

Pre-Leaving Certificate Examination 2026

Mathematics

(Option A)

Paper 1

Higher Level

2 hours 30 mins

300 marks

Name:
School:
Address:
Class:
Teacher:

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

Running total	
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Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer questions as follows:

- **any five** questions from Section A – Concepts and Skills
- **any three** questions from Section B – Contexts and Applications

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet.

If you need to use it, label any extra work clearly with the question number and part.

The superintendent may give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination.

In general, diagrams are not to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

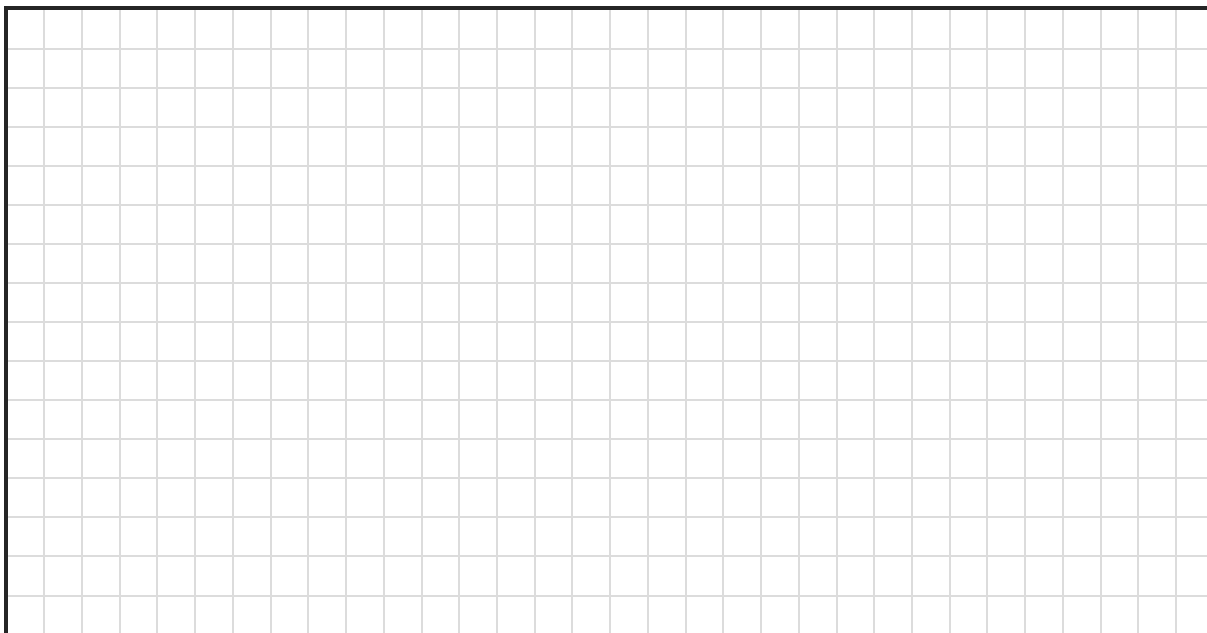
Write the make and model of your calculator(s) here:

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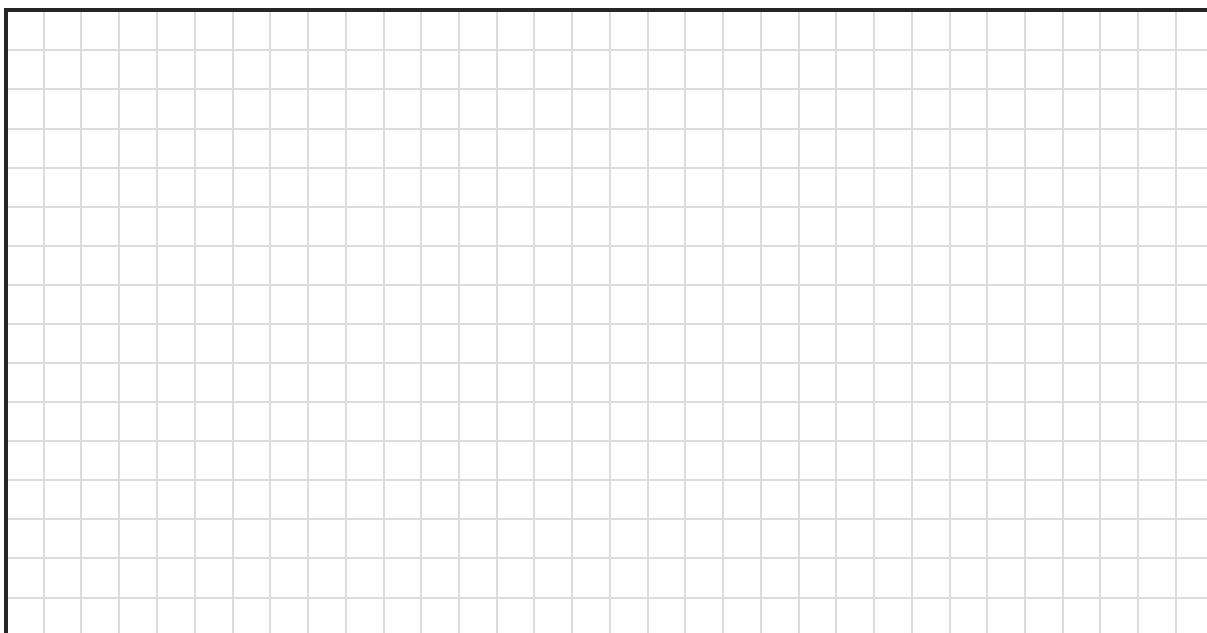
Answer **any five questions** from this section.

Question 1**(30 marks)**

- (a) Fully factorise $a^2 + 2ab + b^2 - 49$.



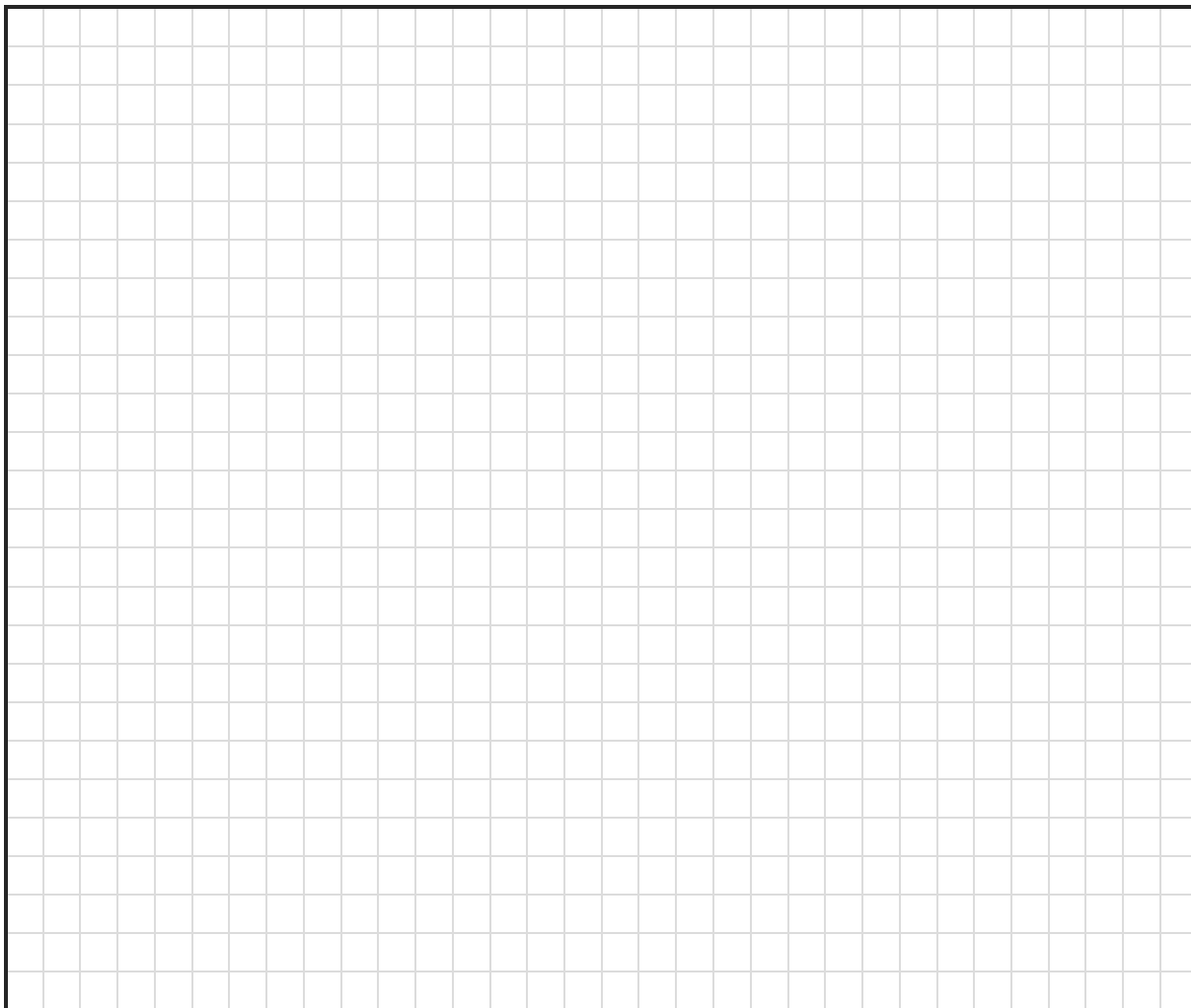
- (b) Fully factorise $3m - 8n - 2 + 12mn$.



(c) Solve the simultaneous equations for $x, y \in \mathbb{R}$:

$$x^2 + 2xy - 8 = 0$$

$$\frac{2}{x} - \frac{2y}{x} = 1 \text{ when } x \neq 0.$$



Question 2**(30 marks)**

The function $f(x)$ is defined for $x \in \mathbb{R}$ by:

$$f(x) = 5 - \frac{1}{e^x}$$

- (a) Find the value of $f(5)$. Give your answer correct to 4 decimal places.

- (b) Find the value of $\lim_{x \rightarrow \infty} f(x)$.

