Lab 4: Least Square Fitting

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1. Introduction

In this lab, we analyze distributions of radiation counts from an eluted Ba-137 solution, which has a relatively short half life of 2-3 minutes. This solution is created through the beta decay of Cs-137 as it is flushed with a weak solution.

The beta decay releases gamma photons which are represented as counts as they are released in the Geiger counter. By controlling for background radiation and calculating a least square fit linear model, we're able to determine the initial decay rate, mean lifetime, and half-life with high precision.

2. Methods

As depicted in Figure 1 and Figure 2, we use a Geiger counter attached to the Logger Pro Interface via BNC cable, which itself is attached to a computer with Logger Pro. The Geiger counter sits on top of a metal box with slits along

the side, which allows us to slide in a tray-like aluminum plate with a cavity containing five drops of a meta-stable Ba-137 solution eluted from a Cs-137 source at various controlled distances from the Geiger counter. In our experiment, we configured Logger Pro to measure for 10 minutes at 1 sample every 10 seconds, for a total of 600 data points. After collecting data for the Ba-137 solution, we collected data for an equivalent sample time/rate/size of pure background radiation in the room.

Figure 1: Picture of experimental setup.

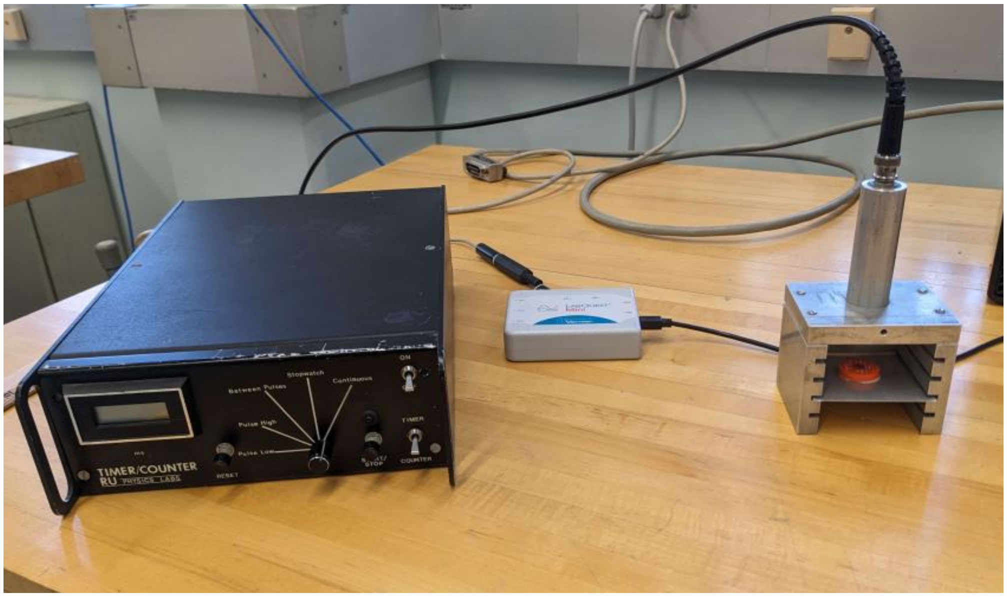
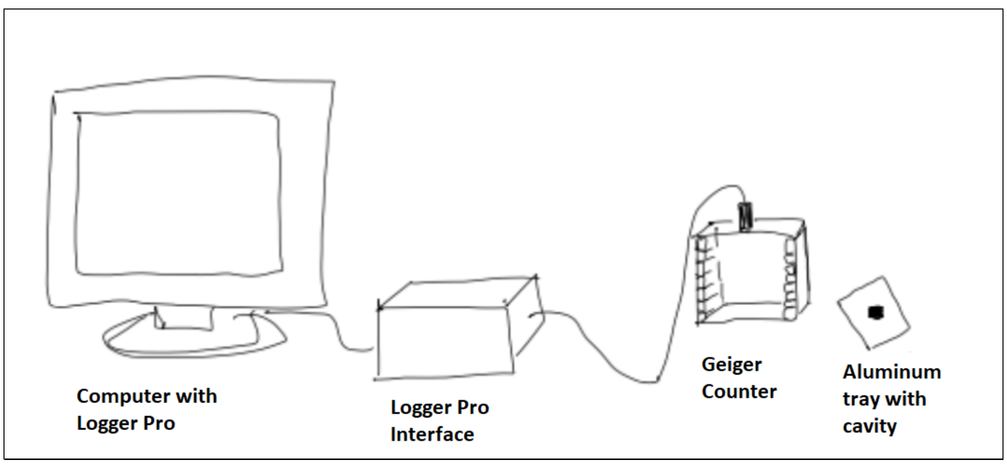


