

**INTRODUCTION TO CONTINUOUS CONTROL SYSTEMS**  
**COLUMBIA UNIVERSITY MECHANICAL AND ELECTRICAL ENGINEERING**  
**DEPARTMENTS: E3601**

*Homayoon Beigi<sup>†</sup>*

1340 Mudd Building  
Columbia University, New York City, NY 10027  
hb87@columbia.edu

## Homework 5

**Problem 1** (Controllable Canonical Form).

Show that the general  $n^{th}$  order differential equation with constant coefficients given by equation 1 may be represented as an array of first order differential equations given by equations 2 and 3.

$$\frac{d^n y(t)}{dt^n} + p_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + p_1 \frac{dy(t)}{dt} + p_0 y(t) = q_{n-1} \frac{d^{n-1} u(t)}{dt^{n-1}} + \cdots + q_1 \frac{du(t)}{dt} + q_0 u(t) \quad (1)$$

$$\ddot{\vec{x}}(t) = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & \cdot \\ -p_0 & -p_1 & \dots & -p_{n-2} & -p_{n-1} \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 0 \\ 0 \\ \cdot \\ 1 \end{bmatrix} u(t) \quad (2)$$

$$y(t) = [ q_0 \ q_1 \ \dots \ q_{n-2} \ q_{n-1} ] \vec{x}(t) \quad (3)$$

---

COPYRIGHT HOMAYOON BEIGI, 2025 THIS DOCUMENT IS COPYRIGHTED BY HOMAYOON BEIGI AND MAY NOT BE SHARED WITH ANYONE OTHER THAN THE STUDENTS REGISTERED IN THE COLUMBIA UNIVERSITY EEME-E3601 COURSE.

<sup>†</sup>Homayoon Beigi is Professor of Professional Practice in the department of mechanical engineering and in the department of electrical engineering at Columbia University