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# Quantitative Aptitude for CAT

Trishna Knowledge Systems

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# Table of Contents


*Preface* *ix*
*CAT Pattern Analysis 2015–2017* *xi*
*Quantitative Aptitude Analysis 2015–2017* *xiv*

## UNIT 1: INTRODUCTION

### Chapter 1: Building Blocks

Classification of Numbers 1.3
Number Line 1.4
Rules for Divisibility 1.6
Prime Factors 1.8
L.C.M. and H.C.F. 1.8
Some Important Points 1.9
The Equation of Division 1.10
Rules Related to the Behaviour of Remainders 1.10
Algebraic Identities and Numerical Expressions 1.12

### Chapter 2: Speed Maths

Importance of Calculation Speed 1.25
Additions, Subtractions and Multiplications 1.26
Fractions and Percentages 1.27
Reciprocals and Its Multiples 1.27
Comparison of Fractions 1.28
Approximations 1.29
BODMAS—Hierarchy of Arithmetic Operations 1.30
Powers of 2 and 3 1.34

## UNIT 2 : NUMBERS

### Chapter 1: Numbers 2.3

Basic Arithmetic Operations 2.3
Classification of Real Numbers 2.4
Hierarchy of Arithmetic Operations 2.5
Recurring Decimals 2.5
Integers 2.6
Factors and Co-Primes of a Number 2.9
Least Common Multiple (L.C.M.) and Highest Common Factor (H.C.F.) 2.10
Successive Division 2.13
Algebraic Identities 2.14
Some Important Points 2.14
Last Digit of Any Power 2.15
Last Digit of a Sum or Product 2.15
Last Two Digits of $a^m$ 2.17
Some Important Theorems 2.18
Some Important Identities 2.19

### Chapter 2: Number Systems 2.104

Base 2.104
Representation 2.105
Conversions 2.105
Binary Arithmetic 2.106

### Chapter 3: Simple Equations 2.126

One Equation in One Unknown 2.126
Two Equations in Two Unknowns 2.126
Three Equations in Three Unknowns 2.127
Additional Cases in Linear Equations 2.127

## UNIT 3: ARITHMETIC

### Chapter 1: Ratio, Proportion and Variation 3.3

Ratio 3.3
Proportion 3.4
Variation 3.4

### Chapter 2: Percentages, Profit and Loss and Partnerships 3.45

Percentage 3.45
Percentage Points 3.46
Profit and Loss 3.46
Partnerships 3.47
Stocks and Shares 3.48

## Chapter 3: Special Equations 3.102

## Chapter 4: Simple Interest and Compound Interest 3.134

Interest 3.134
Present Value 3.136
Repayment in Equal Instalments—Compound Interest 3.137

## Chapter 5: Time and Distance 3.174

Speed 3.174
Average Speed 3.175
Boats and Streams 3.175
Races and Circular Tracks 3.176
Clocks 3.177

## Chapter 6: Time and Work 3.244

Pipes and Cisterns 3.245

## Chapter 7: Averages, Mixtures and Alligations 3.305

Averages 3.305
Mixtures 3.307
Alligations 3.308

# UNIT 4: GEOMETRY

## Chapter 1: Geometry 4.3

Angles and Lines 4.3
Parallel Lines 4.4
Ratio of Intercepts 4.4
Geometric Centres of a Triangle 4.6
Similarity of Triangles 4.8
Congruence of Triangles 4.8
Some More Useful Points about Triangles 4.9
Quadrilaterals 4.9
Trapezium 4.10
Parallelogram 4.10
Rhombus 4.11
Rectangle 4.11
Square 4.11
Polygon 4.11
Circles 4.12
Arcs and Sectors 4.14

## Chapter 2: Mensuration 4.94

Areas of Plane Figures 4.94

## Chapter 3: Coordinate Geometry 4.164

Rectangular Coordinates 4.164
Ordered Pair 4.165
X-coordinate and Y-coordinate 4.165
The Straight Line 4.166
Equations of Lines 4.167
Angle Between Two Lines 4.168
Some Formulae to Remember 4.168
Locus 4.169
Pair of Straight Lines 4.169

## Chapter 4: Trigonometry 4.209

Measurement of Angles 4.209
Basic Conversions 4.209
Important Conversions 4.210
Other Standard Results 4.212
Heights and Distances 4.213

# UNIT 5: ALGEBRA

## Chapter 1: Sequences and Series 5.3

Progressions 5.3
Some Important Results 5.5
Sequences and Series 5.5

## Chapter 2: Indices and Surds 5.45

Indices 5.45
Surds 5.46

## Chapter 3: Logarithms 5.81

## Chapter 4: Quadratic Equations 5.106

Quadratic Equations 5.106
Finding the Roots by Factorization 5.107
Finding the Roots by Using the Formula 5.107
Sum and Product of Roots of a Quadratic Equation 5.107
Nature of the Roots 5.108
Signs of the Roots 5.108
Constructing a Quadratic Equation 5.108
Maximum or Minimum Value of a Quadratic Expression 5.109

**Chapter 5: Inequalities and Modulus 5.158**

Symbols and Notations 5.158
Absolute Value 5.159
Interval Notation 5.160
Some Useful Models 5.160

**Chapter 6: Functions 5.193**

Sets 5.193
Relations 5.195
Functions 5.195

**Chapter 7: Graphs 5.222**

Line Graphs 5.223
Graphs of Commonly Used Functions 5.224
Exponential Function ($e^{ax}$) 5.225
Signum Function 5.225

**Chapter 8: Operator-Based Questions 5.266**

## UNIT 6: MODERN MATHS

**Chapter 1: Permutations and Combinations 6.3**

Permutations 6.3
Combinations 6.3
Circular Permutations 6.6
Some Aditional Points 6.8
Selections 6.8
Arrangements 6.8

**Chapter 2: Probability 6.40**

Probability 6.40
Addition Theorem of Probability 6.41
Expected Value 6.43

**Chapter 3: Statistics 6.81**

Measures of Central Tendencies 6.82
Measures of Dispersion 6.83

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# Preface

Congratulations on arming yourself with the Trishna book on Quantitative Aptitude for CAT. You are now geared to prepare for one of the most challenging careers today!

Of course, success will come only after cracking one of the toughest competitive examinations—CAT, which means your preparation should be nothing short of top class, where each mark will decide your future. You must aim not only to maximize your percentile but also cross the cut-off in each section of the examination. Meeting such stringent criteria calls for a champion-like preparation.

The Quantitative Aptitude section accounts for one-third of the total marks in CAT and an equal percentage in other management entrance examinations as well. This book is your ideal preparation resource with a wide range of questions, including models of problems, that appeared over the last few years in many competitive examinations.

A notable feature of the book is the large number of solved examples provided in each chapter, followed by exercises. The detailed solutions will not only help you comprehend relevant concepts easily but also clarify the different models of problems in each chapter. Once you go through all the solved examples, working out the exercises becomes easier.

Each chapter has a set of four exercises at the end, besides a fifth exercise on data sufficiency has also been provided where applicable. Each exercise covers a different level of difficulty. The first exercise contains easy questions. It is the ideal starting point (even for those only looking at the top institutes in the country). These would bring you up to the level required for the average entrance examination.

The second exercise has questions of a moderate level of difficulty. These would help you upgrade your ability and cope with the examinations considered as tough. The third set of questions is of a higher level of difficulty. The fourth exercise (masters) has the toughest questions. The third and the fourth exercises are to hone your skills and make you ready for the toughest of examinations, such as CAT, XAT, and IIFT.

This book comes with three free AIMCATs. You can access these by following the instructions given on the last page of the book. These will allow students to assess their level of learning and get them prepared for the actual examination. Students can hone their test-taking abilities and strategies by themselves, which are very important aspects in taking any management entrance examination.

This compendium of class-tested content, extensive practice resources, time-tested strategies, and practical guidelines is the result of the collective effort of a team of well-qualified faculty members.

Our content team has extensive experience of teaching and developing high-quality study materials to aid preparations for various competitive examinations. They have guided more than 21 lakh students in the last 26 years, helping them gain admission in some of the top management institutions in India and worldwide.

The extensive experience of the dedicated team at our institute allows us to say that nobody understands the needs of students and the nature of entrance examinations better than we do.

Although this book focuses on helping you prepare for the CAT and other major MBA entrance examinations (OMETs), it will also guide you to build the right foundation to develop and hone your strategies and skills necessary for career advancement in business.

The Editorial Team
Trishna Knowledge Systems

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# CAT Pattern Analysis 2015–2017

## ❒ CAT Journey So Far

We have come a long way from the long-drawn-out 40 slot-25 day window to a 2 slots—a one-day affair.

The CAT paper now comprises three sections, namely

- Verbal Ability and Reading Comprehension (VARC),
- Data Interpretation and Logical Reasoning (DILR), and
- Quantitative Ability/Aptitude (QA), each section has a sectional time limit of 60 minutes with 34, 32, and 34 questions, respectively.

The surprise element in 2015 was the introduction of Non-Multiple Choice Questions (Non-MCQs) across the three sections. It was noted that as many as one-third of the questions were non-MCQs (i.e. 33 questions out of 100 questions), which contributed to increasing the 'difficulty level' of the paper and applied brakes on all those who took chances and marked answers based on random guesses.

In terms of the test interface, there was a slight departure from what used to be shown in the sample test.

- Within the VARC section, VA and RC questions were now grouped separately and given under two separate tabs.
- Similarly, in the DILR section, the DI and LR questions appeared under two separate tabs. This helps students easily access the type of questions they would like to answer.
- Students were also able to look at their performance in the previous sections at any time during the test by clicking on the respective tab for that section.
- The number of questions attempted, left out, and marked for review were also displayed.
- Furthermore, towards the end of the test, a similar snapshot was provided for all the 3 sections.

An overall pattern analysis is provided to help students understand the changes that occurred over the last three years:

| Section | Subject | 2015 | | | 2016 | | | 2017 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | MCQ | Non-MCQ | Level of difficulty | MCQ | Non-MCQ | Level of difficulty | MCQ | Non-MCQ | Level of difficulty |
| I | Verbal Ability and Reading Comprehension (VA & RC) | 24 | 10 | Moderate-difficult | 24 | 10 | Moderate-difficult | 27 | 7 | Moderate-difficult |
| II | Data Interpretation and Logical Reasoning (DI & LR) (one set = 4 questions) | 24 | 8 | Very difficult | 24 | 8 | Difficult | 24 | 8 | Very difficult |
| III | Quantitative Aptitude (QA) | 19 | 15 | Moderate | 27 | 7 | Moderate-difficult | 23 | 11 | Easy |
| | **Total** | **67** | **33** | | **75** | **25** | | **74** | **26** | |

# ❒ PATTERN ANALYSIS

## ❒ 2015

The difficulty of the paper, across sections, was largely similar in both the slots. It was observed that the QA section was relatively much easier compared to the other two, followed by VARC.

With 24 RC questions and only 10 VA questions, the VARC section was moderate to difficult. All the 10 questions on VA were non-MCQs, and this moved the difficulty level of the section up by a significant level.

The RC passages were between 350–550 words long and were not too tough to read. However, with 5 passages (3 passages with 6 questions each and 2 passages with 3 questions each), students found it difficult to attempt all of them.

DILR was the section that troubled many with an unexpectedly high level of difficulty across multiple sets. This was true for both the slots. However, there was a marginal respite because the calculator was available.

This led to two things—As the section was difficult overall, the number of attempts dropped for all the test takers; and with CAT announcing that scores will be normalized across sections, the impact of the section on the overall score was expected to be moderated.

Another significant observation on the DILR section was that while separate tabs were provided for the DI and LR areas, there was no clear-cut segregation. There were both reasoning-based DI sets in the DI area and quantitative-based LR sets in the LR area.

The third section, QA, was moderate in difficulty. The presence of 15 non-MCQs contributed to the increased difficulty level of this section, which otherwise had many direct questions, albeit tricky ones.

| Section description | No. of questions | No. of MCQs | No. of non-MCQs | Difficulty level | No. of attempts for 95%ile | No. of attempts to cross 99%ile |
|---|---|---|---|---|---|---|
| VARC | 34 | 24 | 10 | Moderate-difficult | 22–24 | 26–28 |
| DILR | 32 | 24 | 8 | Very difficult | 11–12 | 14–15 |
| QA | 34 | 19 | 15 | Moderate | 21–23 | 25–27 |
| **Total** | **100** | **67** | **33** | **Difficult** | **54–59** | **65–70** |

*Note*: We are assuming an accuracy of 80% for the above estimates.

## ❒ 2016

In line with the expectations that students had about the CAT exam throwing surprises at them, the CAT 2016 did amaze many. There were quite a few doable questions across sections, as there were last year. However, the number of tough questions went up significantly. The order of questions and options for the questions was different for different students. The presence of easy questions made some students feel that the section was not very tough. However, many felt it to be an arduous task to push their overall attempts beyond a certain level. This was because any further move beyond an easy question was blocked by difficult ones that were present aplenty across the sections.

One significant observation that was gathered from our expert analysis was that the level of difficulty of each of the three sections was very close across the two slots. This was unlike CAT 2015, where there was observable difference in difficulty level in two of the sections (DILR and VARC).

Let's look at the test pattern.

| Section | No. of questions | No. of non-MCQs | Difficulty level |
|---|---|---|---|
| Verbal Ability and Reading Comprehension | 34 | 10 | Moderate-difficult |
| Data Interpretation and Logical Reasoning | 32 | 8 | Difficult |
| Quantitative Ability | 34 | 7 | Moderate-difficult |
| **Total** | **100** | **25** | **Moderate-difficult** |

## ❒ 2017

In line with the expectations that students have about the CAT exam, CAT 2017 did not surprise them much as far as the difficulty level of the paper was concerned.

There were quite a few doable questions across VARC and QA. However, in DILR section, the sets were quite tough to crack, with only a few doable questions. Hence, many felt it to be really difficult to push their attempts in DILR and subsequently the overall attempts beyond a certain level. The order of questions and options for the questions was different for different students.

The level of difficulty was broadly similar across both the slots. However, a relatively higher number of students from the 2nd slot felt positive about their DILR performance than those in the 1st slot. This positive feeling did not exactly translate into better scores/percentile because of the process of equating and scaling that the IIMs have been following over the past few years.

Before we get into the detailed analysis, let us quickly look at the test pattern.

| Section | No. of questions | No. of non-MCQs | Difficulty level |
|---|---|---|---|
| Verbal Ability and Reading Comprehension | 34 | 7 | Moderate-difficult |
| Data Interpretation and Logical Reasoning | 32 | 8 | Very difficult |
| Quantitative Ability | 34 | 11 | Easy |
| **Total** | **100** | **26** | **Moderate-difficult** |

# Quantitative Aptitude Analysis 2015–2017

**Analysis—2015**

The quantitative section could be considered to be of 'Moderate' level of difficulty. A few innocuous questions ended up being trickier on account of the way the questions were framed. Many questions involved application of basic concepts and only well-prepared students would have found quite a few to be outright easy. However, there were a large number of non-MCQs, which prevented students from going for half guesses. Students who covered the basics extensively reported a higher number of attempts, which reinforces the importance of concept building.

**Analysis—2016**

The quantitative section had 34 questions with around 8 questions of non-MCQ type. The number of questions on Geometry was on the higher side and some of them were considered to be of moderate-difficult. This was offset by the large number of Arithmetic questions which helped aspirants increase their attempts.

The low weightage trend for numbers continued this year too. Overall, this section could be classified as being slightly higher than moderate level (maybe a moderate plus).

| Area | Topic | Description | No. of questions | Good attempts |
|---|---|---|---|---|
| Quantitative Ability | Geometry and Mensuration | | 7 | 3–4 |
| | Progressions | | 4 | 2 |
| | Numbers | | 3 | 2 |
| | Arithmetic | AMA, TW, CI, TD, P & L, ratio, and equations* | 10 | 6–7 |
| | Algebra | QE, logs, inequalities, progressions, coordinate geometry, and surds* | 9 | 5 |
| | P & C | | 1 | 1 |

*AMA = Averages, Mixtures and Alligations
TW = Time and Work
TD = Time and Distance
CI = Compound Interest
P & L = Profit and Loss
QE = Quadratic Equations

**Afternoon Slot:** The quantitative section had 34 questions with around 6 questions of non-MCQ type. Permutation & Combination had a very strong comeback along with Numbers, which was unlike what was seen in the first slot. More number of questions were from Arithmetic, and most of which could be quickly solved by a student with moderate-good level of preparation.

There were, however, a few cases where the mathematical symbols could be interpreted incorrectly.

| Area | Topic | Description | No. of questions | Good attempts |
|---|---|---|---|---|
| Quantitative Ability | Geometry and Mensuration | | 6 | 2–3 |
| | Progressions | | 2 | 1 |
| | Numbers | | 6 | 2–3 |
| | Arithmetic | AMA, TW, CI, TD, P & L, ratio, and equations* | 13 | 8–9 |
| | Algebra | QE, logs, inequalities, progressions, coordinate geometry, and surds* | 4 | 2 |
| | P & C | | 3 | 1 |

*AMA = Averages, Mixtures and Alligations
TW = Time and Work
TD = Time and Distance
CI = Compound Interest
P & L = Profit and Loss
QE = Quadratic Equations

**Analysis—2017**

The surprise element in CAT 2017 was saved for the last, in the form of an unexpectedly easier QA section. The number of questions from basic arithmetic was noticeably and pleasantly on the higher side, making life easier for many students who were weak in QA. There were very few uncrackable questions in the paper, pushing the number of attempts higher in this section. This was observed in both the slots. However, there were a few questions in the section that had cleverly devised traps built into them and needed to be treaded with caution.

# Unit 1

# Introduction

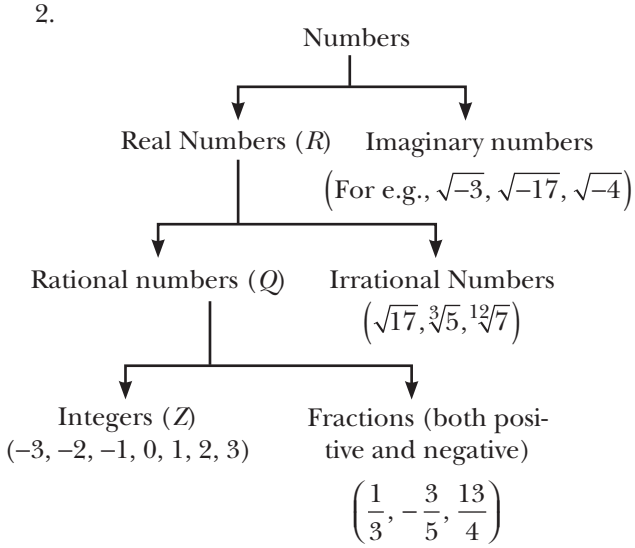
Chapter 1 Building Blocks
Chapter 2 Speed Maths

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# 1 Building Blocks

## ❒ CLASSIFICATION OF NUMBERS

1. Use the following diagram to understand the classification of numbers.
2.




2. (i) Further,
   (a) The set of integers consists of
      (i) the negative integers ($Z^-$),
      (ii) the number zero and
      (iii) the positive integers ($Z^+$) or $N$ also called Natural Numbers.
   (b) The set of natural numbers together with the number zero, is called the set of whole numbers (denoted by $W$)

   (ii) The following are to be noted
   (a) Every rational number can be represented as $(p/q)$, where $q$ is a non-zero integer and $p$ is any integer.
   (b) Any number that cannot be represented in the form of $(p/q)$, where $q$ is a non-zero integer and $p$ is an integer, is called an irrational number.

**NOTE**

$p$ and $e$ are irrational numbers

   (c) All real numbers (Rational and irrational) can be represented in a decimal form i.e., $5 = 5.0$, $\frac{1}{2} = 0.5$, $\frac{1}{3} = 0.333\ldots$, $\sqrt{3} = 1.732\ldots$ and so on.
      (i) Every such decimal form of a real number can be classified on the basis of two features:
         (a) recurring (a set of one or more digits repeat) or non-recurring.
         (b) terminating (i.e., all the digits to the right of any digit are 0's) or non-terminating.
      (ii) The decimal form of any rational number is either terminating or recurring $\left(\frac{1}{6} = 0.16666.., \frac{1}{4} = 0.25\right)$
      (iii) The decimal form of any irrational number is non-recurring as well as non-terminating. (e.g., $\pi = 3.14159265...$; $\sqrt{2} = 1.414\ ...$)

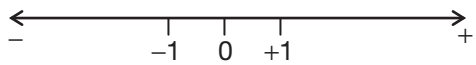
**NOTE**

$\sqrt{-1}$ is denoted by $i$ and is called an imaginary number.

3. Types of integers:
   (a) Even and Odd integers.
   All numbers (including the negative numbers and zero), which are divisible by the number 2 are called even numbers. They are in the form of $2n$, where $n$ is an integer.
   If a number is not divisible by 2, that number is called an odd number. They are in the form of $(2n+1)$, where $n$ is an integer.
   Rules regarding the sums and products of even and odd numbers are given below.
   (i) The sum of any number of even numbers is always even.
   (ii) The sum of an even number of odd numbers is always even while the sum of an odd number of odd numbers is always odd.
   (iii) If the sum of two numbers is even, either both the numbers are even or both the numbers are odd.
   (iv) If the sum of two numbers is odd, one of the numbers is even while the other is odd.
   (v) If the sum of three numbers is odd, all the three numbers are odd or any two numbers are even while the third number is odd.
   (vi) If the sum of three numbers is even, all the three numbers are even or any two numbers are odd while the third number is even.
   (vii) The product of any number of numbers is even only if at least one of the numbers is even, else it will be odd.
   (b) Prime and Composite numbers.
   (i) With the exception of the number 1, any natural number which does not have any factor other than itself and 1 is called a prime number. 2, 3, 5, 7, 11… are prime numbers.
   (ii) A natural number, which has factors other than itself and 1, is called a composite number. (4, 6, 8, 9 …)
   The following are to be noted
   (a) The number 1 is (by definition) neither composite nor prime.
   (b) The number 2 is the only prime number which is even.
   (i) If the sum of two distinct prime numbers is odd, one of the two prime numbers has to be 2.
   (ii) If the sum of two distinct prime numbers is even, then both the primes are odd numbers—neither of them can be equal to 2.
   (iii) If the sum of three distinct prime numbers is even, then the number 2 has to be one of the three prime numbers.
   (iv) If the sum of four distinct prime numbers is even, then all the four numbers are odd, none of them can be equal to 2.
   (c) Every composite number can be written as the product of prime numbers.
   (d) Every prime number which is greater than 3, can be represented either by $(6k+1)$ or $(6k-1)$ for a suitable value of $k$.
   (e) If two numbers do not have a factor other than 1 in common to them, they are called co-primes or relative primes.

## ❒ NUMBER LINE

1. The number line is a straight line on which the entire set of real numbers is represented to a specific scale. As we move on the number line the value of the number increases from the left to the right.




2. This is an effective way of understanding ranges of real values.
   Let us look at four contiguous ranges as shown above and understand the behaviour of certain mathematical operations in these four ranges.
   The ranges, in the increasing order of values, are:
   (i) $-\infty$ to $-1$
   (ii) $-1$ to 0
   (iii) 0 to +1 and
   (iv) +1 to $+\infty$
3. The behaviour of simple functions can be studied and represented on the number line. We consider the behaviours of some simple functions of variables in each of the ranges above.

(a) $x$ and $(-x)$

(i) As $x$ increases from to $-1$, $(-x)$ decreases from $+\infty$ to $+1$, and $-x > x$.

(ii) As $x$ increases from $(-1)$ to 0, $(-x)$ decreases from $+1$ to 0, and $-x > x$.

(iii) When $x$ increases from 0 to $+1$, $(-x)$ decreases from 0 to $-1$ and $x > -x$.

(iv) When $x$ increases from $+1$ to $\infty$, $(-x)$ decreases from $-1$ to $-\infty$, and $x > -x$.

**NOTE**

For $x = 0$ (not considered above) $x = -x$.

(b) $x$ and $\frac{1}{x}$

(i) As $x$ increases from $-\infty$ to $-1$, $\frac{1}{x}$ decreases from 0 to $-1$, and $x < \frac{1}{x}$.

(ii) As $x$ increases from $-1$ to 0, $\frac{1}{x}$ decreases from $-1$ to $-\infty$, and $x > \frac{1}{x}$.

(iii) As $x$ increases from 0 to 1, $\frac{1}{x}$ decreases from $+\mu$ to 1, and $x < \frac{1}{x}$.

(iv) As $x$ increases from 1 to $+\infty$, $\frac{1}{x}$ decreases from 1 to 0, and $x > \frac{1}{x}$.

**NOTE**

For $x = 1$ or $-1$ (not considered above), $x = \frac{1}{x}$.

(c) $x$ and $x^2$

(i) As $x$ increases from $-\infty$ to $-1$, $x^2$ decreases from $+\infty$ to 1, and $x < x^2$.

(ii) As $x$ increases from $-1$ to 0, $x^2$ decreases from 1 to 0, and $x < x^2$.

(iii) As $x$ increases from 0 to 1, $x^2$ increases from 0 to 1, and $x > x^2$.

(iv) As $x$ increases from 1 to $+\infty$, $x^2$ increases from 1 to $\infty$, $x < x^2$.

**NOTE**

For $x = 0$ or 1 (not considered above), $x = x^2$.

(d) $x$ and $\sqrt{x}$

Since there are no square roots for negative numbers, $\sqrt{x}$ is not defined for the first two ranges, i.e., for $-\infty < x < 0$.

(i) As $x$ increases from 0 to 1, $\sqrt{x}$ increases from 0 to 1, and $x < \sqrt{x}$.

(ii) As $x$ increases from 1 to $\infty$, $\sqrt{x}$ increases from 1 to $\infty$ and $x > \sqrt{x}$.

**NOTE**

For $x = 0$ or 1 (not considered above) $x = \sqrt{x}$.

(e) $x$ and $x^3$

(i) As $x$ increases from $-\infty$ to $-1$, $x^3$ increases from $-\infty$ to $-1$ and $x > x^3$.

(ii) As $x$ increases from $-1$ to 0, $x^3$ increases from $-1$ to 0 and $x < x^3$.

(iii) As $x$ increases from 0 to 1, $x^3$ increases from 0 to 1 and $x > x^3$.

(iv) As $x$ increases from 1 to $\infty$, $x^3$ increases from1 to $\infty$ and $x < x^3$.

**NOTE**

For $x = 0, -1, 1$ (not considered above) $x = x^3$.

4. Further, it will be useful to look at the relative values of two quantities like $x$ and $y$ and compare these with the relative values of $x^2$ and $y^2$, $\sqrt{x}$ and, $\sqrt{y}$, $\frac{1}{x}$ and $\frac{1}{y}$ etc.

The following table gives some such useful comparisons.

| $x$ vs $y$ | $x^2$ vs $y^2$ | $\sqrt{x}$ vs $\sqrt{y}$ | $x^3$ vs $y^3$ | $-x$ vs $-y$ | $1/x$ vs $1/y$ |
|---|---|---|---|---|---|
| $-\infty < x, y < 0$ and $x < y$ | $x^2 > y^2$ | $\sqrt{x}$ and $\sqrt{y}$, are not defined | $x^3 < y^3$ | $-x > -y$ | $1/x > 1/y$ |

| $x$ vs $y$ | $x^2$ vs $y^2$ | $\sqrt{x}$ vs $\sqrt{y}$ | $x^3$ vs $y^3$ | $-x$ vs $-y$ | $1/x$ vs $1/y$ |
|---|---|---|---|---|---|
| $0 < x, y < \mu$ and $x < y$ | $x^2 < y^2$ | $\sqrt{x} < \sqrt{y}$ | $x^3 < y^3$ | $-x > -y$ | $1/x > 1/y$ |
| $x < 0$ & $y > 0$ | $x^2 \gtrless y^2$ depending on the actual values of $x$ and $y$ | $\sqrt{x}$ is not defined | $x^3 < y^3$ | $-x > -y$ | $1/x < 1/y$ |
| $x > 0$ & $y < 0$ | $x^2 \gtrless y^2$ depending on the actual values of $x$ and $y$ | $\sqrt{y}$ is not defined | $x^3 > y^3$ | $-x < -y$ | $1/x > 1/y$ |

## ❒ RULES FOR DIVISIBILITY

In a number of situations, we will need to find the factors of a given number. Some of the factors of a given number can, in a number of situations, be found very easily either by observation or by applying simple rules. We will look at some rules for divisibility of numbers.

**Divisibility by 2:** A number divisible by 2 will have an even number as its last digit (For example, 128, 246, 2346, etc.)

**Divisibility by 3:** A number is divisible by 3 if the sum of its digits is a multiple of 3.

For example, take the number 9123, the sum of the digits is 9 + 1 + 2 + 3 = 15 which is a multiple of 3. Hence, the given number 9123 is divisible by 3. Similarly 342, 789, etc,. are all divisible by 3. If we take the number 74549, the sum of the digits is 29 which is not a multiple of 3. Hence, the number 74549 is not divisible by 3.

**Divisibility by 4:** A number is divisible by 4 if the number formed with its last two digits is divisible by 4.

For example, if we take the number 178564, the last two digits form 64. Since this number 64 is divisible by 4, the number 178564 is divisible by 4.

If we take the number 476854, the last two digits form 54 which is not divisible by 4 and hence, the number 476854 is not divisible by 4.

**Divisibility by 5:** A number is divisible by 5 if its last digit is 5 or zero (e.g., 15, 40, etc.)

**Divisibility by 6:** A number is divisible by 6 if it is divisible both by 2 and 3 (18, 42, 96, etc.)

**Divisibility by 7:** If the difference between the number of tens in the number and twice the units digit is divisible by 7, then the given number is divisible by 7. Otherwise, it is not divisible by 7.

Take the units digit of the number, double it and subtract this figure from the remaining part of the number. If the result so obtained is divisible by 7, then the original number is divisible by 7. If that result is not divisible by 7, then the number is not divisible by 7.

For example, let us take the number 595. The units digit is 5 and when it is doubled, we get 10. The remaining part of the number is 59. If 10 (which is the units digit doubled) is subtracted from 59 we get 49. Since this result 49 is divisible by 7, the original number 595 is also divisible by 7.

Similarly, if we take 967, doubling the units digit gives 14 which when subtracted from 96 gives a result of 82. Since 82 is not divisible by 7, the number 967 is not divisible by 7.

If we take a larger number, the same rule may have to be repeatedly applied till the result comes to a number which we can make out by observation whether it is divisible by 7. For example, take 456745, We will write down the figures in various steps as shown below.

| Col(1) Number | Col (2) Twice the units digit | Col (3) Remaining part of the number | Col(3) – Col(2) |
|---|---|---|---|
| 456745 | 10 | 45674 | 45664 |
| 45664 | 8 | 4566 | 4558 |
| 4558 | 16 | 455 | 439 |
| 439 | 18 | 43 | 25 |

Since 25 in the last step is not divisible by 7, the original number 456745 is not divisible by 7.

**Divisibility by 8:** A number is divisible by 8, if the number formed by the last 3 digits of the number is divisible by 8.

For example, the number 3816 is divisible by 8 because the last three digits form the number 816, which is divisible by 8. Similarly, the numbers 14328, 18864 etc., are divisible by 8. If we take the number 48764, it is not divisible by 8 because the last three digits' number 764 is not divisible by 8.

In general, if the number formed by the last $n$ digits of a number is divisible by $2^n$, the number is divisible by $2^n$.

**Divisibility by 9:** A number is divisible by 9 if the sum of its digits is a multiple of 9.

For example, if we take the number 6318, the sum of the digits of this number is 6 + 3 + 1 + 8 which is 18. Since this sum 18 is a multiple of 9, the number 6318 is divisible by 9. Similarly, the numbers 729, 981, etc., are divisible by 9. If we take the number 4763, the sum of the digits of this number is 20 which is not divisible by 9. Hence, the number 4763 is not divisible by 9.

**Divisibility by 10:** A number divisible by 10 should end in zero.

**Divisibility by 11:** A number is divisible by 11 if the sum of the alternate digits is the same or they differ by multiples of 11—that is, the difference between the sum of digits in odd places in the number and the sum of the digits in the even places in the number should be equal to zero or a multiple of 11.

For example, if we take the number 132, the sum of the digits in odd places is 1 + 2 = 3 and the sum of the digits in even places is 3. Since these two sums are equal, the given number is divisible by 11.

If we take the number 785345, the sum of the digits in odd places is 16 and the sum of the digits in even places is also 16. Since these two sums are equal, the given number is divisible by 11.

If we take the number 89394811, the sum of the digits in odd places is 8 + 3 + 4 + 1, which is equal to 16. The sum of the digits in even places is 9 + 9 + 8 + 1, which is equal to 27. The difference between these two figures is 11 (27 – 16), which is a multiple of 11. Hence, the given number 89394811 is divisible by 11.

The number 74537 is not divisible by 11 because the sum of the digits in odd places is 19 and the sum of the digits in even places is 7 and the difference of these two figures is 12 and this is not a multiple of 11.

Divisibility by numbers like 12, 14, 15 can be checked out by taking factors of the number which are relatively prime and checking the divisibility of the given number by each of the factors. For example, a number is divisible by 12 if it is divisible both by 3 and 4.

The next number that is of interest to us from divisibility point of view is 19.

**Divisibility by 19:** If the sum of the number of tens in the number and twice the units digit is divisible by 19, then the given number is divisible by 19. Otherwise it is not.

Take the units digit of the number, double it and add this figure to the remaining part of the number. If the result so obtained is divisible by 19, then the original number is divisible by 19. If that result is not divisible by 19, then the number is not divisible by 19.

For example, let us take the number 665. The units digit is 5 and when it is doubled, we get 10. The remaining part of the number is 66. If 10 (which is the units digit doubled) is added to 66 we get 76. Since this result 76 is divisible by 19, the original number 665 is also divisible by 19.

Similarly, if we take 969, doubling the units digit gives 18 which when added to 96 gives a result of 114. Since 114 is divisible by 19, the number 969 is divisible by 19.

If we take 873, double the units digit ($2 \times 3 = 6$) added to the remaining part of the number (87), we get 93 which is not divisible by 19. Hence, the original number 873 is not divisible by 19.

If we take a larger number, the same rule may have to be repeatedly applied till the result comes to a number which we can make out by observation whether it is divisible by 19. For example, take 456760. We will write down the figures in various steps as shown below.

| Col(1) Number | Col (2) Twice the units digit | Col (3)Remaining part of the number | Col(3) + Col(2) |
|---|---|---|---|
| 456760 | 0 | 45676 | 45676 |
| 45676 | 12 | 4567 | 4579 |
| 4579 | 18 | 457 | 475 |
| 475 | 10 | 47 | 57 |

Since 57 in the last step is divisible by 19, the original number 456760 is divisible by 19.

Let us take another example, the number 37895. Let us follow the above process step by step till we reach a manageable number 37895.

Double the units digit 5 and add the 10 so obtained to 3789, we get 3799.

Double the units digit 9 and add the 18 so obtained to 379, we get 397. Double the units digit 7 and add the 14 so obtained to 39, we get 53.

Since 53 is not divisible by 19, 37895 is not divisible by 19.

## ❒ PRIME FACTORS

Prime factors of a number are the factors of the number which are prime. Prime factorization is the process of expressing a number as a product of its prime factors.

1. **Factorization:** Resolving a given number into factors
   For example, consider 5,544.
   To resolve 5,544 into factors, we can make use of the divisibility rules. We can say that 5,544 is divisible by 9 (sum of the digits is 18). Also 5,544 is divisible by 4 as last two digits 44 is divisible by 4.
   $\therefore 5{,}544 = 9 \times 616 = 9 \times 4 \times 154 = 36 \times 154$
   Consider 15,750
   As last two digits are 50, it is divisible by 25 as well as 2. $15750 = 50 \times 315$
2. **Prime factorization:** Resolving a given number into prime factors.
   For example, consider $23{,}800 = 238 \times 100$
   $= 2 \times 119 \times 4 \times 25$
   $= 2 \times 7 \times 17 \times 2^2 \times 5^2$
   $= 2^3 \times 5^2 \times 7^1 \times 17^1$.
   In this case the number is having 2, 5, 7 and 17 as factors which are distinct prime numbers.
   $\therefore$ The number of distinct prime factors of 23,800 is 4. Also in this case, sum of the powers of prime factors is 7.
   $\therefore$ The number of prime factors of 23,800 is 7.

## ❒ L.C.M. AND H.C.F.

1. L.C.M. of two or more numbers is the lowest (or least) common multiple of the numbers.
   For example, the multiples of 2 are 2, 4, 6, 8 … and the multiples of 3 are 3, 6, 9, 12 …
   The common multiples of 2 and 3 are 6, 12, 18…
   Among them the least is 6. $\therefore$ The L.C.M. of 2 and 3 is 6. As this is a multiple of each of the given set of numbers (i.e., 2 and 3) the L.C.M. is divisible by each of the given numbers, i.e., Consider the numbers 8, 12 and 16. The L.C.M. is 48. It can be seen that 48 is divisible by 8, 12 as well as 16.
2. **Determining the L.C.M.:** The following are two common methods to find the L.C.M.
   (a) Division method
   (b) Factorization method.
   Let us find the L.C.M. of 36 and 54 using each method.
   (a) **Division method:** Start dividing the numbers with the least possible prime numbers and continue the process successively till there is no common factor.

| | |
|---|---|
| 2 | 36, 54 |
| 3 | 18, 27 |
| 3 | 6, 9 |
| | 2, 3 |

   L.C.M. $= 2 \times 3 \times 3 \times 2 \times 3 = 108$
   (b) **Factorization method:** Using this method, we should express the numbers as product of prime factors and multiply the prime factors with the highest powers.
   $36 = 2^2 \times 3^2$; $54 = 2^1 \times 3^3$
   $\therefore$ L.C.M. $= 2^2 \times 3^3 = 108$
   It should be noted that the approach to be used for finding out the L.C.M. will depend on the two numbers and the relationship they hold. This can be understood with the help of the following examples.
   (i) **Consider 12 and 18:** Take the larger number 18 and consider its multiples.
   The multiples of 18 are 18, 36, 54, 72…
   Check one by one of these for divisibility by the second number 12. The first one that is divisible by 12 is the L.C.M. In this case, 36 is divisible by 12. Hence, the L.C.M. of 12 and 18 is 36.
   (ii) **Consider 7 and 8:** As 7 and 8 are co-primes their L.C.M. will be equal to the product of 7 and 8 i.e., 56.
   (iii) **Consider 108 and 45:** 9 is evidently is a factor of both 108 and 45. Also from 45, 5 is the factor which is not there in 108, hence, the L.C.M. should be $108 \times 5 = 540$.
   (iv) **Consider 54 and 72:** We can observe that 18 is a factor of both 54 and 72 and that these two numbers can be written as $18 \times 3$ and $18 \times 4$, respectively. As 3 and 4 are co-primes L.C.M. will be $18 \times 3 \times 4 = 216$.

If the numbers for which L.C.M. has to be found out do not come into any of the above categories, then we can use the factorization method or division method to find out the L.C.M.

3. H.C.F. of two or more numbers is the highest common factor of the numbers. This is also known as the greatest common divisor (GCD).
For example, the factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.
The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36.
The common factors of 24 and 36 are1, 2, 3, 4, 6 and 12. Among them the highest is 12.
∴The H.C.F. of 24 and 36 is 12.
H.C.F. is a factor of each of the given set of numbers. Hence, it (or GCD) divides all the numbers of the given set.
For the set of numbers 24, 36 and 54, the numbers 2, 3 and 6 are factors common to 24, 36 and 54. Among the three common factors, 6 is the highest. Hence, 6 is the H.C.F. (or GCD) of 24, 36 and 54. Each of the numbers 24, 36 and 54 is divisible by 6.
4. Determining the H.C.F.: The following are two common methods to find the H.C.F.
   (i) Factorization method
   (ii) Long Division method
   (a) **Factorization method:** Consider the set of numbers 18, 24 and 48.
   $18 = 3^2 \times 2$
   $24 = 3 \times 2^3$
   $48 = 3 \times 2^4$
   Using this method, we should express the numbers as product of prime factors and multiply the prime factors with the least powers. If any prime factor is not present in any number, it is understood to be having an index of zero in that number.
   H.C.F. of 18, 24 and 48 = $3^1 \times 2^1 = 6$.
   Similarly the case of say 14, 35 and 91 can be taken where H.C.F. = 7.
   (b) **Long-division method:** Take two numbers. Divide the greater by the smaller; then divide the divisor by the remainder; divide the divisor of this division by the next remainder and so on until the remainder is zero. The last divisor is the H.C.F. of the two numbers taken.
   By the same method find the H.C.F. of this H.C.F. and the third number. This will be the H.C.F. of the three numbers.

Consider the case of the numbers 108 and 28.

$$\begin{array}{r} 28\overline{)108}(3 \\ \underline{84} \\ 24\overline{)28}(1 \\ \underline{24} \\ 4\overline{)24}(6 \\ \underline{24} \\ 0 \end{array}$$

Hence, the H.C.F. of 108 and 28 is 4.

Similarly the example of 217 and 341 can be taken, where H.C.F. = 31.

5. L.C.M. and H.C.F. of fractions.

In order to find L.C.M. and H.C.F. of fractions, they have to be in the simplest form. L.C.M. and H.C.F. can then be found using the formulas below.

(a) L.C.M. of a given set of fractions is

$$\frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominatros}}$$

(b) H.C.F. of a given set of fractions is

$$\frac{\text{H.C.F. of numerators}}{\text{L.C.M. of denominators}}$$

Example:
Find the L.C.M. and H.C.F. of $\frac{2}{3}$ and $\frac{4}{9}$

$$\text{L.C.M.} = \frac{\text{L.C.M.}\ (2,4)}{\text{H.C.F.}(3,9)} = \frac{4}{3}$$

$$\text{H.C.F.} = \frac{\text{H.C.F.}(2,4)}{\text{L.C.M.}(3,9)} = \frac{2}{9}$$

## ❒ SOME IMPORTANT POINTS

1. If $k$ is the L.C.M. of $a$, $b$, $c$ … etc., then $nk$ is the L.C.M. of $na$, $nb$, $nc$, etc.
For example, the L.C.M. of 12, 18 and 20 will be 180 and the L.C.M. of 3 × 12, 3 × 18 and 3 × 20, i.e., of 36, 54 and 60 will be 3 × 180 = 540.
2. Similar result holds good for H.C.F. also.
For example, the H.C.F. of 18, 24 and 36 will be 6 and the H.C.F. of 2 × 18, 2 × 24 and2 × 36, i.e., of 36, 48, 72 will be 2 × 6 = 12.
3. If $h$ is the H.C.F. of $a$, $b$, then the H.C.F. of any one of the numbers and their sum will be the same. i.e., H.C.F. of $a$ (or $b$) and (a + $b$) is $h$.
4. For two numbers, $a$ and $b$, L.C.M. × H.C.F. = $a \times b$.

## ❒ THE EQUATION OF DIVISION

1. In any division there are four different components that need to be identified. Namely, the dividend ($N$), divisor ($D$), quotient ($Q$) and the remainder ($R$).

   For example consider the division

| | 2 |
|---|---|
| 5 | 13 |
| | 10 |
| | 3 |

   In the above division, 13 is the dividend, 5 is the divisor, 2 is the quotient and 3 is the remainder. Also, when $N$ is less than $d$, $Q$ is zero and $R = N$ itself.
2. In general, the equation of division is written as $N = dQ + R$.
   In the example above, $13 = 5 \times 2 + 3$.
3. The family of numbers $N$ which when divided by a divisor $d$, leave a remainder of $R$ is $dQ + R$, where $Q$ is the quotient of $\frac{N}{d}$. Generally, $0 \le R \le d - 1$

Example: 'The family of numbers that leave a remainder of 5 when divided by 7', will be denoted as $N = 7Q + 5$, where $N$ takes different values as $Q$ takes different values.

i.e., if $Q = 0, 1, 2, 3\ldots$, etc., $N$ will assume the respective values of 5, 12, 19, 26… etc.

(i) $N - R = dQ$
$N - R$ is a number divisible by $d$.
$N - R + d = d(Q + 1)$
$N - R + d$ is also divisible by $d$.

Example: If $N = 13$, $d = 5$, $Q = 2$, $R = 3$, then $N - R = 13 - 3 = 10$ is a multiple of 5 and so is $N + (d - R) = 13 + (5 - 3) = 15$.

$d - R$ is also termed as the 'complement of the remainder', (C).

Examples:

| D | R | Complement of R (C) |
|---|---|---|
| 9 | 6 | 9 – 6 = 3 |
| 7 | 2 | 7 – 2 = 5 |

(ii) The remainder $R$ of a divisor can be negative. The corresponding positive remainder is $d - R$. The remainder $r$ by convention, is positive (or zero). The number then is $r$ more than a multiple of $d$. It is also $d - r$ less than the next multiple of $d$. Hence, we can say the remainder is $-(d - r)$.

Example: If the divisor is 7 and the remainder is –1, then the remainder is 6.

(iii) If the remainder is longer than the divisor, then the remainder must be decreased by a number equal to the divisor as many times as required until a remainder that is less than the divisor is attained.

Example: If $R = 7$, $d = 3$, the remainder $= 7 - 3 - 3 = 1$

(iv) In order for a given number to be made divisible by a given divisor, we have to

(a) subtract Remainder ($R$) from $N$.
Example: To make 17 divisible by 4, subtract the remainder in 17/4, i.e., 1 from 17.
$\therefore 17 - 1 = 16$, which is divisible by 4.
(OR)

(b) add the complement ($C$) to $N$
Example: To make 17 divisible by 4, add the complement of the remainder $(4 - 1 = 3)$, to 17.
$\therefore 17 + 3 = 20$, which is divisible by 4.

## ❒ RULES RELATED TO THE BEHAVIOUR OF REMAINDERS

1. If two or more dividends, $N1$, $N2$, … are divided by the same divisor, $d$, and leave respective remainders of $R1$, $R2$, …, then the remainder of $(N1 + N2\ldots.)$ when divided by $d$ will be same as the sum of the remainders themselves. If $(R1 + R2 \ldots) = R_s$ (say) is greater than $d$, $R_s$ will not be an acceptable remainder. Any acceptable remainder must lie in the interval 0 to 1 less than the divisor. In order to obtain the acceptable remainder, we have to decrease $R_s$ by the divisor repeatedly until we obtain a value in the acceptable remainder range. This value would be the acceptable remainder.

Example:

(i) $\frac{11}{5} \Rightarrow R = 1$ and $\frac{13}{5} \Rightarrow R = 3$

$\therefore \frac{11 + 13}{5} \Rightarrow R = 1 + 3$

(ii) $\frac{13}{5} \Rightarrow R = 3$ and $\frac{14}{5} \Rightarrow R = 4$

$\therefore \frac{13 + 14}{5} \Rightarrow R = 3 + 4 = 7$

Since, $R > d$, i.e., $7 > 5$.

Remainder = 2, since ${}^{7}_{5} \Rightarrow R = 2$.

2. The concept explained above (in 1) is applicable to the difference of $N1$ and $N2$.

Example:

(i) $\frac{13}{5} \Rightarrow R = 3$, $\frac{11}{5} \Rightarrow R = 1$

$\therefore \frac{13-11}{5} \Rightarrow R = 3 - 1 = 2$

(ii) $\frac{23}{5} \Rightarrow R = 3$, $\frac{14}{5} \Rightarrow R = 4$

$\therefore \frac{23-14}{5} \Rightarrow R = 3 - 4 = -1$

(according to the rule, but we know $\frac{23-14}{5} = \frac{9}{5} \Rightarrow R = 4$)

Since $R$ is negative, true $R = 5 - 1 = 4$.

3. Let us now find the remainder when $k$ times $N$ is divided by $d$. This is by extending Rule 1 regarding addition as follows:

$k$ times $N = N + N + \ldots$ $k$ times

Example:

(i) $\frac{19}{9} \Rightarrow R = 1$, and say $k = 3$

$R$ of $\frac{3 \times 19}{9} = 3 \times 1 = 3$

(ii) If $\frac{21}{9} \Rightarrow R = 3$ and say $k = 4$

$R$ of $\frac{4 \times 21}{9} = 4 \times 3 = 12$ (according to rule, but we know $R$ of $\frac{84}{9} = 3$)



**NOTE**

There is no particular rule for finding the remainder in the case when $N$ is divided by any other number, say $m$.

4. The above rule can be extended to find the remainder of exponents (or powers) of $N$ when divided by $d$. This can be done as follows.

If $K$ in above rule = $N$ itself remainder of

$KN$ = Remainder of $N.N = N^2$ ($\because K = N$)

$\therefore$ If $N$ is squared (or raised to the power of two), then the remainder $R$ also becomes $N$ times

But $N = dQ + R$.

$\therefore$ The Remainder = $R(dQ + R) = RdQ + R^2$

$\therefore$ The first part of this sum ($RdQ$) is divisible by $d$.

$\therefore$ The remainder will be $R^2$.

Hence, if $N$ is squared (or cubed or raised to any power), $R$ also is squared (or cubed or raised to that power)

Example:

(i) $R$ of $\frac{8}{6} = 2$

$\Rightarrow R$ of $\frac{(8)^2}{6} = 2^2 = 4$

(ii) $R$ of $\frac{8}{6} = 2$

$R$ of $\frac{(8)^3}{6} = 2^3 = 8$

$\because 8 > 6$, $R$ of $\frac{8}{6} = 2$.

5. If $R$ of $\frac{N_1}{d} = R_1$ and $R$ of $\frac{N_2}{d} = R_2$

Then $R$ of $\frac{N_1 \times N_2}{d} = R_1 \times R_2$

This can be verified by substituting $N_1 = dQ_1 + R_1$ and $N2 = dQ_2 + R_2$ and expanding $N_1 \times N_2$ to find that all the terms in the expansion except $R_1 R_2$ will be divisible by $d$. Hence, $R_1 R_2$ will be the remainder.

Example:

If $N_1 = 7$, $d = 5$, $R = 2$ $N_2 = 8$, $d = 5$, $R = 3$

$\Rightarrow R$ of $\frac{N_1 \times N_2}{d} = R$ of $\frac{7 \times 8}{5} = 2 \times 3 = 6$

6. If $R$ is the remainder when $N$ is divided by $d$, (i.e., $N = dQ + R$), the remainder when $N$ is divided by any factor of $d$, say $d_f$, is same as the remainder of $R$ when divided by $d_f$

$\Rightarrow R$ of $\left(\frac{N}{d_f}\right) = R$ of $\left(\frac{R}{d_f}\right)$

Example: $R$ of $\frac{14}{8} = 6$

Now 4 is a factor of 8

$R$ of $\frac{14}{4} = R$ of $\frac{6}{4} = 2$

7. If $R$ is the remainder when $N$ is divided by $d$, (i.e., $N = dQ + R$), then

$R$ of $\left(\frac{k \,.\, N}{k \,.\, d}\right) = kR$, when $k$ is any natural number.

**NOTE**

Usually the remainder in the above case is mistakenly understood as $R$. The correct value is $kR$.

Example: $R$ of $7/4 = 3$ (say $k = 2$)

$$R \text{ of } \frac{2 \times 7}{2 \times 4} = 2 \times 3 = 6.$$

## ❒ ALGEBRAIC IDENTITIES AND NUMERICAL EXPRESSIONS

The following algebraic formulae which are very useful should be learnt by heart.

Group I:

(1) $(a+b)^2 = a^2 + 2ab + b^2$

(2) $(a-b)^2 = a^2 - 2ab + b^2$

(3) $(a+b)\ (a-b) = a^2 - b^2$

(4) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$= a^3 + b^3 + 3ab\ (a+b)$

(5) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
$= a^3 - b^3 - 3ab\ (a-b)$

Group II:

(1) $a^3 + b^3 = (a+b)\ (a^2 - ab + b^2)$
$= (a+b)^3 - 3ab\ (a+b)$

(2) $a^3 - b^3 = (a-b)\ (a^2 + ab + b^2)$
$= (a-b)^3 - 3ab\ (a-b)$

(3) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(4) $a^3 + b^3 + c^3 - 3abc = (a+b+c)\ (a^2 + b^2 + c^2 - ab - bc - ca)$

If $(a+b+c) = 0$, $a^3 + b^3 + c^3 = 3abc$.

**NOTE**

Formulae in group I are important as they are more frequently used.

In the entrance exams, sometimes there will be problems involving simplification by direct application of one or more of these formulae.

These formulae can be used effectively in finding certain powers of some numbers as well as carrying out some multiplications.

For example,

$112^2 = (100 + 12)^2 = 100^2 + (2 \times 100 \times 12) + 12^2$
$= 12544$

$89^2 = (100 - 11)^2 = 7921$

$17 \times 23 = (20 - 3)(20 + 3) = 20^2 - 3^2 = 391$

$17^2 = (17 + 3)(17 - 3) + 3^2 = 20 \times 14 + 9 = 289$

$39^2 = (39 + 1)\ (39 - 1) + 1^2 = 40 \times 38 + 1^2$
$= 1520 + 1 = 1521$

When a numerical expression has the structure of an algebraic identity, the numerical expression can be simplified by using the algebraic identity.

Example: Simplify

(i) $(0.3)^3 + (0.5)^3 + (0.4)^3 + 3(0.7)(0.9)(0.8)$.

The above expression is in the form of $a^3 + b^3 + c^3 + 3(a+b)\ (b+c)\ (c+a)$, and hence, equal to $(a+b+c)^3$; which is $(0.3 + 0.5 + 0.4)^3$
$= 1.728$

(ii) $47 \times 53 = (50 - 3)\ (50 + 3) = 50^2 - 3^2 = 2491$

**Summations:** In entrance exams, questions involving direct or indirect applications of the formulae of $\Sigma n$, $\Sigma n^2$, $\Sigma n^3$ are asked.

$1 + 2 + 3 + 4 + \ldots + n = \Sigma n.$

$1^2 + 2^2 + 3^2 + \ldots\ n^2 = \Sigma n^2$

$1^3 + 2^3 + 3^3 + \ldots\ n^3 = \Sigma n^3$

The formulae for $\Sigma n$, $\Sigma n^2$, $\Sigma n^3$ are.

$$\Sigma_n = \frac{n(n+1)}{2}$$

$$\Sigma_n{}^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\Sigma n^3 = \frac{n^2(n+1)^2}{4} = \left(\frac{n(n+1)^2}{2}\right) = (\Sigma n)^2$$

## SOLVED EXAMPLES

**1.1.** In how many ways can you deal out a total amount of 35 roubles using a total of exactly 14 notes having a value of 1, 3 or 5 roubles each?

**Sol:** Since the denomination of each note is an odd number, i.e., 1, 3 or 5 and there are an even number of notes, i.e., 14 notes in all, one will always have the sum of an even number of odd numbers as even. Now, since 35 is an odd number, we will not be able to deal out that amount using an even number of notes, each of an odd denomination.

**1.2.** Sandhya bought a note book containing 124 sheets and numbered the pages in them from 1 to 248. Sandeep tore out 35 sheets out of that book and added the 70 numbers he found on the pages. Is it possible for Sandeep to have got 2004 as the sum?

**Sol:** No. Sandeep would have obtained exactly 35 odd numbers and 35 even numbers. The sum of which will always be odd. Hence, Sandeep could not have got 2004 as the sum.

**1.3.** $X$ is a number formed by the first 101 digits of the number $N$ which is formed by writing 100 natural numbers one after another as follows 1234567891011… Find the remainder when $X$ is divided by 4.

(A) 0 (B) 1 (C) 3 (D) 2

**Sol:** The remainder of $X$ divided by 4 is equal to the remainder when the number formed by the last 2 digits of $X$ is divided by 4. We tabulate below the numbers, the number of numbers, the number of digits and the total number of digits in $X$

| Numbers | Number of Numbers | Number of Digits | Total number of Digits |
|---|---|---|---|
| 1 – 9 | 9 | 9 | 9 |
| 10 – 55 | 46 | 92 | 101 |

We see that the number formed by the last 2 digits of $X$ is 55.

**1.4.** What are the prime factors of 3003?

**Sol:** Sum of the digits is 6 (a multiple of 3).

$3003 = 3 \times 1001$

Sum of the odd digits = the sum of the even digits

$3003 = 3 \times 11 \times 91$

$= 3 \times 11 \times 7 \times 13$

**NOTE**

Remember $1001 = 7 \times 11 \times 13$, it will be useful later.

**1.5.** The H.C.F. of two numbers is 12 and their L.C.M. is 72. If one of the numbers is 24, the other is

**Sol:** Let the other number be $x$

H.C.F. $(24, x) \times$ L.C.M. $(24, x) = (24)(x)$

$12 \times 72 = 24(x)$

$x = 36$

**1.6.** The L.C.M. of two natural numbers $x$ and $y$ is $z$. What is the L.C.M. of $3x$ and $3y$?

(A) $3z$ (B) $z$ (C) $9z$ (D) 27

**Sol:** Given that L.C.M. $(x, y) = z$

L.C.M. $(3x, 3y) = 3z$. This is because both the numbers have become three times their original value.

**1.7.** What is the sum of the squares of the first 30 natural numbers?

(A) 9145 (B) 9455 (C) 9715 (D) 9355

**Sol:** Sum of the squares of first $n$ numbers is given by $\frac{n(n+1)(2n+1)}{6}$.

$1^2 + 2^2 + \ldots + 30^2 = \frac{(30)(31)(61)}{6} = 9455$

**1.8.** $N1$ when divided by $d$ leaves a remainder of 19 and $N2$ when divided by $d$ leaves a remainder of 23. If $(N1 + N2)$ leaves a remainder of 5 when divided by $d$, what is the value of $d$?

**Sol:** Given Remainder of $\left(\frac{N_1}{d}\right) = 19$ and Remainder of $\left(\frac{N_2}{d}\right) = 23$

Remainder of $\left(\frac{N_1 + N_2}{d}\right) = 5 < (23 + 19)$

$\Rightarrow d$ is a factor of $(42 - 5) = 37$ and greater than each of the two remainders (19 and 23). $\therefore d = 37$

**1.9.** A number $N$ when divided by $d$ leaves a remainder of 13. When $9N$ is divided by $d$, it leaves a remainder of 22. What is the value of $d$?

**Sol:** Remainder of $\left(\frac{N}{d}\right) = 13$

$\Rightarrow$ Remainder of $\left(\frac{9N}{d}\right)$ = Remainder $\left(\frac{13 \times 9}{d}\right)$

= Remainder of $\left(\frac{117}{d}\right)$

$\therefore 117 = kd + 22 \ \therefore \ kd = 95$

Since $d > 22$, $d = 95$ and $k = 1 \ \therefore \ d = 95$

**1.10.** What is the sum of the first 25 natural numbers?

**Sol:** $n = 25$

$\Sigma_n = \frac{n(n+1)}{2} = \frac{25 \times 26}{2} = 325$

(b) What is the sum of the first 60 even natural numbers?

**Sol:** $2 + 4 + 6 + \ldots 2 \times 60 = 2(1 + 2 + \ldots 60) = 2 \Sigma 60$

$= \frac{2(60)(61)}{2} = 3660$

**1.11.** What is the sum of the squares of the first 30 natural numbers?

(A) 9145 (B) 9455

(C) 9715 (D) 3025

**Sol:** Sum of the squares of first $n$ natural numbers is $\frac{n(n+1)(2n+1)}{6}$.

$12 + 22 + \ldots + 302 = \frac{(30)(31)(61)}{6} = 9455$

## EXERCISE-1

***Directions for questions 1 to 15:*** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** If A is the set of all rational numbers between 0 and 5 (excluding both) and B is the set of all real numbers not greater than 5, which of the following is true?
(A) The greatest element in A is 4.9999.
(B) The least element in A is 0.00001.
(C) The greatest element in B is 5.
(D) More than one of the above.

**2.** If, $p+\sqrt{5}$, $q$, as well as $\left(p+\sqrt{5}\right)\left(q+\sqrt{7}\right)$ are rational numbers, then $\sqrt{5}pq$ is equal to
(A) $5q$ (B) $\sqrt{5q}$ (C) $-\sqrt{5q}$ (D) $-5q$

**3.** If the sum of four distinct prime numbers is 63, one of the prime numbers must be _____.

**4.** Which of the following is not a rational number?
(A) $\frac{1}{3}$ (B) $212324-1$
(C) $\frac{2.437}{8.92323}$ (D) $\sqrt{2}$

**5.** The product of two real numbers is a rational number. If one of them is a rational number, the other number is
(A) also a rational number (B) an irrational number
(C) a natural number (D) None of these

**6.** The sum of six natural numbers is odd. Which of the following statements is/are definitely true?
I. The sum of all the possible products of the numbers taken two at a time is even.
II. The product of all six numbers is odd.
(A) Only I (B) Only II
(C) Both I and II (D) Neither I nor II

**7.** In typing the first 399 natural numbers using a computer keyboard, how many times are the numbered keys used?
_____

**8.** If $a$ is a prime number less than 100 such that $b=a+2$, in how many cases are both $a$ and $b$ primes?
(A) 10 (B) 8 (C) 9 (D) 7

**9.** $P$ and $Q$ are odd integers, while $R$ and $S$ are even integers. Which of the following statements are true?
I. $(P-Q-R)\ (P+R-S)$ is even.
II. $(P-R)\ (Q+S)$ is even.
III. $(P+Q)\ (R-S)$ is odd.
IV. $P+Q+R-S$ is even.
(A) Only II and IV (B) Only III and II
(C) Only I and III (D) Only I and IV

**10.** There are ten natural numbers $A, B, C \ldots I, J$. Which of the following statements are false, if the product of these ten numbers is odd?
I. $2A+3B+4C+5D+\ldots+11J$ is even.
II. $5A+10B+15C+\ldots+45I+50J$ is even.
III. $(A+B+C+\ldots+I+J)$ is even.
(A) Only I and II (B) Only II and III
(C) Only I and III (D) I, II and III

**11.** If the product of ten natural numbers $A, B, C, \ldots I, J$ is even, which of the following statements is/are true?
I. The sum of the ten numbers is even.
II. $(A-B)\ (B-C)\ (C-D)\ (D-A)$ is even.
(A) Only I (B) Only II
(C) Both I and II (D) None of these

**12.** If $P$, $Q$ and $R$ are three distinct primes satisfying $P+Q+1=2n-R$, where $n$ is a natural number, how many of the following statements are true? _____
I. $P-R$ is odd.
II. $P+Q-R$ is odd.
III. $R$ is of the form $2m+1$, where $m$ is a positive integer.
IV. $PQR$ is odd.

**13.** A square grid of $8\times 8$ squares having 64 unit cells is taken. The unit cells in the first row are numbered 1 to 8, the second row as 9 to 16, the third row as 17 to 24, and so on till the last row. In the first round, Mr. A painted every alternate unit cell starting from the second unit cell. In the next round, Mr. B then painted every third unit cell starting from the third. Mr. C then painted every fifth unit cell starting from the fifth unit cell and so on. In how many rounds were all the unit cells except the first one painted? _____

**14.** If $p$, $q$ and $r$ are three integers, which of the following is not a possible value of $(p+q)(q-r)(p+r)$?
(A) 240 (B) –476 (C) 225 (D) 336

**15.** The average of four distinct prime numbers is 12.25. Which of the following must always be one of the numbers?
(A) 11 (B) 13 (C) 2 (D) 17

## EXERCISE-2

***Directions for questions 1 to 10*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** When $y \geq y^3$, which of the following is always true?
(A) $y \leq 0$ (B) $y \leq 1$
(C) $y \leq -1$ or $0 \leq y \leq 1$ (D) $|y| \geq 1$

**2.** If $y = 0.5$, which of the following is the largest?
(A) $1/y$ (B) $y^{1/5}$ (C) $y^{-1/5}$ (D) $y^5$

**3.** If, $\frac{1}{y^2} < \frac{-1}{y^5}$, then $y$ can belong to which of the following ranges?
(A) $0 < y < 1$ (B) $-\infty < y < -1$
(C) $1 < y < \infty$ (D) None of these

**4.** If $y^3 > y^6$, $x > x^2$ and $y^2 < x^2$, which of the following holds true?
(A) $0 < y < x < 1$ (B) $0 < x < y < 1$
(C) $-1 < x < y < 1$ (D) Either (A) or (B)

**5.** If $y^2 > y^3$ and $y^3 > y$, then
(A) $-1 < y < 0$ (B) $-\infty < y < -1$
(C) $0 < y < 1$ (D) $1 < y < \infty$

**6.** If $y^4 > y^2$, which of the following can be concluded?
(A) $y$ is an integer not equal to zero
(B) $|y| > 1$
(C) $y^2 < 1$
(D) more than one of the above

**7.** (i) If $y^6 > y > y^5$ and $y^5 < y^4$, then $y$ belongs to which of the following ranges?

I. $0 < y < 1$

II. $-\infty < y < -1$

III. $1 < y < \infty$

(A) Only I (B) Only II
(C) Only III (D) All the above

(ii) If $x^4 > x^3$ and $x^3 < x^2$, then $x$ can belong to which of the following ranges?

I. $-\infty < x < -1$

II. $-1 < x < 0$

III. $0 < x < 1$

IV. $1 < x < +\infty$

(A) Only I and II (B) Only I and IV
(C) Only II and IV (D) Only I and III

**8.** If, $\frac{1}{x} > \frac{1}{y}$, where $x$ and $y$ are integers and $x \neq 0$, $y \neq 0$, then
(A) $x > y$ (B) $x < y$
(C) $x \leq y$ (D) None of these

**9.** If $x^2 > x$, then $x$ cannot be a
(A) positive integer
(B) positive proper fraction
(C) negative fraction
(D) negative integer

**10.** (i) If $y^5 > y^4$ or, $\frac{1}{y^3} < \frac{1}{y^5}$, then $y$ can belong to which of the following ranges?

I. $-\infty < y < -1$

II. $-1 < y < 0$

III. $0 < y < 1$

IV. $1 < y < \infty$

(A) Only I (B) Only II
(C) I and IV (D) I, III and IV

(ii) If $x^2 > x^3$, then $x$ can belong to which of the following ranges?

I. $0 < x < 1$

II. $-1 < x < 0$

III. $-\infty < x < -1$

(A) Only I and III (B) Only II
(C) Only III (D) I, II and III

## EXERCISE-3

***Directions for questions 1 to 15*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** In a garden there are 180 apple trees, 216 mango trees and 324 banana trees. They are arranged in '$n$' rows in such a manner that each row has the same number of trees and of the same variety. Find the minimum value of $n$.
(A) 12 (B) 15 (C) 36 (D) 20

**2.** What could be the total number of soldiers in a battalion which can arrange itself into 12, 16 or 20 equal rows?
(A) 280 (B) 360
(C) 420 (D) 480

**3.** The L.C.M. and H.C.F. of 5/8, 15/16 and 25/36 are
(A) 75/8 and 5/72 (B) 75/4 and 5/144
(C) 75/2 and 5/36 (D) 75/16 and 5/18

**4.** Find the L.C.M. of 39, 65 and 143 _____.

**5.** The H.C.F. of two numbers is 4 and their L.C.M. is 48. How many such pairs of numbers are there? _____

**6.** How many distinct prime factors does the number 786000 have?
(A) 4 (B) 2
(C) 5 (D) 3

**7.** There are three bells in a church. The first bell tolls every 5 minutes, the second bell tolls every 4 minutes and the third bell tolls every 6 minutes. If the bells toll together at 10:39 a.m., when will the bells toll together for the first time again?
(A) 11:15 a.m. (B) 11:39 a.m.
(C) 1:00 p.m. (D) 10:54 a.m.

**8.** A fruit vendor has 144 oranges, 198 apples and 252 grapes, which are to be packed in a certain number of boxes. Each box must have the same number of fruits and they all must be of the same variety. What is the minimum number of boxes needed for packing the fruits?
(A) 33 (B) 34 (C) 18 (D) 36

**9.** The L.C.M. of two co-primes $x$ and $y$ is 24. If $x$ is doubled and $y$ is tripled, their L.C.M. becomes 144. Find $x + y$.
(A) 25 (B) 11
(C) 14 (D) None of these

**10.** The L.C.M. and the H.C.F. of two 3-digit numbers $a16$ and $30b$ are 3672 and 18, respectively. Find $a$ and $b$.
(A) 6, 4 (B) 3, 4 (C) 4, 6 (D) 2, 6

**11.** The L.C.M. of two numbers is 196 and their H.C.F. is 7. If the difference of the two numbers is 21, find the larger of the two numbers.
(A) 28 (B) 35 (C) 42 (D) 49

**12.** Four blocks of chocolates of weights $6\frac{1}{8}$ kg, $10\frac{1}{2}$ kg, $8\frac{3}{4}$ kg and $3\frac{15}{16}$ kg, respectively were bought for a birthday party. The blocks were divided into pieces such that all the pieces are of the same weight. What is the least number of pieces that can be obtained? _____

**13.** What is the minimum number of identical square tiles required to completely cover a floor of dimensions 8 m 70 cm by 6 m 38 cm? _____

**14.** The difference between the L.C.M. and H.C.F. of two natural numbers $a$ and $b$ is 57. What is the minimum value of $a + b$?
(A) 22 (B) 27 (C) 31 (D) 58

**15.** If three numbers are in the ratio 3: 4: 5, and their L.C.M. is 480, then find the sum of the three numbers.
(A) 96 (B) 72 (C) 84 (D) 108

## EXERCISE-4

***Directions for questions 1 to 20*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** What is the sum of the first 20 odd natural numbers?
_____

**2.** Find the sum of all the perfect squares lying between 4000 and 7000.
(A) 108710 (B) 108612
(C) 108598 (D) 108816

**3.** The sum of the first 30 even natural numbers is _____ .

**4.** Which of the following is not the sum of the cubes of first $n$ natural numbers where $n$ is a positive integer?
(A) 8281 (B) 14400
(C) 9252 (D) 3025

**5.** Consider the given square grid of size $6 \times 6$. It consists of 36 identical $1 \times 1$ square cells. How many squares of all possible sizes are there in the figure below?
(A) 70 (B) 75 (C) 81 (D) 91

**6.** If $x + \frac{1}{x} = 4$ where $x > 1$, find the value of. $x^2 + \frac{1}{x^2}$
(A) 14 (B) 12
(C) 16 (D) None of these

**7.** What is the sum of the squares of all the odd numbers in the first 60 natural numbers? _____

**8.** The sum of all the odd-numbered pages in a story book is 196. Find the sum of all the even-numbered pages in the book.
(A) 210 (B) 224
(C) 256 (D) 240

**9.** Find the sum of the squares of the first 20 even natural numbers.
(A) 11,580 (B) 11,480
(C) 12,620 (D) 13,720

**10.** If $x = 32 + (2 + 4 + 6 + \ldots 34)$ and $y = 19 + (1 + 2 + 3 + \ldots + 17)$, what is the value of $x - y$? _____

**11.** A number when divided by 54 leaves a remainder of 31. Find the remainder when the number is divided by 27.
(A) 4
(B) 23
(C) Neither (A) nor (B)
(D) Cannot be determined

**12.** Two numbers when divided by a divisor leave remainders of 248 and 372. The remainder obtained when the sum of the numbers is divided by the same divisor is 68. Find the divisor.
(A) 276 (B) 552
(C) 414 (D) Cannot be determined

**13.** A number when divided by $D$ leaves a remainder of 7 and when divided by $3D$ leaves a remainder of 20. What is the remainder left when twice the number is divided by $3D$?
(A) 1 (B) 20
(C) 13 (D) Cannot be determined

**14.** A number when divided by $D$ leaves a remainder of 9 and when divided by $3D$ leaves a remainder of 35. What is the remainder left when twice the number is divided by $3D$?
(A) 17 (B) 31
(C) 70 (D) Cannot be determined

**15.** A number divided by a certain divisor leaves a remainder of 11, whereas the square of the number when divided by the same divisor, leaves a remainder of 1. How many such divisors are possible?
(A) 2 (B) 4 (C) 8 (D) 16

**16.** Find the value of the expression below

$$\frac{(0.68)^3 + (0.67)^3 - (0.5)^3 + (0.68)(0.67)(1.5)}{(0.68)^2 + (0.67)^2 + (0.5)^2 - (0.68)(0.67)(0.5) + (0.68)(0.5)}$$

(A) 1.85 (B) 0.51
(C) 0.49 (D) 0.85

**17.** $\dfrac{10.23^3 - 4.77^3}{10.23^2 + 4.77^2 + (10.23)(4.77)} =$ _____ .

**18.** $\dfrac{[(3.7) + (5.2)]^2 + [(3.7) - (5.2)]^2}{(3.7)^2 + (5.2)^2} =$

(A) 4.0704 (B) 1 (C) 7.2482 (D) 2

**19.** Find the remainder when $1750 \times 1752 \times 1754$ is divided by 13.
(A) 9 (B) 11 (C) 7 (D) 5

**20.** If $a + b = 10$, and $ab = 9$, then find the value of $a^3 + b^3$.
(A) 700 (B) 731 (C) 720 (D) 730

## ANSWER KEYS

### Exercise-1

| | | | | |
|---|---|---|---|---|
| 1. (C) | 4. (D) | 7. 1089 | 10. (A) | 13. 18 |
| 2. (D) | 5. (D) | 8. (B) | 11. (D) | 14. (C) |
| 3. 2 | 6. (D) | 9. (D) | 12. 3 | 15. (C) |

### Exercise-2

| | | | | | |
|---|---|---|---|---|---|
| 1. (C) | 3. (D) | 5. (A) | 7. (i) (B) | 8. (D) | 10. (i) (D) |
| 2. (A) | 4. (A) | 6. (B) | (ii) (A) | 9. (B) | (ii) (D) |

### Exercise-3

| | | | | | |
|---|---|---|---|---|---|
| 1. (D) | 4. 2145 | 6. (A) | 9. (B) | 12. 67 | 15. (A) |
| 2. (D) | 5. Two pairs. | 7. (B) | 10. (D) | 13. 165 | |
| 3. (B) | | 8. (A) | 11. (D) | 14. (B) | |

### Exercise-4

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. 400 | 4. (C) | 7. 35990 | 10. 166 | 13. (A) | 16. (D) | 19. (B) |
| 2. (A) | 5. (D) | 8. (A) | 11. (A) | 14. (D) | 17. 5.46 | 20. (D) |
| 3. 930 | 6. (A) | 9. (B) | 12. (B) | 15. (C) | 18. (D) | |

## SOLUTIONS

## EXERCISE-1

**1.** $A = \{x / 0 < x < 5 \text{ and } x \in Q\}$

$B = \{x / x \leq 5\}$

We consider the choices one by one.

(1) The greatest element in A is $4.\overline{9}$ and it is more than 4.9999.

(2) The least element in A is 0.00000 ... 1 and it is less than 0.00001.

(3) The greatest element in B is 5.

**2.** Given $p+\sqrt{5}$ is a rational number. $q+\sqrt{7}$ is an irrational number, as $q$ itself is a rational number.

$\therefore (p+\sqrt{5})(q+\sqrt{7})$ is the product of a rational number and an irrational number. This has to be irrational unless $p+\sqrt{5}$ is zero. As it is given that the product is rational,

$p+\sqrt{5}=0$ or $p=-\sqrt{5}$,

$\therefore \sqrt{5}pq=-5q$.

**3.** Every prime number except 2 is odd and if 2 is not present, sum of 4 prime numbers will be even, and hence, it cannot be 63.

**4.** Among the given options, except $\sqrt{2}$, all others are rational numbers.

**5.** If the product of two real numbers is rational such that one of them is a rational number, the other may be rational, if the first is non-zero. But if the first number is zero, the second number may be irrational also. Hence, we cannot say anything about the second number.

**6.** Any of the combinations of six numbers can give an odd sum.

Case (i): 1 odd + 5 even

Case (ii): 3 odd + 3 even

Case (iii): 5 odd + 1 even

**Statement I:** Now consider a case of 1 odd and 5 even. There are no odd products and the sum will be even.

Consider another case of 3 odd numbers and 3 even numbers. From 3 odd numbers, for the product of any two, we will get three odd products and the rest of the products will be even, as they will contain at least one even number. Thus the sum of three odd products and a set of even products is odd.

**Statement II:** Atleast one of the 6 numbers has to be even (from the above cases). Therefore, their product is always going to be even. Therefore options (ii) and (iii) can be eliminated.

**7.** The number of times the numbered keys are used

$= 9 \times 1 + 90 \times 2 + 300 \times 3 = 1089$

**8.** Given, $a$ is a prime number less than 100 and $b = a + 2$.

If $a = 2$, then $b = 4$ and in this case only $a$ is prime. But if $a = 3$, then $b = 5$, in this case both $(a, b)$ are primes. This holds true for all the prime numbers which have a gap of 2. In other words, they have to be twin primes. In the first 100 natural numbers, there are eight possibilities for this.

**9. Statement I:** $P$ and $Q$ are both odd integers. As the difference between two odd integers is always even, $P - Q$ is even.

As $R$ is an even integer, $P - Q - R$ will be the difference of two even integers. As the difference of two even integers will always be even, $P - Q - R$ is even.

The product of two or more integers is even, when at least one of them is even. As $P - Q - R$ is even, $(P - Q - R)(P + R - S)$ is even. Hence, Statement I is true.

**Statement II:** $P$ is odd and $R$ is even. The difference of an odd integer and an even integer is always odd. Hence, $P - R$ is odd.

$Q$ is odd and $S$ is even. The sum of an odd integer and an even integer is always odd. Hence, $(P - R)(Q + S)$ is the product of two odd integers. The product of two or more integers is always odd when all of them are odd. Hence, $(P - R)(Q + S)$ is odd. Hence, Statement II is false.

**Statement III:** $P$ and $Q$ are both odd. The sum of two odd integers is always even. Hence, $P + Q$ is even. As the difference of two even integers is always even, $R - S$ is always even. As the product of two or more integers is always even when at least one of them is even, $(P + Q)(R - S)$ is even.

Hence, Statement III is false.

**Statement IV:** As both $P$ and $Q$ are odd, $P + Q$ is even. As both $R$ and $S$ are even, $R - S$ is even. Hence, Statement IV is true.

**NOTE**

After solving Statement I, which is true, from the choices. Choice (C) or choice (D) is true. By solving only one of the statements we can arrive at an answer. i.e., if Statement II is true, then choice (C) is the answer, else choice (D).

**10.** **Statement I:** As all the numbers from $A, B, C, D \ldots J$ are odd, $2A + 3B + \ldots + 11J$ is the sum of 5 even numbers ($2A, 4C, 6E, 8G$ and $10I$) and 5 odd numbers ($3B, 5D, 7F, 9H$ and $11J$).

Hence, the sum of the numbers is the sum of 5 evens and 5 odds. As this is odd, sum of all the numbers will be odd.

**Statement II:** $5A + 10B + 15C + 20D + 25E + 30F + 35G + 40H + 45I + 50J$

$\Rightarrow 5\ (A + 2B + 3C + \ldots + 10J)$. This will be odd if $A + 2B + 3C + \ldots + 10J$ is odd. As $A + 3C + 5E + 7G + 9I$ is the sum of 5 odds and $2B + 4D + 6F + 8H + 10J$ is the sum of 5 evens, the overall sum is odd

**Statement III:** As all the numbers from $A$ to $J$ are odd, $A + B + C + D + \ldots J$ is the sum of 10 odd numbers, i.e., even. Hence, statement III is true.

**11.** As the product of the natural numbers $A, B, C, D \ldots J$ is even, at least one of them must be even.

**Statement I:** If an even number of numbers from $A$ to $J$ are even, the sum of all the numbers is even otherwise their sum is odd. Hence, Statement I is not necessarily true.

**Statement II:** If $A - B, B - C, C - D, D - A$, are all odd, i.e., when $A, C, E$, are odd, while the other numbers are even, their product is odd. Hence, Statement III is not necessarily true.

**12.** $2n - 1$ is always odd. Hence, $P + Q + R$ is always odd. Hence, $P, Q$ and $R$ must all be odd or two of $P, Q$ and $R$ must be even and the third odd. As $P, Q$ and $R$ are distinct primes, this is not possible.

**Statement I:** As $P$ and $R$ are of the same parity, $P - R$ is even. Hence, Statement I is false.

**Statement II:** As $P, Q$ and $R$ are odd, $P + Q - R$ will always be odd. Hence, Statement II is true.

**Statement III:** If $R$ is of the form $2m + 1$, $R$ must be odd. As $P, Q$ and $R$ are odd, Statement III is true.

**Statement IV:** $P, Q$ and $R$ are all odd.

$\therefore$ PQR is odd. Statement IV is true.

**13.** From the given information, in every round all the multiples of a prime number are painted. As there are 18 prime numbers in the first 64 natural numbers, the whole process is completed in 18 rounds.

**14.** $p, q, r$ are integers so either all of them can be even or odd or two out of three can be even and one can be odd or two out of three can be odd and one can be even.

In all the cases, at least one of the three expressions, $p + q, q - r$ and $p + r$ is even. Hence, the expression $(p + q)\ (q - r)\ (p + r)$ will be definitely even and can never be odd.

**15.** The sum of the four distinct prime numbers = $12.25 \times 4 = 49$. As the sum of even number of prime numbers is given as odd, all cannot be odd. Hence, one of them must be even, i.e., 2

## EXERCISE-2

**1.** Given that $y \geq y^3$, so $0 \leq y \leq 1$, or $y \leq -1$. So, choice (C) is always true.

**2.** $1/y = 1/0.5 = 2$

$y^{1/5} = (1/2)^{1/5} = \dfrac{1}{\sqrt[5]{2}}$

As $\sqrt[5]{2} > \sqrt[5]{1}$

$\dfrac{1}{\sqrt[5]{2}} < \dfrac{1}{\sqrt[5]{1}} \Rightarrow \dfrac{1}{\sqrt[5]{2}} < 1$

$y^{-1/5} = \sqrt[5]{2} > \sqrt[5]{1}$

$\sqrt[5]{2} < \sqrt[5]{32} \Rightarrow \sqrt[5]{2} < 2$

$y^5 = (1/2)^5 = 1/32$, which is less than 1.

Hence, the maximum of the options is $1/y$.

**3.** As $\dfrac{1}{y^2} < -\dfrac{1}{y^5}$, multiplying both sides by $y^2$,

$1 < -\dfrac{Y^2}{Y^5} \Rightarrow \dfrac{-1}{Y^3} > 1.$

This is only possible when $-1 < y < 0$.

**4.** Given $y^3 > y^6$, since $y^6$ is always positive $y^3$ cannot be negative, hence, $y > 0$. If $y > 0$, $y^3 \leq y^6$

for all $y \geq 1$ and $y^3 > y^6$ only for $y < 1$.

$0 < y < 1.$ (1)

Similarly given $x > x^2$, since $x^2$ is always positive $x$ cannot be negative, hence, $x > 0$.

Using similarly reasoning as above $0 < x < 1$ (2)

Hence, both $x$ and $y$ are positive fractions. Now, since it is also given that $y^2 < x^2$,

it can be concluded that $|y| < |x|$ (3)

But as already observed in (1) and (2) $x$ and $y$ are both positive.

Hence, (3) $\Rightarrow y < x$ (3)

$\therefore$ From above $0 < y < x < 1$.

**5.** As $y^2 > y^3$, $-\infty < y < 1$

As $y^3 > y$, $-1 < y < 0$

In order to satisfy both $y^2 > y^3$ and $y^3 > y$, $-1 < y < 0$.

**6.** $y^4 > y^2 \Rightarrow y^2 > 1$

$\Rightarrow y^2 - 1 > 0$

$\Rightarrow (y + 1)\ (y - 1) > 0$

i.e., $y > 1$ or $y < -1$ $\therefore y > 1$.

Hence, choice (B) is true.

**7.** (i) As $y^6 > y^5$, $\Rightarrow -\infty < y < 0$ or $1 < y < \infty$ (1)

As $y > y^5$, $-\infty < y < -1$ or $0 < y < 1$ (2)

From (1) and (2), we get $-\infty < y < -1$ (3)

Also, As $y^5 < y^4$, $-\infty < y < 0$ or $0 < y < 1$ (4)

From (3) and (4), we get $-\infty < y < -1$.

(ii) I. $-\infty < x < -1$

say $x = -2$

$\Rightarrow x^4 > x^3 \because 16 > -8$ and $x^3 < x^2 \because -8 < 4$

I satisfies

II. $-1 < x < 0$

Say $x = -\frac{1}{2} \Rightarrow x^4 > x^3 \because \frac{1}{16} > -\frac{1}{8}$

and $x^3 < x^2 \because -\frac{1}{8} < \frac{1}{4}$

$\therefore$ II also satisfies. $\therefore$ From the choices I and II are true. Using numerical examples the other ranges in III and IV can be shown to not satisfy both the conditions simultaneously.

**8.** When $\frac{1}{x} > \frac{1}{y}$, the following possibilities show the relationship between $x$ and $y$.

Case 1:

Let $x = 2$, $y = 3$ $\frac{1}{2} > \frac{1}{3}$

In this case $x < y$. This is true when both $x$ and $y$ are negative also.

Case 2:

Let $x = 2$, $y = -3$

$\frac{1}{2} > \frac{1}{-3}$

In this case, $x > y$.

Hence, it is possible that $x < y$ or $x > y$.

**9.** $x^2 - x > 0$

$x(x - 1) > 0$

$x > 0$ and $x - 1 > 0$ or $x < 0$ and $x - 1 < 0$

$\Rightarrow x > 0$ and $x > 1$ or $x < 0$ and $x < 1$

$\Rightarrow x > 1$ or $x < 0$. $x$ cannot lie between 0 and 1, i.e., it cannot be a positive proper fraction.

**10.** (i) Given $y^5 > y^4$ or $\frac{1}{y^3} < \frac{1}{y^5}$

$y^5 > y^4$ only for $y > 0$ (since $y^4$ is always + ve) also, $y$ cannot be a fractional value, i.e., $y$ cannot lie between 0 and 1.

$\Rightarrow 1 < y < \infty$ (1)

If $\frac{1}{y^3} < \frac{1}{y^5}$, then assuming that both LHS and RHS are positive $y^3 > y^5$. This is true only for $0 < y < 1$ (2)

If one assumes that both L.H.S. and R.H.S. are negatives, then $y^3 > y^5$, but only when $y$ does not be between $-1$ and 0, i.e., $-1 > y > -\infty$ (3)

Hence, from (1) (2) and (3), we get the ranges I, III and IV as possible.

(ii) I. $0 < x < 1$

say $x = 0.5$; then $x^2 > x^3$

II. $-1 < x < 0$

say $x = -0.5$; then $x^2 > x^3$

III. $-\infty < x < -1$

say $x = -2$; then $x^2 > x^3$

$\therefore$ All three ranges are possible.

## EXERCISE-3

**1.** Let the number of trees in each row be $N$.

Number of rows of apple trees + number of rows of mango trees + number of rows of banana trees

$= 180/N + 216/N + 324/N$

This will be minimum when $N$ is maximum.

As the total number of rows is an integer, $N$ must be a factor of 180, 216 and 324.

Thus $N$ must be the H.C.F. of 180, 216, 324 = 36

Hence, the minimum number of rows

$= 180/36 + 216/36 + 324/36 = 5 + 6 + 9 = 20$

**2.** The possible number of soldiers has to be a multiple of the L.C.M. (12, 16, 20), i.e., = 240. It can only be 480.

**3.** We have the following definitions for L.C.M. and H.C.F. of fractions.

L.C.M. of fractions = L.C.M. of the numerators/H.C.F. of the denominators

H.C.F. of fractions = H.C.F. of the numerators/L.C.M. of the denominators

Hence, L.C.M. of 5/8, 15/16 and 25/36

= L.C.M. (5, 15, 25)/H.C.F. (8, 16, 36) = 75/4

H.C.F. of 5/8, 15/16 and 25/36

= H.C.F. (5, 15, 25)/L.C.M. (8, 16, 36) = 5/144.

**4.** $39 = 3 \times 13$

$65 = 5 \times 13$

$143 = 11 \times 13$

The L.C.M. of these three numbers is the number with the highest power of each prime factor in the numbers. Hence, the L.C.M. $= 3 \times 5 \times 11 \times 13 = 2145$

**5.** Let the numbers be $x$ and $y$

H.C.F. $(x, y)$. L.C.M. $(x, y) = x.y$

$4.48 = x.y$

$x.y = 192$

As H.C.F. = 4, each of $x$ and $y$ must be divisible by 4.

Let $x = 4a$ and $y = 4b$

$(4a)\ (4b) = 192$

$ab = 12$

$(a, b) = (1, 12), (2, 6), (3, 4), (4, 3)\ (6, 2)$ or $(12, 1)$

But H.C.F. $(x, y)$ will be 4 only when $a$ and $b$ are co prime.

$\therefore\ (a, b) = (1, 12), (3, 4), (4, 3)$ or $(12, 1)$

The numbers are (4.1 and 4.12) or (4.3 and 4.4) two pairs are there.

**6.** $786000 = 786 \times 53\ x\ 23$

$= 3 \times 262 \times 53 \times 23 = 3 \times 131 \times 24 \times 53$

Hence, 786000 has 4 distinct prime factors.

**7.** The three bells will toll after a time period given by the L.C.M. of the respective time periods of each bell.

$\therefore$ They will toll after L.C.M. (5, 4, 6) = 60 minutes.

Hence, at 11:39 a.m. they toll together for the first time.

**8.** Since the number of boxes needed is minimum, the number of fruits in each box must be maximum

So, the number needed is the highest factor common to 144, 198 and 252.

i.e., H.C.F. (144, 198 and 252) = 18

$\therefore$ The boxes needed $= \dfrac{144 + 198 + 252}{18}$

$= 8 + 11 + 14 = 33$

**9.** As $x$, $y$ are co-primes, they have to be 1, 24 or 3, 8. Only when they are 8, 3, is L.C.M. $(2x, 3y)$ equal to 144.

$\therefore\ x + y = 11.$

**10.** Given, L.C.M. = 3672 and H.C.F. = 18

As the H.C.F. is 18, the numbers will be the multiples of 18.

$(a16)$ and $(30b)$ are multiples of 18.

$\therefore\ a16 = 216$ and $30b = 306$.

$\Rightarrow a = 2$ and $b = 6$

**11.** Since the H.C.F. of the two numbers is 7, we have $7x$ and $7y$ as the two numbers where $x$ and $y$ are co-primes. $7x - 7y = 7(x - y) = 21$, $x - y = 21/7 = 3$, $x = y + 3$

The L.C.M. of the two numbers is $7xy = 196$

$xy = 196/7 = 28 \Rightarrow (y + 3)y = 28 \Rightarrow y2 + 3y = 28$

$y2 + 3y - 28 = 0 \Rightarrow (y + 7)\ (y - 4) = 0$

Since $y$ can't be negative, $y = 4$

$x = 28/y = 28/4 = 7$

Hence, the larger of the two numbers is $7x = 7 \times 7 = 49$.

**Alternate method:**

Going by the options, option (A) says the larger number is 28. The smaller number would then be 28 – 21 = 7. L.C.M. of 28 and 7 is 28. Option (B) says the larger number is 35. Since 196 is not a multiple of 35, option (B) is ruled out. Option (C) says the larger number is 42; smaller number would then be 42 – 21 = 21.

L.C.M. of 42 and 21 is 42. Hence, not possible.

Option (D) says the larger number is 49. Smaller number would then be 49 – 21 = 28.

L.C.M. of 49 and 28 is 196.

**12.** Weight of each piece (in kg)

$$= \text{H.C.F.}\left(6\frac{1}{8}, 10\frac{1}{2}, 8\frac{3}{4}, 3\frac{15}{16}\right)$$

$$= \text{H.C.F.}\left(\frac{49}{8}, \frac{21}{2}, \frac{35}{4}, \frac{63}{16}\right) = \frac{\text{H.C.F.}(44, 21, 35, 63)}{\text{L.C.M.}(8, 2, 4, 16)} = \frac{7}{16}$$

$$\text{Number of pieces obtained} = \frac{\frac{49}{8} + \frac{21}{2} + \frac{35}{4} + \frac{63}{16}}{\frac{7}{16}} = 67$$

**13.** If side of each identical square tile is $x$, then the number of tiles required

$$= \frac{\text{Area of the floor}}{\text{Area of each squaretile}} = \frac{870\text{ cm} \times 638\text{ cm}}{x \times x}.$$

The number of identical square tiles will be minimum if area of each identical square tile ($x^2$) is maximum. Also, to completely each of the cover the floor, the side of the tile should be a factor of the dimensions of the room.

$\Rightarrow x$ must be the H.C.F. of 870 and 638

$\therefore x = 58$

Hence, minimum number of identical square tiles

$$= \frac{870\text{ cm} \times 638\text{ cm}}{58\text{ cm} \times 58\text{ cm}} = 165$$

**14.** Let the L.C.M and H.C.F of $a$ and $b$ be $l$ and $h$, respectively.

Given $l - h = 57$.

Let $k$ times the $h$ be $l$

$\therefore kh - h = 57 \Rightarrow h(k-1) = 57$

Now, 57 can be expressed as product of two numbers in the following ways.

(i) (1, 57) (ii) (57, 1)

(iii) (3, 19) (iv) (19, 3)

Here, (57, 1) and (19, 3) can be eliminated because, the higher the H.C.F., the higher is the sum and minimum sum is required.

From (i), H.C.F. = 1 and L.C.M.= 58

$\therefore (a, b)$ can be (1, 58) or (2, 29)

From (III), H.C.F. = 3 and L.C.M. = 60

$\therefore (a, b)$ can be (12, 15) or (3, 60)

$\therefore$ Minimum possible sum for $a$ and $b$ is 12 + 15 = 27

**15.** Let the numbers be $3x$, $4x$ and $5x$.

L.C.M. $(3x, 4x, 5x) = x$ L.C.M. $(3, 4, 5) = 60x$

Given $60x = 480$

$x = 8$

sum $3x + 4x + 5x = 12x = 96$

## EXERCISE-4

**1.** Consider the first 40 natural numbers.

In this there are 20 even numbers whose sum is twice the sum of the first 20 natural numbers.

$\therefore$ sum of first 20 odd numbers

= sum of the first 40 natural numbers

–2 (sum of the first 20 natural numbers)

$$= \frac{40 \times 41}{2} - \frac{2 \times 20 \times 21}{2} = 820 - 420 = 400.$$

**Alternate Method:** Sum of first $n$ odd numbers is given by $n2$.

$\therefore$ Required sum is 202 = 400.

**2.** The smallest perfect square greater than 4000 is $64^2 = 4096$

The greatest perfect square lesser than 7000 = $83^2 = 6889$

$\therefore$ The sum of all the perfect squares lying between 4000 and 7000 is

$$\sum_{n=64}^{83} n^2 = \sum_{n=1}^{83} n^2 - \sum_{n=1}^{63} n^2$$

$$= \frac{1}{6}(83 \times 84 \times 167) - \frac{1}{6}(63 \times 64 \times 127)$$

$= (83)(14)(167) - (21)(32)(127)$.

The last digit of the product (14) (83) (167) as well as (21) (32) (127) is 4. Hence, the units digit of the result

must be 0. As only choice (A) ends with 0, choice (A) is the answer.

**Alternate Method:** Units digit of sum of the squares of any 10 consecutive numbers is always five. That way we can say (642 + 652 + … 732) will end with a five. Same is the case with (742 + 752 + … + 832) and the sum of two numbers ending with five will have zero in its units place. Hence, Choice (A) has to be the answer.

**3.** Sum of the first 30 even numbers = 2 + 4 + 6 + 8 + … + 60
= 2(1 + 2 + 3 + 4 + … + 30)

Sum of the first '$n$' natural numbers = $n(n+1)/2$

So, 2 (1 + 2 + 3 + 4 + … + 30) = 2(30) (31)/2 = 930

**4.** The sum of the cubes of first $n$ natural numbers is given by $\left[\frac{n(n+1)}{2}\right]^2$. This is obviously a perfect square.

No perfect square ends with 2. ∴ 9252 cannot be a perfect square.

∴ Sum of the cubes of first $n$ natural numbers cannot be 9252.

**5.**

| | | | | | |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 7 | 8 | 9 | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 | 17 | 18 |
| 19 | 20 | 21 | 22 | 23 | 24 |
| 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 |

Number of squares of size 1 × 1 cell = 6 × 6 = 36

Number of squares of size 2 × 2 cells = 5 × 5 = 25 (because two continuous unit cells along the length can be chosen in 5 ways and similarly two continuous unit cells along the breadth can be chosen in 5 ways).

By similar calculations, Total number of squares visible
= 36 + 25 + 16 + 9 + 4 + 1 = 91

**6.** $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 16 - 2 = 14$

**7.** The sum of the squares of all the odd numbers in the first 60 natural numbers =
(12 + 22 + 32 + ... + 602) – (22 + 42 + 62 + ... + 602)
= (60) (61) (121)/6 – 4 (30) (31) (61)/6 = 35990

**8.** The sum of the odd page numbers is 196 which is 142, i.e., sum of the first 14 odd numbers. Hence, the sum of the remaining pages should be the sum of the first 14 even numbers, i.e., 14 (14 + 1) = 210.

**9.** Sum of the squares of the first 20 even natural numbers
= 22 + 42 + 62 + … + 402
= 22(12 + 22 + … + 202)
= 4 × (20 × 21 × 41)/6
280 × 41 which is equal to 11480

**10.** $x$ = 32 + (2 + 4 + 6 +…+ 34)
= 32 + 2(1 + 2 + 3 +…+ 17)
$y$ = 19 + (1 + 2 + 3 +…+ 17)
$x - y$ = 13 + (1 + 2 + 3 +…+ 17)
= 13 + 153
= 166

**11.** Let the number be $N$. Let the quotient obtained, when the number is divided by 54, be $q$.

$N = 54q + 31$

When $N$ is divided by 27, the quotient is $2q + 1$ and the remainder is 4.

**12.** Let the two numbers be $N_1$ and $N_2$. Let the divisor be $D$.

$N_1 = Q_1D + 248$ (1)

$N_2 = Q_2D + 372$ (2)

Adding (1) and (2), we get
$N_1 + N_2 = D(Q_1 + Q_2) + 620 = D\left(Q_1 + Q_2 + \frac{552}{D}\right) + 68$

As $Q_1 + Q_2 + \frac{552}{D}$ must be an integer, $D$ must be a factor of 552. As any divisor is greater than the remainder, $D >$ 372 (from (1)), $D = 552$.

Hence, 552 is the divisor.

**13.** A number when divided by $D$ leaves a remainder of $r$. The possible remainders when the same number is divided by $nD$ are $r$, $(D + r)$, $(2D + r)$, $(3D + r)$, ..., $[(n-1)D + r]$.

Let the number be $N$. Let the remainder left when $N$ is divided by $D$ be represented by Rem $\left(\frac{N}{D}\right)$.

Given that Rem $\left(\frac{N}{D}\right) = 7$ (1)

and Rem $\left(\frac{N}{3D}\right) = 20$ (2)

Rem $\left(\frac{N}{3D}\right)$ could be either $(D + 7)$ or $(2D + 7)$

Since $2D + 7 \neq 20$ when $D$ is a natural number, $D + 7 = 20$

$\Rightarrow D = 13$

$\therefore 3D = 39.$

$\text{Rem}\left(\frac{2N}{3D}\right) = \text{Rem}\frac{40}{3D} = \text{Rem}\frac{40}{39} = 1$ (from (2))

**14.** Let $N$ be the number.

Given that $\text{Rem}\left(\frac{N}{D}\right) = 9$ (1)

and $\text{Rem}\left(\frac{N}{3D}\right) = 35$ (2)

$\Rightarrow$ Either $D + 9 = 35$ or $2D + 9 = 35$.

$\Rightarrow D = 13$ or $26$

$\Rightarrow 3D = 39$ or $78$

$\text{Rem}\left(\frac{2N}{39}\right) = \text{Rem}\left(\frac{2 \times 35}{39}\right) = 31$

$\text{Rem}\left(\frac{2N}{78}\right) = 2 \times (35) = 70$

$\therefore \text{Rem}\left(\frac{2N}{3D}\right) = 31$ or $70$.

**15.** Let the number be $N$ and the divisor be $d$.

$N = dK + 11$

Now $N^2 = N.N$

$= (dK + 11)\ (dK + 11)$

$= d^2K^2 + 2.11dK + 120 + 1$

$= d(dK^2 + 22K) + 120 + 1 = dk_1 + 1$

So, 120 is divisible by the divisor, therefore the divisor must be a factor of 120.

$120 = (3)\ (5)\ (23)$

120 has 16 factors.

The divisor must be a factor of 120 greater than 11.

The divisor can take any of the following 8 values.

12, 15, 20, 24, 30, 40, 60 and 120

**16.** The given expression is of the form

$$= \frac{a^3 + b^3 - c^3 + 3abc}{a^2 + b^2 + c^2 - ab + bc + ca}$$

where $a = 0.68$, $b = 0.67$ and $c = 0.5$

$$\frac{a^3 + b^3 - c^3 + 3abc}{a^2 + b^2 + c^2 - ab + bc + ca} = a + b - c$$

$= 0.68 + 0.67 - 0.5 = 0.85$

**17.** $\frac{10.23^3 - 4.77^3}{10.23^2 + 4.77^2 + (10.23)(4.77)} = 10.23 - 4.77 = 5.46$

$\left(\because \frac{a^3 - b^3}{a^2 + b^2 + ab} = a - b\right)$

**18.** We know that, $(a + b)^2 + (a - b)^2 = 2\ (a^2 + b^2)$

$\therefore [(3.7) + (5.2)]^2 + [(3.7) - (5.2)]^2 = 2[(3.7)^2 + (5.2)^2]$

$\therefore$ Given expression becomes $\frac{2\left[(3.7)^2 + (5.2)^2\right]}{(3.7)^2 + (5.2)^2} = 2$

**19.** 1742 is divisible by 13.

Required remainder = Remainder of (1742 + 8) (1742 + 10) (1742 + 12) divided by 13 = Remainder of (8) (10) (12) divided by 13 = 11.

**20.** $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$= (10)^3 - 3 \times 9 \times 10$

$= 1000 - 270 = 730$

# 2 Speed Maths

## ❒ IMPORTANCE OF CALCULATION SPEED

Calculation speed plays a very important role in almost all the competitive exams—more so in MBA entrance exams. Some people have the natural ability to do calculations fast but, those of us who do not have good calculation speeds need not envy such people for their inborn talent. It is very easy to develop good calculation speed in a relatively short period of time. All it requires is taking care of one basic factor—that is spending a certain amount of time regularly practising calculations.

**How does one improve calculation speed?**

Spend just about 15 minutes a day over a three to six month period on calculation practice and you will find the difference in your calculation speed. The practice involves basic additions, subtractions, multiplications, percentage calculations, comparing fractions and calculating squares.

This practice does not need any material in the form of printed exercises or test papers. Take any figures that you can think of and work out the calculations (additions, subtractions, multiplications, etc.) mentally. What you should certainly try to ensure is that you are doing the calculations mentally wherever possible. Put away your calculators and avoid doing your calculations on paper to the extent possible.

**What does this chapter consist of?**

While you can always take figures at random for the purpose of practicing calculations mentally, in this chapter, we have put together a number of exercises which you can use for calculation practice.

**How to gain from this chapter:**

Before you move to the next unit, make sure that you revise the techniques discussed in this chapter. You should also make sure that you are thorough with the following:

Multiplication Tables (up to $20 \times 10$)
Squares (up to 25)
Cubes (up to 12)
Powers of 2 (up to 12)
Powers of 3 (up to 6)
Reciprocals of numbers (up to 12)
Complements of 100 (i.e., the difference between 100 and the given two-digit number).

While taking each exercise/test paper, please follow the instructions given below:

1. Check the starting time and keep in mind the time that has been allotted for that particular exercise.
2. Do not use a calculator.
3. Write as little as possible on paper. You should try doing as much of the calculation as possible mentally.
4. If you have to do rough work, do it in the book on the same page as the question that you are answering and not at any other place in the chapter.
5. Some questions require precise calculations whereas some other questions require only approximate calculations. Please remember that the level of accuracy to which you should work out the calculations will depend on the answer choices given in the question paper. So, do not spend more time than is necessary on each question.

6. Stop the exercise/test as soon as the prescribed time is over.
7. After you complete each exercise, spend time working out the questions that you could not complete in the given time. Then, check for the correctness of your answers. Rework all the questions in the test to see whether the method that you adopted was the best/shortest.
8. Even after you use up all the exercises given in this booklet, you should continue similar calculation practice on a regular basis to ensure that your calculation speed does not drop.

For any of the MBA entrance and similar other exams you will be appearing for, there are three areas that you have to take care of:

1. **Knowledge:** It is essential to have a certain level of knowledge in every area. It is not that a very high level of knowledge is required. A tenth or twelfth standard student should be able to answer these papers very comfortably, but nevertheless, some minimum level of knowledge is required.
2. **Speed:** One very important factor which determines success in MBA entrance exams is speed. The number of questions one can attempt correctly makes all the difference between the one who gets selected and the one who does not get selected. Speed in all areas of these exams is very important.
3. **Approach:** Knowledge alone is not sufficient to do well in these exams. For example, you cannot afford to leave out 20 questions out of 30 in a section and still hope to get selected. A person who does not take care of all the areas may not get through. This is where what we refer to as 'approach' is important in tackling the test papers. When you take comprehensive test papers, we will discuss this issue of 'approach' to test-taking.

Here, we will discuss the second of the three aspects mentioned above—speed. We will concentrate on certain speed methods of calculations which will be of great use to you in most of these exams.

As far as calculations are concerned, these exams do not allow the use of calculators or any other calculating aids. The ability to perform calculations faster is an advantage and you will solve more questions than the others in the given time. Even in your day-to-day work where you need to perform calculations, try not to use a calculator. This is a habit that you have to cultivate. If you continue using calculating aids like calculators, it is difficult to improve your calculation speed. However, please remember that any of the methods discussed in this chapter are useful only if you practice these methods regularly as well as consciously use such methods in calculations in your day-to-day work also.

## ❒ ADDITIONS, SUBTRACTIONS AND MULTIPLICATIONS

In this chapter, we will show you a number of calculations and take you through the different steps involved in each of the calculations. These steps are put down on paper here for the purpose of explanation but, when you are performing the calculations, you should do all these steps *mentally*.

### ❒ Some Ways of Simplifying Calculations

1. For multiplication by 5, you should multiply the figure given by 10 and then divide it by 2.
   E.g., $6493 \times 5 = 64930/2 = 32465$. This is a very simple method. You may feel that adopting this method will only save 5 seconds and wonder how you will benefit by it. If you adopt such methods at a number of places in the full paper and you can save even 4 to 5 minutes it will help you attempt at least 4/5 more questions. This itself may make all the difference to your chances of selection.
2. For multiplication by 25, you should multiply the figure given by 100 and divide it by 4. E.g., $6493 \times 25 = 649300/4 = 162325$.
3. For multiplication by 125, you should multiply the figure given by 1000 and divide by 8, e.g., $6493 \times 125 = 6493000/8 = 811625$.
   Alternatively, you can treat 125 as 100 + 25. So, multiplication by 125 can be treated as multiplication by 100 and add to this figure one-fourth of itself (because 25 is one-fourth of 100).
4. For multiplication by 11, the rule is 'for each digit add the right hand digit and write the result as the corresponding figure in the product'. For the purpose of applying the rule, it will be easier if you assume that there is one 'zero' on either side of the given number. E.g., $7469 \times 11 \rightarrow 0|7469|0 \rightarrow 82159$.

5. For multiplication by 12, the rule is 'double each digit and add the right hand digit and write the result as the corresponding digit of the product' e.g., 0|7469|0 × 12 = 89628.

   The carry forward digit has to be added to the subsequent step for multiplication by 11 or 12.
6. For multiplication by 13, the rule is 'three times each digit added to the right hand digit gives the corresponding digit in the product'. E.g., 0|92856|0 × 13 = 1207128.
7. Multiplication by 19, can be treated as multiplication by (20 – 1); e.g., 92856 × 19 = 92856 × 20 – 92856 = 1764264

   The important point to note here is that all the above calculations, after one or two examples each, should be done orally, and hence, the students also should practise accordingly. Only when large numbers are dealt with should the student put part of the figures on paper.

## ❒ Multiplying Two Numbers Both of Which are Close to the Same Power of 10

Suppose we want to multiply 97 with 92. The power of 10 to which these two numbers are close is 100. We call 100 as the base. Write the two numbers with the difference from the base, i.e., 100 (including the sign) as shown below.

97 → –3 (because 97 is obtained as 100 – 3)

92 → –8 (because 92 is obtained as 100 – 8)

Then, take the sum of the two numbers (including their signs) along either one of the two diagonals (it will be the same in both cases). In this example, the diagonal sum is 97 – 8 = 92 – 3 = 89. This will form the first part of the answer.

The second part of the answer is the product (taken along with the sign) of the difference from the power of 10 written for the two numbers – in this example, it is the product of –3 and –8 which is 24.

Hence, putting these two parts 89 and 24 together one next to the other, the answer is 8924, i.e., the product of 97 and 92 is 8924.



**NOTE**

The product of the two deviations should have as many digits as the number of zeros in the base. For example, in this case, the product of –8 and –3 has 2 digits which is the same as the number of zeroes in 100.

**Two-digit/three-digit multiplication method:** The usual process of multiplying two digit and three digit numbers is time consuming. For example, consider the multiplication 234 × 186 = 43524

$$\begin{array}{r} 234 \\ 186 \\ \hline 1404 \\ 1872 \\ 234 \\ \hline 43524 \end{array}$$

In the above method, we observe that in order to find the product of 234 and 186, which is 43524, we wrote three steps (1404, 1872, 234) that are not required. By avoiding these steps we could have saved some amount of time. The amount of time saved may be only 10 seconds per calculation. However, as there will be a large number of such calculations in the exam, you will end up saving a significant amount of time by using this method.

## ❒ FRACTIONS AND PERCENTAGES

While solving questions on simplification, sometimes, we may come across simplification of fractions. Simplification of fractions may involve addition, subtraction, multiplication and division. In Addition as well as Subtraction of fractions, we may come across fractions with different denominators. In such cases, the denominators are to be made equal by converting the denominators to their L.C.M.

## ❒ RECIPROCALS AND ITS MULTIPLES

We come across a number of calculations of percentages in data interpretation and in some parts of quant. To do the calculations faster, if we can remember the reciprocals and its multiples, then we can do the calculations at a faster rate. For example, if we want to calculate 37.5% of 896, we can do it faster if we remember 37.5% (as 3/8) = 3/8 × 896 = 3 × 112 = 336.

The important reciprocals are from 1/2 to 1/12 and their multiples. Once we memorize these, upto 12, remembering its multiples is not that difficult. For example,

1/ 8 = 12.5 %; 2/8 = 2 × 1/8 = 2 × 12.5% = 25%; 3/8 ⇒ 3 × 1/8 = 3 × 12.5 = 37.5%; 4/8 ⇒ 4 × 1/8 = 4 × 12.5 = 50% or 4/8 = 1/2 = 50%; 5/8 ⇒ 5 × 1/8 = 5 × 12.5 = 62.5%; 6/8 ⇒ 3/4 = 75%; 7/8 ⇒ 7 × 1/8 = 7 × 12.5 = 87.5%.

Similarly, we can remember all the multiples of reciprocals upto 12.

Conversion of fractions to percentages

1/2 = 50%, 1/3 = 33.33%, 1/4 = 25%,
2/3 = 66.66%, 3/4 = 75%,

1/5 = 20%, 1/6 = 16.66%, 1/7 = 14.28%
2/5 = 40%, 5/6 = 83.33%, 2/7 = 28.57%,
3/5 = 60%, 3/7 = 42.85%,
4/5 = 80%, 4/7 = 57.13%,
5/7 = 71.42%,
6/7 = 85.72%,

1/8 = 12.5%, 1/9 = 11.11%, 1/11 = 9.09%,
3/8 = 37.5%, 2/9 = 22.22%, 2/11 = 18.18%,
5/8 = 62.5%, 4/9 = 44.44%, 3/11 = 27.27%,
7/8 = 87.5%, 5/9 = 55.55%, 4/11 = 36.36%,
7/9 = 77.77%, 5/11 = 45.45%,
8/9 = 88.88%, 6/11 = 54.54%,
7/11 = 63.63%,
8/11 = 72.72%,
9/11 = 81.81%,
10/11 = 90.9%,

1/12 = 8.33%,
5/12 = 41.66%,
7/12 = 58.33%,
10/12 = 83.33%,
11/12 = 91.66%

It will be very useful to memorize all the above values as it will help us to do the calculations very fast.

## ❒ Percentage Calculations

In calculating the percentage value of a number, we usually go for multiplication. But that does not give the answer easily and quickly in most cases. Hence, an easier method called 10% concept, is suggested. In this approach, we take 10% of the denominator. To get close to the answer take further values like 1% and 0.1%.

For example: 23% = 10% × 2 + 1% × 3

43.2% = 10% × 4 + 1% × 3 + 0.1% × 2.

The following is the illustration of the same.

How to calculate the value of 36% of 1325?

Here, explain the concept of 10% and 1%. Therefore, for any value, say 1264, 10% of the value is obtained by simply shifting the decimal point by one place (or digit) to the left. Note that 1264 = 1264.0

10% of 1264.0 = 126.40 (i.e., the decimal point moves to the left by one place (or digit)). Similarly, 1% of 1264.0 will be obtained by shifting the decimal point by two places to the left. Hence, 1% of 1264.0 = 12.640.

Hence, 36% of 1325 = (40% – 4%) of 1325 = (4 × 10% – 4 × 1%) of 1325 = (4 × 132.5 – 4 × 13.25) = 530 – 53 = 477.

Similarly, consider another example, say, 18% of 3250 = (20% – 2%) of 3250

= (2 × 10% – 2 × 1%) of 3250 = (2 × 325 – 2 × 32.5) = 585.

If there is a 10% increase, then the new value will become 1.1 times the old value and in general if there is an increase of $p$%, the new value will become $\left(1 + \frac{p}{100}\right)$ times the old value. But sometimes converting the percentage into fraction maybe easier than this if there is an increase of 33.33%, then the new value will be 4/3 times the old value. Calculating in this way converting $33^1/_3$ into a fraction and simplifying is faster.

Whenever percentage increase cannot easily be converted into a convenient fraction, then the approximate percentage increase $p$, in integer form, must be found and then 1.$p$ has to be used.

## ❒ COMPARISON OF FRACTIONS

Comparison of fractions will be required in a number of problems in Data Interpretation and Quantitative Ability. Let us study some of the common methods of identifying out the largest or smallest of a given set of fractions.

**Type 1:** When two or more fractions have the same numerators and different denominators, the fraction with the largest denominator is the smallest.

**Type 2:** When the numerators are different and the denominators are same, the fraction with the largest numerator is the largest.

**Type 3:** The fraction with the largest numerator and the smallest denominator is the largest.

**Type 4:** When the numerators of two fractions are unequal, we try and equate them by suitably cancelling factors or by suitably multiplying the numerators. Thereafter we compare the denominators as in Type 1.

**Type 5:** *A*: *For a fraction less than* 1.

If the difference between the numerator and the denominator is same, then the fraction with the larger values of numerator and denominator will be the largest.

*B*: *For a fraction greater than* 1.

If the difference between the numerator and denominator is same, then the fraction with the smaller values will be the largest.

**Type 6:** Another method of comparing fractions is by comparing the percentage changes in denominators and numerators. The important points to remember are that when two fractions are compared, if the percentage increase in the numerator is more than the percentage increase in the denominator (where the first fraction is taken as reference), then the second fraction is greater than the first fraction. Instead, if the percentage increase in the denominator is greater than that in the numerator, then the second fraction is smaller than the first.

## ❒ APPROXIMATIONS

'Approximate calculation' is one of the approaches in solving a problem / arriving at the answer to a question at a faster rate. With the help of approximate calculations, one can save a lot of time and this can be utilized in other areas.

In most cases in various exams, the approach towards a question depends on the answer choices. From the answer choices, one should decide which method to follow—actual calculations or approximate calculations.

In most exams, for solving questions based on simplifications, data interpretation, ratios, percentages, etc., the use of approximations is very handy for solving the question at a faster pace. Approximation in any calculation depends on the degree of accuracy required. The closer the given answer choices, the greater the need for closer approximation.

Example: For which of the following values is the increase the highest?

(a) 3164 to 4072
(b) 2422 to 3218
(c) 4234 to 5866
(d) 1876 to 2761

In order to solve the above question, if we calculate accurately, we will get 4072 – 3164 = 908; 3218 – 2422 = 796; 5866 – 5234 = 632; 2761 – 1876 = 885.

If we try approximate calculations,

Rounded off to 4072 4100

Rounded off to 3164 3200

Here, the subtraction is very simple.

4100 – 3200 = 900.

During the process of rounding off, if the last two digits are 50 or greater than 50, then the figure must be rounded off to the next highest hundred. Otherwise, it should be rounded off to the next lowest hundred. Thus, 3164 would be rounded off to 3200, while 4072 would be rounded off to 4100. In fact, in such calculations, even the hundreds (i.e., the last two zeroes) need not be considered since the two zeroes are present in every case. This means that the above calculation would be further simplified by mentally treating it as 41 – 32 = 9. Other calculations can be done in a similar manner.

In case of multiplications like 389 × 1456, suppose the answer choices are given as below:

(A) 564322 (B) 565400
(C) 566384 (D) 572356

We can go for 390 × 1450 which is 565500 whereas the actual answer here is 566384. This is far from the actual answer. But as none of the answer choices lie between these values, this is the required answer. In the above calculation, only one answer ends with 4; hence, without actually calculating, we can say that Choice (C) is the answer.

Suppose the answer choices are closer, like

(A) 565424 (B) 566644
(C) 566384 (D) 572354

then the above approximation will not be useful. Then, the approach should be 1455 × 389 which gives us 565995. Hence, the answer should be very much close to 565995 but should be more than that which is 566384.

Suppose we have to calculate 37.22% of 1384.

The actual calculation takes around 45 seconds to 75 seconds, depending on the speed of the person.

Suppose the answer choices for the above calculation are mentioned as

(A) 564 (B) 515 (C) 529 (D) 542

As the answer choices are not very close, calculating for 40% and reducing that by 3% may be sufficient for answering the question.

10% of 1384 = 138.4. Four times that is ~554 and 3% is approximately equivalent to 42. Hence, the answer is ~554 – 42, i.e., ~512.

Therefore, the closest answer is 510.

Suppose the answer choices for the above calculation are mentioned as:

(A) 510.264 (B) 515.124
(C) 519.316 (D) 522.356

It is clear that the answers are very close. But if you approximate 37.22% equivalent to 37.5%, you can simply convert the calculation into 3/8 of 1384. As 1384 goes 173 times, the answer is 519. Hence, 519.316 cannot be the answer, as it should be less than 519 but very close to 519, which is 515.124.

If the answer choices are even closer than the above example, we go for subtraction of 28%, which is approximately $1/400^{th}$ part of 1384. Hence, by using approximate calculations we can answer questions at a faster rate.

Now, let us consider another question.

Find the value of $\frac{5843}{31200} \times 100$

(A) 17.56 (B) 18.38 (C) 18.72 (D) 16.96

This ratio can be calculated faster by two very useful methods than by conventional division.

The first approach can be called the '*ten percent method*'. In this method, 10% of the denominator is first obtained by simply shifting the decimal point in the denominator it by one place to the left. Similarly, 1% of the denominator, 0.1%, etc., can also be successively obtained by shifting the decimal point to the left by one more place in each successive step. Then, the numerator is expressed as the nearest possible multiple of 10% of the denominator along with some excess or shortfall.

For example,

10% of 31200 = 3120

and

5843 = 1 × 3120 + 2723 (excess)

Again, the excess of 2723 can be expressed as a multiple of say,

5% of denominator plus some excess.

Half of 10% of 31200 = 5% of 31200 = 1560

Hence, 2723 = 5% of 31200 + 1163

Further, 1163 is slightly less than 4 times (1% of 31200), i.e., 1248.

Thus,

5843 ≅ (10% + 5% + 4%) of 31200 ≅ 19% of 31200.

The correct figure must be slightly less than 19% of 31200. Hence, from the choices, the answer can be Choice (C).

The second approach to quickly calculate the ratio $\frac{5843}{31200}$ is by using of the decimal equivalent values of the reciprocals of the first few natural numbers. In this approach, the numerator and denominator are first approximated as $\frac{5800}{31200}$, which is further approximated to $\frac{5.8}{31.2}$ which is close to $\frac{5.8}{3 \times 10.4}$

(i.e., 4% less than 19.3),

i.e., 18.54. But since in the first approximation we had taken 5800 instead of 5843, the answer has to be slightly more than 18.54. Thus, Choice (C) is the answer.

Find the value of $\frac{6164}{26879} \times 100$

(A) 21.68 (B) 22.16 (C) 22.93 (D) 23.37

By observing the given choices, we understand that the answer should be close to 22.22% (i.e., 2/9) (Here, one should remember reciprocals and their multiples). The calculation is:

$\frac{2}{9} \times 26879 = 5973$. As 5973 is about 190 less than 6164, we need to add about 190 to 5973. But as we are interested in percentages, 190 forms slightly more than 0.5% but less than 1%. The answer should be more than 22.7% but less than 23.22%. From the choices, only choice (C) is satisfied.

Find the value of $(2911 / (3784 \times 4)) \times 100$.

(A) 17.86 (B) 18.15 (C) 21. 76 (D) 19.23

$$\text{Required value} = \frac{\left(\frac{6695 - 3784}{3784}\right) \times 100}{4}$$

This is approximated as:

$$\frac{6700 - 3800}{3800 \times 4} \times 100$$

$$= \frac{2900}{38 \times 4} = \frac{2900}{152}$$

$\frac{2900}{152}$ is slightly less than $\frac{3000}{150} = 20$

Thus, 19.23% is close to 20%. Therefore Choice (D) is correct.

Approximations for divisions can be done in two ways. The first one is cross multiplication. The examples show how to solve an approximation problem using cross multiplication.

Therefore, we understand that approximations are very useful in additions, subtractions, multiplications, divisions, percentage calculations, etc.

## ❒ BODMAS–HIERARCHY OF ARITHMETIC OPERATIONS

To simplify arithmetic expressions, which involve various operations like brackets, multiplication, addition, etc. a particular sequence of the operations has to be

followed. For example, $2 + 3 \times 4$ has to be calculated by multiplying 3 with 4 and the result 12 added to 2 to give the final result of 14 (you should not add 2 to 3 first to take the result 5 and multiply this 5 by 4 to give the final result as 20). This is because in arithmetic operations, multiplication should be done first before addition is taken up.

The hierarchy of arithmetic operations are given by a rule called BODMAS rule. The operations have to be carried out in the order in which they appear in the word BODMAS, where different letters of the word BODMAS stand for the following operations:

(V Vinculum)
B Brackets
O Of
D Division
M Multiplication
A Addition
S Subtraction

There are four types of brackets:

1. **Vinculum:** This is represented by a bar on the top of the numbers. For example,

   $2 + 3 - \overline{4 + 3}$; Here, the figures under the vinculum have to be calculated as 4 + 3 first and the 'minus' sign before 4 is applicable to 7. Thus the given expression is equal to 2 + 3 − 7 which is equal to –2.

2. **Simple Brackets:** These are represented by ( )
3. **Curly Brackets:** These are represented by { }
4. **Square Brackets:** These are represented by [ ]

The brackets in an expression have to be opened in the order of vinculum, simple brackets, curly brackets and square brackets, i.e., [ { ( ¯ ) } ] to be opened from inside outwards.

After brackets is *O* in the BODMAS rule standing for 'of' which means multiplication. For example, 1/2 of 4 will be equal to 1/2 *x* 4 which is equal to 2.

After *O*, the next operation is *D* standing for division. This is followed by *M* standing for multiplication. After Multiplication, *A* standing for addition will be performed. Then, *S* standing for subtraction is performed.

## ❒ Squares and Cubes

In competitive examinations, there can be questions on direct application of squares, cubes, square-roots and cube-roots. For example, there can be a question which asks you to find the tens-digit of a four-digit perfect square. Also, an understanding of squares and cubes of useful while performing calculations.

Remembering squares (upto first 25 natural numbers), cubes (upto first 12 natural numbers) is very important in calculations. By remembering these (squares upto 25), one can calculate squares of any natural number from 26 to 125 in no time, which in turn will help in solving some other questions too. Similarly, by remembering cubes (upto 12) one can calculate cubes of any two-digit number with greater speed. Given below are some methods for finding squares and cubes of numbers.

How to find the square of a number ending in 5:

Getting the square of a number ending in 5 is very simple. If the last digit of the number is 5, the last two digits of the square will be 25. Consider the earlier part of the number and multiply it with one more than itself and that product will be the first part of the answer. (The second part of the answer will be 25 itself.)

$35^2 = 1225$ (Here, $3 \times 4 = 12$, so, the answer is 1225)
$45^2 = 2025$
$55^2 = 3025$
$75^2 = 5625$
$95^2 = 9025$
$125^2 = 15625$
$175^2 = 30625$
$195^2 = 38025$
$235^2 = 55225$
$245^2 = 60025$

So, now we know the squares of numbers 35, 45, 55, 75, etc. If we want to find the square of any other number ending in 5, we can find it using these squares which we already know.

To find the square of a number which is one more than the number whose square we already know:

For $26^2$, we will go from $25^2$; for $31^2$ we go from $30^2$ and so on.

One way is by writing $26^2 = (25+1)^2$. But we need not even calculate $(a + b)^2$ by adopting the following method;

$26^2 = 25^2 + 26^{th}$ odd number, i.e., 625 + 51 = 676
$(a + b)^2 = a^2 + 2ab + b^2$
$26^2 = (25 + 1)^2$
$(25)^2 + 2(25 \times 1) + (1)^2$
625 + 50 + 1 = 625 + 51 = 676

But we will look at a different method which will enable the student perform the calculations for squares mentally.

$1^2 = 1 = 1$
$2^2 = 4 = 1 + 3$
$3^2 = 9 = 1 + 3 + 5$

$4^2 = 16 = 1 + 3 + 5 + 7$

$5^2 = 25 = 1 + 3 + 5 + 7 + 9$

i.e., to get $n^2$, we add up the first $n$ odd numbers. If we want $13^2$, it will be the sum of the FIRST 13 odd numbers.

$n^{th}$ odd number is equal to $(2n - 1)$.

Suppose we want to find out $6^2$, knowing what $5^2$ is, we can move from $5^2$ to $6^2$.

$6^2$ will be the sum of $1^{st}$ 6 odd numbers. But the sum of the first 6 odd numbers can be written as 'sum of the first 5 odd numbers' + 'sixth odd number'. Since we already know that the sum of the first 5 odd numbers is $5^2$, i.e., 25, we need to add the sixth odd number, i.e., $(2 \times 6 - 1 =)$ 11 to 25 to give us $6^2 = 36$.

Similarly

$31^2 = 900 + 31$st odd number $= 900 + 61 = 961$

$36^2 = 1225 + 36^{th}$ odd number $= 1225 + 71$

$= 1296$ (Since $35^2 = 1225$)

$41^2 = 1600 + 81 = 1681$

$46^2 = 2025 + 91 = 2116$

$126^2 = 15625 + 251 = 15876$

$196^2 = 38025 + 391 = 38416$

$216^2 = 46225 + 431 = 46656$

We have now seen how to find the squares of numbers which are one more than those numbers whose squares we already know (e.g., 25, 30, 35, etc.)

To find the square of a number which is one less than the number whose squares we already know

Similarly, we can find the squares of numbers which are one less than the numbers whose squares are known. For example,

$29^2 = 30^2 - 30^{th}$ odd number

$= 900 - 59 = 841$

$39^2 = 40^2 - 40^{th}$ odd number $= 1600 - 79 = 1521$

$34^2 = 1225 - 69 = 1156$

$54^2 = 3025 - 109 = 2916$

$74^2 = 5625 - 149 = 5476$

$94^2 = 9025 - 189 = 8836$

$214^2 = 46225 - 429 = 45796$

Thus, we have seen how to arrive at the squares of numbers which are one more or one less than the numbers whose squares we already know (i.e., 25, 30, 35, 40, 45, 50, 55, etc.).

To find the square of a number which is 2 more than the number whose squares we already know:

Now, we will see how to get the squares of numbers which are 2 more (or less) than the numbers whose squares we already know.

$27^2 = 26^2 + 27^{th}$ odd number $= 25^2 + 26^{th}$ odd number $+ 27^{th}$ odd number.

The sum of the $26^{th}$ odd number and $27^{th}$ odd number is the same as 4 times 26. Hence,

$27^2 = 25^2 + 4 \times 26 = 625 + 104 = 729$

$57^2 = 3025 + 224$ (4 times 56) $= 3249$

$77^2 = 5625 + 304$ (4 times 76) $= 5929$

$97^2 = 9025 + 384$ (4 times 96) $= 9409$

To find the square of a number which is 2 less than the number whose squares we already know

Similarly, we can find out the squares of numbers

which are 2 less than the numbers whose squares we know.

$28^2 = (30^2 - 4$ times 29) $= 900 - 116 = 784$

$53^2 = (55^2 - 4$ times 54) $= 3025 - 216 = 2809$

$93^2 = 9025 - 376 = 8649$

$243^2 = 60025 - 976 = 59049$

$143^2 = (145^2 - 4 \times 144) = 21025 - 576 = 20449$

To find the square of a number from 26 to 50

The squares of numbers from 26 to 50 can be calculated by writing down and adding two parts as explained below:

The first part is as many times 100 as the number is more than 25, for example in finding $31^2$, as 31 is 6 more than 25, the first part is $100 \times 6 = 600$.

The second part is the square of the number that is as much less than 25 as the number is more than 25, i.e., in finding $31^2$, the second part is the square of 6 less than 25, i.e., $(25 - 6)^2 = 19^2 = 361$.

Hence, $31^2$ = First part + Second part = 600 + 361 = 961.

The above method can be summarized as

1. Finding $31^2$
   - (i) $31 = 25 + 6$
   - (ii) $25 - 6 \rightarrow 19^2 \rightarrow 361$
   - (iii) $31^2 = 6\ x\ 100 + 361 = 961$
2. Finding $33^2$
   - (i) $33 = 25 + 8$
   - (ii) $25 - 8 \rightarrow 17^2 \rightarrow 289$
   - (iii) $31^2 = 8\ x\ 100 + 289 = 1089$

To find the square of a number from 51 to 75:

The squares of numbers from 51 to 75 can be calculated by writing down two parts, each of which is a two-digit number, adjacent to each other as explained below:

The second part is the two-digit number formed by the two digits which are to the extreme right of the square of the number by which the given number is more than 50.

For example, in finding $63^2$, as 63 is 13 more than 50, the second part will be the two digits to the extreme right of $13^2$ (= 169), i.e., 69. Since there are more than two digits in $13^2$, the digit to the extreme left, i.e., 1, is taken as carry forward and is to be added to the first part.

The first part is the sum of (i) the carry forward, if any, from the second part and (ii) the sum of 25 (for this range (i.e., 51 to 75) 25 is taken as the base) and the number by which the given number is more than 50.

As 63 is 13 more than 50, the first part will be 25 + 13 + 1 (carry forward from the second part) = 39

Therefore $63^2$ = 39 69

The above can be summarized as

First Part Second Part

1. $63^2 = (25 + 13) / 13^2 = 38 / 69 = 38 + 1 / 69 = 39\ 69$ (C.F)

First Part Second Part

2. $61^2 = (25 + 11) / 11^2 = 36 / 21 = 37\ 21$ (C.F)

First Part Second Part

3. $56^2 = (25 + 6) / 6^2 = 31/36 = 31\ 36$

**NOTE**

This process is faster for squares in the range of 50 to 60, as there is no carry forward.

To find the square of a number from 76 to 100:

The squares of numbers from 76 to 100 can be calculated by writing down two parts, each of which is a two-digit number, adjacent to each other as explained below:

The second part is the two-digit number formed by the two digits to the extreme right of the square of the number by which the given number is less than 100.

For example in $88^2$, as 88 is 12 less than 100, the second part will be the two digits to the extreme right of $12^2$(= 144), i.e., 44. Since there are more than two digits in $12^2$, the digit to the extreme left, i.e., 1, is taken as carry forward and is to be added to the first part.

The first part is the sum of (i) the carry forward, if any, from the second part and (ii) the difference between the given number and the number by which the given number is less than 100. As 88 is 12 less than 100, the first part will be 88 – 12 + 1(carry forward from the second part) = 77.

Therefore, $88^2$ = 77 44

The above can be summarized as

First Part Second Part

1. $88^2 = (88 - 12) / 12^2 = 76 / 44 = (76 + 1) / 44 = 77\ 44$ (C.F)

First Part Second Part

2. $89^2 = (89 - 11) / 11^2 = 78 / 21 = 79\ 21$ (C.F)

First Part Second Part

3. $96^2 = (96 - 4) / 4^2 = 92/16 = 92\ 16$

**NOTE**

This process is faster for squares in the range of 90 to 100, as there is no carry forward.

To find the square of a number from 101 to 125:

The squares of numbers from 101 to 125 can be calculated by writing down two parts, each of which is a two-digit number, adjacent to each other as explained below:

The second part is the two-digit number formed by the two digits to the extreme right of the square of the number by which the given number is more than 100.

For example, finding $112^2$, as 112 is 12 more than 100, the second part will be the two digits of $12^2$(= 144), i.e., 44. Since there are more than two digits in $12^2$, the digit to the extreme left, i.e., 1, is taken as carry forward and is to be added to the first part.

The first part is the sum of (i) the carry forward, if any from the second part and (ii) the sum of the given number and the number by which the given number is more than 100. As 112 is 12 more than 100, the first part will be 112 + 12 + 1 (carry forward from the first part) = 125.

Therefore, $88^2$ = 125 44

The above can be summarized as

First Part Second Part

1. $112^2 = (112 + 12) / 12^2 = 124 / 44 = (124 + 1) / 44 = 125\ 44$ (C.F)

2. $113^2 = (113 + 13) / 13^2 = 126 / 169 = 127\ 69$ (C.F)

3. $106^2 = (106 + 6) / 6^2 = 112/36 = 112\ 36$

By observing and remembering a few properties regarding the behaviour of the last digits of numbers and of their squares and cubes, it is sometimes possible to

solve certain kinds of questions. Hence, some of the important properties of the last digits of numbers are given below:

Last digit of any number 0 1 2 3 4 5 6 7 8 9
Last digit of its square 0 1 4 9 6 5 6 9 4 1
Last digit of its cube 0 1 8 7 4 5 6 3 2 9

Some important observations:

1. The square of a number can never end with 2, 3, 7 or 8.
2. Any power of any number ending in 0, 1, 5 or 6 ends with 0, 1, 5, 6, respectively.
3. If the last digits of two numbers are 10's complements, then the last digits of their squares will be equal. Hence, if the last digit of the square of a number is given, it is not possible to determine the last digit of that number uniquely. For example if $n^2$ ends with 9, $n$ may end with 3 or 7.
4. We can uniquely determine the last digit of a number given the cube of that number, for example $(\ldots)^3 = __3$, the number can end only in 7.
5. If the last digits of two numbers are 10's complements, then last digits of their cubes will be also 10's complements.
6. The square of a number with only $n$ 1's, where $n$ is a single digit number, will always be a palindrome. For example:
   112 = 121; 1112 = 12321; 11112 = 1234321. In general, $(1111\ldots\ n \text{ times})^2 = 123\ldots n^{n-1}\ldots 1$
7. The last two digits of any power of a number ending in 25 or 76 always end in 25 and 76, respectively.
8. (a) If the square of any number is ending in 1, then the ten's digit of that square should be an even number.
   (b) If the square of any number is ending in 4, then the ten's digit of that square should be an even number.
   (c) If the square of any number is ending in 5, then the ten's digit of that square should be 2.
   (d) If the square of any number is ending in 6, then the ten's digit of that square should be an odd number.
   (e) If the square of any number is ending in 9, then the ten's digit of that square should be an even number.

## ❒ Powers of 2 and 3

Remembering powers of 2 up to 12 and powers of 3 up to 8 will be of great help. It has been observed that various competitive examinations have direct questions on the application of these.

Property for the powers of 2:

$2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$
$2^4 = 16$, $25 = 32$, $2^6 = 64$, $2^7 = 128$
$2^8 = 256$, $2^9 = 512$, $2^{10} = 1024$, $2^{11} = 2048$

By observing the following, we can see that

$2^0 + 2^1 = 3 = 2^2 - 1$
$2^0 + 2^1 + 2^2 = 7 = 2^3 - 1$
$2^0 + 2^1 + 2^2 + 2^3 = 15 = 2^4 - 1$

Similarly,

$2^0 + 2^1 + 2^2 + 2^3 + 2^4 = 31 = 2^5 - 1$

That is, the sum of powers of 2 from 0 to any number $k$ will be equal to $2^{k+1} - 1$.

The above concept can be used in the following example:

For example, the sum $2^0 + 2^1 + \ldots 2^n$ is equal to $(2^{n+1} - 1)$. This can help us arrive at the answer to a question like 'If ten brothers have some marbles each, such that every brother, except the youngest, has twice the number of marbles than that the brother immediately younger to him has, then find the least possible total number of marbles with the ten brothers'.

To have the least total, the youngest should have the least number of marbles, i.e., only one marble.

The second youngest will have 2 (i.e., $2^1$), the next brother will have 4 (i.e., $2^2$) and so on.

The eldest will have $2^9$. The sum of all the marbles with them will be $2^0 + 2^1 + 2^2 + \ldots 2^9 = 2^{9+1} - 1 = 2^{10} - 1$
$= 1024 - 1 = 1023$.

Property for the powers of 3:

$3^0 = 1$, $3^1 = 3$, $3^2 = 9$, $3^3 = 27$
$3^4 = 81$, $3^5 = 243$, $3^6 = 729$, $3^7 = 2187$
$3^0 + 3^1 + 3^2 + 3^3 = 40$

Using a combination of these numbers, each occurring at the most once, we can obtain all the numbers from 1 to 40 by using the operation of only addition and/or subtraction.

The above concept can be used in the following example.

## SOLVED EXAMPLES

**2.01.** 342 + 557 + 629 + 746 + 825 =?

**Sol:** When we are adding three-digit numbers, first add two-digits at a time (units and tens place).
42 + 57 + 29 + 46 + 25 = 199.
To add 42 and 57, mentally treat 57 as 50 + 7 (50 would facilitate quick addition).
Thus, 42 + 57 = (42 + 50) + 7 = 92 + 7 = 99.
Similarly, 99 + 29 = (99 + 20) + 9 = 128.
128 + 46 = (128 + 40) + 6 = 174.
174 + 25 = (174 + 20) + 5 = 199.
The last two digits (the units place and the tens place) of the addition are 99, while the digit 1 is to be carried forward).
Now add
1 (carried) + 3 + 5 + 6 + 7 + 8 = 30.
∴ The result of the addition is 3099.
The same logic can be extended to four-digit additions.

**2.02.** 6965 + 3246 + 1234 + 9847 + 8238 =?

**Sol:** Part II

| | | |
|---|---|---|
| [2* + 69 ] | = 71 | 69 |
| [(71 + 30) + 2] | = 103 | 32 |
| [(103 + 10) + 2] | = 115 | 12 |
| [(115 + 90) + 8] ** | = 213 | 98 |
| [(213 + 80) + 2] | = 295 | 82 |
| | | 295 |

Part I

| | | |
|---|---|---|
| 65 | | |
| 46 | [(65+40)+ 6] | = 111 |
| 34 | [(111+30) + 4] | = 145 |
| 47 | [(145+40) + 7] | = 192 |
| 38 | [(192+30)+ 8] | = 230 from here, we carry forward 2 |
| 30 | | |

[* The 2 shown here is the carry forward indicated at bottom-right].
[** Alternatively, this calculation can be performed as 115 + 100 – 2 = 215 – 2 = 213].

**2.03.** 1598 + 5423 + 4627 +7953 + 8675 =?

**Sol:** Part II

| | | |
|---|---|---|
| (2* + 15) | = 17 | 15 |
| [(17 + 50) + 4] | = 71 | 54 |
| [(71 + 40) + 6] | = 117 | 46 |
| [(117 + 70) + 9] | = 196 | 79 |
| [(196 + 80) + 6] | = 282 | 86 |
| | | 282 |

Part I

| | | |
|---|---|---|
| 98 | | |
| 23 | [(98 + 20) + 3] | = 121 |
| 27 | [(121 + 20) +7] | = 148 |
| 53 | [(148 + 50) + 3] | = 201 |
| 75 | [(201 + 70) + 5] | = 276 |
| 76 | | |

**2.04.** 987 – 256 =?

**Sol:** Instead of taking a single digit at a time, subtractions would be faster by taking two digits i.e.,
87 – 56 = 31.
900 – 200 = 700
∴ The result of 987 – 256 = 731

**2.05.** 824 – 587 =?

**Sol:** Take 100s complement of 87 (i.e., 100 – 87) which is 13 and add it to 24. The result is 37. This gives the units and tens digits of the result. Since 24 < 87, we have actually subtracted 87 from 124, i.e., we have borrowed 1 from 8 (of 824). Therefore we now do (7 – 5) = 2. The result is 237.

**2.06.** 9217 – 858 =?

**Sol:** Adding 100s complement of 58 (which is 42) to 17, we get (42 + 17) = 59 which gives the units and 10s digits of the result.
Since 58 is greater than 17, we have to borrow 1 from 92 which leaves us with 91. So, the first part of the answer is 91– 8(= 83)
Hence, the result is 8359.

**2.07.** 934 – 286 + 847 – 798 =?

**Sol:** When we have a combination of additions and subtractions, first add all the numbers with + sign before them and add all the numbers with – sign before them.
i.e., (934 + 847) – (286 + 798) = 1781 – 1084.
By applying the method explained in previous examples, 1781 – 1084 = 697.

**2.08.** Find the product of 113 and 118.
Here, both the numbers are greater than 100 and the base here is 100. Taking the difference of the two numbers 113 and 118 from the base, we get +13 and +18 and write them as below.

| | | |
|---|---|---|
| 113 | → | + 13 |
| 118 | → | + 18 |
| 131 | | 234 |

The first part of the answer is the cross-total of 113 and +18 which is 131. The second part of the answer, i.e., the product of the deviations (+13 and +18) is equal to 234. But we said there should be as many digits in this product as the number of zeroes in the base (which is 100 here). Since the base has two zeroes, the second part of the answer should also have two digits. Since 234 has three digits, we should retain two digits 4 and 3 and carry forward the third digit 2 to the first part of the answer. Hence, the first part of the answer now becomes 133 and the second part is 34. The product of 113 and 118 is thus equal to 13334.

**2.09.** Find the product of 109 and 93.
Here, one number is greater than 100 and the other is less than 100. Write the differences from 100 (the closest power of 10) along with the sign of the deviation.

| | | | |
|---|---|---|---|
| 109 | → | +9 | |
| 93 | → | –7 | |
| 102 | | –63 | Ans. 10137 |

The first part of the answer is the cross-total (of 109 and –7 or of 93 and +9) 102. The second part of the answer is the product of +9 and –7 which is –63. Since we cannot have a negative figure as a part of the answer, we need to convert this to a positive number. For this purpose, we borrow the necessary figure from the first part of the answer. Each unit borrowed from the first part of the answer, when it is brought to the second part, becomes equal in value to the base used. If we borrow 1 from the first part (102 here), we are left with 101 for the first part and the 1 that is borrowed becomes 100 for the second part. The second part now is 100 (borrowed) plus -63 (originally there) which is equal to 37. The final result is obtained by putting the first and the second part together. Hence, the product of 109 and 93 is 10137.

**2.10.** Find the product of 117 and 88.

| | | | |
|---|---|---|---|
| 117 | → | +17 | |
| 88 | → | –12 | |
| 105 | | –204 | Ans. 10296 |

Please note that to take care of –204 of the second part, borrowing a 1 from the first part is not sufficient (because the 100 it becomes when it comes to the second part is not numerically greater than –204). So, we should borrow 3 from 105 (leaving 102 as the first part) which becomes 300 in the second part to which –204 should be added giving us 96. Hence, the product of 117 and 88 is 10296.

**2.11.** Find the product of 997 and 983.
Here, both the numbers are close to 1000 – they are both less than 1000.

| | | | |
|---|---|---|---|
| 997 | → | –3 | |
| 983 | → | –17 | |
| 980 | | +51 | Ans. 980051 |

The second part 51 has only two digits whereas the base 1000 has three zeroes—so, 51 will be written as 051. Hence, the product is 980051.

**2.12.** Find the product of 1013 and 981.

| | | | |
|---|---|---|---|
| 1013 | → | +13 | |
| 981 | → | –19 | |
| 994 | | –247 | Ans. 993753 |

The second part is –247 and if we borrow 1 from the first part (the first part itself will then become 993), it becomes 1000 in the second part. So, the second part will effectively be 1000 – 247 = 753. Since the base is 1000, the second part should have three digits and 753 has three digits. Hence, the product of 1013 and 981 is 993753.

We can also extend this method to find the product of two numbers which may not be close to a power of 10 but both of which are close to a multiple of a power of 10. This requires a little bit of modification to the method as discussed in the examples below.

**2.13.** Find the product of 297 and 292.
Here, the numbers are not close to any power of 10 but are close to 300 which is a multiple of 100 which itself is a power of 10. So, we adopt 300 as a 'temporary base'. This temporary base is a multiple (or a sub-multiple) of the main base 100. Here, the temporary base 300 = 3 × 100. Then, the procedure of finding out the deviation from the base, getting the cross-totals and the product of the deviations should be done in a manner similar to the previous cases except that the deviations will be taken from the temporary base.

| | | | |
|---|---|---|---|
| 297 | → | –3 | (289 × 3 = 867) |
| 292 | → | –8 | |
| 289 | | +24 | Ans. 86724 |

We have got the first part of the answer as 289 and the second part of the answer as 24. But before we put these two parts together to get the final result, one more step is involved. The first part of the answer is not the final figure—this is an intermediate stage of the first part. This first part should be multiplied by the same figure with which the power of 10 is multiplied to get the temporary base. In this case, we multiplied 100 (which is the power of 10) by 3 to get the temporary base 300. So, the intermediate stage figure of the first part (289) will also have to be multiplied by 3 to get the final figure for the first part. Hence, the first part will be 867 (= 3 × 289). Now putting the first and the second parts together, the product of 297 and 292 is 86724 (Please note that the product of the deviations should still have as many digits as the number of zeroes in the base—in this case two because 100 has two zeroes).

**2.14.** Find the product of 287 and 281.

| | | | |
|---|---|---|---|
| 287 | → | –13 | (268 × 3 = 804) |
| 281 | → | –19 | |
| 268 | | 247 | Ans. 80647 |

Here, the product of the deviations is 247—there are three digits in this whereas the base has only two zeroes. So, the digit 2 has to be carried forward to the first part of the answer but this carrying forward should be done only after the intermediate stage figure of the first part is multiplied suitably to get the final figure of the first part (in this case, 268 multiplied by 3 gives 804 as the first part of the answer). To this add 2 which is the carry forward digit from the second part and we get 806. Hence, the product of 287 and 281 is 80647.

**2.15.** Find the product of 317 and 291.

| | | | |
|---|---|---|---|
| 317 | → | +17 | (3 × 308 = 924) |
| 291 | → | –9 | |
| 308 | | –153 | Ans. 92247 |

Here, since one number is greater than 300 and the other is less than 300, the product of the deviations is negative. To make the second part positive, we need to borrow from the first part. But the borrowing should be done only after the intermediate stage figure of the first part is multiplied by the suitable digit to get the final figure of the first part. In this case, we get 308 × 3 = 924 as the final form of the first part. Now to take care of the negative second part of –153, we need to borrow 2 from the first part because the main base is 100, 2 borrowed becomes 200). The final form of the second part is 200 – 153 = 47. So, the product of 317 and 291 is 92247.

**2.16.** Find the product of 513 and 478.

| | | | |
|---|---|---|---|
| 513 | → | +13 | (491 × 5 = 2455) |
| 478 | → | –22 | 300 – 286 = 14 |
| 491 | | –286 | Ans. 245214 |

We can look at one more extension of this method where the numbers are not close to the same power of 10 but are close to two different powers of 10. We can multiply such numbers by making a simple modification to this method.

**2.17.** Find the product of 979 and 92.
(by adding 0 to the number 92, it becomes 920)

| | | | |
|---|---|---|---|
| 979 | → | –21 | |
| 920 | → | –80 | |
| 899 | | +1680 | Ans. 900680 |

Here, 979 is close to 1000 and 92 is close to 100. For finding the product, we force 92 also close to 1000 by taking it as 920.Then, apply our regular method and find the product of 979 and 920. From the resulting product drop the zero at the units place to give the correct result for the product of 979 and 92.

So, drop the 0 in units place. Hence, the product of 979 and 92 is 90068.

In some cases, the algebraic rule $a^2 - b^2 = (a - b)(a + b)$ will be very helpful to find the product of two numbers. For example, if we have to find the product of 132 and 118, rather than applying the method discussed in detail above, we can use the algebraic rule discussed just now.

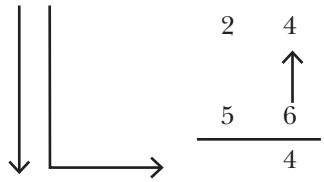
132 can be written as (125 + 7) and 118 can be written as (125 – 7). So, the product of 132 and 118 will be $125^2 - 7^2$. Since we have already discussed methods for calculating squares faster, this method can thus prove to be of immense help in a number of situations provided the student practices sufficiently.

There will be other short cut methods also for a variety of calculations, but the student has to note that none of these will be useful to him in an examination situation unless regular practice is there in using such methods. The student himself should take figures and keep applying various methods for practice on a regular basis.

**2.18.** Find the product of 24 and 56.

**Sol:** Step 1:

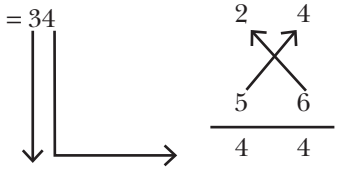
$6 \times 4 = 24$




to be carried forward (C.F.) to the next step.

Step 2:

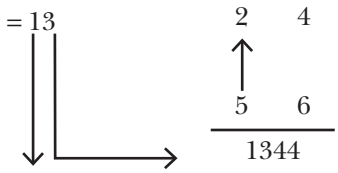
$(2 \times 6) + (4 \times 5) + 2$ (C.F.)




to be carried forward (C.F.) to the next step.

Step 3:

$(5 \times 2) + 3$ (C.F.)




∴ The product of 24 and 56 is 1344.

By observing the above calculation, we summarise the calculations as:

Step 1: Multiply the right most digits vertically (i.e., 6 4)

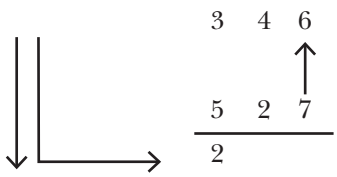
Step 2: Cross multiply and add the carry forward (C.F.) number $(6 \times 2 + 5 \times 4 + \text{C.F.})$

Step 3: Multiply the left most digits vertically and add the C.F. (i.e., $5 \times 2 + \text{C.F.}$)

**2.19.** Find the product of 346 and 527.

**Sol:** Step 1:

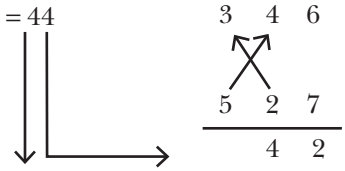
$7 \times 6 = 42$




to be carried forward (C.F.) to the next step.

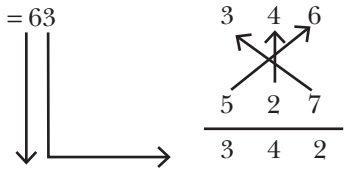
Step 2:

$(7 \times 4) + (2 \times 6) + (\text{C.F.})$



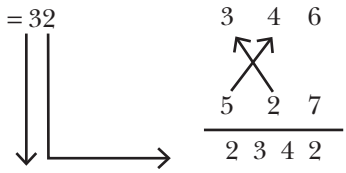

Step 3:

$7 \times 3 + 2 \times 4 + 5 \times 6 + 4$ (C.F.)




Step 4:

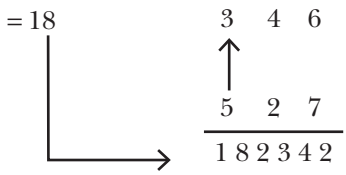
$2 \times 3 + 5 \times 4 + 6$ (C.F.)




to be carried forward (C.F.) to the next step.

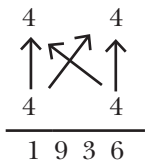
Step 5:

$5 \times 3 + 3$ (C.F.)




∴ The product of 346 and 527 is 182342.

With the help of the above methods, we can also find the square of any number. For example to find the square of 44,




**2.20.** $\frac{4}{9} + \frac{13}{18} + \frac{7}{54} = ?$

**Sol:** The L.C.M. of the denominators 9, 18 and 54 is 54. [The L.C.M. should be calculated mentally] Let us find the numerators.

As 9 has to be multiplied by 6 to get 54, the numerator 4 is multiplied by 6, i.e., $4 \times 6 = 24$.

Similarly $13 \times 3 = 39$ and $7 \times 1 = 7$.

$$\therefore \frac{4}{9} + \frac{13}{18} + \frac{7}{54} = \frac{24}{54} + \frac{39}{54} + \frac{7}{54}$$

$$= \frac{24 + 39 + 7}{54} = \frac{70}{54} = \frac{35}{27}$$

**2.21.** $\frac{7}{18} - \frac{11}{24} + \frac{13}{36} = ?$

**Sol:** The L.C.M. of the denominators 18, 24 and 36 is 72.
36 is divisible by 18, so, the L.C.M. of 18 and 36 is 36. To find the L.C.M. of 24 and 36, take the larger number, i.e., 36 and its multiples 72, 108, etc. 36 is not divisible by 24. So, L.C.M. is not 36. 72 is divisible by 24. So, the L.C.M. is 72. The denominator of the resultant fraction is 72.

$$\frac{7}{18} - \frac{11}{24} + \frac{13}{36} = \frac{28}{72} - \frac{33}{72} + \frac{26}{72} = \frac{21}{72} = \frac{7}{24} =$$

**2.22.** 37.5 % of 1248 =

**Sol:** $37.5\% = 3/8$

$\therefore 37.5\%$ of $1248 = 3/8 \times 1248$

$= 3 \times 156 = 468$

**2.23.** 42.85% of 2114 =

**Sol:** $42.85\% = 3/7$

$\therefore 42.85\%$ of $2114 = 3/7 \times 2114 = 3 \times 302 = 906$

**2.24.** 63.63% of 2233 =

**Sol:** $63.63\% = 7/11$

$\therefore 63.63\%$ of $2233 = 7/11 \times 2233 = 7 \times 203 = 1421$

**2.25.** 58.33% of 2184 =

**Sol:** $58.33\% = 7/12$

$\therefore 58.33\%$ of $2184 = 7/12 \times 2184 = 7 \times 182 = 1274$

**2.26.** 44.44% of 8127 =

**Sol:** $44.44\% = 4/9$

$\therefore 44.44\%$ of $8127 = 4/9 \times 8127 = 4 \times 903 = 3612$

**2.27.** What is 20% of 1205?

**Sol:** Method 1
$20\% = 1/5$
20% of 1205 = 1/5 of 1205 = 241
Method 2

$$10\% = \frac{10}{100} = 0.1$$

10% of 1205 = (0.1) (1205) = 120.5
$\therefore$ 20% of 1205 = $120.5 \times 2 = 241$

**2.28.** Find 22% of 4568

**Sol:** 20% (10% × 2) = $456.8 \times 2 = 913.6$
+ 2% = 1/10 × 20% = 91.36
22% = 1004.96

**2.29.** Find 36% of 183.5

**Sol:** Method 1
30% (10% × 3) = $183.5 \times 3 = 550.5$
+ 6% = 1/5 × 30% = 110.1
36% = 660.6
Method 2
40% (10% × 4) = $183.5 \times 4 = 734$
– 4% = 1/10 of 40% = – 73.4
36% = 660.6

**2.30.** Find the value of 26% of 496.

**Sol:** 26% = 25% + 1%
25% of 496 = 1/4 of 496 = 124

| | |
|---|---|
| + | + |
| 1% of 496 | = 4.96 |
| 26% of 496 | = 128.96 |

**2.31.** Find the value of 35.6% of 928.

**Sol:**

| | |
|---|---|
| 10% of 928 | = 92.8 |
| 30% of 928 | = 92.8 × 3 = 278.4 |
| 5% of 928 | = 46.4 |
| 0.1% of 928 | = 0.928 |
| 35.6% | = 30% = 278.4 |
| +5% | = 46.4 |
| +0.5% | = 4.6 |
| +0.1% | = 0.9 |
| | 330.3 |

30% + 5% + 0.5% + 0.1% =
278.4 + 46.4 + 4.6 + 0.9 = 330.3

**2.32.** 39 is what percent of 186?

**Sol:** The number that follows 'of' should always come in the denominator.

So, $\frac{39}{186} \times 100$ is to be calculated.

10% of the denominator is 18.6
20% of the denominator is 18.6 × 2 = 37.2
1% of the denominator is 1.86
21% of the denominator is 37.2+ 1.86 ~ 39

$$\therefore \frac{39}{186} \simeq 21\%$$

**2.33.** 457 is what percent of 1382?

**Sol:** $1/3 \times 1382 \cong 461 = 33.33\%$
$461 - 457 = 4 \cong 3$ *x* $1.38 = 0.3\%$

$$\therefore \frac{457}{1382} = 33.33\% - 0.3\% = 33.03\%$$

**2.34.** Which of the following fractions is the smallest?

$$\frac{3}{5}, \frac{3}{7}, \frac{3}{13}, \frac{3}{8}$$

**Sol:** 13 is the largest denominator, hence, 3/13 is the smallest fraction. 5 is the smallest denominator, hence, 3/5 is the largest fraction.

**2.35.** Which of the following fractions is the smallest?

$$\frac{7}{5}, \frac{9}{5}, \frac{4}{5}, \frac{11}{5}$$

**Sol:** As 4 is the smallest numerator, the fraction 4/5 is the smallest.
As 11 is the largest numerator, the fraction 11/5 is the largest.

**2.36.** Which of the following fractions is the largest?

$$\frac{19}{16}, \frac{24}{11}, \frac{17}{13}, \frac{21}{14}, \frac{23}{15}$$

**Sol:** As 24 is the largest numerator and 11 is the smallest denominator, $\frac{24}{11}$ is the largest fraction.

**2.37.** Which of the following fractions is the largest?

$$\frac{64}{328}, \frac{28}{152}, \frac{36}{176}, \frac{49}{196}$$

**Sol:** $\frac{64}{328} = \frac{32}{164} = \frac{16}{82} = \frac{8}{41} \simeq \frac{1}{5}$

$\frac{28}{152} = \frac{14}{76} = \frac{7}{38} \simeq \frac{1}{5.5}$

$\frac{36}{176} = \frac{18}{88} = \frac{9}{44} \simeq \frac{1}{5}$

$\frac{49}{196} = \frac{7}{28} = \frac{1}{4}$

As all the numerators are 1 and the least denominator is 4, the fraction $\frac{49}{196}$ is the largest.

**2.38.** Which of the following fractions is the largest?

$$\frac{71}{181}, \frac{214}{519}, \frac{429}{1141}$$

$$\frac{71}{181} = \frac{71 \times 6}{181 \times 6} = \frac{426}{1086}$$

$$\frac{214}{519} = \frac{214 \times 2}{519 \times 2} = \frac{428}{1038}$$

**Sol:** The numerators are now all almost equal (426, 428 and 429). The smallest denominator is 1038. Hence, the largest fraction must be

$\frac{428}{1038}$, i.e., $\frac{214}{519}$

**2.39.** Which of the following fractions is the largest?

$$\frac{31}{37}, \frac{23}{29}, \frac{17}{23}, \frac{35}{41}, \frac{13}{19}$$

**Sol:** The difference between the numerator and the denominator of each fraction is 6. Therefore, the fraction with the largest numerals, i.e., 35/41 is the greatest and the fraction with the smallest numerals, i.e., 13/19 is the smallest.

**2.40.** Which of the following fractions is the largest?

$$\frac{31}{27}, \frac{43}{39}, \frac{57}{53}, \frac{27}{23}, \frac{29}{25}$$

**Sol:** As the difference between the numerator and the denominator is same, the fraction with the smallest values, i.e., $\frac{27}{23}$, is the largest.

We can also compare fractions as follows.
For example, to compare 5/13 and 9/20 make the numerator 1 for all the fractions by approximately dividing the denominator with the respective numerator (upto first decimal place).

$$\therefore \frac{5}{13} = \frac{1}{2.6} \text{ and } \frac{9}{20} \cong \frac{1}{2.2}$$

Now, clearly $\frac{1}{2.6} < \frac{1}{2.2}$ (from rule (ii) above)

$$\Rightarrow \frac{5}{13} < \frac{9}{20}$$

**2.41.** Which of the following fractions is the largest?

$$\frac{15}{17}, \frac{23}{29}, \frac{31}{34}, \frac{11}{15}$$

**Sol:** Comparing fractions

$$\frac{15}{17} \text{ and } \frac{23}{29}$$

The numerator of the fraction has increased from 15 to 23, i.e., $\frac{8}{15}$, i.e., a little more than 50%. The denominator of the fraction has increased from 17 to 29, i.e., 12/17, i.e., well over 50%. As the percentage increase in the numerator is less than the percentage increase in the denominator, the

fraction $\frac{15}{17} > \frac{23}{29}$ Now compare. $\frac{15}{17}$ with $\frac{31}{34}$

As the change in the numerator is more than double (15 to 31), and the change in the denominator is exactly double, the fraction $\frac{15}{17} < \frac{31}{34}$.

$\left(\text{Alternately, } \frac{15}{17} = \frac{30}{34} \;\; \frac{15}{17} < \frac{31}{34}\right)$

Now compare $\frac{11}{15}$ and $\frac{31}{34}$.

The numerator has almost tripled from 11 to 31 whereas the denominator has just over doubled from 15 to 34. Since the increase in numerator is greater than the increase in the denominator,

$\frac{11}{15} < \frac{31}{34}$

So, $\frac{31}{34}$ is the largest fraction.

**2.42.** Find the value of $x$.

$\frac{38}{154} = \frac{x}{190}$

**Sol:** $x = \frac{38 \times 190}{154} = \frac{19}{77} \times 190 \simeq \frac{1}{4} \times 190 \simeq 47.5$

The second method is to find the approximate ratio of the numerators or denominators and arrive at the solution. This is illustrated in the following two examples.

**2.43.** Find the value of $x$.

$\frac{54}{238} = \frac{11}{x}$

**Sol:** $x = \frac{119}{27}(11) = \frac{108+11}{27}(11) = 44 + \frac{121}{27}$

$= 44 + 4\frac{13}{27} \cong 48.5$

**2.44.** Find the value of $x$

$\frac{125}{220} = \frac{176}{x}$

175 is 40% more than 125

$\therefore$ $x$ is ~40% more than 220, i.e., 308.

**2.45.** $16 + \frac{3}{4}$ of

$\left[32 - 16 \div 4 \times 6 + \overline{23-11} + 3 - 2 \times 6\right] = ?$

(A) 89/4 (B) 77/4
(C) 97/4 (D) 81/4

**Sol:** $16 + \frac{3}{4}$ of $[32 - 24 + 12 + 3 - 12] = 16 + \frac{3}{4}$ of $[1]$

$= 16 + \frac{3}{4} \times 11 = 16 + \frac{33}{4} = \frac{97}{4}$

**2.46.** $\frac{1.7 \times 0.0028}{0.068 \times 0.014} = ?$

(A) 5 (B) 10 (C) 20 (D) 15

**Sol:** $\frac{1.7 \times 0.0028}{0.06 \times 0.012} = \frac{17 \times 28}{68 \times 14} \times 10 = 5$

**2.47.** $3\frac{2}{9} + 5\frac{1}{4}\left(16\frac{2}{3} \div 13\frac{4}{6}\right) \div 6\frac{3}{4} = ?$

(A) 3140/369 (B) 1342/369
(C) 1456/369 (D) 1539/369

**Sol:**
$3\frac{2}{9} + 5\frac{1}{4}\left(16\frac{2}{3} \div 3\frac{4}{6}\right) \div 6\frac{3}{4}$

$= \frac{29}{9} + \frac{21}{4}\left(\frac{50}{3} \times \frac{6}{82}\right) \div \frac{27}{4}$

$= \frac{29}{9} + \frac{21}{4} \times \frac{50}{3} \times \frac{6}{82} \times \frac{4}{27}$

$= \frac{29}{9} + \frac{350}{9 \times 41} = \frac{1539}{369}$

**2.48.** 40% of $\left[\left\{\left(\overline{16-8} + \overline{18-12}\right) \times 5 - 6\right\} \times 2 + 3\right] = ?$

(A) 262/5 (B) 271/5
(C) 267/5 (D) 313/5

**Sol:** 40% of $[\{(8+6) \times 5 - 6\} \times 2 + 3]$

$= \frac{2}{5}$ of $[64 \times 2 + 3] = \frac{2}{5} \times 131 = \frac{262}{5}$

**2.49.** $5\frac{7}{6} + 16\frac{2}{3} + 18\frac{4}{9} - 13\frac{5}{6} = ?$

(A) 187/9 (B) 247/9
(C) 319/9 (D) 419/9

**Sol:** $5\frac{7}{6} + 16\frac{2}{3} + 18\frac{4}{9} - 13\frac{5}{6} = \frac{37}{6} + \frac{50}{3} + \frac{166}{9} - \frac{83}{6}$

$= \frac{54}{6} + \frac{166}{9} = \frac{247}{9}$

**2.50.** Find the cube of 12.

Step 1: Cube the left most digit, i.e., 1 in this case, and write it down on the extreme left.

Step 2: Write three more numbers to its right such that the ratio of successive pairs of numbers is same as the ratio of the digits (1 : 2) in the original number. We get the following 1 2 4 8. (1 : 2 = 2 : 4 = 4 : 8)

Step 3: Double the second number (i.e., 2) and the third number (i.e., 4) of the above four numbers and write the result (i.e., $2 \times 2 = 4$ and $2 \times 4 = 8$) under the respective numbers.

Step 4: Add the two rows—one column at a time—such that each column contributes only one digit to the total. (If any column gives more than one digit, the additional digits are carried forward)

| | | | | |
|---|---|---|---|---|
| | 1 ← carry forward | | | |
| | 1 | 2 | 4 | 8 |
| | | 4 | 8 | |
| $12^3 =$ | 1 | 7 | 2 | 8 |

**2.51.** Find the cube of 23.

Step 1 : Cube the left most digit (i.e., $2^3 = 8$) and write it down on the extreme left.

Step 2 : Write three numbers next to the above, such that the ratio between any two successive numbers is the same as the ratio of the digits of the given number. (Therefore, in the number 23, the ratio of the digits is 2 : 3). We get 8 12 18 27

Note: It may sometimes be difficult to find the numbers, i.e., 12, 18 and 27. Note that these numbers are obtained, as $12 = 8 \times 3/2$; $18 = 12 \times 3/2$; $27 = 18 \times 3/2$. Therefore, to get any number, multiply the previous number by the units digit value (i.e., 3) and divide by the ten's digit value (i.e., 2).

Step 3 : Double the 2nd number (i.e., 12) and the 3rd number (i.e., 18) and write them down below the respective numbers.

| | | | |
|---|---|---|---|
| 8 | 12 | 18 | 27 |
| | 24 | 36 | |

Step 4: Add all the numbers, column wise, as shown below, each time carrying forward all digits except the units digit.

| | | | | |
|---|---|---|---|---|
| 4 | 5 | 2 ← | carry forward | |
| 8 | 12 | 18 | 27 | |
| | | 24 | 36 | |
| 12 | 1 | 6 | 7 | |

$\therefore 23^3 = 12167$

**2.52.** Find the cube of 37.

| | | | | |
|---|---|---|---|---|
| 23 | 47 | 34 ← | carry forward | |
| 27 | 63 | 147 | 343 | |
| | 126 | 294 | | |
| 50 | 6 | 5 | 3 | |

$\therefore 37^3 = 50653$

**2.53.** A trader uses only five weights which together weigh 31 kg. With these five weights he can measure all integer weights from 1 kg to 31 kg, with the weight kept only in one pan of the weighing scale. Find the individual weights of the five pieces.

**Sol:** For measuring all integer weights up to 31 kg, the individual weights needed are the powers of 2, i.e., $2^0$, $2^1$, $2^2$, $2^3$ and $2^4$

$\therefore$ if we have weights of 1 kg, 2 kg, 4 kg, 8 kg and 16 kg, we can measure all integer weights upto 31 kg.

For example, if we have to measure 23 kg, we have to use the weights 16 kg, 4 kg, 2 kg and 1 kg on one pan.

If you want to write any number from 1 to *M* as a sum of one or more of the integers of a given set of integers (each integer being used at the most once), it can be done by using the powers of 2. The set of integers we can use consists of all the powers of 2 starting from 1 (i.e., $2^0$) to the largest power of 2 less than or equal to *M*. For example, if you want to build all the integers upto 255, the numbers 1, 2, 4, 8, 16, 32, 64, 128 are sufficient.

**2.54.** A trader uses only four weights, which together weigh 40 kg. With the four weights he could measure all integer weights from 1 kg to 40 kg, placing weights in both the pans. Find the weights of the four pieces.

**Sol:** For measuring all weights upto 40 kg, the weights needed are the powers of 3 whose sum adds upto 40, i.e., $3^0$, $3^1$, $3^2$ and $3^3$ … if we have weights of 1 kg, 3 kg, 9 kg and 27 kg, we can measure all weights from 1 to 40 kg

For example, if we have to measure 33 kg, we have to keep the 27 kg and 9 kg weights on one pan and 3 kg weight on the other, i.e., 27 + 9 – 3 = 33 kg.

## EXERCISE-1

***Directions for questions 1 to 55*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the blank space provided.

**1.** $8563 + 3947 + 5760 + 5691 =$ _____ .

**2.** $99786 - 5584 - 934 - 88 - 9 =$ _____ .

**3.** $35408 + 81563 - 41341 - 51464 =$ _____ .

**4.** $1012 \times 98 =$ _____ .

**5.** $1372 \times 125 =$ _____ .

**6.** $2113 \times 2117 =$ _____ .

**7.** $239 \times 251 =$ _____ .

**8.** $7869 \times 982 =$ _____ .

**9.** 14.28% of 322 = _____ .

**10.** What percentage of 751 is 362 _____ .

**11.** 433 is what percentage of 1444 _____ .

**12.** $106 \times 812$ is what percent of $464 \times 4$ _____ .

**13.** 128.57% of 1694 = (to the nearest integer)_____.

**14.** 84.71% of 742 = _____ .

**15.** $11 \times 4\%$ of $18 \times 2$ is what percentage more than $14 \times 1\%$ of $12 \times 8$ _____ .

**16.** $\frac{5}{24} + \frac{7}{36} + \frac{11}{48} =$ _____ .

**17.** $\frac{12}{17} - \frac{11}{15} + \frac{18}{19} =$ _____ .

**18.** $\frac{5}{7} + \frac{9}{10} + \frac{11}{14} + \frac{8}{35} =$ _____ .

**19.** $\frac{x}{540} = \frac{237}{681}$. Find $x$ _____ .

**20.** $120.01 \times 4.99 + 5.99 \times 80.01 =$
(A) 1020 (B) 1040 (C) 1060 (D) 1080

**21.** $\sqrt[3]{91} \times 162\%$ of $48 =$ _____$^3$
(A) 4 (B) 5 (C) 7 (D) 9

**22.** $\frac{(8.99)(7.01)(2.05) + (17.05)(29.88)(15.01)}{(2.01)(16.01)(19.9)} =$
(A) 14 (B) 10 (C) 11 (D) 12

**23.** $534.95 - 15.23 + 35 \times 6.78 + 40\%$ of $478 =$
(A) 256 (B) 354 (C) 478 (D) 956

**24.** $\sqrt[3]{216400} + \sqrt{280} + \sqrt{322} =$
(A) 651 (B) 361 (C) 85 (D) 95

**25.** $\frac{\sqrt{532.69} + \sqrt{230.15}}{\sqrt{290.96} + \sqrt{364.56}} =$
(A) 1 (B) 3 (C) 5 (D) 6

**26.** $78927.95 \div 448.29 + 3425.6925 =$
(A) 1600 (B) 2600 (C) 4600 (D) 3600

**27.** $\sqrt{36.1} \times 34 + 15\sqrt{8.92} =$ _____ $\times$ (14.28% of 217)
(A) 10 (B) 6 (C) 8 (D) 11

**28.** _____$^3 + 23456 - 21246) \div 31421 = 25$
(A) 65 (B) 80 (C) 69 (D) 92

**29.** $\frac{1}{8}$ of $\frac{1}{7}$ of $\frac{7}{3}$ of $144 +$ _____ $= 13^2 - 5^2$
(A) 140 (B) 132 (C) 134 (D) 138

**30.** $[8 - (7 \text{ of } 16 \div 8 - 10 + 7)] \times \left[\frac{36 \times 14 \times 81}{7 \times 72 \times 3} + 30\right] =$
(A) –171 (B) –59 (C) –3 (D) –161

**31.** $\left\{\frac{1^2 + 2^2 + 3^2 - 11}{78 - (4^2 + 5^2 + 6^2 + 2)}\right\}\left\{\frac{-15 - (16 - \overline{12+12})2}{18 \times 4 - 2 \times 6^2 - 1}\right\}$
(A) –1 (B) 3 (C) –1/3 (D) 1

**32.** 60% of 1300 = _____ % of $1700 + 30^2 \div 90 - 20\%$ of 400
(A) 30 (B) 50 (C) 60 (D) 80

**33.** $\frac{2}{5}$ of $\left\{\left(36 \div \overline{28 \div 7} \times \frac{1}{9}\right) + \frac{1}{9}\right\} =$
(A) $\frac{1}{9}$ (B) $\frac{2}{9}$ (C) $\frac{5}{9}$ (D) $\frac{4}{9}$

**34.** 45% of 60% of 80% of $\left[\frac{714}{8} \div \frac{17}{64}\right] \div 378 \times 1125 =$
(A) 116 (B) 126 (C) 184 (D) 216

**35.** $\overline{(24-20)^3 + (24-24)^3} \times 16\frac{2}{3}\%$ of _____ $= 84$
(A) 4 (B) 6 (C) 8 (D) 2

**36.** $[12 - (9 \text{ of } 24 \div 12 - 14 + 5)] \times \left[\frac{48 \times 22 \times 108}{11 \times 96 \times 9} + 36\right] =$

(A) 126 (B) 132 (C) 138 (D) 144

**37.** $\left\{\frac{30(7+4-12)}{-5+6+9}\right\} \div \left\{\frac{(8 \times 9 - 32)3}{(17+15-31)10}\right\}$

(A) –4 (B) –1 (C) –1 (D) –1/4

**38.** $\frac{9}{15} \text{ of } \frac{45}{81}\left\{\frac{49}{6} \times \left(\frac{16}{7} - 2\right)\right\} \text{ of } \frac{24}{5} \div \frac{16}{15} =$

(A) 7/2 (B) 2/7 (C) 3/7 (D) 5/9

**39.** $\sqrt[3]{\phantom{32768}} = \frac{15}{32}$

(A) 3375 (B) 3500 (C) 4560 (D) 4975

**40.** $\sqrt{42025} \times \sqrt{3481} - (83)^2 = \sqrt{\phantom{x}} + (72)^2$

(A) 488 (B) 484 (C) 464 (D) 488

**41.** $\sqrt{46656} + \sqrt{4096} \times 52 = (\phantom{x})^2 + 115$

(A) 48 (B) 52 (C) 57 (D) 68

**42.** $\sqrt{9218} \times \sqrt{2210} \div \sqrt{1028} =$

(A) 104 (B) 114 (C) 141 (D) 144

**43.** $(225)^2 \div \sqrt[3]{15625} = \phantom{x}^2$

(A) 25 (B) 35 (C) 45 (D) 55

**44.** $19^3 - 18^3 =$ _____ .

**45.** $(84)^3 - (84)^2 =$

(A) 585468 (B) 558468 (C) 558648 (D) 585648

**46.** $324^2 + 576^2 + 324 \times 576 =$ _____ .

**47.** AB is a two digit number $(AB)^2 = CDA$, which is a three-digit number. How many values can AB take?

(A) 3 (B) 0 (C) 1 (D) 2

**48.** Among the options given below, which pair does not represent the combination of a number and its cube?

(A) 45 and 91125 (B) 72 and 373248
(C) 78 and 474552 (D) 87 and 658483

**49.** Which of the following is a perfect square?

(A) 4021025 (B) 1170875
(C) 1130375 (D) 9030025

**50.** A perfect square is added to twice of itself. The resulting number will

(A) never end with 8.
(B) never be a perfect square.
(C) never end with a 6.
(D) satisfy more than one of the above.

**51.** $P$ and $Q$ are natural numbers satisfying the equation $P^2 - Q^2 = 889$. How many integral values are possible for $(P, Q)$?

(A) 1 (B) 2 (C) 3 (D) infinite

**52.** Find the smallest natural number with which 9000 is to be multiplied to make it a perfect square.

(A) 10 (B) 2 (C) 5 (D) 45

**53.** What is the smallest natural number with which 1080 should be multiplied to make it a perfect cube?

(A) 50 (B) 75 (C) 100 (D) 25

**54.** $(132)^2 =$ _____ .

**55.** $(10.12)^2 =$ _____ .

## ANSWER KEYS

### Exercise-1

| | | | | |
|---|---|---|---|---|
| 1. 23961 | 12. 23% | 23. (D) | 34. (D) | 45. (D) |
| 2. 93171 | 13. 2178 | 24. (D) | 35. (C) | 46. 623376 |
| 3. 24166 | 14. 628.55. | 25. (A) | 36. (D) | 47. (D) |
| 4. 99176 | 15. 15% | 26. (D) | 37. (D) | 48. (D) |
| 5. 171500 | 16. 91/144 | 27. (C) | 38. (A) | 49. (D) |
| 6. 4473221 | 17. 4457/4845 | 28. (D) | 39. (A) | 50. (D) |
| 7. 59989 | 18. 92/35 | 29. (D) | 40. (B) | 51. (B) |
| 8. 7727358 | 19. 188 | 30. (A) | 41. (C) | 52. (A) |
| 9. 45.98 | 20. (D) | 31. (B) | 42. (C) | 53. (D) |
| 10. 48.20% | 21. (C) | 32. (B) | 43. (C) | 54. 17424 |
| 11. 29.98% | 22. (D) | 33. (D) | 44. 1027 | 55. 102.4144 |

## SOLUTIONS

## EXERCISE-1

**1.** The sum is 23961.

**2.** The answer is 93171.

**3.** The answer is 24166.

**4.** $1012 \times 98$

$= (1000 + 12)\ (100 - 2)$

$= 100000 + 1200 - 2000 - 24 = 99176$

**5.** $1372 \times 125 = 1372 \times \frac{1000}{8}$

$= \frac{1372}{8} \times 1000 = 171.5 \times 1000 = 171500 = 171.5$

**6.** $2113 = 2110 + 3$ and $2117 = 2110 + 7$

The base for 2113 and 2117 is 2110.

And he sum of the units digits is $3 + 7 = 10$

For such numbers, the ten's digit and units digit will be $3 \times 7 = 21$

The other five digits starting from the ten lakh's place to the hundred's place will be $211 \times 212 = 44{,}732$

(211 is common to both the numbers. Therefore the product of 211 and its successive positive integers should be considered)

$\therefore$ The product of 2113 and 2117 will be 4473221.

**7.** $239\ (250 + 1)$

$= 239 \times \frac{100}{4} + 239 \times 1$

$= 59750 + 239 = 59989$

**8.** $7869 \times 982 = 7869 \times (1000 - (20 - 2))$

$= 7869000 - 141642 = 7727358$

***Solutions for questions 9 to 11*:**

By using ten percent one percent concept, we can calculate the required percentage values.

**9.** 14.28% of 322 is 45.98

**10.** 362 is close to half of 751.

$\therefore$ By using the ten percent one percent concept, we can find that 362 is 48.20% of 751.

**11.** $\frac{1}{4} \times 144 = 361$

Now, 433 – 361, i.e., 72 is very close to 5% of 1440.

$\therefore$ The required answer is 29.98%

**12.** 25% of $464 \times 4 = 116 \times 10$

$106 \times 812$ is $9 \times 288$ less than $464 \times 4$. As $9 \times 288$ is 2% of $464 \times 4$, $106 \times 812$ is 23% of $464 \times 4$.

**13.** 128.57% of 1694 = 100% of 1694 + 28.57% of 1694

$= 1694 + \frac{2}{7} \times 1694 = 1694 + 484 = 2178$

**14.** 84.71% of 742

$= (85.71 - 1)\%$ of $742 = 628.55$.

**15.** $11 \times 4\%$ of $18 \times 2 = \frac{114}{1000} \times 18.2$

$= \frac{114}{1000} \times (2 \times 9 + 0.2)$

$= \frac{228 \times 9 + 228}{1000} = 2.0748$

$14 \times 1\%$ of $12 \times 8$

$= \frac{141}{1000} \times (10 + 2 + 0.8)$

$= \frac{1410 + 282 + 11.28}{1000} = 1.804$

$= \frac{2.07 - 1.80}{1.80} \times 100 = 15$ (approximately)

**16.** The L.C.M. of 24 and 48 is 48.

By prime factorization $36 = 2^2 \times 3^2$

And $48 = 2^4 \times 3^1$

The L.C.M. of 36 and 48 $= 2^4 \times 3^2 = 144$

$\frac{5}{24} = \frac{5 \times 6}{24 \times 6} = \frac{30}{144}$

$\frac{7}{36} = \frac{7 \times 4}{38 \times 4} = \frac{28}{144}$

$$\frac{11}{48}=\frac{11\times3}{48\times3}=\frac{33}{144}$$

$$\therefore \frac{30}{144}+\frac{28}{144}+\frac{33}{144}=\frac{91}{144}$$

**17.** $\frac{12}{17}-1+\frac{4}{15}+1-\frac{1}{19}$

$$=\frac{4}{15}+\frac{12}{17}-\frac{1}{19}=\frac{4457}{4845}$$

**18.** $\frac{5}{7}=\frac{5\times10}{7\times10}=\frac{50}{70}$

$$\frac{9}{10}=\frac{9\times7}{10\times7}=\frac{63}{70}$$

$$\frac{11}{14}=\frac{11\times5}{14\times5}=\frac{55}{70}$$

$$\frac{8}{35}=\frac{8\times2}{35\times2}=\frac{16}{70}$$

$$\frac{50}{70}+\frac{63}{70}+\frac{55}{70}+\frac{16}{70}=\frac{184}{70}=92/35.$$

**19.** $\frac{681}{540}\cong\frac{680}{540}=1.26$

Let $x\times1.25=237$

$$x=237\times\frac{4}{5}=189.6$$

Since $x\times1.25=237$

$x\times1.26<237$

Hence, the answer will be approximately 188.

**20.** $?=120\times5+6\times80=600+480=1080$

**21.** $?^3\cong4.5\times77.76$

$?^3=351$

$\Rightarrow ?^3\cong7^3\Rightarrow\therefore ?=7.$

**22.** $\frac{(8.99)(7.01)(2.05)+(17.05)(29.88)(15.01)}{(2.01)(16.01)(19.9)}$

$$\cong\frac{9\times7\times2+17\times30\times15}{2\times16\times20}$$

$$=\frac{7776}{640}\cong12.$$

**23.** $?\cong535-15+35\times7+191$

$\Rightarrow ?=956.$

**24.** $\sqrt[3]{216400}+\sqrt{280}+\sqrt{322}$

$=60+16.5+18=94.5\cong95$

**25.** $\frac{\sqrt{532.69}+\sqrt{230.15}}{\sqrt{290.96}+\sqrt{364.56}}=?$

$$\Rightarrow ?\simeq\frac{23+15}{17+19}=\frac{38}{36}$$

$\Rightarrow ?\cong1.$

**26.** $?=176.06+3423.6925$

$\Rightarrow ?=3599.75\cong3600.$

**27.** $\sqrt{36.1}\times34+15\sqrt{8.92}=?\times(14.28\%\text{ of }217)$

$\Rightarrow 6\times34+15\times3=?\times1/7\times217$

$\Rightarrow 249=?\times31\Rightarrow ?=\frac{249}{31}=8.$

**28.** $(?^3+2210)=25\times31421$

$\Rightarrow ?^3=785525-2210$

$\Rightarrow ?^3=783315\Rightarrow ?=\sqrt[3]{783315}$

$\therefore ?\cong43.$

**29.** $?=13^2-5^2-\frac{1}{8}\text{ of }\frac{1}{7}\text{ of }\frac{7}{3}\text{ of }144$

$$169-25-\frac{1}{8}\times\frac{1}{7}\times\frac{7}{3}\times144$$

$=144-6=138$

**30.** $8-\{7\text{ of }16\div8-10+7\}\times\left[\frac{36\times14\times81}{7\times72\times3}+30\right]$

$$\Rightarrow\left[8-\left(\frac{7\times16}{8}-10+7\right)\right]\times57=-171$$

**31.** $\left(\frac{-15\left(16-\overline{12+12}\right)2}{18\times4-2\times6^2-1}\right)\times\frac{1^2+2^2+3^2-11}{78-\left(4^2+5^2+6^2+2\right)}$

$$=\left(\frac{15-(16-24)2}{72-72-1}\right)\times\frac{3}{-1}$$

$=-1\times-3=3$

Exercise 1

**32.** $\frac{60}{100}\times 1300 = \frac{?}{100}\times 1700 + \frac{900}{90} - \frac{20}{100}\times 400$

$\Rightarrow ? \times 17 + 10 - 80 = 780$

$\Rightarrow ? \times 17 = 780 + 70 = 850$

$\Rightarrow ? = \frac{850}{17} = 50$

**33.** $? = \frac{2}{5}$ of $\left\{\left(36 \div 4 \times \frac{1}{9}\right) + \frac{1}{9}\right\}$.

$\Rightarrow ? \frac{2}{5} \times \left\{1 + \frac{1}{9}\right\}$

$\Rightarrow ? \frac{2}{5} \times \frac{10}{9} = 4/9$.

**34.** $? = \frac{45}{100}\times\frac{60}{100}\times\frac{80}{100}\times\left[\frac{714}{8}\times\frac{64}{17}\right]\times\frac{1}{378}\times 1125$

$\Rightarrow ? = \frac{9}{20}\times\frac{3}{5}\times\frac{4}{5}\times[42\times 8]\times\frac{1}{378}\times 1125$

$\Rightarrow ? = 24 \times 9 = 216.$

**35.** $\sqrt{(24-20)^3 + (24-25)^3} + (24-25)^3 \times 16\frac{2}{3}\%$ of $? = 84$

$\Rightarrow (64 - 1) \times \frac{1}{6}$ of $? = 84 \Rightarrow ? = 8$

**36.** $? = [12 - (216 \div 12 - 14 - 5)] \times \left[\frac{48}{96}\times\frac{22}{11}\times\frac{108}{9} + 36\right]$

$[12 - (18 - 14 + 5)] \times \left[\frac{1}{2}\times 2 \times 12 + 36\right]$

$= [12 - 9] \times [12 + 36]$

$= 3 \times 48 = 144$

**37.** $\frac{30(7+4-12)}{-5+6+9} \div \frac{(8\times 9 - 32)3}{(17+15-31)10}$

$= -3 \div 12 = -1/4.$

**38.** $\frac{9}{15}$ of $\frac{5}{9}\left\{\frac{49}{6}\times\frac{2}{7}\text{ of }\frac{24}{5}\times\frac{15}{16}\right\} = ?$

$\Rightarrow ? = \frac{9}{15}\times\frac{5}{9}\left\{\frac{7}{3}\times\frac{9}{2}\right\}$

$\Rightarrow ? = 7/2$.

**39.** $\frac{?}{32768} = \left(\frac{15}{32}\right)^3 = \frac{3375}{32768}$

$\therefore ? = 3375.$

**40.** $205 \times 59 - 6889 = \sqrt{?} + 5184$

$\Rightarrow \sqrt{?} = 12095 - 6889 - 5184$

$\Rightarrow \sqrt{?} = 22 \Rightarrow ? = (22)^2$

$\therefore ? = 484.$

**41.** $\sqrt[3]{46656} + \sqrt{4096} \times 52 = (?)^2 + 115$

$\Rightarrow (?)^2 + 115 = 36 + 64 \times 52$

$\Rightarrow (?)^2 = 3364 - 115 = 3249$

$\Rightarrow ? = \sqrt{3249} = 57.$

**42.** $\frac{\sqrt{9218}\times\sqrt{2210}}{\sqrt{1028}} = ? \Rightarrow \frac{\sqrt{9216}\times\sqrt{2209}}{\sqrt{1024}} = ?$

$\Rightarrow ? = \frac{96\times 47}{32} = 141.$

**43.** $(?)^2 = 225 \times 225 \times \frac{1}{25}$

$\Rightarrow (?)^2 = 2025 \Rightarrow ? = \sqrt{2025}$

$\therefore ? = \pm 45.$

**44.** $19^3 - 18^3$ is of the form $a^3 - b^3$ where $a = 19$ and $b = 18$.

$a^3 - b^3$ is defined as $(a - b)\ (a^2 + b^2 + ab)$.

Hence, $19^3 - 18^3 = (19 - 18)$

$(19^2 + 18^2 + 19 \times 18) = 1027$

**45.** $(84)^3 - (84)^2 = ?$

$\Rightarrow ? = (84)^2\ [84 - 1] = 84 \times 83$

$\therefore ? = 585648.$

**46.** $324^2 + 576^2 + 324 \times 576$

$= 324^2 + 576^2 + 2 \times 324 \times 576 - 324 \times 576$

$= (324 + 576)^2 - (18^2 \times 24^2) = 900^2 - 432^2$

$= (900 + 432) \times (900 - 432) = 1332 \times (400 + 60 + 8)$

$= 623376$

**47.** $(AB)^2 = CDA$

$CDA$ is a three digit perfect square.

$\therefore A$ cannot be 2, 3, 7, or 8 (1)

($\because$ No Perfect square ends in 2, 3, 7, or 8)

$(AB)^2 < 1000$. $\therefore$ $AB \leq 31$. (2)

From (1) and (2), $A = 1$,

$(1B)^2 = CD1$.

$B = 1$ or 9.

$AB = 11$ or 19.

$\therefore$ $AB$ can take 2 values

**48.** Going by the options, as the numbers whose cubes are given are multiples of 3, the cubes of the numbers must be multiples of $3^3$, i.e., 27.

Hence, the cubes must be multiplies of 9.

The sum of the digits of 91125, 373248 and 474552 are multiples of 9 whereas the sum of the digits of 658483 is not a multiple of 9.

Hence, 658483 is not a cube of 87.

**49.** Any perfect square ending with a 5 must end with 25. Only choices (A) and (D) have the last two digits as 25.

Checking 4021025, using the rule of finding squares of numbers ending in 5, we need to find factors of the form $(n) \times (n + 1)$ [where $n$ is a natural number] for 40210. Similarly for 90300. $200 \times 201 = 40200$. Hence, choice (A) is eliminated. But $90300 = 300 \times 301$. Therefore $9030025 = (3005)^2$.

**50.** Consider perfect square $X^2$

$X^2 + 2X^2 = 3X^2$

$X^2$ can end with 0, 1, 4, 5, 6, 9

$\therefore$ $3X^2$ can end with

0, 3, 2, 5, 8, 7

$\therefore$ Choice (C) is true, and choice (A) is false.

Besides the product of a non-perfect square and a perfect square can never be a perfect square.

$\therefore$ Choice (B) is also true.

**51.** $P^2 - Q^2 = 889$

$\therefore$ $(P + Q)\ (P - Q) = 127 \times 7 = 889 \times 1$

These are the only two ways of expressing 889 as a product of two natural numbers

Case 1: $P + Q = 127$, $P - Q = 7$

$\Rightarrow$ $P = 67$, $Q = 60$

Case 2: $P + Q = 889$, $P - Q = 1$

$P = 445$, $Q = 444$

**52.** $9000 = 3^2 \times 5^3 \times 2^3$

The smallest natural number to be multiplied with to make it a perfect square $= 5 \times 2 = 10$.

**53.** $1080 = 108 \times 10$

$= 18 \times 6 \times 10 = 2 \times 3^2 \times 2 \times 3 \times 2 \times 5 = 2^3 \times 3^3 \times 5$

The least natural number to be multiplied to make it a perfect cube $= 5 \times 5 = 25$.

**54.** $(132)^2 = (100 + 32)^2$

$= 10{,}000 + 6400 + 1024 = 17424$

**55.** $(10.\ 12)^2$

$= (10)^2 + (0.12)^2 + 2(10)\ (0.12)$

$= 100 + 0.0144 + 2.4 = 102.4144$

# Unit 2

# Numbers


Chapter 1 Numbers

Chapter 2 Number Systems

Chapter 3 Simple Equations

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# 1 Numbers

CHAPTER

## Learning Objectives

*In this chapter, you will*:

- Understand the concept of number system with different types of numbers and their classification
- Understand the hierarchy of arithmetic operations – BODMAS rule
- Learn about various divisibility rules and their applications
- You will understand all about factors of a number including:
  - Finding the number of factors
  - Number of ways of expressing a given number as a product of two factors
  - Sum of all the factors of a number
  - Product of all the factors of a number
  - Number of ways of writing a number as product of two co-primes
  - Number of co-primes to N, that are less than N
  - Sum of co-primes to N, that Are Less than N
- Understand different methods to calculate H.C.F, L.C.M of some numbers such as:
  - Factorization method
  - Long Division method for H.C.F
- Learn about the L.C.M and H.C.F models
- Master successive division
- Acquire skills to find the index of the greatest power of a number in N!
- Understand how to calculate the last digit and the last two digits using cyclicity/pattern method
- Learn about Remainder Theorem and its applications
- Familiarize yourself with useful theorems to calculate remainders of complex expressions such as
  - Binomial Theorem
  - Fermat's Little Theorem
  - Wilson's Theorem

NUMBERS is one of the most important topics required for competitive entrance exams. In this chapter, we have put together a number of models of problems—mainly based on various problems that have been appearing in different exams.

## ❒ BASIC ARITHMETIC OPERATIONS

**Addition** is the most basic operation. We have an intuitive understanding of the operation. It is the process of finding out the single number or fraction equal to two or more quantities taken together. The two (or more) numbers that are added are called addends and the result of the addition is called the sum. **For two numbers *A* and *B*, this is denoted as *A* + *B*.**

**Subtraction** is the process of finding out the quantity left when a smaller quantity (number or fraction) is reduced from a larger one. This is called the difference of the two numbers. The word difference is taken to mean a positive quantity, i.e., the difference of 10 and 8 is 2. The difference of 8 and 10 is also 2. This is also referred to as the remainder.

**Multiplication** is repeated addition. The number that is added repeatedly is the multiplicand. The number of times it is added is the multiplier. The sum obtained is the product.

For example, in the multiplication $3 \times 4 = 12$, 3 is the multiplicand, 4 is the multiplier and 12 is the product.

**Division** is repeated subtraction. From a given number, we subtract another repeatedly until the remainder

is less than the number that we are subtracting. The number from which we are subtracting the second one is the dividend. The number that is subtracted repeatedly (the second one) is the divisor. The number of times it is subtracted is the quotient. The number that remains after we are done subtracting is the remainder. Division can also be thought of as the inverse of multiplication. ***A/B* is that number with which B has to be multiplied to get *A*.**

For example, in the division 32/5, 32 is the dividend, 5 is the divisor, 6 is the quotient and 2 is the remainder.

**Involution** (or raising to the power $n$) is repeated multiplication. Therefore, $a^n$ is the the product of $n$ $a$'s. Here, $a$ is the base, $n$ is the index and $a^n$ is the $n$th power of $a$.

**For example, $a \times a = a^2$, which is the second power of $a$ and $a \times a \times a = a^3$, which is the third power of $a$.**

**Evolution** is the inverse of involution. The $n$th root of a number is that number whose $n$th power is the given number. The root of any number or expression is that quantity which when multiplied by itself the requisite number of times produces the given expression.

For example, the square root of $a$, $\sqrt{a}$ when multiplied by itself two times, gives a; similarly, the cube root of $a$, $\sqrt[3]{a}$ when multiplied by itself three times, gives $a$.

All the above operations are performed in Algebra also. Algebra treats quantities just as Arithmetic does, but with greater generality, for algebraic quantities are denoted by symbols which may take any value we choose to assign them as compared to definite values usually used in arithmetic operations.

## ❒ Rule of Signs

**The product of two terms with like signs is positive; the product of two terms with unlike signs is negative.**

**Example: $-1 \times -1 = +1$; $+1 \times -1 = -1$; $+1 \times +1 = +1$; $-1 \times +1 = -1$**

## ❒ CLASSIFICATION OF REAL NUMBERS

Real Numbers are classified into rational and irrational numbers.

## ❒ Rational Numbers

**A number which can be expressed in the form $p/q$ where $p$ and $q$ are integers and $q \neq 0$ is called a rational number.**

For example, 4 is a rational number since 4 can be written as 4/1 where 4 and 1 are integers and the denominator $1 \neq 0$. Similarly, the numbers 3/4, –2/5, etc. are also rational numbers.

Recurring decimals are also rational numbers. **A recurring decimal is a number in which one or more digits at the end of a number after the decimal point repeats endlessly** (For example, 0.333..., 0.111111..., 0.166666..., etc. are all recurring decimals). Any recurring decimal can be expressed as a fraction of the form $p/q$, and hence, it is a rational number. We will study in another section in this chapter the way to convert recurring decimals into fractions.

Between any two numbers, there can be infinite number of other rational numbers.

## ❒ Irrational Numbers

**Numbers which are not rational but which can be represented by points on the number line are called irrational numbers**. Examples for irrational numbers are $\sqrt{2}, \sqrt{3}, \sqrt[4]{5}, \sqrt[3]{9}$, etc.

Numbers like $\pi$, $e$ are also irrational numbers.

Between any two numbers, there are infinite numbers of irrational numbers.

**Another way of looking at rational and irrational numbers is as follows:**

**Terminating decimals and recurring decimals are both rational numbers.**

**Any non-terminating, non-recurring decimal is an irrational number.**

## ❒ Integers

All integers are rational numbers. Integers are classified into negative integers, zero and positive integers. Positive integers can be classified as Prime Numbers and Composite Numbers. In problems on Numbers, we very often use the word 'number' to mean an 'integer.'

## ❒ Prime Numbers

**A number other than 1 which does not have any factor apart from one and itself is called a prime number.**

Examples for prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, etc.

There is no general formula that can give prime numbers. Every prime number greater than 3 can be written in the form of $(6k + 1)$ or $(6k - 1)$ where $k$ is an integer. For the proof of this, refer to 4th point under 'Some important points to note' given later on in this chapter.

## ❒ Composite Numbers

**Any number other than 1, which is not a prime number is called a composite number.** In other words, a composite number is a number which has factors other than one and itself.

Examples for composite numbers are 4, 6, 8, 9, 10, 14, 15, etc.

**NOTE**

The number 1 is neither prime nor composite.
The only prime number that is even is 2.
There are 15 prime numbers between 1 and 50 and 10 prime numbers between 50 and 100. So, there are a total of 25 prime numbers between 1 and 100.

## ❒ Even and Odd Numbers

**Numbers divisible by 2 are called even numbers whereas numbers that are not divisible by 2 are called odd numbers.**

Examples for even numbers are 2, 4, 6, 8, 10, etc. Examples for odd numbers are 1, 3, 5, 7, 9, etc.

**NOTE**

- Every even number ends in 0, 2, 4, 6 or 8.
- The sum of any number of even numbers is always even.
- The sum of odd number of odd numbers (i.e., the sum of 3 odd numbers, the sum of 5 odd numbers, etc.) is always odd whereas the sum of even number of odd numbers (i.e., the sum of 2 odd numbers, the sum of 4 odd numbers, etc.) is always even.
- The product of any number of odd numbers is always odd.
- The product of any number of numbers where there is at least one even number is even.

## ❒ Perfect Numbers

**A number is said to be a perfect number if the sum of ALL its factors excluding itself (but including 1) is equal to the number itself.**

For example, 6 is a perfect number because the factors of 6, i.e., 1, 2 and 3 add up to the number 6 itself.

Other examples of perfect numbers are 28, 496, 8128, etc.

## ❒ HIERARCHY OF ARITHMETIC OPERATIONS

To simplify arithmetic expressions, which involve various operations like brackets, multiplication, addition, etc. a particular sequence of the operations has to be followed. For example, $2 + 3 \times 4$ has to be calculated by multiplying 3 with 4 and the result 12 added to 2 to give the final result of 14 (you should not add 2 to 3 first to take the result 5 and multiply this 5 by 4 to give the final result as 20). This is because in arithmetic operations, multiplication should be done first before addition is taken up.

The hierarchy of arithmetic operations are given by a rule called BODMAS rule. The operations have to be carried out in the order in which they appear in the word BODMAS, where different letters of the word BODMAS stand for the following operations:

**FORMULA**

| | |
|---|---|
| B | Brackets |
| O | Of |
| D | Division |
| M | Multiplication |
| A | Addition |
| *S* | Subtraction |

There are four types of brackets:

(i) **Vinculum:** This is represented by a bar on the top of the numbers. For example,

$2 + 3 - \overline{4 + 3}$; Here, the figures under the vinculum have to be calculated as $4 + 3$ first and the 'minus' sign before 4 is applicable to 7. Therefore, the given expression is equal to $2 + 3 - 7$ which is equal to $-2$.

(ii) **Simple brackets:** These are represented by ( )

(ii) **Curly brackets:** These are represented by { }

(iv) **Square brackets:** These are represented by [ ]

The brackets in an expression have to be opened in the order of vinculum, simple brackets, curly brackets and square brackets, i.e., [ { ( ¯ ) } ] to be opened from inside outwards.

After brackets is O in the BODMAS rule standing for 'of' which means multiplication. For example, 1/2 of 4 will be equal to $1/2 \times 4$ which is equal to 2.

After O, the next operation is D standing for division. This is followed by M standing for multiplication. After Multiplication, A standing for addition will be performed. Then, *S* standing for subtraction is performed.

Two operations that have not been mentioned in the BODMAS rule are taking powers and extracting roots, viz, involution and evolution, respectively. When these operations are also involved in expressions, there is never any doubt about the order in which the steps of the simplification should be taken. The sign for root extraction is a variant of the vinculum and for powers, brackets are used to resolve ambiguities in the order.

## ❒ RECURRING DECIMALS

**A decimal in which a digit or a set of digits is repeated continuously is called a recurring decimal.** Recurring decimals are written in a shortened form, the digits which are repeated being marked by dots placed over the first and the last of them, therefore,

$$\frac{8}{3} = 2.666..... = 2.\dot{6} \text{ or } 2.\bar{6};$$

$$\frac{1}{7} = 0.142857142857142857... = 0.\overline{142857}$$

In case of 1/7, where the set of digits 142857 is recurring, the dot is placed on top of the first and the last digits of the set or alternatively, a bar is placed over the entire set of the digits that recur.

A recurring decimal like $0.\dot{3}$ is called a pure recurring decimal because all the digits after the decimal point are recurring.

A recurring decimal like $0.1\dot{6}$ (which is equal to 0.16666...) is called a mixed recurring because some of the digits after the decimal are not recurring (in this case, only the digit 6 is recurring and the digit 1 is not recurring).

A recurring decimal is also called a 'circulator'. The digit, or set of digits, which is repeated is called the 'period' of the decimal. In the decimal equivalent to 8/3, the period is 6 and in 1/7 it is 142857.

As already discussed, all recurring decimals are rational numbers as they can be expressed in the form $p/q$, where $p$ and $q$ are integers. The general rule for converting recurring decimals into fractions will be considered later. Let us first consider a few examples so that we will be able to understand the rule easily.

We can now write down the rule for converting a pure recurring decimal into a fraction as follows:

A pure recurring decimal is equivalent to a vulgar fraction which has the number formed by the recurring digits (called the period of the decimal) for its numerator, and for its denominator the number which has for its digits as many nines as there are digits in the period.

Therefore, $0.\overline{37}$ can be written as equal to $\frac{37}{99}$; $0.\overline{225}$ can be written as equal to $\frac{225}{999} = \frac{25}{111}$;

$0.\overline{63} = \frac{63}{99}\ \frac{7}{11}$.

A mixed recurring decimal becomes the sum of a whole number and a pure recurring decimal, when it is multiplied by suitable power of 10 which will bring the decimal point to the left of the first recurring figure. We can then find the equivalent vulgar fraction by the process as explained in case of a pure recurring decimal.

Now, we can write the rule to express a mixed recurring decimal into a (vulgar) fraction as below:

In the numerator write the entire given number formed by the (recurring and non-recurring parts) and subtract from it the part of the decimal that is not recurring. In the denominator, write as many nines as the period (i.e., as many nines as the number of digits recurring) and then place next to it as many zeroes as there are digits without recurring in the given decimal.

i.e., $0.1\overline{56} = \frac{156-1}{990} = \frac{155}{990} = \frac{31}{198}$

$0.7\dot{3} = \frac{73-7}{90} = \frac{66}{90} = \frac{11}{15}$

## ❒ INTEGERS

A number of problems are based on the operation of division and the relation between the quantities involved in division.

### ❒ Properties of Division

Before we take up the next area, the following simple points should be kept in mind.

1. A number when divided by $d$ leaving a remainder of $r$ is of the form $dq + r$ where $q$ is some integer from 0, 1, 2, …
   For example, a number when divided by 4 leaving a remainder of 3 can be written in the form $(4q + 3)$; a number when divided by 7 leaving a remainder of 4 can be written in the form $(7q + 4)$
2. When a number $N$ is divided by divisor $d$ if the remainder is $r$, then the number $N - r$ is exactly divisible by $d$ or in other words, when $N - r$ is divided by $d$ the remainder is 0.
   For example, when the number 37 is divided by 7, the remainder is 2; if this remainder 2 is subtracted from the number 37, the resulting number 35 is exactly divisible by 7.
3. When a number $N$ is divided by a divisor $d$, if the remainder is $r$, then:
   (a) the largest multiple of $d$ which is less than or equal to $N$ is obtained by subtracting $r$ from $N$, i.e., $N - r$ will be the largest multiple of $d$ which is less than or equal to $N$.
   For example, when 27 is divided by 5, the remainder is 2; so 27 – 2, i.e., 25 is the largest multiple of 5 less than 27.
   (b) the smallest multiple of $d$ which is greater than or equal to $N$ is obtained by adding $(d - r)$ to $N$, i.e., $N + (d - r)$ will be the smallest multiple of $d$ which is greater than $N$.
   For example, when 49 is divided by 8, the remainder is 1; hence, the smallest multiple of 8 which is greater than 48 is 49 + (8 – 1) = 56
4. When a division is split into a sum of two divisions (with the same divisor as the original divisor), the original remainder will be equal to the sum of the remainders of the two individual divisions. Similarly, when a division is split into difference

of two divisions, the original remainder will be equal to the difference of the remainders of the two divisions.

For example, if we take the division 15/6 (where the remainder is 3), and write it as a SUM of two divisions 8/6 and 7/6 (where the remainders are, respectively, 2 and 1), the original remainder is equal to the SUM of the two remainders 2 and 1.

$$\frac{15}{6} = \frac{8}{6} + \frac{7}{6}$$

Remainder 3 = 2 + 1

If we take the division 15/6 and write it as the difference of two divisions 29/6 and 14/6 (where the respective remainders are 5 and 2), then the original remainder 3 is equal to the difference of the two remainders 5 and 2.

$$\frac{15}{6} = \frac{29}{6} - \frac{14}{6}$$

Remainder 3 = 5 – 2

5. If the remainder in a division is negative, then add the divisor repeatedly to the negative remainder till we get a positive remainder.
   For example, let us take the division 15/6 (where the remainder is 3) and split into difference of two divisions 25/6 and 10/6. The remainders of the two divisions are 1 and 4, respectively. The **difference** of these two remainders is 1 – 4 which is equal to –3 and this should be equal to the original remainder. Since this remainder is negative, add the divisor 6 to this negative remainder –3 to get the correct remainder 3.

$$\frac{15}{6} = \frac{25}{6} - \frac{10}{6}$$

   Remainders are 3, 1, –4.
   Remainder 1 – 4 = –3 which is same as –3 + 6 = 3
6. In a division, if the dividend (the number which is being divided) is multiplied by a certain factor and then divided by the same divisor, then the new remainder will be obtained by multiplying the original remainder by the same factor with which the dividend has been multiplied.
   For example, when 11 is divided by 8, the remainder is 3. When the dividend 11 is multiplied by 2, we get 22 and when this number is divided by 8, the remainder is 6 which is same as the original remainder 3 multiplied by 2.
7. If the remainder is greater than the divisor, it means division is not complete. To get the correct remainder keep subtracting the divisor from the remainder till you obtain the positive remainder which is less than the divisor.

## ❒ Factors, Multiples and Co-primes

**Factors:** If one number divides a second number exactly, then the first number is said to be a factor of the second number. For example, 5 is a factor of 15; 3 is a factor of 18. Factors are also called sub-multiples or divisors.

**Multiples:** If one number is divisible exactly by a second number, then the first number is said to be a multiple of the second number. For example, 15 is a multiple of 5; 24 is a multiple of 4.

**Co-Primes:** Two numbers are said to be relative primes or co-primes if they do not have any common factor other than 1. For example, the numbers 15 and 16 do not have any common factors and hence, they are relative primes. Please note that none of the two numbers may individually be prime and still they can be relative primes. Unity is a relative prime to all numbers.

## ❒ Rules for Divisibility

In a number of situations, we will need to find the factors of a given number. Some of the factors of a given number can, in a number of situations, be found very easily either by observation or by applying simple rules. We will look at some rules for divisibility of numbers.

**Divisibility by 2:** A number divisible by 2 will have an even number as its last digit (For example 128, 246, 2346, etc)

**Divisibility by 3:** A number is divisible by 3 if the sum of its digits is a multiple of 3.

For example, take the number 9123, the sum of the digits is 9 + 1 + 2 + 3 = 15 which is a multiple of 3. Hence, the given number 9123 is divisible by 3. Similarly 342, 789, etc., are all divisible by 3. If we take the number 74549, the sum of the digits is 29 which is not a multiple of 3. Hence, the number 74549 is not divisible by 3.

**Divisibility by 4:** A number is divisible by 4 if the number formed with its last two digits is divisible by 4.

For example, if we take the number 178564, the last two digits form 64. Since this number 64 is divisible by 4, the number 178564 is divisible by 4.

If we take the number 476854, the last two digits form 54 which is not divisible by 4 and hence, the number 476854 is not divisible by 4.

**Divisibility by 5:** A number is divisible by 5 if its last digit is 5 or zero (e.*g.*, 15, 40, etc.)

**Divisibility by 6:** A number is divisible by 6 if it is divisible both by 2 and 3 (18, 42, 96, etc.)

**Divisibility by 7:** If the difference between the number of tens in the number and twice the units digit is divisible by 7, then the given number is divisible by 7. Otherwise, it is not divisible by 7.

Take the units digit of the number, double it and subtract this figure from the remaining part of the number. If the result so obtained is divisible by 7, then the original number is divisible by 7. If that result is not divisible by 7, then the number is not divisible by 7.

For example, let us take the number 595. The units digit is 5 and when it is doubled, we get 10. The remaining part of the number is 59. If 10 (which is the units digit doubled) is subtracted from 59 we get 49. Since this result 49 is divisible by 7, the original number 595 is also divisible by 7.

Similarly, if we take 967, doubling the units digit gives 14 which when subtracted from 96 gives a result of 82. Since 82 is not divisible by 7, the number 967 is not divisible by 7.

If we take a larger number, the same rule may have to be repeatedly applied till the result comes to a number which we can make out by observation whether it is divisible by 7. For example, take 456745, We will write down the figures in various steps as shown below.

| Col (1) Number | Col (2) Twice the units digit | Col (3) Remaining part of the number | Col (3) – Col (2) |
|---|---|---|---|
| 456745 | 10 | 45674 | 45664 |
| 45664 | 8 | 4566 | 4558 |
| 4558 | 16 | 455 | 439 |
| 439 | 18 | 43 | 25 |

Since 25 in the last step is not divisible by 7, the original number 456745 is not divisible by 7.

**Divisibility by 8:** A number is divisible by 8, if the number formed by the last 3 digits of the number is divisible by 8.

For example, the number 3816 is divisible by 8 because the last three digits form the number 816, which is divisible by 8. Similarly, the numbers 14328, 18864 etc. are divisible by 8. If we take the number 48764, it is not divisible by 8 because the last three digits' number 764 is not divisible by 8.

In general, if the number formed by the last $n$ digits of a number is divisible by $2^n$, the number is divisible by $2^n$.

**Divisibility by 9:** A number is divisible by 9 if the sum of its digits is a multiple of 9.

For example, if we take the number 6318, the sum of the digits of this number is 6 + 3 + 1 + 8 which is 18. Since this sum 18 is a multiple of 9, the number 6318 is divisible by 9. Similarly, the numbers 729, 981, etc. are divisible by 9. If we take the number 4763, the sum of the digits of this number is 20 which is not divisible by 9. Hence, the number 4763 is not divisible by 9.

**Divisibility by 10:** A number divisible by 10 should end in zero.

**Divisibility by 11:** A number is divisible by 11 if the sum of the alternate digits is the same or they differ by multiples of 11, that is, the difference between the sum of digits in odd places in the number and the sum of the digits in the even places in the number should be equal to zero or a multiple of 11.

For example, if we take the number 132, the sum of the digits in odd places is 1 + 2 = 3 and the sum of the digits in even places is 3. Since these two sums are equal, the given number is divisible by 11.

If we take the number 785345, the sum of the digits in odd places is 16 and the sum of the digits in even places is also 16. Since these two sums are equal, the given number is divisible by 11.

If we take the number 89394811, the sum of the digits in odd places is 8 + 3 + 4 + 1, which is equal to 16. The sum of the digits in even places is 9 + 9 + 8 + 1, which is equal to 27. The difference between these two figures is 11 (27 – 16), which is a multiple of 11. Hence, the given number 89394811 is divisible by 11.

The number 74537 is not divisible by 11 because the sum of the digits in odd places is 19 and the sum of the digits in even places is 7 and the difference of these two figures is 12 and this is not a multiple of 11.

Divisibility by numbers like 12, 14, 15 can be checked out by taking factors of the number which are relatively prime and checking the divisibility of the given number by each of the factors. For example, a number is divisible by 12 if it is divisible both by 3 and 4.

The next number that is of interest to us from divisibility point of view is 19.

**Divisibility by 19:** If the sum of the number of tens in the number and twice the units digit is divisible by 19, then the given number is divisible by 19. Otherwise it is not.

Take the units digit of the number, double it and add this figure to the remaining part of the number. If the result so obtained is divisible by 19, then the original number is divisible by 19. If that result is not divisible by 19, then the number is not divisible by 19.

For example let us take the number 665. The units digit is 5 and when it is doubled, we get 10. The remaining part of the number is 66. If 10 (which is the units digit doubled) is added to 66 we get 76. Since this result 76 is divisible by 19, the original number 665 is also divisible by 19.

Similarly, if we take 969, doubling the units digit gives 18 which when added to 96 gives a result of 114. Since 114 is divisible by 19, the number 969 is divisible by 19.

If we take 873, double the units digit ($2 \times 3 = 6$) added to the remaining part of the number (87), we get 93 which is not divisible by 19. Hence, the original number 873 is not divisible by 19.

If we take a larger number, the same rule may have to be repeatedly applied till the result comes to a number which we can make out by observation whether it is divisible by 19. For example, take 456760. We will write down the figures in various steps as shown below.

| Col(1) Number | Col (2) Twice the units digit | Col (3) Remaining part of the number | Col(3) + Col(2) |
|---|---|---|---|
| 456760 | 0 | 45676 | 45676 |
| 45676 | 12 | 4567 | 4579 |
| 4579 | 18 | 457 | 475 |
| 475 | 10 | 47 | 57 |

Since 57 in the last step is divisible by 19, the original number 456760 is divisible by 19.

Let us take another example, the number 37895. Let us follow the above process step by step till we reach a manageable number.

37895 Double the units digit 5 and add the 10 so obtained to 3789. We get,

3799 Double the units digit 9 and add the 18 so obtained to 379. We get 397 Double the units digit 7 and add the 14 so obtained to 39. We get 53.

Since 53 is not divisible by 19, 37895 is not divisible by 19.

## ❒ FACTORS AND CO-PRIMES OF A NUMBER

### ❒ Number of Factors of a Number

If $N$ is a composite number such that $N = a^p, b^q, c^r, \ldots$ where $a, b, c$ are prime factors of $N$ and $p, q, r, \ldots$ are positive integers, then the number of factors of $N$ is given by the expression

**FORMULA**

$$(p+1)\ (q+1)\ (r+1)\ \ldots$$

For example, $140 = 2^2 \times 5^1 \times 7^1$

Hence, 140 has $(2+1)(1+1)((1+1)$, i.e., 12 factors.

Please note that the figure arrived at by using the above formula includes 1 and the given number $N$ also as factors. Hence, if you want to find the number of factors the given number has excluding 1 and the number itself, we find out $(p+1)\ (q+1)\ (r+1)$ and then subtract 2 from that figure.

In the above example, the number 140 has 10 factors excluding 1 and itself.

### ❒ Number of Ways of Expressing a Given Number as a Product of Two Factors

The given number $N$ (which can be written as equal to $a^p, b^q, c^r, \ldots$ where $a, b, c$ are prime factors of $N$ and $p, q, r...$ are positive integers) can be expressed as the product of two factors in different ways.

The number of ways in which this can be done is given by the expression

**FORMULA**

$$1/2\ \{(p+1)(q+1)(r+1)...\}$$

So, 140 can be expressed as a product of two factors in 12/2 or 6 ways {because $(p+1)\ (q+1)\ (r+1)$ in the case of 140 is equal to 12}

If $p, q, r$, etc. are all even, then the product $(p+1)\ (q+1)\ (r+1)\ \ldots$ becomes odd and the above rule will not be valid since we cannot take 1/2 of an odd number to get the number of ways. If $p, q, r, \ldots$ are all even, it means that the number $N$ is a perfect square. This situation arises in the specific cases of perfect squares because **a perfect square can also be written as {square root × square root}**. So, two different cases arise in case of perfect squares depending on whether we would like to consider writing the number as {square root × square root} also as one of the ways.

Therefore, to find out the number of ways in which a perfect square can be expressed as a product of 2 factors, we have the following 2 rules:

**IMPORTANT FACTS**

1. As a product of two different factors: $1/2\{(p+1)(q+1)(r+1)\ldots -1\}$ ways (excluding $\sqrt{N}\times\sqrt{N}$).
2. As a product of two factors (including $\sqrt{N}\times\sqrt{N}$) in $1/2\{(p+1)(q+1)(r+1)\ldots +1\}$ ways.

## ❒ Sum of All the Factors of a Number

If a number $N = a^p, b^q, c^r \ldots$where $a, b, c\ldots$ are prime numbers and $p, q, r \ldots$ are positive integers, then, the sum of all the factors of $N$ (including 1 and the number itself) is:

**FORMULA**

$$\left(\frac{a^{p+1}-1}{a-1}\right)\cdot\left(\frac{b^{q+1}-1}{b-1}\right)\cdot\left(\frac{c^{r+1}-1}{c-1}\right)\ldots$$

The above can be verified by an example.

Consider the number 48, when resolved into prime factors, $48 = 2^4 \times 3^1$. Here, $a = 2$, $b = 3$, $p = 4$, $q = 1$.

Hence, sum of all the factors $= \left(\frac{2^{4+1}-1}{2-1}\right)\left(\frac{3^{1+1}-1}{3-1}\right)$

$= \frac{31}{1}\times\frac{8}{2} = 124$

The list of factors of 48 is 1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

If these factors are added, the sum is 124 and tallies with the above result.

## ❒ Product of All the Factors of a Number

The following examples explain the method of finding the product of all the factors of a number.

## ❒ Number of Ways of Writing a Number as Product of Two Co-primes

Using the same notation and convention used earlier.

If $N = a^p, b^q, c^r, \ldots$, then, the number of ways of writing $N$ as a product of 2 co-primes is $2^{n-1}$, where '$n$' is the number of distinct prime factors of the given number $N$.

Taking the example of 48, which is $2^4\times 3^1$, the value of '$n$' is 2 because only two distinct prime factors (i.e., 2 and 3 only) are involved.

Hence, the number of ways $= 2^{2-1} = 2^1 = 2$, i.e., 48 can be written as product of 2 co-primes, in two different ways. They are (1, 48) and (3, 16).

## ❒ Number of Co-primes to *N* That Are Less Than *N*

If $N$ is a number that can be written as $a^p, b^q, c^r \ldots$ then, the number of co-primes of $N$, which are less than $N$, represented by $\phi(N)$ is,

**FORMULA**

$$N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)\ldots$$

For example if, 48 is considered,
$N = a^p, b^q, c^r, \ldots$, i.e., $48 = 2^4, 3^1$.
Hence, $a = 2$, $b = 3$, $p = 4$, $q = 1$.

$$\phi(48) = 48\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right) = 48\times\frac{1}{2}\times\frac{2}{3} = 16.$$

**NOTE**

If numbers less than 48 are listed, and co-primes to 48 are spotted, the count of co-primes will be 16.

## ❒ Sum of Co-primes to N That Are Less Than N

The sum of the co-primes of $N$, that are less than $N$ is $\frac{N}{2}\cdot\phi(N)$. If we consider the above example, then already we have $\phi(48) = 16$.

Hence, sum of co-primes of 48 that are less than 48 $= \frac{N}{2}\cdot\phi(N) = \frac{48}{2}\times 16 = 384$.

Note: After listing the co-primes of 48 that are less than 48, they can be added and the sum can be verified.

## ❒ LEAST COMMON MULTIPLE (L.C.M.) AND HIGHEST COMMON FACTOR (H.C.F.)

**Least Common Multiple (L.C.M.) of two or more numbers is the least number which is divisible by each of**

**these numbers** (i.e., leaves no remainder; or remainder is zero). The same can be algebraically defined as 'L.C.M. of two or more expressions is the expression of the lowest dimension which is divisible by each of them, i.e., leaves no remainder; or remainder is zero.'

**Highest Common Factor (H.C.F.) is the largest factor of two or more given numbers.** The same can be defined algebraically as ' H.C.F. of two or more algebraical expressions is the expression of highest dimension which divides each of them without remainder.

**H.C.F. is also called GCD (Greatest Common Divisor).**

**FORMULA**

Product of two numbers = L.C.M. × H.C.F.
L.C.M. is a multiple of H.C.F.

For finding L.C.M. and H.C.F. of fractions, first reduce each fraction to its simplest form, i.e., cancel out any common factors between the denominator and numerator and then apply appropriate formula from the following:

**FORMULA**

$$\text{H.C.F. of fractions} = \frac{\text{H.C.F. of numerators}}{\text{L.C.M. of denominators}}$$

$$\text{L.C.M. of fractions} = \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$$

**L.C.M. and H.C.F. can each be found by either one of two methods:**

**(1) Factorization (2) Long Division**

We will look at both the methods.

## ❒ L.C.M. by Factorization

Resolve the numbers into prime factors. Then multiply the product of all the prime factors of the first number by those prime factors of the second number, which are not common to the prime factors of the first number.

This product is then multiplied by those prime factors of the third number, which are not common to the prime factors of the first two numbers.

In this manner, all the given numbers have to be dealt with and the last product will be the required L.C.M.

In other words, take the product of ALL the prime factors of all the numbers except where a factor is occuring in more than one number, it is taken only ONCE in the product. This product is the L.C.M. of all the numbers.

## ❒ L.C.M. by Division

Select any one prime factor common to at least two of the given numbers. Write the given numbers in a line and divide them by the above prime number. Write down the quotient for every number under the number itself. If any of the numbers is not divisible by the prime factor selected, write the number as it is in the line of quotients.

Repeat this process for the line of quotients until you get a line of quotients, which are prime to each other (i.e., no two 'quotients' should have a common factor).

The product of all the divisors and the numbers in the last line will be the required L.C.M.

## ❒ H.C.F. by Factorization

Resolve the given number into prime factors. The product of the prime factors common to all the numbers will be the required H.C.F.

## ❒ H.C.F. by Long Division

Take two numbers. Divide the greater by the smaller; then divide the divisor by the remainder; divide the divisor of this division by the next remainder and so on until the remainder is zero. The last divisor is the H.C.F. of the two numbers taken.

By the same method find the H.C.F. of this H.C.F. and the third number. This will be the H.C.F. of the three numbers.

## ❒ L.C.M. and H.C.F. Models

**L.C.M.—Model 1:** In this model of problem, you will need to find out the smallest number (or number in a specified range like the largest five-digit number) which when divided by 2 or more other numbers (i.e., divisors) leaves the same remainder in all cases.

The basic distinguishing feature of this model of problems is that the remainder will be the same in all the cases (and that remainder will also be given).

The smallest such number will be the remainder itself. The next higher number that satisfies the given conditions is the L.C.M. of the given numbers (i.e., divisors) plus the remainder given, i.e., add the remainder (which is the same in all cases) to the L.C.M. of the given numbers (i.e., divisors).

To find any larger number that satisfies a given condition, we will first need to find out a multiple of the L.C.M. in that range and add the remainder to this multiple of the L.C.M.

The general rule can be written as follows:

Any number which when divided by $p$, $q$ or $r$ leaving the same remainder $s$ in each case will be of the form $k$ (L.C.M. of $p$, $q$ and $r$) + $s$

where $k = 0, 1, 2, ...$

If we take $k = 0$, then we get the smallest such number.

**L.C.M.—Model 2:** In this model, the remainders in the divisions given will not be the same but the difference between the divisor and the remainder (i.e., the complement of the remainder) will be the same in each case. For example, you may be asked to find out 'the smallest number which when divided by 4 or 6 gives respective remainders of 3 and 5.' Here, the remainders are not the same as in L.C.M.—Model 1; but the difference between the divisor and the remainder is same in each case. In the first case the difference between the divisor and the remainder is $1 (= 4 - 3)$. In the second case also the difference between the divisor and the remainder is $1 (= 6 - 5)$.

The smallest such number is L.C.M. minus constant difference (the constant difference being the difference between the divisor and the corresponding remainder in all cases).

Similarly, any multiple of the L.C.M. minus the constant remainder also will satisfy the same condition.

In the example considered above, the L.C.M. of 4 and 6 is 12 and hence, the required number is 11 (which is equal to $12 - 1$).

The general rule can be written as follows:

Any number which when divided by $p$, $q$ or $r$ leaving respective remainders of $s$, t and u where $(p - s) = (q - t) = (r - u) = v$ (say), will be of the form

$k$ (L.C.M. of $p$, $q$ and $r$) - v

The smallest such number will be obtained by substituting $k = 1$.

**L.C.M.—Model 3:** In this model the remainders will not be the same and even the differences between each of the given divisors and the corresponding remainders also will not remain the same.

Let us take an example and see how to solve this type of problem.

Find out the smallest number which when divided by 7 gives a remainder of 3 and when divided by 5 gives the remainder of 2.

Solution Here, the remainders are not the same. The difference between the divisor and the remainder in the first case is 4 and in the second case, is 3.

Take the larger of the two given divisors –7 in this case. The required number, when divided by 7 gives a remainder of 3. We know that a number when divided by 7 giving a remainder of 3 is of the form $7k + 3$, which means we are looking for a number of the form $7k + 3$.

Since the same number, when divided by 5 gives a remainder of 2, this number $(7k + 3)$ when divided by 5 gives a remainder of 2. We know that if there is a remainder in a division, by subtracting the remainder from the given number, the resulting number will then be exactly divisible by the divisor. This means, if 2 is subtracted from $(7k + 3)$, the resulting number, i.e., $7k + 1$ will be exactly divisible by 5. We should now give values of 0, 1, 2, … to $k$ and find out for what value of $k$, $7k + 1$ will be divisible by 5.

The smallest value of $k$ which satisfies the above condition, we notice, is 2 and hence, $k = 2$ will give us a number that we are looking for. Since the number, we said, is $7k + 3$ the number is $7 \times 2 + 3$, i.e., 17. Hence, 17 is the smallest number which satisfies the two given conditions.

The next higher number which satisfies this condition is obtained by adding L.C.M. of 7 and 5 to the smallest number 17 found above. In this manner by adding multiples of 35 (which is L.C.M. of the two given numbers) to 17, we get a series of numbers that satisfy the given conditions. In other words any number of the form $(35m + 17)$ will satisfy the given conditions.

From this, we can also find out the smallest 4 digit number, largest 5 digit number, etc. that will satisfy the given conditions.

For example, let us find out the largest five-digit number that satisfies the conditions that the remainders are 3 and 2, respectively, when divided by 7 and 5.

Since we know that any number that satisfies the above condition will be of the form $(35m + 17)$ and we want the largest 5-digit such number, we need to find a number close to 99999, i.e., $35m + 17 = 99999 \Rightarrow 35m = 99982 \Rightarrow$ we need to find a multiple of 35 which

less than or equal to 99982 (and we have already learnt how to find the multiple of a given number which is less than or equal to another given number). A multiple of 35 less than or equal to 99982 is 99960 (i.e., $35m = 99960$). Hence, the required number which is $35m + 17$ will then be equal to 99960 + 17, i.e., 99977

**H.C.F.—Model 1:** In this model, we have to identify the largest number that exactly divides the given dividends (which are obtained by subtracting the respective remainders from the given numbers).

**The largest number with which the numbers $p$, $q$ or $r$ are divided giving remainders of $s$, t and u respectively will be the H.C.F. of the three numbers $(p - s)$, $(q - t)$ and $(r - u)$.**

**H.C.F.—Model 2:** In this model, the problem will be as follows:

'**Find the largest number with which if we divide the numbers $p$, $q$ and $r$, the remainders are the same.**'

**Take the difference between any two pairs out of the three given numbers. Let us say we take the two differences $(p - q)$ and $(p - r)$. The H.C.F. of these numbers will be the required number.**

**Here, the required number = H.C.F. of $(p - q)$ and $(p - r)$ = H.C.F. of $(p - q)$ and $(q - r)$ = H.C.F. of $(q - r)$ and $(p - r)$**

## ❒ SUCCESSIVE DIVISION

If the quotient of a division is taken and this is used as the dividend in the next division, such a division is called '**successive division**.' A successive division process can continue upto any number of steps – until the quotient in a division becomes zero for the first time, i.e., the quotient in the first division is taken as dividend and divided in the second division; the quotient in the second division is taken as the dividend in the third division; the quotient in the third division is taken as the dividend in the fourth division and so on.

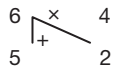
If we say that 2479 is divided successively by 3, 5, 7 and 2, then the quotients and remainders are as follows in the successive division.

| Dividend | Divisor | Quotient | Remainder |
|---|---|---|---|
| 2479 | 3 | 826 | 1 |
| 826 | 5 | 165 | 1 |
| 165 | 7 | 23 | 4 |
| 23 | 2 | 11 | 1 |

Here, we say that when 2479 is successively divided by 3, 5, 7 and 2 the respective remainders are 1, 1, 4 and 2.

**Alternate Method:**

Divisors:




**Remainders:** The smallest number satisfying the given conditions is found by using the following method. Each divisor and the remainder it leaves are written as shown above. Starting with the last remainder, each remainder is multiplied with the previous divisor and added to that divisor's remainder. This procedure is carried out until the divisor's remainder is the first remainder.

Smallest possible value of the number = (6) (2) + 5 = 17

General form of the number = $k$ (6 × 4) + 17 = 24$k$ + 17 where $k$ is any whole number.

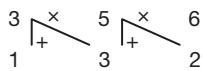
The number would be the largest two-digit number when $24k + 17 < 100$ and $k$ is maximum, i.e., $k < 3^{11}/_{24}$ and $k$ is maximum, i.e., $k = 3$.

∴ Largest two-digit number = 89

∴ required remainder = 8

**Alternate Method:**

Divisors:




**Remainders:** Smallest possible value of the number = ([(5 × 2) + 3] × 3) + 1 = 40

General form of the number = $k \times (3 \times 5 \times 6) + 40 = 90\,k + 40$, where $k$ is any whole number.

If $90\,k + 40 < 1000$, $k < 10\frac{2}{3}$

∴ $k$ has 11 possibilities (i.e., 0 to 10)

## ❒ Factorial

Factorial is defined for positive integers. It is denoted by !. 'Factorial $n$' is written as $n!$ is defined as the product of all the integers from 1 to $n$.

Therefore, $n! = 1 \times 2 \times 3 \ldots \times (n - 1) \times n$

0! is defined to be equal to 1.

0! = 1 and 1! is also equal to 1.

## ❒ IGP of a Divisor in a Number

Very often we would like to know how many times we can divide a given number by another and continue to get integral quotients. We first consider prime divisors and then other divisors.

If a single number is given we simply represent it in its canonical form (the simplest and must convenient form). For example, consider $N = 258{,}048$.

By trial, we express $N = 2^{12}3^27^1$. We see immediately that $N$ can be divided by 2 a total of 12 times, by 3 two times and by 7 just once. In other words the Index of the Greatest Power (IGP) of 2 in $N$ is 12, of 3 is 2 and of 7 is 1.

## ❒ IGP of a Number in N!

This model involves finding the Index of the Greatest Power (IGP) of a divisor that divides the factorial of a given number (say $N$). (The statement '$a$ divides $b$' means the remainder of $b$ divided by '$a$' is 0. In this case, we also say '$b$ is divisible by $a$'.)

Please note that this method is applicable only if the number whose greatest power is to be found out is a prime number.

If the divisor (say D) is not a prime number, then we resolve it into its prime factors. Let $D = p^mq^n$ (where $p$, $q$ are primes and $m$, $n$ are positive integers). We first determine the IGP of $p$ that divides $N$ and the IGP of $q$ that divides $N$. Let these be $a$ and $b$, respectively. Therefore, the IGP of $p^m$ that divides $N$ is $\left\lfloor \frac{a}{m} \right\rfloor$ and the IGP of $q^n$ that divides $N$ is $\left\lfloor \frac{b}{n} \right\rfloor$. Finally, the IGP of D that divides $N$ is the smaller of $\left\lfloor \frac{a}{m} \right\rfloor$ and $\left\lfloor \frac{b}{n} \right\rfloor$. [$\lfloor x \rfloor$ is the greatest integer less than or equal to $x$.]

The following two results will prove to be extremely useful in problems on IGPs.

Let the IGP of $p$ in A and B be $m$ and $n$, respectively.

(1) The IGP of $p$ in AB is $m + n$.

(2) (a) If $m \neq n$, the IGP of $p$ in A + B is the smaller of $m$ and $n$.

(b) If $m = n$, the IGP of $p$ in A + B is at least $m$. It could be more. (For example the IGP of 2 in 58 is 1 and the IGP of 2 in 6 is also 1. But the IGP of 2 in 58 + 6 is 6.)

To understand these type of problems, please refer to the solved examples

## ❒ ALGEBRAIC IDENTITIES

There are a number of identities that we have studied in lower classes. We consolidate them here. We can classify them on two criteria

1. the number of symbols that are used and
2. the degree of each term in the identity.

**Identities with two symbols (degree 2)**

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$a^2 - b^2 = (a+b)(a-b)$

**Identities with two symbols (degree 3)**

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

**Identities with three symbols (degree 2)**

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

$(a+b-c)^2 = a^2 + b^2 + c^2 + 2ab - 2ac - 2bc$

$(a-b-c)^2 = a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$

$(x+a)(x+b) = x^2 + x(a+b) + ab$

**Identities with three symbols (degree 3)**

$(a+b)(b+c)(c+a) = a(b^2 + c^2) + b(c^2 + a^2) + c(a^2 + b^2) + 2abc$

$= a^2(b+c) + b^2(c+a) + c^2(a+b) + 2abc$

$= ab(a+b) + bc(b+c) + ca(c+a) + 2abc$

$(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc$

**Examples:**

1. $112^2 = (100 + 12)^2$
   $= 100^2 + (2 \times 100 \times 12) + 12^2 = 12544$
2. $89^2 = (100 - 11)^2$
   $= 100^2 - (2 \times 100 \times 11) + 11^2 = 7921$
3. $17 \times 23 = (20 - 3)(20 + 3) = 20^2 - 3^2 = 391$
4. $17^2 = (17 + 3)(17 - 3) + 3^2$
   $= 20 \times 14 + 9 = 289$
5. $39^2 = (39 + 1)(39 - 1) + 1^2 = 40 \times 38 + 1^2$
   $= 1520 + 1 = 1521$
6. $13^3 = (10 + 3)^3$
   $= 10^3 + 3^3 + (3 \times 10 \times 3)(10 + 3) = 2197$

## ❒ SOME IMPORTANT POINTS

Please note the following points which will be very useful in solving problems on Numbers.

1. When any two consecutive integers are taken, one of them is odd and the other is even. Hence, the product of any two consecutive integers is always even, i.e., divisible by 2.

Two consecutive integers can be written in the form of $n$ and $n-1$ or $n$ and $n+1$. Hence, any number of the form $n(n-1)$ or $n(n+1)$ will always be even.

2. Out of any 3 consecutive integers, one of them is divisible by 3 and at least one of the three is definitely even. Hence, the product of any 3 consecutive integers is always divisible by 6.
   Three consecutive integers can be of the form $(n-1)$, $n$ and $(n+1)$. The product of 3 consecutive integers will be of the form $(n-1)n(n+1)$ or $n(n^2-1)$ or $(n^3-n)$. Hence, any number of the form $(n-1)n(n+1)$ or $n(n^2-1)$ or $(n^3-n)$ will always be divisible by 6.
3. Out of any $n$ consecutive integers, exactly one number will be divided by $n$ and the product of $n$ consecutive integers will be divisible by $n$!
4. Any prime number greater than 3 can be written in the form of $6k+1$ or $6k-1$. The explanation is: Let $p$ be any prime number greater than 3. Consider the three consecutive integers $(p-1)$, $p$ and $(p+1)$.

Since $p$ is a prime number greater than 3, $p$ CANNOT be even. Since $p$ is odd, both $(p-1)$ and $(p+1)$ will be even, i.e., both are divisible by 2.

Also, since, out of any three consecutive integers, one number will be divisible by 3, one of the three numbers $(p-1)$, $p$ or $(p+1)$ will be divisible by 3. But, since $p$ is a prime number, that too greater than 3, $p$ cannot be divisible by 3. Hence, either $(p-1)$ or $(p+1)$, one of them and only one of them, is definitely divisible by 3.

If $(p-1)$ is divisible by 3, since it is also divisible by 2, it will be divisible by 6, i.e., it will be of the form $6k$. If $(p-1)$ is of the form $6k$, then $p$ will be of the form $(6k+1)$.

If $(p+1)$ is divisible by 3, since it is also divisible by 2, it will be divisible by 6, i.e., it will be of the form $6k$. If $(p+1)$ is of the form $6k$, then $p$ will be of the form $(6k-1)$.

Hence, any prime number greater than 3 will be of the form $(6k+1)$ or $(6k-1)$.

## ❒ LAST DIGIT OF ANY POWER

The last digits of the powers of any number follow a cyclic pattern, i.e., they repeat after certain number of steps. If we find out after how many steps the last digit of the powers of a number repeat, then we can find out the last digit of any power of any number.

Let us look at the powers of 2.
Last digit of $2^1$ is 2.
Last digit of $2^2$ is 4.
Last digit of $2^3$ is 8.
Last digit of $2^4$ is 6.
Last digit of $2^5$ is 2.

Since last digit of $2^5$ is the same as the last digit of $2^1$, then onwards the last digit will start repeating, i.e., digits of $2^5$, $2^6$, $2^7$, $2^8$ will be the same as those of $2^1$, $2^2$, $2^3$, $2^4$. Then the last digit of $2^9$ is again the same as the last digit of $2^1$, and so on. So, we have been able to identify that for powers of 2 the last digits repeat after every 4 steps. In other words, whenever the power is a multiple of 4, the last digit of that number will be the same as the last digit of $2^4$.

Suppose we want to find out the last digit of $2^{67}$, we should look at a multiple of 4 which is less than or equal to the power 67. Since 64 is a multiple of 4, the last digit of $2^{64}$ will be the same as the last digit of $2^4$.

Then the last digits of $2^{65}$, $2^{66}$ and $2^{67}$ will be the same as the last digits of $2^1$, $2^2$ and $2^3$, respectively. Hence, the last digit of $2^{67}$ is the same as the last digit of $2^3$, i.e., 8.

Similarly, we can find out the last digit of $3^{74}$ by writing down the pattern of the powers of 3.
Last digit of $3^1$ is 3.
Last digit of $3^2$ is 9.
Last digit of $3^3$ is 7.
Last digit of $3^4$ is 1.
Last digit of $3^5$ is 3.

The last digit repeats after 4 steps (like in the case of powers of 2).

To find the last digit of $3^{74}$, we look for a multiple of 4 which is less than or equal to 74. Since 72 is multiple of 4, the last digit of $3^{72}$ will be the same as that of $3^4$. Hence, the last digit of $3^{74}$ will be the same as the last digit of $3^2$, i.e., 9.

## ❒ LAST DIGIT OF A SUM OR PRODUCT

The problem consists of finding the last digit of the sum of two numbers each of which is a power of some integer. For example, you may be asked to find out the last digit of the sum $2^{67} + 3^{74}$.

In general, when we want to find out the last digit of the sum of two numbers, we can just take the last digit of the two numbers and add them up. That will be the last digit of the sum. The last digit of 243 + 456 will be the same as the sum of the last digits of the two numbers, i.e., the sum of 3 and 6, which is 9. Similarly,

in the case of $2^{67} + 3^{74}$ also, the last digit will be equal to the sum of the last digits of the two terms $2^{67}$ and $3^{74}$.

We have already looked at finding out the last digit of powers like $2^{67}$ and $3^{74}$. Hence, the last digit of $2^{67} + 3^{74}$ is 8 + 9, i.e., 7.

Similarly, the last digit of a product will be equal to the last digit of the product of the last digits of the two given numbers.

For example, the last digit of the product $2^{67} \times 3^{74}$ will be equal to the last digit of the product of the last digit of $2^{67}$ and the last digit of $3^{74}$, i.e., the last digit of $8 \times 9$, i.e., 2. Hence, the last digit of $2^{67} \times 3^{74}$ is 2.

## ❒ Finding the Remainder in Divisions Involving Powers of Numbers

It is explained below with the help of an example.

For example, to find the reminder of $3^{43}$ when divided by 4.

Let us find the pattern that the remainders follow when the successive powers of 3 are divided by 4.

Remainder of $3^1$ when divided by 4 = 3

Remainder of $3^2$ when divided by 4 = 1

Remainder of $3^3$ when divided by 4 = 3

∴ The remainder repeats after 2 steps and it is 3 when the exponent of 3 is odd and it is 1 when the exponent of 3 is even.

∴ Required remainder = 3 (since the power of 3 is odd)

**Pattern Method:** Similar to the last digit of the powers of a number repeating in a certain pattern, the remainders of powers of a number also follow a certain pattern. If we identify the pattern in which the remainders repeat, we can find out the remainder of any division given.

To solve the example given above, let us find the pattern that remainders follow when various powers of 2 are divided by 7.

Remainder when $2^1$ is divided by 7 is 2.

Remainder when $2^2$ is divided by 7 is 4.

Remainder when $2^3$ is divided by 7 is 1.

Remainder when $2^4$ is divided by 7 is 2.

We find that the remainder repeats in the fourth step, i.e., after 3 steps. So,

- the remainder of $2^4$ when divided by 7 is the same as that when $2^1$ is divided by 7, i.e., 2.
- the remainder of $2^5$ when divided by 7 is the same as that when $2^2$ is divided by 7, i.e., 4.
- the remainder of $2^6$ when divided by 7 is the same as that when $2^3$ is divided by 7, i.e., 1.
- the remainder of $2^7$ when divided by 7 is the same as that when $2^1$ is divided by 7, i.e., 2.

And so on.

If we take $2^{54}$, since 54 is divisible by 3, then $2^{54}$ itself completes a cycle of 3 steps, and hence, the remainder when $2^{54}$ is divided by will be the same as that when $2^3$ is divided by 7. Hence, the remainder is 1.

**Remainder Theorem:** We can apply Remainder Theorem to find the remainder in problems like the one discussed above. Let us first look at Remainder Theorem and understand it.

Remainder Theorem states that when $f(x)$, a polynomial function in $x$ is divided by $x - a$, the remainder is $f(a)$.

A polynomial function in $x$ is a function where $x$ will appear only in the form of $x^n$ and not in any other form, where $n$ is a positive integer.

Let us take an example to understand Remainder Theorem.

When the function $x^2 + 2x - 3$ is divided by $x - 1$, the remainder will be $f(1)$. This is because, as per Remainder Theorem, when the divisor is $(x - a)$, the remainder is $f(a)$. Here, the divisor is $x - 1$ and hence, the remainder is $f(1)$. To get $f(1)$, we should substitute $x = 1$ in the given equation. As we get $f(1) = 0$, the remainder in this case is 0. {Note that when $f(x)$ is divided by $x - a$, if the remainder is 0, then $x - a$ will be a factor of $f(x)$. So, in this case, $(x - 1)$ is a factor of $x^2 + 2x - 3$}.

When the function $x^2 + 2x + 3$ is divided by $x + 1$, the reminder will be $f(-1)$ which is $(-1)^2 + 2(-1) + 3$, i.e., 2.

Now, let us take the example of finding the remainder when $2^{54}$ is divided by 7 (which was solved by the Pattern Method above) and solve it by Remainder Theorem Method.

In the division $2^{54}/7$, the dividend is $2^{54}$ and the divisor is 7. Since the numerator is in terms of powers of 2, express the denominator also in terms of powers of 2. In this case, 7 can be written as 8 – 1 which is $2^3 - 1$. So, now the denominator is in terms of $2^3$, the numerator, i.e., the dividend should be rewritten in terms of $2^3$ which will be $(2^3)^{18}$. Now, the given problem reduces to finding out the remainder when $(2^3)^{18}$ is divided by $2^3 - 1$. Here, if we consider $2^3$ as $x$, it is equivalent to finding out the remainder when $x^{18}$ is divided by $(x - 1)$ which, as per Remainder Theorem, is $f(1)$, i.e., the remainder is obtained by substituting 1 in place of $x$. So, the remainder will be $(1)^{18}$, i.e., 1.

**NOTE**

When a negative remainder is obtained, add the divisor to get the equivalent positive remainder.

**Remainder Theorem Method**

$$\frac{2^{201}}{5} = \frac{2\left(2^2\right)^{100}}{2^2-(-1)}$$

By remainder theorem, required remainder = $2(-1)^{100} = 2$

It is evident that the remainder theorem is more suited to cases where the denominator (i.e., the divisor) can be written in the form of one more or one less than some power of the base in the numerator. For example, in case of $2^{54}/7$, since the base in the numerator is 2, the denominator 7 has to be written as one more or one less than some power of 2. In this case it can be written as $2^3 - 1$. In cases where it is not possible to write it in this manner, then applying the Pattern Method is the easiest method.

## ❒ LAST TWO DIGITS OF $a^m$

The terms of any Geometric progression (GP) leave a cyclic pattern of remainders when divided by any divisor. The sequence of powers of the base '$a$' is a GP with common ratio equal to '$a$'.

If we take the divisor as 100, the remainder is simply the last two digits. We'll find it convenient to consider the following 4 cases separately.

(1) The base ends in 0.
(2) The base ends in 5.
(3) The base ends in 1, 3, 7, or 9.
(4) The base ends in 2, 4, 6, or 8.

The first two cases are very simple.

(1) If $a$ ends in 0, the square and all higher powers end in at least 2 zeroes.
(2) If $a$ ends in 5, the powers either all end in 25 or end alternately in 25 and 75, depending on whether the tens digit of $a$ is even or odd.
(3) If the base ends in 1, 3, 7 or 9, there is a cycle of at the most 20 distinct remainders. The twentieth power ends in 01. (The cycle length could also be some factor of 20, i.e., 1, 2, 4, 5 or 10)
(4) If the base ends in 2, 4, 6 or 8, there is a cycle of at the most 20 distinct remainders. The twentieth power ends in 76. The cycle length could also be some factor of 20.
   (i) Moreover, if $a = 4k$, the second set and all the subsequent sets of 20 remainders are exactly the same as the first set.
   (ii) But if $a = 4k + 2$, it is not possible to get $4k + 2$ as the last two digits in any higher power. All such powers are multiples of 4. Consequently, of the forty 'two-digit' numbers (02, 04, 06, 08, 12, 14, 16, 18, …, 92, 94, 96, 98) only twenty, viz 04, 08, 12, 16, 24, …, 92, 96 can occur as the last two digits in the higher powers. If 02, 06, 14 etc do occur, they can occur only as the first power. We find that the last two digits of $a^{21}$ are obtained by adding 50 to $4k + 2$ (For example, $2^{21}$ ends in 52, $6^{21}$ ends in 56, $14^{21}$ ends in 64, etc.). Therefore, while the second set of 20 remainders is almost the same as the first set (differing only in the first remainder), all subsequent sets are exactly the same as the second set.

The examples below will illustrate these points.

Consider point (3) above.

The last two digits of successive powers of 13 are 13, 69, 97, 61 … 01; 13, 69, 97, 61…01, etc.

Consider point (i) above.

The last two digits of successive powers of 4 are 04, 16, 64, 56, … 76; 04,16, … 76; etc.

Consider point (ii) above.

The last two digits of successive powers of 2 are 02, 04, 08, 16, …, 76; 52,04,08,16, …, 76 (instead of the 02, we get 52).

These 6 points, 1, 2, 3, 4, 4.1, 4.2 (whichever is applicable) should be used in all problems on the last two digits.

For example, to find the remainder when $N = 817^{673}$ is divided by 100. Alternatively, find the last two digits of $N$.

We are interested only in the last two digits of $N$. We need to consider only the last two digits of 817, i.e., 17. Successive powers of 17 (or any other number) show a cyclic pattern, when divided by 100 (or any other divisor). We can list these remainders until we discover the point, where the repetition starts.

| | | | | |
|---|---|---|---|---|
| 17 | 57 | 97 | 37 | 77 |
| 89 | 69 | 49 | 29 | 09 |
| 13 | 73 | 33 | 93 | 53 |
| 21 | 41 | 61 | 81 | 01 |

$17^1 = 17$

To get the next number, we take only the last two digits of $17^2$, i.e., 89. To get the next number, we take only the last two digits of 17(89). We need not perform the complete multiplication. We need only the units

and tens digits. The units digit is 3 and there are 3 parts to the tens digit – the carry over of 6, the units digit of 9(1) and 8(7), i.e., 6 + 9 + 6. Again we need only the units digit of this which is 1.

∴ The last two digits of $17^3$ are 13. Similarly, we can work out the other numbers.

It is convenient to break the column after every 4 steps (the units digit is found to be the same in each row. This serves as a check to our calculations). After we get 01, the next 20 powers show the same pattern.

In the given example, as 673 = 20 (33) + 13.

∴ The 13*th* number in the list, i.e., 37, is our answer.

In general, we find that if we are interested in the last 2 digits, we need to go up to at most 20 steps. In some cases the period may be some factor of 20 (1, 2, 4, 5 or 10).

Consider the powers of 01. The pattern is 01; 01, etc. The period is 1.

Consider powers of 49, 51 or 99. The patterns are:

49, 01; 49, 01; etc.

51, 01; 51, 01; etc.

99, 01; 99, 01; etc., i.e., the period is 2.

Consider powers of 07, 43, 57 or 93. The patterns are:

07, 49, 43, 01, etc.

43, 49, 07, 01, etc.

57, 49, 93, 01, etc.

93, 49, 57, 01, etc., i.e., the period is 4.

Consider powers of 21.

The pattern is 21, 41, 61, 81, 01, etc. The period is 5.

Consider powers of 29, 71 or 79. The patterns are:

29, 41, 89, 81, 49, 21, 09, 61, 69, 01, etc.

71, 41, 11, 81, 51, 21, 91, 61, 31, 01, etc.

79, 41, 39, 81, 99, 21, 59, 61, 19, 01, etc.

The period is 10.

## ❒ SOME IMPORTANT THEOREMS

### ❒ Binomial Theorem

For any natural number $n$, $(a+b)^n$.

$= {}^nC_0 a^n + {}^nC_1\, a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_{n-1}\, ab^{n-1} + {}^nC_n b^n$.

where ${}^nC_r$ is the number of ways of choosing $r$ objects out of $n$ distinct objects and is given by

$$ {}^nC_r = \frac{n(n-1)(n-2)\ldots(n-r+1)}{1(2)(3)\ldots(r)} = \frac{n!}{r!\,(n-r)!} $$

It can be observed that $(a+b)^n = a^n$ + (A multiple of $b$) = (A multiple of $a$) + $b^n$.

### ❒ Fermat's Little Theorem

If $p$ is prime and H.C.F. $(a, p) = 1$, then $a^{p-1} - 1$ is a multiple of $p$.

For example, take $p = 5$, $a = 3$. From the theorem, $3^4 - 1$ or 80 is a multiple of 5.

If we take successive powers of 3, then we get all the possible remainders.

$3^1 = 3$, $3^2 = 4$, $3^3 = 2$, $3^4 = 1$ (also, $3^5 = 3$, $3^6 = 3^2$, $3^7 = 3^3$, etc.). At a certain stage, we get a remainder of 1, and after that, the pattern repeats. In this example, the pattern is 3, 4, 2, 1; 3, 4, 2, 1; etc. The pattern length is 4. In general, it would be $(p-1)$ or some factor of $(p-1)$.

### ❒ Wilson's Theorem

If $p$ is prime, $(p-1)! + 1$ is a multiple of $p$.

For example,

$(2-1)! + 1 = 2(1)$, $(3-1)! + 1 = 3(1)$, $(5-1)! + 1 = 5(5)$, $(7-1)! + 1 = 721 = 7(103)$, and so on.

### ❒ Remainder of a Number When Divided by $10^n \pm 1$

This is best illustrated with the below examples:

For example, find the remainder when 123, 123, … (up to 300 digits) is divided by 999.

To find the remainder when some number (say $N$) is divided by 9 (or $10^1 - 1$), we add up all the digits of $N$ to get (say $S_1$) and divide $S_1$ by 9.

Similarly, to find the remainder when $N$ is divided by 99 (or $10^2 - 1$), we start at the right end of $N$, group the digits two at a time and add up all the groups to get, say $S_2$. Then we find the remainder of $S_2/99$.

In general to find the remainder when $N$ is divided by $D_n$ = 99 … 9 ($n$ nines) or $(10^n - 1)$, we start at the right end of $N$, group the digits $n$ at a time and add up all the groups to get say $S_n$.

$$ \text{Rem}\ \frac{N}{D_n} = \text{Rem}\ \frac{S_n}{D_n} $$

Similarly, we can start with the remainder rule for 11 and work out the corresponding rules for 101, 1001, 10001, etc. All this is an application of Remainder theorem.

Here, $N = 123, 123, \ldots, 123$ (a total of 300 digits or 100 groups) $= 123\ (1000^{99}) + 123\ (1000^{98}) + \ldots + 123\ (1000^{1}) + 123$

Now, let $N = f(1000)$; When $N$ or $f(1000)$ is divided by 999 or $(1000 - 1)$, the remainder is $f(1)$, i.e., 123(100) by remainder theorem. [i.e., $S_3 = 123\ (100)$]

$$\therefore \text{Rem}\ \frac{N}{999} = \text{Rem}\ \frac{12300}{999} = \text{Rem}\ \frac{12+300}{999} = 312$$

## ❒ Rules Pertaining to $a^n + b^n$ or $a^n - b^n$

Sometimes, there will be problems involving numbers that can be written in the form $a^n + b^n$ or $a^n - b^n$ which can be simplified using simple rules. Let us first look at the rules pertaining to both $a^n + b^n$ and $a^n - b^n$, $a$, $b$ and $n$ being positive integers.

The following rules should be remembered for numbers in the form of $a^n - b^n$.

1. It is always (i.e.,when $n$ is even as well as odd) divisible by $a - b$.
2. When $n$ is even it is also divisible by $a + b$.
3. When $n$ is odd it is divisible by $a + b$, if $a + b$ is a factor of $2.b^n$.

The following rules should be remembered for numbers in the form of $a^n + b^n$.

1. When $n$ is odd it is divisible by $a + b$.
2. When $n$ is odd, it is divisible by $a - b$, when $a - b$ is a factor of $2.b^n$.
3. When $n$ is even, it is divisible by $a + b$, if $a + b$ is a factor of $2b^n$.

## ❒ SOME IMPORTANT IDENTITIES

$a^N - b^N = (a - b)\ (a^{N-1} + a^{N-2}\ b + a^{N-3}\ b^2 + \ldots + a^2\ b^{N-3} + ab^{N-2} + b^{N-1})$ for all positive integer values of $N$.

$a^N + b^N = (a + b)\ (a^{N-1} - a^{N-2}\ b + a^{N-3}\ b^2 - a^{N-4}\ b^3 + \ldots - a^3\ b^{N-4} + a^2\ b^{N-3} - ab^{N-2} + b^{N-1})$ for all odd positive integer values of $N$.

## SOLVED EXAMPLES

**1.01.** Simplify $[5 + 1/12$ of $\{38 - (10 + \overline{7-3}) + 1/2$ of $24\} - 3]$.

**Sol:** By applying the BODMAS rule,

$[5 + 1/12$ of $\{38 - (10 + \overline{7-3}) + 1/2$ of $24\} - 3]$

$= [5 + 1/12$ of $\{38 - (10 + 4) + 1/2$ of $24\} - 3]$
$= [5 + 1/12$ of $\{38 - 14 + 12\} - 3]$
$= [5 + 1/12$ of $\{36\} - 3] = [5 + 3 - 3] = 5$

**1.02.** Express $0.\overline{6}$ in the form of a fraction.

**Sol:** $0.\overline{6} = 0.666\ldots$

Let $x = 0.66$ (1)
As the period is of one digit, we multiply the given number by $10^1$, i.e., 10.
Therefore, $10x = 6.666$ (2)
(2) − (1) gives $9x = 6$
$\Rightarrow x = 6/9 = 2/3$

**1.03.** Express $0.\overline{81}$ in the form of a fraction.

**Sol:** $0.\overline{81} = 0.818181$ .....

Let $x = 0.8181$ (1)
As the period is containing 2 digits, we multiply by $10^2$, i.e., 100.
Therefore, $100\ x = 81.8181$ (2)
(2) − (1) gives $99\ x = 81$
$\Rightarrow x = 81/99 = 9/11$

**1.04.** Express the recurring decimal $0.\overline{024}$ in the form of a fraction.

**Sol:** $0.\overline{024} = 0.024024024$

Let $x = 0.024024$ (1)
As the period contains 3 digits, we multiply with $10^3$, i.e., 1000; therefore,
$1000\ x = 24.024024$ (2)
(2) − (1) gives $999x = 24$
$\Rightarrow x = 24/999 = 8/333$

**1.05.** Express $0.2\overline{7}$ as a fraction.

**Sol:** Let $x = 0.2\overline{7}$, then $10x = 2.\overline{7} = 2 + 0.\overline{7}$

$= 2 + 7/9$ (since $0.\overline{7} = 7/9$)

$\Rightarrow 10x = 25/9$
$\Rightarrow x = 25/90 = 5/18$

**1.06.** Express $0.2\overline{79}$ in the form of a fraction.

**Sol:** Let $x = 0.2\overline{79}$

$10x = 2.\overline{79} = 2 + 79/99 = 277/99$

$x = 277/990$

**1.07.** Simplify:

$$\left[4+\frac{1}{9}\text{ of }\left\{30-\left(19+\overline{8-6}\right)+\frac{1}{2}\text{of }54\right\}+3\right]$$

**Sol:** $$\left[4+\frac{1}{9}\text{ of }\left\{30-\left(19+\overline{8-6}\right)+\frac{1}{2}\text{of }54\right\}+3\right]$$

$$=\left[4+\frac{1}{9}\text{ of }\left\{30-(19+2)+\frac{1}{2}\text{of }54\right\}+3\right]$$

$$=\left[4+\frac{1}{9}\text{ of }\{30-21+27\}+3\right]$$

$$=\left[4+\frac{1}{9}\text{ of }\{36\}+3\right]=[4+4+3]=11$$

**1.08.** Express $0.\overline{6}$ in the form of a fraction.

**Sol:** $0.\overline{6} = 0.666\ldots\ldots\ldots\ldots$

Let $x = 0.66$ (1)
As the period is of one digit, we multiply the given number by $10^1$, i.e., 10
Therefore,
$10x = 6.666$ (2)
(2) – (1) gives, $\Rightarrow 9x = 6$
$\Rightarrow x = 6/9 = 2/3$

**1.09.** Express $0.\overline{81}$ in the form of a fraction.

**Sol:** $0.\overline{81} = 0.818181\ldots\ldots\ldots\ldots$
Let $x = 0.8181$ (1)
As the period is containing 2 digits, we multiply by $10^2$, i.e., 100
Therefore, $100\,x = 81.8181$ (2)
(2) – (1) gives, $99\,x = 81$
$\Rightarrow x = 81/99 = 9/11$

**1.10.** Express the recurring decimal $0.\overline{024}$ in the form of a fraction.

**Sol:** $0.\overline{024} = 0.024024024$

Let $x = 0.024024$ (1)
As the period contains 3 digits, we multiply with $10^3$, i.e., 1000; therefore,
$1000\,x = 24.024024$ (2)
(2) – (1) gives, $999x = 24$
$\Rightarrow x = 24/999 = 8/333$

**1.11.** Express $0.2\overline{7}$ as a fraction.

**Sol:** Let $x = 0.2\overline{7}$, then $10x = 2.\overline{7} = 2 + 0.\overline{7}$

$= 2 + 7/9$ (since $0.\overline{7} = 7/9$)

$\Rightarrow 10x = 25/9$
$\Rightarrow x = 25/90 = 5/18$

**1.12.** Express $0.2\overline{79}$ in the form of a fraction.

**Sol:** Let $x = 0.2\overline{79}$

$10x = 2.\overline{79} = 2 + 79/99 = 277/99$

$x = 277/990$

**1.13.** Find the number of factors of 1225.

**Sol:** If a number can be expressed as a product of prime factors like
$a^p \times b^q \times c^r \times \ldots$
where $a$, $b$ and $c$ are the prime numbers, then the number of factors of the number is $(p + 1)$ $(q + 1)$ $(r + 1)\ldots$
First express 1225 as a product of its prime factors. (Note that to express a given number as a product of its prime factors, we first need to identify the prime factors of the given number by applying the rules of divisibility.)
$1225 = 5 \times 7 \times 5 \times 7 = 5^2 \times 7^2$
Hence, the number of factors 1225 has is $(2 + 1)$ $(2 + 1) = 9$.

**1.14.** How many divisors excluding 1 and itself does the number 4320 have?

**Sol:** Note that the two terms factors and divisors are used interchangeably. First express 4320 in terms of its prime factors.
$4320 = 18 \times 24 \times 10$
$= 3 \times 3 \times 2 \times 3 \times 2 \times 2 \times 2 \times 2 \times 5$
$= 3^3 \times 2^5 \times 5^1$
Hence, 4320 has $(3 + 1)$ $(5 + 1)$ $(1 + 1) = 48$ factors. Excluding 1 and itself, the number has $(48 - 2) = 46$ factors.

**1.15.** In how many ways can 3420 be written as a product of two factors?

**Sol:** By prime factorisation, $3420 = 2 \times 5 \times 2 \times 19 \times 3^2$
$= 2^2 \times 3^2 \times 5^1 \times 19^1$
If a number is expressed as product of prime factors, like $a^p \times b^q \times c^r \times \ldots$, where $a$, $b$, $c\ldots$ are prime

numbers, then the number of ways in which the number can be expressed as a product of two factors = $1/2\ [(p+1)\ (q+1)\ (r+1)\ \ldots]$.
Hence, 3420 can be written as product of two factors in ½ [(2 + 1) (2 + 1) (1 + 1) (1 + 1)] = 18 ways.

**1.16.** In how many ways can the number 52900 be written as a product of two different factors?

**Sol:** First expressing 52900 as a product of its prime factors, we get $52900 = 23^2 \times 2^2 \times 5^2$. Since all the powers are even, the given number is a perfect square. (Remember we can look at writing the number as a product of two factors either including or excluding the 'square root × square root'. Since we have to find the number of ways of writing the number as a product of two 'different' factors, we cannot consider square root × square root)
So, required number of ways is:
$1/2\ \{(2+1)\ (2+1)\ (2+1) - 1\} = 1/2\{27 - 1\} = 13$

**1.17.** What is the product of all the factors of 180?

**Sol:** $180 = 4(45) = 2^2 3^2 5^1$. There are (2 + 1) (2 + 1) (1 + 1) or 18 factors.
If the given number is not a perfect square, at least one of the indices is odd and the number of factors is even. We can form pairs such that the product of the two numbers in each pair is the given number (180 in this example).
∴ The required product is $180^{9}$.
In general, if $N = p^a q^b r^c$ (where at least one of $a$, $b$, $c$ is odd), then the product of all the factors of $N$ is $N^{\frac{d}{2}}$, where $d$ is the number of factors of $N$ and is given by $(a+1)(b+1)(c+1)$.

**1.18.** Let us see what happens when $N$ is a perfect square. Find the product of all the factors of 36.

**Sol:** $36 = 2^2 3^2$ (there are 9 factors)
$1(36) = 2(18) = 3(12) = 4(9) = 6(6)$
∴ The product of all the factors is $36^4$ (6).
In general, let $N = p^a q^b r^c$ where each of $a$, $b$, $c$ is even.
There are $(a + 1)\ (b + 1)\ (c + 1)$ say $d$ factors. We can form $\frac{d-1}{2}$ pairs and we would be left with one lone factor, i.e., $\sqrt{N}$. The product of all these factors is $N^{\frac{d-1}{2}}\ (\sqrt{N}) = N^{\frac{d}{2}}$
∴ Whether or not $N$ is a perfect square, the product of all its factors is $N^{\frac{d}{2}}$, where $d$ is the number of factors of $N$.

**1.19.** What is the product of all the factors of 1728?

**Sol:** The product of the factors of a positive integer $N$ is $N^{k/2}$, where $k$ is the number of factors of $N$.
Now, $1728 = 12^3 = 2^6 3^3$ and $k = (6+1)\ (3+1) = 28$
∴ The product of all the factors of $1728 = 1728^{14}$

**1.20.** Find the number of factors of 1225.

**Sol:** If a number can be expressed as a product of prime factors like $a^p \times b^q \times c^r \times \ldots$
where $a$, $b$, $c$, … are the prime numbers, then the number of factors of the number is $(p + 1)\ (q+1)\ (r+1)\ldots$
First express 1225 as a product of its prime factors. (Note that to express a given number as a product of its prime factors, we first need to identify the prime factors of the given number by applying the rules of divisibility.)
$1225 = 5 \times 7 \times 5 \times 7 = 5^2 \times 7^2$
Hence, the number of factors 1225 has is (2 + 1) (2 + 1) = 9

**1.21.** How many divisors excluding 1 and itself does the number 4320 have?

**Sol:** Note that the two terms factors and divisors are used interchangeably. First express 4320 in terms of its prime factors.
$4320 = 18 \times 24 \times 10$
$= 3 \times 3 \times 2 \times 3 \times 2 \times 2 \times 2 \times 2 \times 5$
$= 3^3 \times 2^5 \times 5^1$
Hence, 4320 has (3 + 1) (5 + 1) (1 + 1) = 48 factors. Excluding 1 and itself, the number has (48 – 2) = 46 factors.

**1.22.** In how many ways can 3420 be written as a product of two factors?

**Sol:** By prime factorisation, $3420 = 2 \times 5 \times 2 \times 19 \times 3^2$
$= 2^2 \times 3^2 \times 5^1 \times 19^1$
If a number is expressed as product of prime factors, like $a^p \times b^q \times c^r \times \ldots$ where $a$, $b$, $c$, … are prime numbers, then the number of ways in which the number can be expressed as a product of two factors = $1/2\ [(p+1)\ (q+1)\ (r+1)\ldots]$.
Hence, 3420 can be written as product of two factors in ½[(2 + 1) (2 + 1) (1 + 1) (1 + 1)] = 18 ways.

**1.23.** In how many ways can the number 52900 be written as a product of two different factors?

**Sol:** First expressing 52900 as a product of its prime factors, we get $52900 = 23^2 \times 2^2 \times 5^2$. Since all the powers

are even, the given number is a perfect square. (Remember we can look at writing the number as a product of two factors either including or excluding the 'square root × square root'. Since we have to find the number of ways of writing the number as a product of two 'different' factors, we cannot consider square root × square root)
So, required number of ways is $1/2\ \{(2+1)\ (2+1)\ (2+1) - 1\} = 1/2\{27-1\} = 13$.

**1.24.** What is the product of all the factors of 180?
**Sol:** $180 = 4(45) = 2^2 3^2 5^1$.
There are (2+1) (2+1) (1+1) or 18 factors.
If the given number is not a perfect square, at least one of the indices is odd and the number of factors is even. We can form pairs such that the product of the two numbers in each pair is the given number (180 in this example).
∴ The required product is $180^9$.
In general, if $N = p^a q^b r^c$ (where at least one of $a$, $b$, $c$ is odd), the product of all the factors of $N$ is $N^{\frac{d}{2}}$, where $d$ is the number of factors of $N$ and is given by $(a+1)(b+1)(c+1)$.

**1.25.** Let us see what happens when $N$ is a perfect square. Find the product of all the factors of 36.
**Sol:** $36 = 2^2 3^2$ (there are 9 factors)
$1(36) = 2(18) = 3(12) = 4(9) = 6(6)$
∴ The product of all the factors is $36^4$ (6).
In general, let $N = p^a q^b r^c$ where each of $a$, $b$, $c$ is even.
There are $(a+1)(b+1)(c+1)$ say $d$ factors. We can form $\frac{d-1}{2}$ pairs and we would be left with one lone factor, i.e., $\sqrt{N}$. The product of all these factors is $N^{\frac{d-1}{2}} (\sqrt{N}) = N^{\frac{d}{2}}$
∴ Whether or not $N$ is a perfect square, the product of all its factors is $N^{\frac{d}{2}}$, where $d$ is the number of factors of $N$.

**1.26.** What is the product of all the factors of 1728?
**Sol:** The product of the factors of a positive integer $N$ is $N^{k/2}$, where $k$ is the number of factors of $N$.
Now, $1728 = 12^3 = 2^6 3^3$ and $k = (6+1)\ (3+1) = 28$
∴ The product of all the factors of $1728 = 1728^{14}$.

**1.27.** Find the largest four-digit multiple of 31.
**Sol:** We take the largest four-digit number possible, i.e., 9999 and divide it by 31. We get a remainder of 17. This remainder 17 is then subtracted from 9999 giving 9982 which is a multiple of 31. Therefore, 9982 is the largest four-digit multiple of 31.

**1.28.** Find the smallest five-digit multiple of 17.
**Sol:** First we consider 10,000, the smallest five- digit number. Dividing 10,000 by 17 we get the remainder 4. We take the difference between the divisor 17 and the remainder 4 which is 13 and add this 13 to 10,000. We get 10,013 which is the smallest five-digit multiple of 17.

**1.29.** Find the L.C.M. of 144, 504 and 720.
**Sol:** Write each number in terms of its prime factors.
$144 = 3^2 \times 2^4$
$504 = 3^2 \times 2^3 \times 7$
$720 = 2^4 \times 3^2 \times 5$
L.C.M. is $2^4 \times 3^2 \times 7 \times 5 = 5040$

**1.30.** Find the L.C.M. of 12, 18 and 27.
**Sol:** By division method

| | |
|---|---|
| 2 | 12, 18, 27 |
| 3 | 6, 9, 27 |
| 3 | 2, 3, 9 |
| | 2, 1, 3 |

L.C.M. = $2 \times 3 \times 3 \times 2 \times 1 \times 3 = 108$

**1.31.** Find the H.C.F. of 324, 576 and 784.
**Sol:** First of all, resolve all the numbers into their prime factors
$324 = 3^4 \times 2^2$
$576 = 3^2 \times 2^6$
$784 = 7^2 \times 2^4$
Then take the product of the factors common to all the numbers.
H.C.F. here will be $2^2 = 4$

**1.32.** Find the H.C.F. of 2223 and 3762.
**Sol:**
2223)3762(1
2223
1539)2223(1
1539
684)1539(2
1368
171)684(4
684
0
Hence, H.C.F. of 2223 and 3762 is 171.

**1.33.** Find the largest three digit multiple of 32.
**Sol:** Largest three digit number = 999. When 999 is divided by 32, the remainder is 7
∴ 999 – 7 = 992 is the largest three digit multiple of 32.

**1.34.** Find the smallest four digit multiple of 32.

**Sol:** Smallest four digit multiple of 32
= Largest three digit multiple of 32 + 32
= 992 + 32 = 1024.

**1.35.** Find the H.C.F. and L.C.M. of 288, 432 and 768 using factorisation method.

**Sol:** 288 = (24) (12)
432 = (24) (18) and 768 = (24) (32)
H.C.F. 288, 432, 768 = 24 × H.C.F. (12, 18, 32)
= 24 × 2 = 48
L.C.M. (288, 432, 768) = 24 × L.C.M. (12, 18, 32)
= 24 × 288 = 6912

**1.36.** Find the H.C.F. of 288 and 432 using long division method. Also find their L.C.M. by division method.

**Sol:** H.C.F.: Let us first find H.C.F. of 288 and 432

| | | |
|---|---|---|
| 1 | | |
| 288 | 432 | |
| 288 | | |
| 2 | | |
| | | |
| | 144 | 288 |
| 288 | | |
| | | 0 |

∴ H.C.F. of 288 and 432 is 144.
L.C.M.:

| | | |
|---|---|---|
| | 288, | 432 |
| 2 | 144, | 216 |
| 2 | 72, | 108 |
| 2 | 36, | 54 |
| 2 | 18, | 27 |
| 3 | 6, | 9 |
| 3 | 2, | 3 |

L.C.M. of 288 and 432
= (2) (2) (2) (2) (3) (3) (2) (3)
$= 2^5 \times 3^3$ = (32) (27) = 864

**1.37.** Find the H.C.F. of 1363 and 1457.

**Sol:**
$$1363\overline{)1457}$$
$$1363$$
$$94\overline{)1362}$$
$$1316$$
$$47\overline{)94}$$
$$94$$
$$0$$
∴ H.C.F. (1367, 1457) = 47

**1.38.** Find the smallest number which when divided by 5 or 11 leaves a remainder of 4 and is greater than the remainder.

**Sol:** Set of such numbers are of the form K [L.C.M. (5, 11)] + 4 where K is a whole number.
We get the required number when K = 1
∴ Smallest number = L.C.M (5, 11) + 4
= 55 + 4 = 59.

**1.39.** Find the smallest number which when divided by 5 or 8, leaves a remainder of 2 in each case and the number being greater than the two divisors.

**Sol:** The L.C.M. of 5 and 8 is 40.
Hence, the required number is 40 + 2 = 42.

**1.40.** Find the largest three-digit number which when divided by 4 or 7 leaves a remainder of 3 in each case.

**Sol:** The L.C.M. of 4 and 7 is 28. Since we are interested in the largest three-digit number, we should first find out the largest three-digit multiple of 28. This can be obtained by first dividing 999 by 28, which leaves a remainder of 19. Hence, 999 – 19 = 980 is the largest three-digit multiple of 28. Now, add the remainder 3 to the number to get 983, which is the required number.

**1.41.** Find the smallest seven-digit number which when divided by 8 or 13 leaves a remainder of 5 in each case.

**Sol:** We need to find the smallest seven-digit multiple of 104 (104 is the L.C.M. of 8 and 13) and add the remainder of 5 to that multiple to get the required number. Take the smallest seven-digit number 1000000 and divide by 104. We get a remainder of 40.
Take the difference between the divisor 104 and the above remainder 40, which is 64. This is added to 1000000 to give 1000064, which is the smallest seven-digit multiple of 104.

Now, add the remainder 5 to get the required number as 1000069

**1.42.** Find the smallest number greater than the divisors, which when divided by 6, 13 and 17 leaves a remainder of 7 in each case.

**Sol:** The smallest number that satisfies the above condition is obtained by taking the L.C.M. of the numbers 6, 13 and 17 and adding the remainder of 7 to it. L.C.M. of 6, 13 and 17 is 1326.
Hence, the required number is 1326 + 7 = 1333

**1.43.** Find the smallest number which when divided by 19 and 23 gives remainders of 13 and 17, respectively.

**Sol:** The L.C.M. of 19 and 23 is 437. The difference between the divisor and the remainder in each case is the same, i.e., 19 – 17 = 6 and 23 – 17 = 6. Hence, the number that satisfies the given conditions will be equal to 437 – 6 = 431.

**1.44.** Find the largest four-digit number which when divided by 7 and 12 gives remainders 5 and 10, respectively.

**Sol:** The difference between the divisor and the remainder is the same in each case, i.e., 7 – 5 = 2 and 12 – 10 = 2. The L.C.M. of 7 and 12 is 84. We will first find the largest four-digit multiple of 84 and subtract 2 from it. The largest four-digit number 9999 when divided by 84 leaves a remainder of 3. The largest four-digit number divisible by 84 is hence, 9999 – 3 = 9996. Hence, the required number is 9996 – 2 = 9994

**1.45.** Find the smallest number which, when divided by 7, 13 and 23 leaves respective remainders of 5, 11 and 21.

**Sol:** The difference between the divisor and the remainder is the same in each case, it is 2. The smallest number satisfying the given condition can be obtained by subtracting 2 from the L.C.M. of the given divisors. The L.C.M. of 7, 13 and 23 is 2093. Hence, the required number is 2093 – 2 = 2091

**1.46.** Find the smallest six-digit number which when divided by 8 leaves a remainder of 3 and when divided by 14 leaves a remainder of 9.

**Sol:** Here, again the difference between the divisor and the remainder in each case is 5, (8 – 3 = 5 and 14 – 9 = 5).
First find the smallest six-digit multiple of 56, the L.C.M. of 8 and 14. 100000 leaves a remainder of 40 when divided by 56. Take the difference between 56 and 40, which is 16 and add it back to 100000 to give us the smallest six-digit multiple of 56 which is 100016. Hence, the required number is 100016 – 5 = 100011.

**1.47.** Find the smallest number which, when divided by 6 leaves a remainder of 2 and when divided by 13 leaves a remainder of 6.

**Sol:** The required number will be in the form of $(6k + 2)$ because when divided by 6 it leaves a remainder of 2. The same number when divided by 13 leaves a remainder of 6. Subtracting this remainder from the number $(6k + 2)$, the resulting number $(6k – 4)$ should be divisible by 13.
Trying out values of 0, 1, 2, for $k$, when $k = 5$, $(6k – 4)$ will be 26 which is divisible by 13. Hence, the required number is $6k + 2 = 6(5) + 2 = 32$

**1.48.** Find the largest number, with which, when 425 and 373 are divided, respective remainders of 2 and 4 are left.

**Sol:** Since 425 when divided by the number gives a remainder of 2 it means 425 – 2 = 423 is exactly divisible by that number. Similarly, 373 – 4 = 369 is also exactly divisible by that number. This means that the number we are looking for is the largest number which will divide 369 and 423 exactly. That will be the H.C.F. of 369 and 423, which is 9.

**1.49.** Find the largest number, with which, when 394 and 658 are divided, leave remainders of 1 and 3, respectively.

**Sol:** As discussed in the previous example, the required number is the H.C.F. of (394 – 1) and (658 – 3), i.e., H.C.F. of 393 and 655. The H.C.F. of 393 and 655 is 131.

**1.50.** Find the largest number, with which, when 472, 832 and 1372 are divided the remainders are the same.

**Sol:** Take the difference between any two numbers out of the three given numbers.
832 – 472 = 360
1372 – 832 = 540
The required number is the H.C.F. of these two differences, i.e., H.C.F. of 360 and 540 which is 180.

**1.51.** Find the largest number, with which, when 247, 457 and 1087 are divided, the remainder in each case is the same.

**Sol:** Taking the difference of two of the numbers at a time, we get 457 – 247 = 210 and 1087 – 457 = 630
The required number is the H.C.F. of the two differences, i.e., H.C.F. of 210 and 630 which is 210.

**1.52.** Find the largest three-digit number which when divided by 8 or 12 leaves a remainder of 2 in each case.

**Sol:** Required number must leave a remainder of 2 when divided by L.C.M (8, 12) = 24.
$\therefore$ It must be of the form $24K + 2$, where K is the largest natural number satisfying $24K + 2 < 1000$.
$\Rightarrow K < 41\frac{7}{12}$; $\therefore K = 41$.
$\therefore$ Largest number = 986.

**1.53.** Find the smallest number which when divided by 4, 11 or 13 leaves a remainder of 1 and is greater than the remainder.

**Sol:** Required number = L.C.M (4, 11, 13) + 1 = 573.

**1.54.** Find the smallest number which when divided by 9 and 11 leaves remainders of 7 and 9, respectively.

**Sol:** Required number = L.C.M (9, 11) − 2 = 97.

**1.55.** Find the largest four-digit number which when divided by 9 and 11 leaves remainders of 7 and 9, respectively.

**Sol:** Required number must be in the form L.C.M. (9, 11) $K - 2$, i.e., $99K - 2$, where $K$ is the largest natural number satisfying $99K - 2 < 10000$.

$\therefore K < 101\frac{1}{33}$

$\therefore K = 101$
$\therefore$ Largest number = 9997

**1.56.** Find the smallest six-digit number which leaves a remainder of 10 when divided by 13 and leaves a remainder of 4 when divided by 7.

**Sol:** Required number must be in the form:
L.C.M. (13, 7) $K - 3$, i.e., $91K - 3$, where $K$ is the smallest natural number satisfying $91K - 3 > 100000$

$$K > \frac{100003}{91} = 1098\frac{85}{91}$$

$\therefore K = 1099$
$\therefore$ Smallest number = 100006

**1.57.** Find the smallest number which leaves a remainder of 7 when divided by 11 and leaves a remainder of 12 when divided by 13.

**Sol:** Let the number be in the forms $11K_1 + 7$ and $13K_2 + 12$ where $K_1$ and $K_2$ have the least possible values.
$11K_1 + 7 = 13K_2 + 12$

$$K_1 = K_2 + \frac{2K_2 + 5}{11}$$

As $K_1$ is an integer, $2K_2 + 5$ must be divisible by 11. Hence, $K_2 = 3$.
$\therefore$ Smallest number = 51.

**1.58.** Find the largest number which leaves remainders of 2 and 3 when it divides 89 and 148, respectively.

**Sol:** Largest number = H.C.F (89 − 2, 148 − 3) = 29

**1.59.** Find the largest number which divides 444, 804 and 1344 leaving the same remainder in each case.

**Sol:** Largest number = H.C.F (804 − 444, 1344 − 804)
= H.C.F (360, 540) = 180

**1.60.** A number when divided by 6 and 4 successively leaves remainders of 5 and 2, respectively. Find the remainder when the largest such two digit number is divided by 9.

**Sol:** Let the quotients obtained when the number is divided by 6 and 4 successively be $q_1$ and $q_2$, respectively.
Number = $6q_1 + 5$
In successive division, the quotient obtained for each division starting from the first, forms the dividend for the next division.
$\therefore q_1 = 4q_2 + 2$
$\therefore$ number = $6(4q_2 + 2) + 5 = 24q_2 + 17$
Largest two-digit number satisfying the given conditions is obtained when $24q_2 + 17 < 100$ and $q_2$ is maximum, i.e., $q_2 < 3\frac{11}{24}$ and it is maximum, i.e., $q_2 = 3$.
$\therefore$ number = 89. Required remainder = 8

**1.61.** A number when divided by 3, 5 and 6 successively leaves remainders of 1, 3 and 2, respectively. Find the number of possible values it can assume which are less than 1000.

**Sol:** Let the quotients obtained when the number is divided by 3, 5 and 6 successively be $q_1$, $q_2$ and $q_3$, respectively.
Number = $3q_1 + 1$
$q_1 = 5q_2 + 3$
$q_2 = 6q_3 + 2$
$\therefore$ number = $3(5q_2 + 3)$
$= 3(5(6q_3 + 2) + 3) + 1 = 90q_3 + 40$
$90q_3 + 40 < 1000$

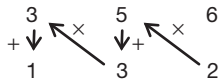
$q_3 < 10\frac{2}{3}$

$\therefore q_3$ has 11 possibilities, i.e., 0 to 10.

**1.62.** A number when divided by 3, 5 and 6 successively leaves remainders of 1, 3 and 2, respectively. Find

the remainders if its smallest possible value is divided successively by 6, 5 and 3.

**Sol:** Divisors




Remainders

Smallest possible number

$= ((2)\ (5) + 3)\ 3 + 1 = 40$

Required remainders are 4, 1 and 1.

**1.63.** Find the IGP of 7 that can divide 256!, without leaving any remainder. (This can be concisely stated as find the IGP of $P$ in $N$!)

**Sol:** First we shall take a look at the detailed explanation and then look at a simple method for solving the problem. When we write $N = 256!$ in its expanded form, we have $256 \times 255 \times 254 \times \ldots \times 3 \times 2 \times 1$

When we divide 256! by a power of 7, we have the first 256 natural numbers in the numerator. The denominator will have only 7's. The 256 numbers in the numerator have 36 multiples of 7 which are 7, 14, 21, …, 252. Corresponding to each of these we can have a 7 in the denominator which will divide $N$ completely without leaving any remainder, i.e., $7^{36}$ can definitely divide 256! Further, every multiple of 49 after cancelling out 7 as above, will still have one more 7 left. Hence, for every multiple of 49 $N$ we can have an additional 7 in the denominator. There are 5 multiples of 49 in 256! Hence, we can have a $7^5$ in the denominator. As $7^{36+5} = 7^{41}$, 41 is the IGP. The above calculation is summarised below. Successively dividing 256 by 7, we get:

| | |
|---|---|
| 7 | 256 |
| 7 | 36 |
| 5 | 2, 3, 9 |

Add all the quotients to get $36 + 5 = 41$.

Hence, the IGP of 7 contained in 256! is 41.

**1.64.** Find the IGP of 3 in 599!

**Sol:** Divide 599 successively by 3

| | |
|---|---|
| 3 | 599 |
| 3 | 199 quotient |
| 3 | 66 quotient |
| 3 | 22 quotient |
| 3 | 7 quotient |
| | 2 |

Add all the quotients,

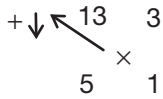
$199 + 66 + 22 + 7 + 2 = 296$

Hence, 296 is the largest power of 3 that divides 599! without leaving any remainder.

**1.65.** A number when divided successively by 13 and 3 gives respective remainders of 5 and 1. What will be the remainder when the largest such two-digit number is divided by 12?

**Sol:** We write down the divisors one after the other and their respective remainders below them.

Divisors




Remainders

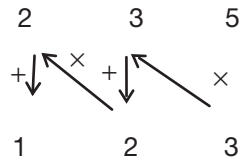
Then starting from the last remainder, we go diagonally left upwards to the first row multiplying and then come down directly adding the figure already obtained. We continue this process till we reach the figure on the extreme left in the second row, so we get $(1 \times 13) + 5 = 18$. So, the number is of the form $(d_1.d_2.k + 18)$ where $d_1$, $d_2$ are divisors. In this case, it is $= 39k + 18$, for $k = 0, 1, 2, \ldots$. Hence, the largest two-digit number is $39(2) + 18 = 96$.

This when divided by 12 leaves a remainder of 0.

**1.66.** A number when successively divided by 2, 3 and 5 leaves respective remainders of 1, 2 and 3. How many such numbers are there below 1000?

**Sol:** Let us write down all the divisors and their respective remainders as shown below:

Divisors




Remainders

We start at the bottom right corner 1 and go from $2^{nd}$ row to $1^{st}$ row diagonally to the left, multiplying. We get $3 \times 3 = 9$, then we come down to the $2^{nd}$ row adding we get, $9 + 2 = 11$. Again, multiplying diagonally left upwards, we get $11 \times 2 = 22$ and coming down to $2^{nd}$ row, adding we get $22 + 1 = 23$.

$\therefore$ The smallest number that satisfies the given condition is 23. The general form of the numbers that satisfy the given condition is got by adding the multiples of the PRODUCT of the

divisors (which may be greater than or equal to the L.C.M.) which is 30, to the smallest value obtained, which is 23.
Therefore, the general form is $30k + 23$.
For $k = 0, 1, 2, \ldots 32$ the number is less than 1000.
Hence, there are 33 numbers less than 1000 that satisfy this condition.

**1.67.** A number when successively divided by 9, 5 and 4 leaves respective remainders of 2, 1 and 3. What will be the remainders when the least such number is divided successively by 7, 3 and 4?

**Sol:** Here, again, we will first find the smallest number which satisfies the given condition.

| Divisors | 9 | 5 | 4 |
|---|---|---|---|
| Remainders | 2 | 1 | 3 |

The smallest number is $[\{(3 \times 5) + 1\} \times 9 + 2] = 146$.
When 146 is successively divided by 7, 3 and 4, the results are

| Dividend | Divisor | Quotient | Remainder |
|---|---|---|---|
| 146 | 7 | 20 | 6 |
| 20 | 3 | 6 | 2 |
| 6 | 4 | 1 | 2 |

The remainders are 6, 2 and 2, respectively.

**1.68.** A number when successively divided by 9, 5 and 4 leaves respective remainders of 2, 1 and 3. What will be the remainders when the largest such three-digit number is divided successively by 7, 3 and 4?

**Sol:** From the previous example, we know that the smallest number which satisfies the given conditions is 146. The number itself is of the form $146 + (9)(5)(4)\ k$, viz $146 + 180k$ where $k = 0, 1, 2, \ldots$
The largest three-digit number is 866 when $k = 4$. When 866 is successively divided by 7, 3 and 4 the results are:

| Dividend | Divisor | Quotient | Remainder |
|---|---|---|---|
| 866 | 7 | 123 | 5 |
| 123 | 3 | 41 | 0 |
| 41 | 4 | 10 | 1 |

The remainders are 5, 0 and 1, respectively.

**1.69.** Find the IGP of 7 that can divide 256! without leaving any remainder. (This can be concisely stated as find the IGP of $P$ in $N!$)

**Sol:** First we shall look at the detailed explanation and then look at a simple method for solving the problem. When we write $N = 256!$ in its expanded form, we have $256 \times 255 \times 254 \times \ldots \times 3 \times 2 \times 1$
When we divide 256! by a power of 7, we have the first 256 natural numbers in the numerator. The denominator will have only 7's. The 256 numbers in the numerator have 36 multiples of 7 which are 7, 14, 21 … 252. Corresponding to each of these we can have a 7 in the denominator which will divide $N$ completely without leaving any remainder, i.e., $7^{36}$ can definitely divide 256! Further, every multiple of 49 after cancelling out 7 as above, will still have one more 7 left. Hence, for every multiple of 49 $N$ we can have an additional 7 in the denominator. There are 5 multiples of 49 in 256! Hence, we can have a $7^5$ in the denominator. As $7^{36+5} = 7^{41}$, 41 is the IGP. The above calculation is summarised below. Successively dividing 256 by 7, we get

| | |
|---|---|
| 7 | 256 |
| 7 | 36 |
| | 5 |

Add all the quotients to get $36 + 5 = 41$.
So, the IGP of 7 contained in 256! is 41.

**1.70.** Find the IGP of 3 in 599!

**Sol:** Divide 599 successively by 3

| | |
|---|---|
| 3 | 599 |
| 3 | 199 quotient |
| 3 | 66 quotient |
| 3 | 22 quotient |
| 3 | 7 quotient |
| | 2 |

Add all the quotients, $199 + 66 + 22 + 7 + 2 = 296$
Hence, 296 is the largest power of 3 that divides 599! without leaving any remainder.

**1.71.** Find the IGP of 10 that can divide 890!

**Sol:** Here, we cannot apply the successive division method as 10 is not a prime number. We know 10 can be written as $2 \times 5$ and these are prime numbers. So, we find the largest powers of 2 and 5, respectively, that can divide 890! and the smaller of the two indices is the index of the required power.

| | |
|---|---|
| 2 | 890 |
| 2 | 445 |
| 2 | 222 |
| 2 | 111 |
| 2 | 55 |
| 2 | 27 |

| | |
|---|---|
| 2 | 13 |
| 2 | 6 |
| 2 | 3 |
| | 1 |

Sum of the quotients = 883

| | |
|---|---|
| 3 | 890 |
| 5 | 178 |
| 5 | 35 |
| 5 | 7 |
| | 1 |

Sum of the quotients = 221

Since the largest power of 5 is the smaller, the largest power of 10 (i.e., $2 \times 5$) is 221.

**1.72.** Find the IGP of 12 in 50!

**Sol:** $12 = 2^2 3$.

The IGP of 2 in 50! is obtained by successive division as shown below.

| **Number/ Quotient** | 50 | 25 | 12 | 6 | 3 | 1 |
|---|---|---|---|---|---|---|
| **Divisor** | 2 | 2 | 2 | 2 | 2 | |

The IGP of 2 in 50! is $25 + 12 + 6 + 3 + 1 = 47$

The IGP of $2^2$ in 50! is $\left\lfloor \frac{47}{2} \right\rfloor = 23$

The IGP of 3 in 50! is $16 + 5 + 1 = 22$

$\therefore$ The IGP of 12 in 50! is the smaller of 23 and 22, viz 22.

**1.73.** Find the IGP of 2 in $31! + 32! + 33! + \ldots + 40!$

**Sol:** The IGP of 2 in 31! is $15 + 7 + 3 + 1$, viz 26.

The IGP of 2 in 32! is $16 + 8 + 4 + 2 + 1$, viz 31.

The IGP of 2 in the other terms is 31 or more.

$\therefore$ The IGP of 2 in the given expression is 26.

**1.74.** Find the H.C.F. of 1311 and 1653.

**Sol:** By division method,

1311) 1653 (1
1311
342) 1311 (3
1026
285) 342 (1
285
57) 285 ( 5
285
0

Therefore, H.C.F. of 1653 and 1311 is 57

**1.75.** Find the H.C.F. of 1891 and 2257.

**Sol:** By division method,

1891) 2257 1
1891
366) 1891 5
1830
61) 366 (6
366
0

Therefore, the H.C.F. is 61

**1.76.** Find the L.C.M. of 2/9, 5/8 and 7/10.

**Sol:** Each fraction is in its simplest form.

Hence, L.C.M. of the fractions

$$= \frac{\text{LCM of the numerators}}{\text{HCF of the denominators}} = 70/1 = 70$$

**1.77.** Find the H.C.F. of 3/7, 5/9 and 11/10.

**Sol:** As each fraction is in its simplest form,

$$\text{H.C.F. of fractions} = \frac{\text{HCF of the numerators}}{\text{LCM of the denominators}}$$

$= 1/630$

**1.78.** Arrange the following in ascending order: 2/5, 6/11, 5/13.

**Sol:** Take the L.C.M. of the denominators and then compare the numerators

$$\frac{2}{5} = \frac{2 \times 143}{5 \times 143} = \frac{286}{715}$$

$$\frac{6}{11} = \frac{6 \times 65}{11 \times 65} = \frac{390}{715}$$

$$\frac{5}{13} = \frac{5 \times 55}{13 \times 55} = \frac{275}{715}$$

Comparing, we get $6/11 > 2/5 > 5/13$

Ascending order is 5/13, 2/5, 6/11

**Alternate Method:**

$$\frac{2}{5} = 0.40$$

$$\frac{6}{11} = 0.54$$

$$\frac{5}{13} = 0.38$$

The ascending order is $\frac{5}{13}, \frac{2}{5}$ and $\frac{6}{11}$

**1.79.** Test for divisibility of 2, 3, 4, 5, 6, 9, 10, 11 and 19 on the following numbers:
(a) 672,
(b) 703 and
(c) 2310

**Sol:** (a) 672 → It is even hence, is divisible by 2. sum of the digits = 15, which is divisible by 3. Hence, the number is divisibly by 3. The last two digits form the number 72. Hence, the number is divisible by 4. It does not end with 5 or 0 hence, is not divisible by 5 or 10 Number is divisible by 2 as well as 3. Hence, is divisible by 6. Sum of the digits is not divisible by 9, and hence, the number 672 is not divisible by 9.

Difference between the sum of the alternate digits of the number = 1, hence, not divisible by 11. Number of tens in the number + twice the unit's digit = 67 + 4 = 71, 7 + 2(1) = 9, hence, is not divisible by 19.

(b) 703 → The number is not even, hence, is not divisible by 2, 4, or 6. Sum of the digits is 10, hence, is not divisible by 3 or 9. It does not end with 5 or 0, hence, not divisible by 5 or 10, difference of alternate number is 10, hence, not divisible by 11. Number of tens in the number + twice the units digit = 70 + 6 = 76, 7 + 12 = 19. Hence, divisible by 19.

(c) 2310 → The number is even, so divisible by 2. Sum of the digits is 6. Hence, divisible by 3, but not by 9. Ends in 0, so divisible by both 5 and 10. The last two digits number which is 10 is not divisible by 4 hence, the number is not divisible by 4. Difference between the alternate digits = 0.
Hence, divisible by 11
Number of tens + twice the unit's digit
= 231 + 0 = 231
23 + 2 = 25
Therefore, not divisible by 19.

**1.80.** Simplify the expression [2/3 of 4/5 {(9 × 3) – (6 × 2)} + 1/4 – 1/2}] using BODMAS rule.

**Sol:** [2/3 of 4/5 {(9 × 3) – (6 × 2)} + 1/4 – 1/12]
Applying BODMAS rule,
= [2/3 of 4/5 {27 – 12} + 1/4 – 1/12]
= [2/3 of 12 + 1/4 – 1/12]
= 8 + 1/4 – 1/12 = 49/6

**1.81.** Arrange 12/7, 14/9 and 9/5 in descending order.

**Sol:** L.C.M. of the denominators = 315

$$\frac{12}{7}=\frac{12\times45}{7\times45}=\frac{540}{315}$$

$$\frac{14}{9}=\frac{14\times35}{9\times35}=\frac{490}{315}$$

$$\frac{9}{5}=\frac{9\times63}{5\times63}=\frac{567}{315}$$

Descending order is 567/315, 540/315, 490/315
∴ Descending order is 9/5, 12/7, 14/9

**1.82.** Simplify: $\dfrac{5\frac{4}{9}\times\frac{19}{7}}{2\frac{5}{7}-\frac{9}{14}}$.

**Sol:** $5^4/_9 = 49/9$
$2^5/_7 = 19/7$
Hence, the numerator = 49/9 × 19/7
= 133/9
The denominator = 19/7 – 9/14
= 38/14 – 9/14 = 29/14

$$\text{Given fraction} = \frac{\frac{133}{9}}{\frac{29}{14}}$$

$$=\frac{133\times14}{9\times29}=\frac{1862}{261}$$

**1.83.** Simplify: 3.56 × 3.56 × 3.56 – 1.06 × 1.06 × 1.06 – 3 × 3.56 × 3.56 × 1.06 + 3 × 3.56 × 1.06 × 1.06

**Sol:** The given expression is in the form of $a^3 - b^3 - 3a^2b + 3ab^2$
where $a = 3.56$ and $b = 1.06$.
The above expression is equal to $(a - b)^3$.
Hence, the simplified value is $(3.56 - 1.06)^3 = (2.50)^3 = 15.625$.

**1.84.** Simplify: $3.66^3 + 3 \times 3.66 \times 1.34 \times 1.34 + 3 \times 3.66 \times 3.66 \times 1.34 + 1.34^3$

**Sol:** The given expression is in the form of $a^3 + 3ab^2 + 3a^2b + b^3$ where $a = 3.66$ and $b = 1.34$.
The above expression is equal to $(a + b)^3$.
Hence, the simplified value is $(3.66 + 1.34)^3 = 5^3 = 125$.

**1.85.** Find the IGP of 10 that can divide 890!.

**Sol:** Here, we cannot apply the successive division method as 10 is not a prime number. We know 10 can be written as $2 \times 5$ and these are prime numbers. Hence, we find the largest powers of 2 and 5, respectively, that can divide 890! and the smaller of the two indices is the index of the required power.

| | |
|---|---|
| 2 | 890 |
| 2 | 445 |
| 2 | 222 |
| 2 | 111 |
| 2 | 55 |
| 2 | 27 |
| 2 | 13 |
| 2 | 6 |
| 2 | 3 |
| | 1 |

Sum of the quotients = 883

| | |
|---|---|
| 3 | 890 |
| 5 | 178 |
| 5 | 35 |
| 5 | 7 |
| | 1 |

Sum of the quotients = 221

Since the largest power of 5 is the smaller, the largest power of 10 (i.e., $2 \times 5$) is 221.

**1.86.** Find the IGP of 12 in 50!

**Sol:** $12 = 2^2 3$.

The IGP of 2 in 50! is obtained by successive division as shown below.

| **Number/Quotient** | 50 | 25 | 12 | 6 | 3 | 1 |
|---|---|---|---|---|---|---|
| **Divisor** | 2 | 2 | 2 | 2 | 2 | |

The IGP of 2 in 50! is 25 + 12 + 6 + 3 + 1 = 47

The IGP of $2^2$ in 50! is $\left\lfloor \frac{47}{2} \right\rfloor = 23$

The IGP of 3 in 50! is 16 + 5 + 1 = 22

∴ The IGP of 12 in 50! is the smaller of 23 and 22, viz 22.

**1.87.** Find the IGP of 2 in 31! + 32! + 33! + … + 40!.

**Sol:** The IGP of 2 in 31! is 15 + 7 + 3 + 1, viz 26.

The IGP of 2 in 32! is 16 + 8 + 4 + 2 + 1, viz 31.

The IGP of 2 in the other terms is 31 or more.

∴ The IGP of 2 in the given expression is 26.

**1.88.** Find the H.C.F. of $\frac{3}{5}, \frac{6}{10}$ and $\frac{9}{20}$.

**Sol:** To find the L.C.M. or H.C.F. of fractions, first express all the fractions in their simplest form.

H.C.F. (fractions) =

$$\frac{\text{HCF(numerators)}}{\text{LCM(denominators)}} = \frac{\text{HCF}(3,3,9)}{\text{LCM}(5,5,20)} = \frac{3}{20}$$

**1.89.** Find the L.C.M. of $\frac{3}{5}, \frac{6}{10}$ and $\frac{9}{20}$.

**Sol:** To find the L.C.M. or H.C.F. of fractions, first express all the fractions in their simplest form.

L.C.M. (fractions)

$$= \frac{\text{LCM(numerators)}}{\text{HCF(denominators)}} = \frac{\text{LCM}(3,3,9)}{\text{HCF}(5,5,20)} = \frac{9}{5}$$

**1.90.** Arrange the following in ascending order:

$\frac{5}{7}, \frac{3}{4}$ and $\frac{7}{10}$

**Sol:** L.C.M. (10, 7, 4) = 140

$$\frac{7}{10} = \frac{98}{140}$$

$$\frac{5}{7} = \frac{100}{140}$$

$$\frac{3}{4} = \frac{105}{140}$$

$$\therefore \frac{7}{10} < \frac{5}{7} < \frac{3}{4}$$

**1.91.** Test whether the number 12320 is divisible by 2, 3, 4, 5, 6, 9, 10, 11 and 19.

**Sol**: The number has its last two digits divisible by 4 and ends with a 0.

∴ it is divisible by 4 and hence, by 2 and also by 10 and 5.

The sum of the digits of the number is 8

∴ it is not divisible by 3. Hence, it is neither divisible by 6 nor by 9.

The sum of the digits in the odd places = The sum of the digits in even places.

∴The number is divisible by 11.

Number of tens in the number + 2 (units digit of the number) = 1232 which is not divisible by 19.

∴ The number is not divisible by 19.

**1.92.** Simplify: $\dfrac{(3\cdot 69+2\cdot 16)^2+(3\cdot 69-2\cdot 16)^2}{3\cdot 69^2+2\cdot 16^2}$.

**Sol:** The given expression is in the form $\left(\dfrac{(a+b)^2+(a-b)^2}{a^2+b^2}\right)$

where $a = 3\times 69$ and $b = 2.16$

$$\frac{(a+b)^2+(a-b)^2}{\left(a^2+b^2\right)}=\frac{2\left(a^2+b^2\right)}{a^2+b^2}=2.$$

**1.93.** Simply: $\dfrac{[3.19\times 3.19\times 3.19+2.23\times 2.23\times 2.23 +1.58\times 1.58\times 1.58-9.57\times 2.23\times 1.58]}{[3\cdot 19^2+2\cdot 23^2+1\cdot 58^2-(3\cdot 19)(2\cdot 23) -(3\cdot 19)(1\cdot 58)-(2\cdot 23)(1\cdot 58)]}$

**Sol:** The given expression is in the form

$$\frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca}$$

where $a = 3\times 19$, $b = 2\times 23$ and $c = 1\times 58$

$$\frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca}=a+b+c$$

$\therefore$ The given expression equals 7.

**1.94.** Simplify: $1.42^2 + 2.33^2 + 4.25^2 + (2.84)\ (2.33) + (4.66)\ (4.25) + (8.5)\ (1.42)$

**Sol:** The given expression is in the form $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

where $a = 1.42$, $b = 2.33$ and $c = 4.25$

$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2$

$\therefore$ The given expression equals 64.

**1.95.** Simplify: $4.56 \times 4.56 \times 4.56 + 3.44 \times 3.44 + 3.44 + 13.68 \times 4.56 \times 3.44 + 10.32 \times 4.56 \times 3.44$

**Sol:** The given expression is in the form $a^3 + b^3 + 3a^2b + 3ab^2$

where $a = 4.56$ and $b = 3.44$

$a^3 + b^3 + 3a^2b + 3ab^2 = (a + b)^3$

$\therefore$ The given expression equals 512.

(ii) Simplify: $4.56 \times 4.56 \times 4.56 - 0.56 \times 0.56 \times 0.56 - 13.68 \times 4.56 \times 0.56 + 1.68 \times 4.56 \times 0.56$

The given expression is in the form $a^3 - b^3 + 3ab^2 - 3a^2b$

where $a = 4.56$ and $b = 0.56$.

$a^3 - b^3 + 3ab^2 - 3a^2b = (a - b)^3$

$\therefore$ The given expression equals 64.

**1.96.** Find the last digit of $2^{412} \times 4^{428}$

**Sol:** Writing down the powers of 2 and 4 to check the pattern of the last digits, we have

Last digit of $2^1$ – 2
Last digit of $2^2$ – 4
Last digit of $2^3$ – 8
Last digit of $2^4$ – 6
Last digit of $2^5$ – 2
Last digit of $4^1$ – 4
Last digit of $4^2$ – 6
Last digit of $4^3$ – 4
Last digit of $4^4$ – 6

We find that the last digit of powers of 2 repeat after 4 steps, the last digit of any power of 4 is 4 for an odd power and 6 for an even power. The last digit of $2^{412}$ will be the same as $2^4$ as 412 is a multiple of 4. Hence, the last digit of $2^{412}$ is 6

Last digit of $4^{428}$ is 6. Since the power of 4 is even. Hence, the last digit of $2^{412} \times 4^{428}$ will be equal to the last digit of $6 \times 6 = 6$

**1.97.** Find the remainder of the division $5^{64}/6$.

**Sol:** Let us find the pattern that remainders follow when successive powers of 5 are divided by 6.

Remainder of $5^1/6$ is 5.
Remainder of $5^2/6$ is 1.
Remainder of $5^3/6$ is 5.
Remainder of $5^4/6$ is 1.

We find that the remainders are repeated after every two powers. So, remainder of $5^{64}$ when divided by 6 is the same as $5^2$ when divided by 6, since 64 is a multiple of 2. Hence, the remainder is 1.

**1.98.** Find the remainder of the division $2^{34}/5$.

**Sol:** In the division, since the numerator is in terms of power of 2, the denominator should also be expressed in terms of power of 2, i.e., as $(2^2 + 1)$. Now, as the denominator is in terms of $2^2$, the numerator should also be rewritten in terms of $2^2$ as $(2^2)^{17}$. The problem reduces to finding the remainder when $(2^2)^{17}$ is divided by $2^2 - (-1)$. This remainder, as per the Remainder Theorem is $(-1)^{17} = -1$; and $-1 + 5 = 4$ (the divisor is added to get a positive remainder).

**1.99.** Find the remainder of the division $2^{56}/31$.

**Sol:** In this division, since the numerator is in terms of powers of 2, the denominator 31 should also be expressed in terms of 2, as $2^5 - 1$. Now, as the

denominator is in terms of $2^5$, the numerator $2^{56}$ should also be rewritten in terms of $2^5$ as $(2^5)^{11} \times 2^1$. The problem now reduces to finding the remainder when $2(2^5)^{11}$ is divided by $2^5 - 1$. This remainder as per the Remainder Theorem is $2(1)^{11} = 2$

**1.100.** Find the remainder of the division $2^{58}/24$.

**Sol:** Pattern method

The remainders of powers of 2 when divided by 24 are as follows:

The remainder when $2^1$ is divided by 24 is 2.
The remainder when $2^2$ is divided by 24 is 4.
The remainder when $2^3$ is divided by 24 is 8.
The remainder when $2^4$ is divided by 24 is 16.
The remainder when $2^5$ is divided by 24 is 8.
The remainder when $2^6$ is divided by 24 is 16

The remainder repeats in such a way (excluding the remainder when $2^1$ and $2^2$ are divided by 24) that the remainder is 16 when an even power of 2 is divided by 24 and 8. When an odd power of 2 is divided by 24. When $2^{58}$ is divided by 24, the remainder is 16.

**1.101.** Find the remainder of the division $3^{98}/10$.

**Sol:** Pattern method

The remainder when 3 is divided by 10 is 3.
The remainder when $3^2$ is divided by 10 is 9.
The remainder when $3^3$ is divided by 10 is 7.
The remainder when $3^4$ is divided by 10 is 1.
The remainder when $3^5$ is divided by 10 is 3.

Since the remainder is repeating after 4 steps, the remainder of $3^{98}/10$ is the same as remainder of $3^2/10$ (since $98 = 4 \times 24 + 2$)

Remainder Theorem Method

In the division $3^{98}/10$, the numerator is in terms of powers of 3, so the denominator can be written as $3^2 + 1$. Since the denominator is written in terms of $3^2$, the numerator is expressed as $(3^2)^{49}$. So, the remainder of $(3^2)^{49}$ divided by $(3^2 + 1)$, as per the Remainder Theorem, is $(-1)^{49} = -1$

Hence, remainder is $-1 + 10 = 9$

**1.102.** Find the remainder of the division $7^{93}$ divided by 10.

**Sol:** Pattern method

The remainders of powers of 7 when divided by 10 are as follows:

Remainder when $7^1$ is divided by 10 is 7.
Remainder when $7^2$ is divided by 10 is 9.
Remainder when $7^3$ is divided by 10 is 3.
Remainder when $7^4$ is divided by 10 is 1.
Remainder when $7^5$ is divided by 10 is 7.

Since the remainder is repeating after 4 steps, the remainder of $7^{93}/10$ is the same as that of $7^1/10 = 7$ (Since $93 = 4 \times 23 + 1$).

**1.103.** Find the units digit of $14^{124} \times 29^{123}$.

**Sol:** Units digit of $(14^{124} \times 29^{123})$
= Units digit of $(4^{124} \times 9^{123})$

The units digit of any power of 4 is 4 if the exponent is odd and 6 if the exponent is even. The units digit of any exponent of 9 is 9 if the exponent is odd and 1 if the exponent is even.

$\therefore$ The required units digit is 4.

**1.104.** Find the remainder when $2^{99}$ is divided by 9.

**Sol:** $$\frac{2^{99}}{9} = \frac{\left(2^3\right)^{33}}{2^3 - (-1)}$$

By remainder theorem, remainder is $(-1)^{33} = -1$
$\Rightarrow -1 + 9 = 8$

**1.105.** Find the remainder when $2^{70}$ is divided by 7.

**Sol:** $$\frac{2^{70}}{7} = \frac{2\left(2^3\right)^{23}}{2^3 - 1}$$

By remainder theorem, remainder is $2\ (1)^{23} = 2$.

**1.106.** Find the remainder when $2^{97}$ is divided by 15.

**Sol:** Let us find the pattern that the remainders follow when successive powers of 2 are divided by 15.

Remainder when $2^1$ is divided by 15 = 2.
Remainder when $2^2$ is divided by 15 = 4.
Remainder when $2^3$ is divided by 15 = 8.
Remainder when $2^4$ is divided by 15 = 1.
Remainder when $2^5$ is divided by 15 = 2.

$\therefore$ The remainder repeats after 4 steps.

$\therefore$ Required remainder = Remainder of $\frac{2^1}{15} = 2$ (since 97 is 4 (24) + 1)

**1.107.** Find the remainder when $2^{201}$ is divided by 5.

**Sol:** From the above example, the units digit of powers of 2 repeats after 4 steps.

$\therefore$ Required remainder = Remainder of $\frac{2^1}{5} = 2$

**1.108.** Find the remainder when $3^{101}$ is divided by 10.

**Sol:** Let us find the pattern that remainders follow when the successive powers of 3 are divided by 10.

Remainder when $3^1$ is divided by 10 is 3
Remainder when $3^2$ is divided by 10 is 9
Remainder when $3^3$ is divided by 10 is 7
Remainder when $3^4$ is divided by 10 is 1
Remainder when $3^5$ is divided by 10 is 3
$\therefore$ The remainder repeats after 4 steps.

$$\therefore \text{Required remainder} = \text{Remainder of } \frac{3^1}{10} = 3$$

**1.109.** Find the remainder when 123, 123, … (up to 300 digits) is divided by 999.

**Sol:** To find the remainder when some number (say $N$) is divided by 9 (or $10^1 - 1$), we add up all the digits of $N$ to get (say $S_1$) and divide $S_1$ by 9.
Similarly, to find the remainder when $N$ is divided by 99(or $10^2 - 1$), we start at the right end of $N$, group the digits two at a time and add up all the groups to get, say $S_2$. Then we find the remainder of $S_2/99$.
In general, to find the remainder when $N$ is divided by $D_n = 99 \ldots 9$($n$ nines) or $(10^n - 1)$, we start at the right end of $N$, group the digits $n$ at a time and add up all the groups to get say $S_n$.

$$\text{Rem } \frac{N}{D_n} = \text{Rem } \frac{S_n}{D_n}$$

Similarly, we can start with the remainder rule for 11 and work out the corresponding rules for 101, 1001, 10001, etc. All this is an application of Remainder theorem.
Here, $N$ = 123, 123, …, 123 (a total of 300 digits or 100 groups) = 123 $(1000^{99})$ + 123 $(1000^{98})$ + … + 123 $(1000^1)$ + 123
Now, let $N = f(1000)$; When $N$ or $f(1000)$ is divided by 999 or (1000 – 1), the remainder is $f(1)$, i.e., 123(100) by remainder theorem. [i.e., $S_3$ = 123 (100)]

$$\because \text{Rem } \frac{N}{999} = \text{Rem } \frac{12300}{999} = \text{Rem } \frac{12+300}{999} = 312$$

**1.110.** Let $N$ = 345345345 …. up to 300 digits. What is the remainder when $N$ is divided by 999? Also find the remainder when $N$ is divided by 1001.

**Sol:** $N$=345, 345, … 345 (upto 300 digits or 100 groups of 3 digits) = 345 $[10^{3(99)} + 10^{3(98)} + 10^{3(97)} + \ldots + 10^3 + 1]$

$$\text{Rem } \frac{N}{999} = \text{Rem } \left( \frac{N}{(10^3 - 1)} \right) \text{Rem } \frac{(345)(100)}{999}$$

($\because$ By remainder theorem)

$$= \text{Rem } \frac{34,500}{999} N = \text{Rem } \frac{34+500}{999} = 534$$

To get $\text{Rem } \frac{N}{1001}$, we need U and Th, where U is the sum of all the alternate groups starting with the rightmost (the group containing the units digit) and Th is the sum of all the alternate groups starting with the second rightmost (the group consisting of the thousands digit)
U = 345(50) = 17250 and Th = 345(50) = 17250

$$\therefore \text{Rem } \frac{N}{1001} = \text{Rem } \frac{U - Th}{1001} = 0$$

**1.111.** What are the last two digits of $37^{12345}$?

**Sol:** $N = 37^{12345} = 37^{12340}\ 37^5 = 37^{20(617)}.37^5$
$37^{20}$ ends in 01,
while $37^5 \equiv (37)^4\ 37 \equiv (1369)^2\ (37) \equiv (69)^2\ (37)$
$\equiv (4761)\ (37)$
$\equiv (61)\ (37) = 2257$. $\therefore$ $N$ ends in 57
Note: $a \equiv b$ means $a - b$ is divisible by the considered divisor.

**1.112.** Find the remainder when $N = 817^{673}$ is divided by 100. Alternatively, find the last two digits of $N$.

**Sol:** We are interested only in the last two digits of $N$. We need to consider only the last two digits of 817, i.e., 17. Successive powers of 17 (or any other number) show a cyclic pattern, when divided by 100(or any other divisor). We can list these remainders until we discover the point, where the repetition starts.

| | | | | |
|---|---|---|---|---|
| 17 | 57 | 97 | 37 | 77 |
| 89 | 69 | 49 | 29 | 09 |
| 13 | 73 | 33 | 93 | 53 |
| 21 | 41 | 61 | 81 | 01 |

$17^1 = 17$
To get the next number, we take only the last two digits of $17^2$, i.e., 89. To get the next number, we take only the last two digits of 17(89). We need not perform the complete multiplication. We need only the units and tens digits. The units digit is 3 and there are 3 parts to the tens digit —the carryover of 6, the units digit of 9(1) and

8(7), i.e., 6 + 9 + 6. Again, we need only the units digit of this which is 1.
∴ The last two digits of $17^3$ are 13. Similarly, we can work out the other numbers.
It is convenient to break the column after every 4 steps (the units digit is found to be the same in each row. This serves as a check to our calculations). After we get 01, the next 20 powers show the same pattern.
In the given example, as 673 = 20 (33) + 13.
∴ The $13^{th}$ number in the list, i.e., 37, is our answer.

**1.113.** What are the last two digits of $37^{12345}$?

**Sol:** $N = 37^{12345} = 37^{12340}\ 37^5 = 37^{20(617)}.37^5$
$37^{20}$ ends in 01,
while $37^5 \equiv (37)^4\ 37 \equiv (1369)^2\ (37) \equiv (69)^2\ (37)$
$\equiv (4761)\ (37)$
$\equiv (61)\ (37) = 2257$. ∴ $N$ ends in 57
Note: $a \equiv b$ means $a - b$ is divisible by the considered divisor

**1.114.** Find the remainder when $164^{359}$ is divided by 100

**Sol:** $N = 164^{359}$. We need the last two digits.
These digits for successive powers form a pattern of cycle length 20. As 359 = 340 + 19 and 164 = 100 + 64, we can think of $64^{19} = 2^{114}$
Now, $2^{14} = 16384$. It ends in 84.
∴ $N$ also ends in 84

**1.115.** Find the last two digits of $282^{822}$

**Sol:** $N = 282^{822}$. We can think of $82^2 = 6724$
∴ $N$ ends in 24

**1.116.** Find the remainder when $164^{359}$ is divided by 100.

**Sol:** $N = 164^{359}$ We need the last two digits.
These digits for successive powers form a pattern of cycle length 20. As 359 = 340 + 19 and 164 = 100 + 64, we can think of $64^{19} = 2^{114}$.
Now, $2^{14} = 16384$. It ends in 84.
∴ $N$ also ends in 84

**1.117.** Find the last two digits of $282^{822}$.

**Sol:** $N = 282^{822}$. We can think of $82^2 = 6724$
∴ $N$ ends in 24.

**1.118.** Show that $(a+b)^7 - a^7 - b^7$ is a multiple of 7 for all positive integral values of $a$ and $b$.

**Sol:** $(a+b)^7$
$= {}^7C_0 a^7 + {}^7C_1\ a^6\ b + {}^7C_2\ a^5\ b^2 + {}^7C_3\ a^4\ b^3 + \ldots + {}^7C_7\ b^7$
∴ $(a+b)^7 - a^7 - b^7$
$= {}^7C_1\ a^6\ b + {}^7C_2\ a^5\ b^2 + {}^7C_3\ a^4\ b^3 + \ldots + {}^7C_7\ b^7$ (1)
If $p$ is any prime number,

$${}^pC_r = \frac{p(p-1)\ldots(p-r+1)}{1(2)\ldots(r)} = p(\text{an integer}) \text{ for all } r < p.$$

∴ The RHS of (1) (and hence, the LHS of (1)) is a multiple of 7.
Note: When $n$ is prime, $(a+b)^n = a^n + b^n +$ (a multiple of $n$)

**1.119.** Find the remainder when $2^{1000}$ is divided by 33

**Sol:** $2^{1000} = (2^5)^{200} = (33 - 1)^{200} = (33^{200} + {}^{200}C_1\ (33)^{199}\ (-1) + {}^{200}C_2\ (33)^{198}(-1)^2 + \ldots + {}^{200}C_{199}\ (33)\ (-1)^{199} + (-1)^{200} = M\ (33) + (-1)^{200} =$ (A multiple of 33) + 1

**1.120.** Show that $(a+b)^7 - a^7 - b^7$ is a multiple of 7 for all positive integral values of $a$ and $b$.

**Sol:** $(a+b)^7$
$= {}^7C_0 a^7 + {}^7C_1\ a^6\ b + {}^7C_2\ a^5\ b^2 + {}^7C_3\ a^4\ b^3 + \ldots + {}^7C_7\ b^7$
∴ $(a+b)^7 - a^7 - b^7$
$= {}^7C_1\ a^6\ b + {}^7C_2\ a^5\ b^2 + {}^7C_3\ a^4\ b^3 + \ldots + {}^7C_7\ b^7$ (1)
If $p$ is any prime number,

$${}^pC_r = \frac{p(p-1)\ldots(p-r+1)}{1(2)\ldots(r)} = p \text{ (an integer) for all } r < p.$$

∴ The RHS of (1) (and hence, the LHS of (1)) is a multiple of 7.
Note: When $n$ is prime, $(a+b)^n = a^n + b^n +$ (a multiple of $n$)

**1.121.** Find the remainder when $2^{1000}$ is divided by 33

**Sol:** $2^{1000} = (2^5)^{200} = (33 - 1)^{200} = (33^{200} + {}^{200}C_1\ (33)^{199}\ (-1) + {}^{200}C_2\ (33)^{198}(-1)^2 + \ldots + {}^{200}C_{199}\ (33)\ (-1)^{199} + (-1)^{200}\ M\ (33) + (-1)^{200} =$ (A multiple of 33) + 1

**1.122.** What is the remainder when $5^{119}$ is divided by 59?

**Sol:** $N = 5^{119}$ We need $\text{Rem}\dfrac{N}{59}$

By Fermat's Little Theorem, $5^{58} = 59k + 1$ (where $k$ is a natural number)
$5^{59} = 59\ (5k) + 5$ or $5^{59} \equiv 5$
∴ $5^{118} \equiv 25$ and $5^{119} \equiv 125 \equiv 7$

**1.123.** Find the remainder when $26^{57}$ is divided by 29

**Sol:** $$\text{Rem}\frac{26^{57}}{29} = \text{Rem}\frac{(26)\,(26)^{56}}{29}$$

$$= \left\{\text{Rem}\frac{26}{29}\right\}\left\{\text{Rem}\frac{26^{56}}{29}\right\}$$

$= \{26\}\{1\} = 26.$

$\therefore$ The remainder is 26.

**1.124.** What is the remainder when 28! is divided by 29?

**Sol:** By Wilson's theorem, $\text{Rem}\dfrac{28!+1}{29} = 0$

$\Rightarrow \text{Rem}\dfrac{28!}{29} = -1 \text{ or } -1+29 = 28$

**1.125.** What is the remainder when $5^{119}$ is divided by 59?

**Sol:** $N = 5^{119}$ We need $\text{Rem}\dfrac{N}{59}$

By Fermat's Little Theorem, $5^{58} = 59k + 1$ (where $k$ is a natural number)

$5^{59} = 59\,(5k) + 5$ or $5^{59} \equiv 5$

$\therefore 5^{118} \equiv 25$ and $5^{119} \equiv 125 \equiv 7$

**1.126.** Find the remainder when $26^{57}$ is divided by 29

**Sol:** $\text{Rem}\dfrac{26^{57}}{29} = \text{Rem}\dfrac{(26)(26)^{56}}{29}$

$= \left\{\text{Rem}\dfrac{26}{29}\right\}\left\{\text{Rem}\dfrac{26^{56}}{29}\right\}$

$= \{26\}\{1\} = 26.$ $\therefore$ The remainder is 26.

**1.127.** What is the remainder when 28! is divided by 29?

**Sol:** By Wilson's theorem, $\text{Rem}\dfrac{28!+1}{29} = 0$

$\Rightarrow \text{Rem}\dfrac{28!}{29} = -1 \text{ or } -1+29 = 28$

**1.128.** Let $N = 345345345$ …. upto 300 digits. What is the remainder when $N$ is divided by 999? Also find the remainder when $N$ is divided by 1001.

**Sol:** $N$ = 345, 345, … 345 (upto 300 digits or 100 groups of 3 digits) = $345\,[10^{3(99)} + 10^{3(98)} + 10^{3(97)} + \ldots + 10^3 + 1]$

$\text{Rem}\dfrac{N}{999} = \text{Rem}\left(\dfrac{N}{(10^3-1)}\right) = \text{Rem}\dfrac{(345)(100)}{999}$

($\therefore$ By remainder theorem)

$= \text{Rem}\dfrac{34{,}500}{999} = \text{Rem}\dfrac{34+500}{999} = 534$

To get $\text{Rem}\dfrac{N}{1001}$, we need U and Th, where U is the sum of all the alternate groups starting with the rightmost (the group containing the units digit) and Th is the sum of all the alternate groups starting with the second rightmost (the group consisting of the thousands digit)

U = 345(50) = 17250 and Th = 345(50) = 17250

$\therefore \text{Rem}\dfrac{N}{1001} = \text{Rem}\dfrac{\text{U} - \text{Th}}{1001} = 0$

**1.129.** Which of the following statements is true about $15^n + 1$?

(a) It is divisible by 16, when n is even.

(b) It is always divisible by 16.

(c) It is never divisible by 16.

(d) It is never divisible by 14.

**Sol:** $15^n + 1$ is in the form of $a^n + b^n$.

The divisor 16 is 15 + 1, which is in the form of $a + b$.

Hence, 16 divides $15^n + 1$ only when $n$ is odd.

Hence, first three options are wrong.

14 = 15 – 1 and hence, is in the form $a - b$.

$a^n + b^n$ is divisible by $a - b$ when $a - b$ is a factor of $a + b$. Here, $a - b = 14$ and $a + b = 16$. 14 is not a factor of 16. Hence, $15^n + 1$ is not divisible by 14.

The fourth option is correct.

**1.130.** Which of the statements is true about $31^n - 1$?

(a) It is always divisible by 32.

(b) It is divisible by 32 when n is odd.

(c) It is never divisible by 30.

(d) It is always divisible by 30.

**Sol:** $31^n - 1$ is in the form of $a^n - b^n$.

$(a - b)$ is a factor of $a^n - b^n$ for even as well as odd values of $n$.

Hence, $31^n - 1$ is always divisible by (31 – 1) which is 30. Option $(d)$ is correct.

Note: 32 = 31 + 1; i.e., it is in the form of $a + b$.

$a^n - b^n$ is divisible by $a + b$ when

(i) $n$ is even or

(ii) $n$ is odd and $2b^n$ is a multiple of $(a + b)$. As neither condition is satisfied, options (a) and $(b)$ are not correct.

## EXERCISE-1

***Directions for questions 1 to 100***: For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answers in the space provided.

**1.** If $x = 56^y + 1$, where $y$ is a natural number, then $x$ is divisible by 19 for
(A) any odd value of $y$. (B) any even value of $y$.
(C) any value of $y$. (D) no value of $y$.

**2.** The product of a seven-digit number, a ten-digit number and a twelve-digit number is a _____ digit number.
(A) 27 (B) 28
(C) 28 or 29 (D) 27 or 28 or 29

**3.** The L.C.M. of $2^3 \times 3^2 \times 5$ and $3^3 \times 5^2 \times 2^3$ is.
(A) $2^2 \times 3^2 \times 5$ (B) $2^3 \times 3^3 \times 5^2$
(C) $2^3 \times 5 \times 3^2$ (D) $2^3 \times 3^2 \times 5$

**4.** The product of 45 even numbers is
(A) even
(B) odd
(C) either even or odd

**5.** Find the greatest number which leaves the same remainder when it divides 93 or 131 or 188.
(A) 17 (B) 19 (C) 23 (D) 29

**6.** The greatest number which divides $38^{2n} - 11^{2n}$, where $n$ is a natural number among the following is.
(A) 26 (B) 441
(C) 676 (D) 1323

**7.** The twelve-digit number 1986$y$5721378 is divisible by 11. Find $y$.
(A) 7 (B) 6 (C) 8 (D) 9

**8.** Find the units digit of $3^{200} \times 4^{500}$. _____

**9.** Find the smallest four-digit number which is a multiple of 112.
(A) 896 (B) 1008 (C) 1120 (D) 1024

**10.** A number, when divided by 32, leaves a remainder of 29. Find the remainder when the number is divided by 64.
(A) 29 (B) 61
(C) Either (A) or (B) (D) Neither (A) nor (B)

**11.** The number of digits in the square root of a thirteen-digit number is _____.

**12.** The eight-digit number 7654321$A$ is divisible by 9 where A is a single digit. Find A. _____

**13.** Find the smallest number which leaves respective remainders of 4, 5, 6, 7 and 8 when divided by 5, 6, 7, 8 and 9.
(A) 2519 (B) 1259
(C) 2521 (D) 1261

**14.** Two numbers have a H.C.F. of 18 and a product of 3240. Find their L.C.M.
(A) 180 (B) 270 (C) 240 (D) 210

**15.** A number when divided by 28 leaves a remainder of 7. Which of the following could be the remainder when it is divided by 35?
(A) 21 (B) 28
(C) 10 (D) More than one of the above

**16.** The number of digits in $(2PQR)^4$ where $2PQR$ is a four-digit number is.
(A) 13 (B) 14 (C) 15 (D) 14 or 15

**17.** $C = 41^D + 7^D$ where D is an odd natural number. Which of the following need not be a factor of C?
(A) 24 (B) 12 (C) 16 (D) 36

**18.** The L.C.M. and H.C.F. of two numbers are 264 and 2, respectively. If one of them is 22, the other is _____.

**19.** Two numbers have their H.C.F. equal to their L.C.M. Which of the following must be true?
(A) The numbers must be coprime and composite.
(B) The numbers must be coprime and prime.
(C) Either (A) or (B)
(D) The numbers must be equal.

**20.** Find the units digit of $(13687)^{3265}$. _____

**21.** L.C.M. $\left(\frac{5}{6}, \frac{9}{10}, \frac{8}{9}\right)$ = _____.

**22.** H.C.F. $\left(\frac{5}{6}, \frac{9}{10}, \frac{8}{9}\right)$ = _____.

**23.** Find the index of the greatest power of 2 which exactly divides 256! _____

**24.** Find the smallest number which when divided by 17 and 24 leaves remainders of 12 and 19, respectively.
(A) 403 (B) 413 (C) 393 (D) 419

**25.** Find the remainder when 1643276569 is divided by 25.
_____

**26.** Find the least natural number by which $(3^8)(5^{13})(7^{19})$ must be multiplied so that the product is a perfect square. _____

**27.** If $y! = 20(y-2)!$, $y =$
(A) 3 (B) 4 (C) 5 (D) 6

**28.** $\dfrac{(2.35)^3 + 7.05(1.45)^2 + 4.35(2.35)^2 + (1.45)^3}{(2.35)^2 + (4.70)(1.45) + (1.45)^2} =$
(A) 3×8 (B) 3×9 (C) 3×7 (D) 3×6

**29.** The number of digits in the cube root of a 29-digit number is
(A) 10 (B) 9 (C) 8 (D) 9 or 10

**30.** A number when divided successively by 7, 4 and 2 leaves respective remainders of 3, 2 and 1. Find the smallest such number.
(A) 45 (B) 42 (C) 49 (D) 56

**31.** Find the remainder when 367543216 is divided by 9. _____

**32.** The smallest positive integer divisible by $(2^2)(3^3)(5)$, $(3^2)(5^3)(7)$ and $(5^2)(7^3)(11)$ is $N$. How many distinct prime factors does $N$ have?
(A) 4 (B) 5 (C) 6 (D) 3

**33.** Find the H.C.F. of $\frac{7}{18}, \frac{11}{48}$ and $\frac{3}{5}$.
(A) $\frac{1}{360}$ (B) $\frac{1}{480}$ (C) $\frac{1}{720}$ (D) $\frac{1}{600}$

**34.** Find the remainder when $18^{18}$ is divided by 19.
(A) 1 (B) 18 (C) 11 (D) 4

**35.** Find the units digit of $3^{42} \times 8^{56} \times 6^{40}$.
(A) 2 (B) 4 (C) 6 (D) 8

**36.** Find the last digit of the product of any 10 consecutive odd natural numbers. _____

**37.** Find the least natural number to be added to 395 so that the sum is a perfect cube. _____

**38.** How many of the following numbers are divisible by 12?
7908, 14772, 13562, 12256, 15492
(A) 2 (B) 3 (C) 1 (D) 4

**39.** Find the area (in sq.cm) of the smallest square which can be formed using rectangular blocks of dimensions 7 cm × 5 cm.
(A) 1225 (B) 4900
(C) 11025 (D) None of these

**40.** What is the units digit of $6^{25} + 9^{16} + 5^{40}$?
(A) 2 (B) 4 (C) 5 (D) 7

**41.** $M = 7^{2N} - 1$ where $N$ is a natural number. M is divisible by 6 for
(A) any odd value of $N$ (B) any even value of $N$.
(C) any value of $N$ (D) no value of $N$.

**42.** Find the length of the longest possible scale (in cm) that can be used to measure the lengths 1 m 50 cm, 8 m 10 cm, 1 m 35 cm and 1 m 95 cm exactly.
(A) 5 (B) 15 (C) 25 (D) 35

**43.** Find the least 4-digit number which leaves a remainder of 10 when divided by 36. _____

**44.** $P$ is a prime number and $Q$ is a positive integer less than $P$. Find the H.C.F. of $Q$ and $Q + P$.
(A) $Q$ (B) $P - Q$ (C) $Q + P$ (D) 1

**45.** Find the number of three-digit natural numbers divisible by 8, 12 and 15. _____

**46.** A number when successively divided by 9 and 14 leaves remainders 8 and 5, respectively. Find the remainder when it is divided by 18.
(A) 17 (B) 15 (C) 13 (D) 11

**47.** Find the least natural number which when divided by 5 and 6, leaves remainders of 3 and 5, respectively. _____

**48.** What is the largest number that divides the product of any three successive even natural numbers?
(A) 48 (B) 24 (C) 12 (D) 64

**49.** $\dfrac{89 \times 89 \times 89 + 11 \times 11 \times 11}{89 \times 89 - 89 \times 11 + 11 \times 11} =$ _____.

**50.** How many trailing zeros does 100! have?
(A) 24 (B) 20 (C) 18 (D) 14

**51.** Find the tens digit of the product of the first 14 natural numbers. _____

**52.** Find the remainder when $7^{645}$ is divided by 10.
(A) 1 (B) 3 (C) 9 (D) 7

**53.** $N = (31)(32)(33) \ldots (39)$. Find the units digit of $N$.
(A) 0 (B) 2 (C) 4 (D) 6

**54.** Any 6 consecutive natural numbers will have their product divisible by
(A) 600 (B) 2160 (C) 480 (D) 720

**55.** Find the least natural number by which $(5^9)(3^6)(17)$ has to be multiplied to obtain a perfect square.
(A) 5 (B) 17 (C) 85 (D) 221

**56.** Find the largest 4-digit number, which when divided by 19 leaves a remainder of 6.
(A) 9984 (B) 9978
(C) 9999 (D) 9981

**57.** $P$ and $Q$ are integers. $P \times Q$ denotes the remainder obtained when $P \times Q$ is divided by 18. Find $(7 \times 6) \times 5$.
(A) 6 (B) 12 (C) 9 (D) 15

**58.** Find the index of the greatest power of 3 in 100!.
(A) 48 (B) 24 (C) 92 (D) 96

**59.** Find the remainder when $2^{2400}$ is divided by 9.
(A) 3 (B) 5 (C) 6 (D) 1

**60.** Find the least natural number that should be added to 54321677 to make it divisible by 8. _____

**61.** Three natural numbers are in the ratio 3:4: 5. Their L.C.M. is 480. Find the least of the three numbers.
(A) 18 (B) 24
(C) 36 (D) 27

**62.** $P$ is a prime number greater than 3. The remainder of $P^2$ divided by 24 is _____.

**63.** How many factors does 2400 have other than itself?
(A) 30 (B) 29 (C) 35 (D) 36

**64.** Find the last remainder when 192 is successively divided by 7, 2 and 4. _____

**65.** What is the remainder when $5^{37}$ is divided by 6? _____

**66.** Four traffic lights change colours at intervals of 45 seconds, 60 seconds, 75 seconds and 90 seconds. They changed colours simultaneously at 11 a.m. Find the next time when they would change colours simultaneously.
(A) 11: 10 a.m. (B) 11: 15 a.m.
(C) 11: 20 a.m. (D) 11: 25 a.m.

**67.** Find the number of ways in which $3^3 \times 2^3 \times 5^2$ can be expressed as a product of two factors.
(A) 46 (B) 24 (C) 48 (D) 96

**68.** What is the remainder when $3^{91}$ is divided by 11?
(A) 3 (B) 9 (C) 5 (D) 4

**69.** A number when divided by 891 leaves a remainder of 57. What is the remainder when the same number is divided by 27?
(A) 1 (B) 3 (C) 2 (D) 5

**70.** The product of 20 prime numbers is.
(A) even
(B) odd
(C) either even or odd

**71.** $B = 13^{2A} - 4^A$, where A is a natural number. B must be divisible by.
(A) 11 (B) 15
(C) Neither (A) nor (B) (D) Both (A) and (B)

**72.** Find the number of factors of $3^8 \times 6^4$. _____

**73.** Is H.C.F. [H.C.F. $(p, q)$, H.C.F. $(r, s)$] equal to H.C.F. [H.C.F. $(p, r)$, H.C.F. $(q, s)$]?
(A) Yes
(B) No
(C) Cannot say

**74.** H.C.F. $(x, y, z) = 1$. Is L.C.M. $(x, y, z) = (x)(y)(z)$?
(A) Yes
(B) No
(C) Cannot say

**75.** Find the least natural number (greater than 10) which leaves a remainder of 2 when divided by 55 or 12.
(A) 656 (B) 662 (C) 678 (D) 684

**76.** If L.C.M. $(x, y, z) = (x)(y)(z)$, H.C.F. $(y, z) =$
(A) 1 (B) 2
(C) $x$ (D) Cannot say

**77.** If $y - \frac{1}{y} = 3$, $y^2 + \frac{1}{y^2} =$ _____.

**78.** Is L.C.M.
$\left[\text{LCM}(p, q), \text{LCM}(r, s)\right]. = \text{L.C.M.}\left[\text{LCM}(p, r), \text{LCM}(q, s)\right]$?
(A) Yes (B) No (C) Cannot say

**79.** Is the ten-digit number PQRSTU6736 divisible by 32?
(A) Yes (B) No (C) Cannot say

**80.** If $x + \frac{1}{x} = 4$, $x^2 + \frac{1}{x^2}$.
(A) 8 (B) 10 (C) 12 (D) 14

**81.** Is $(3^3)(7^7)(21^5)$ a perfect square?
(A) Yes (B) No (C) Cannot say

**82.** Find the last digit of $4^n + 7^n$, where $n = 99$. _____

**83.** Which of the following numbers is divisible by 11?
(A) 1111111 (B) 77777
(C) 246642 (D) 7654321

**84.** If $N = 3^p \times 2^q \times 3^{2r}$, how many natural numbers less than $N$ are coprime to it?
(A) $\frac{N}{4}$ (B) $\frac{N}{3}$ (C) $\frac{2N}{3}$ (D) $\frac{N}{2}$

**85.** Find the sum of the factors of $(2^4)(3^3)$.
(A) 930 (B) 1240 (C) 1085 (D) 808

**86.** Which of the following is a prime number?
(A) 437 (B) 323 (C) 567 (D) 241

**87.** $\dfrac{5.71\times5.71\times5.71-3.21\times3.21\times3.21}{5.71\times5.71+5.17\times3.21+3.21\times3.21}=$ _____

**88.** A number has an odd number of factors. Is it a perfect cube?
(A) Yes (B) No (C) Cannot say

**89.** Find the sum of the factors of 437. _____

**90.** The number of distinct prime factors of 13013 is _____.

**91.** Find the remainder when $3^{48}$ is divided by 82.
(A) 81 (B) 1 (C) 3 (D) 9

**92.** How many natural numbers less than $2^{14}$ are coprime to it?
(A) $2^{12}$ (B) $2^{22}$ (C) $2^{23}$ (D) $2^{13}$

**93.** A number has an even number of factors. Is it a perfect square?
(A) Yes (B) No (C) Cannot say

**94.** Find the remainder of the division $2^{189}/5$. _____

**95.** How many prime numbers are there between 70 and 100?
(A) 5 (B) 19 (C) 1 (D) 6

**96.** If the sum of 35 distinct prime numbers is even, one of them has to be _____.

**97.** Find the sum of all the possible remainders when $3^n$ is divided by 5 where $n$ is any natural number. _____

**98.** The remainder of the square of any prime number more than 3 divided by 6 is.
(A) 1 (B) 3
(C) 5 (D) Either (A) or (C)

**99.** The sum of 50 odd numbers is.
(A) even (B) odd (C) Cannot say

**100.** What is the remainder when $4^{65}$ is divided by 7?
(A) 4 (B) 2 (C) 1 (D) 3

## EXERCISE-2

***Directions for questions 1 to 100*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answers in the space provided.

**1.** If $x = 0.38 + 0.0038 + 0.000038 + 0.00000038 + \ldots \infty$, its fractional value is $\frac{\quad}{99}$.

**2.** C is a composite number with an even number of factors. Consider the following statements:

(i) C has a factor lying between 1 and $\sqrt{c}$.

(ii) C has a factor lying between $\sqrt{c}$ and C.

Which of the following can be concluded?
(A) Both (i) and (ii) are true.
(B) Both (i) and (ii) are false.
(C) (i) is true but (ii) is false.
(D) (i) is false but (ii) is true.

**3.** A natural number $N$ when divided by 12 leaves a remainder of 7. Find the remainder when $N$ is divided by

(i) 6.

(ii) 24. _____ or

(iii) 18. _____ or _____ or _____

**4.** Find the sum of all the positive integers coprime to 420 and less than it. _____

**5.** Find the units digit of $19^{14568}$. _____

**6.** Find the last two digits of

(i) $(486)^{486}$.

(ii) $(675)^{239}$.

(iii) $(734)^{1054}$.

**7.** (i) $(557)^{269}$.

(ii) $(1043)^{375}$.

**8.** Find the remainder when $3^{83}$ is divided by 41.

**9.**

| | 90 | 1680 |
|---|---|---|
| (i) No. of distinct prime factors | | |
| (ii) No. of odd factors | | |
| (iii) No. of even factors | | |

**10.** How many odd natural numbers upto 1400 are divisible neither by 5 nor by 7? _____

Difficulty Level-2: Moderate

**11.** Find the last digit of $123^{39} \times 117^{41} \times 125^{48} \times 112^{38}$. _____

**12.** Deepa typed the first $n$ natural numbers on a keyboard without any gaps. If she had to press the numbered keys 1692 times, find $n$. _____

**13.** Find the remainder when $87^{82}$ is divided by 17. _____

**14.** If $a$, $b$ and $c$ are distinct odd positive integers, which of the following statements can be concluded?
(A) $a^3b^2c$ is odd.
(B) $(a-b)^2c^3$ is even.
(C) $(a+b+c)^2(a-b)$ is even.
(D) All of the above

**15.**
$$\frac{(0.58)^3+(0.52)^3+(0.6)^3-1.74\times 0.52\times 0.6}{(0.58)^2+(0.52)^2+(0.6)^2-(0.58)(0.52)-(0.58)(0.6)-(0.52)(0.6)}$$
= _____.

**16.** Find the last digit of $6^{6^{6^6}}$. _____

**17.**
$$N=\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2}}}}}$$
Find the value of $N$

(A) $\frac{8}{13}$ (B) $\frac{13}{8}$ (C) $\frac{13}{21}$ (D) $\frac{21}{13}$

**18.** A natural number $N$ when divided by 12 leaves a remainder of 7. Find the remainder when
(i) $2N$ is divided by 12.
(ii) $N^2$ is divided by 12.

**19.** Find the remainder when $3^{40}$ is divided by 11.
(A) 3 (B) 9 (C) 5 (D) 1

**20.** The area of a rectangle is 24. Find the sum of the perimeters of all such rectangles whose both sides are integers.

**21.** $N = 4831 \times 4833 \times 4835$. What is the remainder when $N$ is divided by 24?
(A) 9 (B) 23 (C) 21 (D) 11

**22.** Find the greatest number which divides 394 and 658, leaving remainders 1 and 3, respectively. _____

**23.** Find the last two digits of $49^{784}$. _____

**24.** If the seven-digit number $8a2b3c5$ is divisible by 99, find the number of possible values of $a+b+c$. _____

**25.** Find the last two digits of $2^{1053}$. _____

**26.** If $k(N)$ denotes the number of ways of expressing $N$ as a difference of two perfect squares, which of the following has the least value?
(A) $k(187)$ (B) $k(120)$ (C) $k(110)$ (D) $k(105)$

**27.** How many 2-digit numbers have exactly 3 factors? _____

**28.** $X$ and $Y$ are positive integers. $X$ leaves a remainder of 1 when divided by 40. $Y$ leaves a remainder of 2 when divided by 40.

Consider the following statements:

I. $8^Y + 8Y$ is divisible by 5.

II. $2^X - 2X$ is divisible by 5.

Which of the following can be concluded?
(A) Only I is true
(B) Only II is true
(C) Both I and II are true
(D) Neither I nor II is true

**29.** Find the greatest number, by which when 472, 832 and 1372 are divided, the remainders are the same. _____

**30.** Let A be the set of prime numbers less than 50. We multiply all the elements of A to obtain a number B. With how many consecutive zeros will B end?
(A) 11 (B) 5 (C) 2 (D) 1

**31.** Find the remainder when the 100-digit number formed by writing consecutive natural numbers starting from 1 next to each other is divided by 8. _____

**32.** A number when divided by 16 or 12 leaves a remainder 3 in each case. When the number is divided by 24, what is the remainder? _____

**33.** Find the remainder when $48^{50} + 50^{50}$ is divided by 49.
(A) 2 (B) 1 (C) 0 (D) 48

**34.** How many positive integers are factors of exactly one of 1125 and 1800? _____

**35.** Anil has some toffees with him. If he distributes them equally among 20 or 30 or 50 children, he would be left with 16 toffees in each case. Instead, if he distributes them equally among 14 children, he would be left with no toffees. Find the minimum number of toffees he has.
_____

**36.** Find the remainder when $15^{1700} - 1716$ is divided by 16.
(A) 9 (B) 3 (C) 13 (D) 7

**37.** What is the least natural number by which 1920 must be multiplied so that the product is

(i) a perfect square?

(ii) a perfect cube?

**38.** $X$ is an integer and $Y$ is an odd positive integer. If $3^Y + X^2 = (2^4)\ (5^4)$, how many values can $X$ take?
(A) 0 (B) 1
(C) 2 (D) More than 2

**39.** Find the greatest number which divides 425 and 373, leaving remainders 2 and 4, respectively.

40. Find the ratio of the last two digits of $2^{974}$ to that of $2^{696}$. _____

**41.** The L.C.M. and H.C.F. of (64, 80, $x$) are 960 and 16, respectively. Which of the following could be the value of $x$?
(A) 96 (B) 112 (C) 32 (D) 128

**42.** The remainder when $(17^3 + 19^3 + 21^3 + 23^3)$ is divided by 80 is
(A) 10 (B) 20 (C) 40 (D) 0

**43.** A number when divided by 15 leaves 5 as the remainder and when divided by 20 leaves 10 as the remainder. When the number is divided by 60, what is the remainder? _____

**44.** Find the units digit of $34563^{20359} + 2358^{784}$.
(A) 3 (B) 4 (C) 5 (D) 6

**45.** Find the greatest number which divides 247, 457 and 1087 leaving the same remainder in each case. _____

**46.** Find the remainder when $3^{135}$ is divided by 67.
(A) 3 (B) 1 (C) 27 (D) 66

**47.** Find the H.C.F. and L.C.M. of 72, 108 and 162. _____

**48.** Find the remainder when 51! is divided by 53.
(A) 52 (B) 51 (C) 2 (D) 1

**49.** Find the number of zeros at the end of 175! _____

**50.** If 1! + 2! + 3! + 4! +..... + 50! is divided by 5!, the remainder is
(A) 3 (B) 33 (C) 13 (D) 93

**51.** Six bells ring together at 11 am and after that they ring at intervals of 5, 10, 15, 20, 25 and 30 seconds. How many times will they ring together from 11:00 am to 12:30 *pm* on the same day? _____

**52.** Find the remainder when 10! is divided by 11. _____

**53.** Find the index of the greatest power (IGP) of 2 in 10! _____

**54.** Find the remainder when 92879287 ... up to 360 digits is divided by 101. _____

**55.** What is the least natural number by which $(3^9)(5^{13})$ must be divided so that the remainder is zero and the quotient is:

(i) a perfect square?

(ii) a perfect cube?

**56.** How many two-digit numbers when divided by 4 leave a remainder 1 and when divided by 5 leave a remainder 2? _____

**57.** Find the remainder when $(24^{23} + 21^{23})$ is divided by

(i) 15

(ii) 45

**58.** Find the remainder when $7^{71}$ is divided by 35. _____

**59.** In an examination centre, 60 students were to write the Mathematics test, 36 students were to write the Biology test and 24 students were to write the History test. Find the minimum number of rooms required, if in each room the same number of students are to be seated and in each room a test for only one subject is to be conducted. _____

**60.** Find the remainder when 212121... up to 120 digits is divided by 99. _____

**61.**

| | 72 | 1560 |
|---|---|---|
| (i) No. of ways of expressing as a product of two factors | | |
| (ii) No. of ways of expressing as a product of two co-prime factors. | | |

**62.** If $p$, $q$ and $r$ are prime numbers satisfying $p = q + 2 = r + 4$, how many combinations exist for $p$, $q$, $r$? _____

**63.** Find the index of the greatest power of 8 in 120! _____

**64.**

| | 80 | 2340 |
|---|---|---|
| (i) Number of factors | | |
| (ii) Sum of factors | | |
| (iii) Product of factors | | |

**65.** Which of the following is not a factor of $(4^{12} - 1)$?
(A) 5 (B) 63 (C) 255 (D) 127

**66.** Find the smallest number which when divided by 6 leaves a remainder 2 and when divided by 13 leaves a remainder 6. _____

**67.** Using remainder theorem, find the remainder of $2^{59}$ when divided by

(A) 9 (B) 13 (C) 17

**68.** A number when divided successively by 6 and 7 leaves remainders of 4 and 5, respectively. Find the remainder when the number is divided by 21. _____

**69.** The respective values of the L.C.M. and H.C.F. of $\frac{3}{7}, \frac{5}{9}, \frac{4}{10}$ and $\frac{8}{9}$ are _____ and _____.

**70.** Which of the following is the smallest number which when decreased by 3 is divisible by 54, 216 and 121?

(A) 26139 (B) 23179
(C) 25136 (D) 26172

**71.** Simplify: $2+3\times\frac{1}{3}$ of $6-\left(5-\left\{\frac{5}{8}+3\frac{3}{8}-\frac{1}{8}\right\}\right)$.

(A) $\frac{53}{8}$ (B) $\frac{7}{8}$ (C) $\frac{47}{8}$ (D) $\frac{55}{8}$

**72.** Rani has 24 notebooks, 72 erasers and 60 pencils. She gives them to $n$ children in such a way that each type of item has been distributed equally among them. What is the smallest figure for the total number of items that each child could have got?

(A) 8 (B) 12
(C) 14 (D) None of these

**73.** The product of all the factors of a number is equal to the square of the number. If the sum of all the factors of the number other than the number itself is 21, find the number of possible values for the number.

(A) 0 (B) 1 (C) 2 (D) More than 2.

**74.** The number 81A84B6C, where each of the letters A, B and C represents a distinct digit, is divisible by each of 5, 8, 9. Find the value of A + B if B is not a prime.

(A) 6 (B) 8 (C) 10 (D) 9

**75.** If $t_i = i$ where $i$ is a natural number less than 100, for how many values of i is $t_i + t_{i+1}$ a perfect square?

(A) 5 (B) 4 (C) 6 (D) More than 6

**76.** $N = 3! + 4! + \ldots. + 64!$

Which of the following statements is true regarding $N$?

(A) $N$ is a perfect square. (B) $N$ is a perfect cube.
(C) Both (A) and (B) (D) Neither (A) nor (B)

**77.** Which of the following must be subtracted from 214369 so that the resulting number is a multiple of 11?

(A) 4 (B) 2 (C) 3 (D) 1

**78.** $N$ = 0.abcabc … where $N$ is a recurring decimal and at most two of $a$, $b$, $c$ are zero. Which of the following numbers necessarily results in an integer, when multiplied by $N$?

(A) 1000 (B) 2997 (C) 3333 (D) 9999

**79.** How many integer pairs are there which satisfy the condition that the sum of the integers is equal to the product of the integers?

(A) 2 (B) 1 (C) 0 (D) 3

**80.** Raju was given a problem of adding a certain number of consecutive natural numbers starting from 1. By mistake, he added a number twice. He obtained the sum as 825. Find the number that he added twice.

(A) 20 (B) 15 (C) 10 (D) 5

**81.** If the 11-digit number 853$x$426071$y$ is divisible by 8 and 9, what are the respective values of the digits $x$ and $y$?

(A) 2, 7 (B) 7, 2 (C) 6, 2 (D) 5, 4

**82.** The H.C.F. and L.C.M. of a pair of numbers are 11 and 1001, respectively. If the sum of the two numbers is 220, find the smaller of the two numbers. _____

**83.** How many natural numbers from 31 to 47 are coprime to 73?

(A) 10 (B) 12 (C) 18 (D) 17

**84.** $S$ = {5, 11, 17….509}. T is a subset of S. The sum of no two elements in T exceeds 449. What is the maximum possible number of elements in T?

(A) 36 (B) 37 (C) 38 (D) 42

**85.** Find the remainder of $(1!)^2 + (2!)^2 + (3!)^2 + (4!)^2 + \ldots + (50!)^2$ divided by 10.

(A) 1 (B) 3 (C) 7 (D) 9

**86.** Find the sum of the remainders obtained when a number $n$ is divided by 9 and 7 successively given that $n$ is the smallest number that leaves respective remainders of 4, 6 and 9 when divided successively by 13, 11 and 15.

(A) 4 (B) 5 (C) 9 (D) 6

**87.** How many numbers below 180, other than unity, are such that the H.C.F. of the number and 180 is unity?

(A) 54 (B) 47 (C) 132 (D) 48

**88.** If $x$ and $y$ are even integers, the number of solutions of $x^2 - y^2 = 7389746232$ is

(A) 0 (B) 1
(C) 2 (D) more than 2

**89.** How many positive integers are factors of exactly one of 2700 and 1134? _____

**90.** How many natural numbers less than 150 leave a remainder 5 when divided by 7 and leave a remainder 3 when divided by 6? _____

**91.** Find the largest number which divides 606, 732 and 915 leaving remainders 6, 12 and 15, respectively.
(A) 60 (B) 75 (C) 90 (D) 120

**92.** What will the last digit of the number $(348)^{8236}(739)^{4581}$ be?
(A) 8 (B) 2 (C) 4 (D) 6

**93.** When 5, 8 and 12 divide a multiple of 13, the remainders left are 3, 6 and 10, respectively. Find the least such number.
(A) 169 (B) 598
(C) 1298 (D) 1602

**94.** What is the least natural number that should be subtracted from 7575 so that the result leaves a remainder of 5 when divided by 8, 9, 14 and 20? _____

**95.** If the number 2484*x*36*y* is divisible by 36, find the minimum value of $x - y$.
(A) 0 (B) 1 (C) 2 (D) –7

**96.** The difference of any 30-digit number and its reverse is always divisible by
(A) 9 but not necessarily by 11
(B) 11 but not necessarily by 9
(C) 99
(D) neither 9 nor 11

**97.** The product of two natural numbers is 576 and their H.C.F. is 4. How many such pairs of numbers exist?

**98.** If $N = 5 \times 10 \times 15 \times \ldots \times 50$, how many zeros does $N$ end with? _____

**99.** Find the number of positive integers less than 3600 and coprime to it.
(A) 480 (B) 960
(C) 1440 (D) 1800

**100.** Express $1.\overline{137}$ as a fraction _____.

## EXERCISE-3

***Directions for questions 1 to 100*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Find the index of the greatest power of 24 which divides 150!.
(A) 48 (B) 38 (C) 54 (D) 60

**2.** Consecutive positive integers starting from 1 are written on a board. A student erased one of these integers. The total of the remaining integers is 4035. What was the integer erased by the student? _____

**3.** Find the remainder when $(14!)^{38}$ is divided by 17. _____

**4.** How many natural numbers less than 2000 are neither perfect squares nor perfect cubes?
(A) 1916 (B) 1928 (C) 1940 (D) 1946

**5.** Find the last digit of $14^{14^{14}}$. _____

**6.** F($N$) is the number of factors of the positive integer $N$. $P$ and $Q$ are positive integers each having at most two prime factors. F($P$)F($Q$) = F(PQ)

Consider the following statements:

I. $P$ and $Q$ are coprime
II. $P$ and $Q$ are both prime

Which of the following is/are possible?
(A) Only I
(B) Only II
(C) Both I and II
(D) Neither I nor II

**7.** Which of the following is not a factor of $80^{26} - 1$?
(A) 79 (B) 81 (C) 6399 (D) 6401

**8.** A number can be expressed as $2^p5^q$ where $p$ and $q$ are natural numbers. If the sum of all the factors of the number is 42, the ordered pair $(p, q)$ is.
(A) (2, 1) (B) (1, 2) (C) (3, 2) (D) (2, 3)

**9.** Find the sum of all the factors of 24696.
(A) 41300 (B) 42400 (C) 42300 (D) 78000

**10.** If $n!$ is divisible by $(1001)^2$, find the least possible value of $n$. _____

**11.** $X$ = (22 … up to $n$ digits)$^2$ + (444 …up to $n$ digits) + 1
When $n = 10$, $\frac{9}{2}\left(\sqrt{x} - 1\right) =$

(A) $10^{10} + 1$ (B) $10^{10} - 1$
(C) $10^9 + 2$ (D) $10^9 - 8$

**12.** What is the remainder when 987987 ... up to 600 digits is divided by 999?
(A) 231 (B) 348 (C) 597 (D) 420

**13.** $a$ is the greatest integer less than or equal to $a$.
$F(a,b) = [a/b]$
If $F(F(F(F(N,2),3),4),5),6) = 1$, the maximum value of $N$ is.
(A) 719 (B) 1440 (C) 720 (D) 1439

**14.** Find the number of odd natural numbers less than or equal to 1200 which are neither divisible by 3 nor by 5.
(A) 600 (B) 320 (C) 300 (D) 280

**15.** How many natural numbers up to 9999 are divisible by 5 or 13 or 17? _____

**16.** What is the remainder when 406417406417 ... up to 300 digits is divided by 1001?
(A) 440 (B) 645 (C) 795 (D) 550

**17.** $E = (3p + 6q - 9r)^3 + (3p - 6q + 9r)^3 + (-3p + 6q + 9r)^3 - 3(3p + 6q - 9r)(3p - 6q + 9r)(-3p + 6q + 9r)$, where $p$, $q$, $r$ satisfy $p + 2q = -3r$.
Consider the following statements:
I. E is at least zero.
II. E is at most zero.
Which of the following can be concluded?
(A) Only I
(B) Only II
(C) Both I and II
(D) Neither I nor II

**18.** The L.C.M. of $3^{2003} - 1$, $3^{2003}$, $3^{2003} + 1$ is
(A) $\left(\frac{3^{4006} - 1}{2}\right)\left(3^{2003}\right)$ (B) $(3^{4006} - 1)(3^{2003})$
(C) $(3^{4006} - 1)(3^{2004})$ (D) $\left(\frac{3^{4006} - 1}{2}\right)\left(3^{2004}\right)$

**19.** The sum of all the positive integers between $2^{2N}$ and $2^{2N+3}$, where $N$ is a positive integer, is divisible by 162.
Consider the following statements:
I. $N$ leaves a remainder of 2 when divided by 3
II. $N$ leaves a remainder of 1 when divided by 3
Which of the following can be concluded?
(A) Only I is possible.
(B) Only II is possible.
(C) Both I and II are possible.
(D) Neither I nor II is possible.

