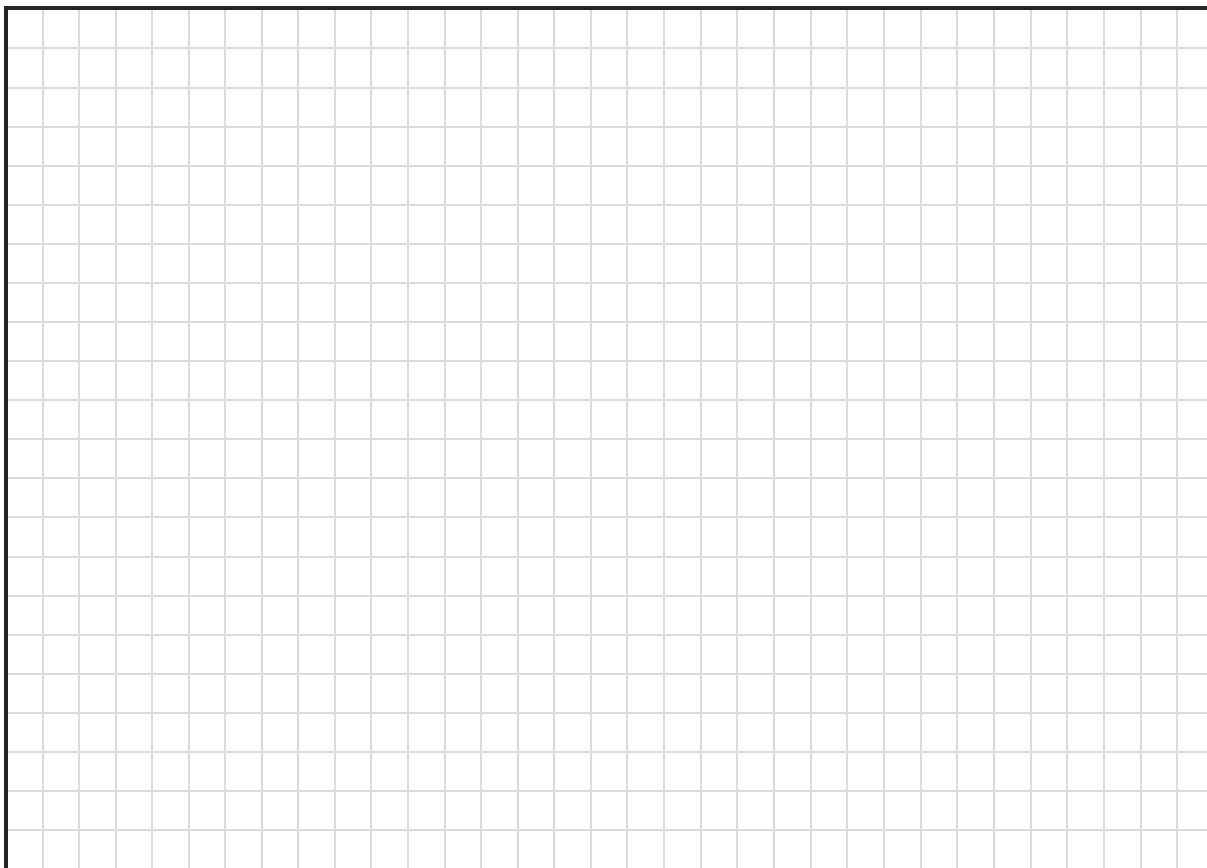


(b) There are **two positive** values of h for which the volume of the cone is $\frac{192\pi}{3}$.

(i) Show that one of the values of h is 2 cm.



(ii) Find the non-integer value of h , in cm, correct to 2 decimal places.

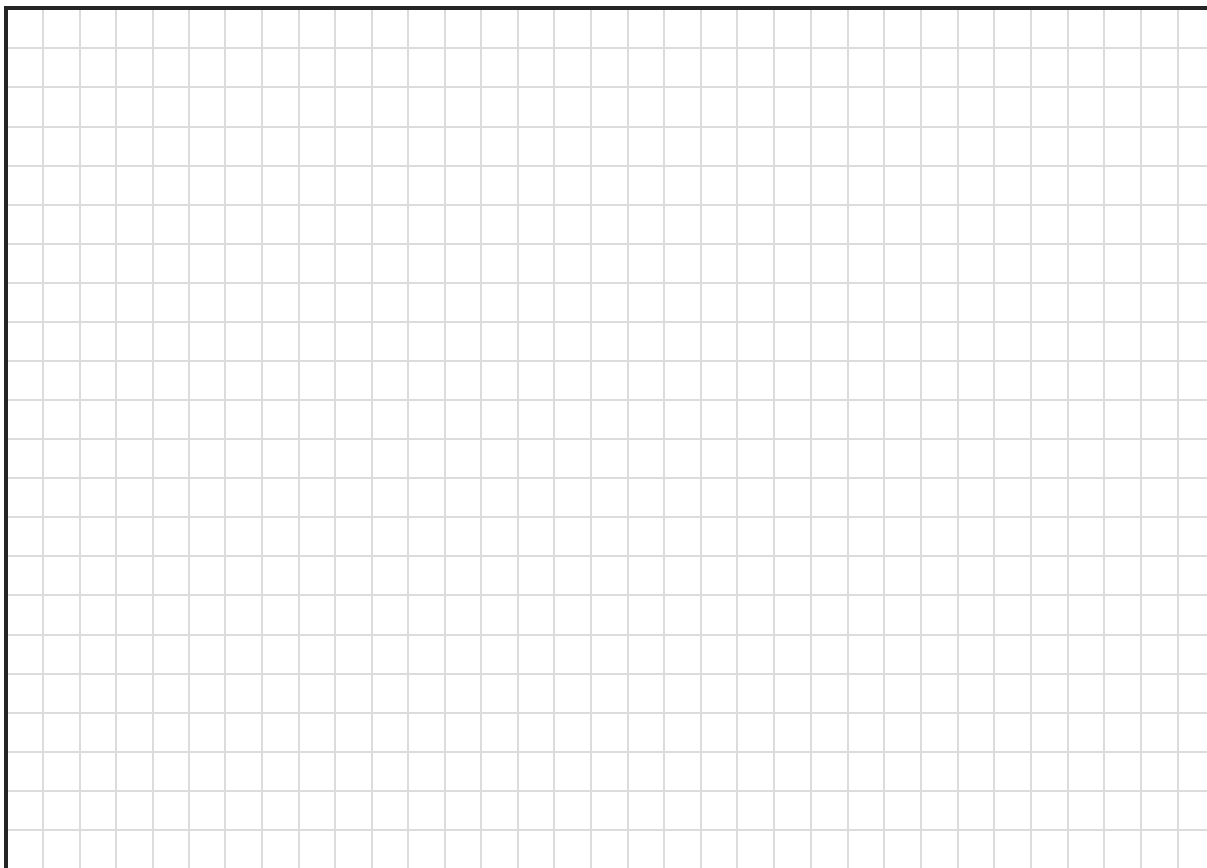


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- (c) Find the maximum possible volume of the cone.
Give your answer in cm^3 , correct to the nearest whole number.