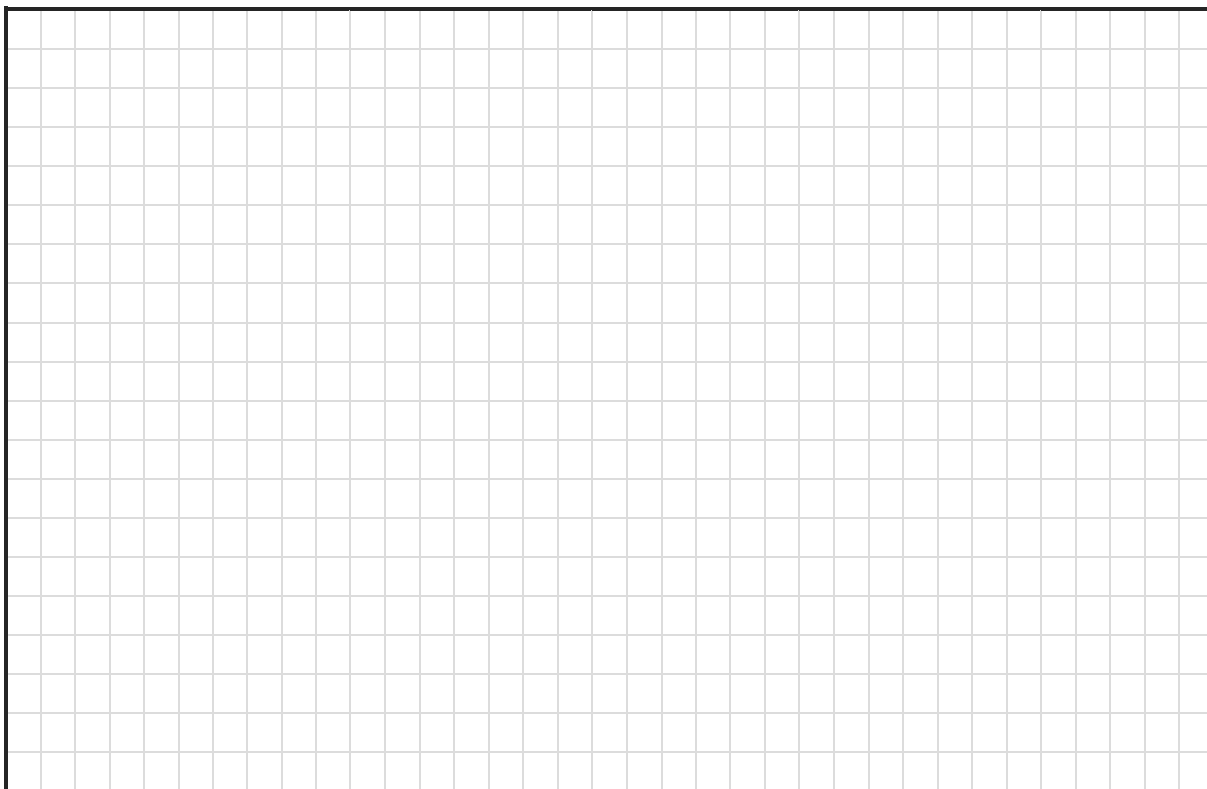
- (c) Solve the following equation in x :

$$f(x) = \frac{1}{5}$$

Give your answer in the form $\ln a - \ln b$, where $a, b \in \mathbb{N}$.

(d) Solve for x , where $x \in \mathbb{R}$:

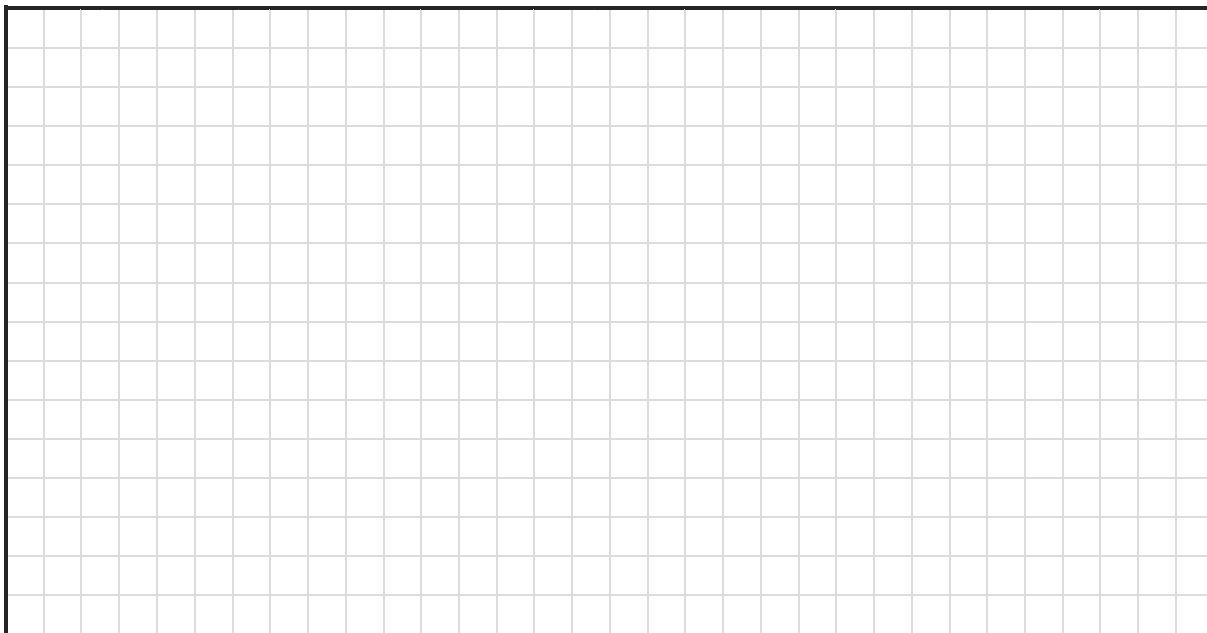
$$\log_2(x + 2) + \log_2(x - 2) = 5$$



Question 3**(30 marks)**

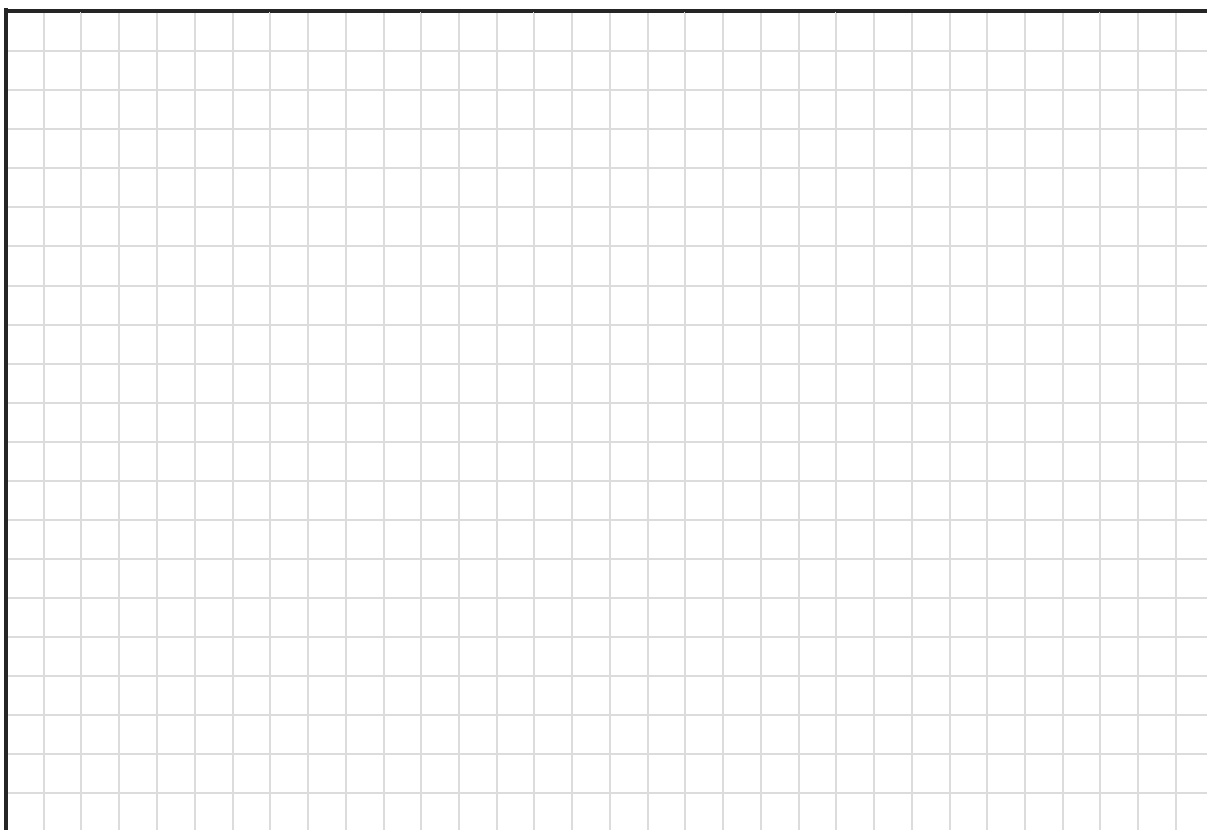
In this question, $i^2 = -1$.

- (a) Simplify $\left(\frac{5+2i}{2-5i}\right)$ and hence find the value of $\left(\frac{5+2i}{2-5i}\right)^5$.