**20.** If $x$ is a natural number greater than 10, the remainder when $x^{550} + x^{275} + x^{55} + x^{11}$ is divided by $(x + 1)$ is
(A) $x - 3$ (B) $x - 2$ (C) $x$ (D) $x - 1$

**21.** The sum of all the factors of 323323 is _____.

**22.** What is the remainder when 15! is divided by 34? _____

**23.** If $1 \le P \le 40$, how many values of $P$ exist such that $(P - 1)!$ is not divisible by $P$?
(A) 11 (B) 10 (C) 12 (D) 13

**24.** The Oxford University Press compiled a 2000-page dictionary. Just before the dictionary went for printing, it was discovered that there were no page numbers. How many times should a typist press numbered keys from 0 to 9 on the keyboard so as to number the dictionary from 1 to 2000?
(A) 6889 (B) 6883 (C) 6879 (D) 6893

**25.** What is the remainder when 909890989098... up to 602 digits is divided by 101?
(A) 1 (B) 100 (C) 10 (D) 19

**26.** What is the remainder when $122^{230}$ is divided by 25?
_____

**27.** What is the remainder when $483^{483}$ is divided by 100?
_____

**28.** Find the remainder when 8! is divided by 11. _____

**29.** Find the sum of the last two digits of $(7776)^{1240}(21)^{1240}$.
(A) 7 (B) 13 (C) 9 (D) 15

**30.** What is the remainder when $(29^6 + 30^6 + 31^6 + 32^6 + 33^6 + 34^6 + 35^6)$ is divided by 32?
(A) 13 (B) 25 (C) 30 (D) 20

**31.** The signboard outside the department store 'Ram and Shyam' lights up as described below. When the switch is turned on, all the three words light up and remain lit for 3 seconds. After that, the first word is switched off for $7^5/_6$ seconds, the second word is switched off for $1^1/_3$ seconds and the third word is switched off for $5^2/_3$ seconds. Then each word is again switched on for 3 seconds and then switched off for the time duration mentioned. This process continues repeatedly. After how many seconds of switching on the signboard will the entire board be switched on for the second time for 3 seconds?
(A) $40\frac{1}{3}$ (B) $41\frac{2}{3}$ (C) $42\frac{2}{3}$ (D) $43\frac{1}{3}$

**32.** $P$ is a positive integer not more than 20. If $2^P + P^3$ is divisible by 5, how many values can $P$ take?
(A) 3 (B) 4 (C) 5 (D) 6

**33.** $[x]$ is the greatest integer less than or equal to $x$

$$\left[\frac{2^0}{7}\right]+\left[\frac{2^1}{7}\right]+\left[\frac{2^2}{7}\right]+\ldots+\left[\frac{2^{600}}{7}\right]=$$

(A) $\frac{1}{7}(2^{601}-1395)$ (B) $\frac{1}{7}(2^{601}-1409)$

(C) $\frac{1}{7}(2^{601}-1416)$ (D) $\frac{1}{7}(2^{601}-1402)$

**34.** A box contains 1000 tokens numbered 1 to 1000. Amar takes all the tokens that are numbered with the multiples of 2. Out of the remaining Bimal takes all the tokens that are numbered with the multiples of 5. What is the sum of the numbers on the tokens left in the box?
(A) 401000 (B) 200000
(C) 200500 (D) 400000

**35.** What is the remainder when 19! is divided by 289? _____

**36.** Akash had a notebook containing 240 pages or 120 leaves made up of 60 sheets, folded at the middle and stapled along the fold. The pages were numbered 1 to 240. Ganga took 15 sheets from the middle of the book. The sum of the remaining page numbers is _____.

**37.** If $\times$ is the least integer greater than or equal to $x$, and $y$ is any positive integer greater than 5, $\left[\frac{12^{2y}}{13}\right]+\left[\frac{12^{2y+1}}{13}\right]$ equals
(A) $12^{2y}+10$ (B) $12^{2y}+9$
(C) $12^{2y}-1$ (D) $12^{2y}+1$

**38.** Find the remainder when $2^{216}$ is divided by 105. _____

**39.** Find the product of all the odd factors of $N = (2^5)(3^3)(5^2)$.
(A) $3^{24}\,5^9$ (B) $3^{18}\,5^6$ (C) $3^{18}\,5^{12}$ (D) $3^{12}\,5^9$

**40.** If $A$, $B$ and $C$ are positive integers such that the last two digits of $A^B$ are not the same as the last two digits of $A^{B+{}^{20}C}$, which of the following must be true?
(A) A must be even.
(B) B must be 1.
(C) Both (A) and (B)
(D) Neither (A) nor (B)

**41.** In how many ways can $45^2$ be written as a sum or difference of two non-zero perfect squares?
(A) 8 (B) 9 (C) 10 (D) 11

**42.** Find the least positive odd integer which has the same number of factors as 540.
(A) 4725 (B) 8505 (C) 3465 (D) 3245

**43.** What is the sum of all the even factors of 2160?
(A) 10,800 (B) 3,600 (C) 7,200 (D) 14,400

**44.** Five blocks of chocolate weighing $2^1/_3$ kg, $4^2/_3$ kg, $4^1/_5$ kg, $5^5/_6$ kg and $6^1/_8$ kg, respectively, were bought for a birthday party. The five blocks must be cut into equal pieces. What is the minimum number of pieces obtained? _____

**45.** If $n$ is a natural number, the remainder of $4^{6n-6} - 6^{4n-4} - 3$ divided by 100 is
(A) 91 (B) 67
(C) 97 (D) dependent on $N$

**46.** $N$ is the number formed by writing all the positive integers from 46 to 105 next to each other. Find the remainder when $N$ is divided by 9.
(A) 0 (B) 3 (C) 4 (D) 6

**47.** Find the remainder when $32^{101}$ is divided by 15. _____

**48.** If $P(a, b, c) = 27 - 9(a + b + c) + 3(ab + bc + ca) - abc$, then $P\left(\frac{3}{2},\frac{3}{2},1\right)$ = _____.

**49.** If $p$ and $q$ are positive integers, find the number of ordered pairs $(p, q)$ that satisfy $p^2 - q^2 = 238$.
(A) 8 (B) 2 (C) 0 (D) 5

**50.** What is the product of all the factors of $4! \times 5!$?
(A) $120^8 \times 24^4$ (B) 2880
(C) $2880^{21}$ (D) $2880^{42}$

**51.** What is the remainder when $3^{1000}$ is divided by 73?
(A) 1 (B) 8 (C) 16 (D) 72

**52.** A number when divided by $d$ leaves 5 as the remainder. The number when divided by $3d$ leaves 25 as the remainder. How many values of $d$ satisfy these conditions? _____

**53.** What is the units digit of the number $(2^9)(3^8)(4^7)(5^6)(6^5)(7^4)(8^3)(9^2)$ after all the trailing zeroes at the end are stripped off? _____

**54.** $N$ = 123123123… (1000 digits)
Find the remainder when $N$ is divided by 143. _____

**55.** If $n$ is a natural number, which of the following is a factor of $n^7 - n$?
(A) 13 (B) 9 (C) 7 (D) 5

**56.** The greatest power of 3 that is a factor of 58 + 59 + 60 + 61 + 62 + 63 + …+90 is _____.

**57.** A number when divided by 7 leaves a remainder 4, when divided by 19 leaves a remainder 8 and when divided by 39 it leaves a remainder 12. Find the least possible such number.
(A) 4305 (B) 4302 (C) 4263 (D) 9489

**58.** Find the remainder when $5^{80}$ is divided by 187. _____

**59.** The H.C.F. of $3^p5^{q+4}7^711^5$ and $3^{q+5}5^{p+4}7^x11^x$ is $3^x5^x7^x11^x$ where $p$ and $q$ are natural numbers. Find the value of $p$.
_____

**60.** The remainder of $484^{484}$ divided by 529 is
(A) 440 (B) 484 (C) 510 (D) 45

**61.** The L.C.M. of 35, 85 and a number $k$ is 7735 and their H.C.F. is 5. What is the least possible value of $k$?
(A) 65 (B) 25 (C) 325 (D) 13

**62.** Some saplings are planted in rows and columns, such that the number of rows is equal to the number of columns. What is the least number of saplings if it is known that the number of saplings is a multiple of 15, 18 and 24? _____

**63.** A number when divided by a divisor leaves 7 as the remainder. The number when divided by twice the divisor leaves 47 as the remainder. How many divisors satisfy these conditions and what are these divisors? _____

**64.** What is the remainder when 53575357… up to 900 digits is divided by 999? _____

**65.** The 600-digit number 1223334444 … is divided by 16. Find the remainder.
(A) 4 (B) 8 (C) 0 (D) 12

**66.** Two numbers are such that their H.C.F. is 16 and their sum is 144. How many such pairs of numbers are possible?
(A) 4 (B) 3 (C) 2 (D) 5

**67.** Find the remainder when $81^{82}$ is divided by 17. _____

**68.** A natural number when divided successively by 5, 4, and 3 leaves remainders 3, 2, and 1, respectively. Find the smallest such four-digit number and find the number of such four-digit numbers. _____

**69.** Let $\lceil x \rceil$ denote the least integer greater than or equal to $x$.
Let $A(x, y) = \lceil 2x \rceil + \lceil 2y \rceil$ and $B(x, y) = \lceil x \rceil + \lceil y \rceil + \lceil x + y \rceil$
Which of the following is false?
(A) $A(x, y) = B(x, y)$ (B) $A(x, y) > B(x, y)$
(C) $A(x, y) < B(x, y)$ (D) $A(x, y) > B(x, y)$

**70.** Find the remainder when $2^{54}$ is divided by 24. _____

**71.** If the index of the greatest power of 5 in $n!$ is 31, what is the index of the greatest power of 11 in $n!$? _____

**72.** $S$ is a set of positive integers such that each element $x$ of $S$ satisfies the following conditions.
(i) Each digit of $x$ is even.
(ii) $2100 \le x \le 2300$.
How many elements of $S$ are divisible by 3?

**73.** Find the remainder when $37^{157}$ is divided by 79. _____

**74.** If $N$ has 4 factors, what is the difference between the minimum and the maximum number of factors that $N^2$ can have? _____

**75.** The L.C.M. of $N$ distinct natural numbers is 576. The maximum value of $N$ is
(A) 18 (B) 20 (C) 21 (D) 23

**76.** A boy invests some money in a fixed deposit. The amount in rupees is numerically a five-digit palindrome, which when divided by 633, 1055 and 1477 leaves respective remainders of 600, 1022 and 1444. Find the amount invested in rupees. _____

**77.** In an office, there are 80 employees. All the employees visited an orphanage having 80 orphans. The first employee donated ₹1000 to every orphan. The second employee donated ₹1000 to every second orphan starting from the second orphan. The third employee donated ₹1000 to every third orphan starting from the third orphan and so on. How many orphans received donations from an odd number of employees? _____

**78.** Which of the following is prime?
(A) $2^{70}+1$ (B) $2^{96}+1$
(C) $2^{160}+1$ (D) None of these

**79.** Two numbers when divided by a certain divisor leave remainders 238 and 342, respectively. When the sum of the two numbers is divided by the same divisor, the remainder is 156. Find the divisor.
(A) 384 (B) 404
(C) 424 (D) Cannot be determined

**80.** Find the remainder when $(35)^{128}$ is divided by 50. _____

**81.** There are three lights which are switched on at intervals of 54 seconds, 48 seconds and 36 seconds, respectively. Each light is kept on for 3 seconds after which it is switched off. If all the lights are switched on simultaneously at 5.00 a.m., how many times will they be switched on together between 6.00 a.m. and 7.00 a.m.? _____

**82.** Find the remainder when $2^{73}$ is divided by 21. _____

**83.** How many odd natural numbers less than 90 can be expressed as a difference of squares of two natural numbers in exactly one way? _____

**84.** Find the remainder when 324516324516 …… upto 600 digits is divided by 1001. _____

**85.** If the index of the greatest power of 7 in ***n***! is 7, how many distinct values can ***n*** take? _____

**86.** The L.C.M. of $2^{2016}$, $2(2^{2015}-1)$, $2(2^{2015}-2)$ is
(A) $2^{2016}(2^{2015}-1)(2^{2015}-2)$
(B) $2^{2015}(2^{2015}-1)(2^{2015}-2)$
(C) $2^{2017}(2^{2015}-1)(2^{2015}-2)$
(D) $2^{2018}(2^{2015}-1)(2^{2015}-2)$

**87.** Ravi distributed all the marbles with him equally among 8 children and found that 5 marbles were left. Had he distributed the marbles equally among 12 or 18 children, he would have still had 5 marbles left with him. If the number of marbles Ravi distributed was less than 200, how many marbles did he initially have? _____

**88.** How many integers from 250 to 750 are coprime to 200? _____

**89.** Find the index of the greatest power of 30 in 150! _____

**90.** $N = 67^{60} - 780$

Consider the following statements:

I. The remainder of $N$ divided by 13 is 1.

II. The remainder of $N$ divided by 11 is 4.

Which of the following can be concluded?
(A) Only I is true.
(B) Only II is true.
(C) Both I and II are true.
(D) Neither I nor II is true.

**91.** M=3(3!)+4(4!)+ … +15(15!) _____.

What is the remainder when M-15 is divided by 14!–2?

**92.** Find the index of the greatest power of 7 in $\frac{1000!}{500!}$.

(A) 74 (B) 90 (C) 82 (D) 98

**93.** Find the number of zeros at the end of 1000! _____

**94.** If the index of the greatest power of 5 in ***n***! is 4, how many distinct values can ***n*** take? _____

**95.** How many of the following can be a perfect square?

1. ABC86 2. 1X2Y75 3. AB56

4. B36 5. AB9 6. A6B

**96.** A number when divided successively by 5, 6 and 8 leaves remainders 1, 3 and 4, respectively. What is the remainder when the number is divided by 96? _____

**97.** Find the remainder when 459459459 … upto 900 digits is divided by 999. _____

**98.** What is the number of integers between 150 and 750 (both excluded) that are divisible by 7 or 9 but not 11? _____

**99.** A four-digit number, $N$, when successively divided by 5, 6 and 8 leaves remainders 4, 2 and 1, respectively. What is the greatest value of $N$?
(A) 9844 (B) 9994 (C) 9644 (D) 9884

**100.** There are four prime numbers written in ascending order. The product of the first three prime numbers is 2431 and that of the last three is 4199. Find the greatest of them. _____

## EXERCISE-4

***Directions for questions 1 to 50:*** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Let $P$ be $xyz$, which is a three-digit number. $G(P) = 5^x 7^y 11^z$. $Q$ and $R$ are three-digit numbers satisfying $605\,[G(Q)] = 175\,[G(R)]$. Find $R - Q$.
(A) – 96 (B) – 100 (C) – 104 (D) – 108

**2.** $A = 3^{a^2-6a+45}$ and $B = 3^{a^2}$ where $a$ is a positive integer. L.C.M. (A, B) = A. How many values can $a$ take?
(A) 6 (B) 7 (C) 8 (D) None of above

**3.** $N$ is a natural number obtained by adding 16 to the product of four consecutive even natural numbers. How many of the following statements are always true?

(1) $N$ is divisible by 32.

(2) $N$ is divisible by 16.

(3) $N$ is divisible by 64.

(4) $N$ is a perfect square.
(A) 0 (B) 1 (C) 2 (D) 3

**4.** A three-digit number $N$ leaves the same remainder upon dividing 68488 and 67516. How many possible values does $N$ have? _____

**5.** The sum of all the factors of 11111111 is _____.

**6.** $Y$ is an even natural number satisfying $Y \;\; 4$. $X = Y^2 + 2Y$. The largest natural number that always divides $X^2 - 8X$ is
(A) 96 (B) 144 (C) 384 (D) 192

**7.** Find the index of the greatest power (IGP) of 24 contained in 360!.
(A) 108 (B) 118 (C) 128 (D) 178

**8.** Find the remainder when 105! is divided by 214.
(A) 168 (B) 108 (C) 196 (D) 172

**9.** Find the remainder when 70! is divided by $67^2$.
(A) 4163 (B) 4087 (C) 61 (D) 423

**10.** Find the remainder when $2222^{7777} + 7777^{2222}$ is divided by 9. _____

**11.** $P$ is a single digit positive integer. For how many values of $P$ is $(8^P + 8P)$ a multiple of 10? _____

**12.** $(1 + x + x^2)^N = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots + a_{2N}x^{2N}$ where $N$ is a positive integer.

Consider the following statements:

I. If $N$ is odd, $a_0 + a_2 + a_4 + \ldots a_{2N}$ is even.

II. If $N$ is even, $a_0 + a_2 + a_4 \ldots a_{2N}$ is odd.

Which of the following is/are true?
(A) Only I (B) Only II
(C) Both I and II (D) Neither I nor II

**13.** If $N$ = 1234 ..... up to 1000 digits, the remainder when $N$ is divided by 3125 is
(A) 568 (B) 2947 (C) 2239 (D) 2443

**14.** $T_N = N(N!)$ and $S(N) = \sum_{i=1}^{N} T_i$.

Find the remainder when $(S(12))^2$ is divided by 13.
(A) 12 (B) 1 (C) 6 (D) 7

**15.** $N$ is a positive integer not more then 25. For how many values of $N$ is the difference of $13^N$ and $N^3$ divisible by 5?
_____

**16.** The remainder when $2^{32N+16} - 3^{12N+6} - 5$ (where $N$ is a positive integer) is divided by 283 is _____.

**17.** Karim wanted to add all the natural numbers from 1 to $N$. Accidentally, he missed out a number. He then counted the numbers that he actually added and computed their average as $27\frac{3}{13}$. Find the number that he missed.
_____

**18.** The number of positive integers not greater than 120 which are divisible by 3 or 4 or 5 is
(A) 72 (B) 80 (C) 84 (D) 76

**19.** Find the index of the greatest power of 3 in 63! + 64! + ... + 100! _____

**20.** The King of Patiala allocated 33,274 gold coins for distribution among his subjects while the Maharaja of Mysore allocated 30,905 gold coins for distribution among his subjects. Each king distributed $p$ coins to each of his subjects and surprisingly, both the kings were left with the same number of gold coins. If $p$ is a 2-digit number, what is the difference in the number of subjects of the two kings?
(A) 309 (B) 103
(C) 23 (D) cannot be determined

**21.** A natural number $N$ has 6 factors and it can be written as a product of two natural numbers (out of which only one is prime) in two ways. If $N$ is the smallest such number, what is the sum of all the natural numbers less than $N$ and co-prime to it? _____

**22.** A positive integer is formed by writing all the integers from 80 to 124 one after another.

Consider the following statements:

I. The positive integer is divisible by 3.

II. The positive integer is divisible by 9.

Which of the following is /are true?
(A) Only I (B) Only II
(C) Both I and II (D) Neither I nor II

**23.** Each of the numbers 3946, 4687 and 5599 leaves the same remainder when divided by natural number $N$. How many values can $N$ take?
(A) 4 (B) 5 (C) 6 (D) More than 6

**24.** Three bells $X$, $Y$ and $Z$ ring at regular intervals and ring simultaneously 24 times every day. If $Y$ rings less frequently than $X$ but more frequently than $Z$, then what could be the minimum number of times that $Y$ rings in a day?
(A) 48 (B) 36 (C) 54 (D) 72

**25.** How many natural numbers between 50 and 1500 are neither perfect squares nor perfect cubes?
(A) 1406 (B) 1409 (C) 1412 (D) 1415

**26.** If $P$ and $Q$ are positive even integers such that $P^2 - Q^2 = 1772968$, then the number of values that $(P, Q)$ can take is _____.

**27.** The H.C.F. of two numbers is 12 and their sum is 588. Among all such pairs of numbers, the maximum value of the product of the two numbers is _____.
(A) 51840 (B) 83520 (C) 86400 (D) 87840

**28.** A four-digit number when divided by 47 leaves a remainder of 6 and when divided by 97 leaves a remainder of 3. What could be the number?
(A) 4659 (B) 4563 (C) 3659 (D) 4569

**29.** A natural number $N$ (less than 100) has the sum of its digits and the product of its digits denoted by $S(N)$ and $P(N)$, respectively, and $N + 2S(N) - 6P(N) = 0$. How many values can $N$ take? _____

**30.** How many even factors of $2^9 3^{11} 5^7$ are perfect squares?
(A) 4 (B) 24 (C) 96 (D) 120

**31.** How many positive integers less than or equal to 3725 are divisible by 2 or 3 or 5? _____

**32.** What is the sum of all the factors of 333333? _____

**33.** M is the set of all the three-digit numbers from 200 to 900 which are divisible by 9. $N$ is the set of all the three digit numbers from 100 to 700 which are divisible by 7. $S$ is the set of all the possible ordered pairs $(a, b)$ where $a \in$ M and $b \in N$, and $a + b$ is odd. How many elements does $S$ have?
(A) 3246 (B) 3354 (C) 3468 (D) 3582

**34.** $P = 2^{y^2+7y+1}$ and $Q = 2^{y^2+y+48}$. If the H.C.F. of $P$ and $Q$ is $P$, how many positive integer values can $y$ take? _____

**35.** The L.C.M. of $2^{2014} - 2$, $2^{2014}$, $2^{2014} + 2$ is
(A) $2^{2014}\,(2^{4028} - 4)$
(B) $2^{2013}\,(2^{4028} - 4)$
(C) $2^{2012}\,(2^{4028} - 4)$
(D) $2^{2011}\,(2^{4028} - 4)$

**36.** The sum of the digits of an eight – digit number is 70. How many of them are divisible by 11?
(A) 4 (B) 32 (C) 8 (D) 16

**37.** A leaf was torn from a novel. The sum of the page numbers on the remaining leaves of the novel is 10000. The number of pages in the novel lies
(A) between 130 and 140
(B) between 140 and 150
(C) between 150 and 160
(D) between 160 and 170

**38.** How many factors of 10! leave a remainder of 2 when divided by 3? _____

**39.** M and $N$ are whole numbers. If H.C.F. $(2^M 3^{N+3} 5^3 7^7, 2^{N+3} 3^{M+3} 5^a 7^a)$ is $2^a 3^a 5^a 7^a$, how many values can $N$ take?
(A) 4 (B) 3 (C) 2 (D) 1

**40.** $\lfloor x \rfloor$ represents the greatest integer less than or equal to $x$.

$$\left\lfloor \frac{2^0}{3} \right\rfloor + \left\lfloor \frac{2^1}{3} \right\rfloor + \left\lfloor \frac{2^2}{3} \right\rfloor + \ldots \left\lfloor \frac{2^{100}}{3} \right\rfloor =$$

(A) $\dfrac{2^{101} - 151}{3}$ (B) $\dfrac{2^{101} - 154}{3}$

(C) $\dfrac{2^{101} - 152}{3}$ (D) $\dfrac{2^{101} - 155}{3}$

**41.** Find the remainder when $36^{1052}$ is divided by 3798. _____

**42.** $N = 10^{51} - 769$

Consider the following statements:

I. $N$ is divisible by 77.

II. $N$ is divisible by 63.

Which of the following is/are true?
(A) Only I (B) Only II
(C) Both I and II (D) Neither I nor II

**43.** Find the remainder when 100010011002 _____ 1099 is divided by 99.
(A) 10 (B) 11 (C) 19 (D) 40

**44.** Find the remainder when 199! is divided by $67^3$
(A) 4489 (B) 6528
(C) 7142 (D) 8978

**45.** Find the remainder when 579579 _____ up to 999 digits is divided by 91.

**46.** Find the number of coprime numbers to $70^n$ between 140 and $70^n$.
(A) $70^n$ (B) $70^n - 48$
(C) 24 $(70^{n-1} - 2)$ (D) None of these

**47.** $N$ = 101010.....1 ($n$ zeroes alternating with $n + 1$ ones). The least value of $n$ for which $N$ is divisible by 9999 is
(A) 196 (B) 197
(C) 198 (D) 199

**48.** Find the remainder when $13(12^{600})$ is divided by 2197. _____

**49.** $N$ is a set of the squares of 1200 consecutive natural numbers. How many elements of $N$ leave a remainder of 1 when divided by 24?
(A) 300
(B) 400
(C) 500
(D) Cannot be determined

**50.** Find the remainder when $2^{181}$ is divided by 210. _____

## EXERCISE-5 Based on Data Sufficiency

***Directions for questions 1 to 45:*** Each question is followed by two statements, I and II. Answer each question based on the following directions:

Choose (A) If the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.

Choose (B) If the question can be answered by using either statement alone.

Choose (C) If the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (D) If the question cannot be answered even by using both the statements together.

**1.** What is the value of integer $x$, if $100 < x < 265$?

I. When $x$ is divided by 15, the remainder is 11.

II. When $x$ is divided by 11, the remainder is 7.

**2.** If $x$ is a negative number, is $xyz < 0$?

I. At least one out of $y$ and $z$ is negative.

II. Sum of $y$ and $z$ is positive.

**3.** If abc 0, is $a^3 + b^3 + c^3 = 0$?

I. $a^2 + b^2 + c^2 = ab + bc + ca$.

II. $a + b + c = 0$.

**4.** Is $x$ an even integer?

I. $x$ is the square of an integer.

II. $x$ is the cube of an integer.

**5.** The number of soldiers in a parade is less than 250. How many soldiers are there?

I. The soldiers can arrange themselves in rows of 3, 5 and 7.

II. The number of soldiers is an even number.

**6.** If $x$, $y$ and $d$ are positive integers and $d$ is odd, are both $x$ and $y$ divisible by $d$?

I. $x + y$ is divisible by $d$.

II. $x - y$ is divisible by $d$.

**7.** For non-zero integers $a$, $b$ and $c$, $\frac{a}{b-c} = 1$. What is the value of $\frac{b-c}{b}$?

I. $a/b = 3/5$.

II. $a$ and $b$ have no common factors greater than 1.

**8.** If $n$ is an integer and $10 < 3^n < 300$, then what is the value of $n$?

I. $n$ is the square of an integer.

II. $3^n$ is the square of an integer.

**9.** What is the smallest number among a set of five distinct natural numbers?

I. The sum of the five natural numbers is 16.

II. The product of the five natural numbers is 120.

**10.** What is the Greatest Common Divisor of the integers $a$ and $b$?

I. The Greatest Common Divisor of $2a$ and $2b$ is 10.

II. Both $a$ and $b$ are odd.

**11.** Is $N$ the H.C.F. of two numbers $x$ and $y$?

I. $N$ divides $x$ and $y$.

II. $2N$ divides $x/2$ and $y/4$.

**12.** Is $x$ an odd integer?

I. If $x$ is divided by five, the remainder is an odd integer.

II. If $x$ is divided by four, the remainder is an odd integer.

**13.** When integer $x$ is divided by 2, the remainder is 1. What is the remainder when $x$ is divided by 4?

I. When $x$ is divided by 8, the remainder is 3.

II. $x$ is a multiple of 5.

**14.** If $a$, $b$ and $c$ are positive integers, is the sum of $(2a + 4b)$ and $(a - b + c)$ divisible by 3?

I. $(a + b)$ is divisible by 3.

II. $c$ is divisible by 3.

**15.** $p^q = r^q$ where $q$ is a whole number. Is $p = r$?

I. $q$ is divisible by 3.

II. $q$ is odd.

**16.** $x$, $y$ and $z$ are three consecutive prime numbers. What are the values of $x$, $y$, $z$?

I. $y - z = 4$, $x - y = 6$

II. $x < 60$

**17.** A number $k$ has three prime factors 2, 5 and 3. What is the value of $k$?

I. The number of factors of $k$ is 12

II. $k$ is a multiple of 4

**18.** If $a$, $b$, $c$ and $d$ are integers, is $(a+b)^2 > (c+d)^2$?

I. $a < c$, $b < d$

II. $(a+b) < (c+d)$

**19.** A stick of length $l$ is broken into 3 parts of length $a$, $b$ and $c$. Can the 3 parts form a triangle?

I. $l = 6$, $a - b = b - c = 1$

II. $l = 19$, $a = 8\ (b - c)$, $b = 2\ (a - c)$

**20.** It is given that $k$ and $l$ are odd numbers. Is $7\frac{1}{2} + \frac{k}{3} + \frac{l}{6}$ an integer?

I. $k = l$ II. $k = 5l$

**21.** If $a$ and $b$ are natural numbers, is $a + b < ab$?

I. $a = b$ II. $a = 1$

**22.** What is the value of natural number $x$?

I. $x$ leaves remainders 2, 3, 7 when divided successively by 3, 4, 9.

II. $x$ is a multiple of 29.

**23.** $N$ is a two-digit integer greater than 70. What is the value of $N$?

I. $N$ is a product of two distinct single digit integers.

II. $N$ can be expressed as a product of three distinct prime numbers

**24.** The number of seeds with a farmer is less than 400. Find the number of seeds.

I. The seeds can be planted in rows of 3, 5 or 11, with no seeds remaining in each case.

II. The number of seeds is even.

**25.** $x$ is a two-digit number whose square is a three-digit number. What is the value of $x$?

I. The units digit of $x$ is the same as that of $x^2$.

II. The digits in the units place and the hundreds place of $x^2$ are the same.

**26.** If $ab > 0$, is $a + b > 0$?

I. $a^2 > b$

II. $b^2 a < 0$

**27.** Is $(x - y + z) > (x + y - z)$?

I. $4xy < 0$

II. $y < (x + z)$

**28.** Is the sum of $(2a - b)$ and $(2a + 5b - 4c)$ divisible by 3?

I. $c$ is negative, while $a > 0$ and $b > 0$

II. $a$, $b$, $|c|$ are successive integers.

**29.** A box contains ₹ 23. It only contains 25-paise, 50-paise and ₹ 1 coins. How many coins are there in the box?

I. If half the 50 paise coins are replaced by 25 paise coins and the other half by ₹ 1 coins, the total goes up by ₹ 1 25.

II. If half the 25 paise coins are replaced by ₹ 1 coins, the amount goes up by ₹ 9.

**30.** Is $N$ the H.C.F. of $x$ and $y$?

I. Both $x$, $y$ are multiples of $N$.

II. $x - y = 2N$

**31.** A number K is multiplied with $0.\overline{ab}$. Is the product an integer?

I. K is a multiple of 9.

II. K is a multiple of 11.

**32.** It is given that $x + y = z$ and $z < 50$. Both $y$ and $z$ are prime numbers and $x$ is a natural number. What is the value of $x$?

I. $x$ is a multiple of 14.

II. L.C.M. of $y$, $z$ is 527.

**33.** $x$ and $y$ are integers. Is $y$ an odd number?

I. xy is an odd number.

II. $x + y$ is an odd number.

**34.** From a pack of cards, four cards are missing. What are the numbers on the numbered cards that are missing?

I. If all the numbered cards are separated from the face cards (king, queen, jack and ace), it is found that there are 16 cards which bear numbers that are multiples of two.

II. If all the numbered cards are separated from face cards (kings, queens, jacks, aces), there are 8 cards which bear numbers that are multiples of three.

**35.** 810$A$4$B$6$C$ is an 8-digit number where A, B, C represent distinct digits. Find A + B + C.

I. The number is divisible by 5, 8 and 9.

II. Both A, B are composite.

**36.** When $X$, a 3-digit number is divided by any natural number from 2 to 7, the remainder is one less than the divisor. What is the value of $X$?

I. When $X$ is divided by 42 it leaves a remainder of 41

II. When $X$ is divided by 13 it leaves a remainder of 7

**37.** Is the natural number $X$, which is greater than 189, a prime number?

I. The number has only five multiples less than 1000

II. The number is odd and does not end in 5.

**38.** Find the units digit of $(6ab)^{64}$ where $a$ and $b$ are the tens digit and the units digit, respectively.

I. $(a)(b) = 24$ II. $b > 4$

**39.** Six employees A, B, C, D, E, F and G working with STC Company, have taken 4, 6, 8, 9, 12, 16 and 18 days leave in a month, though not necessarily in the same order. How many leaves did B take in that month?

I. F took 6 more days of leave than D did.

II. Number of days that D had taken as leave is 4/3 times that of B.

**40.** In a set of 6 numbers, five are 36, 60, 72, 68 and 48. What is the sixth number?

I. The sixth number is greater than two-thirds of the numbers in the set, and it's a prime number.

II. The average of the highest four numbers in the set is 69.

**41.** If the product of three positive integers is 40, how many of them are odd?

I. The sum of the three positive integers is odd.

II. If the three integers are $a$, $b$ and $c$, $ab + bc + ca$ is odd.

**42.** Is the three-digit number abc divisible by 90?

I. When abc is reversed, it forms a 2-digit number.

II. There exists a 4-digit number such that abc = efgh – hgfe, where $a, b, c, e, f, g$ and $h$ are all single digits.

**43.** Three ordinary dice coloured red, blue and green are rolled. The numbers which turn up are denoted by R, B, G, respectively. What is the value of the three-digit number RGB?

I. The two-digit number RB is divisible by 9 and $R$ is greater than B.

II. The three-digit number RBG is an even multiple of 3 and $R$, G and B are all distinct.

**44.** What is the remainder when $N^2 - 30N + 200$ is divided by 50?

I. $N$ is the smallest natural number which leaves a remainder of 4 when divided by 12, 13 or 14.

II. $N$ is the smallest natural number which leaves a remainder of 6, 7, respectively, when divided by 16 and 17.

**45.** $X$ is a two-digit prime number which when divided by the sum of its digits leaves a remainder which is a prime number. What is the value of $X$?

I. When divided by 12 and 18, $X$ leaves a remainder which is equal to the sum of its digits in each case.

II. When X is divided by 4, 6 and 8, it leaves a remainder which is 1 less than the divisor in each case.

## ANSWER KEYS

### Exercise-1

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (A) | 16. (B) | 31. 1 | 46. (A) | 61. (B) | 76. (A) | 91. (B) |
| 2. (D) | 17. (D) | 32. (B) | 47. 23 | 62. 1 | 77. 11 | 92. (D) |
| 3. (B) | 18. 24 | 33. (C) | 48. (A) | 63. (C) | 78. (A) | 93. (B) |
| 4. (A) | 19. (D) | 34. (A) | 49. 100 | 64. 1 | 79. (C) | 94. 2 |
| 5. (B) | 20. 7 | 35. (B) | 50. (A) | 65. 5 | 80. (D) | 95. (D) |
| 6. (D) | 21. 360 | 36. 5 | 51. 0 | 66. (B) | 81. (A) | 96. 2 |
| 7. (D) | 22. 1, 90 | 37. 117 | 52. (D) | 67. (B) | 82. 7 | 97. 10 |
| 8. 6 | 23. 255 | 38. (B) | 53. (A) | 68. (A) | 83. (C) | 98. (A) |
| 9. (B) | 24. (A) | 39. (A) | 54. (D) | 69. (B) | 84. (B) | 99. (A) |
| 10. (C) | 25. 19 | 40. (A) | 55. (C) | 70. (C) | 85. (B) | 100. (B) |
| 11. 7 | 26. 35 | 41. (C) | 56. (D) | 71. (D) | 86. (D) | |
| 12. 8 | 27. (C) | 42. (B) | 57. (B) | 72. 65 | 87. 2.5 | |
| 13. (A) | 28. (A) | 43. 1018 | 58. (A) | 73. (A) | 88. (C) | |
| 14. (A) | 29. (A) | 44. (D) | 59. (D) | 74. (C) | 89. 480 | |
| 15. (D) | 30. (A) | 45. 8 | 60. 3 | 75. (B) | 90. 3 | |

## Exercise-2

1. 38
2. (A)
3. (i) 1 (ii) 7 or 19 (iii) 1 or 13 or 17
4. 96
5. 1
6. (i) 36 (ii) 75 (iii) 36
7. (i) 57 (ii) 07
8. 27
9. (i) 3, 4 (ii) 6, 8 (iii) 6, 32
10. 480
11. 0
12. 600
13. 4
14. (D)
15. 1.7
16. 6
17. (A)
18. (i) 2 (ii) 1
19. (D)
20. 120
21. (C)
22. 131
23. 01
24. 1
25. 92
26. (C)
27. 2
28. (C)
29. 180
30. (D)
31. 1
32. 3
33. (A)
34. 30
35. 616
36. (C)
37. (i) 30 (ii) 900
38. (A)
39. 9
40. 7
41. (A)
42. (D)
43. 50
44. (A)
45. 210
46. (C)
47. 18, 648
48. (D)
49. 43
50. (B)
51. 19
52. 10
53. 8
54. 55
55. (i) 15 (ii) 5
56. 5
57. 0, 0
58. 28
59. 10
60. 72
61. (i) 6, 16 (ii) 4, 8
62. 1
63. 38
64. (i) 10, 36 (ii) 186, 7644 (iii) $80^5$, $2340^{18}$
65. (D)
66. 32
67. 5, 7, 8
68. 13
69. 120, 1, 315
70. (A)
71. (D)
72. (D)
73. (C)
74. (D)
75. (C)
76. (D)
77. (D)
78. (B)
79. (A)
80. (D)
81. (B)
82. 77
83. (D)
84. (C)
85. (C)
86. (D)
87. (B)
88. (A)
89. 40
90. 3
91. (A)
92. (C)
93. (B)
94. 10
95. (D)
96. (A)
97. 2
98. 8
99. (B)
100. 563, 495

## Exercise-3

1. (A)
2. 60
3. 4
4. (D)
5. 6
6. (C)
7. (D)
8. (A)
9. (D)
10. 26
11. (B)
12. (C)
13. (D)
14. (B)
15. 3050
16. (D)
17. (C)
18. (A)
19. (B)
20. (D)
21. 483840
22. 18
23. (D)
24. (D)
25. (A)
26. 24
27. 87
28. 5
29. (B)
30. (D)
31. (D)
32. (B)
33. (D)
34. (B)
35. 255
36. 21690
37. (D)
38. 1
39. (C)
40. (B)
41. (A)
42. (C)
43. (C)
44. 397
45. (C)
46. (B)
47. 2
48. 4.5
49. (C)
50. (C)
51. (B)
52. 2
53. 4
54. 87
55. (C)
56. 29
57. (B)
58. 1
59. 5
60. (B)
61. (A)
62. 3600
63. 1, 40
64. 666
65. (A)
66. (B)
67. 16
68. 1053, 150
69. (B)
70. 16
71. 12
72. 9
73. 37
74. 2
75. (C)
76. 22122
77. 8
78. (D)
79. (C)
80. 25
81. 8
82. 2
83. 26
84. 181
85. 0
86. (B)
87. 77 or 149
88. 200
89. 37
90. (A)
91. 459
92. (C)
93. 249
94. 5
95. 3
96. 88 or 40
97. 837
98. 131
99. (D)
100. 19

## Exercise-4

1. (D)
2. (B)
3. (C)
4. 6
5. 1249 9488
6. (C)
7. (B)
8. (B)
9. (B)
10. 0
11. 3
12. (C)
13. (D)
14. (B)
15. 5
16. 278
17. 15
18. (A)
19. 30
20. (B)
21. 24
22. (C)
23. (A)
24. (A)
25. (C)
26. 0
27. (C)
28. (A)
29. 1
30. (C)
31. 2732
32. 663936
33. (B)
34. 7
35. (C)
36. (D)
37. (B)
38. 26
39. (D)
40. (C)
41. 1296
42. (A)
43. (A)
44. (D)
45. 33
46. (C)
47. (B)
48. 1872
49. (B)
50. 2

## Exercise-5

1. (C)
2. (C)
3. (B)
4. (D)
5. (C)
6. (C)
7. (A)
8. (B)
9. (B)
10. (A)
11. (A)
12. (A)
13. (A)
14. (A)
15. (A)
16. (D)
17. (C)
18. (D)
19. (B)
20. (A)
21. (A)
22. (D)
23. (B)
24. (C)
25. (D)
26. (A)
27. (D)
28. (C)
29. (C)
30. (D)
31. (C)
32. (A)
33. (A)
34. (C)
35. (C)
36. (A)
37. (C)
38. (C)
39. (C)
40. (A)
41. (B)
42. (C)
43. (C)
44. (B)
45. (A)

## SOLUTIONS

## EXERCISE-1

**1.** $x = 56^y + 1 = 56^y + 1^y$ which is divisible by 57 only when $y$ is odd ($\because$ $a^N + b^N$ is divisible by $a + b$ only when $N$ is odd). $\therefore$ It is divisible by any factor of 57 only when $y$ is odd.

$\therefore$ It is divisible by 19 when $y$ is odd.

**2.** The number of digits in the product must be at least the number of digits in $(10^6)$ $(10^9)$ $(10^{11})$ and less than the number of digits in $(10^7)$ $(10^{10})$ $(10^{12})$.

$\therefore$ The number has at least 27 digits and less than 30 digits.

**3.** The L.C.M. of the given expression is $2^3 \times 3^3 \times 5^2$.

**4.** The product of two or more even numbers is always even.

**5.** Let the greatest number be $N$.

Let the remainder be $r$. $93 - r$, $131 - r$ and $188 - r$ must be divisible by $N$

$\therefore$ $131 - r - (93 - r)$ and $188 - r - (131 - r)$ must be divisible by $N$.

$\therefore$ 38 and 57 must be divisible by $N$.

$N$ is the greatest possible number satisfying this condition

$\therefore$ $N$ = H.C.F. (38, 57) = 19.

**6.** $38^{2n} - 11^{2n} = (38^2)^n - (11^2)^n = (1444)^n - (121)^n$. This is always divisible by $1444 - 121 = 1323$. The greatest number which divides it among the choices is 1323.

**7.** The sum of the alternate digits starting from the units digit = 8 + 3 + 2 + 5 + 6 + 9 = 33.

The sum of the alternate digits starting from the tens digit is $7 + 1 + 7 + y + 8 + 1 = 24 + y$. The 11's remainder of the number is equal to the 11's remainder of $33 - (24 + y) = 9 - y$. This is 0 as the number is divisible by 11.

$\therefore$ $y = 9$.

**8.** $3^{200} = 3^{4 \times (50)}$ As the index of the power of 3 is divisible by 4, $3^{200}$ has the same units' digit as $3^4$, i.e., 1.

$4^{500}$ has an even index.

Its units' digit is 6.

$\therefore$ Units' digit of $(3^{200})$ $(4^{500})$ is 6.

**9.** The smallest four-digit number is 1000. If 1000 is divided by 112, the remainder is 104.

$112 - 104 = 8$. If 8 is added to 1000, the resulting number will be the smallest four-digit number that is a multiple of 112.

**10.** Let the number be $N$. Let the quotient obtained when the number was divided by 32 be $q$.

$N = 32q + 29$

When $q$ is of the form $2k + 1$ where $k$ is any whole number, then $N = 64k + 32 + 29 = 64k + 61$. When $64k + 61$ is divided by 64 the remainder is 61. When $q$ is of the form $2k$ where $k$ is a whole number then $N = 64k + 29$. When $64k + 29$ is divided by 64 the remainder is 29.

$\therefore$ The remainder is either 61 or 29.

**11.** Suppose a number $x$ has $m$ digits, i.e., $10^{m-1} \le x < 10^m$

$\therefore$ $10^{2m-2} \le x^2 < 10^{2m}$, i.e., $x^2$ has $2m$ or $2m - 1$ digits. Conversely, if a number has $2m - 1$ or $2m$ digits, its square root has $m$ digits. Therefore, if a number has 13 digits, its square root has 7 digits.

**12.** Sum of the digits of 7654321A = 28 + A, so it must be divisible by 9. As $0 \le A \le 9$, $28 \le 28 + A \le 37$. Only when $28 + A = 36$ is the number divisible by 9.

$\therefore$ A = 8.

**13.** Divisors = 5, 6, 7, 8, 9

Remainders = 4, 5, 6, 7, 8.

The difference between the respective divisors and remainders is constant.

The required number = (L.C.M. of divisors) – Common difference = 2520 – 1 = 2519

**14.** (H.C.F.) (L.C.M.) = Product

$\therefore$ 18 (L.C.M.) = 3240

$\therefore$ L.C.M. = 180

**15.** 63 and 91 are numbers that leave a remainder of 7, when divided by 28. When divided by 35, they leave remainders of 28 and 21, respectively.

**16.** $(2PQR)^4$ must be at least $(2000)^4$ and less than $(3000)^4$. $(2000)^4$ as well as $(3000)^4$ have 14 digits.

$\therefore$ $(2PQR)^4$ also has 14 digits.

**17.** $D$ is odd.

$\therefore$ $41^D + 7^D$ must be divisible by $41 + 7 = 48$

$\therefore$ It is divisible by any factor of 48. Only is not a factor of 48.

**18.** Let the other number be $x$.

(L.C.M.) (H.C.F.) = product of the numbers

$(264)\ (2) = (22)\ (x) \Rightarrow x = 24$

**19.** Let the numbers be $a$ and $b$ where $a \le b$. We consider the possibilities $a < b$ and $a = b$.

Possibility 1: $a < b$ (1)

If $b$ is divisible by $a$, L.C.M. = $b$ and H.C.F. = $a$.

Otherwise L.C.M. > $b$ and H.C.F. < $a$

$\therefore$ L.C.M. $\ge b$ and H.C.F. $\le a$

(1) $\Rightarrow$ L.C.M. $\ge b > a \ge$ H.C.F.

L.C.M. > H.C.F.

Possibility 2: $a = b$

L.C.M. = H.C.F. = each number

The numbers must be equal for H.C.F. = L.C.M. to hold true.

**20.** Units digit of $(13687)^{3265}$ is the same as units digit of $7^{3265} = 7^{4\,(816)+1}$

$\therefore$ Units digit of $7^{3265}$ is the same as that of $7^1$, i.e., 7.

**21.** $\text{L.C.M.}\left(\frac{5}{6}, \frac{9}{10}, \frac{8}{9}\right) = \frac{\text{L.C.M.}(5, 9, 8)}{\text{H.C.F.}(6, 10, 9)} = \frac{360}{1} = 360.$

**22.** $\text{H.C.F.}\left(\frac{5}{6}, \frac{9}{10}, \frac{8}{9}\right) = \frac{\text{H.C.F.}\,(5,9,8)}{\text{H.C.F.}\,(6,10,9)} = \frac{1}{90}$

**23.** Dividing 256 successively by 2, we get

| | |
|---|---|
| 2 | 256 |
| 2 | 128 |
| 2 | 64 |
| 2 | 32 |
| 2 | 16 |
| 2 | 8 |
| 2 | 4 |
| 2 | 2 |
| | 1 |

$\therefore$ The number of twos in 256! is $1 + 2 + 4 + \ldots + 128$

$= 2^8 - 1 = 255.$

**24.** Let the smallest such number be $x$.

$x$ must have the forms $17q_1 + 12$ and $24q_2 + 19$ where $q_1$ and $q_2$ are whole numbers.

$\therefore\ x = 17\ (q_1 + 1) - 5 = 24\ (q_2 + 1) - 5$

$x + 5 = 17\ (q_1 + 1) = 24\ (q_2 + 1)$

$\therefore\ x + 5$ must be divisible by both 17 and 24 and hence, by their L.C.M. As $x$ is the smallest, $x + 5$ is also the smallest

$\therefore\ x + 5 = \text{L.C.M.}\ (17, 24)$

$\therefore\ x = \text{L.C.M.}\ (17, 24) - 5 = (17)\ (24) - 5 = 408 - 5 = 403.$

**25.** The remainder when any number is divided by 25 is the remainder when the number formed by the last two digits of that number (i.e., 69) is divided by 25 which is 19.

**26.** The index of each prime factor must be even. If we multiply the number by (5) (7), i.e., 35, the resulting indices are all even.

**27.** $y! = y\ (y - 1)\ (y - 2)!$

Given

$y! - 20\ (y - 2)! = 0$

$(y - 2)!\ (y.\ (y - 1) - 20) = 0$

$(y - 2)!\ (y^2 - y - 20) = 0$

$(y - 2)! \ge 1$, i.e., it is $\ne 0$

$\therefore\ y^2 - y - 20 = 0$

$(y - 5)\ (y + 4) = 0$

$\because\ y > 0$

$y = 5.$

**28.** The numerator is of the form $a^3 + 3ab^2 + 3a^2b + b^3$, while the denominator is of the form $a^2 + 2ab + b^2$ where $a = 2.35$ and $b = 1.45$

$$\frac{a^3 + 3ab^2 + 3a^2b + b^3}{a^2 + 2ab + b^2} = \frac{(a+b)^3}{(a+b)^2} = a + b = 3.8$$

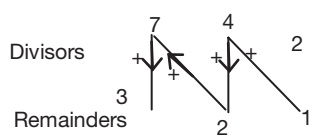
**29.** Suppose a number $x$ has $m$ digits.

$10^{m-1} \le x < 10^m$

$\therefore\ 10^{3m-3} \le x^3 < 10^{3m}$

i.e., $x^3$ has $3m - 2$, $3m - 1$ or $3m$ digits, so if a number has 28, 29 or 30 digits, its cube root has 10 digits.

**30.**



By arrow method, the required number = $(1 \times 4 + 2) \times 7 + 3 = 45$

**31.** The remainder, when any number is divided by 9, is the remainder when the sum of its digits is divided by 9. In the given problem, the sum of the digits of the number = 37. Remainder, when it is divided by 9, is 1.

**32.** $N$ = L.C.M. $(2^2 \times 3^3 \times 5, 3^2 \times 5^3 \times 7, 5^2 \times 7^3 \times 11)$

$= 2^2 \times 3^3 \times 5^3 \times 7^3 \times 11$

Number of distinct prime factors it has = 5.

**33.** H.C.F. (fractions) $= \dfrac{\text{H.C.F. (Numerators)}}{\text{L.C.M. (Denominatros)}}$

$\therefore$ H.C.F. $= \dfrac{\text{H.C.F.}(7, 11, 3)}{\text{L.C.M.}(18, 48, 5)} = \dfrac{1}{\text{L.C.M.}(18, 48, 5)}$

L.C.M. (18, 48) = 144

$\therefore$ L.C.M. (18, 48, 5) = L.C.M. (144, 5) = (144) (5) = 720

$\therefore$ H.C.F. $= \dfrac{1}{720}$.

**34.** The remainder of $a^{b-1}$ divided by $b$ when $a$ and $b$ are co-primes is 1. In the given problem, 18 and 19 are co-primes

$\therefore$ The remainder is 1.

**35.** $3^{42} = 3^{4(10)+2}$ Its units digit = That of $3^2 = 9$ ($\because$ cycle of 4)

$8^{56} = 8^{4(14)}$. Its units digit = That of $8^4 = 6$ ($\because$ cycle of 4)

$6^{40}$ ends with 6 ($\because$ Any positive power of 6 ends with 6)

$\therefore$ Units digit = That of 9.6.6 = 4.

**36.** Any 10 consecutive odd natural numbers have a multiple of 5 among them.

$\therefore$ The product ends with a 5.

**37.** Least perfect cube greater than 395 is 512. 117 should be added to 395 to obtain 512.

**38.** If a number is divisible by both 3 and 4, the number will be divisible by 12.

If the last two digits are divisible by 4, the number is divisible by 4. If the sum of the digits is divisible by 3, the number will be divisible by 3.

By applying the divisibility tests, we find 7908, 14772 and 15942 are divisible by 12.

**39.** The length (in cm) of the side of the smallest square must be divisible by 7 as well as by 5.

$\therefore$ It must be L.C.M. (7, 5) or 35 cm.

Its area = $35^2$ cm$^2$ = 1225 cm$^2$.

**40.** We know that,

$6^{25}$ ends with 6,

$9^{16}$ ends with 1 and

$5^{40}$ ends with 5.

Hence, $6^{25} + 9^{16} + 5^{40}$ ends with the units digit of 6 + 1 + 5, i.e., 2.

**41.** $M = (7^2)^N - 1 = 49^N - 1^N$ which is always divisible by 48 ($\because a^N - b^N$ is always divisible by $a - b$ if $N$ is a natural number).

$\therefore$ It is always divisible by any factor of 48 and hence, by 6.

**42.** 1 m 50 cm = 150 cm

8 m 10 cm = 810 cm

1 m 35 cm = 135 cm

1 m 95 cm = 195 cm

The length of the longest scale required (in cm)

= H.C.F. of (150, 810, 135, 195)

= H.C.F. (15 . 10, 15 . 54, 15 . 9, 15 . 13)

= 15 H.C.F. (10, 54, 9, 13) = 15 (1) = 15

**43.** The remainder, when 1000 is divided by 36, is 28.

$\therefore$ 1000 – 28 = 972 is the largest 3-digit number divisible by 36.

$\therefore$ 972 + 36 = 1008 is the least 4-digit number divisible by 36.

$\therefore$ The least 4-digit number which leaves a remainder of 10 = 1008 + 10 = 1018.

**44.** Let the H.C.F. be $x$. Then $Q$ and $Q + P$ would be divisible by $x$. Then $Q + P - Q$ must be divisible by $x$, i.e., $P$ must be divisible by $x$. But $P$ is prime.

$\therefore$ $x = P$ or 1

$Q < P$

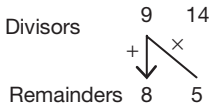
$\therefore$ H.C.F. cannot be $P$. $x \neq P$.

$\therefore$ $x = 1$, i.e., H.C.F. = 1.

**45.** Number of three digit natural numbers divisible by 8, 12 and 15 = Number of three digit natural numbers divisible by L.C.M. (8, 12, 15), i.e., 120.

There are 8-three digit natural numbers divisible by 120, viz.,120 (1), 120(2), ...120(8).

**46.**




The least possible value of the number = 5(9) + 8 = 53
General form of the number = [(product of the divisors) $k$ + Its least possible value] where $k$ is any whole number
= $126\boldsymbol{k}$ + 53

$126k + 53 = 18\ (7k + 2) + 17$

$$\therefore \text{Rem}\left(\frac{126k+53}{18}\right) = 17\ .$$

**47.** Numbers which leave a remainder of 3 when divided by 5 are 3, 8, 13, 18, 23, 28, …

Numbers which leave a remainder of 5 when divided by 6 are 5, 11, 17, 23, 29, …

Therefore, numbers of the form kL.C.M. (5, 6) + 23 satisfy both the conditions. Putting $k$ = 0, gives the least natural number.

**48.** Let the three successive even natural numbers be $2k$, $2k$ + 2, $2k$ +4.

Product ($P$) = $(2k)\ (2(k + 1))\ (2(k + 2)) = 8k\ (k + 1)\ (k + 2)$

$k$, $k + 1$ and $k + 2$ are three consecutive integers. One of them is even and one of them is divisible by 3. So, their product is divisible by both 2 and 3, and hence, by 6.

$P$ = 8 (A number divisible by 6)

= 8 $(6k)$ (say) = 48 $k$

$P$ is divisible by 48.

**49.** $\dfrac{a^3 + b^3}{a^2 - ab + b^2} = a + b$

∴ Required quantity = 89 + 11 = 100.

**50.** The number of zeros that $N!$ ends with = The index of the greatest power of 5 in it.

| | |
|---|---|
| 5 | 100 |
| 5 | 20 |
| | 4 |

∴ 100! ends with 20 + 4 or 24 zeros.

**51.** The largest power of 5 in 14! Is 2.

∴ 14! ends with 2 zeros. The tens digit is 0.

**Alternate Solution:**

Product of the first 14 natural numbers = 1. 2. 3. 4 … 14 = (10) (2) (5) (1.3.4.6.7.8.9.11.12.13.14) = 100 (1.3…….14)

The product is divisible by 100. Its last two digits are 0.

**52.** The remainder when any positive integer $N$ is divided by 10 is the last digit of $N$, i.e., the units digit of $N$.

$7^{645} = 7^{4 \times (161) + 1}$

The units digits of $7^n$ has a cycle of 4.

The units digit of $7^{645}$ is the same as the units digit of $7^1$.

The remainder when $7^{645}$ is divided by 10 is 7.

**53.** $N$ = 31.32.33.34.35…….39

$34 \times 35$ ends with units digit of $4 \times 5$, i.e., 0.

∴ $N$ ends with 0.

**54.** The product of any 6 consecutive natural numbers is always divisible by 6!, i.e., 720.

**55.** For any perfect square the index of the power for each of its prime factors should be even. In the given number the power of 5 and 17 have odd indices, while 13 has an even index.

∴ The least number = (5) (17) = 85

**56.** When 10000 is divided by 19, the remainder is 6. ∴ 10000 − 19 = 9981 is the largest 4 digit number which leaves a remainder of 6 when divided by 19.

**57.** $7 \oplus 6 = \text{Rem}\left(\dfrac{7 \times 6}{18}\right) = \text{Rem}\left(\dfrac{42}{18}\right) = 6$

$6 \oplus 5 = \text{Rem}\left(\dfrac{6 \times 5}{18}\right) = \text{Rem}\left(\dfrac{30}{18}\right) = 12$

**58.** 100 should be divided by 3 and the quotients should be further successively be divided by 3.

$\frac{100}{3}$ = 33 (quotient)

$\frac{33}{3}$ = 11 (quotient)

$\frac{11}{3}$ = 3 (quotient)

$\frac{3}{3}$ = 1 (quotient), the highest power of 3 in 100!

= 33 + 11 + 3 + 1= 48

**59.** $2^{2400} = (2^3)^{800}$

$9 = 2^3 + 1$

Taking $x = 2^3$ and $f(x) = 2^{2400}$,

$f(x) = x^{800}$

$9 = x + 1$

When $f(x)$ is divided by $x - a$ where $a$ is any real number, the remainder is given by $f(a)$.

The remainder for the problem is $f(-1) = (-1)^{800} = +1$.

**60.** The remainder when a number is divided by $2^n$ is equal to the remainder when the 'tail' (the number formed by the last $n$ digits of the given number) is divided by $2^n$. Here, $n = 3$ and the 'tail' is 677. Therefore, the remainder is 5. Therefore, the least number to be added such that the sum is divisible by 8 is 3.

**61.** Let the numbers be $3x$, $4x$ and $5x$. Least = $3x$

L.C.M. = $x$ (L.C.M. (3, 4, 5) = $x$ (60) = $60x$

$60x = 480$

$x = 8$, $3x = 24$

**62.** It can be verified that for any value of $p$ we have Rem $\left(\frac{P^2}{24}\right) = 1$.

**63.** $2400 = 24(100) = (2^3)(3)(2^2)(5^2) = 2^5(3^1)(5^2)$

The number of divisors it has = (5 + 1) (1 + 1) (2 + 1) = 36

∴ It has 35 factors other than itself.

**64.** The successive division is shown below.

| Number / Quotient | 192 | 27 | 13 | 3 |
|---|---|---|---|---|
| Division | 7 | 2 | 4 | |
| Remainder | 3 | 1 | 1 | |

The last remainder is 1.

**65.** Remainder, when $5^1$ is divided by 6 = 5.

Remainder, when $5^2$ is divided by 6 = 1.

Remainder, when $5^3$ is divided by 6 = 5.

Remainder, when $5^4$ is divided by 6 = 1.

37 is odd.

∴ Remainder when $5^{37}$ is divided by 6 = 5.

**66.** Next time = 11 a.m. + L.C.M. (Time intervals) = 11 a.m. + L.C.M. (45 sec, 60 sec, 75 sec, 90 sec) = 11 a.m. + 900 sec

= 11 a.m. + 15 min = 11:15 a.m. (∵L.C.M. (45, 90) = 90

∴ L.C.M. (45, 60, 90) = L.C.M. (60, 90) = 180

∴ L.C.M. (45, 60, 75, 90) = L.C.M. (75, 180)

= L.C.M. (15.5, 15.12) = 15 L.C.M. (5, 12) = 15 (60) = 900).

**67.** If $a^p \times b^q \times c^r$ is the number, the number of ways it can be resolved into two factors.

$= (p + 1)(q + 1)(r + 1)/2$

$= (3 + 1)(3 + 1)(2 + 1)/2 = 24$ ways

**68.** Remainder, when $3^1$ is divided by 11 = 3

Remainder, when $3^2$ is divided by 11 = 9

Remainder, when $3^3$ is divided by 11 = 5

Remainder, when $3^4$ is divided by 11 = 4

Remainder, when $3^5$ is divided by 11 = 1

Remainder, when $3^6$ is divided by 11 = 3

∴ Remainder, when $3^1$ is divided by 11 = Remainder, when $3^6$ is divided by 11 = Remainder, when $3^{5x1+1}$ is divided by 11.

∴ Remainder, when $3^{91}$ is divided by 11 = Remainder, when $3^{5x18+1}$ is divided by 11 = Remainder, when $3^1$ is divided by 11 = 3.

**69.** The number is of the form 891(K)+ 57 = (27)(33)K + (27)(2) + 3

If this number is divided by 27 the remainder will be 3.

**70.** If at least one of them is 2, the product is even, otherwise the product is odd.

∴ We cannot say.

**71.** $B = 13^{2A} - (2^2)^A = 13^{2A} - 2^{2A}$

$a^p - b^p$ must be divisible by $a + b$ and $a - b$ when $P$ is an even natural number. $2A$ is even.

∴ B must be divisible by 13 + 2 = 15 and 13 – 2 = 11

**72.** $(3^8) \times (6^4) = (3^8) \times (2)^4 \times (3)^4 = (3^{12}) \times (2^4)$

Number of factors of $(3^8)$ $(6^4)$ = (12 + 1) (4 + 1) = 65.

**73.** Yes, both expressions are equal to H.C.F. $(p, q, r, s)$

**74.** H.C.F. (2, 3, 5) = 1 and L.C.M. (2, 3, 5) = 2 (3) (5)

But H.C.F. (2, 3, 6) = 1 and L.C.M. (2, 3, 6) ≠ 2 (3) (6)

∴ We cannot say.

**75.** Least number divisible by 55 and 12 = L.C.M. (55, 12)

= (55) (12) = 660

Least number leaving a remainder of 2 which is greater than 10 = 660 + 2 = 662.

**76.** If the L.C.M. of two or more numbers equals their product, the numbers must be coprime; hence, the H.C.F. of any two numbers would be 1. In the given problem,

L.C.M. $(x, y, z) = x.y.z$

$\therefore$ H.C.F. $(y, z) = 1$.

**77.** $\left(y-\frac{1}{y}\right)^2 = y^2+\frac{1}{y^2}-2 = 3^2$ (given)

$\therefore\ y^2+\frac{1}{y^2} = 11$

**78.** Yes, both expressions are equal to L.C.M. $(p, q, r, s)$.

**79.** The number formed by the last 5 digits of PQRST*U*6736 must be divisible by 32. When U = 1, this condition is satisfied. When U = 2, this condition is not satisfied.

$\therefore$ We cannot say.

**80.** $\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2 = 4^2$ (given)

$\therefore\ x^2+\frac{1}{x^2} = 14$

**81.** $(3^3)\ (7^7)\ (21^5) = (3^3)\ (7^7)\ (7\times 3)^5 = (3^8)\times(7^{12})$

The index of each prime factor is even.

$(3^3)\ (7^7)\ (21^5)$ is a perfect square.

**82.** Last digit of $4^{99}$:

99 is odd

$\therefore$ last digit of $4^{99}$ is 4

Last digit of $7^{99}$.

Last digit of $7^1 = 7$

Last digit of $7^2 = 9$

Last digit of $7^3 = 3$

Last digit of $7^4 = 1$

Last digit of $7^5 = 7$

$\therefore$ last digit of $7^1$ = Last digit of $7^5$

= Last digit of $7^{4\times 1+1}$

$\therefore$ last digit of $7^{99}$ = Last digit of $7^{24\times 4+3}$

= Last digit of $7^3 = 3$

$\therefore$ Last digit of $4^{99} + 7^{99}$ is 4 + 3 = 7.

**83.** A number divisible by 11 must have the sum of its odd digits and the sum of its even digits equal, or else their difference should be a multiple of 11. Only satisfies this condition.

**84.** $N = 3^{p+2r}\,2^q$

Number of numbers less than $N$ and co-prime to it = $N\left(1-\frac{1}{3}\right)\left(1-\frac{1}{2}\right) = \frac{N}{3}$.

**85.** Sum of the factors of $(2^4)\ (3^3) = \frac{2^5-1}{2-1}\cdot\frac{3^4-1}{3-1} = 1240$.

**86.** 437 = (19) (23)

323 = (19) (17)

567 = 7 (81)

241 is prime.

**87.** Substituting 5.71 = $a$ and 3.21 = $b$,

the expression to be simplified becomes $\frac{a^3-b^3}{a^2+ab+b^2}$ which is equal to $a-b$.

As $a = 5.71$ and $b = 3.21$,

$a - b = 2.5$

**88.** As the number has an odd number of factors, when the number is expressed as a product of powers of prime factors, each index is even. If each index is divisible by 6, then the number is a perfect cube. Otherwise it is not a perfect cube.

We cannot say.

**89.** Sum of the factors of 437 or (19) (23)

$= \frac{19^2-1}{19-1}\cdot\frac{23^2-1}{23-1} = 480$.

**90.** 13013 = 13 (1001) = 13 (13) (11) (7)

$= 13^2\times(11)\times(7)$

$\therefore$ 13013 has 3 distinct prime factors.

**91.** $\text{Rem}\left(\frac{3^{48}}{82}\right) = \text{Rem}\,\frac{\left(3^4\right)^{12}}{3^4-(-1)}$

Let $x = 3^4$ and $f(x) = x^{12}$

By remainder theorem, required remainder

$= f(-1) = (-1)^{12} = 1$

($\because$ (−1) raised to any even number = 1).

**92.** There are $2^{13}$ odd numbers less than $2^{14}$.

$\therefore$ There are $2^{13}$ numbers coprime to it.

**93.** A perfect square has an odd number of factors.

$\therefore$ The number is not a perfect square.

**94.** Pattern Method:

The remainders of various powers of 2 when divided by 5 are as follows:

Remainder when $2^1$ is divided by 5 is 2.

Remainder when $2^2$ is divided by 5 is 4.

Remainder when $2^3$ is divided by 5 is 3.

Remainder when $2^4$ is divided by 5 is 1.

Remainder when $2^5$ is divided by 5 is 2.

The remainder is repeated for every 4 steps. So, the remainder when $2^{189}$ is divided by 5 is the same as $2^1$ divided by 5, i.e., 2.

**Remainder Theorem Method:** In the division $2^{189}/5$, the numerator is in powers of 2, so the denominator should also be in terms of powers of 2 as $(2^2 + 1)$. Now, as the denominator is in terms of $2^2$ the numerator is also rewritten as $2(2^2)^{94}$. Hence, the division is now reduced to $\frac{2.(2^2)^{94}}{(2^2+1)}$. The remainder of this division, as per Remainder Theorem is $2\ (-1)^{94} = 2\ (1) = 2$.

**95.** The required prime numbers are 71, 73, 79, 83, 89 and 97. There are six numbers.

**96.** 35 is odd; if the sum of an odd number of prime numbers is even, then one of them is always 2.

**97.** Any number when divided by 5 will leave a remainder of its last digit divided by 5.

$3^n$ ends with 1 or 3 or 9 or 7. Hence, the possible remainders when $3^n$ is divided by 5 are 1 or 3 or 4 or 2. The sum of these remainders equals 10.

**98.** Any prime number more than 3 must be of the form $6k+1$ or $6k-1$ where $k$ is a natural number.

$(6k+1)^2 = 36k^2 + 12k + 1 = 6k\ (6k+2) + 1$

$$\text{Rem}\left(\frac{(6k+1)^2}{6}\right) = 1$$

$(6k-1)^2 = 36k^2 - 12k + 1 = 6k\ (6k-2) + 1$

$$\text{Rem}\left(\frac{(6k-1)^2}{6}\right) = 1$$

$\therefore$ The required remainder is 1.

**99.** The sum of an even number of odd numbers is always even.

**100.** $$\frac{4^{65}}{7} = \frac{2^{130}}{2^3-1} = \frac{\left(2^3\right)^{43}\times 2}{2^3-1}$$

Remainder theorem: When $f(x)$ is devided by $x-a$, the remainder of the division is $f(a)$.

Rem $\left(\frac{\left(2^3\right)^{43}\times 2}{2^3-1}\right)$ can be found using remainder theorem as follows.

Let $x = 2^3$, $f(x) = 2x^{43}$, $a = 1$

Remainder $= f(a)\ 2\ a^{43} = 2(1)^{43} = 2$

## EXERCISE-2

**1.** Given $x = 0.38 + 0.0038 + 0.000038 + \ldots \infty = 0.383838\ldots$

$\therefore x = 0.\overline{38}$ $\quad \therefore x = \frac{38}{99}$.

**2.** Any natural number having an even number of factors is not a perfect square. Any natural number that is not a perfect square can be written as a product of two factors where one of the factors lies between 1 and its square root and the other factor lies between its square root and itself.

$\therefore$ In the given problem, both (i) and (ii) are true.

**3.** (i) $\{N = 12q + 7$; $12q$ is divisible by 6 and 7 when divided by 6 leaves a remainder of 1$\}$

(ii) $\{N = 12q + 7$; $q$ can be either even or odd. $N = 12(2k)+7$ or $12(2k+1)+7 = 24k+7$ or $24k+19$.

$N$ leaves a remainder 7 or 19

(iii) $\{N = 12q + 7$; $q$ will be of the form $3k$, or $3k+1$ or $3k+2$.

$N = 12(3k)+7$ or $12(3k+1)+7$ or $12(3k+2)+7$

= $36k+7$ or $36k+19$ or $36k+31$.

$N$ would leave a remainder of 1 or 13 or 7 {depending on the value of $q$}

**4.** Number of positive integers coprime to 420 and less than it, i.e., $2^2.3.5.7 = 420\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)\left(1-\frac{1}{7}\right) = 96$

**5.** The last digit of a power of 9 depends on the parity of the power. It's 9 for odd and 1 for even powers. As the power is even it ends with 1.

**6.** (i) $486^{486}$ has its last two digits same as the last two digits of $86^6$.

$86^1 \equiv 86; 86^2 \equiv 96; 86^4 \equiv 16$

$\therefore 86^6 \equiv 86^2 \times 86^4 \equiv 36$

(ii) $(675)^{239} \equiv 75^{39}$.

75 raised to any odd power ends with 75 and any even power of 75 ends with 25. Hence, $675^{239}$ ends in 75.

(iii) $734^{1054} \equiv 34^{14}$.

$34^1 \equiv 34; 34^2 \equiv 56; 34^4 \equiv 36$

$34^{14} \equiv (34^1 \times 34^2 \times 34^4)^2 \equiv (34 \times 56 \times 36)^2 \equiv (44)^2 \equiv 36$.

**7.** (i) $(557)^{269} = 57^9$

$57^1 = 57; 57^2 = 49\ 57^4 = 01;$

$57^9 = (57)^4 \times (57)^4 \times (57)^1 = (01) \times (01) \times (57) = 57$

(ii) $1043^{375} = 43^{15}$

$43^1 = 43; 43^2 = 49; 43^4 = 01; 43^8 = 01$

$\therefore 43^{15} = 43^8 \times 43^4 \times 43^3 = (01)(01)(07) = 07$

**8.** $\text{Rem}\frac{\left(3^4\right)^{20}}{41} \times \text{Rem}\frac{3^3}{41} = (82-1)^{20} \times 27$

$(82-1)^{20} = (82-1)$ .... 20 times $= 82k + (-1)^{20} = 82k+1.$

$(82-1)^{20} \times 27 = (82k+1)\ (27) = 82k_1 + 27$

$\therefore$ Remainder is 27.

**9.** (i) $90 = 2^1.3^2.5^1$

$\therefore$ It has a total of 3 distinct prime factors.

$1680 = 2^4\ 3^1\ 5^1\ 7^1$

$\therefore$ It has a total of 4 distinct prime factors.

(ii) $90 = 2^1.3^2.5^1$

No. of odd factors = 6 (Only 1, 3, 5, $3^2$, $3^1 \times 5^1$ and 45)

$1680 = 2^4\ 3^1\ 5^1\ 7^1$

No. of odd factors = 8 (Only 1, 3, 5, 7, 15, 21, 35 and 105)

(iii) $90 = 2^1.3^2.5^1$

No. of even factors = Total no. of factors – No. of odd factors = (4) (3) – 6 = 6

$1680 = 2^4\ 3^1\ 5^1\ 7^1$

No. of even factors = Total no. of factors – No. of odd factors = (5) (2) (2) (2) – 8 = 32

**10.** Up to 1400, there are 200 multiples of 7. There are 280 multiples of 5. There are 40 multiples of both 5 and 7 (i.e., of L.C.M. (5, 7) is 35).

$\therefore$ Up to 1400, there are 100 odd numbers divisible by 7, 140 odd numbers divisible by 5, and 20 odd number divisible by both 5 and 7.

There are 140 + 100 – 20, i.e., 220 odd numbers which are divisible by either 5 or 7.

$\therefore$ The remaining 700 – 220, i.e., 480 odd numbers are divisible by neither 5 nor 7.

**11.** The product is an even multiple of 5.

$\therefore$ It ends with 0.

**12.** Number of times Deepa pressed the numbered keys to type first $n$ natural numbers is 1692

Of them, number of presses needed to type the single digit numbers (i.e., 1 to 9) = $1 \times 9 = 9$,the two digit numbers (i.e., 10 to 99) = $2 \times 90 = 180$

Let the number of three digit numbers typed be $x$.

Then, $9 + 180 + 3x = 1692$

$\therefore\ x = 501$

Hence, $n = 600$.

**13.** $87^{82} = (17 \times 5 + 2)^{82}$

$= 2^{82}/17$

$2^{82}/17 = \frac{2^{82}}{2^4+1} = \frac{2^2 x(2^4)^{20}}{2^4+1}$

$R = (-1)^{20} \times 4 = 4$

**14.** From the options, $a^3\ b^2\ c$ is odd.

Therefore, it is true.

$(a-b)^2\ c^3$ is even ($Q\ a-b$ is even)

Therefore, it is true.

$(a+b+c)^2\,(a-b)$ is even as $a-b$ is even.

Therefore, it is true.

**15.** Let $a = 0.58$, $b = 0.52$ and $c = 0.6$

The required value is $\dfrac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca}$

$$=\frac{\left(a^2+b^2+c^2-ab-bc-ca\right)(a+b+c)}{\left(a^2+b^2+c^2-ab-bc-ca\right)}$$

$\therefore$ Required value = a + b + c = 1. 7

**16.** The last digit of any power of 6 is always 6.

**17.** $$N=\frac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{2}}}}}=\frac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{2}{3}}}}=\frac{1}{1+\dfrac{1}{1+\dfrac{3}{5}}}$$

$$=\frac{1}{1+\frac{5}{8}}=\frac{1}{\frac{13}{8}}=\frac{8}{13}.$$

**18.** (i) $2N = 24\,q + 14$

Let $R\left(\dfrac{N}{a}\right)$ denote the remainder of $N$ divided by $a$.

$$R\left(\frac{2N}{12}\right)=R\left(\frac{24q}{12}\right)+R\left(\frac{14}{12}\right)=0+2=2.$$

(ii) Method 1

$N^2 = (12k+7)^2 = 144\,k^2 + 168k + 49$

When $N^2$ is divided by 12, the quotient is $12k^2 + 14k + 4$ and the remainder is 1.

Method 2

$$\text{Rem}\left(\frac{AB}{x}\right)=\text{Rem}\left(\text{Rem}\left(\frac{A}{x}\right)\text{Rem}\left(\frac{B}{x}\right)\right)$$

$$=\text{Rem}\left(\frac{\text{Rem}\left(\frac{N}{12}\right).\text{Rem}\left(\frac{N}{12}\right)}{x}\right)$$

$$=\text{Rem}\left(\frac{7\times 7}{12}\right)=1$$

**19.** Let us try to find the remainder using pattern method.

$$\text{Rem}\left(\frac{3^1}{11}\right)=3.\text{Rem}\left(\frac{3^2}{11}\right)=9$$

$$\text{Rem}\left(\frac{3^3}{11}\right)=5.\text{Rem}\left(\frac{3^4}{11}\right)=4$$

$$\text{Rem}\left(\frac{3^5}{11}\right)=1.\text{Rem}\left(\frac{3^6}{11}\right)=3$$

$\text{Rem}\left(\dfrac{3^7}{11}\right)=9$. Hence, the pattern is found to repeat after a cycle of 5. Hence, the required remainder $=\text{Rem}\left[\dfrac{3^{40}}{11}\right]=\text{Rem}\left[\dfrac{3^{(5\times 7)}+5}{11}\right]=\text{Rem}\left[\dfrac{3^5}{11}\right]=1$ divided by 11 is 1.

**20.** The rectangles can have

| | | | | |
|---|---|---|---|---|
| Length | 24 | 12 | 8 | 6 |
| Breadth | 1 | 2 | 3 | 4 |

Sum of the perimeters

= 2{1 + 2 + 3 + 4 + 6 + 8 + 12 + 24}

= 2 {Sum of factors} = 120

**21.** When 4831, 4833 and 4835 are divided by 24, the remainders are 7, 9, 11, respectively.

$\therefore N = (4831)\,(4833)\,(4835)$ and $(7)\,(9)\,(11) = 693$ have the same remainder.

When 4831, 4833 and 4835 are divided by 24, the remainders are 7, 9, 11, respectively.

$\therefore N = (4831)\,(4833)\,(4835)$ and $(7)\,(9)\,(11) = 693$ have the same remainder.

And when 693 is divided by 24, the remainder is 21.

**22.** The required number is the H.C.F. of (394 – 1) and (658 – 3), i.e., H.C.F. of 393 and 655. The H.C.F. of 393 and 655 is 131.

**23.** $(49)^{784} = (2401)^{392} = (2400 + 01)^{392}$

Expanding this we get all the terms containing 100 except the last term which is 01. Hence, the last two digits are 01.

**24.** The given number $8a2b3c5$ is divisible by 99.

$\therefore$ It is divisible by 11 and 9.

As the number is divisible by 9, $8 + a + 2 + b + 3 + c + 5$, i.e., $(a + b + c + 18)$ must be divisible by 9 (1)

Also, $(8 + 2 + 3 + 5) - (a + b + c)$, i.e., $18 - (a + b + c)$

must be divisible by 11 (2)

$a + b + c$ must be divisible by 9 ( $\therefore$From (1))

$\therefore\ a + b + c = 9, 18$ or 27 ($\because$ $a, b, c$ are single digit whole numbers)

$\therefore a, b, c$ cannot exceed 9.

$18 - (a + b + c) = 9$ or 0 or $-9$. $\therefore a + b + c = 18$.

$\therefore$ Only one value is possible.

**25.** $2^{1053} = 2^{20x52+13}$ (Since in the case of powers of 2, last two digits repeat in a cycle of 20 powers) $2^{13}$ ends in 92. Hence, the last two digits of $2^{1053}$ are 92.

**26.** $N = a^2 - b^2 = (a + b)(a - b)$ $[a > b]$

If $a$ and $b$ are natural numbers, $a + b$ and $a - b$ must be of the same parity.

$\therefore$ We need to identify $N$ for which the number of ways of expressing $N$ as a product of 2 factors of the same parity is the least.

$187 = 1 \times 187 = 11 \times 17$

$\therefore\ k(187) = 2$

$120 = 2 \times 60 = 4 \times 30 = 6 \times 20 = 10 \times 12$

$\therefore\ k(120) = 4$

$k(110) = 0$

$105 = 3 \times 35 = 5 \times 21 = 7 \times 15$

$\therefore\ k(105) = 3$

$\therefore \boldsymbol{k}(110)$ which is equal to 0, is the least.

**27.** Only squares of prime numbers have exactly 3 factors.

The two-digit squares of prime numbers are 25 and 49. Hence, 2 such numbers exist.

**28.** $X$ and $Y$ have the respective forms $40\ k_1 + 1$ and $40\ k_2 + 2$.

$\therefore$ $X$ and $Y$ have the respective forms $4\ k_3 + 1$ and $4\ k_4 + 2$.

The units digits of $2^N$ and $8^N$ have cycles of 4 each.

$\therefore$ $2^x$ and $8^y$ have respective units digits same as those of $2^1$ and $8^2$, i.e., 2 and 4.

I. As $Y$ has the form $40\ k_2 + 2$, its units digit is 2.

$\therefore$ The units digit of $8Y$ is 6

$(8^Y + 8Y)$ ends with 0. $\therefore$ It is divisible by 5.

II. As $X$ has the form $40\ k_1 + 1$, its units digit is 1.

$\therefore$ The units digit of $2X$ is 2

$(2^X - 2X)$ ends with 0. $\therefore$It is divisible by 5.

II is true.

**29.** Take the difference between any two numbers out of the three given numbers. (H.C.F. model 2)

$832 - 472 = 360$

$1372 - 832 = 540$

The required number is the H.C.F. of these two differences, i.e., H.C.F. of 360 and 540 which is 180.

**30.** A = {2, 3, 5, 7, 11, 13, 17, …, 47}, i.e., A has only one even integer, and only one integer ending with 5. A has no integer ending with 0.

$\therefore$ The product of elements of A will end with only one zero.

$\therefore$ B ends with one zero.

**31.** Let the 100 digit number be denoted by $x$.

$x = 12345\ldots\ldots$

The first 9 digits must be the first 9 single digit numbers. The next 90 digits must be the first 45 two digit numbers (i.e., 10 to 54). The 100th digit is the first digit of 55, i.e., 5. The remainder of any number when divided by 8 is the remainder of the last three digits of the number divided by 8. Hence, the remainder obtained is the remainder of 545 divided by 8, i.e., 1.

**32.** $N = 16q_1 + 3 = 12\ q_2 + 3$ ($\because$L.C.M. model 1)

$N - 3 = 16q_1 = 12\ q_2$

$N - 3$ is divisible by 16 and 12 and hence, by

L.C.M. (16, 12), i.e., 48

$N - 3 = 48\ k$

$N = 48\ k + 3 = 24\ (2k) + 3$

When $\boldsymbol{N}$ is divided by 24, the quotient is $2\boldsymbol{k}$ and the remainder is 3.

**33.** $48^{50} = (49 - 1)^{50} = 49\ k_1 + (-1)^{50} = 49\ k_1 + 1$

$50^{50} = (49 + 1)^{50} = 49\ k_2 + 1$

$48^{50} + 50^{50} = 49\ (k_1 + k_2) + 2$

$$\text{Rem}\left(\frac{48^{50} + 50^{50}}{49}\right) = 2$$

**34.** $1125 = (5)\ (225) = 5^3 3^2$

Number of factors of $1125 = (3 + 1)\ (2 + 1) = 12$

$1800 = 8(225) = 2^3(3^2)\ (5^2)$.

Number of factors of $1800 = (3 + 1)\ (2 + 1)\ (2 + 1) = 36$

Number of common factors of 1125 and 1800 = Number of factors of H.C.F. (1125, 1800), i.e., of 225 (i.e., $3^2 5^2$)

$= (2 + 1)\ (2 + 1) = 9$.

Number of factors of 1125 which are not factors of 1800 $= 12 - 9 = 3$.

Number of factors of 1800 which are not factors of 1125 $= 36 - 9 = 27$.

Number of factors of only one of 1125 and 1800 $= 3 + 27 = 30$.

**35.** Let the number of toffees Anil had be $N$.

L.C.M. of (20, 30, 50) = 300.

The minimum possible number is 300 + 16 = 316. But 316 is not divisible by 14. Hence, $N$ is of the form $300k + 16$.

We have to verify for what value of $k$ $(300k + 16)$ is divisible by 14. It is divisible by 14 for $k = 2$

$\therefore$ The minimum number of toffees he had is $300 \times 2 + 16 = 616$.

**36.** $15^{1700} = (16 - 1)^{1700} = 16k + (-1)^{1700} = 16k + 1$

$$\text{Rem}\ \frac{15^{1700} - 1716}{16}$$

$$= \text{Rem}\ \frac{(16k+1) - [16(107) + 4]}{16}$$

$$= \text{Rem}\ \frac{16(k - 107) - 3}{16} \equiv -3 \equiv 13$$

**37.** Given number is $(2^7)\ (3)\ (5)$.

(i) A perfect square when expressed in the prime factorised form must have an even index of each prime factor. A perfect cube when expressed in the prime factorised form must have each index of the form $3k$.

Hence, the given number must be multiplied by $(2^1)\ (3^1)\ (5^1)$, i.e., 30 to make it a perfect square.

(ii) It should be multiplied by $2^2 \times 3^2 \times 5^2$, i.e., 900 to make it a perfect cube. ($\therefore$ Any perfect cube must be of the form $p_1^{\ 3k^1} \times p_2^{\ 3k^2} \times \ldots\ldots\ldots$)

**38.** $3^Y + X^2 = (2^4)\ (5^4)$

RHS is $10^4$. $\therefore$ RHS ends with 0.

$\therefore$ LHS also has to end with 0 (1)

$Y$ is odd. $\therefore$ It has the form $4k + 1$ or $4k + 3$

If $Y$ has the form $4k + 1$, $3^Y$ ends with 3.

If $Y$ has the form $4k + 3$, $3^Y$ ends with 7.

$3^Y$ ends with 3 or 7 (2)

From (1) and (2), $X^2$ ends with 7 or 3.

$X$ is an integer whose square ends with 7 or 3. But there is no integer whose square ends with 7 or 3. $\therefore$ $X$ has no value.

**39.** Since 425 when divided by the number leaves a remainder of 2 it means $425 - 2 = 423$ is exactly divisible by that number. Similarly $373 - 4 = 369$ is also exactly divisible by that number. This means that the number we are looking for is the largest number which will divide 369 and 423 exactly. That will be the H.C.F. of 369 and 423, which is 9. (H.C.F. model 1)

**40.** Last 2 digits of $2^{974}$ is same as $2^{14}$ which is 84. Last 2 digits of $2^{696}$ is same as $2^{16}$ which is 36. Hence, the required ratio is 84:36, i.e., 7:3.

**41.** Expressing 64, 80 and 960 as a product of prime factors, we have the following.

$64 = 2^6$; $80 = 2^4 \times 5^1$; $960 = 2^6 \times 3^1 \times 5^1$

As 960 is a multiple of 3, 64 and 80 are not multiples of 3, $x$ should be a multiple of 3 as well as 16. So, it is a multiple of 48 such that the L.C.M. is 960.

As 96 is a multiple of 48, it is the required answer.

**42.** $17^3 + 19^3 + 21^3 + 23^3 = (17^3 + 23^3) + (19^3 + 21^3)$

$= (17 + 23)\ [17^2 + 17(23) + 23^2] + (19 + 21)\ [19^2 + 19(21) + 21^2]$

$= 40\ [17^2 + 19^2 + 21^2 + 23^2 + 17x23 + 19x21]$

The bracket is the sum of 6 odd numbers, i.e., an even number. Therefore, the given expression is a multiple of 80 and hence, the remainder when divided by 80 is 0.

**43.**

| $N$ | $D$ | $R$ | $D - R$ |
|---|---|---|---|
| $N$ | 15 | 5 | 10 |
| $N$ | 20 | 10 | 10 |

$N = 60k - 10$ ($\therefore$ L.C.M. model 2)

When $N$ is divided by 60, the remainder is –10. This is equivalent to the corresponding positive remainder of –10, i.e., –10 + 60 =50.

**44.** Last digit of $34563^{20359}$ = last digit of $3^{20359}$

= last digit of $3^3 = 7$ (20359 is of the form $4k + 3$)

Last digit of $2358^{784}$ = last digit of $8^{784}$

= last digit of $8^4 = 6$ (784 is of the form $4k + 4$)

∴ The last digit of $(34563^{20359} + 2358^{784})$ = the last digit of (67 + 6) = 3.

**45.** Taking the difference of two of the numbers at a time, we get 457 – 247 = 210 and 1087 – 457 = 630

The required number is the H.C.F. of the two differences, i.e., H.C.F. of 210 and 630 which is 210.

**46.** 67 is prime and 3 is coprime to 67 (1)

Fermat's little theorem: If $P$ is a prime number and $a$ is coprime to $P$, the remainder of $a^{P-1}$ divided by $P$ is 1.

$3^{135} = 3^{66 \times 2 + 3}$

∴ The remainder of $3^{66}$ divided by 67 is 1.

$$\therefore \text{Rem}\left(\frac{3^{135}}{67}\right) = \text{Rem}\left(\frac{(67k+1)^2 (3)^3}{67}\right) = \text{Rem}\left(\frac{3^3}{67}\right) = 27.$$

**47.** $72 = (3^2)\ (2^3)$

$108 = (3^3)\,(2^2)$

$162 = 3^4(2)$

H.C.F. = $3^{\min(2,3,4)} \times 2^{\min(3,2.1)} = 3^2 \times 2^1 = 18$

L.C.M. = $3^{\max(2,3,4)} \times 2^{\max(3,2,1)} = 3^4 \times 2^3 = 648$

**48.** 52! + 1 is a multiple of 53. (Wilson's theorem)

$52! = 53k + 52$

52! is a multiple of 52. ∴ $53k + 52$ is a multiple of 52.

∴ $K$ is a multiple of 52.

$52! = 53\ (52k_1) + 52$

Dividing both sides by 52, we have

$51! = 53k_1 + 1$

The remainder when 51! is divided by 53 is 1.

**49.** The number of zeros at the end of 175! is same as the greatest power of 5 in 175!. 175! has 35 fives, 7 twenty-fives and 1 one twenty five.

∴ Index of the greatest power of 5 in 175! is (35 + 7 + 1) = 43.

**50.** ! + 2! + 3! + …. +(50!) when divided by 5! leaves the same remainder as 1!+ 2! + 3! + 4! when divided by 5! (as 5! onwards all the numbers are divisible by 5!)

Therefore, the remainder is 1! + 2! + 3! + 4!, i.e., 33.

**51.** Time taken for the bells to toll together for the first time = L.C.M. of 5, 10, 15, 20, 25, 30

$5 = 5^1$ $10 = 5^1 \times 2$ $15 = 5^1 \times 3$

$25 = 5^2$ $30 = 5^1 \times 3^1 \times 2^1$

∴ L.C.M. of 5, 10, 15, 20, 25, 30 = $5^2 \times 3 \times 2^2 = 300$.

It means that the bells ring together, once every 5 min. They ring at 11:00, 11:05, 11:10 ... 12:30, a total of 19 times.

**52.** Since $10! + 1 = 11k$, the remainder when 10! is divided by 11 has to be –1, which is same as –1 + 11 = 10.

**53.** $10! = 1 \times 2 \times 3 \times 2^2 \times 5 \times (2 \times 3) \times 7 \times 2^3 \times 3^2 \times (2 \times 5) = 2^8 \times 3^4 \times 5^2 \times 7^1$

The index of the greatest power of 2 in 10! = 8

Use successive division method:

| | |
|---|---|
| 2 | 10 |
| 2 | 5 |
| 2 | 30 |
| 2 | 2 |
| | 1 |

$n = 5 + 2 + 1 = 8$

Here, we stop the division when the quotient becomes less than the divisor and add the quotients to obtain the index of the greatest power of $k$ in $N$!

**54.** Given number is 92879287 …. up to 360 digits and the divisor is 101.

As $101 = 10^2 + 1$, divide the given dividend into two digits each and get the remainder.

$92(100^{179}) + 87(100^{178}) + \ldots 92(100^3) + 87(100^2) + 92(100^1)$

$+ 87(100^\circ)$, i.e., $92(-1)^{179} + 87(-1)^{178} \ldots\ldots + 92(-1)^3 + 87(-1)^2 +$

$92(-1) + 87 = (-92 + 87)\ 90 = -450$

When –450 is divided by 101, remainder = –46.

The actual remainder is 101–46, i.e., 55.

**55.** $(3^9)\ (5^{13})$

(i) If the number of 3's and 5's are decreased by 1 each, the resulting number is a perfect square.

Hence, the given number must be divided by $(3^1)(5^1)$, i.e., 15.

(ii) It should be divided by 5 to make it a perfect cube.

**56.** Let the numbers satisfying the given conditions be denoted by $N$.

$10 \le N \le 99$.

$\text{Rem}\left(\frac{N}{4}\right) = 1$ and $\text{Rem}\left(\frac{N}{5}\right) = 2$

$N = 20k - 3$ ($\because$ L.C.M. model 2)

$N + 3 = 20k$

Also $13 \le N + 3 \le 102$

$\therefore N + 3 = 20$ or 40 or 60 or 80 or 100

$\therefore N = 17$ or 37 or 57 or 77 or 97.

$\therefore$ ***N*** has five possibilities.

**57.** $24^{23} + 21^{23}$ is divisible by $(24 + 21)$, i.e., 45.

Hence, it is divisible by factors of 45, i.e., 15 also.

**58.** The pattern of remainders when successive powers of 7 are divided by 35 is as follows:

| | Remainder |
|---|---|
| $7^1 \div 35$ | 7 |
| $7^2 \div 35$ | 14 |
| $7^3 \div 35$ | 28 |
| $7^4 \div 35$ | 21 |
| $7^5 \div 35$ | 7 |

Hence, the remainders repeat after every 4 powers.

$$\therefore \frac{7^{71}}{35} = \frac{7^{4\times 17+3}}{35} = \frac{7^3}{35} = 28$$

**59.** In each room, the same number of students is to be seated. Also in each room, a test for only one subject can be conducted.

Let the number of students to be seated in each room be $N$. Let the numbers of rooms in which the mathematics test, Biology test and History test are conducted be M, B and H, respectively. NM = 60, NB =36, NH =24.

$$\text{Number of rooms required } (R) = M + B + H = \frac{60}{N} + \frac{36}{N} + \frac{24}{N}.$$

We need the minimum value of $R$. In order for $R$ to be minimum, M, B, H must be minimum, i.e., $\frac{60}{N}, \frac{36}{N}, \frac{24}{N}$ must all be minimum.

$\therefore N$ must be maximum (1)

$M$, $B$ and $H$ must be integers. $\therefore N$ must divide 60, 36, 24 exactly. (2)

Number of students in each room = H.C.F. of 60, 36 and 24 = 12 [$\therefore$From (1) and (2)]

Minimum number of rooms required = min $(R)$

$= 60/12 + 36/12 + 24/12 = 10$

**60.** Given number is 212121… up to 120 digits and divisor is 99.

As $99 = 100 - 1$, i.e., $10^2 - 1$, divide the given dividend into two digits each and use Remainder theorem to get the remainder.

$21(100^{59}) + \ldots 21(100^2) + 21(100^1) + 21(100^0)$

$= 21(1)^{59} + \ldots 21(1)^2 + 21(1) + 21(1) = 21 \times 60$, i.e., 1260

But as $1260 > 99$, divide 1260 by 99. Hence, the remainder is 72.

**61.** (i) $72 = 2^3\ 3^2$

No. of ways of expressing 72 as a product of two of its factors $= (4 \times 3) / 2 = 6$

$1560 = 2^3\ 3^1\ 5^1\ 13^1$

No. of ways of expressing 1560 as product of two of its factors= $(4)(2)(2)(2) / 2 = 16$.

(ii) If a number has '$p$' prime factors, it can be expressed as a product of two coprimes in $2^{p-1}$ ways. As 72 has 2 prime factors and 1560 has 4 prime factors, they can be expressed in 2 ways and 8 ways, respectively.

**62.** $p = q + 2 = r + 4$. $\therefore p = q + 2$, $q = r + 2$.

Each prime number greater than 3 is of the form $6k + 1$.

If $r$ is of the form $6k + 1$, than $q$ is of the form $6k + 3$ and then $q$ is divisible by 3. Also $q$ is prime. $\therefore$ Only possible value of $q$ is 3. But then $r$ (= 1 and hence) will not be prime.

If $r$ is of the form $6k - 1$, $p$ is of the form $6k + 3$. Only possible value of $p$ is 3. But then $q$ (= 1 and hence) will not be prime.

$r$ is neither of the form $6k + 1$ or $6k - 1$. $\therefore$ $r$ is not a prime number greater than 3.

$r = 2$ or 3.

If $r = 2$, then $q = 4$ which is not prime.

$\therefore r = 3$ and $q = 5$ and $p = 7$.

Only one combination exists for $p$, $q$, $r$.

**Alternate Solution:**

$r, q, p$ is an increasing AP with common difference 2.

For any 3 terms in an AP, with common difference which is not a multiple of 3, one of the numbers is a multiple of 3, another leaves a remainder of 1 and the third leaves a remainder of 2 (when divided by 3). The only way in which all 3 can be prime is when the multiple of 3 is 3 itself, i.e., $(r, q, p) = (3, 5, 7)$. We can also consider $(-7, -5, -3)$.

As primes are considered to be positive, we have only one combination.

**63.** $8 = 2^3$. The index of the greatest power of 2 in 120! is A (say). The index of the greatest power of 8 in 120! is that of $2^3$ in 120! This is equal to the integral portion of $\frac{A}{3}$.

| | |
|---|---|
| 2 | 120 |
| 2 | 60 |
| 2 | 30 |
| 2 | 15 |
| 2 | 7 |
| 2 | 3 |
| | 1 |

$A = 60 + 30 + 15 + 7 + 3 + 1$, i.e., $= 116$

The index of the greatest of 8 in 120! is the integral portion of $\frac{116}{3}$ i.e., $38$.

**64.** (i) $80 = 2^4\, 5^1$.

The number of factors $= 5 \times 2 = 10$

$2340 = 2^2\, 3^2\, 5^1\, 13^1$

The number of factors $= 3 \times 3 \times 2 \times 2 = 36$

(ii) $80 = 2^4\, 5^1$

Sum of the factors $= \{[2^5 - 1]/(2 - 1)\}\{[5^2 - 1]/(5 - 1)\} = 31 \times 6 = 186$

$2340 = 2^2\, 3^2\, 5^1\, 13^1$.

Sum of the factors $= \{[2^3 - 1]/(2 - 1)\}\{[3^3 - 1]/(3 - 1)\}\{[5^2 - 1]/(5 - 1)\}\{[13^2 - 1]/(13 - 1)\} = 7644$

(iii) $80 = 2^4 . 5^1$

Product of factors $= 80^{\frac{(4+1)(1+1)}{2}}$, i.e. $80^5$.

$2340 = 2^2.3^2.5^1.\ 13^1$.

Product of factors $= (2340)^{36/2}$, i.e., $(2340)^{18}$

**65.** For even values of $n$, $a^n - b^n$ is divisible by $a + b$.

$4^{12} - 1$ can be written as:

$4^{12} - 1$ (Divisible by $4 + 1 = 5$)

$(4^3)^4 - 1$ (Divisible by $64 - 1 = 63$)

$(4^4)^3 - 1$ (Divisible by $256 - 1 = 255$)

It is not divisible by 127.

**66.** The required number will be in the form $(6k + 2)$ because when divided by 6 it leaves a remainder of 2. The same number when divided by 13 leaves a remainder of 6. Subtracting this remainder from the number $(6k + 2)$, the resulting number $(6k - 4)$ should be divisible by 13.

Trying out values of 0, 1, 2,..... for $k$, when $k = 5$, $(6k - 4)$ will be 26 which is divisible by 13. Hence, the

required number is $6k + 2 = 6(5) + 2 = 32$

**Alternate Solution:**

Number should be in $13k+6$ form, i.e., $12k + 6 + k$, and the remainder obtained when divided by 6 is $k$ which is 2 (given).

$\therefore 13(2) + 6 = 32$, the required number.

**67.** The respective remainders when $2^{59}$ is divided by (i) 9 (ii) 13 and (i) 17 are (i) 5 (ii)7 and (iii) 8, respectively.

**68.** Let the number be $N$. Let the quotient when the number is divided by 6 be $q_1$,

$N = 6q_1 + 4$.

Let the quotient be $q_2$ when $q_1$ is divided by 7.

$q_1 = 7q_2 + 5$

$N = 6\,(7q_2 + 5) + 4$

$= 42q_2 + 34$.

When $N$ is divided by 21, the quotient is $2q_2 + 1$ and the remainder is 13.

**69.** The given fractions are $\frac{3}{7}, \frac{5}{9}, \frac{2}{5}, \frac{8}{9}$

$$\text{L.C.M. of fractions} = \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$$

$$= \frac{\text{L.C.M.}(3, 5, 2, 8)}{\text{H.C.F.}(7, 9, 5, 9)} = \frac{120}{1} = 120.$$

H.C.F. of fractions = $\frac{\text{H.C.F. of numerators}}{\text{L.C.M. of denominators}}$

$= \frac{\text{H.C.F.}(3,5,2,8)}{\text{L.C.M.}(7,9,5,9)} = \frac{1}{315}$

**70.** Let the smallest number be $x$

$x - 3 = \text{L.C.M.}(54, 216, 121) = 26136$

$\Rightarrow x = 26136 + 3 = 26139.$

**71.** The given expression to be simplified is $2 + 3 \times \frac{1}{3}$ of $6 - \left(5 - \left\{\frac{5}{8} + 3\frac{3}{8} - \frac{1}{8}\right\}\right)$

Following the rules for simplification, the expression becomes $2 + 3 \times \frac{1}{3}$ of $6 - \left(5 - \left\{\frac{31}{8}\right\}\right)$

$= 2 + 3 \times \frac{1}{3}$ of $6 - \frac{9}{8} = 2 + 6 - \frac{9}{8} = \frac{55}{8}.$

**72.** Total number of items each child gets $= \frac{24}{n} + \frac{72}{n} + \frac{60}{n}$

$= \frac{156}{n}.$

This is minimum when $n$ is maximum. Hence, as $\frac{24}{n}, \frac{72}{n}$ and $\frac{60}{n}$ must be integers and $n$ is maximum, $n$ = H.C.F. (24, 72, 60) = 12.

Therefore, number of items each child can get = $\frac{156}{12} = 13.$

**73.** Let the number be $N$.

Product of the factors of $N = N\phi = N^2$.

Therefore, $N$ has 4 factors.

We can express a number having 4 factors in two ways, i.e., $P^3$ or $PQ$ (where $P$, $Q$ are primes.)

The factors of $P^3$ are 1, $P$, $P^2$, $P^3$.

Now, $1 + P + P^2 = 21$

$P(P + 1) = 4(5)$

Now, $P$ cannot be 4 as $P$ must be a prime number.

The factors of $PQ$ are 1, $P$, $Q$ and $PQ$.

$1 + P + Q = 21 \Rightarrow P + Q = 20$

The only possible values are 3 + 17 and 7 + 13.

Therefore, $N = 3(17)$ or $7(13)$

$N$ can be 51 or 91.

Two values are possible for the number.

**74.** Number is divisible by 5, 8 $\Rightarrow$ number is divisible by 40.

$\therefore$ Number should end with 0.

$\Rightarrow$ C = 0, $B60$ is divisible by 8

$\Rightarrow$ B = 1, 3, 5, 7, 9, B $\neq$ prime $\Rightarrow$ B = 1 or 9.

Adding all the digits 27 + A + B + C is divisible by 9

$\Rightarrow$ A + B + C is divisible by 9 (Q 27 is divisible by 9)

$\Rightarrow$ A + B is divisible by 9 (Q C = 0)

$\Rightarrow$ A + B = 9 or 18 because A and B are digits.

But A + B = 18 as A $\neq$ B (the digits are given as distinct)

$\therefore$ A + B = 9.

**75.** $t_i$ and $t_{i+1} = i + i + 1 = 2i + 1$ which is an odd number. If $i$ = 1, $t_i + t_{i+1} = i + (i + 1) = 3$. If $i = 100$, $t_i + t_{i+1} = 2i + 1 = 201$.

Among these natural numbers, there are 14 perfect squares of which only 9, 25, 49, 81,121, 169 are odd.

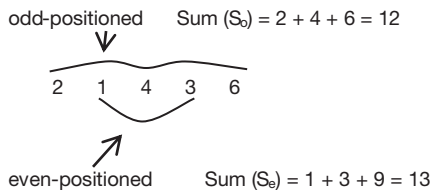
Therefore, there are 6 such perfect squares. They are 4 + 5, 12 + 13, 24 + 25, 40 + 41, 60 + 61 and 84 + 85.

**76.** The last two digits of 3! + 4! + 5! + …… 64! can be obtained by considering the last two digits of 3! + 4! + ….. 9! as from 10! onwards all the numbers end in 00.

Now, 3! + 4! + 5! + ….. +9! = 4037910. $\therefore$ $N$ can neither be a perfect square nor a perfect cube. (Squares end in 00, cubes in 000).

**77.** A number is divisible by 11 whenever difference of the sum of its digits, in the odd positions and sum of its digits in the even positions is 0 or a multiple of 11.

Odd-positioned Sum $(S_0)$ = 2 + 4 + 6 = 12




Remainder when 214369 is divided by 11.

$= S_e - S_0 = 13 - 12 = 1$

Hence, 1 is to be subtracted from 214369 to make it a multiple of 11.

**78.** $N = 0{\cdot}abc.abc\ldots$ (1)

multiply $N$ by 1000.

$\Rightarrow 1000\,N = abc.abc\ldots$ (2)

(2) – (1), $999N = \text{abc}$

where abc is a 3-digit number.

$\therefore$ When $N$ is multiplied by 999 or multiples of 999 it produces an integer.

From the options, 2997 is a multiple of 999.

**79.** Let the integer pair be $(p, q)$ then $p + q = pq \Rightarrow q = \frac{p}{p-1}$.

Only two pairs that satisfy the above equation are (0, 0) and (2, 2) because $p - 1$ divides $p$ if and only if $p - 1 = 1$ or $-1$, i.e., $0 + 0 = 0(0)$ and $2 + 2 = 2(2)$

**80.** Let the number of natural numbers the teacher told Raju to add be $n$. Let the natural number added twice be $k$.

$\therefore \frac{n(n+1)}{2} + k = 825$

when $n = 40$, $\frac{n(n+1)}{2} = 820$,

when $n = 41$, $\frac{n(n+1)}{2} > 825$, hence, $n = 40$

$k = 825 - 820 = 5$

**81.** The given number is $A = 853x426071y$

This number is divisible by 8, if $71y$ is divisible by 8.

If $71y$ is divisible by 8, $y = 2$.

$\therefore$ The sum of the digits of the given number is $(38 + x)$

If A is to be divisible by 9, $38 + x = 45 \Rightarrow x = 7$

$\therefore$ The values of $x$ and $y$ are 7 and 2, respectively.

**82.** Let the numbers be $11x$ and $11y$, where $x$ and $y$ are relative primes and $x \le y$.

L.C.M. of $11x$ and $11y$ is $11xy$.

$11xy = 1001$

$\Rightarrow xy = 91$

$\therefore x = 1$ and $y = 91$

or $x = 7$ and $y = 13$

$\therefore$ The numbers could be 11, 1001 or 77, 143.

Since the sum of the two numbers is 220, the required number is 77.

**83.** To any prime number all the numbers less than it are coprime to it. Hence, to 73, all the numbers from 31 to 47 will be coprime to it. Hence, all the 17 numbers are coprime to 73 in the given range.

**84.** 5, 11, 17, ……., 509 is an AP with the common difference as 6. Let the number of terms in this AP be $n$.

$509 = 5 + (n - 1)\ 6 \Rightarrow n = 85$

$5 + (m - 1)\ 6 \le 449 \Rightarrow m \le 75$

If we take the first 75 terms in the above series we get the series 5, 11, 17, ……, 449

The two middle terms in the above series are

$T_{37} = 5 + (37 - 1)\ 6 = 221$ and $T_{38} = 5 + (38 - 1)\ 6 = 227$

Now, for each term among the first 38 terms there will be at least one term among the other terms such that the sum exceeds 449, but for any two terms among the first 38 terms the sum does not exceed 449. Therefore, the cardinal value of $T$ is 38.

**Alternate Solution:**

The numbers in $S$ are in AP.

$S = \{5,11, 17, ….. 509\}$

T is a subset such that the sum of no two elements in $T$ exceeds 449. We need to look for there consecutive numbers $t_{n-1}$, $t_n$ and $t_{n+1}$ such that while $t_{n-1} + t_n \le 449$, $t_n + t_{n+1} > 449$. We can then include the first $n$ numbers in $T$. Now, $t_1 = 6 - 1$, $t_2 = 2(6) - 1$. $\therefore t_{n-1} = 6(n - 1) - 1 = 6n - 7$, $t_n = 6n - 1$ and $t_{n+1} = 6n + 5$.

$\therefore (6n - 7) + (6n - 1) \le 449$, whereas $(6n + 1) + (6n + 5) > 449$, i.e., $12n \le 457$ and $12n > 443$, i.e., $n \le 38\frac{1}{12}$ and $n > 36\frac{11}{12}$

$\therefore$ The greatest possible value of $n$ is 38.

Note: let $a$ and $b$ be any two elements of $T$ such that $a \le b$.

$a + a \le a + b$, i.e., $2a \le 449$, i.e., $a \le 224.5$. Also, $a$ is an element of $t$. $\therefore$ greatest $a = 221$. Next element of $S$ after greatest $a = 227$. As $221 + 227 = 448$, greatest $b = 227$.

$\therefore T = \{5, 11, 17,…, 221, 227\}$. $\therefore$ Its cardinality is 38.

**85.** $1! = 1$, $2! = 2$, $3! = 6$, $4! = 24$. For $n \ge 5$, $n!$ ends with 0.

5!, 6!,……,50! are all divisible by 10

$\therefore (5!)^2, (6!)^2, …..,(50!)^2$ are all divisible by 10.

$$\text{Rem}\left(\frac{(1!)^2 + (2!)^2 + (3!)^2 + (4!)^2 + \ldots + (50!)^2}{10}\right) =$$

$$\text{Rem}\left(\frac{(1!)^2 + (2!)^2 + (3!)^2 + (4!)^2}{10}\right) =$$

$$\text{Rem}\left(\frac{1 + 4 + 36 + 576}{10}\right) = 7$$

**86.** The smallest number '$n$' which when divided by 13, 11 and 15 successively leaves remainders of 4, 6 and 9 is obtained as shown below:

| | | |
|---|---|---|
| 13 | 11 | 15 |
| 4 | 6 | 9 |

As shown, the divisors are in the order of which division is carried out and their respective remainder is written below. Since we want to the minimum value of 11, we take the last quotient as 0. So, we can ignore the last divisor, i.e., 15. We start with the last remainder. We multiply it with its previous divisor and add the previous remainder. We multiply the result of this operation with the previous divisor and add it to the previous remainder. We carry on with this procedure until we have no more divisors and no more remainders left.

Hence, the number is $(9 \times 11 + 6) \times 13 + 4 = 1369$. The quotient and remainder when 1369 is divided by 9 are 152 and 1, respectively. The quotient and remainder when 152 is divided by 7 are 21 and 5, respectively, sum of the remainders is $1 + 5 = 6$.

**87.** We need to find the number of co-primes of 180 less than $180 = (2^2)\ (3^2)\ (5)$.

Multiples of 2: 90

Multiples of 3: 60

Multiples of 5: 36

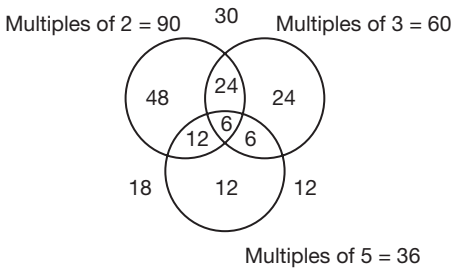
Multiples of 6: 30

Multiples of 10: 18

Multiples of 15: 12

Multiples of 30: 6

Drawing a Venn diagram for the multiples of its prime factors.




$\therefore$ Relative primes to 180 under it.

$= 180 - (48 + 24 + 24 + 12 + 6 + 6 + 12) = 180 - 132 = 48$

Excluding one, we get 47.

**Alternate Solution:**

We can find the number of co-primes by using the formula $N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)$, where $a$, $b$ and $c$ are primes, i.e., $180 = 2^2 \times 3^2 \times 5$

$\therefore$ Required

Answer is $180 \times \frac{1}{2}\left(\frac{2}{3}\right)\left(\frac{4}{5}\right) = 48$

As 1 is one of these 48 numbers, the required

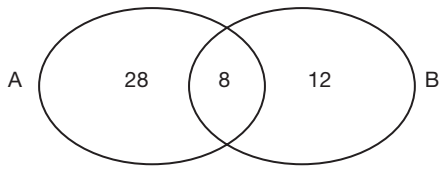
Answer is 47.

**88.** As $x$, $y$ are even, they are of the form $4k$ or $4k + 2$. Their squares are of the form $16k^2$ or $16k^2 + 16k + 4$. Therefore, $x^2 - y^2$ is of the form $16p$, $16p - 4$ or $16p + 4$. It cannot be of the form $16p + 8$, while 7389746232 is of the form $16p + 8$. Therefore, there are no even integers $x$, $y$ such that $x^2 - y^2 = 7389746232$.

**89.** Let $A = 2700 = 2^2 3^3 5^2$ and $B = 1134 = 2(567) = 5(9)\ (63) = 5(81)\ (7) = 3^4 5^1 7^1$

A has $3(4)(3)$ viz 36 factors. B has 20 factors.

H.C.F. (A, B) $= 3^3 5^1$. This has 8 factors. These conclusions are shown in the venn diagram below.




$\therefore$ 36 + 20 – 16, i.e., 40 numbers divide only A or only B.

**90.** Let the numbers satisfying the given conditions be denoted by $N$.

$$\text{Rem}\left(\frac{N}{7}\right) = 5 \text{ and } \text{Rem}\left(\frac{N}{6}\right) = 3$$

$\therefore$ $N$ has the forms $7a + 5$ and $6b + 3$ where $a$ and $b$ are whole numbers.

$N = 7a + 5 = 6b + 3$

R.H.S $= 3\ (2b + 1) = 3$ (an odd number) = an odd number divisible by 3.

$\therefore$ $7a + 5$ must be an odd number divisible by 3.

$\therefore$ $7a$ must be even.

$\therefore$ $a$ is even.

Least possible value of $a = 0$. Then $N = 5$ which is not divisible by 3.

When $a = 2$, $7a + 5 = 19$, which is not divisible by 3.

When $a = 4$, $7a + 5 = 33$, which is divisible by 3.

$\therefore$ The least value of $N = 33$.

The given problem is of the model 'L.C.M. model 3'.

The general solution of only 'L.C.M. model 3' problem for the number is:

Its least value + $k$ L. C. M (Divisors) where $k$ is a whole number.

$\therefore N$ must have the form $33 + k$ LC M $(7, 6) = 33 + 42k$

$N < 150$

$\therefore 33 + 42k < 150$

$42k < 107$

$k < 2\frac{23}{42}$. $\therefore k = 0$ or 1 or 2

$\therefore N = 33$ or 75 or 117

$\therefore \boldsymbol{N}$ has three possibilities.

**91.** Let the largest number satisfying the condition be $N$.

$606 = Na + 6$ (1)

$732 = Nb + 12$ (2)

$915 = Nc + 15$ (3)

In the equations (1), (2) and (3), $a$, $b$ and $c$ are the respective quotients when $N$ divides 606, 732 and 915.

$606 - 6 = 600 = Na$

$732 - 12 = 720 = Nb$

$915 - 15 = 900 = Nc$

As $N$ is the largest possible number satisfying the above three equations,

$N$ = H.C.F. (600, 720, 900) = 60.

**92.** Required last digit = Last digit of $8^{8236} \times 9^{4581}$ = last digit of $8^{4 \times 2059} \times 9^{2 \times 2290 + 1}$

$\therefore$ The last digit of $8^{8236}$ is same as the last digit of $8^4$ which is 6 ($\because$ Units digit cycle of power of 8 is 4)

The last digit of $9^{4581}$ is same as the last digit of $9^1$ which is 9 ($\because$ cycle of 2)

Last digit of $8^{8236} \times 9^{4581}$ = last digit of is $6 \times 9 = 54 = 4$

**93.** If we observe the divisors and remainders, i.e., 5 & 3, 8 & 6, 12 & 10, respectively, we find that there is a common difference of 2 between the divisor and the remainder.

$\therefore$ The number should be of the form

$k \times$ L.C.M. $(5, 8, 12) - 2$.

Since the number is a multiple of 13.

$13k_1 = k(120) - 2$

If we split up $118k$ into the nearest multiple of 13, we get $13k_1 = 117k + 3k - 2$

$\therefore$ for $118k - 2$ to be a multiple of 13,

$(3k - 2)$ must be a multiple of 13, i.e., $3k = 15$, $k = 5$ has the least such value.

$\therefore$ The number is $(120 \times 5) - 2 = 598$.

**94.** Let the least number be $x$.

$\therefore 7575 - x = k$ L.C.M. $(8, 9, 14, 20) + 5$

$= k$ L.C.M. $(8, 9, 14, 20) + 5 = 2520k + 5$

Examining the choices, we take $k = 3$.

$\therefore x = 10$

**95.** The number being a multiple of 36 must be divisible by 9 and 4 (factors of 36). For the number to be divisible by 4, the last two digits of the number must be divisible by 4. Hence, $y$ can be 0 or 4 or 8. For the number to be a multiple of 9, the sum of its digits must be a multiple of 9. Sum of the digits of the number = $27 + x + y$. This is a multiple of 9 only if $x + y$ is a multiple of 9. As $x$ and $y$ are single digit integers, $x + y \leq 18 \setminus x + y$ can be 18 only if $x = y = 9$. As $y \neq 9$, $x + y \neq 18$, $x + y = 9$. Hence, $(x, y)$ can be (9, 0), (5, 4) or (1, 8) or (0, 0). As $x - y$ is least $x - y = -7$.

**96.** The difference of '$ab$' and 'ba' is $9(a - b)$. This is divisible by 9.

The difference of 'abc' and 'cba' is $99(a - c)$. This is divisible by both 9 and 11.

The difference of any $N$ – digit number and its reverse is always divisible by 9. It is also divisible by 11 when $N$ is odd. When $N$ is even, the difference is not necessarily divisible by 11.

**97.** Let the numbers be $A$ and $B$.

$A = 4a$, $B = 4b$

$a$ and $b$ are co-primes.

$576 = 16ab \Rightarrow ab = 36 = 2^2 \times 3^2$

$2^{2-1}$, i.e., 2 such pairs are possible.

Note: The number of ways of writing a natural number $N$ as a product of two co-primes is $2^{p-1}$ where $p$ is the number of distinct prime factors of $N$.

**98.** Given $N = (5)(10)(15)\ (50)$

$= 5^{10}\ (1)(2)(3)(4)(5)(6)(7)(8)(9)(10) = 5^{10}\ (101!)$

As the highest power of 5 in $N$ is 12, and the highest power of 2 is 8, there are 8 zeros.

**99.** $3600 = 36 \times 100 = 4 \times 9 \times 4 \times 25 = 2^4 \times 3^2 \times 5^2$

When $N = a^p \times b^q \times c^r \times \ldots$ where $a, b, c, \ldots$ are primes.

The number of natural numbers less than $N$ that are co-prime to $N$ is given by

$$\phi(N) = N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)\ldots$$

Here, $N = 3600 = 2^4 \times 3^2 \times 5^2$

$$\therefore \phi(N) = 3600\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)$$

$$= 3600 \times \frac{1}{2} \times \frac{2}{3} \times \frac{4}{5} = 960$$

**100.** $1.\overline{137} = 1 + 0.\overline{137} = 1 + \frac{137-1}{990} =$

$$= 1 + \frac{136}{990} = \frac{990+136}{990} = \frac{1126}{990} = 563/495$$

## EXERCISE-3

**1.** $24 = 2^3 \times 3$

Largest power of 2 which divides 150! is found as shown below.

| | | | |
|---|---|---|---|
| 2 | 150 | = | 75 |
| 2 | 75 | = | 37 |
| 2 | 37 | = | 18 |
| 2 | 18 | = | 9 |
| 2 | 9 | = | 4 |
| 2 | 4 | = | 2 |
| 2 | 2 | = | 1 |
| | 1 | | 146 |

Largest power of 2 which divides 150! is 146. Largest power of $2^3$ which divides 150! will be the quotient of 146/3, i.e., 48. Largest power of 3 can be found as shown in a way similar to that shown above to be 72. As we have $48 - 8s$ and $72 - 3s$, we can only combine $48 - 8s$ with $48 - 3s$ to form $48 - 24s$. Hence, the largest power of 24 must be 48 [i.e., min (48, 72)].

**2.** Let the number of positive integers written be $N$. Let the integer erased by the student be $x$.

$$\frac{N(N+1)}{2} - x = 4035.$$

$$x = \frac{N(N+1)}{2} - 4035$$

$$x > 0 \therefore \frac{N(N+1)}{2} > 4035 \qquad (1)$$

Least $N$ satisfying (1) is 90.

If $N = 90$, $x = 60$

Only possible $(N, x)$ is (90, 60).

**3.** We have $16! = 14!\ (15 \times 16)$

So, $16! \mid 17 = (14! \mid 17)\ (15 \times 16 \mid 17)$

or $16 = (14! \mid 17) \times 2$

or $(14! \mid 17) = 16/2 = 8$

Now, $(14!)^{38} \mid 17 = (8)^{38} \mid 17 = 2^{114} \mid 17$

Applying Fermat's theorem,

$2^{114} \mid 17 = 4(2^{16})^7 \mid 17 = 4$

**4.** From 1 to 2000, $1^2, 2^2 \ldots 44^2$ are the perfect squares.

From 1 to 2000, $1^3, 2^3, \ldots 12^3$ are the perfect cubes.

From 1 to 2000, 1, 64 and 729 are perfect squares as well as perfect cubes.

From 1 to 2000, there are 44 perfect squares, 12 perfect cubes (of which 3 numbers are both perfect squares as well as perfect cubes) ($\therefore$ There 44 + (12–3) or 53 numbers should not be counted. Also, the number 2000 should not be counted).

Between 1 and 2000, 1946 numbers are neither perfect squares nor perfect cubes.

**5.** Any even power of 14 will have the last digit as 6.

**6.** Let $P = p_1^{\,a}\, p_2^{\,b}$ and $Q = p_3^{\,c}\, p_4^{\,d}$ where $p_1, p_2, p_3, p_4$ are distinct prime numbers and $a, b, c\ d$ are whole numbers. ($\because b + d + 1 > 0,\ a + c + 1 > 0$)

$F(P)\ F(Q) = F(PQ)$

$(a+1)\ (b+1)\ (c+1)\ (d+1) = F(PQ)$

If $P$ and $Q$ have no common prime factor,

$F(PQ) = (a+1)\ (b+1)\ (c+1)\ (d+1)$

$\therefore$ (1) is satisfied.

Suppose $P$ and $Q$ have exactly one prime factor in common. Suppose $p_1 = p_3$. Then $F(PQ) = (a + c + 1)\ (b + 1)\ (d + 1)$

$(1) \Rightarrow (a + 1)\ (b + 1)\ (c + 1)\ (d + 1) = (a + c + 1)\ (b + 1)\ (d + 1)$

$\Rightarrow (a + 1)\ (c + 1) = (a + c + 1) \Rightarrow ac = 0$

$\therefore$ $P$ and $Q$ cannot have exactly one common prime factor.

Suppose $P$ and $Q$ have both prime factors common. Then $F(PQ) = (a + c + 1)\ (b + d + 1)$

$(1) \Rightarrow (a + 1)\ (b + 1)\ (c + 1)\ (d + 1) = (a + c + 1)\ (b + d + 1)$

$\Rightarrow (ac + a + c + 1)\ (bd + b + d + 1) = (a + c + 1)\ (b + d + 1)$

$ac(b + d + 1) + bd\ (a + c + 1) = 0$

This is only possible if $ac = bd = 0$

$F(P) = (a+1)\ (b+1)$, $F(Q) = (c+1)\ (d+1)$ and $F(PQ) = (a+1)\ (b+1)\ (c+1)\ (d+1)$

But then $P$ and $Q$ cannot have any common prime factor.

$P$ and $Q$ must be co-prime numbers. If $P$, $Q$ are different primes, they are anyway coprimes.

Both I and II are possible.

**7.** $a^n - b^n$ is always divisible by $a - b$ when $n$ is a natural number, and by $a + b$, when $n$ is even. As 26 is even, $80^{26} - 1$ is always divisible by $80 - 1 = 79$ and $80 + 1 = 81$. As 79 and 81 are coprimes, it is also divisible by $81 \times 79 = 6399$.

**8.** The sum of all the factors of number $2^p \times 5^q$ is given by:

$$\left(\frac{2^{p+1}-1}{2-1}\right)\left(\frac{5^{q+1}-1}{5-1}\right) = 42$$

Now, $42 = 2\ (3)\ (7)$

By substitution, we can see that

$$\frac{2^{p+1}-1}{2-1} = 7 \text{ and } \frac{5^{q+1}-1}{5-1} = 6.$$

$\therefore 2^{p+1} = 8 \Rightarrow p = 2$ and $5^{q+1} = 25 \Rightarrow q = 1$

**9.** $24696 = 2^3 \times 3^2 \times 7^3$

Sum of the factors of 24696

$$= \frac{2^4-1}{2-1} \times \frac{3^3-1}{3-1} \times \frac{7^4-1}{7-1} = (15)(13)(400)$$

$= 195\ (400) = 78000$

**10.** $1001 = 13 \times 11 \times 7$.

$\therefore (1001)^2 = 13^2 \times 11^2 \times 7^2$.

For a factorial to be divisible by $(1001)^2$, it should contain 2–13$s$, 2–4$s$ and 2-7$s$ which is possible only when $n$ is at least 26.

**11.** $X = (22\ldots2)^2 + (44\ldots4) + 1 = (2222222223)^2$

$(\because a^2 + 2a + 1 = (a + 1)^2)$

$\sqrt{x} = 2222222223$ $\therefore \sqrt{x} - 1 = 2222222222$

And $\frac{9}{2}(\sqrt{X} - 1) = 9999999999 = 10^{10} - 1$

**12.** Let us denote the given number by $N$.

$N = 987\ (1 + 10^3 + 10^6 + 10^9 + \ldots + 10^{597})$ ($N$ = The block 987 repeated 200 times.)

$$\text{Rem}\left(\frac{N}{999}\right)\ \text{Rem}\left(\frac{N}{10^3-1}\right)$$

$$\text{Rem}\left(\frac{987\left(1+10^3+\left(10^3\right)^2+\ldots+\left(10^3\right)^{199}\right)}{10^3-1}\right)$$

$$= \text{Rem}\frac{M}{10^3-1}$$

Where $M = 987\ (1 + 1 + \ldots 1) = 987(200) = 197{,}400$

$$\text{Rem}\frac{197{,}400}{999} = 197 + 400 = 597$$

**Alternate Method:**

Any number can be expressed as (divisor × quotient) + (remainder)

In the given problem, the number is 987987…. up to 600 digits The sum of its digits = 24(600) = 14400. This is divisible by 9.

Also the divisor, i.e., 999 is divisible by 9.

Therefore, the quotient must be divisible by 9.

As both the number and the divisor × quotient are divisible by 9, even the remainder must be divisible by 9. Only satisfies this condition.

**13.** $F(a, b) = [a/b]$ = greatest integer less than or equal to $\frac{a}{b}$.
Let $[a/b] = n$, i.e., $n \le \frac{a}{b} < n + 1 \Rightarrow nb \le a < (n + 1)b$.

If $a, b$ are integers, then we can write

$nb \le a \le (n + 1)b - 1$ (1)

Let $F(N, 2) = N_1$, $F(N_1, 3) = N_2$, $F(N_2, 4) = N_3$, $F(N_3, 5) = N_4$, $F(N_4, 6) = 1$.

We want the maximum value of $N$. Therefore, we have to work out the maximum value of $N_4$, then of $N_3$, then of $N_2$, then of $N_1$ and finally of $N$.

$$F(N_4, 6) = 1 \Rightarrow 1 \le \frac{N_4}{6} < 2 \Rightarrow 6 \le N_4 \le 11$$

$$F(N_3, 5) = 11 \Rightarrow 11 \le \frac{N_3}{5} < 12 \Rightarrow 55 \le N_3 \le 59$$

$$F(N_2, 4) = 59 \Rightarrow 59 \le \frac{N_2}{4} < 60 \Rightarrow 236 \le N_2 \le 239$$

$$F(N_1, 3) = 239 \Rightarrow 239 \le \frac{N_1}{3} < 240 \Rightarrow 717 \le N_1 \le 719$$

$$F(N, 2) = 719 \Rightarrow 719 \le \frac{N}{2} < 720 \Rightarrow 1438 \le N \le 1439$$

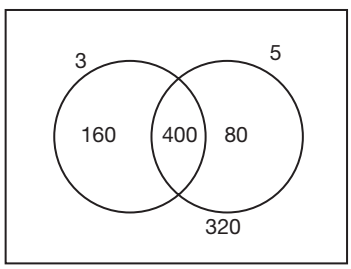
The maximum value of $N$ is 1439.

**14.** Number of odd natural numbers less than or equal to 1200 = 600

Number of odd numbers divisible by 3, $n(3) = 200$

Number of odd numbers divisible by 5, $n(5) = 120$

Number of odd number divisible by both 3 and 5, $n(15) = 40$




Therefore, of the 600 odd natural numbers less than or equal to 1200, there are 280 numbers which are divisible by 3 or 5. Therefore, (600 – 280), i.e., 320 odd numbers in the given range are divisible by neither 3 nor 5.

**15.** L.C.M. (5, 13, 17) = 1105

Any number up to 1105 which is not coprime to 1105 is divisible by 5 or 13 or 17. The greatest four-digit number divisible by 1105 is $1105 \times 9$, i.e., 9945.

There are $1105\left(1-\frac{1}{5}\right)\left(1-\frac{1}{13}\right)\left(1-\frac{1}{17}\right)$ or 768 numbers up to 1105 which are coprime to 1105. If we consider numbers up to 9(1105), viz. 9945, there are 9(768) viz. 6912 numbers which are neither divisible by 5, nor 13 nor 17.

$\therefore$ All the remaining numbers up to 9945 are divisible by at least one of 5, 13 or 17.

Number of numbers up to 9945 divisible by 5 or 13 or 17

$= 9945 - 6912 = 3033$

From 9945 to 9999, there are 10 multiples of 5, 4 multiples of 13 and 3 multiplies of 17. There are no multiples of any two of 5, 13, 17.

From 9945 to 9999, there are 17 numbers divisible by 5 or 13 or 17.

There are 3050 numbers up to 9999 which are divisible by 5 or 13 or 17.

**16.** Let us denote the given number by $N$.

$N = 406417\ (1 + 10^6 + 10^{12} + 10^{18} + \ldots\ 10^{294})$

($\because$ $N$ = The block 406417 repeated 50 times)

$$\text{Rem}\left(\frac{N}{1001}\right) = \text{Rem}\left(\frac{N}{10^3 - (-1)}\right)$$

$$= \text{Rem}\left(\frac{406417\left(1+\left(10^3\right)^2+\left(10^3\right)^4+\ldots\left(10^3\right)^{98}\right)}{10^3-(-1)}\right)$$

By remainder theorem, this equals 406417 (1 + 1 + … 1), i.e., 406417 (50).

But this is greater than 1001. $\therefore$ Remainder

$$= \text{Rem}\left(\frac{406417(50)}{1001}\right) =$$

$$\text{Rem}\left(\frac{(400400+6006+11)(50)}{1001}\right)$$

$$= \text{Rem}\left(\frac{(11)(50)}{1001}\right) = 550$$

**Alternate Solution:**

Let $N = 406417, 406417, \ldots$ (up to 300 digits or 50 blocks)

$$\therefore \text{Rem}\frac{N}{1001} = \text{Rem}\frac{50(417-406)}{1001} = 550$$

**17.** Let $x = 3p + 6q - 9r$, $y = 3p - 6q + 9r$, $z = -3p + 6q + 9r$.

$E = x^3 + y^3 + z^3 - 3xyz$, where $x + y + z = 3p + 6q + 9r = 3(p + 2q + 3r) = 0$

As $x + y + z = 0$, $x^3 + y^3 + z^3 = 3xyz$

$\therefore E = 0$

$E$ is at least zero as well as at most zero.

**18.** The 3 given numbers are consecutive integers of the form $4k-2$, $4k-1$, $4k$. ($3^{2003} + 1$ is a multiple of 4.)

The first and third numbers have exactly one common factor, which is 2.

There are no common factors between the first and second or between the second and third (other than 1).

$\therefore$ The first number is $2(2k-1)$.

$\therefore$ The second number is $4k - 1$.

The third number is $4k$.

Their L.C.M. is $(2k-1)(4k-1)(4k)$ or half their product.

$\therefore$ The L.C.M. of the given numbers $= \dfrac{(3^{2003}-1)(3^{2003})(3^{2003}+1)}{2} = \dfrac{(3^{4006}-1)(3^{2003})}{2}$.

**19.** The positive integers between $2^{2N}$ and $2^{2N+3}$ are in arithmetic progression and the common difference of this progression is 1.

$$\text{Sum} = \frac{2^{2N+3} - 2^{2N} - 1}{2}\left[2^{2N} + 2^{2N+3}\right]$$

$$= \frac{7\left(2^{2N}\right) - 1}{2}\left[9\left(2^{2N}\right)\right] = \left[7\left(2^{2N}\right) - 1\right]\left[9\left(2^{2N-1}\right)\right]$$

The sum is divisible by 162.

$\therefore$ It is divisible by both 81 and 2.

$\therefore$ $7(2^{2N}) - 1$ must be divisible by 9.

$7(2^{2N})$ must be 1 more than a multiple of 9.

Remainder of $2^N$ divided by 9 follows a cycle of 6.

$\therefore$ Remainder of $2^{2N}$ divided by 9 follows a cycle of 3.

$$\text{Rem}\frac{\left(2^2 \times 7\right)}{9} = 1, \text{Rem}\left(\frac{2^4 \times 7}{9}\right) = 4, \text{Rem}\left(\frac{2^6 \times 7}{9}\right) = 7$$

In general, if $N = 3k+1$, then $2N = 6k+2$ and Rem $= \dfrac{7\left(2^{2N}\right)}{9} = 1$. (Otherwise, the remainder is not 1.)

$\therefore$ $2N$ must have the form $6K + 2$.

$\therefore$ $N$ must have the form $3K + 1$.

Only II is possible.

**20.** By remainder theorem, Rem $\left(\dfrac{x^{550}}{x+1}\right)$ is $(-1)^{550} = 1$.

While the remainder of each of $x^{275}$, $x^{55}$ and $x^{11}$ divided by $x + 1$ is $-1$, the remainder of $x^{550} + x^{275} + x^{55} + x^{11}$ divided by $(x + 1)$ is $1 + 3(-1)$, i.e., $-2$. This is equivalent to a remainder of $x - 1$.

**21.** $323323 = 323000 + 323 = 323\ (1000+1) = 323(1001)$

$= (17)\ (19)\ (7)\ (11)\ (13)$

Sum of all the factors of 323323

$$= \frac{17^{1+1}-1}{17-1} \cdot \frac{19^{1+1}-1}{19-1} \cdot \frac{7^{1+1}-1}{7-1} \cdot \frac{11^{1+1}-1}{11-1} \cdot \frac{13^{1+1}-1}{13-1}$$

$= (18)\ (20)\ (8)\ (12)\ (14)$

$= (360)\ (1344) = 483840$

**22.** If $p$ is any prime number, then $(p-1)! + 1$ must be divisible by $p$.

$272 = (17)\ (16)$

As 17 is prime, $16! + 1$ must be divisible by 17.

Suppose $16! + 1 = 17k$

$16! = 17k - 1$

$\therefore$ Rem $\left(\dfrac{16!}{17}\right) = -1 = 16$, i.e., $16!$ has the form

$17k_1 + 16$

$\therefore$ $k_1$ must be divisible by 16.

($\because$ Both $16!$ and 16 are divisible by 16.)

Let $k_1 = 16k_2$

Dividing both sides by 16, we get

$15! = 17\ (k_2) + 1$

As $15!$ is even, $k_2$ must be odd. Let $k_2 = 2k_3 + 1$.

$15! = 17\ (2k_3 + 1) + 1 = 34k_3 + 18$

$\therefore$ Rem $\left(\dfrac{15!}{34}\right) = 18$

**23.** If $p$ is any prime number, then $(p-1)!$ is not divisible by $p$.

$1 \le P \le 40$

Each prime value of $P$ satisfies the given condition.

There are 12 such values.

When $P$ is 1, $(P-1)! = 1$ (which is divisible by $P$).

The only composite number satisfying the condition is 4.

There are 13 values of $P$ satisfying the condition.

**24.** From 1 to 2000, there will be single digit, 2-digit, 3-digit and 4-digit page numbers.

Number of single digit page numbers, i.e., 1 to 9 = 9.

$\Rightarrow$ Number of times a key has to be pressed = $9 \times 1 = 9$.

Number of double digit page numbers, i.e., 10 to 99 = 99 – 10 + 1 = 90

$\Rightarrow$ Number of times a key has to be pressed = $90 \times 2 = 180$

Number of three digit page numbers, i.e., 100 to 999 = 999 – 100 + 1 = 900

$\Rightarrow$ Number of times a key has to be pressed = $900 \times 3 = 2700$

Number of four digit page numbers, i.e., 1000 to 2000 = 2000 – 1000 + 1 = 1001

$\Rightarrow$ Number of times a key has to be pressed = $1001 \times 4 = 4004$

Total Number of times a key has to be pressed = 9 + 180 + 2700 + 4004 = 6893.

**25.** Let $N$ = 9098,9098 … (up to 602 digits)

= 90,9890,9890, … up to 602 digits. (i.e., 2 + 150 groups of 4 digits)

$$\text{Rem}\frac{N}{101} = \text{Rem}\frac{90+90(150)-98(150)}{101}$$

$$= \text{Rem}\frac{90-1200}{101} = -\text{Rem}\frac{1110}{101} = -(-1) = 1.$$

**26.** $122^{230}$ has the same last two digits as $22^{230}$. The last two digits of $a^N$ where $N$ is any positive integer has a cycle of 20.

As $22^{230} = 2^{20\,(11)\,+\,10}$, its last two digits are the same as those that of $22^{10}$.

$22^2 \equiv 84$, $84^2 \equiv 56$, $56^2 = 36 \setminus 22^8 \equiv 36$

$22^{10} \equiv 22^8\, 22^2 \equiv (36)\,(84) \equiv 24$ (where '=' me. The last two digits of the number on the left side are equal to those of the number on the right side.)

$$\therefore \text{Rem}\frac{122^{230}}{25} = 24$$

**27.** $$\text{Rem}\frac{483^{483}}{100} = \text{Rem}\frac{83^{483}}{100} = \text{Rem}\frac{83^{480+3}}{100} = \text{Rem}\frac{83^3}{100}$$

[$\because$ Rem $\dfrac{a^n}{100} = \text{Rem}\dfrac{a^{20k+n}}{100}$, provided a does not have just one 2 or just one 5 as factors. If 4 divides $a$ or $a$ ends in 25, 75 or 00, the Statement Is once again true. (For $a$ ending in 50, the Statement Is false, as $50^{21}$ ends in 00.) For odd numbers which are not multiples of 5 (like the given number), there are no exceptions]

$$= \text{Rem}\frac{17^2(83)}{100} = -\text{Rem}\frac{17^3}{100} = -(13) \equiv 87$$

**28.** We have 10! = 8! (10) (9)

So, 10! | 11 = (8! | 11) (10 × 9 | 11)

or 10 = (8! | 11) . 2

or (8! | 11) = 10/2 = 5

**29.** Any number of the form (—— 76)$^k$, where $k$ is any positive number, ends in 76.

Therefore, $(7776)^{1240}$ = (_____76)

$(21)^{1240} = (20+1)^{1240} = {}^{1240}C_0\ 20^{1240} + {}^{1240}C_1\ (20)^{1239} + \ldots + {}^{1240}C_{1239}\ (20) + {}^{1240}C_{1240}$

$= 100k + 1240\ (20) + 1$

$= 100k + 24800 + 1$

= _____01.

Now, $(7776)^{1240}\ (21)^{1240}$ = (—76) (—01) = (—76)

Therefore, the sum of the last two digits is 7 + 6 = 13.

**30.** $29^6 = (32-3)^6$ = (32 – 3) multiplied 6 times = The sum of a number divisible by 32 and $(-3)^6$

Similarly, $35^6$ is the sum of a number divisible by 32 and $(3)^6$. $\therefore$ $\text{Rem}\left(\dfrac{29^6+35^6}{32}\right) = \text{Rem}\left(\dfrac{(-3)^6+3^6}{32}\right)$

$$= \text{Rem}\left(\frac{2\left(3^6\right)}{32}\right)$$

Similarly $\text{Rem}\left(\dfrac{30^6+34^6}{32}\right) = \text{Rem}\left(\dfrac{2\left(2^6\right)}{32}\right)$

and $\text{Rem}\left(\dfrac{31^6+33^6}{6}\right) = \text{Rem}\left(\dfrac{2\left(1^6\right)}{32}\right)$

Required remainder = $\text{Rem}\left(\frac{2\left(3^6+2^6+1^6\right)+32^6}{32}\right)$

$= \text{Rem}\left(\frac{2\left(3^6+0+1\right)+0}{32}\right) = \text{Rem}\left(\frac{1460}{32}\right) = 20$

**31.** Required time = L.C.M. $\left(3+7\frac{5}{6}, 3+1\frac{1}{3}, 3+5\frac{2}{3}\right)$

$= \text{L.C.M.}\left(\frac{65}{6}, \frac{13}{3}, \frac{26}{3}\right)$

$= \frac{130}{3} = 43\frac{1}{3}$ seconds

**32.** $2^P + P^3$ is divisible by 5. ∴It ends with 0 or 5.

Suppose $2^P + P^3$ ends with 0. Then, it is even. Also, $2^P$ is even.

∴ $P^3$ must be even. ∴ $P$ is even. ∴ $P$ has the form $4k$ or $4k+2$.

If $P$ has the form $4k$, $2^P$ ends with 6. ∴ $P^3$ must end with 4.

If $P$ has the form $4k+2$, $2^P$ ends with 4.

∴ $P^3$ must ends with 6. ∴$P$ must end with 6. $P$ is not more than 20.

∴ $P = 4$ or 6

Suppose $2^P + P^3$ ends with 5. Then, $2^P+P^3$ is odd. As $2^P$ is even, $P^3$ is odd.

∴ $P$ is odd. ∴ $P$ has the form $4k+1$ or $4k+3$.

If $P$ has the form $4k+1$, then $2^P$ ends with 2. ∴ $P^3$ must end with 3. ∴$P$ must end with 7.

If $P$ has the form $4k+3$, then $2^P$ ends with 8. ∴$P^3$ must end with 7.

∴ $P$ must end with 3.

$P$ is not more than 20.

∴ $P = 17$ or 3

$P = 4, 6, 3$ or 17

**33.** $\left[\frac{2^N}{7}\right]$ is the quotient of $2^N$ divided by 7.

Remainders of $\frac{2^N}{7}$ follow a cycle of 3.

$\text{Rem}\left(\frac{2^{3k+1}}{7}\right) = 2, \text{Rem}\left(\frac{2^{3k+2}}{7}\right) = 4, \text{Rem}\left(\frac{2^{3k}}{7}\right) = 1$

$\Rightarrow \left[\frac{2^{3k+1}}{7}\right] = \frac{2^{3k+1}-2}{7}, \left[\frac{2^{3k+2}}{7}\right] = \frac{2^{3k+2}-4}{7}, \left[\frac{2^{3k}}{7}\right] = \frac{2^{3k}-1}{7}$

The required sum $= \frac{2^0-1}{7} + \frac{2^1-2}{7} + \frac{2^2-4}{7}$

$+\frac{2^3-1}{7} + \frac{2^4-2}{7} + \frac{2^5-4}{7} + \frac{2^6-1}{7} + \ldots\ldots$

$+\frac{2^{598}-2}{7} + \frac{2^{599}-4}{7} + \frac{2^{600}-1}{7}$

$= \frac{1}{7}(2^{601} - 1 - 1401) = \frac{1}{7}(2^{601} - 1402)$

**34.** $1000 = 2^3 \times 5^3$, i.e., 2 and 5 are the only prime factors of 1000.

If the multiple of 2 and 5 are excluded from the first 1000 natural numbers, the remaining numbers are co-prime to 1000 and definitely less than 1000.

So, the numbers of tokens left in the box $\phi$ (1000)

$= 1000\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right) = 400$

Sum of the numbers on the token left in the box

= sum of all the co-primes of 1000 less than 1000.

$= \frac{\phi(1000)\times 1000}{2} = \frac{400\times 1000}{2} = 200000$

**35.** $\text{Rem}\frac{19!}{17^2} = 17$ $\text{Rem}\frac{16!(18)(19)}{17} = 17$

$\text{Rem}\frac{16(1)(2)}{17} = 17(15) = 255$

**36.** The 15 sheets (which are removed from middle) will have the pages numbered from 91 to 150.

∴The sum of numbers on remaining pages = (1 + 2 + 3 … 90 + 151 + 152 + … 240) = 90 (241)

= 21690 (∵ 1 + 240 = 2 +239 = 3 + 238 = … 90 +151 = 241).

**37.** By remainder theorem, $\text{Rem}\left(\frac{12^{2y}}{13}\right) = 1$.

$12^{2y} = 13k + 1$

$$\left\lceil \frac{12^{2y}}{13} \right\rceil + \left\lceil \frac{12^{2y+1}}{13} \right\rceil = \left\lceil \frac{13k+1}{13} \right\rceil + \left\lceil \frac{12(13k+1)}{13} \right\rceil$$

$$= \left\lceil k + \frac{1}{13} \right\rceil + \left\lceil 12k + \frac{12}{13} \right\rceil$$

$= (k + 1) + (12k + 1) = (13k + 1) + 1 = 12^{2y} + 1.$

**38.** $105 = 3\ (5)\ (7)$

Let us first find the remainders when $2^{216}$ is divided by 3, 5, and 7.

$2^{216} = (3 - 1)^{216} =$

$= 3k + (-1)^{216} = 3k + 1$

$2^{216} = 4^{108} = (5 - 1)^{108} = 5k_1 + 1$

$2^{216} = 8^{72} = (7 + 1)^{72} = 7k_2 + 1$

$2^{216}$ leaves a remainder of 1 when divided by 3, 5 or 7.

$\therefore 2^{216} - 1$ is divisible by 3, 5, 7 and hence, by L.C.M. (3, 5, 7), i.e., 105.

$$\therefore \text{Rem}\left(\frac{2^{216}}{105}\right) = 1.$$

**39.** $N = 2^5\ 3^3\ 5^2$

The odd factors of $N$ are tabulated below

| | | |
|---|---|---|
| $3^0\ 5^0$ | $3^0\ 5^1$ | $3^0\ 5^2$ |
| $3^1\ 5^0$ | $3^1\ 5^1$ | $3^1\ 5^2$ |
| $3^2\ 5^0$ | $3^2\ 5^1$ | $3^2\ 5^2$ |
| $3^3\ 5^0$ | $3^3\ 5^1$ | $3^3\ 5^2$ |

The product of all these factors is $(3^6\ 5^0)\ (3^6\ 5^4)\ (3^6\ 5^8)$

$= 3^{6(3)}\ 5^{4(3)} = 3^{18}5^{12}$

**40.** If $A$ is a multiple of 2 (but not 4), then the last two digits of $A^1$ and of $A^{21}$ are not the same. Similarly, if A is a multiple of 5 (not ending in 25, 75 or 00), then the last two digits of $A^1$ and of $A^{21}$ are not the same. Therefore, $A$ could be 2, 6, 10, … or 5, 15, 35. However, $B$ has to be 1.

If $B \neq 1$, $A^B$ and $A^{B+20}$ would have the same last two digits.

**41.** We know that $5^2 = 4^2 + 3^2$

Multiplying this by $9^2$, we get $\Rightarrow 45^2 = 36^2 + 27^2$.

This is the only way of expressing $45^2$ as a sum of two non-zero perfect squares.

Let us try to express $45^2 = 2025 = 3^4 \times (5^2)$ as the difference of two non-zero perfect squares. $45^2$ (i.e., $3^4\ 5^2$ can be written as a product of two numbers in $\frac{(4+1)(2+1)+1}{2} = 8$ ways which includes $45 \times (45)$.

Let $A^2 - B^2 = (A + B)\ (A - B) = 45^2$

Since both $A$ and $B$ are positive, $(A + B)$ has to be greater than $(A - B)$. In each way of expressing of $45^2$ as a product of two factors, take the larger factor as $(A + B)$ and the smaller as $(A - B)$.

For example, $(A + B)\ (A - B) = 5 \times (405)$

So, take $A + B = 405$ and $A - B = 5 \Rightarrow A = 205$, and $B = 200$

$(A + B)$ and $(A - B)$ both are taken as 45, $B = 0$. This violates the condition that $A$ and $B$ are non-zero perfect squares.

Therefore, $45^2$ can be expressed as the difference of two perfect squares in $(8 - 1) = 7$ ways and as a sum of two perfect squares in only one way.

So, it can be expressed as the sum or difference of non-zero squares in a total of 8 ways.

**42.** $540 = 3^3(2^2)5$

The number of factors of $540 = (3 + 1)\ (2 + 1)\ (1 + 1) = 24$.

The least positive odd integer having the same number of factors as 540 must be of the form $p_1{}^a p_2{}^b \ldots$ where $p_1$, $p_2$, … are odd primes and $a$, $b$, … are whole numbers, where $(a + 1)\ (b + 1) \ldots = 24$.

Any positive odd integer having 24 factors has the form. The number 24 can be expressed as a single number, the product of 2 numbers, (all greater than 1) as 2(12), 3(8), 4(6), the product of 3 numbers (all greater than 1) as 2(2)(6), 2(3)(4) or as the product of 4 number (all greater than 1) as 2(2)(2)(3).

The least positive odd integer having 24 factors cannot be of the form $p^{23}$ or $p^{11}\ q^1$. It must have one of the other forms.

$\therefore$ It must be the minimum of $3^7 5^2$, $3^5 5^3$, $3^5 5\ (7)$, $3^3\ 5^2 7$ or $3^2\ 5\ (7)\ (11)$; the least positive odd integer having 24 factors is $3^2\ (5)\ (7)\ (11) = 3465$.

**43.** $2160 = 16(135) = 16(27)(5) = 2^4 3^3 5^1$

$\therefore$ The sum of all the factors of 2160

$$= \frac{(2^5 - 1)}{1}\ \frac{(3^4 - 1)}{2}\ \frac{(5^2 - 1)}{4} = (31)(40)(6)$$

and the sum of all the odd factors of 2160

$$= \frac{(3^4 - 1)}{2}\ \frac{(5^2 - 1)}{4} = (40)(6)$$

∴ The sum of all the even factors of 2160 = (30) (40) (6) = 7200.

**44.** The minimum number of pieces would be obtained when the weight of each equal part is maximum.

The weight of each equal part is equal to the H.C.F. of

$2\frac{1}{3}, 4\frac{2}{3}, 4\frac{1}{5}, 5\frac{5}{6}, 6\frac{1}{8},$

i.e., $\frac{7}{3}, \frac{14}{3}, \frac{21}{5}, \frac{35}{6}, \frac{49}{8}$

$$\text{H.C.F.}\left[\frac{7}{3}, \frac{14}{3}, \frac{21}{5}, \frac{35}{6}, \frac{49}{8}\right]$$

$$= \frac{\text{H.C.F}\,(7, 14, 21, 35, 49)}{\text{L.C.M.}\,[3, 5, 6, 8]} = \frac{7}{120}\ \text{kg}$$

∴ Total number of pieces

$$= \frac{\frac{7}{3} + \frac{14}{3} + \frac{21}{5} + \frac{35}{6} + \frac{49}{8}}{\frac{7}{120}}$$

$$= \frac{280 + 560 + 504 + 700 + 735}{7} = 397.$$

**45.** $4^{6n-6} - 6^{4n-4} = (4^3)^{2n-2} - (6^2)^{2n-2} = (64)^{2n-2} - (36)^{2n-2}$

$a^n - b^n$ is divisible by $a - b$ for all values of $n$ and by $a + b$ only when $n$ is even.

$(64)^{2n-2} - (36)^{2n-2}$ is divisible by 64 +36, i.e., 100 since $2n - 2$ is even.

$$\text{Rem}\left(\frac{4^{2n-2} - 6^{4n-4} - 3}{100}\right) = \text{Rem}\left(\frac{-3}{100}\right) = -3.$$

The equivalent positive remainder is 97.

**46.** $N$ = 46 47 48 … 99 100 101 102 103 104 105

The remainder of a block of two or more consecutive positive integers divided by 9 is the same as the remainder of the sum of those positive integers divided by 9.

The remainder when $N$ is divided by 9 is the same as that of 46 + 47 + … +105 divided by 9.

46 + 47 + … + 105 = 1 + 2 + … + 105 – (1 + 2 + … + 45)

$$= \frac{(105)(106)}{2} - \frac{(45)(46)}{2}$$

= (105) (53) – (45) (23)

(105) (53) = (99 + 6) (53) = (99) (53) + 318

The remainder of (105) (53) divided by 9 is the same as that of 318 divided by 9, which is 3.

(45) (23) is divisible by 9.

(105) (53) – (45) (23) leaves a remainder of 3 when divided by 9.

The remainder of $N$ divided by 9 is 3.

**Alternate Solution:**

$N$ = 4647 ….. 105

Let M = 46 + 47 + … + 105

M = 46 + 47 + … + 54

+55 + 56 + … + 63

. . . .

+ 91 + 92 + … + 99 +

(100 + 101 + 102 + 103 + 104 + 105)

$$\text{Rem}\ \frac{N}{9} = \text{Rem}\ \frac{M}{9} = \text{Rem}\ \frac{100 + 101 + .... + 105}{9}$$

$$\text{Rem}\ \frac{1 + 2 + ... + 6}{9} = \text{Rem}\ \frac{21}{9} = 3$$

**Note:** Let $S_n$ be the sum of $n$ consecutive integers. If $n$ is odd, Rem $\frac{S_n}{n} = 0$. In this problem $n = 9$

(Also, If $n$ is even, Rem $\frac{S_n}{n} = \frac{n}{2}$ )

**47.** $$\frac{32^{101}}{15} = \frac{(30 + 2)^{101}}{15} = \frac{2^{101}}{15}$$

$$\Rightarrow \frac{2^{101}}{2^4 - 1} = \frac{\left(2^4\right)^{25} \times 2^1}{2^4 - 1} = (1)^{25} \times 2^1 = 2$$

**48.** $P(a, b, c) = 27 - 9\,(a + b + c) + 3(ab + bc + ca) - abc$

$= (3 - a)\,(3 - b)\,(3 - c)$

$$\therefore\ P\left(\frac{3}{2}, \frac{3}{2}, 1\right) = \left(3 - \frac{3}{2}\right)\left(3 - \frac{3}{2}\right)(3 - 1) = 2\left(\frac{3}{2}\right)^2 = \frac{9}{2} = 4.5$$

**49.** $p^2 - q^2 = 238 \Rightarrow (p + q)\,(p - q) = 238$

Given that $p$ and $q$ are positive integers, so $p + q$ and $p - q$ will both be even or both be odd. As 238 is even, both $p + q$ and $p - q$ should be even. However, if both are even, $p^2 - q^2$ should be divisible by 4 but 238 is not divisible by

4. So, no solution is possible.

**50.** $4!\ 5! = 24\ (120) = 2^3(3)\ 2^3(3)\ (5) = 2^6\ 3^2\ 5^1$

The number of factors is $7(3)(2) = 42$

The product of all these factors is $[24(120)]^{21} = (2880)^{21}$

**51.** Consider the divisor 730 instead of 73.

$$\text{Let } R = \text{Rem}\left(\frac{3^{1000}}{730}\right) = \text{Rem}\left[\frac{3^4(3^6)^{166}}{730}\right]$$

$$= \text{Rem}\left(\frac{3^4}{730}\right)\text{Rem}\left(\frac{729^{166}}{729-(-1)}\right) = 34 \times (-1)^{166} = 81$$

Since 73 is a factor of 730

$$\text{Rem}\left(\frac{3^{1000}}{73}\right) = \text{Rem}\left[\frac{R}{73}\right] = \text{Rem}\left(\frac{81}{73}\right) = 8.$$

**52.** $N = dq + 5$

$N = 3dq_1 + 25$

$dq + 5 = 3dq_1 + 25$

$dq - 3dq_1 = 20$

$d(q - 3q_1) = 20$

$d = 20$ (or) $10$ ($d$ is a factor of 20. Also $d > 5$)

**53.** In the given number $(2^3)(3^8)(4^7)(6^5)(7^4)(8^3)(9^2)(10^6)$ there are 6 zeroes at the end. When all these zeroes are discarded, the units digit of the remaining number is simply the units digit of $(2^3)(3^8)(4^7)(6^5)(7^4)(8^3)(9^2)$ which is the units digit of $(8)(1)(4)(6)(1)(2)(1)$ which is 4.

**54.** The divisor is $143 = 11(13)$.

It is more convenient to think of $7(11)(13) = 1001$.

$N = 123123123\ldots\ldots\ldots\ldots\ldots 1 = 10M + 1$

Where M = 1230 ..... 0 (996 zeros) + 1230 ..... 0 (993 zeros) + ..... + 123 (0 zeros)

$= 123(1000^{332} + 1000^{331} + 1000^1 + 1000^0)$

Rem $M/143$ can be obtained from Rem $\frac{M}{1001}$, which is

$\text{Rem}(123)[(-1)^{332} + (-1)^{331} + \ldots.(-1)^2 + (-1) + 1] = 123$

$(\therefore \text{Rem}\ \frac{M}{143}$ is also 123 )

$$\therefore \text{Rem}\ \frac{N}{143} = \text{Rem}\ \frac{1231}{143} = \text{Rem}\ \frac{230}{143} = 87$$

**55.** $n^7 - n = n(n^6 - 1) = n(n^3 - 1)\ (n^3 + 1)$

$= n(n-1)\ (n^2 + n + 1)\ (n+1)\ (n^2 - n + 1)$

When $n = 1$, $n^7 - n = 0$ is divisible by all numbers.

When $n = 2$, $n^2 + n + 1 = 7$

$\Rightarrow n^7 - n$ is divisible by 7

When $n = 3$, $n^2 - n + 1 = 7 \Rightarrow n^7 - n$ is divisible by 7.

Similarly for $n = 4$, 5, 6, and 7, $n^7 - n$ is divisible by 7.

From $n = 8$ onwards the same pattern repeats.

$\therefore n^7 - n$ is always divisible by 7.

**56.** $58! + 59! + 60! + \ldots..90!$

$= 58!(1 + 59 + (59)(60) + (59)(60)(61) + \ldots\ldots)$

$= 58!(60 + (59)(60) + (59)(60)(61) + \ldots\ldots)$

$= 58!(60)(1 + 59 + (59)(61) + (59)(61)(62) + (59)(61)(62)(63)\ldots.)$

$= 58!\ (60)(60 + (59)(61)(1 + 62) + (59)(61)(62)(63)\ldots.)$

$= 58!\ (60)\ (60 + (59)(61)(63) + (59)(61)(62)(63)\ldots.)$

$= 58!(60)(3(20) + 9k_1)$

$= 58!\ (60)(3)[20 + 3k_1]$

The IGP of 3 in 58! Is 19 + 6 + 2, i.e., 27 and 60, 3 contribute a 3 each.

IGP of 3 in 58! $(60)(3)\ (20 + 3k_1) = 27 + 1 + 1 = 29$.

**Alternate Solution:**

The following results about the IGPs in the product and sum of two or more terms are useful.

(1) If the IGPs of a prime number $p$ in the $n$ numbers $x_1, x_2, x_3, \ldots X_n$ are $m_1, m_2, m_3 \ldots m_n$, respectively, then the IGP of $p$ in the product $x_1\ x_2\ x_3 \ldots x_n$ is $m_1 + m_2 + m_3 + \ldots + m_n$.

(2) If the IGP of $p$(not necessary a prime) in A and B are $a$ and $b$, respectively, then

(i) if $a \neq b$, the IGP of $p$ in $A + B$ is the smaller of $a$ and $b$.

(ii) if $a = b$, the IGP of $p$ in $A + B$ could be $a$ or more.

(iii) Consider the sum of several numbers, say $S = T_1 + T_2 + \ldots + T_n$. Let the IGPs of $p$ in the terms be $m_1, m_2, m_3, \ldots, m_n$ not necessarily, respectively.

This process has to be repeated as often as necessary until we get a single term, in which the IGP is the minimum. This minimum value is the IGP for the entire sum $S$. The proof of each of the 3 statements above can be worked out by the student.

We apply these to the given problem.

$S = 58! + 59! + \ldots + 90!$

The IGP of 3 in the first 6 terms are worked out by successive division (for 58! it is 19 + 6 + 2, viz., 27). These are tabulated below.

| 58! | 59! | 60! | 61! | 62! | 63! |
|---|---|---|---|---|---|
| 27 | 27 | 28 | 28 | 28 | 30 |

We consider the first two terms. The sum is 58! (60). The IGP of 3 in this is 27 + 1, viz., 28. (from statement 1 above)

We need to consider the sum $S_1 = 58! + \ldots + 62!$

If the IGP of 3 in $S_1$ is $m$ and $m$ is less than 30, the IGP of 3 in $S$ is also $m$. If $m$ turns out to be 30, we would need to combine more terms (go up to 66!)

$S_1 = 58!\,[1 + 59 + 59(60) + 59(60)(61)(62)] = 58!\,T$ (say)

We have to work out the IGP of 3 in $T$. If it is 1 or 2 the IGP of 3 in $S_1$ would be 27 + 1 or 27 + 2, respectively. If it turns out to be 3, we would need to combine more terms. We can focus on the Rem $\frac{T}{27}$ (If we consider Rem $\frac{T}{9}$, we would conclude that Rem $\frac{T}{9} = 0$, T could be $27k$, $27k + 9$ or $27k+18$.)

Now, Rem $\frac{T}{27}$ = Rem $[1 + 5\ 5(6) + 5(6)(7) + 5(6)(7)(8)] / 27$

= Rem $[36 + 3(7) + 30(56)] / 27$

= Rem $[9 + 3(7) + 3(2)] / 27 = 9$

$\therefore S_1 = 58!\,(27k + 9)$

The IGP of 3 in $S_1$ is 29.

**57.** The number is of the form $7a + 4$, $19b + 8$, $39c + 12$.

So, $7a + 4 = 19b + 8$

$\Rightarrow a = \frac{19b + 4}{7}$, By trial, setting $b = 2$ gives an integral value for $a$, viz. $a = 6$.

$\therefore$ The number is of the form $133p + 46$

Now, $133p + 46 = 39c + 12$

$133p + 34 = 39c$

$c = \frac{133p + 34}{39} = 3p + \frac{16p + 34}{39}$

Now, $\frac{16p + 34}{39}$ should be an integer.

By trial, we get $p = 32$

So, the

Answer is 4302.

**Alternate Method:**

Trying from

Choices, 4305 – 4 as well as 4263–4 are not divisible by 7. Hence, they can be eliminated. Now, 4302 – 8 is divisible by 19 and 4302 – 12 is divisible by 39. Hence, 4302 is the least number.

**58.** Since $187 = 11 \times 17$, (co-prime)

We first find the remainders for $N = 5^{80}$ divided by 11 and by 17 separately.

Rem $5^1/11 = 5$

$5^2/11 = 3$

$5^3/11 = 3 \times 5 = 15 = 4$

$5^4/11 = 4 \times 5 = 20 = 9$

$5^5/11 = 9 \times 5 = 45 = 1$

$\text{Rem}\left(\frac{5^{80}}{11}\right) = \text{Rem}\frac{(5^5)^{16}}{11} = 1.$

Rem $5^1/17 = 5$

$5^2/17 = 8$

$5^3/17 = 6$

$5^4/17 = 6 \times 5 = 30 = 13$

$5^5/17 = 13 \times 5 = 65 = 14$

$5^6/17 = 14 \times 5 = 70 = 2$

$5^7/17 = 2 \times 5 = 10$

$5^8/17 = 10 \times 5 = 50 = (-1)$

$\text{Rem}\frac{5^{80}}{17} \equiv \text{Rem}\frac{\left(5^8\right)^{10}}{17}$

As the power is even, the remainder will be +1.

Hence, $N/11 = 1$ which is a L.C.M. model 1 question

$N/17 = 1$

$\therefore N = k.$ L.C.M. $(11, 17) + 1 = 187k + 1$

Clearly Remainder $\left(\frac{N}{187}\right) = 1.$

**59.** H.C.F. of $3^p\,5^{q+4}\,7^7\,11^5$ and $3^{q+5}\,5^{p+4}\,7^x\,11^x$ is $3^x\,5^x\,7^x\,11^x$.
We can say $x \le 5$ (1)

Consider the indices of 3 and 5. If between $p$ and $q + 5$,

$q + 5$ is the one which is not greater, then between $p + 4$ and $q + 4$, $q + 4$ would be the one which is definitely smaller. As the H.C.F. = $3^x 5^x 7^x 11^x$, $q + 5 = x$ and $q + 4 = x$.

This is not possible.

$\therefore$ Between $p$ and $q + 5$, $p$ is the one which is not greater, i.e., $p = x$. Between $q + 4$ and $p + 4$, $p + 4$ cannot be the index of 5 in the H.C.F.

($\because$ that index is $x$ and $x = p$).

$\therefore$ $q + 4 = x$. As $q \geq 1$, it follows that $x = q + 4 \geq 5$ (2)

(1), (2) $\Rightarrow x = 5$

$\therefore$ $p = 5$.

($\because$ $x = \min(5, x)$).

$p, q \geq 1$.

$\therefore$ $p + 4, q + 4 \geq 5$.

$x = \min(p + 4, q + 4) \Rightarrow x \geq 5$ (2)

From (1) and (2), $x = 5$.

Also $x = \min(p, q + 5)$ and $q + 5 \geq 6$.

$\therefore$ Only possibility is $p = 5$.

**60.** $484^{484} = (22^2)^{484} = 22^{968} = (23 - 1)^{968}$ and $529 = 23^2$

$(23 - 1)^{968} = 23^{968} + {}^{968}C_1(23)^{967}(-1) + {}^{968}C_2(23)^{966}(-1)^2 +$

${}^{968}C_3(23)^{965}(-1)^3 + \ldots\ldots + {}^{968}C_{966}(23)^2(-1)^{966}$

$+ {}^{968}C_{967}(23)(-1)^{967} + (-1)^{968}$.

In the expansion above, all the terms except the last two are divisible by 529.

$$\text{Remainder} = \text{Rem}\left(\frac{{}^{968}C_{967}(23)(-1)^{967} + (-1)^{968}}{529}\right)$$

$$= \text{Rem}\left(\frac{-(968)(23)+1}{529}\right) = -22263$$

Now, $22263 = 529(42) + 45$.

$-22263$ is equivalent to $-45$ or $529 - 45 = 484$.

**61.** Prime factors of 35 = 5 (7)

$85 = 5 \times 17$

L.C.M. = 7735 = 5 (7) (17) (13)

Since H.C.F. = 5, Let $k = 5a$

L.C.M. $(35, 85, 5a) = 5$ L.C.M. $(7, 17, a) = 5$ (7) (17) (13)

$\Rightarrow a = 13$

$\therefore$ The least possible value of $k = 5 \times 13 = 65$.

**62.** The required number of saplings is a perfect square and a multiple of L.C.M. (15, 18, 24). Required number

$= 360 \times 10 = 3600$.

**63.** Let $N = dq + 7$

$N = 2dq_1 + 47$

$dq + 7 = 2\ dq_1 + 47$

$dq - 2\ dq_1 = 40$

$d(q - 2\ q_{1)} = 40$

$d = 40$

Other factors of 40 are not possible. Because $d = 20$, $2d$ will be 40, then, you cannot have a remainder which is greater than 40. Hence, other factors are not possible.

**64.** To obtain the remainder when a long number is divided by 999, we must group the digits 3 at a time.

There are 300 blocks of three digits. Of these

75 are the group 535.

75 are the group 753.

75 are the group 575.

and 75 are the group 357.

The sum of these 300 three digit numbers = 75 (2220) = 166500.

$$\text{Rem}\frac{N}{999} = \text{Rem}\frac{166500}{999} = \text{Rem}\frac{166+500}{999} = 666$$

**65.** Let the 600-digit number be $N$. The first positive integer occurs once in $N$, the second occurs twice, the third thrice and so on.

The number has 600 digits.

$\therefore$ The last (positive) integer occurring in $N$ is a two-digit number or part of a two-digit number.

Let us work out how many digits we would get if we go up to 25.

Total number of digits in the number up to 99 … 9 (nine times) = 1 + 2 + … 9 = 45.

Total number of digits in the number from 1010 … 10 (ten times) to 2525 … 25 (twenty five times) = 2(10 + 11 + … 25) = 560.

Total number of digits would be 605. We need to leave out the last 5 digits.

The number = (1223334444…) (2525…)2.

There are twenty two 25's in the second bracket above.

The last four digits of the number are 5252.

$\therefore$ Remainder of the number divided by 16 is equal to that of 5252 divided by 16, i.e., 4.

**66.** Let the two numbers be $16x$ and $16y$.

$x, y$ are coprimes. $16x + 16y = 144$

$\therefore x + y = 9$

So, the unordered pairs are (8, 1), (7, 2), (6, 3) and (5, 4)

But (6, 3) has to be rejected as $x, y$ have to be coprimes. So, there are 3 pairs.

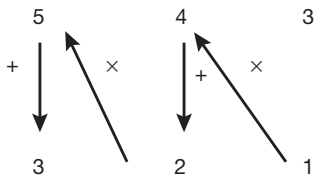
**67.** $81^{82} = (17 \times 4 + 13)^{82} = 13^{82} \mid 17$

As 17 is prime and coprime to 13, we can use Fermat's theorem.

$\therefore 13^{16} \mid 17 = 1$

$(13^{16})^5 \cdot 13^2 \mid 17 = 1.169 = \dfrac{169}{17} = 16$

**68.**




Divisors remainders $N = 5(4(1) + 2) + 3 = (5 \times 6) + 3 = 33$.

Hence, the number is of the form (5.4.3) $k + 33 = 60k + 33$

The smallest four digit number satisfying the condition is $60(17) + 33 = 1053$.

The greatest four-digit number satisfying this condition is $60(166) + 33$, i.e., 9993.

Hence, a total of 150 numbers are there.

**69.** We have to compare $\lceil x \rceil + \lceil y \rceil + \lceil x + y \rceil$ with $\lceil 2x \rceil + \lceil 2y \rceil$

If the fractional part of $x$ is less than 1/2, $\lceil 2x \rceil = 2\lceil x \rceil + 1$

If the fractional part of $x$ is equal to or greater than 1/2 $\lceil 2x \rceil = 2\lceil x \rceil + 2$.

Let $a$ be the greatest integer less than or equal to $x$ and $b$ be the greatest integer less than or equal to $y$.

Let $f$ denote a fraction less than 1/2 and $g$ a proper fraction equal to or greater than 1/2. We tabulate the possibilities for $x, y$, A = $\lceil 2x \rceil + \lceil 2y \rceil$ and B = $\lceil x \rceil + \lceil y \rceil + \lceil x + y \rceil$.

| A | B | $\lceil 2x \rceil + \lceil 2y \rceil$ | $\lceil x \rceil + \lceil y \rceil + \lceil x + y \rceil$ |
|---|---|---|---|
| $a + f$ | $b + f$ | $(2a + 1) + (2b + 1)$ | $(a + 1) + (b + 1) + (a + b + 1)$ |
| $a + f$ | $b + g$ | $(2a + 1) + (2b + 2)$ | $(a + 1) + (b + 1) + (a + b + 1)$ or $(a + b + 2)$ |
| $a + g$ | $b + f$ | $(2a + 2) + (2b + 1)$ | $(a + 1) + (b + 1) + (a + b + 1)$ or $(a + b + 2)$ |
| $a + g$ | $b + g$ | $(2a + 2) + (2b + 2)$ | $(a + 1) + (b + 1) + (a + b + 2)$ |

We see that B can be greater than A or equal to but not less than A.

**70.** Here, $24 = 2^3\, 3$

$$\text{Rem}\left(\frac{2^{54}}{24}\right) = 2^3\ \text{Rem}\left(\frac{2^{51}}{3}\right)$$

$$= 8\left(\text{Rem}\ \frac{(-1)^{51}}{3}\right) = 8(2) = 16.$$

**71.**

| Dd | D | Q | R |
|---|---|---|---|
| 125 | 5 | $25k$ | 0 |
| $25k$ | 5 | $5k$ | 0 |
| $5k$ | 5 | $k$ | 0 |
| | | $31k$ | |

If $k = 1$, $31k = 31$;

The minimum value of $n = 125$.

'$n$' can take the values – 125, 126, 127, 128, 129. In all these cases the index of the greatest power of 11 is 12.

**72.** As each digit of $S$ is even, the first two digits must be 2 each. Their sum is 4. The other two digits, being even, must have an even sum. As the sum of the digits of $S$ is divisible by 3, the sum of the last two digits can be 2 or 8 or 14. (The last two digits are even, each at most must be 8.

$\therefore$ Their sum at most can be 16).

If the sum of the last two digits is 2, the third digit must be 2 and the fourth digit must be 0 or vice versa.

$x$ has 2 possibilities. If the sum of the last two digits is 8, (third digit, fourth digit can be (0, 8), (2, 6), (4, 4), (6, 2) or (8, 0)

$\therefore$ $x$ has five possibilities.

If the sum of the last two digits is 14, the third digit can be 6 and the fourth digit can be 8 or vice versa.

$\therefore$ $x$ has 2 possibilities.

$\therefore$ $x$ has a total of 9 possibilities.

**73.** $\dfrac{37^{157}}{79}$

Using Fermat's Theorem, if $p$ is a prime such that $a$, $p$ are co-primes, then $a^{p-1}$ leaves a remainder of 1 when divided by $p$.

$$\frac{37^{157}}{79} = \frac{37^1.37^{156}}{79} = \frac{37^1.\left(37^{78}\right)^2}{79} = 37(1)^2 = 37.$$

**74.** Let $N = p_1^{\,p} \cdot p_2^{\,q} \cdot p_3^{\,r} \ldots$ Given $(p+1)(q+1)(r+1) = 4$

$= (1)(4) = (1)\ (2)\ (2)$.

Exactly one of $p, q, r \ldots$ must be 3 and the others must all be 0 or exactly two of $p, q, r \ldots$ must be 1 and the others must all be 0.

$N$ is the cube of a prime number or the product of two distinct primes. In the first case, $N^2$ is the sixth power of a prime number ($p^6$/(say)). In the second case, $N^2$ is the product of the squares of two distinct primes ($p_1^{\,2}p_2^{\,2}$(say)).

If $N^2$ is $p^6$, it has seven factors.

If $N^2$ is $p_1^{\,2}p_2^{\,2}$, it has nine factors.

$N^2$ has seven or nine factors.

**75.** The L.C.M. of all the factors of 576 is 576.

$576 = [2^3\ (3)]^2 = 2^6(3^2)$.

The number of factors of 576 is (6+1) (2+1), i.e., 21.

There are 21 distinct factors of 576. If we take 22 or more natural numbers whose L.C.M. is 576, each of these numbers must be a factor of 576. $\therefore$ At least two of these numbers would be equal.

The maximum value of $N$ is 21.

Note: In general, the maximum number of natural numbers whose L.C.M. is $N$ is the number of factors of $N$.

**76.** The amount deposited in the bank

= L.C.M. of [633, 1055, 1477] – $k$ where $k$ = (633 – 600)

= (1055 – 1022) = 1477 – 1444 = 33

$\therefore$ The required amount = 22155 – 33= 22122

**77.** The employee whose number is $x$ donated ₹1000 to all the orphans whose number is a multiple of $x$.

$\therefore$ The orphan whose number is $y$ received donations from all the employees whose number is a factor of $y$, i.e., received as many donations as the number of factors of $y$.

For any number $N$ which has one of its factors equal to $a$, $\dfrac{N}{a}$ is (also) one of its factors. If $a \neq \dfrac{N}{a}$, i.e., $N \neq a^2$ then all the factors of the number would occur in pairs.

Hence, the number would have an even number of factors. Otherwise, except $\sqrt{N}$ all the factors of the numbers would occur in pairs. Hence, the number would have an odd number of factors.

For any number $N$ to have an odd number of factors, it must be a perfect square. The orphan whose number is $y$ would receive an odd number of donations only when $y$ is a perfect square.

The orphans who received donations from an odd number of employees were those whose numbers are perfect squares up to 80. There are eight such orphans.

**78.** For odd values of $n$, $a^n + b^n$ is divisible by $a + b$; since each of 70, 96 and 160 can be written as the products of two numbers, where one is odd and the other is even none of them is prime. $2^{70} + 1 = (2^{14})^5 + 1^5 \therefore$ It is divisible by $2^{14} + 1$.

Please note that $2^2 + 1$, $2^4 + 1$, $2^8 + 1$, $2^{16} + 1$, etc. are all prime numbers of the form $(2^{2^n} + 1)$.

**79.** Required divisor = 238 + 342 – 156 = 424

**80.** $\dfrac{(35)^{128}}{50} = \dfrac{(1225)^{64}}{50} = \dfrac{(25)^{64}\,(49)^{64}}{50}$

The remainder is same as that of the division $\dfrac{(25)^{64}}{50}$ ($\because 49^{64}$ or $(-1)^{64}$ leaves a remainder of 1), which is 25. Alternatively, any power of a number ending with 5 has to end with 25 or 75. In either case the remainder when divided by 50 is 25.

**81.** Since the lights are switched on at intervals of 54 sec, 48 sec and 36 sec, all the three lights will be switched on simultaneously at intervals of (L.C.M. (54, 48, 36), i.e., 432 sec.

The number of times they are switched on together between 6:00 am and 7:00 am

= quotient of $\left(\dfrac{7200}{432}\right)$ – quotient of $\left(\dfrac{3600}{432}\right)$

= 16 – 8 = 8

Note: In this case, the 3 seconds after which the lights are switched off is redundant.

**82.** Multiplying the dividend and the divisor by 3, the given division is equivalent to $\frac{3(2^{73})}{63} = \frac{3(2^{73})}{2^6-1} = \frac{(6).(2^6)^{12}}{2^6-1}$.

Let $2^6 = x$. When $6x^{12}$ is divided by $x-1$, the remainder is 6.

$\therefore$ 6 is the remainder for equivalent division. This is 3 times the actual remainder for the given division the remainder is 2.

**83.** Let $N$ be an odd natural number less than 90. Let the two natural numbers whose difference of the squares equals $N$ be $x$ and $y$ where $x > y$. $N = x^2 - y^2 = (x+y)(x-y)$.

If $N$ is prime, then $x + y = N$ and $x - y = 1$ ($\because$ $x > y$. both $x + y$ and $x - y$ must be positive).

Each odd prime can be expressed as a difference of squares of two natural numbers in only one way.

If $N$ is composite, then it must be the square of a prime number in order to be expressed as a difference of squares in only one way.

There are 24 odd primes upto 100 and hence, 23 upto 90.

$N$ can be any of the 23 odd primes upto 90 or $3^2$, $5^2$, $7^2$.

**84.** Given number is 324516324516324516 …. up to 600 digits, and the divisor is 1001.

As $1001 = 10^3 + 1$, divide 324516324516 …. up to 600 digits into 3 digits each and use Remainder theorem to get the remainder,

$324(1000^{199}) + \ldots 324(1000^3) + 516(1000^2) + 324(1000^1) + 516(1000^0)$, i.e., $324(-1)^{199}$ …. $516(-1)^2 + 324(-1) + 516$

= (516 – 324) × 100

= 19200

Dividing 19200 by 1001, the remainder is 181.

**85.** The index of the greatest power of 7 in 48! is 6.

The index of the greatest power of 7 in 49! is 8.

'$n$' does not take any value.

Note: The index of the greatest power of a prime number '$p$' in $n!$ can never be '$p$'.

**86.** $2^{2016} = 2\,(2^{2015})$

The L.C.M. of $ka, kb, kc, \ldots$ Is $k$ times the L.C.M. of $a, b, c \ldots$.

L.C.M. of $2^{2016}$, $2^{2015} - 1$, $(2^{2015} - 2)$ is twice the L.C.M. of $2^{2015}$, $2^{2015} - 1$, $2^{2015} - 2$.

$2^{2015}$ is divisible by 4 while $2^{2015} - 2$ is not (but it is even).

$\therefore$ H.C.F. of $2^{2015}$ and $2^{2015} - 2$ is 2.

$$\text{L.C.M.}\,(2^{2015}, 2^{2015}-2) = \frac{2^{2015}\left(2^{2015}-2\right)}{\text{HCF}\left(2^{2015}, 2^{2015}-2\right)}$$

$$= \frac{2^{2015}\left(2^{2015}-2\right)}{2} = 2^{2014}\,(2^{2015}-2).$$

$2^{2015} - 1$ and $2^{2015} - 2$ are consecutive integers. Hence, they are coprime.

Any divisor of $2^{2015} - 1$ and $2^{2014}$ is also a divisor of $2^{2015} - 1$ and $2^{2015}$ and hence, is a divisor of $2^{2015} - (2^{2015} - 1)$, i. e., of 1. $\therefore 2^{2015} - 1$, $2^{2014}$ are coprime.

L.C.M. $(2^{2015} - 1, 2^{2014}\,(2^{2015} - 2)) = (2^{2015} - 1)\,(2^{2014}\,(2^{2015} - 2))$

L.C.M. of the given numbers = $2\,[(2^{2015} - 1)(2^{2014}\,(2^{2015} - 2))]$

= $2^{2015}\,(2^{2015} - 1)\,(2^{2015} - 2)$

**Alternate Solution:**

We can avoid long exponential expressions by using symbols. Let $^{22016}$ be $X$. The 3 numbers are

$X - 4$, $X - 2$, $X$

All 3 have a common factor, which is 2. Let $X = 2Y$ ie $Y = 2^{2015}$. Dividing each number by 2, we get $Y - 2$, $Y - 1$, $Y$.

As $Y$, $Y - 2$ have a common factor of 2, their L.C.M. is $\frac{(Y-2)Y}{2}$.

L.C.M. $[Y - 2, Y - 1, Y] = (Y - 2)(Y - 1)\,Y/2$,

and hence, L.C.M. $[X - 4, X - 2, X] = (Y - 2)\,(Y - 1)\,Y = (2^{2015} - 2)(2^{2015} - 1)2^{2015}$

**87.** The number of marbles Ravi had = $K \times$ [L.C.M. (8, 12, 18)] + 5, where $K$ is any natural number= $72K + 5$.

Since the number of marbles he had is less than 200, $72K + 5 < 200 \Rightarrow K = 1$ or 2.

The number of marbles Ravi had could be 77 or 149.

**88.** A number being coprime to 200 means that it should not be divisible by any number which is a factor of 200. Also, $200 = 25 \times 8 = 5^2 \times 2^3$. So, the prime factors are 5 and 2. So, if a number is co-prime to 200, it simply means that it is neither divisible by 5 nor 2. As with the previous question, we first find out the number of integers divisible by 5 or 2 and then subtract them from the total numbers.

What do we understand from 'in the range'? does it mean 250 and 750 are included or they are not? It is the former.

So, if we do a

$$N/2 = \left[\frac{750}{2}\right] - \left[\frac{250}{2}\right] = 375 - 125 = 250.$$

We will have removed 250 from the selection and we will need to add back $a$ +1. So,

$N/2 = 250 + 1 = 251$ {The +1 is applicable only when the lower number in the range is perfectly divisible by the divisor}

$$N/5 = \left[\frac{750}{5}\right] - \left[\frac{250}{5}\right] + 1$$

$= 150 - 50 + 1$

$= 100 + 1$ {since 250 is divisible by 5}

$$N/(2\&5) = \left[\frac{150}{2}\right] - \left[\frac{50}{2}\right] + 1 \text{ (since 50 is divisible by 2).}$$

$= 75 - 25 + 1 = 51$

So, $N/(2 \text{ or } 5) = [N/2] + [N/5] - [N/(2\&5)]$

$= 251 + 101 - 51 = 301.$

Now, numbers that are co-prime with 200

= Total number of numbers – Nos. divisible by 2 or 5.

$= 501 - 301 = 200$

**89.** Index of the greatest power of 30, i.e., (2) (3) (5) in any factorial is powers of 2, 3, 5 in that factorial. This is the equal to the index of the greatest power of 5 in that factorial.

| | |
|---|---|
| 5 | 150 |
| 5 | 30 |
| 5 | 6 |
| | 1 |

Hence, the index of the greatest power of 5 in 150! = 30 + 6 + 1 = 37

**90.** I: $67^{60} = (65 + 2)^{60}$

$(65 + 2)^{60} = (65 + 2)\ (65 + 2) \ldots\ldots$ 60 times $= 65m + 2^{60}$

$2^{60} = (2^6)^{10} = 64^{10} = (65 - 1)^{10} = 65m_1 + (-1)^{10} = 65m_1 + 1$

$67^{60} = 65(m + m_1) + 1 = 13(5)\ (m + m_1) + 1$

The remainder of $67^{60}$ divided by 13 is 1.

780 is divisible by 13.

The remainder of $N$ divided by 13 is 1.

I is true

II: $67^{60} = (66 + 1)^{60}$

$(66 + 1)^{60} = (66 + 1)\ (66 + 1)$ .....60 times $= 66M + 1^{60}$
$= 66m + 1$

The remainder of $67^{60}$ divided by 11 is 1.

The remainder of 780 divided by 11 is 10.

$67^{60}$ is 1 more than a multiple of 11 and 780 is 10 less than a multiple of 11.

$N$ is 9 less than a multiple of 11.

Any positive integer which is 9 less than a multiple of 11 leaves a remainder of 2 when divided by 11. $N$ leaves a remainder of 2 when divided by 11.

II is false.

**91.** $M = 3(3!) + 4(4!) + \ldots\ldots + 15(15!)$

$= (4 - 1)3! + (5 - 1)4! + \ldots\ldots + (16 - 1)15!$

$= (4! + 5! + \ldots\ldots + 16!) - (3! + 4! + \ldots. + 15!)$

$= 16! - 3!$

$= 16! - 6$

So, $M - 15 = 16! - 21$

When the number is divided by 14! – 2, we can substitute 14! with 2 (Remainder Theorem)

Rem = $15 \times 16 \times 2 - 21 = 480 - 21 = 459$

**92.** Index of the highest power of 7 in $\frac{1000!}{500!}$ = Index of the highest power of 7 in 1000! – Index of the highest power of 7 in 500!

Index of the highest power of 7 in $N!$ (say I) is obtained as follows:

| | | | |
|---|---|---|---|
| Number of multiples of 7 in $N!$ (say $a$) | Number of multiples of $7^2$ in $N!$ (say $b$) | | I |
| Quotient of $\frac{N}{7}$ | Quotient of $\frac{a}{7}$ | ..... | $a + b + \ldots$ |

In 1000!, there are 142 multiples of 7, 20 multiples of $7^2$, 2 multiples of $7^2$, no multiple of $7^4$ or a higher power of 7. Index of the highest power of 7 in 1000! = 142 + 20 + 2 = 164

Index of the highest power of 7 in 500! = 82

Index of the highest power of 7 in $\frac{1000!}{500!}$ = 164 – 82 = 82.

**93.** Number of zeros in any factorial is equal to the number of 5*s* in that factorial.

Using successive division, the number of 5's in 1000!

= 200 + 40 + 8 + 1 = 249

**94.** $n$ must be at least 20.

If $n \geq 25$, index of the greatest power of 5 in $n!$ is 6. $n$ can be 20, 21, 22, 23 or 24. $\therefore n$ can take 5 values.

**95.** 1. AB*C*86: The second last digit of a perfect square having last digit 6 should be odd. Hence, it cannot be a perfect square.

2. 1*X*2*Y*75: The square of a number ending in 5 ends with 25. Hence, it cannot be a perfect square.

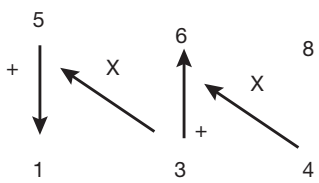
3. *AB*56: $66^2 = 4356$; Not necessarily true. Hence, it can be a perfect square.

4. *B*36: None of the three digit perfect squares end with 36. Hence, it cannot be a perfect square.

5. A*B*9: As 169, 529, 729 end with 9 and are perfect squares, hence, *AB*9 can be a perfect square.

6. *A*6*B*: As169 has the middle digit as 6 and is a perfect square, *A*6*B* can be a perfect square.

**96.**




Number $(N) = (((4 \times 6) + 3)\ 5 + 1) + 5.6.8k = 136 + 240k$

If $k$ is odd, then Rem $\left(\frac{N}{96}\right) = 88$.

If $k$ is even, then Rem $\left(\frac{N}{96}\right) = 40$.

Remainder is 88 or 40.

**97.** Given number is 459459459…. up to 900 digits and divisor is 999.

As $999 = 10^3 - 1$ divide the given dividend into three digits each and use Remainder theorem to get the remainder.

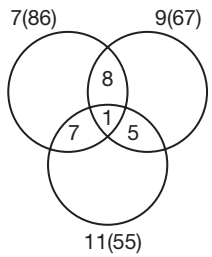
$459(1000)^{299} \ldots + 459(1000^1) + 459(1000^0)$

$= 459(1000)^{299} \ldots 459(1) + 459(1)$

$= 459 \times 300$, i.e., $137{,}700 \equiv 137(1) + 700 = 837$

Dividing 837 by 999, the remainder is 837.

**98.**




There are 86 multiples of 7,67 multiples of 9, total 55 multiples of 11 from 150 to 750. Of these, 9 are multiples of both 7 and 9 (i.e., L.C.M. (7, 9) = 63)

$\therefore$Number of multiples of either 7 or 9 = 86 +67 – 9 = $N$ (Say). Of the multiples of either 7 or 9, 7 are multiple of 7 and 11 but not 9 and 5 are multiples of 9 and 11 but not 7. There are 12 numbers which are multiples 9 and of 11 and exactly one of 7 and 9.

Number of integers divisible by 7 or 9 but not 11 = Number of multiples of either 7 or 9 – 12 = $N$=12 = 131.

**99.** Let $k$ be the last quotient.

Then $N = 5\{6(8k + 1) + 2\} + 4 = 240k + 44$

Since $N$ is a four-digit number, $240k + 44 < 9999$

When $k = 40$, $240k + 44 = 9644$

When $k = 41$, $240k = 44 = 9644 + 240 = 9884$

When $k = 42$, $240\ k + 44 = 9884 + 240$ 10000 which is not permitted. $\therefore k \leq 41$

$\therefore$ Maximum $N$ occurs when $k = 41$ and

$\therefore N = 9884$

**100.** Let the four prime numbers be $a$, $b$, $c$ and $d$.

Given $a \times b \times c = 2431$ and $b \times c \times d = 4199$

$$\therefore \frac{a \times b \times c}{b \times c \times d} = \frac{2431}{4199} \Rightarrow \frac{a}{d} = \frac{11}{19} \quad \therefore d = 19$$

## EXERCISE-4

**1.** $605\ G(Q) = 175\ G(R)$

Let $Q = abc$ and $R = def$

$\therefore 5(11^2)(5^a\ 7^b\ 11^c) = (5^2 7^1)\ (5^d\ 7^e\ 11^f)$

$\therefore 1 + a = d + 2. \Rightarrow a = d + 1$

$b = e + 1$

$2 + c = f \Rightarrow c = f - 2$

$R - Q = 100(d - a) + 10(e - b) + f - c = 100\ (-1) + 10\ (-1) + 2 = -108.$

**2.** $A = 3^{a^2-6a+45}$ and $B = 3^{a^2}$

L.C.M. (A, B) = A

$\therefore$ A/B is an integer.

$\therefore\ 3^{-6a+45}$ is an integer. Also $a$ is a positive integer. If $a > 7$, $3^{-6a+45}$ will not be an integer.

$\therefore a \le 7. \therefore a$ has 7 possible values.

**3.** Let the even natural numbers be $2k$, $2k + 2$, $2k + 4$ and $2k + 6$.

$N = 16 + (2k)\ (2k + 2)\ (2k + 4)\ (2k + 6)$

$= 16(1 + k(k + 1)\ (k + 2)\ (k + 3))$

$= 16(1 + k(k + 3)\ (k + 1)\ (k + 2))$

$= 16(1 + (k^2 + 3k)\ (k^2 + 3k + 2)$

$= 16(1 + (k^2 + 3k)^2 + 2(k^2 + 3k))$

$= 16(k^2 + 3k + 1)^2$

$k^2 + 3k + 1$ is odd for any positive integral value of $k$.

$k \Rightarrow (k^2 + 3k + 1)^2$ is also odd.

$\therefore 16(k^2 + 3k + 1)^2$ is a perfect square divisible by 16.

Hence, only (B) and (D) are true.

**4.** Let the number be $N$.

$68488 = N.K_1 + R$ and

$67516 = NK_2 + R$ where $K_1$ and $K_2$ are natural numbers and $R$ is the remainder $68488 - 67515 = N(K_1 - K_2)$.

$972 = N(K_1 - K_2)$

$N$ must be a factor of 972.

$972 = 1 \times 972 = 2 \times 480$

$= 3 \times 324$

$= 4 \times 243 = 6 \times 162 = 9 \times 108$

$\therefore N$ has 6 possibilities.

**5.** $11111111 = 11(1010101) = 11(101)\ (10001)$

$= 11(101)\ (11025 - 1024) = 11(101)\ (105^2 - 32^2)$

$= 11(101)\ (137)\ (73)$

Sum of all the factors $= \frac{11^2 - 1}{11 - 1} \cdot \frac{101^2 - 1}{101 - 1} \cdot \frac{137^2 - 1}{137 - 1}$.

$\frac{73^2 - 1}{73 - 1} = 12\ (102)\ (138)\ (74) = 12499488$

**6.** $X^2 - 8X = (Y^2 + 2Y)^2 - 8(Y^2 + 2Y) = (Y^2 + 2Y)\ (Y^2 + 2Y - 8)$

$= Y(Y + 2)\ (Y + 4)\ (Y - 2)$

Let $Y = 2a$, where $a$ is a natural number.

$\therefore X^2 - 8X = 2a(2a + 2)\ (2a + 4)\ (2a - 2)$

$= 16(a - 2)\ (a - 1)\ a(a + 2)$

$= 16$ (Product of 4 consecutive natural numbers).

The product of 4 consecutive natural numbers is always divisible by 24.

$\therefore X^2 - 8X$ is always divisible by 384 but not always divisible by 384(2) or 768, e.g., when $a = 1$, $X^2 - 8X = 384$, which is not divisible by 768.

**7.** $24 = 2^3 \times 3$. The largest power of 3 contained in 360! can be calculated by the method indicated below.

| | | |
|---|---|---|
| 3 | 360 | = 120 |
| 3 | 120 | = 40 |
| 3 | 40 | = 13 |
| 3 | 13 | = 4 |
| 3 | 4 | = 1 |

Total = 178

Hence, the largest power of 3 in 360! is 178.

Similarly we can calculate the largest power of 2 in 360!, by the method indicated below.

| | | |
|---|---|---|
| 2 | 360 | = 180 |
| 2 | 180 | = 90 |
| 2 | 90 | = 45 |
| 2 | 45 | = 22 |
| 2 | 22 | = 11 |
| 2 | 11 | = 5 |
| 2 | 5 | = 2 |
| 2 | 2 | = 1 |

Total = 356

Hence, the largest power of 2 in 360! is 356. The largest power of $2^3$ in 360! is the quotient of 356/3 = 118.

Hence, the largest power of $2^3 \times 3$ in 360! is 118 which is the largest common power of $(2^3 \times 3)$ contained in 360!

**8. Wilson's theorem:** If $p$ is any prime number, then $(p-1)! + 1$ is a multiple of $p$.

106! + 1 is a multiple of 107. Let 106! + 1 = 107$k$.

106! = 107$k$ – 1

= 107 $(k-1)$ + 106

Dividing both sides by 106, $105! = 107\left(\frac{k-1}{106}\right) + 1$

$\frac{k-1}{106}$ must be an integer. Also, 105! is even.

$\therefore$ $\frac{k-1}{106}$ must be an odd integer.

Let $k$-1= 106 $(2k_1 + 1)$

105! = 107 $(2k_1 + 1)$ + 1 = 214 $k_1$ + 108

$\text{Rem}\left(\frac{105!}{214}\right) = 108$

Alternately, we can apply the concept of L.C.M. Model – 3 and check the remainders by 2, 107 to get the same result.

**9. Wilson's theorem:** If $p$ is any prime number, then $(p-1)! + 1$ is a multiple of $p$.

66! + 1 is a multiple of 67. Let 66! + 1 = 67$k$

66! = 67$k$ – 1

Multiplying by (67) (68) (69) (70) on both sides, we get

70! = $67^2$ [(68)(69) (70)$k$] – [(67) (68) (69) (70)]

= $67^2$ [(68) (69) (70)$k$] – [67 (67 + 1) (67 + 2) (67 + 3)]

The second bracket can be written as 67 $(67k_1 + 6)$

= $67^2$ [(68) (69) (70) $k - k_1$] – (67) (6)

$\text{Rem}\left(\frac{70!}{67^2}\right)$ = (–67) (6)

The corresponding positive remainder = (–67) (6) + $67^2$ = 4087.

**Alternate Solution:**

$$\text{Rem}\left(\frac{70!}{67^2}\right) = \text{Rem}\frac{(66!\times 67\times 68\times 69\times 70)}{67\times 67}$$

$$= 67\ \text{Rem}\frac{(66!\times 68\times 69\times 70)}{67}$$

$$= 67\left[\text{Rem}\left(\frac{66!}{67}\right)\times(1)(2)(3)\right]$$

= 67[(–1) (1) (2) (3)]

= 67[–6]

The equivalent positive remainder is –6(67) + $67^2$, i.e., 4087.

**10.** 2222 and 7777 leave respective remainders of 8 and 1 when divided by 9.

2222 is 1 less than a multiple of 9.

$\therefore$ $2222^{7777} = (9k-1)^{7777} = 9k_1 + (-1)^{7777} = 9k_1 - 1$

$7777^{2222} = (9m+1)^{2222} = 9m_1 + 1^{2222} = 9m_1 + 1$

$2222^{7777} + 7777^{2222}$ is a multiple of 9.

$\therefore$ The remainder of the sum divided by 9 is 0.

**11.** $8^P + 8P$ is a multiple of 10, i.e., it ends with 0.

$8P$ ends with 8, 4, 2 or 6, following a cycle of 4, while $8P$ has a units digit of 8, 6, 4, 2, 0 (a cycle of 5)

let the units digits of $8P$ and $8P$ be $a$ and $b$, respectively.

$P$ is a single digit positive integer.

For $P$ = 2, $(a, b)$ = (4, 6)

$P$ = 8, $(a, b)$ = (6, 4)

$P$ = 9, $(a, b)$ = (8, 2)

$P$ has three possible values.

**12.** Set $x = 1$, $3^N = a_0 + a_1 + a_2 + \ldots a_{2N}$ (1)

Set $x = -1$, $1 = a_0 - a_1 + a_2 - a_3 \ldots - a_{2N-1} + a_{2N}$ (2)

Adding (1) and (2): $3^N + 1 = 2(a_0 + a_2 + a_4 + \ldots a_{2N})$

$$\frac{3^N+1}{2} = a_0 + a_2 + a_4 \ldots a_{2N}$$

If $N$ is odd, $3^N+1$ will be a multiple of 4.

$\frac{3^N+1}{2}$ will be even.

If $N$ is even, then $3^N + 1$ will be an odd multiple of 2.

$\therefore \frac{3^N+1}{2}$ will be odd.

Both I and II are true.

**13.** $3125 = 5^5$

$N$ has 1000 digits.

$\therefore N$ contains all the single digit numbers and all the two digit numbers.

($\because$ total number of digits in $N$ upto the last two digit number = 9 + 2 (90) = 189)

Number of remaining digits in $N$ = 811.

They have to come from 270 three-digit numbers and one digit from the next number.

$\therefore$ The 270th three-digit number in $N$ is 369.

$\therefore N$ = 1234567891011.....3683693

Remainder of $N$ divided by 3125 is that of the last 5 digits of $N$ divided by 3125. This equals 2443.

**14.** $S(12) = \sum_{i=1}^{12} T_i = 12.12! + 11.11! + 10.10! + \ldots 1.1!$

$12.12! = (13-1)12! = 13! - 12!$

$11.11! = (12-1)11! = 12! - 11!$

$10.10! = (11-1)10! = 11! - 10!$ and so on

$1.1! = (2-1)\ 1! = 2! - 1!$

$S(12) = 13! - 12! + 12! - 11! + 11! - 10! + \ldots 2! - 1!$

$= 13! - 1!$

$(S(12))^2 = (13!)^2 - 2(13!) + 1$

$(S(12))^2$ when divided by 13 leaves a remainder of 1

(Q $(13!)^2$ and $2(13!)$ are both divisible by 13.)

**15.** The difference of $13^N$ and $N^3$ is divisible by 5. $\therefore$ the difference ends with 0 or 5.

If the difference ends with 0, the units digits of $13^N$ and $N^3$ is the same, i.e., of $3^N$ and $N^3$ is the same. This is only possible when $N$ is odd.

$\therefore N = 4k + 1$ or $4k + 3$.

If $N = 4k + 1$, $3^N$ ends with 3 and hence, $N^3$ must end with 7.

If $N = 4k + 3$. $3^N$ ends with 7 and hence, $N^3$ must end with 3.

If $N = 4k + 1$, $N$ can only be 17.

If $N = 4k + 3$, $N$ can be 3 or 23.

If the difference ends with 5, (is odd), $N^3$ must be even, i.e., $N$ is even. $\therefore$ $13^N$ ends with 9 or 1.

If $13^N$ ends with 9, $N$ must be $4k + 2$. Also $N^3$ must end with 4 (and hence, $N$ ends with 4). $\therefore N = 14$.

If $13^N$ ends with 1, $N$ must be $4k$. Also $N^3$ must end with 6 (and hence, $N$ ends with 6). $\therefore N = 16$

$N$ = 17, 3, 23, 14 or 16.

**16.** $E = 2^{32N+16} - 3^{12N+6} - 5$ (say)

$= (2^8)^{4N+2} - (3^3)^{4N+2} - 5$

$= (256)^{4N+2} - (3^3)^{4N+2} - 5$

$4N + 2$ is even.

$\therefore (256)^{4N+2} - (27)^{4N+2}$ is divisible by 256 + 27, i.e., 283

$$\text{Rem}\left(\frac{E}{283}\right) = \text{Rem}\left(\frac{283k-5}{283}\right) = -5 \equiv 278$$

**17.** The actual sum of the numbers

$$= 1 + 2 + 3 + \ldots + N = \frac{N(N+1)}{2}$$

The sum obtained by Karim was $\frac{N(N+1)}{2} - x$, where he missed the number $x$.

Now, the average of the numbers obtained by him was

$$\frac{\frac{N(N+1)}{2} - x}{N-1} = 27\frac{3}{13} = \frac{354}{13}$$

When we find the average of the first $N$ natural numbers starting from 1, the average will be very close to the middle value. It will be the middle value when there are odd number of numbers and it will be the average of the two middle values, when there are even number of numbers.

Even when one number is subtracted from the total and the average is obtained, it will be close to the middle value.

Therefore, the actual number of values present when one of the numbers was not counted was a multiple of 13 close to (27)2 which is 52. Therefore, $(N - 1)$ = 52, implying $N$ was 53.

$$\frac{\frac{N(N+1)}{2}-x}{N-1}=\frac{354\times4}{13\times4}=\frac{1416}{52}$$

Now, $\frac{N(N+1)}{2}-x=1416$

For $N=53$, $\frac{N(N+1)}{2}=1431$

Therefore, the number that he missed out was $1431-1416=15$

**18.** The number of positive integers not greater than 120, which are divisible by $3=\frac{120}{3}=40$

Those divisible by $4=\frac{120}{4}=30$

Those divisible by $5=\frac{120}{5}=24$

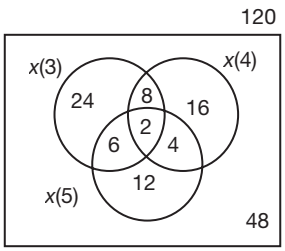
Those divisible by 3 and 4 (i.e., by 12) $=\frac{120}{12}=10$

Those divisible by 4 and 5 $=\frac{120}{20}=6$

Those divisible by 5 and 3 $=\frac{120}{15}=8$

Those divisible by all the three $=\frac{120}{60}3.=2$

The data can be represented in the form of a venn diagram.




$\therefore$ The required number is $40+16+4+12=72$.

**19.** $63!+64!+65!+\ldots+100!=63!\,(1+64+64\times65+64\times65\times66+\ldots)$

$=63!\,(65+64\times65+64\times65\times66+\ldots)$

$=63!\,(65^2+66k)$

$65^2+66k$ is not divisible by 3.

The IGP of 3 in the given sum is same as that in 63!, i.e., 30.

**20.** Let there be $m$ subjects in Patiala and $n$ in Mysore. Each subject got $p$ coins and let's say the number of coins left with either king is $r$.

$\therefore mp+r=33{,}274$

and $np+r=30{,}905$

$\therefore (m-n)\,p=2369=(23)\,(103)$

As $p$ is a 2-digit number, $p=23$ and $m-n=103$.

**21.** Given $N$ has six factors and it can be expressed as the product of two numbers out of which only one is prime in two ways. Hence, it has two prime factors. Also one of the prime factors has an index 2 and the other has one. $N$ is the least of this type, i.e., $N_2=2^2\times3^1$.

1, 5, 7 and 11 are coprime to $N$ and their sum is 24.

**22.** In order to check whether a huge number is divisible by 3 or 9, we add up all the digits. Instead of focusing directly on the individual digits, we can also apply the following procedure. We break up the number into a certain number of parts as per our choice.

We then add up the numbers represented by the various parts. If and only if the sum obtained is divisible by 9, the number itself is divisible by 9. If and only if the sum obtained is divisible by 3, the number itself is divisible by 3.

The positive integer, in the given problem, is 808182……124. Let us break it up as 80, 81, 82,…124. The sum of the numbers represented by the parts is 80 + 81 + … 124. This is the sum of the terms of an AP and hence, equals $\frac{45}{2}$ [80 + 124], i.e., 45 [102]. This sum is divisible by 9. $\therefore$ The positive integer is also divisible by 9.

$\therefore$ The positive integer is divisible by 3 too.

Both I and II are true.

**23.** Let the integer be denoted by $N$.

Let the remainder in each case be $r$.

$N$ divides $3946-r$, $4687-r$ and $5599-r$, exactly without leaving remainders.

$\therefore$ $N$ divides $(4687-r)-(3946-r)$ as well as $(5599-r)-(4687-r)$ exactly, i.e., 741 and 912 exactly.

$\therefore$ $N$ must be a common factor of (741, 912)

H.C.F. (741, 912) = 19(3)

$\therefore$ 19(3) has 4 factors.

$\therefore$ $N$ has 4 possible values.

**24.** Every day, the three bells toll together 24 times, or once in every hour or 60 min.

Let $X$, $Y$, $Z$ ring once every $x$, $y$, and $z$ min, respectively. The maximum value of $z$ is 60 and $y < 60$. Since $y$ is a factor of 60, its greatest value is 30.

Hence, if $Y$ rings once every 30 min, the number of times it tolls in a day will be the minimum, which is

$$= \frac{(24)(60)}{30} = 48$$

**NOTE**

If the time interval between two successive instances when the 3 bells toll simultaneously (say $T$) is slightly more than 60 min, the bells will toll simultaneously 24 times every day, perhaps for several days, but eventually, one day they will toll simultaneously only 23 times. Similarly, if $t = (60 - \delta)$ min, the 3 bells may toll simultaneously 24 times every day for several days, but eventually one day they will toll simultaneously 25 times.

**25.** There are 1499 – 50, viz, 1449 numbers between 50 and 1500. Of these, $8^2$, $9^2$, …, $38^2$ ($38^2 = 1444$ and $39^2 = 1521$) are perfect squares and $4^3$, $5^3$… $11^3$ are perfect cubes. (Note that $4^3 = 8^2$ and $9^3 = 27^2$). Therefore, 31 + 8 – 2, viz., 37 numbers are perfect squares or cubes. The other 1449 – 37, viz., 1412 numbers are neither perfect squares nor perfect cubes.

**26.** Each of $P$ and $Q$ is a multiple of 4 or an odd multiple of 2.

If both $P$ and $Q$ are multiples of 4 or an odd multiple of 2, $P^2 - Q^2$ is divisible by 16.

If exactly one of $P$ and $Q$ is a multiple of 4, $P^2 - Q^2$ leaves a remainder of either 4 or 12 when divided by 16.

The remainder of $P^2 - Q^2$ divided by 16 is 0, 4 or 12.

1772968 leaves a reminder of 8 when divided by 16.

$\therefore$ No solution exists.

**27.** Let the two numbers be $12a$ and $12b$, where $a$ and $b$ are coprimes. Given $12a + 12b = 588$

$\Rightarrow a + b = 49$

Product of the two numbers will be maximum when $a$ and $b$ are as close to each other as possible, i.e., 24 and 25.

$\therefore$ The largest possible product.

$= (12)\ (24) \times (12)\ (25) = 86400.$

**28.** As there are two different remainders and the difference between the divisors and the respective remainders is also different, it is L.C.M. model 3.

Therefore, the solution for this is (L.C.M. of 47, 97) + the smallest possible number that satisfies the given condition.

As the number leaves a remainder of 3 when divided by 97, it will be of the form $97k + 3$. When $97k + 3$ is divided by 47 it leaves a remainder of 6, $97k - 3$ is exactly divisible by 47.

$\therefore$ $97k - 3$ is divisible by 47 for $k = 1$

$\therefore$ The required number is of the form M × L.C.M. (47, 97) + 100, i.e., $4559M + 100$.

$\therefore$ it could be 4659 or 9218. Since 4659 is there in the choice, it is the answer.

Alternatively, when 4659 is divided by 47, the remainder is 6 which is not true for other numbers.

**29.** If $N$ is a single digit number then $N + 2N - 6N = 0$.

$\Rightarrow N = 0$, which is not possible.

Suppose, $N = 10a + b$

$\Rightarrow 10a + b + 2(a + b) - 6ab = 0$

$\Rightarrow 12a + 3b = 6ab$

$\Rightarrow 4a + b = 2ab$

$$b = \frac{4a}{2a - 1}$$

$4a$ must be divisible by $2a - 1$, i.e., $2a - 1$ is a factor of $4a$. $a$ and $2a - 1$ are always coprime.

$2a - 1$ must be a factor of 4. Also it is odd.

$2a - 1 = 1$

$a = 1$, $\therefore b = 4$.

$ab = 14$

**30.** Any factor of $N = 2^9\ 3^{11}\ 5^7$ is of the form $2^a\ 3^b\ 5^c$ If the factor is a perfect square, $a$, $b$, $c$, are even. Therefore, $a$ can take 5 even values (0, 2, 4, 6, 8), $b$ can take 6 even values and $c$ can take 4 even values. As we want the even factors, $a$ cannot take the value 0. Therefore, the number of even factors of $N$ that are perfect squares is $4(6)\ (4) = 96$.

**31.** 3720 is divisible by 2, 3, 5

Number of positive integers upto 3720 which are not divisible by 2 or 3 or 5 is the number of coprimes of 2, 3 and 5 upto 3720.

If $N$ is a positive integer equal to ${p_1}^a \times {p_2}^b \times {p_3}^c \times \ldots\ldots\ldots\ldots$ where $p_1, p_2, p_3, \ldots\ldots\ldots\ldots$ are prime numbers and $a, b, c, \ldots$ are whole numbers, it follows that up to $N$ there are

$\left(1-\frac{1}{p_1}\right)$ numbers coprime to $p_1$

$\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)$ numbers coprime to $p_1$ and $p_2$

$\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\left(1-\frac{1}{p_3}\right)$ numbers coprime to $p_1$, $p_2$ and $p_3$

Number of positive integers upto 3720 which are not divisible by 2 or 3 or 5

$= 3720\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right) = 992$

$\therefore$ The remaining 2728 positive integers upto 3720 are divisible by 2 or 3 or 5.

A total of 2732 positive integers are less than or equal to 3725 and are divisible by 2 or 3 or 5.

**32.** $333333 = 333(1001) = 3(111)\ 7(11)\ (13) = 3^2(37)\ (7)\ (11)\ (13)$

The sum of the factors is $(1 + 3 + 3^2)\ (1 + 37)\ (1 + 7)\ (1 + 11)\ (1 + 13)$

$= 13\ (38)\ (8)\ (12)\ (14) = 663936$

**33.** M is the set of the three-digit numbers from 200 to 900 which are divisible by 9. These are 207, 216, 225, ……, 900.They form an AP with a common difference of 9. Let the number of terms in M be $m$.

$900 = 207 + (m-1)9 \Rightarrow m = 78$

$N$ is the set of the three-digit numbers from 100 to 700 which are divisible by 7. These are 105, 112, 119, ……700, an AP with a common difference of 7. Let the number of terms in $N$ be $n$.

$700 = 105 + (n-1)7 \Rightarrow n = 86$

M has 39 odd elements and 39 even elements. $N$ has 43 odd elements and 43 even elements.

$S$ is the set of all the possible ordered pairs $(a, b)$ where $a \in$ M and $b \in N$, and $a + b$ is odd.

$\therefore$ one of $a$ and $b$ is even and the other is odd. If $a$ is even, $a$ can be any of the 39 even elements of M and $b$ can be any of the 43 odd elements of $N$.

If $a$ is odd, $a$ can be any of the 39 odd elements of M and $b$ can be any of the 43 even elements of $N$.

Number of elements of $S = 2(39)\ (43) = 3354$.

**34.** H.C.F. of $P$ and $Q$ is $P$.

$\therefore \frac{Q}{P}$ is an integer.

$\frac{2^{y^2+y+48}}{2^{y^2+7y+1}}$ is an integer.

$\therefore 2^{y^2+y+48-(y^2+7y+1)}$ is an integer.

$2^{-6y+47}$ is an integer.

This is only possible if $-6y + 47$ is a whole number.

$\therefore 47 \geq 6y$

$y \leq 7\frac{5}{6}$

$y$ can be 1, 2, 3, … or 7.

**35.** $2^{2014} - 2$ and $2^{2014}$ are consecutive even positive integers. Any two consecutive even positive integers have an H.C.F. of 2.

$\therefore$ H.C.F. $(2^{2014} - 2, 2^{2014}) = 2$

L.C.M. $(2^{2014} - 2, 2^{2014}) = \frac{\text{product } (2^{2014} - 2, 2^{2014})}{\text{H.C.F.}(2^{2014} - 2, 2^{2014})}$

$= 2^{2013}\ (2^{2014} - 2)$

$2^{2013}\ (2^{2014} - 2)$ is divisible by $2^{2013}$ while $2^{2014} + 2$ is not even divisible by $2^2$ (1)

Any common factor of $2^{2014} + 2$ and $2^{2014} - 2$

must also divide their difference, i.e., 4.

(1) implies this common factor cannot be 4.

$\therefore$ It has to be 2 ($\because$ common factor must be even).

H.C.F. $(2^{2013}\ (2^{2014} - 2), 2^{2014} + 2) = 2$

$\therefore$ L.C.M. $(2^{2013}\ (2^{2014} - 2), 2^{2014} + 2)$

$= \frac{2^{2013}\left(2^{2014} - 2\right)\left(2^{2014} + 2\right)}{2} = 2^{2012}\left(2^{4028} - 4\right)$

**36.** Consider any 8-digit number $X$. Let the sum of the digits be $S$. (Given $S = 70$). Let the sum of the digits in the alternate places, starting from the units digit be U and the sum of the alternate digits, starting from the ten

place be T. The elevens remainder of $X$ is equal to the eleven remainder of U – T.

As this has to be 0, U – T, = 0, ±11, ±22, …

$\therefore$ U + T = 70

U – T = 0 (1)

i.e., U = 35, T = 35. If U – T = ±11 U and T would not be integers. We can consider U + T = 70, U – T = 22, i.e., U = 46, T = 24 …..(21) (By setting U – T = –22, we would get U = 24, T = 46, (U/T) is the sum of only 4 digits, it cannot be 46. Therefore, we have to consider only (1), i.e., U = T = 35. In U (or T) there are 3 nines and 1 eight. The can be selected in 4 ways in U, and for the each of these choices an 8 can be selected in 4 ways in T. Therefore, the number of numbers is 4(4), viz., 16.

**37.** The page numbers on the leaf must be consecutive.

Let these be $x$ and $x+1$ and let their sum be $L$, i.e., $L = 2x+1$

and let the number of pages in the novel be $N$.

Also, $(3 \le L \le 2N-1)$

$$\frac{N(N+1)}{2} - 3 \ge 10000 \ge \frac{N(N+1)}{2} - (2N-1)$$

$N^2 + N \ge 20006$ and $N^2 - 3N + 2 \le 20000$

As $N^2 + N$ is nearly equal to $N^2$ for large values of $N$, $N^2 \ge 20006$ and $N \ge \sqrt{20006}$

$N \ge 141$ and $N \le 143$

$N$ could be 141, 142, 143.

$\therefore$ $N$ lies between 140 and 150.

**NOTE**

$N$ can only be 141 and 142 but not 143. We'd know this upon checking further.

**38.** $10! = 2\ (3)\ 2^2\ (5)\ 2\ (3)\ 7\ (2^3)\ (3^2)2\ (5) = 2^8\ 3^4\ 5^2\ 7^1$

The factors ($P$) comparing only one prime, which leave remainders of 1 and 2 when divided by 3 are tabulated below.

| Rem $\left(\frac{P}{3}\right) = 1$ | | | Rem $\left(\frac{P}{3}\right) = 2$ | |
|---|---|---|---|---|
| $2^0$<br>$2^2$ | $5^0$<br>$5^2$ | $7^0$<br>$7^1$ | $2^1$<br>$2^3$ | $5^1$ |
| $2^4$<br>$2^6$ | | | $2^5$<br>$2^7$ | |
| $2^8$ | | | | |

Any factor $P$ of 10!, which is a multiple of 3, is of the form $2^a\ 5^b\ 7^c$ = QRS (say). If Rem $\left(\frac{P}{3}\right) = 2$, exactly 1 or all 3 of $Q$, $R$, $S$ are of the form $3k+2$. As $S$ (or $7^c$) cannot be of this form, only $Q$ or only $R$ are of this form. The possible factors are listed below.

| Q | R | S |
|---|---|---|
| $2^1$<br>$2^3$<br>$2^5$<br>$2^7$ | $5^0$<br>$5^2$ | $7^0$<br>$7^1$ |
| $2^0$<br>$2^2$<br>$2^4$<br>$2^6$<br>$2^8$ | $5^1$ | $7^0$<br>$7^1$ |

$\therefore$ The total number of factors of $P$ such that Rem $P/3 = 2$ is

4 (2) (2) + 5(1) (2) = 16 + 10 = 26.

**39.** The two numbers are $2^M\ 3^{N+3}\ 5^3\ 7^7$ and $2^{N+3}\ 3^{M+3}\ 5^a\ 7^a$

The H.C.F. of the given numbers is $2^a\ 3^a\ 5^a\ 7^a$.

$\therefore$ $a = \min\ (M, N+3) = \min\ (N+3, M+3)$

Comparing the indices of 5 and 7, we see that $a = 0, 1, 2,$ or 3 (1)

Comparing the indices of 2 and 3,

$a = \min\ (M, N+3) = \min\ (N+3, M+3)$

If the value of '$a$' from the first bracket is $M$, from the second it has to be $N+3$.

If the value of '$a$' from the first bracket is $N+3$, from the second too it is $N+3$. In either case $a = N+3$.

$N$ is a whole number, i.e., $N = 0, 1, 2$ …

$\therefore$ $a = N+3 = 3, 4, \ldots$ (2)

From (1) and (2), the only possibility is $a = 3$, i.e., $N = 0$.

$\therefore$ $N$ can have only one value.

**40.** $\frac{2^N}{3}$ leaves a remainder of 2 when $N$ is odd and a remainder of 1 when $N$ is even.

$$\left[\frac{2^0}{3}\right] = \frac{2^0 - 1}{3}, \quad \left[\frac{2^1}{3}\right] = \frac{2^1 - 2}{3}$$

$$\left[\frac{2^2}{3}\right] = \frac{2^2 - 1}{3}, \quad \left[\frac{2^3}{3}\right] = \frac{2^3 - 2}{3}$$

$$\left\lfloor \frac{2^{99}}{3} \right\rfloor = \frac{2^{99}-2}{3}$$

$$\left\lfloor \frac{2^{100}}{3} \right\rfloor = \frac{2^{100}-1}{3}$$

Required sum $= \frac{1}{3}(2^0 + 2^1 + 2^2 + \ldots + 2^{100})$

$$-51\left(\frac{1}{3}\right) - 50\left(\frac{2}{3}\right) = \frac{2^{101}-1}{3} - \frac{151}{3}$$

($\because 2^0 + 2^1 + 2^2 + \ldots 2^{100}$ represents the sum of the terms of a G.P. whose first term is 1 and common ratio is 2).

$$= \frac{2^{101}-152}{3}$$

**41.** Fermat's little theorem: If $p$ is any prime number and H.C.F. $(a, p) = 1$, then $a^{p-1}-1$ is a multiple of $p$.

From the theorem, it also follows that $a^{k(p-1)} - 1$ is a multiple of $p$. (1)

$3798 = 211\ (18)$

$$\text{Rem}\left(\frac{36^{1052}}{211}\right) = \text{Rem}\left(\frac{36^{1050}\ 36^2}{211}\right) = \text{Rem}\left(\frac{36^2}{211}\right)$$

[$\because 1050 = (210)\ (5)$]

$\therefore$ From (1), it follows that $\text{Rem}\left(\frac{36^{1050}}{211}\right) = 1$]

$\therefore 36^{1052} - 36^2$ is divisible by 211

$36^{1052} - 36^2$ is also divisible by 18

$\therefore 36^{1052} - 36^2$ is divisible by L.C.M (211, 18)

i.e., 3798.

$$\therefore \text{Rem}\left(\frac{36^{1052}}{3798}\right) = \text{Rem}\left(\frac{36^2}{3798}\right) = 1296$$

**42.** $N = 10^{51} - 769$

$10^{51} = (11-1)^{51} = 11k + (-1)^{51} = 11\ k - 1$ (say)

$\therefore N = 11k - 770 = 11\ (k - 70)$

$\therefore N$ is divisible by 11.

$10^{51} = (10^3)^{17} = (27 \times 37 + 1)^{17}$

$= [(28-1)\ 37 + 1]^{17} = [(28)\ (37) - 35 - 1]^{17}$

$= (7k-1)^{17}$ (say) = A multiple of $7 - 1 = 7k_1 - 1$ (say)

$N = 7k_1 - 770 = 7\ (k_1 - 110)$

$\therefore N$ is divisible by 7 and by 11. I is true.

$N$ is divisible by 7 but not by 9.

[$10^{51} = (9+1)^{51} = 9k + 1$ (say)

$10^{51} - 769 = 9k - 768$ $\therefore$ 768 is not divisible by 9.

$\therefore N$ is not divisible by 9.]

$\therefore N$ is not divisible by 63.

II is not true.

**Alternate Solution:**

The remainder of $10^{51}$ and 769, when divided by 7, 9 and 11 are tabulated below.

| Number \ Div | 7 | 9 | 11 |
|---|---|---|---|
| $10^{51}$ | 6 | 1 | 10 |
| 769 | 6 | 4 | 10 |

$\therefore 10^{51} - 769$ is divisible by 7 and 11, but not by 9, i.e., by 77 but not by 63. Only I is true.

Note: Successive powers of 10 (or 3) when divided by 7 leave the remainders 3, 2, 6, 4, 5, 1; 3, 2, 6, 4, 5, 1;....

$$\therefore \text{Rem}\ \frac{10^{51}}{7} = \text{Rem}\ \frac{106^{k+3}}{7} = \text{Rem}\left(\frac{10^3}{7}\right) = 6$$

$\therefore$ The L.C.M. has to be greater than or equal to 21. We have to check whether 21, 22, 23, 24, ... are possible values of the L.C.M.

**43.** 100010011002 ..... 1099 is a 400-digit number.

It equals $10\ (10^2 + 10^6 + 10^{10} + \ldots + 103^{98}) + 99 + 98\ (10^4) + 97\ (10^8) + \ldots + 01(10^{392})$

$99 = 10^2 - 1$

Applying remainder theorem, we have the remainder of the given division as that of $10\ (1 + 1^3 + 1^5 + \ldots + 1^{199}) + 99 + 98\ (1^2) + 97\ (1^4) + \ldots + 01\ (1^{196})$ divided by 99, i.e., that of $10(100) + \frac{99(100)}{2}$ divided by 99.

Remainder is 10.

**Alternate Solution:**

Let $N$ = 100010011002 ...1099. We want Rem $N/99$. $\therefore$ We have to group the digits two at a time. We can do that as shown below. The most significant figures are the top.

| | |
|---|---|
| 10 | 00 |
| 10 | 01 |
| | |
| | |
| 10 | 99 |

The sum of all these groups is $(0 + 1 + 2 + \ldots + 99) + (10 + 10 + .. + 10) = 4950 + 1000 = 5950$.

$$\text{Rem}\frac{5950}{99} = \text{Rem}\frac{59+50}{99} = 10$$

**44.** $199! = (199 \times 198 \times \ldots\ldots\ldots\ldots 135) \times 134 \times (133 \times 132 \times \ldots\ldots..68) \times 67 \times (66 \times 65 \times \ldots\ldots\ldots\ldots 1)$

$66 \times 65 \times \ldots\ldots\ldots 1 = 66!$

$66! + 1$ is divisible by 67 ($\because$ Wilson's theorem)

$\therefore \text{Rem}\left(\frac{66!}{67}\right) = -1$. The corresponding positive remainder is 66.

$133 \times 132 \times \ldots 68 = (67 + 66)\ (67 + 65)\ldots\ldots\ldots(67 + 1) = 67\ k + 66!$ (say). This leaves a remainder of 66 when divided by 67.

$199 \times 198 \times \ldots 135 = (67 \times 2 + 65)\ (67 \times 2 + 64)\ldots\ldots(67 \times 2 + 1) = 67k + 65!$ (say)

$66! = 67k_1 + 66$ (say) (1)

$\therefore k_1$ must be divisible by 66.

$\therefore$ Dividing both sides of (1) by 66, we have $65! = 67\ k_2 + 1$ where $k_2 = \frac{k_1}{66}$

$$\text{Rem}\left(\frac{66 \times 65 \ldots 1}{67}\right) = \text{Rem}\left(\frac{133 \times \ldots \times 68}{67}\right) = 66$$

$$\text{Also Rem}\left(\frac{199 \times 198 \times \ldots 135}{67}\right) = 1$$

$$\therefore \text{Rem}\left(\frac{199!}{67^3}\right) = \text{Rem}\left(\frac{1 \times 134 \times 66 \times 67 \times 66}{67^3}\right)$$

$$= \text{Rem}\ \frac{(67 \times 2)\left((67-1)\,67\,(67-1)\right)}{67^3}$$

$$= \text{Rem}\ \frac{2\left(67^2\right)}{67^3} = 2\left(67^2\right) = 8978$$

**45.** 579579 ……up to 999 digits $= 579\ (1 + 10^3 + 10^6 + \ldots 10^{996})$

$91 = 7\ (13)$ and $91 \times 11 = 10^3 + 1$

It would be convenient to find the remainder of the number divided by $10^3 + 1$ and then by 91.

Applying remainder theorem, we have the remainder of 579579… up to 999 digits divided by $10^3 + 1$ as that of $\frac{5791 + (-1) + (-1)^2 + \ldots + (-1)^{166}}{10^3 - (-1)}$, i.e., that of $\frac{(579)(1)}{10^3 - (-1)}$ i.e., 579.

The number has the form $1001\ k + 579$. i.e., $91(11k + 6) + 33$.

$\therefore$ The remainder of the number divided by 91 is 33.

**Alternate Solution:**

Let $N$ = 579579…579 (999 digits or 333 groups of 3 digits).

We want $\text{Rem}\frac{N}{91}$. We recall that $91(11) = 1001$.

To obtain $\text{Rem}\ \frac{N}{1001}$, we group the digits of $N$, 3 at a time as shown below, the least significant figures are at the top.

| | |
|---|---|
| 579 | 579 |
| – | – |
| | |
| | |
| 579 | 579 |
| | 579 |

There are 333 groups of 3 digits, 166 on the left and 167 on the right.

$$\therefore \text{Rem}\frac{N}{1001} = 167\ (579) - 166\ (579) = 579.$$

$$\text{And Rem}\ \frac{N}{91} = \text{Rem}\ \frac{579}{91} = 33.$$

**46.** The required number of coprimes

$$= 70^n\left(\frac{1}{2}\right)\left(\frac{4}{5}\right)\left(\frac{6}{7}\right) - 140\left(\frac{1}{2}\right)\left(\frac{4}{5}\right)\left(\frac{6}{7}\right)$$

$$= 70^{n-1}(24) - 48$$

$$= 24\ [70^{n-1} - 2]$$

**47.** $N = 101010\ldots\ldots1$ ($n$ zeros alternating with $n+1$ ones)

$= 100\ (1+ 10^2 + 10^{2n-2}) + 1$ (The greatest term in the expansion has $2n+1$ digits).

$9999 = 10^4 - 1$.

The last index, viz, $2(n-1)$ may be of the form $4k+2$ or $4k+4$.

Accordingly, Rem $\dfrac{N}{9999}$ =

From the remainder theorem,

Rem $\dfrac{100\left(1+10^2+1+10^2+\ldots+1+10^2\right)+1}{10^4-1}$ or

Rem $\dfrac{100\left(1+10^2+1+10^2+\ldots+1+10^2+1\right)+1}{10^4-1}$,

i.e., that of

$\dfrac{100\left[\dfrac{n}{2}(1)+\dfrac{n}{2}(100)\right]+1}{10^4-1}$ or

$\dfrac{100\left[\dfrac{n+1}{2}(1)+\dfrac{n-1}{2}(100)\right]+1}{10^4-1}$

i.e., that of $\dfrac{50(101n)+1}{10^4-1}$ or $\dfrac{50(101n-99)+1}{10^4-1}$

i.e., that of $\dfrac{50(101n)+1}{10^4-1}$ or $\dfrac{101(50n-49)}{10^4-1}$

[$\because -50(99) + 1 = -101\ (49)$]

In the first case, the remainder cannot be 0 since $10^4 - 1$ is divisible by 101 while the numerator is not. In the second case, $50n - 49$ must be divisible by $10^2 - 1$ in order that the remainder may be 0.

$50\ n - 49 = (10^2 - 1)\ k$ (say)

$50\ n = 99k + 49$

Adding 50 both sides,

$50\ (n+1) = 99\ (k+1)$

$\therefore\ n+1$ must be divisible by 99.

Also $k+1$ must be even.

$\therefore$ Least $(n+1, k+1)$ is (198, 100)

$\therefore$ Least $n$ is 197.

**Alternate Solution:**

$N = 1010\ \ldots1$ ($n$ zeroes alternating with $n+1$ ones)

$N$ has to be divisible by 9999, i.e., by 9(11) (101)

$N$ is divisible by 9. $\therefore\ n+1$ is a multiple of 9 (1)

$N$ is divisible by 11. $\therefore\ n+1$ is a multiple of 11. (2)

closing an initial D in $N$, $N = 01, 01, 01, \ldots 01$.

We see that every consecutive pairs of 01*s* is 101. As $N$ is divisible by 101, we must have an integer number of consecutive pairs, i.e., 4 $k$ digits. Number of digits is also $2(n+1)$.

$\therefore\ n+1 = 2k$

$n+1$ is divisible by 9, 11, 2 and hence, by L.C.M. 19, 11, 21, i.e., 198.

Least $n+1 = 198$

Least $n = 197$.

**48.** Let $N = 13(12^{600}) = 13(12-1)^{600}$ and $2197 = 13^3$

$12(13-1)^{600} = 13\ [13^{600} + {}^{600}C_1(13)^{599}\ (-1) + {}^{600}C_2\ (13)^{598}\ (-1)^2 + \ldots + {}^{600}C_{598}\ (13)^2\ (-1)^{598} + {}^{600}C_{599} + (13)\ (-1)^{599} + (-1)^{600}]$

There are 601 terms in the expansion of $(13-1)^{600}$ 1- M (Say). The sum of the first 599 terms is divisible by $13^2$. The sum of the last two terms =

${}^{600}C_{599}\ (13)\ (-1)^{599} + (-1)^{600} = -600\ (13) + 1$

$= -(650 - 50)\ (13) + 1 = -650\ (13) + 650 + 1 = -13^2\ (50) + 13^2\ (3) + 144$

$= -13^2(47) + 144$

$\therefore$ Rem $\dfrac{N}{13^3} = 13$ Rem $\left(\dfrac{M}{13^2}\right)$

$= 13(144) = 1872$

**49.** If we take any AP and consider the squares (or any other power) of successive numbers, we find that they leave a cyclic pattern of remainder, when they are divided by 24 (or any other divisor). In this case the AP could be any set of 1200 consecutive numbers or the set of the first 1200 numbers. Let us consider 1, 2, 3,...1200. The remainder that the squares of these numbers leave when divided by 24 are 1, 4, 9, 16, 1, 12, 1, 16, 9, 4, 1, 0 and then the pattern repeats, i.e., among the squares of the first 12 numbers there are 4 numbers whose squares that are of the form $24k+1$. Therefore, for the entire set $N$, there are 400 such numbers.

**50.** Rem $\dfrac{2^{181}}{210} = 2.\text{Rem}\left(\dfrac{2^{180}}{105}\right)$

$105 = (3)\ (5)\ (7)$

By remainder theorem,

$$\text{Rem}\left(\frac{2^{180}}{3}\right)=\text{Rem}\left(\frac{2^{180}}{5}\right)\text{Rem}\left(\frac{2^{180}}{7}\right)=1$$

$2^{180}$ is of each of the forms $3k + 1$, $5k + 1$, $7k + 1$, and hence, of the form $k$ L.C.M. $(3, 5, 7) + 1$, i.e., $105\ k + 1$.

$$\text{Rem}\left(\frac{2^{180}}{105}\right)=1$$

$\therefore$ Required remainder is 2.

## EXERCISE-5

**1.** From I, some of the values of $x$ are 101, 116, … From II, some of the values of $x$ are 106, 117, … From I and II, the difference between divisors and remainders is constant, i.e., 4, hence, $(15 \times 11) - 4 = 161$ is the only number in the range of (100 and 265).

**2.** $x < 0$.

From Statement I, at least one of $y$ and $z$ is $< 0$

If $y$ and $z$ are negative, then $xyz < 0$

If only one of $y$ and $z$ is negative, then $xyz > 0$.

Hence, Statement I alone is not sufficient.

From Statement II, $y + z > 0 \Rightarrow$ at least one of $y$ and $z$ is positive or both are positive.

If both are positive then $xyz < 0$.

If only one is positive then $xyz > 0$, so Statement II alone is not sufficient.

Combining statements I and II, between $y$ and $z$, one is negative and other is positive. Hence, $xyz > 0$.

**3.** From I, $a^2 + b^2 + c^2 = ab + bc + ca$

$\Rightarrow 2(a^2 + b^2 + c^2) = 2(ab + bc + ca)$

$\Rightarrow (a^2 + b^2 - 2ab) + (a^2 + c^2 - 2ca) + (b^2 + c^2 - 2bc) = 0.$

$\Rightarrow (a - b)^2 = (b - c)^2 = (c - a)^2 = 0.$

$\therefore\ a = b = c$

As $abc \neq 0$, $a^3 + b^3 + c^3 \neq 0$.

If $a + b + c = 0$, $a^3 + b^3 + c^3 = 3abc$. As $abc \neq 0$, $a^3 + b^3 + c^3 \neq 0$.

$\therefore$ Either Statement Is sufficient to answer the question.

**4.** From Statement I, $x = n^2$.

From Statement II, $x = k^3$.

Combining statements I and II

If $x = 729$ it is a perfect square and a cube.

If $x = 64$ it is also a perfect square and a cube.

$\therefore x$ can be even or odd.

Hence, both statements together are also not sufficient.

**5.** Let $x$ be the number of soldiers.

From Statement I, $x$ is a multiple of the L.C.M. of 3, 5 and 7.

$\therefore x = 105\ k \Rightarrow x$ can be 105 or 210.

Hence, Statement I alone is not sufficient.

From Statement II, $x$ is even.

Combining both the statements, we get $x = 210$.

**6.** From Statement I, $x + y = dk_1$ (1)

From Statement II, $x - y = dk_2$ (2)

Using both the statements,

Adding equation (1) and (2) we get

$2x = d\ (k_1 + k_2)$

Since $d$ is odd, $k_1 + k_2$ is even.

$$\frac{x}{d}=\frac{k_1+k_2}{2}=\text{integer.}$$

$\therefore\ x$ is divisible by $d$.

Similarly, $y$ is also divisible by $d$.

**7.** $\dfrac{a}{b-c}=1\Rightarrow a=b-c$ so we have to find $\dfrac{a}{b}$.

$\therefore$ Statement I alone is sufficient.

From Statement II, $a$ and $b$ are co-primes.

Hence, $\dfrac{a}{b}$ may be $\dfrac{3}{5}$ or $\dfrac{7}{9}$ or any other such value.

Hence, unique value is not possible.

**8.** $10 < 3^n < 300$ so $n = 3$, 4, or 5

From Statement I, $n$ is the square of an integer.

$\therefore n = 4$.

From Statement II, $3^n$ is the square of an integer.

$\therefore 3^n = 81 \Rightarrow n = 4$.

$\therefore$ Either statement alone is sufficient.

**9.** From Statement I,

$1 + 2 + 3 + 4 + 6 = 16$ (the only possibility)

So, I alone is sufficient.

From Statement II, $1 \times 2 \times 3 \times 4 \times 5 = 120$ (the only possibility)

So, II alone is also sufficient.

**10.** GCD of $(2a, 2b) = 10 \Rightarrow$ Let $2a = 10k$ and $2b = 10m$, where $k$ and $m$ are co-primes.

$\Rightarrow a = 5k$; $b = 5m$

$\therefore$ GCD of $a$ and $b$ is 5.

From Statement I alone, we can answer.

Statement II does not give any information to solve.

**11.** From Statement I, $x = Nk$ and $y = Nr$

Only if $k$ and $r$ are co-primes, then $N$ is the H.C.F. of $x$ and $y$ other wise not. $\therefore$ I alone is not sufficient.

From Statement II, $\frac{x}{2} = 2Nk_1$ and $\frac{y}{4} = 2Nr_1$.

$\therefore 4N$ divides $x$ and $y$.

$\therefore N$ is not H.C.F. of $x$ and $y$.

$\therefore$ Statement II alone is sufficient.

**12.** From Statement I, $x = 5k +$ odd positive integer where $k$ is a non-negative integer.

If $k = 1$ then $x$ is even.

If $k = 2$ then $x$ is odd.

Statement I alone is not sufficient.

From Statement II, $a = 4P +$ odd positive integer, where $P$ is a non-negative integer.

If $P$ is odd or even, then $x$ is always odd.

Hence, Statement II alone is sufficient.

**13.** From Statement I, when $x$ is divided by 8 the remainder is 3. Hence, $x = 8k + 3$, where $k$ is a whole number, when $8k + 3$ is divided by 4 the remainder is 3. Hence, Statement I alone is sufficient.

From Statement II, when $x = 5$ the remainder when $x$ is divided by 4 is 1 but when $x = 10$ the remainder is 2.

So, the question cannot be answered by Statement II alone.

**14.** $2a + 4b + a - b + c = 3(a + b) + c$.

From Statement I, we do not know whether $c$ is divisible by 3 or not, so we cannot answer the question.

From Statement II, $c$ is divisible by 3.

$\therefore 3(a + b) + c$ is divisible by 3.

Statement II alone is sufficient.

**15.** $p^q = r^q$

From Statement I, if $q = 3$, $p = r$. If $q = 6$, $p = \pm r$. I is not sufficient.

From Statement II, $p = r$ as $q$ is odd II is sufficient.

**16.** It is given that $x, y, z$ are three successive prime numbers.

From Statement I

$x - y = 6$, $y - z = 4$

$\therefore x - z = 10$

$(x, y, z)$ could be (29, 23, 19) or (53, 47, 43).

Statement I alone is not sufficient.

From Statement II, there is a limit given which is $x < 60$. Independently this Statement Is not sufficient.

If we combine both statements, we get two possibilities, i.e., 19, 23, 29 and 43, 47, 53. Hence, data is insufficient.

**17.** From Statement I,

When $k = 2^2 \times 3^1 \times 5^1$ or $2^1 \times 3^2 \times 5^1$ or $2^1 \times 3^1 \times 5^2$ the number of factors of $k$ is 12.

Hence, we cannot determine $k$ uniquely.

from statement (II),

when $k = 2^2 \times 5^1 \times 3^1$ or few of its multiples then $k$ is multiple of 4. Again, we do not get a unique value of $k$. Statement II alone is not sufficient.

Using both statements, $k = 2^2 \times 5^1 \times 3^1$

**18.** From Statement I, it is given that $a < c$, $b < d$, but we do not know if $a + b$ is a positive integer. If $(a + b) > 0$, as $a + b < c + d$. It follows that $(a + b)^2 < (c + d)^2$.

If $c + d < 0$, then we cannot say which of the two expressions $(a + b)^2$ and $(c + d)^2$ is greater. Hence, Statement I is insufficient; Statement II is also insufficient for the same reason. Even if both the statements are used we cannot answer the question.

**19.** For $a, b, c$ to form a triangle, the sum of the smaller two has to be greater than the greatest number.

Using Statement I we find $a, b, c$, are 3, 2, 1

As $b + c = a$, they do not form a triangle.

Hence, Statement I alone is sufficient.

From Statement II, $l = 19$, $a = 8\ (b - c)$ and $b = 2\ (a - c)$

This given $a: b: c = 8: 6: 5$

$\therefore$ This does form a triangle.

Hence, Statement II is also sufficient.

**20.** From Statement I, $7\frac{1}{2} + k\left(\frac{1}{3} + \frac{1}{6}\right)$

$\therefore\ 7\frac{1}{2} + k\left(\frac{1}{2}\right)$

$\therefore$ this sum is an integer if $k$ is odd. Hence, Statement I alone is sufficient.

From Statement II, $7\frac{1}{2} + l\left(\frac{5}{3} + \frac{1}{6}\right) = 7\frac{1}{2} + l\left[\frac{11}{6}\right]$.

$\therefore$ it is an integer if $l$ is a multiple of 3. But we do not know whether $l$ is a multiple of 3 or not. Hence, we cannot answer the question.

**21.** From Statement I, $a = b$.

$\therefore\ a + b = 2b$ and $ab = b^2$

$b^2 < 2b$ if $b = 1$ and $b^2 = 2b$ if $b = 2$

otherwise $b^2 > 2b$

Hence, Statement I alone is insufficient.

From Statement II, $a = 1$

$\therefore\ a + b = b + 1$ and $ab = b$

As, $b + 1 > b$ is definitely true, $a + b > ab$.

Hence, Statement II alone is sufficient.

**22.** From Statement I, the minimum value of $x$ is $[\{(7 \times 4) + 3\} \times 3 + 2] = 95$.

The number $x$ is of the form $= 3 \times 4 \times 9 + 95 = 108k + 95$

Hence, this statement alone is not sufficient.

From Statement II, $x$ is given that it is a multiple of 29. This alone is definitely not sufficient.

Using both the statements the value of $x$ could be 203 or 3335. Combining both also, we cannot answer the question.

**23.** From Statement I, $N$ is a product of two different single digit numbers and $N > 70$. Hence, $N$ is $9 \times 8 = 72$.

Hence, Statement I alone is sufficient.

From Statement II, we can say that the two digit number greater than 70 which is a product 3 distinct primes is 78 $(2 \times 3 \times 13 = 78)$. Hence, Statement II alone is sufficient.

**24.** From Statement I, the seeds can be planted in rows of 3, 5 or 11, i.e., the number of seeds is multiple of the L.C.M. of 3, 5 or 11, i.e., it is of the form $165n$. Hence, number of seeds can be 165 or 330. Statement I alone is not sufficient.

Statement II says, the number is even. This alone is not sufficient. Using both statements, the number of seeds is 330.

**25.** From Statement I, The units digit of $x$ and $x^2$ is the same. Hence, the units digit of $x$ is 0, 1, 5, or 6. So $x$ could be 10, 11, 15, 16, 20, 21, 25, 26, 30, or 31.

Statement I, alone is not sufficient. From Statement II, $x^2$ can be 121, 484 or 676. Hence, $x$ is either 11 or 26.

Statement II alone is also not sufficient.

Using both statements also we $x$ can be 11 or 26. Hence, we cannot answer the question.

**26.** $ab > 0$ so both $a$ and $b$ are positive or both are negative.

From Statement I,

when $a = 4$ and $b = 2$, then $a^2 > b$ and $a + b > 0$.

when $a = -2$ and $b = -2$, then $a^2 > b$ but $a + b < 0$.

Statement I, alone is not sufficient.

From Statement II, $b^2\ a < 0$.

As $b^2$ is positive, $a$ must be negative.

Both $a$ and $b$ are negative (as $ab > 0$), $a + b$ is not greater than 0.

From Statement II alone, we can answer the question,

**27.** We have to determine whether $z > y$.

Statement I does not mention $z$. It is not sufficient.

From Statement II, if $x > 0$, we can infer that $z > y$.

But we do not know whether $x$ is positive.

Even by combining the statements, we cannot answer the question.

**28.** The sum of $(2a - b)$ and $(2a + 5b - 4c)$ is $= 4a + 4b - 4c$.

From Statement I, $c < 0$, while $a, b > 0$. We can conclude that $4a + 4b - 4c$ is positive. But, this is not sufficient.

From Statement II, if $c > 0$, the number $a, b, c$ are $a, a + 1, a + 2$ and $4(a + b - c)$ is $4(a - 1)$, which may or may not be divisible by 3.

If $c < 0$, the number $a, b$ and $c$ are $a, a + 1, -a - 2$ and $4(a + b - c) = 4[a + (a + 1) + (a + 2)]$ which is always divisible by 3.

$\therefore$ Statement II alone is not sufficient, but Statement I and II, taken together are sufficient $a, b, |c|$ are successive numbers.

**29.** Let the number of 25 paise, 50 paise and ₹1 coins be $x$, $y$ and $z$, respectively.

We have $\frac{x}{4}+\frac{y}{2}+z = 23$ (1)

From Statement I,

$$\frac{y}{2}\left(\frac{1}{4}\right)+\frac{y}{2}(1)-y\left(\frac{1}{2}\right)=1.25=\frac{5}{4}$$

$\Rightarrow y = 10$ (2)

From (1) and (2), $x + 4z = 72$

From Statement II, $\frac{x}{2}(1)-\frac{x}{2}\left(\frac{1}{4}\right)=9$

$\Rightarrow x = 24$

$\therefore$ By combining statements I and II we get $x = 24$, $z = 12$ and $y = 10$

$\therefore x + y + z = 46$

**30.** Is $N$ the H.C.F. of two numbers $X$, $Y$.

From Statement I, if $N$ is the H.C.F. it is definitely factor of $x$, $y$. But there may be more common factors of $x$, $y$.

$\therefore$ I alone is insufficient.

from Statement II, let $N = 5$, $x = 30$ and $y = 20$.

$\therefore x - y = 2N = 10$ and the H.C.F. is 10 which is $2N$. If $N = 5$, $x = 25$ and $y = 15$, $x - y = 2N$ and H.C.F. $(x, y) = N$

Hence, (2) alone is insufficient. Even if (1) & (2) are used together, we cannot answer the question.

**31.** $K \times 0 \cdot \overline{ab} = K \times \frac{ab}{99}$

$\therefore$ The product of K and $0 \cdot \overline{ab}$ is an integer if K is a multiple of both 9 and 11.

Both Statements I and II are required to answer the question.

**32.** From Statement I, we have, as a multiple of 14, then the possibilities for $(x\ y,\ z)$ are (28, 3, 31), (42, 5, 47). So, the value of $x$ is not unique. Hence, Statement I alone is insufficient.

From Statement II. L.C.M. of $y$ and $z$ as 527. Here, 527 can be expressed as 17(31), both of which are prime. Hence, $x$ is 31 – 17 = 14.

$\therefore$ Statement II alone is sufficient.

**33.** From Statement I, $x$ and $y$ are integers. If xy is odd, then both $x$ and $y$ are odd. Hence, Statement I alone is sufficient.

From Statement II, $x + y$ is odd so either $x$ or $y$ is odd and the other is even. Hence, we cannot answer the question. Hence, Statement II alone is insufficient.

**34.** From Statement I, even numbered cards include 2, 4, 6, 8, 10. Since there are 4 sets, there are 20 even numbered cards, but in this pack there are only,16 even numbered cards. It is known that 4 are missing. Hence, all 4 are even numbered cards. But we cannot state which are the 4 cards missing hence, Statement I alone is not sufficient. From Statement II, there are three multiples of 3 in every set, i.e., 3, 6, 9. In the 4 sets there are 12 cards which are multiples of 4. But here there are 8 cards which bear multiples of 3 but we cannot say which numbered cards are missing. Statement II alone is not sufficient.

Using Statement I and II, we have 4 cards missing and they bear even numbers that are also multiples of three. Hence, the 4 sixes are missing.

**35.** The given number is 810$A$4$B$6$C$. We need to find A + B + C.

From Statement I, as the number is a multiple of 5 and 8, C = 0 and B is odd.

As the number is a multiple of 9, the possible values of A and B are as listed below.

| 8 | 1 | 0 | A | 4 | B | 6 | C |
|---|---|---|---|---|---|---|---|
| – | – | – | 7 | – | 1 | – | 0 |
| – | – | – | 5 | – | 3 | – | 0 |
| – | – | – | 3 | – | 5 | – | 0 |
| – | – | – | 1 | – | 7 | – | 0 |
| – | – | – | 8 | – | 9 | – | 0 |

From Statement II, both A and B are non prime. There are many possibilities for A, B and C.

Combining both (A, B, C) = (8, 9, 0)

$\therefore$ A + B + C = 17

**36.** $X$ is a 3 digit number that leaves a remainder one less than the divisor when divided by any number form 2 to 7. i.e., it is 1 less than the L.C.M. of 2, 3, 4, 5, 6, 7 = 420 – 1

$\therefore$ The numbers are of the form 420 $(k)$ – 1

i.e., $X = 420\ (k) - 1$

If $k = 1$ $X = 419$

If $k = 2$ $X = 839$

If $k = 3$ $X = 1259$

But since $X$ is a 3 digit number, $X = 419$ or 839.

From Statement I, when divided by 42, $X$ leaves a remainder of 41. This is true for both 419 and 839. Statement I alone is not sufficient.

From Statement II, when $X$ is divided by 13, it leaves a remainder of 7. 419 divided by 13 gives a remainder of 3 while 839 divided 13 leaves a remainder of 7. Hence, $X$ is 839. Hence, Statement II alone is sufficient.

**37.** $X$ is a natural number greater than 189.

From Statement I, the number has only five multiples less than 1000, i.e., it is less than 200.

$\therefore X \in \{190, 191, \ldots, 199\}$

$\therefore$ This statement alone does not determine $X$.

From Statement II, the number is odd and does not end in 5.

$\therefore X \in \{191, 193, 197, 199, \ldots\}$

$\therefore$ This statement alone is also not sufficient.

Combining statements I and II, there are 4 possible values of $X$ which are {191, 193, 197, 199} All the above numbers are prime, so $X$ is definitely a prime.

**38.** From Statement I, we have $a \times b = 24$

$\therefore (a, b) = (8, 3)\ (3, 8), (6, 4), (4, 6)$

There are 4 possible values.

$\therefore b \in \{3, 8, 4, 6\}$

The last digit of $(6ab)^{64}$ can be 1 or 6

Statement I alone is insufficient.

From Statement II, given $b > 4$. From this we cannot answer the question.

Using both statements,

$\therefore b$ could be 6 or 8. Now, the last digit could only be 6.

On combining the two statements, we can answer the question.

**39.** From Statement I, we have $F - D = 6$

If $F = 18$, $D = 12$ and if $F = 12$, $D = 6$.

Statement I alone is insufficient.

From Statement II, we have $D = \frac{4}{3}B. \Rightarrow B = \frac{3}{4}D$.

$\therefore B = 6$ if $D = 8$; $B = 9$ if $D = 12$ and $B = 12$ if $D = 16$

Again Statement II alone is not sufficient.

Using Statement I and Statement II we get $D = 12$

$\therefore B = 9$

Hence, both statements together are sufficient.

**40.** From Statement I, the sixth number is more than two-thirds of the set of numbers so the sixth number is more than 68. Hence, it can be 71 or 73 or any other prime number, Statement I alone is not sufficient.

from Statement II,

let the sixth number be $x$.

The average of 48, 60, 68, 72 is 62. Hence, the sixth number must be more than 48.

Hence, $\frac{60 + 68 + 72 + x}{4} = 69 \Rightarrow x = 76$.

Statement II alone is sufficient.

**41.** Let the three integers be $a$, $b$ and $c$.

Given, $abc = 40 \Rightarrow$ At least one of $a$, $b$, $c$ is even.

Statement I: $a + b + c$ is odd. Possibilities: 1 odd or 3 odd. As at least one is even, exactly 1 is odd.

Statement II: $ab + bc + ca$ is odd. Possibilities: 2 (of $a$, $b$, $c$) are odd or all (of $a$, $b$, $c$) are odd. As at least 1 ($a$, $b$, $c$) is even, exactly 2 are odd.

We can answer the question from either statement.

**42.** From Statement I, when abc is reversed it forms a two digit number. This means $c$ is 0. But $a$, $b$ are unknown. Hence, Statement I is insufficient.

From Statement II, $(1000e + 100f + 10g + h) - (1000h + 100g + 10f + e)$, i.e., abc $= 999e + 90f - 90g - 999h$

$\therefore$ abc is definitely divisible by 9. But we cannot say it is divisible by 90 or not. Statement II alone is not sufficient.

Combining the statements I and II, since it is divisible by both 10 and 9, it is divisible by 90. Using both statements we can answer.

**43.** From Statement I, we have RB is divisible by 9 and $R >$ B.

$\therefore$ (RB) = (6, 3) or (5, 4)

But we cannot say anything about G.

Hence, Statement I alone is insufficient.

From Statement II, RBG is an even multiple of 3.

$\therefore R$ + B + G would be divisible by 3 and G would be an even number. This would yield many possibilities.

So, Statement II alone not sufficient.

Using statements I and II together, we get that $R$ + B is a multiple of 9, i.e., (6, 3) or (5, 4) and $R$ + G + B is a even multiple of 3, i.e., G has to be 6. As $R$, G, B are distinct.

RGB has to be 546.

**44.** From Statement I, $N$ is the smallest number that leaves a remainder of 4 when divided by 12, 13 or 14.
Such numbers are of the form (L.C.M. of 12, 13, 14)

$k + 4 = 1092k + 4$ and the smallest such number is 1096.

Statement I so alone is sufficient.

From Statement II, $N$ is the smallest number of the form $k$L.C.M.(16, 17) – 10 or $272k – 10$, which is 262.

Hence, $N = 262$

Again this alone is sufficient. Hence, each statement alone is sufficient to answer the question.

**45.** $X$ is a prime number which when divided by the sum of its digits, leaves a prime number as remainder. (1)

Consider Statement I

When $X$ is divided by 12 or 18, it leaves the same remainder in each case.

$\Rightarrow 36 < x < 48$ or $72 < x < 84$. And the remainder so obtained is equal to the sum of the digits of $x$.

$\Rightarrow 40 \le X \le 47$ or $80 \le X \le 83$.

Now, the first part of the condition (1) says that $x$ is a prime number.

$\Rightarrow x = 41, 43, 47$ or 83

And the second part of the condition (1), says, $X$ when divided by the sum of its digits leaves a remainder which is a prime.

41, 43, 47 and 83 when divided by 5, 7, 11 and 11, respectively, leaves remainders of 1, 1, 3 and 6, respectively, of these remainders, only 3 is prime.

$\therefore X = 47$

Hence, Statement I alone is sufficient.

Let us now consider Statement II. It says when the number is divided by 4, 6, 8 it leaves a remainder which is 1 less than the divisor. It is of the form $24k – 1$, i.e., 23, 47, 71 and 95.

Of these 23, 47 and 71 satisfy the condition (1). Hence, Statement II alone is not sufficient.

# 2 Number Systems

CHAPTER

## Learning Objectives

*In this chapter, you will*:

- Learn about the representation of numbers in different bases
- Learn to convert
  - Decimal base to other bases
  - Other bases to decimal bases
- Understand about various elements of binary arithmetic, such as:
  - How to solve addition in equations
  - How to solve subtraction in equations
  - How to solve multiplication in equations

NUMBER SYSTEM is a method in which numbers are represented. The most widely used number system is the decimal number system, that we use in our day to day life. *Deca* is a Greek term which means ten. In the decimal number system, all numbers are formed using the ten symbols, namely 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. These ten symbols represent the first ten whole numbers in the decimal number system, thereafter any other number is just a combination of these ten digits. The role played by '10' in the decimal system is termed as the '**base**' of the system. Hence, decimal numbers are also referred to as the base 10 numbers. However, other such number systems also exist depending on the number of single digits used. For instance, If we consider the number 526 in the decimal system, it means $5 \times 10^2 + 2 \times 10^1 + 6 \times 10^0$. Likewise, 85.67 means $8 \times 10^1 + 5 \times 10^0 + 6 \times 10^{-1} + 7 \times 10^{-2}$. In the octal system all numbers are generated using eight digits viz., 0, 1, 2, 3, 4, 5, 6 and 7. In this chapter, we will study different number systems and arithmetic operations in the binary system.

## ❒ BASE

It is a number which decides the place value of a symbol or a digit in a number. Alternatively, it is the number of distinct digits/symbols that are used in that number system.

**NOTE**

(a) The base of a number system can be any integer greater than 1.
(b) Base is also termed as radix or scale of notation.

The following table lists some number systems along with their respective base and symbols.

| Number system | Base | Digits/Symbols |
|---|---|---|
| Binary | 2 | 0,1 |
| Septenuary | 7 | 0,1,2,3,4,5,6 |
| Octal | 8 | 0,1,2,3,4,5,6,7 |
| Decimal | 10 | 0,1,2,3,4,5,6,7,8,9 |
| Duo-decimal | 12 | 0,1,2,3,4,5,6,7,8,9,A,B |
| Hexa-decimal | 16 | 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F |

A = 10, B = 11, C = 12, D = 13, E = 14, F = 15. Some books denote 10 as 'E' and 11 as 'e'.

## ❒ REPRESENTATION

Let $N$ be any integer, r be the base of the system and let $a_0, a_1, a_2, \ldots, a_n$ be the required digits by which $N$ is expressed. Then,

$N = a_n r^n + a_{n-1} r^{n-1} + a_{n-2} r^{n-2} + \ldots + a_1 r + a_0$, where $0 \le a_i < r$.

We now look into some representations and their meaning in decimal system.

**Examples**

(i) $(100011)_2$
$= 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$
$= 32 + 0 + 0 + 2 + 1 = (35)_{10}$

(ii) $(1741)_8$
$= 1 \times 8^3 + 7 \times 8^2 + 4 \times 8^1 + 1 \times 8^0$
$= 512 + 448 + 32 + 1$
$= 993_{10}$

(iii) $(A3D)_{16}$
$= A \times 16^2 + 3 \times 16^1 + D \times 16^0$
$= 10 \times 256 + 48 + 13 = 2621_{10}$

## ❒ CONVERSIONS

**1. Decimal to binary:**

(i) $(253)_{10} = (11111101)_2$

Working:

| | | | |
|---|---|---|---|
| 2 | 253 | | |
| 2 | 126 | – | 1 |
| 2 | 63 | – | 0 |
| 2 | 31 | – | 1 |
| 2 | 15 | – | 1 |
| 2 | 7 | – | 1 |
| 2 | 3 | – | 1 |
| | 1 | – | 1 |

**NOTE**

The remainders are written from bottom to top.

(ii) $(37.3125)_{10} = (100101.0101)_2$

Working: The given decimal number has two parts:

(a) Integral part 37

(b) Fractional part 0.3125

(a) **Conversion of the integral part:**

| | | | |
|---|---|---|---|
| 2 | 37 | | |
| 2 | 18 | – | 1 |
| 2 | 29 | – | 0 |
| 2 | 4 | – | 1 |
| 2 | 2 | – | 0 |
| | 1 | – | 0 |

$\therefore (37)_{10} = (100101)_2$

(b) **Conversion of the fractional part:** Multiply the decimal part with 2 successively and take the integral part of all the products starting from the first.

Binary digits
$0.3125 \times 2 = 0.6250$ 0
$0.6250 \times 2 = 1.2500$ 1
$0.2500 \times 2 = 0.500$ 0
$0.5000 \times 2 = 1.0$ 1
$\therefore (0.3125)_{10} = (0.0101)_2$

**NOTE**

We should stop multiplying the fractional part by 2, once we get 0 as a fraction or the fractional part is non-terminating. It can be decided depending on the number of digits in the fractional part required.

**2. Binary to decimal:**

(i) $(101011011)_2 = (347)_{10}$

Working:
$(101011001)_2 = 1 \times 2^8 + 0 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$
$= 256 + 0 + 64 + 0 + 16 + 8 + 0 + 2 + 1$
$= (347)_{10}$

(ii) $(0.11001)_2 = (0.78125)_{10}$

Working:
$(0.11001)_2 = 1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 0 \times 2^{-4} + 1 \times 2^{-5}$
$= 1/2 + 1/4 + 1/32 = 25/32 = (0.78125)_{10}$

**3. Decimal to octal:**

(i) $(2595)_{10} = (5043)_8$

Working:

| | | | |
|---|---|---|---|
| 8 | 2595 | | |
| 8 | 324 | – | 3 |
| 8 | 40 | – | 4 |
| | 5 | – | 0 |

$\therefore (2595)_{10} = (5043)_8$

**4. Octal to decimal:**

(i) $(4721)_8 = (2513)_{10}$

Working:
$(4721)_8$
$= 4 \times 8^3 + 7 \times 8^2 + 2 \times 8^1 + 1 \times 8^0$
$= 2048 + 448 + 16 + 1 = (2513)_{10}$

(ii) $(365.74)_8 = (245.9375)_{10}$

Working:

**(a) Integral part:**

$(365)_8$

$= 3 \times 8^2 + 6 \times 8^1 + 5 \times 8^0$

$= 192 + 48 + 5 = 245$

$\therefore (365)_8 = (245)_{10}$

**(b) Fractional part:**

$(0.74)_8 = 7 \times 8^{-1} + 4 \times 8^{-2}$

$= \frac{56+4}{64} = \frac{60}{64} = 0.9375$

$\therefore (365.74)_8 = (245.9375)_{10}$

**5. Decimal to hexa-decimal:**

(i) $(47239)_{10} = (B887)_{16}$

Working:

| | | | |
|---|---|---|---|
| 16 | 47239 | | |
| 16 | 2952 | – | 7 |
| 16 | 184 | – | 8 |
| | 11 | – | 8 |

Recall: 11 is B, in hexa-decimal system.

$\therefore (47239)_{10} = (B887)_{16}$

(ii) $(30014)_{10} = (753E)_{16}$

Working:

| | | | |
|---|---|---|---|
| 16 | 30014 | | |
| 16 | 1875 | – | 14 = E |
| 16 | 117 | – | 3 |
| | 7 | – | 5 |

$\therefore (30014)_{10} = (753E)_{16}$

**6. Hexa-decimal to decimal:**

$(52B)_{16} = (1323)_{10}$

Working:

$(52B)_{16}$

$= 5 \times 16^2 + 2 \times 16^1 + B \times 16^0$

$= 1280 + 32 + 11$

$= (1323)_{10}$

$\therefore (52B)_{16} = (1323)_{10}$

**7. Decimal to duo-decimal or duodenary (base 12):**

$(948)_{10} = (66C)_{12}$

Working:

| | | | |
|---|---|---|---|
| 12 | 948 | | |
| 12 | 78 | – | 12 or C |
| | 6 | – | 6 |

$\therefore (948)_{10} = (66C)_{12}$

**8. Duo-decimal to decimal:**

$(5BC)_{12} = (864)_{10}$

Working:

$(5BC)_{12} = 5 \times 12^2 + B \times 12^1 + C \times 12^0$

$= 720 + 132 + 12 = (864)_{10}$

**9. Binary to octal:**

8 being the base of octal system and 2 being the base of binary system, there is a close relationship between both the systems. One can just club three digits of a binary number into a single block and write the decimal equivalent of each group (left to right).

**Example:**

(i) $(100101111)_2 = (100)_2\ (101)_2\ (111)_2 = (457)_8$

$\therefore (100101111)_2 = (457)_8$

(ii) $(11111110)_2 = (011)_2\ (111)_2\ (110)_2 = (376)_8$

$\therefore (11111110)_2 = (376)_8$



**NOTE**

Introduce leading zeros to form a block of 3 without changing the magnitude of the number.

**10. Binary to hexa-decimal:**

This is similar to the method discussed for octal; instead of clubbing 3, we club 4 digits.

**Example:**

$(10111110)_2 = (1011)_2\ (1110)_2 = (11)\ (14) = (BE)_{16}$

$\therefore (10111110)_2 = (BE)_{16}$



**NOTE**

If the number of digits is not a multiple of 4, introduce leading zeros as done earlier for octal conversion.

## ❒ BINARY ARITHMETIC

**Addition:**

Elementary Rules

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 10$

$1 + 1 + 1 = 11$

(1 will be regarded as carry as we do in decimal system)

Examples of binary addition:

**1.** $(110101)_2 + (110)_2$

1 → carry

1 1 0 1 0 1

0 0 0 1 1 0 (Introduce leading zeros)

1 1 1 0 1 1

**2.** $(101111)_2 + (111011)_2$

1 1 1 1 1 → carry

1 0 1 1 1 1

1 1 1 0 1 1

1 1 0 1 0 1 0

**3.** $(110)_2 + (100)_2 + (010)_2$

| | | |
|---|---|---|
| 1 | → | carry |
| 1 1 0 | | |
| 1 0 0 | | |
| 0 1 0 | | |
| 1 1 0 0 | | |

**Subtraction:**

1. Subtract 1101 from 11010.

| | |
|---|---|
| | 2 |
| | 0 0 2 0 2 |
| | 1 1 0 1 0 |
| | – 1 1 0 1 |
| result→ | 1 1 0 1 |

Explanation: Say $N = 11010$,

As 1 cannot be subtracted from 0, we borrow 2 from the next place. This gives $2 - 1 = 1$, as the right most digit of the result. The penultimate digit of $N$ would become 0. A similar calculation gives the 3rd digit of the result from the right as 1 and the 4th digit of $N$ from the right becomes 0.

We now borrow a 2 from the 5th digit of $N$, this makes the 4th digit of $N$ as 2, thereby resulting in $2 - 1 = 1$ as the 4th digit of the result.

**2.** Subtract 11011 from 111001

| | | |
|---|---|---|
| 2 2 1 | | |
| 0 0 2 2 | → | Borrow |
| 1 1 1 0 0 1 | | |
| –1 1 0 1 1 | | |
| 1 1 1 1 0 | | |

## SOLVED EXAMPLES

**2.01.** Show that the binary number (11101010010001) is equal to $(35221)_8$ and $(3A91)_{16}$.

**Sol:** (a) We keep forming blocks of 3 from left to right. Also, if the number of digits is not a multiple of 3, then we introduce leading zeros (since the inclusion of zero to the left will not affect the value).

$(011\ 101\ 010\ 010\ 001)_2$

$= ((011)_2\ (101)_2\ (010)_2\ (010)_2\ (001)_2)_8$

$= (35221)_8$.

(b) We keep forming blocks of 4 from left to right (introduce lead zeros)

$(0011\ 1010\ 1001\ 0001)_2$

$= ((0011)_2\ (1010)_2\ (1001)_2\ (0001)_2)_{16}$

$= (3A91)_{16}$.

**2.02.** Show that 121 is a perfect square in any base greater than 2.

**Sol:** Let $n$ be the base of a number system ($n \geq 3$).

$(121)_n = n^2 + 2n + 1 = (n+1)^2$.

Now $(n + 1)^2$ being a perfect square, for any value of $n$

$\therefore$ 121 is a perfect square in any base greater than 2.

**2.03.** If $f(x, y, z) = (x+y)\ (y+z)\ (x+z)$, where $x, y$ and $z$ are decimal numbers, then find $f((13)_4, (11)_8, (17)_{10})$.

**Sol:** $(13)_4 = 4^1 \times 1 + 4^0 \times 3 = 4 + 3 = (7)_{10}$

$(11)_8 = 8^1 \times 1 + 8^0 \times 1 = 8 + 1 = (9)_{10}$

$(17)_{10} = (17)_{10}$

Now, the numbers are in common base of 10.

$f(x, y, z) = (x+y)\ (y+z)\ (z+x)$

$f(7, 9, 17) = 16 \times 26 \times 24 = (9984)_{10}$

**2.04.** Find the base $k$ of the number system, if $(543)_6 = (317)_k$.

**Sol:** $(543)_6 = 5 \times 6^2 + 4 \times 6 + 3 \times 6^0 = (207)_{10}$

i.e., $(207)_{10} = (317)_k$

$207 = 3k^2 + k + 7 \Rightarrow 3k^2 + k - 200 = 0$

$\Rightarrow 3k^2 - 24k + 25k - 200 = 0$

$\Rightarrow (3k + 25)\ (k - 3) = 0$

$\Rightarrow k = 8$ or $-25/3$ (not possible)

$\Rightarrow k = 8$

$\therefore$ The base is 8.

**2.05.** Find the hexa-decimal equivalent of the number $(174356)_8$

**Sol:** We initially convert the number $(174356)_8$ into binary system, by converting each digit into a triplet in terms of binary digits.

| 1 | 7 | 4 | 3 | 5 | 6 |
|---|---|---|---|---|---|
| ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |

$(001)_2\ (111)_2\ (100)_2\ (011)_2\ (101)_2\ (110)_2$

$(1111100011101110)_2$

Now, we group four digits into a single block (left to right) to get the hexa-decimal equivalent.

$(1111)_2\ (1000)_2\ (1110)_2\ (1110)_2 = (F8EE)_{16}$

**2.06.** Multiply $(134)_5$ and $(220)_5$.

**Sol:** We convert each number into base 10. After computing the product in base 10, we convert it back to base 5.

$(134)_5 = 1 \times 5^2 + 3 \times 5 + 4 \times 1$

$= 25 + 15 + 4 = (44)_{10}$

$(220)_5 = 2 \times 5^2 + 2 \times 5 + 0 \times 1$

$= 50 + 10 + 0 = (60)_{10}$

$44 \times 60 = (2640)_{10}$

| | | |
|---|---|---|
| 5 | 2640 | |
| 5 | 528 | – 0 |
| 5 | 105 | – 3 |
| 5 | 21 | – 0 |
| | 4 | – 1 |

so $(2640)_{10} = (41030)_5$
$\therefore (134)_5 \times (220)_5 = (41030)_5$

**2.07.** Which of these weights among 1, 2, 4, 8, 16, etc., kgs are used in weighing 250 kgs, if, not more than one weight of each denomination is used for weighing?

**Sol:** The denominations are powers of 2, so we express 250 as sum of some powers of 2
Accordingly, 250 = 128 + 64+ 32 + 16 + 8 + 2.
Thus, on expressing 250 in binary scale, we get 11111010, the place value of 1's are the weights required for weighing.

**2.08.** Subtract $(23644)_7$ from $(41066)_7$.

**Sol:** 41066
23644
14122
Explanation: 6 – 4 = 2 for the two rightmost digits.
As we cannot deduct 6 from 0, we now borrow 1 (= 7) from the next place and add it to 0. As (0 + 7) = 7, we deduct 6 from 7 which is 1. As one is borrowed from 1, it becomes 0. Again, since 3 cannot be deducted from 0, we borrow 1 from 4. Similarly, we can proceed.

**2.09.** Find the binary equivalent of the fraction 0.325.

**Sol:** Multiply the fractional part by 2. If we get any integer part, we take1, otherwise, 0 is the binary digit. Each time, we multiply by 2 and take the fraction part for the next time. Once the fractional part becomes 0, we stop, and the binary equivalent of the fraction in 1's or 0's is taken in order from top to bottom as it is obtained in each step.
Also, if the fraction does not terminate, we can stop the process after a certain number of times. The binary equivalent obtained will be the approximate value of the fraction.
So, $(0.325)_{10}$

| Steps: | binary |
|---|---|
| (1) $0.325 \times 2 = 0.65$ | 0 |
| (2) $0.65 \times 2 = 1.3$ | 1 |
| (3) $0.3 \times 2 = 0.6$ | 0 |
| (4) $0.6 \times 2 = 1.2$ | 1 |
| (5) $0.2 \times 2 = 0.4$ | 0 |
| (6) $0.4 \times 2 = 0.8$ | 0 |

As the fractional part is small, we can stop.
$\therefore (0.325)_{10} = (0.010100)_2$

**2.10.** A non-zero number in base 5 is such that twice the number equals the number formed by reversing the digits. Find the number.

**Sol:** Let the number be $(xy)_5$, where $0 \le x, y < 5$
The number formed by reversing the digits is $(yx)_5$
Now $(xy)_5 = (5x + y)_{10}$ and $(yx)_5 = (5y + x)_{10}$
Given $2(xy)_5 = (yx)_5$
So, $2(5x+ y) = 5y + x$
$\Rightarrow 9x = 3y$
$\Rightarrow 3x = y$
As the number is non - zero, $x = 1$ and $y = 3$.
Then, the number is 13.

**2.11.** Show that the binary number 101011011001101 is equal to $(53315)_8$ and $(56CD)_{16}$.

**Sol:** (i) To represent any binary number in base 8 we divide it into blocks of 3 starting from the left. If the number of digits in it is not a multiple of 3, leading zeros are introduced in it since inclusion of 0 to the left does not affect its value.
$(101011011001101)_2$
$= [(101)_2\ (011)_2\ (011)_2\ (001)_2\ (101)_2]$
$= (53315)_8$
(ii) To represent any binary number in base 16 we divide it into blocks of 4 starting from the left. If the number of digits in it is not a multiple of 4, leading zeros are introduced in it since inclusion of 0 to the left does not affect its value.
$(0101011011001101)_2$
$= [(0101)_2\ (0110)_2\ (1100)_2\ (1101)_2]_{16}$
$= (56CD)_{16}$ ($\because 1100_2 = 12 = C$
and $1101_2 = 13 = D$)

**2.12.** Show that 144 is a perfect square in any base more than 4.

**Sol:** Let $n$ be the base of a number system ($n > 4$).
$(144)_n = n^2 + 4n + 4 = (n + 2)^2$
which is a perfect square for all values of $n$, and hence the given number is perfect square for all values of $n > 4$

**2.13.** If a $(p, q, r) = (p + q)\ (q + r)\ (p + r)$, find a $[(32)_4, (22)_8, (16)_{10}]$.

**Sol:** $(32)_4 = (4)\ (3) + 2 = (14)_{10}$
$(22)_8 = (8)\ (2) + 2 = (18)_{10}$
$(16)_{10} = (16)_{10}$
$a((32)_4, (22)_8, (16)_{10}) = a(14, 18, 16) = (14 + 18)\ (18 + 16)\ (14 + 16) = (32)\ (34)\ (30) = 32640$

**2.14.** If $(624)_7 = 312_k$ then find $k$.

**Sol:** $(624)_7 = (6)\ (7^2) + (2)\ (7) + (4)\ (7)^0 = 312$.

$(312)_k = 3k^2 + k + 2$

Given $(312)_k = (624)_7$

$3k^2 + k + 2 = 312$

$3k^2 + k - 310 = 0$

$(k - 10)\ (3k + 31) = 0$

$k > 0$

$\therefore\ k = 10$

**2.15.** Find the hexadecimal equivalent of the number $(234567)_8$.

**Sol:** $(234567)_8$

$= (10\quad 011\quad 100\quad 101\quad 110\quad 111)_2$

$= (0001\ 0011\quad 1001\quad 0111\quad 0111)_2$

$= (1\quad 3\quad 9\quad 7\quad 7)_{16}$

$= (13977)_{16}$

**2.16.** Subtract $(124368)_{11}$ from $(987654)_{11}$.

**Sol:** We write the larger number on top and the smaller number at the bottom and align the digits from the right. If the lower digit is smaller or equal to the upper digit, we carry out at the usual subtraction. If the lower digit is greater, we borrow from the immediate neighbour on the left. But we have to remember that a 'loan' of 1 from the left neighbour represents not ten but the base of the system, in this case – 11.

| | | | | | |
|---|---|---|---|---|---|
| 9 | 8 | 7 | 6 | 5 | 4 |
| 1 | 2 | 4 | 3 | 6 | 8 |
| 8 | 6 | 3 | 2 | 9 | 7 |

**2.17.** Which of these weights (all in kg) among 1, 2, 4, 8, …, etc., are used in weighing 500 kg if not more than one weight of each denomination can be used for the weighing?

**Sol:** $500 = 256 + 128 + 64 + 32 + 16 + 4$

Thus, expressing 500 in binary scale, we get 111110100.

The place values of 1's are the weights required for weighing.

**2.18.** Multiply $(123)_4$ and $(330)_4$.

**Sol:** We convert each number to base 10 and then multiply the results.

$(123)_4 = (1)\ (4^2) + (2)\ (4) + (3)\ (1) = (27)_{10}$

$(330)_4 = (3)\ (4^2) + (3)\ (4) + (0)\ (1) = (60)_{10}$

$(27)\ (60) = (1620)_{10}$

| | | |
|---|---|---|
| 4 | 1620 | |
| 4 | 405 | 0 |
| 4 | 101 | 1 |
| 4 | 25 | 1 |
| 4 | 6 | 1 |
| | 1 | 2 |

$(1620)_{10} = (121110)_4$

**2.19.** Find the binary equivalent of the fraction 0.625.

**Sol:** Consider a fraction $x$, i.e., $0 \le x < 1$.

If $0 \le x < 1/2$, the first digit after the point (in base 2 representation) is 0.

If $1/2 \le x < 1$, the first digit after the point is 1.

i.e., if $0 \le 2x < 1$, the first digit is 0 and if $1 \le 2x < 2$, it is 1.

After this part (the integral part of $2x$) of the fraction is represented (either as 0 or 1), we can proceed along the same lines and represent smaller and smaller parts of the residue.

If the denominator of the fraction is a power of 2, we get a terminating binary fraction. If there are other factors in the denominator, we get a non-terminating binary fraction. For the given fraction successive digits are calculated in the table below.

| $n$ | Residue | 2 (Residue) | Digit |
|---|---|---|---|
| 1 | 0.625 | 1.25 | 1 |
| 2 | 0.25 | 0.5 | 0 |
| 3 | 0.5 | 1 | 1 |

$\therefore\ 0.625 = (0.101)_2$

**2.20.** A non-zero number in base 8 is such that twice the number is the number formed by reversing its digits. Find the number.

**Sol:** Let the number be $(xy)_8$,

where $0 \le x, y < 8$.

The number formed by reversing its digits is $(yx)_8$.

$2(xy)_8 = (yx)_8$

$2(8x + y) = 8y + x$

$$\frac{x}{y} = \frac{2}{5}$$

$x = 2$ and $y = 5$ is the only possibility.

$\therefore\ (xy)_8 = (25)_8$.

**2.21.** If $(23)_4 + (100)_2 = (x)_{10}$, then find the value of $x$.

**Sol:** $(23)_4 = 2\ (4) + 3 = 11$

$(100)_2 = 4$

$\therefore x = (23)_4 + (100)_2 = 11 + 4 = 15$.

**2.22.** If $(46)_7 - (13)_5 = (y)_{10}$, then find $y$.

**Sol:** $(46)_7 = ((7)\ (4) + 6)_{10} = (34)_{10}$

$(13)_5 = ((5)\ (1) + 3)_{10} = (8)_{10}$

$\therefore y = 26$.

## EXERCISE-1

***Directions for questions 1–35:*** For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** The binary equivalent of the decimal 132 is
(A) $(1001000)_2$ (B) $(10001100)_2$
(C) $(10000100)_2$ (D) $(10001000)_2$

**2.** The binary equivalent of the decimal number 502 is
(A) $(111010011)_2$ (B) $(101111001)_2$
(C) $(100111111)_2$ (D) $(111110110)_2$

**3.** The decimal equivalent of the binary number 1000001 is _____.

**4.** The septenary equivalent of the decimal number 235 is
(A) $(445)_7$ (B) $(454)_7$ (C) $(544)_7$ (D) $(444)_7$

**5.** The septenary equivalent of the decimal number 532 is _____.

**6.** The duo-decimal equivalent of the decimal number 1364 is
(A) $(958)_{12}$ (B) $(598)_{12}$ (C) $(985)_{12}$ (D) $(589)_{12}$

**7.** The duo-decimal equivalent of the decimal number 1463 is
(A) $(A19)_{12}$ (B) $(AB1)_{12}$
(C) $(A1B)_{12}$ (D) $(BA1)_{12}$

**8.** The largest four-digit octal number is
(A) $(6666)_8$ (B) $(7000)_8$
(C) $(7666)_8$ (D) $(7777)_8$

**9.** The octal equivalent of the decimal number 239 is
(A) $(753)_8$ (B) $(75)_8$ (C) $(57)_8$ (D) $(357)_8$

**10.** The minimum number of bits required to represent the decimal number 432 in binary system is
(A) 9 (B) 8 (C) 10 (D) 7

**11.** A decimal number when represented in the binary system has its last three digits as zero. The number (in decimal system) can be
(A) 100 (B) 5 (C) 18 (D) 8

**12.** The decimal equivalent of the hexadecimal number AEB is
(A) $(2595)_{10}$ (B) $(2795)_{10}$
(C) $(2790)_{10}$ (D) $(2790)_{10}$

**13.** The decimal equivalent of the hexadecimal number BAD is
(A) 2998 (B) 2899 (C) 2989 (D) 2798

**14.** The hexa-decimal equivalent of the decimal number 1734 is
(A) $(CC6)_{16}$ (B) $(BC6)_{16}$
(C) $(C6C)_{16}$ (D) $(6C6)_{16}$

**15.** $(352)_6 \times (13)_6$ is
(A) $(5500)_6$ (B) $(4500)_6$
(C) $(1200)_6$ (D) $(5400)_6$

**16.** In the duodecimal system, the numerical value of A is _____.

**17.** $(234)_8 + (432)_8 =$
(A) $(666)_{10}$ (B) $(666)_8$ (C) $(666)_7$ (D) $(776)_8$

**18.** The binary equivalent of the octal number 437 is
(A) $(10010111)_2$ (B) $(100011111)_2$
(C) $(100111)_2$ (D) $(10101111)_2$

**19.** The number of digits required to represent the decimal number 213 in base – 5 system is
(A) 6 (B) 3 (C) 5 (D) 4

**20.** Which of the following is equivalent to $(99)_{10}$?
(A) $(243)_6$ (B) $(201)_7$
(C) $(143)_8$ (D) All the above

**21.** The last four bits in the binary representation of a multiple of 16 could be
(A) 0100 (B) 1000 (C) 1100 (D) 0000

**22.** The hexadecimal equivalent of the binary number 1110011110100 is
(A) $(1C4F)_{16}$ (B) $(14CF)_{16}$
(C) $(1FC4)_{16}$ (D) $(1CF4)_{16}$

**23.** The binary equivalent of the decimal number 17 is
(A) 10010 (B) 10001
(C) 10100 (D) 11000

**24.** To express a number in the binary system, the digits we use are.
(A) 0, 1, 2, 3 (B) 0, 1, 2
(C) 0, 1 (D) 0, 1, 2, 3, 4

**25.** The decimal equivalent of the binary number 100001 is
(A) 34 (B) 33 (C) 65 (D) 17

**26.** The largest three-digit septenary number is _____.

**27.** If $(152)_k = (86)_{10}$, then $k =$
(A) 6 (B) 8 (C) 7 (D) 9

**28.** The decimal equivalent of the binary number 1. 011 is
(A) 1.0375 (B) 0.0375 (C) 1.3075 (D) 1.375

**29.** The square of $(321)_7$ is
(A) $(163341)_7$ (B) $(163531)_7$
(C) $(136341)_7$ (D) $(143631)_7$

**30.** Which of the following is equivalent to $(88)_{10}$?
(A) $(154)_7$ (B) $(224)_6$
(C) $(107)_9$ (D) All the above

**31.** The cube root of $(224)_5$ in base 3 is _____.

**32.** The largest three-digit duo-decimal number is
(A) $(BBB)_{12}$ (B) $(AAA)_{12}$
(C) $(999)_{12}$ (D) $(777)_{12}$

**33.** The binary representation of decimal number 129 is
(A) $(11000000)_2$ (B) $(10000010)_2$
(C) $(1000001)_2$ (D) $(10000001)_2$

**34.** The cube root of the nonary number 1331 is
(A) $(10)_{10}$ (B) $(11)_{10}$
(C) $(10)_8$ (D) $(11)_8$

**35.** $(B21)_{12} - (398)_{12}$ is
(A) $(823)_{11}$ (B) $(723)_{12}$
(C) $(745)_{12}$ (D) $(754)_{12}$

## EXERCISE-2

***Directions for questions 1–35***: For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** The binary equivalent of the decimal number 213 is
(A) $(11010101)_2$ (B) $(101010101)_2$
(C) $(10110101)_2$ (D) $(11011011)_2$

**2.** The decimal equivalent of $(3AC)_{13}$ is _____.

**3.** The duo-decimal equivalent of the decimal number 123456 is
(A) $(45B5)_{12}$ (B) $(5B540)_{12}$
(C) $(511540)_{12}$ (D) $(5B54)_{12}$

**4.** The octal equivalent of the decimal number 147 is
(A) 143 (B) 233 (C) 223 (D) 221

**5.** The octal equivalent of the number $(100101011)_2$ is _____.

**6.** $(231)_{16} - (231)_8 =$
(A) $(305)_9$ (B) $(525)_{13}$
(C) $(143)_{11}$ (D) $(341)_{11}$

**7.** The duo-decimal equivalent of the decimal number 1357 is
(A) 233 (B) 543 (C) 951 (D) 781

**8.** The hexa-decimal equivalent of the octal number 1752 is
(A) 3*AE* (B) 3*EA* (C) 5*BC* (D) 5*CC*

**9.** Compute $(110110)_2 - (10001)_2$.
(A) $(112)_4$ (B) $(25)_7$ (C) $(104)_5$ (D) $(211)_4$

**10.** $(101101)_2 + (201)_8 + (453)_{10} =$
(A) $(528)_{11}$ (B) $(766)_9$
(C) $(344)_{12}$ (D) $(3611)_8$

**11.** The hexa-decimal equivalent of the decimal number 546 is
(A) 112 (B) 242 (C) 322 (D) 222

**12.** The octal equivalent of the number $(10010001)_2$ is
(A) 441 (B) 212 (C) 221 (D) 122

**13.** If $(125)_k = (68)_{10}$, then $k =$ _____.

**14.** $(215)_8 + (476)_8 =$
(A) $(713)_{10}$ (B) $(713)_8$
(C) $(691)_{10}$ (D) $(731)_8$

**15.** If f $(x, y, z) = (x + 2y)\ (2y + z)\ (z + x)$, then find the value of f$((A)_{16}, (11)_2, (13)_8)$ _____.

**16.** If '0' is concatenated to the rightmost digit of a positive binary integer, the number thus formed is
(A) same as the original number.
(B) half that of the original number.
(C) double the original number.
(D) None of these

**17.** The square of $(112)_6$ is
(A) $(12544)_6$ (B) $(12344)_6$
(C) $(21523)_6$ (D) $(21544)_6$

**18.** The LCM of $(54)_8$ and $(20)_8$ is
(A) $(54)_8$ (B) $(260)_8$
(C) $(120)_8$ (D) $(360)_8$

**19.** In which of the following scales is the number 1654 a perfect square?
(A) 8 (B) 7 (C) 11 (D) 12

**20.** The LCM of $(310)_4$ and $(110)_4$ is
(A) $(2021)_5$ (B) $(1112)_6$
(C) $(10011)_4$ (D) $(265)_{10}$

**21.** The numbers $(62)_8$, $(144)_8$ and $(226)_8$ are in
(A) AP (B) GP
(C) HP (D) Both AP and GP

**22.** The minimum number of bits required to represent the decimal number 281 in binary system is _____.

**23.** If $(a)_{10} \Delta (b)_{10} = (2a + b - 2)_{10}$, then $(101)_2 \Delta (100)_2 =$
(A) $(1100)_2$ (B) $(10)_{10}$
(C) $(101)_{10}$ (D) $(1001)_2$

**24.** The product of $(45)_6$ and $(20)_4$ expressed in duodenary system is
(A) $(248)_{12}$ (B) $(174)_{12}$
(C) $(147)_{12}$ (D) $(900)_{12}$

**25.** Compute $(10101)_2 - (1010)_2$.
(A) 0 (B) 1
(C) $(1010)_2$ (D) $(1011)_2$

**26.** If the radix of the number $x$ is $n$, then $(n-1)_n + (n-2)_{n-1} + \ldots\ldots\ldots + (1)_2 =$
(A) $\left(\frac{n(n+1)}{2}\right)_{10}$ (B) $(n+1)_{10}$
(C) $(n)_{10}$ (D) $\left(\frac{n(n-1)}{2}\right)_{10}$

**27.** If $a + b + c + d + e$ leaves a remainder of 3 when divided by $n$, $(n > 6)$ then the remainder of $(abcde)_{n+1}$ when divided by $n$ is
(A) 1 (B) 2 (C) 3 (D) n – 3

**28.** If the arithmetic mean of $(39)_{11}$ and $(62)_9$ is $(144)_n$, then the sum of $(32)_4$ and $(21)_5$ in a system with radix $n$ is _____.

**29.** If $(346)_n = (1211)_5$, then $(235)_{10}$ in a system with radix $n$ is _____.

**30.** If 4 and 6 are the roots of the quadratic equation $x^2 - ax + 22 = 0$, in a certain system, find the radix of the system. $(a = 10)$
(A) 18 (B) 19 (C) 11 (D) 12

**31.** The square of $(132)_4$ is
(A) $(4242)_7$ (B) $(10230)_4$
(C) $(2424)_7$ (D) $(32012)_4$

**32.** The number $(10111001)_2$ in hexa-decimal scale is
(A) *AB* (B) *B*9
(C) *BC* (D) *CE*

**33.** The square root of the octal number 341 is
(A) $(15)_8$ (B) $(17)_8$
(C) $(17)_{10}$ (D) $(15)_7$

**34.** In which of the following scale is the number 3120 a perfect cube?
(A) 4 (B) 5 (C) 6 (D) 7

**35.** If the arithmetic mean of $(12)_6$ and $(33)_7$ is 22, then the radix of 22 is
(A) 6 (B) 9
(C) 8 (D) 7

## EXERCISE-3

***Directions for questions 1–35:*** For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** The duo-decimal equivalent of the decimal number 2156 is
(A) $(12B8)_{12}$ (B) $(18B)_{12}$
(C) $(1B18)_{12}$ (D) $(111B)_{12}$

**2.** The decimal equivalent of $(DB3)_{16}$ is
(A) 5307 (B) 3500 (C) 3057 (D) 3507

**3.** $(243)_{16} - (243)_8 + (243)_6 =$
(A) $(4030)_5$ (B) $(4003)_5$
(C) $(4300)_5$ (D) $(3040)_5$

**4.** The square root of the octal number (551) is
(A) $(27)_8$ (B) $(23)_8$ (C) $(26)_8$ (D) $(21)_8$

**5.** If the LCM of $(41)_k$ and $(36)_{k+2}$ is $(150)_{10}$ and GCD is $(5)_{10}$, then $k$ is
(A) 6 (B) 5
(C) 8 (D) 7

**6.** $(386)_{12} - (177)_{12} =$
(A) $(206)_{12}$ (B) $(20B)_{12}$
(C) $(209)_{12}$ (D) $(2BB)_{12}$

**7.** The binary equivalent of $(57.140625)_{10}$ is
(A) $(110101.001001)_2$ (B) $(111001.001001)_2$
(C) $(111001.1000100)_2$ (D) $(110101.001101)_2$

**8.** All 7, 8 or 9-digit numbers in base $m$ can be represented as 5 or 6-digit numbers in base $n$. Which of the following is a possible value of $(m, n)$?
(A) (2, 3) (B) (3, 6)
(C) (3, 7) (D) (4, 8)

**9.** A decimal number, which is represented by the scales of 3, 4, 5, and 7 has 1, 2, 3, and 5, respectively, as the digits on its extreme right.
The smallest such positive number is _____.

**10.** The five numbers $a, b, c, d$ and $e$ are $(26)_7$, $(104)_6$, $(88)_9$, $(120)_{10}$ and $(114)_{12}$, respectively. Choose the correct statement.
(A) $a, b, c$ are in AP (B) $b, c, d$ are in GP
(C) $c, d, e$ are in GP (D) $a, c, e$ are in AP

**11.** If $(a)_{10}$ \$ $(b)_{10} = (3a + 2b - 5)_{10}$, then $(17)_8$ \$ $(51)_8$ is
(A) $(271)_8$ (B) $(217)_8$ (C) $(172)_8$ (D) $(127)_8$

**12.** Each of the 6, 7 or 8 digit number in base $m$ can be represented as 4 or 5 digit numbers in base $n$. Which of the following is a possible value of $(m, n)$?
(A) (10, 12) (B) (4, 10) (C) (4, 7) (D) (2, 5)

**13.** If $f(x, y, z) = 2x + 3y - z$, then $f((27)_8, (27)_{10}, (27)_{16}) =$
(A) 86 (B) 87 (C) 88 (D) 127

**14.** The remainder obtained when $(101010)_2$ is divided by $(101)_2$ is
(A) $(10)_2$ (B) $(100)_2$ (C) $(101)_2$ (D) $(101)_3$

**15.** The octal equivalent of the binary number $(10001001)_2$ is
(A) $(122)_8$ (B) $(211)_8$ (C) $(121)_8$ (D) $(221)_8$

**16.** If the LCM of $(51)_k$ and $(50)_{k+2}$ is $(180)_{10}$ ,and their GCD is $(9)_{10}$, then $k =$ _____.

**17.** If $f(a, b, c) = 3a + 2b - c$, then $f[(23)_{10}, (21)_8, (23)_5] =$
(A) $(231)_8$ (B) $(1011010)_2$
(C) $(315)_6$ (D) $(7B)_{14}$

**18.** $(43916)_{10} = (\ldots\ldots\ldots)_{16}$
(A) AB8C (B) BAC8
(C) ABC8 (D) CA8B

**19.** Add $(2354)_7$ and $(4235)_7$. _____ (7)

**20.** Subtract $(45026)_7$ from $(51231)_7$.= _____(7)

**21.** The number $(7316)_9$is divisible by
(A) 7 (B) 8 (C) 9 (D) 11

**22.** $(B)_{12} + (BB)_{12} + (BBB)_{12} + (BBBB)_{12} =$ _____ 10.

**23.** The number of three-digit numbers in septenary (base 7) system is _____.

**24.** The product of $(34)_7$ and $(31)_8$ is
(A) $(441)_{12}$ (B) $(443)_{12}$
(C) $(421)_{12}$ (D) $(431)_{12}$

**25.** The hexadecimal equivalent of the octal number 23516 is
(A) $(247E)_{16}$ (B) $(27E4)_{16}$
(C) $(274E)_{16}$ (D) $(427E)_{16}$

**26.** $(1010101)_2 + (10101)_4 + (101)_8 =$
(A) $(1A7)_{16}$ (B) $(A71)_{16}$
(C) $(17A)_{16}$ (D) $(72A)_{16}$

**27.** In which of the following scales is the number 2333 a perfect cube?
(A) 4 (B) 8 (C) 7 (D) 5

**28.** If '0' is concatenated to the rightmost digit of a positive binary integer, the number thus formed is
(A) same as the original number.
(B) half that of the original number.
(C) double the original number.
(D) None of these

**29.** If $(1002)_n = (345)_{10}$, then find the value of $n$ _____.

**30.** The square of the number $(325)_8$ is
(A) $(130471)_8$ (B) $(120473)_8$
(C) $(111476)_8$ (D) $(170473)_8$

**31.** A trader wants to have three identical sets of 3 distinct weights (9 weights altogether). His intention is to measure all integral weights from 1 to $N$ kgs using a simple balance. What is the maximum value of '$N$' that he can measure, if he
(i) puts weights only in one of the pans of the balance.
(ii) puts weights in both the pans of the balance.
(i) _____ (ii) _____

**32.** A four-digit number $N_1$ is written in base 13. A new four-digit number $N_2$ is formed by rearranging the digits of $N_1$ in any order. The difference between $N_1$ and $N_2$ is always divisible by _____.

**33.** A trader wants to measure (all integral weights) from 1 to 121 kg using a common balance where weights can be kept in both the pans. What is the minimum number of weights needed? _____

***Directions for questions 34 and 35*:** Read the following data and attempt the questions based on the given data.
A milk vending machine gives milk in the quantities of 1litre, 2litres, 4litres, 8 litres, 16 litres, 32 litres, 64 litres, 128 litres or 256 litres. A person wants to buy 400 litres.

**34.** What is the minimum number of times he has to use the machine to obtain the milk? _____

**35.** The machine develops a snag and cannot give 256 litres in one go. What is the minimum number of times he has to use the machine to obtain the milk? _____

## EXERCISE-4

***Directions for questions 1–25:*** For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** 1331 is a perfect cube in which of the following bases?
(A) 3 (B) 7
(C) 6 (D) More than one base

**2.** A trader wants to measure all weights (integral) from 1–220 kg using a simple balance in which weights are placed in one of the pans. What is the minimum number of weights needed? _____

**3.** How many five-digit numbers in base 7 are four-digit numbers in base 8 and six-digit numbers in base 5?
(A) 970 (B) 971 (C) 1695 (D) 972

**4.** In an island called Rich, they use a currency called Mint (M). It is available in notes of four denominations –1M, 12M, 144M, 1728M. What is the minimum number of notes required to pay an exact amount of $(12^5-1)$M?
(A) 144 (B) 174 (C) 176 (D) 208

**5.** Numbers 1–63 are written in binary form on a board. For example, 3 is written as 11 and 8 is written as 1000. Find the number of 1s on the board. _____

**6.** A decimal number ($N$) can be represented in base 9 using the same three digits as the digits used when representing it in base 11, but in the reverse order. Find the number of possible values of $N$.
(A) 2 (B) 1 (C) 4 (D) 3

**7.** Find the scale in which the numbers 8, 71 and 628 are in geometric progression. _____

**8.** Which of the following represents the square root of the largest three-digit perfect square in base 12?
(A) $(41)_{12}$ (B) $(35)_{12}$ (C) $(42)_{12}$ (D) $(37)_{12}$

**9.** If $(1111)_n + (8)_{10}(50)_n = (13)_{10}(100)_n + (31)_{10}$, then $n$ = _____.

**10.** What is the number of two-digit numbers which are perfect squares in the duo-decimal system?
(A) 11 (B) 8 (C) 9 (D) 1

**11.** How many three-digit numbers in base 10 are four-digit numbers in base 5 and three-digit numbers in base 6 as well as base 7?
(A) 81 (B) 91 (C) 187 (D) 123

**12.** How many four-digit numbers in base 9 are perfect squares?
(A) 54 (B) 53 (C) 55 (D) None of these

**13.** Find the fifth root of $(15AA51)_{19}$.
(A) $(20)_{19}$ (B) $(121)_{19}$
(C) $(11)_{19}$ (D) $(12)_{19}$

**14.** How many three-digit numbers in base 10 are four-digit numbers in base 6 and three-digit numbers in base 7 as well as in base 8? _____

**15.** In which of the following scales is the nonary number $(27367)_9$ represented as 44141?
(A) 7 (B) 8 (C) 9 (D) 10

**16.** How many two-digit numbers in the duodecimal system are even perfect squares in base 10? _____

**17.** $N$, a perfect square in decimal system, is a three-digit number in base 14. Find the square root of the largest such $N$.
(A) $(3A)_{14}$ (B) $(A3)_{14}$ (C) $(3B)_{14}$ (D) $(B3)_{14}$

**18.** Consider the following statements:
I. 10201 is a perfect square in any base greater than 2.
II. 10101 is a composite number in any base greater than or equal to 2.
Which of the following is/are true?
(A) Only I (B) Only II
(C) Both I and II (D) Neither I nor II

**19.** A sequence of digits in base 10, when considered as a number in base 20, has a value which is twice the value it would have, if considered as a number in base 13. If the number is less than 1000, how many possible values does it have?
(A) 1 (B) 2 (C) 3 (D) More than 3

***Directions for questions 20 and 21*:** These questions are based on the data given below.
Ajay wanted to weigh 456kg of rice. The weights are available in denominations of 1 kg, 2 kg, 4 kg, 8 kg, 16 kg ... He decides not to use more than one weight of each denomination.

**20.** How many weights does he use in all if only one side of the balance is used for weighing? _____

**21.** The smallest weight used when weighing 456 kg weight _____ is kg.

***Directions for questions 22 and 23*:** These questions are based on the data given below.
Rajesh found a decimal number which when represented in bases 2, 3, 4, 5 and 6 ends in 1, 2, 3, 4 and 5, respectively.

**22.** What is the smallest positive decimal number satisfying this property? _____

**23.** How many such three-digit decimal numbers are possible?

_____

**24.** If the difference of the numbers 6555 and 777 in a certain number system is 5556, what is the sum of the numbers 5666 and 457 in the same system?
(A) $(3564)_9$ (B) $(5653)_{12}$ (C) $(4644)_{12}$ (D) $(6345)_8$

**25.** $(11)_2 + (11)_3 + (11)_4 + \cdots + (11)_n =$

(A) $n^2 + 3n$ (B) $n^2 + 3n - 4$

(C) $\frac{n^2 + 3n}{2}$ (D) $\frac{n^2 + 3n - 4}{2}$

## ANSWER KEYS

### Exercise-1

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (C) | 6. (A) | 11. (D) | 16. 10 | 21. (D) | 26. 666 | 31. 11 |
| 2. (D) | 7. (C) | 12. (B) | 17. (B) | 22. (D) | 27. (C) | 32. (A) |
| 3. 65 | 8. (D) | 13. (C) | 18. (B) | 23. (B) | 28. (D) | 33. (D) |
| 4. (B) | 9. (D) | 14. (D) | 19. (D) | 24. (C) | 29. (C) | 34. (A) |
| 5. 1360 | 10. (A) | 15. (A) | 20. (D) | 25. (B) | 30. (D) | 35. (C) |

### Exercise-2

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (A) | 6. (D) | 11. (D) | 16. (C) | 21. (A) | 26. (D) | 31. (C) |
| 2. 649 | 7. (C) | 12. (C) | 17. (A) | 22. 9 | 27. (C) | 32. (B) |
| 3. (B) | 8. (B) | 13. 7 | 18. (B) | 23. (A) | 28. 100 | 33. (B) |
| 4. (C) | 9. (D) | 14. (B) | 19. (B) | 24. (B) | 29. 454 | 34. (A) |
| 5. 453 | 10. (B) | 15. 5712 | 20. (B) | 25. (D) | 30. (C) | 35. (D) |

### Exercise-3

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (A) | 6. (B) | 11. (C) | 16. 7 | 21. (B) | 26. (A) | 31. 171 |
| 2. (D) | 7. (B) | 12. (B) | 17. (B) | 22. 22616 | 27. (D) | 32. 12 |
| 3. (A) | 8. (D) | 13. (C) | 18. (A) | 23. 294 | 28. (C) | 33. 5 |
| 4. (B) | 9. 418 | 14. (A) | 19. 6622 | 24. (A) | 29. 7 | 34. 3 |
| 5. (A) | 10. (B) | 15. (B) | 20. 3202 | 25. (C) | 30. (A) | 35. 4 |

### Exercise-4

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (D) | 5. 192 | 9. 6 | 13. (C) | 17. (A) | 21. 8 | 25. (D) |
| 2. 8 | 6. (A) | 10. (B) | 14. 127 | 18. (C) | 22. 59 | |
| 3. (B) | 7. 9 | 11. (B) | 15. (B) | 19. (A) | 23. 15 | |
| 4. (C) | 8. (B) | 12. (A) | 16. 4 | 20. 4 | 24. (D) | |

## SOLUTIONS

## EXERCISE-1

***Solutions for questions 1–35:***

**1.**

| | | |
|---|---|---|
| 2 | 132 | |
| 2 | 66 | – 0 |
| 2 | 33 | – 0 |
| 2 | 16 | – 1 |
| 2 | 8 | – 0 |
| 2 | 4 | – 0 |
| 2 | 2 | – 0 |
| | 1 | – 0 |

$\therefore (132)_{10} = (10000100)_2$.

**2.** We have $502 = 256 + 128 + 64 + 32 + 16 + 4 + 2$

$= 2^8 + 2^7 + 2^6 + 2^5 + 2^4 + 2^2 + 2^1$

$\therefore (502)_{10} = (111110110)_2$

**3.** $(1000001)_2 = 1 \times 2^6 + 1 \times 1 = 65$

**4.**

| | | |
|---|---|---|
| 7 | 235 | |
| 7 | 33 | – 4 |
| | 4 | – 5 |

$\therefore (235)_{10} = (454)_7$.

**5.**
| | | | |
|---|---|---|---|
| 7 | 532 | | |
| 7 | 76 | – | 0 |
| 7 | 10 | – | 6 |
| 7 | 1 | – | 3 |
| | 0 | – | 1 |

$\therefore (532)_{10} = (1360)_7$

**6.**
| | | | |
|---|---|---|---|
| 12 | 1364 | | |
| 12 | 113 | – | 8 |
| | 9 | – | 5 |

$\therefore (1364)_{10} = (958)_{12}$.

**7.**
| | | | |
|---|---|---|---|
| 12 | 1463 | | |
| 12 | 121 | – | 11($B$) |
| 12 | 10 | – | 1 |
| | 0 | – | 10($A$) |

$\therefore (1463)_{10} = (\text{A1B})_{12}$

**8.** The largest four-digit number in octal system is $(7777)_8$.

**9.**
| | | | |
|---|---|---|---|
| 8 | 239 | | |
| 8 | 29 | – | 7 |
| | 3 | – | 5 |

$\therefore (239)_{10} = (357)_8$

**10.** We know that $2^8 = 256$, $2^9 = 512$
$\therefore 256 < 432 < 512$
Hence, 9 bits are required.

**11.** Let $N$ be the number and $(a_k\ a_{k-1} \ldots a_4\ a_3\ 000)_2$
$= a_k \cdot 2^k + a_{k-1} \cdot 2^{k-1}$
$+ \ldots\ldots\ldots\ldots + a_4.2^4 + a_3.2^3$
$+ 0.2^2 + 0.2^1 + 0.2^0$
$= a_k.2^k + a_{k-1}.2^{k-1} + \ldots..$
$+ a_4.2^4 + a_3.2^3$
Clearly, $N$ is divisible by 8. Among the choices, only 48 is divisible by 8.

**12.** $(AEB)_{16} = A \times 16^2 + E \times 16 + B$ X 1
$= 10 \times 256 + 14 \times 16 + 11 \times 1 = (2795)_{10}$

**13.** $(BAD)_{16} = D(16°) + A(16^1) + B(16^2)$
$13(1) + 10(16) + 11(256)$
$13 + 160 + 2816 = 2989$.

**14.** 16 1734
16 108 – 6
16 6 – $C$
0 – 6
$\therefore (1734)_{10} = (6C6)_{16}$.

**15.** $352_{(6)}$
$\times\ 13_{(6)}$
1 5 4 0
3 5 2
$5\ 5\ 0\ 0_{(6)}$

$\therefore (352)_6\ (13)_6 = (5500)_6$.

**16.** In any number system in which the base is at least 11, the numerical value of $A$ is 10. In the duodecimal system, the base is 12. The numerical value of $A$ is 10.

**17.** 2 3 4 (8)
4 3 2 (8)
$(6\ 6\ 6)_8$

**18.** $(437)_8$
When every digit in the above is expressed in the binary system, we get
$4 = (100)_2$ $7 = (111)_2$ $3 = (11)_2$
$437 = (100011111)_2$.

**19.** We know that $5^3 = 125$, $5^4 = 625$
$\therefore$ The number of bits required is 4.

**20.** We have
$(243)_6 = 2 \times 6^2 + 4 \times 6 + 3 \times 1 = (99)_{10}$
$(201)_7 = 2 \times 7^2 + 0 \times 7 + 1 \times 1 = (99)_{10}$
and $(143)_8 = 1 \times 8^2 + 4 \times 8^1 + 3 \times 1 = (99)_{10}$

**21.** The binary representation of any multiple of 16 ends with 0000.

**22.** $(1110011110100)_2$
From right to left, group four digits as 1 unit as shown below: $(1)_2\ (1100)_2\ (1111)_2\ (0100)_2$
$= (1\ (12)\ (15)\ 4)_{16} = (1\text{CF4})_{16}$.

**23.** 17
| | | | |
|---|---|---|---|
| 2 | 17 | | |
| 2 | 8 | – | 1 |
| 2 | 4 | – | 1 |
| 2 | 2 | – | 0 |
| | 1 | – | 0 |

$\therefore (17)_{10} = (10001)_2$.

**24.** To express a number in base $B$, the digits we use are 0, 1, 2,…$B$ – 1.
To express a number in binary, the digits we use are 0 and 1.

**25.** $(100001)_2 = 2^0(1) + 2^5(1) = 1 + 32 = 33.$

**26.** The largest three-digit septenary number is 666.

**27.** $(152)_k = (86)_{10}$
$2 \times k^\circ + 5 \times k + 1 \times k^2 = 86$
$2 + 5k + k^2 = 86$
$k^2 + 5k - 84 = 0$
$(k + 12)\ (k - 7) = 0$
Since $k$ is a positive integer.
$\therefore\ k = 7.$

**28.** The decimal equivalent of the binary number 1. 011 is
$1(2^0) + 0(2^{-1}) + 1(2^{-2}) + 1(2^{-3})$
$= 1 + 0 + \frac{1}{4} + \frac{1}{8} = 1\frac{3}{8} = 1.375$

**29.** The square of $(321)_7$
$= 3\ 2\ 1$
$\times\ 3\ 2\ 1$
$3\ 2\ 1$
$6\ 4\ 2$
$1\ 2\ 6\ 3$
$1\ 3\ 6\ 3\ 4\ 1$
$\therefore\ (321)_7\ (321)_7 = (136341)_7.$

**30.** 7 88 6 88
7 12 – 4 6 14 – 4
1 – 5 2 – 2
$\therefore\ (88)_{10} = (154)_7 \qquad (88)_{10} = (224)_6$

| | | | |
|---|---|---|---|
| 9 | 88 | | |
| 9 | 9 | – | 7 |
| | 1 | – | 0 |

$\therefore\ (88)_{10} = (107)_9$
All are true.

**31.** $(224)_5 = 5^2(2) + 5(2) + 5^0(4) = 64$. Its cube root is 4.
$(4)_{10} = (11)_3.$

**32.** The largest three-digit duo-decimal number is $(BBB)_{12}$.

**33.** $129 = 128 + 1 = 2^7 + 2^\circ = (10000001)_2.$

**34.** $(1331)_9 = [1(9^\circ) + 3(9) + 3(9^2) + 1(9)^3]_{10}$
$= 1 + 27 + 243 + 729 = (1000)_{10}$
$\therefore$ The cube root of $(1331)_9 = (10)_{10}.$

**35.** $(B21)_{12} - (398)_{12}$
$B\ 2\ 1_{(12)}$
$3\ 9\ 8_{(12)}$
$(7\ 4\ 5)_{12}$

## EXERCISE-2

***Solutions for questions 1–35:***

**1.**

| | | | |
|---|---|---|---|
| 2 | 213 | | |
| 2 | 106 | – | 1 |
| 2 | 53 | – | 0 |
| 2 | 26 | – | 1 |
| 2 | 13 | – | 0 |
| 2 | 6 | – | 1 |
| 2 | 3 | – | 0 |
| 2 | 1 | – | 1 |
| | 0 | – | 1 |

$\therefore\ (213)_{10} = (11010101)_2.$

**2.** $(3AC)_{13} = C\,(13^0) + A\,(13^1) + 3\,(13^2)$
$= 12\ (A) + 10\ (13) + 3\ (169)$
$= 12 + 130 + 507$
$= (649)_{10}$

**3.**

| | | | |
|---|---|---|---|
| 12 | 123456 | | |
| 12 | 10288 | – | 0 |
| 12 | 857 | – | 4 |
| 12 | 71 | – | 5 |
| | 5 | – | $B$ |

$\therefore\ (123456)_{10} = (5B540)_{12}$

**4.** $(147)_{10}$

| | | | |
|---|---|---|---|
| 8 | 147 | | |
| 8 | 18 | – | 3 |
| 8 | 2 | – | 2 |
| | 0 | – | 2 |

$\therefore\ (147)_{10} = (223)_8.$

**5.** $(100\ 101\ 011)_2 = [(100)_2\ (101)_2\ (011)]_8 = (453)_8$

**6.** $(231)_{16} = 2\ (16^2) + 3\ (16^1) + 1\ (16^0)$
$= 2\ (256) + 48 + 1$
$= 512 + 49 = (561)_{10}$
$(231)_8 = 2\ (8^2) + 3\ (8) + 1\ (8^0)$
$= 2\ (64) + 24 + 1$
$= 128 + 25$
$= (153)_{10}$
$\therefore (231)_{16} - (231)_8 = (561 - 153)_{10} = (408)_{10}$

| | | | |
|---|---|---|---|
| 11 | 408 | | |
| 11 | 37 | – | 1 |
| | 3 | – | 4 |

$\therefore (408)_{10} = (341)_{11}$

**7.** $(1357)_{10}$

| | | |
|---|---|---|
| 12 | 1357 | |
| 12 | 113 | – 1 |
| 12 | 9 | – 5 |
| | 0 | – 9 |

$\therefore (1357)_{10} = (951)_{12}$.

**8.** $(1752)_8$
$(1752)_8 = (001\ 111\ 101\ 010)_2$
$= (0011\ 1110\ 1010)_2 = (3EA)_{16}$.

**9.** 110110
–10001
100101
$(100101)_2 = 32 + 4 + 1 = 37$
When any number in any base is divided by the base, it leaves a remainder which is equal to the units digit. For example, choice (A), i.e., $(112)_4$ is of the form $4a + 2$. Similarly, the other choices are of the form $7b + 5$, $5c + 4$ and $4d + 1$.
$\therefore$ We have to consider only choice D. $(211)_4$
$= 2(16) + 1(4) + 1 = 37$.

**10.** $(101101)_2 = 1\ (2^5) + 0\ (2^4) + 1(2^3) + 1\ (2^2) + 0\ (2^1) + 1\ (2^0)$
$= 32 + 0 + 8 + 4 + 1$
$= (45)_{10}$
$(201)_8 = 2\ (8^2) + 0\ (8^1) + 1\ (8^0)$
$= 128 + 0 + 1$
$= (129)_{10}$
$(453)_{10} = (453)_{10}$
$\therefore (101101)_2 + (201)_8 + (453)_{10}$
$= (45 + 129 + 453)_{10}$
$= (627)_{10}$

| | | |
|---|---|---|
| 9 | 627 | |
| 9 | 69 | – 6 |
| | 7 | – 6 |

$\therefore (627)_{10} = (766)_9$

**11.**

| | | |
|---|---|---|
| 16 | 546 | |
| 16 | 34 | – 2 |
| 16 | 2 | – 2 |
| | 0 | – 2 |

$\therefore (546)_{10} = (222)_{16}$.

**12.** $(10010001)_2$
$= (010\ 010\ 001)_2$ (include one '0' at the left end)
$= ((010)_2\ (010)_2\ (001)_2)_8 = (221)_8$.

**13.** Given, $(125)k = (68)_{10}$
$\Rightarrow k^2 + 2k + 5 = 68 \Rightarrow k^2 + 2k - 63 = 0$
$\Rightarrow (k + 9)\ (k - 7) = 0$
$\Rightarrow k = -9, 7$
But, $k$ cannot be negative.
$\therefore k = 7$

**14.** 11
(215) 8
+ (476) 8
(713) 8
$\therefore (215)_8 + (476)_8 = (713)_8$
Note that the above addition is carried on in the base-8 system.

**15.** We have, $(A)_{16} = 10$
$(11)_2 = 3$
$(13)_8 = 11$
$f(x, y, z) = (x + 2y)\ (2y + 2)\ (z + x)$
$= (16) \times (17) \times (21) = (5712)_{10}$

**16.**

| Old number | New number |
|---|---|
| $10\downarrow = 2$ | $10\ \underline{0} = 4$ |
| $100\downarrow = 4$ | $100\ \underline{1} = 8$, etc. |

Hence, the number gets doubled.

**17.** $(112)_6 = (44)_{10}$
$44 \times 44 = (1936)_{10} = (12544)_6$.

**18.** $(54)_8 = (44)_{10}$
$(20)_8 = (16)_{10}$
LCM $(44, 16) = (176)_{10}$.

| | | |
|---|---|---|
| 8 | 176 | |
| 8 | 22 | – 0 |
| 8 | 2 | – 6 |
| | 0 | – 2 |

$\therefore (176)_{10} = (260)_8$.

**19.** Let the scale of the number be $n$
$\Rightarrow 1654 = n^3 + 6n^2 + 5n + 4n^0 = n^3 + 6n^2 + 5n + 4$ (1)
From the choices, substituting $n = 7$ in (1),
we get 676 which is a perfect square.

**20.** $(310)_4 = 3\ (4^2) + 1\ (4^1) + 0\ (4^0)$
$= 48 + 4 + 0 = (52)_{10}$
$(110)_4 = 1\ (4^2) + 1\ (4^1) + 0\ (4^0) = (20)_{10}$

| | | |
|---|---|---|
| 4 | 52, | 20 |
| | 13, | 15 |

$\therefore$ LCM of $(310)_4, (110)_4 = 13\ (20) = (260)_{10}$
The first three choices are of the form $5a + 1$, $6b + 2$ and $4c + 1$
We need to consider only choice B, i.e., $(1112)_6 = 216 + 36 + 6 + 2 = 260$.

**21.** We have, $(62)_8 = (50)_{10}$
$(144)_8 = 4 + 32 + 64 = (100)_{10}$
and $(226)_8 = 6 + 16 + 128 = (150)_{10}$
$\therefore$ $(62)_8$ and $(144)_8$ and $(226)_{10}$ are clearly in arithmetic progression.

**22.** $(281)_{10} < 2^n$
$(281)_{10} = 256 + 25$
$= 2^8 + 25$
$\therefore (281)_{10} < 2^n$
$\Rightarrow (2^8 + 25) \le 2n$

$\Rightarrow n \ge 9$

$\therefore$ Number of bits required = 9 bits.
It can be noted that $256 = (100000000)_2$ and $(111111111)_2 = 511$ is the largest 9 bit number. So any number that lies between 256 and 511 would require a minimum of 9 bits to represent it in binary.

**23.** $(a)_{10} \Delta (b)_{10} = (2a + b - 2)_{10}$
$(101)_2 \Delta (100)_2 = (5)_{10} \Delta (4)_{10}$
$= (10 + 4 - 2)_{10} = (12)_{10} = (1100)_2$.

**24.** $(45)_6 = (29)_{10}$
$(20)_4 = (8)_{10}$
$29 \times 8 = (232)_{10} = (174)_{12}$.

**25.** $(10101)_2 - (1010)_2$

$$\begin{array}{r} 10101 \\ -\ 1010 \\ \hline 1011 \\ \hline \end{array}$$

**26.** $(n-1)_n = (n-1)_{10}$
$\therefore (n-1)_n + (n-2)_{n-1} + \ldots\ldots + (1)_2$
$= (n-1)_{10} + (n-2)_{10} + \ldots\ldots (n-(n-1))_{10}$

$$= \left(\frac{(n-1)(n)}{2}\right)_{10}$$

**27.** The remainder when $(abcde)_{10}$ is divided by 9 is equal to the remainder when $a + b + c + d + e$ is divided by 9. In general, the remainder when $(abcde)_{n+1}$ is divided by $n$ is equal to the remainder when $(a + b + c + d + e)$ is divided by $n$.
$\therefore$ The required remainder is 3.

**28.** $(39)_{11} = 3 \times 11 + 9 \times 11^0 = 33 + 9 = (42)_{10}$
$(62)_9 = 6 \times 9 + 2 = 54 + 2 = (56)_{10}$
$\therefore$ Arithmetic mean of $(39)_{11}$ and $(62)_9$

$$= \left(\frac{42+56}{2}\right)_{10} = \left(\frac{98}{2}\right)_{10} = (49)_{10}$$

Given that $(49)_{10} = (94)_n$
$\Rightarrow 49 = 9_n + 4 \Rightarrow n = 5$

$(32)_4 + (21)_5 = (14)_{10} + (11)_{10}$
$= (25)_{10} = (100)_5$

**29.** $(346)_n = (1211)_5$
$\Rightarrow 3n^2 + 4n + 6 = 125 + 50 + 6$
$\Rightarrow 3n^2 + 4n - 175 = 0$
$\Rightarrow 3n^2 + 25n - 21n - 175 = 0$
$\Rightarrow n(3n + 25)$ -7 $(3n+25) = 0$
$\Rightarrow (n - 7)\ (3n + 25) = 0$

$$\Rightarrow n = 7 \text{ or } \frac{-25}{3}$$

As radix cannot be a fraction, $n = 7$

$$\begin{array}{r|l} 7 & 235 \\ 7 & 33 \quad -\ 4 \\ \hline & 4 \quad -\ 5 \end{array}$$

**30.** Let the radix be $n$.
The equation $x^2 - 10x + 22 = 0$ in decimal system becomes $y^2 - 10y + (2x + 2) = 0$
where $(x)_n = (y)_{10}$
So, $y^2 - 10y + (2n + 2) = 0$ (1)
Now $(4)_n = (4)_{10}$ and $(6)_n = (6)_{10}$
Substituting $y = 4$ or $y = 6$ in (1), we get
$(4)^2 - 10\ (4) + (2n + 2) = 0$
$\Rightarrow n = 11$
Likewise, on substituting $y = 6$,
we get $n = 11$.

**31.** $(132)_4 = 1 \times 4^2 + 3 \times 4 + 2 \times 4^0$
$= 16 + 12 + 2$
$= (30)_{10}$

$(30)^2_{10} = (900)_{10}$

The choices are of the form $7a + 2$, $4b$, $7c + 4$, $4d + 2$.
We need to consider only choices 2 and 3
$(10230)_4 = 256 + 2(16) + 3(4)$ and
$(2424)_7 = 2(343) + 4(49) + 2(7) + 4$
$= 686 + 196 + 14 + 4 = 900$.

**32.** $(1011\ 1001)_2 = ((1011)_2\ (1001)_2) = (B9)_{16}$.

**33.** $(341)_8 = 3 \times 8^2 + 4 \times 8^1 + 1 \times 8^0 = 192 + 32 + 1 = (225)_{10}$

$\sqrt{(225)_{10}} = (15)_{10} = (17)_8$.

**34.** Let the scale be $n$.
So, $(3120)_n = 3n^3 + n^2 + 2n = n\ (3n^2 + n + 2)$
From choices, we see that $n = 4$, gives
$n(3n^2 + n + 2) = 216$.

**35.** $(12)_6 = (8)_{10}$
$(33)_7 = (24)_{10}$

$\text{AM}\ (8, 24) = \dfrac{8+24}{2} = 16$

Now $(22)\,n = (16)_{10}$
$\Rightarrow 2n + 2 = 16$
$\Rightarrow 2n = 14$
$\Rightarrow n = 7.$

## EXERCISE-3

***Solutions for questions 1–35:***

**1.**
| | | | |
|---|---|---|---|
| 12 | 2156 | | |
| 12 | 179 | – | 8 |
| 12 | 14 | – | 11(*B*) |
| | 1 | – | 2 |

$\therefore (2156)_{10} = (12B8)_{12}.$

**2.** $(DB3)_{16}$
$3(16^0) + B(16^1) + D(16^2) = 3(1) + 11(16) + 13 \times 256$
$= 3 + 176 + 3328 = 3507.$

**3.** $(243)_{16} = 3 + 4(16) + 2(16^2) = 3 + 64 + 512 = 579$
$(243)_8 = 3 + 4(8) + 2(8^2) = 3 + 32 + 128 = 163$
$(243)_6 = 3 + 4(6) + 2(6^2) = 3 + 24 + 72 = 99$
$(243)_{16} - (243)_8 + (243)_6 = 579 - 163 + 99 = 515$
$(515)_{10} = (4030)_5.$

**4.** $(551)_8 = 1 + 5(8) + 5(8^2) = 1 + 40 + 320 = 361$
The square root of (361) = 19

| | | | |
|---|---|---|---|
| 8 | 19 | | |
| | 2 | – | 3 |

The square root of $(551)_8 = (23)_8.$

**5.** $(41)_k = 1 + 4k$
$(36)_{k+2} = 6 + 3(k + 2) = 3k + 12$
The two given numbers are $4k + 1$ and $3k + 12$
Their LCM and GCD are 150 and 5
$(4k + 1)\ (3k + 12) = 150(5)$
$(4k + 1)\ (k + 4) = 50(5)$
$4k^2 + 17k + 4 - 250 = 0$
$4k^2 + 17k - 246 = 0$
$(k - 6)\ (4k + 41) = 0$

$k = 6$ or $k = -\dfrac{41}{4}$

Since, $k$ is a positive integer $k = 6.$

**6.** $(386)_{12} = (534)_{10}$
$(177)_{12} = (235)_{10}$
$\therefore (386)_{12} - (177)_{12} = (299)_{10}$

Now,

| | | | |
|---|---|---|---|
| 12 | 299 | | |
| 12 | 24 | – | *B* |
| | 2 | – | 0 |

$\therefore (299)_{10} = (20B)_{12}$

**7.** We have
$57 = 32 + 16 + 8 + 1 = 2^5 + 2^4 + 2^3 + 2^0$
$\therefore (57)_{10} = (111001)_2$
Also,
$0.140625 \times 2 = 0{\cdot}28125$
$0.28125 \times 2 = 0{\cdot}5625$
$0.5625 \times 2 = 1{\cdot}125$
$0.125 \times 2 = 0{\cdot}25$
$0.25 \times 2 = 0{\cdot}50$
$0.5 \times 2 = 1{\cdot}0$
$\therefore (57{\cdot}143251)_{10}$ is equal to $(111001.\ 001001)_2$

**8.** Let $a$, $b$, $c$ be 7-digit, 8-digit and 9-digit numbers, respectively, in base $m$.
i.e., $m^6 \le a < m^7$
$m^7 \le b < m^8$
$m^8 \le c < m^9$
Let $x$, $y$ be a 5-digit number and a 6-digit number, respectively, in base $n$, i.e., $n^4 \le x < n^5$
$n^5 \le y < n^6$
$\therefore$ The range $m^6$ to $m^9$ is a subset of the range $n^4$ to $n^6$.
We tabulate these four values for the given options (all in base 10)

| | | A | B | | | |
|---|---|---|---|---|---|---|
| $m$ | $n$ | $m^6$ $m^4$ | $n^4$ $n^6$ | | A ⊂ B | |
| 2 | 3 | 64 | 512 | 81 | 729 | False |
| 3 | 6 | 729 | 19, 683 | 1296 | – | False |
| 3 | 7 | 729 | – | 2401 | – | False |
| 4 | 8 | $2^{12}$ | $2^{18}$ | $2^{12}$ | $2^{18}$ | True |

**9.** LCM (3, 4, 5, 7) = 420
$\therefore 420 - 2 = 418$ is the required positive integer

**10.** $(26)_7 = 2\ (7) + 6\ (7^0) = 14 + 6 = (20)_{10}$
$(104)_6 = 1\ (6^2) + 0\ (6^1) + 4 = (40)_{10}$
$(88)_9 = 8\ (9) + 8\ (9^0) = 72 + 8 = (80)_{10}$
$(40)^2 = 1600 = 80\ (20)$
i.e., the numbers satisfy the condition $b^2 = ac$
Hence, they are in G.P.

**11.** $(17)_8 = 7 + 8 = 15$
$(51)_8 = 1 + 8.5 = 41$
Given $(a)_{10}\ \$\ (b)_{10} = (3a + 2b - 5)_{10}$
$\therefore (15)_{10}\ \$\ (41)_{10} = 3(15) + 2(41) - 5 = (122)_{10} = (172)_8.$

**12.** If $x$ is a 6, 7, or 8 digit number in base $m$, then

$m^5 \le x < m^8$ (1)

Four or five digit numbers ($y$) in base $n$ satisfy the condition

$n^3 \le y < n^5$ (2)

All $x$s should satisfy (2), i.e., among the options, we should check whether $n^3 < m^5 < m^8 < n^5$

The values of $m$, $n$, $n^3$, $m^5$, $m^8$, $n^5$ are tabulated below for the five options.

| $m$ | $n$ | $n^3$ | $m^5$ | $m^8$ | $n^5$ | Possible / not possible |
|---|---|---|---|---|---|---|
| 10 | 12 | 1728 | 100,000 | $10^8$ | 248,832 | × |
| 4 | 10 | 1000 | 1024 | 65,536 | 100,000 | ✓ |
| 4 | 7 | 343 | 1024 | 65,536 | 16,807 | × |
| 2 | 5 | 125 | 32 | | | × |
| 3 | 6 | 125 | 243 | 6561 | 3125 | × |

**Alternate Solution:**

If $x$ is a 6, 7 or 8 digit number, (in base $m$) $m^5 \le x < m^8$ (1)

If $y$ is a 4 or 5 digit number in base $n$, $n^3 \le y < n^5$ (2)

As all $x$s satisfy (2), $n^3 < m^5 < m^8 < n^5$

or 3 log $n$< 5 log $m$ and 8 log $m$< 5 log $n$, i.e.,

$$\frac{8}{5} < \frac{\log n}{\log m} < \frac{5}{3} \Rightarrow 1.6 < \frac{\log n}{\log m} < 1.67$$

The values of $m$, $n$ which appear in the options and their corresponding values of log n / log m are tabulated below:

| $M$ | $n$ | log $n$ / log $m$ |
|---|---|---|
| 10 | 12 | $\log_{12} \approx 1.08$ |
| 4 | 10 | $1/0.602 \approx \frac{10}{6.02} < 1.67$ |
| 4 | 7 | $0.845 / 0.602 \approx 1.25$ |
| 2 | 5 | $0.7 / 0.3 \approx 2.33$ |
| 3 | 5 | $0.7 / 0.48 = \frac{70}{48} = \frac{35}{24} < 1.5$ |

We see that only $(m, n) = (4, 10)$ satisfies the condition.

**13.** $f(x, y, z) = 2x + 3y - z$

$(27)_8 = 7 + 2(8) = 23$

$(27)_{16} = 7 + 2(16) = 39$

$(27)_8, (27)_{10}, (27)_{16} = (23)_{10}, (27)_{10}, (39)_{10}$

$= 2(23) + 3(27) - (39) = 46 + 81 - 39 = 88.$

**14.** $(101010)_2 = 0 + 1(2) + 0 + 1\ (2^3) + 0 + 2^5$

$= 2 + 8 + 32 = 42$

$(101)_2 = 1(2^2)\ (1) = 5$

The remainder on dividing 42 by 5 is 2.

$\therefore (10)_2$.

**15.** $(10001001)_2 =$

Divide the given number in to groups of three digits, from right to left as shown.

$(10,001,001)_2 = (211)_8$.

**16.** $(51)_k = (5k + 1)_{10}$

$(50)_{k+2} = (5k + 10)_{10}$

Given, GCD is $(9)_{10}$ and LCM is $(180)_{10}$.

We have the product of two numbers as equal to product of their LCM and GCD.

$\therefore (5k + 1)\ (5k + 10) = 9 \times 180$

$\Rightarrow k = 7$.

**17.** $(21)_8 = 2\ (8^1) + 1\ (8^0) = (17)_{10}$

$(23)_5 = 2\ (5^1) + 3\ (5^0) = 10 + 3 = (13)_{10}$

$\therefore f[(23)_{10}, (21)_8, (23)_5] = f[(23)_{10}, (17)_{10}, (13)_{10}]$

$= [3(23) + 2(17) - 13]$

$= 69 + 34 - 13$

$= (90)_{10}$

| | | | |
|---|---|---|---|
| 2 | 90 | | |
| 2 | 45 | – | 0 |
| 2 | 22 | – | 1 |
| 2 | 11 | – | 0 |
| 2 | 5 | – | 1 |
| 2 | 2 | – | 1 |
| | 1 | – | 0 |

$\therefore (90)_{10} = (1011010)_2$

The choices are of the form $8a + 1$, $2b$, $6c + 5$ and $14d + 11$

We have to consider only choice B.

**18.**

| | | | |
|---|---|---|---|
| 16 | 43916 | | |
| 16 | 2744 | – | |
| 16 | 171 | – | 8 |
| | $A$ | – | $B$ |

$\therefore (43916)_{10} = (AB8C)_{16}$

**19.** 2 3 5 4

+ 4 2 3 5

6 6 2 2

In the first place, 4 + 5 = 9. But, our base is 7. What do we do in a decimal system when we get 7 + 5 = 12? We write '2' as the number and '1' as a carry. This is obtained by dividing 12 by 10, writing the remainder in its place and quotient as carry. Therefore ,in the present case, 9

divided by 7 leaves a remainder of 1 and quotient '2'. So, we write '2' and keep a carry of 1, so, for the second place, 5 + 3 + 1 = 9 which again is 12 in base 7, so we place 2 and carry 1 again. For the third place, we have 1 + 3 + 2 = 6 which remains as 6 as it is less than 7. For the final place, 4 + 2 = 6.
So, the sum is $(6622)_7$.
$(2354)_7 + (4235)_7 = (6622)_7$

**20.** 5 1 2 3 1
– 4 5 0 2 6
0 3 2 0 2
We cannot subtract 6 from 1, so we need 1 (borrow) from the earlier digit. After that, 3 becomes 2 and 1 becomes 1 + 7 = 8. So, 8 – 6 = 2. For the next digit, 2 – 2 = 0, then 2 – 0 = 2. Again, borrow 1 from 5 and the next subtraction becomes 8 – 5 = 3. So, the answer is $(3202)_7$.

**21.** In base 10, a number is divisible by 9 if the sum of the digits is divided by 9,
Similarly, in base 9 the number is divisible by 8 if the sum of the digits is divisible by 8.
The reason is as follows:
Take a number,
say $(3751)_9 = 3 \times 9^3 + 7 \times 9^2 + 5 \times 9 + 1$. Using remainder theorem, the remainder ,when divided by 8 will be, 3 + 7 + 5 + 1 = 16.
As 16 is divisible by 8, the remainder is zero.
Now, in the given question, the sum of the digits is 24. As 24 is divisible by 8, the number is divisible by 8.

**22.** $(B)_{12} + (BB)_{12} + (BBB)_{12} + (BBBB)_{12}$
$= 12 - 1 + 12^2 - 1 + 12^3 - 1 + 12^4 - 1$
$= 12 + 144 + 1728 + 20736 - 4 = 22616.$

**23.** The least three-digit number in base 7 = $(100)_7 = 7^2 = 49$
The greatest three-digit number in base 7 = $(666)_7 = 7^3 - 1 = 342$.
∴ The total number of three-digit numbers in base 7 system = 342 – 49 + 1 = 294.

**24.** We have, $(34)_7 = (25)_{10}$
$(31)_8 = (25)_{10}$
∴ $(34)_7 \times (31)_8 = (625)_{10} = (441)_{12}$

**25.** $(23516)_8 = (010\ 011\ 101\ 001\ 110)_2$
$= (0010\ 0111\ 0100\ 1110)_2$
$= (274E)_{16}$

**26.** $(1010101)_2 = 1 + 2^2 + 2^4 + 2^6 = 85$
$(10101)_4 = 1 + 4^2 + 4^4 = 273$
$(101)_8 = 1 + 8^2 = 65$
$(1010101)_2 + (10101)_4 + (101)_8 = 85 + 273 + 65 = 423$

16 | 423
16 | 26 – 7
1 – 10(A)

∴ $(423)_8 = (1A7)_{16}$.

**27.** $(2333)_n = 3 + 3n + 3n^2 + 2n^3 = 2n^3 + 3n^2 + 3n + 3$
When $n = 5$, the above value is a perfect cube.

**28.** Old number New number
10↓ = 2 10 0 = 4
100↓ = 4 100 1 = 8, etc.
Hence, the number gets doubled.

**29.** Given $(1002)_n = (345)_{10}$
$\Rightarrow 2 + n^3 = 345$
$\Rightarrow n^3 = 343$
$\Rightarrow n = 7$

**30.** We have, $(325)_8 = (213)_{10}$
$(213)^2 = (45369)_{10} = (130471)_8$

**31.** The trader definitely needs 1 kg weight. He will have 3 numbers of 1 kg weight with this he can weight 1 kg, 2 kg, 3 kg. For the next weight, he depends on whether he can keep weights on both the pans or only one pan.

**Case 1**. Weights on one pan

| | 1st set | 2nd set | 3rd set |
|---|---|---|---|
| Wts | 1 kg | 1 kg | 1 kg |
| Wts | 4 kg | 4 kg | 4 kg |
| Wts | 16 kg | 16 kg | 16 kg |

With these weights, he can weigh any weight from 1 kg to 63 kg. Let us say he want to weigh 51 kg. He can use 3 '16 kg' weights and 3 '1 kg' weights. He wants to weigh 42 kg. 16 + 16 + 4 + 4 + 1 + 1 = 42 kg.
**Case 2.** Weights on both pans with 3 '1kg' weights can weigh 1 kg, 2 kg and 3 kg. If he has 7 kg, he can measure 4 kg (7 – 3), 5 kg(7 – 2), 6 kg (7–1),8 kg (7+1), 9 kg (7 + 2), 10kg (7 + 3), 11kg (7 + 3 + 1), 12kg (7 + 3 + 2) 13 kg (7 + 3 + 2 + 1).
How do we get 7 'kg'?
With 3 '1kg' weights, we can weigh 1 kg, 2 kg and 3kg.
Next the trader needs to weigh 4 kg.
Let us add 3 + 4 = 7
That is how 7 kg is arrived at.

| 1st set | 2nd set | 3rd set |
|---|---|---|
| 1 kg | 1 kg | 1 kg |
| 7 kg | 7 kg | 7 kg |

With the weights, we can weigh any weight from 1 to 24 kg.

The next weight to weigh is 25 kg.
Adding 24 + 25, we get 49. The next weight he needs is 49 kg.

| | | |
|---|---|---|
| 1 kg | 1 kg | 1 kg |
| 7 kg | 7 kg | 7 kg |
| 49 kg | 49 kg | 49 kg |

The trader can weigh any (integrands) weights from 1 to 171 kg.

**32.** In the decimal system, the difference between a number and any number formed by rearranging the digits is divisible by 9, here, it will be divisible by (13 – 1), i.e., 12 or any factor of 12.
The reason is as follows:
Let the digits be $a, b, c, d$ in base 13. So, the number is $N_1 = a \times 13^3 + b \times 13^2 + c \times 13 + d$.
Let us say, after rearranging the number becomes
$N_2 = c \times 13^3 + d \times 13^2 + b \times 13 + a$
Thus, $N_1 - N_2$ is given as: $N_1 - N_2$
$= a(13^3 - 1) + b(13^2 - 13) + c(13 - 13^3) + d(1 - 13^2)$
$= a(13^3 - 1) + b \times 13(13 - 1) - c \times 13(13^2 - 1) - d(13^2 - 1)$
All the terms are divided by 12

**33.** To measure 1 kg, the trader must have a 1 kg weight. Now as he can put weights on both sides of the pan, he can measure the sum as well as the difference between the weights. So, to measure 2 kg, instead of using a 2 kg weight, he will take a new weight as 3 kg because 3 – 1 = 2. As the trader has 1 kg and 3 kg weights, he can measure 4 kg also.
Now, he can measure anything up to 4 kg. To measure 5 kg, a new weight will be taken such that the difference between the new weight and 4 is 5. Hence, the next weight must be 5 kg.
If he has 9 kg weight, he can also measure 6, 7, 8, 9, 10, 11, 12, 13 kg.

| Sl. No. | Wt. | Upper limit for measurements |
|---|---|---|
| 1. | 1 kg | 1 kg |
| 2. | 3 kg | 4 kg |
| 3. | 9 kg | 13 kg |
| 4. | 27 kg | 40 kg |
| 5. | 81 kg | 121 kg |

Minimum number of weights = 5.

**34.** We have, 400 = 256 + 128 + 16
∴ The minimum number of times he needs to use the machine is 3.

**35.** Instead of one 256, he can use the quantity 128 twice, this implies, 400 = 128 + 128 + 128 + 16
∴ The number of times he has to use the machine is 4.

## EXERCISE-4

***Solutions for questions 1–25:***

**1.** $(1331)_a = 1 + 3a + 3a^2 + a^3 = (1 + a)^3$
1331 is a perfect cube in any base which is more than 3; so, the answer is option (D).

**2.** To measure 1 kg, the trader must have a 1 kg weight. Similarly, to measure 2 kg he must have a 2 kg weight. If the trader has 1 kg and 2 kg weights, he can measure 3kg. He needs 4 kg weight to measure 4 kg and one 4 kg weight is available. He can measure 5 kg, 6 kg and 7 kg as 5 = 4 + 1,6 = 4 + 2 and 7 = 4 + 2 + 1 and so on.

| Sl.No. | Wts | Upper limit for measurement |
|---|---|---|
| 1. | 1 kg | 1 kg |
| 2. | 2 kg | 3 kg |
| 3. | 4 kg | 7 kg |
| 4. | 8 kg | 15 kg |
| 5. | 16 kg | 31 kg |
| 6. | 32 kg | 63 kg |
| 7. | 64 kg | 127 kg |
| 8. | 93 kg | 220 kg |

Eight weights are needed.
$220 = 2^8 - 36$
Hence, 8 weights are needed.

**3.** Let $a$ number that satisfies the three properties be denoted by $X$.
$X$ has 6 digits in base 5
$\therefore 5^5 \le X < 5^6$, i.e., $3125 \le X < 15625$
$X$ has 5 digits in base 7
$\therefore 7^4 \le X < 7^5$, i.e., $2401 \le X < 16807$
$X$ has 4 digits in base 8
$\therefore 8^3 \le X < 8^4$, i.e., $512 \le X < 4096$
As $X$ satisfies all the above three conditions,
$3125 \le X < 4096$
The smallest value of $x$ is 3125.
The greatest value of $x$ is 4095.
∴ The number of values of $x$ is 4095 – 3124 = 971.

**4.** The denominations are powers of 12. We can express the amount to be paid in base 12.

$12^5$is the smallest 6-digit number in base 12, and $12^5$–1 is the greatest 5-digit number in base 12.

$12^5 - 1 = (BBBBB)_{12}$.

For the purpose of the explanation below, we can differentiate the five digits as follows. $(B_4B_3B_2B_1B_0)$. Actually, all the digits are the same. The value of $B$ (in base 10) is 11.The denominations which are available are $1^0$, $12^1$, $12^2$, $12^3$. But one denomination is not available—$12^4$.

If there is a one in the place of $B_4$, we need 12 notes of denomination $12^3$. As there is 11, we need 11(12) notes of denomination $12^3$.

For each of the amounts represented by $B_3$ ,$B_2B_1B_0$, we need 11 notes of the corresponding denomination.

∴ We need 11(12) + 4(11) or 176 notes to make the payment.

**5.** The numbers 1–63 and their binary forms are listed below.

| Number | Number Binary From | Number | Number Binary From | Number | Number Binary From |
|---|---|---|---|---|---|
| 1 | 1 | | | | |
| 2<br>3 | 10<br>11 | 8<br>-<br>15 | 1000<br>----<br>1111 | 32<br>- | 100000<br>------- |
| 4<br>5<br>6<br>7 | 100<br>101<br>110<br>111 | 16<br>-<br>31 | 10000<br>-----<br>11111 | 63 | 111111 |

We see that for each group (1–digit to 6–digit), the first digit in each number is 1 and in each of the other columns half of the digits are 1 (while the other half are 0); so the number of 1s is = 1 + 3 + 8 + 20 + 48 + 112 = 192.

The actual expression which we get is

$$1+\left[2+\frac{2(1)}{2}\right]+\left[2^2+\frac{2^2(2)}{2}\right]+\left[2^3+\frac{2^3(3)}{2}\right]$$
$$+\left[2^4+\frac{2^4(4)}{2}\right]+\left[2^5+\frac{2^5(5)}{2}\right]$$

**Alternate Solution:**

All the numbers upto 63 can be expressed as 6-digit numbers with the appropriate number of initial zeros.

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 2 | 0 | 0 | 0 | 0 | 1 | 0 |
| . | - | - | - | - | - | - |
| 63 | 1 | 1 | 1 | 1 | 1 | 1 |

Half of these 6 (64) digits are 0s and the other half are 1s.

The number of 1s = $\frac{6(64)}{2} = 64(3) = 192$

**6.** Let the base 9 representation of $N$ be $(abc)_9$, then the number $N$ in base 11 is $(cba)_{11}$

$81a + 9b + c = 121c + 11b + a$

$\Rightarrow 80a = 120c + 2b$

$\Rightarrow 40a = 60c + b$. ∴ $b$ has to be 0 and $2a = 3c$

Since $a$ and $c$ are integers, $a$ should be a multiple of 3 and $c$ should be a multiple of 2, also $a, c \le 8$.

∴ $a = 3$, $c = 2$, and $a = 6$, $c = 4$ are the only possible combinations.

We cannot consider $(a, c) = (9, 6)$, ∵ 9 is not a digit in base 9.

**7.** Let the scale be '$n$'(∵ $n>8$). $a = 8_n = 8$;

$b = 71_n = 7n + 1$ and $c = (628)_n = 6n^2 + 2n + 8$

Since $a$, $b$ and $c$ are in a GP, $b^2 = ac$

$(7n + 1)^2 = 8\,(6n^2 + 2n + 8)$. Hence, $n = 9$.

**8.** The largest three-digit number in base 12 is $(BBB)_{12} = (1727)_{10}$ (i.e., 1728 – 1)

Now, any three-digit number in base 12 will be less than or equal to $(BBB)_{12}$.

The nearest perfect square, less than $(1727)_{10}$ is $(1681)_{10}$, We convert the square root of $(1681)_{10}$, i.e., $(41)_{10}$ to base 12.

Required result = $(41)_{10} = (35)_{12}$

**9.** $(n^3 + n^2 + n + 1 + 8)(5n) = 13\,n^2 + 31$

$\Rightarrow n^3 - 12n^2 + 41n - 30 = 0$

$\Rightarrow (n-1)\,(n-5)(n-6) = 0$

As $n \neq 1$ and $n \neq 5$, $n = 6$.

**10.** As $(10)_{12} = (12)_{10}$; $(BB)_{12} = (143)_{10}$ and between 12 and 143 there exist 8 perfect squares, the duodecimal system will contain 8 two-digit perfect squares.

**11.** The least four-digit number in base 5 is $(1000)_5$.

$(1000)_5 = (125)_{10}$ (1)

The greatest four-digit number in base 5 is $(4444)_5$. This is 1 less than the least five-digit number in base 5, i.e., is $(1000)_5 - 1$, i.e., $((625-1)_{10} = (624)_{10}$ (2)

From (1) and (2), every four-digit number in base 5 is a three-digit (decimal) number. When every three-digit number in base 6 as well as that in base 7 are represented as three digit (decimal) numbers, the ranges of the three-digit numbers in bases 6 and 7 are

$(36)_{10} - (215)_{10}$ – and $(49)_{10} - (342)_{10}$ (3)

From (1), (2) and (3), the three-digit numbers which satisfy all the given conditions are 125 to 215., i.e., 91 such numbers.

**12.** The four-digit numbers in base 9 are $9^3$ to $(9^4 - 1)$ both inclusive, i.e., 729 to $(81^2 - 1)$, i.e., $27^2$ to $(81^2 - 1)$
The perfect squares in this range are $27^2, 28^2, \ldots 79^2, 80^2$, i.e. there are 80 – 27 + 1, i.e., 54 perfect squares.

**13.** We have, $(15AA51)x$
$= x^5 + 5x^4 + A.\ x^3 + A.\ x^2 + 5.x + 1$
$= (x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1) = (x + 1)^5$
$\therefore$ fifth root of $(15AA51)_{19} = (19 + 1)_{10} = (20)_{10} = (11)_{19}$

**14.** The four-digit numbers in base 6 range from 1000 to 5555, i.e., from $6^3$ to $6^4 - 1$ or 216 to 1295 in base 10 (1)
The three-digit numbers in base 7 range from 100 to 666, i.e., from 49 to 342 in base 10 (2)
The three-digit numbers in base 8 range from 100 to 777, i.e., from 64 to 511 in base 10 (3)
From (1), (2), (3), the three-digit numbers in base 10 satisfying the given conditions are 216 to 342. A total of 127 three-digit numbers satisfy the given conditions.

**15.** $(27367)_9 = 9^4\,(2) + 9^3\,(7) + 9^2\,(3) + 9\,(6) + 7 = 18529$.
Let this be 44141 in base $b$.
$18529 = 4b^4 + 4b^3 + b^2 + 4b + 1$
$18528 = 4b^4 + 4b^3 + b^2 + 4b$ (1)
RHS is divisible by $b$.
$\therefore$ LHS is also divisible by $b$.
$b$ is a factor of 18528.
Among the given choices, only 8 is a factor of 18528. We can verify that (1) is satisfied for $b = 8$.

**16.** The two-digit numbers in the duodecimal system range from 10 to BB, i.e., from 12 to $12^2 - 1$ in base 10. Of these, $4^2, 6^2, 8^2, 10^2$ are even perfect squares.

**17.** The largest three-digit number in base 14 is DDD. Its decimal equivalent is $14^3 - 1$, i.e., 2743.
$N$ is a perfect square. The largest perfect square less than 2743 is 2704 which is $52^2$.
$\therefore$ Square root of this is 52. This is 3A in base 14.

**18.** I: 10201 in base $b$ equals $b^4 + 2b^2 + 1$ which is $(b^2+1)^2$
$\therefore$ It is a perfect square in any base greater than 2. but the base cannot be 2 since one of the digits of the number is 2.
II. 10101 in base $b$ is $b^4 + b^2 + 1$ which is $(b^2+1)^2 - b^2$.
$\therefore\ (10101)_b = (b^2 + 1 + b)\ (b^2 + 1 - b)$. As $b \geq 2$, $b^2 + 1 - b \geq 3$ and $b^2 + 1 + b \geq 7$
$\therefore\ (10101)_b$ is composite.
Both I and II are true.

**19.** Let the number be denoted as $(abc)_{10}$.
$(a\ b\ c)_{20} = 2\ (abc)_{13}$
$400a + 20b + c = 2(169a + 13b + c)$
$62a = 6b + c$
$b, c \leq 9$. $\therefore\ 6b + c \leq 63$
$\therefore\ 62a \leq 63$
$\therefore\ a \leq 1$.
If $a = 1$, $62 = 6b + c$.
$c$ must be even
We see that the only possible $(b, c)$ is $(9, 8)$.
The number has to be 198.

**20.**

| | | |
|---|---|---|
| 2 | 456 | |
| 2 | 228 | – 0 |
| 2 | 114 | – 0 |
| 2 | 57 | – 0 |
| 2 | 28 | – 1 |
| 2 | 14 | – 0 |
| 2 | 7 | – 0 |
| 2 | 3 | – 1 |
| | 1 | – 1 |

$(456)_{10} = (11001000)_2$
$\therefore$ 4 weights can be used.

**21.** The minimum weight is 8 kg

**22.** L.C.M of 2, 3, 4, 5 and 6 is 60
$\therefore$ The required number is
L.C.M (2, 3, 4, 5, 6) – 1 = 60 – 1 = 59.

**23.** The smallest three-digit number is 60 + 59 = 119.
All these numbers differ by 60.
$t_n = a + (n - 1)\ d$
$119 + (n - 1)\ 60 < 1000$
$60n < 1000 - 59$
$60n < 941$

$$n < \frac{941}{60}$$

$n < 15.6$
$\therefore$ The value of $n$ is 15.
$\therefore$ The number of three digit numbers = 15.

**24.** Given $(6555)_x - (777)_x = (5556)_x$
$\Rightarrow (6555)_x = (5556)_x + (777)_x$
Consider unit digit, we know 6 + 7 = 13 but we have 5 in unit digit, k, 13 – 8 = 5, $\therefore x$ should be 8.
$\therefore\ (5666)_8 + (457)_8 = (6345)_8$

**25.** $(11)_2 + (11)_3 + (11)_4 + \ldots (11)n$
$3 + 4 + 5 + \ldots + n + 1$
$= 1 + 2 + 3 + \ldots + n + 1 - 3$

$$= \frac{(n+1)(n+2)}{2} - 3$$

$$= \frac{n^2 + 3n + 2 - 6}{2} = \frac{n^2 + 3n - 4}{2}$$

# 3 Simple Equations

CHAPTER

**Learning Objectives**

*In this chapter, you will*:

- Learn how to solve an equation when only one unknown variable is present
- Learn how to solve two equations when two unknown variables are present
- Learn to solve three equations when three unknowns are present
- Learn about some special cases in linear equations called determinate and indeterminate equations

There will be linear equations of one or two unknowns invariably in every problem. A **linear equation** is one where each variable occurs only in its first power and not in any higher powers. A simple linear equation or linear equation is a first-degree equation with one or more unknowns. In general, to solve them we need as many equations as the variables.

So, for solving for the values of two unknowns, we need two equations (or two conditions given in the problem) and for solving for the values of three unknowns, we need three equations (and hence, the problem should give three conditions from which we can frame three equations). Solving the equations by itself is not a difficult task. The most important part of the problem is framing the equation/equations. Once the equations are framed, solving them is very easy. In this chapter, we will deal with problems involving as many equations (of first-degree equations where the power of the variables is one) as the number of unknowns. Later on, we will look at equations of second degree (Quadratic Equations) and linear equations where the number of equations will be less than that of the number of variables (under the chapter Special Equations).

## ❒ ONE EQUATION IN ONE UNKNOWN

An equation like $2x + 4 = 26$ is an equation in one unknown. We have only one variable $x$ whose value we have to find out. The steps in solving this are:

**Step I:** Take all quantities added to (or subtracted from) the $x$ term (term with the unknown) to the right side with a change of sign.
i.e., $2x = 26 - 4 = 22$.

**Step II:** Take the co-efficient of $x$ from left hand side and divide right hand side with this term to get the value of $x$.
i.e., $x = 22/2 = 11$. Therefore, $x = 11$.

## ❒ TWO EQUATIONS IN TWO UNKNOWNS

A set of equations like

$$2x + 3y = 8 \quad (1)$$

$$5x + 4y = 13 \quad (2)$$

is called system of simultaneous equations in two unknowns. Here, we have two variables (or unknowns) $x$ and $y$ whose values we have to find out. This can be

done using the two given equations. The steps for this are as follows:

**Step I:** Using both the equations we first eliminate one variable (so that we can then have one equation in one unknown).

For this purpose, we multiply equation (1) with 5 (the co-efficient of $x$ in the second equation) and multiply equation (2) with 2 (the co-efficient of $x$ in the first equation) to eliminate $x$. Thus, we have

$(1) \times 5 \Rightarrow 10x + 15y = 40$ (3)

$(2) \times 2 \Rightarrow 10x + 8y = 26$ (4)

Now, subtracting equation (4) from equation (3), we have $7y = 14$ (5)

This is one equation in one unknown.

**Step II:** Solve for the value of one variable from the equation (in one unknown) obtained from Step I above. Therefore, $y = 2$.

**Step III:** Substitute this value of the variable in one of the two equations to get the value of the second variable.

Substituting the value of $y$ in equation (1) or equation (2), we get $x = 1$. Therefore, the values of $x$ and $y$ that satisfy the given set of equations are $x = 1$ and $y = 2$.

## ❒ THREE EQUATIONS IN THREE UNKNOWNS

A set of equations like

$x + 2y + 3z = 14$ (1)

$2x + y + 2z = 10$ (2)

$3x + 3y + 4z = 21$ (3)

is a system of three equations in three unknowns.

Here, we have three unknowns $x$, $y$ and $z$ which we have to solve for from the three given equations. The procedure for the same is as follows:

**Step I:** Take two out of the three equations [say, equation (1) and (2)] and eliminate one variable (say $x$) so that we get an equation in two unknowns ($y$ and $z$ in this case).

For this purpose, take equations (1) and (2). Multiply equation (1) by 2 and subtract equation (2) from it.

Equation $(1) \times 2 \Rightarrow 2x + 4y + 6z = 28$

$$\underline{2x + y + 2z = 10}$$

$$3y + 4z = 18 \quad (4)$$

**Step II:** Repeat Step I for two other equations [say equations (2) and (3)] and eliminate the same variable ($x$ in this case) so that we get one more equation in two unknowns ($y$ and $z$).

For this purpose, take equations (2) and (3). Multiply equation (2) by 3 and from that subtract equation (3) multiplied by 2.

Equation $(2) \times 3 \Rightarrow 6x + 3y + 6z = 30$

Equation $(3) \times 2 \Rightarrow \underline{6x + 6y + 8z = 42}$

$$-3y - 2z = -12 \quad (5)$$

**Step III:** Now the equations in two unknowns that have been obtained from the above two steps have to be solved as discussed previously (in Two Equations in Two Unknowns) to get the values of two of the three variables ($y$ and $z$ in this case).

In this case, solving equations (4) and (5), we get $y = 2$ and $z = 3$.

**Step IV:** Substitute these values of the two variables in one of the three equations to get the value of the third variable.

Substitute the value of $y$ and $z$ in equation (1) to get the value of $x = 1$.

Thus, the values of the three variables $x$, $y$ and $z$ that satisfy the three given equations are $x = 1$; $y = 2$ and $z = 3$

## ❒ ADDITIONAL CASES IN LINEAR EQUATIONS

1. If the number of equations is less than the number of unknowns, then we say the variables are 'indeterminate' or we have an 'indeterminate' system of equations. Here, we cannot uniquely determine the values of all the variables. There will be infinite sets of solutions that satisfy the equations.

For example, if we take the following two equations in three unknowns,

$x + y + 2z = 8$

$2x - y + 3z = 13$

this system of equations have infinite number of solutions and no unique solution is possible. For any value we take for $x$, we can find a corresponding set of values for $y$ and $z$.

2. However, even in case of indeterminate equations, say, of three variables, it is possible that the value of one of the variables may be uniquely determined, i.e., if we have two equations and three unknowns, we may be still able to determine the value of one variable uniquely but the other two variables will have infinite number of values. This will happen if the ratio of the coefficients of two variables in one equation is the same as the ratio of the coefficients of the same two variables in the second equation.

This depends on the equations given. Example 1.08 in the solved example section will clarify this aspect.

3. Even in case of indeterminate equations, when some additional conditions are either implicitly

built into the problem or explicitly imposed by specifying some constraints on the values of the variables, we may sometimes be able to determine the values of the variables uniquely or find out a finite set of values that the variables may take. Such problems are separately considered under the chapter 'Special Equations.'

4. Sometimes, even if we have equations less in number than the number of variables (i.e., indeterminate equations), while we cannot find out the values of all the variables uniquely, it may be possible to find out the value of some specific combination of the variables.
5. Sometimes, even if we have three equations in three unknown, we may not be able to uniquely determine the values of the variables if the equations are not 'independent,' i.e., one of the given equations can be written as a 'linear combination' of the other two equations.

For example, let us take the following system of three equations in three unknowns.

$3x + 5y + 7z = 12$ (1)

$x - 3y + 9z = 16$ (2)

$9x + 8y + 31z = 54$ (3)

If we try to solve these equations, we will find that we cannot get a unique solution. That is because these equations are not independent. In this case, equation (3) can be obtained by multiplying equation (1) by 2.5 and equation (2) by 1.5 and adding them.

**If there are three equations $l_1$, $l_2$ and $l_3$ in three unknowns, we say that they are linearly dependent if one of the three equations can be written as a linear combination of the other two, i.e., $l_3 = l_1 + kl_2$ where $k$ is any constant.**

In such a case, the system of equations will have infinite number of solutions.

If it is not possible to write the three equations in the form above, then they are linearly independent and the system of equations will have a unique solution.

6. Sometimes, we can have 'inconsistent' equations. For example, if we know that $x + 2y = 4$, then the value of $2x + 4y$ has to be 8. The expression $(2x + 4y)$ cannot take any other value. If it is given any other value, there will be inconsistency in the data because then we will effectively be saying that $x + 2y = 4$ and at the same time $x + 2y \neq 4$.

So, if we have the system of equations

$x + 2y = 4$ and $2x + 4y = k$, this system of equations will be consistent only If the value of $k = 8$. For any other value of $k$, the system of equations will be inconsistent.

In the above system of equations, when $k = 8$, there will be infinite number of solutions (and not a unique solution).

## SOLVED EXAMPLES

**3.01.** If 3 tables and 4 chairs together cost ₹2800 and 2 tables and 3 chairs together cost ₹1950, find the cost of each table and chair, respectively.

**Sol:** Let the cost of each table be $x$ and the cost of each chair be $y$, then we have the following equations from the given data.

$3x + 4y = 2800$ (1)

$2x + 3y = 1950$ (2)

To solve these two equations, multiply equation (1) by 2 and equation (2) by 3 and then subtracting one from the other, we get, $y = ₹\,250$

Substituting the value of $y$ in equation (1) we get

$3x + 1000 = 2800 \Rightarrow x = ₹\,600$

Therefore, the cost of each table is ₹600 and the cost of each chair is ₹250.

**3.02.** Arjun, Balu and Charu went to a shop to purchase pencils, sharpeners and erasers. Arjun bought 5 pencils, 2 sharpeners and 3 erasers for ₹13.50. Balu bought 4 pencils, 3 sharpeners and 2 erasers for ₹12. Charu bought 6 pencils, 2 sharpeners and 4 erasers for ₹16. Find the cost of 3 pencils, 4 sharpeners and 8 erasers.

**Sol:** Let the price of each pencil, sharpener and eraser be $p$, $s$ and $e$, respectively. From the data given we get,

$5p + 2s + 3e = 13.50$ (1)

$4p + 3s + 2e = 12$ (2)

$6p + 2s + 4e = 16$ (3)

Let us take equations (1) and (3) and eliminate the variable $s$ by subtracting (1) from (3)

$p + e = 2.50$ (4)

Then take equations (1) and (2), multiply (1) with 3 and (2) with 2 and subtract one from the other.

We get

$7p + 5e = 16.50$ (5)

(5) – [5 × (4)] gives $2p = 4$
$p = 2$; substituting the value of $p$ in (4),
$e = 0.50$; substituting these values in (1)
$s = 1$
Using these values, we find that 3 pencils, 4 sharpeners and 8 erasers cost
$(3 \times 2 + 4 \times 1 + 8 \times 0.5) =$ ₹ 14.

**3.03.** The sum of the digits of a two-digit number is 12. If the digits are interchanged, the resulting number is 18 more than the original number, find the original number.

**Sol:** Let us consider the two-digit number as $xy$, where $x$ is the tens digit and $y$ is the units digit. Hence, the number itself is equal to $10x + y$.
Since the sum of the digits is given as 12,
$x + y = 12$ (1)
When the digits are interchanged $y$ becomes the tens digit and $x$ the units digit. The number then becomes $(10y + x)$.
Since this number is 18 more than the original number, we have $(10y + x) - (10x + y) = 18$.
$\Rightarrow 9y - 9x = 18$
$\Rightarrow y - x = 2$ (2)
On adding (1) and (2), we get $y = 7$
and substituting $y$ in (2), we get $x = 5$
Hence, the number is 57.