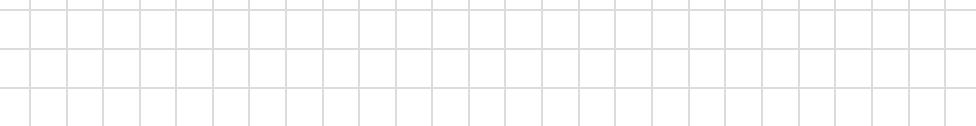


- (d) Find the **exact** value of θ , in radians, for which the volume is a maximum.



(50 marks)

- How much income tax does Pádraig pay for the year?

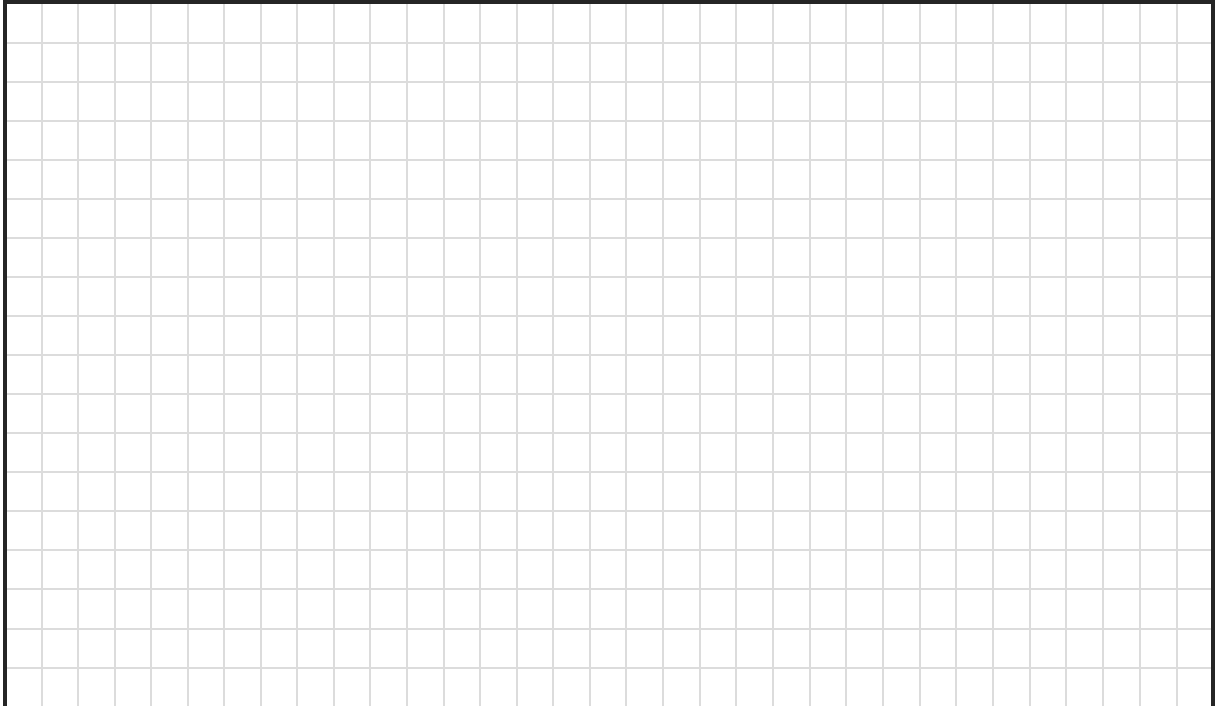


- | Rate | Income band |
|-------------|------------------------------------|
| 0.5% | Up to €12,012 |
| 2% | From €12,012.01 to €27,382 |
| 4% | From €27,382.01 to €70,044 |
| 8% | €70,044.01 and over |
| 11% | Self-employed income over €100,000 |

A full-page sheet of white graph paper with a light gray grid. The grid consists of small squares, approximately 1 cm by 1 cm each. There are 20 columns and 20 rows of squares. A thick black border surrounds the entire grid area.

18

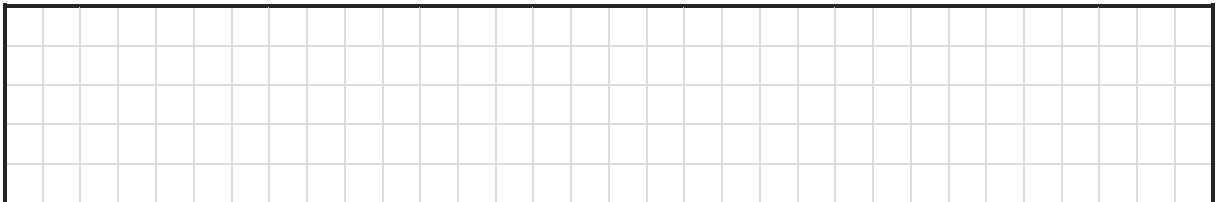
- (c) Pádraig also pays Pay Related Social Insurance (PRSI) at a rate of 4% of his gross salary. What is his net annual income after income tax, USC and PRSI have been paid?