Figure 2: Drawing of experimental setup, labelling key components.



3. Results and Discussion

All data analysis, calculations, and plotting can be recreated by running the following Python notebook in browser: Google Colab (also attached as .ipynb file).

3.1. Background Radiation

The background radiation of the room, with no radioactive samples nearby, is

$$D_b = 4.45 \pm 0.27 \text{ counts/interval.}$$

This calculation can be verified from the supplemental Python notebook, in section "[1] Background Radiation".

3.2. True Decay Rate

We subtract the mean background radiation, D_b, from the dataset of measured counts/interval including background,
D^*, to produce a dataset representing the true decay rate of our sample:

$$D = D^* - D_b.$$

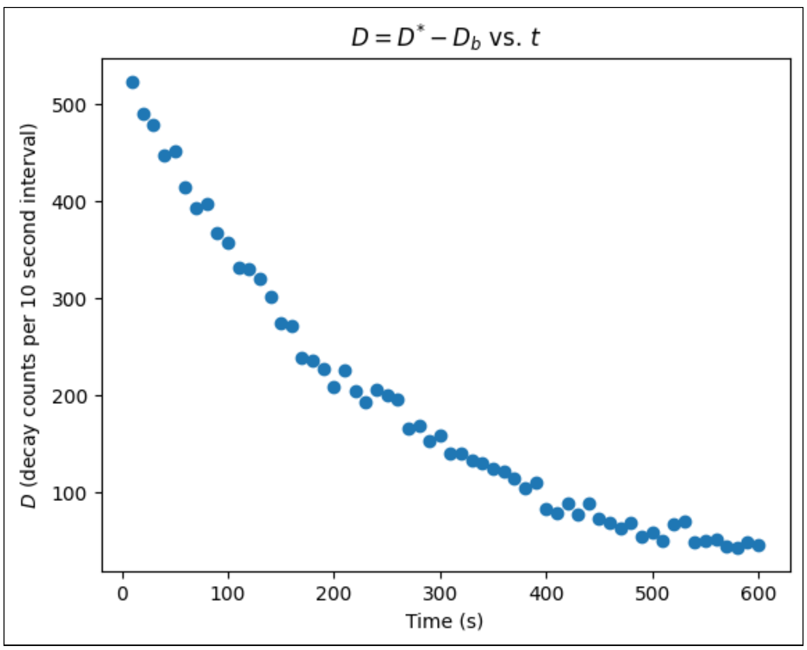
The uncertainty associated with each value of D is given by

$$\begin{aligned} \delta D_i &= \sqrt{\left(\frac{\partial D}{\partial D^*} \cdot \delta D_i^*\right)^2 + \left(\frac{\partial D}{\partial D_b} \cdot \delta D_b\right)^2} \\ &= \sqrt{(1 \cdot 0.5)^2 + (1 \cdot 0.27)^2} \\ &= 0.6, \end{aligned} \tag{1}$$

where $\delta D_i^* = 0.5$ is the instrumental uncertainty of the Geiger counter, which is half of the smallest interval.

We can now plot D versus time, including uncertainty, in Figure 3 below.