**3.04.** Ten years from now, the age of Raja's father will be twice Raja's age. Ten years ago, the age of Raja's father was thrice Raja's age. Find the present age of Raja and his father.

**Sol:** Let Raja's age and his father's age now be $x$ years and $y$ years, respectively.
Ten years from now, Raja's age will be $x + 10$ and his father's age will be $y + 10$.
$y + 10 = 2(x + 10)$
$\Rightarrow y = 2x + 10$ (1)
Ten years ago, Raja's age was $x - 10$
and his father's age was $y - 10$
Given: $y - 10 = 3(x - 10)$
$y = 3x - 20$ (2)
Equating the values of $y$ in (1) and (2),
we have $2x + 10 = 3x - 20$
$\Rightarrow x = 30$.
By substituting $x = 30$ in (1), we get $y = 70$
$\therefore$ Raja's present age is 30 years and his father's present age is 70 years.

**3.05.** The present age of a father is thrice the age of his son. Fifteen years later, the father's age will be twice the son's age. How many years ago was the age of the father six times the age of the son?

**Sol:** Let the present ages of the father and son be $x$ years and $y$ years, respectively.
Given, $y = 3x$ (1)
$\Rightarrow y + 15 = 2(x + 15) = 2x + 30$ (2)
Substituting $y = 3x$ in (2), we get
$3x + 15 = 2x + 30$,
$\Rightarrow x = 15$,
Substituting $x = 15$ in (1), we get $y = 3x$
$= 3 \times 15 = 45$.
Let us say $p$ years ago, age of the father was six times the age of his son.
$45 - p = 6\ (15 - p)$
Solving, we get, $p = 9$
$\therefore$ 9 years ago, father's age was six times the son's age.

**3.06.** If the numerator and the denominator of a fraction are each decreased by 3, the fraction becomes 2/3. If both the numerator and the denominator are increased by 7, the fraction becomes 3/4. Find the fraction.

**Sol:** Let the fraction be $\frac{x}{y}$
When both the numerator and the denominator are decreased by 3, we have, $\frac{x-3}{y-3} = \frac{2}{3}$
$\Rightarrow 3x - 2y = 3$ (1)
When both the numerator and the denominator are increased by 7, we have $\frac{x+7}{y+7} = \frac{3}{4}$
$\Rightarrow 4x - 3y = -7$ (2)
Multiplying equation (2) by 3 and equation (1) by 4 and subtracting one from the other, we have $y = 33$, putting $y = 33$ in (1) we get $x = 23$.
$\therefore$ The required fraction is $\frac{23}{33}$

**3.07.** Find the values of $x$ and $y$ from the following equations.
$\frac{20}{x+y} + \frac{12}{x-y} = 8$ and $\frac{30}{x+y} - \frac{4}{x-y} = 1$

**Sol:** Let $\frac{1}{x+y} = p$ and $\frac{1}{x-y} = q$.
Substituting these in the given equations,
we get $20p + 12q = 8$ (1)
and $30p - 4q = 1$ (2)
Multiplying equation (2) by 3 and adding to equation (1), we get $110p = 11$

$\Rightarrow P = \frac{11}{110} = \frac{1}{10}$

Substituting $p$ in (1), we get $12q = 6$

$\Rightarrow q = \frac{1}{2}$

$\therefore \frac{1}{x+y} = \frac{1}{10}$ and $\frac{1}{x-y} = \frac{1}{2}$

$\Rightarrow x + y = 10$ (3)

$\Rightarrow x - y = 2$ (4)

Adding (3) and (4), we get

$2x = 12, \Rightarrow x = 6$

Substituting $x = 6$ in (3), we get $y = 4$

**3.08.** Two books, four pens and five files cost ₹ 50. Three books, six pens and seven files cost ₹ 70. Find the cost of each file.

**Sol:** Let $x$, $y$ and $z$ be the cost of each book, pen and file, respectively.

Then we have,

$2x + 4y + 5z = 50$ (1)

$3x + 6y + 7z = 70$ (2)

Here, the coefficients of $x$ and $y$ in equation (1) are in the ratio 1 : 2 which is the same as that of the ratio in equation (2).

As the ratio of these coefficients are same, we can find the value of variable $z$.

If we multiply (1) by 3 and (2) by 2 and subtract one from the other, the variables $x$ and $y$ are eliminated and we get the value of $z$ as 10.

$\therefore$ The cost of each file is ₹ 10

**3.09.** If Ramesh eats 5 vadas, 4 idlis and 5 kachoris, the bill amounts to ₹ 131. If he eats 8 vadas, 6 idlis and 10 kachoris, the bill amounts to ₹ 210. If he eats 6 vadas, 4 idlis and 10 kachoris, how much should Ramesh pay?

**Sol:** Let the cost of each vada, idli and kachori be $V$, $I$, and $K$, respectively, then we have

$5V + 4I + 5K = 131$ (1)

$8V + 6I + 10K = 210$ (2)

While there are only two equations in three unknowns we can see that by taking the difference of the two equations

we get $3V + 2I + 5K = 79$

The cost of 6 vadas, 4 Idlis and 10 kachoris is

$6V + 4I + 10K$

$= 2(3V + 2I + 5K) = 2(79) =$ ₹ 158.

**3.10.** For what value of $k$ will the following system of equations be consistent?

$2x - 3y = 4$; $6x - 9y = k$

**Sol:** In the two equations, the ratio of the coefficients of $x$ terms is same as the ratio of the coefficients of $y$ terms (which is 1 : 3). Hence, the ratio of the constant terms should be the same, for the equations to be consistent. For the constant terms to be in the ratio 1 : 3, the value of $k$ must be 3(4) = 12. Hence, the value of $k$ for which the equations are consistent is 12.

**3.11.** The cost of 3 tables and 4 chairs is ₹ 2500. The cost of 4 tables and 3 chairs is ₹ 2400. Find the costs of each table and each chair.

**Sol:** Let the cost of each table be ₹ $x$.

Let the cost of each chair be ₹ $y$.

$3x + 4y = 2500$ (1)

$4x + 3y = 2400$ (2)

**Method 1:**

Multiplying (1) by 3 and subtracting it from (2) multiplied by 4, we get

$7x = 2100$

$x = 300$

Substituting $x = 300$ in (1),

$y = 400$

**Method 2:**

Adding both the equations (1) and (3), we get

$7(x + y) = 4900$

$x + y = 700$ (3)

Subtracting (2) from (1),

$-x + y = 100$ (4)

Adding (3) and (4), $2y = 800$

$y = 400$

Substituting $y = 400$ in either (3) or (4), $x = 300$

**3.12.** Raju bought 6 pens, 5 erasers and 4 sharpeners for ₹ 32. Had he bought 4 pens, 3 erasers and 5 sharpeners, his total expenditure would have been ₹ 23. Had he bought 7 pens, 2 erasers and 6 sharpeners, his total expenditure would have been ₹ 31. Find the cost of 1 pen, 1 eraser and 2 sharpeners.

**Sol:** Let the prices of each pen, each eraser and each sharpener be ₹ $p$, ₹ $e$ and ₹ $s$, respectively.

$6p + 5e + 4s = 32$ (1)

$4p + 3e + 5s = 23$ (2)

$7p + 2e + 6s = 31$ (3)

Multiplying (1) by 2 and subtracting from (2) multiplied by 3, $-e + 7s = 5$ (4)

Multiplying (3) by 4 and subtracting it from (2) multiplied by 7, $13e + 11s = 37$ (5)

Multiplying (4) by 13 and adding it to (5), $102s = 102$

$s = 1$

Substituting $s = 1$ in (4),
$e = 2$
Substituting values of $e$ and $s$ in (1), $p = 3$

**3.13.** In a two digit number, the digits differ by 2. 10 times the number exceeds 5 times the sum of the number formed by reversing its digits and the sum of its digits by 90. Find the number.

**Sol:** Let the number be $xy$. Hence, the value of the number is $10x + y$.
$x - y = 2$ or $y - x = 2$ (1)
$10(10x + y) - 5(10y + x + x + y) = 90$
$90x - 45y = 90$
$2x - y = 2$
$2x - (x \pm 2) = 2$
$x = 4$ or $0$
As $x$ cannot be 0, $x = 4$
$\therefore y = 6$
$\therefore$ the number is 46

**3.14.** The age of Fahim 4 years ago was 4 times of Sachin's age. Fahim's age three years ago was thrice Sachin's age. How many times Sachin's age was Fahim's age 5 years ago?

**Sol:** Let the present ages of Fahim and Sachin be $f$ years and $s$ years, respectively.
$f - 4 = 4\ (s - 4) \Rightarrow f = 4s - 12$
$f - 3 = 3\ (s - 3) \Rightarrow f = 3s - 6$
$f = 4s - 12 = 3s - 6$
$s = 6, f = 12$
$f - 5 = 7\ (s - 5)$
$\therefore$ 5 years ago Fahim's age was 7 times Sachin's age.

**3.15.** The age of a man 15 years ago was 5 times his son's age. His age 10 years ago was thrice his son's age. After how many years from now will their combined age become 80 years?

**Sol:** Let the present age of the man and his son be $f$ years and $s$ years, respectively.
$f - 15 = 5\ (s - 15) \Rightarrow f = 5s - 60$
$f - 10 = 3\ (s - 10) \Rightarrow f = 3s - 20$
$f = 5s - 60 = 3s - 20$
$s = 20, f = 40$
Their combined present age is 60 years. For the combined present age to become 80 years, the age of each of them must increase by 10 years.
$\therefore$ Their combined age will become 80 years after 10 years.

**3.16.** If the numerator and the denominator of a fraction are both increased by 1, the fraction becomes $\frac{3}{5}$. If both are decreased by 1, it becomes $\frac{5}{9}$. Find the fraction.

**Sol:** Let the fraction be $\frac{x}{y}$.

$$\frac{x+1}{y+1} = \frac{3}{5}$$

$\Rightarrow 5x + 5 = 3y + 3$
$5x + 2 = 3y$ (1)

$$\frac{x-1}{y-1} = \frac{5}{9}$$

$\Rightarrow 9x - 9 = 5y - 5$
$9x - 4 = 5y$ (2)
Multiplying (1) by 5 and subtract it from (2) after multiplying by 3, $5\ (5x + 2) = 3\ (9x - 4)$
$\Rightarrow x = 11$
Substituting $x = 11$ in (1), $y = 19$

$\therefore$ The fraction $= \frac{11}{19}$

**3.17.** Find the values of $x$ and $y$ satisfying the equations below.

$$\frac{35}{x+y} + \frac{18}{x-y} = 11$$

$$\frac{28}{x+y} + \frac{33}{x-y} = 15$$

**Sol:** Let $\frac{1}{x+y} = p$ and $\frac{1}{x-y} = q$
$35p + 18q = 11$ (1)
$28p + 33q = 15$ (2)
Multiplying (1) by 4 and subtracting it from (2) multiplied by 5, $93q = 31$

$$\Rightarrow q = \frac{1}{3}$$

Substituting $q = \frac{1}{3}$ in (1),

$$p = \frac{1}{7}$$

$$\therefore \frac{1}{x-y} = \frac{1}{3} \text{ and } \frac{1}{x+y} = \frac{1}{7}$$

$\therefore x - y = 3$ and $x + y = 7$.
Solving these equations, $x = 5$ and $y = 2$.

**3.18.** Tarun bought 2 shirts, 4 trousers and 5 pairs of shoes for ₹3600. Had he bought 6 shirts, 5 trousers and 15 pairs of shoes, his total expenditure would have been ₹8700. Find the price of each trouser.

**Sol:** Let the prices of each shirt, each trouser and each pair of shoes be ₹ $x$, ₹ $y$ and ₹ $z$, respectively.

$2x + 4y + 5z = 3600$ (1)

$6x + 5y + 15z = 8700$ (2)

Multiplying (1) by 3 and subtracting (2) from it,

$7y = 2100$

$\therefore y = 300$

**3.19.** The cost of 3 dosas, 5 idlis and 7 vadas is ₹154. The cost of 5 dosas, 8 idlis and 11 vadas is ₹246. Find the total cost of one idli, one dosa and one vada.

**Sol:** Let the cost of each dosa, each idli and each vada be ₹ $d$, ₹ $i$ and ₹ $v$, respectively.

$3d + 5i + 7v = 154$ (1)

$5d + 8i + 11v = 246$ (2)

Multiplying (1) by 3 and subtracting it, from twice (2), $d + i + v = 30$

**3.20.** Find the value of $k$ for which the following system of equations will be consistent.

$2x - 5y = 10$ and $6x - 15y = k$

**Sol:** In the given system of equations, the ratio of the coefficients of $x$ equals the ratio of the coefficients of $y$.

$\therefore$ They would be consistent only if this ratio equals the ratio of the constant terms.

$\therefore$ If $\frac{10}{k} = \frac{2}{6} = \frac{-5}{-15}$,

i.e., if $k = 30$, the given system of equations would be consistent.

## EXERCISE-1

***Directions for questions 1 to 50*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** A three-digit number is 9 more than ten times the sum of its digits, if the hundreds digit is 1, the units digit is _____.

**2.** Five sharpeners and six erasers cost ₹28. Six sharpeners and five erasers cost ₹27. Find the cost (in ₹) of each sharpener and each eraser, respectively.

(A) 3, 2 (B) 2, 3 (C) 1, 4 (D) 4, 1

**3.** How many pairs of $x$ and $y$ satisfy the equations $6x + 5y = 16$ and $8x + 7y = 22$?

(A) 0 (B) 1

(C) ∞ (D) None of these

**4.** $P$, $Q$ and $R$ have ₹6000 among themselves. $R$ has two-thirds of the total amount with $P$ and $Q$. The amount with $R$ is _____.

**5.** Ashok's age, 30 years hence, will be twice his age five years ago. Find his present age (in years). _____

**6.** Amar has an amount of ₹190 in denominations of ₹2 and ₹5 coins. He has 10 more ₹5 coins than ₹2 coins. The number of ₹2 coins he has is _____.

**7.** The difference between a three-digit number and the number formed by reversing its digits is always divisible by

(A) only 9 (B) only 11 (C) 101 (D) 99

**8.** In a certain zoo, there are some rabbits and some peacocks. The total number of heads is 78 and the total number of legs is 240. Find the number of peacocks and rabbits. _____

**9.** If $x + \frac{x}{2} + \frac{x}{3} + 4 = 26$, then $x =$ _____.

**10.** 2 chocolates, 4 cakes and 3 milk shakes cost ₹17. One chocolate and 2 cakes cost ₹4. What is the cost of a milk shake?

(A) ₹1 (B) ₹2 (C) ₹3 (D) ₹4

**11.** Three pens and four erasers cost ₹18. Four pens and three erasers cost ₹17. Find the cost of 14 pens and 14 erasers.

(A) ₹70 (B) ₹60

(C) ₹50 (D) ₹40

**12.** Three chocolates, four biscuits and five cakes cost ₹34. Six chocolates and eight biscuits cost ₹38. Find the cost of each cake (in ₹). _____

**13.** A man travelled a total distance of 1800 km by plane, train and bus. He travelled one-third of the whole trip by plane and the distance travelled by train is three–fifth of the distance travelled by bus. Find the distance travelled by bus.

(A) 450 km (B) 850 km

(C) 1200 km (D) 750 km

**14.** Twice a number is 12 more than half the number. What is the number?
(A) 7 (B) 6 (C) 8 (D) 9

**15.** In 5 hours $A$ walks 2 km less than the distance $B$ walks in 9 hours In four hours A walks 11 km more than that covered by $B$ in three hours. The distance that $A$ can walk in 2 hours is _____.

**16.** 2 pencils and 3 pens cost ₹ 8. 2 pens and 3 pencils cost ₹ 7. What is the cost of 5 pencils and 5 pens? (in ₹)
(A) 15 (B) 12.50 (C) 17.50 (D) 20

**17.** When the numerator and the denominator of a fraction are increased by 4 each, the fraction becomes 2/3. When the denominator is decreased by 1, it becomes 3/5. Find the fraction.
(A) $\frac{5}{8}$ (B) $\frac{7}{11}$ (C) $\frac{9}{13}$ (D) $\frac{6}{11}$

**18.** Pradeep receives ₹ 5000 in 500 rupee notes and 100 rupee notes. If he receives 18 notes in total, find the number of 500 rupee notes.
(A) 12 (B) 10 (C) 8 (D) 6

**19.** Solve the following pair of equations for $x$ and $y$, respectively.
$5x - 4y - 5 = 0$ and $3x - 5y + 10 = 0$
(A) 3, 4 (B) 5, 5 (C) 4, 3 (D) 6, 4

**20.** Rajani's present age is 9 years more than half of her present age. What is Rajani's present age? (in years)
(A) 18 (B) 20 (C) 22 (D) 24

**21.** At present, the age of a father is five years more than thrice his son's age. Fifteen years hence, the father's age will be twice his son's age at that time. The present age of the son is _____ years.

**22.** Solve for $x$. $3(x + 4) + 8 = 5x$ _____.

**23.** Three years ago, Anurag was thrice as old as Bhargav was two years hence, Anurag's age will be twice that of Bhargav's age. What is the present age of Anurag? (in years)
(A) 16 (B) 18 (C) 20 (D) 22

**24.** When 2 is added to half of one-third of one-fifth of a number, the result is one-fifteenth of the number. The number is _____.

**25.** Thrice a number exceeds three-fourth of it by 36. Find the number. _____

**26.** $P$, $Q$ and $R$ are integers satisfying the equations $P + Q = 12$, $Q + R = 18$ and $P + R = 24$. Find their product.
(A) 567 (B) 405 (C) 243 (D) 390

**27.** In a class, $2/5^{th}$ of the students are girls and the rest are boys. A Physics exam was conducted in the class. Half of the boys passed in the exam and 12 girls failed in the exam. If the number of students who passed in the exam is 30, how many students are there in the class? _____

**28.** The sum of the digits of a two-digit number is 17. The tens digit of the number is 1 more than the units digit. The two-digit number is _____.

**29.** The difference between a three-digit number and the number formed by reversing its digits is divisible by
(A) 9 (B) 11
(C) Both 9 and 11 (D) Neither 9 nor 11

**30.** One samosa and two puffs cost ₹ 14. Three samosas and one puff cost ₹ 17. What is the cost of 5 samosas and 5 puffs? (in ₹)
(A) 40 (B) 45 (C) 50 (D) 55

**31.** The number of solutions of the system of linear equations $5x + 7y = 3$ and $15x + 21y = 24$ is
(A) 0 (B) 1 (C) 2 (D) $\infty$

**32.** The digits of a two-digit number differ by six. Find the difference of the number and the number formed by reversing its digits. _____

**33.** The total cost of one chair and one table is ₹ 500. The total cost of two chairs and three tables is ₹ 1300. The cost of one table is ₹ _____.

**34.** How many pairs of $x$ and $y$ satisfy $3x + 6y = 18$ and $9x + 18y = 57$?
(A) 2 (B) 1
(C) 0 (D) None of these

**35.** Ten consecutive integers are arranged in ascending order. The sum of the first five integers is 100. What is the sum of the other five consecutive integers? _____

**36.** Goutam is four times as old as Girija. Four years hence, the age of Goutam will be thrice that of Girija. What is the present age of Goutam? (in years)
(A) 20 (B) 24 (C) 28 (D) 32

**37.** The cost of one pen and two books together is ₹ 70. The cost of three pens and nine books together is ₹ 300. Find the difference between the cost of a book and a pen.
(A) ₹ 30 (B) ₹ 20
(C) ₹ 10 (D) None of these

**38.** The sum of the digits of a two-digit number is 7. If the digits are interchanged, the resulting number is 27 more than the original number. Find the original number. _____

**39.** Solve the following pair of equations for $x$ and $y$, respectively.

$12x - 10y - 2 = 0$ and $10x - 10y + 20 = 0$

(A) 13, 11 (B) 11, 13 (C) 12, 10 (D) 10, 12

**40.** Three sharpeners and four erasers cost ₹25. Four sharpeners and three erasers cost ₹24. What are the respective costs of each sharpener and each eraser?

(A) ₹4, ₹3 (B) ₹3, ₹4
(C) ₹3, ₹3 (D) ₹4, ₹4

**41.** The sum of a two-digit number and its reverse is $k$ times the sum of its digits. Find the value of k.

(A) 9 (B) 10
(C) 11 (D) Cannot be determined

**42.** The age of a man twenty five years hence will be four times his age twenty years ago. The present age of the man is _____ years.

**43.** The two digits of a two-digit number differ by 4. What is the difference between the number and the number formed by reversing its digits?

(A) 72 (B) 54 (C) 36 (D) 18

**44.** How many pairs of $x$ and $y$ satisfy the equations $4x + 6y = 16$ and $6x + 9y = 24$?

(A) 0 (B) 1
(C) ∞ (D) None of these

**45.** In a three-digit number, the hundreds digit is 2 more than the tens digit and the units digit is 2 less than the tens digit. If the sum of the digits is 18, the number is _____.

**46.** The first and the last digits of a three-digit number differ by 4. Find the difference of the number and the number formed by reversing its digits. _____

**47.** The sum of the digits of a two-digit number is 12. The difference of the digits is 6. Find the number.

(A) 93 (B) 39
(C) Either (A) or (B) (D) None of these

**48.** The cost of two dosas and three idlis is ₹46. The cost of a dosa and two idlis is ₹26. Find the cost of four dosas and four idlis. (in ₹)_____

**49.** The cost of 3 chairs and 4 tables is ₹2,500. The cost of 4 chairs and 3 tables is ₹2,400. Find the total cost of 6 chairs and 6 tables. (in ₹)_____

**50.** Nine times the tens digit of a two-digit number when added to eight times the units digit is equal to the two-digit number. The number is _____.

## EXERCISE-2

***Directions for questions 1 to 50*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** If 2 tables and 3 chairs together cost ₹1800 and 3 tables and 4 chairs together cost ₹2600, then find the costs of each table and each chair separately.
(both in ₹)_____.

**2.** Venkat takes 2 hours more than Vatsa to cover a distance of 600 km. Instead if Venkat doubles his speed, he would reach the destination 4 hours before Vatsa. Find Vatsa's speed.

(A) 100 km/hr (B) 50 km/hr
(C) 60 km/hr (D) 120 km/hr

**3.** If Rahul takes six marbles from Kunal, then the number of marbles with both of them would be equal. Instead if Kunal takes one marble from Rahul, Kunal would have twice the number of marbles as Rahul has. What is the total number of marbles with the two of them? _____

**4.** The number of chocolates with Seoni is $7/9^{th}$ of the number of chocolates with Varsha. If Varsha has 14 chocolates more than Seoni, then find the total number of chocolates with them.

(A) 48 (B) 80 (C) 96 (D) 112

**5.** The cost of three pens, four rulers and five refills is ₹75 while that of ten refills, six pens and seven rulers is ₹138. Find the cost of three pens, one ruler and five refills.

(A) ₹39 (B) ₹42
(C) ₹44 (D) Cannot be determined

**6.** Some apples were distributed evenly among some boys. If eight more apples were distributed, each boy would have got four apples. If the number of apples distributed had been two less, each boy would have got three apples. How many apples were distributed? _____

**7.** The cost of each pencil is ₹2 and the cost of each pen is ₹5. There are some pencils and pens costing ₹50. If the total number of pens and pencils is 16, what is the number of pencils in the box?

(A) 5 (B) 8 (C) 10 (D) 15

Difficulty Level-2: Moderate

**8.** If the equations $3x + 4y = 24$ and $15x + 20y = 8k$ are consistent, find the value of $k$.
(A) 5 (B) 120 (C) 30 (D) 15

**9.** Ten years ago, a man's age was 35 years more than twice his son's age. After how many years from now will the man's age be twice his son's age? _____

**10.** Gopi, Murthy and Hari had some amount of money. Gopi gives half his amount to Murthy, who then gives half of what he now has to Hari. Hari gives half of what he now has to Gopi, who, now has exactly what he started with. If the sum of Murthy's initial amount and twice Hari's initial amount is ₹45, what was the amount (in ₹) Gopi started with? _____

**11.** Four years from now, Prakash's age will be4 times his son's age. Twelve years from now he will be 2½ times his son's age. Find their present ages. (both in years) _____

**12.** A man had enough money to purchase 16 apples or 10 mangoes. If the man buys four apples and five mangoes and is left with ₹20, then what is the difference in the prices of an apple and a mango?
(A) ₹2 (B) ₹3 (C) ₹4 (D) ₹6

**13.** Mr. Ram distributed a total of 225 chocolates among his sons—A, B, C and D. The number of chocolates he gave to A and D together is twice the number of chocolates he gave to B and C together. If B received 15 more chocolates than C, find the number of chocolates C received. _____

**14.** The sum of the digits of a three-digit number ($N$) is 9. When 99 is subtracted from it, the resulting number is a three-digit number formed by reversing the digits of $N$. What is the least value of the number? _____

**15.** Govind is four times as old as Ganesh is. 20 years hence, Govind's age will be twice that of Ganesh's age. Find Ganesh's present age. (in years)
(A) 20 (B) 10 (C) 15 (D) 30

**16.** In an organization, two-thirds of the employees are software professionals and one-fifth of them are women. If 240 men are software professionals, then the number of employees in the organization is
(A) 450 (B) 540 (C) 750 (D) 360

**17.** The present average age of Ram and his wife Sita and their daughter is 35 years. Fifteen years from now, the age of Sita will be equal to the sum of the present ages of Ram and the daughter. Find the present age. (in years) of Sita _____

**18.** Seven burgers and eight pizzas cost ₹780 while twelve burgers and five pizzas cost ₹945. Find the cost of each pizza.
(A) ₹60 (B) ₹40 (C) ₹45 (D) ₹50

**19.** A question paper consists of 50 questions. Each correct answer fetches three marks and one mark is deducted for each wrong answer. A student who attempted all the questions scored 90 marks. Find the number of questions answered correctly by him. _____

**20.** If the system of equations below has infinite solutions, find $k$
$4x + ky = 2 + 10y$ and $kx + 24y = 8$
(A) –10 (B) 16
(C) –16 (D) Cannot be determined

**21.** The present age of a father is 10 times his son's age. In 6 years time his age will be four times his son's age. In how many years will the father be twice as old as his son? _____

**22.** Ours is a big family. I have thrice as many brothers as sisters and my sister Bharathi has four times as many brothers as sisters. How many children do my parents have? _____

**23.** A boy has a total of ₹14 in denominations of 25 paise and 20 paise coins. If the numbers of coins of the two denominations were swapped, the total value of coins would be ₹1 less. Find the total number of coins. _____

**24.** If $4x - 3y + 6 = 0$ and $ky - 8x + 12 = 0$ are inconsistent, find the value of '$k$'. _____

**25.** Two boys and two girls went to a movie. They found that there were only two tickets available in the counter and they bought them. For purchasing the remaining two tickets (in black), they spent ₹50 more for each ticket than the actual price. At the end they found that each person had spent ₹60 for the ticket as his/her share. Find the actual price (in ₹) of each ticket. _____

**26.** Kunal has only 25 paise and 50 paise coins with him. The total amount in 50 paise denomination is ₹4 more than the total amount in 25 paise denomination. The number of 25 paise coins is 20 more than the number of 50 paise coins. What is the total amount with Kunal?
(A) ₹32 (B) ₹36 (C) ₹40 (D) ₹24

**27.** A rope is 64 m long. It is cut into 3 pieces. The longest piece is 3 times as long as the middle sized piece and the shortest piece is 34 m shorter than the longest piece. Find the length of the middle sized piece.
(A) 14 m (B) 16 m (C) 18 m (D) 22 m

**28.** If $2x + 3y + 2z = 23$ and $3x - 2y + 3z = 28$, then what is the value of $x + z$? _____

**29.** Praveen's present age is twice that of Mahesh's age four years ago. Eight years hence, Praveen would be twice as old as Mahesh is today. Find the sum of their present ages.
(A) 36 years (B) 44 years
(C) 64 years (D) Cannot be determined

**30.** $P$, $Q$ and $R$ are consecutive positive even integers in the ascending order. Four times $R$ is 4 more than five times $P$. What is the value of $Q$?
(A) 12 (B) 14 (C) 16 (D) 10

**31.** Solve for $x$, $y$, $z$: $x + 2y + 3z = 14$

$2x + y + 2z = 10$

$3x + 3y + 4z = 21$

**32.** Rohan went to the market to buy 10 kg of each of oranges, mangoes, bananas and grapes. The cost of 5 kg oranges and 2 kg mangoes together was ₹310. The cost of 3 kg mangoes and 3.5 kg bananas together was ₹230. The cost of 1.5 kg bananas and 5 kg grapes together was ₹160. Find the total amount spent by Rohan. (in ₹) _____

**33.** A two-digit number is such that the sum of its digits is thrice the difference of its digits. If the number exceeds the number formed by reversing its digits by 36, find the number. _____

**34.** The cost of three pencils, five rulers and seven erasers is ₹49. The cost of five pencils, eight rulers and eleven erasers is ₹78. Find the cost of one pencil, one ruler and one eraser.
(A) ₹8 (B) ₹9
(C) ₹7 (D) Cannot be determined

**35.** The sum of the ages of Ajay and Bala, 20 years ago was five-ninth the sum of their present ages. Ajay's present age exceeds that of Bala by 20 years. Find the present age of Ajay. (in years) _____

**36.** For what value of $k$ are these two equations not inconsistent? _____

$2x + 3y = 5$

$6x + 9y = k$

**37.** Four years ago, a man was thrice as old as his son. Eight years hence, the man will be twice as old as his son. What is the present age (in years) of the son? _____

**38.** There is some money with Amit and some money with Sunil. If Amit gives ₹40 to Sunil, then the amounts with them would be equal. If instead, Sunil gives ₹10 to Amit, then Amit would have ₹100 more than Sunil has. Find the amount with Sunil.
(A) ₹140 (B) ₹100
(C) ₹120 (D) Cannot be determined

**39.** The sum of the present ages of $P$ and $Q$ is 84 years. Six years ago, the age of $P$ was twice that of $Q$. What is the difference between the present ages of $P$ and $Q$? (in years) _____.

**40.** The combined present age of a girl's parents is five times that of the girl's age. After 8 years, their combined age would be four times that of the age of the girl. What was the combined age of the parents when the girl was born?
(A) 80 years (B) 64 years
(C) 48 years (D) 72 years

**41.** How many two-digit numbers with tens digit greater than units digit are such that have the sum of the digits equals to twice the difference of the digits? _____

**42.** The age of a woman is thrice that of her daughter. When the woman was 29 years old, her only son, who is three years younger to her only daughter, was born. What is the present age of her son?
(A) 8 years (B) 9 years
(C) 10 years (D) 11 years

**43.** If the numerator and the denominator of a fraction are increased by 2, the fraction becomes 3/5. If the numerator and the denominator are increased by 1, the fraction becomes 1/2. Find the fraction. _____

**44.** If the cost of one pen, one pencil and three erasers is ₹140 and the cost of five pens, three pencils and an eraser is ₹320, then find the cost of three pens, two erasers and two pencils.
(A) ₹460 (B) ₹230
(C) ₹250 (D) Cannot be determined

**45.** In $t$ minutes, the time would be 8:00 a.m. If 40 minutes ago, the time was $3t$ minutes past 2:00 a.m., then find the present time.
(A) 6:20 a.m. (B) 6:40 a.m.
(C) 5:20 a.m. (D) 5:40 a.m.

**46.** Katrina wants to buy 6 kg of tomatoes and 7 kg of potatoes which together would cost her ₹190. In the market as tomatoes were very good, she decided to buy 2 kg more tomatoes and 6 kg less potatoes and spent only ₹170. What is the price of 1 kg of tomatoes?
(A) ₹10 (B) ₹15 (C) ₹20 (D) ₹25

**47.** The difference between the digits of a two-digit number is 4. The sum of the number and the number formed by reversing its digits is 110. Find the two-digit number. _____ or _____

***Directions for questions 48 and 49***: These questions are based on the following data.

Three cups of ice-cream, two burgers and four soft drinks together cost ₹128. Two cups of ice-cream, one burger and two soft drinks together cost ₹74.

**48.** What is the cost of each cup of ice-cream?
(A) ₹20 (B) ₹30
(C) ₹25 (D) Cannot be determined

**49.** What is the cost of five burgers and ten soft drinks?
(A) ₹160 (B) ₹128
(C) ₹170 (D) Cannot be determined

**50.** Natasha was asked to find $4/7^{th}$ of a number. She calculated 4/17 of the number instead and got an answer, which was 840 less than the correct answer. What is the original number?
(A) 2600 (B) 2599 (C) 2499 (D) 2737

## EXERCISE-3

***Directions for questions 1 to 50***: For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Bala had three sons. He had some chocolates which he distributed among them. To his eldest son, he gave 3 more than half the number of chocolates with him. To his second eldest son he gave 4 more than one-third of the remaining chocolates with him. To his youngest son he gave 4 more than one-fourth of the remaining chocolates with him. He was left with 11 chocolates. How many chocolates did he initially have?
(A) 180 (B) 78 (C) 144 (D) 120

**2.** Ramu has some chocolate boxes with him to sell. He sells them either as full boxes or half boxes. The first customer buys half a box more than half the number of boxes with Ramu. The second customer buys half a box more than half the remaining number of boxes with him. Ramu continues to sell in this manner to eight other customers. He is left with no boxes to sell after that. How many chocolate boxes did Ramu have in the beginning?
(A) 511 (B) 513 (C) 1023 (D) 1025

**3.** Twice the difference between a two digit number and the number formed by reversing its digits is two less than the sum of the number and the number formed by reversing the digits of the number. If the original number is greater than the number formed by reversing the digits, what is the original number? _____

**4.** At a certain stationery store, four varieties of pens and four varieties of pencils are sold. The price of a set of one of each of the four varieties of pens is ₹45. The price of a set of one of each of the four varieties of pencils is ₹12. The price of each of the cheapest pencils is ₹1 and the combined price of three of the varieties of the pens is exactly twice the combined price of the other 3 varieties of pencils. What is the price of each of the costliest pens?
(A) ₹24 (B) ₹23 (C) ₹22 (D) ₹25

***Directions for questions 5 and 6***: These questions are based on the information given below.

Rohan went to a stationery shop to purchase pens, erasers and rulers. He purchased more number of pens than erasers and more number of erasers than rulers. He purchased at least 10 items of each type. The total number of items purchased is 35.

**5.** How many rulers did Rohan purchase? _____

**6.** If each pen cost ₹20, each ruler cost ₹2 and each eraser cost ₹5, find the minimum amount (in ₹) that Rohan spent for purchasing the items. _____

**7.** Alok went to a casino to play a card game. He played 10 rounds of that game. In each round, he doubled his amount and then gave ₹ $x$ to his friend. After 10 rounds, he had ₹1023. Find the sum of the digits of the least possible value of $x$. (All the amounts involved (in ₹) are integers). _____

***Directions for questions 8 and 9***: These questions are based on the information given below.

A shopkeeper sold a two-digit number of toys all priced at a certain value (also a two-digit number when expressed in rupees). By mistake he reversed the digits of both, the number of items sold and the price (in ₹) of that item while entering in the computer. So, the stock which was shown as left at the end of the day in the computer showed 81 items more than what it actually was.

**8.** How many possibilities exist for the actual number of toys sold? _____

**9.** If the faulty calculations show a total sale of ₹486, what was the actual selling price (in ₹) of each toy? _____

**10.** The difference between a three-digit number and the number formed by reversing its digits is 297. The sum of the units and the tens digits is the same as the difference of the hundreds and the units digits. Also, the hundreds digit is twice the units digit. Find the number.
(A) 242 (B) 342 (C) 603 (D) 884

**11.** Which of the following cannot be the sum of a 6-digit number and the number formed by reversing the digits?
(A) 1999998 (B) 357753
(C) 947749 (D) 906706

**12.** If $x/4$ years ago, Alok was 14 years old and $x/4$ years from now he will be $4x$ years old, how old will he be $5x$ years from now? (in years) _____.

***Directions for questions 13 to 14***: These questions are based on the data given below.

A shopkeeper sold a certain number of toys all at a certain price. The number of toys that he sold is a three-digit number in which the tens digit and units digit are the same and are non-zero, and the price of each toy is a two-digit number when expressed in rupees. By mistake he reversed the digits of both, the number of items sold and the price of each item. In doing so, he found that his stock account at the end of the day showed 792 items more than what it actually was.

**13.** What could be the actual number of toys sold?
(A) 911 (B) 119 (C) 199 (D) 991

**14.** If the faulty calculations show a total sale of ₹5117, what was the actual selling price of each toy?
(A) ₹43 (B) ₹37 (C) ₹75 (D) ₹34

**15.** Using the information in the above question, what are the actual sales?
(A) ₹39,173 (B) ₹30,974
(C) ₹4,046 (D) ₹4,064

**16.** If the following three equations form a system of dependent equations, what is the value of $p$?

I. $3x + 2y - 7z = 56$

II. $5x + 3y + z = 16$

III. $px + 12y - 19z = 200$ _____.

**17.** The sum of the thousands and the units digits of a four-digit number is equal to the sum of its other two digits. The sum of the hundreds and the units digits of the number is equal to four times the sum of the other two digits of the number. The sum of the digits of the number is more than 10. The hundreds digit of the number cannot be
(A) 9 (B) 8 (C) 7 (D) 6

**18.** Laxmilal and Kuberjain have some silver pendants with them. Laxmilal says to Kuberjain 'If you give me a certain number of silver pendants, I'll have five times as many silver pendants as you will be left with. If I give the same number of pendants to you, we both will have equal number of silver pendants.' If they have a total of 36 silver pendants, how many silver pendants does Laxmilal have?
(A) 12 (B) 27 (C) 30 (D) 24

**19.** X says to Y, 'I am twice as old as you were when I was as old as you are.' The sum of their present ages is 63 years. Find the present age of X.
(A) 24 years (B) 39 years
(C) 36 years (D) 42 years

**20.** Which one of the following conditions must $a$, $b$ and $c$ satisfy so that the following system of linear equations has at least one solution such that $a + b + c \neq 0$?

$x + 3y - 4z = a$

$4x + y - 5z = b$

$x + y - 2z = c$

(A) $2a + 3b - 5c = 0$ (B) $3a + 2b - 5c = 0$
(C) $8a + 2b - 11c = 0$ (D) $3a + 2b - 11c = 0$

**21.** Shreya told Lata 'I am thrice as old as you were when I was as old as you are'. Lata told Shreya 'The sum of our ages today is 80 years'. After listening to them, Raman, a mathematician, said 'Shreya is _____ years elder to Lata'.

**22.** A two-digit number is obtained by either subtracting 12 from four times the sum of its digits or by adding 6 to twice the difference of its digits. Find the number.
(A) 16 (B) 28
(C) 39 (D) Cannot be determined

**23.** Beso is a currency which is available only in three denominations—1 Beso, 5 Besos and 20 Besos. A person had to settle a bill which amounted to 49 Besos. If he uses at least one note of each denomination, in how many ways can he settle the bill?
(A) 5 (B) 6 (C) 7 (D) 8

**24.** A so-called great gambler started playing a card game with a certain amount of money. In the first round he tripled his amount and he gave away ₹p to his friend. In the

second round he doubled the amount with him and gave away ₹3p to his friend. In the third round he quadrupled the amount with him and gave away ₹2p to his friend and was finally left with no money. If he gave away a total of ₹360 to his friend, then what was the amount of money that he started with (in ₹)? _____

**25.** Find $k$ if the given system of equations has infinite solutions. $2x + ky = 1 + 2y$ and $kx + 12y = 3$. _____

26. A tells B, 'When I was $3/5^{th}$ your present age, you were $5/7^{th}$ of my present age'. If the present age of A is 28 years, what is the present age of B? (in years) _____.

**27.** A person starts with $x$ rupees and $y$ paisa. He spent ₹8.40 and was left with $2y$ rupees and $2x$ paisa. If $x < 50$ and $y < 100$, what is the amount that he started with? (in ₹) _____

***Directions for questions 28 and 29***: These questions are based on the following data.

The roses in the garden of Manas started blooming from the month of January. In every alternate month starting from January, 60 more flowers appeared than there were at the end of the previous month. In every alternate month starting from February, the number of flowers became half the number there were at the end of previous month. 120 roses bloomed in the month of June.

**28.** How many roses bloomed in the month of January? _____

**29.** How many more or less roses bloomed in the month of September than the number of roses that bloomed in the month of June?

(A) 90 more (B) 30 less
(C) 45 more (D) 30 more

**30.** Prakash, Sameer, Ramesh and Tarun have a total of ₹240 with them. Prakash has half the total amount of what the others have. Sameer has one-third of the total amount of what the others have. Ramesh has one-fourth of the total amount of what the others have. Find the amount with Tarun. (in ₹) _____

**31.** A test has 120 questions. A student gets 4 marks for every correct answer, loses 2 marks for every wrong answer, and loses 1 mark for every unattempted question. If the net score of a student, who took that test is 228 marks, then the number of questions which he answered wrong could be at the most _____.

**32.** Solve: $\frac{15}{x+2y} - \frac{11}{3x+4y} = 2$

$\frac{5}{2(x+2y)} + \frac{22}{3x+4y} = \frac{5}{2}$

(A) $x = 3, y = 1$ (B) $x = 1, y = 2$
(C) $x = 2, y = 1$ (D) $x = 1, y = 3$

**33.** Mr. and Mrs. Smith have only three children, Andy, Bandy and Candy. Seven years ago, Mr. Smith's age was twice the sum of the ages of his three children put together. Now, the combined age of Mr. and Mrs. Smith is equal to twice the sum of the ages of their three children put together. If Bandy is two years elder to Candy and three years younger to Andy, what is the age of Mrs. Smith? years _____.

**34.** A shopkeeper had a weighing balance with uneven pans. The left and the right pans of the balance weighed 0.6 kg and 0.95 kg, respectively. When the shopkeeper placed some rice in the left pan and standard weights in the right pan until the pans leveled, he had to use '$ab$' kg (where '$ab$' is a two-digit number) of the standard weights. If instead, he placed the rice in the right pan and standard weights in the left pan until the pans leveled, he had to use ('$ba$' + 18.7) kg of the standard weights. The actual weight of rice placed in the pans can be

(A) 76.35 kg (B) 85.35 kg
(C) 35.35 kg (D) 53.35 kg

**35.** When a three-digit number is reversed, the number decreases by 396. The difference between the digit at the units place and the digit at the tens place is the same as the difference between the digits at the tens place and the hundreds place. If the product of the digits of the three digit number is non-zero, then how many such three-digit numbers are possible? _____

**36.** Find the value of $k$ if the equations $4x + 5y = 32$ and $12x + 15y = 2k$ are not inconsistent. _____

**37.** An exam has 120 questions. Each correct answer carries 1 mark. Each wrong answer is penalized by $\frac{1^{rd}}{3}$ of a mark and each unanswered question is penalized by $\frac{1^{th}}{6}$ of a mark. A student who attempted the exam scored 60 marks. The minimum number of answers that the student could have got wrong is _____.

**38.** In a four-digit number with distinct digits, the sum of the middle digits equals the sum of the extreme digits. The sum of its second and fourth digits equals five times the

sum of its other two digits. If the sum of its digits is 18, what is the sum of all the possible values of the hundreds digit?
(A) 21 (B) 24 (C) 27 (D) 30

**39.** Once in ten years the government of a country counts all the people living in the country. The following data about two neighbouring villages of Badasansthan and Chotasansthan was complied. Badasansthan has 5114 more males than Chotasansthan. Chotasansthan has twice as many females as males. Badasansthan has 3,004 more males than females. Badasansthan has 9118 females fewer than Chotasansthan. The number of females in Badasansthan is
(A) 11,228 (B) 16,342 (C) 13,628 (D) 13,338

**40.** $p+q+r+s+t+3a=25$

$p+q+r+s+3t+u=23$

$p+q+3r+3s+t+u=21$

$p+q+3r+s+t+u=19$

$p+3q+r+s+t+u=17$

$3p+q+r+s+t+u=15$

Find the value of $(p)(q)(r)(s)(t)(u)$. _____

**41.** A bag has a total of 120 notes in denominations of ₹2, ₹5 and ₹10. The total value of the notes in the bag is ₹760. If there were twice as many ₹5 notes, the total value of the notes would be ₹960. Find the number of ₹10 notes in the bag. _____

**42.** In a three-digit number, the difference between hundreds digit and the tens digit is equal to the difference between the tens digit and the units digit. If the sum of the digits is 9, how many numbers satisfy the given condition? _____

**43.** Considering the equations $2x-3y=8$ and $px-qy=66$, answer the following questions:

1. Find $4(p+q)$ if the equations above have infinite solutions. _____
2. Find $p$ if $q=9$ and the equations above have no solution. _____

**44.** Solve for $x, y, z$: $3x+5y+7z=12$

$x-3y+9z=16$

$9x+8y+31z=54$

(A) 0, 1, 1 (B) 2, $\frac{1}{2}$, $\frac{1}{2}$

(C) 1, $\frac{3}{4}$, $\frac{3}{4}$ (D) Cannot be determined

**45.** Let $x_1, x_2, \ldots, x_6$ be real numbers. The following equations hold true.

$x_1+4x_2+9x_3+16x_4+25x_5+36x_6=9$

$4x_1+9x_2+16x_3+25x_4+36x_5+49x_6=98$

$9x_1+16x_2+25x_3+36x_4+49x_5+64x_6=987$

Find the value of $16x_1+25x_2+36x_3+49x_4+64x_5+81x_6$.
(A) 2676 (B) 2798 (C) 2542 (D) 2754

**46.** The cost of two pencils, one eraser and three sharpeners is ₹23. The cost of six pencils, three erasers and one sharpener is ₹29. The cost of 14 pencils, seven erasers and seven sharpeners is ₹91. Find the cost of each pencil.
(A) ₹3 (B) ₹5
(C) ₹4 (D) Cannot be determined

**47.** There are some oranges and mangoes in a basket and there are a certain number of plates in a cupboard. If each orange is put in one plate, one orange is left without a plate to put it in. If two mangoes are put in each plate, then one plate is left without any mango to put in it. If the difference between the number of oranges and the number of mangoes is 3, what is the number of plates in the cupboard? _____

**48.** If $a$, $b$, $c$ and $d$ satisfy the equations

$a+7b+3c+5d=0$,

$8a+4b+6c+2d=-16$,

$2a+6b+4c+8d=16$ and

$5a+3b+7c+d=-16$, then $(a+d)(b+c)$ equals
(A) 0 (B) 16
(C) –16 (D) –64

**49.** Eswar, Ganesh and Harish played a game of three rounds. The person who lost in a round had to double the amount of money with each of the others. Each of the three lost a round. At the end of three rounds, each of them had ₹60. The amounts with the three persons at the beginning of the first round were
(A) ₹93.5, ₹50.5, ₹39 (B) ₹97.5, ₹52.5, ₹30
(C) ₹93.5, ₹53.5, ₹33 (D) ₹101.5, ₹56.5, ₹22

**50.** Six years ago, the age of a person was two years more than five times the age of his son. Four years hence, his age will be two years less than three times the age of his son. After how many years from now will their combined age be 100 years?
(A) 48 years (B) 14 years
(C) 19 years (D) 38 years

## EXERCISE-4

***Directions for questions 1 to 30*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

***Directions for questions 1 and 2*:** These questions are based on the following data.

Three persons *A*, *B* and *C* started playing a game. They had money in the ratio 3 : 2 : 1 at the beginning of the game. It was decided that, if A lost the round, he would give $1/4^{th}$ of the money that he had prior to that round equally to *B* and *C*. If *B* lost, he would give half of his money equally to *A* and *C* and if *C* lost, he would give $2/3^{rd}$ of his money equally to *A* and *B*. *B* lost the first round, *C* lost the next round, *A* lost the third round and *B* lost the fourth round. *A* gained ₹10 over the four rounds.

**1.** What was the total money with the three of them initially?
(A) ₹60 (B) ₹120 (C) ₹150 (D) ₹240

**2.** After which round should the game have been stopped for *A* to gain the maximum amount?
(A) Round 1 (B) Round 2
(C) Round 3 (D) Cannot be determined

**3.** A taxi driver charged his customers in the following manner. If his customers hire his taxi for not more than 4 hours, he charged them at ₹7/km or ₹40 per hour, whichever was more. If his customers hire his taxi for more than 4 hours, he charged them ₹8/km or ₹48 per hour whichever is more. For how many hours was his taxi rented by a customer who was driven by the driver 25 km and was charged ₹240 for the same?
(A) 4 (B) 7 (C) 5 (D) 6

**4.** In a three-digit number, the number obtained when twice the tens digit is subtracted from thrice the hundreds digit, is equal to the number obtained when the units digit is subtracted from thrice the hundreds digit which in turn is equal to the number obtained when twice the tens digit is subtracted from the units digit. How many such three-digit numbers are possible?
(A) 0 (B) 1 (C) 2 (D) 3

***Directions for questions 5 and 6*:** These questions are based on the following data.

I was visiting my friend's house at Tuticorin for the first time. I knew that he had five children—two daughters and three sons. The daughters were Sonali and Monali and the sons were Surya, Sagar and Prithvi. When I stepped into the house was greeted by Sagar and Sonali. Sagar introduced himself and told me that he was twice as old as Sonali. The next to meet me was Monali who said that she and Sonali together were twice as old as Sagar. Then Surya walked in and introduced himself and said that he and Sagar were twice as old as their sisters together. In the afternoon Prithvi came to my room when I was relaxing and said 'Sorry uncle, I was busy with my friends. As it is my $21^{st}$ birthday today I could not meet you in the morning.' When I greeted him and said that it was fine, he said, 'You know uncle, the three of us Sagar, Surya and I are five times as old as our sisters together,' saying which he went away.

**5.** How much older than Sonali is Monali?
(A) 3 years (B) 3 years 6 months
(C) 5 years 6 months (D) 7 years

**6.** After how long would Surya be twice as old as Sagar?
(A) 2 years (B) 2 years 6 months
(C) 3 years 6 months (D) 4 years

**7.** Four brothers—Ram, Lakshman, Bharath and Shathrugna—bought a car for ₹2,40,000. Ram paid an amount equal to half of the sum of the amounts paid by his three brothers. Shathrugna paid one-fifth of the total amount paid by the other three brothers. The amount paid by Bharath is five-ninteenth of the total amount paid by his three brothers. How much did Lakshman pay?
(A) ₹80,000 (B) ₹40,000
(C) ₹70,000 (D) ₹60,000

**8.** Raman had a certain number of chocolates with him. He distributed these among his three sons in the following manner. To his eldest son, he gave 4 less than half of the chocolates with him. To his second son, he gave 3 less than one-fourth of the remaining chocolates. To his youngest son, he gave 2 less than two-thirds of the remaining chocolates. Find the number of chocolates Raman initially had, if he is finally left with 18 chocolates.

———

***Directions for questions 9 and 10*:** These questions are based on the following data.

Nanda and Parekh are two of my neighbours. Each son of Nanda had five times as many sisters as the number of brothers, while each son of Parekh had as many sisters as the number of brothers. Each daughter of Nanda had twice the number of sisters as the number of brothers, but each daughter of Parekh had twice the number of brothers as the number of sisters.

**9.** How many children does Parekh have?
(A) 7 (B) 4 (C) 5 (D) 6

**10.** What is the ratio of the number of sons to that of the daughters in the two families together?
(A) 4 : 3 (B) 3 : 4 (C) 5 : 2 (D) 2 : 5

**11.** Ajay told Bharat, 'I am four times as old as you were when I was as old as you are'. Bharat told Ajay 'Ten years ago, I was nine years younger to you'. Find the sum of the present ages of Ajay and Bharat.
(A) 39 years (B) 36 years
(C) 42 years (D) 52 years

**12.** Five three-digit numbers including *N*, were to be added. While adding, the reverse of *N* was added by mistake instead of *N*. Hence, the sum increased by 11 times the sum of the digits of *N*. Eight times the difference of N's units and hundreds digits is 6 more than twice its hundreds digit. Find its tens digit.
(A) 4 (B) 6 (C) 8 (D) 2

**13.** In a four-digit number, the sum of the middle two digits is twice the units digit. The sum of the hundreds digit and six times the thousands digit is twice the sum of the other two digits. The sum of the units digit and five times the thousands digit is twice the hundreds digit. How many values can the four-digit number assume?

———

**14.** Amar, Bhavan, Chetan and Dinesh each had some money. Dinesh doubled the amounts with the others. Chetan then doubled the amounts with the others. Bhavan then doubled the amounts with the others. Amar then doubled the amounts with the others. At this stage, each of them has ₹80. Find the initial amount with Chetan (in ₹).
(A) 45 (B) 65 (C) 95 (D) 85

***Directions for questions 15 and 16*:** These questions are based on the following data.

Rohan entered the 'Good World Super market' to buy pens, pencils and erasers for his younger brothers and sisters. He bought at least 11 pieces of each of the items and more pens than pencils and more pencils than erasers. He bought a total of 38 pieces.

**15.** How many erasers did Rohan buy?
(A) 11 (B) 10
(C) 12 (D) Cannot say

**16.** If the number of pencils cannot be divided equally among his four brothers, how many pens did he buy?
(A) 12 (B) 11 (C) 15 (D) 14

***Directions for questions 17 and 18*:** These questions are based on the following data.

Smitha attempts all the questions in a management entrance test which has questions of three different levels of difficulty—A, B and C. For the questions of various difficulty levels, the marks awarded for answering the questions correctly and those deducted for answering the questions incorrectly are as follows:

| Difficulty level | Marks given for answering correctly | Marks deducted for answering incorrectly |
|---|---|---|
| A | 3 | 2 |
| B | 2 | 1 |
| C | 1 | 1/2 |

No marks are deducted for unanswered questions.

There are 60 questions of A level, 60 questions of B level and 80 questions of C level in the test.

**17.** If Smitha gets 60 marks overall and she answered 40 questions of C level difficulty correctly, what is the maximum number of questions she could have answered incorrectly overall?
(A) 74 (B) 116 (C) 108 (D) 102

**18.** In the same test, Mishra attempted 30 questions of each level of difficulty. If he answered 30 questions wrong overall and 46 marks are deducted for them, how many marks did Mishra score?
(A) 108 (B) 126
(C) 144 (D) Cannot be determined

**19.** Varun presented a cheque at his bank. The teller intended to give him the amount in notes of 2 denominations, ₹50 and ₹500. He planned to give him $x$ 50-rupee notes and $y$ 500-rupee notes. But while paying, he was distracted and gave Varun $x$ 500-rupee notes and $y$ 50-rupee notes. Varun pocketed the cash without verifying. He had no money other than what the teller had given him. After spending ₹350, Varun found that he was left with twice the amount for which he had presented the cheque. If the amount of the cheque was ₹P, which of the following could be true?
(A) $1000 < P < 1500$ (B) $1500 < P < 2000$
(C) $2000 < P < 2500$ (D) $2500 < P < 3000$

**20.** Find the condition that $a$, $b$ and $c$ must satisfy so that the equations $p + 3q - 4r = a$, $3p + 2q + 5r = b$ and $5p + 8q - 3r = c$ have at least one solution such that $a + b + c \neq 0$.
(A) $a + b - 2c = 0$ (B) $2a - b + c = 0$
(C) $2a + b - c = 0$ (D) $2a + 2b - c = 0$

**21.** The price of onions was ₹400 on 1st April of year *X*. Every day, for the next 200 days, the price increased by ₹1. After that the price remained constant till the 31st March of the following year. The price of tomatoes on 1st April of the year *X* was ₹200. Every day after that the price increased by ₹1.60. On which day of the year *X* were the prices of onion and tomatoes equal? (All the prices given are per quintal)
(A) 6th Dec (B) 7th Dec
(C) 1st Dec (D) 2nd Dec

**22.** Mohan and Nitin have certain chocolates with them. Mohan gave Nitin three-fourths of his chocolates and Nitin returned him three-fourths of the chocolates he has then. This is called a transaction and it is observed that after any whole number of such transactions the ratio of the number of chocolates with Mohan and Nitin is always the same. If Mohan initially has 324 chocolates more than Nitin, how many chocolates did Nitin initially have?
(A) 81 (B) 108
(C) 162 (D) Cannot be determined

***Directions for questions 23 and 24*:** These questions are based on the following data.

The villagers of Panigarh were facing acute shortage of water this year. When the minister for water resources visited the village, he promised that a water tank would be constructed. As promised by the minister, a tank was built which can contain enough water for the entire village for 50 days.

The villagers were very happy as for the next 50 days there would not be any shortage of water. However, their joy was short lived as they discovered that there was a leak at the bottom of the tank which drained out 20000 litres of water every day. Because of the leak, the water lasted only 40 days for the entire village.

**23.** What is the total requirement of water (in litres) of the village per day?
(A) 40000 (B) 50000
(C) 75000 (D) 80000

**24.** If the leak was plugged partially and only 5000 litres of water was getting drained out per day, on which day would the tank get emptied?
(A) 45th (B) 46th (C) 47th (D) 48th

**25.** Him and Haw, to do their MBA program at Roseiur Institute, appeared for the RAT entrance exam, and got 7 and 24 marks, respectively. Each question answered correctly gets one mark but the negative marks for each of the first five wrong attempts is different from that of the remaining wrong attempts. Him attempted 22 questions and Haw attempted 45 questions and they answered 50% and 66.66% questions correctly, respectively. Find the negative marks for each mistake made, beyond the first 5 mistakes.
(A) 1/3 (B) 1/2 (C) 1/5 (D) 1/4

**26.** Hariharan's expenditure is equal to Lezy Louis' salary and his savings are half of Lezy Louis' savings. If Lezy Louis' salary is ₹25000, which of the following is true?
(A) The sum of twice Hariharan's salary and Lezy Louis' expenditure is ₹75000.
(B) Hariharan's salary is not less than ₹25000 but not more than ₹37500
(C) If Hariharan's salary is more than ₹37500, then Lezy Louis' expenditure could be ₹0.
(D) Both (A) and (B)

**27.** In country *X*, the local currency is Alto. The only notes printed in *X* are 1 Alto, 5 Altos and 15 Altos. At a certain shop, a man bought some items. He received a bill of 41 Altos for the purchase. In how many ways can he settle the bill by using at least one note of each denomination?
_____

**28.** In 2014, the female population of town A was 7671 more than that of town B. The male population of B was twice that of the female population of B. The female population of A was 4506 more than its male population and the male population of A was 13677 less than that of B. The male population of A was _____.

***Directions for questions 29 and 30*:** These questions are based on the following data.

On the 13th of a month I kept some money in a magical purse in which, on every alternate day, starting from the 14th, the money decreases by ₹10 when compared to the closing amount on the evening of the previous day. But on every alternate day, starting from the 15th, the money doubles with respect to the closing amount on the evening of the previous day.

I had ₹500 on the 19th and the magical purse kept on decreasing and multiplying the money for me till the 25th of that month.

**29.** How much money did I put initially in the purse?
(A) ₹50 (B) ₹80
(C) ₹140 (D) Cannot say

**30.** Had I started with ₹100, how much more money would I have than what I had on the 25th of the month?
(A) ₹400 (B) ₹760
(C) ₹1,280 (D) ₹1,640

## EXERCISE-5 Based on Data Sufficiency

***Directions for questions 1 to 30*:** Each question is followed by two statements, I and II. Answer each question based on the following directions:

Choose (A) If the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.

Choose (B) If the question can be answered by using either statement alone.

Choose (C) If the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (D) If the question cannot be answered even by using both the statements together.

**1.** Find the value of $2x + 3y$.

I. $6x + 12y = 18$.

II. $9y + 6x = 12$.

**2.** Is the cost of Musk melon more than that of Jack fruit?

I. The sum of the costs of 5 Musk melons and 15 Jack fruits is ₹400.

II. The sum of the costs of 7 Musk melons and 21 Jack fruits is ₹560.

**3.** A box contains one-rupee coins and 50 paise coins. What is the total amount in the box?

I. Total number of coins is 120.

II. If ten 50 paise coins are replaced by equal number of one-rupee coins, the amount increases by ₹5.

**4.** A piece of wood is cut into 3 pieces A, B and C. Are they of equal length?

I. The sum of the lengths of A and B is $2/3^{rd}$ of the length of the original piece.

II. B and C are of same length.

**5.** What is the difference between the costs of 12 shirts and 36 pairs of pants?

I. Total cost of 12 shirts and 36 pairs of pants is ₹1200.

II. If the cost of each shirt is doubled, 12 shirts will cost the same as that of 72 pairs of pants.

**6.** What is the age of a man, who was blessed with a son at the age of 28 years?

I. Man's age is 8 times the son's age.

II. Man's wife's age is 7 times the son's age.

**7.** A sum of ₹160 is shared among A, B and C. How much of it does B get?

I. A and B together get ₹80.

II. B and C together get ₹100.

**8.** Is Sunil older than Gopal?

I. Gopal is 3 years younger than Sekhar.

II. Sekhar is five years older than Sunil.

**9.** What is my age now?

I. My brother is 2 times as old as I was 4 years ago.

II. I am now three times as old as I was 12 years ago.

**10.** How many questions did Aarti attempt in the Data Sufficiency test?

I. There were 25 questions in the test.

II. Aarti got 21 marks in the test, in which every correct answer fetched her 1 mark and for every incorrect answer 1/3 mark was deducted from the total.

**11.** Is the age of a person equal to the sum of the ages of his son and wife?

I. The person is two years older than his wife.

II. The age of his son is twelve years, and he was born when his wife was 30 years old.

**12.** What is the number of students who opt for History or Chemistry?

I. There are twice as many History students as there are Chemistry students.

II. Twenty students opted for Chemistry.

**13.** How many doors are there in Manish's house?

I. There are 36 doors and windows in total in his house.

II. There are twice as many windows as the number of doors.

**14.** Which car gives more mileage, Uno or Zen?

I. Uno gives a mileage of 45 miles per 10 litres of petrol, which is three-fourths of the mileage of Alto.

II. The mileage of Palio is four-fifths of the mileage of Alto, which is two miles less than that of Zen.

**15.** Is the price of gold 300 times more than that of silver per unit weight?

I. The price of gold is ₹4000 per ten grams, which is exactly half of the price of silver per kg.

II. The price of 100-gram gold is twice that of the American diamond.

**16.** If $x + 2y + 3z = 14$, then find the value of z.

I. $2x + 3y + z = 14$

II. $3x + y + 2z = 11$

**17.** If $3x - y + 2z = 11$, then find the value of $7x + y - 2z$.

I. $2x + 3y = 13$

II. $x - 2y + 4z = 12$

**18.** Guru has ₹500 in one-rupee coins, 50-rupee notes and 100-rupee notes. He exchanged all his coins for 50-rupee and 100-rupee notes (not by value only by number). How many 50-rupee notes did he have after the exchange?

I. He has not more than 6 notes after the exchange.

II. He has not less than 6 notes after the exchange.

**19.** What is my age?

I. Five years back my sister's age was half of my age.

II. Five years from now my sister's age will be three-fourths of my age.

**20.** Does $12x + 18y = c$ have at least one integral solution for $x$ and y?

I. $c$ is an even multiple of 9.

II. $c^2 + 4c - 396 = 0$

**21.** What is the value of $\frac{7a + 9b}{4a + 5b} = ?$

I. $b + a = \frac{1}{2}(6a - b)$

II. $3a + 4b = 5$

**22.** What is $A$'s speed?

I. $A$ covers 10 km more per hour than $B$.

II. If $B$ doubles his speed, then $B$ covers 20 kmph more than $A$.

**23.** Amit bought three apples and a mango for ₹14. What is the price of the mango?

I. The price of five apples and two mangoes is ₹13 less than six apples and four mangoes.

II. The price of an apple is equal to the price of two mangoes.

**24.** If $3x + 7y = 19$, then find the value of $y$.

I. $6x + 14y = 38$ II. $9x - 20y = 16$

**25.** In a town T, each person reads The Hindu or The Times of India. How many readers from town T read only The Hindu?

I. In town T, 1500 people read The Hindu and 900 people read The Times of India.

II. The population of town T is 2000.

**26.** An apple costs ₹7 and an orange costs ₹5. Amisha bought some oranges and apples. Did she buy equal number of oranges and apples?

I. Amisha paid a total of ₹65 for the apples and the oranges she has bought.

II. Had she bought 4 apples more and 3 oranges more, she would have paid a total of ₹108.

**27.** Find the cost of a pencil, a sharpener and an eraser together.

I. The cost of five pencils, seven erasers and fifteen sharpeners is ₹49.

II. The cost of eight pencils, eleven erasers and twenty-three sharpeners is ₹77.

**28.** Find the value of the four-digit number.

I. The sum of the first three digits of the number is equal to the last digit which is also thrice the first digit.

II. The second digit is twice the first digit and the third digit is 2 less than the first digit.

**29.** Amit says, 'After $x$ years, my age will be 2 times the age of Bimal'. Bimal says, 'After $2x$ years, my age will be same as the current age of Amit'. What is the value of $x$?

I. The difference of the present ages of Amit and Bimal is 5 years.

II. $2x$ years hence, Amit will be five times as old as Bimal is now.

**30.** Ram and Shyam have a total of ₹200 with them. Does Ram have more than ₹100?

I. If Ram gives ₹10 to Shyam, the difference of the amounts with them would be ₹20.

II. If Shyam gives ₹5 to Ram, the difference of the amounts with them would be ₹10.

## ANSWER KEYS

### Exercise-1

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (9) | 9. (12) | 17. (D) | 25. (16) | 33. (300) | 41. (C) | 49. (4200) |
| 2. (B) | 10. (C) | 18. (C) | 26. (B) | 34. (C) | 42. (35) | 50. (71) |
| 3. (B) | 11. (A) | 19. (B) | 27. (60) | 35. (125) | 43. (C) | |
| 4. (2400) | 12. (3) | 20. (A) | 28. (98) | 36. (D) | 44. (C) | |
| 5. (40) | 13. (D) | 21. (10) | 29. (C) | 37. (B) | 45. (864) | |
| 6. (20) | 14. (C) | 22. (10) | 30. (B) | 38. (25) | 46. (396) | |
| 7. (D) | 15. (10) | 23. (B) | 31. (A) | 39. (B) | 47. (C) | |
| 8. (42) | 16. (A) | 24. (60) | 32. (54) | 40. (B) | 48. (80) | |

### Exercise-2

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (600), (200) | 8. (D) | 16. (A) | 24. (6) | 31. (1, 2 & 3) | 38. (D) | 46. (C) |
| 2. (C) | 9. (25) | 17. (45) | 25. (35) | 32. (1400) | 39. (24) | 47. (73, 37) |
| 3. (42) | 10. (30) | 18. (C) | 26. (A) | 33. (84) | 40. (C) | 48. (A) |
| 4. (D) | 11. (28, 4) | 19. (35) | 27. (A) | 34. (B) | 41. (3) | 49. (C) |
| 5. (A) | 12. (B) | 20. (B) | 28. (10) | 35. (55) | 42. (C) | 50. (C) |
| 6. (32) | 13. (30) | 21. (24) | 29. (D) | 36. (15) | 43. (1) | |
| 7. (C) | 14. (261) | 22. (21) | 30. (B) | 37. (16) | 44. (B) | |
| | 15. (B) | 23. (60) | | | 45. (B) | |

### Exercise-3

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (B) | 9. (45) | 17. (D) | 25. (6) | 33. (35) | 41. (50) | 47. (6) |
| 2. (C) | 10. (C) | 18. (D) | 26. (30) | 34. (D) | 42. (9) | 48. (C) |
| 3. (82) | 11. (D) | 19. (C) | 27. (37.14) | 35. (5) | 43. (i) (165) | 49. (B) |
| 4. (B) | 12. (35) | 20. (D) | 28. (600) | 36. (48) | (ii) (6) | 50. (C) |
| 5. (10) | 13. (A) | 21. (16) | 29. (D) | 37. (3) | 44. (D) | |
| 6. (340) | 14. (D) | 22. (A) | 30. (52) | 38. (B) | 45. (A) | |
| 7. (6) | 15. (B) | 23. (B) | 31. (42) | 39. (D) | 46. (D) | |
| 8. (1) | 16. (19) | 24. (55) | 32. (B) | 40. (0) | | |

### Exercise-4

| | | | | | |
|---|---|---|---|---|---|
| 1. (B) | 6. (C) | 11. (A) | 16. (D) | 21. (B) | 26. (D) |
| 2. (B) | 7. (C) | 12. (B) | 17. (B) | 22. (B) | 27. (7) |
| 3. (C) | 8. (112) | 13. (2) | 18. (D) | 23. (D) | 28. (20007) |
| 4. (B) | 9. (A) | 14. (D) | 19. (B) | 24. (D) | 29. (B) |
| 5. (B) | 10. (B) | 15. (A) | 20. (C) | 25. (B) | 30. (C) |

### Exercise-5

| | | | | | |
|---|---|---|---|---|---|
| 1. (A) | 6. (A) | 11. (C) | 16. (C) | 21. (A) | 26. (B) |
| 2. (D) | 7. (C) | 12. (D) | 17. (A) | 22. (C) | 27. (C) |
| 3. (D) | 8. (C) | 13. (C) | 18. (A) | 23. (B) | 28. (C) |
| 4. (C) | 9. (A) | 14. (C) | 19. (C) | 24. (A) | 29. (A) |
| 5. (A) | 10. (D) | 15. (A) | 20. (A) | 25. (C) | 30. (A) |

## SOLUTIONS

## EXERCISE-1

**1.** Let the three-digit number be $100a + 10b + c$.

$100a + 10b + c = 10(a + b + c) + 9$

$\Rightarrow 90a = 9c + 9$

$c = \dfrac{90a - 9}{9} = 10a - 1$

As $a = 1$, $c = 9$

**2.** Let the costs of each sharpener and each eraser be ₹ $s$ and ₹ $e$, respectively.

$5s + 6e = 28$ (1)

$6s + 5e = 27$ (2)

Adding both equations, $11(s + e) = 55$

$\Rightarrow s + e = 5$ (3)

Subtracting (2) from (1), we get $-s + e = 1$ (4)

Adding (3) and (4), we get $e = 3$

Substituting $e = 3$ in (3) or (4), we get $s = 2$.

**3.** Two equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ where $x$ and $y$ are variables and $a_1$, $b_1$, $a_2$, $b_2$, $c_1$ and $c_2$ are all constants will have

(i) no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$;

(ii) Infinite solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ and

(iii) a unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

In the given problem, as $\frac{6}{8} \neq \frac{5}{7}$ the equations have a unique solution.

**4.** Let the amount with $R$ be $r$.

$r = \frac{2}{3}$ (total amount with $P$ and $Q$)

$r = \frac{2}{3}(6000 - r)$

$\Rightarrow 3r = 12000 - 2r$

$\Rightarrow 5r = 12000$

$\Rightarrow r = 2400$

**5.** Let Ashok's present age be $x$ years.

His age 30 years hence, is twice his age 5 years ago.

$\therefore x + 30 = 2(x - 5)$

$x = 40$

**6.** Let the number of ₹2 coins with Amar be $a$.

Number of ₹5 coins with him = $a + 10$.

$5(a + 10) + 2a = 190$

$\Rightarrow 7a = 140 \Rightarrow a = 20$

7. Let the three digit number be $100x + 10y + z$. When the digits are reversed it becomes $100z + 10y + x$.

The difference is $(100x + 10y + z) - (100z + 10y + x)$

$= 99x - 99z = 99(x - z)$

$\therefore$ The difference is always divisible by 99, irrespective of values of $x$, $y$ and $z$.

8. Let the no. of peacocks be $P$.

Let the no. of rabbits be $R$.

$P + R = 78$ (1)

$2P + 4R = 240$ (2)

Solving equations (1) and (2), we get $P = 36$ and $R = 42$.

**Alternate Solution:**

If all the animals were peacocks then we would have a total of 156 legs. However total number of legs is $(240 - 156) = 84$ more. The excess in the total number of legs is due to the two additional legs of each rabbit.

$\therefore$ Number of rabbits $= \frac{84}{2} = 42$.

**9.** $x + \frac{x}{2} + \frac{x}{3} + 4 = 26 \Rightarrow x + \frac{x}{2} + \frac{x}{3} = 22$

$\Rightarrow \frac{11x}{6} = 22 \Rightarrow x = 12$

**10.** Let the cost of each chocolate, cake and milk shake be ₹ $a$, ₹ $b$ and ₹ $c$, respectively.

From the data,

$2a + 4b + 3c = 17$ (1)

$a + 2b = 4$ (2)

The corresponding coefficients of $a$ and $b$ in both the equations are in the same ratio.

$\therefore$ We can find the value of $c$.

(1) $- 2 \times$ (2) $\Rightarrow$

$2a + 4b + 3c = 17$

$2a + 4b \quad = 8$

$- \quad - \quad - \quad -$

$3c = 9$

$\Rightarrow c = 3$

$\therefore$ The cost of each milk shake is ₹3

**11.** Let the cost of each pen and each eraser be ₹ $x$ and ₹ $y$, respectively.

$3x + 4y = 18$

$4x + 3y = 17$

Adding both equations, we get

$7x + 7y = 35$

Cost of 14 pens and 14 erasers

$= 14x + 14y = 2(7x + 7y) =$ ₹70

**12.** Le $t$ the costs of each chocolate, each biscuit and each cake be ₹ $x$, ₹ $y$ and ₹ $z$, respectively.

$3x + 4y + 5z = 34.$

$6x + 8y = 38 \Rightarrow 3x + 4y = 19$

$5z = 34 - (3x + 4y) = 15$

$z = 3$

**13.** Total distance travelled = 1800 km

Distance travelled by plane = 600 km

Let distance travelled by bus = $x$

$\therefore$ Distance travelled by train = $\frac{3x}{6}$

$\Rightarrow x + \frac{3x}{5} + 600 = 1800$

$\Rightarrow \frac{8x}{5} = 1200$

$\Rightarrow x = 750$ km

**14.** Let the number be $x$.

$2x = \frac{x}{2} + 12 \Rightarrow \frac{3x}{2} = 12 \Rightarrow x = 8$

$\therefore$ The number is 8.

**15.** Let the distance that $A$ and $B$ can walk in 1 hour be $a$ km and $b$ km, respectively.

$5a = 9b - 2$ (1)

$4a - 3b = 11$ (2)

Solving (1) and (2) we have $a = 5$ and $b = 3$.

Distance that $A$ can walk in 2 hours = $2a = 10$ km.

**16.** Let the cost of each pencil and each pen be ₹ $x$ and ₹ $y$, respectively. From the data,

$2x + 3y = 8$ (1)

$3x + 2y = 7$ (2)

Adding (1) and (2), we get $5x + 5y = 15$

$\therefore$ The cost of 5 pencils and 5 pens is ₹ 15

Note that in some cases it is not necessary to solve individually for $x$ and $y$.

**17.** Let the numerator and denominator of fraction be $n$ and $d$, respectively.

$\frac{n+4}{d+4} = \frac{2}{3} \Rightarrow 3n + 12 = 2d + 8$

$\Rightarrow 2d - 3n = 4$ (1)

$\frac{n}{d-1} = \frac{3}{5} \Rightarrow 5n = 3d - 3 \Rightarrow 3d - 5n = 3$ (2)

Multiply equation (1) by 3 and equation (2) by 2 and subtract it from resultant equation (1), we get, $n = 6$

Substitute $n = 6$ in equation (1) or (2),

we get, $d = 11$

So, the fraction is 6/11.

**18.** Let $x$ be the number of ₹ 500 notes and $y$ be the number of ₹ 100 notes.

$500x + 100y = 5000$ (1)

$x + y = 18$ (2)

Solving the equations (1) and (2), we get $x = 8$.

**19.** $5x - 4y - 5 = 0 \Rightarrow 5x - 4y = 5$ (1)

$3x - 5y + 10 = 0 \Rightarrow 3x - 5y = -10$ (2)

Multiplying equation (1) by 3 and equation (2) by 5 and subtracting the resultant equation (1) from resultant equation (2), we get, $13y = 65$

$\Rightarrow y = 5$

Substituting $y$ in either equation (1) or (2) we get, $x = 5$

**20.** Let Rajani's age be $P$ years.

From the data, $R = 9 + \frac{R}{2}$

$\Rightarrow \frac{R}{2} = 9 \Rightarrow R = 18$

$\therefore$ Rajani's age is 18 years.

**21.** Let the present ages of the father and the son be $F$ and $S$, respectively, $F = 3S + 5$.

$F + 15 = 2\ (S + 15)$

Substituting $F$ as $3S + 5$ in the above equation, we get $3S + 5 + 15 = 2S + 30$.

$S = 10$ years.

**22.** $3(x + 4) + 8 = 5x \Rightarrow 3x + 20 = 5x$

$\Rightarrow x = 10.$

**23.** Let the present age of Bhargav be '$b$'.

Let the present age of Anurag be '$a$'.

From the data,

$(a - 3) = 3(b - 3)$

$\Rightarrow 3b - a = 6$ (1)

$(a + 2) = 2\ (b + 2)$

$\Rightarrow a - 2b = 2$ (2),

By solving, $b = 8$

Substituting $b = 8$ in (1) $\Rightarrow a = 18$

$\therefore$ The present age of Anurag is 18 years.

**24.** Let the number be $a$.

$$2+\frac{1}{2}\left(\frac{1}{3}\left(\frac{a}{5}\right)\right)=\frac{a}{15}$$

$$\Rightarrow 2=\frac{a}{30}\Rightarrow a=60$$

**25.** Let the number be $x$. Given, $3x-\frac{3}{4}x=36$

$\therefore x=16$.

**26.** $P+Q=12$, $Q+R=18$ and $P+R=24$ (1)

Adding these, we get $2(P+Q+R)=54$

$\therefore P+Q+R=27$.

(1) $\Rightarrow R=15$, $P=9$ and $Q=3$.

$P.Q.R=405$.

**27.** Let the number of students in the class be $x$.

$\frac{2}{5}x$ are girls and $\frac{3}{5}x$ are boys.

It is given that 1/2 of the boys passed.

$\therefore \frac{3}{5}x\times\frac{1}{2}=\frac{3x}{10}$ boys passed.

Given

$$\frac{3x}{10}+\frac{2x}{5}-12=30\Rightarrow x=60$$

$\therefore$ The total number of students in the class is 60.

**28.** Let the two-digit number be $10a+b$

$a+b=17$ (1)

$a-b=1$ (2)

Adding (1) and (2), we get $2a=18\Rightarrow a=9$

Substitute $a=9$ in equation (1) or (2), we get, $b=8$

Hence, the two-digit number is 98.

**29.** The difference between a three-digit number and the number formed by reversing its digits is always divisible by 99 and hence by both 9 and 11.

**30.** Let the cost of each Samosa and Puff be ₹ $S$ and ₹ $P$, respectively. From the data,

$S+2P=14$ (1)

$3S+P=17$ (2)

$2x$ (1) + (2) $\Rightarrow 2S+4P=28$

$3S+P=17$

$5S+5P=17$

$\therefore$ The cost of 5 Samosa and 5 Puffs is ₹ 45.

**31.** Given linear equations are $5x+7y=3$ and $15x+21y=24$.

Clearly, the given lines are parallel.

So, no common solution.

**32.** The difference of a two digit number and the number formed by reversing its digits is given by 9 (difference of its digits). In the given problem, as the difference of the digits is 6, the difference of the number and its reverse = 9 (6) = 54.

**33.** Let the cost of each chair and each table be $C$ and $T$, respectively.

$C+T=500$ (1)

$2C+3T=1300$ (2)

Multiplying equation (1) by 2 and subtracting it from equation (2), we get $T=300$.

**34.** Multiplying the first equation by 3, we get $9x+18y=54$.

Combining this with the second equation, the conclusion is 54 = 57, which can never be true.

$\therefore$ The given equations have no solution.

**35.** Let the first five consecutive integers be $(x-2)$, $(x-1)$, $x$, $(x+1)$ and $(x+2)$.

By adding these five numbers we get $5x=100$, $x=20$.

The five consecutive integers are 18, 19, 20, 21 and 22.

The next five consecutive integers are 23, 24, 25, 26 and 27. Their sum = 125.

Alternatively, each number in the next five numbers is 5 more than the corresponding number in the given series. Therefore, their total = 100 + 25 = 125.

**36.** Let the age of Goutam be '$m$' and that of Girija be '$n$'.

From the data,

$m=4n$ (1)

$(m+4)=3(n+4)$ (2)

substituting (1) in (2)

$\Rightarrow 4n+4=3n+12$

$\Rightarrow n=8$

$\therefore m=4n=4\times 8=32$

$\therefore$ The age of Goutam is 32 years.

**37.** Let the price of book be $y$ and the price of pen be $x$.

$\therefore x+2y=70$ (1)

$3x+9y=300$ (2)

By solving the equations, we get $x = 10$ and $y = 30$.

$\therefore$ The difference between the cost of a book and a pen is $30 - 10 = ₹20$.

**38.** Let us take the two digits as $x$ and $y$. $x$ is ten's digit and $y$ is the unit's digit, hence, the number itself is equal to $(10x + y)$.

Since sum of the digits is 7,

$x + y = 7$ (1)

When the digits are interchanged, $y$ becomes the $10x + y$ ten's digit and $x$ the units digit. The number then becomes $(10y + x)$, since this number is 27 more than the original number, we get $(10y + x) - (10x + y) = 27 \Rightarrow 9y - 9x = 27$

$\Rightarrow y - x = 3$ (2)

On adding (1) and (2), we get $y = 10/2 = 5$.

By substituting the value of $y$, we get $x = 2$.

Thus, the number is 25.

**39.** $12x - 10y - 2 = 0$ (1)

$10x - 10y + 20 = 0 \Rightarrow 10x = 10(y - 2)$

Dividing both sides by 10, $x = y - 2$ (2)

Substituting $x$ as $y - 2$ in equation (1),

$12(y - 2) - 10y - 2 = 0$

$\Rightarrow 12y - 24 - 10y - 2 = 0$

$\Rightarrow 2y = 26 \Rightarrow y = \frac{26}{2} = 13$.

From (2), $x = 11$.

**NOTE**

Suppose there are two linear equations in two variables, $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where $a_1$, $b_1$, $c_1$, $a_2$, $b_2$, $c_2$ are the coefficients of $x$ and y.

(i) The set of equations has a unique solution when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) The set of equations has infinite solutions, when $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(iii) The set of equations has no solutions, when $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

**40.** Let the cost of each sharpener and eraser be $S$ and $E$, respectively.

From the data,

$3S + 4E = 25$ (1)

$4S + 3E = 24$ (2)

(1) + (2) $\Rightarrow$

$7S + 7E = 49$

$\Rightarrow 7(S + E) = 49 \Rightarrow S + E = 7$ (3)

(1) – (3) × (3) $\Rightarrow$

$3S + 4E = 25$

$3S + 3E = 21$

$E = 4$

Substituting $E = 4$, in (3), we get

$3 + 4 = 7 \Rightarrow S = ₹3$

$\therefore$ The cost of each sharpener is ₹3 and that of each eraser is ₹4.

**41.** Let the two-digit number be $10a + b$.

Given $(10a + b) + (10b + a) = k(a + b)$

$\Rightarrow 11(a + b) = k(a + b) \Rightarrow k = 11$ ($\because a + b \neq 0$).

**42.** Let $x$ be the present age of the man.

20 years ago it was $(x - 20)$ years.

Given $x + 25 = 4(x - 20)$

$\Rightarrow 3x = 105 \Rightarrow x = 35$ years.

**43.** Let the two digit number be $10x + y$. When its digits are reversed, it becomes $10y + x$.

The difference is $(10x + y) - (10y + x)$.

$= 9x - 9y = 9(x - y)$

Given that the two digits differ by 4, $(x - y) = 4$

$\therefore$ The difference $= 9(x - y) = 9 \times 4 = 36$

**44.** Multiplying the first equation by 3/2 the second equation is obtained.

$\therefore$ We have one equation with two unknowns.

$\therefore$ $(x, y)$ has infinite values.

**45.** Let the three-digit number be $100a + 10b + c$.

$a = b + 2$

$c = b - 2$

$a + b + c = 3b = 18 \Rightarrow b = 6$

so $a = 8$ and $c = 4$

Hence, the three-digit number is 864.

**46.** The difference between a three-digit number and the number formed by reversing its digits = 99 (difference of its first and last digits). As the difference of its first and last digits is 4, the difference of the number and the number formed by reversing its digits = 99 (4) = 396.

**47.** Let the two digit number be $10a + b$.

$a + b = 12$ (1)

If $a > b$, $a - b = 6$,

If $b > a$, $b - a = 6$

If $a - b = 6$, adding it to equation (1), we get

$2a = 18 \Rightarrow a = 9$

so $b = 12 - a = 3$

$\therefore$ Number would be 93.

if $b - a = 6$, adding it to equation (1),

$2b = 18 \Rightarrow b = 9$

$a = 12 - b = 3$.

$\therefore$ Number would be 39.

$\therefore$ Number would be 39 or 93.

**48.** Let the cost of each dosa and each idli be ₹ $d$ and ₹ $i$, respectively.

$2d + 3i = 46$ (1)

$d + 2i = 26$ (2)

(1) – (2) : $d + i = 20$

The cost of 4 idlis and 4 dosas viz $4(d + i) = 4(20)$, i.e., ₹80

**49.** Let the costs of a chair and a table be $c$ and $t$, respectively. Required cost = $6c + 6t$.

$3c + 4t = 2500$

$4c + 3t = 2400$

By adding the equations and dividing the resulting equation by 7 we get the value of $1c + 1t$.

$6c + 6t = 6(c + t) = 6(700) = 4200$.

**50.** Let the two-digit number be $10a + b$.

$9a + 8b = 10a + b$

$7b = a$

$0 < a \leq 9$ and $0 < 7b \leq 9$ so $b = 1$

Hence, $b$ is 1, then a = 7, hence, the number is 71.

## EXERCISE-2

**1.** Let cost of each table be $x$ and cost of each chair be $y$. Then we have the following equations from the given data.

$2x + 3y = 1800$ (1)

$3x + 4y = 2600$ (2)

To solve these two equations, multiply the first equation by 3 and the second by 2 and then subtract one from the other.

We get, $y = 200$

Substituting $y$ in (1), we get

$2x + 600 = 1800$

$\therefore x = 600$

$\therefore$ The cost of each table is ₹600 and the cost of each chair is ₹200.

**2.** Let, Venkat's speed be $x$ km/hr and that of Vatsa be $y$ km/hr.

Given, $\frac{600}{x} - \frac{600}{y} = 2$

$\Rightarrow \frac{1}{x} - \frac{1}{y} = \frac{1}{300}$ (1)

and $\Rightarrow \frac{600}{y} - \frac{600}{2x} = 4$

$\Rightarrow \frac{1}{y} - \frac{1}{2x} = \frac{2}{300}$ (2)

(1) + (2) $\Rightarrow \frac{1}{x} - \frac{1}{2x} = \frac{1}{300} + \frac{2}{300}$

$\Rightarrow x = 50$

From $\frac{1}{50} - \frac{1}{y} = \frac{1}{300} \Rightarrow \frac{1}{y} = \frac{5}{300} = \frac{1}{60}$

$\therefore y = 60$

$\therefore$ Vasta's speed is 60 km/hr.

**3.** Let the number of marbles with Rahul and Kunal be $x$ and $y$, respectively.

Given $x + 6 = y - 6$

$x - y = -12$ (1)

$y + 1 = 2\ (x - 1)$ (2)

By solving, we get $x = 15$ and $y = 27$.

Total number of marbles = 15 + 27 = 42.

**4.** Let the numbers of chocolates with Seoni and Varsha be $7x$ and $9x$, respectively.

Given, $9x - 7x = 14 \Rightarrow x = 7$

Total number of chocolates

$= 7x + 9x = 16x = 16\ (7) = 112$

$\therefore$ Total number of chocolates with them is 112.

**5.** Let, the cost of each pen, each ruler and each refill be $x$, $y$ and $z$, respectively.

$3x + 4y + 5z = 75$ (1)

$6x + 7y + 10z = 138$ (2)

$2 \times (1) - (2); \Rightarrow y = 12$

$\therefore\ 3x + y + 5z = (3x + 4y + 5z) - 3y$

$= 75 - 3\ x\ 12 = 75 - 36 = 39$

**6.** Let the number of apples be $a$.

If 8 more apples were distributed, number of boys $= \dfrac{a+8}{4}$

Similarly, from the next sentence, number of boys $= \dfrac{a-2}{3}$

$\Rightarrow \dfrac{a+8}{4} = \dfrac{a-2}{3} \Rightarrow a = 32.$

**7.** Let the numbers of pencils and pens be $x$ and $y$, respectively.

$\therefore\ 2x + 5y = 50$ (1)

and $x + y = 16$ (2)

$(1) - 5 \times (5) \Rightarrow 2x + 5y = 50$

$5x + 5y = 80$

$-3x = -30. \quad \therefore\ x = 10$

**8.** $3x + 4y = 24$

Multiply both the sides with 5. Then,

$15x + 20y = 120$ (1)

$15x + 20y = 8k$ (2)

To have (1) and (2) as consistent,

$\Rightarrow 120 = 8k, \quad \therefore\ k = 15$

**9.** Let the present ages of the man and his son be $m$ and $s$, respectively.

Given, $m - 10 = 2(s - 10) + 35$

$\Rightarrow m - 2s = 25$

or $m = 2s + 25$

i.e., present age of the man is 25 years more than twice his son's present age.

Since we need to find out after how many years will the man's age be twice his son's age, let us assume that it happens after $x$ years.

$\therefore$ After $x$ years $m + x = 2(s + x)$

Substitute $m = 2s + 25$ in above equation

$\Rightarrow 2s + 25 = 2s + x$

$\Rightarrow x = 25$

$\therefore$ After 25 years from now, the age of the man will be twice his son's age.

**10.** Let the initial amount with Gopi, Murthy and Hari in rupees be $x$, $y$ and $z$, respectively.

Gopi gives $\dfrac{x}{2}$ to Murthy

They now have $\dfrac{x}{2}$ , $\dfrac{x}{2} + y$ and $z$

Murthy gives half of his amount to Hari

They now have

$\dfrac{x}{2}; \dfrac{x}{4} + \dfrac{y}{2}; \dfrac{x}{4} + \dfrac{y}{2} + z$

Hari gives half his amount to Gopi.

Gopi now has $\left(\dfrac{x}{2} + \dfrac{x}{8} + \dfrac{y}{4} + \dfrac{x}{4} + \dfrac{z}{2}\right)$ and this is equal to $x$.

$\Rightarrow x = \dfrac{x}{2} + \dfrac{x}{8} + \dfrac{y}{4} + \dfrac{z}{2}$

$\Rightarrow \dfrac{3x}{8} = \dfrac{y}{4} + \dfrac{z}{2}$

$\Rightarrow 3x = 2y + 4z = 2 \times 45 = 90$, as $(y + 2z)$ is given equal to 45.

$\Rightarrow x = 30$

**11.** Let Prakash and his son's present ages be $x$ years and $y$ years, respectively.

Four years from now, Prakash will be $(x + 4)$ years and son will be $(y + 4)$ years.

$\therefore\ x + 4 = 4(y + 4)$

$\Rightarrow x - 4y = 12$ (1)

Twelve years from now, Prakash will be $x + 12$ and his son $y + 12$.

$\therefore\ x + 12 = \dfrac{5}{12}(y + 12)$

$\Rightarrow 2x - 5y = 36$ (2)

$[(1) \times 2] - (2)$ gives $-3y = -12$, $y = 4$

By substituting $y = 4$ in (1), we get $x = 28$.

**12.** After purchasing 4 apples and 5 mangoes, the man will be left with 1/4 of what he initially had, which is ₹20. He had ₹80 to start with.

With ₹80, the man can purchase 16 apples. Each apple costs $\frac{80}{16} = ₹5$

With ₹80, if the man can purchase 10 mangoes, each mango costs = $\frac{80}{10} = ₹8$.

∴ The difference in the prices of an apple and a mango is ₹3.

**13.** Let the number of chocolates received by A, B, C and D be *a*, *b*, *c*, and *d*, respectively.

$a + b + c + d = 225$ (1)

$a + d = 2\ (b + c)$

Substituting $a + d$ as $2(b + c)$ in equation (1), we get

$3(b + c) = 225 \Rightarrow b + c = 75$

As $b = c + 15$,

$c + 15 + c = 75$

$c = 30$.

**14.** Let the three-digit number be *xyz*.

$x + y + z = 9$ (1)

$xyz - 99 = zyx$

$\Rightarrow 99x - 99z = 99$

$\Rightarrow x - z = 1$ (2)

(1) + (2) $\Rightarrow 2x + y = 10$.

∴ $(x, y)$ can be (1, 8), (2, 6), (3, 4), (4, 2) or (5, 0).

∴ *xyz* can be 180, 261, 342, 423 or 504. The least number satisfying the second condition is 261.

**15.** Let Ganesh's present age be *g* years.

Govind's present age = $4g$ years.

$4g + 20 = 2\ (g + 20)$

$g = 10$

**16.** Let the total number of employees be *x*.

Given two thirds of them are software professionals and of these four-fifths are males

i.e., $\frac{4}{5}\left(\frac{2}{3}x\right) = 240 \Rightarrow x = 450$.

**17.** Let the present ages of Ram, Sita and their daughter be *r*, *s* and *d*, respectively.

$\frac{r + s + d}{3} = 35$

$\Rightarrow r + s + d = 105$ (1)

$s + 15 = r + d$

Substituting $r + d$ as $s + 15$ in (1), we get $2s + 15 = 105$

$\Rightarrow 2s = 90 \Rightarrow s = 45$ years

**18.** Let the cost of a burger be ₹*p*, and that of a pizza be ₹*q*.

$7p + 8q = 780$ (1)

$12p + 5q = 945$ (2)

$12 \times (1) - 7 \times (2) \Rightarrow 84p + 96q = 9360$

$84p + 35q = 6615$

$61q = 2745$

∴ $q = 45$

∴ The cost of each pizzas is ₹45.

**19.** Let the number of questions he answered correctly be *x*. Then for $50 - x$ questions were answered wrongly.

Total marks scored by the student

$= x \times 3 - (50 - x)1 = 90 \Rightarrow x = 35$.

**20.** The equations have infinite solutions.

$\frac{4}{k} = \frac{k - 10}{24} = \frac{2}{8}$

$\frac{4}{k} = \frac{2}{8} \Rightarrow k = 16$

**21.** Let the present ages of the father and son be *x* years and *y* years, respectively.

We now have,

$x = 10y$ (1)

$x + 6 = 4(y + 6) = x - 4y = 18$

Solving (1) and (2), we get $x = 30$ and $y = 3$.

Let father's age be twice the son's age in '*p*' years.

Then we have $30 + p = 2(p + 3) \Rightarrow p = 24$

So, in 24 years, the father will be twice as old as his son.

**22.** Let the number of boys be *B* and the number of girls be *G*. Since there is a difference in the number of brothers for the narrator and his sister, the narrator has to be a boy.

For a boy, no of brothers = $b - 1$ and number of sisters = *g*; so $b - 1 = 3g$.

For a girl, no of brothers = $b$ and the number of sisters = $g-1$; so $b = 4(g-1)$. Solving the 2 equations, we get the $(b, g) = (16, 5)$. $\therefore\ b+g = 21$.

**23.** Let the number of 25 paise and 20 paise coins be $a$ and $b$, respectively.

$25a + 20b = 1400$

$25b + 20a = 1300$

Adding both equations, $45\ (a+b) = 2700$

$\Rightarrow a + b = 60$

**24.** If we multiply $4x - 3y + 6 = 0$ with $-2$, we get $6y - 8x - 12 = 0$ which differs from the second equation, w.r.t., constant. $\therefore$ for $k = 6$ the system is inconsistent.

**25.** Let the counter price of each ticket be $x$.

Cost of 2 counter tickets = $2 \times x$.

Cost of 2 extra tickets = $2(x + 50)$

Total amount = $4x + 100$

Total money they spent = $4 \times 60 = 240$.

$\therefore\ 4x + 100 = 240$.

$\Rightarrow x = 35$.

**26.** Let the number of 25 paise and 50 paise coins with Kunal be $x$ and $y$, respectively.

Given, $a - b = 20$ (1)

and $50b - 25a = 400$ (2)

Solving (1) and (2),

$a = 56$ and $b = 36$

$\therefore$ The amount with Kunal is $25(56) + 50(36) = 3200$ paise, i.e., ₹ 32.

**27.** Let the length of the middle sized piece be $x$ m

Given: length of the longest piece = 3(length of middle sized piece) = $3x$

Also, the length of the shortest piece = (length of the longest piece) – 34 m = $3x - 34$

As the length of the rope is 64 m,

$3x + x + 3x - 34 = 64 \Rightarrow 7x = 98 \Rightarrow x = 14$

**28.** $2x + 3y + 2z = 23$ (1)

$3x - 2y + 3z = 28$ (2)

From (1) and (2) $13y = 13 \Rightarrow y = 1 \Rightarrow x + z = 10$.

**29.** Let Praveen's present age be $x$ years and that of Mahesh be $y$ years.

$x = 2(y-4) \Rightarrow x - 2y + 8 = 0$ (1)

$x + 8 = 2y \Rightarrow x - 2y + 8 = 0$ (2)

Since the two equations are the same, the sum of their present ages cannot be uniquely determined.

**30.** $P$, $Q$ and $R$ are successive even positive integers in the ascending order.

$\Rightarrow R - Q = 2$ and $Q - P = 2$

$\Rightarrow R - P = 4$

$4R = 5P + 4$

$\Rightarrow 4R - 5P = 4$

$\Rightarrow 4R - 4P - P = 4$

But $R - P = 4$

$\therefore 4(R-P) - P = 4$

$\Rightarrow 4(4) - P = 4$

$\Rightarrow P = 12$

$\therefore\ Q = P + 2 = 12 + 2 = 14$

**31.** $x + 2y + 3z = 14$ (1)

$2x + y + 2z = 10$ (2)

$3x + 3y + 4z = 21$ (3)

Multiply equation (1) with 2 and subtract equation (2) from the resulting equation to eliminate '$x$' and obtain an equation in '$y$' and '$z$'.

$2x + 4y + 6z = 28$

$2x + y + 2z = 10$

$3y + 4z = 18$ (4)

Now from any two other equations eliminate $x$.

For example take equations (1) and (3)

Multiply equation (1) with 3 and subtract equation (3) from the resulting equation

$3x + 6y + 9z = 42$

$3x + 3y + 4z = 21$

$3y + 5z = 21$ (5)

Now solve equations (4) and (5) for '$y$' and '$z$'

$3y + 4z = 18$

$3y + 5z = 21$

$-z = -3$

Substituting $z$ in (4), we get $y = 2$

Substitute $y = 2$ and $z = 3$ in any of (1), (2) or (3) to get the value of $x$.

$x + 2(2) + 3(3) = 14$

$\Rightarrow x + 4 + 9 = 14$

$\Rightarrow x + 13 = 14 \Rightarrow x = 1$

**32.** Let the price per kg of Oranges, Mangoes, Bananas and Grapes be ₹ $R$, ₹ $M$, ₹ $B$ and ₹ $G$, respectively.

Given that

$5R + 2M = 310$ (1)

$3M + 3.5B = 230$ (2)

$1.5B + 5G = 160$ (3)

$(1) + (2) + (3) \Rightarrow 5R + 5M + 5B + 5G = 700$

$\therefore 10R + 10M + 10B + 10G = 2 \times 700 = 1400$

**33.** Let the number be $10x + y$

$x + y = 3(x - y)$ ($\because xy > yx$) $\Rightarrow 2x = 4y$

or, $x = 2y$ (1)

Also, $(10x + y) - (10y + x) = 36$

$\Rightarrow 9x - 9y = 36 \Rightarrow x - y = 4$ (2)

$\therefore y = 4$ and $x = 8$

$\therefore$ The number is 84.

**34.** Let the costs of each pencil, each ruler and each eraser be $p$, $r$ and $e$, respectively.

$3p + 5r + 7e = 49$ (1)

$5p + 8r = 11e = 78$ (2)

$2\ x\ (2) - 3\ x\ (1) \Rightarrow$

$10p + 16r + 22e = 156$

$9p + 15r + 21e = 147$

$p + r + e = 9$

$\therefore$ The cost of 1 pencil, 1 ruler and 1 eraser is ₹ 9

**35.** Let the present ages of Ajay and Bala be a years and $b$ years, respectively.

$a - 20 + b - 20 = \frac{5}{9}$ (a + b)

$a + b = 90$ (1)

$a - b = 20$ (2)

Solving (1) and (2), $a = 55$

**36.** Here, $a_1/a_2 = b_1/b_2$

Thus for any other value of $k$ except 15, we will have a case of

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

So, the only value of $k$ that makes the equations consistent is 15.

**37.** Let, the present age of the man be $x$ years and that of the son be $y$ years.

$x - 4 = 3(y - 4) \Rightarrow x - 3y + 8 = 0$ (1)

and $x + 8 = 2(y + 8) \Rightarrow x - 2y - 8 = 0$ (2)

Solving (1) and (2), we get

$x = 40$ and $y = 16$

**38.** Let, $x$ and $y$ be the amounts with Amit and Sunil, respectively. Given,

$x - 40 = y + 40$

$\Rightarrow x - y = 80$ (1)

$x + 10 = y - 10 + 100$

$\Rightarrow x - y = 80$ (2)

Since the two equations are the same, the value of $x$ or $y$ cannot be uniquely determined.

**39.** Let the present ages of $P$ and $Q$ be $x$ and $y$ years, respectively.

Then, $x + y = 84$ (1)

Also, $x - 6 = 2(y - 6)$

$x - 2y = -6$ (2)

By solving the equations (1) and (2), we get $x = 54$ and $y = 30$

$\therefore$ Difference $= x - y = 24$.

**40.** Let the combined present age of the parents of the girl be $x$ and the present age of the girl be $g$.

Given, $x = 5g$

Also, $(x + 16) = 4(g + 8)$

$5g + 16 = 4g + 32$

$g = 16$

$\therefore x = 5\ (16) = 80$

The combined age of the parents when the girl was born $= 80 - (16 + 16) = 80 - 32 = 48$ years.

**41.** Given that, $(a + b) = 2(a - b)$ (1)

$a = 3b$, $a$ must be a multiple of 3 (as $b$ is an integer). $(a, b)$ can be (3, 1), (6, 2) or (9, 3). So, there are three numbers which satisfy the given condition.

**42.** When the woman was 29, the son was born and at that time the daughter was 3. Therefore, the woman was (and is) 26 years older than her daughter. As the woman is 3 times as old as the daughter, she is 39 and the daughter is 13. Therefore, the son is 10.

**43.** Let the fraction be $\frac{x}{y}$

When both the numerator and the denominator are increased by 2 each, we have

$$\frac{x+2}{y+2}=\frac{3}{5}\Rightarrow 5(x+2)=3(y+2)$$

$$\Rightarrow 5x-3y=-4 \quad (1)$$

When both the numerator and the denominator are increased by 1 each, we have

$$\frac{x+1}{y+1}=\frac{1}{2}\Rightarrow 2(x+1)=y+1$$

$$\Rightarrow 2x-y=-1 \quad (2)$$

$[(1)-(2)\times(3)]$ gives $-x=-4+3\Rightarrow x=1$. Substituting this value in (2), $y=3$ is obtained.

Hence, the fraction is 1/3.

**44.** Let the costs of a pen, a pencil and an eraser be $a$, $b$ and $c$, respectively.

Given: $a+b+3c=₹140$ (1)

$5a+3b+c=₹320$ (2)

We need the value of $3a+2b+2c$.

$\frac{1}{2}[(2)+(1)]$ gives

$\Rightarrow 3a+2b+2c=₹230$

**45.** The duration from 2:00 a.m to 8: 00 a.m , which is 360 minutes, is the sum of 3 parts – $t$ minutes, 40 minutes and $3t$ minutes.

$\therefore\ t+40+3t=360$ min

$4t+40=360$ min

$4t=320$ min $\Rightarrow t=80$ min

Present time = 8:00 a.m – 80 min = 6:40 a.m

**46.** Let the cost of 1 kg of tomatoes be ₹ $x$ and the cost of 1 kg of potatoes be ₹ $y$

$6x+7y=190$ (1)

$8x+y=170$ (2)

$(1)\times 4-(2)\times 3$

$24x+28y=760$ (3)

$24x+3y=510$ (4)

(3) – (4) $25y=250$, $y=10$.

Substitute $y=10$ in $-(-2)$, $x=20$.

$\therefore$ The cost of 1 kg of tomatoes is ₹ 20.

**47.** Let the two digit number be $xy$.

Using the first statement,

$x-y=4$ (1)

or $y-x=4$ (2)

Using the second statement, $10x+y+10y+x=110$.

$\therefore\ 11x+11y=110$

$x+y=10$ (3)

From (1) and (3), $xy=73$.

From (2) and (3), $xy=37$.

$xy=73$ or 37.

***Solutions for questions 48 and 49:***

Let the costs (in ₹) of a cup of ice-cream, a burger and a soft drink be $C$, $B$ and $S$, respectively.

**48.** $3C+2B+4S=128-\text{I}$

$2C+1B+2S=74-\text{II}$

Multiply II by 2 and then subtract I from it

i.e., $4C+2B+4S=148$

$-(3C+2B+4S=128)$

$\therefore\ C=20$

$\therefore$ The cost of 1 cup of ice-cream = ₹ 20

**49.** Substituting the value of $C$ in equation II of the above problem, we have,

$B+2S=34-\text{III}$

5 times equation III is $5B+10S=170$

$\therefore$ The cost of 5 burgers and 10 soft drinks = ₹ 170

**50.** Let the number to be multiplied be $x$.

$$\frac{4}{7}x-\frac{4}{17}x=840\Rightarrow 4x\left[\frac{1}{7}-\frac{1}{17}\right]=840$$

$$x\left[\frac{17-7}{119}\right]=210\Rightarrow 10x=210\ x\ 119$$

$\therefore\ x=2499$

## EXERCISE-3

**1.** The number of chocolates with Balu at different stages can be tabulated as below.

| | Numbers given | Balu has |
|---|---|---|
| | | 78 |
| To eldest son half + 3 | 39 + 3 | 36 |
| To second eldest son (one third + 4) | 12 + 4 | 20 |
| To youngest son (one fourth + 4) | 5 + 4 | 11 |

After giving 4 more than one-fourth to the youngest son he is left with 11 which means that Balu was left with 4 less than three – fourths. (11 + 4) is three fourths and hence, 5 is one fourth. He has 20 chocolates before giving to his youngest son. Similarly he has 36 and 78 before giving to his second eldest son and his eldest son, respectively.

**2.** Let the number of chocolate boxes with Ramu in the beginning be $x$.

He sold $\left(\frac{1}{2}x+\frac{1}{2}\right)$ boxes to the first customer. He would be left with $\left(\frac{x}{2}-\frac{1}{2}\right)$ boxes. He sold $\frac{1}{2}\left(\frac{1}{2}x-\frac{1}{2}\right)+\frac{1}{2}=\left(\frac{1}{4}x+\frac{1}{4}\right)$ boxes to the second customer. He would be left with $\left(\frac{1}{4}x-\frac{3}{4}\right)$ boxes. He sold $\frac{1}{2}\left(\frac{1}{4}x-\frac{3}{4}\right)+\frac{1}{2}=\left(\frac{1}{8}x+\frac{1}{8}\right)$ boxes to the third customer. He would be left with $\left(\frac{1}{8}x-\frac{7}{8}\right)$ boxes.

It can be seen that to the $n^{th}$ customer he would have sold $\left(\frac{1}{2}\right)^n(x+1)$ boxes. After that he would be left with $\left(\frac{1}{2}\right)^n x-\left(1-\left(\frac{1}{2}\right)^n\right)$ boxes.

Since $n = 10$, $\frac{x}{1024}=1-\frac{1}{1024}$

$\Rightarrow x = 1023$

**3.** Let $x$ and $y$ be the units and ten's digits of the two digit number.

Given $y > x$ and $10x + y + 10y + x = 2[9\ (y - x)] + 2$

$\Rightarrow 11x + 11y = 18y - 18x + 2$

$29x = 7y + 2$

$x = 2$ and $y = 8$ is the only solution set that satisfies this equation

(both lie between 0 and 9)

$\therefore$ the number is 82

**4.** Let the prices of the four varieties of pens be $p_1$, $p_2$, $p_3$ and $p_4$ and that of the four varieties of pencils be $c_1$, $c_2$, $c_3$ and $c_4$.

If $c_1$ is the price of the cheapest pencil and $c_2$, $c_3$ and $c_4$ are the prices of three of the varieties of the pens, we have

$p_2 + p_3 + p_4 = 2(c_2 + c_3 + c_4)$

As $c_1 = 1$, $c_2 + c_3 + c_4 = ₹11$

$\therefore p_2 + p_3 + p_4 = 22$

As $p_1 + p_2 + p_3 + p_4 = 45$, we have $p_1 = ₹23$, which has to be the highest price (as the sum of the other 3 prices is 22).

***Solutions for questions 5 and 6:***

Let, the number of pens, erasers and rulers be $p$, $e$ and $r$, respectively.

$p > e > r$ and $p \geq 10$, $e \geq 10$, $r \geq 10$

$p + e + r = 35$

If $r = 10$, there are two possibilities, $p = 14$, $e = 11$ or $p = 13$, e = 12.

$r$ cannot take a value greater than or equal to 11, since if $r = 11$, $p + e = 24$, which is not possible.

$\therefore r = 10$

**5.** $r = 10$.

**6.** Minimum amount spent

$= 13 \times 20 + 12 \times 5 + 10 \times 2 = 340$

7. All the amounts are in rupees. Let us assume that he started the game with rupees $A$. Amount with him at the end of the first round = ₹ $(2A - x)$.

Amount with him at the end of the second round = ₹ $2(2A - x) - x =$ ₹ $(4A - 3x)$

Amount with him at the end of the third round = ₹ $2(4A - 3x) - x$ = ₹ $(8A - 7x)$.

It can be seen that the amount with him at the end of the nth round = ₹ $(2nA - (2n-1)x)$

The amount with him at the end of the $10^{th}$ round = $1024A - 1023\,x = 1023 \Rightarrow \frac{A}{x+1} = \frac{1023}{1024}$

$\therefore$ $x + 1$ must be divisible by 1024

$\therefore$ least possible value of $x = 1023$

Sum of its digits is 6.

***Solutions for questions 8 and 9:***

Let the actual number of toys sold be '$ab$'.

As the stock left showed 81 items more than what it actual was, the mistaken number of items sold must be 81 less than the actual number sold.

$\therefore$ '$ab$' – '$ba$' = $81 \Rightarrow (10a + b) - (10b + a) = 81$

$\Rightarrow a - b = 9$

$\therefore$ $a = 9$ and $b = 0$

**8.** There is only *one* possibility.

**9.** Actual selling price = reverse of $\frac{486}{\text{'}ba\text{'}}$

= Reverse of $\frac{486}{9}$ = ₹ 45

**10.** Let the three digit number be $100x + 10y + z$

Now, $(100x + 10y + z) - (100z + 10y + x) = 297$

$\Rightarrow 99(x - z) = 297$

$\Rightarrow x - z = 3$ (1)

Also, $y + z = x - z = 3$

$x = 2z$ (2)

Substituting (2) in (1) gives $z = 3 \Rightarrow x = 6$ and $y = 0$

$\therefore$ The required number is 603.

**11.** Let '$abcdef$' be any 6-digit number and let $N$ be the reverse.

$M = 10^5a + 10^4b + 10^3c + 10^2d + 10^1e + f$

$N = 10^5f + 10^4e + 10^3d + 10^2c + 10^1b + a$

$\therefore$ $M + N = (10^5 + 1)a + (10^4 + 10)b + (10^3 + 10^2)c + (10^2 + 10^3)d + (10 + 10^4)e + (1 + 10^5)f$

$= 1000{,}001\,(a + f) + 10{,}010(b + e) + 1100\,(c + d)$

$\therefore$ $M + N$ has to be a multiple of 11.

The 11's remainder is the same as that of $(6 + 7 + 0) - (0 + 6 + 9) = -2$ or 9.

All the other numbers are multiples of 11. It is possible for $M + N$ to be any of the other numbers. But it cannot be 906706.

**12.** $14 + \frac{x}{4} + \frac{x}{4} = 4x$

$\Rightarrow 4x - \frac{x}{2} = 14$

$\Rightarrow \frac{7x}{2} = 14$

$\Rightarrow x = 4$

$\therefore$ Present age = $14 + \frac{x}{4} = 15$

$5x$ years from now, his age will be $15 + 5 \times 4 = 35$ years.

***Solutions for questions 13 to 14:***

Let the number of toys actually sold = $100x + 10y + z$

Now $(100x + 10y + z) - (100z + 10y + x) = 792$

$\Rightarrow 99(x - z) = 792 \Rightarrow x - z = 8$

$\therefore$ $x = 9$, $y = 1$ is the only possible solution set, since $y$ is a non-zero digit.

And also as $y = z$, $z = 1$.

$\therefore$ The required number is 911.

**13.** From the above, the number of toys actually sold is 911.

**14.** Faulty selling price = $\frac{5117}{119} = 43$.

$\therefore$ Actual selling price of each toy = ₹ 34

**15.** Actual total sales = $911 \times 34$ = ₹ 30, 974

**16.** The equation dependent on the first two given equations can be written as $(3x + 2y - 7z - 56) + k(5x + 3y + z - 16) = 0$. This can be written as

$x(3 + 5k) + y(2 + 3k) + z(-7 + k) + (-56 - 16k) = 0$ (1)

As the given equations are dependent, the corresponding coefficients of the third equation and equation (1) are proportional.

$\frac{p}{3 + 5k} = \frac{12}{2 + 3k} = \frac{-19}{-7 + k} = \frac{-200}{-(56 + 16k)}$

$\Rightarrow \frac{12}{2 + 3k} = \frac{-19}{-7 + k} \Rightarrow k = 2/3.$

Put the value of $k$ in (1), we get,

$x\left(3 + \frac{10}{3}\right) + y\left(2 + \frac{6}{3}\right) + z\left(-7 + \frac{2}{3}\right) - \left(56 + \frac{32}{3}\right) = 0.$

$19x + 12y - 19z = 200$. So, $p = 19$.

**17.** Let the four digit number be $abcd$

$a + d = b + c$ (1)

$b + d = 4(a + c)$ (2)

$a + b + c + d > 10$, i.e., $b + d + a + c > 10$

$4(a + c) + a + c > 10 \Rightarrow a + c > 2$.

Least $a + c$ is 3. When $a + c$ is 3, $b + d$ is 12.

$\therefore$ a + b + c + d is 15. But then (1) would be violated $\therefore a + c \neq 3$.

If $a + c$ is 4, $b + d$ is 16. $\therefore$ a + b + c + d is 20.

and $a + d = b + c = 10$

$(a, b, c, d) = (a, 6 + a, 4 - a, 10 - a)$

If $a + c$ is 4, $b = 6 + a$.

$\therefore b$ is 7, 8 or 9

(but not 6 as $a \neq 0$)

If $a + c$ is 5, $b + d = 20$. But this is not possible ($\because b \leq 9$ and $d \leq 9$. $\therefore$ max $(b + d) = 18$)

$a + c$ cannot be 5 or more.

$a + c$ must be 4.

$\therefore b$ can be 7, 8 or 9 but not 6.

**18.** Let the number of silver pendants with Laxmilal be $x$.

Then the number of silver pendants with Kuberjain will be $36 - x$. Also, let Kuberjain give $y$ pendants to Laxmilal.

Then $x + y = 5\ (36 - x - y)$

$\Rightarrow x + y = 30$ (1)

If Laxmilal gives $y$ pendants to Kuberjain, then

$x - y = 36 - x + y$

$\Rightarrow x - y = 18$ (2)

From (1) and (2), $x = 24$.

**19.** Let the present age of 'X' be $x$ years.

$\therefore$ present age of Y = $(63 - x)$ years.

Present age of Y = past age of 'X' = $63 - x$

The difference between their past and present ages

$= x - (63 - x)$

$= 2x - 63$

$\therefore$ Past age of Y = Present age of Y – difference of ages.

$= (63 - x) - (2x - 63) = 126 - 3x$

Present age of X = 2(past age of Y)

$x = 2(126 - 3x) = 252 - 6x$

$\Rightarrow 7x = 252 \Rightarrow x = 36$.

**20.** Given equations are

$x + 3y - 4z = a$

$4x + y - 5z = b$

x + $y - 2z = c$

If $a$, $b$ and $c$ can be expressed one in terms of the other, then they will have at least one solution. By observation of choices,

$11c = 11x + 11y - 22z$

$3a = 3x + 9y - 12z$

$2b = 8x + 2y - 10z$

$\therefore 11c = 3a + 2b$

i.e., $3a + 2b - 11c = 0$.

**21.** Let the ages of Shreya and Lata be $s$ and $l$, respectively. Shreya was as old as Lata is exactly $(s - l)$ years ago. Then Lata would have been $l - (s - l) = 2l - s$ years old

Given, $s = 3(2l - s) \Rightarrow s = \frac{3}{2}l$ (1)

and $s + l = 80$ (2)

Solving (1) and (2), $s = 48$, $l = 32$

$\Rightarrow s - l = 16$

**22.** Let the number be $ab$

$ab = 4(a + b) - 12$

$10a + b = 4(a + b) - 12$

$2(a + 2) = b$

If $a = 1$, $b = 6$

If $a = 2$, $b = 8$

If $a \geq 3$, the digit $b$ exceeds 9.

$\therefore$ab could be 16 or 28

Only 16 satisfies the second condition given.

**23.** Let us say the person paid a 1 Besos, $b$ 5 Besos and $c$ 20 Besos to settle the bill. Then, the bill amount (in Besos) $= a + 5b + 20c = 49$

Since he has at least one of each type, so $a$, $b$ $c \geq 1$

If $c = 1$, a + $5b = 29$.

If $c = 2$,

$a + 5b = 9$.

If $c \geq 3$, $a + 5b$

would be negative which is not possible.

$\therefore$ $c = 1$ or 2. We consider each of these possibilities below.

*Possibility 1: c = 1*

$a + 5b = 29$

$\therefore$ $(a, b) = (24, 1), (19, 2), (14, 3), (9, 4), (4, 5)$

$\therefore$ $(a, b)$ has 5 possible values.

Possibility 2: $c = 2$

$a + 5b = 9$

$\therefore$ $(a, b) = (4, 1)$

$\therefore$ $(a, b)$ has 1 possibility.

$(a, b)$ has a total of 6 possibilities.

**24.** Let the gambler start with an amount $x$ and after the first round he had $(3x - p)$.

After second round he had $[2(3x - p) - 3p]$. After third round he had $4\,[2(3x - p) - 3p] - 2p$ viz 0 (given). (1)

$p + 2p + 3p = 360$ (given) (2)

from (2) $\Rightarrow p = ₹60$ (3)

From (3) and (1) $x = ₹55$

**25.** For the equations to have infinitely many solutions.

$\frac{2}{k} = \frac{k-2}{12} = \frac{1}{3}$ must be satisfied

$\frac{2}{k} = \frac{1}{3} \Rightarrow k = 6$

**26.**

| | A | B |
|---|---|---|
| Present | 28 | $x$ |
| Past | $3/5\ x$ | 20 |

$28 - 3/5\ x = x - 20$

Thus, $x + 3/5\ x = 48$

$8/5x = 48$

$x = 30$

**27.** We have to decide whether $y$ is greater than 40 or less than 40.

$2y < 50$. $\therefore$ $y < 25$.

$y - 40 + 100 = 2x$

$y + 60 = 2x$ (1)

$x - 8 - 1 = 2y$

$x - 9 = 2y$ (2)

Solving (1) and (2), $x = 37$ and $7y = 14$.

The amount is ₹37.14.

**28.** Let the number of roses blooming in the month of January be $x$. The number for successive months

| Month | Number | Month | Number |
|---|---|---|---|
| Jan | $x$ | Apr | $\frac{x}{4} + 30$ |
| Feb | $\frac{x}{2}$ | May | $\frac{x}{4} + 90$ |
| Mar | $\frac{x}{2} + 60$ | June | $\frac{x}{8} + 45$ |

$\therefore$ Number of roses blooming in June is

$\frac{x}{8} + 45 = 120 \Rightarrow x = 600$

**29.** As number of roses that bloomed in June is 120, in successive

| June | July | August | September |
|---|---|---|---|
| 120 | 180 | 90 | 150 |

$\therefore$ 30 more flowers bloomed in September.

**30.** Let the amounts with Prakash, Sameer, Ramesh and Tarun be ₹$p$, ₹$s$, ₹$r$ and ₹$t$, respectively.

$p + s + r + t = 240$

$p = \frac{1}{2}(s + r + t) = \frac{240 - p}{2}$

$p = \frac{1}{3}(240) = 80$

So, half of the total amount with the others has become one-third of the total amount.

Similarly, $s = \frac{1}{4}(240) = 60$ and $r = \frac{1}{5}(240) = 48$

$t = 240 - (p + s + r) = 52$

**31.** Let the number of correct answers wrong answers, unattempted questions be $C$, $W$, $U$, respectively.

Then, $C + W + U = 120$ (1)

$4C - 2W - U = 228$ (2)

$4 \times (1) - (2) \Rightarrow 6W + 5U = 252$. As $W$ should be maximum, $U$ should be minimum.

For $U = 0$, $W = 42$.

Hence, the number of questions for which he gave a wrong answer can at the most be 42.

**32.** Let $\frac{1}{x + 2y} = p$ and $\frac{1}{3x + 4y} = q$

Then, $15p - 11q = 2$ (1)

$\frac{5}{2}p + 22q = \frac{5}{2}$

$\Rightarrow 5p + 44q = 5$ (2)

(1) – (2) × 3

$\Rightarrow \quad 15p - 11q = 2$

$15p + 132q = 15$

$-143q = -13$

$\Rightarrow q = 1/11$

From (1), $15p - 11 \times \frac{1}{11} = 2$

$15p = 2 + 1 \Rightarrow p = 1/5$

But $p = \frac{1}{x+2y}, q = \frac{1}{3x+4y}$

$\Rightarrow \frac{1}{x+2y} = 1/5 \Rightarrow x + 2y = 5$ (3)

$= \frac{1}{3y+4y} 1/11 \Rightarrow 3x + 4y = 11$ (4)

(4) – 2 × (3)

$\Rightarrow \quad 3x + 4y = 11$

$2x + 4y = 10$

$x = 1$

$3\ (1) + 4y = 11 \Rightarrow y = 2$

$\therefore x = 1, y = 2$

**33.** Let $s$ be the present age of Mr. Smith and $a$, $b$, $c$ be the present ages of Andy, Bandy and Candy, respectively. Let $y$ be the present age of Mrs. Smith.

$2\ [(a + b + c) - 21] = s - 7$ (1)

$2\ (a + b + c) = s + y$ (2)

Subtracting (1) from (2)

$42 = y + 7 \Rightarrow y = 35$ years

**34.** The left pan weighs 0.6 kg and the right one weighs 0.95 kg. When the pans level, the total weights on the two sides are equal.

The two weighings are shown in the tables below. Let the actual weight of the rice be $r$ kg.

| L | R | L | R |
|---|---|---|---|
| 0.6 | 0.95 | 0.6 | 0.95 |
| $r$ | $ab$ | $ba + 18.7$ | $r$ |

'$ab$' is 0.35 less than $r$.

'$ba$' +18.7 is 0.35 more than $r$.

$\therefore$ '$ba$' +18.7 is 0.7 more than $ab$

i.e., '$ba$' + 18 is equal to '$ab$' (i.e., $a > b$)

$\therefore 10b + a + 18 = 10a + b$

$\Rightarrow a - b = 2.$

($\therefore$ $ab$ could be 20 ,31,42,53,64,75,86, or 97)

The weight of the rice is $ab + 0.35$, i.e., it could be 20.35, 31.35,... or 97.35.

From the options given, it can be 53.35.

**35.** Let the three digit number be $abc$. Given, $abc - cba = 396$.

Hence, $99(a - c) = 396 \Rightarrow a - c = 4$

Also, $a - b = b - c$ ($a - b \neq c - b$ because $a \neq c$) $\Rightarrow a + c = 2b$.

If $b$ is less than or equal to 2, we get $c$ as zero or negative. For $b$ = 3, 4, 5, 6 or 7, we've the numbers 531, 642, 753, 864 and 975, respectively. Hence, there are five such possible numbers.

**36.** $4x + 5y = 32$ (1)

and $6x + 7.5y = k$ (2)

As (1) and (2) are not inconsistent, i.e., consistent, equation (1) × 1.5 must be equal to equation (2).

$\therefore$ Value of $k = \frac{32 \times 3}{2} = 48$

**37.** Let the number of questions answered correctly, answered wrongly and unanswered by the student be $C$, $W$ and $U$, respectively.

$C + W + U = 120$ and $C - \frac{1}{3} W - \frac{1}{6} U = 60.$

$C + W + U - (C - \frac{1}{3} W - \frac{1}{6} U) = 120 - 60$, i.e., $\frac{4}{3} W + \frac{7}{6} U = 60$

Multiplying both sides by 6, we get $8W + 7U = 360$.

As 360 is divisible by 8, $8W + 7U$ must also be divisible by 8.

$\therefore$ $7U$ must be divisible by 8. If $U$ is not divisible by 8, $7U$ will not be divisible by 8. But $7U$ is divisible by 8.

$\therefore$ $U$ must be divisible by 8 (1)

$7U$ cannot exceed 360.

$\therefore$ $U$ cannot exceed $\frac{360}{7}$, i.e., $51\frac{3}{7}$ (2)

From (1) and (2), maximum value of $U$ is 48. Also, when $U$ is maximum, $W$ is minimum.

$\therefore$ Min $(W) = \frac{360 - 7(48)}{8} = 3.$

**38.** Let the four-digit number be *abcd*.

$b + c = a + d$ (1)

$b + d = 5(a + c)$ (2)

$\Rightarrow a + b + c + d = 6(a + c)$

Given that $a + b + c + d = 6(a + c) = 18$

$\Rightarrow a + c = 3 \Rightarrow b + d = 15$

$\Rightarrow (b, d) = (6, 9), (7, 8), (8, 7), (9, 6)$ and $(a, c)$

$= (1, 2), (2, 1), (3, 0)$ as $a \neq 0$.

But according to (1), we have $a + d = b + c$

$\Rightarrow (a, b, c, d) = (1, 7, 2, 8), (2, 8, 1, 7), (3, 9, 0, 6)$

$\therefore$ The hundreds digit can be 7, 8 or 9.

So, their sum is $7 + 8 + 9 = 24$

**39.** Let the number of males in Chotasansthan be $m$. Then, number of males in Badasansthan is $x + 5114$.

Also, the number of females in Chotasansthan is $2x$.

Number of females in Badasansthan is $2x - 9118$. Also, when compared to males, as females are 3004 less in Badasansthan, it has $(x + 5114) - 3004$ females, i.e., $x + 2110$.

$\therefore 2x - 9118 = x + 2110 \therefore x = 11{,}228$.

$\therefore$ Number of females in Badasansthan

$= 11{,}228 + 2110 = 13{,}338$.

**40.** Adding the given equations, we have $8(p + q + r + s + t + u) = 120$

$\therefore p + q + r + s + t + u = 15$

$3p + q + r + s + t + u - (p + q + r + s + t + u) = 0$

$\therefore P = 0$

$\therefore (p)(q)(r)(s)(t)(u) = 0$

**41.** Let the number of ₹2, ₹5 and ₹10 notes in the bag be denoted by $a$, $b$ and $c$, respectively.

$a + b + c = 120$ (1)

$2a + 5b + 10c = 760$ (2)

$2a + 5(2b) + 10c = 960$ (3)

$\Rightarrow 2a + 10b + 10c = 960$

$\Rightarrow 2a + 10(120 - a) = 960$

$a = 30$

Subtracting (2) from (3),

$5b = 200$

$b = 40$

Substituting $a$ and $b$ in (1), $c = 50$.

**42.** Let $100a + 10b + c$ be the number

$\therefore a - b = b - c$ (or) $a - b = c - b$

$\Rightarrow a + c = 2b$ (or) $a = c$

When $a + c = 2b$ and $a + b + c = 9$, $b = 3$ and $a + c = 6$.

Hence, a can have values 1 to 6, i.e., six possible numbers.

When $a = c$, the possible numbers are 171, 252, 333 and 414, i.e., 4 possible numbers. But 333 is common to both.

The number of possible numbers = 6 + 4 –1 = 9

**43.** (i) If the equations given have infinite solutions,

$\frac{p}{2} = \frac{q}{3} = \frac{66}{8}$

$\therefore p = \frac{66}{4}$ and $q = \frac{99}{4}$

$\therefore 4(p + q) = 165$.

(ii) The equation given will have no solution if

$\frac{p}{2} = \frac{q}{3}$ and neither $\frac{p}{2}$ nor $\frac{q}{3}$ is $\frac{66}{8}$

As $q = 9$, $p = 6$

**NOTE**

The equations given will have a unique solution if $\frac{p}{2} \neq \frac{q}{3}$.

**44.** Eliminate '$x$' from equations (1) and (2)

The three given equations are as follows:

$3x + 5y + 7z = 12$ (1)

$x - 3y + 9z = 16$ (2)

$9x - 8y = 31z = 54$ (3)

Eliminate '$x$' from equations (1) and (3)

$9x + 15y + 21z = 36$

$9x + 8y + 31z = 54$

$10z - 7y = 18$ (5)

Equations (4) and (5) are same.

As equations 4 and 5 are same, it is not possible to find unique values of $x$, $y$ and $z$.

**45.** $x_1 + 4x_2 + 9x_3 + 16x_4 + 25x_5 + 36x_6 = 9$ (1)

$4x_1 + 9x_2 + 16x_3 + 25x_4 + 36x_5 + 49x_6 = 98$ (2)

$9x_1 + 16x_2 + 25x_3 + 36x_4 + 49x_5 + 64x_6 = 987$ (3)

(2)–(1): $3x_1 + 5x_2 + 7x_3 + 9x_4 + 11x_5 + 13x_6 = 89$ (4)

(3)–(2): $5x_1 + 7x_2 + 9x_3 + 11x_4 + 13x_5 + 15x_6 = 889$ (5)

(5)–(4): $2(x_1 + x_2 + x_3 + x_4 + x_5 + x_6) = 800$

$16X_1 + 25X_2 + 36X_3 + 49X_4 + 64X_5 + 81X_6$

$= 9X_1 + 16X_2 + 25X_3 + 36X_4 + 49X_5 + 64X_6$

$+ (5X_1 + 7X_2 + 9X_3 + 11X_4 + 13X_5 + 15X_6)$

$+ 2(X_1 + X_2 + X_3 + X_4 + X_5 + X_6) = 2676.$

**46.** $2P + E + 3S = 23$ (1)

$6P + 3E + 1S = 29$ (2)

$14P + 7E + 7S = 91$ (3)

$(1) \times 3 - (2)$

$8S = 40 \Rightarrow S = 5$

Substituting $S$ in (3)

$14P + 7E = 56 \Rightarrow 2P + E = 8$

Also, from (1) $\Rightarrow 2P + E = 8$

So, cannot be determined.

**47.** Let the number of Oranges be $R$, Mangoes be $M$ and plates be $P$

$R = P + 1$ (1)

$M = 2(P - 1)$ (2)

Given that $R \sim M = 3$

Case (i)

$R - M = 3$

$P + 1 - 2P + 2 = 3$

$-P = 0$

$\Rightarrow P = 0$ which cannot be true, hence ignored.

Case (ii)

$M - R = 3$

$(2P - 2) - (P + 1) = 3$

$P - 3 = 3 \Rightarrow P = 6.$

**48.** $a + 7b + 3c + 5d = 0$ (1)

$8a + 4b + 6c + 2d = -16$ (2)

$2a + 6b + 4c + 8d = 16$ (3)

$5a + 3b + 7c + d = -16$ (4)

Adding (1) and (4), we have

$6(a + d) + 10(b + c) = -16$

Adding (2) and (3), we have $10(a + d) + 10(b + c) = 0.$

$\therefore a + d = 4.$ And $(b + c) = -4.$

$\therefore (a + d)(b + c) = -16.$

**49.** Let the amounts with Eswar, Ganesh and Harish at the beginning of the first round be $e$, $g$ and $h$, respectively

Each person lost a round. Suppose Eswar lost the first round, Ganesh lost the second, Harish lost the third. Then, the persons who double the amounts with the others in the first, second and third rounds are Eswar, Ganesh and Harish, respectively

The table below shows the amounts with each at the various

| Final (amounts) | Eswar | Ganesh | Harish |
|---|---|---|---|
| | 60 | 60 | 60 |
| Before Harish doubled | 30 | 30 | 120 |
| Before Ganesh doubled | 15 | 105 | 60 |
| Before Eswar doubled | 97.5 | 52.5 | 30 |

The amounts with the three persons at the beginning of the first round were ₹97.5, ₹52.2 and ₹30. Even if the order of the losers in the three rounds was different, only the order of the amounts with the three persons at the beginning of the first round will change.

The amounts with the three persons at the beginning of the first round were ₹97.5, ₹52.5, ₹30.

**50.** Let the present age of the person be $f$, and the son be $s$.

$(f - 6) = 2 + 5(s - 6) \Rightarrow f - 5s = -22$ (1)

$f + 4 = -2 + 3(s + 4)$

$f - 3s = 6$ (2)

On solving, $s = 14, f = 48$

$f + s + 2x = 100$

$62 + 2x = 100 \Rightarrow x = 19$

## EXERCISE-4

***Solutions for questions 1 and 2:***

Let $A$ have ₹$3k$, $B$ ₹$2k$ and $C$ ₹$k$ with them initially. The loser and the amounts with each of them are tabulated below.

| | | A | B | C | |
|---|---|---|---|---|---|
| Round | Loser | $3k$ | $2k$ | $K$ | (0) |
| 1 | $B$ | $3k + \frac{k}{2}$ | $2k - k$ | $k + \frac{k}{2}$ | |
| | | $3.5k$ | $k$ | $1.5k$ | (1) |
| 2 | $C$ | $4k$ | $1.5k$ | $0.5k$ | (2) |
| 3 | $A$ | $3k$ | $2k$ | $k$ | (3) |
| 4 | $B$ | $3.5k$ | $k$ | $1.5k$ | |
| | | | | (same as that after Round 1) | |

Now, $3.5k - 3k = 10 \Rightarrow k = 20$

1. Total money = $3k + 2k + k = 120$

2. After round 2, $A$ would have gained ₹20, which is the maximum gain for him in the course of the 4 rounds.

**3.** At ₹7 per km, the cost of travelling 25 km would be ₹175 and at ₹8 per km, the cost would be ₹200 (less than ₹240).

Hence, the taxi is hired for more than 4 hours, as the charge is ₹240.

Number of hours for which the taxi is hired = 240/48

= 5 hours.

**4.** Let the three digit number be $xyz$.

Given $3x - 2y = 3x - z = z - 2y$

$\Rightarrow 2y = z$ and $z = 3x$

$\Rightarrow x : y : z = 2 : 3 : 6$

$\therefore$ Only one number is possible under the given conditions, which is 236.

***Solutions for questions 5 and 6:***

Let Sonali's age be $x$ years

$\therefore$ Sagar's age is $2x$ years.

Let Monali's age be $y$ years

$\therefore y + x = 2\ (2x) \Rightarrow y = 3x$

Let Surya's age be $z$ years

$\therefore z + 2x = 2\ (x + 3x) \Rightarrow z = 6x$

Prithvi's age = 21 years

Now, $21 + 6x + 2x = 5(x + 3x) \Rightarrow 12x = 21 \Rightarrow x = 1^3/_4$

$\therefore$ Sonali's age = $1^3/_4$ years = 1 year 9 months

Sagar's age = $2\ (1^3/_4) = 3^1/_2$ years

Monali's age = $3\ (13/4) = 5^1/_4$ years

= 5 years and 3 months

Surya's age = $6\ (1^3/_4) = 10^1/_2$ years = 10 years 6 months

**5.** 5 years 3 months – 1 year 9 months = 3½ years

**6.** Let Surya be twice as old as sagar in '$t$' years.

$10^1/_2 + t = 2(3^1/_2 + t) \Rightarrow 10^1/_2 + t = 7 + 2t \Rightarrow t = 3^1/_2$

**7.** Let the amounts paid by Ram, Lakshman, Bharath and Shatrugna be $a$, $b$, $c$ and $d$, respectively.

$\therefore a + b + c + d =$ ₹2,40,000 (1)

Given: $a = \frac{1}{2}(b + c + d)$

$\Rightarrow b + c + d = 2a$ (2)

From (1) and (2), $a =$ ₹80,000

Now, $d = \frac{1}{5}(a + b + c)$

$\Rightarrow a + b + c = 5d$ (3)

From (1) and (3), $d =$ ₹40,000

Also, $c = \frac{5}{19}(a + b + d)$

$\therefore a + b + d = \frac{19c}{5}$ (4)

From (1) and (4), $c =$ ₹50,000

$\therefore b = 2,40,000 - 80,000 - 50,000 - 40,000$

= ₹70,000

**8.** Let the number of chocolates Raman had just before giving to the youngest son be $x$.

As he gave $(2/3)x - 2$ chocolates to the younger son, he would be left with $(1/3)x + 2$ chocolates $(1/3)x + 2 = 18$

$\Rightarrow x = 48$

Assume Raman had $y$ chocolates before giving to the second son.

As he gave $(1/4)y - 3$ chocolates to the second younger, he would be left with $3/4y + 3$

$(3/4)y + 3 = 48$, $y = 60$

Assume the number of chocolates Raman originally had was $z$.

As he gave $(1/2)z - 4$ chocolates to his eldest son, he would left with $1/2z + 4$ chocolates

$1/2z + 4 = 60$

$z = 112$

***Solutions for questions 9 and 10:***

Let the number of daughters in the Nanda family be $x$ and that of sons be $y$.

$5(y - 1) = x$ (1)

and $x - 1 = 2y$ (2)

Solving (1) and (2), we get

$x = 5$, $y = 2$

Let the number of daughters in the Parekh family be $a$ and that of the brothers be $b$.

$b - 1 = a$ (3)

$2(a - 1) = b$ (4)

Solving (3) and (4), we get; $a = 3$, $b = 4$

**9.** Parekh had 3 + 4 = 7 children.

**10.** Required ratio = (2 + 4) : (5 + 3) = 3 : 4

**11.** Let the present age of Ajay be $x$ years

Some time in the past, Bharat was $x/4$ years old.

The age of Ajay at that time is Bharat's present age.

Bharat's present age is $(x-9)$ years.

As their difference of ages is constant,

$x-(x-9)=(x-9)-x/4$

$x=24$

Sum of their present ages = $2x-9=39$ years

**12.** Let the number $N$ be '$abc$'.

Let the sum of the remaining numbers be $R$.

$R+cba=R+abc+11(a+b+c)$

$\Rightarrow 99(c-a)=11(a+b+c)$

$\Rightarrow 8c-10a=b$ (1)

As $b\geq 0$ and $c\geq \frac{5}{4}a$, $c>$ a

$\therefore$ 8 (Difference of $a$ and $c$)

$=8(c-a)=6+2a$

$8c-10a=6$ (2)

From (1) and (2), $b=6$

**13.** Let the number be '$abcd$'

Given that,

$b+c=2d$ (1)

$b+6a=2(c+d)$ (2)

$d+5a=2b$ (3)

Let us that the equations (1), (2) and (3) as the linear equations in $a$, $b$ and $c$ and express the values of $a$, $b$ and $c$ in terms of $b$.

By (2) – (1), we get $3c=6a\Rightarrow a=\frac{c}{2}$

By substituting $c=2a$ in (1), it becomes

$2d-2a=b$ (4)

Subtracting (4) from $2\times$ (3), we get

$12a=3b\Rightarrow a=\frac{b}{4}$

As $c=2a$, $c=\frac{b}{2}$

By substituting $a=\frac{b}{4}$ in (4), we get $2d=\frac{3b}{2}\Rightarrow d=\frac{3b}{4}$

$\therefore a:b:c:d=1:4:2:3$

$\therefore$ '$abcd$' can be 1423 or 2846.

**14.** Before doubling, the amounts with Bhavan, Chetan and Dinesh, each of them must have had $\frac{80}{2}$ = ₹40.

$\therefore$ Amar must have then had ₹80 + ₹120, i.e., ₹200. Similarly we can work out the amounts with each of them before the other doubled the amounts. The results are summarized below.

| | Amar | Bhavan | Chetan | Dinesh |
|---|---|---|---|---|
| Finally | 80 | 80 | 80 | 80 |
| Before Amar doubles | 200 | 40 | 40 | 40 |
| Before Bhavan doubles | 100 | 180 | 20 | 20 |
| Before Chetan doubles | 50 | 90 | 170 | 10 |
| Before Dinesh doubles | 25 | 45 | 85 | 165 |

***Solutions for questions 15 and 16***:

Let the number of erasers, pencils and pens be $x$, $y$ and $z$, respectively.

$x+y+z=38$ and $z>y>x$ and $x, y, z\geq 11$.

$\therefore x=11$, and $y+z=27$

There are two cases possible,

$\Rightarrow y=12, z=15$ or $y=13, z=14$

Let, $x=12$, then $y+z=26$

which is not possible because, if $y>x$, and $y=13$, then $z=13$ which would make $y=z$ which cannot be considered as $z>y$. Any other value of $y>13$ would make $y>z$ which also is not acceptable.

$\therefore$ Two sets of values are possible.

$\therefore$ $x=11, y=12, z=15$ and $x=11, y=13, z=14$

**15.** Let the number of pens, pencils and erasers purchased be $a$, $b$ and $c$ respectively. It is given that $a>b>c\geq 11$. Also, it is known that he bought a total of 38 pieces.

$a+b+c=38$

If $c=11$, the following cases are possible:

$b=12$ and $a=15$

$b=13$ and $a=14$

If $c=12$, the least value that $b$ can take is 13 and the least value that a can take is 14, hence the sum will exceed 38. So, this case is ruled out.

Hence, $c=11$.

**16.** If the number of pencils cannot be divided equally among the four brothers, then the number of pencils should not be a multiple of 12.

$\therefore$ The number of pencils = 13

$\therefore$ The number of pens = 14

**17.** We want the net score of Smitha to be 60 with maximum possible number of incorrect answers. Given that Smitha

answered all the questions and got exactly 40 correct answers in level C. So, the remaining 40 questions of level C are incorrect and the net score in level C is $(1 \times 40) - (1/2 - 40) = 20$. Since we want the maximum number of incorrect answers, the incorrect answers in level B must be made maximum possible. Say all the 60 questions of level B are wrong. Then the net score becomes $20 - (1 \times 60) = -40$. But the required net score is 60. So from level A, a net score of $60 - (-40) = 100$ must be scored. Let $a$ be the number of incorrect answers in level A, then the number of correct answers is $(60 - a)$.

$\Rightarrow 3(60 - a) - 2a = 100 \Rightarrow 5a = 80 \Rightarrow a = 16$

Hence, the total (maximum possible) number of incorrect answers = 40 (in level C) + 60 (in level B) + 16 (in level A) = 116

**18.** Let the number of questions of A, B, C levels of difficulty for which the answers were wrong be $a$, $b$ and $c$, respectively.

Given: $a + b + c = 30$; $2a + b + \frac{c}{2} = 46$.

$\Rightarrow a = \frac{62 - b}{3}$ and $c = \frac{28 - 2b}{3}$

$(a, b, c) = (20, 2, 8)$ or $(19, 5, 6)$ or $(18, 8, 4)$ or $(17, 11, 2)$ or $(16, 14, 0)$. As there is more than one possibility for the values of $a$, $b$ and $c$, the score cannot be determined.

**19.** The amount on the cheque was $50x + 500y$. Varun actually got $500x + 50y$. After spending ₹350, he had double of the amount on the cheque

$500x + 50y - 350 = 2(50x + 500y)$

$\Rightarrow 400x - 950y = 350$

$\Rightarrow 8x - 19y = 7$

$\Rightarrow \text{Rem } \frac{3y}{8} = 1$

$\Rightarrow y = 3 \Rightarrow x = 8$

The other solutions are linked below.

$7 = 8(8) - 19(3)$

$= 8(27) - 19(11)$

$= 8(46) - 19(19)$

$= 8(65) - 19(27)$

$\therefore$ The amount on the cheque could have been

$50(8) + 500(3) = 1900$ or $50(27) + 500(11) = 6850$ or $50(46) + 500(19) = 11{,}800$ and so on.

$\therefore$ Among the options only choice (B) can be true

**20.** Given $p + 3q - 4r = a$ (1)

$3p + 2q + 5r = b$ (2)

$5p + 8q - 3r = c$ (3)

Multiply the equation (1) with 2 and add it to equation (2), we get equation (3)

So, $2a + b - c = 0$

When $2a + b - c = 0$ the equations have at least one solution.

**21.** Onions lead over tomato by 200 per quintal. Tomatoes gain on onion by Rs0.60 per day. Assuming that the increase rates are constant, tomatoes need $\frac{200}{0.60}$ of $333\frac{1}{3}$ days to catch up. (i.e., more than 200 days). But after 200 days, the price of onions remain constant at ₹600.

Tomatoes need $\frac{600 - 200}{1.6}$ or 250 days to catch up.

Starting from Apr 2, 250th day would be 7th Dec.

**22.** Let the number of choclates with Mohan and Nitin be $x$ and $y$, respectively.

After the first transaction the number of choclates with Mohan

$= \frac{x}{4} + \left(y + \frac{3x}{4}\right)\frac{3}{4} = \frac{x}{4} + \left(\frac{3x + 4y}{4}\right)\frac{3}{4}$

$= \frac{x}{4} + \frac{9x + 12y}{16} = \frac{4x + 9x + 12y}{16} = \frac{13x + 12y}{16}$

Since the ratio of chocolates with them is the same after any number of transactions and the total number of chocolates with them is also equal after any number of transactions, the number of chocolates with Mohan (or Nitin) initially is equal to the number of chocolates with Mohan (as Nitin) after the first (or any) transaction.

So, $\frac{13x + 12y}{16} = x$

$\Rightarrow 3x = 12y \Rightarrow x = 4y$

So, $4y - y = 324 \Rightarrow y = 108$.

The number of chocolates with Nitin = 108.

***Solutions for questions 23 and 24:***

Let the requirement for the entire village per day be $x$ kilo litres. (Note: 1 kilo litre = 1000 litres)

Volume of the tank = $50x$

Also, volume of the tank = $40(x + 20) = 40x + 800$

$\therefore 50x = 40x + 8{,}000 \Rightarrow x = 80$ kilo litres

**23.** The total requirements of the village per day is 80 kilo litres.

**24.** Let the number of days be $n$.

$50\ (80) = n\ (80 + 5) \Rightarrow n = \dfrac{50(800)}{85} = 47^{1}/_{17}$

The tank would be emptied on the $48^{th}$ day.

**25.** The number of questions attempted by Him = 22

The number of questions Him got right = 11 (since his strike rate is 50%)

Let the negative marking for first 5 mistakes be $x$ and for the remaining mistakes be $y$.

$\therefore 7 = 11 - 5x - 6y$

$5x + 6y = 4$ (1)

Similarly, the number of questions attempted by Haw = 45

The number of questions he got right = 30 (since his strike rate is 66.66%)

$\therefore 24 = 30 - 5x - 10y$

$5x + 10y = 6$ (2)

Solving (1) and (2), we get,

$(x, y) = \left(\dfrac{1}{5}, \dfrac{1}{2}\right)$

$\therefore$ The negative marks for each mistake made for more than 5 mistakes is $y = \dfrac{1}{2}$.

**26.** Hariharan Lezy Louis

$I$ $x$ 25000

$E$ 25000 $y$

$S$ $p$: 2p

$\dfrac{x - 25000}{25000 - y} = \dfrac{p}{2p}$

$2x + y = 75000$

$\therefore$ Option (A) is definitely correct.

$2x + y = 75000$ and $x = 25000 + p$

If $y = 0$, $2x = 75000$

$x = 37500$, otherwise it will be less than ₹37,500.

As $p$ can be 0, $x \geq 25{,}000$.

$\therefore$ Both (A) and (B) are correct.

**27.** Let the number of 1 Alto, 5 Alto, 15 Alto notes used to settle the bill be $a$, $b$, and $c$, respectively.

$1a + 5b + 15c = 41$ (1)

Each of $a$, $b$ and $c$ must be at least one (2)

From (1) and (2), $15c < 41$. $\therefore c < 2\dfrac{11}{15}$.

$c = 1$ or 2

If $c$ is 1, $a + 5b = 26$.

$\therefore (b, a) = (1, 21), (2, 16), (3, 11), (4, 6), (5, 1)$

If $c$ is 2, $a + 5b = 11$. $\therefore$ $(b, a) = (1, 6), (2, 1)$

$(a, b, c)$ has 7 possible values.

$\therefore$ The bill can be settled in 7 ways.

**28.** Let the male and female population of A and B be denoted as $m_A, f_A, m_B, f_B$, respectively.

$f_A = 7671 + f_B$ (1)

$m_B = 2f_B$ (2)

$f_A = 4506 + m_A$ (3)

$m_A = m_B - 13677$ (4)

From (1), (2) (3) $f_A = 7671 + \dfrac{m_B}{2} = 4506 + m_A$

$3165 = m_A - \dfrac{m_B}{2}$ (5)

Solving (4) and (5), $m_A = 20007$.

**29.** The amount for the different days can be worked out sequentially by starting with the amount for the $19^{th}$.

| Dates | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|
| Money | 80 | 70 | 140 | 130 | 260 | 250 | 500 | 490 | 980 | 970 |

| Dates | 23 | 24 | 25 |
|---|---|---|---|
| Money | 1,940 | 1,930 | 3,860 |

**30.** Dates 13 14 15 16 17 18 19 20 21

Money 100 90 180 170 340 330 660 650 1,300

| Dates | 22 | 23 | 24 | 25 |
|---|---|---|---|---|
| Money | 1,290 | 2,580 | 2,570 | 5,140 |

$\therefore$ I would have received 5140 – 3860, i.e., ₹1,280 more.

## EXERCISE-5

**1.** From Statement I, $6x + 12y = 18$,

So, we can get $x + 2y = 3$ but, not $2x + 3y$.

Statement I is not sufficient.

From Statement II, $9y + 6x = 12$, i.e., $3y + 2x = 4$.

$\Rightarrow$ The value of $2x + 3y$ can be found, so Statement II is sufficient.

**2.** Let the cost of Musk melon be ₹ $x$ and that of Jack fruit be ₹ $y$

From Statement I, $5x + 15y = 400$ (1)

From Statement II, $7x + 21y = 560$ (2)

Multiplying the first equation by 7/5 we get equation (2). So, we have two unknowns and one equation, which has infinite solutions. So, the question cannot be answered.

**3.** Since the composition of coins is not given, the question cannot be answered.

**4.** From Statement I, $la + lb = 2/3\ (la + lb + lc)$

$\Rightarrow la + lb = 2lc$

From Statement II, $lb = lc$

Combining Statements I and II, $la = lb = lc$.

**5.** From Statement II, if the cost of 12 shirts is doubled, it will be the same as that of 72 pants $\Rightarrow$ cost of 12 shirts is the same as the cost of 36 pants. So, only Statement II is sufficient and the difference is zero.

**6.** Let the father's age be $x$ and the son's age be $y$.

Then, $x - y = 28$.

From Statement I, $x = 8y$.

$8y - y = 28$, $y = 4$, $x = 32$.

Statement I alone is sufficient.

Statement II alone is irrelevant.

**7.** From Statement I, A + B = 80 $\Rightarrow$ C gets ₹ 80.

From Statement II, B + C =100 $\Rightarrow$ A gets ₹ 60.

Combining both the statements, B gets ₹ 20.

**8.** Either of the statements alone is not sufficient as information about Sunil and Gopal is given in different Statements.

Using both the statements,

Let $x$ be Sunil's age, $y$ be Gopal's age and $z$ be Sekhar's age.

$y + 3 = z$

$z = x + 5$

$\therefore\ y = x + 2$

Hence, it can be answered using both the Statements.

**9.** Let my present age be $x$ years.

Statement I

$\Rightarrow$ My brother's age = $2(x - 4)$, from which the value of $x$ cannot be found.

Statement II $\Rightarrow x = 3(x - 12)$

$\Rightarrow 2x = 36 \Rightarrow x = 18$.

**10.** There are two possibilities

(i) 21 attempts all correct.

(ii) 25 attempts 22 correct.

Hence, the question cannot be answered even after using both the Statements.

**11.** From Statement I, we have

man's age = wife's age + 2 years

But to know whether man's age = wife's age + son's age, we should know the ages of at least two of them.

$\therefore$ Statement I alone is not sufficient.

From Statement II, we have

son's present age = 12 years

wife's present age = 42 years

But to know whether man's age = wife's age + son's age or not, we should know the relationship between the ages of the man and wife or man and the son.

$\therefore$ Statement II alone is not sufficient.

Using both the statements, we have

man's age = 42 + 2 = 44 years

$\therefore$ Clearly man's age < wife's age + son's age.

**12.** Let the number of students who opted for only History, only Chemistry and both History and Chemistry be $x$, $z$ and $y$, respectively.

From Statement I, $(x + y) = 2\ (y + z)$.

From Statement II, $y + z = 20$.

2 equations in 3 unknowns. It cannot be solved.

**13.** From Statement I, let $x$ be the number of doors and $(36 - x)$ the number of windows. This information is not sufficient to answer the question.

From Statement II, the number of windows and the number of doors are in the ratio of 2 : 1. This information alone is not sufficient to answer the question. Using I and II, $36 - x = 2x \Rightarrow x = 12$.

**14.** As Statements I and II individually do not relate to Uno and Zen, they are not sufficient.

Using both, we have Palio = 4/5 (Alto);

Alto = Zen – 2

Also, Alto gives $45 \times \frac{4}{3} = 60$ km

∴ Zen = 62 km

∴ Zen gives more mileage.

**15.** From Statement I, we have the price of gold per ten grams = ₹ 4000.

Price of silver per 1 kg = ₹ 8000

∴ The price of silver per 10 grams = ₹ 8

∴ The price of gold per unit weight = 500 times of price of silver per unit weight

∴ Statement I alone is sufficient

Statement II does not give any relevant information.

**16.** Two equations $a_1x + b_1y + c_1z = k_1$ and $a_2x + b_2y + c_2z = k_2$ have a unique value for $z$ only if $\frac{a_1}{a_2} = \frac{b_1}{b_2}\left(\neq \frac{C_1}{C_2}\right)$.

Combining the given equations with the equation in either Statement, the above condition is not satisfied. Combining both Statements, as we have three equations and three unknowns. A unique solution in $z$ is possible.

Hence, both Statements taken together are sufficient.

**17.** Using Statement I, multiplying the equation in I by 5 and subtracting the given equation from it, the value of $7x + 16y - 2z$ is obtained. $7x + y - 2z = 7x + 16y - 2z - 15y$. As $y$ is unknown, $7x + y - 2z$ cannot be found.

∴ Statement I alone is not sufficient.

Using Statement II, $3(3x - y + 2z) - 2(x - 2y + 4z)$

$= 7x + y - 2z$

$\therefore 7x + y - 2z = 3(11) - 2(12) = 9$

∴ Statement II alone is sufficient.

**18.** After the coins are exchanged the possible combinations of notes that Guru has are as follows.

| ₹ 50 | ₹ 100 | Total |
|---|---|---|
| 2 | 4 | 6 |
| 4 | 3 | 7 |
| 6 | 2 | 8 |
| 8 | 1 | 9 |

From Statement I, It follows that he has two 50 rupee notes.

∴ Statement I alone is sufficient

From Statement II, it follows that he has four, six or eight 50 rupee notes.

∴ Statement II alone is not sufficient.

**19.** Let my current age be $x$ years and my sister's current age be $y$ years.

Using either statement, we get one equation in two unknowns.

∴ $x$ cannot be found.

∴ Either statement alone is not sufficient.

Using both statements, as we have two equations in two unknowns, $x$ and $y$ can be found

**20.** Using Statement I, $12x + 18y = 18a \Rightarrow 2x + 3y = 3a$

∴ It has integral solutions (Eg: $x = 0$, $y = a$)

∴ Statement I alone is sufficient.

Using Statement II,

$c^2 + 4c - 396 = 0$

$(c - 18)\ (c + 22) = 0$

If $c = 18$, there are integral solutions

If $c = -22$, there are no integral solutions

∴ Statement II is not sufficient.

**21.** Dividing both numerator and denominator of $\frac{7a + 9b}{4a + 5b}$ by $b$, it becomes $\frac{7\frac{a}{b} + 9}{4\frac{a}{b} + 5}$. Value of $\frac{a}{b}$ is sufficient to find $\frac{7a + 9b}{4a + 5b}$.

From Statement I, we have $b + a = \frac{1}{2}(6a - b)$

$\Rightarrow {}^{a}/_{b} = {}^{3}/_{4}$. The value of the expression can be found.

∴ Statement I is sufficient.

From Statement II, we have $3a + 4b = 5$.

By simplifying $3a + 4b = 5$, $\frac{a}{b}$ cannot be found.

∴ Statement II is not sufficient.

**22.** Using either of the Statements, we get one equation in two unknowns. Therefore, A's speed cannot be found. Combining both Statements, as we have two equations in two unknowns A can be found.

**23.** Let the cost of an apple and a mango be ₹ $x$ and ₹ $y$, respectively.

$3x + y = 14$ (1)

Using Statement I, $5x + 2y + 13 = 6x + 4y$

$\Rightarrow x + 2y = 13$ (2)

Solving (1) and (2), $y = 5$.

Using Statement II, $x = 2y$

Substituting $x = 2y$ in (1),

$y = 2$.

∴ Statement II alone is sufficient.

**24.** Two equations $a_1\ x + b_1\ y = c_1$ and $a_2\ x + b_2\ y = c_2$ have a unique solution if and only if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. Combining the given equations with equation in Statement I, the above condition is not satisfied. Combining it with equation in Statement II, this condition is satisfied.

∴ Statement II alone is sufficient.

**25.** Let $y$ people read both.

Using Statement I, 1500 read The Hindu and 900 read The Times of India. We can't determine the number of people who read only The Hindu.

∴ Statement I is not sufficient.

Statement II is clearly not sufficient as nothing is said about any newspaper.

Combining both Statements, $1500 + 900 - y = 2000y = 400$

**26.** From Statement I,

If Amisha purchases equal number of apples and oranges, (say $n$ of each) she must pay ₹ $12n$

Since $65 \neq 12n$, she has not bought equal number of apples and oranges. Hence, I alone is sufficient.

From Statement II alone, had she bought 3 oranges and 4 apples more, she would have paid ₹ 108.

So, she actually paid $108 - 3 \times 5 - 4 \times 7 = 65$.

This is now same as Statement I and hence, II alone is sufficient.

**27.** Neither of the Statements is independently sufficient, as we have three unknowns and two equations from the two Statements. Using both the Statements, we have the following equations.

$5P + 7E + 15S = 49$ (1)

$8P + 11E + 23S = 77$ (2)

By multiplying the first equation by 3 and subtracting the second equation multiplied by 2 from the resulting equation, will give us the cost of $1P + 1S + 1E$.

**28.** Let the four-digit number be abcd.

Using Statement I, $a + b + c = d = 3a$

Using Statement II, $b = 2a$ and $c = a - 2$

Using both the Statements,

$a + b + c = a + 2a + (a - 2)$

$4a - 2 = 3a$ as $a + b + c = 3a$

$\Rightarrow a = 2$, $b = 4$ and $c = 0$

$d = \text{a} + b + c = 6$

Hence, the number is 2406

**29.** Let the present age of Amit be $a$ years

Let the present age of Bimal be $b$ years

Given

$a + x = 2(b + x)$ (1)

Also,

$b + 2x = a$ (2)

From (1) and (2), we have

$b + 3x = 2b + 2\text{x}$

$b = x$

$x + 2x = a$ so $a = 3x$

From Statement I

$a - b = 5$

Thus, $3x - x = 5$

Hence, $x$ can be found.

So, Statement I alone is sufficient.

From Statement II, $a + 2x = 5b$

As $a = 3x$ and $b = x$, we get $5x = 5x$.

From this, $x$ can't be found out.

Statement II alone is not sufficient.

**30.** Let the amounts with Ram and Shyam be ₹ $R$ and ₹ $S$, respectively. Given that $R + S = 200$

If Ram has more than ₹ 100, he would have more than Shyam. If he does not have more than ₹ 100, he would not have more than Shyam.

Using Statement I, if Ram gives ₹ 10 to Shyam, Ram and Shyam would have ₹ $(R-10)$ and ₹ $(S+10)$, respectively.

There are two possibilities.

(1) $R-10 > S+10$. In this case,

$R-10-(S+10) = 20$, i.e.,

$R-S = 40$

(2) $R-10 < S+10$. In this case, $S+10-(R-10)$

$= 20$, i.e., $S = R$

$\therefore R-S = 40$ or $R = S$.

In the first case, Ram has more than Shyam. In the second case, he does not have more than Shyam. I is not sufficient.

Using Statement II, $S-5-(R+5) = 10$ or $R+5-(S-5)$ $= 10$, i.e., $S-R = 20$ or $S = R$. In both cases, Ram does not have more money than Shyam. II is sufficient.

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# Unit 3

# Arithmetic


Chapter 1 Ratio, Proportion and Variation

Chapter 2 Percentages, Profit and Loss and Partnerships

Chapter 3 Special Equations

Chapter 4 Simple Interest and Compound Interest

Chapter 5 Time and Distance

Chapter 6 Time and Work

Chapter 7 Averages, Mixtures and Alligations

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# 1 Ratio, Proportion and Variation

CHAPTER

## Learning Objectives

*In this chapter, you will*:

- Understand about ratio
- Understand the effect of change in quantities on the ratio of the quantities
- Learn how to divide any quantity into two or more parts, based on the ratio in which it is to be divided
- Understand the concept of proportion, and how to apply componendo and dividendo
- Learn how to find out mean proportional, second proportional, etc.
- Understand variation between two or more quantities and how to apply it in varied situations in time and work along with time and distance problems in situations which are
  - Direct variation
  - Inverse variation
  - Joint variation

## ❒ RATIO

Ratio proportions is a useful tool of solving questions from chapters like Alligations, Time and Distance, Time and Work etc. Also, every year, one or two question from this chapter appear in the CAT.

If the values of two quantities A and B are 4 and 6, respectively, then we say that they are in the ratio 4:6 (read as 'four is to six'). **Ratio is the relation in which one quantity bears to another of the same kind, the comparison being made by considering what multiple, part or parts, one quantity is of the other.** The ratio of two quantities '$a$' and '$b$' is represented as $a:b$ and read as '$a$ is to $b$'. Here, '$a$' is called antecedent, '$b$' is the consequent. Since the ratio expresses the number of times one quantity contains the other, it's *an abstract* quantity.

Ratio of any number of quantities is expressed after removing any common factors that all the terms of the ratio have. For example, if there are two quantities having values of 4 and 6, their ratio is 4:6, i.e., 2:3 after taking the common factor 2 between them out. Similarly, if there are three quantities 6, 8, and 18, there is a common factor among all three of them. So, dividing each of the three terms by 2, we get the ratio as 3:4:9.

If two quantities whose values are A and B, respectively, are in the ratio $a:b$, since we know that some common factor $k(>0)$ would have been removed from A and B to get the ratio $a:b$, we can write the original values of the two quantities (i.e., A and B) as $ak$ and $bk$, respectively. For example, if the salaries of two persons are in the ratio 7:5, we can write their individual salaries as $7k$ and $5k$, respectively.

A ratio $a:b$ can also be expressed as $a/b$. So, if two items are in the ratio 2:3, we can say that their ratio is 2/3. If two terms are in the ratio 2, it means that they are in the ratio of 2/1, i.e., 2:1.

'A ratio is said to be a ratio of greater or less inequality or of equality according as antecedent is greater than, less than or equal to consequent'. In other words,

1. the ratio $a:b$ where $a > b$ is called ratio of greater inequality (example 3:2).

2. the ratio $a:b$ where $a < b$ is called ratio of less inequality (example 3:5).
3. the ratio $a:b$ where $a = b$ is called ratio of equality (example 1:1).

From this, we can find that a ratio of greater inequality is diminished, and a ratio of less inequality is increased by adding same quantity to both terms, i.e., in the ratio $a:b$, when we add the same quantity $x$ (positive) to both the terms of the ratio, we have the following results:

1. If $a < b$ then $(a+x):(b+x) > a:b$.
2. If $a > b$ then $(a+x):(b+x) < a:b$.
3. If $a = b$ then $(a+x):(b+x) = a:b$.

This idea can also be helpful in questions on data interpretation when we need to compare fractions to find the larger of two given fractions.

If two quantities are in the ratio $a:b$, then the first quantity will be $a/(a+b)$ times the total of the two quantities and the second quantity will be equal to $b/(a+b)$ times the total of the two quantities.

## ❒ PROPORTION

When two ratios are equal, then the four quantities involved in the two ratios are said to be proportional. i.e., if $a/b = c/d$, then $a$, $b$, $c$ and $d$ are proportional.

This is represented as $a:b::c:d$ and is read as '$a$ is to $b$ (is) as $c$ is to $d$'.

When $a$, $b$, $c$, and $d$ are in proportion, then $a$ and $d$ are called extremes and $b$ and $c$ are called means. We also have the relationship:

Product of the means = Product of the extremes, i.e., $b\,c = adv$

If $a:b = c:d$ then

$$b:a = d:c \quad \text{(A)}$$

$$a:c = b:d \quad \text{(B)}$$

$$(a+b):b = (c+d):d \quad \text{(C)}$$

(obtained by adding 1 to both sides of the given relationship)

$$(a-b):b = (c-d):d \quad \text{(D)}$$

(obtained by subtracting 1 from both sides of the given relationship)

$$(a+b):(a-b) = (c+d):(c-d) \quad \text{(E)}$$

{obtained by dividing relationship (C) above by (D)}

Relationship (A) above is called invertendo
Relationship (B) is called alternendo
Relationship (C) is called componendo
Relationship (D) is called dividendo
Relationship (E) is called componendo–dividendo.

The last relationship, i.e., componendo–dividendo is very helpful in simplifying problems. By this rule, whenever we know $a/b = c/d$, then we can write

$$(a+b)/(a-b) = (c+d)/(c-d).$$

The converse of this is also true - whenever we know that $(a+b)/(a-b) = (c+d)/(c-d)$, then we can conclude that $a/b = c/d$.

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} \ldots$, then each of these ratios is equal to $\frac{a+c+e+\ldots}{b+d+f+\ldots}$.

If three quantities $a$, $b$, and $c$ are such that $a:b::b:c$, then we say that they are in continued proprotion. We also get $b^2 = ac$. In such a case, $c$ is said to be the third proportional of $a$ and $b$. Also, $b$ is said to be the mean proportional of $a$ and $c$.

## ❒ VARIATION

Two quantities $A$ and $B$ may be such that as one quantities changes in value, the other quantity also changes in value bearing certain relationship to the change in the value of the first quantity.

### ❒ Direct Variation

One quantity $A$ is said to vary directly as another quantity $B$ if the two quantities depend upon each other in such a manner that if $B$ is increased in a certain ratio, $A$ also increases in the same ratio and if $B$ is decreased in a certain ratio, $A$ also decreases in the same ratio.

This is denoted as $A \propto B$ ($A$ varies directly as $B$).

If $A \propto B$ then $A = kB$, where $k$ is a constant. It is called constant of proportionality.

For example, when the quantity of sugar purchased by a housewife doubles from the normal quantity, the total amount she spends on sugar also doubles, i.e., the quantity and the total amount increase (or decrease) in the same ratio.

From the above definition of direct variation, we can see that when two quantities $A$ and $B$ vary directly with each other, then $A/B = k$ or the ratio of the two quantities is a constant. Conversely, when the ratio of two quantities is a constant, we can conclude that they vary directly with each other.

If $X$ varies directly with $Y$ and we have two sets of values of the variables $X$ and $Y - X_1$ corresponding to $Y_1$ and $X_2$ corresponding to $Y_2$, then, since $X \propto Y$, we can write down

$$\frac{X_1}{Y_1} = \frac{X_2}{Y_2} \text{ or } \frac{X_1}{X_2} = \frac{Y_1}{Y_2}$$

## ❒ Inverse Variation

A quantity $A$ is said to vary inversely as another quantity $B$ if the two quantities depend upon each other in such a manner that if $B$ is increased in a certain ratio, $A$ gets decreased in the same ratio and if $B$ is decreased in a certain ratio, then $A$ gets increased in the same ratio.

It is the same as saying that $A$ varies directly with $1/B$. It is denoted as $A \propto 1/B$, i.e., $A = k/B$ where $k$ is $k$ the constant of proportionality.

For example, as the number of men doing a certain work increases, the time taken to do the work decreases and conversely, as the number of men decreases, the time taken to do the work increases.

From the definition of inverse variation, we can see that when two quantities $A$ and $B$ vary inversely with each other, then $AB$ = a constant, i.e., the product of the two quantities is a constant. Conversely, if the product of two quantities is a constant, we can conclude that they vary inversely with each other.

If $X$ varies inversely with $Y$ and we have two sets of values of $X$ and $Y - X_1$ corresponding to $Y_1$ and $X_2$ corresponding to $Y_2$, then since $X$ and $Y$ are inversely related to each other, we can write down

**FORMULA**

$$X_1 Y_1 = X_2 Y_2 \text{ or } \frac{X_1}{X_2} = \frac{Y_2}{Y_1}$$

## ❒ Joint Variation

If there are three quantities $A$, $B$ and $C$ such that $A$ varies with $B$ when $C$ is constant and varies with $C$ when $B$ is constant, then $A$ is said to vary jointly with $B$ and $C$ when both $B$ and $C$ are varying. i.e., $A \propto B$ when $C$ is constant and $A \propto C$ when $B$ is a constant;
$\Rightarrow A \propto BC$

$A \propto BC \Rightarrow A = kBC$ where $k$ is the constant of proportionality.

In these types of problems on variation, there are typically three parts:

1. The relationship between different variables is defined to frame an equation involving the variables and the constant of proportionality.
2. One set of values of all the values of all the variables is given to enable us to find the value of the constant of proportionality.
3. The values of all but one variable of a second set are given and we are asked to find the value of the one variable whose value is not given.

The problems involving ratio and proportion are just different forms of the models of the basic problems we saw above. For example, in place of variables, the problems can be reframed using mangoes, apples, number of days worked, work done etc. Here, practice and perseverance pay you a lot. In entrance exams, there will be either direct problems on ratio, proportion and variation or indirect problems of application of these concepts just discussed to areas like time and work or time and distance.

## SOLVED EXAMPLES

**1.01.** The sum of two numbers is 84. If the two numbers are in the ratio 4:3, then find the two numbers.

**Sol:** As the two numbers are in the ratio 4:3, let their actual values be $4x$ and $3x$.
As the sum of two numbers is 84, we have $4x + 3x = 84$.
$\Rightarrow 7x = 84$
$\Rightarrow \quad x = (84/7) = 12$
Hence, $4x = 48$ and $3x = 36$.
Alternatively, the two numbers are $(4/7) \times 84$ and $(3/7) \times 84$, i.e., 48 and 36, respectively since the ratio of the two numbers is 4:3.

**1.02.** If $4a = 3b$, then find $(7a + 9b):(4a + 5b)$.

**Sol:** It is given that $4a = 3b$.
Hence, $(a/b) = (3/4)$
$\Rightarrow a = 3k$ and $b = 4k$, where $k$ is the common factor of $a$ and $b$.
Required expression $(7a + 9b):(4a + 5b)$
$= [(7 \times 3k) + (9 \times 4k)] : [(4 \times 3k) + (5 \times 4k)]$
$= (21k + 36k):(12k + 20k)$
$= 57k:32\ k = 57:32$

**1.03.** The number of red balls and green balls in a bag are in the ratio 16:7. If there are 45 more red balls than green balls, find the number of green balls in the bag.

**Sol:** Since the ratio of number of red and green balls is 16:7, let the number of red balls and green balls in the bag be $16x$ and $7x$. So, the difference of red and green balls is $9x$.
$16x - 7x = 9x = 45 \Rightarrow x = 5$
$\Rightarrow$ Hence, the number of green balls
$= 7x$, i.e., 35
Alternatively, $7x = (7/9)\ (9x)$

$= (7/9)\ (45) = 35.$

Hence, there are 45 green balls in the bag.

**1.04.** What least number must be added to each of a pair of numbers that are in the ratio 7 : 16 so that the ratio between the terms becomes 13 : 22?

**Sol:** Let the number to be added to each number be $a$. Let the actual values of the numbers be $7x$ and $16x$, since their ratio is 7 : 16.

Given that,

$$\frac{7x + a}{16x + a} = \frac{13}{22}$$

$\Rightarrow 154x + 22a = 208x + 13a \Rightarrow 9a = 54x$

$\Rightarrow a = 6x$. When $x = 1$, $a$ is the least number required and is equal to 6.

**1.05.** A number is divided into four parts such that 4 times the first part, 3 times the second part, 6 times the third part, and 8 times the fourth part are all equal. In what ratio is the number divided?

**Sol:** Let the four parts into which the number is divided be $a$, $b$, $c$, and $d$.

$4a = 3b = 6c = 8d$. Let the value of each of these equal to $e$.

$$a = \frac{e}{4},\ b = \frac{e}{3},\ c = \frac{e}{6},\ \text{and}\ d = \frac{e}{8}.$$

Hence, $a : b : c : d = \frac{e}{4} : \frac{e}{3} : \frac{e}{6} : \frac{e}{8}$

$$= \frac{6}{24} : \frac{8}{24} : \frac{4}{24} : \frac{3}{24}$$

(where 24 is the L.C.M of the denominators)

$= 6 : 8 : 4 : 3.$

Hence, the ratio of the parts into which the number is divided is 6 : 8 : 4 : 3.

**1.06.** Divide 3150 into four parts such that half of the first part, a third of the second part, a fourth of the third part is equal to one-twelfth of the fourth part.

**Sol:** Let the four parts into which 3150 is divided be $a$, $b$, $c$, and $d$.

Given that,

$$\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{d}{12};$$

Let each of the above equal $k$.

Then, $a = 2k$, $b = 3k$, $e = 4k$ and $d = 12k$

As $a + b + c + d = 3150$, the equation becomes, $(2k + 3k + 4k + 12k) = 3150$;

$\Rightarrow 21k = 3150 \Rightarrow k = 150.$

Hence, the four parts in the order, are:

300, 450, 600 and 1800.

**1.07.** If $x : y = 4 : 3$, $y : z = 2 : 3$, find $x : y : z$.

**Sol:** As $y$ is common to both the ratios, make $y$ in both ratios equal. This is done by making $y$ have the value equal to the L.C.M of the two parts corresponding to $y$ in the two ratios, i.e., L.C.M $(3, 2) = 6$.

If $y = 6$, $x = (4/3) \times 6 = 8$, $z = (3/2) \times 6 = 9$

Hence $x : y : z = 8 : 6 : 9$.

**1.08.** If $\frac{a}{b} = \frac{4}{5}$, then find $\frac{2a^2 + 3b}{7a + 6b^2}$.

**Sol:** It is given that $(a/b) = (4/5)$.

Hence $a$ and b can be taken as $4k$ and $5k$, where $k$ is the common factor of $a$ and $b$.

Substituting the values in given expression, the expression is $(2a^2 + 3b) : (7a + 6b^2)$

$[2(4k)^2 + 3(5k)] : [7(4k) + 6(5k)^2]$

$(32k^2 + 15k) : (28k + 150k^2)$

$k\,(32k + 15) : k\,(28 + 150k)$

$(32k + 15) : (150k + 28)$

As the value of $k$ is not known, the value of the required expression cannot be determined.

**1.09.** Two numbers are in the ratio 4 : 5. If 7 is added to each, the ratio between the numbers becomes 5 : 6. Find the numbers.

**Sol:** Let the numbers be $x$ and $y$.

$$\frac{x}{y} = \frac{4}{5} \Rightarrow x = \frac{4}{5}y$$

$$\frac{x + 7}{y + 7} = \frac{5}{6} \Rightarrow 6\left(\frac{4}{5}y + 7\right) = 5(y + 7)$$

(on cross multiplication and substituting for $x$)

$$\Rightarrow \frac{24}{5}y + 42 = 5y + 35$$

$$\Rightarrow \frac{y}{5} = 7 \Rightarrow y = 35$$

$$x = \frac{4}{5}y = 28.$$

**Alternative Method:**

Let the numbers be $4k$ and $5k$, where $k$ is the common factor.

$$\frac{4k + 7}{5k + 7} = \frac{5}{6}$$

$24k + 42 = 25k + 35$

$\Rightarrow k = 7.$

The numbers are $4k = 28$ and $5k = 35$.

**1.10.** The scores of Mohan and Sohan in a test are in the ratio 5 : 4. If their total score is 135, find Mohan's score.

**Sol:** As their scores are in the ratio of 5 + 4, let their scores be $5k$ and $4k$.

Given that the sum of their scores = $5k = 4k = 135$
$\Rightarrow 9k = 135$
$\Rightarrow k = 15$
$\therefore$ Mohan's score = $5k = 5 \times 15 = 75$
**Alternative Method:**

$$\text{Mohan's score} = \frac{5}{5+4}(135) = 75$$

**1.11.** If $a:b = 3:4$, find $3a + 4b : 4a + 5b$.

**Sol:** $3a + 4b : 4a + 5b$

$$= \frac{3a+4b}{4a+5b} = \frac{\frac{3a+4b}{b}}{\frac{4a+5b}{b}}$$

$$= \frac{3\left(\frac{a}{b}\right)+4}{4\left(\frac{a}{b}\right)+5} = \frac{3\left(\frac{3}{4}\right)+4}{4\left(\frac{3}{4}\right)+5} = \frac{25}{32}$$

**1.12.** The ratio of the number of marbles with Ram and Shyam is 19:13. If Ram gives Shyam 30 marbles, both will have equal number of marbles. Find the number of marbles with Ram.

**Sol:** Let the number of marbles with Ram and Shyam be $19x$ and $13x$, respectively. Total number of marbles with them = $32x$
If Ram gives Shyam 30 marbles each will have $\frac{32x}{2} = 16x$ marbles.

$\therefore 19x - 16x = 30$
$x = 10$
$19x = 190$

**1.13.** Two numbers are in the ratio 3:4. What part of the larger number must be added to each number so that their ratio becomes 5:6?

**Sol:** Let the two numbers be $3x$ and $4x$.

$$\frac{3x+k}{4x+k} = \frac{5}{6}$$

$18x + 6k = 20x + 5k$
$k = 2x$
$\therefore$ Half of the larger number must be added to each number.

**1.14.** 1400 is divided into 4 parts such that twice the first part, thrice the second part, 4 times the third part and 12 times the last part are all equal. Find the 4 parts.

**Sol:** Let the first part, second part, third part, and last part be $a$, $b$, $c$, and $d$, respectively.
$2a = 3b = 4c = 12d$
$a = 6d,\ b = 4d,\ c = 3d$
Given, $a + b + c + d = 1400$
$\therefore 6d + 4d + 3d + d = 1400$
$\Rightarrow d = 100$
$\therefore a = 600,\ b = 400,\ c = 300$

**1.15.** 1400 is divided into 4 parts such that half of the first part, one third of the second part, one fourth of the third part and $\frac{1}{12}$th of the last part are all equal. Find the 4 parts.

**Sol:** Let the first part, second part, third part, and fourth part be $a$, $b$, $c$, and $d$, respectively.

$$\frac{1}{2}a = \frac{1}{3}b = \frac{1}{4}c = \frac{1}{12}d$$

$$d = 6a,\ b = \frac{3}{2}a,\ c = 2a$$

Given, $a + b + c + d = 1400$

$$\Rightarrow a + \frac{3}{2}a + 2a + 6a = 1400$$

$$\Rightarrow a = \frac{400}{3}$$

$$\therefore b = 200,\ c = \frac{800}{3},\ d = 800$$

**1.16.** If $a:b = b:c = 2:3$, find $a:b:c$

**Sol:** $\frac{a}{b} = \frac{b}{c} = \frac{2}{3} \Rightarrow a = \frac{2}{3}b$ and $b = \frac{2}{3}c$

$$\therefore a = \frac{2}{3}\left(\frac{2}{3}c\right) = \frac{4}{9}c$$

$$\therefore a:b:c = \frac{4}{9}c : \frac{2}{3}c : c = 4:6:9$$

**Alternative Method:**
As $b$ is common to both ratios and since it is divisible by 3 (from the first ratio) and it is divisible by 2 (from the second ratio), it is divisible by L.C.M (3, 2), i.e., 6. Hence, if $b = 6$, $a = 4$, and $c = 9$
$\therefore a:b:c = 4:6:9$

**1.17.** There are 2 classes A and B. If 10 students leave class A and join class B, then the ratio of the number of students in class A and class B would reverse. Find the difference in the numbers of students in class A and class B.

**Sol:** Let the numbers of students in class A and class B be $ax$ and $bx$, respectively.

Given, $\frac{ax - 10}{bx + 10} = \frac{b}{a}$

$a^2x - 10a = b^2x + 10b \Rightarrow a^2x - b^2x - 10a - 10b = 0$
$\Rightarrow (ax - bx - 10)(a + b) = 0$
$\therefore ax - bx = 10$

**1.18.** A husband's age exceeds that of his wife by 6 years. Before 10 years, the ratio of their ages was 5:4. Find the present age of the husband.

**Sol:** Let the present age of the husband be $x$ years.
$\Rightarrow$ Present age of the wife = $(x-6)$ years.
10 years ago, the ages of the husband and the wife will be $(x-10)$ years and $(x-16)$ years, respectively.

Given $x-10=\frac{5}{4}(x-16)$

$\therefore x=40$

**Alternative Method:**
Let the age of the husband 10 years ago be $5x$ years. Age of his wife at that time = $4x$ years.
The husband would then also be 6 years older than his wife.
$\therefore 5x=4x+6 \Rightarrow x=6$
Hence, the present age of the husband = $5x+10$, i.e., 40 years.

**1.19.** Find $x$, if $x+2:4x+1::5x+2:13x+1$.

**Sol:** In a proportion, product of means = product of extremes
$(x+2)\ (13x+1)=(4x+1)\ (5x+2)$
$\Rightarrow 13x^2+x+26x+2=20x^2+8x+5x+2$
$\Rightarrow 13x^2+27x+2=20x^2+13x+2$
$\Rightarrow 7x^2-14x=0 \Rightarrow 7x\ (x-2)=0$
$\Rightarrow x=0$ or 2.

**1.20.** If $x$ varies directly as $y^4+9$ and $x=3$ when $y=3$, find $x$ when $y=9$.

**Sol:** $x \propto (y^4+9)$.
Hence, $x=c\ (y^4+9)$ where $c$ is a constant.

$c=\frac{x}{y^4+9}$.

when $x=3$, $y=3$ (given)

Hence, $c=\frac{3}{3^4+9}=\frac{3}{90}=\frac{1}{30}$;

and $x=\frac{1}{30}(y^4+9)$

When $y=9$

$x=\frac{1}{30}(y^4+9)=\frac{1}{30}(6561+9)=219.$

**1.21.** Find the value(s) of $x$ if $\frac{2x+5}{x+1}=\frac{x+2}{x-1}$.

**Sol:** $(2x+5)\ (x-1)=(x+2)\ (x+1)$
$2x^2+5x-2x-5=x^2+2x+x+2$
$\Rightarrow x^2=7$
$\therefore x=\pm\sqrt{7}$

**1.22.** $X$ varies directly with $Y^2+18$. When $Y=18$, $X=18$. Find $Y$ when $X=1$.

**Sol:** $\frac{X_1}{X_2}=\frac{Y_1^2+18}{Y_2^2+18}$

$\frac{18}{1}=\frac{18^2+18}{Y_2^2+18}$

$Y_2^2+18=19$

$Y_2=\pm 1$

**1.23.** The curved surface area of a cylinder jointly varies directly with the height and the radius. When the height of the cylinder is 36 cm and the radius of the cylinder is 10 cm, the curved surface area of the cylinder is $720\pi \text{cm}^2$. Find the curved surface area of the cylinder when the height of the cylinder is 54 cm and the radius of the cylinder is 15 cm.

**Sol:** Let the curved surface area of the cylinder be denoted by $s$. Let the radius and height of the cylinder be denoted by $r$ and $h$.
$s \propto rh$. Hence, $s=c\,r\,h$ where $c$ is a constant.
$c=s/rh$
When $s=720\pi$ sq.cm $r=36$ cm and $h=10$ cm.

Hence, $c=\frac{720\pi}{36\times 10}=2\pi.$

Surface area of the cylinder when $r=48$ cm and $h=15$ cm is $2\pi\times 54\times 15=1620\pi\,\text{cm}^2$.

**Alternative Method:**
As both radius and height become 3/2 times their original values, the curved surface area, being proportional to $rh$, becomes $\left(\frac{3}{2}\right)^2$, i.e., (9/4) times its original value.
Hence, it is $(9/4)\times 720\pi=1620\pi\,\text{cm}^2$.

**1.24.** The total monthly sales of two companies A and B are in the ratio 2:3 and their total monthly expenditures are in the ratio 3:4. Find the ratio of the profits of the two companies given that company A's profit is equal to a fifth of its sales.

**Sol:** Let the total monthly sales of companies A and B be $2x$ and $3x$ and their total monthly expenditures be $3y$ and $4y$.

Given that A's profit = $\frac{1}{5}$ of sales = $(2x/5)$.

$\therefore 2x-3y=\frac{1}{5}(2x)$

$$\Rightarrow \frac{4}{5}(2x) = 3y$$

$$\Rightarrow y = \frac{8}{15}x$$

Profit of company B

$$= 3x - 4y = 3x - 4\left(\frac{8}{15}x\right) = \frac{13x}{15}$$

Hence, the ratio of the profits of the two companies are $\frac{2}{5}x : \frac{13x}{15} = 6:13$

**1.25.** Given that $x$ varies directly as $y$, verify whether $(x+y)^3$ varies directly with $(x-y)^3$.

**Sol:** This is a model of a problem where a certain relationship is given, and we are asked to check the relationship between different forms of combinations of the two variables.

As $x$ varies directly with $y$.

$x = Ry$ where $R$ is a constant.

$$\frac{(x+y)^3}{(x-y)^3} = \frac{(Ry+y)^3}{(Ry-y)^3}$$

$$= \frac{(R+1)^3}{(R-1)^3}$$

As R.H.S. of above equation is also a constant, $(x+y)^3$ varies directly with $(x-y)^3$.

**1.26.** A part of the monthly expenses of Amar, a marketing executive is fixed, and the remaining part varies with the distance travelled by him. If he travels 200 km in a month, his total expenditure is ≠3300. If he travels 500 km in a month, his total expenditure is ≠3900. Find his total expenditure, if he travels 800 km in a month.

**Sol:** Let the total expenses be $T$, $F$ be the fixed part and $V$ be the variable part. Given that, $T = F + V$.

As $V$ varies with the distance travelled, if distance travelled is denoted by $d$,

$V = Rd$ where '$R$' is the proportionality constant.

Hence, $T = F + Rd$

From the given data,

$3300 = F + 200\ R$ (1)

$3900 = F + 500\ R$ (2)

Subtracting (1) from (2)

$600 = 300\ R.\ (\Rightarrow R = 2)$

Total expenditure if he travels 800 km

$= F + 800\ R = F + 500\ R + 300\ R$

$= 3900 + 600 = ₹4500$

**1.27.** The volume of a cone varies jointly as the area of its base and its height. The volume of a cone whose base area is A $cm^2$ and height is 10 cm is 270 $cm^3$. Find the volume of a cone whose base area is 2A $cm^2$ and height is 30 cm.

**Sol:** $V \propto A\ h$

$$\frac{V_1}{V_2} = \frac{A_1}{A_2}\frac{h_1}{h_2}$$

$$\frac{270}{V_2} = \left(\frac{A}{2A}\right)\frac{10}{30}$$

$V_2 = 1620\ cm^3$

Note that there should be consistency of the units used for the variables, i.e., whatever be the units used to express the variables when the constant of proportionality is being calculated, the same units should be used for different variables later on also when finding the value of the variable which we are asked to find out.

**1.28.** The ratio of the monthly incomes of A and B is 4:3. The ratio of their monthly expenditures is 5:4. If A saves one fourth of his monthly income, find the ratio of their monthly savings.

**Sol:** Let the monthly incomes of A and B be ₹4$x$ and ₹3$x$, respectively.

Let the monthly expenditures of A and B be ₹5$y$ and ₹4$y$ respectively. Monthly savings of A = ₹$(4x - 5y)$. Monthly savings of B = ₹$(3x - 4y)$.

Given that $4x - 5y = \frac{1}{4}(4x)$

$3x = 5y$.

∴ Monthly saving of B $= 3x - 4y = 5y - 4y$, i.e., ₹$y$.

Required ratio $= \frac{1}{4}(4x) : y = x : y = 5:3$

**1.29.** If $x$ varies directly with $y$, check whether $x^3 + y^3$ varies directly with $x^3 - y^3$.

**Sol:** Let $\frac{x}{y} = K$, where $K$ is a constant.

$x = Ky$

$x^3 + y^3 = y^3(K^3 + 1)$

$x^3 - y^3 = y^3(K^3 - 1)$

$$\frac{x^3 + y^3}{x^3 - y^3} = \frac{K^3 + 1}{K^3 - 1}\ a\ \text{constant}$$

∴ $(x^3 + y^3)$ varies directly with $(x^3 - y^3)$

**1.30.** The monthly expenses of Raja on his car are partly constant and partly vary with the number of kilometres he travels in a month. If he travels 100 km in a month his total car expenses will be ₹3500. If he travels 200 km in a month, his total car expenses will be ₹4000. If he travels 250 km in a month, what will be his total car expenses.

**Sol:** Let his total car expenses be ₹$T$. Let the fixed expense be ₹$F$. Let the variable expense be ₹$V$.

$T = F + V$

If he travels $D$ km in a month, $\frac{V}{D} = K$, where $K$ is a constant.

$\therefore T = F + KD$

Given that

$3500 = F + 100K$ (A)

$4000 = F + 200K$ (B)

Solving (A) and (B),

$F = 3000$ and $K = 5$.

Total car expenses if he travels 250 km

$= F + 250K = ₹4250$.

## EXERCISE-1

***Directions for questions 1 to 50*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Raja divided 35 sweets among his daughters Rani and Sita in the ratio 4:3. How many sweets did Rani get? _____

**2.** The force of attraction between two objects varies directly with the product of their masses and inversely with the square of the distance between them. When the product of the masses (taken in kg) of two objects is 12 and the distance between them is 2 m, the force of attraction between them is 18 Newtons. The force of attraction between two objects whose product of masses (taken in kg) is 18 and whose separation is 3 m is _____ Newtons.

**3.** The area of a circle varies as the square of its radius. Given that the area of a circle whose radius is 7 ft is 196 sq ft, then the area (in sq ft) of a circle whose radius is 8 ft is _____.

**4.** Find the numbers that are in the ratio 3:2:4 such that the sum of the first and the second numbers added to the difference of the third and the second numbers is 21.

(A) 12, 8, 16 (B) 6, 4, 8

(C) 9, 6, 24 (D) 9, 6, 12

**5.** If $2.7p = 0.09q$, $\frac{q + 6p}{q - 6p} =$

(A) $\frac{4}{3}$ (B) $\frac{3}{2}$

(C) $\frac{5}{4}$ (D) $\frac{6}{5}$

**6.** A varies directly with B when C is constant and inversely with C when B is constant. A is 16, when B is 28 and C is 7. Find the value of A, when B is 9 and $C$ is 6.

(A) 6 (B) 7

(C) 8 (D) 9

**7.** The number of men and women in a conference hall are in the ratio 5:4. If three men and six women join the conference, then the number of men and women in the conference hall will be in the ratio 7:6. Find the original number of men and women in the conference hall.

(A) 48, 60 (B) 60, 48

(C) 30, 24 (D) 40, 32

**8.** Quantities $a$ and $b$ are inversely proportional to each other. When $a = 8$, $b = 240$, find $b$ when $a = 6$ _____.

**9.** Calculate the fourth proportional to the numbers 0.8, 1.6, and 1.6

(A) 32.4 (B) 2.34

(C) 3.2 (D) 25.6

**10.** If $\frac{X + Y}{2X + Y} = \frac{4}{5}$, then find $\frac{2X + Y}{3X + Y}$.

(A) $\frac{4}{5}$ (B) $\frac{5}{6}$

(C) $\frac{6}{7}$ (D) $\frac{3}{4}$

**11.** An article worth ₹6400 breaks into two pieces whose weights are in the ratio 3:5. If the value of the article is proportional to the square of its weight, the loss incurred due to the breakage is ₹ _____.

**12.** $x$ varies directly as the square of $y$. When $y = 8$, $x = 192$. Find $x$ when $y = 10$.

(A) 100 (B) 30

(C) 300 (D) 200

**13.** The marks obtained by Raju in Maths, Physics, and Chemistry are in the ratio 2:3:4. If the total marks that Raju obtained in these three subjects is 189, how many marks did Raju score in Maths?

(A) 21 (B) 42

(C) 63 (D) 84

**14.** If P: $Q = 3:4$, find 5P:7Q.

(A) $\frac{20}{37}$ (B) $\frac{3}{4}$

(C) $\frac{15}{28}$ (D) $\frac{20}{37}$

**15.** An amount of ₹1560 was divided among A, B and C, in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. The share of C is _____.

**16.** A fort had provisions for 150 men for 45 days. After ten days, 25 men left the fort. How long will the food last at the same rate for the remaining men?
(A) 40 days (B) 28 days
(C) 50 days (D) 42 days

**17.** In a class of 30 students, which of the following can't be the ratio of boys and girls?
(A) 2:3 (B) 1:5
(C) 4:5 (D) 2:1

**18.** The sum of the present ages of Anil and his wife is 88 years. After 8 years, the ratio of their ages will be 7:6. Find Anil's present age (in years).
(A) 48 (B) 46
(C) 50 (D) 52

**19.** Ratio of two numbers is 3:5 and their sum is 40. Find the smaller of the two numbers _____.

**20.** $x$ varies directly with the square of $y$. When $y$ is 12, $x$ is 452. Find $x$ when $y$ is 18 _____.

**21.** $x$ varies inversely with the square of $y$. When $y$ is 2, $x$ is 48. Find $x$ when $y$ is 4 _____.

**22.** If $x+y+z=120$ and $x=\frac{1}{2}y$ and $y=\frac{2}{3}z$, find $z$ _____.

**23.** Salary of A in a month varies directly with the number of working days in the month. A got a salary of ₹10,000, in a month which had 25 working days. What salary will he get in a month which has 26 working days?
(A) ₹10,100 (B) ₹10,200
(C) ₹10,300 (D) ₹10,400

**24.** The extension of a spring is directly proportional to the force applied. The extension is 3 cm when the force applied is 30 *N*. Find the extension when the force applied is 20 *N*.
(A) 2 cm (B) 1.5 cm
(C) 1 cm (D) 3 cm

**25.** If $a:b=2:3$ and $b:c=5:7$, then find $a:b:c$.
(A) 10:15:21 (B) 10:21:15
(C) 9:12:14 (D) 12:7:18

**26.** If $P$, $Q$, $R$ and $S$ are in proportion, which of the following follows?
(A) $S$, $R$, $Q$ and $P$ are in proportion.
(B) $Q$, $S$, $P$ and $R$ are in proportion.
(C) Both (A) and (B)
(D) Neither (A) nor (B)

**27.** Two numbers A and B are in the ratio 10:13. If 90 is subtracted from each, the resulting numbers will be in the ratio 7:10. Find A.
(A) 290 (B) 280
(C) 310 (D) 300

**28.** There are two variables $x$ and $y$, where $x$ varies directly as the cube root of $y$. When $y=8$, $x=2$. Find $x$ when $y=216$.
(A) 6 (B) 12
(C) 18 (D) 24

**29.** If $a:b=4:1$, find $\frac{a-3b}{2a-b^2}$.
(A) $\frac{2}{7}$ (B) $\frac{1}{7}$
(C) $\frac{3}{7}$ (D) Cannot be determined

**30.** If $a+b:b+c:c+a=3:4:5$, find $a:b:c$.
(A) 1:2:3 (B) 2:1:3
(C) 2:3:1 (D) 1:3:2

**31.** If $a:b=7:3$, find $a+b:a-b$.
(A) 5:2 (B) 2:5
(C) 7:3 (D) 3:7

**32.** The ratio of the ages of Arun, Brahma, and Chari is 5:4:3. If Brahma's age is 28 years, then the sum of the ages of the three persons is _____.

**33.** If $P^2:Q=R:P$ and $27Q^2=R$, find $P:Q$.
(A) 1:9 (B) 9:1
(C) 3:1 (D) 1:3

**34.** At a party, there are total of 28 adults. If $x$ ladies join the party, the ratio of the number of ladies to that of gents will change from 3:4 to 5:4. Find $x$ _____.

**35.** There are total of 30 employees in a company. The ratio of male employees to the female employees is 8:7. How many female employees have to be recruited so that the ratio becomes 1:1?
(A) 5 (B) 2
(C) 3 (D) 4

**36.** If $x:y=4:9$, $\frac{4\sqrt{x}+5\sqrt{y}}{6\sqrt{x}+7\sqrt{y}}=$
(A) $\frac{21}{31}$ (B) $\frac{19}{29}$
(C) $\frac{27}{37}$ (D) $\frac{23}{33}$

**37.** Quantity $A$ varies directly with the sum of the quantities $B$ and $C$. If $B$ increases by 2 and $C$ increases by 4, by how much does $A$ increase?
(A) 2 (B) 4
(C) 6 (D) Cannot be determined

**38.** Three numbers are in the ratio of 2:3:5. Given that the product of the extremes is 90. The difference between the largest and the smallest of them is _____.

**39.** Find the following:
(a) Duplicate ratio of 3:4.
(A) 3:8 (B) 6:4
(C) 5:7 (D) 9:16

(b) Triplicate ratio of 2:3.
(A) 6:9 (B) 3:2
(C) 8:27 (D) 5:8
(c) Sub-duplicate ratio of 16:9.
(A) 2:3 (B) 4:3
(C) 4:9 (D) 8:3
(d) Mean proportional of 16 and 4.
(A) 64 (B) 16
(C) 8 (D) 14

**40.** If $x:y = 3:7$ and $y:z = 7:4$, find $x:z$.
(A) 4:3 (B) 3:2
(C) 2:3 (D) 3:4

**41.** If $a+b:a-b = 3:2$, find $a:b$.
(A) 5:1 (B) 1:5
(C) 3:5 (D) 5:3

**42.** Quantity $A$ varies directly with the product of $B$ and $C$. $A = 300$ when $B = 20$ and $C = 50$. What is the value of $B$ when $A = 900$ and $C = 60$?
(A) 40 (B) 45
(C) 50 (D) 60

**43.** Which of the following must be true?
(A) If $x$ varies directly with $\sqrt{y}$, $x^2$ varies directly with $y$.
(B) If $\sqrt{x}$ varies inversely with $\sqrt{\frac{1}{y}}$, $\sqrt{y}$ varies inversely with $\sqrt{\frac{1}{x}}$
(C) Both (A) and (B)
(D) Neither (A) nor (B)

**44.** If three numbers are in the ratio 1:3:5 and their sum is 108, then the largest number is _____.

**45.** The monthly salaries of X and Y are in the ratio 3:4. The monthly expenditures of X and Y are in the ratio 4:5. Find the ratio of the monthly savings of X and Y.
(A) 5:3 (B) 4:7
(C) 3:5 (D) Cannot be determined

**46.** If $a = \frac{50b}{100}$ and $b = \frac{25c}{100}$, then find $a:c$.
(A) 1:2 (B) 1:4
(C) 3:4 (D) 1:8

**47.** There are two positive numbers in the ratio 5:8. If the larger number exceeds the smaller by 15, the smaller number is _____.

**48.** Quantity $P$ varies inversely with the product of $Q$ and $R$. When $Q = 6$ and $R = 12$, P = 75. Find $P$ when $Q = 5$ and $R = 10$ _____.

**49.** The mean proportional of
(a) 6 and 24 is _____.
(b) 50 and 512 is _____.

**50.** The present ages of Rohit and Sunil are in the ratio 3:5. 10 years hence, the ratio of their ages will be 4:5. Find the present age of Rohit. (in years) _____.

## EXERCISE-2

***Directions for questions 1 to 34:*** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Eighteen men take 30 days to complete a piece of work working 10 hours a day. Find the time taken by 25 men to complete five times as much work working 12 hours a day.
(A) 30 days (B) 36 days
(C) 72 days (D) 90 days

**2.** The cost of 3 kg tea powder is equal to the cost of 6 kg of sugar, the cost of 24 kg of sugar is equal to the cost of 21 kg of oil, the cost of 56 kg of oil is equal to the cost of 12 kg of ghee. What is the cost of 3 kg of ghee if the cost of 1 kg tea powder is ₹72?
(A) ₹576 (B) ₹288
(C) ₹192 (D) ₹436

**3.** A student took 6 papers in an examination, where the maximum marks were the same for each paper. In all papers together, the candidate obtained 58% of the total marks. His marks in these papers were in the ratio of 12:13:14:15:16:17. Then, the number of papers in which he got more than 55% is
(A) 6 (B) 5
(C) 4 (D) 3

**4.** The cost of a precious stone varies directly as the square root of its weight. A certain precious stone broke into 3 pieces whose weights are in the ratio 1:4:4. As a result, its value went up by ₹12000. Find its initial value.
(A) ₹9000 (B) ₹12000
(C) ₹15000 (D) ₹18000

**5.** Nine farmers take 6 hours to plough 18 acres of land. How many acres of land can 16 farmers plough in 21 hours? (Assume that the rate of work of each farmer is the same) _____

**6.** If $a:b = 4:5$ then find the value of $\frac{4a+3b}{3a+2b}$.

_____
22

**7.** If $x:y = 6:7$ then find the value of $\frac{3x^2+4y^2}{2x+3y}$.

(A) $\frac{304}{33}$ (B) $\frac{152}{33}$

(C) $\frac{608}{33}$ (D) Cannot be determined

**8.** In a school, there are 650 students. The ratio of the number of boys to that of the girls is 8:5. How many more girls should join the school so that the ratio becomes 4:3? _____

**9.** If heat radiated by a certain body per unit time varies directly with the square root of the excess of the temperature of the body over the ambient temperature. The heat radiated by the body in 1 second is 12 joules, when the temperature of the body is 34°C. Find the temperature of the body when the heat radiated in 1 second was 20 joules. (Assume the ambient temperature to be 25°C).

(A) 60°C (B) 45°C

(C) 40°C (D) 50°C

**10.** Find $a:b:c$, given $3a+2b=7c$ and $b=a+c$.

(A) 3:8:5 (B) 1:6:5

(C) 1:2:1 (D) 3:10:7

**11.** A purse contains 10 paise, 20 paise, and 50 paise coins in the ratio 5:2:1. The total value of all the coins in the purse is ₹140. How many 20 paise coins are in the purse? _____.

**12.** The total income and the total expenditure of two persons A and B are in the ratio 22:17. The incomes of A and B are in the ratio 5:6 and their expenses are in the ratio 7:10. What is the ratio of the savings of A and B?

(A) 3:2 (B) 2:3

(C) 7:12 (D) 10:9

**13.** If $\frac{p+q}{r}=\frac{q+r}{p}=\frac{p+r}{q}=k$, then find $k$.

(A) 1 (B) –1

(C) 2 (D) Either (B) or (C)

**14.** In a shop, the quantities of three kinds of commodities sold on a particular day were in the ratio 3:4:5. The total sales proceeds was ₹4,000. If the prices of the three commodities were also in the same ratio, what was the total amount received by selling the commodity which fetched the maximum sales revenue?

(A) ₹2,500 (B) ₹2,000

(C) ₹3,000 (D) ₹1,500

**15.** A man divided ₹62500 among his four sons such that four times the share of his first son, three times the share of his second son, two times the share of his third son and the share of his fourth son are all equal. Find the share of each of his sons. (in ₹) _____, _____, _____, _____

**16.** The distance travelled by a freely falling body is directly proportional to the square of the time for which it falls. A body fell 95 m in the 10th second. Find the distance (in m) it fell in the 14th second _____.

**17.** There are five identical glasses containing milk in the ratio 3:4:5:6:7. How many glasses are at least half full of milk if the total volume of milk in the glasses is three-fifth of the total volume of the five glasses? _____

**18.** A string is cut into two parts such that the ratio of the lengths of the complete string and the smaller part is 20 times the ratio of the lengths of the smaller part and the larger part. Find the ratio of the length of the string and the square of the length of the smaller part (taken in cm) if the longer part is 4 cm long

(A) 5:3 (B) 5:4

(C) 5:2 (D) 5:1

**19.** A purse contains 72 coins comprising one rupee, 50 paise and 25 paise coins, their values being in the ratio 10:15:8. Find the number of 50 paise coins _____.

**20.** A machine with a power of 18 units can lift an object of a maximum weight of 9 units. If the power of a machine varies directly as the square root of the maximum weight of an object that it can lift, then the machine with 24 units of power can lift an object weighing a maximum of _____ units.

**21.** What number must be subtracted from both the numerator and denominator of the fraction $\frac{22}{37}$ so that it becomes 2:7? _____

**22.** Ten years ago, the ratio of the ages of a woman and her daughter was 3:2. Which of the following cannot be the ratio of their ages 5 years from now?

(A) 6:5 (B) 7:3

(C) 8:7 (D) 11:9

**23.** If $a$, $b$, $c$ and $d$ are in proportion, then which of the following is equal to $(a-b)(a-c)/a$?

(A) $a+b+d-c$ (B) $a-b-c-d$

(C) $a+d-b-c$ (D) $a+b-c+d$

**24.** If $\frac{a}{b}=\frac{3}{4}$, $\frac{b}{c}=5$, $\frac{c}{d}=\frac{4}{3}$, $\frac{d}{e}=2$ and $\frac{e}{f}=\frac{1}{4}$ then $\frac{def}{abc}=$

(A) $\frac{40}{9}$ (B) $\frac{400}{9}$

(C) $\frac{9}{40}$ (D) $\frac{9}{400}$

**25.** A writer gets a fixed amount for his book apart from the royalty he gets per book sold. He gets ₹22000 and ₹46000 for 6000 books sold and 18000 books sold, respectively. Find his income per book when 25000 books are sold _____.

**26.** If $\frac{p}{q+r-p}=\frac{q}{p+r-q}=\frac{r}{p+q-r}$, each of these equals _____ or _____.

**27.** A piece of land is to be divided between two men in the ratio of 5:11. Instead, if the land is divided in the ratio 11:5, what fraction of the whole piece of land does the second man lose?
(A) 3/8 (B) 6/11
(C) 5/6 (D) 10/11

**28.** The volume of a cylinder varies jointly as its height and the area of its base. When the area of the base is 64 sq.ft. and the height is 10 ft, the volume is 640 cu.ft. What is the height of the cylinder (in ft), whose volume is 360 cu.ft and the area of the base is 72 sq.ft.? _____

**29.** The time taken ($T$) by on automobile to cover a certain distance ($D$) varies directly as the distance when its engine capacity ($C$) is constant. $T$ varies inversely as $C$ when $D$ is constant. $P$ and $Q$ are two automobiles. The engine capacities of $P$ and $Q$ are 1500 cc and 1200 cc, respectively. To cover 800 km, P takes 16 hours. Find the time taken by $Q$ to cover 600 km. (in hours)
(A) 15 (B) 17
(C) 20 (D) $9\frac{1}{2}$

**30.** The volume of a solid figure is proportional to the square of its radius when its height is constant and to its height when its radius is constant. The solid figure A has a radius of 7 units, a height of 9 units and a volume of 1386 cubic units. Find the volume of a solid figure (in cubic units) whose radius is 14 units and height is 3 units _____.

**31.** The ratio of the number of sparrows on the left and right branches of a tree is 7:13. If 12 sparrows shift to the left branch from the right, then the number of sparrows on the left branch is equal to that on the right branch. How many sparrows were there on the left branch, before the shift? (Assume that the branches of the tree are classified as either left branches or right branches.)
(A) 84 (B) 14
(C) 28 (D) 40

**32.** If $\frac{x}{2a+b}=\frac{y}{2b+c}=\frac{z}{2c+a}=8$ and $\frac{x}{2a}=\frac{y}{2b}=\frac{z}{2c}=k$ where $a+b+c\neq 0$, then $k=$ _____.

**33.** The annual incomes of Varun and Vikram are in the ratio 8:3 and their annual expenditures are in the ratio 4:1. If each saves ₹2000 per annum, then what is the annual income of Varun? (in ₹) _____

**34.** If $\frac{ka}{b+c}=\frac{kb}{c+a}=\frac{kc}{a+b}=l$ and $k\neq 0$, $a+b+c\neq 0$, then what is the value of '$l$'?
(A) $k$ (B) $\frac{k}{3}$
(C) $\frac{k}{2}$ (D) $\frac{k}{4}$

***Directions for questions 35 and 36:*** These questions are based on the information given below.

There are two colleges in the town – college A and college B. There are 500 more students in college A than in college B. The ratio of the boys to that of the girls in college A is 3:2 and that in college B is 4:1. The ratio of the number of Science, Humanities, and Commerce students in college A and college B are 2:5:3 and 2:3:3, respectively. The number of Commerce students in both the colleges is the same.

**35.** How many students are there in college A? _____

**36.** How many girls are there in the two colleges together? _____

***Directions for questions 37 to 50:*** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**37.** If $a$, $b$ and $c$ are in continued proportion, then which of the following is equal to $a:c$?
(A) $a^2:b^2$ (B) $(a^2+b^2):(b^2+c^2)$
(C) $b^2:c^2$ (D) All of these

**38.** The ratio of the present ages of a man and his wife is 5:4. Which of the following cannot be a possible ratio of their ages 20 years ago?
(A) 7:5 (B) 3:2
(C) 13:10 (D) 6:5

**39.** The monthly tariff of a DTH provider consists of two parts – a fixed part for providing the service and a variable part which varies with the number of channels opted for. The monthly tariffs of two customers who opted for 30 channels and 50 channels are ₹350 and ₹450, respectively. Find the monthly tariff of a customer who opted for 60 channels (in ₹) _____.

**40.** There are five vessels, with equal capacities, each containing some milk. The quantities of milk in the 5 vessels are in the ratio 4:5:6:7:8. The total quantity of milk in the five vessels is equal to 75% of the total capacities of the 5 vessels. How many of the vessels are at least 64% full of milk?
(A) 2 (B) 1
(C) 4 (D) 3

**41.** $3x+y-5z=0$ and $4x+5y-14z=0$. Find $x:y:z$.
(A) 1:1:1 (B) 2:1:1
(C) 1:2:1 (D) 1:1:2

**42.** A garrison of 2,000 men has provisions for 20 weeks at the rate of 2.5 kg per day per man. After 4 weeks, 500 more men join the garrison. For how many more weeks, will the remaining provisions last at the rate of 2 kg per day per man?
(A) 12 (B) 15
(C) 16 (D) 20

**43.** A sum of money that was supposed to be divided between A and B in the ratio of 3:5 was divided among A, B and C

in the ratio of 6:5:4. Due to this, A gained ₹3,000. What was the loss incurred by B?
(in ₹) _____

**44.** The volume of a sphere varies directly as the cube of its radius. If three cubes of radii 3 cm, 4 cm and 5 cm are melted and recast into one sphere, then find the radius of the sphere _____.

**45.** Arjun buys 5 erasers for every 3 pencils bought by him. For every 24 articles of the combined lot of erasers and pencils bought by him, he buys 2 geometry boxes. If the total number of all three types of articles bought by him is 104, what is the number of pencils bought by him?
(A) 65 (B) 36
(C) 8 (D) 60

**46.** The ratio of incomes of A and B is 3:4. Each of them spends a part of his income and saves the rest. The ratio of their expenditures is 2:3. Whose savings, as a proportion of his income, is higher? _____

**47.** The monthly telephone bill has a fixed tariff for up to 50 outgoing calls. Outgoing calls in excess of 50 are charged at a certain fixed rate per call. The monthly bills of Ramesh and Suresh who made 98 outgoing calls and 218 outgoing calls, respectively, were ₹300 and ₹450, respectively. Find the monthly bill of a person who has made 160 outgoing calls (in ₹) _____.

**48.** A stone is dropped from a height of one km. The distance it falls through varies directly with the square of the time taken to fall through that distance. If it travels 64 m in 4 seconds, find the distance it covers in the 5th second.
(A) 36 m (B) 24 m
(C) 28 m (D) 44 m

**49.** A and B are two numbers. If 10 is the mean proportional of A and B and 10000 is the third proportional of A and B, what will be the value of the larger of A and B?
_____

**50.** The ratio of the incomes of A and B is 5:4 and ratio of their expenditures is 3:2. If B saves one-third of his income, find the ratio of their savings.
(A) 7:8 (B) 11:12
(C) 5:6 (D) 3:4

## EXERCISE-3

***Directions for questions 1 to 50*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** If $a:b = 2:3$, $b:c = 6:5$, $c:d = 10:13$ and $e:d = 2:1$, then find $abc:ed^2$. $\frac{480}{\quad}$

**2.** $p$, $q$, $r$, $s$ and $t$ are five integers satisfying $p = 3q = 4r$ and $2q = 5s = 12t$. Which of the following pairs contains a number that can never be an integer?
(A) $(2p/15, q/t)$ (B) $(p/t, 4r/t)$
(C) $(p/4, rs/180)$ (D) $(p/8, s/r)$

**3.** A quantity $Q$ is obtained by adding three quantities. The first is a constant, the second varies directly as the square root of $y$, and the third varies directly as the cube root of $y$. When $y = 1$, $Q = 90$, when $y = 64$, $Q = 450$, and when $y = 729$, $Q = 1270$. Find the constant.
(A) 5 (B) 10
(C) 15 (D) 20

**4.** A salesman for a company gets an incentive for every unit of product he sells apart from his fixed salary. He gets ₹8,000 and ₹9,000 for 150 units and 200 units he sold, respectively. If he sells 400 units, what is his income per unit?
(A) ₹32.50 (B) ₹30
(C) ₹27.50 (D) ₹20

**5.** $A$ varies directly as the sum of the two quantities $B$ and $C$. $B$ in turn varies directly as $x$ and $C$ varies inversely as $x$. When $x = 2$, $A = 6$ and when $x = 4$, $A = 9$. Find the value of A when $x = 16$.
(A) 2½ (B) 1
(C) 8½ (D) 32¼

**6.** There are three unequal quantities $x$, $y$ and $z$ in continued proportion. Which of the following equals $z:x$?
(A) $y^2:x^2$ (B) $\frac{z^2 - y^2}{y^2 - x^2}$
(C) $z^2:y^2$ (D) All of the above

**7.** A certain sum is divided among A, B and C in a manner that for every rupee that A gets, B gets 75 paise and for every rupee that B gets, C gets 50 paise. If C's share in the total sum is ₹420, then find the share of A (in ₹) _____.

**8.** If $b/a$ is a proper fraction satisfy the equation $15a^2 - 26ab + 8b^2 = 0$, find $b:a$.
(A) 2:5 (B) 1:5
(C) 4:5 (D) 3:4

**9.** A number is divided into five parts. Twice the first part, thrice the second part and four times the fourth part are equal. Twice the second part, five times the third part and six times the last part are equal. Which of the following is always true if all the parts are integers?

(A) The first part is a multiple of 72.
(B) The second part is divisible by the fourth part.
(C) The first part is a factor of the last part.
(D) The product of the first and fourth parts is divisible by 30.

**10.** Balloons A and B are fitted with special valves and placed in an observation chamber. Initially, the volume of helium in balloon A was seven times that in balloon B. The helium in balloon A leaks at a constant rate and half of the helium which leaks from balloon A at any given time enters balloon B. After 3 hours, the ratio of the volumes of helium in balloons A and B is 13:7. After how many hours from the start, will the helium in balloons A and B be in the ratio 7:37?
(A) 7 (B) 4
(C) 11 (D) 14

***Directions for questions 11 to 14*:** These questions are based on the data given below.

There are two software companies in a city – 'Smart Softcom' and 'Fast Softcom'. The ratio of the number male employees to the number of female employees in 'Smart Softcom' is 5:3 and that in 'Fast Softcom' is 7:5. The ratio of number of employees in the age groups (in completed years) 21 to 30, 31 to 40 and 41 to 50 in 'Smart Softcom' and 'Fast Softcom' are 4:5:7 and 2:1:3 ,respectively. The number of employees in the age group 21 to 30 in both the companies is the same and Smart Softcom has 400 employees more than 'Fast Softcom'.

**11.** How many employees are there in Fast Softcom?
(A) 1600 (B) 1200
(C) 800 (D) 600

**12.** How many male employees are there in the two companies together?
(A) 1100 (B) 1300
(C) 1500 (D) 1700

**13.** What is the difference in the number of employees in the age group 41 to 50 in Smart Softcom and Fast Softcom?
(A) 300 (B) 200
(C) 100 (D) 160

**14.** What is the ratio of the number of male employees in Fast Softcom to the number of female employees in Smart Softcom?
(A) 7:6 (B) 2:1
(C) 1:2 (D) 7:10

**15.** The electricity bill for a month varies directly as the number of units consumed. The charge per unit is ₹1.35 up to 50 units used. If the number of units consumed is more than 50, then the cost of each additional unit is ₹2.70. If the consumption in the first month is 97 units, then what should the consumption in the second month be such that the average bill for the two months combined is ₹135 per month?
(A) 46 (B) 56
(C) 59 (D) 53

**16.** The volume of a gas is inversely proportional to the pressure acting on it when the temperature is constant and directly proportional to the temperature when the pressure acting on it is constant. When the temperature is 40 and the pressure acting on it is 64, its volume is 200. Find the pressure when the temperature and its volume are 50 and 400 respectively.
(A) 32 (B) 40
(C) 44 (D) 50

**17.** The earnings of A and B are in the ratio 3:7 and that of B and C is 4:9 and that of D and C is 7:6. If the sum of the earnings of A, B, C and D is ₹52950, then what are the earnings of D?
(A) ₹21,050 (B) ₹22,050
(C) ₹23,050 (D) ₹24,050

**18.** Three friends, Aravind, Bharath and Chandu are about to have their breakfast. Aravind has 7 apples, Bharath has 5 apples and Chandu has no apples but has 12 coins. He offers to pay for some apples. They agree to share the 12 apples equally among themselves and agree that Chandu would pay 12 coins for his share. Bharath suggests that he be paid 5 coins and Aravind be paid 7 coins. Aravind says that he should get more than 7 coins. How much should Aravind get?
(A) 11 coins (B) 10 coins
(C) 12 coins (D) 9 coins

**19.** The kinetic energy of a moving body varies directly with its mass when its velocity is constant and with the square of its velocity when its mass is constant. A body has a mass of 7.2 kg and a velocity of 0.2 m/sec and a kinetic energy of 0.144 joules. Find the kinetic energy of a body having a mass of 3.6 kg and a velocity of 0.8 m/sec (in joules) _____.

**20.** If $a:b = 2:3$ and $p:q = 3:2$, what is the value of $(2a^2p^3 + 3b^2q^3):(3abpq^2 + 4a^2p^2q)$?
(A) 1:1 (B) 2:3
(C) 6:7 (D) Cannot be determined

**21.** If $\frac{a+b}{c+d} = \frac{a-b}{c-d}$, then which of the following is true?
(A) $ab + cd = 0$ (B) $ac + bd = 0$
(C) $ad - bc = 0$ (D) $ab - cd = 0$

**22.** The radii of cylinders of equal heights vary directly with the square root of their volumes. The radii of cylinders of equal volumes vary inversely as the square root of their heights. The radius of a cylinder is 10 cm, its volume is 1500 $cm^3$ and height is 5 cm. Find the radius of the cylinder whose volume is 2400 $cm^3$ and height is 2 cm.
(A) 10 cm (B) 15 cm
(C) 20 cm (D) 25 cm

**23.** If $a \neq c$ and $\frac{a^2+b^2}{a+b} = \frac{b^2+c^2}{b+c} = k$, find $k$.

(A) $a+c$ (B) $a-c$

(C) $c-a$ (D) $\frac{ac}{a+c}$

**24.** A product can be manufactured using 4 different processes – *A*, *B*, *C*, and *D*. In each process there is a fixed component to the cost and another component that varies with the number of units produced. The following table gives the costs incurred in the four processes.

| | A | B | C | D |
|---|---|---|---|---|
| Fixed cost (in ₹) | 10,000 | 12,000 | 15,000 | 20,000 |
| Cost per unit (in ₹) | 20 | 15 | 10 | 8 |

If 500 units have to be produced, which process will result in the least cost?

(A) *A* (B) *B*

(C) *C* (D) *D*

**25.** The monthly telephone bill has a fixed tariff of ₹250 for up to 50 outgoing calls. For over 50 calls, there is a charge of ₹1.25 per call. The ratio of the bills paid by Aravind and Prasad for a particular month is 2 : 3 and the number of outgoing calls made by Aravind is 90. What is the number of outgoing calls made by Prasad?

(A) 210 (B) 250

(C) 160 (D) 180

**26.** A machine's output varies directly with its effective input in kilos when its efficiency is constant and it varies directly with its efficiency when its effective input is constant. The effective input is the input minus the waste material. The machine produces an output of 1008 kg when its input is 2400 kg, efficiency is 70% and waste material is 40% of the input. Find the output of the machine if its input is 1680 kg, efficiency is 80% and waste material is 30% of the input (in kg) _____.

**27.** The expenses for yoga classes in a colony are partly constant and partly varying with the number of members. If there are 50 members, then each of the members has to bear ₹220 per month and if there are 10 more members, then the share of each of the members comes down by ₹15 per month. How many members would be there if the share of each member is ₹160?

(A) 150 (B) 130

(C) 90 (D) Cannot be determined

**28.** The kinetic energy that a body acquires when it falls freely for a time $t$ varies directly with the square of $t$ for a given mass. For bodies of different masses for a given value of $t$, the kinetic energy varies directly as the mass. The kinetic energy of a body of mass 5 kg which is under free fall for 10 seconds is 25 kilojoules. Find the kinetic energy of a body with mass 2.5 kg which is under free fall for 3 seconds (in kilojoules)

(A) 102.5 (B) 1.175

(C) 125 (D) 1.125

**29.** If $(x+y)$ varies directly as $(x-y)$, then $(x^2+y^2)$ will vary as

(A) $x^2-y^2$ (B) $xy$

(C) Both (A) and (B) (D) None of these

**30.** The total surface area of a cylinder having a certain height is the sum of two parts. One of the parts varies directly with the radius and the other parts varies directly with the square of the radius. The total surface area is 7200 units when the radius is 30 units and 3600 units when the radius is 20 units. Find the total surface area when the radius is 10 units.

(A) 1040 (B) 1200

(C) 1260 (D) 1170

**31.** The lateral surface area of a right square pyramid varies directly with the edge of its base when the slant height (the shortest distance from the vertex to any one of its edges) is constant. Also, the lateral surface area of the pyramid varies directly with the slant height when the edge of its base is constant. The edge of the base of a right square pyramid is 7 cm, its slant height is 14 cm and its lateral surface area is 196 $cm^2$. Find the lateral surface area of a right square pyramid which has the edge of its base as 12 cm and slant height as 20 cm.

(A) 240 $cm^2$ (B) 360 $cm^2$

(C) 480 $cm^2$ (D) 640 $cm^2$

**32.** The sum of the present ages of a woman and her daughter is 60 years. When the woman attains her husband's present age, the ratio of the ages of her husband and her daughter will be 2 : 1. Find the present age (in years) of her daughter.

(A) 10 (B) 15

(C) 20 (D) 25

**33.** The consumption of diesel per hour of a bus varies directly as the square of its speed. When the bus is travelling at 40 km/h its consumption is 1 *l*/h. If each litre costs ₹40 and other expenses per hour cost ₹40, then what would be the minimum expenditure required to cover a distance of 400 km?

(A) ₹600 (B) ₹700

(C) ₹800 (D) ₹900

**34.** If $\frac{ax^2+by^2}{ax^2-by^2} = \frac{77}{13}$ and $\frac{xy^2+ab^2}{xy^2-ab^2} = \frac{17}{7}$, what is the value of $x^2 : b^2$?

(A) 1 : 4 (B) 9 : 4

(C) 2 : 3 (D) 49 : 64

**35.** The cost of a bars of a precious metal varies directly as the square of the weight of the bar. Metal bars of weights in the ratio 4:5:6 were bought from three different places and melted together to form a big bar. The big bar costed ₹4440 more than the total cost of the metal bars. What was the cost of the lightest bar?
(A) ₹750 (B) ₹1080
(C) ₹480 (D) ₹600

***Directions for question 36*:** These questions are based on the data given below.

Thomas and Lala who live in a remote location, use special services under a particular scheme from a special provider in which the bill depends on the number of outgoing calls made and the number of incoming calls received. For every outgoing call made, the charge is ₹2.50 and for every incoming call received the charge is ₹1. In a particular month, the number of calls received by Thomas was 10 more than that received by Lala.

**36.** (a) If the number of outgoing calls made by Lala was 70 and the total number of calls (i.e.,incoming calls received and outgoing calls made) for Lala was 150, what was the bill for Lala?
(A) ₹255 (B) ₹250
(C) ₹245 (D) ₹260
(b) If Thomas received a bill of ₹250 and the number of calls received by Lala is 60, then what is the total number of calls (i.e., incoming calls received and outgoing calls made) for Thomas and Lala together, if the number of calls made by Lala is 20 more than that made by Thomas?
(A) 324 (B) 314
(C) 304 (D) 294

**37.** The cost ($C$) of a certain type of marble varies directly as the square root of its surface area ($A$) when its thickness ($T$) is constant and $C$ varies directly as the square of $T$ when $A$ is constant. If the cost of a piece of a marble of surface area 1600 sq.cm. and thickness 3 cm is ₹360, what is the cost of a marble piece of area 900 sq.cm. and thickness 4 cm?
(A) ₹360 (B) ₹480
(C) ₹540 (D) ₹600

**38.** If the equations $px + qy + r = 0$ and $lx + my + n = 0$ yield a unique solution set for the values of $x$ and $y$, then which of the following is/are necessarily true?
I. $(pm \neq ql)$ II. $(qn \neq rm)$ III. $(rl \neq pn)$
(A) III alone (B) I alone
(C) I, II and III (D) II alone

**39.** If $l$, $m$, $n$ are non-zero and satisfy $l^2 + 8m^2 + 9n^2 = 4m(l + 3n)$, then $l:m:n =$
(A) 1:2:3 (B) 6:3:2
(C) 1:4:5 (D) 3:2:1

**40.** The mean proportional between two numbers is 12. The third proportional of the same numbers is 96. Find the greater of the two numbers.
(A) 18 (B) 24
(C) 12 (D) 36

**41.** Five kg sugar costs the same as 2 $l$ cooking oil. 8 kg rice can be bought with the same amount of money that can buy 3 litres cooking oil. Two kg dal and 5 kg rice cost the same. If 1 $l$ cooking oil costs ₹40, then what is the difference between the price of 1 kg dal and 1 kg sugar?
(A) ₹21.50 (B) ₹22.50
(C) ₹12.50 (D) ₹25

**42.** The time taken by a group of workers to complete a piece of work varies directly with the amount of work to be done by them when the number of workers is constant and inversely as the number of workers in the group when the amount of work is constant. If 8 workers take 1/2 a day to plough 2 acres of a field, find the time taken by 16 workers to plough 8 acres of the field.
(A) 1/2 days (B) 1/4 days
(C) 3/4 days (D) 1 day

**43.** The total surface area of a special cylinder having a certain height is the sum of 2 parts. One of the parts varies directly with its radius. The other part varies directly with the square of its radius. The total surface areas of two cylinders having the same height whose radii are 10 cm and 20 cm are 720 sq.cm and 2640 sq.cm, respectively. Find the total surface area (in sq.cm) of a cylinder of the same height whose radius is 15 cm
_____.

**44.** Four brothers – Ram, Lakshman, Bharath and Shathrugna – bought a car for ₹2,40,000. Ram paid an amount equal to half of the sum of the amounts paid by his three brothers. Shathrugna paid one-fifth of the total amount paid by the other three brothers. The amount paid by Bharath is five-nineteenth of the total amount paid by his three brothers. How much did Lakshman pay?
(A) ₹80,000 (B) ₹40,000
(C) ₹70,000 (D) ₹60,000

**45.** Spice Airline has a free luggage allowance of $f$ kg. If any passenger carries excess luggage, it is charged at a constant rate per kg. The total luggage charge paid by two passengers Mohan and Sohan is ₹1050. If each of Mohan and Sohan had carried twice the luggage he actually did, their luggage charges would have been ₹2400 and ₹900 respectively. Find Mohan's luggage charge
(A) ₹750 (B) ₹900
(C) ₹1200 (D) ₹1500

**46.** Each of four girls, A, B, C and D had a few chocolates. A first gave 1/3rd of the chocolates with her to B. B then gave 1/4th of what she had to C and C then gave 1/5th of what she had to D. Finally, all the four girls had an equal number of chocolates. If A had 80 chocolates more

than B initially, find the ratio between the number of chocolates that C and D initially had _____.

**47.** The weight of a solid right circular cone of a certain material varies directly as the square of its radius when its height is constant and varies directly as its height when its radius is constant. The weight of one such cone is 12 kg, its radius is 2 cm and its height is 4 cm. Find the weight of another such cone whose radius is 4 cm and whose height is 3 cm (in kg) _____.

**48.** A person reads 20 books, each having the same number of pages, completely in 25 days at the rate of 80 pages per day. The number of days he takes to read such books varies directly as the number of books he reads when his reading rate is constant and inversely as his reading rate (in pages/day) when the number of books he reads is constant. How many more such books can he read in 150 days reading 20 pages per day?

(A) 30 (B) 20
(C) 15 (D) 10

**49.** The engine of a railway train can cover a distance of 30 km in 20 minutes, when no wagons are attached to it. When wagons are attached, the decrease in speed is directly proportional to the cube root of the number of wagons attached. When 8 wagons are attached, it covers a distance of 14 km in 12 minutes. What is the speed of the train (in km/h) when 64 wagons are attached to the engine?

(A) 60 (B) 70
(C) 50 (D) 30

**50.** The expenses of a hostel are partly fixed and partly variable, varying with the number of occupants. If there are 20 occupants, then each of the occupants has to bear ₹650 per month and if there are 5 more occupants, then the share of each of the occupants comes down by ₹50 per month. How many occupants were there if the share of each occupant is ₹500?

(A) 35 (B) 40
(C) 45 (D) 50

## EXERCISE-4

***Directions for questions 1 to 25*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

***Directions for questions 1 and 2*:** These questions are based on the following data.

Three persons A, B, and C played four rounds of a game. At the start of the game, B had twice as much money as C and C had one-third as much as A had. It was decided that, if A lost a round he would give 1/4th of the money that he had prior to that round equally to B and C. If B lost, he would give half of his money equally to A and C and if C lost, he would give 2/3rd of his money equally to A and B. B lost the first round, C lost the next round, A lost the third round, and B lost the fourth round. A gained ₹10 over the four rounds.

**1.** What was the total money (in ₹) with the three of them initially? _____

**2.** After which of the four rounds, should the game have ended for A to gain the maximum amount?

(A) Round 1 (B) Round 2
(C) Round 3 (D) cannot be determined

**3.** If $\frac{A+B+C-10}{A+B-C+10} = \frac{A-B+C+10}{A-B-C-10}$, which of the following is true?

(A) $BC = -10A$ (B) $BC = 10A$
(C) $AB = -10C$ (D) $AB = 10A$

**4.** The scores of five candidates Anil, Bala, Chetan, Dinesh, and Eswar in an exam are positive integers. The maximum score that any candidate could have got in the exam is 100. The scores of Anil, Bala, Chetan are in the ratio 11:10:13. The score of either Chetan or Dinesh is 95. The scores of Bala, Dinesh, and Eswar are in the ratio 12:19:18. The total score of the five candidates is _____.

***Directions for questions 5 and 6*:** These questions are based on the following data.

A, B, C, and D are four salesmen. In the first month, they received a total commission of ₹4,200 from their company and divided it in the ratio 2:3:4:5. In the second month, the commission doubled, the amount was divided in the ratio 3:4:5:2. In the third month, the commission tripled when compared to the first month and they shared it in the ratio 4:5:3:2 and in the fourth month the commission became half of that of the previous month and they shared it in the ratio 4:3:5:2.

**5.** What was the average monthly earning of C over the period?

(A) ₹2,287.50 (B) ₹2,552.50
(C) ₹1,600.25 (D) ₹1869.75

**6.** How much more should B have earned so that his average monthly earning would have been ₹3,500?

(A) ₹2,400 (B) ₹3,250
(C) ₹4,850 (D) ₹5,750

**7.** The inhabitants of Warawa island (where the currency is Waras) are taxed in a funny manner. They must pay a fixed sum irrespective of their income level. In addition to this they have to pay a sum which is proportional to the excess of their annual salary over Wr.50,000. Mr. Wakhln pays a total tax of Wr.6,200 per annum when his annual salary is Wr.60,000 and his wife pays a total tax of Wr.7,700 per annum when her annual salary is Wr.75,000. What is the annual salary (in Waras) of Mr. Sangma who pays a total tax of Wr.8,200 per annum?

(A) Wr.80,000 (B) Wr.82,000
(C) Wr.90,000 (D) Wr.81,000

**8.** The price of onions was ₹400 on 1st April of year *X*. Every day, for the next 200 days the price increased by ₹1. After that, the price remained constant till the 31st March of the following year. The price of tomatoes on 1st April of the year *X* was ₹200. Every day after that the price increased by ₹1.60. On which day of the year *X*, were the prices of onion and tomatoes equal? (All the prices given are per quintal)

(A) 6th Dec (B) 7th Dec
(C) 1st Dec (D) 2nd Dec

**9.** A garrison of 900 soldiers had food-stock sufficient for 30 days when the rate of consumption is 2.5 kg/day/soldier. After some days of consumption at that rate, 300 soldiers were transferred to another garrison and the balance food lasted for 25 days for the remaining soldiers. If the rate of consumption of the remaining soldiers was 3.0 kg/day/soldier, after how many days from the start, were the soldiers transferred?

(A) 12 (B) 10
(C) 8 (D) 15

**10.** The amount collected per month from a consumer of electrical power consists of two parts – a fixed charge for providing the service and a variable charge which is directly proportional to the number of units of power consumed. An amount of ₹700 is collected from the consumer when he consumed 50 units in a month. It is also noticed that when the consumption increases from 100 units per month to 200 units per month, the bill amount increases to 5/4 times that of the former. How much is the fixed charge per month?

(A) ₹600 (B) ₹350
(C) ₹500 (D) ₹400

**11.** The rate of cooling of a body is directly proportional to the fourth power of the temperature difference between the surroundings and the body as well as to the surface area of the body. A body in the shape of a cube is melted to form a sphere and during this process the temperature difference of the body and the surroundings increased by 10%. Find the approximate ratio ($R$) of the new rate of cooling to the initial rate of cooling.

(A) $0 < R < 1$ (B) $1 \le R < 2$
(C) $2 \le R < 3$ (D) None of these

***Directions for question 12*:** This question is based on the following data.

Prathima has coins of denominations 25 ps, 50 ps, and ₹1 in the ratio 12:8:5. If she removes 24 of the 25p coins and replaces the equivalent amount in denominations of 50p and ₹1 in ratio of 2:1 by value, she would have a total of $x$ coins. Instead, if she removes 24 of the 50 ps coins and replaces the equivalent amount in denominations of 25ps and ₹1 coins in the ratio 2:1 by value, she would have $2x - 85$ coins.

**12.** (a) How many coins did Prathima have initially?
(b) How many coins will be there if 24 ₹1 coins are removed and the equivalent amount is replaced in denominations of 25 ps and 50 ps in the ratio 2:1.

(A) 181 (B) 149
(C) 165 (D) 137

**13.** The National Agency for Sales and Auctions had put up a lunar rock for sale. The selling price commanded by a piece of lunar rock is proportional to the cube of its weight. Incidentally, an intelligent customer figured out that the single rock was in fact a cleverly crafted ensemble of five smaller rocks whose masses were in the ratio 1:2:3:4:5. When he pointed this out to the manager of sales, he was asked not to divulge the secret to the public. Further the rocks are sold to him at the correct price, with the largest of the five being given for free. If the customer thus saved himself ₹32,750, then what was the price initially quoted?

(A) ₹35,089 (B) ₹38,750
(C) ₹35,750 (D) ₹33,750

**14.** None of the digits of a three-digit number is 0. Exactly two of its digits are equal. $R$ is the ratio of the three-digit number and the sum of its digits. Consider the following statements.

I. $R$ is more than 10.
II. $R$ is less than 82.

Which of the following statement(s) is/are true?

(A) Only I (B) Only II
(C) Both I and II (D) Neither I nor II

**15.** The present age of Alok is more than that of Bala, which is more than that of Chetan. When Bala attains Alok's present age, the ratio of the ages of Alok and Chetan will be 18:13. When Alok was as old as Chetan is, the ratio of the ages of Bala and Chetan was 7:5. Find the ratio of the present ages of Alok and Bala.

(A) 5:4 (B) 4:3
(C) 6:5 (D) 7:6

**16.** Beta Airlines allows every passenger to carry a luggage up to certain weight for free. For each passenger,

it charges at a fixed rate per kg, for excess luggage. Two passengers, Anil and Balu, have a total of 130 kg of luggage. If Anil had half the luggage as he actually had, the total luggage charge paid by the two would have been ₹4500. If Balu had one-third the luggage that he actually had, the total luggage charge paid by the two would have been ₹4200. If the entire luggage was with one passenger, the luggage charge would have been ₹7200 per passenger. It is also known that Anil's luggage is greater than twice the free luggage allowance and Balu's luggage is greater that thrice the free luggage allowance. Find the free luggage allowance of Beta Airlines.

(A) 5 kg (B) 15 kg
(C) 10 kg (D) 20 kg

**17.** *A*, *B*, *C*, *D*, and *E* are positive quantities. A varies directly with $B^2$ when *C*, *D*, *E* are constant. *B* varies inversely with $C^{3/2}$ when *A*, *D*, *E* are constant. *C* varies directly with $D^2$ when *A*, *B*, *E* are constant. *C* varies directly with $E^{8/3}$ when *A*, *B*, *D* are constant. If each of *B*, *C*, *D*, *E* double, the ratio of the old to the new values of *A* is

(A) $2^8$ (B) $2^9$
(C) $\frac{1}{2^8}$ (D) $\frac{1}{2^9}$

***Directions for questions 18 and 19*:** These questions are based on the following data.

A company that produces cricket balls pays its workers in the following manner:

The workers are paid a fixed sum for producing up to 600 balls. If a worker produces more than 600 but less than or equal to 810 balls, an excess amount which is proportional to the excess of balls produced over 600 is paid to them. Worker A is paid ₹24,800 for 680 balls and worker B is paid ₹25,500 for 750 balls.

**18.** How many balls did worker C produce if he was paid ₹25,200?

(A) 600 (B) 120
(C) 960 (D) 720

**19.** If a worker produces in excess of 810 balls he is paid an amount proportional to the square of the excess of 810 balls apart from the usual payment for the first 810 balls. How much must worker D be paid for 830 balls if worker E was paid ₹34,100 for 850 balls (in ₹)?

(A) 28100 (B) 27100
(C) 28000 (D) 21000

**20.** Murali served in the Navy for 9 years more than Mohan and got ₹1500 more pension than Mohan. The pension paid by the Navy is directly proportional to the square root of the number of years of service an employee puts in it. Had Mohan put in $4\frac{1}{4}$ years more service in the Navy than what he actually did, then their pension would have been in the ratio of 10 : 9. How long did Murali serve in the Navy?

(A) 16 (B) 17
(C) 18 (D) 25

***Directions for questions 21 and 22*:** These questions are based on the following data.

A and B had a certain number of marbles with them, not necessarily the same. First A gave 60% of his total marbles to B. Next B gave to A 60% of the total he had then. This process of A giving 60% of his marbles to B and then B giving 60% of his marbles back to A is considered one transaction. The ratio of the number of marbles with A and B after each such transaction remains the same.

**21.** What was the ratio of the total number of marbles with A and B initially?

(A) 2 : 1 (B) 3 : 2
(C) 5 : 2 (D) 4 : 3

**22.** If the difference between the number of marbles with A and B initially was 36, then find the total number of marbles with them.

(A) 48 (B) 54
(C) 60 (D) 84

**23.** The sum of the present ages of P and Q is 84 years. Six years ago, the age of P was twice that of Q. What is the difference between the present ages of P and Q? (in years) _____

**24.** In a chase in the time, a hare takes five leaps a jackal takes four leaps. If the distance covered in four leaps of the hare is equal to that covered in five leaps of the jackal, find the ratio of their speeds.

(A) 4 : 5 (B) 16 : 25
(C) 25 : 16 (D) 1 : 1

**25.** Ajay went to a market to buy a total by 90 apples, oranges and bananas. He bought an equal number of oranges and bananas. The ratio of the number of apples and oranges he bought is 5 : 2. If the price of each orange was equal to that of each apple, he could have skipped the purchase of bananas and instead purchased the same number of apples and oranges as he actually bought for the same total amount. If the prices (in ₹) are all integers, find the minimum possible total expenditure he could have incurred (in ₹).

(A) 120 (B) 130
(C) 140 (D) 150

## EXERCISE-5 Based on Data Sufficiency

***Directions for questions 1 to 20:*** Each question is followed by two Statements, I and II. Answer each question based on the following directions:

Choose (A) If the question can be answered by using one of the statements alone but cannot be answered by using the other statement alone.
Choose (B) If the question can be answered by using either statement alone.
Choose (C) If the question can be answered by using both the statements together but cannot be answered by using either statement alone.
Choose (D) If the question cannot be answered even by using both the statements together.

**1.** Companies X and Y earned profits in 2015. Is the profit of X less than that of Y in 2015?
I. The ratio of the sales of X and Y is 4 : 5.
II. The ratio of the expenditures of X and Y is 3 : 4.

**2.** What is the ratio of the savings of A and B?
I. The incomes of A and B are in the ratio 4 : 5.
II. The ratio of the expenditures of A and B is 4 : 5.

**3.** What is the value of $\frac{7a+9b}{4a+5b}$?
I. $b+a=\frac{1}{2}(6a-b)$
II. $3a+4b=5$

**4.** Find $x:y:z$.
I. $\frac{x}{y}=\frac{y}{z}$ II. $\frac{y}{z}=\frac{z}{x}$

**5.** ₹510 was spent on diesel. What was the total money spent on petrol, diesel and CNG?
I. The amounts required to buy 1 $l$ of petrol, 2$l$ of diesel and 3$l$ of CNG are equal.
II. Equal quantities of each fuel were purchased.

**6.** A bowl contains ₹1, 50 paise and 25 paise coins. What is the total amount in the bowl?
I. The total number of coins in the bowl is 20.
II. The total value of ₹1 coins is ₹6 and the number of 50 paise coins and 25 paise coins are in the ratio 6 : 1.

**7.** The ratio of technical staff and non-technical staff in a company is 3 : 10. What percentage of the staff are graduates?
I. 80% of the technical staff and 40% of the non-technical staff are graduates.
II. The ratio of the number of graduates to the total staff in the company is 32 : 65.

**8.** How many girls are there in the class?
I. The ratio of the number of boys to girls is 5 : 4.
II. If four boys leave the class, the ratio of number of boys to girls in the class will be 1 : 1.

**9.** What is the percentage of students who are not intelligent?
I. The ratio of number of boys and girls is 3 : 2.
II. 20% of the boys and 20% of the girls are intelligent.

**10.** If a cake is distributed among two boys A and B and three girls X, Y, and Z, who received the largest part?
I. X received one-eighth part of the cake, which is two-thirds of B's share.
II. A received twice B's share, which is equal to thrice the share of Y.

**11.** The ratio of the prices of three different types of cars is 3 : 2 : 5. Find the price of the car which is neither the costliest nor the cheapest.
I. The difference between the prices of the costliest and the cheapest cars is ₹30,000.
II. The cheapest car costs ₹20,000.

**12.** Last year, was A's salary more than B's salary?
I. Last year, A's savings were more than B's savings.
II. Last year, the ratio of the expenditures of A and B was the same as the ratio of their savings.

**13.** What is the value of the ratio $p:q$?
I. $3p=2q$.
II. $2p+q=6$.

**14.** Is the number of boys in the class more than the number of girls?
I. The ratio of the number of boys to the total number of students in the class is more than the ratio of the number of girls to the number of boys.
II. The ratio of the number of boys to the number of girls in the class is more than the ratio of the number of girls to the number of students in the class.

**15.** A bag contains a total of 1000 apples, mangoes, and oranges. Which fruit is present in the highest quantity in the bag?
I. The number of apples is $\frac{2}{3}$rd of the total number of mangoes and oranges.
II. The number of mangoes is $\frac{3}{7}$th of the total number of apples and oranges.

**16.** If $c \neq -d$ and $d \neq 0$, is $k=\frac{a}{c}$?
I. $k=\frac{a+b}{c+d}$ II. $k=\frac{b}{d}$

**17.** Find the ratio of the savings of Singhal and Thakur.
I. The ratio of the incomes of Singhal and Thakur is 4 : 3.
II. The ratio of the expenditures of Singhal and Thakur is 3 : 2.

**18.** Is the expenditure of A more than that of B?
I. The incomes of A and B are in the ratio 13 : 10.
II. The savings of A and B are in the ratio 11 : 10.

**19.** The ratio of incomes of A and B is 6 : 5. What is the ratio of savings of A and B?
I. The ratio of expenditures of A and B is 4 : 3.
II. A spends two-thirds of his income.

**20.** Rohit purchased a pen, a book and a sharpener for ₹30. What is the cost of the pen?
I. The price of a book is as much more than the price of the pen as the price of the pen is more than that of the sharpener.
II. The prices of the book, the pen and the sharpener are in the ratio of 3 : 2 : 1.

## ANSWER KEYS

### Exercise-1

1. 20
2. 12
3. 256
4. (D)
5. (B)
6. (A)
7. (B)
8. 320
9. (C)
10. (B)
11. 3000
12. (C)
13. (B)
14. (C)
15. 360
16. (D)
17. (C)
18. (A)
19. 15
20. 1017
21. 12
22. 60
23. (D)
24. (A)
25. (A)
26. (C)
27. (D)
28. (A)
29. (D)
30. (B)
31. (A)
32. 84
33. (C)
34. 8
35. (B)
36. (D)
37. (D)
38. 9
39. (a) (D) (c) (B) (d) (C)
40. (D)
41. (A)
42. (C)
43. (C)
44. 60
45. (D)
46. (D)
47. 25
48. 108
49. (a) 12 (b) 160
50. 6

### Exercise-2

1. (D)
2. (A)
3. (C)
4. (D)
5. 112
6. 31
7. (D)
8. 50
9. (D)
10. (C)
11. 200
12. (A)
13. (D)
14. (B)
15. 7,500, 10,000, 15,000, 30,000
16. 135
17. 3
18. (D)
19. 30
20. 16
21. 16
22. (B)
23. (C)
24. (D)
25. 2.40
26. 1, –0.5
27. (A)
28. 5
29. (A)
30. 1848
31. (C)
32. 12
33. 12,000
34. (C)
35. 2,500
36. 1,400
37. (D)
38. (D)
39. 500
40. (D)
41. (C)
42. (C)
43. 35,000
44. 6
45. (B)
46. (A)
47. 377.50
48. (A)
49. 100
50. (D)

### Exercise-3

1. 2197
2. (D)
3. (B)
4. (A)
5. (D)
6. (D)
7. 1120
8. (D)
9. (D)
10. (A)
11. (B)
12. (D)
13. (C)
14. (A)
15. (D)
16. (B)
17. (B)
18. (D)
19. 1.152
20. (C)
21. (C)
22. (C)
23. (A)
24. (B)
25. (A)
26. 940.8
27. (A)
28. (D)
29. (C)
30. (B)
31. (C)
32. (C)
33. (C)
34. (B)
35. (C)
36. (a) (A) (b) (D)
37. (B)
38. (B)
39. (B)
40. (B)
41. (A)
42. (D)
43. 1530
44. (C)
45. (B)
46. 20
47. 36
48. (D)
49. (C)
50. (D)

### Exercise-4

1. 120
2. (B)
3. (A)
4. 389
5. (A)
6. (C)
7. (A)
8. (B)
9. (B)
10. (A)
11. (B)
12. (a) 125 (b) (A)
13. (D)
14. (A)
15. (A)
16. (C)
17. (B)
18. (D)
19. (A)
20. (D)
21. (C)
22. (D)
23. 24
24. (C)
25. (C)

### Exercise-5

1. (D)
2. (C)
3. (A)
4. (C)
5. (C)
6. (C)
7. (B)
8. (C)
9. (A)
10. (C)
11. (B)
12. (C)
13. (A)
14. (A)
15. (C)
16. (C)
17. (D)
18. (C)
19. (C)
20. (B)

## SOLUTIONS

## EXERCISE-1

**1.** Number of sweets that Rani got $= \frac{4}{7}(35) = 20$.

**2.** Given $F \propto M_1 M_2$ and $F \propto \frac{1}{D^2} \Rightarrow F = \frac{K M_1 M_2}{D^2}$

$\Rightarrow 18 = \frac{K.12}{4} \Rightarrow K = 6 \Rightarrow F = \frac{6 M_1 M_2}{D^2}$

$F = \frac{6.18}{9} = 12$ Newtons

**3.** Let the radius and the area of the circle be represented by $r$ and $A$, respectively. As $A \propto r^2$, $A = Kr^2 \Rightarrow 196 = K.7^2$
$\Rightarrow K = 4A = 4r^2 \Rightarrow A = 4 \times 8^2 = 4 \times 64 = 256$ sq.ft

**4.** Let the numbers be $a$, $b$ and $c$.
Given that $a$, $b$ and $c$ are in the ratio 3:2:4.
$a : b : c = 3 : 2 : 4$
Let, $a = 3x$, $b = 2x$ and $c = 4x$
Given, $(a + b) + (c - b) = 21$
$\Rightarrow a + b + c - b = 21 \Rightarrow a + c = 21$
$\Rightarrow 3x + 4x = 21 \quad \Rightarrow 7x = 21$
$\Rightarrow x = 3$
$a$, $b$, $c$ are $3x$, $2x$, $4x$.
$\therefore$ $a$, $b$, $c$ are 9, 6, 12.

**5.** $2.7p = 0.09q \Rightarrow 30p = q$

$\therefore \frac{q + 6p}{q - 6p} = \frac{36p}{24p} = \frac{3}{2}$.

**6.** Given that $A \propto B$ and $A \propto \frac{1}{C}$.

$A \propto \frac{B}{C} \Rightarrow A = \frac{KB}{C}$

Given that $A = 16$ when $B$ is 28 and $C$ is 7.

So, $16 = K \times \frac{28}{7} \Rightarrow 16 = K \times 4 \Rightarrow K = 4$.

If $B$ is 9 and $C$ is 6, then

$A = K\frac{B}{C} = 4 \times \frac{9}{6} = 6$.

**7.** Let the number of men and women in the conference hall be $5x$ and $4x$, respectively.
When 3 men and 6 women join in the conference, the ratio will become 7 : 6.

$\Rightarrow \frac{5x + 3}{4x + 6} = \frac{7}{6}$

$6(5x + 3) = 7(4x + 6)$
$\Rightarrow 30x + 18 = 28x + 42$
$\Rightarrow 30x - 28x = 42 - 18$
$\Rightarrow 2x = 24 \Rightarrow x = 12$
$\therefore$ Number of men $= 5x = 5 \times 12 = 60$
Number of women $= 4 \times 12 = 48$.

**8.** $a \propto \frac{1}{b}$

$a_1 b_1 = a_2 b_2$
If $a_1 = 8$, $b_1 = 240$ and $a_2 = 6$,

$b_2 = \frac{a_1 b_1}{a_2} = \frac{(8)(240)}{6} = 320$

**9.** The fourth proportional of $a$, $b$, $c$ is given by

$\frac{bc}{a} = \frac{1.6 \times 1.6}{0.8} = 3.2$

**10.** Given that, $\frac{x + y}{2x + y} = \frac{4}{5}$

$\Rightarrow 5(x + y) = 4(2x + y)$
$\Rightarrow 5x + 5y = 8x + 4y \Rightarrow y = 3x$

Now, $\frac{2x + y}{3x + y}$

$= \frac{2x + 3x}{3x + 3x} = \frac{5x}{6x} = \frac{5}{6}$.

**11.** Cost of article 6400.
The ratio of weight of broken pieces 3 : 5.
Let the weights of the broken pieces be $3x$ and $5x$.
Total weight $3x + 5x = 8x$
Cost of the article is proportional to the square of its weight.
$V \propto w^2$
$V = Kw^2$
$V = 6400$
When weight is $8x$,
$6400 = k(8x)^2$
$6400 = k\,64x^2$
$kx^2 = 100$
Sum of the costs of the two broken pieces $= k(3x)^2 + k(5x)^2$
$= 34kx^2$
$= 34 \times 100 = 3400$
Loss incurred = 6400 – 3400 = ₹3000.

**12.** $x \propto y^2$

$\frac{x_1}{y_1^2} = \frac{x_2}{y_2^2}$

$\frac{192}{(8)^2} = \frac{x_2}{(10)^2}$

$x_2 = 300$.

**13.** Let the marks obtained in maths, physics and chemistry be $2x$, $3x$, and $4x$, respectively.
$\therefore$ Total marks $= 2x + 3x + 4x = 9x = 189 \Rightarrow x = 21$
$\therefore$ The marks obtained in maths is $2x = 2 \times 21 = 42$

**14.** $\frac{p}{q}=\frac{3}{4}$

$\frac{5p}{7q}=\frac{5}{7}\left(\frac{3}{4}\right)=\frac{15}{28}$

**15.** Let the shares of A, B and $C$ be $a$, $b$, and $c$, respectively.

$a:b:c=\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$

Let us express each term with a common denominator of the least number divisible by the denominators of each term, i.e.,12.

$a:b:c=\frac{6}{12}:\frac{4}{12}:\frac{3}{12}=6:4:3$

Share of $C=\frac{3}{13}\times 1560=₹360$.

**16.** Number of days left = 45 – 10 = 35

Number of men left = 150 – 25 = 125

$m_1\ d_1=m_2\ d_2$

$150\times 35=125\times x$

$x=42$ days

**17.** Among the given options, 4 : 5 can't be a ratio of boys and girls because 30 is not divisible by (4 + 5) viz 9.

**18.** Let the present ages of Anil and his wife be $a$ years and $w$ years, respectively.

$a+w=88$ (1)

$\frac{a+8}{w+8}=\frac{7}{6}$

(1) $\Rightarrow w=88-a$

$\therefore\ \frac{a+8}{88-a+8}=\frac{7}{6}$

$6a+48=7(6-a)\Rightarrow 6a+48=672-7a$

$\Rightarrow 13a=624\Rightarrow a=48.$

**19.** Let the numbers be $3x$ and $5x$.

$3x+5x=40$

$x=5$

Smaller of the two numbers = $3x=15$

Alternately, smaller number = $\frac{3}{8}(40)=15.$

**20.** Since $x$ is proportional to square of $y$, their ratio is constant i.e.,

$x_1/y^2{}_1=x_2/y_2{}^2$. Substituting the given values, we get

$452/(12)^2=x_2/(18)^2$

$x_2=452\times 18\times 18/(12\times 12)=1017$

**21.** Since $x$ is proportional to square of $y$, we have $x_1y_1{}^2=x_2y_2{}^2$

$\Rightarrow 48\times 4=x_2\times 16\Rightarrow x_2=12.$

**22.** $x=\frac{1}{2}y\Rightarrow\frac{x}{y}=\frac{1}{2}$

$y=\frac{2}{3}z\Rightarrow\frac{y}{z}=\frac{2}{3}$

So, $x:y:z=1:2:3$

So, $z=\frac{3}{6}\times 120=60.$

**23.** Let the number of working days in a month and the salary in that month be $x$ and $y$, respectively.

$y\propto x\Rightarrow y=kx.$

$\Rightarrow 25=k\times 10{,}000$

$10{,}000=k.25$

$k=400$

If $x=26$, $y=x\times k=26\times 400=₹10{,}400$

**24.** $E\ a\ \frac{f}{w}\Rightarrow\frac{E_1}{E_2}=\frac{f_1}{f_2}$

$\frac{3}{E_2}=\frac{30}{20}\Rightarrow E_2=2$ cm

**25.** $a:b:c=(5\times 2):(5\times 3):7\times 3=10:15:21.$

**26.** $P, Q, R, S$ are in proportion $\therefore\ PS=QR.$

Choice (A) $\Rightarrow PS=QR$; Choice (B) $\Rightarrow QR=PS$

$\therefore$ Both (A) & (B) are true

**27.** Let the numbers be $10x$ and $13x$.

$\frac{10x-90}{13x-90}=\frac{7}{10}\Rightarrow 100x-900=91x-630\Rightarrow 9x=270$

$\Rightarrow x=30$

The smaller number = $10x=300$

**28.** $x=A\sqrt[3]{y}$ where $A$ is a constant.

$A=\frac{x}{\sqrt[3]{y}}=\frac{2}{\sqrt[3]{8}}=1$

$\therefore\ x=\sqrt[3]{y}$, when $y=216$, $x=6$

**Alternative Solution:**

$\frac{x_1}{x_2}=\sqrt[3]{\frac{y_1}{y_2}}$ (1)

If $x_1=2$, $y_1=8$ and $y_2=216$, $x_2$ can be found to be 6.

**29.** Given that $a:b=4:1$.

$\frac{a}{b}=\frac{4}{1}$

$a=4b$

$\frac{a-3b}{2a-b^2}=\frac{4b-3b}{2(4b)-b^2}\ (\because\ a=4b)$

$=\frac{b}{8b-b^2}=\frac{1}{8-b}.$

But as we do not know the value of '$b$', the question cannot be answered.

**30.** Let $a+b=3K$ (1)

$b+c=4K$ (2)

and

$c+a=5K$ (3)

$a+b+b+c+c+a=3K+4K+5K$

$2\ (a+b+c)=12K$

$a+b+c=6K$

(1) $\Rightarrow c = 3K$
(2) $\Rightarrow a = 2K$
(3) $\Rightarrow b = K$
$\therefore a:b:c = 2:1:3$

**31.** Let $a = 7x$ and $b = 3x$.
$a + b = 10x$ and $a - b = 4x$.
$a + b : a - b = 10x : 4x = 5:2$.

**32.** Let the ages of Arun, Brahma and Chari be $5x$, $4x$ and $3x$ years, respectively.
$4x = 28 \Rightarrow x = 7$.
Sum of the ages of three persons
$= 12x = 12 \times 7 = 84$ years.

**33.** $P^2 : Q = R : P$ or $\frac{P^2}{Q} = \frac{R}{P}$
$\therefore P^3 = RQ$ and $R = 27Q^2$
$\therefore P^3 = RQ = 27Q^3$
$P = \sqrt[3]{(27Q)^3} = 3Q$
$P : Q = 3:1$.

**34.** Number of gents at the party do not change.
It is $= \frac{4}{7}(28) = 16$.
Initial number of ladies $= (3/7)\ 28 = 12$
After $x$ ladies join, the number of ladies would be $5/4$ (Number of gents) $= 20$
$\therefore x = 20 - 12 = 8$

**35.** Let the number of male employees be $8x$ and the number of female employees be $7x$
$\therefore$ Total number of employees
$= 8x + 7x = 15x = 30 \Rightarrow x = 2$
$\therefore$ Number of male employees is 16 and that of female employees is 14.
If two female employees are recruited, then the total number of female employees will be $14 + 2 = 16$, which is equal to that of male employees. Then, the ratio becomes 16 : 16 i.e., 1 : 1.

**36.** Let $x = 4a$ and $y = 9a$
$\sqrt{x} = 2\sqrt{a}$ and $\sqrt{y} = 3\sqrt{a}$
$$\frac{4\sqrt{x} + 5\sqrt{y}}{6\sqrt{x} + 7\sqrt{y}} = \frac{4(2\sqrt{a}) + 5(3\sqrt{a})}{6(2\sqrt{a}) + 7(3\sqrt{a})} = \frac{23\sqrt{a}}{33\sqrt{a}} = \frac{23}{33}.$$

**37.** $A \propto (B + C)$
Let $A = K(B + C)$ where $K$ is the proportionality constant.
If $B_2 = B_1 + 2$ and $C_2 = C_1 + 4$,
$A_1 = K(B_1 + C_1)$ and $A_2 = K(B_1 + 2 + C_1 + 4)$
$A_2 - A_1 = 6K$
As $K$ is unknown, $A_2 - A_1$ cannot be found.

**38.** Let the numbers be $2x$, $3x$ and $5x$. Given,
$2x \times 5x = 10x^2 = 90 \rightarrow x^2 = 9 \rightarrow x = 3$
Therefore, the difference between the largest and the smallest numbers.
$= 5 \times 3 - 2 \times 3 = 9$

**39.** (a) Duplicate ratio of $a:b$ is $a^2:b^2$
$\Rightarrow$ Duplicate ratio of 3 : 4 is $3^2 : 4^2 = 9:16$.
(b) Triplicate ratio of $a:b$ is $a^3:b^3$
$\Rightarrow$ Triplicate ratio of 2 : 3 is $2^3 : 3^3 = 8:27$.
(c) Sub-Duplicate ratio of $a:b$ is $\sqrt{a} : \sqrt{b}$
$\Rightarrow$ Sub-duplicate ratio of 16 : 9 is $\sqrt{16} : \sqrt{19} = 4:3$.
(d) Mean proportion of $a$, $b$ is $\sqrt{ab}$.
$\Rightarrow$ Mean proportion of 16, 4 is $\sqrt{16 \times 4} = 8$.

**40.** $x : y = 3:7$ (1)
$y : 3 = 7:4$ (2)
from (1) and (2) we have
$x : y : z = 3:7:4$, since $y = 7$ in both the ratios.
$\therefore x : z = 3:4$

**41.** $\frac{a+b}{a-b} = \frac{3}{2} \Rightarrow 2a + 2b = 3a - 3b$
$5 = \frac{a}{b} \Rightarrow \frac{a}{b} = \frac{5}{1} \Rightarrow a:b = 5:1$

**42.** $A \propto B.C \Rightarrow A = k.BC$
$300 = k(20)(50) \Rightarrow k = \frac{3}{10}$
When $A = 900$, $C = 60$
$900 = \frac{3}{10} \times B \times 60 \Rightarrow B = 50$

**43.** Choice (A)
If $x$ varies directly with $\sqrt{y}$, $\frac{x}{\sqrt{y}} =$ constant.
Let this constant be $k$.
$$\frac{x}{\sqrt{y}} = k$$
Squaring both sides, $\frac{x^2}{y} = k^2$
$k^2$ is a constant.
$\frac{x^2}{y} =$ a constant.
$\therefore x^2$ varies directly with $y$.
Choice (A) must be true.
Choice (B)
If $\sqrt{x}$ varies inversely with $\sqrt{\frac{1}{y}}$, $\sqrt{x}\sqrt{\frac{1}{y}} =$ a constant.
Let this constant be $k$.
$$\frac{\sqrt{x}}{\sqrt{y}} = k;\ \frac{\sqrt{y}}{\sqrt{x}} = \frac{1}{k}$$
$$\sqrt{y}.\left(\frac{1}{\sqrt{x}}\right) = \frac{1}{k}$$
$\frac{1}{k}$ is a constant.

$\therefore \sqrt{y}.\left(\frac{1}{\sqrt{x}}\right)$ = a constant.

$\therefore \sqrt{y}$ varies inversely with $\sqrt{\frac{1}{x}}$.

Choice (B) must be true.

**44.** Let the three numbers be $x$, $3x$, $5x$.
Given, $x + 3x + 5x = 108 \Rightarrow x = 12$
$\therefore$ The largest number is $5x = 5 \times 12 = 60$

**45.** Let the monthly salaries of X and Y be ₹$3x$ and ₹$4x$, respectively. Let the monthly expenditures of X and Y be ₹$4y$ and ₹$5y$, respectively.
Ratio of the monthly savings of X and Y

$$= \frac{3x - 4y}{4x - 5y} = \frac{3\frac{x}{y} - 4}{4\frac{x}{y} - 5}.$$

As $\frac{x}{y}$ is unknown, the ratio cannot be found.

**46.** $a = \frac{50b}{100} = \frac{b}{2}$

$b = \frac{25c}{100} = c/4 \therefore c = 4b.$

$a : c = \frac{b}{2} : 4b = 1 : 8$

**47.** Let the two positive numbers be $5x$ and $8x$, respectively.
$8x - 5x = 15$
$3x = 15 \Rightarrow x = 5.$
$\Rightarrow$ Smaller number $= 5x = 25.$

**48.** $P \propto \frac{1}{QR}$

$$\frac{P_1}{P_2} = \frac{Q_2R_2}{Q_1R_1}$$

If $P_1 = 75$, $Q_1 = 6$, $R_1 = 12$, $Q_2 = 5$

and $R_2 = 10$, $P_2 = \frac{P_1Q_1R_1}{Q_2R_2}$.

$= \frac{(75)(6)(12)}{(5)(10)} = 108.$

**49.** Mean proportional of two numbers $a$ and $b$ is defined as $\sqrt{ab}$.
(a) $a = 6$, $b = 24$
Mean proportional $= \sqrt{6(24)} = 12.$
(b) $a = 50$, $b = 512$
Mean proportional $= \sqrt{(50)(512)} = 160.$

**50.** Let the present ages of Rohit and Sunil be $3x$ years and $5x$ years, respectively. The ratio of their ages 10 years hence $= \frac{3x + 10}{5x + 10} = \frac{4}{5}$

$15x + 50 = 20x + 40$
$2 = x$
Therefore, Rohit's present age is 6 years.

## EXERCISE-2

***Solutions for questions 1 to 34:***

**1.**

| Men ($M$) | Days ($D$) | Hours/ Day | Work |
|---|---|---|---|
| 18 | 30 | 10 | $W$ |
| 25 | $x$ | 12 | 5W |

Let the work done by 18 men be denoted by $W$.
The work to be done varies directly with the number of men and the time for which they work.
Hence, $M_1D_1H_1/M_2D_2H_2 = W_1/W_2$ (18) (30) (10)/(25) ($x$) (12) = 1/5, $x = 90$.

**2.** The cost of 3 kg of tea = The cost of 6 kg of sugar
$T/S = 6/3$
Similarly, sugar/oil = 21/24
Oil/ghee = 12/56
Tea/sugar × sugar/oil × oil/ghee
Tea/ghee = 6/3 × 21/24 × 12/56 = 3/8
3 × ghee = 8 × tea
3 × ghee = 8 × 72 = ₹576

**3.** The marks are in the ratio 12:13:14:15:16:17.
Let the marks be $12x$, $13x$, $14x$, $15x$, $16x$, and $17x$.
Then the sum of the marks in all the papers together
$= 12x + 13x + 14x + 15x + 16x + 17x = 87x$
But given average of the total works obtained = 58%.
Let the full marks for a paper be 100
Then, $87x = 58$ (6) $\Rightarrow x = 4$
$\therefore$ The marks obtained in all 6 papers are 48, 52, 56, 60, 64, 68. He got more than 55% of the full marks in 4 papers.

**4.** Let $V = K\sqrt{w}$ where $K$ is a proportionality constant.
Let the weights of the pieces be $x$, $4x$ and $4x$ units.
Gain $= K\left(\sqrt{x} + \sqrt{4x} + \sqrt{4x} - \sqrt{x + 4x + 4x}\right)$
$= 2K\sqrt{x} = 12000$
Initial value $= 3K\sqrt{x} = 18000$

**5.** Nine farmers take six hours to plough 18 acres.
$\therefore$ Rate of work of each farmer $= \frac{18}{9(6)}$ acre/hr $= \frac{1}{3}$ acre/hr.

Number of acres 16 farmers can plough in 21/hours

$= \frac{1}{3}(16)(21) = 112$

**6.** Since $a:b = 4:5$, let $a = 4x$; $b = 5x$

$\therefore \frac{4a+3b}{3a+2b} = \frac{4(4x)+3(5x)}{3(4x)+2(5x)}$

$= \frac{16x+15x}{12x+10x} = \frac{31x}{22x} = 31/22$

**7.** $\frac{x}{y} = \frac{6}{7}$

Let $x = 6k \therefore y = 7k$

$\therefore \frac{3x^2+4y^2}{2x+3y} = \frac{3(6k)^2+4(7k)^2}{2(6k)+3(7k)} = \frac{3(36)+4(49)}{2(6)+3(7)}k$

This depends on $k$. $\therefore$ The given expression cannot be uniquely determined.

**8.** Number of boys $= \frac{8}{13} \times 650 = 400$

$\therefore$ Number of girls = 250

Let $x$ more girls be admitted

$\therefore \frac{400}{250+x} = \frac{4}{3}$

$\Rightarrow 1200 = 1000 + 4x \Rightarrow x = 50$

**9.** Let the temperature of the body be $T$°C and the heat radiated by the body in 1 second be $H$ joules.

$$H \propto \sqrt{T-25} \Rightarrow H = K\sqrt{T-25}$$

Where $K$ is the proportionality constant.

When $T = 34$, $H = 12$ $\therefore K = \frac{H}{\sqrt{T-25}} = \frac{12}{\sqrt{9}} = 4$

$\therefore$ Let us assume the new temperature is $T$° c

$\therefore K = 4 = \frac{20}{\sqrt{T-25}} = 4$

$\Rightarrow 20/4 = 5 = \sqrt{T-25} \Rightarrow T - 25 = 25$

$\Rightarrow T = 50$

**10.** Substituting $b$ with $a + c$ in the equation $3a + 2b = 7c$ and simplifying, we get $a = c$.

Substituting $a = c$ in one of the two equations, we get $b = 2c$

Hence, Choice (C) follows.

**11.** Let the number of 10 paise, 20 paise, 50 paise coins in the bag be $5x$, $2x$, and $x$, respectively.

$\therefore 5x(10) + 2x(20) + x(50) = 14000$

$140x = 14000 \Rightarrow x = 100$

Number of 20 paise coins $= 2x = 200$.

**12.** Let the incomes of A and B be ₹$5x$ and ₹$6x$, respectively, and their expenses be ₹$7y$ and ₹$10y$, respectively.

Let the total income of A and B be $22k$, and the total expenditure of A and B be $17k$.

Given, $5x + 6x = 22k$

and $7y + 10y = 17k$

$\Rightarrow x = 2k$ and $y = k$

$\therefore$ Required ratio

$= (5x - 7y):(6x - 10y)$

$= (5(2k) - 7k):(6(2k) - 10k)$

$= 3k:2k = 3:2$

**13.** Given $\frac{p+q}{r} = \frac{q+r}{p} = \frac{p+r}{q} = k$

we know that If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$,

each of these ratios is equal to $\frac{a+c+e}{b+d+f}$

$\therefore k = \frac{p+q+q+r+p+r}{p+q+r}$

$(p+q+r)k = (p+q+r)2$ (1)

$\therefore (p+q+r)(k-2) = 0$ (2)

$\Rightarrow p+q+r = 0$ or $k = 2$

If $p+q+r = 0 \Rightarrow p+q = -r$

$\frac{p+q}{r} = -1 = k$

$\therefore k = -1$ or $k = 2$ from (2)

$\therefore$ There are two solutions.

**NOTE**

Generally, we tend to cancel the expression $(p + q + r)$ on LHS and RHS of (1), thereby we lose a case of $p + q + r = 0$, where we get $k = -1$.

**14.** Let the number of commodities sold be $3x$, $4x$ and $5x$ and the prices of the commodities be $3y$, $4y$ and $5y$.

Total sales proceeds

$= 3x \times 3y + 4x \times 4y + 5x \times 5y$

$= 9xy + 16xy + 25xy = 50xy$

The sale proceeds of the commodity which fetches the maximum sales revenue $= 25xy$

Its share of the total sales proceeds $= (25xy/50xy) \times$ ₹4000

= ₹2000

**15.** If we assume the shares of first son, second son, third son, fourth son are $p$, $q$, $r$, $s$, respectively, it follows that

$4p = 3q = 2r = s$

Individually considering we get

$4p = s \Rightarrow p = s/4$

$3q = s \Rightarrow q = s/3$

$2r = s \Rightarrow r = s/2$

We know that the total of shares of all four sons is equal to ₹62500

$\therefore p+q+r+s=62500$
$\Rightarrow s/4+s/3+s/2+s=62500$
$\Rightarrow s[1/4+1/3+1/2+1]=62500$
$\Rightarrow s[3+4+6+12/12]=62500$
$\Rightarrow s=62500\times 12/25$
$\therefore s=30000$
The shares of the four sons are as follows
$p=₹7500$; $q=₹10000$; $r=₹15000$; $s=₹30000$
**Alternative Solution:**
$4p=3q=2r=s$
Dividing by L.C.M of 1,2,3,4, i.e., 12 throughout, we get
$p/3=q/4=r/6=s/12$
Thus, $p:q:r:s=3:4:6:12$.
So, $p=3/(3+4+6+12)$ of the total amount $=3/25$th of $62500=7500$ and so on.

**16.** Let $D=Kt^2$ where $K$ is a proportionality constant.
Distance that the body fell in the 10th second
= Distance that it fell in the first 10 seconds – Distance that it fell in the first 9 seconds
$=K(10^2-9^2)=19K=95$, $K=5$
Required distance $=K(14^2-13^2)=27K=135$ m.

**17.** Let the quantities of milk in the glasses be $3x$ ml, $4x$ ml, $5x$ ml, $6x$ ml, and $7x$ ml. Let the volume of each glass be 100 ml. Total volume of the glasses = 500 ml
$3x+4x+5x+6x+7x=0.6(500)$
$\Rightarrow x=12$
The quantities of milk in the glasses are 36 ml, 48 ml, 60 ml, 72 ml and 84 ml
i.e., each of 3 glasses are filled with milk to at least 50% of their capacity.

**18.** Let the length of the smaller part be a cm and the length of the string be $l$ cm.
$\therefore$ Larger part = 4 cm.
Given that, $l/a=20a/4$
$\Rightarrow l=5a^2$
$\Rightarrow l/a^2=5:1$

**19.** Let us assume the bag has only ₹33 (10 + 15 + 8).
The value of one-rupee coins $\Rightarrow$ ₹10, i.e., 10 coins
The value of 50ps. coins $\Rightarrow$ ₹15, i.e., 30 coins
The value of 25ps. coins $\Rightarrow$ ₹8, i.e., 32 coins
$\therefore$ Number of 50ps. coins are 30 [$\because 10+30+32=72$]
Alternately,
Ratio of number of coins $=\dfrac{\text{value}}{\text{face value}} \Rightarrow$ Ratio of number of coins of 1₹, 50 paise, 25 paise $=\dfrac{10}{100}:\dfrac{15}{50}:\dfrac{8}{25}$
$=10:30:32$.
$\therefore$ Number of 50 paise coins $=\dfrac{30(72)}{10+30+32}=30$

**20.** Let the power of the machine and the maximum weight of the object be $p$ and $w$, respectively.

$p\propto\sqrt{w}\Rightarrow p/\sqrt{w}=k$ (1)
$p_1\propto\sqrt{w_1}\Rightarrow p_1/\sqrt{w_1}=k$ (2)
Equating both the values of $k$,
$p/\sqrt{w}=p_1/\sqrt{w_1}$
$18/\sqrt{9}=24/\sqrt{w_1}$
$\Rightarrow\sqrt{w_1}=24\times 3/18=4$
$w_1=4^2=16$ units

**21.** Let $x$ be the number $\dfrac{22-x}{37-x}=\dfrac{2}{7}$
$\Rightarrow 7(22-x)=2(37-x)$
$\Rightarrow 154-7x=74-2x$
$\Rightarrow 5x=80\Rightarrow x=16$

**22.** The data is tabulated below.

| | 10 years ago | Present | 5 years from now |
|---|---|---|---|
| Woman | $3a$ | | $3a+15$ |
| Daughter | $2a$ | | $2a+15$ |

If $\dfrac{N}{D}>1$, then for all $k>0$, $\dfrac{N}{D}>\dfrac{N+k}{D+k}>1$.
$\therefore$ The ratio of the future ages can be $\dfrac{6}{5},\dfrac{8}{7},\dfrac{11}{9}$ but it cannot be $\dfrac{7}{3}\left(\because \dfrac{7}{3}>\dfrac{3}{2}\right)$

**23.** Given $ad=bc$
$\therefore \dfrac{(a-b)(a-c)}{a}=\dfrac{a^2-a(b+c)+bc}{a}$
$=\dfrac{a^2-a(b+c)+ad}{a}$ ($\because ad=bc$)
$=a-(b+c)+d$

**24.** Given $\dfrac{a}{b}=\dfrac{3}{4}$, $\dfrac{b}{c}=5$, $\dfrac{c}{d}=\dfrac{4}{3}$, $\dfrac{d}{e}=2$ and $\dfrac{e}{f}=\dfrac{1}{4}$
We can express all the unknowns in terms of $f$.
$\dfrac{a}{b}.\dfrac{b}{c}.\dfrac{c}{d}.\dfrac{d}{e}.\dfrac{e}{f}=\dfrac{3}{4}.5.\dfrac{4}{3}.2.\dfrac{1}{4}$
$\dfrac{a}{f}=\dfrac{5}{2}\Rightarrow a=\dfrac{5f}{2}$
$b=\dfrac{4}{3}a=\dfrac{4}{3}.\dfrac{5}{2}f=\dfrac{10f}{3}$
$c=\dfrac{b}{5}=\dfrac{10}{3\times 5}f=\dfrac{2}{3}f$
$d=\dfrac{3c}{4}=\dfrac{3}{4}.\dfrac{2}{3}f=\dfrac{1}{2}f$
$e=\dfrac{1}{4}f$
$\therefore \dfrac{def}{abc}=\dfrac{\frac{1}{2}f.\frac{1}{4}.f.f}{\frac{5f}{2}.\frac{10f}{3}.\frac{2f}{3}}=\dfrac{9}{400}$

**25.** Let the fixed amount be $F$ and the variable amount be $KB$, where $K$ is a proportionality constant.
Total income = $F + KB$
$22000 = F + 6000K$ (1)
$46000 = F + 18000K$ (2)
Solving (1) and (2), $K = 2$ and $F = 10000$

$$\text{Required income} = \frac{10000 + 25000(2)}{25000} = ₹2.40$$

**26.** Let each of the ratios be equal to $k$.
$p = k(q + r - p)$, $q = k(p + r - q)$, $r = k(p + q - r)$
Adding these, $(p + q + r)(1 - k) = 0$.
$p + q + r = 0$ or $k = 1$
If $p + q + r = 0$, $q + r = -p$.

$$\therefore k = \frac{p}{-p - p} = \frac{-1}{2}.$$

$$k = 1 \text{ or } \frac{-1}{2}.$$

**27.** In total, there are 16 parts (5 + 11) of land and the second man was to get 11 parts. But they share the land in the ratio of 11:5. So, the second man loses 6 parts. That is, he loses 6/16 = 3/8

**28.** Let $V$ be the volume, $a$ = area of the base and $h$ = height.
$V = mah$ ($m$ is proportionality constant)
we know $a = 64$, $h = 10$ and $V = 640$
$640 = m.\ (64)\ (10)$

$$\Rightarrow m = 1 \ \therefore V = ah$$

Therefore, $360 = 72 \times h$

$$h = \frac{360}{72} \Rightarrow h = 5 \text{ ft}$$

Hence, height of the cylinder is 5 ft.

**29.** Time ($T$) $\propto$ Distance ($D$) when capacity($C$) is constant and $T \propto \frac{1}{C}$ when D = constant.

$$\therefore T = K\frac{D}{C} \text{ where } k \text{ is a constant, i.e., } \frac{TC}{D} = K$$

$$TC/D = T_1C_1/D_1$$

$$\frac{16 \times 1500}{800} = \frac{T_1 \times 1200}{600}$$

$$T_1 = \frac{16 \times 1500}{800} \times \frac{600}{1200}$$

$T_1$=15 hours

**30.** $v \propto r^2$ when $h$ is constant (1)
$v \propto h$ ($r^2$ is constant and hence) when $r$ is constant (2)
From (1) and (2) $v \propto r^2h$
$\therefore v/r^2h$ = constant
$\therefore v_1/r_1^2h_1 = v_2/r_2^2h_2$
$386/(7)^2 \times 9 = v_2/(14)^2 \times 3$

$$\Rightarrow v_2 = \frac{1386 \times 3 \times 14 \times 14}{9 \times 7 \times 7}$$

$\therefore v_2 = 1848$ cubic units.

**31.** The ratio of the number of sparrows is:
Left branch : Right branch = 7 : 13
The sparrows are to be equally distributed, by the transfer of sparrows from the right to the left. The existing difference in the ratio – numbers = 13 – 7 = 6
For $E$ or equal distribution, half of this difference shall be transferred to the left (1)
It is known that, in reality, 12 sparrows were transferred to the left (2)
From (1) and (2), the conclusion is that, 3 parts of the ratio 7 : 13 is equal to 12;

$$\Rightarrow 1 \text{ part of the ratio} = \frac{12}{3} = 4 \quad (3)$$

Sparrows on the left branch, before transfer = 7 parts of the ratio = $7 \times 4 = 28$ (from (3)).

**32.** $x = 8(2a + b)$, $y = 8(2b + c)$, $z = (2c + a)$
$\Rightarrow x + y + z = 8(3(a + b + c))$
$\Rightarrow x + y + z = 24(a + b + c)$ (1)
if $x/2a = y/2b = z/2c$ = k
$\Rightarrow x + y + z = 2k(a + b + c)$ (2)
$\Rightarrow$ From (1) and (2),
$24(a + b + c) = 2k\ (a + b + c)$
$(a + b + c)\ (24 - 2k) = 0$
$a + b + c \neq 0$
$\therefore 24 - 2k = 0$, i.e., $k = 12$

**33.** Let the incomes of Varun and Vikram be $8x$ and $3x$, respectively.
As income – savings = expenditure $\frac{8x - 2000}{3x - 2000} = \frac{4}{1}$
$\Rightarrow 8x - 2000 = 12x - 8000$
$\Rightarrow 6000 = 4x$
$\Rightarrow x = 1500$
Income of Varun = $8x$ = ₹12000.

**34.** $\frac{ka}{b+c} = \frac{kb}{c+a} = \frac{kc}{a+b} = l$

$$\Rightarrow \frac{ka}{b+c} = l; \frac{kb}{c+a} = l; \frac{kc}{a+b} = l$$

So, $ka = lb + lc$, $kb = lc + la$, $kc = la + lb$
Now, $k(a + b + c) = l(b + c + c + a + a + b)$
$\Rightarrow k(a + b + c) = 2l(a + b + c)$
$(k - 2l)\ (a + b + c) = 0$
As $a + b + c \neq 0$, $k - 2\ l = 0$

$$\therefore l = \frac{k}{2}$$

**35.** Number of students in college A = 2000 + 500 = 2500

Difficulty Level-2: Moderate

**36.** Number of girls in college A $= \frac{2}{5} \times 2500 = 1000$

Number of girls in college B $= \frac{1}{5} \times 2000 = 400$

$\therefore$ Total number of girls in the two colleges together $= 1000 + 400 = 1400$

**37.** $\frac{a}{b} = \frac{b}{c} = k$ (say)

$\frac{a}{c} = \left(\frac{a}{b}\right)\left(\frac{b}{c}\right) = k^2$

Choice A: $\frac{a^2}{b^2} = \left(\frac{a}{b}\right)^2 = k^2$

Choice B: $\frac{a^2+b^2}{b^2+c^2} = \frac{(bk)^2+b^2}{(ck)^2+c^2} = \frac{b^2(k^2+1)}{c^2(k^2+1)} = \frac{b^2}{c^2} = k^2$

**38.** Let their present ages be $5x$ years and $4x$ years, respectively.

Ratio of their ages 20 years ago

$= \frac{5x-20}{4x-20} = \frac{5(x-5)+5}{4(x-5)}$ which is more than $\frac{5}{4}$.

Only choice (D) violates this condition.

**39.** Let the fixed part be $f$ and the variable part per channel be $v$.

Given, $f + 30v = 350$ and $f + 50v = 450$

Solving these, $v = 5$ and $f = 200$

Required expression $= f + 60v = 200 + 60(5) = ₹500$

**40.** Let the quantities of milk in the 1st to 5th vessels be $4x$, $5x$, $6x$, $7x$ and $8x$, respectively.

Total quantity of milk in the vessels is $30x$

Total capacity of the 5 vessels $= 30x(100/75) = 40x$

Capacity of each vessel is $8x$.

64% of $8x = 5.12x$.

The number of vessels which contain at least $5.12x$ of milk is three.

**41.** $3x + y - 5z = 0$ (1)

$4x + 5y - 14z = 0$ (2)

Multiplying (1) by 5 and then

Subtracting (2) from it, we get $11x = 11z$

$\Rightarrow x = z$.

From (1), $y = 2z$

$\therefore x : y : z = 1 : 2 : 1$

**42.** After 4 weeks, 500 more men joined the garrison. If no man joined the garrison, then the 2000 men can survive for 16 (= 20 – 4) weeks at the rate of 2.5 kg per day per man.

The number of men $(M) \propto \frac{1}{\text{Weeks}(W)}$

The number of men $(M) \propto \frac{1}{\text{Rate}(R)}$

$M \propto \frac{1}{W \times R}$

$\frac{M_1}{M_2} = \frac{W_2}{W_1} \times \frac{R_2}{R_1}$

$\frac{2000}{2500} = \frac{x}{16} \times \frac{20}{25}$

$\Rightarrow x = \frac{2000 \times 16 \times 2.5}{2500 \times 2} = 16$ weeks

**43.** Let the sum to be distributed be $x$.

A's share in the initial case = $3x/8$ and that in the second case = $6/15(x)$.

Given, $2x/5 - 3x/8 = 3{,}000 \Rightarrow x/40 = 3{,}000$

$\Rightarrow x = 120000$

Hence, difference in the shares of B

$= 5/8(120000) - 5/15(120000) = ₹35{,}000$.

**44.** $V \propto r^3 \Rightarrow V = kr^3$

$V_1 = k(3)^3 = 27k$

$V_2 = k(4)^3 = 64k$

$V_3 = k(5)^3 = 125k$

$\therefore$ Volume of the resulting sphere

$= V_1 + V_2 + V_3 = (27 + 64 + 125)\ k = 216k$

$\therefore kr^3 = 216k \Rightarrow r = 6$

**45.** The ratio of erasers and pencils bought = 5 : 3

The ratio of erasers, pencils and total articles of the lot $= 5 : 3 : (5 + 3) = 5 : 3 : 8$

The ratio of the combined lot and geometry boxes = 24 : 2

By combining the above two ratios,

The ratio of erasers, pencils, and geometry boxes $= 15 : 9 : 2$

The total number of articles bought, as per ratio $= 15 + 9 + 2 = 26$

The number of pencils for a total of 104 articles

$= \frac{9}{26} \times 104 = 36$

**46.** As the ratio of incomes of A and B is 3 : 4, let their incomes be $3x$ and $4x$, respectively.

As ratio of expenses of A and B is 2 : 3, let their expenses be $2y$ and $3y$, respectively.

Hence, the savings of A and B are $3x - 2y$ and $4x - 3y$, respectively.

A's savings as a fraction of his income = $(3x - 2y)/3x$

B's savings as a fraction of his income = $(4x - 3y)/4x$

Between $(3x - 2y)/3x$ and $(4x - 3y)/4x$, as $3y/4x$ is greater than $2y/3x$, clearly A's savings as a fraction of his income are more than that of B.

**47.** Let the fixed tariff be $F$ and the number of outgoing calls be $C$. If $C > 50$, the charge is in the form $K(C - 50)$.

$300 = F + K(48)$ (1)
$450 = F + K(168)$ (2)
Subtracting (1) from (2), $150 = 120\ K$
$K = \frac{5}{4}$
Required monthly bill $= F + K(110)$
$= F + 48K + 62K$
$= 300 + 62\left(\frac{5}{4}\right) = ₹377.50$

**48.** Let the distance travelled by the body be $d$ m in $t$ seconds.
$d \propto t^2$
Hence, $d = kt^2$ where k is the proportionality constant.
$k = d/t^2 = 64/4^2 = 4$
Distance travelled by the body in the 5th second = distance travelled by the body in the first 5 seconds – distance travelled by the body in the first 4 seconds $= k(5)^2 - k(4)^2$
$= 9\ k$ or 36 m.

**49.** If 10 is mean proportional of $A$ and $B$, then $A$, 10, $B$ are in continued proportion.
Hence, $10^2 = A*B$; so $A = 10^2/B$
If 10000 is the third proportional of $A$ and $B$, then $A$, $B$, 10000 are in continued proportion. Hence, $B^2 = A*10000$
So, $B^2 = (10^2/B)*(10000)$
$B^3 = 10^6$
$B = 100$
Since $A = 10^2/B$, $A = 1$
So, larger of $A$ and $B$ is 100.

**50.** Since the ratio of the incomes of A and B is 5 : 4, let us assume A's income as $5x$ and B's income as $4x$.
Ratio of their expenditures is 3 : 2 Let their expenditures be $3y$ and $2y$.
Savings = Income – expenditure
Savings of A/Savings of B $= 5x - 3y/4x - 2y$ (1)
We know that savings of B = 1/3 (B's income)
$\Rightarrow 4x - 2y = 1/3\ (4x)$
$\Rightarrow 8x = 6y$
$x = \frac{3}{4}y$ (2)
From (1) and (2),
Savings of A/savings of B $= 5\left(\frac{3}{4}\right)y - 3y/4\left(\frac{3}{4}\right)y - 2y$
$= \frac{3}{4}$
$\therefore$ Ratio of savings = 3 : 4
**Alternative Solution:**
**Step 1:**

| | A | B |
|---|---|---|
| Income | $5x$ | $4x$ |
| Savings | | $4x/3$ |
| Expenditure | | $8x/3$ |

(As expenditure = income – savings)
**Step 2:**

| | A | B |
|---|---|---|
| Expenditure | $12x/3$ | $8x/3$ |

(As the ratio is 3 : 2)
**Step 3:**

| | A | B |
|---|---|---|
| Income | $5x$ | $4x$ |
| Expenditure | $12x/3$ | $8x/3$ |
| Savings | $3x/3$ | $4x/3$ |

The ratio of savings of $A$ and B = 3 : 4.

## EXERCISE-3

**1.** Given $a : b = 2 : 3$, $b : c = 6 : 5$ and $c : d = 10 : 13$. Making the $b$ term equal in $a : b$ and $b : c$, we get a : b : c = 4 : 6 : 5
Making the $c$ term equal in $a : b : c$ and $c : d$, we get $a : b : c : d$ = 8 : 12 : 10 : 13
As $e = 2d$, $a : b : c : d : e = 8 : 12 : 10 : 13 : 26$.
Let $a = 8k$, $b = 12k$, $c = 10k$, $d = 13k$, $e = 26k$

$$abc : ed^2 = \frac{(8k)(12k)(10k)}{(26k)(13k)^2} = \frac{960k^3}{13^2(2)k^3} = \frac{480}{2197}$$

**2.** $p = 3q = 4r \Rightarrow 2p = 6q = 8r$ (1)
$2q = 5s = 12t \Rightarrow 6q = 15s = 36t$ (2)
From (1) and (2), $2p = 6q = 8r = 15s = 36t$
Let $k$ = L.C.M (2, 4, 8, 15, 36) = 360
and
let $\frac{2p}{360} = \frac{6p}{360} = \frac{8r}{360} = \frac{15s}{360} = \frac{36t}{360} = c$
$\Rightarrow p = 180c$, $q = 60c$, $r = 45c$, $s = 24c$, $t = 10c$
Now going from the choices only the pair $\left(\frac{p}{8}, \frac{s}{r}\right)$ contains a number that can never be an integer as it equals $\left(\frac{45c}{2}, \frac{24}{45}\right)$.

**3.** Let the constant be $A$, $Q = A + B\sqrt{y} + C\sqrt[3]{y}$ where $B$ and $C$ are the proportionality constants.
$90 = A + B + C$ (1)
$450 = A + 8B + 4C$ (2)
$1270 = A + 27B + 9C$ (3)
From (2) –4 (1) and (3) –9 (1), we get,
$4B - 3A = 90 - (4)$ and $9B - 4A = 230 - (5)$
From 4 (5) – 9 (4), we get $11A = 110 \Rightarrow A = 10$

**4.** Let the salesman's fixed salary be ₹$x$
and the incentive per unit sold be ₹$y$.
$\therefore x + 150y = 8000$ (1)
$x + 200y = 9000$ (2)
(2) – (1)
$\Rightarrow 50y = 1000 \Rightarrow y = 20$
$\therefore x = 9000 - 200\ (20) = 9000 - 4000 = 5000$
$\therefore$ His total income when he sold 400 units
$= x + 400y = 5000 + 400\ (20) = 13000.$

His income per unit $= \dfrac{13000}{400} = ₹32.5$

**5.** $A \propto B + C \Rightarrow A = k(B + C)$
Also, $B \propto x \Rightarrow B = k_1 x$
and, $C \propto 1/x$

$\Rightarrow C = \dfrac{k_2}{x}$

$\therefore A = kk_1 x + \dfrac{kk_2}{x}$

$= p_1 x + \dfrac{p_2}{x}$ (where $p_1$ and $p_2$ are constants)

$\Rightarrow 2p_1 + \dfrac{p_2}{2} = 6$

$\Rightarrow 4p_1 + p_2 = 12$ (1)

and, $4p_1 + \dfrac{p_2}{4} = 9$

$\Rightarrow 16p_1 + p_2 = 36$ (2)

Solving (1) and (2), we get; $p_1 = 2$ and, $p_2 = 4$

$\therefore A = 2x + \dfrac{4}{x}$

When $x = 16$, $A = 32 + \frac{1}{4} = 32\frac{1}{4}$

**6.** As $x$, $y$, and $z$ are in continued proportion, $y^2 = xz$
Option (A) is $y^2 : x^2 = xz : x^2 = z : x$

Option (B) is $\dfrac{z^2 - y^2}{y^2 - x^2}$

$= \dfrac{z^2 - xz}{xz - x^2} = \dfrac{z(z-x)}{x(z-x)}$

As two of the options are $z : x$, only possibility is all of the above.

**7.** $A : B = 1 : 3/4 = 4 : 3$
$B : C = 1 : 1/2 = 2 : 1$
$\therefore A : B : C = 8 : 6 : 3$
Given $C$'s share = ₹420

$A$'s share $= \dfrac{8}{3} \times 420 = ₹1120$

**8.** $15a^2 - 26ab + 8b^2 = 0$
$\Rightarrow 15a^2 - 20ab - 6ab + 8b^2 = 0$
$\Rightarrow 5a\,[3a - 4b] - 2b\,[3a - 4b] = 0$
$\Rightarrow [5a - 2b]\ [3a - 4b] = 0$
$\Rightarrow 5a - 2b = 0$ and $3a - 4b = 0$
$\dfrac{b}{a} = \dfrac{5}{2}$ or $\dfrac{3}{4}$
As $b/a$ is a proper fraction, it is equal to 3 : 4

**9.** Let the first, second, third, fourth and fifth parts be $a$, $b$, $c$, $d$ and $e$, respectively.
$2a = 3b = 4d$ (1)
$2b = 5c = 6e$ (2)
From (1) and (2)
$4a = 6b = 15c = 8d = 18e$ (3)
Let us divide (3) by the L.C.M of the coefficients.
i.e., L.C.M (2, 6, 15, 8, 18) = 360

Let $\dfrac{2a}{360} = \dfrac{6b}{360} = \dfrac{15c}{360} = \dfrac{8d}{360} = \dfrac{18c}{360} = k$

(an integer)
$a = 180k$, $b = 60k$, $c = 24k$, $d = 45k$, $e = 20k$
Consider choice (A)
$a = 180k$
$a$ is not a multiple of 72 for odd values of $k$. Hence, Choice (A) is not always true.
Consider choice (B)
$\dfrac{b}{d} = \dfrac{60k}{45k} = \dfrac{4}{3}$
so, $b$ is not divisible by $d$.
Hence, choice (B) is not true.
Consider choice (C)
As $a$ is not a factor of $e$, choice (C) is also not true.
Consider choice (D)
$\dfrac{ad}{30} = \dfrac{(180k)(45k)}{30} = 270k$ (an integer)
Hence, choice (D) is always true.

**10.** Let the volumes of helium in balloons A and B be $7v$ and $v$, respectively. Let $2x$ be the hourly rate at which balloon A leaks and $x$ be hourly rate at which balloon B is filled.
Given, $\dfrac{7v - 3(2x)}{v + 3(x)} = \dfrac{13}{7} \Rightarrow x = \dfrac{4v}{9}$

After $n$ hours, $\dfrac{7v - n(2x)}{v + n(x)} = \dfrac{7}{37}$

$\dfrac{7v - n\left(\dfrac{8v}{9}\right)}{v + n\left(\dfrac{4v}{9}\right)} = \dfrac{7}{37} \Rightarrow n = 7$

***Solutions for questions 11 to 14:***

For Smart Softcom:
Let the number of employees in the age group of 21 to 30 years be $4x$.
Then the number of employees in the age groups 31 to 40 and 41 to 50 are $5x$ and $7x$, respectively.
Share total number of employees = $16x$

∴ Number of male employees = (5/8) (16$x$) = 10$x$ and Number of Female employees = 16$x$ – 10$x$ = 6$x$

For Fast Softcom:

Let the number of male employees be 7$y$. Then the number of female employees will be 5$y$.

∴ Total number of employees = 12$y$

∴ Number of employees in the age groups 21 to 30, 31 to 40 and 41 to 50 are (2/6) (12$y$), (1/6) (12$y$) and (3/6) (12$y$), i.e., 4$y$, 2$y$, and 6$y$, respectively.

Given number of employees in the age group 21 to 30 in both the companies is the same.

∴ $4x = 4y \Rightarrow x = y$

Also given, the 'Smart Softcom' has 400 employees more than the 'Fast Softcom'

∴ $16x - 12y = 400$ [∵ $x = y$]

i.e., $16x - 12x = 400$

$\Rightarrow x = 100 \Rightarrow y = 100$

**11.** Number of employees in Fast Softcom
= 12 $y$ = 1200

**12.** Number of male employees in the two companies together
= 10$x$ + 7$y$ = 1000 + 700 = 1700

**13.** Difference in the number of employees in the age group 41 to 50 in two companies
= 7$x$ – 6$y$ = 700 – 600 = 100

**14.** The ratio of the number of male employees in Fast Softcom to the number of female employees in Smart Softcom = 7$y$ : 6$x$ = 7$y$ : 6$x$ = 7 : 6 as $x = y$.

**15.** Number of units consumed in the first month
= 97 = 50 + 47. The bill for the first month
= 50 (1.35) + 47 (2.7) = 67.5 + 126.9 = 194.4
The average bill for two months = ₹135
Sum of the two bills = 2 (135) = ₹270
The second month bill is = 270 – 194.4 = ₹75. 6
which is greater than ₹67.5
∴ In the second month also the number of units consumed is more than 50.
∴ The charge for consumed in excess of 50 units
= ₹75.6 – ₹67.5 = ₹8.1

Number of additional units = $\frac{8.1}{2.7}$ = 3

∴ Total number of units consumed in the second month = 53

**16.** $V \propto 1/P$, when $T$ is constant

$V \propto T$, when $P$ is constant $\Rightarrow V \propto T/P$

$$\Rightarrow \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\Rightarrow \frac{64 \times 200}{40} = \frac{P_2 \times 400}{50}$$

$\Rightarrow P_2 = 40$

**17.** Let the earnings of A, B, C, and D be $a$, $b$, $c$, and $d$, respectively.

The ratio of $a$, $b$, $c$, $d$ are tabulated below.

| | $a$ | $b$ | $c$ | $d$ | Total |
|---|---|---|---|---|---|
| | 3 | 7 | | | |
| | | 4 | 9 | | |
| | | | 6 | 7 | |
| ∴ | $\frac{12}{7}$ | 4 | 9 | 10.5 | |
| or | 12 | 28 | 63 | 73.5 | 176.5 |
| | | | | | 52.950 |

As $\frac{52.950}{176.5}$ = 300, D's earnings are (73.5) (300)

= ₹22,050

**18.** As they together have 12 apples, each person gets a share of 4 apples. Chandu took 4 apples as his share and paid 12 coins. Of the 4 apples, 1 apple is given by Bharath and 3 apples are given by Aravind.

∴ The ratio of apples with Aravind and Bharath = 3 : 1

∴ They must be paid in that ratio i.e., 3 : 1

Hence, the number of coins that Aravind should get

$= \frac{3}{4}$ (12) = 9.

**19.** Let the kinetic energy of a body, its mass and its velocity be denoted by $K$ joules, $M$ kg and $V$ m/sec, respectively.

$K \propto MV$

$$\frac{K_1}{K_2} = \frac{M_1}{M_2}\frac{V_1^2}{V_2}$$

Taking $K_1$ = 0.144, $M_1$ = 7.2, $V_1$ = 0.2, $M_2$ = 3.6 and $V_2$ = 0.8 and substituting them in the equation above, $K_2$ = 1.152 joules.

**20.** Given that, $a : b = 2 : 3$ and $p : q = 3 : 2$

Consider $\frac{2a^2p^3 + 3b^2q^3}{3abpq^2 + 4a^2p^2q}$

By dividing numerator and denominator by $b^2q^3$,

we get $$\frac{2\left(\frac{a}{b}\right)^2\left(\frac{p}{q}\right)^3 + 3}{3\left(\frac{a}{b}\right)\left(\frac{p}{q}\right) + 4\left(\frac{a}{b}\right)^2\left(\frac{p}{q}\right)^2}$$

$$= \frac{2\left(\frac{2}{3}\right)^2\left(\frac{3}{2}\right)^3 + 3}{3\left(\frac{2}{3}\right)\left(\frac{3}{2}\right) + 4\left(\frac{2}{3}\right)^2\left(\frac{3}{2}\right)^2}$$

$$= \frac{3+3}{3+4} = \frac{6}{7}$$

**21.** If $\frac{p}{q}=\frac{r}{s}$, each of these $=\frac{p+r}{q+s}$ and also $=\frac{p-r}{q-s}$.

$\therefore \frac{a+b}{c+d}=\frac{a-b}{c-d} \Rightarrow$ Each of these $=\frac{2a}{2c}=\frac{a}{c}$ and also $=\frac{2b}{2d}=\frac{b}{d}$

$\therefore \frac{a}{c}=\frac{b}{d}. \therefore ad=bc$, i.e., $ad-bc=0$.

**22.** Let the radius of the cylinder be denoted by $r$.
Let the volume and the height be denoted by $v$ and $h$, respectively.

$$r\propto\sqrt{\frac{v}{h}} \Rightarrow \frac{r_1}{r_2}=\frac{\sqrt{\frac{v_1}{h_1}}}{\sqrt{\frac{v_2}{h_2}}}$$

Taking $r_1=10$, $v_1=1500$, $h_1=5$, $v_2=2400$ and $h_2=2$ and substituting these values in the above equation, $r_2=20$.

**23.** If $\frac{p}{q}=\frac{x}{y}=k$, then $k=\frac{p-x}{q-y}$

$$\therefore \frac{a^2+b^2-\left(b^2+c^2\right)}{a+b-(b+c)}=\frac{a^2-c^2}{a-c}=\frac{(a+c)(a-c)}{a-c}$$

$=a+c$

**24.** Costs using process (all in ₹)
A = 10,000 + 20 (500) = 20,000
B = 12,000 + 15 (500) = 19,500
C = 15,000 + 10 (500) = 20,000
D = 20,000 + 8 (500) = 24,000.

**25.** The number of outgoing calls made by Aravind is 90.
$\therefore$ The bill = 250 + (90 – 50) (1.25)
= 250 (40) (1.25) = 250 + 50 = 300.
The bills paid by Aravind and Prasad are in the ratio 2 : 3.

$\Rightarrow$ The bill paid by prasad $=300\left(\frac{3}{2}\right)=₹450$

which is greater than ₹250.
$\therefore$ Number of calls Prasad made

$$=50+\frac{(450-250)}{1.25}=50+\frac{200}{1.25}$$

= 50 + 160 = 210.

**26.** The data and calculations are tabulated below.

| Input | Waste | Effective input | Efficiency | Output |
|---|---|---|---|---|
| 2400 | 40% | 1440 | 70% | 1008 |
| 1680 | 30% | 1176 | 80% | 940.8 |

**27.** Let the fixed expenses be ₹$x$ and the variable expenses per head be ₹$y$.
$x+50y=50\times220$
$\Rightarrow x+50y=11000$ (1)
Also, $x+60y=60\times205$
$\Rightarrow x+60y=12300$ (2)
From (1) and (2), we get
$x=4500$ and $y=130$
Let the number of members when the share of each member is ₹160 be $n$.
$4500+130n=160n \therefore n=150$.

**28.** Let the kinetic energy of the body be denoted by $E$.
Let the mass and time for which the body falls be denoted by $m$ and $t$, respectively.
$E\propto m$ when $t$ is fixed
$E\propto t^2$ when $m$ is fixed
Hence, $E=kmt^2$ where $k\neq0$.
$k=E/mt^2=25/5(10)^2=1/20$

Kinetic energy of the body $=\frac{mt^2}{20}$

$=\frac{1}{20}(2.5)\ (3)^2$ kJ = 1.125 kJ

**29.** $(x+y)\propto(x-y)$
$\Rightarrow (x+y)=k(x-y)$, when $k$ is a constant

$\Rightarrow \frac{x+y}{x-y}=k$

Applying componendo and dividendo,

$\frac{x}{y}=\frac{k+1}{k-1}=c$ (Say)

Squaring both sides; $x^2/y^2=c^2$
Applying componendo and dividendo

$$\frac{x^2+y^2}{x^2-y^2}=\frac{c^2+1}{c^2-1}$$

$\Rightarrow (x^2+y^2)=(c^2+1/c^2-1)\ (x^2-y^2)$
$\Rightarrow (x^2+y^2)=$ (a constant) $(x^2-y^2)$
$\therefore (x^2+y^2)\propto(x^2-y^2)$

Also, $\frac{x^2+y^2}{xy}=\frac{y^2\left(1+c^2\right)}{(cy)y}$ (Since $x=cy$)

$=\frac{1+c^2}{c}$

$\therefore x^2+y^2=(1+c^2/c)xy=$ (a constant) $xy$
$\therefore (x^2+y^2)\propto xy$
$\therefore$ Both (A) and (B) are true.

**30.** Let the total surface area of the cylinder be $T$.
Let the two parts be $Ar$ and $Br^2$ where $A$ and $B$ are constants.
$T=Ar+Br^2$
$7200=30A+900B$ (1)
$3600=20A+400B$ (2)

The total surface area when the radius is 10 units is $10A+100B$.

This can be written as 1/2 of equation (2)'s RHS – $100B$.

As RHS of equation (2) has a value of 3600, if $B$ is found $10A+100B$ can be found.

Multiplying equation (1) by 2 and subtracting the from it equation (2) multiplied by 3, $B$ is found to be 6.
Hence, $10A + 100B = 1/2\ [3600] - 600 = 1200$

**31.** Let the edge of the base and slant height be $s$ and $l$, respectively.
Given that,
LSA $\propto s$ (when l is constant)
LSA $\propto l$ (when s is constant)
Given that LSA = k $(s)$ (l) where $k \neq 0$
$196 = k\ (7)\ (14) \Rightarrow k = 2$
When $s = 12$ and $l = 20$
LSA = 2 (12) (20) or 480 $cm^2$.

**32.** Let the present ages of the woman, her husband and her daughter be $y$ years, $x$ years, and $z$ years, respectively.
$y + z = 60$ (1)
The woman would attain her husband's age after $(x - y)$ years.
$\therefore \dfrac{x + x - y}{z + x - y} = 2$
$y = 2z$ (2)
From (1) and (2), $z = 20$

**33.** Let, the speed of the bus be $x$ km/h and the consumption of diesel per hour be $y$ litres.
Given, $y \propto x^2 \Rightarrow y = kx^2$
Given $1 = k\ (40)^2 \Rightarrow k = \dfrac{1}{1600}$
$\therefore y = \dfrac{x^2}{1600}$
Let the required speed be $x$ km/h
Time taken to cover 400 km at $x$ km/h = $\dfrac{400}{x}$ hours
$\therefore$ Consumption of diesel in $\dfrac{400}{x}$ hours = $\dfrac{400y}{x}$ litres
$\therefore$ Cost of diesel = $\dfrac{400y}{x}(40) = ₹\ \dfrac{1600y}{x}$
$= \left(\dfrac{16000}{x}\right)\left(\dfrac{x^2}{1600}\right) = 10x \left[\because y = \dfrac{x^2}{16000}\right]$
and other expenses for $\dfrac{400}{x}$ hours
$= \dfrac{400}{x}(40) = ₹\ \dfrac{16000}{x}$
$\therefore$ Total expenditure $= 10x + \dfrac{16000}{x} = 400\left[\dfrac{x}{40} + \dfrac{40}{x}\right]$
For $a > 0$, the minimum value of $a + 1/a = 2$.
So, the minimum value of $\dfrac{x}{40} + \dfrac{40}{x}$ is 2
$\therefore$ The minimum value of $400\left[\dfrac{x}{40} + \dfrac{40}{x}\right] = 400\ (2) = 800.$
Hence, the minimum expenditure to cover of a distance of 400km is ₹800.

**34.** Given, $\dfrac{ax^2 + by^2}{ax^2 - by^2} = \dfrac{77}{13}$. By componendo and dividendo, we get $\dfrac{ax^2}{by^2} = \dfrac{90}{64} = \dfrac{45}{32}$ (1)
Given, $\dfrac{xy^2 + ab^2}{xy^2 - ab^2} = \dfrac{17}{7}$. By componendo and dividendo, we get $\dfrac{xy^2}{ab^2} = \dfrac{17 + 7}{17 - 7} = \dfrac{12}{5}$ (2)
If (1) and (2) are multiplied, we get $\dfrac{ax^2}{by^2} \times \dfrac{xy^2}{ab^2} = \dfrac{45}{32} \times \dfrac{12}{5}$
$\Rightarrow \dfrac{x^3}{b^3} = \dfrac{27}{8} \Rightarrow \dfrac{x}{b} = \dfrac{3}{2}$ and $\dfrac{x^2}{b^2} = \dfrac{9}{4}$

**35.** Let the weights of the three bars be $4x$, $5x$, $6x$. Then the weight of the large bars that is obtained after the melting is $4x + 5x + 6x = 15x$
Sum of the costs of three bars (take the constant of popularity as 1)
$= (4x)^2 + (5x)^2 + (6x)^2 = 16x^2 + 25x^2 + 36x^2 = 77x^2$
The cost of the big bar obtained
$= (15x)^2 = 225x^2$
Increase in worth = $225x^2 - 77x^2 = 148x^2 = ₹4440$
$\Rightarrow x^2 = 30$
$\therefore$ The cost of lightest bar
$= (4x)^2 = 16x^2 = 16\ (30) = 480.$

**36.** (a) Number of outgoing calls made by Lala = 70
Number of incoming calls received by Lala = 150 – 70 = 80
$\therefore$ The bill for Lala = (70) (2.50) + 80 (1)
= 175 + 80 = ₹255.
(b) The number of calls received by Lala is 60. Then the number of calls received by Thomas = 60 + 10 = 70
Let the number of outgoing calls made by Thomas be $y$.
Then, the bill for Thomas = ₹250 = $y$ (2.50) + 70 (1)
$\Rightarrow y\ (2.50) = 180 \Rightarrow y = 72$
$\therefore$ Number of outgoing calls made by Lala = 72 + 20 = 92
$\therefore$ The total number of calls
= (70 + 72) + (60 + 92) = 294.

**37.** Given that $C \propto \sqrt{A}$, when ($T$ is constant)
$C \propto T^2$, when ($A$ is constant)
$\Rightarrow C \propto (\sqrt{A})\ (T^2), \Rightarrow C = k\ (\sqrt{A})\ (T^2)$ or
$\dfrac{C_1}{C_2} = \left(\sqrt{\dfrac{A_1}{A_2}}\right)\left(\dfrac{T_1}{T_2}\right)^2$
Given that $C_1$ = ₹360, $A_1$ = 1600.sq.cm, $T_1$ = 3 cm
$C_2$ = ?, $A_2$ = 900. sq.cm, $T_2$ = 4 cm
$\Rightarrow \dfrac{360}{C_2} = \sqrt{\dfrac{1600}{900}} \times \left(\dfrac{3}{4}\right)^2$
$\Rightarrow C_2 = 360 \times 3/4 \times 16/9 = 480.$

**38.** The values of $x$ and $y$, for the given equation are:

$px + qy + r = 0$ (1)

$lx + my + x = 0$ (2)

(1) $x\ m$ – (2) $x\ q$ (given),

$$x = \frac{qn - rm}{pm - lq} \quad (3)$$

and (1) $x1$ – (2) $x\ p$, given

$$y = \frac{rl - pn}{mp - lq} \quad (4)$$

The denominator in both the cases is $(mp - lq)$. As long as this is not equal to zero, $x$ and $y$ will have unique values. Hence, $mp \neq lq$ is the required condition.

**39.** $l^2 + 8m^2 + 9n^2 = 4m(l + 3n)$

$\Rightarrow l^2 + 8m^2 + 9n^2 - 4ml - 12mn = 0$

$\Rightarrow (l^2 + 4m^2 - 4ml) + (4m^2 - 12mn + 9n^2) = 0$

$\Rightarrow (l - 2m)^2 + (2m - 3n)^2 = 0$

$\Rightarrow l - 2m = 0$ and $2m - 3n = 0$

So $l = 2m$ and $2m = 3n$

$$\frac{l}{m} = \frac{2}{1} \text{ and } \frac{m}{n} = \frac{3}{2}$$

$l : m = 2 : 1$ and $m : n = 3 : 2$

$\Rightarrow l : m = 6 : 3$ and $m : n = 3 : 2$

$l : m : n = 6 : 3 : 2$.

**40.** Let the two numbers be $a$ and $b$ (where $a < b$)

$\therefore$ $a$, 12, $b$, $br$, 96 are in continued proportion (or geometric progression with common ratio $r$).

Relating the two known terms, 12 and 96 we can see that $12\ (r^3) = 96$ or $r = 2$.

$\therefore$ $a = 6$ and $b = 24$

**41.** The relations among the prices of the commodities are:

$5S = 2L$; $8R = 3L$; $2D = 5R$

where $S$, $R$, $D$ are prices per kg and $L$ is price per litre of sugar, rice, dal and oil, respectively.

The three relations can be combined into a single relation among the four prices as shown below.

$3(5S) = 3(2L)$; $2(8R) = 2(3L)$; $2D = 5R$

$\Rightarrow 15S = 6L = 16R$; $2D = 5R$

$\Rightarrow 5[15S = 6L = 16R]$; $16(2D) = 16(5R)$

$\Rightarrow 75S = 30L = 80R = 32D$ (1)

Given that $L = ₹40$

$75S = 30 \times 40 \Rightarrow S = 16$ (2)

It also follows that

$32D = 30 \times 40 \Rightarrow D = 37.5$ (3)

The difference of $S$ and $D = 21.5$

**42.** This problem can be solved using the method shown in the two previous questions.

Another method is illustrated below.

Let the number of days taken by the group of men to complete the work be denoted by $N$.

Let the number of workers and the work to be completed by denoted by $M$ and $W$, respectively.

$$M \propto \frac{W}{N} \quad \therefore \quad \frac{M_1 N_1}{W_1} = \frac{M_2 N_2}{W_2}$$

We take $M_1 = 8$, $W_1 = 2$, $N_1 = 1/2$, $M_2 = 16$ and $W_2 = 8$ and find $N_2$ to be 1.

**43.** Let $T = Ar + Br^2$

$720 = 10A + 100B \Rightarrow A + 10B = 72$ (1)

$2640 = 20A + 400B \Rightarrow A + 20B = 132$ (2)

Required area $= 15A + 225B$

$= 15\ (A + 15B)$

$$= 15\left(\frac{2A + 30B}{2}\right) = 15\left(\frac{72 + 132}{2}\right) [\because \text{ From (1) and (2)}]$$

$= 1530$

**44.** Let the amounts paid by Ram, Lakshman, Bharath and Shatrugna be $a$, $b$, $c$, and $d$, respectively.

$\therefore$ $a + b + c + d = ₹2,40,000$ (1)

Given, $a = \frac{1}{2}(b + c + d)$

$\Rightarrow b + c + d = 2a$ (2)

From (1) and (2), $a = ₹80,000$

Now, $d = \frac{1}{5}(a + b + c)$

$\Rightarrow a + b + c = 5d$ (3)

From (1) and (3), $d = ₹40,000$

Also, $c = \frac{5}{19}(a + b + d)$

$$\therefore\ a + b + d = \frac{19c}{5} \quad (4)$$

From (1) and (4), $c = ₹50,000$

$\therefore$ $b = 2,40,000 - 80,000 - 50,000 - 40,000$

$= ₹70,000$

**45.** Let the luggage weights carried by Mohan and Sohan be $m$ kg and $s$ kg, respectively. Let the rate of excess luggage charge be $₹e$ per kg.

Excess luggage of Mohan $= (m - f)$ kg. Excess luggage of Sohan $= (s - f)$ kg

$e(2m - f) = 2400$ (1)

$e(2s - f) = 900$ (2)

(1) + (2) $\Rightarrow e(2m + 2s - 2f) = 3300$

i.e., $e(m + s - f) = 1650$

(3) $\Rightarrow e(m + s - f) - ef = 1050$

i.e., $1650 - ef = 1050$, i.e., $ef = 600$

Excess luggage charge for Mohan $= e(m - f)$

$$= \frac{e(2m - 2f)}{2} = \frac{e(2m - f) - ef}{2}$$

(1) $\Rightarrow$ this is $\frac{2400 - 600}{2} = ₹900$.

**46.** The data is tabulated below. At each stage if a person gives $1/n$ of the number of chocolates with himself

(or herself), we assume that at that stage the number of chocolates with him (or her) is $nk$. ($k = a, b, \ldots$). Also, let the number of chocolates with $D$ be $d$.

| A | B | C | D |
|---|---|---|---|
| $3a$ | $(4b - a)$ | $(5c - b)$ | $(d)$ |
| | $4b$ | | |
| | | $5c$ | |
| $2a$ | $3b$ | $4c$ | $d + c$ |

The initial number of chocolates with $B$ is $4b - a$.

$\therefore 3a - (4b - a) = 80$ (1)

(1) $\Rightarrow a - b = 20$

Also, $2a = 3b = 4c = c + d$.

i.e., $\frac{a}{b} = \frac{3}{2}$ ($\therefore a = 60$, $b = 40$, $c = 30$, $d = 90$)

$C$ initially had $5c - b = 110$

$D$ initially had 90.

The difference is 20.

**47.** $W \propto r^2$, when '$h$' is constant

and, $W \propto h$, when '$r$' is constant

$\Rightarrow W \propto r^2 h$

$\Rightarrow \frac{W_1}{r_1^2 h_1} = \frac{W_2}{r_2^2 h_2}$

$\frac{12}{(2)^2 \times 4} = \frac{W_2}{(4)^2 \times 3}$

$\Rightarrow W_2 = \frac{12}{(2)^2 \times 4} \times (4)^2 \times 3 = 36$ kg

**48.** Number of days $\propto$ Number of books and when Rate is constant

$(D) \qquad (B) \qquad (R)$

$D \propto \frac{1}{R}$ When $B$ = constant.

Combining both conclusions we get $D = k\frac{B}{R}$ when $k$ = constants.

$\therefore \quad \frac{D}{B} \times R = k$

$\therefore$ We get $\frac{D \times R}{B} = \frac{D_1 \times R_1}{B_1}$

$\frac{25 \times 80}{20} = \frac{150 \times 20}{B_1}$

$B_1 = 30$

He can read $30 - 20 = 10$ more books.

**49.** The speed of the train when no wagons are attached = 30 km/20 minutes = 90 km/h (1)

If $d_s$ is the decrease in the speed, it is given that $d_s \propto \sqrt[3]{n}$, where $n$ is the number of wagons attached;

$\Rightarrow d_s = k \cdot \sqrt[3]{n}$, where $k$ is constant of proportionality (2)

The speed of the train when 8 wagons are attached = 14 km/12 minutes = 70 km/h (3)

From (1) and (3), decrease in speed ($d_s$) = 90 – 70 = 20 km/h (4)

From (2) and (4), $20 = k \cdot \sqrt[3]{8}$, $\Rightarrow 20 = 2k$,

$\Rightarrow k = 10$ (5)

Decrease in speed ($d_s$) when 64 wagons are attached is $d_s = k\sqrt[3]{n}$, or $d_s = 10.\sqrt[3]{64} = 40$

Hence, the speed of the train when 64 wagons are attached = 90 – 40 = 50 km/h

**NOTE**

Working can be reduced by considering:

$$\frac{\text{decrease in speed with 8 wagons}}{\text{decrease in speed with 64 wagons}} = \frac{k\sqrt[3]{8}}{k\sqrt[3]{64}} = \frac{2}{4} = \frac{1}{2}.$$

Because the decrease in the first case is 90 – 70 = 20, the decrease in the second class = 2 × 20 = 40. Hence, the speed of train (in km/hr) with 64 wagons = 90 – 40 = 50.

**50.** Let the fixed expense be ₹$x$ and the variable expense be ₹$y$ per head.

$x + 20y = 20 \times 650 \Rightarrow x + 20y = 13000$ (1)

and, $x + 25y = 25 \times 600 \Rightarrow x + 25y = 15000$ (2)

From (1) and (2), we get;

$x = 5000$ and $y = 400$

Let, the required number of occupants be $n$.

$5000 + 400n = 500n$

$\Rightarrow 100n = 5000 \Rightarrow n = 50$

## EXERCISE-4

***Solutions for questions 1 and 2:***

Let A, B, and $C$ have ₹$3k$, ₹$2k$ and ₹$k$ with them initially. The loser at the end of each round and the amounts with each of them are tabulated below.

| | | $A$ | $B$ | $C$ | |
|---|---|---|---|---|---|
| Round | Loser | $3k$ | $2k$ | $k$ | (0) |
| 1 | B | $3k + \frac{k}{2}$ | $2k - k$ | $k + \frac{k}{2}$ | |

| | | | | | |
|---|---|---|---|---|---|
| | | $3.5k$ | $k$ | $1.5k$ | (1) |
| 2 | C | $4k$ | $1.5k$ | $0.5k$ | (2) |
| 3 | A | $3k$ | $2k$ | $k$ | (3) |
| 4 | B | $3.5k$ | $k$ | $1.5\,k$ | |

(same as that after Round 1)

Now, $3.5k - 3k = 10 \Rightarrow k = 20$

Total money $= 3k + 2k + k = 120$

**2.** After round 2, A would have gained ₹20, which is the maximum gain for him in the course of the 4 rounds.

**3.** $\dfrac{A+B+C-10}{A+B-C+10} = \dfrac{A-B+C+10}{A-B-C-10}$

Applying componendo–dividendo (CD) we get

$\dfrac{2(A+B)}{2(C-10)} = \dfrac{2(A-B)}{2(C+10)}$

$\Rightarrow \dfrac{A+B}{A-B} = \dfrac{C-10}{C+10}$

Applying *CD* again, we get $\dfrac{A}{B} = \dfrac{C}{-10}$

$\Rightarrow BC = -10A$

**4.** Let the scores of Anil, Bala, Chetan, Dinesh, Eswar in the exam be *a, b, c, d, e,* respectively

| $a$ | $b$ | $c$ | $d$ | $e$ |
|---|---|---|---|---|
| 11 | 10 | 13 | | |
| | 12 | | 19 | 18 |
| 66 | 60 | 78 | 95 | 90 |

*a, b, c, d, e* are positive integers.

$\therefore$ *c* is a multiple of 13. $\therefore c \neq 95$. $\therefore d = 95$

Total score = 389

Alternatively, once we get the ratio of *a, b, c, d, e.* since the maximum score is 100 $(a, b, c, d, e) = (66, 60, 78, 95, 90)$

***Solutions for questions 5 and 6:***

The table below shows the shares of A, B, C, and D in the four months.

| | A | B | C | D |
|---|---|---|---|---|
| First month | $\frac{2}{14} \times 4{,}200 = 600$ | $\frac{3}{14} \times 4{,}200 = 900$ | $\frac{4}{14} \times 4{,}200 = 1{,}200$ | $\frac{5}{14} \times 4{,}200 = 1{,}500$ |
| Second month | $\frac{3}{14} \times 8{,}400 = 1{,}800$ | $\frac{4}{14} \times 8{,}400 = 2{,}400$ | $\frac{5}{14} \times 8{,}400 = 3{,}000$ | $\frac{2}{14} \times 8{,}400 = 1{,}200$ |
| Third month | $\frac{4}{14} \times 12{,}600 = 3{,}600$ | $\frac{5}{14} \times 12{,}600 = 4{,}500$ | $\frac{3}{14} \times 12{,}600 = 2{,}700$ | $\frac{2}{14} \times 12{,}600 = 1{,}800$ |
| Fourth month | $\frac{4}{14} \times 6{,}300 = 1{,}800$ | $\frac{3}{14} \times 6{,}300 = 1{,}350$ | $\frac{5}{14} \times 6{,}300 = 2{,}250$ | $\frac{2}{14} \times 6{,}300 = 900$ |
| Total | 7,800 | 9,150 | 9,150 | 5,400 |

**5.** Average monthly earning of C $= ₹\dfrac{9{,}150}{4} = ₹2{,}287.50$

**6.** B should have earned (3,500 (4) – 9,150) = 4,850 more, to make his average monthly earning ₹3500.

**7.** Tax $= F + K$ (Salary – 50,000)

$6{,}200 = F + K\,(60{,}000 - 50{,}000)$

$6{,}200 = F + 10{,}000K$ (1)

$7{,}700 = F + K\,(75{,}000 - 50{,}000)$

$7{,}700 = F + 25{,}000K$ (2)

From (1) and (2)

$F = 5{,}200$ and $K = 1/10$

Now $T = 8{,}200$

$8{,}200 = 5{,}200 + \dfrac{1}{10}$ (Salary – 50,000)

Annual Salary = Wr.80,000

**8.** Onions lead over tomato by 200 per quintal. Tomatoes gain on onion by Rs0.60 per day. Assuming that the increase rates are constant, tomatoes need $\dfrac{200}{0.60}$ or 333 $\dfrac{1}{3}$ days to catch up. (i.e., more than 200 days). But after 200 days, the price of onions remains constant at ₹600.

Tomatoes need $\dfrac{600 - 200}{1.6}$ or 250 days to catch up.

Starting from Apr 2, 250$^{th}$ day would be 7$^{th}$ Dec.

**9.** The quantity of the balance of food after the transfer is such that

(900 – 300) = 600 soldiers, consumed at the rate of 3 kg/day/soldier, for 25 days (1)

If the soldiers were not transferred, 900 soldiers would have consumed it at the rate of 2.5kg/day/soldier, the same food. (2)

The data can be tabulated as:

| Soldiers | Consumption rate | Number of days |
|---|---|---|
| 600 | 3.0 | 25 |
| 900 | 2.5 | How many? |

Number of soldiers and the number of days for which food lasts are inversely proportional. The number of soldiers increased; hence, number of days decreases. Hence, multiplication factor is (600/900).

Consumption rate and number of days are also inversely proportional. Hence, multiplication factor is 3.0/2.5

Applying the above rates of variation, the number of days
$= 25 \times (600/900) \times (3.0/2.5) = 20$ days
The initial stock was to last for 30 days.
$\Rightarrow$ Soldiers were transferred after $30 - 20 = 10$ days

**10.** Fixed charge = $x$, unit rate = $y$, $x$ and $y$ are in Rupees.
$5/4(x + 100y) = x + 200y$
$5x + 500y = 4x + 800y$ (1)
$x + 50y = 700$ (2) (data)
$y = 2$, $x = 600$

**11.** The rate of cooling is proportional to the fourth power of the temperature difference and the surface area of the body.

The temperature difference increases by 10%, i.e., it gets multiplied by a factor of $\left(\frac{11}{10}\right)$, while the area gets multiplied by a certain factor $K$.

$\therefore R = \left(\frac{11}{10}\right)^4 K$ (As $K < 1$, the choice is between (A) and ($B$))

When the cube of side a is melted and cast into a sphere of radius $r$ the surface area changes by the factor

$$K = \frac{4\pi r^2}{6a^2}, \text{ where } a^3 = \frac{4}{3}\pi r^3, \text{ i.e., } \left(\frac{r}{a}\right)^3 = \frac{3}{4\pi}$$

$$\therefore K = \frac{4\pi}{6}\left(\frac{3}{4\pi}\right)^{\frac{2}{3}} = \frac{1}{2}\left(\frac{4\pi}{3}\right)^{\frac{1}{3}} = \left(\frac{\pi}{6}\right)^{\frac{1}{3}}$$

$$\therefore K \approx \left(\frac{11}{21}\right)^{\frac{1}{3}} \approx (0.52)^{\frac{1}{3}} > (0.512)^{\frac{1}{3}} = 0.8$$

$$\therefore R > \left(\frac{11}{10}\right)^4 (0.8) = (1.4641)\ (0.8) \approx 1.17$$

***Solutions for question 12:***

Let the number of 25 ps, 50 ps, and ₹1 coins be $12k$, $8k$, and $5k$, respectively. The amounts of 25 ps, 50 ps and ₹1 coins are $12k\,(25) = 300k$; $8k\,(50) = 400k$ and $5k\,(100) = 500k$. There are $x$ coins after removing 24 of the 25 ps coins and replacing them by 50 ps and ₹1 coins in 2 : 1 ratio.

$$\therefore \frac{300k - 600}{25} + \frac{400k + 400}{50} + \frac{500k + 200}{100} = x \Rightarrow 25k - 14 = x \quad (1)$$

Also 85 coins less than twice $x$ are there if 24 of the 50 ps coins are replaced by 25 ps and ₹1 coins in 2 : 1 ratio by value

$$\therefore \frac{300k + 800}{25} + \frac{400k - 1200}{50} + \frac{500k + 400}{100} = 2x - 85$$

$\Rightarrow 25k + 12 = 2x - 85 \rightarrow (2)$ from (1) and (2), $x = 111$ and $k = 5$

**12.** (a) Total coins initially $= 12k + 8k + 5k = 25\ (5) = 125$

(b) If 24 ₹1 coins are removed, ₹16 is to be placed in the form of 25 ps coins and ₹8 is to be placed in the form of 50 ps. coins.

Number of 25 ps coins $= 60 + 64 = 124$
Number of 50 ps coins $= 40 + 16 = 56$
Number of ₹1 coins $= 25 - 24 = 1$
$\therefore$ Total coins are 181

**13.** Let the cost be equal to $k(w)^3$, where $w$ is the weight of the rock
$\therefore$ initially quoted cost $= k(1 + 2 + 3 + 4 + 5)^3$
$= 3375k$
Final selling price
$= k(1)^3 + k(2)^3 + k(3)^3 + k(4)^3 = 100\ k$
$\therefore$ Savings $= 3275k$
given $3275k = 32750 \Rightarrow 3375\ k = 33750$

**14.** Let the number be $abc$.

$$R = \frac{abc}{a+b+c} = \frac{100a + 10b + c}{a+b+c}$$

$$\text{If } a = b,\ R = \frac{110a + c}{2a + c} = 55 - \frac{54}{2\frac{a}{c} + 1}$$

$$\text{If } a = c,\ R = 50.5 - \frac{40.5}{2\frac{a}{b} + 1}$$

$$\text{If } b = c,\ R = 100 - \frac{189}{\frac{a}{c} + 2}$$

The greatest value of $R$ occurs when $b = c$ and $\frac{a}{c}$ is maximum, i.e., $a = 9$, $c = 1$

The greatest value is $100 - \frac{189}{11} \approx 82.8$

$\therefore$ The least value of $R$ should occur when $abc = 199$ or 119 (when $\frac{a}{c}$ is minimum)

When $abc = 199$, $R = 10\frac{9}{19}$

When $abc = 119$, $R = 10\frac{9}{11}$

$\therefore\ 10\frac{9}{19} \le R \le 82\frac{9}{11}$

Only I is true.

**15.** Let the present ages of Alok, Bala, Chetan be $a$ years, $b$ years, $c$ years, respectively.

Bala will attain Alok's present age $(a - b)$ years from now. The ages of Alok and Chetan at that time will be $(2a - b)$ years and $(c + a - b)$ years, respectively.

$$\frac{2a - b}{c + a - b} = \frac{18}{13} \quad (1)$$

When Alok was as old as Chetan is, the ages of Bala and Chetan were $[b - (a - c)]$ and $c - (a - c)$ years, respectively.

$$\frac{b - (a - c)}{c - (a - c)} = \frac{7}{5}, \text{ i.e., } \frac{b + c - a}{2c - a} = \frac{7}{5} \quad (2)$$

(1) $\Rightarrow$ $18c = 8a + 5b$

(2) $\Rightarrow$ $2a + 5b = 9c$

$18c = 8a + 5b = 2(2a + 5b) \Rightarrow \frac{a}{b} = \frac{5}{4}$

**16.** Let the luggage with Anil be $2a$ and that with Balu be $3b$. Let the free luggage be $x$ kg. The data is tabulated below.

| Anil | $2a$ | $a$ | $2a + 3b$ |
|---|---|---|---|
| Balu | $b$ | $3b$ | – |
| Chargeable luggage | $2a + b - 2x$ | $a + 3b - 2x$ | $2a + 3b - x$ |
| Charge | 4200 | 4500 | 7200 |
| Ratio | 14 | 15 | 24 |

If we take the differences of the terms in the third row and the terms in the fifth row (after doubling the terms in the last column), we can eliminate $x$.

$$\frac{(a+3b-2x)-(2a+b-2x)}{15-14} = \frac{(4a+6b-2x)-(a+3b-2x)}{48-15}$$

$$\Rightarrow \frac{2b-a}{1} = \frac{a+b}{11}$$

$$\Rightarrow 22b - 11a = a + b$$

$$\Rightarrow 21b = 12a \Rightarrow \frac{a}{b} = \frac{21}{12} = \frac{7}{4}$$

$\therefore \frac{2a}{3b} = \frac{2}{3}\frac{7}{4} = \frac{7}{6} = \frac{70}{60}$, i.e., Anil has 70 kg and Balu has 60 kg.

($\because$ $2a + 3b = 130$) Also $a = 35$, $b = 20$

$$\frac{2a + b - 2x}{14} = \frac{a + 3b - 2x}{15}$$

$$\frac{2(35) + 20 - 2x}{14} = \frac{35 + 3(20) - 2x}{15}$$

$$\Rightarrow \frac{90 - 2x}{14} = \frac{95 - 2x}{15} \Rightarrow \frac{90 - 2x}{95 - 2x} = \frac{14(5)}{15(5)}$$

$\therefore$ $90 - 2x = 14$ (5) (if $\frac{N_1}{D_1} = \frac{N_2}{D_2}$ and $N_1 - D_1 = N_2 - D_2$, then $N_1 = N_2$ and $D_1 = D_2$)

$\Rightarrow$ $x = 10$

**17.** $A$ varies directly with $B^2$ (when $C$, $D$, $E$ are constant)

$B^2$ varies inversely with $C^3$.

$C$ varies directly with $D^2$.

$C^3$ varies directly with $E^8$.

From the above, $C^3$ varies directly with $D^6 \times E^8$, and hence,

$A = \frac{KB^2C^3}{D^6E^8}$ where $K$ is a constant.

If each of $B$, $C$, $D$, and $E$ double, $A$ gets multiplied by $\frac{1}{2^9}$.

**18.** Let the fixed amount paid to the workers for the first 600 balls be $x$.

For 680 balls worker A would be paid $(x + 80k)$ where $k$ is the constant of proportionality.

$\therefore$ $x + 80k = 24{,}800$ (1)

For 750 balls worker B would be paid $(x + 150k)$

$\therefore$ $x + 150k = 25500$ (2)

From (1) and (2), we get

$k = 10$ and $x = 24{,}000$.

If $C$ had produced $(600 + n)$ balls,

he is paid $24000 + kn$

$\therefore$ $25200 = 24000 + kn$

$\Rightarrow 10n = 1200$

$n = 120$

So, worker $C$ produced $(600 + 120) = 720$ balls.

**19.** Amount that would be paid to worker $E$ for producing 850 balls

$= 24000 + (810 - 600)\ k + (850 - 810)^2\ l$

$\Rightarrow 34100 = 24000 + 2100 + 1600\ l$

$\Rightarrow l = 5$

Hence, worker $D$ will be paid

$24000 + 2100 + (20)^2\ l$

$= 28100$.

**20.** Pension $\propto \sqrt{n}$, where $n$ = number of years of service

Pension $= K\sqrt{n}$

Let the number of years of service for Mohan be $m$.

Murali's pension $= K\sqrt{m+9}$

Mohan's pension $= K\sqrt{m}$

Had Mohan put in $4\frac{1}{4}$ yrs. more his pension would be $K\sqrt{m + 4.25}$

Now, it is given that $\frac{K\sqrt{m+9}}{K\sqrt{m+4.25}} = \frac{10}{9}$

$$\Rightarrow \frac{m+9}{\frac{4m+17}{4}} = \frac{100}{81}$$

$$\Rightarrow \frac{m+9}{4m+17} = \frac{25}{81} \text{ or } 81m + 729 = 100m + 425$$

$\Rightarrow 19m = 304 \Rightarrow m = 16$

So Murali served in the Navy for 16 + 9 or 25 years

**21.** Let the total number of marbles that A and B initially had be $25a$ and $25b$, respectively.

| | **A** | **B** |
|---|---|---|
| Initial | $25a$ | $25b$ |
| After A gives | $10a$ | $15a + 25b$ |
| After B gives | $19a + 15b$ | $9a + 10b$ |

$19a + 15b = 25a \Rightarrow \frac{a}{b} = \frac{15}{6} = \frac{5}{2}$

Initial ratio of the number of marbles with A and $B$
$= \frac{25a}{25b} = \frac{a}{b} = \frac{5}{2}$

**22.** From the above table, the number of marbles with A and B initially was $5K$ and $2K$, respectively.

| A | B | Difference | Sum |
|---|---|---|---|
| $5K$ | $2K$ | $3K$ | $7K$ |

Now $3K = 36$, therefore $K = 12$
So, $7K = 7(12) = 84$

**23.** Let the present ages of P and Q be $x$ and $y$ years, respectively.
Then, $x + y = 84$ (1)
Also, $x - 6 = 2(y - 6)$
$x - 2y = -6$ (2)
By solving the equations (1) and (2), we get $x = 54$ and $y = 30$
$\therefore$ Difference $= x - y = 24$.

**24.** In a unit time,
Number of leaps taken by the hare = 5
Number of leaps taken by the jackal = 4
Given that
4 leaps of the hare = 5 leaps of the jackal
$\therefore$ 1 leap of the hare = 5/4 leaps of the jackal
$\therefore$ In the unit time,

The hare takes $\frac{5}{4} \times 5$ and the jackal takes 4 steps

$\therefore$ Ratio of their speeds $= \frac{25}{4} : 4 = 25 : 16$

**25.** The data is tabulated below

| | Apples | Oranges | Bananas | Total |
|---|---|---|---|---|
| **Number** | 50 | 20 | 20 | 90 |
| **Price** | $a$ | $r$ | $b$ | |

Actual amount spent $= 50a + 20r + 20b$
If $r = a$, amount spent $= 70a$
Given $50a + 20r + 20b = 70a \Rightarrow r + b = a$
Amount spent $= 70a$. Minimum value of a is 2, corresponding to $b = 1$, $r = 1$.
$\therefore$ Minimum possible amount = ₹140

## EXERCISE-5

**1.** Using either of the statements, we cannot find the ratio of profits as the data about sales and expenditures are given in different statements.

Combining both statements, let $4a$ and $5a$ be the sales and let $3b$ and $4b$ be the expenditures.

$\therefore$ Ratio of profits $= \frac{4a - 3b}{5a - 4b} = \frac{(5a - 4b) - (a - b)}{5a - 4b}$
$= 1 - \frac{a - b}{5a - 4b}$.

$\frac{a - b}{5a - 4b}$ may or may not be positive, depending on whether $a > b$ or $a < b$ ($\because 4a \geq 3b$). So, the ratio of profits may or may not be greater than 1 and hence, which company got a higher profit can't be determined even using both the statements together.

**2.** Neither statement alone is sufficient as the information about income and expenditure is given in different statements. Combining both statements, let $4x$ and $5x$ be the incomes and let $4y$ and $5y$ be the expenditures.
$\therefore$ Ratio of savings $= (4x - 4y) : (5x - 5y) = 4(x - y) : 5(x - y)$
$= 4 : 5$

**3.** Dividing both numerator and denominator of $\frac{7a + 9b}{4a + 5b}$ by $b$, it becomes $\frac{7\frac{a}{b} + 9}{4\frac{a}{b} + 5}$.

Value of $\frac{a}{b}$ is sufficient to find $\frac{7a + 9b}{4a + 5b}$.

From Statement I, we have $b + a = \frac{1}{2}(6a - b)$
$\Rightarrow a/b = 3/4$
$\therefore$ Statement I is sufficient.
From Statement II, we have $3a + 4b = 5$.

By simplifying $3a + 4b = 5$, $\frac{a}{b}$ cannot be found.

$\therefore$ Statement II is not sufficient.

**4.** Using Statement I, $y^2 = xz$. From this we cannot find the ratio of $x$, $y$, $z$.
$\therefore$ Statement I alone is not sufficient.
For a similar reason as above Statement II alone is not sufficient.

Combining both the statements, $\frac{x}{y} = \frac{y}{z} = \frac{z}{x} = k$ (say)
$x = yk = (zk)k = (xk)k^2 = k^3x$.
$\Rightarrow k^3 = 1$
$\Rightarrow k = 1$
$\therefore x : y : z = 1 : 1 : 1$.

**5.** Using Statement I, price of petrol = 2 (price of diesel) = 3 (price of CNG). As the quantities of the fuels bought is unknown, total money spent cannot he found.
$\therefore$ Statement I is not sufficient.

Using Statement II, equal quantities of each fuel were purchased. As the price of any fuel is not known, total money spent cannot be found.
∴ Statement II is not sufficient.
Combining both statements, since ₹510 is spent on diesel, money spent on petrol is (2) (510) = ₹1020 and money spent on CNG is (1/3) (1020) = ₹340.
Total money spent = ₹1870.

**6.** Let 1₹ coins, be $x$; 50 paise coins be $y$ and 25 paise coins be $z$.
From Statement I, $x + y + z = 20$.
From Statement II, $x = 6$ and $y : z = 6 : 1$.
Combining Statements I and II, we get $x = 6$, $y = 12$, $z = 2$.

**7.** Let the number of technical staff be $3x$ and non-technical staff be $10x$.
From Statement I,

$$\text{Total number of graduates} = \frac{80}{100}(3x) + \frac{40}{100}(10x)$$
$$= 2.4x + 4x = 6.4x$$

$$\text{Percentage of graduates} = \frac{6.4x}{10x} \times 100 = 64\%$$

So Statement I alone is sufficient.
From Statement II,

$$\text{The required ratio} = \frac{32x}{65x} \times 100\%$$

So Statement II alone is also sufficient.

**8.** Let $x$ be the number of boys and $y$ be the number of girls.

From Statement I, $\frac{x}{y} = \frac{5}{4} \Rightarrow 4x = 5y$

From Statement II, $\frac{x-4}{y} = 1 \Rightarrow x = y + 4$

Combining both the statements, we get the value of $y$ which is the number of girls.

**9.** From Statement I, we do not know what percentage of boys and what percentage of girls are intelligent, so we cannot answer the question.
From Statement II,
20% of boys and 20% of girls are adults, so (100 – 20)% = 80% of students are not intelligent, whatever be the ratio of boys and girls.

**10.** From Statement I, we have $x = \frac{1}{8}$ kg (assuming total as 1 kg)
Also $B = \frac{3}{16}$ kg, i.e., $\left(\frac{3}{2} \times \frac{1}{8} \text{ kg}\right)$
As we do not know about others, we cannot find who received least from this statement alone.
From Statement II, we have

$$A = \frac{3}{8} \text{ kg}; \; Y = \frac{1}{8} \text{ kg}$$

Again Statement II alone is not sufficient, as we do not know about others.
Using both, we know about four persons out of five. We can find how much $Z$ received, and hence, also the one who received the largest part.

**11.** If the prices are $3x$, $2x$, $5x$, from Statement I, we get $5x - 3x = 30{,}000$; hence, we get $x$ and the price of the car which is neither the costliest nor the cheapest. From Statement II, we get $2x = 20{,}000$ hence $x$ and then the required price. Hence, each statement itself is sufficient.

**12.** From Statement I alone, $\text{Sav}_A > \text{Sav}_B$; nothing can be decided about salaries.
From Statement II alone,
Since $E_A : E_B = \text{Sav}_A : \text{Sav}_B$; nothing can be decided.
When both the statements are combined, since Sal = $E$ + Sav, and $\text{Sav}_A > \text{Sav}_B$, $\text{Sal}_A > \text{Sal}_B$.

**13.** Rewriting Statement I, $p/q = 2/3$. Statement I alone is sufficient. We cannot find the ratio $p : q$ from Statement II as the equation in $p$ and $q$ has a constant also.

**14.** Let the number of boys be $b$
and the number of girls be $g$.
From Statement I:

$$\frac{b}{b+g} > \frac{g}{b} \text{ and}$$

$$\text{as } \frac{g}{b} > \frac{g}{g+b} \text{ (always), we have}$$

$$\Rightarrow \frac{b}{b+g} > \frac{g}{b} > \frac{g}{b+g} \Rightarrow \frac{b}{b+g} > \frac{g}{b+g}$$

$$\Rightarrow b > g$$

Statement I alone is sufficient.

From Statement II, $\frac{b}{g} > \frac{g}{b+g}$

Let $b/g = k$.

$$\text{As } \frac{g}{b+g} = \frac{1}{\frac{b+g}{g}} = \frac{1}{\frac{b}{g}+1} \Rightarrow k > \frac{1}{k+1}$$

$$\text{If } k > 1, \; k > \frac{1}{k+1} \;\; \therefore b > g$$

$$\text{If } k = 0.9, \; k > \frac{1}{k+1} \text{ is satisfied but then } b < g.$$

II alone is not sufficient.

**15.** Let number of apples be $a$
Number of mangoes be $m$
Number of oranges be $o$
$1000 = a + m + o$
From Statement I, $a = 2/3(m + o)$
$\Rightarrow a = 2/3\ (1000 - a)$
$\Rightarrow 5a = 2000 \Rightarrow a = 400$
Statement I alone is not sufficient as the relationship between '$m$' and '$o$' is not known.

From Statement II, $m = 3/7\ (a + o)$
$\Rightarrow m = 3/7(1000 - m)$
$\Rightarrow 10m = 3000$
$\Rightarrow m = 300$
II alone is also not sufficient.
Using both the statements
$a = 400,\ m = 300$
So, $o = 1000 - (400 + 300) = 300$
So, the apples are maximum in quantity.

**16.** Using either statement, $k$ may or may not be equal to $\frac{a}{c}$
Combining both statements, $k = \frac{a+b}{c+d} = \frac{b}{d}$
$\Rightarrow ad + bd = bc + bd$
$\Rightarrow ad = bc$
$\Rightarrow \frac{a}{c} = \frac{b}{d} = k$

**17.** Either statement alone is not sufficient as one talks about only incomes and other about only expenditures.

From I and II together also we cannot find the ratio of savings, as the ratio of income to expenditure is not known.

Thus, both statements together are also not sufficient.

**18.** Neither of the statements is independently sufficient as the income and expenditure ratios are given in two different statements.

Combining both the statements, we have income ratio as 13:10 and savings ratio as 11:10.

Let the incomes of A and B be $13x$ and $10x$, respectively, and their expenditures be $11y$ and $10y$, respectively.
$\therefore$ Savings of A is $13x - 11y$ and that of B is $10x - 10y$.
As savings is a positive quantity,
$10x - 10y = 10(x - y)$ is positive;
$\Rightarrow x > y$
As $x > y$, $13x - 11y > 10(x - y)$

**19.** As the ratio of incomes of A and B is 6:5, let the actual incomes be $6x$ and $5x$, respectively.

Statement I alone is not sufficient to find the savings ratio, as the relationship between incomes and expenditures is not known.

But from this statement, we can assume the expenditures of A and B as $4y$ and $3y$, respectively.

Statement II alone is not sufficient as there is no information about B.

Combining both the statements, we have A spends $\frac{3}{4}^{\text{th}}$ of his income.

i.e., $\frac{3}{4}(6x) = 4y \Rightarrow y = \frac{9x}{8}$.

As $x$ and $y$ relationship is known, savings ratio of $k$ and B can be found.

**20.** Let the price of each pen, each sharpener and each book be $p$, $s$ and $b$ respectively.
Then $p + b + s = 30$ (1)
From statement I, we have $b - p = p - s$
$\Rightarrow 2p = b + s$ (2)
Solving (1) and (2), we can find the price of the pen.
From Statement II, as the ratio of the prices of the book, pen and sharpener are known, the price of the pen can be found using $\frac{2}{(1+2+3)} \times 30$.

# 2 Percentages, Profit and Loss and Partnerships

CHAPTER

**LEARNING OBJECTIVES**

*In this chapter, you will*:

- Understand the concept of percentage, expressing percentage as a fraction and vice-versa
- Understand about percentage changes—increase, decrease, percentage points
- Learn to apply various application of percentages to solve profit & loss questions including:
  - Profit/Loss percentage
  - Discounts
- Apply various application of percentages to solve Partnerships questions involving same time period and different time periods
- Learn to apply these applications to also solve stocks and shares based questions

## ❒ PERCENTAGE

PERCENT implies for every hundred. This concept is developed to make the comparison of fractions easier by equalising the denominators of all fractions to hundred.

For example, 7/11 as percentage is represented as

$$\frac{7}{11}=\frac{7\times100}{11\times100}=\frac{(7\times100)/11}{100}=\frac{63.63}{100}=63.63\%$$

Percentages can also be represented as decimal fractions. In such a case it is effectively equivalent to the proportion of the original quantity.

For example, 20% is the same as $\frac{20}{100}$, i.e., 0.2.

Any percentage can be expressed as a decimal fraction by dividing the percentage figure by 100 and conversely, any decimal fraction can be converted to percentage by multiplying it by 100.

percentage increase or decrease in a quantity is the ratio, expressed in percentage, of the actual increase or decrease in the quantity to the original amount of the quantity, i.e.,

$$\textbf{Percentage Increase} = \frac{\textbf{Actual increase}}{\textbf{Original quantity}} \times 100$$

$$\textbf{Percentage Decrease} = \frac{\textbf{Actual decrease}}{\textbf{Original quantity}} \times 100$$

For example, if the production of rice went up from 225 MT in 1993 to 242 MT in 1994, then the percentage increase in rice production from 1993 to 1994 is calculated as follows:

Actual increase = 242–225 = 17 MT

Percentage increase

$$= \frac{\text{Quality increase from 1993 to 1994}}{\text{Actual production of rice in 1993}} \times 100$$

$$= \frac{17}{225} \times 100 = 7^{5}/_{9}\%$$

Ratio of any two quantities also can be expressed as percentage.

For example, if the ratio of $A$ and $B$ is 3 : 2, we can say the ratio of $A$ : $B$ is 60% : 40%.

Whenever there is any percentage increase or decrease in a quantity, we can directly calculate the new value of the quantity instead of calculating the actual increase/decrease and then adding to/subtracting from the original quantity.

For example, if the increase on a value of 350 is 15%, the new quantity is $1.15 \times 350 = 402.5$ (where 1.15 = 1 + 0.15, 0.15 being the decimal equivalent of 15%).

If the production in 1994 is given as 400 MT and the increase from 1993 to 1994 is given to be 25%, then the production in 1993 will be equal to 400/1.25 = 320 MT (where 1.25 = 1 + 0.25, 0.25 being the decimal equivalent of 25%).

Similarly, if there is a decrease of 12% on a quantity of 225, then the new quantity will be equal to $225 \times 0.88$ (where 0.88 = 1– 0.12, 0.12 being the decimal equivalent of 12%).

If the production in 1994 is given as 400 MT and it is a decrease of 13% from 1993, then the production in 1993 will be equal to 400/0.87 (where 0.87 = 1 – 0.13, 0.13 being the decimal equivalent of 13%).

On the basis of percentage increase, we can write down how many times the old value gives the new value. For example, if the percentage increase is 100%, then we can conclude that the new value is 2 times the old value. If the percentage increase is 300%, the new value is 4 times the old value. If the percentage increase is 450%, then the new value is 5.5 times the old value. In general, if the percentage increase is $p$%, then the new value is $\left(\frac{p}{100} + 1\right)$ times the old value.

Conversely, if we know how many times the old value gives the new value, we can find out the percentage increase in the old value to get the new value. For example, if the new value is 3 times the old value, the percentage increase in the old value to get the new value is 200%. If the new value is 4.25 times the old value, then the percentage increase is 325%. In general, if the new value is $k$ times the old value, then the percentage increase is $(k - 1) \times 100$.

We can generalize each of the three cases as below:

**Case 1:** If the value of an item goes up/down by $x$%, the percentage reduction/increment to be now made to bring it back to the original level is $\frac{100x}{(100 \pm x)}$ %

**Case 2:** If $A$ is $x$% more/less than $B$, then $B$ is $\frac{100x}{(100 \pm x)}$ % less/more than $A$.

**Case 3:** If the price of an item goes up/down by $x$%, then the quantity consumed should be reduced/increased by $\frac{100x}{(100 \pm x)}$ % so that the total expenditure remains the same.

## ❒ PERCENTAGE POINTS

The concept of percentage points is important in the usage of percentages. Percentage points is the difference of two percentage figures.

Let us understand this with an example.

Suppose that rice forms 20% of total food grain production in Year I and 30% of total food grain production in Year II.

If we are asked to find out the percentage increase in the production of rice, calculating percentage increase from 20 to 30 as $\frac{30 - 20}{20} \times 100$ and saying it is 50% increase is not correct. with the available data, we cannot find out the percentage increase in the production of rice from Year I to Year II. We can only say that the production of rice as a percentage of total food grain production went up by 10 percentage points (the 10 being the increase from 20 to 30—both percentage figures).

We can see by taking the following figures that the percentage increase in rice production need not be 50%.

| | Year I | Year II |
|---|---|---|
| Rice | 1000 | 960 |
| Total foodgrains | 5000 | 3200 |
| Rice as percent of | 20% | 30% |
| Total foodgrains | | |

Here, while rice is 20% of total food grains in Year I and 30% of total food grains in Year II, we find that the actual production of rice has not even increased—it decreased from 1000 in Year I to 960 in Year II.

## ❒ PROFIT AND LOSS

In any business/commercial environment the most important concern is about the profit/loss of the transaction conducted.

The Selling Price (S.P.) and the Cost Price (C.P.) of an article determine the profit or loss made on the particular transaction.

The computation is done as follows:

Profit = Sale Price – Cost Price = S.P. – C.P.

$$\text{Percentage Profit} = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100 = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$\text{Loss} = \text{C.P.} - \text{S.P.}$$

$$\text{Percentage Loss} = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

It is customary to express Profit/Loss as percentage of Cost Price. However, in some problems it may specifically be given that profit/loss percentage has been calculated on the selling price or the student may be asked to calculate the profit/loss percentage on the selling price. Unless such specific directions are given, the profit/loss percentage is always to be calculated on the cost price.

Given Profit/Loss percentage along with S.P., C.P. can be found out and similarly, given Profit/Loss percentage along with C.P., S.P. can be found out by using the concepts discussed at the beginning of this chapter (where, if percentage increase or decrease is given, we can find out the new value from the old value or the old value from the new value).

The following simple rules can be remembered for this purpose.

**Rule 1:** Given the cost price (C.P.) and profit percentage $p\%$, the selling price will be given by S.P. $= \text{C.P.} \times \frac{(100+p)}{100}$

**Rule 2:** Given the cost price (C.P.) and loss percentage $p\%$, the selling price will be given by S.P. $= \text{C.P.} \times \frac{(100-p)}{100}$

**Rule 3:** Given the selling price (S.P.) and profit percentage $p\%$, the cost price will be given by C.P. $= \text{S.P.} \times \frac{100}{(100+p)}$

**Rule 4:** Given the selling price (S.P.) and loss percentage $p\%$, the cost price will be given by C.P. $= \text{S.P.} \times \frac{100}{(100-p)}$

When two articles are sold at the same price (i.e., their s.p. is the same) such that there is a profit of $p\%$ on one article and a loss of $p\%$ on the other (i.e., common profit or loss percentage), then, irrespective of what the s.p. actually is, the net result of the transaction is loss. this percentage loss is given by

**FORMULA**

Loss percentage $= \frac{(\text{Common profit or loss})^2}{100} = \frac{p^2}{100}$

Marked price or list price is the price that is indicated or marked on the product or it is the price which is given in the price list. this is the price at which the product is intended to be sold. however, there can be some discount given on this price and consequently, the actual selling price of the product may be less than the marked price.

**FORMULA**

Selling Price = Marked Price – Discount

The amount of discount given can also be expressed as a percentage. discount is always expressed as a percentage of the marked price or the list price.

discount percent

$$= \frac{\text{Marked Price} - \text{Selling Price}}{\text{Marked Price}} \times 100$$

$$= \frac{\text{Discount}}{\text{Marked price}} \times 100$$

Certain discount is given on an article whose selling price is S.P. If further discounts are given on this discounted price, such discounts are referred to as successive discounts. If the successive discounts are $p\%$, $q\%$ and $r\%$, on a product whose selling price is S.P., then the effective price after all the discounts is given by

**FORMULA**

Discounted price $= \text{S.P.} \times \frac{(100-p)(100-q)(100-r)}{100 \times 100 \times 100}$

## ❒ PARTNERSHIPS

Two or more people can get together to do business by pooling their resources. The money put in by each of the partners is called his 'investment' or 'capital.'

All the people who have invested money in the partnership are called partners.

While two or more partners would have invested money, it is not necessary that all of them should be involved in the day-to-day running of the business. The partners involved in the day-to day activities of the business are called working partners and the others are called sleeping partners or dormant partners.

The profits left after paying the working partners' remuneration/commission are shared amongst all the partners.

Sometimes, the partners also take interest on their investments and only the remaining profits are shared by the partners.

Sharing of profits among the partners also depends on the understanding between the partners. However, if no special scheme of sharing the profits is specified (in a problem), then the profits are shared based on the investments of the partners. There are three different possibilities that exist here.

- If the partners invest different amounts each for the same period of time, then the profits at the end of the year are shared in the ratio of their investments.
- If the partners invest the same amounts for different periods of time, then the profits at the end of the year are shared in the ratio of the time periods for which their respective investments have been in business.
- If the partners invest different amounts and the time periods for which their investments are in the business are also different, then the profits at the end of the year are shared in the ratio of the products of (investment *x* time period) calculated for each partner.

There can be problems that are modelled along the sharing of profits in partnerships. An example of this type is where a particular facility (like renting a tractor for ploughing their fields by three different people) is used by more than one party and the rent has to be shared by all the concerned parties—similar to sharing of profits in a partnership.

## ❒ STOCKS AND SHARES

A limited company raises capital by floating shares. It is also referred to as stock. The capital required is divided into small units called shares. In India, the generally accepted value for such a unit is ₹10 or ₹100. This is called face value or par value.

The shares of a public limited company are traded in the market place and depending on the demand for the share, the price fluctuates. The rate at which a share is bought or sold in the market is the Market Value of the share. This fluctuates. If the market value is more than the face value of the share, then we say that such a share is quoting at a **premium**. If the market value is less than the face value of the share, then we say that such a share is quoting at a **discount**.

The people who are holding the shares are called **shareholders**. The company distributes a part of its profits from its operations as **dividend** to the shareholders. The dividend is expressed as a percentage of the Par Value. Whenever any company quotes a dividend percentage figure, it goes without saying that it is a percentage of the face value.

$$\% \text{ of dividend} = \frac{\text{Dividend Amount}}{\text{Par Value}} \times 100$$

Dividend is always calculated only on the 'face value' or the 'par value' irrespective of the price at which the share was purchased.

The government also deals with stock where it issues bonds or other form of stock with a certain face value and a certain assured rate of interest. This stock is then traded in the market as per the regulations of the government. Since the government stock comes with fixed rate of return, the stock is normally referred to by the percentage of the return. For example, if 5% is the rate of return (of stock whose face value is ₹100), then such stock is referred to as 5% stock. The face value of the government bond is normally ₹100. Supposing this stock yielding 5% return (on face value) is purchased by somebody at ₹95, then we say that person has purchased '5% stock at 95'. Instead, if he purchases it at ₹108, then we say that he has purchased '5% stock at 108'.

In the case where he purchased 5% stock at 95, to buy one unit of that stock, he pays ₹95. But since the face value is ₹100, the return or income he gets at the end of the year will be 5% of 100, i.e., ₹5.

In this case, since he receives an income of ₹5 per year by investing ₹95, his rate of return is $\frac{5}{95} \times 100$ which is $5\frac{5}{19}\%$

To compare two investments (i.e., investments in two different stocks), we compare the rate of return for both investments and whichever gives a higher rate of return is a better investment.

If somebody is holding ₹1000 worth of stock, it means that the face value of stock he is holding is

₹1000. If the face value of the stock is ₹100, that person will be holding 10 units of such stock.

Typical problems in shares and stocks may include finding as to which out of given investment is a better one or finding the annual income or change in income from a certain investment or change in portfolio, etc.

These problems are very similar to problems in Profit and loss percentages except for involving the terminology as given above.

For all the examples we are going to look at, the face value of the stock is to be taken as ₹100 unless otherwise specified.

## SOLVED EXAMPLES

**2.01.** The number of tourists visiting a country increased by 80% from 1990 to 1991. From 1991 to 1992, there was a 50% increase. Find the percentage increase in the number of tourists visiting the country from 1990 to 1992.

