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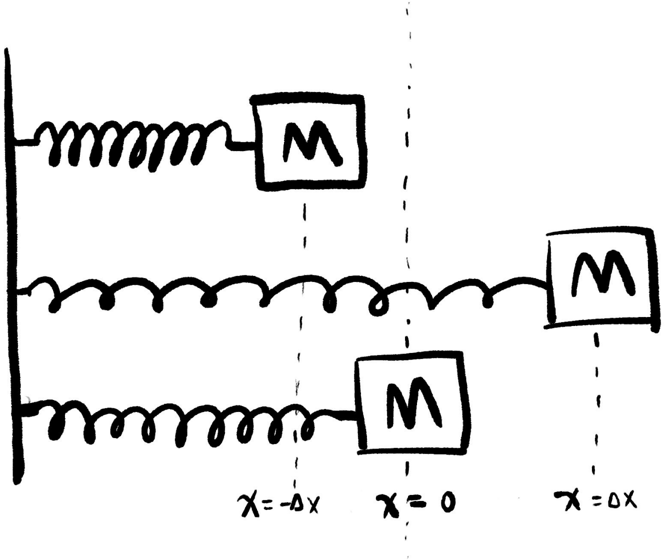
Simulation of the physical execution of arbitrary 2 qubit entanglement gates

The Simple Harmonic Oscillator



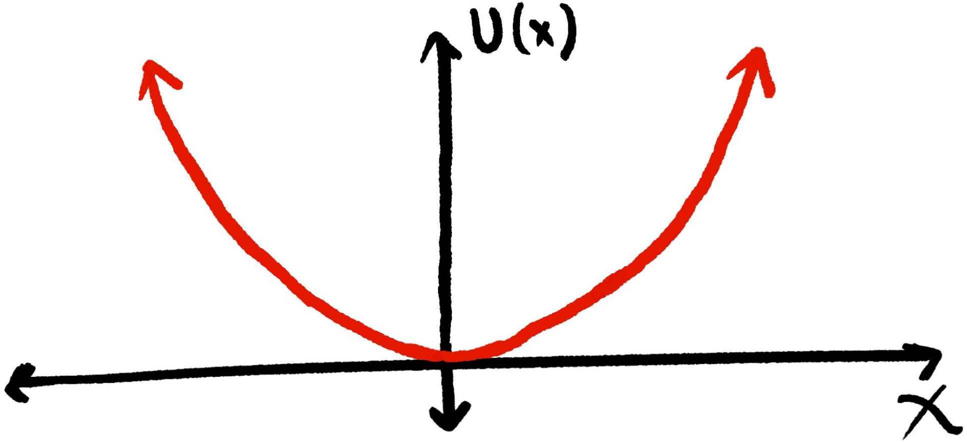
$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$U(x) = \frac{1}{2}kx^2 \quad KE = \frac{1}{2}mv^2$$

$$x(t) = A\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}}$$

The Simple Harmonic Oscillator



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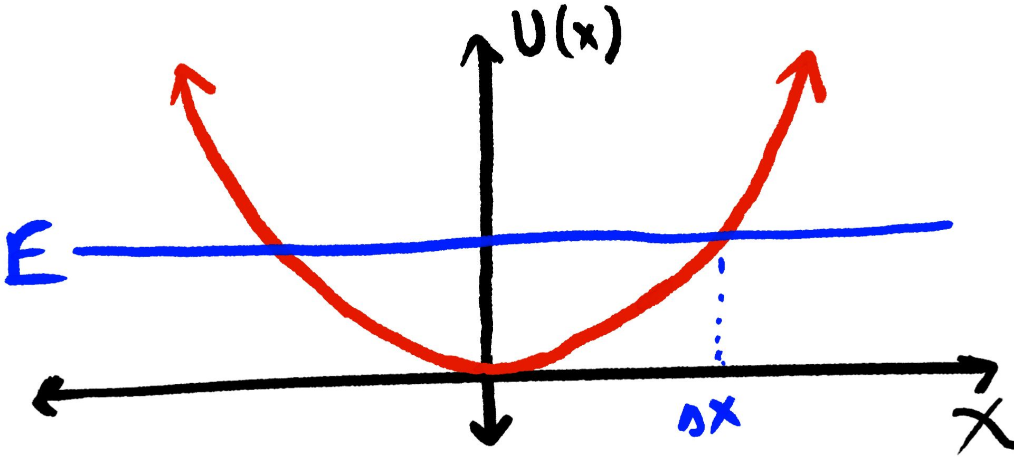

