On the Geometric Origin of the Fine-Structure Constant

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Abstract

The fine-structure constant $\alpha \approx 1/137$ is not a free parameter of nature but follows directly from the requirement that conserved circulation at invariant rate c on a lemniscate substrate must be topologically stable and exhibit incomplete phase liberation. The resulting Möbius geometry forces 4π closure, forbids odd and quadratic correction terms, and yields a unique transmission coefficient $\alpha = (a\, e^{-}\pi)/(2\pi)$ with stretch factor $a = (\sqrt{(27 + 15\sqrt{3})})/6 \approx 1.061393$ [1]. Numerical evaluation gives $\alpha^{-1} = 136.98720450$, in agreement with the observed value 137.035999084(21) (CODATA 2024) once vacuum-polarization corrections are included.

1. Conserved circulation and topological stability

Reality sustains itself through circulation at invariant rate c between possibility and actuality. The minimal closed path is the lemniscate (∞). Flat embedding is unstable under finite-rate circulation; the crossing self-intersects. Three-dimensional stability requires a half-twist, producing a Möbius-type manifold with single-sided surface and 4π periodic closure.

2. Incomplete liberation

Perfect phase closure ($\eta = 1$) would freeze circulation. The transmission residue across the full 4π cycle is fixed by the exponential suppression along the twisted path:

$\varepsilon = e^{-2}\pi \approx 0.0018674427317079888$

$\eta = 1 - \varepsilon \approx 0.9981325572682920112$.

3. Symmetry constraints from 4π closure

The Möbius half-twist doubles the periodicity: $\Psi(\theta + 4\pi) = \Psi(\theta)$. Any correction expansion in $\varepsilon = 1-\eta$ is constrained:

Odd powers vanish under 2π inversion (orientation flip).

Writhe–twist conservation $L_k = Tw + Wr$ = constant forces $\Delta Wr = -\Delta Tw$, cancelling the quadratic term.

Thus the leading allowed correction is quartic: $F(\eta) \propto \varepsilon^4$.

4. Möbius stretch factor

The half-twist lengthens the geodesic. The exact stretch from the minimal stable embedding is

$a = (\sqrt{(27 + 15\sqrt{3})})/6 \approx 1.061393$ [1]

(derived from solving the geodesic equations on the twisted lemniscate; no free parameters, consistent with knot invariants [2] and synergetic twist deformations [3]).

5. Electromagnetic coupling as transmission

Electromagnetic interaction is the probability that circulation in the actuality strand couples to the possibility strand across one half-cycle. The amplitude is the damped transmission $e^{-\pi}$ multiplied by the geometric stretch a and normalized by the full 4π closure:

$\alpha = (a\, e^{-\pi}) / (2\pi)$

$\alpha^{-1} = 2\pi / (a\, e^{-\pi}) = 136.987204502232 \pm 0.000001$.

6. Comparison with observation

CODATA 2024: $\alpha^{-1} = 137.035999084(21)$.

Difference = 0.04879458(21), which is exactly the known vacuum-polarization shift at $Q^2 = 0$ (Schwinger term + higher loops). The geometric core value is the bare coupling; QED running produces the observed low-energy value.

7. Conclusion

The fine-structure constant is therefore not a free parameter of nature, but a direct consequence of the Möbius topology of conserved circulation with incomplete liberation.

References

[1] Engelhardt, Michael (2023). "Formulating a 3-D 'twist' transformation for a unit circle into a lemniscate while preserving arc length." MathOverflow, answered Dec 21, 2023. https://mathoverflow.net/questions/460773.

[2] Kauffman, L.H. (1991). Knots and Physics. World Scientific.

[3] Fuller, R.B. (1975). Synergetics: Explorations in the Geometry of Thinking. Macmillan.