**Sol:** Let the number of tourists visiting the country in 1990 be 100. As the number of visitors increased by 80% from 1990 to 1991, the number of visitors increased by 80% of 100, i.e., 80. Hence, the number of visitors will be 180 in 1991. Then, there was 50% increase from 1991 to 1992. This means, in 1992 the number of tourists to the country will be 180 + 90 (50% of 180) = 270. So, the number of tourists to the country went up from 100 in 1990 to 270 in 1992, an increase of 170 from the initial number of tourists of 100. Hence, the percentage increase

$$= \frac{\text{Increase}}{\text{Initial}} \times 100 = \frac{170}{100} \times 100 = 170\%$$

**2.02.** The percentage increase in the value of exports of apples of a country is as follows:
2001–2002 → 25%
2002–2003 → 20%
2003–2004 → 10%
What is the percentage increase in the value of exports of apples of the country from 2001 to 2004?

**Sol:** Let the value of exports in 2001 be 100 units. In 2002, it will become 125 units. Then there is a 20% increase in 2003, hence in 2003, value of exports = 150 units. As there is a 10% increase in 2004, value of exports in 2004 = 165 units.
Hence, overall increase in exports = 65 units starting from 100 units.
⇒ Therefore, there is a 65% increase.

**2.03.** The price of a clock imported from Switzerland was ₹1200. Due to devaluation of rupee, its value increased to ₹2100. What is the percentage increase in the price of the clock due to devaluation of rupee?

**Sol:** Percentage increase is given by

$$= \frac{\text{Final price} - \text{Initial price}}{\text{Initial price}} \times 100$$

∴ The required percentage increase

$$= \frac{2100 - 1200}{1200} \times 100 = \frac{3}{4} \times 100 = 75\%$$

**2.04.** In 1991, Anjana's salary was ₹30000 and Bhavana's salary was ₹40000. In 1990, Anjana's salary was 40% of the sum of the salaries of Anjana and Bhavana in 1991. What was the percentage increase in Anjana's salary from 1990 to 1991?

**Sol:** Anjana's salary in 1990
= (30000 + 40000) × 0.4 = ₹28000
Percentage increase in Anjana's salary

$$= \frac{30000 - 28000}{28000} \times 100$$

$$= (1/14) \times 100 = 7.14\%$$

**2.05.** 56% of a number is 448. What is 98% of the same number?

**Sol:** Let the number be $n$.
Given $(56/100) \times n = 448 \Rightarrow n = 800$
∴ 98% of 800 = (98/100) (800) = 784
**Alternate Method:**
Given 56% of a number is 448.

$$\therefore 98\% \text{ of the number} = \frac{98 \times 448}{56} = 784$$

**2.06.** The ratio of the salaries of Mehta and Dixit is 20: 21. By what percentage is Dixit's salary greater than that of Mehta?

**Sol:** The given ratio = 20: 21
The salary of Dixit is 21 parts when the salary of Mehta is 20 parts. Percentage by which Dixit's salary is greater than Mehta's

$$= \frac{21-20}{20}(100) = 5\%$$

**2.07.** The length of a rectangle is increased by 10% and the breadth is increased by 20%. What is the consequent percentage increase in its area?

**Sol:** Let the initial length be $l$ and initial breadth be $b$.
Area of the rectangle = $lb$
Increase in length = $10l/100$
Increase in breadth = $20b/100$
New area of the rectangle

$$= \left(l + \frac{10}{100}l\right)\left(b + \frac{20}{100}b\right)$$

$$= (1.1l)\ (1.2b) = 1.32lb$$

Percentage increase in area

$$= \frac{1.32lb - lb}{lb} \times 100 = 32\%$$

**2.08.** If the price of an item goes up by 10%, by what percentage should the new price be reduced to bring it down to the original price?

**Sol:** Let the original price be ₹100.
New price = ₹110, due to 10% increase. Now, to bring this down to the original price, we have to effect a reduction of ₹10 from ₹110. Hence, percentage reduction $\frac{10}{110} \times 100 = 9.09\%$

**NOTE**

[In this problem the percentage reduction can be written as $\frac{10}{(100+10)} \times 100$] and can be generalized as $\{(100x)/(100+x)\}$ %

**2.09.** If Dinesh's salary is 20% less than Eswar's salary, then by what percentage is Eswar's salary more than Dinesh's salary?

**Sol:**

**NOTE**

In general, the object or quantity which follows the phrase MORE THAN or LESS THAN in assumed as 100, to facilitate the working.

Let Eswar's salary be 100
Dinesh' salary being 20% less, it will be 80
Eswar's salary is more than Dinesh's salary by

$$\frac{100-80}{80} \times 100 = 25\%$$

**2.10.** If the price of tea goes up by 10%, then what should be the percentage decrease in the quantity consumed so that the total expenditure on tea remains the same?

**Sol:** Let $p$ be the price of tea and $q$ be the quantity consumed initially.
Since the price is increased by 10%, the new price is 1.1 $p$. Let the new quantity consumed be $z$.
As the total expenditure remains the same, we have $p \times q = 1.1p \times z$.

$$\Rightarrow z = \frac{q}{1.1}$$

$\therefore$ Percentage reduction in consumption

$$= \frac{q - \frac{q}{1.1}}{q} \times 100 = 9\frac{1}{11}\%$$

**NOTE**

In this problem, the percentage reduction in the quantity can be written as $\frac{10}{100+10} \times 100 = 9\frac{1}{11}\%$

**2.11.** A shopkeeper bought a table for ₹500 and sold it for ₹600. What is his profit percentage?

**Sol:** Given selling price of the table = ₹600
Given cost price of the table = ₹500
Profit = S.P. – C.P. = 600 – 500 = ₹100
Percentage of profit

$$= \frac{\text{Profit}}{\text{C.P}} \times 100 = \frac{100}{500} \times 100 = 20\%$$

**2.12.** A merchant gains 20% by selling a book for ₹12. Find the merchant's percentage of gain or loss if he sold the book at

(a) ₹9 (b) ₹15
(c) ₹8 (d) ₹18

**Sol:** Given selling price = ₹12 and profit = 20%
⇒ Cost price × 1.2 = 12
⇒ Cost price = ₹10
(a) Selling price = ₹9,
Cost price = ₹10

$$\text{Loss \%} = \frac{10-9}{10}\times 100 = 10\%$$

(b) Selling price = ₹15,
Cost price = ₹10

$$\text{Profit \%} = \frac{15-10}{10}\times 100 = 50\%$$

(c) Selling price = ₹8,
Cost price = ₹10

$$\text{Loss \%} = \frac{10-8}{10}\times 100 = 20\%$$

(d) Selling price = ₹18,
Cost price = ₹10

$$\text{Profit \%} = \frac{18-10}{10}\times 100 = 80\%$$

**2.13.** By selling a chair at ₹300, Soman makes a profit of 20%. Find the cost price of the chair.

**Sol:** Cost price of the chair

$$= 300\left(\frac{100}{100+20}\right)\times = ₹250$$

**2.14.** The selling price of 12 pens is equal to the cost price of 20 pens. Find the profit percentage.

**Sol:**

**NOTE**

In such problems where there is no amount specified for the cost price or the sales prices, the best approach is to assume cost of each unit to be Re.1 and proceed.

Let the cost price of each pen be ₹ $x$
Cost price of 12 pens = ₹ $12x$
Selling price of 12 pens = Cost price of 20 pens = ₹ $20x$

$$\text{Percentage of profit} = \frac{20x-12x}{12x}\times 100$$

$$= 66\frac{2}{3}\ \%$$

**2.15.** The cost price of 6 oranges is equal to the selling price of 8 oranges. Find the profit or loss percentage.

**Sol:** Let the cost price of each orange be ₹ $x$
Cost price of 8 oranges = ₹ $8x$
Selling price of 8 oranges = cost price of 6 oranges = ₹ $6x$

$$\text{Percentage of loss} = \frac{8x-6x}{8x}\times 100 = 25\%$$

**2.16.** A trader cheats his customers to make profit by announcing that he sells the goods at cost price but gives 200 gm less for every 1 kg. Find the profit percentage of the trader.

**Sol:** From the given data, selling price of 800 gm = cost price of 1000 gm
Let the cost price of each gram be ₹ $x$, then cost price of 800 gm = ₹ $800x$
Selling price of 800 gm = cost price of 1000 gm = ₹ $1000x$
Profit percentage

$$= \frac{1000x-800x}{800x}(100) = 25\%$$

**2.17.** A sells a suitcase to B at 10% profit. B sells it to C at 30% profit. If C pays ₹2860 for it, then what is the price at which A bought the suitcase?

**Sol:** Let the price at which A bought the suitcase be $x$.
B's C.P = $x$ (110/100) = $1.1x$
C's C.P = $(1.1x)$ (130/100) = $(1.1x)$ (1.3)
C bought the suitcase at ₹2860
⇒ $(1.1x)$ (1.3) = 2860
⇒ $x$ = 2860/ (1.1) (1.3)
⇒ $x$ = ₹2000

**2.18.** Ajay sold his bag at a loss of 6%. Had he sold it for ₹42 more he would have made a profit of 8%. Find the cost price of the bag.

**Sol:** Let the cost price be ₹100.
Since he sold the bag at a loss of 6%, selling price would be ₹94.
Had he sold the bag at 8% profit, selling price would have been ₹108.
Difference in the selling prices is ₹14, when the C.P. is ₹100
But, it is given that difference in the selling prices is ₹42. Hence, the cost price

$$= \frac{42\times 100}{14} = 300$$

**Alternate Method:**

Loss on the sale of the bag = 6%

If sold at ₹42 more, gain = 8%

$\therefore [8\% - (-6\%)] \times$ C.P. = 42

$\Rightarrow 14\% \times$ C.P. = 42 $\Rightarrow$ C.P. = ₹300

**2.19.** Rathod sells a chair at a loss of 15%. Had he sold it for ₹70 more he would have made a profit of 5%. What is the cost price of the chair?

**Sol:** The difference in profit and loss percentages is $5 - (-15) = 20\%$

Let the C.P of the chair be $C$

$\therefore 20\%$ of $C$ = ₹70

$C = 70/20\% =$ ₹350

**2.20.** Praveen bought 18 kg of rice for ₹360. He was forced to sell it at a loss which is as much as the selling price of 4.5 kg. At what price did he sell the rice?

**Sol:** Let S.P. per kg be $S$. Given loss = $4.5S$

As Loss = C.P. – S.P.

$\Rightarrow 4.5S = 360 - 18S \Rightarrow S =$ ₹16

**2.21.** Ajit buys a certain number of bananas at 8 for ₹10 and an equal number at 10 for ₹15. If he sells them at 15 for ₹20, does he gain or lose and by what percentage?

**Sol:** Let the number of bananas that Ajit bought be $2x$, i.e., $x$ at each of the two prices.

The cost price of $x$ bananas at 8 for ₹10 = $x(10/8)$ = ₹$5x/4$

Similarly, cost price of $x$ bananas at 10 for ₹15 = $x(15/10)$ = ₹$3/2x$

Cost price of the bananas = $5x/4 + 3x/2$ = ₹$11x/4$

Selling price of the bananas = $(2x)\ (20/15)$ = ₹$8x/3$

As $\frac{8x}{3} < \frac{11x}{4}$, there is loss.

$$\text{Percentage of loss} = \frac{\frac{11x}{4} - \frac{8x}{3}}{\frac{11x}{4}} \times 100 = \frac{100}{33}\%$$

Hence, Ajit incurred a loss of $\frac{100}{33}\% = 3.03\%$

**2.22.** If goods are purchased for ₹120, and one-third of them sold at a loss of 5%, then at what profit percentage should the rest be sold to obtain an overall profit percentage of 5%?

**Sol:** Cost price of one-third of goods

= 1/3 (120) = ₹40

Selling price of these goods at 5% loss

= (40) (0.95) = ₹38

Let selling price of the rest of the goods be $x$.

5% profit on ₹120 gives S.P. as ₹126

$\Rightarrow 126 = x + 38 \Rightarrow x =$ ₹88

As cost price of the remaining goods is ₹80, required profit %

$$= \frac{88 - 80}{80} \times 100 = 10\%$$

**2.23.** Prakash calculates his profit on selling price and finds it to be 20%. What is his actual profit percentage?

**Sol:** Let's assume selling price as ₹100

Since profit = 20% of selling price, actual profit = 20/100 (100) = ₹20

Cost price = 100 – 20 = ₹80

$\therefore$ Actual profit percentage, i.e., with reference to the cost price = $20/80 \times 100 = 25\%$

**2.24.** Ashok sells two radio sets at the same price, one at a loss of 12% and the other at a profit of 12%. Find the overall loss or gain percentage.

**Sol:** When 2 items are sold at the same selling price, one at $p$% profit and the other at $p$% loss, irrespective of what the selling price is, the net result is always a loss and the loss percentage = $(p^2/100)$.

Here the common profit or loss = 12%

$\therefore$ Net loss = $12^2/100 = 1.44\%$

So, Ashok incurs 1.44% loss overall.

**2.25.** If Ajay sells an item at three-fourths of its selling price he incurs a loss of 4%. What will be the profit or loss percentage if he sells it at the actual selling price?

**Sol:** Let the cost price be ₹100

When sold at $3/4^{th}$ of the selling price, the loss is 4%. This means selling price in this case = ₹96 = 3/4 times the actual selling price

Hence, 96 = (3/4) [Actual selling price]

$\Rightarrow$ Actual selling price

= (96) (4/3) = ₹128

If Ajay sells at the actual S.P, then he makes a profit of ₹28 on a cost price of ₹100, i.e., 28% profit.

**2.26.** A trader marks his product 15% above the cost price and offers a discount of 20%. Find the loss percentage incurred by the trader.

**Sol:** Let the cost price be ₹100, then marked price = ₹115

Since a discount of 20% is allowed on ₹115, selling price = (80/100) × 115 = ₹92

$$\text{Loss \%} = \frac{100-92}{100} \times 100 = 8\%$$

**2.27.** A trader marks his goods at a certain percentage over his cost price and then gives a 30% discount, thereby making 5% profit. What is the mark up percentage?

**Sol:** Let cost price be ₹100, then
selling price = ₹105
Since 30% discount was given,
70% of MP = sale price = 105, where MP is the marked price.
Marked price = ₹150.
Hence, the marked price is ₹50 above the cost price, i.e., 50% above the cost price of ₹100.

**2.28.** A and B invest ₹27000 and ₹22500 respectively in a business and at the end of the year, they make a profit of ₹35200. Find their individual shares in the profit.

**Sol:** Since their investments are there in the business for the same duration (1 year) profits will be shared in the ratio of their investments, i.e., 27000: 22500 = 6 : 5
∴ A's share = 6/11 (35200) = ₹19200
B's share = 5/11 (35200) = ₹16000

**2.29.** Ali starts a business with ₹36000. Three months later Akbar joins him with ₹21000. At the end of the year in what ratio should they share the profits?

**Sol:** As the investments of Ali and Akbar are for different time periods, sharing of profits will be in the ratio of investments multiplied by the respective time periods.
Hence, ratio is (36000 × 12): (21000 × 9)
= 48: 21 = 16: 7

**2.30.** Dinesh started a business with ₹20000 and after 5 months, Dhiraj joined him with ₹45000. Dinesh received ₹45250 including 10% of the profits as commission for managing the business. What amount did Dhiraj receive?

**Sol:** Ratio of the shares of profits is
(20000 × 12): (45000 × 7) = 240: 315 = 16: 21.
Let the total profit be *P*. As Dinesh receives 10% of this as commission, the remaining 90% of *P*, is shared in the ratio of 16: 21. Hence, Dinesh's receipts will be $16/37^{th}$ of 90% of the total profit plus his commission.
$0.1p + 16/37\ (0.9p) = 45250$
$\Rightarrow p = ₹92500$
Dhiraj's share = Total profit – payment to Dinesh
= 92500 – 45250 = ₹47250

**2.31.** A started a business with ₹30000. After 4 months B joined him with ₹40000. C joined them after some more time with ₹50000. If C gets ₹15000 as his share at the end of the year out of a total profit of ₹49000, how many months after B joined the business did C join?

**Sol:** The ratio of the shares of profits is
(30000 × 12): (40000 × 8): (50000 × *x*).
(Here *x* is the number of months that C was with the business) = 36: 32: 5*x*

$$\text{C's share } \frac{5x}{36+32+5x} = \frac{5x}{68+5x}$$

$$\text{Given, } \frac{5x}{68+5x} = \frac{15000}{49000} \Rightarrow x = 6$$

∴ C joined the business 2 (i.e., 6 – 4) months after B joined the business.

**2.32.** Girish started a business with ₹25000. After three months, Harish joins with an investment of ₹30000 and Girish withdraws ₹5000 out of his capital. Three months later, Harish brought in ₹10000 more. At the end of the year what should be the ratio in which they share the profits?

**Sol:** Girish has invested ₹25000 for 3 months and then since he withdrew ₹5000, his investment was only ₹20000 for the rest of the nine months. The term of the ratio that will represent Girish's share will be
(25000 × 3) + (20000 × 9) = 255000
Harish joined with ₹30000 which remained unchanged for 3 months and then since he brought in ₹10000 more, his investment was ₹40000 for the rest of the 6 months. The term of the ratio that will represent Harish's share will be (30000 × 3) + (40000 × 6) = 330000
Ratio in which they share their profits
= 255000: 330000 = 51: 66 = 17: 22

**2.33.** The commission paid to the working partner of a business is equal to 30% of the profit left after his commission is paid. If his commission is known to be ₹24000, then find the total profit.

**Sol:** Let the total profit be *p*. The profit left after the working partner's commission of ₹24000 is paid is (*p* – 24,000). 30% of this is the working partner's commission. So, we have 0.3 (*p* – 24,000) = 24,000

$0.3p = ₹31200$

$p = ₹104000$

**2.34.** What is the annual income from ₹41000 invested in 4% stock at 2.5% premium?

**Sol:** 2.5% premium means the market value is ₹102.50 when the face value of the share is ₹100. Since ₹41000 is invested in this stock, the number of units purchased = 41,000/102.5 = 400. Since this is 4% stock, each unit of this stock will yield 4% interest on ₹100 at the end of the year. Therefore 400 units will have an annual income of $400 \times 4 = ₹1600$.

**2.35.** Which of the following is a better investment 6% stock at 125 or 5% stock at 114?

**Sol:**

**NOTE**

We can calculate the rate of return for each of these investments and decide which is better. Another approach is to take a certain amount as invested in each of these two stocks and calculate the income from each stock. For this purpose, instead of taking any arbitrary amount, if we take the amount invested as the product of the market value of both the stocks, then calculations become simple.

Let the amount invested be equal to the product of the market value of both the stocks, i.e., $125 \times 114 = ₹14250$.

In 6% stock we get $\frac{125 \times 114}{125} \times 6 = ₹684$

The same amount in 5% stock we get $\frac{125 \times 114}{114} \times 5 = ₹625$

Since the annual income from 6% stock is higher, it is a better investment.

**2.36.** A man owned ₹12500 worth of 3% stock. When it was quoting at ₹216, he sells it and uses the proceeds to buy 5% stock quoting at ₹160, so that his annual income doubles. How much money was he left with or how much more money was he required to bring in?

**Sol:** ₹12500 worth of stock means it refers to face value, which is ₹100 for 1 unit. Hence, he owned 125 units. When he sells at ₹216, his sales proceeds will be $216 \times 125 = ₹27000$. Since each unit sold gave him ₹3 as income per annum his annual income was $125 \times 3 = ₹375$

With the investment in the second stock, his income doubled, new income = $2 \times 375 = ₹750$.

To get the annual income of ₹750, he must have bought 750/5 = 150 units. (∵ he is buying 5% stock)

Market price of 5% stock was ₹160.

To buy 150 units, he will need to pay $160 \times 150 = ₹24000$

The sale proceeds from 3% stock was ₹27000.

Hence, the difference between ₹24000 and ₹27000, i.e., ₹3000 is the amount he is left with after the transaction.

**2.37.** A person invests ₹28200 in 10% stock at 94. He then sells it when it is quoting at 106. He then reinvests the money in 6% stock at 100 which he sells when the stock is quoting at 107. Find the overall profit of the transaction.

**Sol:** At ₹94 per unit, ₹28200 will get him 28200/94 = 300 units. These when sold at 106 give him ₹31800. This is invested in 6% stock at 100. This gets him 31800/100 = 318 units. This stock is then sold when it is quoting at 107 giving him $318 \times 107 = ₹34026$

His overall profit = 34026 – 28200 = ₹5826

**2.38.** Rice production in a country increased by 25% from 2000 to 2004. It increased by 20% from 2004 to 2008. Find the percentage increase in the rice production from 2000 to 2008.

**Sol:** Let the rice production in 2000 be 100 tonnes.

Rice production in 2004

$= 100 + \frac{25}{100}(100) = 125$ tonnes

Rice production in 2008

$= 125 + \frac{20}{100}(125) = 150$ tonnes

∴ Percentage increase in the rice production from 2000 to 2008 is $\frac{150-100}{100} \times 100$, i.e., 50%.

**2.39.** The population of a country increased by 10% from 2001 to 2002. It increased by 20% from 2002 to 2003. It increased by 30% from 2003 to 2004. Find the simple average yearly percentage increase in the population from 2001 to 2004.

**Sol:** Let the population of the country in 2001 be 100 million. Population in 2002 = 110 million.

Population in 2003 = $110 \times 1.2 = 132$ million.

Population in 2004 = $132 \times 1.3$

= 171.6 million

Simple average percentage increase

$$= \frac{171.6-100}{\frac{100}{3}}(100) = 23\frac{13}{15}\ \%$$

**2.40.** The price of a Swiss watch was ₹10000 in 2001. Due to devaluation of the rupee it becomes ₹12000 in 2002. Find the percentage increase in its price from 2001 to 2002.

**Sol:** Percentage increase

$$= \frac{\text{Final price} - \text{Initial price}}{\text{Initial price}}(100)$$

$$= \frac{12000-10000}{10000}(100) = 20\%$$

**2.41.** In 2002, Rakesh's salary was ₹24000. In 2001, his salary was equal to that of Ramesh. Rakesh's salary in 2001 was 20% less than his salary in 2002. Ramesh's salary in 2002 was 20% more than his salary in 2001. By what percentage is the sum of the salaries of Ramesh in both the years more / less than that of the Rakesh in both the years?

**Sol:** Salary of Ramesh in 2001 = Salary of Rakesh in 2001

$$= 24000\left(1-\frac{20}{100}\right) = 19200$$

Salary of Ramesh in 2002

$$= 19200\left(1+\frac{20}{100}\right) = ₹23040.$$

Total salary of Rakesh in both the years = 19,200 + 24,000 = ₹43200. Total salary of Ramesh in both the years = 19200 + 23040 = ₹42240, which is ₹960 less than that of Rakesh.

∴ Required percentage

$$= \frac{960}{43200}(100) = 2\frac{2}{9}\%$$

**2.42.** 64% of a number is 416. Find 85% of that number.

**Sol:** Let the number be $x$

$$\frac{64}{100}x = 416$$

$$x = 650$$

$$\frac{85}{100}x = 552\cdot5$$

**Alternative Method:**

64% = 416

$$85\% = \frac{85}{64}(416) = 552\cdot5$$

**2.43.** The ratio of the salaries of A and B is $2:2\frac{1}{7}$. By what percentage is B's salary greater than A's salary?

**Sol:** Let the salary of A be $2x \Rightarrow$ Salary of B = $₹2\frac{1}{7}x$

∴ The salary of B is more than the salary of A by

$$\frac{\frac{1}{7}x}{2x}(100) = 7\frac{1}{7}\%$$

**2.44.** The height of a triangle as well as its base are increased by 30%. Find the percentage increase in its area.

**Sol:** Let the original height as well as the base be 100 cm. Original area = $\frac{(100)(100)}{2} = \frac{100^2}{2}$

New Height = New Base = 130 cm

$$\text{New area} = \frac{(130)(130)}{2}$$

$$= \frac{1}{2}\left((1.3)(100)^2\right) = \frac{1.69\left(100^2\right)}{2}$$

= 1.69 (original area)

∴ The area increased by 69%.

**2.45.** The price of an article is increased by 25%. By what percentage must this price be reduced to bring it down to the original price?

**Sol:** Let the original price be ₹100

New price = 100 × 1.25 = ₹125

To bring back the price to ₹100 it has to be reduced by ₹25

$$\text{Percentage reduction} = \frac{25}{125}(100) = 20\%$$

**2.46.** If Ram's salary is 20% less than Shyam's salary, by what percentage is Shyam's salary more than Ram's salary?

**Sol:** Let Shyam's salary be ₹100

Ram's salary = ₹80

Shyam's salary is more than Ram's salary by

$$\frac{20}{80}(100) = 25\%$$

**2.47.** If the price of tea goes up by $33\frac{1}{3}\%$, what should be the percentage by which its consumption must be reduced so that the expenditure on it remains unchanged?

**Sol:** Let the original price be ₹300 per kg and the original consumption be 100 kg. Original expenditure = New expenditure = ₹30000.

$$\text{New price} = 300\left(1+\frac{1}{3}\right) = ₹400$$

$$\text{New consumption} = \frac{30000}{400}, \text{ i.e., } 75 \text{ kg.}$$

Percentage reduction in consumption

$$= \frac{25}{100}(100) = 25\%$$

**NOTE**

In the above three examples, if the percentage given initially is $x$, what is asked to be found is $\frac{100x}{(100+x)}$.

**2.48.** Alok bought a watch for ₹250 and sold it for ₹300. Find his profit percentage.

**Sol:** Given that

S.P = ₹300 C.P = ₹250

$\Rightarrow$ Profit = S.P–C.P = ₹300 – ₹250 = ₹50

$$\text{Profit } \% = \frac{50}{250}(100) = 20\%$$

**2.49.** Anand gained 20% by selling a book at ₹30. Find his gain percentage if he sells it for ₹36.

**Sol:** Let his cost price be ₹$x$

$$\text{Given, } \left(1+\frac{20}{100}\right)x = 30 \Rightarrow x = 25$$

Gain percent when sold at ₹36

$$= \frac{36-25}{25}(100) = 44\%$$

**2.50.** Ajay calculated his profit / loss percentage on his selling prices. Find his actual profit/loss percentage if he calculated

(a) his profit percentage to be 25%

(b) his loss percentage to be 25%

**Sol:** Let his selling price be ₹100

(a) Profit = ₹25 $\Rightarrow$ C.P. = S.P.–Profit = ₹75

Actual profit percentage

$$= \frac{25}{75}(100) = 33\frac{1}{3}\%$$

(b) Loss = ₹25 $\Rightarrow$ C.P. = S.P. + Loss = ₹125

Actual loss percentage

$$= \frac{25}{125}(100) = 20\%$$

**2.51.** The cost of 4 apples equals the selling price of 3 apples. Find the profit/loss percentage.

**Sol:** Let the cost of each apple be ₹$x$

Cost of 3 apples = ₹$3x$

Selling price of 3 apples = Cost price of 4 apples = ₹$4x$

$\therefore$ Profit on selling 3 apples = ₹$x$

$$\text{Profit } \% = \frac{x}{3x}(100) = 33\frac{1}{3}\%$$

**2.52.** The profit made by selling 5 m of a cloth equals the selling price of 2 m of that cloth. Find the profit percentage made.

**Sol:** S.P. (5 m) = C.P. (5 m) + Profit (5 m)

As Profit (5 m) = S.P. (2 m),

S.P. (5 m) = C.P. (5 m) + S.P. (2 m)

$\Rightarrow$ S.P. (3 m) = C.P. (5 m)

$$\Rightarrow \frac{\text{S.P.}}{\text{C.P.}} = \frac{5}{3}$$

$$\therefore \text{Profit } \% = \frac{5-3}{3}\times 100 = 66\frac{2}{3}\%$$

**2.53.** A trader promised his customers to sell at cost price. But he cheats his customers by giving 100 gms less for every kg that he sells. Find his profit percentage.

**Sol:** Let the cost of each gm to the trader be ₹1. Cost price of 1000 gms = Selling price of 900 gms.

ost of 900 gms = ₹900

elling price of 900 gms = ₹1000

$$\text{rofit percentage} = \frac{1000-900}{900}(100) = 11\frac{1}{9}\%.$$

**2.54.** A sold a chair to B at 10% profit. B sold it to C at 20% profit. If C bought it for ₹660, find the price at which A bought the chair.

**Sol:** Let the cost price of A be ₹$x$

Selling price of A = Cost price of B = ₹$1.1x$

Selling price of B = Cost price of C = ₹$1.32x$

Given that $1.32x = 660 \Rightarrow x = 500$

$\therefore$ A bought it at ₹500

**2.55.** Kiran sold a table at 20% loss. If he sold it at 10% profit, he would have earned ₹300 more. Find the cost price of Kiran.

**Sol:** Let the cost price of Kiran be ₹ $x$
Selling price of Kiran = ₹ $0.8x$
If he sold it at 10% profit, his selling price = ₹ $1.1x$
Given that
$1.1x = 0{\cdot}8x + 300$
$\therefore x = 1000$

**2.56.** Ashok bought 15 kg of cashew nuts for ₹ 540. He was forced to sell them at a loss equal to the amount he would have realized by selling 3 kg. Find his selling price per kg.

**Sol:** C.P. = S.P. + Loss
$\Rightarrow$ C.P. (15 kg) = S.P. (15 kg) + S.P. (3 kg)

$\Rightarrow$ C.P. (15 kg) = S.P. (18 kg) = ₹ 540 (given)

$\therefore$ selling price per kg = $\frac{₹540}{18}$ = ₹ 30

**2.57.** Shyam purchased some goods for ₹ 1800. He sold one-third of the goods purchased at 20% loss. Find the profit percentage at which the rest of the goods must be sold to realize an overall profit of 20%.

**Sol:** Suppose that Shyam purchased 3 kg for ₹ 1800
Cost price of 1 kg = ₹ 600
He sold one kg at a loss of 20%. Hence, his selling price of this kg = ₹ 480
His total selling price must be (1.2) × 1800 = ₹ 2160
Hence, selling price of the remaining 2 kg must be ₹ 1680. Cost price of 2 kg = 2 × 600 = ₹ 1200

$$\therefore \text{Profit percentage} = \frac{1680-1200}{1200}(100) = 40\%$$

**2.58.** Usha bought a certain number of chocolates at the rate of 16 chocolates for ₹ 12 and the same number of chocolates at the rate of 24 chocolates for ₹ 20. She sold all of them at the rate of 30 chocolates for ₹ 30. Find her gain / loss percentage.

**Sol:** Let $2x$ be the total number of chocolates that she bought.
otal cost price of Usha

$$\frac{12}{16}(x) + \frac{20}{24}(x) = ₹\frac{19}{12}x$$

Total selling price of Usha = $\frac{30}{30}(2x) = ₹\ 2x$

Hence, the gain% of Usha

$$= \frac{2x - \frac{19}{12}x}{\frac{19}{12}x}(100) = \frac{500}{19}\% = 26\frac{6}{19}\%$$

**2.59.** Anwar bought two radios at the same price. He sold one at $x$% profit and the other at $y$% loss. Find his overall profit/loss percentage, if
(a) $x = y = 10$
(b) $x = 20, y = 10$
(c) $x = 10, y = 20$

**Sol:** Let the cost price of each radio be ₹ 1000.
Total cost price = ₹ 2000
(a) Selling price of the radio sold at profit = 1.1 × 1000 = ₹ 1100
Selling price of the radio sold at loss = 0.9 × 1000 = ₹ 900
Total selling price = Total cost price = ₹ 2000
$\therefore$ Neither profit nor loss is made in the entire transaction.
(b) Selling price of the radio sold at profit = 1.2 × 1000 = ₹ 1200
Selling price of the radio sold at loss = 0.9 × 1000 = ₹ 900
Total selling price = ₹ 2100
Overall profit = ₹ 100

$$\text{Overall profit \%} = \frac{100}{2000}(100) = 5\%$$

(c) Similar to a method as in (ii), it can be shown that overall loss = 5%

**NOTE**

Overall profit/loss percentage is always half of the difference of $x$ and $y$.

**2.60.** Balu sold two TV sets, one at 10% profit and the other at 10% loss. Find his overall profit/loss percentage if he sold both the sets at the same price.

**Sol:** When two items are sold at the same selling price with one being sold at $x$% profit and the other being sold at $x$% loss, on the overall a loss is always made and the overall loss percentage is given by $\frac{x^2}{100}$%. As Balu sold both TV sets at the same price and $x$ = 10, he must have made an overall loss of $\frac{10^2}{100}$, i.e., 1%

**2.61.** If Ram sold an article at four-fifth of its actual selling price, he would have incurred a loss of 40%. Find his actual profit / loss percentage.

**Sol:** Let the cost price of Ram be ₹ 100
Given that four-fifth of its actual selling price 0.6 × 100 = ₹ 60. Hence, its actual selling price

is $\frac{5}{4} \times 60 = ₹75$

$\therefore$ Ram made a loss of 25%.

**2.62.** A trader marked his goods 40% above his cost price. He then gave a discount of 20%. Find his profit percentage.

**Sol:** Let the cost price be ₹100.
Marked price = $1.4 \times 100 = ₹140$.
Selling price = $0.8 \times 140 = ₹112$.

$\therefore$ Profit % = $\frac{112-100}{100} \times 100 = 12\%$.

**2.63.** Sachin gave a discount of 20% on the marked price of his watch and then sold it. He made a profit of 25%. By what percentage did he mark the watch above its cost price?

**Sol:** Let the cost price of the watch be ₹100
Let the marked price be ₹ $x$.
Selling price = ₹ $0{\cdot}8x$
Also given that the selling price = ₹125
$\Rightarrow 0{\cdot}8x = 125$

$x = \frac{625}{4} = 156{\cdot}25$

$\therefore$ Percentage of mark up = 56·25

**2.64.** Alex sold his goods after announcing two successive discounts of 30% each. Find his effective discount percentage.

**Sol:** Let the marked price be ₹100. Price after the first discount = ₹70. Price after the second discount $0.7 \times 70 = ₹49$
$\therefore$ Effective discount percentage is 100 – 49 = 51%.

**2.65.** Ram and Shyam invested ₹18000 and ₹21600 respectively in a business and at the end of the year shared a profit of ₹22000. Find the profit share of Ram.

**Sol:** Ratio of profits when ivested for the same periods = ratio of investments = 18000 : 21600 = 5 : 6

Profit share of Ram = $\frac{5}{11}(22000) = ₹10000$

**2.66.** David started a business with ₹40000. Three months later, Edward joined him with ₹50000. Find the ratio in which they must share the annual profit.

**Sol:** Ratio of the profits of David and Edward = The ratio of the product of their investments and the period of investment.
= (40000) (12) : (50000) (9) = 16 : 15

**2.67.** Sachin started a business with ₹20000 and after 4 months Sunil joined him with ₹40000. Sachin received ₹39000 as his annual profit share which included a salary of 16% of the annual profit. Find the annual profit share of Sunil.

**Sol:** Let the total annual profit be ₹ $x$
Ratio of the part of the total annual profit which is shared in the ratio of the (investments × time)
= (20000) (12) : (40000) (8) = 3 : 4.
Hence, the share of Sachin

$= \frac{3}{7}\left(\frac{84}{100}x\right) + \frac{16x}{100} = 39000$

$x = 75000$
$\therefore$ Annual profit share of Sunil
= ₹75000 – ₹39000 = ₹36000

**2.68.** A started a business with ₹20000. After 3 months, B joined him with ₹40000. After some more months, C joined them with ₹100000. B received ₹18000 out of the total annual profit of ₹55000. How many months after A started the business did C join?

**Sol:** Let us say C joined after $x$ months.
Profit is shared in the ratio
(20000) (12) : (40000) (9) : 100000 (12 – $x$)
= 24 : 36 : 10 (12 – $x$)

Given $\frac{36}{180-10x} = \frac{18}{55} = \frac{36}{110}$

$180 - 10x = 110$
$\therefore\ x = 7$

**2.69.** Mohan started a business with ₹20000. After 4 months, Sachin joined him with ₹30000. At the beginning of the fifth month, Mohan added ₹10000. Find the ratio in which the annual profit will be shared.

**Sol:** Ratio of the profit shares
= [(20000) (12) + (10000) (8)] : [30000 (8)]
= 4 : 3

**2.70.** The salary of a working partner equals 20% of the annual profit remaining after his salary is paid. If his salary is ₹10000, find the annual profit.

**Sol:** Let the annual profit be ₹ $x$

Given, $10000 = \frac{20}{100}(x - 10000)$

$\Rightarrow x = 60000$

**2.71.** What is the annual income from ₹32400 invested in 12% stock at 8% premium?

**Sol:** Market value = ₹108

Number of units purchased = $\frac{32400}{108} = 300$

Each unit purchased gives ₹12 as income.

∴ Annual income = (12) (300) = ₹3600

**2.72.** Which of the following is a better investment – 6% stock at 84 or 8% stock at 96?

**Sol:** In the first investment, ₹84 must be invested to obtain ₹6 as income.

∴ ₹14 must be invested to obtain ₹1 as income.

In the second investment, ₹96 must be invested to obtain ₹8 as income.

∴ ₹12 must be invested to obtain ₹1 as income.

∴ The second investment is a better investment.

**2.73.** Sastry invests half his salary in 4% stock at 95 and the other half in 8% stock at 105. Sharma invests the same amounts in 4% stock at 90 and 8% stock at 110. Who receives a better percentage yield?

**Sol:** Let the total Amount be (2 × 95 × 105 × 90 × 110);

(Common multiple of prices)

Sastry makes $(4 \times 105 \times 90 \times 110) + (8 \times 95 \times 90 \times 110) = 90 \times 110 \times 4\ (105 + 190)$ (1)

Rate of return = $\frac{295 \times 90 \times 110 \times 4}{2 \times (95 \times 105 \times 90 \times 110)} \times 100$

= 5.09%

Sharma makes

$(4 \times 105 \times 95 \times 110) + (8 \times 105 \times 95 \times 90) = (110 + 180) \times 4 \times 105 \times 95$ (2)

Rate of return = $\frac{290 \times 4 \times 105 \times 95}{2 \times 110 \times 90 \times 95 \times 105} \times 100$

= 5.08%.

**NOTE**

Even without calculating the rates of return, the numbers under (1) and (2) can be compared and the result obtained.

## EXERCISE-1

***Directions for questions 1 to 60*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** What is the annual income of a person who invested ₹20,000 in a 5% stock at par? (in ₹)
(A) 1000 (B) 1100 (C) 1200 (D) 900

**2.** 62.5% of 960 = _____ .

**3.** Anil and Sunil started a business with investments of ₹20000 each. Anil left after eight months and Sunil stayed for an year. If the annual profit for that year is ₹2000, Anil's share (in ₹) is _____ .

**4.** A man spends 75% of his income. If he saves ₹2000, then his income in rupees is _____ .

**5.** The population of a village this year is twice the population of the same village last year. Find the percentage increase in the population of the village from last year to this year. _____

**6.** A man invested ₹17,400 in 6% stock at 13% discount. What is his yield percent approximately?
(A) 7% (B) 6.5% (C) 5.5% (D) 7.5%

**7.** A fruit vendor purchased 21 apples for ₹180. Because of higher temperature, $33^1/_3$% of apples are rotten and had to be thrown away. The price that the trader should charge per apple if he wants to earn a profit of $16^2/_3$% is _____ . (in ₹)

**8.** Sameer started a business with an investment of ₹9000. After three months, Tarun joined with an investment of ₹12000. If the annual profit for that year is ₹6000, Sameer's share (in ₹) is _____ .

**9.** If a and *b* are positive numbers satisfying 30% of *a* = 40% of *b*, then by what percentage is *a* more/less than *b*?
(A) 50% more (B) $33^1/_3$% less
(C) $33^1/_3$% more (D) 50% less

**10.** 80 is what percent of 64? _____

**11.** The successive discounts of 20% and 15% are equivalent to a single discount of
(A) 35% (B) 38%
(C) 32% (D) 29%

**12.** If 20% of a number *n* is 30 more than 12.5% of 400, then what is 36% of *n*?
(A) 320 (B) 144 (C) 180 (D) 160

**13.** The fraction equivalent of 23.33% is
(A) 7/30 (B) 16/75
(C) 4/15 (D) 9/40

**14.** Ashok's salary increased by 25% this year. By what percentage was his last year salary less, when compared to this year's salary? _____

**15.** If Ajay scored 30% more than Robin in a test, find the ratio of the scores of Robin and Ajay in the test.
(A) 5 : 7 (B) 10 : 13 (C) 13 : 10 (D) 7 : 5

**16.** Find the value of 40% of 20 – 80% of 40 + 30% of 10 – 4% of 100.
(A) 25 (B) –26 (C) –25 (D) –27

**17.** If the rate of sales tax is reduced from 12.5% to 8%, how much less has one to pay for purchasing an item whose price is ₹ 40,000?
(A) ₹ 18000 (B) ₹ 3600 (C) ₹ 36000 (D) ₹ 1800

**18.** The cost prices of two shirts are equal, one shirt is sold for 20% profit and the other is sold for 10% loss. Find the overall profit percentage. _____

**19.** In an examination, A scored 90 marks which is 50% more than the pass mark. What is the pass mark in the examination?
(A) 35 (B) 40 (C) 50 (D) 60

**20.** Ajay got 30% of the maximum marks in a test and failed by 15 marks. Bala got 40% of the maximum marks in it and got 5 marks more than the pass mark. Find the maximum marks and also the pass marks.
(A) 200; 75 (B) 200; 60
(C) 250; 75 (D) 300; 90

**21.** Ajay invested ₹ 3,120 in a 8% stock at ₹ 104. Sujan invested ₹ 3,800 in a 5% stock at ₹ 95. Who will get more income at the end of the year?
(A) Sujan (B) Ajay
(C) Both get equal incomes (D) Cannot say

**22.** If 35% of a fraction $\frac{3n}{5}$ is 10 more than 10% of 110, then 10% of $n$ is _____ .

**23.** If A got 80 marks and B got 60 marks, then what percent of A's marks is B's marks?
(A) 60% (B) 80% (C) 65% (D) 75%

**24.** 27% of 600 – 15% of 840 =
(A) 60 (B) 36 (C) 23 (D) 16

**25.** A radio is purchased for ₹ 55,000 and is sold for ₹ 44,000. Find the loss percentage.
(A) 15% (B) 25% (C) 20% (D) 33%

**26.** If the length and breadth of a rectangle are increased by 10% each, then by what percent will the area increase?
(A) 15% (B) 10%
(C) 21% (D) 20%

**27.** Which of the following fractions is nearest to 83.33%?
(A) 8/9 (B) 4/9
(C) 5/7 (D) 5/6

**28.** The monthly salaries of two persons are equal. If the monthly salary of one of them is increased by 20% and that of the other is decreased by 20%, find the percentage change in the total salary of the two persons. _____

**29.** A trader, by mistake, calculates his profit on the selling price and expresses it as 20%. What is the correct figure of his profit?
(A) 22% (B) 25%
(C) 22¼% (D) 23½%

**30.** If $16^2/_3$% of 600 = 12½% of $x$, then $x$ =
(A) 800 (B) 1000 (C) 1200 (D) 1600

**31.** Find the annual income (in ₹) of a person who invested ₹ 19300 in a 6% stock at 3.5% discount. _____

**32.** The ratio of two positive numbers is 4 : 9. By what percentage should the first number be increased so that both the numbers will be equal?
(A) 125% (B) 80%
(C) 55.55% (D) 44.44%

**33.** A profit of 12% is made by selling an article for ₹ 50.40. Find the cost price of the article.
(A) ₹ 46 (B) ₹ 56.45
(C) ₹ 44.35 (D) ₹ 45

**34.** In a test, Karan got 40% of the maximum marks and Kiran got 50% of the maximum marks. Karan's score was less than Kiran's score by _____ percentage points.

**35.** Sun, Moon and Star invest ₹ 50,000, ₹ 60,000 and ₹ 70,000 respectively in a business. Find the share of Moon in the total profit of ₹ 5,40,000.
(A) ₹ 1,20,000 (B) ₹ 1,80,000
(C) ₹ 1,45,000 (D) ₹ 1,60,000

**36.** If $A$% of $\frac{A}{B} + \frac{A}{B}$% of $A = C$% of $\frac{A}{B}$, $C = A$. _____ .

**37.** Rohit bought a cassette for ₹ 40. He sold it at 20% profit to Rakesh. Rakesh sold it at $33^1/_3$% profit to Suresh. Find Suresh's cost price (in ₹). _____

**38.** The income of A is more than that of B. A spends 30% of his income on food and B spends 25% of his income on food. Who spends more on food? _____

**39.** In the above question, if the income of A is less than that of B, then who spends more on food?
(A) A
(B) B
(C) Both spend equal amounts on food
(D) Cannot be determined

**40.** $11\frac{1}{9}\%$ of 918 = _____ .

**41.** The price of an item P is twice the price of an item Q. If the price of P is decreased by 25% and the price of Q is increased by 50%, find the percentage increase in the sum of the prices of the two items. _____

**42.** The cost price of a company to manufacture a product is ₹60. The company sells the product to a dealer for ₹70. The dealer sells the product to a shopkeeper for ₹85 and the shopkeeper sells it to a person for ₹102. What is the percentage of profit for the company?
(A) $16^{2}/_{3}\%$ (B) 20%
(C) $21^{3}/_{7}\%$ (D) 70%

**43.** In problem 42, who made the highest profit on selling the product?
(A) Company (B) Dealer
(C) Shopkeeper (D) Customer

**44.** By selling 80 oranges a man gains the cost price of 20 oranges. Find the gain percentage. _____

**45.** Antony, Ben and Charles enter into a business with investments in the ratio of 3 : 4 : 6. At the end of one year, they shared the profits in the ratio of 6 : 4 : 3. Find the ratio of the time periods for which they invested.
(A) 1 : 2 : 4 (B) 4 : 2 : 1
(C) 4 : 1 : 2 (D) 1 : 4 : 2

**46.** When a number is decreased by 20%, it becomes 192. What is the original number?
(A) 96 (B) 120 (C) 144 (D) 240

**47.** The selling price of a tube of toothpaste is decreased by 20%. It now sells for ₹20. What was the original selling price (in ₹) of the tube of toothpaste? _____

**48.** 44.44% of 2790 = _____ .

**49.** If 6% of $x$ and 24% of 380 are equal, then $x$ is _____ .

**50.** 13.6% of 750 = _____ .

**51.** By selling 60 oranges a man gains the selling price of 20 oranges. Find the gain percentage. _____

**52.** 57.14% of 784 = _____ .

**53.** For a family, the savings per month are 30% of the total income. If the expenditure on education is increased from 30% to 40% of the total income, then find the savings as a percentage of the new expenditure on education, provided there is no change in the savings.
(A) $133^{1}/_{3}\%$ (B) 75%
(C) 100% (D) Cannot be determined

**54.** The cost price of a table is ₹330. It is sold at a profit of ₹30 after giving a 10% discount. Find its marked price. (in ₹) _____

**55.** If 60% of $x$ is equal to 75% of 1200, then $x$ = _____ .

**56.** A man invested ₹18600 in a 5% stock at ₹93. If he sells it when it is quoting ₹105, find his profit (in ₹). _____

**57.** $\frac{3}{20}$ is what percent of $\frac{12}{25}$?
(A) 42.75% (B) 31.25%
(C) 35.5% (D) 40.5%

**58.** The salaries of 30 employees of a company are increased by 10% each and that of the remaining 20 employees of the same company are increased by 20% each. What is the percentage increase in the total salary of the 50 employees?
(A) 15% (B) 26%
(C) 14% (D) Cannot be determined

**59.** Girish bought a cycle for ₹600 and sold it for ₹900. Find the percentage of profit he made. _____

**60.** In an examination, 38% of the students passed. If the number of students passed is 60 less than the number of students failed, then find the total number of students who appeared for the examination.
(A) 160 (B) 180 (C) 240 (D) 250

## EXERCISE-2

***Directions for questions 1 to 60*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** In an office, 30% of the employees are men and the remaining are women. 60% of the men are married and 66% of the employees are married. What percentage of women are not married?
(A) $17^{1}/_{7}\%$ (B) 30% (C) $31^{3}/_{7}\%$ (D) 20%

**2.** A and B invest ₹8,000 and ₹7,000 respectively in a business. After one year, they make a profit of ₹4,500.

(a) Find the ratio of the shares of A and B in the profit.

$\frac{\quad}{7}$

(b) Find the respective shares of A and B in the profit. (in ₹) _____, _____ .

**3.** The price of sugar increased by 25%. By what percentage should the consumption be reduced so that the expenditure on sugar remains the same? _____ %

**4.** An item was marked up by 40% and after that a discount of 20% is given on it. If the item costs ₹225, then what profit (in ₹) is earned by selling it? _____

**5.** When an article is sold at $3/5^{th}$ of the actual selling price, there is a profit of 20%. The profit on selling 45 such articles at the actual selling price is equal to the
(A) cost price of 45 articles
(B) selling price of 45 articles
(C) cost price of 30 articles
(D) selling price of 30 articles

**6.** A man purchased a stock of goods worth ₹6,000. He sold $2/5^{th}$ of it at 30% profit. If he wants to gain 20% on the whole, then at what profit percentage should the remaining goods be sold?
(A) 12.5% (B) 16.66%
(C) 13.33% (D) 14.28%

**7.** A discount of 40% on the marked price of a trouser enables a man to purchase a shirt also, which costs him ₹320. How much did the man pay (in ₹) for the trouser? ____

**8.** A started a business with ₹30,000. After two months B joined him with ₹20,000. A left after 6 months whereas B stayed on till the end of the year.

(a) Find the ratio of their shares in the year-end profit.
(A) 4 : 5 (B) 14 : 15
(C) 9 : 10 (D) 19 : 20

(b) By what percentage is B's share in the profit more/less than A's share in the profit? $\frac{\quad}{9}\%$

**9.** In 2004, 40% of the sales of the product A is equal to 60% of the sales of product B. If the total sales of A and B in 2004 (in lakhs of rupees) is 9000 and the sales of A and B increased by 10% and 20% respectively in 2005, then the difference in the sales of A and B in 2005 (in lakhs of rupees) is
(A) 2520 (B) 1860
(C) 1620 (D) 2120

**10.** When the price of an article is increased by 30%, its sales decreased by 20%. What is the effect on the sales revenue? _____ % increase/decrease.

**11.** A, B and C have certain number of mangoes with them. B has 10% less mangoes than A and C has 20% less than A. By what percentage is the number of mangoes with B more than those with C? _____

**12.** The population of a country increases by 10% every year, and the national income increases by 21% every year. What is the percentage change in the per capita income of the country every year?

$\left(\text{Per capitalincome} = \frac{\text{NationalIncome}}{\text{Population}}\right)$

(A) 10% (B) 11% (C) 2.1% (D) 20%

**13.** A person sold two refrigerators each at ₹5060. On one he gained 10% and on the other he lost 8%. Find his overall profit/loss.
(A) ₹20 loss (B) ₹20 profit
(C) ₹10 loss (D) ₹10 profit

**14.** A student gets 60% of the maximum marks. If he gets 50% more than the pass mark, then the pass mark is what percentage of the maximum marks?
(A) 75% (B) 37.5% (C) 40% (D) 50%

**15.** Two items were sold both at the same price. The first item was sold at a profit of 40% and the second at a profit of 60%. If the customer paid a total sum of ₹12000 for the two items, then at what price (in ₹) did the shopkeeper purchase the cheaper item? _____

Difficulty Level-2: Moderate

**16.** Which of the following options is the best investment?
(A) 2% stock at ₹75 (B) 3% stock at ₹150
(C) 3% stock at ₹100 (D) 4% stock at ₹175

**17.** Abdul started a business with ₹10,000 and 4 months later Basker joined in with ₹15,000. At the end of the year, out of the total profit, Abdul received a total amount of ₹4,400, which included a salary of 10% of the profit. What was the amount Basker received? (in ₹) _____ .

**18.** A man purchased 120 kg of sugar and was forced to sell it at a loss equal to the selling price of 30 kg of sugar. If he purchased each kilogram of sugar for ₹15, then at what price (in ₹) did he sell each kilogram of sugar? _____

**19.** A and B run a business in a certain year. The salary of A turns out to be equal to 20% of the balance of the profit left after his salary was paid. If A finally received a total of ₹35,000 after splitting the remaining profit equally between the two, then find the total profit made in that year. (in ₹). _____

**20.** In a business, the investments of Ram and Shyam are in the ratio 3 : 2. Ram invests for ten months while Shyam invests for the entire year. The annual profit is ₹8,00,000. If Ram's salary is included, the ratio of their shares will be 3 : 2. Find the salary of Ram. (in ₹) (Assume that Ram is a working partner). _____

**21.** The population of a colony of ants increases by 20% everyday. If one Monday the population is 3000. On which day of the week is it 5184?
(A) Wednesday (B) Tuesday
(C) Thursday (D) Friday

**22.** The price of an item went up by 25% and a family decided to reduce its consumption so that the total expenditure would increase by only 8% as compared to the previous expenditure. If 25 kg of the item was consumed previously, then how many kg of the item should be consumed now? _____

**23** The price of an article increased by 20% and its sales decreased by 20%. What is the effect on the sales revenue? _____ % increase/decrease

**24.** A wholesale vegetable vendor sold potatoes, marked at ₹1000. Four successive discounts of 10% each, instead of the promised 40% discount were offered. By what amount did the vendor defraud the customer?
(A) ₹58.10 (B) ₹56.10 (C) ₹65.10 (D) ₹66.10

**25.** If $a : b = 2 : 3$, $b : c = 4 : 5$ and $c : d = 1 : 3$, find $a : b : c : d$.
(A) 8 : 12 : 25 : 75 (B) 8 : 18 : 25 : 75
(C) 8 : 12 : 15 : 45 (D) 16 : 24 : 27 : 81

**26.** In a joint venture, A and B invested ₹32000 and ₹56000 respectively. A received ₹1000 per month as salary for managing the business and the remaining profit was divided in the ratio of their investments. At the end of the year, A received a total of ₹20000. How much did B receive at the end of the year (in ₹)? _____

**27.** Amar went to a shop to buy a shirt. The amount he had with him was equal to the list price of one of the shirts in the shop. As a result of a 30% discount that he received on the shirt, he was able to buy one more shirt worth ₹600 and still save ₹270. The initial amount with Amar is ₹ _____

**28.** The factory equipment cost a certain factory ₹5,00,000. If the equipment depreciates 15% in value in the first year, 13·5% in the next year, 12% in the third year and so on, what will be its value at the end of 10 years (All percentages applying to the original cost of equipment)?
(A) ₹82,500 (B) ₹84,500
(C) ₹87,500 (D) ₹90,000

**29.** By selling an article for ₹816, a shopkeeper incurs a loss of 20%. At what price (in ₹) should he sell the article so as to gain 10%? _____

**30.** How many shares can be purchased for an amount of ₹1,21,000 if the face value of each share is ₹100, and they are available at a premium of 10% each? _____

**31.** Two schools A and B participated in a talent search exam. Schools A and B sent 200 and 300 students respectively. 36% of the students from school A and 44% of the students from school B cleared the exam. What percent of the total number of students from schools A and B cleared the exam? _____%

**32.** If the payment per hour of a part-time employee increased by 25% but his working time reduced by 20%, then find the net percentage change in his income. _____

**33.** Roopa sold 300 pens in a day. She made neither a profit nor a loss on one out of every four pens that she sold. If she earned an average profit of 20% on the remaining pens, find her overall profit percentage. _____

**34.** What is the dividend income from a 4% stock worth ₹4,00,000? (in ₹) _____ .

**35.** The income of Ananth is 50% more than that of Bhargav. The expenditure of Bhargav is equal to the savings of Ananth. If the savings of Ananth is 300% of that of Bhargav, then what percentage of his income does Ananth save? _____

**36.** In the U.S. opinion polls, 30% favoured George Bush and the rest favoured Bill Clinton. Following a debate between the two groups, 20% of the supporters of each shifted their loyalty to the other. Find the percentage of the total electorate which must shift from Clinton to Bush so that both have equal number of supporters. _____

**37.** In a library, 40% of the books are in English. 30% of the remaining books are in Hindi. One-fifth of the remaining books are in Sanskrit and the remaining 1680 books are in other Indian languages. How many books are there in the library? _____

**38.** A trader marks his washing machine at 30% above the cost price and offers a discount. Find the discount percentage if the trader makes a net profit of 17%. _____ %

**39.** David sold a calculator for ₹400 after offering a discount of ₹100 and he observed that the profit made by him on selling the calculator for ₹400 would be the same as the loss he would have made in selling it at a 25% loss. By how much percentage his marked price was above his cost price? _____

**40.** Andy scored 75 out of 150 marks in his mid-term exam and 105 out of 150 in his end-term exam. Find the percentage increase/decrease in his marks. _____

**41.** Find the increase in his marks in terms of percentage points. _____

**42.** The Indian cricket team played 25 one-day matches in a particular season of an year and won 40% of their matches. A new coach was then brought in, who wanted a success rate of 75% of the matches played by the end of the year. What is the minimum number of additional matches that should be played to achieve that target? _____

**43.** Two successive discounts of 20% each is equivalent to a single discount of how much percentage? _____

**44.** A man sold his computer at a loss of 5%. Had he sold it for ₹2250 more, he would have made a profit of 10%. What is the cost price of his computer?
(A) ₹15,000 (B) ₹16,000
(C) ₹20,000 (D) ₹24,000

**45.** In any given month, a salesman earns a commission of a% on the first ₹1000 worth of his sales. He earns b% on all further sales during that month. He made a commission of ₹900 from the sales of ₹3000 in January and ₹1300 from the sales of ₹4000 in February. Find the value of *a*. _____

**46.** Which is a better investment, 6% stock at 108 or 7% stock at 109? _____ % stock

**47.** Mohan purchased a certain amount of rice. If he sold one-third of the rice at a profit of 20%, then find the profit percent he needs to make on the remaining rice to make an overall profit of 25%. _____

**48.** Two candidates contested in an election. The candidate favoured by 38% of the votes is rejected by a majority of 18924 votes. Find the total number of valid votes. _____

**49.** The price of a radio at the beginning of an year was ₹3000. It increased by *x*% and then decreased by *y*% over the year. It increased by *x*% and then decreased by y% over the next year too. At the end of the second year, the price of the radio was ₹3000. Which of the following can be concluded?
(A) $x > y$ (B) $x = y$
(C) $x < y$ (D) None of these

**50.** X sold an article to Y at 25% profit. Y sold the article to Z at 20% profit. The difference of the prices at which Y and Z bought the article is ₹80. Find the price at which X bought the article.
(A) ₹240 (B) ₹320 (C) ₹400 (D) ₹160

**51.** An article was marked at ₹350. After successive discounts of 20% and d%, it was sold for ₹210. Find *d*. _____

**52.** Avinash buys a certain number of apples at 3 for ₹16 and sells them at 5 for ₹28. Find his profit/loss percentage in the transaction.
(A) 4% loss (B) 4% profit
(C) 5% profit (D) 5% loss

**53.** Akbar and Birbal started a business with investments of ₹30000 and ₹40000 respectively. At the end of one year, they gave 10% of the total profit of ₹7000 to charity. Find the difference in the profit shares of both out of the remaining profit. (in ₹) _____

**54.** If the discount on an article is increased from 20% to 35%, the money realized from the sale decreases by ₹27. What is the marked price of the article? (in ₹) _____

**55.** The ratio of three positive quantities *P*, *Q* and *R* is $\frac{1}{4}:\frac{1}{5}:\frac{1}{6}$ By what percent is *Q* less than $(P+R)$? _____

**56.** Three partners A, B and C run a business. The ratio of their shares in the annual profit is 3:2:6. If A's capital is less than B's capital and more than C's capital, whose capital is invested for the maximum period?
(A) A (B) C
(C) B (D) Cannot be determined.

**57.** Ram spends 80% of his annual salary. If his annual salary increased by 14% and his annual expenditure increased by 10%, his annual savings would increase by _____ %

Difficulty Level-2: Moderate

**58.** A and B start a business by investing some capital each. At the end of the year, A received 20% of the total profit as salary on while the rest of the profit is shared in the ratio of their respective investments. A finally receives 50% more than B does. If A's capital is ₹10,576, then B's capital is
(A) ₹14,101 (B) ₹8,813
(C) ₹6,610 (D) ₹10,576

**59.** In a town, 40% of the population are minors, 35% of the minors are educated, 40% of the majors are uneducated. The total educated population is 48,000. What is the population of the town? _____

**60.** I had enough money to purchase either 40 apples or 50 oranges. I decided to spend only 80% of the money and purchased 20 oranges and some apples. How many apples did I purchase? _____

## EXERCISE-3

***Directions for questions 1 to 60*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

***Direction for question 1*:** In the beginning of 2010, Anil had two dozen cows with him. Starting from 2010, in each year, Anil sold q% of the cows he had at the beginning of the year and then bought p% of the cows that he was left with. At the end of 2011, Anil had three dozen cows.

Conside0r the following statements.

I. $p = q$ II. $p < q$

**1.** Which of the following is/are possible?
(A) Only I (B) Only II
(C) Both I and II (D) Neither I nor II

***Direction for questions 2 and 3*:** These questions are based on the following data.

In an examination every question correctly answered fetches two marks and for every question wrongly answered one mark is deducted. Anushk and Preetham took the examination. Anushk attempted a certain number of questions and 20% of them went wrong. Preetham attempted a certain number of questions and 10% of the questions attempted by him went wrong. Anushk got 33 marks more than the pass mark and Preetham got 48 marks more than the pass mark. Anushk and Preetham together attempted 100 questions.

**2.** What is the pass mark of the examination?
(A) 59 (B) 43 (C) 47 (D) 37

**3.** The total marks scored by Anushk and Preetham is how much more or less than the maximum marks of the examination?
(A) 0 (B) 8 more
(C) 12 less (D) Cannot be determined

**4.** A total expenditure of ₹900 was incurred by a trader in purchasing two varieties of rice. If the selling price of each variety of rice was ₹10 /1kg more than the cost price, then the profit would be ₹540. If the cost price of the first variety is ₹5 /1kg less than the second one and the respective quantities purchased are in the ratio 2 : 1 respectively, what is the cost price of the second variety of rice? (in ₹ 1kg) _____

**5.** Andy sells a scooter to Bandy for a profit of 20%, and Bandy sells it to Candy for a profit of ₹6,000, making 20% profit. Find Andy's cost price. (in ₹)_____

**6.** If the price of tea decreased by 20%, then by what percentage should the consumption be increased so that the total expenditure is increased by 10%? _____ .

***Directions for questions 7 and 8*:** Anita has the choice of three schemes to invest her surplus cash for a certain week. She wants the maximum returns on her investment.

Scheme 1: A fixed deposit account which promises a guaranteed 0.15% return.

Scheme 2: A mutual fund account which offers a 5.5% return when the market is bearish and a - 4% return when the market is bullish.

Scheme 3: A mutual fund account which results in a –3.5% return when the market is bearish and offers 3% return when the market is bullish.

**7.** Anita has a maximum guaranteed return of
(A) 0.15% (B) $\frac{5}{32}$ % (C) $\frac{7}{32}$ % (D) $\frac{5}{16}$ %

**8.** The investment strategy for which the guaranteed return is maximum is
(A) 100% investment in scheme 1.
(B) investing equally in the 3 schemes.
(C) investing $\frac{1300}{32}$ % in scheme 2 and $\frac{1900}{32}$ % in scheme 3.
(D) investing $\frac{1900}{32}$ % in scheme 2 and $\frac{1300}{32}$ % in scheme 3.

**9.** A shopkeeper bought an article for ₹1,000 and marked its price as ₹2,160. He gave three successive discounts of $a\%$, $b\%$, and $c\%$, where $a + b + c = 60$. If he made a profit of $x\%$ finally, which of the following ranges contain all the possible values of $x$?
(A) $0 < x < 10$ (B) $14 < x < 20$
(C) $20 < x < 24$ (D) $24 < x < 26$

**10.** What should come in the place of the following blanks?
_____

| | C. P | S.P | Profit/loss | Profit/loss percentage |
|---|---|---|---|---|
| (a) | ₹400 | | | 20% |

**11.** What should come in the place of the following blanks?
_____

| | C. P | S.P | Profit/loss | Profit/loss percentage |
|---|---|---|---|---|
| (b) | | ₹600 | | 25% |

**12.** What should come in the place of the following blanks?
_____

| | C. P | S.P | Profit/loss | Profit/loss percentage |
|---|---|---|---|---|
| (c) | | | ₹300 | 20% |

**13.** What should come in the place of the following blanks?
_____

| | C. P | S.P | Profit/loss | Profit/loss percentage |
|---|---|---|---|---|
| (d) | | ₹270 | | –10% |

**14.** A village has 10000 people, 68% of the people are males. The total literate population of the village is 6000. There are 1200 more male illiterates than female illiterates. What percent of the female population are illiterates? _____

**15.** The hourly percentage increase in the number of bees in a colony increases by 10 percentage points every hour. In the first hour, the increase in the number of bees is 20%. Find the effective increase in the number of bees in the 2nd hour, if there are 1248 more bees in the third hour when compared to the second hour. _____

**16.** Ravi invested a certain amount in a 4% stock at ₹120. He invested an equal amount in a 6% stock at ₹96. Had Ravi invested the total amount invested in the two stocks in the 6% stock at ₹96, he would have realized ₹350 more as annual dividend. Find the total amount invested in the two stocks. (in ₹) _____

**17.** A and B started a venture with different investments. It was decided that A would get 20% of the profit as salary and the remaining profit would then be equally divided between the two of them. Had the entire profit been divided in the ratio of their investments, B would have received ₹5200 more as his share of the profit. The respective earnings of A and B are ₹16800 and ₹11200. Find the investment of A, if B invested ₹82000.
(A) ₹44000 (B) ₹58000
(C) ₹68000 (D) ₹77000

**18.** The second column in the following partially filled table gives the year-on-year changes in the number of foreign tourists visiting a country from 1995 to 1999.The third column gives the change in these percentage figures over the previous year. The (+) sign indicates increase and the (–) indicates decrease. By what percentage did the number of tourists change from 1994 to 1999?

| Year | Change as a Percentage | Change in percentage points |
|---|---|---|
| 1995 | +25% | _____ |
| 1996 | _____ | $+8^1/_3$ |
| 1997 | –25% | _____ |
| 1998 | _____ | –25 |
| 1999 | +20% | _____ |

(A) –25% (B) –20% (C) +20% (D) –5%

**19.** At a certain shop, the marked price of a mobile phone was 60% of that of a TV. A customer requested for 10% discount on the mobile phone and 20% discount on the TV. The shopkeeper accidentally offered 10% discount on the TV and 20% discount on the mobile phone. Find the ratio of the amount that the customer actually paid and the amount which the customer planned to pay.
_____
67

**20.** Merchant A sold an article after giving a 30% discount. Merchant B sold the same article after giving three successive discounts of 10% each. Merchant C sold the same article after giving successive discounts of 15%, 10% and 5%. If the marked prices of the three merchants were the same, find the ascending order of their selling prices.
(A) ACB (B) ABC (C) BCA (D) BAC

**21.** A man invests a certain sum in a 5% stock at 120 and twice the sum in a 4% stock at 110. Had he invested the entire sum in a 8% stock at 99, his income would have been ₹845 more. How much did he invest in the 4% stock?
(A) ₹6600 (B) ₹13200
(C) ₹15700 (D) ₹20440

**22.** By giving a discount of 30%, a trader made a profit of 20%. If the trader had given a discount of ₹400, he would have incurred a loss of 20%. Find the marked price of the article.
(A) ₹600 (B) ₹750 (C) ₹900 (D) ₹800

**23.** A dishonest merchant weighs 100 gms less for every 1 kg that he sells.

Find his gain percentage if the merchant claims that his selling price is the same as his cost price. $\frac{}{9}$

**24.** Find his overall gain percentage, if his selling price is 20% more than his cost price. $\frac{}{3}$

***Directions for questions 25 and 26:*** These questions are based on the information given below.

In manufacturing a certain item, 40% of the expenditure is on account of raw materials, 20% on account of labour charges, 20% on account to fixed charges and the rest on miscellaneous. The item is sold at a profit of 25%. The price of the raw materials went up by 15% and the labour charges went up by 20% and the cost on the miscellaneous heads went up by 50% while the fixed costs remained unchanged.

**25.** If the selling price remained unchanged, then what is the profit percentage?
(A) 2% (B) 4%
(C) $4^{1}/_{6}\%$ (D) 10%

**26.** If the manufacturer wants a $13^{7}/_{11}\%$ profit, then by what percentage should he reduce his expenditure on raw materials (at the increased price) as to achieve that target, the selling price remaining the same?
(A) $17^{17}/_{23}\%$ (B) $20^{14}/_{23}\%$
(C) $21^{17}/_{23}\%$ (D) $24^{14}/_{23}\%$

**27.** A printer undertook the task of printing 3000 pages for a client. The client provided the paper. The dimensions of each page were 34 cm × 24 cm. On each page, a 2 cm gap was left on all the four sides of the margin. The cost of printing was ₹6 per $m^2$. If the printer wanted to make 6% profit, how much should he have charged per page (in ₹)? _____

**28.** The population of a colony of ants increases by 20% everyday. If one Monday the population is 3000. On which day of the week is it 5184?
(A) Wednesday (B) Tuesday
(C) Thursday (D) Friday

**29.** The price of a pencil is twice that of an eraser. The price of an eraser is thrice that of a sharpener. If the price of a pencil increases by 10%, price of an eraser increases by 30% and the price of a sharpener increases by 20%, find the percentage increase in the price of 30 pencils, 30 erasers and 30 sharpeners.
(A) 13% (B) 17%
(C) 21% (D) Cannot be determined

**30.** Mr. Mohit imported a camera after paying a 6 percent customs duty on the value of the camera in excess of ₹1,20,000. If the amount of customs duty that Mohit paid was ₹5670, what was the value of the camera? (in ₹) _____ .

**31.** A total expenditure of ₹900 was incurred by a trader in purchasing two varieties of rice. If the selling price of each variety of rice was ₹10 kg more than the cost price, then the profit would be ₹540. If the cost price of the first variety is ₹5 less than the second one and the quantities purchased are in the ratio 2 : 1, what is the cost price of the second variety of rice? (in ₹ /1kg) _____ .

**32.** A shopkeeper defrauds both the dealer and the customer by measuring weights incorrectly. When he is purchasing items from the dealer, he takes 20% more than the indicated weight and when he is selling them to the customer, he gives 20% less than the indicated weight. If the price that the shopkeeper charges his customer is the same as what the dealer charges the shopkeeper, then what profit percentage does the shopkeeper make? _____

**33.** A shopkeeper sold three articles A, B, and C at 30%, 40%, and 60% profit respectively. The overall profit percentage made on selling B and C is 48% and that on A and C is 40%. Find the overall profit percentage made on selling all the three articles. _____

**34.** A shopkeeper defrauds both the dealer and the customer by measuring the weights incorrectly. When he is purchasing items from the dealer, he takes 20% more than the indicated weight and while selling he gives to the customer a quantity, such that if 20% is added to it, the indicated weight is obtained. If the price that the shopkeeper charges his customer is the same as what the dealer charges the shopkeeper, what is the shopkeeper's profit percentage? _____

**35.** The savings of Achyut are 30% of his earnings while the savings of Ranjit are 150% of the savings of Achyut. The combined earnings of Achyut and Ranjit as a percentage of Achyut's earnings cannot be
(A) 150 (B) 160 (C) 120 (D) 180

**36.** Akshay owned ₹36,000 worth of 5% stock on 1/4/2012. On 1/4/2013, he sold the stock, when it was quoting ₹150, which was 25% more than the price he paid. He wanted to invest an amount in 7.5% stock quoting ₹135, which

would yield an annual income which was double of what he had got earlier. For this purpose, he intended to invest from the sale proceeds that he received on 1/4/2013, and an additional amount, if necessary. How much money was he left with (from the sale proceeds) or what was the additional amount he had to bring in? (in ₹) _____ .

**37.** The ratio of the populations of cities X, Y and Z in 2008 was 3 : 5 : 6. The percentage increases in the populations of X, Y and Z from 2008 to 2009 were 10%, 20% and 15% respectively. Find the approximate percentage increase in their total population from 2008 to 2009.
(A) 11.14% (B) 15.71%
(C) 19.85% (D) 25.28%

**38.** The population of a village is 7500. 42% of the population are females and the rest males. The total number of literate persons in the village is 2370 while there are 90 more male illiterates compared to female illiterates. What is the percentage of literate males out of the total number of males in the village? _____

**39.** Ajay spends 16.66% of his income on rent, 25% of the remaining on food, 60% of the remaining on education and 50% of the remaining on others. If he saves ₹4500 per month, find his income. (in ₹) _____

**40.** Arvind sold a pen at a certain price. Had he sold the pen at two-thirds of the selling price, he would have incurred a loss of 10%. What was the actual profit percentage that Arvind made? _____

**41.** A shopkeeper sells 120 articles each at a price of ₹250. To the persons who do not bargain, he does not give any discount and makes 50% profit on each article. To those who bargain, he gives a discount of 20%. If one in four customers does not bargain, find his profit percentage on selling all the 120 articles.
(A) 25% (B) 30% (C) 37×5% (D) 27.5%

***Direction for question 42*:** These questions are based on the information given below.

In manufacturing a certain item, 40% of the expenditure is on account of raw materials, 20% on account of labour charges, 20% on account to fixed charges and the rest on miscellaneous. The item is sold at a profit of 25%. The price of the raw materials went up by 15% and the labour charges went up by 20% and the cost on the miscellaneous heads went up by 50% while the fixed costs remained unchanged.

**42.** An item was sold at a price after giving two successive discounts of 30% and 50%. If the selling price of the item was ₹448, then what was the marked price of the item (in ₹)? _____

***Directions for questions 43 and 44*:** These questions are based on the data given below.

Ajay, Bharat, Chandan and Deepak are 4 friends. Ajay purchases a bicycle for ₹1000. He sells it to Bharat at a certain gain percentage. Bharat in turn sells it for ₹1,500 to Chandan who in turn sells it to Deepak. The gain percentages of Ajay, Bharat and Chandan are in arithmetic progression. Had Bharat sold the bicycle to Deepak directly for the same rate for which Chandan sells the cycle, then he would have made a profit of $62^1/_2\%$.

**43.** What profit did Bharat make on selling the cycle to Chandan? (in ₹)
(A) 450 (B) 250 (C) 200 (D) 300

**44.** What profit did Chandan make on selling the cycle to Deepak? (in ₹)
(A) 300 (B) 450 (C) 200 (D) 650

**45.** If after giving a discount of 12%, a profit of 10% was made on an article, then by what percentage was the price marked up? _____

**46.** Alok and Bunty invest ₹20000 and ₹10,000 respectively for a business at the start of a year. In each of the next six months after the first month Alok keeps on removing ₹1,000 while Bunty keeps on adding ₹1,000. In the months that remain Alok keeps on adding ₹1,000 while Bunty keeps on removing ₹1,000 every month. Chandan joined them with ₹25000 three months after the start and continued till the end of the year. What will be the difference in the shares of Chandan and Bunty after a year if the total profit at the end of the year is ₹1,17,000?
(A) ₹12,900 (B) ₹26,800
(C) ₹13,800 (D) ₹22,600

**47.** A shopkeeper hiked the price of an article by $x\%$ and then gave $x\%$ discount and the price of the article decreased by ₹2000. Once again the price of the article was increased by $\frac{x}{2}\%$ and then decreased by $\frac{x}{2}\%$ and it was finally sold at ₹47520. What was the initial price of the article? (In ₹)
(A) 48,500 (B) 50,000
(C) 51,000 (D) 46,000

**48.** A salesman used to get 8% on the total sales as commission. Under a new policy he now gets a fixed amount ₹2000 and 5% commission on any amount of sales in excess of ₹10000. The salesman thus got ₹540 more this month. What were his total sales for this month?
(A) ₹20000 (B) ₹26000
(C) ₹32000 (D) ₹35000

**49.** Prakash used to sell readymade shirts in his stall at the local market. He purchased 160 shirts from a company outlet at ₹ $x$ per shirt. If he sells 20% of the shirts at 15% profit and 37.5% of the remaining shirts at 40% profit, then at what percentage profit should he sell the remaining to make a profit of 25% on the whole?
(A) 6.25% (B) 20%
(C) 25% (D) 32.5%

**50.** A and B started a business by investing ₹35,000 and ₹13,000 respectively. At the end of every month, A withdraws a certain amount from his investment and B invests the same amount as A has withdrawn. At the end of the year, they share the profits in the ratio 1 : 1. Find the amount withdrawn by A every month. _____

***Direction for question 51*:** A, B, C and D have equal amounts of money with them. The following transactions take place among them in the following sequence.

A gives D 10% of what A has.

B gives A 50% of what B has.

C gives D 30% of what C has.

D gives B 25% of what D has.

C gives A 30% of what C has.

D gives B 20% of what D has.

**51.** After the above transactions take place, which of the following represents the descending order of the amounts they have?
(A) ABCD (B) BADC
(C) ABDC (D) None of these

**52.** A company imported two components X and Y. It imported X from U.K. and Y from France. It then assembled those components with other components to form a machine used in a chemical process. X made up to 20% of the production cost. Y made up to 40% of the production cost. Their regular practice was to sell the machine at 25% above the production cost. The U.K. pound became 40% costlier and the French franc became 30% costlier. Owing to these reasons, the company increased its selling price by 8%. Find the profit percentage, now.
(A) 10% (B) 12.5% (C) 15% (D) 8%

**53.** Ankush bought a plot of land for ₹50000. After two years he found that the market value of land increased by 300%. At this stage, he has two options- he can sell at the market rate or invest some more money on marketing and then get a better price. He estimated that if he spent ₹10,000 on marketing, he would get 25% more than the market value. Find the difference between the profit percentages in the two options (in percentage points).
(A) 10 (B) $12\frac{1}{2}$
(C) $14\frac{2}{7}$ (D) $16\frac{2}{3}$

**54.** If the selling price of 10 oranges is equal to the cost price of 14 oranges, which, in turn, is equal to one-third of the discount offered on 10 oranges, then find the profit/loss percentage when the markup percentage is halved and the discount percentage is decreased by 5 percentage points. _____

**55.** A school had raised 60% of the amount it needed for a new building by receiving an average donation of ₹300 from the people it had already solicited. The people already solicited represent 80% of the people the school intends to ask for donations. The school wants to raise the exact amount it needs for the new building. Find the average donation it should receive from the remaining people to be solicited (in ₹).
(A) 600 (B) 700 (C) 800 (D) 500

**56.** A milk vendor who sells milk at ₹10.50 per half a litre, finds a milk can lying by the roadside unclaimed and unattended. He picks up the milk can and tests the milk. He finds it fresh and of the same quality as the milk he already has with him. He initially has 52 litres of milk, while the can that he finds has 13 litres. He mixes the two quantities and sells at a reduced price but maintains his profit percentage. By what proportion does he decrease the price per litre?
(A) $(1/4)^{th}$ (B) $(1/5)^{th}$
(C) $(1/6)^{th}$ (D) $(1/3)^{rd}$

**57.** Mr. Singh purchased 40,000 apples at ₹7 each. He found that 30% of those were spoilt and hence were found unfit to be sold. At what price should he sell each of the remaining apples so as to get an overall profit of 25%?
(A) ₹8.75 (B) ₹10
(C) ₹11.25 (D) ₹12.50

**58.** A motorist uses 12% of his fuel to cover 18% of his total journey for non-city driving conditions. He knows that he has to cover another 24% of his total journey in non-city driving conditions. What should be the percentage decrease in his fuel efficiency, for city driving over non-city driving, so that he just completes his entire journey without a refill?
(A) 25% (B) 30% (C) 32% (D) 46%

**59.** A man invests a certain sum in a 4% stock at 96 and an equal amount in a 5% stock at 102. Another man invests

a certain sum in a 8% stock at 120 and an equal amount in a 3% stock at 125. If the investment of the first person is ₹960 more than that of the second person and the difference of their earnings is ₹44, then find the investment made by the first man.
(A) ₹24480 (B) ₹48960
(C) ₹97920 (D) None of these

**60.** A trader purchases two sofa sets at prices which are in the ratio 5 : 3. He made $x$% loss on the first set and $2x$% profit on the second set. when he sold them. In the entire transaction, the trader makes a profit of 2.5%.Find $x$. _____

## EXERCISE-4

***Directions for questions 1 to 30*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** There are a certain number of female students and 2·5 times as many male students who planned to go to the Himalayas in search of Shangri-la. On the way a certain number of cookies were to be distributed. Every female student was to get 3 cookies and every male student was to get half as many cookies as there were male students. Just before starting, some more male students joined the group, increasing their strength by 20%. If the distribution was to take place according to the same formula the number of cookies that were needed increased by $\frac{5500}{131}$% . Find the number of male students in the original group.
(A) 40 (B) 20 (C) 50 (D) 30

**2.** In 1999, Martden, a chain of supermarkets gained 20% of the customers of its competitor—Duck Walt (in 1998) but lost 10% of its customers (in 1998) to Duck Walt. From 1998 to 1999, the customer base of the two supermarkets increased by 40%.In 1998, the customer base of Martden was the same as that of Duck Walt and in 1999, Martden has a share of 50% of the total customers of the two competitors. What percentage of the new customers patronized Duck Walt in 1999?
(A) 37.5% (B) 25% (C) 62.5% (D) 50%

**3.** When a liquor dealer shipped a crate of 800 beer bottles to a customer a few of the bottles broke due to careless handling. The dealer had offered a discount of 10% and the customer paid ₹11520. But because of the loss in transit, despite the discount, the customer gets beer bottles, whose value is ₹160 less than what he pays. What is the value (in ₹) of the broken bottles? (Assume that the value of the beer bottles is proportional to its marked price).
(A) 1500 (B) 1600 (C) 1728 (D) 1440

**4.** The length, the breadth and the height of a cuboid are in the ratio 5 : 4 : 3. If the length increases by $x$%, the breadth increases by $y$% and the height decreases by $z$%, the total length of all the edges of the cuboid will increase by $16^2/_3$%. The volume of the cuboid will then increase by 40.4% and the lateral surface area of the cuboid will then increase by 13%. Find the value of $z$.
(A) 10 (B) 15 (C) 20 (D) 25

**5.** 2000 persons voted on a resolution. 12.5% of the votes were invalid. The supporters of the resolution called for a repoll, to which the opponents of the resolution agreed. This time, 20% of the votes were invalid. The opponents decreased by $16^2/_3$% and the motion was passed by a margin which was 100% more than that by which it was previously rejected. How many persons voted against the resolution in the repoll?
(A) 900 (B) 850 (C) 800 (D) 750

**6.** The cost price and the marked price of an article are in the ratio 4 : 5. The article was sold after a discount. The discount percentage offered and the profit/loss percentage made on the article are in the ratio 4 : 5. Consider the following statements.

I. The article was sold at a profit.

II. The discount percentage offered is 20%.

Which of the following is/are true?
(A) Only I (B) Only II
(C) Both I and II (D) Neither I nor II

**7.** A, B and C start a venture with different amounts. A started with ₹1000 and increased his investment by ₹1,000 every month. B started with ₹2,000 and increased his investment by ₹2000 every 2 months. C started with ₹3000 and increased his investment by ₹3000 every 3 months. The ratio in which A, B and C share the profit for the first year is
(A) 15 : 14 : 13 (B) 13 : 14 : 15
(C) 14 : 13 : 15 (D) 15 : 13 : 14

**8.** The pollution level at a point 50 *m* above the main chimney of a factory should be less than 0.02%. Due to a leak in a factory, the pollution level increased to 10%. If 40% of the pollutants are cleaned up every day, then in how many days would the pollution come back to the acceptable level once again?

(Take log 2 = 0.3010 and log 3 = 0.4771)

(A) 8 (B) 10 (C) 11 (D) 13

**9.** Mr. Reddy, before shifting from the US sold few of his articles. He sells his bicycle for 30% profit, lamp for 20% loss and M.P. 3 player for 20% profit. He made neither a profit nor a loss on the lamp and the M.P. 3 player taken together. There is a net profit of 8% when the bicycle and lamp are taken together. Find the overall profit or loss on all the three items taken together.

(A) 11.66% loss (B) 20% profit

(C) 20% loss (D) 11.66% profit

**10.** An article is marked at 80% above the cost price. What is the maximum number of successive discounts of 10% each, which can be offered, before selling the article such that a loss is not incurred?

(A) 4 (B) 6 (C) 5 (D) 7

**11.** Pradeep bought a puppy for ₹ *x*. He sold it to his neighbour at a profit of *y*%. The profit realised by him is ₹ *y* in the transaction. After a couple of days, the neighbour sold it back to Pradeep at 20% loss. Finally, Pradeep is left with the puppy and ₹ 0.275*x*. Find *y*.

(A) 30 (B) 37.5

(C) 27.5 (D) 32.50

**12.** Two candidates Ram and Rajat are appearing for an examination. Each question answered correctly fetches 1 mark, while for every wrong answer 1 mark is deducted. The examination has a fixed pass mark. Ram attempts a certain percentage of questions and gets 10% of his attempts wrong. Consequently, he scored a net score of 64% of the total, which is 22 marks more than the pass mark. Rajat attempts a different percentage of questions and gets 20% of his attempts wrong. Consequently, he secures a net score of 54% of the total and passes by 12 marks. What percent of questions did Rajat attempt? _____

**13.** A retail fruit vendor purchases two varieties of oranges at the rates of 12 oranges for ₹ 18 and 18 oranges for ₹ 12. He mixes the two varieties in the ratio 2 : 3 and sells the mixed stock at a price of ₹ 144 for 10 dozens. What percentage of profit or loss does he make?

(A) $16^2/_3\%$ (B) 20%

(C) $33^1/_3\%$ (D) 25%

**14.** In an exam, P, Q and R scored 32%, 40% and 48% of the total marks respectively. Q and R passed the exam and P failed the exam. One of Q and R got 37 marks more than the pass mark. P failed by 23 marks. If the total marks of the exam are more than 300, find the pass mark of the exam.

(A) 143 (B) 263

(C) 203 (D) Cannot be determined

**15.** A cement wholesaler bought cement for ₹ 12 lakhs from a cement factory. Each day, he transports 8 lorry loads of cement from the factory to his godown at ₹ 1,250 per lorry load. From the $11^{th}$ day onwards, the factory charged him ₹ 1500 per day for not clearing their godown. It took the wholesaler 20 days to transport all the cement to his godown. If the wholesaler sold the cement to a retailer for ₹ 15.565 lakhs, find his profit percentage. _____ %

**16.** The members of a committee were asked to vote for a president. Each member voted for one of the 28 candidates. The number of votes received by each candidate was at least 1 more than the percentage of votes received by the candidate. The minimum number of members of the committee lies between

(A) 120 and 125 (B) 115 and 120

(C) 125 and 130 (D) 130 and 135

**17.** $X = 4p + 5q^2 + 6r^3$. If *p* increases by 119.7%, $q^{4/3}$ increases by 69% and $r^2$ increases by 69%, the percentage increase in X will be

(A) 107.9% (B) 119.7%

(C) 131.3% (D) None of these

**18.** A and B started a venture with different investments. It was decided that A would get 20% of the profit as salary and the remaining profit would then be equally divided between the two of them. Had the entire profit been divided in the ratio of their investments, B would have received ₹ 5200 more as his share of the profit. The respective earnings of A and B are ₹ 16800 and ₹ 11200. Find the investment of A, if B invested ₹ 82000.

(A) ₹ 44000 (B) ₹ 58000

(C) ₹ 68000 (D) ₹ 77000

**19.** A trader marks an article at p% above the cost price and sells it after offering a $\frac{2}{3}q\%$ discount at a profit of q%. Another trader marks the same article at $\frac{p}{3}\%$ above the cost price and sells it after offering a discount of $\frac{8}{5}q\%$, at a loss of $\frac{6}{5}q\%$. Find the discount percentage offered by the first trader.

(A) $16^2/_3\%$ (B) 20% (C) 25% (D) $12^1/_2\%$

**20.** A shelf in a book rack in a library is stacked with a series of $n$ volumes of an encyclopaedia. In each volume, the total thickness of the covers is 5% of the total thickness of the volume without covers. The vacant space in the shelf is 10% of the total shelf length. All the hard-bound volumes are replaced by their paper back editions, in which the total cover thickness is only 1% of the total thickness of the volume without covers and there is no change in either the number or thickness of the pages. Find the percentage increase in the vacant space.
(A) 4% (B) 30%
(C) 800/210% (D) 240/7%

**21.** A and B invested certain amounts in a business venture. Later they employed C who managed the business. At the end of the year 1, they made a certain profit and agreed to share it as follows.C would be paid a salary which was 14.28% of the profit that remains after paying C his salary and the remaining profit would be shared between A and B in the ratio of their investments. B's share in the profit was 100% more than C's salary and he had invested ₹48,000 in the business. What was the investment made by A?
(A) ₹24,000 (B) ₹48,000
(C) ₹64,000 (D) ₹1,20,000

**22.** Anil has 3 race horses A, B and C. If A wins a race, Anil makes 15 % of the money he has before the race. If B wins, he makes 25 %. If C wins, he makes 10 % and if any of his horses lose, he loses 5 % of the money he has before the race. In each race he enters only 2 horses. On a certain day there are 4 races. B wins race 1 and race 3. A wins race 2 and C wins race 4. He ends up with ₹16,632. In which race did Anil make the most money?
(A) race 1 (B) race 2
(C) race 3 (D) race 4

**23.** A book shop owner sells three books A, B and C and incurs loss of 21%, 11% and 10% respectively. The overall loss percentage on selling A and B books is 14·33% and that of the B and C books is 10·4%. Find the overall loss percentage on selling the three books.
(A) 15% (B) 12.16%
(C) 13.4% (D) 10%

**24.** A trader claims to sell his goods at a certain nominal gain percentage, but he cheats his customers by giving 200 gm less for every kg. Therefore, his actual gain percentage is more than that. Find his nominal gain percentage, if his actual gain percentage is 37.5%.
(A) 10% (B) 20%
(C) 25% (D) 15%

**25.** One morning, a tree had 10 kg of apples. Each afternoon, a bird comes to the tree and eats $x$ kg of apples. Each night, the apples on the tree increase by 10%. Find the maximum value of $x$ for which the bird can sustain itself indefinitely on the apples of the tree.
(A) 5/11 kg (B) 9/11 kg
(C) 15/11 kg (D) 10/11 kg

**26.** Praveen, Ramu and Shashi entered into a partnership with capitals of ₹62,000, ₹93,000 and ₹1,24,000 respectively. Praveen manages the business for a certain number of months while Ramu manages the business for the rest of the year. The persons who manage the business get a total commission equal to 10% of the profits. If the total income of Praveen at the end of the year is ₹2,900 and the total profit is ₹12,000, for how many months did Praveen manage the business?
(A) 5 (B) 8 (C) 7 (D) 9

**27.** Mutual Fund XYZ offers three schemes, in which one can invest for a period of 30 days.

Scheme A: The money is invested in debt funds. The scheme gives a steady return of 1.2%.

Scheme B: It caters to a bearish view of the market, offering a return of –6% if the market goes up and +9% return if the market goes down.

Scheme C: It caters to a bullish view of the market, offering a return of 13% if the market goes up and a return of –7% if the market goes down.

Find the maximum guaranteed return that a person can expect if he invested in one or more of these schemes.
(A) $\frac{13}{7}$% (B) $\frac{15}{7}$% (C) $\frac{16}{7}$% (D) $\frac{17}{7}$%

**28.** A city has a population of 182,000. It is to be divided into 16 voting districts such that the population of any of the districts does not exceed that of any of the other districts by more than 10%. Find the least population that any of the districts can have.
(A) 10,834 (B) 10,400
(C) 11,700 (D) 7,804

**29.** A fruit vendor bought some apples and some oranges. He bought the apples at the rate of 5 apples for ₹80 and 3 times as many oranges at the rate of ₹50 for half a dozen. One-sixth of the apples and one-fifth of the oranges got spoilt. He sold the remaining apples at ₹60 for 3 and the remaining oranges at ₹100 for 8. Find his profit or loss percentage approximately.
(A) 14 (B) 16 (C) 12 (D) 18

**30.** Ketan Parekh purchased 5% shares of Centasoft at ₹92 for a total sum of ₹46,000. He then sold ₹25,000 worth of the stock when the stock was quoted at ₹95, another 200 shares when the price of the stock had fallen to ₹90 and the remaining shares he sold at no gain, no loss. If he retained the shares for one dividend date before selling any of them what is his overall profit or loss?

(A) ₹4,850 (B) ₹2,850
(C) ₹2,500 (D) Data inadequate

## EXERCISE-5 Based on Data Sufficiency

***Directions for questions 1 to 30*:** Each question is followed by two statements, I and II. Answer each question based on the following directions:

Choose (A) If the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.

Choose (B) If the question can be answered by using either statement alone.

Choose (C) If the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (D) If the question cannot be answered even by using both the statements together.

**1.** Was the discount percentage offered on item *x* less than 16%?

I. Its marked price was at most ₹30 and the profit made was 25%.

II. Its cost price was ₹20.

**2.** What is the cost price of the cycle?

I. The cost price is ten times the profit, and the selling price is ₹165

II. The profit is 10% of ₹160, the selling price.

**3.** Did the transaction result in loss or gain?

I. Selling price of the article is ₹320.

II. Selling price of 18 articles is equal to the cost price of 21 articles.

**4.** What percentage of income does the man save?

I. He spends 35% of income on house rent.

II. He spends 75% of his income remaining after paying house rent on other items.

**5.** A and B are in a business, in which B joined A after *x* months. What is the value of *x*?

I. They shared the profits equally at the end of the year.

II. The capitals of A and B are in the ratio 7 : 12.

**6.** In a partnership business, did A and B invest equal capitals, given that they are in the business for the same period where A did not receive any remuneration?

I. B looked after the business and got a remuneration of 25% of the profit for it.

II. A and B got equal amounts at the end of the year.

**7.** What is the percentage of literacy in females?

I. Out of the total population, 60% are males. 25% of the males are literates and 25% of the total population are literates.

II. For every four females one is literate.

**8.** What were the sales of a salesman in a month?

I. The salesman earns ₹1500 in that month.

II. Every month the salesman gets a flat commission of 7.5% on all the sales and a bonus of 2.5% on all the sales exceeding ₹9000.

**9.** In a certain store, item X sells for 10% less than item Y. What is the ratio of the store's revenue from the sales of item X to that from the sales of item Y?

I. The store sells 20% more units of item Y than item X.

II. The store's revenue from the sales of item X is ₹6000 and from the sales of item Y is ₹8000.

**10.** If the original price of an article was ₹120, and the original price was reduced by ₹ *d*, then *d* is what percent of the reduced price?

I. *d* is 25% of the original price.

II. *d* is $^1/_3^{rd}$ of the reduced price.

**11.** A certain sum was divided among P, Q and R. Who got the least share?

I. P got 10% of the total amount

II. Q's share is 80% of R's share

**12.** Is there any improvement in Sudhir's score in Maths from test 1 to test 2?

I. In Maths he scored 60% marks in test 1.

II. In Maths he scored 70 marks in test 2.

**13.** What is the profit percent made by selling a television set?

I. 50% of the cost price of the television set is equal to 40% of its selling price.

II. The cost price of the television set is 20% less than its selling price.

**14.** What is the profit percent made by selling each banana?

I. Bananas are bought at ₹ $x$ per gross but sold at ₹ $x/8$ per dozen (1 gross = 12 dozens).

II. By selling $x$ dozens of bananas at ₹ $x$ per dozen, there is a profit of ₹ 300.

**15.** What is the profit percent made by selling two houses?

I. One house is sold at 20% profit and the other house is sold at 20% loss.

II. The cost price of each of the houses is the same.

**16.** What is the discount percentage offered on the calculator?

I. The marked price of the calculator is ₹ 800

II. The ratio of the selling price and the marked price of the calculator is 5 : 8.

**17.** A coat was marked at ₹ 600 above the cost price. What is the profit made by selling the coat?

I. 50% discount on the marked price leads to a loss of ₹ 200.

II. Discount offered on the coat is ₹ 200.

**18.** Is $x > y$?

I. 20% of $x$ is equal to 35% of $y$

II. 12% of $x$ is less than 8% of $y$.

**19.** What is the number of employees in the material department in TEAM organization which has a total of 240 employees?

I. The number of persons in the material department is 25% of the number of marketing employees. The number of marketing employees is 40% of the number of employees of TEAM.

II. If 60 employees leave the marketing department, then the number of employees in departments other than material department decreases by 20 percentage points.

**20.** What is the percentage increase in the area of the square?

I. The side of the square increased by 10%.

II. The length of the diagonal of the square increased by 10%.

**21.** What is the discount percent offered?

I. The selling price is four times the discount.

II. Five times the marked price is equal to six times the selling price.

**22.** What is the percentage increase in the volume of the cube?

I. The diagonal of the cube increased by 17%.

II. The lateral surface area of the cube increased by 56.25%.

**23.** Find the profit on article A.

I. 15% of the selling price is ₹ 3 more than 18% of its cost price.

II. 9% of the selling price exceeds 10% of the cost price by ₹ 9.

**24.** If 35% of all employees of a company are men, what percentage of the employees attended a meeting?

I. 20% of the men in the company attended the meeting.

II. 40% of the women attended the meeting.

**25.** An article was sold for ₹ 144. What was the price at which the article was bought?

I. The percentage profit made was numerically equal to the cost price in rupees.

II. Profit made on the article was 45% of the selling price.

**26.** Profits earned by a company in 1995 were ₹ 1,00,000. What was the profit earned in 1994?

I. In 1995, the income was 30% more than that in 1994.

II. The expenditure in 1995 was 10% less than that in 1994.

**27.** Is the selling price of an article more than the cost price, given that the article is sold at a discount of 20%?

I. The marked price of the article was 30% more than the cost price.

II. Had the article been sold at a discount of 40%, there would have been a loss of ₹ 50.

**28.** Two years from now, will the population of city X exceed the population of city Y?

I. The population of city X increases at the rate of 20% per annum and the population of city Y increases at the rate of 10% per annum.

II. The present population of city X is less than the present population of city Y.

**29.** The cost of a train ticket is 20% greater than the cost of a bus ticket. What percentage of the cost of the bus ticket is the cost of the air ticket?

I. The air ticket is 10% costlier than the train ticket.

II. The cost of the air ticket is as much higher than the train ticket as the train ticket is higher than the bus ticket.

**30.** In a college election, only Vinod and Varma contested. The total number of valid votes is 300. Who won in the elections?

I. Among the voters whose votes are valid, 79% of the postgraduate voters and 43% of the undergraduate voters voted in favour of Varma.

II. The ratio of the post graduates to the undergraduates in the college is 2 : 1.

## ANSWER KEYS

### Exercise-1

1. (A)
2. 600
3. 800
4. 8000
5. 100
6. (A)
7. 15
8. 3000
9. (C)
10. 64
11. (C)
12. (B)
13. (A)
14. 20
15. (B)
16. (C)
17. (D)
18. 5
19. (D)
20. (A)
21. (B)
22. 10
23. (D)
24. (B)
25. (C)
26. (C)
27. (D)
28. 0
29. (B)
30. (A)
31. 1200
32. (A)
33. (D)
34. 10
35. (B)
36. 2
37. 64
38. (A)
39. (D)
40. 102
41. 0
42. (A)
43. (C)
44. 25
45. (B)
46. (D)
47. 25
48. 1240
49. 1520
50. 102
51. 50
52. 448
53. (B)
54. 400
55. 1500
56. 2400
57. (B)
58. (D)
59. 50
60. (D)

### Exercise-2

1. (C)
2. (a) 8 (b) 2400, 2100
3. 20
4. 27
5. (A)
6. (C)
7. 480
8. (a) (C) (b) 100
9. (C)
10. 4
11. 12.5
12. (A)
13. (B)
14. (C)
15. 3750
16. (C)
17. 3600
18. 12
19. 60,000
20. 80,000
21. (C)
22. 21.6
23. 4
24. (B)
25. (C)
26. 14,000
27. 2,900
28. (C)
29. 1122
30. 1100
31. 40.8
32. 0
33. 15
34. 16,000
35. 50
36. 12
37. 5,000
38. 10
39. 56.25
40. 40
41. 20
42. 35
43. 36
44. (A)
45. 10
46. 7
47. 27.5
48. 78850
49. (A)
50. (B)
51. 25
52. (C)
53. 900
54. 180
55. 52
56. (B)
57. 30
58. (D)
59. 96,000
60. 16

### Exercise-3

1. (D)
2. (D)
3. (D)
4. 20
5. 25,000
6. 37.5
7. (B)
8. (C)
9. (A)
10. 480, 80
11. 480, 120
12. 1,500, 1,800
13. 300, 30
14. 43.75
15. 720
16. 24,000
17. (B)
18. (A)
19. 69
20. (A)
21. (B)
22. (B)
23. 100
24. 100
25. (C)
26. (C)
27. 0.3816
28. (C)
29. (B)
30. 2,14, 500
31. 20
32. 50
33. 40
34. 44
35. (C)
36. 9,000
37. (B)
38. 40
39. 36,000
40. 35
41. (D)
42. 1280
43. (D)
44. (B)
45. 25
46. (C)
47. (B)
48. (C)
49. (B)
50. 2000
51. (C)
52. (B)
53. (D)
54. 1
55. (C)
56. (B)
57. (D)
58. (D)
59. (B)
60. 20

### Exercise-4

1. (C)
2. (C)
3. (D)
4. (A)
5. (D)
6. (A)
7. (B)
8. (D)
9. (D)
10. (C)
11. (B)
12. 90
13. (B)
14. (D)
15. 10
16. (D)
17. (B)
18. (B)
19. (A)
20. (D)
21. (D)
22. (C)
23. (B)
24. (A)
25. (D)
26. (A)
27. (B)
28. (B)
29. (A)
30. (B)

### Exercise-5

1. (D)
2. (B)
3. (A)
4. (C)
5. (C)
6. (C)
7. (B)
8. (C)
9. (B)
10. (B)
11. (C)
12. (D)
13. (B)
14. (A)
15. (C)
16. (A)
17. (A)
18. (B)
19. (B)
20. (B)
21. (B)
22. (B)
23. (C)
24. (C)
25. (B)
26. (D)
27. (A)
28. (D)
29. (B)
30. (A)

## SOLUTIONS

## EXERCISE-1

**1.** Annual income is equal to the interest accrued/dividend paid $= 20000 \times \frac{5}{100} = ₹1000$

**2.** 62.5% of 960 $= \frac{5}{8}(960) = 600$

**3.** Ratio of Anil's and Sunil's profits = Ratio of time periods of Anil and Sunil = 8 : 12 = 2 : 3

Anil's share $= \frac{2}{5}(2000) = ₹800$

**4.** Let the man's income be ₹ $x$.

75% of $x = \frac{3x}{4}$

$\therefore$ Savings $= x - \frac{3x}{4} = \frac{4x-3x}{4} = \frac{x}{4}$

Given, $\frac{x}{4} = 2000$

$\Rightarrow x = ₹8000$

**5.** Let the population of India last year be 100.

Population of India this year = 200.

Percentage increase $= \frac{200-100}{100}(100)\% = 100\%$

**6.** 13% discount means the market value ₹87 where as the face value is ₹100

$\therefore$ The number of units of this stock purchased $= \frac{17400}{87} = 200$

Because, the money invested in 6% stock, every stock of face value of ₹100 gives an income of ₹6 at the end of the year.

$\therefore$ 200 units will give $200 \times 6 = ₹1200$.

$\therefore$ Yield percent $= \frac{1200}{17400} \times 100 \sim 6.9\% \sim 7\%$

**7.** Cost Price of 21 apples = ₹180

But $33^1/_3\%$ of apples are rotten.

$\therefore$ Number of apples to be sold = 21 – ($33^1/_3\%$ of 21) = 14

Selling price of 14 apples = 180 + ($16^2/_3\%$ of 180)

= 180 + (1/6 × 180) = ₹210

$\therefore$ Selling price of 1 apples $= \frac{210}{14} = ₹15$.

**8.** Ratio of profits of Sameer and Tarun

$= (9000 \times 12) : (12000 \times 9) = 1 : 1$

Sameer's share $= \frac{6000}{2} = ₹3000$

**9.** 30% of $a$ = 40% of $b$

$\Rightarrow a/b = 4/3$

Clearly, $\frac{a-b}{b} = \frac{1}{3}$

$\therefore$ Required percentage $= \frac{a-b}{b} \times 100$

i.e., $1/3 \times 100 = 33^1/_3\%$ more.

**10.** Let $x$ percent of 64 be 80

$\therefore 64 \times \frac{x}{100} = 80$

$\Rightarrow x = \frac{80 \times 100}{64} \Rightarrow x = 125$

$\therefore$ 80 is 125% of 64.

**11.** Let the cost price of an article be ₹100

Given that successive discounts are 20% and 15%

$\therefore$ S.P. = 85% of 80% of 100

$= \left(\frac{85}{100}\right)\left(\frac{80}{100}\right)(100) \Rightarrow$ S.P. = ₹68

Clearly, single discount is 32%.

**12.** Given, 20% of $n$ – 12.5% of 400 = 30

$\Rightarrow$ 20% of $n = 30 + 1/8 \times 400 \Rightarrow$ 20% of $n = 80$

$\Rightarrow n = 400$

$\therefore$ 36% of $n = 36 \times 4 = 144$

**13.** 23.33% = 33.33% – 10%

= 1/3 – 1/10 = 7/30

**14.** Required percentage $= \frac{25}{125} \times 100 = 20\%$

**15.** Let Robin's score = $x$

Ajay's score = $1.3x$

Robin's score : Ajay's score = $x : 1.3x = 10 : 13$

**16.** Given 40% of 20 – 80% of 40 + 30% of 10 – 4% of 100

$= \frac{40}{100} \times 20 - \frac{80}{100} \times 40 + \frac{30}{100} \times 10 - \frac{4}{100} \times 100.$

$= 8 - 32 + 3 - 4 = -25$

**17.** Difference in the sales tax

= 12.5% of 40,000 – 8% of 40,000 = 4.5% of 40000 = 1800.

One has to pay 1800 less for purchasing the item.

**18.** Let the cost price of each shirt be ₹100

Profit on 1 shirt = ₹20

Loss on the other shirt = ₹10

An overall profit of ₹10 is made

Profit percentage = $\frac{10}{200}(100)\% = 5\%$ Note: If two articles are bought for the same price and one is sold at $x\%$ profit and the other is sold at $y\%$ loss, then

(i) an overall profit of $\frac{x-y}{2}\%$ is made if $x > y$.

(ii) an overall loss of $\frac{y-x}{2}\%$ is made if $x < y$.

(iii) neither profit nor loss is made if $x = y$.

**19.** Let the pass marks be $p$.

Given, 150% of $p = 90$

$\Rightarrow p = 60$

**20.** Let the maximum marks be $M$.

**Method 1:**

Ajay's marks = $\frac{30}{100}M$ and Bala's marks = $\frac{40}{100}M$

Ajay failed by 15 marks and Bala got 5 marks more than the pass marks.

$\therefore$ Pass marks = $\frac{30}{100}M + 15 = \frac{40}{100}M - 5$

$20 = \frac{10}{100}M$

$M = 200$.

Also pass marks = 30% of 200 + 15 = 75

**Method 2:**

Let the pass marks be $P$

Bala's marks – Ajay's marks = $P + 5 - (P - 15) = 20$

$\frac{40}{100}M - \frac{30}{100}M = 20$

$M = 200$

**21.** Number of units that Ajay purchased = $\frac{3120}{104} = 30$

Income of Ajay at the end of the year = $30 \times 8 = ₹240$

Number of units that Sujan purchased = $\frac{3800}{95} = 40$

Income of Sujan at the end of the year = $40 \times 5 = ₹200$

$\therefore$ Ajay will get more income than Sujan.

**22.** 35% of $\frac{3n}{5} = \frac{35}{100} \times \frac{3n}{5} = \frac{21n}{100}$

10% of 110 = $\frac{10}{100} \times 110 = 11$

According to the problem $\frac{21n}{100} - 11 = 10$

$\Rightarrow \frac{21n}{100} = 21 \Rightarrow n = 100$

$\therefore$ 10% of $n$ is 10.

**23.** A's marks = 80

B's marks = 60

Let $x\%$ of A = B

$\Rightarrow \frac{x}{100} \times 80 = 60 \Rightarrow x = \frac{60 \times 100}{80} = 75$

$\therefore$ B's marks is 75% of A's marks.

**24.** 27% of 600 – 15% of 840

$= 27 \times 6 - 15 \times 84/10$

$= 162 - 126 = 36$

**25.** Loss percentage = $\frac{55000 - 44000}{55000} \times 100 = 20\%$

**26.** Let the length and breadth of the rectangle be 10 units each.

Area = $10 \times 10 = 100$ sq. units.

Given length and breadth are increased by 10%.

$\therefore$ Their new length & breadth are 11 units each

Area = $11 \times 11 = 121$ sq. units

i.e., 21% more than 100

$\therefore$ The area of the rectangle is increased by 21%.

**27.** $83.33\% = \frac{83.33}{100} = 0.833$

$= 0.8 + 0.033$

$= \frac{8}{10} + \frac{1}{30} = \frac{24+1}{30} = \frac{25}{30} = \frac{5}{6}$

**28.** Let the initial monthly salary of each person be ₹100.

After the changes, monthly salary of one of them becomes ₹20 more while that of the other becomes ₹20 less.

$\therefore$ There is no change in the total salary of the 2 persons.

Percentage change = 0%.

**29.** Let S.P. be ₹100 $\Rightarrow$ C.P. be ₹80

In the above case, if profit is calculated on selling price, then profit is 20%.

But profit percentage is to be calculated on cost price.

$\therefore$ Actual percentage of profit = $\frac{20}{80} \times 100 = 25\%$

**30.** $16^2/_3\%$ of 600 = 12½% of $x$

$\Rightarrow 1/6 \times 600 = 1/8 \times x$

$\Rightarrow x = 800$

**31.** Cost of each unit of stock = Face value – Discount = ₹ 100 – ₹ 3.5 = ₹ 96.50.

Annual income of the person = $\frac{19300}{96.50}(6) = ₹\,1200$

**32.** First number should be increased by

$\frac{9-4}{4} \times 100 = 125\%$

So that both the numbers will be equal.

**33.** Let C.P. = $x$

$1.12x = 50.40$

$x = \frac{50.40}{1.12} = ₹\,45$

**34.** Karan's score was less than Kiran's score by (50 – 40) or 10 percentage points.

**35.** Ratio of investments of Sun, Moon and Star = 5 : 6 : 7

Moon's share in profit = $\frac{6}{5+6+7} \times 5{,}40{,}000$

= 1,80,000

**36.** $A\%$ of $\frac{A}{B} + \frac{A}{B}\%$ of

$A = \frac{A}{100}\left(\frac{A}{B}\right) + \frac{\frac{A}{B}}{100}(A)$

$= \frac{A^2}{100B} + \frac{A^2}{100B} = \frac{A^2}{50B}$

C% of $\frac{A}{B} = \frac{C}{100}\left(\frac{A}{B}\right) = \frac{AC}{100B}$

$\frac{A^2}{50B} = \frac{AC}{100B}$

$C = \frac{\frac{A^2}{50B}}{\frac{A}{100B}} = 2A$

**37.** Rohit's profit = ₹ 8

Rohit's selling price = ₹ 48

Rakesh's cost price = ₹ 48

Rakesh's profit = ₹ 16

Rakesh's selling price = ₹ 64

Suresh's cost price = ₹ 64

**38.** A's income is more than B's and he spends a higher % of income (30% > 20%) on food. Hence, A spends more on food than B.

**39.** As a percentage, 30% > 25%. But here we are taking higher % (i.e., 30%) of a lower base (i.e., A). This may be more or less than 25% of B. Hence, answer is cannot be determined.

**40.** $11\frac{1}{9}\%$ of 918

$= \frac{100}{9}\%$ of 918

$= \frac{100}{9} \times \frac{1}{100} \times 918 = 102$

**41.** Let the initial price of Q be ₹ $q$ per kg. Initial price of $P$ = ₹ $2q$ per kg.

Decrease in the price of P = ₹ $0.50q$ per kg

Final price of P = ₹ $1.50q$ per kg

Increase in the price of Q = ₹ $0.50q$ per kg

Final price of Q = ₹ $1.50q$ per kg

Final sum of the prices of P and Q

= Initial sum of the prices of P and Q.

∴ Percentage increase = 0%

**42.** Cost price for the company is ₹ 60

Selling price for the company is ₹ 70

Profit = 70 – 60 = ₹ 10

Percentage of profit $\frac{10}{60} \times 100 = 16^2/_3\%$

**43.** Cost price for the company is ₹ 60

Selling price for the company is ₹ 70

Profit for the company = 70 – 10 = ₹ 10

Similarly, Profit for the dealer = 85 – 70 = ₹ 15 and

Profit for the shopkeeper = 102 – 85 = ₹ 17

∴ The shopkeeper got highest profit.

**44.** Gain percentage = $\frac{\text{C.P of 20 oranges}}{\text{C.P of 80 oranges}} \times 100$

$= \frac{20}{80} \times 100 = 25\%$

**45.** Ratio of time periods for which Antony, Ben and Charles stayed = $\frac{\text{Profit}}{\text{Investment}}$ of respective partners. = $\frac{6}{3} : \frac{4}{4} : \frac{3}{6}$

= 4 : 2 : 1

**46.** Let the number be $x$. Given, 80% of $x$ is 192

$\frac{80}{100}x = 192 \Rightarrow x = 240$

**47.** Let the original price of the tube of toothpaste be ₹ $x$.

Decrease in its price = ₹ $0.2x$

$x - 0.2x = 20 \Rightarrow x = 25$.

**48.** $44.44\%$ of $2790 = \frac{4}{9}(2790) = 1240$

**49.** Given 6% of $x$ = 24% of 380

$\Rightarrow \frac{6}{100} \times x = \frac{24}{100} \times 380$

$\Rightarrow 6x = 24 \times 380$

$\Rightarrow x = 4 \times 380$

$\Rightarrow x = 1520$

**50.** 13.6% of 750 = 136% of 75 = 75% of 136

$= \frac{3}{4} \times 136 = 102.$

**51.** C.P. of 60 oranges = S.P. of 60 oranges – Gain on 60 oranges

= S.P. of 60 oranges – S.P. of 20 oranges

= S.P. of 40 oranges

$\therefore$ Gain percentage $= \frac{20SP_1}{40SP_1} \times 100 = 50\%$

**52.** 57.14% of 784

$= \frac{57.14}{100} \times 784 = \frac{4}{7} \times 784$

$= 4 \times 112 = 448$

**53.** Let the total income of the family be ₹ 100

$\Rightarrow$ The savings = ₹ 30

The expenditure = 100 – 30 = ₹ 70

Given that expenditure on Children education is increased by 10% of the total income, i.e., ₹ 10 and the new expenditure on children education = ₹ 40

Since the savings is constant, savings as a percentage of expenditure towards education $= \frac{30}{40} \times 100 = 75\%$

**54.** Let the marked price be ₹ $x$

Discount = ₹ $0.1x$

Selling price = ₹ $0.9x$

$0.9x = 330 + 30 = 360 \Rightarrow x = 400$

**55.** 60% of $x$ = 75% of 1200 $\Rightarrow x = 1500$

**56.** Number of units of stock he purchased $= \frac{18600}{93} = 200$

If he sells each unit at ₹ 105,

His profit = 200 (105 – 93) = ₹ 2400

**57.** Let the required percentage be $x$%.

$\therefore \frac{3}{20} = x\%$ of $\frac{12}{25}$

$\Rightarrow \frac{3}{20} = \frac{x}{100} \times \frac{12}{25} \Rightarrow 12x = \frac{300 \times 25}{20} \Rightarrow x = \frac{25 \times 25}{20}$

$\Rightarrow x = 31.25$

**58.** Let the total salary of first 30 employees before hike be ₹ $x$

$\therefore$ The total salary of these 30 employees after 20% increase will be $x \times (1.1) = 1.1x$

Let the total salary of remaining 20 employees before like be ₹ $y$

$\therefore$ The total salary of these 20 employees after 20% increase will be $y \times (1.2) = 1.2\ y$

Total percentage change in the salary = $p$%

$= \frac{0.1x + 0.2y}{x + y} \times 100$

As $p$ depends on the values of $x$ and $y$ we cannot find the percentage increase in the total salary.

**59.** Girish's profit = ₹ 300

Girish's profit percentage $= \frac{300}{600}(100)\% = 50\%$

**60.** Let the number of students be $x$.

Number of students passed = 38% of $x$

Number of students failed

= (100 – 38)% of $x$ = 62% of $x$

So, (62% of $x$) – (38% of $x$) = 60

24% of $x$ = 60 $\Rightarrow x = 250$

## EXERCISE-2

**1.** Let the number of employees in the office be 100.

The men/women, married/unmarried break-up is shown below.

| | Men | Women | |
|---|---|---|---|
| Unmarried | 12 | 22 | |
| Married | 18 | 48 | 66 |
| | 30 | 70 | |

The percentage of women who are unmarried is

$\frac{22}{70} = \frac{11}{35} = \frac{1100}{35}\% = \frac{220\%}{7} = 31\frac{3}{7}\%$

**2.** (a) Since the time period of investment is the same for both, the profit would be shared in the ratio of their investments, i.e., 8000 : 7000 = 8 : 7

(b) Share of A = ₹ $\frac{8}{15}(4500)$ = ₹ 2400

Share of B = ₹ $\frac{7}{15}(4500)$ = ₹ 2100

**3.** As price is increased by 25%, new price is 125%, i.e., $\frac{5}{4}$ times the initial price.

$\therefore$ Consumption has to become $\frac{4}{5}$ times so that expenditure remains unchanged. The consumption has to become 80% or decrease by 20%.

**4.** C.P. = 225

M.P. = $1.4 \times 225 = 315$

S.P. = $0.8 \times 315 = 252$

$\therefore$ Profit = 252 – 225 = ₹27

5. Let the C.P. of the article be ₹100. Then, the new S.P. is ₹120. But this new S.P. is $3/5^{th}$ of the actual S.P. Hence, the original S.P. = ₹ $\frac{5}{3}$ (120) = ₹200.

As S.P. is double the C.P., profit on each article is equal to C.P. of each article.

$\therefore$ Profit on 45 articles = C.P. of 45 articles.

**6.** Let the worth of the stock be 100 then S.P. of $\frac{2^{th}}{5}$ of the stock = $\frac{2}{5}(100)\left(\frac{130}{100}\right) = 52$

Required S.P. of the stock = 120

$\therefore$ S.P. of remaining stock = 120 – 52 = 68.

$\therefore$ Required profit percentage = $\frac{8 \times 100}{\frac{3}{5}(100)} = 13.33\%$.

**7.** Let the listed price of the trouser be $x$.

$\frac{40}{100} \times = 320 \Rightarrow x = 800$

$\therefore$ I paid 800 – 320 = ₹480 for the trouser.

**8.** (a) Since the time of investment for A and B is 6 months and 10 months respectively, the ratio of the profit shares of A and B is = 30,000 (6) : 20,000 (10) = 18 : 20 = 9 : 10

(b) B gets 1/9 or $\frac{1}{9}(100)\% = 11.11\%$ more than A.

**9.** $\frac{40}{100}$ A = $\frac{60}{100}$ B

$\Rightarrow 2A = 3B$

But A + B = 9000 (lakhs)

$\Rightarrow A + \frac{2A}{3} = 9000 \Rightarrow A = 5400 \Rightarrow B = 3600$

Sales of A in 2005 = 5400 (1.1) = 5940

Sales of B in 2005 = 3600 (1.2) = 4320

$\therefore$ Difference in sales(in lakhs) = 5940 – 4320 = 1620.

**10.** As revenue = price (sales),

New revenue = 130% of old price (80% of old sales)

= 104 % of (old price) (old sales)

= 104 % of original revenue.

Hence, new revenue is 4% more than the original revenue.

**11.** Let the number of mangoes with A be 100

$\therefore$ Number of mangoes with B = $0.9 \times 100 = 90$ and number of mangoes with C = $0.8 \times 100 = 80$

$\therefore$ Required percent = $\frac{90 - 80}{80} \times 100 = 12.5\%$

**12.** Let $x$ be the income and $y$ be the population this year

$\therefore$ Per capita income = $x/y$.

The next year, the income will be $1.21x$ and the population will be $1.1y$

$\therefore$ Per capita income = $\frac{1.21x}{1.1y} = 1.1\frac{x}{y}$

$\therefore$ Percentage increase in per capita income

= $\left(\frac{1.1-1}{1}\right)(100\%) = 10\%$

**13.** C.P. of first article = $\frac{5060}{1.1}$ = ₹4600

C.P. of second article = $\frac{5060}{0.92}$ = ₹5500

Overall S.P. = ₹10120

Overall C.P. = ₹10100

$\therefore$ Profit = ₹20

**14.** Let the maximum marks be 100.

Then the marks scored by the student are 60.

$\therefore$ The pass mark = $\frac{60}{150}(100) = 40$

$\therefore$ Pass mark is 40% of the maximum marks.

**15.** Let the shopkeeper's cost prices be $x$ and $y$. The selling price of each item will be ₹6000.

$x = \frac{6000}{1.4} = 4285.7$

$y = \frac{6000}{1.6} = 3750$

**16.** As the price of each share is a multiple of 25, we can divide all the prices by 25. The modified prices are ₹3, ₹6, ₹4 and ₹7 respectively. Let the required amount of investment be ₹3 (6) (4) (7), i.e., ₹504. The returns for the different stocks are tabulated below.

| Stock | Return |
|---|---|
| 2% Stock at 3 | $\frac{2(504)}{3} = 336$ |
| 3% Stock at 6 | $\frac{3(504)}{6} = 252$ |
| 3% Stock at 4 | $\frac{3(504)}{4} = 378$ |
| 4% Stock at 7 | $\frac{4(504)}{7} = 228$ |

$\therefore$ 3% stock at 100 is the best option.

**17.** The ratio of their shares is

10000 (12) : 15000 (8) = 120 : 120 = 1 : 1

The profit (after Abdul's salary is deducted) will be divided equally. Let the total profit be $x$. Of this, 10%, i.e. $0.1x$ is paid to Abdul as salary. The remaining profit, i.e., $0.9x$ is divided equally among them, i.e., $0.45x$ to each.

$\therefore$ Amount received by Abdul

$= 0.45x + 0.1x = 4400 \Rightarrow 0.55x = 4400$

$\Rightarrow x = \dfrac{4400\,(100)}{55}$

$\therefore$ Amount received by Basker

$= 0.45x = ₹0.45\ (8000) = ₹3600$

**18.** S.P. of 150 kg = C.P. of 120 kg

$\Rightarrow$ S.P. of 150 kg = $(15 \times 120)$

$\therefore$ S.P. of 1 kg of sugar $= \dfrac{15 \times 120}{150} = ₹12$

**19.** Let the total profit be $x$ and the salary paid to A be $y$.

$y = 20\%$ of $(x - y) \Rightarrow y = 0.20x - 0.2y$

$\Rightarrow 0.2x = 1.2y \Rightarrow x = 6y \Rightarrow y = \dfrac{x}{6}$

Profit remaining after A's salary is paid $= x - \dfrac{x}{6} = \dfrac{5x}{6}$

This is divided equally among A and B.

A gets $\dfrac{5x}{12} + \dfrac{x}{6} = 35{,}000$

$\Rightarrow \dfrac{7x}{12} = 35{,}000 \Rightarrow x = 60{,}000$

**20.** Ratio of profit shares of Ram and Shyam = 3 × 10 : 2 × 12, i.e., 5 : 4.

After Ram is paid a salary $s$, the remaining profit is distributed among the two in the ratio 5 : 4.

So, Ram gets a total of $₹\left[s + \dfrac{5}{9}\,(8{,}00{,}000 - s)\right]$.

Given, $\dfrac{s + \dfrac{5}{9}(8{,}00{,}000 - s)}{\dfrac{4}{9}(8{,}00{,}000 - s)} = \dfrac{3}{2}$

Upon solving, we get $s = ₹80{,}000$.

**21.** Let the initial population be P

After $n$ days it is P $(1.2)^n$, i.e., P = 3000

P $(1.2)^n = 1.728 \Rightarrow n = 3$

On Thursday the population would be 5184.

**22.** Let initial price be $x$

Let initial consumption be $y$.

$\therefore$ Expenditure = $xy$

New price = 1.25 $x$

New expenditure = 1.08 $xy$

$\therefore$ New consumption $= \dfrac{1.08\,xy}{1.25x} = \dfrac{108}{125}\,y$

$y$ = initial consumption = 25 kg (given)

$\therefore$ New consumption $= \dfrac{108}{125} \times 25 = 21.6$ kg

**23.** Required percentage = (1.2) (0.8) × 100% = 96%

$\therefore$ The sales revenue decreased by 4%.

**24.** Four successive discounts of 10%, 10%, 10%, and 10% mean the final offer is

(0·9) (0·9) (0·9) (0·9) M (where M is the marked price)

= 0.6561M

The promised offer was 0·6M.

The excess amount charged was

(0·0561) (1000) = 56.1

**25.** Let $b = 3\ x\ 4$

$a = 2\ x\ 4$ and $c = 5\ x\ 3$. a : $b$ : $c$ = 8 : 12 : 15

$d = 3c$

a : $b$ : $c$ : $d$ = 8 : 12 : 15 : 45.

**26.** $20000 - 1000 \times 12 = 8000$ = A's share of profit other than salary

Let B's share of the profit = $x$

$\dfrac{8000}{x} = \dfrac{32000}{56000} \Rightarrow x = ₹14000$

**27.** Let the initial amount with Amar be ₹ $x$.

List price of the shirt = $x$.

Discount he received = $0.3x$.

As a result of the discount, he bought one more shirt costing ₹600 and was able to save ₹270. $\therefore$ $0.3x = 870$

$\therefore$ initial amount is ₹2,900.

**28.** The price of the equipment after 10 years

$= (100 - (15 + 13{\cdot}5 + 12 + \ldots. + 1.5)) \ / \ 100 \times 50{,}000$

$= \dfrac{100 - 825}{100} \times 500{,}000 = ₹87{,}500$

**29.** C.P. of the article = 816/0.8 = 1020

$\therefore$ Required S.P. = 1.1 × 1020 = ₹1122

**30.** Market value of the share = Face value + Premium

= ₹100 + 10% of ₹100 = ₹110

$\therefore$ No. of shares that can be bought $= \dfrac{1{,}21{,}000}{110} = 1100$

**31.** Required percentage

$= \dfrac{36\% \text{ of } 200 + 44\% \text{ of } 300}{500}(100\%) = 40.8\%$

**32.** Let us say previously the payment per hour of the employee was ₹100 and his working time was 10 hours

$\therefore$ His income was $100 \times 10 =$ ₹1000

Present payment per hour of the employee

$= 1.25 \times 100 =$ ₹125

Total working time $= 0.8 \times 10 = 8$ hours

$\therefore$ His income now is $125 \times 8 =$ ₹1000

$\therefore$ There is no change in his income

(OR)

$x$ ₹/hr, $y$ hrs income = ₹$xy$

$1.25x$ ₹/hr, $0.8\ y$ hrs, income = ₹$xy$. No change in the income occurs.

**33.** Roopa made no profit on one in four pens, i.e., $\dfrac{300}{4}$

$= 75$ pens

On the rest 225 pens she made a 20% profit.

Overall profit $\% = \dfrac{225\ x.2}{300} = 15\%$

**34.** The dividend from a 4% stock worth ₹4,00,000

$= \dfrac{4}{100}(4,00,000) =$ ₹16,000

**35.** Let the income of Ananth and Bhargav be $3x$ and $2x$ respectively. Let the expenditure of Bhargav be $y$, then savings of Ananth will be $y$.

Given $\dfrac{y}{2x-y} = \dfrac{3}{1}$

$\Rightarrow y = 6x - 3y$

$\Rightarrow 4y = 6x \Rightarrow x = \dfrac{2y}{3}$

$\therefore$ Expenditure of Ananth $= 3x - y = y$

$\therefore$ Expenditure of Ananth as a percentage of his income

$= \dfrac{y}{3x}(100)\%$ i.e. $\dfrac{y(3)}{3(2y)}(100)\% = 50\%$.

$\therefore$ Ananth saves 50% of his income.

**36.** Let us assume that there were 100 voters. Before the debate, 70 favoured Bush and 30 favoured Clinton.

Number of voters who shifted their loyalty to Bush

$= 20\%$ of $70 = 14$

Number of voters who shifted their loyalty to Clinton

$= 20\%$ of $30 = 6$

$\therefore$ After the debate, Clinton would have $70 - 14 + 6 = 62$ supporters and Bush would have $30 - 6 + 14 = 38$ supporters.

$\therefore$ 12% of the supporters must shift from Clinton to Bush, so that both have equal number of supporters.

**37.** Let the total number of books in the library be $x$. As the books in other Indian languages are represented by 80% of 70% of 60% of total books, we have 80% of 70% of 60% of $x = 1680$

$\Rightarrow \left(\dfrac{4}{5}\right)\left(\dfrac{7}{10}\right)\left(\dfrac{3}{5}\right)x = 1680 \Rightarrow x = 5000.$

**38.** If we assume the C.P. to be ₹100, then the M.P. will be ₹130.

The net profit is 17%, i.e., the S.P. will be ₹ (100 + 17% of 100) = ₹117. Now, as we know the S.P. and M.P., the discount percentage can be calculated as

Percentage discount $= \dfrac{\text{M.P.} - \text{S.P.}}{\text{M.P.}}(100\%)$

$= \dfrac{130 - 117}{130}(100\%) = \dfrac{13}{130}(100\%)$

**39.** Let the cost price be ₹$x$

Loss made in selling it at 25% loss = ₹ $\dfrac{x}{4}$

$400 - x = \dfrac{x}{4}$

$x = 320$

Discount = ₹100

Marked price of David = ₹500

Percentage at which his marked price was above his cost price = 56.25%

**40.** %increase $\dfrac{105 - 75}{75}(100\%) = \dfrac{30}{75}(100\%) = 40\%.$

**41.** Percentage marks in mid term exam

$= \dfrac{75}{150}(100\%) = 50\%$

Percentage marks in end term exam

$= \dfrac{105}{150}(100\%) = 70\%$

$\therefore$ Change in percentage points is $70\% - 50\% = 20$

**42.** $\dfrac{40}{100} \times 25 = 10$

$\therefore$ So far the Indian cricket team has won 10 matches. The minimum number of matches would be in the situation where India wins all the remaining matches. Let, there be $x$ number of matches required to achieve the target.

$\therefore \dfrac{10 + x}{25 + x} = \dfrac{75}{100}$

i.e., $\frac{10+x}{25+x} = \frac{3}{4}$

$\Rightarrow 40 + 4x = 75 + 3x$

$\Rightarrow x = 35$

**43.** Let the M.P. be 100.

The first discount is 20%. ∴ The first offer price is 80. The second discount is 20%. ∴ The second (and final) offer price is $\frac{80}{100}(80) = 64$.

∴ Effective discount = $\frac{100-64}{100}(100\%) = 36\%$

**44.** The difference between 5% loss and 10% profit is 15% of the cost price which is ₹2250.

∴ The cost price is $2250\left(\frac{100}{15}\right) = 15{,}000$ .

**45.** $\frac{a}{100}(1000) + \frac{b}{100}(3000-1000) = 900$ (1)

$\frac{a}{100}(1000) + \frac{b}{100}(4000-1000) = 1300$ (2)

Subtracting (1) from (2) $\frac{b}{100}1000 = 400$

$\frac{b}{100} = 0.4$

Substituting $\frac{b}{100}$ in (1) or (2),

$a = 10$.

**46.** In order to compare investments, we have to compare rate of returns.

i.e., $\frac{6}{108}$ and $\frac{7}{109}$, i.e., 5.56% and 6.42%

The rate of return on the 7% stock at 109 is better and hence, it is the better investment.

**47.** Let total C.P. be 300. Overall profit is 25%.

Total S.P. = 300 + 75 = 375

Now C.P.$_1$ = 100 (C.P. of $\frac{1}{3}^{rd}$ of the rice)

∴ C.P.$_2$ = 200 (C.P. of $\frac{2}{3}^{rd}$ of the rice)

S.P.$_1$ = 120 ∴ S.P.$_2$ = 375 – 120 = 255

∴ Profit% = $\frac{255-200}{200}(100\%) = 27.5\%$

**48.** Let the total number of votes polled be $x$.

Majority = 18924

i.e., 62% of $x$ – 38% of $x$ = 18924

$\Rightarrow$ 24% of $x$ = 18924

$\Rightarrow \frac{24}{100} \times x = 18924 \Rightarrow x = \frac{18924 \times 100}{24}$

$\Rightarrow x = 78850$

∴ The total number of votes polled were 78850.

**49.** Price of the radio at the end of the first year

$= 3000\left(1+\frac{x}{100}\right)\left(1-\frac{y}{100}\right)$

Price of the radio at the end of the next year

$= 3000\left(1+\frac{x}{100}\right)\left(1-\frac{y}{100}\right)\left(1+\frac{x}{100}\right)\left(1-\frac{y}{100}\right)$

$= 3000\left(1+\frac{x}{100}\right)^2\left(1-\frac{y}{100}\right)^2$

This equals 3000, when, $x = y$, this price equals

$3000\left[\left(1+\frac{y}{100}\right)\left(1-\frac{y}{100}\right)\right]^2$

$3000\left[1-\left(\frac{y}{100}\right)^2\right]^2$, i.e., less than 3000.

∴For this price to equal 3000, $x > y$ must hold true.

**50.** Let the price at which $x$ bought the article be $a$.

Price at which $y$ bought the article from $x = a + \frac{25}{100}a$

$= \frac{5}{4}a$

Price at which $z$ bought the article from $y = \frac{5}{4}a + \frac{1}{5}\left(\frac{5}{4}a\right) = \frac{3}{2}a$

$\frac{3}{2}a - \frac{5}{4}a = 80 \Rightarrow a = 320$.

**51.** Selling price = ₹210 = ₹350$\left(1-\frac{20}{100}\right)\left(1-\frac{d}{100}\right)$

$210 = 350\left(\frac{4}{5}\right)\left(1-\frac{d}{100}\right)$

$\frac{3}{4} = 1-\frac{d}{100}$

$d = 25$

**52.** We should consider the C.P. and S.P. of the same number of articles. The C.P. of 15 articles = ₹80

The S.P. of 15 articles = ₹84

∴The profit percentage = $\left(\frac{4}{80}\right)(100\%)$

= 5%

**53.** Profit shared by both = $\frac{90}{100}(7000)$ = ₹6300

Akbar and Birbal share this in the ratio 3 : 4.

Difference in profit shares of both is $\frac{1}{7}(6300)$

= ₹900.

**54.** Discount is calculated as % of M.P. ∴ Difference between 20% discount. and 35% discount.

= 35% M.P. – 20% M.P. = ₹27

⇒ 15% M.P. = ₹27

⇒ M.P. = ₹ $\frac{27}{15}(100)$ = ₹180

**55.** Given $P : Q : R = \frac{1}{4} : \frac{1}{5} : \frac{1}{6}$

= 15 : 12 : 10

$P + R$ is 13 more than $Q$.

∴ Percent by which $(P + R)$ is more than $Q$

= $\frac{13}{25} \times 100 = 52\%$

**56.** C gets the maximum share in the profit though his capital is the least. Therefore, his capital was invested for the longest period.

**57.** Let the annual salary of Ram be ₹100.

The amount spent by Ram = ₹80

Savings of Ram = 100 – 80 = ₹20.

If his annual salary increased by 14% and his expenditure increased by 10%, his annual salary would become ₹114 and his expenditure would become ₹88. His new savings

= 114 – 88 = ₹26.

Percentage increase in his savings = $\frac{6}{20} \times 100\%$

= 30%

**58.** Let the total profit = ₹100

A takes ₹20 as salary.

Let the remaining ₹80 be divided as a and $b$.

∴ $a + b = 80$ (1)

and $a + 20 = (b + 50/100b)$ (2)

Solving (1) and (2)

$b = 40$, $a = 40$

⇒ the ratio $a : b$, i.e.,the ratio of the capitals is 1 : 1

∴ B's capital = 10,576

**59.** Let the total population be $x$

Population of minors = $40x/100$

Population of majors = $60x/100$

Educated population

= $(35/100)\ (40x/100) + (60/100)\ x\ (60x/100)$

= 48,000

⇒ $x$ = 96,000.

**60.** I can buy 20 oranges with $\frac{20}{50} \times 100$, i.e., 40% of the money.

I am now left with (80 – 40), i.e., 40% of the money.

With 40% of the money, I can purchase

$\frac{40}{100} \times 40 = 16$ apples

## EXERCISE-3

**1.** Number of cows that Anil had at the beginning of 2010 = 24

Starting from 2010, in each year, Anil sold $q$% of the cows he had in the beginning of the year and bought $p$% of the cows that he was left with.

Number of cows Anil had after the sales in 2010 = 24 $(1 - \frac{q}{100})$

Number of cows Anil had after the purchase in 2010 = 24 $\left(1 - \frac{q}{100}\right)\left(1 + \frac{p}{100}\right)$

Number of cows Anil had at the end of 2011 = 24 $\left(1 - \frac{q}{100}\right)\left(1 + \frac{p}{100}\right)\left(1 - \frac{q}{100}\right)\left(1 + \frac{p}{100}\right)$

= 24 $\left(1 + \frac{p}{100}\right)^2 \left(1 - \frac{q}{100}\right)^2$

24 $\left(1 + \frac{p}{100}\right)^2 \left(1 - \frac{q}{100}\right)^2$ = 3 dozen = 36

If $p = q$, 24 $\left(1 + \frac{p}{100}\right)^2 \left(1 - \frac{q}{100}\right)^2 = 24 \left(1 + \frac{q}{100}\right)^2$

$\left(1 - \frac{q}{100}\right)^2 = 24\left(1 - \frac{q^2}{100^2}\right)^2$

This is always less than 24. ∵ $\left(1 - \frac{q^2}{100^2}\right)^2$ is always less than 1

If $p < q$, let $p = p_1$.

$24\left(1+\frac{p}{100}\right)^2\left(1-\frac{q}{100}\right)^2 = 24\left(1+\frac{p_1}{100}\right)^2\left(1-\frac{q}{100}\right)^2$. This is less than $24\left(1+\frac{q}{100}\right)^2\left(1-\frac{q}{100}\right)^2$, which is less than 24.

$24\left(1+\frac{p_1}{100}\right)^2\left(1-\frac{q}{100}\right)^2$ is less than 24.

Neither $p = q$ nor $p < q$ is possible.

2. Let the number of questions attempted by Anushk and Preetham be $x$ and $y$ respectively. Then the number of questions wrongly answered by Anushk and Preetham are $0.2x$ and $0.1y$ respectively.

Let the pass mark be $y$.

Given, $(0.8x) \times 2 - (0.2x)1 = P + 33$.

$(0.9y) \times 2 - (0.1y)1 = P + 48$.

$\Rightarrow 1.7y - 1.4x = 15$ (1)

Also, $x + y = 100$ (2)

Solving (1) and (2), we have $x = 50$ and $y = 50$.

$\therefore$ The pass mark is $(1.4) \times 50 = P + 33$

$\Rightarrow P = 37$.

3. As the total number of questions in the examination are not known, the question cannot be answered.

**4.** Let the cost price of the first variety of rice be ₹ $x$ per kg. Then the price of the second variety is ₹ $(x + 5)$. Let the quantities of rice of the two varieties purchased be $2y$ and $y$ respectively.

$x(2y) + (x + 5)y = 900$ (1)

$(x + 10)(2y) + (x + 15)y = 1440$ (2)

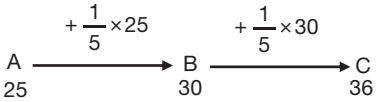
(2) – (1) $\Rightarrow y = 18$ and $x = 15$.

$\therefore$ Price of second variety of rice is ₹ 20 per kg.

**5.** $20\% = \frac{1}{5}$

We have to increase value by 20 %

Let C.P. for A be 25




Profit for B is 6 parts ≡ 6000

$\therefore$ C.P. for A is 25 parts ≡ ₹ 25,000.

**6.** As expenditure = price (consumption)

New expenditure = 110% of initial,

New price = 80% of initial price

110% of E = 80% of $P(x)$ (where $x$ is the new consumption)

$x = \frac{110}{80} = \frac{E}{P}$ As $\frac{11}{8} = 1 + \frac{3}{8}$, i.e., the new consumption is 137.5% of initial consumption.

Hence, new consumption should be 37.5% more than the initial consumption.

***Solutions for questions 7 and 8:***

The first scheme ($S_1$) offers a certain return of 0.15 %.

The second one ($S_2$) offers a gain of 5.5% or a loss of 4%.

The third one ($S_3$) offers a loss of 3.5% or a gain of 3%.

If Anita splits her investment between $S_2$ and $S_3$, she may get a guaranteed return (the smaller of the two possible returns) of more than 0.15%.

Let's assume she invests $x$ fraction of her total amount in $S_2$ and $1 - x$ in $S_3$. If the market is bearish, she would get (as a percentage figure) $5.5x - 3.5(1 - x)$.

If it is bullish she would get $-4x + 3(1 - x)$.

i.e., she gets either $9x - 3.5$ or $3 - 7x$.

For smaller values of $x$, $9x - 3.5 < 3 - 7x$

For greater values of $x$, $9x - 3.5 > 3 - 7x$.

For a critical value of $x$, $9x - 3.5 = 3 - 7x$.

This is also the value for which the guaranteed return would be the maximum.

$\therefore 9x - 3.5 = 3 - 7x \Rightarrow x = 6.5/16 = 13/32$

And guaranteed return = $\frac{9(13)}{32} - \frac{112}{32} = \frac{5}{32} = 0.15625 > 0.15$.

$\therefore$ The best option for her is to invest 13/32 in $S_2$ and 19/32 in $S_3$

**7.** She would get a guaranteed return of $\frac{5}{32}$ %.

**8.** The option which results in maximum guaranteed return is the investment where $x = 13/32$

i.e., when $\frac{1300}{32}$ % of the money is invested in $S_2$ and the rest, i.e., $\frac{1900}{32}$ % is invested in $S_3$.

9. The extreme cases for $a$, $b$ and $c$ will be when they are all equal and when two of them are nearly zero.

If all are equal, then $a = b = c = 20$.

Net discount = 48.8% which is approx. 50%. So, S.P. = 2160/2 = ₹ 1080, meaning a profit of 8%.

If two of them are nearly equal to zero, the effective discount will be 60%, so S.P. = 2160 × 0.4 = 864; therefore. there is a loss. Since the problem specifies that there is a profit, we can assume that the range of profit will be from 0 % to 8%.

Clearly, the answer is (A).

**10.** As profit is 20%,

S.P. = 120% of C.P.

= 120% of 400 = ₹480 and profit is ₹80.

**11.** Given S.P. = 600, profit = 25%, i.e., S.P. = 125% of C.P.

125% of C.P. = 600

⇒ C.P. = ₹480 and profit is ₹120.

**12.** Given, profit = 300 = 20%

20% of C.P. = 300

⇒ C.P. = ₹1500

and S.P. = ₹1800.

**13.** As loss is 10%, S.P. = 90% of C.P.

90% of C.P. = 270 ⇒ C.P. = ₹300

Hence, loss = ₹30.

**14.** Number of males = 6800

Number of females = 3200

If $x$ males are literates, $6000 - x$ females are literates.

Number of male illiterates = $6800 - x$.

Number of female illiterates = $3200 - (6000 - x)$

$= x - 2800$.

$\therefore 6800 - x = x - 2800 + 1200$

$x = 4200$

Percentage of female illiterates = $\frac{x-2800}{3200}(100)$

$= \frac{1400}{3200}(100) = 43.75\%$

**15.** Let there be 100 bees initially

| | 1st hour | 2nd hour | 3rd hour |
|---|---|---|---|
| Number of bees | 100 + 20 = 120 | 120 + 36 = 156 | 156 + 62.4 = 218.4 |

Given 62.4 – 1248

36 – ?

$? = \frac{36 \times 1248}{62.4} = 720$

∴ Effective increase = 720.

**16.** Let the total amount invested in the two stocks be ₹ $x$.

Amount invested in each stock = $\frac{x}{2}$

Annual dividend from the 4% stock = $\frac{4}{120}\left(\frac{x}{2}\right) = \frac{x}{60}$

Annual dividend from the 6% stock = $\frac{6}{96}\left(\frac{x}{2}\right) = \frac{x}{32}$

Had $x$ been invested in the 6% stock at 96, annual dividend realizable = $\frac{6}{96}x = \frac{x}{16}$

$\frac{x}{16} = \frac{x}{60} + \frac{x}{32} + 350$

$x = 24000$.

**17.** Had the profits been divided in the ratio of their investments, then profit of A would have been

(16,800 – 5,200), i.e., ₹11600 and that of

B (11,200 + 5,200), i.e., ₹16400

∴ Ratio of their investments = 11,600 : 16,800

∴ A's investment = $\frac{29}{41} \times 82{,}000 = ₹58000$

**18.** We can fill up the table and enter a new column as shown below. The entries that have been inserted in brackets are the conclusions that we draw from the other entries.

| Year | Change as a Percentage | Change in Percentage Points | Number of Tourists |
|---|---|---|---|
| 94 | — | — | $x$ (say) |
| 95 | 25% | — | $\left(\frac{5}{4}\right)x$ |
| 96 | $\left(33\frac{1}{3}\%\right)$ | $8\frac{1}{3}$ | $\left(\frac{4}{3}\right)\left(\frac{5}{4}\right)x$ |
| 97 | –25% | — | $\left(\frac{3}{4}\right)\left(\frac{4}{3}\right)\left(\frac{5}{4}\right)x$ |
| 98 | (–50%) | –25 | $\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{4}{3}\right)\left(\frac{5}{4}\right)x$ |
| 99 | 20% | | $\left(\frac{6}{5}\right)\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{4}{3}\right)\left(\frac{5}{4}\right)x = \frac{3x}{4}$ |

∴ There is a net 25% decrease from 1995 to 1999.

**19.** Let the marked price of the TV be ₹ $x$.

Marked price of the mobile phone = ₹0.6$x$

The amount that the customer actually paid = $x\left(1 - \frac{10}{100}\right)$

$+ 0.6x\left(1 - \frac{20}{100}\right) = ₹1.38x$

The amount that the customer should have paid = $x$

$\left(1-\frac{20}{100}\right) + 0.6x\left(1-\frac{10}{100}\right) = ₹1.34x$

Ratio = 69 : 67.

**20.** Let the marked price of each merchant be ₹100

Discount offered by A = ₹30

Selling price of A = ₹70

First discount offered by B = ₹10

Price after discount = ₹90

Second discount = ₹9

Price after second discount = ₹81

Third discount offered by B = ₹8·10

Selling price of B = ₹72·90

It can be similarly shown that the selling price of C = ₹72·675

∴Ascending order of the selling prices is ACB.

**21.** Let the entire sum of money be $3x$.

$\left(\frac{3x}{99}\times 8\right) - \left(\frac{2x}{110}\times 4 + \frac{x}{120}\times 5\right) = 845$

$\Rightarrow (24/99 - 4/55 - 1/24)\ x = 845$

$\Rightarrow \frac{169}{1320}x = 845$

$\Rightarrow x = \frac{845\times 1320}{169} = 6600$

∴Amount invested in the 4% stock = $2 \times 6600$

= ₹13,200

**22.** 0.7 M.P. = 1.2 C.P.

$\Rightarrow \frac{\text{M.P.}}{\text{C.P.}} = \frac{12}{7}$

$(\text{M.P.} - 400) = \frac{80}{100}\text{C.P.}$

$\text{M.P.} - 400 = \frac{4}{5}x\frac{7}{12}\text{M.P.} = \frac{7}{15}\text{M.P.}$

$\frac{8}{15}\text{M.P.} = 400$

M.P. = 750

**23.** Let the C.P. per gm be ₹1

| | **Bills 1000gm** | **Gives 900 gm** |
|---|---|---|
| Price/gm | 1 | 1 |
| | 1000 = S.P. | 900 = C.P. |

$P\% = \frac{1000-900}{900}(100\%) = \frac{100}{9}\%$

**24.** If his S.P. is 20 % more than C.P., then his S.P. = C.P. of 1200 gm.

$\therefore P\% = \frac{1200-900}{900}(100\%) = \frac{100}{3}\%$

***Solutions for questions 25 and 26:***

Let the cost of the item be ₹100

Raw materials – ₹40

Labour charges – ₹20

Fixed charges – ₹20

Miscellaneous expenses – ₹20

∴ Selling price = $1.25 \times 100$ = ₹125

After the increase the following expenditures are incurred;

Raw materials –$1.15 \times 40$ = ₹46

Labour charges –$1.2 \times 20$ = ₹24

Fixed charges = ₹20

Miscellaneous expenses = $1.5 \times 20$ = ₹30

∴ Total cost of the item = ₹120

**25.** Required profit percentage = $\frac{125-120}{120}\times 100$

$= 4^{1}/_{6}\%$

**26.** New cost price of the item

$= \frac{125}{100+13\frac{7}{11} = 100+\frac{150}{11} = \frac{1250}{11}}\times 100 = 110$

Reduction in expenditure on raw materials required = ₹10

∴Percentage reduction required = $\frac{10}{46}\times 100$

$= 21^{17}/_{23}\%$

**27.** Area each page = (34 – 2 (2)) (24 – 2 (2)) = 600 $cm^2$ = 0.06$m^2$

Cost of printing each page = $0.06 \times 6$ = ₹ 0.36

In order to get a 6% profit ,he should charge (0.36) (1.06) per page, i.e., ₹ 0.3816 per page

**28.** Let the initial population be $P$

After $n$ days it is P $(1.2)^n$, i.e., $P = 3000$

$P(1.2)^n = 1.728 \Rightarrow n = 3$

On Thursday the population would be 5184.

**29.** Let the prices of a pen, an eraser and a sharpener be ₹ $p$, ₹ $e$ and ₹ $s$ respectively.

$p = 2e$

$e = 3s$

Prices of a pen, an eraser and a sharpener after the percentage increases would be $2e(1.1) = 2.2e$, $e(1.3)$

$= 1.3\ e$ and $\frac{e}{3}(1.2) = 0.4e$ respectively.

Percentage increase in the price of 30 pencils, 30 erasers and 30 sharpeners

$$= \frac{30\left[(2.2e+1.3e+0.4e)-\left(2e+e+\frac{e}{3}\right)\right]}{30\left(2e+e+\frac{e}{3}\right)} = 17\%$$

**30.** Let the price of the camera be ₹ $x$.

Given, 6% of $(x - 120000) = 5670$

$\Rightarrow x = 2{,}14{,}500.$

**31.** Let the cost price of the first variety of rice be ₹ $x$ per kg. Then the price of the second variety is ₹ $(x + 5)$. Let the quantities of rice of the two varieties purchased be $2y$ and $y$ respectively.

$x(2y) + (x + 5)y = 900$ (1)

$(x + 10)\ (2y) + (x + 15)y = 1440$ (2)

(2) – (1) $\Rightarrow y = 18$ and $x = 15$.

$\therefore$ Price of second variety of rice is ₹ 20 per kg.

**32.** Let shopkeeper have ₹ 100

He buys goods worth $1.2 \times 100 =$ ₹ 120

He sells goods worth $0.8 \times 100 =$ ₹ 80

$\therefore$ Profit $= \frac{40}{80} \times 100 = 50\%$

**33.** Let the cost prices of the three articles A, B and C be $a$, $b$ and $c$ respectively.

Then, their selling prices could be 1.3a, $1.4b$ and $1.6c$ respectively.

Given, $1.3a + 1.6c = 1.4(a + c) \Rightarrow a = 2c$

Also, $1.4b + 1.6c = 1.48(b + c) \Rightarrow 2b = 3c$.

From these, we see that the cost prices of A, B and C are in the ratio of 4 : 3 : 2.

Hence, the overall profit percentage

$= (0.3a + 0.4b + 0.6c) / (a + b + c) \times 100 = 40\%$.

**34.** Let shopkeeper have ₹ 100.

He buys goods worth $1.2 \times 100 =$ ₹ 120.

He sells goods worth $\frac{100}{1.2} = \frac{100 \times 5}{6} =$ ₹ $\frac{250}{3}$

$$\therefore \text{Profit } \% = \frac{120 - \frac{250}{3}}{\frac{250}{3}} \times 100$$

$$= \frac{110}{250} \times 100 = 44\%$$

**35.** Let the earning of Achyut be ₹ 100. His savings will be ₹ 30 and the saving of Ranjit = ₹ 45. Since Ranjit's saving is ₹ 45, his income must be at least ₹ 45, meaning their combined income must be at least 145. So, the ratio of their combined earning to Achut's earning can not be less than 145%.

**36.** The share is quoting ₹ 150, which is 25% more than the price that Akshay paid. $\therefore$ Akshay paid ₹ (4/5) (150) or ₹ 120 per share.

No. of shares owned by Akshay $= \frac{36000}{120} = 300$

His dividend income in 2012 = 300 (5% of 100) = 1500

By selling the 300 shares at ₹ 150 each, he gets ₹ 45,000.

The dividend income he wants for 2013 = 2(₹ 1,500) or ₹ 3,000.

Hence, the number of shares he should purchase

$= \frac{3000}{7.5} = 400$

$\therefore$ The amount that has to be invested in purchasing these shares is 400 (135), i.e., ₹ 54,000.

The additional amount required = ₹ (54,000 – 45,000)

= ₹ 9,000

**37.** Let their respective populations be 3 k, 5 k and 6 k. Therefore, total population is 14 k. Increments are equal to .3 k, k and 0.9 k, respectively. Total increase = 2.2 k.

Percentage increase = 2.2 k/14 k × 100 + 15.71%.

**38.** Number of females = $0.42 \times 7500 = 3150$

$\therefore$ Number of males = 7500 – 3150 = 4350

Let the number of literate males and females be $x$ and $y$ respectively

$\therefore$ The number of illiterate males and females will be $(4350 - x)$ and $(3150 - y)$ respectively

Now, $x + y = 2370$ (1)

And $(4350 - x) - (3150 - y) = 90$

$\Rightarrow 1200 + y - x = 90$

$\Rightarrow x - y = 1110$ (2)

Solving (1) and (2), we get

$x = 1740$

$\therefore$ Required percentage $= \frac{1740}{4350} \times 100 = 40\%$

**39.** Let Ajay's income be ₹ $x$.

Given his savings = ₹ 4500.

i.e., 50% of 40% of 75% of 83.33% of $x$ = 4500.

$\Rightarrow x = 36{,}000$.

**40.** Let C.P. = 100

New S.P. = 90 (as loss = 10%)

But New S.P. $= \frac{2}{3}$ (Actual S.P.)

$90 = \frac{2}{3}$ (Actual S.P.)

Actual S.P. $= 90\left(\frac{3}{2}\right) = 135$

Hence, the profit is 35%.

**41.** Out of every 4 customers, three people bargain and one does not, so 90 articles were sold on discount and 30 articles without discount. S.P. of each article = ₹ 250

C.P. (1·5) = 250 $\Rightarrow$ C.P. = ₹ $\frac{500}{3}$.

S.P. of article after discount = 250 − 50 = ₹ 200

$\therefore$ Profit on 90 articles $= 90\left(200 - \frac{500}{3}\right) =$ ₹ 3000

$\therefore$ Profit on 30 articles $= 30\left(250 - \frac{500}{3}\right)\% =$ ₹ 2500

$\therefore$ Overall profit = ₹ 5500

$\therefore$ Overall profit percentage $= \frac{5500}{\frac{500}{3}(120)}(100\%) = 27.5\%$.

***Solution for question 42*:**

Let the cost of the item be ₹ 100

Raw materials – ₹ 40

Labour charges – ₹ 20

Fixed charges – ₹ 20

Miscellaneous expenses – ₹ 20

$\therefore$ Selling price = 1.25 × 100 = ₹ 125

After the increase the following expenditures are incurred;

Raw materials -1.15 × 40 = ₹ 46

Labour charges -1.2 × 20 = ₹ 24

Fixed charges = ₹ 20

Miscellaneous expenses = 1.5 × 20 = ₹ 30

$\therefore$ Total cost of the item = ₹ 120

**42.** Let marked price be ₹ $x$.

$(1 - 0.5) \times (1 - 0.3)x = 448$

$\Rightarrow x = \frac{448}{0.5 \times 0.7} = \frac{896}{0.7} = 1280$

***Solutions for questions 43 and 44*:**

C.P. of Ajay = 1000

S.P. of Ajay = C.P. of Bharat = $B$ (Say)

Gain% $= \frac{B - 1000}{1000} \times 100$

S.P. of Bharat = C.P. of Chandan

₹ 1500 $\Rightarrow$ gain % $= \frac{1500 - B \times 100}{B}$

S.P. of Chandan = C.P. of Deepak = $D$ (say)

$\Rightarrow$ gain% $= \frac{D - 1500}{1500} \times 100$

$\therefore$ gain % are in A.P.

$\therefore \frac{B - 100}{1000} + \frac{D - 1500}{1500} = 2\left(\frac{1500 - B}{B}\right)$

$\Rightarrow \frac{3B - 3000 + 2D - 3000}{3000} = \frac{3000 - 2B}{B}$

$\Rightarrow (3B + 2D - 6000)$

$B = 3000\ (3000 - 2B)$ (1)

Also, $\frac{D - B}{B} \times 100 = 62\frac{1}{2}$

$\Rightarrow \frac{D - B}{B} = \frac{5}{8}$

$\Rightarrow 8D - 8B = 5B \Rightarrow 8D = 13B$

$\Rightarrow \frac{D}{B} = \frac{13}{8}$ (2)

Put (2) in (1) then $B = 1200$

$\Rightarrow D = 1950$

**43.** $C - B = 1500 - 1200 =$ ₹ 300.

**44.** $D - C = 1950 - 1500 =$ ₹ 450.

**45.** Let, M.P. = $x$ and, C.P. = $y$

$0.88x = 1.1y$

$\Rightarrow x = \frac{1.1}{0.88}$ y $\Rightarrow x = 1.25y$

$\therefore$ The article was marked up by 25%.

**46.** Alok's investment: 1000 [20 + (19 + 18 + 17 + 16 + 15 + 14) + (15 + 16 + 17 + 18 + 19)]

= 1000 [20 + 14 + 2 (15 + … 19)]

= 1000 [34 + 2 (85)] = ₹ 204000

Bunty's investment: 1000 [10 + (11 + 12 + 13 + 14 + 15 + 16) + (15 + 14 + 13 + 12 + 11)]

= 1000 [10 + 16 + (11 + …. 15)]

= 1000 [26 + 65] = ₹1,56,000

Chandans investment: = 25000 × 9 = ₹225000

∴ Ratio of profits: 204000 : 91000 : 225000

= 204 : 156 : 225

Difference in the shares of Chandan and Bunty

$$= \left(\frac{225-156}{225+156+204}\right) \times 1{,}17{,}000 = \frac{69}{585} \times 1{,}17{,}000$$

= ₹13,800

**47.** Let the price of the article be ₹ $P$. When the price is increased and decreased successively by $x$%, the effective decrease is $\left(\frac{x^2}{100}\right)$ % on the list price. This is ₹2000,

After that, (P –2000) is increased by $\frac{x}{2}$% and decreased by $\frac{x}{2}$% . The effective decrease must be less than ₹500.

∴ The total decrease is in the range ₹2000 to ₹2500

∴ $47520 + 2000 < P < 47520 + 2500$.

Among the two given choices only ₹50,000 lies in that range. We can verity that this is correct. $\frac{x^2}{100} = 4$ or $x$ = 20. After the first increase/decrease the price is ₹48,000. After the second increase/decrease, there would be net decrease of 1% or 480. The final price would be ₹47,520.

**48.** Let the total sales be worth ₹ $x$

$$2000 + \frac{5}{100}(x - 10{,}000) - \frac{8}{100}x = 540$$

$$\Rightarrow 2000 + \frac{5}{100}x - 500 - \frac{8}{100}x = 540$$

$$\Rightarrow \frac{3}{100}x = 960$$

$$\Rightarrow x = 32000.$$

**49.** Total money spent by Prakash to buy the shirts = $160x$ By selling 20% of the shirts at 15% profit, he received 20% (160) $(1.15x)$ = $36.8x$

By selling 37.5% of the remaining shirts at 40% profit he received = 37.5% (160 –20% of 160) × (1.4) = $67.2x$

Total money received by him by selling 20% + 37.5% of 80%, i.e., 50% of the shirts = $36.8x + 67.2x = 104x$

To make an overall profit of 25%, his total sales must be $160x\ (1.25) = 200x$.

Therefore, he must sell the remaining 50% of the shirts for $200x - 104x = 96x$.

Profit percentage at which he must sell the remaining shirts = $\frac{96x - 80x}{80x}(100\%) = 20\%$

**NOTE**

The question can be solved by assuming the total money spent as y instead of 160 $x$. So, the number of shirts involved is redundant.

**50.**

| | Investment of A | Investment of B |
|---|---|---|
| 1st month | 35000 | 13000 |
| 2nd month | 35000 – $x$ | 13000 + $x$ |
| 3rd month | 35000 – $2x$ | 13000 + $2x$ |
| 12th month | 35000 – $11x$ | 13000 + $11x$ |

Given, profit was shred in the ratio 1:1.

It means that the investments were also in the 1:1.

$$35{,}000 + \ldots\ 35{,}000 - 11x = \frac{1}{1}$$

$$13{,}000 + \ldots 13{,}000 + 11x$$

$$35000\ (12) - 66x = 13000\ (12) + 66x$$

$$12\ (22000) = 132x$$

$$\Rightarrow x = 2{,}000$$

**51.** The amount with A, B, C, D at the different stages are tabulated below.

| Initial | A | B | C | D |
|---|---|---|---|---|
| | **100** | **100** | **100** | **100** |
| After A gives | 90 | 100 | 100 | 110 |
| After B gives | 140 | 50 | 100 | 110 |
| After C gives | 140 | 50 | 70 | 140 |
| After D gives | 140 | 85 | 70 | 105 |
| After C gives | 161 | 85 | 49 | 105 |
| After D gives | 161 | 106 | 49 | 84 |

Initially all of them start with equal amounts say $x$. Since all the amount involved are given as percentages, we can assume any convenient value for $x$. Let $x = 100$.

∴ The 4 of them, listed in the descending order of the amounts with them are ABDC.

**52.** Let the production cost be ₹100, before the currencies become costlier.

Cost of X = ₹20

Cost of Y = ₹40

Selling price of the machine = ₹125.

After the currencies become costlier, increase in the cost of X = 40% of 20 = ₹8. Increase in the cost of Y = 30% of 40 = ₹12.

∴ Product cost increases by ₹20 to become ₹120. Selling price of the machine becomes 8% more, i.e., ₹10 more, i.e., ₹135.

Profit percentage now $= \frac{135-120}{120}(100)\% = 12.5\%$

**53.** The value of land 2 yrs back was ₹50000

So, its present value will be ₹200000 (as it increases by 300%)

Amount invested on marketing = ₹10000

∴ Price that can be realised after marketing

= 200000 + 50000 = ₹250000

Profit percentage in option 2

$= \frac{250000-60000}{60000} \times 100\% = 316^{2}/_{3}\%$

∴ The required percentage difference (in percentage points) $= 16^{2}/_{3}$

**54.** Let the cost price of an orange be 100.

10 (S.P.) = 14 (100). So, S.P. = 140

$14\,(100) = \frac{1}{3}\,(M-140) \times 210 \Rightarrow M = 560$

Discount on each orange ($D$) = 560 – 140 = 420.

$\text{Discount}\% = \frac{D}{M}(100\%) = 75\%$

Markup % = 460%.

When the markup % is halved, new $M$ will be 100 + 230, i.e., 330. The discount % then is decreased by 5 percentage points.

∴ New discount is 70%.

New S.P. = 330(0.3) = 99.

∴ Loss percentage = 1%.

**55.** Let the amount the school needed for the new building be ₹ $T$. Let the total of people already solicited and to be solicited be N.

$$\frac{60}{100}T = 300\left(\frac{80}{100}N\right) \quad (1)$$

If the average donation the school should receive from the remaining people to be solicited is ₹ $x$,

$$\frac{40}{100}T = x\left(\frac{20}{100}N\right) \quad (2)$$

Dividing (2) by (1),

$$\frac{2}{3} = \frac{x}{300}\left(\frac{1}{4}\right)$$

$\Rightarrow x = 800$

**56.** The milk vendor's stock of milk gets multiplied by 5/4. He aims to get only the same amount from this increased stock, as he would have got from the initial stock.

∴ He can reduce the price per litre to $\frac{4}{5}$ of his normal price, i.e., he can reduce the price by $\left(\frac{1}{5}\right)$

**57.** Number of apples that can be sold = (1 – 0.3) 40,000

= 28,000

Selling price of 28,000 apples = (40,000 × 7) × 1.25

∴ Selling price of each apple

$= \frac{40000 \times 7 \times 1.25}{28000} = ₹12.5$

**58.** Let the total distance be $d$ and the total fuel capacity be $f$.

For covering 18% of $d$ as 12% of $f$ is used, for covering 24% of $d$. 16% of $f$ is consumed.

Hence, for the remaining journey of 58% of $d$, 72% of $f$ is available.

∴ Fuel efficiency for the remaining journey $= \frac{58\% \text{ of } d}{72\% \text{ of } f}$

∴ The required percentage

$$= \frac{\frac{18\% \text{ of } d}{12\% \text{ of } f} - \frac{58\% \text{ of } d}{72\% \text{ of } f}}{\frac{18\% \text{ of } d}{12\% \text{ of } f}} \times 100$$

$$= \frac{50\% \text{ of } d}{72\% \text{ of } f} \times \frac{12\% \text{ of } f}{18\% \text{ of } d} \times 100 = 46\%.$$

**59.** Let us say the first man invested ₹ $x$ in each stock and the second man ₹ $y$ in each stock

$2x - 2y = 960 \Rightarrow x - y = 480$ (1)

$[(x/96) \times 4 + (x/102) \times 5) - \{(y/120) \times 8 + (y/125 \times 3)\}] = 44$

$\Rightarrow \frac{37x}{408} - \frac{34y}{375} = 44$

$\Rightarrow 4625x - 4624y = 2244000$ (2)

Solving (1) and (2), we get; $x = 24{,}480$ and $y = 24{,}000$

$\therefore$ The investment made by the first man $= 2 \times 24{,}480$

$= ₹\,48960$

**60.** Let the cost prices be ₹ 500 and ₹ 300 respectively. Let the loss percent on the first be $x$.

$\therefore$ S.P.$_1 = 500\ (1 - x/100)$

S.P.$_2 = 300(1 + 2x/100)$

S.P.$_1$ + S.P.$_2 = 800 + x$

Given $x = 2.5\%$ of $800 \Rightarrow x = 20$

## EXERCISE-4

***Solutions for questions 1 to 30:***

**1.** Let the number of female students be $x$.

So, number of male students $= \frac{5x}{2}$

No. of cookies female students get $= 3x$

No. of cookies male students get

$= \frac{5x}{2} \times \frac{5x}{4} = \frac{25x^2}{8}$

Total number of cookies $= \frac{25x^2}{8} + 3x$

Number of male students who joined later

$= \frac{20}{100} \times \frac{5x}{2} = \frac{x}{2}$

Additional number of cookies that would be required

$= \frac{3x \times 3x}{2} - \frac{25x^2}{8} = \frac{11x^2}{8}$

Given, $\frac{11x^2/8}{3x + \frac{25x^2}{8}} = \frac{55}{131} \Rightarrow 131(11x) = 55(25x + 24)$

$\Rightarrow 66x = 1320 \Rightarrow x = 20.$

Hence, the original number of male students $= 2.5 \times 20 = 50$

**2.** Let the customers of Martden and Duck Walt in 1998 be 100 each.

$\therefore$ Total customers of Martden and Duck Walt in 1999 $= 200(1.4) = 280$. Out of 280, share of Martden is 50%, i.e., 50% of 280 = 140. But at the end of 1998, Martden lost 10 of its customers to Duck Walt but gained 20 customers from Duck Walt, i.e., without new customers it was 100 + 20 – 10 = 110. As on the whole it has 140 customers, remaining 30 are new customers.

$\therefore$ Of the new customers, 50 became inclined towards Duck Walt.

$\therefore$ Required percentage $= \frac{50}{80} \times 100 = 62.5\%$

**3.** Marked price of the crate of the bottle $= ₹\,\frac{11520}{0.9}$ $= ₹\,12800$. The crate can hold 800 bottles.

$\therefore$ The price of each bottle $= ₹\,\frac{12800}{800} = ₹\,16$

Inspite of the discount the customer gets beer bottles whose value is ₹ 160 less than what he pays. No. of beer bottles that he actually paid for $= \frac{11520}{16} = 720$

No. of beer bottles equivalent to ₹ 160

$= \frac{160}{16} = 10$ So, the number of beer bottles broken = 800 – 720 + 10 = 90

Value of the broken bottles $= 90 \times 16 = ₹\,1440$

**4.** Let the length, breadth and height of the cuboid be $5a$, $4a$, $3a$ respectively.

$$\frac{4\left[5a\left(1+\frac{x}{100}\right)+4a\left(1+\frac{y}{100}\right)+3a\left(1-\frac{z}{100}\right)\right]}{4[5a+4a+3a]} = 1+\frac{16\frac{2}{3}}{100}$$

$5x + 4y - 3z = 200$ (1)

$$\frac{5a\left(1+\frac{x}{100}\right)4a\left(1+\frac{y}{100}\right)3a\left(1-\frac{z}{100}\right)}{(5a)(4a)(3a)} = 1+\frac{40.4}{100}$$

$\Rightarrow (100 + x)\ (100 + y)\ (100 - z) = 1404000$ (2)

$$\frac{2(3a)\left(1-\frac{z}{100}\right)\left[5a\left(1+\frac{x}{100}\right)+4a\left(1+\frac{y}{100}\right)\right]}{2(3a)[5a+4a]} = 1+\frac{13}{100}$$

$(100 - z)\left[9 + \frac{5x}{100} + \frac{4y}{100}\right] = 101700$, i.e., $(100 - z)$ [900 + 200 + 3z] = 101700 — (3) (From (1))

From (3), $3z^2 + 800z - 8300 = 0$

$(z - 10)\ (3z + 830) = 0$

$z$ must be positive. $\therefore\ z = 10$

Note: (2) is redundant.

**5.** In the first poll, 250 votes were invalid.

$\therefore$ 1750 votes were valid.

In the repoll, 400 votes were invalid.

$\therefore$ 1600 votes were valid.

| | Votes for | Votes against |
|---|---|---|
| First vote | $x$ | $1750 - x$ |
| Repoll | $1600 - y$ | $\frac{5}{6}(1750 - x)$ (=y (say)) |

In the repoll, the motion was passed by $1600 - 2y$ votes. In the first vote, the motion was rejected by $1750 - 2x$ votes.

$1600 - 2\left(\frac{5}{6}\right)(1750 - x) = 2\ (1750 - 2x) \Rightarrow x = 850$

$\therefore$ 750 people voted against the resolution in the re-poll.

**6.** Let the cost price and the marked price of the article be ₹ $4x$ and ₹ $5x$ respectively.

Let the discount percentage offered and the profit/loss percentage made be $4y$% and $5y$% respectively.

$$\text{Discount} = \frac{4y}{100}(5x) = \frac{20xy}{100}$$

$$\text{Profit/loss} = \frac{5y}{100}(4x) = \frac{20xy}{100}$$

We see that the discount equals the profit/loss. This is only possible if a profit is made.

I. This is true.

II. Selling price $= 4x + \frac{20xy}{100} = 5x - \frac{20xy}{100}$

$\Rightarrow\ y = 2.5$

Discount percentage = $4y\% = 10\%$

II is false

**7.** A's investment is $1000\left[\frac{12}{2}\{2(1) + (12-1)1\}\right] = ₹\,78000$

B's investment is $2000 \times 2\left(\frac{6}{2}\{2(1) + (6-1)1\}\right) = ₹\,84{,}000$

C's investment is $3000 \times 3\left[\frac{4}{2}\{2(1) + (4-1)1\}\right] = ₹\,90{,}000$

$\therefore$ The ratio of investments of A, B and C is 78000 : 84000 : 90000, i.e., 13 : 14 : 15.

**8.** Let the number of days required be $n$.

On the $n^{th}$ day the level of pollution (as a percentage) would become $10\ (0.6)^n$

This should be less than 0.02.

$\Rightarrow 10\ (0.6)^n < 0.02$

Taking log on both the sides

$\Rightarrow \log 10 + n\log 0.6 < \log 2 + \log(1/100)$

$\Rightarrow 1 + n\log 6 - n\log 10 < \log 2 + \log\left(\frac{1}{100}\right)$

$\Rightarrow n\ (\log 6 - 1) < 0.3010 - 2 - 1$

$\Rightarrow n\ (\log 2 + \log 3 - 1) < -2.699$

$\Rightarrow n\ (-0.2219) < -2.699 \Rightarrow n > \frac{2.699}{0.2219} \Rightarrow n > 12.15$

$\therefore$ The pollution would come back to the acceptable level on the $13^{th}$ day.

**9.** Let the cost of Mr. Reddy's bicycle be ₹ $x$.

Let the cost of lamp be ₹ $y$.

Let the cost of M.P. 3 player be ₹ $z$

Given:

$y + z = 0.8y + 1.2z \Rightarrow y = z$ (1)

Also, $(x + y)\ (1.08) = 1.3x + 0.8y$

$\Rightarrow 0.22x = 0.28y \Rightarrow x/y = 14/11.$

Overall profit $= -0.2y + 0.2z + 0.3x = 0.3x$

Overall profit %

$$= \frac{0.3x}{x + y + z} \times 100 = \frac{0.3 \times \frac{14}{11}y}{2y + \frac{14}{11}y} \times 100 = \frac{4.2}{36} \times 100$$

$$= \frac{0.7}{6} \times 100 \quad = 11.66\%$$

**10.** Let the cost price of the article be ₹ 100

Marked price = ₹ 180

Let the number of successive discounts offered be $N$.

Selling price $= 180(0.9)^N$. This must be at least 100.

$(0.9)^N \geq \frac{5}{9}$ (i.e., $0.\bar{5}$).

Greatest value of $N$ satisfying this inequality is 5.

**11.** Let the profit percent be $y$ %. $\therefore$ The profit is ₹ $y$.

$\frac{y}{x} \times 100 = y \Rightarrow x = 100$

So, S.P. = 100 + $y$. Therefore, C.P. for the neighbour = 100 + $y$

Selling at 20% loss implies selling at $\frac{4}{5}(100 + y)$

Now the part refunded to Pradeep = $\frac{27.5}{100}(100) = 27.5$

$(100 + y) - \frac{4}{5}(100 + y) = 27.5$

$\Rightarrow 100 + y = 137.5 \Rightarrow y = 37.5.$

Therefore, he sold the puppy to his neighbour at 37.5% profit.

***Solutions for question 12*:**

Let Ram attempt $x$ questions

He gets 10% of $x$ wrong

$\therefore$ His net score

$= (0.9x) - (0.1x) = 0.8x$

Let T be the total no of questions.

$0.8x = 0.64T$

$\therefore x = 80\%$ of total

Similarly, say Rajat attempts $y$ questions. He gets 20% of $y$ wrong

$\therefore$ His net score $= 0.8y - 0.2\ y = 0.6y$

$0.6y = 0.54T$

$\therefore y = 90\%$ of total

now let $p$ be pass mark

$0.64T = P + 22$ (1)

$0.54\ T = P + 12$ (2)

By solving (1) and (2)

$0.1\ T = 10$

$T = 100$

$x = 80,\ y = 90$

$P = 42$

**12.** Rajat attempts 90% of questions

**13.** The prices of the two varieties are:

Variety 1 – 12 oranges for ₹18 or ₹3/2 per orange.

Variety 2 – ₹12 for 18 oranges or ₹2/3 per orange.

The two varieties are mixed in the ratio 2 : 3.

i.e. 2 oranges of variety 1 and 3 oranges of variety 2 are mixed. The total cost price of the 5 oranges $= (2 \times 3/2) + (3 \times 2/3) =$ ₹5.

Hence, the cost price of oranges in the mixed group is ₹1 per orange. Sale price is ₹144 for 10 dozen.

The cost of price of 10 dozen oranges of the mixed group $= 10(12)$ ₹1 = ₹120.

Hence, percentage of profit $= \left(\frac{144-120}{120}\right) \times 100\% = 20\%.$

**14.** Let the total marks of the exam be $M$ and the pass mark be $L$.

$M > 300$

One of Q and R got P + 37 marks

Case I: R got P + 37 marks.

P's marks $= \frac{32}{100} M = p - 23$

R's marks $= \frac{48}{100} M = p + 37$

$\frac{48}{100} M - \frac{32}{100} M = 60$

$\Rightarrow M = 375.\ \therefore p = 143.$

The marks of P, Q, R are 120, 150, 180

This case is possible.

Case II: Q got $p + 37$ marks.

$\frac{32}{100} M = p - 23$

$\frac{40}{100} M = p + 37$

$\frac{40}{100} M - \frac{32}{100} M = 60$

$M = 750.\ \therefore p = 263$

The marks of P, Q, R are 240, 300, 360.

This case is also possible.

The pass mark cannot be uniquely determined

**15.** The factory godowns were cleared in 20 days.

The factory started charging from the 11th day at 1500 per day.

$\therefore$ Its total charge $= (1500)\ (10)$

Total cost price of the dealer $= 12\ l + 8\ (1250)\ (10) + (1500)\ (10) + (8)\ (1250)\ (10) = 14.15\ l$

Total selling price $= 15.565\ l$

$\therefore$ profit% = 10%

**16.** Let the number of members of the committee be $M$.

Let us say the candidate $i$ received $v_i$ votes

$V_i \geq 1 + \frac{V_i}{M}(100)$

$\therefore \sum V_i \geq i \sum \left(1 + \frac{V_i}{m}(100)\right)$

$M \geq 28 + 100$, i.e., $M \geq 128$

(If each candidate has to get at least 5 votes, we need at least 140 members)

$\therefore V_i \geq \frac{128}{28} \approx 4.5$

$\therefore$ Each candidate has to get at least 5 votes.

But this is not possible, as 28 (5) = 140 while $M < 140$

If $M = 129$, $V_i \geq \frac{129}{29}$

If $M = 130$, $V_i \geq \frac{130}{30}$

If $M = 134$, $V_i \geq \frac{134}{34}$

For each $M$ from 128 to 133, $V_i$ is at least 5. $\therefore$ Each of these values of $M$ is ruled out. (if each candidate has to get at least 5 votes, we need at least 140 members)

If $M = 134$, $V_i \geq 3.94$, i.e., $V_i \geq 4$.

This is possible. (We can have 6 candidates receiving 4 votes each and 22 candidates receiving 5 votes each).

[6 (4) + 22 (5) = 134]

The least value of $M$ is 134.

**Alternate Solution:**

The number of votes each candidate got was at least 1 more than the percentage of votes that he got, i.e., if a candidate got $n$ votes, and the total number of votes is $T$ (i.e., the number of members is $T$), then $n \geq \frac{n}{T}100 + 1$
We denote this condition as C.

$C \Rightarrow n\left(1 - \frac{100}{T}\right) \geq 1$

$\Rightarrow 1 - \frac{100}{T} \geq \frac{1}{n}$

$\Rightarrow 1 - \frac{1}{n} \geq \frac{100}{T}$

$\Rightarrow T \geq \frac{100n}{n-1}$. ($\therefore n$ cannot be 1)

The values of $n$, lower bounds for $T$ and the values of $28n$ are tabulated below.

| $n$ | T | $28n$ |
|---|---|---|
| 2 | T ≥ 200 | 56 |
| 3 | T ≥ 150 | 84 |
| 4 | $T \geq 133\frac{1}{3}$ | 112 |
| 5 | T ≥ 125 | 140 |
| 6 | T ≥ 120 | 168 |

We want the minimum value of $T$. Also, the condition $C$ is satisfied for every candidate. We would get the least value of $T$, if we select the greatest possible value of $n$. This would have to be the name for every candidate.

If we have to provide 5 votes for each of the 28 candidates, $T$ has to be 140.

If we provide 4 votes for each of the 28 candidates, $T \geq 112$.

If we take $T = 134$, and distribute the votes 4 to every candidate and a fifth one for 22 of the candidates, the condition C is satisfied for every candidate.

If $T < 134$, each candidate has to get at least 5 votes. This is not possible ($\because$ 28 (5) = 140)

$\therefore$ The minimum value of T is 134. This lies between 130 and 135.

**17.** $X = 4p + 5q^2 + 6r^3$

$p$ increases by 119.7%

$q^{\frac{4}{3}}$ increases by 69%

$\therefore q^2 = \left(q^{4/3}\right)^{3/2}$

$\therefore \frac{q_2^2}{q_1^2} = \left(\frac{q_2^{4/3}}{q_1^{4/3}}\right)^{3/2} = (1.69)^{\frac{3}{2}} = 2.197$

$\therefore q^2$ increases by 119.7%

Similarly $r^2$ increases by 69%, it follows that $r^3$ increases by 119.7%

All of $p$, $q^2$, $r^3$ increase by 119.7%

$\therefore X$ will also increase by 119.7%

**18.** Had the profits been divided in the ratio of their investments, then profit of A would have been

(16,800 – 5,200), i.e., ₹ 11600 and that of

B (11,200 + 5,200), i.e., ₹ 16400

$\therefore$ Ratio of their investments = 11,600 : 16,400

$\therefore$ A's investment = $\frac{29}{41} \times 82{,}000 = ₹ 58000$

**19.** Let the cost prices of the first and the second traders be ₹ $x$ and ₹ y respectively.

$$x\left(1 + \frac{p}{100}\right)\left(1 - \frac{\frac{2}{3}q}{100}\right) = x\left(1 + \frac{q}{100}\right)$$

$$\therefore \frac{p - \frac{5}{3}q}{100} = \frac{\frac{2}{3}pq}{100^2}. \therefore 100\left(p - \frac{5}{3}q\right) = \frac{2}{3}pq \quad (1)$$

$$y\left(1 + \frac{\frac{p}{3}}{100}\right)\left(1 - \frac{\frac{8}{5}q}{100}\right) = y\left(1 - \frac{\frac{6}{5}q}{100}\right)$$

$$\frac{\frac{p}{3}-\frac{2}{5}q}{100}=\frac{\frac{8}{15}pq}{100^2}. \therefore 100\left(\frac{p}{3}-\frac{2}{5}q\right)=\frac{8}{15}pq \quad (2)$$

Dividing (1) by (2), we get $\frac{p-\frac{5}{3}q}{\frac{p}{3}-\frac{2}{5}q}=\frac{5}{4}$

$\therefore p = 2q$

(1) $100\frac{q}{3}=\frac{2}{3}(2q)(q)$

$q = 0$ or 25

$q$ cannot be 0. $\therefore$ $q$ is 25

$\therefore$ Discount percentage offered by the first trader

$= \frac{2}{3}q = 16\frac{2}{3}\%$

**20.** There are $n$ volumes on each shelf. Let the thickness of the pages of each volume be a. The total cover thickness is a/20.

Total length of the shelf occupied by the books is $n(a + a/20)$ Vacant space = 10% of total shelf length (say L) = L/10 $\therefore$ occupied space = 9L/10

i.e., $na(21/20) = 9L/10$ or $na(21) = 18L$

If all the volumes are replaced by the paperback editions, total space occupied is

$n(a + a/100) = na(101/100) = (18L/21)\ (101/100)$

$= (606/700)L$. Vacant space = 94L/700

Initial vacant space = 70L/700

Increase in vacant space = 24L/700

Percentage increase in vacant space

$= [24L/70L]\ (100)\% = 2400/70\% = 240/7\%$

**21.** Let A's investment in the business be ₹ $x$ and let the profit be ₹ $P$

A B

$x$ 48000

Let S be the salary paid to C

S = 14.28% (P – S)

$S = \frac{1}{7}(P - S)$

P = 8S

| A | B | C |
|---|---|---|
| 5S | 2S | S |

Thus, B gets $S$ + 100% of $S = 2S$

So, A gets $P - (2S + S) = 8S - 3S = 5S$

Thus the ratio of the investments made by A and B was 5 : 2. Since B invested ₹ 48000,

A invested $\frac{5}{2}(48000) = 1,20,000$

**Solutions for question 22:**

Let the initial amount that Anil has be $x$. He gains 15%, 25% and 10% if A, B, C win, respectively.

| Race | Winner | Percentage gain due to winning horse | Percentage loss due to losing horse | Net gain | Money at the end of the race |
|---|---|---|---|---|---|
| 1 | B | 25% | 5% | 20% | $1.2x$ |
| 2 | A | 15% | 5% | 10% | $1.32x$ |
| 3 | B | 25% | 5% | 20% | $1.584x$ |
| 4 | C | 10% | 5% | 5% | $1.6632x$ |

$1.6632\ x = 16,632$

$x = 10,000$

**22.** Race Gain

1 2000

2 1200

3 2640

4 792

$\therefore$ Race 3 results in the maximum gain

**23.** Let the cost of the book A be ₹ $x$

Let the cost of the book B be ₹ $y$

Let the cost of the book C be ₹ $z$

S.P. of the book A = 0.79 $x$

S.P. of the book B = 0.89 $y$

S.P. of the book C = 0.9 $z$

Overall loss percentage on the 1st two books = 14.33

$\therefore \frac{0.21x + 0.11y}{x + y} = 0.1433 \quad (1) \Rightarrow \frac{x}{y} = \frac{1}{2}$

Overall loss percentage on the 2nd & 3rd books = 10.4%

$\therefore \frac{0.11y + 0.1z}{y + z} = 0.104 \quad (2) \Rightarrow \frac{y}{z} = \frac{2}{3}$

Overall loss percentage:

$$\frac{0.21x + 0.11y + 0.1z}{x + y + z} \times 100$$

$$= \frac{1(0.21) + 2(0.11) + 3(0.1)}{1 + 2 + 3} = 0.1216 = 12.16\%$$

**24.** By back substitution, if the trader's nominal gain is 10%, he gets the price of 1,100 grams for every kg. As he measures 800 gm per kg, his overall gain percentage $\frac{300}{800}(100) = 37.5\%$

**Alternative Solution:**

Let the cost price of 1 kg be ₹ 1,000 and the nominal selling price be ₹ $(1000 + x)$.

But the trader is only giving 800 gm instead of 1 kg. His real cost price is ₹ 800 and his profit is ₹ $(200 + x)$.

$\frac{200 + x}{800}(100) = 37.5\% = \frac{3}{8} = \frac{300}{800} \Rightarrow x = 100$, i.e., his nominal profit is 10%.

**25.** Let us consider the day on which the tree had 10 kg of apples in the morning to be day 1.

On the night of day 1, after the increase, the tree had $(10 - x)$ 1.1 apples.

If $(10 - x)$ 1.1 is more than 10, the apples on the tree effectively increased on day 1.

∴ The apples on the tree after the bird eats on day 2 will be more than that on day 1. ∴ After the increase on day 2, the apples on the tree will be more than that on day 1

∴ Every day, the apples on the tree will increase.

If $(10 - x)$ 1.1 is equal to 10, every day the apples on the tree will be maintained.

If $(10 - x)$ 1.1 is less than 10, every day the apples on the tree will decrease.

For the bird to sustain itself indefinitely with the apples of the tree, $(10 - x)$ 1.1 must be at least 10.

∴ $x$ must be at most $\frac{10}{11}$.

**26.** The ratio of the capitals of Praveen, Ramu and Shashi = 62,000 : 93,000 : 1,24,000 = 2 : 3 : 4. Since 10% of the profit was paid as a commission for managing the business, the remaining 90% is the net profit. Profit share of the profit = $\frac{2}{9}\left(\frac{90}{100}\right)12000 = 2400$, i.e., the remaining ₹ 500 was paid for managing the business.

Since ₹ 500 out of a total annual commission of ₹ 1200 was paid to Praveen, he managed the business for $\frac{500}{1200}(12)$, i.e., for 5 months.

**27.** If any person invested in scheme A, he is assured a 1.2% return. If he invested in scheme B or scheme C, he gets different returns depending on stock market conditions. Suppose he invests $x$ in B and $y$ in C. Then his returns would be ₹ $\left(\frac{-6}{100}x + \frac{13}{100}y\right)$ if the market goes up and $\left(\frac{9x}{100} - \frac{7y}{100}\right)$ otherwise. His guaranteed return would then be minimum $\left(\frac{-6}{100}x + \frac{13y}{100}, \frac{9}{100}x - \frac{7}{100}y\right)$. This would be maximum when

$\frac{-6}{100}x + \frac{13}{100}y = \frac{9}{100}x - \frac{7}{100}y$, i.e., when $\Rightarrow x = \frac{4}{3}y$.

In this case, his maximum guaranteed returns

$= \left[\frac{13}{100}y - \frac{6}{100}\left(\frac{4}{3}y\right)\right] = ₹\ \frac{5y}{100}$

∴ His maximum guaranteed return percentage

$= \frac{\frac{5y}{100}}{x + y}(100)\% = \frac{\frac{5y}{100}}{\frac{7y}{3}}(100)\% = \frac{15}{7}\%$

**28.** Let the population that the least populated district can have after the division be $x$. The population of any of the districts after the division must not exceed that of any of the others by more than 10%. Also the total population of the districts is fixed. ∴ The least value of $x$ occurs when the population of each of the other districts is 10% more than $x$.

$x + 15\ (1.1x) = 182{,}000 \Rightarrow x = 10400$

**29.** The number of apples, oranges, their cost price and selling price are tabulated below

| | **Apples** | **Oranges** |
|---|---|---|
| C | 80 480 $n$ | 50 750 $n$ |
| Number | 5 30 $n$ | 6 90 $n$ |
| S | 60 500 $n$ | 100 900 $n$ |
| Numbers | 3 25 $n$ | 8 72 $n$ |

The total cost = 480 $n$ + 750 $n$ = 1230 $n$

The total sales proceeds = 500 $n$ + 900 $n$ = 1400 $n$

∴ Profit percentage = $\frac{170 \times 100}{1230} = \frac{1700}{123}\%$

$= 13.8\% \simeq 14\%$.

**30.** Total number of shares purchased = 46,000/92 = 500

He sold ₹ 25,000 worth

Stock = 250 shares at ₹ 95 each

So, he will get 250 (95) = ₹ 23,750

200 shares at 90, so he will get

200 (90) = ₹ 18,000

and the remaining 50 shares at no profit, no loss, i.e., at ₹ 92.

∴ He will get 50 (92) = 4,600

Since he retained the shares for one dividend date, he will get one dividend = 500 × 5 = 2,500

Total realization = 23,750 + 18,000 + 4,600 + 2,500 = 48,850

Profit = 48850 – 46000 = 2,850

## EXERCISE-5

**1.** Each statement is not sufficient.

Combining statements I and II,

S.P. = $20 + \frac{25}{100} \times 20 = 25$.

M.P. ≤ 30. ∴ Discount ≤ 5.

Hence, the discount percentage ≤ $\frac{5}{30} \times 100 = 16\frac{2}{3}\%$

∴ Discount percentage may or may not be less than 16%

**2.** From Statement I, profit = S.P. – C.P. so

C.P. = 10 [S.P. – C.P.] ⇒ C.P. = 10(165 – C.P.)

so we can find the cost price of the cycle

From Statement II, profit = $\frac{10}{100} \times 160 = ₹16$

So, the cost price of the cycle = 160 – 16 = ₹144

∴ Statement II alone is sufficient.

**3.** Cost price of the article is not known so we can't find the profit percentage, so Statement I alone is not sufficient

From Statement II,

18 Selling Price = 21 Cost Price

Profit percentage = $\frac{21-18}{18} \times 100$ .

Statement II alone is sufficient.

**4.** Combining Statements I and II, the percentage of income saved is $\frac{25}{100} \times \frac{65}{100} \times 100$.

**5.** Combining Statements I and II,

$(7 \times 12) = 12 \times (x) \Rightarrow x = 7$

(as they share equal profits)

∴ B joined after 12 – 7 = 5 months.

**6.** Combining Statements I and II, we can say that they did not invest equal capitals because inspite of B getting 25% remuneration, they got equal profits.

**7.** From Statement I,

Let the population be $x$, and the female literacy rate be $a$%.

Males = $0.6x$ so females = $0.4x$

$\frac{0.6x \times 25}{100} + \frac{0.4x \times a}{100} = \frac{25}{100}x$, solving, we can get the value of $a$.

This can also be obtained by Alligation the rule of from Statement II,

Female literacy = $\frac{x}{4x} \times 100\% = 25\%$

So, Statement II alone is also sufficient.

**8.** Either of the statements alone is not sufficient as earning and the rate of commission is given in different statements.

Combining Statements I and II,

let the sales values be ₹ $x$

$1500 = \frac{7.5}{100} \times (x) + [x - 9000] \times \frac{2.5}{100}$ .

Solving we can get the value of $x$.

**9.** Let the selling price of item Y be ₹ $a$

then the selling price of item X be ₹ $0.9a$

From Statement I,

Let the number of units of item Y sold be $k$

Then the number of units of item X sold be $1.2k$

Revenue from item Y = $ak$

Revenue from X = $0.9a \times 1.2k = 1.08ak$

Required ratio = $ak : 1.08 = 25 : 27$

From Statement II,

Ratio of the revenues is $\frac{6000}{8000} = \frac{3}{4}$

Hence, either statement alone is sufficient.

**10.** Reduced price = $120 - d$

From Statement I, $d = 120 \times \frac{25}{100} = 30$

Required percentage = $\frac{30}{120-30} \times 100\%$

So, Statement I alone is sufficient.

From Statement II,

$d = (1/3)\ (120 - d) \Rightarrow 4d = 120 \Rightarrow d = 30$

Required percentage = $\frac{30}{120-30} \times 100\%$

So, Statement II alone is also sufficient.

**11.** Either of the statements alone is not sufficient as the information about P, Q and R is not given in one statement.

Using both statements, let total sum be ₹ $x$

P's share = $\frac{10}{100} \times = \frac{x}{10}$

Q's share = $\frac{80}{100}$ R's share = $\frac{4}{5}$ R's share.

Q's share + R's share = $x - \frac{x}{10} = \frac{9x}{10}$

$\frac{4}{5}$ R's share + R's share = $\frac{9x}{10}$. So, R's share = $\frac{x}{2}$

Q's share = $\frac{x}{2} \times \frac{4}{5} = \frac{2x}{5}$

So, P gets the least share.

**12.** Either statement alone is not sufficient as the information about test 1 and test 2 is not given in one statement. Using both statements, in test 2 he scored 70 marks but we do not know the maximum marks in test 1 and test 2. Hence, the question cannot be answered.

**13.** From Statement I, 50% of C.P. = 40% of S.P.

C.P. : S.P. = 4 : 5

Let C.P. = $4x$, S.P. = $5x$

Profit percent = $(x/4x) \times 100\% = 25\%$

Statement I alone is sufficient.

From Statement II, C.P. = 80% of S.P.

C.P. : S.P. = 4 : 5

So, profit percent can be calculated.

Statement II alone is sufficient.

**14.** From Statement I, C.P. of one dozen bananas = $x/12$ and S.P. of one dozen bananas = $x/8$

As we know C.P. and S.P. we can find profit percentage

(Percentage profit = $\frac{\frac{x}{8} - \frac{x}{12}}{\frac{x}{12}} \times 100\% = 50\%$)

Statement I alone is sufficient

From Statement II we have

S.P. – C.P. = ₹ 300, i.e.,

$x \times x - x \times y = 300$ (where $y$ is the cost price)

$\Rightarrow x(x - y) = 300$

As we do not know $x$ and $y$ values or their relationship, we cannot answer the question.

**15.** From Statement I, we do not know the cost price or selling price of each house. Hence, we cannot find the profit percent.

Statement I alone is not sufficient.

From Statement II, only by knowing the cost prices, we can't find the profit percent.

Using both the statements, let the cost price of each house be ₹ $x$.

Total selling price of two houses = $120x/100 + 80x/100 = 2x$

Total cost price is ₹ $2x$ and total selling price is ₹ $2x$, so the profit percent is zero.

**16.** From Statement I, we don't have any information about the selling price, so we can't find the discount percent.

Statement I alone is not sufficient.

From Statement II, let the marked price and the selling price be ₹ $8x$ and ₹ $5x$ respectively.

Discount percent = $\frac{8x - 5x}{8x} \times 100\% = 37.5\%$

Statement II alone is sufficient.

**17.** The coat was marked at ₹ 600 above the cost price. Using Statement I, we know that if there is a discount of 50%, a loss of ₹ 200 is incurred. Let the cost price of the coat be ₹ $x$.

i.e., $(x + 600)\frac{1}{2} = x - 200$.

i.e., $x = 1000$, but we can't find the profit as we do not know the selling price.

Statement I alone is not sufficient.

From Statement II, selling price was ₹ 200 less than the marked price.

$\therefore$ Profit = 600 – 200 = ₹ 400.

Statement II alone is sufficient.

**18.** From Statement I,

$\frac{20}{100}x = \frac{35}{100}y$

$\Rightarrow 4x = 7y \Rightarrow \frac{x}{y} = \frac{7}{4}$

So, $x$ is greater than $y$ as $x$ and $y$ are positive.

Statement I alone is sufficient

From Statement II,

$\frac{12}{100}x \quad \frac{8}{100}y \Rightarrow 3x \quad 2y$

x and $y$ are positive. So, $x$ is less than $y$.

Statement II alone is sufficient.

**19.** There are totally 240 employees in TEAM

From Statement I, 40% of the employees work as the marketing employees (i.e., $\frac{40}{100} \times 240 = 96$ employees)

Of these 25% work in material department

i.e., $\frac{96 \times 25}{100} = 24$

24 employees work in material development section.

Statement I alone is sufficient.

From Statement II let the total number of employees in material department be $x$. Number of employees in other departments is $240 - x$.

Given $\left[\left(\frac{240-x}{240}\right) - \left(\frac{180-x}{180}\right)\right] \times 100 = 20.$

$x$ can be found $x$ is unique

Hence, Statement II alone is sufficient .

**20.** From Statement I, let side of square be $x$ units.

$\therefore$ area = $x^2$ sq. units.

When the side is increased by 10%, then area of square

$= \left(\frac{110}{100}x\right)^2 = 1.21x^2$

Percentage increase in area $= \frac{1.21x^2 - x^2}{x^2} \times 100\% = 21\%$

Statement I alone is sufficient.

From Statement II, let the length of diagonal be $d$ units.

Area of square $= \frac{1}{2}d^2$ sq. units.

When diagonal increased by 10% then the area of the square

$= 1/2\left(\frac{110}{100}d\right)^2 = \frac{1.21}{2}d^2$

Percentage increase in area

$= \frac{\frac{1.21}{2}d^2 - \frac{10d^2}{20}}{\frac{1}{2}d^2} \times 100\% = 21\%$

Statement II alone is sufficient

**21.** We know that discount percentage = discount / M.P. $\times 100$

Using Statement I alone S.P. = $4d$

M.P. = S.P. + $d$

M.P. = $5d$

Since M.P. is known in terms of discount, discount percentage can be found. Statement I alone is sufficient.

Using Statement II alone, 5M.P. = 6S.P.

$\frac{MP}{SP} = \frac{6}{5}$.

Let M.P. be $6x$ and S.P. be $5x$.

Discount = $6x - 5x = x$

As discount and M.P. are found in terms of $x$,

Discount percentage can be found.

$\therefore$ Either Statement I alone or Statement II alone is sufficient.

**22.** From Statement I, If the diagonal of the cube increases by 17% then its side (or edge) also increases by 17%. So, the percentage increase in the volume can be found.

From Statement II, when the lateral surface area increases by 56.25%, then its side increases by 25%. So, the percentage increase in the volume can be found.

**23.** From Statement I,

We have 15% of S.P. = 18% of C.P. + 3.

Profit = S.P. – C.P. But from the above equation, we cannot uniquely determine (S.P. – C.P.). Hence, Statement I alone is not sufficient.

From Statement II, 9% of S.P. = 10% of C.P. + 1.

So, we cannot find the profit. Hence, Statement II alone is not sufficient.

Using I and II together, we have two independent linear equations in S.P. and C.P. So, we can find S.P. and C.P., hence, we can find the profit. I and II together are sufficient.

**24.** Given that 35% are men $\Rightarrow$ 65% women.

From Statement I, 20% of 35% of the total = 7% of the total. But, we do not know what percentage of the women employees attended the meeting.

Hence, Statement I alone is not sufficient.

From Statement II, 40% of 65% of the total = 26% of the total. But, we do not know what percentage of the men attended the meeting. Hence, Statement II alone is not sufficient.

But, if we take both the statements together, we have (7% + 26%) of the total, i.e., 33% of the total employees attended the meeting.

**25.** Given S.P. = ₹ 144, if $c$ is the cost price.

From Statement I, $c(1 + c/100) = 144$

$100c + c^2 = 14{,}400$

(or)

$c^2 + 100c - 14{,}400 = 0$ or $c = 80$ or $-180$. As $c$ cannot be negative $c$ is 80.

From Statement II,

$\frac{\text{S.P.} - \text{C.P.}}{\text{S.P}} \times 100 = 45$

$\frac{144 - \text{C.P.}}{144} \times 100 = 45$

So, C.P. can be calculated.

Either of the statements is self-sufficient.

**26.** Let the income and expenditure in 1994 be I and E respectively. Then given I – E = 1,00,000

From Statement I, $I_{1995} = 1.3$ I

From Statement II, $E_{1995} = 0.9$ E

Even after combining both the statements,

Profit = 1.3 I – 0.9 E cannot be calculated.

**27.** From Statement I

M.P. = 1.3 C.P.

Discount = 20%

S.P. = (100 – 20) % of M.P.

⇒ S.P. = (0.8) (M.P.)

⇒ S.P. = (0.8) (1.3) C.P.

⇒ S.P. = 1.04 C.P

Thus, the article is sold at a profit.

So, Statement I alone is sufficient.

From Statement II

If D = 40%

Loss = ₹ 50

When discount is 40% S.P. = 0.6M.P.

C.P. – S.P. = 50 so C.P. – 0.6M.P. = 50

However, if D = 20%

S.P. = 0.8M.P.

C.P. – 0.8M.P. (which can be negative or non negative).

So, Statement II alone is not sufficient.

**28.** Let the present population of city X and city Y be $x$ and $y$ and rates of increase in these two cities be $a$ and $b$.

From Statement I, we do not get any information about $x$ and $y$. Hence, Statement I is insufficient.

From Statement II, we do not get any information about a and $b$.

Hence, Statement II, is insufficient.

From both statements, $a = 5$ and $b = 4$

$a(120/100)^2$ and $b(110/100)^2$

represent the population of cities X and Y after two years.

Let us consider two cases

**Case 1:**

The population of city X is 20 million and the population of city Y is 22 million.

The population of city X and city Y after two years would be 28.8 million and 26.62 million respectively. Hence, city X would have more population than city Y.

**Case 2:**

The population of city X is 10 million and the population of city Y is 20 million.

The population of city X and city Y after two years would be 1,44,000 and 2,42,000.

Hence, city X would have less population than city Y.

**29.** Let the bus ticket be ₹ 100 and the cost of air ticket be ₹ $x$. Then the train ticket will be ₹ 120. From Statement I, air ticket will be 120(1.1) = ₹ 132. Hence, air ticket is 132% of air ticket. Hence, Statement I alone is sufficient.

From Statement II, $x - 120 = 120 - 100$

$\Rightarrow x = 140$

$\therefore$ we can find $x/100$.

$\therefore$ Statement II alone is sufficient.

**30.** From Statement I, let the male voters voted in favour of Varma be $x$.

Total votes casted to Varma = $\frac{79x}{100} + \frac{43(300 - x)}{100}$

79 is a prime number so $x$ must be divisible by 100.

So, $x$ is either 100 or 200.

If $x$ is 100 or 200 the total votes received by Varma is more than 150.

So, from Statement I, we can say that Varma won in the elections.

Statement II, alone is not sufficient as nothing about the number of votes is given.

# 3 Special Equations

CHAPTER

## Learning Objectives

*In this chapter, you will*:

- Understand what special equations are
- Learn how to solve one equation with two unknown variables
- Learn how to solve two equations with three unknown variables
- Understand what polynomial equations are
- Apply understanding to solve polynomial equations in 2 variables using one equation

SPECIAL EQUATIONS are equations which cannot be solved using the regular techniques learnt in simple equations as the number of variables is higher than the number of equations. We therefore need a certain set of external conditions to arrive at an answer. In the chapter of Simple Equations, we learnt about simultaneous equations in two and three unknowns. When we have two independent equations in two unknowns or three independent equations in three unknowns, we can solve for the variables. These types of equations are called determinate equations.

The variables in Determinate Equations have unique values. However, if we have only one equation in two unknowns, or two equations in three unknowns, such equations are called 'Indeterminate Equations'. The variables here do not have unique values but take more than one value–in general, infinite number of values.

If we impose certain other conditions on these variables, then such indeterminate equations also can yield unique values for the variables. We take such conditions also into account while solving such equations.

In our earlier chapter on equations, it has been discussed that the system of equations where the equations are less than the unknowns is indeterminate, i.e., we cannot determine the values of all the unknowns uniquely. Rather, the system has an infinite set of solutions.

Consider $4x + y = 15$. This being one equation in two unknowns, is indeterminate. Suppose, we impose a condition that both $x$ and $y$ are positive integers. With this condition, the possibilities are reduced to a finite number $x = 1$, $y = 11$; $x = 2$, $y = 7$; $x = 3$, $y = 3$. If we further impose the condition that $x$ should be equal to $y$, then there is a unique solution $x = 3$, $y = 3$. So, even though we have one equation, because of additional conditions, it may have finite or sometimes even a unique solution. The conditions that we have, could be explicitly mentioned as above or could be in-built into the problem as we see in the following solved examples.

## SOLVED EXAMPLES

**3.01.** Abhinav purchased a certain number of pencils and a certain number of pens spending ₹ 15 on the whole. If each pen costs him ₹ 4 and each pencil ₹ 1, then how many pens and pencils could he have bought?

**Sol:** Solving this problem is basically solving the equation $4x + y = 15$, where $x$ and $y$, respectively, denote the number of pens and pencils purchased, here from the context we know that $x$ and $y$ should be both positive integers, and hence, this

equation has exactly the three solutions given by $(x, y) = (1, 11)$ (or) $(2, 7)$ (or) $(3, 3)$, as discussed earlier.

**3.02.** Sachin asked Anil for his birthday. Anil replied, 'Take the date and the month of my birthday. Multiply the date with 12 (which is the number of months in a year) and the month with 31 (which is the number of days in a month) and add up the two products. The sum is 452. Find Anil's birthday.

**Sol:** If we denote the date as $D$ and month as $M$, we have $12D + 31M = 452$. Here we have only one equation with two unknowns. This is an indeterminate equation. However, we have the following additional information.

1. $D$ is a positive integer less than or equal to 31.
2. $M$ is a positive integer less than or equal to 12.

To solve this equation and in fact to solve any equation in two variables, we shall

(i) first divide the entire equation by the least coefficient.

(ii) get all the fractional values on to one side, say left and all the integral values on to the other side of the equation.

(iii) replace the whole of right-hand side by $k$, where $k$ is an integer.

The following is the sequence of steps:

$12D + 31M = 452$.

(i) $\dfrac{12D + 31M}{12} = \dfrac{452}{12}$

(ii) $D + 2M + \dfrac{7M}{12} = 37 + \dfrac{8}{12}$

(iii) $\dfrac{7M - 8}{12}$ $37 - 2M - D = k$ (k an integer)

$\Rightarrow \quad M = \dfrac{12k + 8}{7}$

By trial, we can find out the value of $k$, which gives a proper value of $M$ ($M$ being a positive integer less than or equal to 12). We see that $k = 4$ gives $M = 8$. Further, values of $k$ are not feasible. Using this, we now find $D$, which is 17. Thus, the date is $17^{th}$ August. Here we get a unique solution for the equation.

We can write the same solution concisely, by focussing only on the remainder.

$12D + 31M = 452$

We select the smaller of the two coefficients, i.e., 12, as a divisor, because the number of possible remainders is less. When the LHS is divided by 12, the remainder is equal to the remainder when $7M$ is divided by 12.

$\therefore$ Rem $\left(\dfrac{7M}{12}\right)$ = Rem $\left(\dfrac{452}{12}\right) = 8$

$\Rightarrow 7M = 12x_1 + 8 \Rightarrow$ Rem $\left(\dfrac{5x_1}{7}\right) = 6$

$\Rightarrow 5x_1 = 7x_2 + 6 \Rightarrow$ Rem $\left(\dfrac{2x_2}{5}\right) = 4$

$\Rightarrow 2x_2 = 5x_3 + 4 \Rightarrow$ Rem $\left(\dfrac{x_3}{2}\right) = 0$

$\therefore$ $x_3$ can be 0, 2, 4, etc. or –2, –4, …

$x_3 = 0 \Rightarrow x_2 = 2 \Rightarrow x_1 = 4 \Rightarrow M = 8$

Substituting in the original equation, we get $D = 17$

The other values for $D$ are obtained by adding (or subtracting) the coefficient of $M$ (i.e., 31) from this value of $D$.

The corresponding value of $M$ is obtained by subtracting (or adding) the coefficient of $M$ to the, value of $M$, i.e., $(D, M)$ could be (17, 8) (48, –4) (79, –16) and (–14, 20) (–45, 32) (–76, 44), etc.

**Alternative Method:**

$12D + 31M = 452$ (1)

We notice that 4 is a factor of $12D$ and 452

$\therefore$ It is a factor of $31M$ (or $M$), i.e., $M = 4$, 8 or 12.

We also notice that

Rem $\left(\dfrac{31M}{12}\right)$ = Rem $\left(\dfrac{7M}{12}\right)$ = Rem $\left(\dfrac{452}{12}\right) = 8$

If $M = 4$, Rem $\left(\dfrac{7M}{12}\right) = 4$

If $M = 8$, Rem $\left(\dfrac{7M}{12}\right) = 8$

If $M = 12$, Rem $\left(\dfrac{7M}{12}\right) = 0$

$\therefore$ $M = 8$. Substituting in (1), we get $D = 17$, i.e., the date is 17 August.

**3.03.** Samir bought two varieties of pens, the first variety costing ₹ 17 each and the second variety costing ₹ 10 each, spending ₹ 207 in total. In how many different combinations, could he have purchased the pens?

**Sol:** Let $x$ be the number of pens of the first variety and $y$ be the number of pens of the second variety. Then,

$17x + 10y = 207$.

Proceeding as discussed in the last example,

$$x + \frac{7x}{10} + y = 20 + \frac{7}{10}$$

$$\frac{7x - 7}{10} = k \text{ (integer)}$$

$$x = \frac{10k + 7}{7}$$

As $x$ is a positive integer, the values of $k$ can be 0, 7, 14, … when, $k = 0$, $x = 1$ and $y = 19$; when $k = 7$, $x = 11$ and $y = 2$, when $k = 14$ $x = 21$, which is not possible as the amount spent on the first variety ($17 \times 21$) in this case will be more than the total amount spent.

Hence, Samir can buy these pens in exactly two different combinations.

We can write this solution briefly by focussing only on the remainders.

$17x + 10y = 207$

$7x = 10x_1 + 7 \Rightarrow \text{Rem}\left(\frac{3x_1}{7}\right) = 0$

$x_1$ can be 0, 7, 14, 21, etc. or –7, –14, etc.

$x_1 = 0 \Rightarrow x = 1 \Rightarrow y = 19$

The other values of $x$ are obtained by adding (or subtracting) 10 successively and those of $y$ are obtained by subtracting (or adding) 17 successively, i.e., $(x, y)$ could be (1, 19), (11, 2), (21, –15) or (–9, 36) (–19, 53), etc.

As $x > 0$, $y > 0$ only (1, 19) and (11, 2) are acceptable solutions.

**3.04.** I bought two different varieties of ice-creams, the first variety costing ₹ 7 per ice-cream and the other costing ₹ 13 per ice-cream. If I had paid a total amount of ₹ 134 for both varieties of ice-creams together, how many of each variety did I buy?

**Sol:** If $p$ is the number of ice-creams costing ₹ 7 per piece and $q$ is the number of ice-creams costing ₹ 13 per piece, we have $7p + 13q = 134$ (1)

The various steps involved in solving this equation are explained below.

**Step 1:** Dividing the equation with the least coefficient separating all fractions on to the left and integers on to the right and then denoting the right-hand side by $k$, we get

$$p + q + \frac{6q}{7} = 19 + \frac{1}{7}$$

$$\frac{6q - 1}{7} = k \text{ (where } k \text{ is an integer)}$$

**Step 2:** Rewrite this relationship such that the coefficient of the variable ($q$ in this case) is 1. To do this, multiply both sides of the relationship with a constant, such that the coefficient of $q$ will then be 1 more than a multiple of the denominator (the denominator being 7 in this case). By observation, we can see that 36 (which is a multiple of the coefficient 6) is 1 more than 35 (which is the multiple of the denominator 7). Hence, to get 36, we multiply both sides of the relationship with 6.

$$\frac{36q - 6}{7} = 6k \Rightarrow \frac{35q}{7} + \frac{q}{7} - \frac{6}{7} = 6k$$

$$\Rightarrow \frac{q - 6}{7} = 6k - 5q.$$

Here, again the R.H.S. is the sum or the difference of integers, and hence, will be an integer.

We call this $k_1$, and thus, we get $\frac{q - 6}{7} = k_1$

(Note that this step has to be done mentally. However, we completely skipped this step in the two examples we took earlier in this chapter).

**Step 3:**

Write the variable in the equation in terms of $k_1$. In this case, the variable $q$ is written in terms of $k_1$ as $q = 7k_1 + 6$.

**Step 4:** Substitute this value of one variable, in terms of $k$, in the original equation to express the other variable also in terms of $k$.

Here, substituting $q = 7k_1 + 6$ in equation (1) we get $p = 8 - 13k_1$.

**Step 5:** On the basis of the values of the two variables (expressed in terms of $k_1$), identify what values $k_1$ can take to ensure that the variables are positive integers.

$q = 7k_1 + 6$ and $p = 8 - 13k_1$

To ensure that $p$ is a positive integer, $k_1 \leq 0$

To ensure that $q$ is a positive integer, $k_1 \geq 0$

The only value of $k_1$ that satisfies both these conditions is $k_1 = 0$.

For $k_1 = 0$, $q = 6$ and $p = 8$. Thus, this problem has a unique solution.

**3.05.** $9x - 7y = 17$

$$\text{Rem}\left(\frac{2x}{7}\right) = \text{Rem}\left(\frac{17}{7}\right) = 3$$

**Sol:** $2x = 7x_1 + 3 \Rightarrow \text{Rem}\left(\frac{x_1}{2}\right) = 1$

$\therefore x_1 = 1, 3, 5$, etc., or $-1, -3, -5$, etc.

$x_1 = 1 \Rightarrow x = 5 \Rightarrow y = 4$

$\therefore$ In this example, because there is a negative sign, values of both $x$ and $y$ will increase together or decrease together, i.e., $(x, y) = (5, 4)$, $(12, 13)$ $(19, 22)$ etc or $(-2, -5)$ $(-9, -14)$, etc.

**3.06.** Raju bought '$e$' erasers and '$s$' sharpeners spending ₹ 24 on the whole. Each eraser cost ₹ 2 and each sharpener cost ₹ 3. Find the number of possible values of $e$.

**Sol:** Given the numbers of erasers and sharpeners bought are $e$ and $s$, respectively.

$2e + 3s = 24$

$$s = 8 - \frac{2e}{3}$$

$s$ and $e$ must be positive integers.

$\therefore e$ must be divisible by 3.

The possible values of $e$ and $s$ are given below.

$\therefore (e, s) = (3, 6), (6, 4)$ or $(9, 2)$.

$\therefore e$ has 3 possibilities.

**3.07.** Rohan asked Sohan when his birthday was. Sohan replied, 'Take the date and month of my birthday. Multiply the date by the number of months in a year. Multiply the month by 31.The sum of the products would be 639'.Find Rohan's birthday.

**Sol:** If we denote the date of birth by $D$ and the month of birth by $M$, we have $12D + 31M = 639$ (1)

As 639 as well as $12D$ are divisible by 3,

$31M$ (and hence $M$) must also be divisible by 3.

As $\text{Rem}\left(\frac{639}{12}\right) = 3$, $\text{Rem}\left(\frac{31M}{12}\right)$ is also 3.

For, $M = 3, 6, 9, 12$, $\text{Rem}\left(\frac{31M}{12}\right)$ is 9, 6, 3, 0, respectively.

When $M = 9$ then $12D = 639 - 31\ (9) = 360$

$D = 30$

Sohan's date of birth is $30^{th}$ of September.

**3.08.** Lokesh bought two varieties of books – $A$ and $B$. Each book of variety $A$ cost ₹ 30 and each book of variety $B$ cost ₹ 40. He spent a total of ₹ 720 for purchasing these books. How many different combinations are possible for his purchase?

**Sol:** Let us say Lokesh bought $a$ books of variety $A$ and $b$ books of variety $B$.

$30a + 40b = 720$

$\therefore 3a + 4b = 72$ (1)

We see that $4b$ (and hence $b$) has to be a multiple of 3 and $3a$ (and hence $a$) has to be a multiple of 4.

If $a = 4$, $b = 15$. Each subsequent solution is obtained by adding 4 (the coefficient of $b$) to the value of $a$ and subtracting 3 (the coefficient of $a$) from the value of $b$.

i.e., $3a + 4b = 72$ the values of $a$ and $b$ that satisfy the equation are listed below.

$3(4) + 4(15) = 72$

$3(8) + 4(12) = 72$

$3(12) + 4(9) = 72$

$3(16) + 4(6) = 72$

$3(20) + 4(3) = 72$

$\therefore$ There are 5 possible values for $(a, b)$.

**3.09.** Rohit bought bars of two varieties of ice cream. The first variety cost ₹ 9 per bar. The second variety cost ₹ 11 per bar. He paid a total of ₹ 227 for his purchase. How many different combinations are possible for his purchase?

**Sol:** Let the numbers of the first variety and second variety of ice cream bars she bought be $x$ and $y$, respectively.

$9x + 11y = 227$

dividing both sides by 9,

$$x + \frac{11}{9}y = 25 + \frac{2}{9}$$

$$\frac{11y - 2}{9} = 25 - x$$

Let $25 - x = k$ (1)

$k$ is an integer.

$$y = \frac{9k + 2}{11}$$

$y$ is an integer.

$\therefore 9k + 2$ must be divisible by 11.

$\therefore k$ can be 1, 12, 23, 34,……

If $k > 23$, $x < 0$

$\therefore k$ has 3 possibilities

$(x, y)$ has 3 possibilities.

**3.10.** $13x - 11y = 43$ where $x$ is a positive integer. Find the number of possible values of $(x, y)$, if $16 < x < 40$.

**Sol:** Given $13x - 11y = 43$

Dividing both sides by 11,

$$x + \frac{2}{11}x - y = 3 + \frac{10}{11}$$

$$-x+y+3=\frac{2x-10}{11}$$

Let $y+3-x=k$

$k$ is an integer.

$$x=\frac{11k+10}{2}=\frac{11}{2}k+5 \qquad (1)$$

$$16<\frac{11}{2}k+5<40.$$

$$2<k<\frac{70}{11}=6\frac{4}{11}$$

To satisfy (1), $k$ must be even.

$\therefore$ $k$ has 2 possibilities.

$\therefore$ $x$ has 2 possibilities.

**Alternate Solution:**

$13x-11y=43$

We first find one solution. The other solutions can be obtained by successively adding 11 to the value of $x$ and adding 13 to the value of $y$. We see that $\text{Rem}\left(\frac{43}{11}\right)=10$ and

$\text{Rem}\left(\frac{13x-11y}{11}\right)$

$=\text{Rem}\left(\frac{13x}{11}\right)$. As $\text{Rem}\left(\frac{13x}{11}\right)=2$, $x=5$ satisfies the equation. Different values of $x$, $y$ that satisfy the equation are listed below.

$13(5)-11(2)=43$

$\therefore$ $13(16)-11(15)=43$

and $13(27)-11(28)=43$

and $13(38)-11(41)=43$

$\therefore$ There are two values of $x$ such that $16<x<40$, that satisfy the equation.

## EXERCISE-1

***Directions for questions 1 to 40*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** If $4x+5y=27$ where $x, y \in Z^+$, then possible values of $x, y$ are

(A) 2, 3 (B) 3, 5 (C) 3, 4 (D) 3, 3

**2.** If $5x+16y=100$ where $x, y \in Z^+$ the values that $y$ can take are

(A) multiples of 10 (B) multiples of 4

(C) multiples of 5 (D) None of these

**3.** If remainder $\left(\frac{4Q}{5}\right)=4$, $Q$ is of the form

(A) $4k+1, k \in Z$ (B) $5k, k \in Z$

(C) $5k+1, k \in Z$ (D) $5k-1, k \in Z$

**4.** The number of non-negative integral solutions of the equation $3x+5y=20$ is _____.

**5.** $x$ and $y$ are positive integers such that $5x+7y=135$, which of the following is a possible value of $y$?

(A) 7 (B) 12 (C) 15 (D) 20

**6.** The value of $(x, y)$ if $14x+5y=57$ and $xy>0$ is

(A) (2, 2) (B) (3, 3) (C) (4, 4) (D) (5, 5)

**7.** The number of positive integral solutions of $5x+3y=34$ is _____.

**8.** If the remainder of $\left(\frac{4a}{7}\right)=2$, then the least possible positive integer value of $a$ is _____.

**9.** Which of the following is the value of $x$ that satisfies the equation $3x+4y=29$ and the condition $0<y<x$?

(A) 5 (B) 3 (C) 6 (D) 7

**10.** If $x>0$ and the remainder of $\left(\frac{3x}{5}\right)=3$, then which of the following set could be the values of $x$?

(A) {1, 6, 11, 16, …} (B) {1, 3, 6, 9, …}

(C) {1, 6, 9, 12, …} (D) None of these

**11.** Given $3a+7b=72$, where $a, b$ are positive integers. Which of the following is a possible value of $b$?

(A) 2 (B) 4 (C) 5 (D) 6

**12.** If the remainder obtained when $4p$ is divided by 9 is 5, then a possible value of $p$ is

(A) 3 (B) 4 (C) 6 (D) 8

**13.** The number of non-negative integral solutions of the equation $12x+7y=35$ is

(A) 0 (B) 1 (C) 2 (D) 3

**14.** In the above problem, the number of ball point pens the student purchases is

(A) 0 (B) 1 (C) 2 (D) 3

**15.** The number $3x$ divided by 7 leaves 6 as its remainder. The values of $x$ form an arithmetic progression whose common difference is
(A) 7 (B) 3 (C) 2 (D) 6

**16.** If the remainder obtained when $4S$ is divided by 7 is 3 then a possible value of $S$ where $S$ is a positive integer is
(A) 5 (B) 6 (C) 7 (D) 4

**17.** $a$ and $b$ are two positive integers such that $5a + 8b = 78$. Which of the following is a possible value of '$a$'?
(A) 3 (B) 9 (C) 14 (D) 11

**18.** A person purchased three items $a$, $b$ and $c$ spending a total amount of ₹101. Each of the items $a$, $b$ and $c$ cost ₹80, ₹3 and ₹2, respectively. Find the maximum number of items altogether he could have purchased. _____

**19.** $3x + 7y = 37$ where $x, y \in Z^+$ then the number of combinations of $x$, $y$ that satisfy the given equation is
(A) 1 (B) 2 (C) 3 (D) 0

**20.** Ramesh bought some pencils and erasers spending an amount of ₹15. If a pencil cost him ₹4 and an eraser ₹3, then how many pencils did he buy?
(A) 3 (B) 4
(C) 1 (D) Cannot be determined

**21.** Deepika purchased a total of $n$ oranges and apples costing ₹6 and ₹14 each, respectively. If she spent an amount of ₹200, then the least possible value of $n$ is _____.

**22.** Which of the following is/are a possible value of $a + b$ for which $7a - 3b = 20$?
(A) 0 (B) 20
(C) –20 (D) All the previous choices

**23.** Mr Raghu purchased toys of two types – $T_1$ and $T_2$ –costing ₹11 and ₹17, respectively. Raghu spent an amount of ₹123 for purchasing them. Find the maximum number of toys he could have purchased if he purchased at least one of each type.
(A) 7 (B) 5 (C) 9 (D) 10

**24.** Which of the following is a possible solution or are possible solutions for the equation $12x - 5y = 19$?
(A) (7, 13) (B) (12, 25)
(C) Neither (A) nor (B) (D) Both (A) and (B)

**25.** The values of $x$ that satisfy the condition that remainder of $\left(\frac{7x}{9}\right) = 7$, form an arithmetic progression with common difference _____.

**26.** The number of non-negative integral solutions of the equation $4x + 5y = 60$ is
(A) 1 (B) 2 (C) 3 (D) 4

**27.** If $7x + 4y = 64$ and $x$, $y$ are positive integers, then the values of $x$ are
(A) multiples of 7 (B) multiples of 4
(C) multiples of both 4 and 7 (D) any integer

**28.** If the remainder of $\left(\frac{4x}{5}\right) = 4$, one value of $x$ that satisfies the given equation is 1, and $k$ is any integer then the other values of $x$ are
(A) $5k + 1$ (B) $4k - 1$ (C) $5k - 1$ (D) $4k + 1$

**29.** The number of positive integer solutions of the equation $2x + 3y = 15$ is
(A) 0 (B) 1 (C) 2 (D) 3

**30.** Which of the following is a solution of $3x - 7y = 28$?
(A) (3, –8) (B) (7, 1) (C) (8, 3) (D) (14, 2)

**31.** Shakuntala has some parrots and rabbits with her. If 4 times the number of parrots added to 7 times the number of rabbits is equal to 29, then how many parrots does she have? _____

**32.** Kashyap purchased some cricket balls and shuttle cocks. The price of each cricket ball is ₹10 and the price of each cock is ₹11. If Kashyap spends a total amount of ₹130, then how many cocks did he purchase? _____

**33.** Ritish has some coins in denominations of two rupees and five rupees. If the total amount with him is ₹13, then how many coins does he have? _____

**34.** Satwick purchased some scales and charts. Each scale cost him ₹7 and each chart ₹5. If he has spent a total amount of ₹42, then how many items did he purchase?
(A) 8 (B) 7
(C) 9 (D) Cannot be determined

**35.** The number of solutions of the equation $2x + 3y = 14$ in which both $x$ and $y$ are positive integers is _____.

**36.** A student purchases gel pens, ballpoint pens and pencils by spending a total of ₹28. Each gel pen, ball point pen and pencil cost ₹15, ₹5, and ₹3, respectively. In how many combinations, can he purchase them, if he buys at least one of each item?
(A) 1 (B) 2 (C) 3 (D) 0

**37.** The sum of a two-digit number and the number formed by reversing its digits is equal to 88. The number of such numbers, in which the tens digit is greater than the units digit is
(A) 1 (B) 2 (C) 3 (D) 4

**38.** In the above problem, find the minimum number of toys the salesman could have sold.
(A) 6 (B) 4 (C) 5 (D) 8

**39.** Seven times a number plus eleven times another is equal to 61. Then, a possible value of one of the numbers is
(A) 4 (B) 2
(C) 1 (D) 6

**40.** On a particular day, a salesman sold three types of toys. Each toy of the 3 varieties costs ₹100, ₹50, and ₹25, respectively. If the total sale on that day was ₹300 and the salesman sold at least one toy of each variety, find the maximum number of toys he could have sold.
(A) 4 (B) 5
(C) 6 (D) 8

## EXERCISE-2

***Directions for questions 1 to 40***: For the multiple choice questions, select the correct alternative from the given choices. for the non-multiple choice questions, write your answer in the space provided.

**1.** Ravi spent a total of ₹65 on buying pens and markers. The price of each pen is ₹3 and that of each marker is ₹5. Find the maximum number of articles purchased. _____

**2.** How many positive integer solutions are possible for the equation?
(A) $3x + 4y = 60$ (B) $3x - 4y = 60$

**3.** Given, $7x + 13y = 157$

where $x$ and $y$ are positive integers, find how many solutions exist _____.

**4.** Given, $3x - 4y = 28$ where $x$, $y$ are positive integers. What is the least possible value of $y$? _____

**5.** Ramesh purchased stamps of 3 denominations ₹5, ₹3 and ₹2. He spent a total of ₹65. He purchased a minimum of 5 stamps of each denomination. In how many combinations could he have purchased the stamps? _____

**6.** The average percentage of a group of '$n$' students is 75. Three students who got 75, 85, and 99% join the group and one student whose percentage is between 94 and 100 leaves the group. The average percentage of the group now goes up by 2. If initially, the number of students is a multiple of 5, then what is the number of students in the group now? _____

**7.** In the above question, for how many days did the craftsman report for work?
(A) 29 (B) 24 (C) 28 (D) 30

**8.** A fruit-seller wants to sell some fruits that he has and get exactly ₹100. He has some mangoes which cost ₹5 each and some apples which cost ₹6 each. In how many possible combinations, can he sell the fruits to obtain the desired amount if he sells at least one fruit of each kind?
_____

**9.** In the month of June, a craftsman takes up an assignment on a daily-wage basis. If he completes the day's task, he earns ₹50 or else he earns only ₹30 per day. At the end of the month, if he earns ₹1430, on how many days did he not complete the task given to him?
(A) 28 (B) 1 (C) 15 (D) 5

**10.** Ramu told Ravi '8 times the date of my birth added to 15 times the month of my birth is 240'. In which month, was Ramu born?
(A) January (B) March
(C) September (D) August

***Directions for questions 11 and 12***: These questions are based on the data given below.

Mungeri Lal of Zaveri and Zaveri Sons bought three types of precious stones–emerald, jade, and topaz–totalling 27 and costing totally ₹60,000. The prices of which being ₹1500, ₹2000, and ₹2500 in that order. He has bought more than 3 stones of each variety and he has not bought the same number of stones of any two varieties.

**11.** How many emeralds and jades together did Mungerilal buy?
(A) 10 (B) 15 (C) 11 (D) 23

**12.** Which precious stone was bought in maximum number?
(A) Emerald (B) Jade
(C) Topaz (D) None of these

**13.** In a triangle, all the angles are acute and 19 times an angle equals 15 times the other. What is the least possible angle in the triangle in degrees?
(A) 57 (B) 60 (C) 44 (D) 30

**14.** Eatwell Chocolates, sell three types of candies, namely eclairs, caramel, and mint priced at ₹10, ₹2, and ₹1 for each candy. Rakshitha purchased these candies worth ₹100. She bought 4 times as many mints as caramel. What is the maximum number of éclairs she must have bought?
(A) 6 (B) 7 (C) 8 (D) 5

**15.** How many 3-digit numbers leave a remainder of 4, when divided by 21 and a remainder of 8 when divided by 13?
(A) 2 (B) 3 (C) 5 (D) 4

**16.** Ram and Ravi, two magicians, present a magic show together. Ram triples every red flower in 10 s and Ravi doubles every yellow flower in 10 s. If after a minute, there were 1049 red and yellow flowers, how many flowers were there initially? _____

**17.** A fruit vendor packs less than 21 dozen fruits into more than 15 boxes. He finds that if he packs 5 fruits less per box, he needs 10 boxes more to pack the fruits. How many fruits does he pack?
(A) 208 (B) 168 (C) 132 (D) 100

***Directions for questions 18 and 19*:** These questions are based on the information given below.

A cashier collects a sum of ₹700 in 27 notes in the denominations of ₹50, ₹10, and ₹5. The number of 5-rupee notes is less than the number of 50-rupee notes by almost 2.

**18.** How many ₹10 notes does the cashier have?

**19.** If he loses two ₹50 notes, then what would be the total number of ₹50 notes with him?

**20.** Madhavi has 5 times as many one-rupee stamps as three-rupee stamps. She also has some five-rupee stamps with her. The total value of these stamps is ₹75. If she has at least 20 one-rupee stamps, then how many stamps does she have in all?
(A) 36 (B) 37 (C) 38 (D) 39

**21.** The average weight of a group of $n$ persons is 64 kg. If 3 people whose average weight is between 70 kg and 80 kg joined the group and a man of weight 58 kg left the group, the average increases by 3 kg. If the original number of people in the group is a perfect square, then find the average weight (in kg) of the people joining the group _____.

**22.** Lara found that if 18 times the date of his birth is subtracted from 25 times the month of his birth, the result is 210. In which month was Lara born?
(A) April (B) October
(C) December (D) September

**23.** Chaitanya has ₹330 with which he has to buy stamps of U.S., England, and Russia, each costing ₹16, ₹18, and ₹24, respectively. He has promised to give 5 U.S. stamps, 4 English stamps, and 5 Russian stamps to his brother Kaushik. In how many combinations (assume that he has spent the entire amount on buying the stamps), can he buy the stamps? _____

**24.** A man stocks 2 different types of VCD players A and B priced at ₹8,000 and ₹13,000, respectively. If he has got an order of ₹2,90,000 for the VCD players, what is the minimum number of pieces he could have sold? _____.

**25.** Larry works in a circus, whose owner has laid down the following rules. If the show that Larry does on a given day is a success, he gets ₹500 from the owner. If the show is a partial failure, Larry pays the owner ₹150 and if the show is a total failure, Larry pays the owner ₹240. If Larry has 1 show on each day and if he earns ₹4,530 for the month of June, on how many days was Larry's show a success?
_____

**26.** How many 2-digit numbers leave a remainder of 1 when divided by 17 and a remainder of 2 when divided by 3?
_____

***Directions for questions 27 and 28*:** These questions are based on the information given below.

A gift shop owner buys some gifts of two kinds A and B, spending a total of ₹4000. Each piece of the kind A costs ₹100 and each piece of the kind B costs ₹20. Instead, if he buys as many pieces of B as he actually bought of A and as many pieces of A as he actually bought of B, he would spend less than half of what he actually spent.

**27.** In how many different combinations, could he have bought the gifts? _____

**28.** If he buys at least 10 pieces of each variety, then how many gifts must he have bought in total? _____

**29.** Ram bought 2 kinds of books – arts and science. Each art book costs ₹72 and each science book costs ₹110. If he has tendered exactly ₹1020 for the purchase, how many books could he have bought in total?
(A) 10 (B) 9 (C) 11 (D) 14

**30.** Ravi bought a total of 40 pens of three varieties A, B, and C. Each pen of variety A, B, and C costs ₹24, ₹50, and ₹30, respectively. He spent an amount of ₹1420. If he has bought the greatest number of pens of variety B that satisfies these conditions, then how many pens of varieties A and C together did he buy?
(A) 23 (B) 26 (C) 24 (D) 25

**31.** Mahta Tools Ltd, a manufacturer of tools produces a certain type of pliers. Adam, a worker in the packaging section, is supposed to pack less than 275 pliers into boxes. Adam finds that if he packs nine more pliers in each box, he can pack them in five boxes less. If he has to pack more than 10 boxes, find the number of pliers he can actually pack.
(A) 160 (B) 270 (C) 240 (D) 260

**32.** How many ordered pairs of integers $(x, y)$, where $x > 0$ satisfy the equation $\frac{1}{x} + \frac{7}{y} = \frac{1}{21}$?

(A) 14 (B) 24 (C) 16 (D) 12

**33.** How many integral values of $(x, y)$ satisfy the equation $x^2 - y^2 = 220$?

(A) 4 (B) 8 (C) 12 (D) 6

**34.** How many ordered pairs of integers satisfy the equation $\frac{13}{x} - \frac{7}{y} = \frac{1}{4}$?

(A) 39 (B) 19 (C) 38 (D) 40

**35.** How many integral values of $(x, y)$ satisfy the equation $x^2 - y^2 = 627$?

(A) 20 (B) 10 (C) 16 (D) 8

**36.** In an acute triangle, the angles are all positive integers, when measured in degrees. Thirteen times one of the angles is equal to 17 times another. What is the measure in degrees of the least possible angle in the triangle?

_____

**37.** A question paper contains a certain number of 10-mark, 5-mark, and 2-mark questions with a total of 35 questions in it. The maximum marks that a student can secure in the paper are 100. How many 5-mark questions are there in the paper?

(A) 30 (B) 2 (C) 3 (D) 4

**38.** An organization has 14 employees belonging to three different sections A, B, and C. The monthly salaries of each of the employees of the sections A, B, and C are, respectively, ₹11000, ₹6000, and ₹2000. If the total salaries paid to the employees of all the three sections is ₹84000, then the number of employees in the section A is _____.

**39.** On a certain winter day, a dealer sold some woollen jackets, sweaters, and gloves. He sold each jacket at ₹300, each sweater at ₹175 and each pair of gloves at ₹100. If the dealer sold at least one piece of each variety and made a total collection of ₹1175 by the end of the day, then how many sweaters did he sell?

(A) 1 (B) 5 (C) 4 (D) 2

**40.** Jasmine wants to buy some apples, oranges, and jackfruits. Each apple costs ₹8, each orange costs ₹3 and each jackfruit costs ₹13. If she wants to spend an amount of ₹112 and if she wants to buy a minimum of 4 fruits of each variety, then in how many different combinations can she buy the fruits?

(A) 1 (B) 2 (C) 3 (D) 4

## EXERCISE-3

***Directions for questions 1 to 40***: For the multiple choice questions, select the correct alternative from the given choices. for the non-multiple choice questions, write your answer in the space provided.

**1.** The average weight of a certain group of $n$ men is 72 kg. When two men whose weights are 73 kg and 67 kg join the group, and one man whose weight is more than 58 kg leaves the group, the average goes up by 3 kg. If the initial number of men in the group is an even number, then the weight of the man who left the group is kg _____.

**2.** Kushal takes up an assignment, wherein each working day he is given a target. For each day, he meets the target he is paid ₹105 and for each day he does not meet the target he is paid ₹18 less. If by the end of a month, he is paid a total of ₹2988, then for how many days does he meet the target?

(A) 21 (B) 9 (C) 18 (D) 19

**3.** Supriya asked Rajesh 'What is the date of your birth'? Rajesh replied, '25 times the date of my birth added to 9 times the month of my birth is 563'. In which month was Rajesh born?

(A) September (B) July
(C) February (D) November

**4.** The 'Friends Club' purchased some shuttlecocks and cricket balls. Each shuttlecock cost ₹8 and each cricket ball cost ₹15. In how many different ways, could the club have bought the items if it spent a total amount of ₹769?

_____

**5.** John and Rosy have a collection of less than 90 coins together. If Rosy gives John a certain number of coins, then John would have 4 times as many coins as Rosy. Instead, if John gives the same number of coins to Rosy, then John would have 3 times as many coins as Rosy. The number of coins with John and Rosy together can be

(A) 80 (B) 29 (C) 35 (D) 85

**6.** Bhanu purchased some pens and erasers for ₹126. Had he interchanged the number of pens and erasers, it would not have cost him more than ₹63. If each pen and each

eraser cost him ₹6 and ₹0.75, respectively, then the total number of pens and erasers he actually purchased can be
(A) 20 (B) 28 (C) 24 (D) 26

**7.** Two friends Bharathi and Pranathi had a certain number of 5-rupee coins. If Pranathi gives a certain number of coins to Bharathi, then Bharathi will have ₹50 more than Pranathi. Instead, if Bharathi gives the same number of coins to Pranathi, then Pranathi will have ₹30 more than Bharathi. Find the total number of coins the two can have.
(A) 22 (B) 21 (C) 23 (D) 19

**8.** Kumar plays a game in which a dice is cast. Whenever one shows up, he receives ₹50. When three shows up, he receives ₹75 and when five shows up, he has to pay ₹30. He played the game ten times and every time he played, only 1, 3, or 5 showed up. He got a total of ₹360. Find the number of times he received ₹75.
(A) 1 (B) 2 (C) 3 (D) 4

**9.** In an acute angled triangle, thirteen times of an angle is equal to seventeen times of another. The angles, measured in degrees, are integers. One angle is less than or equal to 30°. The difference between the other two angles is
(A) 16° (B) 12° (C) 22° (D) 20°

**10.** A three-digit number leaves a remainder of 16 when divided by 25 and a remainder of 5 when divided by 21. How many such numbers exist?
(A) 1 (B) 3 (C) 2 (D) 4

***Directions for questions 11 and 12***: These questions are based on the information given below.

Kishan Lal, a jeweller, bought a total of 27 precious stones of types A, B, and C for a total of ₹30,000. The prices of each stone of types A, B, and C are ₹750, ₹1000, and ₹1250, respectively. He bought more than 3 stones of each variety and he did not buy the same number of stones of any two varieties.

**11.** How many stones of the types A and B together did he buy?
(A) 11 (B) 12 (C) 13 (D) 14

**12.** Which variety of stones did he buy the most?
(A) A (B) B (C) C (D) A or B

**13.** A and B together have a collection of less than 60 chocolates. If A gives a certain number of chocolates to B, then B would have thrice the number of chocolates left with A. Instead, if B gives the same number of chocolates to A, then both will have the same number of chocolates. The number of chocolates with A can be
(A) 6 (B) 16 (C) 19 (D) 14

**14.** Sangamithra goes to a stationery shop to buy some scales, pencils, and crayons. She decides to buy twice as many crayons as pencils and at least one pencil more than scales. Also, pencils are thrice as expensive as scales, and scales are sold at half the price of crayons. If each scale costs ₹2 and the total amount spent is ₹94, how many items did she purchase in all?
(A) 23 (B) 25 (C) 27 (D) 29

**15.** How many ordered pairs of positive integers $(x, y)$ satisfy the equation $\frac{1}{x}+\frac{1}{y}=\frac{1}{7}$?
(A) 1 (B) 2 (C) 3 (D) more than 3

**16.** Dhanunjay has 72 coins of two rupee, one-rupee and fifty paise denominations. The total value of the coins is ₹66. The number of coins of each denomination is a multiple of 6 greater than 10. Find the number of fifty paise coins he has.
(A) 12 (B) 18 (C) 36 (D) 24

**17.** Saroja purchases three different varieties of pens–Add gel, Montex, and Luxes–for a total of ₹285. She decides to buy twice as many Luxes pens as Addgel pens and more Montex pens than Addgel pens but at the most, 15 more Montex pens than Addgel pens. Montex pens are one and half times as expensive as Luxes pens and Addgel pens are two and half times as expensive as Luxes pens. The cost of each Luxes pen is ₹2. Find the maximum number of Montex pens she could have bought for the amount she spent.
(A) 30 (B) 35 (C) 32 (D) 38

***Directions for questions 18 and 19***: These questions are based on the data given below.

Arun plays a game wherein he casts a dice and scores six points if an odd number turns up and 9 points if an even number turns up. A total of exactly 117 points is required to win the game.

**18.** In how many ways, can Arun win the game?
(A) 7 (B) 3 (C) 4 (D) 6

**19.** Arun casts the dice exactly 16 times and wins. What is the maximum number of times a 2 or a 4 could have turned up?
(A) 9 (B) 7 (C) 8 (D) 6

**20.** The average marks of a group of $n$ students in a subject is 65. Three students with marks 48, 66, and 56 leave the group and one student with marks between 55 and 65 join the group. As a result, the average of the group goes up by 3. If the number of students in the group initially

is an odd perfect square, then the marks of the student who joined the group are
(A) 58 (B) 64 (C) 61 (D) 62

**21.** Ravi wants to buy some stationery. He wants to buy pencils, pens, and sheets of paper spending a total of ₹50. If each pen costs ₹6, each pencil cost ₹5 and each sheet cost ₹3, and if Ravi wants to buy at least 3 items of each kind, then in how many combinations can he purchase the items?
(A) 4 (B) 1 (C) 2 (D) 3

**22.** In an organization, there are a total of 36 employees belonging to three different departments–A, B, and C with each department having a minimum of 10 employees. The organization decides to pay a bonus of ₹5000, ₹3000, and ₹2000 to each of the employees of the departments A, B, and C, respectively. If the total amount paid to the employees as bonus is ₹1,22,000, then the maximum possible number of employees in department C is _____.

**23.** On a certain day, a book dealer sold some books on art, some books on science, and some magazines. The cost of each book on art is ₹100, the cost of each book on science is ₹120 and the cost of each magazine is ₹25. If at the end of the day the total sales are ₹685, then what is the maximum possible number of books and magazines he could have sold that day, if he has sold at least one of each kind? _____

**24.** In how many ways, can 149 be divided into two positive parts such that one part is divisible by 5 and the other part is divisible by 8? _____

**25.** Eat-well Candies sells three types of chocolates–eclairs, caramels, and mints. Each eclair costs ₹3, each caramel costs ₹2, and each mint costs ₹1. Ravi purchased chocolates worth a total of ₹45. He bought twice as many mints as caramels. How many eclairs could he have bought?
(A) 4 (B) 7 (C) 6 (D) 10

***Directions for questions 26 and 27***: These questions are based on the data given below:

Pradyumna, who is interested in numismatics, came across some old coins consisting of one rupee, 8-annas, and 4-anna coins totalling 36 and their total value being 19 rupees and 8-annas. If the number of 4-annas is at the most two more than the eight-anna coins and there are at least four 4-anna coins more than one-rupee coins (16 annas make a rupee).

**26.** How many 4-anna coins does Pradyumna have with him?
(A) 14 (B) 15 (C) 16 (D) 12

**27.** If Pradyumna lost two of 4 anna-coins, then which of the coins did he have in equal number?
(A) one-rupee and 8-anna (B) 8-anna and 4-anna
(C) one-rupee and four-anna (D) All three

**28.** Jadhu Shyam and Jadhu Ram are two renowned magicians who presented a magic show in the annual ABCRACADABRA magic festival. Jadhu Shyam using his tricks could double every red hat in 10 s, while Jadhu Ram could triple every black hat in every 10 s. After 40 s, the total number of hats were 161. How many red and black hats together were there at the beginning of this item?
(A) 11 (B) 10 (C) 8 (D) 6

**29.** Airtite Bolts and Nuts Inc, manufacturer of precision bolts have a unit which produces 15 mm bolts. Turner, a worker in that unit is supposed to pack less than 252 bolts into boxes. Turner finds that if he can pack 5 bolts less per box, he can pack 10 boxes more per day. How many bolts does he pack per day given that he packs at least 16 boxes per day?
(A) 234 (B) 238 (C) 252 (D) 208

**30.** Sangamithra goes to a stationary shop to buy some scales, pencils and crayons. She decided to buy twice as many crayons as pencils and at least one pencil more than scales. Also, she finds that pencils are thrice as expensive as scales and scales are at half the price of crayons. If each scale cost ₹1 and Sangamithra spends a total of ₹47, how many items did she purchase in all?
(A) 23 (B) 25 (C) 19 (D) 18

**31.** A two-digit number $N$, $(20 < N < 50)$ is such that when divided by 6, it leaves a remainder 5 and when divided by 5, it leaves a remainder 2. The number is _____.

**32.** Tinku wants to purchase some flower pots and sparklers. Each flower pot costs ₹12 and each sparkler costs ₹8. If Tinku has ₹96 and she wants to buy at least one of each, find the maximum number of flower pots she can buy.
(A) 4 (B) 8 (C) 2 (D) 6

**33.** Dhanush wants to offer samosas, cutlets, and kachoris to his friends, each of which costs ₹6, ₹9, and ₹7, respectively. Dhanush orders at least two of each item and spends exactly ₹74. Find the maximum number of kachoris he could have ordered.
(A) 3 (B) 4 (C) 5 (D) 6

**34.** Radha went to a stationery shop to purchase pens and pencils each costing ₹12 and ₹5, respectively. She wants to spend an amount of ₹97. Find the number of combinations of pens and pencils she can buy. _____

**35.** A bag contains 26 coins in denominations of ₹5, ₹2, and ₹1. The money in the bag amounts to ₹57. The number of one-rupee coins is more than the total number of other coins. The number of ₹2 coins in the bag is _____.

**36.** Mandavi has some stamps with her. She has 8 times as many one-rupee stamps as three-rupee stamps. She also has some five-rupee stamps. The total amount in these denominations together is ₹200. If she has more than 10 three-rupee stamps, how many stamps does she have in all?
(A) 210 (B) 132 (C) 142 (D) 150

**37.** $P$ and $Q$ are natural numbers such that $P \geq Q$. $P + 2Q = 60$. Find the number of possibilities for $(P, Q)$.
(A) 18 (B) 10 (C) 20 (D) 21

**38.** Ramesh went to a bank with a cheque. He had no money with him. He handed over the cheque to one of the bank tellers. The bank teller was in a confused state of mind and cashed the cheque. He gave Ramesh as many rupees as those of paise he had to give and as many paise as those of rupees he had to give. Ramesh took the amount and left the bank. He then bought a chocolate for 75 paise from a shop nearby. He then observed that he was left with an amount that was the excess of the amount on the cheque over the amount he received from the teller. Find the sum of the number of rupees and paise on the cheque.
(A) 60 (B) 75 (C) 90 (D) 120

**39.** How many ordered pairs of integers $(x, y)$, where $y > 0$ satisfy the equation $\frac{5}{x} + \frac{1}{y} = \frac{1}{15}$?
(A) 16 (B) 12 (C) 24 (D) 15

**40.** How many positive integral values of $(x, y)$ satisfy the equation $x^2 - y^2 = 357$?
(A) 4 (B) 8 (C) 6 (D) 3

## EXERCISE-4

***Directions for questions 1 to 20***: For the multiple choice questions, select the correct alternative from the given choices. for the non-multiple choice questions, write your answer in the space provided.

**1.** There are two strains of bacteria in a culture. One strain of bacteria doubles its number every 15 s and the other increases its number by 50% every 15 s. If, after 60 s, the total number of bacteria is 113, find the difference in the number of bacteria of the two strains at the beginning.
(A) 16 (B) 18 (C) 20 (D) 14

***Directions for questions 2 and 3***: These questions are based on the following data.

In a triangle, the measures of all angles, in degrees, are integers. Eleven times of one angle is equal to 13 times of another angle.

**2.** If one of the angles lies between 90° and 100°, the difference between the greatest and the smallest angles of the triangle is
(A) 75° (B) 69° (C) 79° (D) 65°

**3.** Find the number of acute-angled triangles that satisfy the given conditions.
(A) 1 (B) 2 (C) 3 (D) 4

**4.** If $\frac{7y}{17} - \frac{5x}{17} = 1$, where $x$ and $y$ are positive integers, the maximum value of $y - x$ is _____.

**5.** Given that $3x + 13y = 85$, how many integral solutions exist for $-50 \geq x \geq 50$? _____

**6.** How many ordered pairs of positive integers $(x, y)$ satisfy the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{18}$?
(A) 7 (B) 15 (C) 8 (D) 10

**7.** How many ordered pairs of integers $(x, y)$, where $x > 0$, satisfy the equation $\frac{1}{x} + \frac{3}{y} = \frac{1}{29}$?
(A) 6 (B) 4 (C) 8 (D) 12

**8.** How many ordered pairs of integers $(x, y)$ satisfy the equation $\frac{13}{x} - \frac{5}{y} = \frac{1}{8}$?

**9.** If $x^2 - y^2 = 60$, how many integral values can $(x, y)$ take?
(A) 16 (B) 4 (C) 2 (D) 8

**10.** If $x^2 - y^2 = 385$, how many integral values can $(x, y)$ take?
_____

**11.** If 12 times the date of my birth is added to 31 times the month of my birth, the sum is 316. In which month was I born?
(A) May (B) June
(C) April (D) July

**12.** The number 228 is to be divided into two positive parts – one a multiple of 7 and the other a multiple of 9. In how many ways, can the number be so divided?
(A) 3 (B) 5 (C) 6 (D) 4

**13.** The equation $5x + 7y = k$ has 7 solutions in which both $x$ and $y$ are non-negative integers. What is the minimum value of $k$?
(A) 210 (B) 280 (C) 223 (D) 290

**14.** Deepti has 49 stamps, which are of one-rupee, 50-paise, and 25-paise denominations. The total value of her stamps is ₹23.50. The number of 25-paise stamps is greater than the number of 50-paise stamps and one-rupee stamps together, and the number of one-rupee stamps is less than that of 50-paise stamps. The number of 25-paise stamps with Deepti is how much more than the number of one-rupee stamps with her?
(A) 15 (B) 14 (C) 1 (D) 26

**15.** Karan whose pockets were empty, presented a cheque for a certain amount to a teller at his bank. The teller intended to give the amount only in denominations of ₹50 and ₹500. He figured that he had to give $x$ 50-rupee notes and $y$ 500-rupee notes. However, while giving, he made an error and instead gave Karan $y$ 50-rupee notes and $x$ 500-rupee notes. After spending ₹50, Karan counted the money he had, and found that it was three times the amount that was written on the cheque. The amount (in rupees) on the cheque could be
(A) more than 3000 and less than 4000
(B) more than 5000 and less than 8000
(C) more than 14,000 and less than 16,000
(D) more than 16,000 and less than 18,000

***Directions for question 16***: The question is based on the following data.

Kapil is playing a video game, which has 36 levels and each level would give him 4 points or 2 points or 1 point depending upon its complexity. A player gets 78 points when he clears all the levels. The number of levels that give him 4 points is at most 2 greater than the number of levels that give him 2 points and the number of levels that give him 1 point is at least 7 greater than the number of levels that give him 2 points.

**16.** If Kapil starts the game and completes 2 four-point levels, he has an equal number of which of the levels left?
(A) 2-point levels and 1-point levels
(B) 4-point levels and 1-point levels
(C) 4-point levels and 2-point levels
(D) Cannot be determined

**17.** Ice cream is made of only two ingredients–ice and cream. Ice costs ₹27 per kg, while cream costs ₹22 per kg. The total amount spent on acquiring the two ingredients is ₹304. If the amount of ice as well as cream used are integral number of kilograms, what is the weight of ice cream obtained (in kilograms)? _____

**18.** A man sells 3 types of widgets A, B, and C, whose costs are ₹32, ₹19, and ₹16, a piece, respectively. The money he earns by selling these widgets is ₹386. If he sold at most 2 widgets more of type B than that of type A and if he sold at least 3 widgets more of type C than that of type B, find the number of widgets of type C that he sold _____.

**19.** How many integral values of $(x, y)$ satisfy the equation $x^2 - y^2 = 1309$? _____

**20.** How many integral values of $(x, y)$ satisfy the equation $x^2 - y^2 = 240$? _____

## ANSWER KEYS

### Exercise-1

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (D) | 7. 2 | 13. (B) | 19. (B) | 25. 9 | 31. 2 | 37. (D) |
| 2. (C) | 8. 4 | 14. (C) | 20. (A) | 26. (D) | 32. 10 | 38. (C) |
| 3. (C) | 9. (D) | 15. (A) | 21. 16 | 27. (B) | 33. 5 | 39. (A) |
| 4. 2 | 10. (A) | 16. (B) | 22. (D) | 28. (A) | 34. (A) | 40. (D) |
| 5. (C) | 11. (D) | 17. (C) | 23. (C) | 29. (C) | 35. 2 | |
| 6. (B) | 12. (D) | 18. 10 | 24. (D) | 30. (D) | 36. (A) | |

### Exercise-2

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. 21 | 6. 7 | 12. (C) | 18. 5 | 24. 25 | 30. (A) | 36. 30 |
| 2. (a) 4 | 7. (A) | 13. (C) | 19. 10 | 25. 15 | 31. (B) | 37. (B) |
| (b) ∞ | 8. 3 | 14. (B) | 20. (B) | 26. 2 | 32. (C) | 38. 4 |
| 3. 2 | 9. (B) | 15. (D) | 21. 73 | 27. 2 | 33. (B) | 39. (A) |
| 4. 2 | 10. (D) | 16. 6 | 22. (C) | 28. 48 | 34. (A) | 40. (B) |
| 5. 7 | 11. (C) | 17. (A) | 23. 1 | 29. (C) | 35. (C) | |

## Exercise-3

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (59) | 7. (A) | 13. (A) | 19. (B) | 25. (B) | 31. (47) | 37. (C) |
| 2. (A) | 8. (D) | 14. (A) | 20. (C) | 26. (A) | 32. (D) | 38. (B) |
| 3. (B) | 9. (D) | 15. (C) | 21. (B) | 27. (B) | 33. (C) | 39. (A) |
| 4. (6) | 10. (C) | 16. (C) | 22. (12) | 28. (D) | 34. (2) | 40. (A) |
| 5. (A) | 11. (A) | 17. (B) | 23. (13) | 29. (D) | 35. (3) | |
| 6. (B) | 12. (C) | 18. (A) | 24. (4) | 30. (A) | 36. (C) | |

## Exercise-4

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (D) | 4. (1) | 7. (C) | 10. (16) | 13. (A) | 16. (C) | 19. (16) |
| 2. (C) | 5. (8) | 8. (56) | 11. (C) | 14. (A) | 17. (12) | 20. (24) |
| 3. (C) | 6. (B) | 9. (D) | 12. (D) | 15. (C) | 18. (9) | |

## SOLUTIONS

## EXERCISE-1

**1.** Clearly choice D alone satisfies the given equation.

**2.** Given $5x + 16y = 100$

Clearly, $x$ and 100 are multiples of 5

$\therefore$ $y$ must also be multiple of 5.

**3.** Given Rem $\left(\frac{4Q}{5}\right) = 4$

4 Rem $\left(\frac{Q}{5}\right) = 4$

$\Rightarrow$ Rem $\left(\frac{Q}{5}\right) = 1 \Rightarrow Q = 5k + 1$

**4.** The required solutions are: (0, 4) and (5, 1)

$\therefore$ The number of solutions = 2.

**5.** Given, $5x + 7y = 135$

As $5x$ and 135 are divisible by 5, $7y$ should also be divisible by 5

$\Rightarrow y = 15$ or 20

But, when $y = 20$, $7y = 140 > 135$.

$\therefore$ $y = 15$

**6.** Given $14x + 5y = 57 \Rightarrow y = \frac{57 - 14x}{5}$

Solutions of the above are $x$ : .... 8, 3, –2, ...

$y$ : ... –11, 3, 17, ...

$\therefore xy > 0 \Rightarrow (x, y) = (3, 3)$

**7.** Given $5x + 3y = 34 \Rightarrow y = \frac{34 - 5x}{3}$ (1)

We notice that $x = 2 \Rightarrow y = 8$

The other possible values of $x$ are 5, 8, 11, ...

But from (1), $x \geq 6$

$\therefore$ The possible values of $x$ and $y$ are (2, 8), (5, 3)

$\therefore$ Number of positive integral solutions is 2.

**8.** Remainder of $\left(\frac{4a}{7}\right) = 2$

when $a = 4$, remainder is 2

**9.** Given $3x + 4y = 29 \Rightarrow y = \frac{29 - 3x}{4}$

The following are the possible solutions of this equation

$x$ : 3, 7, 11, ...

$y$ : 5, 2, –1, ...

When $x = 7$, $y = 2$, i.e., $0 < y < x$

**10.** Remainder of $\left(\frac{3x}{5}\right) = 3$

One possible value of $x$ is 1. This value and the remaining values of $x$ form an arithmetic progression with common difference 5. The set of possible values of $x$ is {1, 6, 11, 16, ...}.

**11.** Given $3a + 7b = 72$. As $3a$ and 72 are multiples of 3, $b$ is also a multiple of 3

$\therefore$ From the options, when $b = 6$, a = 10

**12.** Rem $\left(\frac{4p}{9}\right) = 5 \Rightarrow 4p = 9p_1 + 5$.

Rem $\frac{5}{4} = 1$, $\therefore$ Rem $\frac{9p_1}{4} = 3$ or Rem $\frac{p_1}{4} = 3$, i.e.,

$p_1 = 3, 7, 11, \ldots$ and $p = 8, 17, 26, \ldots$

$p = 8$ satisfies the above condition.

**13.** The given equation is $12x + 7y = 35$

As Rem $\frac{35}{7} = 0$, Rem $\frac{x}{7}$ is also 0.

When $x = 0$, $y = 5$. This is the only non-negative solution.

$\therefore$ There exists only one non-negative integral solution.

**14.** In the problem above, the number of ball point pens purchased, $y$ is 2.

**15.** Rem $\frac{3x}{7} = 6$ and one of the values of $x$ that satisfies this equation is 2. The possible values of $x$ are 2, 9, 16, 23, 30,… They form an arithmetic progression with a common difference of 7.

**16.** Given Rem $\left(\frac{4S}{7}\right) = 3$

By trial and error method when $S = 6$, then Rem $\left(\frac{4S}{7}\right) = 3$.

$\therefore$ The possible values of $S$ are 6, 13, 20, etc.

**17.** By trial and error method, we can find that only when $a = 14$, $b$ is a positive integer.

**18.** Given $80a + 3b + 2c = 101$.

Clearly $a \geq 1 \Rightarrow a = 1$ ($\because$ $a, b, c$ are natural numbers)

Amount left for purchasing the items $b$ and $c$ is $101 - 80 = 21$.

Possible values are $b = 3$, $c = 6$.

$\therefore$ Total items purchased $= 1 + 3 + 6 = 10$.

**19.** Given $3x + 7y = 37$

When $y = 1$, $x = 10$

again when $y = 4$, $x = 3$ ($\because$ $x, y \in z^+$)

only two combinations possible.

**20.** Let the number of pencils bought be $x$ and the number of erasers bought be $y$.

Then $4x + 3y = 15$ $3y$ and 15 are divisible by 3

$\Rightarrow 4x$ should also be divisible by 3

$\Rightarrow x = 3, 6, 9, \ldots$.But for $x \geq 6$, $4x > 15$

$\therefore x = 3$

Hence, the number of pencils bought by the Ramesh is 3

**21.** Let the number of oranges and apples, Deepika purchased be $x$ and $y$, respectively (i.e., $x + y = n$)

$\therefore 6x + 14y = 200$ or $3x + 7y = 100$

Dividing both sides by 3 we have Remainder of $\left(\frac{7y}{3}\right)$

$=$ Remainder $\left(\frac{100}{3}\right) \Rightarrow R\left(\frac{y}{3}\right) = 1$

Possible values of $y$ and $x$ are listed below

$y$ : 1 4 7 10 13

$x$ : 31 24 17 2 10

of all $x + y = 13 + 3 = 16$ is the least value

$\therefore$The minimum value of $n$ is 16

**22.** Given $7a - 3b = 20 \Rightarrow b = \frac{7a-20}{3}$

Possible values of $a$, $b$ that satisfy this equation are listed below.

$a$ : 2, 5, 8, –1, –4, …

$b$ : –2, 5, 12, –9, –16, …

Possible values of $a + b$ are 0, 10, 20, –10, –20, …

**23.** Let the number of toys purchased be $t_1$ and $t_2$, respectively.

$11t_1 + 17t_2 = 123$

Remainder $\left(\frac{17t_2}{11}\right)$ Remainder $\left(\frac{123}{11}\right)$

$\Rightarrow R\left(\frac{6t_2}{11}\right) = 2$

$t_2 = 4$ satisfies the above. Correspondingly, $t_1 = 5$ (If $t_2 = 4 + 11$, $t_1 = 5 - 17$, which is negative)

$\therefore$ The total number of toys Mr Raghu purchased $= 5 + 4 = 9$

**24.** Given $12x - 5y = 19 \Rightarrow y = \frac{12x-19}{5}$

Both options A, B satisfy this equation.

**25.** Remainder $\left(\frac{7x}{9}\right) = 7$

The values of $x$ that satisfy the above equation are 1, 10, 19, 28, … which is an arithmetic progression with common difference 9.

**26.** Given $4x + 5y = 60$. By inspection, $x$ is a multiple of 5 and $y$ is multiple of 4.

The values of $x$, $y$ are tabulated below

| $y$ | 0 | 4 | 8 | 12 |
|---|---|---|---|---|
| $x$ | 15 | 10 | 5 | 0 |

$\therefore$ 4 combinations of values are possible.

**27.** Given, $7x + 4y = 64$, $x, y \in Z^+$

$4y$ and 64 are multiples of 4

$\therefore$ Values of $x$ are also multiples of 4.

**28.** Rem $\left(\frac{4x}{5}\right) = 4 \Rightarrow 4x = 5x_1 + 4$

$\Rightarrow x_1 = 0, 4, 8, \ldots$

and $x = 1, 6, 11, \ldots$

$\therefore x = 5k + 1$, where $k$ is any integer.

**29.** Given $2x + 3y = 15$, $x, y \in Z^+$. $\therefore$ $x$ is a multiple of 3

When $x = 3$, $y = 3$ and when $x = 6$, $y = 1$

Only two combinations of positive integral solutions are possible.

**30.** Given $3x - 7y = 28$ and $x, y > 0$

$\therefore$ From options $x = 14$, $y = 2$ satisfy the given equation.

**31.** Let the number of parrots be $x$ and the number of rabbits be $y$.

Then, $4x + 7y = 29$

Dividing though out the equation by 4, we get

$$\text{Rem}\left(\frac{4x+7y}{4}\right) = \text{Rem}\left(\frac{29}{4}\right) \Rightarrow \text{Rem}\left(\frac{3y}{4}\right) = 1$$

By trial $y = 3$ satisfies the above condition and the other values of y can be obtained at an interval of 4 thereof.

$\therefore y = 3, 7, 11, \ldots$ But for $y \geq 7$, $7y > 29$

$\therefore y = 3 \Rightarrow x = 2$

Hence, the number of parrots with Shakuntala is 2.

**32.** Let $x, y$ be the number of cricket balls and shuttle cocks purchased, respectively, $10x + 11y = 130$.

As $10x$ and 130 are divisible by 10, $11y$ should also be divisible by 10

$\Rightarrow y = 10, 20, 30, \ldots$ But for $y \geq 20$, $11y > 130$

$\therefore y = 10 \Rightarrow x = 2$

Hence, the number of shuttle cocks purchased is 10.

**33.** Let $x, y$ be the number of ₹2 and ₹5 coins with Ritish. Then, $2x + 5y = 13$

Dividing throughout the equation by 2, we get

$$\text{Rem}\left(\frac{2x+5y}{2}\right) = \text{Rem}\left(\frac{13}{2}\right) \Rightarrow \text{Rem}\left(\frac{y}{2}\right) = 1$$

So, $y = 3$ satisfies the above condition and the other values of $y$ can be obtained at an interval of 2 thereof.

$y = 1, 3, 5$

but when $y \geq 3$ $5y \geq 13$

$\therefore y = 1 \Rightarrow x = 4$

Hence, the number of coins with Ritish is $1 + 4 = 5$

**34.** Let the number of scales and charts purchased be $x$ and $y$, respectively, $7x + 5y = 42$.

As $7x$ and 42 are divisible by 7, $5y$ should also be divisible by 7.

$\Rightarrow y = 7, 14, 21, \ldots$

But for $y \geq 14$, $5y > 42$

$\therefore y = 7 \Rightarrow x = 1$

Hence, the number of items purchased is $1 + 7 = 8$

**35.** The required solutions are: (1, 4) and (4, 6)

$\therefore$ The number of positive integral solution = 2.

**36.** Let the number of gel pens, ball point pens and pencils, the student purchased be $x$, $y$, and $z$, respectively.

Total amount spent is $15x + 5y + 3z = 28$

As $x > 0$, $x = 1$ and $\therefore$ $y = 2$ and $z = 1$

$\therefore$ Only one combination is possible.

**37.** Let the number be $10x + y$ where $(x > y)$

The sum of the number and the number formed by reversing the digits is $10x + y + 10y + x = 88$

$11x + 11y = 88$ or $x + y = 8$

$\therefore$ Possible two-digit numbers such that $x > y$ are 80, 71, 62, 53. There are 4 such numbers.

**38.** In order that the total number of toys is the minimum, toys of the first variety have to be bought for remaining ₹125

He can buy one toy each of the first and third varieties for ₹125

$\therefore$ Minimum number of toys he can sell is = 2 + 1 + 2 or 5.

**39.** Let the number be $x$ and $y$, respectively, $7x + 11y = 61$

Rem $\frac{61}{7} = 5$ $\therefore$ Rem $\frac{4y}{7} = 5$ or $4y = 5y_1 + 7$

Rem $\frac{7}{4} = 3$ $\therefore$ Rem $\frac{y_1}{4} = 1$ or $y_1 = 1, 5, 9\ldots$

$y_1 = 1 \Rightarrow y = 3 \Rightarrow x = 4$.

**40.** Let the number of toys of each variety sold be $x$, $y$, $z$, respectively

The total cost is $100x + 50y + 25z = 300$

He sold at least one of each variety

$\therefore$ Amount received by selling one of each toy is

$100 + 50 + 25 = 175$

Amount left with him $300 - 175 = 125$

For the number of toys sold to be the maximum he has to sell toys of the third variety for the remaining amount 125.

For ₹125 he can sell $\frac{125}{25} = 5$ toys of this variety.

∴ The maximum number of toys he can sell

$= 1 + 1 + 6 = 8$

## EXERCISE-2

**1.** Let the number pens and markers purchased be $x$ and $y$, respectively.

Given $3x + 5y = 65 \Rightarrow y = \frac{65 - 3x}{5}$

Possible values of $x$ and $y$ are listed below

| $x$ | $y$ |
|---|---|
| 0 | 13 |
| 5 | 10 |
| 10 | 7 |
| 15 | 4 |
| 20 | 1 |

∴ Maximum number he purchased = 20 + 1 = 21.

i.e., 20 pens and 1 marker.

**2.** (a) $3x + 4y = 60$

$3x = 60 - 4y$

$x = 20 - \frac{4}{3}y$

Now, to get an integer, solutions for $x$ value of $y$ must be divisible by 3.

| $x$ | 16 | 12 | 8 | 4 |
|---|---|---|---|---|
| $y$ | 3 | 6 | 9 | 12 |

Four solutions are possible.

**Note:** We observe that the value of $x$ changes by 4 (i.e., coefficient of $y$) and the value of $y$ changes by 3 (i.e., coefficient of $x$). If $x$ increases, $y$ decreases.

(b) $3x - 4y = 60$

$3x = 60 + 4y$

$x = 20 + \frac{4}{3}y$

Here also to get an integer solution for $x$, $y$ must be divisible by 3.

| $x$ | 24 | 28 | 32 | 36 | 40 | … | … | … |
|---|---|---|---|---|---|---|---|---|
| $y$ | 3 | 6 | 9 | 12 | 15 | … | … | … |

We realize that there are infinite solutions.

**Note:** Here also, we observe a similar pattern as the previous question. But here, one of the coefficients is negative. Therefore, the values of both $x$ and $y$ increase.

**3.** We will first try to the one solution. Keep the smaller coefficient term as it is and take all other terms to the other side.

$7x = 157 - 13y$

$\Rightarrow x = \frac{157 - 13y}{7}$

From the numerator, remove as many multiples of 7 as possible.

$x = \frac{(154 - 7y) + 3 - 6y}{7}$

$\Rightarrow x = \frac{154 - 7y}{7} + \frac{3 - 6y}{7}$

$\Rightarrow x = (22 - y) + \frac{3 - 6y}{7}$

Now, as $x$ and $y$ are integers, $3 - 6y$ must be divisible by 7. Using this, we can find the possible values of $y$:

The first value of $y$ is obtained by trial and error as $y = 4$.

$3 - 6 \times 4 = -21$ is divisible by 7.

at $y = 4$, $x = 15$.

Now the value of x will change by 13 and that of $y$ by 7. But one will increase, and another will decrease.

| $y$ | 4 | 11 | 18 | … |
|---|---|---|---|---|
| $x$ | 15 | 2 | -ve | … |

Thus, all values of $y$ after 18 onwards will give negative value of $x$ and hence are neglected. So, we have only 2 values.

$x = 15, y = 4$

and $x = 2, y = 11$

**4.** Given $3x - 4y = 28$

$\Rightarrow y = \frac{3x - 28}{4}$

As $x$, $y$ are positive integers $3x - 28 > 0 \Rightarrow x > 9$

The possible value of $x$, $y$ are listed below.

$x$: 12, 16, 20,...

$y$: 2, 5, 8,...

of the above the least possible value of $y$ is 2.

**5.** Let the number of stamps of each of ₹5, ₹3 and ₹2 purchased be $x$, $y$ and $z$.

$\Rightarrow 5x + 3y + 2z = 65$

To purchase 5 stamps of each denominations, he spent an amount of ₹50.

The amount left with him is ₹15.

The possible combinations are listed below:

| ₹5 | ₹3 | ₹2 |
|---|---|---|
| 3 | 0 | 0 |
| 2 | 1 | 1 |
| 1 | 2 | 2 |
| 0 | 3 | 3 |
| 1 | 0 | 5 |
| 0 | 1 | 6 |
| 0 | 5 | 0 |

$\therefore$ He could have purchased stamps in 7 different combinations.

**6.** Let the number of students in the group be '$n$' and let the percentile of the person who left the group be $x$.

Then,

$$\left(\frac{75n + 75 + 85 + 99 - x}{n + 2f}\right) = 77$$

$75n + 259 - x = 77n + 154$

$\Rightarrow 105 = 2n + x$

$\Rightarrow x = 105 - 2n$ But, $94 < x < 100$

$94 < 105 - 2n < 100$

$\Rightarrow -11 < -2n < -5$

$\Rightarrow 5.5 > n > 2.5$

$\Rightarrow 2.5 < n < 5.5$

Since $n$ is given to be a multiple of 5, $n = 5$

Therefore, the number of students in the group now is

$= (5 + 2) = 7$.

**7.** The craftsman reported for work for 28 + 1 = 29 days.

**8.** Let the number of mangoes sold be $m$ and the number of apples sold be $a$. We have $5m + 6a = 100$

Dividing by the least coefficient, we have

$m + a + (a/5) = 20$

$a/5$ must be an integer. $\therefore$ a must be divisible by 5

$\therefore a = 5, 10, 15, 20, \ldots$

But, when $a \geq 20$, $6a \geq 120$, which is not possible. Therefore, only three values of a are possible, viz., $a = 5$, 10, and 15 and the corresponding values of $m$ are 14, 8, and 2. Therefore, he can sell the fruits in three combinations.

**9.** Let the number of days on which he completed the task be $x$ and number of days on which he did not complete the task be $y$.

We have, $x + y \geq 30$ (1)

and $50x + 30y = 1430$

$\Rightarrow 5x + 3y = 143$

Dividing by the least coefficient, we have

Rem $(2x/3) = 2 \Rightarrow x = 1, 4, 7, 10, \ldots 28$ and the corresponding values of $y$ are 46, 41, 36, 31, ..., 1

But considering the inequality (1), only one combination is possible that is $x = 28$ and $y = 1$.

**10.** Let the date on which Ramu was born be $d$ and the month be $m$.

We have $8d + 15m = 240$

Dividing by the least coefficient, we get

$$d + m\left(\frac{7m}{8}\right) = 30$$

In the equation above, $\frac{7m}{8}$ must be an integer.

$\therefore$ $m$ must be divisible by 8.

$\therefore$ $m$ can only be 8.

Therefore, the month in which Ramu was born was August.

*Solutions for questions 11 and 12:*

We note that $e, j, t > 3$ and $e \neq j$, $j \neq t$ and $e \neq t$

$e + j + t = 27$ (1)

$15e + 20j + 25t = 600$ (in thousands)

or $3e + 4j + 5t = 120$ (2)

Eliminating '$e$' from (1) and (2), we get

$j + 2t = 39$

or $j = 39 - 2t$

$\Rightarrow t \geq 17$ (as $j > 3$)

Substituting for $j$ in (1) we get

$e = 21 - (j + t) = 27 - (39 - t) = t - 12$

$\Rightarrow t \geq 16$ ( as $e > 3$)

when $t = 17$, we get $j = 5$ and $t = 5$

which is contrary to the premise given.

Thus, $t = 16$, $e = 4$, and $j = 7$

**11.** Hence, emeralds and jades together are 11.

**12.** Topaz is bought in maximum number.

**13.** Let $x$, $y$, and $z$ be the three angles in the triangle

Now, $19x = 15y$

$x = 15y/19$

This means y is a multiple of 19

We know that sum of 3 angles is 180°. Also, $x$, $y$, and $z$ should be acute. We enlist the possibilities as follows:

| $y$ | $x$ | $z$ |
|---|---|---|
| 19 | 15 | > 90 |
| 38 | 30 | > 90 |
| 57 | 45 | 78 |
| 76 | 60 | 44 |

The least angle possible is 44°.

**14.** $10e + 2c + m = 100$ (1)

$m = 4c$ (2)

$\Rightarrow 10e + 6c = 100 \Rightarrow 5e + 3c = 50$ (dividing by 2)

$\Rightarrow 5e + 50 - 3c \Rightarrow 5e = 50 - 3c$

$\Rightarrow c$ should be a multiple of 5

For $e$ to be maximum, $c$ should be minimum.

$\therefore c = 5$

So $5e + 3(5) = 50 \Rightarrow e = 7$

**15.** Although this is a Numbers problem, the same can be solved with the help of special equations concept also.

The number can be expressed as $21x + 4$ or $13y + 8$

Thus, $21x + 4 = 13y + 8 \Rightarrow 21x - 13y = 4$

Dividing by least coefficient and separating integer and fraction parts, we get

$$\frac{8x}{13} - \frac{4}{12} = k \text{ (where } k \text{ is an integer)}$$

We multiply by a number, so that the coefficient of $x$ becomes one more than a multiple of 13. Accordingly, we multiply by 5.

$40x - 20/13 = 5k$

Once again we separate integer and fraction parts and we get, $x = 13k + 7$, where $k$ is an integer.

Using this in $21x + 4$, we get the number to be of the form $273k + 151$.

For $k = 0, 1, 2, 3$ the number is a three digit number.

For $k = 4$ we will not get a 3 digit number.

Thus, there are 4 such 3 digit numbers.

**16.** Let the initial number of red flowers be $x$, and that of yellow flowers be $y$.

Due to Ram's magic, the number of red flowers at the end of one minute is $729x$.

Due to Ravi's magic, the number of yellow flowers at the end of one minute is $64y$.

Given, $729x + 64y = 1049$ where $x$ and $y$ are integers.

For $x > 2$, $729x > 1049$,

$\therefore x = 1$. The corresponding value of $y$ is $(1049 - 729)/64$

$= 320/64 = 5$

Therefore, the total number of flowers initially is $x + y$, i.e., 6.

**17.** Let $x$ be the number of boxes packed and y be the number of fruits packed per box.

Then, $(x)\ (y) < (21)\ (12)$, i.e., $(x)\ (y) < 252$ (1)

Also, $x \geq 16$ (2)

and $xy = (x + 10)\ (y - 5)$

$\Rightarrow x = 2y - 10$

The possible values of $x$ and $y$ (for $x > 15$) and the corresponding value of $(x)\ (y)$ are tabulated below

| $x$ | $y$ | $(x)\ (y)$ |
|---|---|---|
| 16 | 13 | 208 |
| 18 | 14 | 252 |
| 20 | 15 | 300 |

$\therefore$ Only when $(x, y) = (16, 13)$, $(x)\ (y)$ is less than 252 and in this case $(x)\ (y) = 208$

***Solutions for questions 18 and 19:***

Let the number of 50 rupee, 10 – rupee and 5 – rupee notes be $x$, $y$, and $z$, respectively.

Then, $x + y + z = 27$ (1)

and $50x + 10y + 5z = 700$ (2)

Also, $x - z \geq 2$ and (3)

(i) $\Rightarrow 2x + 2y + 2z = 54$

(ii) $10x + 2y + z = 140$ (1)

$\Rightarrow 8x - z = 86$, i.e., $x = \dfrac{86 + z}{8}$

(iii) $\Rightarrow \frac{z+86}{8} - z \geq 2 \therefore z \geq 10$ (1)

$y = 27 - (x + z) = \frac{130 - 9z}{8}$

$y > 3$

$\therefore z < \frac{106}{9} = 11\frac{5}{9}$ As $x$ must be an integer, $z$ must be even

$\therefore z \geq 10$ (2)

From (1) and (2), $z = 10$

$\therefore x = 12$ and $y = 5$.

**18.** The number of ₹10 notes is 5.

**19.** If the cashier loses two 50 – rupee notes, then the number of 50 – rupee notes with him is 10.

**20.** Let $r$, $t$, and $f$, respectively, denote the number of one-rupee, three-rupee, and five-rupee stamps

Then, $r + 3t + 5f = 75$ (1)

$r \geq 20$ and $5t = r$ (2)

$\therefore$ From (1) and (2), we have $8t + 5f = 75$

Dividing by the least coefficient, we have

Rem $(3t/5) = 0$

$\Rightarrow t = 5, 10, 15 \ldots 75$

But $8t \geq 75$, $\Rightarrow$ the only possible value of $t$ is 5

For $t = 5$, $f = \frac{(75 - 40)}{5} = \frac{35}{5} = 7$ and $r = 25$ [$\because$ from (1)]

$\therefore$ The total number of stamps with Madhavi is

$5 + 7 + 25 = 37$.

**21.** The average weight of '$n$' men is 64 kg. The total weight of the group is $64n$. Let $x$ be the average weight of the men who joined the group.

We have $70 < x < 80$

Now $\frac{64n + 3x - 58}{n + 3 - 1} = 67$

$64n + 3x - 58 = 67n + 134$

$\Rightarrow 3x - 3n = 192$

$x = 64 + n$

As we know that $70 < x < 80$

$70 < 64 + n < 80$

$\Rightarrow 6 < n < 16$

The only perfect square between 6 and 16 is 9.

$\therefore n = 9$

$\therefore x = 64 + 9 = 73$ kg.

**22.** Let the day on which Lara was born be $D$ and the month be $M$.

Then, we have $25M - 18D = 210$.

Divide the equation by the least coefficient 18, and collect all fractions on left and all integers on the right.

Now denoting the combination of all integers on the right by '$K$' where $K$ is an integer, we get

$\frac{7M}{18} - \frac{12}{18} = K$

$M = \frac{18k + 12}{7}$

By trial and error, we get

$K = 0$ $M = \frac{12}{7}$

$K = 1$ $M = \frac{30}{7}$

$K = 2$ $M = \frac{48}{7}$

$K = 3$ $M = \frac{66}{7}$

$K = 4$ $M = \frac{84}{7} = 12$

The next possible value of $K = 11 \Rightarrow M = 30$

Since $M$ denotes a month, $M = 30$ is not valid

$\therefore M = 12$.

Hence, Lara was born in December.

**23.** Suppose Chaitanya buys $x$ U.S. stamps, $y$ English stamps, and '$z$' Russian stamps.

Then $16x + 18y + 24z = 330$

Since he has to give 5 U.S., 4 England, 5 Russian stamps, this is the minimum number of stamps he has to buy in each category. Thus, $16 \times 5 + 18 \times 4 + 24 \times 5 = 80 + 72 + 120 =$ ₹272 have already been spent and with the remaining ₹58, it is only possible to buy 1 U.S. 1 English, 1 Russian stamp.

Hence, it is unique combination of

6 U.S. stamps, 5 English stamps, 6 Russian stamps.

**24.** Let the number of pieces of A sold be $x$ and the number of pieces of B sold be $y$. Then

$8000x + 13000y = 290{,}000$

$\Rightarrow 8x + 13y = 290$

Dividing by the least coefficient, i.e., 8 and collecting all fractions on left and all integers on right (and denoting the combination of integers on the right as $k$), we get

$\frac{5y}{8} - \frac{2}{8} = k$

$y = \frac{8k+2}{5}$

By trial and error, we get

For $k = 1$ $y = 2$ $\Rightarrow$ x = 33

For $k = 6$ $y = 10$ $\Rightarrow$ x = 20

For $k = 11$ $y = 18$ $\Rightarrow$ x = 7

The next possible value of $k$ is $k = 16$ for which $y = 26$. But this is not possible as in this case $13y = 338$ while $8x + 13y = 290$.

Of the three possible pairs of solutions, the least number of total pieces sold is 18 + 7 = 25

**25.** Let $x$ be the number of days on which the show is a success. Let $y$ be the number of days on which the show is a partial failure.

Let $z$ be the number of days on which the show is a total failure. Now,

$x + y + z = 30$ (1)

(Since the month is June and he has a show on each day)

Also $500x - 150y - 240z = 4530$

$\Rightarrow 50x - 15y - 24z = 453$ (2)

Solving (1) and (2), we get

$50x - 15y - 24z = 453$

$15x + 15y + 15z = 450$

$65x - 9z = 903$ (3)

Dividing (3) by the least coefficient, i.e., 9 and keeping all fractions to the left while taking all integers to the right, and denoting the RHS as $k$, we get

$\frac{2x}{9} - \frac{3}{9} = k$ ; i.e., $\frac{2x-3}{9} = k$

$\Rightarrow x = \frac{9k+3}{2}$

By trial and error, we get

$k = 1 \Rightarrow x = 6$

For $x = 6$, we get $z$ as a negative number, which is not possible.

$k = 3 \Rightarrow x = 15$

For $x = 15$, $9z = 65 \times 15 - 903 = 72$

$\Rightarrow z = 8$

$\Rightarrow y = 30 - (x + z) = 7$

for $k = 5 \Rightarrow x = 24$

for $x = 24$, $9z = 65 \times 24 - 903 = 657$

$\Rightarrow z = 73$.

This is not possible as $x + y + z = 30$.

Hence, the only possibility is $x = 15$, $y = 7$, $z = 8$

Hence, the number of days on which the show was a success is 15.

**26.** Let the number be $n$

Then $n = 17x + 1$ and $n = 3y + 2$

$\therefore 17x + 1 = 3y + 2$

$\Rightarrow 17x - 3y = 1$

Dividing by the least coefficient, we have

$\text{Rem}\left(\frac{2x}{3}\right) = 1$

$\Rightarrow x = 2, 5$ (only) (because when $x = 8$, $(17x + 1)$ is not a two-digit number)

$\therefore$ There are two numbers of the required kind.

**27.** Suppose the owner purchased $x$ gift articles of the kind A and $y$ gift articles of the kind B. Then, $100x + 20y = 4000$

$5x + y = 200$ (1)

Also, $100y + 20x < 2000$

$5y + x < 100 \Rightarrow 5(200 - 5x) + x < 100$

$x > \frac{900}{24} = 37.5$

$\Rightarrow x \geq 38$

But for $x \geq 40$, $5x \geq 200$.

So, $x = 38$, or $x = 39$, which implies that he can buy the items in two possible combinations.

**28.** When $x = 38$, $y = 10$, and when $x = 39$, $y = 5$

Now since a minimum of 10 pieces of each variety are purchased, the only combination possible is $x = 38$ and $y = 10$.

So, the total number of gifts the shop owner must have bought is 48.

**29.** Let the number of art books purchased be $x$ and the number of science books purchased be $y$.

Then, $72x + 110y = 1020$, i.e., $36x + 55y = 510$

We note that $55y$ and 510 are multiples of 5. So, $36x$ must also be a multiple of 5. This means $x$ should be a multiple of 5.

The possible values of $x$ and the corresponding values of $y$ are listed below

$36x + 55y = 510$, i.e., $36(5) + 55(6) = 510$

$36(60) + 55(-30) = 510$

After getting one value for $(x, y)$, the other values are obtained by increasing $x$ by 55, (the coefficient of $y$) and decreasing y by 36, (the coefficient of $x$). We see that only (5, 6) is a feasible value for $(x, y)$. Therefore, the total number of books bought by Ram is 5 + 6 = 11.

**30.** Given, $a + b + c = 40$ (1)

$24a + 50b + 30c = 1420$ (2)

Eliminating a from the above equations, we get

$13b + 3c = 230$ (3)

$\Rightarrow \text{Rem}\left(\frac{13b}{3}\right) = 2$

$\Rightarrow b = 2, 5, 8, 11, 14$ and 17

$\therefore$ The maximum possible value of $b$ is 17

$\therefore$ A and C together would buy $(40 - 17) = 23$ pens

**31.** Let Adam pack $x$ boxes and $y$ tools in each box

Number of tools = $xy < 275$

When 9 tools are increased in each box the number of boxes is five less

$\Rightarrow (x - 5)\ (y + 9) = xy$

$xy + 9x - 5y - 45 = xy \Rightarrow 9x - 5y = 45$

By inspection $y$ is a multiple of 9. When $y = 9$, $x = 10$ the possible values of $x$ and $y$ are listed below.

| $x$ | 10 | 15 | 20… |
|---|---|---|---|
| Y | 9 | 18 | 27 … |

As $xy < 275$ and $x > 10 \Rightarrow x = 15$ and $y = 18$

And the number of tools packed = 15 (18) = 270.

**32.** $\frac{1}{x} + \frac{7}{y} = \frac{1}{21} \Rightarrow 21(y + 7x) = xy$

$xy - 21y - 147x = 0$, i.e., $y(x - 21) - 147x = 0$

Adding (147) (21) both sides $(x - 21)\ (y - 147) = 3087 = 3^2 7^3$.

If $x - 21$ and $y - 147$ are both positive, each of them must be a factor of 3087. $\therefore$ $(x - 21, y - 147) = (1, 3087), (23, 1029), (7, 441), (9, 343), (21, 147), (49,. 63), (63, 49), (147, 21), (343, 91), (441, 7), (1029, 3)$ or $(3087, 1)$.

If $x - 21$ and $y - 147$ are both negative, $x - 21$ must be more than $-21$ (since $x$ must be greater than zero)

$\therefore$ $x - 21$ can be $-1, -3, -7$ or $-9$.

If $x - 21$ and $y - 147$ are both positive, $(x, y)$ has 12 possible values. If $x - 21$ and $y - 147$ are both negative, $(x, y)$ has 4 possible values.

$(x, y)$ has 16 possible values.

**33.** $x^2 - y^2 = 220$

$(x + y)\ (x - y) = 220$ (1)

The R.H.S of (1) is even. $\therefore$ At least one of $x + y$ and $x - y$ must be even. Also $x + y$ and $x - y$ must have the same party. $\therefore$ Both $x + y$ and $x - y$ must be even

Let $x + y = 2k_1$ and $x - y = 2k_2$, where $k_1$ and $k_2$ are integers

$(2k_1)\ (2k_2) = 220$

$K_1 k_2 = 55$

As the product of $k_1$ and $k_2$ is positive, $k_1$ and $k_2$ must have the same sign

$(k_1, k_2) = (\pm 1, \pm 55), (\pm 5, \pm 11), (\pm 11, \pm 5)$ or $(\pm 55, \pm 1)$

$(k_1, k_2)$ has 8 possible values

$(x, y)$ has 8 possible values.

**34.** $\frac{13}{x} - \frac{7}{y} = \frac{1}{4}$ (1)

$\Rightarrow 4(13y - 7x) = xy$

$\Rightarrow xy - 52y + 28x = 0$, i.e., $y(x - 52) + 28x = 0$

Adding $28(-52)$, $(x - 52)\ (y + 28) = 28(-52) = -1456$ (2)

As the product of $x - 52$ and $y + 28$ is negative, $x - 52$ and $y + 28$ have opposite signs.

If $x - 52$ is positive and $y + 28$ is negative, $(x - 52, y + 28)$

$= (1, -1456), (2, -728), (4, -364), (7, -208), (8, -182), (13, -112), (14, -104), (16, -91), (26, -56), (28, -52), (52, -28)$, ……or $(1456, -1)$.

If $x - 52$ is positive and $y + 28$ is negative, $(x, y)$ has 20 possible values. If $x - 52$ is negative and $y + 28$ positive, $(x, y)$ has 20 possible values.

$(x, y) = (0, 0)$ is a solution of (2). But since $x$ and $y$ are denominators in the given equation (1), $(x, y)$ cannot be $(0, 0)$. $\therefore$ $(x - 52, y + 28)$ cannot be $(-52, 28)$. $\therefore$ Of the 20 possible values of $(x, y)$, when $x - 52$ is negative and $y + 28$ is positive, we have 19 valid solutions

$(x, y)$ has 39 possible values.

**35.** $x^2 - y^2 = 627 \Rightarrow (x + y)\ (x - y) = 627$ (1)

To satisfy (1), $x + y$ and $x - y$ must have the same sign

If $x + y$ and $x - y$ are both positive, each of them must be a factor of 627. $\therefore$ $(x + y, x - y) = (1, 627), (3, 209), (11, 57), (19, 33), (33, 19), (57, 11), (209, 3)$ or $(627, 1)$.

If $x + y$ and $x - y$ are both negative, $(x + y, x - y)$

$= (-1, -627), (-3, -209), (-11, -57), (-19, -33), (-33, -19), (-57, -11), (-209, -3)$, or $(-627, -1)$

$(x, y)$ has 16 possible values.

**36.** Let $A$, $B$, and $C$ be the angles of the triangle such that

$13A = 17B \Rightarrow A = \dfrac{17B}{13}$

As $A$, $B$, and $C$ are integers, $B$ must be a multiple of 13.

So, the possible values of $A$, $B$, and $C$ are listed below:

| $B$ | 13 | 26 | 39 | 52 | 65 | 78 |
|---|---|---|---|---|---|---|
| $A$ | 17 | 34 | 51 | 68 | 85 | X |
| $C$ | X | X | X | 60 | 30 | X |

$\therefore$ The least possible angle in the triangle is 30°.

**37.** Let $x$, $y$ and $z$ be the number of 10 marks, 5 marks and 2 marks questions, respectively.

Then, $x + y + z = 35$ (1)

and $10x + 5y + 27z = 100$ (2)

(2) – 2 × (1) gives, $8x + 3y = 30$.

Since $3y$ and 30 are multiples of 3, $8x$ must be a multiple of 3.

$\Rightarrow x = 3, 6, 9, 12, \ldots$ But for $x \geq 6$, $8x \geq 30$.

$\therefore x = 3$

$\Rightarrow y = 2$ and $z = 30$. Hence, the number of 5 marks questions in the paper is 2.

**38.** Let $x$, $y$ and $z$ be the number of employees in the sections A, B, and C, respectively.

Then, $x + y + z = 14$. (1)

and $11000x + 6000y + 2000z = 84000$

$\Rightarrow 11x + 6y + 2z = 84$ (2)

(2) – 2(1) gives $9x + 4y = 56$.

Clearly $x = 4$ and $y = 5$ satisfy the above equation and there is no other solution for the equation. Hence, the number of employees in the section A is 4.

**39.** Let $x$, $y$, and $z$ be the number of woollen jackets, sweaters and gloves, respectively, sold by the dealer. Then,

$300x + 175y + 100z = 1175$

$\Rightarrow 12x + 7y + 4z = 47$.

Dividing the equation by 4, we get

$\text{Rem}\left(\dfrac{12x + 7y + 4z}{4}\right) = \text{Rem}\left(\dfrac{47}{4}\right) \Rightarrow \text{Rem}\left(\dfrac{3y}{4}\right) = 3.$

By trial, $y = 1$ satisfies the above equation, and the other values of $y$ can be obtained at by adding 4 successively.

$\therefore y = 1, 5, 9, 13, \ldots$ but for $y \geq 9$, $7y > 47$.

$\therefore y = 1, 5$. When $y = 5$, $12x + 3y = 12$

This equation does not have positive integral solutions.

$\therefore y = 1$. Hence, only one sweater is sold by the dealer.

**40.** Let $x$, $y$, and $z$ be the number of apples, oranges, and jack fruits, respectively, bought by Jasmine.

Then, $8x + 3y + 13z = 112$

Put $x = X + 4$, $y = Y + 4$ and $z = Z + 4$

We get, $8(X + 4) + 3(Y + 4) + 13(Z + 4) = 112$.

$\Rightarrow 8X + 3Y + 13Z = 16$.

By trial and error, we can find that the solutions of the above equation are $X = 0$, $Y = 1$, $Z = 1$; $X = 2$, $Y = 0$, $Z = 0$. Hence, Jasmine can buy the fruits in two different combinations.

## EXERCISE-3

**1.** Let the weight of the man who left the group be $k$ kg.

Then, $72n + (140 - k) = 75(n + 1) \Rightarrow k = 65 - 3n$

when $n = 2$, $k = 59$, and when $n = 3$, $k = 56$.

But $k > 58$. $\therefore k = 59$.

**2.** Let the number of days on which Kushal met the target be x and the number of days on which he did not meet the targets be y.

Then, $105x + 87y = 2988$

$\Rightarrow 35x + 29y = 996$

$\Rightarrow \text{Rem}\left(\dfrac{6x}{29}\right) = \text{Rem}\left(\dfrac{996}{29}\right) = 10 \Rightarrow 6x = 29x_1 + 10$

$\Rightarrow \text{Rem}\left(\dfrac{5x_1}{6}\right) = 2 \Rightarrow 5x_1 = 6x_2 + 2$

$\Rightarrow \text{Rem}\left(\dfrac{x_2}{5}\right) = 3 \therefore x_2 = 3, 8, 13$, etc.

$x_2 = 3 \Rightarrow x_1 = 4 \Rightarrow x = 21 \Rightarrow y = \dfrac{996 - (35)(21)}{29} = 9$

$\therefore$ Other values of $x$ are obtained by adding 29 successively while those of y are obtained by adding 35 successively. But as $x \geq 31$, $y \geq 31$, $(x, y) = (21, 9)$, i.e., Kushal met the target on 21 days.

**3.** Let $D$ be the date of birth and $M$ be the month of birth of Rajesh.

Then, $25D + 9M = 563$ (1)

Dividing the equation by 9, we get

$$\text{Rem}\left(\frac{25D+9M}{9}\right) = \text{Rem}\left(\frac{563}{9}\right),$$

(where Rem $\left(\frac{x}{y}\right)$ is the remainder when $x$ is divided by y) $\Rightarrow$ Rem $\left(\frac{7D}{9}\right) = 5$

By trial, $D = 2$ satisfies the above condition and the other values of $D$ can be obtained by adding 9 successively.

$\therefore D = 2, 11, 20$

The corresponding values of $M$ are 57, 32, 7 –18, …

As $0 < M \geq 12$, $M = 7$.

**4.** Let $x$, $y$ be the number of shuttlecocks and cricket balls purchased. Then, $8x + 15y = 769$

Dividing the equation by 8, we get

$$\text{Rem}\left(\frac{8x+15y}{8}\right) = \text{Rem}\left(\frac{769}{8}\right) \Rightarrow \text{Rem}\left(\frac{7y}{8}\right) = 1$$

By trial, $y = 7$ satisfies the above condition, and the other values of y can be obtained by adding 8 successively.

$\therefore y = 7, 15, 23, 31, 39, 47, 55, \ldots$

But for $y \geq 55, 15y > 769$. So, $y < 55$.

The corresponding values of $x$ are 83, 68, 53, 38, 23, 8.

Hence, the items can be purchased in 6 different combinations.

**5.** Let the number of coins with John and Rosy be $x$ and $y$, respectively. Then, $x + k = 4(y - k)$

$\Rightarrow x - 4y = -5k$ (1)

and $x - k = 3(y + k)$

$\Rightarrow x - 3y = 4k$ (2)

Eliminating $k$ from (1) and (2), we get $9x - 31y = 0$

$\Rightarrow x = \frac{31y}{9} \Rightarrow y = 9, 18, 27, 36, \ldots$ and the corresponding values of $x$ are 31, 62, 93, 124, …

But $x + y < 90$ (given).

$\therefore$ The possible number of coins with John and Rosy together is 9 + 31, 18 + 62, i.e., 40 or 80.

6. Let the number of pens and erasers purchased be $x$ and $y$, respectively. $6x + \frac{3}{4}y = 126$

$\Rightarrow 2x + \frac{y}{4} = 42$ or $8x + y = 168$ (1)

Had he interchanged the number of pens and erasers, the total cost is

$6y + \frac{3}{4}x \leq 63 \Rightarrow 8y + x \leq 84$ (2)

Eliminating $y$ from (2) by using (1), we have

$8(168 - 8x) + x \geq 84 \Rightarrow 1344 - 64x + x \geq 84$

$\Rightarrow -63x \geq -1260 \Rightarrow x \geq 20$

When $x > 20$ then $y \geq 20$

$\therefore x = 20$ is the only solution and when $x = 20$, $y = 8$

$\therefore$ The number of pens and erasers together which he actually purchased is 28.

**7.** The number of coins with Bharathi (B) and Pranathi (P), in the two hypothetical cases (second and third rows) and actually (first row) are tabulated below. The data in the first statement appears in the second row. The inference from this and the second statement appears in the third row. The inference from these two statements appears in the first row.

| | B | P | Total |
|---|---|---|---|
| Actually | $x + 6$ | $x + 4$ | |
| If P gives | $x + 10$ | $x$ | $2x + 10$ |
| If B gives | $x + 2$ | $x + 8$ | |

The total number of coins is of the form $2x + 10$, i.e., it has to be even.

**8.** Let one, three, five occur $x$, $y$ and $z$ times, respectively.

$\therefore x + y + z = 10$ (1)

and $50x + 75y - 30z = 360$, i.e., $10x + 15y - 6z = 72$

Eliminating $z$ from (1) and (2) we get

$10x + 15y - 6(10 - x - y) = 72$ (2)

$16x + 21y = 132$

Rem $\frac{132}{16} = 4$. $\therefore$ Rem $\frac{5y}{16}$ is also 4, i.e., $5y = 16y_1 + 4$

Rem $\frac{y_1}{5} = 1$

$y_1 = 1, 6, 11, \ldots$ and $y = 4, 20, 36, \ldots$

when $y = 4$ we get $x = 3$ and $z = 3$

$\therefore$ Kumar received ₹ 75 four times.

**9.** Let the angles (in degrees be) A, B, C

13A = 17B

$A = \frac{17}{13} B$

The angles of the triangle are acute and integers (when measured in degrees). The following table gives the possible values of the angles.

| A | B | C | |
|---|---|---|---|
| 17 | 13 | >90 | × |
| 34 | 26 | >90 | × |
| 51 | 39 | 90 | × |
| 68 | 52 | 50 | ✓ |
| 85 | 65 | 30 | ✓ |
| > 90 | 78 | Not possible | × |

The least possible angle is 30°. In this case the difference between the other two angles is 85° – 65° = 20°.

**10.** Given $N = 25x + 16 = 21y + 5 \Rightarrow 21y - 25x = 11$

Rem $\frac{4x}{21} = 10 \Rightarrow 4x = 21x1 + 10$

$\therefore$ Rem $\frac{x_1}{4} = 2 \Rightarrow x1 = 2, 6, 10, \ldots$

$x_1 = 2 \Rightarrow x = 13 \Rightarrow N = 341$. The other values of N are obtained by adding the LCM (21, 25) or 525 successively to 341.

$\therefore$ The 3 digit values of N are 341 and 866.

***Solutions for questions 11 and 12:***

Let the number of precious stones purchased of the varieties *A*, *B* and *C* be *a*, *b* and *c* respectively

Then, $a + b + c = 27$ (1)

$750a + 1000b + 1250c = 30000$

$3a + 4b + 5c = 120$ (2)

Eliminating '*a*' from (1) and (2), we get

$b + 2c = 39$

$\Rightarrow b = 39 - 2c \Rightarrow c \geq 17$ (as $b > 3$)

Substituting for b in (i), we get

$a + 39 - 2c + c = 27 \Rightarrow c = 12 + a \quad a \geq 4$

When $c = 17$, we get $a = b = 5$, which contradicts the condition that $a \neq b$). When $c = 16$, $a = 4$ and $b = 7$.

When $c \geq 15$, $a \geq 3$ which is not possible.

$\therefore$ $c = 16$, $a = 4$ and $b = 7$ is the only feasible solution.

**11.** The total number of pieces of A and B purchased together is 11.

**12.** C is bought in the greatest possible number.

**13.** Let the number of chocolates with *A* be a, and with *B* be b. Then, $a + b < 60$

Let the number of chocolates which A gives to B be equal to *x*.

Then, $3(a - x) = b + x$ and $a + x = b - x$

$\Rightarrow 2x = b - a$ and $4x = 3a - b$

$\Rightarrow 3a - b = 2b - 2a$,

$\Rightarrow 5a = 3b$

$b = \frac{5a}{3} \Rightarrow$ a has to be a multiple of 3.

So, choice (A) is the right option.

**14.** Let the number of scales bought be s, number of pencils be p and number of crayons be c.

Then, $c = 2p$ and $p \geq s + 1$

Cost of each scale is ₹2

Cost of each pencil is ₹6

Cost of each crayon is ₹4

$6p + 2s + 4c = 94$

$\Rightarrow 3p + s + 2c = 47$

$s + 4p + 3p = 47$

$\Rightarrow s = 47 - 7p$

$\Rightarrow p \geq 6$ (1)

Also $p \geq s + 1 \Rightarrow p \geq 47 - 7p + 1$

$\Rightarrow 8p \geq 48$

$\Rightarrow s > 0 \Rightarrow p \geq 6$ (2)

$\therefore$From (1) and (2), p = 6 $\Rightarrow$ c = 12 and s = 5

So, s + p + c = 5 + 6 + 12 = 23.

**15.** $\frac{1}{x} + \frac{1}{y} = \frac{1}{7} \Rightarrow xy - 7x - 7y + 49 = 49$

$\Rightarrow (x - 7)(y - 7) = 49$

As $x > 0$, $y > 0$, it follows that $x - 7$ and $y - 7 > -7$

$\therefore (x - 7, y - 7) = (1, 49), (7, 7)$ or $(49, 1)$

$\Rightarrow (x, y) = (8, 56)$ (14, 14) or (56, 8)

$(x, y)$ has 3 possible values.

**16.** Let the number of ₹2, Re.1 and ₹0.50 coins be $x$, $y$, $z$, respectively

$\therefore x + y + z = 72$ (1)

and $2x + y + \frac{1}{2} z = 66 \Rightarrow 4x + 2y + z = 132$ (2)

Eliminating $z$ from (2) using (1) we have

$4x + 2y + 72 - x - y = 132$

$\Rightarrow 3x + y = 60 \Rightarrow y = 60 - 3x$

Since the number of coins of each denomination is 12, 18, 24…., the only possible values of $x$, $y$ and $z$ are $x = 12$, $y = 24$, $z = 36$

$\therefore$ The number of fifty paise coins he has = 36.

17. Let the number of Addgel, Montex and Luxes pens purchased be $A$, $M$ and $L$

$\therefore L = 2A$ and $0 \geq M - A \geq 15$

And cost of each Luxes pen is ₹2, Montex pen is ₹3 and Add gel pen = ₹5

So $5A + 3M + 2L = 285 \Rightarrow 5A + 3M + 2.2A = 285$

$9A + 3M = 285 \Rightarrow 3A + M = 95$

As Rem $\frac{95}{3} = 2$, Rem $\frac{M}{3}$ is also 2, i.e., M = 2, 5, 8

The following table gives all possible values of A, M and L which satisfy the condition $M - A \geq 15$

| | | | | | |
|---|---|---|---|---|---|
| A | : | 23 | 22 | 21 | 20 |
| M | : | 26 | 29 | 32 | 35 |
| L | : | 46 | 44 | 42 | 40 |

$\therefore$ From the above table the maximum number of Montex pens Saroja can buy is 35.

***Solutions for questions 18 and 19:***

Let an odd number turn up $x$ times and an even number turn up y times. If Arun wins,

$6x + 9y = 117 \Rightarrow 2x + 3y = 39$

39 and 3y are multiples of 3. Therefore, $x$ is also a multiple of 3. The possible values of $x$, $y$ are listed below.

| $x$ | 0 | 3 | 6 | 9 | 12 | 15 | 18 |
|---|---|---|---|---|---|---|---|
| $y$ | 13 | 11 | 9 | 7 | 5 | 3 | 1 |

**18.** The number of ways of scoring 117 points is 7.

**19.** When $x + y = 16$, $x = 9$, $y = 7$, i.e., an even number turns up 7 times. Therefore, the maximum number of times a 2 or a 4 could have turned up is 7.

**20.** The average marks of the $n$ students is 65

Total marks of n students = $65n$

When three students whose marks are 48, 66 and 56 left the total marks of the remaining n – 3 students is

$65n - (48 + 66 + 56) = 65n - 170$

Let the marks of the student who joined the group be $x$, $(55 < x < 65)$

$\therefore$ The average of these $(n-2)$ students is $\frac{65n - 170 + x}{n - 2}$

This is 68.

i.e., $\frac{65n - 170 + x}{n - 2} = 68 \Rightarrow 3n = x - 34$

and $n = \frac{x - 34}{3}$. As $55 < x < 65$ the values of $x$ (that give integral values of $n$) are 58, 61 and 64 and the corresponding values of $n$ are 8, 9 and 10. The initial value of $n$ is an odd perfect square.

$\therefore n = 9$ the marks of the person who joined the group is $3(9) = x - 34 \Rightarrow x = 61$.

**21.** Suppose Ravi buys a pens, b pencils, and c notebooks

Then, $6a + 5b + 3c = 50$.

Since he has to buy at least 3 items of each kind, ₹42 has been spent and with the remaining ₹8, it is only possible to buy 1 pencil and 1 note book. Thus, there is a unique combination in which the items can be bought

**22.** Let the number of employees in each of the departments $A$, $B$ and $C$ be $x$, $y$ and $z$, respectively.

Then, $x + y + z = 36$ (1)

and $5000x + 3000y + 2000z = 1{,}22{,}000$

$5x + 3y + 2z = 122$ (2)

Eliminating $z$ from (1) and (2), we have

$3x + y = 50$

Considering possible values,

$x = 10$, $y = 20$, $z = 6$ (not possible $\because$ each department has at least 10 employees)

$x = 11$, $y = 17$, $z = 8$ (not possible)

$x = 12$, $y = 14$, $z = 10$ (possible)

$x = 13$, $y = 11$, $z = 12$ (possible)

$x = 14$, $y = 8$, $z = 14$ (not possible)

Therefore, we have two possible combinations.

The maximum possible number of people in the department C is 12.

**23.** Let the number of art books sold be a, science books sold be s, and magazines sold be m.

Then,

$120s + 100a + 25m = 685 \Rightarrow 24s + 20a + 5m = 137$

Dividing by the least coefficient, we have

Rem $\left(\frac{4s}{5}\right) = 2 \Rightarrow s = 3, 8, 13 \ldots$

But '$s$' cannot be greater than 3 in the possible set

$\therefore s = 3$

$\Rightarrow 100a + 25m = 685 - 360 \Rightarrow 100a + 25m = 325$

$4a + m = 13$

The possible values of '$a$' and '$m$', such that $a > 0$ and $m > 0$ are

$a = 1,\ m = 9\ \ a = 2,\ m = 5\ \ a = 3,\ m = 1$

$\therefore$ The maximum possible number of books that could have been sold is $3 + 1 + 9 = 13$

**24.** Let $p$ and $q$ be the two parts such that $p = 5x$ and $q = 8y$. Then, $5x + 8y = 149$ Rem $(3y/5) = 4$

The possible values of $y$ are 3, 8, 13, 18 and the corresponding values of $x$ are 25, 17, 9 and 1.

Hence, we have 4 possible ways.

**25.** Let the number of éclairs, caramels and mints purchased be $x$, $y$ and $z$, respectively.

Then, $3x + 2y + z = 45$ and $z = 2y$

$\therefore 3x + 4y = 45$

$\Rightarrow$ Rem $\left(\frac{y}{3}\right) = 0$ (Dividing by the least coefficient)

$\Rightarrow y = 3, 6, 9$ and corresponding $x = 11, 7, 3$.

Therefore, a possible value of the number of eclairs bought is 7.

***Solutions for questions 26 and 27:***

We note that

16 anna = ₹1

8 anna = ₹½

and 4 anna = ₹¼

Let $r$ $e$, $f$ denote the number of one-rupee 8-anna and 4-anna coins, respectively

$r + e + f = 36$ ... (1)

$r + 1/2e + 1/4f = 19½$

$\Rightarrow 4r + 2e + f = 78$ ... (2)

eliminating $r$ from (1) and (2), we get,

$3r + e = 42$

$\Rightarrow e = 42 - 3r$

$\Rightarrow r \geq 15$

substituting for $e$ in (1),

we get $r + (42 - 3r) + f = 36$

$\Rightarrow f = 2r - 6$

$r \geq 4$

We also have $f \geq e + 2$ and $f \geq r + 4$

We get, $2r - 6 \geq 42 - 3r + 2$ and $2r - 6 \geq r + 4$

$\Rightarrow r \geq 10$ and $r \geq 10$

$\Rightarrow r = 10$. Consequently $e = 12$ and $f = 14$

**26.** Hence, 4-anna coins are 14.

**27.** On loosing 2 4 anna-coins, Pradyumna now has with him 12 of 4 anna-coins and 12 of 8 anna-coins which are equal.

**28.** Let '$r$' be the number of red hats and '$b$' be the number of black hats at the beginning. We have,

| | **Red hats** | **Black hats** |
|---|---|---|
| at the beginning | $r$ | $b$ |
| at the end of 10 secs | $2r$ | $3b$ |
| at the end of 20 secs | $4r$ | $9b$ |
| at the end of 30 secs | $8r$ | $27b$ |
| at the end of 40 secs | $16r$ | $81b$ |

Now $16r + 81b = 161$

Clearly, $b = 1$ while $r = \frac{161 - 81}{16} = 5$

Hence, the number of hats at the beginning are 6.

**29.** Let x be the number of boxes packed per day and y be the number of bolts packed per box.

We have $xy < 252$ ... (1)

$x \geq 16$ ... (2)

$xy = (x + 10)\ (y - 5)$ ... (3)

$\Rightarrow x = 2y - 10$

when $x = 16$, $y = 13$

we have $16(13) < 252$

or $208 < 252$ true

when $x = 17$; $y = 13.5$, which is not possible as $y$ is an integer when $x = 18$; $y = 14$, but 18 x 14 < 252.

so 208 bolts must be packed on any day.

**30.** Let $c$ = number of crayons

$s$ = number of scales

$p$ = number of pencils

we have,

Cost of 1 scale = ₹1

Cost of 1 crayons = ₹2

Cost of 1 pencil = ₹3

The conditions are $c = 2p$ and $p \geq s + 1$

we also have the equation for cost as

$s + 2c + 3p = 47$

Incorporating the conditions that are above mentioned

$s + 2(2p) + 3P = 47$

$s = 47 - 7p$

$p \geq 6$ ... (1)

Also $p \geq s + 1 \Rightarrow p \geq 47 - 7p + 1$

$8p \geq 48$

$p \geq 6$ ... (2)

From (i) and (ii), $p = 6 \Rightarrow c = 2(6) = 12$

and $s = 5$

so $s + p + c = 5 + 6 + 12 = 23$

**31.** As per the given conditions

$N = 6x + 5$ and $N = 5y + 2$

$\Rightarrow 6x + 5 = 5y + 2$

$6x - 5y = -3. \Rightarrow y = \dfrac{6x + 3}{5}$

When $x = 2, 7$ or 12, N is 15, 47, 77 correspondingly.

As $20 < N < 50$, $N = 47$.

**32.** Let the number of flower pots and sparklers purchased be $x$ and $y$, respectively

$\therefore 12x + 8y = 96 \qquad y = \dfrac{24 - 3x}{2}$

Possible values of $x$ and $y$ are $x$: 2 4 6 8

$y$: 9 6 3 0

As Tinku wants at least one of each, the maximum number of flower pots she can purchase is 6.

**33.** Total amount spent = 74.

Amount spent for 2 of each item = 2 (6 + 9 +7) = 44

Amount left = 74 – 44 = 30

With ₹30, Dhanush can order 4 kachoris. But as he spent exactly ₹74 (and hence, exactly ₹30 on the items over and above 2 of each kind), he must have ordered 3 kachoris and 1 cutlet.

$\therefore$ The maximum number of kachoris he could have ordered = 2 + 3 = 5

**34.** Let the number of pens and pencils Radha purchases be $x$ and $y$, respectively

$\therefore 12x + 5y = 97 \; y = \dfrac{97 - 12x}{5}$

Possible values of $x$, $y$ are $x$: 1 6

$y$: 17 5

$\therefore$ She can purchase two combinations.

**35.** Let the number of ₹5, ₹2 and ₹1 coins in the bag be $x$, $y$ and $z$, respectively. Also $x = y + z$

$x + y + z = 26$ (1) and $5x + 2y + z = 57$ (2)

Eliminating $z$ from (1) and (2), we have $5x + 2y + 26 - x - y = 57$

$\Rightarrow 4x + y = 31 \Rightarrow y = 31 - 4x$

Possible values of $x$, $y$, $z$ are $x$: 2 3 4 5 6 7

$y$: 23 19 15 11 7 3

$z$: 1 4 7 10 13 16

Only when x = 7, $y = 3$ and $z = 16$ is $x + y < z$. In all other cases, $x + y \geq z$.

$\therefore$ The number of ₹2 coins in the bag y = 3.

**36.** Let $r$, $t$, $f$, respectively, denote the number of one rupee, three rupees and five rupees stamps.

$r + 3t + 5f = 200$ ... (1)

$r = 8t$ ... (2)

from (1) and (2)

$(8t + 3t) + 5f = 200$

$11t + 5f = 20$

Clearly, $t$ is a multiple of 5.

As $t > 10 \Rightarrow$ t = 15, 20,…

But t = 20 is not acceptable as

$11(20) = 220 > 200$

Thus, $t = 15 \Rightarrow f = 7$, $r = 120$

Thus, there are 15 + 7 + 120 = 142 stamps in all.

**37.** $P + 2Q = 60$ (1)

Since $2Q$ is even number, $P$ must be even number.

$P \geq Q \Rightarrow 2Q \geq 2P \Rightarrow$

$P + 2Q \geq P + 2P \Rightarrow 60 \geq 3P \Rightarrow P \geq 20.$

$P$ must be a maximum of 58.

$\therefore$ $P$ has 20 possibilities,

$\therefore$ $(P, Q)$ has 20 possibilities.

**38.** Let the amount on the cheque be ₹$x$ and y paise, i.e., $(100x + y)$. Ramesh received an amount of ₹$y$ and $x$ paise $(100y + x)$. After buying the chocolate, he was left with an amount of $(100y + x - 75)$ paise.

This will be equal to $(100x + y) - (100y + x)$.

$100y + x - 75 = 100x + y - (100y + x)$

$\Rightarrow 199y = 98\, x + 75$

$98\,(2y - x) = 3\,(25 - y)$

To satisfy the equation above, $25 - y$

must be divisible by 98. (1)

Since $0 < y < 100$, $25 - y = 0$, i.e., $y = 25$

$\Rightarrow y = 25$, then $x = 50$

$\therefore$ Amount on the cheque = ₹ 50 and 25 paise

$\therefore$ Required sum = $x + y = 75$.

**39.** $\frac{5}{x} + \frac{1}{y} = \frac{1}{15} \Rightarrow 15(5y + x) = xy$.

$\Rightarrow xy - 15x - 75y = 0$, i.e., $x(y - 15) - 75y = 0$.

Adding (75) (15) both sides, $(y - 15)\ (x - 75) = 1125 = 3^2 5^3$.

If $y - 15$ and $x - 75$ are both positive, each of them must be a factor of 1125. $\therefore$ $(y - 15, x - 75) = (1, 1125)$, $(3, 375)$, $(5, 225)$, $(9, 125)$, $(15, 75)$, $(25, 45)$, $(45, 25)$ $(75, 15)$, $(125, 9)$, $(225, 5)$, $(375, 3)$, or $(1125, 1)$.

If $y - 15$ and $x - 75$ are both negative, $y - 15$ must be more than $-15$ since $y$ must be greater than zero).

$y - 15$ can be $-1$, $-3$, $-5$ or $-9$.

If $y - 15$ and $x - 75$ are both positive, $(x, y)$ has 12 possible values. If $y - 15$ and $x - 75$ are both negative, $(x, y)$ has 4 possible values

$(x, y)$ has 16 possible values.

**40.** $x^2 - y^2 = 357 \Rightarrow (x + y)\ (x - y) = 357$ (1)

If $x$ and $y$ are positive, $x + y$ will be positive. $\therefore$ To satisfy (1), $x - y$ must also be positive.

$x + y$ and $x - y$ must be factors of 357. Also $x + y$ cannot be less than $x - y$.

$(x + y, x - y) = (357, 1)$, $(119, 3)$, $(51, 7)$ or $(21, 17)$

$(x, y)$ has 4 possible values.

## EXERCISE-4

**1.** Let the number of bacteria of the first strain be $x$ and the number of bacteria of the second strain be y at the beginning. As per the conditions given, we have the number of bacteria at different intervals of time as listed below.

| Time | Strain 1 | Strain 2 |
|---|---|---|
| 0 s | $x$ | $y$ |
| 15 s | $2x$ | $\frac{3}{2}y$ |
| 30 s | $4x$ | $\frac{9}{4}y$ |
| 45 s | $8x$ | $\frac{27}{8}y$ |
| 60 s | $16x$ | $\frac{81}{16}y$ |

Given after 60 s, total bacteria = 113

$\Rightarrow 16x + \frac{81}{16}y = 113$

Clearly, the number of bacteria is a positive integer.

$\Rightarrow y$ must be a multiple of 16. If $y = 32, 48$, $x$ is negative.

$\therefore y = 16$ and $x = 2$

$\therefore$ The difference in the number of bacteria at the beginning is $16 - 2 = 14$

***Solutions for questions 2 and 3:***

Let $A$, $B$, $C$ be the angles of a triangle. Given 11 times of an angle = 13 times of other.

Let $11A = 13B \Rightarrow A = \frac{13}{11}B$

Clearly, $B$ must be a multiple of 11. The following table gives us the possible angles (in degrees) of the triangle.

| A | B | C |
|---|---|---|
| 13 | 11 | 156 |
| 26 | 22 | 132 |
| 39 | 33 | 108 |
| 52 | 44 | 84 |
| 65 | 55 | 60 |
| 78 | 66 | 36 |
| 91 | 77 | 12 |

**2.** In the last row of the above table, the angles of the triangle are $91^o$, $77^o$, and $12^o$.

In this case alone, one of the angles lies between 90° and 100°.

$\therefore$ The difference between the greatest and the least angles is $91^o - 12^o = 79^o$.

**3.** From the above table, three acute-angled triangles are possible.

**4.** $\frac{7y}{17} - \frac{5x}{17} = 1$

$7y - 5x = 17$

The least value of $y$ satisfying the equation is 6. The corresponding value of $x$ is 5.

The family of values of $(x, y)$ satisfying the equations are $(5 + 7k, 6 + 5k)$, where $k$ is any whole number.

$y - x = 1 - 2k$

If $k = 0$, $y - x = 1$

If $k > 0$, $y - x < 1$

$y - x \le 1$

Max $(y - x) = 1$

**5.** Given $3x + 13y = 85$, if integral solutions exist for this equation, then the minimum positive value of $y$ satisfying the equation is such that $1 \ge y \ge 3$, similarly minimum possible positive value of $x$ is such that $1 \ge x \ge 13$.

Since $y$ has a lesser range, by trial and error we find the minimum positive value of $y$. This comes out to be 1.

For $y = 1$, $x = 24$.

$\therefore$ Possible values of $x$ are in the form $13k + 24$; $k \in Z$

(13 is coefficient of $y$).

for $-50 \ge x \ge 50$

$-50 \ge 13k + 24 \ge 50$

$-76 \ge 13k \ge 26$

$-5 \times 8 \ge k \ge 2 \Rightarrow k = -5, -4, -3, -2, -1, 0, 1, 2$

$\therefore$ 8 such solutions exist.

**6.** $\frac{1}{x} + \frac{1}{y} = \frac{1}{18}$

$\frac{y + x}{xy} = \frac{1}{18} \Rightarrow 18x + 18y = xy$

$xy - 18x - 18y = 0$

$xy - 18x - 18y + 324 = 324$

$(y - 18)\ (x - 18) = 4\ (81)$ (1)

$(y - 18)\ (x - 18) = 2^2\ (3^4)$

The number of factors of the RHS is 15.

$\therefore$ $(y - 18)\ (x - 18)$ can be expressed in 15 ways.

$\therefore$ The number of ordered pairs of positive integers $(x, y)$ is 15.

Note: Although $x$ and $y$ are positive integers, $(y - 18)$ and $(x - 18)$ can be negative. But those negative values do not result in positive integer values of $x$ and $y$.

$\therefore$ They can be ignored. Had (1) been $(y - 19)\ (x - 20)$ $= (-18, -18)$ would give a possible $(y, x)$ as $(1, 2)$.

**7.** $\frac{1}{x} + \frac{3}{y} = \frac{1}{29} \Rightarrow (x - 29)\ (y - 87) = 87(29) = 3(29)^2$

If we add 87 (29) to both sides, we can factorize the LHS.

The number of positive factors of $3(29)^2$ is 6. We also have 6 negative factors, so total number of factors of $3(29)^2$ is 12.

But $x > 0$; $x - 29 > -29$

The possible values of $x - 29$ can be $-3$, $-1$, 1, 3, 29, $3(29)$, $(29)^2$, $3(29)^2$, i.e., 8 values

$y$ has the 8 corresponding values.

$\therefore$ Total number of solutions is 8.

**8.** $\frac{13}{x} - \frac{5}{y} = \frac{1}{8}$

$\frac{13y - 5x}{xy} = \frac{1}{8}$

$104y - 40x = xy$

$xy + 40x - 104y = 0$

$x(y + 40) - 104y - 40.104 = -40.104$

$x(y + 40) - 104(y + 40) = -40(104)$

$(x - 104)\ (y + 40) = -40(104)$

$= -8.5.8.13 = -2^6.5.13$

The number of positive factors of $2^6 5.13$ is (7) (2) (2), i.e., 28. We also have 28 negative factors.

$\therefore$ Total number of factors is 56.

$\therefore$ The total number of ordered pairs of integers is 56.

**9.** $x^2 - y^2 = 60$

$(x + y)\ (x - y) = 60 = (4)\ (3)\ (5) = 2^2\ (3)\ (5)$

The total number of factors is 3(2) (2), i.e., 12

since $x$, $y$ are integers.

$(x + y)$ and $(x - y)$ must be both even numbers.

$\therefore$ The possible values of $(x + y)$ are 2, 6, 10, 30, –2, –6, –10, –30. Correspondingly, $x - y$ would be 30, 10, 6, 2, –30, –10, –6, –2. Hence, the number of possible integral solutions is 8.

**10.** $x^2 - y^2 = 385 \Rightarrow (x - y)\ (x + y) = (5)\ (7)\ (11)$

The number of positive factors of 5(7) (11) is 8. We also have 8 negative factors.

$\therefore$ Total number of factors is 16. There are 16 possible values of $x + y$ and correspondingly 16 values of $(x, y)$

**11.** Let the date of birth be $x$. $\therefore$ $1 \ge x \ge 31$.

and the month of birth be $y$. $\therefore$ $1 \ge y \ge 12$

$12x + 31y = 316$ (Given)

Dividing throughout by 12 (least coefficient) we get

$7y = 4 + 12k$

$y = 4$ satisfies the above equation

$\therefore$ when $y = 4$, $x = 16$

The month that I was born in is April.

**12.** Let $7x + 9y = 228$

$\therefore \text{Rem}\left(\frac{9y}{7}\right) = \text{Rem}\left(\frac{228}{7}\right) = 4$

$\Rightarrow \text{Rem}\left(\frac{2y}{7}\right) = 4$

$y = 2$ satisfies the equation and the corresponding value of $x$ is 30. The other values of $x$ and $y$ are given below.

| $x$ | 30 | 21 | 12 | 3 |
|---|---|---|---|---|
| $y$ | 2 | 9 | 16 | 23 |

$\therefore$ The number can be divided in the required way in 4 ways.

**13.** $5x + 7y = k$

Dividing throughout by 5, we get

$\text{Rem}\left(\frac{7y}{5}\right) = \text{Rem}\left(\frac{k}{5}\right)$

$\text{Rem}\left(\frac{2y}{5}\right) = \text{Rem}\left(\frac{k}{5}\right)$

$y = p$ satisfies the above equation and the corresponding value of $x$ is $q$.

$\therefore$ for remaining values of $y$ and $x$, $y$ values are increased by 5 and $x$ values are decreased by 7. This gives the following table.

| $x$ | $q$ | $q-7$ | $q-14$ | $q-21$ | $q-28$ | $q-35$ | $q-42$ |
|---|---|---|---|---|---|---|---|
| $y$ | $p$ | $p+5$ | $p+10$ | $p+15$ | $p+20$ | $p+25$ | $p+30$ |

The number of solutions is 7.

$\therefore$ for the minimum value of $k$, $q = 42$ ; p = 0

$k = 5(42) + 7(0) = 210$

The minimum value of $k = 210$

**14.** Let $x$, $y$, and $z$ be the one-rupee, 50 paise, and 25 paise stamps with Deepthi.

Given $x + y + z = 49$ (1)

$x + \frac{1}{2}y + \frac{1}{4}z = 23\frac{1}{2}$, i.e., $4x + 2y + z = 94$ (2)

Eliminating $z$ from (1) and (2), we get $3x + y = 45$.

i.e., $y = 45 - 3x$.

Further, given that $z > x + y$ and $x < y$.

Hence, $4 + 2x > x + 45 - 3x$ and $x < 45 - 3x$

i.e., $4x > 41$ and $4x < 45$

$\Rightarrow x > 10.25$ and $x < 11.29$

$\Rightarrow x \geq 11$ and $x \geq 11$.

Hence, $x = 11$, $y = 12$ and $z = 26$

Hence, Deepti has got 15 of 25-paise stamps more than one-rupee stamps.

**15.** Let the number of 50-rupee notes and the 500-rupee notes that the teller intended to give be $x$ and $y$. He made a mistake and gave $y$ 50-rupee notes and $x$ 500-rupee notes.

$\therefore 50y + 500x - 50 = 3(50x + 500y)$

$\Rightarrow 350x - 1450y = 50 \Rightarrow 7x - 29y = 1$

$\text{Rem}\frac{29y}{7} = 6 \Rightarrow \text{Rem}\frac{y}{7} = 6$

This and the other values of $(x, y)$ are listed below. The corresponding values of $50x + 500y$ are also listed.

| $x$ | $y$ | $50x + 500y$ |
|---|---|---|
| 25 | 6 | 4250 |
| 54 | 13 | 9200 |
| 83 | 20 | 14,150 |
| 112 | 27 | 19,100 |

We see that the amount on the cheque could be more than 14,000 and less than 16,000.

***Solutions for question 16:***

Let $x$, $y$, and $z$ be the number of levels with 4 points, 2 points, and 1 point, respectively.

Given $x + y + z = 36$ (1)

$4x + 2y + z = 78$ (2)

Eliminating $z$ from (1) and (2), we get $3x + y = 42$

$\Rightarrow y = 42 - 3x$

Substituting for $y$ in (1), we get $z = 2x - 6$

we are also given that

$x - y \geq 2$ and $z - y \geq 7$

Thus, $x - (42 - 3x) \geq 2$ and $(2x - 6) - (42 - 3x) \geq 7$

$4x \geq 44$ and $5x \geq 55 \Rightarrow x \geq 11$ and $x \geq 11$

$\therefore x = 11$. Consequently, $y = 9$ and $z = 16$

**16.** Given, Kapil completes 2 four-point levels, he now has 9 of these left. Hence, he now has equal number of four-point levels and two-point levels left.

**17.** Suppose $x$ kg of ice and $y$ kg of cream are bought for a total of ₹304.

Then $27x + 22y = 304$.

Dividing by the least coefficient and collecting all fractions on the left and all integers on the right (and denoting the combination of integers on the right by $k$), we get $\frac{5x}{22} - \frac{18}{22} = k$

$\Rightarrow x = \frac{22k + 18}{5}$

when $k = 1$

$x = \frac{22 + 18}{5} = \frac{40}{5} = 8$

when $k = 6$

$x = \frac{132 + 18}{5} = \frac{150}{5} = 30$

This is not possible as the amount spent on ice in such case will be $27 \times 30 >$ total amount spent.

Hence, $x = 8$ is the only possibility

$\Rightarrow 22y = 304 - 216 = 88 \Rightarrow y = 4$

$\therefore$ 8 kg of ice and 4 kg of cream are bought and a total of 12 kg ice cream is obtained.

**18.** Let $x$, $y$, and $z$ denote the number of widgets of type A, B, and C, respectively.

then, $32x + 19y + 16z = 386$.

Dividing the entire equation by the least coefficient, i.e., 16 and keeping all fractional values to the left and all integral values to the right and denoting them as $k$ we get.

$\frac{3y}{16} - \frac{2}{16} = k \; ; \Rightarrow ; y = \frac{16k + 2}{3}$

By trial and error, we get

when $k = 1$, $\quad y = 6$.

when $k = 4$, $\quad y = 22$

For $y = 22$ we get $x$, $z$ as negative values.

Hence, $y = 22$ is not possible. $\therefore y = 6$

$\Rightarrow 32x + 16z = 272 \Rightarrow 2x + z = 17$

Further, it is given that

$y - x \geq 2$ and $z - y \geq 3$

$\therefore x \geq 4$ and $z \geq 9$

using $2x + z = 17$, we see that if $z = 9$ $x = 4$ and $z = 10$, $x = 7/2$, which is not possible

If $z = 11$, $x = 3$, which again is not possible, as $x \geq 4$

$\therefore z = 9$, $x = 4$ and $y = 6$

$\therefore$ Number of widgets of type C sold is 9.

**19.** $x^2 - y^2 = 1309$, i.e., $(x + y)\ (x - y) = 1309$ ... (1)

To satisfy (1), $x + y$ and $x - y$ must have the same sign.

If $x + y$ and $x - y$ are both positive, each of them must be a factor of 1309. Then $(x + y, x - y) = (1, 1309), (7,187), (11, 119), (17, 77), (77, 17), (119, 11), (187, 7)$ or $(1309, 1)$.

If $x + y$ and $x - y$ are both negative, each of them must be a negative factor of 1309. Then $(x + y, x - y) = (-1, -1309), (-7, -187), (-11, -119), (-17, -77), (-77, -17), (-119, -11), (-187, -7)$ or $(-1309, -1)$.

For each of the 16 values of and distinct value of $(x + y, x - y)$, there exists a unique $(x, y)$.

$\therefore (x, y)$ has 16 possible values.

**20.** $x^2 - y^2 = 240$, i.e., $(x + y)\ (x - y) = 240$ ... (1)

To satisfy (1), $x + y$ and $x - y$ must have the same sign. Also at least one of $x + y$ and $x - y$ must be even

$x + y$ and $x - y$ have the same parity. Also as at least one of them must be even, both must be even.

Let $x + y = 2k$, and $x - y = 2\ k_2$

$(2k_1)\ (2k_2) = 240$

$k_1\ k_2 = 60 = 2^2 3^1 5^1$.

There are 12 positive factors and 12 negative factors of 60 $k_1$ can have any of these 24 values $k_2$ would have the corresponding value.

For each $(k_1, k_2)$, i.e., for each $(x + y, x - y)$, there exists a unique and distinct value of $(x, y)$.

$(x, y)$ has 24 possible values.

# 4 Simple Interest and Compound Interest

CHAPTER

**LEARNING OBJECTIVES**

*In this chapter, you will:*

- Learn about key terms such as Interest, Principal, Time, Rate of interest and Amount, etc.
  - Learn about simple interest and how to calculate interest for a given sum
- Understand what is compound interest
  - Learn to calculate compound interest for a given sum
  - Learn how to compound interest, more than once a year
- Understand present value concept
- Learn how to calculate repayment in instalments

## ❒ INTEREST

INTEREST is money paid to the lender by the borrower for using his money for a specified period of time. Various terms and their general representation are as follows:

1. **Interest:** Money paid by borrower for using the lender's money. Denoted by $I$.
2. **Principal:** The original sum borrowed. Denoted by $P$.
3. **Time:** Time for which money is borrowed. Denoted by $n$. ($n$ is expressed in number of periods, which is normally one year.)
4. **Rate of interest:** Rate at which interest is calculated on the original sum. Denoted by $r$ and is expressed as a percentage or decimal fraction.
5. **Amount:** Sum of principal and interest. Denoted by $A$.

## ❒ Simple Interest

When interest is calculated every year (or every time period) on the original principal, i.e., the sum at the beginning of first year, such interest is called simple interest.

Here, year after year, even though the interest gets accumulated and is due to the lender, this accumulated interest is not taken into account for the purpose of calculating interest for latter years.

$$\text{Simple Interest} = \frac{Pnr}{100}$$

where $P$, $n$, $r$ are as explained above.

$$\text{Total amount } A = P + \frac{Pnr}{100}$$

$$= P\left(1 + \frac{nr}{100}\right)$$

## ❒ Compound Interest

Under compound interest, the interest is added to the principal at the end of each period to arrive at the new principal for the next period.

In other words, the amount at the end of first year (or period) will become the principal for the second year (or period); the amount at the end of second year (or period) becomes the principal for the third year (or period) and so on.

If $P$ denotes the principal at the beginning of Period 1, then principal at the beginning of Period 2

$= P\left(1+\frac{r}{100}\right)$

$= PR$ = Amount at the end of Period 1
where

$R = \left[1+\left(\frac{r}{100}\right)\right]$

$P$ at the beginning of Period 3

$= P\left(1+\frac{r}{100}\right)^2$

$= PR^2$ = Amount at the end of Period 2

$P$ at the beginning of Period $(n+1)$

$= P\left(1+\frac{r}{100}\right)^n = PR^n$

= Amount at the end of Period $n$
Hence, the amount after $n$ years (periods)
$= PR^n = A$
Interest $= I = A - P = P[R^n - 1]$

The following table gives an example of how simple interest and compound interest operate, i.e., how the Principal is for various years under simple interest and compound interest. A principal at the beginning of 1st year, of ₹100 and a rate of 10% p.a. are considered. The details are worked out for three years and shown below.

(All figures pertaining to principal, interest and amount are in Rupees)

| | Under simple interest | | | | Under compound interest | | | |
|---|---|---|---|---|---|---|---|---|
| **Year** | **Principal at the beginning of the year** | **Interest for the year** | **Interest till the end of the year** | **Amount at the end of the year** | **Principal at the beginning of the year** | **Interest for the year** | **Interest till the end of the year** | **Amount at the end of the year** |
| 1 | 100 | 10 | 10 | 110 | 100 | 10 | 10 | 110 |
| 2 | 100 | 10 | 20 | 120 | 110 | 11 | 21 | 121 |
| 3 | 100 | 10 | 30 | 130 | 121 | 12.1 | 33.1 | 133.1 |

As can be seen from the table,

In case of simple interest,
- the principal remains the same every year
- the interest for any year is the same as that for any other year.

In case of compound interest,
- the amount at the end of an year is the principal for the next year
- the interest for different years is not the same

The compound interest for the first year (where compounding is done every year) is the same as the simple interest for one year.

## ❒ Compounding More Than Once a Year

We just looked at calculating the amount and interest when the compounding is done once a year. But, compounding can also be done more frequently than once a year. For example, the interest can be added to the principal every six months or every four months and so on.

If the interest is added to the principal every six months, we say that compounding is done twice a year. If the interest is added to the principal every four months, we say that compounding is done thrice a year. If the interest is added to the principal every three months, we say that compounding is done four times a year.

The formula that we discussed above for calculating the amount will essentially be the same.



**FORMULA**

i.e., Amount $= P\left(1+\frac{r}{100}\right)^n$

where $r$ = rate% *per annum* and $n$ = number of *years*, but the rate will *not* be for one year but for the time period over which compounding is done and the power to which the term inside the bracket is raised ($n$ in the above case) will *not* be the number of years

but the number of years multiplied by the number of times compounding is done per year (this product is referred to as the total number of time periods).

For example, if a sum of ₹10000 is lent at the rate of 10% p.a. and the compounding is done for every four months (thrice a year), then the amount will be equal to

$$10000\left(1+\frac{10}{3}\times\frac{1}{100}\right)^{2\times3}$$

Here, the dividing factor of 3 in the rate and the multiplying factor of 3 in the power (multiplying the number of years)—are nothing but the number of times compounding is done in a year.

If compounding is done $k$ times a year (i.e., once every 12/k months), at the rate of $r$% p.a., then in $n$ years, the principal of $P$ will amount to $= P\left(1+\frac{r}{k\cdot100}\right)^{kn}$

When compounding is done more than once a year, the rate of interest given in the problem is called nominal rate of interest.

We can also calculate a rate of interest which will yield simple interest in one year equal to the interest obtained under the compound interest at the given nominal rate of interest. The rate of interest so calculated is called effective rate of interest.

If the number of times compounding is done in a year is increased to infinity, we say that the compounding is done every moment and then the amount is given by $P\times e^{nr/100}$, where $r$ is the rate % p.a. and $n$ is the number of years.

The following points should also be noted which are helpful in solving problems.

The difference between the compound interest and simple interest on a certain sum for two years is equal to the interest calculated for one year on one year's simple interest.

In mathematical terms, the difference between compound interest and simple interest for two years will be equal to $P(r/100)^2$, which can be written as $P(r/100)(r/100)$. In this, $Pr/100$ is the simple interest for one year and when this is multiplied by $r/100$ again, it gives interest for one year on $Pr/100$, i.e., interest for one year on one year's simple interest.

The difference between the Compound Interest for $k$ years and the Compound Interest for $(k+1)$ years is the interest for one year on the amount at the end of $k^{th}$ year.

This can also be expressed in terms of the amount as follows:

The difference between the amount for $k$ years and the amount for $(k+1)$ years under compound interest is the interest for one year on the amount at the end of the $k^{th}$ year.

The difference between the Compound Interest for the $k^{th}$ year and the Compound Interest for the $(k+1)^{th}$ year is equal to the interest for one year on the compound interest for the $k^{th}$ year.

## ❒ PRESENT VALUE

Consider a given sum $P$ and a rate of interest $r$.

We have seen that interest is cost of using the money over a period of time. That means a sum at the beginning of a period is always higher than the same amount after a period greater than or equal to 1.

Let the sum $P$ that is being considered at a rate of interest $r$% p.a., becomes $Y$ at the end of Year 1 and $Z$ at the end of Year 2 (i.e., $Y$ and $Z$ are the amounts at the end of first and second years respectively on a principal of $P$).

Then, we can say that what is $P$ today is equal to $Y$ at the end of one year and equal to $Z$ at the end of the second year. In other words, if an amount of $Y$ were to come at the end of one year from now, its value today is equal to $P$. Similarly, if an amount of $Z$ were to come at the end of two years from now, its value today is equal to $P$.

So, $P$ is the present value of $Y$ coming at the end of one year and $P$ is the present value of $Z$ coming at the end of two years.

Similarly, if we consider $n$ years (or $n$ periods in general), and $X$ is the amount that $P$ will become in $n$ periods, then we say that $P$ is the present value of $X$ coming at the end of $n$ periods.

If we consider a series of payments $Y_1$ at the end of first year, $Y_2$ at the end of second year and so on, the present value of the series of payments will then be equal to the sum of the present values of each of the payments calculated separately. If $Z_1$ is the present value of $Y_1$, $Z_2$ is the present value of $Y_2$ and so on, then the present value of the series of payments $Y_1$, $Y_2$, ... is equal to $Z_1 + Z_2 + ...$

Present value can be looked at both under Simple Interest and Compound Interest.

If an amount of $Y$ whose present value is $P_1$ comes at the end of Year 1, and an amount of $Z$ whose present value is $P_2$ comes at the end of Year 2, then the present value of both the amounts together will be equal to $(P_1 + P_2)$, i.e., the present value of the stream of payments that come at different points of time is equal to the sum of the present values of the individual amounts coming in at various points of time.

## ❒ Present Value Under Simple Interest

The principal $P$ is amounting to $X$ in $n$ periods. From this, we know that

$$X = P\left(1+\frac{nr}{100}\right) \Rightarrow P = \frac{X}{\left(1+\frac{nr}{100}\right)}$$

Hence, in general, the present value $P$ of an amount $X$ coming (or due) after $n$ periods is given by

**FORMULA**

$$P = \frac{X}{\left(1+\frac{nr}{100}\right)}$$

where $r$ is the rate percent per time period.

## ❒ Present Value Under Compound Interest

The principal $P$ is amounting to $X$ in $n$ periods. From this, we know that

$$X = P\left(1+\frac{r}{100}\right)^n \Rightarrow P = \frac{X}{\left(1+\frac{r}{100}\right)^n}$$

Hence, in general, the present value $P$ of an amount $X$ coming (or due) after $n$ periods is given by

**FORMULA**

$$P = \frac{X}{\left(1+\frac{r}{100}\right)^n}$$

where $r$ is the rate percent per time period.

## ❒ REPAYMENT IN EQUAL INSTALMENTS–COMPOUND INTEREST

If a sum $P$ borrowed is repaid in $n$ equal instalments compound interest being calculated at $r\%$ per period of instalment, we can find out the value of each instalment. Let us consider the case of $n$ equal ANNUAL instalments (even if the instalments are not annual, but monthly, the approach will remain the same except that the rate of interest taken should then be the rate per month and not rate per annum).

Let each instalment (i.e., the amount paid at the end of each year) be $X$.

Instalment $X$ paid after year 1 gives a present value of $\frac{X}{\left(1+\frac{r}{100}\right)}$.

Instalment $X$ paid at the end of year 2 gives a present value of $\frac{X}{\left(1+\frac{r}{100}\right)^2}$.

Similarly, instalment $X$ paid for $n^{\text{th}}$ period (at the end of year $n$) gives a present value of $\frac{X}{\left(1+\frac{r}{100}\right)^n}$.

The sum of all these present values would be equal to the loan amount $P$ (because only if the amount borrowed is equal to the amount repaid can we say that the loan is repaid).

$$\frac{X}{\left(1+\frac{r}{100}\right)} + \frac{X}{\left(1+\frac{r}{100}\right)^2} + \ldots + \frac{X}{\left(1+\frac{r}{100}\right)^n} = P$$

Call $\frac{X}{\left(1+\frac{r}{100}\right)} = k \Rightarrow k = \frac{100}{100 + r}$

The above equation can then be rewritten as
$X\{k + k^2 + \ldots + k^n\} = P$

The terms within the brackets form a G.P. with first term $k$ and common ratio $k$.

The sum of this G.P. $= \frac{k\,(k^n - 1)}{(k-1)}$;

Thus, $\frac{X \cdot k\,(k^n - 1)}{(k-1)} = P \Rightarrow X = \frac{P\,(k-1)}{k\left(k^n - 1\right)}$

$$= \frac{\left\{P\left[100/(100+r)\right]-1\right\}}{\left\{\left[100/(100+r)\right]\right\}\left\{\left[100/(100+r)\right]^{n}-1\right\}}$$

$$= \frac{P \cdot r}{100\left\{1-\left[100/(100+r)\right]^{n}\right\}}$$

**FORMULA**

$$\text{Each Instalment} = \frac{P \cdot r}{100\left\{1-\left[100/(100+r)\right]^{n}\right\}}$$

## SOLVED EXAMPLES

**4.01.** Find the simple interest on a principal of ₹2400 at 5% p.a. for a period of 4 years.

**Sol:** Simple interest on a sum of ₹$P$ at $r$% p.a. for $n$ years is given by $\frac{Pnr}{100}$

Here, $P$ = ₹2400; $n$ = 4 years; $r$ = 5%

$\therefore$ S.I. $= \frac{2400 \times 4 \times 5}{100} =$ ₹480

**4.02.** If ₹4000 becomes ₹4800 in 2 years, what will ₹6000 become at the end of 6 years at the same rate of interest, under simple interest?

**Sol:** Amount = Principal + Interest

Principal ($P$) = ₹4000 and amount ($A$) = ₹4800.

$\therefore$ Interest ($I$) = 800

$\Rightarrow 800 = \frac{4000 \times 2 \times r}{100} \Rightarrow r = 10\%$.

For $P$ = ₹6000, $n$ = 6, and $r$ = 10%,

$I = \frac{6000 \times 6 \times 10}{100} =$ ₹3600

Therefore, ₹6000 becomes ₹6000 + ₹3600 = ₹9600 at the end of 6 years.

**4.03.** What is the amount under compound interest at the end of three years on a sum of ₹12000 at 10% p.a.?

**Sol:** If $P$ is the principal, $r$ is the rate of interest and $n$ is number of years, then the amount under compound interest is given by

$$= P\left(1+\frac{r}{100}\right)^{n} = 12000\left(1+\frac{10}{100}\right)^{3}$$

$$= 12000\left(1+\frac{1}{10}\right)^{3} = 12000\left(\frac{11}{10}\right)^{3}$$

$= 12\ (1331) \Rightarrow$ ₹15972

**4.04.** What principal would amount to ₹9600 in 6 years at the rate of 10% p.a. simple interest?

**Sol:** Amount under simple interest $= P\left(1+\frac{nr}{100}\right)$

$$9600 = P\left(1+\frac{(6)(10)}{100}\right)$$

$9600 = P(1.6)$

$P = \frac{9600}{1.6} = 6000$

**4.05.** What sum would amount to ₹29282 in three years at the rate of 10% p.a. compound interest?

**Sol:** Amount $= P\left(1+\frac{r}{100}\right)^{n}$

$$29282 = P\left(1+\frac{10}{100}\right)^{3} \Rightarrow 29282 = P\,(1.1)^{3}$$

$$\Rightarrow P = \frac{29282}{(1.1)^{3}} \Rightarrow P = \frac{29282}{1.1 \times 1.1 \times 1.1} =$$ ₹22000

**4.06.** If a certain sum doubles in five years under simple interest, in how many years would it become 6 times itself at the same rate of interest?

**Sol:** If $P$ is the sum, the amount at the end of five years will be $2P$ (as the sum doubles).

Since Amount = Principal + Interest, the interest for five years is $P$.

If the sum has to become 6 times itself the amount should be $6P$, out of which $P$ is the principal and the balance $5P$ is the interest. Since the interest is $P$ for five years, we get the interest of $5P$ in 25 years (as interest is same every year under simple interest).

$\therefore$ The sum becomes 6 times itself in 25 years.

**4.07.** A certain sum triples in 3 years under compound interest at a certain rate of interest. In how many years would the sum become 9 times itself at the same rate under compound interest?

**Sol:** Let $P$ be the sum. Then amount at the end of 3 years is $3P$. Therefore, we get, $P\left(1+\frac{r}{100}\right)^3 = 3P$

If $\left(1+\frac{r}{100}\right) = R$, then we have $R^3 = 3$.

Now, since the sum has to become 9 times, the amount should be $9P$, i.e., $P\left(1+\frac{r}{100}\right)^n = 9P \Rightarrow R^n = 3^2$.

Since $R^3 = 3$, the 3 on the right-hand side can be replaced by $R^3$ giving us $R^n = (R^3)^2 = R^6$

$\Rightarrow n = 6$.

Thus, the sum will become 9 times itself in 6 years.

**4.08.** If ₹8000 has been lent at 10% p.a. the interest being compounded annually, what is the interest for the fifth year?

**Sol:** The amount at the end of the $4^{th}$ year will be the principal for the fifth year.

Amount at the end of the $4^{th}$ year

$= 8000\left(1+\frac{10}{100}\right)^4 = 8000 \times 1.4641$

$=$ ₹11712.8

Hence, interest for the $5^{th}$ year

$= 11712.8(0.1) =$ ₹1171.28.

**4.09.** How much will ₹20000 approximately amount to in 2 years at 15% p.a., the interest being compounded every 6 months?

**Sol:** If interest is compounded half yearly, then the amount is given by $P\left(1+\frac{r}{100}k\right)^{kn}$, where $P$ is principal, $r$ is the rate of interest, $k$ is the number of compounding periods and $n$ is the number of years. Since the interest is compounded every 6 months, the number of time periods is 4.

Amount at the end of 2 years

$= P\left(1+\frac{1}{2\times100}\right)^4 = 20{,}000\left(1+\frac{15}{200}\right)^4$

$= 20{,}000\left(1+\frac{15}{200}\right)^4$

$=$ ₹26709 (ignoring the decimal part)

**4.10.** If the rate of interest is 20% p.a., where interest is compounded every 4 months, what is the effective rate of interest per annum?

**Sol:** Let the principal be 100. At 20% p.a. rate of interest compounded every 4 months, the amount at the end of one year will be

$= 100\left(1+\frac{20}{3\times100}\right)^3 = 100\ (1.066)^3 =$ ₹121.36

(rounding off to two decimal points).

An interest rate of 21.36% p.a. will also give an amount of ₹121.36 on a principal of ₹100 when compounding is done annually. Hence, this 21.36% p.a. is called effective rate of interest.

**4.11.** A certain sum amounts to ₹7200 after 2 years and to ₹8640 after 3 years, interest being compounded annually. Find the principal and the rate of interest which are whole numbers.

**Sol:** We know that the difference between the amounts at the end of the $n^{th}$ year and $(n+1)^{th}$ year is the interest for the $(n+1)^{th}$ year.

$\therefore$ The difference between ₹8640 and 7200, i.e., 1440 is the interest for the $3^{rd}$ year.

This is equal to the interest for one year on 7200.

Therefore, $7200 \times r/100 = 1440 \Rightarrow r = 20\%$ p.a.

If $P$ is the principal, $P\left(1+\frac{20}{100}\right)^2 = 7200$

$\Rightarrow P = \frac{7200}{1.44} =$ ₹5000

**4.12.** The difference between the compound interest and the simple interest for two years on a certain sum at a certain rate of interest is ₹64. Find the principal if the compound interest for two years is ₹704.

**Sol:** Simple interest for two years = compound interest for two years – difference between C.I. and S.I. for two years = 704 – 64 = ₹640. We know that the difference between the compound interest and the simple interest for 2 years is equal to the interest for one year on first year simple interest. Since simple interest for two years is ₹640, for one year it is ₹320. Hence, interest for one year on first year's simple interest

$= \frac{r(320)}{100} = 64 \Rightarrow r = 20\%$ p.a.

Also, $\frac{P\times2\times20}{100} = 640 \Rightarrow P =$ ₹1600

**4.13.** The compound interest on a certain sum at a certain rate of interest for the second year is ₹2400 and for the third year is ₹2880. Find the principal and the rate of interest.

**Sol:** Note that what is given is the compound interest for the second and the third years but NOT for two and three years. We know that the difference between the compound interest for $(R+1)^{th}$ year and that for the $R^{th}$ year is equal to the interest for $(R+1)^{th}$ year on the compound interest for the $R^{th}$ year. Hence, the difference between the two figures ₹2400 and ₹2880, which is ₹480 will be equal to the interest for one year on ₹2400. Therefore, if the rate of interest is $r$, then

$$2400 \times \frac{r}{100} = 480 \Rightarrow r = 20\% \text{ p.a.}$$

The compound interest for the 2nd year will be equal to the difference between the amount at the end of the 2nd year and the amount at the end of 1st year,
i.e.,
$\Rightarrow P(1+r)^2 - P(1+r) = 2400$
$\Rightarrow P(1+r)(1+r-1) = 2400$

$$\Rightarrow P(1.2)(0.2) = 2400 \Rightarrow P = \frac{2400}{0.24} = ₹10000$$

Therefore, principal and rate of interest are ₹10000 and 20% p.a. respectively.

**4.14.** Sanjay borrowed ₹14000 at 10% p.a. He repaid ₹8000 at the end of the 1st year. What amount should he pay at the end of the 2nd year to completely discharge the loan, compound interest being reckoned annually?

**Sol:** At 10% p.a., amount at the end of the 1st year is ₹15400, out of which ₹8000 is repaid. The balance ₹7400 is the principal for the second year, which amounts to ₹8140 at the end of the 2nd year. Hence, to discharge the loan completely at the end of the 2nd year, Sanjay has to repay ₹8140.

**4.15.** Hussain borrowed ₹15000 at the rate of 10% p.a. rate of compound interest, compound annually. He repaid a certain amount at the end of the first year. Then he paid ₹12100 at the end of the 2nd year to completely discharge the loan. What amount did he repay at the end of the 1st year?

**Sol:** ₹15000 borrowed at 10% p.a. will become ₹16500 at the end of the 1st year. Let $k$ be the amount repaid at the end of the first year. Then the balance is ₹16500 – $k$ will become the principal for the second year.
$\Rightarrow (16500 - k)\ 1.1 = 12100$
$\Rightarrow 16500 - k = 11000 \Rightarrow k = ₹5500$

**4.16.** Pavan borrowed ₹21000 at the rate of 10% p.a. compound interest. If this amount has to be repaid in two equal annual instalments, find the value of each instalment.

**Sol:** Let the value of each instalment be $x$. ₹21000 at 10% p.a. will become ₹23100 at the end of the 1st year. At this point an amount of $x$ is repaid. Therefore, balance at this point is ₹$(23{,}100 - x)$. This sum at 10% p.a. will become $1 \times 1\ (23{,}100 - x)$ at the end of the 2nd year. Another amount of $x$ is to be paid to discharge the loan completely.
$\therefore\ 1.1\ (23{,}100 - x) = x \Rightarrow 1.1 \times 23{,}100 = 2 \times 1x$

$$x = \frac{1.1 \times 23100}{2.1} = ₹12100.$$

**4.17.** Find the present value of the payments of ₹8400 to be received at the end of the first year and ₹8640 to be received at the end of the second year, if compound interest at 20% p.a. is reckoned.

**Sol:** If a sum of ₹$z$ is going to be received at the end of the $n$ years, then the present value of that amount is $\dfrac{z}{\left(1+\frac{r}{100}\right)^n}$

Hence, present value of the payments to be received is $\dfrac{8400}{1.2} + \dfrac{8640}{(1.2)^2}$ $\dfrac{8400}{1.2} + \dfrac{8640}{1.44}$

$= 7000 + 6000 = ₹13000$

**4.18.** Find the simple interest on a sum of ₹1000 at 10% p.a. for 4 years.

**Sol:** Simple interest $= \dfrac{PNR}{100}$

$$\text{Interest} = \frac{(1000)(4)(10)}{100} = ₹400$$

**4.19.** A sum of ₹4000 becomes ₹4500 in 2 years under simple interest. In how many years will ₹5000 become ₹5625 under simple interest at the same rate of interest?

**Sol:** Let the rate of interest be $R$% p.a.
Interest on ₹4000 = ₹500

$$500 = (4000)\left(\frac{R}{100}\right)(2)$$

$R = 6 \times 25$
Interest on ₹5000 = ₹625
Let the required time be $T$ years.

$$625 = (5000)\left(\frac{6 \cdot 25}{100}\right) T \Rightarrow T = 2$$

**4.20.** Find the value that ₹1000 would amount to under compound interest at 20% p.a., interest being compounded annually in 3 years.

**Sol:** Amount $= P\left(1+\frac{R}{100}\right)^N$

$$= 1000\left(1+\frac{20}{100}\right)^3 = ₹1728$$

**4.21.** Find the sum that would amount to ₹6600 under simple interest in 4 years at 8% p.a.

**Sol:** Let the sum be ₹$P$.

$$\text{Given that } P\left(1+4\left(\frac{8}{100}\right)\right) = 6600$$

$$P = 5000$$

**4.22.** If a sum triples in 4 years under simple interest, find the time that it would take to become 5 times itself at the same rate of interest.

**Sol:** If the sum triples, the interest obtained will be twice the sum. This takes 4 years. If the sum becomes 5 times, the interest must be four times the sum.
$\therefore$ This takes a total of 8 years.

**4.23.** A sum triples in 4 years under compound interest at a certain rate of interest, interest being compounded annually. Find the time it would take to become 9 times itself.

**Sol:** The sum triples in 4 years. If it becomes 9 times itself, it has tripled twice.
$\therefore$ This takes 8 years.
Let the sum of ₹$P$, triple in 4 years at $R$% p.a.

$$\Rightarrow P\left(1+\frac{R}{100}\right)^4 = 3P$$

$$\Rightarrow \left(1+\frac{R}{100}\right)^4 = 3 \qquad (1)$$

Let it takes $K$ years to become 9 times.

$$P\left(1+\frac{R}{100}\right)^K = 9P \Rightarrow \left(1+\frac{R}{100}\right)^K = 9$$

$$\Rightarrow \left[\left(1+\frac{R}{100}\right)^4\right]^{\frac{K}{4}} = 3^2$$

from (1),

$$3^{\frac{K}{4}} = 3^2 \Rightarrow \frac{K}{4} = 2$$

$\therefore K = 8$

**4.24.** If ₹4000 is lent at 10% p.a., interest being compounded annually, find the interest for the fourth year.

**Sol:** Interest for the fourth year = Amount at the end of the first 4 years − Amount at the end of the first 3 years

$$= 4000\left(1+\frac{10}{100}\right)^4 - 4000\left(1+\frac{10}{100}\right)^3$$

$= 4000\ (1.4641 - 1.3310)$
$= 4000\ (0.1331)$, i.e., ₹532.40

**4.25.** Find the value that ₹8000 will amount to in 2 years at 20% p.a., interest being compounded half yearly.

**Sol:** Rate of interest = 10% per half year
Number of time periods (i.e., half years) = 4

$$\text{Amount} = 8000\left(1+\frac{10}{100}\right)^4 = 8000\ (1.4641);\ \text{i.e.,}$$

₹11712.80

**4.26.** The interest on a sum is compounded every 3 months. If the rate of interest is 40% p.a., find the effective rate of interest per annum.

**Sol:** Let the sum be ₹100
Amount at the end of a year

$$= 100\left(1+\frac{40}{4(100)}\right)^4 = ₹146.41$$

$\therefore$ effective rate of interest = 46.41%

**4.27.** A sum amounts to ₹28800 in two years and to ₹34560 in three years under compound interest, interest being compounded annually. Find the sum and the rate of interest.

**Sol:** Let the sum be ₹$P$ and the rate of interest be $R$% p.a.

$$P\left(1+\frac{R}{100}\right)^2 = 28800 \qquad (1)$$

$$P\left(1+\frac{R}{100}\right)^3 = 34560 \qquad (2)$$

Dividing (2) by (1), we get

$$1+\frac{R}{100} = 1.2 \Rightarrow R = 20$$

Substituting $R = 20$ in (1) or (2),
$P = 20000$

**4.28.** The compound interest and the simple interest on a sum at certain rate of interest for 2 years are ₹2760 and ₹2400 respectively. Find the sum and the rate of interest.

**Sol:** Let the sum be ₹$P$ and let the rate of interest be $R$% p.a.
Difference between the compound interest and the simple interest = ₹360

$$\therefore\ P\left(\frac{R}{100}\right)^2 = 360 \qquad (1)$$

$$P(2)\left(\frac{R}{100}\right) = 2400 \Rightarrow \frac{PR}{100} = 1200 \qquad (2)$$

$$\therefore\ \frac{PR}{100}\left(\frac{R}{100}\right) = 1200\left(\frac{R}{100}\right) = 360$$

$R = 30$
Substituting $R$ in (1) or (2),
$P = 4000$

**4.29.** The compound interest on a certain sum for the 2nd year and the 3rd year are ₹3300 and ₹3630 respectively. Find the sum and the rate of interest.