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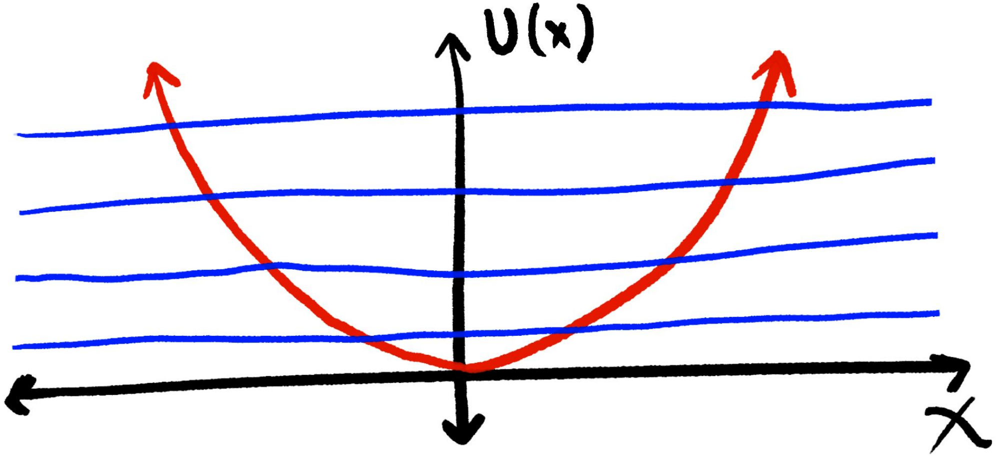
The QUANTUM Simple Harmonic Oscillator



Discrete possible total energies that are allowed under quantum mechanics.

$$x(t) = A\cos(\omega t + \phi) \qquad \omega = \sqrt{\frac{k}{m}}$$

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

The QUANTUM Simple Harmonic Oscillator

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

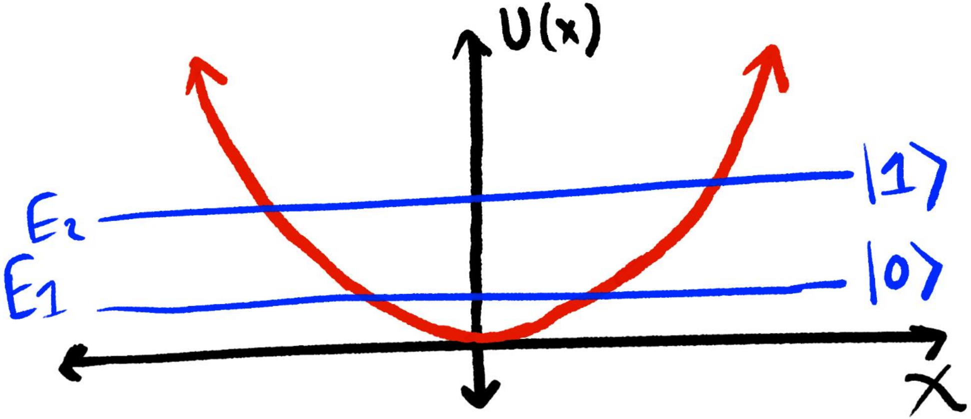
$$E = hf \qquad \hbar \equiv \frac{h}{2\pi}$$

$$f_n = \frac{E_n}{h} = \frac{\hbar\omega\left(n + \frac{1}{2}\right)}{h} = \frac{\omega}{2\pi}\left(n + \frac{1}{2}\right)$$

Via the Plank relation, these energies have a corresponding frequency.

In order to get a quantum state from one energy state to another, it requires applying energy at the frequency difference between the two states.

The QUANTUM Simple Harmonic Oscillator



$$E_1 - E_0 = \hbar\omega$$

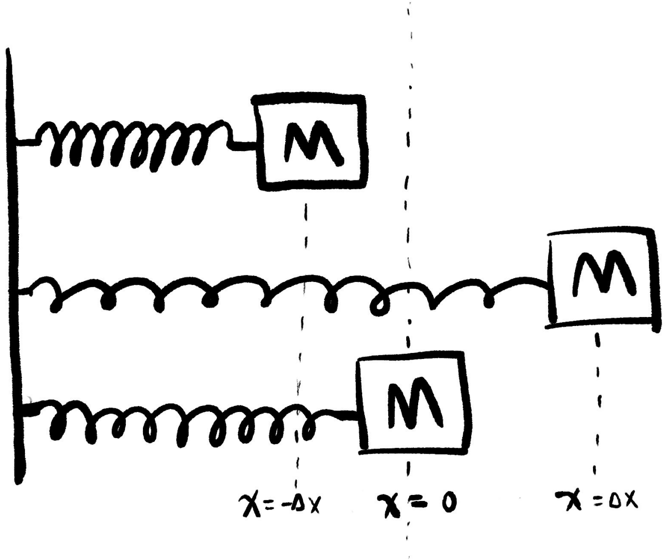
$$f_{\text{drive}} = \frac{E_1 - E_0}{h} = \frac{\omega}{2\pi}$$

The **QUANTUM** Simple Harmonic Oscillator

All this to say:

1. There is a set energy difference between energy levels of this quantum oscillator.
2. That energy difference is associated with a transition frequency via the plank equation.
3. In order to transition from one energy level to another, we must apply energy at **this transition frequency**

