PHYS 2601: Classical and Quantum Waves Final Exam

Professor James McIver

December 19, 2024.

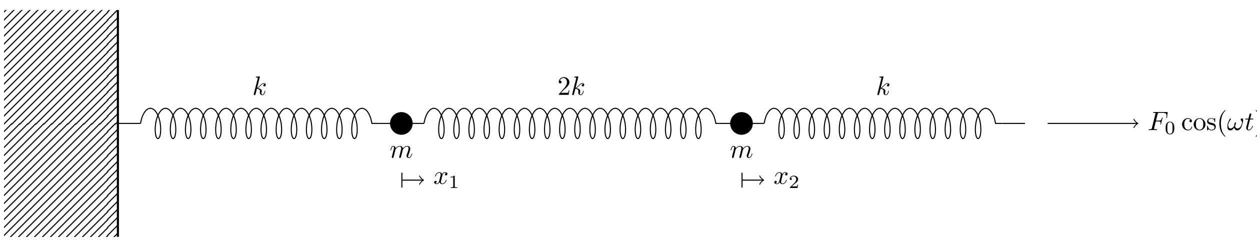
Problem 1: Damped Oscillators (30 points)

According to classical electromagnetic theory, an accelerating electron radiates energy at a rate $\frac{Ke^2a^2}{c^3}$, where a is the acceleration, e is the electronic charge, c is the speed of light and K is a constant. Suppose that the motion of the electron during 1 cycle of its motion can be represented by the expression:

$$x = A \sin \omega t$$

a) Solve for the energy radiated away by the electron during 1 cycle. (10 points)
b) Solve for the quality factor Q of this oscillator. (10 points)
c) How many periods of oscillation would elapse before the energy of the motion was down to half its initial value? Express your answer symbolically rather than as a numerical value. (10 points)

Problem 2: Driven Coupled Oscillators (30 points)



Suppose we have a driven coupled oscillator system as modeled above. The left and right springs have spring constant k while the middle spring has spring constant $2k$. The masses are identical. At the time $t = 0$ a driving force is applied on the rightmost spring, given by $F(t) = F_0 \cos \omega t$.
a) Write the differential equation of motion for x_1 (make sure the signs are correct). (5 points)
b) Write the differential equation of motion for x_2 (make sure the signs are correct). (5 points)
c) Using the results from parts **a** and **b**, write the uncoupled equations of motion for $q_1 = x_2 + x_1$ and $q_2 = x_2 - x_1$. What are the resonant angular frequencies ω_1 and ω_2 for q_1 and q_2? (5 points)
d) Suppose our system approaches a steady-state solution. Solve for $q_1(t)$ and $q_2(t)$ to get solutions for $x_1(t)$ and $x_2(t)$. (10 points)
e) Describe the motion of the masses as the driving angular frequency ω approaches ω_1 and ω_2. (5 points)

Problem 3: Standing Waves (15 points)

The tension in the A string of the violin is adjusted to produce a fundamental frequency of 440 Hz.
a) What are the frequencies of the second and third harmonics? Does the wave velocity change between these harmonics? (5 points)
b) The hearing range of the violinist extends to 15 kHz. What is the total number of harmonics of the string the violinist can hear? (5 points)
c) If the violin string is $32cm$ long, how far from the end o the string should the violinist place their finger to play the note C (523 Hz)? (5 points)

Problem 4: Traveling Waves (25 points)

Two points on a string are observed as a traveling wave passes them. The points are at $x_1 = 0$ and $x_2 = 1m$. The transverse motions of the two points are found to be as follows:

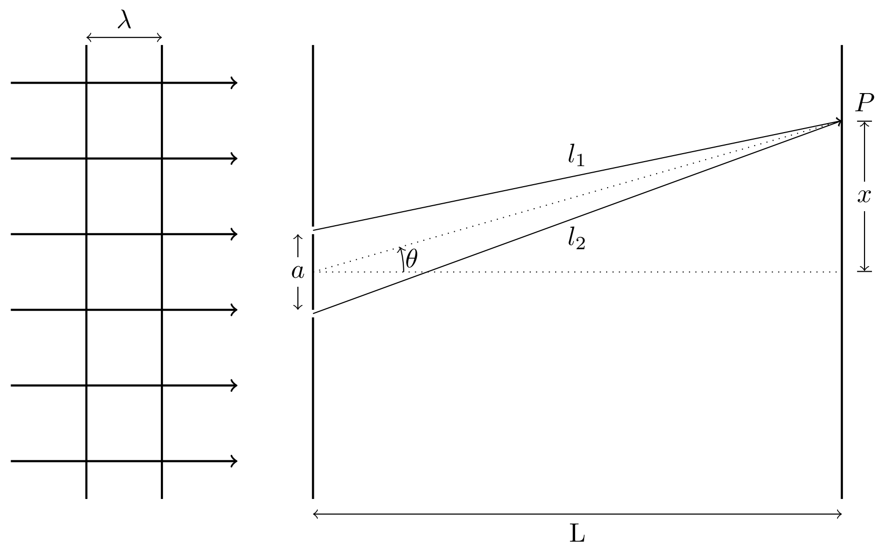
$$y_1 = 0.2sin(3\pi t)$$
$$y_2 = 0.2sin(3\pi t + pi/8)$$