**Sol:** Let the sum be ₹$P$
Let the rate of interest be $R$% p.a.
Interest for the 2nd year

$$= P\left(1+\frac{R}{100}\right)^2 - P\left(1+\frac{R}{100}\right) = 3300$$

$$P\left(\frac{R}{100}\right)\left(1+\frac{R}{100}\right) = 3300 \qquad (1)$$

Interest for the 3rd year

$$= P\left(1+\frac{R}{100}\right)^3 - P\left(1+\frac{R}{100}\right)^2 = 3630$$

$$P\left(\frac{R}{100}\right)\left(1+\frac{R}{100}\right)^2 = 3630 \qquad (2)$$

Dividing (2) by (1), we get

$$1+\frac{R}{100} = 1.1 \Rightarrow R = 10$$

Substituting $R$ in (1) or (2),
$P = 30000$.

**4.30.** Sanjay borrowed ₹15000 at 20% p.a. under compound interest, interest being compound annually. He repaid ₹12000 at the end of the first year. Find the amount he must repay at the end of the second year to clear the loan.

**Sol:** Value of ₹15000 at the end of the first year = ₹18000
As Sanjay repaid ₹12000, he has to repay the value of the balance of ₹6000 at the end of the second year. He has to repay $6000 \times 1.2$
i.e., ₹7200.

**4.31.** Praveen borrowed ₹26400 at 20% p.a. under compound interest, interest being compounded annually. If he has to repay this in two equal annual instalments, find the value of each instalment.

**Sol:** Let each instalment be ₹$x$
Value of ₹26400 at the end of the first year $= 26400 \times 1.2$, i.e., ₹31680
As Praveen repaid ₹$x$ at the end of the first year, he has to repay the value of the balance of ₹$(31680 - x)$ at the end of the second year.
$\Rightarrow 1.2\ (31680 - x) = x$
$\therefore\ x = 17280$

**4.32.** Find the sum of the present values of the payments received at 10% p.a. under compound interest, interest being compounded annually, if ₹7700 and ₹7260 are received at the end of the first year and second year respectively.

**Sol:** Sum of the present values of the payments received

$$= \frac{7700}{1+\frac{10}{100}} + \frac{7260}{\left(1+\frac{10}{100}\right)^2} = ₹13000$$

## EXERCISE-1

***Directions for questions 1 to 40*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** The amount that Kiran would receive if he invests ₹18,000 at 15% p.a. simple interest for four years is _____ (in ₹).

**2.** The difference between the interests earned on a principal under a certain rate of compound interest in $p^{th}$ year and $(p+1)^{th}$ year is more than that in the $q^{th}$ year and $(q+1)^{th}$ year if
(A) $p > q$ (B) $p < q$ (C) $p = q$ (D) Can't say

**3.** The amount that Kiran receives if he invests ₹2,400 at 20% p.a. compound interest for 1 year, compounding done semi-annually is _____ (in ₹).

**4.** If ₹4,000 becomes ₹4,560 in 2 years time, what will ₹5,000 become at the end of 4 years at the same rate of interest under simple interest? _____ (in ₹)

**5.** A sum of money doubles at compound interest in three years. It becomes eight times itself in _____ years.

**6.** A sum amounts to 2.197 times of itself in three years at a certain rate of compound interest. Find the rate of interest.
(A) 30% p.a. (B) 13% p.a.
(C) 39.9% p.a. (D) 23% p.a.

**7.** A sum doubles in 8 years at compound interest. In how many years will the sum become 4 times the original sum if the interest is compounded annually? _____

**8.** A sum of ₹4,800 is invested at compound interest for three years, the rate of interest being 10% p.a., 20% p.a. and 25% p.a. for 1st, 2nd and 3rd years respectively. The interest received at the end of 3 years is (in ₹) _____.

**9.** What is the amount under compound interest at the end of three years on a sum of ₹8,000 at 5% per annum? (in ₹) _____

**10.** If ₹2,000 amounts to ₹2,500 in 2 years at simple interest, what is the rate of interest per annum?
(A) 8% (B) 37.5% (C) 25% (D) 12.5%

**11.** A sum invested at 20% p.a. compound interest yields an interest of ₹5,544 in two years. The sum invested (in ₹) _____.

**12.** Find the effective rate of interest (per annum), if the normal rate of interest is 10% p.a. and the interest is compounded every six months _____.

**13.** What is the interest earned in the first year on ₹200 at 10% per annum compound interest, interest compounded every 6 months?
(A) ₹20 (B) ₹20.50 (C) ₹25 (D) ₹23.20

**14.** The compound interest on ₹18,000 at 15% p.a. for two years is (in ₹) _____.

**15.** A man saves ₹15,000 at the beginning of each year and puts the money in a bank that pays 10% interest per year, interest being compounded annually. What would be the total savings of the man at the end of 6 years? [$1.1^6 = 1.772$]
(A) ₹1,27,380 (B) ₹1,67,340
(C) ₹1,37,380 (D) ₹1,57,320

**16.** A sum becomes 7/5 times itself in three years at simple interest. Find the rate of interest.
(A) $13^2/_3$% p.a. (B) $12^2/_3$% p.a.
(C) $13^1/_3$% p.a. (D) $12^1/_3$% p.a.

**17.** A sum of ₹3,700 amounts to ₹5,476 in four years invested at simple interest. If the rate of interest is 6% p.a. more, then, in the same time period, the sum becomes (in ₹) _____.

**18.** The interest on a certain amount under compound interest for the fifth year is ₹1,280 and that for the sixth year is ₹1,408. What is the annual rate of interest?
(A) 10% (B) 15% (C) 20% (D) 21%

**19.** A certain sum yields ₹840 more interest if it is invested at 18% p.a. than at 12% p.a. for two years. The sum is (in ₹) _____.

**20.** A certain sum is lent at 4% p.a. for the first four years, 8% p.a. for the next three years and 5% p.a. for the remaining period. If for a period of nine years, the simple interest obtained is ₹8,500, find the sum.
(A) ₹25,500 (B) ₹18,000
(C) ₹12,750 (D) ₹17,000

**21.** Find the interest earned in the first year on ₹400 at 20% p.a. compound interest, the interest being compounded half yearly (in ₹) _____.

**22.** The simple interest on a certain sum for three years with the rate of interest being 7% p.a., 10% p.a. and 12% p.a., for the first year, second year and third years respectively is ₹8,178. The sum is (in ₹) _____.

**23.** What principal would amount to ₹8,880 in 4 years at 12% p.a. rate of simple interest? (in ₹) _____

**24.** The difference between the compound interest on ₹12,000 at 20% p.a. for one year when compounded yearly and half yearly is (in ₹) _____.

**25.** A sum of money amounts to four times itself in eight years at simple interest. In how many years will the same sum amount to 17.5 times itself?
(A) 35 (B) 44 (C) 30 (D) 28

**26.** Find the interest on a sum of ₹88,000 at $9^{1}/_{11}$% p.a. simple interest for 45 months.
(A) ₹40,000 (B) ₹30,000
(C) ₹36,000 (D) ₹28,000

**27.** The sum that will yield a simple interest of ₹1,368 in three years at 12% p.a. is (in ₹) _____.

**28.** The simple interest and compound interest on a certain sum at a certain rate of interest for two years are ₹800 and ₹960 respectively.

Find the rate of interest as well as the sum.
(A) 30% p.a.; ₹2,000 (B) 40% p.a.; ₹2,000
(C) 40% p.a.; ₹1,000 (D) 45% p.a.; ₹1,000

**29.** A sum becomes ₹24,200 in 2 years and ₹29,282 in 4 years at a certain rate of compound interest, compounded annually. Find the rate of interest.
(A) 12% p.a. (B) 11% p.a.
(C) 13% p.a. (D) 10% p.a.

**30.** Compound interests earned on a sum for second and third years are ₹1,200 and ₹1,440 respectively. Find the rate of interest. Also find the sum.
(A) 18% p.a.; ₹6,000 (B) 20% p.a.; ₹5,000
(C) 20% p.a.; ₹6,000 (D) 24% p.a.; ₹5,000

**31.** In how many years does a sum become four times itself at 25% p.a. simple interest?
(A) 12 (B) 15 (C) 10 (D) 17

**32.** Find the simple interest earned (in ₹) on ₹20,000 for 2 years at 10% p.a. _____.

**33.** What rate of interest per annum does a person get if he invests at 40% p.a. compound interest, interest being compounded half-yearly?
(A) 42% (B) 48% (C) 44% (D) 46%

**34.** If ₹2,000 amounts to ₹2,880 in 2 years at compound interest, what is the rate of interest per annum if the interest is being compounded annually?
(A) 10% (B) 20% (C) 15% (D) 25%

**35.** By selling 45 metres of cloth, a trader gains a profit equal to the selling price of 15 metres of cloth. Find the gain percentage.
(A) 25% (B) 20% (C) 33.33% (D) 50%

**36.** A certain sum becomes ₹20,720 in four years and ₹24,080 in six years at simple interest. Find the sum and rate of interest.
(A) ₹12,000; 12% p.a. (B) ₹14,000; 12% p.a.
(C) ₹12,000; 15% p.a. (D) ₹14,000; 15% p.a.

**37.** If the interest is compounded quarterly, which of the following is necessarily true regarding the effective annual rate of interest?
(A) It is same for every year.
(B) It becomes 4 times that of the previous year.
(C) It increases by 4% every year.
(D) None of these

**38.** A sum triples in three years at a certain rate of interest, compounded annually. In how many years will the same sum amount to eighty one times itself at the same rate?
(A) 18 (B) 9 (C) 12 (D) 27

**39.** The interest for the 3rd year on a certain sum at a certain rate of simple interest is ₹3,000. Find the sum of the interests accrued on it (in ₹) in the 6th, 7th and 8th years _____.

**40.** A certain sum becomes three times itself at simple interest in six years. It will become eight times itself in years _____.

## EXERCISE-2

***Directions for questions 1 to 40***: For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** A certain loan amount, under compound interest, compounded annually earns an interest of ₹1,980 in the second year and ₹2,178 in the third year. How much interest did it earn in the first year? (in ₹) _____

**2.** In what time will a sum of money become fifteen times itself at 21% p.a. simple interest?
(A) 72 years 6 months (B) 66 years 8 months
(C) 60 years 9 months (D) 75 years 3 months

Difficulty Level-2: Moderate

**3.** What will be the compound interest accrued on an amount of ₹12,000 at 20% p.a. for the second year if the interest is compounded half-yearly? (in ₹) _____

**4.** Raju took a loan at 8% per annum simple interest for a period of 5 years. At the end of five years, he paid ₹10,640 to clear his loan. How much loan did he take?
(A) ₹8,500 (B) ₹8,000
(C) ₹7,700 (D) ₹7,600

**5.** The compound interest on ₹18,000 at 4% p.a. for a certain period is ₹1,468.80. Find the time period (in years) _____.

**6.** A man borrowed ₹80,000 at the rate of 10% p.a. compound interest, interest being compounded annually. How much amount should he have repaid at the end of the first year, if by repaying ₹55,000 at the end of the second year he can clear the loan?
(A) ₹38,000 (B) ₹40,000
(C) ₹45,000 (D) ₹50,000

**7.** What is the present worth of ₹5,324 due in 3 years at 10% per annum compound interest, interest being compounded annually?
(A) ₹5,000 (B) ₹4,000
(C) ₹3,600 (D) ₹3,800

**8.** A person invested half of the money he has at a rate of 10% p.a. compounded annually and the remaining half at a rate of 20% p.a. simple interest. After 2 years he received a total of ₹2,610 from both the investments. How much will he receive after 3 years?
(A) ₹2,300 (B) ₹2,931 (C) ₹2,642 (D) ₹2,800

**9.** The difference in compound interests earned on a certain sum, for which the interest is compounded annually, in the first and the second years is ₹140. If the rate of interest becomes thrice the original rate, then the difference in the amounts (in ₹) would be _____.

**10.** Anil lent a total of ₹24,000 to two persons at simple interest, the rates of interest being 12% p.a. and 9% p.a. respectively. The interest received from the first person in four years is equal to the interest received from the second person in three years. How much amount was lent to the second person? ₹ _____

**11.** The value of an electronic device depreciates at the rate of 5% per annum. If its present value is ₹32,490, what was its value two years ago? (in ₹) _____

**12.** A sum was lent for an year, another sum was lent for 2 years and a third sum was lent for 3 years. Each sum was lent at 8% p.a. compound interest. Each sum amounted to the same value. The ratio of the first, second and third sums is
(A) 729 : 675 : 625 (B) 25 : 27 : 29
(C) 29 : 27 : 25 (D) 625 : 675 : 729

**13.** A sum was lent at a certain rate of compound interest. It amounted to ₹15,600 at the end of 4 years and to ₹18,720 at the end of 8 years. Find the sum (in ₹) _____.

**14.** Find the present worth of ₹1,749.60 due in 2 years at 8% p.a. compound interest.
(A) ₹1,200 (B) ₹1,400
(C) ₹1,500 (D) ₹1,650

**15.** A total of ₹$p$ is lent out to three people at simple interest at the rates of 10% p.a., 20% p.a. and 14% p.a., for 6 years, 5 years and 2 years respectively such that the amount received from each of them is ₹6,400. Find the value of $p$ _____.

**16.** Under simple interest, due to the fall in the interest rate by 0.5 percentage point, my yearly income from savings came down by ₹884. Find the savings (in ₹) _____.

**17.** Giri borrowed ₹36,000 from Hari at 10% p.a. simple interest. He lent the sum he borrowed to Karthik at 20% p.a. compound interest. Karthik repaid the loan at the end of two years. Had Karthik repaid the loan at the end of three years, the additional profit of Giri would have been
(A) ₹6,148 (B) ₹7,244
(C) ₹6,588 (D) ₹6,768

**18.** The compound interest on a certain sum for the third year and the fourth year is ₹1,815 and ₹1,996.50 respectively. What is the rate of interest per annum? _____%

**19.** The difference between the compound interest, interest being compounded annually and the simple interest for two years on the same sum and at the same rate of interest is ₹25.60. Find the sum (in ₹) if the rate of interest is 8% p.a. _____.

**20.** A person borrowed ₹1,000 at the rate of 10% per annum simple interest. What is the amount he has to pay after 5 years to clear the debt?
(A) ₹1,564 (B) ₹1,532
(C) ₹1,500 (D) ₹1,550

**21.** If interest is compounded half yearly, what would be the interest earned, if a person invests ₹2,000 at 40% p.a. for one year? (in ₹)_____

**22.** If interest is compounded quarterly? (in ₹)_____

**23.** 1. $x$ varies directly with the square of $y$. When $y$ is 12, $x$ is 452, find $x$ when $y$ is 18.

2. $x$ varies inversely with the square of $y$. When $y$ is 2, $x$ is 48. Find $x$ when $y$ is 4.

**24.** A person invested ₹p at a certain rate of simple interest. After 1, 2 and 3 years, it amounted to ₹3/2p, ₹2p, ₹5/2p respectively. What will it amount to at the end of 20 years?
(A) ₹6p (B) ₹10p (C) ₹11p (D) ₹7p

**25.** Find the effective annual rate of interest if the rate of interest is 18% p.a., and the interest is compounded once every 4 months _____%.

**26.** Dolly deposited ₹20,000 at 10% p.a. simple interest for a period of $n$ years. Lilly deposited ₹18,000 at the same rate at compound interest for the same period. After $n$ years, the amount received by Dolly is ₹2,042 more than the amount received by Lilly. Find the value of $n$ _____.

**27.** Find the difference between the simple interest and compound interest (compounded annually) on ₹500 at 10% p.a. for 2 years (in ₹)_____.

**28.** Janardhan deposited a certain sum of money in a fixed deposit account at $k$% p.a., interest being compounded annually. If the interest accrued for the 3rd and the 4th years is ₹5,000 and ₹6,250 respectively, what is the total interest accrued for the first two years? (in ₹)_____

**29.** A certain sum of money increased by 72.8% at a certain rate in three years with interest being compounded annually. If the same sum is lent at simple interest at the same rate of interest, in how many years would it become four times itself?
(A) 5 (B) 8 (C) 11 (D) 15

**30.** Shyam borrowed ₹18,000 at 15% p.a. compound interest, compounded annually. He repaid ₹10,700 at the end of the 1st year. What is the amount he should pay at the end of the 2nd year to completely discharge the loan? (in ₹) _____

**31.** If compound interest for third year is ₹1,000 on a certain sum at the rate of 10% p.a. compounded annually, then what is the compound interest for 7th year? (in ₹)_____

**32.** Amar borrows ₹2,100 at the rate of 10% p.a. compounded annually and has to repay it in two equal instalments in 2 years. Each instalment is paid at the end of an year. What is the value of each instalment? (in ₹)_____

**33.** If the difference between simple interest for 2 years at 5% p.a. on a certain sum and compound interest for 2 years at 6.25% p.a. on the same sum compounded annually is ₹37, find the sum (in ₹) _____.

**34.** A man borrowed ₹50,000 at simple interest with the rate of interest not remaining constant for the entire period. He repaid the entire amount after 8 years. The rate of interest for the first two years is 8% p.a., for the next three years it is 10% p.a., for the next two years, it is 5% p.a. and 7% for the last year. How much amount did he repay to clear his loan at the end of the period (in ₹)? _____

**35.** Kavitha borrowed ₹72,000 from Karuna and Kalyani. Karuna charges 12% p.a. compound interest and Kalyani charges 7% p.a. compound interest. At the end of an year, Kavita paid ₹6,120 as interest. Find the sum borrowed from Kalyani.
(A) ₹48,600 (B) ₹50,400
(C) ₹49,200 (D) ₹51,300

**36.** A sum of money under compound interest doubles in 4 years. In how many years will it become 16 times itself?
_____

**37.** The value of a motorcycle is ₹80,000 and its value depreciates by 20% every year, with respect to its value at the beginning of the year. What is the profit earned by selling the motorcycle at the end of the 2nd year at ₹53,600?
(A) ₹2,200 (B) ₹2,400 (C) ₹2,300 (D) ₹2,100

**38.** Find the compound interest earned on a sum of ₹32,000 at 25% p.a. for 4 years (in ₹)_____.

**39.** A loan is taken today and repaid in two annual instalments (each paid at the end of an year) of ₹2,662 each. The rate of interest is 10% p.a. and it is compounded annually. Find the sum borrowed.
(A) ₹4,540 (B) ₹4,620 (C) ₹2,848 (D) ₹2,152

**40.** The compound interests earned in the third and the fourth years on a certain sum of money are ₹576 and ₹691.2 respectively. Find the sum (in ₹) _____.

Difficulty Level-2: Moderate

## EXERCISE-3

***Directions for questions 1 to 40*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Find the sum of the present values of the payments received at 10% p.a. under compound interest, interest being compounded annually if ₹7,700 and ₹7,260 are received at the end of the 1st year and 2nd year respectively (in ₹) _____.

**2.** Two sums of money in the ratio 2 : 3 are lent for a period of 5 years each. The rates of interest on the two sums are in the ratio 1 : 2 and the total simple interest on the two sums is ₹2,400. Find the difference in the simple interests on the two sums (in ₹) _____.

**3.** Pradeep borrowed ₹12,600 at 10% p.a. rate of compound interest. If this amount has to be repaid in two equal annual instalments, find the value of each instalment. (Each instalment is paid at the end of an year.) (in ₹) _____.

**4.** What equal annual instalment (in ₹) will discharge a loan of ₹72,800 borrowed at 20% p.a. compounded annually in 3 years?
(A) 34,560 (B) 35,560
(C) 35,580 (D) 34,480

**5.** Ravi bought a car worth ₹7,55,000. He paid ₹1,05,000 as down payment and paid the remaining amount including interest, in 3 equal instalments. Interest was charged at 14% p.a. compounded annually. If he paid the first, the second and the third instalments at the ends of the first, second and third years respectively, the value of each instalment he paid (in ₹) is _____. (Assume $\frac{1}{(1.14)^3} = 0.675$).

**6.** Kashi borrowed ₹12,000 at 10% p.a. rate of compound interest. He repaid a certain amount at the end of the first year. Then, he paid ₹10,120 at the end of the 2nd year to completely discharge the loan. What amount did he repay at the end of 1st year? (in ₹) _____

**7.** A sum of ₹6,440 is due to be repaid over four years. It has to be repaid in four equal annual instalments (the instalments being payable at the ends of the 1st, 2nd, 3rd and 4th years) at 10% p.a. simple interest. Find the value of each instalment.
(A) ₹1,200 (B) ₹1,400 (C) ₹1,600 (D) ₹1,500

**8.** A person deposited ₹100 in a bank which pays at $r$ % p.a. with interest being compounded annually. The same person invested another ₹100 in another bank which pays at $2r$% p.a. simple interest. In how many years will the two deposits amount the same?
(A) 2 years (B) 3 years
(C) 4 years (D) Depends on the value of $r$.

**9.** A sum of money yields an interest of ₹2,016 for the second year and an interest of ₹2,257.92 for the third year at a certain rate of interest, compounded annually. What is the sum? (in ₹) _____

**10.** How much will ₹10,000, approximately amount to in 2 years at 15% p.a., with the interest being compounded every 6 months? (in ₹) _____

**11.** A lender had received his amount in three annual equal instalments at ₹2,662 per instalment at a rate of 10% p.a., interest being compounded annually. Find the sum (in ₹) that the lender had rent _____.

**12.** If the rate of interest is 10% p.a. where interest is compounded every 4 months, what is the effective annual rate of interest?_____%.

**13.** Geet borrows ₹1,00,000 from a bank at 10% p.a. rate of compound interest and clears the debt in five annual instalments. If the instalments paid at the ends of the second, third, fourth and fifth years are ₹20,000, ₹18,800, ₹21,600 and ₹44,000 respectively, what was the amount paid at the end of the first year?
(A) ₹20,000 (B) ₹30,000
(C) ₹36,000 (D) ₹48,000

**14.** The compound interest on a certain sum at a certain rate of interest for the 2nd year is ₹2,200 and that for the 3rd year is ₹2,420. Find the principal and the rate of interest (in ₹, %, p.a.) _____, _____.

**15.** A sum was borrowed at 20% p.a. compound interest. It was repaid in three annual instalments at the end of one year, two years and three years respectively. The first, second and third instalments were ₹2,400, ₹2,304 and ₹5,184 respectively. Find the sum borrowed (in ₹) _____.

**16.** A sum of ₹2,000 was split into two parts. One of the parts was invested at 8% p.a. simple interest and the other part was invested at 5% p.a. compound interest. Each part was invested for two years. The total interest realized from the two parts was ₹251. Find the value of the part invested at simple interest.
(A) ₹600 (B) ₹750 (C) ₹900 (D) ₹800

**17.** Ramu borrowed ₹62,500 from a bank at 20% p.a., interest being compounded annually. He repaid it in two annual instalments. The amount he paid at the end of the second year was ₹2,000 more than that paid at the end of the first year. Find the amount (in ₹) he paid at the end of the first year.
(A) 37,500 (B) 42,000
(C) 35,000 (D) 40,000

**18.** The compound interest for the $4^{th}$ year on a certain sum is ₹2,000 and the compound interest for the $7^{th}$ year on the same sum is ₹2,662. If interest is compounded annually, what is the annual rate of interest?
(A) 9% (B) 10% (C) 15% (D) 20%

**19.** What annual instalment will discharge a debt of ₹1,450 due after five years at 8% p.a. simple interest?
(A) ₹320 (B) ₹450
(C) ₹250 (D) ₹400

**20.** Prakash invested a certain amount in a six-year fixed deposit scheme, interest being compounded annually. The interests accrued on this deposit for the fourth and fifth years, respectively, are ₹1,331 and ₹1,464.10. If George deposited ₹12,000 in the same scheme, how much interest would be accrued on this deposit for the first two years?
(A) ₹1,320 (B) ₹1,452 (C) ₹2,520 (D) ₹1,440

**21.** The difference in the simple interest and the compound interest on ₹1,600 for 2 years at a certain rate of interest is ₹16. Find the simple interest on that sum of money for 2 years (in ₹) _____.

**22.** If ₹6,000 has been lent at 10% p.a., the interest being compounded annually, what is the interest for the $4^{th}$ year? (in ₹) _____

**23.** ₹2,000 was lent at compound interest, interest being compounded annually for 3 years. The respective rates of interest for the first, the second and the third years are 10% p.a., 20% p.a. and 30% p.a. Had the sum of ₹2,000 been lent at 20% p.a. simple interest for 3 years, how much more/less interest would have been realized?
(A) ₹116 more (B) ₹116 less
(C) ₹232 less (D) ₹232 more

**24.** In which year will a sum of money invested at 10% p.a. compound interest (compounded annually) double? (Given $\log 1.1 = 0.0414$)
(A) $6^{th}$ (B) $7^{th}$ (C) $8^{th}$ (D) $9^{th}$

**25.** Mohan borrowed ₹16,000 from a bank at simple interest at a certain rate of interest. After four years, he borrowed ₹28,880 from the same bank. Starting from the fifth year, the bank charged compound interest on the total amount repayable on the two loans at 10% p.a. At the end of six years, Mohan repaid a total of ₹60,500 and thereby cleared both loans. Find the rate of interest on the first loan for the first four years (in % p.a.) _____.

**26.** A person deposited a certain amount in a bank which is offering 10% p.a., compound interest for the first two years and for the next two years, each year the rate of interest is 10% points more than that in previous year. The value of his investment at the end of the $3^{rd}$ year is ₹4,840 more than that at the end of the second year. Find the total amount received by the person at the end of the $4^{th}$ year.
(A) ₹37,752 (B) ₹38,572
(C) ₹38,752 (D) Cannot be determined

**27.** Raghu borrowed ₹4,000 at a certain rate of simple interest payable at the end of 7 years. After 2 years, he repaid ₹3,000 towards the principal and after 5 more years, he repaid ₹3,600 to settle his loan. Find the rate of interest.
(A) 10% p.a. (B) 12.5% p.a.
(C) 20% p.a. (D) 16.66% p.a.

**28.** A man borrowed a sum of ₹42,000 at 10% p.a. compound interest and initially planned to repay it in two equal annual instalments (the instalments being paid at the end of each year). He actually repaid it in three equal annual instalments. What is the difference between each instalment initially planned and each instalment actually paid (rounded off to the nearest rupee)?
(A) ₹6,226 (B) ₹7,311 (C) ₹9,331 (D) ₹4,993

**29.** A person invested a sum of ₹91.3. In how many years will the sum become ₹5,000, if it is compounded every moment at the rate of 100% p.a.? (Given $e = 2.72$)
(A) 2 (B) 4 (C) 6 (D) 1

**30.** Mani gave ₹30,000 and ₹50,000 as loans to Ajay and Vijay respectively for two years under compound interest, compounded annually. For Ajay, the rate of interest was 6% for the first year and then 10% on the total amount due for the second year. For Vijay, the rate of interest was 8% for the first year and 12% on the total amount due for the second year. If Vijay had returned ₹30,000 at the end of the first year and the Ajay returned the entire amount at the end of second year, how much more interest did Vijay pay than Ajay?
(A) ₹1,900 (B) ₹6,880 (C) ₹4,980 (D) ₹4,000

**31.** A person invested a certain sum for five years at the rate of $r$% per annum simple interest. If he had invested the same sum at the same rate, compounded half yearly for one year, he would have received the same amount. Find the value of $r$ _____.

**32.** Anand and Batra borrowed sums of money from Keertilal at different rates of interest. Anand borrowed the money at simple interest and Batra at compound interest, compounded annually. If Anand cleared the loan at the end of five years by paying double the money he borrowed and Batra cleared the loan at the end of four years by paying double the money he borrowed, then what is the rate of interest at which Batra borrowed?
(Given antilog of 0.07525 = 1.19)
(A) 16% p.a. (B) 15% p.a.
(C) 18% p.a. (D) None of these

**33.** A person invested ₹20,000 in a bank which is offering 10% per annum simple interest. After two years, he withdrew the money from the bank and deposited the entire amount in another bank which offers an interest rate of $r$% p.a. compounded annually. After 2 years he received an amount of ₹2,460 more than what he had invested in that bank. Find the value of $r$ _____.

**34.** Given, the difference between simple interest for 2 years and compound interest for 2 years on the same sum and at the same rate of interest compounded annually is ₹120. The difference between simple interest for 3 years and compound interest for 3 years on the same sum and at the same rate of interest is ₹366. Find the rate of interest. % p.a. _____.

**35.** A person started a business with a capital of ₹80,000. In the first year, he made 20% profit and added it to the capital. In the second year, he made 15% profit and again added it to the capital. In the third year, he incurred 10% loss and he set off the loss from the capital. Had he invested the same capital in a bank which pays 10% p.a. compound interest, instead of in the business, how much more would he have received?
(A) ₹8,240 (B) ₹7,120 (C) ₹6,320 (D) ₹9,560

**36.** Mukesh took a loan at 10% p.a. compound interest. He cleared the loan in 5 years by paying an annual instalment of ₹1,61,051 at the end of each year. Find the amount of loan he took (in rupees) _____.

**37.** A man borrowed ₹30,000 from a bank at 10% p.a. compound interest, interest being compounded annually. At the end of every year he paid ₹9,000. At the end of the third year, he wanted to clear the loan. How much should he pay to clear the loan?
(A) ₹19,000 (B) ₹19,140
(C) ₹18,180 (C) ₹18,560

**38.** The rate of interest on a sum of money lent at simple interest, (in % per annum) is $\frac{5}{2}$ times the period (in years) for which it is lent. If the simple interest for the given period on the sum lent is 10% of the sum, find the period (in years) for which the sum is lent _____.

**39.** A person deposited a sum of ₹10,000 in a bank for a period of $n_1$ years at a rate of 20% p.a. compounded annually. The same person deposited a sum of ₹11,520 in another bank for a period of $n_2$ years at a rate of 25% p.a. simple interest. The amounts received from the two banks are equal and the total amount is ₹34,560. Find $n_1$ and $n_2$.
(A) $n_1 = 3, n_2 = 2$ (B) $n_1 = 2, n_2 = 2$
(C) $n_1 = 1, n_2 = 3$ (D) Cannot be determined

**40.** Raju borrowed ₹15,000 from Mahesh at the rate of 15% p.a. at simple interest for 3 years. Raju lent some part of money borrowed at 20% p.a. at simple interest for 3 years and the remaining part at 12% p.a. at simple interest for 3 years. If the interest received by Raju on the money, he lent is equal to the interest payable by him, then find the sum he lent at 20% p.a. simple interest (in ₹) _____.

## EXERCISE-4

***Directions for questions 1 to 20*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** A man bought a VCD in an instalment scheme. The rate of interest charged in the instalment scheme was 15% p.a., interest being compounded annually. He paid ₹800 as down payment and the remaining amount was paid in two instalments. The first instalment was ₹1,000 and the second was ₹966. The first and the second instalments were paid at the end of the first and the second years respectively. Find the cost (in ₹) of the VCD _____.

**2.** *P* borrowed a certain sum from a bank at compound interest, compounded annually. *Q* borrowed an equal sum from another bank under simple interest. The interest on the sum borrowed by *P* for the third year is equal to

the interest on the sum borrowed by *Q* for the third year. If *P* and *Q* repaid their loans at the end of 5 years, the interest paid by *P* is
(A) less than the interest paid by *Q*.
(B) equal to the interest paid by *Q*.
(C) more than the interest paid by *Q*.
(D) either (A) or (B).

**3.** Madhavi deposited a certain sum at a certain rate of interest, interest being compounded every quarter. If the interest for the 5th and 6th quarters is ₹2,100 and ₹2,200 respectively, what is the approximate total interest for the first and the second quarters?
(A) ₹3,970 (B) ₹3,770 (C) ₹3,570 (D) ₹3,270

**4.** Vijay lent ₹$P_1$, ₹$P_2$ and ₹$P_3$ to three persons X, Y and Z respectively for two years. He lent his money to X and Y at the rate of 10% p.a. and 15% p.a. simple interest and to Z at the rate of 10% p.a. compound interest, interest being compounded annually. If the total interest at the end of two years from all the three persons is ₹2,415 and the sum of 20% of $P_1$ and 30% of $P_2$ is 1,312.50, find the value of $P_3$ (in ₹).
(A) 5,250 (B) 5,600 (C) 5,450 (D) 5,750

**5.** A man invests ₹35,000 at the beginning of each of 6 consecutive years in a bank at 10% p.a. compound interest, interest being compounded annually. The total value of the man's investment at the end of 6 years is approximately (assume $1.1^6 = 1.772$)
(A) ₹2,95,440 (B) ₹2,97,220
(C) ₹2,84,680 (D) ₹3,06,420

**6.** Geet borrows ₹1,00,000 from a bank at 10% p.a. rate of compound interest and clears the debt in five annual instalments. If the instalments paid at the end of the second, third, fourth and fifth years are ₹20,000, ₹18,800, ₹21,600 and ₹44,000 respectively, what was the amount paid at the end of the first year?
(A) ₹20,000 (B) ₹30,000
(C) ₹36,000 (D) ₹48,000

**7.** A sum was split into two equal parts. One of the parts was lent at *R*% p.a. simple interest and the other part was lent at *R*% p.a. compound interest. Both parts were lent for 3 years. The difference of the interest accrued on the two parts at the end of two years and at the end of three years are in the ratio 10 : 31. Find *R*.
(A) 5 (B) 8 (C) 10 (D) 2

**8.** Mohan borrowed ₹16,000 from a bank at simple interest at a certain rate of interest. After four years, he borrowed ₹28,880 from the same bank. Starting from the fifth year, the bank charged compound interest on the total amount repayable on the two loans at 10% p.a. At the end of six years, Mohan repaid a total of ₹60,500 and thereby cleared both loans. Find the rate of interest on the first loan for the first four years (in % p.a.) _____.

**9.** Mr. X invested a certain amount for 15 years in a term deposit scheme. An interest of ₹1,250 was accrued for the 5th year whereas ₹5,000 was accrued for the 11th year. If the interest is compounded annually, then what would be the maturity value of a sum of ₹10,000, invested today, at the end of 15 years?
(A) ₹32,000 (B) ₹16,000
(C) ₹21,500 (D) None of these

**10.** A person borrowed a sum of ₹6,000 at 10% p.a., interest compounded annually. If the money is to be repaid in three equal annual instalments, each payable at the end of the year, then what is the value of each instalment?
(A) ₹2,000 (B) ₹2,413 (C) ₹2,314 (D) ₹2,662

**11.** Rekha borrows a sum of ₹3,00,000 at $16^2/_3$% p.a. rate of compound interest for 3 years. She repays two instalments of ₹1,40,000 each at the end of the first and second year. What amount does she repay at the end of the third year to clear the debt?
(A) ₹1,30,000 (B) ₹1,05,000
(C) ₹1,22,500 (D) ₹1,40,000

**12.** A certain amount of money lent at compound interest becomes 2.25 times itself in 3 years. If the rate of interests for the first, second and the third years are in the ratio 4 : 5 : 10, find the rate of interest for the third year given that the interest is compounded annually?
(A) 30% (B) 45% (C) 50% (D) 55%

**13.** Arjun borrowed some money from Bharath at 8% p.a. annum simple interest and twice that amount from Charan at 9% p.a. simple interest. He then added some more money of his own and lent the entire amount to Dheeraj at 10% p.a. simple interest. At the end of the year Arjun gained ₹1,400. If Arjun lent ₹40,000 to Dheeraj, then how much did Arjun borrow from Bharath?
(A) ₹25,000 (B) ₹20,000
(C) ₹15,000 (D) ₹10,000

**14.** A few years back Bart borrows a certain sum of money at a certain rate of simple interest. Now the same sum becomes 200% of what he borrowed. After 2 years, the amount will increase by $12 \times 5$% compared to the present value. After how many years from now will the amount be 5 times the sum?
(A) 32 (B) 24 (C) 25 (D) 40

**15.** Arun borrowed ₹2 lakhs from a bank for 2 years at 10% p.a. compound interest. He split this amount into four equal parts. He lent each part to a different person for 2 years. From all the persons, he charged 12% p.a. compound interest, interest being compounded every 8 months. Approximately, what is the profit that Arun would make?
(A) ₹8,000 (B) ₹9,000 (C) ₹10,000 (D) ₹11,000

**16.** Mukesh took a loan at 10% compound interest. He cleared the loan in 5 years by paying an annual instalment of ₹1,61,051 at the end of each year. Find the amount of loan he took (in rupees) _____.

**17.** Naresh and Prabhat borrowed ₹50,000 and ₹70,000 respectively from a co-operative bank at compound interest. While Naresh was charged 6% p.a. and 8% p.a. for the first and second years respectively, Prabhat was charged 5% and 9% p.a. for the first and second years respectively. Prabhat repaid ₹35,000 at the end of the first year, while Naresh did not make any such partial repayment. Both cleared their loans at the end of the second year. How much more or less interest did Prabhat pay, compared to Naresh?
(A) ₹325 more (B) ₹325 less
(C) ₹275 more (D) ₹275 less

**18.** A sum doubles in six years at a certain rate of compound interest in one scheme. In another scheme, the same sum doubles in six years at a different rate of simple interest. In either scheme, a customer has the choice of switching after a minimum of two years, in which case he would be given interest for the period for which the amount is invested. If every three years, a customer switches between these two schemes, starting with the simple interest scheme, in how many years, will the sum amount to 4.5 times its value? _____

**19.** If the interest compounded annually on a certain sum at a certain rate of interest for 2 years is equal to 55% of the simple interest on the same amount at the same interest rate for twice the time period, find the rate percentage.
(A) 12% (B) 20% (C) 24% (D) 18%

**20.** A man can purchase a flat from a co-operative credit society for ₹2,75,000 cash or ₹1,10,000 as down payment and the remaining amount along with the interest in two equal annual instalments. If the society charges interest at 20% p.a. compounded annually, and the man purchased a flat on instalment basis, how much more did he pay as compared to an outright purchase?
(A) ₹55,000 (B) ₹51,000
(C) ₹59,000 (D) ₹61,000

## EXERCISE-5 Based on Data Sufficiency

***Directions for questions 1 to 20:*** Each question is followed by two statements, I and II. Answer each question based on the following directions:

Choose (A) If the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.

Choose (B) If the question can be answered by using either statement alone.

Choose (C) If the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (D) If the question cannot be answered even by using both the statements together.

**1.** What is the compound interest on a certain sum for 2 years, compounded every 3 months?

I. The simple interest on the same sum for 2 years is ₹1,000 at the same rate of interest.

II. The rate of interest is 5% p.a.

**2.** In how many years will a certain amount $A$ increase by ₹205 at $x$% p.a. compound interest?

I. $x = 5$.

II. $A$ is ₹205.

**3.** Which scheme, between $P$ and $Q$, doubles the investment faster?

I. Scheme $P$ - 16% per annum with interest compounded quarterly.

II. Scheme $Q$ - 18% per annum with interest compounded half-yearly.

**4.** Find the difference between the compound interest on a sum of money for the fourth year and that for the sixth year.

I. The compound interest for the fourth year is ₹400 and that for the fifth year is ₹480.

II. The amount earned on the sum at the end of the first three years is ₹13,310.

**5.** Part of a sum was lent at 6% p.a. and the remaining was lent at 4% p.a. Find the amount lent at 6% p.a.

I. Total sum lent was ₹7,000.

II. Total simple interest was ₹1,600.

**6.** What is the difference between compound interest and simple interest at the end of two years?

I. The difference between compound interest and simple interest for 2 years on double the principal and at half the rate of interest would be ₹400.

II. The principal for the second year in compound interest is 120% of the principal for the second year in simple interest.

**7.** At what rate of interest was ₹$P$ invested?

I. The simple interest on ₹$P$ for four years was ₹800.

II. The difference between the simple interest and the compound interest on ₹$P$ for two years was ₹80, interest being compounded annually.

**8.** A scooter is purchased by making a down payment plus five equal instalments. What is the total amount paid by the customer for the scooter?

I. The down payment is 40% of the total amount paid by the customer.

II. Each instalment is ₹1,272.

**9.** What is the principal deposited by Krishna, INA bank?

I. The interest earned on the fixed deposit for 2 years is ₹4,830

II. The bank offers 10% interest compounded annually on fixed deposits.

**10.** Sohini invests in three schemes which offer her simple interests of 8% p.a., 10% p.a. and 20% p.a., respectively. She invested in each scheme for one year. What is the sum that Sohini invested?

I. The interest she earned on each of the schemes is the same.

II. The total interest she earns is ₹1,200.

**11.** What is the compound interest earned on ₹20,000 for 2 years?

I. If the rate of interest had been 3 percentage points more, an additional interest of ₹1,302 would have accrued for the two years.

II. ₹98 is the difference between the simple interest and the compound interest at the same rate for two years.

**12.** A person gets an interest of ₹30 more for two years by lending an amount at compound interest rather than at simple interest when the interest is compounded, its compounded at a certain frequency. What is the rate of interest per annum?

I. The amount lent is ₹12,000.

II. The interest is compounded annually

**13.** What is the rate of interest per annum?

I. The interest is ₹500 for two years.

II. At twice the rate of interest, the interest for four years is ₹2,000.

**14.** What is the rate of simple interest on the deposit?

I. The interest earned on the deposit for 8 years is ₹3,600.

II. The amount at the end of 8 years is double the principal.

**15.** A man invests a total of ₹$x$ on the names of his three sons A, B and C in such a way that they get the same amount after 2, 3 and 4 years respectively. What is the ratio of amounts invested on A, B and C?

I. $x = 15{,}860$.

II. The man invested at the rate of 5%. p.a. simple interest.

**16.** What is the rate of compound interest per annum?

I. ₹100 becomes ₹125 in three years compounded annually.

II. ₹$x$ becomes ₹$1.5x$ in 3 years compounded annually.

**17.** What is the total compound interest accrued on a sum of money after five years?

I. Rate of interest is 6% p.a.

II. The total simple interest on the same amount after 5 years at the same rate will be ₹600.

**18.** A certain sum was deposited in a bank at a simple interest. What is the period for which it was deposited?

I. The numerical value of $r$ and $n$ is same.

II. The interest accrued is $^1/_4{}^{th}$ the original amount.

**19.** Did Nakul earn an interest which is equal to the principal?

I. Nakul deposited 20,000 in a bank at 5% interest.

II. The amount was deposited for 20 years.

**20.** A finance company offers only 10% or 11% interest and either SI or CI on all deposits. Is the interest simple or compounded?

I. The interest collected for the second year on ₹10,000 is ₹1,100.

II. Total interest on ₹15,000 is ₹3,150 after 2 years.

## ANSWER KEYS

### Exercise-1

1. 28,800
2. (A)
3. 2,904
4. 6,400
5. 9
6. (A)
7. 16
8. 3,120
9. 9,261
10. (D)
11. 12,600
12. 10.25
13. (B)
14. 5,805
15. (A)
16. (C)
17. 6,364
18. (A)
19. 7,000
20. (D)
21. 84
22. 28,200
23. 6,000
24. 120
25. (B)
26. (B)
27. 3,800
28. (C)
29. (D)
30. (B)
31. (A)
32. 4,000
33. (C)
34. (B)
35. (D)
36. (B)
37. (A)
38. (C)
39. 9,000
40. 21

### Exercise-2

1. 1,800
2. (B)
3. 3,049.2
4. (D)
5. 2
6. (A)
7. (B)
8. (B)
9. 1,260
10. 15,360
11. 36,000
12. (A)
13. 13,000
14. (C)
15. 12,200
16. 1,76,800
17. (D)
18. 10
19. 4,000
20. (C)
21. 880
22. 928.20
23. (i) 1,017
    (ii) 12
24. (C)
25. 19.1016
26. 3
27. 5
28. 7,200
29. (D)
30. 11,500
31. 1,464.1
32. 1,210
33. 1,280
34. 81,500
35. (B)
36. 16
37. (B)
38. 46,125
39. (B)
40. 2,000

### Exercise-3

1. 13,000
2. 1,200
3. 7,260
4. (A)
5. 2,80,000
6. 4,000
7. (B)
8. (D)
9. 15,000
10. 13,354
11. 6,620
12. 10.34
13. (B)
14. 20,000 & 10
15. 6,600
16. (D)
17. (D)
18. (B)
19. (C)
20. (C)
21. 320
22. 798.60
23. (C)
24. (C)
25. 8
26. (A)
27. (C)
28. (B)
29. (B)
30. (A)
31. 1,600
32. (D)
33. 5
34. 5
35. (B)
36. 6, 10, 510
37. (B)
38. 2
39. (A)
40. 5,625

### Exercise-4

1. 2,400
2. (C)
3. (C)
4. (A)
5. (B)
6. (B)
7. (C)
8. 8
9. (A)
10. (B)
11. (C)
12. (C)
13. (D)
14. (B)
15. (C)
16. 6, 10, 510
17. (D)
18. 12
19. (B)
20. (B)

### Exercise-5

1. (C)
2. (C)
3. (C)
4. (A)
5. (D)
6. (A)
7. (C)
8. (C)
9. (C)
10. (C)
11. (B)
12. (C)
13. (D)
14. (A)
15. (A)
16. (B)
17. (C)
18. (C)
19. (D)
20. (A)

## SOLUTIONS

## EXERCISE-1

**1.** Simple interest $= \frac{18000 \times 4 \times 15}{100} =$ ₹10,800

Amount $= P + I = 18{,}000 + 10{,}800 =$ ₹28,800

**2.** Under compound interest, at the end of each year the principal amount increases.

$\therefore$ The interest in $n^{th}$ year will be greater than that in $(n-1)^{th}$ year.

Also this difference increases as the value of $n$ increases.

$\therefore$ Only when $p > q$, the given situation arises.

**3.** Here, $P = 2400$

$R = 10\%$, and $n = 2$

$$A = P\left(1+\frac{R}{100}\right)^n = 2400\left(1+\frac{10}{100}\right)^2 = ₹2{,}904$$

**4.** Amount = Principal + Interest

Principal = 4000 and Amount = 4560

$\therefore$ Interest = 560

$$560 = \frac{4000 \times 2 \times r}{100} \Rightarrow r = 7\%$$

$\therefore$ For $P = 5000$ and $n = 4$

$$I = \frac{5000 \times 4 \times 7}{100} = 1400$$

$\therefore$ ₹5,000 becomes ₹6,400 at the end of 4 years.

**5.** Let the principal be ₹$x$. Then, amount = ₹$2x$

So, $2x = x\left(1+\frac{R}{100}\right)^3 \Rightarrow \left(1+\frac{R}{100}\right)^3 = 2$

If the sum becomes 8 times, therefore amount = ₹$8x$

Let it becomes 8 times itself in $n$ years.

$$\therefore\ 8x = x\left(1+\frac{R}{100}\right)^2$$

$$\Rightarrow \left(1+\frac{R}{100}\right)^n = 8 \Rightarrow \left(1+\frac{R}{100}\right)^n = 2^3$$

$$\Rightarrow \left(1+\frac{R}{100}\right)^n \Rightarrow \left(1+\frac{R}{100}\right)^9$$

$$\Rightarrow \left(1+\frac{R}{100}\right)^n = \left(1+\frac{R}{100}\right)^9$$

$\therefore$ $n = 9$ years

**6.** Let the sum be ₹$x$.

Then, the amount after three years = ₹$2.197 \times x$

$$\Rightarrow 2.197 \times x = x\left(\frac{100+r}{100}\right)^3$$

$$\Rightarrow (1.3)^3 = \left(\frac{100+r}{100}\right)^3$$

$$\Rightarrow 1.3 = \frac{100+r}{100}$$

$$\Rightarrow 130 = 100 + r \Rightarrow r = 30\%$$

**7.** For the sum to become 4 times the original sum. it can be considered as the sum getting doubled and then getting doubled again. As the sum takes 8 years to get doubled, the time taken for it to become 4 times = 2 (8) = 16 years.

**8.** Let $A$ be the amount received at the end of three years.

$$A = 4800\left(1+\frac{10}{100}\right)\left(2+\frac{20}{100}\right)\left(1+\frac{25}{100}\right)$$

$$A = \frac{4800 \times 11 \times 6 \times 5}{10 \times 5 \times 4}$$

$\therefore$ $A =$ ₹7920

So, interest = 7920 – 4800 = ₹3,120

**9.** Amount $= P(1 + r/100)^n = 8000\ (1 + 0.05)^3$

$= 8000\ (1.157625) =$ ₹9,261.

**10.** Let the rate of interest be $R\%$ p.a.

Interest = 2500 – 2000 = ₹500

$$500 = 2000\ (2)\ \left(\frac{R}{100}\right)$$

$$R = 12.5$$

**11.** Let the principal be ₹$x$.

Given: $x\left(1+\frac{20}{100}\right)^2 - x = 5544$

$$\frac{36x}{25} - x = 5544\ ,\ \frac{11x}{25} = 5544$$

$\Rightarrow$ $x =$ ₹12,600.

**12.** Effective rate of interest = 5% for half a year

$$= \left[\left(1+\frac{5}{100}\right)^2 - 1\right]100 = 10.25\%\ \text{p.a.}$$

**13.** Interest for first six months $= \dfrac{200 \times \frac{1}{2} \times 10}{100} = ₹10$

Principal for next six months = 200 + 10 = ₹210

Interest for next six months $= \dfrac{200 \times \frac{1}{2} \times 10}{100} = ₹10.5$

$\therefore$ Total interest = 10 + 10.5 = ₹20.5

**14.** $A = 18000\left(1+\dfrac{15}{100}\right)^2$

$= 18000\left(1+\dfrac{3}{20}\right)^2$

$= \dfrac{18000 \times 23 \times 23}{20 \times 20} = ₹23,805$

$\therefore$ Compound interest = 23805 – 18000 = ₹5,805

**15.** The first ₹15,000 would become 15,000 $(1.1)^6$ after 6 years, the second will become $15,000(1.1)^5$, the third 15,000 $(1.1)^4$, the fourth 15,000 $(1.1)^3$, the fifth 15,000 $(1.1)^2$ and the sixth 15,000 (1.1)

Total amount = 15,000 $[(1.1) + (1.1)^2 + (1.1)^3 + (1.1)^4 + (1.1)^5 + (1.1)^6]$ = (15,000) (1.1) $[1 + (1.1) + (1.1)^2 + (1.1)^3 + (1.1)^4 + (1.1)^5] = (16,500)\dfrac{(1.1)^6 - 1}{1.1 - 1} = ₹1,27,380$

**16.** Let the principal be ₹*P*.

Amount $= \dfrac{7P}{5}$

$\therefore$ Interest $= \dfrac{7P}{5} - P = \dfrac{2P}{5}$

$\therefore\ r = \dfrac{100 \times 2P/5}{P \times 3}$

$\Rightarrow r = \dfrac{100 \times 2}{5 \times 3} = 13\frac{1}{3}\%$p.a.

**17.** Interest = 5476 – 3700 = ₹1,776

So, $r = \dfrac{100 \times 1776}{3700 \times 4} = 12\%$p.a

Now, the rate of interest is (12 + 6)% p.a.

So, interest $= \dfrac{3700 \times 4 \times 18}{100} = ₹2,664$

Amount = 3700 + 2664 = ₹6,364

**18.** The interests accrued under compound interest are in the geometric progression of common ratio $= \left(1+\dfrac{r}{100}\right)$.

Hence, $\left(1+\dfrac{r}{100}\right) = \dfrac{1408}{1280} = 1.1$

$\Rightarrow r = 10\%$

**Alternative Solution:**

The difference in the interests accrued in the fifth year and the sixth year is the interest on the interest accrued in the fifth year.

$\therefore$ 1,408 – 1,280 = ₹128 is interest accrued on ₹1,280

$\therefore$ Rate of interest $= \dfrac{128}{1280} \times 100 = 10\%$

**19.** Let the sum be ₹$x$

$\dfrac{x \times 18 \times 2}{100} - \dfrac{x \times 12 \times 2}{100} = 840$

$\Rightarrow \dfrac{36x}{100} - \dfrac{24x}{100} = 840$

$\Rightarrow \dfrac{12x}{100} = 840 \Rightarrow x = 840 \times \dfrac{100}{12} = 7000$

**20.** Let the sum be ₹$x$

$\dfrac{x \times 4 \times 4}{100} + \dfrac{x \times 8 \times 3}{100} + \dfrac{x \times 5 \times 2}{100} = 8500$

$\Rightarrow x \times \dfrac{50}{100} = 8500 \Rightarrow x = ₹17,000$

**21.** Half yearly rate of interest $= \dfrac{20\%}{2} = 10\%$

Interest earned in the first year = total interest earned in two half years

$= 400\left[\left(1+\dfrac{20}{2}\right)^2 - 1\right] = ₹84$

**22.** Let sum be ₹$x$

$8178 = \dfrac{x \times 1 \times 7}{100} + \dfrac{x \times 1 \times 10}{100} + \dfrac{x \times 1 \times 12}{100}$

$8178 = \dfrac{7x \times 10x \times 12x}{100}$

$8178 = \dfrac{29x}{100}$

$x = \dfrac{8178 \times 100}{29} = ₹28,200$

**23.** Amount = $P(1 + nr/100)$

$= P(1 + 4 \times 12/100) \Rightarrow 8880 = P(1.48)$

$\Rightarrow P = \dfrac{8880}{1.48} = 6000$

So, the principal is ₹6,000.

**24.** When compounded annually, interest

$$= 12000\left(1+\frac{20}{100}\right)^1 - 12000 = ₹2{,}400$$

When compounded semi-annually, interest

$$= 12000\left(1+\frac{10}{100}\right)^1 - 12000 = ₹2{,}520$$

Required difference = 2520 – 2400 = ₹120

**25.** Let the sum be ₹100

Amount after eight years = ₹400

Interest = ₹300

17.5 times of sum = ₹1,750

Interest = ₹1,650

| Interest (₹) | Number of years |
|---|---|
| 300 | 8 |
| 1650 | ? |

$$? = \frac{1650\times 8}{300} = 44$$

**26.** Interest $= \dfrac{88000\times 100\times 45}{11\times 100\times 12} = ₹30{,}000$

**27.** Let the sum be $P$.

$$P = \frac{100\times 1368}{3\times 12} = ₹3{,}800$$

**28.** As simple interest for two years is ₹800, simple interest for each year is ₹400.

So, compound interest for first year is ₹400 and for second year is 960 – 400 = ₹560

So, ₹560 – 400 = ₹160 is the interest on ₹400 for 1st year at the same rate of interest

$$\therefore \text{Rate of interest} = \frac{100\times 160}{400\times 1} = 40\% \text{ p.a.}$$

$$\text{The sum} = \frac{800\times 100}{40\times 2} = ₹1{,}000.$$

**29.** Let the principle be ₹$P$

$$24200 = P\left(1+\frac{r}{100}\right)^2 \qquad (1)$$

$$29282 = P\left(1+\frac{r}{100}\right)^4 \qquad (2)$$

Dividing equation (2) by equation (1), we get

$$\left(1+\frac{r}{100}\right)^2 = \frac{121}{100}$$

$$\left(1+\frac{r}{100}\right)^2 = \left(\frac{11}{10}\right)^2 \Rightarrow 1+\frac{r}{100} = \frac{11}{10}$$

$$\Rightarrow \frac{r}{100} = \frac{1}{10} \Rightarrow r = 10\% \text{ p.a.}$$

**30.** ₹1,440 – 1,200 = ₹240 is the interest on ₹1,200 for 1 year.

$$\text{Rate of interest} = \frac{100\times 240}{1200\times 1} = 20\% \text{ p.a.}$$

If the sum is ₹100, then interest for second year is ₹24.

Given: ₹24 → 1200,

100 → ?

$$? = \frac{100\times 1200}{24} = ₹5{,}000$$

**31.** Let the principal be ₹$x$.

As amount = $4x$, interest is $4x - x = 3x$

$$T = \frac{100\times 3x}{x\times 25} = 12 \text{ years}$$

**32.** Simple interest $= (20000)(2)\left(\dfrac{10}{100}\right) = ₹4{,}000$

**33.** Let the principal be ₹$x$

$$\text{Amount} = x\left(1+\frac{20}{100}\right)^2$$

Amount = ₹1.44$x$

So, interest = $1.44x - x = 0.44x$

$$\text{Rate of interest} = \frac{0.44x\times 100\%}{x} = 44\%$$

**34.** Let the rate of interest be $R$% p.a.

$$2000\left(1+\frac{R}{100}\right)^2 = 2{,}880$$

$$\left(1+\frac{R}{100}\right)^2 = 1.44 = (1.2)^2$$

$$1+\frac{R}{100} = 1.2$$

$$\Rightarrow R = 20$$

**35.** $45S - 45C = 15S$

$30S = 45C$

$S = \dfrac{45}{30}C = C + \dfrac{C}{2}$, i.e., 50% gain

**36.** Let interest for 1 year be $x$.

As Amount = Principal + Interest, we have

$$P + 4x = 20720 \qquad (1)$$

$$P + 6x = 24080 \qquad (2)$$

(where $P$ is the principal).

Solving the equations (1) and (2), we can get

$\therefore P = ₹14{,}000$ and $x = ₹1{,}680$

Interest for 1 year on ₹14,000 is ₹1,680

So, $R = \dfrac{100 \times 1680}{14000 \times 1} = 12\%$ p.a.

**37.** When interest is compounded many times a year, the equivalent rate of interest that will fetch the same amount of interest, if the interest was calculated annually, is called effective rate of interest.

The effective rate of interest is constant for every year.

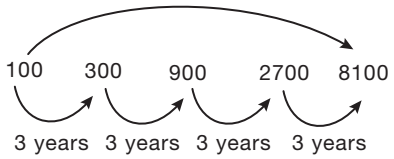
**38.** Let the sum be ₹100

Amount after three years at C.I. = ₹300

Amount after six years at C.I. = ₹900

Amount after nine years at C.I. = ₹2,700

Amount after twelve years at C.I. = ₹8,100




$\therefore$ Time period is twelve years

**39.** A sum lent at simple interest has the same interest on it each year. Sum of the interests accrued on it in the $6^{th}$, $7^{th}$ and $8^{th}$ years $= 3(3000) = ₹9{,}000$

**40.** Let the sum be ₹$x$, then it becomes ₹$3x$ in 6 years

So, ₹$2x$ is the interest on $x$ for 6 years.

So, $R = \dfrac{100 \times 2x}{x \times 6} = \dfrac{100}{3}\%$

If the sum becomes 8 times itself, then interest is $7x$.

The required time period $= \dfrac{100 \times 7x}{x \times 100/3}$

$= \dfrac{100 \times 7x \times 3}{x \times 100} = 21$ years

## EXERCISE-2

**1.** Rate of interest $= \dfrac{(2178 - 1980) \times 100}{1980} = 10\%$

Let the interest earned in the first year be $x$.

$1.1x = 1980 \Rightarrow x = 1800$

**2.** Let the sum be ₹$x$

Amount after $x$ years = ₹$15x$

Simple interest = ₹$14x$

$\therefore 14x = \dfrac{x(21)n}{100}$ (where the time is $n$ years)

$\Rightarrow n = 66\dfrac{2}{3}$

$\therefore$ The period is 66 years 8 months.

**3.** In two years the interest is compounded 4 times when interest is compounded half yearly and the rate of interest also reduced to half.

$\therefore$ Compound interest for 2 years

$= 12000\left(1 + \dfrac{10}{100}\right)^4 - 12000$

$= 12000((1.1)^4 - 1) = 12000\ ((1.21)^2 - 1^2)$

$= 12000 \times 2.21 \times 0.21$

$= 120 \times 21 \times 2.21 = ₹5{,}569.20.$

Also, compound interest for one year

$= 12000\left(1 + \dfrac{10}{100}\right)^2 - 12000 = ₹2{,}520.$

$\therefore$ Interest for $2^{nd}$ year $= 5569.20 - 2520$

$= ₹3{,}049.20$

**4.** Let the loan amount be ₹100

$\therefore$ Amount to be repaid $= 100 + \dfrac{100 \times 8 \times 5}{100} = ₹140$

If the amount to be repaid is ₹140, then the loan amount is ₹100

$\therefore$ When the amount to be repaid is ₹10,640, the loan amount will be $10{,}640 \times \dfrac{100}{140} = ₹7{,}600$

**Alternative Method:**

$P\left(1 + \dfrac{8 \times 5}{100}\right) = 10{,}640$

$P = 10640 \times \dfrac{5}{7} = 7600$

**5.** Let the time period be $T$ years

$18000\left(\left(1 + \dfrac{4}{100}\right)^T - 1\right) = 1468.8$

$\Rightarrow 18000\left(1 + \dfrac{4}{100}\right)^T = 19468.8$

$\Rightarrow (1.04)\,T = 1.0816 = (1 + 0.04)^2$, i.e., $(1.04)\,T = (1.04)^2$

Equating the powers of $T$ both sides, $T = 2$.

**6.** At the end of the first year the loan amounts to ₹88,000 at 10% p.a.

Let $x$ be the amount repaid at the end of the first year.

Then, $(88000 - x)\,(1.1) = 55000$

$\Rightarrow 1.1x = 41800$

$\Rightarrow x = 38{,}000$

$\therefore$ ₹38,000 was repaid at the end of the first year.

**7.** Let the present worth be ₹$x$.

$x(1 + 10/100)^3 = 5324$

$x = \dfrac{5{,}324}{1{,}331} = 4000$

**8.** A person has invested half of the money he has at C.I. at 10% p.a. and the other half at S.I. at a rate of 20% p.a. for 2 years.

$\Rightarrow \dfrac{P}{2}\,[1.1]^2 + \dfrac{P}{2}\,[1.4] = 2610$

$\Rightarrow \dfrac{P}{2}\,[1.21] + \dfrac{P}{2}\,[1.4] = 2610$

$\Rightarrow \dfrac{P}{2}\,[1.21 + 1.4] = 2610$

$\Rightarrow P\,[2.61] = 5220 \Rightarrow P = \dfrac{5220}{2.61} = 2000$

So, at the end of 3rd year he will receive

$= \dfrac{2000}{2}\,(1.1)^3 + \dfrac{2000}{2}\,(1.6)$

$= 1000 \times 1.331 + 1000 \times 1.6 =$ ₹2,931

**9.** If the interest increased from $r$ to $3r$, the interest earned in the first year would be 3 times the present interest, which then would have earned interest at the rate of $3r$, implying that the difference would become $3 \times 3$, i.e., 9 times the previous difference.

$\therefore$ New difference $= 9 \times 140 =$ ₹1,260

**10.** Let the amounts lent to the first person and the second person be '$a$' and '$b$' respectively.

Given that, $\dfrac{a \cdot 412}{100} = \dfrac{b \cdot 3(9)}{100}$

$\Rightarrow \dfrac{a}{b} = \dfrac{27}{48}$, i.e., $\dfrac{9}{16}$

The amount lent to the second person $= \dfrac{16}{9+16} \times$ ₹24,000

= ₹15,360.

**11.** Let the value of the device two years ago be $P$.

$32490 = P\left[1 - \dfrac{R}{100}\right]^2$

$32490 = P\left[1 - \dfrac{5}{100}\right]^2 \Rightarrow 32490 = P\left[\dfrac{20-1}{20}\right]^2$

$\Rightarrow 32490 = P\left(\dfrac{19}{20}\right)\left(\dfrac{19}{20}\right)$

$\Rightarrow P = 36000$

**12.** Let the first, second and the third sums be $P_1$, $P_2$, and $P_3$ respectively

Each sum amounted to the same value. Let us say each sum amounted to $A$

$P_1\left(1 + \dfrac{8}{100}\right) = P_2\left(1 + \dfrac{8}{100}\right)^2 = P_3\left(1 + \dfrac{8}{100}\right)^3 = A$

$P_1 = \dfrac{A}{1.08}, P_2 = \dfrac{A}{(1.08)^2}, P_1 = \dfrac{A}{(1.08)^3}$

$P_1 : P_2 : P_3 = \dfrac{27}{25} : \dfrac{25^2}{27^2} : \dfrac{25^3}{27^3} = 1 : \dfrac{25}{27} : \dfrac{25^2}{27^2}$

$= 729 : 675 : 625.$

**13.** Let the sum be ₹ $P$.

Let the rate of interest be $R$% p.a.

$P\left[1 + \dfrac{R}{100}\right]^4 = 15600$ (1)

and $P\left[1 + \dfrac{R}{100}\right]^8 = 18720$ (2)

$(2)/(1) \Rightarrow \left[1 + \dfrac{R}{100}\right]^4 = 1.2$

$\therefore\ P = \dfrac{15600}{1.2} = 13{,}000$

**14.** Let the present worth be ₹$x$

$x\,(1 + 8/100)^2 = 1749.6$

$\Rightarrow x = \dfrac{1749.6}{(1.08)^2} = 1{,}500$

**15.** Let the three parts be $100x$, $100y$ and $100z$.

Given:

$160x = 6400;\ 200y = 6400;\ 128z = 6400$

$x = 40;\ y = 32;\ z = 50$

$\therefore\ p = 100(x + y + z)$, i.e., 12,200.

**16.** Let ₹$x$ be the savings

$\frac{(1/2)}{100}x = 884$

$\Rightarrow x = (884)\frac{100}{1/2} = 1{,}76{,}800.$

**17.** Actual gain of Giri = Amount repaid by Karthik – Amount repaid by Giri $= 36000\left(1+\frac{20}{100}\right)^2 - 36000\left(1+\frac{2(10)}{100}\right)$

$= 36000\ (1.44 - 1.2)$

Had Karthik repaid at the end of three years the gain of Giri would have been $360000\left(1+\frac{20}{100}\right)^3 - 36000\left(1+\frac{3(10)}{100}\right)$

$= 36000\ (1.728 - 1.3)$

Extra gain of Giri = 36000 (1.728 – 1.3) – 36000 (1.44 – 1.2)

= 6,768.

**18.** Let the compound interest for the third year and fourth year be denoted by $I_3$ and $I_4$ respectively and the rate of interest (expressed as a fraction) be $r$. (As a percentage, it would be $100r\%$)

$\therefore I_3 = 1815$

$I_4 = 1996.5$

As $I_4 = I_3\ (1 + r)$, it follows that $1 + r = \frac{I^4}{I^3} = \frac{1996.5}{1815} = 1.1$

$\therefore r = 0.1 = 10\%$

**19.** Let the sum be 100.

S.I for the two years = $2 \times 8 = 16$

C.I for the two years = $8 + 8 + \frac{8}{100} \times 8 = 16.64$

When the difference is 0.64, principal is ₹100

$\therefore$ When the difference is ₹25.6, the principal will be

$\frac{100}{0.64} \times 25.6 =$ ₹4,000

**20.** Principal = 1,000

Rate = 10% p.a.

Period, $N = 5$ years

Amount, $A = P + \frac{PNR}{100}$

Amount $= 1000 + \frac{1000 \times 5 \times 100}{100} =$ ₹1,500

**21.** Let the principal be $P$.

(a) $A = 2000\left(1+\frac{40}{2\times100}\right)^{2\times1}$

$A = 2880$

$\therefore$ Interest earned = $A - P = 2880 - 2000 = 880$

**22.** $A = 2000\left(1+\frac{40}{4\times100}\right)^{4\times1}$

A = 2928.20

$\therefore$ Interest earned = 2928.20 – 2000 = 928.20

**23.** (i) Since $x$ is proportional to square of $y$, their ratio is constant, i.e.,

$x_1/y^2_1 = x_2/y_2^2$. Substituting the given values, we get

$452/(12)^2 = x_2/(18)^2$

$x_2 = 452 \times 18 \times 18\ /\ (12 \times 12\ ) = 1{,}017$

(ii) Since $x$ is proportional to square of $y$, we have $x_1 y_1^{\ 2} = x_2\, y_2^{\ 2}$

$\Rightarrow 48 \times 4 = x_2 \times 16 \Rightarrow x_2 = 12.$

**24.** As it is S.I., one year interest = $2p - 3p/2 = p/2$

Interest for 20 years = $20 \times p/2 = 10p$

After 20 years, he will receive $10p + p = 11p$

**NOTE**

The data that ₹$p$ amounted to ₹$\frac{5}{2}p$ after 3 years is redundant.

**25.** $A = P\left(1+\frac{18}{3\times100}\right)^{3\times1} = 1.191016P$

Hence, $0.191016P$ is the interest earned on $P$.

$\therefore$ The effective rate of interest = 19.1016% p.a.

**26.** $20000\left(1+\frac{10\times n}{100}\right) - 18000\left(1+\frac{10}{100}\right)^n = 2042$

$20000\left(\frac{10+n}{10}\right) - 18000\ (1.1)^n = 2042$

From options, $n = 3$ satisfies the equation. So, $n = 3$

**27.** Difference between compound interest and simple interest on $P$ at $R\%$ p.a. for 2 years is $P\left(\frac{R}{100}\right)^2 = Pr^2$ where $r$ is the rate of interest, expressed as a fraction or $J_2 - I_2$

[where $J_n$ is the compound interest for $n$ years and $I_n$ is the simple interest for $n$ years $= 500\left(\frac{10}{100}\right)^2 = 5$].

**28.** Let the principal be 1 and the rate of interest (expressed as a fraction) be $r$. The compound interest for the first 4 years is $r$, $r(1+r)$, $r(1+r)^2$, $r(1+r)^3$, respectively.

$\therefore Pr(1+r)^2 = 5000$ (1)

and $Pr(1+r)^3 = 6250$ (2)

$(2)/(1)$: $1 + r = 1.25 \Rightarrow r = 0.25$

Substituting in (1), we get $P\left(\frac{1}{4}\right)\left(\frac{25}{16}\right) = 5000$

$P = 200(4)(16) = 12800$

The total interest for the first two years is $P[(1+r)^2 - 1]$

$= P(2r + r^2) = 12800\left(\frac{1}{2} + \frac{1}{16}\right) = 12800\left(\frac{9}{16}\right) = 7{,}200$

**29.** $P(1 + r/100)^3 = 1.728P$

$\Rightarrow (1 + r/100)^3 = (1.2)^3$

$\Rightarrow 1 + \frac{r}{100} = 1.2 \Rightarrow r = 20\%$

Let, the required time period be $t$ years

$\frac{P \times 20 \times t}{100} = 3P$ (since, interest $= 4P - P = 3P$)

$\Rightarrow t = \frac{300}{20} = 15$ years

**30.** Given:

$P = ₹18000$, $R = 15\%$

Amount after one year $= 18000\left(1 + \frac{15}{100}\right) = 20{,}700$

If Shyam pays ₹10,700, then the remaining amount is ₹10,000.

Now, $P = ₹10{,}000$, $R = 15\%$

$A = 10{,}000\left(1 + \frac{15}{100}\right) = 11{,}500$

$\therefore$ The amount he has to pay at the end of the 2nd year is ₹11,500.

**31.** As there are 4 compounding periods from end of 3rd year to end of 7th year, compound interest for 7th year

$= \left(1 + \frac{10}{100}\right)^4 (1000) = 1464.1$

**32.** Amar borrows ₹2,100 at the rate of 10% p.a. compounded annually and has to repay it in 2 equal instalments in 2 years:

| Borrowed amount | Interest due | Amount due | Instalment paid | Balance |
|---|---|---|---|---|
| 2100 | 210 | 2310 | $x$ | $(2310 - x)$ |
| $(2310 - x)$ | $\left(\frac{2310 - x}{10}\right)$ | $(2310 - x)\ \frac{11}{10}$ | $x$ | 0 |

$\therefore x = (2310 - x)\ \frac{11}{10}$

$\Rightarrow \frac{10x}{11} = 2310 - x$

$\Rightarrow \frac{21}{11}x = 2310$

$x = \frac{2310(11)}{21} = 110(11) = 1210$

**33.** Let the amount invested under simple interest be $X$ and the amount invested under compound interest be $Y$.

Then, $Y - X = 37$

$P\left[1 + \frac{6.25}{100}\right]^2 - P\left[1 + \frac{2(5)}{100}\right] = 37$

$\Rightarrow P\left[\left(\frac{17}{6}\right)^2 - \frac{11}{10}\right] = 37$

$P = 1280$

**Alternate Method:**

$5\% = \frac{5}{100} = \frac{1}{20}$ and $6.25\% = \frac{6.25}{100} = \frac{1}{16}$.

Since compounding is for 2 years, take a number which is divisible by $16^2$, say, 256. Since number should be divisible by 20 as well, assume principal as $256 \times 20 \times k$.

$\therefore P = 5120k$

| | Simple interest | Compound interest |
|---|---|---|
| 1st year | $256k$ | $320k$ |
| 2nd year | $256k$ | $340k\ \left(\because 320k + \frac{1}{16} \times 320k\right)$ |
| Total: | $512k$ | $660k$ |

$\therefore$ Difference between interests is

$660k - 512k = 148k = 37 \Rightarrow k = 1/4$

$\therefore P = 5120\left(\frac{1}{4}\right) = 1{,}280$

**34.** Total interest payable

$$= \frac{50000\times 8\times 2}{100} + \frac{50000\times 10\times 3}{100} + \frac{50000\times 5\times 2}{100}$$

$$+ \frac{50000\times 7\times 1}{100} = ₹31{,}500$$

$\therefore$ Amount payable = 50000 + 31500 = ₹81,500

'or'

The effective rate of interest becomes equivalent to $(8\times 2 + 10\times 3 + 5\times 2 + 7\times 1)$

i.e., 63% for 1 year

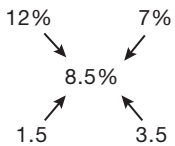
$\therefore$ Interest = $\frac{50000\times 63}{100} = 31{,}500$

$\therefore$ Amount = 50000 + 31500 = ₹81,500

**35.** Average rate of interest she paid

$$= \frac{6120\times 100}{72000\times 1} = 8.5\%$$

By alligation method




The ratio of sums borrowed = 1.5 : 3.5 = 3 : 7

Sum borrowed from Kalyani $= \frac{7}{10}\times 72000 = ₹50{,}400$

**36.** Let principal = $p$

Amount = $2p$

$\therefore p(1 + r/100)^4 = 2p$

$\Rightarrow (1 + r/100)^4 = 2$

Now, if the amount is $16p$, then

$p(1 + r/100)^t = 16p$

$\Rightarrow (1 + r/100)^t = 16 = 2^4 = [(1 + r/100)^4]^4$

$\therefore t = 16$

**Alternate Method:**

The sum doubles in 4 years

| | sum | | | in years |
|---|---|---|---|---|
| doubles | $x$ | – | $2x$ | 4 |
| | $2x$ | – | $4x$ | 4 |
| | $4x$ | – | $8x$ | 4 |
| | $8x$ | – | $16x$ | 4 |
| | $x$ | – | $16x$ | 16 years |

**37.** Value of the motor cycle at the end of first year

$= 80000\times\frac{80}{100} = ₹64{,}000$

Value of the motor cycle at the end of second year

$= 64000\times\frac{80}{100} = ₹51{,}200$

Profit = 53600 – 51200 = ₹2,400

**38.** Compound interest $= 32000\left[\left(1+\frac{25}{100}\right)^4 - 1\right]$

$= 32000\left[\left(\frac{5}{4}\right)^4 - 1\right]$

$= 32000\left(\frac{625-256}{256}\right) = 32000\left(\frac{369}{256}\right) = 46{,}125$

**39.** Let the sum borrowed be ₹$P$.

Given: $(1.1P - 2662)\ 1.1 = 2662$

$\Rightarrow 1.21P = 2.1\times 2662$

$\Rightarrow P = 4{,}620$

**40.** Rate of interest = $\frac{691.2-576}{576}\times 100 = 20\%$

Let the sum be 100

$\therefore$ Interest earned on the third year

$= 100\times(1.2)^3 - 100\ (1.2)^2 = ₹28.8$

When interest is ₹28.8, sum is ₹100

When interest is ₹576, sum would be $\frac{100}{28.8}\times 576 = ₹2{,}000$

## EXERCISE-3

**1.** Let us suppose that the principal $P$ amounts to $X$ in $n$ periods, then we write the relationship as

$$X = P\left(1+\frac{r}{100}\right)^n.$$

This can be re-written as $P = \frac{X}{\left(1+\frac{r}{100}\right)^n}$.

$\therefore$ Sum of the present values of the payments received

$$= \frac{7700}{\left(1+\frac{10}{100}\right)} + \frac{7260}{\left(1+\frac{10}{100}\right)^2} = 13{,}000$$

**2.** Let the two sums of money be ₹$2x$ and ₹$3x$ and the rates of interest on the two sums be $y$% p.a. and $2y$% p.a.

$$(2x)\left(\frac{y}{100}\right)(5) + (3x)\left(\frac{2y}{100}\right)(5) = \frac{40xy}{100} = 2{,}400.$$

The difference in the simple interests on the two sums

$$= (3x)\left(\frac{2y}{100}\right)(5) - 2x\left(\frac{y}{100}\right)(5) = \frac{20xy}{100}$$

$$= \frac{2400}{2} = ₹1{,}200$$

**3.** Let value of each instalments be '$Z$'

₹12,600 at 10% will become ₹13,860 at the end of the 1st year. At this point, an amount of $Z$ is repaid.

$\therefore$ Balance at this point is $(13860 - Z)$

This sum at 10% will become $(13860 - Z)(1.1)$, at the end of the 2nd year. Another amount of Z is repaid to completely discharge the loan.

So, $(13860 - Z)\ (1.1) = Z \Rightarrow 15246 = (2.1)Z$

$\Rightarrow Z = 7{,}260.$

Hence, value of each instalment is ₹7,260.

**4.** Let '$x$' be the value of each instalment

$x + 1.2x + 1.44x = (1.2)^3(72800)$

$$= 1x = \frac{1.728(72800)}{3.64} = 34{,}560$$

**5.** The down payment was 1,05,000. The remaining amount paid in instalments = 7,55,000 – 1,05,000 = 6,50,000.

Let each instalment be $e$.

$$e = \frac{65000\left(\frac{14}{100}\right)}{1-\left(\frac{1}{1.14}\right)^3} \approx \frac{65000\left(\frac{14}{100}\right)}{1-0.675} = 2{,}80{,}000.$$

**6.** ₹12,000 borrowed at 10% p.a. will become ₹13,200 at the end of the 1st year.

Let '$k$' be the amount repaid at this point.

Hence, balance due at the point is $(13{,}200 - k)$

At 10% p.a., this will become

$(1.1)(13{,}200 - k) = 14{,}520 - 1.1k = 10{,}120$

$\Rightarrow 1.1k = 4400 \Rightarrow k = 4{,}000$

**7.** Let each instalment be ₹$x$.

After four years, the first instalment (paid at the end of first year) will amount to $x + x(3)\left(\frac{10}{100}\right)$, i.e., $x\ (1.3)$

Similarly, the second, third and fourth instalments will amount to $x(1.2)$, $x(1.1)$ and $x$ respectively.

$\therefore$ Total amount paid = $x(1.3) + x(1.2) + x(1.1) + x = x(4.6)$

Given the sum that has to be repaid = ₹6,440.

$\Rightarrow x(4.6) = 6{,}440$

$\therefore\ x = ₹1{,}400$

**8.** Given that $100\left(1+\frac{r}{100}\right)^n = 100\left(1+\frac{n \cdot 2r}{100}\right)$

$$\left(1+\frac{r}{100}\right)^n = \left(1+\frac{nr}{50}\right)$$

Since, the value of $r$ is not known, $n$ cannot be found. So, the data is insufficient and it can't be determined.

**9.** Let $r$ be the rate of interest and $p$ be the principal.

Given: $p(1 + r/100)(r/100) = 2{,}016$ (1)

$p(1 + r/100)^2(r/100) = 2{,}257.92$ (2)

$(2)/(1) \Rightarrow (1 + r/100) = 1.12$

$\Rightarrow r = 12\%$ p.a.

If the sum is ₹100, at 12% p.a., interest for 2nd year is ₹13.44.

Hence, the sum is $\frac{2016}{13.44} \times 100$, i.e., ₹15,000.

**10.** Since the interest is compounded every 6 months, the number of time periods is 4.

$\therefore$ Amount at the end of 2 years

$$= P\left[1+\frac{r}{2\times 100}\right]^4 = 10{,}000\left[1+\frac{15}{200}\right]^4$$

= ₹13,354 (ignoring the decimal part)

**11.** Each annual instalment $(x) = \dfrac{P \cdot r}{100\left[1-\left\{\frac{100}{100+r}\right\}^n\right]}$

$x = 2662$, $r = 10\%$ and $n = 3$, so

$$2662 = \frac{P \times 10}{100\left[1-\left(\frac{100}{100+10}\right)^3\right]}$$

$$\Rightarrow 2662 = \frac{P}{10\left(1-\left(\frac{10}{11}\right)^3\right)} \Rightarrow 2662 = \frac{P}{10\left(\frac{331}{1331}\right)}$$

$$\Rightarrow P = \frac{2662 \times 10 \times 331}{1331} \Rightarrow P = ₹6{,}620$$

**12.** Let the principal be 100. At 10% p.a. rate of interest compounded every 4 months, the amount at the end of one year will be $100\left[1+\frac{10}{3\times 100}\right]^{1/3} = 100\ (1.033)^3$

$= 110.34$ (rounding up to two decimal taken)

An interest rate of 10.34 will also give an amount of 110.34 on a principal of 100 when compounding is done annually. Hence, this 10.34% p.a. is what is called effective rate of interest.

**13.** Let the instalment paid in the first year be ₹$x$

Sum = ₹1,00,000

Amount at the end of the first year

$= 100000 \times \frac{110}{100}$ = ₹1,10,000

Amount after first instalment is paid

= ₹ $(110000 - x)$

Amount at the end of the second year

$= (110000 - x) \times \frac{110}{100}$.

Amount after second instalment is paid

$= 101000 - 1.1x$.

Amount at the end of the third year

$= 1.1(101000 - 1.1x)$

Amount after third instalment is paid

$= 92300 - 1.21x$

Amount at the end of the fourth year

$= (92300 - 1.21x)1.1$

Amount after fourth instalment is paid

$= 79930 - 1.331x$

Amount at the end of the fifth year

$= (79930 - 1.331x)1.1 = 87923 - 1.4641x =$ ₹44,000

$\Rightarrow x = 30{,}000$.

**14.** Note that what is given is the compound interest for the second and the third years but NOT for two and three years.

We know that the difference between the compound interest for $(k+1)^{th}$ year and that for the $k^{th}$ year is equal to the interest for one year on the compound interest for the $k^{th}$ year.

Hence, the difference between the 2 figures 2,420 and 2,200 which is 220 will be equal to interest for one year on 2,200.

$\therefore 2200 \times r/100 = 220 \Rightarrow r = 10\%$

The compound interest for the 2nd year will be equal to the difference between the amount at end of 2nd year and the amount at the end of 1st year, i.e.,

$P(1+r)^2 - P(1+r) = 2200$

$\Rightarrow P(1+r)\ (1+r-1) = 2200$

$\Rightarrow Pr(1+r) = 2200 \Rightarrow P\,0.1(1.1) = 2200$

$\Rightarrow P(0.11) = 2200$

$\therefore P = 20{,}000$

$\therefore$ The principal and the rate of interest is ₹20,000 and 10% p.a. respectively.

**15.** Present value of the first instalment $= \frac{2400}{1+\frac{20}{100}} = 2000$

Present value of the second instalment $= \frac{2304}{\left(1+\frac{20}{100}\right)^2}$
$= 1600$

Present value of the third instalment $= \frac{5184}{\left(1+\frac{20}{100}\right)^3} = 3000$

Sum borrowed = Total of the present values of the three instalments = 2000 + 1600 + 3000 = 6,600.

**16.** Let the value of the part invested at simple interest be $x$.

Value of the part invested at compound interest = $2000 - x$

$$(x)\ (2)\left(\frac{8}{100}\right) + (2000-x)\left[\left(1+\frac{5}{100}\right)^2 - 1\right] = 251$$

$x = 800$.

**17.** Let the amount paid at the end of the first year be ₹$x$

$\therefore [62500(1.2) - x](1.2) = x + 2000$

$\Rightarrow 90000 - 2000 = 2.2x$

$\Rightarrow x = 40{,}000$

**18.** Let $P$ be the sum and $r$ be rate of interest (as a fraction rather than a percentage figure)

Interest for the $4^{th}$ year will be

$$P(1+r)^4 - P(1+r)^3 = 2000 \quad (1)$$

Interest for the $7^{th}$ year will be

$$P(1+r)^7 - P(1+r)^6 = 2662 \quad (2)$$

$(2)/(1)$ gives

$(1+r)^3 = 1.331$

$\Rightarrow 1 + r = 1.1$

$r = 0.1 = \dfrac{10}{100} = 10\%.$

**19.** Let the annual instalment be ₹$x$.

Let $(FV)$ denote the value of the $i^{th}$ instalment at the end of the fifth year.

Sum of future values of all the instalments = ₹1,450

$(FV)_1 = x + \dfrac{(x)(8)(4)}{100}$

$(FV)_2 = x + \dfrac{(x)(8)(3)}{100}$

$(FV)_3 = x + \dfrac{(x)(8)(2)}{100}$

$(FV)_4 = x + \dfrac{(x)(8)(1)}{100}$

$(FV)_5 = x$

$\therefore 5x + \dfrac{x}{100}\ (8)\,(4+3+2+1) = 1450$

$\Rightarrow 5x + 0.8x = 1450$

$\Rightarrow x = \dfrac{1450}{5.8} = 250$

**Alternate Method:**

Each instalment must be less than ₹$\left(\frac{1}{5}\text{th of } 1450\right)$ = ₹(290). From among the choices given, only Choice (C) satisfies the condition.

**20.** When compounded annually, the interest accrued for the $n^{th}$ year is $r\%$ more than the interest accrued for the $(n-1)^{th}$ year, where $r$ is the annual rate of interest.

$\therefore I_5 = \left(1 + \dfrac{r}{100}\right) I_4$

$\Rightarrow 1464.1 = \left(1 + \dfrac{r}{100}\right)(1331) \Rightarrow \left(1 + \dfrac{r}{100}\right) = 1.1$

$\Rightarrow r = 10\%$

Interest accrued for the first two years when ₹12000 is invested in the same scheme

$= 1200\left[\left(1 + \dfrac{10}{100}\right) - 1\right] = 12000[(1.1)^2 - 1]$

$= 12000 \times 0.21 = 2520$

**21.** The difference between the simple interest and compound interest on a sum of money of ₹$P$ lent at $R\%$ p.a. for 2 years is given by ₹$P\left(\dfrac{R}{100}\right)^2$

$\therefore 1600\left(\dfrac{R}{100}\right)^2 = 16. \Rightarrow \dfrac{R}{100} = \dfrac{1}{10} \Rightarrow R = 10$

Simple interest on the sum of money for two years

$= ₹(1600)\left(\dfrac{10}{100}\right)(2) = ₹320$

**22.** The amount of the end of the $3^{rd}$ year will be the principal for the $4^{th}$ year.

Amount at the end of $3^{rd}$ year

$= P\left[1 + \dfrac{r}{100}\right]^3 = 6000\ (1.1)^3 = 7986$

Hence, interest for the $4^{th}$ year

$= (7986)\ (0.1) = ₹798.60$

**23.** The value that the sum of ₹2,000 would have amounted to at the end of 3 years $= 2000\left(1 + \dfrac{10}{100}\right)\left(1 + \dfrac{20}{100}\right)\left(1 + \dfrac{30}{100}\right) = 2000\ (1.716)$

The value that the sum would have amounted to at the end of 3 years had it been lent at 20% pa simple interest

$= 2000\left(1 + \dfrac{3(20)}{100}\right) = 2000\ (1.6)$

The difference of the interests in the two cases = The difference of the amounts in the two cases = 2000 (1.716 – 1.6) = 2000 (0.116) = 232

The sum amounts to a higher value when lent at compound interest than when lent at simple interest

Had the sum been lent at 20% p.a. simple interest for 3 years, the interest realized would have been ₹232 less.

**24.** Let the sum be ₹$P$ and let it double in $n$ years, then

$2P = P\left(1 + \dfrac{10}{100}\right)^n$

$(1.1)^n = 2$

$n\log(1.1) = \log 2$

$\Rightarrow n = \dfrac{0.3010}{0.0414} = 7.27$ years.

$\therefore$ The sum will double in the eighth year.

**25.** Let the rate of interest be $R\%$ p.a.

Let the total amount repayable at the beginning of the fifth year be $P$.

$$P\left(1+\frac{10}{100}\right)^2 = 60500 \Rightarrow P = 50000$$

Amount repayable at the end of four years on the first loan $= P - 28880 = 21120$

$$16000\left(1+\frac{4R}{100}\right) = 21120 \Rightarrow R = 8.$$

**26.** $P\left(1+\dfrac{20}{100}\right)\left(1+\dfrac{10}{100}\right)\left(1+\dfrac{10}{100}\right)$

$-P\left(1+\dfrac{10}{100}\right)\left(1+\dfrac{10}{100}\right) = 4840$

$P(1.2)(1.21) - P(1.21) = 4840$

$P(1.21)(1.2-1) = 4840$

$P(1.21)(0.2) = 4840$

$P = 20000$

For the 4th year, rate of interest is 30% p.a.

So, amount at the end of the 4 years $= P(1.30)(1.2)(1.21)$.

$= 20000 \times 1.3 \times 1.2 \times 1.21 = 37{,}752$

**27.** Let the rate of interest be $R\%$

$$4000\left(1+\frac{2R}{100}\right) - 3000 + (4000-3000)\left(\frac{5R}{100}\right) = 3600$$

$\Rightarrow R = 20.$

**28.** Let us say the man paid $x$ as annual instalment in the first case.

Then, $(42{,}000 \times 1.1 - x)\ (1.1) = x$

$\Rightarrow 46{,}200 \times 1.1 = 2.1x$

$\Rightarrow x = \dfrac{46{,}200 \times 1.1}{2.1} = ₹24{,}200$

Let us say the man paid $y$ as annual instalment in the second case.

Then $\{[(42{,}000 \times 1.1 - y)1.1] - y\}\ (1.1) = y$

$= [(46{,}200 \times 1.1 - 1.1y) - y]\ (1.1) = y$

$= 50{,}820 \times 1.1 = 3.31y$

$\Rightarrow y = ₹16{,}888.82$

$\cong ₹16{,}889.$

$\therefore$ Required difference $= 24200 - 16889 = ₹7{,}311.$

**29.** $P = 91.3$

$A = 5000$

$A = P \times e^{nr/100}$

$5000 = 91.3 \times e^{nx\frac{100}{100}}$

$\dfrac{5000}{91.3} = e^n$

$= (2.72)^n = 54.76 = (2.72)^n = (2.72)^4$

$n = 4$ years

**30.** Interest paid by Ajay

$$= \left[30000\left(\frac{6}{100}\right)\right] + \left[(30000)\left(\frac{6}{10}\right) + 30000\right]\left(\frac{10}{100}\right)$$

$= ₹4{,}980$

Interest paid by Vijay

$$= 50000\left(\frac{8}{100}\right) + [50000\left(\frac{8}{100}\right) + 50000 - 30000]$$

$$\times \left(\frac{12}{100}\right) = 4000 + 24000\left(\frac{12}{100}\right) = 4000 + 2880 = ₹6{,}880$$

Vijay paid $(6880 - 4980) = ₹1{,}900$ more as interest.

**31.** $$P\left(1+\frac{r}{2\times 100}\right)^{1\times 2} = P\left(1+\frac{5r}{100}\right)$$

$$\left(1+\frac{r}{200}\right)^2 = 1+\frac{5r}{100} = 1+\frac{r}{20}$$

$$1+\frac{r^2}{200^2}+\frac{2r}{200} = 1+\frac{r}{20}$$

$r = 1600$

**32.** Let the sum borrowed by Anand be ₹$x$ and that of Batra be ₹$y$.

Anand paid ₹$2x$ to clear his loan at the end of five years. Hence, Anand paid, an interest of ₹$x$.

$$\therefore x = \frac{(x)(5)(R)}{100}$$

$\Rightarrow R = 20$

Batra paid ₹$2y$ at the end of four years to clear his loom.

$$\therefore 2y = y\left(1+\frac{r}{100}\right)^4$$

$\therefore \log 2 = 4 \log \left(1+\frac{r}{100}\right)$

$\frac{0.3010}{4} = \log\left(1+\frac{r}{100}\right)^4$

$\therefore \log\left(\frac{P_1 \times 2 \times 10}{100}\right)+\left(\frac{P_2 \times 2 \times 15}{100}\right)+\left\{P_3\left[\left(1+\frac{10}{100}\right)^2-1\right]\right\}$

$= 0.07525$

$\Rightarrow \left(1+\frac{r}{100}\right) = \text{Antilog of}(0.07525)$

$= 1.19$ (Given)

$\Rightarrow \frac{r}{100} = 0.19$

$\Rightarrow r = 19$

$\therefore$ Batra borrowed at 19%

**33.** He invested ₹20,000 at S.I. for 2 years at 10%

$A = 20000\left(1+\frac{2\times10}{100}\right) = 20000\ (1.2) = 24{,}000$

Then, he invested this total amount in another bank which is giving a rate of interest $r\%$ p.a. compounded annually. After two years, he received 2460 more than the investment.

$24000 + 2460 = 26{,}460$

$24000\left(1+\frac{r}{100}\right)^2 = 26460$

$\left(1+\frac{r}{100}\right)^2 = \frac{26460}{24000} \Rightarrow \left(1+\frac{r}{100}\right)^2 = \frac{441}{400}$

$\left(1+\frac{r}{100}\right)^2 = \left(\frac{12}{20}\right)^2$

$= 1+\frac{r}{100} = \frac{21}{20}, \frac{r}{100} = \frac{1}{20}$

$r = 5\%$ p.a.

**34.** Let the principal be 1 and the rate of interest (expressed as a fraction) be $r$.

The S.I. for 2 years is $2r$, while the C.I. is $2r + r^2$.

The difference is $r^2$

The S.I. for 3 years is $3r$, while the C.I. is $3r + 3r^2 + r^3$.

The difference is $3r^2 + r^3$.

$\therefore Pr^2 = 120$ (1)

and $P(3r^2 + r^3) = 366$ (2)

$(2)/(1) \Rightarrow 3 + r = \frac{366}{120} = 3 + \frac{1}{20} \Rightarrow r = \frac{1}{20} = 5\%$

**35.** As the profit for the first year is 20%, capital at the end of first year

$= 80000(1.2) = ₹96{,}000$

Similarly, the capital at the end of second year

$= 96000(1.15) = ₹1{,}10{,}400$

Also capital at the end of third year

$= 110400(0.9) = ₹99{,}360.$

If ₹80,000 is invested at 10% p.a. compound interest, then amount at the end of three years

$= 80000(1.1)^3 = ₹1{,}06{,}480$

$\therefore$ Required difference = 106480 – 99360 = ₹7,120

He would have earned ₹7,120 more from the bank.

**36.** Let $r$ be the rate of interest, expressed as a fraction rather than a percentage figure.

We can write

$P(1+r)^5 = I + I(1+r) + I(1+r)^2 + I(1+r)^3 + I(1+r)^4$

$\therefore P = \frac{1}{(1+r)^5}+\frac{1}{(1+r)^4}+\frac{1}{(1+r)^3}+\frac{1}{(1+r)^2}+\frac{1}{1+r}$

$r = 0.1$

$P = 161051\left(\frac{1}{1.1^5}+\frac{1}{1.1^4}+\frac{1}{1.1^3}+\frac{1}{1.1^2}+\frac{1}{1.1}\right)$

$= 100000 + 110000 + 121000 + 133100 + 146410$

$= 6{,}10{,}510$

**37.** Amount outstanding at the end of the first year

$= 1.1(30000) = 33{,}000$

Amount outstanding before the second year

$= 33000 - 9000 = 24{,}000$

Amount outstanding at the end of the second year

$= 1.1(24000) = 26{,}400$

Amount outstanding before the third year

$= 26400 - 9000 = 17{,}400$

Amount outstanding at the end of the third year

$= 1.1(17400) = 19{,}140$

$\therefore$ He should pay ₹19,140 to clear the loan at the end of the third year.

**38.** Let the time period of the loan be $T$ years and the sum of money be ₹$P$.

$$\frac{P(2.5T)(T)}{100} = \frac{P}{10}$$

$\Rightarrow T^2 = 4 \Rightarrow T = 2.$

**39.** ₹10000 becomes $\frac{34560}{2} = 17280$ at 20% p.a. C.I. in $n_1$ years. ₹11520 becomes ₹17280 at 25% p.a. S.I. in $n_2$ years.

$$17280 = 10000\ \frac{4300}{1.2}$$

$$17280 = 10000\left(\frac{6}{5}\right)^{n_1}$$

$$\left(\frac{6}{5}\right)^{n_1} = 1.728$$

$(1.2)^{n_1} = (1.2)^3 \Rightarrow n_1 = 3$ years

$$17280 - 11520 = \frac{11520 \times n_2 \times 25}{100}$$

$$\Rightarrow 5760 = \frac{11520 \times n_2 \times 25}{100}$$

$\Rightarrow n_2 = 2$ years

**40.** The sum borrowed by Raju = ₹15,000

After 3 years, Raju has to pay

$$15000\left[1 + \frac{3(15)}{100}\right] = 15000\left(\frac{145}{100}\right) = 21{,}750$$

Interest gained by Mahesh = 21750 – 15000 = 6,750

Let the sum lent by Raju at 20% be $x$ then

the sum lent at 12% is $(15000 - x)$.

$$\frac{x(20)(3)}{100} + \frac{(15000 - x)(12)(3)}{100} = 6{,}750$$

$$\left[20x + (15000 - x)(12)\right]\frac{3}{100} = 6{,}750$$

$\Rightarrow 8x + 15000\ (12) = 225000$

$8x + 180000 = 225000$

$\Rightarrow 8x = 45000 \Rightarrow x = 5625$

$\therefore$ The sum lent at 20%, p.a. is ₹5,625.

## EXERCISE-4

**1.** Present value of the instalments

$$= \frac{1000}{1.15} + \frac{966}{(1.15)^2} \approx 869.56 + 730.43 \approx ₹1{,}600$$

Down payment = ₹800

Cost of the VCD = Present value + Down payment

= ₹2,400.

**2.** Let the compound interest for the sum borrowed by $P$ for the first, second, third, fourth and fifth year be $I_1$, $I_2$, $I_3$, $I_4$ and $I_5$ respectively.

Let the total interest for the sum taken by $P$ for the five years be C.I.

$\therefore$ C.I. $= I_1 + I_2 + I_3 + I_4 + I_5$ (1)

Under compound interest, the difference between the interests for the third year and for the first year is less than the difference between the interests for the fifth year and for the third year.

i.e., $I_3 - I_1 < I_5 - I_3$

$\Rightarrow 2I_3 < I_5 + I,$ (2)

Similarly, $I_3 - I_2 < I_4 - I_3$

$\Rightarrow 2I_3 < I_2 + I_4$ (3)

(2) + (3)

$\Rightarrow 2I_3 + 2I_3 < I_1 + I_5 + I_2 + I_4$

Adding $I_3$ on both sides of the inequality, we get

$5I_3 < I_1 + I_2 + I_3 + I_4 + I_5$

From (1), we have

$I_1 + I_2 + I_3 + I_4 + I_5 =$ C.I.

$\therefore 5I_3 <$ C.I. (4)

Let the simple interest for the sum borrowed by $Q$ for one year be $I$.

Let total interest for the sum taken by $Q$ for the 5 years be S.I.

$\therefore$ S.I. $= 5I$ (5)

Given the interest for third year for the sum borrowed by $P$ is equal to the interest for the third year for the sum borrowed by $Q$.

i.e., $I_3 = I$ (6)

From (4), (5) and (6),

we have, S.I. < C.I.

i.e., C.I. > S.I.

**3.** The amounts after successive periods (in this case, quarters) are in geometric progression. The difference of successive terms (i.e., the interest earned in successive periods are also in a geometric progression with the same common ratio, which is equal to $\frac{2200}{2100} \approx 1.048$)

Let ₹$I$ be the interest for the first quarter

The interest for the second, fifth and sixth quarters is $IK$, $IK^4$ and $IK^5$ respectively, where $K = 1.048$ as calculated above.

$IK^4 + IK^5 = (I + IK)K^4 = 2100 + 2200 = 4300$

$\Rightarrow (I + IK)(1.048)^4 = 4300$

$\Rightarrow (I + IK)(1.2) = 4300$

$\Rightarrow I + IK \approx \frac{4300}{1.2} \approx 3583$

$\Rightarrow$ First quarter + Second quarter interest is closest to 3583. From the choices, 3570 is the nearest.

**NOTE**

If $x < 1$ ($x$ is very small compared to 1),
$(1 + x)\,n \approx 1 + nx$
$\therefore (1.048)^4 \approx 1 + 4(0.048) = 1.192 \approx 1.2$

**4.** Total interest for two years =

$$\left(\frac{P_1 \times 2 \times 10}{100}\right) + \left(\frac{P_2 \times 2 \times 15}{100}\right) + \left\{P_3\left[\left(1 + \frac{10}{100}\right)^2 - 1\right]\right\}$$

$= 2415$

$$\Rightarrow \frac{20}{100}P_1 + \frac{30}{100}P_2 + P_3\left(\frac{21}{100}\right) = 2415 \qquad (1)$$

Also given $\frac{20}{100}P_1 + \frac{30}{100}P_2 = 1312.5$ (2)

From (1) – (2), we get

$P_3\left(\frac{21}{100}\right) = 2415 - 1312.5 = 1102.5$

$P_3 = 1102.5 \times \frac{100}{21} = 5250.$

**5.** Total value of the investment = 35000

$$\left[\left(1 + \frac{10}{100}\right)^6 + \left(1 + \frac{10}{100}\right)^5\left(1 + \frac{10}{100}\right)^4 + \ldots + \left(1 + \frac{10}{100}\right)\right]$$

$= 35000\,[1.1 + (1.1)^2 + (1.1)^3 + (1.1)^4 + (1.1)^5 + (1.1)^6]$
$= 35000$ (sum of the 6 terms of the GP whose first term is 1 and common ratio is 1.1)

$$= \frac{35000(1.1)\left[(1.1)^6 - 1\right]}{1.1 - 1} = \frac{38500}{0.1}(1.772 - 1) = 2,97,220$$

**6.** Let the instalment paid in the first year be ₹$x$

Sum = ₹1,00,000

Amount at the end of the first year

$= 100000 \times \frac{110}{100} =$ ₹1,10,000

Amount after first instalment is paid

$=$ ₹$(110000 - x)$

Amount at the end of the second year

$= (110000 - x) \times \frac{110}{100}$.

Amount after second instalment is paid

$= 121000 - 1.1x - 20000 = 101000 - 1.1x.$

Amount at the end of the third year

$= 1.1(101000 - 1.1x)$

Amount after third instalment is paid

$= 92300 - 1.21x$

Amount at the end of the fourth year

$= (92300 - 1.21x)1.1$

Amount after fourth instalment is paid

$= 79930 - 1.331x$

Amount at the end of the fifth year

$= (79930 - 1.331x)1.1 = 87923 - 1.4641x =$ ₹44,000

$\Rightarrow x = 30,000.$

**7.** The principal, interest for the $i^{th}$ year, interest for $i$ years and the amount at the end of $i$ years for $I = 1, 2, 3$, is tabulated below both for S.I. and C.I.

| S.I. | | | | C.I. | | | |
|---|---|---|---|---|---|---|---|
| **Principal** | **Int. for $i^{th}$ year** | **Int. for $i$ years** | **Amt. at end of $I$ year** | **Principal** | **Int. for $i^{th}$ years** | **Int. for $i$ years** | **Amt. at end of $i$ years** |
| 1 | $r$ | $r$ | $1 + r$ | 1 | $r$ | $R$ | $1 + r$ |
| 1 | $r$ | $2r$ | $1 + 2r$ | $1 + r$ | $(1 + r)r$ | $2r + r^2$ | $(1 + r)^2$ |
| 1 | $r$ | $3r$ | $1 + 3r$ | $(1 + r)^2$ | $(1 + r)^2 r$ | $3r + 3r^2 + r^3$ | $(1 + r)^3$ |

Difference between C.I. and S.I. for 2 years = $Pr^2$

Difference between C.I. and S.I. for 3 years = $P(3r^2 + r^3)$

Given: $\frac{r^2}{3r^2 + r^3} = \frac{10}{31} \Rightarrow \frac{1}{3+r} = \frac{1}{3+0.1} \Rightarrow r = 0.1 = 10\%$

$\therefore R = 10$

**8.** Let the rate of interest be $R\%$ p.a.

Let the total amount repayable at the beginning of the fifth year be $P$.

$P\left(1+\frac{10}{100}\right)^2 = 60500 \Rightarrow P = 50{,}000$

Amount repayable at the end of four years on the first loan = $P - 28880 = 21120$

$16000\left(1+\frac{4R}{100}\right) = 21120 \Rightarrow R = 8.$

**9.** Let the interest accrued in the $n^{th}$ year be $I_n$

Given, $I_5 = 1250$, $I_{11} = 5{,}000$

$\frac{I_{11}}{I_5} = \left(1+\frac{r}{100}\right)^6 = 4$

Maturity value $= 10000\left(1+\frac{r}{100}\right)^{15}$

$= 10000\ (4^{1/6})^{15} = 32{,}000$

**10.** Sum borrowed = ₹6,000

Interest = 10% at compounded annually and

time = 3 years

Let $x$ be the amount paid at the end of each of the 3 years

$\Rightarrow x(1.1)^2 + x(1.1) + x = 6000\ (1.1)^3$

$\Rightarrow x = 2412.68$

$\therefore$ Each annual instalment = ₹2,413

**Alternate Method:**

Each annual instalment $= \frac{Pr}{100\left[1-\left(\frac{100}{100+r}\right)^n\right]}$

Where $P$ = Sum borrowed; $r$ = rate of interest and $n$ = number of annual instalments.

**11.** Let the amount that Rekha has to repay at the end of the third year, to clear the loan be ₹100$P$. The worth of the borrowed amount and the 3 repayments, at the end of the 3 years is equal. The rate of interest as a fraction is 1/6.

$\therefore 300\left(\frac{7}{6}\right)^3 = 140\left(\frac{7}{6}\right)^2 + 140\left(\frac{7}{6}\right) + P$

$\therefore P = 300\left(\frac{7}{6}\right)^3 - 140\left(\frac{7}{6}\right)^2\left[1+\frac{6}{7}\right]$

$= \frac{300}{216}(343) - 140\left(\frac{49}{36}\right)\left(\frac{13}{7}\right) = \frac{25}{18}(343) - \frac{10}{18}(49)(13)$

$= \frac{8575}{18} - \frac{6370}{18} = \frac{2205}{18} = \frac{245}{2} = 122.5$

$\therefore$ The amount to be repaid = $1000P = 1{,}22{,}500$

**12.** Let the rate of interests for the 1st, 2nd and 3rd year be $4x\%$, $5x\%$ and $10x\%$ respectively

Given that, $P\left[1+\frac{4x}{100}\right]\left[1+\frac{5x}{100}\right]\left[1+\frac{10x}{100}\right] = 2.25P$

$(100 + 4x)(100 + 5x)(100 + 10x) = 225 \times 100 \times 100$

$4 \times 5 \times 10 \times (25 + x)(20 + x)(10 + x) = 225 \times 100 \times 100$

$(25 + x)(20 + x)(10 + x) = 450 \times 25$

$(25 + x)(20 + x)(10 + x) = 30 \times 25 \times 15$

$x = 5$

Hence, the rate of interest for the third year is $10x\%$, i.e., 50%.

**Alternate Method:**

Choosing option (A), the rate of interests will be 12%, 15% and 30%. The net interest in this case will be definitely less than 125%

Similarly choosing option (C), the rate of interests will be 20%, 25% and 50%

The net interest in this case will be 125%

**13.** Let the sum borrowed by Arjun from Bharath be ₹$x$.

Then, the sum borrowed from Charan = ₹$2x$.

Arjun lent ₹40,000 to Dheeraj at 10% p.a. Therefore, at the end of 1 year, he gained.

$0.1(40000) - 0.08x - 0.09(2x)$ which is equal to 1,400

$\Rightarrow 0.26x = 4000 - 1400$

$\Rightarrow x = \frac{2600}{0.26} = 10{,}000$

$\therefore$ The money borrowed from Bharath is ₹10,000.

**14.** Let Bart's principal be '$p$'.

Present amount = $2p$

After 2 years the present amount increased by 12.5%. Simple interest in these 2 years

$= \frac{p \times r \times 2}{100} = \frac{2pr}{100}$ (where $r$ is the rate of interest.)

Given: 12.5% of $2p = r = 12.5\%$.

Let the time at 12.5% be 8 years, for $p$ to become $2p$. This present amount becomes 5 times the principal in time '$t$' (say). As amount is $5p$, interest is $4p$.

$\therefore\ 4p = \frac{p \times t \times 12.5}{100} \Rightarrow t = \frac{400}{12.5} = 32$ years.

$\therefore$ From now, it is $32 - 8 = 24$ years.

**15.** Amount to be repaid by Arun (in lakhs) $= 2\left(1+\frac{10}{100}\right)^2 = 2.42$.

Amount realized in total due to lending (in lakhs) $= 2\left(1+\frac{8}{100}\right)^{\frac{24}{8}} \approx 2.52$ (For 8 months, the rate of interest is 8%.)

Arun made a profit of approximately ₹10,000.

**16.** Let $r$ be the rate of interest, expressed as a fraction rather than a percentage figure.

We can write

$P(1+r)^5 = I + I(1+r) + I(1+r)^2 + I(1+r)^3 + I(1+r)^4$

$\therefore P = \frac{I}{(1+r)^5} + \frac{I}{(1+r)^4} + \frac{I}{(1+r)^3} + \frac{I}{(1+r)^2} + \frac{I}{1+r}$

$r = 0.1$

$P = 161051\left(\frac{1}{1.1^5} + \frac{1}{1.1^4} + \frac{1}{1.1^3} + \frac{1}{1.1^2} + \frac{1}{1.1}\right)$

$= 100000 + 110000 + 121000 + 133100 + 146410$

$= 610510$

**17.** Naresh's loan amounts to $50000(1.06)(1.08) = 57{,}240$. He has to pay an interest of ₹7,240. Prabhat's loan amounts to $70000(1.05)(1.09) = 80{,}115$. But Prabhat pays 35,000 at the end of the first year which amounts to $35000(1.09) = 38{,}150$ at the end of the second year.

$\therefore$ He has to pay $80115 - 38150$ or 41,965 at the end of the second year to discharge his loan.

$\therefore$ He pays an interest of $35000 + 41965 - 70000 = ₹6{,}965$, i.e., ₹275 less than Naresh.

**18.** Let sum = ₹100

At compound interest, ₹100 becomes ₹200 in six years.

In three years, the sum becomes $\sqrt{2}$ times of itself.

At a different rate of interest under simple interest, ₹100 becomes ₹200 in six years.

In three years, the sum becomes 1.5 times of itself 4.5

$= 1.5 \times \sqrt{2} \times 1.5 \times \sqrt{2}$

$\therefore$ The interest is calculated with simple interest for three years for two times and compound interest for three years for two times. Hence, the number of years $= 3 + 3 + 3 + 3 = 12$ years.

**19.** Let, $P$ be the sum and $r$ be the rate of interest (expressed as a decimal fraction); we have

$P(1+r)^2 - P = 0.55(4rP)$

$P(1 + 2r + r^2 - 1) = (0.55)\ (4)\ (Pr) = 2.2Pr$

$\Rightarrow 2r + r^2 = 2.2r \Rightarrow r^2 = 0.2r$

$\Rightarrow r = 0.2$, i.e., 20%.

**20.** Remaining amount = ₹275000 – 110000 = ₹1,65,000

This is paid in two equal instalments of ₹$x$ at 20% p.a. compounded annually

$[165000(1.2) - x]1.2 = x$

$\Rightarrow \frac{2.24}{1.44} = 165000 \Rightarrow x = 1{,}08{,}000$

$\therefore$ The total amount paid by the person

$= 110000 + 2(108000) = ₹3{,}26{,}000$

$\therefore$ He pays 32600 – 275000 = ₹51,000 more on the instalment plan.

## EXERCISE-5

**1.** Statement I gives an equation in terms of Principal and the rate of interest. Statement II gives the rate of interest. Principal can be found using the Statements I and II. As the principle and the rate of interest can be found, compound interest can be calculated.

**2.** To solve this, both the rate and the principal amount are required. $A + 205 = A(1 + x/100)^n$.

$A$ and $x$ are required. As they are given in two different statements, both the statements are required.

**3.** From statement I alone,

In Scheme $P$ money grows at $\left(1+\frac{4}{100}\right)^2 = (1.04)^2$

$= 1.0816$, i.e., the investment becomes 1.0816 times ever

half year. But nothing is mentioned about Scheme *Q*, hence I alone is not sufficient.

From Statement II alone,

In Scheme *Q*, money grows at $\left(1+\frac{9}{100}\right)^2$, i.e., the investment becomes 1.09 times every half year. But nothing is mentioned about Scheme *P*, hence II alone is not sufficient.

From I and II together, we can answer the question as the growth rate in both the Schemes is known.

**4.** Let principal be *P* and rate of interest be *R*.

From Statement I, the difference between the compound interest between the $n^{th}$ year and $(n+1)^{th}$ year is equal to the interest earned on the compound interest on the $n^{th}$ year.

Hence, ₹480 – 400 = ₹80 is the interest earned on ₹400

If rate of interest is *R*, then

$400R/100 = 80$

$\Rightarrow R = 20\%$

Therefore, interest for the $6^{th}$ year is $480 + \frac{20}{100}(480)$ = ₹576 and the difference between the interest for the $4^{th}$ and $6^{th}$ year be found. So, we can answer the question from I alone.

From Statement II, $P(1 + R/100)^3 = 13310$ (2)

AS *P* and *R* are not known, we cannot find

$P(1 + r/100)^6 - P(1 + R/100)^4$.

As *P* and *R* are not known, we cannot find

$P(1 + R/100)^6 - P(1 + R/100)^4$.

**5.** Since the number of years for which the sum is lent is not given, the question cannot be answered.

**6.** The difference between C.I. and S.I. for a principal of *P*, at a rate of interest *R* is $PR^2/100^2$

The implication of Statement I is

$(2P)(R/2)^2 \times 1/100^2 = 400$,

$\Rightarrow PR^2/100^2 = 400 \times 4/2 \Rightarrow$ values can be calculated.

Hence, the question can be answered by this statement.

Statement II implies that

$P(1 + R/100) = 1.2P$; *R* can be calculated from this, but $PR^2/100^2$ cannot be calculated as *P* is not known. Hence, this statement is not sufficient.

**7.** Neither of the statements is independently sufficient to find the rate of interest.

Combining both the statements, we have

$$\frac{P(4)(r)}{100} = 800 \quad (1)$$

$$\text{And } P\left(\frac{r}{100}\right)^2 = 80 \quad (2)$$

Solving (1) and (2), *r* can be found.

**8.** Statement I says the down payment is 40% of the total amount paid by the customer. But we do not know about the instalment.

Statement II says ₹1,272 is the value of each instalment.

5 instalments add upto ₹1,272 × 5 = ₹6,360. But we do not know the down payment. Hence, Statement II is also insufficient.

If we combine both statements, we can determine the total amount paid by the customer.

**9.** From Statement I, we do not know anything about the interest rate.

∴ Statement I alone is not sufficient.

From Statement II, we do not know the interest earned.

∴ Statement II alone is not sufficient.

By combining (I) and (II),

$$P\left(1+\frac{10}{100}\right)^2 - P = 4830$$

Hence, we can find the value of *P*

**10.** To find out the total money Sohini invested, we need to know the interest earned.

From Statement I, the interest she earns from each of her investments is the same. This alone is insufficient.

From Statement II, the total interest she earned is ₹1,200. But considering this statement alone, we can not solve the problem. Hence, Statement II alone is not sufficient.

If we consider both Statements I and II, we know that

$\frac{1200}{3}$ = ₹400 is earned on each investment.

Hence, combining both I and II, we get the answer.

**11.** The C.I. earned on ₹20,000 for 2 years depends on the rate of interest. Let us assume rate of interest is *r*.

From Statement I,

$$P\left[\left(1+\frac{r+3}{100}\right)^2 - \left(1+\frac{r}{100}\right)^2\right]$$

= ₹1,302.

$\Rightarrow 20000\left[\left(2+\frac{2r+3}{100}\right)\left(\frac{3}{100}\right)\right]=₹1{,}302$

$\Rightarrow 2+\frac{2r+3}{100}=\frac{1302}{600}\Rightarrow\frac{2r+3}{100}=\frac{102}{600}$

$\Rightarrow 2r+3=17\Rightarrow 2r=14\Rightarrow r=7$

$\therefore$ Statement I alone is sufficient.

From Statement II,

$P\left(1+\frac{r}{100}\right)^2-P-\frac{P(2r)}{100}=98.$

$\therefore\left(1+\frac{r}{100}\right)^2-\left(1+\frac{2r}{100}\right)=\frac{98}{20{,}000}$

$\Rightarrow 1+\frac{2r}{100}+\frac{r^2}{10000}-1-\frac{2r}{100}=\frac{49}{10{,}000}$

$\Rightarrow r^2=49$ or $r=7$.

Hence, Statement II alone is also sufficient.

**12.** Let $P$ be the amount lent. $P=12000$

$\Rightarrow 12000\left[\left(1+\frac{r}{100k}\right)^{2k}-1-\frac{2r}{100}\right]=30$

where $k$ is the frequency of compounding.

But we do not know at what frequency the interest is being compounded.

$\therefore$ Statement I alone is not sufficient

II is not sufficient as it gives no numerical data

By combining I and II, we get

$12000\left[\left(1+\frac{r}{100}\right)^2-\frac{2r}{100}-1\right]=30\Rightarrow\left(\frac{r}{100}\right)^2=\frac{1}{400}$

$\Rightarrow r=5$

**13.** Statements I and II give the same information.

$\therefore$ The question cannot be answered as the principal is not given.

**14.** From Statement I, $\frac{p(8)R}{100}=36{,}000$

So, Statement I alone is not sufficient.

From Statement II, $\frac{p(8)R}{100}=p.$

$\therefore R=12.5\%$.

Statement II alone is sufficient.

**15.** Statement I is not sufficient

From Statement II,

Let the sums be $a$, $b$ and $c$, then

$\frac{a(100+2\times5)}{100}=\frac{b(100+3\times5)}{100}=\frac{c(100+4\times5)}{100}=k$

$\frac{11a}{10}=\frac{23b}{20}=\frac{6c}{5}=k$

$a:b:c=276:264:253$

**16.** From Statement I,

$125=100\left(1+\frac{R}{100}\right)^3$ ; so we can find the rate of interest $(R)$.

From Statement II,

$1.5x=x\left(1+\frac{R}{100}\right)^3\Rightarrow 1.5=\left(1+\frac{R}{100}\right)^3$

So, we can find the rate of interest.

So, each statement alone is sufficient.

**17.** Either of the statements alone is not sufficient as the rate of interest and the simple interest earned in 5 years is given in different statements.

Combining statements I and II,

$\frac{p(5)(6)}{100}=600$, so we can get the value of $p$.

Compound Interest $=\left[p\left(1+\frac{6}{10}\right)^5-p\right]$

**18.** From Statement I, $r=n$

$\therefore$ S.I. $=\frac{Pr^2}{100}$

But we do not know anything about $P$ and S.I.

Hence, statement I alone is insufficient.

From Statement II,

S.I. $=\frac{P}{4}$

$\therefore\ \frac{Pm}{100}=\frac{P}{4}$

$rn=25$.

Since $r$, $n$ are both unknown, Statement II is insufficient.

But if we consider both statements

we have $r^2=25$, i.e., $r=5$.

**19.** From Statement I, we do not know the time for which the amount was deposited.

$\therefore$ Statement I alone is not sufficient

From Statement II, we do not know the rate of interest

∴ Statement II alone is not sufficient

By combining I and II, we still do not know whether the interest is simple or compound.

**20.** From Statement I, interest collected for the second year on ₹10,000 is ₹1,100. In case of an S.I. a 11% S.I. yields ₹1,100 as the interest. A 10% C.I. for the second year yields ₹1100.

Hence, Statement I alone is not sufficient.

From Statement II, interest collected after 2 years on ₹15,000 is 3,150.

Now, $15000(1.1)^2 = 18150$

∴ Interest after two years is ₹3,150.

Hence, Statement II alone is sufficient.

# 5 Time and Distance

CHAPTER

**LEARNING OBJECTIVES**

*In this chapter, you will*:

- Learn to identify the nature of the relation between Time, Speed and Distance
- Understand the variation concept of Time, Speed and Distance
- Learn about conversion of speed from kmph to m/sec and vice-versa
- Understand average speed and its applications in different scenarios
- Understand relative speed, when bodies move in the same direction as well as when they move in opposite directions
- Master skills in applying relative speed concepts in solving questions based on trains, boats and streams, escalators, linear races
- Understand circular motion, find out times of meeting, points of meeting when runners run in the same direction as well as in opposite directions
- Understand the concept of clocks and solve problems on clocks by relative speed concept

In this chapter, we will look at problems in the following different areas:

1. General problems on Time, Speed and Distance
2. Relative Speed
3. Boats and Streams
4. Races and Circular Tracks

Before we look at problems in various areas, let us first look at some basic concepts pertaining to speed, time and distance.

## ❒ SPEED

**Distance covered per unit time is called speed.**

**i.e., Speed = Distance/time**

The above relationship between the three variables distance, speed and time can also be expressed as follows:

**FORMULA**

Distance = Speed × Time or
Time = Distance/Speed

1. If two bodies travel with the same speed, Distance covered ∝ Time (Direct Variation).
2. If two bodies travel for the same period of time, Distance covered ∝ Speed (Direct Variation).
3. If two bodies travel the same distance,

**FORMULA**

$$\text{Time} \propto \frac{1}{\text{Speed}} \quad \text{(Inverse Variation)}$$

Distance is normally measured in kilometres, metres or miles; time in hours or seconds and speed in km/hr (also denoted by kmph), miles/hr (also denoted by mph) or metres/second (denoted by m/s).

To convert speed in kmph to m/sec, multiply it with 5/18.

To convert speed in m/sec to kmph, multiply it with 18/5.

In the case of moving trains, three different situations need to be considered:

1. When a train passes a stationary point, the distance covered (in the passing) is the length of the train.
2. If the train is crossing a platform (or a bridge), the distance covered by the train (in the crossing) is equal to the length of the train plus the length of the platform (or bridge).
3. If two trains pass each other (travelling in the same direction or in opposite directions), the total distance covered (in the crossing or the overtaking, as the case may be) is equal to the sum of the lengths of the two trains.

## ❒ AVERAGE SPEED

Average speed of a body travelling at different speeds is defined as follows:



**FORMULA**

$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Please note that the Average speed of a moving body is NOT equal to the Average of the speeds.

1. If a body travels from point $A$ to point $B$ with a speed of $p$ and back to point $A$ (from point $B$) with a speed of $q$, then the average speed of the body can be calculated as $2pq/(p+q)$. Please note that this does not depend on the distance between $A$ and $B$.
2. If a body covers part of the journey at speed $p$ and the remaining part of the journey at speed $q$ and the distances of the two parts of the journey are in the ratio $m : n$, then the average speed for the entire journey is $(m+n)\text{pq}/(mq+np)$.
3. In general, if a person travelling between two points reaches $p$ hours late travelling at a speed of $u$ kmph and reaches $q$ hours early travelling at $v$ kmph, the distance between the two points is given by $\frac{vu}{v-u}(p+q)$.

## ❒ Relative speed

**The speed of one (moving) body in relation to another moving body is called the relative speed of these two bodies**, i.e., it is the speed of one moving body as observed, from the second moving body.

Two important points to remember:

1. If two bodies are moving in the same direction, the relative speed is equal to the difference of the speeds of the two bodies.
2. If two bodies are moving in opposite directions, the relative speed is equal to the sum of the speeds of the two bodies.

## ❒ BOATS AND STREAMS

Problems related to boats and streams are different in the computation of relative speed from those of trains/cars.

1. When a boat is moving in the same direction as the stream or water current, the boat is said to be moving with the stream or current.
2. When a boat is moving in a direction opposite to that of the stream or water current, it is said to be moving against the stream or current.
3. If the boat is moving with a certain speed in water that is not moving, the speed of the boat is then called the speed of the boat in still water.
4. When the boat is moving upstream, the speed of the water opposes (and hence reduces) the speed of the boat.
5. When the boat is moving downstream, the speed of the water aids (and thus adds to) the speed of the boat.

Thus, we have



**IMPORTANT FACTS**

Speed of the boat against stream = Speed of the boat in still water – Speed of the stream

Speed of the boat with the stream = Speed of the boat in still water + Speed of the stream

These two speeds, the speed of the boat against the stream and the speed of the boat with the stream, are relative speeds.

If $u$ is the speed of the boat down the stream and $v$ is the speed of the boat up the stream, then we have the following two relationships.

Speed of the boat in still water = $(u+v)/2$

Speed of the water current = $(u-v)/2$

In problems, instead of a boat, it may be a swimmer but the approach is exactly the same. Instead of boats/swimmers in water, it could also be a cyclist cycling against or along the wind. In some problems it can be person(*s*) going up/down an ascending or descending escalator. The approach to solving the problems still remains the same.

## ❒ RACES AND CIRCULAR TRACKS

When two persons *P* and *Q* are running a race, they can start the race at the same time or one of them may start a little later than the other. In the second case, suppose *P* starts the race and after 5 seconds, *Q* starts. Then we say *P* has a 'start' of 5 seconds. Alternatively, in a race between *P* and *Q*, *P* starts first and then when *P* has covered a distance of 10 metres, *Q* starts. Then we say that *P* has a 'start' of 10 metres.

In a race between *P* and *Q* where *Q* is the winner, by the time *Q* reaches the winning post, if *P* still has another 15 metres to reach the winning post, then we say that *Q* has won the race by 15 metres. Similarly, if *P* reaches the winning post 10 seconds after *Q* reaches it, then we say that *Q* has won the race by 10 seconds.

In problems on races, we normally consider a 100 m race or a 1 km race. The length of the track

need not necessarily be one of the two figures mentioned above but can be as given in the problem.

**When two or more persons running around a circular track (starting at the same point and at the same time), then we will be interested in two main issues:**

1. When they will meet for the first time and
2. When they will meet for the first time at the starting point

To solve the problems on circular tracks, you should keep the following points in mind.

**When two persons are running around a circular track in opposite directions**

1. The relative speed is equal to the sum of the speeds of the two individuals and
2. From one meeting point to the next meeting point, the two of them together cover a distance equal to the length of the track.

**When two persons are running around a circular track in the same direction**

1. The relative speed is equal to the difference of the speeds of the two individuals and
2. From one meeting point to the next meeting point, the faster person covers one complete round more than the slower person.

We can now tabulate the time taken by the persons to meet for the first time ever or for the first time at the starting point in various cases.

**When two people are running around a circular track**

Let the two people *A* and *B* with respective speeds of *a* and *b* (a > *b*) be running around a circular track (of length *L*) starting at the same point and at the same time. Then,

| | **When the two persons are running in the same direction** | **When the two persons are running in opposite directions** |
|---|---|---|
| Time taken to meet for the first time ever | $\frac{L}{(a-b)}$ | $\frac{L}{(a+b)}$ |
| Time taken to meet for the first time at the starting point | L.C.M. of $\left\{\frac{L}{a}, \frac{L}{b}\right\}$ | L.C.M. of $\left\{\frac{L}{a}, \frac{L}{b}\right\}$ |

Please note that when we have to find out the time taken by the two persons to meet for the first time at the starting point, what we have to do is to find out the time taken by each of them to complete one full round and then take the L.C.M. of these two timings ($L/a$ and $L/b$ are the timings taken by the two of them, respectively to complete on full round).

**When three people are running around a circular track**

Let the three people *A*, *B* and *C* with respective speeds of *a*, *b* and *c* (*a* > *b* > *c*)) be running around a circular track (of length *L*) starting at the same point at the same time. In this case we consider the three persons running in the same direction as the general case.

| | |
|---|---|
| Time taken to meet for the first time ever | L.C.M. of $\left\{\frac{L}{(a-b)}, \frac{L}{(b-c)}\right\}$ |
| Time taken to meet for the first time at the starting point | L.C.M. of $\left\{\frac{L}{a}, \frac{L}{b}, \frac{L}{c}\right\}$ |

The logic in obtaining the above is as follows:

*A* and *B* will be together with a time gap of $L/(a - b)$; *B* and *C* will be together with a time gap of $L/(b - c)$; for *A*, *B* and *C* to be together, *A* and *B* should be together as well as *B* and *C* should be together. Hence, the L.C.M. of the two timings $L/(a - b)$ and $L/(b - c)$ will give the time when *A*, *B* and *C* will all be together.

When we have to find out the time taken by the three persons to meet for the first time at the starting point, what we have to do is to find out the time taken

by each of them to complete one full round and then take the L.C.M. of these three timings ($L/a$, $L/b$ and $L/c$ are the timings taken by the three of them, respectively to complete one full round).

Even if we are given a case where three persons are running around a circular track with two persons running in the same direction and the third in the opposite direction, we can work out the time taken by them to meet for the first time ever and for the first time at the starting point by extending the above logic.

## ❒ CLOCKS

**The hours and minutes hands of a clock move in relation to each other continuously and at any given point of time, they make an angle between 0° and 180° with each other.**

If the time shown by the clock is known, the angle between the hands can be calculated. Similarly, if the angle between two hands is known, the time shown by the clock can be found out.

When we say angle between the hands, we normally refer to the acute/obtuse angles (upto 180°) between the two hands and not the reflex angle (> 180).

For solving the problems on clocks, the following points will be helpful.

1. Minutes hand covers 360° in 1 hour, i.e., in 60 mins. Hence, minutes hand covers 6° per minute.
2. Hours hand covers 360° in 12 hours. Hence, hours hand covers 30° per hour or hours hand covers 1/2° per minute
3. All angles are measured in the clockwise direction starting from the vertical line at 12 o' clock.

### FORMULA

We can also solve the problems on clocks by the route of 'Relative Speed'.

In 1 minute, Minutes hand covers 6° and Hours hand covers 1/2°.

Therefore, Relative Speed = 6 – 1/2

= 5½° per minute.

Alternately, in 1 hour, minutes hand covers 60 minute divisions whereas hours hand covers 5 minute divisions.

∴ Relative Speed = 60 – 5 = 55 minute divisions per hour.

However, taking the route of actual angles covered is by far the simplest and easy to understand as well as helpful in solving all the different models of problems on clocks. Hence, we will look at mainly that method only and not the relative speed method. This will not create any confusion.

The following additional points should also be remembered.

In general, every angle is made twice in one hour by the two hands of the clock.

In a period of 12 hours, the hands make an angle of

1. 0° with each other ( i.e., they coincide with each other) 11 times, and hence, the time gap between two successive coincidences is 12/11 hours, i.e., $1^{1}/_{11}$ hours, i.e., $65^{5}/_{11}$ minutes.
2. 180° with each other ( i.e., they lie on the same straight line) 11 times.
3. 90° or any other angle with each other 22 times.

## SOLVED EXAMPLES

**5.01.** Express a speed of 20 m/s in kmph?

**Sol:** $20 \times \frac{18}{5} = 72$ km/hr

**5.02.** A car can cover 600 km in 8 hours. If the speed is increased by 25 kmph, how much time does the car take to cover 800 km?

**Sol:** Speed = $\frac{\text{Distance}}{\text{Time}} = \frac{600}{8} = 75$ kmph

Now this is increased by 25 kmph. Hence, new speed is 100 kmph. At this speed, time taken to cover 800 km = 800/100 = 8 hours

**5.03.** A person covers a certain distance at a certain speed. If he decreases his speed by 20%, then he takes 16 minutes more to cover the same distance. Find the time taken by him to cover the distance at original speed.

**Sol:** When the speed decreases by 20%, the new speed is 80% of the original speed, it is (4/5) times the original speed. Since speed and time are inversely related, if the speed is (4/5) times the original

speed, then the time will be (5/4) times the original time. Let the time taken to cover the distance at original speed be $t$. Then, $\frac{5}{4}t - t = \frac{1}{4}t = 16$
$\Rightarrow t = 4 \times 16 = 64$ minutes.

**5.04.** A car covers a certain distance at a speed of 90 km/hr while going and returns to the starting point at a speed of 60 km/hr. Find the average speed of the car for the whole journey.

**Sol:** We know that the average speed is $2pq/(p+q)$ where $p$ and $q$ are the speeds for covering two equal distances.
Therefore, the average speed
$= (2 \times 90 \times 60) / (90 + 60)$

**5.05.** What is the time taken by a train of length 360 m to cross a pole at a speed of 72 kmph?

**Sol:** Time taken by a train to cross a pole $= \frac{\text{Length of the train}}{\text{Speed of the train}} = \frac{360}{72 \times (5/18)} = 18$ seconds

**5.06.** How long will a train 200 m long travelling at a speed of 54 kmph take to cross a platform of length 100 m?

**Sol:** Distance covered by the train = Length of the train + length of the platform
= 200 + 100 = 300 m
Speed of the train
= 54 kmph = 54 × (5/18) = 15 m/sec
∴Time taken by the train
= 300/15 = 20 seconds

**5.07.** Find the length of a train running at 45 kmph which can cross a bridge of length 250 m in 36 seconds.

**Sol:** Speed of the train = 45 kmph
= 45 × (5/18) = 12.5 m/s
Distance covered by the train = 12.5 × 36
= 450 m.
Length of the train = Distance covered – length of the Bridge = 450 – 250 = 200 m

**5.08.** A worker reaches his work place 20 minutes late by walking at 3 km/hr. The next day he increases his speed by 2 km/hr and reaches on time. Find the distance from his house to his work place.

**Sol:** Let the distance be $x$ km. Then, time taken on the 1st day = x/3 hrs
Time taken on the 2nd day = x/5 hrs
Given that the travel time of the 1st day is 20 minutes more than that of the 2nd day.

$\frac{x}{3} - \frac{x}{5} = \frac{20}{60} \Rightarrow x = 2.5$ km

**5.09.** A person travelling at 6 kmph reaches his office 15 minutes late. Had he travelled at 8 kmph he would have been 25 minutes early. Find the distance the person has to travel to reach his office.

Using the formula, $d = \frac{vu}{v-u}(p+q)$,

**Sol:** Distance $= \frac{6 \times 8}{8-6}\left(\frac{15}{60} + \frac{25}{60}\right) = 16$ km

**5.10.** Find the time taken by a train of length 100 m running at a speed of 72 kmph to cross another train of length 200 m running at a speed of 63 kmph in the same direction.

**Sol:** Total distance covered = sum of the lengths of the two trains = 100 + 200
= 300 m.
Relative speed of the two trains = 72 – 63 = 9 kmph (Since the trains are running in the same direction, the relative speed will be the difference of the speeds)

$= 9 \times \frac{5}{18} = \frac{5}{2}$ m/s

Therefore, time taken = 300/ (5/2) = 120 seconds

**5.11.** A train crosses two persons cycling in the same direction as the train, in 12 seconds and 15 seconds, respectively. If the speeds of the two cyclists are 10 kmph and 12 kmph, respectively, then find the length and the speed of the train.

**Sol:** Let the speed of the train be $s$ kmph.
Relative speed in overtaking the first cyclist
$= (s - 10)$ kmph. Time taken to overtake the first cyclist = 12 seconds.
Hence, the length of the train
$= 12 \times (s - 10) \times 5/18$ (1)
Similarly, considering the case of the overtaking the second cyclist, length of the train
$= 15 \times (s - 12) \times 5/18$ (2)
Equating (1) and (2),
$12 \times (s - 10) \times 5/18 = 15 \times (s - 12) \times 5/18$
$\Rightarrow 12s - 120 = 15s - 180$
$60 = 3s \Rightarrow s = 20$ kmph
Length of the train $= 12 \times (s - 10) \times 5/18$
$= 12 \times 10 \times 5/18 = (100/3)$ metres

**5.12.** Two trains running at 63 kmph and 45 kmph cross each other in 15 seconds, when they run in

opposite direction. When they run in the same direction, a person in the faster train observes that he crossed the other train in 40 seconds. Find the lengths of the two trains.

**Sol:** Let $p$, $q$ be the lengths of the slow and faster trains, respectively. When the trains are travelling in the opposite direction, relative speed = 63 + 45 = 108 kmph = 30 m/sec.
Distance covered = sum of the lengths of the two trains
$\Rightarrow p + q = 30 \times 15 = 450$ m (1)
When trains are travelling in the same direction, since we are given that the time noted by a person in the faster train as 40 seconds to cross the slower train, the distance covered is equal to the length of the slower train, distance covered = $q$
Relative speed = 63 – 45 = 18 kmph
= 5 m/sec
$q = (5)\ (40) = 200$ m (2)
From (1) and (2), we get
$p = 450 - q = 250$ m

**5.13.** Two trains of length 100 m and 250 m run on parallel lines. When they run in the same direction it will take 70 seconds to cross each other and when they run in opposite direction, they take 10 seconds to cross each other. Find the speeds of the two trains.

**Sol:** Let the speeds of the two trains be $p$ and $q$ with $p > q$.
Total distance covered = sum of the lengths of the two trains = 100 + 250 = 350 m.
When they run in the same direction, relative speed is given by
$p - q = 350/70 = 5$ (1)
When they run in opposite direction relative speed is given by
$p + q = 350/10 = 35$ (2)
Solving (1) and (2)
we get, $p = 20$ m/s and $q = 15$ m/s.
Therefore, speeds of the trains are 72 kmph and 54 kmph, respectively.

**5.14.** A boat travels 36 km upstream in 9 hours and 42 km downstream in 7 hours. Find the speed of the boat in still water and the speed of the water current.

**Sol:** Upstream speed = 36/9 = 4 kmph
Downstream speed = 42/7 = 6 kmph
Speed of the boat in still water
= (4 + 6)/2 = 5 kmph
Speed of the water current = (6 – 4)/2 = 1 kmph

**5.15.** A man can row at 10 kmph in still water. If it takes a total of 5 hours for him to go to a place 24 km away and return, then find the speed of the water current.

**Sol:** Let the speed of the water current be $y$ kmph.
Upstream speed = $(10 - y)$ kmph
Downstream speed = $(10 + y)$ kmph

$$\text{Total time} = \frac{24}{10+y} + \frac{24}{10-y} = 5$$

$$\text{Hence, } \frac{480}{100-y^2} = 5 \Rightarrow y^2 = 4$$

Thus $y = 2$ kmph

**5.16.** A man rows 12 km in 1 hour in still water and in 80 minutes against the current. Find the time taken by him to row 45 km with the current and return to the starting point.

**Sol:** Speed in still water = 12/1 = 12 kmph
$\because$ 80 min = 4/3 hrs
Speed against the current = 12/(4/3) = 9 kmph
Therefore, the speed of water current = 12 – 9 = 3 kmph
Therefore, the speed with the current = 12 + 3 = 15 kmph
Hence, time taken to row 45 km with the current and return to the starting point
$= \frac{45}{15} + \frac{45}{9} = 8$ hours

**5.17.** In a given time, a boat can travel down the stream a distance, which is $2^1/_2$ times that of the upstream travel. If the speed of the boat in still water is 14 kmph, find the speed of the stream.

**Sol:** If the distance covered down the stream is $2^1/_2$ times that covered up the stream, then the speed down the steam will also be $2^1/_2$ times the speed up the stream.
Let the speed of the stream = $v$

$$\text{We get } \frac{14+v}{14-v} = \frac{5}{2} \Rightarrow V = 6 \text{ kmph}$$

**5.18.** A man can row $3/7^{th}$ of a kilometre upstream in 36 minutes and return in 12 minutes. Find the speed of the water current.

**Sol:** Upstream speed = (3/7)/ (36/60) = (5/7) kmph
Downstream speed = (3/7)/ (12/60) = (15/7) kmph
Speed of the water current

$$= (1/2)\left[(15/7) - (5/7)\right] = (5/7)\text{kmph}$$

**5.19.** An escalator is moving downwards. Sam takes 30 steps to reach the bottom of the escalator from the top and John takes 70 steps to reach the top from the bottom. The speeds of Sam and John are equal. What is the number of steps in the escalator when it is stationary?

**Sol:** Let $e$ be the number of steps the escalator moved for ten steps of Sam or John.
Let the number of steps in the escalator when it is stationary be $N = 30 + 3e$
Also, total steps $= 70 - 7e$
$\Rightarrow 30 + 3e = 70 - 7e$
$\Rightarrow 10e = 40 \Rightarrow e = 4$
$N = 30 + 3 \times e = 42$.

**5.20.** A takes 60 steps to reach the bottom of an escalator which is moving upwards, but $B$ takes 30 steps to reach the top of the escalator. If $A$ takes two steps for every step of $B$, then find the number of steps in the escalator when it is stationary?

**Sol:** Let the escalator move '$e$' steps when $A$ took 60 steps (or) $B$ took 30 steps $\therefore$ Number of steps in the escalator $= 60 - e$.
Also, the number of steps in the escalator $= 30 + e$
$\therefore 60 - e = 30 + e \Rightarrow 2e = 30 \Rightarrow e = 15$
$\therefore$ Number of steps $= 30 + 15 = 45$.

**5.21.** Ajay runs $(5/3)$ times as fast as Arjun. In a race if Ajay beats Arjun by 40 meters find the length of the race.

**Sol:** Since Ajay runs $(5/3)$ times as fast as Arjun, if the length of the race is a metres, $\frac{a}{a-40} = (5/3)$
$\Rightarrow 3a = 5a - 200 \Rightarrow a = 100$ m
Hence, the length of the race is 100 m

**5.22.** In a 1600 m race, $A$ beats $B$ by 160 m and in the same race $A$ beats $C$ by 340 m. By what distance did $B$ beat $C$ in the same race?

**Sol:** In the time $A$ runs 1600 m,
$B$ runs $1600 - 160 = 1440$ m
and $C$ runs $1600 - 340 = 1260$ m
When $B$ runs 1600 m,

$C$ would have run $\frac{1260 \times 1600}{1440} = 1400$ m

Hence, $B$ beats $C$ by $1600 - 1400 = 200$ m

**5.23.** In a race of 100 m, $A$ beats $B$ by 40 m or 10 seconds. Find the time taken by $A$ to complete the race and the speed of $B$.

**Sol:** Since $A$ beats $B$ by 40 meters, it means by the time $A$ reaches the winning point, $B$ is 40 m away and as $A$ beats $B$ by 10 seconds, it means $B$ takes 10 seconds more time than $A$ to reach the winning point.
Hence, $B$ covers 40 m in 10 seconds, i.e., $B$'s speed is $(40/10) = 4$ m/s
Time taken by $B$ to complete the race $= 100/4 = 25$ sec
Time taken by $A$ to complete the race $= 25 - 10 = 15$ seconds

**5.24.** In a 750 m race, the ratio of speeds of two runners $A$ and $B$ is 2: 3. $A$ has a head start of 150 m. Who wins the race and by what distance?

**Sol:** Since $A$ has a start of 150 m at the time $B$ starts at the starting point, $A$ is already 150 m ahead, and hence, he has only $750 - 150 = 600$ m to cover.
In the time $B$ covers 750 meters, $A$ would have covered $(2/3) \times 750 = 500$ m
Hence, he would be 100 m from the finishing point. Hence, $B$ wins the race and at the time he finishes the race the distance between him and $A$ would be 100 metres

**5.25.** In a circular race along a track of length of 3600 m, $A$ and $B$ start from the same point and at the same time with speeds of 36 km/hr and 45 km/hr. Find when they will meet for the first time on the track when they are running
(i) in the same direction.
(ii) in the opposite directions.

**Sol:** Length of the track $(L) = 3600$ m

Speed of $A = 36 \times 5/18 = 10$ m/s

Speed of $B = 45 \times 5/18 = 12.5$ m/s

(i) Same direction:

$$\text{Time} = \frac{\text{L}}{\text{Relative speed}}$$

$$= \frac{3600}{(12.5 - 10)} = 1440 \text{ seconds}$$

(ii) Opposite direction:

$$\text{Time} = \frac{\text{L}}{\text{Relative speed}}$$

$$= \frac{3600}{(12.5 + 10)} = \frac{3600}{22.5} = 160 \text{ seconds}$$

**5.26.** In a circular race along a track of length 3600 m, $A$ and $B$ run with speeds of 27 km/hr and 36 km/

hr, respectively starting at the same time from the same point. When will they meet for the first time at the starting point, if they run
(i) in the same direction.
(ii) in the opposite directions.

**Sol:** $L = 3600$ m

Speed of $A = 27 \times 5/18 = 7.5$ m/sec

Speed of $B = 36 \times 5/18 = 10$ m/sec

Time taken by $A$ to complete one round

$$= \frac{36000}{75} = 480 \text{ sec}$$

Time taken by $B$ to complete one round

$$= \frac{3600}{10} = 360 \text{ sec}$$

**(i) Same direction:** They will meet at the starting point at a time which is the L.C.M. of the timings taken by each of them to complete one full round, i.e., the L.C.M. of 480 sec and 360 sec which is 1440 seconds.

**(ii) Opposite direction:** They will meet at the starting point at a time which is the L.C.M. of the timings taken by each of them to complete one full round, i.e., the L.C.M. of 480 seconds and 360 seconds which is 1440 seconds.

(Please note that the time taken by them to meet at the starting point does not change in the two cases, i.e., it does not depend on whether the persons are running in the same direction or in opposite direction).

**5.27.** $A$, $B$ and $C$ with respective speeds of 18 km/hr, 27 km/hr and 36 km/hr run along a circular track 3600 m long. They start at the same time from the same point and run in the same direction. When will they meet for the first time?

**Sol:** $L = 3600$ m

Speed of $A$ $(a) = 18 \times 5/18 = 5$ m/s

Speed of $B$ $(b) = 27 \times 5/18 = 7.5$ m/s

Speed of $C$ $(c) = 36 \times 5/18 = 10$ m/s

They will meet for the first time at a time which is the L.C.M. of $\frac{L}{b-a}$ and $\frac{L}{c-b}$

$$\frac{L}{b-a} = \frac{3600}{7.5-5} = 1440 \text{ seconds}$$

$$\frac{L}{c-b} = \frac{3600}{10-7.5} = 1440 \text{ seconds}$$

Therefore, they will meet for the first time after 1440 seconds, i.e., 24 minutes after they start.

**5.28.** $P$, $Q$ and $R$ run along a circular track 3600 m long at respective speeds of 18 km/hr, 27 km/hr and 36 km/hr. If they start at the same point and at the same time in the same direction when will they meet again at the starting point?

**Sol:** L = 3600 m

Speed of $P$ $(p) = 18 \times 5/18 = 5$ m/s

Speed of $Q$ $(q) = 27 \times 5/18 = 7.5$ m/s

Speed of $R$ $(r) = 36 \times 5/18 = 10$ m/s

They will meet for the first time at the starting point after a time interval

which is the L.C.M. of $\frac{L}{p}, \frac{L}{q}$ and $\frac{L}{r}$

$$\frac{L}{p} = \frac{3600}{5} = 720 \text{ seconds}$$

$$\frac{L}{q} = \frac{3600}{7.5} = 480 \text{ seconds}$$

$$\frac{L}{r} = \frac{3600}{10} = 360 \text{ seconds}$$

L.C.M. of $\frac{L}{p}, \frac{L}{q}$ and $\frac{L}{r}$ is 1440 seconds.

Hence, they meet at the starting point for the first time after 1440 seconds, i.e., 24 minutes from the time they start.

**5.29.** Find the angle between the hands of the clock when the time is 4:40.

**Sol:** At exactly 4 O' clock the minute hand is at the vertical line. (on the marking 12).

From then till 4:40, it travels for 40 minutes. Since the minute hand travels 6° every minute, it covers 6° × 40 = 240° in 40 minutes. At exactly 4 O' clock the hour hand makes 120° with the vertical line (on the marking 4). We know that the hour hand covers 1/2° per minute. Hence, in 40 minutes it covers 20°. So, at 4:40, the hour hand makes an angle of 120° + 20° = 140° with the vertical line.
Hence, the angle between the minute hand and the hour hand = 240° – 140° = 100°.

**5.30.** At what time between 5 O' clock and 6 O' clock will the two hands of the clock be at an angle of 30° with each other?

**Sol:** Let the time be $P$ minutes after 5 O' clock when the hands make an angle of 30°.
The angle made by the hour hand with the vertical line when the time is 5 O' clock is 150°. Since $P$ minutes later the hour hand would have covered $\frac{P^\circ}{2}$ additionally, at $P$ minutes after 5 O' clock, the hour hand makes an angle of $\left(150+\frac{P}{2}\right)$ with the vertical line.
The angle made by the minute hand with the vertical line when the time is $P$ minutes after 5 O' clock is $6P^\circ$. For the angle between the two hands to be 30°, the difference between the two angles considered above should be equal to 30°.
Thus, we get

$$6P-\left(150+\frac{P}{2}\right)=30 \qquad (1)$$

$$\left(150+\frac{P}{2}\right)-6P=30 \qquad (2)$$

From (1) and (2) we get $P = 32\frac{8}{11}$ or $P = 21\frac{9}{11}$ min. Hence, the angle between the two hands of the clock will be 30° at $5:32\frac{8}{11}$ O' clock and $5:21\frac{9}{11}$ O' clock.

**5.31.** Find the time at which the hands of a clock are at right angles between 2 O' clock and 3 O' clock.

**Sol:** Let the time be $P$ minutes after 2 O' clock where the angle is 90°. Then we have two equations as found in the pervious example, (at exactly 2 O' clock the hour hand makes 60° with the vertical line)

$$6P-\left(60+\frac{P}{2}\right)=90 \qquad (1)$$

$$\left(60+\frac{P}{2}\right)-6P=90 \qquad (2)$$

From (1) we get $P = 27\frac{3}{11}$

From (2), 60 – 90 = –30
As the position of (–30°) is same as (+330°) ( i.e., – 30° + 360°)
The equation becomes, + 330° = $(11p/2) \Rightarrow p$ = 60.
This implies that at 2 hours + 60 minutes, which is same as 3 O' clock the angle is 90°.
As the question is about the time between 2 O' clock and 3 O' clock, at only $27\frac{3}{11}$ minutes past 2 O' clock will the hands of the clock be at right angles.

**5.32.** Find the time at which the hands of a clock are exactly in opposite direction between 9 O' clock and 10 O' clock.

**Sol:** The hands are in opposite direction when the angle between them is 180°. Let this happen $P$ minutes after 9 O' clock. We have the angle made by the hour hand with the vertical line to be 270° at 9 O' clock.

Hence, $\left(270+\frac{P}{2}\right)-6P=180 \qquad (1)$

$$6P-\left(270+\frac{P}{2}\right)=180 \qquad (2)$$

From (1) and (2), we get

$P = +16\frac{4}{11}$ (possible) and $P = 81\frac{1}{11}$

(not possible as the hands coincide after 10 O' clock)

Hence, required time is $9:16\frac{4}{11}$ O' clock.

**5.33.** At what time between 1 O' clock and 2 O' clock do the hands of a clock coincide with each other?

**Sol:** To coincide with each other, the angle between them should be 0°. Let this happen $P$ minutes after 1 O' clock. We have the angle made by the hour hand with the vertical line as 30° at 1 O' clock.

Hence, $\left(30+\frac{P}{2}\right)-6P=0 \Rightarrow P=5\frac{5}{11}$ minutes

Hence, the two hands will coincide at 1:05 $\frac{5}{11}$ O' clock.

**5.34.** In 12 hours, how many times do the hands of a clock make an angle of 180°?

**Sol:** Between any two hours, the two hands of the clock make an angle of 180° for one time. In 12 hours the hands of the clock make an angle of 180° only 11 times, since, between 5 O' clock and 7 O' clock, the hands of the clock will be on a straight line for only one time, i.e., at 6 O' clock.

**5.35.** If the hands of a clock coincide for every 64 minutes, then how much time does the clock gain or lose per day?

**Sol:** A clock which shows correct time has its hands coinciding every $(65\frac{5}{11}$ minutes), if the hands of a clock coincide for every 64 minutes, it means $65\frac{5}{11}$ minutes of that clock = 64 minutes of the correct time.

Hence, the clock gains $65\frac{5}{11}-64=\frac{16}{11}$ minutes for every 64 minutes.

Hence, in one day it gains $\frac{24\times 60\times \frac{16}{11}}{64}$

$=\frac{360}{11}=32\frac{8}{11}$ minutes.

**5.36.** Express 72km/hr in m/sec.

**Sol:** $72 \text{ km/hr} = (72)\left(\frac{5}{18}\right)$, i.e., 20 m/sec

**5.37.** A car can cover 90 km in 3 hours. If its speed is increased by 15 kmph, find the time taken by it to cover this distance.

**Sol:** Speed of the car = 30 kmph

If its speed is increased by 15 kmph,

time taken $=\frac{90}{30+15}=2$ hours.

**5.38.** Ashok covered a certain distance at a certain speed. If his speed was 20% more, he would take 10 minutes less to cover the same distance. Find the time he takes to cover the distance.

**Sol:** If his speed was 20% more, it would be 120%, i.e., $\frac{6}{5}$ times his actual speed.

$\therefore$ Time taken would be $\frac{5}{6}$ times his actual time.

Reduction in time $=\frac{1}{6}$ (actual time)

= 10 minutes

$\therefore$ Actual time = 60 minutes.

**5.39.** A car covered a certain distance at 90 kmph and returned back at 60 kmph. Find its average speed for the entire journey.

**Sol:** Let $x$ km be the distance to be covered, each way.

Total time of travel (in hours)

$=\frac{x}{90}+\frac{x}{60}=\frac{5x}{180}=\frac{x}{36}$

Average speed (in km / hr)

$=\frac{\text{Total distance travelled}}{\text{Total time taken}}=\frac{2x}{\frac{x}{36}}=72$

**5.40.** Find the time taken by a 200 m long train running at 36 kmph to cross a boy standing on a platform.

**Sol:** Speed of the train $=(36)\left(\frac{5}{18}\right)=10$ m / sec

Distance to be travelled by the train to cross the stationary boy = length of the train

Time taken by the train to cross the boy

$=\frac{200}{10}=20$ sec

**5.41.** Find the time taken by a train, 100 m long, travelling at a speed of 63 kmph to cross a platform of length 250 m.

**Sol:** Speed of the train $=(63)\left(\frac{5}{18}\right)=\frac{35}{2}$ m / sec

Distance to be travelled by the train to cross the platform = length of the train + length of the platform.

Time taken to cross the platform $=\frac{100+250}{\frac{35}{2}}=$ 20 seconds

**5.42.** Find the length of the platform which a train 400 m long, travelling at 45 kmph can cross in 40 seconds.

**Sol:** Speed of the train $=(45)\left(\frac{5}{18}\right)=12\cdot 5$ m / sec

Length of the platform = distance travelled by the train – length of the train
= (12×5) (40) – 400 = 100 m

**5.43.** Ashok would reach his office 15 minutes early if he walked at 4 kmph from his house. He would reach it 45 minutes late if he walked at 3 kmph from his house. Find the distance between his house and office.

**Sol:** Let the distance be $x$ km. Time taken by Ashok if he walked at 4 kmph = $\frac{x}{4}$ hours.

Time taken by Ashok if he walked at 3 kmph = $\frac{x}{3}$ hours. In this case he would take one hour more to reach his office compared to the time taken if he had walked at 4 kmph.

$\therefore \frac{x}{3} - \frac{x}{4} = 1 \Rightarrow x = 12.$

**5.44.** Find the time taken by a train 175 m long running at a speed of 54 kmph to overtake another train 75 m long running at a speed of 36 kmph.

**Sol:** Relative speed = 18 kmph = 5 m/sec
Time taken for the faster train to overtake the slower train =

$$\frac{(\text{Length of the faster train}) + (\text{Length of the slower train})}{\text{Their relative speed}}$$

$$= \frac{175+75}{5} = 50 \text{ seconds}$$

**5.45.** A train overtakes two persons, cyling at 9 kmph and 18 kmph in 40 seconds and 48 seconds, respectively. Find its length and speed.

**Sol:** Let the length and the speed of the train be $l$ m and $s$ kmph, respectively.

$$l = 40(s-9)\frac{5}{18} = 48(s-18)\frac{5}{18}$$

$$\Rightarrow \frac{s-9}{s-18} = \frac{48}{40} \Rightarrow s = 63$$

$$\therefore l = 40\ (63-9) \times \frac{5}{18} = 600 \text{ m}$$

**5.46.** Two trains running at 36 kmph and 45 kmph cross each other in 20 seconds when they run in the opposite directions. When the trains run in the same direction, a person in the faster train observed that he overtook the slower train in 48 seconds. Find the lengths of the trains.

**Sol:** Let the lengths of the faster and the slower trains be $f$ and $s$, respectively.

Given that, $\frac{f+s}{(36+45)\frac{5}{18}} = 20$

$f + s = 450$ (1)

When the trains run in the same direction, the time taken by the person in the faster train to overtake the slower train = $\frac{s}{(45-36)\frac{5}{18}} = 48$

$s = 120$
From (1), $f = 330$

**5.47.** Two trains have lengths of 300 m and 200 m. When they run in the same direction, the faster train will take 100 seconds to cross the slower train. When they run in the opposite directions, they will take 20 seconds to cross each other. Find the speeds of the trains.

**Sol:** Let the speeds of the trains be $x$ m/sec and $y$ m/sec where $x > y$.

$$\frac{300+200}{x-y} = 100$$

$5 = x - y$ (1)

$$\frac{300+200}{x+y} = 20$$

$25 = x + y$ (2)

Solving (1) and (2)
$x = 15$ and $y = 10$

**5.48.** A boat travels 30 km upstream in 5 hours and 100 km downstream in 10 hours. Find the speed of the boat in still water and the speed of the stream.

**Sol:** Upstream speed = $\frac{30}{5} = 6$ kmph

Downstream speed = $\frac{100}{10} = 10$ kmph

Speed in still water = $\frac{6+10}{2} = 8$ kmph

Speed of the stream = $\frac{10-6}{2} = 2$ kmph

**5.49.** Anand can row 20 km in 2 hours in still water. If the speed of the stream is 6 kmph, he would take 3.75 hours to cover a round trip journey. Find the distance that he would then cover each way.

**Sol:** Speed of the boat in still water = $\frac{20}{2} = 10$ kmph

Let the total distance covered be $2x$ km.

Given that, $\frac{x}{10+6} + \frac{x}{10-6} = 3 \cdot 75$

$x = 12$

**5.50.** Arun rowed a distance of 36 km in still water in 3 hours. He rowed 30 km in 2 hours with the current. Find the time he would take to row 27 km against the current.

**Sol:** Let the speed of his boat in still water and the speed of the current be $x$ kmph and $y$ kmph, respectively

$x = \frac{36}{3} = 12$

Downstream speed = $x + y = \frac{30}{2} = 15$

$\Rightarrow y = 3$ (As $x = 12$)

Required time = $\frac{27}{x-y} = \frac{27}{12-3} = 3$ hours

**5.51.** In a certain time, a boat can cover either a certain distance upstream or $\frac{5}{2}$ times that distance downstream. If the speed of the current is 6 kmph, find the speed of the boat in still water.

**Sol:** Let the speed of the boat in still water be $x$ kmph.

$\frac{x+6}{x-6} = \frac{5}{2} \Rightarrow 2x + 12 = 5x - 30$

$\therefore x = 14$

**5.52.** Balu can row 2 km upstream in 20 minutes and can return in another 15 minutes. Find his average speed for the entire journey.

**Sol:** Average speed of Balu

$= \frac{2+2}{\frac{20}{60} + \frac{15}{60}} = \frac{48}{7}$ kmph

**5.53.** An escalator is moving downwards. Anil takes 60 steps to reach the bottom of the escalator starting from the top. Bala takes 140 steps to reach the top of the escalator starting from the bottom. The speeds of Anil and Bala are equal. Find the number of steps visible when the escalator is stationary.

**Sol:** Let the number of steps that the escalator moves for ten steps of Anil or Bala be $e$.
Let the number of steps visible when the escalator is stationary be $N$.
$N = 60 + 6e = 140 - 14e$
$20e = 80$
$e = 4$
$N = 60 + 6e = 84$

**5.54.** $A$ takes 120 steps to reach the bottom of an escalator which is moving upwards. $B$ takes 60 steps to reach the top of the escalator. If $A$ takes two steps for every step of $B$, find the number of steps visible when the escalator is stationary.

**Sol:** Let us say the number of steps that the escalator takes when $A$ takes 120 steps is $e$. In the time that $A$ takes 120 steps, $B$ takes 60 steps.
Let the number of steps on the escalator when it is stationary be $N$.
$N = 120 - e = 60 + e$
e = 30
$N = 90$

**5.55.** In a 200 m race, $A$ beats $B$ by 10 m or 2 seconds. Find B's speed and A's speed.

**Sol:** A beats $B$ by 10 m or 2 seconds.
$\Rightarrow$ When $A$ reached the finishing line $B$ was 10 m behind the finishing line and took 2 seconds to cover it.

$\therefore$ B's speed = $\frac{10}{2} = 5$ m/sec

Time taken by $B$ to complete the race = $\frac{200\,\text{m}}{5\,\text{m/s}}$
= 40 seconds
$\therefore$ Time taken by $A$ to complete the race
= 38 seconds

A's speed = $\frac{200}{38} = \frac{100}{19}$ m/sec

**5.56.** Ramu is 50% faster than Somu. In a race, Ramu gave Somu a head start of 200 m. Both finished the race simultaneously. Find the length of the race.

**Sol:** Let the length of the race be $x$ m.

$\frac{x}{x-200} = \frac{150}{100}$

$\Rightarrow x = 600$.

**5.57.** In a 1200 m race, Ram beats Shyam by 300 m. In the same race, Shyam beats Tarun by 400 m. Find the distance by which Ram beats Tarun.

**Sol:** Let the speeds of Ram, Shyam and Tarun be $r$ m/sec, $s$ m/sec, and $t$ m/sec, respectively

$$\frac{r}{s}=\frac{1200}{1200-300}=\frac{4}{3}$$

$$\frac{s}{t}=\frac{1200}{1200-400}=\frac{3}{2}$$

$$\frac{r}{t}=\left(\frac{r}{s}\right)\left(\frac{s}{t}\right)=2$$

$\Rightarrow$ By the time Ram covers 1200 m, Tarun covers 600 m.

$\therefore$ Ram beats Tarun by (1200 – 600), i.e., by 600 m.

**5.58.** In a 600 m race, $P$ gives $Q$ a start of 200 m. Ratio of the speeds of $P$ and $Q$ is 5 : 4. Who wins the race? By what distance does the winner beat the loser?

**Sol:** Q has to run 400 m to finish the race. In the time $Q$ runs 400 m, $P$ can run 500 m.

$\therefore$ When $Q$ finished the race, $P$ would have another 100 m to run.

$\therefore$ $Q$ wins the race and he beats $P$ by 100 m.

**5.59.** On a circular track of length 1800 m, $X$ and $Y$ start from the same point simultaneously with speeds of 36 kmph and 45 kmph, respectively. Find the minimum time after which they will meet if they are running in

(i) the same direction.

(ii) opposite direction to each other.

**Sol:** (i) Time taken to meet for the first time

$$=\frac{1800}{(45-36)\frac{5}{18}}=720 \text{ seconds}$$

(ii) Time taken to meet for the first time

$$\frac{1800}{(45+36)\frac{5}{18}}=80 \text{ seconds}$$

**5.60.** In the previous example, find the time after which they will meet at the starting point for the first time.

**Sol:** Time taken to meet at the starting point for the first time

$$=\text{L.C.M}\left(\frac{1800}{(36)\left(\frac{5}{18}\right)},\frac{1800}{(45)\left(\frac{5}{18}\right)}\right)$$

= L.C.M. [180, 144] = 720 seconds

**5.61.** On a circular track of length 1200 m, $P$, $Q$ and $R$ start from the same point simultaneously with speeds of 18 kmph, 27 kmph and 36 kmph, respectively. Find the minimum time after which they will meet if they are running in the same direction.

**Sol:** If three runners with speeds of $a$ m/sec, $b$ m/sec and $c$ m/sec, respectively start from the same point in the same direction on a circular track and $a > b > c$, they will meet for the first time after

$$\text{L.C.M.}\left(\text{any two of } \frac{L}{a-b},\frac{L}{b-c},\frac{L}{a-c}\right) \text{seconds}$$

In the problem given, the required time

$$=\text{L.C.M.}\left(\frac{1200}{(36-27)\frac{5}{18}},\frac{1200}{(27-18)\frac{5}{18}}\right)$$

= L.C.M. [480, 480] = 480 seconds

**5.62.** Find the angle between the hands of a clock when the time is 3:40 p.m.

**Sol:** From 12:00 p.m. to 3:00 p.m. the minute hand would effectively cover 3 (0°) = 0°. In the next 40 minutes, it would cover 40 (6°) = 240°

$\therefore$ The minute hand would have covered a total of 240° effectively by 3:40 p.m. Similarly the hour hand would cover a total of 110° effectively by 3:40 p.m. from the 12:00 noon.

$\therefore$ The angle between the hands is 240° – 110°, i.e., 130° at 3:40 p.m.

**5.63.** At what time between 5 O' clock and 6 O' clock in the evening will the hands of a clock be at an angle of 90° with each other?

**Sol:** Let the time be 5:$x$ p.m.

Angle effectively covered by the hour hand by 5:$x$ p.m. $=\left(150+\frac{x}{2}\right)^{\circ}$

Angle effectively covered by the minute hand by 5:$x$ p.m. = 6 x°

$$\left(150+\frac{x}{2}\right)^{\circ}-6x^{\circ}=90^{\circ} \quad \text{(or)}$$

$$6x^{\circ}-\left(150+\frac{x}{2}\right)^{\circ}=90^{\circ}$$

$$x=\frac{120}{11} \text{ or } \frac{480}{11}$$

$\therefore$ The required time is

$$5:10\frac{10}{11} \text{ p.m. or } 5:43\frac{7}{11} \text{ p.m.}$$

**5.64.** At what time between 8 O' clock and 9 O' clock in the evening will the hands of a clock be in a straight line and in opposite directions?

**Sol:** Let the time be 8:$x$ p.m.

Angle between the hands = 180°

Angle effectively covered by the hour hand by 8:$x$ p.m. $=\left(240+\frac{x}{2}\right)$

Angle effectively covered by the minute hand by 8:$x$ p.m. = 6x°

$$\left(240+\frac{x}{2}\right)^{\circ}-6x^{\circ}=180^{\circ} \quad \text{(or)}$$

$$6x^{\circ}-\left(240+\frac{x}{2}\right)^{\circ}=180^{\circ}$$

$$\frac{11}{2}x=60^{\circ} \text{ or } 420$$

$$x=\frac{120}{11} \text{ or } \frac{840}{11}$$

If $x=\frac{840}{11}$, $x>60$ which is not possible.

$$\therefore x=\frac{120}{11}=10\frac{10}{11}$$

$\therefore$ The required time is $8:10\frac{10}{11}$ p.m

**5.65.** In 24 hours, how many times do the hands of a clock coincide?

**Sol:** In 12 hours, the hands of a clock coincide 11 times. In 24 hours, they coincide (11) (2) = 22 times.

**5.66.** If the hands of a clock coincide every 66 minutes, how much time does the clock gain/lose per day?

**Sol:** In a clock showing correct time the hands coincide after every $65\frac{5}{11}$ minutes. If the hands coincide every 66 minutes, it take 66 minutes to cover $65\frac{5}{11}$ minutes of correct time.

$\therefore$It is losing time. It loses $\frac{6}{11}$ minutes every 66 minutes

$$\therefore \text{In a day it loses } \frac{\frac{6}{11}}{66}(24)(60)=11\frac{109}{121} \text{ minutes}$$

**5.67.** At a certain time between 9 a.m. and 10 a.m. the angle between the hands of a clock is 6°. This time can be _____.

**Sol:** Let all possible times be denoted by 9 : $x$ a.m. From 12 midnight to 9 : $x$ a.m, the hour hand would have covered $\left(270+\frac{x}{2}\right)^{\circ}$, from 9:00 a.m. to 9:$x$ a.m. the minute hand would have covered $(6x)^{\circ}$.

$$(6x)-\left(270+\frac{x}{2}\right)=6 \Rightarrow \frac{11}{2}x=276$$

$$\Rightarrow x=50\ \frac{2}{11} \text{ or}$$

$$270+\frac{x}{2}-6x=6 \Rightarrow \frac{11x}{2}=264 \Rightarrow x=48$$

$\therefore$The time can be $9:50\ \frac{2}{11}$a.m (or 9 : 48 a.m.)

## EXERCISE-1

***Directions for questions 1 to 59:*** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Convert the following speeds into metres per second

(a) 54 kmph = _____ m/sec

(b) 108 kmph = _____ m/sec

(c) 21.6 kmph = _____ m/sec

**2.** Convert the following speeds into kilometres per hour.

(a) 20 m/s = _____ kmph.

(b) 45 m/s = _____ kmph.

(c) 12.5 m/s = _____ kmph.

(*d*) 13/36 m/s = _____ kmph.

**3.** Gopal travelled first half of his journey time at an average speed of 40 kmph and the remaining half at an average speed of 50 kmph. Find his average speed (in kmph) for the journey. _____

**4.** A train takes 40 seconds to overtake another train. If the slower train doubles its speed, it would overtake the other train in 80 seconds. Find the ratio of the speeds of the faster train and the slower train.

(A) 5 : 3 (B) 5 : 4 (C) 3 : 2 (D) 4 : 3

**5.** In a day, how many times will the hour hand and minute hand be at an angle of 73°?

(A) 22 (B) 44

(C) Neither (A) nor (B) (D) Cannot be determined

**6.** A gives *B* a start of 10 m or 2 seconds in a 100 m race. Find *A*'s speed if both finish simultaneously.

(A) $3\frac{1}{3}$ m/sec (B) $5\frac{5}{9}$ m/sec

(C) $6\frac{5}{9}$ m/sec (D) $4\frac{1}{9}$ m/sec

**7.** Pavan travelled for 11 hours. He covered the first half of the distance at 30 kmph and the remaining half of the distance at 25 kmph. The distance travelled by Pavan is _____ . (in km).

**8.** In a 600 m race, *A* gives *B* a start of 200 m and beats him by 120 m. Find the ratio of the speeds of *A* and *B*.

(A) 15 : 4 (B) 15 : 7 (C) 10 : 3 (D) 12 : 5

**9.** The minute and hour hands of a clock coincide after every 65 minutes. Is the clock gaining time?

(A) Yes (B) No (C) Cannot say

**10.** The downstream speed of a boat is 2 kmph more than its upstream speed. Find the speed of the stream.

(A) 2 kmph (B) 1 kmph

(C) 4 kmph (D) 6 kmph

**11.** A boat takes 4 hours more to travel a distance of 40 km upstream than to travel the same distance downstream. If the speed of the boat in still water is thrice the speed of the stream, find the speed of the stream.

(A) 2 kmph (B) 2.5 kmph

(C) 3 kmph (D) 4 kmph

**12.** The ratio of the speeds of *A* and *B* is 3 : 7. If *B* takes 20 minutes less than *A* to cover a certain distance, then what is the time taken (in minutes) by *A* to cover the distance?

_____

**13.** Three cyclists with respective speeds of 5 m/sec, 10 m/sec and 20 m/sec are cycling around a circular track of length 100 metres. If they are cycling in the same direction, after how many seconds will all the three meet for the first time?

(A) 60 (B) 30 (C) 40 (D) 20

**14.** If the speed of a train is doubled, it would have crossed a 800 m long bridge in 40 seconds. Find the usual speed of the train given that the length of the train is 200 m.

(A) 11 m/sec (B) 12.5 m/sec

(C) 13 m/sec (D) 50 m/sec

**15.** Three cyclists start cycling simultaneously from the same point on a circular track 900 m long in the same direction with speeds of 10 m/sec, 20 m/sec and 15 m/sec, respectively. How long will they take (in seconds) before they meet for the first time?

(A) 90 (B) 180 (C) 360 (D) 45

**16.** In question 15, how long will the cyclists take (in seconds) to meet at the starting point for the first time? _____

**17.** A train takes 40 seconds to cross a stationary pole. Find the time it will take to cross a platform whose length is equal to that of the train.

(A) 1 minute

(B) 1 minute 20 seconds

(C) 1 minute 30 seconds

(D) 1 minute 40 seconds

**18.** In a day, how many times do the hour hand and the minute hand coincide?
(A) 22 (B) 44
(C) Neither (A) nor (B) (D) Cannot be determined

**19.** Three cars have the ratio of their speeds as 5 : 6 : 7. Find the ratio of the times they would take to travel a certain distance.
(A) 7 : 6 : 5 (B) 30 : 35 : 42
(C) 5 : 6 : 7 (D) 42 : 35 : 30

**20.** Travelling at a speed of 8 kmph, a student reaches school from his house 10 minutes early. If he travels at 6 kmph, he is late by 20 minutes. The distance between the school and the house is _____.

**21.** Travelling at three-fifth of his usual speed a man is late by 20 minutes. What is the usual time (in minutes) to cover the same distance? _____

**22.** A person sitting in a train *P* observes that a train *Q* with thrice the speed of the train *P* overtakes him in 20 seconds. If the trains *P* and *Q* are travelling in opposite directions find the time taken by the train *Q* to cross him.
(A) 15 seconds (B) 20 seconds
(C) 10 seconds (D) 30 seconds

**23.** By travelling at an average speed of 40 kmph, a person reaches his destination on time. He covered two-third of the total distance in one-third of the total time. The speed he should maintain for the remaining distance to reach his destination on time is _____ Kmph.

**24.** Anand covered 2*x* km at a speed of 20 kmphand 3*x* km at a speed of 30 kmph. Find the average speed of Anand for the entire journey.
(A) 24 kmph (B) 22 kmph
(C) 28 kmph (D) 25 kmph

**25.** A car reaches its destination one hour late, if it travels at 80 kmph. If its speed was 100 kmph which is its usual speed, it reaches on time. Find the usual time (in hours) it takes to travel to its destination.
(A) 3.5 (B) 4 (C) 4.5 (D) 5

**26.** *A* and *B* cover a certain distance at respective speeds of 30 kmph and 20 kmph. If *A* took three hours less than *B* to cover the distance, find the distance travelled.
(A) 180 km (B) 120 km
(C) 150 km (D) 210 km

**27.** Kiran travels from *A* to *B* by car and returns from *B* to *A* by cycle in 7 hours. If he travels both ways by car, he saves 3 hours. The time taken to cover both ways by cycle is _____ hours.

**28.** In a race, *A* beats *B* by 10 seconds and *B* beats *C* by 20 seconds. By how many seconds did *A* beat *C*?
(A) 10 (B) 20
(C) 30 (D) Cannot be determined

**29.** Two men start simultaneouslyfrom a point in opposite directions at speeds of 1.2 m/sec and 1.5 m/sec. Find the distance (in m) between them after 10 seconds.
(A) 27 (B) 7.5 (C) 45 (D) 65

**30.** If a student cycles from his house to his school at a speed of 4 kmph, then he is late by 10 minutes. If he cycles at a speed of 6 kmph, then he is early by 20 minutes. What is the distance between his house and his school?
(A) 8 km (B) 6 km (C) 10 km (D) 4 km

**31.** At 3:30 p.m., what is the angle between the hour hand and the minute hand of a clock?
(A) 75° (B) 90° (C) 105° (D) 60°

**32.** Find the number of times that the hands of a clock are 180° apart in a day. _____

**33.** The distance between Hyderabad and Tirupati is 600 km. A person 'X' leaves Hyderabad for Tirupati at 8:00 a.m. at a speed of 60 kmph and another person 'Y' leaves Tirupati for Hyderabad at 10:00 a.m. at a speed of 20 kmph. At what time will the two persons meet?
(A) 2:00 p.m. (B) 4:00 p.m.
(C) 3:00 p.m. (D) 1:00 p.m.

**34.** A train, 250 m long, is running at 30 kmph. The time in which the train will cross another train of length 150 m, running at a speed of 42 kmph, in the opposite direction to the train is _____ sec.

**35.** While covering a certain distance, if a person increases his speed from 50 kmph to 60 kmph,he saves 5 minutes. The distance covered by him is _____ km.

**36.** A 180 m long train is running at a speed of 40 kmph. In how much time will it cross a person moving at a speed of 4 kmph in the same direction?
(A) 15 sec (B) 12 sec
(C) 21sec (D) 18 sec

**37.** A train 400 m long travels at a speed of 36 kmph. Find the time (in seconds) it would take to cross a platform 600 m long. _____

**38.** By doubling his speed, Ajay reaches his office half an hour earlier than usual. Find the time taken by him to travel to his office at thrice his usual speed.
(A) 30 minutes (B) 40 minutes
(C) 20 minutes (D) 25 minutes

**39.** In question 28, how long will the cyclists take (in seconds) to meet at the starting point for the first time? _____

**40.** Ajay covered half of a certain distance at 2 km/hr, one-fourth of the distance at 4 km/hr and the remaining one-fourth of the distance at 1 km/hr. His average speed (in km/hr) for the entire journey is

(A) $\frac{16}{9}$ (B) $\frac{9}{4}$ (C) $\frac{8}{7}$ (D) $\frac{16}{13}$

**41.** The ratio of the speeds of $A$ and $B$ is 3 : 2. $A$ and $B$ start simultaneously towards each other from $P$ and $Q$, respectively. If they meet 20 km from $Q$, then the distance between $P$ and $Q$ is _____ km.

**42.** Kalyan leaves Chennai at 8:00 a.m. at a speed of 40 kmph. After 2 hours, Murali leaves Chennai in the same direction at a speed of 60 kmph. When will they meet each other?

(A) 12:00 noon (B) 1:00 p.m.
(C) 2:00 p.m. (D) 3:00 p.m.

**43.** The speeds of three runners A, B and C are in the ratio 1 : 2 : 3. If A takes 2 hours more than C to cover a certain distance, find the time taken by B to cover the same distance.

(A) 2 hours (B) $\frac{3}{2}$ hours
(C) $\frac{5}{2}$ hours (D) 3 hours

**44.** Ajay and Vijay start running simultaneously from a point on a circular track 1000 m long with speeds of 5 m/sec and 3 m/sec, respectively. Find the time (in seconds) taken by them to meet for the first time, if they are running in

(i) _____ the same direction.
(ii) _____ opposite directions.

**45.** A train travelling at 8 m/sec takes 30 seconds to overtake a boy. If the speed of the boy was 1 m/sec less, it would have taken 25 seconds to overtake him. Find the speed of the boy.

(A) 3 m/sec (B) 4 m/sec
(C) 2 m/sec (D) 1 m/sec

**46.** A train running at a speed of 36 kmph crosses an electric pole in 12 seconds. The time it will take to cross a platform 350 m long is _____ sec.

**47.** Karuna travelled from A to B at a speed of 20 kmph and returned to A from B at a speed of 60 kmph. What is the average speed of Karuna for the entire journey?

(A) 40 kmph (B) 25 kmph
(C) 35 kmph (D) 30 kmph

**48.** The ratio of speeds of Tarun and Varun is 4 : 3. Tarun starts to chase Varun who is at a distance of 10 km from Tarun and is running away from him. In order to catch Varun, how many kilometres should Tarun cover?

(A) 30 km (B) 40 km
(C) 50 km (D) Cannot be determined

**49.** Anand can row his boat in still water at a speed of 6 kmph. If the speed of the stream is 4 kmph, how long (in hours) will he take to row a distance of 30 km downstream? _____

**50.** A train would take 30 seconds to cross a stationary pole. If the train was 200 m longer, it would have taken 10 seconds more to cross the pole. Find the speed of the train.

(A) 20 m/sec (B) 18 m/sec
(C) 15 m/sec (D) 12 m/sec

**51.** $P$ beats $Q$ by 125 m in a kilometer race. Find $Q$'s speed (in m/sec) if $P$'s speed is 16 m/sec. _____

**52.** In a clock, after every how many minutes do the hour hand and minute hand coincide?

(A) $64\frac{6}{11}$ (B) $55\frac{6}{11}$
(C) $55\frac{5}{11}$ (D) $65\frac{5}{11}$

**53.** $P$ started running towards North at 6:00 a.m. $Q$ started running towards North at 8:00 a.m. At what time do they meet, if their speeds are in the ratio 3 : 5?

(A) 11:00 a.m. (B) 10:00 a.m.
(C) 12:00 a.m.. (D) Cannot be determined

**54.** In a 100 m race, $A$ beats $B$ by 10 m and $B$ beats $C$ by 10 m. By what distance does $A$ beat $C$ (in m)? _____

**55.** A train crosses a 300 m long bridge in 40 seconds. If its speed is 11 m/sec, find its length.

(A) 130 m (B) 120 m
(C) 140 m (D) 100 m

**56.** A train, 375 m long, is running at 50 kmph. The time in which the train will cross a person moving at 4 kmph in the opposite direction to the train is _____ sec.

**57.** In a clock, for every how many minutes is the angle between the hour hand and the minute hand 90°?

(A) $32\frac{8}{11}$ (B) $33\frac{1}{11}$ (C) $33\frac{3}{11}$ (D) $34\frac{6}{11}$

**58.** A travelled for 3 hours at a speed of 60 kmph, and then for 4 hours at a speed of 95 kmph. What is the average speed of the journey?

(A) 155 kmph (B) 77.5 kmph
(C) 80 kmph (D) 100 kmph

**59.** Ganesh and Girish are running along a circular track 600 m long starting from the same point simultaneously. If their speeds are 8 m/sec and 4 m/sec, respectively, how many more rounds than Girish will Ganesh complete in an hour? _____

## EXERCISE-2

***Directions for questions 1 to 60*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Find the time at which the hands of a clock are at right angles between 7 and 8 O' clock.

(A) $7 : 16\ \frac{4}{11}$ and $7 : 53\ \frac{7}{11}$

(B) $7 : 09\ \frac{1}{11}$ and $7 : 58\ \frac{2}{11}$

(C) $7 : 21\ \frac{9}{11}$ and $7 : 54\ \frac{6}{11}$

(D) $7 : 18\ \frac{2}{11}$ and $7 : 52\ \frac{8}{11}$

**2.** Arpita and Bipasha start from $P$ towards $Q$ simultaneously with their speeds in the ratio 3 : 5. Bipasha reaches $Q$, turns back and meets Arpita at a distance of 15 km from $Q$. When Arpita reaches $Q$, how far from $P$ will Bipasha be?
(A) 12 km (B) 20 km (C) 24 km (D) 40 km

**3.** Two trains $T_1$ and $T_2$ of the same length are moving in opposite directions $T_1$ and $T_2$ crossed a standing man in 4 seconds and 6 seconds, respectively. If the speed of $T_2$ is 90 km/hr, find the speed of $T_1$.(in km/hr). _____

**4.** A train P leaves station A at 4:00 a.m. and reaches station B at 8:00 a.m. Another train Q leaves station B at 5:00 a.m. and reaches station A at 11:00 a.m. At what time did the two trains cross each other? _____: _____ a.m.

**5.** A person travels a distance $AB$ of 15.625 m in 5 seconds. In the last four seconds, every second he covers a distance equal to one-fourth of the total distance covered upto the end of the previous second. Find the distance covered in the 1st second. (in m) _____

**6.** If A travels at 50 kmph, he reaches his destination at 4 p.m. and if he increases his speed to 60 kmph, he reaches one hour earlier. Find the distance he has to travel to reach his destination. _____ km

**7.** $P$ and $Q$ are running around a circular track of length 400 m in opposite directions with initial speeds of 10 m/s and 40 m/s, respectively. Whenever they meet, $P$'s speed doubles and $Q$'s speed halves. After what time (in seconds) from the start will they meet for the third time? _____

**8.** $A$ and $B$ start walking simultaneously from point $P$ towards point $Q$. The ratio of the speeds of $A$ to $B$ is 7 : 1. After reaching $Q$, $A$ starts walking back towards $P$ and meets $B$. If $PQ = 200$ m, find the distance of the meeting point from $P$. _____(in m)

**9.** Raj made an upstream journey of 30 km in a certain time. If his speed in still water was 2 kmph less and the speed of the stream was 2 kmph more, he would have taken ten hours more for that journey. Find his upstream speed (in kmph). _____

**10.** At what time between 4 O'clock and 5 O'clock, will the hands of a clock bein a straight line?

(A) $4 : 45\ \frac{5}{11}$ (B) $4 : 54\ \frac{6}{11}$

(C) $4 : 58\ \frac{2}{11}$ (D) $4 : 47\ \frac{3}{11}$

**11.** A person covers a total distance $x$ in the following manner. He covers the first one-third of the distance at 50 kmph, 40% of the remaining at 40 kmph and the remaining distance at 60 kmph. If he covers a total distance of 260 km, find his average speed over the entire journey. _____ kmph

**12.** Three men start at the same point on a 3 km long circular track with speeds of 6 km/hr, 4 km/hr and 5 km/hr and walk in the same direction. After how much time do they meet for the first time at the starting point? (in hours) _____.

**13.** A train takes 20 seconds to cross a 100 m long bridge. It takes 30 seconds to cross a 200 m long bridge. Find the time that the train takes to pass an electric pole. (in seconds) _____

**14.** Two points $A$ and $B$ are diametrically opposite points on a circular road of circumference 12 km. $A$ cyclist started from $A$ and made three rounds. He made the first round

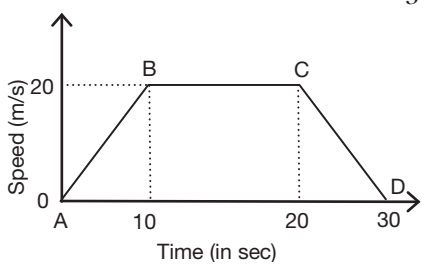
with a speed of 12 kmph and decreased his speed by 3 kmph after every round. What is the interval between the first time he passes through *B* and the third time he passes through *B*?

(A) 200 minutes (B) 100 minutes
(C) 85 minutes (D) 170 minutes

**15.** A bus $B_1$, left a town $T_1$ for $T_2$ at 6:00 a.m. at a speed of 36 kmph while another bus $B_2$ left $T_2$ for $T_1$ at 7:30 a.m. at speed of 24 kmph. At what time can they be 12 km apart if the distance between $T_1$ and $T_2$ is 72 km?

(A) 7:32 a.m. (B) 7:36 a.m.
(C) 7:40 a.m. (D) 7:48 a.m.

**16.** The following graph shows the relationship between speed and time of a moving car. Find the average speed of the car in the first 30 seconds. (in m/s) _____




**17.** At what time between 4 O'clock and 5 O'clock, will the hands of a clock becoinciding with each other?

(A) 4 : 15 $\frac{5}{11}$ (B) 4 : 21 $\frac{9}{11}$
(C) 4 : 26 $\frac{4}{11}$ (D) 4 : 14 $\frac{6}{11}$

**18.** Mohan started from *A* towards *B* at 8:00 a.m. at 6 km/hr. Sohan started from *A* towards *B* at 8:45 a.m. at 12 km/hr. After reaching their respective destinations, each of them returned to their starting points and *AB* = 6 km. Find the time when Mohan met Sohan for the first time.

(A) 9:05 a.m. (B) 9:10 a.m.
(C) 9:15 a.m. (D) 9:20 a.m.

**19.** At what time between 4 O'clock and 5 O'clock, will the hands of a clock beat right angles with each other?

4:_____ $\frac{5}{11}$ or 4: _____ $\frac{2}{11}$

**20.** A man travels from his house to his office. If he travelled at 5 kmph, he would reach his office 2 minutes late. If he travelled at 6 kmph, he would reach his office 5 minutes early. Find the distance between his house and his office (in kilometres). Also find the speed at which he should travel to reach his office exactly on time. (in kmph)
_____, _____

**21.** In a kilometer race, P beats Q by 1 minute. Q beats R by 30 seconds. If P beats R by 200 m, find the time R takes to run the race. (in seconds) _____

**22.** In a running race, Sunita gives Asha a headstart of 350 m and still beats her by 50 m. If Sunita's speed is 1¼ times Asha's speed, what is the length of the race (in m)? _____

**23.** A, B and C run around a circular track of length600 m, starting at the same time, from the same point, in the same direction, with speeds of 3 m/s, 4 m/s and 5 m/s, respectively. Which of the following statements is true?

(A) A and B meet before B and C.
(B) B and C meet before A and B.
(C) A and B meet before A and C.
(D) A and C meet before A and B.

**24.** Lucky is a thief. He steals a TV from the house of Mr. Batra and escapes on his Lambretta at 6:00 a.m. at a speed of 40 kmph. At 8:00 a.m., Mr. Batra realizes that there had been a burglary and immediately starts chasing Lucky on his Luna at a speed of 60 kmph.

At what time will Mr. Batra catch up with Lucky? _____ : _____ p.m.

**25.** What is the distance covered by Lucky before he was caught? (in km) _____.

**26.** Two cyclists simultaneously start from *A* to *B* and *B* to *A*, respectively. They cross each other after a time *t* hours. The first person reaches *B* in another $t_1$ hours while the second person reaches *A* in another $t_2$ hours. Then

(A) $t = \frac{t_1 + t_2}{2}$ (B) $t = \frac{2t_1t_2}{t_1 + t_2}$
(C) $t = \sqrt{t_1 t_2}$ (D) $t = \frac{t_2 - t_1}{2}$

**27.** Meena and Neena start simultaneously from *A* towards *B*. When Meena has covered 27 km, Neena doubles her speed and 6 hours later she overtakes Meena. What is Neena's original speed, if the speed of Meena is 4.5 km/hr?

(A) 2 km/hr (B) 3 km/hr
(C) 6 km/hr (D) 4 km/hr

**28.** X and Y are two stations 630 km apart. P starts from X and moves towards Y at 90 km/hr. Q starts from Y at the same time and moves towards X at 120 km/hr. Find the extra distance travelled by Q when they meet. (in km)
_____

**29.** Ram, Shyam and Tarun started from *A* towards *B* with speeds 20 km/hr, 30 km/hr and 40 km/hr, respectively.

Shyam started 3 hours after Ram. Shyam and Tarun overtook Ram simultaneously. Find the time interval between the starting times of Tarun and Ram. (in hours) _____

**30.** *PQ* is a tunnel. *A* dog sits at a distance of 5/11 of *PQ* from *P*. The train's whistle coming from any end of the tunnel would make the dog run. If a train approaches *P* and the dog runs towards *P*, the train would hit the dog at *P*. If the dog runs towards *Q* instead, it would hit the dog at *Q*. Find the ratio of the speeds of the train and the dog.
(A) 5 : 2 (B) 16 : 5 (C) 11 : 1 (D) 34 : 3

**31.** If a person increases his speed by 25%, he reaches office 12 minutes early. Find the time taken by the person to reach his office if he decreases his speed by 25%. (in minutes) _____

**32.** Two gunshots were fired from the same place with a gap of 12 minutes. *A* person in a train approaching the place hears the second shot 11 minutes after he hears the first. Find the speed of the train given that the speed of sound is 330 m/s.
(A) 81 km/hr (B) 90 km/hr
(C) 99 km/hr (D) 108 km/hr

**33.** Manish and Nitin start running simultaneously from a point on a circular track 900 m long in opposite directions. The speeds of Manish and Nitin are 5 m/s and 1 m/s, respectively. After how much time (in seconds) will they meet for the first time? _____

**34.** The average speed for an entire journey is 60 km/hr without considering the stoppages. When the stoppages are considered, the average speed becomes 48 km/hr. How many minutes per hour on an average were the stoppages?
(A) 10 minutes (B) 12 minutes
(C) 16 minutes (D) 18 minutes

**35.** Two trains of length 150 m each take 6 seconds to cross each other when they are travelling in opposite directions and take 30 seconds to cross each other when they are travelling in the same direction. Find the speed of the faster train. (in kmph) _____

**36.** Two stations Mumbai and Goa are 465 km apart. A train leaves Mumbai for Goa at 11 a.m. at a speed of 60 kmph. Another train leaves Goa for Mumbai on a parallel track at noon at a speed of 75 kmph. They will cross each other at
(A) 3 : 00p.m. (B) 4 : 00p.m.
(C) 5 : 00p.m. (D) 4 : 30 p.m.

**37.** Everyday an aircraft starts from country *X* at 8:00 a.m. (country X's standard time) and reaches country *Y* at 10 a.m (country Y's standard time) on the same day. It halts there for 4 hours and flies back to country *X* and reaches country *X* at 4:00 p.m. (country X's standard time). If the distance between the countries is 1000 km, find the average speed of the aircraft. (in kmph) _____

**38.** Two cars *P* and *Q* start simultaneously from points *A* and *B*, respectively at different speeds and travel towards each other. They meet in two hours, after which *P* takes 3 hours less to reach *B* than the time *Q* takes to reach *A*. If the distance between *A* and *B* is 540 km, find the speed of *Q* (in kmph).
(A) 90 (B) 120 (C) 60 (D) 180

**39.** A parachutist, before he opens his parachute, falls for a time $t_1$ and covers a distance of $5t_1^2$ and after he opens his parachute, he falls for a time $t_2$ and covers a distance $Vt_2$. *V* is the velocity attained just before the parachute is opened and is given by $5t_1$. After what time (in seconds) did he open the parachute, if the total distance covered by the parachutist is 1500 m and the total time taken is 30 seconds? _____

**40.** A person travels the first one-fourth of his journey time at 80 kmph, and the remaining at 60 kmph. If he covers a total distance of 260 km, find his average speed over the entire journey. (in kmph) _____

**41.** By hiring boat *X*, Rohan took 12 hours to travel a round trip between two points that are 40 km apart in a river. By hiring boat *Y*, whose speed in still water is twice that of boat *X* in still water, he took $3^3/_4$ hours for the round trip. Find the speed (in km/hr) of boat *X* in still water.
_____

**42.** A boat can travel at 12 kmph in still water. It travelled 135 km upstream in a river and back taking altogether 24 hours. Find the rate at which the stream flows. (in kmph) _____

**43.** Arundhati and Beena are 12.5 km apart. If they start simultaneously and travel in the same direction, they meet after 2.5 hrs and if they travel in opposite directions they meet after 30 minutes. If Beena travels faster than Arundhati, then Beena's speed is .
(A) 10 kmph (B) 15 kmph
(C) 20 kmph (D) 25 kmph

**44.** The driver of a car, going at a speed of 38 kmph observes a bus 400 m ahead of him, travelling in the same direction. After 200 seconds, the bus is 600 m behind him. The speed of the bus is
(A) 10 kmph (B) 20 kmph
(C) 15 kmph (D) 18 kmph

**45.** Ram and Shyam started simultaneously in opposite directions from towns *R* and *S*, respectively. Each of them travelled towards the starting point of the other person. After crossing each other, Ram and Shyam took 40 minutes and 90 minutes, respectively to reach their destinations. Find the time taken by them to meet each other (in minutes). _____

**46.** Ankita and Bhoomika start running simultaneously on a circular track. Ankita finishes one lap in 14 minutes and Bhoomika finishes one lap in 10 minutes. After how many minutes from the start will Bhoomika have covered exactly 2 laps more than those by Ankita?
(A) 45 minutes (B) 70 minutes
(C) 60 minutes (D) 40 minutes

**47.** In a race, Ashok gave Bharat a head start of 400 m. If the ratio of speeds of Ashok and Bharat is 5 : 3 and both reach the finishing point simultaneously, find the length of the race. (in metres) _____

**48.** A 100 m long train running at a speed of 40 kmph crosses a bridge in 180 seconds.*A* 200 m long train running at a speed of 80 kmph will cross the same bridge in how many seconds?
(A) 86.5 seconds (B) 90 seconds
(C) 94.5 seconds (D) 87 seconds

**49.** A person cycled 18 km along a straight road in the direction of the wind blowing at a constant rate for 3 hours. Suddenly the direction of the wind reversed, he further travelled 18 km in 6 hours. How much time would he take to come back to the starting point, if the speed of the wind becomes negligible? (in hrs) _____.

**50.** Three runners *X*, *Y* and *Z* run a race at uniform speeds. *X* beats *Y* by 12 m and beats *Z* by 24 m. Y beats *Z* by 15 m. Find the length of the race. (in m) _____

**51.** Samuel takes 6 hours to row up the stream from *A* to *B* and returns to *A* in 3 hours. How much time does he take to row a distance *AB* in still water?
(A) 3.6 hours (B) 3.75 hours
(C) 4 hours (D) 4.50 hours

**52.** A building had two floors. There were 90 steps from the first floor to the second floor. An escalator was moving up from the first floor to the second floor. Ram took nine seconds to walk from the first floor to the second floor on it. He took 45 seconds to return to the first floor on it. Find the time he would have taken to return if the escalator had been switched off. (in seconds) _____

**53.** Manish and Sheetal start running simultaneously from *P* and *Q*, towards *Q* and *P*, respectively They meet each other after a certain time '*t*'. Thereafter,Manish and Sheetal take 16 seconds and 25 seconds to reach their respective destinations. If Manish's speed was 12 m/s, find the distance between *P* and *Q*. (in m) _____

**54.** *P* and *Q* are 2 stations. *A* single track runs betweenthem. Train *A* leaves everyday from *P* at 6:00 a.m. and reaches *Q*. Another train *B* starts from *Q* as soon as *A* reaches *Q*. *B* reaches *P* at 7:00 a.m. The speed of *B* is four times that of *A*. One day, *A* started from *P* 10 minutes late as per the schedule. In order to maintain *B*'s right arrival time at *P*, both trains travel at an increased speed. If *B* had doubled its speed, find the ratio of the new speeds of *A* and *B*.
(A) 3 : 22 (B) 22 : 3 (C) 38 : 3 (D) 44 : 9

**55.** A person travels a certain distance at a speed of 40 km/hr and returns to the starting point at a speed of 60 km/hr. What is his average speed for the entire trip? (in km/hr) _____.

**56.** A flight which leaves city *P* at 3:00 a.m. reaches city *Q* at 4:00 p.m. on the same day. Another flight which leaves city *Q* at 7:00 p.m. reaches city *P* at 3:00 p.m. on the next day. If both flights travel non-stop at 700 km/hr and all times mentioned are local times, find the distance between city *P* and city *Q* (in km).
(A) 11,550 (B) 10850
(C) 6,650 (D) 5,775

**57.** *A* and *B* are two points on a river. Two boats *X* and *Y* started simultaneously from *A* and *B*, respectively towards each other with equal speeds and the distance between them reduces by 32 km in 4 hours. Find the speed of each boat in still water. (in kmph) _____

**58.** An escalator was moving up from floor *X* to floor *Y*. It had 120 steps. Anil took 24 seconds to walk up on it. If his speed doubled and that of the escalator tripled, he would have taken only ten seconds to walk up on it. Find the time he would take to go from floor *Y* to floor *X* by the escalator. (in seconds) _____

**59.** Find the angle (in degrees) between the hands of a clock at 5:15 p.m. _____

**60.** In a kilometre race, *A* beats *B* by 30 seconds. *B* beats *C* in the same race by 30 seconds. If *A* beats *C* by 400 m, find the time taken by *A* to run the race.
(A) 1 minute 20 seconds
(B) 1 minute 30 seconds
(C) 1 minute 40 seconds
(D) 2 minutes

## EXERCISE-3

***Directions for questions 1 to 60*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Lucky starts running at a speed of 10 m/s from a point. Mr. Batra, who is 5000 m behind him, starts chasing him at a speed of 20 m/s. There is a dog running at 30 m/s which starts from Mr. Batra and keeps on running to and fro between him and Lucky till the latter gets caught. Find

(a) thetotal distance travelled by the dog.m _____.

(b) thetotal distance travelled by the dog in the forward direction (assume the direction in which the dog started running is the forward direction)._____m

***Directions for questions 2 and 3*:** These questions are based on the following data.

Towns *P* and *Q* are located in different time zones. *P* is located 3600 km west of *Q*. The table below shows the daily schedule of a plane which operates non-stop between *P* and *Q*. All indicated times are local and refer to the same day.

| Departure | | Arrival | |
|---|---|---|---|
| Town | Time | Town | Time |
| *P* | 3 p.m | *Q* | 11:30 p.m |
| *Q* | 4 a.m | *P* | 9:30 a.m |

Assume that the cruising speed of the plane in each direction is the same. However the effective speed of the plane is influenced by a steady wind blowing from west to east at 75 km/hr

**2.** Find the cruising speed of the plane in each direction (in km/hr).
(A) 575 (B) 475 (C) 625 (D) 525

**3.** Find the time difference between *P* and *Q*.

(A) 1 hour (B) $2\frac{1}{2}$ hours

(C) $1\frac{1}{2}$ hours (D) 2 hours

**4.** Arvind takes 42 minutes to travel from Tarnaka to Chilkur and Prasad takes 56 minutes to travel from Chilkur to Tarnaka. Arvind and Prasad start from Tarnaka and Chilkur, respectively, at 7:30 a.m. and go towards Chilkur and Tarnaka, respectively. If they take the same route, when will they meet?
(A) 7:48 a.m. (B) 7:51 a.m.
(C) 7:54 a.m. (D) 7:58 a.m.

**5.** *A*, *B* and *C* run a race from *P* to *Q*. *C* starts first, five seconds after *C*, *A* starts and five seconds after *A*, *B* starts. All the three reach a point *R* between *P* and *Q* simultaneously. If *B* reaches *Q* 4 seconds before *C*, how much more time does *A* take than *B* to run from *P* to *Q*? _____

**6.** Shiv starts from *P* towards *Q* at a speed of 72 km/hr and decreases his speed by 12 km/hr after every 6 km. If he takes 36 minutes to travel from *P* to *Q*, then what is the distance *PQ*? (in km) _____.

**7.** Ravi and Vikram ran a 9 km race on a circular track of length 900 m. They started running simultaneously from the same point on the track. They completed one round in 9 seconds and 15 seconds, respectively. After what time from the start will the faster person meet the slower person for the last time (in seconds)? _____

**8.** Car *P* started from town *A* towards town *B* at 9 a.m. at 40 km/hr. Car *Q* started from *B* towards *A* at 9 a.m. at 60 km/hr. Each car travelled back and forth between the two towns. The cars met for the first time at 9:40 a.m. At what time will the cars meet for the third time?
(A) 12:40 p.m. (B) 12:20 p.m.
(C) 11:50 a.m. (D) 12:50 p.m.

**9.** Train *X* started from town *A* towards town *B* at 6:00 a.m. and reached *B* at 2:00 p.m. Train *Y* started from *B* towards *A* at 7:00 a.m. and reached *B* at 3:00 p.m. At what time did both the trains meet each other? _____ : _____ a.m

**10.** Chotu starts from a point *P* towards point *Q*, where *PQ* = 90 km. After 1 hour, Motu starts from *P* and catches up with Chotu after 2 more hours. After meeting, they continue to travel towards *Q*. On reaching *Q*, Motu reverses his direction and meets Chotu 6 hours after the first meeting. Find Chotu's speed.

(A) $\frac{45}{7}$ km/hr (B) $\frac{60}{7}$ km/hr
(C) $\frac{120}{7}$ km/hr (D) $\frac{30}{7}$ km/hr

**11.** A motorcyclist travelling on a road at 36 kmphovertakes a cyclist travelling at 20 kmph at 12:00 p.m. He also overtakes a pedestrian walking at $9^1/_3$kmph at 2:00 p.m. At what time would the cyclist overtake the pedestrian? _____ : _____ p.m

**12.** Three boys Amar, Akbar and Anthony decide to run on a circular track. All start simultaneously at the same point *P* and run in the same direction. Amar overtakes Anthony 2 times per round, where the 2nd time is at *P*. Anthony overtakes Akbar 4 times per round, the 4th time at *P*. If the ratio of Amar's speed to Akbar's speed is $n$ : 1, find $n$. _____

**13.** Bhola went to bed at a certain time between 10 p.m. and 11 p.m. When he woke up in the morning between 5 a.m. and 6 a.m, he noticed that each hand was at the position of the other hand when he had gone to bed. For how long was Bhola in bed (in minutes)?
(A) $442^{2}/_{13}$ (B) $443^{1}/_{11}$
(C) $442^{9}/_{11}$ (D) $443^{1}/_{13}$

**14.** *M* and *N* start at the same time from two ends *S* and *T*, at speeds of 3 m/sec and 4 m/sec towards each other, *ST* being 7 m. After reaching the opposite ends, they retrace their path and start moving towards each other. Find the total distance travelled by *M* when he meets *N* for the second time.
(A) 20 m (B) 15 m (C) 9 m (D) 26 m

**15.** Ajit, Bhusan and Charan start simultaneously from a point *P* on a circular track of length 600 m. Their speeds are *n* m/s, (n+1) m/s and (n+2) m/s, respectively where *n* is a positive integer. If all the three move in the same direction, how many times will they meet before they meet at *P* for the first time?
(A) 1 (B) 2
(C) 3 (D) None of these

**16.** In a certain race, *A* beats *B* by 100 m in terms of distance and 10 *s* in terms of time. *B* beats *D* by 50 sec. If the speeds of *A*, *B* and *C* are in the ratio 5:4:3, by what distance does *C* beat *D* in the same race?
(A) 75 m (B) $166\frac{2}{3}$ m
(C) $266\frac{2}{3}$ m (D) 100 m

**17.** A train is travelling towards Kolkata. *A* passenger at the front end of the first coach started walking towards the rear end of the last coach and reached there in 25 minutes. He then started back and reached the front end of the first coach after walking for 15 minutes. In the time in which he completed this round trip, the train had covered a distance of 80 km. Find the speed of the train. (in kmph) _____

**18.** A thief was running along a median on a road at 6 kmph. He crossed a police jeep moving at 9 kmph in the opposite direction. The jeep had to continue for 10 more minutes before it could find a gap in the median and start chasing the thief. Find the total time taken by the jeep to catch the thief from the time it crossed him (in minutes).
(A) 45 (B) 50 (C) 55 (D) 60.

***Directions for questions 19 and 20*:** These questions are based on the following data.

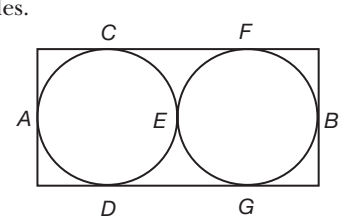
On a certain planet, a week has 8 days – Sunday to Saturday and another day called Maturday. There are 36 hours in a day and each hour has 90 minutes, while each minute has 60 seconds.

**19.** Find the approximate angle between the hands of a clock (in degrees) on the planet when the time is 15:50 a.m.
(A) 1000/9 (B) 2000/9 (C) 190 (D) 130

**20.** At what time between 14 O' clock and 15 O' clock will the two hands of the clock be at an angle of 60°?
(A) $14 : 58^{4}/_{17}$ (B) $14 : 38^{8}/_{17}$
(C) $14 : 25^{3}/_{17}$ (D) $14 : 87^{16}/_{17}$

**21.** 18*t* minutes from now, a faulty clock which gains time uniformly will show 6:00 a.m. The clock gains 60 minutes during this period. 6*t* minutes from now, the clock will show 11:20 p.m. What will be the time in the clock exactly 24 hours from now?
(A) 8:00 p.m. (B) 10:40 p.m.
(C) 9:30 p.m. (D) 9:40 p.m.

**22.** The diagram below shows a jogging park, which has two identical circular tracks touching each other. A rectangular track encloses the circles. Its edges are tangential to the circles.




*P* and *Q* are two friends. They start jogging simultaneously from the points *A* and *B*, respectively. *P* jogs traversing the path of an eight which is *ACEGBFEDA*. *Q* jogs along the rectangular track. Both reach their starting points simultaneously. By what percentage is the distance run by *Q* less than that run by *P*?
(A) $4^{1}/_{11}\%$ (B) $4^{2}/_{11}\%$ (C) $4^{6}/_{11}\%$ (D) $4^{4}/_{11}\%$

**23.** Two cyclists are traveling on a highway which has milestones at every kilometer. Cyclist *A* takes alternatively 7 min and 5 min between successive milestones while cyclist *B* takes alternatively 4 min and 9 min. If they both start from the same milestone at the same time and travel in the same direction, how many times do they meet in all after they start? _____

**24.** *A* race track is 18 km uphill and 18 km downhill. *A* and *B* participate in a race in this track. *A* cycles uphill at 9 kmph and downhill at 15 kmph. *B* cycles uphill as well as downhill at 12 kmph. Who wins the race, and by what distance (in km), if both start the race simultaneously from the bottom of the hill? _____, _____.

**25.** One day, Amar walked along a circular track for 6 hours. He then ran along the track for the time that he would take to walk one round. He covered a total of 18 km. The next day, Amar walked along the same track for 4 hours. He then cycled around the track for the time that he would take to walk one round. He covered a total of 27 km. The length of the track is 3 km. If the speeds at which he ran and cycled were 4.5 km/hr and 10.5 km/hr, respectively, then find his walking speed.
(A) 1.5 km/hr (B) 2.5 km/hr
(C) 1 km/hr (D) 2 km/hr

***Directions for questions 26 and 27:*** These questions are based on the data given below.

Hero and Hercules start simultaneously from *P* towards *Q*. When the distance between Hero and Hercules was 30 km, Champ started from *P* towards *Q*. The distance travelled by Champ to overtake Hercules is twice the distance he travelled to overtake Hero. The ratio of the speeds of Hercules and Hero is 3 : 2.

**26.** What is the ratio of the speed of Champ to that of Hero?
(A) 4 : 1 (B) 3 : 1
(C) 2 : 1 (D) None of these

**27.** What is the distance between Hero and Hercules, when Champ was equidistant from Hero and Hercules?
(A) 45 km (B) 48 km
(C) $51^3/_7$ km (D) Cannot be determined

**28.** Two stations *X* and *Y* are 330 km apart. Train *A* starts from *X* at 9:00 a.m. at 60 kmph towards *Y*. Train *B* starts from *Y* at 10:00 a.m. at 40 kmph towards *X*. At a station 120 km from *Y* and 210 km from *X*, *A* halts for 35 minutes. *B* does not halt anywhere along its journey. Find the time when both trains meet for the first time.
(A) 12:00 p.m. (B) 12:30 p.m.
(C) 1:00 p.m. (D) 1:30 p.m.

**29.** One day, a cheetah sighted a deer at a distance of 70 m. It started chasing the deer at a speed of 20 m/s. After two seconds, the deer saw the cheetah which was nearing it and started running instantaneously, with a speed of 19 m/s. After how much time from the moment the deer saw the cheetah (in sec), will the cheetah catch the deer? _____

**30.** Ram, Shyam and Tarun started simultaneously from town *A* towards town *B*. Ram set off on foot and Shyam took Tarun along with him on a bike. After travelling a certain distance, Shyam dropped Tarun and then turned back to pick up Ram, while Tarun continued towards *B* on foot. Shyam picked up Ram and turned back. All the three reached *B* simultaneously. If the walking speeds of Ram and Tarun were the same and each of these speeds was one-sixth of Shyam's driving speed, then Tarun's average speed for the entire distance is _____ of his walking speed.

(A) $\frac{72}{19}$ (B) $\frac{54}{19}$ (C) $\frac{60}{17}$ (D) $\frac{54}{17}$

**31.** Flight *A* leaves city *X* at 6:00 a.m. and reaches city *Y* at 8:30 a.m. Flight *B* leaves city *Y* at 3:00 p.m. and reaches city *X* at 10:30 p.m. Both flights fly non-stop between *X* and *Y* and all times mentioned are local times. If flight *A* travels at 600 km/hr and flight *B* travels at 400 km/hr, find the distance between city *X* and city *Y* (in km).
(A) 3,600 (B) 3,000
(C) 2,400 (D) 1,800

**32.** A car travelled a distance of 900 km. It developed an engine problem after travelling for some distance. It travelled the remaining distance at $3/5^{th}$ of its speed. It reached 2 hours late. If the engine problem had developed after it had travelled for another 150 km, it would have reached 1 hour earlier than the time it actually reached. Find the distance it travelled without any problem and its speed over that part of the journey. (in km, in kmph).
(A) 500, 100 (B) 400, 75
(C) 600, 100 (D) 700, 75

**33.** The clocks $C_1$ and $C_2$ were set to correct time at noon, Sunday. The hands of $C_1$ coincide every 66 minutes. The hands of $C_2$ coincide every 50 minutes. The difference of the times shown by $C_1$ and $C_2$ at noon, Monday is approximately
(A) 412 min (B) 493 min
(C) 457 min (D) 547 min

**34.** Vijay and his girlfriend started on a boat from a point on a river and began to row upstream. After rowing for two hours, Vijay stopped the boat. He wanted to hand over a flower to his girlfriend but it slipped from his hand into the river and floated downstream. They rowed upstream for two more hours and then turned back. They caught up with the flower, at a point 6 km downstream of the point at which it fell into the river. Find the speed (in km/hr) of the stream.

(A) 4 (B) 3
(C) 1.5 (D) Cannot be determined

***Directions for questions 35 and 36*:** These questions are based on the following data.

Three athletes *A*, *B* and *C* race in a marathon that has three legs: running, cycling and swimming. *A* can cycle thrice as fast as he can run and swim one-third as fast as he can run. *B* can cycle four times as fast as he can run and swim one-fourth as fast as he can run. *C* can swim half as fast as he can run and cycle four times as fast as he can run. *A* can run 10% faster than *B* and 20% faster than *C*. The length of the running track is half of that of the cycling track and is twice that of the swimming track.

**35.** Who takes the least time for the second leg?
(A) *A* (B) *B*
(C) *C* (D) Cannot say

**36.** Who wins the marathon?
(A) *A* (B) *B*
(C) *C* (D) Cannot say

**37.** Ram and Shyam started running a 1.2 km race simultaneously from a point on a circular track 120 m long. They take 6 seconds and 9 seconds, respectively to complete one round. After how many seconds will Ram overtake Shyam for the last time? _____

**38.** *A* and *B* are in a shopping complex. They found an escalator that is moving up at a constant speed. Both use the escalator to climb up but instead of letting the escalator do the complete work, they climb up the escalator at different speeds. *A* has a climb up speed twice that of *B*. *A* reaches the top of the escalator after having climbed 40 steps while Btakes only 30 steps to reach the top. If the escalator was switched off, how many steps would *A* have to take to climb up? _____

**39.** A bus covers a certain distance every day at an uniform speed. One day, after covering 30% of the distance, its speed decreased and the bus reached its destination 10 hours late. Had the decrease in speed occurred after the bus had covered 70 km more, it would have been late by only 9 hours. Find the distance covered by the bus every day.
(A) 800 km
(B) 1000 km
(C) 900 km
(D) Cannot be determined

**40.** One evening, Zombie goes out for a drive, sometime between 4'O clock and 5'O clock. When he gets back, sometime between 7'O clock and 8'O clock, he noticed that the minute and hour hands have interchanged their positions with where they were when he started. For how long was he out for a drive (approximately)?
(A) 2 hours 15 minutes
(B) 1 hour 12 minutes
(C) 2 hours
(D) 2 hours 46 minutes

**41.** Two steam-boats Viraat and Samudra start from a jetty on a river and take the same route. Viraat travels downstream at 10 m/s and Samudra starts 5 minutes later at 5 m/s downstream. Fifteen minutes later, two passengers on Viraat decide that they need to board Samudra. They are left behind on the raft that floats along with the river to be picked up by Samudra. If Viraat can cover 6 m/s upstream, then find the approximate time for which the passengers stayed on the raft.
(A) $49^1/_2$ min (B) $41^2/_3$ min
(C) 50 min (D) 48 min

**42.** In a race, Raja beats Sameer by 30 m and in the same race, Sameer beats Tarun by 45 m. If Raja is thrice as fast as Tarun, find the length of the race. (in m) _____

**43.** Two cars *A* and *B* started from towns *X* and *Y*, respectively in opposite directions towards each other simultaneously. After reaching the other town, they returned to their starting towns using the same route. They travelled at uniform speeds throughout their journeys. They met for the first time at 30 km from *Y*. They met for the second time at 10 km from *X*. Which of the following can be the distance (in km) between *X* and *Y*?
(A) 70 (B) 80 (C) 90 (D) 60

**44.** A train travelling at 36 km/hr takes 48 seconds to cross a bridge. It then crosses a man cycling at 9 km/hr in the same direction in 20 seconds. Find the length of the bridge. (in metres) _____

**45.** Two people can run around a circular track in 30 s and 60 s. If they start simultaneously in the same direction from diametrically opposite points, when will they meet for the second time?
(A) 180 s (B) 90 s (C) 60 s (D) 120 s

**46.** A train is 270 m long. It overtook a motorcyclist travelling at 36 kmph in 27 seconds. An hour later, it overtook a cyclist in 18 seconds. How long after the train overtook the cyclist would the motorcyclist overtake it approximately? (in minutes)
(A) 120 (B) 210 (C) 180 (D) 150

**47.** Two faulty clocks are set at correct time on Tuesday at 10:00 p.m. The first clock loses 3½ minutes per hour,

while the second clock gains 2½ minutes per hour. When will the times in the two clocks be 3 hours apart?
(A) 4:00 a.m. Thursday
(B) 6:00 p.m. Wednesday
(C) 12:00 noon Wednesday
(D) 4:00 a.m. Friday

**48.** Pran needs to reach a destination 130 km downstream on a river. He can row at 4 km/hr in still water. The river flows at a usual rate of 2 km/hr but due to tides every two hours, the speed doubles and then after another two hours it halves. If Pran starts at the change of tide when the river begins to flow at 2 km/hr,then in what time will he reach his destination?

(A) $18^3/_4$ hr (B) $19^1/_4$ hr

(C) $9^2/_7$ hr (D) $18^4/_7$ hr

**49.** Two vehicles – *A* and *B* – start from the same point at the same time but move in opposite directions along a circular track of radius 14 km. The initial speed of *A* is thrice that of *B*. Every time they meet, they reverse their directions and exchange their speeds. Find

(a) the shortest distance between their starting point and the third meeting point. (in km) $\sqrt{2}$ _____.

(b) the distance covered by *A* from its starting point till the third meeting point. (in km) _____.

**50.** Every day, Hari drives his motorcycle at a certain speed and reaches his office exactly on time. One day, after travelling a certain distance, he slowed down to $3/4^{th}$ of his initial speed and arrived at his office 6 minutes late. Had he slowed down to $2/3^{rd}$ of his initial speed 6 km further on, he would still have reached 6 minutes late. What is his initial speed (in km/hr)? _____

**51.** Find the time at which the hands of a clock are exactly in opposite directions between 8 and 9' O clock.

$8 : \frac{10}{11}$ _____.

**52.** At what times between 4 and 5 O'clock will the 2 hands of a clock be at an angle of 60° with each other?

$4 : \frac{10}{11}$ and 4 : _____. $\frac{8}{11}$ _____.

***Directions for question53*:** These questions are based on the information given below.

Train *P* has a length of 300 m and a speed of 72 kmph. Train *Q* has a length of 600 m and a speed of 90 kmph. Both enter a tunnel in opposite directions simultaneously. The length of the tunnel is 600 m.

**53.** (i) Which train exits the tunnel first? What length of the other train is still in the tunnel at that time? (in m)
(A) *P*, 75 (B) *Q*, 75
(C) *P*, 90 (D) *Q*, 90

(ii) Find the distance between the point where the rear ends of the trains cross each other and the point of entry of the slower train (in m).

(A) $398\frac{1}{3}$ (B) $366\frac{2}{3}$

(C) $216\frac{2}{3}$ (D) $233\frac{1}{3}$

**54.** Two cities *X* and *Y* lie on a straight line. Two men *P* and *Q* left simultaneously for *Y* and *X*, respectively from *X* and *Y*, respectively. *P* reaches *Y* and immediately turns around and moves towards *X*. On reaching *X*, again he turns around and moves towards *Y*. This kind of movement continues indefinitely. *Q* also travels in a similar manner. The distance between *X* and *Y* is 1000 m and the ratio of the speeds of *P* and *Q* is 3 : 2. Find the distance travelled by *P* when he meets *Q* for the $4^{th}$ time.
(A) 4000 m (B) 5600 m
(C) 4200 m (D) 4800 m

**55.** At what time between 2 and 3 O 'clock are the hands of a clock together? 2: $\frac{10}{11}$ _____.

**56.** Soumya and Vivian start together from the same point in the same direction and run around a circular race track. They agree that every time either of them overtakes the other, the former would give to the latter as many coins as the latter had. But if one of them overtakes the other at the starting point, then the latter had to give half of what he had to the former. Soumya runs at a speed that is five times the speed of Vivian. Vivian had only one coin initially and Soumya had 182 coins at the end of the race. How many coins could Soumya have had in the beginning?
(A) 245 (B) 246 (C) 207 (D) 208

**57.** Two students started simultaneously from the same point on a circular track and ran at speeds of 20 km/hr and 30 km/hr. Find the number of distinct meeting points where they meet if they are running in the same/opposite direction(*s*). _____, _____

**58.** In a 500 m race, *A* beats *B* by 40 m. In a 1000 m race, *B* beats *C* by 40 m. If *A* beats *C* by 14 6 seconds in a 500 m race, find the time taken (in minutes) by *B* to run 2.4 km. _____

**59.** A man started from Hyderabad at 7 a.m. and reached Bangalore at 6 p.m. The next day he started from Bangalore at 7 a.m. and returned to Hyderabad at 6 p.m. using exactly the same route. The man's speed was not uniform but he moved in one direction, without retracing his path on either day. Which of the following is definitely true?

(A) There were two instances when the man was at the same point at the same time on the two days.

(B) There were more than two instances when the man was at the same point at the same time on the two days.

(C) The man was never at the same point at the same time on the two days.

(D) There was only one instance when the man was at the same point at the same time on the two days.

**60.** The village of Risalgarh has an annual bullock cart race. The final participants are required to cover the shortest path touching all the four entrances to the village, (*S*, *E*, *N*, *W*) located at the south, east, north and west, respectively, spaced equally on the circumference of a circle. One year, there were just two finalists selected after the preliminaries. It was known that one of them was five times as fast as the other. If they start from *S* and go towards *W* (and beyond), where will they meet for the 23rd time after they start?

(A) *N*

(B) *E*

(C) Midway between *E* and *S*

(D) Midway between *N* and *W*

## EXERCISE-4

***Directions for questions 1 to 30*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Bala started from town *Q* at 10:00 a.m. and travelled towards town *P*. Amar started from *P* at 12:24 p.m. and travelled towards *Q*. They crossed each other at 1:36 p.m. Bala reached *P* 66 min before Amar reached *Q*. When did Bala reach *P*?

(A) 3:12 p.m. (B) 3:06 p.m.

(C) 3:00 p.m . (D) 3:24 p.m.

***Directions for question 2*:** The question is based on the following information.

Nishit wanted to travel from City *A* to City *B* by car and calculated that if he drove his car at a certain speed, he would be on time to attend a wedding party in city *B*. After traveling 40% of the total distance at this speed, he met with an accident and damaged his car. He immediately repaired his car and when he resumed his trip, he noted that he had lost 90 minutes in the process. Moreover, now he could drive his car at only 75% of its former speed. On reaching city *B*, he found that he was 3 hours behind schedule. However, had the accident occurred 120 km further towards city *B*, he would have been only 2 ½ hrs behind schedule.

**2.** What is the distance between the cities *A* and *B*?

(A) 480 km (B) 540 km

(C) 600 km (D) 640 km

***Directions for question 3*:** The question is based on the following information.

Cities *P*, *Q* and *A* are in different time zones. *P* and *Q* are located 4500 km east and 4500 km west of *A*, respectively. The table below describes the schedule of an airline operating non-stop flights between *A* and *P*, A and *Q*. All the times indicated are local times and on the same day.

| Departure | | Arrival | |
|---|---|---|---|
| City | Time | City | Time |
| *A* | 7:00 a.m. | *P* | 3:00 p.m. |
| *A* | 9:00 a.m. | *Q* | 12:00 noon |

All flights travel at the same speed but effective speed is influenced by a steady wind blowing from east to west at 75 kmph.

**3.** What is the time difference between cities *A* and *Q*?

(A) 1 hour

(B) $1\frac{1}{2}$ hours

(C) 2 hours

(D) Cannot be determined

**4.** The village of Risalgarh has an annual bullock cart race. The final participants are required to cover the shortest path touching all the four entrances to the village, (*S*, *E*, *N*, *W*) located at the south, east, north and west, respectively different directions spaced equally on the circumference of a circle. One year there were just two finalists selected after the preliminaries. It was known that one of them was five times as fast as the other. If they start from *S* and go towards *W* (and beyond) where will they meet for the 23rd time, after they start?

(A) $N$
(B) $E$
(C) Midway between $E$ and $S$
(D) Midway between $N$ and $W$

**5.** Two soldiers $A$ and $B$ start from different points on a straight line. Initially $A$ is to the south of $B$. They march north at uniform rates. When $A$ covers 2 km, both make an about turn and start marching south at different uniform rates. 3 min later $B$ arrives at A's turning point. $A$ arrives at his starting point, and continues to march south. One minute after A crosses his starting point, $B$ reaches that point. How far south do they travel beyond A's starting point by the time they meet? _____

***Directions for question 6*:** The question is based on the following information.

Three athletes $A$, $B$ and $C$ race in a marathon that has three legs: running, cycling and swimming. $A$ can cycle thrice as fast as he can run and swim one-third as fast as he can run. $B$ can cycle four times as fast as he can run and swim one-fourth as fast as he can run. $C$ can swim half as fast as he can run and cycle four times as fast as he can run. $A$ can run 10% faster than $B$ and 20% faster than $C$. The length of the running track is half that of the cycling track and is twice that of the swimming track.

**6.** Who wins the marathon?
(A) A (B) $B$ (C) C (D) Cannot say

**7.** $M$ and $N$ started running simultaneously around a circular track 8 km long in the same direction. The initial position of $M$ was 2 km behind that of $N$. $M$ met $N$ for the first time after 2000 seconds when both were in their respective second rounds around the track. Had $N$ travelled in the opposite direction, they would have met after 200 seconds. If M's speed is $9\frac{1}{11}$ % less and N's speed is $11\frac{1}{9}$% more and the two start simultaneously from the same point in opposite directions, they would meet after
(A) 800 seconds (B) 400 seconds
(C) 1600 seconds (D) 769 seconds

**8.** Three cyclists Rona, Sona and Tina start from Battisgarh to Chattisgarh at 8.00a.m., 9.00a.m. and 10.00a.m., respectively. The distance Tina travels to meet Sona is twice the distance she travels to meet Rona. By the time Tina reaches Chattisgarh, Sona is twice as far from Rona as she is from Tina. If the ratio of the speeds of Sona and Tina is 2 : 3, at what time does Rona reach Chattisgarh?
(A) 8.00 p.m. (B) 6.00 p.m.
(C) 3.00 p.m. (D) 2.00 p.m.

**9.** A suspect escaped from City $X$ in a car and is going towards City $Y$ on a straight road, at a speed of 60 km/hr. After 15 minutes, a police party $P$ left $X$ to chase the suspect at 65 km/hr. $A$ police party $Q$ left $Y$ at 60 km/hr, at the same time as $P$ left $X$. When the suspect is caught by either $P$ or $Q$, they hold him for interrogation on the spot for 10 minutes. If $XY = 300$ km, which of the following statements is true?
(A) $Q$ reaches 37.5 min after $P$ catches the suspect.
(B) $P$ reaches 37.5 min after $Q$ catches the suspect.
(C) $P$ reaches in less than 3 minutes after $Q$ catches the suspect.
(D) $P$ is 20 km away when the suspect is caught.

***Directions for question 10*:** The question is based on the following information.

Two trains begin to enter a tunnel simultaneously at opposite ends on parallel tracks. While train $A$ takes 10$s$ to completely enter the tunnel, train $B$, takes only 5$s$. The distance between the points where the engines cross each other to the point where the last compartments cross each other is $42\frac{6}{7}$ m. Train $B$ is $\frac{4}{3}$ as fast as $A$.

**10.** How many seconds do the two trains take to cross each other completely?
(A) 6 (B) $8\frac{2}{7}$ (C) $7\frac{1}{7}$ (D) $8\frac{6}{7}$

**11.** Diya and Riya started from $X$ and $Y$ towards $Y$ and $X$, respectively. After reaching their destinations, they return to their respective starting points. After reaching their starting points, they again start towards $Y$ and $X$, respectively. They continue this process infinitely. But every time they meet, both decrease their speed by 30%. The first time they meet at 12 km from $Y$ and the second time they meet at 6 km from $X$. If the speed of the faster one is less than twice the speed of the slower one, where will they meet for the third time?
(A) at $Y$ (B) at $X$
(C) 18 km from $Y$ (D) 18 km from $X$

**12.** Avinash starts from his hostel at 9:30 a.m. and walks towards a neighbourhood sports complex. Sometime later, Bharat starts from the same hostel and walks towards the sports complex. $A$ few minutes after that, exactly at 9:45 a.m., Chetan also sets off from the same hostel and walks to the sports complex, he arrives at the complex at 10:30 a.m., twenty minutes after that Bharat reaches the complex and 10 minutes later, Avinash also reaches the complex. At a certain time, all three happen to be at the same point on the route. When does Bharat start from the hostel?

(A) 9:35 a.m. (B) 9:36 a.m.
(C) 9:40 a.m. (D) Cannot be determined

**13.** *P* and *Q* start at the same time from two different points *X* and *Y* and run back and forth between the two points at their respective speeds. If they meet at only 3 different points on *XY*, what could be the ratio of their speeds?
(a) 2 : 3 (b) 3 : 4 (c) 1 : 3 (*d*) 1 : 2
(A) c (B) a or d (C) b (D) a or c

**14.** Car *A* starts from city *X* at 7:00 a.m. at 50 kmph and travels towards city *Y*. Car *B* starts from *X* at 8:00 a.m. at 40 kmph and travels towards *Y*. Car *C* starts at 9:00 a.m. from *Y* and travels towards *X* at 30 kmph. When is the distance between *A* and *B* equal to that between *A* and *C* if *XY* = 120 km?
(A) 8:10 a.m. (B) 8:20 a.m.
(C) 8:30 a.m. (D) 8:40 a.m.

**15.** Three cyclists start from the same point and move in different directions. The first two cyclists move in a straight line in opposite directions. The first cyclist travels at a constant speed of 12.5 kmph, while the second cyclist starts at a speed of 5 kmph, but increases his speed by 5 kmph every hour. At the end of four hours it is found that the distance travelled by the third cyclist is the average of the distances travelled by the first two cyclists. If the distance between the first and the third cyclist is 80 km, what is the distance between the second and the third cyclist? _____ km

***Directions for question 16:*** The question is based on the following information.

There are two buses which shuttle between a hill-station at the top of a hill and a town at the bottom of the hill. They both start at the same time—one from the bottom and the other from the top of the hill. They both travel along the same route and cross each other at a certain point. After they cross, 8 times the time taken by the bus going downhill is equal to the difference between the time taken by the two buses to reach their destinations from their meeting point.

**16.** The speed of the bus from the bottom of hill to the top is 10 kmph and it takes 8 hours more than the bus from the top to complete its trip. What is the distance between the top and the bottom of the hill?
(A) 120 km (B) 180 km (C) 240 km (D) 300 km

**17.** Darpan lives in a room in which there is a large wall clock on the northern wall and a large mirror on the southern wall. One day, Darpan left the room between 7pm and 8pm and returned, between 10pm and 11pm, just when the hands of the clock interchanged their positions. As Darpan was tired when he returned, he took the time shown in the reflection to be the real time. How long after he had left did he think he returned?

(A) 5 hr 13 $\frac{39}{143}$ min (B) 5 hr 13 $\frac{121}{143}$ min

(C) 5 hr 27 $\frac{39}{143}$ min (D) 5 hr 26 $\frac{121}{143}$ min

**18.** Vicky and Nicky run back and forth between the town hall and the county station at respective speeds of 12 kmph and 18 kmph. They start simultaneously – Vicky from the town hall and Nicky from the county station. If they cross each other for the first time 14 minutes from the start, at what distance from the county station do they cross each other for the fifth time?
(A) 4.2 km (B) 2.8 km (C) 3.6 km (D) 3.5 km

**19.** *P* and *Q* are two stations 330 km apart. Train $T_1$ starts from *P* at 10 a.m. at 60 km/hr and goes towards *Q*. Train $T_2$ starts from *Q* at noon and goes towards *P*. A bird starts flying from *P* at 10 a.m. and goes towards $T_2$. It continues to fly back and forth between the trains until the trains meet. The bird covers 240 km in its first flight to $T_2$ and a total of 272 km. If the bird flew throughout at a uniform speed, then its speed is
(A) 91 km/hr (B) 80 km/hr
(C) 102 km/hr (D) 80 km/hr or 102 km/hr

**20.** Four friends - Dinesh, Eswar, Giri and Hari wanted to travel from town *A* to town *B*. Dinesh and Eswar started on Dinesh's bike, at the same time Giri started on foot and Hari started on his cycle. After some time, Dinesh dropped Eswar on the way and went back to pick up Giri while Eswar proceeded to *B* on foot. Dinesh picked up Giri and reached *B* at the same time as Eswar. Surprisingly, Hari too reached *B* at the same time. Dinesh traveled at 60 km/hr. The speed at which Eswar and Giri walked was 10 km/hr. Find Hari's average speed. (in kmph)

(A) $\frac{480}{17}$ (B) $\frac{600}{23}$ (C) $\frac{900}{29}$ (D) $\frac{540}{19}$

**21.** Alok walked along a circular track for 1 $\frac{1}{2}$ hours, taking *t* hours for each round and he then cycled around the track for 2*t* hours and completed a total of 17 rounds around the track. On another occasion, he walked around the track for 2.5 hours and then rode his motorbike around the track for 4*t* hours, thereby completing a total of 50 rounds around the track. If his speeds on the cycle and motorcycle were in the ratio 1 : 3, find the time taken to complete one round on the motorbike.
(A) 40 sec (B) 50 sec (C) 60 sec (D) 80 sec

**22.** A ship which was 154 km from the shore developed a leak in the hull, through which 4½ tonnes of water started leaking into the ship every 11 minutes. When 184 tonnes of water enter the ship, the ship would start sinking. But the pumps can throw out 1 tonne every five minutes. Find the minimum average speed of the ship so that she may not sink.
(A) 10.5 kmph (B) 12 kmph
(C) 10 kmph (D) 13 kmph

**23.** A person walked along a two-kilometre long circular track for 5 hours. He then cycled for twice as much time as he took to walk one round. He then rode a bike for as much time as he took to walk one round. He covered a total of 15 rounds in cycling and in the first 4 hours of walking. He covered a total of 17 rounds in riding his bike and in 5 hours of walking. If the speeds of his cycle and his bike were in the ratio 3 : 4, find the speed of his cycle.
(A) 12 km/hr (B) 8 km/hr
(C) 9 km/hr (D) 10 km/hr

**24.** Raju, who stays in city *Z* wants to catch a train at city *Y*. The train arrives at *Y* from city *X* which is 720 km directly to the south of *Z*. *ZY* and *XY* make angles of 60° and 30°, respectively with *XZ*. The train starts at *X* at 7:00 a.m. and travels at $90\sqrt{3}$ km/hr, while Raju drives to *Y* at 60 km/hr. If Raju wants to be at *Y* at least 20 minutes before the arrival of the train, what is the latest time at which he can start from *Z*?
(A) 4:50 a.m. (B) 5:10 a.m.
(C) 5:00 a.m. (D) 4:40 a.m.

**25.** *A* and *B* started running simultaneously in the same direction from a certain point on a circular track. They met each other for the third time after *A* covered 15 laps in 5 hours and 30 minutes. Instead, had they run in opposite directions, after how many minutes will they meet each other for the sixth time?
(A) 54 minutes (B) 60 minutes
(C) $73^1/_3$ minutes (D) Either *B* or *C*

***Directions for question 26:*** The question is based on the following information.

There are two cities *P* and *Q* along a straight line 120 km apart. *A* and *B* start at the same time from *P* and *Q*, respectively with speeds 12 km/hr and 18 km/hr, respectively. After they meet for the first time, they reverse their directions and interchange their speeds. When they reach their respective starting points, they reverse their directions once again and start proceeding towards each other.

**26.** What distance does *B* cover till the time he meets *A* for the second time?

Enter your answer as a decimal value, rounded off to one decimal place. _____.

**27.** A deer, a hare and a tortoise start from *X* and go towards *Y*. The deer starts 1 hour after the hare and the hare starts 1 hour after the tortoise. The speeds of the hare and the deer are in the ratio 3 : 4. The time the deer takes to catch up with the hare is 1.5 times the time the hare takes to catch up with the tortoise. By the time the deer reaches *Y*, the hare is 4.5 km away from *Y*. If the deer takes 7.5 hours to cover the complete distance, what is the distance between *X* and *Y* and how much time does the tortoise take to cover the entire distance?
(A) 30 km, 15 hrs (B) 15 km, 30 hrs
(C) 45 km, 20 hrs (D) 20 km, 45 hrs

**28.** Two cities *X* and *Y* lie on a straight line. Two men *P* and *Q* left simultaneously for *Y* and *X* from *X* and *Y*, respectively. *P* reaches *Y* and immediately turns around and moves towards *X*. On reaching *X*, again he turns around and moves towards *Y*. This kind of movement continues indefinitely. *Q* also travels in a similar manner. The distance between *X* and *Y* is 1000 m and the ratio of the speeds of *P* and *Q* is 3 : 2. Find the distance travelled by *P* when he meets *Q* for the 4th time.
(A) 4000 m (B) 5600 m
(C) 4200 m (D) 4800 m

**29.** Ajay and Sanjay start simultaneously from the same point on a circular track. If they travel in opposite directions, they meet at 7 distinct points on the track whereas if they travel in the same direction, then they meet at *n* distinct points on it where *n* is a prime number. If Ajay is faster than Sanjay and Sanjay's speed is *P*% less than that of Ajay, then which of the following can be a value of *P*?
(A) 25 (B) 50
(C) 75 (D) $83^1/_3$

**30.** Three runners *A*, *B* and *C* run around a circular track at different speeds. They start at the same point, but while *A* and *B* run clockwise, *C* runs anticlockwise. All 3 go around the track several times. It is found that between *A* and *B*, the faster overtakes the slower at 3 different points. *A* and *C* cross each other at 7 different points and *B* and *C* cross each other at 10 different points. What could be the ratio of the speeds of *A*, *B* and *C*?
(A) 4 : 3 : 7 (B) 4 : 7 : 3
(C) 3 : 4 : 7 (D) 3 : 7 : 4

## EXERCISE-5 Based on Data Sufficiency

***Directions for questions 1 to 30*:** Each question is followed by two statements, I and II. Answer each question based on the following directions:

Choose (A) If the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.

Choose (B) If the question can be answered by using either statement alone.

Choose (C) If the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (D) If the question cannot be answered even by using both the statements together.

**1.** The distance covered by a bus is 400 km. What is the speed of the bus?

I. If the speed of the bus reduces by 20 kmph, it would take 20 hrs more to cover the distance of 400 km.

II. The speed of the bus was at least 20 kmph and the time taken by it was at least 20 hrs.

**2.** Trains $T_1$ and $T_2$ start simultaneously from $P$ and $Q$, respectively towards each other and meet at 11:30 a.m. Train $T_2$ travels at a speed of 60 kmph. What is the distance between $P$ and $Q$?

I. Train $T_1$ leaves for $Q$ at 9:30 a.m. at a speed of 40 kmph.

II. The sum of the speeds of the trains is 100 kmph.

**3.** A train crosses a bridge in 1 minute at a speed of 20 kmph. What is the length of the bridge?

I. The length of the train is 200 m.

II. A man can cross the same bridge in 5 minutes.

**4.** In a running race along a circular track, $A$ and $B$ start at the same time from the same point. When will they meet for the first time at the starting point?

I. They run in the same direction.

II. $A$ and $B$ take 10 minutes and 12 minutes, respectively to complete one round.

**5.** What time does the clock show?

I. The hour hand is between 8 O' clock and 9 O' clock.

II. The hour hand and the minute hand make an angle of 72° with each other.

**6.** Ram, Ramesh and Rameez run around a circular race track of length 500 m. They start from the same point 'O' and move in the same direction. After how much time will all the three meet at the starting point 'O'?

I. Ram and Ramesh meet for the first time after 10 seconds.

II. Ramesh and Rameez meet for the first time after 10 seconds.

**7.** What is the time taken by $A$ to complete a 1 km race?

I. In a 1-km, race $A$ beats $B$ by 100 m.

II. In a 1-km, race $B$ beats $C$ by 200 m or 25 sec.

**8.** $A$ and $B$ start from two points $P$ and $Q$ and move in the same direction on a straight road. $A$'s speed is 5 m/sec. Find the distance between them after 10 seconds.

I. Their relative speed is 1m/sec.

II. The initial distance between them is 30 m and they meet after 15 seconds.

**9.** $A$, $B$ and $C$ are three persons running around a circular track. $A$ and $B$ run in the same direction while $C$ runs in a direction opposite to that of $A$ and $B$. Find the time taken by all the three to meet for the first time.

I. $A$ and $C$ meet after every 16 minutes

II. $A$ and $B$ meet after every 20 minutes, $B$ and $C$ meet after every 18 minutes.

**10.** $A$, $B$ and $C$ run a race of 120 m. $A$ beats $B$ and $C$ by 30 seconds and 40 seconds, respectively. Find the time taken by $A$ to complete the race.

I. $A$ beats $B$ by 40 m.

II. $A$ gives $C$ a start of 8 m and beats him by 20 m.

**11.** Prasanna travels from $A$ to $B$ and back. What is his average speed for the whole journey?

I. Prasanna totally travels 200 km in a span of 6 hours.

II. Prasanna's speed from $A$ to $B$ is 30 km/hr while his speed during the return journey goes up by $33\frac{1}{3}$ % when compared to the forward journey.

**12.** What is the ratio of the speeds of Pulkit and Angadh?

I. Even if Pulkit starts half an hour earlier than Angadh he takes one hour more than Angadh in covering a certain distance.

II. Angadh and Pulkit start from the two ends of a track at the same time. After crossing each other, Angadh

and Pulkit take $3^1/_3$ hours and $4\frac{4}{5}$ hours to reach opposite ends, respectively.

**13.** What is the speed of the train?

I. To cover a certain distance in 2 hours, Karun travelled 12 km by ship, 45 km by train and 6 km by rickshaw.

II. The ratio of the speeds of the ship, the train and the rickshaw by which Karun travelled is 4 : 15 : 1.

**14.** Aarti and Bhallu participated in a race where they run from *P* to *Q* and back to *P*. Bhallu being the faster of the two, reached *Q* first and met Aarti at *R* on his way to *P*. What is the length of *PR*?

I. $PQ = 20$ km

II. Ratio of speeds of Aarti and Bhallu is 3 : 5

**15.** A train crossed two men moving in the same direction. The speed of one man is 2 m/s more than that of the other. What is the length of the train?

I. The train crosses the slower man in9 seconds and the other man in 10 seconds.

II. The faster man walks at 4 m/s.

**16.** What is the speed of the train?

I. It takes 6 seconds to cross a stationary observer.

II. It takes 25 seconds to cross a 750-m long platform.

**17.** What is the time taken to travel from *A* to *B* at *x* kmph?

I. Time taken to travel from *A* to *B* at *x* kmph is 2 hours more than the time taken at $(x + 10)$ kmph.

II. Time taken to travel from *A* to *B* at *x* kmph is 2 hours less than the time taken to travel from *A* to *B* at $3x/4$ kmph.

**18.** By what distance does Asif beat Alok in a 800-m race?

I. In a 400-m race, Biswas beats Alok by 30 m.

II. In a 800-m race, Biswas beats Asif by 40 m.

**19.** At what time between 3 O'clock and 4 O'clock is the angle between the hands $K°$?

I. $K = 130$.

II. $K = 100$.

**20.** Speed of a motorboat in still water is how many times the speed of the water current?

I. The time taken by the boat to cover a certain distance downstream is half the time taken by it to cover the same distance upstream.

II. The speed of the boat upstream is 10 m/s.

**21.** Find the speed of the train, which crosses a signal post in 10 sec.

I. The length of the train is 200 m.

II. The train crosses a platform in 40 sec.

**22.** In a race, Ram gave Shyam a head start of at least *x* m and was beaten by him by at most *y* m. Was Ram faster than Shyam?

I. $x \geq y$

II. $x \leq y$

**23.** Mohan rowed his boat from a point *A* in a river 12 km upstream and returned to *A*. If the river was moving at a constant rate, then was his speed in still water more than 3 kmph?

I. He took four hours for the upstream journey.

II. He took three hours for the downstream journey

**24.** Two people *A* and *B* drove one car on a 500 Km trip. *A* drove for 5 hours, which was half an hour more than the time *B* drove. What was *B*'s average speed?

I. *B* drove 50 km more than *A*.

II. A's average speed was 5 kmph less than that of *B*.

**25.** A boat started moving from *A* to *B*. What is the speed of boat in still water?

I. Speed of the river current is 2 m/sec.

II. The distance between *A* and the boat is increasing at the rate of 1 m/s.

**26.** What is the distance covered by a car during a certain trip?

I. The average speed of the car is 50 kmph.

II. The car would have covered 100 km more (in the same time) if the average speed increases by 25%.

**27.** If Ram walked from his home to his office at '*a*' kmph, he would be '*a*' minutes early. Instead had he walked at '*b*' kmph, he would be '*b*' minutes early, where *a* and *b* are distinct. Find the usual time he takes to reach his office.

I. $a + b = 10$

II. $a - b = 2$

**28.** Trains *A* and *B* have their lengths in the ratio 3 : 2. A takes *x* seconds to cross platform $P_1$. *B* takes y seconds to cross platform $P_2$. Is $\frac{x}{y} > \frac{5}{4}$?

I. A and *B* have equal speeds.

II. $P_1$ and $P_2$ have equal lengths.

**29.** Train $A$ takes 50 seconds to cross a bridge $B_1$ 500 m long. Train $B$ is twice as long as $A$ and has half its speed. Find the time taken by it to cross bridge $B_2$.

I. Length of $A$ is 300 m.

II. Length of $B_2$ is 1000 m.

**30.** In a kilometre race, $A$ beats $B$ by $x$ m and $B$ beats $C$ by $y$ m. Does $A$ beat $C$ by more than 400 m?

I. $x = 200$; $y \geq 200$

II. $x = 200$; $y \leq 200$

## ANSWER KEYS

### Exercise-1

1. (a) 15 (b) 30 (c) 6
2. (a) 72 (b) 162 (c) 45 (d) 1.3
3. 45
4. (A)
5. (B)
6. (B)
7. 300
8. (B)
9. (A)
10. (B)
11. (B)
12. 35
13. (D)
14. (B)
15. (B)
16. 180
17. (B)
18. (A)
19. (D)
20. 12
21. 30
22. (C)
23. 20
24. (D)
25. (B)
26. (A)
27. 10
28. (C)
29. (A)
30. (B)
31. (A)
32. 22
33. (B)
34. 20
35. 25
36. (D)
37. 100
38. (C)
39. 180
40. (A)
41. 50
42. (C)
43. (B)
44. (i) 500 (ii) 125
45. (A)
46. 47
47. (D)
48. (B)
49. 3
50. (A)
51. 14
52. (D)
53. (D)
54. 19
55. (C)
56. 25
57. (A)
58. (C)
59. 24

### Exercise-2

1. (C)
2. (B)
3. 135
4. 6, 48
5. 6.4 m
6. 300
7. 26
8. 50
9. 6
10. (B)
11. 50
12. 3
13. 10
14. (D)
15. (B)
16. 40
17. (B)
18. (B)
19. 5, 38
20. 3.5, 5.25
21. 450
22. 2000
23. (D)
24. 12,00
25. 240
26. (C)
27. (B)
28. 90 km
29. 4.5
30. (C)
31. 80
32. (D)
33. 150
34. (B)
35. 108
36. (A)
37. 500
38. (A)
39. 10
40. 65
41. 12
42. 3
43. (B)
44. (B)
45. 60
46. (B)
47. 1000
48. (C)
49. 8
50. 60
51. (C)
52. 15
53. 432
54. (A)
55. 48
56. (A)
57. 4
58. 120
59. 67.5
60. (B)

### Exercise-3

1. (a) 15000 (b) 12500
2. (D)
3. (B)
4. (C)
5. 7
6. 27
7. 90
8. (B)
9. 10, 30
10. (B)
11. 500
12. 15
13. (D)
14. (C)
15. (D)
16. (B)
17. 120
18. (D)
19. (A)
20. (A)
21. (B)
22. (C)
23. 6
24. (B), 3
25. (A)
26. (B)
27. (C)
28. (C)
29. 30
30. (B)
31. (C)
32. (C)
33. (C)
34. (C)
35. (B)
36. (C)
37. 54
38. 60
39. (B)
40. (D)
41. (B)
42. 90
43. (B)
44. 330
45. (B)
46. (A)
47. (A)
48. (A)
49. (a) 14 (b) 154
50. 60
51. 10
52. 10, 32
53 (i) (A) (ii) (B)
54. (C)
55. 10
56. (A)
57. 1, 5
58. 9.6
59. (D)
60. (B)

### Exercise-4

1. (A)
2. (C)
3. (C)
4. (B)
5. 1
6. (C)
7. (A)
8. (A)
9. (C)
10. (C)
11. (A)
12. (A)
13. (D)
14. (B)
15. 60
16. (A)
17. (C)
18. (A)
19. (D)
20. (D)
21. (C)
22. (A)
23. (C)
24. (D)
25. (D)
26. 50.4
27. (A)
28. (C)
29. (D)
30. (B)

### Exercise-5

1. (B)
2. (A)
3. (A)
4. (A)
5. (D)
6. (D)
7. (C)
8. (A)
9. (A)
10. (B)
11. (B)
12. (A)
13. (C)
14. (C)
15. (A)
16. (C)
17. (A)
18. (C)
19. (A)
20. (A)
21. (A)
22. (A)
23. (A)
24. (B)
25. (C)
26. (A)
27. (A)
28. (D)
29. (A)
30. (A)

## SOLUTIONS

## EXERCISE-1

**1.** (a) $54\text{kmph} = 54 \times \frac{5}{18}\text{m/s} = 15\text{ m/s}$

(b) $108\text{kmph} = 108 \times \frac{5}{18}\text{m/s} = 30\text{ m/s}$

(c) $21.6\text{ kmph} = 21.6 \times \frac{5}{18}\text{m/s} = 6\text{ m/s}$

**2.** (a) $20\text{ m/s} = 20 \times \frac{18}{5}\text{ kmph} = 72\text{ kmph.}$

(b) $45\text{ m/s} = 45 \times \frac{18}{5}\text{ kmph} = 162\text{ kmph}$

(c) $12 \cdot 5\text{ m/s} = 12 \cdot 5 \times \frac{18}{5}\text{ kmph} = 45\text{ kmph}$

(d) $\frac{13}{36}\text{ m/s} = \frac{13}{36} \times \frac{18}{5} = 1.3\text{ kmph}.$

**3.** As he travelled for equal time durations at different average speeds, his average speed for the journey happens to be the average of the average speeds, i.e., $\frac{40+50}{2} = 45\text{ kmph}.$

**4.** Let the lengths of the faster and slower trains be $L_1$ and $L_2$, respectively. Let the speeds of the faster and slower trains be $S_1$ and $S_2$, respectively.

$$\frac{L_1 + L_2}{S_1 - S_2} = 40 \quad (1)$$

$$\frac{L_1 + L_2}{2S_2 - S_1} = 80 \quad (2)$$

Dividing (1) by (2), $\frac{2S_2 - S_1}{S_1 - S_2} = \frac{1}{2}$

$4S_2 - 2S_1 = S_1 - S_2$

$\frac{S_1}{S_2} = \frac{5}{3}.$

**5.** In a day, any angle $\theta$ $(0° < \theta < 180°)$ between the two hands (hours and minutes) of a clock occurs 44 times.

**6.** $B$ takes 2 seconds to run the first 10 m.

$\therefore$ $B$'s speed $= \frac{10}{2} = 5$ m/sec.

$B$ takes $\frac{100-10}{5} = 18$ seconds to run the race from the point he was at when $A$ starts the race.

$\therefore$ A's speed $= \frac{100}{18} = 5\frac{5}{9} = $ m/sec

**7.** Let the distance travelled be $x$ km.

$$\text{Total time} = \frac{x/2}{30} + \frac{x/2}{25} = 11$$

$$\Rightarrow \frac{x}{60} + \frac{x}{50} = 11 \Rightarrow \frac{5x + 6x}{300} = 11$$

$\Rightarrow x = 300.$

**8.** When $A$ finishes the race, $B$ would have run the length of the race – (the start distance + the distance by which $A$ beats him) = 600 – (200 + 120) = 280 m.

Ratio of speeds of $A$ and $B$ = 600 : 280 = 15 : 7.

**9.** In normal clock for every $65\frac{5}{11}$ minutes, the minute hand and hour hand coincide with each other.

As in this clock, for every 65 minutes, coinciding of both hands occurs, the clock is running fast, i.e., gaining time.

**10.** Let the speed of the boat in still water and the speed of the stream be $u$ and $v$, respectively.

$u + v = u - v + 2$

$2v = 2$ kmph$\Rightarrow v = 1$ kmph

**11.** Let the speed of the stream be $S$ kmph.

Speed of the boat in still water = $3S$.

$$\frac{40}{3S - S} = \frac{40}{3S + S} + 4$$

S = 2.5 kmph.

**12.** Since the ratio of this speeds is 3 : 7. The ratio of the times taken to cover the same distance is 7 : 3.

Let the times taken by $A$ and $B$ be $7t$ min and $3t$ min, respectively

Given that, $7t - 3t = 20$

$\Rightarrow t = 5$

$\therefore$ A takes $7 \times 5$, i.e., 35 minutes to cover the distance.

**13.** Let the three cyclists be $A$, $B$ and $C$.

Let their respective speeds be 5 m/sec, 10 m/sec and 20 m/sec.

Relative speed of $A$ and $B = (10 - 5) = 5$ m/sec.

Length of the track = 100 metres.

The intervals at which $A$ and $B$ meet = 100/5

= 20 seconds (1)

Relative speed of $B$ and $C = (20 - 10) = 10$ m/sec.

The intervals at which $B$ and $C$ meet = 100/10

= 10 seconds (2)

The interval at which $A$, $B$ and $C$ meet

= L.C.M. of (1) and (2).

= L.C.M. of 10 and 20 = 20 seconds.

**14.** If the train is moving at its usual speed, it would take (40) (2) = 80 sec to cross the bridge.

$$\text{Speed} = \frac{800+200}{80} = \frac{100}{8} = 12.5 \text{ m/sec.}$$

**15.** $\text{Time taken} = \text{L.C.M.}\left(\frac{900}{15-10}, \frac{900}{20-10}\right) = 180 \text{ seconds.}$

**16.** $\text{Time taken} = \text{L.C.M.}\left(\frac{900}{10}, \frac{900}{15}, \frac{900}{20}\right) = 180 \text{ seconds.}$

**17.** Let the length and the speed of the train be $L$ and $S$, respectively.

$L = 40\ S$ (1)

Time taken by the train to cross a platform of length '$L$'

$$= \frac{L+L}{S} = \frac{2L}{S} = 80 \text{ seconds } (\because L = 40\ S)$$

**18.** In a day the angle between the two hands is 0° for 22 times and is 180° for 22 times.

$\therefore$ The two hands coincides (i.e., 0≡) for 22 times.

**19.** Let the speeds of the cars be $5x$, $6x$ and $7x$.

Let the distance be $d$.

$$\text{Ratio required} = \frac{d}{5x} : \frac{d}{6x} : \frac{d}{7x} = 42 : 35 : 30.$$

**20.** Let the distance between school and house be $x$ km.

$$\frac{x}{6} - \frac{x}{8} = \frac{30}{60}$$

$$\Rightarrow \frac{4x-3x}{24} = \frac{1}{2} \Rightarrow x = 12.$$

**21.** Let the distance be $d$ km and usual speed be $5x$ kmph.

$$\frac{d}{3x} - \frac{d}{5x} = \frac{20}{60}$$

$$\Rightarrow \frac{5d-3d}{15x} = \frac{1}{3}$$

$$\Rightarrow 2d = 5x.$$

$$\text{Usual time taken} = \frac{d}{5x} = \frac{d}{2d} = \frac{1}{2} \text{ hour} = 30 \text{ min}.$$

**22.** Let the speeds of the trains $P$ and $Q$ be $S$ and $3S$, respectively. Let the length of the train $Q$ be $L$.

$$\frac{L}{3S-S} = 20 \Rightarrow L = 40S.$$

Time taken by the train $Q$ to cross the person if the trains are moving in opposite direction = ——— = 10 seconds.

**23.** Let the time taken reach the destination be $3x$ hours.

Total distance $= 40 \times 3x = 120x$

He covered $\frac{2}{3} \times 120x = 80x$ km in $\frac{1}{3} \times 3x = x$ hours

So, the remaining $40x$ km, he has to cover in $2x$ hours

$$\text{Required speed} = \frac{40x}{2x} = 20 \text{ kmph}.$$

**24.** Total time taken to cover $5x$ km $= \frac{2x}{20} + \frac{3x}{30} = \frac{6x+6x}{60} = \frac{x}{5}$

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time taken}} = \frac{5x}{\frac{x}{5}} = 25 \text{ kmph.}$$

**25.** Let the usual time taken by the car to reach its destination be $t$ hours. Distance travelled by the car = 80 $(t + 1)$ = 100 $t$.

Hence, $80t + 80 = 100t$.

$\Rightarrow t = 4$ hours.

**26.** Let the distance be $x$

$$\frac{x}{20} - \frac{x}{30} = 3$$

$\frac{3x-2x}{60}=3 \Rightarrow x=180$ km.

**27.** Let the times taken to cover from $A$ to $B$ in car and cycle by $x$ hours and $y$ hours, respectively.

$x+y=7$ (1)

$2x=4$ (2)

Solving the both equations we get $y=5$

So, time taken to cover both ways by cycle $=2y$ hours $=10$ hours.

**28.** By the time $B$ finishes the race, $C$ requires 20 more seconds to finish the race.

10 seconds before that, when $A$ finishes the race, $B$ requires 10 seconds more. $C$ requires 30 seconds to finish the race.

$\therefore$ $A$ beats $C$ by 30 seconds.

**29.** Total distance by which they are separated in 1 second = 1.2 + 1.5 = 2.7 m

Total distance of separation in 10 seconds = (2.7) (10) = 27 m.

Hence, they would be 27 m apart after 10 seconds.

**30.** Let the distance between the school and the house be $x$ km.

$\frac{x}{4}-\frac{x}{6}=\frac{30}{60}$

$\frac{3x-2x}{12}=\frac{1}{2} \Rightarrow x=6$ km.

**31.** Consider the line joining the centre of the dial and the number 12 on the dial as the reference line. Measurements of angles made by the two hands, during any specified interval, are measured with reference to their line.

At 12 O' clock, the two hands coincide and the angle made by each of the hands with the reference line is zero degrees.

The angle made by the hour hand at 3:30 p.m.

$=(3\times 30°)+(30\times 1/2°)=105°$ (1)

The angle made by the minute hand, in 30 minutes

$=30\times 6°=180°$ (2)

Hence, at 3:30 p.m., the minute hand leads the hour hand by an angle equal to $(180°-105°)=75°$.

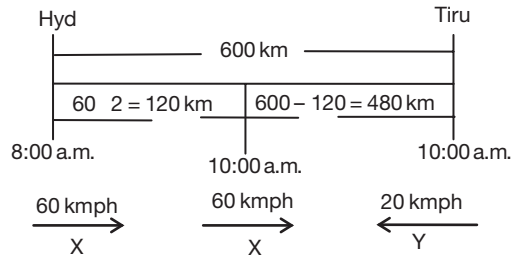
Hence, the required angle is 75°.

**32.** 22 times.

**NOTE**

The hands of a clock are separated by any angle other than 0° and 180° 44 times in a day.

**33.**




$X$ travels 2 hours before $Y$. Distance covered in 2 hours = $2\times 60=120$. At 10:00 a.m. The distance between $X$ and $Y$ is 480 km.

Time taken for $X$ and $Y$ to meet

$\frac{\text{Distance}}{\text{Relative Speed}}=\frac{480}{60+20}=6$ hours

So, $X$ and $Y$ meet 6 hours after 10:00 a.m.

i.e., at 4:00 p.m.

**34.** Time taken to cross each other

$=\frac{\text{sum of the lengths of the two trains}}{\text{relative speed}}$

$=\frac{250+150}{(30+42)\frac{5}{18}}=20\sec.$

**35.** Let the distance be $x$ km

$\frac{x}{50}-\frac{x}{60}=\frac{5}{60}$

$\frac{6x-5x}{300}=\frac{1}{12} \Rightarrow x=\frac{300}{12}=25.$

**36.** Time taken $=\frac{180}{(40-4)\frac{5}{18}}\sec=\frac{180}{10}=18\sec.$

**37.** Time taken $=\frac{400+600}{(36)\left(\frac{5}{18}\right)}=100$ seconds

**38.** Let the time taken by Ajay to reach his office travelling at his usual speed be $t$. If Ajay doubles his speed, he takes half the time he usually takes to travel to his office.

$t - \frac{1}{2}t = \frac{1}{2} \Rightarrow t = 1$ hour.

If Ajay travels to office at thrice his usual speed, he would take one-third of the usual time, i.e., $\frac{1}{3}$ hours.

**39.** Time taken = L.C.M.

$\left(\frac{900}{10}, \frac{900}{15}, \frac{900}{20}\right) = 180$ seconds.

**40.** Let the total distance travelled by Ajay be 16 km. Sum of time durations in which the distances travelled at 2 km/hr, 4 km/hr and 1 km/hr were covered

$= \frac{16/2}{2} + \frac{16/4}{4} + \frac{16/4}{1}$

$= 4 + 1 + 4 = 9$ hours.

Average speed $= \frac{\text{Total distance}}{\text{Total time}} = \frac{16}{9}$ kmph.

**41.**




Ratio of speeds of $A$ and $B$ is 3:2. So, by the time $B$ covers 20 km, $A$ covers 30km.

They totally cover (20 + 30) km to meet

So, distance between $P$ and $Q$ is 50 km.

**42.** Murali leaves Chennai 2 hours after Kalyan, so in two hours Kalyan covers $2 \times 40 = 80$ km.

Kalyan is 80 km away from Murali when Murali starts.

$\text{Time} = \frac{\text{Distance}}{\text{Ralative speed}}$

$\text{Time} = \frac{80}{60 - 40} = 4$ hours

Murali travels for 4 hours and meets Kalyan 4 hours after 10:00 a.m. that is they will meet at 2:00 p.m.

**43.** Let the distance covered by the three runners be $d$. Let the speeds of the runners A, B and C be $x$, $2x$ and $3x$, respectively.

$\frac{d}{x} - \frac{d}{3x} = 2$

$\frac{d}{x} = 3$ hours.

Time taken by $B$ to cover the same distance

$= \frac{d}{2x} = \frac{3}{2}$ hours.

**44.** (i) Time taken $= \frac{1000}{5-3} = 500$ seconds.

(ii) Time taken $= \frac{1000}{5+3} = 125$ seconds.

**45.** Let the speed of the boy and the length of the train be $S$ and $L$, respectively.

$L = 30\ (8 - S) = 25(9 - S)$

$240 - 30S = 225 - 25S$

$S = 3$ m/sec

**46.** Let length train be $x$ m.

When a train crosses an electric pole, then the distance covered is its own length. So,

$x = 12 \times 36 \times \frac{5}{18} m = 120$

Time taken to cross the platform $= \frac{120 + 350}{36 \times \frac{5}{18}} = 47$ sec.

**47.** If a person covers two equal distances at speeds of $x$ kmph and $y$ kmph, respectively, then average speed = $\frac{2xy}{x+y}$ kmph

$\therefore$ Average speed $= \frac{2 \times 20 \times 60}{80} = 30$ kmph.

**48.** Let the speed of Tarun be $4x$ kmph. The speed of Varun is $3x$ kmph.

If Tarun covers a distance of $4x$ km, he gains $x$ km over Varun; $\Rightarrow$ to gain $x$ km, he shall cover $4x$ km.

Distance between Tarun and Varun is 10 km.

In order to gain 10 km over Varun, Tarun has to cover 40 km.

**49.** Downstream speed of Anand = 10 kmph.

Required time $= \frac{30}{10} = 3$ hours.

**50.** Let the length of the train be $L$ m.

Let the speed of the train be $S$ m/sec

$L = 30S$

$L + 200 = 40S$

$\Rightarrow 30S + 200 = 40S$

S = 20 m/sec

**Alternate Method:**

The train takes 10 seconds to cover 200 m of its own length. Hence, its speed is 20 m/sec.

**51.** Ratio of the speeds of $P$ and $Q = 1000 : 1000 - 125 = 8 : 7$.

$Q$'s speed $= \frac{7}{8}(16) = 14$ m/sec.

**52.** For every minute, minute hand gains 5.5° over hour hand. From the time the two hands coincide once, to the next time the two hands coincide, minute hand has to gain 360°.

$\therefore$ The required time $= \frac{360}{5.5} = \frac{720}{11} = 65\frac{5}{11}$ minutes.

**53.** If $Q$ starts from the same point from which $P$ started, then the distance of separation between them is the distance travelled by $P$ in 2 hours, (because 8:00 a.m. – 6:00 a.m. = 2 hours).

The information about the starting points of $P$ and $Q$ is not available. Hence, the question cannot be answered.

**54.** When $A$ ran 100 m, $B$ would have run 90 m.

$\therefore$ $B$'s speed is 0.9 ($A$'s speed).

Similarly, $C$'s speed is 0.9 ($B$'s speed), i.e., 0.81($A$'s speed).

$\therefore$ $A$ beats $C$ by 0.19 (length of the race) = 19 m.

**55.** Let the length of the train be $L$

$L + 300 = 40 \times 11 = 440$ (1)

$L = 140$ m.

**56.** Time taken to cross a moving person $= \frac{\text{length of train}}{\text{relative speed}}$

Time taken $= \frac{375}{(50+4)\frac{5}{18}} = \frac{375}{15}$ sec $= 25$ sec.

**57.** If the angle between the minute hand and hour hand is 90°, then the minute hand has to gain 180°, to get the angle between both the hands again as 90°.

$\therefore$ The time required $= \frac{180°}{5.5°}$ minutes.

$= \frac{360}{11} = 32\frac{8}{11}$ minutes.

**58.** Total distance travelled by $A = 3 \times 60 + 4 \times 95 = 560$ km.

Total time taken = 3 + 4 = 7 hours.

Average speed $= \frac{560}{7} = 80$ kmph.

**59.** Ratio of speeds of Ganesh and Girish = 2 : 1.

$\therefore$ Ganesh will complete as many more rounds than Girish as the number of rounds Girish completes in an hour.

Required number $= \frac{(4)(3600)}{600} = 24$.

## EXERCISE-2

**1.** Let the time be $p$ minutes after 7 O'clock when the angle is 90°.

Then, we have two equations as found in the previous example (at exactly 7 O'clock the hours hand makes 210° with the vertical line)

$6p - [(210) + p/2)] = 90$ (1)

$[(210) + p/2)] - 6p = 90$ (2)

From (1) and (2),

we get $p = 54\,{}^{6}/_{11}$ and $p = 21\,{}^{9}/_{11}$

Hence, the two hands will be at right angles at $7{:}21\,{}^{9}/_{11}$ and $7{:}54\,{}^{6}/_{11}$ of clock.

**2.** Let $x$ km be the distance between $P$ and $Q$.

We know that the ratio of the distance travelled in a given time is equal to the ratio of the speeds.

$\therefore \frac{x-15}{x+15} = \frac{3}{5} \Rightarrow x = 60$ km.

When Arpita travels 60 km, Bipasha travels $60 \times \frac{5}{3} = 100$ km.

As $PQ = 60$, Bipasha is at a distance of 120 – 100 = 20 km from $P$.

**3.** Let the length of each train be $L$. The speeds of the first and the second trains are $x$ km/hr (say) and 90 km/hr, respectively and they take $4s$ and $6s$, respectively to cross the man. The first is $1^1/_2$ times as fast as the second, i.e.,

$x = \left(\frac{3}{2}\right)(90) = 135$

**4.** Let the distance between $A$ and $B$ be $d$ km.

Trains $P$ and $Q$ take 4 hours and 6 hours, respectively to cover $d$ km.

Speed of train $P = d/4$

Speed of train $Q = d/6$

At 5 a.m. the distance between two trains

$= d - (d/4) = 3d/4$

Time taken for the two trains to meet from 5 a.m.

$= \frac{\frac{3d}{4}}{\frac{d}{4}+\frac{d}{6}} = \frac{9}{5}$ Hours = 108 mins

The two trains meet $\frac{9}{5}$ hours from 5 a.m., i.e., at 6:48 a.m.

**5.** Let the person cover $x$ m in the 1st second. In the 2nd second, he covers $x/4$ m. Hence, in 2 seconds, he covers $x + x/4 = 5x/4$ m

In the 3rd second, he covers $1/4(5x/4)$ m

Hence, in 3 seconds, he covers $5x/4 + 5x/16$

$= 25x/16 = (5/4)^2 x$ m

Thus after 5 seconds, the person would have covered $(5/4)^{5-1}x = 625x/256$ m

$625x/256 = 15.625$

$x = 15.625/625 \times 256 = 6.4$ m

**6.** From the first case to the second, the speed becomes 6/5 times, so the time would become 5/6 times, which is a decrease of 1/6 times (of the original time).

$\frac{t}{6} = 1$

$\Rightarrow t = 6$

$\therefore$ Distance $= 6 \times 50$ kmph $= 300$ km.

**7.** Time for the first meeting $= \frac{400}{10+40} = 8\ s$

Speeds after the first meeting

$P = 20$ m/s $Q = 20$ m/s

Time between the first and the second meeting

$\frac{400}{20+20} = 10$ s

Speeds after the second meeting

$P = 40$ m/s $Q = 10$ m/s

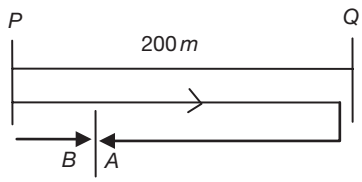
Time between the second and the third meeting

$= \frac{400}{40+10} = 8$ s

Time from the start to the third meeting

$= 8 + 10 + 8 = 26\ s$

**8.** This is a case where time is a constant. Therefore the ratio of distances covered by $A$ and $B$ are in the ratio of their speeds.




From the diagram, we see that total distance covered by $A$ and $B$ together is 400 m.

$\Rightarrow B$ covers $\frac{1}{8} \times 400 = 50$ m

$\therefore$ They meet at a distance of 50 m from $P$.

**9.** Let his speed in still water and the speed of the stream (both in kmph) be $u$ and $v$, respectively.

Given, $\frac{30}{u-2-(v+2)} = \frac{30}{u-v} + 10$

$\Rightarrow \frac{30}{u-v-4} - \frac{30}{u-v} = 10$

Let $W = u - v$

$\Rightarrow \frac{30}{W-4} - \frac{30}{W} = 10$

$\Rightarrow \frac{120}{W(W-4)} = 10$

$\Rightarrow W^2 - 4W - 12 = 0$

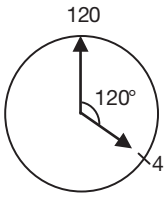
$\Rightarrow (W-6)\ (W+2) = 0$

$\Rightarrow W = 6$ or $-2$.

As $W > 0$, $W = 6$.

**10.** Relative speed of minute hand = $6° - \frac{1}{2}° = 5.5°$ per minute.

At 4 O'clock angle between the hands is $30° \times 4 = 120°$




Two hands will be in a straight line when the angle between them is 180°. For this to happen, the minute hand has to travel (120° + 180°) with reference to the hour hand, i.e., with a relative speed of *A* 5.5° per min. Time required will be $\frac{120+180}{5.5} = \frac{300}{\frac{11}{2}}$

$= \frac{600}{11} = 54\frac{6}{11}$ min after 4 O'clock.

Hence, at $4{:}54\frac{6}{11}$ the two hands will be in the straight line.

**11.** Average Speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

$$= \frac{x}{\frac{\frac{x}{3}}{50} + \frac{\frac{2}{5}\left(\frac{2x}{3}\right)}{40} + \frac{\frac{3}{5}\left(\frac{2x}{3}\right)}{60}} = 50 \text{ kmph.}$$

**12.** The time taken for all the three men to meet at the starting point for the first time in hours

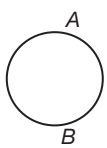
$$= \text{L.C.M.}\left(\frac{3}{6}, \frac{3}{4}, \frac{3}{5}\right) = \text{L.C.M.}\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{5}\right)$$

$$= \frac{\text{L.C.M.}(1,3,3)}{\text{H.C.F.}(2,4,5)} = \frac{3}{1} = 3$$

**13.** The train takes 10 seconds longer to cross the second bridge than it takes to cross the first bridge. The second bridge is 100 meters longer than the first bridge.

From these two inferences, it follows that the train takes 10 seconds to travel 100 meters, i.e., its own length.

**14.**




$T_0$ = starting time = 0

$T_1$ = first pass at *B*

= (6/12) × 60 = 30 minutes

$T_2$ = second pass at *B*

= 30 + 30 + (6/9) *x* 60 = 100 minutes

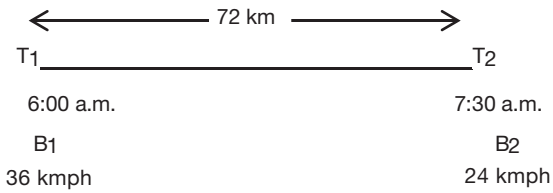
$T_3$ = third pass at *B*

$$= 100 + \frac{6}{9} \times 60 + \frac{6}{6} \times 60 = 140 + 60 = 200 \text{ minutes}$$

Interval between the first pass and third pass

= 170 minutes

**15.**




At 7:30 a.m. $B_1$ is 54 km from $T_1$ and $B_2$ just starts from $T_2$.

$B_1$ and $B_2$ approach each other at 60 kmph (36 + 24) and they are 72 – 54 = 18 km apart.

Since the distance between them has to be 12 km, there are 2 possibilities.

(1) Before they cross each other.

(2) After they cross each other.

**Case 1:** time = $\frac{18-12}{60}$ hr = 6 min

i.e., at 7:36 a.m.

**Case 2:** at 8:00 a.m. $B_1$ is at $T_2$ and $B_2$ is exactly 12 km from $T_2$.

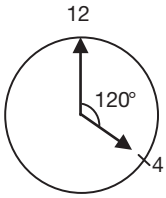
**16.** Area of the graph always represents total distance covered by the moving object, if the graph is plotted in speed and time.

Hence, total distance = Area of the graph = (10 × 20/2) + (10 × 20) + (10 × 20/2) = 400.

Or average speed = 400/30 = 13.33 m/s = $\frac{40}{3}$

**17.** Relative speed of minute hand = $6° - \frac{1}{2}° = 5.5°$ per minute.

At 4 O'clock angle between the hands is $30° \times 4 = 120°$




To coincide, minute hand has to cover 120° with reference to hour hand, i.e., with a relative speed of 5.5° per min.

So, time take will be $\frac{120}{5.5}$

$= 120 \times \frac{2}{11} = \frac{240}{11} = 21\frac{9}{11}$ min.

Hence, at $4 : 21\frac{9}{11}$ the two hands will coincide.

**18.** Distance that Mohan would have covered by 8 : 45 a.m.= (6) $\left(\frac{3}{4}\right) = 4.5$ km.

In the time he would reach *B*, i.e., after covering another 1.5 km, Sohan would have covered 3 km.

∴ Distance between them would be 3 km. They would meet after another $\frac{3}{6+12}$ hours, i.e., 10 minutes for the first time. Sohan would reach *B* in another $\frac{6-\left(3+\frac{10}{60}(12)\right)}{12}$ hours, i.e., 5 minutes. In this time Mohan would have covered 0.5 km.

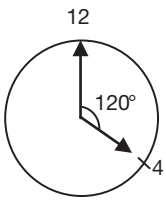
∴ Distance between them would be 1.5 km when Mohan was at *B*. Sohan would overtake Mohan in another $\frac{1.5}{12-6}$ hours, i.e., 15 minutes.

The time at which they meet for the first time

= 9 : 00 a.m.+ 10 minutes = 9 : 10 a.m.

**19.** Relative speed of minute hand = $6° - \frac{1}{2}° = 5.5°$ per minute.

At 4 O'clock angle between the hands is $30° \times 4 = 120°$




The angle between two hands will be 90° before minute hand crossing hour hand as well as after minute hand crossing hour hand. At the first and the second times, the minute hand would have covered (120° – 90°) and (120° + 90°), respectively with a relative speed of 5.5° per min.

$\text{Time} = \frac{120-90}{5.5} = 30 \times \frac{2}{11} = 5\frac{5}{11}$ min

Or

$\text{Time} = \frac{120+90}{5.5} = 210 \times \frac{2}{11} = \frac{420}{11} = 38\frac{2}{11}$ min.

Hence, angle between the hands will be 90° at $4 : 05\frac{5}{11}$ as well as $4 : 38\frac{2}{11}$.

**20.** Let the distance be *d* km

$\frac{d}{5} - \frac{d}{6} = \frac{7}{60}$

$\Rightarrow d = 3.5$ km.

Time taken to cover 3.5 km @ 5 kmph = 42 min.

$\therefore$ Required speed $= \frac{3.5}{40} \times 60 = 5.25$ kmph.

**21.** Let the time taken by P to run the race be *t* seconds. Time taken by Q to run the race is $(t + 60)$ seconds. Time taken by R to run the race is $(t + 90)$ seconds.

Ratio of the speeds of P and R $= \frac{1000}{1000-200} = \frac{5}{4}$

$\therefore \frac{t+90}{t} = \frac{5}{4} \Rightarrow t = 360$

∴ R takes 450 seconds to complete the race.

**22.** Let the length of the race be *x*, Asha's speed be *v* and Sunita's speed be $5/4\ v$

Then, Sunita covers *x* and Asha covers $x - 400$ by the time the race is over

$\frac{x}{\frac{5}{4}y} = \frac{x-400}{y}$

$4x = 5x - 2000;\ x = 2000$ m

**23.** A and B meet for the first time after $= \frac{600}{4-3}$, viz 600 seconds

B and C meet for the first time after $\frac{600}{5-4}$, viz 600 seconds

A and C meet for the first time after $\frac{600}{5-3}$, viz., 300 seconds

So, A and C meet before A and B meet.

**24.** From the data, only Lucky was moving from 6 a.m. to 8 a.m. By this time, distance covered by Lucky is $40 \times 2 = 80$ km. So, he ahead of Mr. Batra by 80 km.

The relative speed of Mr. Batra is 60 – 40 = 20 kmph. So, a distance of 80 km will be covered in 80/20 = 4 hours from 8:00 a.m., i.e., 12:00 p.m.

**25.** Lucky has been running for 6 hours at 40 kmph, so distance covered by him = $6 \times 40 = 240$ km.

**26.** Let the speeds of the two cyclists be $u$ km/hr and $v$ km/hr, respectively.

Let the two cyclists cover the distances $x$ and $y$, respectively in the time $t$.



$x = ut$ $\quad y = ut_1$

$y = vt$ $\quad x = vt_2$

$xy = xy$

$\Rightarrow t = \sqrt{t_1 t_2}$

**27.** Let $V$ be Neena's original speed. To travel 27 km,

Meenatakes $\frac{27}{4.5}$, viz.6 hours

After another 6 hours,(at double the speed) he catches up with Meena.

$6V + 12V = 4.5\ (12)$

$18V = 54 \Rightarrow V = 3$

**28.** Time taken for them to meet = 630/210 = 3 hours

extra distance travelled by Q compared to P when they meet = 3(120 – 90) = 90 km.

**29.** Let the starting time of Ram be $t = 0$ hr. Shyam starts at $t = 3$ hr. The distance between Ram and Shyam at $t = 3$ hr is 60 km. This would decrease to $O$ km in $\frac{60}{30-20}$ hr, viz 6 hr, i.e., at $t = 9$hr. This is alos the time when Tarun catches up with Ram and shyam. Tarun would have takne $\frac{9(20)}{40}$ hr (or $\frac{60(30)}{40}$ hr), viz 4.5 hourse. He must have started at $t = 4\frac{1}{2}$ hr. The time interval between the starting times of Ram and Tarun is $4\frac{1}{2}$ hours.

**30.** Let the length of the tunnel be $x$ m. Let the speeds of the train and the dog be $t$ m/sec and $d$ m/sec, respectively.

Let us say that the train is approaching the tunnel from $P$, and would be ym from $P$ initially.

$$\frac{y}{t} = \frac{\frac{5}{11}x}{d} \quad (1)$$

$$\frac{y+x}{t} = \frac{\frac{6}{11}x}{d} \quad (2)$$

Dividing (2) by (1), we get

$$\frac{y+x}{t} = \frac{6}{5}$$

$y = 5x$

$$\frac{t}{d} = \frac{y}{\frac{5}{11}x} = 11:1$$

**31. Equation Method:**

Let $d \rightarrow$ distance

$t \rightarrow$ usual time (mins)

s$\rightarrow$ usual speed

$\therefore d = s \times t \quad (1)$

Distance being constant,

$d = 1.25\ s \times (t - 12) \quad (2)$

From (1) and (2)

$1.25s \times (t - 12) = s \times t$

$\therefore t = 60$ mins.

As speed decreases by 25%, new speed = 75% of $s$

Hence, time taken becomes 4/3 times the time taken at usual speed.

Required time = $60 \times \frac{4}{3}$, i.e., 80 min

**Ratio Method:**

As the person increases his speed by 25%, i.e., $\frac{1}{4}^{\text{th}}$ of the usual speed, his new speed becomes $\frac{5}{4}^{\text{th}}$ of the usual speed. This implies, time taken becomes $\frac{4}{5}^{\text{th}}$ of the usual time (as $s \propto \frac{1}{t}$).

Hence, he saves $\frac{1}{5}^{\text{th}}$ of the usual time which is 12 mins.

If $\frac{1}{5}$th usual time $\rightarrow$ 12 mins

Usual time $\rightarrow$ 60 mins.

$\therefore$ Time in the required case using the above

$= 60 \times \frac{4}{3}$

$= 80$min

**32.** Had the train been stationary the person in the train would have heard the gunshots at an interval of 12 minutes. Since the train was moving towards the source of the sound, 11 minutes travel of the train could save one minute travel of sound. Therefore, the distance travelled by sound in 1 minute equals the distance travelled by train in 11 minutes

$\therefore 330\ (60) = 11\ (60) \times (V)$ (where the speed of the train is V $\frac{m}{s}$ )

$\Rightarrow V = 30$. The speed of the train is 30 $\frac{m}{s}$, viz 108 $\frac{km}{hr}$

**33.** The time taken by Manish and Nitin to meet for the first time.

$= \frac{\text{Length of track}}{\text{Relative speed}} = \frac{900}{5+1} = 150$ seconds:

**34.** Stoppage time per hour

$= \frac{(60-48)}{60} \times 60 \times 60$ minutes $= 12$ minutes

**35.** Let the speeds of the trains be $S_1$ m/s and $S_2$ m/s

$$\frac{150+150}{S_1+S_2} = 6 \Rightarrow S_1 + S_2 = 50 \qquad (1)$$

$$\frac{150+150}{S_1-S_2} = 30 \Rightarrow S_1 - S_2 = 10 \qquad (2)$$

$\therefore S_1 = 30$ and $S_2 = 20$.

Speed of the faster train $= 30\left(\frac{18}{5}\right) = 108$ kmph

**36.** Distance covered by the first train in one hour = 60 km.

Distance between the two trains (at 12 noon) = 465 – 60 = 405 km.

Time taken to cross each other $= \frac{405}{60+75} = 3$ hours.

They will cross each other at 12.00 noon + 3 hours, i.e., 3.00 p.m.

**37.** (The first impression of a student is that the problem has some data missing).

He can be confused further by asking if he needs any more data. Surely he will ask about the time difference between the two countries.) But tell students that this problem can be solved without any additional data because of the first and last data of the problem.

Aircraft starts at 8 a.m. from country $X$ and finally returns at 4 p.m. on same day. These two timings can be used. Therefore the craft took a total of 8 hours from 8 a.m. to 4 p.m. Now from these 8 hours we can subtract 4 hours of the halt at country $Y$.

So, it was in the sky for 4 hours. We got the duration of the travel.

Total distance is 2000 km, thus the average speed is = $2000/4 = 500$ kmph

**38.** Let the speeds of cars $P$ and $Q$ be $x$ kmph and $y$ kmph, respectively. Distances travelled by $P$ and $Q$ before meeting are $2x$ km and $2y$ km, respectively. After the meeting, time taken by $Q$ to cover $2x$ is $\frac{2x}{y}$ hours and the time taken by $P$ to cover $2y$ is $\frac{2y}{x}$ hours.

$\therefore \frac{2x}{y} - \frac{2y}{x} = 3$

$\frac{x}{y} - \frac{y}{x} = \frac{3}{2} = \frac{2}{1} - \frac{1}{2}$

Comparing both sides,

$\frac{x}{y} = \frac{2}{1} \Rightarrow x = 2y$

Total distance between $A$ and $B = 2(x + y) = 6y$.

$6y = 540 \Rightarrow y = 90$.

**39.** $5t_1^2 + 5t_1t_2 = 1500$

$5t_1\ (t_1 + t_2) = 1500$

$150\ t_1 = 1500;\ t_1 = 10$

**40.** Let the total time be $t$. Then,

$\frac{t}{4} \times 80 + \frac{3t}{4} \times 60 = 260$

$\Rightarrow 65t = 260 \Rightarrow t = 4$

$\therefore$ Average speed $= 260/4 = 65$ kmph

**41.** Let the speeds of his boat in still water and the speed of the river be $x$ kmph and $y$ kmph, respectively.

$$\frac{40}{x+y}+\frac{40}{x-y}=12$$

$$\frac{40(2x)}{x^2-y^2}=12 \qquad (1)$$

$$\frac{40}{2x+y}+\frac{40}{2x-y}=3\frac{3}{4}=\frac{15}{4}$$

$$\frac{40(4x)}{4x^2-y^2}=\frac{15}{4} \qquad (2)$$

Dividing (1) by (2)

$$\frac{4x^2-y^2}{2\left(x^2-y^2\right)}=\frac{48}{15}$$

$15(4x^2-y^2)=48(2(x^2-y^2))$

$y^2=\frac{4}{9}x^2$

$\therefore$ (1) becomes $\frac{40(2x)}{x^2-\frac{4}{9}x^2}$

$\therefore x=12$

**42.** Let the rate at which stream flows be $x$ kmph.

$$\frac{135}{12+x}+\frac{135}{12-x}=24$$

$\Rightarrow x=3.$

**43.** Let Beena's speed be '$b$' kmph and Arundhati's speed be '$a$' kmph.

$$\frac{12.5}{b-a}=2.5$$

$$\frac{12.5}{b+a}=\frac{1}{2}$$

solving for a and $b$, $b=15$.

**44.** Let the speed of the bus be $y$. Distance between them after 20 seconds

$= 400 + 600 = 1000$ m $= 1$ km

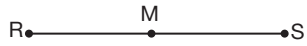
Relative speed $x$ time = Distance

$$[38-y]\times\frac{200}{3600}=1$$

$\Rightarrow y=20$ kmph

**45.** Let the time taken by both to meet be '$t$' minutes.

Let the speeds of Ram and Shyam be $r$ km/min and $s$ km/min, respectively.




Let the meeting point be $M$.

$RM = r.t$ km $= s$ . 90 km $\therefore$ rt $= 90$s (1)

$MS = s.t$ km $= r$ . 40 km $\therefore$ st $= 40$r (2)

(1) . (2) $\Rightarrow rs\ t^2 = 3600\ rs$

$t^2 = 3600$

$t > 0$

$\therefore\ t = 60$

**46.** Speed of Ankita $= x/14$

Speed of Bhoomika $= x/10$

Let Ankita be ahead of Bhoomika by '$2x$'m, where one lap is '$x$' m long.

$$\text{Time taken} = \frac{2x}{x/10-x/14}=\frac{2}{1/10-1/14}$$

$$=\frac{2\times140}{4}=70 \text{ min.}$$

**47.** Let the length of the race be $d$ m.

As both reach the finishing point simultaneously, when Ashok covers $d$ metres, Bharat must have covered

$(d-400)$ metres

$$\therefore \frac{d}{d-400}=\frac{5}{3}$$

$3d = 5\ (d-400);\ 3d = 5d - 2000$

$d = 1000.$

**48.** $\frac{100+L}{40\times5/18}=180$

$\Rightarrow L = 1900$

$$\frac{200+L}{80\times5/18}=\frac{2100}{8\times5/18}=94.5 \text{ sec}$$

**49.** Let the cycling speed of the person be $C$ and the speed of the wind be $W$.

Then, $C + W = \frac{18}{3} = 6$ kmph

Also $C - W = \frac{18}{6} = 3$ kmph

Solving, $C = 4.5$ kmph. Time taken $= \frac{36}{4.5} = 8$ hrs

**50.** Let the length of the race be $d$ m. When $X$ finishes the race, $Y$ and $Z$ would have covered $(d-12)$ m and $(d-24)$ m, respectively. When $Y$ finishes the race, $Z$ would have covered $(d-15)$ m.

Ratio of speeds of $Y$ and $Z = \frac{d-12}{d-24} = \frac{d}{d-15}$.

$(d-12)\ (d-15) = d(d-24)$

$3d = 180$

$d = 60$

**51.** If $t_u$, $t_d$ and $t_s$ be the upstream time, downstream time and time taken on still water to travel the same distance,

then $t_s = \frac{2t_u t_d}{t_u + t_d} = \frac{2\times3\times6}{3+6} = 4$ hours

**52.** Let the speeds of Ram and the escalator (both in m/sec) be $r$ and $e$, respectively.

Then, we have $9\ (r+e) = 45\ (r-e) = 90$.

$\Rightarrow r + e = 10$ and $r - e = 2$

Solving the above two equations for $r$, we get, $r = 6$.

Required time (in seconds) $= \frac{90}{6} = 15$.

**53.** Let Manish and Sheetal meet each other at point $R$ after '$t$' seconds.

$t = \sqrt{t_1 t_2} = \sqrt{16\times25} = 20$

As Manish's speed was 12 m/s, the distance between $P$ and $Q = 12 \times 36 = 432$ m.

**54.** From the given information, normal speeds of $A$ and $B$ are in the ratio 1 : 4.

Normal travel time of $B = \frac{1}{5}(60) = 12$ min

Starting time of $A$ = 6:10 a.m.

Normal total travel time of $A$ and $B$ = 50 minutes.

As $B$ doubled its speed on the day, it would reach $Q$ in 6 minutes on that day. Travel time for $A$ on that day is 44 minutes.

$\therefore$ Ratio of speeds of $A$ and $B$ on that day = 6 : 44 = 3 : 22

**55.** Let the distance be $d$

For his onward journey he took $\frac{d}{40}$

For his return journey he took $\frac{d}{60}$

$\therefore$ Average speed $= \frac{d}{\frac{d}{40}+\frac{d}{60}} = 48$ km/hr.

**56.** From the data, the time at $P$ is ahead of the time at $Q$, ($P$ is to the east of $Q$). Let $P$ be $x$ hours ahead of $Q$. The speeds of both flights are the same ($\because$ Distances traveled by the two flights are the same). The travel times from city $Q$ to city $P$ and from city $P$ to city $Q$ are $(13 + x)$ hours and $(20 - x)$ hours, respectively.

$700\ (20 - x) = 700\ (13 + x)$

$2x = 7 \Rightarrow x = 3.5$

Distance between $P$ and $Q = 700\ (20 - 3.5)$ km

= 11,550 km

**57.** Let the speeds of the boats in still water be $\mu$ kmph each and that of the current be $u$ kmph.

$\therefore 4(\mu + u) + 4\ (\mu - u) = 32$

$\Rightarrow 8\mu = 32$

$\Rightarrow \mu = 4$.

**58.** Let the speeds of Anil and the escalator (both in m/sec) be $a$ and $e$, respectively.

Given, $\frac{120}{a+e} = 24$ and $\frac{120}{2a+3e} = 10$

$\therefore a + e = 5$ and $2a + 3e = 12$

Solving for a and $e$, $a = 3$ and $e = 2$

Required time (in seconds) $= \frac{120}{a-e} = \frac{120}{3-2} = 120$

**59.** The hour hand covers 30° per hour and $\frac{1°}{2}$ per minute

The minute hand covers 360° (i.e., 0) per hour and 6° per minute

The hands coincide at 12 noon

From 12 noon to 5:15 p.m., (i.e., in 5 hours 15 min), the hour hand covers $5(30°) + 15\left(\frac{1°}{2}\right)$, i.e., 157.5°

From 12 p.m. to 5:15 p.m. the minute hand covers 5(0) + 15(6°), i.e., 90°

Angle between the hands at 5:15 p.m. = 157.5° – 90°

= 67.5°.

**60.** If $A$ takes $t$ seconds to run the race, $B$ would take $t + 30$ seconds to run the race. $C$ would take $t + 60$ seconds to run the race.

Ratio of speeds of $A$ and $C$ is the reciprocal of the ratio of the times taken by them to run the race, i.e., $\frac{t+60}{t}$.

When $A$ finishes the race, $C$ would have run 1000 – 400 = 600 m.

$$\frac{t+60}{t} = \frac{1000}{600}$$

$t = 90$ seconds.

## EXERCISE-3

**1.** (a) Mr. Batra will catch Lucky in $\frac{5000}{(20-10)}$ secs = 500 secs.

Thus total distance travelled by the dog = 500 × 30 m = 15000 m

(b) Since the dog is always between Mr. Batra and Lucky, when Mr. Batra catches Lucky, all the three are at the same point. Thus, net distance traveled by the dog = Distance traveled by Mr. Batra = (500 × 20) m = 10,000 m

Net distance covered by the dog = Total Forward Distance – Total Reverse Distance

Thus $F - R = 10{,}000$

Also, from the last question, we know

$F + R = 15000$

Solving $F = 12500$ m.

Total distance travellied by the dog in the forward direction is 12500 m

**2.** $P$ is located to the west of $Q$. $\therefore$ $P$ is time wise behind $Q$

Let the time difference between $P$ and $Q$ be $t$ hours.

Travel time of the planes

From $P$ to $Q$.

$= 11{:}30$ p.m. – $t$ hours – 3 p.m. $= \left(8\frac{1}{2} - t\right)$ hours

Form $Q$ to $P$ = 9:30 a.m. + $t$ hours – 4 a.m. $= \left(5\frac{1}{2} + t\right)$ hours.

The wind blows steadily from west to east.

$\therefore$ It will favour the travel from $P$ to $Q$ and oppose the travel from east to west. $\therefore$ Assuming the cruising speed of the plane in either direction to be $S$ km/hr, we have

$\frac{3600}{S+75} = 8\frac{1}{2} - t$ and $\frac{3600}{S-75} = 5\frac{1}{2} + t$

Adding these, $\frac{3600}{S+75} + \frac{3600}{S-75} = 14$

$$\Rightarrow \frac{3600(2S)}{S^2 - 75^2} = 14$$

$$3600\,S = 7(S^2 - 75^2) \qquad (1)$$

From the choices, $S$ is an integer

$\therefore$ RHS of (1) is an integer divisible by 7

$\therefore$ LHS is divisible by 7. Among the choices, only 525 is divisible by 7 and it satisfies (1).

$\therefore$ $S = 525$.

**3.** $\frac{3600}{525+75} = 8\frac{1}{2} - t \Rightarrow t = 2\frac{1}{2}$.

**4.** Let $d$ be the distance between Tarnaka and Chilkur.

Then, time taken by them to meet after they started

$$\frac{x}{\frac{x}{42} + \frac{x}{56}} = \frac{42 \times 56}{42 + 56} = 24 \text{ minutes}$$

Since they started at 7.30 a.m, they would meet at 7 : 54 a.m.

**5.** Let the time taken by $C$ to run $PR$ be $t_1$ seconds.

$\therefore$ $A$ takes $(t_1 - 5)s$ to run $PR$ and $B$ takes $(t_1 - 10)s$ to run $PR$.

Let the time taken by $C$ to run $RQ$ be $t_2$ $s$

$\therefore$ The time taken by $B$ to run $RQ$ is $(t_2 - 4)$ $s$ and the time taken by $A$ to run $RQ$ would be $(t_2 - 2)$ $s$

$\therefore$ $B$ takes $[(t_1 - 10) + (t_2 - 4)]\,s = (t_1 + t_2 - 14)\,s$.

While $A$ takes $[(t_1 - 5) + (t_2 - 2)\ s = (t_1 + t_2 - 7)\ s$.

$\Rightarrow A$ takes $7s$ more than $B$.

**6.** To travel the first 6 km, Shiv takes $\frac{6}{72}(60)$, i.e., 5 min

To travel the second 6 km, he takes $\frac{6}{60}(60)$, i.e., 6 min.

To travel the third 6 km, he takes $\frac{6}{48}(60)$, i.e., 7.5 min.

To travel the fourth 6 km, he takes $\frac{6}{36}(60) = 10$ min.

i.e., in 28.5 minutes he travels 24 km/hr. He still has 7.5 min more.

The next 6 km he travels at 24, i.e., he requires 15 minutes to travel 6 km, i.e., in 7.5 minutes he can travel 3 km.

So, he can travel a total of 27 km in 36 minutes.

**7.** Ravi and Vikram complete one round of 900 m in 9 $s$ and 15 $s$, respectively

$\therefore$ speeds are 900/9, 900/15

= 100 m/s, 60 m/s, i.e., relative speed of 40 m/s

Hence, Ravi will meet Vikram after every $22^1/_2$ seconds.

But since the race lasts for 90 seconds only (i.e., Ravi finishes 10 rounds)

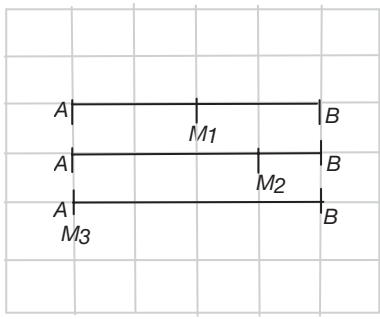
Ravi meets Vikram exactly 4 times and the time taken will be 90 seconds.

**8.** Ratio of the speeds of $P$ and $Q$ is 2 : 3.The two cars started simultaneously $AB = d$ km

Suppose two cars start simultaneously from two points that are $x$ km apart and the cars travel to and fro between the two points. The total distance covered by the cars when they meet for the first time is $x$ km. The total distance covered by the cars between their $n^{th}$ meeting and $(n+1)$ th meeting will be $2x$ km provided the ratio of the speeds of any of the cars and the other lies between $\frac{1}{2}$ and 2.

In the given question, the ratio of the speeds of $P$ and $Q$ lies between $\frac{1}{2}$ and 2.

$\therefore$ The total distance covered by the cars when they meet for the second time will be $3d$ km. The total distance covered by them when they meet for the third time will be $5d$ km




Time taken for the cars to meet for the first time = 40 min

Time taken for the cars to meet for the second time = 3(40 min) = 120 min

Time taken for the cars to meet for the third time

= 5(40) min = 200 min

The cars met for the third time at 12 : 20 p.m.

**9.** Let the speeds of $X$ and $Y$ be $x$ kmph and $y$ kmph, respectively.

Let $AB = d$ km.

Travel time of $X$ = Travel time of $Y$ = 8 hours

$\therefore x = y = \frac{d}{8}$

Distance traveled by $X$ at 7 a.m.= $\frac{d}{8}$km

$\therefore$ Distance between both trains at 7:00 a.m.

$= \left(d - \frac{d}{8}\right)\text{km} = \frac{7d}{8}\text{km}$

Both trains would meet in $\frac{\frac{7d}{8}}{\frac{d}{8}+\frac{d}{8}}$ hours $= 3\frac{1}{2}$ hours

$\therefore$ Meeting time = 10:30 a.m.

**10.** Let the speeds (in km/hr) of Chotu and Motu be $C$ and $M$, respectively. Total distance travelled by Chotu and Motu before meeting is $3C = 2M$. Distance (in km) from the meeting point to $Q$ is $90 - 3C$. In the 6 hours between the first and second meetings, $2(90 - 3C)$ is covered by the two together,

i.e., $2(90 - 3C) = 6(C + M) = 6C + 9C$

$\Rightarrow 180 = 21C \Rightarrow C = 60/7$ km/hr

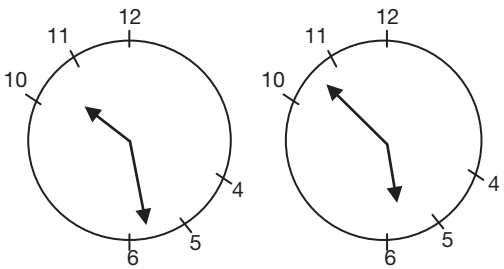
**11.** Distance between the cyclist and the man at 2:00 p.m. = $(36 - 20)\ 2 = 32$ km.

Time at which the cyclist to overtake the pedestrian $= \frac{32}{20 - 9\frac{1}{3}} = 3$ hours after 2:00 p.m., i.e., 5:00 p.m.

**12.** Amar overtakes Antony 2 times per round, i.e., Amar's speed is 3 times that of Antony's. Similarly, Antony's speed = 4 + 1 = 5 times the speed of Akbar.

$\therefore$ Amar's speed is 3 (5) = 15 times the speed of Akbar, i.e., the ratio 15 : 1.

**13.**



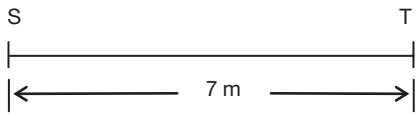

Positions of the two hands of the clock when he went to bed and when he woke up in the morning are shown above.

As the time in the night is less than 10 : 30 p.m. and the time in the morning is more than 5 : 30 a.m., the hour hand moves through slightly more than 7 big divisions. (say $7 + x$).The minutes hand moves through 7 full revolutions and $12 - (7 + x)$ or $(5 - x)$ big divisions.

$\therefore 12(7 + x) = 7(12) + 5 - x \Rightarrow 13x = 5 \Rightarrow x = 5/13.$

$\therefore$ Bhola was in bed for $7^{5}/_{13}$ hours or $443\frac{1}{13}$ min.

**14.**




When $M$ meets 'N' for the 2nd time, sum of the total distances travelled by $M$ and $N = 7 + (7 + 7) = 21$ m.

($M$ covered from $S$ to $T$, 7 m;

$N$ covered from $T$ to $S$, 7 m;

and they together covered $ST$ 7 m.)

The time of travel from the start to the time of second meeting is the same for both. Hence, distance = time $\times$ (sum of the speeds).

Time taken $= \frac{21}{(3+4)} = 3$ sec.

Total distance travelled by 'M' alone $= 3 \times 3 = 9$ m.

**15.** Time taken for them to meet anywhere on the track

$= \text{L.C.M. of } \left(\frac{600}{(n+1)-n}, \frac{600}{(n+2)-n}\right)$

$= \text{L.C.M. of } \left(\frac{600}{1}, \frac{600}{2}\right) = 600$ seconds

Time taken for them to meet at the starting point for the first time $= \text{L.C.M. of } \left[\frac{600}{n}, \frac{600}{n+1}, \frac{600}{n+2}\right]$

$= \frac{600}{GCD\,(n, n+1, n+2)} = \frac{600}{1} = 600$ seconds

So, the first time they meet is at the starting point. So, they will not meet anywhere on the track (meet zero times) before they meet at the starting point for the first time.

**16.** As $A$ beats $B$ by 100 m or 10 sec, speed of

$B = \frac{100}{10} = $ m/sec.

Given that ratio of speeds of $A$, $B$ and $C$ is 5 : 4 : 3.

$\therefore$ The distances covered by $A$, $B$ and $C$ will also be in the ratio of 5 : 4 : 3

As $A$ beats $B$ by 100 m, length of the race is $\frac{5 \times 1(00)}{5-4}$ m = 500 m.

Time taken by $B$ to cover these 100 m $= \frac{500}{10} = 50$ sec

$\therefore$ D takes 100 sec to cover the 500 m.

$\therefore$ D's speed $= \frac{500}{10} = $ m/s $= 5$ m/s

Ratio of speeds of C and D = 7.5 : 5 = 3:2. If C runs 3 parts, he beats D by 1 part.

If $C$ runs 500 m, he beats $D$ by $\frac{500}{3}$m, i.e., $166\frac{2}{3}$m.

**17.** The train covered 80 km in 40 minutes.

$\therefore$ Its speed $= \frac{80}{\left(\frac{40}{60}\right)} = 120$ kmph.

**18.** Distance ran by the thief in 10 minutes = 1 km

Distance moved by the jeep in 10 minutes $= 1 \times 5$ km

Distance between the jeep and the thief when the jeep found a gap in the median = 2.5 km

Additional time in which the jeep would overtake the thief = $\frac{2 \cdot 5}{9-6} = \frac{5}{6}$ hours = 50 minutes

∴ Total time = 60 minutes.

**19.** The angle covered (in degrees, small divisions (sd), big divisions (bd)) by the min hand and the hour hand in 1 min and 1 hour are tabulated below. (We need not fill up all the cells for this particular problem)

| | degrees | | sd | | bd | |
|---|---|---|---|---|---|---|
| | min | hr | min | hr | min | hr |
| 1 min | 4 | 2/9 | 1 | | | |
| 1 hr | 360 | 20 | 90 | 5 | | 1 |

There are 36 hours per day, 18 hours (or big divisions) on the clock dial and the entire dial is divided into 90 small division (each corresponding to 1 min)

The dial of the clock on the planet would have 18 hours (instead of 12 hours), and hence, the hour hand travels 360/18 = 20° per hr, (2/9)° per min and minute hand travels 360° per hr. (4° per min)

(∵ 1 hr = 90 min)

To calculate the angle 15:50 a.m.

At 15:00 a.m. the minute hand is $\frac{15}{18}(360°)$ or 300° 'behind' the hour hand. The relative speed of the minute hand wrt the hour hand is 4 – 2/9 or (34/9)° per min

∴ In 50 min the minute hand covers (50) (34°/9) = $(188^{8}/_{9})°$

∴ The final angle in degrees

= 300 – $188^{8}/_{9} = 111^{1}/_{9} = 1000/9$

**20.** At 14 O' clock the angle is (14/18) (360)° = 280°

If this has to become 60°, then the minute hand covers either 280° + 60° = 340° or

280° – 60° = 220° relative to hour's hand

∴ $\frac{340}{(34/9)} = 90$ min or $\frac{(220)(9)}{34} = 58^{4}/_{17}$ min

At 14 : $58\frac{4}{17}$ the two hands of the clock make a 60° angle.

**21.** In 18$t$ minutes, the clock gains 60 minutes.

∴ Actual time 18$t$ minutes from now = 5:00 a.m.

∴ In 6$t$ minutes, the clock will gain = 20 minutes

∴ 6$t$ minutes from now, actual time = 11:00 p.m.

Let the current time be $T$

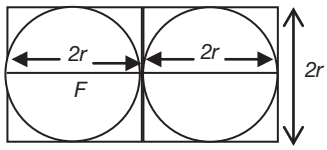
$T + 18t$ = 5:00 a.m.

$T + 6t$ = 11:00 p.m.

$T$ = 8:00 p.m.

Exactly 24 hours later, the clock will gain $24\left(\frac{1}{9}\right)$ hours = $\frac{8}{3}$ hrs = 2 hr 40 minutes and therefore it will show 8 + 2:40 = 10:40 p.m.

**22.**




Let the radius of each circle be $r$ units. The rectangle has a length of 4$r$ units and a breadth of 2$r$ units. In order for both to reach their starting points simultaneously, P must complete running the circumference of the two circles and $Q$ must complete running the rectangle perimeter once simultaneously.

The circumference of each circle = 2$\pi$r units

Perimeter of the rectangle = 12$r$ units.

Required percentage = $\frac{2(2\pi r) - 12r}{2(2\pi r)} \times 100 = 4\frac{6}{11}\%$

**23.** The following table shows the time taken in minutes for the two cyclists to arrive at successive milestones. The last column indicates whether or not a meeting (overtaking) takes place just before or at the milestone.

| Milestone | *A* | *B* | Meeting |
|---|---|---|---|
| 0 | 0 | 0 | ✓ |
| 1 | 7 | 4 | |
| 2 | 12 | 13 | ✓ |
| 3 | 19 | 17 | ✓ |
| 4 | 24 | 26 | ✓ |
| 5 | 31 | 30 | ✓ |
| 6 | 36 | 39 | ✓ |
| 7 | 43 | 43 | ✓ |
| 8 | 48 | 52 | |
| 9 | 55 | 56 | |
| 10 | 60 | 65 | |
| 11 | 67 | 69 | |
| 12 | 72 | 76 | |

As $B$ arrives at milestone 1 ($M1$) before $A$ and at $M2$ after $A$, $A$ overtakes him just before $M2$. Similarly $B$ arrives at $M2$ after $A$, but at $M3$ before $A$, and hence, he must have overtaken $A$ just before $M3$ and so on. The $6^{th}$ meeting takes place at $M7$. After that $B$ begins to trail $A$ and there are no further meetings.

**24.** Time taken by $A$ to cycle 36 km $= \frac{18}{9} + \frac{18}{15} = 3\frac{1}{5}$ hrs.

Time taken by $B$ to cycle 36 km $= \frac{36}{12} = 3$ hrs.

$\therefore$ $B$ wins the race. He wins by $(3\frac{1}{5} - 3)\ (15) = 3$ km.

**25.** Let the walking speed of Amar be $w$ km/hr

The length of the track is 3 km.

On the first day, he covered 6 rounds. On the second day, he covered 9 rounds. Time taken by Amar to walk one round $= \frac{L}{w}$ (where $L$ is the length of the track (say))

$$6w + 4.5\frac{L}{w} = 6L \qquad (1)$$

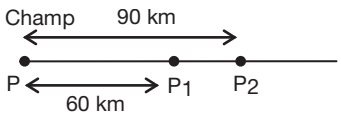
$$4w + 10.5\ \frac{L}{w} = 6L \qquad (2)$$

$$2(1) - 3(2) : \frac{22.5L}{w} = 15L$$

$w = 1.5$.

**26.** Given that $\frac{\text{speed of Hercules}}{\text{speed of Hero}} = \frac{3}{2}$

Let $C : 3$ be the ratio of speeds of champ to that of Hercules. The ratio of the speed of Hercules to the speed of Hero is 3 : 2. When the distance between them is 30 km, Hercules must have travelled 90 km from $P$ and Hero must have travelled 60 km from $P$. This is the time when Champ starts from $P$.



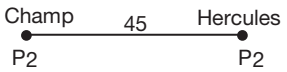

Since the distance travelled by Champ to overtake Hercules is twice the distance travelled by him to overtake Hero, the time taken by Champ to overtake Hercules is twice the time taken by him to overtake Hero.

i.e., $\frac{90}{C-3} = 2\ x\ \frac{60}{C-2} \Rightarrow C = 6$

$\therefore$ Champ : Hero = 6 : 2 = 3 : 1.

**27.** Consider the position of the three when Champ has just met Hero.

Since the ratio of the speeds of Champ, Hero and Hercules is 6 : 3 : 2. When Champ and Hero meet, Champ would have covered 90 km, Hero would have covered 30 km and Hercules would have covered 45 km. Now the situations is as given below




After overtaking Hero and before he overtakes Hercules there will be a position when Champ will be equidistant from Hero and Hercules. Let $x$ be the distance that Hero travels from the position in the figure.

Champ travels $3x$ and Hercules travels $\frac{3}{2}x$. Then, the distance between Hero and Hercules is twice the distance between Hero and Champ.

$$\Rightarrow 2(3x - x) = 45 + \frac{3}{2}x - x$$

$$\Rightarrow x = \frac{45x2}{7} = 12\frac{6}{7}$$

The distance between Hero and Hercules = $45 + x/2$

$$= 45 + \frac{12\frac{6}{7}}{2} = 51\frac{3}{7} \text{ km.}$$

**28.** Distance that $A$ travelled by 10:00 a.m. is 60 km.

Distance between $A$ and $B$ at 10:00 a.m. = 270 km.

Distance of the halting station of $A$ from its location at 10:00 a.m. = 270 – 120 = 150 km.

A would reach its halting station in $\frac{150}{60}$ i.e., $2\frac{1}{2}$ hours.

By then $B$ would have travelled $2\frac{1}{2}$ (40), i.e., 100 km.

$B$ would need to travel another 120 – 100, i.e., 20 km to reach A's halting station.

Time taken for this travel = $\frac{20}{40}$ i.e., $\frac{1}{2}$ hr

As $A$ would halt for 35 minutes, both would meet at the halting station 3 hours after $B$ starts from $Y$, i.e., at 1:00 p.m.

**29.** Given the speed of Cheetah = 20 m/s.

In two seconds it covers 40 m.

Now the distance between Cheetah and the deer = 70 – 40 = 30 m.

The relative speed of Cheetah with respect to the deer = 20 – 19 = 1 m/s.

∴ Time taken by Cheetah to cover 30 m with respect to the deer = $\frac{30}{1}$ = 30 secs

**30.** Let the driving speed of Shyam be $6x$ km/hr Walking speed of each of Ram and Tarun = $x$ km/hr

Let us say Shyam dropped Tarun after travelling for $t$ hours.

When Shyam dropped Tarun, Ram and Shyam would have covered respective distances of $xt$ km and $6xt$ km, respectively. Distance between Ram and Shyam then would be $5xt$ km.

Time taken for Ram and Shyam to meet from the time Tarun was dropped = $\frac{5xt}{x+6x}$ hours

= $\frac{5t}{7}$ hours

Distance covered by Ram in this time = $\frac{5}{7}$ $xt$ km

Walking speeds of Ram and Tarun were the same.

∴ In $\frac{5t}{7}$ hours, Ram as well as Tarun would have covered $\frac{30xt}{7}$ km.

When Ram and Shyam meet, Tarun would be $5xt$ km ahead of the meeting point. Ratio of the speeds of Shyam and Tarun = 6 : 1. Also distances covered by Shyam and Tarun from the meeting time of Ram and Shyam to the time that the three reach $B$ differ by $5xt$ km. ∴ Distances covered by Shyam and Tarun in this time were $6xt$ km and $xt$ km, respectively.

Total distance covered by

Tarun = $(6xt + \frac{5xt}{7} + xt)$ km = $\frac{54}{7}$ $xt$ km

Average speed of Tarun = $\frac{\frac{54}{7}xt}{t+\frac{5t}{7}+\frac{xt}{x}}$

= $\frac{54}{19}$ $x$ = $\frac{54}{19}$ th of Tarun's walking speed.

**31.** From the data $X$ is ahead (to the east) of $Y$.

Let the time difference be $t$ hours.

The distance covered by the two flights is the same.

Travel times of $A$ and $B$ are $(2\frac{1}{2} + t)$ hours and $(7\frac{1}{2} - t)$ hours, respectively (1)

Ratio of the travel times of $A$ and $B$ = Inverse ratio of their speeds = 2 : 3 (2)

$600\ (2.5 + t) = 400\ (7.5 - t)$

$3\ (2.5+t) = 2\ (7.5 - t)$

$7.5 + 3t = 15 - 2t \Rightarrow (5t = 7.5) \Rightarrow t = 1.5$

Distance between $X$ and $Y$ = 600 (2.5 +1.5) km = 2400 km

**32.** Let the distance it travelled without any problem be $x$ km and its speed over that part of the journey be $S$ kmph

$$\frac{x}{s} + \frac{900-x}{\frac{3}{5}S} = \frac{900}{S} + 2 \quad (1)$$

$$\frac{x+150}{S} + \frac{900-(x+150)}{\frac{3}{5}S} = \frac{900}{S} + 1 \quad (2)$$

Subtracting (2) from (1),

$$\frac{150}{S} + \frac{150}{\frac{3}{5}S} = 1$$

S = 100

Substituting $S$ in (1) or (2), $x$ = 600.

**33.** The hands of a normal clock coincide every $65\frac{5}{11}$ min.

The hands of $C_1$ coincide every 66 minutes.

∴ $C_1$ loses $\frac{6}{11}$ minutes every 66 minutes.

∴ It loses $\frac{1}{121}$ minutes every minute. (an $\frac{1}{121}$ hr every hour)

The hands of $C_2$ coincide every 50 minutes.

∴ $C_2$ gains $15\frac{5}{11}$ minutes every 50 minutes.

∴ It gains $\frac{17}{55}$ minutes every minute.

From 12 a.m. Sunday to 12 a.m. Monday (i.e., a time span of 24 hours)

$C_1$ would have lost $\frac{24}{121}$ hours and $C_2$ would have gained $\frac{(17)}{55}$ hours.

The difference of the times shown by $C_1$ and $C_2$ at 12 a.m. Monday is $+\left(\frac{1}{121}24\frac{17}{55}\right)$ hours, i.e., 24 $\frac{(192)}{605}$ hours

i.e., $\frac{192}{605}$ (24) (160) (60 min), i.e., ≈ 457 min.

**34.** All speeds are in kmph. The speed of Vijay's boat in still water is $x$ and the speed of the stream is $y$.

When Vijay is going upstream (after the flower drops) his speed is $x - y$ while the flower floats downstream at $y$.

∴ With respect to the flower, Vijay is going away at $x$. When Vijay turn back his speed is $x + y$, while the flower continues to move at $y$.

∴ Vijay is approaching the flower at $x$, i.e., at the same speed at which he was going away. As he was going away for 2 hours, he needs exactly 2 hours to come back to the flower. In these 4 hours the flower drifts by 6 km.

∴ The speed of the stream ($y$) is 6/4 or 1.5 kmph.

**35.** Let A's running speed be a m/s. The data is tabulated below

| | Running | Cycling | Swimming |
|---|---|---|---|
| Length of the track | $d$ | $2d$ | $\frac{d}{2}$ |
| A's speeds | $a$ | $3a$ | $\frac{a}{3}$ |
| $B$'s speeds | $\frac{a}{1.1}$ | $4\left(\frac{a}{1.1}\right)$ | $\frac{1}{4}\left(\frac{a}{1.1}\right)$ |
| C's speeds | $\frac{a}{1.2}$ | $4\left(\frac{a}{1.2}\right)$ | $\frac{1}{2}\left(\frac{a}{1.2}\right)$ |

Time taken by $A$ to finish cycling is

$$d\left(\frac{1}{a}+\frac{2}{3a}\right)=\left(\frac{d}{a}\right)\frac{5}{3}=1.67\,\frac{d}{a}$$

Similarly, time taken by $B$ to finish cycling

$$=\frac{d(1.1)}{a}\left(1+\frac{2}{4}\right)=\left(\frac{d}{a}\right)(1.1)\left(\frac{3}{2}\right)=1.65\frac{d}{a}$$

And time to taken by $C$ to finish cycling

$$=\frac{d(1.2)}{a}\left(1+\frac{2}{4}\right)=\frac{d}{a}(1.2)\left(\frac{3}{2}\right)=1.8\frac{d}{a}$$

∴ $B$ is the first to finish the cycling leg or the second leg.

**36.** To find who wins the marathon we now compare the total time taken by the 3 athletes.

$$\text{A: } \frac{d}{a}\left(1+\frac{2}{3}+\frac{3}{2}\right)=\left(\frac{d}{a}\right)\left(\frac{19}{6}\right)=(3.17)\left(\frac{d}{a}\right)$$

$$B\text{: } \frac{(d)\,(1.1)}{a}\left(1+\frac{2}{4}+\frac{4}{2}\right)=\left(\frac{d}{a}\right)(1.1)\left(\frac{7}{2}\right)=(3.85)\left(\frac{d}{a}\right)$$

$$\text{C: } \frac{(d)\,(1.2)}{a}\left(1+\frac{2}{4}+\frac{2}{2}\right)=\left(\frac{d}{a}\right)(1.2)\,\frac{10}{4}=(3.00)\left(\frac{d}{a}\right)$$

∴ C finishes first.

**37.** Ratio of the speeds of Ram and Shyam = $\frac{9}{6}=\frac{3}{2}$.

Let the speeds of Ram and Shyam be $3x$ m/sec and $2x$ m/sec. Let us say Ram and Shyam meet after $t$ seconds for the first time.

$3xt - 2xt = 120$

$xt = 120$

$3xt = 3(120)$

∴ Ram would have covered 3 rounds when he meets Shyam for the first time.

∴ After every 3 rounds Ram meets Shyam. But Ram completes the race in 10 rounds. Therefore Ram overtakes Shyam after 9 rounds for the last time.

∴ Time taken = 9 × 6 = 54 seconds.

**38.** Let $N$ = total number of steps on the stationary escalator.

Let the speed of $A$ be 2 steps/sec, the speed of $B$ be 1 step/sec, and the speed of escalator be $E$ steps/sec.

∴To take 40 steps $A$ needs 20 seconds.

Since $N$ = steps taken by $A$ + steps taken by Escalator

i.e., $N = 40 + 20\times E$ (1)

Also, to take 30 steps $B$ needs 30 seconds.

Since $N$ = steps taken by $B$ + steps taken by Escalator, i.e.,$N = 30 + 30 \times E$ (2)

Equations (1) and (2)

$40 + 20E = 30 + 30\,E$

∴E = 1 step/sec

Substitute $E = 1$ in eqn (1) or in eqn (2),

N = 60 steps

Hence, $A$ should take 60 steps on a stationary escalator.

**39.** Let $x$ km be the total distance and $u$ km/hr be the normal speed, $v$ km/hr be the decreased speed

$$\frac{0.3x}{u}+\frac{0.7x}{v}-\frac{x}{u}=10$$

$$\Rightarrow\ 0.7\ x\left(\frac{1}{v}-\frac{1}{u}\right)=10 \qquad (1)$$

$$\frac{0.3x+70}{u}+\frac{0.7x-70}{v}-\frac{x}{u}=9$$

$$(0.\ 7\ x-70)\left(\frac{1}{v}-\frac{1}{u}\right)=9 \qquad (2)$$

(1)/(2)

$$\frac{0.7x}{0.7x-70}=\frac{10}{9}\Rightarrow x=1000\,.$$

**40.** Let us say Zombie started '$p$' minutes after 4 O'clock

Angle between the minute's hand and the vertical line $= 6p°$

Angle between the hour's hand and the vertical line

$= 120° + p/2°$.

Let Zombie come back $q$ minute after 7 O'clock Angle between the hour hand and the vertical line at that time $= 210° + q/2°$.

Angle between the minute's hand and the vertical $= 6q°$

So, $6p° = 210° + q/2°$ (1)

$6q° = 120° + p/2°$ (2)

Subtracting equation (2) from (1)

$$6(p-q)=90-\frac{p-q}{2}\ ;\Rightarrow\frac{13}{2}\ (p-q)=90°$$

$$\Rightarrow p-q=\frac{180}{13}\ \text{minutes}\cong 14\ \text{minutes}$$

Zombie was out for a time of $(7 : q) - (4 : p) = [3$ hours $+ (q - p)]$ minutes $= 2$ hours 46 minutes.

**41.** Viraat can travel 10 m/s downstream

By the time the two passengers on Viraat decide to get off, their boat will have travelled for 20 min

It covers (20) (60) (10) m = 12000 m, while Samudra travels for 15 min and covers (15) (60) (5) m = 4500 m

$\therefore$ The raft is 7500 m ahead of Samudra

Samudra gains (5 - 2) m/s with respect to the raft.

$\therefore$ It takes $7500/3\ s = 2500\ s$

$= 41(2/3)$ min to reach the raft.

**42.** Let the length of the race be $d$ metres

When Raja finishes the race, Sameer would have run $(d - 30)$m. If Sameer finishes the race, then Tarun would have run $(d - 45)$ m

Let the speeds of Raja, Sameer and Tarun be $r$, $s$ and $t$, respectively.

$$\frac{r}{t}=\frac{r}{s}\times\frac{s}{t}$$

$$3=\frac{d}{d-30}\times\frac{d}{d-45}$$

$3\ (d-30)\ (d-45)=d^2$

$3(d^2-75\ d+1350)=d^2$

$2d^2-225\ d+4050=0$

$2d^2-180d-45d+4050=0$

$2d\ (d-90)-45\ (d-90)=0$

$(d-90)\ (2d-45)=0$

$$d=90\ \text{or}\ \frac{45}{2}$$

As Sameer beats Tarun by 45 m, $d > 45$

$\therefore\ d = 90$

**43.** Assume the speeds of $A$ and $B$ are a kmph and $b$ kmph, respectively. Let $XY = d$ km.

When the cars meet for the first time, $B$ will have travelled 30 km. $A$ will have travelled $(d - 30)$ km.

When the cars meet for the second time, $B$ will have travelled $(d + 10)$ km or $(d - 10)$ km. $A$ will have travelled $(2d - 10)$ km or $(2d + 10)$ km

$$\frac{a}{b}=\frac{d-30}{30}=\frac{2d-10}{d+10}$$

$(d-30)\ (d+10)=(2d-10)\ 30$

$d^2-20d-300=60d-300$

$d\ (d-80)=0$

As $d$ cannot be 0, $d = 80$

(or)

$$\frac{a}{b}=\frac{d-30}{30}=\frac{2d+10}{d-10}$$

$d^2 - 40d + 30 = 60d + 30$

$d(d-100) = 0$

As $d$ cannot be 0, $d = 100$

**Alternate Method:**

Suppose two bodies $A$ and $B$ start from two points/places $P$ and $Q$, respectively towards others starting point simultaneously which are initially $d$ km apart. If they travel back to their starting points using the same route after reaching their destinations and meet a km from $Q$ for the first time and $b$ km from $P$ for the second time.

$d = (3a - b)$ (or) $(3a + b)$

As a = 30 and $b = 10$, $d = 80$ or 100

**44.** Speed of the train $= 36 \times \frac{5}{18} = 10$ m/sec

$\therefore$ Sum of the lengths of the train and the bridge

$= 48 \times 10 = 480$ m

Speed of the man = 9 km/hr $= 9 \times \frac{5}{18} = 5/2$ m/s

Relative velocity of the train $= 10 - 5/2 = 15/2$ m/s

$\therefore$ Length of the train $= 20 \times \frac{15}{2} = 150$ m

$\therefore$ Length of the bridge $= 480 - 150 = 330$ m

**45.** For the first meeting, the faster person (say $A$) has to gain half a round with respect to the slower person (say $B$). For the second meeting $A$ has to gain $\frac{3}{2}$ rounds with respect to $B$. $A$ actually gains half a round in 30$s$. Therefore he needs 90 $s$ to gain $\frac{3}{2}$ rounds.

**46.** Let the speeds of the train and the cyclist be tkmph and c kmph, respectively.

$$\frac{270}{(t-36)\frac{5}{18}} = 27$$

$t = 72$

$$\frac{270}{(t-c)\frac{5}{18}} = 18$$

$$\frac{270}{(72-c)\frac{5}{18}} = 18$$

$c = 18$

In an hour, the train can travel 36 km more than the motorcyclist. So, when the train overtakes the cyclist, the cyclist and the motor cyclist will be separated by 36 km.

The motor cyclist would overtake the cyclist in another $\frac{36}{(36-18)}$ hours = 120 minutes

**47.** The difference in the times of the 2 clocks increases by (2 ½ + 3 ½) = 6 minutes every hour.

6 minutes → 1 hour

$\therefore$ 3 hours $\rightarrow \frac{3 \times 60}{6} = 30$ hours.

So, the clocks will be 3 hours apart at 4:00 a.m. on Thursday which is 30 hours after 10:00 p.m. on Tuesday.

**48.** The distance covered in the first 2 hours (starting with the usual rate) is 2 (2 + 4) km = 12 km. But the next 2 hr the speed of the current doubles and the distance travelled in those 2 hr = 2 [2 (2) + 4) = 16 km.

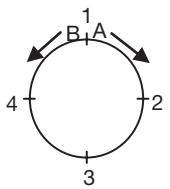
So, every 4 hr, distance travelled = 28 km.

130 = 28 (4) + 18

Now 12 out of 18 km will be covered at the usual rate of flow in 2 hr and another 6 km will take 3/4 hr with the river flowing at 4 km/hr.

$\therefore$ Total time = [(4) (4) + 2 + (3/4)] hr = $18^3/_4$ hr.

**49. (a)**




Let the speed of $A$ = 3 km/hr and then $B$ = 1 km/hr

As they are moving in opposite directions, the relative speed is 3 + 1 = 4 km/hr

To make the problem simpler, divide the circular track into four points 1, 2, 3 and 4 as shown.

By the time $B$ reaches 4 (distance of 1 quadrant) $A$ would have covered 3 quadrants and would have met $B$ at this point.

A covers the arcs 1-2, 2-3 and 2-4

$B$ covers the arc 1-4.

At the first meeting, A's speed changes to 1 and $B$'s speed to 3.

Also they reverse the directions of their movement.

Now $B$ covers the arcs 4-1, 1-2 and 2-3 and in the same time $A$ covers the arc 4-3.

So, the second meeting point is 3.

At the second point of meeting, A's speed becomes equal to 3 and $B$'s speed = 1 and the directions reverse again.

$A$ covers the arcs 3-4, 4-1 and 1-2 and at the same time $B$ covers the arc 3-2.

The third point of their meeting is therefore point 2.

Shortest distance between the starting point and third meeting point.

$= 14\sqrt{(2)}$ km.

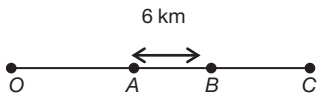
**(b)** Each quadrant is ¼ of the circumference of the circular track.

$$1 \text{ quadrant} = \frac{1}{4} \times 2 \times 14 = 7 \text{ km}.$$

$$7 \text{ quadrants} = 49\,\pi \text{ km} = 154 \text{ km} \left(\pi = \frac{22}{7}\right)$$

Therefore the distance travelled by $A$ from the starting point to the third meeting point is 154 km.

**50.** Let $O$ be the starting point and $A$ be the first slowdown point, $B$ be the second slowdown point and $C$ be the office.




Let $AC = x$ km.

Let $V$ be the speed and $t$ minutes be the time taken to travel from $A$ to $C$ with his initial speed.

To travel from $A$ to $C$ with $3/4^{th}$ of his speed, he would take $t/3$ minutes more time

So, $\frac{t}{3} = 6 \Rightarrow t = 18$

Let $t_1$ minutes be the usual time taken to cover $BC$.

To travel from $B$ to $C$ with $2/3^{rd}$ of his speed, he would take $t_1/2$ more. So, $t_1/2 = 6 \Rightarrow t_1 = 12$. So, his usual time to travel from $B$ to $C$ is 12 min.

So, to travel from $A$ to $B$ (6 km) at his initial speed he would take (18 – 12) or 6 minutes. His initial speed

$$= \frac{6}{(6/60)} \frac{\text{km}}{\text{hr}} = 60 \text{ km/hr}$$

**51.** The hands are in opposite directions when the angle between them is 180°.

Let this happen p minutes after 8 O'clock we get $(240 + p/2) - 6p = 180°$ (1)

(here the second equation will not be valid as in the case, the right hand side should be 0°)

From (1), we get $p = 10\,{}^{10}/_{11}$

∴ The two hands will be exactly in opposite directions at $8{:}10\,{}^{10}/_{11}$ O'clock.

**52.** Let the time be p minutes after 4 O'clock that the hands make an angle of 60°.

The angle made by the minutes hand with the vertical line when the time is P minutes after 4 O'clock is 6p°.

The angle made by the hours hand with the vertical line when time is 4 O'clock is 120°. So, P minutes latter the hours hand would have covered $P/2°$, additionally.

So, at P minutes after 4 O'clock, the hours hand makes an angle of $(120 + p/2)$ with the vertical line.

For the angle between the two hands to be 60°, the difference between the two angles discussed above should be equal to 60°.

Thus we get

$6p - (120 + p/2) = 60°$ (1)

$(120 + p/2) - 6p = 60°$ (2)

From (1) and (2),

we get $p = 32\,{}^{8}/_{11}$ min and $p = 10\,{}^{10}/_{11}$ min.

Hence, the angle between the two hands of the clock will be 60° at $4{:}10\,{}^{10}/_{11}$ O'clock and $4{:}32\,{}^{8}/_{11}$ O'clock.

**53.** (i) Time taken by $P$ to cross the tunnel $= \frac{900}{72 \times \frac{5}{18}} = 45$ seconds

Time taken by $Q$ to cross the tunnel $= \frac{1200}{90 \times \frac{5}{18}}$ = 48 seconds

∴ $P$ exits the tunnel first. When $P$ exits, $Q$ will have travelled a distance of $90 \times \frac{5}{18} \times 45 = 1125$ m

∴ 75 m of $Q$ would still be inside the tunnel.

(ii) The rear ends of the trains will cross each other when the trains completely cross each other.

This will happen after $\frac{1500}{(72+90)\frac{5}{18}}$ seconds.

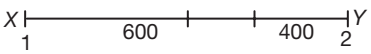
$= \frac{100}{3}$ seconds

Distance travelled by the slower train in this time

$= \frac{100}{3}\ (72)\left(\frac{5}{18}\right) = 666^{2}/_{3}$ m

Distance between the point where the rear ends of the trains cross each other and the point of entry of the slower train $= 666\frac{2}{3} - 300 = 366^{2}/_{3}$ m

**54.**




When P meets $Q$ for the first time, he would have travelled $3/5\ (1000) = 600$ m. Between any two consecutive meetings, any two people separated by a certain distance initially, would be together covering twice the initial distance between them, if they start running simultaneously in opposite directions and move to and fro between their initial starting points.

Hence, $P$ and $Q$ would together cover 2000 m between the 2nd and 3rd as well as the 3rd and 4th meeting. Total distance $P$ covers when he meets $Q$ for the fourth time $= 600 + 3 \times 3/5 \times 2000 = 4200$ m

**55.** To coincide with each other, the angle between them should be 0°. Let this happen p minutes after 2 O'clock.

We have $(60 + p/2) - 6p$

$\Rightarrow p = 120/11 \qquad \Rightarrow p = 10\ ^{10}/_{11}$ minutes

Hence, the 2 hands will coincide at $2{:}10^{10}/_{11}$ O'clock

**56.** Let Soumya and Vivian start at $A$ and run towards $B$. As Soumya is 5 times faster than Vivian, he overtakes him in his (Soumya's) second round at $B$, in his third round at $C$, in his fourth round at $D$ and in his fifth round at $A$. Let the initial number of coins with Soumya be $N$. The points at which Soumya overtakes Vivian and the coins each of them has at each stage are tabulated below.

| Overtaking Number | Location | Coins with Soumya | Coins with Vivian |
|---|---|---|---|
| | $A$ | $N$ | 1 |
| 1 | $B$ | $N-1$ | 2 |
| 2 | $C$ | $N-3$ | 4 |
| 3 | $D$ | $N-7$ | 8 |
| 4 | $A$ | $N-3$ | 4 |
| 5 | $B$ | $N-7$ | 8 |
| 6 | $C$ | $N-15$ | 16 |
| 7 | $D$ | $N-31$ | 32 |
| 8 | $A$ | $N-15$ | 16 |

The number of coins with Soumya is always of the form $N-(2^k-1)$. Finally it is 182

$\therefore N = (2^k - 1) + 182$. For $k = 6$, $N = 63 + 182 = 245$

For $k = 5$, $N = 213$

It can be verified that this value of $N$ occurs at overtaking number 10 and 12.

**57.** Ratio of speeds 2 : 3

No. of meeting points when students are running in the same direction $= 3 - 2 = 1$

No. of meeting points when students are running in the opposite directions $= 3 + 2 = 5$.

**58.** In the time $A$ runs 500 m, $B$ runs 460 m.

In the time $B$ runs 1000 m, $C$ runs 960 m.

In the time $B$ runs 460 m, $C$ runs $\frac{460}{1000}(960) = 441{\cdot}6$ m.

$\therefore$ $A$ would beat $C$ by 58·4 m in a 500 m race.

As $A$ beats $C$ by 14·6 seconds in a 500 m race, the speed of $C = \frac{58.4}{14.6} = 4$ m/sec

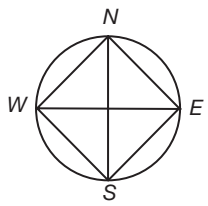
Speed of $B = \frac{460}{441.6}(4)$ m/sec

Time taken by $B$ to run 2·4 km $= \frac{2400}{\frac{460}{441.6}(4)} = 9.6$ minutes.

**59.** The situation is similar to a man coming from Bangalore to Hyderabad and another going from Hyderabad to Bangalore, both having started simultaneously. They will meet at exactly one point somewhere between Hyderabad and Bangalore

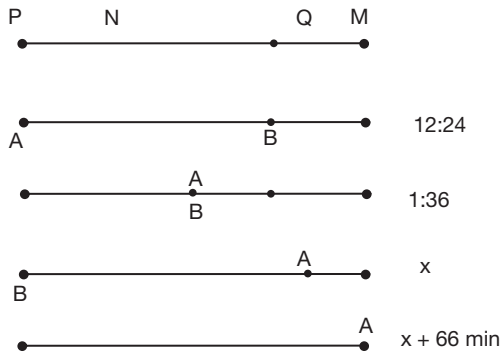
Thus there will be exactly one special point between Hyderabad and Bangalore the man reaches at the same time on both these days.

**60.** The distance $NW = WS = SE = EN$. By the time the slower competitor goes from $S$ to $W$, the faster competitor would have travelled five times the distance, i.e., he would arrive at $W$ exactly. Therefore they meet for the first time at $W$. Similarly, they meet for the second time at $N$, third time at $E$ and for the 23rd time, they meet at $E$.




## EXERCISE-4

**1.**




The 5 events are shown in the diagram above in the order in which they occur.

Let Bala be at $N$ at 12:24, when Amar started from $P$ and let the meeting point be $M$. As Amar started (from $P$) 2.4 hrs later than Bala and arrived (at $Q$) only 1.1 hrs later than Bala (at $P$), Amar is faster. If he is $k$ times faster, the time taken to cover any distance would be only $1/k$ times the time taken by Bala.

Bala covers $QN$ (from 10:00 to 12:24) and $NM$ (from 12:24 to 1:36) $QN = 2NM$. The time taken by the two to cover the 3 parts are tabulated below

| | ***PM*** | ***MN*** | ***NQ*** |
|---|---|---|---|
| Amar | 1.2 | $\frac{1.2}{k}$ | $\frac{2.4}{k}$ |
| Bala | 1.2k | 1.2 | 2.4 |

The time taken by Amar to cover $MQ$ is 66 min more than the time taken by Bala to cover $MP$.

i.e., $1.2k + 1.1 = \frac{3.6}{k} \Rightarrow 12k^2 + 11k - 36 = 0$

$\Rightarrow (4k+9)(3k-4) = 0 \Rightarrow k = \frac{4}{3} (\because k > 0)$

The time taken by Bala to cover $MP$ is $1.2k$ hr

$= 1.2 \times \frac{4}{3} = 1.6$ hr

The crossing took place at 1:36 p.m.

Therefore, Biswas reached $P$ 1.6 hours later, at 3:12 p.m.

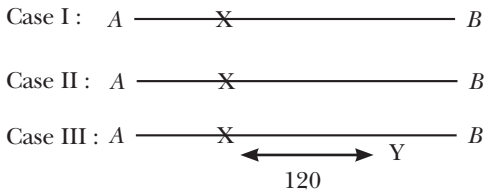
Choice (A)

**2.** Let us consider three cases.

Case I : When no accident takes place.

Case II : When accident takes place at $X$.

Case III: When accident takes place at $Y$, 120 km further down from $X$ towards City $B$.




Let Nishit's initial speed be $V$. Let the time taken by him to cover $XB$ (in case I) at speed $V$ be $T$.

Now at $\frac{3V}{4}$ he will take $\frac{4T}{3} = T + \frac{1}{3}T$

He was late by (3 hours – 90 minutes) = $1\frac{1}{2}$ hours

(Since it took 90 minutes to repair the car, that is his idle time.)

$\Rightarrow \frac{1}{3}T = \frac{3}{2}$ hours

$\therefore T = \frac{9}{2}$ hours

Therefore, he would have taken $\frac{9}{2}$ hours to travel 60% of the distance. To travel $\frac{3}{5}d$ he would have taken $\frac{9}{2}$ hours.

To travel $d$, he would have taken $\frac{15}{2}$ hours.

Considering Case II and Case III.

Travelling at $\frac{3V}{4}$ over 120 km Nishit took an extra 30 minutes (Difference between 3 hr and 2 ½ hr)

Let the time taken by Nishit to cover 120 km at $V$ be $t$ hr.

Time taken to cover 120 km at $\frac{3V}{4} = \frac{4}{3}t = t + \frac{1}{3}t$

$\frac{t}{3} = 30 \text{ min} \Rightarrow t = 90 \text{ min}$

Therefore his normal speed was $\frac{120\text{km}}{90\text{min}} = \frac{120}{3/2\text{hr}} = 80\text{km} = 80$ km/hr.

At 80 km/hr in $\frac{15}{2}$ hours, he would cover $80 \times \frac{15}{2} = 600$ km.

The distance between the two cities $A$ and $B$ is 600 km.

**3.** Let the cruising speed of the plane be $x$ km/hr.

Let the time difference between $A$ and $P$ be $t$ hours (i.e., $P$ is $t$ hours ahead of $A$).

$\therefore$ The time difference between $A$ and $Q$ is $t$ hours ($Q$ is $t$ hours behind $A$)

$\therefore$ we have the following equations.

(from $A$ to $P$) $\frac{4500}{x-75} = 8 - t$ (1)

(from $A$ to $Q$) $\frac{4500}{x+75} = 3 + t$ (2)

(1) + (2) $\Rightarrow 4500\left[\frac{2x}{x^2-75^2}\right] = 11 \Rightarrow 11x^2 - 11(75)^2 = 9000x$

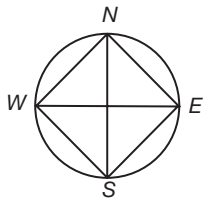
$$\therefore x = \frac{9000 \pm \sqrt{(9000)^2 - 4(11)(-11)75^2}}{2(11)}$$

$$= \frac{9000 \pm (150)\sqrt{60^2+121}}{22} = \frac{9000 \pm 150(61)}{22}$$

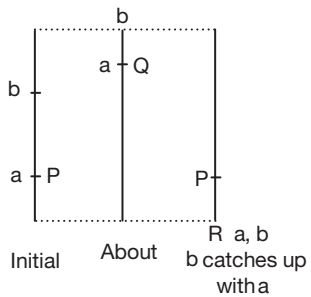
As $x$ is positive, $x = \frac{9000+9150}{22} = 825$

From (1), $\frac{4500}{825-75} = 8 - t \Rightarrow 6 = 8 - t \Rightarrow t = 2$ hours.

**4.** The distance $NW = WS = SE = EN$. By the time the slower competitor goes from $S$ to $W$, the faster competitor would have travelled five times the distance, i.e., he would arrive at $W$ exactly. Therefore they meet for the first time at $W$. Similarly, they meet for the second time at $N$, third time at $E$, fourth time at $S$, fifth time at $W$ and so on. For the 23rd time, they meet at $E$.




**5.** Let's assume that $B$ catches up with $A$ at $R$, after marching for a certain distance. At A's initial starting point, ($P$) $B$ is 1 min later than $A$ and at A's turning point $Q$, he is 3 min later than $A$. Hence, $RQ$ is 3 times $RP$. As $PQ = 2$ km, $RP = 1$ km.




**NOTE**

Soldiers are denoted by lower case letters, points by upper case.

**6.** To find who wins the marathon we now compare the total time taken by the 3 athletes.

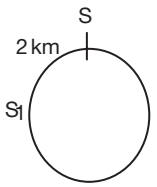
$$A: \frac{d}{a}\left(1+\frac{2}{3}+\frac{3}{2}\right) = \left(\frac{d}{a}\right)\left(\frac{19}{6}\right) = (3.17)\left(\frac{d}{a}\right)$$

$$B: \frac{(d)(1.1)}{a}\left(1+\frac{2}{4}+\frac{4}{2}\right) = \left(\frac{d}{a}\right)(1.1)\left(\frac{7}{2}\right) = (3.85)\left(\frac{d}{a}\right)$$

$$C: \frac{(d)(1.2)}{a}\left(1+\frac{2}{4}+\frac{2}{2}\right) = \left(\frac{d}{a}\right)(1.2)\frac{10}{4} = (3.00)\left(\frac{d}{a}\right)$$

$\therefore$ C finishes first.

**7.**




Let the starting points of $M$ and $N$ be $S$ and $S_1$, respectively.

N met $M$ for the first time in his second round

Let this meeting point be $x$ km from $S_1$.

Let the speeds of $M$ and $N$ be m and $n$, respectively (both in kms/sec)

When $M$ and $N$ meet for the first time, $M$ and $N$ would have covered $(8 + 2 + x)$ km and $(6 + 2 + x)$km, respectively.

$10 + x = 2000m$ and $8 + x = 2000n$

$\therefore\ 18 + 2x = 2000(m + n)$ (1)

Had $N$ travelled in the opposite direction, they would have met after 200seconds.

$\therefore\ 2 = 200(m + n)$ (2)

(1), (2) $\Rightarrow 18 + 2x = 20 \Rightarrow x = 1$

The speeds of $M$ and $N$ are 5.5m/sec and 4.5m/sec, respectively.

If M's speed was $9\frac{1}{11}$ % less and N's speed was $11\frac{1}{9}$ % more, each would have a speed of 5m/sec.

$\therefore$ If they start from the same point and ran in opposite directions, they would meet after $\frac{8000}{10}$, i.e., 800 seconds.

**8.** Let the speeds of Rana, Sona and Tina be $c$, 2 and 3, respectively (considering the ratios of Sona and Tina).

By 10:00 a.m. Rana covers a distance of $2c$ and Sona covers 2.

Since the distance travelled by Tina to meet Sona is twice the distance she travelled to meet Rana, the time taken by Tina to meet Sona is twice the time taken to meet Rana

$$\Rightarrow \frac{2}{3-2} = 2\left(\frac{2c}{3-c}\right) \Rightarrow 3c = 3 \Rightarrow c = 1$$

Let $t$ be the time taken by Tina to reach Chattisgarh.

$\therefore$ Distances travelled by Tina, Sona and Rana are $3t$, $2t + 2$ and $t + 2$.

Given that by the time Tina reaches Chattisgarh, distance between Sona and Rana

= 2 (Distance between Tina and Sona)

$\Rightarrow (2t + 2) - (t + 2) = 2\,[3t - (2t + 2)]$

$\Rightarrow t = 2t - 4 \quad \Rightarrow t = 4$

$\therefore$ Total distance travelled by Tina in 4 hours = 3 (4) = 12

$\therefore$ Time taken by Rana = $\frac{12}{1}$ = 12 hours

$\therefore$ Rana reached Chattisgarh at 8:00 p.m.

**9.** The speed of suspect is 60 km/hr

$\therefore$ In 15 min suspect would have covered 15 km.

The relative speed of party $P$ with respect to (wrt) the suspect is (65 – 60) km/hr = 5 km/hr

$\therefore$ Time taken by party $P$ to catch the suspect if they proceed at their respective speeds = 3hr

The party $Q$ from city $Y$ has to cover 285 km.

The relative speed of $Q$ wrt to $S$ = 120 km/hr

$\therefore$ Time taken by $Q$ to catch $S = \frac{285}{120}$ hr $= 2\frac{3}{8}$ hr hr

Clearly statement (A) is false.

It may seem that $P$ takes

$\left(3 - 2\frac{3}{8}\right)$ hr $= \frac{5}{8}$ hr = 37.5 min more to reach the thief.

But this is not really so. After the party $Q$ catches $S$ after $2\frac{3}{8}$ hr, $S$ is not going to continue towards $Y$ at 60 km/hr.

For statements (C) and (D), we need to know where $P$ is when $Q$ catches $S$. $P$ covers $65\left(2\frac{3}{8}\right)$ km $= \left(130 + 24 + \frac{3}{8}\right)$ km

$= 154\frac{3}{8}$ km from $X$, while $Q$ and $S$ are at $\left[15 + 602\frac{3}{8}\right]$ km

$= \left(15 + 120 + 21 + \frac{3}{2}\right)$ km $= 157\frac{1}{2}$ km from $X$;, i.e., $P$ is $3\frac{1}{8}$ on the $X$ side of $S$ and $Q$. It takes $\frac{25(60)}{8(65)}$ min $= \frac{25(3)}{2(13)}$ min $= \frac{75}{76}$ min < 3 min to reach the point where $Q$ catches $S$. Statement (C) is correct, while statement (D) is false as $P$ is only $3\frac{1}{8}$ km away from $S$ and $Q$.

$\therefore$ Only statement (C) is true.

**10.** Let the length and speed of train A be $l$ and $u$ and those of $B$ be $m$ and $v$. We have $\frac{\ell}{u} = 10\ s$ and $\frac{m}{v} = 5\ s$

Besides $v/u = 4/3$

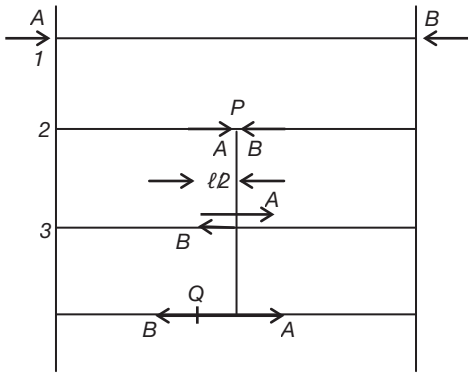
The following diagram shows the positions of the trains at four instants:

(1) When they begin to enter the tunnel

(2) When the engines cross each other at $P$

(3) When train $B$ completely crosses $P$ (the trains are shown along different lines, for the sake of clarity) and

(4) When the last compartments cross each other at $Q$




The time taken by the two trains to cross each other is $t = \frac{l+m}{u+v}$. We have $\frac{l}{u} = 2\frac{m}{v}$ and $v = \frac{4}{3}u$

$$\therefore\ m = \frac{l}{2}\left(\frac{v}{u}\right) = \frac{l}{2}\left(\frac{4}{2}\right) = \frac{2}{3}l$$

$$\therefore\ t = \frac{l+\frac{2}{3}l}{u+\frac{4}{3}u} = \frac{5}{7}\frac{l}{u} = \frac{5}{7}(10)s = \frac{50}{7}s\,7\frac{1}{7}s$$

**11.**




The ratio of Diya's and Riya's speeds does not change at any of the meeting points.

Let $S_1$ and $S_2$ be the initial speeds of Diya and Riya, respectively.

$\therefore$ If the total distance is $d$ km then the ratio of their speeds before the 1st meeting is $(d-12)/12$.

And ratio of their speeds before the 2nd meeting is $(2d-6)/(d+6)$

$$\therefore\ \frac{d-12}{12} = \frac{2d-6}{d+6} \Rightarrow d = 30$$

At the third meeting point, let the distance from $Y$ be '$z$'. The ratio of Diya's speed to Riya's speed at any stage is 3/2.

$$\therefore\ \frac{3}{2} = \frac{3d-z}{2d-z} \Rightarrow \frac{3}{2} = \frac{3(30)-z}{2(30)-z}$$

$$\Rightarrow z = 0$$

$\therefore$ The third meeting point is at $Y$.

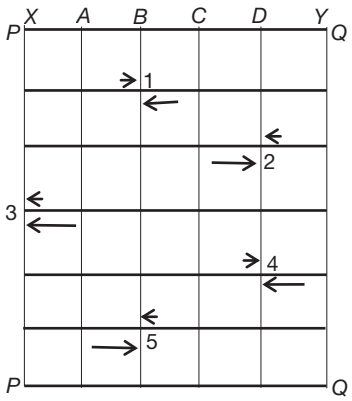
**12.** The 3 people are $A$, $B$ and $C$. Let the hostel and the sports complex be denoted as $H$ and $S$, respectively. At a certain time all 3 happen to be at the same point on the route, say $P$

$A$ starts from $H$ at 9 : 30 and reaches $S$ at 11:00, $C$ starts from $H$ at 9 : 45 and reaches $S$ at 10:30. The difference in departure times from $H$ is 15 minutes and the difference in arrival times at $S$ is 30 minutes. $\therefore\ 2HP = PS$

Also as the difference in arrival times for $B$ and $C$ at $S$ is 20 minutes, the difference in departure times from $H$ is 10 minutes, i.e., $B$ starts 10 minutes before $C$, i.e., at 9 : 35 a. m.

**13.** As the ratio of the speeds is m:n, we should divide the distance $PQ$ into m + $n$ parts. To assess the 4 options, we have to divide into 2 + 3, 3 + 4, 1 + 3 and 1 + 2 (viz., 5, 7, 4 and 3) parts, respectively.

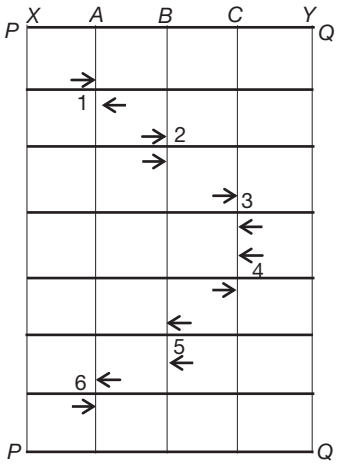
Consider 2 : 3. We divide $XY$ into 5 equal parts. Let the intermediate points be $A$, $B$, $C$ and $D$.




Initially $P$ is at $A$ and $Q$ is at $B$. The first 5 meetings take place at $B$, $D$, $X$, $D$ and $B$, respectively. Then, once again $P$ would be at $X$ exactly when $Q$ is at $Y$. The entire cycle gets repeated exactly.

$\therefore$ $P$ and $Q$ meet at exactly 3 points.

Consider 1 : 3. We have to divide $XY$ into 4 parts. Let the intermediate points be $A$, $B$, and $C$.




The first 6 meetings take place at $A, B, C, C, B, A$, respectively. Then, once again $P$ would be at $X$ exactly when $Q$ is at $Y$. The entire cycle gets repeated exactly.

$\therefore$ The ratio could be 2 : 3 or 1 : 3.

**14.** Let the coordinate of $X$ be $O$ and that of $Y$ be $L$. Let the coordinates of the cars $A, B, C$ be $a, b, c$, respectively.

These coordinates are functions of time, given by different expressions in different time intervals as shown below. The symbol $t$ represents the time (in hours) elapsed after 7:00 a.m.

| | 7 to 8 | 8 to 9 | After 9 |
|---|---|---|---|
| $a$ | $50\ t$ | $50\ t$ | $50\ t$ |
| $b$ | 0 | $40\ (t-1)$ | $40\ (t-1)$ |
| $c$ | $L$ | $L$ | $L-30\ (t-2)$ |

Let the event of $AC$ being equal to $AB$ be denoted by $E$. Depending on in which time interval $E$ occurs, we would write down different equations to determine the time.

Assume that $E$ occurs at $T$ where

**1.** 7 a.m. $< T \le$ 8 a.m.

$L - 50t = 50t - 0 \Rightarrow t = \dfrac{L}{100}$ (A) (if $\dfrac{L}{100} \le 1$, this is the required time)

**2.** $8 < T \le 9$ a.m.

$L - 50t = 50t - 40(t-1)$

$\Rightarrow t = \dfrac{L-40}{60}$ (B)

If $1 \le \dfrac{L-40}{60} \le 2$, this is the required time

i.e., if $100 \le L \le 160$, the required time would be given by $\dfrac{L-40}{60}$ (Note that for $L = 100$, both $A$ and $B$ give the same value of 1)

**3.** 9 a.m. $< T$

$L - 30\ (t-2) - 50t = 50t - 40(t-1)$

$\Rightarrow L + 60 - 40 = 90t \Rightarrow t = \dfrac{L+20}{90}$ … (C)

If $2 \le \dfrac{L+20}{90}$, i.e., if $L \ge 160$, the required time is given by $\dfrac{L+20}{90}$ (Note that for $L = 160$, both $B$ and $C$ give the same value of 2).

As $L = 120$, we have to use (B) $t = \dfrac{120-40}{60} = 1\ \dfrac{1}{3}$.

This represents the time 8:20 a.m.

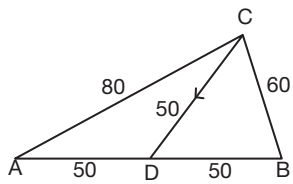
Choice (B)

**15.** In 4 hours, distance travelled by the 1st cyclist

$= 12.5\ (4) = 50$ km and distance travelled by the second cyclist $= 5 + 10 + 15 + 20 = 50$ km

$\therefore$ Distance travelled by the third cyclist $= \dfrac{50+50}{2} = 50$ km

So, it can be concluded that the three cyclists are at the vertices of a right angled triangle (their paths from the starting point $D$ are the radii of the circumcircle and the distance between the first two cyclists is the diameter). $ACB = 90°$, being the angle in a semi circle.



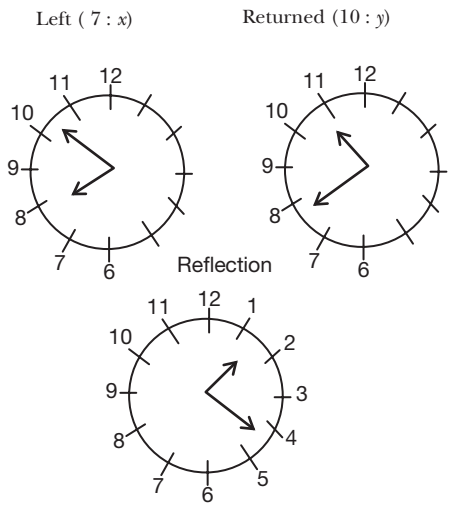

Now, $BC = \sqrt{100^2 - 80^2} = 60$ km

**16.** As $\dfrac{t_2}{t_1} = \dfrac{9}{1}$ and $t_2 - t_1 = 8$, $t_2 = 9$ and $t_1 = 1$ and

$t = \sqrt{t_1 t_2} = \sqrt{1(9)} = 3$

The bus from the bottom takes $(t + t_2)$ hr or $(3 + 9)$ hr and travels at 10 km/hr. $\therefore$ The distance is 120 km.

**17.** The position of the hands of the clock when Darpan left and the actual positions and its reflections when he returns are shown below.




$$210 + \frac{x}{2} = 6y$$

$$6x = 300 + \frac{y}{2}$$

Solving these, $x = \frac{7620}{143}$ and $y = \frac{5640}{143}$

Darpan actually returned at 10:39 $\frac{63}{143}$ p.m.

But he sees the reflection of the clock in the mirror and thinks that the time is 1: 20 $\frac{80}{143}$ a.m.

(1 : 20 ≡ 13:20 ≡ 12 : 80)

Time at which Darpan thinks he returned = 12 : 80 : 80/143

Time at which Darpan left = 7 : 53 : 41/143

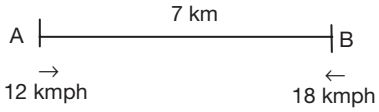
Time for which he thinks he was out = 5 : 27 : 39/143

**18.** Relative speed of Vicky with respect to Nicky

= (12 + 18) kmph = 30 kmph.

Distance = Speed x Time = $30\left(\frac{14}{60}\right)$ or 7 km

∴ The total distance covered by the two of them, when they cross each other for the first time is 7 km.




Let the total distance covered by Vicky and Nicky when they cross each other for the first time be *AB*.

For such a situation, when the ratio of speeds = 3 : 2, it can be observed the total distance covered by the two runners, between any two consecutive meetings is always twice the distance *AB*.

When they cross each other for the 2nd time, they must have covered a distance of *AB* + 2*AB*

When they cross each other for the 3rd time, they must have covered *AB* + 4*AB*.

∴ Total distance covered by them when they cross each other for the fifth time is = *AB* + 4(2*AB*) = 9*AB* = 7(9) or 63 km.

Time taken = $\frac{63}{30}$ or $2\frac{1}{10}$ hr.

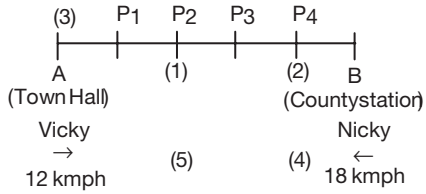
∴ Distance travelled by Nicky when they cross each other for the fifth time = $18\left(\frac{21}{10}\right)$ = 37.8 km.

= 4(7) + 9.8

= 7 + 2.8 = 2.8 km from the town hall.

i.e., 4.2 km from the county station

**Alternative solution:**




Vicky's and Nicky's speeds are in the ratio of 2 : 3. Dividing the total distance between *A* and *B* into 5 equal parts by $P_1$, $P_2$, $P_3$, $P_4$ ($AP_1 = P_1P_2 = P_2P_3 = P_3P_4 = P_4B = x$ say) we see that Vicky and Nicky meet for the first time at $P_2$. After that Vicky covers 4 parts and Nicky 6 parts before they meet again. Their meetings are tabulated below.

| Meeting number | Distance covered by vicky | Distance covered by nicky | Meeting point |
|---|---|---|---|
| 1 | 2*x* | 3*x* | $P_2$ |
| 2 | 6*x* | 9*x* | $P_4$ |
| 3 | 10*x* | 15*x* | *A* |
| 4 | 14*x* | 21*x* | $P_4$ |
| 5 | 18*x* | 27*x* | $P_2$ |

$AB = (12 + 18)\left(\frac{14}{60}\right)$ km = 7 km.

$\therefore BP_2 = \frac{3}{5}(7)$ km = 4.2 km.

**19.** Let the speed of $T_2$ be $t$ km/hr. Let the speed of the bird be $b$ km/hr.

By noon, $T$ would have covered 120 km. Suppose the trains meet in another $x$ hours.

The bird starts at the same time as $T_1$ and flies back and forth until the trains meet.

$\therefore$ It travels for $(2 + x)$ hours.

$PQ = 60\ (2 + x) + tx$

$$330 = 60\left(\frac{272}{b}\right) + t\left(\frac{272}{b} - 2\right) \qquad (1)$$

By the time the bird and $T_2$ meet, the bird covers 240 km.

$$\therefore 2 + \frac{330 - 240}{t} = \frac{240}{b} \qquad (2)$$

$\therefore$ (1) can be written as $330 = \frac{272}{b}(t + 60) - 2t$

$$= \frac{272}{240}\left(2 + \frac{90}{t}\right)(t + 60) - 2t$$

$$330 = \frac{17}{15}\left(2 + \frac{90}{t}\right)(t + 60) - 2t$$

$t^2 - 345t + 22950 = 0$

$(t - 90)\ (t - 255) = 0$

$t = 90$ or 255.

From (2), $b = 80$ or 102.