The **QUANTUM** Simple Harmonic Oscillator



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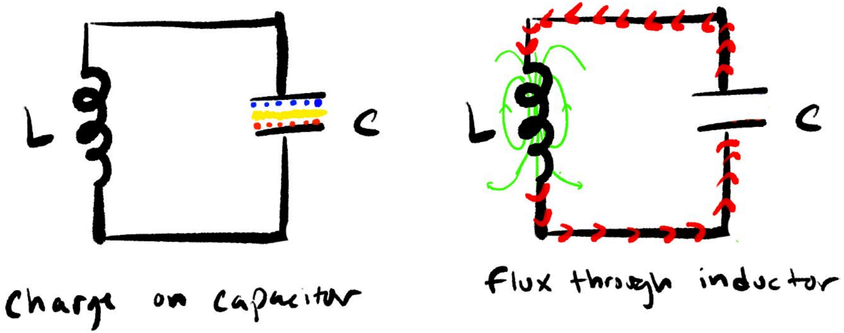


The **QUANTUM** Simple Harmonic Oscillator

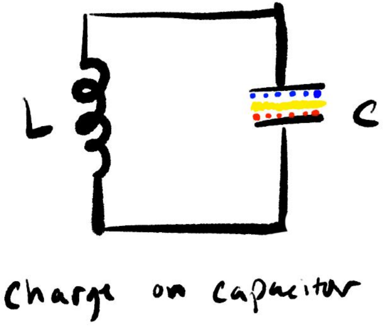
$$\omega = \frac{1}{\sqrt{LC}}$$

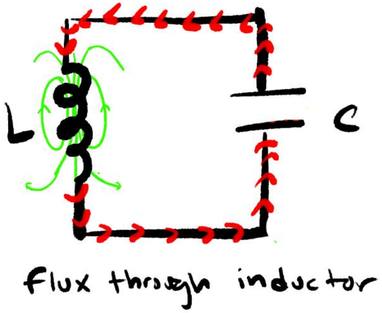
$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

$$\Delta E = E_1 - E_0 = \hbar\omega$$

$$f_{\text{drive}} = \frac{\Delta E}{h} = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$



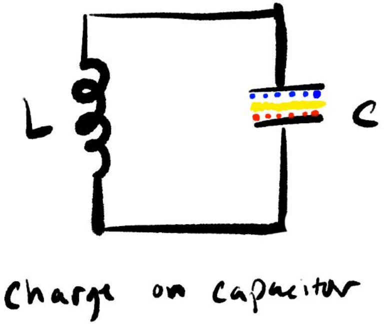


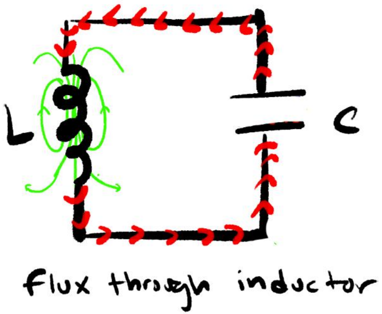
Building a Transmon (Superconducting Qubit)

For SHOs, energy levels represented different total potential energies in the system, which affected the amplitude and phase of the position function.

Here, it is the similar, with amplitude and phase of the oscillations according to discrete possible energies the system can take.

These “energy levels” correspond to information states of the qubit. (|0>, |1>, etc.)



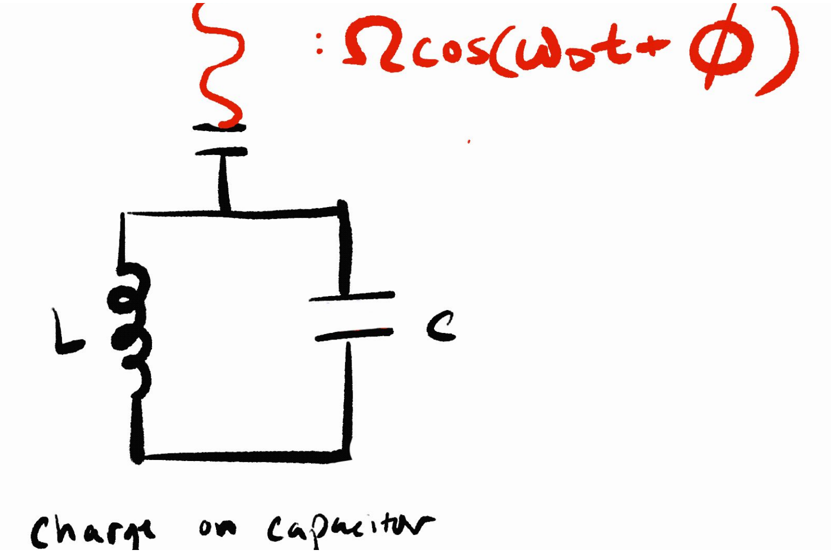


Building a Transmon (Superconducting Qubit)

So we have two energy states that we can transition to by applying energy!

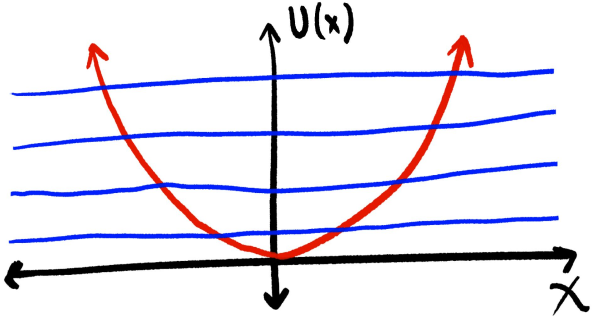
Applying radiation at the drive frequency 2pi * f will drive our energy transitions.