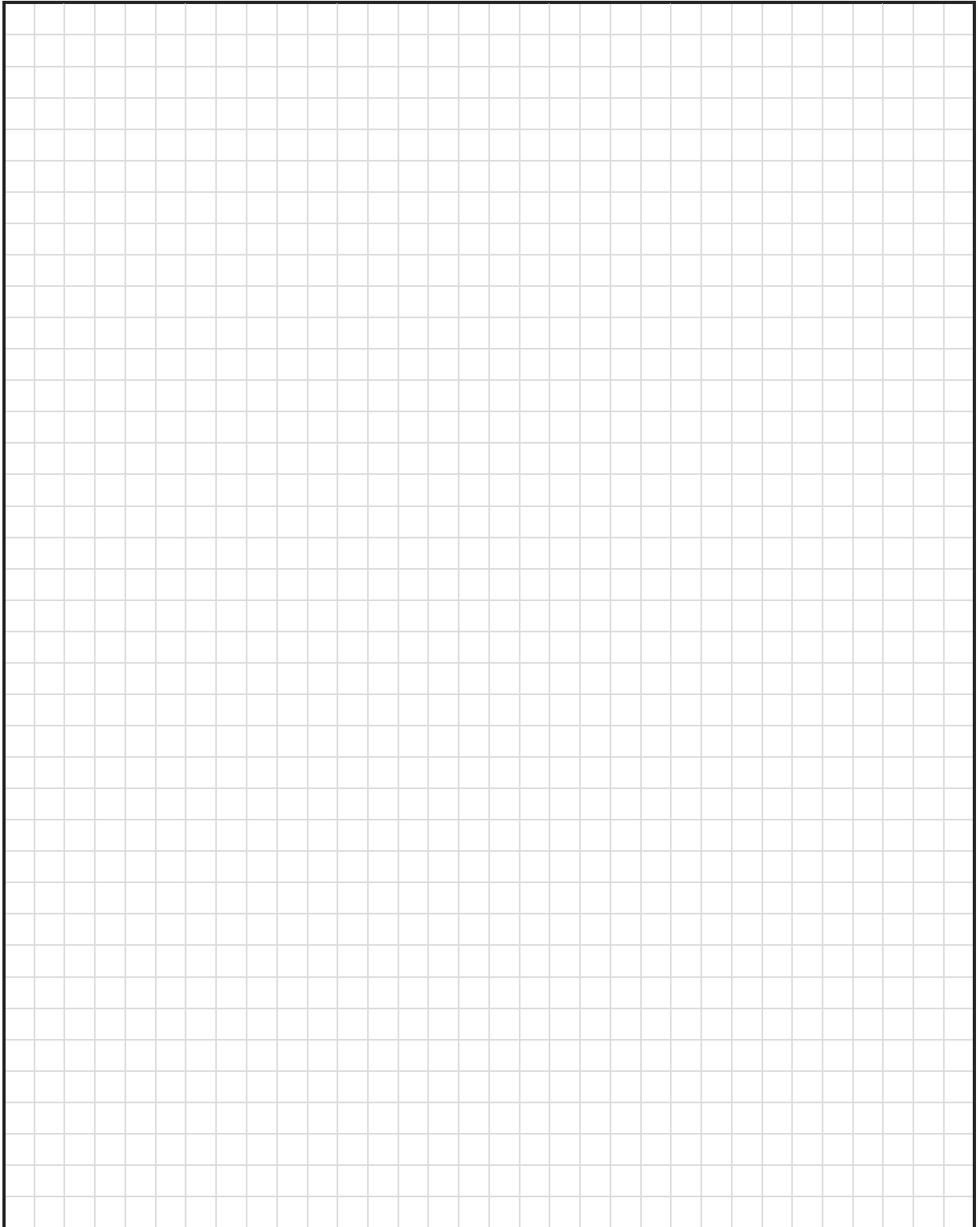
- (b) Use **de Moivre's theorem** and the expression $(\cos\theta + i\sin\theta)^3$ to prove that the following is true, for any angle θ :

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$



- (c) Express $-8 - 8\sqrt{3}i$ in the form $r(\cos\theta + i\sin\theta)$, and hence use de Moivre's theorem to find the **four roots** of the equation $z^4 = -8 - 8\sqrt{3}i$.

Give each of your solutions in the form $a + bi$, where $a, b \in \mathbb{R}$.



Question 4

(30 marks)

- (a) The function $f(x)$ is defined for $x \in \mathbb{R}$ by:

$$f(x) = \frac{x^2 - 1}{x^2 + 1}$$

Find an expression for $f'(x)$, the derivative of $f(x)$, in its simplest form.

- (b) The function is defined for $x \in \mathbb{R}$ by:

$$g(x) = 3e^x - \sin x + 5$$

- (i) Find $g'(0)$, the derivative of $g(x)$, where $x = 0$.

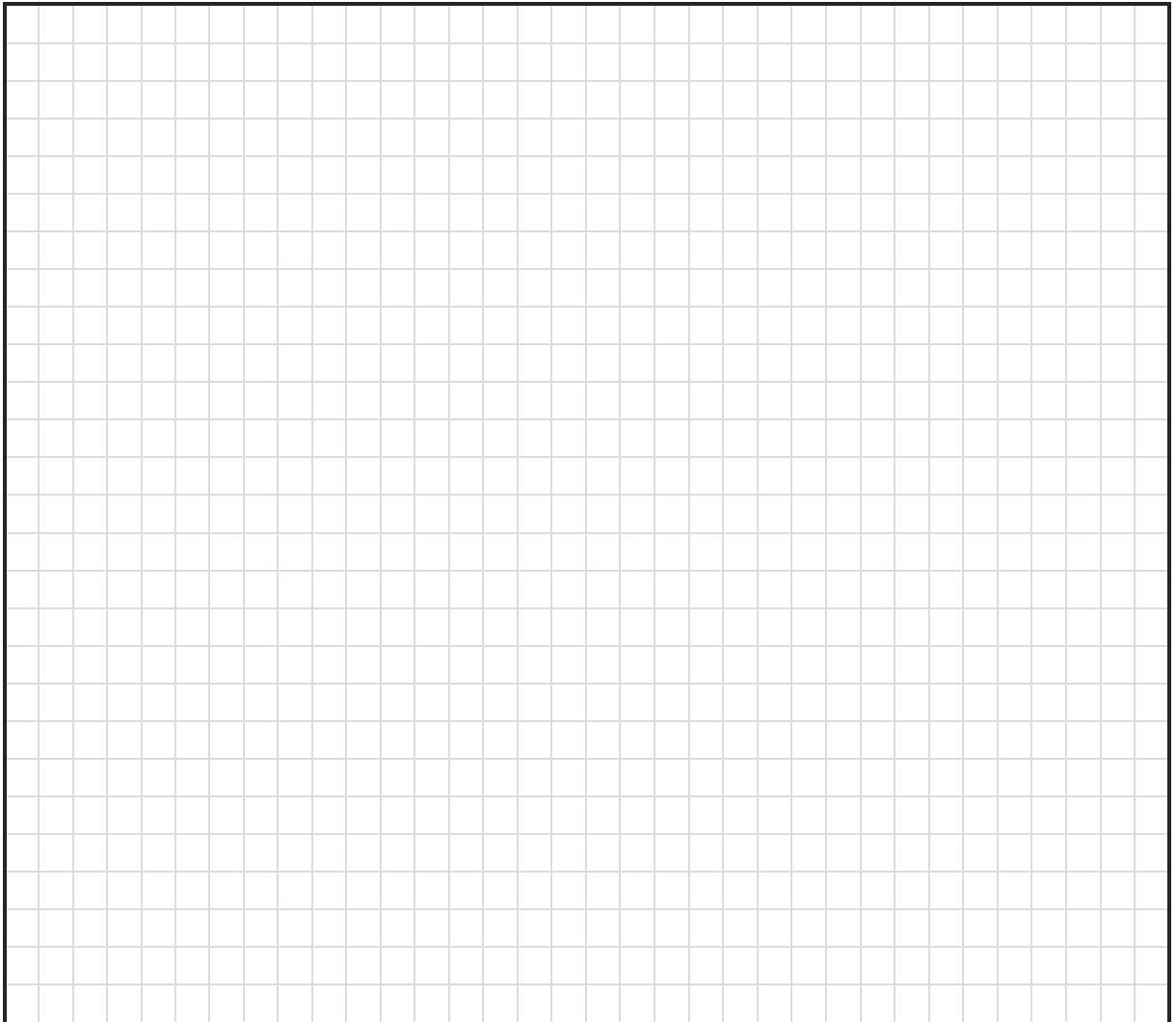
- (ii) Hence, find the **equation** of the tangent to the curve $y = g(x)$ at the point where $x = 0$. Give your answer in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Z}$.

- (c) (i) Use the Product Rule to find the derivative of $x^2 \tan x$.



- (ii) Hence, find $\int (x \tan x + 1)^2 dx$.

Hint: use the identity $\tan^2 x = \sec^2 x - 1$.



Question 5**(30 marks)**

- (a) $\frac{5}{2}$ and $\frac{3}{5}$ are the solutions of the following equation, where $x \in \mathbb{R}$ and $a, b, c \in \mathbb{Z}$:

$$ax^2 + bx + c = 0$$

Find the value of a and the value of b and the value of c .

$a =$	$b =$	$c =$
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- (b) (i) The function $g(x)$ is defined for $x \in \mathbb{R}$ by:

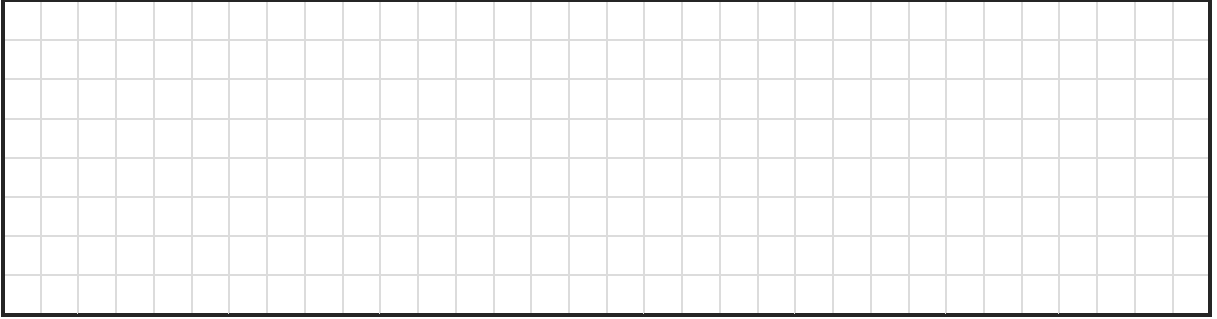
$$2x^2 + 4x - 5$$

Write $g(x)$ in the following form, where $a, h, k \in \mathbb{Z}$ are constants:

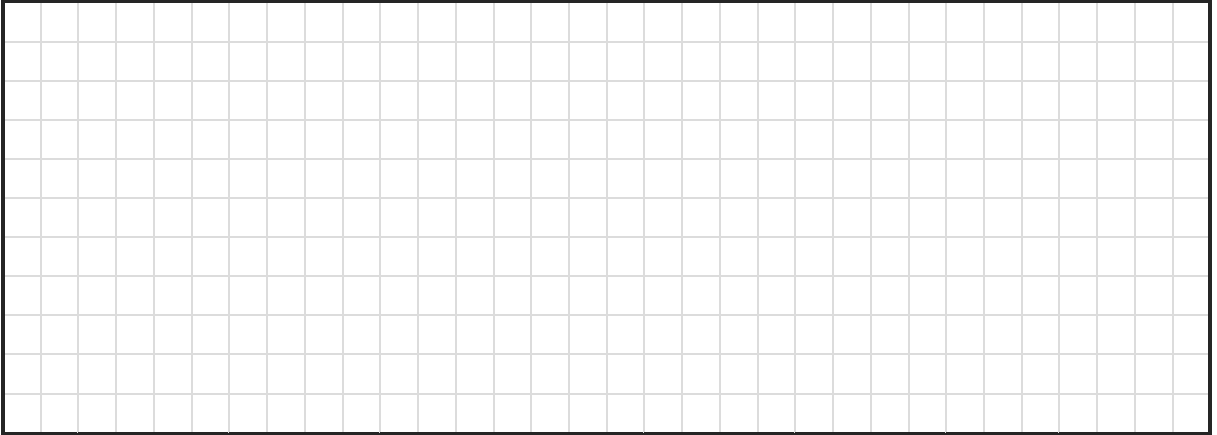
$$g(x) = a(x + h)^2 + k$$

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(ii) Hence, find the minimum point of $g(x)$. Justify your answer.