$(b,t) = (80,90)$ or $(102,255)$ In either case $b < t$ Once the bird meets $T_2$, it turn back but $T_2$, reaches $T_1$ before the bird does. It is stated in the question that the bird flies back and forth between $T_1$ and $T_2$. Actually, it reverses its direction only once.

**20.** Let us say Dinesh turned back to pick up Giri when the latter had walked for $t$ hrs. Dinesh would have covered $60t$ km (then). Giri would have covered $10t$ km. They would meet in $\frac{5t}{7}$ hr. In this time, Giri would have covered $\frac{50t}{7}$ km while Dinesh would have covered $\frac{300t}{7}$ km.

Distance covered by Giri up to the pick up point = $\frac{120t}{7}$ km.

$\therefore$ Distance covered by Eswar from the dropping point = $\frac{120t}{7}$

($\therefore$ Both of them start together and reach together. They cover equal distance on foot and equal distances on the bike)

$$AB = \frac{120t}{7} + \frac{300t}{7} + \frac{120t}{7} = \frac{540t}{7}$$

Hari's average speed (or that of Giri or Eswar)

$$= \frac{\frac{540}{7}t}{t + \frac{5}{7}t + \left(\frac{300t}{7} + \frac{120t}{7}\right)\left(\frac{1}{60}\right)} = \frac{540}{19}\ \frac{\text{km}}{\text{hr}}$$

**21.** Let the number of rounds completed on cycle and motor cycle in time $t$ hours be $x$ and $3x$, respectively. From the 2 given conditions, we get the following equations.

$$\frac{3}{2t} + 2(x) = 17$$

$$\frac{5}{2t} + 4(3x) = 50$$

$$\text{i.e, } \frac{3}{t} + 4x = 34 \qquad (1)$$

$$\frac{5}{t} + 24x = 100 \qquad (2)$$

$$\therefore \frac{1}{t} = \frac{34(24) - 4(100)}{3(24) - 4(5)} = \frac{816 - 400}{52} = 8 \text{ and}$$

$$x = \frac{3(100) - 34(5)}{3(24) - 4(5)} = \frac{130}{52} = \frac{5}{2}$$

$\therefore$ The time taken to complete one round on foot is 1/8hr = 7.5 min. The number of rounds completed on the mobike in 7.5 min is $3x = 7.5$ or the time taken to complete one round on the mobike is 1 min.

**22.** In 11 minutes, 4½ tonnes of water is admitted by the leak. In 1 minute, the leak admits $9/2 \times 1/11$

9/22 tonnes of water.

In 1 minute, the pumps can throw out 1/5 tonnes

The net inflow of water per minute = (9/22) – (1/5) = 23/110 tonnes

Time taken to accumulate 184 tonnes of water = 184/(23/110) = 880 minutes

Average rate of sailing so that the boat may just reach the shore as she begins to sink = 154/880

= 10.5 kmph

**23.** Let the speeds of the cycle and the bike be $3x$ km/hr and $4x$ km/hr, respectively. Let the speed at which the person walks be $w$ km/hr.

$$4(w)+2\left(\frac{2}{w}\right)(3x)=(2)(15) \text{ and } 5(w)+\frac{2}{w}(4x)=(2)\ (17)$$

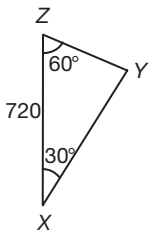
i.e, $4w+\frac{12x}{w}=30$ and $5w+\frac{8x}{w}=34$

Multiplying the first equation by 5 and then subtracting 4 times the second equation from it, $\frac{x}{w}=\frac{1}{2}$

From either equation, $w$ is 6, and hence, $x$ is 3.

$\therefore$ Speed of cycle is $3x$ i.e, 9 km/hr

**24.**




As $ZX = 720$ km, $ZY = 360$ km and $XY = 360\sqrt{3}$ km.

The train starts from $X$ at 7:00 a.m. and takes $360\sqrt{3}$ / $90\sqrt{3}$ or 4 hours to reach $Y$, i.e., it arrives at 11:00 a.m. Raju has to be at $Y$ latest by 10:40 a.m.

He takes 360/60 or 6 hours for the drive. He should start latest by 4:40 a.m.

**25.** When two runners run on a circular track in the same direction, a meeting between two persons will occur each time the faster one takes a lead of an entire lap over the slower one. Now, if $A$ is the faster person, when $A$ covered 15 laps, $B$ must have covered three laps less, i.e., 12 laps. But if $A$ is the slower person, $B$ must have covered (15 + 3) or 18 laps.

| | $A$ | $B$ | Total |
|---|---|---|---|
| **Case I:** No. of laps covered | 15 | 12 | 27 |
| **Case II:** No of laps covered | 15 | 18 | 33 |

We can conclude that $A$ and $B$ together covered either 27 laps or 33 laps in the given time of 330 mins. Therefore, the time taken by them to together cover one lap would be $\frac{330}{27}$ minutes or $\frac{330}{33}$ minutes.

Two runners starting simultaneously on a circular track travelling in opposite directions will meet each other each time they together cover the length of the track. Therefore, they will meet each other for the sixth time after they together cover 6 times the length of the circular track. Therefore they will meet each other for the sixth time after $\frac{330}{27}6$, i.e., $73\frac{1}{3}$ minutes or $\frac{330}{33}\times 6$ = 60 minutes

**26.** Total distance travelled by $B$

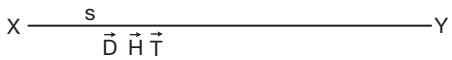
$= [(18)\ (1.2) + 12\ (1.8 + 0.6)]$ km = 50.4 km

**27.** Let the tortoise start from $X$ at $t = 0$

Then, the hare and the deer start from $X$ at $t = 1$ and $t = 2$, respectively.

Let the speeds of the deer, hare and tortoise be $4x$, $3x$ and $s$, respectively.

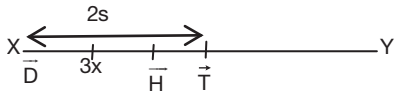
The position of the three at $t = 1$




The time taken by the hare to catch up with the tortoise

$= \frac{s}{3x-s}$

The position of the three at $t = 2$




The time taken by the deer to catch up with the hare

$= \frac{3x}{4x-3x} = 3$

Given that, $3 = 1.5\left[\frac{s}{3x-s}\right] \Rightarrow s = 2x$

So, the ratio of their speeds is 4 : 3 : 2 and the ratio at the time they take to travel $XY$ is $\frac{1}{4}:\frac{1}{3}:\frac{1}{2}$, i.e., 3 : 4 : 6. Since the deer takes 7.5 hr to go from $X$ to $Y$, the tortoise takes $\frac{6}{3}\times 7.5$, i.e., 15 hours and the hare takes $\frac{4}{3}\times 7.5$, i.e., 10 hours for the same.

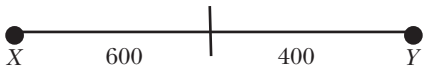
The distance $\overline{XY} = 3x + (7.5)3x + 4.5 = 7.5\ (4x)$

(as travelled by the hare = as travelled by the deer)

$\Rightarrow x = 1$

$\therefore\ \overline{XY} = 7.5\ (4)\ (1) = 30$ km.

**28.**




When P meets $Q$ for the first time, he would have travelled 3/5 (1000) = 600 m. Between any two consecutive meetings, any two people separated by a certain distance initially, would be together covering twice the initial distance between them, if they start running simultaneously in opposite directions and move to and fro between their initial starting points.

Hence, $P$ and $Q$ would together cover 2000 m between the 2nd and 3rd as well as the 3rd and 4th meeting. Total distance $P$ covers when he meets $Q$ for the fourth time = $600 + 3 \times 3/5 \times 2000 = 4200$ m

**29.** If the ratio of the speeds of Ajay and Sanjay in the lowest terms is $\frac{a}{s}$, then Ajay and Sanjay will meet at $(a + s)$ distinct points on the track when travelling in opposite directions and $(a - s)$ distinct points on the track when travelling in the same direction.

It is given that $a + s = 7$

The different possibilities are 6 + 1, 5 + 2 and 4 +3. For $a - s = n$, we get the corresponding possibilities as 6 – 1 = 5, 5 – 2 = 3 and 4 – 3 = 1

As it is given that $n$ is a prime number, $n$ can be 5 or 3. Therefore the ratio of the speeds of Ajay and Sanjay can be $\frac{6}{1}$ or $\frac{5}{2}$.

So, Sanjay's speed can be less than Ajay's by $\frac{6-1}{6} \times 100\%$ or by $\frac{5-2}{5} \times 100\%$, i.e, by $83^1/_3\%$ or 60%.

Thus, the value of $P$ can be $83^1/_3\%$ or 60%.

**30.** If 2 runners run in the same direction on a circular track with their speeds in the ratio m: $n$ (where m>n and $HCF(m, n) = 1$), then whether they start from the same point, the faster one (first runner) overtakes the other at m-n distinct points on the track.

Similarly, if they run in opposite directions with speeds in the ratio m : $n$ (where $HCF(m, n) = 1$), then they cross each other at m + $n$ distinct points on the track.

$A$ and $B$ run clockwise, while $C$ runs anticlockwise. Between $A$ and $B$, the faster overtakes the slower at 3 points. $A$, $C$ cross each at 7 points. $B$, $C$ cross each other at 10 points.

$\therefore$ If the speeds of $A$ and $B$ are in the ratio a : $b$, then $|a - b| = 3$.

We see that only choice $B$ (i.e., 4:7:3) satisfies this condition.

Also, 4 +3 = 7 and 7+3 = 10.

$\therefore$ The ratio could be 4:7:3 but it can't be any of the other three.

## EXERCISE-5

**1.** Let $d$ km be the distance.

$d = 400$

Let the speed of the bus be $s$ kmph and the time taken be $t$ hours.

$400 = st \rightarrow$ (I)

From Statement I, $(s - 20)\ (t + 20) = 400$

$\Rightarrow st + 20s - 20t - 400 = 400$

$\Rightarrow 20(s - t) = 400$

$\Rightarrow s - t = 20 \rightarrow$ (II)

From (I) and (II), we get $(20 + t)\ (t) = 400$

$t^2 + 20t - 400 = 0$

A unique value of $t$ exists, and hence, '$s$' can be determined.

So, Statement I alone is sufficient.

From Statement II, $s \geq 20$ and $t \geq 20$ and $st \geq 400$

From (I) $st = 400$

$\Rightarrow s = 20, t = 20$

Hence, Statement II alone is also sufficient.

Either statement alone is sufficient.

**2.** From Statement I, $PQ$ = relative velocity $x$ time of travel; and the time of travel = 11.30 – 9.30 = 2 hours; relative velocity = (60 + 40) = 100 kmph. So, Statement I is sufficient to answer the question.

From Statement II, we do not know what time the two trains start simultaneously. So, we can't answer the question.

**3.** Time taken to cross the bridge

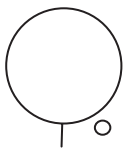
$$= \frac{\text{length of the bridge + length of the train}}{\text{speed of the train}}$$

Statement I alone is sufficient.

**4.** Whether they run in the same or in the opposite direction, they will meet at the starting point after a time equal to the L.C.M. of times taken by them individually to complete one round. The time taken by each individually is given in Statement II, and hence, Statement II alone is sufficient.

**5.** Since the hour hand and the minute hand make an angle of 72° between 8 O' clock and 9 O' clock twice, the answer choice is (D).

**6.**



→ Ram, Ramesh and Rameez

Length of the track = 500 m

From Statement I,

Ram and Ramesh meet after 10 seconds for the first time.

There is no information of Rameez.

So, Statement I alone is insufficient.

From Statement II,

Ramesh and Rameez meet after 10 seconds. There is no information of Ram here. So, Statement II alone is insufficient.

Using both the statements we get that Ram, Ramesh and Rameez, meet for the first time after 10 seconds. But it cannot be deduced whether they are meeting for the first time at the starting point or elsewhere. So, both the statements together are not insufficient.

**7.** From Statement I, we can say that $A$ covers 1000 m in same time as $B$ covers 900 $m$, but time taken by $B$ to complete the race is not known so we can't find time the taken by $A$ to complete the race.

From Statement II, speed of $C$ is 200/25 = 8 m/s and $B$ takes 25 sec less than $C$ to complete the race.

C's speed is known so we can find time taken by $C$ to complete the race and thereby the time taken by $B$ to complete the race, but not the time taken by $A$ to complete the race. So, the second statement alone is not sufficient.

Using both the statements, time taken by $B$ to cover 1000 m is known by the second statement so we can calculate the time taken by him to cover 900 m which is equal to the time taken by $A$ to complete the race.

So, using both the statements we can find the answer.

**8.** Let speed of $B$ be b

Using Statement I, $5 - b = 1$ or $b - 5 = 1$

Hence, $b$ = 4m/sec or 6m/sec

We don't $t$ know the initial distance between $A$ and $B$, nor do we know $B$'s exact speed. Hence, Statement I alone is not sufficient.

From Statement II, the distance between them is decreasing for the first 15 $s$.

In 15 $s$ the distance between them decreases by 30 m. Hence, in 10 $s$ it decreases by 20 m. The distance between them after 10 $s$ is (30 – 20) m = 10 m. We can answer the question from II alone.

**9.** Let the speeds of $A$, $B$ and $C$ be a, $b$ and $c$.

let the length of the track be $L$.

Time taken to meet = L.C.M. $(L/a + c, L/b + c)$ or L.C.M. $(L/a + c, L/a - b)$ or L.C.M. $(L/b + c, L/a - b)$

Using Statement I, Time taken for all the three to meet for the first time cannot be found as $L/a + c$ is known but $L/b + c$ or $L/a - b$ is not known.

Using Statement II, Time taken for all the three to meet for the first time can be found since

$L/a - b$ and $L/b + c$ are both known.

**10.** Let the time taken by $A$ to run the race be $t$.

Times taken by $B$ and $C$ to complete the race are $t + 30$ sec and $t + 40$ sec, respectively.

Using Statement I, $(t + 30)/t = 120/(120 - 40)$

Hence, $t$ can be found.

Using Statement II,

$(t + 40)/t = 120/(120 - 8 - 20)$

Hence, $t$ can be found.

Hence, using either statement, the question can be answered.

**11.** From Statement I, distance travelled is 200 km, and time taken is 6 hours.

Hence, average speed is $\frac{200}{6}$ = 33.33 km/hr.

Statement I alone is sufficient.

From Statement II, his onward journey speed is 30 km/hr and his return journey speed is 40 km/hr. Hence, average speed is $\frac{2\times 30\times 40}{30+40}$ km/hr.

Statement II alone is also sufficient.

**12.** From Statement I, Pulkit takes an hour and a half more than Angadh. But we do not know the time taken to cover the distance by Angadh.

From Statement II, the ratio of speeds of Angadh and Pulkit is $\sqrt{\frac{4\frac{4}{5}}{3\frac{1}{3}}} = \sqrt{\frac{24}{5}\times\frac{3}{10}} = \frac{6}{5}$

Hence, Statement II alone is sufficient.

**13.** From Statement I, we do not know the speeds of ship and riskshaw, and hence, we can't answer the question.

From Statement II, by knowing only ratio of speeds we can't find the speed of train.

So, Statement II alone is not sufficient.

Using both the statements, let the speeds of ship, train and riskhaw be $4x$, $15x$ and $x$, respectively.

Given, $\frac{12}{4x}+\frac{45}{15x}+\frac{6}{x}=2$

So, the speed of the train can be found.

**14.** From Statement I, when Aarti and Bhallu met, the distance covered is 20 x 2 = 40 km. We do not know about their speeds. So, we can't answer the question. Statement I alone is not sufficient.

From Statement II, the ratio of speeds of Aarti and Bhallu is 3 : 5.

Let their speeds be $3x$ and $5x$, respectively.

Now the length $PR$ is equal to $2PQ\times 3x/8x$

We do not know the distance between $P$ and $Q$.

So, Statement II alone is not sufficient.

Using both statements, $PR = 2\ x\ 20\ x\ \frac{3x}{8x} = 15$.

**15.** Let $V_a$ be the velocity of the slower man.

$\therefore$ the faster man will walk at $V_a + 2$ m/s.

Let the train travel at $V_t$ m/s and let its length be $l$.

From Statement I, we have $\frac{\ell}{V_t - V_a} = 9$ (1)

$\frac{\ell}{V_t - V_a - 2} = 10$ (2)

So, $\frac{V_t - V_a}{V_t - V_a - 2} = \frac{10}{9}$

Let $V_t - V_a = x$

$\frac{x}{x-2} = \frac{10}{9}$

So, $x = 20$, i.e., $V_t - V_a = 20$.

Now $\frac{\ell}{20} = 9$

$\therefore l = 180$ m.

Statement I alone is sufficient.

From Statement II, faster man travels at 4 m/s. But without the times the train takes to overtake the men we cannot solve for the length of the train. Statement II alone is not sufficient.

**16.** Let the length of train be $L$ meters. From Statement I the train covers $L$ meters in 6 seconds.

So, speed of train = $L/6$.

But we do not know $L$. Hence, Statement I alone is insufficient. From Statement II, the train crosses $750 + L$ in 25 seconds.

Speed of train = $\frac{750+L}{25}$

Again we do not know $L$, therefore Statement II is also insufficient.

Using both the statements, we have $\frac{750+L}{25} = \frac{L}{6}$

i.e., $25L = 4500 + 6L$.

$\therefore L = \frac{4500}{19}$

So, speed of train is $\frac{4500}{19\times 6}$ m/s

**17.** Let the distance between $A$ and $B$ be $d$ km.

From Statement I, $\frac{d}{x} - \frac{d}{x+10} = 2$

We do not know the value of $d$ or $x$ so we can't find the time taken to travel from $A$ to $B$ at $x$ kmph. Statement I alone is not sufficient.

From Statement II, ratio of speeds is $x : \frac{3x}{4}$ i,e., 4 : 3. So, ratio of time taken to cover the distance at $x$ kmph and $\frac{3x}{4}$ kmph is 3 : 4

$\therefore$ time taken to cover the distance at $x$ kmph

$= \frac{3}{4-3} \times 2 = 6$ hours. Statement II alone is sufficient.

**18.** Either of the statements alone is not sufficient to answer the question as we have information about Alok and Asif in different statements.

Using both statements, $\frac{\text{Biswas}}{\text{Alok}} \times \frac{\text{Asif}}{\text{Biswas}} = \frac{400}{370} \times \frac{760}{800}$

$\frac{\text{Asif}}{\text{Alok}} = \frac{380}{370}$. In a 380 m race, Asif beats Alok by 10 m. So, in a 800 m race, Asif beats Alok by $\frac{10}{380} \times 800$ m.

**19.** The angle between the hands of a clock at 3 O'clock is 90°.

From here on, the angle decreases to 0° and then increases upto 180° and again decreases upto 120° (i.e., the angle at 4 O'clock.)

From Statement I, the angle is 130°. But this angle occurs twice.

$\therefore$ Statement I alone is not sufficient.

From Statement II, the angle is 100°, which occurs only once. Hence, we can find the time.

$\therefore$ Statement II alone is sufficient.

**20.** From Statement I, let the speed of the boat in still water and the speed of the water current be $x$ kmph and $y$ kmph, respectively.

$$\frac{d}{x+y} = \frac{1}{2}\left(\frac{d}{x-y}\right)$$

$\Rightarrow x = 3y$, i.e., speed of the boat in still water is 3 times the speed of the water current.

Statement I alone is sufficient.

Statement II gives the speed of the boat upstream as10 m/s. But this alone is not sufficient.

**21.** From Statement I,

speed of the train is

$\frac{200}{10} = 20$ m/s

From Statement II, we do not know the length of the platform, so we can't find the speed of the train.

So, Statement II alone is not sufficient.

**22.** Let the length of the race be $L$ m.




In the above figure, $A$ and $B$ denote the starting points of Ram and Shyam, respectively. $D$ denotes the finishing points of both. $C$ denotes the position of Ram when Shyam finished.

When Shyam covered $BD$, Ram would have covered $AC$.

$BD = BC + CD = BC + y$

$AC = AB + BC = x + BC$

If Ram had not been faster than Shyam, $AC \le BD$ must have been true.

$\therefore x \le y$ must be true.

From Statement I, if $x = y$, Ram was not faster than Shyam. If $x > y$, Ram was faster than Shyam.

I. alone is not sufficient.

II. alone is sufficient.

**23.** Let the speed of Mohan's boat in still water and the speed of the stream be $x$ kmph and $y$ kmph, respectively.

From Statement I, $12 = 4(x - y)$

$\Rightarrow 3 = x - y$

$\therefore x > 3$

I is sufficient.

From Statement II, $12 = 3\ (x + y)$

$4 = x + y$

For $x \ge y$

$\therefore x \ge 2$

II is not sufficient

**24.** A drove for 5 hours and $B$ drove for 4½ hours.

From Statement I, $d_A + 50 = d_B$ and $d_A + d_B = 500$

$\Rightarrow d_A = 225$ and $d_B = 275$.

$\therefore$ We can find the average speed of $B$.

From Statement II, $\frac{d_A}{5} + 5 = \frac{d_B}{4.5}$

$\Rightarrow \frac{2d_B}{9} - = \frac{d_A}{5} = 5$

$\Rightarrow \frac{10d_B - 9d_A}{45} = 5 \Rightarrow 10d_B - 9d_A = 225$

We know $d_A + d_B = 500$

So, we can find $d_A$ and $d_B$ and then the average speed.

$\therefore$ It can be answered using either statement alone.

**25.** From Statement I, speed of the river current = 2 m/s

From Statement II, speed of the boat relative to the speed of the river is 1 m/s.

Using both the statements,

$V - 2 = 1$ where $V$ is the speed of the boat in still water. $\Rightarrow V = 3$.

**26.** Statement I alone is not sufficient as time taken for the trip is not known.

From Statement II alone, distance travelled in the trip = st where $s$ is the average speed and $t$ is the time taken for the trip.

Given $(1.25s)\,t - st = 100 \Rightarrow 0.25\ st = 100$

$\therefore$ st can be found

Hence, II alone is sufficient.

**27.** Let the usual time be $t$ hours.

Distance between his home and office = $a\left(t - \frac{a}{60}\right)$

$= b\left(t - \frac{b}{60}\right)$

$(a - b)\ t = \frac{a^2 - b^2}{60}$

$t = \frac{a + b}{60}$

I is sufficient.

From II, as we cannot find $a + b$, it is not sufficient.

**28.** Let the length of $A$ be $3z$ m. Length of $B = 2z$ m. Let the speeds of $A$ and $B$ be a m/sec and $b$ m/sec, respectively.

Let the lengths of $P_1$ and $P_2$ be p m and $q$ m, respectively.

$\frac{3z + p}{a} = x$

$\frac{2z + q}{b} = y$

From Statement I, $a = b$

$\therefore \frac{x}{y} = \frac{3z + p}{2z + q}$

Nothing is known about $p$ and $q$.

$\therefore$ The question cannot be answered. I is not sufficient.

From Statement II, $p = q$

$\therefore \frac{x}{y} = \frac{3z + q}{2z + q}\left(\frac{b}{a}\right)$

Nothing is known about a and $b$.

$\therefore$ The question cannot be answered.

II is not sufficient.

Using both statements, we get $a = b$ and $p = q$

$\therefore \frac{x}{y} = \frac{3z+q}{2z+q} = \frac{3}{2} - \frac{q}{2z+q}$. $\therefore 1 < \frac{x}{y} < \frac{3}{2}$

We cannot answer the question.

Both statements even when taken together are not sufficient.

**29.** Let the lengths of $A$ and $B$ be $p$ m and $q$ m, respectively. Let the speeds of $A$ and $B$ be a m/sec and $b$ m/sec, respectively.

$b = \frac{a}{2}$

$\frac{p + 500}{a} = 50$ (1)

Length of $B = 2p$ m

$\frac{2p + b_2}{b} = \frac{2p + b_2}{\frac{a}{2}}$

Required time =

From Statement I, $b_2$ and a are unknown.

$\therefore$ required time cannot be found.

I is not sufficient.

From Statement II, $b_2 = 1000$

$\frac{2p + b_2}{\frac{a}{2}} = 4\left(\frac{p + 500}{a}\right)$

From (1), this can be found. II is sufficient.

**30.** Let the speeds of $A$, $B$ and $C$ be a m/sec, $b$ m/sec and c m/sec, respectively.

$\frac{a}{b} = \frac{1000}{1000 - x}$

$$\frac{b}{c}=\frac{1000}{1000-y}$$

$$\frac{a}{c}=\left(\frac{a}{b}\right)\left(\frac{b}{c}\right)=\frac{1000^2}{1000^2-1000(x+y)+xy}$$

$$\frac{a}{c}=\frac{1000}{1000-\left(x+y-\frac{xy}{1000}\right)}$$

$A$ beats $C$ by $\left(x+y-\frac{xy}{1000}\right)m$

In each statement, $x=200$

When $200+y-\frac{200y}{1000}>400$

$y>250$

Using Statement I, we cannot say that $y > 250$. I is not sufficient. As $y \le 200$, II is sufficient.

# 6 Time and Work

CHAPTER

## Learning Objectives

*In this chapter, you will*:

- Learn about the basic assumptions in time and work
- Apply the variation concept in time and work
- Understand about efficiency, negative work and man days
- Understand alternate days models, multiple persons working simultaneously
- Learn about the distribution of wages based on efficiency/contribution
- Apply the time and work concepts to analogous situations like pipes and cisterns

Work to be done is usually considered as one unit. It may be constructing a wall or a road, filling up or emptying a tank or cistern or eating certain amount of food.

There are some basic assumptions that are made in the problems on Time and Work. These are taken for granted and are not specified in every problem.

1. If a person (or one member of the workforce) does some work in a certain number of days, then we assume (unless otherwise explicitly stated in the problem) that he does the work uniformly, i.e., he does the SAME amount of work everyday.

For example, if a person can do some work in 15 days, he does $1/15^{th}$ of the work in one day.

If a person completes the work in 4 days, he does $1/4^{th}$ of the work on each day and conversely, if a person can complete $1/4^{th}$ of the work in one day, he can complete the work in 4 days.

If a tap can fill a tank in 20 minutes, then in one minute, it can fill $1/20^{th}$ part of the tank.

2. If there is more than one person (or members of 'workforce') carrying out the work, it is assumed that each person (or members of the workforce), unless otherwise specified, does the same amount of work each day. This means they share the work equally.

If two people together can do the work in 8 days it means that one man can do it in 16 days. This, in turn means, each person can do $1/16^{th}$ of the work per day.

If a man works three times as fast as a boy does, the man takes one-third of the time the boy takes to complete the work. If the boy takes 12 days to complete the work, then the man takes 4 days to complete the work.

This method is known as 'Unitary Method', i.e., the time taken per unit work or number of persons required to complete unit work or work completed by unit person in unit time, etc., is what is first calculated.

**We should recollect the fundamentals on variation (direct and inverse) here.**

- **Time remaining constant, Work and Men are directly proportional to each other,** i.e., if the work increases the number of men required to complete the work in the same number of days increases proportionately and vice-versa.
- **Work remaining constant, Men and Days are inversely proportional,** i.e., if the number of men increases, the number of days required to complete the same work decreases in inverse proportion and vice-versa.

- **The number of workingmen remaining constant, Work and Days are directly proportional,** i.e., if the work increases, the number of days required to complete the work with the same number of working men also proportionately increases and vice-versa.

The concept of Mandays is very important and useful here. The number of men multiplied by the number of days that they take to complete the work will give the number of mandays required to do the work. The total number of mandays required to complete a specific task will remain a constant. So, if we change one of the variables—men or days—the other will change accordingly so that their product will remain constant (remember from our knowledge of variation, two variables whose product is a constant are said to be inversely proportional to each other). The two variables—men and days—are inversely proportional to each other, when work is constant.

In general we can say that

**FORMULA**

If $M_1$ men can do $W_1$ work in $D_1$ days working $H_1$ hours per day and $M_2$ men can do $W_2$ work in $D_2$ days working $H_2$ hours per day (where all men work at the same rate), then

$$\frac{M_1\ D_1\ H_1}{W_1} = \frac{M_2\ D_2\ H_2}{W_2}$$

If two persons A and B can individually do some work in $p$ and $q$ days, respectively, we can find out how much work can be done by them together in one day. Since A can do $1/p^{th}$ part of the work in one day and B can do $1/q^{th}$ part of the work in one day, the two of them together do $(1/p + 1/q)^{th}$ part of the work in one day.

From this we can find out the number of days that they take to complete the work.

**FORMULA**

If A can do a piece of work in $p$ days and B can do it in $q$ days, then A and B together can complete the same in $\frac{pq}{p+q}$ days.

In general, money earned should be shared by people doing the work together in the ratio of the share of work done by each of them.

For example, if A does $2/5^{th}$ of the work, then he should get $2/5^{th}$ of the total earnings for the work. If the remaining $3/5^{th}$ of the work is done by B and C in the ratio of 1 : 2, then the remaining $3/5^{th}$ of the earnings (after paying A) should be shared by B and C in the ratio of 1 : 2. Suppose ₹ 500 is paid to A, B and C together for doing the work, then A will get ₹ 200 (which is 2/5 of ₹ 500), B will get ₹ 100 and C, ₹ 200 (because the remaining ₹ 300 after paying A is to be divided in the ratio 1 : 2 between B and C).

When people work for the same number of days each, then the ratio of the total work done will be the same as the work done by each of them per day. Hence, if all the people involved work for the same number of days, then the earnings can directly be divided in the ratio of work done per day by each of them.

For example, let's take the case of three persons A, B and C who work on a project for a certain number of days. It is also known that A is twice as efficient as B, who in turn is thrice as efficient as C. If the total amount paid to the group is ₹ 1,000, we'll try and calculate the share of each of them. Since we know that B is thrice as efficient as C, he'll do thrice as much work as C does in the same time. Since A is twice as efficient as B, he'll do twice as much work as B or six times as much work as C in the same time. As the amounts of work done by A, B and C in the same time are in the ratio 6 : 3 : 1, their wages should also be distributed in the same ratio., i.e. ₹ 600, ₹ 300 and ₹ 100 respectively.

## ❒ PIPES AND CISTERNS

There can be pipes (or taps) filling (or emptying) tanks with water. The time taken by different taps (to fill or empty the tank) may be different. Problems related to these can also be dealt with in the same manner as the foregoing problems on Work have been dealt with.

There is only one difference between the problems on regular Work (of the type seen earlier on in the chapter) and those in Pipes and Cisterns. In Pipes and Cisterns, a filling pipe or tap does positive work and an emptying pipe or a leak does negative work.

In the previous cases concerning two persons working together, we've seen that if A and B can do some work in $p$ days and $q$ days respectively, the work done by both of them on any given day is $1/p + 1/q$. However, when we're dealing with a filling pipe which can fill an empty tank in p hours and an emptying pipe (or a leak, for that matter) which can empty the full tank in q hours, work done per hour will be $1/p - 1/q$, all other calculations remaining same.

## SOLVED EXAMPLES

**6.01.** If 15 men take 60 days to complete a job, in how many days can 10 men complete the same job?

**Sol:** If 15 men can complete the job in 60 days, then the number of man days required for the work is $15 \times 60 = 900$ mandays.

If this work is to be done by 10 men, since the number of man days will be same for same amount of work, the number of days they will take is $900/10 = 90$ days.

**6.02.** Fifteen men take 20 days to complete a job working 8 hours a day. How many hours a day should 20 men work to complete the job in 12 days?

**Sol:** Since 15 men take 20 days at 8 hours per day, the total man hours required for the job is $15 \times 20 \times 8 = 2400$. As 20 men are also working on the same job, the number of man hours required for the job will still be the same. Hence, the number of hours they should work per day is $2400 / (20 \times 12) = 10$ hours.

Therefore, 20 men can complete the work in 12 days working 10 hours per day.

**6.03.** A piece of work can be done by 24 men in 4 days working 10 hours a day. How many men are needed to complete another work which is twice as large as the first one in 48 days working 8 hours a day?

**Sol:** The problem can be solved by using the man days concept as shown in the question above or using the following formula.

$$\frac{M_1 D_1 H_1}{W_1} = \frac{M_2\ D_2\ H_2}{W_2}$$

Here $W_1 = 1$ and $W_2 = 2$, $M_1 = 24$, $H_1 = 10$, $D_1 = 4$ $H_2 = 8$, and $D_2 = 48$

$$\frac{24 \times 10 \times 4}{1} = \frac{M_2 \times 48 \times 8}{2} \Rightarrow M_2 = 5 \text{ men.}$$

**6.04.** A can do a piece of work in 10 days. B can do the same in 15 days. In how many days can the work be completed if A and B work together?

**Sol:** As A can do the work in 10 days in one day he can complete $1/10^{th}$ of the work. Similarly work done by B in one day $= 1/15^{th}$.

$\therefore$ The work done by A and B in one day

$$= \frac{1}{10} + \frac{1}{15} = \frac{1}{6}$$

$\therefore$ They can complete it in 6 days.

Alternately, using the formula,

A and B together can do the work in $\frac{10 \times 15}{10 + 15} = \frac{150}{25}$ = 6 days.

**6.05.** A and B together can do a piece of work in 24 days and A alone can complete the work in 36 days. How long will B alone take to complete the job?

**Sol:** In a day, A and B together can do $1/24^{th}$ part of the work. In a day, A alone can do $1/36^{th}$ part of the work. In one day, work done by B alone is:

(work done by A and B) – (work done by A)

i.e., $\frac{1}{24} - \frac{1}{36} = \frac{1}{72}$.

Therefore, B alone can complete the work in 72 days.

**6.06.** Girish and Harish can complete a job in 24 days working together. Girish alone can complete it in 32 days. Both worked together for 8 days and then Girish left. How long will Harish take to complete the remaining work?

**Sol:** Work done in one day by both together $= 1/24^{th}$

$\therefore$ Work done by them both in 8 days $= 8 \times (1/24) = 1/3^{rd}$

Remaining work $= 1 - \frac{1}{3} = \frac{2}{3}$.

Work done by Harish in 1 day

$$= \frac{1}{24} - \frac{1}{32} = \frac{1}{96}.$$

Since only two-thirds work is left, Harish will do the remaining work in $(2/3) \times 96 = 64$ days.

**6.07.** A and B together can do a piece of work in 36 days, B and C can do it in 48 days and A and C can do it in 72 days. How long would each take to do the job?

**Sol:** Work done by A and B in one day $= 1/36^{th}$ (1)

Work done by B and C in one day $= 1/48^{th}$ (2)

Work done by A and C in one day $= 1/72^{nd}$ (3)

Adding (1) + (2) + (3), we get 2 (A + B + C)'s

$$1 \text{ day's work} = \frac{1}{36} + \frac{1}{48} + \frac{1}{72} = \frac{9}{144} = \frac{1}{16}$$

A, B and C can together finish in 1 day, $\frac{1}{2}\left(\frac{1}{16}\right) = \frac{1}{32^{nd}}$ part of the work.

Work done by A in 1 day

$= \frac{1}{32} - \frac{1}{48} = \frac{1}{96}$.

Therefore, A alone can do it in 96 days.

Work done by B in 1 day

$= \frac{1}{32} - \frac{1}{72} = \frac{5}{288}$.

Therefore, B alone can do it in 288/5, i.e., $57^{3}/_{5}$ days.

Work done by C in 1 day

$= \frac{1}{32} - \frac{1}{36} = \frac{1}{288}$.

Therefore, C alone can do it in 288 days.

**6.08.** A and B can do a work in 18 days. B and C do it in 24 days and A and C in 36 days. They all work together for 4 days and then A left. In how many more days can B and C finish the remaining work?

**Sol:** By using a procedure similar to the previous question, we get 2 (A + B + C)'s 1 day's work

$= \frac{1}{18} + \frac{1}{24} + \frac{1}{36} = \frac{9}{72} = \frac{1}{8}$. A, B and C can finish the work in 16 days. Work done by A, B and C in 4 days

$= 4 \times (1/16) = (1/4)^{th}$.

$\therefore$ Work remaining $= 1 - 1/4 = 3/4$

B and C can finish the remaining work in $(3/4) \times 24 = 18$ days.

**6.09.** A can do a work in 18 days. When he had worked for 2 days, B joined him. If they complete the remaining work in 4 more days, in how many days can B alone finish the whole work?

**Sol:** Work done by A in one day $= 1/18^{th}$.

Number of days A worked $= 2 + 4 = 6$.

Therefore, total work done by A

$= 6 \times (1/18)$

$= (1/3^{rd})$.

The remaining two-third work is done by B in 4 days.

Therefore, B takes $4 \times (3/2) = 6$ days to complete the whole work.

**6.10.** A and B together can do a piece of work in 16 days. B and C together can do the same work in 32 days. After A worked for 4 days, B for 12 days, C takes up and finishes it alone in 48 days. In how many days will each of them complete the work?

**Sol:** Work done by A and B in 1 day $= 1/16^{th}$

Work done by B and C in 1 day $= 1/32^{nd}$

A worked for 4 days, B worked for 12 days and C worked for 48 days to complete the work.

Denoting work done by A, B and C in one day as $a$, $b$ and $c$, we have $4a + 12b + 48c = 1$

$\Rightarrow 4a + 4b + 8b + 8c + 40c = 1$

$4\left(\frac{1}{16}\right) + 8\left(\frac{1}{32}\right) + 40c = 1$

$\frac{1}{4} + \frac{1}{4} + 40c = 1 \; ; \; 40c = \frac{1}{2} \Rightarrow c = \frac{1}{80}$.

C takes 80 days to finish the work.

B's 1 day's work $= \frac{1}{32} - \frac{1}{80} = \frac{3}{160}$

Hence, B takes 160/3, i.e., $53^{1}/_{3}$ days to finish the work.

A's 1 day's work $= \frac{1}{16} - \frac{3}{160} = \frac{7}{160}$.

Hence, A takes 160/7, i.e., $22^{6}/_{7}$ days to finish the work.

**6.11.** To do a certain work, C alone takes thrice as long as A and B together. A alone would take twice as long as B and C together. All three together complete the work in 3 days. How long would each take separately?

**Sol:** As C takes thrice a long as A and B together, (A + B)'s one day work = 3 times of C's one day work.

Adding C's one-day work to both sides, 4 times C's one day work

= (A + B + C)'s one day work, i.e., $1/3^{rd}$.

Hence, C's one day work $= 1/12^{th}$.

As A takes twice as long as B and C work together, (B + C)'s one day work = 2 times of A's one day work. Adding A's one day work to both sides, 3 times A's one day work = (A + B + C)'s 1 day's work, i.e., $1/3^{rd}$.

A's one day work $= 1/9^{th}$.

B's one day work

$= \frac{1}{3} - \left(\frac{1}{12} + \frac{1}{9}\right) = \frac{12 - 3 - 4}{36} = \frac{5}{36}$.

Hence, A alone takes 9 days to complete the work, B alone takes 36/5, i.e., $7^{1}/_{5}$ days to complete the work and C alone takes 12 days to complete the work.

**6.12.** Three men or five women can construct a wall in 68 days. How long will it take for5 men and 3 women to complete the same work?

**Sol:** Given $3m = 5w$ where $m$ is the work done by one man in one day and $w$ is the work done by one woman in one day.
$\Rightarrow 1m = 5w/3$
Now $5m + 3w = 5(5w/3) + 3w$
$= \frac{25w}{3} + \frac{9w}{3} = \frac{34w}{3}$
If 5 women can do the work in 68 days, (34/3) women can finish it in $5 \times 68 \times (3/34) = 30$ days.

**6.13.** If 15 boys and 20 men take 4 days to do a piece of work and 12 boys and 13 men take 6 days to do the same work, find the time taken by 2 men and 8 boys to complete the same work.

**Sol:** Given 15 boys + 20 men can do the work in 4 days and 12 boys + 13 men can do the same work in 6 days. Let $m$ and $b$ be the work done by a man and a boy in one day, respectively. As the number of persons in both the cases are doing the same work, number of mandays will be same.
Therefore, $4(15b + 20m) = 6(12b + 13m)$
$\Rightarrow m = 6b$. Hence, $[15b + 20m]$ is equal to $[15b + 120b]$, i.e., $135b$.
Time taken by 2 men and 8 boys
i.e., $12b + 8b = 20b$, to complete the work.
$= (135 \times 4) / 20 = 27$ days.

**6.14.** A certain number of men can do a work in 24 days. If there were 3 more men, the work can be done in 6 days less. How many men were there initially?

**Sol:** Let the initial number of men be P. Number of days is 24, so number of mandays is 24 P. If there are 3 more men, i.e., P + 3, it is completed in 24 – 6 = 18 days. So, $24P = (P + 3)\ 18 \Rightarrow P = 9$. Therefore, initially there were 9 men.

**6.15.** X is 4 times as fast as Y and is able to complete the work in 90 days less than Y. Find the time in which they can complete the work working together.

Sol: As X is 4 times as fast as Y, if Y does the work in 4 days, X does it in 1 day, i.e. the difference is 3 days.
If the difference is 3 days, X takes 1 day and Y takes 4 days. If the difference is 90 days (i.e., 30 times), X takes 30 days and Y takes 120 days. Therefore, time taken by them working together to complete the work
$= \frac{1}{30} + \frac{1}{120} = \frac{5}{120} = \frac{1}{24}$, i.e., 24 days

**6.16.** Savita, can finish a work in 48 days working 9 hours a day. Rani can finish the same work in 36 days working 16 hours a day. In how many days can they finish it working together for $7^5/_7$ hours a day?

**Sol:** Number of hours taken by Savita
$= 48 \times 9 = 432$.
Number of hours taken by Rani
$= 36 \times 16 = 576$.
Together they will do, $\frac{432 + 576}{432 \times 576}$
$= \frac{7}{1728}$th work in 1 hour.
Hence, they will take (1728/7) hours, i.e., working 1 hour a day, (1728/7) days. Therefore, working $7^5/_7$ hours a day they can complete the work in
$= \frac{1728}{7} \times \frac{7}{54} = 32$ days.

**Alternate Method:**
At 9 hrs/day, Savita takes 48 days. If S is the work/hour of Savita, then total work
$= 48 \times 9 \times S$ (1)
Similarly, total work $= 36 \times 16 \times R$ (2)
where R is the work/hour of Rani.
Equating (1) and (2), the relation is
$48 \times 9 \times S = 36 \times 16 \times R$;
$\Rightarrow 3S = 4R$ (3)
When Savita and Rani work together, work done per hour = (S + R).
They work of $7\frac{5}{7}$ hrs/day.
If they take $d$ days for the work, total work
$= d \times (54 / 7) \times (S + R)$ (3)
Equating (1) and (3), the relation is
$48 \times 9 \times S = d \times (54 / 7) \times (S + R)$
Multiplying with 4, and then replacing 4R by 3S, the relation is:
$4 \times 48 \times 9 \times S = d \times (54/7) \times (4S + 4R)$
$\Rightarrow 7 \times 4 \times 48 \times 9 \times S = d \times 54 \times 7S$;
$\Rightarrow 32 = d$.
Hence, they take 32 days.

**6.17.** A alone can do a work in 36 days and B alone in 54 days. If C takes five times as long as A and B together, how long will B and C together take to complete the same work?

**Sol:** As A can do the work in 36 days and B in 54 days, when they work together, they take $\frac{36 \times 54}{36 + 54} = \frac{108}{5}$ days.

Given, C takes 5 times the time A and B together take; i.e., C takes 5 × (108/5) = 108 days to do the work.
If B and C work together, they can complete the work in $\frac{54\times108}{54+108}=36$ days.

**6.18.** A and B each working alone can do a work in 15 days and 25 days, respectively. They started the work together but B left after sometime and A finished the remaining work in 7 days. After how many days from the start did B leave?

**Sol:** A's work for 7 days = $7\times1/15 = 7/15^{th}$ part of the work. The remaining $8/15^{th}$ part of the work was done by A and B together.
Work done by A and B in a day

$$=\frac{1}{15}+\frac{1}{25}=\frac{8}{75}.$$

Number of days they worked together

$=\frac{8/15}{8/75}=5$ days. Hence, B left after 5 days from the start of work.

**6.19.** A contractor decided to complete a work in 60 days and employed 60 men at the beginning and 30 men additionally after 20 days and got the work completed as per schedule. If he had not employed the additional men, how many extra days would he have needed to complete the work?

**Sol:** Total mandays required for the work
= 60 × 20 + 90 × 40 = 4800
To complete 4800 mandays work, 60 men will take 4800/60 = 80 days.
∴ Extra days needed = 80 – 60 = 20

**6.20.** A group of 45 men is employed to complete some work in 58 days. After 40 days, an additional 15 men are employed and the work is finished three days earlier. If the additional men had not been employed, how many more days would it have taken beyond the expected period?

**Sol:** 60 men complete the remaining work in 58 – 40 – 3 = 15 days.
45 men will do it in 60 × 15/45 = 20 days. Extra time taken = 20 – 18 = 2 days

**6.21.** A and B working separately can do a piece of work in 3 days and 6 days, respectively. They work on alternate days starting with B on the first day. In how many days will the work be completed?

**Sol:** Since they are working on alternate days, let us consider a time period of two days in which B does one day's work and A does the next day's work.
In a period of two days, work done by B and A = $\frac{1}{6}+\frac{1}{3}=\frac{1}{2}$.

Since they complete 1/2 of the work in a period of two days, they need one more period of two days to complete the work remaining, i.e., they take 2 × 2 = 4 days.

**6.22.** A and B working separately can do a piece of work in 12 days and 20 days, respectively. They work on alternate days starting with A on the first day. In how many days will the work be done?

**Sol:** Since they are working on alternate days with A starting the work, let us consider a period of two days. In a period of two days, work done by A and B

$$=\frac{1}{12}+\frac{1}{20}=\frac{5+3}{60}=\frac{2}{15}$$

If we consider 7 such time periods of 2 days (we are considering 7 periods because in the fraction 2/15 the numerator 2 goes 7 times in the denominator 15).
Work done = 7 × (2/15) = (14/15)
Remaining work = 1 – (14/15) = 1/15.
Now it is A's turn since 7 whole number of periods are over.
Time taken by A to finish $1/15^{th}$ part of the work = (1/15) / (1/12) = 4/5 days.
So, total time taken to complete the work is $14^{4}/_{5}$ days.

**6.23.** A and B working separately complete a work in 8 days and 12 days, respectively. They work on alternate days starting with A on the first day. In how many days will the work be completed?

**Sol:** In a period of 2 days, work done by A and B

$$=\frac{1}{8}+\frac{1}{12}=\frac{5}{24}.$$

In 4 such time periods of 2 days, i.e., 8 days. (we are considering 4 periods because in the fraction 5/24 the numerator 5 goes 4 times in the denominator 24), work done is $20/24^{th}$.

Remaining work = $1-\frac{20}{24}=\frac{4}{24}=\frac{1}{6}$.

Now it is A's turn, since 4 whole number of periods are over.
But A can do only $1/8^{th}$ part of the work in a day.
So, the remaining work after A works for 1 day

$$= \frac{1}{6} - \frac{1}{8} = \frac{1}{24}.$$

Now it is B's turn to do $1/24^{th}$ part of the work; and he takes $(1/24) / (1/12) = (1/2)$ a day.
Total number of days taken
$= (4 \times 2) + 1 + 1/2 = 9^{1}/_{2}$ days.
When people work together and earn money for the work, the money has to be shared by all the people doing the work.

**6.24.** A, B and C can do a piece of work in 6 days, 4 days and 3 days, respectively. They completed a work together and received ₹540 for the job. What is C's share?

**Sol:** Since they work for the same number of days, the ratio in which they share the money is the ratio of work done per day.
Ratio of work done per day by A, B and C

$$= \frac{1}{6} : \frac{1}{4} : \frac{1}{3} = 2 : 3 : 4.$$

Hence, C's share = (4/9) (540) = ₹240.

**6.25.** A, B and C contract a work for ₹7,500. A and B together complete $2/5^{th}$ of the work and then C takes over and finishes the work. What is the amount received by C?

**Sol:** Work done by C alone = 1 – (2/5) = 3/5.
Therefore, C's share = (3/5) (7500)
= ₹4,500.

**6.26.** Wages for 80 women for 60 days are ₹43,200. How many men must work for 40 days to earn ₹57,600, if the daily wages for a man is four times that of a woman?

**Sol:** Daily Wages of a woman $= \frac{43,200}{80 \times 60} = ₹9.$

Daily Wages of a man = 4 × 9 = ₹36.

Therefore, number of men $= \frac{57,600}{36 \times 40} = 40$ .

**6.27.** A, B and C together can earn ₹3,240 in 18 days. A and C can earn ₹1,200 in 10 days whereas B and C can earn ₹1,820 in 14 days. Find the daily earnings of C.

**Sol:** Daily Wages of (A + B + C)
= 3240/18 = 180 (1)
Daily Wages of (A + C)
= 1200/10 = 120 (2)
Daily Wages of (B + C)
= 1820/14 = 130 (3)
(2) + (3) – (1), leads to:
(A + C) + (B + C) – (A + B + C)
= 120 + 130 – 180 ⇒ C = 70

**6.28.** Two men undertake a work for ₹480, they can do the work individually in 24 days and 40 days. If they complete the work in 10 days with the help of a boy, how should they divide the money?

**Sol:** The $1^{st}$ man can do $1/24^{th}$ part of the work in 1 day and since he works for 10 days, he does 10 × 1/24 = $5/12^{th}$ part of the work. So, he should get 5/12 × 480 = ₹200
The $2^{nd}$ man can do $1/40^{th}$ part of the work in 1 day and since he works for 10 days, he does 10 × (1/40) = $1/4^{th}$ work. So, he should get 1/4 × 480 = ₹120.
Since the two men earned ₹200 + ₹120 = ₹320, the balance of ₹480 – ₹320 = ₹160 will be paid to the boy.

**6.29.** Two pipes A and B separately can fill an empty tank in 30 minutes and 45 minutes, respectively. If both the pipes are opened simultaneously, how long will it take to fill the empty tank?

**Sol:** Part of the tank filled by A in 1 minute = $1/30^{th}$.
Part of the tank filled by B in 1 minute = $1/45^{th}$.
Part of the tank filled by both A and B in one minute

$$= \frac{1}{30} + \frac{1}{45} = \frac{1}{18}.$$

The tank can be filled in 18 minutes.

**6.30.** Pipe A can fill an empty tank in 24 minutes, pipe B in 36 minutes and pipe C can empty a full tank in 72 minutes. If all of them work together, find the time taken to fill the empty tank.

**Sol:** Work done by the pipes A, B and C together in 1 minute $= \frac{1}{24} + \frac{1}{36} - \frac{1}{72} = \frac{1}{18}.$

So, the empty tank can be filled in 18 minutes.

**6.31.** Two pipes can fill an empty tank in 40 minutes and 60 minutes, respectively. There is an outlet pipe C. If all the 3 pipes are opened simultaneously, the empty tank can be filled in 48 minutes. How much time will it take for C alone to empty the full tank?

**Sol:** Part of the tank emptied by C in one minute $= \frac{1}{40} + \frac{1}{60} - \frac{1}{48} = \frac{1}{48}$.

$\therefore$ C can empty the full tank in 48 minutes.

**6.32.** Two pipes A and B fill an empty tank in 40 minutes and 60 minutes, respectively. If both pipes are opened simultaneously, after how much time should A be closed so that the tank is filled in 36 minutes?

**Sol:** Pipe B works for 36 minutes. In 1 min, B fills $1/60^{th}$ part of the tank, i.e., in 36 minutes, it fills $36(1/60) = 3/5$. The remaining $2/5^{th}$ is filed by A. Since A fills the tank fully in 40 minutes, it takes $(2/5) \times 40 = 16$ minutes to fill $2/5^{th}$ part of the tank. So, A should be closed after 16 minutes.

**6.33.** Three taps A, B and C together can fill an empty tank in 4 hours. After 1 hour, C is closed and the tank is filled in 6 more hours. Find the time in which C alone can fill the empty tank.

**Sol:** Work done by A, B and C in 1 hour = $1/4^{th}$.
Remaining part of the tank = $1 - (1/4) = 3/4$
Time taken by (A and B) to fill this $3/4^{th}$ part of the tank = 6 hours.
$\Rightarrow$ A and B together fill the tank in $6 / (3/4) = 8$ hours. Now, we know A, B and C take 4 hours and A and B take 8 hours.
Part of the tank filled by C in 1 hour

$= \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$.

So, C alone can fill the tank in 8 hours.

**6.34.** If 15 men take 60 days to complete a job, find the time taken by 45 men to complete it.

**Sol:** Number of Mandays required to complete the job = 900. Time taken by 45 men to complete it

$= \frac{900}{45}$, i.e., 20 days

**6.35.** 18 men take 20 days to complete a job working 12 hours a day. Find the number of days that 15 men will take to complete it if they work 9 hours a day.

**Sol:** Total time for which 18 men work = 240 hours.
Number of man hours required to complete the job = (18) (240) man hours.
Number of days taken by 15 men working 9 hours a day to complete it

$= \frac{(18)(240)}{(15)(9)} = 32$

**6.36.** 20 men take 10 days to complete a job working 12 hours a day. Find the number of men required to complete a job, twice as large, in 30 days working 8 hours a day.

**Sol:** Number of man hours required to complete the job = (20) (10) (12) = 2400
Number of men required to complete a job twice as large = $14\frac{2}{5}$ days

**Alternative method:**
$M_1 = 20, D_1 = 10, H_1 = 12$
$D_2 = 30, H_2 = 8$
$W_2 = 2W_1$

$$M_2 = \frac{M_1 D_1 H_1 W_2}{W_1 D_2 H_2} = \frac{(20)(10)(12)(2W_1)}{W_1(30)(8)} = 20.$$

**6.37.** A and B can complete a job in 10 days and 12 days, respectively. Find the time taken to complete it, if both A and B work together.

**Sol:** Time taken by them to complete it

$= \frac{(10)(12)}{10+12} = \frac{60}{11}$ days

**6.38.** A and B together can complete a job in 12 days. A alone can complete it in 24 days. Find the time taken by B to complete it.

**Sol:** Part of the job that A and B can complete in a day $= \frac{₹1320}{6}$

Part of the job that A can complete in a day $= \frac{1}{24}$

Part of the job that B can complete in a day

$= 0\ \frac{1}{12} - \frac{1}{24} = \frac{1}{24}$

$\therefore$ B can complete it in 24 days.

**6.39.** Ajay and Bala working together can complete a job in 16 days. Ajay alone can complete it in 18 days. Both work together for 4 days and then Bala leaves. Find the time taken by Ajay to complete the remaining work.

**Sol:** Part of the job that can be done by both in a day

$= \frac{1}{16}$

Part of the job that can be done by them in 4 days

$= 4 \times \frac{1}{10} + \frac{1}{20} - \frac{1}{40} = \frac{1}{8} = \frac{1}{4}$

Remaining part of the job = $\frac{3}{4}$

Time taken by Ajay to complete it

$= \frac{xy}{x+y} = 12 = 13.5$ days

**6.40.** A and B together can complete a job in 12 days. B and C together can complete it in 20 days. A and C together can complete it in 10 days. Find the times taken by each of A, B and C to complete it.

**Sol:** Suppose that A, B and C take a days, $b$ days and $c$ days, respectively to complete the job.

Given, $\frac{1}{a}+\frac{1}{b}=\frac{1}{12}$ (1)

$\frac{1}{b}+\frac{1}{c}=\frac{1}{20}$ (2)

$Z = \frac{1}{5}$ (3)

Adding (1), (2) and (3),

$2\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=\frac{14}{60}$

$\frac{8000}{5}$ (4)

$\therefore$ By (4) – (1), $\frac{1}{c}=\frac{1}{30}$

$c = 30$

By (4) – (2), $\frac{1}{a}=\frac{1}{15}$

$a = 15$

By (4) – (3), $\frac{1}{b}=\frac{1}{60}$

$b = 60$

**6.41.** P and Q together can complete a job in 12 days. Q and R together can complete it in 15 days. P and R together can complete it in 18 days. All the three work together for 3 days and then R leaves. In how many days can P and Q complete the remaining work?

**Sol:** Part of the job that can be done by P, Q and R in a day $= \frac{1}{4}:\frac{1}{5}:\frac{1}{6} = 15:12:10$

Part of the job completed in 3 days

$= 3\left[\frac{1}{2}\left(\frac{1}{12}+\frac{1}{15}+\frac{1}{18}\right)\right]=\frac{37}{120}$

Remaining part of the job = $\frac{83}{120}$

This can be completed by P and Q in

$\frac{15}{37}(3700) = 8 \times 3$ days

**6.42.** A can complete a job in 16 days. He started the work and after 4 days, B joined him. They completed the job in 4 more days. Find the number of days in which B alone can complete it.

**Sol:** Part of the job done by A in a day = $\frac{1}{16}$

A worked for a total of 8 days

$\therefore$ A completed $(8)\left(\frac{1}{16}\right)=\frac{1}{2}$ of the job.

Hence, B can complete the remaining $\frac{1}{2}$ of the job in 4 days.

$\therefore$ B alone can complete the entire job in 8 days.

**6.43.** P and Q together can complete a job in $14\frac{2}{5}$ days Q and R together can complete it in $20\frac{4}{7}$ days. P and R together can complete it in 16 days. Find the time taken by each of them to complete the job.

**Sol:** Part of the job that P and Q can do in a day = $\frac{5}{72}$

Part of the job that Q and R can do in a day $= \frac{7}{144}$

Part of the job that P and R can do in a day = $\frac{1}{16}$

Let the times taken by P, Q and R to complete the job be $p$ days, $q$ days and $r$ days, respectively.

$\frac{1}{p}+\frac{1}{q}=\frac{5}{72}$ (1)

$\frac{1}{q}+\frac{1}{r}=\frac{7}{144}$ (2)

$\frac{1}{p}+\frac{1}{r}=\frac{1}{16}$ (3)

Adding (1) and (2) and subtracting (3),

$$\Rightarrow \frac{2}{q} = \frac{5}{72} + \frac{7}{144} - \frac{1}{16} = \frac{8}{144}$$

$q = 36$

Substituting $q = 36$, in (1), we get $p = 24$

Substituting $q = 36$ in (2), we get $r = 48$.

**6.44.** To complete a job, P takes half as long as Q and R together take. Q takes 8 times as long as P and R together take. All the three together can complete the job in $\frac{20}{3}$ days. Find the time taken by each of P, Q and R to complete it.

**Sol:** Let the times taken by P, Q and R be $p$ days, $q$ days and $r$ days, respectively.

Given $\frac{1}{p} = 2\left(\frac{1}{q} + \frac{1}{r}\right)$

$$\Rightarrow \frac{3}{p} = 2\left(\frac{1}{p} + \frac{1}{q} + \frac{1}{r}\right) \quad (1)$$

Given $\frac{1}{q} = \frac{1}{8}\left(\frac{1}{p} + \frac{1}{r}\right)$

$$\Rightarrow \frac{8}{q} = \frac{1}{p} + \frac{1}{r}$$

$$\Rightarrow \frac{9}{q} = \frac{1}{p} + \frac{1}{q} + \frac{1}{r} \quad (2)$$

Given $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} = \frac{3}{20}$ (3)

$\therefore$ From (1) and (3), $\frac{3}{p} = \frac{6}{20}$

$p = 10$

From (2) and (3), $\frac{9}{q} = \frac{3}{20}$

$q = 60$

$$\frac{1}{r} = \frac{3}{20} - \left(\frac{1}{10} + \frac{1}{60}\right)$$

$r = 30$

**6.45.** 10 boys or 20 girls can complete a job in 10 days. Find the time taken by 10 boys and 20 girls to complete it.

**Sol:** Part of the job that can be done by a boy in a day $= \frac{1}{100}$.

Part of the job that can be done by a girl in a day $= \frac{1}{200}$.

Part of the job that can be done by 10 boys and 20 girls $= 10\left(\frac{1}{100}\right) + 20\left(\frac{1}{200}\right) = \frac{1}{5}$

$\therefore$ They can complete the job in 5 days.

**Alternative Method:**

10 boys are as efficient as 20 girls.

$\therefore$ 10 boys and 20 girls are as effective as 20 boys.

$\therefore$ They will take $\frac{10}{2} = 5$ days to complete the job.

**6.46.** If there are 4 more men in a work force, they will take 8 days less to complete a job. Instead if there are 8 men more in it, they will take 12 days less to complete it. Find the ratio of the number of men and the time they take (in days) to complete it.

**Sol:** Let the number of men be $x$ and the number of days they take to complete the job be $y$.

$(x + 4)(y - 8) = xy \Rightarrow 4y - 8x = 32$ (1)

$(x + 8)(y - 12) = xy \Rightarrow 8y - 12x = 96$ (2)

solving (1) and (2), we get

$x = 8$ and $y = 24$

$\therefore x : y = 8 : 24 = 1 : 3$

**6.47.** 3 men and 4 women can complete a job in 10 days. 24 men and 2 women can complete it in 2 days. Find the time taken by 11 men and 3 women to complete it.

**Sol:** Let the parts of the job done by a man and a woman each day be $m$ units and $w$ units, respectively.

Given, $3m + 4w = \frac{1}{10}$ (1)

$24m + 2w = \frac{1}{2}$ (2)

Solving for $m$ and $w$,

$m = \frac{1}{50}$ and $w = \frac{1}{100}$

$11m + 3w = \frac{1}{4}$

$\therefore$ Time taken by 11 men and 3 women to complete the job is 4 days.

**6.48.** X is twice as fast as Y and hence takes 30 days less than Y to complete a job. Find the time taken by X and Y together to complete it.

**Sol:** Let the time taken by X to complete the job be $n$ days.

Time taken by Y to complete the job = $2n$ days

$n = 2n - 30 \Rightarrow n = 30$

Time taken by X and Y together to complete it

$= \dfrac{(n)\,(2n)}{n+2n} = 20$ days

**6.49.** Sita can complete a job in 6 days working 8 hours a day. Gita can complete it in 3 days working 12 hours a day. In how many days can they together complete it working 4 hours a day?

**Sol:** Numbers of hours taken by Sita and Gita to complete the job are 48 hours and 36 hours, respectively.

They together can complete $\dfrac{1}{48}+\dfrac{1}{36}=\dfrac{7}{144}$th of the job in an hour.

$\therefore$ They can complete the job in $\dfrac{144}{7}$ hours.

$\therefore$ They can complete the job in $\dfrac{\frac{144}{7}}{4}=\dfrac{36}{7}$ days.

**6.50.** P and Q can complete a job in 20 days and 30 days, respectively. P started it and after 10 days, Q joined. In how many days will they complete the remaining work?

**Sol:** Part of the job completed by P and Q in a day

$= \dfrac{1}{20}+\dfrac{1}{30}=\dfrac{1}{12}$.

Suppose that the remaining work is completed in $x$ days.

Then, $x\left(\dfrac{1}{12}\right)+10\left(\dfrac{1}{20}\right)=1$

$x = 6$

**6.51.** A contractor decided to complete a job in 30 days for which he employed 20 men in the beginning. After 10 days he released that the job could not be completed on time. Hence, he employed 15 more men and thus completed the job on time. Find the number of extra days it would have taken to complete the job if the additional men were not employed.

**Sol:** Number of mandays required to complete the job

= (20) (10) + (20 + 15) (20) = 900 man days.

If the additional men were not employed, number of extra days = $\dfrac{900}{20}-30=15$ days.

**6.52.** A and B can complete a job in 18 days and 36 days, respectively. They work on alternate days with A starting the job. In how many days will the job be completed?

**Sol:** Part of the job completed in the first 2 days

$= \dfrac{1}{18}+\dfrac{1}{36}=\dfrac{1}{12}$

$\therefore$ To complete the job, 12 cycles of 2 days, i.e., a total of 24 days will be required.

**6.53.** P and Q together can complete a job in 8 days and 16 days, respectively. They work on alternate days with Q starting the job. In how many days will the job be completed?

**Sol:** Part of the job completed by P and Q in the first 2 days = $\dfrac{1}{8}+\dfrac{1}{16}=\dfrac{3}{16}$

After 5 cycles of 2 days, i.e., after 10 days, $\dfrac{15}{16}$th of the job will be completed.

Remaining part = $\dfrac{1}{16}$th. Q will work on the $11^{th}$ day and he takes exactly one day to complete the remaining part.

$\therefore$ The job will be completed in 11 days.

**6.54.** A, B and C can complete a job in 4 days, 5 days and 6 days, respectively. They work together and complete it. If their total wage is ₹3700, find A's wage.

**Sol:** Ratio of the wages of A, B and C = Ratio of the work done by A, B and C = Ratio of the daily work done by A, B and C

$= \dfrac{1}{4}:\dfrac{1}{5}:\dfrac{1}{6}=15:12:10$

$\therefore$A's wage = $\dfrac{15}{37}(3700)=₹1500$

**6.55.** X, Y and Z take a job on contract for ₹8000. X and Y started the job and completed $\dfrac{4}{5}$th of the job. Z then took over and completed the remaining work. Find Z's share.

**Sol:** Part of the job completed by $Z = \frac{1}{5}$

$\therefore$ Z's share $= \frac{8000}{5} = ₹1600$

**6.56.** P, Q and R can together earn ₹3100 in 10 days. Q and R together can earn ₹1320 in 6 days. P and R together can earn ₹1050 in 5 days. Find R's daily earning.

**Sol:** Total daily wage of P, Q and R $= \frac{₹3100}{10} = ₹310$

Total daily wage of Q and R $= \frac{₹1320}{6} = ₹220$

Total daily wage of P and R $= \frac{₹1050}{5} = ₹210$

Total daily wage of P, Q and 2R = ₹430

$\therefore$ R's daily wage = ₹120

**6.57.** Two men undertake a job for ₹960. They can complete it in 16 days and 24 days. They work along with a third man and take 8 days to complete it. Find the share that the third man should get.

**Sol:** The amount payable should be proportional to the fraction of work done.

Part of the job done by the third man

$= 1 - \left(\frac{8}{16} + \frac{8}{24}\right) = \frac{1}{6}$.

$\therefore$ The third man should get $\frac{₹960}{6} = ₹160$

**6.58.** Pipes P and Q can fill a tank in 20 minutes and 30 minutes, respectively. If they are opened simultaneously, in how much time can they fill it?

**Sol:** If a pipe takes $x$ hours to fill a tank and another takes $y$ hours to fill it, they can fill it in $\frac{xy}{x+y}$ hours.

In this problem, $x = 20$ and $y = 30$

$\therefore \frac{xy}{x+y} = 12$

**6.59.** Pipes X and Y take 10 minutes and 20 minutes, respectively to fill an empty tank. Pipe Z takes 40 minutes to empty a full tank. Find the time taken to fill the empty tank if all the three pipes are opened simultaneously.

**Sol:** Part of the tank filled per minute when all the three pipes are opened $= \frac{1}{10} + \frac{1}{20} - \frac{1}{40} = \frac{1}{8}$

$\therefore$ They would take 8 minutes to fill the tank.

**6.60.** Pipes P and Q take 24 minutes and 36 minutes, respectively to fill an empty tank. If both take 18 minutes to fill the tank along with an outlet pipe R, find the time R would take to empty the full tank.

**Sol:** Let the time taken by R to empty the tank be $r$ minutes.

$\frac{1}{24} + \frac{1}{36} - \frac{1}{r} = \frac{1}{18}$

$r = 72$

**6.61.** Pipes X and Y can fill a tank in 30 minutes and 60 minutes, respectively. Both pipes are opened simultaneously. After how much time should X be closed so that the tank is filled in 30 minutes?

**Sol:** Let us say pipe X should be closed after $n$ minutes.

i.e., pipe X is in operation for $n$ minutes and pipe Y for all the 30 minutes.

So, $\frac{n}{30} + \frac{30}{60} = 1 \Rightarrow n = 15$

**6.62.** Pipes P, Q and R together can empty a full tank in 6 hours. All the three pipes are opened simultaneously and after 2 hours, P is closed. The tank is emptied in another 6 hours. Find the time in which P can empty the tank.

**Sol:** Part of the tank that can be emptied by P, Q and R per hour $= \frac{1}{6}$.

Part of the tank that was emptied by P, Q and R in 2 hours $= \frac{1}{3}$

Part of the tank which was emptied by Q and R per hour $= \frac{1 - \frac{1}{3}}{6} = \frac{1}{9}$

Time in which P can empty the tank

$= \frac{1}{\frac{1}{6} - \frac{1}{9}}$, i.e., 18 hours.

**6.63.** A tank has a leak at its bottom which empties it at 6 litres/minute. It also has a filling tap which can fill the tank in 6 hours. The tank takes 18 hours to become full. Find the capacity of the tank.

**Sol:** Let the time that would be taken by the leak to empty the full tank be $x$ hours.

$\therefore \frac{1}{6} - \frac{1}{x} = \frac{1}{18}$

$x = 9$

$\therefore$ Capacity of the tank = (6) (9) (60)
= 3240 litres.

## EXERCISE-1

***Directions for questions 1 to 50*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** If 30 men can do a job in 30 days, find the part of the job that can be completed by 1 man in 1 day.

(A) $\frac{1}{30}$ (B) $\frac{1}{90}$

(C) $\frac{1}{900}$ (D) None of these

**2.** A can do a work in 9 days and B can do the same work in 18 days. If they work together, in how many days will they complete the work? _____

**3.** A can do a work in 21 days and B in 28 days. Together they started the work and B left after 4 days. In how many days can A alone do the remaining work?

(A) 12 (B) 10 (C) 16 (D) 14

**4.** A, B and C can do a work in 7, 14 and 21 days, respectively. They completed the work together and got ₹ 220. The share of C is (₹) _____.

**5.** If the ratio of the individual times taken by P, Q and R to complete a job is 2 : 3 : 6, find the ratio of the parts of the jobs done by P, Q and R in an hour.

(A) 2 : 3 : 6 (B) 6 : 3 : 2
(C) 3 : 2 : 1 (D) 1 : 2 : 3

**6.** A, B and C together can do a work in 22 days. B and C together can do $\frac{3}{4}$ of the work in 30 days. In how many days can A alone do the work?

(A) $47\frac{8}{9}$ (B) $48\frac{7}{9}$

(C) $48\frac{8}{9}$ (D) $47\frac{7}{9}$

**7.** X and Y can complete a job in 15 days working together. Y and Z can complete the job in 20 days working together. X and Z can complete the job in 30 days working together. Who is the slowest of the 3 workers?

(A) X (B) Y
(C) Z (D) Cannot be determined

**8.** A, B and C can do a piece of work in 18 days, 24 days and 36 days, respectively. In how many days can they complete the work working together?

(A) 10 (B) 8 (C) 6 (D) 12

**9.** Tap A takes 4 hours to fill a tank. Tap B takes 6 hours to fill the tank. If both the taps were opened simultaneously, by the time the tank was full, what fraction of the tank was filled by tap A?

(A) 3.2 hours, $\frac{2}{5}$ (B) 3.2 hours, $\frac{3}{5}$

(C) 2.4 hours, $\frac{2}{5}$ (D) 2.4 hours, $\frac{3}{5}$

**10.** A and B can do a piece of work in 12 days and 18 days, respectively. They worked together and earned ₹ 300. What is the share of A in this amount?

(A) ₹ 120 (B) ₹ 180 (C) ₹ 150 (D) ₹ 100

**11.** If a man completes a job in 6 days working 6 hours a day, in how many days can he complete the job working 9 hours a day? _____

**12.** A is 30% more efficient than B. If B can do a piece of work in 26 days then in how many days can A alone do the work?

(A) 20 (B) 24 (C) 22 (D) 26

**13.** P can complete a job in 30 days. Q can complete the job in 60 days. If they work on alternate days, the job would be completed in the

(A) least possible time, if P starts the job.
(B) least possible time, if Q starts the job.
(C) same time, irrespective of who starts the job.

**15.** A can do $\frac{3}{4}$ th of a work in 12 days and B can do $\frac{5}{8}$ th of it in 20 days. In how many days can A and B together complete the work?

(A) $11\frac{1}{3}$ (B) $10\frac{1}{3}$ (C) $11\frac{2}{3}$ (D) $10\frac{2}{3}$

**16.** P and Q completed a job working together in $x$ days. P takes $(x + 8)$ days to complete the job, Q takes $(x + 18)$ days to complete the job. Find $x$. _____

**17.** A, B and C together can do a work in 20 days. If A and C together can do the work in 40 and 60 days respectively, B alone can do the work in _____ days.

**18.** Three men and seven boys can do a piece of work in 45 days. In how many days can 9 men and 21 boys do the work?

(A) 15 (B) 12
(C) 18 (D) Cannot be determined

**19.** If the ratio of the work done by $(x + 1)$ men in $(x - 2)$ days to the work done by $(x + 2)$ men in $(x - 4)$ days is 9 : 8, then the value of $x$ is

(A) 12 (B) 14 (C) 10 (D) 16

**20.** 9 men can complete a job in 20 days. If a woman works at one-third the rate of a man, in how many days can 15 women complete the job? _____

**21.** Kavitha can do a piece of work in K days. What part of the work can she complete in X days?

(A) $K/X$ (B) $XK$ (C) $1/XK$ (D) $X/K$

**22.** A can complete a job in 2 days. B can complete the job in 4 days. If they work on alternate days, the job would be completed in the

(A) least possible time, if A starts the job.
(B) least possible time, if B starts the job.
(C) same time, irrespective of who starts the job

**23.** 215 men can construct a 677 m long wall in 13×5 days. How many men are required to construct a wall twice the length of the previous wall in half the time? _____

**24.** Work done by $(k - 1)$ men in $(k - 8)$ days is equal to the work done by $(k + 4)$ men in $(k - 11)$ days. What is the value of $k$?

(A) 22 (B) 26 (C) 24 (D) 28

**25.** 2 men can complete a job in 8 days working 8 hours a day.

(i) How many men are required to complete the job in 4 days working 4 hours a day? _____

(ii) How many men are required to complete twice the job working 4 hours a day for 4 days? _____

**26.** Two pipes can fill a tank in 20 minutes and 26 minutes. An outlet pipe can empty the tank in 13 minutes. If all the pipes are opened simultaneously, in how many minutes will the tank become full?

(A) $87\frac{2}{3}$ (B) $86\frac{1}{3}$ (C) $87\frac{1}{3}$ (D) $86\frac{2}{3}$

**27.** A, B and C can do a work in 15, 30 and 45 days respectively. C started the work. A and B joined after one day. In how many days was the work completed?

(A) 6 (B) 8 (C) 7 (D) 9

**28.** P and Q can complete a job in 12 days working together. Q and R can complete the job in 20 days working together. P and R can complete the job in 15 days working together. In how many days can P, Q and R complete the job working together?

(A) 5 (B) 10 (C) 15 (D) 20

**29.** A is twice as fast as B. If B alone can do the work in 30 days, A and B together can complete the work in _____ days.

**30.** P, Q and R take 20 days, 30 days and 40 days respectively to complete a job. The three work on a rotation basis to complete the job with only 1 person working on a day. Who should start the job so that the job is completed in the least possible time?

(A) P (B) Q
(C) R (D) Any one of the three

**31.** 6 men can do a piece of work in 10 days. If a woman works half as fast as a man, in how many days can 10 women complete the work?

(A) 24 (B) 6 (C) 18 (D) 12

**32.** If A is thrice as fast as B. Together they can do a work in 27 days. A alone can do the work in _____ days.

**33.** Varma can read a book in $k$ minutes. What part of the book can he read in 8 minutes? $(k > 8)$

(A) $8 + k$ (B) $\frac{8}{k}$ (C) $\frac{k}{8}$ (D) $\frac{k-8}{k}$

**34.** Two pipes can fill a tank in 30 minutes and 20 minutes respectively. If they are opened simultaneously, in what time will the tank become full?

(A) 15 minutes (B) 12 minutes
(C) 18 minutes (D) 9 minutes

**35.** P and Q working together can complete a piece of work in 24 days. The ratio of their efficiencies is 2 : 3. In how many days can the faster person complete the work working alone?

(A) 40 (B) 50 (C) 60 (D) 90

**36.** Vinay and Varma can do a work in 30 days and 60 days respectively. If they work on alternate days beginning with Vinay in how many days will the work be completed?

_____

**37.** The ratio of the efficiencies of P, Q and R is 4 : 5 : 6. If they completed a job working together, find the fraction of the job done by P.

(A) $\frac{1}{5}$ (B) $\frac{1}{3}$ (C) $\frac{4}{15}$ (D) $\frac{7}{15}$

**38.** Pipe A can fill a tank in 6 hours. Due to a leak at the bottom, it takes 9 hours to fill the tank. In what time the leak alone can empty the tank?

(A) 16 hours (B) 15 hours
(C) 18 hours (D) 17 hours

**39.** Twenty four men can do a work in 35 days. How many men are required to complete the work in 21 days?

_____

**40.** A and B can do a work in $7\frac{1}{2}$ days and 30 days respectively. In how many days can they complete the work if they work together?

(A) 5 (B) 4 (C) 6 (D) 3

**41.** If A is 20% more efficient than B, then what is the ratio of the work completed by A and B in one hour?

(A) 5 : 6 (B) 6 : 5
(C) 12 : 1 (D) Cannot be determined

**42.** P is twice as fast as Q and R together. Working together, all three can do a work in 21 days. In how many days can Q and R together do the work?

(A) 48 (B) 63 (C) 54 (D) 72

**43.** P and Q working together can complete a piece of work in 6 days. If P alone can complete the work in 15 days, then in how many days can Q alone complete the work?

(A) 10 (B) 12 (C) 20 (D) 18

**44.** An inlet pipe takes 8 hours to fill a tank. An outlet pipe takes 12 hours to empty it. If both pipes are opened simultaneously, in how many hours will the tank be filled?

_____

**45.** $x$ men can do a work in 120 days. If there were 20 men less, the work would have taken 60 days more. The value of $x$ is _____.

**46.** 4 men can complete a piece of work in 4 days working 4 hours a day. In how many hours can eight men complete the same work working 8 hours a day?

(A) 1/2 (B) 2 (C) 4 (D) 1

**47.** A and B working together can complete a job in 30 days. The ratio of their efficiencies is 3 : 2. In how many days can the faster person complete the job? _____

**48.** If one man can eat one fruit in one day, then how many fruits can eight men eat in eight days?

(A) 8 (B) 64
(C) 16 (D) None of these

**49.** If 12 men or 20 women can do a work in 54 days, then 9 men and 12 women can together do the work in _____ days.

**50.** For how many days will 75 kg of ration be sufficient for a family of 6 members if each member consumes 1.25 kg of ration per day? _____

## EXERCISE-2

***Directions for questions 1 to 50***: For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** A, working alone, takes 80 days more than the time taken by A and B working together to complete the work. B, working alone, takes 20 days more than the time taken by A and B working together to complete the work. Find the time taken by A alone to complete the work. (in days)

_____

**2.** A can complete a piece of work in 12 days which B and C can complete in 16 and 20 days respectively. All the three of them started the work, but A left after 4 days, B left 2 days before the completion of the work. In how many days was the work completed?

(A) $7^1/_{27}$ (B) $8^1/_{14}$ (C) $9^1/_{16}$ (D) $10^1/_{27}$

**3.** P, Q and R are 3 small pumps fitted to a tank. S is a large pump fitted to the tank. Q is 50% more efficient than P. R is $33^1/_3$% more efficient than Q. S is 50% more efficient than R. Find the ratio of the time taken by the four pumps working together to fill the tank and the time taken by the large pump to fill it.

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{3}{10}$ (D) $\frac{7}{20}$

Difficulty Level-2: Moderate

**4.** A pipe which can fill a tank in 12 hours, is opened at 6:00 a.m. A leak which can empty the tank in 24 hours was observed at 10:00 a.m. and was immediately stopped. The tank would be full at _____ :_____ p.m.

**5.** A man, a woman and a boy can do a piece of work in 3, 4 and 12 days respectively. How many boys must assist one man and a one woman to complete the work in one and a half days?
(A) 12 (B) 6 (C) 1 (D) 3

**6.** A takes 1 day to complete a job. B takes twice the time as A to complete the job. C takes twice the time as B to complete the job. D takes twice the time as C to complete the job. Two of the four when working together take atmost 40% of the time to complete the job that the other two would take working together. Find the faster pair.
(A) A, D (B) A, C
(C) A, B (D) Cannot be determined

**7.** Three taps A, B and C together can fill a tank in 6 hours. All the taps are opened simultaneously. After 2 hours, C was closed and the tank was filled in 8 more hours. Find the time (in hours) in which C can fill the tank. _____

**8.** Working together, A and B can complete a piece of work in 24 days, B and C can complete the same work in 40 days while A and C can complete it in 30 days. Find the number of days in which each of them can complete the same work. _____, _____, _____

**9.** Three taps P, Q and R can fill a tank in 8, 10 and 12 hours respectively. Tap P is opened at 8:00 a.m., tap Q at 10:00 a.m. and tap R at 11:00 a.m. At what time would the tank be full?
(A) 12:12 p.m. (B) 12:25 p.m.
(C) 12:42 p.m. (D) 12:58 p.m.

**10.** Naveen, Praveen and Lenin are three brothers. The rate of work of Naveen is one-third of that of Praveen and Lenin together. If Praveen can complete the work in 8 days, while Lenin can complete it in 24 days, then in how many days can they together complete the same work? _____

**11.** Pipe A can fill a tank in 12 hours and pipe B can empty the tank in 18 hours. Both pipes are opened at 6 am and after some time B is closed, and the tank is full at 8 p.m. on the same day. At what time was B closed?
(A) 10 am (B) 8 am (C) 9 am (D) 11 am

**12.** The ratio of the rates of work of A, B and C is 1 : 3 : 2. The total wages of A, B and C for working for 15, 20 and 25 days respectively are ₹5000. Find the total wages of the three (in ₹), if A works for 10 days, B for 15 days and C for 12 days. _____

**13.** Working together, A and B can complete a work in 12 days. They work together for 9 days after which B leaves. If A finishes the remaining work in 5 days, then find the number of days that B alone would take to complete the work. _____

**14.** 20 workers can reap 120 acres of land in 30 days working 6 hours a day. How many more acres of land can 30 workers working 12 hours a day reap in 15 days? _____

**15.** P and Q can do a work in 12 and 16 days respectively. If they work on alternate days, beginning with Q, then in how many days will the work be completed?
(A) $13\frac{1}{4}$ (B) $13\frac{2}{3}$ (C) $13\frac{3}{4}$ (D) $13\frac{1}{3}$

**16.** A tank has two inlet pipes which can fill it in 10 hours and 15 hours respectively. It also has an outlet pipe which can empty it in 30 hours. The inlet pipes are opened at 6:00 am. Just when the tank was expected to be full, it was spotted that the outlet pipe had accidentally been left open. The outlet pipe was then closed. After how much more time will the tank be full? (in hours) _____.

**17.** There are three taps P, Q and R. P takes thrice as much time as Q and R together to fill a tank. Q takes twice as much time as P and R together to fill the tank. In how much time (in hours) can R fill the tank individually, if the three together require 10 hours to fill the tank when opened simultaneously? _____

**18.** A can complete a work in 20 days. B can complete the same work in 30 days while C can complete the work in 60 days. They start working together. A leaves after 3 days and B leaves 4 days before the work was originally scheduled to be completed. For how many days did C work?
_____

**19.** Pipe A can fill a tank in half the time in which pipe B can fill the same tank. If both the pipes are opened simultaneously, it takes 8 hours to fill the tank. In how many hours can A alone fill the tank? _____

**20.** Two pipes can fill a tank in 18 minutes and 27 minutes. A third pipe can empty the tank (when full) in 6 minutes. All the three pipes are opened when the tank was $\frac{2}{3}$ rd full. In how many minutes will the tank become empty?
_____

**21.** A certain quantity of work can be completed by A, B, C in 20, 15 and 30 days respectively and an amount of ₹4000 is paid for that work. Another quantity of work can be completed by A, B, C in 15, 25, 10 days respectively and

an amount of ₹10,000 is paid for this work. If both the works are completed by A, B, C together, who got the minimum amount of the three?
(A) A (B) B
(C) C (D) Cannot be determined

**22.** The rate of work of B is twice that of A. A can complete a piece of work in 15 days. A started the work and B joined him after a few days. The work was completed in 11 days. For how many days did A work alone? _____

**23.** Pipes A and B are inlet pipes while pipe C is an outlet pipe. Pipe A supplies water at 50 ltrs/hour. Pipe B can fill a tank in 4 hours while pipe C can empty it in 12 hours. All the pipes are simultaneously opened and the tank gets filled in 1 hour. What is the capacity of the tank?
(A) 120 ltrs (B) 60 ltrs
(C) 54 ltrs (D) None of these

**24.** If four men, or three women, or two children can complete a work in 183 days, then how long would it take for three men, four women, and six children working together to finish the work? _____ days

**25.** 20 cows and 40 goats can be kept for 10 days for ₹460. What would be the cost of keeping 50 cows and 30 goats for 12 days if the cost of keeping 5 goats is the same as the cost of keeping 1 cow?
(A) ₹888 (B) ₹965
(C) ₹1007 (D) ₹1104

**26.** Mahesh, Suresh and Naresh together can complete a work in 4 days. The wages paid to Mahesh, Suresh and Naresh for completing the work are ₹6300, ₹7350 and ₹8400 respectively. Find the number of days in which Naresh alone can complete the work. _____

**27.** A and B can complete a work in 10 days and 20 days respectively. If they work on alternate days, then find the least number of days in which the work will be completed. _____

**28.** Four persons A, B, C and D produced 256 pieces of item P in 8 hours. B is twice as efficient as A. D is 25% less efficient than A but is three times as efficient as C. How many pieces of P did B produce?
(A) 128 (B) 64 (C) 48 (D) 16

**29.** A piece of work can be completed by 20 men in 25 days. 20 men start the work together.

**1.** If starting from the second day, one person leaves at the end of every day, then what fraction of the work was completed? _____ /50

**2.** If starting from the second day, one additional person with the same working capacity joins every day, then in how many days will the work be completed? _____

**30.** 20 men can finish a piece of work in 30 days. After how many days should 5 men leave the work so that it is finished in 35 days? _____

**31.** A rectangular tank has dimensions 6 m × 5 m × 4 m. Three inlet pipes X, Y, Z have filling rates of 4 $m^3$/hr, 5 $m^3$/hr, 6 $m^3$/hr respectively. At 8 a.m., X was opened and at 8.30 a.m., Y was opened. At 10.30 a.m., Z was opened. The time at which the tank would be full is
(A) 4:50 p.m. (B) 5:40 p.m.
(C) 4:20 p.m. (D) 5:10 p.m.

**32.** A, B and C can complete a piece of work in 10, 20 and 40 days respectively. They work on a rotation basis with A working on the first day, B on the second, C on the third, then again A on the fourth day and so on. In how many days will the work be completed? _____

**33.** A boy is trying to cover a distance of 100 m up a ramp. He takes a jump forward and covers 2 m, but every time he jumps forward he also slips backwards by 1 m. In all, how many jumps would be required to cover the distance?
(A) 99 (B) 100
(C) 98 (D) None of these

**34.** A piece of work can be completed by ten men in a certain number of days. If there were two men less, it would have taken three more days to complete the work. In how many days can 24 men complete the same work?
_____

**35.** N filling pipes, $P_1$, $P_2$, … $P_N$ (where N > 10), are attached to a tank. The rate of filling of $P_i$, where $i > 1$, is equal to thrice the combined rate of filling of all the lower numbered pipes. If the time taken by $P_{N-3}$ to fill the tank is 128 minutes, find the time (in minutes) taken by $P_N$ to fill the tank. _____

**36.** Ram, Shyam and Tarun can complete a work in 24 days, 36 days and 30 days respectively. All the three started working together. After $x$ days, Ram left. Shyam left $x$ days before the work was completed. The work was completed at the end of $t$ days. If $x$ and $t$ are integers, the value of $t$ is _____.

**37.** Twenty four men working 9 hours a day can complete a piece of work in 35 days. In how many days can thirty men working 8 hours a day complete twice the original work? _____

**38.** 25 men take 20 days to construct a wall 10 m high. How many men would be required to construct a similar wall, which is 8 m high, if the work is planned to be completed in 10 days? _____

**39.** Shiven and Shalini agreed to do a job for ₹180. They can complete the work together in 12 days. If they complete the work in 9 days with the help of Sonia, then what is the share of Shiven, given that Shalini can complete the entire work on her own in 18 days?
(A) ₹30 (B) ₹45 (C) ₹50 (D) ₹6

**40.** Every day A, B and C work for 6 hours, 8 hours and 10 hours respectively. A sum of money is sufficient to pay for the daily wages of A, B and C for 15 days, 10 days and 5 days respectively. For how many complete days can all of them be paid if they work for 8 hours a day?
(A) 3 (B) 2 (C) 4 (D) 5

**41.** 64 men working 8 hours a day plan to complete a piece of work in 9 days. However 5 days later they find that they had completed only 40% of the work. They now wanted to finish the remaining portion of the work in 4 more days. Find the number of hours per day that they need to work in order to achieve the target. _____

**42.** A man takes 80 days to complete a job. Four men, 8 women and 4 machines take 5 days to complete the job. Four men, 1 woman and 2 machines take 10 days to complete the job. Find the time taken by a woman to complete the job (in days).
(A) 90 (B) 100
(C) 105 (D) 120

**43.** P and Q are filling pipes. If the bottom $\frac{3}{4}$ th of the tank is filled by P and the rest is filled by Q, the tank will be filled in 45 minutes. If the bottom $\frac{3}{4}$ th of the tank is filled by Q and the rest is filled by P, the tank will be filled in 39 minutes. Find the time taken by the two pipes working together to fill the tank.
(A) $18\frac{4}{7}$ minutes (B) $24\frac{6}{7}$ minutes
(C) $20\frac{4}{7}$ minutes (D) $16\frac{2}{7}$ minutes

**44.** 40 boys can earn ₹40,000 by completing a work in 50 days. Some men accompanied 30 boys and the work was completed in 20 days. What is the difference in the total earnings of the men and the boys? (in ₹) _____.

**45.** Six women and two men together take 10 days to complete a work. Four women and eight men together can complete the same work in 5 days. Find the number of days in which six men and eight women together can complete the work.

**46.** 9 men and 24 women can do a work in 8 days. 8 men and 32 boys can do the same work in 9 days. If 30 women can do the work in 12 days, then how many boys can do the same work in 18 days? _____

**47.** Anil and Bala agreed to complete a work in 14 days. They also agreed to forfeit thrice the amount of wages corresponding to the unfinished if they failed to complete the work in 14 days. As they could not complete the work in 14 days, they lost $\frac{3}{8}$ th of the amount that they would have together received had they completed the work in time. If Anil alone can complete the work in 80 days, the time that Bala alone would take to complete the work is
(A) 24 days (B) 20 days
(C) 30 days (D) 40 days

**48.** Two men are as efficient as 3 women who are as efficient as 4 machines. The number of men, women and machines that worked together and completed a job is in the ratio 3 : 4 : 5. They are paid a total of ₹4900 for the job. Find the total share of women (in ₹).
(A) 1200 (B) 1800
(C) 1500 (D) 1600

**49.** A, B and C working alone, can complete a work in 8, 12 and 24 days respectively. All the three of them started the work and then A left after 2 days. B left after another 3 days and C then completed the remaining work. If the total wages earned by the three of them are ₹6000, find the share of A.
(A) ₹1,000 (B) ₹2,500
(C) ₹2,000 (D) ₹1,500

**50.** Sixty men can complete a piece of work in 80 days. They started the work but after 20 days, ten of them left. After 20 more days, another ten men left. In how many days will the remaining men complete the remaining work?
_____

## EXERCISE-3

***Directions for questions 1 to 50***: For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** One man can complete a work in 129 days. One man starts the work and after that on each day one more man joins. With each man joining, the rate of work of every man doubles. In how many days does the work get completed? _____

**2.** R takes 5 times the time taken by P to do a certain work. P takes half the time taken by Q to do the same work. S takes three times the time taken by Q to do the same work. Q, R and S do $(5/16)^{th}$, $(1/4)^{th}$, $(3/16)^{th}$ of the work respectively. P does the remaining work. If it takes 13 days to complete the work, one working after the other, what will be the number of days taken by P, Q, R and S individually to complete the work?
(A) 4, 8, 20, 24 (B) 5, 10, 25, 30
(C) 6, 12, 30, 36 (D) 7, 14, 35, 42

**3.** Ram and Shyam can complete a job in 50 days and 60 days respectively. They worked only on weekdays (i.e., from Monday to Friday). They started working together on $20^{th}$ January 2014, a Monday. On $9^{th}$ February 2014, Ram met with an accident, and hence could not report for work. Shyam continued to work alone and Ram joined him on $17^{th}$ February 2014. On which date was the job completed?
(A) 28 Feb, 2014 (B) 27 Feb, 2014
(C) 26 Feb, 2014 (D) 1 Mar, 2014

**4.** Thirty men can complete a work in 12 days. One man starts the work and thereafter every day one more man joins the work and this continues till the work is completed. If all the men involved work at the same rate, then on which day is the work completed?
(A) $26^{th}$ (B) $27^{th}$ (C) $28^{th}$ (D) $29^{th}$

**5.** There are five oil tanks A, B, C, D and E, each containing 2400 litres of oil. Oil is being transferred simultaneously amongst the five tanks in the following manner.

From B to A @ 70 litres / minute

From E to C @ 100 litres / minute

From C to A @ 60 litres / minute

From D to C @ 120 litres / minute

From A to D @ 90 litres / minute

From E to B @ 30 litres / minute

From A to E @ 50 litres / minute

From B to D @ 80 litres / minute

From C to B @ 70 litres / minute

From D to E @ 110 litres / minute

Which tank gets emptied first and how many minutes does it take to empty that tank, after the oil transfer operation starts?
(A) D, 30 min (B) D, 48 min
(C) B, 60 min (D) D, 40 min

**6.** A house can be painted by 7 men and 2 women in 8 days, if they work for 12 hours a day. Working at the same rate, 6 men and 20 women can do the same job in 4 days. In how many hours can 8 men and 8 women paint the house?
(A) 12 hours (B) 16 hours
(C) 32 hours (D) 64 hours

**7.** In a garrison with 600 men, there are provisions which are sufficient for 24 days. After 18 days, 300 additional men join. In addition, the average consumption per head also changes. As a result, the provisions last for only three more days. What is the percentage increase in the average consumption per day for the last three days?
(A) 25% (B) 50% (C) $33^{1}/_{3}\%$ (D) $66^{2}/_{3}\%$

**8.** Raj, Shyam and Tarun can prepare a total of 8 dosas in one minute. They have to prepare a total of 80 dosas. Raj started preparing dosas and after some time, Shyam and Tarun took over and completed the job. If it took a total of 20 minutes to complete the job and Raj preparing atleast 5 dosas per minute, how long did Raj work alone (in minutes)?
(A) 9 (B) 10 (C) 11 (D) 12

**9.** When A and B work together, they take 16 days less than what A alone takes and 25 days less than what B alone takes to do a certain work. They agree to do the work for a certain sum of money, and completed it, with the help of C, all three working together for the whole time. If the difference between the shares of A and B was one-third the share of C, then in how many days did the work get completed?.
(A) 10 (B) 15 (C) 1 (D) 12

**10.** In the previous question, if B got ₹1250 more than C, then how much money did A get as his share?
(A) ₹2500 (B) ₹3750
(C) ₹5000 (D) ₹6250

**11.** P and Q can do a certain piece of work in 10 and 15 days respectively. P and Q work for alternately till the work is completed. If P starts the work, in how many days will the work be completed?
(A) 6 (B) 9 (C) 18 (D) 12

**12.** A, B and C can individually complete a piece of work in 30, 50 and 75 days, respectively. They worked on one day each, with A starting the work followed by B the next day and C the next day. They continued working like this till the $30^{th}$ day after which the remaining work was completed by B and C working on alternate days with B working on the $31^{st}$ day. In how many days was the work completed?
(A) $49\frac{1}{2}$ (B) 40 (C) 51 (D) 50

**13.** There are 25 workers in a group. Each can do 1 unit of work per day. They start a job of 330 units. After each day, a worker of the same efficiency as each worker in the group joins the group. If the job was completed in $x$ days, find $x$.
(A) 9 (B) 10 (C) 11 (D) 12

**14.** P works 25% more efficiently than Q and Q works 50% more efficiently than R. To complete a certain project, P alone takes 50 days less than Q alone. If in this project P alone works for 60 days and then Q alone works for 125 days, in how many days can R alone complete the remaining work?
(A) 50 days (B) 75 days
(C) 100 days (D) 150 days

**15.** Howard and John take 12 days and 16 days respectively to complete a job. Gerard is at least as fast as John but at most as fast as Howard. Gerard and John work on alternate days and completed the job in $x$ days. Which of the following can be the value of $x$?
(A) 12 (B) 13 (C) 14 (D) 17

**16.** In a tank, there is a leak, which can empty the tank in 20 hours, at $3/4^{th}$ of the height above the base. There are two pipes filling the tank which can fill the tank in 6 and 12 hours respectively. How much time does it take to fill the tank, if both the filling pipes are opened simultaneously?
(A) $3^3/_4$ hours (B) $4^1/_4$ hours
(C) $4^3/_4$ hours (D) $5^1/_4$ hours

***Directions for questions 17 and 18***: These questions are based on the data given below.

A contractor employed some workers to finish a certain project in a specified time. After a few days of work, he realized that the actual time would overshoot the scheduled time by 25%. So, he increased the workforce by a third and managed to finish the project on time.

**17.** Find the fraction of work that was completed by the time the additional workers were employed.
(A) $2/5^{th}$ (B) $3/5^{th}$
(C) $1/4^{th}$ (D) $1/5^{th}$

**18.** What fraction of the scheduled time is the time after which the additional workers were employed?
(A) 2/5 (B) 1/5
(C) 1/4 (D) Cannot be determined

**19.** A cistern can be filled by two pipes in 8 min and 12 min respectively. Both pipes are opened together for a certain time but due to some defects, only $5/6^{th}$ of full quantity of water flows through the former and only $3/4^{th}$ through the latter. The defects are detected and rectified. The cistern is filled in 2 min from that moment. How long was it before the defects were rectified?
(A) 1 min (B) 1.5 min
(C) 2.5 min (D) 3.5 min

**20.** Three men A, B and C working together can do a work in 30 days. They start the work together and A works for 3 days and takes rest on the fourth day, B works for 5 days and takes rest on the next two days and C works for 7 days and takes rest on the next three days. If A, B and C work at the same rate, in how many days will the work be completed?
(A) 39 days (B) 40 days
(C) 41 days (D) 42 days

**21.** A, B and C are three pipes which are attached to a tank. A and B fill the tank in 20 and 30 mins respectively and C empties it in 15 min. If A, B, C are opened in that order for 2 min each till the tank gets filled, then how many minutes will it take for the tank to get filled? _____

**22.** P, Q and R can do a piece of work in 4, 6 and 12 days respectively, working for 12 hours a day. On each day, P starts the work and works for 8 hours, then Q works for 8 hours and then R works for 8 hours. In the same way, they continue until P leaves after 2 days of work. Then Q and R work alternately for 8 hours each till the work is completed. In how many days is the work completed?
(A) $3^1/_3$ days (B) $3^1/_2$ days
(C) $3^2/_3$ days (D) $3^3/_4$ days

**23.** L, M and N are three machines, which produce electronic gadgets, the efficiencies of production of which are in the ratio 2 : 4 : 5. If L works for 7 days, M for 4 days and N for 2 days, they produce 1680 gadgets. How many

gadgets are produced if L, M and N work for 3, 6, 4 days respectively?
(A) 1720 (B) 1860 (C) 2100 (D) 2240

**24.** A contractor employed 20 men to complete a project in 50 days. After 30 days, he observed that only 50% of the work had been completed.

**1.** How many more men should be employed to complete the work in time? _____

**2.** If the work is increased by 20% of the original and still the contractor wants to finish it within 50 days, then at least how many more men need to be employed? _____

**25.** A garrison had provisions for 1500 men for 30 days. After some days, 300 more men joined the garrison. The provisions lasted for a total of 26 days from the beginning. After how many days did the new men join?
(A) 24 (B) 6 (C) 4 (D) 26

**26.** A task is assigned to a group of twenty men, not all of whom have the same capacity to work. Every day exactly two men out of the group work on the task, with no same pair of men working together twice. Even after all the possible pairs have worked once, all the men together had to work for exactly one more day to finish the task. Find the number of days required for all the men working together to finish the task. _____

**27.** Amar, Bhavan, and Chetan can complete a work in 36 days, 48 days, 72 days respectively. They worked one after the other and each completed one-third of the work. Instead, if Bhavan had done an $x^{th}$ fraction of Amar's share and an $x^{th}$ fraction of Chetan's share in addition to his own, the work would have been completed a day earlier. Find the value of $x$.
(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{9}$

**28.** Five men can complete a work in 15 days. Six women can complete the same work in 25 days. Two groups comprising two men and three women respectively work on alternate days, starting with the men. In how many days is the work completed?
(A) $42^{3}/_{4}$ (B) $33^{2}/_{3}$ (C) $6^{41}/_{50}$ (D) $9^{3}/_{5}$

**29.** Two pipes A and B can fill a tank in 20 hours and 30 hours respectively. Both the pipes were opened and just when the tank should have been full, it was noticed that an emptying pipe was also open. The emptying pipe was then closed and it took 6 hours more to fill the tank. In how much time (in hours) does the emptying pipe empty the tank (when full)? _____

**30.** Three friends Aravind, Manoj and Viswanath work together to complete a piece of work. The time it takes for them to do the work together is 2 hours less than what Arvind would have taken working alone, 10 hours less than what Manoj would have taken working alone and one-third the time Viswanath would have taken working alone. How long did it take for them to complete the work working together?
(A) 1 hour (B) 5 hours
(C) 3 hours (D) 2 hours

**31.** Ten painters can paint a house in 10 days. However after starting the work, one painter leaves work at the end of the first day. Another painter leaves work at the end of the second day. This process continues until only one painter is left and he continues till the job is completed. How long does it take to finish the job?
(A) 55 days (B) 45 days
(C) 46 days (D) 56 days

**32.** Irfan, Jay and Komal can complete a job in 1 day, 20 days and 10 days respectively. They started the work together but Irfan was unwell on the first day, therefore, he could not work at his normal capacity and left after a day. When Jay and Komal completed the job, Jay was paid ₹17000 out of the total ₹60000 paid to them. At what percentage of his normal capacity did Irfan work on the first day?
(A) 10% (B) 20% (C) 15% (D) 25%

**33.** Anand, Raju, Suresh and Venkat together produced 392 pieces of an item in 6 hours. Suresh is four times as efficient as Anand and is $1/3^{rd}$ less efficient than Venkat. Raju is half as efficient as Venkat. How many pieces would Raju have produced if he worked for 8 hours?
(A) 84 (B) 96 (C) 28 (D) 112

**34.** A group of 30 women takes 36 days to complete a piece of work for which they are paid a total of ₹60,000. The rate at which a man works as well as his daily wage is double that of a woman. How many men must join 15 women to complete the work in 24 days? How much more money is earned by the men than by the women?
(A) 10 men, ₹15,000 (B) 15 men, ₹20,000
(C) 15 men, ₹30,000 (D) 10 men, ₹30,000

**35.** A tank of dimensions 18 m × 10 m × 8 m is filled to its capacity. A drain pipe can empty the tank in 50 hours. An inlet pipe supplies water at the rate of 6 litres/sec. If both the pipes are opened at the same time, how much time would they take to empty the tank?
(A) 100 hours (B) 150 hours
(C) 300 hours (D) 200 hours

**36.** A job which 94 men take 87 days to complete is given to some robots. The first day one robot whose rate of doing work is same as that of a man starts the work. If on each day starting from the second day, one more robot joins the team of robots, then in how many days does the job get completed, given that the rate of work of the $i^{th}$ robot $(i > 1)$ is twice that of the $(i-1)^{th}$ robot?
(A) 12 days (B) 10 days
(C) 11 days (D) 13 days

**37.** In a tank of dimensions 5 m × 6 m × 7 m, the inlet pipe can supply at 5 $m^3$/hr and the outlet pipe can empty the tank in 30 hrs. If the two pipes are opened simultaneously, how much time does it take to empty the tank when it is half full?
(A) 52.5 hours (B) 105 hours
(C) 210 hours (D) 420 hours

**38.** A frog, which is at the bottom of a 50 m deep well, is trying to come out of it. In every jump it covers 1.25 m but slips 0.75 m. In how many jumps would it come out of the well?
(A) 98 (B) 99 (C) 100 (D) 101

***Directions for questions 39 and 40***: These questions are based on the following information.

There are 4 types of pipes which are connected to an empty tank.

Type A can fill the tank in 20 hours.

Type B can fill the tank in 25 hours.

Type C can empty the tank in 40 hours.

Type D can empty the tank in 50 hours.

**39.** If 3 type-A pipes, 2 type-B pipes, 4 type-C pipes and 5 type-D pipes are used, in how many hours will the tank be filled? $\frac{=}{3}$

**40.** One type-A, one type-B, one type-C and a certain number of type-D pipes are used and the tank is filled in 40 hours. Find the number of type-D pipes used. _____

**41.** A certain work can be done by A and B, B and C and A and C in $4^4/_5$, $6^6/_7$ and $5^1/_3$ days respectively. Find the number of days in which A, B, C can do the work alone respectively.
(A) 8, 12, 16 (B) 9, 12, 18
(C) 9, 15, 18 (D) 12, 16, 18

**42.** A, B and C can together earn ₹1,620 in 9 days. A and C can earn ₹600 in 5 days; where as B and C in 7 days can earn ₹910. Find the daily earnings of C. (in ₹) _____

**43.** It takes 8 women, each working at the same rate a total of 20 days to build a room. If 8 women start to build the room on January 1, 2009 and one man per day is added beginning from January 7, 2009 at the end of which day will the room be completed given that each man can work twice as fast as each woman?
(A) January 12, 2009 (B) January 13, 2009
(C) January 14, 2009 (D) January 16, 2009

**44.** A and B together can do a piece of work in $14^2/_5$ days; B and C together can do the same work in 12 days. After A worked for 8 days and B for 12 days, C takes up and finished it alone in 6 days. In how many days can each of them do the work, working alone? _____, _____, _____

**45.** Pipes P, Q and R are fitted to a tank. Each of Q and R can fill the tank in 6 hours. The capacity of P, which is an emptying pipe, is half of that of Q. If P is fitted at one-fourth of the height of tank from the base and all the pipes are opened simultaneously, the empty tank would be filled in _____ hours.
(A) 4 (B) $3\frac{3}{4}$ (C) $3\frac{1}{2}$ (D) 3

**46.** A is 80% more efficient than B who is 60% more efficient than C. A takes 40 days less than B to complete a work. A starts the work and works for 25 days and then B takes over. B then works for the next 30 days and then C takes over. In how much more time can C complete the remaining work?
(A) 20 days (B) 24 days
(C) 32 days (D) 40 days

**47.** A, B and C are three taps connected to a tank. Time taken by C to fill the tank is 9/2 times the time taken by A and B to fill it. The time taken by A to fill it is 5/6 times the time taken by B and C to fill it. A, B and C take 20/11 hours to fill it. Find the time taken by B to fill it (in hours).
(A) $3\frac{1}{3}$ (B) 5 (C) $6\frac{2}{3}$ (D) 10

**48.** Some workers have been divided into two groups—A and B—depending on their rate of doing work. Three workers from A and six from B take 20 days to complete a job. Eight from A and 4 from B take 10 days to complete it. Find the time taken by one worker from each group to complete it (in days).
(A) 90 (B) 108 (C) 72 (D) 54

**49.** P, Q and R are three pipes which can fill a tank in 6, 8 and 10 hours respectively. All the three are opened simultaneously but P works at $1/3^{rd}$ its normal rate for the first 3 hours and Q works at $1/4^{th}$ its normal rate for the first 2 hours. In how many hours will the tank be filled?

(A) $3\frac{12}{94}$ hours (B) $1\frac{19}{94}$ hours

(C) $3\frac{83}{94}$ hours (D) $1\frac{42}{94}$ hours

**50.** Pipes A and B can fill a tank in 20 minutes and 30 minutes respectively and C can empty it in 15 minutes. A is opened for a minute and then closed. B is then opened for a minute and then closed. C is then opened for a minute and then closed. This process is repeated until the tank is filled. Find the time taken to fill the tank (in minutes).

(A) 169 (B) 170

(C) 167 (D) 166

## EXERCISE-4

***Directions for questions 1 to 30***: For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Four persons P, Q, R and T can complete a work in 8, 12, 16 and 24 days respectively. One of them starts the work and it is continued on a rotation basis, with all the four taking turns and only one person working per day. Any one of the four can work on the first day. Which of the following is the complete set of people, who can work on the first day such that the work begins and ends with the same person?

(A) P, Q (B) Q, R

(C) P, Q, R (D) P, Q, T

**2.** A can complete a piece of work in 10 days, B in 20 days and C in 40 days. A, B and C start the work but B and C work only on the first day, third day, fifth day and so on. In how many days does the work get completed?

(A) 8 days (B) 4 days

(C) 7 days (D) $7^1/_{11}$ days

**3.** A mason employed a certain number of workers to finish constructing a wall in a certain scheduled time. After some time, he realised that the work would get delayed by a fourth of the scheduled time. So, he immediately increased the number of workers by a third and thus managed to finish constructing the wall on time. Some time after the workforce was increased, all the newly added workers left and the remaining workers reduced their rate of work by half. Finally, the work got completed with a delay of 50% of the scheduled time. What fraction of the total work was still incomplete by the end of the scheduled time?

(A) 22.5% (B) 25%

(C) 20% (D) 16.6%

**4.** The time taken by pipe I to fill an empty tank is m times the time taken by pipes II and III working together to fill the same. The time taken by II is $m$ times the time taken to fill the tank by I and III working together. The time taken by III is $m$ times the time taken by I and II working together. Also I, II and III take 6 hours to fill the tank working together. Find the time (in hours) taken by pipe I alone to fill the tank.

(A) 12 (B) 18 (C) 24 (D) 30

***Directions for questions 5 and 6***: These questions are based on the following information.

Three men are hired to dig four identical moats. Initially the three men decide to work together on all the four moats. But after working together to dig the first moat they split up and dig the other three moats working individually. The second moat took 4 1/6 times the number of days that the first moat took to dig. The third moat took 10 days more than 5/2 times the time taken to dig the first moat and the fourth moat took exactly 5/2 times the time taken to dig the first moat.

**5.** Find the total time taken for the digging of all the four moats to be completed. (in days) _____

**6.** If the three men receive ₹10,000 for the four moats, what would the difference between the smallest and the largest shares be? (in ₹) _____.

**7.** In summer, men and women work at certain rates of work, while in winter these rates decrease by 20% and 12.5% respectively. In summer a certain work can be done by 5 men and 4 women in 8 days or by 6 men and 13 women in 4 days. If $p$ men and $q$ women can complete the same work in a single day in winter, find the number of possible values for $(p, q)$. (Assume $p > 0$, $q > 0$). _____

**8.** Three pipes X, Y and Z fitted to an empty tank, are opened simultaneously. The time taken by X to fill the

tank is six times the time taken by all the three pipes to fill the tank. The time taken by Y to fill the tank is 40 hours less than seven times the time taken by all the three pipes to fill the tank. The time taken by Z to fill the tank is 10 hours more than the time taken by all the three pipes to fill the tank. If the time taken to fill the tank is an integer (in hours), then find which pipe is the most efficient.
(A) X (B) Y
(C) Z (D) All are equally efficient

**9.** A, B and C are three taps connected to a tank such that 6 times the time taken by A to fill the tank is 7 times the time taken by B and C together to fill the tank. 3 times the time taken by C to fill the tank is 10 times the time taken by A and B together to fill the tank. If A, B and C together fill the tank in 60/13 hours, find the time taken by B alone to fill the tank.
(A) 10 hours (B) 15 hours
(C) 20 hours (D) 25 hours

**10.** If A and B work together, they take half a day more than what A, B and C together would take to complete the same work. If B and C work together, the number of days they take is 200% more than what A, B, C would take. All three worked together from start to finish and B earned ₹400 as his share out of the total earnings of ₹1800. In how many days did A, B and C together complete the work?
(A) 6 days (B) 4 days
(C) 18 days (D) 8 days

**11.** A boat developed a leak. Immediately after the leak developed, water started entering the boat at a uniform rate. The leak was later detected and by then some water had already accumulated. Immediately after the detection, 12 men worked for 3 hours to pump the boat dry. Had 18 men worked instead of 12 men, they would have taken $1\frac{1}{2}$ hours to pump the boat dry. Had 15 men worked instead of 12 men, they would have pumped the boat dry in
(A) $1\frac{3}{4}$ hours (B) 2hours
(C) $2\frac{1}{2}$ hours (D) $2\frac{1}{4}$ hours

**12.** Ram works on any given day only if Shyam and Tarun also work on that day. Shyam works for the first two days and then does not work for the next two days. He continues in this manner. Tarun works for the first three days and then does not work for the next three days and continues to work in this manner. Ram, Shyam and Tarun started working on an assignment. The assignment was completed in 130 days. On how many days did Ram work?
(A) 33 (B) 32 (C) 34 (D) 35

**13.** Two pipes A and B can fill a cylindrical tank in 10 minutes and 40 minutes respectively. The tank also has two holes, one at half the height and the second at 2/3rd the height which can empty the tank (if they were at the bottom of the tank) in 60 minutes and 120 minutes respectively. If the tank is initially empty and the two pipes A and B are opened simultaneously, in how much time will the tank be full?
(A) $6\frac{21}{23}$ minutes (B) $9\frac{29}{37}$ minutes
(C) $8\frac{34}{39}$ minutes (D) $9\frac{34}{39}$ minutes

**14.** P, Q, R and S are filling pipes fitted to a tank. Their flow rates are in descending order. The greatest part of the tank which can be filled by any three of the pipes in an hour is $\frac{47}{120}$. The least part of the tank which can be filled by any three of the pipes in an hour is $\frac{37}{120}$. The flow rate of P is twice that of S. Find the time (in hours) taken by S to fill the tank.
(A) 9 (B) 12 (C) 18 (D) 24

**15.** Ravi was assigned a job to be completed in 10 days working from 9 a.m. to 6 p.m. every day with a break of 1 hour for lunch. The job would have taken Ravi exactly 80 hours, which were available. But he was also allowed to work after 6 p.m. (which would be considered overtime), if he wanted. On each of the 10 days excluding the 6th and the 7th days, he worked for all the regular 8 hours. On the 6th day, he was ill and did not work at all. On the 7th day, he worked only from 9:00 a.m. to 1:00 p.m. He completed the job by 6 p.m. on the 10th day. For regular working hours on any day he was paid ₹150/hr and for working overtime he was paid ₹180/hr. He must receive a total amount of
(A) ₹13,260 (B) ₹11,820
(C) ₹12,360 (D) ₹14,220

**16.** Two masons were supposed to finish making a certain compound wall around a square playground in a certain number of days. They can do the job in the given time provided they work together. But the second mason did not come for work till the wall on the first side of the playground was built. The work at that point was lagging by 12 days. For the next side of the playground, the second mason worked alone and by the time this part of the

wall was completed, the work was lagging by 39 days. The two masons finished making the other two walls working together. If they received an amount of ₹2,00,000 for the entire work, what would be the share of the second mason in this amount?
(A) ₹1,20,000 (B) ₹80,000
(C) ₹1,10,000 (D) ₹90,000

**17.** A certain tank has two pipes A and B that can either fill it or empty it at the same respective rates depending on the requirements. If pipe A only is used to fill the tank for the time that both pipes together take to fill the tank, then $3/5^{th}$ of the tank would be full. When the tank is empty, pipe A is used to fill the tank while B is used to empty it. What percent of the tank will be empty in half the time it takes to fill the tank by both pipes together when they are used for filling? _____%

**18.** Prakash, Pranay and Pramod working alone can do a piece of work in 20, 30 and 60 days respectively. All three of them start the work together, but after $x$ days Prakash leaves and then after $y$ more days Pranay leaves and Pramod completes the remaining work. If Pranay had not left, Pramod and Pranay would have completed the remaining work in $(y + 6)$ days after Prakash had left. If both Prakash and Pranay had stayed, the work would have been completed in $(x + 6)$ days. What is the total number of days taken to complete the work? _____

**19.** A worker works for 10 days. Every day, he works on two tasks. Starting from the second day, his daily wage for the first task increases by 5% of the first day's wage for the first task, while his daily wage for the second task increases by 10% of the first day's wage for the second task. The total wages of the worker for the first 5 days are ₹800 and that for the entire period are ₹1850. For the first day, the worker was paid
(A) ₹130 (B) ₹150 (C) ₹140 (D) ₹120

**20.** Ram always works on any day only when Shyam and Tarun work on that day. Whenever Shyam works, he works for the first four days and then does not work for the next four days. He continues this manner. Whenever Tarun works, he works for the first five days and then does not work for the next five days and continues to work in this manner. Ram, Shyam and Tarun started working on an assignment. The assignment was completed in 700 days. On how many days did Ram work?
(A) 172 (B) 176 (C) 184 (D) 196

**21.** There are three groups of workers—P, Q and R. 1 worker from P and 1 from Q together take 72 days to complete a job. 4 from P and 6 from R together take 20 days to complete the job. 2 from P, 3 from Q and 8 from R together take 18 days to complete the job. In how many days can 3 workers, 1 from each group together complete the job?
_____

**22.** A tank, of height $3h$ has an inlet pipe A and two outlet pipes B and C fitted to it. B and C are fitted at heights of $2h$ and $h$ respectively from the base. A can fill the tank in 12 hours. The times in which B and C can empty it are in the ratio 1 : 2. All the pipes are opened simultaneously. The tank is filled in 16 hours and 48 minutes. Find the time (in hours) in which B can empty it.
(A) 24 (B) 27 (C) 30 (D) 36

**23.** Anil and Bala can complete a job in 40 days and 50 days respectively. They planned to complete a job, working together, such that they would work only on the weekdays (i.e., from Monday to Friday). They started working on $20^{th}$ Feb 2012 which was a Monday. On the morning of $12^{th}$ March 2012, Bala met with an accident and hence could not work. Anil continued to work alone and Bala joined him on $19^{th}$ March 2012. On which day was the job completed?
(A) $22^{nd}$ March, 2012 (B) $23^{rd}$ March, 2012
(C) $24^{th}$ March, 2012 (D) $25^{th}$ March, 2012

**24.** A tank has N inlet pipes numbered from 1 to N fitted to it and all are opened simultaneously. The rate at which the $n^{th}$ pipe (for $1 \le n \le N$) fills the tank is n lit/min Together, all the N taps take a certain time to fill the tank. If the rate of $n^{th}$ pipe had been $n^2$ lit/min the time taken to fill the tank would have been 80% less. Find the value of N.
(A) 5 (B) 6 (C) 7 (D) 8

**25.** Ram starts reading all the letters of the English alphabet starting from '$a$' at the rate of 1 letter per second. Shyam, his younger brother reads only the vowels starting from '$a$' at the rate of 1 vowel per second. This cycle of reading letters at the uniform rate continues indefinitely. If the two of them start reading together, after how much time (in seconds), will they just complete reading the letter '$u$' together? _____

**26.** A water tank has a capacity V. Two pipes of diameters $D_1$ and $D_2$ are used to fill the tank. The speed at which water flows through the first and the second pipe is $S_1$ and $S_2$ respectively. An emptying pipe of diameter $D_3$ is also connected to the tank and water flows through this at a speed of $S_3$. If all the three pipes are opened simultaneously, the time taken to fill the tank is (Given that the tank, which is initially empty, becomes full)

(A) $\dfrac{V}{\frac{\pi}{4}\left(S_1 D_1^2 + S_2 D_2^2 - S_3 D_3^2\right)}$

(B) $\dfrac{V}{\pi\left(S_1 D_1 + S_2 D_2 - S_3 D\right)}$

(C) $\dfrac{V}{\dfrac{\pi}{4}\left(S_3 D_3^2 - S_1 D_1^2 S_2 D_2^2\right)}$

(D) None of these

**27.** At a certain factory, there are four tanks: P, Q, R and S. Each tank has a capacity of 2000 litres and is filled with 1500 litres of chemical X. X is pumped from one tank to another as follows:

From R to Q @ 60 litres/min

From P to R @ 130 litres/min

From S to Q @ 30 litres/min

From R to S @ 120 litres/min

From Q to P @ 70 litres/min

The pumping continues until a tank is either completely drained or completely full. In such an event, the tank triggers a signal which stops the pumping in the entire network. Which tank triggers the signal and how many minutes after the pumping starts does the signal get triggered?

(A) P, 25 min (B) Q, 75 min

(C) R, 30 min (D) S, $5\frac{5}{9}$ min

**28.** Pipes P, Q and R are fitted to a tank. Each pipe can be used as an inlet pipe or an outlet pipe with the same flow rate. If P is used as an emptying pipe and the other two are used as filling pipes, the tank will be filled in $1/m$ times the time taken by it to fill the tank. If Q is used as an emptying pipe and the other two are used as filling pipes, the tank will be filled in $1/n$ times the time taken by it to fill the tank. If R is used as an emptying pipe and the other two are used as filling pipes, the tank will be filled in 1/p times the time taken by it to fill the tank.

$$\frac{1}{m+2}+\frac{1}{n+2}+\frac{1}{p+2} = _____.$$

**29.** A group consists of $m$ equally efficient workers. A work was started by one of them. The $p^{th}$ worker (where $2 \le p \le m$) joined the work $(p-1)x$ days after the $(p-1)^{th}$ worker joined. The work was completed in $mx$ days after the $m^{th}$ worker joined. The $1^{st}$ worker received a share of ₹5500 out of the total ₹38,500 paid to the group for completing the work. Find $m$.

(A) 9 (B) 10 (C) 11 (D) 22

***Directions for question 30***: The question is based on the following information.

A water tank is provided with four outlet pipes having the same emptying evenly spaced out vertically one above the other. The first pipe is located at the point that is at the bottom level of the tank. The fourth pipe is located at three-fourths of the height of the tank above its base. The first outlet pipe can empty the full tank in four hours. There is also an inlet pipe which can fill the empty tank in one hour. The inlet pipe is opened to fill the empty tank and after one hour it is closed and then all the outlet pipes are opened.

**30.** In how many minutes would the tank be empty?

(A) 60 (B) 125 (C) 120 (D) 105

## EXERCISE-5 Based on Data Sufficiency

***Directions for questions 1 to 35***: Each question is followed by two statements, I and II. Answer each question based on the following directions:

Choose (A) If the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.

Choose (B) If the question can be answered by using either statement alone.

Choose (C) If the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (D) If the question cannot be answered even by using both the statements together.

**1.** A, B and C together can do a piece of work in 15 days. Whose rate of work is the highest?

I. A and C together, can do the same work in 30 days.

II. B and C together, can do the same work in 25 days.

**2.** In how many days does a piece of work get completed, if A and B work on alternate days?

I. A alone can do the work in 20 days, while B can do the same in 30 days.

II. B starts the work.

**3.** Out of A, B and C, which two must be employed to work together to have the work completed with the least expenses? (The rate of work is not always proportional to the rate of earning)

I. The rates of work of A, B and C are in the ratio of 1 : 2 : 3.

II. The daily wages that are given to A, B and C are in the ratio of 3 : 4 : 5.

**4.** Among A, B, C, D and E, exactly one can finish the work in less than 18 days. Who is it?

I. A, B and C can do a work in 12 days; D and E can do it in 20 days.

II. Among A, B and C, C takes the least time and D is the faster between D and E.

**5.** Is the time taken by A to complete the work more than that by C?

I. Time taken by A and C together to complete a work is twice the time taken by A and B together to do the same work.

II. B takes less number of days than C to complete the work.

**6.** Working together A, B and C can complete a piece of work in 10 days. Is A's rate of work the highest among the rates of work of A, B, C?

I. B alone can complete the work in 25 days.

II. A alone can complete the work in 50 days.

**7.** A can complete a piece of work in 20 days. In how many days can A and B together complete the work?

I. In 5 days, B alone can complete 50% of the work.

II. B works twice as fast as A.

**8.** Find the time taken by 6 men, 4 boys and 3 women to complete a job.

I. 2 boys and 3 women OR 10 men and 3 boys can complete the job in five days.

II. 9 men and 3 boys OR 10 boys and 15 women can complete the job in three days.

**9.** A and B together can complete a job in 10 days. How many days does A alone take to complete the job?

I. A, B and C together take 6 days to complete the job.

II. C works twice as fast as A.

**10.** A and C together take 20 days to complete a piece of work. B and C together take 15 days to complete the same work. How many days does C alone take to complete the work?

I. A and B together take 12 days to complete the work.

II. A, B and C together take 10 days to complete the work.

**11.** Two taps P and Q together take 3 minutes 36 seconds to fill an empty tank. How many minutes does a drain pipe R take to empty the tank when it is full?

I. P and R together can fill the tank in 9 minutes.

II. Q alone takes 9 minutes to fill the tank.

**12.** A, B and C together take 15 days to complete the work. In how many days can B alone complete the work?

I. A and C together got half the total amount earned by A, B and C working together.

II. B and C together got 60% of the total amount earned by A, B and C working together.

**13.** How long will it take the two pipes A and B to empty or to fill a tank which is $3/4^{th}$ full?

I. Pipe A can fill the tank (when empty) in 12 minutes.

II. Pipe B can empty the tank (when full) in 8 minutes.

**14.** Find the time in which two persons P and Q can complete a work, working on alternate days.

I. P and Q can complete the work in 12 days and 36 days respectively.

II. P started the work.

**15.** 10 men and 15 women can complete a work in 30 days. In how much time can 24 men and 15 women complete the same work?

I. 18 men and 12 women can complete the work in 20 days.

II. 12 men and 18 women can complete the work in 25 days.

**16.** A water pump functions at a uniform rate. How long does it take to fill the tank?

I. The water pump is switched on at 8:00 a.m.

II. By 9:30 a.m., $3/8^{th}$ of the tank is filled by the pump. By 11.00 a.m. $3/4^{th}$ of the tank is filled by the pump.

**17.** In how many days can a man make 100 pots?

I. A man, a woman and a child together make 10 pots a day, while a woman and a child together can make 15 pots in 3 days.

II. 10 men together take 1 more day than 25 children to make 100 pots. A man can make twice as many pots as a child in a day.

**18.** 10 soldier ants take 6 hours to build an anthill. How long does it take for 5 queen ants to build the anthill?

I. A queen ant works at half the working rate of a soldier ant.

II. 5 soldier ants and 5 queen ants together take 7½ hours to build the ant hills.

**19.** How long did Tarun work to construct the wall?

I. Piyush, Rohan, Saurabh and Tarun can individually construct the wall in 10, 15, 20 and 30 days respectively.

II. Initially Piyush started the work. After three days, Rohan and Saurabh joined him and the three together worked for two days, then Tarun joined them and all of them worked until the wall is constructed.

**20.** Who among Arun, Bhargav and Chadda is the most efficient?

I. To complete a work, Chadda takes 5 times as much time as Arun and Bhargav take working together.

II. To complete a work Arun takes as much time as Bhargav and Chadda take working together.

**21.** How long would Vidya and Asha take to complete a work if they work on alternate days?

I. They work for six days a week and if one of them works on the first day of one week, the other one works on the first day of the next week.

II. Asha working alone on alternate days completes the work in 35 days while Vidya working alone on every third day completes the work in 25 days.

**22.** If Varun and Sameer together completed a project in 25 days and earned ₹5000, then what is Sameer's share?

I. Working individually, Sameer takes thrice the time taken by Varun to complete the project.

II. Varun alone can complete the project in 40 days.

**23.** How long does it take to fill a 330-litre tank?

I. There are a total of 6 taps. First a tap is turned on and then every half an hour an additional tap is turned on till all the taps are turned on.

II. A tap can fill a 20 litre tank in one hour.

**24.** Rishab's cow and Shrayan's cow can together eat the grass on the farm in 30 days. How long does it take for Rishab's cow to eat the grass on the farm?

I. The two cows along with Vashist's cow can eat the grass on the farm in 20 days.

II. Vashist's cow eats twice as fast as Shrayan's cow.

**25.** Pipes A, B, C and D have their filling capacities in descending order. They take $2^{2}/_{7}$ hours to fill a tank. Find the time taken by B and C to fill it.

I. The maximum time in which the tank can be filled by three of the pipes is $3^{9}/_{13}$hours

II. The minimum time in which the tank can be filled by three of the pipes is $2^{2}/_{3}$ hours.

**26.** If ten men take six hours to do a piece of work, then how long will five boys take to do the same work?

I. A boy works at 3/4 the rate of a man.

II. Five men and five boys take ten hours to do the work.

**27.** How long will it take for the tank to get filled when the taps A and B are opened simultaneously?

I. Tap A is a filling tap, and can fill it in two hours.

II. Tap B is an emptying tap, and can empty it in three hours.

**28.** What is the time taken to fill the tank, if the two pipes A and B are turned on alternately for one minute each?

I. The two pipes together fill the tank in six minutes.

II. One pipe can fill the tank in ten minutes.

**29.** A, B and C are paid a total amount of ₹529. How much should A be paid?

I. A and B together completed $19/23^{rd}$ of the work.

II. B and C together completed $8/23^{rd}$ of the work.

**30.** From a tank full of water, the water is discharged into a tub through a pipe. What is the quantity of water discharged from the tank in a minute?

I. The tank was emptied in 30 minutes.

II. The dimensions of the tub are 3 m × 2 m × 2 m.

**31.** There are two groups of people P and Q. In P each person works with the same efficiency. In Q each person works with the same efficiency. Is the time taken by one person from P and one person from Q to complete the job more than 6 days?

I. Two persons from P and three persons from Q can complete the job in $\frac{12}{5}$ days.

II. Three persons from P and two persons from Q can complete the job in $\frac{68}{35}$ days.

**32.** Ajay and Bala completed a job working on alternate days. Find the time taken by them to complete it.

I. Had they worked together they would have completed the job in 20 days.

II. Times taken by Ajay and Bala working individually to complete a similar job are 30 days and 60 days respectively.

**33.** Pipes A and B can fill a tank in $t$ hours. A can fill it in $(t + a)$ hours. B can fill it in $(t + b)$ hours. Find $t$.

I. $4a = 9b$

II. $ab = 144$

**34.** Rohan and Sohan can complete a job in 12 days. Who is the least efficient among Rohan, Sohan and Mohan?

I. Rohan and Mohan can complete the job in 15 days.

II. Mohan and Sohan can complete the job in 20 days.

**35.** Pipes A, B and C can be used for filling or emptying with the same capacity. If A and B are used for filling and C is used for emptying, a tank would be filled in 6 hours. Find the time taken by C to fill it.

I. If all the pipes are used for filling, the tank would be filled in 2 hours.

II. If B and C are used for filling and A is used for emptying, the tank would be filled in 6 hours.

## ANSWER KEYS

### Exercise-1

1. (C)
2. 6
3. (D)
4. 40
5. (C)
6. (C)
7. (C)
8. (B)
9. (D)
10. (B)
11. 4
12. (A)
13. (C)
14. 3000
15. (D)
16. 12
17. 120
18. (A)
19. (B)
20. 36
21. (D)
22. (A)
23. 860
24. (B)
25. (i) 8 (ii) 16
26. (D)
27. (D)
28. (B)
29. 10
30. (A)
31. (D)
32. 36
33. (B)
34. (B)
35. (A)
36. 40
37. (C)
38. (C)
39. 40
40. (C)
41. (B)
42. (B)
43. (A)
44. 24
45. 60
46. (D)
47. 50
48. (B)
49. 40
50. 10

### Exercise-2

1. 120 B = 60 C = 120
2. (A)
3. (A)
4. 8,00
5. (C)
6. (C)
7. 12
8. A = 40
9. (C)
10. 4.5
11. (C)
12. 3160
13. 30
14. 60
15. (C)
16. 1.2
17. 24
18. 39
19. 12
20. 9
21. (B)
22. 9
23. (B)
24. 36
25. (D)
26. 10.5
27. 13
28. (A)
29. 1) 21 2) 500
30. 15
31. (D)
32. 16.5
33. (A)
34. 5
35. 2
36. 15
37. 63
38. 40
39. (B)
40. (B)
41. 15
42. (D)
43. (C)
44. 16000
45. 5
46. 30
47. (B)
48. (D)
49. (D)
50. 65

### Exercise-3

1. 5
2. (A)
3. (A)
4. (B)
5. (D)
6. (D)
7. (C)
8. (B)
9. (B)
10. (D)
11. (D)
12. (D)
13. (C)
14. (B)
15. (C)
16. (B)
17. (D)
18. (C)
19. (D)
20. (C)
21. 154
22. (A)
23. (C)
24. (1) 10 (2) 22
25. (B)
26. 20
27. (B)
28. (A)
29. 24
30. (D)
31. (A)
32. (C)
33. (D)
34. (B)
35. (D)
36. (A)
37. (A)
38. (B)
39. 100
40. 2
41. (A)
42. 70
43. (B)
44. 24, 36, 18
45. (B)
46. (B)
47. (C)
48. (C)
49. (C)
50. (C)

### Exercise-4

1. (C)
2. (C)
3. (C)
4. (B)
5. 186
6. 400
7. 11
8. (C)
9. (B)
10. (B)
11. (B)
12. (A)
13. (C)
14. (B)
15. (C)
16. (D)
17. 90
18. 28
19. (C)
20. (B)
21. 60
22. (D)
23. (B)
24. (C)
25. 125
26. (A)
27. (D)
28. 1
29. (B)
30. (B)

### Exercise-5

1. (A)
2. (A)
3. (C)
4. (C)
5. (D)
6. (A)
7. (B)
8. (A)
9. (C)
10. (B)
11. (C)
12. (A)
13. (C)
14. (A)
15. (A)
16. (A)
17. (B)
18. (B)
19. (C)
20. (A)
21. (A)
22. (B)
23. (C)
24. (C)
25. (C)
26. (B)
27. (C)
28. (A)
29. (A)
30. (C)
31. (A)
32. (B)
33. (A)
34. (C)
35. (A)

## SOLUTIONS

## EXERCISE-1

**1.** Part of the job which can be completed by 30 men in a day $= \frac{1}{30}$.

Part of the job completed by 1 man in a day

$= \frac{1}{(30)(30)} = \frac{1}{900}$.

**2.** One day work of A and B $= \frac{1}{9} + \frac{1}{18} = \frac{2+1}{18} = \frac{1}{6}$

So, the time taken to complete the work is 6 days.

**3.** Let A worked for $x$ days.

$\frac{x}{21} + \frac{4}{28} = 1$

$\Rightarrow \frac{x}{21} = 7 \Rightarrow x = 18$

A worked for 18 days. So, A can complete the remaining work in 18 – 4 = 14 days.

**4.** The ratio of their working rates

$= \frac{1}{7} : \frac{1}{14} : \frac{1}{21} = 6 : 3 : 2$. Since, they work together,

the share of C $= \frac{2}{11} \times 220 = ₹40$

**5.** Ratio of the parts of the jobs done by P, Q and R in an hour is the ratio of the reciprocals of their individual times to complete it.

i.e., $\frac{1}{2} : \frac{1}{3} : \frac{1}{6} = 3 : 2 : 1$

**6.** B and C can do the work in $30 \times \frac{4}{3} = 40$ days

A's one day work $= \frac{1}{22} - \frac{1}{40} = \frac{40-22}{22 \times 40} = \frac{18}{22 \times 40} = = \frac{4}{440}$

A can do the work in $\frac{440}{9} = 48\frac{8}{9}$ days.

**7.** X and Y, Y and Z, Z and X can complete the job in ascending number of days.

∴ When X and Y are together they are faster than Y and Z together who are together faster than X and Z.

∴ Z is slower than X and X is slower than Y.

∴ Z is the slowest of the three.

**8.** One day work of A, B and C

$= \frac{1}{18} + \frac{1}{24} + \frac{1}{36} = \frac{4+3+2}{72} = \frac{1}{8}$ th of total work.

A, B and C together can do the work in

$\frac{8}{1} = 8$ days.

**9.** Time taken by both taps working together to fill the tank

$= \frac{(4)(6)}{4+6} = 2.4$ hours.

Every hour A can fill $\frac{1^{th}}{4}$ of the tank. Fraction of the tank filled by A in 2.4 hours $= \frac{2.4}{4} = \frac{3}{5}$.

**10.** Whenever two or more people work together to complete a piece of work and all work for the same time, the ratio of their wages is given by the ratio of the daily work they do. If all do not work for the same amount of time, the ratio of their wages is given by the ratio of their total work they do. As A and B work together for the same time, the ratio of their wages is the ratio of their daily work, they do.

The ratio of one day's work of A and B

$= \frac{1}{12} : \frac{1}{18} = 3 : 2$

A's share $= \frac{3}{5} \times 300 =$ ₹ 180

**11.** Time taken to complete the job = 36 hours.

Number of days required to complete the job working 9 hours a day $= \frac{36}{9} = 4$.

**12.** B's one day's work $= \frac{1}{26}$ th of total work.

As A is 30% more efficient than B, he can do 30% more work than B in the same time.

A's one day's work $= \frac{1}{26} \times \frac{130}{100} = \frac{1}{20}$ th of total work.

A can do the work in 20 days.

**13.** Part of the job completed in the 1st two days irrespective of the person who starts the job $= \left(\frac{1}{30} + \frac{1}{60}\right)^{th} = \frac{1}{20}^{th}$

∴ The entire job which is 20 times will be completed in (2) (20) = 40 days irrespective of the person who starts the job.

**14.** Part of the job completed by X and Y working together in a day $= \frac{1}{30}+\frac{1}{60}=\frac{1}{20}^{\text{th}}$.

In 10 days, they can complete $\frac{1}{2}$ of the job working together.

∴ Z completes the remaining $\frac{1}{2}$ of the job.

∴ Z gets a share of $\frac{1}{2}$ (6000) = ₹3000.

**15.** A can do the work in $12 \times \frac{4}{3} = 16$ days. B can do the work in $20 \times \frac{8}{5} = 32$ days. One day's work of A and B

$= \frac{1}{16}+\frac{1}{32}=\frac{2+1}{32}=\frac{3}{32}$. Time taken by A and B to complete the work is $\frac{32}{3}=10\frac{2}{3}$ days

**16.** Part of the job completed by P and Q working together in a day $= \frac{1}{x+8}+\frac{1}{x+18}=\frac{1}{x} \Rightarrow \frac{x+18+x+8}{(x+8)(x+18)}=\frac{1}{x}$

$2x^2 + 26x = x^2 + 26x + 144$

As $x > 0$, $x = 12$

**Alternate Method:**

If P takes $(x + a)$ days to complete a job and Q takes $(x + b)$ days to complete the job and both working together takes $x$ days to complete the job, it follows that $x = \sqrt{ab}$. In the given problem, a = 8 and $b = 18$.

∴ $x = 12$

**17.** B's one day work $= \frac{1}{20}-\frac{1}{40}-\frac{1}{60}=\frac{6-3-2}{120}=\frac{1}{120}$

So, B can do the work in 120 days.

**18.** As 9 men and 21 boys have thrice the efficiency of 3 men and 7 boys, the time taken by 9 men and 21 boys to complete the work will be one third of the time taken by 3 men and 7 boys, i.e., $\frac{45}{3}=15$ days

**19.** $(x + 1)\ (x - 2) : (x + 2)\ (x - 4) = 9 : 8$

$\Rightarrow 9\ (x + 2)\ (x - 4) = 8\ (x + 1)\ (x - 2)$

$\Rightarrow 9x^2 - 18x - 72 = 8x^2 - 8x - 16$

$\Rightarrow x^2 - 10x - 56 = 0 \Rightarrow x^2 - 14x + 4x - 56 = 0$

$= x\ (x - 14) + 4\ (x - 14) = 0$

$= (x - 14)\ (x + 4) = 0$

$x - 14 = 0$ or $x + 4 = 0$

So, $x = 14$ or $-4$

As the number of men cannot be negative, $x = 14$.

**20.** Total job = (9) (20) = 180 man days.

= (180) (3) = 540 women days.

Time taken by 15 women to complete the job

$= \frac{540}{15}$ = days

**21.** Kavita's one day's work $= \frac{1}{k}$ Kavita's $x$ day's work $= \frac{x}{k}$

**22.** Part of the job completed in the 1st two days irrespective of the person who starts the job $= \left(\frac{1}{2}+\frac{1}{4}\right)^{\text{th}} = \frac{3}{4}^{\text{th}}$

If A starts the job, then the remaining part of the job, i.e., $\frac{1}{4}^{\text{th}}$ can be completed in 1/2 a day.

But if B starts the job, then he will take 1 full day to complete the remaining $1/4^{\text{th}}$ part of the job.

∴ The job can be completed in the least possible time if A starts the job

**23.** We have $\frac{M_1D_2}{W_1}=\frac{M_2D_2}{W_2}$ (Variation rule)

$$\frac{215\times 13.5}{677}=\frac{M_2\times\frac{13.5}{2}}{677\times 2} \Rightarrow M_2 = 215\times 4 = 860$$

**24.** We have $M_1 D_1 = M_2 D_2$

$(k - 1)\ (k - 8) = (k + 4)\ (k - 11)$

$k^2 - 8k - k + 8 = k^2 - 11k + 4k - 44$

$\Rightarrow k^2 - 9k + 8 = k^2 - 7k - 44$

$\Rightarrow 2k = 52$

$\Rightarrow k = 26$

**25.** (i) Job to be completed = (2) (8) (8) = 128 man hours.

Required number of men $= \frac{128}{(4)(4)} = 8$.

(ii) Required number of men = 2(8) = 16.

**26.** The part of the tank filled by the three pipes in one minute

$= \frac{1}{20} + \frac{1}{26} - \frac{1}{13} = \frac{13+10-20}{260} = \frac{3}{260}$ th

The time taken to fill the tank by the three pipes is $\frac{260}{3} = 86\frac{2}{3}$ minutes

**27.** Let the work be completed in $x$ days

$\frac{x-1}{15} + \frac{x-1}{30} + \frac{x}{45} = 1$

$\frac{6(x-1)+3(x-1)+2x}{90} = 1$

$6x - 6 + 3x - 3 + 2x = 90$

$x = 99 \Rightarrow x = 9$.

**28.** P and Q, Q and R and P and R can complete $\frac{1}{2}^{th}$, $\frac{1}{20}^{th}$ and $\frac{1}{15}^{th}$ of the job respectively working together in a day.

$\therefore$ 2 P's, 2 Q's and 2 R's can together complete $\frac{1}{12} + \frac{1}{20} + \frac{1}{15} = \frac{1}{5}^{th}$ of the job in a day.

$\therefore$ P, Q and R can complete $\frac{1}{2(5)} = \frac{1}{10}^{th}$ of the job in a day. $\therefore$ They will take 10 days.

**29.** A can do the work in $\frac{30}{2}$, i.e., 15 days

A and B's one day work = $\frac{1}{15} + \frac{1}{30} = \frac{2+1}{30} = \frac{1}{10}$

So, A and B together can do the work in 10 days.

**30.** As they work on rotation, the part of the job completed in every period of 3 consecutive days will be constant and is $\frac{1}{20} + \frac{1}{30} + \frac{1}{40} = \frac{13}{120}$.

As the number of such periods $\left(\frac{120}{13}\right)$ is not an integer, the job will be completed in the least possible time if the most efficient person starts the job. As P is the most efficient, he must start the job.

**31.** The ratio of efficiencies of man and women is $1 : \frac{1}{2}$, i.e., 2 : 1.

The number of mandays required = $6 \times 10 = 60$

$\therefore$ The number of womandays required = $60 \times \frac{2}{1} = 120$

$\therefore$ 10 women can complete the work in $\frac{120}{10}$, i.e., 12 days.

**32.** Let A can do the work in $x$ days, then

B can do the work in $3x$ days.

$\frac{1}{x} + \frac{1}{3x} = \frac{1}{27}$

$\frac{3+1}{3x} = \frac{1}{27}$

$\Rightarrow x = 36$.

**33.** Part of the book he can read in 1 minute = $\frac{1}{k}$

$\therefore$ Park of the book he can read in 8 minutes = $\frac{8}{k}$.

**34.** Part of tank filled by both pipes in 1 minute

$= \frac{1}{30} + \frac{1}{20} = \frac{2+3}{60} = \frac{1}{12}$

$\therefore$ Both the pipes can fill the tank in 12 minutes.

**35.** The ratio of efficiencies of the two persons is 2 : 3

$\therefore$ Q is faster. He completes $\frac{3}{2+3}$, i.e., $\frac{3}{5}^{th}$ of the work by the time, the work gets completed.

$\therefore$ In 24 days he completes $\frac{3}{5}^{th}$ of the work

$\therefore$ He completes total work in $24 \times \frac{5}{3} = 40$ days.

**36.** First 2 days work of Vinay and Varma = $\frac{1}{30} + \frac{1}{60}$

$= \frac{2+1}{60} = \frac{1}{20}$

2 days work = $\frac{1}{20}$

$(2 \times 20)$ days work = $\frac{1}{20} \times 20 = 1$

So, the work will be completed in 40 days.

**37.** Fraction of the job done by P = $\frac{4}{4+5+6} = \frac{4}{15}$.

**38.** Let leak can empty the full tank in $x$ hours $\frac{1}{6} - \frac{1}{x} = \frac{1}{9}$

$\Rightarrow \frac{1}{x} = \frac{1}{6} - \frac{1}{9} = \frac{3-2}{18} = \frac{1}{18}$

$\therefore x = 18$.

**39.** We have $M_1D_1 = M_2D_2$

So, $24 \times 35 = M_2 \times 21$

$\Rightarrow M_2 = 40.$

**40.** A and B 's one day work = $\frac{2}{15} + \frac{1}{30} = \frac{4+1}{30} = \frac{1}{6}$

So, A and B together can do the work in 6 days.

**41.** A is 20% more efficient than B.

$\therefore$ The ratio of efficiencies is 120 : 100, i.e., 6 : 5

$\therefore$ The ratio of work done by A and B is 6 : 5

**42.** Let time taken by P to do the work be $x$ days. Q and R together can do the work in $2x$ days.

$\frac{1}{x} + \frac{1}{2x} = \frac{1}{21}$

$\frac{2+1}{2x} = \frac{1}{21}$

$\Rightarrow 2x = 21 \times 3 = 63$ days.

**43.** P and Q together can complete $\frac{1}{6}^{\text{th}}$ of the work in 1 day

P alone can complete $\frac{1}{15}^{\text{th}}$ of the work in 1 day

The work done by Q alone in 1 day

$= \frac{1}{6} - \frac{1}{15} = \frac{1}{10}^{\text{th}}$ of the work

$\therefore$ In 10 days, Q alone can complete the work.

**44.** The inlet pipe can fill $\frac{1}{8}^{\text{th}}$ of the tank in an hour. The outlet pipe can empty $\frac{1}{12}^{\text{th}}$ of the tank in an hour. Both pipes working together can fill $\left(\frac{1}{8} - \frac{1}{12}\right)^{\text{th}} = \frac{1}{24}^{\text{th}}$ of the tank in an hour.

$\therefore$ The tank will be filled in 24 hours.

**45.** We have $M_1D_1 = M_2D_2$

$120x = (x - 20)\ 180$

$\Rightarrow 2x = (x - 20)\ 3 \Rightarrow 2x = 3x - 60$

$\Rightarrow x = 60.$

**46.** Number of manhours required to complete the work

$= 4 \times 4 \times 4 = 64$ manhours

Eight men working 8 hours a day completes $8 \times 8 = 64$ manhours in one day which is required total number of manhours.

$\therefore$ In one day the work will be completed.

**47.** Ratio of the time taken by A and B to complete the job

$= \frac{1}{\text{Ratio of the efficiencies of } A \text{ and } B} = 2:3 \ = 2:3$

Let the time taken by A and B to complete the job be $2x$ days and $3x$ days respectively.

$\frac{(2x)(3x)}{2x + 3x} = 30$

$x = 25$

Time taken by the faster person to complete the job = Time taken by A = 50 days.

**48.** Given one man can eat one fruit in one day

Eating one fruit represents a work of 1 manday by 1 man. In $n$ days, 1 man can eat $n$ fruits. In $n$ days, $m$ men can eat $m \times n$ fruits. They would do a work of $m \times n$ mandays.

$\therefore$ 8 men in 8 days, i.e., in $8 \times 8 = 64$ mandays = 64 fruits.

**49.** Given that $12m = 20w$

$\Rightarrow 3m = 5w$

9 men + 12 women = 15 women + 12 women = 27 women

20 women can do the work in 54 days. So, 27 women can do it in $\frac{20 \times 54}{27} = 40$ days .

**50.** Total consumption of the family members per day

$= (6)\ (1.25) = 7.5$ kg.

Number of days the ration will be sufficient $= \frac{75}{7.5}10$ .

## EXERCISE-2

***Solutions for questions 1 to 50***:

**1.** Let total work be 1 unit.

As A takes $n + 80$ days, A does $\frac{1}{n+80}$ units per day.

Similarly, B does $\frac{1}{n+20}$ units per day.

$\left[\frac{1}{n+80} + \frac{1}{n+20}\right] n = 1$

Upon solving $n = 40$

Hence, A completes it in 40 + 80, i.e., 120 days

**2.** Amount of work done by A, B and C in 4 days

$= 4[1/12 + 1/16 + 1/20] = \frac{47}{60}$

Amount of work completed by C in the last 2 days

$= \frac{2}{20} = \frac{1}{10}$

Amount of work completed by B and C

$= 1 - [47/60 + 1/10] = 7/60$

Amount of work completed by B and C in 1 day

$= \frac{1}{16} + \frac{1}{20} = \frac{9}{80}$

$\therefore$ Time taken by B and C to complete 7/60 of the work

$= \frac{7}{60} \times \frac{80}{9}$

$= 1\frac{1}{27}$ days

$\therefore$ Total time taken $= 4 + 1\frac{1}{27} + 2$

$= 7\frac{1}{27}$ days

**3.** Let the capacity of P to fill be $p$ litres/ hr.

Capacity of Q to fill $= \frac{3p}{2}$ litres/ hr.

Capacity of R to fill $= 2p$ litres/ hr.

Capacity of S to fill $= 3p$ litres/hr

Ratio of the sum of the four capacities and S's capacity

$= \frac{15p}{2} : 3p = 5 : 2$

$\therefore$ Required ratio = 2 : 5

**4.** The extent to which the tank is filled in 4 hours, i.e., from 6:00 a.m. to 10:00 a.m. $= \frac{4}{12} - \frac{4}{24} = \frac{1}{6}$th

The pipe can completely fill the tank in 12 hours. Then $\frac{5}{6}$th of the tank would be filled in $\left(\frac{5}{6} \times 12\right) = 10$ hours.

Therefore, the tank gets filled at 10:00 a.m. + 10 hours. = 8:00 p.m

**5.** Let the total work be 1 unit. Parts of it completed by one man and one woman in $\frac{3}{2}$ days $= \frac{1}{2}$ units and $\frac{3}{8}$ units respectively.

Remaining $= 1 - \left(\frac{1}{2} + \frac{3}{8}\right) = \frac{1}{8}$ units

$\therefore$ work completed by each boy in $\frac{3}{2}$ days $= \frac{1}{8}$

Only one boy will be required.

**6.** Time taken by B to complete the job = 2 days.

Time taken by C to complete the job = 4 days.

Time taken by D to complete the job = 8 days.

As two of the four when working together take less than 40% of the time taken by the other pair, they must have a combined efficiency which is more than $\frac{5}{2}$ times that of the other pair.

$\therefore$ The part of the job completed by them in a day must be more than $\frac{5}{2}$ times of that completed by the other pair. Only A and B satisfy this condition.

**7.** Part of the tank filled by A, B and C in 1 hour

$= A + B + C = \frac{1}{6}$

In the first 2 hours, $\frac{1}{3}$ rd of the tank is filled

Remaining part $= \frac{2}{3}$ rd

This was filled in 8 hours

A and B can together fill the tank in 12 hours.

$\therefore$ Part of the tank filled by C in 1 hr $= \frac{1}{6} - \frac{1}{12} = \frac{1}{12}$ th

C can fill the tank in 12 hours.

**8. Method 1: Unitary**

$1/A + 1/B = 1/24$

$1/B + 1/C = 1/40$

$1/A + 1/C = 1/30$

Adding the three equations:

$2(1/A + 1/B + 1/C) = 1/24 + 1/40 + 1/30 = 1/10$

From equations (i), (ii) and (iii), we get the values of C, A and B as 120, 40 and 60 days respectively.

**Method 2: L.C.M.**

Let us take the work as 120 units which is = L.C.M. of (24, 40, 30)

| A + B | B + C | C + A | A + B + C |
|---|---|---|---|
| Time required | 24 | 40 | 30 |
| Units/day | 5 | 3 | $4\frac{12}{2} = 6$ |

| | A | B | C |
|---|---|---|---|
| Units/day | 3 | 2 | 1 |
| Time required | 40 | 60 | 120 |

**9.** Portion of the tank filled by P in 2 hours = $2 \times 1/8 = 1/4$

Portion of the tank filled by P and Q in 1 hour

$= \frac{1}{8} + \frac{1}{10} = \frac{9}{40}$

Portion of the tank yet to be filled = $1 - [1/4 + 9/40]$ = 21/40

Time taken to fill the remaining portion of the tank

$= \frac{21/40}{1/8 + 1/10 + 1/12}$

$= \frac{21}{40} \times \frac{120}{37} = \frac{26}{37}$

∴ Total time taken to fill the tank = $2 + 1 + 1^{26}/_{37}$

$= 4^{26}/_{37}$ hours = 4 hours 42 minutes 9 sec.

∴ The tank was filled at 12:42 p.m.

**10.** Praveen's and Lenin's one day combined work

$= \left(\frac{1}{8} + \frac{1}{24} = \frac{4}{24} = \frac{1}{6}\right)$

i.e., they take 6 days to complete the total work.

Naveen is one third as fast as Praveen and Lenin together, therefore, he would take = 6 *x* 3 = 18 days to finish the work.

Hence, all the three together can complete the work in $\frac{6 \times 18}{6 + 18}$, i.e., $4\frac{1}{2}$ days.

**11.** Let Pipe B be closed *x* hours after 6 a.m. Pipe A worked from 6 am to 8 p.m., i.e., 14 hours.

$\frac{14}{12} - \frac{x}{18} = 1$

$\frac{x}{18} = \frac{1}{6} \Rightarrow x = 3$

So, pipe B was closed 3 hours after 6 a.m., i.e., at 9 a.m.

**12.** Ratio of the work done by A, B and C

$= (1 \times 15) : (3 \times 20) : (2 \times 25)$

$= 15 : 60 : 50 = 3 : 12 : 10$

Total wage of A = $\frac{3}{3 + 12 + 10} \times 5000 = ₹600$

∴ Daily wages of A = $\frac{600}{15} = ₹40$

Similarly daily wages of B = $\frac{1}{20} \times \frac{12}{25} \times 5000 = ₹120$ and

daily wage of C = $\frac{1}{25} \times \frac{10}{25} \times 5000 = ₹80$

∴ Total wage for A, B and C for the given periods for a work which is twice as big as this work

$= 2[10 \times 40 + 15 \times 120 + 12 \times 80] = ₹6320.$

∴ Total wages = ₹3160

**13.** Since they worked together for 9 days, they will have completed 9/12 of the work. So, remaining work is only $(1 - 9/12) = 1/4$. To complete 1/4 work, A takes 5 days, so he will complete the entire work in 20 days.

Applying the formula time taken by B = 12 *x* 20 / (20-12)

= 30 days.

B can do the same work in 30 days.

**14.**

| Workers | Area | Days | Hrs/day |
|---|---|---|---|
| 20 | 120 | 30 | 6 |
| 30 | *x* | 15 | 12 |

$\frac{20 \times 30 \times 6}{120} = \frac{30 \times 15 \times 12}{x} \Rightarrow x = 180$

∴ 60 more acres can be reaped.

**15.** Part of work completed in the first 2 days

$= \frac{1}{16} + \frac{1}{12} = \frac{3+4}{48} = \frac{7}{48}$

Work done in $(2 \times 6)$ days = $\frac{7}{48} \times 6$

So, in 12 days $\frac{7}{8}$ of work is completed.

Work remaining after 12 days = $1 - \frac{7}{8} = \frac{1}{8}$

On the 13th day Q works and he can do $\frac{1}{16}$ of total work on that day.

Work remaining after 13 days = $\frac{1}{8} - \frac{1}{16} = \frac{1}{16}$

On the 14th day P works and he can complete the work in $\frac{1/6}{1/12} = \frac{3}{4}$ days

Total time taken = $13\frac{3}{4}$ days.

**16.** Let the capacity of the tank be L.C.M.(10, 15, 30) = 30 units. So, the pipes fill/empty 3, 2 and 1 unit per hour

respectively. Now, the two inlet pipes should have taken 30/(2+3) = 6 hours to fill the tank. However, since C was also open it would have drained 6x1 = 6 units of water in this duration. To supply this additional 6 units, A and B will take another 6/(3 + 2) = 1.2 hours = 1 hour 12 min.

**17.** 3 × (portion of the tank filled by P in 1 hour) = Portion of the tank filled by Q and R in 1 hour.

Adding the portion of the tank filled by P in 1 hour to both sides, we get;

4 × (portion of the tank filled by P in 1 hour) = portion of the tank filled by P, Q and R in 1 hour = 1/10

⇒ Portion of the tank filled by P in 1 hour = 1/40

∴ P alone can fill the tank in 40 hours.

Similarly, we can find that Q takes 30 hours to fill the tank.

∴ Time taken by R to fill the tank

$$= \frac{1}{1/10 - [1/40 + 1/30]} = \frac{1}{1/24} = 24 \text{ hours}$$

**18.** Let's say C works alone for $n$ days.

Since the problem states 'B leaves 4 days before the work was originally scheduled to be completed' let us first find the number of days in which the work was originally scheduled to be completed.

A and C will together complete the work in 60/(3+1) = 15 days. This combined entity working with B will complete the work in 30/(2+1) = 10 days.

Now,

| | A | B | C |
|---|---|---|---|
| No. of days | 20 | 30 | 60 |
| Work per day | $\frac{1}{20}$ | $\frac{1}{30}$ | $\frac{1}{60}$ |
| No. of days | 3 | 10 – 4 | $n$ |

$$\therefore \frac{3}{20} + \frac{6}{30} + \frac{n}{60} = 1.$$

Solving we get the value of $n$ as 39 days.

**19.** Let the time taken by A to fill the tank be $x$ hours.

Then, the time taken by B to fill the tank will be $2x$ hours.

Now, $\frac{1}{x} + \frac{1}{2x} = \frac{1}{8}$

$$\Rightarrow \frac{3}{2x} = \frac{1}{8} \Rightarrow x = 12$$

∴ A alone can fill the tank in 12 hours.

**20.** Part of tank emptied in one minute when all the pipes are opened = $\frac{1}{6} + \frac{1}{18} - \frac{1}{27} = \frac{9-3-2}{54} = \frac{2}{27}$

So, $\frac{2}{3}$ of tank will be emptied in $\frac{27}{2} \times \frac{2}{3}$ minutes.

= 9 minutes.

**21.** The ratio of the shares of A, B, C for the first work = $\frac{1}{20} : \frac{1}{15} : \frac{1}{30} = 3 : 4 : 2$

The ratio of the shares of A, B, C for second work

$$= \frac{1}{15} : \frac{1}{25} : \frac{1}{10} = 10 : 6 : 15$$

$$\text{A's total share (in ₹)} = \frac{3}{9}(4000) + \frac{10}{31}(10{,}000)$$

$$\text{B's total share (in ₹)} = \frac{4}{9}(4000) + \frac{6}{31}(10{,}000)$$

$$\text{C's total share (in ₹)} = \frac{2}{9}(4000) + \frac{15}{31}(10{,}000)$$

We can divide all the shares by 4000. We have to compare $\frac{3}{9} + \frac{25}{31}, \frac{4}{9} + \frac{15}{31}, \frac{2}{9} + \frac{37.5}{31}$.

i.e., 93 + 225, 124 + 135, 62 + 337.5

∴ B has the least share.

**22.** Let the total units of work be L.C.M.(11, 15) = 165

A will complete 165/15 = 11 units per day. Since B is twice as efficient as A, he will complete 22 units of work per day.

In 11 days, A will complete 11x11 = 121 units. So, B will need to do the remaining (165 – 121) = 44 units of work. At the rate of 22 units a day, he will take 44/22 = 2 days.

So, B joined A after 9 days.

**23.** Portion of tank filled by pipe A in 1 hour = $1 - \frac{1}{4} + \frac{1}{12} = \frac{5}{6}$

∴ Time taken by A to fill the tank = 6/5 hours

∴ Volume of the tank = 6/5 × 50 = 60 ltrs.

**24.** $4m = 3w = 2c$

$w = \frac{4m}{3}$ and $1c = 2m$

$m_1 d_1 = m_2 d_2$

$$4m \times 183 = \left[3m + 4\left(\frac{4m}{3}\right) + 6(2m)\right] x$$

$$\Rightarrow 4m \times 183 = \left[\frac{61m}{3}\right] x$$

$\therefore x = 36$ days

**25.** 5 goats = 1 cow

$\Rightarrow$ 40 goats = 8 cows and 30 goats = 6 cows

20 cows + 40 goats = 28 cows and

50 cows + 30 goats = 56 cows

Now,

| Cows | Days | Sum |
|---|---|---|
| 28 | 10 | 460 |
| 56 | 12 | $x$ |

$$\therefore \frac{56 \times 12}{x} = \frac{28 \times 10}{460}$$

$$\Rightarrow x = \frac{56 \times 12 \times 460}{28 \times 10} = 1104$$

$\therefore$ Cost of keeping 50 cows and 30 goats for 12 days = ₹1104.

**26.** Let the time in which Mahesh, Suresh and Naresh can complete the work be $m$ days, $s$ days and $n$ days respectively Ratio of the wages of Mahesh, Suresh and Naresh = ratio of the work done by Mahesh, Suresh and Naresh

$$\therefore 6300 : 7350 : 8400 = 4\left(\frac{1}{m}\right) : 4\left(\frac{1}{s}\right) : 4\left(\frac{1}{n}\right)$$

$$\Rightarrow \frac{1}{m} : \frac{1}{s} : \frac{1}{n} = 6 : 7 : 8$$

$$\text{Also, } \frac{4}{m} + \frac{4}{s} + \frac{4}{n} = 1, \text{ i.e., } \frac{1}{m} + \frac{1}{s} + \frac{1}{n} = \frac{1}{4}$$

$$\therefore \frac{1}{n} = \frac{8}{6+7+8}\left(\frac{1}{4}\right) = \frac{1}{10.5}$$

$n = 10.5$.

**27.** Total work = L.C.M. (10, 20) = 20 units.

Then A does $\frac{20}{10}$, i.e., 2 units per day.

B does $\frac{20}{20}$, i.e., 1 unit per day.

Hence, in 2 days, they together complete 3 units of work.

Now to complete 20 units of work they take $\frac{20}{3} = 6\frac{2}{3}$ days - a fraction.

Hence, to complete the work earliest, start the work with A as he is more efficient.

Start with A and take a multiple of 3 which is close to but less than 20 (which is the number of units of work).

$3 \times 6 = 18$ means that in 12 days ($6 \times 2$), they complete 18 units of work.

Now A will do the remaining 2 units of work on $13^{th}$ day.

Hence, the work will be completed earliest by $13^{th}$ day.

**28.** Let us say, A produced $x$ pieces.

$\therefore$ B produced $2x$ pieces, D produced $3x/4$ pieces and C produced $1/3 \times 3x/4$, i.e., $x/4$ pieces.

Now, $x + 2x + 3x/4 + x/4 = 256 \Rightarrow x = 64$.

$\therefore$ B produced $2 \times 64 = 128$ pieces

**29.** Let us assume a man completes 1 unit of work in a day. So, 20 men will complete 20 x 25 = 500 units of work in the given period and this represents the total units of work to be done.

**1.** If 1 person starts leaving from the second day onwards, the units of work done would be 20 on day 1, 19 on day 2, 18 on day 3, etc., till the last day when only 1 unit of work will be done.

Total work done = $(20 + 19 + 18 + \ldots + 1)$

$= (20 \times 21)/2 = 210$ units.

Fraction of work completed = 210 / 500 = 21/50.

**2.** If one more man joins the site starting with the second day the units of work done would be 20 on day 1, 21 on day 2, 22 on day 3, etc., till the entire work is completed.

Total work done = $(20 + 21 + 22 + \ldots)$

$= 20.n + n(n-1)/2 = 500$

**Alternative Solution:**

As can be observed, for $n = 17$, work done = $20 \times 17 + 16.17/2 = 476$;

Work doable on the next day = $(20 + 17)$

$= 37$ units

Remaining work = 500 – 476 = 24

So, time taken = 24/37 day + 17 days

= 17 24/37 days.

**30.** Total work = $30 \times 20 = 600$ man days

$\Rightarrow 600 = 20n + 15(35 - n)$

$n = 15$ days

**31.** By 10.30am, X would have filled 2.5 × 4, i.e., 10 $m^3$ and Y would have filled 5 × 2, i.e., 10 $m^3$.

Total volume filled upto 10.30 a.m. = 20 $m^3$

Remaining volume to be filled = 6 × 5 × 4 – 20 = 100 $m^3$.

Time taken to fill the remaining volume = $\frac{100}{4+5+6}$

= $6\frac{2}{3}$ hours

The tank would be full at 10.30 a.m. + $6\frac{2}{3}$ hours, i.e., 5:10 p.m.

**32.** Let the total work be L.C.M. (10, 20, 40) = 40;

A completes 40/10 = 4 units per day; Similarly B and C will complete 2 and 1 units respectively. So, at the end of one cycle (first 3 days) they will have completed 4 + 2 + 1 = 7 units of work.

They can carry out [40/7] = 5 such cycles thereby completing 35 units of work.

The next day A completes 4 units of work leaving 1 unit of work.

The next day, B will complete the work in 1/2 part of the day.

Total time = 5 × 3 + 1 + 1/2 = 161/2 days.

**33.** In each jump the boy goes forward by (2 –1) = 1 m.

In 98 jumps he covers 98 m.

In $99^{th}$ jump he would have covered the distance of 100 m.

**34.** Let, the number of days be $x$

$10x = 8(x+3) \Rightarrow x = 12$

Now, number of days taken by 24 men to complete the same work = $\frac{10 \times 12}{24} = 5$

**35.** Let the rate of filling of $P_1$ be $x$ lit/min.

Let the rate of filling of $P_k$ where $1 \le k \le N$ be $R_K$

$R_1 = x$

$R_2 = 3R_1 = 3x$

$R_3 = 3(R_1 + R_2) = 12x$

$R_4 = 3(R_1 + R_2 + R_3) = 48x$

$R_5 = 3(R_1 + R_2 + R_3 + R_4) = 192x$

We see that $R_y = 4R_{y-1}$ where $y \ge 3$

As $N > 10$, $N - 3 > 7$.

$\therefore R_N = 4(R_N - 1) = 4(4R_{N-2}) = 4(4(4R_{N-3})) = 64\, R_{N-3}$.

Time taken by $P_N$ to fill the tank = $\frac{1}{64}$ (time taken by $P_{N-3}$ to fill the tank) = $\frac{1}{64}$ (128 min) = 2 min.

**36.** Ram worked for $x$ days, Shyam worked for $(t - x)$ days, Tarun worked for $t$ days.

$$x\left(\frac{1}{24}\right) + (t-x)\left(\frac{1}{36}\right) + t\left(\frac{1}{30}\right) = 1$$

$$\frac{15x + 10(t-x) + 12t}{360} = 1$$

$$22t + 5x = 360 \quad (1)$$

$x > 0$.

$\therefore 22t < 360$

$$T < 16\frac{4}{11} \quad (2)$$

As R.H.S of (1) is divisible by 5, L.H.S of (1) is also divisible by 5.

$\therefore 22t$ must be divisible by 5. $\therefore t$ must be divisible by 5 (3)

$x$ must be less than $t$.

$\therefore 22t + 5x < 27t$, i.e., $360 < 27t$

$$\therefore t > 13\frac{1}{3} \quad (4)$$

From (2), (3), (4) $t$ must be 15.

**37.** Using variation concept, we get $M_1D_1H_1 / W_1 = M_2D_2H_2 / W_2$ i.e., 24 x 35 x 9 / 1 = 30 x $D_2$ x 8 / 2 => $D_2$ = 63 days.

**38.**

| Men | Days | Work |
|---|---|---|
| 25 | 20 | 10 |
| $x$ | 10 | 8 |

Now, $x = \frac{25 \times 20}{10} \times \frac{8}{10} = 40$

$\therefore$ 40 men would be required to complete the work in the given time.

**39.** Shiven & Shalini can complete the work together in 12 days while Shalini alone does it in 18 days.

$\therefore$ Shiven's work in a day = $\frac{1}{12} - \frac{1}{18} = \frac{1}{36}$ th part

Shiven's share is $9 \times \frac{1}{36}$ th part of 180, i.e., ₹45

**40.** Let $x$, $y$, $z$ be hourly wages of A, B and C. respectively.

90 $x$ = 80 $y$ = 50 $z$ = $M$

$\therefore x = \frac{M}{90}, y = \frac{M}{80}, z = \frac{M}{50}$

For 8 hours, for the three of them, we need to totally pay

$= 8\left(\frac{M}{90}+\frac{M}{80}+\frac{M}{50}\right) = \frac{8(40+45+72)}{3600}M = \frac{157}{450}M$

Number of days all of them can be paid $= \frac{M}{\frac{157M}{450}}$

$= \frac{450}{157}$, As $2 < \frac{450}{157} < 3$, all of them can be paid for 2 complete days.

**41.** Number of men hours put in the first 5 days

$= 64 \times 5 \times 8 = 2560$ hours.

For the remaining 60% of the work, men hours required

$= 2560 \times \frac{60}{40} = 3840$

Let number of hours everyday = $x$

$\therefore x = \frac{3840}{64 \times 4} = 15$ hours

**42.** Let the works done by a man, a woman and a machine in a day be '$a$' units, '$b$' units and '$c$' units, respectively.

$(4a + 8b + 4c)5 = (4a + 1b + 2c)10$

$3b = 2a$

$\therefore$ A woman will take 3/2 times the time taken by a man to complete the job, i.e., $\frac{3}{2} \times 80 = 120$ days.

**43.** Let the times taken by the pipes P and Q to independently fill the tank be $p$ minutes and $q$ minutes respectively.

$\frac{\frac{3}{4}}{\frac{1}{p}} + \frac{\frac{1}{4}}{\frac{1}{q}} = 45$, i.e., $\frac{3}{4}p + \frac{1}{4}q = 45$

$\frac{\frac{3}{4}}{\frac{1}{q}} + \frac{\frac{1}{4}}{\frac{1}{p}} = 39$, i.e., $\frac{3}{4}q + \frac{1}{4}p = 39$

Solving the equations above, $p = 48$ and $q = 36$.

Time taken by the pipes working together to fill the tank

$= \frac{1}{\frac{1}{p}+\frac{1}{q}} = 20\frac{4}{7}$ minutes.

**44.** Using variation concept, we have

$M_1D_1 = M_2D_2 = 40 \times 50 = (30 + x) \times 20 \Rightarrow x = 70$ boys

As a man's wage is twice that of a boy, we require 35 men to complete the work. Also for 40 boys, 50 days earnings = ₹ 40,000. $\Rightarrow$ A boy's 1 day earning = ₹ 20

$\therefore$ A man's 1 day earning = ₹ 40

Difference in the earnings of the men and the boys per day: $35 \times 40 - 30 \times 20 =$ ₹ 800

$\therefore$ Total difference $= 800 \times 20 =$ ₹ 16,000

**45.** Let's say a women does the work in $w$ days while a man does the work in $m$ days. So, the equations are

$\frac{6}{w} + \frac{2}{m} = \frac{1}{10}$ (1)

and $\frac{4}{w} + \frac{8}{m} = \frac{1}{5}$ (2)

$\frac{8}{w} + \frac{6}{m} = ?$ (3)

By solving for $w$ and $m$, from (1) and (2) and substituting in (3) we get the answer. Alternately, we observe that (1) + 0.5 × (2) = (3).

So, no. of days $= \frac{1}{10} + \frac{0.5}{5}$

$= \frac{1}{10} + \frac{1}{10} = \frac{2}{10} = \frac{1}{5} \Rightarrow 5$ days.

**46.** $(9m + 24w)\ 8 = (8m + 32b)\ 9$

$\Rightarrow 72m + 192w = 72m + 288b$

$\Rightarrow 192w = 288b \Rightarrow 2w = 3b$

30 women can do the work in 12 days. So, to complete the work in 18 days number of women required

$= \frac{30 \times 12}{18} = 20$

20 women = 30 boys

So, 30 boys can do the work in 18 days.

**47.** Let the time in which Bala alone would complete the work be $b$ days.

Let the amount that the two would have been paid together had they completed the work on time be A.

The two together lost $\frac{3}{8}$A. This is thrice the amount of wages corresponding to the uncompleted part of the work.

$\therefore$ Amount of wages corresponding to the uncompleted part of the work = $\frac{1}{8}$A.

$\therefore \frac{7}{8}$ th of the work was completed in 14 days.

$\therefore$ Time that Anil and Bala would take to complete the work

$= \left(14 \times \frac{8}{7}\right)$ viz 16 days

Anil can complete the work in 80 days.

$\therefore \frac{1}{80} + \frac{1}{b} = \frac{1}{16} \Rightarrow b = 20.$

**48.** Let the job done by each man, each woman and each machine be '$a$' units, '$b$' units and '$c$' units respectively.

$\Rightarrow 2a = 3b = 4c$; Let the numbers of men, women and machines who work together be 3, 4 and 5, respectively.

Job $= 3a + 4b + 5c = 3(2c) + 4\left(\frac{4}{3}c\right) + 5c = \frac{49}{3}c$ units.

Total share of women $= \frac{\frac{16}{3}c}{\frac{49}{3}c}(4900) = ₹1600.$

**49.** A works for 2 days

$\therefore$ He earns $\frac{2}{8} \times 6000 = ₹1500$

**50.** The given work requires 4800 man days. During the first 20 days, 1200 man days of work has been completed. In the next 20 days, 1000 man days of work will be completed. Work remaining is 2600 man days of work. Hence, the time required to complete this is 2600 / 40 = 65 days.

## EXERCISE-3

**1.** Let the work be 129 units.

Total work done is

$1 + 2 \times 2 + 3 \times 4 + 4 \times 8 + 5 \times 16$, i.e., 129.

Hence, it will be completed in 5 days.

**2.** Let P take $x$ days to complete the work individually.

R takes $5x$ days

Q takes $2x$ days and

S takes $6x$ days to complete the work individually.

Amount of work done by

P is $1 - \frac{5}{16} - \frac{1}{4} - \frac{3}{16} = \frac{1}{4}$

$\therefore$ Time taken by P to do his part of work $= x \times \frac{1}{4}$

Time taken by Q to do his part of work $= 2x \times \frac{5}{16}$

Time taken by R to do his part of work $= 5x \times \frac{1}{4}$

Time taken by S to do his part of work $= 6x \times \frac{3}{16}$

Total time taken = 13 days.

$\therefore \frac{x}{4} + \frac{5x}{8} + \frac{5x}{4} + \frac{9x}{8} = 13$

$\frac{2x + 5x + 10x + 9x}{8} = 13$

$\frac{26x}{8} = 13$

$\therefore x = 4$

The time taken by P, Q, R and S (in days) to complete the work individually is 4, 8, 20 and 24 respectively.

**3.** Ram takes 50 days and Shyam takes 60 days.

Let the job be 300 units [L.C.M. (50, 60)].

Ram works at the rate of 6 units/day.

Shyam works at the rate of 5 units/day.

From 20 January 2014 to 7th February 2014, there are 3 weeks or 15 weekdays. Work completed in this period = 165 units.

From 10th February 2014 to 14th February 2014, only Shyam worked. He completed 25 units in this period. Remaining work to be completed at the end of 14th February 2014 is 110 units.

Ram joined Shyam on 17th February 2014. The two would have taken 10 week days to complete the remaining work.

The job was completed on 28 Feb 2014.

**4.** One man can do 1/360th part of the work in one day.

On the first day, work completed = 1/360

Second day = 2/360. Third day = 3/360 and so on.

$\frac{1}{360} + \frac{2}{360} + \frac{3}{360} + \ldots + \frac{(n-1)}{360} + \frac{n}{360} = 1$

or

$\frac{\Sigma(n-1)}{360} < 1 \leq \frac{\Sigma n}{360}$

where $n$ is the day on which the work is completed.

So, $n = 27$, i.e., $\frac{351}{360} < 1 < \frac{378}{360}$

So, on the 27th day the work was completed and 27 men worked on that day.

**5.**

| Tanks | Oil flowing into the tank | Oil flowing out of the tank |
|---|---|---|
| A | 70 + 60 = 130 | 90 + 50 = 140 |
| B | 30 + 70 = 100 | 70 + 80 = 150 |
| C | 100 + 120 = 220 | 60 + 70 = 130 |
| D | 90 + 80 = 170 | 120 + 110 = 230 |
| E | 50 + 110 = 160 | 100 + 30 = 130 |

Effectively per minute, the quantity of oil content of tanks A, B and D decreases by 10 litres, 50 litres and 60 litres respectively.

Since maximum oil flows out of tank D, tank D will get emptied first.

To exhaust 2400 litres at the rate of 60 litres per minute, it would take $\frac{2400}{60}$ = 40 minutes.

Therefore, tank D gets emptied first and it happens in 40 minutes.

**6.** Let the amount of work done by 1 man in 1 day for 12 hours be '$m$' and 1 woman in 1 day for 12 hours be '$w$'.

Total work done by 7 men and 2 women in 8 days working 12 hours a day is 8 $(7m + 2w)$ (1)

Total work done by 6 men and 20 women in 4 days working 12 hours a day is 4 $(6m + 20w)$ (2)

Equating (1) and (2) $56m + 16w = 24m + 80w$

$32m = 64w$

$m = 2w$

Total work = $8\left(7m + 2\left(\frac{m}{2}\right)\right) = 64m$

Number of days 8 men and 8 women take to do the work (working for 12 hours a day) is

$64m/\ (8m + 8w) = 64m\ /\ (8m + 4m) = \frac{64}{12}$ days

$= \frac{64}{12} \times 12 = 64$ hours.

**7.** Let the average consumption be $x$ units per day.

$x_1$ is the increased consumption per day.

600 men $\times$ 24 $\times$ $x$ = 600 $\times$ 18 $\times$ $x$ + 900 $\times$ 3 $\times$ $x_1$

$600 \times 6 \times x = 900 \times 3 \times x_1$ $\frac{x_1}{x} = \frac{4}{3}$

$x_1$ is more than $x$ by $\frac{1}{3} \times 100 = 33\frac{1}{3}$ %

**8.** Let the numbers of dosas which can be made by Raj, Shyam and Tarun in each minute be a, $b$ and $c$, respectively. Let us say Raj worked alone for $t$ minutes.

$a + b + c = 8$; $at + (20 - t)\ (b + c) = 80$

$at + (20 - t)\ (8 - a) = 80$; $(t - 10)\ (2a - 8) = 0$

$t = 10$ or $a = 4$; $a \geq 5$; $\Rightarrow a \neq 4$; $\therefore\ t = 10$

**9.** Let A and B working alone take $t + a$ and $t + b$ days to complete the work, such that A and B working together take $t$ days to complete it.

$$\frac{1}{t+a} + \frac{1}{t+b} = \frac{1}{t};\ \frac{t+a+t+b}{(t+a)(t+b)} = \frac{1}{t}$$

$\Rightarrow 2t^2 + at + bt = t^2 + at + bt + ab$; $\Rightarrow t^2 = ab$; $\Rightarrow t = \sqrt{ab}$

It is given that A takes 16 days more than what A and B together take where as B takes 25 days more than what A and B together take. Therefore, a = 16 and $b$ = 25.

Thus $t = \sqrt{(16)(25)} = 20$.Therefore, A takes 20 + 16 = 36 days and B takes 20 + 25 = 45 days.

| | A | B |
|---|---|---|
| Number of days taken | 36 | 45 |
| Rate of doing work in units | 5K | 4K |

(Since rate of doing work is inversely proportional to the time taken to do the work)

Since A and B together takes 20 days to do the work, total work done = (5 K + 4 K) 20 = 180 K units.

| A | B | Difference between shares of A and B | C |
|---|---|---|---|
| 5K | 4 K | 5 K – 4 K = K | 3 K |

Sum of A, B and C = 5 K + 4 K + 3 K = 12 K units.

To do 180K units, it will take $\frac{180K}{12K}$ days or 15 days.

Thus A, B and C together completed the work in 15 days.

**10.**

| Work done in units by | A | B | C |
|---|---|---|---|
| | 5 K(15) | 4 K(15) | 3 K(15) |
| | 75 K | 60 K | 45 K |

They will share the money in the same ratio.

Difference between B and C = 60 K – 45 K = 15 K units.

If 15 K units = ₹ 1250, 75 K units will be equal to 5(1250) = ₹ 6250

Thus, A got ₹ 6250 as his share.

**11.** P and Q can do $\left(\frac{1}{10}\right)^{th}$ and $\left(\frac{1}{15}\right)^{th}$ of the total work in a day, respectively.

In 6 days, they do $\left(\frac{3}{10}+\frac{3}{15}\right)^{th}$ of the total work, i.e., $\left(\frac{1}{2}\right)$ of the total work

$\therefore$ They complete the work in 12 days.

**12.** In 3 days, A, B and C complete $\frac{1}{30}+\frac{1}{50}+\frac{1}{75}=\frac{1}{15}$ of the work.

In 30 days they do 10/15 = 2/3 of the work.

In 2 days B and C do $\frac{1}{50}+\frac{1}{75}=\frac{1}{30}$ of the work

So, in 20 days they will do 1/3 of the work

So, in 50 days they complete the work.

**13.** Total work done = 25 + 26 + … (25 + $x$ – 1) = 330

Consecutive numbers starting from 25 have to be added until a sum of 330 is obtained. A sum of 330 is obtained when the last number is 35. $\therefore$ 25 + $x$ – 1 = 35 $\Rightarrow$ $x$ = 11.

**14.** Let the amount of work R does in a day be '$x$' units

$\therefore$ Q does 1.5$x$ units of work in a day and P does 1.25 × 1.5$x$ units of work in a day.

If for the given project, Q takes 'y' days then P takes y – 50 days.

$\therefore$ Total work = $(y-50)\times 1\times 25\times 1\times 5x = y\times 1.5x$

$\therefore$ Number of days taken by Q to complete the project y = 250 days $\therefore$ P takes 250 – 50 = 200 days.

And R takes 250 × 1.5 = 375 days

Let the number of days in which R completes the work be '$z$'. Work done

by P in 60 days = $60\times 1.25\times 1.5x$

by Q in 125 days = $125\times 1.5x$

by R in $z$ days = $z\times x$

Total work = $200\times 1.25\times 1.5x$

$250\times 1.5x = 375x$

$\therefore$ $60\times 1.25\times 1.5x + 125\times 1.5x + z\times x = 375x$

$\therefore$ $z = 75$

$\therefore$ R completes the work in 75 days.

**15.** Let Gerard take G days to complete the job.

Given that 12≤ G ≤ 16

Consider, John and Howard working on alternate days starting with Howard (the faster of the two). In every pair of days they do (1/12 + 1/16), i.e., $\frac{7}{48}$ of work and so they complete 42/48, i.e., 7/8 of the work in 6 pairs, i.e., 12 days. Now it is Howard's turn and he does 1/12 of the work on the 13th day.

Finally John completes the remaining part, i.e., $\frac{1}{8}-\frac{1}{12}$ = 1/24 in $\frac{1/24}{1/16}$, i.e., 2/3 days. So, they take $13\frac{2}{3}$ days to complete the work.

$\therefore$ When John and Gerard work on alternate days they take at least $13\frac{2}{3}$ days and at most 16 days.

The minimum days is realized when G = 12 with Gerard starting the work and the maximum days is realized when G = 16.

**16.** The two pipes can fill $\left(\frac{1}{6}\right)^{th}$ and $\left(\frac{1}{12}\right)^{th}$ of the tank in one hour.

Together they can fill $\frac{1}{6}+\frac{1}{12}=\frac{1}{4}^{th}$ of the tank in one hour $\left(\frac{3}{4}\right)^{th}$ of the tank can be filled in 3 hours.

At a height which is $\left(\frac{3}{4}\right)^{th}$ of the total height of the tank, there is a leak, which empties $\left(\frac{1}{20}\right)^{th}$ of the tank in 1 hour.

Therefore, the last $\left(\frac{1}{4}\right)^{th}$ is filled at $\left(\frac{1}{6}+\frac{1}{12}-\frac{1}{20}\right)^{th}$ per hour, i.e., $\left(\frac{1}{5}\right)^{th}$ per hour

$\therefore$ The last $\left(\frac{1}{4}\right)^{th}$ is filled in 1 hour 15 minutes.

$\therefore$ Total time taken to fill the tank is $3+\frac{1}{4}$ hrs = $4\frac{1}{4}$ hrs.

***Solutions for questions 17 and 18***:

Let the scheduled time be 100 days and number of men initially = $n$

$\therefore$ Total man days of work

$= n(100 + 25\% \text{ of } 100) = 125n$

Let the additional workers be employed after $x$ days

$\Rightarrow 125n = nx + n\,(1 + {}^{1}/_{3})\,(100 - x) \Rightarrow x = 25$

i.e.,. 1/4 of scheduled time

**17.** Fraction of work done

$$\frac{\text{Work done}}{\text{Total work}} = \frac{25 \times n}{125 \times n} = \frac{1}{5}$$

**18.** From the above calculations we got the result that the additional workers were employed after 1/4 of the scheduled time.

**19.** Let both the pipes be working at partial capacity for $x$ minutes. In 1 minute, the first pipe fills $1/8^{th}$ of the tank. Since $5/6^{th}$ of the capacity flows, in 1 minute, $1^{st}$ pipe fills only $(1/8)\ (5/6) = 5/48^{th}$ of cistern. In $x$ minutes, it fills $(5x/48)$th part of the cistern. Similarly, the second pipe fills $(1/12)\ (3/4) = (3/48)$th part in 1 min. In $x$ minutes, it fills $(3x/48)$th part of cistern.

When the defects are rectified, in two minutes both the pipes fill $2(1/8 + 1/12) = 5/12^{th}$ part

Now $5x/48 + 3x/48 + 5/12 = 1 \Rightarrow x = 3.5$.

Therefore, the pipes were defective for 3.5 minutes.

**20.** Given three men A, B and C can do the work in 30 days.

Given that A, B, C work at the same rate.

Then, A alone can do the work in 90 days.

B alone can do the work in 90 days.

C alone can do the work in 90 days.

$\therefore$ Total of 90 man days are required to complete the work.

Also given,

A works for first three days and takes rest on the next day.

$\therefore$ In every four days A worked for 3 days.

B works for first five days and takes rest on the next two days.

$\therefore$ In every se days and takes rest on the next three days.

$\therefore$ In every ten days C works for 7 days.

Going by the options, in 39 days A works for $\frac{36}{4} \times 3 + 3$ = 30 days, since, in the last three days A works for 3 days.

B works for $\frac{35}{7} \times 5 + 4 = 29$ days, since, in the last four days B works for 4 days.

C works for $\frac{30}{10} \times 7 + 7 = 28$ days

since, in the last nine days C works for 7 days.

$\therefore$ Total number of man days = 30 + 29 + 28 = 87 man days.

On $40^{th}$ day, A takes rest, B works, and C takes rest.

$\therefore$ On $40^{th}$ day only one man day completed.

On $41^{st}$ day, A works, B takes rest, and C works.

$\therefore$ On $41^{st}$ day only two man days completed.

$\therefore$ In 41 days total man days = 87 + 1 + 2 = 90

$\therefore$ In 41 days the work will be completed.

**21.** In one cycle, all three pipes one opened in turns for 2 min each.

They fill $\frac{2}{20} + \frac{2}{30} - \frac{2}{15} = \frac{6+4-8}{60} = \frac{1}{30}$ th part of the tank. Therefore, in 6 minutes all three pipes can fill $\frac{1}{30}$ th part of the tank.

Let the tank be filled during the $(n + 1)^{st}$ cycle. In $n$ cycles $\frac{n}{30}$ of the tank is filled. During the $(n + 1)^{st}$ cycle at the most, $\frac{6}{60} + \frac{4}{60} = \frac{1}{6}^{th}$ of the tank is filled. We have to determine what part of this last cycle is needed to fill the tank. The first $\frac{5}{6} = \frac{50}{60}^{th}$ of the tank is filled in $\frac{5}{6}(30)$ = 25 cycles, lasting for 150 min. In the next 2 min A fills up $\frac{6}{60}$ and in 2 more min B fills up the remaining $\frac{4}{60}$.

$\therefore$ The tank fills up in $(150 + 2 + 2)$min = 154 min.

**22.** P can complete the work in 4 days or $4 \times 12 = 48$ hours

Similarly Q and R can complete the work in 72 hours and in 144 hours respectively

$\therefore$ In 8 hours P, Q and R can do $\frac{8}{48}, \frac{8}{72}, \frac{8}{144}$, i.e., $\frac{1}{6}, \frac{1}{9}, \frac{1}{18}$ of the work respectively

$\therefore$ In 1 day they can $\frac{1}{6} + \frac{1}{9} + \frac{1}{18} = \left(\frac{1}{3}\right)^{rd}$ of the total work.

By the time P leaves, i.e., after 2 days $\left(\frac{2}{3}\right)^{rd}$ of the total work is completed

Q and R does $\frac{1}{9}+\frac{1}{18}=\frac{1}{6}$ of the total work in 16 hours

$=\frac{2}{3}$ days

$\therefore$ The rest $\left(\frac{1}{6}\right)^{th}$ of the total work is done by Q and R in 16 hours, i.e., $\frac{2}{3}$ days

$\therefore$ Total number of days taken to complete the work is $2+\frac{2}{3}+\frac{1}{3}=3\frac{1}{3}$ days.

**23.** Let L produce $2x$ books in 1 day.

$\therefore$ M and N produce $4x$ and $5x$ books in 1 day.

No. of books produced by

L in 7 days = $14x$

M in 4 days = $16x$

N in 2 days = $10x$

Total no. of books produced is $14x + 16x + 10x$

$= 40x = 1680 \therefore x = 42$

$\therefore$ No of books produced by L, M and N in 3, 6 and 4 days respectively are $2 \times 42 \times 3 + 4 \times 42 \times 6 + 5 \times 42 \times 4$

$= 42\ (6 + 24 + 20) = 2100.$

**24.** 1. If 50% of the work has been completed by 20 men in 30 days, they will take another 30 days to complete the remaining work.

Let $m$ extra men be employed to complete the work in another 20 days. We have

$20\ x\ 30 = (20 + m)\ x\ 20$

Solving $m = 10$ men

**2.** If the work is increased by 20%, then the 20 men will take $60\ x\ 1.2 = 72$ days to complete the entire piece of work. Since they have already worked for 30 days, they will need $72 - 30 = 42$ days to complete the remaining piece of work.

Let $m$ extra men be employed to complete the work in stipulated $(50 - 30 = 20)$ days. We have $20\ x\ 42 = (20 + m)\ x\ 20$

Solving $m = 22$ men.

**25.** Let us say the new men join after '$x$' days.

Given, $1500 \times 30 = 1500 \times 26 + 300\ (26 - x)$

$\Rightarrow x = 6.$

$\therefore$ The new men joined after 6 days.

**26.** Let the ith person be able to do the job in ai days.

Since $n$ people are working in all possible pairs, each person works exactly $(n-1)$ days

$\therefore$ Total work = $(n-1)\ [1/a_1 + 1/a_2 \ldots 1/a_n]$

All then persons working together in a single day would be able to do $w = [1/a_1 + 1/a_2 \ldots 1/a_n]$ of the work

$1 - (n\text{-}1)w = w$

$w = 1/n$

$\therefore$ All persons working together will need exactly $n$ days to finish the work N = 20

**27.** Each person completed one-third of the work.

Amar takes 12 days, Bhavan 16 days and Chetan 24 days, i.e., they take a total of 52 days.

Had Bhavan done extra work equal to the sum of the xth of the work done by Amar and the xth of the work done by Chetan, the extra work he would have done is $x\left(\frac{1}{3}\right) + x\left(\frac{1}{3}\right)$, i.e., $\frac{2}{3}x$

Then Amar and Chetan would have done $(1 - x)$th of the work that they actually did.

The work would have been completed a day earlier.

Time taken by Bhavan to complete $\frac{2}{3}x = \frac{2}{3}x(48) = 32x$

Amar would take $12x$ days less, Chetan would take $24x$ day less and Bhavan would have taken $16(2x)$ days more.

Given $32x - 12x - 24x = -1 \Rightarrow x = \frac{1}{4}$.

**28.** If five men complete in 15 days,

one man can complete in 75 days.

If six women can complete in 25 days

one woman can complete in 150 days.

Three women in 50 days

2 men's work + 3 women's work in 2 days

$=\frac{2}{75}+\frac{1}{50}=\frac{7}{150}$

$\Rightarrow 2 \text{ days} \times 21 = \frac{7}{150} \times 21$

$\Rightarrow 42 \text{ days work} = \frac{147}{150}$

Remaining work $= 1 - \frac{147}{150} = \frac{3}{150} = \frac{1}{50}$

On the last day, two men complete this work in

$\Rightarrow \frac{75}{2} \times \frac{1}{50}$ = 3/4 days

Total work is completed in 42¾ days.

**29.** The two taps should fill the tank in $\frac{30 \times 20}{5}$ = 12 minutes but they take (12 + 6) = 18 minutes.

i.e., they fill 18/12 = 1½ tanks or 1/2 tank extra.

This has been leaked by the emptying pipe in 12 hours.

So, it empties a full tank in 2 × 12

= 24 hours.

**30.** Let the time taken by Aravind, Manoj and Viswanath working alone to complete the work be $x$, $y$ and $z$ hours respectively.

And let the time taken by them working together be $t$ hours.

Given, the time it takes them, working together, to do the work is 2 hours less than Aravind would have taken alone; 10 hours less than Manoj would have taken alone; and one third the time Viswanath would have taken working alone.

i.e., $t = x - 2 \Rightarrow x = t + 2$ (1)

$t = y - 10 \Rightarrow y = t + 10$ (2)

$t = \frac{z}{3} \Rightarrow z = 3t$ (3)

The work done by Aravind, Manoj and Viswanath in one hour is $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$, which is equal to $\frac{1}{t}$

$\therefore \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{t}$ (4)

Substituting the values of $x$, $y$ and $z$ from (1), (2) and (3) in (4), we have

$$\frac{1}{t+2} + \frac{1}{t+10} + \frac{1}{3t} = \frac{1}{t}$$

$$\Rightarrow \frac{1}{t+2} + \frac{1}{t+10} = \frac{2}{3t}$$

$\Rightarrow 3t\,(t + 10 + t + 2) = 2\,(t + 10)\,(t + 2)$

$\Rightarrow 4t^2 + 12t - 40 = 0$

$\Rightarrow (t + 5)\,(4t - 8) = 0$

$t = 2$ or $-5$

$t$ cannot be $-5$, $\therefore t = 2$ hours

**31.** As 10 painters take 10 days to finish the work, one painter would take 100 days to do it.

Work done on the first 9 days

= 10/100 + 9/100 + . . . 2/100 = 54/100

The remaining 46/100 of the work is completed in 46 days by the last painter.

Total time taken = 46 + 9 = 55 days.

**32.** Jay received $\frac{17}{60}$th of the total share.

He worked along with Komal for $\frac{17}{60}(20) = \frac{17}{3}$ days.

Part of the job they would have completed is

$= \frac{17}{3}\left(\frac{1}{20} + \frac{1}{10}\right) = \frac{17}{20}$.

Remaining part, i.e., $\frac{3}{20}$th of the job, would have been done by Irfan on the first day.

Required percentage = $\frac{\frac{3}{20}}{1}(100) = 15\%$.

**33.** Given Suresh in four times as efficient as Anand.

Let Anand's efficiency be $x$.

Then the efficiency of Suresh = $4x$.

Also given Suresh is $33^1/_3\%$ = (1) less efficient than Venkat, i.e., Suresh is (2/3) as efficient as Venkat.

$\therefore$ Venkat is (3/2) as efficient as Suresh, i.e., 150%

$\therefore$ Efficiency of Venkat = $6x$

Also given, Raju is half as efficient as Venkat.

$\therefore$ Efficiency of Raju = $3x$

Sum of efficiencies of Anand, Raju, Suresh and Venkat

= $x + 3x + 4x + 6x = 14x$

$\Rightarrow$ Raju produced $\frac{3}{14}^{\text{th}}$ of the pieces produced by then

in 6 hours, i.e., $\frac{3}{14} \times 392 = 84$ pieces.

$\therefore$ In 8 hours, Raju can produces $84 \times \frac{8}{6}$

i.e., 112 pieces.

**34.** 30 women - 36 days

$\Rightarrow$ ? women - 24 days

The number of women = $30 \times \frac{36}{24} = 45$

$45w = 15w + 30w$

15 women and 15 men ($30w$) are required to do the work in 24 days.

Women's earnings = $\frac{1}{3} \times 60000 = ₹20{,}000$

Men's earnings = $\frac{2}{3} \times 60000 = ₹40{,}000$

The difference between the shares of men and women is ₹20,000.

**35.** Given the dimensions of the tank is 18 m × 10 m × 8 m

∴ The capacity of the tank = $18 \times 10 \times 8\ m^3$

= 1440 × 1000 litre

Given the rate of supply of inlet pipe is 6 litre/sec.

The water supplied by inlet pipe in 1 hour = 6 × 60 × 60

= 21600 litre

Time taken by inlet pipe to fill the empty tank

$= \frac{1440 \times 1000}{21600} = \frac{200}{3}$ hours

Given drain pipe can empty the tank in 50 hours, which is less than $\frac{200}{3}$ hours.

∴ When the two pipes are opened the quantity of water drained out is greater than the quantity of water supplied into the tank, for any given interval of time.

The net effect is draining of the tank.

∴ The fraction of the tank emptied when two pipes are opened = $\frac{1}{50} - \frac{3}{200} = \frac{1}{200}$

∴ It takes 200 hours to empty the full tank.

**36.** Let the work done by a human in one day be 1 unit.

Therefore, total work = (94) (87) units.

On the first day only one robot works who does 1 unit/day.

Work done per day by the $i^{th}$ robot is 2 times the work done by the $(i-1)^{th}$ robot.

Let us denote the robots by $R_1, R_2, R_3, R_4, R_5, \ldots\ldots$

| | $R_1$ | $R_2$ | $R_3$ | $R_4$ | ... | $R_n$ |
|---|---|---|---|---|---|---|
| Work done / day in units | 1 | 2 | 4 | 8 | … | $2^{n-1}$ |
| | Day 1 | Day 2 | Day 3 | Day 4 | ... | Day $n$ |
| Work done on the days | 1 | 1 + 2 | 1 + 2 + 4 | 1 + 2 + 4 + 8 | | $1 + 2 + . + 2^{n-1}$ |
| | 1 | 3 | 7 | 15 | | $2^{n-1}$ |

Now, we have to find for what value of $n$,

$1 + 3 + 7 + 15 + \ldots + 2^n - 1 = (94)\ (87)$

$\Rightarrow (2-1) + (2^2 - 1) + (2^3 - 1) + \ldots + (2^n - 1) = 8178.$

$\Rightarrow 2 + 2^2 + 2^3 + \ldots. + 2^n - n = 8178$

$\Rightarrow 2\ (1 + 2 + 2^2 + \ldots + 2^{n-1}) - n = 8178$

$\Rightarrow 2(2^n - 1) - n = 8178$

$\Rightarrow 2^{n+1} - n = 8180$. For $n = 12$, we get

$2^{13} - 12 = 8192 - 12 = 8180.$

Therefore, the work gets over in 12 days.

**37.** The outlet pipe can empty a volume of 5 × 6 × 7 metres = 210 $m^3$ in 30 hours.

i.e., its rate of emptying is $\frac{210}{30} = 7\ m^3/hr$

Inlet pipe supplies at 5 $m^3$/hr.

∴ The actual rate of emptying is 7 – 5 = 2 $m^3$/ hr

The time taken to empty a tank which is half filled, when both taps are open, is

$\frac{5 \times 6 \times 7}{2} \times \frac{1}{2} = 52.5$ hours.

**38.** In every jump, it covers 1.25 m and slips 0.75 m. So, effectively it covers 0.5 m in 1 jump.

∴ In 98 jumps, it covers 49 m.

In $99^{th}$ jump it covers remaining 1 m and does not slip.

Therefore, it takes 99 jumps to cover 50 m.

**39.** The part of the tank filled if 3 type–A, 2 type–B, 4 type–C and 5 type–D pipes are used in an hour =

$\frac{3}{20} + \frac{2}{25} - \frac{4}{40} - \frac{5}{50} = \frac{15 + 8 - 10 - 10}{100} = \frac{3}{100}$

It will be filled in $33\frac{1}{3}$ hours.

**40.** Let's say $k$ type D pipes are used

$$\frac{1}{20}+\frac{1}{25}-\frac{1}{40}-\frac{k}{50}=\frac{1}{40}$$

$$\frac{k}{50}=\frac{1}{20}-\frac{1}{20}+\frac{1}{25}=\frac{1}{25}\Rightarrow k=2.$$

**41.** Let the work done by A, B & C alone be a units $b$ units and $c$ units respectively

$\therefore$ They can do $\frac{1}{a},\frac{1}{b}$ and $\frac{1}{c}$ units of work in a day.

$\therefore$ Work done by A & B

$$=4\frac{4}{5}\times\left(\frac{1}{a}+\frac{1}{b}\right)=\frac{24}{5}\left(\frac{1}{a}+\frac{1}{b}\right) \quad (1)$$

Work done by B & C

$$=6\frac{6}{7}\times\left(\frac{1}{b}+\frac{1}{c}\right)=\frac{48}{7}\left(\frac{1}{b}+\frac{1}{c}\right) \quad (2)$$

Work done by A & C

$$=5\frac{1}{3}\left(\frac{1}{a}+\frac{1}{c}\right)=\frac{16}{3}\left(\frac{1}{a}+\frac{1}{c}\right) \quad (3)$$

Equating (1), (2) and (3)

$$\frac{24}{5}\left(\frac{1}{a}+\frac{1}{b}\right)=\frac{48}{7}\left(\frac{1}{b}+\frac{1}{c}\right)=\frac{16}{3}\left(\frac{1}{a}+\frac{1}{c}\right)$$

Solving the above equation

$$\frac{1}{a}=\frac{1}{8};\frac{1}{b}=\frac{1}{12};\frac{1}{c}=\frac{1}{16}$$

$\therefore$ A, B and C can do the work alone in 8, 12 and 16 days respectively.

**42.** Daily wages of A + B + C = 1620 / 9 = 180 (1)

Daily wages of A + C = 600 / 5 = 120 (2)

Daily wages of B + C = 910 / 7 = 130 (3)

[(2) + (3)] – (1) gives

[A + B + 2C] – [A + B + C] = C

= (120 + 130) – 180 = ₹ 70

**43.** Given, 8 women take a total of 20 days to build the room.

$\therefore$ 8 × 20 = 160 woman days are required to build the room.

Till January 6, 2009, the number of woman days, completed = 8 × 6 = 48 woman days.

Also given each man works twice as fast as a woman.

$\therefore$ one manday = 2 woman days.

From January 7, 2009 onwards one man per day is added.

Which is equal to the adding of two women per day

$\therefore$ January 7, 2009 onwards 10 woman day, 12 woman days, 14 woman days, ... are completed on successive days.

Number of woman days required to complete the work from January 7, 2009 = 160 – 48 = 112 woman days

$\therefore$ 10 + 12 + 14 + 16 + ... = 112

The above is in arithmetic progression.

Let $10+12\times14+\ldots+[10+(n-1)\,2]\geq112$

Where '$n$' is the number of days required to complete 112 woman day. Consider,

$$\therefore\ \frac{n[10+\left[10+(n-1)\,2\right]}{2}=112$$

$\Rightarrow n^2+9n=112\Rightarrow n^2+9n-112=0$

$(n+16)\ (n-7)=0\ \therefore\ n=-16$ or $n=7$

$n=7$, because $n$ is positive

$\therefore$ Total number of days required to build the room

= 6 + 7 = 13 days

$\therefore$ The room will be ready by the end of January 13, 2009.

**44.** Work done by A and B in 1 day = 5/72

Work done by B and C in 1 day = 1/12

A worked for 8 days, B for 12 days C worked for 6 days to complete the work.

$\Rightarrow 8a+12b+6c=1$ where

$a$, $b$ and $c$ are the works done in one day by A, B and C respectively.

$\Rightarrow (8a+8b)+(4b+4c)+2c=1$

$\Rightarrow 8(5/72)+4(1/12)+2c=1$

$\Rightarrow 40/72+4/12+2c=1$

$\Rightarrow 2c=1-(64/72)=8/72$

$\Rightarrow c=4/72=1/18$

$\therefore$ C alone can complete the work in 18 days.

work done by B in 1 day = $(b+c)-c$

= 1/12 – 1/18 = 1/36

work done by A in 1 day = $(a+b)-b$

= 5/72 – 1/36 = 1/24

$\therefore$ B alone can do it in 36 days and

A alone can do it in 24 days.

**45.** As the efficiency of P is half of Q (or R), P takes 12 hours to empty the full tank. When all the three pipes are opened, the part of the tank filled in one hour = $\frac{1}{6}$ + $\frac{1}{6} - \frac{1}{12} = \frac{1}{4}$. Time taken to fill the tank = 4 hrs.

Time taken to fill $\frac{3}{4}^{\text{th}}$ of the tank = 3 hrs.

The remaining one–fourth cannot be emptied by P and Q and R can fill = $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$ in one hour.

Q and R can fill the tank in 3 hrs.

One–fourth of the tank can be filled in $\frac{3}{4}$ hrs. Therefore, the total time taken is $3\frac{3}{4}$ hrs.

**46.** The ratio of the time taken by A, B and C to complete the job working independently is

$$\frac{1}{1.8\times1.6\times1}:\frac{1}{1.6\times1}:1=\frac{1}{2.88}:\frac{1}{1.6}:1$$

$$=\frac{100}{288}:\frac{10}{16}:1=\frac{25}{72}:\frac{5}{8}:1=25:45:72$$

$\therefore$ A takes $25x$ days, B $45x$ days and C $72x$ days to complete the work.

Now, $45x - 25x = 40 \Rightarrow x = 2$

$\therefore$ Times taken by A, B and C to complete the work are 50 days, 90 days and 144 days respectively.

Portion of work completed by A in 25 days = 25/50 = 1/2

Portion of work completed by B in 30 days = 30/90 = 1/3

$\therefore$ Amount of work remaining = 1 – (1/2 + 1/3) = 1/6

$\therefore$ Time taken by C to complete the remaining work

= 1/6 × 144 = 24 days

**47.** Let the times taken by A, B and C to fill the tank be $a$ hours, $b$ hours and $c$ hours respectively.

$$c=4\frac{1}{2}\left(\frac{ab}{a+b}\right)$$

Taking reciprocals both sides, $\frac{1}{2c}=\frac{1}{9}\left(\frac{1}{a}+\frac{1}{b}\right)$

$$\frac{9}{2c}=\frac{1}{a}+\frac{1}{b} \qquad (1)$$

Similarly $\frac{5}{6a}=\frac{1}{b}+\frac{1}{c}$ (2)

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{11}{20}$$

Adding $\frac{1}{c}$ to both sides of (1),

$$\frac{11}{2c}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{11}{20}\ \ \frac{1}{c}=\frac{1}{10}$$

Adding $\frac{1}{a}$ to both sides of (2),

$$\frac{11}{6a}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{11}{20}$$

$$\frac{1}{a}=\frac{3}{10}$$

$$\frac{1}{b}=\frac{11}{20}-\left(\frac{1}{a}+\frac{1}{c}\right)=\frac{3}{20}$$

$\therefore\ b=6\frac{2}{3}$.

**48.** Let the works done by each worker in group A and each worker in a group B in a day be $a$ units and $b$ units respectively.

Given $3a + 6b = \frac{1}{20}$ (1)

$8a + 4b = \frac{1}{10}$ (2)

Solving (1) and (2) we get $a=\frac{1}{90}$ and $b=\frac{1}{300}$

$$a+b=\frac{1}{72}$$

$\therefore$ One worker from each group take 72 days to complete it.

**49.** Let the total time be $x$ hours.

Volume of tank $p$ that can be filled in 3 hours

$=\frac{1}{6}\times\frac{1}{3}(3)=\frac{1}{6}^{\text{th}}$ of the capacity of the tank.

Volume of tank P that can be filled after 3 hours

$=\frac{1}{6}\times(x-3)^{\text{th}}$ the capacity of the tank.

Volume of tank Q that can be filled in first 2 hours

$=\frac{1}{8}\left(\frac{1}{4}\right)(2)=\frac{1}{16}^{\text{th}}$ of the capacity of the tank.

Volume of tank Q that can be filled after 2 hours

$= \frac{1}{8} \times (x-2)^{th}$ of the capacity of the tank.

Volume of the tank that R = $\frac{1}{10} \times x^{th}$ of the capacity of the tank

$$\frac{1}{6}+\frac{1}{6}(x-3)+\frac{1}{16}+\frac{1}{8}(x-2)+\frac{x}{10}=1$$

$$\frac{1}{6}+\frac{x}{6}-\frac{3}{6}+\frac{1}{16}+\frac{x}{8}-\frac{2}{8}+\frac{x}{10}=1$$

$$\frac{40-120+15-60}{240}+\frac{20X+15X+12X}{120}=1$$

$$-\frac{125}{240}+\frac{47X}{120}=1$$

$$\frac{47X}{120}=\frac{240+125}{240}=\frac{365}{240}$$

$$\therefore X = \frac{365}{240}\times\frac{120}{47}=\frac{365}{94} \text{ hours}$$

Total time taken to fill the tank by P, Q and R is $3\frac{83}{94}$ hrs

**50.** Part of the tank filled in 3 minutes

$$=\frac{1}{20}+\frac{1}{30}-\frac{1}{15}=\frac{1}{60}$$

Part of the tank filled in $55 \times 3$ minutes $= \frac{55}{60}$

Part filled in $(165+1)$ minutes $= \frac{58}{60}$

Part of the tank filled in $(166+1)$ minutes $= \frac{58}{60}+\frac{1}{30}=1$

$\therefore$ It will take 167 minutes

## EXERCISE-4

**1.** Work done by the four persons (each working for 1 day)

$$=\frac{1}{8}+\frac{1}{12}+\frac{1}{16}+\frac{1}{24}=\frac{5}{16}$$

$\therefore$ Work done in 4 days = 5/16

Work done in 12 days = $3\times\frac{5}{16}=\frac{15}{16}$

Remaining work = $1-\frac{15}{16}=\frac{1}{16}$

If P or Q or R start the work on the first day, they can complete the work on the last day. But, T cannot complete the work on that day itself.

**2.** Portion of work done by A, B and C on the first day

= 1/10 + 1/20 + 1/40 = 7/40

Portion of work done by A on the second day = 1/10

$\therefore$ Portion of work done every two days = 1/10 + 7/40 = 11/40

Portion of work completed in 6 days = 3 (11/ 40) = 33/40

On the seventh day, all the three of them work and finish the remaining 7/40 of the work.

**3.** From the time the new workers were added work remaining = (75) (400) mandays

When all the new workers left then we are left with 300 workers with half efficiency ≡ 150 workers

The factor by which effective workers decreased = 3/8

$\therefore$ The work takes 8/3 times the normal time

= 5/3 more than the normal time which is 50 days, i.e., the normal time is 30 days.

The workers must have left at the end of 70 days

Now remaining work = (30) (400) mandays

Only 3/8 of which = (30) (150) will be done in scheduled time.

$\therefore$ Remaining work = $\frac{(30)(250)}{(300)(125)}$ = 20% of total work

**Alternative Method:**

Sometime after the work force is increased the new workers leave. This must have been before the scheduled time for completing the work. If not, the work would be over. But actually, the old workers continue at 50% efficiency and take 50% more than the scheduled time. The initial set of workers complete $\frac{4}{5}$ of the work in the scheduled time ($4t$). In half this time, (beyond the scheduled time) working at 50% efficiency they do

$\frac{1}{4}\left(\frac{4}{5}\right)=\frac{1}{5}$ of the work.

$\therefore$ Work left to be done at the end of scheduled time is 20%.

**4.** Let the work done by pipes I, II and III be $x$, $y$ and $z$ respectively (in liters / hour).

$mx = y + z$ (1)

$my = x + z$

$mz = x + y$

Adding the three equations above, we get $m = 2$.

Adding $x$ on both sides of (1), $(m+1)\,x = x + y + z$

$= \dfrac{\text{Volume of the tank}}{6}$

$x = \dfrac{\text{Volume of the tank}}{6(m+1)} = \dfrac{\text{Volume of the tank}}{18}$

Hence, the tank takes 18 hours to fill.

**5.** Let $x$ be the number of days taken on the first moat, on which all three men are working together

$\therefore$ The second moat took (25/6) days on which only (say) the first man worked.

The 3rd moat took $(5/2)x + 10$ days, on which only the second man worked

And the 4th moat took $(5/2)x$ days, on which only the third man worked.

As all the three men worked on the first moat, the work that they did in 1 day is

$$\frac{6}{52x} + \frac{2}{5x+20} + \frac{2}{5x} = \frac{1}{x} \Rightarrow \frac{2}{5x+20} = \frac{9}{25x}$$

$\Rightarrow 50x = 45x + 180 \Rightarrow x = 36$

$\therefore$ The second moat took $\left(\dfrac{25}{6}\right)x = \left(\dfrac{25}{6}\right)(36) = 150$ days

$\therefore$ The third and fourth moats took 100 days and 90 days respectively.

$\therefore$ Total time for all the moats to be complete

$= 36 +$ maximum of [150, 100, 90]

$= 36 + 150 = 186$ days

**6.** The ratio in which the amount should be split is

$$\frac{1}{150}(36+150) : \frac{1}{100}(36+100) : \frac{1}{90}(36+90)$$

$$= \frac{31}{25} : \frac{34}{25} : \frac{35}{25} = 31 : 34 : 35$$

If amount = 10,000

Then the shares are 3100, 3400, and 3500

$\therefore$ The difference = ₹ (3500 − 3100) = ₹ 400

**7.** Let the amount of work done by each man and each woman be $m$ and $w$ respectively.

$8(5m + 4w) = 4(6m + 13w) \Rightarrow 16m = 20w \Rightarrow m = \dfrac{5w}{4}$.

$\therefore$ Let $w = 4$, i.e., $m = 5$

In winter, let the work done by each man and each woman be $m'$ and $w'$ respectively.

$\therefore$ $m'$ = 80% of $m = 4$ and $w'$ = 87.5% of $w = 3.5$

The work = $8(5m + 4w) = 8[5(5) + 4(4)] = 328$

$p$ men and $q$ women can do this work in a single day in winter

$4p + 3.5q = 328 \Rightarrow 8p + 7q = 656$

$\text{Rem}\left(\dfrac{656}{7}\right) = 5 \Rightarrow \text{Rem}\left(\dfrac{8p}{7}\right) = 5$

$\therefore \text{Rem}\left(\dfrac{p}{7}\right) = 5$ or $p = 7p_1 + 5$

$p_1 = 0 \Rightarrow p = 5 \Rightarrow q = 88$.

This and the other values of $(p, q)$ are listed below.

$8p + 7q = 656$

$= 8(5) + 7(88)$

$= 8(12) + 7(80)$ .....

$= 8(75) + 7(8)$

$\therefore p = 5, 12, \ldots 75$ and

$q = 88, 80, \ldots 8$

There are 11 values of $(p, q)$.

**8.** Let the time taken for the tank to become full be $x$ hours. Time taken by X, Y and Z to fill the tank are $6x$ hours, $7x - 40$ hours and $x + 10$ hours.

Part of the tank filled by X, Y and Z

$$= \frac{1}{x} = \frac{1}{6x} + \frac{1}{7x-40} + \frac{1}{x+10}$$

$$\frac{5}{6x} = \frac{1}{7x-40} + \frac{1}{x+10} = \frac{8x-30}{7x^2-30x-400}$$

$5(7x^2 - 30x - 400) = 6x(8x - 30)$

$13x^2 - 330x + 2000 = 0$

$x = 10$ or $\dfrac{200}{13}$

As $x$ is an integer, $x = 10$

$\therefore 6x = 60$, $7x - 40 = 30$ and $x + 10 = 20$

$\therefore$ Z is the most efficient.

**9.** Let the time taken by A, B and C (in hours) to fill the tank be a, $b$ and $c$ respectively.

$$6a = 7\left(\frac{bc}{b+c}\right)$$

$$6\left(\frac{b+c}{bc}\right) = \frac{7}{a}$$

$$6\left(\frac{1}{b}+\frac{1}{c}\right) = \frac{7}{a} \quad (1)$$

$$3c = 10\left(\frac{ab}{a+b}\right)$$

$$3\left(\frac{a+b}{ab}\right) = \frac{10}{c}$$

$$3\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{10}{c} \quad (2)$$

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c} = \frac{1}{4\frac{8}{13}} = \frac{13}{60} \quad (3)$$

Multiplying (3) by 6

$$6\left(\frac{1}{a}\right)+6\left(\frac{1}{b}+\frac{1}{c}\right) = \frac{78}{60}$$

Subtracting (1) from the above equation

We get $\frac{1}{a} = \frac{1}{10}$

Multiplying (3) by 3

$$3\left(\frac{1}{a}+\frac{1}{b}\right)+\frac{3}{c} = \frac{39}{60}$$

Subtracting the above equation from (2)

we get $\frac{1}{c} = \frac{1}{20}$

$$\frac{1}{b} = \frac{13}{60} - \frac{1}{a} - \frac{1}{c} = \frac{1}{15}$$

$\therefore$ B takes 15 hours to fill the tank.

**10.** If '$x$' is the number of days A, B, C together take to complete the work, then

(A + B) take ($x$ + ½) days and (B + C) take ($x$ + 200% of $x$) days = 3$x$ days. (Work done by A, B in one day) + (work done by B, C in one day) − (work done by A, B, C in one day) = work done by B in one day

$$\Rightarrow \frac{2}{2x+1}+\frac{1}{3x}-\frac{1}{x} = \text{the work done by B per day}$$

$$\Rightarrow \frac{6x+2x+1-6x-3}{3x(2x+1)} = \frac{2x-2}{3x(2x+1)} \quad (1)$$

B's share of the total earnings = 400/1800 = 2/9 (2)

B's share of the earnings as a fraction of the total earnings

$$= \frac{\text{work of B per day}}{\text{work of all persons per day}}$$

Hence, $\frac{2}{9} = \frac{2x-2}{3x(2x+1)} \Big/ \frac{1}{x}$

$$\frac{2}{9} = \frac{(2x-2)x}{3x(2x+1)}$$

$\Rightarrow 9\,(2x-2) = 2(3)(2x+1) \Rightarrow x = 4$

Hence, A, B, C together completed the work in 4 days.

**Alternate Method:**

Let the work done by A, B, C per day be a, $b$, $c$ and let A, B, C take $x$ days to complete the work. As A and B take $x + \frac{1}{2}$ days to finish the work, $\left(\frac{a+b}{2}\right) = cx$ (1)

The work that A and B do in $\frac{1}{2}$ day is the work that C would have done in $x$ days.

As B and C take 3$x$ days to complete the work,

$(b+c)\,(2x) = ax \Rightarrow 2(b+c) = a$ (2)

B's share in the earnings

$$= \frac{b}{a+b+c} = \frac{400}{1800} = \frac{2}{9} \quad (3)$$

$(3) \Rightarrow 7b = 2(a+c) = 2[2b+2c+c]$ (from (2)) $= 4b+6c$

$\Rightarrow b = 2c$

$(2) \Rightarrow a = 2(2c+c) = 6c$

$(1) \Rightarrow x = \frac{a+b}{2c} = \frac{6c+2c}{2c} = 4$

**11.** Let the quantity of water accumulated at the time of detection of the leak be $x$ lts.

Let the quantity of water entering the boat each hour be $q$ liters

12 men worked for 3 hours to pump the boat dry

$\therefore$ They cleared $(x+3q)$ lts in 3 hours.

Let the rate of clearing of each man be K lts/hr

$x + 3q = 3(12)K$ (1)

$$x + 1\frac{1}{2}q = 1\frac{1}{2}(18)K \qquad (2)$$

Solving (1) and (2), $x = 18$ K and $q = 6$ K

Had 15 men worked instead of 12, they would have pumped the boat dry in $t$ hours (say)

$x + tq = (t)\ (15)$K

18K + 6Kt = 15 Kt $\Rightarrow t = 2$

**12.** The days on which Shyam (S) and Tarun (T) work are tabulated below.

| S | T |
|---|---|
| 1, 2 | 1–3 |
| 5, 6 | 7–9 |
| 9, 10 | 13–15 |
| 13, 14 | 19–21 |
| 17, 18 | |

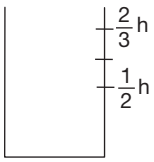
So, in the first 12 days, Shyam and Tarun both work on 1st, 2nd and 9th day, i.e., 3 days.

In the first 120 days Tarun and, Shyam would have worked together for 30 days. In the next 10 days, they would have worked together for 3 days.

∴ Ram worked on 33 days.

**13.** There is no hole till half the tank is filled. Hence, time taken

$$= \frac{1}{2}\left(\frac{1}{\frac{1}{40}+\frac{1}{10}}\right) = 4 \text{ minutes}$$




from $(1/2)h$ to $(2/3)h$, two pipes and 1 leak are in operation and they have to fill $\frac{2}{3}-\frac{1}{2}=\frac{1}{6}$ of the tank.

$$\therefore \text{time taken} = \frac{1}{6}\times\frac{1}{\frac{1}{10}+\frac{1}{40}-\frac{1}{60}} = \frac{20}{13} \text{ minutes}$$

Now, the second leak also comes into operation and the two pipes and two leaks together had to ensure that 1/3 of the tank is filled.

$$\text{So, time taken} = \frac{1}{3}\times\frac{1}{\frac{1}{10}\ \frac{1}{40}-\frac{1}{60}-\frac{1}{120}} = \frac{10}{3} \text{ minutes}$$

$$\text{So, total time taken} = 4+\frac{20}{13}+\frac{10}{3} = 8\frac{34}{39} \text{ minutes}$$

**14.** Let the time taken by P, Q, R and S to fill the tank be $p$ hours, $q$ hours, $r$ hours and $s$ hours respectively.

The flow rates of P, Q, R and S are in descending order.

$$\frac{1}{p}>\frac{1}{q}>\frac{1}{r}>\frac{1}{s}$$

$$\therefore \frac{1}{p}+\frac{1}{q}+\frac{1}{r}=\frac{47}{120} \text{ and}$$

$$\frac{1}{q}+\frac{1}{r}+\frac{1}{s}=\frac{37}{120}$$

$$\therefore \frac{1}{p}+\frac{1}{q}+\frac{1}{r}-\left(\frac{1}{q}+\frac{1}{r}+\frac{1}{s}\right)=\frac{1}{12} \qquad (1)$$

$$\text{Also } \frac{1}{p}=2\left(\frac{1}{s}\right) \qquad (2)$$

From (1) and (2), $s = 12$

**15.** Number of hours Ravi took to complete the job = 80

Regular working hours available = 80

Number of regular hours in which he did not work = 12

∴ Over time = 12

Amount that Ravi must receive

= (68) (150) + (12) (180)

= 80 (150) + 12 (30) = 12360

**Solutions for question 16:**

**16.** The two masons finished the other two sides of the square by working together. They worked for another 18 (2) = 36 days. Now they should share the amount in the ratio in which they did the work

First mason worked for 30 + 36 days

2nd mason for 45 + 36

$$\therefore \text{Ratio} = \frac{(30+36)}{30}:\frac{(45+36)}{45} = \frac{66}{30}:\frac{81}{45} = \frac{11}{5}:\frac{9}{5} = 11:9$$

∴ the second mason would get

$$₹9\ x\ \frac{200000}{(11+9)} = ₹90000$$

**17.** Let the time taken to fill the tank when both pipes are filling it be $t$ minutes. Then the time taken for both pipes together to empty the tank will also be $t$ minutes. It is given that in $t$ minutes only pipe A fills $3/5^{th}$ of the tank. Hence, in $t$ minutes pipe B can fill $2/5^{th}$ of the tank. Now it also follows that pipe A alone can empty $3/5^{th}$ of the tank in '$t$' minutes and pipe B alone would empty $2/5^{th}$

of the tank in '$t$' minutes (when they are used to empty the tank individually). If pipe A is used to fill and pipe B is used to empty for $t/2$ minutes, then the tank will be $(1/2)\ (3/5 - 2/5) = 1/10 = 10\%$ full. Therefore, the tank will be 90% empty.

**18.** The data provides three different combinations of working, to do the work completely.

(a) Combination 1 : –

All three people working for $(x + 6)$ days.

Hence, $(x+6)\left(\frac{1}{20}+\frac{1}{30}+\frac{1}{60}\right) = 1; \Rightarrow x + 6 = 10,$

$\Rightarrow x = 4$ (1)

(b) Combination 2 : –

All three people for $x$ days, and two people for $(y + 6)$ days.

Hence, $x\left(\frac{1}{10}\right)+(y+6)\left(\frac{1}{30}+\frac{1}{60}\right)$

$= 1$; Substituting $x = 4$, the equation becomes $(y + 6)$

$\left(\frac{3}{60}\right) = 1 - \frac{4}{10} = \frac{6}{10}$, $y = 6$ (2)

(c) Combination 3: –

All three for $x$ days, two people for $y$ days and one person for $z$ days (say).

Hence, $x\left(\frac{1}{10}\right) + y\left(\frac{3}{20}\right) + \frac{z}{60} = 1$

Substituting $x = 4$, $y = 12$, and simplifying, $z = 18$ (3)

Total number of days required under combination 3, is $x + y + z = 4 + 6 + 18 = 28$ days.

**Alternate Method:**

Prakash, Pranay and Pramod together complete

$\left(\frac{1}{20}+\frac{1}{30}+\frac{1}{60}\right)^{th}$ of work per day or, $\frac{1}{10}^{th}$ of the work per day. Together, the three take $(x + 6)$ days to complete.

$x + 6 = 10,$

$\Rightarrow x = 4$ (1)

The work remaining after the departure of Prakash can be completed by Pranay and Pramod in $(y + 6)$ days.

$\therefore (y+6)\left(\frac{1}{30}+\frac{1}{60}\right) = \left(1 - \frac{x}{10}\right)$

$\Rightarrow (y + 6)\ 3/60 = [1 - (4/10)]. \Rightarrow y = 6$ (2)

**19.** Let the wages for the two tasks for the first day be A and B respectively. The wages for the two tasks for the 10 days are tabulated below.

| Day | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Task1 | A | 1.05A | 1.1A | 1.15A | 1.2A | 1.25A | 1.3A | 1.35A | 1.4A | 1.45A |
| Task2 | B | 1.1B | 1.2B | 1.3B | 1.4B | 1.5B | 1.6B | 1.7B | 1.8B | 1.9B |

Wages for the first task

(1) for the first 5 days are 5.5 A

(2) for the entire period are 12.25 A

Wages for the second task

(1) for the first 5 days are 6 B

(2) for the entire period are 14.5 B

$\therefore$ 5.5 A + 6 B = 800 $\Rightarrow$ 11 A + 12 B = 1600 and

12.25 A + 14.5 B = 1850 $\Rightarrow$ 49 A + 58 B = 7400

$\therefore A = \frac{1600\,(58) - 12\,(7400)}{11\,(58) - 12\,(49)} = 80$ and

$B = \frac{11\,(7400) - 49\,(1600)}{11\,(58) - 12\,(49)} = 60$

$\therefore$ First day's wages = A + B = 80 + 60 = ₹ 140.

**20.** The days on which Shyam (S) and Tarun (T) work are tabulated below.

| S | T |
|---|---|
| 1–4 | 1–5 |
| 7–12 | 11–15 |
| 17–20 | 21–25 |
| 25–28 | 31–35 |
| 33–36 | |

So, in the first 40 days, Shyam and Tarun both work on days 1–4, 11, 12, 25, 33, 34, 35, i.e., on 10 days.

In the first 40 days Tarun and Shyam would have worked together for 10 days. In the first 680 days Tarun and Shyam would have worked together for 170 days. In the next 20 days, they would have worked together for 6 days.

Shyam would have worked for a total of 176 days along with Tarun.

$\therefore$ Ram worked on 176 days.

**21.** Let the rates of work of each P worker, each Q worker and each R worker be $p$,q and $r$ respectively (all in units/day).

job = $(p + q)\ (72) = (4p + 6r)\,(20) = (2p + 3q + 8r)\,(18)$

(1) (2) (3)

(1) = (2) $\Rightarrow 15r + p = 9q$

(2) = (3) $\Rightarrow 22\,p = 27\,q + 12r$

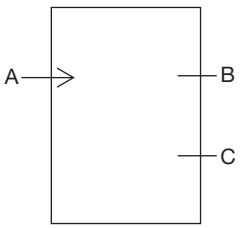
$22(9q - 15r) = 27q + 12r$

$q = 2r \therefore p = 3r$

Job = 72 $(p + q) = 360r$

Time taken by 1 from each group together to complete the job (in days) = $\frac{360r}{p+q+r} = 60.$

**22.** Let the times in which B and C can empty the tank be $x$ hours and $2x$ hours respectively.




Time in which the bottom one third of the tank would be filled = 4 hours.

Part of the tank which A and C can fill each hour

$= \frac{1}{12} - \frac{1}{2x} = \frac{x-6}{12x}.$

$\therefore$ They can fill the tank in $\frac{12x}{x-6}$ hours

$\therefore$ They can fill one-third of the tank in $\frac{4x}{x-6}$ hours

Part of the tank which A, B and C can fill each hour

$= \frac{1}{12} - \frac{1}{x} - \frac{1}{2x} = \frac{x-12-6}{12x} = \frac{x-18}{12x}$

$\therefore$ They can fill the tank in $\frac{12x}{x-18}$ hours

$\therefore$ They can fill one-third of the tank in $\frac{4x}{x-18}$ hours

Total time taken to fill the tank

$= \left(4 + \frac{4x}{x-6} + \frac{4x}{x-18}\right)$ hours $= 16\frac{4}{5}$ hours

$4 + \frac{4x}{x-6} + \frac{4x}{x-18} = 16\frac{4}{5}$

$\frac{4x}{x-6} + \frac{4x}{x-18} = 12\frac{4}{5}$

$\Rightarrow \frac{x}{x-6} + \frac{x}{x-18} = 3\frac{1}{5}$ going by the choices only choice (D) satisfies the equation above.

**23.** Anil works at the rate of 5 units/day and Bala at 4 units/day. They can complete the given job of 200 units in 40 days and 50 days respectively.

From 20 Feb 2012 to 9 Mar 2012, there are 15 weekdays. The work completed is 9(15) = 135 units

From 12 Mar 2012 to 16 Mar 2012, only Anil works and completes 25 units. From 19 Mar 2012 to 22 Mar 2012, both work and complete 36 units. The remaining 4 units gets done on 23 Mar 2012.

**24.** Total capacity of all the pipes in litres/min= $\sum_{i=1}^{N} i$

If the capacity of each filling pipe is numerically equal to the square of its pipe number, then total capacity of all pipes = $\sum_{i=1}^{N} i^2$ and the time taken to fill the tank would be 20% of the normal time.

Hence, $\sum_{i=1}^{N} i^2 = 5\sum_{i=1}^{N} i$

$\Rightarrow \frac{N(N+1)(2N+1)}{6} = \frac{5N(N+1)}{2}$

$\Rightarrow 2N + 1 = 15 \Rightarrow N = 7$

**25.**

| a | e | i | o | u |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |
| 6 | 7 | 8 | 9 | 10 |
| 11 | | | | 15 |

Shyam completes reading '$u$' at intervals of 5 seconds, i.e., at $5k_1$, where $k_1$ is a natural number

| a | b | c | u | z |
|---|---|---|---|---|
| 1 | 2 | 3 | 21 | 26 |
| 27 | 28 | 29 | | |

The letter $u$ is at the 21st position, which is completely read by Ram at the end of the 21st second, and then at the end of the 47th second (after every 26 seconds) or, 21 + 26 $k_2$, where $k_2$ is a natural number.

$\therefore 5k_1 = 21 + 26k_2 \Rightarrow 5k_1 - 26\,k_2 = 21$

$\therefore$ Rem $\frac{21}{5} = 1$

$\therefore \text{Rem}\ \frac{26k_2}{5} = \text{Rem}\ \frac{k_2}{5} = 4$

i.e., $k_2 = 5k_3 + 4$

For $k_3 = 0$, $k_2 = 4$.

$k_1 = 4 + 20 + 1 = 25$

$\therefore$ After $5k_1 = 125$ seconds, they read the letter $u$ together.

**26.** Let the time taken be $t$. The volumetric flow rate $R$ is equal to $\frac{\pi D^2 S}{4}$, where D is the diameter and $S$ is the speed of each pipe.

$$t = \frac{V}{R} = \frac{V}{\frac{\pi}{4}\left(D_1^2 S_1 + D_2^2 S_2 - D_3^2 S_3\right)}$$

**27.** The details of the inflows to and the outflows from each tank are tabulated below. All figures mentioned are in liters/min

| Tank | Inflow from the tank(s) | Outflow to tank(s) | Total inflow | Total outflow | Net inflow |
|---|---|---|---|---|---|
| P | Q(70) | R(130) | 70 | 130 | –60 |
| Q | R(60),S(30) | P(70) | 90 | 70 | 20 |
| R | P(130) | Q(60), S(120) | 130 | 180 | –50 |
| S | R(120) | Q(30) | 120 | 30 | 90 |

In the table, '–' indicates a net outflow.

The tanks to get emptied are P and R. Of these, P empties at a greater rate.

Time taken for P to get emptied = $\frac{1500}{60}$ min = 25 min.

Initially, each tank has 1500 liters of the chemical. The additional capacity in each tank is 500 liters. Tank S would be full in 500/90 min. or $5\frac{5}{9}$ min.

$\therefore$ S triggers the signal, long before P has a chance.

**28.** Let the times taken by P, Q and R to fill (empty) the tank be $x$ hours, $y$ hours and $z$ hours respectively.

$$\frac{1}{y} + \frac{1}{z} - \frac{1}{x} = \text{m}\left(\frac{1}{x}\right) \Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

$$= (m+2)\left(\frac{1}{x}\right) \Rightarrow \frac{1}{x} = \left(\frac{1}{m+2}\right)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$$

$$\frac{1}{x} + \frac{1}{z} - \frac{1}{y} = \text{n}\left(\frac{1}{y}\right) \Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

$$= (n+2)\left(\frac{1}{y}\right) \Rightarrow \frac{1}{y} = \left(\frac{1}{n+2}\right)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$$

$$\frac{1}{x} + \frac{1}{y} - \frac{1}{z} = \text{p}\left(\frac{1}{z}\right) \Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

$$= (p+2)\left(\frac{1}{z}\right) \Rightarrow \frac{1}{z} = \left(\frac{1}{p+2}\right)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$$

$$\therefore \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \left(\frac{1}{m+2} + \frac{1}{n+2} + \frac{1}{p+2}\right)$$

$$\therefore \left[1 - \left(\frac{1}{m+2} + \frac{1}{n+2} + \frac{1}{p+2}\right)\right]\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0$$

As $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ 0, $\frac{1}{m+2} + \frac{1}{n+2} - \frac{1}{p+2} = 1$

**29.** Total time taken to complete the work

$= x + 2x + 3x + 4x + \ldots\ mx = \frac{m(m+1)x}{2}$.

Let the work done by each worker per day be $k$ units.

Total work = Work done by the 1st worker + work done by the 2nd worker + work done by the 3rd worker + … work done by the m th worker

$= k(x + 2x + 3x + 4x + \ldots\ mx) + k(2x + 3x + 4x + \ldots\ mx) + k(3x + 4x + \ldots\ mx) + \ldots\ k(mx)$

$= k(x) + k(2)\ (2x) + k(3)\ (3x) \ldots\ + k(m)\ (mx)$

$= kx\ (1^2 + 2^2 + \ldots m^2)$

$= kx\left(\frac{m(m+1)(2m+1)}{6}\right)$

Part of the total wages obtained by the 1st worker

$$= \frac{kx\left(\frac{m(m+1)}{2}\right)}{kx\left(\frac{m(m+1)}{6}(2m+1)\right)} = \frac{5500}{38500}$$

$$\Rightarrow \frac{3}{2m+1} = \frac{55}{385} \Rightarrow m = 10$$

**30.** If the four outlet pipes are opened simultaneously, they can empty the top $\frac{1}{4}$ th part in $\frac{1}{4}$ hour or 15 minutes. Now, only 3 pipes work. Three pipes can empty the next $\frac{1}{4}$ th part in one hour.

$\frac{1}{4}^{th}$ part of the tank in $\frac{1}{4} \times \frac{4}{3} \times 60 = 20$ minutes.

Now only two pipes work. They can empty half part in one hour.

They can empty the $\frac{1}{4}^{th}$ part in $\frac{1}{2}$ an hour or 30 minutes.

Now the last pipe is left alone to empty $\frac{1}{4}^{th}$ part.

It takes one hour. The total time taken to empty

$= 15 + 20 + 30 + 60 = 125$ minutes.

## EXERCISE-5

***Solutions for questions 1 to 35***:

**1.** From Statement I, the time taken by A and C to complete the work is 30 days, Hence, B also takes 30 days to complete the same work. Hence, we can say that B's rate is the highest.

From Statement II, the question cannot be answered as B or C may be faster than A, or A may be the fastest.

**2.** From Statement I, we have A alone can do the work in 20 days and B alone can do the work in 30 days. So, when A and B work on alternate days,

Work done for the first 2 days $= \frac{1}{20} + \frac{1}{30} = \frac{1}{12}$

Work done for 24 days $= 12 \times \frac{1}{12} = 1$

$\therefore$ They complete the work in 24 days irrespective of who starts the work.

$\therefore$ Statement I alone is sufficient.

**3.** Here, we have to select two persons who take less money per unit work done.

$\therefore$ we need $\left(\frac{\text{rate of work}}{\text{daily wage}}\right)$ value of each of the persons.

So, either statement alone is not sufficient to answer the question.

$\therefore$ Combining I and II, we can say that amount paid per unit work is least for C and the second least for B.

$\therefore$ We can say that B and C can complete the work with least expenses.

**4.** Neither of the statements is independently sufficient to find who can finish the work in less than 18 days.

Combining the two statements, we have (A + B + C)'s 1 day's work = 1/12 and (D + E)'s 1 day's work = 1/20. But as C is fastest among A, B and C, and D is faster between D and E, we have to consider only these two. The least possible time taken by C will be more than 12 days whereas that taken by D is more than 20 days, Hence, C can finish it in less than 18 days but not D.

**5.** Let the work done by A, B, C in one day be a, *b*, *c* respectively.

The question is : Is a *c*?

From Statement I, $a + c = \frac{1}{2}(a + b) \Rightarrow 2a + 2c = a + b$

$\Rightarrow b = 2c + a \Rightarrow b > c$. But we cannot say anything about a and *c*.

From Statement II, $b > c$ But we cannot say anything about a and *c*. Hence, neither statement is sufficient.

**6.** From Statement I, work done by A and C in one day

$= 1/10 - 1/25 = 3/50$

So, A and C together can do the work in

$50/3 = 16\ 2/3$ days.

We don't know the time taken by A and the time taken by C so we can not find who has the highest rate of work. So, Statement I alone is not sufficient.

From Statement II, B and C's one day's work

$= 1/10 - 1/50 = 2/25$

B's one day's work + C's one day's work is 2/25 so either B's one day's work or C's one day's work must be more than 1/50 as the sum of the two fractions which are less than 1/50 is not equal to 2/25.

One of B and C will take less than 50 days to complete the work. So, A's rate of work is not the highest among the rates of work of A, B, C. Statement II alone is sufficient.

**7.** From Statement I alone, since 50% of the work is completed by B in 5 days we can say that work can be completed by B in 10 days. So, we can answer the question. Hence, Statement I alone is sufficient. From Statement II alone, since B works twice as fast as A, he can complete the work in 5 days and hence, we can answer the question. So, Statement II alone is sufficient. Each statement by itself is sufficient.

**8.** Let the works done by a man, boy and woman be *m*, *b* and *w* respectively.

Consider Statement I: $2b + 3w = 1/5$ (1)

$10m + 3b = 1/5$ (2)

We need $6m + 4b + 3w = k$ (3)

To get $6m$, we multiply (2) by $\frac{6}{10}$.

We have $1.8b$ from (2). We need $4b$ in (3).

$\therefore$ We multiply (1) by $\frac{2.2}{2}$ we get $3.3w$ and $3w$, i.e., (3) is not a linear combination of (1) & (2)

Consider Statement II: $9m + 3b = 1/3$ (1)

and $10b + 15w = 1/3$ (2)

We have to see whether we can get

$6m + 4b + 3w = k$ (3)

A linear combination of (1) and (2) gives (3). $6m + 2b$ is obtained by multiplying (1) by $\frac{2}{3}$.

$2b + 3w$ is obtained by dividing (2) by 5.

$\therefore \frac{2}{3}(1) + \frac{1}{5}(2) \Rightarrow 6m + 4b + 3w = \frac{2}{9} + \frac{1}{15}$

$\therefore$ we can not answer the question from I alone, but we can from II alone.

**9.** Given $a + b = 1/10$ (1)

Statement I gives a + $b + c = 1/6$ (2)

Statement II gives $c = 2a$ (3)

Using (1), (2) and (3), the question can be answered.

**10.** Main data gives A + C do 1/20 of the work in 1 day.
(1) B + C do 1/15 of the work in 1 day (2)

Statement I gives A + B do 1/12 in 1 day. We can get C from the three equations and hence, Statement I alone is sufficient. (3)

Statement II gives A + B + C do 1/10 in 1 day. (4)

From (1), (2) and (4), we can get C and hence, Statement II alone is also sufficient.

**11.** Given P + Q = 1/216.

Statement I gives P + R = 1/540. It is clear that Statement I alone is not sufficient as it gives another equation, but not the values of P and R. Statement II gives Q = 1/540. This statement also gives one more equation and hence is not sufficient by itself to find R. When both the statements are taken together, we get three equations and by solving we get P, Q and R and hence can answer the question.

**12.** Let A, B and C be the work done by A, B and C in one day. The problem gives A + B + C = 1/15.

Since A and C get half the amount that A, B, C together get, $(1/15) \times 1/2 = 1/30$ work in one day or A + C = 1/30 and hence, B = 1/30. It implies B takes 30 days to complete the work. Hence, Statement I alone is sufficient. Statement II gives an equation which is not sufficient by itself to answer the question as we don't know how much work is done by B.

**13.** Using both the statements I and II, $(1/12 - 1/8)$th of the work is done in one minute.

**14.** From Statement I, as we know the times taken by P and Q, it follows that the work done in two days is

$$\frac{1}{12} + \frac{1}{36} = \frac{1}{9}$$

As the numerator is one, whoever starts, the work will be completed in $9 \times 2 = 18$ days.

Statement II is irrelevant.

**15.** $10m + 15w$ can do the work in 30 days.

From Statement I,

$18m + 12w$ can do the work in 20 days

so, $30(10m + 15w) = 20(18m + 12w)$

$3(10m + 15w) = 2(18m + 12w)$

$30m + 45w = 36m + 24w$

$21w = 6m$.

So, by converting all men workforce into women workforce we can find $(10m + 15w)$ and $(24m + 15w)$ are equal to how many women.

$(10m + 15w)$ equals those many women and the work done by them in 30 days. So, $24m + 15w$ equals these many women and the time taken by them can be found. So, Statement I alone is sufficient.

From Statement II,

$30(10m + 15w) = 25(12m + 18w)$

$6(10m + 15w) = 5(12m + 18w)$

$60m + 90w = 60m + 90w$

So, how many men equal to how many women is not known. Hence, we can not answer the question. So, Statement II alone is not sufficient.

**16.** Statement I is definitely not sufficient, since it gives no information other than the starting time.

Using Statement II, from 9:30 a.m. to 11:00 a.m., i.e., in $1^1/_2$ hour, $\left(\frac{3}{4}+-\frac{3}{8}\right)$ of the tank is filled, so the time taken to completely fill the tank would be 4 hours. Hence, Statement II alone is sufficient.

**17.** Using Statement I let the number of pots made by a man, woman and child per day be $m$, $w$ and $c$ respectively.

$m + 1w + 1c = 10$ pots/day.

$1w + 1c = 15$pots/3day $= 5$ pots/day

Therefore, from the above two equations we can find the time taken by a man to make 100 pots. Therefore, Statement I alone is sufficient.

Using Statement II, let the number of days taken by a boy to make 100 pots be $x$.

Given $10m(x+1)=25b(x)$and $m = 2b$. We can calculate '$x$'. Therefore, Statement II alone is also sufficient.

**18.** Using Statement I, 10 soldier ants take 6 hours to build anthill, so 5 soldier ants will take 12 hours. As each queen ant is working at half the rate of a soldier ant, 5 queen ants will take 24 hours to build the anthill. Therefore, I alone is sufficient. Using Statement II, from the initial data, we know the time taken by a soldier ant to build anthill. From the given statement the relation between the time taken by a soldier ant and that taken by a queen ant can be found. From these the time taken to build the anthill by five queen ants can be determined. Hence, II alone is also sufficient.

**19.** Using Statement I, we know the time that each individual takes to construct the wall, but we don't know for how long each individual worked in constructing the wall, therefore, we can not answer. Hence, Statement I alone is not sufficient.

Using Statement II, we do not know how long each individual takes to construct the wall. Hence, Statement II alone is also insufficient.

Using I and II, let $x$ be the number of days for which all the four persons worked together

$$\frac{3}{10}+2\left(\frac{1}{10}+\frac{1}{15}+\frac{1}{20}\right)+x\left(\frac{1}{10}+\frac{1}{15}+\frac{1}{20}+\frac{1}{30}\right)=1.$$

From the above equation we can find '$x$', which is the number of days for which Tarun worked. Hence by combining I and II we can answer the question.

**20.** Consider Statement I. Let the time taken by A and B working together be $x$ days. The most efficient of A and B can definitely complete the work in less than $2x$ days. The less efficient can take $5x$ days or more or less C takes $5x$ days. From this, we can conclude that either A or B is the most efficient and A or B or C is the least efficient.

Consider Statement II. To complete the job, A takes as much time as B and C take working together. So, he does more work than either of them in the same time. Therefore, of the three, A is the most efficient.

Statement II is enough to answer the question, but not Statement I.

**21.** Statement I alone is not sufficient as it has no data about the times taken by them to complete the work. From Statement II, Asha works on days 1, 3, 5,…35, i.e., on 18 days to complete the work. On one day she can do 1/18 of the work; Vidya works on days 1,4,7,10,13,16,19, 22 and 25, i.e., on 9 days. On one day, she can do 1/9 of the work.

In a two day period, the part of the work completed by them working on alternate days $= \frac{1}{9}+\frac{1}{18}=\frac{3}{18}=\frac{1}{6}$

$\therefore$ six such periods are required.

Irrespective of who starts the work, the work will be completed in 12 days.

$\therefore$ Statement II alone is sufficient.

**22.** From Statement I, we have the ratio of efficiencies of Varun and Sameer as 1:3. Therefore, we can find the share of Sameer in ₹ 5000. Statement I alone is sufficient.

From Statement II, we can get Sameer's share. The two persons' individual and combined efficiencies and times taken are tabulated below.

| $V$ | $S$ | $VS$ |
|---|---|---|
| Eff. 5 | 3 | 8 |
| No.of Days | 40 | 25 |

$\therefore$ Sameer's salary is $(3/8)(5000) = 1875$.

We can answer the question using Statement II alone

**23.** Statement I tells that the six taps are turned on one after the other at intervals of half an hour. But the flow rates of the taps are not mentioned. This is not enough.

Statement II alone is not sufficient, as we do not know the number of taps. Combining both the statements we have, according to Statement II, each tap can fill a 20 litre tank in 1 hour, i.e., in half an hour it can fill 10 litres. Using this data and the data in Statement I, we can say that the 330 litre tank will be filled in 4 hours.

**24.** Let the number of days in which Rishab's cow, Shrayan's cow and Vashist's cow can individually eat the grass on the farm be R,S and V respectively.

Using Statement I, $\frac{1}{R}+\frac{1}{S}+\frac{1}{V}=\frac{1}{20}$. As we do not have any information about S and V, R cannot be found.

∴ I alone is insufficient.

Using Statement II, as nothing is mentioned about R, the question cannot be answered.

∴ II alone is also insufficient

Using I and II we have

$\frac{1}{R}+\frac{1}{S}=\frac{1}{30}$; $\frac{1}{R}+\frac{1}{S}+\frac{1}{V}=\frac{1}{20}$ and S = 2V.

$\frac{1}{R}+\frac{1}{2V}=\frac{1}{30}$ and $\frac{1}{R}+\frac{1}{2V}+\frac{1}{V}=\frac{1}{20}$

R can be found uniquely from these two equations.

∴ I and II together are sufficient to answer the question.

**25.** Let the time taken by A, B, C and D to fill the tank bea hours, $b$ hours, $c$ hours and $d$ hours respectively.

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}=\frac{1}{2\frac{2}{7}}=\frac{7}{16} \quad (1)$$

$\frac{1}{a}>\frac{1}{b}>\frac{1}{c}>\frac{1}{d}$

Using I, the tank will be filled in the maximum time if B, C and D fill it

$$\therefore \frac{1}{b}+\frac{1}{c}+\frac{1}{d}=\frac{1}{3\frac{9}{13}}=\frac{13}{48} \quad (2)$$

As 1/d is unknown, the question cannot be answered.

I is not sufficient.

Using II, the tank will be filled in the minimum time if A,B and C fill it.

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{2\frac{2}{3}}=\frac{3}{8} \quad (3)$$

As 1/a is unknown, the question cannot be answered

∴ II is not sufficient.

Combining both the statements, adding (2) and (3)

$$\frac{1}{a}+2\left(\frac{1}{b}+\frac{1}{c}\right)+\frac{1}{d}=\frac{31}{48}\ldots \quad (4)$$

Subtracting (1) from (4), $\frac{1}{b}+\frac{1}{c}=\frac{10}{48}$

∴ Time taken by B and C to fill the tank is 48/10 hours

**26.** From Statement I,

1 boy = $\frac{3}{4}$ man ⇒ 5 boys = $\frac{3}{4}\times 5$ men = $\frac{15}{4}$ men

10 men take 6 hours to do the work. So, 5 boys, which is equal to $\frac{15}{4}$ men, can take $\frac{10\times 6}{15/4}$ days to do the work.

∴Statement I alone is sufficient.

From Statement II,

10 men can complete the work in 6 hours. So, 5 men can complete the work in 12 hours.

The amount of work done by 5 boys in one hour

$= \frac{1}{10}-\frac{1}{12}=\frac{1}{60}$

So, 5 boys can do the work in 60 hours.

Statement II alone is also sufficient.

**27.** Either of the statements alone is not sufficient as information about I and II is given in different statements.

Combining statements I and II, the time taken to fill the tank is $\frac{1}{(1/2)-(1/3)}$ = 6 hrs

**28.** From Statement I, work done by the pipes working alternately, in the first 2 minutes = 1/6.

So, the time taken to fill the tank when they are opened alternately = 12 minutes.

So, Statement I alone is sufficient.

From Statement II, we do not know the time taken by the other pipe to fill the tank. So, we can not answer the question.

**29.** From Statement I, it is not known what part of the total work is completed, so we can not answer the question.

From Statement II, B and C did $^{8}/_{23}$ of the work.

∴ A did $1-{}^{8}/_{23}={}^{15}/_{23}$ of the work.

∴ He gets $^{15}/_{23}$ of 529.

∴ Statement II alone is sufficient.

**30.** From Statement I, the question cannot be answered as there is no information about volume of water emptied.

From Statement II, as there is no information about tank, we cannot answer the question.

Combining both the statements, as the tub holds 12 $m^3$ of water, we can say that the tank has a capacity of 1 2 $m^3$ and was emptied in 30 minutes.

**31.** Let the times taken by each person of P and each person of Q to complete the job be $p$ days and $q$ days respectively.

Using Statement I,

$$2\left(\frac{1}{p}\right)+3\left(\frac{1}{q}\right)=\frac{1}{\frac{12}{5}}=\frac{5}{12}$$

$$2\left(\frac{1}{p}+\frac{1}{q}\right)+\frac{1}{q}=\frac{5}{12}$$

$$\frac{1}{p}+\frac{1}{q}=\frac{5}{24}-\frac{1}{2q}$$

$$\therefore \frac{1}{p}+\frac{1}{q}<\frac{5}{24}$$

$\therefore$ Time taken $> \frac{24}{5}$ days.

I is not sufficient.

Using Statement II,

$$3\left(\frac{1}{p}\right)+2\left(\frac{1}{q}\right)=\frac{35}{68}$$

$$3\frac{1}{p}+\frac{1}{q}-\frac{1}{q}=\frac{35}{68}\Rightarrow\frac{1}{p}+\frac{1}{q}=\frac{35}{204}+\frac{1}{3q}$$

$$\frac{1}{p}+\frac{1}{q}>\frac{35}{204}$$

$\therefore$ Time taken $< \frac{204}{35}$ days, i.e., less than 6 days.

$\therefore$ II is sufficient

**32.** Whenever the time taken by two men to complete a job when they work together is an integral number of days, the time taken by them to complete it if they work on alternate days would be twice of that. In the given problem, from Statement I, time taken by both to complete the job is an integral number of days.

$\therefore$ Required time = 2 (20)

= 40 days. I is sufficient

Using II, the time taken when they work together

$$=\frac{30\times 60}{30+60}=20$$

$\therefore$ Required time = 2 (20) = 40 days.

$\therefore$ II is sufficient.

**33.** Given, $\frac{1}{t}=\frac{1}{t+a}+\frac{1}{t+b}$

$$\Rightarrow \frac{(t+b)+(t+a)}{(t+a)(t+b)}=\frac{1}{t}$$

$$\Rightarrow t\,[(t+b)+(t+\mathrm{a})]=(t+\mathrm{a})\ (t+b)$$

$$2t^2+(\mathrm{a}+b)\ t=t^2+(\mathrm{a}+b)\ t+\mathrm{ab}$$

$$t=\sqrt{ab}$$

Using Statement I, ab is unknown.

$\therefore$ $t$ cannot be found

I is not sufficient.

II is sufficient.

**34.** Using Statement I, Rohan and Sohan take less time to complete the job when compared to Rohan and Mohan.

$\therefore$ Sohan is more efficient than Mohan. More information is required about the relation between the efficiencies of Sohan / Mohan and Rohan.

I is not sufficient.

Using Statement II, Rohan and Sohan take less time to complete the job when compared to Mohan and Sohan.

$\therefore$ Rohan is more efficient than Mohan. More information is required about the relation between the efficiencies of Rohan / Mohan and Sohan.

II is not sufficient

Using both statements, Mohan is the least efficient. Both statements are required to answer the question.

**35.** Let the times taken by A, B and C to fill the tank be $a$ minutes, $b$ minutes and $c$ minutes respectively.

$$\frac{1}{a}+\frac{1}{b}-\frac{1}{c}=\frac{1}{6} \qquad (1)$$

Using Statement I,

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{2} \qquad (2)$$

$$(2)-(1)\Rightarrow \frac{1}{c}-\left(\frac{1}{c}\right)=\frac{1}{2}-\frac{1}{6}.$$

$c$ can be found.

Required time = $c$

I is sufficient.

Using Statement II,

$$\frac{1}{b}+\frac{1}{c}-\frac{1}{a}=\frac{1}{6} \quad (3)$$

From (1) and (3),

$$\frac{1}{a}+\frac{1}{b}-\frac{1}{c}=\frac{1}{b}+\frac{1}{c}-\frac{1}{a}$$

$$\frac{1}{a}=\frac{1}{c}$$

$a$ is unknown

$\therefore$ $c$ cannot be found

Required time cannot be found.

II is not sufficient.

# 7 Averages, Mixtures and Alligations

CHAPTER

**LEARNING OBJECTIVES**

*In this chapter, you will*:

- Understand average as a measure of group behaviour
- Learn about various properties of averages, effect of change in individual values on the group average
- Learn real life applications of averages and how to use deviation method to calculate average of a group easily
- Understand the concept of weighted average and about mixtures and their applicability to different parameters
- Apply different approaches to solving mixtures-based questions—Equations and Alligations
- Learn about Alligations—the converse of weighted average
- Learn how to apply mixtures concept to replacement model questions

## ❒ AVERAGES

Average is a very simple but effective way of representing an entire group by a single value.

Average of a group is defined as:

**FORMULA**

$$\text{Average} = \frac{\text{Sum of all items in the group}}{\text{Number of items in the group}}$$

Sum of all the items in the group means sum of the values of all the items in the group.

A batsman's performance can be expressed as the average number of runs scored per innings rather than giving the scores in individual innings. For example, let us say a cricketer scored the following runs in 9 different innings in a year: 35, 56, 124, 29, 0, 87, 98, 45 and 75. Then, his average score (per innings) for the year is

$$\frac{35+56+124+29+0+87+98+45+75}{9} = 61$$

Similarly, if there are 60 students in a class, instead of talking of the height of each individual student, we can talk of average height of the class. The average height of the class of students is equal to the sum of the heights of all the students of the class divided by the number of students in the class.

Average is also called mean or mean value of all the values.

### ❒ Points to Remember

1. If the value of each item is increased by the same value $p$, then the average of the group or items will also increase by $p$.
2. If the value of each item is decreased by the same value $p$, then the average of the group or items will also decrease by $p$.
3. If the value of each item is multiplied by the same value $p$, then the average of the group or items will also get multiplied by $p$.
4. If the value of each item is divided by the same value $p$ ($p \neq 0$), then the average of the group or items will also get divided by $p$.

5. The average of a group of items will always lie between the smallest value in the group and largest value in the group, i.e., the average will be greater than the smallest value and less than the largest value in the group.

## ❒ An Easy Method to Calculate Averages

As already discussed, the average of a group of items whose values are given can be found out by the rule given at the beginning of this section. However, in most of the cases, we do not need to perform such elaborate additions and divisions. The calculation of averages can be simplified greatly by taking some arbitrary number ($P$) as a starting point, take the deviations (differences) of the given items ($Q_i$) from this arbitrary number, find the average of all these deviations ($Q_i - P$) and algebracially add it to the arbitrary number ($P$) to give the correct average of the given items.

If there are $n$ items and they are denoted by $Q_1$, $Q_2$, $Q_3$, ..., $Q_n$, then the average of these $n$ items is given by

**FORMULA**

$$\text{Average} = P + \frac{1}{n}\sum_{i=1}^{n}(Q_i - P)$$

The extent to which this method will simplify the calculation will depend on the selection of the arbitrary value $P$. It should be selected in such a way that the positive and negative deviations cancel out each other to the extent possible. Then, the final figure left for division will be relatively small making the division easier.

For example, the cricketer that we considered above scored the following runs in seven innings: 35, 56, 45, 43, 67, 70 and 48. Now, to find his average, we take an arbitrary figure, say 50 and first find the deviations of each of the scores from this figure. The deviations of the scores from 50 are –15, +6, –5, –7, +17, +20 and –2. The sum of these deviations is +14.

Hence, the average of the cricketer's scores is

$$50 + \frac{14}{7} = 52$$

Please note that the number $P$ (= 50 above) can be any value. Let us work out the same example taking a different value for $P$. Let us take $P$ equal to 45. The deviations of the scores from $P$ are –10, +11, 0, –2, +22, +25 and +3. The sum of these deviations is 49. Hence, the average is 45 + 49/7 = 45 + 7 = 52.

## ❒ Weighted Average

When two groups of items are combined together, then we can talk of the average of the entire group. However, if we know only the average of the two groups individually, we cannot find out the average of the combined group of items.

For example, there are two sections $A$ and $B$ of a class where the average height of section $A$ is 150 cm and that of section $B$ is 160 cm. On the basis of this information alone, we cannot find the average of the entire class (of the two sections). As discussed earlier, the average height of the entire class is

$$\frac{\text{total height of the entire class}}{\text{total number of students in the entire class}}$$

Since we do not have any information regarding the number of students in the two sections, we cannot find the average of the entire class. Now, suppose that we are given that there are 60 students in the section $A$ and 40 students in section $B$, then we can calculate the average height of the entire class which, in this case will be equal to $\frac{60\times150+40\times160}{60+40} = 154$ cm.

This average height 154 cm of the entire class is called 'weighted average' of the class.

The above step in calculating the weighted average of the class can be rewritten as below:

$$\frac{60\times150+40\times160}{60+40}$$

$$= \frac{60}{100}150 + \frac{40}{100}160 = \frac{3}{5}150 + \frac{2}{5}160$$

It is clear from the above step that we would have been able to calculate the average height of the entire class even if we had not been given the number of students in the individual sections but only the ratio of the number of students in the two sections (which in this case is 3 : 2).

Even if there are more than two groups of items to be combined, then also the weighted average can be calculated by the same method. For example, if three sections in a class have their average marks as 75, 76

and 79, respectively and their respective strengths are 30, 35 and 35, then the average mark of the entire class is given by

$$\frac{30 \times 75 + 35 \times 76 + 35 \times 79}{30+35+35} = 76.75$$

The method of deviations we used for calculating averages can be applied to calculate weighted average also. Here, that method will involve finding out deviations from the arbitrarily chosen number and calculating the weighted average of these deviations. In the above example, if we take 70 as the arbitrary figure, then the deviations of the three observed values given from 70 are +5, +6 and +9. The weighted average of these deviations is

$$\frac{30\times 5 + 35\times 6 + 35\times 9}{30+35+35} = \frac{675}{100} = 6.75.$$

Hence, the weighted average will be

$$70 + 6.75 = 76.75$$

The arbitrary figure chosen can be any figure and if it is selected, as in the previous case, between the smallest and largest observed figures, some of the deviations will be positive and some negative making the final division relatively simpler. For example, in the above case, if we take with 76 as the arbitrary figure, the deviations are –1, 0 and +3. Then the weighted average will be

$$\frac{30\times(-1) + 35\times 0 + 35\times(+3)}{30+35+35} = \frac{75}{100} = 0.75$$

Hence, the weighted average will be

$$76 + 0.75 = 76.75$$

'Weighted Average' can be defined or calculated for any *mixture*.

## ❒ MIXTURES

Mixing of two or more qualities of things produces a mixture. When two items of different qualities are thus mixed, the quality of the resultant mixture lies in between the qualities of the original constituent items, i.e., it will be higher than the lowest quality and lower than the highest quality of the items being mixed.

In the above example that we took, the quality that we looked at was the height of the students. We could also have taken their weights or the marks scored by them or any other quality or parameter and calculated the weighted average value of that particular quality for the entire group.

Similarly, if two types of a product of different prices per unit are mixed, the unit price of the resultant mixture will lie between the prices of the two types that form the mixture.

Here, the average quality is essentially the weighted average of the two constituent items.

If $q_1$ is the quantity (or number of items) of one particular item of quality $p_1$, and $q_2$ be the quantity (or number of items) of the second item of quality $p_2$ are mixed together to give a new mixture, then the weighted average value ($p$) of the quality of the mixture is given by

**FORMULA**

$$p = \frac{p_1\ q_1 + p_2\ q_2}{q_1 + q_2}$$

Even if there are more than two groups of items mixed, the weighted average rule can be applied. We will only have to take figures (as shown in the formula for the two groups) for all the groups in the numerator as well as the denominator and calculate the weighted average. For example, if there are four groups of quantities $q_1$, $q_2$, $q_3$ and $q_4$ whose respective qualities are $p_1$, $p_2$, $p_3$ and $p_4$, then the weighted average quality of the group can be written as

**FORMULA**

$$p = \frac{p_1\ q_1 + p_2\ q_2 + p_3\ q_3 + p_4\ q_4}{q_1 + q_2 + q_3 + q_4}$$

*A* mixture can also be a solution, that is, a liquid mixed with another liquid which is normally water. The concentration of the solution is expressed as the proportion (or percentage) of the liquid in the total solution.

For example, if 10 litres of pure alcohol is mixed with 40 litres of water, then in a total solution of 50 litres, there is 10 litres of alcohol. Hence, the concentration of this solution is 0.2 (= 10/50) or 20%.

Similarly, if 30 litres of pure milk is mixed with 10 litres of water, the concentration of this solution can be expressed as 75% (= 30/40) milk or 25% water.

We can also have two solutions mixed together to give a new solution. Such problems can also be handled in the same manner as other mixtures. In the weighted average rule, the quality of the constituents ($p_1$, $p_2$, etc.) will then be the concentrations of various solutions mixed together.

## ❒ ALLIGATIONS

We will take the weighted average rule discussed in the previous section and rewrite the formula such that the quantity terms come on one side and the price terms come on the other side. If we do this we get the rule

$$\frac{q_1}{q_2} = \frac{p - p_2}{p_1 - p}$$

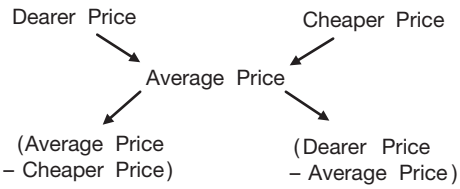
This is called the rule of alligation. This rule connects quantities and prices in mixtures. This can also be written as $\frac{q_1}{q_2} = \frac{p_2 - p}{p - p_1}$

In a descriptive manner, the rule of alligation can be written as $\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}}$

$$= \frac{\text{Rate of Dearer} - \text{Average Rate}}{\text{Average Rate} - \text{Rate of Cheaper}}$$

This rule is a very powerful rule and is useful in problems on weighted averages and mixtures. This rule is also useful in a number of problems which can be treated as mixtures and applied to parameters other than price also. We will take examples where alligation rule can be applied.

In actual practice, to apply alligation rule, we do not need to remember the above formula at all. It can be made very simple by representing the rule pictorially. The above formula can be represented as follows:




**NOTE**

We write the dearer and cheaper prices in one line with some gap in between them. Then, we write the average price in between these two but in the line below the line in which dearer and cheaper prices are written. Then, take the differences of quantities as shown along the arrows and write along the same direction of the arrows continued, i.e., in a diagonally opposite corner. The difference between the top left hand quantity (Dearer Price) and average price has to be written at the bottom right hand corner. Similarly, the difference between the top right hand corner (Cheaper Price) and the average price has to be written at the bottom left hand corner. now the ratio of the two quantities in the bottommost line will give us the ratio of the quantities of dearer and cheaper varieties. please note that since we took dearer price on the top left corner, the ratio of the bottom left figure to that of the bottom right figure will give the ratio of dearer quantity to cheaper quantity.

If there is $P$ volume of pure liquid initially and in each operation, $Q$ volume is taken out and replaced by $Q$ volume of water, then at the end of $n$ such operations, the concentration ($k$) of the liquid in the solution is given by $\left\{\frac{P-Q}{P}\right\}^n = k$

This gives the concentration ($k$) of the liquid as a proportion of the total volume of the solution.

If the concentration has to be expressed as a percentage, then it will be equal to $100k$.

If the volume of the liquid is to be found out at the end of $n$ operations, it is given by kP, i.e., the concentration $k$ multiplied by the total volume $P$ of the solution.

## SOLVED EXAMPLES

**7.01.** In an examination consisting of eight subjects Shyam scored 85, 85, 90, 91, 92 94, 95 and 96 marks. Find his average score.

**Sol:** The average score of Shyam

$$= \frac{\text{sum of the scores}}{\text{Number of subjects}}$$

$$= \frac{85 + 85 + 90 + 91 + 92 + 94 + 95 + 96}{8}$$

$$= \frac{728}{8} = 91 \text{ marks.}$$

**Alternate Method:**
The problem, can be solved using assumed average/mean also.
Let us take 90 as the assumed mean, then the actual average score

$$= 90 + \frac{-5+-5+0+1+2+4+5+6}{8}$$

$$= 90 + \frac{8}{8} = 91 \text{ marks.}$$

**7.02.** The monthly incomes of Raja for four months of a year are ₹15,000, ₹16,000, ₹17,000 and ₹18,000. What is Raja's average income per month for these four months?

**Sol:** Average monthly income

$$= \frac{\text{Total income for the given months}}{\text{Number of months}}$$

$$= \frac{66,000}{4} = ₹16,500.$$

**7.03.** During mid-summer, the average daily temperatures for five consecutive days are 36°c, 39°c, 40°c, 41°c and 44°c. What is the average temperature recorded per day for the five days?

**Sol:** Average temperature =

$$\frac{\text{Total temperature during the given period}}{\text{Number of days}}$$

$$= \frac{36+39+40+41+44}{5} = \frac{200}{5} = 40°C.$$

**7.04.** Chetan purchased two dozen mangoes of one variety at ₹6 per dozen, 4 dozen mangoes of second variety at ₹7 per dozen and one dozen mangoes of third variety at ₹9 per dozen. What is the average cost per dozen of the mangoes purchased by Chetan?

**Sol:** Cost of 2 dozen mangoes of first variety = 2 × 6 = ₹12
Cost of 4 dozen mangoes of second variety= 4 × 7 = ₹28.Cost of 1 dozen mangoes of third variety = 1 × 9 = ₹9
Total cost of 7 dozen mangoes = ₹49
Therefore average cost per dozen of mangoes

$$= \frac{49}{7} = ₹7$$

**7.05.** The average age of a group of 10 friends is 15 years. *A* new friend joins them and their average age increases to 16 years. What is the age of the new friend?

**Sol:** As average $= \frac{\text{Sum of the items}}{\text{Number of items}}$, Sum of the items= (Number of items) × (Average)
Total age of 10 friends = 10 × 15 = 150 years .Total age of 11 friends after a new friend joins = 11 × 16 = 176 years
Age of the new friend = 176 – 150 = 26 years

**7.06.** Eight kgs of rice costing ₹12 per kg is mixed with 7 kgs of rice costing ₹9.60 per kg. What is the average cost per kg of the mixture?

**Sol:** Cost of 8 kg of rice = 8 × 12 = ₹96
Cost of 7 kg of rice = 7 × 9.60 = ₹67.20
Total cost of 15 kg of rice = ₹163.20

$$\text{Average cost of the mixture} = ₹\ \frac{163.20}{15}$$

= ₹10.88 per kg.

**7.07.** A batsman scored an average of 54 runs in the first 6 tests. If the first test is not counted, and the seventh is counted, then his average score goes up to 58 runs. If his score in the first test was 50 runs, find his score in the seventh test.

**Sol:** Total score in the first 6 tests = 6 × 54 = 324 runs
Total score in the last six tests = 6 × 58 = 348 runs.
Score in the seventh test = Total score in all the 7 tests – Total score in the first 6 tests = 348 + 50 – 324 = 398 – 324 = 74

**7.08.** The average weight of all the students in three sections *A*, *B* and *C* of a class are 50 kg, 60 kg and 40 kg, respectively. If the number of students in these three sections is 40, 70 and 90, respectively, then find the average weight of the entire class.

**Sol:** Applying the weighted average rule, the average weight of all the students in three sections of the class is,

$$\frac{(50\times 40)+(60\times 70)+(40\times 90)}{40+70+90}$$

$$= \frac{9800}{200} = 46\text{kg}$$

Alternatively, if we take the arbitrary figure 50 kg as the assumed mean, then the average

$$= 50 + \left(\frac{0\times 40+10\times 70+(-10\times 90)}{40+70+90}\right)$$

$$= 50 + \frac{700 - 900}{200}$$

$$= 50 - \frac{200}{200} = 49 \text{ kg}$$

**7.09.** The rainfall recorded by a city during the seven days of a week was as follows: 5 cm per day on the first 3 days, 8 cm on the fourth day, 10 cm on the fifth day, 12 cm on the sixth day and 7 cm on the seventh day. Find the average rainfall recorded per day during the week.

**Sol:** Average rainfall

$$= \frac{\left[(5 \times 3) + 8 + 10 + 12 + 7\right]}{7}$$

$$= \frac{52}{7} = 7^{3}/_{7} \text{ cm per day.}$$

**7.10.** Suhas earned an average of ₹1500 per month from January through May. Then, he earned ₹1600, ₹1700, ₹1800, ₹1900 and ₹2000 during the months June through October. During November he earned 50% of what he earned in December. If his average earnings for the entire year is ₹1500, then find his earnings in the month of November.

**Sol:** Let the earnings for the month of November be $x$. Then, the earnings in December will be $2x$. Now the total annual earnings will be $(1500 \times 5) + 1600 + 1700 + 1800 + 1900 + 2000 + x + 2x$ = ₹16,500 + 3$x$.

Total earnings for the given year = 12 × 1500 = ₹18,000 which is equal to 16,500 + 3$x$.

$\Rightarrow 16{,}500 + 3x = 18{,}000$

$\Rightarrow 3x = 1500 \Rightarrow x =$ ₹500

Hence, the earnings in the month of November are ₹500.

**7.11.** The average age of employees of a company is 35 years. If five new persons with an average age of 32 years join the company, the average of the entire company becomes 34 years. How many people were there in the company initially?

**Sol:** Let there be $n$ employees in the company initially. The total age of the employees after the five new employees join the company is $35n + (5 \times 32)$. But this is equal to 34 $(n + 5)$.

Therefore, $34(n + 5) = 35n + 160$

$\Rightarrow 34n + 170 = 35n + 160 \Rightarrow n = 10$.

So, there were 10 employees in the company initially.

**7.12.** If 30 kg of rice priced at ₹20 a kg is mixed with 10 kg of a second variety of rice priced at ₹24 a kg, then what is the average cost of the resulting mixture?

**Sol:** Applying the weighted average rule, the average cost of the mixture per kg is

$$\frac{(30 \times 20) + (10 \times 24)}{(30 + 10)} \text{ ₹21}$$

**7.13.** If two kinds of grapes which cost ₹18 a kg and ₹24 a kg are mixed in the ratio of 3 : 2, then find the cost of the mixture per kg.

**Sol:** As the two kinds of grapes are mixed in the ratio 3 : 2, let the actual quantities mixed be 3 $k$ and 2 $k$.

Applying the weighted average rule, the average cost of the mixture can be calculated as

$$\frac{(18 \times 3k) + (24 \times 2k)}{(3k + 2k)} = \frac{102}{5}$$

= ₹20.40 per kg

**7.14.** If 20 litres of 30% milk solution is mixed with 25 litres of 20% milk solution, then find the concentration of milk in the resulting solution.

**Sol:** We can apply the weighted average rule directly taking the concentrations of the 2 mixtures as the parameters in the weighted average rule. Concentration of milk in the resulting solution

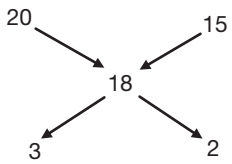
$$= \frac{\frac{30}{100} \times 20 + \frac{20}{100} \times 25}{(20 + 25)} \times 100$$

$$= \frac{11}{45} \times 100 = 24^{4}/_{9}\%$$

**7.15.** How many kilograms of wheat costing ₹20 per kg must be mixed with 28 kg of wheat costing ₹15 per kg, so that the resultant mixture, costs ₹18 per kg?

**Sol:** Applying the rule of alligation,

$$\frac{\text{Quantity costing ₹15 per kg used for mixing}}{\text{Quantity costing ₹20 per kg used for mixing}} = \frac{\text{Dearer price} - \text{Average price}}{\text{Average price} - \text{cheaper price}}$$



$$= \frac{20-18}{18-15} = \frac{2}{3}$$

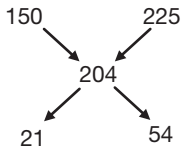
Let the quantity of wheat costing ₹20 per kg to be mixed be $x$.

$$\frac{28}{x} = \frac{2}{3} \Rightarrow x = 28 \times \frac{3}{2} = 42 \text{ kg.}$$

**7.16.** A total amount of ₹204 was collected from boys and girls of a class of 150 students. If each boy contributes ₹1 and each girl contributes ₹1.50, then find the number of boys and girls in the class.

**Sol:** Assuming the class is full of girls total amount collected = 150 × 1.50 = ₹225. Assuming the class is full of boys, total amount collected = 150 × 1 = ₹150.

Ratio of girls to boys can be found by applying the alligation rule.

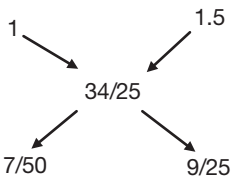



Ratio of girls and boys = 54 : 21

Number of girls = $\frac{54}{75} \times 150$ = 108 Number of boys = $\frac{21}{75} \times 150$ = 42

Alternatively the ratio of boys and girls can be found as follows.Average amount contributed by each student = $\frac{204}{150}$ = ₹34/25

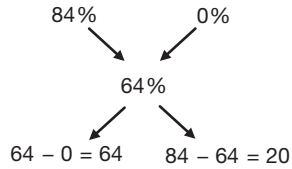
Ratio of boys and girls




i.e., 7 : 18.

**7.17.** There is a 60 litre solution of milk and water in which milk forms 84%. How much water must be added to this solution to make it a solution in which milk forms 64%?

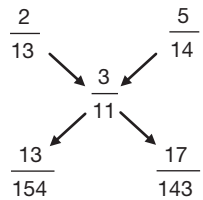
**Sol:** We apply alligation rule here taking 84% milk solution (of milk and water) mixed with a 0% milk solution (or pure water) to give 64% milk solution.




The ratio of the initial solution and water is 64 : 20 = 16 : 5. Since there was 60 litres of solution initially, water to be added is $\frac{60}{16} \times 5 = \frac{75}{4}$ = 18.75 litres

**7.18.** Two vessels contain milk solutions, with milk and water in the ratio of 2 : 11 in the first vessel and in the ratio of 5 : 9 in the second. In what ratio should the contents of these two vessels be mixed such that the resultant mixture has milk and water in the ratio 3 : 8?

**Sol:** Here, we can apply the alligation rule taking the concentration of the mixtures. The concentration of the milk in the first vessel is (2/13) and that in the second vessel is (5/14).




The ratio in which the two mixtures should be mixed is $\frac{13}{154} : \frac{17}{143}$

= 13 × 143 : 17 × 154 = 169 : 238

**7.19.** A vessel has 500 ml of alcohol. 50 ml of alcohol is removed and 50 ml of water is poured into the vessel (bringing the volume of the mixture in the vessel back to 500 ml). If this operation is repeated another 2 times, what is the percentage of alcohol in the vessel at the end?

**Sol:** Using the formula discussed above, concentration of alcohol in the resultant solution

$$= \left(\frac{500-50}{500}\right)^3 = \left(\frac{450}{500}\right)^3 = \left(\frac{9}{10}\right)^3$$

= 0.729

Therefore, percentage of alcohol in the vessel is 72.9%.

**7.20.** Find the average of the scores of the tests taken by Ram given that his scores were 78, 82, 84, 86, 87, 89, 93 and 95.

**Sol:** Average score = $\frac{\text{Total score in all the tests}}{\text{Number of tests taken}}$

$= \frac{78+82+84+86+87+89+93+95}{8} = 86.75$

**7.21.** The monthly incomes of Raja, his wife and their son are ₹6000, ₹4000 and ₹1880, respectively. Find their average monthly income.

**Sol:** Average income

$= \frac{6000+4000+1880}{3} = ₹3960$

**7.22.** Rajiv purchased three dozen mangoes at ₹10 per dozen, two dozen mangoes at ₹15 per dozen and five dozen mangoes at ₹16 per dozen. Find the average cost per dozen of the mangoes that he purchased.

**Sol:** The cost of first three dozen mangoes
= (3) (10) = ₹30
The cost of next two dozen mangoes
= (2) (15) = ₹30
The cost of next five dozen mangoes
= (5) (16) = ₹80
Total cost of the mangoes purchased = ₹140.
Average cost per dozen

$= \frac{\text{Total cost of Mangoes}}{\text{Number of dozens}} = \frac{140}{10} = ₹14.$

**7.23.** The average age of 5 men is 20 years. Their average age increased by 1 year when a new man joined them. Find the age of the new man.

**Sol:** Total age of 5 men = (5) (20) = 100 years
Total age of 6 men = (6) (21) = 126 years
The age of the new man = 126 – 100
i.e., 26 years

**7.24.** Six kilograms of wheat costing ₹18 per kg is mixed with nine kilograms of wheat costing ₹12 per kg. Find the price per kg of the mixture.

**Sol:** Total cost of 6 kg = (6) (18) = ₹108.
Total cost of 9 kg = (9) (12) = ₹108.
Average cost of the mixture

$= \frac{\text{Total cost}}{\text{Total quantity}} = \frac{108\,(2)}{15} = ₹14.40$ per kg.

**7.25.** A batsman had an average of 20 runs in the first 6 tests. In the second to the seventh tests his average score was 21. If he scored a total of 30 runs in the first and the seventh tests, find his score in the seventh test.

**Sol:** Total score in the first 6 tests = (6) (20) = 120.
Total score in the second to the seventh test
= (6) (21) = 126.
Let his scores in the first and seventh tests be $a$ and $b$, respectively. Total score in the second to the seventh test – Total score in the first 6 tests = 6
$\therefore b - a = 6$ (1)
Given, $b + a = 30$ 2)
Solving (1) and (2), $b = 18$.

**7.26.** The average marks of three sections in the tenth class were 90, 120 and 150. If the number of students in these sections are 30, 40 and 50, respectively, find the average mark of the tenth class.

**Sol:** Total mark of the first section = (90) (30) = 2700
Total mark of the second section = (120) (40) = 4800
Total mark of the third section = (150) (50) = 7500

Average mark of class $X = \frac{\text{Total mark}}{\text{Number of students}}$

$= \frac{2700+4800+7500}{30+40+50} = \frac{15000}{120} = 125.$

**7.27.** The sales recorded by a salesman in 6 days of a week are as follows: ₹14,000 per day for the first 3 days, ₹13000 for the fourth day, ₹24000 for the fifth day and ₹26000 for the sixth day. Find his daily average sales over this period.

**Sol:** Total sales for the first 3 days = (3) (14000)
= ₹42000
Sales on the fourth day = ₹13000
Sales on the fifth day = ₹24,000
Sales on the sixth day = ₹26000
Total sales on the 6 days = ₹105000

Average sales per day = $\frac{105000}{6} = ₹17500$

**7.28.** Tarun earned an average of ₹1500 per month from January to April in a year. He earned an average of ₹1600 per month from May to October in that year. His earning in the month of December of that year was ₹300 more than his earning in the month of November of that year. His average monthly earnings for that year was ₹1675. Find his earnings in the month of November.

**Sol:** Total earnings of Tarun from January to April
= (1500) (4) = ₹6000
Total earnings of Tarun from May to October
= (1600) (6) = ₹9600.

Let his earnings in November be ₹ $x$
His earnings in December = ₹ $(x + 300)$

$$\frac{6000+9600+x+x+300}{12}=1675$$

$x = 2100$.

**7.29.** The average age of a group of friends is 37 years. If 6 new friends whose average age is 35 years join them, the average age of the entire group becomes 36 years. How many people were there in the group initially?

**Sol:** Let the initial number of people in the group be $n$.
The total age of the initial group of friends = $37n$ years
The total age of the six friends who joined the group = $35 \times 6$, i.e., 210 years.
Given that, $37n + 210 = 36$ (n+6)
$\Rightarrow 37n - 36n = 216 - 210$
$\therefore n = 6$.

**7.30.** The average age of the boys in a class is 18 years. The average age of the girls in that class is 12 years. If the ratio of the numbers of boys and girls in that class is 3 : 2, find the average age of the class.

**Sol:** Let the number of boys in the class be $3x$.
$\Rightarrow$ Number of girls in that class = $2x$
Total age of the boys = (18) $(3x) = 54x$ years. Total age of the girls = (12) $(2x) = 24x$ years.
Average age of the class

$$=\frac{54x+24x}{3x+2x}=15.6 \text{ years}$$

**7.31.** If 10 litres of 80% alcohol is mixed with 15 litres of 60% alcohol, find the concentration of the resulting solution.

**Sol:** Quantity of alcohol in 10 litres = (0.8) (10) = 8 litres. Quantity of alcohol in 15 litres = (0.6) (15) = 9 litres.
Concentration of the resulting solution

$$=\frac{8+9}{10+15}\times 100=68\%.$$

**7.32.** How many kilograms of rice costing ₹6 per kg must be mixed with 6 kg of rice costing ₹8 per kg so that the resulting mixture costs ₹7×20 per kg?

**Sol:** Let $x$ kg of rice costing ₹6 per kg be used for mixing.
Using alligation rule,

$$\frac{x}{6}=\frac{8-7\cdot 20}{7\cdot 20-6}=\frac{2}{3}\Rightarrow x=4$$

**7.33.** A bag contains a total of 120 coins in the denominations of 50 p and ₹1. Find the number of 50 p coins in the bag if the total value of the coins is ₹100.

**Sol:** Let the number of 50 p coins be $x$.

$$\text{Total value of the coins} = ₹\left[\frac{50x}{100}+1\,(120-x)\right]$$

$$\frac{50x}{100}+120-x=100$$

$$\Rightarrow x=40$$

**Alternative Method:**

$$\text{Average value per coin} = \frac{(100)(100)}{120}$$

$$=\frac{500}{6} \text{ paise.}$$

$$\text{Using allegation rule, } \frac{x}{120-x}=\frac{100-\frac{500}{6}}{\frac{500}{6}-50}=\frac{1}{2}$$

$\Rightarrow 2x = 120 - x \Rightarrow x = 40$.

**7.34.** A vessel has 60 litres of solution of milk and water having 80% milk. How much water should be added to it to make it a solution in which milk is 60%?

**Sol:** Let the quantity of water to be added be $x$ litres.

$$\text{Quantity of milk in the vessel} = \left(\frac{80}{100}\right)(60)$$

= 48 litres $\Rightarrow 48 = 0.6\ (60 + x)$
$\therefore x = 20$

**7.35.** Two vessels contain mixtures of milk and water. The ratios of milk and water are 3 : 10 and 4 : 5 in the first and second vessels, respectively. In what ratio should they be mixed so that the resulting mixture contains milk and water in the ratio 79 : 155?

**Sol:** Milk content in the first milk solution

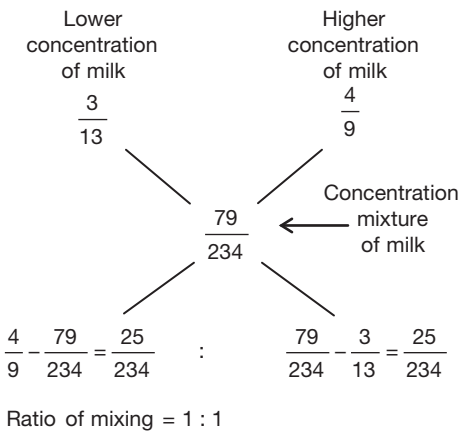
$$=\frac{3}{3+10}=\frac{3}{13}$$

Milk content in the second milk solution

$$=\frac{4}{4+5}=\frac{4}{9}$$

Given that the milk content of mixture of the above two solutions = $\frac{79}{79+155}$

$$=\frac{79}{234}$$

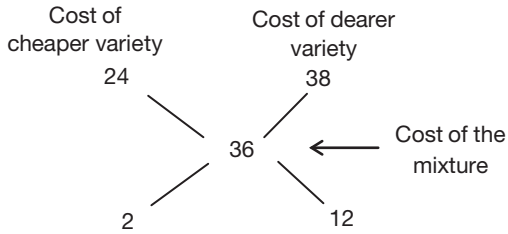



Ratio of mixing = 1 : 1

**7.36.** In what ratio must Anand mix two varieties of barley costing ₹24 per kg and ₹38 per kg so that by selling the mixture at ₹40 per kg he would make $11\frac{1}{9}\%$ profit?

**Sol:** Cost price of the mixture $= \dfrac{40(100)}{100+11\frac{1}{9}}$

= ₹36 per kg




∴ The ratio of the costs of the cheaper to the dearer variety = 2 : 12 = 1 : 6.

**7.37.** A vessel has 400 litres of pure milk. 40 litres of milk is removed from the vessel and replaced by water. 40 litres of the mixture thus formed is replaced by water. This procedure is repeated once again. Find the percentage of milk in the resultant solution.

**Sol:** Let $v$ litre be volume of milk with a concentration of $c_1\%$.

If $x$ litres of the solution is removed and replaced with water, the new concentration is $\left(\dfrac{v-x}{v}\times c_1\right)\%$

Given that initial concentration is 100%. (pure milk), $v$ = 400, $x$ = 40 and the replacement is done thrice.

∴ Concentration of milk in the resultant solution

$= \left(\dfrac{400-40}{400}\right)^3 \times 100 = 72.9\%$

## EXERCISE-1

***Directions for questions 1 to 60*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** The average height of the students in a class is 155 cm. If each student's height is 1 cm more, the average height of the students becomes
(A) 153 cm (B) 158 cm (C) 156 cm (D) 157 cm

**2.** The average age of all the girls in a class of 42 students is 13 years. The average age of all the boys as well as the average age of 23 of the boys of the same class is 16 years. Which of the following could be the average age of the students in the class? (in years)
(A) 13.5 (B) 14 (C) 14.5 (D) 15

**3.** In what ratio should water be mixed with 80% wine so that a 60% wine solution is formed?
(A) 7 : 6 (B) 2 : 3 (C) 5 : 4 (D) 1 : 3

**4.** The sum of five numbers is 655. The average of the first two numbers is 85 and the third number is 125. Find the average of the last two numbers.
(A) 180 (B) 170 (C) 190 (D) 175

**5.** The average number of sweets with six children is six. If another child joined them the average number of sweets with the seven children will become 7. Find the number of sweets with the seventh child. _____

**6.** The average age of seven persons sitting in a row facing east is 28 years. If the average age of the first three persons is 21 years and the average age of the last three persons is 34 years, then find the age of the person sitting in the middle of the row.
(A) 30 years (B) 31 years
(C) 26 years (D) 33 years

**7.** The average of all the even natural numbers less than 50 is _____ .

**8.** The average marks scored by the girls in a class of 45 students in an exam is 90. If the number of boys in the class and their average marks in the same examination are 15 and 60, respectively, what is the average marks of all the students in the class?
(A) 85 (B) 80
(C) 75 (D) Cannot be determined

**9.** Rahim bought 65 books for ₹1150 from one shop and 50 books for ₹920 from another shop. What is the average price he paid per book?
(A) ₹20 (B) ₹18
(C) ₹22 (D) ₹16

**10.** The average monthly income of a man for the first 9 months of a year is ₹12000. His average monthly income for the last 3 months of that year is ₹16000. Find his average monthly income (in ₹) for that year. _____

**11.** The average weight of a group of boys is 30 kg. After a boy of weight 35 kg joins the group, the average weight of the group goes up by 1 kg. Find the number of boys in the group originally.
(A) 5 (B) 4 (C) 6 (D) 7

**12.** In an office, the average salary of the male employees as well as the average salary of the female employees is ₹15000 per month. Find the average salary (in ₹) per month of all the employees.
(A) 1400 (B) 15000
(C) 16000 (D) Cannot be determined

**13.** The average runs scored by a batsman in 20 matches is 40. In the next 10 matches, the batsman scored an average of 13 runs. Find his average score in all the 30 matches is _____ .

**14.** The average weight of 20 students of a class is 25 kg. If one student named Amar leaves the class, the average weight of the class decreases by 0.2 kg. Find the weight of Amar. (in kg) _____

**15.** A vessel contains 25 litres of a mixture of milk and water containing 40% milk. Find the quantity (in litres) of pure milk to be added to the vessel so that the ratio of milk and water in the vessel becomes 1 : 1.
(A) 2.5 (B) 5 (C) 7.5 (D) 10

**16.** ₹6000 is lent out in two parts. One part is lent at 7% p.a simple interest and the other is lent at 10% p.a. simple interest. The total interest at the end of one year was ₹450. Find the ratio of the amounts lent at the lower rate and the higher rate of interest.
(A) 5 : 1 (B) 4 : 1 (C) 3 : 2 (D) 2 : 1

**17.** The average of 11 consecutive natural numbers is $x$. If the sixth number is 12, $x$ = _____ .

**18.** The average marks scored by Mahesh in five out of six subjects in an exam is 60. If Mahesh scored an average of 75 marks in the six subjects, find his score in the sixth subject. _____

**19.** In an exam, Amar scored 64 percent, Bhavan scored 36 percent and Chetan scored 44 percent. The maximum score awarded in the exam is 800. Find the average mark scored by all the three boys.
(A) 384 (B) 364 (C) 324 (D) 404

**20.** A milkman has 20 litres of pure milk. Find the quantity (in litres) of water to be added to it so that he gets 50% profit by selling it at its cost price.
(A) 5 (B) 10 (C) 15 (D) 20

**21.** Find the quantity of tea costing ₹12 per kg to be mixed with 18 kg of tea costing ₹9 per kg to form a mixture costing ₹10.2 per kg.
(A) 24 kg (B) 27 kg (C) 16 kg (D) 12 kg

**22.** The ratio of the number of boys to the number of girls in a class is 3 : 5. If the average weights of the boys and the girls in the class are 20 kg and 16 kg, respectively, find the average weight of the students in the class (in kg).
(A) 17 (B) 18
(C) 17.5 (D) Cannot be determined

**23.** The sales recorded by a salesman in 6 days of a week were as follows:

₹20,000 per day of the first 3 days. ₹16,000 on Thursday, ₹24,000 on Friday and on Saturday it was ₹30,000. Find his average sales per day. (in ₹)
(A) 19333 (B) 25666
(C) 22333 (D) 21667

**24.** The average of the first ten two-digit natural numbers is _____ .

**25.** A bag contains 3 kg of potatoes and 5 kg of tomatoes. If the cost of potatoes is ₹6 per kg and the cost of tomatoes is ₹8 per kg, find the average price (in ₹) per kilogram of the vegetables in the bag. _____

**26.** If two kinds of grapes costing ₹16 a kg and ₹21 a kg are mixed in the ratio of 2 : 3, then find the cost of mixture per kg. (in ₹) _____

**27.** A trader claims to sell jeera at cost price but mixes freely available sand and thereby gains 25%. What is the percentage of sand in the mixture?
(A) $16^2/_3\%$ (B) 20% (C) 25% (D) 12.5%

**28.** Find the average of the numbers 41, 45, 49, 53, 57, 61, 65, 69 and 73. _____

**29.** The average weight of a group of persons increased from 48 kg to 51 kg, when two persons weighing 78 kg and 93 kg join the group. Find the initial number of members in the group.
(A) 21 (B) 22 (C) 23 (D) 24

**30.** 6 kg of sugar costing ₹10 per kg is added to 9 kg of sugar costing ₹15 per kg. At what price (in ₹) should this mixture be sold so that there is no loss or gain? _____

**31.** In an exam, the average mark of the students of a class was calculated as 20. But as the marks of two students were wrongly recorded as 70 and 85 instead of 60 and 77, the actual average was 2 marks less. The number of students who wrote the exam is .

**32.** A total of 57 sweets were distributed among 10 children such that each girl gets 5 sweets and each boy gets 6 sweets. Find the number of girls. _____

**33.** How many kilograms of an ore containing 95% iron and 5% nickel should be mixed with 120 kg of ore having iron and nickel in the proportion 7 : 3 such that the resultant ore contains 4/5 of iron?
(A) 90 (B) 180 (C) 80 (D) 60

**34.** In a month of thirty days, a salesman sells an average of 48 articles per day. If for the first fourteen days, the average sales per day is 80 articles per day, what is his average sales per day for the remaining period of the month?
(A) 24 articles (B) 30 articles
(C) 16 articles (D) 20 articles

**35.** The average monthly income per person in a family of 5 members is ₹9000. In a certain month, each member got ₹900 less than usual. Find the average monthly income (in ₹) per person in that month. _____

**36.** Alok purchased 16 items. The average cost of those 16 items is ₹59. He then returned four items with an average price of ₹60 and three other items costing ₹39, ₹49 and ₹40. Find the average cost of the remaining items. (in ₹) _____

**37.** The average ages (in years) of two groups with the same number of persons are 30 and 40, respectively. What is the average age (in years) of the two groups together?
(A) 30 (B) 35
(C) 40 (D) Cannot be determined

**38.** The average monthly income of a man for the first 4 months, next 4 months and the last 4 months of a year are ₹4000, ₹5600 and ₹6420, respectively. Find his average monthly income (in ₹) for the year.
(A) 5310 (B) 5320 (C) 5340 (D) 5380

**39.** A solution of 80 litres contains 30% alcohol. How many litres of pure alcohol must be added to this mixture so that the concentration of alcohol becomes 65%?
(A) 40 (B) 80 (C) 24 (D) 12

**40.** Two solutions of sulphuric acid are mixed in the ratio of 3 : 7. The first solution contains 20% sulphuric acid and the second solution contains 30% sulphuric acid. Find the concentration (in %) of sulphuric acid in the final mixture. _____

**41.** Vijay purchased 1 dozen mangoes at ₹6 per dozen, 2 dozen mangoes of another variety at ₹10 per dozen and 5 dozen mangoes of a third variety at ₹6 per dozen. Find the average cost per dozen of mangoes purchased by Vijay. (in ₹) _____

**42.** Find the average of (5 + 5 + ... upto 100 times) and (8 + 8 + ... upto 200 times) _____ .

**43.** The average number of chocolates with four children is 4. Another child joins them with a certain number of chocolates. Now, the average number of chocolates with the five children is 5. What is the number of chocolates with the fifth child?
(A) 1 (B) 9 (C) 5 (D) 6

**44.** A vessel contains 10 litres of pure milk. 1 litre of milk is taken out and replaced by an equal amount of water. 1 litre of mixture is then taken out and replaced by an equal amount of water. Find the ratio of milk and water in the final mixture.
(A) 81 : 100 (B) 81 : 19
(C) 19 : 81 (D) 19 : 100

**45.** The average income of a man for first 9 months is ₹10,000 per month, and that for last 3 months is ₹20,000. Which of the following is necessarily true regarding the average income per month for the man?
(A) It is between ₹10,000 and ₹15,000
(B) It is exactly ₹15,000
(C) It is between ₹15,000 and ₹20,000
(D) None of these

**46.** The current average age of a family of four members A, B, C and D is 15 years. Find the average age (in years) of the family after 5 years.
(A) 25 (B) 20 (C) 15 (D) 19

**47.** The average sales of Saraswati Motors from January to June are 150 units. Its average sales from July to October are 240 units. What is the average sales (in units) for the

months November and December if the average sales for the year are 200 units?
(A) 180 (B) 240 (C) 220 (D) 270

**48.** Three numbers have an average of 20. If two of the numbers are 14 and 28, the third number is _____ .

**49.** What is the average cost of manufacturing the article, if the fixed cost of producing 5000 units is ₹40,000 and the variable cost is ₹5 per article?
(A) ₹12 (B) ₹7 (C) ₹5 (D) ₹13

**50.** The average age of a class of 20 students is 20 years. If a new student, whose age is 20 years, joins the class, find the new average age of the class. (in years) _____

**51.** The day temperatures at a place $x$ from Sunday to Wednesday are 45°C, 40°C, 48°C and 52°C, respectively. The day temperatures on Thursday and Friday are 41°C and 42°C. What should be the day temperature on Saturday if the average day temperature of the week is 45°C?
(A) 48°C (B) 47°C (C) 45°C (D) 44°C

**52.** A student scored an average of 80 marks in 3 subjects: Physics, Chemistry and Mathematics. If his average marks in Physics and Mathematics is 90 and that in Physics and Chemistry is 70, what is the marks in Physics? _____

**53.** A vessel contains 20 litres of a mixture of milk and water containing 60% milk. 5 litres of pure milk is added to it. Find the percentage of milk in the new mixture.
(A) 50% (B) 52% (C) 62% (D) 68%

**54.** In a mixture of milk and water, the proportion of milk by weight was 80%. If, in a 180 gm mixture, 36 gms of pure milk is added, what would be the percentage of milk in the mixture formed?
(A) 80% (B) 84%
(C) 87.5% (D) None of these

**55.** In a group there were 19 boys. One boy named Bhavan ate 18 chocolates more than the average number of chocolates eaten by all the boys. If the remaining 18 boys ate an average of 5 chocolates, find the number of chocolates eaten by Bhavan. _____

**56.** The average weights of the boys and the girls in a class are 30 kg and 40 kg, respectively. Find the average weight of all the students in the class (in kg).
(A) 30 (B) 35
(C) 40 (D) Cannot be determined

**57.** The average ages of the students of three colleges A, B and C are 30, 25, and 15 years, respectively. Find the weighted average age of the students if the ratio of the strengths of A, B and C is 1 : 2 : 4.
(A) 40 years (B) 70 years
(C) 20 years (D) 35 years

**58.** A vessel contains 70 litres of a mixture of milk and water containing 90% milk. Find the quantity (in litres) of water to be added to the vessel so that the percentage of milk in the new solution is 87.5%
(A) 1 (B) 2
(C) 3 (D) 4

**59.** A group consists of 11 men. If two men whose ages are 29 and 31 years are replaced by two other men named Amar and Bhavan, the average age of the group drops by one year. The average age of Amar and Bhavan (in years) is _____ .

**60.** In an office, the average height of female employees is 150 cm and the average height of male employees is 160 cm. If there are 300 employees in the office, find the average height (in cm) of all the employees.
(A) 150 (B) 160
(C) 155 (D) Cannot be determined

## EXERCISE-2

***Directions for questions 1 to 60*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** What is the average of all the two-digit numbers which leave a remainder of two when divided by 6? _____

**2.** Two vessels have petrol, diesel and kerosene mixed in the ratio 1 : 2 : 4 and 3 : 5 : 6. If the quantities in the two vessels are mixed in the ratio 1 : 1, what is the ratio of petrol, diesel and kerosene in the resultant mixture?
(A) 5 : 9 : 14 (B) 3 : 6 : 8
(C) 4 : 8 : 11 (D) 4 : 7 : 10

**3.** The average runs scored by a batsman, who has played 52 innings, increased by one after an innings of 126. Find the average runs of the batsman before this innings.
(A) 71 (B) 73 (C) 64 (D) 58

**4.** There are three varieties of rice A, B, and C costing ₹25/ kg, ₹30/ kg, and ₹35/ kg, respectively. A total of 10 kg of A and C and 8 kg of B is mixed to make a mixture costing ₹32/kg. Find the ratio in which A and C are mixed.
(A) 7 : 41 (B) 17 : 31 (C) 7 : 43 (D) 9 : 17

**5.** The heights of five bamboo trees are as follows: 1093 cm, 1103 cm, 1109 cm, 1089 cm and 1096 cm. What is the average height of these trees? (in cm) _____ .

**6.** There are 55 persons. The age of each person (in years) is a two-digit number. The average age of the persons is $A$ years. Had one of the 55 persons, who is ab years old, been ba years old, the average age of the 55 persons would be $1.2A$ years. Find the maximum value of $A$.
(A) 12.1 (B) 13.2
(C) 11 (D) Data inconsistent

**7.** Two varieties of rice are mixed in the ratio 4 : 5. The mixture is sold at ₹60/kg at 20% profit. The prices of the first and the second variety are in the ratio 5 : 6. Find the price of the cheaper variety. (in ₹ per kg) _____

**8.** Two mixtures of milk and water contain milk and water in the ratios 4 : 1 and 3 : 1, respectively. They are mixed in such a way that the resultant mixture contains milk and water in the ratio 77 : 23. Find the ratio in which the two mixtures are mixed.
(A) 3 : 4 (B) 2 : 3
(C) 6 : 7 (D) 8 : 9

**9.** There are three classes—P, Q and R. The number of students in P, Q and R are 18, 24 and 30, respectively. Which of the following is not true?
(A) If a student is shifted from Q to R and the average weight of both the classes increased, then his weight must be between the average weights of the classes.
(B) If the total weight of the students of each class is the same, then the average weight of the three classes is the same as the average weight of Q.
(C) If a student is shifted from P to R and the average weight of both the classes decreased, then his weight must be between the average weights of these classes.
(D) None of these

**10.** The number of students who attended school from Monday to Saturday, in a certain week, were consecutive integers in increasing order. The average number of students attending the school per day that week from Monday to Wednesday was 34. Find the average number of students attending the school per day that week from Monday to Friday. _____

**11.** The average height of a group went up by 5 cm when a person whose height was 160 cm was replaced by another person whose height was 190 cm. Find the number of members in the group.
(A) 6 (B) 7 (C) 8 (D) 9

**12.** The average of $n$ numbers is 32. If three-fourth of the numbers are increased by 4 and the remaining are decreased by 6, what is the new average? _____

**13.** A flask contains pure milk. Five litres of milk is removed and replaced with water. This process is repeated once more. At this stage, the ratio of water and milk is 15 : 49. Find the amount of milk present in the flask at the beginning (in litres). _____

**14.** In an organisation, there are 60 executives and some managers. The average salary of all the employees is ₹15,000. If the average salaries of the executives and the managers are ₹9,000 and ₹45,000, respectively, then find the number of managers. _____

**15.** How many litres of water should a milkman add to 35 litres of milk costing ₹560, so that by selling the milk at ₹14 per litre, he just recovers his cost? _____

**16.** If 10 $cm^3$ of ink can write, on an average, 340 words, how many words can one write with a quarter pint (1 pint = 0.568 litres) of ink?
(A) 4,828 (B) 8,484 (C) 19,312 (D) 24,467

**17.** Fresh grapes contain 80% water by weight. Dry grapes contain 20% water by weight. How many kilograms of fresh grapes will be required to prepare 20 kg of dry grapes? _____

**18.** The strength and the average score of section-1 of a class are respectively equal to the average score and the strength of section-2 of the class. If the strengths of section-1 and section-2 are in the ratio 2 : 1 and the weighted average score of the two sections is $53^1/_3$, which of the following is the average score of section-2?
(A) 60 (B) 80 (C) 40 (D) 100

**19.** A man bought 10 litres of pure milk and added 5 litres of water to it but spilt 2 litres of the mixture. Of the remaining mixture, he sold 3 litres, and again added 2 litres of water to the remaining quantity. Find the approximate percentage of milk in the final mixture.
(A) 56% (B) 55.55%
(C) 56.5% (D) Cannot be determined

**20.** A class has three sections—A, B and C with 34, 51 and 85 students, respectively. If the average marks of the three

sections are 55, 65 and 75, respectively, then what are the average marks of the class? _____

**21.** A vessel is completely filled with milk. 20 litres is withdrawn from the vessel and is replaced with water. 20 litres of the mixture is then replaced with water. The vessel now has 18 litres of milk. Find the capacity of the vessel (in litres)
(A) 70 (B) 50 (C) 60 (D) 40

**22.** An overall profit of 24% is to be made from the sale of two articles. One of them was sold at aloss of 11%. At what profit percentage should the second article be sold, given that the cost prices of the first and the second articles are in the ratio 3 : 5?
(A) 52.5% (B) 45% (C) 35% (D) 40%

**23.** The ace leg spinner Bhayanak Singh Bedi had 1200 runs taken of him in a certain number of matches. In the next match he took 4 wickets conceding 144 runs as a result of which his average (runs conceded / number of wickets taken) increased by 1. How many wickets did he take (in the matches considered) before the last match?
(A) 40 (B) 48 (C) 60 (D) 77

**24.** In a company of 20 employees, each person gets a salary of ₹8000 per month. Ten employees were promoted and got 22.5% increase in their salaries. Two other employees left the company. Find the average income (₹) of the remaining 18 employees.
(A) 9125 (B) 9250 (C) 9500 (D) 9000

**25.** The average of the marks of a group of 10 students is 60. One student leaves the group and the average marks of the group come down by 1. What is the score of the person who leaves the group? _____

**26.** A milkman bought 10 litres of pure milk at ₹10/litre and 10 litres of adulterated milk at ₹6/litre. He mixed both and sold the mixture at ₹10 / litre. Find his profit percentage. _____

**27.** There are 48 students in a class. The age of one of them is twice that of another. If these 2 are replaced by 2 others whose ages are 16 years and 11 years, the average age of the class increases by 1.5 months. Find the age of the younger of the 2 students (in years). _____

**28.** A man purchased a T.V. set and a washing machine for a total of ₹75,000, and made an overall gain of 30% on selling both. If the profits on selling the T.V. set and the washing machine are 20% and 45%, respectively, then find the cost price of the washing machine. (in ₹)
_____

**29.** A vessel had 20 ml of a mixture of milk and water.The mixture has 40% milk. *X* ml of milk was added to the mixture. *Y* ml of water was then added to the mixture. Each of the two additions reverses the ratio of milk and water in the vessel. Find the difference of *X* and *Y*. _____

**30.** Six boys and six girls went to a hotel to have lunch. Each boy paid an equal amount and it is ₹20 more than what each girl paid. The bill amount was ₹1380. Half of the girls as well as half of the boys left after paying their share of the bill. Find the average amount (in ₹) paid by the others. _____

**31.** A vessel contains 10 litres of pure milk. A person takes out 1 litre of milk from the vessel and replaces it with 1 litre of water. Again, 1 litre of the mixture is taken out and replaced with 1 litre of water. This process is repeated one more time. What is the quantity of milk left in the mixture?(in litres) _____

**32.** A's capacity to do a work is the average of C's capacity and twice B's capacity. B's capacity is the average of C's capacity and half of A's capacity. Find the ratio of capacities of A and B.
(A) 5 : 8 (B) 8 : 5 (C) 4 : 5 (D) 5 : 4

**33.** Dev's bowling average (runs conceded per wicket) decreased by 0.5 runs/wicket when he took four wickets and conceded 38 runs, in a match. If his initial average was 23 runs/wicket, how many runs did he concede before this match? _____

**34.** The average age of a group of 10 men decreased by 1 year when a new man aged 23 years joined the group and an existing man left. What is the age of the man who left the group?(in years) _____ .

**35.** Find the average of the first 10 multiples of 50.
(A) 85 (B) 125 (C) 215 (D) 275

**36.** The average weight of Atul, Bhanu, Chinna and Dhanush is 80 kg. Atul's weight is equal to the average weight of the other three. Bhanu's weight equals the average weight of the other three. Find the average weight of Chinna and Dhanush (in kg).
(A) 40 (B) 80 (C) 60 (D) 100

**37.** If 12 kg of rice which costs ₹24/kg is mixed with 6 kg of rice which costs ₹27/kg, then what is the cost of the resulting mixture (in ₹/kg)? _____

**38.** The daily average temperatures in Chennai for the 7 days of a week were recorded. The average temperatures on Sunday, Monday and Tuesday were 20°C, 38°C and 23°C, respectively. The average temperatures on Thursday,

Friday and Saturday were 29°C, 22°C and 30°C, respectively. If the average of average temperatures on the first three days of the week beginning with Sunday equals that of the last four days, find the temperature on Wednesday.
(A) 26°C (B) 25°C (C) 27°C (D) 28°C

**39.** Srikanth earned an average of ₹1,600 per month from January to June. Then, he earned ₹1,500, ₹1,800, ₹1,900 and ₹2,150, respectively during the months July, through October. During November he earned 50% of what he earned in December. If his average earnings for the entire year is ₹1,600. Find his earnings in the month of November. (in ₹) _____

**40.** The average age of a family of five members is the same today as it was five years ago. There is no change in the family except that the elder daughter was replaced by the daughter-in-law. If the age of the elder daughter is 48 years, how old (in years) is the daughter-in-law? _____

**41.** Raju went to the market to buy 1.5 kg of dried peas having 20% water content. He went home and soaked them for some time and the water content in the peas becomes 60%. Find the final weight of soaked peas.
(A) 1.5 kg (B) 0.5 kg
(C) 0.75 kg (D) 3 kg

**42.** A man lent a total of ₹60,000 at simple interest to two persons *P* and *Q* at 12% p.a. and 9% p.a., respectively for two years. If he received a total interest of ₹13,200, then find the sum lent at 9% p.a. (in ₹) _____

**43.** The average age of a couple when they got married was 30 years. Three years after their marriage, a child was born to them. The present average age of the couple and the child is 34 years. Find the present age of the child
(A) 16 years (B) 15 years
(C) 12 years (D) 10 years

**44.** *N* is natural number and *P* is the set {1, 2, 3, …. 2*N*}. *Q* denotes the average of all the odd elements of *P* and *R* denotes the average of all the even elements. Find *R* – *Q*.
(A) 1 (B) 0 (C) 2 (D) 1×5

**45.** There are two vessels *P* and *Q*. *P* containing 120 litres of milk and *Q* containing 120 litres of water. 30 litres is drawn from *P* and poured into *Q* and then 30 litres from *Q* is poured back into *P*. This process is repeated once. At the end of this repetition, what is the ratio of milk and water in *P*? _____

**46.** Two containers contain petrol and diesel in the ratios 4 : 3 and 3 : 1. How many litres from the first container should be mixed with 16 litres from the second so that the ratio in the resultant mixture is 32 : 19? _____

**47.** Raju went to the market to purchase three pens. The cost of the first pen was more than that of the second by 25%, which was more than that of the third by 20%. If the average cost of the first two pens was ₹15 more than that of the last two pens, find the cost of the costliest pen.
(A) ₹60 (B) ₹45 (C) ₹50 (D) ₹90

**48.** In a class there are two sections—A and B. Section A has 50 students and section B has 75 students. The students of both the sections took a maths test. The average marks of sections A and B are 30 marks and 40 marks, respectively. What are the average marks of the whole class?
_____

**49.** The average weight of a group of boys is 30 kg. After 1 more boy, weighing 62 kg, joins the group, the average weight of the group goes up by 2 kg. Find the original number of boys in the group.
(A) 11 (B) 12 (C) 15 (D) 19

**50.** Two varieties of rice are mixed in the ratio 2 : 5 and the mixture is sold at ₹12 per kg at a profit of 20%. If the first variety costs ₹7 per kg more than the second variety, find the cost per kg of the first variety.
(A) ₹7 (B) ₹15
(C) ₹16 (D) None of these

**51.** The average weight of *N* boys in a group is 30 kg. If 5 boys with an average weight of 12 kg join the group the average weight would be the same as if 5 boys with an average weight of 36 kg leave the group. Find *N*
(A) 15 (B) 20 (C) 25 (D) 10

**52.** The average marks obtained by 40 students of a class is 85. The difference between the marks obtained by the students who got the highest mark and the lowest mark is 108. If both these students are removed, the average falls by 1 mark. Find the highest mark.
(A) 144 (B) 158
(C) 172 (D) Cannot be determined

**53.** A shop keeper mixes two varieties of wheat in the ratio 3 : 7, which cost ₹10 per kg and ₹15 per kg, respectively. Find the ratio in which the two varieties of wheat should be mixed when the cost price of the second variety of wheat drops by ₹0.50 per kg and the cost price of the mixture is maintained the same?
(A) 7 : 2 (B) 4 : 5 (C) 2 : 7 (D) 1 : 8

**54.** A certain quantity of rice costing ₹6/kg is mixed with another variety costing ₹8/kg so that the resulting mixture costs ₹7.20/kg. In what ratio were they mixed?
(A) 2 : 3 (B) 3 : 4
(C) 5 : 6 (D) 4 : 5

**55.** If the average marks of first $3/4^{th}$ of the students of a class is $3/4^{th}$ of the average of the entire class, then what is the average marks of the remaining students?

(A) $\frac{4}{3}^{rd}$ of the average of the entire class.

(B) $\frac{7}{4}^{th}$ of the average of the entire class.

(C) $\frac{7}{3}^{rd}$ of the average of the entire class.

(D) $\frac{16}{9}^{th}$ of the average of the entire class.

**56.** A dishonest milkman professes to sell milk at cost price but he makes $33^1/_3\%$ profit by mixing it with water. What is the water content in the milk solution?
(A) 20% (B) $33^1/_3\%$ (C) 25% (D) 30%

**57.** In a class there are two sections—A and B. The average marks obtained by the students of sections A and B in a mathematics test are 30 marks and 40 marks, respectively. If the number of students in sections A and B are in the ratio 2 : 3, what are the average marks of the whole class? _____

**58.** The average of the heights of the students in a class will become 180 cm if three students with in average height of 160 cm join the class and five students with an average height of 168 cm leave the class. If two students whose heights are 172 cm and 182 cm join the class, what will happen to the average height of the class?
(A) It will increase
(B) It will decrease
(C) It will not change
(D) Cannot be determined

**59.** A shopkeeper mixed 3 varieties of wheat costing ₹12 per kg, ₹18 per kg and ₹21 per kg and sold the mixture at ₹15 per kg at 20% profit. Which of the following represents a possible ratio of the varieties mixed?
(A) 60 : 1 : 3 (B) 44 : 1 : 2
(C) 39 : 2 : 1 (D) 27 : 1 : 1

**60.** A mixture contains milk and water in the ratio 3 : 1. After adding 14 litres of water to it, the ratio of milk to water in the resultant solution became 2 : 3. What was the quantity of the original mixture? _____

## EXERCISE-3

***Directions for questions 1 to 60*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** The total cost of producing a certain number of units of an article is partly fixed and partly varies directly with the number of units of that article produced. The average cost of producing 15,000 units of that article is ₹2 while that of producing 25,000 units of that article is ₹1.5. Find the variable cost of each unit of the article produced.
(A) ₹1.25 (B) ₹1.50 (C) ₹0.75 (D) ₹2.50

**2.** A class has 31 students. If the top scorer's score is not considered, the average score of the remaining students falls by 2 marks. Instead, if the lowest scorer's score is not considered, the average score of the remaining students increases by 1 mark. If the top scorer's score and the lowest scorer's score are in the ratio of 10 : 1, what is the score of the top scorer? _____

**3.** The average weight of the students in class *P* is 42 kg and that in class *Q* is 48 kg. The ratio of the number of students in *P* and *Q* is 3 : 4. Find the approximate average weight of the students in classes *P*, *Q* and *R* together given that the average weight of students in *R* is 45 kg and *Q* has 25% more students than *R*.
(A) 45.29 kg (B) 43.42 kg
(C) 46.89 kg (D) 44.17 kg

**4.** Instead of adding the requisite quantity of water to dilute 50% wine to 40% wine, by mistake, the same quantity of pure wine is added. Now, how much water as a fraction of the original volume must be added to achieve the original objective?
(A) $\frac{4}{8}$ (B) $\frac{3}{8}$ (C) $\frac{3}{5}$ (D) $\frac{5}{8}$

**5.** Sixty litres of wine is drawn from 600 lts of wine and replaced with water. Sixty litres of the mixture is then drawn and replaced with water and this procedure is repeated once more. Find the present quantity of wine in the mixture.
(A) 420 *l* (B) 393.66 *l*
(C) 486 *l* (D) 437.4 *l*

**6.** Two cans contain mixtures of milk and water. The first can contains 73% water while the second can contains 41% water. The contents of the two cans are mixed in the ratio 3 : 5. What is the percentage of milk in the new mixture?
(A) 64.25% (B) 22%
(C) 47% (D) 53%

**7.** Roja attempted to add 10 two-digit numbers. One of them, *P*, was the reverse of one of the others. If *P* was replaced by another two-digit number *Q* and its reverse was replaced by the reverse of *Q*, the average of the numbers would be 6.6 more. (The sum of the digits of *Q*) – (The sum of the digits of *P*) = _____ .

**8.** In a class, the number of boys and girls are distinct. The average age of all the students in the class equals the average of the average age of the boys and the average age of the girls. The sum of the average age of the boys and the average age of the girls is 10 years. Find the average age (in years) of the boys of the class.
(A) 10 (B) 5
(C) 15 (D) Cannot be determined

**9.** An educational institution runs six branches. The number of students in each of the branches and the average score of the students in each branch are tabulated below. What is the approximate weighted average score of the students of the institution?

| Branch number | Number of students | Average mark |
|---|---|---|
| 1. | 225 | 60 |
| 2. | 205 | 56 |
| 3. | 180 | 55 |
| 4. | 195 | 70 |
| 5. | 220 | 65 |
| 6. | 250 | 80 |

(A) 55.2 (B) 56.1 (C) 64.96 (D) 70.1

**10.** Three years ago the average age of a couple and their three children was 17 years. Later another child was born. The present average age of the family is again 17 years. If the present age of the husband is 43 years and his wife is a year younger to him, then what could be the maximum possible age of any of the children, at present?
(A) 13 years (B) 9 years
(C) 11 years (D) 7 years

**11.** Nine years ago, the sum of the ages of the members of a family of seven is 209 years. Three years later, one member died at the age of 52 years and a child was born. After two more years, one more member died again at 52 and yet another child was born. The current average age of all the members of the family is nearest to .
(A) 21 years (B) 24 years
(C) 23 years (D) 22 years

**12.** A vessel has 300 ml of pure milk. Thirty millilitres of milk is removed and 30 ml of water is poured into the vessel (bringing the volume of mixture in the vessel back to 300 ml). If this operation is repeated another 2 times, what is the percentage of milk in the vessel at the end? % _____ .

**13.** The average height of group B of 20 people is 156 cm. The average height of the group falls by $1^5/_7$ cm when 6 people leave the group. These 6 people join group C which previously had 30 people and increase its average height by $\frac{5}{6}$ cm. Find the difference between the initial average heights of group B and group C.
_____ cm

**14.** The average monthly salary per head of the entire staff of a college is ₹20,400. The average salary per head of the four Heads of Departments (HOD's) is ₹24,000 while those who are not HODs have an average salary of ₹18,000 per head. The entire staff of the college, barring the HOD's, contribute a months salary to an orphanage of 10,000 children. Out of this contribution, if each boy gets ₹12 and each girl gets ₹8, then how many boys are there in the orphanage?
(A) 7,000 (B) 3,000 (C) 5,000 (D) 4,000

**15.** From a vessel containing only milk, 10 litres are drawn and replaced with water. 10 litres of the mixture is now taken out and replaced with water again. The ratio of milk to water now is 25 : 24. How many litres of milk was there initially?
(A) 20 litres (B) 25 litres
(C) 32 litres (D) 35 litres

**16.** A water purifier filters and processes water and reduces the salt content in it by 50% in 5 minutes. If water with salt content of 5% is poured into it, then in approximately how many minutes will the water be potable? (The salt content in potable water is at the most 0.05%. Assume that the concentration of salt decreases exponentially with time. Take log2 = 0.3010)
(A) 30 (B) 35 (C) 50 (D) 55

**17.** Two varieties of tea powder costing ₹210/kg and ₹300/kg are mixed in certain ratios to form varieties *X* and *Y*. *X* and *Y* are mixed in the ratio 3 : 2 to form variety *Z*

which is sold at 315/kg at 25% profit. If $X$ costs 240/kg, the ratio in which the two varieties are mixed to form $Y$ is
(A) 2 : 3 (B) 3 : 2 (C) 2 : 1 (D) 1 : 2

**18.** A vessel contains 80% alcohol solution. 20% of the solution was removed and replaced with water. If this process is repeated, find the percentage of alcohol which remains in the solution.
(A) 64% (B) 51.2%
(C) 40.96% (D) None of these

**19.** There are three sections A, B and C in class X of a school. They wrote a Talent Search Test. The average scores of A, B and C in that test were 60, 70 and 80, respectively. If the average score of the students of sections A and B together is 64 and that of sections B and C together is 76, then find the average score of the students of sections A and C together.
(A) 68 (B) 70 (C) 72 (D) 75

**20.** In a company there are 252 employees, in which the ratio of the number of men and women is 2 : 1. Some more women are employed and the ratio of men and women becomes 1 : 1. The average age of all the employees is now 22 years and the average age of the women is 2 years less than the average age of the men. Find the average age (in years) of men and women.
(A) 22, 20 (B) 23, 21
(C) 24, 22 (D) 21, 23

**21.** The volumes of two milk solutions are 18 litres and 12 litres. If these solutions are mixed, the concentration of milk in the resultant solution is 40%. If one litre each of the two solutions are mixed, the concentration of milk in the resultant solution is $38^{8}/_{9}$%. Find the respective volumes of milk in the two solutions, in litres.
(A) 10, 8 (B) 6, 4
(C) 12, 10 (D) 8, 4

***Directions for questions 22 and 23*:** These questions are based on the following data.

Alloy I contains 30% zinc, 40% copper and the remaining is gold.

Alloy II contains 40% tin and some zinc and gold. On mixing these two alloys in a certain ratio,

Alloy III, which has 30% zinc and 25% tin is obtained.

**22.** If equal weights of both Alloy I and Alloy III are mixed, then what percentage of the alloy formed will be copper?
(A) 15% (B) 40%
(C) 27.5% (D) 32.5%

**23.** Find the percentage of gold in the new alloy, formed by mixing Alloy II and Alloy III in a certain ratio.
(A) 20% (B) 30%
(C) 40% (D) Cannot be determined

**24.** There are two positive quantities a and $b$ satisfying the relationship $(a + x)\ (b - x) = 64$. Which of the following statements can be true?
(A) The average of $a$ and $b$ is more than 8.
(B) The average of $a$ and $b$ is 8.
(C) The average of $a$ and $b$ is at most 8.
(D) Both (A) and (B)

**25.** Find the average of all the three-digit numbers whose units digit is half the hundreds digit.
(A) 547.5 (B) 550 (C) 500 (D) 524.5

**26.** There is a 40 litres solution of milk and water in which milk forms 72%. How much water must be added to this solution to make it a solution in which milk forms 60%? ( in litres) _____ .

**27.** Two varieties of coffee—$A$ and $B$ are mixed in the ratio 3 : 2. The mixture is sold at ₹100 per kg at 100% profit. If variety $A$ costs ₹10 per kg more than variety $B$, what is the cost of variety $B$ per kg?
(A) ₹34 (B) ₹54 (C) ₹44 (D) ₹64

**28.** In a class, the average height of the boys lies between 100 cm and 120 cm. The average height of the girls lies between 80 cm and 100 cm. If there are 90 boys and 60 girls in the class, the average height of the class (in cm) definitely lies in which of the following ranges?
(A) (90, 115) (B) (100, 125)
(C) (80, 105) (D) (100, 110)

**29.** Two varieties of rice $A$ and $B$ priced at ₹6.75 per kg and ₹9.75 per kg, respectively were mixed and sold at ₹10.80 per kg at a profit of 20%. Find the ratio in which $A$ and $B$ are mixed.
(A) 1 : 4 (B) 1 : 3 (C) 1 : 2 (D) 1 : 5

**30.** Each of the two vessels $X$ and $Y$ has 20 litres of a solution of milk and water. $X$ has 10% water and certain quantity of water is added to it so as to raise the percentage of water to 25%. $Y$ has milk and water in the ratio 3 : 2 and a certain quantity of water is added to this so as to reverse this ratio. What is the difference between the quantity of water added to $X$ and that added to $Y$?
(A) 8 litres (B) 4 litres
(C) 6 litres (D) 2 litres

**31.** The average age of the employees in a company is 40 years. If six new persons with an average age of 30 years

joined and 5 employees with an average age of 40 years left the company, then the average age of the entire company would decrease by 5 years. Find the number of employees in the company. _____

**32.** A 6-litre solution of sulphuric acid had 45% acid. It was mixed with a 5-litre solution of sulphuric acid which had a% acid, where $40 \le a \le 50$. The acid concentration in the resulting mixture is $b$%. Which of the following is not a possible value of $b$?

(A) $42\frac{9}{11}$ (B) 45

(C) $47\frac{4}{11}$ (D) 46

**33.** Two containers $A$ and $B$ contain equal volumes of water and alcohol, respectively. 3 litres of water is taken from $A$ and poured into $B$. From the resulting solution in $B$, 3 litres is taken out and poured into $A$. If the quantity of water in both the containers is the same after the two transfers find the volume of alcohol in $B$ initially (in litres).

(A) 12 (B) 9 (C) 6 (D) 3

**34.** How many litres of 27% alcohol solution are to be added to 20 litres of 36% alcohol solution to obtain a 30% alcohol solution? _____

**35.** The average marks of a class of $n$ students are 64. When eight new students whose average marks are 73 join the class, the new average of the entire class is a whole number. Find the final number of students now in the class, given that $n$ lies between 25 and 60. _____

**36.** The concentration of spirit in three vessels $A$, $B$ and $C$ are 45%, 30% and 25%, respectively. If 4 litres from vessel $A$, 5 litres from vessel $B$ and 6 litres from vessel $C$ are mixed, find the concentration of spirit in the resultant solution.

(A) 25% (B) 31.25%

(C) 37.5% (D) 32%

**37.** A milk solution has milk and water in the ratio 3 : 2. What part of the solution has to be substituted with water so as to reverse the ratio of milk and water in the solution?

(A) Half (B) One-third

(C) One-fourth (D) One-fifth

**38.** The average of 36 numbers is 25. A new average is found after including some numbers from the set {30, 31, 32, …65}. The minimum number of numbers to be included in the order for the new average to exceed 30 is

(A) 4 (B) 5 (C) 6 (D) 7

***Directions for question 39*:** These questions are based on the data given below.

In a manufacturing company, the average salary of all the 6 production managers is ₹8,000 while that of the administrative employees is ₹3,500. The average salary of the production managers and administrative employees is ₹4,000. All the production managers are males and among the administrative employees, the females get an average salary of ₹4,500 and their male-counterparts get an average salary of ₹3,000.

**39. (a)** How many men are there among the administrative employees of the company?

(A) 26 (B) 50 (C) 32 (D) 44

**(b)** By what amount does the total salary of all the males exceed, that of all the females in the company? (in ₹ 000$s$)

(A) 92 (B) 72 (C) 114 (D) 142

**40.** A metal weighs 1500 kg per cubic metre and another metal weighs 2500 kg per cubic metre. Find the weight (in kg) of 5 cubic metres of an alloy formed by mixing 40% of the first metal and 60% of the second metal. _____

**41.** Class $X$ of Vidyaniketan School has four sections—$A$, $B$, $C$ and $D$. The average weights of the students of $A$, $B$, $C$ together and $A$, $C$, $D$ together are 45 kg and 55 kg, respectively, while the average weights of the students of sections $A$, $B$, $D$ together and B, $C$, $D$ together are 50 kg and 60 kg, respectively. Which of the following could be the average weight of the students of all the four sections together?

(A) 47.6 kg (B) 49.9 kg

(C) 53.7 kg (D) 56.5 kg

**42.** Two cans $A$ and $B$ contain 80 litres each of milk and water, respectively. 10 litres of milk is taken from can $A$ and poured into can $B$. Then, 10 litres of solution from can $B$ is removed and poured into can $A$. What are the respective quantities of milk in can $B$ and that of water in can $A$ (both in litres)?

(A) 10/9, 80/90 (B) 100/9, 100/9

(C) 80/9, 80/9 (D) 70/9, 20/9

**43.** Vessel $A$ contains 5 litres of milk and vessel B contains 5 litres of water. One litre of milk is taken from $A$ and is poured into $B$. One litre of the mixture in $B$ is then poured into $A$. If the present quantities of milk in $B$ and water in $A$ are $V_M$ and $V_W$, respectively, then which of the following holds true?

(A) $V_M > V_W$ (B) $V_M = V_W$

(C) $V_M < V_W$ (D) Cannot be determined

**44.** The average age of a group of men is 20 years. If $x$ men whose average age is $K$ years join the group, the average

age of the group would be y years. If $x$ men in the group whose average age is $K$ years leave the group, the average age of the group would be $y$ years. Find the value of $K$.
(A) 20 (B) 25
(C) 30 (D) Cannot be determined

**45.** Vessels $A$ and $B$ have 8 litres milk and 8 litres water, respectively. A jar $C$ is filled with the milk from vessel $A$ and then emptied into $B$. Jar $C$ is then filled with the mixture so formed, and then emptied into vessel $A$. If the ratio of milk to water in vessel $A$ at this stage is 2:1, find the volume of jar $C$ (in litres). _____

**46.** There are two vessels $A$ and $B$ each containing wine and water in the ratio of 2 : 7 and 1 : 4, respectively. When 20 litres of water are added to the contents of vessel $A$ the ratio of wine and water in both the vessels will become equal. If initially the quantities of wine in both the vessels is the same, how much wine should be added to vessel $B$, so that the concentration of wine in the vessel $B$ will equal the initial concentration of wine in vessel $A$?
(A) $2^1/_3$ litres (B) $3^1/_4$ litres
(C) $4^1/_6$ litres (D) None of these

**47.** In what ratio should Ram mix two varieties of barley costing ₹20 and ₹24 per kg so that by selling it at ₹27.60 per kg Ram makes a profit of 20%?
(A) 1 : 2 (B) 3 : 4
(C) 1 : 3 (D) 2 : 3

**48.** Dinku Beora was a chronic alcoholic and because of persistent health problems, he decided to quit drinking. He devised an ingenuous way of doing so. He bought a 750 ml 'goodbye' bottle of Old Monk. On the first day, he drank 5% of the contents in it and replaced that quantity with water. Next day he drank 10% of the contents in the bottle and replaced it with water. Like this, he continued. On the 19th day, he drank 95% of the contents in the bottle and replaced it with water and on the 20th (last day) he drank the entire contents of the bottle. Find the ratio of the total quantities of alcohol and water that he drank in the entire process. _____

**49.** There are three sets of numbers $A$, $B$ and $C$. The averages of the elements of $A$, $B$, $C$ are 31, 38, 35, respectively. The average of the elements of $A$ and $B$ is 35 and the average of the elements of $B$ and $C$ is 37. Find the average of all the elements of A, $B$, and $C$. _____

**50.** There are 10 boys in a group. Four boys leave and five boys join the group. It is found that the average weight of the group remains unchanged. If the average weight of the boys who leave is '$b$', that of the boys who join is '$c$' and the average weight of the initial group is '$x$', which of the following can be true?
(A) $x < b < c$ (B) $b < c < x$
(C) $b < x < c$ (D) $c < x < b$.

**51.** A grocer has two varieties of rice, Rice $A$ and Rice $B$ costing ₹10 per kg and ₹12 per kg, respectively. He sells the total mixture of Rice $A$ and Rice $B$ having 420 kg of Rice $A$ for ₹25,200 with a profit margin of 20%. How much quantity of rice $B$ is present in the mixture sold?
(A) 3500 kg (B) 2800 kg
(C) 7000 kg (D) 1400 kg

**52.** There are two containers with mixtures of Pepsi and Coke. In container 1, Pepsi and Coke are in the ratio 3 : 2 and in container 2 Pepsi and Coke are in the ratio 2 : 3. How many litres of the mixture should be taken from container 1 and mixed with an appropriate quantity of the mixture from container 2 in order to make 20 litres of a mixture containing Pepsi and Coke in the ratio 9 : 11?
(A) 10 (B) 15 (C) 20 (D) 5

**53.** Two vessels $A$ and $B$ of equal volume contain milk and water in the ratio 3 : 2 and 2 : 1 to their brim respectively. Two litres of the solution from vessel $A$ and three litres of the solution from vessel $B$ are poured into a big empty vessel $C$. If the solution in $C$ occupied 40% of the capacity of $C$, what proportion of the volume of vessel $C$ should be the volume of water that shall be added so that the ratio of milk and water in vessel $C$ becomes 1 : 1?
(A) $\frac{21}{125}$ (B) $\frac{2}{25}$ (C) $\frac{4}{75}$ (D) $\frac{14}{125}$

**54.** In a school there are a total of 50 students in class $X$, who are divided into three sections $A$, $B$ and $C$. Sections $A$ and $B$ have an equal number of students. All the students of the class wrote a test. The average marks obtained by the students of sections $A$ and $B$ together is 52.5. The average marks obtained by the students of sections $A$ and $C$ together is 60. The average marks obtained by the students of sections $B$ and $C$ together is 70. The average marks obtained by the students of sections $A$, $B$ and $C$ together is 60. How many students are there in section $C$?
(A) 10 (B) 20
(C) 15 (D) Cannot be determined

**55.** Twelve years ago, the average age of a joint family of 9 members was $28\frac{1}{9}$ years. Five years thereafter, two members, whose ages were 51 years and 59 years passed away and a child was born in that family. Five years

after that, two members whose ages were 57 years and 63 years passed away and another child was born in that family. If no changes occurred in the family from that time until now, the present average age of the family is closest to

(A) 19 years (B) 20 years
(C) 21 years (D) 17 years

**56.** Alloy *X* has 75% aluminium and 25% zinc. *A* certain process decreases the quantity of aluminium by one-fourth and the quantity of zinc by one-eighth. Further applications of the process have the same effect as the first one. The least number of times that the process must be applied to *X* in order to obtain an alloy in which the proportion of aluminum is 60% or less is

(A) 3 (B) 4 (C) 5 (D) 6

**57.** There are three vessels (with equal volumes) filled with mixtures of water and milk in the ratios 1 : 2, 2 : 3 and 3 : 4, respectively. These are all poured into a big vessel. What proportion of the mixture in the big vessel should be substituted by water so that the milk forms 50% of the resulting mixture?

(A) $\frac{71}{244}$ (B) $\frac{71}{386}$
(C) $\frac{71}{325}$ (D) $\frac{71}{193}$

**58.** A set has 100 numbers. *X*, *Y* and *Z* were asked to find the average of the numbers in the set.

Each person found the average of the numbers in the set. As the three persons used different methods, they ended up with different answers.

*X* split up the set of 100 numbers into three groups - a group of 20 numbers, a group of 30 and a group of 50. X's answer for the average of the 100 numbers was the average of the numbers in the first group, the average of the numbers in the second group and the average of the numbers in the third group.

*Y* split up the set of 100 numbers into two groups - a group of 30 numbers and a group of 70 numbers. *Y*'s answer was 4 more than the average of the numbers in the smaller group.

*Z* split up the set of 100 numbers into two groups, a group of 55 numbers and a group of 45 numbers. *Z*'s answer was the sum of $\frac{11}{20}$ th of the average of the numbers in the larger group and $\frac{9}{20}$ th of the average of the numbers in the smaller group.

The person whose answer was correct was

(A) *X* (B) *Y*
(C) *Z* (D) Cannot be determined

**59.** A vessel contains a solution of milk and water containing 90% milk. 10 litres of solution is withdrawn from the vessel and replaced by water. The procedure is repeated one more time. If the final quantity of milk in the vessel is 72.9 litres, then find the initial quantity of the solution in the vessel (in litres).

(A) 150 (B) 200 (C) 120 (D) 100

**60.** Twelve years ago, the average of the ages of the members of a joint family having ten members was 25 years. Four years later, a member aged 50 years died and a child was born in the family that year. Four years after that, another member aged 50 years died and another child was born. Find the present average age of the members of the family (in years). _____

## EXERCISE-4

***Directions for questions 1 to 30*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Find the average of all the three-digit numbers whose units digits are three times their hundreds digits. _____

**2.** An educational institution runs six branches. The number of students in each of the branches and the average score of the students in each branch in a certain test are tabulated below. What is the approximate weighted average score of the students of the institution?

| Branch number | Number of students | Average marks |
|---|---|---|
| 1. | 225 | 60 |
| 2. | 205 | 56 |
| 3. | 180 | 55 |
| 4. | 195 | 70 |
| 5. | 220 | 65 |
| 6. | 250 | 80 |

(A) 55.20 (B) 56.10 (C) 64.96 (D) 70.10

**3.** Jar $A$ has 3 litres of a solution that is 30% acid. Jar $B$ has 6 litres of a solution that is 40% acid. Jar $C$ has 4 litres of a solution that is $x$% acid. A certain quantity was drawn from $C$ and mixed with the entire contents of $A$. The remaining solution in $C$ was mixed with the entire contents of $B$. If each of these mixtures has 50% acid, find $x$. _____

**4.** The mean of $N$ distinct natural numbers is 27. The greatest of the numbers is 375. Find $N$.
(A) 25 (B) 35
(C) 40 (D) Cannot be determined

**5.** Consider the set $S = \{2, 5, 8, \ldots 89\}$; subsets A, $B$, $C$, $D$ and $E$ of $S$ are defined as the sets containing all the numbers of the forms $5k$, $5k + 1$, $5k + 2$, $5k + 3$ and $5k + 4$, respectively, where $k$ is a whole number. The averages of all the numbers in $A$, $B$, $C$, $D$ and $E$ are a, $b$, $c$, $d$ and $e$, respectively. Which of the following is true?
(A) $a < b < c$ (B) $b < c < d$
(C) $c < d <$ e (D) a $< c < d$

**6.** A vessel contains an 80% alcohol solution. 20% of the solution was removed and replaced with water. If this process is repeated, find the percentage of alcohol which remains in the solution.
(A) 64% (B) 51.2%
(C) 40.96% (D) None of these

**7.** A local grocer mixed three qualities of coffee $T_1$, $T_2$ and $T_3$ priced at ₹79 per kg, ₹64 per kg and ₹62 per kg, respectively in the ratio 1 : 2 : 4. To 4 kg of this mixture he adds quantities of $T_1$ and $T_3$ which are in the ratio 1 : 5. He now sells this new mixture for ₹77.88 per kg thereby making a profit of 20%. How much of $T_1$ did he mix with the mixture?
(A) 1 kg (B) 5 kg (C) 6 kg (D) 2 kg

**8.** Vessels $A$ and $B$ contain mixtures of milk and water. $A$ has $a$% milk and $B$ has $b$% milk. Mixture $P$ is formed by mixing a% of vessel A's contents with $(100 - b)$% of vessel B's contents. Mixture $Q$ is formed by mixing $(100 - a)$% of vessel A's contents with $b$% of vessel $B$'s contents. Mixtures $P$ and $Q$ have $k$% milk each. If $a \neq b$, find $a + b$.
(A) 80 (B) 90
(C) 100 (D) Cannot be determined

**9.** $N$ is any four–digit number whose units digit is half its thousands digit and the two middle digits are equal. Find the average of all such numbers. _____

**10.** There are two sets of numbers—$A$ and $B$—with $3n$ and $4n$ numbers, respectively. The average of the numbers in $A$ lies in the interval [20, 30], while the average of the numbers in $B$ lies in the interval [69, 79]. The interval which best describes the average of all the numbers in $A$ and $B$ is
(A) [41, 51] (B) [48, 58]
(C) [41, 58] (D) [48, 57]

**11.** Three vessels contain three different mixtures of milk and water. Volume of each mixture is 12 litres and their respective concentrations of milk are 60%, 40% and 20%. 4 litres of the first, 8 litres of the second and 12 litres of the third are mixed and the mixture is named $A$. The left overs of the three vessels are mixed and this mixture is named $B$. Six litres of $A$ and 9 litres of $B$ are mixed to form mixture $C$.

Which of the following statements is true regarding the milk concentrations of $A$, $B$ and $C$?

I. $A$ has the highest concentration

II. $B$ has the lowest concentration

III. $C$ has the highest concentration

IV. $A$ has neither the highest nor the lowest concentration

(A) I and II (B) II and III
(C) III and IV (D) None of these

**12.** A, $B$, $C$ are three piles of rocks. The average weight of the rocks in pile $A$ is 50 pounds and that in pile $B$ is 60 pounds. The average weight of the rocks in piles $A$ and $B$ combined is 53 pounds. The average weight of the rocks in piles $A$ and $C$ combined is 54 pounds. If the average weight of the rocks in piles $B$ and $C$ combined (in pounds) is an integer, its greatest value is _____ .

**13.** There are 3 sections $A$, $B$, $C$ of class $X$ of a school. The average marks of the three sections in a certain exam taken by all the students of class $X$ are 52, 58, 63, respectively. The average marks of sections $A$ and $B$ together are 56 and those of sections $B$ and $C$ together are 60. Find the average marks of the three sections taken together.
(A) $57\frac{12}{13}$ (B) $57\frac{11}{13}$ (C) $58\frac{2}{13}$ (D) $52\frac{4}{13}$

**14.** In a company ARP Bluefish Pvt. Ltd., there are 3 departments—Marketing, Finance and Production. The average age of the employees of Marketing and Finance together is 25 years. The average age of the employees of Finance and Production together is 30 years and the average age of the employees of Production and Marketing together is 20 years. Find the average age of all the employees of the company together, if the numbers of employees of Marketing and Finance are the same.
(A) $22 \times 5$ years (B) 25 years
(C) $27 \times 5$ years (D) Cannot be determined

**15.** A vessel is filled to its capacity with pure milk. Nine litres are withdrawn from the vessel and replaced with an equal amount of water. Nine litres of the mixture is again withdrawn and then replaced with an equal amount of water. After these changes, the vessel contains 17.1 litres of milk less than it did initially.

(i) Find the capacity of the vessel. (in litres). _____

(ii) What is the least number of such additional replacements required, so that the vessel contains less than 75% milk? _____

***Directions for questions 16 and 17*:** These questions are based on the following information.

There are five solutions *A*, *B*, *C*, *D*, and *E* with concentrations 10%, 20%, 40%, 50%, 80%, respectively. (*X*, *Y*) denotes a mixture of *X* and *Y* in which *X* and *Y* are mixed in some proportion, such that both are included. Similarly (*X*, *Y*, *Z*) denotes a mixture in which *X*, *Y* and *Z* are mixed in some proportion, such that each is included.

**16.** Which of the following cannot have a concentration of 75 %?
(A) ((*A*, *B*), (*B*, *C*), (*C*, *E*))
(B) ((*B*, *C*), (*D*, *E*), (*A*, *E*))
(C) ((*A*, *B*), (*B*, *C*), (*A*, *D*))
(D) ((*A*, *E*), (*B*, *C*), (*C*, *D*))

**17.** There are three containers *P*, *Q*, and *R*. Each is filled with one of the above 5 solutions. By taking some solution from each of the containers *P*, *Q* and *R* and mixing them, a new solution *N* is prepared. The concentration of this new solution *N* is 50%. In how many ways can the solutions be taken in the 3 containers so that it is possible to prepare *N*?
(A) 45 (B) 46 (C) 55 (D) 54

**18.** Two large drums *A* and *W*, have 1000 litres of alcohol and 1000 litres of water, respectively. Two litres of the contents in drum *A* are removed and mixed with the contents of drum *W*. This is the first operation. Two litres of the contents of drum *W* are then mixed with the contents of drum *A*. This is the second operation. The first and the second operations together constitute process *P*. Process *P* is carried out 3 times. After that, only the first operation is done. After this, '*a*' is the alcohol in the first drum and '*w*' is the water in the second drum. What is the relation between a and *w*?
(A) $a \geq 1.1w$ (B) $a < w$
(C) $a = w$ (D) $w < a < 1.1w$

**19.** The volumes of two milk solutions are 18 litres and 12 litres. If these solutions are mixed, the concentration of milk in the resultant solution is 40%. If one litre each of the two solutions are mixed, the concentration of milk in the resultant solution is $38^{8}/_{9}$%. Find the respective volumes of milk in the two solutions, in litres.
(A) 10, 8 (B) 6, 4
(C) 12, 10 (D) 8, 4

***Directions for question 20*:** This question is based on the following information.

Satish wrote 12 tests, each of maximum marks 100. At the end of the year he sat down to check his performance for the entire year. He noticed that he could not trace the answer scripts of tests V, VII, XI and XII and could not recollect the marks that he had scored in those tests. However, he remembered that the average marks that he got in all those 12 tests was 85. The marks that he got in tests I, II, III, IV, VI, VIII, IX and X were respectively 88, 84, 85, 77, 95, 76, 86 and 83. His friend Dinesh remembered that he got 2 marks more in test V than he got in test XI and he got 12 marks more in test XII than in test VII. The marks obtained in test XI was 18 more than those obtained in test VII.

**20.** (a) How much did Satish score in test *V*?
(A) 94 (B) 90 (C) 88 (D) 86

(b) If test XII was reconducted and Satish increased his average by 1 mark, how many marks did Satish get in the retest?
(A) 100 (B) 98 (C) 96 (D) 92

**21.** There are 10 litres of milk in container *A* and 10 litres of water in container *B*. One litre of milk is transferred from *A* to *B*. After that, one litre of the contents of *B* are transferred back to *A*. This entire process is carried out once again. What is the final concentration of milk in *A*?

(A) $\frac{100}{121}$ (B) $\frac{101}{121}$ (C) $\frac{109}{121}$ (D) $\frac{10}{11}$

**22.** There are two vessels, *A* and *B*, of equal capacity. *A* is filled completely with pure benzene and *B* is left empty. Each operation involves transferring some quantity of liquid from *A* to *B* and then refilling *A* with the same quantity of water. At the end of *n* such operations ($n > 1$), it is ensured that *B* is full. If $n = 2$, then what is the maximum possible concentration of water in *B*?
(A) $77^{7}/_{3}$% (B) 50% (C) $28^{1}/_{7}$% (D) 25%

**23.** Alloy A has 80% copper and 20% tin. A certain process when applied repeatedly to the alloy decreases the copper quantity in the alloy by one–fifth and the tin quantity in it by one–tenth, each time it is applied. The minimum number of times the process must be applied so that the concentration of copper in the alloy is less than 70% is
(A) 4 (B) 5 (C) 3 (D) 2

**24.** Two large drums *A* and *W*, have 1000 litres of alcohol and water, respectively. Two litres of the contents of drum *A* are removed and mixed with the contents of drum *W*. This is the first operation. Two litres of the contents of drum *W* are then mixed with the contents of drum *A*. This is the second operation. The first and the second operations together constitute process *P*. Process *P* is carried out 51 times. After that, only the first operation is carried out. Finally, '*a*' is the alcohol in the first drum and '*w*' is the water in the second drum. What is the relation between a and *w*?

(A) $a \geq 1.1w$ (B) $a < w$

(C) $a = w$ (D) $w < a < 1.1w$

**25.** A mixture has 25% salt. Using a filtration process, the mixture can be purified. Each iteration will decrease the salt content in the mixture by 20%. The least number of iterations required for the salt content in the mixture to decrease to 2% or less of the initial salt content is (Assume log 2 = 0.3010)

(A) 17 (B) 18

(C) 19 (D) 16

**26.** Find the average of all the four-digit numbers whose units digit is twice the thousands digit. _____

**27.** *N* is a four-digit number in which the hundreds digit is double the thousands digit and the units digit is 4 times the thousands digit. Find the average of all such numbers.

(A) 1850 (B) 1853 (C) 1851 (D) 1852

**28.** Two cans *P* and *Q* contain *x* liters of pure milk and *x* liters of water respectively. An empty can *R* has a volume of 5 liters. *R* is filled using a part of the contents of *P*. The contents of *R* are now emptied into *Q*. Now, 5 liters of *Q* are transferred to *P*. If the ratio of the volumes of milk and water in *P* is 5 : 1, what is the value of *x*?

(A) 25 (B) 30 (C) 24 (D) 36

**29.** There are *N* liters of milk in container *A* and *N* liters of water in container *B*. *X* liters of milk is transferred from *A* to *B*. After thorough mixing, *X* liters of the contents of *B* are transferred back to *A*. The difference of the final concentrations of milk in *A* and water in *A* is 75 percentage points. Find the final concentration of milk in *B*.

(A) 12.5% (B) $14\frac{2}{7}\%$

(C) $16\frac{2}{3}\%$ (D) $11\frac{1}{9}\%$

**30.** There are *N* liters of milk in container *A* and *N* liters of water in container *B*. *X* liters of milk is transferred from *A* to *B*. After the contents are thoroughly mixed, *X* liters of the contents of *B* are transferred back to *A*. The final concentration of milk in *A* is $\frac{3}{4}$. What is the value of X/*N*?

(A) $\frac{1}{2}$ (B) $\frac{2}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$

## EXERCISE-5 Based on Data Sufficiency

***Directions for questions 1 to 25*:** Each question is followed by two statements, I and II. Answer each question based on the following directions:

Choose (A) If the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.

Choose (B) If the question can be answered by using either statement alone.

Choose (C) If the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (D) If the question cannot be answered even by using both the statements together.

**1.** In what ratio must two varieties of wheat be mixed so that the mixture is worth ₹ 1.54 per kg?

I. First variety of wheat costs ₹ 1.6 per kg.

II. Second variety of wheat costs ₹ 0.15 per kg less than the first.

**2.** In a mixture, the ratio of milk and water is 2 : 1. In order to change the ratio of milk and water by addition of water to 1 : 2, the volume of water to be added is *w* litres. Find *w*.

I. The volume of the initial mixture is 60 litres.

II. The volume of water in the initial mixture is 20 litres.

**3.** The average weight of boys in a class is 28 kg and that of girls is 21 kg. Find the average weight of the class.

I. The ratio of boys to girls is 5 : 2.

II. Number of boys in the class is 30.

**4.** Find the average age of the all the students combined of sections *A*, *B* and *C*.

I. The average age of all the students combined in sections $A$ and $B$ is 25 years .

II. The average age of all the students combined in sections $B$ and $C$ is 25 years.

**5.** Find the average weight of four men $A$, $B$, $C$ and $D$, given that $A$ and $B$ have an average weight of 40 kg.

I. The weight of $C$ is equal to the average weight of the other three.

II. The weight of $D$ is equal to the average weight of the other three.

**6.** Is the average age in a class of 20 students more than 15 years?

I. Eighteen students in the class are each 15 years old.

II. The average age of four of the students in the class is less than 15 years.

**7.** A certain alloy contains lead, copper and tin. How many kilograms of tin is contained in 60 kilograms of the alloy?

I. By weight, the alloy has $2/5^{th}$ lead and $3/16^{th}$ copper.

II. By volume, the alloy has $1/3^{rd}$ lead and $1/3^{rd}$ copper.

**8.** If the arithmetic mean of six numbers is 30, then how many numbers are greater than 30?

I. Three of the six numbers are equal to 40.

II. Three of the six numbers are equal to 20.

**9.** Between two mixtures $x$ and $y$—each containing milk and water, the concentration of which mixture is more?

I. $x$ has three parts of water to seven parts of milk.

II. y has seven parts of water to thirteen parts of milk.

**10.** In a certain class, $x$ boys and $y$ girls took a test. Was the average score of $x$ boys greater than 83?

I. The sum of all the scores was $85(x + y)$.

II. The average score of $y$ girls was 86.

**11.** Variety $A$ of rice costs ₹ 32 per kg. Variety $B$ of rice costs ₹ 22 per kg. The two varieties of rice are mixed in a certain ratio. What is the ratio in which they are mixed?

I. The mixture is sold at ₹ 36 per kg at 20% profit.

II. Weight of the mixture is 12 kg.

**12.** Is the average age of the students taking *SAT* greater than 22 years?

I. Out of the 100 test takers, the average age of 92 students is 14 years.

II. The oldest test taker is 24 years old.

**13.** The average weight of Amar, Bhanu, Chetan and Dinesh is 80 kg. Find the average weight of Amar and Bhanu.

I. The weight of Amar is the average of the weights of the others.

II. The weight of Bhanu is the average of the weights of the others.

**14.** Can $A$ contains milk and water solution and can $B$ contains orange juice and water solution. These solutions are mixed. What is the ratio of water, milk and orange juice in the resultant solution?

I. Can $A$ contains 40% water and can $B$ contains 35% orange juice.

II. Half of the resultant solution is water

**15.** There are $N$ men in a group. Their average age is 20 years. Find $N$

I. If 2 men aged 22 years and 28 years join the group, the average age of the group increases by a prime number.

II. If 2 men aged 22 years and 28 years leave the group, the average age of the group decreases by a prime number

**16.** What is the average age of the four brothers if their ages are in arithmetic progression?

I. The average age of the eldest and the youngest brothers is 21 years.

II. The second brother is 24 years old and the third brother is 18 years old.

**17.** In class $X$ of a school, there are three sections—A, $B$ and $C$. Is the average mark of the class more than 65?

I. The average mark of the sections $A$, $B$ and $C$ are 40, 60 and 90, respectively.

II. Among the three sections, section $A$ has the maximum number of students.

**18.** Is the average age of the students of a school less than 15 years?

I. The average age of the students of class $X$ of the school is 16 years and that of the remaining classes is 14 years.

II. The strength of class $X$ is less than 30% of the strength of the school.

**19.** Is the average age of a group of persons at least 20 years?

I. If four persons of average age 40 years join the group, the average age of the group will increase by 2 years.

II. If four persons of average age 40 years join the group, the average age of the group doubles.

**20.** There are three vessels *A*, *B* and *C* whose capacities are in the ratio 3 : 6 : 8. In vessel *A* there is 12 litres of milk solution in which milk and water are in the ratio 3 : 1. If there are 20 litres and 30 litres of milk solutions in vessels *B* and *C* respectively, how many litres of milk solution can vessel *C* hold?

I. The total quantity of pure milk in these vessels is 45 litres.

II. The total quantity of water in these vessels is 20% of the total capacity of all the three vessels.

**21.** Each of the two groups *A* and *B* consists of some people. Is the average age of group *A* greater than that of group *B*?

I. If one person goes from group *A* to group *B*, the average age of each of the groups decreases.

II. When one person leaves from group *B* to group *A*, the average age of group *B* increases but the average age of group *A* decreases.

**22.** Of the two containers *A* and *B*, *A* contains pure milk and *B* contains water. 50% of *A* is shifted to *B*. After thoroughly mixing it, 50% of the contents in *B* are shifted to *A*. Is the ratio of pure milk to water in *A*, greater than 1?

I. Initial volume of milk in $A \geq$ Initial volume of water in *B*.

II. Initial volume of milk in $A \leq$ Initial volume of water in *B*.

**23.** A trader mixed two varieties of raisins, one costing Rs.96 per kg and the other Rs.112 per kg. How much of the dearer variety raisins did he mix in each pack?

I. The trader packed the raisins in a 700-gm pack.

II. He sold the 700-gm packet at a price of Rs.105 and got 50% profit.

**24.** What percent of *P* is *Q*?

I. *P* is 50% more than *Q*.

II. The sum of *P* and *Q* is 75% more than *P*.

**25.** Shreya got 94 marks in a test while both Shravya and Shriya got more than 81 marks. The maximum marks in the test were 100 and the marks the 3 girls got were all integers. Did Shreya get the highest marks?

I. The average score of the 3 girls was 90 or less.

II. The average score of the 3 girls was 91 or more.

## ANSWER KEYS

### Exercise-1

1. (C)
2. (D)
3. (D)
4. (A)
5. 13
6. (B)
7. 25
8. (B)
9. (B)
10. 13000
11. (B)
12. (B)
13. 31
14. 28.80
15. (B)
16. (A)
17. 12
18. 150
19. (A)
20. (B)
21. (D)
22. (C)
23. (D)
24. 14.5
25. 7.25
26. 19
27. (B)
28. 57
29. (C)
30. 13
31. 9
32. 3
33. (C)
34. (D)
35. 8100
36. 64
37. (B)
38. (C)
39. (B)
40. 27
41. 7
42. 7
43. (B)
44. (B)
45. (A)
46. (B)
47. (D)
48. 18
49. (D)
50. 20
51. (B)
52. 80
53. (D)
54. (D)
55. 24
56. (D)
57. (C)
58. (B)
59. 24.5
60. (D)

### Exercise-2

1. 56
2. (A)
3. (B)
4. (C)
5. 1098
6. (D)
7. 45
8. (B)
9. (D)
10. 35
11. (A)
12. 33.5
13. 40
14. 12
15. 5
16. (A)
17. 80
18. (B)
19. (B)
20. 68
21. (B)
22. (B)
23. (C)
24. (D)
25. 69
26. 25
27. 7
28. 30,000
29. 5
30. 115
31. 7.29
32. (B)
33. 2,392
34. 33
35. (D)
36. (B)
37. 25
38. (C)
39. 750
40. 23
41. (D)
42. 20,000
43. (C)
44. (A)
45. 2.125
46. 35
47. (D)
48. 36
49. (C)
50. (B)
51. (D)
52. (B)
53. (C)
54. (A)
55. (B)
56. (C)
57. 36
58. (B)
59. (C)
60. 16

### Exercise-3

1. (C)
2. 100
3. (A)
4. (D)
5. (D)
6. (C)
7. 6
8. (B)
9. (C)
10. (B)
11. (B)
12. 72.9
13. 1
14. (A)
15. (D)
16. (B)
17. (D)
18. (B)
19. (B)
20. (B)
21. (D)
22. (C)
23. (B)
24. (D)
25. (A)
26. 8
27. (C)
28. (A)
29. (B)
30. (C)
31. 11
32. (C)
33. (D)
34. 40
35. 36
36. (D)
37. (B)
38. (C)
39. (a) (C)
(b) (B)
40. 10, 500
41. (C)
42. (C)
43. (B)
44. (A)
45. 4
46. (D)
47. (C)
48. 9.5
49. 35
50. (B)
51. (D)
52. (D)
53. (D)
54. (A)
55. (D)
56. (C)
57. (B)
58. (C)
59. (D)
60. 27

### Exercise-4

1. 251
2. (C)
3. 80
4. (D)
5. (C)
6. (B)
7. (A)
8. (C)
9. 5497.5
10. (B)
11. (D)
12. 69
13. (C)
14. (B)
15. (i) 90
(ii) 1
16. (C)
17. (C)
18. (B)
19. (D)
20. (a) (A)
(b) (B)
21. (B)
22. (D)
23. (B)
24. (B)
25. (B)
26. 3000
27. (C)
28. (A)
29. (A)
30. (D)

### Exercise-5

1. (C)
2. (B)
3. (A)
4. (D)
5. (C)
6. (C)
7. (A)
8. (C)
9. (C)
10. (C)
11. (A)
12. (C)
13. (C)
14. (C)
15. (A)
16. (B)
17. (C)
18. (C)
19. (A)
20. (C)
21. (A)
22. (A)
23. (A)
24. (B)
25. (A)

## SOLUTIONS

## EXERCISE-1

**1.** If each students height was 1 cm more, the average height of the students, increases by 1 cm and becomes 156 cm.

**2.** As more than half of the strength of the class are boys, the average age of the class will be more than half of the sum of the average ages of boys and girls.

**3.** Let $x$ litres of water be mixed with 1 litre of wine solution.

Quantity of wine in 1 litre of wine solution

$= \frac{80}{100}(1) = 0.8$ litres

Quantity of wine in $(1 + x)$ litres of mixture

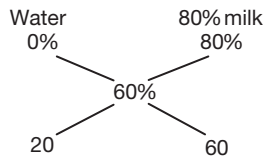
$= \frac{60}{100}(1+x)$ litres

As quantity of wine before and after mixing remains the same, $0.8 = \frac{60}{100}(1+x) = 0.6 + 0.6x$

$1/3 = x$

Required ratio $= x : 1 = 1 : 3$

**Alternative Method:**




Required ratio = 20 : 60 = 1 : 3

**4.** Let the five numbers be $P$, $Q$, $R$, $S$ and $T$.

$\Rightarrow P + Q + R + S + T = 655$;

$\frac{P+Q}{2} = 85$ and $R = 125$

$\therefore P + Q = 170$ and $R = 125$

$\therefore P + Q + R = 295$

$S + T = 655 - (P + Q + R) = 360$

Average of the last two numbers $= \frac{S+T}{2} = 180$

**5.** Total number of sweets with 6 children = (6) (6) = 36

Total number of sweets with 7 children = (7) (7) = 49

The number of sweets with the seventh child

= 49 – 36 = 13

**6.** Total age of seven persons = $(28 \times 7)$ years

Total ages of the first three persons and the last three persons are $(21 \times 3)$ years and $(34 \times 3)$ years, respectively.