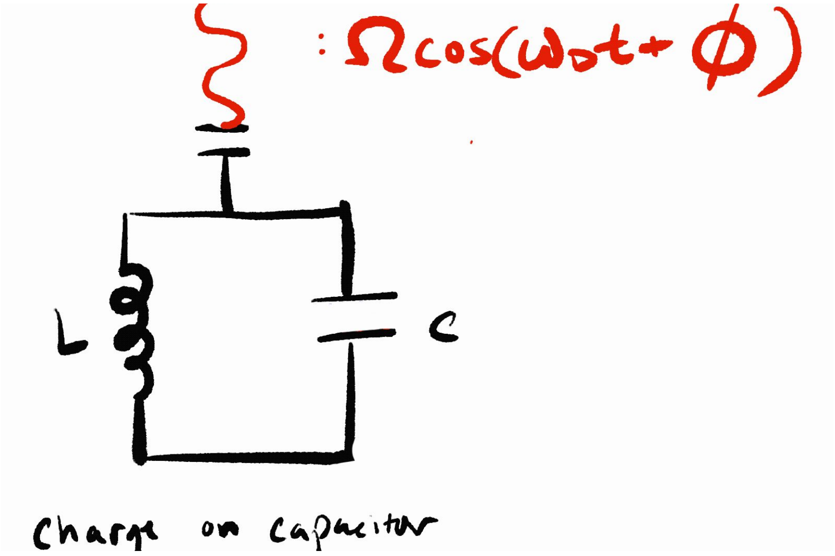
But, there is a problem.



Building a Transmon (Superconducting Qubit)

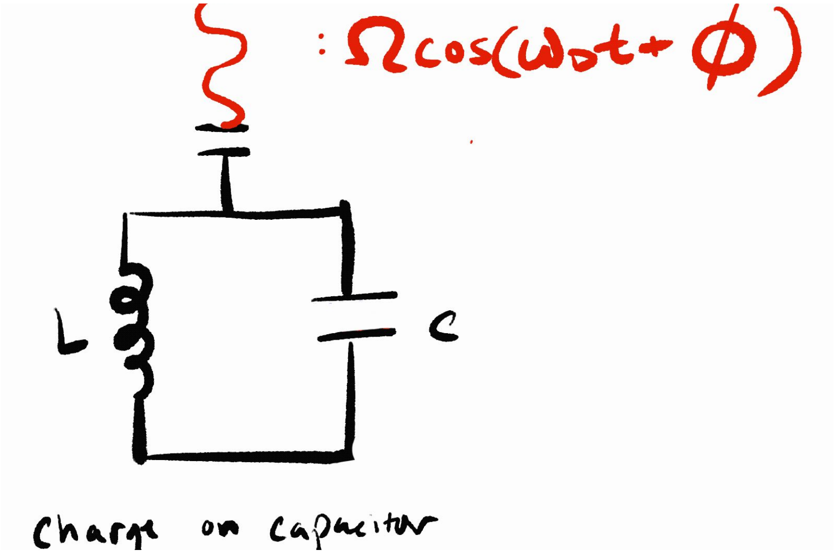
The energy spacing between any two rungs of the ladder (aside from the 0th-1st), is the same. So not only do we drive our system from |0> to |1>, but also from |1> to |2>.





Building a Transmon (Superconducting Qubit)

Qubits need to have **nonlinear energy spacing** so that our drive frequency can be unique to the |0> to |1> transition. LC Circuits won’t work for this. Instead, we replace our inductor with a similar element called the **Josephson Junction.**

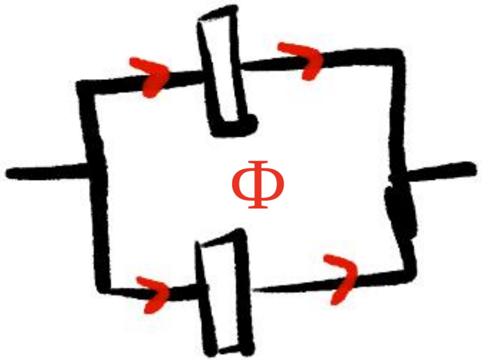


Building a Transmon (Superconducting Qubit)