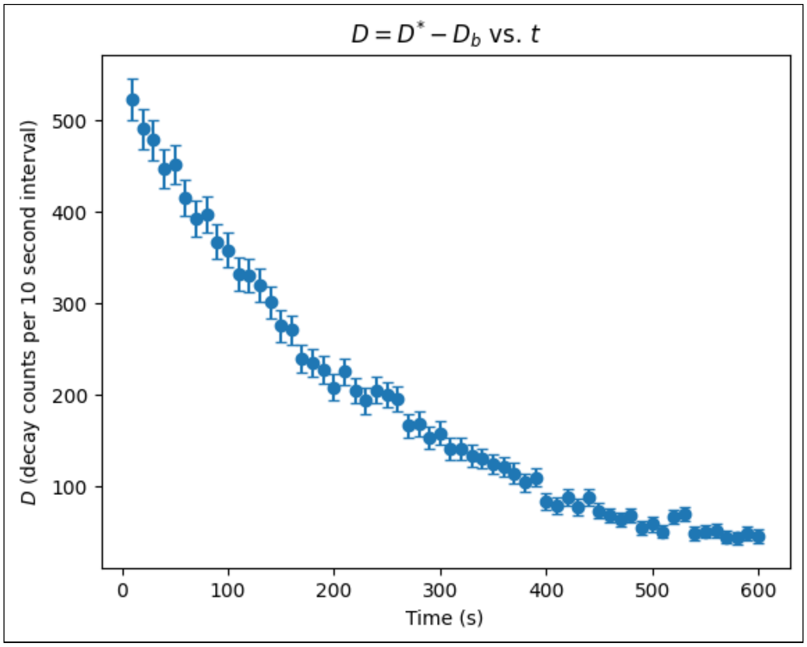
Figure 3: True decay rate (per 10 second interval) versus time for five drops of eluted Ba-137 sample. Uncertainty from propogation of error result from equation (1).



As expected, we observe exponential decay. Uncertainty bars are too small to see on this scale.

If $\delta D_b = 0$, then we take the uncertainty to be $\delta D_i = \sqrt{D_i}$. We can recreate Figure 3 with this new measure of uncertainty in Figure 4 below.

Figure 4: True decay rate (per 10 second interval) versus time for five drops of eluted Ba-137 sample. Uncertainty from assuming $\delta D_i = \sqrt{D_i}$.



We still observe the expected exponential decay, but error bars are much more visible now.

These calculations + plots can be verified from the supplemental Python notebook, in section "[2] True Decay Rate".

3.3. Least Squares Fitting

In order to perform a least square fit, we need a linear relationship. This can be achieved by fitting to $y = \ln(D)$.

The uncertainty associated with each value of $y = \ln(D)$ is given by

$$\begin{aligned}\sigma_i \equiv \delta y_i &= \delta\Big[\ln(D_i)\Big] \\ &= \delta\Big[\ln\Big(D_i^* - D_b\Big)\Big] \\ &= \sqrt{\left(\frac{\partial \ln D}{\partial D^*} \cdot \delta D^*\right)^2 + \left(\frac{\partial \ln D}{\partial D_b} \cdot \partial D_b\right)^2} \\ &= \sqrt{\frac{D_i{}^* + D_b}{\left(D_i^* - D_b\right)^2}}. \end{aligned} \tag{2}$$

We can now use equations 2-3 from the lab manual (which themselves are derived by differentiating the χ^2 equation, restated here in eq (4)) to determine the fitting parameters, which are summarized in Table 1 below.

Table 1: Calculated values to determine fitting constants.