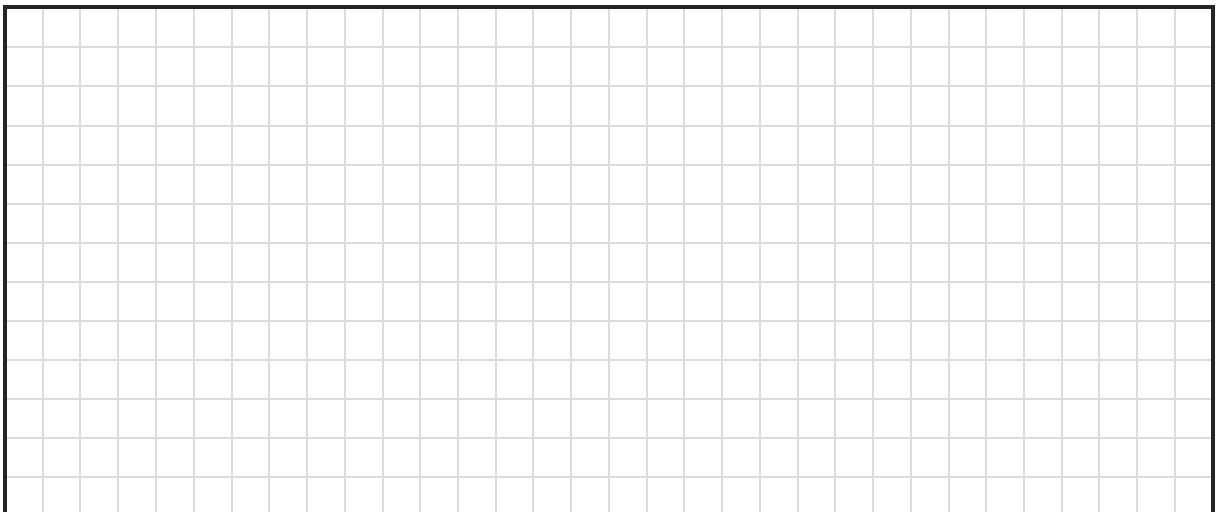
- (d) Pádraig puts money into a savings account and leaves it there for a number of years. The amount of money in the account in euro after t years can be modelled by the function

$$M(t) = 3200e^{0.035t}$$

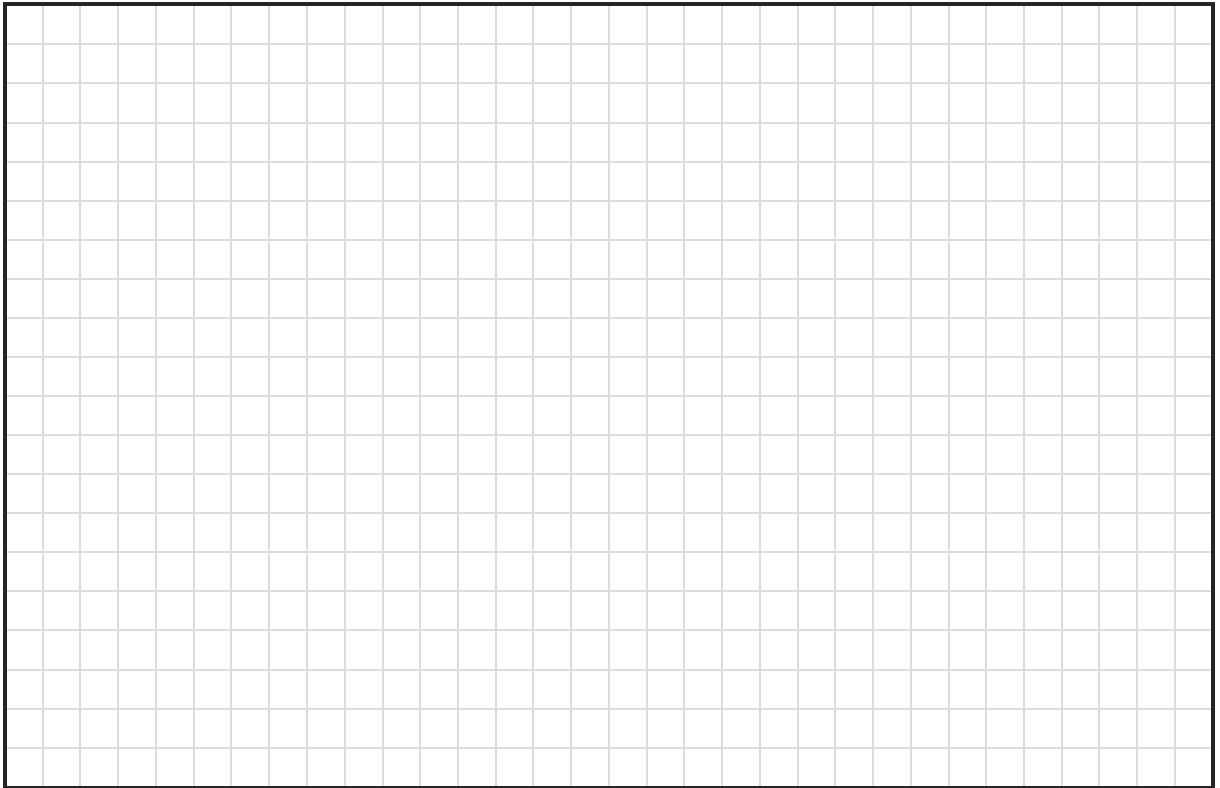
- (i) What was the initial amount of money that Pádraig put into the account?



- (ii) Use differentiation to find the rate at which the amount of money in the account is increasing after 4.5 years. Give your answer correct to the nearest euro per year.



- (iii) Use integration to find the average amount of money in the account over the first 5 years. Give your answer correct to the nearest euro.

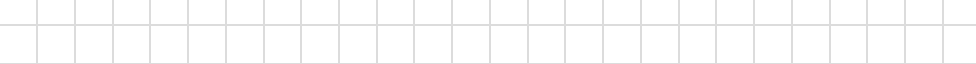


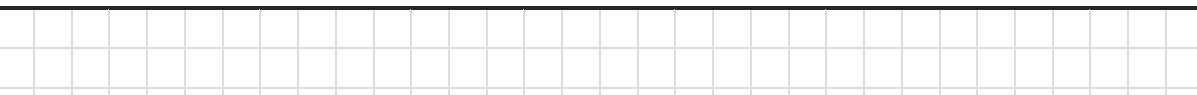
(50 marks)

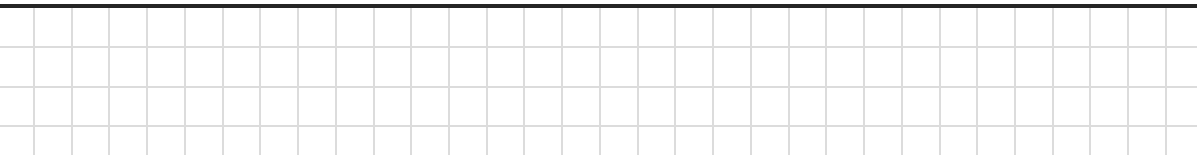
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n (weeks)	1	2	3	4	5
Subscriptions ('000s)		97	94.09	91.267	