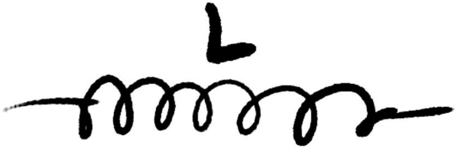
Josephson Junction: Nonlinear Inductor via quantum tunnelling

$$I = I_c \sin\left(2\pi\Phi/\Phi_0\right) \text{ (nonlinear)}$$

$$I = \Phi/L \text{ (linear)}$$



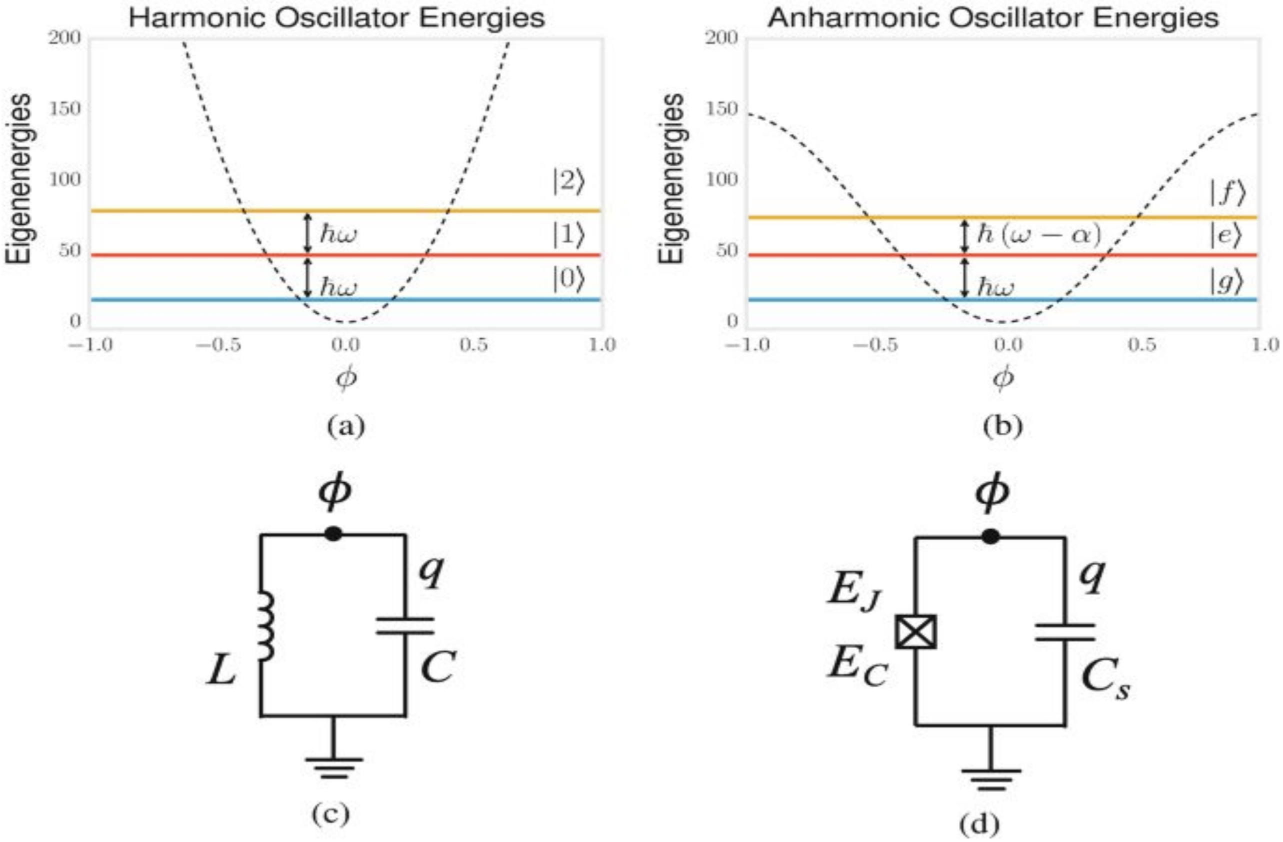
SQUID module
2 Josephson junctions



$$\omega \approx \frac{\sqrt{8E_J E_C}}{\hbar} - \frac{E_C}{\hbar}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

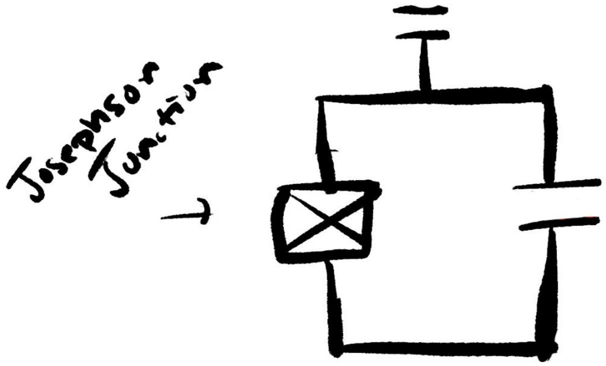
Building a Transmon (Superconducting Qubit)



Building a Transmon (Superconducting Qubit)

The “anharmonicity” just means that the energy gaps are no longer linear. This means we can choose a unique |0> and |1> state to do quantum computation with.