Age of the person sitting in the middle of the row

$= 28 \times 7 - 21 \times 3 - 34 \times 3 = 196 - 63 - 102 = 31$ years

**7.** The average of all even natural numbers less than 50 equals $\frac{\text{Sum of all even natural numbers less than 50}}{\text{Number of even natural numbers less than 50}}$

Sum of all even natural numbers = 2 + 4 + 6 + 8 + … + 48

This represents the sum of the first 24 even natural numbers. The sum of the first $n$ even natural numbers is given by $n(n + 1)$

Their average $= \frac{n(n+1)}{n} = n + 1$.

Hence, average of first 24 even natural numbers = 25 (Taking $n = 24$).

**8.** There are 30 girls in the class

Sum of the marks of the girls = (30) (90) = 2700

Sum of the marks of the boys = (15) (60) = 900

Average marks of all the students in the class

$= \frac{2700 + 900}{45} = 80$

**9.** Average price per book $= \frac{1150 + 920}{65 + 50} = \frac{2070}{115} = ₹18$

**10.** Average monthly income of the man for that year

$= \frac{9(12000) + 3(16000)}{12} = ₹13000$

**11.** Let the number of boys in the group originally be $x$.

Total weight of the boys = $30x$

After the boy weighing 35 kg joins the group, total weight of boys = $30x + 35$

So, $30x + 35 = 31(x + 1)$

$x = 4$

**12.** As the average salary of male employees as well as female employees is the same, irrespective of the number of male and female employees, the average salary of all the employees will be equal to the average salary of the male or female employees.

Here, the required average salary will be ₹15000 per month.

**13.** Total score of the batsman in 20 matches = 800

Total score of the batsman in the next 10 matches = 130.

Total score of the batsman in the 30 matches = 930

Average score of the batsman = $\frac{930}{30} = 31$

**14.** Weight of Amar = Weight of 20 students of the class – Weight of other 19 students of the class = (20) (25) – (19) (24.8) = 28.80 kg

**15.** Quantity of water in the vessel does not change. The vessel contains 60% water, quantity of pure milk to be added must be 20% of the vessel's contents so that the ratio of milk to water becomes 1 : 1.

Quantity of pure milk to be added = 0.2 (25) = 5 litres.

**16.** Let the amount lent at 7% be ₹ $x$

Amount lent at 10% is ₹ $6000 - x$

Total interest for one year on the two sums lent

$= \frac{7}{100}x + \frac{10}{100}(6000 - x) = 600 - \frac{3x}{100}$

$600 - \frac{3}{100}x = 450$

$x = 5000$

Amount lent at 10% = 1000

Required ratio = 5000 : 1000 = 5 : 1

**17.** The average of 11 consecutive numbers is equal to the middle term which is $6^{th}$ term.

**18.** Total score of Mahesh in 6 subjects = (6) (75) = 450

Score of Mahesh in the sixth subject = 450 – (60) (5)

= 150

**19.** Average mark scored by all the three boys

$= \frac{\frac{64}{100}(800) + \frac{36}{100}(800) + \frac{44}{100}(800)}{3} = 384$

**20.** As the milkman wants to get 50% profit by selling it at cost price, the milk man has to add water which is equal to 50% of pure milk, i.e., 50% (20 litres) = 10 litres.

**21.** By alligation rule,

$\frac{\text{Quantity of tea costing ₹ 9 per kg}}{\text{Quantity of tea costing ₹12 per kg}} = \frac{12 - 10.2}{10.2 - 9} = \frac{3}{2}$

Quantity of tea costing ₹ 12 per kg = $\frac{2}{3}(18) = 12kg$

**22.** Let the numbers of boys and the girls in the class be $3x$ and $5x$, respectively.

Average weight of the students in the class

$= \frac{20(3x) + 16(5x)}{3x + 5x} = 17.5\text{kg}$

**23.** Average sales per day

$= [(20{,}000 \times 3) + 16{,}000 + 24{,}000 + 30{,}000]/6$

= ₹ 21,667

**24.** The first ten two digit natural numbers are the natural numbers from 10 to 19.

Their sum is 145.

Their average is $\frac{145}{10} = 14.5$

**25.** Total cost of potatoes = (3) (6) = ₹ 18

Total cost of tomatoes = (5) (8) = ₹ 40

Average price per kilogram of the vegetables in the bag

$= \frac{18 + 40}{8}$ = ₹ 7.25

**26.** Applying the weighted average rule, the average cost of the mixture can be written as $\frac{(16 \times 2) + (21 \times 3)}{(2 + 3)} = 95/5$

= ₹ 19 per kg

**27.** Let there be $x$ kgs of sand in 100 kgs of jeera.

Profit percentage = $\frac{x}{(100 - x)} \times 100 = 25$

$\Rightarrow x = 20$kgs

Percentage of sand in the mixture = $20/100 \times 100 = 20\%$

**28.** If some members are in arithmetic progression, their average is middle term (if the number of terms is odd) and the average of middle two terms (if the number of terms is even). Here, the average will be 57.

**29.** Let the initial number of members in the group be $n$.

Initial total weight of all the members in the group = $n(48)$

From the data,

$48n + 78 + 93 = 51(n + 2) \Rightarrow 51n - 48n = 69$

$\Rightarrow n = 23$

Therefore there were 23 members in the group initially.

**30.** Total cost price of the mixture = (6) (10) + (9) (15) = ₹ 195

Selling price of the mixture (in ₹ per kg) = Cost price of the mixture (in ₹ per kg) = $\frac{195}{6 + 9}$ = ₹ 13

**31.** Let the number of students who wrote the exam be $x$. Calculated total marks of the students = $20x$

Correct value of the total marks of students =

$20x - 70 - 85 + 60 + 77 = 20x - 18$

$20x - 18 = 18x \Rightarrow x = 9$

**32.** Average number of sweets received by each child $= \frac{57}{10} = 5.7$

Let the number of girls be $x$. By alligation rule,

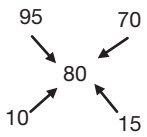
$$\frac{x}{10-x} = \frac{6-5.7}{5.7-5} = \frac{3}{7} = \frac{3}{10-3}$$

Comparing both sides, $x = 3$

**33.** Concentration of iron ore = 95%

Concentration of iron in another ore = 70%

Concentration of iron in resultant ore = 80%




= 2 : 3

$\therefore$ Quantity of 95% iron ore $= \frac{2 \times 120}{3} = 80\ kg.$

**34.** Total sales for the month $= 48 \times 30 = 1440$

Total sales for the first 14 days $= 14 \times 48 = 1120$

Total sales for the remaining 16 days $= 1440 - 1120 = 320$

Average sales per day for the remaining 16 days

$= 320/16 = 20$

**35.** Average monthly income in that month = ₹9000 – 900 = ₹8100

**36.** Total cost of 16 items = ₹ $(16 \times 59)$.

Total cost of the final 9 items

$= ₹\,[(16 \times 59) - (4 \times 60) - (39 + 49 + 40)\ ] = ₹576$

New average = 576/9 = ₹64

**37.** Let the number persons in each group be $x$.

Total age of the first group $= 2 \times 30 = 30x$

Total age of the second group $= x \times 40 = 40x$

Average age of the two groups combined

$= \frac{30x + 40x}{x + x} = \frac{70}{2} = 35$ years

**38.** Average monthly income of the man

$= \frac{(4000) + (5600) + (6420)}{3} = 5340$

**39.** Concentration of alcohol in the mixture = 30%

Concentration of acid = 70%

Quantity of acid in the mixture = 70% of 80 = 56 litres

Let $x$ litres of conc. acid be added to the mixture

$85\%\ (80 + x) = 56 + x \Rightarrow x = 80$

**40.** Let the quantities of the two solutions be $3x$ litres and $7x$ litres

Quantity of the final mixture = $10x$ litres.

Quantity of sulphuric acid in the final mixture

$= 0.2\ (3x) + 0.3(7x) = 2.7x$ litres.

Concentration of sulphuric acid in the final mixture

$= \frac{2.7x}{10x}\ (100) = 27\%$

**41.** Cost of 1 dozen mangoes = (1) (6) = ₹6

Cost of 2 dozen mangoes = (2) (10) = ₹20

Cost of 5 dozen mangoes = (5) (6) = ₹30

Total cost of 8 dozen mangoes = ₹56

Average cost per dozen of mangoes

$= ₹\ \frac{56}{8} = ₹7$

**42.** Required average $= \frac{5(100) + 8(200)}{300} = 7$

**43.** Total number of chocolates possessed by four children = 4(4) = 16

Total number of chocolates with five children together, after fifth child joined = 5(5) = 25

$\therefore$ The number of chocolates with the fifth child = number of, chocolates increased = 25 – 16 = 9

**44.** Ratio of milk and water in the vessel after the first replacement = 9 : 1.

When 1 litre of the mixture is now taken out, remaining mixture will have $\frac{9}{10}$ th of 9 litres as milk, i.e., 8.1 litres.

Final mixture will have milk and water in the ratio 8.1 : (10 – 8.1) = 81 : 19

**Alternate Method:**

$P = 10,\ Q = 1$

Ratio of quantity of milk in the final mixture to the final mixture $= \left(\frac{P-Q}{P}\right)^2 = 81 : 100$

Ratio of milk and water in the final mixture = 81 : 19

**45.** The average income is ₹10,000 per month for 9 months and that is ₹20,000 for 3 months. As ₹10,000 is maintained

for greater period, the average income per month for the entire year lies between ₹ 10,000 and ₹ 15,000.

**46.** As the age of each member would be 5 years more after 5 years, the average age of the family would also be 5 years more, i.e., 20 years.

**47.** Total sales for the year = $200 \times 12 = 2400$ units

Sales form January to June = $150 \times 6 = 900$

Sales form July to October = $240 \times 4 = 960$

Sales of November and December together

= 2400 – 900 – 960 = 540 units

Average sales of November and December

= 540/2 = 270 units

**48.** Sum of the three numbers = (20) (3) = 60

Third number = 60 – (sum of the other two numbers)

= 60 – 42 = 18

**49.** Total manufacturing cost = 40000 + 5 × 5000 = ₹ 65,000

Average manufacturing cost = 65000/5000 = ₹ 13

**50.** As the age of the new student equals the old average age of the class, the new average age of the class will be 20 years.

**51.** Total of day temperatures of the week = $45 \times 7 = 315$

Total of day temperatures for the first six days

= 45 + 40 + 52 + 48 + 41 + 42 = 268 day temperature on Saturday = 315 – 268 = 47°C

**52.** Given $M + P + C = 80 \times 3 = 240$ (1)

$M + P = 90 \times 2 = 180$ (2)

$P + C = 70 \times 2 = 140$ (3)

where $M$, $P$ and $C$ are marks obtained by the student in Mathematics, Physics and Chemistry.

$P = (2) + (3) - (1) = 180 + 140 - 240 = 80$

**53.** Initial quantity of milk in the vessel = 60% (20) = 12 litres

Final quantity of milk in the vessel = 12 + 5 = 17 litres.

Quantity of new mixture = 20 + 5 = 25 litres.

Required percentage = $\frac{17}{25}$ (100) = 68%

**54.** Percentage of milk in the mixture formed

$$= \frac{\frac{80}{100}(180) + 36}{180 + 36} \times 100\% = \frac{144 + 36}{216}.100\%$$

$$= \frac{5}{6}.100\% = 83.\overline{3}\ \%.$$

**55.** Let the average number of chocolates eaten by the boys be $x$.

Number of chocolates eaten by Bhavan = $x + 18$.

Total number of chocolates eaten by the others = (5) (18) = $90 \Rightarrow 19x = x + 18 + 90$

$x = 6$

So, Bhavan ate 6 + 18 = 24 chocolates

**56.** As nothing is known about the ratio of the number of boys to the number of girls in the class, the average weight of the students in the class cannot be found.

**57.** Let the strengths of the students in the three colleges be $x$, $2x$, $4x$, respectively, Weighted average age

$$= \frac{x \times 30 + 2x \times 25 + 4x \times 5}{7x} = 20 \text{ years.}$$

**58.** Quantity of milk in the vessel does not change.

Quantity of milk in the vessel = (0.9) (70) = 63 litres.

After adding water, milk forms 87.5% of the vessel's contents.

$\frac{87.5}{100}$ (Final quantity of mixture) = 63 litres.

Final quantity of the mixture = $(63)\left(\frac{100}{87.5}\right) = 72$ litres.

Quantity of water to be added = 72 – 70 = 2 litres.

**59.** Let the average age of the 11 men in the original group be $A$.

So, $11A = 29 + 31 +$ (sum of ages of rest)

Let the ages of the two men replacing the two men aged 29 years and 31 years be $x$ and $y$, respectively.

$11(A - 1) = x + y +$ (sum of ages of rest)

Subtracting the above equation from the previous equation,

$11 = 60 - (x + y)$

$x + y = 49$

Average age of Amar and Bhavan

$= \frac{x + y}{2} = 24.5$ years

**60.** As nothing is known about the ratio of the number of male employees to the number of female employees in the office, the average height of the employees cannot be found.

## EXERCISE-2

**1.** The two-digit numbers that leave a remainder of two when divided by 6 are 14, 20, 26, ..., 98. As the given series is in arithmetic progression, the required average is $(14 + 98)/2 = 56$.

**2.** Quantity of petrol now $= 1/7 \times k + 3/14 \times k = 5k/14$

Quantity of diesel now $= 2/7 \times k + 5/14 \times k = 9k/14$

Quantity of kerosene now $= 4/7 \times k + 6/14 \times k = 14k/14$.

Ratio of quantities of petrol, diesel and kerosene

$= 5k/14 : 9k/14 : 14k/14 = 5 : 9 : 14$

**3.** Let the average of the batsman before the 53rd innings be $x$. $52x = 53\ (x + 1) - 126 \Rightarrow x = 73$.

**4.** Let X, Z be mixed in the ratio $m : n$. in the 10 kg.

Cost of that variety

$$= \frac{25m + 35n}{m + n}$$

$$\frac{10\left(\frac{25m + 35n}{m + n}\right) + 8(30)}{18} = 32 \text{ (given)}$$

$\Rightarrow m : n = 7 : 43$

**5.** Average $= \dfrac{1093 + 1103 + 1109 + 1089 + 1096}{5}$

Or we observe that the numbers are close to 1100. Let us say, the numbers are

$1093 = 1100 - 7$

$1103 = 1100 + 3$

$1109 = 1100 + 9$

$1089 = 1100 - 11$

$1096 = 1100 - 4$

---

Sum $= 1100 \times 5 - 10$

So, average $= \dfrac{1100 \times 5 - 10}{5} = 1100 - 2 = 1098$

This can be written as

$$\overline{P} = P + \frac{\sum(P_i - P)}{n}$$

Here, it doesn't really make a difference whether we assume the mean as 1100 or 1095 etc.

This method would come in handy while dealing with weighted averages or in *DI*.

**6.** Let the total of the ages of the persons other than the person $ab$ years old be $T$

$T = 55A - ab$ and $55(1.2A) = T + ba$

$\therefore 55(1.2A) = 55A - ab + ba$

$\Rightarrow 55(0.2A) = ba - ab$

$\Rightarrow 11A = (10b + a) - (10a + b)$

$\Rightarrow 11A = 9(b - a)$

All the ages are two digit numbers. $\therefore$ $A$ is a two digit number. But then $b - a$ will also be a two digit number This is not possible,.

**7.** Let the prices of the two varieties be $5x$ and $6x$.

Let the cost price of the mixture be $y$.

$y + \dfrac{20}{100}y = 60y = 50$.

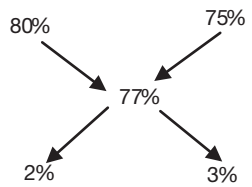
The cost price of the mixture $= \dfrac{4(5x) + 5(6x)}{4 + 5} = 50$

$50x = 50(9) \Rightarrow x = 9$

The cost of the cheaper variety is ₹ 45 / kg

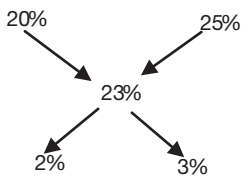
**8.** The question can be solved using the concept of alligation and converting the ratios into percentages. Observing carefully, you can see that the concentration of milk is 80% and in the second mixture the concentration of milk is 75%. The resultant mixture has 77% milk.

Thus the ratio can be found out using the following diagram as




Or in other words the required ratio is nothing but 2 : 3.

Similarly, also by taking the water concentration, the result can be arrived at as below:




**9.** Choice (A):

This is true because it is a property of weighted average.

Choice (B):

Average weight of $P$, $Q$ and $R = \dfrac{3(\text{Total weight of } Q)}{18 + 24 + 30}$

$= \frac{3(\text{Total weight of } Q)}{72} = \text{Average weight of } Q$

$\therefore$ Choice (B) is true.

Choice (C):

For $P$'s average weight to decrease, the student's weight must be more than $P$'s average weight, and for R's average weight to decrease, his weight must be less than R's average weight.

$\therefore$ Choice (C) is true.

**10.** Let the number of students attending classes on Monday be $x$.

Number of students attending classes on Tuesday, Wednesday, Thursday, Friday and Saturday

were $x + 1$, $x + 2$, $x + 3$, $x + 4$ and $x + 5$, respectively

$\frac{x+x+1+x+2}{2} = 34$

$\frac{3x+3}{3} = 34$

$x + 1 = 34$

Required average = $\frac{33+37}{2} = 35$

**11.** Let $n$ be the number of people in the group.

Let a cm be this average height.

Hence, initial total value = $na$ (1)

Net change in total value = –160 + 190 = 30 cm.

Hence, total value of changes = $(na + 30)$ cm (2)

The average = initial average + 5 cm = $(a + 5)$cm.

Hence, total value after changes = $n(a + 5)$ (3).

(2) and (3) are equal.

i.e., $na + 30 = na + 5x; \Rightarrow n = 6$.

**12.** Given that the average of $n$ numbers is 32.

Since $3/4^{th}$ of the numbers are increased by 4, the overall average of the $n$ numbers increases by $\left(4 \times \frac{3}{4}\right) = 3$

So, the average becomes $(32 + 3) = 35$

Since $1/4^{th}$ of the $n$ numbers are decreased by 6, the overall average of the $n$ numbers decreases by $\left(6 \times \frac{1}{4}\right) = 1.5$

So, the new average is $(35 – 1.5) = 33.5$

**13.** Let $x$ be the volume of milk at the beginning. The following table gives the amount of milk, water and the total fluid after the first and the second operations.

| S.No | Milk | Water | Total |
|---|---|---|---|
| 1. | $x - 5$ | 5 | $x$ |
| 2. | $\frac{(x-5)^2}{x}$ | | $x$ |

Given, $\frac{(x-5)^2}{x^2} = \frac{49}{15+49} \Rightarrow \frac{x-5}{x} = \frac{7}{8}$

$\Rightarrow 8x – 40 = 7x \Rightarrow x = 40$.

**Alternative Solution:**

After one replacement, the concentration of milk gets multiplied by $k$ (say). After 2 replacements it gets multiplied by $k^2$.

Initial concentration = 1.

Final concentration =

$$k^2 = \frac{\text{Milk}}{\text{Milk + Water}} = \frac{49}{49+15} = \left(\frac{7}{8}\right)^2$$

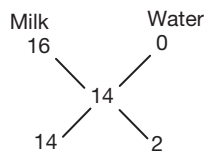
$\therefore k = \frac{7}{8}$, i.e., 5 litres (the quantity removed) is $\frac{1}{8}$ of the volume i.e., the original volume is 40 litres.

**14.** Let the number of managers in the organization be $n$.

Then, $(60 \times 9000 + n \times 45{,}000)/(60 + n) = 15{,}000$.

$\Rightarrow n = 12$

**15.** Cost of each litre of milk = 560/35 = ₹ 16.




$\therefore$ Quantity of water added = $\frac{1}{7} \times 35 = 5$ litres

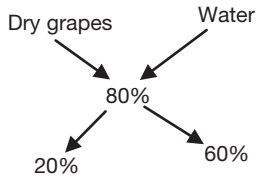
**16.** Quarter pint = $1/4 \times 0.568$ litres = 568/4 = 142 ml.

1 cu.cm = 1 ml.

$\therefore$ 142 ml can write $\frac{340}{10} \times 142 = 4828$ words

**17.** Though this question can be solved using the concepts of percentages, it can be solved in an easier manner using concept of alligation.

Fresh grapes are nothing but (dry grapes + water) the question can be solved in the following manner:




Or in other words 1 kg of dry grapes can be mixed with 3 kgs of water to make (1+3) = 4 kgs of fresh grapes.

Thus the ratio of dry grapes : fresh grapes = 1: 4

Thus for 20 kgs of dry grapes we will require 80 kgs of fresh grapes.

**18.** Ratio of strengths = 2 : 1

Let the values be $2x$ and $x$

Strength of Section 1 = Average score of section 2

$\Rightarrow$ Average Score of Section 2 = $2x$

Average Score of Section 1 = Strength of Section 2 = $x$

Weighted Average

$$= \frac{(2x \times x) + (x + 2x)}{2x + x} = \frac{4x^2}{3x} = \frac{4x}{3}$$

Given that weighted average = $53\ \frac{1}{3} = \frac{160}{3}$

$\frac{4x}{3} = \frac{160}{3}$ ; $x = 40$ and $2x = 80$

**19.** The quantities of milk, water and mixture at each stage are tabulated below.

| | Milk | Water | Total | Decrease |
|---|---|---|---|---|
| Initial Quantity | 10 | | 10 | |
| After water added | 10 | 5 | 15 | |
| After spilt and sold | $\frac{2}{3}(10)$ | $\frac{1}{3}(10)$ | 10 | 5 |
| After water added | $\frac{20}{3}$ | $\frac{10}{3} + 2$ | 12 | |

$x = (2/3)\ 10 = 20/3$

Percentage of milk in final mixture = $\frac{20/3}{12}(100)\%$

$= \frac{500}{9}\% = 55.55\%$

**20.**

| | A | B | C |
|---|---|---|---|
| No. of students | 34 | 51 | 85 |
| Avg. marks | 55 | 65 | 75 |

The question can be solved using the basic rule of weighted average.

$$\Rightarrow \text{Avg. marks} = \frac{34 \times 55 + 51 \times 65 + 85 \times 75}{34 + 51 + 85}$$

$$= \frac{11560}{170} = 68$$

Let us now look at the application of two very important concepts to weighted average.

(i) The number of items can be broken into ratios

So, 34 : 51: 85 = 2 : 3 : 5

(ii) The concept of deviation method of calculation can be used in calculating weighted average

$\Rightarrow$ Let us say the *WA* is 65.

$\Rightarrow$ Deviations are (– 10), (0) and (+ 10)

So, the calculation simplifies to

$$WA = 65 + \frac{(-10).2 + 0.3 + 10.5}{2 + 3 + 5}$$

$$= 65 + \frac{30}{10} = 68.$$

**21.** Let the capacity of the vessel be $x$ litres. After the first replacement, $x - 20$ litres of milk would be present in the vessel.

$\therefore \left(\frac{x-20}{x}\right)$ of the vessel will have milk. After 20 litres is withdrawn, quantity of milk would have then been

$\frac{x-20}{x}(x-20)$ litres.

$\frac{(x-20)^2}{x} = 18$

$x^2 - 58x + 400 = 0$

$x = 50$ or 8

$x$ must be more than 20.

$\therefore x = 50$

**22.** The question can be solved using allegation principle.

Article 1 $\rightarrow$ Loss = 11% = –11 ($P_1$)

Article 2 $\rightarrow$ Profit = ? % $P_2$

Overall from both articles $\rightarrow$ Profit = 24% = 24 ($P$)

The cost price of the articles are in the ratio 3 : 5

$$\Rightarrow \frac{Q_1}{Q_2} = \frac{3}{5}\ \frac{Q_1}{Q_2} = \frac{P_2 - P}{P - P_1} \Rightarrow \frac{3}{5} = \frac{P_2 - 24}{24 - (-11)}$$

$$\Rightarrow \frac{3}{5} = \frac{P_2 - 24}{35} \Rightarrow 7 \times 3 = P_2 - 24\ P_2 = 45$$

Hence, the 2nd article should be sold at 45% profit.

**23.** Let the number of wickets taken before last match be $x$

$$\frac{1200}{x} = \frac{1200+144}{x+4}$$

$$\Rightarrow \frac{1200}{x} + 1 = \frac{1200+144}{x+4}$$

$$\Rightarrow (1200 + x)\ (x + 4) = 1344x$$

$$\Rightarrow x^2 - 140x + 4800 = 0$$

$$\Rightarrow (x - 60)\ (x - 80) = 0$$

$$\therefore x = 60 \text{ or } 80$$

**24.** Total salary = 20 (8000) = 1, 60,000

There is a 22.5% increase for 10 employees.

$\frac{22.5}{100}$ (8000) = 1800.

The new salary of these employees is 8000 + 1800 or 9800.

For 10 employees, the total is 10 (9800) = 98,000

From the remaining 10 employees, 2 left. The total salary of the other eight is 8 (8000) or 64,000

$\therefore$ Total income of all the 18 employees = 1,62,000

$$\therefore \text{The new average} = \frac{162000}{18} = ₹9000$$

**25. Method 1**

Let the marks of the student leaving the group be $x$.

Since the average marks of the 10 students is 60, their total score will be 60 $x$ 10 = 600.

The new average being 59, we can say

$600 - x = 59 \times 9$

so, $x = 69$.

**Method 2**

Let us say, each person in the group had contributed ₹60 to the group. When one person leaves the group, he will take his share of ₹60. Also, since the new average is 1 less, we can say the leaving person has taken Re 1 from each of the remaining members. Since there are 9 members remaining, we get ₹60 + ₹9 = ₹69. The mark of the person who left the group is 69.

The above question could also be framed around a new person joining the group and as a result the average goes up or down. So, if instead of leaving, a new person had joined the group and as a result, the average had gone up by 2, then:

The new person first contributes ₹60 to match the contributions of every one else. Further, to increase the average by 2, he must contribute ₹2 per person present. Now, we have 11 (10 + 1 new person) present. So, additional contribution is ₹11 × 2

= 22. So, the overall mark of the new person is 60 + 22 = 82.

**26.** Total cost price = (10) (10) + (10) (6) = ₹160

Total selling price = (10 + 10) (10) = ₹200

Total profit = ₹40

Profit percent = $\frac{40}{160}$ (100) = 25%

**27.** Let the average age of the class be $A$ years. Let the ages of the replaced students be $2x$ and $x$ years. Total age of the students who are not replaced = $(48A - 3x)$ years.

$$48A - 3x + 16 + 11 = 48\left(A + \frac{1.5}{12}\right)$$

$$\Rightarrow x = 7$$

**28.** Using Alligation Rule, the ratio of the cost prices of the T.V. set and the washing machine

= (45% – 30%) : ( 30% – 20%), i.e., 3 : 2.

Hence, the cost price of the machine = 2 × 75000 / 5 = ₹30,000.

**29.** The mixture of 20ml has 0.4(20), i.e., 8 ml milk and 12 ml water.

Initial ratio of milk and water = 2 : 3.

$$\frac{8+X}{12} = \frac{3}{2} \Rightarrow X = 10$$

$$\frac{8+X}{12+Y} = \frac{2}{3} \Rightarrow Y = 15$$

Difference of $X$ and $Y$ is 5.

**30.** Let the amount spent by each girl be ₹ $a$.

Amount spent by each boy = ₹ $(a + 20)$

$$\therefore 6(a + 20 + a) = 1380 \Rightarrow a = 105$$

$$\text{Average amount} = \frac{1380 - 3(a + a + 20)}{6}$$

$$= \frac{1380 - 3(230)}{6} = ₹115$$

**31.** Assuming that the total quantity of milk initially was $100X$ litres, if 1 litre is replaced every time from 10 litres, then it is equal to $10X$ litres from $100X$ litres.

Thus after the first operation, the amount of milk left will be $90X$ litres.

Thus the ratio of milk to water after this operation is 90:10.

Thus the amount of milk that will go out in the second operation is $10X \times \left(\frac{90}{100}\right) = 9X$

Thus the amount of milk left will be $90X - 9X = 81X$ and ratio of milk to water will be $81X : 19X$.

So, in the 3rd operation, milk going out will be

$10X \times \left(\frac{81}{100}\right) = 8.1X$

Thus the amount of milk left will be $81X - 8.1\ X = 72.9X$

Or in other words every time 10% of the existing amount of milk is going out.

Thus after the first operation the concentration of milk becomes 90% of original, the second operation it becomes 90% of (90% of the original) = 81% of the original and so on.

Thus the multiplying factor becomes 0.9, $0.9^2$, $0.9^3$ after the first, second and third operations, respectively.

This gives the concentration of the liquid as a proportion of the total volume of the solution.

Thus for the above question the amount of milk after the 3rd operation will be $\left(\frac{9}{10}\right)^3 \times 10$ litres = 7.29 litres.

**32.** $A = \frac{2B + C}{2} \Rightarrow 2A = 2\ B + C$ (1)

$B = \frac{\frac{1}{2}(A) + C}{2} \Rightarrow 2B = \frac{A + 2C}{2}$

$\Rightarrow 4B = A + 2C \Rightarrow A = 4B - 2C$ (2)

From (1), $4A = 4B + 2C$ (3)

$2C = 4A - 4B = 4B - A$ ($\because$ From (2) and (3))

$5A = 8B$

$\frac{A}{B} = \frac{8}{5}$

**33.** Bowling average = $\frac{\text{Runs conceded}}{\text{Wickets taken}}$

$\Rightarrow 23 = \frac{R}{W} \Rightarrow R = 23\ W$

Given, $\frac{23W + 38}{W + 4} = 22.5 \Rightarrow W = 104$

$\therefore$ Runs conceded before this match = $23W$

$= 23(104) = 2392$

**34.** Let the original average of the group be a and the age of the man who left be $y$.

Average = $\frac{\text{sum}}{\text{Nos.}} \Rightarrow$ Sum = Average × Nos.

$\Rightarrow a.10 + 23 - y = (a - 1).10 \Rightarrow y = 33.$

**35.** Required average = $\frac{50\,(1 + 2 + 3 + 4 + 5 + \ldots + 10)}{10}$

$= \frac{10}{2}\ (50 + 500)\ \frac{1}{10} = \frac{550}{2} = 275$

**Alternate Method:**

Average of even number of terms in an arithmetic series is equal to average of 1st and last terms.

$\therefore$ Average = $\frac{50 + 500}{2} = 275$

**36.** Let the weights of Atul, Bhanu, Chinna and Dhanush be a kg, $b$ kg, $c$ kg and $d$ kg, respectively.

$\frac{a + b + c + d}{4} = 80$ (1)

$a = \frac{b + c + d}{3}$

$\Rightarrow 3a = b + c + d$ (2)

$b = \frac{a + c + d}{3}$

$\Rightarrow 3b = a + c + d$ (3)

By adding (2) and (3), we get

$3\ (a + b), = a + b + 2\ (c + d)$

$a + b = c + d$; $\therefore$ (1) becomes $\frac{1(c + d)}{4} = 80$

$\frac{c + d}{2} = 80$; Therefore, the average weight of Chinna and Dhanush is 80 kg.

**37.** Total cost of the resulting mixture

$= ₹\,[(12)\ (24) + (6)\ (27)] = ₹\,[288 + 162] = ₹\,450$

Total quantity of the resulting mixture

$= (12 + 6)$ kg = 18 kg

Cost of the resulting mixture (in ₹/kg) = $\frac{450}{18} = 25$

**38.** The average temperature on the first 3 days of the week

$= \frac{20 + 38 + 23}{3} = 27°C$

This is also the average temperature on the last four days of the week.

Let the temperature on Wednesday be $x$.

$\frac{x + 29 + 22 + 30}{4} = 27$, $x = 27°\ C.$

**39.** Let earnings for the month of November be '$x$'. Then, total annual earnings will be

$(1600\ x\ 6) + 1500 + 1800 + 1900 + 2150 + 3x$

$= 16950 + 3x$

Total earnings for the entire year

= 12 x 1600 (given)

= 19200 which is equal to 16950 + 3$x$

16950 + 3$x$ = 19200 $\Rightarrow$ 3$x$ = 2250

$\therefore$ $x$ = ₹750

Hence, the earnings in the month of November is ₹750.

**40.** The total age of the family should have increased by 25 years (5 × 5). Since it did not happen, it implies that the member who has come in is 25 years younger than the member who left the group.

$\therefore$ The age of the daughter-in-law = 48 – 25 = 23 years.

**41.** Weight of dried pees = 1.5 kg

Water content = 20%

Hence, non-water content = 80% of 1.5

= 0.8 × 1.5 = 1.2kg (1)

Let the weight of the soaped pees = $x$ kg

Water content = 60 %

Non-water content = 40% of $x$ kg = 0.4$x$ (2)

(1) and (2) an equal

Hence, 1.2$x$ = 0.4$x$ $\Rightarrow$ $x$ = 3.

**42.** Let the sum lent to $P$ be ₹$P$, then the sum lent to $Q$ will be (60,000 – $P$).

Given, [($P$ x 2 x 12) + (60,000 – $P$) x 2 x 9] / 100 =13,200 $\Rightarrow$ $P$ = 40,000.

Hence, the sum lent at 9% = ₹20,000.

**43.** Let us say the couple got married $K$ years ago.

Let the ages of the couple be $F$ and $M$ at that time.

Present age of the child = $K$ – 3

$$\frac{K+F+K+M+K-3}{3}=34$$

60 + 3$K$ – 3 = 102 (Q $F$ + $K$ = 30 × 2) $K$ = 15.

Present age of Child = $K$ – 3 = 12

**44.** $P$ is the set of the first 2$N$ natural numbers.

$\therefore$ It is the set of the first $N$ odd natural numbers and the first $N$ even natural number. The nth even natural number where $1 \le n \le N$ is 1 more than the $n^{th}$ odd natural number.

$\therefore$ The average of the first $N$ even natural numbers must be 1 more than the average of the first $N$ odd natural numbers.

$\therefore$ $R$ – $Q$ = 1.

**45.** The concentration of milk in $P$ initially = 100%.

**Iteration I:**

When we add 30 litres to $Q$, its volume becomes 150 litres with the milk: water ratio of 1 : 4. So, if we take out 30 litres of the mixture, it will be in the same ratio. So, milk

$= \frac{1}{5} \times 30 = 6$ litres. Added to $P$, the new ratio of milk and water in $P$ becomes 96 : 24 = 4 : 1.

**Iteration II:**

When we take out 30 L from $P$, it will have $30 \times \frac{4}{5} = 24$ litres milk.

Added to $Q$, the ratio of milk : water becomes 48 : 102 = 8 : 17. Again taking out 30 L will get us $30 \times \frac{8}{25} l = 9.6$ litres milk.

So, the ratio of milk : water in $P$ = 81.6/38.4 = 17/8 = 2.125

**46.** Let $x$ litres be taken from the first container.

Total quantity of petrol = $\frac{4}{7}x + \frac{3}{4}x\,16 = \frac{4}{7}x + 12$

Total quantity of diesel = $\frac{3}{7}x + \frac{1}{4}x\,16 = \frac{3}{7}x + 4$

Now, $\frac{\frac{4x}{7}+12}{\frac{3x}{7}+4} = \frac{32}{19} \Rightarrow x = 35.$

**47.** Let the cost of the second pen be $x$, the cost of the first pen = $x$ + 25$x$/100 = 5$x$/4

Let the cost of the third pen be $y$

$x = y + 20y/100 = 6y/5$

$y = 5x/6$

$$\frac{\frac{5}{4}x + x}{2} = \frac{x + \frac{5}{6}x}{2} + 15 = \frac{x + \frac{5}{6}x + 30}{2}$$

$$\left(\frac{1}{4} - \frac{1}{6}\right)x = 6 \Rightarrow x = 12 \times 6 = 72$$

The costliest pen = 5$x$/4 = ₹90.

**48.** For finding out the average mark of the whole class we will need the sum of marks of the whole class.

The sum of marks of all students in section $A$ is 50 × 30 = 1500

The sum of marks of all students in section $B$ is 75 × 40 = 3000

Thus total of marks for the class is (1500 + 3000)

= 4500

So, the average mark of the class is $4500/(50+75) = 36$.

**49.** Let the original number of boys in the group be $N$.

Total weight of boys originally in the group = $30N$ kgs = $30N + 62 = (30+2)(N+1) \Rightarrow 30N + 62 = 32N + 32$

$\Rightarrow 2N = 30 \Rightarrow N = 15$.

**50.** Cost of the mixture = $12/1.2 = ₹10$ per kg.

Let the cost of the first variety be ₹ $x$ kg, that of the second variety would be ₹ $(x-7)$ per kg.

$$\frac{2}{5} = \frac{10-(x-7)}{x-10} \Rightarrow 2x - 20 = 85 - 5x$$

$\Rightarrow 7x = 105 \quad \therefore x = 15$.

**51.** Total weight of boys in the group = $30N$ kg

If 5 boys with an average weight of 12 kg join the group, the average weight becomes $\frac{30N+60}{N+5}$ kg,

If 5 boys with an average weight of 36 kg leave the group, the average weight becomes $\frac{30N-180}{N-5}$ kg.

$$\frac{30N+60}{N+5} = \frac{30N-180}{N-5}$$

$30N^2 - 150N + 60N - 300$

$= 30N^2 - 180N + 150N - 900 = N = 10$.

**52.** Total marks of all the students = $40 \times 85 = 3400$

Total marks of the remaining 38 students

$= 38 \times 84 = 3192$

$\therefore$ Sum of the highest and the least marks

$= 3400 - 3192 = 208$

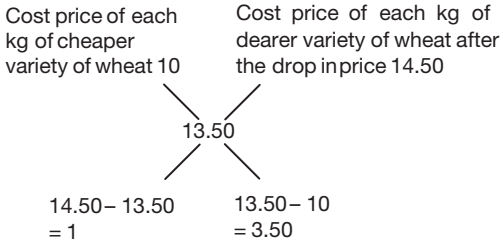
Difference between the two marks = 108

$\therefore$ Highest marks = $\frac{280+108}{2} = \frac{316}{2} = 158$.

**53.** Let the quantities of the cheaper and dearer varieties mixed be $3x$ kg and $7x$ kg.

Cost prices of $3x$ kg and $7x$ kg of the mixture

$$= \frac{3x(10)+7x(15)}{10x} = ₹13.50$$




Quantity of cheaper variety/quantity of dearer variety = $1/3.50 = 2/7$

**54.** Let $x$ kgs of the first variety be mixed with $y$ kgs of the second variety to create the resultant variety.

$$\frac{6x+8y}{x+y} = 7.2$$

Taking all the '$x$' on one side and all the '$y$' on the other side we have

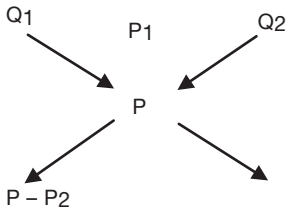
$6x - 7.2x = 7.2y - 8y$

Or $\frac{x}{y} = \frac{8-7.2}{7.2-6} = \frac{0.8}{1.2} = \frac{2}{3}$

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{Rate of dearer} - \text{Average rate}}{\text{Averag rate} - \text{Rate of Cheaper}}$$

or

$$\frac{Q_1}{Q_2} = \frac{P_1 - P}{P - P_2}$$




**55.** Let the total number of students be $x$ and the average marks of the class be $y$.

Let the average of the remaining one fourth of the students be $z$.

$$\therefore xy = \frac{3x}{4}\left(\frac{3y}{4}\right) + \left(\frac{x}{4}\right)(z)$$

$$\Rightarrow xy = \frac{9xy}{16} - \frac{xz}{4} \Rightarrow \frac{7xy}{16} = \frac{zx}{4} \Rightarrow z = \frac{7y}{4}$$

$\therefore$ The required average is $\frac{7}{4}$ th of the average of the entire class.

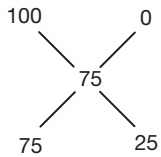
**56.** Let $CP$ of 1 litre of pure milk = ₹100

$\Rightarrow SP$ of 1 litre of milk solution = ₹100

But by selling the milk solution at ₹100 there is a profit of $33^1/_3\%$. Hence, the cost price of the milk solution

$$= \frac{100 \times 3}{4} = ₹75$$

Using alligation rule, we have



The ratio of milk and water as 75 : 25, i.e., 3 : 1

$\therefore$ Percentage of water in the solution

$= \frac{1}{4} \times 100 = 25$

**57.** Average marks = (Sum of the marks obtained by the students of sections $A$ and $B$)/Total number of students.

As the number of students in sections $A$ and $B$ is in the ratio 2 : 3, let the number of students in sections $A$ and $B$ be $2k$ and $3k$, respectively. Hence, the required average $= (2k \times 30 + 3k \times 40)/(2k + 3k)$, i.e., $180k/5k = 36$.

**58.** Let the average height of the class be $H$ cm.

Let the number of students in the class be $N$.

$HN + (3 \times 160) - (5 \times 168) = 180\ (N - 2)$.

$HN - 360 = 180N - 360$. $H = 180$.

If two students whose heights are 172 cm and 182 cm join the class, their average (177) is less than the present average of the class.

$\therefore$ The average height of the class will decrease.

**59.** Let the quantities of the first, second and third varieties used for mixing be $x$ kg, $y$ kg and $z$ kg, respectively.

Total cost of the mixture = ₹ $(12x + 18y + 21z)$

$12x + 18y + 21z = (x + y + z)\ \frac{15 \times 100}{120}$

$\Rightarrow 2(12x + 18y + 21\ z) = 25(x + y + z)$

$\Rightarrow x = 11y + 17z$

Only choice (C) is a possible ratio among the choices.

**60. Method 1:**

| Milk | Water | Total |
|---|---|---|
| $3x$ | $x$ | $4x$ |
| $\frac{0}{3x}$ | $\frac{14}{x+14}$ | $\frac{14}{4x+14}$ |

Thus $3x : (x+14) = 2:3$

So, $x = 4 \Rightarrow 4x = 16$

**Method 2:** No change component




7 parts = 14 litres

8 parts = 16 litres

Since no milk has been added, make that component equal in both the ratios. Thus writing the first ratio as 6 : 2 and the second ratio as 6 : 9, we see that the amount of water has increased from 2 parts to 9 parts that is 7 parts.

Thus 7 parts = 14 litres

So, initial quantity = (6+2) = 8 parts = 16 litres

**Method 3:** Using the concept of variation, the concentration is inversely proportional to the volume of a solution.

Applying this, the concentration of milk becomes

$\frac{\left(\frac{2}{5}\right)}{\left(\frac{3}{4}\right)} = \frac{8^{th}}{15}$ of the original concentration.

So, the volume must have become $\frac{15^{th}}{8}$ of the original volume.

Hence, there is an increase of 7/8 from the original volume which is equal to 14 litres. So, the original amount must be equal to 16 litres.

## EXERCISE-3

**1.** Let fixed cost be $F$ and variable cost per unit of the article produced be $K$.

$30{,}000 = F + 15{,}000\ K$ ($\because$ Total cost $= 2 \times 15000 = 30000$)

$37{,}500 = F + 25{,}000\ K$

Subtracting the first equation from the second, $7500 = 10{,}000\ K \Rightarrow K = 0.75$

Hence, variable cost of each unit produced is ₹0.75.

**2.** Let the average score be $n$, the score of the topper $10x$ and the score of the least scorer $x$.

As we can see

$(31n - 10x)/30 = n - 2$

$(31n - x)/30 = n + 1$

Taking the difference of the two equations, we get

$9x = 90$ or $x = 10$

The score of the topper is 100.

**3.** $P : Q = 3 : 4$ and $Q : R = 5 : 4$

$\Rightarrow P : Q : R = 15 : 20 : 16$

Let $P = 15k$, $Q = 20k$, $R = 16k$

$$\text{Average} = \frac{15k \times 42 + 20k \times 48 + 16k45}{51k} \simeq 45.29\,\text{kg}$$

**4.** Solution 1 : 50% wine, volume unknown.

Solution 2 : 40% wine, obtained by adding water to Solution 1.

Hence, quantity of wine of (1) and (2) is same.

If $V_1$ and $V_2$ are the volumes of solution 1 and solution 2, respectively, $50\%\ V_1 = 40\%\ V_2$

$\Rightarrow V_2 = \frac{5}{4}\ V_1 \Rightarrow \frac{1}{4}\ V_1$ of water needs to be added.

By mistake this much quantity $\left(\frac{V_1}{4}\right)$ of pure wine

is added, and solution 3 is formed.

Solution 3 has $\left(\frac{V_1}{2} + \frac{V_1}{4}\right)$ wine and $\left(\frac{V_1}{2}\text{water}\right)$

i.e., solution 3 has $\frac{3V_1}{4}$ wine out of a total of $\frac{5V_1}{4}$ volume.

By adding water to solution 3, 40% wine ( i.e., solution 4) is to be made.

If $V_4$ is the volume of solution 4, the equation regarding quantity of wine is,

$$\frac{3V_1}{4} = \frac{4}{10} \times V_4 = \frac{3}{4} \times \frac{10}{4} \times V_1$$

$$V_4 = \frac{15}{8} V_1$$

Water added to solution 3

= Volume of solution 4 – Volume of solution 3

$$= \frac{15}{8} V_1 - \frac{5}{4} V_1 = \frac{5}{8} V_1$$

Water added to solution 3 is $\frac{5}{8}$th of the initial volume of solution 1.

**Alternate Method:**

Let us assume the total quantity to be 100 lt. In this 100 lt, 50 lt is wine and the other 50 lt is water. By adding some water, the total concentration of wine should be 40%, i.e., 40% × (Total solution) = 50 lt

$\Rightarrow$ total solution is 125 lt, i.e., 25 lt of water was supposed to be added. But, by mistake, 25 lt of wine is added.

$\therefore$ In this solution, some water has to be added so that in the resulting solution, the concentration of wine is 40%

i.e., 40% (Total new solution) = 75 lt

(= new quantity of wine = 50 + 25)

$\Rightarrow$ Total volume of new solution is 187.5 lt

i.e., 187.5 – 125 = 62.5 lt of water is added. Required fraction is $\frac{62.5}{100} = \frac{5}{8}$.

**5.** After drawing 60 lts of wine from 600 lts of wine, 540 lts of wine is left.

After 60 lts of water is added, ratio of wine: total quantity of mixture = 540: 600 = 9 : 10

After drawing 60 lts from this mixture, 540 lts of mixture is left.

Quantity of wine in this = (9/10) (540) = 486 lts.

After 60 lts of water is added to the mixture, the ratio of wine to total quantity of mixture = 486 : 600

After 60 lts is again drawn from this mixture, 540 lts of mixture is left.

Quantity of wine in this = (486/600) × 540

= 437.4 lts.

After 60 lts of water is added quantity of wine remains the same. Present quantity of wine in the mixture = 437.4lts.

**Alternative Solution:**

All such problems can be solved using the following relation.

(Final concentration) = (Initial concentration) $\left(\frac{V - x}{V}\right)^n$

Where $V$ is the initial volume of the mixture, $x$ is the volume replaced each time, $n$ is the number of replacements.

Hence, final concentration of wine in the mixture

$$= (1)\left(\frac{600 - 60}{600}\right)^3 = \left(\frac{540}{600}\right)^3 = 0.729$$

Hence, quantity of wine = (600) (0.729) = 437.4 ltrs.

**6.** Milk content in the first and second cans is 100 – 73 = 27% and 100 – 41 = 59%, respectively.

Milk concentration in the new mixtures =

$$= \frac{3 \times 27\% + 5 \times 59\%}{8} \times 100 = 47\%$$

**7.** Sum of $P$ and its reverse = 10 a + $b$ + 10 $b$ + $a$ = 11($a$ + $b$)

Sum of $Q$ and its reverse = 10 $c$ + $d$ + 10 $d$ + $c$ = 11($c$ + $d$)

$\therefore 11[(c + d) - (a + b)] = 6.6 \times 10$

$\Rightarrow (c + d) - (a + b) = 6$

**8.** Let the average ages of the boys and the girls in the class be $B$ years and $G$ years, respectively.

Let the number of boys and the number of girls in the class be $b$ and $g$, respectively.

$$\frac{Bb+Gg}{b+g}=\frac{B+G}{2};\ 2Bb+2Gg=Bb+Bg+Gb+Gg;$$

$\Rightarrow (b-g)\ (B-G)=0$; As $b\neq g$, $b-g\neq 0$

$\therefore B-G=0$ (1)

From the second condition, $B+G=10$ (2)

From (1) and (2), $B=5$.

**9.** Average mark of the students in the class =

$$\frac{(225)(60)+(205)(56)+(180)(55)+(195)(70)+(220)(65)+(250)(80)}{225+205+180+195+220+250}$$

Assume the average mark to be 60. Writing the deviation from 60, the average mark of the students in the classes

$$\frac{225(60)+205(60-4)+180(60-5)+195(60+10)+220(60+5)+250(60+20)}{1275}$$

Dividing both numerator and denominator by 5

$= (255)(60)/255 + 1266/225 = 60 + 4.96 = 64.96$

**10.** Total age of 5 members today $(17 + 3)\ (5) = 100$

= After the new member added, the average age = 17

$\Rightarrow$ Total age = 17 (6) = 102

$\Rightarrow$ Age of baby now = (102 – 100) = 2 years

Let $a$, $b$, $c$ be ages of 3 children now

$\Rightarrow 43 + 42 + a + b + c + 2 = 102 \Rightarrow a + b + c = 15$

We have to minimize 2 ages in order to maximize the third.

Minimum possible current age = 3

(Q 3 years ago it was a family of 5 people)

$\Rightarrow a = b = 3$ (say)

$\Rightarrow c = 15 - (3 + 3) = 9$

**11.** The sum of the ages of the seven members, 9 years ago, was 209 years.

The sum of their ages 6 years ago would have been 209 + 7 (3).

Since one member died at the age of 52 and one member was born, the sum of the ages of the members seven member family will be 209 + 7(3) – 52.

Sum of the ages of the members seven member family 4 years ago was (Before the member died and the baby was born)

= 209 + 7(3) – 52 + 7(2).

Again a member died at 52 and another was born, thereby decreasing the sum by 52 years, and number of members in the family would still be seven. $\therefore$ Sum of their ages 4 years ago = 209 + 7(3) – 52 + 7(2) –52.

Sum of the ages of the members of the seven member family currently = 209 + 7(3) – 52 + 7(2) – 52 + 7(4). = 168 years. The current average age of the seven member joint family is $\frac{168}{7}$ = 24 years.

**12.** To solve this problem, we can look at a formula (to be remembered by the students) which will greatly simplify the solution.

In this problem,

$P = 300$ ml, $Q = 30$ ml and $n = 3$.

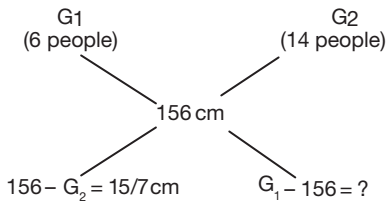
Hence, concentration of milk

$$= \left(\frac{300-30}{300}\right)^3 = \left(\frac{270}{300}\right)^3 = \left(\frac{9}{10}\right)^3 = 0.729$$

$\therefore$ Percentage of milk is 72.9% volume.

**13.** Let the average height of 6 people in group $B$ be $G_1$ and the average height of the remaining 14 people of group $B$ be $G_2$

Using allegations:




$6 : 14 :: 1^5/_7 : (G_1 - 156)$

$$G_1 - 156 = 1\frac{5}{7}\times\frac{14}{6} = 4 \text{ cm}$$

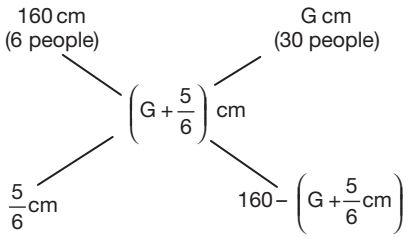
$\Rightarrow G_1 = 160$ cm

Let the average height of 30 people of group $C = G$ cm

$\Rightarrow$ The average height of new group $C$ (of 36 people)

$= G + \frac{5}{6}$ cm

Using allegations:




$6:30::\frac{5}{6}:\left[160-\left(G+\frac{5}{6}\right)\right] \Rightarrow 160-\left(G+\frac{5}{6}\right)=\frac{5}{6}\times\frac{30}{6}$

$\Rightarrow G=159\frac{4}{6}-4\frac{4}{6}=155$

The difference between the averages of $B$ and $C$, before the transfer of $A$ is $(156-155)=1$ cm.

**14.** Say there are $x$ non *HOD*'s

$\Rightarrow 20400=\frac{4\times24000+18000x}{4+x}$

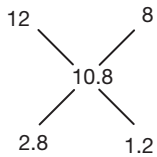
$\Rightarrow 204\ (4+x)=960+180x \Rightarrow 24x=960-816$

$\Rightarrow x=\frac{144}{24}=6$

Their contribution $=18{,}000\times6=₹1{,}08{,}000$

Average contribution received per child $=\frac{1{,}08{,}000}{10{,}000}$
$=₹10.8$

Using alligation rule,




$\therefore$ Ratio of boys and girls is $(2.8):(1.2)=7:3$.

$\therefore$ Number of boys $=\frac{7}{10}\times10{,}000=7{,}000$

**15.** Let initially there be $x$ litres of milk.

Now, $\left(\frac{x-10}{x}\right)^2=\frac{25}{25+24}$

$\Rightarrow \frac{x-10}{x}=\frac{5}{7}$

$\Rightarrow 7x-70=5x \Rightarrow x=35$.

**16.** In 5 minutes, the concentration of salt is reduced by 50%, i.e., to 50% of the initial value. Let it take $n$ intervals of 5 minutes for the concentration to fall below 0.05 %

i.e., $\left(\frac{1}{2}\right)^n 5\% \le 0.05\%$

or $\left(\frac{1}{2}\right)^n \le \frac{1}{100}$

$\Rightarrow n=7$

$\therefore$ It takes 35 minutes

**17.** Let $V_1$ and $V_2$ be the varieties costing ₹210/kg and ₹300/kg, respectively.

$V_1$ and $V_2$ are mixed in certain ratios to form $X$ and $Y$.

$X$ and $Y$ are mixed in the ratio 3 : 2 to form $Z$.

$Z$ is sold at ₹315/kg at 25% profit.

Let the cost of $Z$ be ₹ $c$/kg

$c+\frac{25c}{100}c=315$

$c=252$

Let the cost of $Y$ be ₹ $y$/kg

Le the qualities of $X$ and $Y$ mixed to form $Z$ be $3k$ kg and $2k$ kg, respectively.

Cost of $Z=\frac{3k\,(240)+2ky}{3k+2k}=\frac{720+2y}{5}=c=252$

$y=270$

Let $p$ kg of $V_1$ and $q$ kg of $V_2$ be mixed to form $V$.

Cost of $Y=\frac{p(210)+q\,(300)}{p+q}=y=270$

$\frac{p}{q}=\frac{1}{2}$

**18.** If a solution of alcohol and water contains y% of alcohol and z% of the solution is taken out and replaced by water, percentage of alcohol remaining is given by $\left(\frac{100-z}{100}\right)\times y\%$

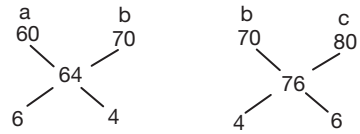
If this process is repeated $n$ times, the resultant concentration of alcohol is $\left(\frac{100-z}{100}\right)^n\times y\%$

For the given problem, $z=20$ and $y=80$.

Percentage of alcohol remaining

$=\left(\frac{100-20}{100}\right)^2\times\frac{80}{100}=51.2\%$

**19.** Let the number of students in section consider $A$, $B$ and $C$ be $a$, $b$ and $c$, respectively. We get




$\frac{a}{b}=\frac{70-64}{64-60}=\frac{6}{4}=\frac{3}{2}$ and $\frac{b}{c}=\frac{80-76}{79-70}=\frac{4}{6}=\frac{2}{3}$

$\therefore a : b : c = 3 : 2 : 3$

Thus the ratio of the number of students in $A$ and $C$ is 1 : 1

The average score of the students of section $A$ and $C$ together $= \frac{1}{2}(60) + \frac{1}{2}(80) = 70$.

**20.** The initial ratio of the number of boys and girls is not important. Finally, it is 1 : 1. The average age of all the students is 22. Therefore, for girls it is $22 - x$ and for boys it is $22 + x$. As $(22 + x) - (22 - x) = 2$, $x = 1$.

$\therefore$ The average ages of boys and girls are 23 and 21 years, respectively.

**21.** Let the two solutions be $A$ and $B$. The data is tabulated below.

| | Water | Milk | Total |
|---|---|---|---|
| A | | | 12 |
| B | | | 18 |
| | 18 | 12 | 30 |

| Water | Milk | Total |
|---|---|---|
| | | 12 |
| | | 12 |
| $14^2/_3$ | $9^1/_3$ | 24 |

If the entire volumes of $A$ and $B$ are mixed, milk constitutes 40% of $30l$ or $12l$.

If only 12 litres of $A$ are mixed with the entire $12l$ of B, milk constitutes $38\frac{8}{9}\%$ (or $\frac{350}{900}$ i.e., $\frac{7}{18}$) of $24l$, which is $9\frac{1}{3}l$.

$\therefore$ The extra six litres of A contain the extra $2\frac{2}{3}l$ of milk.

i.e., The $18l$ of A contain $8l$ of milk and hence B contains $4l$ of milk.

***Solutions for questions 22 and 23:***

The percentage of gold in Alloy I = 100 – (30 + 40) = 30%

Let the weights of Alloy II and Alloy III be $x$ kg and $y$ kg, respectively. As Alloy I does not contain tin, weights of tin in alloys II and III must be equal.

$\therefore \frac{40}{100}(x) = \frac{25}{100}(y)$; $\therefore \frac{x}{y} = \frac{5}{8} \Rightarrow x = \frac{5}{8}y$

Weight of Alloy I $= \frac{3}{8}y\,kg$.

As Alloy II does not contain copper, weights of copper in alloys I and III must be equal.

$\therefore$ Weight of copper in alloy III $= \frac{40}{100}\left(\frac{3}{8}y\right)kg$

As weight of alloy III = $y$ kg, % of copper in alloy III

$= \frac{\frac{40}{100}\left(\frac{3}{8}y\right)kg}{y} \times 100 = 15\%$.

**22.** Whenever equal weights of two or more alloys are mixed together, percentage of an element in the new alloy equals the average of the percentages of that element in the alloys mixed.

$\therefore$ The required percentage $= \frac{40+15}{2}\% = 27.5\%$

**23.** As Alloy I has 30% zinc and also Alloy III has 30% zinc, Alloy II must also have 30% zinc.

$\therefore$ The percentage of gold in Alloy II = 30%

The percentage of gold in Alloy I = 100 – (30 + 40) = 30%.

As both alloys I and II have equal percentage of gold ( i.e., 30%), alloy III must have 30% gold.

As both Alloys II and III have 30% gold, alloy obtained finally will also have 30% gold.

**24.** When the product of the two positive numbers is a constant, their sum is minimum when the numbers are equal. As $a > 0$, $b > 0$ and $(a + x)(b - x) = 64\ (> 0)$, both $(a + x)$ and $(b - x)$ are positive.

As product of $(a + x)$ and $(b - x)$ is constant,

$a + x + b - x$, i.e., $a + b$ is minimum when $(a + x) = (b - x) = \sqrt{64} = 8$.

Hence, $a + b \geq 16 \Rightarrow \frac{a+b}{2} \geq 8$.

**25.** We are interested in 3 – digit numbers of the form xyz where $x = 2z$.

$z$ can take the values 1, 2, 3 and 4. The corresponding values of $x$ are 2,4,6 and 8. $y$ can take all the values from 0 to 9.

The average of all the 3 – digit numbers =

$$\frac{(100x_1 + 10y_1 + z_1) + (100x_2 + 10y_2 + z_2) + \ldots + (100x_N + 10y_N + Z_N)}{N}$$

$$= \frac{100(x_1 + x_2 + \ldots + x_N) + 10(y_1 + y_2 + \ldots + y_N) + (z_1 + z_2 + \ldots + z_N)}{N}$$

$= 100\,A_H + 10\,A_T + A_U$

As each value in the units, tens and hundreds place occurs an equal number of times,

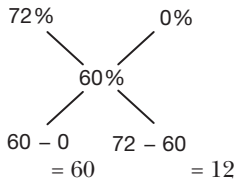
$A_U = \dfrac{1+2+3+4}{4} = 2.5$

$A_T = \dfrac{0+1+2+\ldots 9}{10} = 4.5$

$A_H = \dfrac{2+4+6+8}{4} = 5$

Average of all numbers = 5 (100) + (4.5) (10) + (2.5) (1) = 547.5

**26.** We will apply alligation rule here taking 72% concentration solution (of milk and water) mixed with a 0% concentration solution (pure water to give) 60% concentration solution.




The ratio of initial solution and water is 60 : 12

= 5 : 1

Since there were 40 litres of the solution initially, water to be added is 40/5 = 8 litres.

**27.** As the mixture is sold at ₹ 100 at 100% profit

$\Rightarrow$ we have 2 (*CP*) = 100

$\Rightarrow$ *C.P.* of the mixture = ₹ 50

Let the *C.P.* of variety *A* be ₹ *x*.

$\dfrac{3}{5}(x) + \dfrac{2}{5}(x-10) = 50 \Rightarrow 5x - 20 = 250$

$\Rightarrow 5x = 270 \Rightarrow x = ₹ 54$

$\therefore$ Variety *B* costs = ₹ 44

**28.** Let the average height of the boys and the girls be *B* cm and *G* cm, respectively. Class average = *A* (Say) = $\dfrac{90B + 60G}{150} = 0.6B + 0.4G$

B lies between 100 and 120 and *G* lies between 80 and 100.

*G* lies between 80 and 100.

0.6 (100) + 0.4 (80) < *A* < 0.6 (120) + 0.4 (100)

$\Rightarrow$ 92 < *A* < 112. Only choice *A* includes all these values.

**29.** Let the ratio in which *A* and *B* are mixed be *x* : *y*.

Cost price of the mixture = $\dfrac{10.80}{\left(1+\dfrac{20}{100}\right)}$ = ₹ 9 per kg

By alligation rule,

$\dfrac{x}{y} = \dfrac{9.75-9}{9-6.75} = \dfrac{1}{3}$

**30.** As *x* has 10% water, remaining 90% is milk, i.e., it has 2 litres of water and remaining 18 litres is milk.

Let *x* litres of water be added

$\Rightarrow \dfrac{2+x}{20+x} = \dfrac{25}{100} \Rightarrow 8 + 4x = 20 + x$

$\Rightarrow 3x = 12 \Rightarrow x = 4$ litres

As *y* has milk and water in the ratio 3 : 2, it has 12 litres of milk and 8 litres of water.

Let '*y*' litres of water be added

Then, $\dfrac{2}{3} = \dfrac{12}{8+y}$

$\Rightarrow 16 + 2y = 36 \Rightarrow 2y = 20 \Rightarrow y = 10$

$\therefore$ Required difference = $y - x = 10 - 4 = 6$ litres

**31.** The average of entire company becomes 35 years after 6 new persons join and 5 old persons leave the company.

Let the number of employees be '*n*'

$\dfrac{(n \times 40) + 6(30) - 5(40)}{(n+1)} = 35$

$(n \times 40) + 180 - 200 = 35(n+1)$

$n \times 40 - 20 = 35n + 35$

$5n = 55$

$n = 11.$

**32.** The components of the mixture, their concentration and quantities are tabulated below. We consider the two extreme values of a.

| Conc | $a = 40$ | 45 | 45 | 45 | $a = 50$ |
|---|---|---|---|---|---|
| Quantity | 5 | 6 | 6 | | 5 |

The concentration in the two extreme cases are

$40 + \dfrac{6}{11}(5) = 42\ \dfrac{8}{11}$ and $45 + \dfrac{5}{11}(5) = 47\ \dfrac{3}{11}$.

All values in the range $\left(42\dfrac{8}{11}, 47\dfrac{3}{11}\right)$ are possible.

Among the options, $47\ \dfrac{4}{11}$ is not possible

**33.** Let the containers have *x* litres of water and *x* litres of alcohol initially.

After the first transfer, container *A* has *x* – 3 litres water and container *B* has *x* litres alcohol and 3 litres water.

After the second transfer, container $B$ will have $\left(3-3\left(\frac{3}{3+x}\right)\right)$ litres water and $(x-3)\left(\frac{x}{3+x}\right)$ itres alcohol.

Given, water content is the same in both the containers finally

$$x-3+\frac{3(3)}{3+x}=3-\frac{3(3)}{3+x}$$

$$x+\frac{18}{3+x}=6 \Rightarrow x=3$$

**34.** Let the quantity of the 27% solution of alcohol added be $x$ litres.

Quantity of alcohol in it $=\frac{27}{100}x$

Quantity of alcohol in 20 litres of 36% alcohol solution

$=\frac{36}{100}(20)=7.2$ litres

$$\frac{27}{100}x+7.2=\frac{30}{100}(x+20)$$

$$1.2=\frac{3}{100}x \Rightarrow x=40$$

**35.** Let '$x$' be the increase in the average

$$\Rightarrow x=\frac{(73-64)8}{n+8}$$

$$\Rightarrow x=\frac{9\times 8}{n+8}$$

For '$x$' to be a whole number 72 $(=9\,x\,8)$ should be divisible by $(n+8)$.

It can be said that 24, 36 and 72 are such factors. But 24 and 72 do not lie within the range.

$\therefore$ number of students in class are 36.

**36.** Quantity of spirit in 4 litres drawn from vessel $A$

$=\frac{45}{100}(4)=1.8$ litres.

Quantity of spirit in 5 litres drawn from vessel $B$

$=\frac{30}{100}(5)=1.5$ litres

Quantity of spirit in 6 litres drawn from vessel $C$

$=\frac{25}{100}(6)=1.5$ litres

Hence, in 15 litres of the resultant solution, 4.8 litres of spirit is present.

Concentration of spirit in the resultant solution

$=\frac{4.8}{15}(100)=32\%$

**37.** As milk and water in the solution are in the ratio 3 : 2, let their quantities be $3k$ and $2k$, respectively.

Let $x$ litres of the solution be replaced with water. Then,

$$\frac{3k-3x/5}{2k-(2x/5)+x}=\frac{2}{3}\Rightarrow x=\frac{5k}{3}$$

$\therefore$ Required answer is $\frac{5k/3}{5k}$, i.e., one-third.

**38.** Average of the 36 numbers is 25

$\therefore$ Their total is (36) (25).

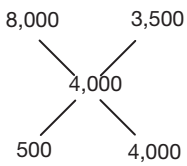
If the new average is 30, the negative deviation is 36 (30 – 25), i.e., 180 ($\therefore$ Each of the included numbers is at least 30. $\therefore$ There will be no negative deviation due to any of these numbers). As the new average must exceed 30, the included numbers must offset the negative deviation of 180.

The number of numbers included must be minimum. $\therefore$ We have to include the numbers with the greatest positive deviations.

The element 65 of the set has the greatest positive deviation. Its positive deviation is 35. If the elements 65, 64, 63, 62, 61 are included,their total positive deviation is 165. We need a total positive deviation of more than 180 $\therefore$ We need to include 60 also, which takes the positive deviation to 195.

Minimum number of numbers to be included is 6.

**39.** (a) Using alligation rule, we have



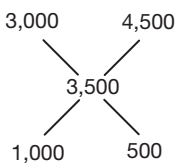

i.e., 5 : 40 = 1 : 8 the ratio of production managers and administrative employees as 1 : 8.

As there are 6 production managers the administrative employees = $6\times 8=48$

Total salary of administrative employees = $48\times 35$

= ₹168 (000$s$)

Using alligation rule, we have.




i.e., 2 : 1, The ratio of males to females in administrative employees as 2 : 1.

$\therefore$ Number of males in administrative employees is $\frac{2}{3} \times 48 = 32$.

(b) The total salary of males in the company

$= 6 \times 8{,}000 + 32 \times 3{,}000 = ₹1,44,000.$

The total salary of female $= 16 \times 4{,}500 = ₹72{,}000$

$\therefore$ Required difference $= 1{,}44{,}000 - 72{,}000 =$ ₹72,000

**40.** In 5 cu.$m$ of the alloy, volume of the first metal

$= \frac{40}{100} \times 5 = 2$ cu.m

and, volume of the second metal = 3 cu.$m$.

$\therefore$ Required weight $= (2 \times 1500) + (3 \times 2500)$

$= 10{,}500$ kg.

**41.** Let the sum of the weights of the students of the sections $A$, $B$, $C$ and $D$ be $P$, $Q$, $R$ and $S$, respectively and similarly the number of students in each section be $p$, $q$, $r$ and $s$, respectively.

From the problem,

$P + Q + R = 45\ (p + q + r)$ (1)

$P + R + S = 55\ (p + r + s)$ (2)

$Q + R + S = 60\ (q + r + s)$ (3)

$P + Q + S = 50\ (p + q + s)$ (4)

Adding the equations, we get

$3(P + Q + R + S) = 150\ p + 155q + 160r + 165s$

Average of the four classes $= (P + Q + R + S)/(p + q + r + s)$

If we take $q = r = s = 0$; average = 50

If we take $p = q = r = 0$; average = 55

[Since the average is defined for the section combinations of $A$, $B$, $C$ as well as $A$, $C$, $D$ the actual numbers of students in these sections can not be 0. However, for the extreme case we can assume a value for these which is close to 0 while the fourth being a very high number and thereby the values tending to the same effect.]

The range of the values for the average is 50 to 55.

**42.** The data is tabulated below. Milk is $M$, water is $W$.

| **A** | | **Part removed** | | **B** | |
|---|---|---|---|---|---|
| $M$ | $W$ | $M$ | $W$ | $M$ | $W$ |
| 80 | – | | | 0 | 80 |
| 70 | – | 10 | – | 10 | 80 |
| $\frac{640}{9}$ | $\frac{80}{9}$ | $\frac{10}{9}$ | $\frac{80}{9}$ | $\frac{80}{9}$ | $\frac{640}{9}$ |

The quantity of milk in $B$ is $\frac{80}{9}$, and water in $A$ is $\frac{80}{9}$.

**43.** One litre of milk when poured into $B$ would result in the ratio of milk and water in $B$ to be 1 : 5.

When 1 litre from $B$ is poured into $A$, $B$ would have $\frac{1}{6}(5) = \frac{5}{6}$ litres of milk.

A would have water only from 1 litre poured into it whose quantity is $\frac{5}{6}(1) = \frac{5}{6}$ liters.

$\therefore V_M = V_W = \frac{5}{6}$ litres

**44.** Let us say there are $N$ men in the group. From the given conditions,

$$y = \frac{20N + Kx}{N + x} \quad (1)$$

$$y = \frac{20N - Kx}{N - x} \quad (2)$$

Equating (1) and (2), $20N + Kx)\ (N - x) = (N + x)\ (20N - Kx)$

$20N^2 + NKx - 20Nx - Kx^2 = 20N^2 + 20Nx - NKx - Kx^2$

$2Nx\ (K - 20) = 0$; As both $N$ and $x$ are positive, $Nx \neq 0$

$\therefore\ K = 20$

**45.** Let the volume of $C$ be $v$ liter

When $C$ is filled with milk and then emptied into vessel $B$, (new) volume of milk in vessel $A = 8 - v$ litres. Also

volume of( milk + water) in vessel $B = 8 + v$ litres

Vessel $B$ has water and milk in the ratio 8: $v$.

$\therefore$ In the $v$ litres of mixture in $B$ which is transferred to $A$, $v\left(\frac{8}{8+v}\right)$ is water and $\left(\frac{8}{8+v}\right)$v is milk

Ratio of milk to water in vessel $A = \dfrac{8 - v + \frac{v^2}{8+v}}{\frac{8v}{8+v}}$ (which is given as 2:1)

$$\frac{8 - v + \frac{v^2}{8+v}}{\frac{8v}{8+v}} = \frac{2}{1} \Rightarrow \frac{8}{v} = \frac{2}{1} \Rightarrow v = 4$$

**46.** Let the quantities of wine and water in vessel $A$ be $2k$ and $7k$, respectively.

Given, $\frac{2k}{7k + 20} = \frac{1}{4} \Rightarrow k = 20.$

$\therefore$ Wine in vessel $A$ is $2k$, i.e., 40 litres.

As vessels $A$ and $B$ have same quantities of wine, wine in vessel $B$ is also 40 litres. Also as vessel $B$ has wine and

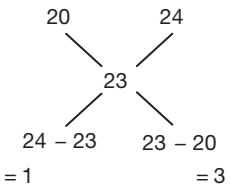
water in the ratio 1 : 4, it has $4 \times 40 = 160$ litres of water.

Let $x$ litres of wine be poured into vessel $B$. Then,

$$\frac{40+x}{200+x} = \frac{2}{9} \Rightarrow \frac{40+x}{160} = \frac{2}{7} \Rightarrow 280 + 7x = 320$$

$$\Rightarrow 7x = 40 \Rightarrow x = 5^5/_7$$

**47.** Ram should make a profit of 20% by selling the mixture at ₹27.6 kg means the average cost price of the mixture should be 27.6/1.2 = ₹23 per kg. By alligation rule, we get




∴ The ratio in which the two varieties of barley must be mixed is 1 : 3.

**48.** Over the course of his regimen, he will drink 750(5% + 10% + 15% + 20% + . . . + 100%) mixture = $750 \times 5 \times 20 \times 21/200 = 7875$ $L$

Of this, only 750 ml is alcohol. So, water = 7875 – 750 = 7125 ml

Ratio of alcohol to water = 750/7125 = 2/19.

**49.** The average of all the elements of $C$ is 35 and the average of all the elements of $A$ and $B$ is also 35. Therefore, the average of all the elements of $A$, $B$ and $C$ is also 35.

**50.** Given that $10x - 4b + 5c = 11x \Rightarrow 5c = x + 4b$

Subtracting $5x$ from both sides,

$5(c - x) = 4(b - x)$.

If $b = x$, $b - x = 0 \Rightarrow c - x = 0$, i.e., $b = c = x$.

Otherwise, $b - x$ and $c - x$ must have the same sign.

If $c - x > 0$, and $b - x > 0$, then $c > x$ and $b > x$ and $c - x < b - x$, i.e., $c < b$.

$\therefore x < c < b$

If $c - x < 0$, and $b - x < 0$, then $c < x$ and $b < x$ and $c - x > b - x$, i.e., $c > b$.

$\therefore b < c < x$.

∴ Only choice (B) is possible.

**51.** The total cost price of the mixture = $\frac{25,200}{1.2}$ = ₹21,000.

The total cost of Rice $A = 420 \times 10$ = ₹4,200.

∴ The total cost of Rice $B$ in the mixture

= 21,000 – 4,200 = ₹16,800.

∴ The quantity of Rice $B$ in the mixture = $\frac{16,800}{12}$

= 1,400 kg.

**52.** The data is tabulated below.

| | I | II |
|---|---|---|
| Pepsi | 3 | 2 |
| Coke | 2 | 3 |

| | I | II |
|---|---|---|
| Pepsi | 3 | 2 |
| Coke | 2 | 3 |

Let the quantity taken from I be $5x$.

∴ The quantity taken from II is $20 - 5x$.

The final quantity of 20 litres contains Pepsi and Coke in the ratio 9 : 11, i.e., 9 litres of Pepsi and 11 litres of Coke.

∴ Amount of Pepsi in final mixture

$= 3x + (2/5)(20 - 5x) = 9$

$\Rightarrow x = 1 \Rightarrow 5x = 5$

**53.** Amount of milk poured into $C$ from vessel $A$ and vessel $B$

$= 2 \times \frac{3}{5} + 3 \times \frac{2}{3} = \frac{16}{5}$ litres.

Also, amount of water poured into $C$ from vessels $A$ and $B = 5 - \frac{16}{5} = \frac{9}{5}$ litres.

Given, 5 litres represent 40% of the capacity of vessel $C$, vessel $C$ has a capacity of = $\frac{5 \times 5}{2} = 12.5$ litres.

To make the quantities of water and milk same in vessel $C$, quantity of water to be added

$= \frac{16}{5} - \frac{9}{5} = \frac{7}{5}$ litres.

∴ Required answer = $\frac{7/5}{12.5} = \frac{14}{125}$

**54.** Let the number of students in $A$, $B$ and $C$ be $b$, $b$ and $c$, respectively. Total marks obtained by the students of class $A$ and $B$, $B$ and $C$ and $A$ and $C$ = 2(Total marks obtained by the students of class $A$, $B$ and $C$) = $2(60(b + b + c))$

$= 120(b + b + c)$

This also equals

$52.5(b + b) + 70(b + c) + 60(b + c)$

$\therefore 120(2b + c) = 235b + 130c$

$b = 2c$

Total number of students of $A$, $B$ and $C = b + b + c = 50$

$2(2c) + c = 50$

$c = 10$

**55.** The data and calculation are tabulated below. $P$ denotes the present year.

| Time | $P$-12 | $P$-7 Before | $P$-7 after | $P$-2 Before | $P$-2 after | $P$ |
|---|---|---|---|---|---|---|
| Av. age | $28\frac{1}{9}$ | | | | $15\frac{3}{7}$ | $17\frac{3}{7}$ |
| No. of members | 9 | 9 | 8 | 8 | 7 | |
| Total age | 253 | 298 | 188 | 228 | 108 | |
| Changes | | 510<br>59 | | 570<br>63 | | |

The present average age is $17\frac{3}{7}$ years = 17 years.

**56.** Let the weight of $X$ be $w$.

$X$ has $\frac{3}{4}w$ aluminium (Al) and $\frac{w}{4}$ zinc (Zn).

If the process is applied once, the resulting alloy will have the quantity of $Al$ in it equal to $\frac{3}{4}w\left(\frac{3}{4}\right)$ and the quantity of $Z_n$ in it equal to $\frac{w}{4}\left(\frac{7}{8}\right)$.

If the process is applied repeatedly $n$ times, the quantity of Al in the resulting alloy would be $\frac{3}{4}w\left(\frac{3}{4}\right)^n$ and the quantity of $Z_n$ in would be it equal to $\frac{w}{4}\left(\frac{7}{8}\right)^n$.

If the proportion of Al is 60%, the ratio the quantities of Al and $Z_n$ would be $\frac{3}{2}$. (If the proportion of Al is less than 60%, the ratio would be less than $\frac{3}{2}$)

$$\therefore \frac{\frac{3}{4}w\left(\frac{3}{4}\right)^n}{\frac{w}{4}\left(\frac{7}{8}\right)^n} \le \frac{3}{2}$$

$$3\left(\frac{6}{7}\right)^n \le \frac{3}{2}, \text{ i.e., } \left(\frac{6}{7}\right)^n \le \frac{1}{2} \qquad (1)$$

Least value of $n$ satisfying (1) is 5 ($\therefore \left(\frac{6}{7}\right)^2 = \frac{36}{49}, \left(\frac{6}{7}\right)^3 = \frac{216}{343}, \left(\frac{6}{7}\right)^4 = \frac{1296}{2401}, \left(\frac{6}{7}\right)^5 = \frac{7776}{16807}$.

**57.** In the new vessel, water and milk are in the ratio

$$\left(\frac{1}{3}+\frac{2}{5}+\frac{3}{7}\right)\left(\frac{2}{3}+\frac{3}{5}+\frac{4}{7}\right)$$

$$= \frac{35+42+45}{105} : \frac{70+63+60}{105} = 122 : 193$$

Mixture with drawn $= \frac{1}{2} \times \frac{193-122}{193} = \frac{71}{386}$

**58.** Each person found the average of the 100 numbers by splitting the set into two or more groups. All the three persons ended up with different answers.

The correct answer for the average of the 100 numbers is the weighted average of the groups.

$Z$'s answer was $\frac{55}{100}$ (Average of the numbers in the larger group) plus $\frac{45}{100}$ (average of the numbers in the smaller group. This equals the weighted average of the two groups.

$\therefore$ $Z$ was the only person whose answer was correct.

**59.** Let the initial quantity of the solution in the vessel be $x$ litres.

If 10 litres is withdrawn, quantity of milk in the remaining solution $= \frac{90}{100}(x-10)$ litres. After water is added, milk would form $\frac{90}{100}\left(\frac{x-10}{x}\right)$ part of the solution mixture. Once 10 litres of the solution is again removed, quantity of milk in the remaining solution

$$= \frac{90}{100}\frac{(x-10)}{x}(x-10) = 72.9$$

$\Rightarrow x^2 - 20x + 100 = 81x$

$\Rightarrow x^2 - 101x + 100 = 0$

$\Rightarrow x = 1$ or 100; As $x = 1$ is not possible, $x = 100$.

**60.** Had there been no alterations, the current average would have been 25 + 12 = 37.

The first person who goes out takes with him 50 years, Similarly, the second person who goes out takes with him another 50 years.

Present average = 37 – (50 + 50)/10 = 37 – 10 = 27 years.

## EXERCISE-4

**1.** Let each number be denoted by $abc$

$c = 3a$

Each number must have the form $1b3$, $2b6$ or $3b9$. There are 10 numbers having each of these forms.

Average of all the numbers

$$= \frac{1}{30}\{(103 + 113 + \ldots + 193) + (206 + 216 + \ldots + 296) + (309 + 319 + \ldots + 399)\}$$

$$= \frac{10(100(1+2+3)+3(10(0+1+\ldots+9))+10(3+6+9)}{30}$$

$$= 100(2) + 10(4.5) + 6 = 251$$

**2.** Average mark of the students in the class =

$$\frac{(225)(60)+(205)(56)+(180)(55)+(195)(70)+(220)(65)+(250)(80)}{225+205+180+195+220+250}$$

Assume the average mark to be 60. Writing the deviation from 60, the average mark of the students in the classes

$$\frac{225(60)+205(60-4)+180(60-5)+195(60+10)+220(60+5)+250(60+20)}{1275}$$

Dividing both numerator and denominator by 5

$$= (255)(60)/255 + 1266/255 \cong 60 + 4.96 = 64.96$$

**3.** From either of the given conditions, $x > 50$.

If the contents of $A$, $B$, $C$ are mixed, the concentration would be 50. The concentration and quantities are tabulated below

| | A | B | ABC | C |
|---|---|---|---|---|
| **Conc (%)** | 30 | 40 | 50 | $x$ |
| **Quantity** | 3 | 6 | 13 | 4 |

The total quantity of the solution in $A$, $B$, $C$ is 13 litres. Of this, 6.5 is acid. The quantity of acid in $A$ is 0.9 and in $B$ is 2.4. Therefore the quantity of acid in $C$ is 6.5 – 3.3, i.e., 3.2. Of the total of 4 litres in $C$, 3.2 litres is acid, i.e., 80% is acid., i.e., $x = 80$.

**4.** The mean of $N$ distinct natural numbers is 27.

One of them is 375 (i.e, it is 348 more than the mean)

The positive deviation from the mean must be offset by the negative deviation. Also, we are not free to choose numbers which are arbitrarily less than the mean. The numbers have to be positive as well as distinct.

If we choose 1, 2, ... 24, the negative deviation from 27 is 3 + ...+ 26, viz., 348, i.e, the mean is 27. The number of numbers selected is 24 + 1.

In addition, if 27 is selected, the mean would still be 27. $\therefore$ $N$ could be 26.

If instead of 27, we select 25 and 29, the mean of the 27 numbers would again be 27.

In addition, if we select 27, the mean of the 28 numbers is again 27.

If instead of 27, we select 26 and 28, the mean would still be 27 and $N$ is 29.

Finally, we can include 27 once again. $N$ would be 30.

These possibilities are tabulated below.

| Numbers | | | | | | | |
|---|---|---|---|---|---|---|---|
| **1–24** | **25** | **26** | **27** | **28** | **29** | **375** | **No of Nos** |
| ✓ | | | | | | ✓ | 25 |
| ✓ | | | ✓ | | | ✓ | 26 |
| ✓ | ✓ | | | | ✓ | ✓ | 27 |
| ✓ | ✓ | | ✓ | | ✓ | ✓ | 28 |
| ✓ | ✓ | ✓ | | ✓ | ✓ | ✓ | 29 |
| ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | 30 |

If we select any more natural numbers, the mean will increase.

The number of numbers could be 25, 26, 27, 28, 29 or 30

**5.** $A = \{5, \ldots \ldots, 80\}$ Average $= \frac{5+80}{2} = 42.5$

$B = \{11, \ldots\ldots, 86\}$ Average $= \frac{11+86}{2} = 48.5$

$C = \{2, \ldots\ldots, 77\}$ Average $= \frac{2+77}{2} = 39.5$

$D = \{8, \ldots\ldots, 83\}$ Average $= \frac{8+83}{2} = 45.5$

$E = \{14, \ldots\ldots\ldots, 89\}$ Average $= \frac{14+89}{2} = 51.5$

$\therefore c < a < d < b < e$

Consider the options. Only (C) is true.

**6.** If a solution of alcohol and water contains $y\%$ of alcohol and $z\%$ of the solution is taken out and replaced by water, percentage of alcohol remaining is given by $\left(\frac{100-z}{100}\right)\times y\%$

If this process is repeated $n$ times, the resultant concentration of alcohol is $\left(\frac{100-z}{100}\right)^n \times y\%$

For the given problem, $z = 20$ and $y = 80$.

Percentage of alcohol remaining

$= \left(\frac{100-20}{100}\right)^2 \times \frac{80}{100} = 51.2\%$

**7.** Cost of 1 kg of mixture obtained by mixing $T_1$, $T_2$ and $T_3$

$= \frac{(79)(1)+64(2)+62(4)}{1+2+4} = 65$

Cost of 4 kg of mixture = ₹ 260.

Let us say he mixes $x$ kg of $T_1$, $4x$ kg of $T_3$ with 4 kg of the mixture.

$\frac{260+79x+62(5x)}{4+6x}$ = cost price of new mixture (1)

But cost price of the new mixture = 77.88/1.2

= 64.9 (2)

Solving these, we get $x = 1$

**8.** Let the quantities of the mixtures in $A$ and $B$ be $x$ ml and $y$ ml, respectively.

For $P$, a% of $x$ ml is mixed with $(100 - b)$ % of $y$ ml.

For $Q$, $(100 - a)$ % of $x$ ml is mixed with $b$% of $y$ ml.

$A$ has a% milk and $B$ has $b$% milk.

Both $P$ and $Q$ have k% milk.

$\frac{ax}{(100-b)y} = \frac{k-b}{a-k}$ and $\frac{(100-a)x}{by} = \frac{k-b}{a-k}$ [ $\because a \neq b$. If $a = b$,

then $a = b = k$ and the *RHS* in both equations would be 0/0]

$\therefore \frac{ax}{(100-b)y} = \frac{(100-a)x}{by}$

$\therefore abxy = xy\,((100 - a)\,(100 - b))$

$xy\,(100^2 - (100)\,(a + b)) = 0$ $xy \neq 0$

$\therefore a + b = 100$

**9.** Let the number be denoted by abbc

$c = \frac{1}{2}\,a$

Each number must have the form 2bb1, 4bb2, 6bb3, or 8bb4

For each of these forms, there are 10 numbers, i.e., a total of 40 numbers. The average of $n$ numbers (not necessarily having the same number of digits, but let us say each has at most 4 digits) is equal to $1000A + 100B + 10C + D$, where $A$, $B$, $C$, $D$ are the averages of the thousands, hundreds, tens and units digits. Here,

$A = \frac{10(2+4+6+8)}{40} = 5$

$B = \frac{4(0+1+\ldots+9)}{40} = 4.5$

$C = \frac{1(0+1+\ldots+9)}{40} = 4.5$

$D = \frac{10(1+2+3+4)}{40} = 2.5$

Average of all the numbers

$= 1000A + 100B + 10C + D$

$= 1000\ (5) + 100\ (4.5) + 10\ (4.5) + 2.5 = 5497.5$

**10.** The calculations are tabulated below.

Lower Limit Upper Limit

| | **A** | | **B** | | **A** | | **B** |
|---|---|---|---|---|---|---|---|
| Av. | 20 | 48 | 69 | | 30 | 58 | 79 |
| No. | $3n$ | | $4n$ | | $3n$ | | $4n$ |

The average of all the $7n$ numbers in $A \cup B$ lies in [48, 58]

**11.** The data can be tabulated as follows.

| | **Vessel 1** | **Vessel 2** | **Vessel 3** |
|---|---|---|---|
| Intial Volume (lt) | 12 | 12 | 12 |
| Concentration | 60% | 40% | 20% |
| Volume taken to form $A$ (lt) | 4 | 8 | 12 |
| Volume left over to form $B$ (lt) | 8 | 4 | 0 |

Mixture $C$ is formed by mixing 6 litres of $A$ and 9 litres of $B$.

Concentration of milk in $A$

$= \frac{4(0.6)+8\,(0.4)+12\,(0.2)}{24} = \frac{2.4+3.2+2.4}{24} = \frac{1}{3}$

Concentration of milk in $B$

$= \frac{(8)(0.6)+(4)(0.4)+(0)(0.2)}{(8+4+0)} = \frac{6.4}{12} = \frac{1.6}{3}$

As $C$ is formed by mixing $A$ and $B$, the resultant concentration lies between the concentrations of $A$ and $B$, i.e., between $\frac{1}{3}$ and $\frac{1.6}{3}$.

Hence, $A$ has the lowest concentration, $B$ has the highest and $C$ has a value in between.

Hence, none of the given statements is true.

**12.** Let the number of rocks in $A$, $B$, $C$ be $a$, $b$, $c$, respectively.

Average weight of the rocks in $A$ and $B = \frac{50a + 60b}{a + b}$.

$$\frac{50a + 60b}{a + b} = 53$$

$$\frac{a}{b} = \frac{7}{3} \text{ let a} = 7k \text{ and } b = 3k$$

$$\frac{50a + A_cC}{a + c} = 54$$

$$A_cc = 28k + 54c$$

The average weight of the rocks in the combined piles $B$ and $C$ (A)

$$= \frac{60b + A_cC}{b + c} = \frac{208k + 54c}{3k + c}$$

$$A = \frac{54(c + 3k) + 46k}{3k + c}$$

$$= 54 + \frac{46k}{3k + c}$$

This is less than $54 + \frac{46k}{3k}$, i.e., $69\ \frac{1}{3}$

Greatest integer value of $A$ is 69.

**Alternative Solution:**

The number of rocks and the average weight (in pounds) in the 3 piles are tabulated below.

| Pile | A | B | C |
|---|---|---|---|
| Average Wt | 50 | 60 | $c$ |
| Number | $7x$ | $3x$ | $r$ |

As the average for $A$, $B$ is 53, the number of rocks in $A$ and $B$ have to be in the ratio 7 : 3, i.e., $7x$ and $3x$, respectively.

The average weight for $AC$ is 54, i.e., $4(7x) = (c - 54)\ r$.

Consider $BC$. The deviation of $B$ from 54 is $6(3x)$. The deviation of $C$ from 54 is $28x$. Therefore the average for $BC$(say $X$) $= 54 + \frac{18x + 28x}{3x + r}$. As $r > 0$, $X < 54 + \frac{46}{3}$

The greatest integral value of $\frac{18x + 28x}{3x + r}$ is 15, which occurs when $3x + r = \frac{46x}{15}$, i.e., $r = \frac{x}{15}$. Now $r$ has to be an integer, but then can be managed by taking $x$ to be a multiple of 15. For example, $x = 15$, $r = 1$. The weights and numbers would be

| | A | B | C |
|---|---|---|---|
| **Wt** | 50 | 60 | 474 |
| **No** | 105 | 45 | 1 |

The greatest integral value of the average weight for $B$, $C$ is 54 + 15, i.e., 69

**13.** The data is shown below:

| A | AB | B | BC | C |
|---|---|---|---|---|
| 52 | 56 | 58 | 60 | 63 |
| 1 | | 2 | | |
| | | 3 | | 2 |
| 3 | | 6 | | 4 |

As the average for $AB$ is 56, the ratio of the number of students in $A$ and $B$ is $\frac{58 - 56}{56 - 52} = \frac{2}{4} = \frac{1}{2}$

The ratio of the number of students in $B$ and $C$ is $\frac{63 - 60}{60 - 58} = \frac{3}{2}$

The ratio for the 3 sections is 3 : 6 : 4

We assume the average to be 58.

Negative deviation = 3(58 – 52) = 18

Positive deviation = 4(63 – 58) = 20

Net deviation = 2

Average deviation = $\frac{2}{13}$

Average = Assumed average + Average deviation

$= 58\ \frac{2}{13}$.

**14.** Let the number of employees of marketing, finance and production be $m$, $f$ and $p$, respectively.

$\therefore$ Twice the total age of all the employees

$= 25\ (m + f) + 30\ (f + p) + 20\ (p + m)$

Average age of all the employees

$$= \frac{45m + 55f + 50p}{2(m + f + p)} = \frac{45}{2} + \frac{5(2f + p)}{2(m + f + p)}$$

As $m = f$, the average age of all the employees

$= \frac{45}{2} + \frac{5}{2} = 25$ years

**15.** Let the capacity of the vessel be $x$ litres.

(i) quantity of milk in the vessel finally

$= x\left(\frac{x-9}{x}\right)^2 = x - 17.1$

$x^2 - 18x + 81 = x^2 - 17.1x \quad x = 90$

(ii) let the number of further replacements be $n$.

$\left(\frac{x-9}{x}\right)^{n+2} \le \frac{75x}{100}$

$x = 90$

$(0.9)^{n+2} \le 0.75 \qquad (1)$

least value of $n$ satisfying (1) is 1

**16.** To get a solution of 75% concentration by mixing two other solutions with distinct concentrations, the concentration of one has to be less than 75% while that of the other has to be more than 75%. Only $E$ has a concentration of more than 75%.

Options $A$, $B$ and $D$ have $E$ in one of the solutions in the mixture. So, we can get 75% concentration but from option (C), we cannot get 75% concentration.

**17.** The concentrations of the 5 solutions are listed below

| $A$ | $B$ | $C$ | $D$ | $E$ |
|---|---|---|---|---|
| 10% | 20% | 40% | 50% | 80% |

We can take the following solutions in $P$, $Q$, $R$.

| | | | | |
|---|---|---|---|---|
| $A, A, E$ | $B, BE$ | $CCE$ | $DDD$ | $AEE$ |
| $A, BE$ | $B, CE$ | $CDE$ | $BEE$ | |
| $A, C, E$ | $B, DE$ | $CEE$ | | |
| $A, DE$ | | | | |

$\therefore$ There are 13 combinations. But the containers $P$, $Q$, $R$ are distinct.

$DDD$ corresponds to only 1 way of filling $P$, $Q$, $R$.

Each of 6 of these combinations ($AAE$, $BBE$, $CCE$, $AEE$, $BEE$, $CEE$) corresponds 3 ways of filling $P$, $Q$, $R$.

Each of the other 6 ways corresponds to 6 ways of filling $P$, $Q$, $R$.

Thus the total number of ways of filling $P$, $Q$, $R$ is

$1 + 6(3) + 6(6) = 55$

**Alternative Solution:**

If we want to get a solution of a certain concentration (say $x$%) by mixing different solutions there are two ways. Either all the constituent solutions have a concentration of $x$% or some of them have a lower concentration and others have a higher concentration –(1).

The given solutions and their concentrations are listed below

| $A$ | $B$ | $C$ | $D$ | $E$ |
|---|---|---|---|---|
| 10% | 20% | 40% | 50% | 80% |

We need to fill up the containers $P$, $Q$, $R$ with these and then prepare $N$ with 50% concentration from $P$, $Q$, $R$.

In all, there are 5(5)(5) or 125 ways of filling up $P$, $Q$, $R$. Of these, consider the ways in which none of $P,Q,R$ is filled with a solution of concentration greater than 50% (in the given situation, this means only $E$ is left out). Of the 4(4)(4) or 64 such ways only from one (each of $P$, $Q$, $R$ is filled with $D$) is it possible to prepare $N$.

Consider the ways in which none of $P,Q,R$ is filled with a solution of concentration less than 50%. In the given situation this means each is filled with $D$ or $E$. Of the 2(2)(2) or 8 such ways only one (each of $P,Q,R$ is filled with $D$) is it possible to prepare $N$.

The total number of ways of filling up $P,Q,R$ such that it is *NOT* possible to prepare $N$ is (63 + 7 = )70

(1) $\Rightarrow$ From the other 125–70 or 55 ways, it is possible to prepare $N$.

**18.** After one execution of process $P$ (two symmetrical) operations the amount of alcohol in drum $A$ and the amount of water in drum $W$ will be the same. Since at the end of any execution the total content of each drum = total quantity of the alcohol in the drums = total quantity of water in the drums = 1000 litres.

Even after 3 executions of $P$ the amount of alcohol in drum $A$ and the amount of water in drum $W$ will be the same. After this, 2 litres of the contents of drum $A$ are mixed with contents of drum $W$. So, the amount of alcohol in drum $A$ will be less than the amount of water in drum $W$ ($\because$ The amount of alcohol in $A$ has decreased while that of water in $W$ has increased). $a < w$.

**19.** Let the two solutions be $A$ and $B$. The data is tabulated below.

| | Water | Milk | Total |
|---|---|---|---|
| A | | | 12 |
| B | | | 18 |
| | 18 | 12 | 30 |

| Water | Milk | Total |
|---|---|---|
| | | 12 |
| | | 12 |
| $14^{2}/_{3}$ | $9^{1}/_{3}$ | 24 |

If the entire volumes of $A$ and $B$ are mixed, milk constitutes 40% of $30l$ or $12l$.

If only 12 litres of $A$ are mixed with the entire $12l$ of B, milk constitutes $38\frac{8}{9}\%$ (or $\frac{350}{900}$, i.e., $\frac{7}{18}$) of $24l$, which is $9\frac{1}{3}l$.

$\therefore$ The extra six litres of A contain the extra $2\frac{2}{3}l$ of milk.

i.e., The $18l$ of A contain $8l$ of milk and hence B contains $4l$ of milk.

***Solutions for question 20:***

Let the marks scored in Test V, Test VII, Test XI and Test XII be $x+2$, $y$, $x$ and $y+12$, respectively.

Now, $(x+2)+y+x+(y+12) = 1{,}020 - 674 = 346$

$\Rightarrow x+y = 166$. Also, $x-y = 18$

$\therefore x = 92$, $y = 74$

**20.** (a) Score in Test $V = x+2 = 92+2 = 94$.

(b) To increase his average by 1.

Satish should have got $(12 \times 1)$, i.e., 12 marks more than that he got presently.

His present score in test XII is 86.

$\therefore$ His score would have been $(86 + 12) = 98$

**21.** The quantities of milk and water in $A$ and $B$ at the 5 different stages are tabulated below.

| A | | B | |
|---|---|---|---|
| **Milk** | **Water** | **Milk** | **Water** |
| 10 | 0 | 0 | 10 |
| 9 | 0 | 1 | 10 |
| $9\frac{1}{11} = \frac{100}{11}$ | $\frac{10}{11}$ | $\frac{10}{11}$ | $9\frac{1}{11} = \frac{100}{11}$ |
| $\frac{90}{11}$ | $\frac{9}{11}$ | $\frac{20}{11}$ | $\frac{101}{11}$ |
| $\frac{990+20}{121} = \frac{1010}{121}$ | $\frac{99}{121} + \frac{101}{121} = \frac{200}{121}$ | $\frac{200}{121}$ | $\frac{1010}{121}$ |

The concentration of milk in $A$ is $\frac{1010}{1010+200} = \frac{1010}{1210} = \frac{101}{121}$.

**22.** Let the total volume of benzene in $A$ be 1.

If '$x$' and '$y$' are the quantities of benzene removed in the course of first and second operations, then $x+y = 1$

And concentration of solution in $A$ reduces to $(1-x)(1-y)$

Hence, the concentration of Benzene in $A$ is equal to the concentration of water in $B$.

To attain maximum concentration in $B$ $x = y = 1/2$

$\therefore$ 25% concentration.

**23.** Let the number of times the process is applied be $N$.

Each application results in the copper quantity dropping by 20% and the tin quantity dropping by 10%. Let the weight of the alloy be 100 kg. This has 80 kg of copper and 20kg of tin.

The process is applied $N$ times, the quantity of copper in the alloy will be $(0.8)^N 80$ and that of tin in it will be $(0.9)^N (20)$.

$$\frac{(0.8)^N (80)(100)}{(0.8)^N (80) + (0.9)^N (20)} < 70$$

$$\Rightarrow \frac{(0.8)^N (80)}{(0.8)^N 8 + (0.9)^N 2} < 7$$

$$\Rightarrow (0.8)^N (24) < (0.9)^N (14)$$

$$\frac{12}{7} < \left(\frac{9}{8}\right)^N \text{ i.e., } \left(\frac{9}{8}\right)^N > 1\frac{5}{7} \approx 1.7$$

The values of $N$ and the approximate values of $\left(\frac{9}{8}\right)^N$ are tabulated below.

| $N$ | $\left(\frac{9}{8}\right)^N$ |
|---|---|
| 1 | 1.125 |
| 2 | 1.27 |
| 3 | 1.42 |
| 4 | 1.60 |
| 5 | 1.80 |

$N$ must be at least 5 in order that the alloy may have less than 70% copper.

**24.** After 2 litres are transferred from $A$ to $W$, drum $A$ has 998 litres of alcohol and drum $B$ has 1000 litres water and 2

litres alcohol. When 2 litres are transferred back to $A$, it will have 998.004 litres of alcohol and 1.996 litres of water. Similarly drum $W$ will have 998.004 litres of water and 1.996 litres of alcohol. After one execution of process $P$ (two symmetrical operations) the amount of alcohol in drum $A$ and the amount of water in drum $W$ will be the same.

Even after 51 executions of $P$ the amount of alcohol in drum $A$ and the amount of water in drum $W$ will be the same. After this, 2 litres of the contents of drum $A$ are mixed with contents of drum $W$. So, the amount of alcohol in drum $A$ will be less than the amount of water in drum $W$ ($\because$ The amount of alcohol in $A$ has decreased while that of water in $W$ has increased). $a < w$.

**25.** Let the initial quantity of mixture be 100

Initial quantity of salt in the mixture = 25

If the filtration process is applied, the quantity of salt in the mixture becomes 0.8 (25)

At the end of $N$ iterations of the process, the quantity of salt becomes $(0.8)^N(25)$. We require the least value of $N$ for which the final quantity is 0.5 or less

$(0.8)^N(25) \le 0.5$

$(0.8)^N \le \frac{1}{50}$

Taking the logs on both sides to base 10,

$N \log 0.8 \le -\log 50$

$N \ge \frac{-\log 50}{\log 0.8}$ ($\because$ both the sides have been divided by a negative number)

$N \ge \frac{-\log(100/2)}{\log(8/10)}$

$N \ge \frac{-(\log 10^2 - \log 2)}{\log 8 - \log 10}$

$N \ge \frac{-(2 - 0.3010)}{3(0.3010) - 1}$

$N \ge 17.515$

$\therefore$ Least number of iterations is 18.

**26.** The numbers are of the form abcd

where $d = 2a$, $b = 0/1/2...9$, $c = 0/1/2/3...9$. a can take the values 1,2,3 and 4 corresponding values of $d$ are 2,4,6 and 8, respectively.

Average of all the 4-digit numbers = $1000\,A_{TH} + 100A_H + 10A_T + A_u$

As all possible values of $a$, $b$, $c$ and $d$ occur the same number of times,

$A_H = \frac{1+2+3+4}{4} = 2.5$

$A_H = \frac{0+1+2+....9}{10} = 4.5$

$A_T = \frac{0+1+2+....9}{10} = 4.5$

$A_U = \frac{2+4+6+8}{4} = 5$

Average of all possible numbers of the form abcd = 2.5 (1000) + 4.5 (100) + 4.5 (10) + 5 = 3,000

**27.** $N$ can have the following values.

12_4, 24_8.

There are 10 of each of the two kinds.

The average of the 10 numbers of the first kind is

$1204 + \frac{45(10)}{10} = 1249$

The average of the 10 numbers of the second kind is

2408 + 45 = 2453.

The average of all the 20 numbers is

$\frac{10(1249 + 2453)}{20} = \frac{3702}{2} = 1851$

**28.** The original compositions of $P$ and $Q$ and the results of the 2 transfers ($P \to Q$, $Q \to P$) are shown in the diagram below ($M$ is milk and $W$ is water):

| P | | R | | Q | |
|---|---|---|---|---|---|
| $M$ / $X$ | $W$ | $M$ | $W$ | $M$ | $W$ / $X$ |
| $X-5$ | | 5 | | 5 | $X$ |
| $\frac{X^2}{X+5}$ | $\frac{5X}{X+5}$ | $\frac{5(5)}{X+5}$ | $\frac{X(5)}{X+5}$ | | |

After the two transfers, the ratio of the milk and water in $P$ is $\frac{x}{5}$. As $\frac{x}{5} = 5$, it follows that $X = 25$.

**29.** The quantities of milk and water in $A$ and $B$ at the 3 different stages are tabulated below

| A | | B | |
|---|---|---|---|
| Milk | Water | Milk | Water |
| $N$ | 0 | 0 | $N$ |
| $N - X$ | 0 | X | $N$ |
| $(N-X) + \frac{X^2}{N+X}$ | $\frac{N(N)}{N+X}$ | $\frac{XN}{N+X}$ | $\frac{N^2}{N+X}$ |

The final concentration of milk in $A = \frac{N}{N+X}$

The final concentration of water in $A = \frac{X}{N+X}$

The difference $= \frac{N-X}{N+X} = \frac{3}{4}$ (given)

$\therefore 4N - 4X = 3N + 3X$ or $N = 7X$

The final concentration of milk in $B$ = The final concentration of water in $A = \frac{X}{N+X} = \frac{X}{8X} = \frac{1}{8} = 12.5\%$.

**30.** The quantities of milk and water in $A$ and $B$ at the 3 different stages are tabulated below

| A | | B | |
|---|---|---|---|
| **Milk** | **Water** | **Milk** | **Water** |
| $N$ | 0 | 0 | $N$ |
| $N - X$ | 0 | $X$ | $N$ |
| $N - X + \frac{X^2}{N+X}$ | $\frac{XN}{N+X}$ | $\frac{NX}{N+X}$ | $\frac{N^2}{N+X}$ |

The concentration of milk in $A$ at this stage is

$\frac{N^2}{N+X}\ \frac{1}{N} = \frac{N}{N+X} = \frac{3}{4} \Rightarrow 4N = 3N + 3X$

$\Rightarrow N = 3X$ or $\frac{X}{N} = \frac{1}{3}$.

## EXERCISE-5

**1.** To require both the prices, we require both the statements.

Since the other mixture is worth 1.45 (1.60 – 0.15), we can tell in what ratio they have to be mixed by applying the rule of alligation.

**2.** If $x$ is the total mixture, then $2x/3$ is milk from the main statement;

From Statement I, we get $(60 \times 2/3) = (60 + y)1/3$, where $y$ is the water added. Hence, $y$ can be determined.

From Statement II, we know that water in the initial mixture is 20 litres and hence, milk should be 40 litres. Now 60 litres of water is to be added to make the ratio of milk and water equal to 1 : 2. Hence, Statement II alone is sufficient.

**3.** Since the ratio of boys and girls is given in Statement I, we get,

average weight = $5/7(28) + 2/7(21) = 26$ kg.

**4.** Let the number of students in sections $A$, $B$, $C$ be $n_a$, $n_b$ and $n_C$, respectively.

Let the average age of students in $A$, $B$, $C$ be $x$, $y$, $z$, respectively.

From Statement I

$\frac{n_a x + n_b y}{n_a + n_b} = 25$

Statement I alone is not sufficient as there is no mention about section $C$.

Statement II

$\frac{n_b y + n_c z}{n_b + n_c} = 25$

Statement II alone is not sufficient.

Using both the statements we need to find,

$\frac{n_a x + n_b y + n_c z}{n_a + n_b + n_c}$

Here, the information regarding number of students in each section is not given, so the average age of the students in the three sections combined can't be determined.

**5.** Let the weights of the four men be $a$, $b$, $c$ and $d$, respectively.

$(a + b)/2 = 40$

$\Rightarrow a + b = 80$

Using Statement I, $c = \frac{a+b+c}{3}$

$\Rightarrow c = \frac{80+d}{3}$

As $d$ is not known, we cannot find $c$ and hence the average weight of the four cannot be found.

Using Statement II, $d = \frac{a+b+c}{3}$

$\Rightarrow d = \frac{a+b+c}{3} = \frac{80+c}{3}$

As $c$ is not known, we cannot find $d$ and hence the average weight of the four cannot be found.

Using both the statements

$3c = 80 + d$ (1)

$3d = 80 + c$ (2)

Adding (1) and (2)

$3c + 3d = 160 + c + d$

$\Rightarrow c + d = 80$

Average weight of the four is equal to $\frac{a+b+c+d}{4}$ and Hence, it can be found.

**6.** From Statement I, 18 students are 15 years old.

From Statement II, the average age of 4 students is less than 15 years.

Combining statements I and II, there are two students whose age is less than 15 years and the remaining 18 students are 15 years old.

So, we can say that the average age of 20 students is less than 15 years.

**7.** Statement I alone is sufficient as tin by weight is

$\left[1-\left[\frac{2}{5}+\frac{3}{16}\right]\right]\times 60$ kg

Statement II alone gives by volume. So, it is not sufficient.

**8.** Either of the statements alone is not sufficient as the information about six numbers is not given in one statement.

Using both the statements,

We can say that three numbers are more than 30.

**9.** Using both statements I and II, we can say that $x$ is more concentrated. (concentration of $x$ is $\frac{4}{7}$ = 57.14% and then concentration of $y$ is $\frac{6}{13} < 50\%$)

**10.** Either of the statements alone is not sufficient as the total score of boys and girls is given in Statement I and total score of girls is given in Statement II.

Using both the statements,

Let the average score of the boys be $k$

$kx + 86y = 85(x + y) \Rightarrow kx = 85x - y$

$y = x(85 - k)$

As $x$ and $y$ are positive, $k$ must be less than 85.

So, the average score of the boys is less than 85 but we can't say whether it is more than 83 or not.

**11.** From Statement I, the cost price of the mixture (in ₹) = (100/120) (36) = 30. As we know the costs of the varieties $A$ and $B$, we can find the required ratio. Statement I alone is sufficient.

From II, we don't know the cost of the mixture, so we cannot answer the question.

**12.** According to Statement I, the average age of 92 test takers is 14. We do not know the ages of the remaining test takers. Hence, Statement I alone is insufficient.

From Statement II, the oldest test taker is 24 years old. But we don't know the ages of the remaining test takers. Statement II alone is insufficient.

Combining the two statements, we can find the average, and hence, we can answer the question.

**13.** Let the weights of Amar, Bhanu, Chetan and Dinesh be $a$ kg, $b$ kg, $c$ kg and $d$ kg, respectively.

$\frac{a+b+c+d}{4} = 80$ . The required average = $\frac{a+b}{2}$

From Statement I, $a = \frac{b+c+d}{3}$

More information is required to find $\frac{a+b}{2}$

$\therefore$ Statement I is not sufficient.

Using Statement II, $b = \frac{a+c+d}{3}$

More information is required to find $\frac{a+b}{2}$

$\therefore$ Statement II is not sufficient.

Using both statements,

$a = \frac{b+c+d}{3} \Rightarrow 3a = b + c + d$ (1)

$b = \frac{a+c+d}{3} \Rightarrow 3b = a + c + d$ (2)

Adding (1) and (2), $a + b = c + d$

$\frac{a+b+c+d}{4} = \frac{2(a+b)}{4} = 80 \Rightarrow \frac{a+b}{2} = 80$

$\therefore$ Both statements are required.

**Alternative Solution:**

From Statement I, Amar's weight is 80 kg.

From Statement II, Bhanu's weight is 80 kg.

$\therefore$ By combining the two statements, their average weight is 80 kg.

**14.** Let the quantities of solutions in can $A$ and can $B$ be $100x$ litres and $100y$ litres, respectively.

From Statement I, we have the required ratio as $(40x + 65y) : 60x : 35y$. But we don't know the relationship between $x$ and $y$. $\therefore$ Statement I alone is insufficient.

From Statement II, we know the part of water in the resultant solution, but we don't know about the other liquids. $\therefore$ Statement II alone is insufficient.

Combining the two statements, we have

$40x + 65y = 1/2\ [100x + 100y]$

From this, we can get the relationship between $x$ and $y$. We can then answer the question.

**15.** Using Statement I, let the increase in the average be $x$ years. $20N + 22 + 28 = (N + 2)\ (20 + x) \Rightarrow 10 = x\ (N+2)$

and as $x$ is prime, $x = 2$ or 5.

If $x = 2$, $N = 3$. If $x = 5$, $N = 0$ which is not possible.

$\therefore$ Statement I is sufficient.

Using Statement II, let the decrease in the average be $x$ years $20N - 22 - 28 = (N - 2)\ (20 - x)$

$10 = x\ (N - 2)$

As $x$ is prime, $x = 2$ or 5

If $x = 2$, $N = 7$. If $x = 5$, $N = 4$

$\therefore$ Statement II is not sufficient.

**16.** Given, the ages of four persons are in arithmetic progression, let their ages be $a - 3d$, a $- d$, $a + d$, $a + 3d$. So, we have 4a as the sum and '$a$' is the average. From Statement I, we get 2a = 42. Hence, a = 21.

$\therefore$ Statement I alone is sufficient.

From Statement II,

$a + d = 24$ (1)

$a - d = 18$ (2)

Solving (1) and (2), a and $d$ can be determined.

$\therefore$ We get the ages to be 30, 24, 18, 12 and the average to be 21.

$\therefore$ Statement II alone is also sufficient.

**17.** As we don't have any information about the number of students, Statement I alone is not sufficient,

Statement II alone is not sufficient, as it has no information about the marks.

Combining statements I and II, let the number of students in section $C$ be $x$.

$\therefore$ Section $A$ has at least $x$ number of students.

As $65 - 40 = 90 - 65$, and B's average is less than the required class average, the average marks of the class cannot be 65 or more.

**18.** Neither of the statements is independently sufficient as the average age and the strength of different classes are mentioned in two different statements.

Combining both the statements, even if class $X$ forms 30% of the total strength, the average age will be less than 15 years. This is illustrated below:

Let the strength of the school be $10k$ and that of the tenth class students be $3k$. In that case the strength of the remaining classes is $7k$. Hence, the average age of the school $= \dfrac{3k \times 16 + 17k \times 14}{10k} = 14.6$ years.

Hence, the average age will be less than 15 years. If the strength of the tenth class is less than 30%, the average age would be less than 14.6 years.

**19.** Let the number of persons in the group be $n$ and the average age of the group be $y$ years.

From I, $\dfrac{ny + 40(4)}{n + 4} = y + 2$

$\Rightarrow ny + 160 = ny + 2n + 4y + 8 \Rightarrow n + 2y = 76$

$y > 20$ or $y = 20$ or $y < 20$

$\therefore$ We cannot answer the question

$\therefore$ Statement I alone is not sufficient

From II, $\dfrac{ny + 40(4)}{n + 4} = 2y$

$\Rightarrow ny + 160 = 2ny + 8y$

$\Rightarrow (n + 8)y = 160$

If $y = 20$, then $n = 0$

$\therefore$ $y$ must be less than 20.

$\therefore$ Statement II alone is sufficient.

**20.** From Statement I, we can't find the quantity of water. Hence, the question cannot be answered.

From Statement II also the question can't be answered as there is no information about the total capacity of the vessels.

Combining both the statements, we have total milk quantity is 45 litres and water quantity as

$(12 + 20 + 30) = 45$, i.e., 17 litres.

Given, 20% of $(3k + 6k + 8k) = 17$ litres

$\Rightarrow \therefore 17k = 17 \times 5\ ; \Rightarrow k = 5$

Hence, the capacity of $C$ can be found.

**21.** From I, we can have the following conclusion.

If a person goes from $A$ to $B$ and the average age of group $A$ decreases, it means that the age of the person is more than the average age of $A$.

When he is joining $B$, he is decreasing the average age of $B$.

$\therefore$His age is less than the average age of $B$.

Average of $B$ > age of person > average of $A$.

From I, we can say average age of $A$ < Average age of $B$.

From II, when a person is shifted from $B$ to $A$, the average age of $B$ increases.

$\therefore$ The age of the person is less than the average age of *B*. Age of person < average age of *B*. When he joins *A*, the average age of *A* decreases, i.e., age of person < Average age of *A*.

It cannot be concluded that the average age of *A* < average age of *B*.

**22.** Let the volume of container *A* be a litres and *B* be *b* litres. The following steps make us understand the transfer

| | Vessel A | | Vessel B | |
|---|---|---|---|---|
| | **Water** | **Milk** | **Water** | **Milk** |
| Initially | 0 | a | $b$ | 0 |
| After 1st transfer | 0 | $\frac{a}{2}$ | $b$ | $\frac{a}{2}$ |
| After 2nd transfer | $\frac{b}{2}$ | $\frac{a}{2}+\frac{a}{4}$ | $\frac{a}{2}+\frac{a}{4}$ | $\frac{a}{4}$ |

$\therefore$ The ratio of milk to water in vessel $A = \frac{\frac{3a}{4}}{\frac{b}{2}} = \frac{3a}{2b}$.

From I, we have $a \geq b \Rightarrow \frac{3a}{2b} > 1$

From II, we have $a \leq b \Rightarrow \frac{3a}{2b}$ may be greater than 1 or less than 1. From I alone we can say that the ratio of pure milk to water in *A* is greater than 1.

**23.** Statement I is not sufficient as nothing is mentioned about the ratio in which they are mixed.

From Statement II, the packet contains 700gm and its cost is ₹70 (as he got a profit of 50% when he sells at ₹105)

$\Rightarrow$ The cost of 1 kg of mixture is 100.

$\therefore$ We can find the ratio in which it is mixed.

**24.** From Statement I,

$P = Q + \frac{50}{100}Q = 1.5\,Q$

Hence, the question can be answered using Statement I.

From Statement II, $P + Q = P + \frac{75}{100}P$.

$\Rightarrow Q = \frac{75}{100}P$

Hence, the question can be answered using Statement II alone also.

**25.** Consider Statement I. Suppose Shreya did not get the highest marks. One of the other girls got at least 95. The other got at least 82. The average would be at least $82 + \frac{12+13}{3}$ or $90\frac{1}{3}$. It can't be 90 or less. Therefore Shreya got the highest marks. Statement I is sufficient.

Consider Statement II, Suppose the average of the three was 91. Two possibilities are 86, 93, 94 and 84, 94, 95.

$\therefore$ We can't say whether Shreya, who got 94 got the highest or not. Statement II is not sufficient.

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# Unit 4

# Geometry


Chapter 1 Geometry

Chapter 2 Mensuration

Chapter 3 Coordinate Geometry

Chapter 4 Trignometry

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# 1 Geometry

CHAPTER

## Learning Objectives

*In this chapter, you will*:

- Revise the basic concepts of line, line segment, ray and angle
- Gain understanding of different types of angles including complementary and supplementary angles
- Understand what are parallel lines, intersecting lines and learn the ratio of intercepts
- Understand and identify different angles formed when a transversal intersects a set of parallel lines
- Learn about the different types of triangles and their properties such as
  - Inequality related to sides of a triangle
  - Classification of triangles based on length of sides and measure of angles
  - Learn about the different geometric centers of a triangle and their properties
  - Similarity of triangles and congruence of triangles
- Learn important geometry theorems, such as Basic Proportionality Theorem, Midpoint Theorem, Apollonius Theorem, Angle Bisector Theorem
- Understand different types of quadrilaterals and polygons and their properties
- Learn how to find out the number of diagonals and the sum of angles in any $n$-sided polygon
- Learn different formulae to find out the area of triangles and polygons
- Understand the properties of a circle, arcs and sectors and their applications

In most competitive exams, the problems relating to Geometry usually cover triangles, quadrilaterals and circles. Even though polygons with more than four sides are also covered, the emphasis on such polygons is not as much as it is on triangles and circles. In this chapter, we will look at some properties as well as theorems and riders on parallel lines, angles, triangles (including congruence and similarity of triangles), quadrilaterals, circles and polygons.

## ❒ ANGLES AND LINES

1. An angle of 90° is a right angle
2. an angle less than 90° is acute angle
3. an angle between 90° and 180° is an obtuse angle
4. an angle between 180° and 360° is a reflex angle.

The sum of all angles made on one side of a straight line $AB$ at a point $O$ by any number of lines joining the line $AB$ at $O$ is 180°. In Fig. 1.1, the sum of the angles $u$, $v$, $x$, $y$ and $z$ is equal to 180°.

When any number of straight lines meet at a point, the sum of all the angles around that point is 360°. In Fig. 1.2, the sum of the angles $u$, $v$, $w$, $x$, $y$ and $z$ is equal to 360°.

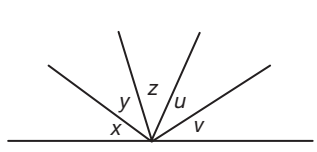



**Fig. 1.1**

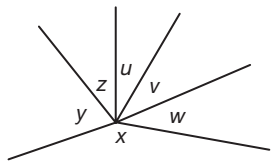



**Fig. 1.2**

Two angles whose sum is 90° are said to be **complementary** angles and two angles whose sum is 180° are said to be **supplementary angles**.

When two straight lines intersect, vertically opposite angles are equal. In Fig. 1.03, $\angle AOB$ and $\angle COD$ are vertically opposite angles and $\angle BOC$ and $\angle AOD$ are vertically opposite angles. So, we have $\angle AOB = \angle COD$ and $\angle BOC = \angle AOD$.

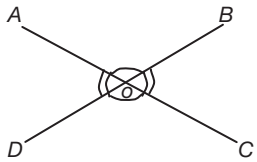



**Fig. 1.3**

Two lines which make an angle of 90° with each other are said to be **perpendicular to each other**.

If a line/line segment $l_1$ passes through the mid-point of another line segment $l_2$, the line/line segment $l_1$ is said to be the **bisector of the line segment** $l_2$, i.e., the line segment $l_2$ is divided into two equal parts.

If a line $l_1$ is drawn at the vertex of an angle dividing the angle into two equal parts, the line $l_1$ is said to be the **angle bisector of the angle**. **Any point on the angle bisector of an angle is equidistant from the two arms of the angle**.

If a line $l_1$ is perpendicular to line segment $l_2$ as well as passes through the mid-point of the line segment $l_2$, the line $l_1$ is said to be the perpendicular bisector of the line segment $l_2$.

Any point on the perpendicular bisector of a line segment is equidistant from both ends of the line segment.

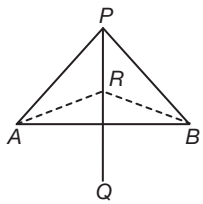



**Fig. 1.4**

In Fig 1.04, line $PQ$ is the perpendicular bisector of line segment $AB$. A point $P$ on the perpendicular bisector of $AB$ will be equidistant from $A$ and $B$, i.e., $PA = PB$. Similarly, for any point $R$ on the perpendicular bisector $PQ$, $RA = RB$.

## ❒ PARALLEL LINES

**When a straight line cuts two or more lines in the same plane, the line is called the transversal.** When a transversal $XY$ cuts two parallel lines $PQ$ and $RS$ [as shown in Fig. 1.05], the following are the relationships between various angles that are formed. [$M$ and $N$ are the points of intersection of $XY$ with $PQ$ and $RS$, respectively].

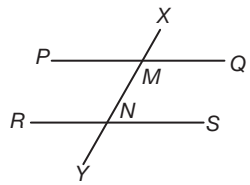



**Fig. 1.5**

1. **Alternate angles are equal,**
   **i.e., $\angle PMN = \angle MNS$ and $\angle QMN = \angle MNR$**
2. **Corresponding angles are equal,**
   **i.e., $\angle XMQ = \angle MNS$; $\angle QMN = \angle SNY$;**
   **$\angle XMP = \angle MNR$; $\angle PMN = \angle RNY$**
3. **Sum of interior angles on the same side of the transversal is equal to 180°,**
   **i.e., $\angle QMN + \angle MNS = 180°$ and $\angle PMN + \angle MNR = 180°$**
4. **Sum of exterior angles on the same side of the transversal is equal to 180°,**
   **i.e., $\angle XMQ + \angle SNY = 180°$; and $\angle XMP + \angle RNY = 180°$**

## ❒ RATIO OF INTERCEPTS

If three or more parallel lines make intercepts on a transversal in a certain proportion, then they make intercepts in the same proportion on any other transversal as well. In Fig. 1.06, the lines $AB$, $CD$ and $EF$ are parallel and the transversal $XY$ cuts them at the points $P$, $Q$ and $R$. If we now take a second transversal, $UV$, cutting the three parallel lines at the points $J$, $K$ and $L$, then we have $PQ/QR = JK/KL$.

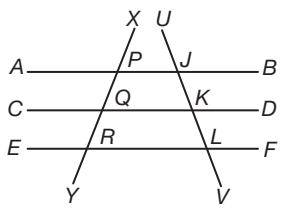



**Fig.1.6**

**NOTE**

If three or more parallel lines make equal intercepts on one transversal, they make equal intercepts on any other transversal as well.

## ❒ Triangles

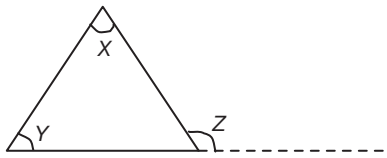



**Fig. 1.7**

**Sum of the three angles of a triangle is 180°**

The exterior angle of a triangle (at each vertex) is equal to the sum of the two opposite interior angles. (Exterior angle is the angle formed at any vertex, by one side and the extended portion of the second side at that vertex).

**A line perpendicular to a side and passing through the midpoint of the side is said to be the perpendicular bisector of the side**. It is not necessary that the perpendicular bisector of a side should pass through the opposite vertex in a triangle in general.

**The perpendicular drawn to a side from the opposite vertex is called altitude to that side**.

**The line joining the midpoint of a side with the opposite vertex is called median drawn to that side**. A median divides the triangle into two equal halves as far as the area is concerned.

**An equilateral triangle is one in which all the sides are equal (and hence, all angles are equal, i.e., each of the angles is equal to 60°). An isosceles triangle is one in which two sides are equal (and hence, the angles opposite to them are equal). A scalene triangle is one in which no two sides are equal.**

In an isosceles triangle, the unequal side is called base. The angle where the two equal sides meet is called vertical angle. In an isosceles triangle, the perpendicular drawn to the base from the vertex opposite the base (i.e., the altitude drawn to the base) bisects the base as well as the vertical angle. That is, the altitude drawn to the base will also be the perpendicular bisector of the base as well as the angle bisector of the vertical angle. It will also be the median drawn to the base.

In an equilateral triangle, the perpendicular bisector, the median and the altitude drawn to a particular side coincide and that will also be the angle bisector of the opposite vertex. If a is the side of an equilateral triangle, then its altitude is equal to $\sqrt{3}.a / 2$

**Sum of any two sides of a triangle is greater than the third side; difference of any two sides of a triangle is less than the third side.**

If the sides are arranged in the ascending order of their measurement, the angles opposite the sides (in the same order) will also be in ascending order (i.e., greater angle has greater side opposite to it); if the sides are arranged in descending order of their measurement, the angles opposite the sides in the same order will also be in descending order (i.e., smaller angle has smaller side opposite to it).

There can be only one right angle or only one obtuse angle in any triangle. There can also not be one right angle and an obtuse angle both present at the same time in a triangle.

**Hypotenuse is the side opposite the right angle in a right-angled triangle.** In a right-angled triangle, the hypotenuse is the longest side. In an obtuse angled triangle, the side opposite the obtuse angle is the longest side.

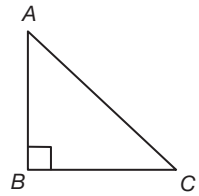



**Fig. 1.8.** Right-angled triangle

In a right-angled triangle, the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides. In Fig. 1.08, $AC^2 = AB^2 + BC^2$

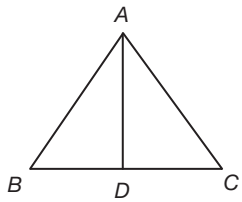



**Fig. 1.9.** Acute angled triangle

In an acute angled triangle, the square of the side opposite the acute angle is less than the sum of the squares of the other two sides by a quantity equal to twice the product of one of these two sides and the projection of the second side on the first side.

In Fig. 1.09, $AC^2 = AB^2 + BC^2 - 2\ BC.BD$

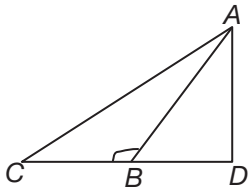



**Fig. 1.10.** Obtuse angled triangle

In an obtuse angled triangle, the square of the side opposite the obtuse angle is greater than the sum of the squares of the other two sides by a quantity equal to twice the product of one of the sides containing the obtuse angle and the projection of the second side on the first side.

**In Fig. 1.10, $AC^2 = AB^2 + BC^2 + 2BC.BD$**

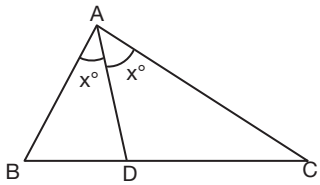



**Fig. 1.11**

In a triangle, the internal bisector of an angle bisects the opposite side in the ratio of the other two sides. In triangle *ABC*, if *AD* is the angle bisector of angle *A*, then $BD/DC = AB/AC$. This is called Angle Bisector Theorem (refer to Fig. 1.11).

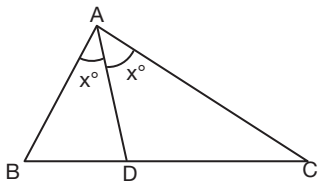



**Fig. 1.12**

In $\Delta ABC$, if *AD* is the median from *A* to side *BC* (meeting *BC* at its mid point *D*), then $2(AD^2 + BD^2) = AB^2 + AC^2$. This is called Apollonius Theorem. This will be helpful in calculating the lengths of the three medians given the lengths of the three sides of the triangle (refer to Fig. 1.12).

## ❒ GEOMETRIC CENTRES OF A TRIANGLE

### ❒ Circumcentre

The three perpendicular bisectors of a triangle meet at a point called circumcentre of the triangle and it is represented by *S*. The circumcentre of a triangle is equidistant from its vertices and the distance of circumcentre from each of the three vertices is called circumradius (represented by *R*) of the triangle. The circle drawn with the circumcentre as centre and circumradius as radius is called cirumcircle of the triangle and passes through all three vertices of the triangle. (Refer to Fig. 1.13)

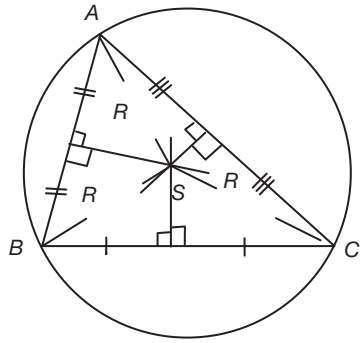



**Fig. 1.13**

### ❒ Incentre and Excentres

The internal bisectors of the three angles of a triangle meet at a point called incentre of the triangle and it is represented by *I*. Incentre is equidistant from the three sides of the triangle i.e., the perpendiculars drawn from the incentre to the three sides are equal in length and this length is called the inradius (represented by *r*) of the triangle. The circle drawn with the incentre as centre and the inradius as radius is called the incircle of the triangle and it touches all three sides on the inside.

**In Fig. 1.14, $\angle BIC = 90° + \frac{1}{2} A$ where *I* is the incentre. $\angle CIA = 90° + \frac{1}{2} B$; and $\angle AIB = 90° + \frac{1}{2} C$.**

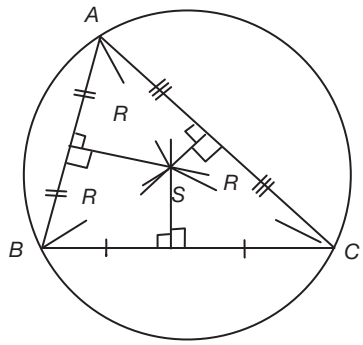



**Fig. 1.14**

If the internal bisector of one angle and the external bisectors of the other two angles are drawn, they meet at a point called excentre**.** There will be totally three excentres for the triangle - one corresponding to the internal bisector of each angle.

### ❒ Orthocentre

The three altitudes meet at a point called orthocentre and it is represented by *O* (refer to Fig. 1.15).

$\angle BOC = 180° - \angle A$; $\angle COA = 180° - \angle B$;

$\angle AOB = 180° - \angle C$.

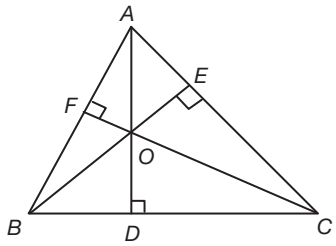



**Fig. 1.15**

## ❒ Centroid

The three medians of a triangle meet at a point called centroid and it is represented by *G* (refer to Fig. 1.16).

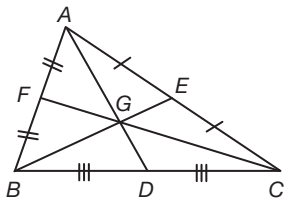



**Fig. 1.16**

**Important points about geometric centres of a triangle:**

1. In an acute angled triangle, the circumcentre lies inside the triangle.
2. In a right-angled triangle, the circumcentre lies on the hypotenuse of the triangle (it is the midpoint of the hypotenuse).
3. In an obtuse angled triangle, the circumcentre lies outside the triangle.
4. In an acute angled triangle, the orthocentre lies inside the triangle.
5. In a right-angled triangle, the vertex where the right angle is formed (i.e., the vertex opposite the hypotenuse) is the orthocentre.
6. In an obtuse angled triangle, the orthocentre lies outside the triangle.
7. In a right-angled triangle the length of the median drawn to the hypotenuse is equal to half the hypotenuse. This median is also the circumradius of the right-angled triangle.
8. Centroid divides each of the medians in the ratio 2 : 1, the part of the median towards the vertex being twice in length to the part towards the side.
9. Inradius is less than half of any of the three altitudes of the triangle.
10. In an isosceles triangle, the centroid, the orthocentre, the circumcentre and the incentre, all lie on the median to the base.
11. In an equilateral triangle, the centroid, the orthocentre, the circumcentre and the incentre, all coincide.

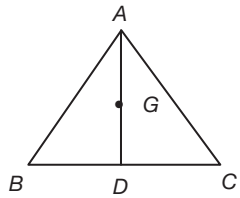



**Fig. 1.17**

Hence, in equilateral triangle *ABC* shown in Fig. 1.17, *AD* is the median, altitude, angle bisector and perpendicular bisector. *G* is the centroid which divides the median in the ratio 2 : 1. Hence, $AG = 2/3\ AD$ and $GD = 1/3\ AD$.

But since *AD* is also the perpendicular bisector and angle bisector and since *G* is the circumcentre as well as the incentre, *AG* will be the circumradius and *GD* will be the inradius of the equilateral triangle *ABC*. Since *AD* is also the altitude, its length is equal to $\sqrt{3}a / 2$ where a is the side of the equilateral triangle. Hence, the circumradius of the equilateral triangle

$= \frac{2}{3} \times \frac{\sqrt{3}}{2}$. a $= a/\sqrt{3}$ and the inradius $= \frac{1}{3} \times \frac{\sqrt{3}}{2}$. $a$

$= a / 2\sqrt{3}$

Since the radii of the circumcircle and the incircle of an equilateral triangle are in the ratio 2 : 1, the areas of the circumcircle and the incircle of an equilateral triangle will be in the ratio 4 : 1.

12. When the three medians of a triangle (i.e., the medians to the three sides of a triangle from the corresponding opposite vertices) are drawn, the resulting six triangles are equal in area and the area of each of these triangles in turn is equal to one-sixth of the area of the original triangle.

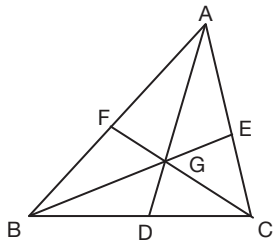



**Fig. 1.18**

In Fig 1.18, *AD*, *BE* and *CF* are the medians drawn to the three sides. The three medians meet at the centroid *G*. The six resulting triangles *AGF*,

*BGF*, *BGD*, *CGD*, *CGE* and *AGE* are equal in area and each of them is equal to $1/6^{th}$ of the area of triangle *ABC*.

## ❒ SIMILARITY OF TRIANGLES

**Two triangles are said to be similar if the three angles of one triangle are equal to the three angles of the second triangle.** Similar triangles are alike in shape only. The corresponding angles of two similar triangles are equal but the corresponding sides are only proportional and not equal.

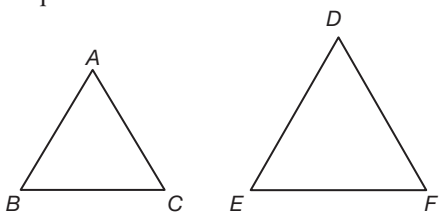



**Fig 1.19**

For example, in Fig 1.19, if $\Delta ABC$ is similar to $\Delta DEF$ where $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$, we have the ratios of the corresponding sides equal, as given below.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

By 'corresponding sides', we mean that if we take a side opposite to a particular angle in one triangle, we should consider the side opposite to the equal angle in the second triangle. In this case, since *AB* is the side opposite to $\angle C$ in $\angle ABC$, and since $\angle C = \angle F$, we have taken *DE* which is the side opposite to $\angle F$ in $\Delta DEF$.

### IMPORTANT FACTS

**Two triangles are similar if,**

1. the three angles of one triangle are respectively equal to the three angles of the second triangle, or
2. the two sides of one triangle are proportional to two sides of the other and the included angles are equal, or
3. if the three sides of one triangle are proportional to the three sides of another triangle.

In two similar triangles,

1. Ratio of corresponding sides = Ratio of heights (altitudes) = Ratio of the lengths of the medians = Ratio of the lengths of the angular bisectors = Ratio of inradii = Ratio of circumradii = Ratio of perimeters.
2. Ratio of areas = Ratio of squares of corresponding sides

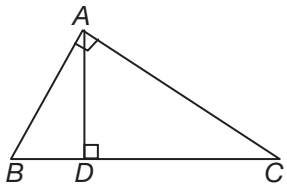



**Fig. 1.20**

In a right-angled triangle, the altitude drawn to the hypotenuse divides the given triangle into two similar triangles, each of which is in turn similar to the original triangle. In triangle *ABC* in Fig.1.20, *ABC* is a right-angled triangle where $\angle A$ is a right angle. *AD* is the perpendicular drawn to the hypotenuse *BC*. The triangles *ABD*, *CAD* and *CBA* are similar because of the equal angles given below.

In triangle *ABC*, $\angle A = 90°$. If $\angle B = \theta$, then $\angle C = 90° - \theta$.

In triangle *ABD*, $\angle ADB = 90°$. We already know that $\angle B = \theta$; hence, $\angle BAD = 90° - \theta$.

In triangle *ADC*, $\angle ADC = 90°$. We already know that $\angle C = 90° - \theta$; hence, $\angle CAD = \theta$.

### FORMULA

We can write down the relationship between the sides in these three triangles. The important relationships that emerge out of this exercise are

1. $AD^2 = BD \cdot DC$;
2. $AB^2 = BC \cdot BD$;
3. $AC^2 = CB \cdot CD$.

## ❒ CONGRUENCE OF TRIANGLES

Two triangles will be congruent if at least one of the following conditions is satisfied:

Three sides of one triangle are respectively equal to the three sides of the second triangle (normally referred to as the S-S-S rule, i.e., the side-side-side congruence).

Two sides and the included angle of one triangle are respectively equal to two sides and the included angle of the second triangle (normally referred to as the S-A-S rule, i.e., side-angle-side congruence).

Two angles and one side of a triangle are respectively equal to two angles and the corresponding side of the second triangle (normally referred to as the A-S-A rule, i.e., angle-side-angle congruence).

Two right-angled triangles are congruent if the hypotenuse and one side of one triangle are respectively equal to hypotenuse and one side of the second right-angled triangle.

In two congruent triangles,

1. the corresponding sides (i.e., sides opposite to corresponding angles) are equal
2. the corresponding angles (angles opposite to corresponding sides) are equal
3. the areas of the two triangles will be equal.

## ❒ SOME MORE USEFUL POINTS ABOUT TRIANGLES

### ❒ Basic Proportionality Theorem

A line drawn parallel to one side of a triangle divides the other two sides in the same proportion. For example, in Fig. 1.21, $PQ$ is drawn parallel to $BC$ in $\Delta ABC$. This will divide the other two sides $AB$ and $AC$ in the same ratio, i.e., $AP/PB = AQ/QC$.

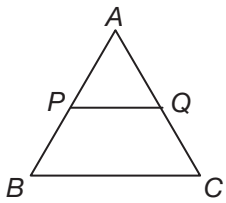



**Fig. 1.21**

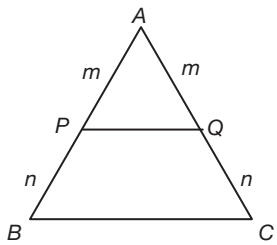



**Fig. 1.22**

Conversely, a line joining two points (each) dividing two sides of a triangle in the same ratio is parallel to the third side. In Fig. 1.22, $P$ divides $AB$ in the ratio $m : n$ and $Q$ divides $AC$ in the ratio $m : n$. Now, the line joining $P$ and $Q$ will be parallel to the third side $BC$ and the length of $PQ$ will be equal to $\frac{m}{m+n}$ times the length of $BC$.

We can say that a line drawn through a point on a side of the triangle parallel to a second side will cut the third side in the same ratio as the first side is divided.

### ❒ Mid-Point Theorem

The line joining the midpoints of two sides of a triangle is parallel to the third side and it is half the third side.

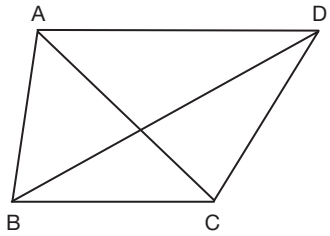



**Fig. 1.22 (a)**

Two triangles having the same base and lying between the same pair of parallel lines have their areas equal [Fig.1.22(a)].

$AD$ is parallel to $BC$. Hence, $\Delta ABC = \Delta DBC$

## ❒ QUADRILATERALS

**Any four-sided closed figure is called quadrilateral.** By imposing certain conditions on the sides and/or angles of a quadrilateral, we can get the figures trapezium, parallelogram, rhombus, rectangle, square.

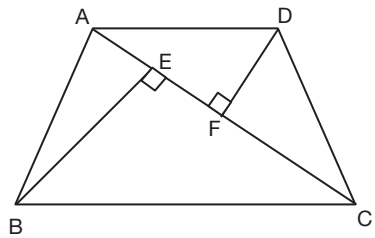



**Fig. 1.23**

The sum of four angles of a quadrilateral is equal to 360°.

The perpendiculars drawn to a diagonal (in a quadrilateral) from the opposite vertices are called 'offsets'. In Fig. 1.23, $BE$ and $DF$ are the offsets drawn to the diagonal $AC$.

If the four vertices of a quadrilateral lie on the circumference of a circle (i.e., if the quadrilateral can be inscribed in a circle) it is called a cyclic quadrilateral (refer to Fig. 1.24). In a cyclic quadrilateral, sum of opposite angles = 180°, i.e., in Fig. 1.24, $A + C = 180°$ and $B + D = 180°$.

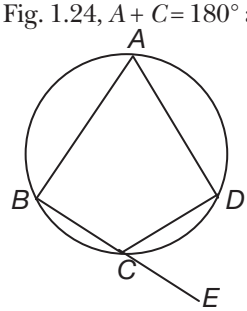



**Fig. 1.24**

Also, in a cyclic quadrilateral, exterior angle is equal to the interior opposite angle, i.e., in Fig.1.24, $\angle DCE$ is equal to $\angle BAD$.

Now, we will look at different quadrilaterals and their properties.

## ❒ TRAPEZIUM

**If one side of a quadrilateral is parallel to its opposite side, then it is called a trapezium. The two sides other than the parallel sides in a trapezium are called the oblique sides.**

In Fig. 1.25, $ABCD$ is a trapezium where $AD$ is parallel to $BC$.

If the midpoints of the two oblique sides are joined, it is equal in length to the average of the two parallel sides, i.e., in Fig. 1.25, $PQ = 1/2\ [AD + BC]$

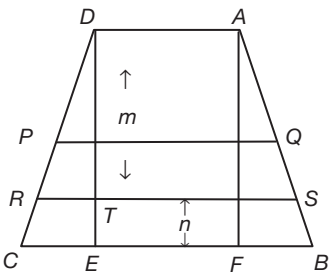



**Fig. 1.25**

In general, if a line is drawn in between the two parallel sides of the trapezium such that it is parallel to the parallel sides and also divides the distance between the two parallel sides in the ratio $m : n$ (where the portion closer to the shorter of the two parallel sides is $m$), the length of the line is given by :

**FORMULA**

$\frac{m}{m+n} \times$ Longer side $+ \frac{n}{m+n} \times$ Shorter side where shorter side and longer side refer to the shorter and longer of the two parallel sides of the trapezium.

In Fig. 1.25, $RS$ is the line parallel to $AD$ and $BC$ and the ratio of the distances $DT$ and $TE$ is $m : n$.

The length of $RS$ is given by

$$\frac{m}{m+n} \times BC + \frac{n}{m+n} \times AD$$

## ❒ PARALLELOGRAM

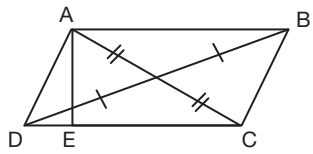



**Fig. 1.26**

**A quadrilateral in which opposite sides are parallel is called a parallelogram.**

In a parallelogram

1. Opposite sides are equal
2. Opposite angles are equal
3. Sum of any two adjacent angles is 180°
4. Each diagonal divides the parallelogram into two congruent triangles
5. The diagonals bisect each other

**Conversely, if in a quadrilateral**

6. The opposite sides are equal or
7. The opposite angles are equal or
8. The diagonals bisect each other or
9. A pair of opposite sides are parallel and equal- such a quadrilateral is a parallelogram.

**If two adjacent angles of a parallelogram are equal, then all four angles will be equal and each in turn will be equal to 90°. Then, the figure will be a rectangle.**

**If any two adjacent sides of a parallelogram are equal, then all four sides will be equal to each other and the figure will be a rhombus.**

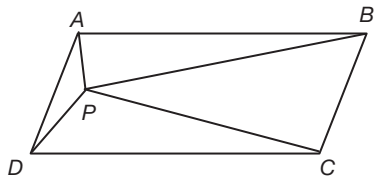



**Fig. 1.27**

If any point inside a parallelogram is taken and is joined to all the four vertices the four resulting triangles will be such that the sum of the areas of opposite triangles is equal. In Fig. 1.27, $P$ is a point inside the parallelogram $ABCD$ and it is joined to the four vertices of the parallelogram by the lines $PA$, $PB$, $PC$ and $PD$, respectively. Then, **Area of triangle *PAB* + Area of triangle *PCD* = Area of triangle *PBC* + Area of triangle *PAD* = Half the area of parallelogram *ABCD*.**

If there is a parallelogram and a triangle with the same base and between the same parallel lines, then the area of the triangle will be half that of the parallelogram.

If there is a parallelogram and a rectangle with the same base and between the same parallel lines, then the areas of the parallelogram and the rectangle will be the same. The figure formed by joining the midpoints of the sides of any quadrilateral taken in order, is a parallelogram.

## ❒ RHOMBUS

**A rhombus is a parallelogram in which a pair of adjacent sides are equal (all four sides of a rhombus are equal).**

Since a rhombus is a parallelogram, all the properties of a parallelogram apply to a rhombus. Further, in a rhombus, the diagonals bisect each other perpendicularly.

Conversely, any quadrilateral where the two diagonals bisect each other at right angles will be a rhombus.

The four triangles that are formed by the two bisecting diagonals with the four sides of the rhombus will all be congruent. In Fig.5.28, the four triangles *PAB, PBC, PCD* and *PAD* are congruent.

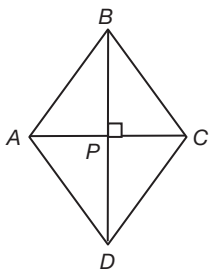



**Fig. 1.28**

**FORMULA**

Side of a rhombus

$= \sqrt{1/4 \times \text{Sum of squares of the diagonals}}$.

## ❒ RECTANGLE

*A* rectangle also is a special type of parallelogram and hence, all properties of a parallelogram apply to rectangles also. **A rectangle is a parallelogram in which two adjacent angles are equal or each of the angles is equal to 90°.**

**The diagonals of a rectangle are equal (and, of course, bisect each other).**

**When a rectangle is inscribed in a circle, the diagonals become the diameters of the circle.**

If $a$ and $b$ are the two adjacent sides of a rectangle, then the **diagonal is given by** $\sqrt{a^2+b^2}$ .

If a rectangle and a triangle are on the same base and between the same parallels, then the area of the triangle will be equal to half the area of the rectangle.

## ❒ SQUARE

**A square is a rectangle in which all four sides are equal (or a rhombus in which all four angles are equal, i.e., all are right angles).** Hence, the diagonals are equal and they bisect at right angles. So, all the properties of a rectangle as well as those of a rhombus hold good for a square.

**FORMULA**

Diagonal = $\sqrt{2} \times$ Side

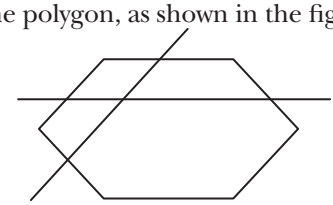
**IMPORTANT FACTS**

1. When a square is inscribed in a circle, the diagonals become the diameters of the circle.
2. When a circle is inscribed in a square, the side of the square is equal to the diameter of the circle.
3. The largest rectangle that can be inscribed in a given circle will be a square.

## ❒ POLYGON

**Any closed figure with three or more sides is called a polygon.**

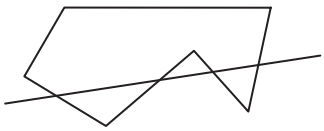
**A convex polygon is one in which each of the interior angles is less than 180°.** It can be noticed that any straight line drawn cutting a convex polygon passes only two sides of the polygon, as shown in the figure below.



Convex Polygon

**Fig. 1.28 (a)**

**In a concave polygon, it is possible to draw lines passing through more than two sides**, as shown in the figure below.



Concave Polygon

**Fig. 1.28 (b)**

**A regular polygon is a convex polygon in which all sides are equal and all angles are equal.** A regular polygon can be inscribed in a circle. The centre of the circumscribing circle (the circle in which the polygon is inscribed) of a regular polygon is called the centre of the polygon.

The names of polygons with three, four, five, six, seven, eight, nine and ten sides are respectively triangle, quadrilateral, pentagon, hexagon, heptagon, octagon, nonagon and decagon.

**FORMULA**

The sum of interior angles of a convex polygon is equal to $(2n - 4)$ right angles where $n$ is the number of the sides of the polygon.

If each of the sides of a convex polygon is extended, the sum of the external angles thus formed is equal to 4 Right Angles (i.e., 360°).

In a regular polygon of $n$ sides, if each of the interior angles is $d°$, then $d = \frac{2n-4}{n} \times 90°$ and each exterior angle $= \frac{360°}{n}$.

It will be helpful to remember the interior angles of the following regular polygons:

Regular pentagon: 108°

Regular hexagon: 120°

Regular octagon: 135°

If the centre of a regular polygon (with $n$ sides) is joined with each of the vertices, we get $n$ identical triangles inside the polygon. In general, all these triangles are isosceles triangles. Only in case of a regular hexagon, all these triangles are equilateral triangles, i.e., in a regular hexagon, the radius of the circumscribing circle is equal to the side of the hexagon.

*A* line joining any two non-adjacent vertices of a polygon is called a diagonal. A polygon with $n$ sides will have $\frac{n(n-3)}{2}$ diagonals.

## ❒ CIRCLES

A circle is a closed curve drawn such that any point on the curve is equidistant from a fixed point. The fixed point is called the centre of the circle and the distance from the centre to any point on the circle is called the radius of the circle.

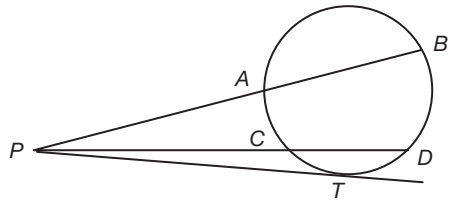



**Fig. 1.29**

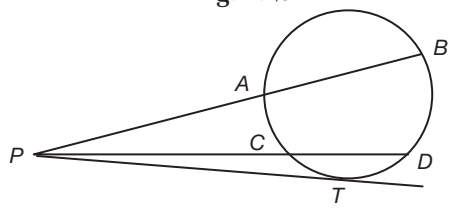



**Fig. 1.30**

Diameter is a straight line passing through the centre of the circle and joining two points on the circle. A circle is symmetric about any diameter.

*A* chord is a point joining two points on the circumference of a circle (*AB* in Fig. 1.30). Diameter is the largest chord in a circle.

*A* secant is a line intersecting a circle in two distinct points and extending outside the circle also.

*A* line that touches the circle at only one point is a tangent to the circle ($R_1R_2$ is a tangent touching the circle at the point $R$ in Fig. 5.30).

If *PAB* and *PCD* are two secants (in Fig. 1.29), then $PA.PB = PC.PD$

If *PAB* and *PCD* are secants and *PT* is a tangent to the circle at *T* (in Fig. 1.29), then $PA.PB = PC.PD = PT^2$.

Two tangents can be drawn to the circle from any point outside the circle and these two tangents are equal in length. In Fig. 1.30, *P* is the external point and the two tangents *PX* and *PY* are equal.

*A* tangent is perpendicular to the radius drawn at the point of tangency (In Fig. 1.30, $R_1R_2 \perp OR$). Conversely, if a perpendicular is drawn to the tangent at the point of tangency, it passes through the centre of the circle.

*A* perpendicular drawn from the centre of the circle to a chord bisects the chord (In Fig. 1.30, *OC*, the perpendicular from *O* to the chord *AB* bisects *AB*) and conversely, the perpendicular bisector of a chord passes through the centre of the circle.

Two chords that are equal in length will be equidistant from the centre, and conversely two chords which are equidistant from the centre of the circle will be of the same length.

One and only one circle passes through any three given non-collinear points.

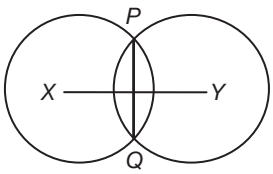



**Fig. 1.31**

When there are two intersecting circles, the line joining the centres of the two circles will perpendicularly bisect the line joining the points of intersection. In Fig. 1.31, the two circles with centres *X* and *Y*, respectively intersect at the two points *P* and *Q*. The line *XY* (the line joining the centres) bisects *PQ* (the line joining the two points of intersection).

Two circles are said to touch each other if a common tangent can be drawn touching both the circles at the same point. This is called the point of contact of the two circles. The two circles may touch each other internally (as in Fig. 1.32) or externally (As in Fig. 1.33). When two circles touch each other, then the point of contact and the centres of the two circles are collinear, i.e., the point of contact lies on the line joining the centres of the two circles.

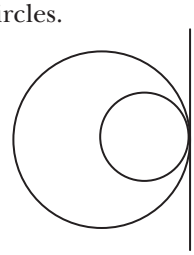


**Fig.1.32**

If two circles touch internally, the distance between the centres of the two circles is equal to the difference in the radii of the two circles. When two circles touch each other externally, then the distance between the centres of the two circles is equal to the sum of the radii of the two circles.

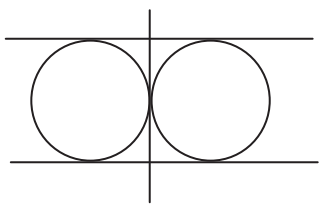


**Fig. 1.33**

*A* tangent drawn common to two circles is called a common tangent. In general, for two circles, there can be anywhere from zero to four common tangents drawn depending on the position of one circle in relation to the other.

If the common tangent is either parallel to the line of centres or cuts the line joining the centres not between the two circles but on one side of the circles, such a common tangent is called a direct common tangent. A common tangent that cuts the line joining the centres in between the two circles is called transverse common tangent.

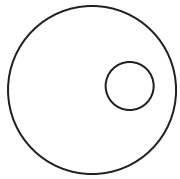


**Fig. 1.34**

If two circles are such that one lies completely inside the other (without touching each other), then there will not be any common tangent to these circles (refer to Fig. 1.34).

Two circles touching each other internally (i.e., still one circle lies inside the other), then there is only one common tangent possible and it is drawn at the point of contact of the two circles (refer to Fig. 1.32).

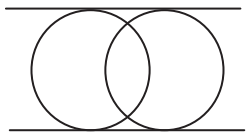


**Fig. 1.35**

Two intersecting circles have two common tangents. Both these are direct common tangents and the two intersecting circles do not have a transverse common tangent (refer to Fig. 1.35).

Two circles touching each other externally have three common tangents. Out of these, two are direct common tangents and one is a transverse common tangent. The transverse common tangent is at the point of contact (Refer to Fig. 1.33).

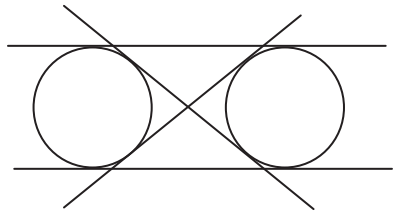


**Fig. 1.36**

Two circles which are non-intersecting and non-enclosing (i.e., one does not lie inside the other) have four common tangents - two direct and two transverse common tangents (Refer to Fig. 1.36).

If $r_1$ and $r_2$ are the radii of the two non-intersecting non-enclosing circles,

Length of the direct common tangent

$$= \sqrt{(\text{Distance between centre})^2 - (r_1 - r_2)^2}$$

Length of transverse common tangent

$$= \sqrt{(\text{Distance between centre})^2 - (r_1 + r_2)^2}$$

Two circles are said to be concentric if they have the same centre. As is obvious, here the circle with smaller radius lies completely within the circle with bigger radius.

## ❒ ARCS AND SECTORS

An arc is a segment of a circle. In Fig. 1.37, *ACB* is called minor arc and *ADB* is called major arc. In general, if we talk of an arc *AB*, we refer to the minor arc. *AOB* is called the angle formed by the arc *AB* (at the centre of the circle).

The angle subtended by an arc at the centre is double the angle subtended by the arc in the remaining part of the circle.

In Fig. 1.37, $\angle AOB = 2 \,.\, \angle AXB. = 2.\angle AYB$

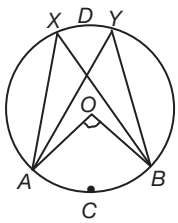



**Fig. 1.37**

Angles in the same segment are equal. In Fig. 1.37, $\angle AXB = \angle AYB$.

The angle between a tangent and a chord through the point of contact of the tangent is equal to the angle made by the chord in the alternate segment (i.e., segment of the circle on the side other than the side of location of the angle between the tangent and the chord). This is normally referred to as the 'alternate segment theorem.' In Fig. 1.38, *PQ* is a tangent to the circle at the point *T* and *TS* is a chord drawn at the point of contact. Considering $\angle PTS$ which is the angle between the tangent and the chord, the angle *TRS* is the angle in the 'alternate segment'. So, $\angle PTS = \angle TRS$.

Similarly, $\angle QTS = \angle TUS$.

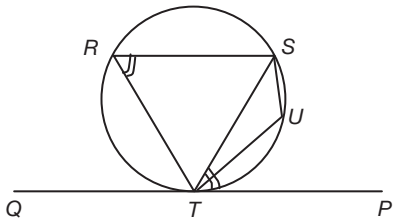



**Fig. 1.38**

We have already seen in quadrilaterals, the opposite angles of a cyclic quadrilateral are supplementary and that the external angle of a cyclic quadrilateral is equal to the interior opposite angle.

The angle in a semicircle (or the angle the diameter subtends in a semicircle) is a right angle. The converse of the above is also true and is very useful in a number of cases-in a right-angled triangle, a semi-circle with the hypotenuse as the diameter can be drawn passing through the third vertex (Refer to Fig. 1.39).

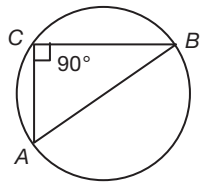



**Fig. 1.39**

The area formed by an arc and the two radii at the two end points of the arc is called sector.

In Fig. 1.40, the shaded figure *AOB* is called minor sector.

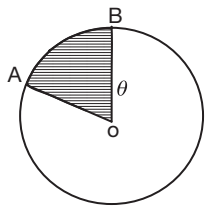
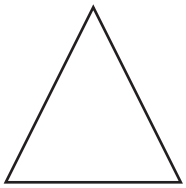
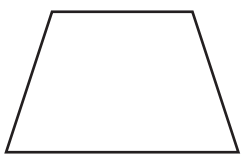


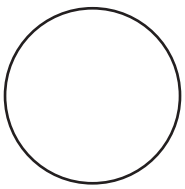
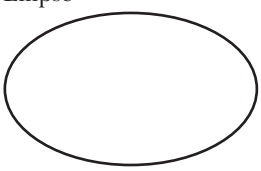

**Fig. 1.40**

## ❒ AREAS OF PLANE FIGURES

Mensuration is the branch of geometry that deals with the measurement of length, area and volume. We have looked at properties of plane figures till now. Here, in addition to areas of plane figures, we will also look at surface areas and volumes of 'solids.' Solids are objects, which have three dimensions (plane figures have only two dimensions).

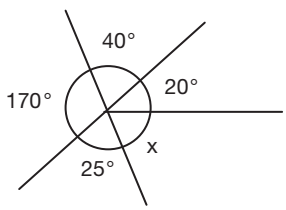
Let us briefly look at the formulae for areas of various plane figures and surface areas and volumes of various solids.

| Name | Formula |
|---|---|
| Triangles  | 1. For any triangle in general,<br>(i) When the measurements of three sides $a$, $b$, $c$ are given,<br>Area = $\sqrt{s(s-a)(s-b)(s-c)}$ where<br>$s = \frac{a+b+c}{2}$<br>This is called Hero's formula.<br>(ii) When base ($b$) and altitude [height ($h$)] to that base are given,<br>Area = ½ × base × altitude = ½ $b.h$<br>(iii) Area = ½ $ab$ . sin$C$ = ½ bc.sin$A$<br>= ½ $ca$.sin$B$<br>(iv) Area = $\frac{abc}{4R}$ where $R$ is the circumradius of the triangle.<br>(v) Area = $r.s$ where $r$ is the inradius of the triangle and $s$, the semi-perimeter.<br>Out of these five formulae, the first and the second are the most commonly used and are also more important from the examination point of view.<br>2. For a right angled triangle,<br>Area = ½ × Product of the sides containing the right angle<br>3. For an equilateral triangle<br>Area = $\frac{\sqrt{3}\cdot a^2}{4}$ where $a$ is the side of the triangle<br>The height of an equilateral triangle = $\frac{\sqrt{3}\cdot a}{2}$<br>4. For an isosceles triangle<br>Area = $\frac{b}{4}\sqrt{4a^2-b^2}$ where '$a$' is length of each of the two equal sides and $b$ is the third side.<br>**Note:** The area of a triangle is represented by the symbol D. For any triangle, the three sides are represented by a, b and c and the angles opposite these sides represented by A, B and C, respectively. |
| Quadrilaterals  | 1. For any quadrilateral<br>Area of the quadrilateral = ½ × One diagonal × Sum of the offsets drawn to that diagonal<br>Hence, for the quadrilateral $ABCD$ shown in Fig. 1.23, area of quadrilateral $ABCD$ = 1/2 × $AC$ × ($BE$ + $DF$)<br>2. For a cyclic quadrilateral where the four sides measure a, $b$, $c$ and $d$, respectively,<br>Area = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$ where $s$ is the semi-perimeter, i.e., $s = (a+b+c+d)/2$<br>3. For a trapezium<br>Area of a trapezium = ½ × Sum of parallel sides × Distance between them<br>= ½ × ($AD$ + $BC$) × $AE$ (refer to Fig. 1.25)<br>4. For a parallelogram<br>(a) Area = Base × Height<br>(b) Area = Product of two sides × Sine of included angle<br>5. For a rhombus<br>Area = ½ × Product of the diagonals<br>Perimeter = 4 × Side of the rhombus |

| | |
|---|---|
| | 6. For a rectangle<br>Area = Length × Breadth<br>Perimeter = $2(l + b)$, where $l$ and $b$ are the length and the breadth of the rectangle, respectively<br>7. For a square<br>(i) Area = Side$^2$<br>(ii) Area = ½ × Diagonal$^2$<br>[where the diagonal = $\sqrt{2}$ × side]<br>Perimeter = 4 × Side<br>8. For a regular hexagon<br>(i) Area = $\frac{3\sqrt{3}}{2}$ (side)$^2$<br>(ii) Perimeter = 6(side)<br>9. For a polygon<br>(i) Area of a regular polygon = ½ × Perimeter × Perpendicular distance from the centre of the polygon to any side<br>(Please note that the centre of a regular polygon is equidistant from all its sides)<br>(ii) For a polygon which is not regular, the area has to be found out by dividing the polygon into suitable number of quadrilaterals and triangles and adding up the areas of all such figures present in the polygon. |
| Circle  | 1. Area of the circle = $\pi r^2$ where $r$ is the radius of the circle<br>Circumference = $2\pi r$<br>2. Sector of a circle<br>Length of arc = $\frac{\theta}{360°} \times 2\pi r$<br>Area = $\frac{\theta}{360°} \times \pi r^2$ where $\theta$ is the angle of the sector in degrees and $r$ is the radius of the circle.<br>Area = $(1/2)lr$; $l$ is length of arc and $r$ is radius.<br>3. Ring is the space enclosed by two concentric circles.<br>Area = $\pi R^2 - \pi r^2 = \pi(R + r)(R - r)$ where $R$ is the radius of the outer circle and $r$ is the radius of the inner circle. |
| Ellipse  | Area = $\pi ab$ where '$a$' is semi-major axis and '$b$' is semi-minor axis.<br>Perimeter = $\pi(a + b)$ |

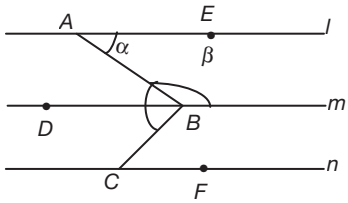
## SOLVED EXAMPLES

**1.01.** Calculate the value of $x$ in the figure given below.




**Sol:** The angles at a point add up to 360°.
Hence, $(20° + 40° + 25° + 170°) + x = 360°$
$\therefore x = 360° - 255° = 105°$

**1.02.** If $l$, $m$ and $n$ are lines parallel to each other, calculate the measures of the angles $\alpha$ and $\beta$ in the given figure. Given that angle $ABC$ is 100° and angle $DBC$ is 75°. Also, $A$, $E$; $D$, $B$; and $C$, $F$ are points on $l$, $m$ and $n$, respectively.




**Sol:** As $\angle DBC = 75°$
$\angle DBA = 100° - \angle DBC = 100° - 75° = 25°$
$\alpha = \angle DBA = 25°$
(As $\alpha$ and $DBA$ are alternate angles, they are equal)
As interior angles on the same side of the transversal are supplementary,
$\alpha + \beta = 180° \Rightarrow \therefore \beta = 180° - 25 = 155°$

**1.03.** How many degrees are there in an angle which is equal to one-fourth of its supplement?

**Sol:** Let the angle be $x$. Then, its supplement is $180° - x$.

Given, $x = \frac{1}{4}(180° - x)$

$\Rightarrow 5x = 180°$
$\Rightarrow x = 36°$.

**1.04.** How many degrees are there in an angle which is equal to one-fifth of its complement?

**Sol:** Let the angle be $x$. Then, its complement is $90° - x$.

Given that, $x = \frac{1}{5}(90° - x)$

$\Rightarrow 6x = 90° \Rightarrow x = 15°$

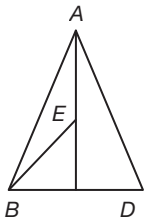
**1.05.** In a right-angled triangle $ABC$, find the length of the altitude $BD$ drawn from $B$ to the hypotenuse $AC$, given $AB = 9$ cm and $BC = 12$ cm.

**Sol:** Since the sides containing the right angle are 9 cm and 12 cm, the area of the triangle $= \frac{1}{2}(9)(12) = 54$ cm$^2$.
As $AB = 9$ cm, $BC = 12$ cm, $AC$ will be 15 cm. Considering $AC$ as the base and $BD$ as the altitude, the area of $\Delta ABC$

$= \frac{1}{2}(AC)(BD) = \frac{1}{2}(15)(BD)$ cm $= 54$ cm$^2$

$BD = \frac{54(2)}{15}\frac{\text{cm}^2}{\text{cm}} = \frac{108}{15}$ cm $= 7.2$ cm.

**1.06.** In the figure given below, $ABC$ is a triangle in which $D$ is the midpoint of the side $BC$ and $E$ is the midpoint of $AD$. What is the ratio of the areas of $\Delta BED$ and $\Delta ABC$?




**Sol:** Since $AE = ED$, $BE$ is a median on $AD$ and the median divides $\Delta ABD$ into halves, such that area of $\Delta BED$ = Area of $\Delta AEB = \frac{1}{2}$ (Area of $\Delta ABD$).
Since $AD$ is the median, it divides $\Delta ABC$ into halves such that area of $\Delta ABD$ = area of $\Delta ADC$ $= \frac{1}{2}$ (Area of $\Delta ABC$).

Hence, the area of $\Delta BED$: Area of $\Delta ABC$

$= \frac{1}{2}\left(\frac{1}{2}(\text{Area of } \Delta ABC)\right)$ : Area of $\Delta ABC$

$= \frac{1}{4} : 1 = 1 : 4$.

**1.07.** Triangles $ABC$ and $DEF$ are similar. If $\angle A = \angle D$, $\angle C = \angle F$, $BC = 3.4$ cm, $DE = 4.8$ cm and $AB = 1.6$ cm, then find the length of $EF$.

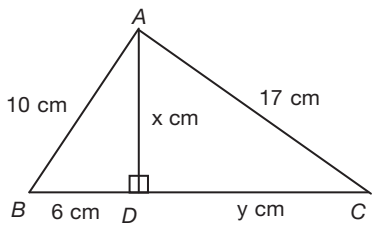
**Sol:** Given that triangles $ABC$ and $DEF$ are similar.
Hence, the corresponding sides are proportional.

$\therefore$ We have, $\frac{AB}{DE} = \frac{BC}{EF}$

Hence, $EF = \frac{(BC)(DE)}{(AB)} = \frac{(3.4)(4.8)}{(1.6)}\frac{\text{cm}^2}{\text{cm}}$

$= 10.2$ cm.

**1.08.** In the figure given below, $AD$ is perpendicular to $BC$ in triangle $ABC$. Find $x$ and $y$.




**Sol:** From $\Delta ABD$, $x^2 = AB^2 - BD^2$
(By Pythagoras theorem)
$x^2 = 10^2 - 6^2 = 64 \Rightarrow x = 8$.
From $\Delta ADC$ $y^2 = AC^2 - x^2$

(By Pythagoras theorem)

$y = \sqrt{17^2 - 8^2} = \sqrt{225} = 15$

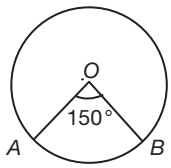
**1.09.** A and B leave a point at the same time. A travels North at 18 km/hr and B travels West at a speed of 24 km/hr. Find the distance between A and B after two hours.

**Sol:** Let the starting point be *O*. After 2 hours, let A be at *X*, which will be $2 \times 18 = 36$ km from O and B be at *Y*, which will be $2 \times 24 = 48$ km from *O* in the direction perpendicular to *OX*. Using Pythagoras theorem, we can find the third side *XY*.

$XY = \sqrt{36^2 + 48^2} = 60$ km. Hence, A and B will be 60 km away from each other.

**1.10.** In the figure given below, if the length of major arc $AB = 154$ cm, find the length of the radius *OA* and find the area of the minor sector *AOB*.

**Sol:**




Angle of the major arc = $360° - 150° = 210°$

Length of the major arc = $\frac{\text{Angle of the arc}}{360°}(2\pi r)$

$\therefore \frac{210°}{360°}(2)\left(\frac{22}{7}\right)(OA) = 154 \text{ cm} \Rightarrow OA = 42 \text{ cm}.$

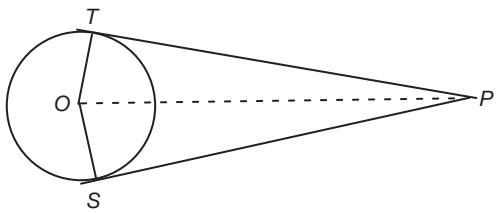
Area of the minor sector

$= \frac{\text{Angle of the sector}}{360°} \times \pi r^2$

$= \frac{150}{360}\left(\frac{22}{7}\right)(42^2) \text{ cm}^2 = 2310 \text{ cm}^2.$

**1.11.** In a circle with centre *O*, *PT* and *PS* are tangents drawn to it from point *P*. If $PT = 24$ cm and $OT = 10$ cm, then find the length of *PO*.

**Sol:**




From the above figure

$\angle OTP = 90°$, (Radius makes an angle of 90° with the tangent at the point of tangency).

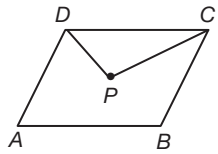
In triangle *OTP*,

$PO^2 = PT^2 + OT^2 = 24^2 + 10^2 \text{ cm}^2$

$PO = \sqrt{24^2 + 10^2} \text{ cm} = 26 \text{ cm}$

**1.12.** In parallelogram *ABCD*, *DP* and *CP* are the angle bisectors of $\angle CDA$ and $\angle DCB$, respectively. Find $\angle CPD$.

**Sol:**




Given *DP* is the angle bisector of $\angle CDA$

$\Rightarrow \angle CDP = \frac{1}{2} \angle CDA \quad (1)$

Given *CP* is the angle bisector of $\angle DCB$

$\Rightarrow \angle DCP = \frac{1}{2} \angle DCB \quad (2)$

In a parallelogram the sum of adjacent angles $= 180°$.

Therefore, $\angle CDA + \angle DCB = 180°$.

From (1) and (2),

$2 \angle CDP + 2 \angle DCP = 180°$

$\Rightarrow \angle CDP + \angle DCP = 90°$

In triangle $\angle PDC$,

$\angle CPD = 180° - (\angle PDC + \angle PCD)$

$= 180° - 90° = 90°$

**1.13.** The number of sides of a regular polygon is 18. Find the interior angle of the polygon.

**Sol:** Exterior angle of a regular polygon

$= 360°/n = 360°/18 = 20°$.

The sum of the interior angle and the exterior angle is 180°.

Interior angle of this polygon $= 180° - 20°$

$= 160°$.

**1.14.** For a polygon of 12 sides, find the sum of the interior angles of the polygon.

**Sol:** Sum of the interior angles of a polygon $= (2n - 4) \times 90°$ where *n* is the number of sides.

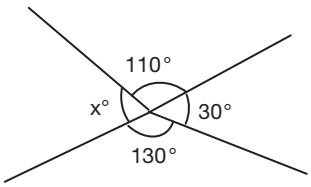
$\therefore$ Sum of the interior angles of this polygon $= [2(12) - 4]\ 90° = 1800°$.

**1.15.** If one angle of a pentagon is 40° and all the other angles are equal, find the measure of each of the other angles.

**Sol:** Sum of the interior angles of a pentagon.
= [(2 (5) – 4)] 90° = 540°
One angle is 40°, the sum of the remaining four equal angles
= 540° – 40° = 500°

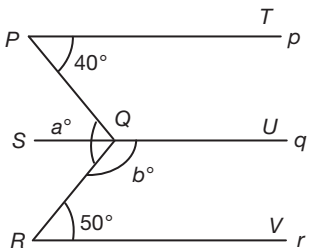
Each of the other four angles $= \frac{500°}{4} = 125°$.

**1.16.** Find the value of $x$ in the figure given below.




**Sol:** The sum of the angles at a point is 360°
$\Rightarrow x° + 110° + 30° + 130° = 360°$
$x = 90$

**1.17.** Find the measures of a° and b° in the figure below, given that the lines p, q and r are parallel to each other.




**Sol:** $\angle PQR = \angle PQS + \angle SQR$
$\angle PQS = \angle QPT$ (Alternate angles are equal)
$\angle SQR = \angle QRV$ (Alternate angles are equal)
$\therefore \angle PQS + \angle SQR = \angle QPT + \angle QRV = 90°$
$\therefore a° = 90°$
b + 50° = 180° (co-interior angles are supplementary)
$b° = 130°$

**1.18.** An angle equals two-thirds of its supplement. Find it.

**Sol:** Let the angle be $x°$. Its supplement is $(180° - x°)$.

Given that $x° = \frac{2}{3}(180° - x°)$

$x° = 72°$

**1.19.** An angle equals two-third of its complement. Find it.

**Sol:** Let the angle be $x°$. Its complement is $(90° - x°)$.

Given that $x° = \frac{2}{3}(90° - x°) \Rightarrow x° = 36°$

**1.20.** In a right angled triangle, one of the sides containing the right angle is 30 cm and hypotenuse is 50 cm. Find its area.

**Sol:** Let the other perpendicular side of the triangle be $h$ cm.

$\sqrt{h^2 + 30^2} = 50$ (Pythogoras theorem)

$\Rightarrow h = 40$

$\therefore$ Area $= \frac{1}{2}$ (40) (30), i.e., 600 sq.cm

**1.21.** Find the area of an equilateral triangle of side 6 cm.

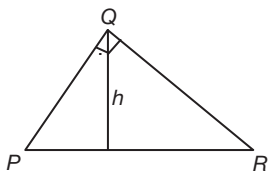
**Sol:** Area of an equilateral triangle

$= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}\left(6^2\right) = 9\sqrt{3}$ sq.cm

**1.22.** Triangle $PQR$ is right angled at $Q$. Find the length of the altitude drawn from $Q$ to $PR$ if $PQ = 8$ cm and $QR = 15$ cm.

**Sol:** Area of triangle $PQR = \frac{1}{2}(8)(15) = 60$ sq.cm

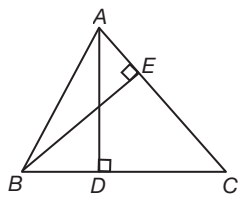
Let the altitude be $h$ cm.




$\frac{1}{2}(PR)(h) = 60$

$\frac{1}{2}\left(\sqrt{8^2 + 15^2}\right)h = 60 \Rightarrow h = \frac{120}{17}$

**1.23.**



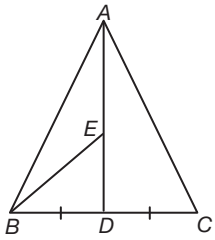

In the figure above,
AD = 4 × 2 cm, BC = 10 × 8 cm and BE = 7 × 2 cm. Find AC.

**Sol:** Area of $\Delta ABC$

$= \frac{1}{2}(AD)(BC) = \frac{1}{2}(BE)(AC)$

$$AC = \frac{(AD)(BC)}{BE} = \frac{(4\cdot 2)(10\cdot 8)}{7\cdot 2} = 6.3 \text{ cm}$$

**1.24.**




In the figure given above, *AD* is the median on BC. BE is the median on *AD*. Find the ratio of the areas of the triangles *AEB*, *BED* and *ABC*.

**Sol:** Since *BE* is the median, Area of $\Delta AEB$ = Area of $\Delta BED$.

Since *AD* is the median, Area of $\Delta ABD$ = Area of $\Delta ADC \Rightarrow$ Area of $\Delta\ AEB$

$$\frac{1}{2}\left[\frac{1}{2}(\text{Area of } \Delta\, ABC)\right]$$

$\therefore$ Areas of $\Delta AEB$, $\Delta BED$ and $\Delta ABC$ will be in the ratio 1 : 1 : 4.

**1.25.** In triangles *XYZ* and PQR, $\angle X = \angle P$ and $\angle Z = \angle R$. $XY = 4 \times 2$ cm, $YZ = 6 \times 4$ cm, $PR = 2 \times 8$ cm, $QR = 3\times2$ cm. Find the lengths of *PQ* and *XZ*.

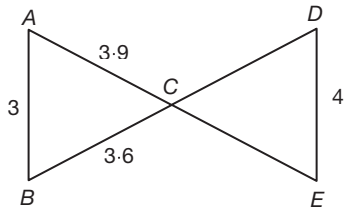
**Sol:** As $\angle X = \angle P$ and $\angle Z = \angle R$, triangles *XYZ* and *PQR* are similar.

$$\therefore \frac{XY}{PQ} = \frac{YZ}{QR} = \frac{XZ}{PR}$$

$$\frac{4\cdot 2}{PQ} = \frac{6\cdot 4}{3\cdot 2} = \frac{XZ}{2\cdot 8}$$

$PQ = 2\times1$ cm and $XZ = 5\times6$ cm

**1.26.**




In the figure above, find CD and CE given $\angle BAC = \angle CDE$.

**Sol:** In triangles ABC and CED, $\angle ACB = \angle DCE$ (vertically opposite angles are equal)

$\angle BAC = \angle CDE$ (given)

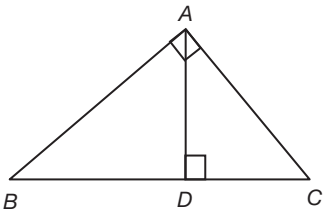
$\therefore$ The Triangles ABC and DEC are similar

$$\frac{BC}{CE} = \frac{AB}{DE} = \frac{AC}{CD}$$

$$\frac{3\cdot 6}{CE} = \frac{3}{4} = \frac{3\cdot 9}{CD}$$

$\therefore$ CE = 4×8 cm and CD = 5×2 cm

**1.27.**




In the figure above,

AC = $4\sqrt{3}$ cm and BD = 8 cm. Find DC.

**Sol:** Let DC = *x* cm

In triangles ADC and BAC, $\angle ADC = \angle BAC = 90°$ (given)

$\angle ACD = \angle ACB$ ($\angle$C is common)

$\therefore$ The triangles BAC and ADC are similar

$$\therefore \frac{AC}{BC} = \frac{AD}{AB} = \frac{DC}{AC}$$

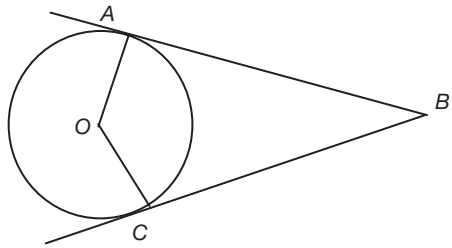
AC2 = (BC) (DC)

$\left(4\sqrt{3}\right)^2 = (x+8)x \Rightarrow$ (12) (4) = (*x* + 8) *x*

$\Rightarrow x^2 + 8\,x - 48 = 0 \Rightarrow x = 4$ or $-8$

As $x > 0$, x = 4.

**1.28.**




In the figure above, BA and BC are tangents to the circle with centre O. AB = 28 cm and OA = 21 cm. Find OB + BC.

**Sol:** $\angle OAB = \angle OCB = 90°$ (AB and BC are tangents)

$\therefore$ $OA^2 + AB^2 = OB^2 = OC^2 + CB^2$

$$OB = \sqrt{21^2 + 28^2} = 35\,\text{cm}$$

AB = BC (tangents to the circle from an external point)

$\therefore$ BC = 28 cm
Hence, OB + BC = 63 cm

**1.29.** The angle subtended by an arc at the centre of a circle is 40°. If the area of the sector formed by the arc and the radii of the circle is $68\frac{4}{9}$ sq.cms, find the radius of the circle. $\left(\text{take } \pi = \frac{22}{7}\right)$

**Sol:** Let the radius of the circle be r cm.

$$\frac{40^\circ}{360^\circ}\left(\frac{22}{7}r^2\right) = \frac{616}{9} \Rightarrow r = 14$$

**1.30.** Two circles of radii 9 cm and 4 cm touch each other externally. Find the length of the direct common tangent drawn to them.

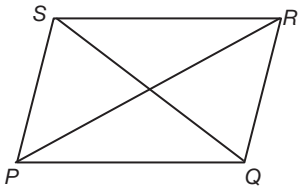
**Sol:** Length of the direct common tangent

$$= \sqrt{(\text{distance between centres})^2 - (\text{difference between radii})^2}$$

$$= \sqrt{(9+4)^2 - (9-4)^2} = 12\,\text{cm}$$

**1.31.** PQRS is a parallelogram, Are the areas of triangles PQR and RQS equal?

**Sol:**




In parallelogram PQRS (any),
$\Delta$ PQR $\equiv \Delta$ RSP and $\Delta$ SPQ $\equiv \Delta$ QRS

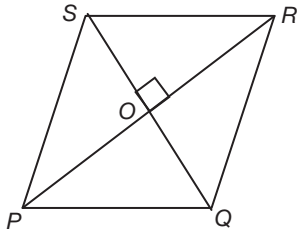
$\Rightarrow$ Area [$\Delta$ PQR] = Area [$\Delta$RSP] = $\frac{1}{2}$ Area [PQRS]

and Area [$\Delta$ SPQ] = Area [$\Delta$QRS] = Area [PQRS]

$\therefore$ Areas of $\Delta$PQR and $\Delta$RQS are equal.

**1.32.** PQRS is a rhombus such that PR = 32 cm and QS = 24 cm. Find the perimeter of PQRS.

**Sol:**




In rhombus, the diagonals bisect each other at right angles.

So, SO = OQ = 12 cm. And

PO = OR = 16 cm.

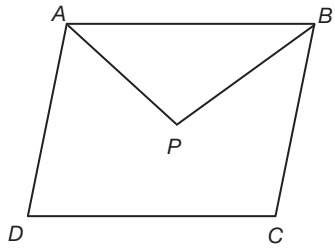
And $\Delta$SOR is right angled at O.

$$\Rightarrow \text{SR} = \sqrt{\left(\frac{32}{2}\right)^2 + \left(\frac{24}{2}\right)^2}$$

$\therefore$ SR = 20 cm
$\therefore$ Perimeter = 80 cm

**1.33.** In parallelogram ABCD, the angle bisectors of $\angle$A and $\angle$B intersect at P. Find $\angle$APB.

**Sol:**




In parallelgram ABCD,

$\angle A + \angle B = 180^\circ$

$$\Rightarrow \frac{\angle A}{2} + \frac{\angle B}{2} = 90^\circ$$

In $\Delta$ABP, $\angle$PAB + $\angle$PBA + $\angle$APB = 180°

$$\frac{\angle A}{2} + \frac{\angle B}{2} + \angle APB = 180^\circ$$

$\angle$APB = 180° − 90° = 90°

**1.34.** In a polygon, the sum of the interior angles is 1980°. Find the number of sides in it.

**Sol:** Let the number of sides be n.
Then, the sum of its interior angles
= 180° $(n - 2)$ = 1980°
$\therefore$ $n = 13$

**1.35.** In a hexagon, one of the interior angles is 100°. If all the other angles are equal, find each of these angles.

**Sol:** In a hexagon the sum of the interior angles
= 180° (6 – 2), i.e., 720°
Each of the remaining angles

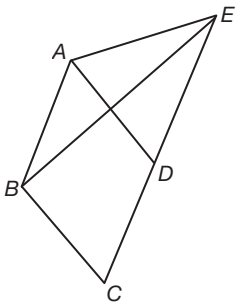
$$= \frac{720^\circ - 100^\circ}{5} = 124^\circ$$

## EXERCISE-1

***Directions for questions 1 to 70*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Find the circumradius of a triangle whose sides are 6 cm, 8 cm and 10 cm. (in cm) _____

**2.**

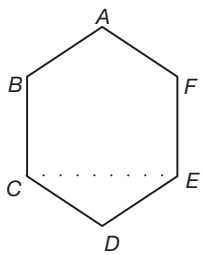



In the figure, $ABCD$ is a rhombus and $ADE$ is an equilateral triangle and $E$ is on $\overline{CD}$ produced. What is the measure of $\angle AEB$?

(A) 40° (B) 30°
(C) 45° (D) 50°

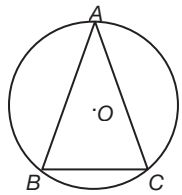
**3.** Among a parallelogram, a rhombus and an isosceles trapezium, how many are necessarily cyclic quadrilaterals?
_____

**4.** In the figure, $ABCDEF$ is a regular hexagon. What is the measure of $\angle CED$ in degrees? _____




**5.** The sides of a triangle are 4 cm, 6 cm and 8 cm. Its circumcentre lies
(A) inside the triangle
(B) on the triangle
(C) outside the triangle
(D) either inside or on the triangle

**6.**




In the above figure, $O$ is the centre of the circle and $OB$ = 6 cm. The perimeter of triangle $BOC$ is 18 cm. Find $\angle BAC$.
(A) 20° (B) 30° (C) 25° (D) 35°

**7.** Find the maximum number of common tangents that can be drawn to two circles which intersect each other.
_____

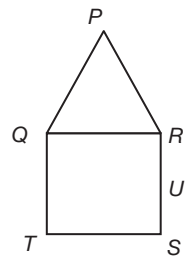
**8.** In triangle $ABC$, $\angle B = 45°$, $\angle C = 45°$. The ratio of the sides opposite to the angles $A$, $B$ and $C$ is

(A) $1:1:\sqrt{2}$ (B) $\sqrt{2}:1:1$

(C) $1:\sqrt{2}:1$ (D) $1:\sqrt{2}:\sqrt{2}$

**9.** In triangle $PQR$, $S$ and $T$ are points on $PQ$ and $PR$, respectively such that $ST$ is parallel to $QR$. $PS$ = 4 cm, $SQ$ = 12 cm and $ST$ = 3 cm. Find $QR$ (in cm).
(A) 9.6 (B) 8.4
(C) 10.8 (D) 12

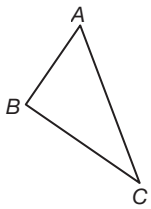
**10.**




In the above figure, triangle $PQR$ is equilateral. $QRST$ is a rectangle. $QU$ bisects $\angle PQT$. Find $\angle QUS$.
(A) 110° (B) 120°
(C) 105° (D) 115°

**11.** In $\Delta ABC$, $AB^2 + AC^2 < BC^2$ and $\angle BAC = x°$, then which of the following is true?
(A) $x < 90°$ (B) $x = 90°$
(C) $x > 90°$ (D) None of these

**12.**




In the above triangle $ABC$, the sides $AB$, $BC$ and $CA$ are 3 cm, $3\sqrt{3}$ cm and 6 cm, respectively. What is the measure of $\angle ABC$ in degrees? _____

**13.** The point of concurrence of angle bisectors of a triangle is called the _____ of the triangle.
(A) incentre (B) circumcentre
(C) centroid (D) orthocentre

**14.** Two chords $AB$ and $CD$ of a circle intersect inside the circle at $P$. $PA \times PB =$
(A) $PC \times PD$ (B) $AB^2$ (C) $CD^2$ (D) $PC \times CD$

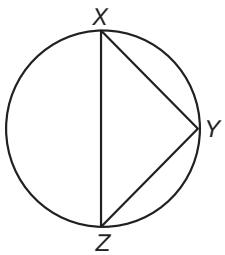
**15.** The midpoints of the sides of a quadrilateral of area 200 sq.cm are joined. What type of quadrilateral is formed and what is its area?
(A) Parallelogram, 100 sq.cm.
(B) Rectangle, 100 sq.cm.
(C) Rectangle, 50 sq.cm.
(D) Parallelogram, 50 sq.cm.

**16.** In triangle $ABC$, $AB = 12$ cm and $BC = 21$ cm. What is the range of the perimeter ($p$) of triangle $ABC$?
(A) 3 cm $< p <$ 21 cm (B) 9 cm $< p <$ 33 cm
(C) 33 cm $< p <$ 66 cm (D) 42 cm $< p <$ 66 cm

**17.** In triangle $PQR$, if $\angle P = 50°$, find $\angle QIR$, where $I$ is the incentre of the triangle. (in degrees) _____

**18.** The point of concurrence of the perpendicular bisectors of the sides of a triangle is called its
(A) centroid. (B) incentre.
(C) orthocentre. (D) circumcentre.

**19.**

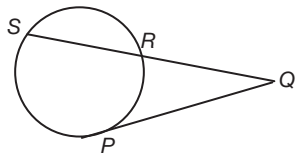



In the figure, $XZ$ is the diameter of the circle. $\angle XZY = 35°$. Find $\angle YXZ$. (in degrees) _____

**20.** One angle of a pentagon is 60°. If all the other angles being equal, find each of the other angles.(in degrees)
_____

**21.** The number of diagonals of a regular decagon is _____.

**22.**



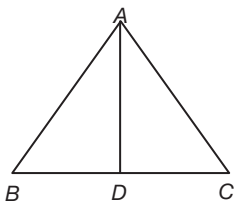

In the above figure, $QP$ is a tangent to the circle at $P$. Which of the following can never be the ratio of the lengths of $QP$ and $QS$?
(A) 2 (B) $\frac{3}{4}$
(C) $\frac{4}{5}$ (D) More than one of the above choices.

**23.** A triangle has its circumcentre on one of its sides. It can be
(A) acute–angled
(B) obtuse–angled
(C) right–angled
(D) acute, obtuse or right-angled

**24.** The sum of the interior angles of a polygon is 1620°. How many sides does the polygon have? _____

**25.** An equilateral triangle has a side of 12 cm. Find the area (in $cm^2$) of the triangle formed by the incentre, centroid and the circumcentre. _____

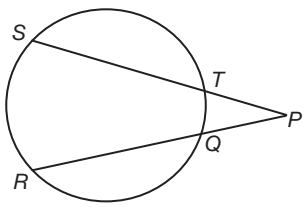
**26.**




In the figure, $AB = 10$ cm, $AC = 12$ cm and $DC = 8$ cm. If $AD$ bisects $\angle BAC$, find $BD$ (in cm).

(A) 5 (B) $5\frac{2}{3}$ (C) $6\frac{1}{3}$ (D) $6\frac{2}{3}$

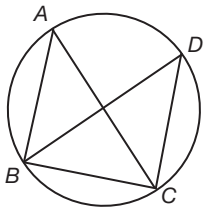
**27.**



In the above figure, $PT = 4$ cm, $TS = 8$ cm and $PQ = 3$ cm. Find $PR$ (in cm).

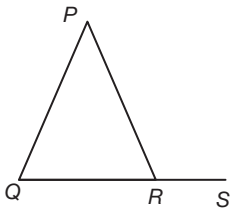
(A) 14 (B) 16 (C) 18 (D) 20

**28.**




In the figure, $\angle BCA = 45°$ and $\angle BDC = 50°$. Find $\angle ABC$. (in degrees) _____

**29.**




In the above triangle, $\angle PRS = 120°$ and $\angle PQR = 2(\angle QPR)$. Find $\angle QPR$.

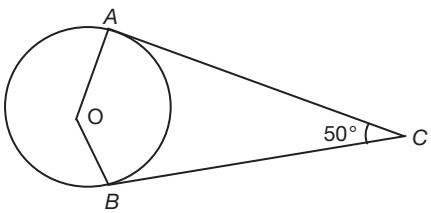
(A) 25° (B) 30° (C) 40° (D) 35°

**30.** In triangle $PQR$, $S$ and $T$ are points on $PQ$ and $PR$, respectively, such that $ST$ is parallel to $QR$. If $PS = 8$ cm, $SQ = 4$ cm and $TR = 6$ cm, find $PT$. (in cm) _____

**31.** In triangle $PQR$, $\angle P - \angle Q = \angle Q - \angle R$. Find $\angle Q$.

(A) 60° (B) 50°
(C) 70° (D) Cannot be determined

**32.**

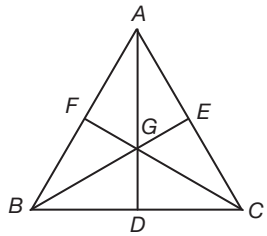



In the figure, $O$ is the centre of the circle. $AC$ and $BC$ are tangents to the circles. If $\angle ACB = 50°$, find $\angle AOB$.

(A) 130° (B) 80°
(C) 100° (D) 90°

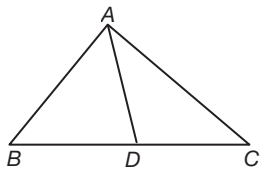
**33.** In parallelogram $ABCD$, $AP$ and $BP$ are the angle bisectors of $\angle DAB$ and $\angle ABC$. Find $\angle APB$. (in degrees) _____

**34.**




In the figure, $AD$, $BE$ and $CF$ are the medians and $G$ is the centroid. If the area of triangle $ABC$ is 24 cm$^2$, find the area of triangle $GFB$. (in cm$^2$) _____

**35.** In the figure, $\angle BAC = 90°$ and $AD$, a median, is 10 cm long. If $AB = 12$ cm, $AC$ (in cm) = _____.




**36.** A regular hexagon has a side of 4 cm. Find its area (in cm$^2$).

(A) $24\sqrt{3}$ (B) $18\sqrt{3}$
(C) $36\sqrt{3}$ (D) None of these

**37.** In a rhombus $ABCD$, $AC = 24$ cm and $BD = 10$ cm. Find the perimeter of the rhombus. (in cm) _____

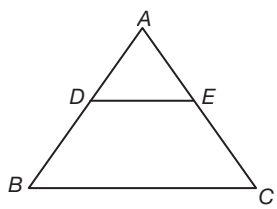
**38.** The radius of a circle is 15 cm. Find the length of a chord which is 9 cm from the centre of the circle. (in cm) _____

**39.** In the above question, find the cirumradius of the triangle (in cm).

(A) $3\sqrt{3}$ (B) $4\sqrt{3}$ (C) $6\sqrt{3}$ (D) $8\sqrt{3}$

**40.** In an isosceles triangle $ABC$, $BC = 40$ cm, $AB = AC = 25$ cm. What is the length of the median drawn from $A$ to $BC$? (in cm) _____

**41.**



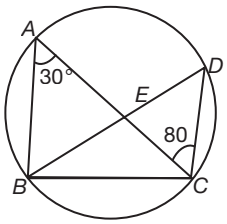
In the given figure, $AD : DB = 1 : 3$ and $DE$ is parallel to $BC$. If the area of the triangle $ABC$ is 432 $cm^2$, what is the area of triangle $ADE$ in $cm^2$? _____

**42.** Find the maximum number of common tangents that can be drawn to two circles which touch each other externally. _____

**43.** In triangle $PQR$, angle bisectors of angles $Q$ and R meet at point $I$. If $\angle P = 80°$, find $\angle QIR$.
(A) 130° (B) 120° (C) 140° (D) 135°

**44.** $PQRS$ is a cyclic quadrilateral $\angle P = 50°$ and $\angle Q = 70°$. Find the respective measures of $\angle R$ and $\angle S$.
(A) 50°, 70° (B) 70°, 50°
(C) 110°, 130° (D) 130°, 110°

**45.** In the following figure, $ABC$ and $BCD$ are two triangles inscribed in the circle. $\angle BAC = 30°$ and $\angle ACD = 80°$. What is the measure of $\angle AEB$ in degrees? _____



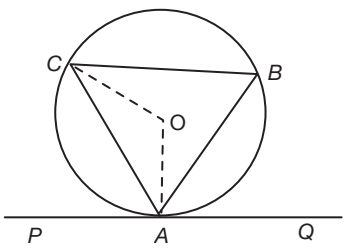

**46.** $P$ and $Q$ are points on the circumference of a circle, with centre $O$. $R$ is a point on the major arc $PQ$. $\angle POQ = 100°$. Find $\angle PRQ$.
(A) 40° (B) 45° (C) 50° (D) 55°

**47.** Two circles of radii 2 cm and 8 cm touch externally. Find the length of their direct common tangent. (in cm) _____

**48.** An equilateral triangle has a side of 18 cm. Find the inradius of the triangle (in cm).

(A) $3\sqrt{3}$ (B) $4\sqrt{3}$ (C) $6\sqrt{3}$ (D) $8\sqrt{3}$

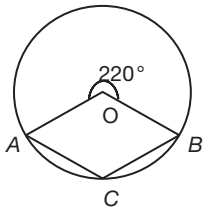
**49.**




In the given figure, $PAQ$ is a tangent to the circle with centre $O$. If $\angle PAC = 80°$, find $\angle AOC$
(A) 160° (B) 120° (C) 130° (D) 140°

**50.** A right circular cone is cut parallel to its base at one-third of its height from the top. Find the ratio of the volume of the smaller piece to that of the original cone.
(A) 1 : 26 (B) 1 : 27 (C) 2 : 25 (D) 1 : 8

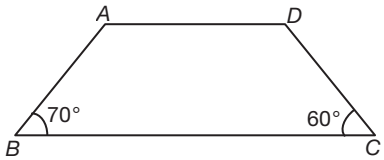
**51.**




In the figure given, $O$ is the centre of the circle. What is the measure of $\angle ACB$ in degrees? _____

**52.** In an equilateral triangle, the orthocentre divides each median into 2 parts. The larger part and the smaller part are in the ratio of
(A) 3 : 2 (B) 3 : 1
(C) 2 : 1 (D) None of these

**53.** In the figure, $ABCD$ is a trapezium. If $AD$ is parallel to $BC$, which of the following is true?

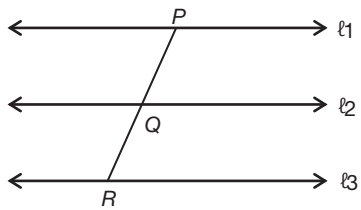



(A) $\angle A + \angle C > \angle B + \angle D$ (B) $\angle A + \angle C = \angle B + \angle D$
(C) $\angle A + \angle C < \angle B + \angle D$ (D) None of the above

**54.** $A$ and $B$ leave a point at the same time. $A$ travels South at $C$ speed of 16 km/hr and $B$ towards East at a speed of 12 km/hr. What is the distance between $A$ and $B$ after 4 hours? (in km) _____

**55.** $XY$ is the diameter of a circle. $Z$ is a point on the circumference of the circle. $XY = 26$ cm and $XZ = 24$ cm. Find $YZ$. (in cm) _____

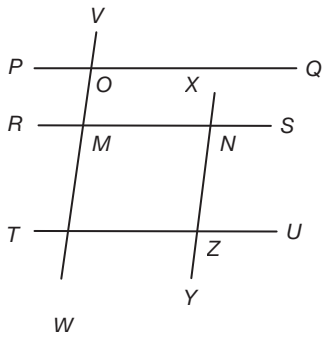
**56.** In the given figure, line $\ell_2$ is parallel to line $\ell_1$ and $\ell_2$ is equidistant from them. If line segment $PR$ measures 12 cm, find the length of $PQ$. (in cm) _____



**57.** Triangle $PQR$ is right angled at $Q$. $QS$ is an altitude to $PR$. Find $QS$ if $PS = 32$ cm and $SR = 18$ cm. (in cm) _____

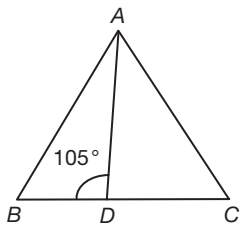
**58.** How many degrees are there in an angle, which is equal to $1/5^{th}$ of its supplement? _____

**59.**



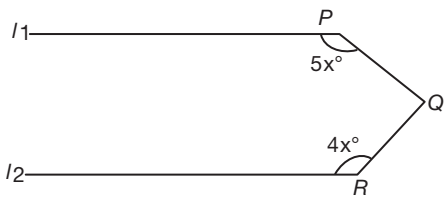

In the figure, $\angle XZT = 130°$. $PQ$, $RS$ and $TU$ are parallel. $VW$ and $XY$ are parallel. Find $\angle VOP$.
(A) 125° (B) 130° (C) 120° (D) 135°

**60.** In the figure, $ABC$ is an equilateral triangle and $\angle ADB = 105°$. What is the measure of $\angle DAC$ in degrees? _____



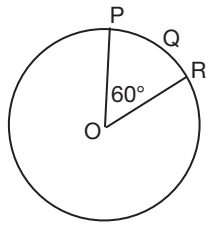

**61.** In a triangle, the smallest altitude is 18 cm. The inradius is always
(A) > 9 cm (B) 9 cm
(C) < 9 cm (D) Either (A) or (B)

**62.**




In the figure, $\ell_1$ and $\ell_2$ are parallel lines and $\angle PQR = 90°$. What is $x$?
(A) 10° (B) 20°
(C) 30° (D) 40°

**63.**

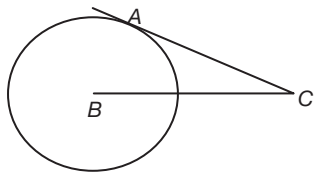



In the figure, $O$ is the centre of the circle and $\angle POR = 60°$. Find $\angle POR$. (in degrees) _____

**64.** How many degrees are there in an angle, which equals $1/3^{rd}$ of its complement? _____

**65.** $ABCD$ is a trapezium. $AB$ is parallel to $CD$. $AB = 12$ cm and $CD = 24$ cm. Find the length of the line $EF$ which is parallel to $AB$ and $CD$ and divides the distance between them in the ratio 2 : 3. (in cm) _____

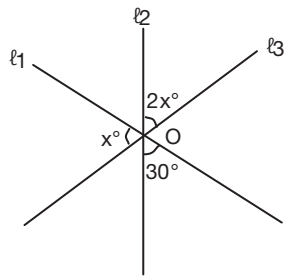
**66.**




In the circle shown with centre $B$, $AC$ is the tangent 24 cm long. If the radius of the circle is 18 cm, what is the length of $BC$ (in cm)? _____

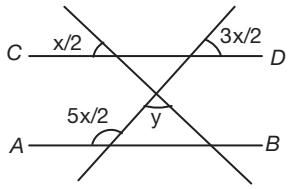
**67.** Find the maximum number of common tangents that can be drawn to two non-intersecting and non-enclosing circles.
(A) 1 (B) 2 (C) 3 (D) 4

**68.** In the figure, $\ell_1$, $\ell_2$ and $\ell_3$ are intersecting at $O$. What is the measure of $x$ in degrees? _____




**69.** The sides of a triangle are 12 cm, 16 cm and 20 cm. Find the sum of the distances from the orthocentre to the vertices of the triangle. (in cm) _____

**70.**




In the above figure, $AB$ and $CD$ are parallel lines. What is the measure of angle $y$?

(A) 30° (B) 45° (C) 60° (D) 90°

## EXERCISE-2

***Directions for questions 1 to 70*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Find the complementary and supplementary angles to the following angles.

| | | Complementary angle | Supplementary angle |
|---|---|---|---|
| (i) | 36° | | |
| (ii) | 72° | | |

**2.** $ABCD$ is a convex quadrilateral. Based on the additional information given in each sub-question, identify the type of quadrilateral.

(i) $AB = CD$, $AD = BC$ _____.

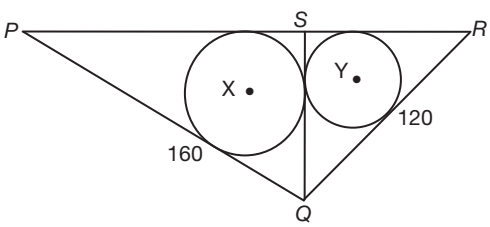
(ii) $AB$ is parallel to $CD$, $AD = BC$ _____.

(iii) $AD$ is perpendicular to $AB$, $AB = CD$ and $AB$ is parallel to $CD$ _____.

(iv) $AC$ is perpendicular to $BD$, $AC = BD$, $AC$ and $B$ $D$ bisect each other _____.

**3.** Points $D$ and $E$ lie on the sides $AB$ and $AC$ of triangle $ABC$ such that $AD = 2DB$, $AE = 2EC$ and $BC = 12$ cm. Find $DE$. (in cm) _____

**4.**



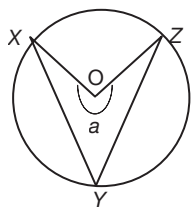

In the figure, triangle $PQR$ is right-angled at $Q$. $QS$ is the altitude to side $PR$. Circles with centres $X$ and $Y$ are inscribed in $\Delta PQS$ and $\Delta RQS$, respectively. Find $XY^2$. _____

**5.** A quadrilateral $ABCD$ is inscribed in a circle centred at $O$. A tangent, drawn to the circle from an external point $P$, touches the circle at $A$. If $\angle CAP = 120°$, find $\angle ADC$. (in degrees) _____ or _____

**6.** Lines $L_1$ and $L_2$ intersect at a point $P$. The four angles formed are $A$, $B$, $C$ and $D$ in the clockwise direction. What will be the type/measure of angles $B$, $C$ and $D$ if angle $A$ is

| | | B | C | D |
|---|---|---|---|---|
| (i) | acute | | | |
| (ii) | 70° | | | |
| (iii) | 89° | | | |
| (iv) | obtuse | | | |

**7.** In the figure below, $O$ is the centre of the circle and $\angle XYZ = 45°$. Find $a$. _____ degrees



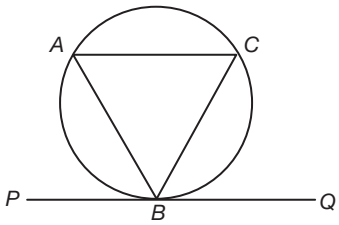

**8.** Two parallel chords of a circle of radius 25 cm have lengths of 30 cm and 48 cm. Find the distance between the two chords (in cm).

(A) 6.5 or 13.5 (B) 13 or 27

(C) 19.5 or 40.5 (D) None of these

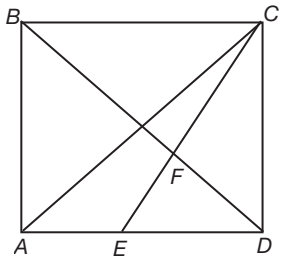
**9.** G is the centroid of triangle $ABC$. $AB = 18$ cm, $BC = 24$ cm and $AC = 30$ cm. If $D$ is the mid-point of $AC$, then find the length of $GD$. (in cm) _____

**10.** In the given figure, $PQ$ is the tangent at the point $B$ and $\angle ABQ = 130°$. Find $\angle ACB$.

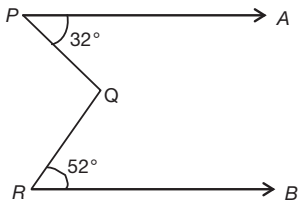



(A) 40° (B) 50°
(C) 60° (D) 70°

**11.** In the figure below, *ABCD* is a square. *E* is a point on *AD* such that ∠*CED* is four times ∠*ACE*. Find ∠*CFB*. _____ degrees

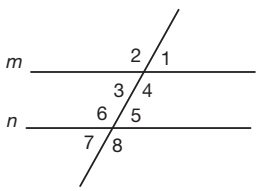



**12.**

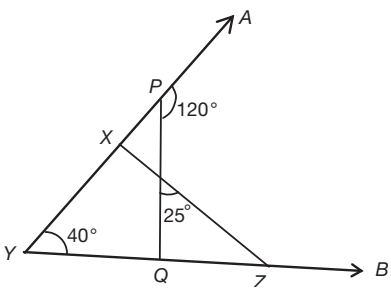



In the given figure, *PA* is parallel to *RB*. Find ∠*PQR*.
(A) 90° (B) 76° (C) 84° (D) 80°

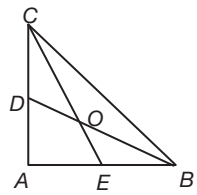
**13.** In the given figure, lines *m* and *n* are parallel and twice the measure of angle 8 is equal to thrice the measure of angle 1. Find the measure of angle 4. _____ degrees



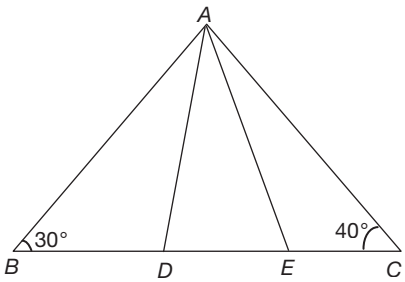

**14.** In the following figure, find ∠*BZX*. (in degrees) _____




**15.** In the figure below, *ABC* is an isosceles triangle, in which *AB* = *AC* and angle *ABC* = 50°. *BD* is the angle bisector of angle *ABC* and *CE* is the angle bisector of angle *ACB*. Find angle *COD*. (in degrees) _____

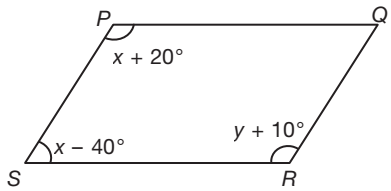



**16.**




In the figure, *AD* = *BD* and *AE* = *CE*. Find ∠*DAE*.
(A) 50° (B) 20° (C) 30° (D) 40°

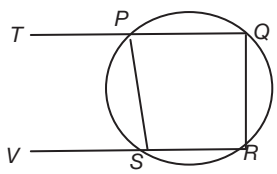
**17.** In the figure given below, *PQRS* is a parallelogram. Find *y* .(in degrees) _____




**18.** A scalene triangle has integral sides and a perimeter of 18 cm. How many such triangles exist? _____

**19.** In a triangle *PQR*, *PQ* = 19×5 cm and *PR* = 14 cm. *PS*, the altitude drawn to *QR* is 6 cm. What is the length (in cm) of the circumradius of triangle *PQR*?
(A) 21 × 25 (B) 22 × 75 (C) 21 × 75 (D) 23 × 25

**20.**



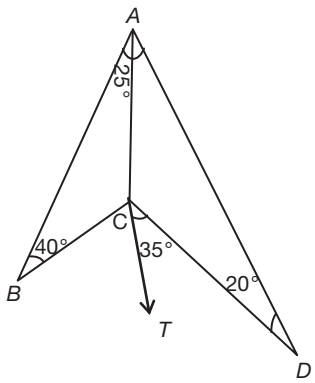

In the above figure (not drawn to scale), *TQ* is parallel to *VR* and 2(∠*TPS*) = 3(∠*PSV*). Find ∠*QRS*. _____ degrees

**21.** The perimeter of a triangular field is 80 m. If two of its sides are 16 m and 30 m, then find the approximate length of the altitude to the longest side.
(A) 34 m (B) 14.1 m (C) 14.9 m (D) 15.9 m

**22.** T is an obtuse angled triangle. Two of its sides are 7 cm and 13 cm. How many possibilities exist for the sides of $T$ such that the third side has an integral measure (in cm)?
(A) 8 (B) 7 (C) 16 (D) 9

**23.** G is the centroid of triangle $ABC$. $AB = 7$ cm, $BC = 24$ cm and $AC = 25$ cm. Find the ratio of the areas of triangle $ABG$ and the triangle $ABC$. $\frac{}{3}$

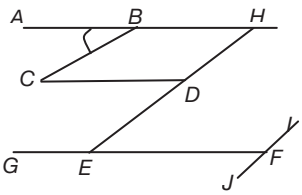
**24.** In the figure $AC = CD$. Find $\angle BCT$. (in degrees)_____




**25.** In the given figure, $O$ is the centre of the circle.$PT$ is a tangent to the circle and $\angle PTA = 55°$. Find $\angle AOT$. (in degrees) _____

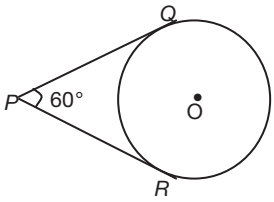
**26.** What is the length of the direct common tangent that is drawn to two circles whose centers are 8 cm apart and whose diameters are10 cm and 6 cm? _____ $\sqrt{15}$ cm

**27.**

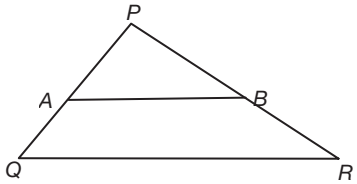



In the given figure $AH$ is parallel to $CD$, $CD$ is parallel to $GF$, $BC$ is parallel to $HE$ as well as $IJ$ and angle $ABC = 30°$. Find the sum of the angles $EFJ$ and $DHB$ _____ degrees.

**28.** In the figure below, if $PQ = 24$ cm and $O$ is the centre of the circle, find the radius of the circle. _____ $\sqrt{3}$cm



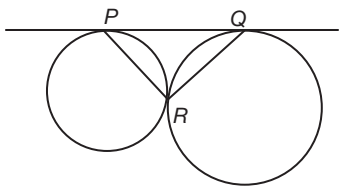

**29.** In the figure given below, $AB$ is parallel to $QR$ and the ratio of the areas of triangles $PAB$ and $PQR$ is 1 : 4. If the perimeter of $PQR$ is 24 cm, what is the perimeter of $PAB$ (in cm)? _____




**30.** The number of diagonals in a regular polygon is 14. Find the sum of the interior angles of the polygon. (in degrees) _____

**31.** In triangle $PQR$, the angle bisector of angle $P$ meets $QR$ at $S$. $PQ = 8$ cm, $QR = 8.5$ cm and $PR = 9$ cm. Find $QS$. (in cm) _____

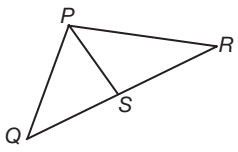
**32.** In the following figure, $R$ is the point of contact of two circles $P$ and $Q$ are the points of contact of the common tangent with the two circles. Find $\angle PRQ$.




(A) 120° (B) 150°
(C) 90° (D) Cannot be determined

**33.** $B$ is a point on a circle with centre $O$ and radius 9 cm. A is a point on the tangent to the circle at $B$. The area of the triangle $ABO$ is 54 sq.cm. Find $AO$. _____ (cm)

**34.** In the given figure, $PS$ is the median drawn to the side $QR$ of triangle $PQR$. If $PQ = 11$ cm, $PR = 13$ cm and $PS = 8$ cm, what is the length of $QS$?



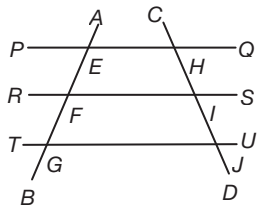
(A) $4\sqrt{14}$ cm (B) 9 cm

(C) $3\sqrt{15}$ cm (D) 10 cm

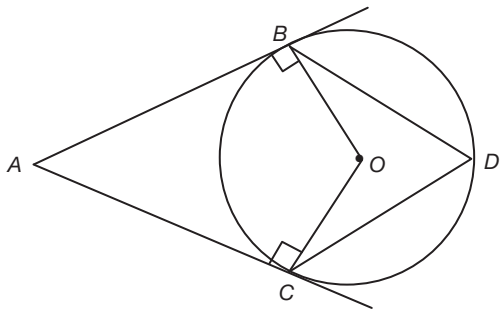
**35.** A set of parallel lines *PQ*, *RS*, and *TU* are cut by transversals *AB* and *CD* as shown in the given figure. Transversal *XY* is drawn perpendicular to *PQ*, which cuts *PQ*, *RS* and *TU* at *M*, *N* and *O*, respectively.

It is known that *EF* = 6 and *FG* = 4. Find *MN* : *MO*.

$\frac{3}{\quad}$

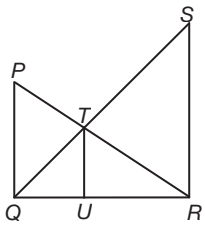



**36.** In the given figure, *AB* and *AC* are two tangents to the circle, whose centre is *O*. If $\angle BAC = 50°$, find $\angle BDC$.




(A) 55° (B) 60° (C) 75° (D) 65°

**37.**

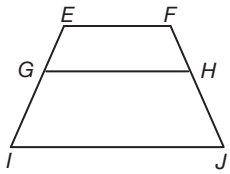



In the figure above, angle *PQR* = angle *QRS* = angle *TUR* = 90°, *PQ* = 8 m, *SR* = 12 m and *UR* = 6 m. Find *TU*. (in m) _____

**38.** In trapezium *ABCD*, *AB* and *CD* are the parallel sides. The diagonals *AC* and *BD* intersect at *X*. The ratio of the area of Δ *AXB* to the area of Δ *CXD* is 25 : 49. Find the ratio of the area of Δ *AXD* and the area of *ABCD*.

(A) $\frac{35}{144}$ (B) $\frac{5}{7}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$

**39.**




In the figure above, *EF* is parallel to *GH* which is parallel to *IJ*. If *EG* : *GI* = 1 : 3, *EF* = 12 cm and *IJ* = 16 cm, find *GH*. _____ cm

**40.** The lengths of the sides of different triangles are given below. Identify the type of triangle in each case.

(i) 5, 12, 13 _____.

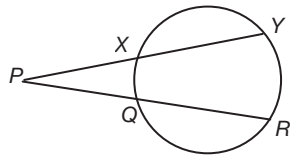
(ii) 7, 8, 13 _____.

(iii) $5, 5, 5\sqrt{2}$ _____.

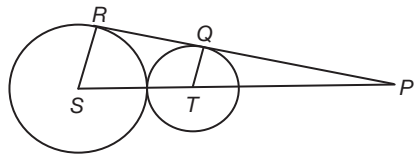
(iv) 6, 8, 7 _____.

(v) 4, 11, 15 _____.

**41.** In the figure below, (not drawn to scale), *PXY* and *PQR* are secants to the circle. If *PX* = 2*PQ*, *XY* = 12 cm and *PQ* = 4 cm, then find *QR*. (in cm) _____




**42.** In the figure given below radii of two circles with centres *S* and *T*, respectively are in the ratio 5 : 3 and *PT* = 12 cm. If *R* and *Q* are points of contacts of the tangent drawn from *P* to the circles with centres *S* and *T*, respectively, then find the areas of the quadrilateral *RQTS* and the triangle *PQT*.




(A) $16\sqrt{15}$ cm$^2$, $9\sqrt{15}$ cm$^2$

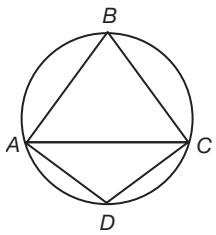
(B) $8\sqrt{15}$ cm$^2$, $4.5\sqrt{15}$ cm$^2$

(C) $24\sqrt{15}$ cm$^2$, $13.5\sqrt{15}$ cm$^2$

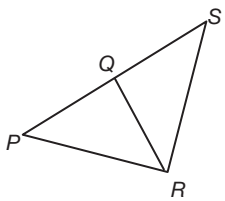
(D) None of these

**43.** The altitude of an isosceles triangle drawn to its base is $4\sqrt{5}$. If each of its equal sides measure 12, find its perimeter. _____

**44.** In the figure below, $ABC$ is an equilateral triangle and $AD = DC$. Find $\angle ACD$. _____ degrees



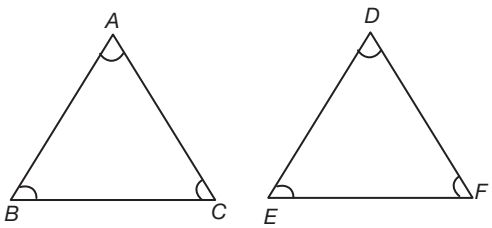

**45.** In the figure below, $PS = 32$ cm, $PQ = 18$ cm, $SR = 25$ cm and $\angle QRP = \angle QSR$.



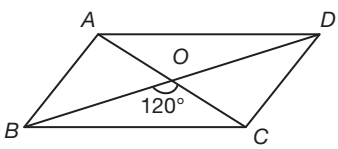

Find the ratio of the perimeters of the triangle $PQR$ and $PRS$.

(A) 3 : 2 (B) 3 : 1 (C) 2 : 1 (D) 3 : 4

**46.** Given $ABC$ and $DEF$ are two triangles such that $AC = 10$, $BC = 12$, and $DE = 3$. Find the length of $EF$. Find the ratio of the areas of the triangles $ABC$ and $DEF$. _____




**47.**




In the given parallelogram, $AO$ is 5 units, $BO$ is 7 units and angle $BOC = 120°$. The area of quadrilateral $ABCD$ is

(A) 35 sq.units (B) 70 sq.units

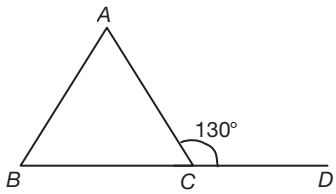
(C) $35\sqrt{3}$ sq.units (D) $70\sqrt{3}$ sq.units.

**48.** $ABCD$ is a parallelogram of perimeter 24 cm. The bisectors of angle $C$ and angle $D$ meet on $AB$ at $E$.

**(i)** Find $AE : BC$. _____

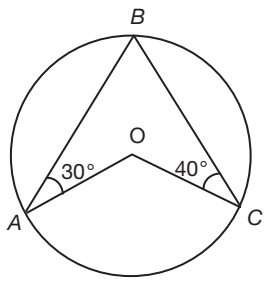
**(ii)** Find $BC$ (in cm). _____

**49.**




In the given figure, $ABC$ is an isosceles triangle and angle $ACD = 130°$. Find the minimum measure of the sum of the angles $BAC$ and $ACB$. (in degrees) _____

**50.** In the given figure, $O$ is the centre of the circle. Find the measure of $\angle AOC$.




(A) 160° (B) 170° (C) 150° (D) 140°

**51.** The sides of triangle $ABC$ are a cm, $b$ cm, and $c$ cm. The sides of triangle $DEF$ are $d$ cm, $e$ cm and $f$ cm.

$a^2 + b^2 + c^2 = 50$

$d^2 + e^2 + f^2 = 50$

$ad + be + cf = 50$

What can be said about the two triangles?

(A) They have the same perimeter.

(B) They have the same area.

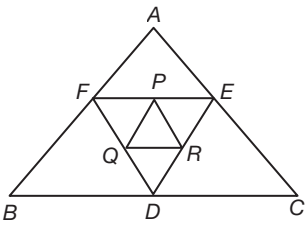
(C) Both (A) and (B)

(D) Neither (A) nor (B)

**52.** In triangle $PQR$, $PQ = 14$, $QR = 48$ and $PR = 50$. Find the length of the altitude drawn to $PR$. _____

**53.** Two parallel chords of lengths 40 cm and 30 cm are on the same side of the centre of a circle. If the radius of the circle is 25 cm, what is the distance between the chords? (in cm) _____.

**54.**



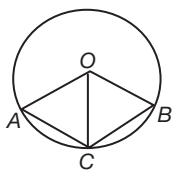

In the above diagram if the area of triangle $ABC$ is 64 $cm^2$, then what is the area of the triangle $PQR$, where $D$, $E$ and $F$ are the midpoints of the sides of $\Delta ABC$ and $P$, $Q$ and $R$ are the midpoints of the sides of $\Delta DEF$ (in $cm^2$)? _____

**55.** Find the number of diagonals in a polygon with nine sides. _____

**56.** In triangle $PQR$, $PA$, $QB$ and $RC$ are the medians. The centroid is at point $G$. What is the ratio of the area of quadrilateral $CQAG$ and the area of triangle $PQR$?
(A) 1 : 4 (B) 1 : 3 (C) 1 : 2 (D) 1 : 6

**57.** In a quadrilateral $ABCD$, angle $CAB$ = angle $DAC$ and angle $BCA$ = angle $ACD$. The area of $ABCD$ is 72 sq.cm. Find the area of the triangle $ACD$. (sq. cm). _____

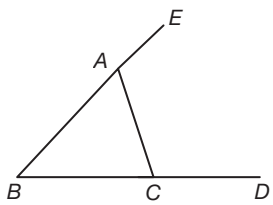
**58.** In the given figure, $O$ is the centre of the circle and $\angle OAC = 60°$ and $\angle OBC = 70°$. Find $\angle AOB$. (in degrees) _____



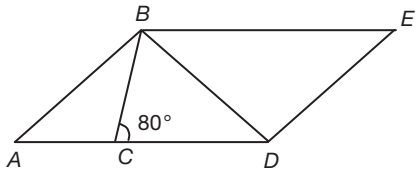

**59.** The longest distance between any two points on a rhombus is equal to twice the length of the shorter diagonal, which is 2 cm long. Find the length of the side of the rhombus.(cm) $\sqrt{\phantom{x}}$_____

**60.** If each angle of a regular polygon of $n$ sides is 144°, find the value of $n$. _____

**61.** In the given figure, the sum of angle $ACD$ andangle $EAC$ is 260°. Angles $ABC$ and $ACB$ are in the ratio 4 : 3. Find angle $ABC$ and angle $CAB$ in the triangle. (in degrees) _____, _____



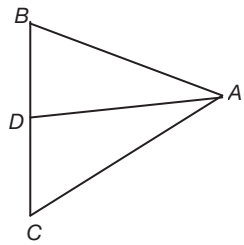

**62.**




In the figure above, $ABED$ is a parallelogram and $AC = BC$. $BD$ is the bisector of angle $D$. Find $\angle DBC$. _____ degrees

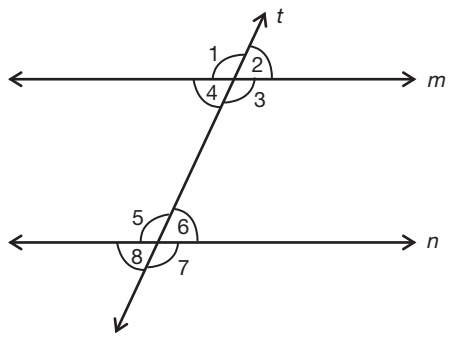
**63.** The total number of diagonals in a regular polygon is 20. What is the interior angle of the polygon?
(A) 90° (B) 115° (C) 120° (D) 135°

**64.** In the given figure (not necessarily to scale), $\angle BDA = 80°$, $\angle DAC = 40°$ and $ABC$ is an isosceles triangle. Which of the following can be the measure of $\angle BAD$ ?




(A) 60° (B) 80° (C) 40° (D) 70°

**65.** (i) In the figure given below, lines $m$ and $n$ are parallel and $t$ is the transversal.



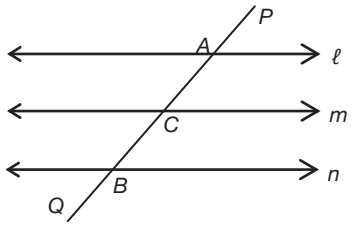

$\angle 8$ is less than $\angle 3$ by 90°. Consider the following statements:

I. $\angle 1 = \angle 3 = \angle 5 = \angle 7 = 135°$

II. $\angle 2 = \angle 4 = \angle 6 = \angle 8 = 45°$

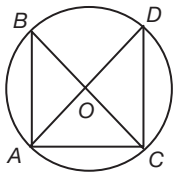
Which of the following statements is/are true?
(A) Only I (B) Only II
(C) Both I and II (D) Neither I nor II

(ii) In the figure given below, $\ell$, $m$ and $n$ are parallel lines. $PQ$ is a transversal intersecting the three lines. The perpendicular distances between ($\ell$ and $m$) and ($m$ and $n$) are in the ratio 3 : 4. If $AC = 12$ cm, what is the length of $AB$ (in cm)?




_____.

**66.** In the figure below, $O$ is the centre of the circle and $\angle ABC = 50°$. Find the measures of $\angle AOC$ and $\angle ADC$. (in degrees) _____, _____




**67.** Bisectors of angle $B$ and angle $C$ in triangle $ABC$ meet at $O$. If angle $BOC = 115°$, find angle $A$. _____ degrees

**68.** $P$ is a regular polygon. Fill the following table.

| Number of sides of $P$ | Name of the polygon | Sum of interior angles ($S$) | Each interior angle ($I$) | Each exterior angle ($E$) |
|---|---|---|---|---|
| 5 | | | | |
| 9 | | | | |
| 12 | | | | |

**69.** It is known that the measure of all the angles of a triangle (in degrees) are integers. If one of the angles of the triangle is 35°, then how many combinations of the angles of the triangle are possible if the triangle is

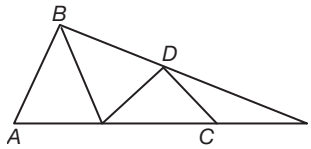
(i) obtuse-angled?

(A) 54 (B) 53 (C) 51 (D) 52

(ii) scalene?

(A) 70 (B) 72 (C) 73 (D) 71

**70.**




In the above figure, $AB = BC = CD = DE = EF$. If $\angle ABC = \angle DFE$, find $\angle DFE$

(A) 20° (B) 45° (C) 50° (D) 60°

## EXERCISE-3

***Directions for questions 1 to 70:*** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** A chord $AB$ in a circle divides the radius drawnperpendicular to it in the ratio 5 : 2. If thediameter of the circle is 14 cm, find the length of the chord given that it is shorter than 10 cm. (in cm) _____ $\sqrt{6}$

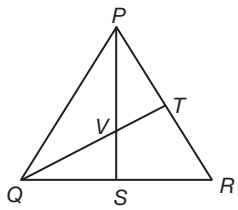
**2.** In $\Delta ABC$, $AB = AC$. $P$ and $Q$ are points on $AB$ and $AC$, respectively such that $PQ$ is parallel to $BC$. $S$ and $R$ are the feet of the perpendiculars from $P$ and $Q$, respectively onto $BC$. If $AP : AB = r : 1$, the ratio of the areas of $APQ$ and $PQRS$ is

(A) $\frac{r}{1-r}$ (B) $\frac{r}{2(1-r)}$

(C) $\frac{r}{3(1-r)}$ (D) $\frac{1-r}{r}$

**3.** I is a point inside the triangle $ABC$, the bisectors of $\angle BIC$, $\angle AIC$ and $\angle AIB$ meet the sides $BC$, $CA$ and $AB$ in points $E$, $F$ and $D$, respectively. The lengths (in cm) of $AD$, $DB$, $AF$, $FC$ and $BE$ are 3, 5, 4, 4, 6, respectively. What is the length of $CE$? _____

**4.** In the figure below, $QS : SR = PV : VS = 4 : 3$, $PT = 8$ cm. Find $PR$. (in cm) _____



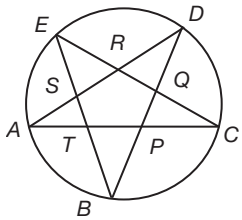

**5.** If $x$ cm and $y$ cm are the length and breadth of a rectangle inscribed in an equilateral triangle of side 6 cm, then which of the following is the value of $y$?

(A) $\frac{\sqrt{3}}{2}(6-x)$ (B) $\frac{\sqrt{3}}{2}(6+x)$

(C) $\frac{\sqrt{3}}{2}(3+x)$ (D) $\frac{\sqrt{3}}{2}(3-x)$

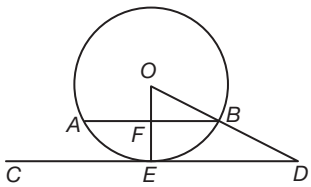
**6.** A point $X$ is chosen inside the triangle $ABC$ such that it is equidistant from the three vertices of the triangle. If $\angle XAB = 25°$ and $\angle BXC = 120°$, then which of the following can be concluded regarding the sides of the triangle?
(A) $AB > AC > BC$ (B) $AB > BC > AC$
(C) $BC > AC > AB$ (D) $AC > BC > AB$

**7.**




In the figure above, $PQRST$ is a regular pentagon. Which of the following is correct?
(A) $\angle RST < \angle AST$ (B) $\angle SAT = \angle AST$
(C) $\angle BPC < \angle RQP$ (D) $\angle SAT = \angle QCP$

**8.** In the figure, $O$ is the center of the circle and $CD$ is a tangent to the circle at $E$. $OE$ bisects the chord $AB$ at $F$. $AB = 16$ cm and $FE = 2$ cm. Find the length of $DE$ (in cm).




(A) 108/15 (B) 118/15
(C) 126/15 (D) 136/15

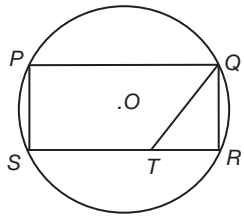
**9.** On side $AB$ of rectangle $ABCD$, $P$ is a point such that $\angle APD : \angle DPC : \angle CPB = 2 : 5 : 5$. Find $AB : BC$.

(A) $\sqrt{3}$ (B) $2\sqrt{3} - 2$

(C) 2 (D) Cannot be determined

**10.** The inradius and the circumradius of a right–angled triangle are 9 and 37.5, respectively. Find the perimeter of the triangle.
(A) 144 (B) 168
(C) 192 (D) Cannot be determined

**11.** In the figure below (not to scale), $PQRS$ is a rectangle inscribed in a circle with centre $O$. $PQ > PS$. Areas of the rectangle $PQRS$ and the circle are in the ratio $2\sqrt{5} : 3\pi$ and $\angle OQP = \angle TQR$. Find $QR : RT$.




(A) $1 : \sqrt{5}$ (B) $2 : \sqrt{5}$ (C) $\sqrt{5} : 1$ (D) $\sqrt{5} : 2$

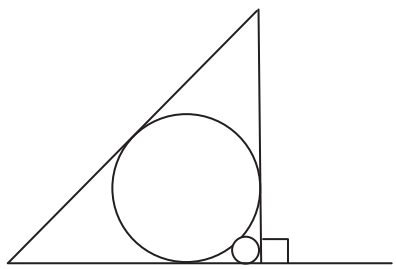
**12.** An equilateral triangle $TPS$ is drawn inside square $PQRS$. Find $\angle QTR$ (in degrees). _____

**13.** A square of the longest possible side is drawn inside the triangle $ABC$ with one side of the square lying on side $BC$. If $AB = 13$, $BC = 21$ and $AC = 20$, then find the side of the square.

(A) $\frac{84}{11}$ (B) 7 (C) $\frac{28}{3}$ (D) $\frac{31}{4}$

**14.** The length of the tangent drawn to a circle from a point $P$ is 10 and the maximum and minimum distances of the point $P$ from the circle are 'a' and 5, respectively. Find the distance of the point $P$ from the centre of the circle.
(A) 12.5 (B) 12
(C) 13.5 (D) None of these

**15.** A circle of radius 4 cm is inscribed in a right triangle. Another smaller circle is drawn, touching the bigger circle as shown in the figure. Find its radius (in cm).



(A) $4(5 - 2\sqrt{2})$ (B) $4(7 - 2\sqrt{2})$

(C) $4(9 - 2\sqrt{2})$ (D) $4(3 - 2\sqrt{2})$

**16.** In a right-angled triangle, the area and the perimeter are equal in magnitude. If its inradius is 20% of its circumradius, then the area of the triangle is

(A) 48 (B) 44
(C) 96 (D) Cannot be determined

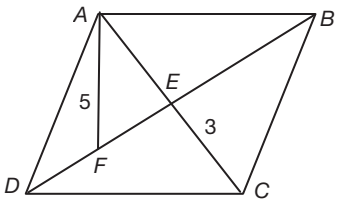
**17.** Sindhu draws a rhombus *ABCD*. She joins the midpoints of the sides *AB*, *BC*, *CD* and *DA* to form a quadrilateral *EFGH*. Then joins the midpoints of the sides *EF*, *FG*, *GH* and *HE* to form a quadrilateral *IJKL* and so on. She continues joining the midpoints of the quadrilaterals formed and names them in the alphabetical order. What is the nature of the quadrilateral *UVWX*?
(A) rhombus (B) rectangle
(C) square (D) parallelogram

**18.** In a triangle *ABC*, $\angle A = \frac{1}{2}\angle B$. If the sides opposite $\angle A$, $\angle B$ and $\angle C$ are denoted by *a*, *b* and *c*, respectively, then which of the following relations is true?
(A) $b^2 - c^2 = a\,b$ (B) $b^2 - a^2 = a\,c$
(C) $a^2 - c^2 = b\,c$ (D) $a^2 - b^2 = b\,c$

**19.** *PQR* is a triangle. $QR = 4$ cm, $PR = 2\sqrt{5}$ cm. If the medians *PS* and *QT* of the triangle are perpendicular to each other, then the area of the triangle is (in sq. cm)

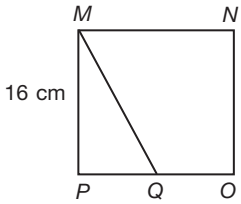
(A) $5\sqrt{6}$ (B) $6\sqrt{5}$ (C) $7\frac{\sqrt{7}}{5}$ (D) $\frac{8\sqrt{11}}{5}$

**20.** Find the length of side *AB* of the rhombus *ABCD* if $AF = 5$ cm; $CE = 3$ cm, $BE = 2EF$.




(A) $\sqrt{52}$ cm (B) $\sqrt{45}$ cm
(C) $\sqrt{73}$ cm (D) None of these

***Directions for questions 21 and 22*:** These questions are based on the information given below.




*MNOP* is a square of side 16 cm. *Q* is the midpoint of *OP*. *R*, *S* and *T* are any three points, on or inside the triangle *PQM*. *U*, *V* and *W* are any three points, on or inside the trapezium *MNOQ*. None of the distances *RU*, *RV*, *RW*, *SU*, *SV*, *SW*, *TU*, *TV* and *TW* is less than 20 cm.

**21.** Which of the following statements is/are necessarily true?
(A) In the figure, the pair of points *T* and *U* is not the closest pair of points.
(B) $ST > TU$.
(C) The shortest distance between any two of the six points *R*, *S*, *T*, *U*, *V*, *W* is either *RS* or *ST* or *RT*.
(D) More than one of the above.

**22.** The largest possible value of *UV* (in cm) is

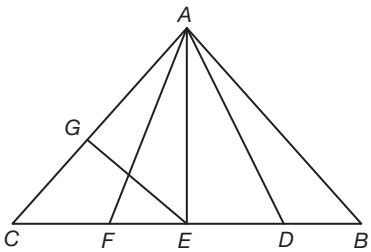
(A) $5\sqrt{2}$ (B) $3\sqrt{2}$ (C) $4\sqrt{2}$ (D) $6\sqrt{2}$

**23.** A circle with centre *O* passes through three points X, *Y*, *Z*. The area of quadrilateral *OXYZ* is $18\sqrt{3}$ cm$^2$ and each of the vertices is equidistant from its adjacent vertices. What is the radius of the circle in cm? _____

**24.** A triangle has sides of lengths *a*, *b*, *c* where $a \le b \le c$ and *a*, *b*, *c* are integers. The perimeter of the triangle is 30. How many such triangles are possible? _____

**25.** *ABC* is a triangle *D* and *E* are points on *AB* and *AC*, respectively. *F* is a point on *BC*. *DF* is parallel to *AC* and *EF* is parallel to *AB*. The areas of the triangles *EFC* and *ABC* are 64 and 400, respectively. Find the area of the quadrilateral *ADFE*. _____

**26.**



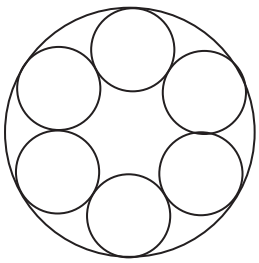

In triangle *ABC*, *AE* is the median from *A* to *BC*. In triangles *AEB* and *AEC*, *AD* and *AF* are the medians to *EB* and *CE*, respectively. In triangle *ACE*, *EG* is the median to *AC*. What is the ratio of the area of triangle *ABC* to that of quadrilateral *GEDA*?
(A) 3 : 1 (B) 4 : 3 (C) 2 : 1 (D) 8 : 7

**27.** Four points *P*, *Q*, *R* and *S* lie on a straight line in the *XY* plane, such that $PQ = QR = RS$ and the length of *PQ* is 5 metres. A man wants to go from *P* to *S*. But there are dogs tethered with 5 metre chains at points *Q* and *R*. The man

would not go within the reach of any dog. The minimum distance in metres the man must travel to reach the point $S$ is

(A) $5(\pi + 1)$ (B) $\frac{20n}{3}$ (C) $\frac{40n}{3}$ (D) $15\sqrt{2}$

**28.**



In the above big circle there are six small circles. If the radius of each small circle is 7cm, then what is the approximate area of the big circle?

(A) 616 $cm^2$ (B) 1386 $cm^2$
(C) 962.5 $cm^2$ (D) 2425.5 $cm^2$

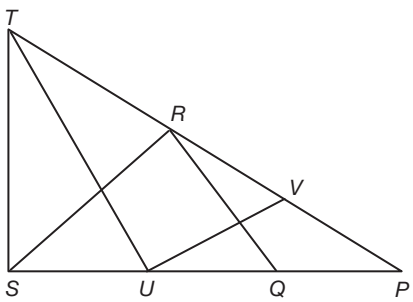
**29.** A trapezium is inscribed in a circle. The longer of the parallel sides is a diameter of the circle. The length of one of the non-parallel sides is equal to half the radius of the circle. Find the ratio of the lengths of the parallel sides of the trapezium.

(A) 7 : 16 (B) 7 : 12
(C) 7 : 8 (D) 7 : 18

**30.** In a triangle $ABC$, $\angle A > \angle B > \angle C$, if the numerical values of the three angles (when expressed in degrees) are distinct prime numbers, then what is the minimum value of $\angle A + \angle C$?

(A) 109° (B) 175°
(C) 95° (D) 113°

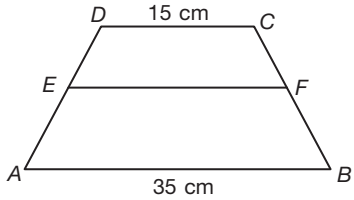
**31.**




In the figure above, $PQ = QR = RS = ST = TU = UV = VP$. $\angle SPT$ is approximately,

(A) 30° (B) 26°
(C) 20° (D) 15°

**32.**




In the figure above $AE : ED = 3 : 2$ and $EF$ is parallel to $AB$. If $ABCD$ is a trapezium, then what is the ratio of the area of the trapezium $ABFE$ to that of $EFCD$?

(A) 9 : 4 (B) 3 : 2
(C) 31 : 14 (D) 87 : 38

**33.** If $a, b, c$ are the sides of a triangle and $a^2 + b^2 + c^2 = K(ab + bc + ca)$, then the range of possible values of $K$ is

(A) $1 + K + 2$ (B) $1 + K < 2$
(C) $1 < K < 2$ (D) $0 + K + 3$

**34.** The lengths of the perpendicular sides of a right angled triangle are integers. If the length of its hypotenuse is 34, the perimeter of the triangle can be

(A) 70 (B) 90 (C) 80 (D) 100

**35.** E and $F$ are the centres of two equal circles. The circles intersect each other at $G$ and $H$. Neither circle has its centre within the region of intersection of the circles. Which of the following cannot be a possible value of $\angle GEF$?

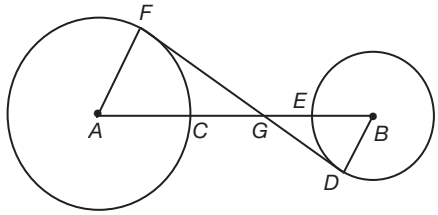
(A) 30° (B) 40° (C) 50° (D) 70°

**36.** Find the length of the common chord of two circles with radii of 26 cm and 25 cm. The distance between the centres is 17 cm. _____

**37.** In triangle $ABC$, $AD$ and $BE$ are medians and $G$ is the centroid. $\angle AGE = 30°$, $AD = 12$, $BE = 18$. Find the area of the triangle. _____

***Directions for questions 38 and 39:*** Answer these questions based on the information given below.

In the figure, $A$ and $B$ are the centers of the circles shown (the figure given is not drawn to scale).




$DF$ is a common tangent to the two circles, at $D$ and $F$. $AF$: $DB = 5{:}4$. $AB = 18$ cm.

**38.** Find $CG : EG$

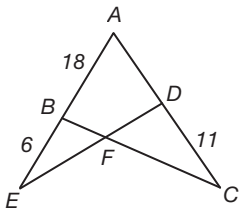
(A) 4 : 5 (B) 5 : 4

(C) 4 : 3 (D) Cannot be determined

**39.** Find $GF$ (in cm), if $AC = 1.5\ CG$. _____

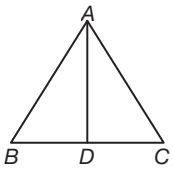
**40.** In the figure $AB$ = 18 cm, $BE$ = 6 cm and $CD$ = 11 cm. If $\angle AED = \angle ACB$, then find the length of $AD$. (in cm)

_____




**41.** In the figure, $AD$ bisects angle $BAC$ and $\frac{AB}{DC} = \frac{AC}{BD} = 2$

What is the sum of the angles $ACD$ and $BAC$? _____ degrees

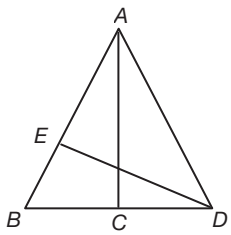



**42.** $PQRS$ is a square. $T$, $U$, $V$ and $W$ are the midpoints of $PQ$, $QR$, $RS$ and $SP$, respectively. $X$ and $Y$ are points on the line joining $T$ and $V$ such that $\angle PXQ = \angle RYS = 120°$. Find the ratio of the area of $PXQRYS$ and the remaining area in $PQRS$.

(A) $2\sqrt{3}+1$ (B) $2\sqrt{3}+2$

(C) $\sqrt{3}+1$ (D) $2\sqrt{3}-1$

**43.**




In the figure, $AC$ and $DE$ are angle bisectors of $\angle BAD$ and $\angle ADB$, respectively. If $AB = 2BC$ and $AD = BD$, find $AE : CD$.

(A) 1 : 1 (B) 1 : 2 (C) 2 : 1 (D) 2 : 3

**44.** In a polygon, each interior angle is either 150° or 210°. Which of the following cannot be the number of angles that measure 150° each?

(A) 10 (B) 18 (C) 14 (D) 15

**45.** Chords $PQ$ and $RS$ of a circle intersect at a point $T$ which is inside it. If $TR = 3$, $TS = 8$ and $PQ = 14$, find the maximum possible length of $PT^2$. _____

**46.** $PQRS$ is a cyclic trapezium and $\angle P = \angle R$. The area of $PQRS$ and the area of the circumcircle of $PQRS$ are in the ratio $240 : 169\pi$. The ratio of the perimeter of $PQRS$ and the sum of the diagonals of $PQRS$ is

(A) 19 : 15 (B) 23 : 17

(C) 17 : 13 (D) cannot be determined

**47.** For a right-angled triangle with a fixed length of hypotenuse, which of the following is/are true?

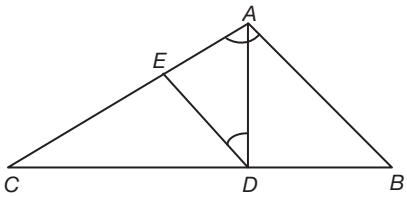
(A) The area of the triangle is the minimum when it is isosceles.

(B) The area of the triangle is the maximum when it is isosceles.

(C) The perimeter of the triangle is the minimum when it is isosceles.

(D) Both (A) and (C)

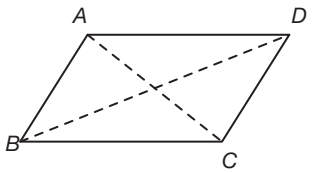
**48.** In the following figure, in $\Delta ABC\ DA^2 = (CA)(AE)$, $\angle CAB = 110°$ and $\angle ADE = 40°$. Find the respective measures of $\angle ACD$ and $\angle B$.




(A) 25°, 30° (B) 50°, 30°

(C) 40°, 30° (D) 30°, 40°

**49.**

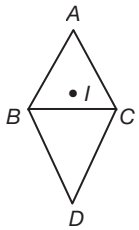



In the given figure, $ABCD$ is a parallelogram and the lengths of $AC$ and $BD$ are 10 units and 12 units, respectively. The sum of the squares of $AB$ and $BC$ is

(A) 120 sq.units (B) 140 sq.units

(C) 122 sq.units (D) 128 sq.units

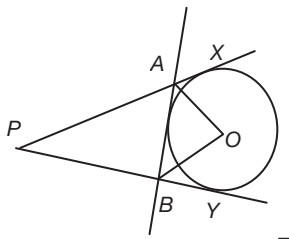
**50.**



In the figure, $I$ is the incentre of $\Delta ABC$. $BC$ bisects $\angle ABD$ and $\angle ACD$. $\angle BIC = 5\angle BDC$. Find $\angle BAC$ (in degrees). _____

**51.** Two angles with different measures are supplementary. Three-fifth of the measure of the smaller angle and one-fourth of the measure of the larger angle differ by 23°. The larger angle can be
(A) 100° (B) 95° (C) 105° (D) 110°

**52.** In the figure, $PX$, $PY$ and $AB$ are tangents to the circle with its centre at $O$. If $\angle APB = 50°$, then find $\angle AOB$ (in degrees).



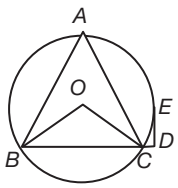

_____.

**53.** $A$, $B$, $C$ and $D$ are points on the circumference of a circle. $ABD$ is equilateral and $AC$ is the diameter of the circle. Find the ratio of the perimeter of $ABCD$ and the length of $AC$.

(A) $1+\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}+\sqrt{3}$

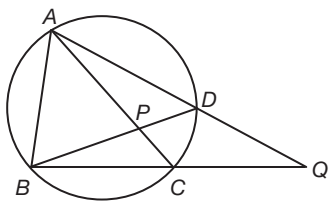
(C) $1+\sqrt{3}$ (D) $2+\sqrt{3}$

**54.** In the figure, $O$ is the centre of the circle of radius 12 cm and $\angle BAC = 30°$. $DE$ is a tangent to the circle at $E$. If $CD = 4$ cm, find the length of $DE$ (in cm).




_____.

**55.**




In the figure, $A$, $B$, $C$ and $D$ are points on a circle. $AC$ and $BD$ intersect inside the circle at $P$ and $AD$ and $BC$ extended intersect outside the circle at $Q$. If triangle $APB$ is equilateral, $\angle Q = 20^o$ and $BD = 7$ cm, find $DQ$.

(A) 6 cm (B) $3\sqrt{3}$ cm

(C) 5 cm (D) None of these

**56.** The sides of a cyclic quadrilateral are 10 cm, 14 cm, 12 cm and 15 cm. If one of its diagonals is 15 cm, then find the other diagonal (in cm). _____

**57.** In triangle $PQR$, $\frac{7\sin Q + 4\sin R}{6\sin Q - 2\sin R} = \frac{13}{6}$ and

$PQ = 18$ cm. The length of $PR$ is _____. cm

**58.** What is the length (in cm) of the longest diagonal in a regular octagon each of whose sides is 10 cm? _____
$\sqrt{4+2\sqrt{2}}$

**59.** Consider the following statements:

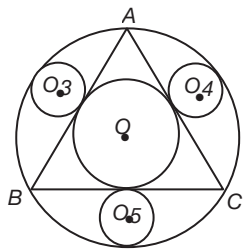
I. The altitudes of a triangle can be 9, 12, 16.

II. The altitudes of a triangle can be 8, 12, 15.

How many of the statements are true? _____

***Directions for questions 60 and 61*:** These questions are based on the following information.

In the figure, $ABC$ is an equilateral triangle drawn inside the circle $C_1$, with centre $O$. Another circle $C_2$, concentric to $C_1$ is inscribed inside the triangle. There are three more circles $C_3$, $C_4$ and $C_5$ with centres at $O_3$, $O_4$ and $O_5$, respectively such that they touch the midpoints of the sides of the triangle $ABC$ and the circle $C_1$. The radius of circle $C_1$ is $R$.




**60.** Find the area of the triangle with the vertices $O_3$, $O_4$ and $O_5$ (in terms of $R$).

(A) $\frac{9\sqrt{3}}{64}R^2$ (B) $\frac{27\sqrt{3}}{64}R^2$

(C) $\frac{18\sqrt{3}}{64}R^2$ (D) $\frac{36\sqrt{3}}{54}R^2$

**61.** Find the ratio of the sum of the perimeters of $C_3$, $C_4$ and $C_5$ to the perimeter of $C_2$.
(A) 3 : 2 (B) 2 : 1 (C) 4 : 3 (D) 9 : 8

Difficulty Level-3: Advanced

**62.** What is the sum of the squares of the sides of triangle $ABC$ (in $cm^2$), if the sum of the squares of its medians is 36 $cm^2$? _____

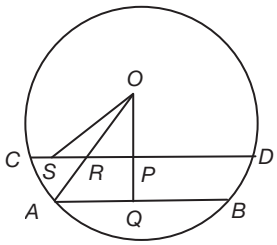
**63.** In triangle $EFG$, angle $F = 90°$. $FH$ is the altitude to $EG$. Which of the following can be concluded?
(A) $FH^2 = (EH)(HG)$ (B) $EF^2 = (EH)(EG)$
(C) $FG^2 = (GH)(EG)$ (D) All the above

**64.** There are 12 uniformly spaced points on a circle with radius 10. Each of these points is joined with the two adjacent points by straight lines. Find the perimeter of the polygon so formed.

(A) $30\left(\sqrt{6}-\sqrt{2}\right)$ (B) $60\left(\sqrt{6}-\sqrt{2}\right)$

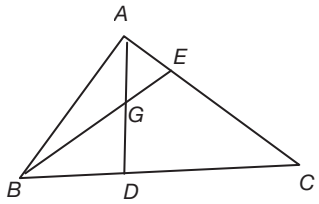
(C) $60\left(\sqrt{6}+\sqrt{2}\right)$ (D) $30\left(\sqrt{6}+\sqrt{2}\right)$

**65.** In the figure below, $AB \| CD$ $P$ and $Q$ are the midpoints of $CD$ and $AB$, respectively. If $OP = AQ$, $PR = 16/5$ $(PQ)$ and $SR = 4/5$ $PQ$, then find the measure of $\angle OSP$ (in degrees). _____




**66.** $AC$, the diameter of a circle, is of length 10 cm. $B$ and $D$ are points lying on the circle on either side of $AC$. $BC = 6$ cm and $AD = 5$ cm. Find $BD$ (in cm). _____ + _____ $\sqrt{3}$

**67.** In the figure, if $\frac{AG}{GD}=\frac{3}{4}$, $\frac{BD}{DC}=\frac{4}{7}$ and $AE = 12$ cm, then find the length of $EC$ (in cm).

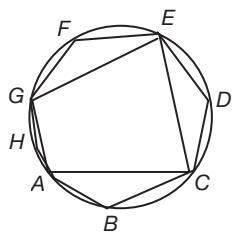



**68.** In triangle $RST$, $RS$, $ST$ and $RT$ are 48, 20 and 52, respectively and $SX$ is the altitude drawn from S to $RT$. Find $(RX)(TX)\dfrac{240^2}{\quad}$.

**69.** $ABCD$ is a cyclic quadrilateral in which $AC$ is a diameter of the circumcircle of the quadrilateral and $AB = AD = \frac{\sqrt{3}}{2}(AC)$. Find $\angle BCD$.

(A) 60° (B) 120° (C) 90° (D) 150°

**70.**




In the above figure, $ACEG$ is a cyclic quadrilateral. $\angle B + \angle D + \angle F + \angle H$ equals
(A) 360° (B) 540° (C) 600° (D) 640°

## EXERCISE-4

***Directions for questions 1 to 40*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** In a circle, $PQ$ and $RS$ are chords intersecting at T. $\angle PRQ = 90°$, $RT = 4$ cm, $TS = 20.25$ cm and $PT - QT = 24$ cm. Find the circumference of the circle (in cm).
(A) $27\pi$ (B) $30\pi$
(C) $33\pi$ (D) $36\pi$

**2.** In triangle $ABC$, $D$ and $E$ are points on $AB$ and $AC$, respectively. $F$ is a point on $BC$. $DF$ is parallel to $AC$ and $EF$ is parallel to $AB$. The areas of the triangles $EFC$ and $ABC$ are 64 and 400, respectively. Find the area of the quadrilateral $ADFE$. _____

**3.** The sides of the triangle $ABC$ are $a$, $b$ and $c$. The sides of the triangle $DEF$ are $d$, $e$ and $f$. $a(b + c - a) = d^2$

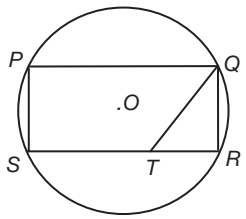
$b(c + a - b) = e^2$

$c(a + b - c) = f^2$

Triangle $DEF$ is
(A) acute angled (B) right angled
(C) obtuse angled (D) either (A) or (C)

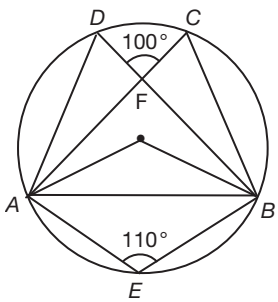
**4.** In rhombus $ABCD$, $AC$ is $\sqrt{2}+1$ times $BD$. Find $\angle ABD$. (You may use the result $\tan 22.5^\circ = \sqrt{2}-1$)
(A) 75° (B) 72° (C) 60° (D) 67.5°

**5.** In the figure below (not to scale), $PQRS$ is a rectangle inscribed in a circle with centre $O$. $PQ > PS$. Areas of the rectangle $PQRS$ and the circle are in the ratio $2\sqrt{5} : 3n$ and $\angle OQP = \angle TQR$. Find $QR : RT$.




(A) $1:\sqrt{5}$ (B) $2:\sqrt{5}$ (C) $\sqrt{5}:1$ (D) $\sqrt{5}:2$

**6.**




In the figure above if $\angle BAD = 65^\circ$, find $\angle CAO$.
(A) 30° (B) 20° (C) 15° (D) 25°

**7.** A scalene triangle has a perimeter of 60 cm. The length of each side is an integral number of centimeters. How many such triangles exist?
(A) 53 (B) 57 (C) 61 (D) 65

**8.** Find the area of the largest rectangle that can be inscribed in a semicircle of radius 45.
(A) 1012.5 (B) 2037 (C) 1018.5 (D) 2025

**9.** $C_1$ is a circle with center $O$ and radius 15 cm. $A$, $B$, $C$, $D$ and $E$ are points on radius $OE$ such that $OA = AB = BC = CD = DE = 3$ cm. Circles $C_2$ and $C_3$ are constructed with centers at $B$ and $D$, respectively and radii equal to 3 cm. Circle $C_4$ is constructed such that it touches $C_2$, $C_3$ externally and $C_1$ internally. Find the radius of $C_4$.
(A) 3.5cm (B) 4 cm
(C) 4.5 cm (D) 5 cm

**10.** In triangle $ABC$, $\angle BAC = 105^\circ$, $\angle ACB = 30^\circ$ and $AB = \sqrt{2}$. Find the perimeter of $ABC$.
(A) $2+\sqrt{3}+\sqrt{2}$ (B) $1+2\sqrt{3}+\sqrt{2}$
(C) $1+\sqrt{3}+\sqrt{2}$ (D) None of these

**11.** $ABCD$ is a trapezium. $AB \parallel CD$. $E$ and $F$ are the midpoints of $AB$ and $CD$, respectively. $\angle C$ and $\angle D$ are complementary. $AB = 11$ and $EF = 3$. Find $CD$. _____

**12.** In triangle $PQR$, $PQ = 18$ and $PR = 24$. A semicircle is drawn within $PQR$. The semicircle, which touches sides $PQ$ and $PR$, has its diameter on $QR$ and is centered at $O$. If $OR = OQ + 5$, $QR$ equals
(A) 28 (B) 30 (C) 35 (D) 40

**13.** The sides of a triangle measure $a$, $b$ and $c$ and the corresponding medians measure $p$, $q$ and $r$.
If $p^2 + q^2 = \frac{9}{20}(a^2 + b^2)$ and $5c^2 = (a^2 + b^2)$, find the value of $\frac{r^2}{c^2}$.
(A) $\frac{9}{4}$ (B) $\frac{4}{9}$ (C) $\frac{3}{2}$ (D) $\frac{2}{3}$

**14.** If the sides of a triangle measure $(p+q)^{1/2}$, $(q+r)^{1/2}$ and $(r+p)^{1/2}$, find the circumference of the circumcircle of the triangle.

(A) $\pi\frac{pqr}{\sqrt{(p+q)(q+r)(r+p)}}$

(B) $\pi\frac{\sqrt{p^2+q^2+r^2}}{\sqrt{p+q+r}}$

(C) $\pi\sqrt{\frac{(p+q)(q+r)(p+r)}{(pq+qr+pr)}}$

(D) $\pi\frac{\sqrt{pq+qr+pr}}{(p+q)(q+r)}$

**15.** In rhombus $ABCD$, $\angle A = 120^\circ$. $E$ and $F$ are points on $DC$ such that $DE = FC < 1/2\ (DC)$. $G$ is a point on $BC$ such that $BG = CF$. Which of the following is true about $r$ where $r = AE/FG$?

(A) $\frac{1}{2} < r \le \frac{3}{4}$ (B) $\frac{3}{4} < r \le 1$

(C) $1 < r \le \frac{5}{4}$ (D) $\frac{5}{4} < r$

**16.** In $\Delta$ *ABC*, *AD* and *BE* are the medians drawn to the sides opposite them, and are perpendicular to each other. If $AD = 4.5\sqrt{105}$ and $BE = 36$, the perimeter of *ABC* is
(A) 162 (B) 135 (C) 180 (D) 204

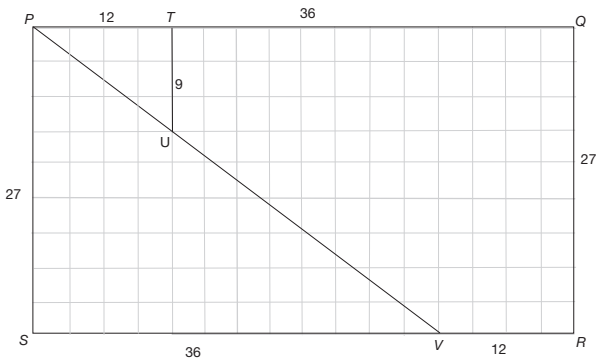
**17.** The sides of an equiangular octagon are 4, 8,12,16, $8\sqrt{2}$ and $16\sqrt{2}$ not necessarily in that order. Find the area of the octagon.

(A) $308\sqrt{2} + 320$ (B) $1296\sqrt{2} + 360$

(C) $324\sqrt{2} + 300$ (D) $284\sqrt{2} + 280$

**18.** In cyclic quadrilateral *ABCD*, $AB = 25$ cm, $BC = 7$ cm and $AD = 15$ cm. Find the length of *CD* if *AB* is the diameter of the circle within which the quadrilateral lies.
(A) 12 cm (B) 15 cm
(C) 17 cm (D) Cannot be determined

**19.** The figure shows a rectangle *PQRS*. The triangles *PTU* and *PSV* are cut out from the rectangle. The triangles and the pentagon *TQRVU* are rearranged to form a square. Find the perimeter of this square.



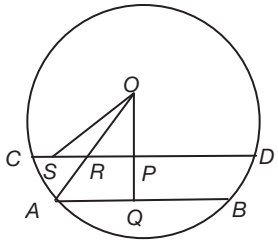

(A) 216 (B) 180 (C) 144 (D) 108

**20.** A piece of paper in the shape of a triangle with sides 10 cm, 17 cm and 21 cm is cut into two triangular regions by a single straight cut. What is the minimum possible sum of the perimeters of the two regions?
(A) 62.14 cm (B) 64.0 cm
(C) 66.0 cm (D) 68.2 cm

**21.** In square *ABCD*, points *P*, *Q*, *R* and *S* lie on sides *DA*, *AB*, *BC* and *CD*, respectively such that $AP = AQ = CR = CS$. *X* and *Y* are points on *AB* and *CD*, respectively, and are equidistant from *P* and *R*. Find $\dfrac{XB + YD}{AP + CR}$.

(A) 0.5 (B) 0.75
(C) 1 (D) Cannot be determined

**22.** In the figure below, $AB \,||\, CD$ while *P* and *Q* are the midpoints of *CD* and *AB*, respectively. If $OP = AQ$, $PR = 16/5$ (*PQ*) and $SR = 4/5$ *PQ*, find the measure of $\angle OSP$ (in degrees). _____

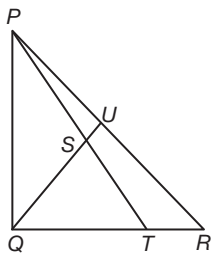



**23.** There are two concentric semicircles, centered at *P* on line *MN*. The diameter *MN* of the bigger semicircle is $2 + 2\sqrt{3}$ while the diameter *AB* of the smaller semicircle is 4. *D* and *C* are points on the inner semicircle such that $AD = DC = CB$. Three congruent semicircles *AXD*, *DYC* and *CZB* are inscribed in the space between the two semicircles where *X*, *Y*, *Z* are points on the outer semicircle. Find the ratio of the sum of the curved lengths of the 3 inscribed semicircles to the curved length of the bigger semicircle.

(A) $\dfrac{3\sqrt{3} - 2}{3}$ (B) $\sqrt{3} - 1$

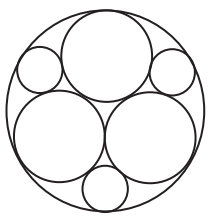
(C) $\dfrac{3\sqrt{3} - 2}{2}$ (D) $\dfrac{3\sqrt{3} - 3}{2}$

**24.**




In the figure given $2PS = 3ST$, $QT = 2TR = \dfrac{4}{3\sqrt{2}}$ $UR = \dfrac{2}{3}$ *PQ*. $\Delta PQR$ is
(A) Right-angled (B) Equilateral
(C) Isosceles (D) Both (A) and (C)

**25.**



Six circles consisting of 3 identical big circles and 3 identical small circles are cut out from a large circle, as shown in the figure above. What is the ratio of the radius of the large circle to that of any of the small circles?

(A) $\sqrt{3}+1$ (B) $1+2\sqrt{3}$

(C) $\sqrt{3}+2$ (D) $3+\sqrt{3}$

**26.** Two circles have their centres at $P$ and $Q$. The radius of the first circle is $r$, while that of the second is $2r$ and $PQ = 4r$. The tangents from $P$ to the second circle intersect the first circle at $A$ and $B$, respectively, while the tangents from $Q$ to the first circle intersect the second circle at $C$ and $D$, respectively. If $AB = 4$, find $CD$.

(A) 8 (B) 5

(C) 6 (D) None of these

**27.** **(a)** In a circle with centre $O$, $AB$ is a diameter and $M$ is the midpoint of $OB$. $CD$ is a chord through $M$, such that $BD < BC$. The perpendicular from $B$ to $CD$ intersects $CD$ at $P$. If $PD = 2$, find the ratio $CM : MD$.

(A) 7 : 4 (B) 3 : 2

(C) 7 : 3 (D) Cannot be determined

(b) In a circle with centre $O$, $AB$ is a diameter and $M$ is the midpoint of $OB$. $CD$ is a chord through $M$, where $BD < BC$. The perpendicular from $B$ to $CD$ intersects $CD$ at $P$. If $PD = PM/2$, find the ratio $CM$: $MD$.

(A) 7 : 4 (B) 3 : 2

(C) 7 : 3 (D) Cannot be determined

**28.** The sum of the squares of the diagonals of a trapezium is 825 and the sum of the squares of its sides is 1450. Find the difference in the lengths of its parallel sides.

(A) 20 (B) 25

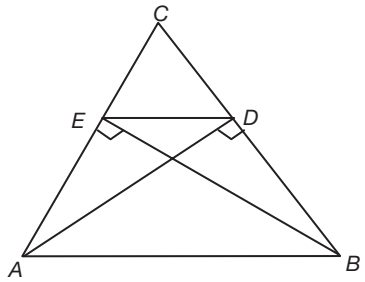
(C) 15 (D) 30

**29.** There is a narrow circular path. A man starts from a point $P$ on the outer edge and walks in a straight line across the path. He just touches the inner edge, and continues walking in a straight line to reach a point $Q$ on the outer edge, covering a total distance of 54 feet. If the diameters of both the edges of the path, in feet, are integers, which of the following could be the diameter of the inner edge (in feet)?

(A) 240 (B) 244

(C) 246 (D) 248

**30.** In $\Delta ABC$, $AD$ and $BE$ are perpendiculars to sides $BC$ and $AC$, respectively. $AD : BE = 13 : 14$, $CD = 5$ cm and $ED$ is $(5/13)$ $AB$. Find the length of $BD$ in cm.




(A) $\frac{99}{13}$ (B) 9 (C) $\frac{168}{13}$ (D) 12

**31.** $T$ is a right-angled triangle of inradius 4 cm and circumradius 13 cm. Find the perimeter of $T$ (in cm).

(A) 56 (B) 66 (C) 60 (D) 72

**32.** In $\Delta ABC$, a rectangle with its length $DE$ on $BC$ and the other two vertices $F$ and $G$ on sides $AC$ and $AB$, respectively is constructed. If the area of the rectangle is 4/9 times the area of the triangle, which of the following is/are possible value(s) of $DE/BC$?

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$

(A) only $c$ (B) only $a$

(C) $a$ or $c$ (D) only $b$

**33.** The lengths of the sides of a triangle are $a$, $b$, $c$ where $a \le b \le c$. The length of each side of the triangle is a multiple of 6 and the perimeter of the triangle is 78. The number of values that $(a, b, c)$ can take is

(A) 5 (B) 6 (C) 7 (D) 4

**34.** In $\Delta ABC$, $P$ is the foot of the altitude from $A$ to $BC$. $D$ and $E$ are points on $BC$, with $D$ being closer to $B$. $F$ is a point on $AC$ and $G$ is a point on $AB$ such that $DEFG$ is a square. A line through $A$ is drawn parallel to $GC$ to intersect $BC$ extended at $H$. Which of the following is true?

(A) $CH < AP$ (B) $CH = AP$

(C) $CH > AP$ (D) Cannot be determined

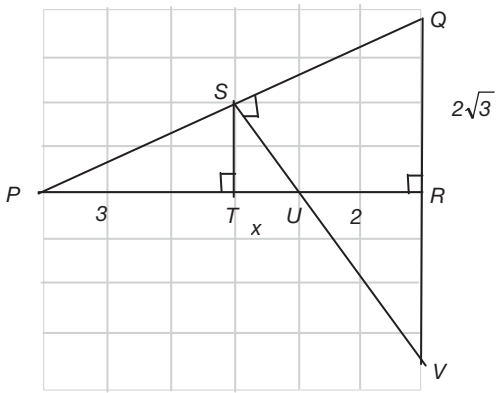
**35.** $ABCD$ is a square. $E$ is a point inside the square such that at least one of $\angle AEB$ and $\angle BEC$ is obtuse. Find the ratio of the area in which $E$ can lie to the area of the square.

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$

(C) $\frac{\pi-2}{8}$ (D) $\frac{\pi+2}{8}$

**36.** The diagonals of $PQRS$ meet at $T$. $PT = TR = 20$, $TS = 21$ and $TQ = 15$. The area of $PQRS$ is 720. Find the perimeter of $PQRS$.
(A) 88 (B) 108
(C) 98 (D) Cannot be determined

**37.**




In the given figure, $\angle PTS = \angle PRQ = \angle VSQ = 90°$.
$PT = 3$, $UR = 2$ and $RQ = 2\sqrt{3}$. Find $TU$.
(A) 1 (B) 2 (C) 1.5 (D) 2.5

**38.** $ABC$ is an acute angled triangle. $D$ and $E$ are the points of trisection of $AC$ with $D$ being closer to $A$. $F$ and $G$ are the points of trisection of $BE$ with $F$ being closer to $B$. $CG$ when extended meets $AB$ at $H$. Find $BH : HA$.
(A) 2 : 3 (B) 1 : 3 (C) 1 : 2 (D) 2 : 5

**39.** In triangle $ABC$ $AB = 15$, $BC = 20$, $AC = 25$. $PQRS$ is any possible rectangle drawn in $ABC$ such that $P$ lies on $AB$, $Q$ lies on $BC$ and $RS$ lies on $CA$. Find the maximum area of $PQRS$.
(A) 90 (B) 75 (C) 105 (D) 60

**40.** $ABC$ is a triangle. $D$ and $E$ are points on $AB$ and $AC$, respectively such that $DE$ is parallel to $BC$. $DE = 7$ cm, $BD = 4$ cm, $EC = 8$ cm, and $BC = x$ cm. How many integer values can $x$ take?
(A) 6 (B) 8 (C) 7 (D) 9

## EXERCISE-5 Based on Data Sufficiency

***Directions for questions 1 to 30:*** Each question is followed by two statements, I and II. Answer each question based on the following directions:

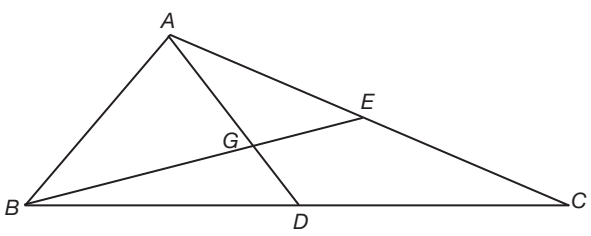
Choose (A) If the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.

Choose (B) If the question can be answered by using either statement alone.

Choose (C) If the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (D) If the question cannot be answered even by using both the statements together.

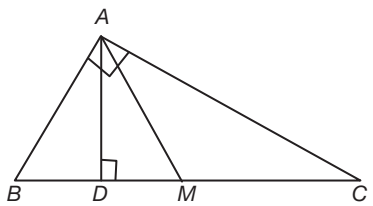
**1.** In the triangle $ABC$, $AD$ and $BE$ are two of the medians. What is the area of the quadrilateral $GDCE$?




I. $AG : GE = 2 : 1$

II. $AB = 10$ cm and $BC = 20$ cm

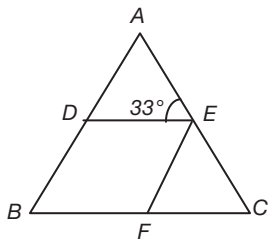
**2.** What is the length of the median $AM$ of the triangle $ABC$, if $\angle BAC = 90°$?




I. $BC = 21$ cm.

II. $AD = 14$ cm.

**3.** In the figure below, $\angle AED = 33°$. If $D$, $E$ and $F$ are the midpoints of the sides $\Delta ABC$, what is the measure of $\angle EFB$?



I. $\angle FEC = 78°$

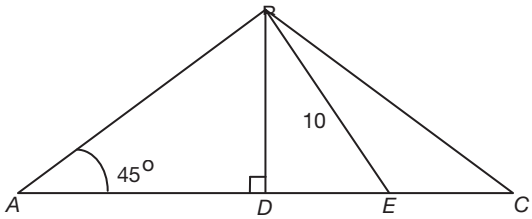
II. $\angle BAC = 78°$

**4.** Find the area of triangle $ABC$.

I. $\angle ABC = 60°$

II. $AB = 10$ cm and $AC = 10$ cm.

**5.** In the figure below, if $BE = 10$ cm, and $\angle A = 45°$, what is the length of $AB$?




I. $\angle BCA = 45°$.

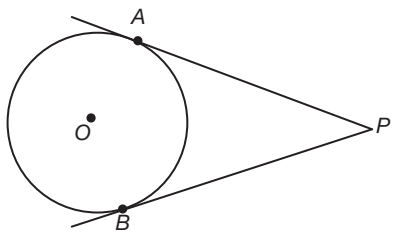
II. $\angle BEA = 60°$.

**6.** $PQRS$ is a rectangle and points $A$, $B$, $C$ lie on $PQ$, $QR$, $RS$, respectively. What is the ratio of the area of the triangle $ABC$ to the area of rectangle $PQRS$?

I. Point $A$ divides $PQ$ in the ratio 1 : 2.

II. $AC$ is parallel to $PS$

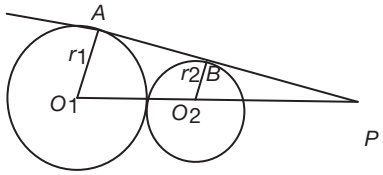
**7.** In the figure below two tangents from $P$ meet the circle at $A$ and $B$. What is the length of $PO$?




I. O is the centre of the circle whose radius is 10 cm.

II. The circumradius of the triangle $AOP$ is 14 cm.

**8.** In the figure below, $A$ and $B$ are the points of contact of the direct common tangent from $P$ to the circles with centres $O_1$ and $O_2$, respectively. If $r_1, r_2$ are the radii of the circles, find the length of $AB$.




I. $r_1 = 15$ cm, $r_2 = 9$ cm

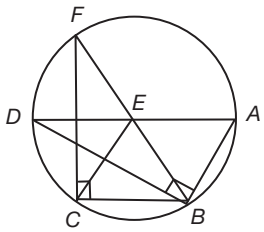
II. $\angle BPO = 30°$

**9.** The points $O_1$ and $O_2$ are the centres of two circles having radii 10 cm and 20 cm, respectively. What is the distance between the points $O_1$ and $O_2$?

I. The two circles have only one common tangent

II. The two circles have three common tangents.

**10.**




What is the measure of $\angle BEA$ in the figure above if $\angle ABD = \angle BCF = 90°$?

I. $\angle BDA = 30°$

II. $\angle CFB = 30°$

**11.** Is the four-sided figure a rectangle?

I. The diagonals are equal.

II. The diagonals are perpendicular to each other.

**12.** In triangle $ABC$, is angle $A$ a right angle?

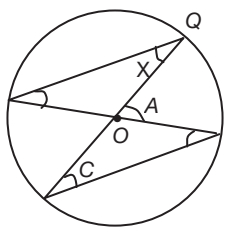
I. The circumcircle of triangle $ABC$ passes through $A$.

II. The circle with $AC$ as diameter passes through $B$.

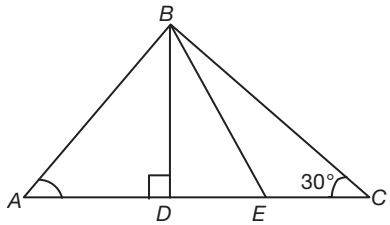
**13.** What is the measure of angle $X$ in the given circle with centre $O$?

I. $\angle A = 60°$

II. $\angle B = 30°$




**14.** In the figure below, what is the perimeter of triangle $ABC$?




Data Sufficiency

I. The length of $AD$ is 2 cm.

II. The length of $EC$ is $\sqrt{3}$ cm.

**15.** What is the number of sides in a regular polygon?

I. Sum of the interior angles of the polygon is $(2n-4) \times 90°$ where $n$ is the number of sides of the polygon.

II. One of the exterior angles of the polygon is 60°.

**16.** Which is the smallest angle of triangle $ABC$?

I. $AB^2 > BC^2 + AC^2$.

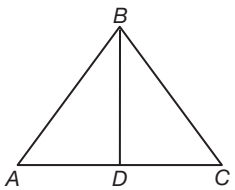
II. $BC^2 < AB^2 + AC^2$.

**17.** What is the area of triangle $ABC$?

I. The angles of the triangle are in the ratio 1 : 2 : 3.

II. The longest side of the triangle is 5 cm.

**18.** Is $ABC$ an isosceles triangle?




I. $\angle BAC = 50°$ and $\angle ABC - \angle BCA = 30°$

II. $AD = DC$ and $BD$ is perpendicular to $AC$.

**19.** Does the perpendicular bisector of the side $BC$ of triangle $ABC$ passes through $A$?

I. $AB + AC = 2BC$ and $AC = BC$.

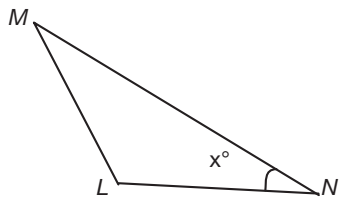
II. $AB = AC$.

**20.** Do the two circles with centres $A$ and $B$ touch each other externally?

I. The distance between $A$ and $B$ is equal to the diameter of the circle with centre $A$.

II. One circle is bigger than the other.

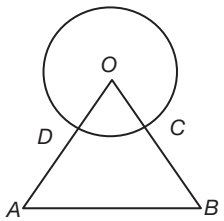
**21.** What is the perimeter of the triangle $LMN$ in the figure?




I. The area of triangle $LMN$ is 6.

II. $x = 45$.

**22.** What is the measure of $\angle CAD$ in the following figure given $O$ is the centre of the circle?




I. $\angle ODB = 90°$.

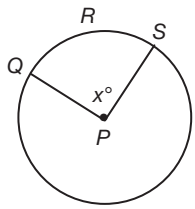
II. $DA = BC$ and $\angle AOB = 60°$.

**23.** $A$, $B$ are two vertical corners of a polygon and $P$ is a point on one of the sides. $AB = 8$, $BP = 5$. What is the area of the polygon?

I. The polygon is a triangle.

II. Another vertex of the polygon is $C$, where $AC = 6$ and $\angle CAB = 90°$

**24.** If $P$ is the centre of the circle as shown in the figure, what is the value of $x$?




I. The length of arc $QRS$ is 1/3 the circumference of the circle.

II. The radius of the circle is 5.

**25.** $A$, $B$ and $C$ are three points on the circumference of a circle with centre $O$. What is the measure of $\angle BOC$?



I. $\angle ABO = 25°$.

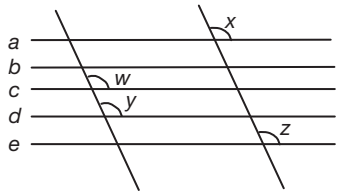
II. $\angle ACO = 35°$.

**26.** In a triangle $PQR$, find the ratio of $\angle P : \angle Q : \angle R$.

I. $PQ / QR = QR / PR$.

II. $PQ > QR \sim PR$.

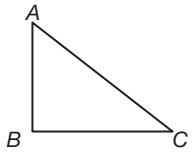
**27.** Is $\angle x = \angle y$?




I. Lines $a$, $c$ and $e$ are parallel to one another.

II. Lines $b$ and $d$ are parallel to each other.

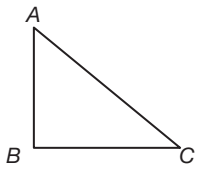
**28.** $ABC$ is a right-angled triangle. Is $AB = 4$?




I. $AB^2 + BC^2 = 25$.

II. $AB + BC = 7$.

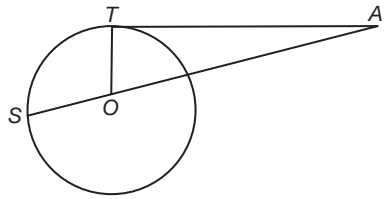
**29.** $ABC$ is a right-angled triangle. What is its perimeter?




I. The hypotenuse is 5 units.

II. The length of either $AB$ or $BC$ is 4 units.

**30.** $AT$ is a tangent to a circle with centre $O$. What is the radius of the circle?




I. $AS = 12$ cm.

II. $AT = 6$ cm.

## ANSWER KEYS

### Exercise-1

1. 5
2. (B)
3. 1
4. 30
5. (C)
6. (B)
7. 2
8. (B)
9. (D)
10. (C)
11. (C)
12. 90
13. (A)
14. (A)
15. (A)
16. (D)
17. 115
18. (D)
19. 55
20. 120
21. 35
22. (A)
23. (C)
24. 11
25. 0
26. (D)
27. (B)
28. 85
29. (C)
30. 12
31. (A)
32. (A)
33. 90
34. 4
35. 16
36. (A)
37. 52
38. 24
39. (C)
40. 15
41. 27
42. 3
43. (A)
44. (D)
45. 70
46. (C)
47. 8
48. (A)
49. (A)
50. (B)
51. 110
52. (C)
53. (C)
54. 80
55. 10
56. 6
57. 24
58. 30
59. (B)
60. 45
61. (C)
62. (C)
63. 150
64. 22.5
65. 16.8
66. 30
67. (D)
68. 50
69. 28
70. (D)

### Exercise-2

1. (i) 54,144 (ii) 18,108
2. (i) Parallelogram (ii) Isosceles trapezium/ parallelogram (iii) Rectangle (iv) Square
3. 8
4. 3200
5. 60, 120
6. (i) obtuse, acute, obtuse (ii) obtuse, acute, obtuse (iii) obtuse, acute, obtuse (iv) acute, obtuse, acute
7. 270
8. (B)
9. 5
10. (B)
11. 75
12. (C)
13. 108
14. 125
15. 50
16. (D)
17. 110
18. 3
19. (B)
20. 108
21. (B)
22. (D)
23. 1
24. 70
25. 110
26. 2
27. 60
28. 8
29. 12
30. 900
31. 4
32. (C)
33. 15
34. (B)
35. 5
36. (D)
37. 4.8
38. (A)
39. 13

40. (i) right-angled
(ii) Scalene and obtuse
(iii) Right-angled isosceles
(iv) Scalene and acute
(v) triangle not possible
41. 36
42. (B)
43. 40
44. 30
45. (D)
46. 16
47. (C)
48. (i) 1
(ii) 4
49. 100
50. (D)
51. (C)
52. 13.44
53. 5
54. 4
55. 27
56. (B)
57. 36
58. 100
59. 5
60. 10
61. 80, 40
62. 30
63. (D)
64. (A)
65. (i) (C)
(ii) 28
66. 100, 50
67. 50
68. Pentagon, 540, 108, 72
Nonagon, 1260, 140, 40
(D)odecagon, 1800, 150, 30
69. (i) (A)
(ii) (D)
70. (A)

## Exercise-3

1. 4
2. (B)
3. 3.6
4. 18.5
5. (A)
6. (B)
7. (D)
8. (D)
9. (C)
10. (B)
11. (C)
12. 150
13. (A)
14. (A)
15. (D)
16. (B)
17. (B)
18. (B)
19. (D)
20. (C)
21. (A)
22. (C)
23. 6
24. 19
25. 192
26. (C)
27. (A)
28. (B)
29. (C)
30. (A)
31. (B)
32. (D)
33. (B)
34. (C)
35. (D)
36. 48
37. 72
38. (B)
39. 8
40. 16
41. 120
42. (D)
43. (A)
44. (A)
45. 144
46. (C)
47. (B)
48. (C)
49. (C)
50. 20
51. (A)
52. 65
53. (C)
54. 8
55. (D)
56. 22
57. 25
58. 10
59. 2
60. (B)
61. (A)
62. 48
63. (D)
64. (B)
65. 45
66. 3, 4
67. 44
68. 169
69. (B)
70. (B)

## Exercise-4

1. (B)
2. 192
3. (A)
4. (D)
5. (C)
6. (C)
7. (C)
8. (D)
9. (B)
10. (D)
11. 17
12. (C)
13. (A)
14. (C)
15. (B)
16. (A)
17. (A)
18. (B)
19. (C)
20. (B)
21. (C)
22. 45
23. (D)
24. (D)
25. (B)
26. (D)
27. (a) (D)
(b) (C)
28. (B)
29. (A)
30. (B)
31. (C)
32. (C)
33. (B)
34. (B)
35. (D)
36. (B)
37. (A)
38. (A)
39. (B)
40. (C)

## Exercise-5

1. (C)
2. (A)
3. (D)
4. (C)
5. (A)
6. (C)
7. (A)
8. (A)
9. (B)
10. (A)
11. (D)
12. (A)
13. (B)
14. (A)
15. (A)
16. (D)
17. (C)
18. (B)
19. (B)
20. (C)
21. (D)
22. (C)
23. (C)
24. (A)
25. (C)
26. (D)
27. (D)
28. (D)
29. (C)
30. (C)

## SOLUTIONS

## EXERCISE-1

**1.** $6^2 + 8^2 = 10^2$

$\therefore$ the triangle is right angle.

$\therefore$ Circumradius $= \dfrac{\text{Hypotenuse}}{2} = 5\,\text{cm}$

**2.** $\angle ADE = \angle DAE = \angle AED = 60°$ (angle of an equilateral triangle) and 'E' is on produced $\overline{CD}$, so in rhombus $ABCD$, $\angle BAD = \angle BCD = 60°$.

In triangle $BAE$, $\angle BAE = 60° + 60° = 120°$.

$AB = AD$ (sides of rhombus)

$AD = AE$ (sides of equilateral triangle)

$\Rightarrow AB = AE$.

$\therefore \angle ABE = \angle AEB$

In triangle $ABE$, $\angle ABE + \angle AEB + \angle BAE = 180°$

$\Rightarrow 2\angle AEB + 120° = 180° \Rightarrow \angle AEB = 30°$.

**3.** Only an isosceles trapezium is necessarily a cyclic quadrilateral.

**4.** The interior angle of a regular

polygon of $n$ sides $= \dfrac{(2n-4)90°}{n}$

For a hexagon, interior angle $= \dfrac{(2\times 6-4)\times 90°}{6} = 120°$

In triangle $CDE$,

$CD = DE$ (sides of a regular hexagon)

$\therefore \angle CDE = \angle CED = 30°$

**5.** $8^2 > 6^2 + 4^2$

$\therefore$ The triangle is obtuse angled.

$\therefore$ Its circumcentre lies outside the triangle.

**6.** $OB = OC$ ($\therefore$ the radii are equal).

$\therefore$ $OB = 6$ cm. The perimeter of triangle $BOC = OB + OC + BC = 18$ cm.

6 cm + 6 cm + $BC = 18$ cm. $\Rightarrow BC = 6$ cm.

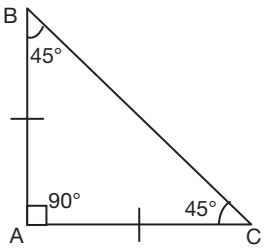
$OB = OC = BC$

$\therefore$ Triangle $BOC$ is equilateral.

Again $\angle BOC = 2\angle BAC$ ($\therefore$ the angle subtended by a chord at the center of a circle is always twice the angle it subtends at any point on the circumference in the same segment as that in which the center lies) $\therefore \angle BAC = 30°$.

**7.** To two circles which intersect each other, two direct common tangents can be drawn. No transverse common tangent can be drawn.

**8.** The ratio of the sides opposite to the angles $A$, $B$ and $C$ is sin $A$: sin $B$ : sin $C$ = sin 90° : sin 45° : sin 45°




$= 1 : \dfrac{1}{\sqrt{2}} : \dfrac{1}{\sqrt{2}} = \sqrt{2} : 1 : 1$

**9.** As $ST$ is parallel to $QR$,

$$\frac{PS}{PQ} = \frac{ST}{QR}$$

$$QR = \frac{PQ}{PS}(ST) = \frac{16}{4}(3)\frac{\text{cm}^2}{\text{cm}} = 12\ \text{cm}$$

**10.** Triangle $PQR$ is equilateral.

$\therefore \angle PQR = 60°$.

$QRST$ is a rectangle.

$\therefore \angle RQT = 90°$.

$\angle PQT = \angle PQR + \angle RQT = 150°$.

$QU$ bisects $\angle PQT$.

$\therefore PQU = \dfrac{\angle PQT}{2} = 75°$.

$\angle RQU = \angle PQU - \angle PQR = 15°$

$\angle QUS = \angle QRU + \angle RQU = 90° + 15° = 105°$

($\because$ the exterior angle of a triangle equals the sum of the interior angles opposite to it).

**11.** Only in an obtuse angled triangle,

$AB^2 + AC^2 < BC^2 \Rightarrow \angle BAC > 90°$

i.e., $x > 90°$

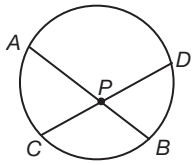
**12.** The ratio of sides $AB$, $BC$ and $CA$ is

$3 : 3\sqrt{3} : 6 = 1 : \sqrt{3} : 2$ so the angles opposite to these sides are 30°, 60°, 90°.

$\therefore$ The angle opposite $\overline{AC}$ is 90°, i.e., $\angle ABC = 90°$

**13.** Incentre.

**14.**



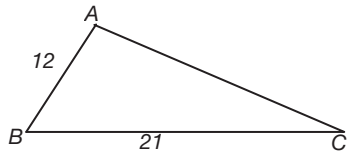

Here, $PA \times PB = PC \times PD$

**15.** The quadrilateral formed by joining the midpoints of another quadrilateral is always a parallelogram.

Its area is always half the area of the outer quadrilateral.

In the given problem, the quadrilateral formed is a parallelogram of area $\frac{200}{2}$ $cm^2$, i.e., 100 $cm^2$.

**16.**




In a triangle, sum of the sides is greater than the third side and third side is greater than the difference between the two sides.

$(21 - 12) < AC < (21 + 12)$

$9 < AC < 33$

$(21 + 12 + 9) <$ perimeter $< (21 + 12 + 33)$

$\Rightarrow 42$ cm $< p < 66$ cm

**17.** $\angle QIR = 90° + \frac{1}{2}\angle QPR = 90° + \frac{50°}{2} = 115°$

**18.** The point of intersection of the perpendicular bisectors is called circumcentre.

**19.** As $XZ$ is the diameter of the circle,

$\angle XYZ = 90°$.

In triangle $XYZ$, $\angle YXZ + \angle XYZ + \angle YZX = 180°$

$\angle YXZ = 180° - (\angle XYZ + \angle YZX)$

$= 180° - (90° + 35°) = 55°$

**20.** Sum of the interior angles of a polygon

$= (2n - 4)\ 90° = (10 - 4)\ 90° = 540°$

One angle is 60°, hence, sum of the angles

$= 480°$.

Each of the other angles $= 480/4 = 120°$

**21.** Number of diagonals $= \frac{(10)(10-3)}{2} = 35$

**22.** $QP$ is a tangent to the circle at $P$ and $QRS$ is a secant.

$\therefore QP^2 = QR \cdot QS$

Dividing both sides by $QS^2$

$$\left(\frac{QP}{QS}\right)^2 = \frac{QR}{QS}$$

$$\because \frac{QR}{QS} < 1$$

$$\therefore \left(\frac{QP}{QS}\right)^2 < 1, \text{ i.e., } \frac{QP}{QS} < 1, \text{ i.e., } QP < QS$$

Only Choice (A) violates this condition.

**23.** A triangle which has its circumcentre on one its sides must have its circumcentre as the midpoint of its longest side. Such a triangle must be right angled.

**24.** We have, sum of the interior angles of a polygon $= (2n - 4) \times 90° = 1620°$

$\Rightarrow 2n - 4 = 1620/90 = 18$

$\Rightarrow 2n = 22 \Rightarrow n = 11$

**25.** The incentre, the centroid and the circumcentre coincide, since the triangle is equilateral.

$\therefore$ The required area is 0.

**26.** $\frac{AB}{AC} = \frac{BD}{DC}$

$$BD = \frac{AB}{AC}(DC) = \frac{10}{12}(8)\frac{cm^2}{cm} = 6\frac{2}{3} \text{ cm.}$$

**27.** $PTS$ and $PQR$ are secants.

$\therefore (PT)\ (PS) = (PQ)\ (PR)$

$PT\ (PT + TS) = (PQ)\ (PR)$

$4\ (4 + 8) = 3(PR)$

$48 = 3PR$

$PR = 16$

**28.** $\angle BAC = \angle BDC = 50°$

(Angles in the same segment are equal).

In triangle $ABC$, $\angle ABC + \angle BAC + \angle BCA = 180°$

$\angle ABC = 180° - (\angle BAC + \angle BCA)$

$= 180° - (50° + 45°) = 85°$

**29.** $\angle PRS = 120°$.

$\angle PQR + \angle QPR = \angle PRS$ ($\therefore$the exterior angle of a triangle is equal to the sum of the opposite interior angles).

$2(\angle QPR) + \angle QPR = 120°$.

$3\ \angle QPR = 120°$.

$\angle QPR = 40°$.

**30.** As $ST$ is parallel to $QR$, $\frac{PS}{SQ} = \frac{PT}{TR}$

$PT = \frac{PS}{SQ}(TR) = \frac{8}{4}(6)\frac{\text{cm}^2}{\text{cm}} = 12$ cm

**31.** $\angle P + \angle Q + \angle R = 180°$

$\therefore \angle P + \angle R = 180° - \angle Q$ (1)

$\angle P - \angle Q = \angle Q - \angle R$

$\angle P + \angle R = 2\angle Q$

(1) $\Rightarrow 180° - \angle Q = 2\angle Q$

$180° = 3\angle Q$

$60° = \angle Q$

**32.** As $AC$ and $BC$ are tangents to the circles,

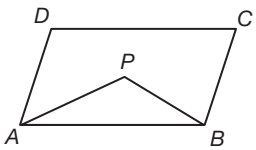
$\angle OAC = \angle OBC = 90°$

In quadrilateral $OABC$, $\angle OAC + \angle ACB + \angle OBC + \angle AOB = 360°$

$\angle AOB = 360° - (\angle OAC + \angle ACB + \angle OBC)$

$= 360° - (90° + 50° + 90°) = 130°$

**33.**




Given $AP$ is the angular bisector of $\angle DAB$,

$\Rightarrow \angle PAB = \frac{1}{2}\angle DAB$ (1)

and $BP$ is the angular bisector of $\angle ABC$,

$\Rightarrow \angle PBA = \frac{1}{2}\angle CBA$ (2)

Now in a parallelogram sum of adjacent angles = 180°

$\therefore \angle DAB + \angle ABC = 180°$

From (1) and (2)

$2\ [\angle PAB + \angle PBA] = 180°$

$\Rightarrow \angle PAB + \angle PBA = 90°$

In triangle $PAB$, $\angle APB$

$= 180° - [\angle PAB + \angle PBA] = 180° - 90° = 90°$

**34.** Area of triangle $GFB = \frac{\text{Area of } \Delta ABC}{6} = \frac{24}{6}\ \text{cm}^2 = 4\ \text{cm}^2$.

**35.** Since $AD$ is the median so the hypotenuse, $BC = 20$ cm.

$AC = \sqrt{20^2 - 12^2} = 16$ cm.

**36.** Area $= \frac{3\sqrt{3}}{2}(4)^2 = 24\sqrt{3}\ \text{cm}^2$.

**37.** In a rhombus, the diagonals bisect each other at right angles. Let $O$ be the point where the two diagonals $AC$ and $BD$ intersect each other.

Since $AC = 24$ and $BD = 10$

$\Rightarrow AO = 12$ cm and $OD = 5$ cm

As the diagonals bisect at right angles, $\Delta AOB$ is right angled.

$\therefore AB^2 = OA^2 + OB^2$

$12^2 + 5^2 = 13^2$; Hence, $AB = 13$ cm.

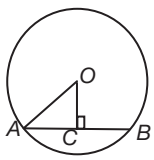
$\therefore$ Perimeter $= 4 \times 13 = 52$ cm.

**38.** Let the centre of the circle be $O$

and the chord be $AB$.

Let $OC$ be the perpendicular line drawn from $O$ to $AB$,

$OC^2 + AC^2 = OA^2$




$AC^2 = OA^2 - OC^2 = (15^2 - 9^2)\ \text{cm}^2$

$\Rightarrow AC = 12$ cm

$AB = 2AC = 24$ cm

**39.** Circumradius $= \frac{18}{\sqrt{3}}$ cm $= 6\sqrt{3}$ cm.

**40.** Let $AD$ be the median to base $BC$.

$BC = 40$ cm $\Rightarrow BD = 20$ cm

$AD = \sqrt{25^2 - 20^2} = 15$ cm.

**41.** In triangles $ADE$ and $ABC$

$\angle A = \angle A$ (common)

$\angle ABC = \angle ADE$, $\angle AED = \angle ACB$ ($\because$ corresponding angles)

Triangle $ADE$ is similar to triangle $ABC$. In similar triangles, areas are proportional to the squares of corresponding sides.

$$\frac{\text{Area of triangle } ADE}{\text{Area of triangle } ABC} = \frac{1^2}{4^2}$$

$$\text{Area of triangle } ADE = \frac{1}{16} \times 432 = 27\text{cm}^2$$

**42.** To two circles which touch each other externally, two direct common tangents and one transverse common tangent can be drawn.

**43.** $\angle QIR = 90° + \frac{1}{2}\angle P = 90 + \frac{1}{2}(80°) = 130°$

**44.** As $PQRS$ is a cyclic quadrilateral,

$\angle R = 180° - \angle P = 130°$

$\angle S = 180° - \angle Q = 110°$

**45.** $\angle ACD = \angle ABD = 80°$ (Angles in the same segment).

In triangle $ABE$,

$\angle A + \angle ABD + \angle AEB = 180°$

$30° + 80° + \angle AEB = 180°$

$\Rightarrow \angle AEB = 70°$

**46.** $\angle POQ = 2\angle PRQ$

$\angle PRQ = \frac{\angle POR}{2} = 50°$

**47.** $t^2 = d^2 - (r_2 - r_1)^2$, where

$t \rightarrow$ length of the direct common tangent

$d \rightarrow$ distance between centres of the radius

$r_1, r_2 \rightarrow$ radii of the two circles

$d = r_1 + r_2 = 8 + 2 = 10$ (as the 2 circles touch externally)

$\therefore t^2 = 10^2 - (6)^2 = 100 - 36 = 64$

$\Rightarrow t = \sqrt{64} = 8$ cm.

**48.** Inradius $= \frac{18}{2\sqrt{3}}$ cm $= 3\sqrt{3}$ cm.

**49.** $\angle PAC = \angle ABC$ (By alternate segment theorem)

$\therefore \angle ABC = 80°$

$\angle AOC = 2\angle ABC = 2 \times 80 = 160°$

**50.** Let $R$ and $H$ represent the radius and the height, respectively of the original cone.

Let $r$ and $h$ represent the radius and the height, respectively of the frustum.

Then it follows that

$$\frac{r}{R} = \frac{H - h}{H}$$

$$h = \frac{2}{3}H$$

$$\therefore r = \frac{1}{3}R$$

$\therefore$ The height of the smaller cone $= \frac{H}{3}$

As the smaller cone and the original cone are similar, the radius of the smaller cone will be $\frac{R}{3}$.

$$\therefore \text{Required ratio} = \frac{1}{3}n\left(\frac{R}{3}\right)^2\frac{H}{3} : \frac{1}{3}nR^2H = 1:27$$

**51.** Bigger arc makes an angle of 220° at the centre of the circle. It makes half the angle in the segment $ACB$.

$\therefore \angle ACB = 110°$

**52.** The larger part and the smaller part are in the ratio of 2 : 1.

**53.** In trapezium $ABCD$,

$\angle DAB + 70° = 180° \Rightarrow \angle DAB = 110°$.

($\because$ Adjacent angles on the same side of the transversal).

Similarly, $\angle ADC + 60° = 180°$

$\Rightarrow \angle ADC = 120°$.

$\angle A + \angle C = 110° + 60° = 170°$

$\angle B + \angle D = 70° + 120° = 190°$

$\angle A + \angle C < \angle B + \angle D$

**54.** Let the starting point be $O$. After 4 hours, $A$ will be at $X$, which will be $4 \times 16 = 64$ km from $O$.

$B$ will be at $Y$, which will be $4 \times 12 = 48$ km from $O$ in direction perpendicular to $OX$.

Using Pythagoras theorem, we can find the third side $XY$ as $\sqrt{64^2 + 48^2} = 80$ km.

Hence, $A$ and $B$ will be 80 km from each other.

**55.** As $Z$ is a point on the circumference and $XY$ is the diameter of the circle, $\angle XZY = 90°$

$\therefore XY^2 = XZ^2 + ZY^2$

$YZ = \sqrt{XY^2 - ZX^2} = \sqrt{26^2 - 24^2}$ cm $= 10$ cm .

**56.** The perpendicular distance from $\ell_1$ to $\ell_2$ is equal to that from $\ell_2$ to $\ell_3$

$\therefore \frac{PQ}{QR} = \frac{1}{1}$

$PQ = QR = 6$ cm

**57.** $QS = \sqrt{(PS)(SR)} = \sqrt{(32)(18)}$ cm $= 24$ cm

**58.** Let the angle be $x$.

Then its supplement is $(180° - x)$

$x = 1/5(180 - x)$ [given] $\Rightarrow 6x/5 = 36 \Rightarrow x = 30°$

**59.** As $RS \parallel TU$, $\angle XZT = \angle XNR$ (1)

(Corresponding angles are equal).

As $VW \parallel XY$, $\angle VMR = \angle XNR$ (2)

(Corresponding angles are equal)

As $PQ \parallel RS$, $\angle VOP = \angle VMR$ (3)

(Corresponding angles are equal)

From (1), (2) and (3), $\angle VOP = \angle XZT = 130°$

**60.** $\angle ADC = 180° - 105° = 75°$

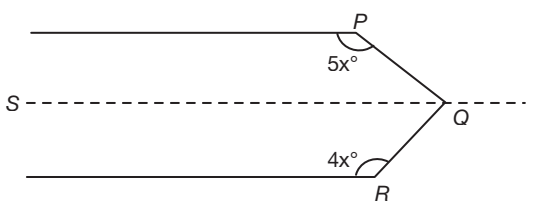
$\angle ACD = 60°$

In triangle $ADC$,

$\angle DAC = 180° - (75° + 60°) = 45°$

**61.** The inradius of a triangle is always less than $\frac{1}{2}$ (smallest altitude in the triangle). In the given problem, as the smallest altitude = 18 cm, the inradius has to be less than 9 cm.

**62.**




By construction, draw a line parallel to the given two lines.

$\angle PQS = 180° - 5x$

($\because$ Sum of the interior angles on the same side of the transversal) and $\angle RQS = 180° - 4x$.

Therefore, $180° - 5x + 180° - 4x = 90°$

$\Rightarrow x = 30°$

**63.** Reflex $\angle POR = 300°$

$2\angle PQR =$ Reflex $\angle POR$

$\therefore \angle PQR = 150°$

**64.** Let the angle be $x$.

Then its complement is $(90° - x)$

$x = 1/3(90 - x)$ [given] $\Rightarrow 4x/3 = 30 \Rightarrow x = 22.5°$

**65.** $EF = \left[\frac{2}{5}(24) + \frac{3}{5}(12)\right]$ cm $= 16.8$ cm

**66.** By construction, join $AB$. $\angle BAC = 90°$

(Radius makes an angle of 90° with the tangent)

Since $ABC$ is a right angled triangle and $AB = 18$ cm, $AC = 24$ cm, $BC = \sqrt{18^2 + 24^2} = 30$ cm

**67.** To two non-intersecting and non-enclosing circles, two direct common tangents and two transverse common tangents can be drawn.

**68.** The angles opposite to $x°$, $2x°$ and $30°$ are also correspondingly $x°$, $2\ x°$ and $30°$.

$\therefore 2(x + 2x + 30) = 360° \Rightarrow x = 50°$

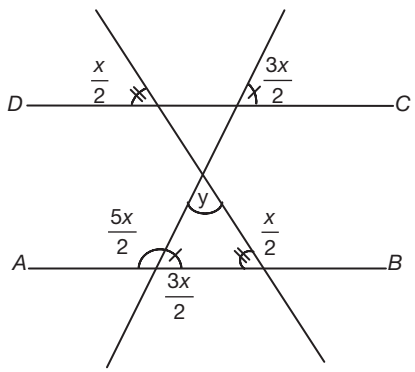
**69.** $12^2 + 16^2 = 20^2$

$\therefore$ The triangle is a right-angled triangle.

$\therefore$ The point of intersection of the perpendicular sides is the orthocentre.

$\therefore$ Sum of the distances from the orthocentre to the vertices of the triangle = (0 + 12 + 16) cm = 28 cm.

**70.**



$\frac{5x}{2}+\frac{3x}{2}=180 \Rightarrow x=45°$

$y=180-\left(\frac{3x+x}{2}\right)=180°-2x=180°-90°=90°$

## EXERCISE-2

**1.** (i) Complementary angle = 54°

Supplementary angle = 144°

(ii) Complementary angle = 18°

Supplementary angle = 108°

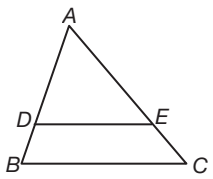
**2.** (i) Parallelogram

(ii) Isosceles trapezium/parallelogram.

(iii) Rectangle

(iv) Square

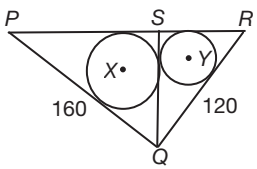
**3.** $\frac{AD}{DB}=\frac{AE}{EC}=2$




$\therefore \frac{AD}{AB}=\frac{AE}{AC}=\frac{2}{3}$ and $DE=\frac{2}{3}(BC)$

$=\frac{2}{3}(12)\text{cm}=8\text{cm}.$

**4.**




In $\Delta PQR$, $PR^2 = PQ^2 + QR^2$

$PR=\sqrt{160^2+120^2}=200$

Let $PS = x$

$SR = 200 - x$

$SQ^2 = PQ^2 - PS^2 = QR^2 - SR^2$

$= 160^2 - x^2 = 120^2 - (200 - x)^2$

$x = 128$

$SQ = 96$, $SR = 72$

Perimeter of $\Delta PSQ = 384$

If the inradius of $\Delta PQS$ is $a$,

$a=\frac{\frac{1}{2}(128)(96)}{\frac{384}{2}}=32$

If the inradius of $\Delta RSQ$ is $b$,

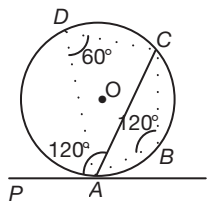
$b=\frac{\frac{1}{2}(72)(96)}{\frac{288}{2}}=24$

$XY^2 = (a + b)^2 - (a - b)^2$

$= 3136 + 64 = 3200$

**5.** Applying alternate segment theorem, $\angle ABC = 120°$

Since, $ABCD$ is a cyclic quadrilateral,



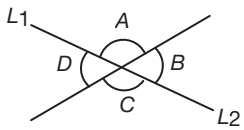

$\angle B + \angle D = 180°$

$\Rightarrow \angle D = 60°$, i.e., $\angle ADC = 60°$

We can interchange the vertices $B$ and $D$ and still all the given conditions are valid in which case angle $ADC$ is 120°.

Hence, angle $ADC$ is 60° or 120°.

**6.**




$\angle A$ and $\angle C$ are vertically opposite angles. $\angle A = \angle C$. $\angle B$ and $\angle D$ are also vertically opposite angles. $\angle B = \angle D$.

(i) If $\angle A$ is acute, $\angle C$ is acute and $\angle B$, $\angle D$ are obtuse.

(ii) If $\angle A = 70°$, $\angle C = 70°$ which is acute and $\angle B = \angle D = 110°$ and both are obtuse.

(iii) If $\angle A = \angle C = 89°$, both are acute and $\angle B = \angle D = 91°$ and both are obtuse.

(iv) If $\angle A$ and $\angle C$ are obtuse, $\angle B$ and $\angle D$ are acute.

**7.** $\angle XYZ = 45°$

$\angle XOZ = 90°$

$\therefore$ Reflex $\angle XOZ$ = a = $360° - 90° = 270°$

**8.** The two parallel chords could be on either side of the centre of the circle or on the same side of the centre of the circle.

Case I: (See in Fig *A*)

The two parallel chords lie on either side of the centre of the circle.

$MN = MO + ON$

$MO = 20$ ($\because$ $AM = 15$, $OA = 25$)

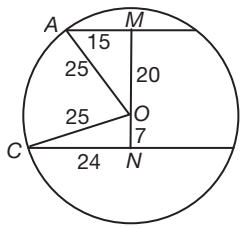



**Fig: A**

$ON = 7$ ($\because$ CN = 24, OC = 25)

$\therefore$ MN = 27

Case II: See Fig *B*

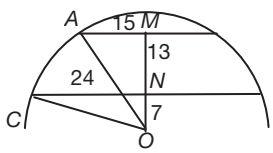
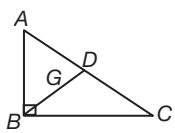



**Fig: B**

$MN = MO - NO = 20 - 7 = 13$

**9.** *ABC* is a right angled triangle, *D* is the circumcentre

$\therefore$ $BD = 15$ cm




$GD = \frac{1}{3}(BD) = \frac{15}{3}\text{ cm} = 5\text{ cm}$

**10.** $\angle ABP = 180° - \angle ABQ$

$= 180° - 130° = 50°$

From the alternate segment theorem,

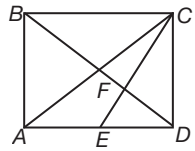
$\angle ACB = \angle ABP = 50°$.

**11.** In triangle *ACE*, let $\angle ACE$ be $\alpha$.

Then $\angle CED = 4\alpha$.

$\therefore$ $\angle CEA = 180 - 4\alpha$.

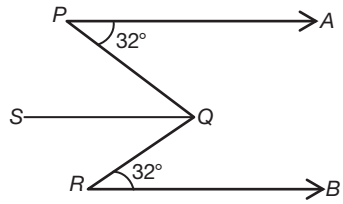
In $\Delta ACE$, $\alpha + 180 - 4\alpha + 45 = 180$




$\Rightarrow \alpha = 15°$

$\therefore$ $\angle CFB = \angle FCD + \angle FDC = 30 + 45$, i.e., 75°.

**12.**




Draw *SQ* parallel to *RB*

$\angle SQP = \angle APQ = 32°$

$\angle SQR = \angle QRB = 52°$

$\therefore$ $\angle PQR = \angle PQS + \angle SQR$

$= 32° + 52° = 84°$.

**13.** Given $2\angle 8 = 3\angle 1$

$\angle 8 + \angle 1 = 180°$

$\Rightarrow \frac{3}{2}\angle 1 + \angle 1 = 180° \Rightarrow \angle 1 = 72°$ and $\angle 8 = 108°$

$\Rightarrow \angle 8$ and $\angle 4$ are corresponding angles

$\angle 8 = \angle 4 = 108°$.

**14.** $\angle PQY + \angle PYQ = \angle QPA$ (The exterior angle of a triangle is equal to the sum of the interior angles opposite to it)

$\angle PQY = 120° - 40° = 80°$

$\therefore \angle ZQR = 180° - 80° = 100°$

$\therefore \angle BZX = \angle RQZ + \angle ZQR$

$= 100° + 25° = 125°$

**15.** As $BD$ and $CE$ are angle bisectors of $\angle B$ & $\angle C$,

$\angle OBC = \angle OCB = 25^\circ$ ($\angle B = 50^\circ$, $\angle B = \angle C$)

$\angle COD = \angle OBC + \angle OCB = 25^\circ + 25^\circ = 50^\circ$.

**16.** $\angle BAD = \angle ABD$ (since $AD = BD$) $\therefore \angle BAD = 30^\circ$

$\angle EAC = \angle ECA$ (since $AE = EC$) $\therefore \angle EAC = 40^\circ$

$\angle BAC = 180^\circ - (\angle ABC + \angle ACB) = 180^\circ - (30^\circ + 40^\circ) = 110^\circ$

But, $\angle BAC = \angle BAD + \angle DAE + \angle EAC$

$110^\circ = 30^\circ + \angle DAE + 40^\circ$

$\therefore \angle DAE = 40^\circ$.

**17.** In a parallelogram, the sum of the adjacent angles is $180^\circ$.

$\Rightarrow x + 20 + x - 40 = 180^\circ \Rightarrow x = 100^\circ$

Opposite angles are equal.

$x + 20 = y + 10$

$100 + 20 = y + 10 \Rightarrow y = 110^\circ$

**18.** Let the sides of the triangle be $a$, $b$, $c$.

Let $a > b > c$.

$a$, $b$, $c$ are integers and have a sum of 18.

Each side of any triangle must be less than its semi perimeter. $\therefore$ a is less than 9. Also a is more than $\left(\frac{18}{3}, \text{i.e.}\right) 6$.

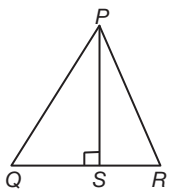
$\therefore$ a is either 7 or 8

If a is 7, ($b$, $c$) can be only (6, 5).

If a is 8, ($b$, $c$) can be only (7, 3) or (6, 4).

Three triangles exist satisfying the given conditions.

**19.** Let the required radius be $R_1$ cm.




Area of $\Delta PQR = \frac{1}{2}(PS)(QR) = \frac{(PQ)(PR)(QR)}{R_1}$

$R_1 = \frac{(PQ)(PR)}{2PS} = \frac{(19.5)(14)}{2(6)}$ cm $= 22.75$ cm.

**20.** $\angle TPS + \angle PSV = 180^\circ$ ($\because TQ \parallel VR$ and interior angles on the same side are supplementary).

$2\angle TPS = 3\angle PSV$

$\therefore \angle PSV = \frac{2}{3}\angle TPS$. $\therefore \angle TPS + \frac{2}{3}\angle TPS = 180^\circ$

$\angle TPS = 108^\circ$

$PQRS$ is a cyclic quadrilateral.

$\therefore \angle QRS = \angle TPS$. [$\because \angle QRS + \angle SPQ = 180^\circ$ and $TPS + \angle SPQ = 180^\circ$]

$\therefore \angle QRS = 108^\circ$.

**21.** The third side = 80 − (16 + 30) = 34 m

The three sides are 16 m, 30 m and 34 m which form triplet of a right angled triangle.

$\therefore$ The area $= \frac{1}{2} \times 16 \times 30 = 240$ sq.m

The largest side is hypotenuse = 34 m

Let the length of the altitude from the opposite vertex onto the hypotenuse be '$a$'

Then the area $= \frac{1}{2} \times a \times 34$

$240 = a \times 17 \Rightarrow a \approx 14.1$ m

$\therefore$ The length of the attitude is 14.1 m. (approx.)

**22.** Let the third side be $x$ cm. There are two possibilities which are dealt with below.

Possibility 1: Third side is the longest side.

Is this case, $x^2 > 7^2 + 13^2 = 49 + 169 = 218$

$\therefore x \geq 15$

By triangle inequality, $x < 7 + 13 = 20$

$\therefore x$ can be 15, 16, 17, 18 or 19.

Possibility 2: The side which is 13 cm is the longest side

In this case, $13^2 > 7^2 + x^2$

$\therefore x^2 < 120$, i.e., $x \leq 10$.

By triangle inequality, $13 < 7 + x$

$\therefore x > 6$

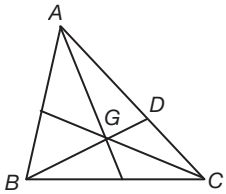
$\therefore x$ can be 7, 8, 9 or 10.

Total number of possible values of $x$ is 5 + 4 or 9.

$\therefore$ 9 possibilities exist.

**23.** A median divides a triangle into two equal parts. The three medians divide it into six triangles of equal area.

$\therefore$ Area of triangle $ABG = \frac{1}{3}$ (area of triangle $ABC$)




**24.** $\angle CAD = \angle CDA = 20°$ (Since $AC = CD$)

$\therefore \angle BAD = 25° + 20° = 45°$

$\therefore \angle BCD = 360° - (40° + 45° + 20°) = 255°$

$\therefore \angle BCT = 360° - (255° + 35°)$

$= 360° - 290° = 70°$

**25.** $\angle PTA = 55°$ and $\angle PTO = 90°$

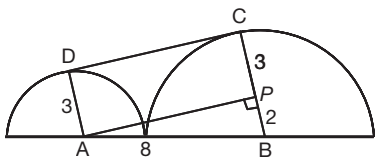
$\therefore \angle ATO = 90° - 55° = 35°$

$OA = OT \Rightarrow \angle ATO = \angle OAT$

$\therefore \angle AOT$

$= 180° - 2(\angle ATO) = 110°$.

**26.** $DCT = \sqrt{D^2 - (r_2 - r_1)^2}$




The direct common tangent

$DC = AP = \sqrt{8^2 - 2^2} = \sqrt{60} = 2\sqrt{15}$

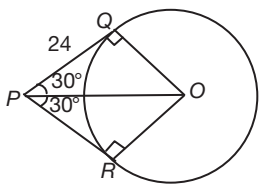
**27.** $\angle DEG = 180° - \angle CDE$, $\angle CDE = \angle BCD = 30°$

$\therefore \angle DEG = 150° \Rightarrow \angle EFJ = 30°$

$\angle DHB = \angle ABC = 30°$

$\therefore$ Required sum = 60°.

**28.**




$\Rightarrow \angle P = 60° \Rightarrow \angle OPQ = \frac{60°}{2} = 30°$

$\tan 30° = \frac{QO}{PQ} \Rightarrow \frac{1}{\sqrt{3}} = \frac{QO}{24}$

$\Rightarrow QO = 8\sqrt{3}$.

**29.** In triangles $PAB$ and $PQR$, $\angle PAB = \angle PQR$ and $\angle PBA = \angle PRQ$

(corresponding angles) and $\angle QAR$ is common.

Since the ratio of areas is 1 : 4, ratio of corresponding sides is $\sqrt{1} : \sqrt{4} = 1 : 2$

$\therefore$ Perimeter of $PAB$ will be half of the perimeter of triangle $PQR$, i.e., 12 cm.

**30.** $\frac{n(n-3)}{2} = 14 \Rightarrow n(n-3) = 28$

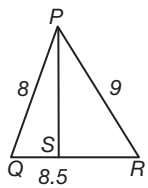
$\Rightarrow n^2 - 3n - 28 = 0$

$\therefore n = 7$ or $-4$

$\therefore n = 7$

Hence, sum of interior angles = $(7 - 2) \times 180°$, i.e., 900°.

**31.** $PS$ bisects $\angle QPR$

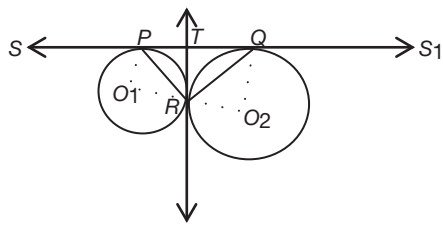



$\therefore \frac{QS}{SR} = \frac{PQ}{PR} = \frac{8}{9}$

Also $QR = QS + SR = 8.5$ cm

$\therefore QS = \frac{8}{8+9}(QS + SR)$ 4 cm.

**32.**




$RT$ is a common tangent to both the circles.

Now, $PT = TR$ and $QT = TR$

Let $\angle RPQ = a$ and $\angle RQP = b$

$\therefore \angle PRT = a$ and $\angle QRT = b$

Now $\angle PRQ = \angle PRT + \angle QRT = a + b$

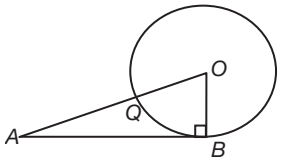
In $\Delta PQR$, $\angle RPQ + \angle RQP + \angle PRQ = 180°$

$\Rightarrow a + b + a + b = 180$

$\Rightarrow a + b = 90°$

$\Rightarrow \angle PRQ = a + b = 90°$.

**33.**




$O$ is the centre of the circle.

$BO = 9$ cm $= r$

Area of $ABO = \frac{1}{2}(BO)(AB)$

$\frac{1}{2}(9)(AB)\ \text{cm}^2 = 54\ \text{cm}^2$

$\Rightarrow AB = 12$ cm

$AO = \sqrt{(AB)^2 + (BO)^2} = 15\,\text{cm}$

**34.** As per Apollonious theorem,

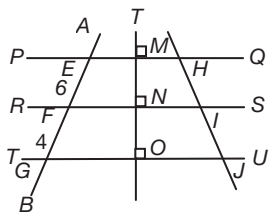
$PQ^2 + PR^2 = 2(PS^2 + QS^2)$

$\therefore QS^2 = \frac{PQ^2 + PR^2}{2} - PS^2 = \left[\frac{121 + 169}{2} - 64\right]\text{cm}^2$

$= 81\ \text{cm}^2$

$\Rightarrow QS = 9$ cm

**35.** For any set of transversals intersecting any particular set of parallel lines, the ratio of intercepts will be the same.




$\frac{EF}{FG} = \frac{MN}{NO} \Rightarrow \frac{EF}{EG} = \frac{MN}{MO}$

$\frac{EF}{EF + FG} = \frac{MN}{MO} = \frac{6}{10} = \frac{3}{5}$.

**36.** $\angle BOC = 360° - [90° + 90° + \angle BAC]$

$= 360° - (180° + 50°) = 130°$

$\angle BDC = \frac{1}{2}\angle BOC = \frac{1}{2}(130°)$

$\therefore \angle BDC = 65°$.

**37.** In $\Delta TUR$ and $\Delta PQR$,

$\angle R$ is common.

$\angle TUR = \angle PQR = 90°$

$\therefore \Delta TUR \sim \Delta PQR$.

$\therefore \frac{TU}{PQ} = \frac{UR}{QR}$ (1)

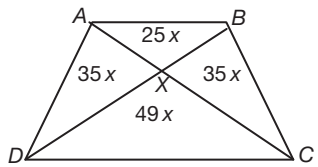
Similarly $\Delta TUQ \sim \Delta SRQ$.

$\therefore \frac{TU}{SR} = \frac{QU}{QR}$ (2)

$\frac{(1)}{(2)} \Rightarrow \frac{SR}{PQ} = \frac{UR}{QU} \Rightarrow \frac{12}{8} = \frac{6}{QU} \Rightarrow QU = 4$

$\Rightarrow \frac{TU}{12} = \frac{4}{10} \Rightarrow TU$ is 4.8 m.

**38.**




$\Delta AXB \sim \Delta DXC$

$\frac{BX}{XD} = \sqrt{\frac{25}{49}} = \frac{5}{7}$

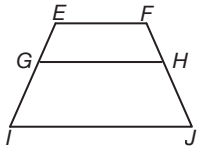
Area of $\Delta ADX = \frac{7}{5}(25x) = 35x$

Ratio of the areas of $\Delta AXD$ and trapezium $ABCD$

$= \frac{35x}{35x + 25x + 35x + 49x} = \frac{35}{144}$

**39.** Let the line through $F$, parallel to $EI$ intersect $GH$ at $K$ and $IJ$ at $L$.

$GH = GK + KH$



$= EF + \frac{1}{4}(LJ) = 12 + \frac{1}{4}(16-12) = 13$

**40.** (i) Right angled triangle.

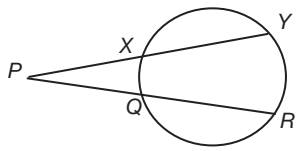
(ii) Scalene, obtuse triangle.

(iii) Right-angled isosceles triangle.

(iv) Scalene, acute triangle.

(v) As $AB + BC = AC$, no triangle is possible. It can also be seen as a degenerate triangle of zero area.

**41.**



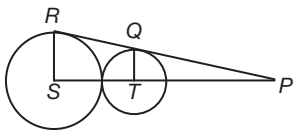

$\frac{PX}{PQ} = \frac{2}{1}$. As $PQ = 4$, it follows that $PX = 8$

$PX.\ PY = PQ.PR$

$(8)\ (8 + 12) = (4)\,(PR)$

$\Rightarrow PR = 40,\ \therefore\ QR = 36$ cm.

**42.**




Given that, $RS : QT = 5 : 3$. Clearly $\Delta PQT$ and $\Delta PRS$ are similar triangles. $\therefore \frac{PT}{PS} = \frac{QT}{RS} \Rightarrow \frac{12}{(12+ST)} = \frac{3}{5}$

$\Rightarrow ST = 8$ cm. $\therefore\ RS = 5$ cm and $QT = 3$ cm.

Now, the length of the common tangent $RQ$

$= \sqrt{(ST)^2 - (RS - QT)^2} = \sqrt{64-4}$.

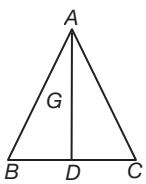
$RQ = 2\sqrt{15}$ cm. $\Rightarrow QP = 3\sqrt{15}$ cm.

(i) Clearly $RQTS$ is a trapezium.

$\therefore$ Its area $= \frac{1}{2}(2\sqrt{15})\ (5+3) = 8\sqrt{15}$ cm$^2$.

(ii) Area of $\Delta PQT = \frac{1}{2}\left(3\sqrt{15}\right)(3) = \frac{9\sqrt{15}}{2}$ cm$^2$.

**43.**




Let the triangle be $ABC$ with $AB = AC$. Let $AD$ be $\perp BC$

$AD = 4\sqrt{5}$.

In an isosceles triangle, the altitude to base is also the median.

$\therefore\ AD$ is the median to $BC$.

In triangle $ABD$,

$BD^2 + AD^2 = AB^2$

$BD^2 + \left(4\sqrt{5}\right)^2 = 12^2$

$\Rightarrow BD = 8$

$\Rightarrow DC = 8$

Hence, perimeter of triangle $ABC$ is 40

**44.** $ABCD$ is a cyclic quadrilateral, and $ABC$ is equilateral. $\therefore$ $\angle ABC = 60^\circ \Rightarrow \angle ADC = 120^\circ$

As $AD = DC$,

$AEC$ is an isosceles triangle.

$\therefore\ \angle CAD = \angle ACD = 30^\circ$.

**45.** In $\Delta PQR$ and $\Delta PRS$,

$\angle QPR = \angle RPS$

$\angle QRP = \angle RSP$

As two pairs of corresponding angles of $\Delta PQR$, $\Delta PRS$ are equal, the third pair of angles must also be equal.

$\therefore\ \Delta PQR$ and $\Delta PSR$ are similar.

Raito of corresponding sides of $\Delta PQR$ and $\Delta PSR$

$= \frac{QR}{SR} = \frac{PQ}{PR} = \frac{PR}{PS}$

$PR^2 = (PS)\ (PQ) = (32)\ (18)$ cm$^2$

$PR = \sqrt{(32)(18)}$ cm $= 24$ cm

Ratio of perimeters of $\Delta PQR$ and $\Delta PRS = \frac{PQ}{PR} = 3 : 4$.

Ratio of perimeters of $\Delta PQR$ and $\Delta PRS = 3 : 4$.

**46.** Since the three angles of triangle $ABC$ are equal to the corresponding angles of triangle $FDE$, the corresponding sides will be proportional,

$\frac{AC}{BC} = \frac{EF}{DE} \Rightarrow \frac{10}{12} = \frac{EF}{3}$

$EF = \frac{10 \times 3}{12} = 2.5$ inches

Ratio of areas $= \left(\frac{AC}{EF}\right)^2\ 16 : 1 = 16$

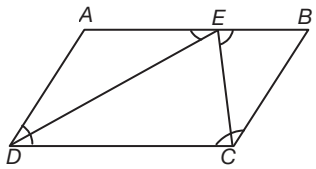
**47.** We know that the area of a quadrilateral is

$= \frac{1}{2} d_1 d_2 \sin\theta$

$= \frac{1}{2}(10)(14)\sin 60° = 70\left(\frac{\sqrt{3}}{2}\right) = 35\sqrt{3}$ sq.units.

$[\because \sin(180° - \theta) = \sin\theta]$

**48.**




$CE$ bisects $\angle C$ and $DE$ bisects $\angle D$.

$\therefore \angle BCE = \angle ECD = \alpha$

and $\angle ADE = \angle EDC = \beta$ (1)

$\angle CEB = \alpha$ and $\angle AED = \beta$ ($\because AB \| DC$, angles)

$\therefore \angle ECD = \angle CEB$ and $\angle EDC = \angle AED$) (2)

(1) and (2) $\Rightarrow \Delta AED$ and $\Delta EBC$ are isosceles triangles.

$\therefore AD = AE$ and $EB = BC$

Since $ABCD$ is a parallelogram, $AD = BC$.

$\therefore AE = EB = AD = BC$

(i) $AE : BC = 1 : 1$.

(ii) $2(AB + BC) = 24$ cm

$\Rightarrow AB + BC = 12$ cm

$\Rightarrow AE + EB + BC = 12$ cm

$\Rightarrow BC + BC + BC = 12$ cm $\Rightarrow BC = 4$ cm.

**49.** $\angle C = 180° - 130° = 50°$

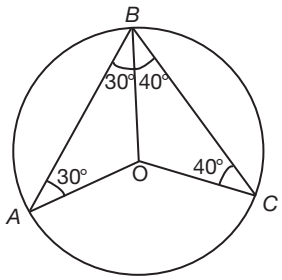
If $AB$ and $AC$ are the equal sides, then $\angle B = \angle C$. $\angle B = 50°$. Also $\angle A = 80°$

If $AC$ and $BC$ are equal sides, then $\angle A = \angle B = 65°$.

If $AB$ and $BC$ are equal sides, then $\angle A = \angle C = 50°$.

Hence, the minimum sum of the angles is 100°.

**50.**




$\angle OBA = \angle OAB = 30°$ [$\because$ isosceles triangle]

$\angle OBC = \angle OCB = 40°$

$\angle ABC = 30° + 40° = 70°$

$\therefore \angle AOC = 140°$ (since $\angle AOC = 2\angle ABC$).

**51.** $a^2 + b^2 + c^2 = 50$ (1)

$d^2 + e^2 + f^2 = 50$ (2)

$ad + be + cf = 50$ (3)

Adding (1) and (2) and subtracting (2) (3) from the result, we get

$(a - d)^2 + (b - e)^2 + (c - f)^2 = 0$

$\therefore a = d$, $b = e$ and $c = f$

$\therefore$ The two triangles are congruent.

$\therefore$ They will have the same perimeter and the same area.

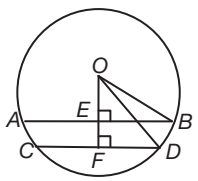
**52.** (14, 48, 50) is a Pythagorean triplet.

$\therefore \angle Q = 90°$. Let the altitude drawn to $PR$ be $QS$.

$\therefore \frac{1}{2} \times 14 \times 48 = \frac{1}{2} \times QS \times 50$

$\Rightarrow QS = 13.44$

**53.**




Let $AB = 40$ cm and $CD = 30$ cm.

As $OB = 25$ cm and $EB = 20$ cm,

$OE = \sqrt{25^2 - 20^2} = 15$ cm

Similarly, $OF = \sqrt{25^2 - 15^2} = 20$ cm

$EF = OF - OE = 5$ cm.

$\therefore$ the chords are 5 cm apart.

**54.** Given that

$D$, $E$ and $F$ are midpoints of $BC$, $CA$ and $AB$

and $P$, $Q$ and $R$ are midpoints of $EF$, $FD$ and $DE$

we know that, Area of $\Delta ABC = 4$ Area of $\Delta DEF$

But area of $ABC = 64$ sq. units

4 Area of $\Delta DEF = 64$

Area of $\Delta DEF = \frac{64}{4}$

Area of $\Delta DEF$ = 16 sq. units.

Area of $\Delta DEF$ = 4 Area of $\Delta PQR$

4 Area of $\Delta PQR$ = 16

Area of $\Delta PQR = \frac{16}{4} = 4$

Area of $\Delta PQR$ = 4 sq. units.

**55.** A polygon having $n$ sides will have $\frac{n(n-3)}{2}$ diagonals.

So, a polygon having 9 sides has $\frac{9(9-3)}{2}$

= 27diagonals.

**56.** The three medians divide the triangle into six triangles of equal areas.

Quadrilateral $CQAG$ consists of two such triangles.

The required ratio is 2 : 6 or 1 : 3

**57.** As two angles $\angle CAB$ and $\angle BCA$ of $\Delta ABC$ are equal to the angles $\angle DAC$ and $\angle ACD$ of $\Delta ACD$, respectively and $AC$ is common to both triangles, they are congruent and hence, their areas are equal.

$\therefore$ Area of triangle $ACD = \frac{1}{2}(72)\ \text{cm}^2 = 36\ \text{cm}^2$.

**58.** In triangle $OAC$,

$\angle OAC = 60°$, $OA = OC$ $\therefore$ $\angle OCA = 60°$ and

$\angle AOC = 60°$ (1)

In triangle $BOC$

$\angle OBC = 70°$, $OB = OC$

$\therefore$ $\angle OCB = 70°$

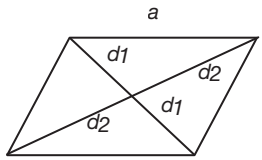
$\therefore$ $\angle COB = 40°$ (2)

$\angle AOB = \angle AOC + \angle COB$

= 60° + 40° (From (1) and (2)) = 100°.

**59.** The maximum distance between any two points on a rhombus is the length of the longer diagonal (say $2d_2$). The shorter one is say $2d_1$ ($2d_1 = 2$)

$\therefore$ $2d_2 = 2\ (2d_1) = 4 \Rightarrow d_2 = 2$




$a = \sqrt{d_1^2 + d_2^2} = \sqrt{1^2 + 2^2} = \sqrt{5}$.

**60.** Let, number of sides = $n$

Now, $\frac{n-2}{n}(180) = 144$

$\Rightarrow 180n - 360 = 144n \Rightarrow 36n = 360$

$\therefore n = 10$

**61.** As angles $ABC$ and $ACB$ are in the ratio 4 : 3, let them be $4\alpha$ and $3\alpha$.

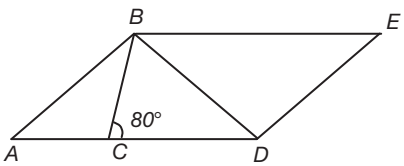
Given, $\angle EAC + \angle ACD = 260°$

$\Rightarrow 7\alpha + 180 - 3\alpha = 260°$

$\Rightarrow \alpha = 20°$

$\angle A = 40°$, $\angle ABC = 80°$

**62.**




From the figure, we get

$\angle ACB = 180° - \angle BCD = 180° - 80° = 100°$

$AC = BC$ (Given) $\Rightarrow \angle CAB = \angle ABC$

$\Rightarrow 2\angle ABC + \angle ACB = 180° \Rightarrow 2\angle ABC = 80°$

$\Rightarrow \angle ABC = \angle CAB = 40°$

As $ABED$ is a parallelogram, angle $DAB$ = angle $DEB = 40°$

Hence, angle $ABE = 140°$. As $BD$ is the bisector of $\angle D$, angle $CDB = 70°$. (Since diagonal bisects the angle).

$\therefore$ angle $\angle DBC = 180° - 80° - 70° = 30°$

**63.** $\frac{n(n-3)}{2} = 20$, where $n$ is the number of sides.

$\Rightarrow n = 8$.

Interior angle $= \frac{(2n-4)}{n}(90°) = \frac{12}{8}(90)$

= 135°

**64.** $\angle BDA = 80°$

$\Rightarrow \angle ADC = 100°$

$\angle DAC = 40°$

$\therefore \angle DCA = 40°$

Since $ABC$ is isosceles, the only two possibilities are $AB = AC$ or $AC = BC$.

I. If $AB = AC$, $\angle ABD = \angle ACD \Rightarrow \angle ABD = 40°$

$\therefore \angle BAD = 180 - (80 + 40) = 60°$.

II. If $AC = BC$, $\angle DBA = \angle BAC = 70°$

$\therefore \angle BAD = 30°$

**65.** (i) $\angle 1 = \angle 3$ (vertically opposite angles)

$\angle 5 = \angle 7$ (vertically opposite angles)

$\angle 2 = \angle 4$ (vertically opposite angles)

$\angle 6 = \angle 8$ (vertically opposite angles)

$\angle 4 = \angle 6$ and $\angle 3 = \angle 5$ (alternate angles)

$\angle 1 + \angle 8 = 180°$ (sum of the exterior angles on the same side of the transversal) (1)

$\angle 3 - \angle 8 = 90°$ given

$\angle 1 - \angle 8 = 90°$ (2) $\{\because \angle 1 = \angle 3\}$

$\Rightarrow$ solving equations (1) and (2),

we get $\angle 1 = 135°$ and $\angle 8 = 45°$

$\therefore \angle 1 = \angle 3 = \angle 5 = \angle 7 = 135°$

and $\angle 2 = \angle 4 = \angle 6 = \angle 8 = 45°$

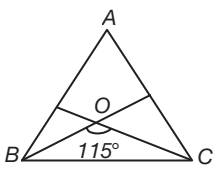
(ii) The perpendicular distances between ($l$ and $m$) and ($m$ and $n$) are in the ratio 3 : 4, so $\frac{AC}{BC} = \frac{3}{4}$.

$\therefore AB = \frac{7}{3}(12)$ cm = 28 cm $\{\because AC = 12$ cm$\}$

**66.** $\angle ABC = 50°$

$\angle AOC = 2\angle ABC = 100°$

$\angle ADC = \frac{\angle AOC}{2} = 50°$




**67.** $\angle BOC = 115°$

$\angle BOC = 90° + \frac{1}{2}\angle A$

$\therefore 115° = 90° + \frac{1}{2}\angle A$

$\angle A = 50°$.

**68.** Each angle of $p = \frac{(n-2)(180°)}{n}$ where $n$ = number of sides of the polygon.

| Number of sides of $P$ | Name of the polygon | Sum of interior angles ($S$) | Each interior angle ($I$) | Each exterior angle ($E$) |
|---|---|---|---|---|
| 5 | Pentagon | 540° | 108° | 72° |
| 9 | Nonagon | 1260° | 140° | 40° |
| 12 | Dodecagon | 1800° | 150° | 30° |

**69.** (i) $\angle A = 35°$ $\angle B + \angle C = 145°$

54 combinations are possible. These are (35°, 91°, 54°), (35°, 92°, 53°),...(35°, 144°, 1°).

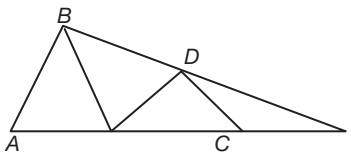
(ii) $\angle A = 35°$, $\angle B + \angle C = 145°$. 71 combinations are possible. These are (35°, 1°, 144°), (35°, 2°, 143°) .. (35°, 34°, 111°), (35°, 36°, 109°), .. (35°, 72°, 73°).

**70.** Let $\angle DFE = y$

As $DE = EF$, $\angle EDF = y$

Hence, $\angle DEF = 180° - 2y$

$\angle DEC = 180° - \angle DEF = 2y$




As $CD = DE$,

$\angle DCE = \angle DEC = 2y$

$\angle CDE = 180° - (\angle DCE + \angle DEC) = 180° - 4y°$

$\angle CDB = 180 - (\angle CDE + \angle EDF)$

$= 180 - (180 - 4y + y) = 3y$

$\angle CBD = \angle CDB = 3y$

$\angle BCD = 180 - 6y$

$\angle BCA = 180 - (\angle BCD + \angle DCE)$

$= 180 - (180 - 6y + 2y) = 4y$

$\therefore \angle BAC = \angle BCA = 4y$

$\angle ABC = 180 - 8y$

$\because \angle ABC = \angle DFE$ (given)

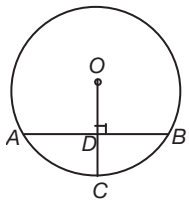
$180 - 8y = y$

$9y = 180$

$y = 20°$ $\therefore \angle DFE = y = 20°$.

## EXERCISE-3

**1.** $OA = OB = 7$ cm




Either $OD : DC = 5 : 2$

(or) $OD : DC = 2 : 5$

If $OD : DC = 5 : 2$,

$OD = 5,\ OB = 7$

$\Rightarrow BD = \sqrt{OB^2 - OD^2} = 2\sqrt{6}$

$AB = 4\sqrt{6}$

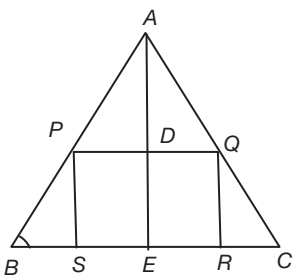
If $OD : DC = 2 : 5$

$OD = 2,\ OB = 7$

$\Rightarrow BD = \sqrt{7^2 - 2^2} = 3\sqrt{5}$ and $AB = 6\sqrt{5}$

But as $6\sqrt{5} > 10$, it should be $4\sqrt{6}$ cm .

**2.**




Area of $APQ = \frac{1}{2}(PQ)(AD)$

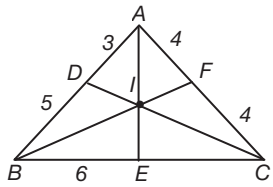
$= \frac{1}{2}(r\,BC)(r\,AE) = \frac{r^2}{2}(BC)(AE)$

Area of $PQRS = (PQ)\ (PS)$

$= (r\,BC)\ [(1 - r)(AE)] = r(1 - r)(BC)(AE)$

$\frac{\text{Area of } APQ}{\text{Area of } PQRS} = \frac{r}{2(1-r)}$

**3.**




In $\Delta AIB$, $ID$ is bisector of $\angle AIB$

$\therefore AD/DB = IA/IB$ (1)

In $\Delta BIC$, $IE$ is bisector of $\angle BIC$

$\therefore BE/EC = BI/IC$ (2)

In $\Delta\ CIA$, $IF$ is the bisector of $\angle CIA$

$\therefore CF/FA = CI/IA$ (3)

Multiplying (1), (2), (3) we get

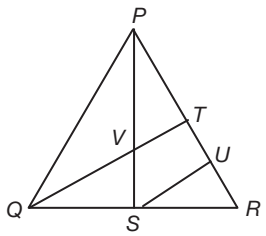
$(AD/DB)\ (BE/EC)\ (CF/FA) = 1$

$\Rightarrow (AD)\ (BE)\ (CF) = (DB)\ (EC)\ (FA)$

$\therefore$ Substituting the values we have

(3) (6) (4) = (5) (4) $(CE) \Rightarrow CE = 3.6$

**4.** Let $U$ be a point on $PR$ such that $SU$ is parallel to $QT$.




As $QT \parallel SU$,

$\frac{PT}{TU} = \frac{PV}{VS} = \frac{4}{3}$ (given) (1)

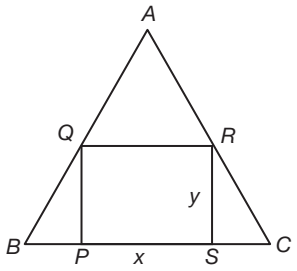
and $\frac{TU}{UR} = \frac{QS}{SR} = \frac{4}{3}$ (given) (2)

From (1), $TU = \frac{3}{4}PT = 6$ cm

From (2), $UR = \frac{3}{4}TU = 4.5$ cm

$\therefore PR = PT + TU + UR = 18.5$ cm

**5.**



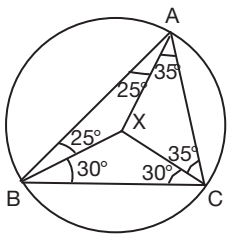

$\angle C = 60°$

Side $CS = \left(\frac{6-x}{2}\right)$ cm

$y = \frac{\sqrt{3}}{2}(6 - x)$

**6.** The point equidistant from the three vertices of a triangle is the circumcentre for the triangle

In the figure below




$XA = XB = XC = R$ (The circumradius)

Now $\angle XAB = \angle XBA = 25°$

$\therefore \angle AXB = 180° - (25° + 25°) = 130°$. Also $\angle BXC = 120°$ (given)

$\therefore \angle AXC = 360° - (120° + 130) = 110°$

The angle subtended by any arc (or chord) at any point on the circle is half the central angle.

$\therefore \angle ABC = 55°$, $\angle ACB = 65°$ and $\angle BAC = 60°$

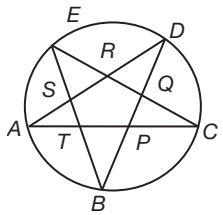
In a triangle, the side opposite the greatest angle is the longest and the side opposite the smallest angle is the shortest [The sides are proportional to the sine of the angle opposite them]. $\therefore AB > BC > AC$.

Note: we can see from the given solution that $\angle AXB > \angle BXC > \angle AXC$ (1)

In a circle, greater the angle subtended at the centre by a chord, the greater would be the chord length.

$\therefore AB > BC > AC$.

**7.**




Since $PQRST$ is a regular pentagon,

$\angle P = \angle Q = \angle R = \angle S = \angle T = 108°$

$\angle RST = 108°$, $\angle AST = 180° - 108° = 72°$

$\therefore$ (A) is false

In triangle $SAT$, $\angle AST = 180° - 108° = 72°$

$\angle SAT = 180° - (72° + 72°) = 36° \therefore$ (B) is false

$\angle BPC = 108°$, $\angle RQP = 108° \therefore$ (C) is false

$\angle SAT = \angle QCP = 36°$

**8.** As $OF$ bisects $AB$, $\angle OFB = 90°$

$OF = OE - FE = r - 2$,

where the radius of the circle is $r$ cm

$OB^2 = OI^2 + FB^2$

$r^2 = (r-2)^2 + (16/2)^2 = (r-2)^2 + 8^2$

$r^2 - (r-2)^2 = 4r - 4 = 64 \Rightarrow r = 17$ cm

In triangles $OFB$ and $OED$,

$\angle OFB = \angle OED = 90°$

(as $CD$ is a tangent of the circle)

$\angle FOB = \angle EOD$ (common angle)

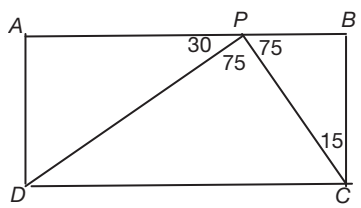
As two angles are equal the third angle of both the triangles must also be equal.

Hence, the triangles $FOB$ and $EOD$ are similar

Hence, $OF/OE = BF/DE$

$DE = (OE/OF)(BF) = (17/15)(8)$ cm $= 136/15$ cm

**9.**



As $\angle APD : \angle DPC : \angle CDB = 2 : 5 : 5$

$\angle APD = 30°, \angle DPC = 75°, \angle CPB = 75°$

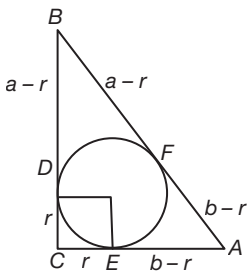
$\therefore \angle PCD = 75°$, and $PD = DC$

In $\Delta\, DAP$, $\angle A = 90°$, $\angle P = 30°$

$\therefore PD = 2AD$ hence, $DC = 2\, AD$

Or $AB : BC = 2$.

**10.**




Let the triangle be $ABC$. Let $\angle C = 90°$. The inradius $r = 9$ and the circumradius $R = 37.5$

$BC$ and $AC$ are tangents to the incircle at the points of contact of the incircle with the sides of the triangle. Let these points be $D, E, F$ as shown.

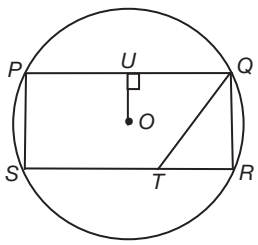
$BD = BF = a - r$, $CD = CE = r$, $AE = AF = b - r$.

$\therefore\ c = a + b - 2r \Rightarrow a + b = 2R + 2r = 2(37.5 + 9) = 93$

Perimeter $= 2(a + b) - 2r$

Perimeter $= 2\ (93) - 2(9) = 168$

**11.**




Let $U$ be the point on $PQ$ such that $OU \perp PQ$

Consider $\Delta OUQ$ and $\Delta TRQ$

$\angle OUQ = \angle TRQ = 90°$ and $\angle URO = \angle RQT$

As two pairs of angles of $\Delta UQO$ and $\Delta RQT$ are equal, the third pair of angles must also be equal.

$\therefore\ \Delta UQO$ is similar to $\Delta RQT$

$$\therefore\ \frac{QR}{TR} = \frac{QU}{OU} = k \text{ (say)} \qquad (1)$$

Area of the rectangle $PQRS = (PQ)\ (QR) = (2UQ) \times (2OU) = 4k\ (OU)^2$

(Q As $O$ is the centre of the circle, $PQ = 2UQ$ and $QR = 2OU$).

Area of the circle $= \pi\ (OU^2\ (1 + k^2))$

Given $\dfrac{\text{Area of rectangle}}{\text{Area of circle}} = \dfrac{2\sqrt{5}}{3\pi}$

$$\therefore\ \frac{4k\,(OU)^2}{\pi\left(k^2+1\right)(OU)^2} = \frac{2\sqrt{5}}{3\pi} \Rightarrow 12k = 2\sqrt{5}\,k^2 + 2\sqrt{5}$$

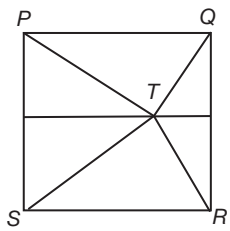
$$\Rightarrow \sqrt{5}\,k^2 - 6k + \sqrt{5} = 0 \Rightarrow (\sqrt{5}\,k - 1)(k - \sqrt{5}) = 0$$

$$\Rightarrow k = \sqrt{5} \text{ or } 1/\sqrt{5}$$

As $PQ > PS$, $UQ > OU$ and $k = QU/OU > 1$.

$\therefore\ k = \sqrt{5}$

**12.**




$\angle QPS = 90^o$ ($\because$ $QPSR$ is a square)

$\angle TPS = 60^o$ ($\because$ $\Delta\, TPS$ is equilateral) (1)

$\therefore\ \angle QPT = 30^o$

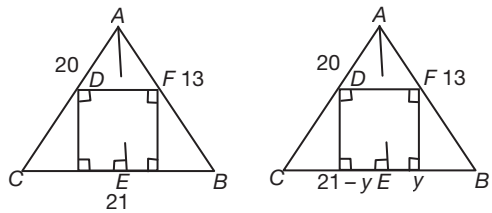
In $\Delta\, PQT$, $PQ = PT$ ($\because$ $PT = PS$ and $PS = PQ$)

$\therefore\ \angle PTQ = \angle PQT = 75^o$ (2)

Similarly $\angle STR = 75^o$ (3)

From (1), (2), (3) $\angle QTR = 150^o$ ($\because$ the sum of all the non overlapping angles at a point is $360^o$).

**13.**




Let the side of the square be $x$

Let $EB = y \therefore CE = 21 - y$

Now, $AE^2 = AB^2 - EB^2 = AC^2 - CE^2$

$\therefore 13^2 - y^2 = 20^2 - (21 - y)^2$

$\Rightarrow (21 - y)^2 - y^2 = 20^2 - 13^2$

$\Rightarrow (21 - 2y)\ (21) = 7\ (33) \Rightarrow y = \dfrac{21-11}{2} = 5$

Thus $AE = \sqrt{13^2 - 5^2} = 12$

$\Delta AGF$ is similar to $\Delta ACB$

$$\frac{AD}{AE} = \frac{GF}{CB}$$

$$\Rightarrow \frac{AE - DE}{AE} = \frac{GF}{BC}$$

$$\Rightarrow \frac{12 - x}{12} = \frac{x}{21}$$

$$\Rightarrow 252 - 21x = 12x$$

$$\therefore x = \frac{252}{33} = \frac{84}{11}$$

The side of the square is $\dfrac{84}{11}$ units.

**14.** From the figure $PA$ and $PB$ are the maximum and the minimum distances of the point $P$ from the circle. $\overline{PD}$ is the tangent from $P$ to the circle.

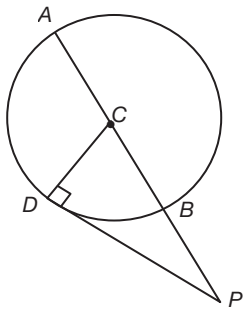
Given, $PA = a$, $PB = 5$, $PD = 10$

$AB = PA - PB = a - 5$

Radius of the circle:

$$CD = CB = \frac{a-5}{2}, \quad PC = PB + BC$$

$$= 5 + \frac{a-5}{2} = \frac{5+a}{2}$$




In right $\Delta PDC$,

$PC^2 = PD^2 + CD^2$

$$\left(\frac{5+a}{2}\right)^2 = 10^2 + \left(\frac{a-5}{2}\right)^2 \Rightarrow (a+5)^2 - (a-5)^2 = 2^2(10^2)$$

$$\Rightarrow 4(a)(5) = 2^2(10^2) \Rightarrow a = 20$$

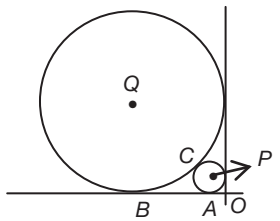
$$\therefore PC = \frac{5+a}{2} = \frac{25}{2} = 12.5$$

**Alternative Solution:**

$(PA)(PB) = PD^2 \Rightarrow 5a = 10^2 \Rightarrow a = 20$

Now, $PC = PB + \dfrac{(PA - PB)}{2} = 5 + \left(\dfrac{20-5}{2}\right) = 12.5$

**15.** Let $Q$ and $P$ be the centres of the bigger and the smaller circles, respectively and $C$ be the point of contact.

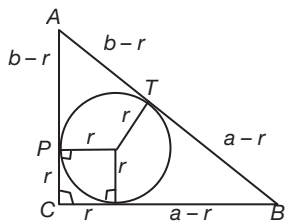



Let $PA = r$ and $QB = R$, $OQ = \sqrt{2}\ QB = \sqrt{2}\ R$ and $OQ = OP + PC + CQ = \sqrt{2}\ r + r + R$

$\therefore R(\sqrt{2} - 1) = r(\sqrt{2} + 1)$

$$\Rightarrow r = \frac{\sqrt{2}-1}{\sqrt{2}+1} R = (3 - 2\sqrt{2})\, R = 4\ (3 - 2\sqrt{2})$$

**16.**




Let $AB = c$, $BC = a$ and $AC = b$

$AP = AT = b - r$ and $BQ = BT = a - r$

$AB = AT + BT = a + b - 2r$

$\therefore c = a + b - 2r \Rightarrow 2r = a + b - c$

or, $2r = a + b + c - 2c$ or $r = \dfrac{a+b+c}{2} - c = s - c$

Since $ABC$ is right angled, $c = 2R$ where $R$ is the circum-radius.

Now, area of the triangle $ABC = rs$

$= r(r + c) = r(r + 2R)$

It is given that the area and the perimeter are equal in magnitude

$\therefore rs = 2s \Rightarrow r = 2\ (\because s \neq 0)$

Now, $r = 20\%\ R \Rightarrow R = 5r = 10$

The area of the triangle = $r(r + 2R) = 2(2 + 20) = 44$.

**17.** As *ABCD* is a rhombus,

*EFGH* is a rectangle.

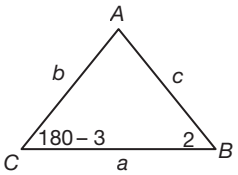
*IJKL* is a rhombus,

*MNOP* is a rectangle,

*QRST* is a rhombus and

Hence, *UVWX* is a rectangle

**18.**




In the triangle $\angle B = 2\angle A$

Let $\angle A$ be equal to $\theta°$

$\angle B = 2\theta°$ and $\angle C = 180° - (\angle A + \angle B) = 180° - 3\theta°$

Now using the sine rule, we get

$$\frac{a}{\sin\theta} = \frac{b}{\sin 2\theta} = \frac{c}{\sin(180° - 3\theta)} = \frac{c}{\sin 3\theta}$$

$$\frac{a}{\sin\theta} = \frac{b}{2\sin\theta\cos\theta} = \frac{c}{\sin 3\theta}$$

$$\frac{a}{\sin\theta} = \frac{b}{2\sin\theta\cos\theta} \Rightarrow \cos\theta = \frac{b}{2a} \qquad (1)$$

Again $\dfrac{c}{3\sin\theta - 4\sin^3\theta} = \dfrac{a}{\sin\theta}$.

$$\therefore \frac{c}{3 - 4\sin^2\theta} = a$$

$$\Rightarrow 3 - 4(1 - \cos^2\theta) = \frac{c}{a} \Rightarrow 4\cos^2 = \frac{c}{a} + 1$$

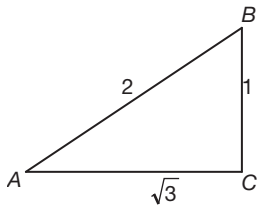
$$\Rightarrow \cos^2\theta = \frac{a + c}{4a} \qquad (2)$$

$$(1), (2) \Rightarrow \left(\frac{b}{2a}\right)^2 = \frac{a + c}{4a} \Rightarrow b^2 = a(a + c) \Rightarrow b^2 - a^2 = ac.$$

**Alternate Solution:**

We can consider one particular triangle in which the condition is satisfied and check for the relations given in the options. Let $\angle A = 30°$, $\angle B = 60°$ and $\angle C = 90°$.

