Introduction to Experiment

Objective: To gain familiarity with some of the measurement tools you will use in lab this semester. To learn how to measure distance with a motion sensor and force with a force sensor. To acquire experimental data, present it both statistically and graphically and become skilled at its analysis and interpretation.

Apparatus: Motion sensor, force sensor, LabPro interface, computer & keyboard, meter stick, chalk/masking tape (optional)

Introduction

This lab serves as an introduction to the tools you will use to gather and analyze experimental data. Data analysis is important in any scientific, technical or health-care discipline, where it is used for critical decisions and thinking. In addition, the familiarity you develop with these instruments and software will come in handy when they are used many times throughout the semester.

In the first part of the lab you will generate data by measuring your own reaction time; your lab partner will then repeat the experiment and you will analyze the data and compare your results. In the second part of the lab, you will tinker with a motion sensor and a force sensor to become familiar with them.

Theory of Uncertainty and Measurement

A scientist or engineer collects experimental data by taking measurements. You have no doubt collected experimental data in your life – perhaps by finding out how much you weigh, how tall you are, or how fast you have run a certain distance. Let's assume that you know your height to be 5'6" (168 cm), your weight 140 lb (64 kg) and your time in the 100-meter dash 12.16 seconds – how valid are these numbers? Are you *exactly* 168 cm tall, and not 168.0625 cm (5'6 $^1/_8$") tall? How does your 140 lb weight as measured on your $20 bathroom scale compare to your weight measured on a calibrated pharmaceutical scale used to weigh the ingredients at a manufacturing plant? To answer these questions, you will need to understand three basic concepts – uncertainty, precision and accuracy. You may have used the last two interchangeably in everyday life, but here in the laboratory we will make an important distinction between the two. Precision and accuracy are two different things - you can have data with high precision but low accuracy, and vice versa.

Uncertainty:

There are two types of uncertainties you will encounter in the lab: *Systematic Uncertainties and Random Uncertainties***.**

Systematic uncertainties cause a measurement to be skewed in a certain direction, i.e., consistently large or consistently small. For example, weighing yourself repeatedly on a bathroom scale that has an initial reading of 20lbs will result in your always appearing to be heavier than you actually are. This form of uncertainty can be removed, once identified – in this case you can just zero the scale using the little ridged knob (calibration).

Random uncertainties are *variations* in measurement that linger even after systematic uncertainties have been eliminated. If you weighed yourself on ten bathroom scales around your neighborhood (or weighed yourself ten times on your bathroom scale) over the period of a few minutes, you would probably record ten different values even though you know your weight is essentially not changing. This sort of uncertainty cannot be eliminated but can be reduced by making lots of measurements and averaging.

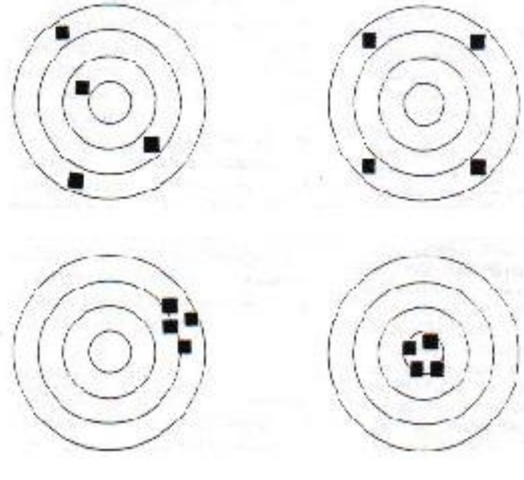
Precision:

Measurement A is more *precise* than Measurement B if the random uncertainty in A is less than that of B. If you weighed yourself 10 times on a bathroom scale, the difference between your highest and lowest readings may be as much as 5 pounds. However, at the doctor's office, the same measurement may only yield a difference of perhaps half a pound; we say that the latter measurement is more precise. Precision is affected by random uncertainty.

Accuracy:

Measurement A is more *accurate* than Measurement B if the result you get from A is closer to the generally accepted, or *standard* value, than B is. When you zeroed your bathroom scale, you improved the accuracy of the measurement. Accuracy is affected by systematic uncertainty.