(c) Explain why $g(x) = 0$ must have two real roots.



(30 marks)

- $$g(x) = x^3 - 6x^2 + 9x - 5$$

(i) the local minimum point of g

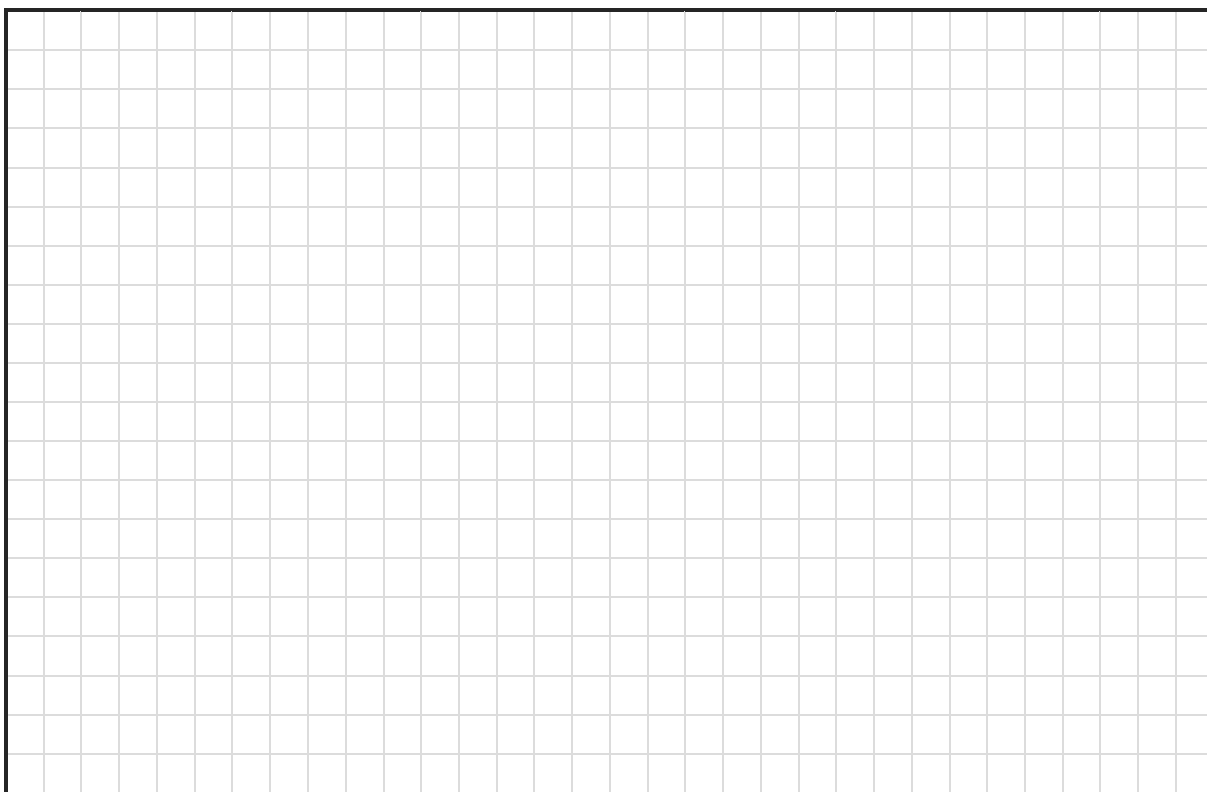
(ii) the local maximum point of g

(iii) the point of inflection of g

(i)	Local minimum:	$(\quad , \quad)$	
(ii)	Local maximum:	$(\quad , \quad)$	
(iii)	Point of inflection:	$(\quad , \quad)$	

(b) Differentiate the following function from first principles, with respect to x :

$$f(x) = 5x^2 - 7x$$



Answer **any three questions** from this section.

Question 7

(50 marks)

A beekeeper is starting a new colony of bees. The population of bees in a colony, in thousands, can be modelled by

$$b(t) = 4.5e^{1.4t}$$

where t is the number of years since the establishment of the colony.



- (a) Find the initial population of the bee colony.

[illegible]

- (b)** Find the increase in the population of the bee colony in the first six months.

[illegible]

- (c) Find the **rate of population growth** two years after the establishment of the colony.

[illegible]

- (d) After how many years will the rate of population growth be 60 000 bees/year?
Give your answer in years, correct to 2 decimal places.

After three years, the beekeeper notices that the number of bees begins to decline. The declining population, d , in thousands, has the form $d(t) = Ae^{kt}$, where t is the number of years since the start of the decline.

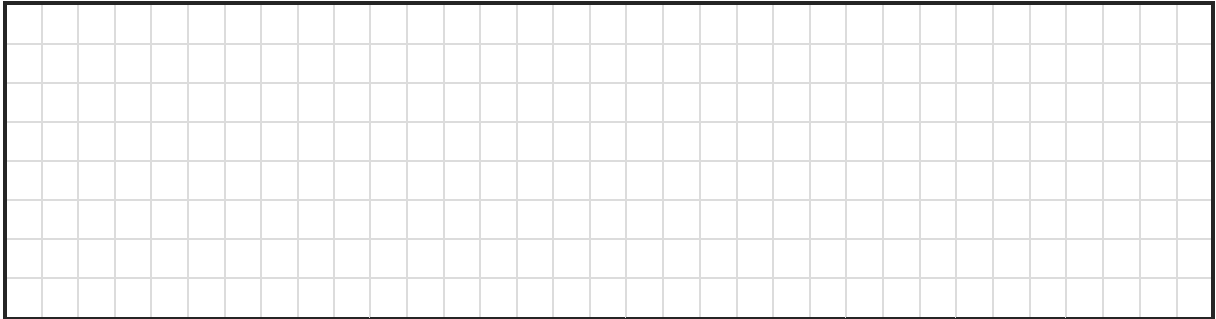
- (e) (i) Find the population of bees at the start of the decline in numbers and use this to find the value of A , correct to the nearest whole number.

- (ii) After one year, the population of bees is found to have reduced to 180,000.
Use this to find the value of k , correct to 4 decimal places.

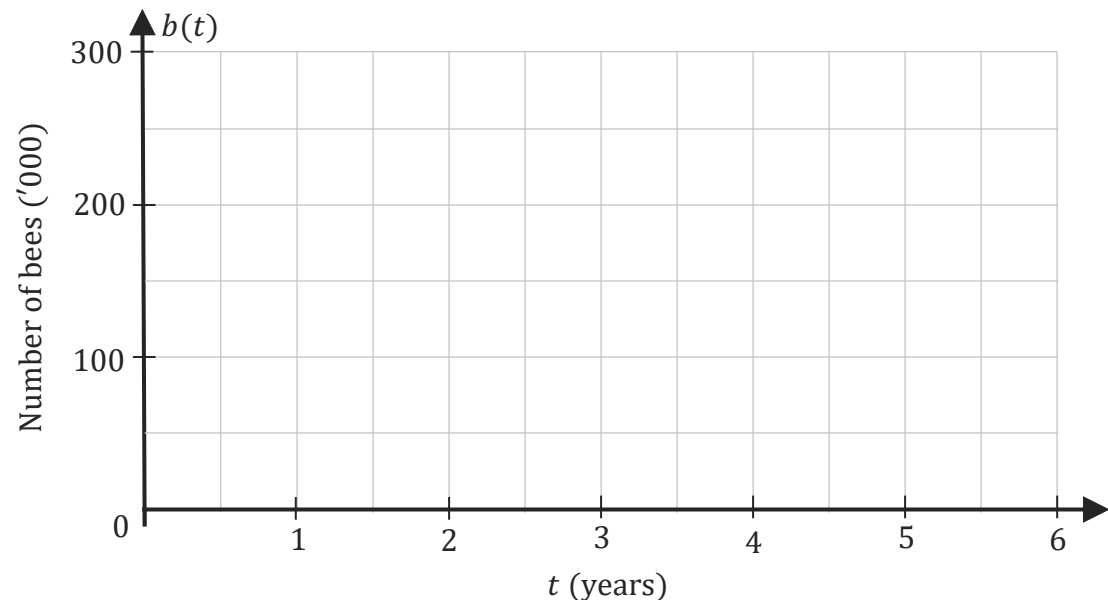
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- (f) (i) Fill in the table below to show the values of $b(t)$ for the given values of t , up to $t = 6$. Give each value correct to 1 decimal place, where appropriate.

t (year)	0	1	2	3	4	5	6
$b(t) = 300e^{-0.5108t}$				64.8			



- (ii) Hence, draw the graph of the function $y = b(t)$ on the axes below, for $0 \leq t \leq 6$, $t \in \mathbb{R}$.