This is a transmon!
(The physical circuit is the system and the qubit is encoded in the quantum state of the collective electromagnetic mode)

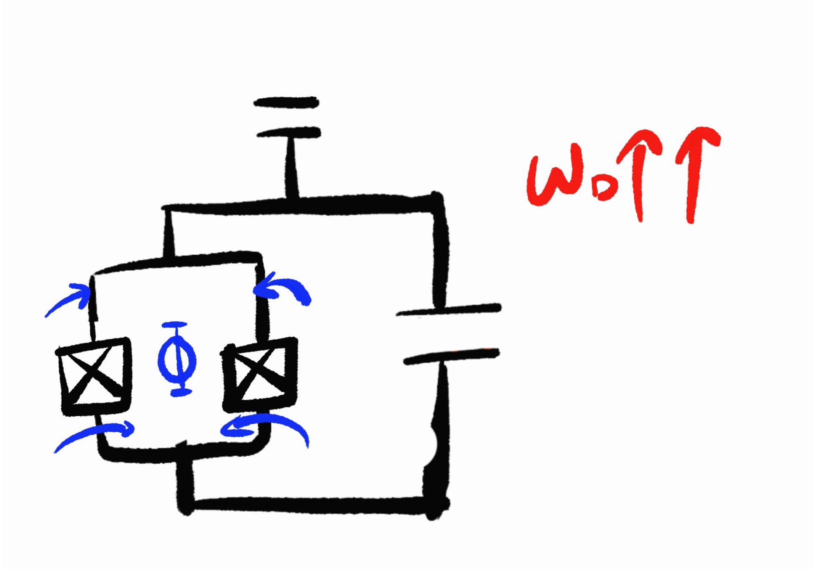


Building a Transmon (Superconducting Qubit)

There is one last step. It will be useful to us later to be able to change this energy transition to whatever we want.

Instead of one josephson, we use two (that act as one) and apply a magnetic flux* through it’s area to change its inductive effects and affect the drive frequency.

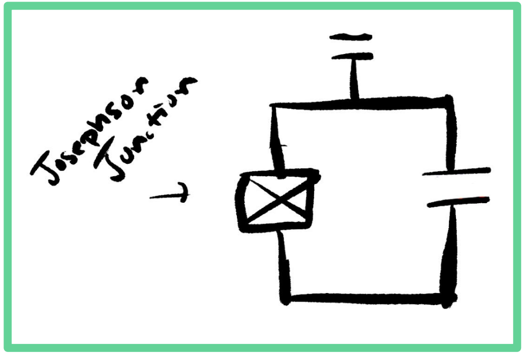
This is a tunable transmon!



*measure of how much magnetic field passes through that surface

Building a Transmon (Superconducting Qubit)

One final step to minimize noise, thermal energy, and make Josephson Junctions work.



This makes our qubit (the circuit) superconductive!

So where is the bloch sphere?

We have:

- Built a tunable transmon qubit (our circuit!) where we can define a |0> and |1> state.
- A method of transitioning between these states by applying microwave radiation at the drive frequency

But where do we get quantum superpositions?

So where is the bloch sphere?

Saying: **“We can transition between these states by applying microwave radiation at the drive frequency”** is an oversimplification.

Applying radiation with a certain **amplitude, frequency, and phase** yields a complex energy “hamiltonian” that represents how the state will evolve. With some approximation steps ignored, we yield:

$$H_{\text{rot}} = \frac{\hbar\Omega}{2}\left(\cos\phi\,\sigma_x + \sin\phi\,\sigma_y\right)$$

So where is the bloch sphere?

$$H_{\text{rot}} = \frac{\hbar\Omega}{2}\left(\cos\phi\,\sigma_x + \sin\phi\,\sigma_y\right)$$

$\boldsymbol{\Omega}$: Rabi Frequency—a parameter we control via the amplitude of applied radiation that sets the speed of rotation of a quantum state around a chosen axis in the bloch sphere.

ϕ: Phase, parameter we control via the phase of applied radiation. When its sine and cosine are multiplied to the pauli X and pauli Y rotation matrices, creates a rotation around a desired axis in the XY plane.

So where is the bloch sphere?

$$H_{\text{rot}} = \frac{\hbar\Omega}{2}\left(\cos\phi\,\sigma_x + \sin\phi\,\sigma_y\right)$$

In class, we learned that we can apply any desired 2x2 Unitary transformation around the bloch sphere. This is directly related to this hamiltonian as well as time. The hamiltonian sets the rotation axis and speed, and after applying it for a certain amount of time, we reach the desired U.

$$U(t) = e^{-iHt/\hbar}$$

Single-Qubit Gates

- Thus far, we have shown that we can create an arbitrary desired single-qubit gate by altering parameters of our input radiation: phase and amplitude (corresponding to the Rabi Frequency).
- The frequency of input radiation is not a parameter that we should vary—in order to apply our gate, we need this to be very close to the frequency required for our desired energy state transition.

$$U(t) = e^{-iHt/\hbar}$$

Two Qubit Gates?

But what about 2 Qubit Gates?