Note that you can have a high-precision, low-accuracy measurement, such as a doctor's scale that hasn't been calibrated. Conversely, you can have a low-precision, high accuracy scale, such as a cheap bathroom scale that happens to yield an average weight close to your true weight. Below is a representation of the permutations of accuracy and precision:



Can you discuss the level of both accuracy *and* precision for each of the above dart games?

MATHEMATICAL DESCRIPTION OF RANDOM UNCERTAINTIES

The first concept is the average (arithmetic mean) of a number of measurements:

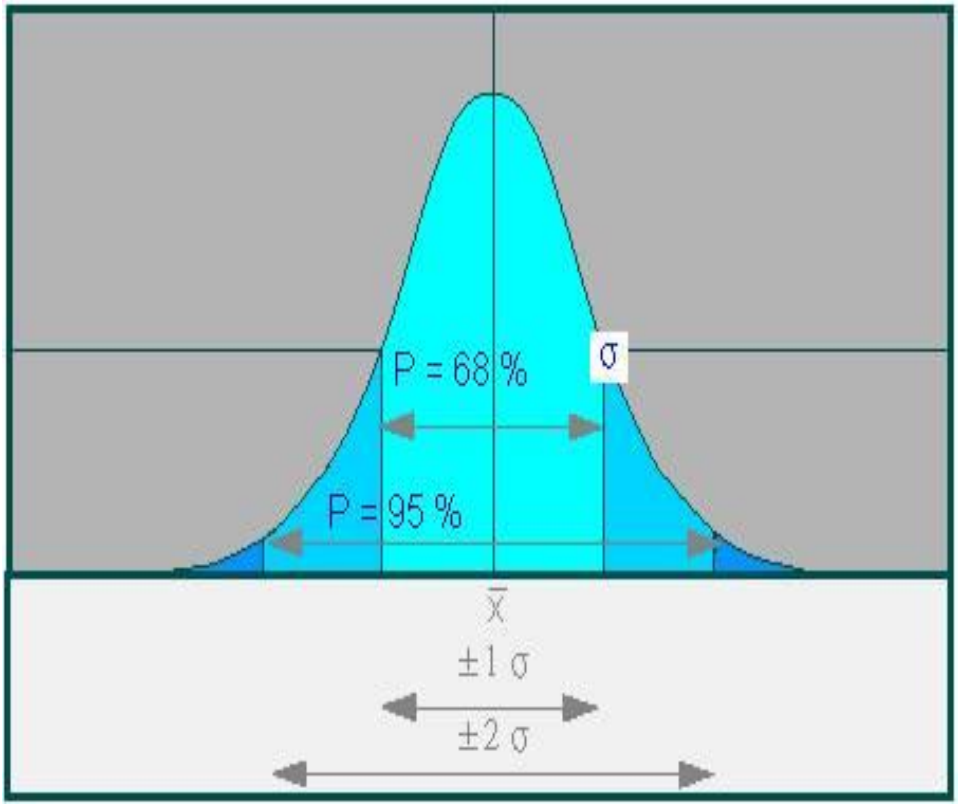
$$\overline{x} = \frac{\sum_{i=1}^{N} x_i}{N} \qquad (1)$$

This formula says: Add the N measurements x_1 , x_2 , x_3 , etc., up to x_N . This sum is written as $\sum_{i=1}^{N} x_i$. Now divide by N to get the mean value of x. Another important statistical parameter is the standard deviation, σ, which is a measure of the spread of the individual data points. A large value for σ implies a large spread in the values whereas a smaller value implies a small or narrow spread. We call σ (calculated from a set of measurements as shown below) the standard deviation for repeated individual measurements, x_i. The standard deviation $\overline{\sigma}$ of the mean $\overline{x}$ of measurements (also called the *standard error*) gives the spread in the means of repeated sets of measurements $\overline{x}_1, \overline{x}_2, \overline{x}_3, \ldots, \overline{x}_N$. Mathematically, these are different:

$$\overline{\sigma} = \sqrt{\frac{\sum_1^N \left(x_i - \overline{x}\right)^2}{[N(N-1)]}} \qquad (2a)$$

$$\sigma = \sqrt{\frac{\sum_1^N \left(x_i - \overline{x}\right)^2}{(N-1)}} \qquad (2b)$$

