

**INTRODUCTION TO CONTINUOUS CONTROL SYSTEMS**  
**COLUMBIA UNIVERSITY MECHANICAL AND ELECTRICAL ENGINEERING**  
**DEPARTMENTS: E3601**

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## Homework 3

**Problem 1** (Analyticity of the Trigonometric Functions).

*Prove that the following functions are analytic everywhere in the  $\mathbb{C}$  plane, except at some special points, and that  $\sin(s)$  and  $\cos(s)$  are periodic with period  $2\pi$ .*

*Trigonometric Functions of complex variable  $s$  are defined in terms of the Exponential Function,  $e^s$ , as follows,*

$$\sin(s) \triangleq \frac{e^{is} - e^{-is}}{2i} \tag{1}$$

$$\cos(s) \triangleq \frac{e^{is} + e^{-is}}{2} \tag{2}$$

$$\csc(s) \triangleq \frac{1}{\sin(s)} \tag{3}$$

$$\sec(s) \triangleq \frac{1}{\cos(s)} \tag{4}$$

$$\tan(s) \triangleq \frac{\sin(s)}{\cos(s)} \tag{5}$$

$$\cot(s) \triangleq \frac{\cos(s)}{\sin(s)} \tag{6}$$

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