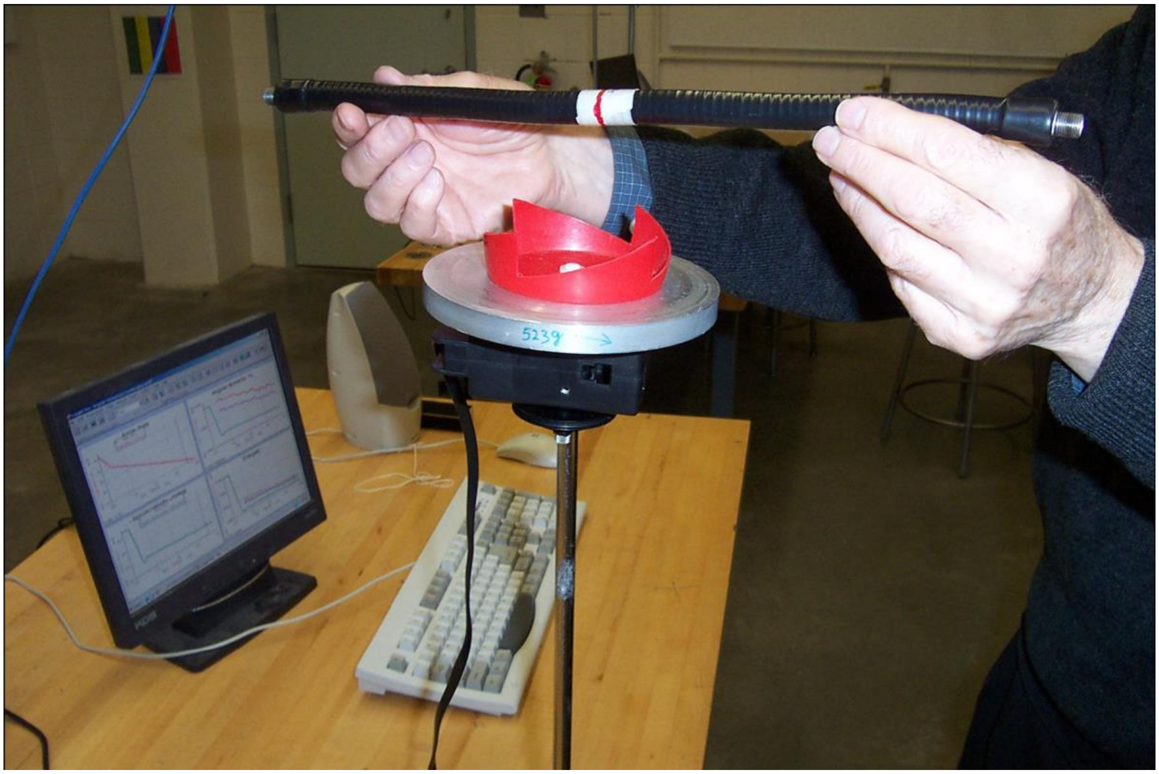
Rotational Dynamics



Objective: To investigate the behavior of a rotating system subjected to internal torques and external torques. To apply knowledge gained from the linear momentum lab to its rotational analog. To see how total energy is a function of linear and rotational energy by re-visiting the loop-the-loop part of the Work, Energy & Circular Motion experiment.

Apparatus: Angular motion sensor, circular aluminum disc with pulleys of varying diameter on other side, meter stick, 20g or 50g mass on a string, flexible goose neck mass.

Introduction:

A few weeks ago you investigated the interaction of a system of two masses without any external forces, namely the conservation of momentum of two colliding carts. In this lab you will investigate a similar "collision", but there will be no linear motion, only rotational. Below are some useful equations you may remember from lecture (all scalar).