(Some definitions use *N* for σ, instead of *N-1* which is more accurate.) A bin plot (histogram) of the number n_i of individual measurement differences from the mean (in units of the standard deviation $\overline{\sigma}$), has a bell-shaped "normal" distribution. A **bin** contains the number of data points that are specified between the minimum and maximum value of the bin.



Notice that the bell-shaped/normal or *Gaussian* distribution seen above is made from data whose events are both random (cannot be predicted) and independent (one data point has no relationship with the one that preceded it or succeeded it).

The area under the normal curve centered around the mean between (x $+\sigma$) and x $-\sigma$) is 68.3% of the total area under the curve. This means that about two-thirds of the measurements, x_i, fall between (x $+\sigma$) and (x $-\sigma$). The area between (x + 2σ) and (x - 2σ) is 95.4% of the total area. And, $\pm 3\sigma$ covers 99.7%. The standard deviation σ is used as a measure of the random uncertainty expected for an individual measurement. We write 15.5 ± 0.2 m for the mean and spread of individual measurements. This implies that the probability is about 66% that the true value lies between 15.3 and 15.7. However, in most science and engineering applications, you will need to calculate $\overline{x} \pm \overline{\sigma}$, the mean and the standard deviation of the mean. To illustrate the procedure, we will work out the mean value $\overline{x}$ and the standard deviation σ of a set of 21 individual data points, and then the predicted uncertainty, $\overline{\sigma}$, of the set's mean.

What does all this mean? In Psychology, someone with an IQ score within 1σ of the mean is said to have average intelligence. Those scoring above $+1\sigma$ are considered to have above-average intelligence. Anything over $+2\sigma$ is termed *gifted*. For example, if the average IQ is 100, and the standard deviation is 15, then the average category consists of people scoring between 85-115 (68.3% of the population). Those scoring above 130 (+2σ) comprise 2.3% of the population – notice that this is half of 100% - 95.4% = 4.6% because we are only considering the part of the population with above-average intelligence.

Note that Reaction Time distribution appears *skewed* to the right/positive side), resulting in a long “tail”. A simplified explanation is that this happens because there are more possible large (long reaction times) values since there are more possible reaction times than there are small (short reaction times) ones. Therefore, t**he distribution is strictly not a Gaussian (Normal) one**. Functions such as the ex-Gaussian, the ex-Wald, the Weibull and the Gamma describe the skewed behavior better, but they are more complicated mathematically. For this reason, in this lab **we are treating the Reaction Time Data as a Gaussian distribution.**

CALCULATION OF MEAN AND STANDARD DEVIATION
USING INDIVIDUAL LENGTH MEASUREMENTS

x_i	$(x_i - \overline{x})$	$(x_i - \overline{x})^2$
15.68	0.15	0.0225
15.42	-0.11	0.0121
15.03	-0.50	0.2500
15.66	0.13	0.0169
15.17	-0.36	0.1296
15.89	0.36	0.1296
15.35	-0.18	0.0324
15.81	0.28	0.0784
15.62	0.09	0.0081
15.39	-0.14	0.0196
15.21	-0.32	0.1024
15.78	0.25	0.0625
15.46	-0.07	0.0049
15.12	-0.41	0.1681
15.93	0.40	0.1600
15.23	-0.30	0.0900
15.62	0.09	0.0081
15.88	0.35	0.1225
15.95	0.42	0.1764
15.37	-0.16	0.0256
15.51	-0.02	0.0004
326.08 m	**(-0.05 m)**	**1.6201 m^2**