- Some 2 Qubit Gates (4x4, Unitary) can be broken down into a tensor product of two single qubit gates (2x2, Unitary).
- Physically, we can apply single qubit gates individually to any set of 2 qubits and achieve any desired rotation on each just by modifying the input radiation to the respective drive capacitor. Unlike with single qubit gates, we cannot necessarily achieve any 2 Qubit gate with this set of physical operations (adding microwave pulses to both qubits).
- **A gate that cannot be broken down into two single qubit gates in this way is known as an entangling gate.**

Non-entangling Gate:

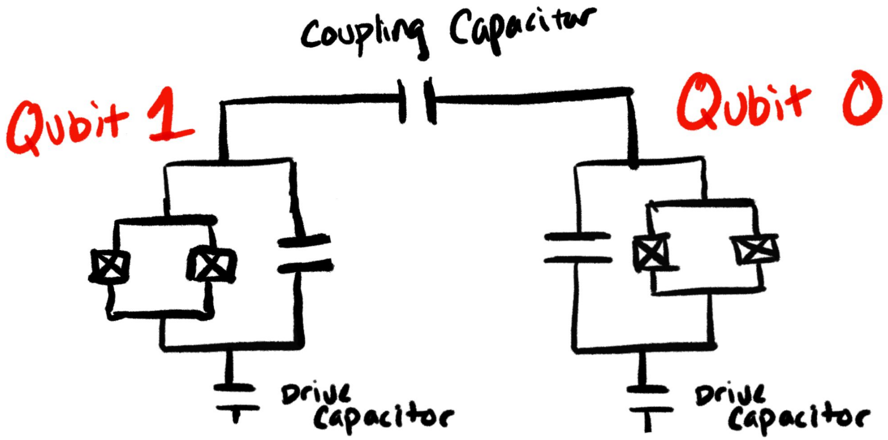
$$H \otimes H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

Entangling Gate:

$$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Two Qubit Gates?

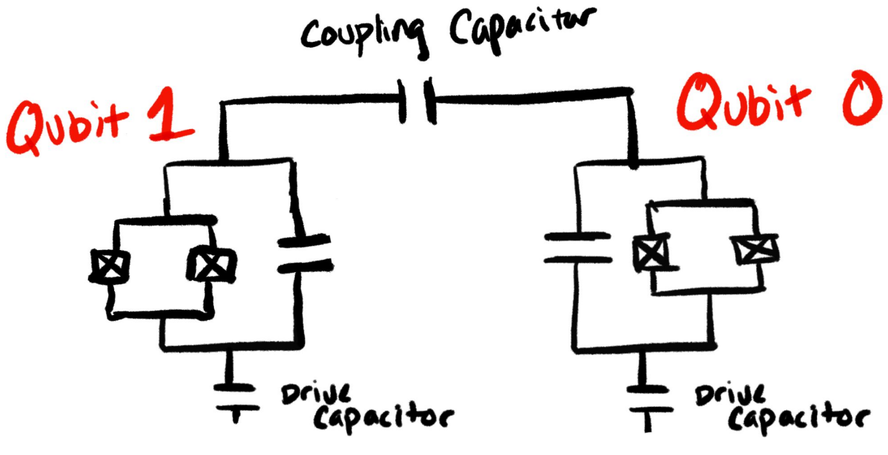
- To allow the qubits to interact with one another, we add a coupler to the system. In our case, we use a capacitor as our coupler.
- When two transmons are coupled, an “interaction Hamiltonian” is created that affects the evolution of both quantum states.
- Here, g represents the effective coupler strength, which is characteristic of the physical system.



$$H_{\text{int}} = \frac{\hbar g}{2}\left(\sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y\right)$$

Two Qubit Gates?

- When we "connect" 2 qubits through an coupler, they naturally begin interacting with one another over time.
- In other words, **entanglement is the natural interaction between two coupled transmon qubits.**



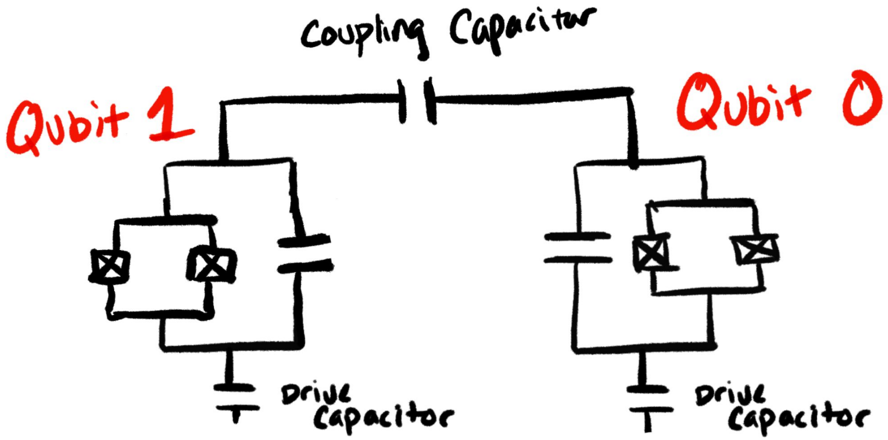
The "natural interaction":

$$U(t) = e^{-iH_{\text{int}}t/\hbar}$$

$$H_{\text{int}} = \frac{\hbar g}{2}\left(\sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y\right)$$

Two Qubit Gates?