Definition of angular velocity:

$$\omega = v/r$$

Moment of Inertia of a point mass about an axis of rotation r away:

$$I_m = mr^2$$

Moment of Inertia of a circular disc about a perpendicular axis through its center:

$$I_{disk} = \frac{1}{2} m r^2$$

Moment of Inertia of a rod about an axis through its center (perpendicular to its length):

$$I_{(rod,center)} = \frac{1}{12} m l^2$$

Definition of angular momentum:

$$L = I\omega$$

Kinetic energy of rotation:

$$KE_{rotational} = \frac{1}{2} I w^2$$

Total energy of an object undergoing both translational and rotational motion:

$$KE_{total} = KE_{translational} + KE_{rotational} + PE_{translational} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2 + mgh_{C.M.}$$

Newton's Law in rotational form (tau is the external torque; alpha is the *angular acceleration)*:

$$\tau = I\alpha$$

Equation of motion in rotational form (constant angular acceleration):

$\theta = \theta_0 + \omega t + \frac{1}{2} \alpha t^2$

Time-independent version of equation above:

$$\omega^2 = \omega_0^2 + 2\alpha \Delta\theta$$

In addition, the Moment of Inertia of an object of known I_0, when taken about another axis parallel to the axis used to calculate the known I (when the two axes are a distance h away) is:

$$I = I_0 + mh^2$$

This is also known as the *Parallel Axis Theorem.*

You will perform two experiments:

1. Gently drop a non-rotating rod (which can either be straight or bent into an approximate circle) onto a circular disc rotating at low angular momentum. The disc is permanently mounted on an angular sensor which will give output angular position and speed to Logger Pro. This is the rotational equivalent of the linear collision you performed with the two Pasco carts on the Pasco track. In this experiment the system (disc + rod) experiences no external torques, just as the two carts experienced no external forces; all interactions are internal.

2. Use a small mass, connected to a pulley with a string, to rotate the circular disc. Here you are

using the same equipment as in (1) above, except that you have re-oriented the setup 90 degrees. This is the rotational equivalent of a force causing a previously stationary mass to accelerate linearly. In this experiment there *is* an external torque, just as a cart that is pulled by a mass hanging over a pulley experienced a force.

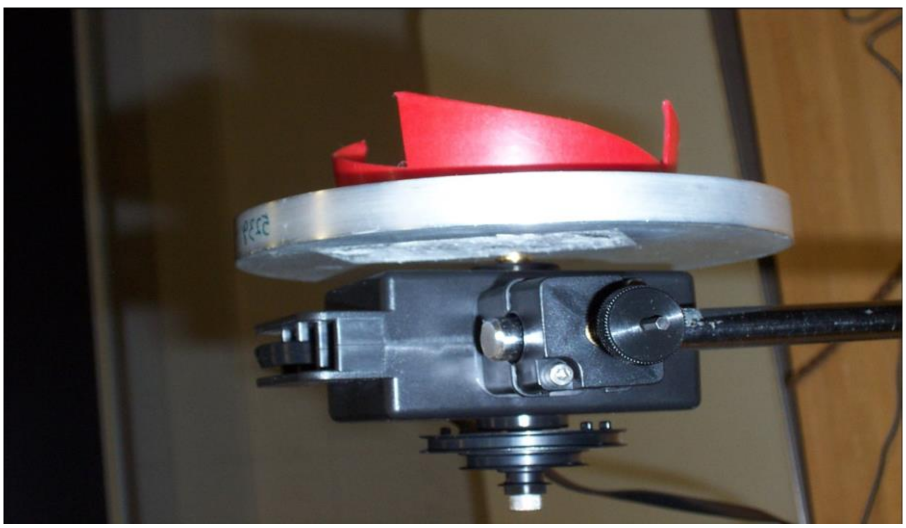
Procedure

As usual, write your lab in Google Docs and share it with lab partner(s), TA and LA.