From the above table we can make the following calculations for N = 21 measurements

$$\overline{x} = \frac{\sum_{i=1}^{N} x_i}{N} = \frac{326.08}{21} = 15.53m$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \overline{x})^2}{(N-1)}} = \sqrt{\frac{1.6201}{20}} = 0.29m$$

$$\overline{\sigma} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \overline{x})^2}{[N(N-1)]}} = \sqrt{\frac{1.6201}{[(21)(20)]}} = 0.062m$$

Hence $\overline{x} \pm \overline{\sigma}$ = 15.53m ± 0.062 m. This says the average is 15.53m, which average has an uncertainty of 0.062m. But the uncertainty or spread in *individual* measurements is σ = 0.29m. Remember, when you calculate a non-zero value for σ or $\overline{\sigma}$ random, uncertainty is present; the values of σ tell you how large the magnitude is.

Increasing the number of individual measurements reduces the statistical uncertainty (random uncertainties); this improves the "precision". On the other hand, more measurements do not diminish systematic uncertainty in the mean because these are always in the same direction; the "accuracy" of the experiment is limited by systematic uncertainties.

Often, we must compare different measurements. Consider two measurements

$$A \pm \overline{\sigma}_A \qquad\qquad B \pm \overline{\sigma}_B$$

If you want to compare, say, their difference, the expected uncertainty $\overline{\sigma}$ is given by

$$\overline{\sigma}^2 = \overline{\sigma}_A^2 + \overline{\sigma}_B^2$$

The reason we sum the squares is that we must sum the squares of the variances $(x_i - \overline{x})^2$, in order to get the standard deviation. (Look again at the above equations for the standard deviation.) We should expect that $|\overline{A} - \overline{B}| < \overline{\sigma}$, if the two data sets are statistically the same. If they diverge greatly from the expected standard deviation,

$|\overline{A} - \overline{B}| > \overline{\sigma}$, they are then statistically different. Often, a value of $2\overline{\sigma}$ is used as a simplified reference for being significantly different.

Propagation of Uncertainty

There is variation in most measurements, e.g., reaction time, body weight, height, but what if you making measurements that figure into a *calculated* quantity – such as Volume – what would be the uncertainty in the V? The measured dimensions of, say, a box would be properly reported as the mean of each dimension ± the uncertainty in that dimension:

$$\overline{L} \pm \overline{\sigma}_L \qquad \overline{H} \pm \overline{\sigma}_H \qquad \overline{W} \pm \overline{\sigma}_W$$

To find the uncertainty in the the Volume V, Partial Differential Calculus is required – but this course is an Algebra-based one. Luckily, for calculated quantities that are the product of two or more quantities, one can *estimate* the uncertainty without calculus by adding the fractional uncertainties (the uncertainty divided by the mean) of each dimension; for example, the fractional length of the box would be $\bar{\sigma}_L/\bar{L}$. The fractional height and fractional width can be calculated similarly. To calculate the uncertainty in the volume, you only need to add the fractional uncertainties of the 3 dimensions involved in the calculation. The mean volume, when multiplied by the **sum** of the fractional uncertainties, gives the uncertainty in the volume:

$$\bar{\sigma}_V = \bar{V}\left(\frac{\bar{\sigma}_L}{\bar{L}} + \frac{\bar{\sigma}_H}{\bar{H}} + \frac{\bar{\sigma}_W}{\bar{W}}\right)$$

The volume would then be reported as $\bar{V} \pm \bar{\sigma}_V$.

Note that this estimation does not work well for functions such as trigonometric ones (sin, cos, tan). However, for non-linear functions that have exponents like x^2 the uncertainty is obtained by multiplying the uncertainty in *x* by the exponent *2*.

Measurement and Limitations

Stopwatch (*Hsu-Chang Lu & Moshe Levy*) is online software that measures your reaction time, specifically, the time interval between a visual cue from your computer's monitor and your hitting its keyboard's space bar.

A **motion sensor** is a device that continuously emits sound pulses which bounce off an object and detects them when they return to the sensor. By timing the time interval between the departure and arrival of the pulse (and knowing the speed of sound), the distance to the object can be determined continuously. The motion sensor you will use can measure objects between the range of 0.15 to 8 meters and has a resolution (discussed later) of 0.001m (one millimeter).