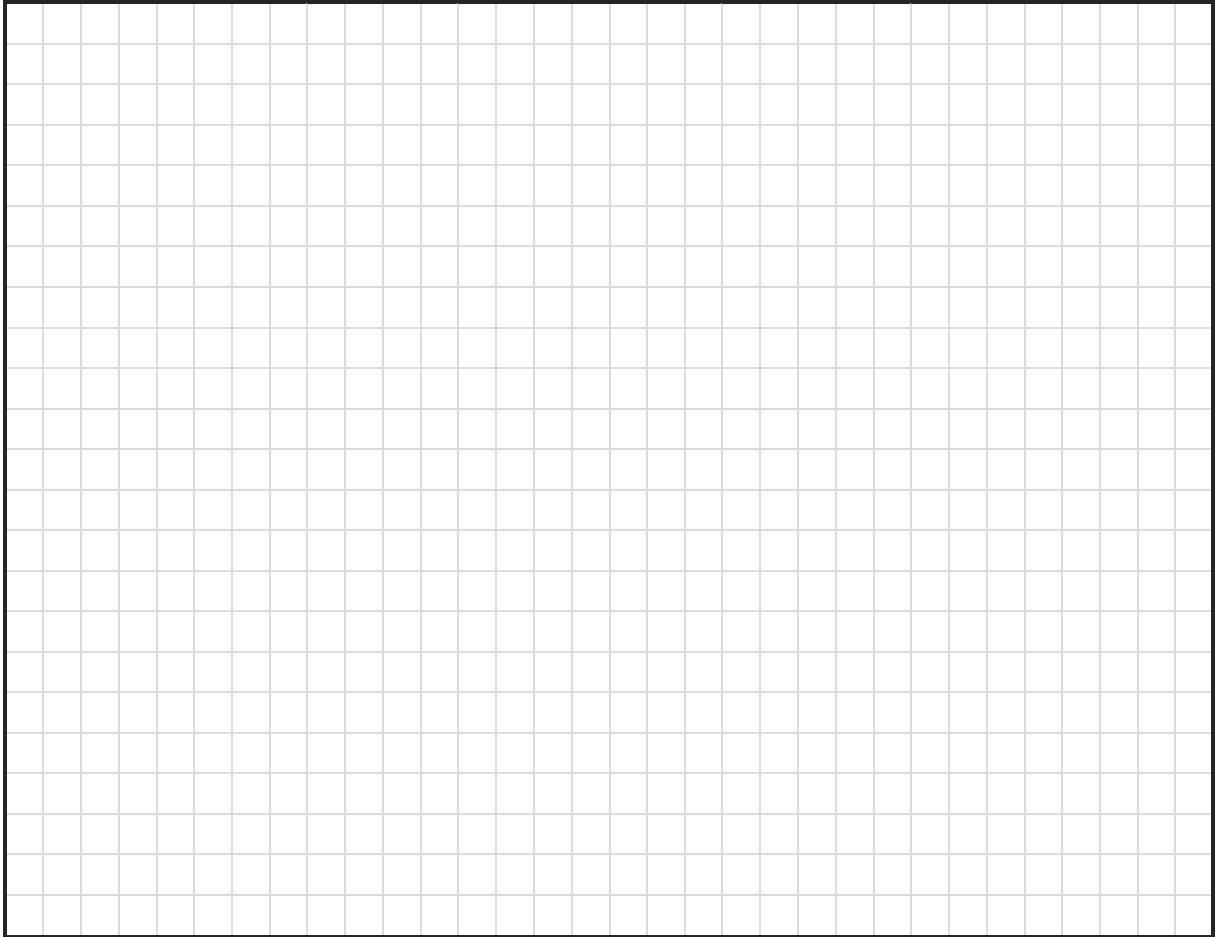
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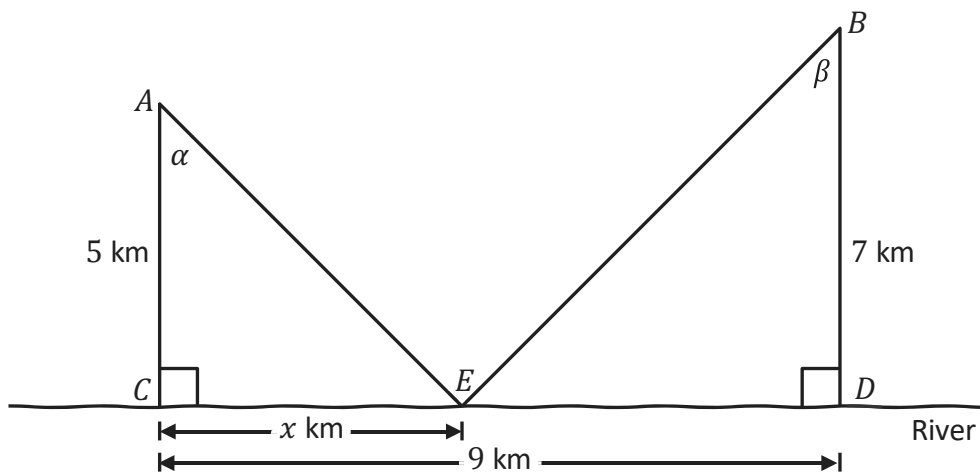
- (iii) The streaming service will need to maintain 8.5 million total subscriptions if it is to be considered sustainable. Find how many weeks it will take to reach this target using the second loyalty program.