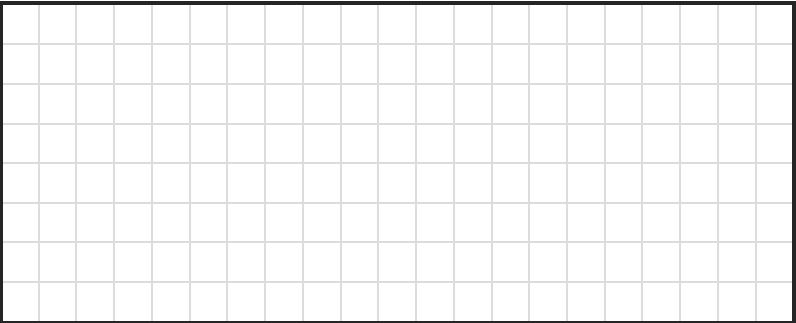
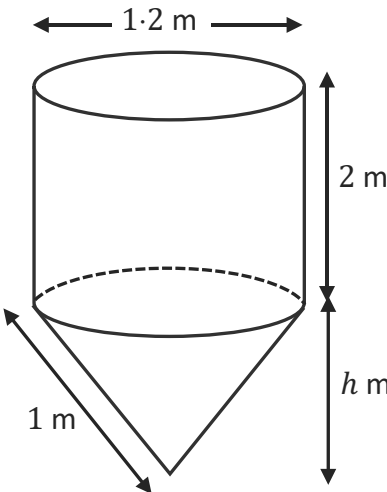


Question 8

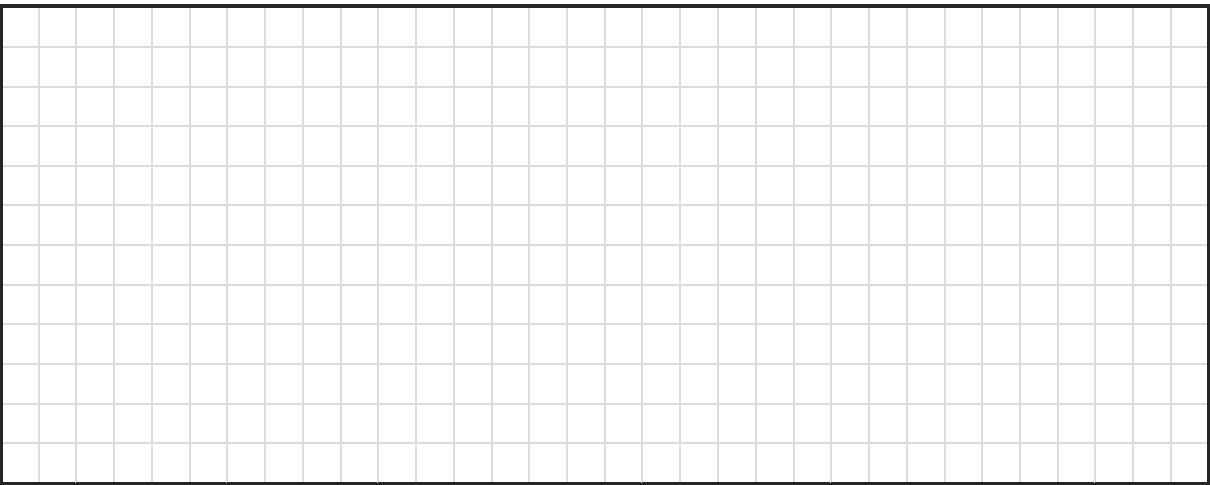
(50 marks)

(a) A large water tank is formed from a cylinder and a cone as shown.

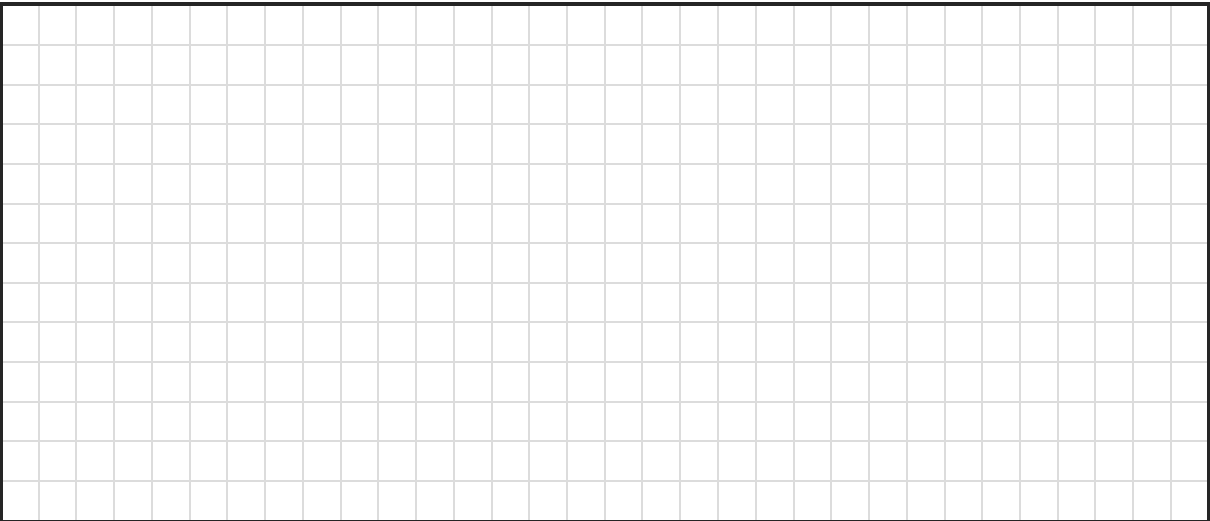
(i) Show that h , the height of the conical section of the tank, is 0.8 m.



(ii) The inside of the tank is sprayed with a mould-resistant paint. If the paint can cover 3 square metres per can, how many cans of paint are required to paint the total interior of the tank, including the lid?



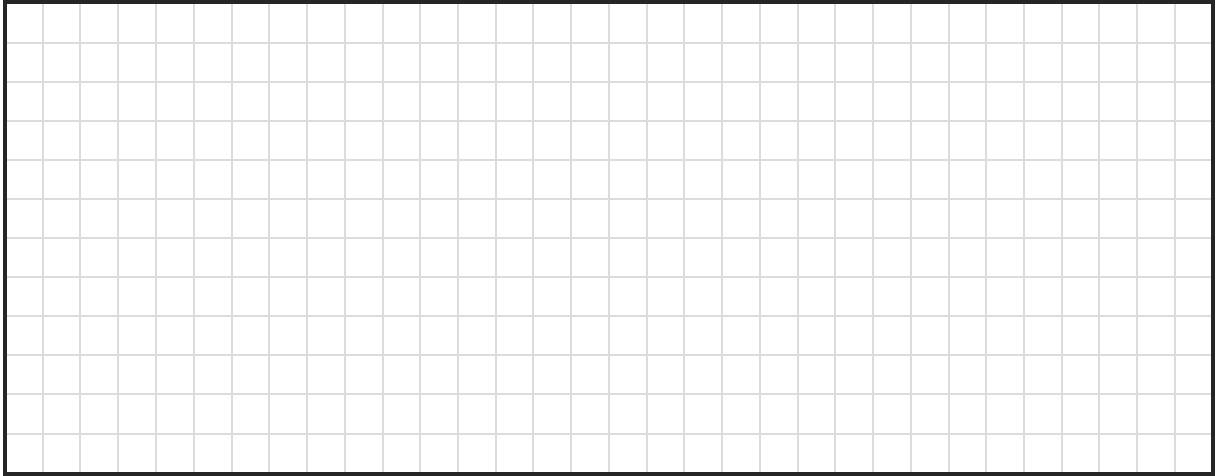
(iii) Find the volume of the water tank.
Give your answer in cubic metres, correct to 2 decimal places.



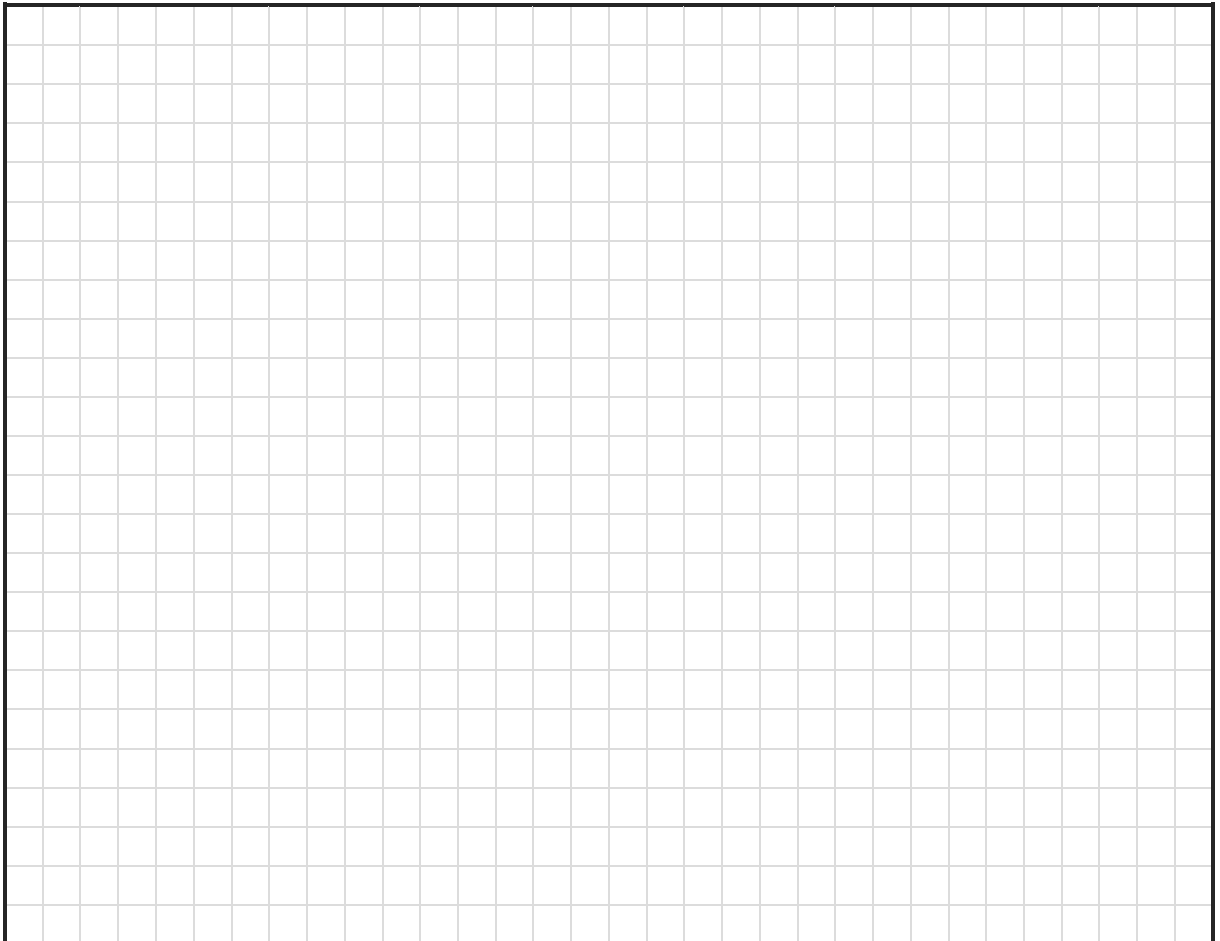
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Water is pumped into the tank at a rate of $1.2 \text{ m}^3/\text{min}$, and the water hasn't reached the cylindrical part.