A **force sensor** is a device that continuously monitors the force exerted on it. There is a metal strip inside the sensor that deforms slightly in response to the force (stress) on it. Since the metal's electrical resistance varies as its dimensions change (strain), the electrical signal can be converted to a value for force – this is basically a *strain gauge*. The force sensor you will use has a range of ±10N and a resolution of 0.01N in low range and a range of ±50N and a resolution of 0.05N in high range. You may remember from high school that a 1kg mass weighs 9.8N (2.2 lbs) at the surface of the Earth.

Procedure (Reaction Time) (50 pts)

*What's this all about: here you will measure the time it takes for you to respond to a visual (or aural) cue. You will do this repeatedly, find your mean, minimum and maximum reaction times, plot the distribution of reaction times, and note the spread (standard deviation) of the data, as well as the standard deviation of the mean, which is a measure of how **reliable** the mean value is.*

Your lab report, on which you and your lab partners will record all your data and calculations, and answer all the questions in this write-up, will be written on Google Docs. Make sure the lab partner who creates it for the group shares it with all partners, the TA and the LA. Note that lab partners and collaborate on the document simultaneously

For reaction time data, each partner should determine and compare her/his reaction time distributions and their means for the Space bar key on the keyboard.

1. Go to https://rutgers-stopwatch.glitch.me/. When you are ready and feeling alert, press the Reset button, then click anywhere else on the webpage. Then position your fastest finger over any key of your computer. Once you see the picture Albert Einstein appear, press the key immediately; your reaction time (in milliseconds) will be recorded as the first data point (Trial 1). You should be be ready for the next time Einstein appears, so that you can again press the key and record the second data point. You will do this for 5 straight minutes, to collect dozens of data points; you shouldn't be doing anything else on your computer (like switching to another tab) or you may lose all the data you've taken up to that point. If this inadvertently happens, Refresh the webpage to start all over again.

2. When you are finished with your last data point, there is no need to press any button – and certainly **do not press Reset or you will lose all your data**. You can either select your data simply by clicking and dragging (both Trial and Reaction Time columns) with the mouse/trackpad, until you have gotten all the points. Once they have been highlighted, copy them to your computer's clipboard (Edit→Copy, or use Right-Click / Control-Click)

Another option is to press the Export button; doing so will generate and download a .CSV file, which is readable in Microsoft Excel, Google Sheets, or other spreadsheet software.

3. **Create a Google Sheets or Microsoft Excel document**. Since Google accounts are ubiquitous, it might be preferable to choose the first options since sharing data with lab partners would likely be easier. You can label the first two (or three) columns with the lab partners' names ("Lisa", "Pankaj", etc.). Paste the data taken by the first lab partner, taken either from the Stopwatch webpage or the CSV file downloaded, into the first column of the spreadsheet. (This part is worth 5pts)

4. Plot the data. Remove any outliers (points which are obviously bad, such as those resulting from not being ready to click the button), remove them from the data column by highlighting the cell and pressing the Delete key. Using included functions in Google Sheets, or by writing your own formula, determine the Minimum, Maximum, Mean and Standard Deviation. *Note that this Standard Deviation is σ and not $\overline{\sigma}$, but that you can*

get the latter from the former with minimal calculation by carefully examining the mathematical definitions for both. In Google Sheets the Minimum, Maximum, Mean, and Standard Deviation can be found by typing in =MIN(DATA), =MAX(DATA), =AVERAGE(DATA), and =STDEV(DATA), respectively, where DATA represents your data set. The input for DATA should look something like startCell:endCell, e.g. B2:B35 if your data points are from B2 and B35. There are several other standard deviation commands that work, like STDEV.S and STDEVA which will give the same value as STDEV, but be careful not to use STDEVP, STDEV.P or STDEVPA which use N in the denominator instead of N-1. Record these values in your lab report. (10 pts)

