

INTRODUCTION TO CONTINUOUS CONTROL SYSTEMS
COLUMBIA UNIVERSITY MECHANICAL AND ELECTRICAL ENGINEERING
DEPARTMENTS: E3601

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Homework 3

Problem 1 (Analyticity of the Trigonometric Functions).

Prove that the following functions are analytic everywhere in the \mathbb{C} plane, except at some special points, and that $\sin(s)$ and $\cos(s)$ are periodic with period 2π .

Trigonometric Functions of complex variable s are defined in terms of the Exponential Function, e^s , as follows,

$$\sin(s) \triangleq \frac{e^{is} - e^{-is}}{2i} \quad (1)$$

$$\cos(s) \triangleq \frac{e^{is} + e^{-is}}{2} \quad (2)$$

$$\csc(s) \triangleq \frac{1}{\sin(s)} \quad (3)$$

$$\sec(s) \triangleq \frac{1}{\cos(s)} \quad (4)$$

$$\tan(s) \triangleq \frac{\sin(s)}{\cos(s)} \quad (5)$$

$$\cot(s) \triangleq \frac{\cos(s)}{\sin(s)} \quad (6)$$

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