Question 10

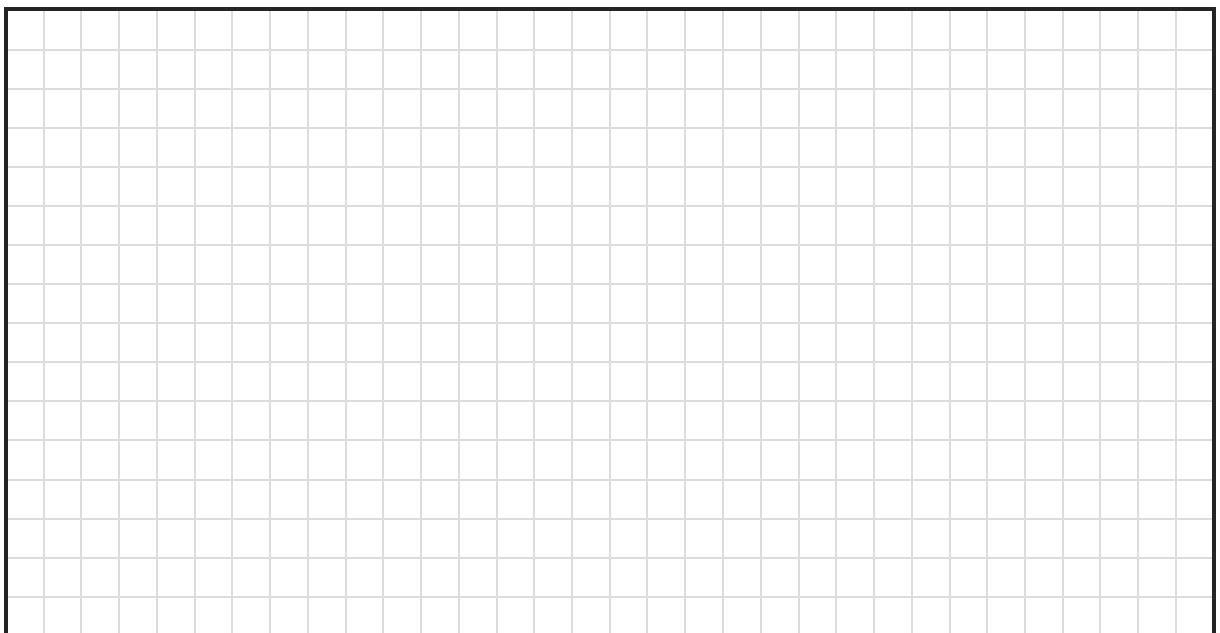
(50 marks)

- (a) Robert's home is at point A and his school is at point B . A straight river runs nearby. The point on the river closest to A is point C , which is 5 km from A . The point on the river closest to B is point D , which is 7 km from B . The distance from C to D is 9 km. Robert cycles from home directly to point E on the river, x km from C , before cycling directly to school at B , as shown in the diagram below.



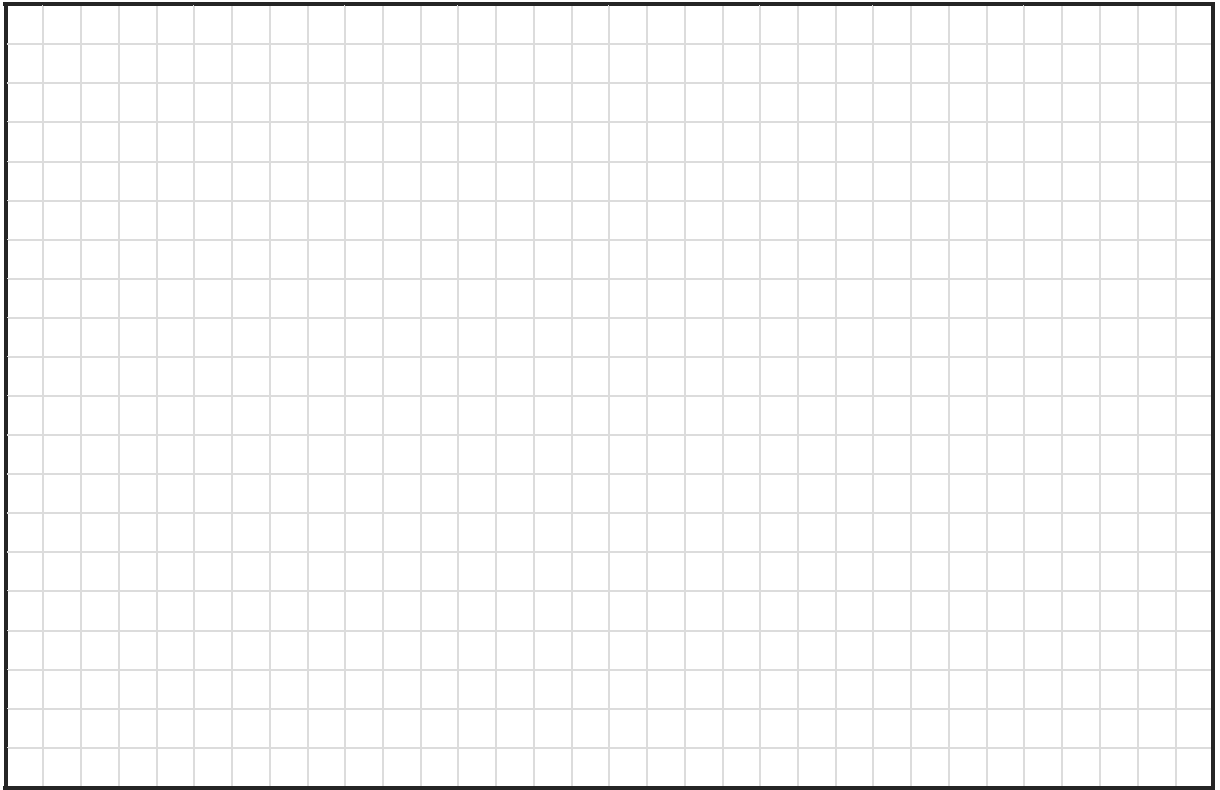
The total distance Robert cycles from home to school is k km.

- (i) Show that $k = \sqrt{x^2 + 25} + \sqrt{49 + (9 - x)^2}$.



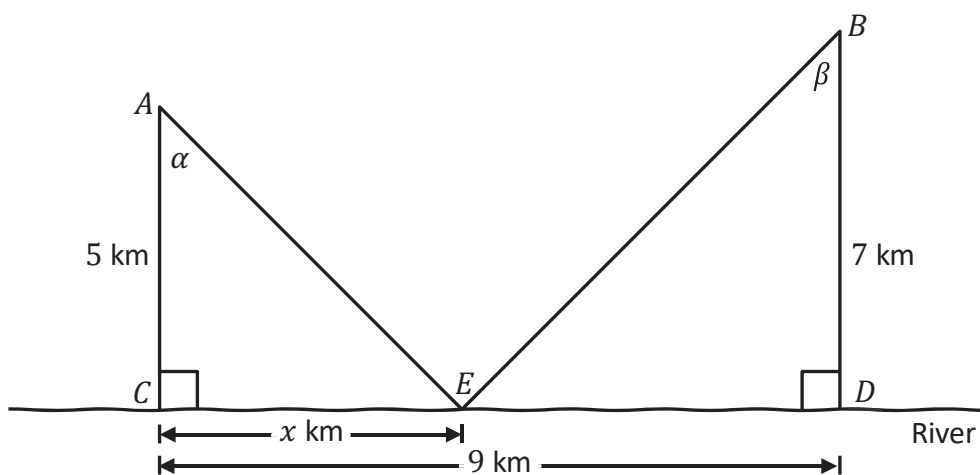
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(ii) Show that $\sin \alpha = \sin \beta$ when $\frac{dk}{dx} = 0$.

