

INTRODUCTION TO CONTINUOUS CONTROL SYSTEMS
COLUMBIA UNIVERSITY MECHANICAL AND ELECTRICAL ENGINEERING
DEPARTMENTS: E3601

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Homework 5

Problem 1 (Controllable Canonical Form).

Show that the general n^{th} order differential equation with constant coefficients given by equation 1 may be represented as an array of first order differential equations given by equations 2 and 3.

$$\frac{d^n y(t)}{dt^n} + p_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + p_1 \frac{dy(t)}{dt} + p_0 y(t) = q_{n-1} \frac{d^{n-1} u(t)}{dt^{n-1}} + \cdots + q_1 \frac{du(t)}{dt} + q_0 u(t) \quad (1)$$

$$\dot{\vec{x}}(t) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ -p_0 & -p_1 & \cdots & -p_{n-2} & -p_{n-1} \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} u(t) \quad (2)$$

$$y(t) = \begin{bmatrix} q_0 & q_1 & \cdots & q_{n-2} & q_{n-1} \end{bmatrix} \vec{x}(t) \quad (3)$$

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