5. Create a histogram, which is a plot of 'Frequency of Value' vs. 'Value'. Ideally you should see a bell curve shape – if your histogram looks too "blocky", decrease the Bin Size (the resolution of the histogram). You can create a histogram in Google Sheets by first highlighting your data, and then selecting "chart" in the insert tab at the top of the sheet, right below the title. This should produce a chart, but not the type you are looking for. From there a tab titled "chart editor" should appear automatically, but if it does not double click on the new chart. In the tab click on "setup" and then change the chart type to "histogram". To change the bin sizes double click on your histogram, then click on "Customize". There should be a tab called "Histogram" which has a box for "bucket size". Manually input whatever values you would like the bin sizes to be. (5 pts)

6. Does your histogram resemble a Gaussian distribution? Why should it? (5 pts) (Hint: see the graphic (in blue) of the Normal/Gaussian distribution and read the sentence directly below it.)

7. The other lab partner(s) should repeat Steps 1-5, adding their data to the Google Sheets document. (15 pts)

Partner 1 Times

$\bar{x}$	x_{MIN}	x_{MAX}	σ

Partner 2 Times

$\bar{x}$	x_{MIN}	x_{MAX}	σ

Calculate the difference in the reaction times of you and one lab partner - that is, ***the difference in their means***. (5 pts) Then, estimate the uncertainty in the difference. (5 pts)

(HINT: You can determine $\bar{\sigma}$ from σ if you can find a simple relationship between their mathematical definitions - **see equations 2a and 2b**).

Procedure (Propagation of Uncertainty) (20 pts)

Calculate the Mean Volume and the Uncertainty in the Volume of a box with the following dimensions: (20pts)

	Length (cm)	Height (cm)	Width (cm)
Trial 1	58.6	42.3	19.2
Trial 2	59.1	42.1	19.2
Trial 3	59.4	42.5	19.1
Trial 4	58.8	42.4	18.9
Trial 5	59.0	42.0	19.0

Procedure (Motion and Force Sensor) (30 pts)

Here you will familiarize yourselves with tools that you will use a lot for the remainder of the semester in lab. Many measurements will be taken with the motion and force sensor; the former can measure position, displacement, velocity and acceleration and the latter various types of forces such as frictional, gravitational and normal. This section will show you how to use them, with the aid of software, and will show you their limitations.

Motion Sensor

1. Open "Motion PRACTICE.CMBL", which is a Logger Pro file in the same folder as this write-up (Course Folders-->205-->Intro. To Experimentation). **If you get a message warning you that there is data stored in the LabPro interface, choose the option to Ignore/Erase the data.** You should see a window (Distance) on the left and a data Table Window on the right. Once you start collecting data, the table on the right will fill with Distance data. The plot on the left will also be drawn. Note that the time scale for the graph ranges from 0 to 10 seconds – the software will take data for 10 seconds at a time.

2. Position the motion sensor near the side of the table – as close as possible to the edge, but far enough such that it still lands on the table in the event it tips over. Rotate the head so that the grilled circle points horizontally. There is a Narrow/Wide switch on the other side of the sensor – set it to Wide, if it isn't already in that position.