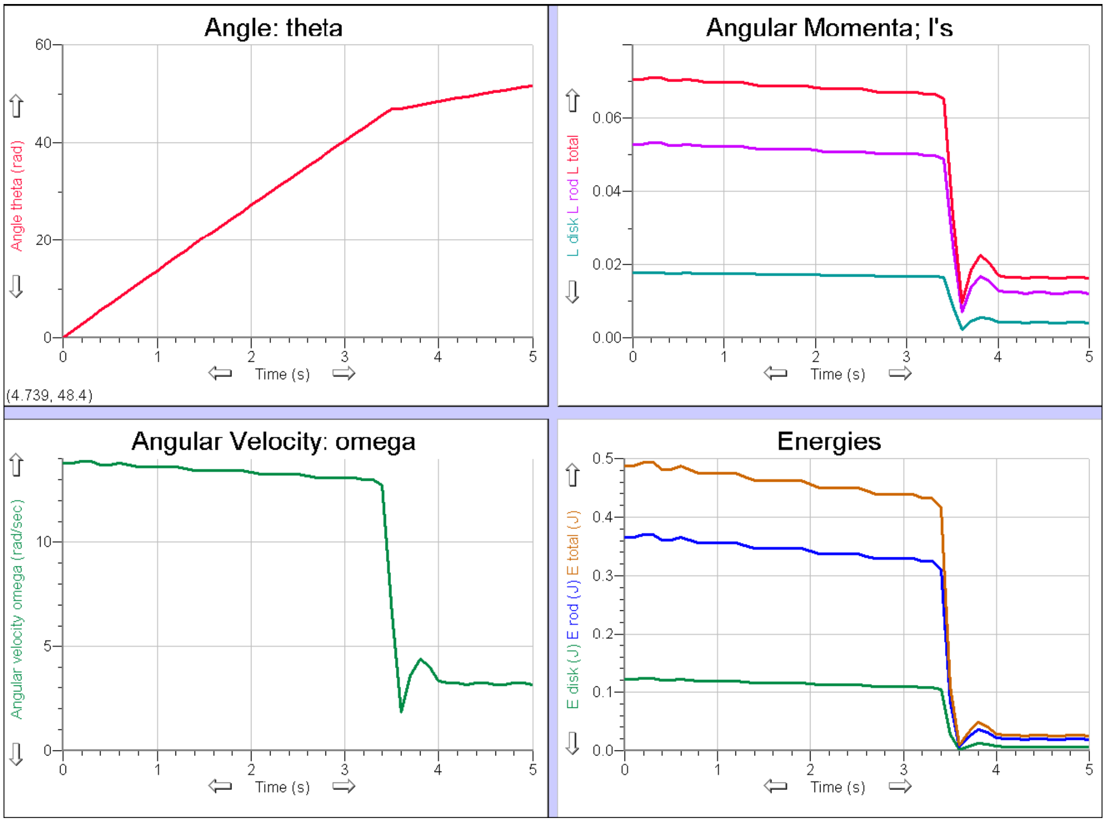
I. Gently drop a non-rotating rod onto a circular disc (no external torques) (65 pts)

1. Open the Logger Pro file Angular Momentum Conservation.cmbl, which is a Logger Pro template file in the same folder as this write-up. Scroll through the different pages by clicking on the arrows to the left and right of page selector near the upper left-hand corner: Page 1, Page 2, Page 3. On Page 3 you will see the data table. Double click on "L rod" and examine the way it is calculated (currently $I_{(rod,center)} = \frac{1}{12} ml^2$). As with the Linear Momentum lab, you will have to change some parameters to their exact values: look at the handwritten mass value on the aluminum circular disc and change this in Logger Pro if necessary. Note that the 1/12 coefficient will have to be changed if you alter the shape of the rod from its straight shape. Also measure the length of the rod and change that parameter in Logger Pro if needed. You will therefore be changing the values for L rod and Rod Energy.

2. Make sure the apparatus is positioned as in the photo below. The plane of the circular disc should be horizontal, with the red toothed "catch" facing upwards:



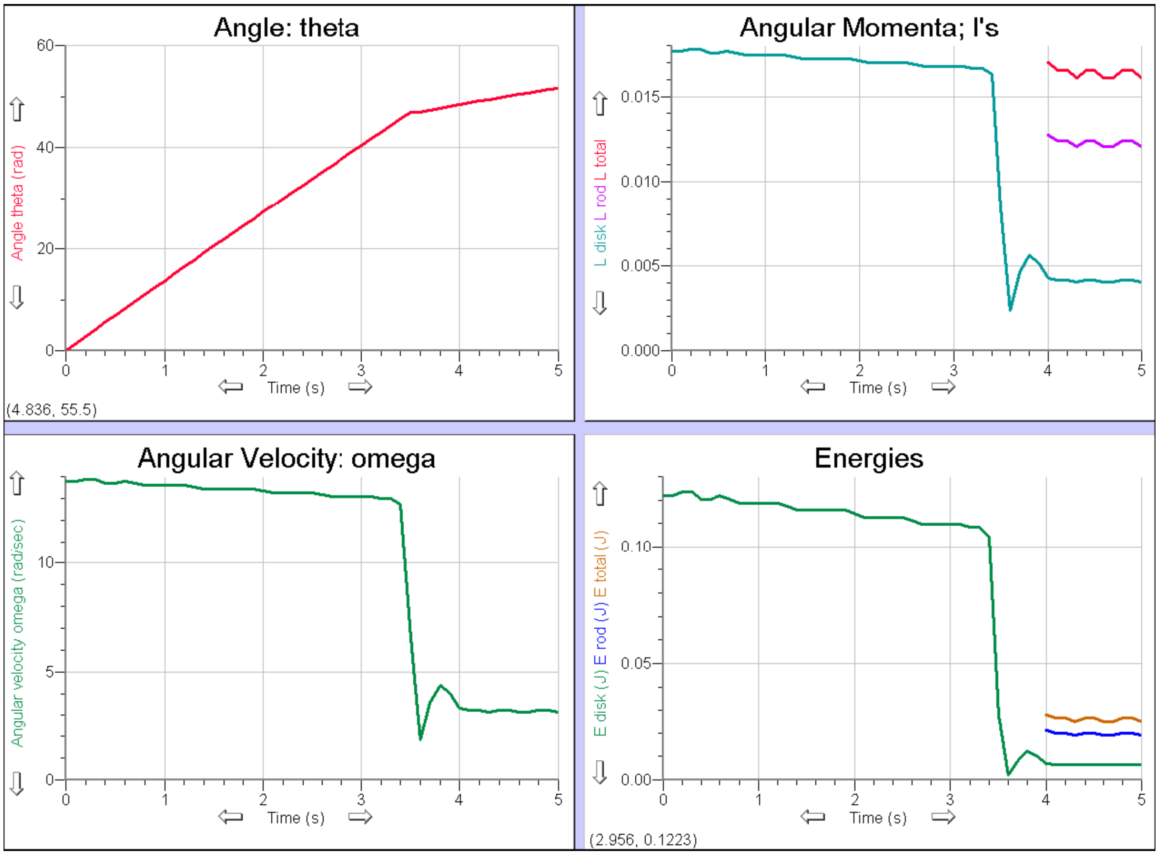
3. Go back to Page 1. Press the Collect button in Logger Pro and practice spinning the disc at low angular speed (you must spin counterclockwise; software is configured for CCW spin) and gently dropping the straight rod onto the red toothed catch. It should be centered - the middle of the rod should land on the center of the disc. Remember to drop the rod gently just a centimeter or two above the disk to minimize vibrations and energy losses. Look at the resulting graph and visually check if your results make sense. You should see something like the photo below:



Plots, with all data, of rod being dropped onto disk. Note that on some sensors, clockwise rotation corresponds to *decreasing* angle (downward slope on Theta, upward jump on Omega. <u>If this is the case, go to Page 3 in Logger Pro, double-click on the Omega (Angular Velocity) column, and put a minus sign in front of the expression in the Equation box</u>.

It is critical to remember that the only thing that the instrumentation will directly measure is the angular position and speed of the rotary sensor (connected to the disc); the angular velocity of the rod should be the same as the disc after the collision since they spin in unison. **Before** the collision, the rod's angular speed (and the resulting calculated angular momentum and rotational kinetic energy) should be **zero**; the software displays a non-zero value since the software algorithm assumes that the rod *always* spins with the disc. **You will then have to eliminate these false data points by highlighting the L rod and KE rod points in Logger Pro and going to Edit--> Strike Through Data Cells**. Note that the points you will be striking through should correspond to the time interval before the rod and disc have settled into a fairly constant (yet slowly dropping) angular speed. ***Identify the points just before the "collision" and just after it.*** (5 pts)

Examine the numbers in the data table for L rod - they will help you choose where to put the strike-through cutoff by showing periods of mostly constant values. After you strike through the pre-collision and collision data point, your graph should appear as follows:



Plots, with pre-dropping data removed, of rod being dropped onto disk. Note that on some sensors, clockwise rotation corresponds to *decreasing* angle (downward slope on Theta, upward jump on Omega. <u>If this is the case, go to Page 3 in Logger Pro, double-click on the Omega (Angular Velocity) column, and put a minus sign in front of the expression in the Equation box</u>.