- So, it turns out we can create an entanglement gate by simply coupling the circuits and waiting for entanglement to occur.
- But if this entanglement always occurs, how can we control it?
- We need to modify something about the system to "switch" the entanglement on/off.



$$H_{\text{int}} = \frac{\hbar g}{2}(\sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y)$$

Detuning and Resonance

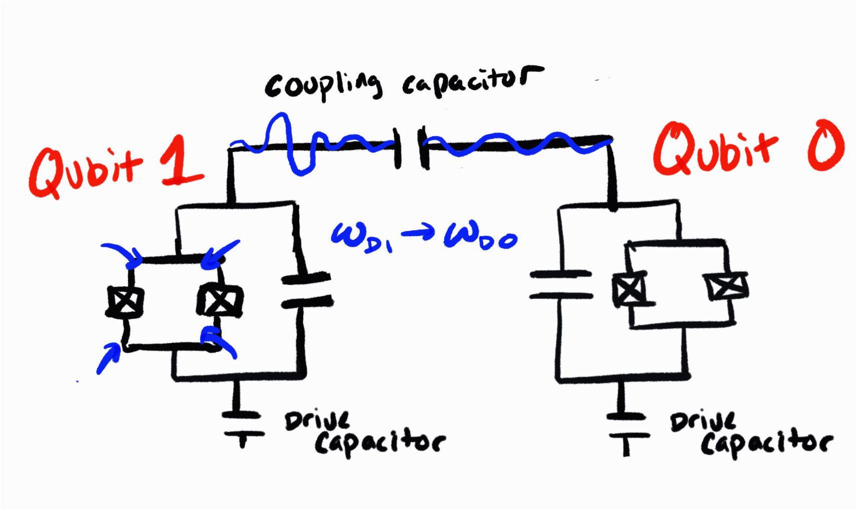
- The effective coupler strength (g) is highly sensitive to the detuning (△ – difference in frequency) between the 2 circuits.
- When $\omega_1 \approx \omega_2$, g_{eff} reaches its maximal value and the 2 circuits become resonant with one another.
- By tuning the circuits to very different frequencies, we can make g_{eff} very small so that the circuits are essentially "isolated".

$$g_{\text{eff}} \approx \frac{g_{\text{phys}}}{\sqrt{1 + (\Delta / g_{\text{phys}})^2}}$$

$$\Delta = |\omega_1 - \omega_2|$$

Detuning and Resonance

- By applying a magnetic flux (from some external device) to the SQUID in each transmon circuit, we can change the intrinsic frequency of either qubit.
- So by applying a magnetic flux to one of the qubits so they become resonant, we can effectively “switch on/off” the natural entanglement interaction between qubits.



$$\Delta = |\omega_1 - \omega_2|$$

$$\omega(\Phi) \approx \frac{\sqrt{8E_{J0}\cos\left(\frac{\pi\Phi}{\Phi_0}\right)E_C}}{\hbar} - \frac{E_C}{\hbar}$$

Creating the Entanglement Gate

- To create a standard entanglement gate, we define our time interval to be a set amount of time, given by t = π/(2g).
- Recall our equations for a 2-qubit through the natural entanglement interaction:

$$U(t) = e^{-iH_{\text{int}}t/\hbar} \qquad H_{\text{int}} = \frac{\hbar g}{2}\left(\sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y\right)$$

- Now, we plug in the Hamiltonian and t = π/(2g) to find our entanglement gate:

$$U\left(\tfrac{\pi}{2g}\right) = e^{-i\frac{\pi}{4}(\sigma_x\otimes\sigma_x+\sigma_y\otimes\sigma_y)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \text{iSWAP}$$

- This is known as the iSWAP gate!

Decomposing any gate?

- Now that we have access to any combination of single-qubit gates and an entanglement gate, it turns out that we have a **fundamental gate set** for any 2-qubit quantum gate!
- We can decompose any such gate as the following, where A's and B's are single-qubit gates:

$$U = (A_1 \otimes B_1) \cdot \text{iSWAP} \cdot (A_2 \otimes B_2) \cdot \text{iSWAP} \cdot (A_3 \otimes B_3) \cdot \text{iSWAP} \cdot (A_4 \otimes B_4)$$

- To make matters slightly more straightforward, it also turns out that we can express any single-qubit gate as a combination of rotations about Y and Z axes (times a global phase), so we have a fundamental gate set out of {iSWAP, $R_y(\theta)$, $R_z(\phi)$}:

$$U = e^{i\alpha} R_z(\beta) R_y(\gamma) R_z(\delta)$$

(Only for single–qubit gates)

Demo Preparation

- In order to test our demo, let’s try creating one of the fundamental entangled states – the Bell |ϕ+> state.
- First we apply a Hadamard gate to qubit 0 (which can be broken down into R_y and R_z), and then a CNOT gate, which can be broken down into 2 iSWAP’s and a selection of R_y and R_z:

$$H \otimes I = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \qquad \mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

- Applying to the |00> state, we expect the following (which can be verified by matrix multiplication):

$$|\Phi^+\rangle = (\mathrm{CNOT} \cdot (H \otimes I))|00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

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Demo!