- (b) (i) Show that in the conical section of the tank $r = 0.75h$.



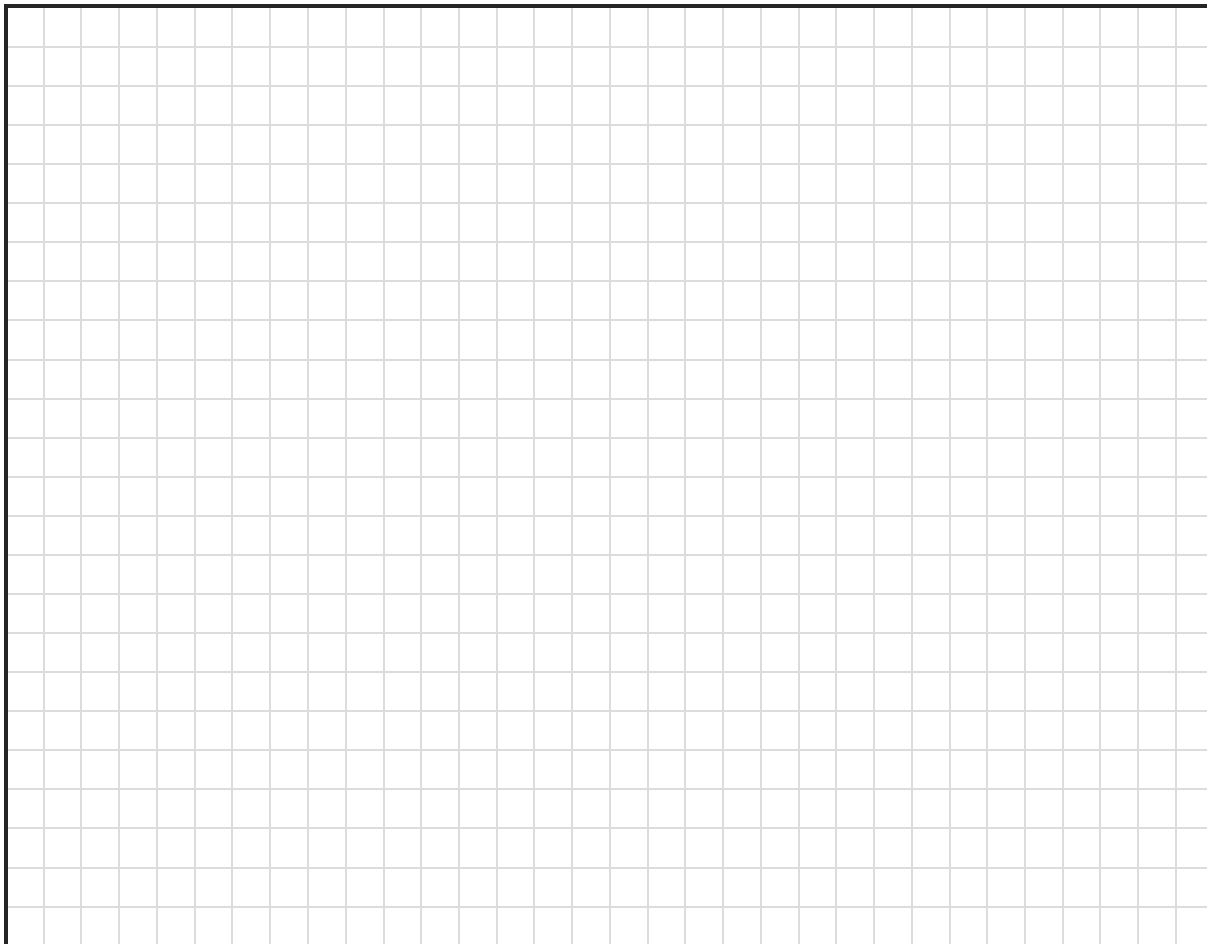
- (ii) How fast is the water level rising in the conical base when the depth of water is 0.5 metres? Give your answer in m/minute, correct to 2 decimal places.



- (c) Water is filling the conical base, such that the radius of the water surface is increasing at a rate of **0.04 m/min**.

How fast is the volume of water changing when the radius is **0.5 metres**?

Give your answer in cubic metres per minute, correct to 3 decimal places.



Question 9

(50 marks)

- (a) An elderly patient receives a 20 mg dose of a drug in tablet form every day at the same time. Each day, the amount of the drug remaining in the body decreases by 30%. So, the amount of the drug, in mg, left in the patient's body n days after a single tablet is given by:

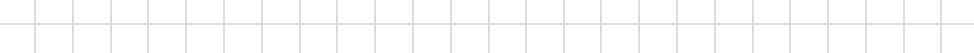
$$20(0.7)^n$$

where $n \in \mathbb{R}$.

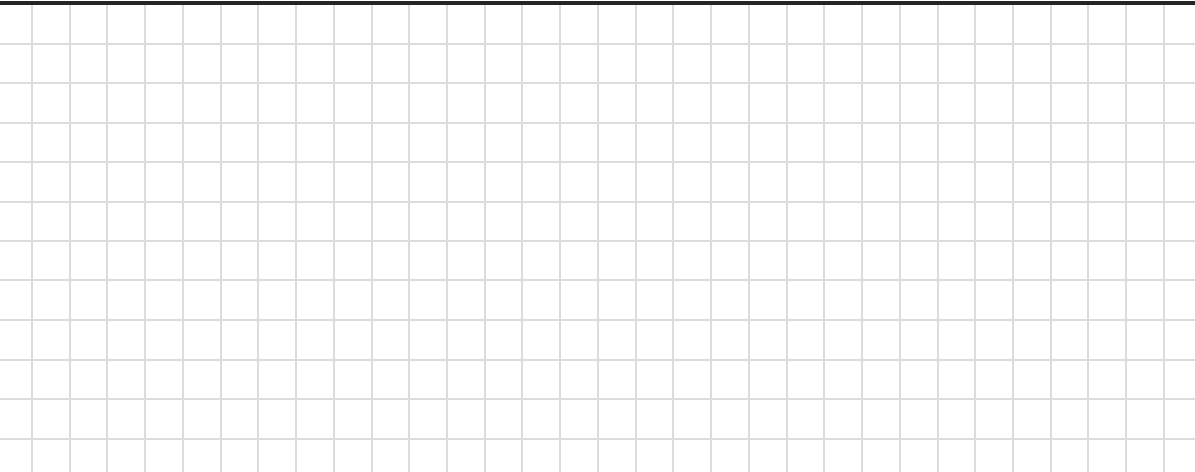


- (i) What is the amount of the drug left in the body 5 days after the tablet is taken?

Give your answer in mg, correct to 2 decimal places.

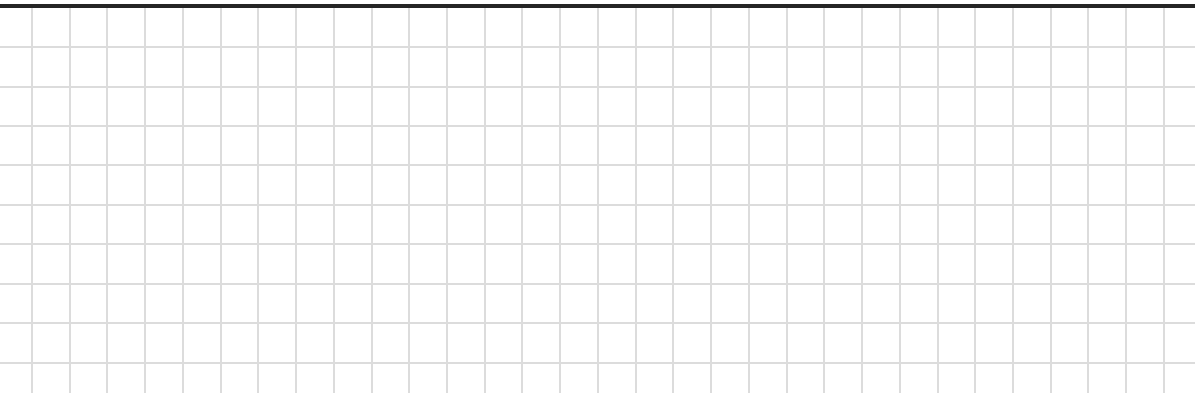


- (ii) How many days after a single tablet has been taken will there be less than 1 mg of the drug left in the patient's body?

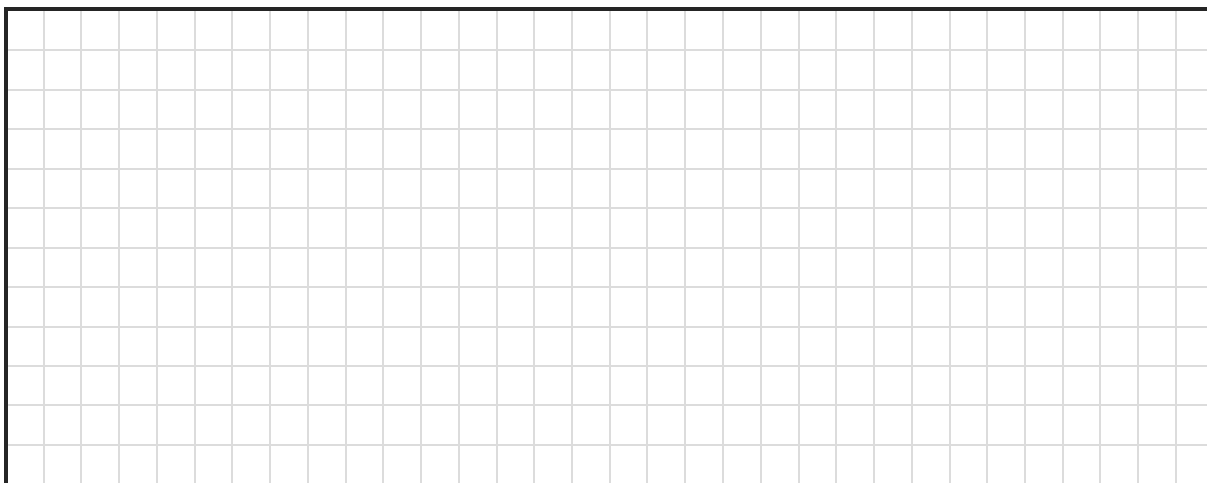
A large grid of graph paper, consisting of 20 columns and 10 rows of squares, intended for drawing a picture.