a) What is the frequency in hertz? (5 points)
b) What is the wavelength? (5 points)
c) With what speed does the wave travel? (10 points)
d) Which way is the wave traveling? (5 points)

Problem 5: Traveling Waves II (20 points)

Two strings are smoothly joined at $x = 0$ and held under constant tension $T = 12N$. The left string has mass per unit length, $\mu_1 = 0.04\ kg\ m^{-1}$. The right string has mass per unit length, $\mu_2 = 0.09\ kg\ m^{-1}$. If the incident wave begins in the left string and has amplitude $A_1 = 0.04m$ and $\lambda_1 = \pi\ m$. Find the following:
a) The wavenumber k_1 of the wave traveling on the left string. (2 points)
b) The frequency w_1 of the wave traveling on the left string. (2 points)
c) The wavenumber and frequency (k_2 and w_2) of the wave traveling on the right string. (5 points)
d) The amplitude and wavelength (A_2 and λ_2) of the wave traveling through the right string. (2 points)
e) The wavenumber and frequency (k_3 and w_3), of the reflected wave. (2 points)
f) The reflection coefficient and the transmission coefficient. (2 points)
g) The fraction of the wave power reflected at the boundary of the two strings. (5 points)

Problem 6: Young's double-slit experiment (20 points)

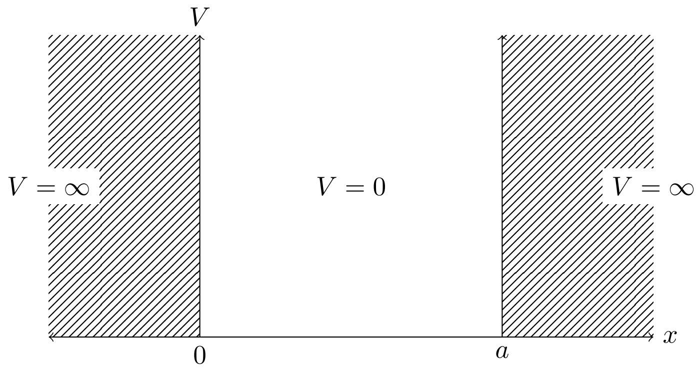


Above is a diagram of Young's double-slit experiment. Using your knowledge of this experiment, answer the following questions:

a) We find that ten bright interference fringes span a distance of 1.8 cm on a screen placed 1.0 m away while the separation of the two slits is 0.30 mm. What is the wavelength λ of the light? (10 points)

b) A $\lambda = 500$nm green laser is used to illuminate slits at a distance of 0.5mm apart with a screen distance of 2m. What is the distance between adjacent bright interference fringes? What is the distance between adjacent dark interference fringes (10 points)?

Problem 7: Quantum particle in a box (30 points)



A quantum particle is placed inside a 1 dimensional box, such that the potential V is as follows:

$$V(x) = \begin{cases} 0 & 0 \leq x \leq a \\ \infty & \text{otherwise} \end{cases}$$

a) Solve the time-independent Schrödinger equation to find the allowed energies of a particle inside the box for any nonzero integer n, where n denotes the energy level. (hint: use boundary conditions) (10 points)
b) What are the corresponding allowed $\Psi_n(x)$ inside the box? (10 points)
c) Find the normalization factor for $\Psi(x)$ (hint: the probability of finding the particle somewhere inside the box must be 1) (5 points)
d) What's the probability of finding the particle in the ground state n=1 between $x = a/2$ and $x = 3a/4$? (5 points)

Problem 8: Extra credit (10 points)

Write a brief essay on the interpretations of quantum mechanics.