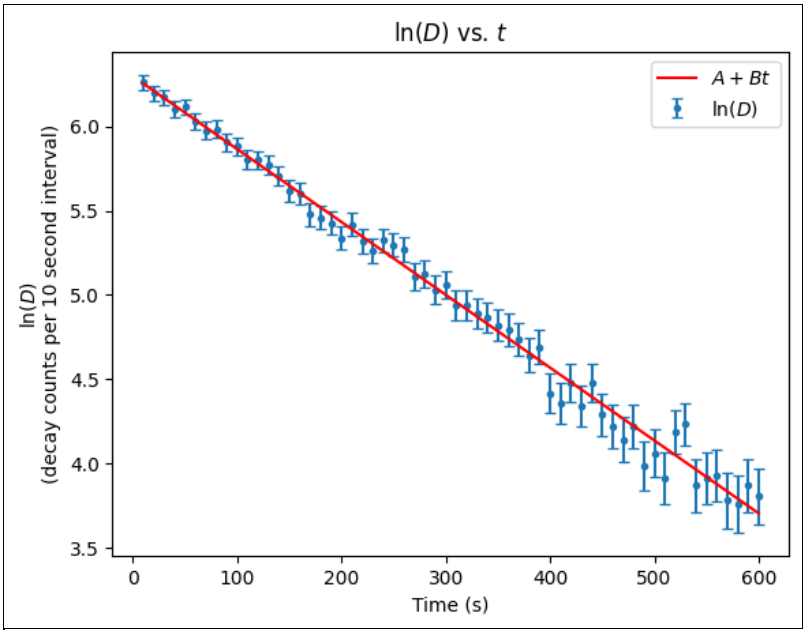
Parameter	**Value**
M_{11}	1.08317×10^{4}
M_{12}	1.97698×10^{6}
M_{21}	1.97698×10^{6}
M_{22}	5.89428×10^{8}
V_1	5.96619×10^{4}
V_2	9.90312×10^{6}
Δ	2.47605×10^{12}
A	6.29557
B	-4.31451×10^{-3}
σ_A	1.54289×10^{-2}
σ_B	6.61407×10^{-5}

Thus, our least square fit is given by

$$Y_i = A + Bt_i \begin{cases} A = (6.29557) & \pm\left(1.54289 \times 10^{-2}\right) \\ B = \left(-4.31451 \times 10^{-3}\right) & \pm\left(6.61407 \times 10^{-5}\right) \end{cases}. \tag{3}$$

To evaluate the effectiveness of this fitted model, we can plot $\ln(D)$ vs. time with uncertainty (calculated here in eq (2)), and superimpose the calculated least square fit, in Figure 5 below.

Figure 5: Log plot of true decay rate versus time, with superimposed least square fit.



Visually we observe great agreement between experimental values and our least square fit linear model, but this agreement can be more rigorously quantified by using equation 1 from the lab manual:

$$\begin{aligned}\chi^2 &= \sum \left(\frac{(y_i - Y_i)^2}{\sigma_i^2} \right) \\ &= 29.84. \end{aligned} \tag{4}$$

These calculations + plots can be verified from the supplemental Python notebook, in section "[3] Least Squares Fitting".

3.4. True Half Life

The characterstic exponential decay of a sample is given by

$$D = D_0 \exp\left(-\frac{t}{\tau}\right),$$

which can be rearranged as

$$\begin{aligned}\ln D &= \ln D_0 - \left(\frac{1}{\tau}\right) t \\ &= A \qquad + B \quad t.\end{aligned}$$

We've previously determined values for A and B in eq (3). Thus we can calculate our final values for

$$\boxed{\begin{aligned} D_0 &= 542.16 \pm 1.10 \ \big(\text{counts per 10 seconds}\big), \\ \tau &= 232 \pm 10 \text{ seconds}, \\ t_{1/2} &= 162 \pm 7 \text{ seconds}. \end{aligned}}$$

These calculations can be verified from the supplemental Python notebook, in section "[4] True Half Life".

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4. Conclusion

The given value for half life is 2.55 minutes or 153 seconds, which is within 5.00% error of our calculated value, and just barely falls within our range of error. Thus we have achieved our goal of using discrete and randomized distribution of data, in the form of random decays of a radioactive sample, in order to extract underlying physical values using a least square fitting linear model.