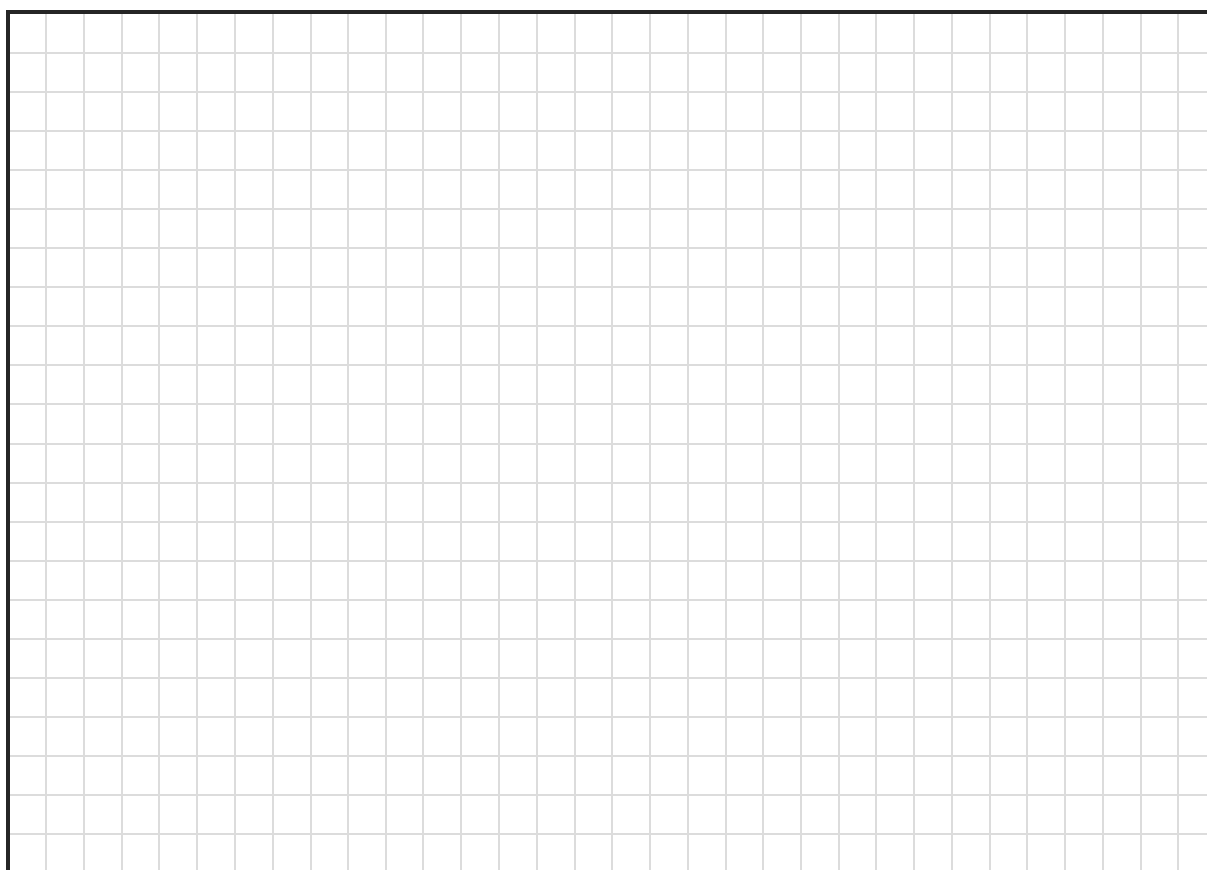


(iii) Show that triangle ACE and triangle BDE are similar when $\frac{dk}{dx} = 0$.





- (iv) Hence, or otherwise, find the value of x , in kilometres, that makes $\sin \alpha = \sin \beta$.



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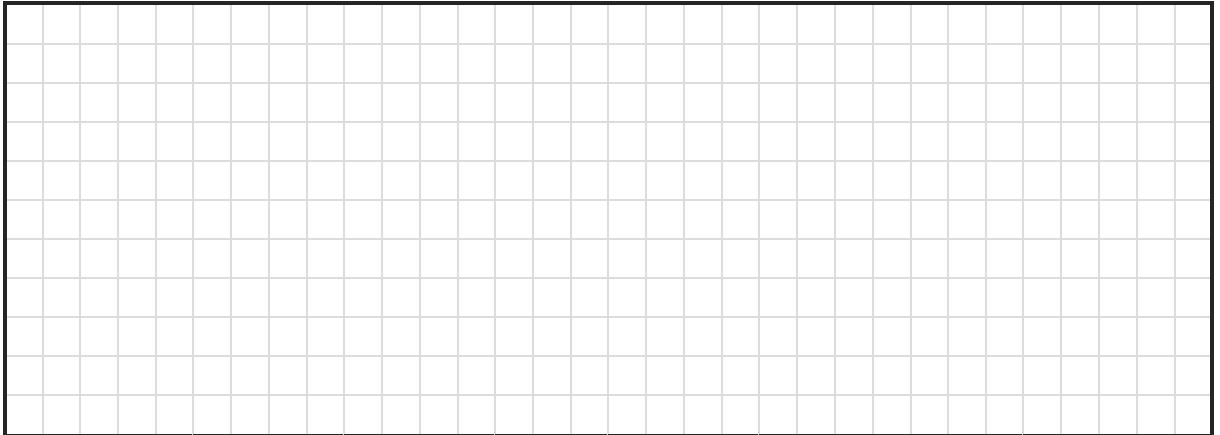
- (b) On the weekends, Robert does long-distance cycling outside of the town. His velocity, in kilometres per hour, can be modelled using the function:

$$v(t) = \frac{30t}{e^t}$$



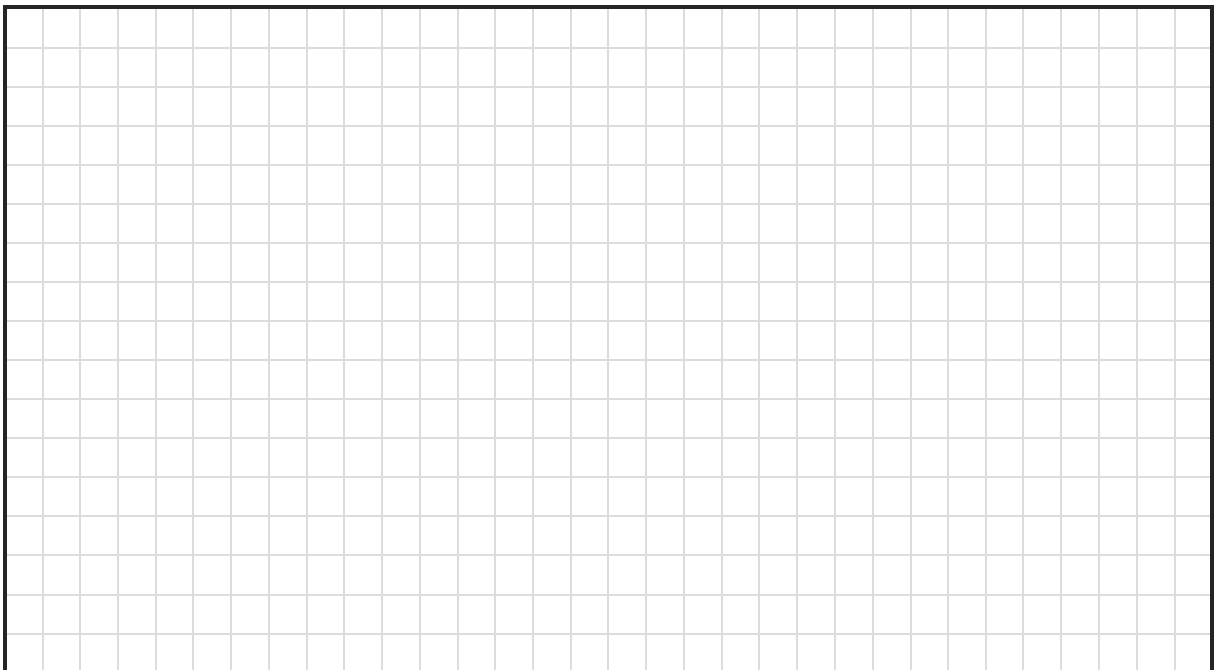
where t is in hours.

- (i) Find his velocity after two hours.
Give your answer correct to 2 decimal places.

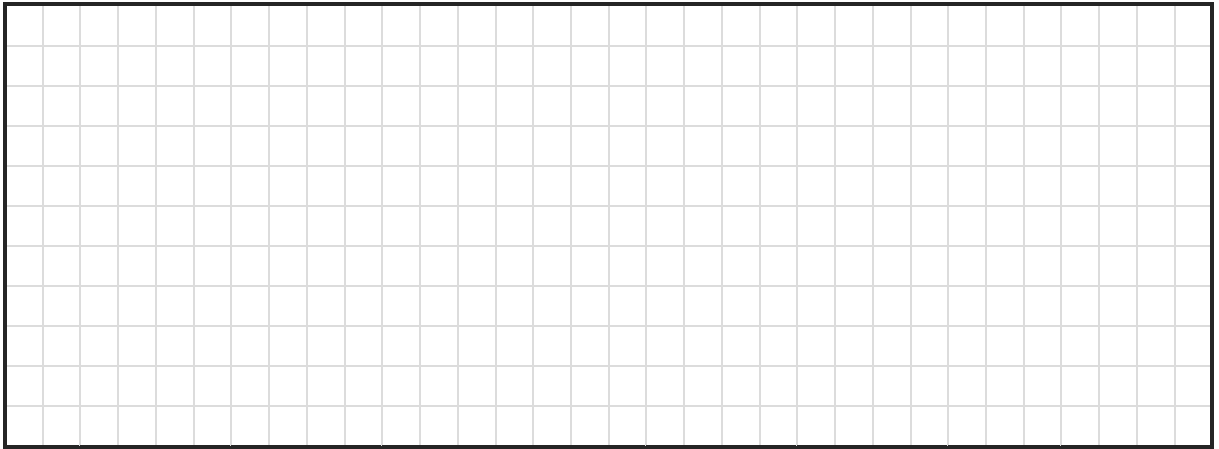


- (ii) Show that his acceleration, in metres/second², can be expressed as:

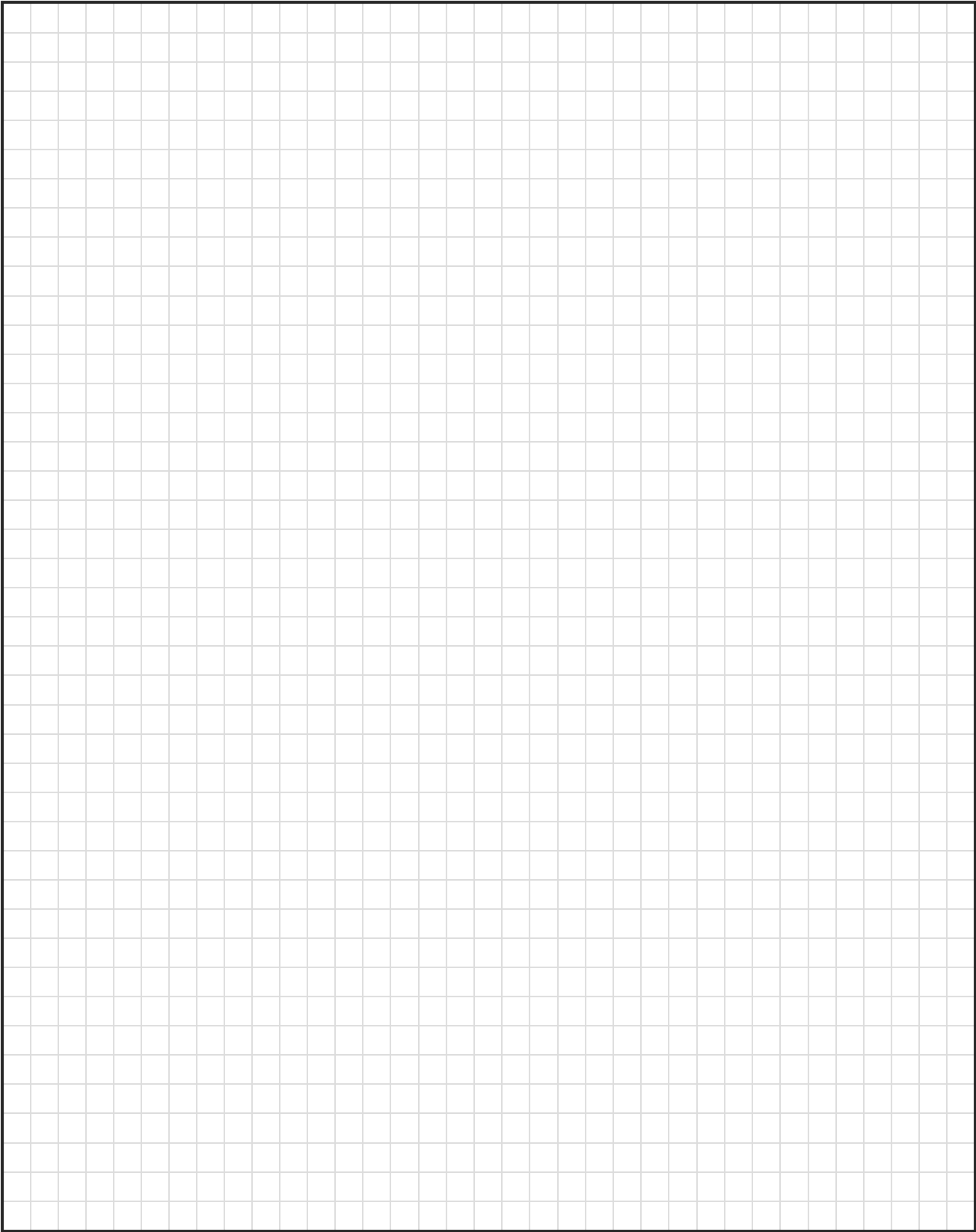
$$a(t) = \frac{30(1-t)}{e^t}$$



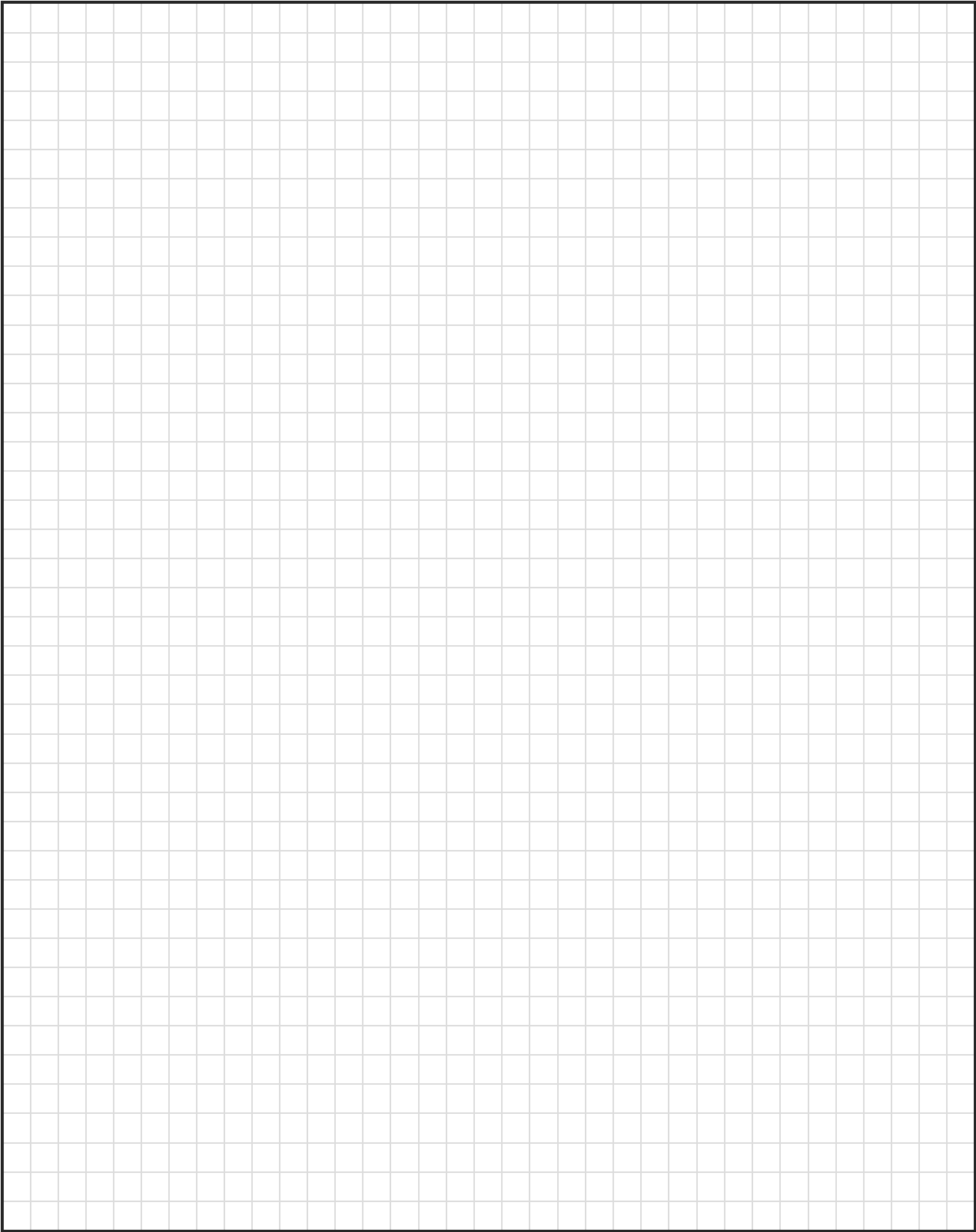
- (iii) Find Robert's maximum velocity for the journey.
Give your answer correct to 2 decimal places.



Page for extra work.
Label any extra work clearly with the question number and part.



Page for extra work.
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