Notice that before the collision, only the disc has angular velocity, momentum and kinetic energy; the rod's should be zero. After the collision, all three quantities are present in both rod and disc. For your lab report, include the graphs you obtain (as above) and also record (see below) the following : total energies (initial i, final f, ratio f/i); total angular momenta(i, f, f/i). (20 pts)

	DISC	ROD	DISC + ROD
Initial L			
Final L			
Initial E			
Final E			

Ratio of $(L_{total})^{final} / (L_{total})^{initial}$ = ____________

Ratio of $(E_{total})^{final} / (E_{total})^{initial}$ = ____________

Note that **initial** refers to just before the collision, and **final** refers to just after the collision. Use your experience from Linear Momentum to select the appropriate points.

4. Now bend the rod into a roughly circular shape and repeat the experiment. For the red to retain a circular shape, you will have to tape the ends together with one or two, short strips of tape oriented along the length of the rod – **no need to wind the tape around the diameter of the rod (which would make it difficult for to unwind when you are done)**. See this photo:

The moment of inertia of circular hoop rotating about an axis perpendicular to and going through its center is:$I_{(cicular hoop,center)} = mr^2$. If you cannot bend (and tape) the rod into a perfect circle, you can try either:

a) measuring the *semi-major axis of the elliptical rod* (the "radius" of the long diameter of the ellipse) and average that with the *semi-minor axis of the elliptical rod* (the "radius" of the short diameter of the ellipse) to find the effective ***r*** in the above equation, or

b) using the equation for $I_{(elliptical\ hoop,\ center)} = mab$, where **a** is the *semi-major axis of the elliptical rod* and **b** is the *semi-minor axis of the elliptical rod* (as described above)

Don't forget to change parameters – as you did in the Linear Momentum lab by double clicking on the data column header in Logger Pro - for rod mass, rod radius (when bent in circle; use average between long radius and short radius) and moment of inertia coefficient (1/12 for straight rod, 1 for circular rod) in Logger Pro. Also remember to enter the rod **radius** instead of the rod length. As with (3), include the graphs you obtain (as above) and also record (see below) the following: total energies (initial i, final f, ratio f/i); total angular momenta(i, f, f/i). (30 pts)

	DISC	**ROD**	**DISC + ROD**
Initial L			

Final L			
Initial E			
Final E			

Ratio of $(L_{total})^{final}/(L_{total})^{initial}$ = ______________
Ratio of $(E_{total})^{final}/(E_{total})^{initial}$ = ______________