- (iii) Explain why the total amount of the drug, in mg, in the patient's body immediately after the 5th tablet is taken, is given as the geometric series:

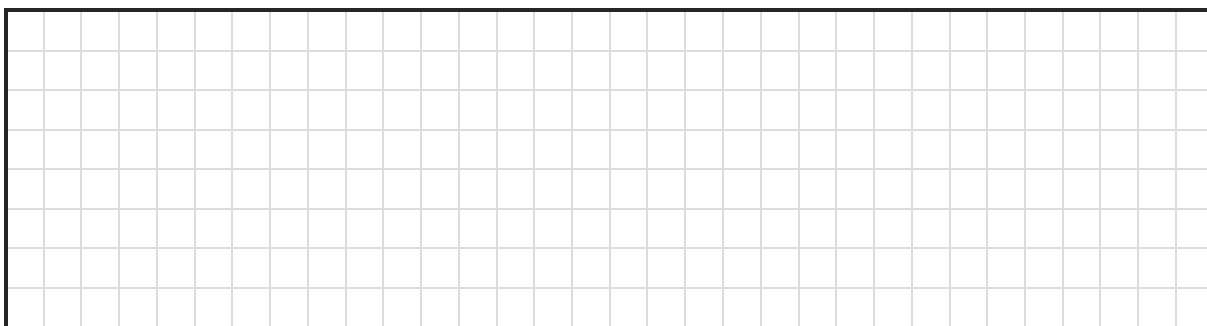
$$20 + 20(0.7) + 20(0.7)^2 + 20(0.7)^3 + 20(0.7)^4$$

A large grid of graph paper, consisting of 20 columns and 10 rows of squares, intended for drawing a picture.

- (iv) Find the amount of the drug in the patient's body immediately after one week, that is after the 7th tablet has been taken.
Give your answer in mg, correct to 2 decimal places.



- (v) Use the formula for the sum to infinity of a geometric series to estimate the amount of the drug in the patient's body after a long period of time taking the tablets daily.
Give your answer in mg, correct to 2 decimal places.

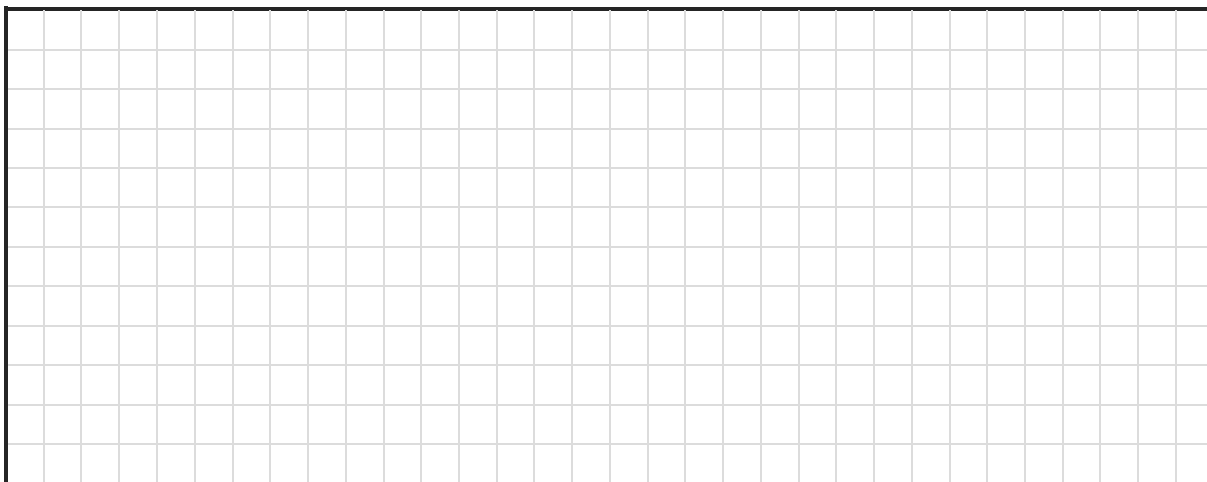


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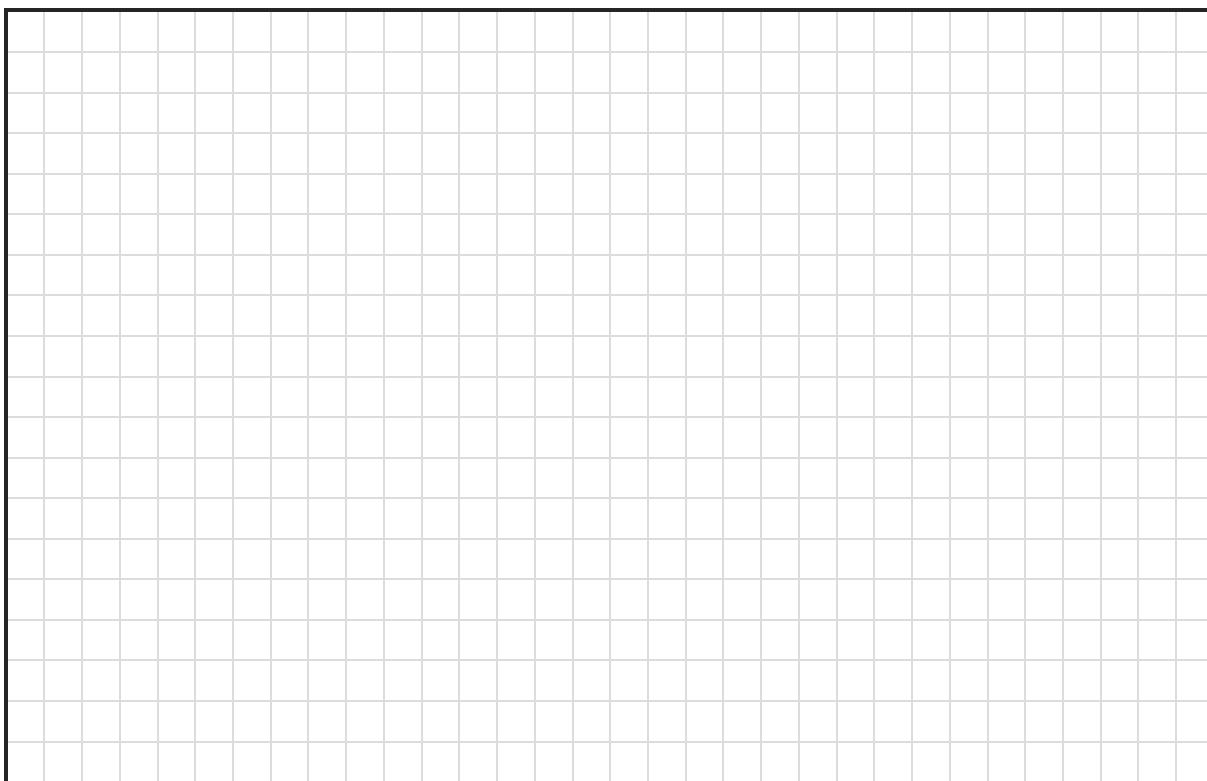
The patient takes another tablet at the same time every day to maintain their cholesterol level. There is k mg of the drug in each tablet, where $k \in \mathbb{R}$. Each day, the amount of the drug left in the patient's body from this tablet **decreases** by 35%.

- (b) (i) Use the sum of a geometric series to show that the total amount of the drug, in mg, in the patient's body immediately after the n th tablet is taken, where $n \in \mathbb{N}$, is:

$$\frac{20k(1 - 0.65^n)}{7}$$



- (ii) Immediately after the 6th tablet is taken, there are 45 mg of the drug in the patient's body.
Find the amount of the drug in one of the patient's tablets.
Give your answer correct to the nearest mg.



(50 marks)

- $$T(x) = \frac{\sqrt{64 + x^2}}{5} + \frac{20 - x}{15}$$

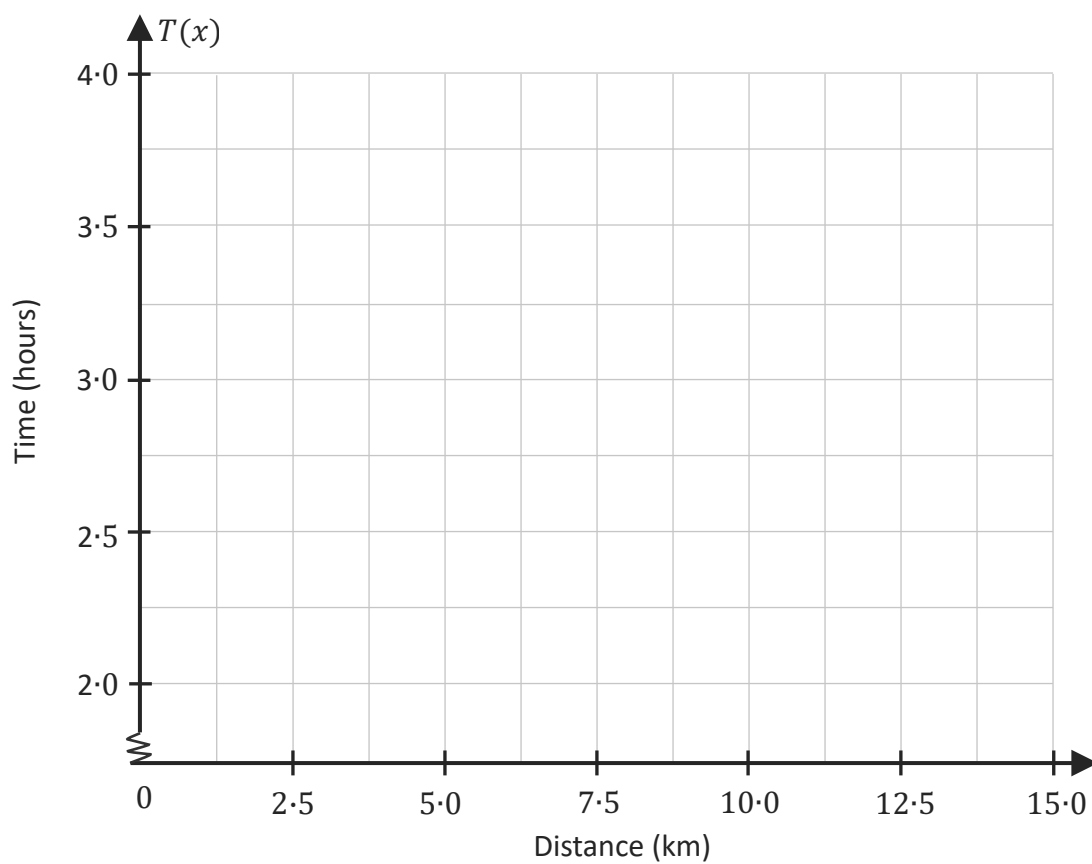
[illegible]