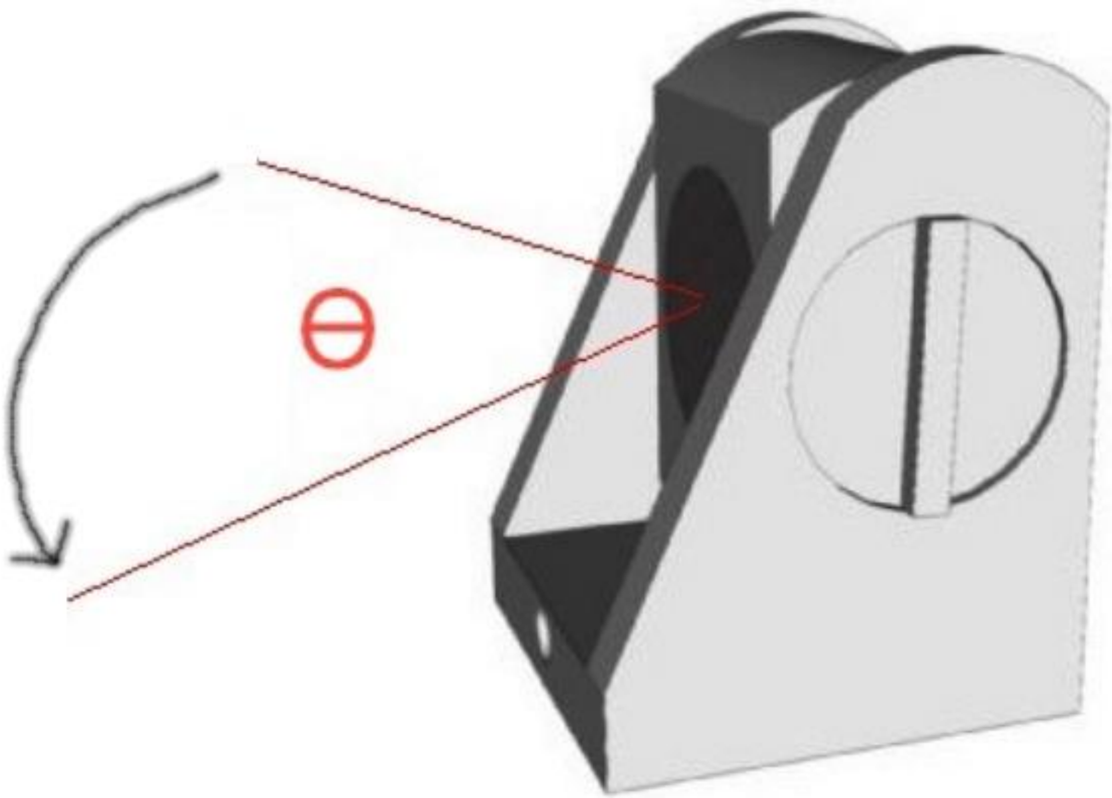
3. *Play around with the apparatus and software.* Still in Motion PRACTICE.CMBL, Press the Collect button in Logger Pro (upper right-hand side of window) to collect data. You will hear rapid clicking, which is the sound of the pulses coming out of the motion sensor. Have your partner move back and forth in front of the motion sensor in a random fashion.

4. After the 10 second collection period, examine your graphs. In the menu bar, go to Analyze--> Examine. Now activate any of the three graphs by clicking on it, then moving the cursor around to trace the plot – you should see the coordinates displayed on the top left hand corner of the graph, and they will change as you move around the plot. **If you are not getting any data, make sure your motion sensor is plugged into the DIG/SONIC1 input on the LabPro (translucent aqua blue covered flat box).**

5. Mark off two lines on the floor corresponding to two positions along the beam line

that you will position yourself on. Using both the motion sensor and a meter stick (separately), measure the distance between these lines. (5 pts) By how much (in percentage) do they differ? Show all work. (5 pts)

7. Calculate the **horizontal** field of view (the angle of the beam θ) of the sensor when the Narrow/Wide selector is set to Wide by slowly moving an object in front of it (as long as it's not closer than 15cm), perpendicular to the beam until it appears on the Logger Pro graph, and then disappears from it – anything inside this "cone" will be detected by the sensor and anything outside will not. You should use two meter sticks. (5 pts) After you do this, again calculate the field of view (beam angle) when the Narrow/Wide selector is set to Narrow. (5 pts)



Note that in this graphic above, the angle sweep is ***horizontal*** and not vertical as it may suggest.

Force Sensor

1. Close down Motion PRACTICE.CMBL. Open "Force PRACTICE.CMBL", which is a Logger Pro file in the same folder as this write-up (Course Folders-->205-->Measuring Motion).

2. Press the Collect button, as you did with the motion sensor, gently pull or press on the

hook at the end of the sensor and watch the graph on the screen. Which way is the force positive and which way is it negative? (5 pts) With nothing touching the hook, hold the sensor so that the hook is oriented dowwards and collect data – is the reading higher or lower than when the hook is upward? (5 pts) Explain why this is the case (5 pts). What about when the sensor is laying on the table – how does this reading compare to hook-up and hook down? Again, explain. (5pts)