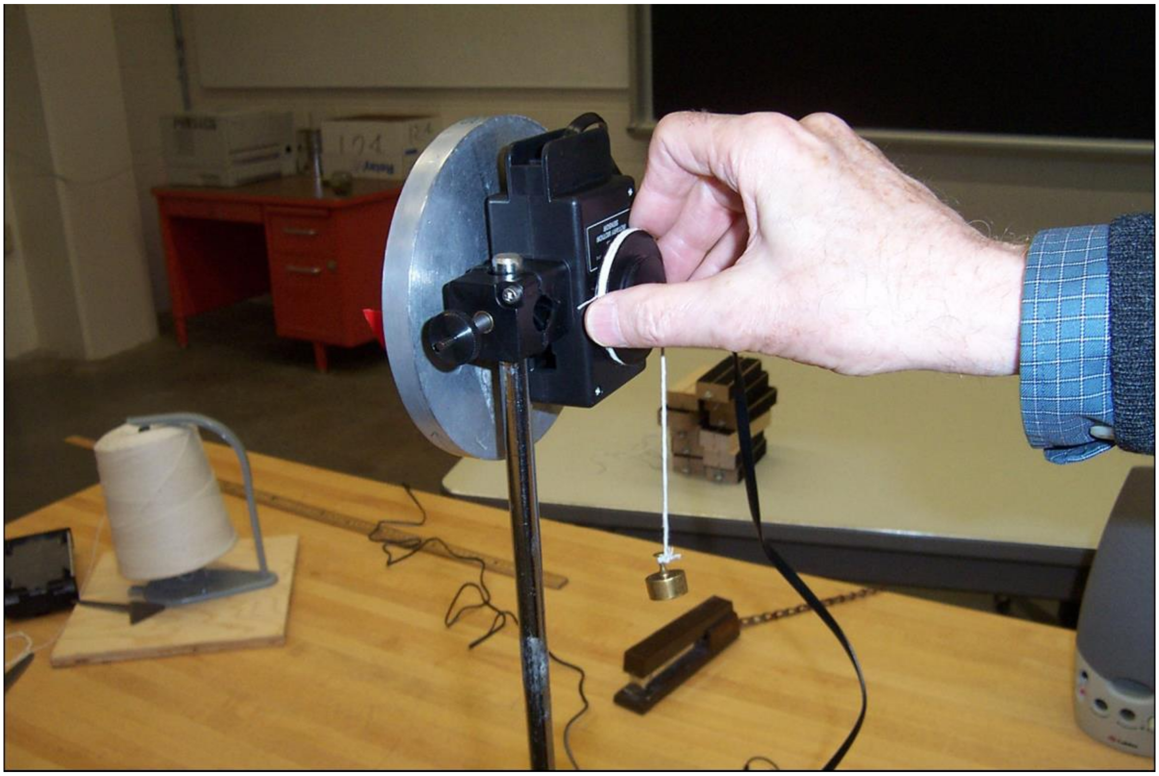
*****REMOVE THE MASKING TAPE FROM THE ROD AFTER YOU ARE DONE*****

2. Use a small mass to rotate the circular disc (external torque) (35pts)

1. Open up the Logger Pro template file **Constant Torque Rotational Motion.cmbl.** Unscrew the angular sensor/disc from the ring stand and re-orient it so that the plane of the disc is now vertical (see photo below). Anchor a thread or string through a pulley slot and wrap a thread or string with small weight suspended around one of the drive pulleys. The free end of the string must be the first thing to wrap around the pulley. Also, **the string must be wound tightly enough not to slip, and that you should start winding by putting the knot at the end of the string through the notch in the pulley**. Then the external torque exerted is approximately $\mathbf{mgR_{pulley}}$ (0.025m for the larger pulley; remember to change this in the parameters if necessary).

Question (not for credit): why is the external torque only approximately $\mathbf{mgR_{pulley}}$?

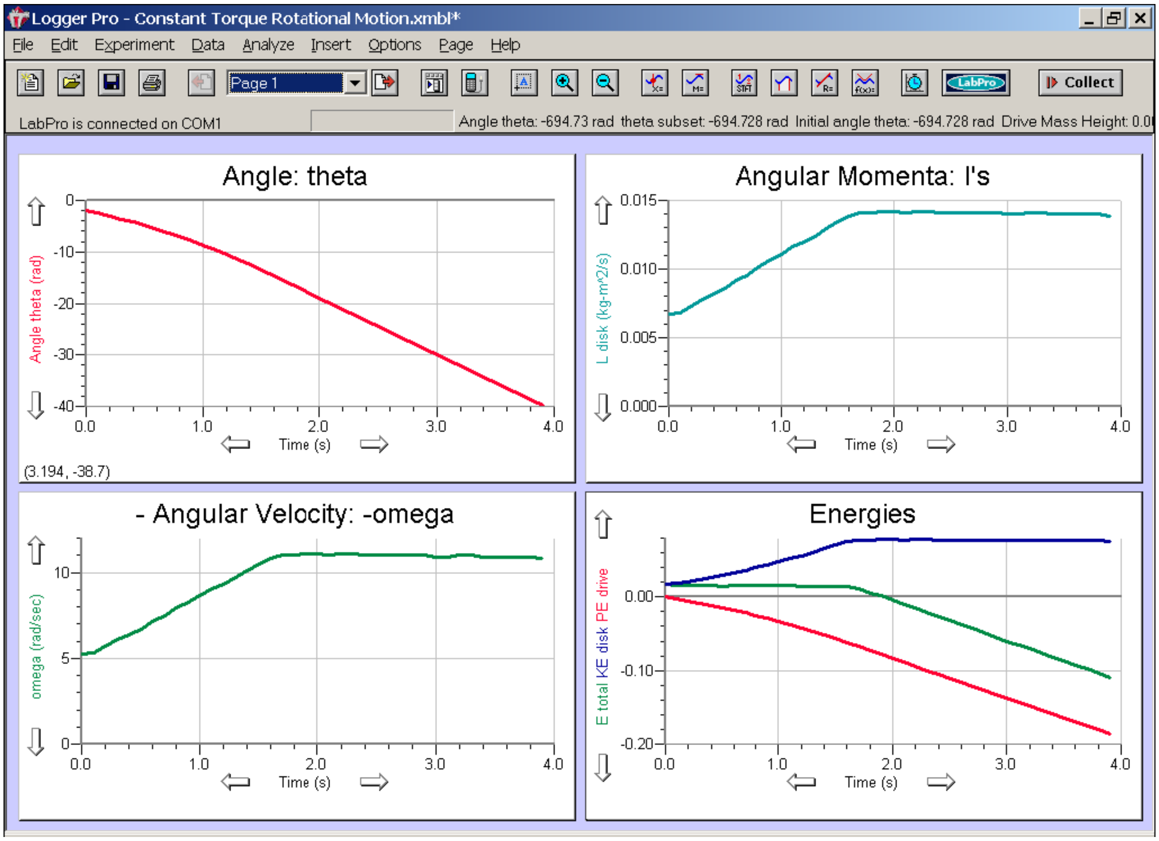
The smaller pulley will have a radius of 0.0125m. This being constant (we neglect any variation in earth gravitational pull with distance from earth center), the constant angular acceleration equations apply (see equations in introduction). These have the same form as those for constant force in translational acceleration. In the photo below, the string is wrapped around the larger (0.025m radius) pulley:



The drop mass PE is set to zero by subtraction of the time-zero height. This is accomplished with the two data columns "theta sub" (subset) and "init theta".

(This is not a "before and after" collision test. We are interested in the continuous time interval from time zero until dropping drive mass hits. We are not interested in any data after drop mass hits the table. Strike through subsequent data only if it makes you feel better.)

2. Practice winding the drive mass string around the pulley and collecting data. **Wind the string around the pully such that the disk rotates in the same direction as in Part I.** The picture and LP graph below show the operation and result, for 20g drop mass on larger pulley.



NOTE: You would expect potential energy to decrease as the 20g mass falls; if Logger Pro shows the opposite, simply wind the string around the pulley in the *opposite* direction.

The 20g weight hit sthe table at about 1.6 seconds; algorithms fail for later data (the PE no longer changes). Most of the drive mass gravitational PE (red) is converted to disk rotational KE (blue); the KE of the falling drive mass is relatively very small (both theoretically and experimentally). Total energy (green) appears to decrease slightly.

(Note, drive mass PE would be linear if plotted vs. height h, but this is a time plot.)

If the string slips while the drive mass falls, assumptions are invalid!

3. From theory, calculate and include in your report, the following quantities. Include a ***FBD***:

a) The torque exerted on the disk by the drive mass. (5 pts) N*ote that the force associated with this torque should be the T in the string which, as you learned last week, is* ***not*** *equal to because it is accelerating – but you can choose to neglect a if you can justify that this is a good approximation. See "Newton's Laws II" write-up for reference.*

b) The moment of inertia of the disk (5 pts)
c) The calculated angular acceleration of the disk (5 pts)
d) The change in theta (angle) and omega (angular velocity) from the time the drive mass is released to when it hits the table. (5 pts) You may find these equations useful:

Kinematics equation without time variable:

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

Relationship between drop height and change in angle:

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

Make sure to put the proper subscript on the variable to distinguish between different objects, i.e., r_{disk} vs. r_{pulley} . Errors in labeling may lead to incorrect calculations.

4. Include a graph for one good example, as in the first experiment (collision). Label the points where the mass was released and when it hit the table. Enter numerical values, and ratios of experimental to theoretical, for Angular Momentum and Energy (disc) for when the drive mass hits the table. (10 pts)

5) Compare your value for angular acceleration (part **c** in step 3 above) to your experimental value (values from the Logger Pro plots/tables) and give your analysis on how close (or far) these values are using Percent Difference. (5 pts) Remember that you can obtain the angular acceleration value from the angular velocity vs. time plot, just as you did with linear acceleration and the linear velocity vs. time plot.

QUESTIONS

1. What if the rod (straight or bent) were already rotating at the same angular velocity as the disc before you dropped the former onto the latter – would the disc's angular velocity change after dropping? Why or why not? (5 pts)

2. If you were a bug (instead of the falling mass) hanging from the string wrapped around the same pulley you used in the lab, and you wanted to descend to the lab table as softly as possible (smallest possible acceleration) so as not to squash yourself. You have your choice of discs – the usual one, or one with half the mass but twice the radius – which one would you choose? (Remember – you are only swapping the disc (aluminum) part of the setup). (5 pts)