- | x (km) | 0 | 2.5 | 5.0 | 7.5 | 10.0 | 12.5 | 15.0 |
|--------------|---|-----|------|------|------|------|------|
| Time (hours) | | | 2.89 | 3.03 | | | |

[illegible]

24

- (c) Hence, draw the graph of the function $y = T(x)$, on the axes below, for $0 \leq x \leq 15$, $x \in \mathbb{R}$.

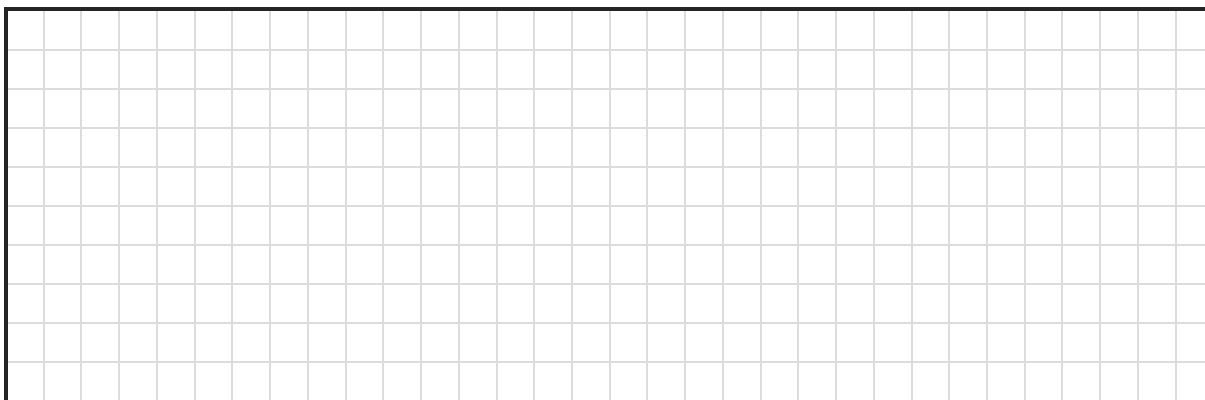


- (d) Show that $T'(x)$, the derivative of $T(x)$, can be written as:

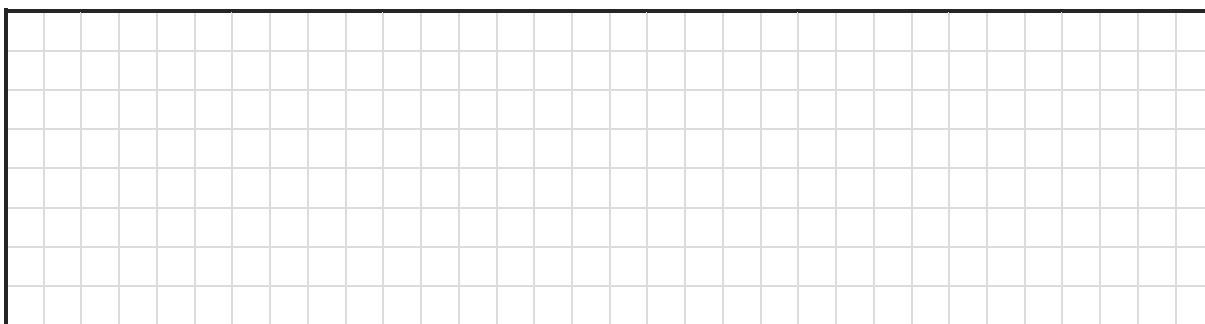
$$T'(x) = \frac{x}{5\sqrt{64 + x^2}} - \frac{1}{15}$$



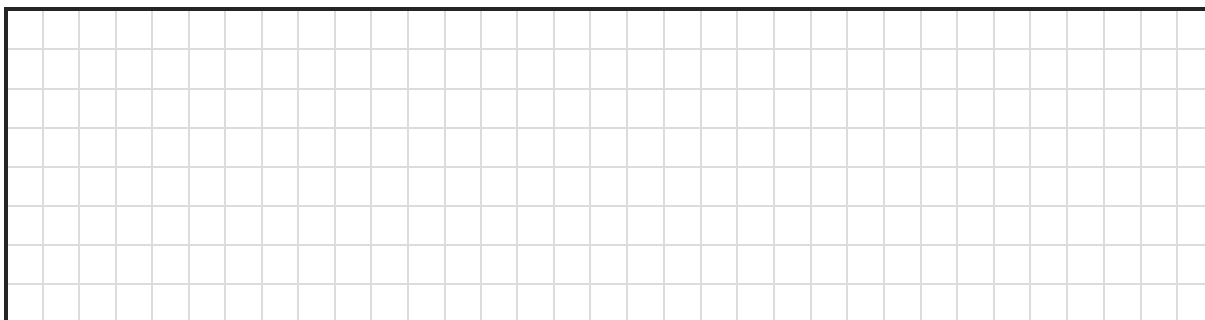
- (e) Show that the time taken for Kate to complete the course has its minimum value when $x = 2\sqrt{2}$.



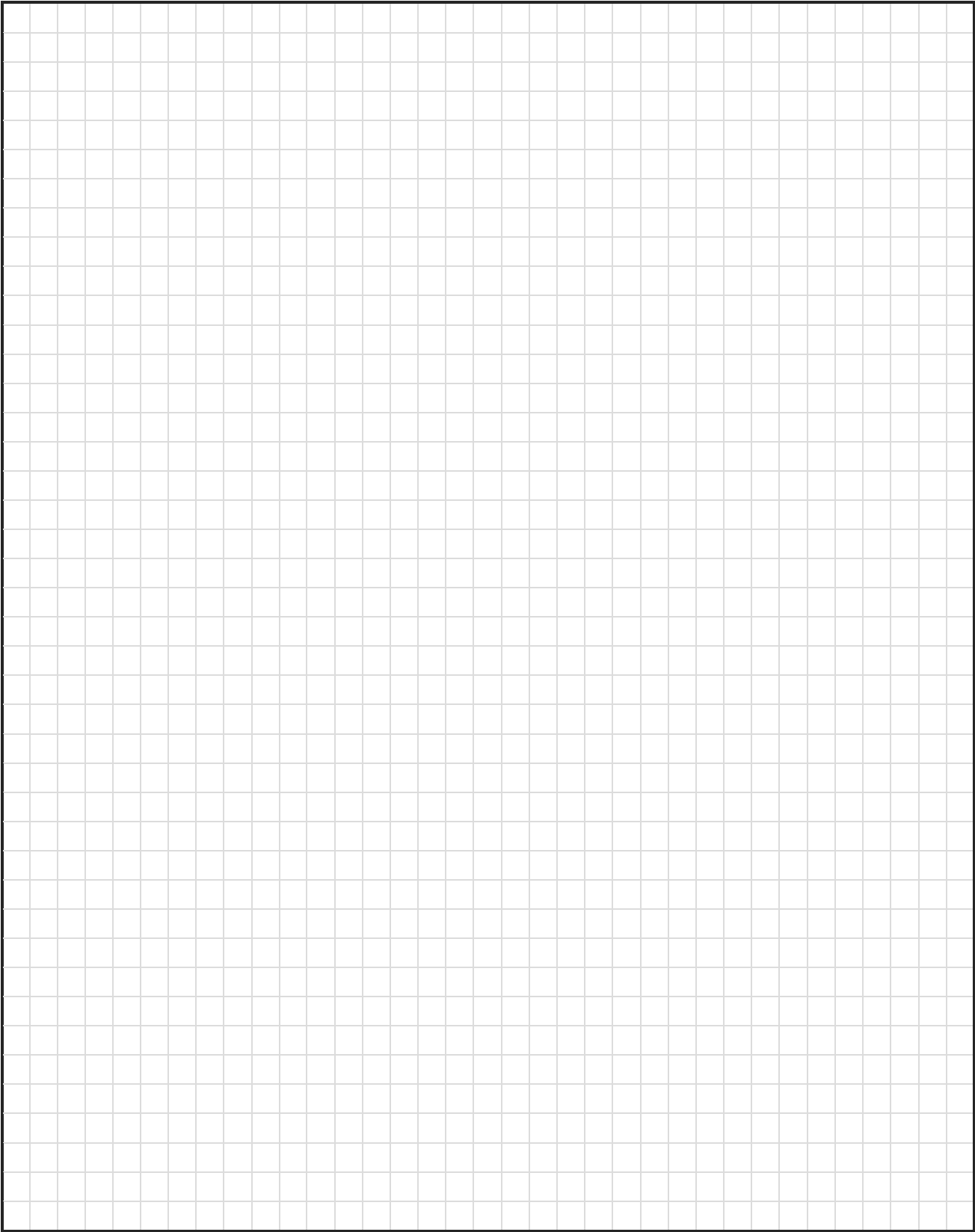
- (f) Find the time when this minimum value is reached.
Give your answer correct to the nearest minute.



- (g) What value of x minimises the time taken if $|OF|$ is 2 km instead of 20 km?



Page for extra work.
Label any extra work clearly with the question number and part.





P 6

Page for extra work.

Label any extra work clearly with the question number and part.