$\therefore a = 1$, $b = \sqrt{3}$, $c = 2$ or $a^2 = 1$, $b^2 = 3$, $c^2 = 4$.




Consider the options.

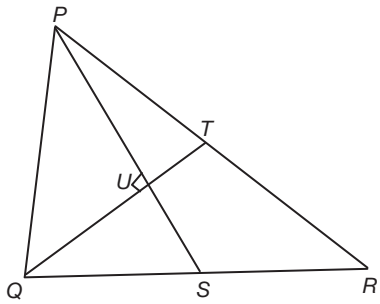
(A) $b^2 - c^2 = -1$ while $ab = \sqrt{3}$

(B) $b^2 - a^2 = 2$ and $ac = 2$

(C) $a^2 - c^2 = -3$ while $bc = 2\sqrt{3}$

(D) $a^2 - b^2 = -2$ while $bc = 2\sqrt{3}$

**19.**




Let *U* be the point of intersection of *PS* and *QT*.

Let $QT = 3x$ and $PS = 3y$.

$QU^2 + US^2 = QS^2$ and $PU^2 + UT^2 = PT^2$

$(2x)^2 + y^2 = 2^2$ and $(2y)^2 + x^2 = \left(\sqrt{5}\right)^2$

Solving these, $x = \sqrt{\dfrac{11}{15}}$ and $y = \sqrt{\dfrac{16}{15}}$

Area of $\Delta QUS = \dfrac{1}{2} \times 2x \times y = 4\dfrac{\sqrt{11}}{15}$

Area of $\Delta PQR = 6\left(\frac{4\sqrt{11}}{15}\right) = \frac{8\sqrt{11}}{5}$

**20.** $AB^2 = AE^2 + EB^2$ (Since diagonals of a rhombus bisect each other perpendicularly)

$AB^2 = AE^2 + 4EF^2$ (1)

$AE = ED = 3$

$AF = 5$

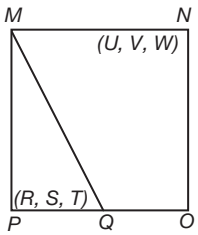
$\therefore EF^2 = AF^2 - AE^2 = 25 - 9 = 16 \Rightarrow EF = 4$

From (1)

$AB^2 = 9 + (4)\ (16) = 73$

$AB = \sqrt{73}$

***Solutions for questions 21 and 22:***




$PN = 16\sqrt{2} = 16\,(1.414) = 22.624.$

**21.** If a point from the group *R, S, T* is joined to a point from the group *U, V, W,* that distance is not less than 20 cm, i.e., the distance is greater than 20 cm.

As the diagonal is 22.624 cm, the group *R, S, T* and the group *U, V, W* will lie close to diagonally opposite vertices, i.e., if *R, S, T* are close to *P*, then *U, V, W* are close to *N*, and if *R, S, T* are close to *M*, then *U, V, W* are close to *O*.

As *R, S, T* cluster near, either *P* or *M*, *RS, ST, RT* are definitely less than *TU* (as $TU \geq 20$ cm).

Hence, option one is necessarily true.

Note: The explanation about the remaining options is as follows:

Option (B) : As explained above, $ST < TU$.

$\therefore$ Option (B) is false

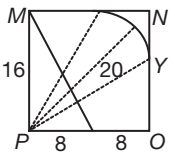
Option (C) : As the cluster *U, V, W* is near either *N* or *O*, we cannot say which of the group of pairs *ST, TR, SR* and *UV, VW, UW* is closest pair of points.

$\therefore$ Option (C) is not necessarily true.

Option (D) : As option (A) is necessarily true, option (D) cannot be the answer.

**22.** The largest possible value of *UV, VW, UW* is to be calculated, that means *U, V, W* must be spread in as large area as possible. This happens when *R, S* and *T* coincides with one of the vertices, either *P* or *M*. As the given distance is not less than 20 cm, draw an arc with *P* (or *M*) as the centre and radius 20 cm. Let this arc cut the sides *MN* and *ON* at *X* and *Y*, respectively.

Now the area bounded between the arc and the straight lines *NX, OY* is the area in which *U, V* lie. The largest possible value for the distance between any two points in this figure is *XY*, which can be calculated from the right angle triangle *XNY*.




$PX = PY = 20$ cm.

$MX^2 = OY^2 = PY^2 - OP^2 = 20^2 - 16^2 = (12)^2$

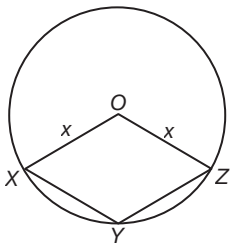
$\therefore MX = OY = 12$ cm.

$\therefore NX = NY = ON - OY = 16 - 12 = 4$ cm.

$\therefore XY^2 = XN^2 + NY^2 = 4^2 + 4^2 = 32$

$\therefore XY = \sqrt{32} = 4\sqrt{2}$ cm.

**23.**




The quadrilateral here is a rhombus

*OX, OY* are the radius of the circle $(= x)$ (say)

Also since $OY = OX = XY$, $\Delta\ OXY$ is an equilateral triangle. The area of the rhombus $= 2(\text{Area of } \Delta\ OXY) = 2(1/4)\left(\sqrt{3}\right)\ (x^2) = 18\sqrt{3}$ cm$^2$ $\therefore\ x = 6$ cm

**24.** $a + b + c = 30$

Each side of a triangle must be less than its semiperimeter $\therefore$ Each of *a, b, c* is less than 15.

Also $a \leq b \leq c$. $\therefore$ c ranges from 10 to 14

When $c$ is 10, $a + b = 20$. $\therefore (a, b) = (10, 10)$

When $c$ is 11, $a + b = 19$. $\therefore (a, b) = (9, 10), (8, 11)$

When $c$ is 12, $a + b = 18$. $\therefore (a, b) = (9, 9), (8, 10), (7, 11), (6, 12)$

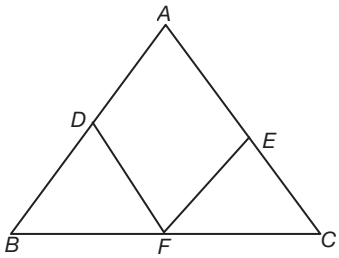
When $c$ is 13, $a + b = 17$. $\therefore (a, b) = (8, 9,), (7, 10), (6, 11), (5, 12), (4, 13)$

When $c$ is 14, $a + b = 16$. $\therefore (a, b) = (8, 8),$

$(7, 9), (6, 10), (5, 11), (4, 12), (3, 13), (2, 14)$

There are 19 such triangles.

**25.**




(1) $DF \parallel AC$ and $EF \parallel AB$.

Since $DF \parallel AC$, triangles $DBF$ and $ABC$ are similar ($\because \angle B$ is common to the two triangles. Corresponding angles are equal)

Since $EF \parallel AB$, triangles EFC and ABC are similar.

Ratio of the areas of DBF and ABC is $\left(\frac{BF}{BC}\right)^2$. Ratio of the areas of EFC and ABC is $\left(\frac{CF}{CB}\right)^2$.

$\left(\frac{CF}{CB}\right)^2 = \frac{64}{400}. = \frac{4}{25} \therefore \frac{CF}{CB} = \frac{2}{5}. \therefore \frac{BF}{BC} = \frac{3}{5}.$

$\therefore$ Ratio of the areas of DBF and ABC is $\frac{9}{25}$. $\therefore$ Area of DBF is 144.

Area of ADFE = 400 – (144 + 64) = 192

**26.** Let the area of triangle $ABC$ be 8.

Area of triangle $ACE$ = Area of triangle $ABE$ = 4.

Area of triangle $CGE = \frac{4}{2} = 2$ = Area of triangle $ADB$

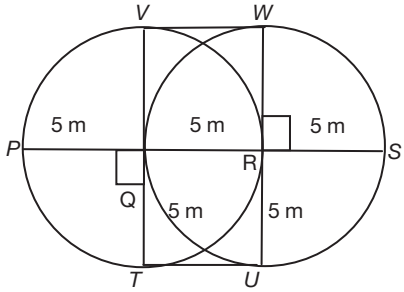
Area of quadrilateral $GEBA = 8 - 2 = 6$

Area of quadrilateral or $GEDA = 6 - 2 = 4$

Area of triangle $ABC$ : Area of Quadrilateral $GEDA$

$= 8 : 4 = 2 : 1$

**27.** Consider the following figure:




The man can go along either of the two shortest ways – $PVWS$ or $PTUS$.

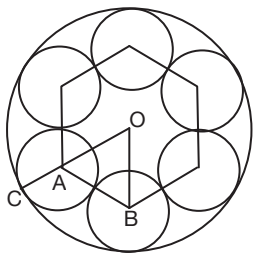
Length of arc $PT = \frac{1}{4} \times 2\pi (5) = 2 \times 5\pi$ metres.

Length of $TU = 5$ metres

Length of $US = \frac{1}{4} \times 2\pi (5) = 2 \times 5\pi$ metres

$\therefore PTUS = 2 \times 5\ \pi + 5 + \ 2 \times 5\pi \ = 5\ (\pi + 1)$ meters.

**28.**




By joining the centres of the small circles, a hexagon is formed. If the centres of adjacent two small circles, $A$ and $B$ and the centre of the big circle $O$ are joined, $\angle AOB = \frac{360°}{6} = 60°$.

Since $OA = OB$, the triangle $OAB$ will be an equilateral triangle.

$AB = (7 + 7)$ cm = 14 cm.

Hence, $OA = OB = 14$ cm

$OA$ extended meets the big circle at $C$.

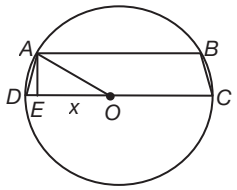
$AC = 7$ cm (radius of the small circle)

$OC = OA + AC = 14 + 7 = 21$ cm

Area of the big circle = $\pi R^2$

$\approx \frac{22}{7}\ (21)\ (21)\ \text{cm}^2 = 1386\ \text{cm}^2$

**29.** Let the trapezium be $ABCD$.




Let $CD$ be the diameter. Let $AE$ be the perpendicular from $A$ to $CD$. Given $AD = \frac{r}{2}$ where $r$ is the radius. Let $O$ be the centre of the circle.

By Pythagoras theorem,

$AD^2 - DE^2 = AE^2$

$AO^2 - OE^2 = AE^2$

$\therefore AD^2 - DE^2 = AO^2 - OE^2$

Let $OE$ be $x$.

$DE = r - x$

$$\left(\frac{r}{2}\right)^2 - (r-x)^2 = r^2 - x^2$$

$r(4x - 7r) = 0.$

As $r$ cannot be 0, $x = \frac{7r}{8}$.

$$\therefore AB = 2(OE) = 2x = \frac{7r}{4}$$

$$AB : CD = \frac{7r}{4} : 2r = 7 : 8$$

**30.** In the triangle $ABC$, $\angle A + \angle B + \angle C = 180°$

Since $\angle A > \angle B > \angle C$, and the three angles are numerically prime numbers, $\angle C = 2°$, the smallest prime number. If $\angle C > 2°$, and all three angles are numerically primes, their sum has to be odd. Hence, only $\angle A$ and $\angle B$ are odd prime numbers.

$\angle B < 88°$ and $\angle A > 88°$

Let $\angle B = 83°$, a prime number.

$\Rightarrow \angle A = 180° - (83° + 2°) = 95°$ not a prime number.

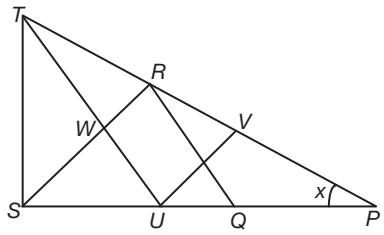
By trial, we get

$\angle B = 71°$; $\angle A = 107°$

Both are prime

$\therefore \angle A + \angle C = 107° + 2° = 109°$

**31.**




Given, $PQ = QR = RS = ST = TU = UV = VP$

Let $\angle QPR = x \Rightarrow \angle QRP = x$ [$QPQ = QR$]

and $\angle VUP = x$ [$Q\ UV = VP$]

$\angle TVU = \angle VUP + \angle VPU = x + x = 2x$ (1)

$RQS = \angle QRP + \angle QPR = x + x = 2x$ (2)

(Since the exterior angle of a triangle at a vertex is equal to the sum of the two opposite interior angles)

$\Rightarrow \angle UTV = \angle UVT = 2x$ [$\therefore\ TU = UV$] (3)

$\Rightarrow \angle RSQ = \angle RQS = 2x$ [$\therefore\ QR = RS$] (4)

$\angle TUS = \angle UPT + \angle UTP = x + 2x = 3x$ (5)

[Since, the exterior angle of a triangle at a vertex is equal to the sum of the two opposite interior angles]

$\angle TSU = \angle TUS = 3x$ [$\therefore\ ST = TU$] (6)

$\angle TRS = \angle RSP + \angle RPS = 2x + x = 3x.$

But, $\angle TRS = \angle RTS$, because $SR = TS$.

$\therefore \angle RTS = 3x$ (7)

Hence, $\angle PTS + \angle TPS + \angle PST = 3x + 2x + 3x = 7x = 180°$

$\Rightarrow 3x + 2x + 3x = 180°$

$$\therefore x = \frac{180}{7} = 25.7° \text{ ; approximated to } 26°.$$

**32.** $EF = \frac{2}{5}(35) + \frac{3}{5}(15) = 14 + 9 = 23$ cm

Area of $ABFE$ : Area of $EFCD$

$$= \frac{1}{2}(35 + 23)\ \frac{3}{5}x : \frac{1}{2}(23 + 15)\ \frac{2}{5}\ x$$

where $x$ is the distance between the parallel lines $CD$ and $AB$,

$$= \left(\frac{1}{2}\right)(58)\left(\frac{3}{5}\right) : \left(\frac{1}{2}\right)(38)\left(\frac{2}{5}\right) = 87 : 38$$

**33.** $(a-b)^2+(b-c)^2+(c-a)^2 \geq 0$

$a^2+b^2+c^2-ab-bc-ca \geq 0$

$a^2+b^2+c^2 \geq ab+ca+bc$

$\frac{a^2+b^2+c^2}{ab+bc+ca} \geq 1$

$a+b>c \Rightarrow ac+bc>c^2$

$b+c>a \Rightarrow ba+ca>a^2$

$c+a>b \Rightarrow bc+ab>b^2$

$\therefore 2(ab+bc+ca) > a^2+b^2+c^2$

$\frac{a^2+b^2+c^2}{ab+bc+ca} < 2$

$1 \leq K < 2$.

**34.** Let the length of the perpendicular sides be $a$ and $b$.

$a^2+b^2 = 34^2 = [(2)(17)]^2 = 2^2(17^2) = 2^2(8^2+15^2)$

$= 16^2+30^2$

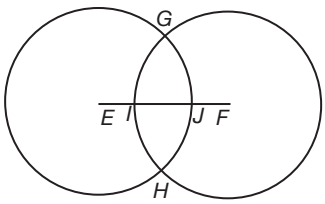
$(a, b)$ can be (16, 30) or (30, 16). In either case, $a+b=46$

$\therefore$ Perimeter = 46 + (hypotenuse)

$= 46 + 34 = 80$

Perimeter can be 80.

**35.** As the circles are equal, the figures formed by the circles is a symmetric one (1)




Suppose the figure was as shown above. As $E$ moves closer to the intersection region, $\angle GEF$ increases. (1) $\Rightarrow$ $F$ must also move closer to the intersection region. Neither circle can have its centre within the intersection region.

$\therefore \angle GEF$ will be maximum when $E$ coincides with $I$, i.e., when $J$ coincides with $F$. In this case, $IJ$ would be the common radius for both circles.

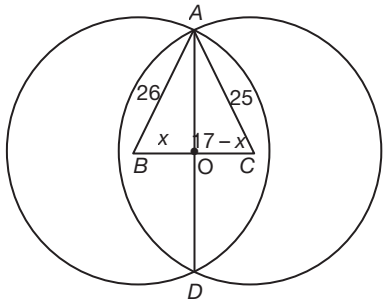
$\therefore GI = IJ = GJ$.

$\therefore \Delta GIJ$ is equilateral

$\therefore \angle GIJ = 60°$ (2)

(2) $\Rightarrow \angle GEF \leq 60°$ only Choice (D) violates this condition.

**36.**




$AB = 26$

$AC = 25$

Let $BO = x$

$OC = 17 - x$

$AO^2 = AB^2 - BO^2 = AC^2 - OC^2$

$\Rightarrow 26^2 - x^2 = 25^2 - (17-x)^2$

$\Rightarrow 34x = 340$

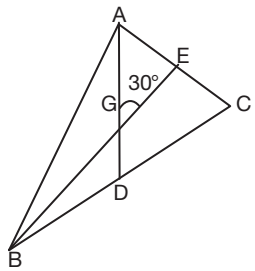
$\Rightarrow x = 10$

$\therefore AO = \sqrt{26^2-10^2} = 24$

Length of the chord,

$AD = 2\ (AO) = 48$

**37.**




$AG = \frac{2}{3}(12) = 8$ and $GE = \frac{1}{3}(18) = 6$

Area of $AGE = \frac{1}{2}(8)(6)\sin 30° = 12$

Area of triangle = 12(6) = 72

***Solutions for questions 38 and 39:***

Triangles $AGF$ and $BGD$ are similar.

$\therefore \frac{AF}{BD} = \frac{AG}{BG}$

**38.** $\frac{AG}{BG} = \frac{5}{4}$

$AG = \frac{5}{4} BG$ (1)

$AF = \frac{5}{4} BD$ (2)

Subtracting (2) from (1),

$CG = \frac{5}{4} EG$

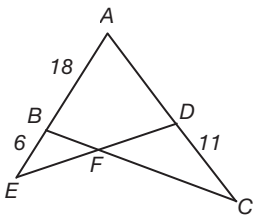
**39.** $AB = 18$, $AG = 10$

And $AC : CG = 3:2$

$\therefore AC = AF = 6$

$GF = \sqrt{AG^2 - AF^2} = \sqrt{10^2 - 6^2} = 8$ cm

**40.** The triangles $AED$ and $ACB$ are similar. Since $\angle A$ is common to the two triangles, $\angle AED = \angle ACB$, so the remaining third angles in the two triangles must also be equal.




$\therefore \frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}$

or $\frac{18}{AD} = \frac{AD+11}{24}$

or $AD^2 + 11AD - 432 = 0$

or, $(AD + 27)\ (AD - 16) = 0$

Since $AD$ is positive, $AD = 16$ cm.

**41.** $AD$ bisects $\angle BAC$

$\therefore \frac{AB}{BD} = \frac{AC}{DC}$ (1)

$\frac{AB}{DC} = \frac{AC}{BD} = 2$ (2)

$\frac{(1)}{(2)} \Rightarrow \frac{DC}{BD} = \frac{BD}{DC} \Rightarrow (DC)^2 = (BD)^2$

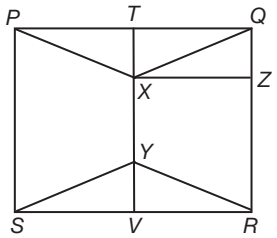
$\therefore DC = BD$.

$(1) \Rightarrow AB = AC$

$(2) \Rightarrow AB = 2DC = DC + DC = BD + DC = BC$

$\therefore \Delta ABC$ is equilateral. Hence, the required sum of the angles is 120°.

**42.**




Triangles $RVY$ and $SVY$ have a common side $VY$.

$\angle YVS = \angle YVR = 90°$

Also $V$ is the mid point of $RS$

$\therefore SV = VR$

$\therefore$ By $RHS$ property, the two triangles are congruent

$\therefore \angle VYR = \frac{\angle RYS}{2} = \frac{120°}{2} = 60°$

$\tan \angle VYR = \sqrt{3} = \frac{VR}{YV}$

$\therefore YV = \frac{VR}{\sqrt{3}} = \frac{\frac{SR}{2}}{\sqrt{3}} = \frac{SR}{2\sqrt{3}}$

$\therefore$ Area of $\Delta SYR = \frac{1}{2}(YV)(SR) = \frac{SR^2}{4\sqrt{3}}$

Similarly area of $\Delta PXQ = \frac{PQ^2}{4\sqrt{3}} = \frac{SR^2}{4\sqrt{3}}$

$\therefore$ Sum of the areas of $\Delta SYR$ and $\Delta PXQ = \frac{SR^2}{2\sqrt{3}}$

Area of $PXQRYS$ = Area of $PQRS$ – (sum of the areas of $\Delta SYR$ and $\Delta PXQ$) $= SR^2 - \frac{SR^2}{2\sqrt{3}} = \frac{SR^2\left(2\sqrt{3}-1\right)}{\sqrt{3}}$

$SR^2 - \frac{SR^2}{2\sqrt{3}} \quad \frac{SR^2\left(2\sqrt{3}-1\right)}{\sqrt{3}}$

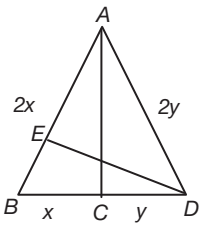
$\therefore$ Required ratio $= \frac{\frac{SR^2\left(2\sqrt{3}-1\right)}{\sqrt{3}}}{\frac{SR^2}{\sqrt{3}}} = 2\sqrt{3} - 1.$

**Alternative Solution:**

$$\frac{\text{Area of } PXQRYS}{\text{Area of } (PXQ + SYR)} = \frac{\text{Area of } XYRQ}{\text{Area of } (XTQ + YVR)}$$

$$\frac{\text{Area of } XYRZ}{\text{Area of } TXZQ} = \frac{XV}{TX} = 2\sqrt{3} - 1$$

(Q If $TX = 1$, $TQ = \sqrt{3}$, $QR = 2\sqrt{3}$ and $ZR = XV = 2\sqrt{3} - 1$).




**43.** Let $BC = x$ and $CD = y$

$AC$ bisects $\angle BAD$

$AB = 2x$, As $\frac{AB}{BC} = \frac{AD}{CD}$,

$AD = \frac{2x}{x}(y) = 2y$

As $AD = BD$, $2y = x + y$

$\Rightarrow y = x$.

$\therefore AB\ (2x) = AD\ (2y) = BD(x + y)$

The triangle $ABD$ is equilateral and as $AC$ and $DE$ are the bisectors $AE = CD$.

Hence, $AE : CD = x : x = 1 : 1$

**44.** Let the number of 150° and 210° angles in the polygon be $x$ and $y$, respectively. Sum of angles in the polygon

$= (150\ x + 210y)°$

As the polygon would have $x + y$ sides, sum of the angles in the polygon $= [180(x + y - 2)]°$

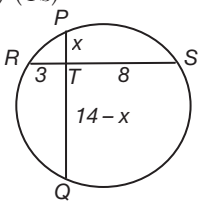
$\therefore\ 150x + 210y = 180\ (x + y - 2)$

$y + 12 = x$ (1)

$x$ has a minimum value of 12, when $y = 0$ as $y$ cannot be negative.

$\therefore$ $x$ cannot be 10

**45.** $(PT)\ (TQ) = (RT)\ (TS)$




$\Rightarrow 24 = x(14 - x)$

where $x = PT$

$24 = 14x - x^2 \Rightarrow x = 2, 12$

Hence, the maximum possible value of $PT^2 = 144$.

**46.** $PQRS$ is a cyclic trapezium. Therefore, it is isosceles. Also $\angle P = \angle R \therefore \angle P = \angle R = 90^0$ irrespective of which two sides are parallel, $\angle Q = \angle S = 90^0$

$PQRS$ is a rectangle.

$$\frac{(PQ)(QR)}{\pi\left(\frac{PR}{2}\right)^2} = \frac{240}{169\pi}$$ ($\because PR$ is the circumdiameter of $PQRS$)

$$\frac{(PQ)(QR)}{\frac{\pi}{4}\left((PQ)^2 + (QR)^2\right)} = \frac{240}{169\pi}$$

$$\frac{(PQ)(QR)}{(PQ)^2 + (QR)^2} = \frac{60}{169}$$

$60\ (PQ)^2 - 169\ (PQ)\ (QR) + 60\ (QR)^2 = 0$

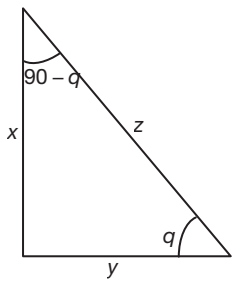
$(5\ PQ - 12\ QR)\ (12\ PQ - 5\ QR) = 0$

$$\frac{PQ}{QR} = \frac{12}{5} \text{ or } \frac{5}{12}$$

Ratio of the perimeter of $PQRS$ and sum of the diagonals of $PQRS = \frac{2(PQ + QR)}{2PR} = \frac{PQ + QR}{PR}$

$= \frac{12K + 5K}{13K} = \frac{17}{13}$

**47.**




The area of a right angled triangle $= \frac{1}{2}xy$

Where $x$ and $y$ are the lengths of the perpendicular sides and $z$ is the length of hypotenuse.

We have $x = z\sin\theta$, $y = z\cos\theta$,

$$\text{Area} = \frac{xy}{2} = \frac{z^2 \sin\theta\cos\theta}{2}$$

$$\Rightarrow \frac{xy}{2} = \frac{z^2}{4}(2\sin\theta\cos\theta) = \left(z^2 \sin 2\theta\right)/4$$

As $z$ is a fixed quantity, area depends on the value of $\sin(2\theta)$.

$\sin(2\theta)$ is maximum when $2\theta = 90°$, or $\theta = 45°$.

Hence, area is maximum when it is a isosceles right triangle.

**48.** As $DA^2 = (CA)\ (AE)$

$$\Rightarrow \left(\frac{DA}{AE}\right) = \left(\frac{CA}{AD}\right) \text{and } \angle A \text{ is common,}$$

$\Delta DAE$ and $\Delta CAD$ are similar.

As $\angle ADE = \angle ACD$,

$\angle ACD = 40°$

In $\Delta\ ABC\ \angle A + \angle B + \angle C = 180°$

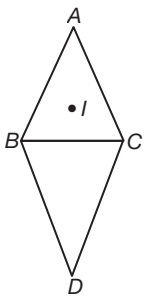
$\Rightarrow \angle B = 180° - 40° - 110° = 30°$

**49.** $AC^2 + BD^2 = 2(AB^2 + BC^2)$.

$\therefore 2(AB^2 + BC^2) = 10^2 + 12^2$ sq. units

and $AB^2 + BC^2 = 122$ sq.units.

**50.**




$BC$ bisects $\angle ABD$ and $\angle ACD$

Hence, $\angle ABC = \angle CBD = x°$

Also, $\angle ACB = \angle BCD = y°$

Hence, $\angle BDC = 180° - (x° + y°)$

In $\Delta BAC$, $\angle BAC = 180° - (x° + y°)$

$\therefore \angle BDC = \angle BAC = z°$ (let us say) (1)

It is given that $\angle BIC = 5\ \angle BDC$

$$\Rightarrow 90 + \left(\frac{z}{2}\right) = 5z\ ; \Rightarrow 90 = \frac{9z}{2}\ ,$$

$\Rightarrow z = 20; \Rightarrow BAC = 20°$

**51.** Let the larger and the smaller angles (in degrees) be $l$ and $s$, respectively.

$l + s = 180°\ \therefore\ l = 180° - s$

$$\frac{1}{4}\ l = 45 - \frac{s}{4}$$

$$\therefore \frac{3}{5}\ s - \left(45 - \frac{s}{4}\right) = 23°$$

$$OR\ (45 - \frac{s}{4}) - \frac{3}{5}\ s = 23°$$

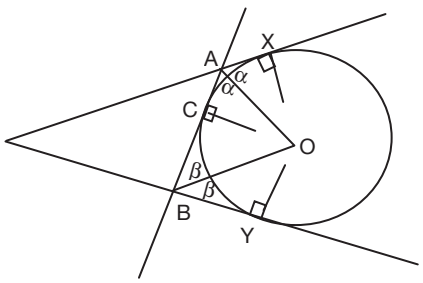
$$\frac{17}{20}\ s = 68° \text{ or } \frac{17}{20}\ s = 22°$$

$$s = 80° \text{ or } s = \frac{440°}{17}$$

$$\therefore\ l = 100° \text{ or } l = 180° - \left(\frac{440}{17}\right)^{\circ}$$

$\therefore\ l$ can be 100°.

**52.**




$\angle\ APB = 50°$

$\angle\ OXA = \angle OYB = 90°$ (since radius drawn to the point of contact is perpendicular to the tangent)

In the quadrilateral $PXOY$, $\angle XOY = 360° - 50° - 90° - 90° = 130°$

$\angle CAO = \angle OAX = \alpha$ (say) and $\angle CBO = \angle OBY = \beta$(say)

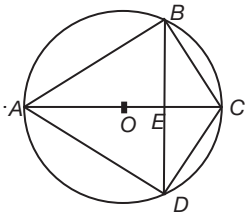
In the pentagon $OXABY$, $\angle XAC + CBY = 540 - (130° + 90° + 90°)$

$2\alpha + 2\beta = 230°$

$\Rightarrow \alpha + \beta = 115°$

$\angle AOB = 180° - (\alpha + \beta) = 180° - 115° = 65°$

**53.**




If an equilateral triangle is inscribed in a circle, the centre of the circle is the incentre (also the orthocentre, centroid and circumcentre) of the triangle. Hence, $AC$ bisects $\angle A$ or $\angle BAO = 30°$ As $\angle ABC$ is an angle in a semicircle, $\angle ABC = 90°$.

$\therefore$ Angles in $\Delta ABC$ (and similarly $\Delta ADC$) are 30°, 60° and 90°, and the ratio of sides $BC : AB : AC = 1 : \sqrt{3} : 2$

$\therefore$ The required ratio $= \dfrac{AB + BC + CD + DA}{AC}$

$= \dfrac{\sqrt{3} + 1 + 1 + \sqrt{3}}{2} = 1 + \sqrt{3}$

**54.** $\angle BOC = 2\angle BAC = 2\ (30°) = 60°$

$\angle OBC = \angle OCB = \dfrac{180° - BOC}{2} = 60°$

Hence, the triangle $BOC$ is equilateral and $BC = 12$

$DE^2 = (DC)\ (DB)$

$DE = \sqrt{4(4+12)}$ cm = 8 cm

**55.** From the chord $CD$, $\angle CAD = \angle CBD$.

Let $\angle CAD = \theta$.

In triangle $ABQ$, $\angle AQB + \angle BAQ + \angle ABQ = 180°$

$\Rightarrow 20° + 60° + \theta + 60° + \theta = 180°$ [$\because \angle BAP = \angle ABP = 60°$].

$\Rightarrow \theta = 20°$

In triangle $BDQ$, $\angle BQD = \angle DBQ = 20°$

$\therefore DQ = BD = 7$ cm.

**56.** In a cyclic quadrilateral, if $a$, $b$, $c$ and $d$ are the lengths of the four consecutive sides and the diagonals are $d_1$ and $d_2$, then $d_1\ d_2 = ac + bd$ (Ptolemy's theorem)

$\Rightarrow d_2 = \dfrac{(10)(12) + (14)(15)}{15} = 22$

**57.** $\dfrac{7\sin Q + 4\sin R}{6\sin Q - 2\sin R} = \dfrac{13}{6}$

$42\ \sin Q + 24\ \sin R = 78\ \sin Q - 26\ \sin R$

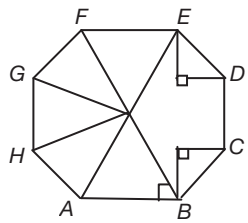
$50\ \sin R = 36\ \sin Q \Rightarrow \dfrac{\sin R}{\sin Q} = \dfrac{18}{25}$

From Sine rule,

$\dfrac{\sin R}{\sin Q} = \dfrac{PQ}{PR}$

$\therefore \dfrac{PQ}{PR} = \dfrac{18}{25}$. As $PQ = 18$, $PR = 25$.

**58.** Each internal angle in a regular octagon is 135°. Also $\angle ABE = 90°$




$\therefore AE = \sqrt{AB^2 + BE^2}$

$BE = CD + BC\ \text{Sin}45° + ED\ \text{Sin}45°$

$= 10 + \dfrac{10}{\sqrt{2}} + \dfrac{10}{\sqrt{2}} = 10\left(\sqrt{2} + 1\right)$

$AE = \sqrt{10^2 + \left(10\left(\sqrt{2} + 1\right)\right)^2} = 10\sqrt{4 + 2\sqrt{2}}$

**59.** I: Let the area of the triangle be $A$.

Let the sides to which the altitudes 9, 12, 16 are drawn be $a$, $b$, $c$, respectively

$\dfrac{1}{2}$ (a) (9) $= \dfrac{1}{2}$ $(b)$ (12) $= \dfrac{1}{2}$ $(c)$ (16) $= A$

$a = \dfrac{2A}{9}$, $b = \dfrac{2A}{12}$, $C = \dfrac{2A}{16}$

Sum of any two of $a$, $b$, $c$ is more than the third I is true

II: Let the area of the triangle be $A$

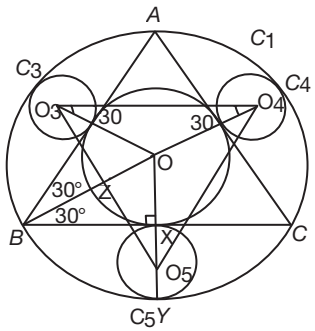
The sides of the triangle are $\frac{2A}{8}, \frac{2A}{12}, \frac{2A}{15}$

Sum of any two of the sides is more than the third

II is true.

***Solutions for questions 60 and 61:***

**60.**




Let the radius of $C_1 = OB = R$.

Radius of $C_2 = OB \sin 30° = R/2$.

Radius of $C_5$ or $C_4$ or $C_3 = \frac{OY - OX}{2} = \frac{R - \frac{R}{2}}{2} = \frac{R}{4}$.

Side of equilateral triangle $O_3\ O_4\ O_5$

$= \sqrt{3}\ OO_3 = \sqrt{3}\ \left(\frac{R}{2} + \frac{R}{4}\right) = \frac{3\sqrt{3}}{4} R$.

Area of $\Delta O_3O_4O_5 = \frac{\sqrt{3}}{4}\left(\frac{3\sqrt{3}}{4} R\right)^2 = \frac{27\sqrt{3}}{64} R^2$.

Note: $C_1$ is the circumcircle of $\Delta ABC$ while $C_2$ is the incircle of $\Delta ABC$. $\therefore$ The ratio of their radii = 2 : 1.

**61.** Sum of perimeters of $C_3$, $C_4$, $C_5$ / perimeter of $C_2$ $= \frac{1+1+1}{2} = \frac{3}{2}$

**62.** In a triangle $ABC$ let $D$, $E$ and $F$ be the midpoints of $BC$, $CA$ and $AB$, respectively. Then

$$AB^2 + AC^2 = 2AD^2 + \frac{BC^2}{2} \quad (1)$$

$$BC^2 + AC^2 = 2CF^2 + \frac{AB^2}{2} \quad (2)$$

$$AB^2 + BC^2 = 2BE^2 + \frac{AC^2}{2} \quad (3)$$

Adding (1), (2), (3) we get $3(AB^2 + BC^2 + CA^2)$

$= 4(AD^2 + BE^2 + CF^2)$

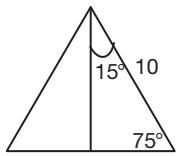
In any triangle, three times the sum of the squares of its sides is equal to four times the sum of the squares of its medians.

$\therefore$ Required sum $= \frac{4}{3}(36)\ \text{cm}^2 = 48\ \text{cm}^2$

**63.** In the right-angled triangle, if $FH$ is the perpendicular drawn the hypotenuse $EG$, then the following relations hold (1) $FH^2 = EH.\ HG$ (2) $EF^2 = EH.EG$ and (3) $FG^2 = GH.EG$.

$\therefore$ All the statements are true.

**64.** Side of the polygon $= 2(10) \cos 75°$




$= 2(10)\ \frac{\sqrt{3}-1}{2\sqrt{2}}$

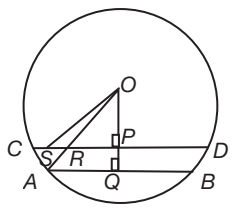
$= 5\left(\sqrt{6} - \sqrt{2}\right)$

Perimeter of polygon

$= 60\left(\sqrt{6} - \sqrt{2}\right)$.

**65.** $OP = AQ = x$ (say) and $PQ = y$ (say) $\therefore PR = \frac{16}{5}(PQ) = \frac{16y}{5}$ and $SR = \frac{4y}{5}$

$\angle OPC = \angle OQA = 90°$ (since the line joining the center of a circle and the midpoint of a chord is perpendicular to the chord)




As $\Delta OAQ$ is similar to $\Delta ORP$, $\frac{AQ}{OQ} = \frac{RP}{OP}$

i.e., $\frac{x}{x+y} = \frac{16/5\,y}{x}$

$\Rightarrow 5x^2 = 16xy + 16y^2$

$\Rightarrow 5x^2 - 16xy - 16y^2 = 0$

$\Rightarrow 5x^2 - 20xy + 4xy - 16y^2 = 0$

$\Rightarrow (x - 4y)\ (5x + 4y) = 0$

$\therefore x = 4y\ [\because x, y > 0]$

In $\Delta OSP$, $\tan \angle OSP = \dfrac{OP}{SP} = \dfrac{OP}{SR + RP}$

$= \dfrac{x}{\frac{4}{5}y + \frac{16}{5}y} = \dfrac{x}{4y} = \dfrac{4y}{4y} = 1\ [\because x = 4y]$

$\therefore \tan \angle OSP = 1$

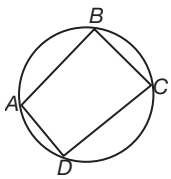
$\therefore \angle OSP = 45°$.

**66.** $AC = 10,\ BC = 6$

$\Rightarrow AB = \sqrt{AC^2 - BC^2} = 8$

$AC = 10,\ AD = 5$

$\Rightarrow DC = \sqrt{AC^2 - AD^2} = 5\sqrt{3}$




In a cyclic quadrilateral whose sides are $a$, $b$, $c$, $d$ and whose diagonals are $d_1$ and $d_2$,

$d_1 d_2 = ac + bd$ (Ptolemy's theorem)

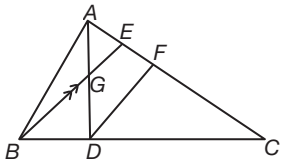
As $ABCD$ is a cyclic quadrilateral,

$(AC)\ (BD) = 8\left(5\sqrt{3}\right) + 6(5) = 30 + 40\sqrt{3}$

$BD = 3 + 4\sqrt{3}$

**67.** Construct $DF$ parallel to $BE$.

$\Delta AGE$ is similar to $\Delta ADF$ and $\Delta CDF$ is similar to $\Delta CBE$.




$\dfrac{AG}{GD} = \dfrac{3}{4} \qquad \therefore \dfrac{AE}{EF} = \dfrac{3}{4}$

$\dfrac{BD}{DC} = \dfrac{4}{7} \qquad \therefore \dfrac{EF}{FC} = \dfrac{4}{7}$

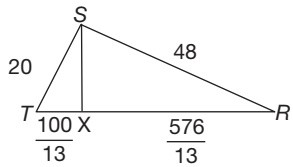
$\therefore AE : EF : FC = 3 : 4 : 7$

As $AE$ (3 parts) is 12 cm, $EC$ (11 parts) is 44 cm.

**68.** (20, 48, 52) is a pythagorean triplet.

$\therefore RT$ is the hypotense of $\Delta RST$ $SX$ is the altitude drawn to $RT$

$\therefore SX^2 = RX.\ TX$ (or)

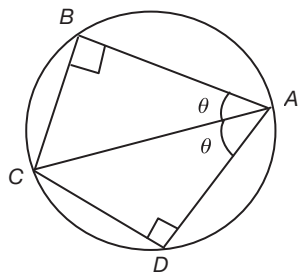



$\dfrac{1}{2}\ (SX)\ (RT) = \dfrac{1}{2}\ (RS)\ (ST)$

$\Rightarrow SX = (240/13)$

$\therefore RX.TX = \left(\dfrac{240}{13}\right)^2$.

**69.**

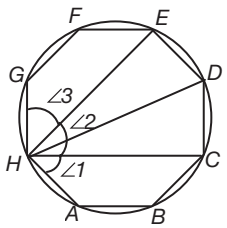



$\cos\theta = \dfrac{AB}{AC} = \dfrac{\sqrt{3}}{2}$

$\theta = 30°$

$\angle BCD = 360° - 180° - 60° = 120°$.

**70.**



In the cyclic quadrilateral $AHCB$, $\angle 1 + \angle B = 180°$

$\angle 2 + \angle D = 180°$ (In the cyclic quadrilateral $DCHE$).

$\angle 3 + \angle F = 180°$ (In cyclic quadrilateral $EFGH$).

$\angle 1 + \angle B + \angle 2 + \angle D + \angle 3 + \angle F = 540°$

But $\angle H = \angle 1 + \angle 2 + \angle 3$

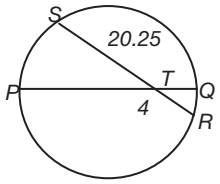
$\Rightarrow \angle B + \angle D + \angle F + \angle H = 540°$

## EXERCISE-4

**1.** $PQ$ and $RS$ are chords intersecting at $T$.

$\therefore (PT)\ (TQ) = (RT)\ (TS)$

$\therefore (PT)\ (TQ) = (4)\ (20.25) = 81$ (1)




$\angle PRQ = 90° \Rightarrow PQ$ is the diameter of the circle.

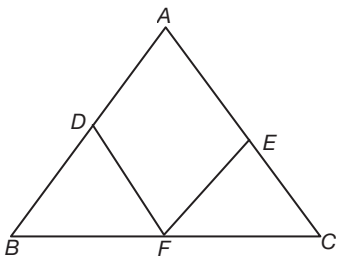
$PT = QT + 24$ cm (2)

Solving (1) and (2), $PT = 27$ cm and $TQ = 3$ cm

$\therefore PQ = 30$ cm

Circumference of the circle $= \pi(PQ) = 30\pi$ cm

**2.**




(1) $DF \parallel AC$ and $EF \parallel AB$.

Since $DF \parallel AC$, triangles $DBF$ and $ABC$ are similar ($\because \angle B$ is common to the two triangles. Corresponding angles are equal)

Since $EF \parallel AB$, triangles $EFC$ and $ABC$ are similar.

Ratio of the areas of $DBF$ and $ABC$ is $\left(\frac{BF}{BC}\right)^2$. Ratio of the areas of $EFC$ and $ABC$ is $\left(\frac{CF}{CB}\right)^2$.

$\left(\frac{CF}{CB}\right)^2 = \frac{64}{400} = \frac{4}{25} \therefore \frac{CF}{CB} = \frac{2}{5}. \therefore \frac{BF}{BC} = \frac{3}{5}.$

$\therefore$ Ratio of the areas of $DBF$ and $ABC$ is $\frac{9}{25}$. $\therefore$ Area of $DBF$ is 144.

Area of $ADFE = 400 - (144 + 64) = 192$

**3.** $d^2 + e^2 - f^2 = 2ab - a^2 - b^2 + c^2 = c^2 - (a-b)^2$

For any triangle, each side must be more than the difference of the other two sides.

$c > |a - b| \Rightarrow c^2 > (a - b)^2$

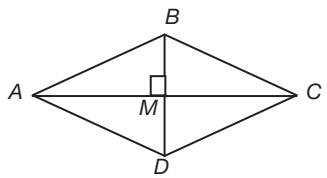
$d^2 + e^2 - f^2 > 0$ i.e, $d^2 + e^2 > f^2$

$\therefore$ The angle opposite to $f$ in $\Delta DEF$, i.e., $\angle F$ is acute.

Similarly, it follows that the angles $\angle D$ and $\angle E$ are acute.

$\therefore \Delta DEF$ is an acute-angled triangle.

**4.**

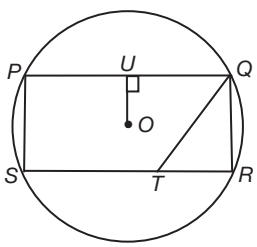



Let the diagonals intersect at $M$.

As $AC = (\sqrt{2} + 1)\ BD$, it follows that $\frac{AM}{EM} = \sqrt{2} + 1$

Or $\frac{BM}{AM} = \sqrt{2} - 1$. $\therefore \angle BAM = 22.5°$ and $\angle ABD = 67.5°$.

**5.**




Let $U$ be the point on $PQ$ such that $OU \perp PQ$

Consider $\Delta OUQ$ and $\Delta TRQ$

$\angle OUQ = \angle TRQ = 90°$ and $\angle URO = \angle RQT$

As two pairs of angles of $\Delta UQO$ and $\Delta RQT$ are equal, the third pair of angles must also be equal.

$\therefore \Delta UQO$ is similar to $\Delta RQT$

$$\therefore \frac{QR}{TR} = \frac{QU}{OU} = k \text{ (say)} \qquad (1)$$

Area of the rectangle $PQRS$ = $(PQ)$ $(QR)$ = $(2UQ) \times (2OU) = 4k\,(OU)^2$

(Q As $O$ is the centre of the circle, $PQ = 2UQ$ and $QR = 2OU$).

Area of the circle = $\pi\,(OU^2\,(1 + k^2))$

$$\text{Given } \frac{\text{Area of rectangle}}{\text{Area of circle}} = \frac{2\sqrt{5}}{3\pi}$$

$$\therefore \frac{4k\,(OU)^2}{\pi\left(k^2+1\right)(OU)^2} = \frac{2\sqrt{5}}{3} \Rightarrow 12k = 2\sqrt{5}\,k^2 + 2\sqrt{5}$$

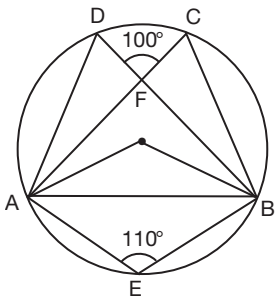
$$\Rightarrow \sqrt{5}\,k^2 - 6k + \sqrt{5} = 0 \Rightarrow (\sqrt{5}\,k - 1)\,(k - \sqrt{5}) = 0$$

$$\Rightarrow k = \sqrt{5} \text{ or } 1/\sqrt{5}$$

As $PQ > PS$, $UQ > OU$ and $k = QU/OU > 1$.

$\therefore k = \sqrt{5}$

**6.**




Reflex $\angle AOB = 2$

$\angle AEB = 2\,(110°) = 220°$

$\angle AOB = 360° - 220° = 140°$

$$\angle ACB = \angle ADB = \frac{140°}{2} = 70°$$

Also, $\angle DFC = \angle AFB = 100°$

$\angle DFA = \angle CFB = 180° - 100° = 80°$

Then $\angle DAF = 180° - (70° + 80°) = 30°$

As $AOB$ is an isosceles triangle,

$$\angle OAB = \frac{1}{2}\,(180° - \angle AOB) = 20°$$

Now, $\angle DAF + \angle CAO + \angle OAB = 65°$

$\Rightarrow \angle CAO = 65° - (30° + 20°) = 15°$

**7.** Let the sides of the triangle be $a$ cm, $b$ cm and $c$ cm where $a < b < c$. All of $a$, $b$, $c$ are integers

$a + b + c = 60$

Any side of any triangle must be less than its semi-perimeter.

$\therefore a, b, c < 30 \qquad (1)$

The greatest side of a scalene triangle must be more than one third the perimeter

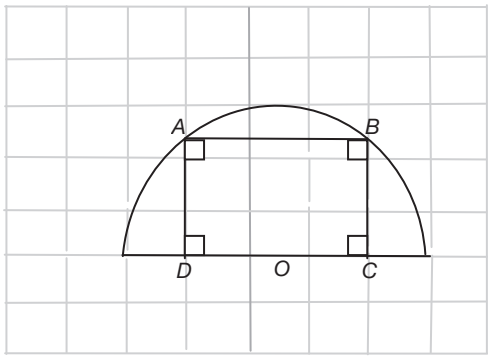
$\therefore c > 20 \qquad (2)$

The table below shows the possibilities for $(a, b)$ for each possible value of $c$.

| $c$ | $(a, b)$ |
|---|---|
| 21 | (19, 20) |
| 22 | (17, 21) (18, 20) |
| 23 | (15, 22)...(18, 19) |
| 24 | (13, 23)...(17, 19) |
| 25 | (11, 24)...(17, 18) |
| 26 | (9, 26)...(16, 17) |
| 27 | (7, 26)…(16,17) |
| 28 | (5, 27)...(15, 17) |
| 29 | (3, 28)...(15, 16) |

The total number of triangles of the required kind is $1 + 2 + 4 + 5 + 7 + 8 + 10 + 11 + 13 = 61$

**8.** Let $O$ be the centre of the semicircle.




$OA = OB$ = Radius

Triangles $OAD$ and $OBC$ are congruent (by $RHS$)

$\therefore OD = OC$ and $\angle DOA = \angle COB$. Let $\angle COB$ be $\theta$

$OD = OC =$ (Radius) $(\cos\theta)$ and $AD = BC =$ (Radius) $(\sin\theta)$

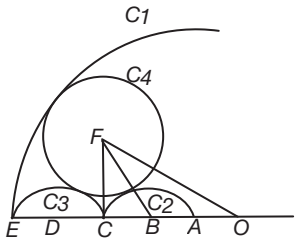
Area of $ABCD = 2(OD)(AD) = 2(\text{Radius})^2 (\sin\theta)(\cos\theta)$

$= 45^2 (\sin 2\theta)$

The area is maximum when $\sin 2\theta$ is 1

$\therefore$ Maximum area is 2025.

**9.** Let the centre of $C_4$ be $F$ and the radius be $r$.

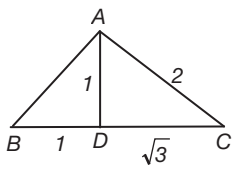



In $\Delta BCF$, $BC = 3$, $BF = r + 3$. $\therefore$ $FC^2 = r^2 + 6r$ (applying $FC^2 + BC^2 = BF^2$)

In $\Delta OCF$, $OC = 9$, $OF = 15 - r$.

$\therefore$ $81 + (r^2 + 6r) = r^2 - 30r + 225 \Rightarrow r = 4$

**10.**




Let $AD$ be the perpendicular drawn from $A$ to $BC$.

$\angle ABC = 45°$, $\angle BAD = 45°$, $\angle DAC = 60°$

$BD = AD = \dfrac{AB}{\sqrt{2}} = 1.$

Triangle $ADC$ is a $30° - 60° - 90°$ triangle.

$\therefore$ $AD : DC : AC = 1 : \sqrt{3} : 2$. $\therefore$ $DC = \sqrt{3}$, $AC = 2$

Perimeter of $ABC = 3 + \sqrt{3} + \sqrt{2}$

**11.** $AB$ and $CD$ are the parallel sides. $CD$ is the longer parallel side or the shorter parallel side.

$\angle C$ and $\angle D$ are complementary, i.e., $\angle C + \angle D = 90°$

$\therefore$ Both are acute.

Hence, $CD$ must be the longer parallel side.

If $\angle C = \angle D$, each would be $45°$. Then the offsets of the perpendiculars drawn from $A$ and $B$ to $CD$ will be equal. Also each of these would be equal to the height of $ABCD$.

If $\angle C = \angle D$, $EF$ would be equal to the height of $ABCD$.

$\therefore$ $EF =$ Each perpendicular offset $= \dfrac{CD - AB}{2}$

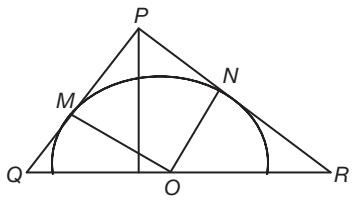
$3 = \dfrac{CD - 11}{2}$

$CD = 17$

It can be shown that even if $\angle C \neq \angle D$, $CD = 17$.

**12.** Let the semicircle touch $PQ$ at $M$ and $PR$ at $N$, and let $OM = ON = r$. From the sine rule, $\sin Q / \sin R = PR / PQ$

Area of $\Delta PQR = \dfrac{1}{2}(PQ)(QR)\sin\angle Q = \dfrac{1}{2}(PR)(RQ)\sin\angle R$.




Also $\sin Q = r / OQ$, $\sin R = r / OR$

$\therefore$ $\dfrac{\sin Q}{\sin R} = \dfrac{OR}{OQ}$

$\therefore$ $\dfrac{OR}{OQ} = \dfrac{PR}{PQ} \Rightarrow \dfrac{OQ + 5}{OQ} = \dfrac{24}{18} = \dfrac{4}{3} \Rightarrow OQ = 15$. $\therefore OR = 20$ and $QR = 35$

**13.** In any triangle, $p^2 + q^2 + r^2 = \dfrac{3}{4}(a^2 + b^2 + c^2)$ (1)

In the given triangle $p^2 + q^2 = \dfrac{9}{20}(a^2 + b^2)$ (2)

And $c^2 = \dfrac{1}{5}(a^2 + b^2)$ (3)

In (1), we substitute for $p^2 + q^2$ (using (2))

Thus $\dfrac{9}{20}(a^2 + b^2) + r^2 = \dfrac{3}{4}(a^2 + b^2) + \dfrac{3}{4}c^2$

$\Rightarrow r^2 = \dfrac{3}{4}c^2 + \dfrac{3}{10}(a^2 + b^2) = \dfrac{3}{4}c^2 + \dfrac{3}{10}(5c^2) = \dfrac{9c^2}{4}$

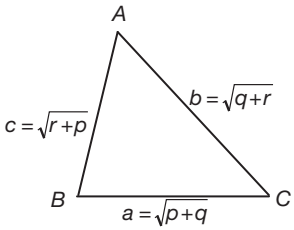
i.e., $\dfrac{r^2}{c^2} = \dfrac{9}{4}$

**14.** Let $ABC$ be the triangle with sides $a$, $b$ and $c$.

From the cosine rule, $\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

$= \dfrac{(p+q)+(q+r)-(r+p)}{2\sqrt{(p+q)(q+r)}}$

$= \dfrac{q}{\sqrt{(p+q)(q+r)}}$




$\therefore \cos^2 C = \dfrac{q^2}{(p+q)(q+r)}$

$1 - \cos^2 C = 1 - \dfrac{q^2}{(p+q)(q+r)}$

$\Rightarrow \sin^2 C = \dfrac{pq+pr+qr}{(p+q)(q+r)}$

$\therefore \sin C = \dfrac{\sqrt{(pq+qr+pr)}}{\sqrt{(p+q)(q+r)}}$

From the sine rule $\dfrac{c}{\sin C} = 2R$ where $R$ is the circumradius of the $\Delta ABC$.

$\therefore R = \dfrac{c}{2\sin C} = \dfrac{\left(\sqrt{r+p}\right)\sqrt{(p+q)(q+r)}}{2\sqrt{(pq+qr+pr)}}$

$= \dfrac{\sqrt{(p+q)(q+r)(r+p)}}{2\sqrt{(pq+qr+pr)}}$

The circumference of circumcircle = $2\pi R$

$= \pi\sqrt{\dfrac{(p+q)(q+r)(p+r)}{(pq+qr+rp)}}$

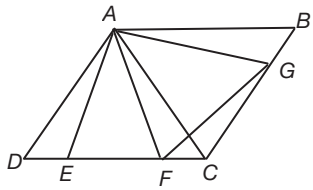
Let $p = q = r = 1$. Then we have an equilateral triangle whose side is $\sqrt{2}$.

$\therefore$ Circumradius of this triangle = $\dfrac{\sqrt{2}}{\sqrt{3}}$.

$\therefore$ Circumference of the circumcircle = $2\pi\dfrac{\sqrt{2}}{\sqrt{3}}$.

Only choice (C) evaluates to this value when $p = q = r = 1$.

**15.**




$\Delta ABG \cong \Delta ACF$ ($AB = AC$ in the given rhombus, $BG = CF$, $\angle B = \angle ACF$)

$\therefore AG = AF$ and $\angle BAG = \angle CAF$ ($\therefore \angle GAF = \angle GAC + \angle BAG = 60°$)

$\Delta AFG$ is isosceles with the vertex angle equal to 60° $\therefore$ It is equilateral, i.e., $FG = AF$ (1)

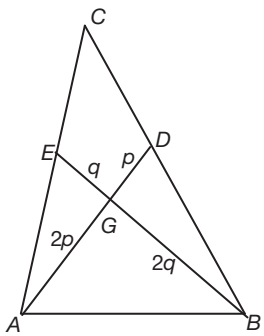
$\Delta ADE \cong \Delta ACF$ ($AD = AC$ in the given rhombus, $\angle D = \angle ACF$, $DE = CF$)

$\therefore AE = AF$ (2)

$\therefore FG = AE$ (from (1) and (2))

$\therefore r = \dfrac{AE}{FG} = 1$

**16.**




Two of the medians are perpendicular. The lengths of these medians are given. We can find the lengths of the sides. In $\Delta ABC$, let the length of the median $AD$ be $3p$, and of median $BE$ be $3q$.

$\therefore c^2 = 4(p^2 + q^2)$, i.e., $\dfrac{c^2}{4} = p^2 + q^2$

$\dfrac{a^2}{4} = p^2 + 4q^2$

$\frac{b^2}{4} = 4p^2 + q^2$

As $p = 1.5\sqrt{105}$ and $q = 12$,

$\therefore \frac{c^2}{4} = 2.25(105) + 144 = 236.25 + 144 = 380.25$

$\frac{a^2}{4} = 2.25\ (105) + 4(144) = 236.25 + 576 = 812.25$

$\frac{b^2}{4} = 9(105) + 144 = 945 + 144 = 1089$

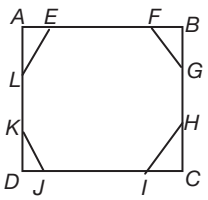
i.e., $\frac{c}{2} = 19.5$, $\frac{a}{2} = 28.5$, $\frac{b}{2} = 33$

The semiperimeter is 81 and the perimeter is 162.

**17.** The octagon is equiangular

$\therefore$ Each exterior angle of the octagon is $\frac{360°}{8}$, i.e., 45°

The octagon can be obtained by cutting four right angled isosceles triangles from the corners of a rectangle. Let *ABCD* be this rectangle and *EFGHIJKL* be the octagon.




Suppose $LK = GH = 14$

$AL + KD = BG + HC$.

$\frac{LE}{\sqrt{2}} + \frac{KJ}{\sqrt{2}} = \frac{FG}{\sqrt{2}} + \frac{HI}{\sqrt{2}}$, i.e., $LE + KJ = FG + HI$

This is only possible if *LE, KJ, FG, HI* are 4, 8, 12, 16 in some order. Also $LE + KJ$ and $FG + HI$ must be 20 each.

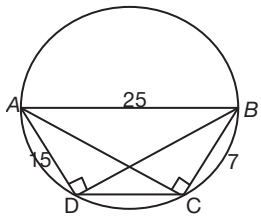
If $LE = 4$, $KJ = 16$. As *AB* must equal *CD*, the only possibility is $(EF, FG, HI, IJ)$ is $(16\sqrt{2}, 8, 12, 8\sqrt{2})$. $\therefore AB = 22\sqrt{2}$ and $BC = 14 + 10\sqrt{2}$

Even if $EF = IJ = 14$, the rectangle will have dimensions of $22\sqrt{2}$ and $14+10\sqrt{2}$. But in any other case, we do not obtain the equiangular octagon.

Area of the octagon = Area of *ABCD* – (Total area of the right angled isosceles triangles) = $(22\sqrt{2})\ (14+10\sqrt{2})$

$- \frac{1}{2}\left(\left(2\sqrt{2}\right)^2 + \left(4\sqrt{2}\right)^2 + \left(6\sqrt{2}\right)^2 + \left(8\sqrt{2}\right)^2\right) = 308\sqrt{2} + 320.$

**18.**




In the $\Delta ABC$, $\angle BCA = 90°$

$\therefore AC = \sqrt{AB^2 - BC^2} = \sqrt{25^2 - 7^2}$ or 24 cm

In $\Delta ADB$, $\angle ADB = 90°$

$\therefore BD = \sqrt{AB^2 - AD^2} = \sqrt{25^2 - 15^2}$ or 20 cm

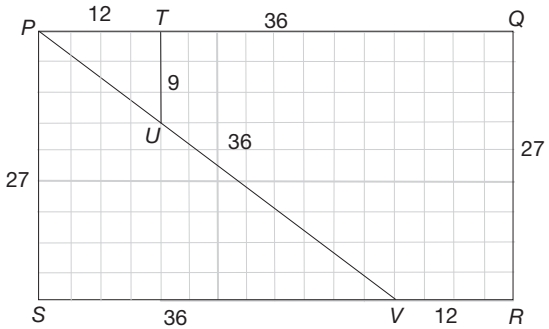
Now, since *ABCD* is a cyclic quadrilateral,

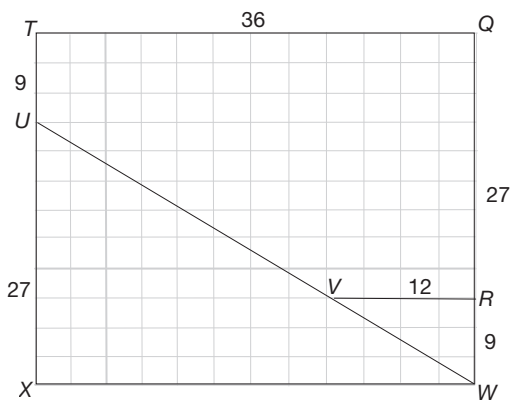
$AB\ (CD) + AD\ (BC) = AC\ (BD)$ (Ptolemy's Theorem)

*OR*, $25\ (CD) + 15(7) = 24\ (20)$

Or, $CD = \frac{375}{25}$ or 15 cm

**19.**

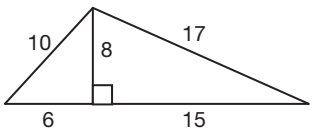


Let us slide $\Delta PSV$ along $PV$ to the right until $PS$ is in line with $TU$. Then by placing the triangle $PTU$ such that $PU$ coincides with $VW$, $PT$ coincides with $VR$ and $TU$ coincides with $RW$, we will end up with the square $TQWX$.

The side of $TQWX$ is 36. $\therefore$ Its perimeter is 144.

**20.**

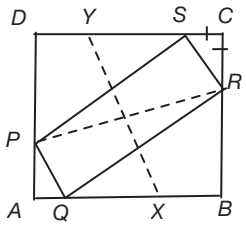



If the triangle is to be divided into two triangular regions by a single cut, the cut must pass through a vertex.

If the sum of the perimeters of the two triangles thus formed is to be the least, the cut itself should be of the smallest possible length; i.e., it should be the altitude to the longest side, which can be seen to be 8 cm.

$\therefore$ The minimum possible sum of the perimeters of the two regions is $[10 + 17 + 21 + 2(8)]$ cm = 64 cm

**21.**




As $AP = AQ = CR = CS$, each acute angle in $\Delta APQ$ and $\Delta CRS$ is 45°. Similarly each acute angle in $\Delta DPS$ and $\Delta BQR$ is 45°, *i.e*, $PQRS$ is a rectangle.

Also as $AP = CR$ and $AP \parallel CR$,

$APCR$ (not shown) is a parallelogram and the diagonals bisect each other, i.e., the midpoint of $PR$ is the centre of the square.

If $U$ is a point on $AB$ such that $BU = AP$, $\Delta PAU \cong \Delta UBR$ and $PU = UR$, i.e., $U$ is equidistant from $P$ and $R$

(no other point on $AB$ can be equidistant from $P$ and $R$)

$\therefore$ The perpendicular bisector of $PR$ (say $L$) intersects $AB$ at $U$.

$U$ lies on $L$ and $AB$

$X$ lies on $L$ and $AB \Rightarrow X = U$

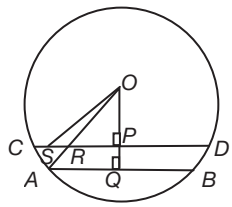
Also $DY = BX$ and

$$\frac{XB + YD}{AP + CR} = \frac{2XB}{2AP} = \frac{XB}{AP} = 1.$$

**Alternately,** we can assume $P$, $Q$, $R$ and $S$ as the midpoints of the respective sides and we'll get the same result as above.

**22.** $OP = AQ = x$ (say) and $PQ = y$ (say) $\therefore PR = \frac{16}{5}(PQ) = \frac{16y}{5}$ and $SR = \frac{4y}{5}$

$\angle OPC = \angle OQA = 90°$ (since the line joining the center of a circle and the midpoint of a chord is perpendicular to the chord)




As $\Delta OAQ$ is similar to $\Delta ORP$, $\frac{AQ}{OQ} = \frac{RP}{OP}$

i.e., $\frac{x}{x+y} = \frac{16/5\,y}{x}$

$\Rightarrow 5x^2 = 16xy + 16y^2$

$\Rightarrow 5x^2 - 16xy - 16y^2 = 0$

$\Rightarrow 5x^2 - 20xy + 4xy - 16y^2 = 0$

$\Rightarrow (x - 4y)\ (5x + 4y) = 0$

$\therefore x = 4y\ [\because x, y > 0]$

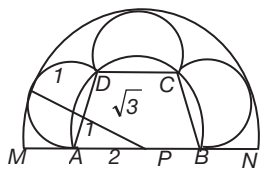
In $\Delta OSP$, $\tan \angle OSP = \frac{OP}{SP} = \frac{OP}{SR + RP}$

$$= \frac{x}{\frac{4}{5}y + \frac{16}{5}y} = \frac{x}{4y} = \frac{4y}{4y} = 1\ [\because x = 4y]$$

$\therefore \tan \angle OSP = 1$

$\therefore \angle OSP = 45°$.

**23.**



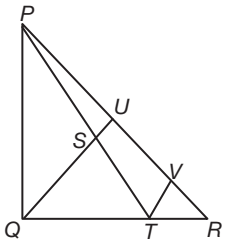

$ABCD$ is an isosceles trapezium with $AD = DC = CB = 2$ and $AB = 4$. The radius of each of the 3 small semicircles is 1,

while that of the big semicircle (on $MN$) is $1+\sqrt{3}$

$\therefore$ The required ratio

$$= \frac{3(1)}{1+\sqrt{3}} = \frac{3}{(\sqrt{3}+1)} \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)} = \frac{3\sqrt{3}-3}{2}$$

**24.**




Let $V$ be the point on $PR$ such that $QU \parallel TV$.

Then $\frac{PS}{ST} = \frac{PU}{UV}$ and $\frac{QT}{TR} = \frac{UV}{VR}$

$$2PS = 3ST \Rightarrow \frac{PS}{ST} = \frac{3}{2} \therefore \frac{PU}{UV} = \frac{3}{2} \quad (1)$$

$$QT = 2TR \Rightarrow \frac{QT}{TR} = 2 \therefore \frac{UV}{VR} = \frac{2}{1} \quad (2)$$

(1) and (2) $\Rightarrow PU : UV : VR = 3 : 2 : 1$

Let $VR = x$. Then $UV = 2x$ and $PU = 3x$

$\therefore PR = 6x$ and $UR = 3x$

$$QT = \frac{4}{3\sqrt{2}} UR = \frac{2}{3} PQ \text{ (given)}$$

$$\therefore QT = 2\sqrt{2}\, x \text{ and } PQ = \sqrt{2}\ UR = 3\sqrt{2}\, x \text{ and}$$

$$TR = \sqrt{2}\, x \left[\because TR = \frac{QT}{2}\right]$$

$$\therefore QR = 3\sqrt{2}\, x$$

$PQ = QR = 3\sqrt{2}\, x \therefore \Delta PQR$ is isosceles

$PQ^2 + QR^2 = 36x^2$

$PR^2 = 36x^2 \therefore PQ^2 + QR^2 = PR^2$

$\therefore \Delta PQR$ is right angled

**Alternate Solution:**

We see 4 relations.

The first gives $PS : ST$. (1)

The second gives $QT : TR$. (2)

The third relates $UR$ to $TR$. (3)

The fourth relates $PQ$ to $TR$. (4)

We should be able to get the sides of $\Delta PQR$ in terms of $TR$ (say)

$(2) \Rightarrow QT = 2a. \therefore QR = 3a.$

Let $V$ be the point on $PR$ such that $TV \parallel QU$.

$$\frac{PU}{UV} = \frac{PS}{ST} = \frac{3}{2}$$

Also $\frac{PU}{UV} = \frac{PS}{ST} = \frac{3}{2}$

$\therefore PU : UV : VR = 3 : 2 : 1. \therefore PU = UR = 3VR$

$$(3) \Rightarrow UR = \frac{3TR}{\sqrt{2}} = \frac{3}{\sqrt{2}}\ a$$

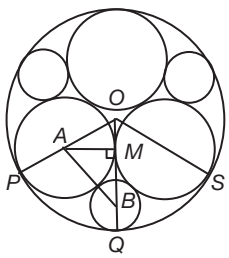
$$\therefore PU = \frac{3}{\sqrt{2}}\ a \text{ and } PR = 3\sqrt{2}\ a$$

$(4) \Rightarrow PQ = 3a$

$\therefore$ In $\Delta PQR$, $PQ = 3a$, $QR = 3a$ and $PR = 3\sqrt{2}\ a$.

It is an isosceles right-angled triangle.

**25.**




In the above figure, let the radii of the big circles be $R$ and that of the small circles be $r$.

In $\Delta AMB$, $AB^2 = AM^2 + MB^2$ $[\because \angle AMB = 90°]$

$AB = R + r$, $AM = R$ and $MB = OQ - OM - BQ$

$$(\because \angle POS = \frac{360}{3} = 120° \text{ by symmetry})$$

Also by symmetry $\angle AOM = \frac{\angle POS}{2} = 60°$

$\therefore OM = \frac{AM}{\sqrt{3}} = \frac{R}{\sqrt{3}}$ and $OA = \frac{2R}{\sqrt{3}}$

$\therefore PO = OQ = OA + PA = \frac{2R}{\sqrt{3}} + R = \frac{R(2+\sqrt{3})}{\sqrt{3}}$

Thus, $MB = \left(\frac{2R}{\sqrt{3}} + R\right) - \frac{R}{\sqrt{3}} - r = \frac{R(\sqrt{3}+1)}{\sqrt{3}} - r$

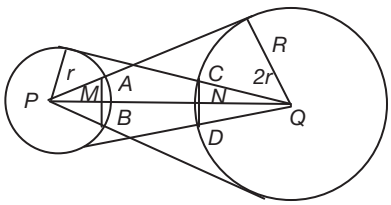
$\therefore (R+r)^2 = R^2 + \left(R\frac{(\sqrt{3}+1)}{\sqrt{3}} - r\right)^2$

$2Rr = R^2\ \frac{(4+2\sqrt{3})}{3} - 2\ Rr\ \frac{(\sqrt{3}+1)}{\sqrt{3}}$

$\Rightarrow r\ \frac{(2\sqrt{3}+1))}{\sqrt{3}} = R\frac{(2+\sqrt{3})}{3}$

$\frac{OQ}{BQ} = \frac{R(2+\sqrt{3})}{\sqrt{3}r} = \mathbf{1 + 2\sqrt{3}}$

**26.**




The radius of the first circle is $r$. Let the radius of the second be $R = 2r$. Let the line $PQ$ intersect $AB$ and $CD$ at $M$ and $N$, respectively. It can be seen that $M$ is the midpoint of $AB$ and $N$ is the midpoint of $CD$.

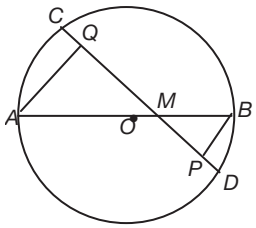
$AB = 2\ AM = 2r \sin \angle APQ = \frac{2rR}{d}$ (where $d = PQ$)

$CD = 2\ CN = 2R \sin \angle CQP = 2R\ \frac{r}{d}$

We see that $AB = CD$, no matter how $R$ and $r$ are related

$\therefore$ Whether $R = 2r$ as given or $R = kr$ (where $k$ is any constant), $CD = AB = 4$

**27. (a)**

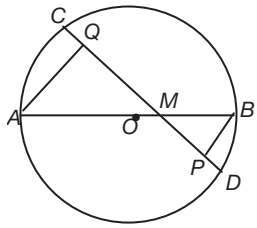



Let the perpendicular from $A$ to $CD$ intersect $CD$ at $Q$ and let the perpendicular from $O$ to $CD$ intersect $CD$ at $R$. ($R$ and $OR$ are not shown in the figure, for the sake of clarity.) Now $RC = RD$. Also $RQ = RP$ (The 3 parallel lines $AQ$, $OR$ (which is not shown) and $BP$ cut equal intercepts on $AB$. $\therefore$ They cut equal intercepts on $CD$)

$\therefore CQ = PD$. $PD$ is given to be 2 $\therefore CQ = 2$. But $QM$ or $MP$ are not given. We can conclude that $QM = 3\ MP$, but we cannot get the absolute values.

$\therefore$ We cannot determine $CM : MD$

(b)

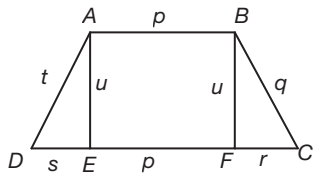



Let the perpendicular from $A$ to $CD$ intersect $CD$ at $Q$ and let the perpendicular from $O$ to $CD$ intersect $CD$ at $R$. ($R$ and $OR$ are not shown in the figure, for the sake of clarity.) Now $RC = RD$. Also $RQ = RP$ (The 3 parallel lines $AQ$, $OR$ (which is not shown) and $BP$ cut equal intercepts on $AB$. $\therefore$ They cut equal intercepts on $CD$)

$\therefore CQ = PD$. $PD$ is given to be $PM/2$. Also we can show that $MQ = 3\ MP$.

Let $PD = 1$ $\therefore PM = 2$, $MQ = 6$, $QC = 1$ and $CM : MD = (CQ + QM) : (MP + PD) = (1 + 6) : (2 + 1) = 7 : 3$

**28.**




Let the trapezium be $ABCD$ as shown above.

$AC^2 + BD^2 = 825$ (1)

$AB^2 + BC^2 + CD^2 + DA^2 = 1450$ (2)

The parallel sides are $AB$ and $CD$. $AB < CD$

$\therefore$ The required difference = $CD - AB = DE + FC$

(1) $\Rightarrow AE^2 + EC^2 + BF^2 + FD^2 = 825$

$u^2 + (p+r)^2 + u^2 + (p+s)^2 = 825$

$u^2 + p^2 + r^2 + 2pr + u^2 + p^2 + s^2 + 2ps = 825.$

(2) $\Rightarrow p^2 + q^2 + (r+p+s)^2 + t^2 = 1450$

$\Rightarrow p^2 + (u^2 + r^2) + (r^2 + p^2 + s^2 + 2rp + 2ps + 2rs) + (u^2 + s^2) = 1450$

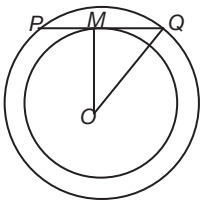
(2) – (1) $\Rightarrow 2p^2 + 2u^2 + 2r^2 + 2s^2 + 2rp + 2ps + 2rs - (2p^2 + 2u^2 + r^2 + s^2 + 2pr + 2ps) = 1450 - 825$

$\Rightarrow r^2 + s^2 + 2rs = 625$

$(r + s)^2 = 625$

$r + s = 25$.

**29.**




Let the diameters of the outer and inner circles be $D$ and $d$, respectively, where $D$ and $d$ are positive integers (given).

From the figure, $OQ^2 - OM^2 = MQ^2$

$$\Rightarrow \frac{D^2}{4} - \frac{d^2}{4} = \frac{54^2}{4}$$

$\Rightarrow D^2 - d^2 = 54^2 \Rightarrow (D + d)\ (D - d) = 54^2$

Since $D$ and $d$ are positive integers, both $(D + d)$ and $(D - d)$ are even (and positive)

$\Rightarrow (D + d)\ (D - d) = 1458 \times 2 = 486 \times 6$

$= 162 \times 18$

$\Rightarrow (D, d) = (730, 728), (246, 240), (90, 72)$

The diameter of the inner edge could be 240.

**30.** $\Delta CDA \sim \Delta CEB$ and the corresponding sides are in the ratio 13 : 14

i.e., $CD : DA : AC = CE : EB : BC$

$$CD = 5 \Rightarrow CE = \frac{14}{13}(5) = \frac{70}{13}$$

$\Delta CDE \sim \Delta CAB$

(As $\angle AEB = \angle BDA = 90°$, quadrilateral $ABDE$ is cyclic.

$\angle ABD$ and $\angle AED$ are supplementary; $\angle AED$ and $\angle CED$ are supplementary

$\therefore \angle CED$ of $\Delta CDE = \angle CBA$ of $\Delta CAB$)

$\therefore CD : DE : EC = CA : AB : BC$.

It's given that $DE : AB = 5 : 13$.

$$CD = 5 \Rightarrow CA = \left(\frac{13}{5}\right)(5) = 13$$

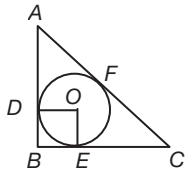
As $EABD$ is a cyclic quadrilateral,

$CE \cdot CA = CD \cdot CB$

$$\Rightarrow \left(\frac{70}{13}\right)(13) = 5\ (CB) \Rightarrow CB = 14.$$

$\therefore BD = CB - CD = 14 - 5 = 9$.

**31.**




Let the triangle be $ABC$ where $\angle B = 90°$

Let the incentre be $O$.

Let $D$ and $E$ be the points of contact of the in circle with $AB$ and $BC$, respectively.

$\angle ODB = \angle OEB = 90°$

($\because$ $AB$ and $BC$ are tangents to the incircle at $D$ and $E$, respectively.)

$\therefore \angle DOE = 360° - (\angle B + \angle ODB + \angle OEB) = 360° - 3(90°) = 90°$

$\therefore$ Each angle of $BEOD$ is 90° and also $EO = OD = 4$ cm

$\therefore$ $ODBE$ is a square

$BD = OE = BE = OD = 4$ cm

$AD$ and $AF$ are tangents to the incircle from $A$.

$\therefore AD = AF$ (1)

Also $CF$ and $CE$ are tangents to the incircle from $C$.

$\therefore CF = CE$ (2)

(1) and (2) $\Rightarrow AD + EC = AF + CF$

Also $AC = 2$(circum radius) = 26 cm

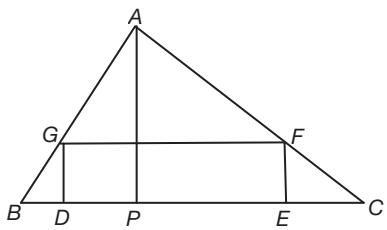
$\therefore AF + CF = 26$ cm

$\therefore AD + EC = 26$ cm

$\therefore AD + EC + AF + CF = 52$ cm

$\therefore$ Perimeter of $T = AD + EC + AF + CF + BD + BE = 60$ cm.

**32.**



Let $GD = x\,AP$ (where $AP$ is the altitude)

$\therefore BG = x\,BA$, $AG = (1-x)\,AB$ and $GF = (1-x)\,BC$ (1)

$\therefore (GD)\,(GF) = x(1-x)\,(BC)\,(AP) = 2x(1-x)\,\dfrac{(BC)(AP)}{2}$

$= 2x(1-x)\,\Delta$ where $\Delta$ = Area of $\Delta ABC$

$2x(1-x) = \dfrac{4}{9} \Rightarrow 9x(1-x) = 2 \Rightarrow 9x^2 - 9x + 2 = 0$

$\Rightarrow (3x-1)\,(3x-2) = 0 \Rightarrow x = 1/3$ or $2/3$

$(1) \Rightarrow \dfrac{DE}{BC} = \dfrac{GF}{BC} = \dfrac{(1-x)BC}{BC} = 1 - x = \dfrac{2}{3}$ or $\dfrac{1}{3}$

The length is along $BC$, i.e., $GD < DE$ or x$AP < (1-x)\,BC$

As the ratio of $AP/BC$ is not given, we can take suitable values so that whether $x = \dfrac{1}{3}$ or $\dfrac{2}{3}$, we can ensure that $GD < DE$. Therefore, both $\dfrac{1}{3}$ and $\dfrac{2}{3}$ are possible values of $x$ (and hence, both $\dfrac{2}{3}$ and $\dfrac{1}{3}$ are possible values of $1 - x = DE/BC$)

**33.** Let $a = 6K_1$, $b = 6K_2$, $c = 6K_3$

$6(K_1 + K_2 + K_3) = 78$

$K_1 + K_2 + K_3 = 13$ (1)

The sum of the any two sides of a triangle must be more than the third side.

$a \le b \le c$. As $c$ is the maximum of $a$, $b$ and $c$, $a + c > b$ and $b + c > a$. $\therefore$ The condition to be satisfied is $a + b > c$, i.e., $6K_1 + 6K_2 > 6K_3$, i.e., $K_1 + K_2 > K_3$ (2)

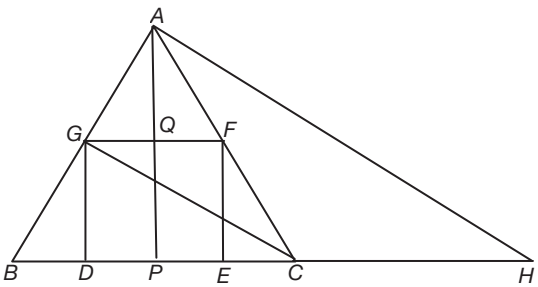
from (1) and (2), $(K_1, K_2, K_3) = (1, 6, 6)$, $(2, 5, 6)$ $(3, 3, 7)$ $(3, 4, 6)$, $(3, 5, 5)$ or $(4, 4, 5)$

$(K_1, K_2, K_3)$ has six values.

$(a, b, c)$ has six values.

**34.** Let $AB = c$, $BC = a$, $BG = c_1$; $CH = a_1$;

$AP = h$, $DE = EF = FG = GD = h_1$, $AQ = h_2$,




We can relate $c/c_1$ to $a$ and $a_1$ by considering $\Delta BCG$ and $\Delta BHA$. Also, we can relate $c/c_1$ to $h$ and $h_1$ by considering $\Delta AGQ$ and $\Delta ABP$

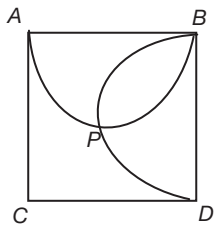
Thus, $\dfrac{c}{c_1} = \dfrac{BH}{BC}$ (Q $GC$ is parallel to $AH$) $= \dfrac{a + a_1}{a}$

Also $\dfrac{c}{c_1} = \dfrac{AP}{QP} = \dfrac{h}{h_1} = \dfrac{h_1 + h_2}{h_1} = 1 + \dfrac{h_2}{h_1} = 1 + \dfrac{AQ}{GF}$

$= 1 + \dfrac{AP}{BC} = 1 + \dfrac{h}{a} = \dfrac{a + h}{a}$

$\therefore \dfrac{a + a_1}{a} = \dfrac{a + h}{a} \Rightarrow a_1 = h$

**35.**




$ABCD$ is a square.

$\because$ Each of its sides subtends a right angle at $P$, the point of intersection of the diagonals

At least one of the angles $\angle AEB$ and $\angle BEC$ is obtuse (1)

If a semicircle is drawn with $AB$ as the diameter, $AB$ subtends a right angle at any point on the semicircle. It subtends an obtuse angle at any point inside the semicircle and an acute angle at any point outside the semicircle.

(1) implies $E$ must lie inside one of the semicircles (whose diameters are $AB$ and $BD$)

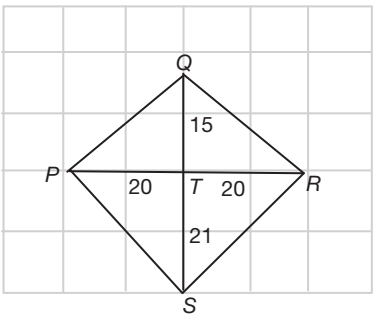
Area of the region in which $E$ can lie = Combined area of the semicircles – Area of the common region

$= \dfrac{\pi\left(\dfrac{AB}{2}\right)^2}{2}(2) - 2\left(\dfrac{\text{Area of the semicircle} - \text{Area of } \Delta APB}{2}\right)$

$= \dfrac{\pi AB^2}{4} - \left(\dfrac{\pi\left(\dfrac{AB}{2}\right)^2}{2} - \dfrac{AB^2}{4}\right) = \pi\dfrac{AB^2}{8} + \dfrac{AB^2}{4}$

Required ratio $= \dfrac{\pi\dfrac{AB^2}{8}+\dfrac{AB^2}{4}}{AB^2} = \dfrac{\pi+2}{8}$

**36.**



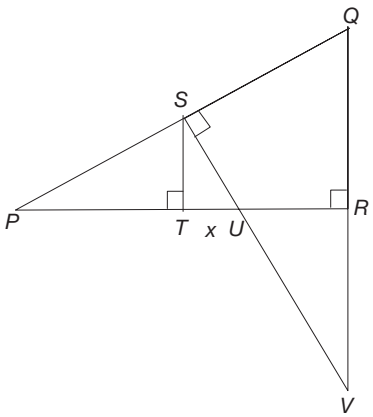

$PQRS = Ar\,PQS + Ar\,RQS$

$\le \dfrac{1}{2}[PT\,(QS) + RT\,(QS)]$

$= (PT)\ (QS) = 20\ (36) = 720$

As the area is given to be 720, we conclude that $PR$ is perpendicular to $QS$. The perimeter $= PQ + QR + RS + SP = 2(PQ + PS) = 2(25 + 29) = 108$

**37.** Let $TU = x$




$\angle PSU = 90°$. $\therefore$ $ST$ is the altitude to the hypotenuse $PU$ in the triangle $PSU$.

$\therefore\ ST^2 = (PT)\ (TU)$

$ST = \sqrt{3x}$

$SU = \sqrt{ST^2 + TU^2} = \sqrt{3x + x^2}$

$UR = 2$

$\angle SUT = \angle RUV$. $\therefore$ Triangles $TUS$ and $RUV$ are similar.

$\therefore\ RV = \dfrac{UR}{TU}(ST) = \dfrac{2}{x}\left(\sqrt{3x}\right)$ and $UV = \dfrac{2}{x}\left(\sqrt{3x + x^2}\right)$

Triangles $PRQ$ and $PTS$ are similar

$\therefore\ QR = \dfrac{PR}{PT}(ST) = \dfrac{x+5}{3}\left(\sqrt{3x}\right)$ and $PQ = \dfrac{x+5}{3}(PS)$

$= \dfrac{x+5}{3}\sqrt{9+3x}$

$QV^2 = QS^2 + SV^2$

$$\left(\frac{x+5}{3}\sqrt{3x} + \frac{2}{x}\sqrt{3x}\right)^2 = \left(\frac{x+5}{3}\sqrt{9x+3x} - \sqrt{9x+3x}\right)^2$$

$$+ \left(\sqrt{3x+x^2}\right) + \frac{2}{x}\left(\sqrt{3x+x^2}\right)^2$$

Going by the choices only choice (A) satisfies the equation above.

**Alternate Solution:**

Let $TU = x$. As $ST$ is the altitude to the hypotenuse in $\Delta$ $PSU$, $ST^2 = PT\,(TU)$. $\therefore$ $ST = \sqrt{3x}$, and $PS = \sqrt{9+3x}$

In $\Delta$ $PRQ$, $PR + PT + TU + UR = 3 + x + 2 = 5 + x$.

Also $RQ = 2\sqrt{3}$ (given) and we can work out $PQ$ in terms of $x$.

As $\Delta$ $PTS \sim \Delta$ $PRQ$, $\dfrac{PT}{PR} = \dfrac{PS}{PQ}$

$\therefore\ PQ = \dfrac{PR}{PT}PS = \dfrac{5+x}{3}\sqrt{9+3x}$

$\therefore\ (5+x)^2 + \left(2\sqrt{3}\right)^2 = \left[\dfrac{5+x}{3}\sqrt{9+3x}\right]^2$

$\Rightarrow x^2 + 10x + 37 = \left(\dfrac{x^2+10x+25}{9}\right)(9+3x)$

$\Rightarrow 3x^2 + 30x + 111 = (x^2 + 10x + 25)\ (x+3)$

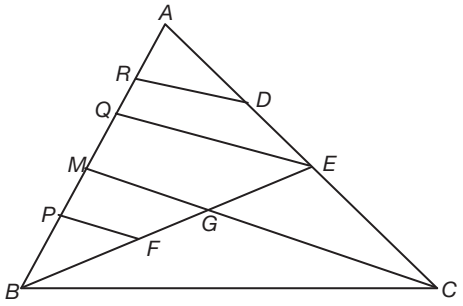
$\Rightarrow 3x^2 + 30x + 111 = x^3 + 13x^2 + 55x + 75$

$\Rightarrow x^3 + 10x^2 + 25x - 36 = 0$

By inspection $x = 1$, satisfies the equation. (The other two roots are complex.)

**38.** Let us draw lines parallel to $CGM$ passing through $F, E, D$

The intercepts formed are proportional.



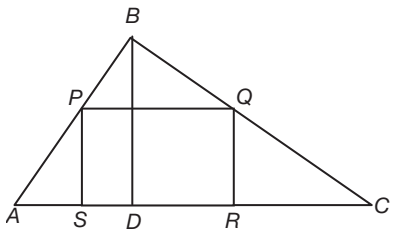

$BP = PM$ ($\because$ $BF = FG$)

$PM = MQ$ ($\because FG = GE$)

$MQ = QR = RA$ ($\because$ $CE = ED = DA$)

$$\frac{BM}{MA} = \frac{2}{3}$$

**39.** In $\Delta ABC$, $AB = 15$, $BC = 20$, $AC = 25$




$AB^2 + BC^2 = AC^2$. $\therefore$ Triangle is right angled at $B$.

$$\therefore \text{ Area of triangle } ABC = \frac{1}{2}(AB)(BC) = \frac{1}{2}(15)(20) = 150$$

Altitude $BD$ to $AC = \dfrac{2(150)}{25} = 12$. $\therefore$ $AD = \sqrt{AB^2 - BD^2} = 9$

We need the maximum value of the area of $PQRS$ we can first get $SR$ in terms of $PS$. For this we need $BS$ and $RQ$ in terms of $PS$.

Triangles $ABD$ and $APS$ are similar.

$$\therefore \frac{AS}{AD} = \frac{PS}{BD}. \therefore AS = \frac{AD}{BD}PS = \frac{9}{12}PS. = \frac{3}{4}PS.$$

Similarly $CR = \dfrac{4}{3}$ $QR = \dfrac{4}{3}PS.$

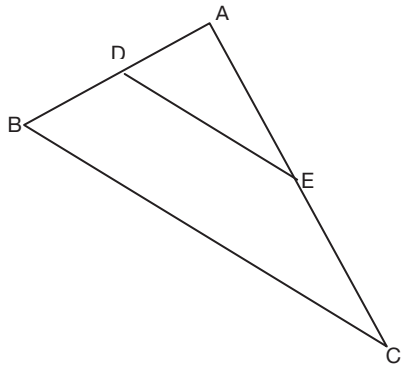
$$SR = AC - (AS + CR) = 25 - \frac{25}{12}PS.$$

$$\text{Area of } PQRS = PS\left(25 - \frac{25}{12}PS.\right)$$

$= \dfrac{25}{12}(36 - (6 - PS)^2)$. This is maximum when $PS$ is 6.

Maximum area is 75.

**40.**




$DE$ is parallel to $BC$.

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \text{ ($\because$ Intercepts are proportional)}$$

$$\frac{AD}{4} = \frac{AE}{8} \Rightarrow AE = 2\,AD \qquad (1)$$

Each side of a triangle is more than the difference of the other two sides and less than the sum of those two sides.

$AE - AD < DE < AE + AD$, i.e., $AD < 7 < 3AD$ (from (1))

$$\therefore \frac{7}{3} < AD < 7$$

Triangles $ADE$ and $ABC$ are similar ($\because$ $DE \parallel BC$)

$$\therefore \frac{BC}{DE} = \frac{AB}{AD}$$

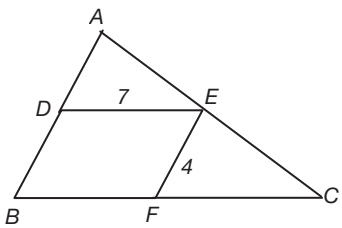
$$BC = \frac{AD + DB}{AD}(DE) = \left(1 + \frac{4}{AD}\right)(7) = 7 + \frac{28}{AD} \qquad (2)$$

$$\frac{1}{7} < \frac{1}{AD} < \frac{3}{7}. \therefore 4 < \frac{28}{AD} < 12.$$

$11 < BC < 19$, i.e., $11 < x < 19$.

$x$ can take the integer values 12,13,14,15,16,17,18

**Alternative Solution:**



Through $E$, draw a line parallel to $AB$ to intersect $BC$ at $F$.

In $\Delta FEC$, $FE = 4$, $EC = 8$ $\therefore$ $8 - 4 < FC < 4 + 8$

i.e., $FC$ can be 5, 6, 7, 8, 9,10 or 11.

$\therefore BC$ can be 12, 13, 14, 15, 16, 17 or 18. It can have 7 integral values.

## EXERCISE-5

***Solutions for questions 1 to 30:***

**1.** As $AD$ and $BE$ are two of the medians, $G$ must be the centroid of the triangle.

$\therefore AG : GD = 2 : 1$ and $BG : GE = 2 : 1$.

Statement I alone is not sufficient as it gives no lengths.

Statement II alone is not sufficient, as it gives the information about only two sides

From I and II, we have, $AB = 10$ cm, $BC = 20$ cm.

$AG : GE = 2 : 1$,

$\therefore GD = GE$ and $BG = AG$.

$\Rightarrow AD = BE$.

In a triangle, if two medians are equal, then the sides to which these medians are drawn must be equal.

$\therefore BC = AC$

$\therefore AC = 20$ cm.

As we know the three sides of the triangle, we can find the area of the triangle.

$\therefore$ We can answer the question, using both the statements.

**2.** Given, $\angle BAC = 90°$

$\therefore BC$ is the diameter of the circumcircle of the triangle

From Statement I, $BC = 21$cm

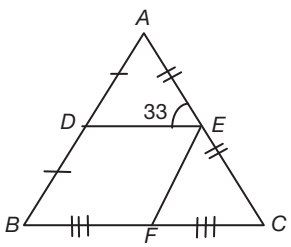
Hence, $AM$ would be the circumradius.

$\therefore AM = 1/2BC$

$\therefore$ Statement I alone is sufficient.

From Statement II, $AD$ alone cannot yield the length of $AM$. Statement II alone is not sufficient.

**3.**




As $D$, $E$, $F$ are the midpoints of $AB$, $AC$ and $BC$, respectively, the triangles $AED$, $ECF$, $DFB$ and $FDE$ are congruent.

$\therefore \angle FCE = \angle AED = 33°$.

From Statement I, $\angle FEC = 78°$.

$\therefore \angle BFE = \angle FEC + \angle ECF$

$= 78 + 33 = 111^o$

So, $\angle BFE = 111°$

Hence, Statement I alone is sufficient .

From Statement II, $\angle BAC = 78°$.

$\therefore \angle FEC = \angle BAC = 78°$.

$\therefore$ We can find $\angle BFE$.

Hence, Statement II alone is sufficient.

**4.** Statement I alone is not sufficient, as it gives no lengths.

Statement II alone is not sufficient, as we don't know the information about the other side or any of the angles of the triangle.

From I and II, we have

$\angle ABC = 60°$ and $AB = 10$ cm, $AC = 10$ cm.

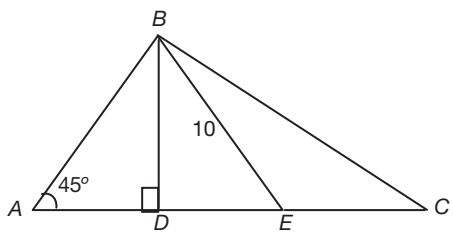
$\therefore \angle BCA = \angle ABC = 60°$

$\therefore \angle BAC = 180° - (60° + 60°) = 60°$

$\therefore BC = 10$ cm

We know all the sides, therefore, we can find the area.

**5.**




From Statement I, $\angle BCA = 45^o$. So, triangle $ABC$ is a right angled triangle, but we do not know the lengths of $AC$ or $BC$, so Statement I is not sufficient.

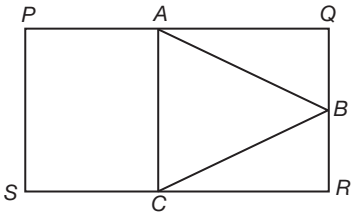
From Statement II, we have $\angle BEA = 60^o$ As triangle $BDE$ is a right angled triangle, $BD = BE \sin(60°)$

$BD = \frac{10\sqrt{3}}{2} = 5\sqrt{3}$ cm.

$AB = \frac{BD}{\text{Sin } 45°}$ [$\because \Delta ABD$ is a right triangle].

$\therefore AB = 5\sqrt{6}$ cm. Hence, Statement II alone is sufficient.

**6.**




From Statement I, we do not know the information about the point *C*, so we can't answer the question. Statement I alone is not sufficient.

From Statement II, by knowing *AC* is parallel to *PS* we can't answer the question as we don't know the information about *B*. Statement II alone is not sufficient.

Using both the statements,

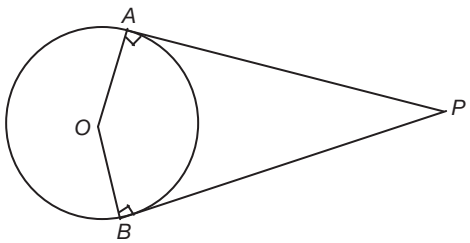
we have $AC = PS$

and $\frac{2}{3}PQ = AQ$

$\therefore \frac{\text{area of the triangle } ABC}{\text{area of the rectangle } PQRS} = \frac{\frac{1}{2} \times AC \times AQ}{PS \times PQ} = \frac{1}{3}$

Using both the statements we can answer the question.

**7.**



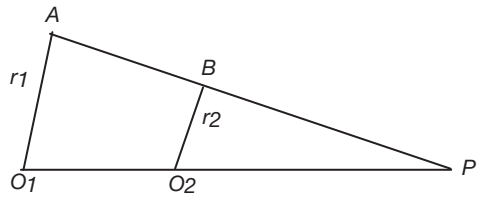

From the Statement I, by knowing the radius of circle we can't find the length of *PO*, because to find *PO*, we need *OA* as well as *AP*.

Statement I alone is insufficient.

From Statement II, we have circumradius of triangle *AOP* as 14 cm. As $\angle OAP = 90°$, the triangle *AOP* is a right angled triangle. As *OP* is the hypotenuse it must be twice the circumradius.

Statement II alone is sufficient.

**8.**




From Statement I, we have,

$r_1 = 15$ cm and $r_2 = 9$ cm.

Also, $O_1 O_2 = r_1 + r_2 = 24$ cm.

The two triangles, $AO_1P$ and $BO_2P$ are similar.

$\therefore \frac{AO_1}{BO_2} = \frac{O_1P}{O_2P}$.

$\Rightarrow \frac{15}{9} = \frac{O_2P + O_1O_2}{O_2P}$

$\Rightarrow 5O_2P = 3O_2P + 3 \times 24$

$\Rightarrow O_2P = 36$ cm.

As $\angle O_2BP = \angle O_2AP = 90°$

$BP = \sqrt{O_2P^2 - O_2B^2}$

$AP = \sqrt{O_1P^2 - O_2A^2}$

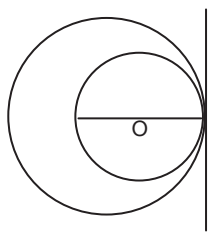
As we know

$O_2P$, $O_1P$, $O_2B$, $O_1A$, we can find *AP* and *BP*, and thereby *AB*.

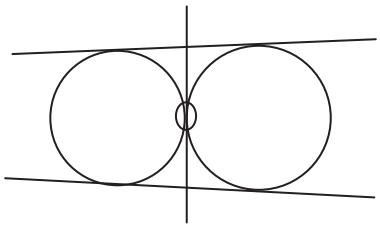
Statement I alone is sufficient.

Statement II alone is not sufficient, as it does not have any length.

**9.**

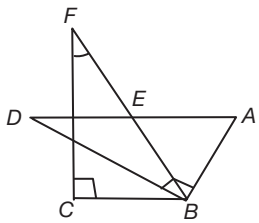


From Statement I, the two circles have only one common tangent therefore, one circle lies inside the other, which means the distance between the centres is equal to the difference between the radii. So, Statement I alone is sufficient.



From Statement II, when two circles having only three common tangents the circles touch each other externally. Therefore, the distance between centres is equal to the sum of the radii. Statement II alone is also sufficient.

**10.**




As $\angle ABD = \angle BCF = 90°$, $AD$ and $BF$ are diameters of the circle. $\therefore$ $E$ must be the centre of the circle.

$\therefore \angle CEB = 2\angle CFB$ and $\angle BEA = 2\angle BDA$.

[$\because$Angle at centre is twice the angle subtended by the arc at any point on the circumference of the circle].

From Statement I, $\angle BDA = 30°$. $\therefore \angle BEA = 60°$.

$\therefore$ Statement I alone is sufficient.

From Statement II, $\angle CFB = 30°$.

$\therefore \angle CEB = 60°$

But we cannot find $\angle BEA$.

$\therefore$ Statement II alone is not sufficient.

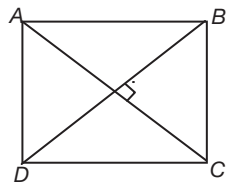
**11.** Statement I: The diagonals can be equal even in a non-rectangle. For example, diagonals are equal in an isosceles trapezium.

(If the figure is a rectangle, it must be a square).

I is not sufficient.

Statement II alone is not sufficient as it is not given whether the diagonals are equal and bisect each other.

Using both the statements also we can't answer the question as the four sided figure may be as given below.



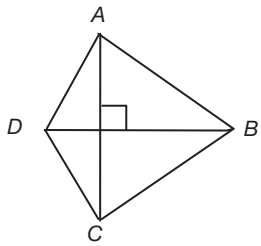




Figure (1) is a rectangle but figure (2) is not a rectangle.

**12.** If circumcircle passes through $A$ then we can't say whether $\angle A$ is 90° or not. So, Statement I alone is not sufficient.

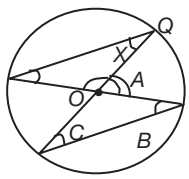
From Statement II, $\angle B = 90° \Rightarrow \angle A$ is not equal to 90°.

$\therefore$ Statement II alone is sufficient.

**13.** From Statement I, $\angle A = 60°$

As $OP = OQ =$ radii

$\angle P = \angle Q = \angle X$ and $\angle POQ = 180° - 60° = 120°$




$\therefore \angle X + \angle X + 120° = 180°$

$\Rightarrow \angle X$ can be determined.

$\therefore$ Statement I alone is sufficient.

From Statement II, $\angle B = 30° = \angle C$

$\therefore \angle A = 60°$. $\therefore \angle X = 30°$

$\therefore$ Statement II alone is sufficient.

**14.** From Statement I, $AD = 2$.

$\Rightarrow BD = 2$, because $\angle DAB = \angle ABD = 45°$.

$\therefore AB = 2\sqrt{2}$

$$BD = 2 \Rightarrow DC = 2\sqrt{3} \left[\because \tan 30° = \frac{BD}{DC} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}\right]$$

$BC = \sqrt{4+12} = 4$

$\therefore$ Perimeter $= 2\sqrt{2} + 4 + 2 + 2\sqrt{3}$.

$\therefore$ Statement I alone is sufficient.

But Statement II alone is not sufficient as nothing is known about the point $E$.

**15.** Statement I is always true for any regular polygon.

From Statement II, one of its exterior angles = 60°

$\therefore$ The number of sides $= \frac{360°}{60} = 6$

$\therefore$ Statement II alone is sufficient.

**16.** From Statement I, $AB^2 > BC^2 + AC^2$ so $\angle C$ is the largest, but we can't say which is the smallest angle. So, Statement I alone is not sufficient.

From Statement II, $BC^2 < AB^2 + AC^2$ so we can say that $\angle A$ is not obtuse, but we can't say which is the smallest angle.

Using both statements also we can't say which is the smallest angle in triangle $ABC$.

**17.** From Statement I, the angles are 30°, 60° & 90° and hence, the ratio of the sides can found but not the area. From Statement II, the longest side is 5

Using both the statements, in the right angled triangle the hypotenuse is known and the two acute angles are also known. So, we can find the remaining two sides and thereby the area.

**18.** From Statement I,

$\angle BAC + \angle BCA + \angle ABC = 180°$

$50° + \angle BCA + 30° + \angle BCA = 180°$

$\angle BCA = 50°$

So, the given triangle is an isosceles triangle.

Statement I alone is sufficient.

From Statement II, the altitude bisects the base. Hence, it is isosceles.

**19.** From Statement I, $AB + AC = 2BC$ and $AC = BC$

So, $AB = AC = BC$, so triangle $ABC$ is an equilateral triangle.

In an equilateral triangle the perpendicular line drawn from $A$ bisects $BC$. So, Statement I alone is sufficient.

From Statement II, $AB = AC$. So, triangle $ABC$ is an isosceles triangle. $\therefore$ The perpendicular line drawn from $A$ bisects $BC$. So, Statement II alone is also sufficient.

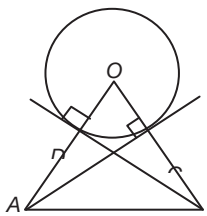
**20.** From Statement I, we don't know whether the diameter of circle $B$ is more or less than the diameter of circle $A$. So, we can't answer the question. II is not sufficient.

Using both the statements, if the circle with centre $A$ is bigger than the other circle the two circles will not touch each other. If the circle with centre $B$ is bigger than the other circle they will intersect each other, but the two circles never touch each other externally.

**21.** An infinite number of different triangles can be drawn having the same given area, and each of them will have a different perimeter. So, Statement I by itself cannot enable us to answer the given question.

Again, an infinite number of triangles can be drawn with one angle being 45°. So, Statement II by itself cannot enable us to answer the given question. Even using both the statements I and II, an infinite number of triangles having both properties can be drawn. So, both the statements I and II together also do not help us to answer the given question.

**22.** Neither of the statements is independently sufficient as complete information about the figure is not given.




Using both the statements,

$OAB$ is an equilateral triangle,

because $OA = OD + DA = OC + CB = OB$

and $\angle AOB = 60°$ (1)

$\Delta ODC$ is also an equilateral, for similar reasons (2)

As $\angle ODB = 90°$ $DB$ is a tangent. By symmetry $AC$ is also a tangent.

$\therefore \angle OCA = 90°$.

In $\Delta OAC$, $\angle CAD = 180° - \angle AOB - \angle OCA$

$= 180° - 60° - 90° = 30°$.

**23.** From Statement I, though we know that the given polygon has 3 sides, the triangle is not fully defined. Also we do have any other information to find its area.

From Statement II, we do not know whether the polygon is a triangle or not. Hence, this statement alone is not sufficient.

From I and II together, we know the two adjacent sides of the triangle and the included angle. Hence, its area can be found.

**24.** From Statement I, $x = 360/3 = 120°$.

So, Statement I by itself is sufficient for answering the given

question. Since the length of the radius of the circle is irrelevant for calculating the value of $x$, Statement II by itself cannot help us to answer the given question.

**25.** $\angle BOC = 2\angle BAC = 2(\angle ABO + \angle ACO)$

From Statement I alone, we cannot find $\angle BOC$ as we do not know $\angle ABO$. II is not sufficient.

From I and II together,

Join $OA$ from Statement I, since $AOB$ is an isosceles triangle (because $OA = OB$ = radius), $\angle BAO$ is also equal to 25°.

Similarly from Statement II, $\angle CAO = 35°$.

By taking both the statements together,

$\angle BAC = \angle BAO + \angle CAO = 60°$

Hence, $\angle BOC = 2 \times 60 = 120°$.

**26.** From Statement I alone, $\frac{PQ}{QR} = \frac{QR}{PR}$, we cannot find the ratio of the sides from information, hence cannot find the ratio of $\angle P$, $\angle Q$, and $\angle R$.

So, I alone is not sufficient.

From Statement II alone, we cannot find the ratio of the sides and what is given here is a redundant information (it is true for all triangles).

So, II alone is not sufficient.

From statements I and II together, we cannot find $\angle P : \angle Q : \angle R$ as I alone is not sufficient and II alone is a redundant statement.

**27.** It is given that $a$, $c$ and $e$ are parallel and $b$, $d$ are parallel. Even after combining both the statements, we cannot conclude that a is parallel to $d$, which is necessary to conclude that $x = y$.

**28.** $AB = 4$; $BC = 3$ or $AB = 3$; $BC = 4$ satisfies both the statements I and II. We cannot tell with certainty whether $AB = 3$ or $AB = 4$.

**29.** For a given hypotenuse, there can be more than one combination of the other two sides, so we cannot compute a unique value of the perimeter with Statement I alone. However, with another side as 4 units, the third side can be computed and hence, the perimeter.

**30.** Since the tangent is perpendicular to the radius at the point of contact, radius $OT = \sqrt{AT^2 - OA^2}$.

Taking both the statements together,

we have $AO = 12 - r$ and $AT = 6$, where $r$ is the radius of the circle.

# 2 Mensuration

CHAPTER

**LEARNING OBJECTIVES**

*In this chapter, you will*:

- Learn various formulae for finding the area of plane geometric figures like
  - Triangles—different types of triangles
  - Quadrilaterals—trapezium, parallelogram, rhombus, rectangle, square, circle, and ellipse
  - Hexagons, octagons and other regular polygons
- Learn about Euler's rule for convex polyhedrons
- Learn various formulae to calculate area and volume of solid geometric shapes—prism, cuboid, cube, cylinder, pyramid, cone, cone frustum, torus, sphere, hemisphere, etc.

## ❒ AREAS OF PLANE FIGURES

MENSURATION is the branch of mathematics which deals with the study of geometric shapes, their area, volume and related parameters. We have looked at properties of plane figures till now. Here, in addition to areas of plane figures, we will also look at surface areas and volumes of solids. Solids are objects, which have three dimensions (plane figures have only two dimensions).

Let us briefly look at the formulae for areas of various plane figures and surface areas and volumes of various solids.

### ❒ Triangles

The area of a triangle is represented by the symbol Δ. For any triangle, the three sides are represented by $a$, $b$ and $c$ and the angles opposite these sides represented by $A$, $B$ and $C$, respectively.

1. **For any triangle in general**
   (i) When the measurements of three sides $a$, $b$, $c$ are given:



**FORMULA**

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

**where**

$$s = \frac{a + b + c}{2}$$

**This is called Hero's formula.**

(ii) When base ($b$) and altitude (height) to that base are given:



**FORMULA**

$$\text{Area} = \tfrac{1}{2} \times \text{base} \times \text{altitude} = \tfrac{1}{2}\, b.h$$

(iii) When any two of the sides $a$, $b$ and $c$ are given and the measure of the angle included between the sides is given:

**FORMULA**

Area $= \frac{1}{2}\, ab \cdot \sin C = \frac{1}{2}\, bc.\ \sin A$
$= \frac{1}{2}\, ca \cdot \sin B$

(iv) When sides $a$, $b$, $c$ and the circumradius $R$ is given:

**FORMULA**

$$\text{Area} = \frac{abc}{4R}$$

(v) When the radius $r$ and the semi perimeter $s$ is known

**FORMULA**

Area $= r.s$

Out of these five formulae, the first and the second are the most commonly used and are also more important from the examination point of view.

2. **For a right-angled triangle:**

**FORMULA**

Area = ½ × Product of the sides containing the right angle

3. **For an equilateral triangle:**

**FORMULA**

$$\text{Area} = \frac{\sqrt{3} \times a^2}{4}$$

where $a$ is the side of the triangle

The height of an equilateral triangle $= \frac{\sqrt{3} \cdot a}{2}$

4. **For an isosceles triangle:**

**FORMULA**

$$\text{Area} = \frac{b}{4}\sqrt{4a^2 - b^2}$$

where '$a$' is length of each of the two equal sides and '$b$' is the third side.

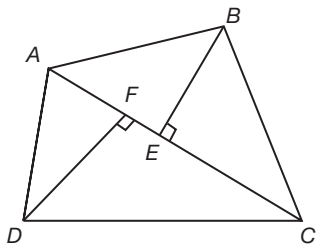
## ❒ Quadrilaterals

1. **For any quadrilateral:**

**FORMULA**

Area of the quadrilateral = ½ × One diagonal × Sum of the offsets drawn to that diagonal

Hence, for the quadrilateral $ABCD$ shown in below figure, area of quadrilateral $ABCD = 1/2 \times AC \times (BE + DF)$.




2. **For a cyclic quadrilateral where the four sides measure *a*, *b*, *c* and *d*, respectively:**

**FORMULA**

$$\text{Area} = \sqrt{(s-a)\,(s-b)\,(s-c)\,(s-d)}$$

where $s$ is the semi-perimeter, i.e., $s = (a + b + c + d)/2$

3. **For a trapezium:**

**FORMULA**

Area of a trapezium = ½ × Sum of parallel sides × Distance between them
$= \frac{1}{2} \times (AD + BC) \times AE$

4. **For a parallelogram:**

**FORMULA**

(i) Area = Base × Height
(ii) Area = Product of two sides × Sine of included angle

5. **For a rhombus:**

**FORMULA**

Area = ½ × Product of the diagonals
Perimeter = 4 × Side of the rhombus

6. **For a rectangle:**

**FORMULA**

Area = Length × Breadth
Perimeter = $2(l + b)$

where $l$ and $b$ are the length and the breadth of the rectangle, respectively

7. **For a square:**

**FORMULA**

(i) Area = $\text{Side}^2$
(ii) Area = ½ × $\text{Diagonal}^2$

[where the diagonal = $\sqrt{2}$ × side]

Perimeter = 4 × Side

8. **For a polygon:**

**FORMULA**

Area of a regular polygon = ½ × Perimeter × Perpendicular distance from the centre of the polygon to any side

**Please note that the centre of a regular polygon is equidistant from all its sides.**

**FORMULA**

Area of a regular hexagon = $\frac{3\sqrt{3}}{2}(\text{side})^2$

**IMPORTANT FACTS**

For a polygon which is not regular, the area has to be found out by dividing the polygon into suitable number of quadrilaterals and triangles and adding up the areas of all such figures present in the polygon.

## ❒ Circle

**FORMULA**

Area of the circle = $\pi r^2$

where $r$ is the radius of the circle

Circumference = $2\pi r$

1. **Sector of a circle:**

Length of arc = $\frac{\theta}{360°} \times 2\pi r$

**FORMULA**

Area = $\frac{\theta}{360°} \times \pi r^2$

where θ is the angle of the sector in degrees and $r$ is the radius of the circle.

Area = $(1/2)lr$; $l$ is length of arc and $r$ is radius.

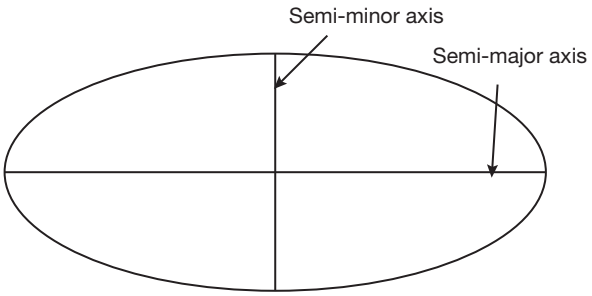
2. **For a ring is the space enclosed by two concentric circles:**

**FORMULA**

Area = $\pi R^2 - \pi r^2 = \pi(R + r)(R - r)$

where $R$ is the radius of the outer circle and $r$ is the radius of the inner circle.

## ❒ Ellipse




**FORMULA**

Area = $\pi ab$ where $a$ is semi-major axis and $b$ is semi-minor axis.

Perimeter = $\pi (a + b)$

## ❒ Areas and Volumes of Solids

Solids are three-dimensional objects which, in addition to area, have volume also. For solids, two different types of areas are defined

1. **Lateral surface area or curved surface area**
2. **Total surface area**

As the name itself indicates, lateral surface area is the area of the *lateral* surfaces of the solid. Total surface area includes the areas of the top and the bottom surfaces also of the solid. Hence, Total surface area = Lateral surface area + Area of the top face + Area of the bottom face

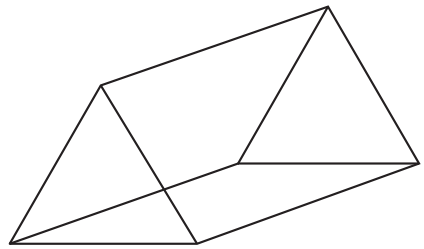
In solids (like cylinder, cone, sphere) where the lateral surface is curved, the lateral surface area is usually referred to as the '**curved surface area**'.

For any solid, whose faces are regular polygons, there is a definite relationship between the number of vertices, the number of sides and the number of edges of the solid. This relationship is given by '**Euler's Rule**'.

**FORMULA**

Number of faces + Number of vertices
= Number of edges + 2(Euler's Rule)

## ❒ Prism



*A* right prism is a solid whose top and bottom faces (bottom face is called base) are parallel to each other and are identical polygons (of any number of sides) that are parallel. The faces joining the top and bottom faces are rectangles and are called lateral faces. There are as many lateral faces as there are sides in the base. The distance between the base and the top is called height or length of the right prism.

In a right prism, if a perpendicular is drawn from the centre of the top face, it passes through the centre of the base.

For any prism:

**FORMULA**

Lateral Surface Area = Perimeter of base × Height of the prism

Total Surface Area = Lateral Surface Area + 2 × Area of base

Volume = Area of base × Height of the prism

## ❒ Cuboid or Rectangular Solid

*A* right prism whose base is a rectangle is called a rectangular solid or cuboid. If $l$ and $b$ are, respectively the length and breadth of the base and $h$, the height, then

**FORMULA**

Volume = $lbh$

Lateral Surface Area = $2(l + b) \cdot h$

Total Surface Area = $2(l + b)h + 2lb$

$= 2(lb + lh + bh)$

Longest diagonal of the cuboid = $\sqrt{l^2 + b^2 + h^2}$

## ❒ Cube

A right prism whose base is a square and height is equal to the side of the base is called a cube.

**FORMULA**

Volume = $a^3$ where a is the edge of the cube
Lateral Surface Area = $4a^2$
Total Surface Area = $6a^2$

The longest diagonal of the cube (i.e., the line joining one vertex on the top face to the diagonally opposite vertex on the bottom face) is called the diagonal of the cube. **The length of the diagonal of the cube is** $a\sqrt{3}$ .

## ❒ Cylinder

A cylinder is equivalent to a right prism whose base is a circle. A cylinder has a single curved surface as its lateral faces. If $r$ is the radius of the base and $h$ is the height of the cylinder,

**FORMULA**

Volume = $\pi r^2 h$
Curved Surface Area = $2\pi rh$
Total Surface Area = $2\ \pi rh + 2\pi r^2 = 2\pi r(h + r)$

A hollow cylinder has a cross-section of a ring.

Volume of the material contained in a hollow cylindrical ring = $\pi (R^2 - r^2) h$ where $R$ is the outer radius, $r$ is the inner radius and $h$, the height.

## ❒ Pyramid

A solid whose base is a polygon and whose faces are triangles is called a pyramid. The triangular faces meet at a common point called vertex. The perpendicular from the vertex to the base is called the height of the pyramid.

A pyramid whose base is a regular polygon and the foot of the perpendicular from the vertex to the base coincides with the centre of the base, is called a right pyramid.

The length of the perpendicular from the vertex to any side of the base (please note that this side will be the base of one of the triangular lateral faces of the prism) along the slant lateral surface is called the slant height of the prism.

**FORMULA**

Volume of a pyramid = $^1/_3$ × Area of base × Height

Lateral Surface area = $^1/_2$ × Perimeter of the base × Slant height

Total Surface Area = Lateral Surface Area + Area of the base.

## ❒ Cone

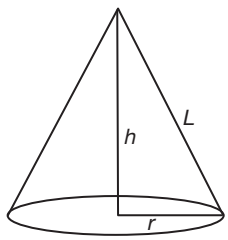



Fig. 2.1

A cone is *equivalent* to a right pyramid whose base is a circle. The lateral surface of a cone does not consist of triangles like in a right pyramid but is a single curved surface.

If $r$ is the radius of the base of the cone, $h$ is height of the cone and' $l$ is the slant height of the cone, then we have the relationship (Fig. 2.1)

**FORMULA**

$l^2 = r^2 + h^2$
Volume = $^1/_3\ \pi r^2 h$
Curved Surface Area = $\pi r.l$
Total Surface Area = $\pi rl + \pi r^2 = \pi r(\ l + r)$

A cone can be formed by taking the sector of a circle and joining together its straight edges. If the radius of the sector is $R$ and the angle of the sector is $\theta°$, then we have the following relationships between the length of the arc and area of the sector on the one hand and base perimeter of the cone and curved surface area of the cone on the other hand.

Radius of the sector = Slant height of the cone
i.e., $R = l$

Length of the arc of the sector = Circumference of the base of the cone

i.e., $\frac{\theta}{360} \times 2 \times \pi R = 2\pi r \Rightarrow r = \frac{\theta}{360} \times R$

and Area of the sector = Curved surface area

(Actually, from this last equation, substituting the values from the first two equations, we can get the curved surface area of the cone, which is what is given previously as equal to $\pi rl$.)

## ❒ Cone Frustum

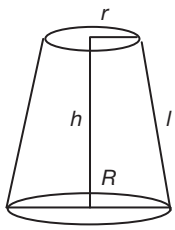



Fig. 2.2

If a cone is cut into two parts by a plane parallel to the base, the portion that contains the base is called the frustum of a cone.

If $r$ is the top radius; $R$, the radius of the base; $h$ the height and $l$ the slant height of a frustum of a cone (Fig. 2.2), then,

**FORMULA**

Lateral Surface Area of the cone = $\pi\, l(R + r)$
Total Surface Area = $\pi\,(R^2 + r^2 + R.l + r.l)$
Volume = $1/3\ \pi\, h\,(R^2 + Rr + r^2)$
$l^2 = (R - r)^2 + h^2$

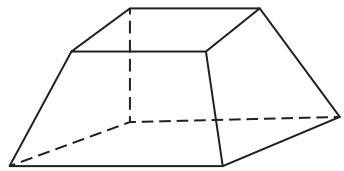
If $H$ is the height of the complete cone from which the frustum is cut, then from similar triangles, we can write the following relationship.

**FORMULA**

$$\frac{r}{R} = \frac{H - h}{H}$$

A bucket that is normally used in a house is a good example of the frustum of a cone. The bucket is actually the inverted form of the frustum that is shown in the figure 2.2.

## ❒ Frustum of a Pyramid



A pyramid left after cutting of a portion at the top by a plane parallel to the base is called a frustum of a pyramid.

If $A_1$ is the area of the base; $A_2$ the area of the top and $h$, the height of the frustum,

**FORMULA**

Volume of frustum =
$^1/_3 \times h \times \left(A_1 + A_2 + \sqrt{A_1 A_2}\right)$

Lateral Surface Area = ½ × (Sum of perimeters of base and top) × Slant height

Total Surface Area = Lateral Surface Area + $A_1$ + $A_2$

## ❒ Torus

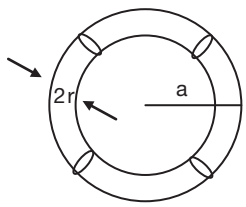



Fig. 2.3

*A* torus is a three-dimensional figure produced by the revolution of a circle about an axis lying in its plane but not intersecting the circle. The shape of the rubber tube in a bicycle (when it is inflated fully) is an example of a torus. If $r$ is the radius of the circle that rotates and a is the distance between the centre of the circle and the axis of revolution,

**FORMULA**

Surface Area of the torus = $4\pi^2 ra$
Volume of the torus = $2\pi^2 r^2 a$

A torus is also referred to as a solid ring. (Fig. 2.3)

## ❒ Sphere

Any point on the surface of a sphere is equidistant from the centre of the sphere. This distance is the radius of the sphere.

**FORMULA**

Surface Area of a sphere = $4\pi r^2$
Volume of a sphere = $(4/3)\pi r^3$

The curved surface area of a hemisphere is equal to half the surface area of a sphere, i.e., $2\pi r^2$

**NOTE**

Among the solids as discussed above, pyramid, frustum of a pyramid and torus are not important from the point of view of the entrance exams, and hence can be ignored if you so wish. Similarly, among the plane figures, Ellipse may be ignored if you so wish.

The following examples cover various properties / theorems discussed in Geometry as well as areas and volumes discussed in Mensuration. You should learn all the properties of triangles, quadrilaterals and circles as well as areas/volumes of plane figures and solids thoroughly before starting with the worked out examples and the exercise that follows the worked out examples.

## SOLVED EXAMPLES

**2.01.** The base of a right-angled triangle is 7 cm and its area is 84 $cm^2$. Find its hypotenuse.

**Sol:** Given base = 7 cm.

Area of the triangle = $\frac{1}{2}bh$, (where $b$ is the base and $h$ is the height)

Given, $\frac{1}{2}bh = 84\ cm^2$,

$$h = \frac{2(84)}{7}\ \frac{cm^2}{cm} = 24\ cm$$

In a right-angled triangle, the sum of the squares of perpendicular sides = square of hypotenuse,
∴ Length of the hypotenuse

$= \sqrt{7^2 + 24^2}\,cm = 25\ cm$.

**2.02.** The sides of a triangle are 7 cm, 8 cm and 9 cm long. Find its area.

**Sol:** Area of a triangle whose sides are given as $a$, $b$ and $c$ is $\sqrt{s(s-a)(s-b)(s-c)}$,

where $s = \frac{a+b+c}{2}$.

As a = 7 cm; $b$ = 8 cm, $c$ = 9 cm ⇒ $s$ = 12 cm.

Hence, area = $\sqrt{12(5)(4)(3)\ cm^4} = 12\sqrt{5}\ cm^2$.

**2.03.** If the area of an equilateral triangle is $64\sqrt{3}\ cm^2$, then find its side.

**Sol:** Area of an equilateral triangle of side $a = \frac{\sqrt{3}}{4}a^2$.

$$\therefore \frac{\sqrt{3}}{4}a^2 = 64\sqrt{3}\ cm^2$$

$\Rightarrow a^2 = 256\ cm^2$

$\Rightarrow a = 16$ cm

Hence, side is 16 cm.

**2.04.** Find the number of revolutions made by a wheel to cover a distance of 352 m, if its diameter is 40 cm.

**Sol:** Number of revolutions

$$= \frac{\text{Distance travelled}}{\text{Circumference of the wheel}}$$

$$= \frac{(352 \times 100)\text{ cm}}{2\left(\frac{22}{7}\right)\left(\frac{40}{2}\right)\text{cm}} = 280.$$

**2.05.** A piece of wire is bent in the form of a circle of radius 42 cm. An equal length of wire is bent and made a square. Find the difference in the area of the square and the circle.

**Sol:** Perimeter of the circular wire

$= (2)\,(22 / 7)\,(42)$ cm $= 264$ cm.

If an equal length of wire is bent and made a square, perimeter of the square is equal to that of the circle.

Perimeter of the square

$= 4(\text{side}) = 264$ cm.

$\therefore$ The side of the square $= 66$ cm.

Area of the square $= 66^2$ cm$^2 = 4356$ cm$^2$.

Area of the circle $= \pi\,(42)^2$ cm$^2$

$= (22 / 7)\,(42)\,(42)$ cm$^2 = 5544$ cm$^2$.

Difference in the areas of the square and the circle $= 1188$ cm$^2$.

**2.06.** If the circumference of one circle is 5/2 times that of the other, how many times the area of the smaller circle is the bigger one?

**Sol:** Let the radius of smaller circle $= r$

As the circumference of the bigger circle is (5/2) times that of the smaller circle, radius of the bigger circle will be $(5r/2)$.

Area of the smaller circle $= \pi r^2$.

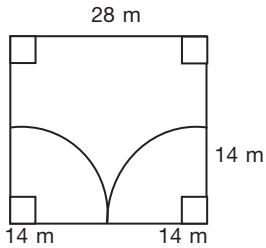
Area of the bigger circle $= \pi\left(\frac{5r}{2}\right)^2 = \frac{25}{4}\pi r^2$.

Therefore, ratio of the area of the bigger circle to that of the smaller circle

$$= \frac{\frac{25}{4}\pi r^2}{\pi r^2} = \frac{25}{4}$$

**2.07.** Two goats are each tied to two adjacent corners of a square plot of side 28 m with ropes 14 m long. Find the area of the plot over which the goats cannot graze.

**Sol:**




Area of the square $= (28)\ (28)$ m$^2 = 784$ m$^2$.

Area over which the goats can graze

$= 2\left(\frac{90°}{360°}\right)\left(\frac{22}{7}\right)(14)\,(14)$ m$^2 = 308$ m$^2$

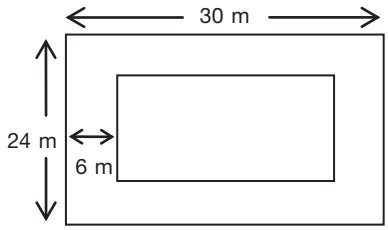
Area over which the goats cannot graze

$= (784 - 308)$ m$^2$

$= 476$ m$^2$.

**2.08.** A path 6 metres wide was laid all around and inside a rectangular plot of length 30 metres and breadth 24 metres. Find the area of the rectangular plot, not covered by the path.

**Sol:**




Length of the inner rectangular plot

$= [30 - 2(6)]$ m $= 18$ m.

Breadth of the inner rectangular plot

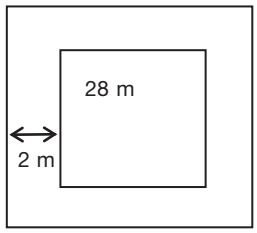
$= [24 - 2(6)]$ m $= 12$ m.

Area of the rectangular plot not covered by the path = Area of the inner rectangle $= (18)\,(12)$ m$^2$

$= 216$ m$^2$.

**2.09.** A path of uniform width of two metres runs around and outside a square plot of side 28 metres. If the path is to be covered with tiles at the rate of ₹15 per square metre, then find the total cost of the work.

**Sol:** Let the area of the path $= A$




Area of the outer square plot

$= [28 + 2\ (2)]^2\ m^2 = 1024\ m^2$.
Area of the inner square plot $= 28^2\ m^2$
$= 784\ m^2$
Total cost of the work in rupees
$= (1024 - 784)\ (15) = 240(15) = 3600$.

**2.10.** The angle subtended by an arc at the centre of a circle is 70°. If the circumference of the circle is 132 cm, then find the area of the sector formed.

**Sol:** Let the radius of the circle be $r$ cm.
Given $2\pi r = 132 \Rightarrow r = 21$.
Area of the sector

$$= \frac{70°}{360°}\left(\frac{22}{7}\right)(21^2)\ cm^2 = 269.5\ cm^2.$$

**2.11.** Two circles of radii 9 cm and 4 cm touch each other externally. Find the length of their direct common tangent.

**Sol:** As the circles touch each other externally, distance between centers = sum of the radii = 13 cm.
Length of the direct common tangent ($L$)

$$= \sqrt{\left(\begin{matrix}\text{Distance between}\\ \text{the centres}\end{matrix}\right)^2 - \left(\begin{matrix}\text{Difference in the}\\ \text{radii of the two circles}\end{matrix}\right)^2}$$

The difference in the radii is 5 cm.

$$\therefore L = \sqrt{\left(13^2 - 5^2\right) cm^2} = 12\ cm$$

**2.12.** If the base of a parallelogram is 9 cm and the height of the parallelogram is 6 cm, find the area of the parallelogram.

**Sol:** The area of the parallelogram= base × height = $(9)\ (6)\ cm^2 = 54\ cm^2$.

**2.13.** $ABCD$ is a parallelogram. Are the areas of $\Delta ABC$ and $\Delta ADB$ equal?

**Sol:** $ABCD$ is a parallelogram. Therefore, $AB$ is parallel to $CD$. $\Delta ABC$ and $\Delta ADB$ lie on the same base $AB$. They also lie between the same parallels $AB$ and $DC$. Hence, their heights are equal. Therefore, $\Delta ABC$ and $\Delta ADB$ are equal in area.

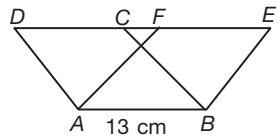
**2.14.** In a rhombus $ABCD$, $AC = 24$ cm and $BD = 20$ cm. Find the area of the rhombus.

**Sol:** Area of the rhombus

$= (1/2)$(Product of the diagonals).

$= (1/2)\ (AC)\ (BD) = (1/2)\ (24)\ (20)\ cm^2 = 240\ cm^2$

**2.15.** In the figure given below, if the area of parallelogram $ABCD$ is 208 $cm^2$, then find the height of the parallelogram $ABEF$.




**Sol:** Parallelograms $ABCD$ and $ABEF$ lie on the same base $AB$ and between the same parallels $AB$ and $DE$. Hence, they are equal in area. Therefore, area of $ABEF$ = 208 $cm^2$, height = Area/base

$$= \frac{208\ cm^2}{13\ cm} = 16\ cm$$

**2.16.** A rectangle has its length 10 cm more than twice the side of a square and breadth 12 cm less than the side of the square. If the rectangle has the same perimeter as that of the square, find the area of the square.

**Sol:** Let the side of the square be $S$ cm, Length of the rectangle is $(2S + 10)$ cm and breadth is $(S - 12)$ cm
$= S - 12$.
$2\ (2S + 10 + S - 12) = 4S$.
$6S - 4 = 4S \Rightarrow S = 2$
Area of the square $= 2^2\ cm^2 = 4\ cm^2$.

**2.17.** Find the lateral surface area and volume of a cuboid whose length is 12 cm, breadth is 10 cm and height is 8 cm.

**Sol:** Lateral surface area of a cuboid
$= 2h\ (l + b) = 2\ (8)\ (12 + 10)\ cm^2 = 352\ cm^2$.
Volume of a cuboid $= l$ bh
$= 12(10)\ (8) = 960\ cm^3$

**2.18.** The cross section of a canal is a trapezium of 5 m width at the top, 3 m at the bottom and the depth is 2 m. Find the quantity of earth dug out in digging a 150 m long canal.

**Sol:** Area of cross section of the canal = Area of trapezium $= \frac{h}{2}\ (a + b) = \frac{2}{2}\ (5 + 3)\ m^2 = 8\ m^2$

Volume of the earth dug out = area of cross section (length) $= 8\ (150)\ m^3 = 1200\ m^3$

**2.19.** A wall of dimensions 12 m × 8 m × 36 m is to be constructed with bricks of dimensions 10 cm × 9 cm × 8 cm. Find the number of bricks required to construct the wall.

**Sol:** Volume of the wall $= 1200(36)\ (800)\ cm^3$
Volume of brick $= 10(9)\ (8)\ cm^3$

$$\text{Number of bricks} = \frac{1200(36)\ (800)}{10(9)\ (8)} = \frac{cm^3}{cm^3}$$
$= 48{,}000$

**2.20.** The cost of flooring a room 20 m long at ₹7 per square metre is ₹2520. Find the breadth of the room.

**Sol:** Area of the room $= \frac{₹\ 2520}{₹7/\text{m}^2} = 360\ \text{m}^2$.

Breadth of the room $= \frac{360}{20}\frac{\text{m}^2}{\text{m}} = 18\ \text{m}.$

**2.21.** A cylindrical vessel of diameter 48 cm has water to a height of 10 cm. *A* metal cube of 14 cm edge is immersed in it. Calculate the height to which the water level rises.

**Sol:** Let the increase in the water level be $h$. Volume of water that rises $= \pi r^2 h$. Since the volume of water that rises is equal to the volume of the cube,

$$(22/7)(24)(24)(h)\ \text{cm}^2 = 14(14)(14)\ \text{cm}^3$$

$$h = \frac{14(14)(14)}{(22/7)(24)(24)}\frac{\text{cm}^3}{\text{cm}^2} = 1.516\ \text{cm}.$$

**2.22.** The external radius of a steel pipe is 1.6 cm and its thickness is 1 cm. If 1 $\text{cm}^3$ of steel weighs 20 gms, then find the weight of the steel pipe of length 28 cm.

**Sol:** External radius, $R = 1.6$ cm.
Internal radius, $r = (1.6 - 1)$ cm $= 0.6$ cm.
Volume of the steel pipe of length 28 cm

$$= \frac{22}{7}\left(1.6^2 - 0.6^2\right)(28)\ \text{cm}^3.$$

Weight of the steel pipe

$$= \left(\frac{22}{7}\right)\left(1.6^2 - 0.6^2\right)(28)(20)\ \text{cm}^3\ \frac{\text{gm}}{\text{cm}^3}$$

$= 3872$ gm $= 3.872$ kg.

**2.23.** A conical cup is completely filled with ice cream and topped with a hemispherical scoop on its open end. Find the volume of ice cream, if the radius of the base of the cone is 7 cm and the vertical height of the cone is 12 cm.

**Sol:** Volume of ice cream inside the cone= volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\left(\frac{22}{7}\right)(7)(7)(12)\ \text{cm}^3 = 616\ \text{cm}^3$$

Volume of the hemispherical scoop of ice cream

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3}\left(\frac{22}{7}\right)(7)(7)(7)\ \text{cm}^3 = \frac{2156}{3}\ \text{cm}^3$$

Total volume of ice cream $= \left(616 + \frac{2156}{3}\right)\text{cm}^3$

$$= \frac{4004}{3}\ \text{cm}^3$$

**2.24.** If the radius of a right cylinder is increased by 20% and its height is decreased by 10%, find the percentage increase in its volume.

**Sol:** Original volume $= \pi r^2 h$.

Increase in radius $= \frac{20}{100}r = 0 \cdot 2r$

Decrease in height $= \frac{10}{100}h = 0.1h.$

New volume $= \pi\,(r + 0{\times}2r)^2\,(h - 0{\times}1h)$
$= \pi\,(1.2r)^2\,(0.9h) = \pi \times 1{\times}44\ r^2 \times 0{\times}9h$
$= 1{\times}296\ \pi r^2 h.$
Percentage increase in the volume of the cylinder

$$= \frac{1 \cdot 296\pi r^2 h - \pi r^2 h}{\pi r^2 h}(100\%) = 29.6\%.$$

**2.25.** A swimming pool 150 m long and 50 m wide is 1 m deep at the shallow end and 6 m deep at the deep end. Find the volume of the pool.

**Sol:** Area of cross section perpendicular to the width
$= (150)\ (1/2)\ (1 + 6)\ \text{m}^2 = 525\ \text{m}^2$
Volume = Area of cross section × Width
$= 525\ (50)\ \text{m}^3 = 26{,}250\ \text{m}^3$

**2.26.** A metallic solid cylinder of 14 cm diameter and 32 cm height is cast into 77 solid cubes of equal size. What is the edge of each cube thus formed?

**Sol:** Volume of the cylinder

$$= \frac{22}{7}\left(\frac{14}{2}\right)^2(32)\ \text{cm}^3 = 4298\ \text{cm}^3 \qquad (1)$$

If the edge of each cube is $a$, total volume of the cubes is $77a^3$ (2)
$77a^3 = 4928\ \text{cm}^3$

$$\Rightarrow a = \sqrt[3]{64}\ \text{cm} = 4\ \text{cm}.$$

Hence, edge of the cube formed is 4 cm.

**2.27.** A cone has its height equal to a third of its diameter. If the height measures $p$ cm, find its volume, in terms of $p$.

**Sol:** Volume of the cone = $\frac{1}{3}\pi r^2 h \text{ cm}^3$

$h = \frac{2r}{3} = p \Rightarrow r = \frac{3}{2}p$ (Here, $h$, $r$, $p$ represent only numbers.)

Volume of the cone

$= \frac{1}{3}\pi\left(\frac{3}{2}p\right)^2 p \text{ cm}^3 = \frac{3}{4}\pi p^3 \text{ cm}^3$

**2.28.** A metallic cone of diameter 48 cm and height 18 cm is melted into identical spheres each of radius 2 cm. How many such spheres can be made?

**Sol:** Volume of each sphere = $\frac{4}{3}\pi r^3 \text{ cm}^3 = \frac{4\pi}{3}(2^3) \text{ cm}^3$ (1)

Volume of the cone $= \frac{1}{3}\pi\left(\frac{48}{2}\right)^2 (18) \text{ cm}^3$ (2)

Dividing (2) by (1), number of spheres $= \frac{24(24)(18)}{4(8)} = 324$

**2.29.** A solid is in the form of a cylinder surmounted by a cone. The diameter of the cone is 28 cm. The height of the cylindrical part is 16 cm and that of the conical part is10 cm. Find the volume of the solid.

**Sol:** Volume of the solid = Volume of the cylinder + volume of the cone

$= \frac{22}{7}(14)^2(16) + \frac{1}{3}\left(\frac{22}{7}\right)(14)^2(10) \text{ cm}^3$

$= \frac{22}{7}(14^2)\left(16 + \frac{10}{3}\right) \text{ cm}^3$

$= \frac{22}{7}(14^2)\left(\frac{58}{3}\right) \text{ cm}^3 = 11{,}909 \text{ cm}^3$ (rounded off to the nearest unit)

**2.30.** The area of the base of a right circular cone is 154 $\text{cm}^2$ and its height is 24 cm. Find its volume and curved surface area.

**Sol:** Area of the base = $\pi r^2 = 154 \text{ cm}^2$

$\Rightarrow r^2 = \frac{7}{22}(154) \text{ cm}^2 = 49 \text{ cm}^2$.

Slant height = $l = \sqrt{r^2 + h^2}$

$= \sqrt{7^2 + 24^2} \text{ cm} = \sqrt{49 + 576} \text{ cm} = 25 \text{ cm}$.

Curved surface area = $\pi r l$

$= \frac{22}{7}(7)(25) \text{ cm}^2 = 550 \text{ cm}^2$.

Volume of the cone = $\frac{1}{3}(\pi r^2 h)$

$= \frac{1}{3}(154)(24) \text{ cm}^3 = 1232 \text{ cm}^3$

**2.31.** Find the volume of the largest sphere which can be cut from a cube having edges of 7 cm length.

**Sol:** For the largest sphere, the diameter is equal to the edge of the cube. Hence, radius $r$ of this sphere = $\frac{7}{2}$ cm.

Volume of this sphere

$= \frac{4}{3}\left(\frac{22}{7}\right)\left(\frac{7}{2}\right)^3 \text{ cm}^3 = \frac{539}{3}\text{cm}^3$.

**2.32.** What is the maximum number of spherical balls of radius 4 cm that can be packed in a box of size 40 cm × 24 cm × 8 cm?

**Sol:** Diameter of the spherical ball = 8 cm.
Number of spherical balls that can be adjusted along the length = 40/8 = 5; along the breadth = 24/8 = 3; and along the height = 8 cm/8 = 1.
There will be only one layer.
Total number of spherical balls = 5(3)(1) = 15.

**2.33.** The area of the floor of a conical tent is 616 $\text{ft}^2$. If its height is $2\sqrt{15}$ ft, then find the area of the canvas required for the tent.

**Sol:** Area of the base = $\pi r^2 = 616 \text{ ft}^2$

$r^2 = \frac{616(7)}{22}\text{ft}^2$

$r = \sqrt{28(7)\text{ ft}^2} = 14 \text{ ft}$.

Slant height $(l) = \sqrt{14^2 + \left(2\sqrt{15}\right)^2}$ ft

$= \sqrt{196 + 60}$ ft $= 16$ ft.

Canvas required = $\pi r l = \frac{22}{7}(14)(16) = 704 \text{ ft}^2$.

**2.34.** A roller is 8 m long and has a diameter of 0.7 m. It takes exactly 1000 rotations of the roller to level a road. If the cost of using the roller is ₹2 per square metre, then find the total cost of levelling the road.

**Sol:** Radius of the roller = $0.7 / 2\ m = 0.35\ m$

Length of the roller = 8 m.
Curved surface area of the roller = $2\pi rh = 2(22/7)$ $(0.35)$ $(8)$ $m^2 = 17.6\ m^2$.
In 1 rotation the roller covers 17.6 $m^2$. In 1000 rotations, the roller covers 17600 $m^2$.

Hence, the cost = $(17600)(2)\ m^2 \dfrac{Rs}{m^2} =$ ₹35,200

**2.35.** A cylinder has its height equal to its diameter. If the diameter measures $p$ cm, find its volume in terms of $p$.

**Sol:** Volume of the cylinder = $\pi r^2 h$
As diameter is $p$ cm, height is also $p$ cm.

Volume = $\pi\left(\dfrac{p}{2}\right)^2 p\ \text{cm}^3 = \pi\dfrac{p^3}{4}\ \text{cm}^3$.

**2.36.** The sides of a triangle are 12 cm, 18 cm and 24 cm. Find its area.

**Sol:** The semi – perimeter (s)

$= \dfrac{12+18+24}{2} = 27$

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(27)(27-12)(27-18)(27-24)}$

$= 27\sqrt{15}$ sq. cm

**2.37.** The wheel of a motorcar makes 800 revolutions in covering 880 m. Find its diameter

$\left(\text{Take } \pi = \dfrac{22}{7}\right)$.

**Sol:** Let the diameter be $d$ m.
Distance covered = (Number of revolutions) (circumference of the wheel).

$\therefore 880 = (800)\left(\dfrac{22}{7}\right)(d)$

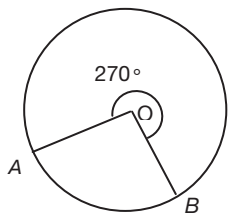
$d = 0.35$

**2.38.** A copper wire would enclose an area of 484 sq.cm if it was bent in the shape of a square. If it was bent in the form of a circle, find the radius of the circle. $\left(\text{Take } \pi = \dfrac{22}{7}\right)$

**Sol:** Area of the square = 484. Hence, perimeter of the square = 88. If the wire was bent in the form of a circle, it would have its circumference as 88 cm.

Hence, radius = $\dfrac{88}{(2)\left(\dfrac{22}{7}\right)} = 14$ cm

**2.39.**




In the figure above, O is the centre of the circle. Find the length of the minor arc AB if OA = 7 cm.

$\left(\text{Take } \pi = \dfrac{22}{7}\right)$

**Sol:** $\angle AOB = 360° - 270° = 90°$

Length of the minor arc $AB = \dfrac{\theta}{360°} \times 2\pi r$

$= \left(\dfrac{90°}{360°}\right)\left((2)\left(\dfrac{22}{7}\right)(7)\right) = 11$ cm

**2.40.** In the previous example, find the area of the minor sector AOB $\left(\text{Take } \pi = \dfrac{22}{7}\right)$.

**Sol:** Area of the sector = $\dfrac{\theta}{360°} \times \pi r^2$

$= \left(\dfrac{90°}{360°}\right)\left(\dfrac{22}{7}\right)(7)^2 = 38{\times}5$ sq.cm

**2.41.** The circumference of a circle is $2\dfrac{1}{2}$ times that of another circle. How many times the area of the smaller circle is the area of the larger circle?

**Sol:** Let the radius of the smaller circle be $r$ cm.
Radius $\alpha$ circumference.

Hence, radius of the larger circle = $\frac{5}{2}$ (Radius of the smaller circle) = $\frac{5}{2}r$ cm.

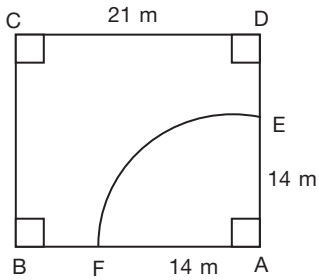
Area = $\pi$ (radius)$^2$

$\therefore$ Area of the larger circle = $\pi\left(\frac{5}{2}r\right)^2$

= $\frac{25}{4}\left[\pi r^2\right] = \frac{25}{4}$ (Area of the smaller circle)

**2.42.** A goat is tied to a corner of a square field of side 21 m with a rope of length 14 m. Find the area of the square field that the goat cannot graze $\left(\text{Take } \pi = \frac{22}{7}\right)$.

**Sol:**




Area that the goat cannot graze = (Area of ABCD) – (Area of sector AEF)

= (212) – $\left(\frac{90°}{360°}\pi(14)^2\right)$ = 287 sq. m

**2.43.** A circular garden has a diameter of 56 m. It has a circular path running all around and outside it. The difference between the circumferences of the larger and the smaller gardens is 44 m. Find the width of the path $\left(\text{Take } \pi = \frac{22}{7}\right)$

**Sol:** Difference between the circumferences = $2\pi$ (radius of the larger garden) – $2\pi$ (radius of the smaller garden) = $2\pi$ (width of the path)

$(2)\left(\frac{22}{7}\right)$ (width of the path) = 44

Width of the path = 7 m

**2.44.** A circular garden has a radius of 15 m. It is surrounded by a circular path of width 7 m. If the path is to be polished at a rate of ₹15 per sq. m, find the total cost of polishing the path. $\left(\text{Take } \pi = \frac{22}{7}\right)$

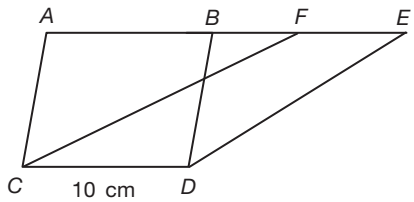
**Sol:** Total cost of polishing = (15) (Area to be covered)

= $15\left(\frac{22}{7}\left(22^2 - 15^2\right)\right)$ = ₹12210

**2.45.** The area of parallelogram *ABCD*, whose height on side AB is 9 cm, is 108 sq.cm. Find length of side AB.

**Sol:** Area = (AB) (height) = 108 = (AB) (9)
$\therefore$ AB = 12 cm

**2.46.**




In the figure above, *ACDE* is a trapezium. If the area of parallelogram *ABCD* is 100 sq.cm, find the area of parallelogram *CDEF*.

**Sol:** Area of *ABCD* = Area of *CDEF* (the parallelograms are between the same parallel lines and hence their areas will be equal).
$\therefore$ Area of *CDEF* = 100 sq.cm

**2.47.** The areas of a rectangle and a square are in the ratio 3 : 4. The length of the rectangle is 8 cm more than that of the square. The breadth of the rectangle is 8 cm less than that of the square. Find the perimeter of the square.

**Sol:** Let the side of the square $a$ cm. Length of the rectangle and its breadth are $(a + 8)$ cm and $(a - 8)$ cm, respectively. Given that

$(a + 8)\ (a - 8) = \frac{3}{4}a^2$

$\Rightarrow a^2 - 64 = \frac{3}{4}a^2$

$\Rightarrow a = 16$

Hence, perimeter of the square = $4a$ = 64 cm

**2.48.** If the square on the diagonal of a rectangle is four the area of the rectangle, find the ratio of its length and breadth.

**Sol:** Let the length and breadth of the rectangle be $l$ and $b$, respectively ($l > b$). Its diagonal = $\sqrt{l^2 + b^2}$

Given that $\left(\sqrt{l^2 + b^2}\right)^2 = 4(lb)$

$\Rightarrow l^2 + b^2 - 4lb = 0$

By dividing throughout with $lb$ and taking $\frac{l}{b}$ As $K$, we get $K^2 - 4K + 1 = 0$

$\Rightarrow K = \frac{4 \pm \sqrt{16+4}}{2} = 2 + \sqrt{5}$

As $K > 1$, $K = 2 \pm \sqrt{5}$

**2.49.** Find the total surface area and the volume of a cuboid whose length, breadth and height are 20 cm, 15 cm and 12 cm, respectively.

**Sol:** Total surface area = $2(l\,b + bh + lh)$
= 2 ((20) (15) + (15) (12) + (20) (12))
= 1440 sq.cm
Volume = $lbh$ = (20) (15) (12) = 3600 cubic cm

**2.50.** A wall of measurements 30 m × 12 m × 4 m was constructed with bricks of dimensions 8 cm × 6 cm × 6 cm. If 80% of the wall consists of bricks, find the number of bricks used for the construction.

**Sol:** Volume of the wall
= (30) (100) (12) (100) (D) (100) cubic cm.
Total volume of the bricks = 0.8 [volume of the wall]
Volume of each brick = (8) (6) (6) cubic cm
Number of bricks required

$= \frac{(0.8)(30)(100)(12)(100)(4)(100)}{(8)(6)(6)} = 4000000$

**2.51.** A certain type of wood costs ₹500 per $m^3$. A solid cubical block of this wood costs, ₹108. Find its volume as well as its length.

**Sol:** Volume = $\frac{108}{500} = 0.216\ m^3$

As the block is in the shape of a cube, its length = $\sqrt[3]{\text{volume}} = 0.6$ m

**2.52.** A cylinder has a diameter of 36 cm. It has water upto a height of 22 cm. A metal cuboid which has a length of 22 cm and each of whose lateral face is a square of side 18 cm is immersed in it. Find the rise in the height of the water level $\left(\text{Take } \pi = \frac{22}{7}\right)$

**Sol:** Let the rise in the water level be $h$ cm.

$\frac{22}{7}\left(\frac{36}{2}\right)^2 h = (22)\left(18^2\right) \Rightarrow h = 7$

**2.53.** A steel pipe has an external diameter of 1 × 6 cm and a thickness of 1 mm. Each cubic cm of it weights 8 gms. Find its weight if its length is 70 cm $\left(\text{Take } \pi = \frac{22}{7}\right)$

**Sol:** External radius = $\frac{1.6}{2} = 0.8$ cm

Internal radius = 0.8 − 0.1 = 0.7 cm

Volume = $\frac{22}{7}(70)\,(0.8^2 - 0.7^2) = 33$

Weight = (33) (8) = 264 gm

**2.54.** The radius as well as the height of a right circular cone increases by 20%. Find the percentage increase in its volume.

**Sol:** Let the radius as well as the height of the cone be 100.

Initial volume = $\frac{1}{3}\pi(100^2)(100) = \frac{1}{3}\pi(100^3)$

New radius = New height = 120

Final volume = $\frac{1}{3}\pi(1.2(100))^3$

$= 1.728\left(\frac{1}{3}\pi(100^3)\right)$

$\therefore$ Volume increased by $72 \times 8\%$

**2.55.** A swimming pool is 200 ft long and 60 ft wide. It is 2 ft deep at the shallow end of the length and is 6 ft deep at the deep end of the length. Find the volume of the water contained in it.

**Sol:** Area of cross section

$= \frac{1}{2}(200)\,(2+6) = 800$ sq.m

Volume = (Area of cross section) (width)
= 48000 cubic m

**2.56.** A metallic solid cylinder has a diameter of 24 cm and a height of 96 cm. It is melted and made into 48 solid spheres of equal size. Find the diameter of each sphere.

**Sol:** Volume of the cylinder = $\frac{22}{7}\left(\frac{24}{2}\right)^2 (14)$

As the cylinder is melted and recasted into a 48 indentical solid spheres, the volume of the cylinder is equal to the total volume of all the 48 spheres.
Let the diameter of each sphere be d cm

$$48\left[\frac{4}{3}\left(\frac{22}{7}\right)\left(\frac{d}{2}\right)^3\right] = \frac{22}{7}(12)^2 \ (96)$$

$d = 12$

**2.57.** A cylinder has its height equal to twice its diameter. If its radius is r cm, find its volume in terms of $r$.

**Sol:** Height = 2 (diameter)
= 2 (2$r$) = 4$r$ cm
Volume = $\pi r2$ (4$r$)
= 4$\pi r3$ cubic cm

**2.58.** A solid is in the form of a cylinder surmounded by a cone. The diameter of the cone is 14 cm The heights of the cylinder and the cone are 12 cm and 6cm, respectively. Find its volume. $\left(\text{Take } \pi = \frac{22}{7}\right)$.

**Sol:** Volume = Volume of cylinder + Volume of cone

$$= \pi\left(\frac{14}{2}\right)^2 (12) + \frac{1}{3}\pi\left(\frac{14}{2}\right)^2 (6)$$

$$= 49\pi\,(14)$$

$$= (49)\left(\frac{22}{7}\right)(14)$$

= 2156 cubic cm.

**2.59.** The area of the base of a right circular cone is 1386 sq.cm. Its height is 20 cm. Find its volume and curved surface area $\left(\text{Take } \pi = \frac{22}{7}\right)$.

**Sol:** Let the radius be $r$ cm.

$$\frac{22}{7}r^2 = 1386 \Rightarrow r = 21$$

$$\text{Volume} = \frac{1}{3} \times \frac{22}{7}\ (21^2)\ (20)$$

= 9240 cubic cm
Let the slant height be l cm.

$$l = \sqrt{21^2 + 20^2} = 29 \text{ cm}$$

Curved surface area

$$= \frac{22}{7}(21)(29) = 1914 \text{ sq.cm.}$$

**2.60.** A conical cup is filled with ice-cream. The ice cream forms a hemispherical shape on its open top. The height of the hemispherical part is 7 cm. The radius of the hemispherical part equals the height of the cone. Find the volume of the ice cream. $\left(\text{Take } \pi = \frac{22}{7}\right)$

**Sol:** As the radius of the hemispherical part equals the height of the cone, radius of the part = height of the part = height of the cone = 7 cm
Volume of the ice cream

$$= \frac{1}{3}\pi\,(7^2)(7) + \frac{2}{3}\pi(7^3)$$

$$= \frac{22}{7}(7^3) = 1078 \text{ cubic cm}$$

**2.61.** Find the volume of the largest right circular cylinder which can be cut from a cube of side 7 cm. $\left(\text{Take } \pi = \frac{22}{7}\right)$

**Sol:** The largest right circular cylinder will have its height as well as its diameter equal to the side of the cube.
∴ Its volume

$$= \frac{22}{7}\left(\left(\frac{7}{2}\right)^2\right)(7) = 269 \cdot 5 \text{ cubic cm}$$

**2.62.** A ten rupee note measures 15 cm × 8 cm and a bundle of 90 such notes is 1 cm thick. Find the

value of the ten rupee notes that can be contained in a box of size 48 cm × 36 cm × 30 cm, if the bundles are tightly packed in it without any empty space.

**Sol:** Volume of the box = (48 × 36 × 30) cubic cm
Volume of each bundle = (15 × 8 × 1) cubic cm
Number of bundles in the box

$$= \frac{(48)\,(36)\,(30)}{(15)(8)(1)} = 432$$

Value of each bundle = ₹900
∴ Total value = (900) (432) = ₹388800

**2.63.** The area of the floor of a conical tent having a circular base is 616 sq. m. Find the canvas required for the tent if its height is 48 cm. $\left(\text{Take } \pi = \frac{22}{7}\right)$

**Sol:** Let the radius of the tent be r m.

$$\frac{22}{7} r^2 = 616$$

$r = 14$
Let the slant height be l m.

$$1 = \sqrt{14^2 + 48^2} = 50$$

Canvas required = $\frac{22}{7}$ (14) (50) = 2200 sq. m.

**2.64.** A roller has a length of 3 m. Its diameter is 0×7 m. It requires 600 revolutions of the roller to level a road. The cost of usage of the roller is ₹5 per sq. m. Find the total cost of levelling the road. $\left(\text{Take } \pi = \frac{22}{7}\right)$

**Sol:** Curved surface area of the roller

$$= 2\left(\frac{22}{7}\right)\left(\frac{0\cdot 7}{2}\right)(3) = 6 \times 6 \text{ sq. m}$$

Cost of levelling the road
= (600) (6 × 6) (5) = ₹19800

**2.65.** A sphere and a hemisphere have the same radius. Find the ratio of their
(i) volumes.
(ii) curved surface areas.
(iii) total surface areas.

**Sol:** (i) Ratio of volumes = $\frac{4}{3}\pi r^3 : \frac{2}{3}\pi r^3 = 2 : 1$

(ii) Curved surface area of a sphere = Its total surface area = $4\pi r^2$
∴ Ratio of curved surface areas
= $4\pi r^2 : 2\pi r^2 = 2 : 1$

(iii) Ratio of total surface areas
= $4\pi r^2 : (2\pi r^2 + \pi r^2) = 4 : 3$

## EXERCISE-1

***Directions for questions 1 to 60*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

1. The lengths of two sides of a triangle are 6 cm and 8 cm. The angle between them is 30°. Find the area of the triangle. (in $cm^2$) _____

2. What is the breadth of a cuboid having length and height equal to 6 cm and 4 cm, respectively and the total surface area of 148 sq. cm? (in cm)
(A) 5 (B) 4 (C) 3 (D) 2

3. A parallelogram and a triangle lie on a common base. One of the vertices of the triangle lies on the side parallel to its base. Find the ratio of the areas of the parallelogram and the triangle.
(A) 3 : 1 (B) 4 : 1 (C) 3 : 2 (D) 2 : 1

4. A circle has a radius of 7 cm with centre *O*. *AOB* represents a sector where $\angle AOB = 90°$. Find the area (in $cm^2$) of the sector *AOB*. _____

5. What is the total surface area of a solid hemisphere having radius equal to *r*?
(A) $\pi r^2$ (B) $2\pi r^2$ (C) $3\pi r^2$ (D) $4\pi r^2$

6. *N* spherical balls were melted and cast into a cylinderical rod whose height equals its radius. The radius of the rod equals four times the radius of each sphere. Find *N*.
(A) 32 (B) 54 (C) 48 (D) 64

7. Find the area of the triangle (in $cm^2$), whose sides are 13 cm, 13 cm and 10 cm. _____

8. What is the volume of a sphere having radius equal to 3 cm?
(A) 36 $\pi$ $cm^3$ (B) 27 $\pi$ $cm^3$
(C) 72 $\pi$ $cm^3$ (D) 54 $\pi$ $cm^3$

**9.** Find the area (in $cm^2$) of a cyclic quadrilateral whose sides are 8 cm, 10 cm, 12 cm and 16 cm.

(A) $\sqrt{15015}$ (B) $\sqrt{30030}$

(C) $\sqrt{10010}$ (D) None of these

**10.** The radii of two spheres are in the ratio 2 : 3. Find the ratio of their curved surface areas.

(A) 2 : 3 (B) 4 : 9 (C) 1 : 3 (D) 5 : 9

**11.** If a right cone is inscribed in a hollow cylinder, then what fraction of the cylinder is not covered by the right cone? Ignore the thickness of the cylinder.

(A) $1/3^{rd}$ (B) $2/3^{rd}$

(C) $1/6^{th}$ (D) 1/2

**12.** What is the area of the triangle whose inradius and semi perimeter are $r$ and $s$ respectively?

(A) $\frac{rs}{2}$ (B) $\frac{rs}{4}$ (C) $2rs$ (D) $rs$

**13.** What is the total surface area of a pyramid having a slant height of 6 cm and the base of which is a square of side 4 cm?

(A) 64 sq. cm (B) 48 sq. cm

(C) 36 sq. cm (D) 24 sq. cm

**14.** $R_1$ and $R_2$ are two ropes. $R_1$ can make 49 rounds on the circumference of the base of a cylinder whose diameter is 60 cm. $R_2$ can make 70 rounds on the circumference of the base of a cylinder whose diameter is 42 cm. Find the difference between the lengths of $R_1$ and $R_2$ (in cm).

(A) 140 (B) 70 (C) 40 (D) 0

**15.** Find the area of a trapezium (in $cm^2$) whose shorter and longer parallel sides have lengths 12 cm and 18 cm respectively and are separated by 15 cm.

(A) 180 (B) 195 (C) 210 (D) 225

**16.** What is the total surface area of a cone having slant height and radius equal to 5 cm and 4 cm respectively?

(A) 24 $\pi$ sq. cm (B) 36 $\pi$ sq. cm

(C) 48 $\pi$ sq. cm (D) 60 $\pi$ sq. cm

**17.** The volume of a right circular cone is $196\pi$ $cm^3$. If the area of its base is 154 $cm^2$, what is the vertical height of the cone (in cm)? _____

**18.** Find the perimeter (in cm) of a semicircle of radius 14 cm. _____

**19.** What is the lateral surface area of the frustum of a cone having top radius, radius of the base and slant height equal to 4 cm, 6 cm and 5 cm respectively?

(A) 50 $\pi$ sq. cm (B) 48 $\pi$ sq. cm

(C) 24 $\pi$ sq. cm (D) 36 $\pi$ sq. cm

**20.** Find the longest diagonal of a cuboid having the length, breadth and the height of $l$, $b$ and $h$ respectively.

(A) $\sqrt{l^2 + b^2 + h^2}$ (B) $2\sqrt{l^2 + b^2 + h^2}$

(C) $\frac{\sqrt{l^2 + b^2 + h^2}}{2}$ (D) None of these

**21.** A picture measures 80 cm × 50 cm. It has a frame of uniform width of 10 cm all around and outside it. What is the area of the frame (in $cm^2$)? _____

**22.** Find the area of an equilateral triangle (in $cm^2$), whose side is 6 cm.

(A) $6\sqrt{3}$ (B) $12\sqrt{3}$ (C) $8\sqrt{3}$ (D) $9\sqrt{3}$

**23.** The total surface area of a frustum of a square pyramid with side of top as 4 cm, the side of the base as 8 cm and slant height as 6 cm is

(A) 176 sq. cm. (B) 196 sq. cm.

(C) 224 sq. cm. (D) 49 sq. cm.

**24.** A trapezium has parallel sides of 20 cm and 10 cm, and the distance between them is 15 cm. Find its area in $cm^2$.

_____

**25.** The circumference of a circle and the perimeter of a rectangle are in the ratio $\pi : 2$. Find the ratio of their areas if the radius of the circle equals one of the dimensions of the rectangle.

(A) $\pi : 1$ (B) $\pi : 2$ (C) $\pi : 3$ (D) $\pi : 4$

**26.** What is the slant height of a cone, given the height of the cone is 8 cm and the diameter of the base is 12 cm?

(A) 10 cm (B) 7 cm (C) 14 cm (D) 16 cm

**27.** The volume of a prism (in $cm^3$) whose height is 8 cm and whose base is an equilateral triangle of side 4 cm is $x\sqrt{3}$. Find $x$. _____

**28.** The radius of a garden that is in the shape of a circle is 35 m. There is a path of uniform width of 7 m all around and outside the garden. What is the area of the path (in $m^2$)? _____

**29.** A triangle has sides $a$, $b$ and $c$ and a circumradius of $R$. Find its area.

(A) $\frac{abc}{R}$ (B) $\frac{abc}{2R}$ (C) $\frac{abc}{4R}$ (D) $\frac{abc}{8R}$

**30.** What is the volume of a frustum of a cone having top radius, radius of the base and height as $r$, $R$ and $h$, respectively?

(A) $\frac{1}{3}\pi h\,(r^2 + R^2 - Rr)$ (B) $\frac{1}{3}\pi h\,(r^2 - R^2 - Rr)$

(C) $\frac{1}{3}\pi h\,(r^2 + R^2 + Rr)$ (D) $\frac{1}{3}\pi h\,(r^2 + R^2 + 2Rr)$

**31.** A cubical box $B$ has a side of 30 m. Find the maximum number of cubical boxes of side 5 m each that can be placed inside the box $B$.
(A) 162 (B) 36 (C) 108 (D) 216

**32.** Find the area (in $cm^2$) of a rhombus whose diagonals are 6 cm and 8 cm. _____

**33.** A right circular cone is cut parallel to its base at half of its height. What is the ratio of the volume of the smaller cone formed to that the original cone?
(A) 7 : 8 (B) 1 : 8 (C) 6 : 7 (D) 3 : 5

**34.** In the question 33, what is the ratio of the curved surface area of smaller cone formed to that of the original cone?
(A) 3 : 4 (B) 2 : 3 (C) 1 : 4 (D) 2 : 5

**35.** In the question 33, what is the ratio of the slant height of the frustum formed to that of the original cone?
(A) 1 : 3 (B) 2 : 3 (C) 1 : 2 (D) 3 : 4

**36.** A total of twenty cubic metres of aluminium was used to make a sheet of area 1600 sq m. Find the thickness of the sheet. (in cm) _____

**37.** The length, breadth and the height of a cuboid are 12 cm, 10 cm and 9 cm respectively. Find the volume of the cuboid. (in $cm^3$) _____

**38.** In triangle $PQR$, $PQ = 3$ cm, $PR = 4$ cm and $QR = 6$ cm. Find the length of the median drawn from $P$ to $QR$. (in cm).
(A) $\sqrt{2.5}$ (B) $\sqrt{1.5}$ (C) $\sqrt{3.5}$ (D) $\sqrt{4.5}$

**39.** A metallic sphere of radius 12 cm is melted and recasted into a cylinder, whose radius is 16 cm. What is the height of the cylinder? (in cm) _____ .

**40.** Find the area enclosed by a ring if the radii of the inner circle and the outer circle are 6 cm and 8 cm respectively (in $cm^2$). (Take $\pi = 22/7$)
(A) 66 (B) 88 (C) 77 (D) 99

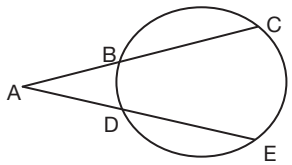
**41.** A right circular cone is cut parallel to its base at $1/3^{rd}$ of its height from the base what is the ratio of the volume of the smaller cone formed to that of the frustum formed?
(A) 8 : 27 (B) 19 : 27
(C) 11 : 27 (D) 8 : 19

**42.** Three circles of radii 2 cm, 4 cm and 6 cm are tangent to one another. What is the area of the triangle formed by joining the centers of the three circles (in $cm^2$)? _____

**43.** The areas of an equilateral triangle and a square are equal. Which of the following can be concluded?
(A) The triangle and the square have the same perimeter.
(B) The perimeter of the square exceeds that of the equilateral triangle.
(C) The perimeter of the square is less than that of t h e triangle.
(D) None of these

**44.** The radius of a wheel is 28 cm. What is the distance covered by the wheel in making 500 revolutions (in m)? _____ .

**45.**




In the figure, $AB = 2$ cm, $BC = 22$ cm and $DE = 8$ cm. Find $AD$. (in cm) _____

**46.** Two cubes are joined to form a cuboid. The edge of each cube is 40 cm. Find the total surface area of the cuboid formed (in sq. cm.).
(A) 12,000 (B) 13,600
(C) 20,000 (D) 16,000

**47.** Find the lateral surface area of a cuboid having length, breadth and height of $l$, $b$ and $h$ respectively.
(A) $2h(l+b)$ (B) $h(l+b)$
(C) $3h(l+b)$ (D) $4h(l+b)$

**48.** The curved surface area of a cylinder is thrice the area of its base. Find the ratio of its height and its radius.
(A) 2 : 1 (B) 2 : 3 (C) 1 : 2 (D) 3 : 2

**49.** The ratio of the areas of a square and an equilateral triangle is $3\sqrt{3} : 4$ If $s$ and $t$ are the perimeters of the square and the triangle, then what is the order relation between $s$ and $t$?
(A) $s > t$ (B) $s = t$
(C) $s < t$ (D) Cannot be determined

**50.** What is the volume of a rectangular box of dimensions 20 cm × 8 cm × 15 cm (in $cm^3$)? _____

**51.** Two adjacent sides of a parallelogram have lengths of 6 cm and 10 cm. The angle between them is 30°. Find the area of the parallelogram. (in $cm^2$) _____

**52.** A triangle has a perimeter of 36 cm. Two of its sides are 12 cm and 15 cm. Find its area (in sq. cm.).
(A) 36 (B) 54 (C) 42 (D) 60

**53.** What is the total surface area of a cube with side 8 cm (in $cm^2$)? _____

**54.** The height of a rectangular prism is 8 cm. The length and the breadth of the base are 4 cm and 2 cm respectively. Find the lateral surface area of the prism. (in $cm^2$) _____

**55.** A wire in the form of a circle of radius 3.5 m is bent in the form of a rectangle, whose length and breadth are in the ratio of 6 : 5. What is the area of the rectangle (in $cm^2$)? _____

**56.** A circle has a diameter of 14 cm. Find the length of the tangent (in cm) drawn to it from a point outside the circle which is at a distance of 25 cm from the centre. _____

**57.** The length and breadth of a rectangular cardboard are 40 cm and 35 cm respectively. If the largest possible square is cut out from this, what is the area of the rectangle unused (in $cm^2$)? _____

**58.** Find the total surface area of a prism having a height of 10 cm and whose base is a square of side 6 cm (in $cm^2$). _____

**59.** The area of a square is equal to five times the area of a rectangle of dimensions 125 cm × 64 cm. What is the perimeter of the square (in cm)? _____

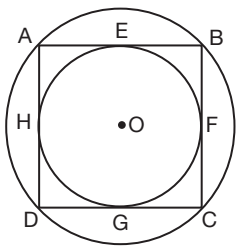
**60.** *ABCD* is a quadrilateral. $AC = 12$ cm. The perpendicular distances from *B* and *D* to *AC* are 4 cm and 6 cm respectively. Find the area (in $cm^2$) of the quadrilateral *ABCD*.
(A) 30 (B) 45
(C) 60 (D) 90

## EXERCISE-2

***Directions for questions 1 to 60*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** The perimeter of the sector of a circle of radius 14 cm is 41 cm. Find the area of the sector.
(A) 85 sq. cm (B) 93 sq. cm
(C) 91 sq. cm (D) 95 sq. cm

**2.** In the figure below, there are two concentric circles with centre *O*. *ABCD* is a square inscribed in the outer circle and circumscribing the inner circle *E*, *F*, *G* and *H* are the points of contact of *AB*, *BC*, *CD* and *DA* respectively with the inner circle. Find the ratio of the areas of the outer circle and the polygon *EFGH*.




(A) $\pi : 1$ (B) $\pi : 2$ (C) $2\pi : 3$ (D) $3\pi : 4$

**3.** *D* and *E* are two points respectively on sides *AB* and *AC* of triangle *ABC* such that *DE* is parallel to *BC*. If the ratio of area of triangle *ADE* to that of the trapezium *DECB* is 25 : 144 and $DE = 5$ cm, then find the length of *BC*.
(A) 12 cm (B) 13 cm
(C) 14 cm (D) 11 cm

**4.** **(a)** Each side of a triangle is 6 cm less than the sum of the other two sides. Find the area of the triangle. (in sq. cm.) _____ $\sqrt{3}$

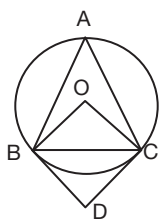
(b) Find the circumradius of an equilateral triangle whose area is $36\sqrt{3}$ sq. cm. (in cm) _____ $\sqrt{3}$

**(c)** Find the area of the circle which is inscribed in a triangle whose sides are 13, 14, and 15. _____ $\pi$

**5.** The length, the breadth and the height of a cuboid are in the ratio of 5 : 4 : 3. The longest rod that can be placed in the cuboid is $10\sqrt{2}$ cm. What is the volume of the cuboid (in $cm^3$)? _____

**6.** If the ratio of the magnitudes of volume of a sphere to the surface area of the same sphere is 1 : 1, then what is the volume of the sphere (in cubic units)?
(A) $36\pi$ (B) $27\pi$
(C) $18\pi$ (D) $42\pi$

**7.**




In the figure, *O* is the centre of the circle. If $OB = 48$ cm and $CD = 14$ cm, find the area of quadrilateral *OBDC* (in $cm^2$).
(A) 672 (B) 608
(C) 702 (D) Cannot be determined

Difficulty Level-2: Moderate

**8.** A cylindrical vessel having base radius 7 cm and height 6 cm is surmounted by a cone of height 24 cm. The curved surface area (in $cm^2$) of the resultant solid is _____ .

**9.** For a right angled triangle, the circumference of the circumcircle is $50\pi$ cm. If the perimeter of the triangle is 112 cm, then find the area of the triangle.
(A) 150 $cm^2$ (B) 240 $cm^2$
(C) 300 $cm^2$ (D) 336 $cm^2$

**10.** *ABC* is a right-angled triangle with two sides of length 14 cm each. Three circles each of radius 7 cm are drawn with the vertices of the triangle as their centres. Find the area (in $cm^2$) of the portion of the triangle which is not included in any of the circles. (Assume $\pi = \frac{22}{7}$) _____

**11.** A toy consists of a hemisphere surmounted by a cone. The radius of the hemisphere is 21 cm and the height of the toy is 28 cm. Find its volume. (in $cm^3$) _____

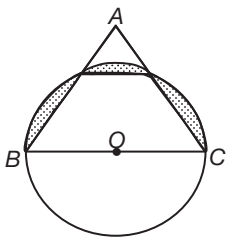
**12.** The cost of painting the total outside surface of a closed cylinder at ₹3 per sq. cm is ₹2,772. If the height of the cylinder is 2 times the radius, find its volume.
(A) 34,312 cu. cm. (B) 3,342 cu. cm.
(C) 2,144 cu. cm. (D) 2,156 cu. cm.

**13.** In triangle *ABC*, *P*, *Q* and *R* are points on *AB*, *BC* and *CA* such that $AP : PB = BQ : QC = CR : RA = 2 : 1$. Find the ratio of the areas of triangles *ABC* and *PQR*.
(A) 3 : 1 (B) 5 : 3 (C) 3 : 2 (D) 7 : 4

**14.** **(a)** A solid metallic cube of side 154 cm is melted to form a wire. If the length of the wire is 23,716 cm, what is the curved surface area of the wire? (in sq. cm.) _____ .

**(b)** A metallic cube of side 16 cm is melted and then recast into small cubes each of side 4 cm. Find the number of such cubes formed. _____

**15.**

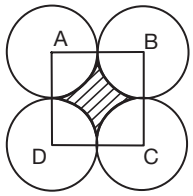



In the given figure *ABC* is an equilateral triangle of side 2 cm and the radius of the circle with centre O is 1 cm. Find the area of the shaded region.

(A) $\left(\frac{\pi}{2} - \frac{\sqrt{3}}{4}\right) cm^2$ (B) $\left(\frac{\pi}{2} - \frac{3\sqrt{3}}{4}\right) cm^2$

(C) $\left(\frac{\pi}{2} - \frac{5\sqrt{3}}{4}\right) cm^2$ (D) $\left(\frac{\pi}{2} + \frac{3\sqrt{3}}{4}\right) cm^2$

**16.**




Four circles each of radius 7 cm, are drawn with each circle touching two adjacent circles as shown. A square is formed by joining the centres of the circles *A*, *B*, *C* and *D*. The area of the shaded region is approximately what percent of the area of the square?
(A) 78% (B) 18%
(C) 25% (D) 21%

**17.** The radius and the height of a right cylinder are each increased by 10%. Find the percentage increase in its volume. _____

**18.** Three cubes, each having an edge of *x* cm and 3 more cubes each having an edge of $2x$ cm are melted to form a cube. If the body diagonal of the new cube is $12\sqrt{3}$ cm, find the value of *x*.
(A) 8 (B) 6 (C) 12 (D) 4

**19.** *CD* is the angle bisector of angle *BCA* in triangle *ABC*, where *BC* = 17.5, *AC* = 18 and *AB* = 17. Find the ratio of the area of triangle *CAD* to the area of triangle *CBD*.
$\overline{\overline{35}}$

**20.** The product of the surface areas (in $cm^2$) of three mutually adjacent faces of a cuboid is 4096. If the area of the base of the cuboid is 16 $cm^2$, find its height. (in cm)
_____

**21.** Uma and Suma wanted to go, from one end of a rectangular park to its diagonally opposite end. Uma walked along two adjacent sides to reach the opposite end while, Suma walked diagonally to reach the opposite end. The distance walked by Uma exceeds that walked by Suma by $(1/4)^{th}$ of the larger side of the park. Find the ratio of the length and breadth of the park.
(A) 4 : 3 (B) 12 : 5
(C) 15 : 8 (D) 24 : 7

**22.** *ABCD* is a parallelogram having an area of $40\sqrt{3}$ sq.cm . *E*, *F*, *G* and *H* are the midpoints of *AB*, *BC*, *CD* and *DA* respectively. Find the area (in sq. cm) of *EFGH*.

(A) $16\sqrt{3}$ (B) $20\sqrt{3}$ (C) $24\sqrt{3}$ (D) $10\sqrt{3}$

**23.** A metallic cone of diameter 32 cm and radius 9 cm is melted and made into identical spheres each of radius 2 cm. How many such spheres can be made? _____

**24.** The total surface area of a cube equals the surface area of a sphere. Find the ratio of the volumes of the cube and the sphere. $\frac{\sqrt{\pi}}{\sqrt{\quad}}$

**25.** The sum of the radius of the base of a solid cylinder and the height of the cylinder is 15 cm. If the total surface area of the cylinder is 660 sq. cm, then find the volume of the cylinder.

(A) 1232 cu. cm (B) 1256 cu. cm
(C) 1296 cu. cm (D) 1272 cu. cm

**26.** The sum of the squares of the length, the breadth, and the height of a cuboid is half its total surface area. Find the ratio of the length to the breadth. _____

**27.** What is the length of the pendulum which swings through an angle of 30° and describes an arc of 66 cm in length?

(A) 134 cm (B) 140 cm
(C) 136 cm (D) 126 cm

**28.** *ABCD* and *EFGH* are opposite faces of a cube with *AE*, *BF*, *CG* and *DH* as its edges. A triangle is formed with lengths of its sides equal to *AG*, *DF* and *CE*. The radius of the circle circumscribing this triangle will be equal to

(A) $1/\left(2\sqrt{3}\right)$ times the edge of the cube.

(B) $1/\sqrt{3}$ times the edge of the cube.

(C) $\sqrt{3}$ times the edge of the cube.

(D) None of these

**29.** A sphere and a cube have the same volume. Find the ratio of the total surface area of the cube to that of the sphere.

(A) $\sqrt{6}:\sqrt{\pi}$ (B) $\sqrt[3]{6}:\sqrt[3]{\pi}$

(C) $\sqrt{\pi}:\sqrt{6}$ (D) $\sqrt[3]{\pi}:\sqrt[3]{6}$

**30.** The length, the breadth and the height of a cuboid are in the ratio 6 : 5 : 4. If the length was 20% more, the breadth was 25% more and the height was 25% less, then the total surface area of the cuboid would have been approximately

(A) 15% more (B) 10% more
(C) 13% less (D) 8% less

**31.** A rectangular reservoir is 80 m long and 35 m wide. At what speed must water flow into it through a pipe of cross-section of area 625 $cm^2$ so that water rises by 2 m in 10 hours?

(A) $8\frac{24}{25}$ kmph (B) $8\frac{12}{25}$ kmph

(C) $12\frac{24}{25}$ kmph (D) $9\frac{12}{24}$ kmph

**32.** *PQRS* is a rhombus. The vertices *P*, *Q* and *R* lie on circumference of a circle whose centre is at '*S*'. Find the area of the circle, if area of the rhombus is $18\sqrt{3}$ sq.cm

(A) $9\pi$ sq. cm (B) $18\pi$ sq. cm
(C) $26\pi$ sq. cm (D) $36\pi$ sq. cm

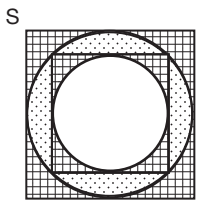
**33.** One of the diagonals of a rectangle is $5\sqrt{2}$ cm long. Find the maximum area (in sq. cm.) that the rectangle can have _____ .

**34.** Bricks of dimensions 10 cm × 8 cm × 4 cm have to be used to construct a platform of dimensions 15 m × 10 m × 8 m. 10% of the volume of the platform is occupied by mortar. If 100 bricks cost ₹400, find the cost of the bricks required for constructing the platform in crores of rupees.

(A) 0.80 (B) 1.35
(C) 1.80 (D) None of these

**35.** Three mutually adjacent faces of a cuboid have areas 20, 30 and 24. Find the volume of the cuboid.

**36.**




In the figure, a circle is inscribed in the square *S*. A smaller square is inscribed in this circle and a smaller circle is inscribed in the small square. Find the ratio of the areas of the regions marked by lines and marked by dots.

(A) $\frac{4(4-\pi)}{3(\pi-2)}$ (B) $\frac{3(4-\pi)}{4(\pi-2)}$

(C) $\frac{3(\pi-3)}{2(\pi-2)}$ (D) $\frac{3(4-\pi)}{2(\pi-2)}$

**37.** The area of the floor of a circular based tent is 308 sq. m. If its height is $7\sqrt{2}$ m, then find the area of the canvas required for the tent. (in sq. m)

(A) $154\sqrt{3}$ (B) 308

(C) $308\sqrt{2}$ (D) $385\sqrt{2}$

**38.** **(a)** A circle is centered at $O$. The area of the sector $AOB$ of the circle, is $12\pi$. If $\angle AOB = 120°$, find the radius of the circle.

**(b)** An arc of a circle subtends 150° at the centre of the circle. The length of the arc is $20\pi$. Find the radius of the circle.

**39.** What is the circumradius of the triangle whose sides are 102.6 cm, 136.8 cm and 171 cm?
(A) 68.4 cm (B) 51.3 cm
(C) 85.5 cm (D) 34.4 cm

**40.** Find the area of a regular hexagon (in sq. cm.) in which the perpendicular drawn from the centre to any side is 12 cm long.

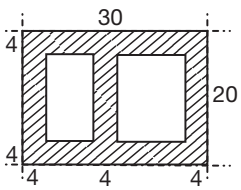
**41.** The area of a trapezium is 132 sq. cm. One of the parallel sides is 9 cm longer than the other. Find the longer of the two parallel sides, if the distance between them is 8 cm.
(A) 12 cm (B) 15 cm (C) 21 cm (D) 9 cm

**42.** A lawn mower is in the shape of a cylinder. The radius of crosssection is 14 cm and the length is 1 m. What is the area covered by the lawn mower in making 200 revolutions? (in $m^2$) _____ .

**43.** The area of a rhombus is 21 $cm^2$. If the perimeter of the rhombus is 40 cm, then find the sum of its diagonals. (in cm) _____

**44.** The dimensions of a cuboidal box are in the ratio 9 : 8 : 6 and the difference between the costs of covering it fully with sheet of paper at the rate of ₹ $4 \times 50$ per sq. m. and ₹ $6 \times 25$ per sq. m. is ₹ 5481. Find the volume of the box.
(A) 10664 cu. m (B) 11664 cu. m
(C) 10244 cu. m (D) 12884 cu. m

**45.**




In the given figure, the shaded portion is a path paved with tiles inside a rectangular plot. The dimensions of the plot are 30 m × 20 m. The width of the path is 4 m. What is the area of the path? _____ sq. m

**46.** The length and the breadth of a cuboid are increased by one-eighth and one-third rrespectively. What should be the percentage increase in the height such that the volume which was 150 cu. cm earlier, becomes 300 cu. cm?
(A) 50% (B) 25% (C) $16^2/_3$% (D) $33^1/_3$%

**47.** In triangle $QUR$, a line drawn parallel to $UR$, cuts $QU$ and $QR$ at $S$ and $T$ respectively. $QS$ =10 and $SU$ = 8. Find the area of triangle $QST$, if the area of triangle $QUR$ is 54 sq. units. (in sq units) $\frac{\quad}{3}$

**48.** **(a)** Find the area of a parallelogram which has two of its sides as 6 and 12 and one of its angles as 30° _____ .

**(b)** Find the perimeter and area of a rhombus whose diagonals are 16 and 12. _____, _____ .

**49.** The radii of a cylinder and a cone are equal, their heights are also equal. The curved surface area of the cone to that of the cylinder is 17 : 16. Find the ratio of the radius to the height.
(A) 8 : 15 (B) 15 : 8 (C) 17 : 8 (D) 8 : 17

**50.** The perimeter of a rectangle is equal to that of a square. One of the dimensions of the rectangle and the side of the square are in the ratio 2 : 3. Find the ratio of the areas of the rectangle and the square. $\frac{8}{\quad}$

**51.** An equilateral triangle is inscribed in a circle of radius 8 cm and another equilateral triangle is circumscribed about the circle. Find the difference between the areas of the two triangles (in sq. cm). _____ $\sqrt{3}$

**52.** Two of the medians of a triangle are perpendicular. If the lengths of these medians are 12 and 9, the area of the triangle is _____ .

**53.** The total surface area of a cuboid is 280 sq. cm. The areas of two of its adjacent surfaces are 72 sq. cm and 36 sq. cm. Find the length of the cuboid if it is the greatest of the dimensions.
(A) 12 cm (B) 6 cm (C) 9 cm (D) 8 cm

**54.** If the side of an equilateral triangle is increased by 5 cm, its area increases by $\frac{225\sqrt{3}}{4}$ sq. cm. Find the area of the original triangle. (in sq cim) $100\sqrt{\quad}$

**55.** Three solid metallic cubes whose edges are in the ratio 3 : 4 : 5 are melted together and recast into one cube. If the diagonal of the cube formed is $12\sqrt{3}$ cm, then find the edge of the smallest of the initial cubes. (in cm) _____

**56.** In triangle $PQR$, $\angle PRQ = 60°$. $S$ is a point on $PR$ and $QS$ is the perpendicular bisector of $PR$. If $PR$ = 10 cm, find the area of triangle $PQR$ (in $cm^2$).
(A) $20\sqrt{3}$ (B) $25\sqrt{3}$
(C) $30\sqrt{3}$ (D) $40\sqrt{3}$

**57.** In a triangle, if the length of each side is subtracted from the sum of the lengths of the other two sides, the results are 10 cm, 20 cm and 30 cm. Find the area of the triangle. (in $cm^2$) _____ .

**58.** The diagonal *AC* of a quadrilateral *ABCD* measures 12 cm. The lengths of the perpendiculars to it from B and *D* measure 6 cm and 8 cm respectively. Find the area of the quadrilateral. (in $cm^2$) _____

**59.** The radius of a cylindrical box is 14 cm and its height is 9 cm. By decreasing the height or radius by *x* cm, the decrease in volume is the same. What is the value of *x*? (given that, $x \neq 0$)

(A) $\frac{42}{9}$ cm (B) $\frac{56}{9}$ cm (C) $\frac{30}{7}$ cm (D) $\frac{21}{4}$ cm

**60.** A square, a regular hexagon and a circle have the same area. If the perimeters of the square, the hexagon and the circle are *S*, *H* and *C* respectively, which of the following must be true?

(A) S < *H* < C (B) S > *C* > *H*
(C) S < *C* < H (D) S > *H* > *C*

## EXERCISE-3

***Directions for questions 1 to 60*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** A right-angled isosceles triangle has a perimeter of $8\sqrt{2} + 8$. The largest possible quadrant that can be cut out from the triangle is cut out from it. Find the area of the remaining region. (Take $\pi = 22/7$).

(A) $\frac{12}{7}$ (B) $\frac{24}{7}$ (C) $\frac{18}{7}$ (D) $\frac{36}{7}$

**2. (a)** Each side of a triangle is 6 cm less than the sum of the other two sides. Find the area of the triangle. (in sq. cm.) _____ $\sqrt{3}$

**(b)** Find the circumradius of an equilateral triangle whose area is $36\sqrt{3}$ sq. cm. (in cm) _____ $\sqrt{3}$

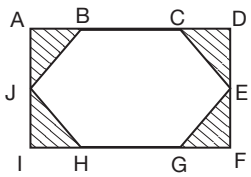
**(c)** Find the area of the circle which is inscribed in a triangle whose sides are 13, 14, and 15. _____ $\pi$

**3.** Around a circle of radius 3 cm, circumscribing regular polygons are drawn. $C(n)$ represents a circumscribing regular polygon of $n$ sides and $CP(n)$ the perimeter of that polygon where $n$ = 4, 5, 6, ... A sequence of regular polygons, $I(n)$ inscribed in the circle are also drawn. $IP(n)$ is the perimeter of $I(n)$ where $n$ = 4, 5, 6 ... then, $\frac{CP(6)+6\pi}{IP(8)}$, where both $CP(6)$ and $IP(8)$ are in cm, is

(A) greater than 1 and less than 2
(B) less than 1
(C) greater than 2
(D) Either (B) or (C)

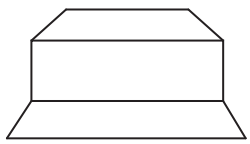
**4.** A roller is 4 m long and has a diameter of 0.7 m. It takes exactly 500 rotations of the roller to level a road. If the cost of using the roller is ₹ 5 per sq. m., then find the total cost of levelling the road. (in ₹) _____

**5.** *ADFI* is a rectangle of breadth 8 cm. *BCEGHJ* is a regular hexagon drawn in it. Find the area of the shaded region (in $cm^2$).



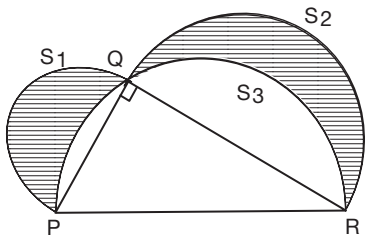

(A) $16\sqrt{3}$ (B) $\frac{64}{\sqrt{3}}$ (C) $\frac{32}{\sqrt{3}}$ (D) $32\sqrt{3}$

**6.** The figure shows the front view of a model of a building. The top and the bottom parts of the model are frustums of cones. The middle part of the model is a cylinder. The diameters of the upper frustum are 4 cm and 6 cm. The bottom diameter of the lower frustum is 8 cm. The heights of the frustums are 1.5 cm each. The height of the cylinder is 3 cm. Find the volume of the model.



(A) $45\pi$ cu. cm. (B) $65\pi$ cu. cm.
(C) $55\pi$ cu. cm. (D) $75\pi$ cu. cm.

**7.**



In the figure, $\angle PQR$ is a right angle. The diameters of semicircles $S_1$, $S_2$, $S_3$ are $PQ$, $QR$, $PR$ respectively. The area of the shaded region is 30. Find the area of the triangle.

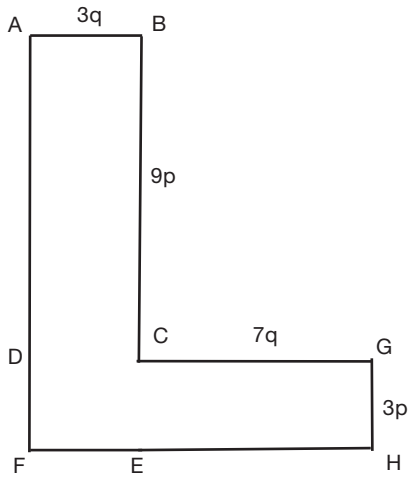
(A) 15 (B) 30
(C) 45 (D) Cannot be determined

**8.** The lengths of the sides of a quadrilateral are 21, 25, 25 and 35. If the sides measuring 21 and 35 are parallel, the area of the quadrilateral is _____ .

**9.** $PQ$ is the diameter of a circle. $R$ is a point on the circle. $RS$ is an altitude of triangle $PQR$. $RS = 12$ cm and $SQ = 24$ cm. Find the area of the circle (in sq. cm)

(A) 196 $\pi$ (B) 225 $\pi$ (C) 256 $\pi$ (D) 289 $\pi$

**10.**




The perimeter of the figure is 300. Find the area of the figure, in terms of $p$.

(A) $\frac{171p(25-2p)}{5}$ (B) $\frac{159p(25-2p)}{5}$

(C) $\frac{153p(15-2p)}{5}$ (D) $\frac{183p(25-2p)}{5}$

**11.** In triangle $ABC$, $AB = 26$, $AC = 30$ and $BC = 28$. A square is drawn with one of its sides lying on $BC$ and the other two vertices lying on sides $AB$ and $AC$ respectively. The area of the square rounded off to the nearest integer is _____ .

**12.** A solid sphere is cut into 8 identical pieces by three mutually perpendicular cuts. By what percent-age is the sum of the total surface areas of the eight pieces more than the total surface area of the original sphere?

(A) 125% (B) 150%
(C) 175% (D) 200%

**13.** There is a fenced rectangular plot in a large field. The dimensions of the plot are 40 m ×14 m. A cow is tethered outside the plot at one corner by a rope 21 metres long. Find the total area of the field, over which it can graze. (in $m^2$) _____

**14.** There are 3 objects, each of volume 38,808 $cm^3$. One is a cube, another is a cuboid while the third is a sphere. The outer surfaces of all the objects have to be painted. The cost of painting is ₹ 20 per sq. cm.

Consider the following statements.

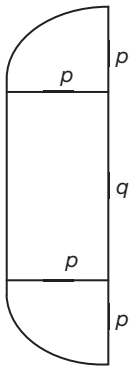
I. The cost of painting the cube is less than the cost of painting the cuboid.

II. The cost of painting the sphere is less than the cost of painting the cube.

Which of the following is / are true?

(A) Only I. (B) Only II.
(C) Both I. and II. (D) Neither I. nor II.

**15.**




The perimeter of the figure is 128. The area of the figure, in terms of $p$, is (Take $\pi = \frac{22}{7}$)

(A) $72p - p^2$ (B) $56p - p^2$
(C) $64p - p^2$ (D) $80p - p^2$

**16.** A drum in the shape of a frustum of a cone with radii 24 ft and 15 ft and height 5 ft is full of water. The drum is emptied into a rectangular tank of base dimensions 99 ft × 43 ft. Find the rise in the water level in the tank.

(A) $\frac{4}{7}$ ft (B) $1\frac{1}{4}$ ft

(C) $1\frac{3}{4}$ ft (D) None of these

**17.** The area of the triangle whose sides are $\sqrt{9p^2+16q^2}$, $\sqrt{9r^2+16q^2}$ and $3\sqrt{p^2+r^2}$ is

(A) $3\sqrt{\frac{3}{4}p^2r^2+4q^2p^2+4q^2r^2}$

(B) $3\sqrt{2p^2r^2+4q^2p^2+4q^2r^2}$

(C) $3\sqrt{6p^2r^2 + 4q^2p^2 + 4q^2r^2}$

(D) $3\sqrt{\frac{9}{4}p^2r^2 + 4q^2p^2 + 4q^2r^2}$

**18.** The base of a regular pyramid is a square of area $A$. The height of the pyramid is five times the semi-perimeter of the square. The area of any of the triangular regions of the pyramid is $S$. Find $A$.

(A) $\frac{2S}{\sqrt{101}}$ (B) $\frac{2S}{\sqrt{401}}$ (C) $\frac{S}{\sqrt{101}}$ (D) $\frac{4S}{\sqrt{401}}$

**19.** $ABC$ is a triangle. $D$ is a point on $BC$ and $E$ is a point on $AD$. The areas of the triangles $AEB$, $BED$ and $AEC$ are 16, 12 and 24 respectively. Find the area of the triangle $CED$.

_____

**20.** In a regular octagon $ABCDEFGH$, a quadrilateral $CDGH$ is formed by joining $CH$ and $DG$. The side of the octagon is $12\sqrt{2}$. Find the area of $CDGH$.

(A) 144 (B) $144\left(1+\sqrt{2}\right)$
(C) 288 (D) None of these

**21.** O is the centre of a circle having a radius of 10 cm. Find the area (in $cm^2$) of the region enclosed by the chord $AB$ and the minor arc $AB$ if $\angle AOB = 120°$.

(A) $\frac{100\pi}{3} - 25\sqrt{3}$ (B) $\frac{50\pi}{3} - 20\sqrt{3}$
(C) $\frac{40\pi}{3} - 15\sqrt{3}$ (D) $\frac{49\pi}{3} - 10\sqrt{3}$

**22.** In $ABCD$, a square is inscribed in a circle. The equilateral triangle $BEF$ is also inscribed in the circle. $F$ and $C$ lie on the same side of $BD$. If the radius of the circle is 12 cm, what is the area of the $\Delta BFC$? ($\sin 15° = \sqrt{2-\sqrt{3}}/2$) (in sq. cm.)

(A) $36\sqrt{2}$ (B) $36\left(2+\sqrt{3}\right)$
(C) $36\sqrt{6}$ (D) $36(3-\sqrt{3})$

**23.** The sides of a triangle are 8, 15, 17. The triangle is rotated about its longest side. Find the volume of the figure generated.

(A) $\frac{4800\pi}{17}$ (B) $\frac{2400\pi}{17}$
(C) $\frac{3600\pi}{17}$ (D) $\frac{5400\pi}{17}$

**24.** A circle has a radius of 1 cm. It has 8 sectors. The second sector is adjacent to the first, the third to the second and so on with the eighth adjacent to the seventh. The sum of the lengths of all the arcs of the sectors is $(1/10)^{th}$ of the circumference of $C$. Moreover, the arc length of the $k^{th}$ sector is twice that of the $(k-1)^{th}$ sector, where $2 \le k \le 8$. Find the central angle of the $1^{st}$ sector.

(A) $\frac{\pi}{2555}$ (B) $\frac{\pi}{1275}$ (C) $\frac{\pi}{1270}$ (D) $\frac{\pi}{635}$

**25.** In a large field there is a shed in the shape of a regular pentagon of side 12 m. A goat is tied at one vertex of this shed by a rope of length 16 m. The goat cannot enter the shed. Find the area (in $m^2$) of the field that the goat can graze upon.

(A) $160\pi$ (B) $175\pi$
(C) $179\frac{1}{5}\pi$ (D) $185\frac{3}{5}\pi$

**26.** Find the volume of the largest right circular cone that can be cut out of a cube of side 12 cm.

(A) $452\frac{4}{7}cm^3$ (B) $462\frac{6}{7}cm^3$
(C) $482\frac{2}{7}cm^3$ (D) $505\frac{5}{7}cm^3$

**27.** A right circular cone has a base radius of 6 cm and a height of 21 cm. A cylinder was placed inside the cone with one of its flat surfaces resting on its base. Find the maximum possible total surface area of the cylinder. (in sq. cm.)

(A) $\frac{441\pi}{4}$ (B) $\frac{441\pi}{8}$ (C) $\frac{441\pi}{10}$ (D) $\frac{441\pi}{5}$

**28.** Find the area of a regular pentagon (in $cm^2$), whose side is 20 cm. (Take $\cot 36° = 1.376$) _____

**29.** Rectangular tiles each of size 60 cm by 40 cm must be laid horizontally on a rectangular floor of size 120 cm by 170 cm, such that the tiles do not overlap. A tile can be placed in any orientation so long as its edges are parallel to the edges of the floor. No tile should over shoot any edge of the floor. The maximum number of tiles that can be accommodated on the floor is

(A) 6 (B) 7 (C) 8 (D) 9

**30.** A cylindrical beaker has a base diameter of 7 cm and a height of 18 cm. It is filled with water to a height of 3 cm. If $N$ identical marbles of diameter 1.4 cm each are dropped into the beaker, the height of the water in the beaker would rise to 11.4 cm. Find the value of $N$. _____

**31.** A box has dimensions of 30 m × 20 m × 15 m. Find the maximum number of boxes with each having dimensions of 6 m × 5 m × 3 m, which can be placed in it. _____

**32.** $PQR$ is a triangle right-angled at $Q$. A semicircle is drawn with $PQ$ as diameter. Another semicircle is drawn with $QR$ as diameter. The two semi-circles lie outside $PQR$. A

third semicircle is drawn with $PR$ as diameter such that it passes through $Q$. The sum of the areas of the region between the first and the third semicircles and the region between the second and the third semicircles is $A$.

Consider the following statements:

I. The area of $PQR$ is at least $A$.

II. The area of $PQR$ is at most $A$.

Which of the following is/are true?

(A) Only I (B) Only II
(C) Both I and II (D) Neither I nor II

**33.** A cylindrical hole is drilled along the axis of a solid cone of radius 10 cm and height 10 cm. The radius of the hole is 5 cm. Find the volume of the remaining portion of the solid. (in cu. cm.)

(A) $\frac{200}{3}\pi$ (B) $100\,\pi$

(C) $\frac{250}{3}\pi$ (D) $\frac{500}{3}\pi$

**34.** A tank, in the shape of a frustum of a cone is full of water. The radii of the bases of the frustum are 6 m and 3 m and its height is 14 m. The tank was emptied into another tank, which is conical with a base radius 7 m. If the second tank is just full, find its height (in m). _____

**35.** Two equal circles intersect each other. The circumference of each passes through the centre of the other. What part of the area of each circle is the area of their intersecting region?

(A) $\frac{1}{3}-\frac{\sqrt{3}}{4\pi}$ (B) $\frac{2}{3}+\frac{\sqrt{3}}{2\pi}$

(C) $\frac{4}{3}-\frac{\sqrt{3}}{2\pi}$ (D) $\frac{2}{3}-\frac{\sqrt{3}}{2\pi}$

**36.** In a swimming pool the depth at one end is three metres and at the other end it is ten metres. The width of the pool is 6 m. If the volume of the water that can be filled in the pool is 3120 cu. m, then its length is _____ m

**37.** The interior of a building is in the form of a cylinder of diameter 6 m, surmounted by a cone of vertical angle 60° and having an over-all height of $12\sqrt{3}$ m. Find the volume of the building.

(A) $282\frac{6}{7}$ cu.cm. (B) $\sqrt{3}\left(282\frac{6}{7}\right)$ cu. cm.

(C) $\sqrt{2}\left(282\frac{6}{7}\right)$ cu. cm. (D) $565\frac{5}{7}$ cu.cm.

***Directions for question 38*:** These questions are based on the following data.

In a big field, there is a small plot fenced on all sides. The plot $ABC$ is in the form of an equilateral triangle, length of each side being 40 m. A goat is tied at $A$. Assume that the goat cannot graze inside the plot but can graze outside the plot as for as the rope allows it.

**38.** Find the area that can be grazed by the goat, if it is tied with a rope of length 24 m.

(A) $576\pi$ sq. m. (B) $288\pi$ sq. m.
(C) $480\pi$ sq. m. (D) $240\pi$ sq. m.

**39.** Find the area that can be grazed by the goat, if it is tied with a rope of length 50 m.

(A) $2000\pi$ sq. m. (B) $2083\frac{1}{3}\pi$ *sq.m*

(C) $2150\pi$ sq. m. (D) $2100\pi$ sq. m.

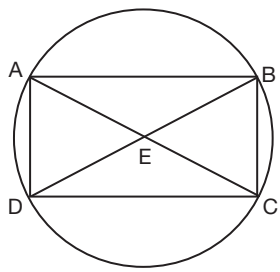
**40.** A square sheet has a side of 30 cm. From each of its corners, four squares each of side $y$ cm are cut. The remaining sheet is folded to form an open cuboid. Find the value of $y$ which maximizes the volume of the cuboid formed.

(A) 5 (B) 6 (C) 3 (D) 10

**41.** A layer of hemispherical blocks is laid on the base of a cuboidal box with a square base of side 42 cm and height 70 cm. An identical layer is then laid and this process is repeated till the blocks reach the top of the box. Find the empty volume in the box, if the radius of each block is 7 cm. (Assume $\pi = \frac{22}{7}$) _____

**42.** A conical tent has a base radius 7 m and a height24 m. What is the length of the canvas required tocover the conical portion, if it is stitched from a rectangular sheet of breadth 11 m? _____ m

**43.** In the figure given below, $ABCD$ is a cyclic quadrilateral and its diagonals $AC$ and $BD$ intersect at $E$. If $AE = 8$, $BE = 12$ and $CE = 16$, find the ratio of the areas of triangles $AED$ and $BEC$.




(A) 2 : 3 (B) 4 : 9 (C) 1 : 2 (D) 1 : 4

**44.** A five-rupee note measures 12 cm × 6 cm and a bundle of such five rupee notes 100 notes is 1 cm thick. What is the value of the five rupee notes contained in a box of size 36 cm × 24 cm × 3 cm, if the bundles are tightly packed in the box without any empty space?

₹ _____ .

**45.** Cylinder $C$ is inscribed in sphere $S$. The volume of C is $12\sqrt{3}\,\pi$ cm$^3$. If $C$ is the largest cylinder that can be inscribed in $S$, the radius of $S$ is

(A) 1.5 cm (B) 6 cm (C) 4.5 cm (D) 3 cm

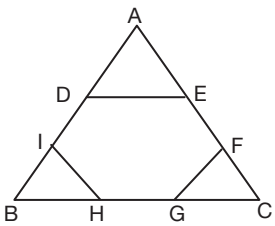
**46.** $ABCDEF$ is a regular hexagon. The sum of the areas of $ACE$ and $BDF$ is 108 cm$^2$. The area of $ABCDEF$ is _____ cm$^2$.

**47.** In triangle $ABC$, $D$, $E$, $F$ are points on $BC$, $CA$, $AB$ respectively, such that $BD : DC = 1 : 2$, $CE : EA = 1 : 2$ and $AF : FB = 1 : 2$. Find the ratio of the areas of $\Delta DEF$ and $\Delta ABC$.

(A) 2 : 5 (B) 1 : 4 (C) 1 : 3 (D) 2 : 7

**48.** The slant height of a cone equals the difference of its radius and twice its height. Find the ratio of its curved surface area and total surface area $\frac{5}{____}$

**49.**




In the figure, $DEFGHI$ is a hexagon inscribed in the triangle $ABC$ $AE : EF : FC = 3 : 1 : 4$, $BH : HG : GC = 3 : 5 : 4$ and $AD : DI : IB = 2 : 1 : 3$. Find the ratio of the area of triangle $ABC$ and that of the hexagon.

(A) 72 : 53 (B) 72 : 43
(C) 64 : 47 (D) 12 : 7

**50.** The sum of the slant height and the base radius of a cone equals twice the difference of the base diameter and the slant height. Find the ratio of the base radius and the height of the cone.

(A) $1 : 2\sqrt{6}$ (B) $2\sqrt{6} : 1$
(C) $1 : \sqrt{6}$ (D) $\sqrt{6} : 1$

**51.** The sum of the slant height and the height of a cone is twice the difference of its slant height and twice its height. If the volume of the cone is $192\pi$ cm$^3$, find the height of the cone (in cm).

(A) $\sqrt{6}$ (B) $\sqrt{3}$ (C) $2\sqrt[3]{3}$ (D) $3\sqrt[3]{3}$

**52.** Find the volume of the largest right circular cone that can be cut out of a cube of edge 16 cm.

(A) $1072\frac{5}{21}$ cu. cm (B) $1072\frac{11}{21}$ cu. cm
(C) $1072\frac{16}{21}$ cu. cm (D) $1072\frac{10}{21}$ cu. cm

**53.** The largest possible right circular cylinder is cut out from a wooden cube of edge 7 cm. Find the volume of the cube left over after cutting the cylinder (in cm$^3$). _____

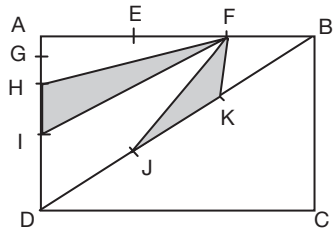
**54.** The areas of a regular hexagon and a rectangle are equal. If the side of the hexagon is equal to one of the dimensions of the rectangle, find the ratio of the perimeter of the hexagon to that of the rectangle.

(A) $6 : (2\sqrt{3} + 3)$ (B) $6 : (3\sqrt{3} + 2)$
(C) $4 : (\sqrt{3} + 1)$ (D) $12 : (3\sqrt{3} + 4)$

**55.** A room is in the shape of a cuboid. Its length, breadth and height are in the ratio 6 : 4 : 3. If its length and breadth are tripled and its height is halved its lateral surface area will

(A) increase by $33\frac{1}{3}\%$. (B) increase by $16\frac{2}{3}\%$.
(C) increase by 50%. (D) increase by $66\frac{2}{3}\%$.

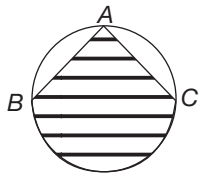
**56.** Find the area (in sq. cm) of the shaded region where $E$, $F$ and $K$, $J$ are the points of trisection of lines $AB$ and $BD$ respectively and $AG = GH = HI = 1/4AD$, if the area of the rectangle is 180 sq. cm.




(A) 30 (B) 25 (C) 60 (D) 50

**57.** An isosceles triangle has each of its equal sides equal to 25. The area of the triangle is 168. Find the length of the altitude drawn to the base. _____ or _____

**58.**



In the above figure, the circle has a radius of 7 cm. $AB$ and $AC$ are chords of equal length and $\angle BAC$ is 90°. The area of the shaded region is

(A) 196 $cm^2$ (B) 203 $cm^2$
(C) 308 $cm^2$ (D) 126 $cm^2$

**59.** The girth of a cylindrical tree is 660 cm and its height is 2×5 m. Wood sells at ₹2000 per cu. ft. If there was a wastage of 10% in cutting the tree, then what is the total realisation from the sale of the wood? (1 inch = 2×5 cm, 1 ft = 12 inch)

(A) ₹57750 (B) ₹5,77,500
(C) ₹65,650 (D) ₹6,56,500

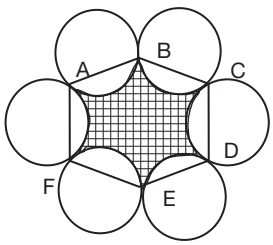
**60.** Find the volume of the largest right circular cone, which can be cut from a cube of side 4 cm. (in cu. cm.) $\frac{}{21}$

## EXERCISE-4

***Directions for questions 1 to 30***: For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

***Directions for questions 1 and 2***: These questions are based on the following data.

Six non-overlapping circular plots of land, each of radius 35 m are situated as shown below. The points of contact of the circles are $A$, $B$, $C$, $D$, $E$ and $F$, and $ABCDEF$ is a regular hexagon.




**1.** Krishna walks along the outer periphery of the circular plots without coming inside the hexagonal area. Approximately, what distance does he travel before coming back to the starting point?

(A) 570 m (B) 640 m
(C) 770 m (D) 880 m

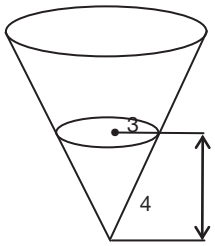
**2.** What is the approximate area of the circular segments inside the hexagon? (Take $\pi = 3.1416$ and $\sqrt{3} = 1.73211$)

(A) 2300 $m^2$ (B) 3000 $m^2$
(C) 3800 $m^2$ (D) 4500 $m^2$

**3.** Two poles $AB$ and $CD$ of heights 9 m and 16 m are standing upright with the distance separating them being 26 m. Ropes are tied from the top of each pole to a nail on the ground situated at $P$, a point on the line joining the feet of the poles such that the angle made by the two ropes is $90^0$. If $A$ and $C$ denote the tops of the two poles, and $K = \frac{\text{Area of } \Delta APC}{\text{Area of } \Delta ABP \text{ of } \Delta CDP}$, find the value of $K$.

(A) $\frac{29}{36}$ (B) $\frac{2}{5}$
(C) $\frac{36}{29}$ (D) Either (A) or (C)

**4.**

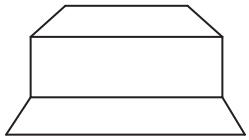



An inverted cone which is sealed has a liquid up to a height of 4 cm from the vertex as shown in the figure and the radius of the surface of the liquid is 3 cm. The radius (in cm) and height (in cm) of the bigger cone are both integers less than 10. What is the height of the liquid level, if the cone is inverted to have the base at the bottom?

(A) 2 cm (B) 1.5 cm
(C) 1 cm (D) None of these

**5.** A cone $C$ of height $H$ rests on its base. A cut, $C_1$, was made parallel to the base at a height $\frac{8H}{9}$ and another cut, $C_2$, was made parallel to the base at a height $\frac{5H}{9}$. The volumes of the frustums so formed differ by 4816 $cm^3$. Find the volume of $C$, in $cm^3$. _____

**6.** A layer of hemispherical blocks is laid on the base of a cuboidal box with a square base of side 42 cm and height 70 cm. An identical layer is then laid, and this process is repeated till the blocks reach the top of the box. Find the empty volume in the box, if the radius of each block is 7 cm. (Assume $\pi = \frac{22}{7}$) _____



**7.** The figure shows the front view of a model of a building. The top and the bottom parts of the model are frustums of cones. The middle part of the model is a cylinder. The diameters of the upper frustum are 4 cm and 6 cm. The bottom diameter of the lower frustum is 8 cm. The heights of the frustums are 1.5 cm each. The height of the cylinder is 3 cm. Find the volume of the model.

(A) $45\pi$ cu. cm. (B) $65\pi$ cu. cm.
(C) $55\pi$ cu. cm. (D) $75\pi$ cu. cm.

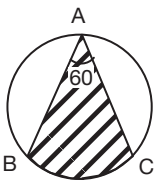
**8.** The volume of air that an exhaust fan can evacuate per unit time is proportional to the square of its speed in revolutions per minute. If it takes 20 minutes for such a fan running at a certain speed to evacuate all the air from a room of dimensions 2 m × 3 m × 4 m, then find the number of solid boxes each of dimensions 1/2 m x 1/2 m x 1 m that should be placed in a room of dimensions 6 m × 8 m × 12.5 m, if the same fan must evacuate the room within 10 minutes, when running at five times its initial speed.

(A) 1200 (B) 1250 (C) 1600 (D) 1260

**9.** A sphere has a radius of 1. Find the volume of the largest cylinder that can be inscribed in the sphere.

(A) $\frac{4\pi}{3\sqrt{3}}$ (B) $\frac{8\pi}{3\sqrt{3}}$ (C) $\frac{2\pi}{\sqrt{3}}$ (D) $\frac{2\pi}{3\sqrt{3}}$

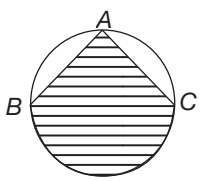
**10. (i)**




In the circle above, chords *AB* and *AC* have the same length. Find the approximate percentage of the area of the circle that is not shaded.

(A) 28% (B) 39%
(C) 48% (D) 60%

**(ii)**



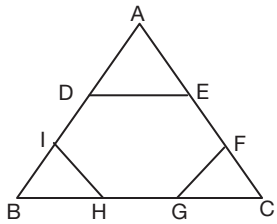

In the above figure, the circle has a radius of 7 cm. *AB* and *AC* are chords of equal length and $\angle BAC$ is 90°. The area of the shaded region is

(A) 196 $cm^2$ (B) 203 $cm^2$
(C) 308 $cm^2$ (D) 126 $cm^2$

**11.** *ABCD* is a square. *E*, *F*, *G*, *H* are the midpoints of the sides. *J* and *K* are points on *HG* and *GF* such that *HJ* = (2/3) *HG* and *GK* = (1/2) *GF*. *L* is a point on *EF* such that *LF* = (1/4) *EF*. What is the ratio of the area of triangle *JKL* to that of the square *ABCD*?

(A) 3 : 64 (B) 5 : 96
(C) 7 : 96 (D) 13 : 128

**12.**




In the figure, *DEFGHI* is a hexagon inscribed in the triangle *ABC* *AE* : *EF* : *FC* = 3 : 1 : 4, *BH* : *HG* : *GC* = 3 : 5 : 4 and *AD* : *DI* : *IB* = 2 : 1 : 3. Find the ratio of the area of triangle *ABC* and that of the hexagon.

(A) 72 : 53 (B) 72 : 43
(C) 64 : 47 (D) 12 : 7

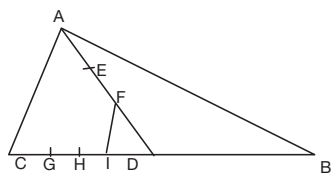
**13.** Identical black and white square tiles completely cover a rectangular floor. The tiles on the edges are all black and those in the interior are all white. The number of black tiles is half of the number of white tiles. The number of tiles along any of the edges of the floor can be

I. 7 II. 12 III. 30
(A) I or II (B) I or III
(C) II or III (D) I, II or III

**14.** Apoorva must pack cartons with cakes of soap. She has 24,000 cakes of soap. The cakes were individually of the dimensions 5 cm × 3 cm × 4 cm while the carton was 51 cm broad, 52 cm high and 35 cm long. She is left with 2240 cakes after using exactly 16 cartons. At this rate, how many cartons more than the minimum possible number of cartons does she use?

(A) 1 (B) 2 (C) 3 (D) 4

**15.**

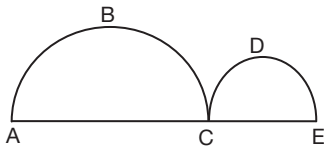


The area of $\Delta ABC = 162$ sq. cm. Find the area (in sq. cm) of $\Delta FID$ if $BD = (2/3)\ BC$, $AE = EF = FD$ and $CG = GH = HI = ID$. _____

**16.** A hemispherical dome rests on its circular base. A cylinder is placed inside the dome with one of its bases resting on the base of the dome. Find the radius of the largest possible cylinder that can be placed, if the radius of the hemisphere is $10\sqrt{\frac{3}{2}}$.

(A) 10 (B) 5 (C) $5\sqrt{2}$ (D) $10\sqrt{2}$

**17.**




In the figure above, $AC = 10$ cm, $CE = 5$ cm, and $ABC$, $CDE$ are semicircles. Find the volume (in $cm^3$) of the solid formed when the figure is rotated about $AE$.

(A) $375\pi$ (B) $\frac{375}{4}\pi$

(C) $\frac{125}{2}\pi$ (D) $\frac{375}{2}\pi$

**18.** The perimeter of a right-angled triangle is 56 cm and its inradius is 3 cm. All the three sides, in cm, are integers. If the triangle is rotated about its hypotenuse, what is the volume of the solid so generated, in $cm^3$?

(A) 739200 (B) 792810
(C) 1182.72 (D) 1268.50

**19.** Crowe the crow was very thirsty, and it found a bucket with a little water in it. The bucket was in the shape of a frustum of a cone. The base had a circumference of 88 cm and the top had a circumference of 44 cm and the height of the bucket was 20 cm. The water level was only half the height of the bucket. Crowe needed the water level to reach 80 percent of the height. How many sphere shaped pebbles of diameter 2 cm should Crowe drop into the water so that the level rises to the required height?

(A) 4562 (B) 526 (C) 404 (D) 622

***Directions for questions 20 and 21:*** These questions are based on the following data.

There is a rectangle $R_0$. The biggest possible square $S_0$ is cut out from $R_0$ leaving behind a rectangle $R_1$. The biggest possible square $S_1$ is cut out from $R_1$ leaving behind a rectangle $R_2$. Continuing this procedure, from the rectangle $R_{n-1}$ the biggest possible square $S_{n-1}$ is cut out leaving behind a rectangle $R_n$. The above process is terminated when $R_n$ obtained is a square.

**20.** If the squares $S_0$, $S_1$, $S_2 \ldots S_9$ obtained in the process are of distinct areas and the area of $R_{10}$ is same as that of $S_9$, what is the ratio of the length and breadth the rectangle $R_0$?

(A) 100 : 99 (B) 55 : 34
(C) 34 : 21 (D) 144 : 89

**21.** If $k$ (greater than 1) is the ratio of length and breadth of the rectangle $R_0$ and $n$ is the number of times the procedure is repeated, which of the following is true?

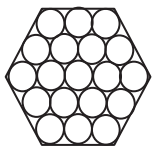
I $n$ is finite for all rational values of $k$.

II $n$ is infinite only for one particular value of $k$.

III $n$ is infinite for all irrational values of $k$.

(A) Only II is true
(B) Only I and II are true
(C) Only I and III are true
(D) Only I is true.

**22.**



Nineteen spherical balls are tightly arranged in a hexagonal box in a single layer as shown. The radius of each ball is 2 cm. What is the approximate volume in the box in $cm^3$ which is not occupied by the balls?

(A) $\frac{1664 + 1536\sqrt{3}}{3} - \frac{608\pi}{3}$

(B) $\frac{416 + 384\sqrt{3}}{3} - \frac{608\pi}{3}$

(C) $608 + 752\sqrt{3} - 205\pi$

(D) $384 + 416\sqrt{3} - \frac{608\pi}{3}$

**23.** There is a large trough in the shape of a frustum of a rectangular pyramid. The base of the trough is a 2 m × 3 m rectangle, while the open top is a 4 m × 6 m rectangle. The height is 1 m. Water flows into the trough through a pipe whose diameter is 25 cm. If water flows through the pipe at 30 m/min, which of the following is true about '$t$', the time (in minutes) it takes for the water level to reach half the height of the trough?

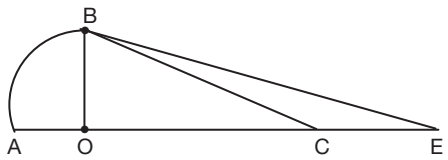
(A) $t < 3$ (B) $3 < t < 3.5$
(C) $3.5 < t < 4$ (D) $4 < t < 4.5$

**24.** $ABCD$ is an isosceles trapezium in which $DA = AB = BC = \frac{1}{2}CD$. $M$ is the midpoint of $AB$, while $P$ and $Q$ are the feet of the perpendiculars to $CD$ from $A$ and $B$ respectively. Diagonal $BD$ intersects $MP$ and $MQ$ at $S$ and $R$ respectively. If the area of $ABCD$ is 60 and the area of the quadrilateral $PQRS$ is $x$, which of the following is true about $x$?

(A) $16 < x \leq 18$ (B) $18 < x \leq 20$

(C) $20 < x \leq 22$ (D) $22 < x \leq 24$

**25.**




In the given figure, $ABCOA$ represents a plot of land which has a quadrant adjoining a triangle to be covered with tiles. A certain number of tiles, which are just sufficient to tile 70% of the plot are available. However, it was observed that there was a mistake in locating $C$. It should have been at $E$ where $CE = 0.05(AC)$. $OC = 3(OA)$ where $AO$ is the radius of the quadrant $AOB$. Approximately, what percent of the actual field $ABEOA$ can be tiled with the given tiles?

(A) 50% (B) 55% (C) 67% (D) 45%

**26.** A circle is centered at $O$. $P_1$, $P_2$, … $P_{24}$ are 24 points on the circle. The lengths of the arcs $P_1P_2$, $P_2P_3$, … $P_{23}P_{24}$ and $P_{24}P_1$ are the same. The area of the sector formed by the line segment $OP_1$, the line segment $OP_9$ and one of the arcs $P_1P_9$ is 462. Find the area of the sector formed by the line segment $OP_3$, the line segment $OP_{15}$ and the arc $P_3P_{15}$.

(A) 693 (B) 346.5

(C) 1039.5 (D) Either (A) or (B)

**27.** $PQRS$ is any convex quadrilateral in which $PR = 24$ and $QS = 34$. Find the area of the largest such quadrilateral.

**28.** The sides $XY$, $YZ$ and $XZ$ of triangle $XYZ$ are 8, 15 and 17 respectively. A semicircle with its diameter lying on $XZ$ is drawn, touching the sides $XY$ and $YZ$ at $P$ and $Q$ respectively. Find the area of the region bounded by $PY$, $YQ$ and the arc $PQ$.

(A) $\left(\frac{64}{23}\right)^2\left(1-\frac{\pi}{4}\right)$ (B) $\left(\frac{120}{23}\right)^2\left(1-\frac{\pi}{4}\right)$

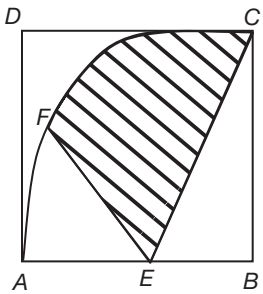
(C) $\left(\frac{120}{23}\right)^2\left(1-\frac{\pi}{8}\right)$ (D) $\left(\frac{64}{23}\right)^2\left(1-\frac{\pi}{4}\right)$

**29.** A solid right circular cone is cut into 4 parts of equal height, with each cut being parallel to its base. If the semi-vertical angle of the original cone is 30°, what is the ratio of the total surface area of the original cone to the sum of the total surface areas of the four parts?

(A) 24 : 31 (B) 18 : 31

(C) 12 : 19 (D) 15 : 19

**30.**




$ABCD$ is a square of side 18 cm. $F$ is a point inside the square, such that $BCF$ forms an equilateral triangle. $CFA$ is a quarter circle with center $B$. $E$ is the point on $AB$ such that the area of the region $EFC$ is half that of the area of the quarter circle. Find the length of $AE$ (in cm).

(A) $3\pi$ (B) $18 - 3\pi$

(C) 9 (D) $18 - 6\sqrt{3}$

## EXERCISE-5 Based on Data Sufficiency

***Directions for questions 1 to 20***: Each question is followed by two Statements I and II. Answer each question based on the following directions:

Choose (A) If the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.

Choose (B) If the question can be answered by using either statement alone.

Choose (C) If the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (D) If the question cannot be answered even by using both the statements together.

Data Sufficiency

**1.** Find the area of a scalene triangle $ABC$, whose sides in centimetres are natural numbers.

I. The length of the median drawn to the longest side is 10 cm and its square is equal to one-fourth of the sum of squares of the sides containing this median.

II. The perimeter of the triangle is 24 cm.

**2.** An equilateral triangle of side $a$ is constructed on one of the equal sides of an isosceles triangle, which also measures $a$. What is the length of the unequal side of the isosceles triangle?

I. The perimeter of the equilateral triangle is 45 cm.

II. The perimeter of the isosceles triangle is 40 cm.

**3.** What is the area of a parallelogram?

I. The perimeter of the parallelogram is 3 times the longer side.

II. The longer side of the parallelogram is 8 cm.

**4.** Find the ratio of the perimeters of the regular hexagon and the rectangle.

I. The areas of the hexagon and the rectangle are equal, with one of the dimensions of the rectangle being equal to the side of the hexagon.

II. The length of the rectangle, the breadth of the rectangle and the side of the hexagon are in the ratio 2 : 1 : 3.

**5.** Find the volume of the cuboid.

I. The surface areas of three mutually adjacent faces of the cuboid are 8 $cm^2$, 16 $cm^2$ and 8 $cm^2$.

II. The sum of the squares of the dimensions of the cuboid (when considered in cm) is 36. The total surface area of the cuboid is 64 $cm^2$.

**6.** The circumference of the base of a cylinder is 44 cm. What is the total surface area of the cylinder?

I. The sum of the radius of the base and the height of the cylinder is 21 cm.

II. The curved surface area of the cylinder is616 $cm^2$.

**7.** What is the area of a regular polygon?

I. The length of the longest diagonal of the polygon is 20 cm.

II. One of the exterior angles of the polygon is $60^o$.

**8.** What is the area of the triangle $ABC$?

I. $\angle BAC = 30°$.

II. $AC = 16$ cm and $BC = 12$ cm.

**9.** What is the total surface area of a cuboid?

I. The length of body diagonal of the cuboid is 36 cm.

II. The sum of the length, breadth and the height of the cuboid is 38 cm.

**10.** The radius of the base of a cone is 3.5 cm. What is the volume of the cone?

I. The radius of the base subtends an angle of 60° at the vertex of the cone.

II. The ratio of the total surface area (in sq. cm.) to the volume (in cu. cm) is 3 : 2.

**11.** What is the perimeter of a rhombus?

I. The area of the rhombus is 24 $cm^2$.

II. One of the diagonals of the rhombus is 6 cm.

**12.** Find the length of the diagonal of a cube.

I. Total surface area of the cube is 96 sq. cm.

II. Volume of the cube is 64 cc.

**13.** What is the volume of the sphere?

I. The surface area is 120 sq. cm.

II. The sphere can be reformed into a cuboid of total surface area of 120 sq. cm. by melting.

**14.** What is the volume of a right circular cone?

I. The height is equal to half the radius of the base.

II. The radius is equal to one side of a square of area 4 $cm^2$.

**15.** What is the ratio of the volumes of right circular cylinders $A$ and $B$?

I. The ratio of the heights of the right circular cylinders $A$ and $B$ is 1 : 2.

II. The ratio of the radii of the bases of $A$ and $B$ is 1 : 4.

**16.** The area of a rectangle is 77 $cm^2$. What is the perimeter of the rectangle?

I. The breadth of the rectangle is 7 cm.

II. The length of the rectangle is 11 cm.

**17.** What is the area of the rectangle?

I. The longer side of the rectangle is 16 cm.

II. One side of the rectangle (in cm) is equal to the square of its perpendicular side (in cm).

**18.** What is the area of the rectangle?

I. One side of the rectangle is 9 cm.

II. One side of the rectangle (in cm) is the square of its perpendicular side (in cm).

**19.** What is the area of the rectangle?

I. One side of the rectangle (in cm) is equal to the square of its perpendicular side (in cm).

II. The longer side of the rectangle is $p$ cm.

**20.** Find the length of the shorter side of the rectangle.

I. The area of the maximum square that can be cut out of the rectangle is 64 $cm^2$.

II. After a square of maximum possible area is cut from the rectangle, the area of the remaining portion of the rectangle is 32 $cm^2$.

## ANSWER KEYS

### Exercise-1

1. 12
2. (A)
3. (D)
4. 38.5
5. (C)
6. (C)
7. 60
8. (A)
9. (A)
10. (B)
11. (B)
12. (D)
13. (A)
14. (D)
15. (D)
16. (B)
17. 12
18. 72
19. (A)
20. (A)
21. 3000
22. (D)
23. (C)
24. 225
25. (A)
26. (A)
27. 32
28. 1694
29. (C)
30. (C)
31. (D)
32. 24
33. (B)
34. (C)
35. (C)
36. 1.25
37. 1080
38. (C)
39. 9
40. (B)
41. (D)
42. 24
43. (C)
44. 880
45. 4
46. (D)
47. (A)
48. (D)
49. (B)
50. 2400
51. 30
52. (B)
53. 384
54. 96
55. 30
56. 24
57. 175
58. 312
59. 800
60. (C)

### Exercise-2

1. (C)
2. (A)
3. (B)
4. (a) 9
   (b) 4
   (c) 16
5. 480
6. (A)
7. (A)
8. 814
9. (D)
10. 21
11. 22,638
12. (D)
13. (A)
14. (a) 10,43, 504
    (b) 64
15. (B)
16. (D)
17. 33.1
18. (D)
19. 36
20. 4
21. (D)
22. (B)
23. 72
24. 6
25. (A)
26. 1
27. (D)
28. (D)
29. (B)
30. (A)
31. (A)
32. (D)
33. 25
34. (B)
35. 120
36. (D)
37. (C)
38. (a) 6
    (b) 24
39. (C)
40. 288 $\sqrt{3}$ $cm^2$
41. (C)
42. 176
43. 22
44. (B)
45. 384
46. (D)
47. 50
48. (a) 36
    (b) 40, 96
49. (B)
50. 9
51. 144
52. 72
53. (C)
54. 3
55. 6
56. (B)
57. 150
58. 84
59. (B)
60. (D)

### Exercise-3

1. (B)
2. (a) 9
   (b) 4
   (c) 16
3. (C)
4. 22000
5. (C)
6. (C)
7. (B)
8. 672
9. (B)
10. (A)
11. 167
12. (B)
13. 1078
14. (C)
15. (C)
16. (D)
17. (D)
18. (D)
19. 18
20. (D)
21. (A)
22. (D)
23. (A)
24. (B)
25. (D)
26. (A)
27. (D)
28. 688
29. (C)
30. 225
31. 100
32. (C)
33. (D)
34. 18
35. (D)
36. 80
37. (B)
38. (C)
39. (C)
40. (A)
41. 58800
42. 50
43. (B)
44. 18000
45. (D)
46. 108
47. (C)
48. 8
49. (D)
50. (A)
51. (C)
52. (C)
53. 73.5
54. (B)
55. (C)
56. (B)
57. 7, 24
58. (D)
59. (B)
60. 352

### Exercise-4

1. (D)
2. (D)
3. (D)
4. (D)
5. 5832
6. 58800
7. (C)
8. (A)
9. (A)
10. (i) (B) (ii) (D)
11. (C)
12. (D)
13. (D)
14. (B)
15. 4.5
16. (A)
17. (D)
18. (C)
19. (C)
20. (D)
21. (C)
22. (D)
23. (B)
24. (A)
25. (C)
26. (D)
27. 408
28. (B)
29. (C)
30. (B)

### Exercise-5

1. (A)
2. (C)
3. (D)
4. (B)
5. (A)
6. (B)
7. (C)
8. (C)
9. (C)
10. (B)
11. (C)
12. (B)
13. (A)
14. (C)
15. (C)
16. (B)
17. (C)
18. (D)
19. (D)
20. (A)

## SOLUTIONS

## EXERCISE-1

**1.** Area of the triangle $= \frac{1}{2}\ (6)\ (8)\ \sin 30° = 12\ \text{cm}^2$.

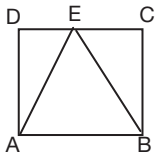
**2.** The total surface area of a cuboid $= 2\ (l\ + b)h + 2\ l\ \text{b}$

$= 2\ (l\ \text{b} + bh + h\,l\ )$. This area is $148\ \text{cm}^2$.

$\therefore 148 = 2\ (4b + 6b + 24)$

$\Rightarrow 10b = 74 - 24 \Rightarrow b = 5$ cm.

**3.** Let the parallelogram and the triangle be $ABCD$ and $ABE$ respectively where $AB$ is the common base of both.




$AB \perp CD$ and $AB = CD$. Again $E$ lies on $CD$.

Area of $ABCD = (AB)\ (AD)$.

Area of $ABE = 1/2\ (AB)\ (AD)$ (the height of $\Delta ABE = AD$).

The ratio of the areas of $ABCD$ and $ABE = \dfrac{(AB)(AD)}{\frac{1}{2}(AB)(AD)}$

$= 2 : 1$.

**4.** Area of the sector $AOB = \dfrac{90}{360}\pi\ (7)^2\ \text{cm}^2$

$= 38.5\ \text{cm}^2$

**5.** Total surface area of a sphere $= 4\pi r^2$

Hemisphere is half of the sphere and its surface area $= r^2\ \pi + \pi r^2 = 3\pi r^2$ [$\because \pi r^2$ is due to she surface area of the base].

**6.** Let the radius of each sphere be $r$ cm

Volume of each sphere $= \frac{4}{3}\ \pi\ r^3$ cu. cm.

Total volume of the spheres $= N.\ \frac{4}{3}\ \pi r^3$ cu. cm.

Radius of the cylinder = Its height $= 4r$ cm.

Its volume $= \pi\ (4r)^2\ (4r)$ cu. cm.

$N.\ \frac{4}{3}\ \pi r^3 = \pi\ (4r)^2\ (4r)$

$N = \dfrac{\pi(64\,r^3)}{\frac{4}{3}\pi r^3} = \dfrac{64}{\frac{4}{3}} = 48.$

**7.** Area of an isosceles triangle of base $b$ cm and equal sides a cm each $= \dfrac{b}{4}\sqrt{4a^2 - b^2}$.

In the given problem, $b = 10$ and a $= 13$.

Area of the triangle $= \dfrac{10}{4}\sqrt{4(13)^2 - 10^2} = 60\ \text{cm}^2$.

**8.** The volume of a sphere with radius $r$ is $\left(\frac{4}{3}\right)\pi r^3$

$\therefore$ Volume $= \dfrac{4}{3}\pi(3)^3 = 36\,\pi\,\text{cm}^3$.

**9.** Semi Perimeter $= 23$ cm.

Area $= \sqrt{(23-8)(23-10)(23-12)(23-16)}\ \text{cm}^2$

$= \sqrt{(15)(13)(11)(7)}\ \text{cm}^2 = \sqrt{15015}\ \text{cm}^2$.

**10.** Let the radii of the first and the second spheres be $2x$ and $3x$ respectively.

Curved surface area of the first sphere $= 4\,\pi\ (2x)^2$

Curved surface area of the second sphere $= 4\,\pi\ (3x)^2$

Required ratio $= \dfrac{4\pi(2x)^2}{4\pi(3x)^2} = \left(\dfrac{2x}{3x}\right)^2 = \left(\dfrac{2}{3}\right)^2 = \dfrac{4}{9}$

**11.** The volume of a cylinder is $\pi r^2 h$, when $r$ is the radius and $h$ is the height.

The volume of the right circular cone is $\frac{1}{3}\pi r^2 h$

$\therefore$ The fraction of the cylinder that is not covered by the cone is $\dfrac{\pi r^2 h - \frac{1}{3}\pi r^2 h}{\pi r^2 h} = 2/3.$

**12.** If the inradius and semiperimeter a triangle are known, then the area of the triangle is given by product of the inradius and semiperimeter i.e.,

Area $= rs$

**13.** Total surface area of a pyramid

= Lateral surface + Area of the base.

Lateral surface area $= \frac{1}{2}$ (perimeter of the base) (slant height.)

$\therefore$ Total surface area $= 48 + (4)\ (4) = 64$ sq. cm.

**14.** The length of each rope = (the circumference of the cylinder on which it makes rounds) (the number of rounds it can make on that cylinder).

The length of $R1 = (\pi 60)\ (49)$ cm.

The length of $R2 = (\pi 42)\ (70)$ cm.

The length of $R1$ – the length of $R2 = (\pi.60)\ (49)$ cm – $(\pi.42)\ (70)$ cm. = 2,940 $\pi$ cm – 2,940 $\pi$cm = 0 cm.

**15.** Area = $\frac{1}{2}(12 + 18)\ (15)\ \text{cm}^2 = 225\ \text{cm}^2$.

**16.** Total surface area of a cone

= curved surface area + area of the base

Curved surface area = $\pi$(radius) (slant height)

$= \pi \times 4 \times 5 = 20\ \pi$ sq. cm.

Total surface area = $20\ \pi + \pi\ (4)^2 = 36\ \pi$ sq. cm.

**17.** $\frac{1}{3}\pi r^2 h = 196\pi$

$\Rightarrow r^2 h = 196 \times 3$ (1)

Area of the base = $\pi r^2 = 154 \Rightarrow r^2 = 49$

$\Rightarrow r = 7\text{cm} \Rightarrow h = \frac{196 \times 3}{49} = 12$ {from (1)}

**18.** Perimeter = $[\pi(14) + 2(14)]$ cm $\approx \left[\frac{22}{7}(14) + 2(14)\right]$ cm = 72 cm.

**19.** Lateral surface area of a cone frustum = $(\pi)$ (slant height) (sum of the top radius and the base radius)

$= \pi(5)\ (4 + 6) = 50\ \pi$ sq. cm.

**20.** Longest diagonal = $\sqrt{l^2 + b^2 + h^2}$

**21.** External length of the picture (with frame)

= internal length + 2 × width = 80 + 2 × 10 = 100 cm

and external breadth

= 50 + 2 × 10 = 70 cm

$\therefore$ Area of the frame = external area – internal area

$= (100 \times 70) - (80 \times 50) = 3000\ \text{cm}^2$

**22.** Area of the triangle = $\frac{\sqrt{3}}{4}(6)^2\ \text{cm}^2 = 9\sqrt{3}\ \text{cm}^2$.

**23.** Total surface area of frustum of a pyramid

= Lateral surface + sum of the surface area of top and that of base.

Lateral surface area = $\left(\frac{1}{2}\right)$ (sum of perimeters of the base and top) (Slant height)

$= \left(\frac{1}{2}\right)[4(4) + 4(8)](6) = 144$ sq. cm.

Total surface area = 144 + (4) (4) + (8) (8)

= 144 + 16 + 64 = 224 sq. cm.

**24.** Area of a trapezium = 1/2 (sum of parallel sides) × perpendicular distance between them

$= \frac{1}{2}(20 + 10)(15)$

$= 225\ \text{cm}^2$

**25.** Let the radius of the circle be $r$ cm.

Let the length and the breadth of the rectangle be $l$ cm and $b$ cm respectively.

$\frac{\pi}{2}[2(l + b)] = 2\pi r$

$l + b = 2r$

If $l = r$, $b = r$

If $b = r$, $l = r$

In either case, $l = b = r$

Required ratio = $\pi r^2 : lb = \pi : 1$

**26.** Slant height = $\sqrt{\text{radius}^2 + \text{height}^2}$

$= \sqrt{6^2 + 8^2} = 10$ cm.

**27.** Volume of the Prism = (Area of the base) (Height of the Prism) = $\frac{\sqrt{3}}{4}(4)^2(8) = 32\sqrt{3}$ cu. cm.

**28.** Given $r = 35$ m

where $r$ is the radius of the garden.

$R = r$ + width = 35 + 7, $R = 42$

Area of the path

$= \pi R^2 - \pi r^2$

$\frac{22}{7} \times (42)^2 - \frac{22}{7}(35)^2$

$= 1694\ \text{m}^2$

**29.** Area of the triangle = $\frac{abc}{4R}$.

**30.** The volume of the frustum of the come

$= \frac{1}{3}\ \pi h\ (R^2 + Rr + r^2)$.

**31.** The maximum number of cubical boxes which can be placed = $\frac{\text{Volume of the big box}}{\text{Volume of each small box}} = \frac{30^3}{5^3} = \left(\frac{30}{5}\right)^3 = 6^3$

= 216.

**32.** Area = $\frac{1}{2}(6)\ (8)\ \text{cm}^2 = 24\ \text{cm}^2$.

**33.** Volume of the smaller cone $= \frac{1}{3}\pi\left(\frac{R}{2}\right)^2\left(\frac{h}{2}\right)$

Volume of the original cone $= \frac{1}{3}\pi R^2 h$

$\therefore$ The required ratio $\frac{\frac{1}{3}\pi\left(\frac{R}{2}\right)^2\left(\frac{h}{2}\right)}{\frac{1}{3}\pi R^2 h} = 1:8.$

**34.** The height of the smaller cone $= \frac{h}{2}$,

Radius of the smaller cone $= \frac{R}{2}$

Let $l$ be the slant height of the original cone.

$\therefore\ l = \sqrt{h^2+R^2}$

The slant height of the smaller cone

$= \sqrt{\left(\frac{h}{2}\right)^2+\left(\frac{R}{2}\right)^2} = \frac{\sqrt{h^2+R^2}}{2} = \frac{\ell}{2}$

$\therefore$ The ratio of the curved surface area of the smaller cone to that of the original cone $= \frac{\pi\left(\frac{R}{2}\right)\left(\frac{\ell}{2}\right)}{\pi R\ell} = 1:4.$

**35.** The slant height of the frustum $= \sqrt{\left(R-\frac{R}{2}\right)^2+\left(\frac{h}{2}\right)^2}$

[$\because$ The slant height of the frustum is the square root of the sum of the squares of height of the frustum and the difference of the radii]

$= \sqrt{\left(\frac{R}{2}\right)^2+\left(\frac{h}{2}\right)^2} = \frac{\sqrt{R^2+h^2}}{2} = \frac{\ell}{2}$

$\therefore$ The ratio $= \frac{\ell/2}{\ell} = 1:2.$

**36.** Let the thickness of the sheet be $t$ m.

$(1600)\ (t) = 20.$

$t = \frac{1}{80}.$

$\frac{1}{80}\ m\ = 1.25$ cm

**37.** Volume $= (12)\ (10)\ (9) = 1080$ cu. cm.

**38.** Let $PS$ be the median from $P$ to $QR$.

By Apollonius theorem, $2(PS^2 + QS^2) = PQ^2 + PR^2$

$PS^2 = \frac{3^2+4^2}{2} - \left(\frac{6}{2}\right)^2$ cm$^2 = 3.5$ cm$^2$

$PS = \sqrt{3.5}$ cm.

**39.** Volume of the wire (Cylinder) is equal to the volume of the sphere.

$\pi(16)^2 \times h = \frac{4}{3}\pi(12)^3 \Rightarrow h = 9$ cm

**40.** Area enclosed by a ring whose inner circle radius is $r$ and outer circle is $R$ is given by $\pi(R^2 - r^2)$.

In the given problem, $R = 8$ cm and $r = 6$ cm

$\therefore$ Required area $= \pi(8^2 - 6^2)$ cm$^2 = 28\ \pi$ cm$^2 = 88$ cm$^2$

**41.** Let $h$, $l$, $R$ be the height, slant height and radius of the original cone

The height of the frustum $= \frac{h}{3}$

$\therefore$ The height of the smaller cone $= h - \frac{h}{3} = \frac{2h}{3}$

If $l_3$, $R_3$ are the slant height and radius of the smaller cone then from the property of similar triangles,

$\frac{R_3}{R} = \frac{2(h/3)}{h} = \frac{2}{3}\ \therefore$ Rs $= \frac{2R}{3}$

Top radius of the frustum $= \frac{2R}{3}$

$\therefore$ The volume of the smaller cone

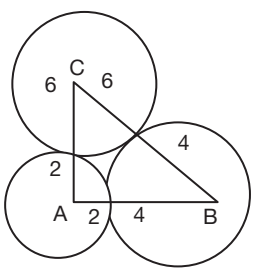
$= \frac{1}{3}\pi\left(\frac{2R}{3}\right)^2\left(\frac{2h}{3}\right) = \frac{\pi\ 8R^2h}{81}$

The volume of the frustum

$= \frac{1}{3}\pi\left(\frac{h}{3}\right)\left(R^2 + R\left(\frac{2R}{3}\right) + \left(\frac{2R}{3}\right)^2\right) = \frac{\pi\ h}{81}(19R^2)$

$\therefore$ The ratio $= \frac{3\pi R^2 h/81}{19\pi R^2 h/81} = 8:19.$

**42.**



Let $A$, $B$ and $C$ be the centres of the circles.

$AB = 6$ cm, $BC = 10$ cm and $CA = 8$ cm

Semi perimeter$(s)$ of the circle $= \frac{6+8+10}{2}$ cm $= 12$ cm

Area of the triangle $= \sqrt{12(12-8)(12-6)(12-10)}$ cm$^2$

24 cm$^2$.

**NOTE**

We observe that the above triangle is a right triangle.
∴ Area = 1/2(6)(8) cm$^2$ = 24 cm$^2$.

**43.** Let the sides of the triangle and the square be $s$ and $a$ respectively.

Area of the triangle $= \frac{\sqrt{3}}{4} s^2$.

Area of the square $= a^2$.

$\frac{\sqrt{3}}{4} s^2 = a^2$.

Perimeter of the triangle $= 3s$.

Perimeter of the square $= 4a$.

$(4a)^2 = 16a^2 = 16(\frac{\sqrt{3}}{4} s^2) = 4\sqrt{3}\, s^2$

Now, $4\sqrt{3}\, s^2 < 4\ (2)\ s^2 < 9s^2$

$(4a)^2 < (3s)^2$

∴ $4a < 3s$. Choice (C) follows.

**44.** In one revolution, the distance covered by the wheel is its circumference. Distance covered in 500 revolutions

$= 500 \times 2 \times \frac{22}{7} \times 28 = 88000$ cm $= 880$ m

**45.** $AC$ and $AE$ are secants. Let $AD = X$ cm

$(AB)\ (AC) = (AD)\ (AE)$

$(2)\ (2 + 22) = (X)\ (X + 8) \Rightarrow X = 4$

∴ $AD = 4$ cm

**46.** Each cube has an edge of 40 cm.

∴ The cuboid formed on joining both cubes will have a length of 80 cm, a breadth of 40 cm and a height of 40 cm.

Its surface area (in sq. cm.)

$= 2\ ((80)\ (40) + (80)\ (40) + (40)\ (40)$

$= 2\ (3{,}200 + 3{,}200 + 16{,}00) = 2\ (8{,}000) = 16{,}000.$

**47.** Lateral Surface Area $= 2h\ (l + b)$

**48.** Let the radius and the height of the cylinder be $r$ and $L$ respectively.

$2\pi r L = 3\ \pi r^2$

$\pi r\ (2L - 3r) = 0.$

$\because \pi r \neq 0$

$\Rightarrow 2L - 3r = 0$, i.e., $2L = 3r.$

$\frac{L}{r} = \frac{3}{2}$.

**49.** Let the area of the square be $3\sqrt{3}\ k$ and that of the triangle be $4k$

$S(S) = 3\sqrt{3}\, k \Rightarrow S = (3\sqrt{3}\, k)^{1/2}$

$\Rightarrow \frac{\sqrt{3}}{4} a^2 = 4k$

$\Rightarrow a = \left(\frac{16k}{\sqrt{3}}\right)^{1/2}$

Perimeter of the square $= 4\,(3\sqrt{3}k)^{1/2}$

Perimeter of the triangle $= 3\,\frac{(4\sqrt{k})}{(\sqrt{3})^{1/2}}$

The required ratio is $= 4(3\sqrt{3}k)^{1/2} : 3\frac{(4\sqrt{k})}{(\sqrt{3})^{1/2}}$

$= 1 : 1$

**50.** Volume of a cuboid $= l$bh

$20 \times 8 \times 15 = 2400$ cm$^3$

**51.** Area $= (6)\ (10)\ \sin 30°$ cm$^2$ $= 30$ cm$^2$.

**52.** The third side of the triangle $= 36$ cm $-$ (12 cm + 15 cm) $=$ 9 cm.

∴ the sides satisfy Pythagoras theorem.

∴ the triangle is right–angled and has perpendicular sides of lengths 12 cm and 9 cm.

∴ Its area (in sq. cm.)

$= 1/2\ (12)\ (9) = 54.$

**53.** The total surface are a of a cube

$= 6(S)^2$, where $S$ is the side

∴ Surface area $= 6\ (8)^2 = 384$ cm$^2$

**54.** Lateral Surface Area of the prism = (Perimeter of the base) (Height) $= 2\ (4 + 2)\ (8) = 96$ cm$^2$.

**55.** The circumference of the circle is equal to the perimeter of the rectangle.

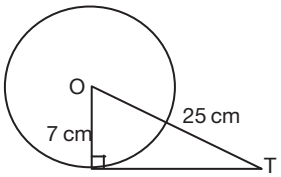
Let $l = 6x$ and $b = 5x$

$2(6x + 5x) = 2 \times \frac{22}{7} \times 3.5 \Rightarrow x = 1$

$\therefore l = 6$ cm and $b = 5$ cm

Area of the rectangle $= 6 \times 5 = 30$ cm$^2$

**56.**




Let $O$ be the centre of the circle and $T$ be the point from which the tangent is drawn.

Length of the tangent $= \sqrt{OT^2 - (Radius)^2}$

$= \sqrt{25^2 - 7^2}$ cm $= 24$ cm

**57.** The sides of the biggest square in the rectangle will be 35 cm.

Area unused from the rectangle

= (Area of the rectangle) – (Area of the square)

$= (40 \times 35) - (35 \times 35)$

$= 35\ (40 - 35) = 35 \times 5 = 175$ cm$^2$

**58.** Lateral Surface Area $= (4)\ (6)\ (10) = 240$ cm$^2$.

Total Surface Area = Lateral Surface Area + 2(Base Area)

$= 240 + 2(6)^2 = 312$ cm$^2$.

**59.** Area of the square $= s \times s = 5(125 \times 64)$

$\Rightarrow s = 25 \times 8 = 200$ cm

$\therefore$ Perimeter of the square $= 4 \times 200 = 800$ cm

**60.** Area of $ABCD$ = Area of triangle $ABC$ + Area of triangle $ACD = \left[\frac{1}{2}(4)(12) + \frac{1}{22}(6)(12)\right]$ cm$^2 = 60$ cm$^2$.

## EXERCISE-2

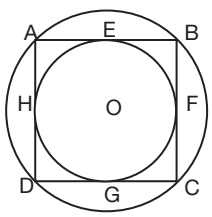
**1.** The perimeter of a sector $= l + 2r = 41$ cm

$\Rightarrow l = 41 - 2 \times 14 = 13$

Area of the sector

$= \frac{1}{2} \times l \times r = 13 \times \frac{14}{2} = 91$ sq.cm $= 91$ sq. cm.

**2.**




Let $OF = 1$. $\therefore EF = \sqrt{2}$ and area of square $EFGH = 2$

Radius of bigger circle $OB = \sqrt{2}$ . $\therefore$ Area $= 2\pi$

Required ratio $= 2\pi : 2 = \pi : 1$

**3.** $\frac{\text{Area of triangle } ADE}{\text{Area of triangle } DECB} = \frac{25}{144}$

(As area of triangle $ABC$ = Area of triangle $ADE$ + Area of trapezium $DECB$)

$\therefore \frac{\text{Area triangle } ADE}{\text{Area of triangle } ABC} = \frac{25}{169}$

In similar triangles, the ratio of the area of triangles is equal to the ratio of squares of corresponding sides of triangle.

$\frac{25}{169} = \frac{DE^2}{BC^2}$, as the two triangles are similar.

But $DE = 5$ cm $\therefore BC^2 = 5^2, \frac{169}{25}$

$BC^2 = 169$ $\therefore BC = 13$ cm.

**4. (a)** Let the sides of the triangle be $a$, $b$, and $c$.

$a = b + c - 6$, $b = c + a - 6$, $c = a + b - 6$

$6 = b + c - a = c + a - b = a + b - c$

$a = b = c = 6$ cm.

$\therefore$ The triangle is equilateral.

Its area is $9\sqrt{3}$ cm$^2$.

**(b)** Let the side of the triangle be a

$$\frac{\sqrt{3}}{4}a^2 = 36\sqrt{3}$$

$$a = 12 \Rightarrow \text{Circumradius} = \frac{a}{\sqrt{3}} = 4\sqrt{3} \text{ cm}$$

**(c)** Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)} = 84$.

Semi perimeter of the triangle = $s = 21$.

In radius of the triangle = $r = 4$. ( $\Delta = rs$)

Required area = $16\pi$ .

**5.** Let the length, the breadth and the height of the cuboid be $5x$, $4x$, and $3x$, respectively

Longest rod's length $\sqrt{(5x)^2 + (4x)^2 + (3x)^2} = 10\sqrt{2}$ cm

$50x^2 = \left(10\sqrt{2}\right)^2$ cm$^2 \Rightarrow x = 2$ cm

Volume of the cuboid = $(5x)\ (4x)\ (3x) = 480$ cm$^3$

**6.** Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of a sphere = $4\pi r^2$

Where '$r$' is the radius

$$\therefore \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{1}{1}$$

$\Rightarrow r = 3$

$\therefore$ The volume = $\frac{4}{3}\pi(3)^3 = 36\pi$ cu.units.

**7.** Area of the quadrilateral $OBDC$ = Area of the $\Delta OBD$ + Area of the $\Delta OCD$

= 1/2 $(OB)$ $(BD)$ + 1/2 $(OC)$ $(CD)$

1/2 (48) (14) + 1/2 (48) (14) cm$^2$ = 672 cm$^2$

**8.** Slant height of cone = 25 cm

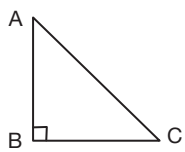
Curved surface area of solid

= C.S.A. of cylinder + C.S.A. of cone

= 2 $\pi$ (7) (6) + $\pi$ (7) (25) cm$^2$

= 22(12 + 25) cm$^2$ = 814 cm$^2$

**9.**




For any right angled triangle, the cirumradius is equal to half the hypotenuse.

$$\therefore \frac{AC}{2} = R$$

It is given that the circumference of the cirumcircle is $= 50\pi$

$\therefore R = 25$

$\therefore$ The hypotenuse is $25 \times 2 = 50$

The perimeter of the triangle = $AB + BC + 50 = 112$

$\therefore AB + BC = 62$ (1)

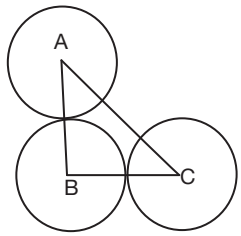
$AB^2 + BC^2 = 50^2$ (2)

Area of the triangle = $\frac{1}{2}(AB)\ (BC)$

$$= \frac{(AB + BC)^2 - \left(AB^2 + BC^2\right)}{2.2} = \frac{62^2 - 50^2}{4} \text{cm}^2$$

$$= \frac{(62+50)(62-50)}{4} \text{ cm}^2 = 112.3 \text{ cm}^2 = 336 \text{ cm}^2.$$

**10.** Given that $AB = BC = 14$ cm




$\angle A = \angle C = 45°$

$\therefore$ Area of triangle which is included in three circles is given by

$$\frac{(45+45+90)}{360}\pi r^2 = \frac{180}{360} \times \frac{22}{7}(7)^2 = 77 \text{ cm}^2$$

Area of triangle = $\frac{1}{2}(AB)(BC) = \frac{1}{2}(14)(14) = 98$ cm$^2$

$\therefore$ Left over area = [98 – 77] cm$^2$ = 21 cm$^2$

**11.** Height of the conical part = 7 cm

Volume of the toy = Volume of Cone + Volume of Hemi-sphere

$= \frac{\pi}{3}(21)(21)(7) + \frac{2}{3}\pi(21)(21)(21)\ \text{cm}^3$

$= \frac{\pi}{3}(21)(21)[7 + 21(2)]\ \text{cm}^3$

$= 462\,[49]\ \text{cm}^3 = 22{,}638\ \text{cm}^3.$

**12.** $h = 2r$

Total surface area $= 2\pi r(r+h) = 2\pi r(r+2r) = 6\pi r^2$

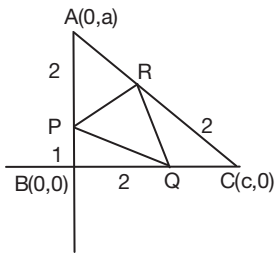
Cost of painting $= 6\pi r^3 \times 3$

$\therefore\ 18\pi r^2 = 2772 \Rightarrow r^2 = \frac{2772(7)}{18(22)} \Rightarrow r = 7$ cm

$\therefore\ h = 14$ cm $\therefore$ The volume $= \pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 14 = 2156$ cu. cm.

**13.**




As shown in the figure, let the sides of $\overline{AB}$ and $\overline{BC}$ be aligned with the $y$ – and $x$ – axis respectively and the vertex $B$ coincide with the origin.

Let A = (0, a), $C = (c, 0)$

Area of the $\Delta ABC = \Delta_1 = \frac{1}{2}ac$

Given, $\frac{AP}{PB} = \frac{BQ}{QC} = \frac{CR}{RA} = \frac{2}{1}$

From the section formula,

$P = \left(0, \frac{a}{3}\right), Q = \left(\frac{2c}{3}, 0\right), R = \left(\frac{c}{3}, \frac{2a}{3}\right)$

Area of the $\Delta PQR$

$\Delta_2 = \frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_2 - x_3 \\ y_1 - y_2 & y_2 - y_3 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 0 - \frac{2c}{3} & \frac{2c}{3} - \frac{c}{3} \\ \frac{a}{3} - 0 & 0 - \frac{2a}{3} \end{vmatrix} = \frac{1}{2}.ac\begin{vmatrix} \frac{-2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{-2}{3} \end{vmatrix}$

$= \frac{1}{2}ac\left(\frac{4}{9} - \frac{1}{9}\right) = \frac{1}{2} \cdot \frac{3ac}{9} = \frac{1}{6}ac$

Now, $\Delta_1 : \Delta_2 = \frac{\frac{1}{2}ac}{\frac{1}{6}ac} = 3 : 1.$

**Alternate Solution**:

Let the area of $\Delta ABC$ be $\Delta$.

For each of the 3 small triangles marked off from $\Delta ABC$, one side is $\frac{1}{3}^{rd}$ the corresponding side of $\Delta ABC$ and the other is $\frac{2}{3}$.

$\therefore$ The area is $\left(\frac{2}{9}\right)\Delta$. The total area of the triangle is $\left(\frac{6}{9}\right)\Delta$.

$\therefore$ Area of $\Delta PQR = \left(\frac{3}{9}\right)\Delta$.

$\therefore$ Area of $\Delta ABC$/ Area of $\Delta PQR = 3 : 1$.

**14. (a)** As the cube is melted to form a wire,

Volume of cube = Volume of wire

$\Rightarrow 154\,(154)\,(154) = \frac{22}{7}\,(r)\,(r)\,(\text{length})$

$\Rightarrow 154\,(154)\,(154) = \frac{22}{7}\,(r^2)\,(23716)$

$\Rightarrow r^2 = \frac{154\,(7)}{22} \Rightarrow r^2 = 7^2 \therefore r = 7$

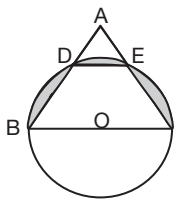
C.S.A. $= 2\,\frac{22}{7}(7)(23716) = 10{,}43{,}504.$

**(b)** Volume of the cube = sum of the volumes of the $n$ small cubes.

$16\,(16)\,(16) = n\,(4)\,(4)\,(4)$

$n = \frac{16\,(16)\,(16)}{64};\ n = 64.$

**15.**



As $BC = 2$ cm and the radius of the circle is 1 cm, $BC$ is the diameter of the circle.

Let $O$ be the centre of the circle and $D$ and $E$ be the respective points of intersection of $AB$ and $AC$ with the circle.

As $OB = OD$, and $\angle ABO = 60°$, $\Delta BDO$ is an equilateral triangle.

Similarly, $ECO$ is an equilateral triangle.

As $\angle BOC = 180°$, $\angle DOE = 60°$.

The triangles $BDO$, $DOE$ and $EOC$ are congruent.

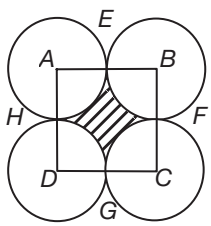
The area of the shaded region = Area of semi-circle– 3(Area of each equilateral triangle)

$$= \left[\frac{\pi}{2}(1)^2 - 3x\frac{\sqrt{3}}{4}(1)^2\right]$$

$$= \left(\frac{\pi}{2} - 3\frac{\sqrt{3}}{4}\right) \text{sq.cm.}$$

**16.** Let $E, F, G, H$ be the points of contacts of circles and the square.

Area of the shaded region = Area of the square $ABCD$ – (Area of the sector $AEH$ + Area of the sector $BEF$ + Area of the sector $FCG$ + Area of the sector $GDH$) = $(7+7)^2 - 4\ x\ 90/360\ x\ \pi(7)^2 = (196 - 49\pi)$ cm$^2$ = 42 cm$^2$




$$\frac{\text{Area of the shaded region}}{\text{Area of the square}} = \frac{42}{196} = 0.21 = 21\%.$$

**17.** Original volume = $\pi r^2 h$

increase in radius = 10%

New radius = $r + \frac{r}{10} = \frac{11r}{10}$

New height = $h + \frac{h}{10} = \frac{11h}{10}$

New volume = $\pi \times \left(\frac{11r}{10}\right)^2 \times \left(\frac{11h}{10}\right)$

Increase in volume

$$= \pi \times \left(\frac{121r^2}{100}\right) \times \left(\frac{11h}{10}\right) - \pi r^2 h$$

$$\pi r^2 h\left(\frac{1331}{1000} - 1\right) = \frac{331}{1000}\pi r^2 h$$

Percentage increase

$$= \frac{331\pi r^2 h/1000}{\pi r^2 h} \times 100 = 331/10 = 33.1\%$$

**18.** Given edges of the cubes as $x$ cm and $2x$ cm.

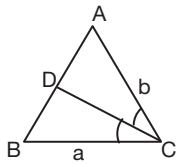
Edge of the new cube = $\frac{12\sqrt{3}}{\sqrt{3}} = 12$ cm. (since diagonal = $\sqrt{3}a$ )

$\therefore 3\ (x)^3 + 3\ (2x)^3 = 12^3 \Rightarrow 3x^3\ [1^3 + 2^3] = 12^3$

$\Rightarrow x^3 = \frac{12^3}{3 \times 9} = 4^3 \therefore x = 4.$

**19.** $\frac{\text{Area}(\Delta CAD)}{\text{Area}(\Delta CBD)} = \left(\frac{AD}{BD}\right) = \left(\frac{AC}{BC}\right)$

$= \frac{18}{17.5} = \frac{36}{35}$.




**20.** Let the length, breadth and height be denoted by $l$, $b$ and $h$. (all in cm)

Given that $(lb)\ (bh)\ (lh) = l^2b^2h^2 = 4096 \Rightarrow lbh = 64$

$h = 64/lb = 64/16 = 4$

**21.** Let the length and breadth of the rectangular park be $l$ and $b$ respectively.

Distance walked by Uma = $l + b$

Distance walked by Suma = $\sqrt{l^2 + b^2}$

It is given that $(l + b) = \sqrt{l^2 + b^2} + \frac{l}{4}$

Hence, $\frac{3}{4}l + b = \sqrt{l^2 + b^2}$

Squaring on both sides = $\frac{9}{16}l^2 + b^2 + \frac{3}{2}lb = l^2 + b^2$

$$\frac{7}{16}l^2 = \frac{3}{2}lb \Rightarrow \frac{l^2}{lb} = \frac{48}{14}$$

$\therefore\ l : b = 24 : 7.$

**22.** *EFGH* forms another parallelogram. The area of the parallelogram formed by joining the midpoints of the sides of a parallelogram is always half the area of the original parallelogram.

Hence, area of $EFGH = \dfrac{40\sqrt{3}}{2} = 20\sqrt{3}$ sq. cm

**23.** Volume of each sphere $= 4/3\pi r^3$

$$= \frac{4}{3}\times\frac{22}{7}\times 8 = \frac{704}{21} \qquad (1)$$

Volume of cone $= 1/3\ .\ \pi\ .\ r^2 h$

$$= \frac{1}{3}\times\frac{22}{7}\times 16\times 16\times 9$$

$$= \frac{22\ \times\ 256\times\ 3}{7} = \frac{256\times 66}{7} \qquad (2)$$

$$\text{Number of spheres} = \frac{(256\ \times\ 66)\ /\ 7}{(704)\ /\ 21}$$

$$= \frac{256\ \times\ 66\ \times\ 3}{704} = 72.$$

**Alternative Solution:**

Volume of the cone $= 1/3\ \pi r^2 h$

and the volume of sphere $= 4/3\ \pi R^3$

If '$n$' spheres are formed by melting the cone, then

$$n\times\frac{4}{3}\times\pi R^3 = \frac{1}{3}\pi r^2 h$$

$\Rightarrow 4n\ R^3 = r^2 h$

Substituting, $R = 2$, $r = 16$ and $h = 9$,

$4\times n\times 8 = 16\times 16\times 9 \Rightarrow n = 72.$

**24.** Let the side of the cube be a. Let the radius of the sphere be $r$.

$$6a^2 = 4\pi r^2 \Rightarrow \frac{a}{r} = \sqrt{\frac{2\pi}{3}}$$

$$\text{Ratio of the volumes} = \frac{3a^3}{4\pi r^3} = \frac{3}{4\pi}\left(\sqrt{\frac{2\pi}{3}}\right)^3 = \frac{\sqrt{\pi}}{\sqrt{6}}$$

**25.** Let radius $= r$ and height $= h$

$r + h = 15$ and $2\pi r\,(r + h) = 660$

$$\therefore\ 2\times\frac{22}{7}\times r\times 15 = 660 \Rightarrow r = 7 \text{ cm}$$

$\therefore\ h = 15 - r = 8$ cm

$\therefore$ Volume $= \pi r^2 h$

$$= \frac{22}{7}\times 7\times 7\times 8 = 22\times 56 = 1232 \text{ cu. cm.}$$

**26.** $l^2 + b^2 + h^2 = \dfrac{1}{2}\left(2\left(lb + lh + bh\right)\right)$

$\Rightarrow (l - b)^2 + (b - h)^2 + (l - h)^2 = 0$

$l = b = h$

Ratio $= 1 : 1 . = 1$

**27.** The length of the pendulum is equal to the radius of the sector.

$$\text{Length of the arc} = 2\pi r\times\frac{\theta}{360^\circ} = \pi\times\frac{\theta}{180^\circ}$$

$$\Rightarrow \frac{22}{7}\times\frac{30^\circ}{180^\circ}\times r = 66 \Rightarrow r = 126 \text{ cm.}$$

**28.** The triangle formed is a triangle with each side equal to the body diagonal of the cube. If the cube has an edge of a cm, the body diagonal $= \sqrt{3}$ a cm. The triangle will be equilateral.

$$\text{Its circumradius} = \frac{\text{Side of the triangle}}{\sqrt{3}} = a\,\text{cm.}$$

**29.** Let each side of the cube be '$a$' units and the radius of the sphere '$r$' units.

$$\frac{4}{3}\pi r^3 = a^3$$

$$\Rightarrow \frac{a^3}{r^3} = \frac{4}{3}\pi,\ \frac{a}{r} = \left(\frac{4\pi}{3}\right)^{\frac{1}{3}}$$

$$\frac{\text{Total surface area of the cube}}{\text{Suface area of the sphere}} = \frac{6a^2}{4\pi r^2} = \frac{6}{4\pi}\left(\frac{4\pi}{3}\right)^{\frac{2}{3}}$$

$$= \sqrt[3]{6} : \sqrt[3]{\pi}.$$

**30.** Let the length, the breadth and the height be $6x$, $5x$, $4x$ respectively.

Total surface of the cuboid = $2(l\,b + bh + l\,h) = 148x^2$

If the length was 20% more, the breadth was 25% more and the height was 25% less, the new length, breadth, height values will be 1.2 $l$, 1.25$b$, 0.75$h$, respectively.

$\therefore$ New total surface area = $170.7x^2$

$\therefore$ Total surface area will increase.

It will be increase approximately 15%.

**31.** Let rate of flow per hour = $x$ m/$s$

Volume of water flowing through the pipe per second

$$= x\left(\frac{25}{100}\right)\left(\frac{25}{100}\right)\left(\frac{x}{16}\right)\text{m}^3$$

Volume of water flown in 10 hours = $80(35)(2) = 5600\ m^3$

$\therefore$ Volume of water flown into the tank per second

$$= \frac{5600}{10(60)(60)}m^3 = \frac{7}{45}m^3$$

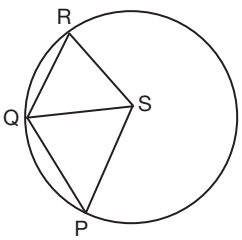
$$\therefore \frac{x}{16} = \frac{7}{45}$$

$$\Rightarrow x = \frac{7(16)}{45} = \frac{112}{45}$$

The flow rate in $\frac{km}{hr}$ is

$$\frac{112}{45}\left(\frac{3600}{1000}\right) = 8\frac{24}{25}$$

**32.**




In the given figure $PQ = QR = RS = SP = QS$.

Let the sides be '$x$' units each.

Then, the area of the rhombus

= 2 × (Area of the equilateral triangle $RQS$)

$$= 2 \times \frac{\sqrt{3}x^2}{4} = \frac{\sqrt{3}x^2}{2}$$

Now, $\frac{\sqrt{3}}{2}x^2 = 18\sqrt{3} \Rightarrow x = 6$ cm

$\therefore$ Area of the circle = $\pi x^2 = 36\pi$ sq. cm.

**33.** Let the length and the breadth of the rectangle be $l$ and $b$ respectively.

$$\sqrt{\ell^2 + b^2} = 5\sqrt{2} \Rightarrow l^2 + b^2 = 50$$

Subtracting 2$l$b from both sides, $(l - b)^2 = 50 - 2l$b

L.H.S. is always non-negative.

$\therefore$ R.H.S. is also non-negative.

$\therefore$ $l\,b \le 25$, i.e., maximum area is 25 sq. cm.

**34.** Number of bricks required

$$= \frac{0.9(1500)(1000)(800)}{10(8)(4)} = 3{,}375{,}000$$

Now cost of 3,375,000 bricks (in ₹)

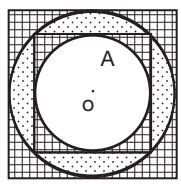
$$= 3{,}375{,}000\left(\frac{400}{100}\right) = 13{,}500{,}000$$

$\therefore$ The cost of the bricks is ₹ one crore, thirty five lakhs.

**35.** Three mutually adjacent faces of the cuboid have areas $lb$, lh and $bh$ in some order.

$(lb)$ (lh) $(bh) = (20)\ (30)\ (24) \Rightarrow lbh = 120$.

**36.**




Let the radius of the smaller circle (say $D$) be $r$.

The side of the smaller square (say $T$) = $r + r = 2r$

The radius of the bigger circle (say $C$) = $\sqrt{2}\,r$

The side of the bigger square (say $S$) = $2\sqrt{2}\,r$

The area shaded by lines is $(S - C) + (T - D)$.

The area shaded by dots is $(C - T)$.

$$\text{The required ratio} = \frac{4r^2 - \pi r^2 + 8r^2 - 2\pi r^2}{2\pi r^2 - 4r^2}$$

$$= \frac{12r^2 - 3\pi r^2}{2\left(\pi r^2 - 2r^2\right)} = \frac{3(4 - \pi)}{2(\pi - 2)}$$

**37.** Area of base = $\pi r^2 = 308$

$$\Rightarrow r^2 = \frac{308 \times 7}{22} = 98 \Rightarrow r = 7\sqrt{2}\text{ m}$$

$l = \sqrt{r^2 + h^2} = \sqrt{\left(7\sqrt{2}\right)^2 + \left(7\sqrt{2}\right)^2}$

$l = \sqrt{196} = 14$ m

Curved surface area = $\pi rl$

$= \frac{22}{7} \times 7\sqrt{2} \times 14 = 308\sqrt{2}$ sq. m.

$\therefore$ Canvas required = $308\sqrt{2}$ sq. m.

**38. (a)** Let the radius of the circle be $r$.

Then, $\frac{120}{360}\pi r^2 = 12\pi$

$\therefore r = 6$.

**(b)** Let the radius of the circle be $r$

Then, $\frac{150}{360} 2\pi r = 20\pi \therefore r = 24$.

**39.** $102.6 = 34.2 \times 3$

$136.8 = 34.2 \times 4$

$171 = 34.2 \times 5$

As the sides are in the ratio 3 : 4 : 5 the given triangle is a right angled triangle and the hyypotenuse is 171 cm which is also the diameter of the circumcircle.

$\therefore$ The radius of the circum circle $= \frac{1}{2} \times 171 = 85.5$ cm

**40.** If each side in a regular hexagon is '$a$', its area

$\Delta = \frac{3\sqrt{3}a^2}{2}$.

The perpendicular drawn from the centre to any side

$= \frac{a\sqrt{3}}{2} = 12$ (given) a $= 8\sqrt{3}$.

$\therefore \Delta = \frac{3\sqrt{3}}{2}\left(8\sqrt{3}\right)^2 \text{cm}^2 = 288\sqrt{3}\ \text{cm}^2$.

**41.** Let the parallel sides be $p$ cm and $(p + 9)$ cm

Area $= \frac{1}{2}\left\{p + \left(p + 9\right)\right\} \times 8 = 132$

$\Rightarrow 2p + q = 33$

$\Rightarrow p = 12$

$\therefore$ The longer side = 12 + 9 = 21 cm.

**42.** The area covered by the lawn mower in one revolution is its curved surface area.

$\therefore$Area covered in 200 revolutions

$= 200(2\pi rh) = 200(2)\left(\frac{22}{7}\right)\left(\frac{14}{100}\right)(1)\ \text{m}^2 = 176\ \text{m}^2$

**43.** ½ $d_1\ d_2 = 21$

$d_1\ d_2 = 42$

Also, side of the rhombus = 40/4 = 10 cm

$\therefore \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2 = 10^2 \Rightarrow {d_1}^2 + {d_2}^2 = 400$

Now, $(d_1 + d_2)^2 = {d_1}^2 + {d_2}^2 + 2d_1d_2$

$= 400 + (2 \times 42) = 484 = (22)^2$

$\therefore d_1 + d_2 = 22$

**44.** Let the dimensions be $8x$, $9x$ and $6x$.

Total surface area = $2\ (6x \times 8x + 6x \times 9x + 8x \times 9x)$

$= 348x^2$ sq. cm

Now, $6.25 \times 348x^2 - 4.5 \times 348x^2 = 5481$

$\Rightarrow \frac{7}{4} \times 348x^2 = 5481 = x^2 = \frac{4 \times 5481}{7 \times 348} = 9$

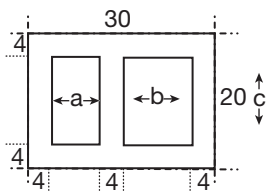
$\Rightarrow x = 3$ cm

$\therefore$ Volume of the box

$= (6 \times 3)\ (8 \times 3)\ (9 \times 3) = 11664$ cm

$\therefore$Volume of the box = 11664 cu. cm.

**45.**




$4 + a + 4 + b + 4 = 30 \Rightarrow a + b = 18$

$4 + c + 4 = 20 \Rightarrow c = 12$

Area of the paved portion = 30 (20) – 18 (12) sq. m.

= 384 sq. m.

**46.** The volume is 150cu. cm

It becomes 300 cu. cm, i.e., the volume of doubles

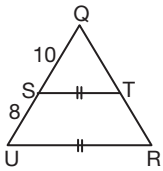
actual volume $V = lbh$

New volume $V = 9l/8 \times 4b/3 \times h_1 = 2lbh$

$\Rightarrow 3/2\ lbh_1 = 2lbh \Rightarrow h_1 = 4/3h$

The height increases by one-third, i.e., $33^1/_3\%$.

**47.**




From similarity of triangles, $\frac{QS}{QU}=\frac{QT}{QR}=\frac{10}{18}=\frac{5}{9}$

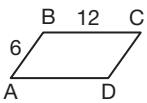
$\therefore \frac{\text{Area of } \Delta QUR}{\text{Area of } \Delta QST}=\left(\frac{9}{5}\right)^2$

$\therefore$ Area of $\Delta QST = 54\times\frac{25}{81}$ sq. units $=\frac{50}{3}$ sq. units.

**48. (a)** Let $ABCD$ be the parallelogram.

$AB = CD$ and $BC = AD$.

Let $AB = 6$ and $BC = 12$.




$\angle A = \angle C = 30°$ and $\angle B = \angle D$

$= 150°$ or vice versa.

Area of the parallelogram = Area of $\Delta ABC$ + Area of $\Delta ACD$ = 2(Area of $\Delta ABC$)

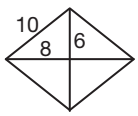
$=2\left(\frac{1}{2}(AB)(BC)\text{Sin}30°\right)$ or $2\left(\frac{1}{2}(AB)(BC)\,\text{Sin}\,150°\right)$

$= 36$.

**(b)** Side of the rhombus $=\sqrt{8^2+6^2}=10$.

Perimeter of the rhombus = 40.

Area $=\frac{1}{2}(16)(12)=96$.




**49.** Let the radius of the cylinder = radius of the cone = $r$ and the height of the cylinder = height of the cone = $h$.

Curved surface area of the cone

$= \pi rl = \pi r\sqrt{r^2+h^2}$

Curved surface area of the cylinder = $2\pi rh$

$\frac{\pi r\sqrt{r^2+h^2}}{2\pi rh}=\frac{17}{16}$, as per data.

$\Rightarrow \frac{\sqrt{r^2+h^2}}{h}=\frac{17}{8}$

Squaring on both sides $\frac{r^2+h^2}{h^2}=\frac{289}{64}$

$\Rightarrow \frac{r^2}{h^2}=\frac{289}{64}-1=\frac{289-64}{64}=\frac{225}{64}=\frac{15^2}{8^2}$

$\therefore$ The ratio $r : h = 15 : 8$.

**50.** Let the length and the breadth of the rectangle be $l$ and $b$ respectively.

Let the side of the square be a.

$2(l+b) = 4a \Rightarrow l+b = 2a$. As $b < l$, it follows that $b < a < l$

Given, $\frac{1}{a}=\frac{2}{3}$ or $\frac{b}{a}=\frac{2}{3}$

But $l$ cannot be less than $a$

$\therefore b = \frac{2}{3}$ a consequently, $l = \frac{4}{3}$a. The required ratio

$= 8 : 9$.

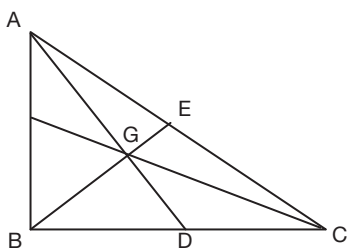
**51.** For a circle of radius $R$, the side of the inscribed equilateral triangle $T_1$ is $\sqrt{3}R$, while the side of the circumscribed equilateral traingle $T_2$ is $2\sqrt{3}R$. Therefore the area of $T_2$ is 4 times the area of $T_1$ and the difference is 3 times that of $T_1$.

Area of $T_1 = \frac{\sqrt{3}}{4}(\sqrt{3}R)^2=\frac{\sqrt{3}}{4}(8\sqrt{3})^2$ cm$^2$

$= 48\sqrt{3}$ cm$^2$

Difference in areas = $144\sqrt{3}\,cm^2$.

**52.**




Let $AD$ and $BE$ be the perpendicular medians.

The point of intersection of any two medians of a triangle is the same. $AD$, $BE$ and the median through $C$

intersect at $G$, the centroid. As $G$ trisects $AD$ and $BE$, $BG = \frac{2}{3}(BE) = 6$ and $GD = \frac{1}{3} AD = 4$.

The medians of a triangle divide the triangle into six triangles of equal area.

Area of $ABC = 6$ (Area of $BGD$)

$= 6\left(\frac{1}{2}(BG)(GD)\right) = 6\left(\frac{1}{2}(6)(4)\right) = 72.$

**53.** Let the length, breadth and height of the cuboid be $l_1$ $b$ and $h$ respectively.

$280 = 2(72 + 36 + bh)$

$\Rightarrow bh = 32$ (1)

$lb = 72$ (2)

$lh = 36$ (3)

From (2) and (3)

$\frac{b}{h} = \frac{2}{1}$

$\Rightarrow b = 2h$

Substituting in (1), we get

$(2h)(h) = 32 \Rightarrow h = 4$

$\therefore b = 2 \times 4 = 8$

$\therefore l = \frac{72}{8} = 9.$

**54.** Let the original side of the equilateral triangle be $x$ cm

Given $\frac{\sqrt{3}}{4}(x+5)^2 - \frac{\sqrt{3}}{4}x^2 = \frac{225\sqrt{3}}{4} \Rightarrow x = 20.$

$\therefore$ Area of original triangle $= \frac{\sqrt{3}}{4}(20)^2$ sq. cm.

$= 100\sqrt{3}$ sq. cm.

**55.** Let the edges of the cubes be $3x$, $4x$ and $5x$ cm

Side of the new cube $= \frac{12\sqrt{3}}{\sqrt{3}} = 12$ (Diagonal $= \sqrt{3}$ a)

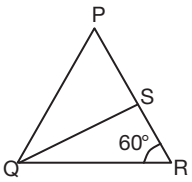
$\therefore (3x)^3 + (4x)^3 + (5x)^3 = 12^3$

$\Rightarrow (27 + 64 + 125)\, x^3 = 1728$

$\Rightarrow x^3 = \frac{1728}{236} = 8 \therefore x = 2$

$\therefore$ Edge of the smallest cube will be $3 \times 2 = 6$ cm

**56.** $PS = SR = 10/2 = 5$cm as $QS$ is the perpendicular bisector of $PR$.




$\angle SQR = 180 - (90 + 60) = 30°$

In triangle $RQS$, $\angle RQS$, $\angle SRQ$, $\angle QSR$ are 30°, 60°, 90° respectively.

$SR : QS : QR = 1 : \sqrt{3} : 2$

$QS = \sqrt{3}SR = 5\sqrt{3}$ cm

Area of triangle $PQR$

$= \frac{1}{2} x 5\sqrt{3} x 10 \text{ cm}^2 = 25\sqrt{3} \text{ cm}^2$

**57.** Let the sides of the triangle be a cm, $b$ cm and $c$ cm

Let $a + b - c = 10$, $b + c - a = 20$ and $c + a - b = 30$.

Adding these equations, we get $a + b + c = 60$.

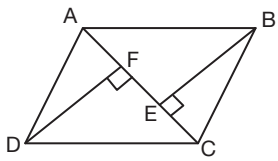
Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{\frac{a+b+c}{2}\left(\frac{a+b+c-a}{2}\right)\left(\frac{a+b+c-b}{2}\right)\left(\frac{a+b+c+-c}{2}\right)}$$

$$= \sqrt{\frac{(a+b+c)(b+c-a)(a+c-b)(a+b-c)}{16}}$$

$$= \sqrt{\frac{(60)(10)(20)(30)}{16}} = 150 \text{ cm}^2.$$

**58.**




The lengths of the perpendiculars $BE$ and $DF$ are 6 cm and 8 cm respectively.

Area of the quadrilateral $= 1/2\ AC\ (BE + DF)$

$= 1/2\ x\ 12(6 + 8) = 84 \text{ cm}^2$

**59.** Given that $\pi(14 - x)^2 \times 9 = \pi(14)^2 \times (9 - x)$

$\Rightarrow (196 - 28x + x^2)\ 9 = 196\ (9 - x)$

$\Rightarrow 9x^2 = -196x + 252x$

$\Rightarrow x = \frac{56}{9}$ or 0

But it is given that $x \neq 0$

$\therefore\ x = \frac{56}{9}$ cm.

**60.** Let the side of the square, the side of the hexagon and the radius of the circle be $a$, $h$ and $r$ respectively.

$a^2 = \frac{3\sqrt{3}}{2}h^2 = \pi r^2$ ($\because$ The area of a hexagon is the total of the areas of six equilateral triangles which together form the hexagon) $S = 4a$, $H = 6h$, $C = 2\pi r$.

$\therefore S^2 = 16a^2$, $H^2 = 36h^2$, $C^2 = 4\pi^2 r^2$

$\frac{S^2}{H^2} = \frac{16\left(\frac{3\sqrt{3}}{2}h^2\right)}{36h^2} = \frac{2}{\sqrt{3}}$ which is more than 1

$\therefore\ S^2 > H^2$. $S > H$ similarly $H > C$ ($\because \frac{H^2}{C^2} = \frac{2\sqrt{3}}{\pi} \approx \frac{3.46}{3.14}$ )

$\therefore\ S > H > C$

## EXERCISE-3

***Solutions for questions 1 to 60:***

**1.** Let the equal sides of the triangle be a each.

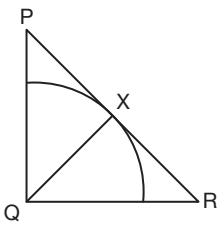
Perimeter of the triangle = $2a + a\sqrt{2}$

$2a + a\sqrt{2} = 8\sqrt{2} + 8$

$\Rightarrow a = 4\sqrt{2}$

Let the radius of the largest possible quadrant cut out be $r$.

The hypotenuse of the triangle is a tangent to the quadrant at $X$.




$\therefore r = QX$ = Altitude drawn to the hypotenuse.

$r = \frac{a}{\sqrt{2}} = 4$

Area of the remaining region = Area of the triangle – Area of the quadrant

$= \frac{1}{2}a^2 - \frac{\pi r^2}{4} \approx \frac{1}{2}\left(4\sqrt{2}\right)^2 - \frac{22}{7}\frac{(4)^2}{(4)}(4)^2 = \frac{24}{7}$.

**2.** (a) Let the sides of the triangle be $a$, $b$, and $c$.

$a = b + c - 6$, $b = c + a - 6$, $c = a + b - 6$

$6 = b + c - a = c + a - b = a + b - c$

$a = b = c = 6$ cm.

$\therefore$ The triangle is equilateral.

Its area is $9\sqrt{3}$ cm$^2$.

**(b)** Let the side of the triangle be $a$

$\frac{\sqrt{3}}{4}a^2 = 36\sqrt{3}$

a = 12 $\Rightarrow$ Circumradius = $\frac{a}{\sqrt{3}} = 4\sqrt{3}$ .cm

**(c)** Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)} = 84$.

Semi perimeter of the triangle = $s = 21$.

In radius of the triangle = $r = 4$. ($\because \Delta = rs$)

Required area = $16\pi$ .

**3.** The circumference of a circle is always greater than the perimeter of any inscribed polygon and the perimeter of any circumscribing polygon is always greater than the circumference of the circle.

As $r = 3$ cm, $2\pi r = 6\pi$.

$CP(n) > 6\pi$ and $IP(n) < 6\pi$ for all $n > 3$.

Hence, $CP(6) > 6\pi$

$\Rightarrow CP(6) + 6\pi > 6\pi + 6\pi$; $\Rightarrow CP(6) + 6\pi > 12\pi$

$IP(8) < 6\pi$.

$\therefore \frac{CP(6) + 6\pi}{IP(8)}$ is greater than 2 as the numerator is greater than $12\pi$ and the denominator is less than $6\pi$.

**4.** Radius of the roller = 0.7/2 = 0.35 m

Length of the roller = 4 m

Curved surface area of roller = $2\pi rh$

$= 2 \times \frac{22}{7} \times (0.35)\ (4) = 8.8$ sq. m.

In one revolution, the roller covers 8.8 sq. m., so in 50 revolutions, the area covered

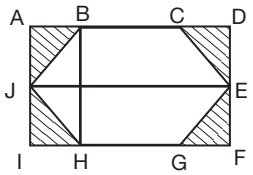
$= 500 \times 8.8 = 440$ sq. m.

Hence, the cost (at ₹5/sq. m.)

$= 4400 \times 5 =$ ₹22,000

**5.** Let the side of the hexagon be a cm.

The breadth of the rectangle is a $\sqrt{3}$ cm and the length is 2a cm



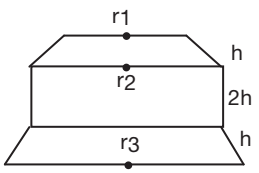

Given that $a\sqrt{3} = 8 \Rightarrow a = \frac{8}{\sqrt{3}}$

$\Rightarrow AB = \frac{2a-a}{2} = \frac{a}{2} = \frac{4}{\sqrt{3}}$ and $AJ = \frac{\sqrt{3}a}{2} = 4$

The area of the shaded region $= 4\left(\frac{1}{2}\right)(AB)\ (AJ)$

$= 4\left(\frac{1}{2}\right)(4)\frac{4}{\sqrt{3}}\text{cm}^2 = \frac{32}{\sqrt{3}}\text{cm}^2$

**6.**




In the figure, $r_1 = 2$ cm. $r_2 = 3$ cm, $r_3 = 4$ cm and $h = 1.5$ cm

Volume of the upper frustum

$= \frac{\pi}{3}h\left(r_1^2 + r_1r_2 + r_2^2\right) = \frac{\pi(1.5)}{3}\left(2^2 + 6 + 3^2\right)$ cm$^3$.$= 9.5\ \pi$ cm$^3$.

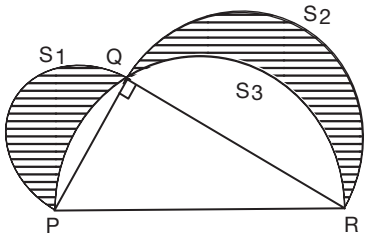
Volume of the lower frustrum $= \frac{1.5\pi}{3}[3^2 + 12 + 4^2]$ cm$^3$.

$= 18.5\pi$ cm$^3$.

Volume of the cylinder $= \pi\ (3)^2(3)$ cm$^3$ $= 27\pi$ cm$^3$.

Volume of the model = Sum of the volumes of the three parts $= 55\pi$ cm$^3$.

**7.** Let the shaded areas in $S_1$ and $S_2$ be $A_1$ and $A_2$, respectively.




Let the shaded and unshaded regions in $S_1$ be a and $b$ respectively.

Let the shaded and unshaded regions in $S_2$ be $c$ and $d$ respectively and let the area of $\Delta PQR$ be $e$.

As $PQ^2 + QR^2 = PR^2$ $\quad (\because \angle Q = 90°)$

$S_1 + S_2 = S_3$

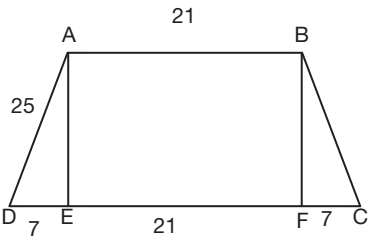
i.e., $(a + b) + (c + d) = (b + e + d)$

$\therefore\ a + c = e$

As $a + c = 30$, $e$ is also 30.

**8.** The sides 21 and 35 are parallel.

$\therefore$ The quadrilateral is a trapezium. Also since the other two sides are both 25, the quadrilateral is an isosceles trapezium.




$AB = 21$, $AD = BC = 25$, $DC = 35$

Area of $ABCD = \frac{1}{2}\ (AE)\ (AB + DC)$

$AE = h$(say), $BF = h$

$DE = FC = \sqrt{25^2 - h^2}$ ($\because \Delta s\ AED$ and $BFC$ are right angled)

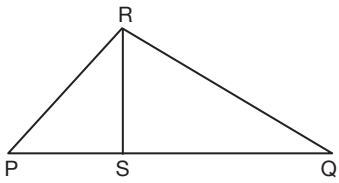
$DE = FC = \frac{DC - AB}{2} = 7$

$h = \sqrt{25^2 - 7^2} = 24$

Area of $ABCD = \frac{1}{2}\ (24)\ (21 + 35) = 672$

**9.** $PQ$ is the diameter $\therefore \angle PRQ$ is the angle in a semicircle

$\therefore \angle PRQ = 90°$ (1)




$RS$ is an altitude of $\Delta PRQ$, i.e., $RS \perp PQ$

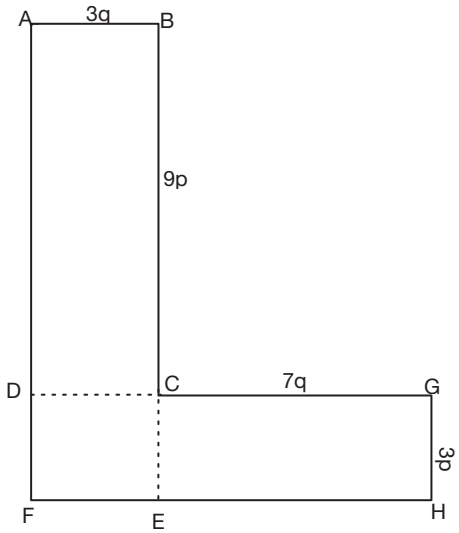
(1) $\Rightarrow RS^2 = (PS)\ (SQ)$

$$PS = \frac{12^2}{24} = 6 \text{ cm}$$

$\therefore PQ = 30$ cm

$$\therefore \text{Area of the circle} = \pi\left(\frac{30}{2}\right)^2 = 225\pi \text{ sq.cm}.$$

**10.**




Area of the figure $(A)$ = Area of $ABCD$ $(A_1)$ + Area of $DCEF$ $(A_2)$ + Area of $CGHE(A_3)$

$A_1 = (3q)\ (9p) = 27pq$

$A_2 = (3q)\ (3p) = 9pq$

$A_3 = (7q)\ (3p) = 21pq$

$A = A_1 + A_2 + A_3 = 57pq$

Perimeter of the figure $= AB + BC + CG + GH + HE + EF + FA = 3q + 9p + 7q + 7q + 3q + 12q = 20q + 24p$

$20q + 24p = 300$

$5q + 6p = 75$

$$A = 57pq = 57p\left(\frac{75-6p}{5}\right) = 57p\left(\frac{3(25-2p)}{5}\right)$$

$$= \frac{171p(25-2p)}{5}$$

**11.** $AB^2 - BY^2 = AY^2$ and $AC^2 - YC^2 = AY^2$

$\therefore AB^2 - BY^2 = AC^2 - (BC - BY)^2$

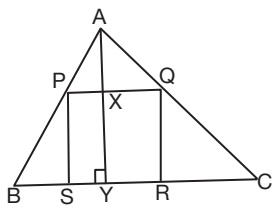
$\Rightarrow 26^2 - BY^2 = 30^2 - (28 - BY)^2$

$\Rightarrow (28 - BY)^2 - BY^2 = 30^2 - 26^2$

$\Rightarrow (28 - 2BY)\ (28) = 4\ (56) \Rightarrow 28 - 2BY = 8$

$\Rightarrow BY = 10$ and $AY = 24$ $[\therefore AY = \sqrt{AB^2 - BY^2}\ ]$

From similar triangles $\frac{AX}{AY} = \frac{PQ}{BC}$




$$\frac{AY - x}{24} = \frac{x}{28} \text{ (where } x \text{ is the side of the square)}$$

$$\frac{24 - x}{24} = \frac{x}{28}$$

$$\Rightarrow x = \frac{168}{13}$$

Therefore, the area of the square is approximately 167.

**12.** If 3 mutually perpendicular cuts divide the sphere into 8 identical pieces, then each cut must necessarily pass through the centre of the sphere.

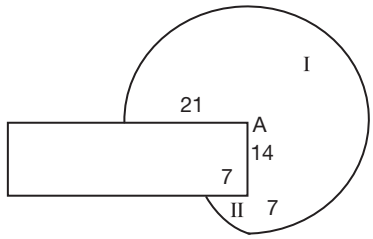
Each cut through centre produces 2 new surfaces each of which is a circle of radius equal to the radius of the sphere. So, $n$ cuts through the centre produce an additional surface area which is equal to the area of $2n$ circles. So, with 3 cuts an additional surface area equal to the area of 6 circles is produced.

Original $TSA$ of the sphere is $4\pi R^2$ and the new area produced by 3 cuts is $6\pi R^2$.

$\therefore$ The percentage increase in the surface area

$$= \frac{6\pi R^2}{4\pi R^2} \times 100 = 150\%.$$

**13.** The cow can graze the shaded areas numbered I and II.




Let the cow be tied at *A*.

Area of the region I = $\frac{22}{7} \times 21 \times 21 \times \frac{270}{360} = 1039^1/_2$ sq. m.

Area of the region II = $\frac{22}{7} \times 7 \times 7 \times \frac{90}{360} = 38^1/_2$ sq. m.

$\therefore$ Total area that the cow can graze

$= 1039^1/_2 + 38^1/_2 = 1078$ sq. m.

**14.** The cost of painting the vessel is determined by its outer surface area.

I: When a cube has the same volume as a cuboid of distinct dimensions, the total surface area of the cube will be less than that of the cuboid. $\therefore$ I is true

II: Let the side of the cube be a and the radius is the sphere be *r*.

$a^3 = 38808$ and $\frac{4}{3}\left(\frac{22}{7}\right) r^3 = 38808$

$a^3 = 8 \times 4851$ and $r^3 = 9261 \Rightarrow r = 21$

(Outer / Total) Surface area of the cube = $6a^2 = 6936$.

Surface area of the sphere = $4\pi r^2 = 5544$.

II is true.

Both I and II are true.

**15.** Perimeter of the figure = Combined length of the arcs of the two quadrants each of radius $p+2q+2p$.

$= \pi p + 2q + 2p = 128$ (given)

$$\text{Area} = pq + \frac{\pi p^2}{2} = p\left(\frac{128-2p-\pi p}{2}\right) + \frac{\pi p^2}{2}$$

$$= 64p - p^2 - \frac{\pi p^2}{2} + \frac{\pi p^2}{2} = 64p - p^2$$

**16.** Volume of the drum = $\frac{\pi h}{3}(R^2 + r^2 + Rr)$

$= \frac{22}{7}\left(\frac{5}{3}\right)(24^2 + 15^2 + 24 \times 15) = 42570/7$ cu.ft

Let the rise in the water level be *H* ft.

$99(43)H = 42570/7$

$\therefore H = \frac{42570}{7(99)(43)} = \frac{10}{7} = 1\frac{3}{7}$

**17.** Area of the triangle (A) = $\sqrt{s(s-a)(s-b)(s-c)}$

$$A = \sqrt{\frac{a+b+c}{2}\left(\frac{a+b+c}{2}-a\right)\left(\frac{a+b+c}{2}-b\right)\left(\frac{a+b+c}{2}-c\right)}$$

$$= \frac{1}{4}\sqrt{(a+b+c)(b+c-a)(a+c-b)(a+b-c)}$$

$$= \frac{1}{4}\sqrt{(a+b+c)(a+b-c)(b+c-a)(a+c-b)}$$

$$= \frac{1}{4}\sqrt{\left((a+b)^2-c^2\right)\left(c^2-(a-b)^2\right)}$$

Let $a = \sqrt{9p^2+16q^2}, b = \sqrt{9r^2+16q^2}, c = 3\sqrt{p^2+r^2}$

$a^2 = 9p^2 + 16q^2$, $b^2 = 9r^2 + 16q^2$, $c^2 = 9(p^2 + r^2)$

$$A = \frac{1}{4}\sqrt{\left((a+b)^2-c^2\right)\left(c^2-(c-b)^2\right)}$$

$$4A = \sqrt{\left((a+b)^2-c^2\right)\left(c^2-(c-b)^2\right)}$$

$16A^2 = ((a+b)^2 - c^2)\ (c^2 - (a-b)^2)$

$= 2c^2(a^2 + b^2) - c^4 - (a^2 - b^2)^2$

$= 2(9p^2 + 9r^2)(9p^2 + 9r^2 + 32q^2) - (9(p^2 + r^2)^2 - (9p^2 - 9r^2)^2$

$= (9(p^2 + r^2))^2 + 576q^2p^2 + 576q^2\ r^2 - (9p^2 - 9r^2)^2$

$= 81p^4 + 81r^4 + 162p^2\ r^2 + 576q^2p^2 + 576q^2\ r^2 - (81p^4 + 81p^4 - 162p^2r^2)$

$= 324p^2r^2 + 576q^2p^2 + 576q^2r^2$

$$A^2 = \frac{81}{4}\ p^2r^2 + 36q^2p^2 + 36q^2r^2$$

$$A = 3\sqrt{\frac{9}{4}p^2\ r^2 + 4q^2\ p^2 + 4q^2\ r^2}$$

**Alternate Method:**

Suppose $p = q = r = 1$

The sides of the triangle then would be 5, 5, $3\sqrt{2}$ .

The altitude to the side $3\sqrt{2}$ would be $\sqrt{\frac{41}{2}}$ .

Area of the triangle = $\frac{1}{2} 3\sqrt{2} \sqrt{\frac{41}{2}} = \frac{3}{2}\sqrt{41}$

Only choice $D$ is equal to this value when $p = q = r = 1$.

**18.** Let the side of the square be $a$.

Height of the pyramid = $5(2a) = 10a$

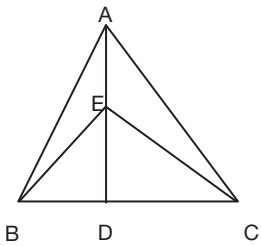
Let the length of the perpendicular drawn from the vertex to any of the bases of the triangular regions be $h$.

$h^2 = (10a)^2 + (\frac{a}{2})^2 = \frac{401a^2}{4} \Rightarrow h = \frac{\sqrt{401}}{2} a$

$S = \frac{1}{2} ah = \frac{\sqrt{401}}{4} a^2 = \frac{\sqrt{401}}{4} A$

$A = \frac{4S}{\sqrt{401}}$

**19.**




Consider triangles $AEB$ and $BED$:

In these triangles if $AE$ and $ED$ are considered as bases, the altitudes drawn to these bases would be equal.

$\therefore$ Ratio of the areas of $AEB$ and $BED$ = Ratio of the bases $AE$ and $ED$

$\frac{AE}{ED} = \frac{4}{3}$

Now consider triangles $AEC$ and $CED$:

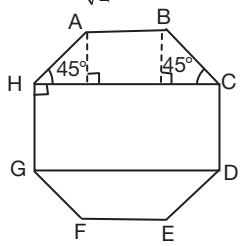
Ratio of the areas of $AEC$ and $CED = AE : ED$

$\frac{24}{\text{Area of } CED} = \frac{AE}{ED} = \frac{4}{3}$

Area of $CED = 18$

**20.** The length of the rectangle $CH = AB + 2AH \cos 45°$

$CH = 12\sqrt{2} + 2\left(12\sqrt{2}\right) \frac{1}{\sqrt{2}}$




$CH = 12\sqrt{2} + 24$

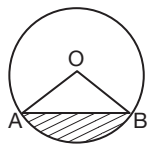
The breadth of the rectangle, $HG = 12\sqrt{2}$

Therefore the area of the rectangle $CDGH = 12\sqrt{2}\left(12\sqrt{2} + 24\right)$

$= 288 + 288\sqrt{2} = 288\left(1 + \sqrt{2}\right)$.

**21.** Required region = Area of the shaded region

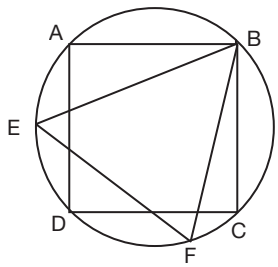
$= \frac{1}{3} \times \pi (10)^2 - \frac{1}{2} (OA)(OB) \sin \angle AOB$




$= \frac{100\pi}{3} - \frac{50\sqrt{3}}{2} \text{ cm}^2$

$= \left(\frac{100\pi}{3} - 25\sqrt{3}\right) \text{cm}^2$

**22.**




In the figure above,

$\angle ABC = 90°$ and $\angle DBA = \angle DBC = 45°$

From symmetry $\angle EBD = \angle DBF = 30°$

$\therefore \angle FBC = \angle DBC - \angle DBF = 45° - 30° = 15°$

Let the radius of the circle be $r$.

Side of the square = $\sqrt{2}\,r$ .

Side of the equilateral triangle is $\sqrt{3}\,r$ (Q the circle circumscribes the triangle)

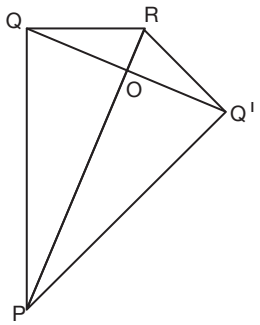
Area of $\Delta BFC = \frac{1}{2}\,(BF)\,(BC)\sin 15°$

$= \frac{1}{2}\,\left(12\sqrt{3}\right)\left(12\sqrt{2}\right)\,\frac{\sqrt{2-\sqrt{3}}}{2}$

$= 36\sqrt{6}\,\left(\sqrt{2-\sqrt{3}}\right) = 36\,\sqrt{3}\,\sqrt{4-2\sqrt{3}}$

$= 36\sqrt{3}\,\left(\sqrt{3}-1\right) = 36(3-\sqrt{3})$.

**23.**




$8^2 + 15^2 = 17^2$.

$\therefore$ The triangle is right-angled.

By rotating the triangle about its hypotenuse, we get a double cone.

Volume of the figure generated = Volume of the double cone = Volume of the upper cone ($U$) +Volume of the lower cone ($L$)

Each of the two cones has a base diameter of $QQ^1$. $O$ is the centre of the common circular base of either cone.

$\therefore$ $QO = Q^1O$ = Radius of the common circular base

$= r$ (say)

$QQ^1 \perp PR$

$U = \frac{1}{3}\,\pi(r^2)\,(RO)$ and $L = \frac{1}{3}\,\pi(r^2)\,(OP)$

$U + L = \frac{1}{3}\,\pi r^2\,(RO + OP) = \frac{1}{3}\,\pi r^2(RP)$

$r$ = Length of the altitude drawn from $Q$ to $PR$.

Area of $PQR = \frac{1}{2}\,(8)\,(15) = \frac{1}{2}\,17r$

$\Rightarrow r = \frac{120}{17}$ .

Volume of the figure generated $= \frac{1}{3}\,\pi\left(\frac{120}{17}\right)^2\,(17)$

$= \frac{4800\pi}{17}$ .

**24.** Let the arc length of 1st sector be $x$ cm. Arc lengths of 2nd, 3rd, 4th, 5th, 6th, 7th and 8th sectors are $2x$ cm, $4x$ cm, $8x$ cm, $16x$ cm, $32x$ cm, $64x$ cm and $128x$ cm respectively.

Sum of the arc lengths of the sectors = $255x$ cm

$\therefore 255x = \frac{2\pi(1)}{10}$

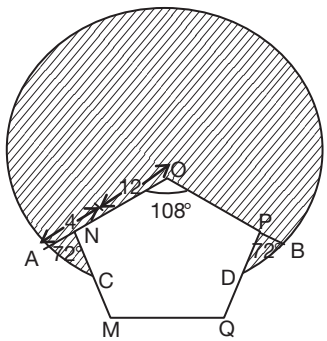
$\Rightarrow x = \frac{\pi}{1275}$

If the central angle of the 1st sector is $\theta$,

$\frac{\theta}{2\pi}\left(2\pi(1)\right) = \frac{\pi}{1275}$

$\theta = \frac{\pi}{1275}$

**25.**




The area that can be accessed majorly by the goat is shown as the shaded area = The area of the sector $OAB$ + the area of the sectors $NAC$ and $PDB$

Therefore, the required area (in $m^2$)

$= \frac{360-108}{360}\,(\pi 16^2) + 2.\,\frac{72}{360}\,(\pi 4^2)$

$= \frac{7}{10}\,\pi\,(16)^2 + \pi(4)^2$

$= \frac{16\pi}{10}\,(112+4) = \frac{\pi}{5}\,(928) = 185\,\frac{3}{5}\,\pi$.

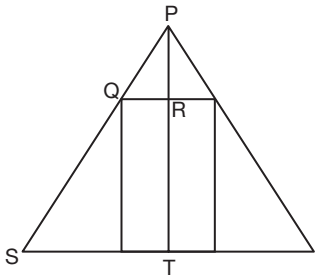
**26.** Radius of the cone = $^1/_2$ (12) cm = 6 cm

Height of the cone = 12 cm

$\therefore$ Volume = $\frac{1}{3}\left(\frac{22}{7}\right)(6)(6)(12)$ cm$^3$

= $452\frac{4}{7}$ cm$^3$

**27.**




In $\Delta QPR$ and $\Delta SPT$,

$\angle QPR = \angle SPT$ (1)

$\angle PRQ = \angle PTA = 90°$ (2)

(1) and (2) $\Rightarrow$ third angle or $\Delta QPR$ = third angle of $\Delta SPT$

$\therefore QPR \sim \Delta SPT$

$\therefore \frac{PR}{QR} = \frac{PT}{TS}$

Let $QR = x$ cm

$\frac{PR}{QR} = \frac{21}{6} = \frac{7}{2}$

$PR = \frac{7}{2}x$ cm

$\therefore RT = \left(21 - \frac{7}{2}x\right)$ cm

Total surface area of the cylinder (in sq. cm)

$= 2\pi x\left(x + 21 - \frac{7}{2}x\right) = 2\pi x.\left(21 - \frac{5x}{2}\right) = 2\pi\left(21x - \frac{5}{2}x^2\right)$

This is maximum when $21x - \frac{5}{2}x^2$ is maximum

$21x - \frac{5}{2}x^2$ is a quadratic expression. Its maximum value

$= \frac{4\left(\frac{-5}{2}\right)(0) - 21^2}{4\left(\frac{-5}{2}\right)} = \frac{441}{10}$

($\because$ maximum value of the quadratic expression $ax^2 + bx + c$ is $\frac{4ac - b^2}{4a}$).

Required area (in sq. cm.) = $2\pi\left(\frac{441}{10}\right) = \frac{441\pi}{5}$.

**28.** Area of the pentagon = $5 \times \frac{(20)^2}{4} \times \cot\left(\frac{180°}{5}\right)$

= $500 \times 1.376 = 688$ cm$^2$

**29.** Size of each tile is 60 cm by 40 cm

$\therefore$ Area of each tile = $60 \times 40 = 2400$ cm$^2$

Size of the room is 120 cm by 170 cm

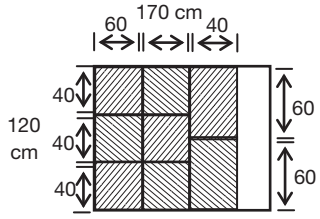
$\therefore$ Area of the room = $170 \times 120 = 20400$ cm$^2$

No. of tiles = $\frac{20400}{2400} = 8.5$

$\therefore$ Maximum of 8 tiles can be accommodated.

**NOTE**

Whether 8 tiles can be actually fitted to be verified with the help of diagrams.




**30.** Radius of the beaker = 3.5 cm

Radius of each marble = 0.7 cm

Total volume of the $N$ marbles

= Volume of the water displaced by the $N$ marbles

= (Base area) (Rise in level)

$\therefore N\left(\frac{4}{3}\pi(0.7)^3\right) = \pi(3.5)^2(8.4)$

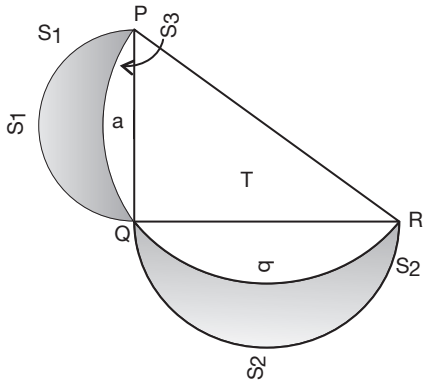
$\Rightarrow N = 225$

**31.** Each dimension (in m) of the larger box is a multiple of the same dimension (in m) of each of the smaller boxes.

$\therefore$ Maximum number = $\frac{\text{volume of the larger box}}{\text{volume of each small box}}$

$$\frac{30(20)(15)}{6(5)(3)} = 100.$$

**32.** Let $S_1$, $S_2$, $S_3$ be the first, second and third semicircles respectively.




Let $A$ be the area of the region between $S_1$ and $S_3$

and $B$ be the area of the region between $S_2$ and $S_3$

Let $S_1 = A + a$ and $S_2 = B + b$. and let the area of $\Delta PQR$ be $T$.

$\Delta PQR$ is right angled at $Q$.

$\therefore PQ^2 + QR^2 = PR^2$

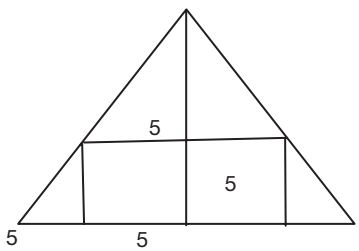
Multiplying both sides by $\frac{\pi}{4}$, we get $S_3 = S_1 + S_2$

$a + b + T = A + a + B + b$

$\therefore T = A + B$

Both I and II follow.

**33.**




Volume of the solid left

$$= \frac{\pi}{3}(10^2)(10) - \frac{\pi}{3}(5^2)5 - \pi(5^2)(5)$$

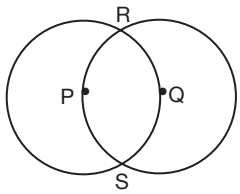
$$= \frac{500}{3}\pi$$

**34.** Volume of the water in the conical tank is equal to the volume of the water in the frustum.

$$\Rightarrow \frac{1}{3}(\pi)(7)^2(h) = \frac{1}{3}\pi[6^2 + (6)(3) + 3^2]14.$$

$$\Rightarrow h = \frac{(36+18+9)(14)}{49} = 18 \text{ m}.$$

**35.**




Let $P$ and $Q$ be the centres of the circles shown above. $PQ$ is a radius of both circles.

$\therefore$ The two circles have equal radii.

$\therefore PR = RQ = PQ$ and $PS = SQ = PR$

$\therefore \Delta PRQ$ and $\Delta PSQ$ are equilateral

$\therefore \angle RPQ = \angle QPS = 60°$

$\therefore \angle RPQ = 120°$

Area of the intersecting region $RSQR$ = 2(Area of the region $RQS$) = 2 (Area of the sector $PRS$ – Area of the $\Delta PRS$)

$$= 2\left[\left(\frac{120}{360}\right)\pi(PQ)^2 - \frac{1}{2}(PQ)^2 \sin 120°\right]$$

$$= 2(PQ)^2\left(\frac{\pi}{3} - \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)\right) = 2(PQ)^2\left(\frac{4\pi - 3\sqrt{3}}{12}\right)$$

$$\therefore \text{Required ratio} = \frac{2(PQ)^2\left(\frac{4\pi - 3\sqrt{3}}{12}\right)}{\pi(PQ)^2} = \frac{2}{3} - \frac{\sqrt{3}}{2\pi}.$$

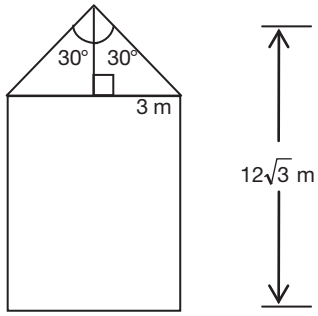
**36.** The volume of the water that can be filled in the swimming pool

= (Area of the cross section at the mid point along the length) × (Length of the pool) $= 6\left(\frac{d_1 + d_2}{2}\right) l$

(where $d_1 = 3$ m is the depth at the shallow end and $d_2 =$ 10 m is the depth at the deep end) and the length is $l$ m.

$= 6\left(\frac{13}{2}\right) l = 3120$ given $\Rightarrow l = 80$.

**37.**




Height of the cone = $\sqrt{3} \times$ radius of the cone = $3\sqrt{3}$ m

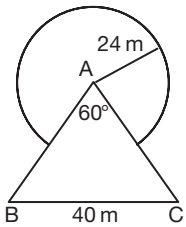
Height of the cylindrical portion = $12\sqrt{3} - 3\sqrt{3} = 9\sqrt{3}$ m.

Volume of the building

$$= \frac{22}{7}(3)^2\left(9\sqrt{3}\right) + \frac{1}{3}\left(\frac{22}{7}\right)(3)^2\left(3\sqrt{3}\right)$$

$$= \frac{1980\sqrt{3}}{7} = \sqrt{3}\left[282\frac{6}{7}\right] \text{cu. cm.}$$

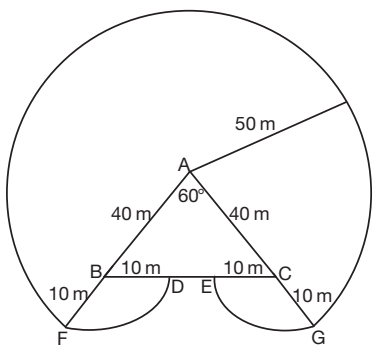
**38.**




Area grazed by the goat is a sector of angle 300° (i.e. 360° – 60°) and of radius 24 m

Hence, area grazed = $\left(\frac{300}{360}\right)\pi(24)^2 = 480\pi$ sq. m.

**39.** The goat can graze the sector of angle of 300° and of radius 50 m at vertex *A*. The goat can graze the sector of angle of 120° and of radius 10 m at vertex *B*. The goat can graze the sector of angle 120° and of radius 10 m at vertex *C*.




Area in which goat can graze is the sum of the areas of the three sectors shown.

$$= \pi\left[\frac{300}{360}(50)^2 + 2 \times \frac{120}{360}\left(10^2\right)\right] = 2150\pi \text{ sq. m.}$$

**40.** The length and breadth of the cuboid formed are both equal to $(30 - 2y)$ cm each.

Volume of the cuboid = $(30 - 2y)\ (30 - 2y)\ y$

$= 2(15 - y)\ (15 - y)\ (2y)$

The sum of $15 - y$, $15 - y$ and $2y$ is constant. (i.e., 30)

Their product is maximum when $15 - y = 15 - y = 2y$

i.e., $y = 5$

$\therefore$ the volume of the cuboid is maximum, when $y = 5$.

**41.** Number of hemispherical blocks

$$= \left(\frac{42}{14}\right)\left(\frac{42}{14}\right)\left(\frac{70}{7}\right) = 90$$

Volume of the cuboid = (42) (42) (70) c.c. = 123480 c.c.

Volume occupied by the hemispherical blocks

$$= 90\left(\frac{2}{3}\right)\left(\frac{22}{7}\right)(7)^3 \text{ c.c.} = 64680 \text{ c.c.}$$

$\therefore$ Empty space = (123480 – 64680)c.c.

= 58800 c.c.

**42.** Curved surface area of the cone = Area of the canvas. Given $r = 7$ m and $h = 24$ m.

$\therefore$ Slant height = $\sqrt{r^2 + h^2}$

$l = \sqrt{7^2 + 24^2}$ m = 25 m.

Let the length of the canvas be $x$ m

$$\Rightarrow \frac{22}{7}(7)\ (25) = x\ (11) \Rightarrow x = 50.$$

**43.** In triangles *AED* and *BEC*,

$\angle AED = \angle BEC$, as both are vertically opposite angles.

$\angle DAE = \angle CBE$ and $\angle ADE = \angle BCE$ (as angles in the same segment are equal)

$\therefore$ Triangles *AED* and *BEC* are similar.

As *AE* and *BE* are corresponding sides and they are 8 and 12 respectively, their ratio is 2 : 3.

The ratio of areas of triangles *AED* and *BEC*

$$= \left(\frac{AE}{BE}\right)^2 = \left(\frac{2}{3}\right)^2 = 4 : 9.$$

**44.** Volume of the box = $36 \times 24 \times 3$

Volume of one bundle of 5 rupee notes = $12 \times 6 \times 1$

Number of bundles in the box = $\dfrac{36 \times 24 \times 3}{12 \times 6 \times 1} = 36$

Since each bundle is of value ₹ 500,

total amount in the box = $36 \times 500 =$ ₹ 18,000.

**45.** Let the radius of $S$ be $R$ cm.

Let the radius and the heights of $C$ be $r$ and $h$ respectively.

Let A be a point on the top surface of the cylinder. Let $B$ be the point diametrically opposite to $A$. Let $C$ be the point on the bottom surface of the cylinder directly below $B$.

$AC$ = Diameter of $S$

$\sqrt{(2r)^2 + h^2} = 2R \Rightarrow 4r^2 + h^2 = 4R^2$

Volume of the cylinder $(V) = \pi r^2 h$

$V^2 = \pi^2 (r^2)^2 h^2 = \pi^2 (r^2)^2 (4R^2 - 4r^2)$

C is the largest cylinder that can be inscribed in $S$.

$\therefore$ $V$ is maximum.

As $V$ is maximum, $V^2$ is also maximum.

$\therefore$ $(r^2)^2 (4R^2 - 4r^2)$ is maximum.

$(r^2)^2 (4R^2 - 4r^2) = \dfrac{1}{4} (2r^2)(2r^2)(4R^2 - 4r^2)$

$2r^2, 2r^2, 4R^2 - 4r^2$ have a sum of $4R^2$, which is a constant.

When the sum of two or more positive quantities (which vary in such a way that it is possible for all of them to be equal) is a constant, their product is maximum when all the quantities are equal.

$(2r^2)(2r^2)(4R^2 - 4r^2)$ is maximum when $2r^2 = 4R^2 - 4r^2$

i.e., when $r^2 = \dfrac{2}{3} R^2$.

$\text{Max}(V^2) = \pi^2 \left( \left( \dfrac{2}{3} R^2 \right)^2 \left( 4R^2 - 4\left( \dfrac{2}{3} R^2 \right) \right) \right)$

$= \pi^2 \left( \dfrac{4}{9} R^4 \right) \left( \dfrac{4}{3} R^2 \right) = \pi^2 \left( \dfrac{16}{27} R^6 \right)$

$\text{Max}(V) = \pi \left( \dfrac{4}{3\sqrt{3}} R^3 \right)$

$\text{Max}(V) = 12\sqrt{3}\pi$ (given)

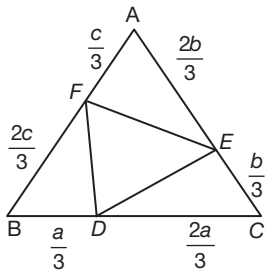
$\therefore \pi \left( \dfrac{4}{3\sqrt{3}} R^3 \right) = 12\sqrt{3}\pi \Rightarrow R^3 = 27 \Rightarrow R = 3.$

**46.** The area of a triangle formed by joining any three alternate vertices of a regular hexagon is half the area of the hexagon.

Area of $ACE$ = Area of $BDF = \dfrac{\text{Area of } ABCDEF\,(A)}{2}$

A = 2 (Area of $ACE$) = Area of $ACE$ + Area of $BDF$ = 108.

**47.** Let $AB = c$; $BC = a$, $AC = b$




Area of $ABC = \dfrac{1}{2} bc \sin A = \dfrac{1}{2} ab \sin C = \dfrac{1}{2} ac \sin B$

Area of $CDE = \dfrac{1}{2} \left( \dfrac{2a}{3} \right) \left( \dfrac{b}{3} \right) \sin C = \dfrac{2}{3} \Delta$

Area of $AFE = \dfrac{1}{2} \left( \dfrac{2b}{3} \right) \left( \dfrac{c}{3} \right) \sin A = \dfrac{2}{9} \Delta$

Similarly area of $BDE = \dfrac{2\Delta}{9}$

Area of $DEF = \Delta - 3\left( \dfrac{2\Delta}{9} \right) = \dfrac{\Delta}{3}$

The ratio of the areas of $DEF$ and $ABC = \dfrac{\Delta / 3}{\Delta} = 1 : 3$

**48.** $l$ = Difference $(r, 2h)$

$l > r - h$ $(\therefore l > r - 2h)$

$\therefore$ $l = 2h - r$

$\sqrt{r^2 + h^2} = 2h - r$

$r^2 + h^2 = 4h^2 - 4hr + r^2 \Rightarrow r = \dfrac{3}{4} h$ $(\therefore r : h : l = 3 : 4 : 5)$

$$\frac{C.S.A.}{T.S.A.} = \frac{\pi rl}{\pi rl + \pi r^2}$$

$$\frac{1}{1+r} = \frac{5}{5+3} = \frac{5}{8}$$

**49.** Let the area of triangle $ABC$ be $X$.

Let $AE = 3x$, $EF = x$ and $FC = 4x$.

Let $BH = 3y$, $HG = 5y$ and $GC = 4y$.

Let $AD = 2z$, $DI = z$ and $IB = 3z$

$AC = 8x$, $BC = 12y$ and $AB = 6z$.

The area of triangle $ABC$

$= \frac{1}{2}(AC)(BC)\sin\angle C = \frac{1}{2}(AB)(BC)\sin\angle B$

$= \frac{1}{2}(AB)(AC)\sin\angle A$.....

$\therefore$ The area of triangle $ABC$

$= 48xy\sin\angle C = 36yz\sin\angle B = 24xz\sin\angle A = X$

The area of the hexagon = the area of the $ABC$ – the sum of areas of triangles in triangle $ABC$.

The area of triangle $ADE = \frac{1}{2}(AD)(AE)\sin\angle A = \frac{1}{8}X$

The area of triangle $BIH = \frac{1}{2}(BI)(BH)\sin\angle B = \frac{1}{8}X$

The area of triangle $FCG = \frac{1}{2}(FC)(GC)\sin\angle C = \frac{1}{6}X$

The area of hexagon $= X - \left(\frac{1}{8}X + \frac{1}{8}X + \frac{1}{6}X\right) = \frac{7}{12}X$

The ratio of areas = 12 : 7

**50.** Let the radius and the slant height be $r$ units and $l$ units respectively.

$l + r = 2$ (Difference of $2r$ and $l$)

If $2r = l$, $l + r = 0$ which is not possible.

If $2r > l$, $l + r = 2(2r - l)$

$l = r$ which is not possible.

If $2r < l$, $l + r = 2(l - 2r)$

$l = 5r$

$\sqrt{r^2 + h^2} = 5r$

Squaring both sides, $h^2 = 24r^2$

$h = 2\sqrt{6}r$

**51.** Let the slant height and the height of the cone be $l$ cm and $h$ cm respectively.

If $l > 2h$, the difference of $l$ and $2h$ is $l - 2h$

$l + h = 2(l - 2h) \Rightarrow 5h = l$

If $2h > l$, the difference of $l$ and $2h$ is $2h - l$

$l + h = 2(2h - l) \Rightarrow h = l$

But $l$ has to be greater than $h$, so, $h = l$ is not possible.

$\therefore l = 5h$

The radius of the cone ($r$) is given by $r^2 = l^2 - h^2 = 24h^2$

and volume of the cone $= \frac{1}{3}\pi(24h^2)h = 192\pi$ (given)

$\Rightarrow h = \sqrt[3]{24} = 2\sqrt[3]{3}$ cm.

**52.** Radius of the base of the cone $= \frac{1}{2} \times 16 = 8$ cm

Height of the cone = 16 cm

$\therefore$ Volume $= \frac{1}{3} \times \frac{22}{7} \times 8 \times 8 \times 16$

$= \frac{22528}{21} = 1072\frac{16}{21}$ cu. cm.

**53.** Volume of the cube $= (7)^3\ m^3 = 343\ m^3$.

Volume of the largest right circular cylinder $= \pi r^2 h$

$= \frac{22}{7}(3.5)^2(7)\ \text{cm}^3 = 269.50\ \text{cm}^3$.

$\therefore$ The volume of the metal which is not used

$= 343 - 269.5\ \text{cm}^3 = 73.5\ \text{cm}^3$.

**54.** Let the side of the hexagon be a cm

So, a cm is one of the dimensions of the rectangle and let the other dimension be $x$ cm. Since areas of the hexagon and the rectangle are equal,

$\frac{3\sqrt{3}}{2}a^2 = xa \Rightarrow x = \frac{3\sqrt{3}}{2}a$

Ratio of the perimeters $= 6a : 2\left(\frac{3\sqrt{3}a}{2} + a\right)$

$= 6 : (3\sqrt{3} + 2)$

**55.** Let the length of the room be $6x$. Its breadth $= 4x$ and its height $= 3x$. Its lateral surface area $= 2(3x)(6x + 4x) = 60x^2$.

If its length and breadth are tripled, its length and breadth would become $18x$ and $12x$ respectively. If its height is halved, its height would become $\frac{3x}{2}$. Its lateral surface area would be $2\left(\frac{3x}{2}\right)(18x+12x) = 90x^2$

$\therefore$ Its lateral surface area increases by

$$\frac{90x^2 - 60x^2}{60x^2}(100)\% = 50\%.$$

**56.** In rectangle $ABCD$. As $BF = 1/3AB$, $\alpha = 2/3AB$

Area of $\Delta AFD = 2/3 \times \Delta ABD$

As $HI = 1/4\ AD$

Area of $\Delta\ HIF = 1/4\ \Delta\ AFD$

$= 1/4 \times 2/3 \times \Delta ABD = 1/4 \times 2/3 \times 1/2\ \angle ABCD$

$= 1/2\ \angle ABCD$

Area of $\Delta FKJ = 1/3 \times 1/3 \times 1/2 \times \angle\ ABCD$

$= 1/18\ \angle ABCD$

Area of shaded region

$$= \left(\frac{1}{12} + \frac{1}{18}\right)\angle ABCD = 5/36 \times 180 = 25\text{sq. cm}$$

**57.** Let the base be $b$.

Let the perpendicular drawn to the base be $h$.

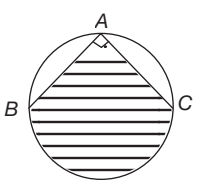
$$\sqrt{\left(\frac{b}{2}\right)^2 + h^2} = 25 \Rightarrow \therefore b^2 + 4h^2 = 2500$$

$$\frac{1}{2}(b)(h) = 168 \Rightarrow \therefore b^2\ h^2 = 336^2$$

$$\frac{336^2}{h^2} + 4h^2 = 2500$$

$$h^2 = 49 \text{ or } 576 \Rightarrow h = 7 \text{ or } 24.$$

**58.** The figure is shown below




Since, any angle in a semicircle is 90°, we can conclude that $BC$ is a diameter of the circle. The shaded area is equal to the sum of the area of the triangle and the area of the semicircle.

$$\text{Area of the semicircle} = \frac{\pi r^2}{2} = \frac{\frac{22}{7}(7)(7)\text{ cm}^2}{2} = 77\text{ cm}^2$$

In the triangle $ABC$, $AB = AC = 7\sqrt{2}$ cm.

$$\text{Area of the triangle} = \frac{1}{2}(7\sqrt{2})\left(7\sqrt{2}\right) = 49\text{ cm}^2$$

Therefore the shaded area = (49 + 77) $\text{cm}^2$ = 126 $\text{cm}^2$.

**59.** Let the radius be $r$ cm

$$2\pi r = 660 \Rightarrow 2 \times \frac{22}{7} \times r = 660 \Rightarrow r = 105\text{ cm}$$

$$\therefore \text{Volume} = \pi r^2 h = \frac{22}{7} \times 105 \times 105 \times 250$$

$= 8662500$ cu. cm

1ft $= 12 \times 2 \times 5 = 30$ cm

1 cu.ft $= 30 \times 30 \times 30 = 27000$ cu. cm

$$\therefore \text{Volume} = \frac{8662500}{27000} = \frac{28875}{90}\text{ cu. ft}$$

$$\text{Volume of wood that can be sold} = 0.9 \times \frac{28875}{90} = \frac{28875}{100}$$

$$\therefore \text{Total realisation} = \frac{28875}{100} \times 2000 = ₹5,77,500.$$

**60.** The largest right circular cone will be such that the diameter of the base of the cone will be equal to the side of the cube and the height of the cone will be equal to the side of the cube.

Radius of base of cone = 4/2 = 2 cm

Height = 4 cm

$\therefore$ Volume = $1/3\ \pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 2 \times 2 \times 4 = \frac{352}{21}\text{ cm}$$

## EXERCISE-4

***Solutions for questions 1 and 2:***

Each side of the hexagon subtends 60° at the centre of the hexagon.

Each side of the hexagon also subtends 120° at the centre of each outer circle.

**1.** Distance travelled by Krishna

$$= 6 \times \left(\frac{240}{360}\right)(2)\left(\frac{22}{7}\right) \times 35 \text{ m} = 880 \text{ m}$$

**2.** Area of the minor segment of each circle (in $m^2$) =

$$\frac{1}{3}\pi r^2 - \frac{r^2}{2}\sin 120° = r^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) = 35^2\left(\frac{3.14}{3} - \frac{1.732}{4}\right)m^2$$

$$= 1225\ (1.047 - 0.4330)$$

$$= 1225\ (0.614)$$

$\therefore$ Area of the 6 segments = $6(1225)\ 0.614\ m^2 \approx 4500\ m^2$

**3.** Let $BP$ be equal to $x$ m and let $\angle APB = \theta$

$\therefore PD = (26 - x)$ m

$\angle CPD = 90° - \angle APB = 90° - \theta$

i.e., $\tan\theta = \frac{9}{x}$ and $\tan(90 - \theta) = \frac{16}{26 - x}$

$$\Rightarrow \cot\theta = \frac{16}{26 - x}$$

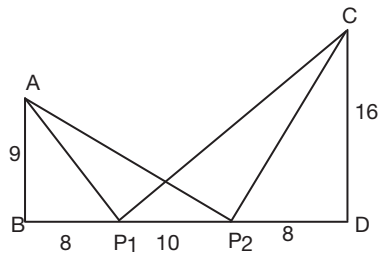
Now, $(\tan\theta)\ (\cot\theta) = 1$, i.e., $\left(\frac{9}{x}\right)\left(\frac{16}{26 - x}\right) = 1$

$$x^2 - 26x + 144 = 0$$

$$\Rightarrow (x - 18)(x - 8) = 0$$

Therefore $x = 8$ or $x = 18$.

These two positions of $P$ are shown in the figure as $P_1$ and $P_2$.

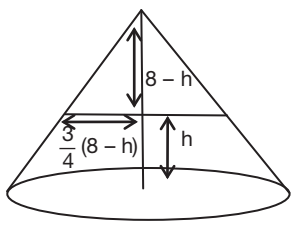



The value of $x$ and the corresponding area of triangles $ABP$, $CDP$, $APC$ and the value of $K$ are tabulated below.

| x | Area of $\Delta ABD$ | Area of $\Delta CDP$ | Area of $\Delta APC$ | $K$ |
|---|---|---|---|---|
| 8 | $\frac{1}{2}(8)(9) = 36$ | $\frac{1}{2}(18)(16) = 144$ | $\frac{1}{2}\left(\sqrt{145}\right)\left(\sqrt{580}\right) = 145$ | 29/36 |
| 18 | $\frac{1}{2}(18)(9) = 81$ $\frac{1}{2}(18)(9) = 81$ | $\frac{1}{2}(8)(16) = 64$ | $\frac{1}{2}\left(\sqrt{405}\right)\left(\sqrt{320}\right) = 180$ | 36/29 |

Therefore the value of $K$ is either 29/36 or 36/29

**4.** As the radius ($R$) and height ($H$) in cm are both integers less than 10, $R = 6$ cm and $H = 8$ cm.




When, the cone is inverted again, i.e., the cone on its base, let the liquid be at height of '$h$' cm.

Volume of the liquid = $(1/3)\ \pi\ (3^2)\ (4) = 12\ \pi$

$\therefore (1/3)\ (\pi)\ (6^2)\ (8) - (1/3)\ (\pi)\ \{(3/4)\ (8 - h)\}^2\ (8 - h)$

$= 12\ \pi$

$$288\pi - \frac{9}{16}\pi(8 - h)^3 = 36\pi$$

$$(8 - h)^3 = 16(28)$$

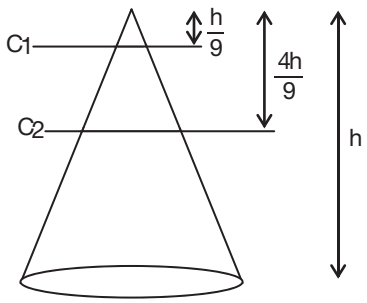
$h = 8 - 4 \quad \sqrt[3]{7} \approx 8 - 7.6 \cong 0.4$ cm.

**5.** Let the volume of $C$ be $V$.

Let the height of $C$ be $h$.

The cut $C_1$ was made parallel to the base of $C$ and $\frac{h}{9}$ from the vertex of $C$.

The cut $C_2$ was made parallel to the base of $C$ and $\frac{4h}{9}$ from the vertex of $C$.




$$\frac{\text{Volume of the top cone formed } (V_1)}{\text{Volume of } C} = \left(\frac{\frac{h}{9}}{h}\right)^3$$

($\because$ The top cone formed and $C$ are similar)

$$V_1 = \frac{1}{729}V$$

$$\frac{\text{Volume of (top cone + upper frustrum) } (V_2)}{\text{Volume of } C} = \left(\frac{\frac{4h}{9}}{h}\right)^3$$

$$V_2 = \frac{64}{729}V$$

Volume of the upper frustum $(V_3) = V_2 - V_1 = \frac{64}{729}V$

Volume of the lower frustum $= V - (V_1 + V_3) = \frac{665}{729}V$

$$\frac{665}{729}V - \frac{63}{729}V = 4816$$

$$\frac{602}{729}V = 4816$$

V = 5832

**6.** Number of hemispherical blocks

$$= \left(\frac{42}{14}\right)\left(\frac{42}{14}\right)\left(\frac{70}{7}\right) = 90$$

Volume of the cuboid = (42) (42) (70) c.c. = 123480 c.c.

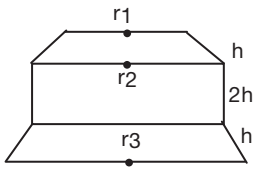
Volume occupied by the hemispherical blocks

$$= 90\left(\frac{2}{3}\right)\left(\frac{22}{7}\right)(7)^3 \text{ c.c.} = 64680 \text{ c.c.}$$

$\therefore$ Empty space = (123480 – 64680)c.c.

= 58800 c.c.

**7.**




In the figure, $r_1 = 2$ cm, $r_2 = 3$ cm, $r_3 = 4$ cm and $h = 1.5$ cm

Volume of the upper frustum

$$= \frac{\pi}{3}h\left(r_1^2 + r_1 r_2 + r_2^2\right) = \frac{\pi(1.5)}{3}\left(2^2 + 6 + 3^2\right) \text{ cm}^3 = 9.5\,\pi \text{ cm}^3.$$

Volume of the lower frustrum $= \frac{1.5\pi}{3}[3^2 + 12 + 4^2]$ cm$^3$.

$= 18.5\pi$ cm$^3$.

Volume of the cylinder $= \pi(3)^2(3)$ cm$^3$ $= 27\pi$ cm$^3$.

Volume of the model = Sum of the volumes of the three parts $= 55\pi$ cm$^3$.

**8.** Given that the volume of air that an exhaust fan can evacuate per unit time is proportional to the square of its speed in rev. per min $\Rightarrow v \propto n^2$

given it takes 20 min. to evacuate 2 m × 3 m × 4 m

i.e., $v_1 = 24$ cu. m.

If the same fan is running 5 times its speed it can evacuate $5^2$ times $v_1$.

i.e., 25 × 24 = 600 cu. m. in 20 min

But in 10 min it will evacuate only 300 cu. m.

$\therefore$ The total volume of air in the room should be 300 cu. m.

Total volume of the room = 6 × 8 ×12.5 = 600 cu. m.

$\therefore$ Volume of the boxes = 600 – 300 = 300 cu. m.

but the volume of each box = 1/2 × 1/2 × 1 = 1/4 cu. m.

$\therefore$ One needs 300 × 4 = 1200 such boxes.

**9.** Let the radius and the height of the largest cylinder that can be inscribed in the sphere be $r$ and $h$ respectively.

Distance between any point $P$ on the top surface of the cylinder and the point $R$ on the bottom surface that lies directly below $Q$, which is diametrically opposite to $P$.

$= \sqrt{(2r)^2 + h^2} = D$ (say). This is also the diameter of the sphere, ie $D = 2$

Volume of the cylinder $= \pi r^2 h = V$(say)

$V$ is maximum $\therefore$ $V^2$ is maximum

$\therefore \pi^2 r^4 (D^2 - 4r^2) = \pi^2 r^4 (4 - 4r^2) \Rightarrow r^4 (1 - r^2)$ is maximum

$\therefore r^4(1 - r^2) = \left[\frac{r^2}{2}\frac{r^2}{2}\left(1 - r^2\right)\right]4$ is maximum

When the sum of two or more positive quantities is constant, their product is maximum when all the quantities are equal. The sum of $\frac{r^2}{2}, \frac{r^2}{2}$ and $1 - r^2$ is a constant

$\therefore$ Their product is maximum when $\frac{r^2}{2} = 1 - r^2$ or $r^2 = \frac{2}{3}$

$$V = \pi\left(\frac{2}{3}\right)\sqrt{2^2 - 4\left(\frac{2}{3}\right)} = \frac{4\pi}{3\sqrt{3}}$$

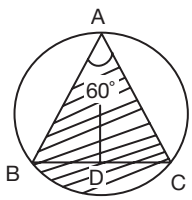
**10.** (i) Let $O$ be the centre of the circle.

Let us denote the radius of the circle by $r$.

Let $D$ be the mid point of $BC$.

$$BD = OB\cos 30° = \frac{r\sqrt{3}}{2}$$

Side of the triangle,




$BC = 2BD = r\sqrt{3}$

Since $\angle A = 60°$,

$$\angle B = \angle C = \frac{180° - \angle A}{2} = 60° \ (\because AB = AC)$$

$$\text{Area of } \Delta ABC = \frac{\sqrt{3}}{4}\left(r\sqrt{3}\right)^2 = \frac{3\sqrt{3}r^2}{4}$$

Area of all the three segments lying outside the triangle

= Area of the circle – Area of the triangle $= \pi r^2 - \frac{3\sqrt{3r^2}}{4}$

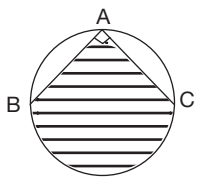
$$\text{Unshaded area (or two segments)} = \frac{2}{3}\left[\pi r^2 - \frac{3\sqrt{3r^2}}{4}\right]$$

The percentage of the area of the circle that is not shaded $= \frac{\text{unshaded area}}{\text{area of the circle}}(100)\% = \frac{\frac{2}{3}r^2\left[\pi - \frac{3\sqrt{3}}{4}\right]}{\pi r^2}(100\%)$

$$= \frac{2}{3}\frac{\left[4\pi - 3\sqrt{3}\right]}{4\pi}(100\%) = \frac{2}{3}\frac{[12.566 - 5.196]}{12.566}(100\%)$$

$\approx 39\%$.

**(ii)** The figure is shown below




Since, any angle in a semicircle is 90°, we can conclude that $BC$ is a diameter of the circle. The shaded area is equal to the sum of the area of the triangle and the area of the semicircle.

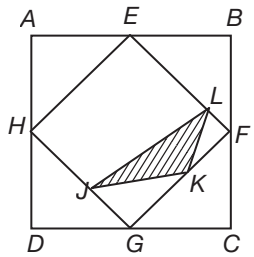
$$\text{Area of the semicircle} = \frac{\pi r^2}{2} = \frac{\frac{22}{7}(7)(7)\text{ cm}^2}{2} = 77\text{ cm}^2$$

In the triangle $ABC$, $AB = AC = 7\sqrt{2}$ cm.

$$\text{Area of the triangle} = \frac{1}{2}\left(7\sqrt{2}\right)\left(7\sqrt{2}\right) = 49\text{ cm}^2$$

Therefore the shaded area = (49 + 77) $\text{cm}^2$ = 126 $\text{cm}^2$.

**11.**




Let $AB = \sqrt{2}$

$\therefore$ Then, the area of the square $ABCD = 2$

$HG = \frac{AB}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} = 1$ and $HGFE$ is a square.

Area of $JKL$ ($\Delta$) = Area of $HGFE$ – (Area of $HJLE$ + Area of $JGK$ + Area of $KFL$)

$HJ = HG\left(\frac{2}{3}\right) = 1\left(\frac{2}{3}\right) = \frac{2}{3} \Rightarrow JG = 1 - \frac{2}{3} = \frac{1}{3}$

$GK = KF = \frac{GF}{2} = \frac{1}{2}$

$LF = \frac{EF}{4} = \left(\frac{1}{4}\right)1 = \frac{1}{4}$

$EL = 1 - \frac{1}{4} = \frac{3}{4}$

$\therefore \Delta = (1)\ (1) -$

$$\left[\frac{1}{2}(1)\left(\frac{2}{3}+\frac{3}{4}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\right]$$

$$= 1 - \left(\frac{17}{24}+\frac{1}{12}+\frac{1}{16}\right) = 1 - \frac{41}{48} = \frac{7}{48}$$

$\therefore$ The ratio of the area of $\Delta JKL$ to that of the square $ABCD = \frac{7}{48} : 2 = 7 : 96$

**12.** Let the area of triangle $ABC$ be $X$.

Let $AE = 3x$, $EF = x$ and $FC = 4x$.

Let $BH = 3y$, $HG = 5y$ and $GC = 4y$.

Let $AD = 2z$, $DI = z$ and $IB = 3z$

$AC = 8x$, $BC = 12y$ and $AB = 6z$.

The area of triangle $ABC$

$= \frac{1}{2}(AC)\ (BC) \sin\angle C = \frac{1}{2}\ (AB)\ (BC) \sin\angle B$

$= \frac{1}{2}\ (AB)\ (AC) \sin\angle A$

$\therefore$ The area of triangle $ABC$

$= 48xy \sin\angle C = 36yz \sin\angle B = 24xz \sin\angle A = X$

The area of the hexagon = the area of $ABC$ – the sum of areas of triangles in triangle $ABC$.

The area of triangle $ADE = \frac{1}{2}\ (AD)\ (AE) \sin\angle A = \frac{1}{8}X$

The area of triangle $BIH = \frac{1}{2}\ (BI)\ (BH) \sin\angle B = \frac{1}{8}X$

The area of triangle $FCG = \frac{1}{2}\ (FC)\ (GC) \sin\angle C = \frac{1}{6}X$

The area of hexagon $= X - \left(\frac{1}{8}X + \frac{1}{8}X + \frac{1}{6}X\right) = \frac{7}{12}X$

The ratio of areas = 12 : 7

**13.** Let there be $a + 2$ tiles along the breadth and $b + 2$ along the length.

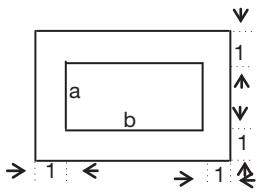
Number of white tiles = $ab$

Number of black tiles = $2a + 2b + 4$

Given ab = $4a + 4b + 8$

$\Rightarrow ab - 4a - 4b + 16 = 24$

$\Rightarrow (a - 4)\ (b - 4)$




= 1 (24)

= 2 (12)

= 3 (8)

= 4 (6)

$\therefore$ (a + 2, $b + 2$) = (7, 30), (8, 18), (9, 14) or (10, 12)

The number of tiles along an edge can be 7, 12 or 30.

**14.** The data is tabulated below.

| | **Width (cm)** | **Height (cm)** | **Length (cm)** |
|---|---|---|---|
| Carton | 51 | 52 | 35 |
| Cake | 5 | 3 | 4 |

We see that the width, height and length of the carton are multiples of the height, length, and width of the cake respectively. Therefore, we can pack the cakes in the cartons with zero wastage of space. The minimum number of cartons would correspond to this packing. The number of cakes that can be placed this way is

$\left(\frac{51}{3}\right)\left(\frac{52}{4}\right)\left(\frac{35}{5}\right)$

= (17) (13) (7) = 1547.

To pack 24000 cakes, we need at least $\frac{24000}{1547}$

= 15.51 or 16 cartons.

Apoorva is actually left with 2240 cakes unpacked after she uses 16 cartons, i.e., she has packed 24,000 – 2240 or 21,760 cakes in 16 cartons or 1360 cakes per carton.

For the remaining 2240 cakes she needs $\frac{2240}{1360}$ = 1.65 or 2 more cartons.

**15.** As $BD = \frac{2}{3} BC$, Area of $\Delta ABD = \frac{2}{3}$ (Area of $\Delta ABC$)

As $FD = \frac{1}{3} AD$, Area of $\Delta BFD = \frac{1}{3}$ Area of $\Delta ABD = \frac{2}{9}$ (Area of $\Delta ABC$)

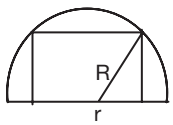
$CG = GH = HI = ID = \frac{1}{4} CD = \left(\frac{1}{4}\right)\left(\frac{1}{3}\right) BC$

$= \left(\frac{1}{4}\right)\left(\frac{1}{3}\right)\left(\frac{3}{2}\right) BD = \frac{1}{8} BD$

As $ID = \frac{1}{8} BD$, Area of $\Delta FID = \frac{1}{8}$ Area of $\Delta BFD = \left(\frac{1}{8}\right)\left(\frac{2}{9}\right)$ (Area of $\Delta ABC$)

$= \left(\frac{1}{36}\right)$ (Area of $\Delta ABC$) $= \left(\frac{1}{36}\right)$ (162) = 4.5

**16.**




Let the radius of the sphere and radius of the cylinder be $R$ and $r$ respectively. The height would be

$h = \sqrt{R^2 - r^2}$

volume $(V) = \pi r^2 h = \pi r^2 \sqrt{R^2 - r^2}$

or

$\frac{2V^2}{\pi^2} = 2r^4 (R^2 - r^2) = 2r^2 r^2 (R^2 - r^2)$

$= r^2 r^2 (2R^2 - 2r^2)$

We can consider the R.H.S. to be the product of 3 factors whose sum is $2R^2$ (a constant)

$\therefore$ The product is maximum, when the factors are all equal ie $r^2 = 2R^2 - 2r^2$

$\Rightarrow r^2 = (2/3) R^2$

or, $r = \sqrt{\frac{2}{3}} R = \sqrt{\frac{2}{3}} \sqrt{\frac{3}{2}} 10 = 10$

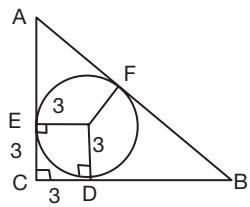
**17.** As $AC$ and $CE$ are the diameters, the volume of the solid generated is the total volume of 2 spheres.

Volume of the solid generated

$= \frac{4}{3}\pi\left(5^3\right) + \frac{4}{3}\pi\left(\frac{5}{2}\right)^3$

$= \frac{4}{3}\pi\ (5^3)\ \left(1 + \frac{1}{8}\right) = \frac{375\pi}{2}$

**18.**




By property of tangents, $AE = AF$ and $BD = BF$.

Also $AF + BF = 2R$ where $R$ is the circumradius.

$\therefore AE + BD = 2R$.

$\therefore$ Perimeter = $2(3) + 2(2R) = 6 + 4R = 56$ cm

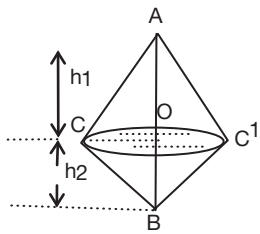
$2R = 25$ cm, i.e., c = 25cm

$\therefore a^2 + b^2 = 25^2$ (1)

and $a + b = 56 - 25 = 31$ (2)

Solving (1) and (2), we get the sides of the triangle as 7, 24 and 25.

By rotating the triangle about the hypotenuse, i.e., about $AB$, we get two cones.




The volume of the figure so formed = $\frac{1}{3}\pi r^2 (h_1 + h_2)$

Now $OC = OC^1 = r$

$(AB)\ r = AC\ (BC)$ = twice the area of the triangle $ABC$

$\Rightarrow (24)\ (7) = (25)\ (r) \Rightarrow r = \dfrac{168}{25}$ cm

Thus the volume of the solid generated

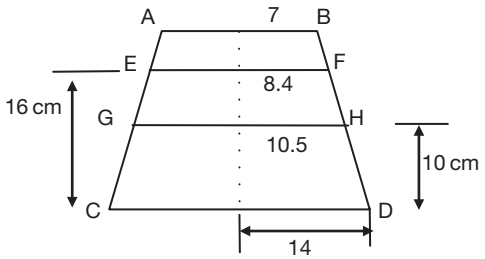
$= \dfrac{1}{3}\ \pi \left(\dfrac{168}{25}\right)^2 (25)\text{cm}^3 = \dfrac{1}{3}\ \left(\dfrac{22}{7}\right)\ \left(\dfrac{168}{25}\right)^2\ (25)\ \text{cm}^3$

$= 1182.72\ \text{cm}^3$.

**19.** The volume of water in the bucket when the height is half would be

$\dfrac{1}{3}\pi \times \dfrac{h}{2} \times (R^2 + Rr + r^2)$

$R = 14$ cm; $r = 7$ cm, $h = 20$ cm




Initial height = 20/2 = 10 cm

Radius at this height = 10.5 cm

Base radius = 14 cm. When height = 80%, i.e., 16 cm,

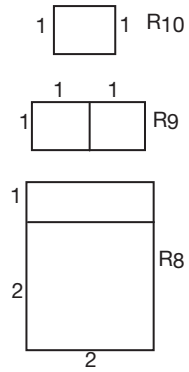
then radius = $14 - 0.8\ x\ (14 - 7) = 8.4$ cm

∴Required volume is the volume of the frustum of dimensions $R = 10.5$ cm, $r = 8.4$ cm and $h = 16 - 10 = 6$ cm

$\therefore$ Number of pebbles required $= \dfrac{\frac{1}{3}\pi h\left(R^2 + Rr + r^2\right)}{\frac{4}{3}\pi(1)^3}$

= 404 pebbles.

**20.** The breadth and length of the 11 rectangles are tabulated below. It is convenient to fill up the table (or to read it) from the bottom to the top. The rectangle $R_9$ is obtained from $R_{10}$ (which is a square) just by doubling it. After that, $R_i$ is obtained from $R_{i+1}$ by joining the square upon the length. (Because if the square is joined on the breadth then $S_i$ and $S_{i+1}$ will become equal in area). Therefore, the breadth of $R_i$ is the length of $R_{i+1}$ and the length of $R_i$ is the sum of the length and breadth of $R_{i+1}$.

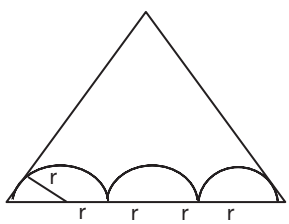



| Rectangle | Breadth | Length |
|---|---|---|
| $R_0$ | 89 | 144 |
| $R_1$ | 55 | 89 |
| $R_2$ | 34 | 55 |
| $R_3$ | 21 | 34 |
| $R_4$ | 13 | 21 |
| $R_5$ | 8 | 13 |
| $R_6$ | 5 | 8 |
| $R_7$ | 3 | 5 |
| $R_8$ | 2 | 3 |
| $R_9$ | 1 | 2 |
| $R_{10}$ | 1 | 1 |

∴The length to breadth ratio for $R_0$ is 144 : 89.

**21.** If $k$ is a rational number, i.e., $k = p/q$ where $p$, $q$ are natural numbers the process would terminate. The side of the square that is finally obtained is the *GCD* of $p$ and $q$. If $k$ is an irrational number, the process would never terminate because, if the process terminates at $R_x$, the sides of $R_0$ can be expressed as some rational combination of the sides of $R_x$, i.e., the length to breadth ratio of $R_0$ would be rational. Therefore, only Statements I and III are true.

**22.**




$\dfrac{r}{\sqrt{3}}$

As can be seen from the figure, if the radius of the ball is $r$, each side of the hexagonal box a = $\left(4+\frac{2}{\sqrt{3}}\right)r$

Height of the box = 4 cm

$\therefore$ Side of the base of the box = $\left(8+\frac{4}{\sqrt{3}}\right)$ cm

Area of the base = $6\left(\frac{\sqrt{3}}{4}\right)\left(8+\frac{4}{\sqrt{3}}\right)^2$ cm$^2$

= $(96 + 104\ \sqrt{3}\ )$ cm$^2$

Volume of the box = $4\ (96 + 104\ \sqrt{3}\ )$ cm$^3$
= $384 + 416\ \sqrt{3}$

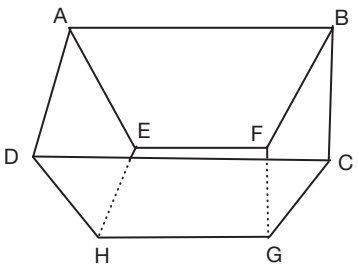
Volume occupied by 19 balls = $19\left(\frac{4}{3}\right)\pi\,(2^3)$ cm$^3$

= $\frac{608}{3}\pi$ cm$^3$

Volume of the box unoccupied by the balls

= $\left(384+416\sqrt{3}-\frac{608}{3}\pi\right)$ cm$^3$

**23.**




The trough is shown in the figure above.

$ABCD$ is the open top

$AD = 4$ m, $AB = 6$ m

$EFGH$ is the base. $EH = 2$ m. $EF = 3$ m (area of base = 6 m$^2$)

If water fills up to half the height of the frustum, the water surface would be a rectangle of dimensions 3 m × 4.5 m (and area (13.5 m$^2$))

Volume of water = $\frac{h}{3}\ (A_1 + \sqrt{A_1\ A_2}\ + A_2)$,

where $h = \frac{1}{2}$ m, $A_1 = 6$ m$^2$, $A_2 = 13.5$ m$^2$

Volume of water = $\frac{1}{6}\ \left(6+\sqrt{6(13.5)}+13.5\right)$ m$^3$

= $\frac{28.5}{6}$ m$^3$ (1)

Rate at which water flows = $Av$

where $A$ = area of cross section of the pipe and

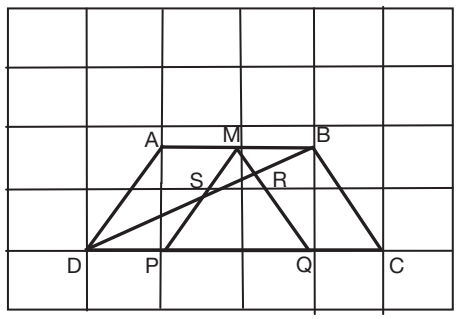
$v$ = speed at which water flows

$\therefore$ Rate = $\pi\left(\frac{25}{2}\right)^2\left(\frac{3000}{60}\right)\ \frac{\text{cm}^3}{\text{s}}$ (2)

$\therefore\ t = \frac{(1)}{(2)} = 193.6s = 3.2$ min.

$\therefore\ 3 < t < 3.5$.

**24.** The trapezium is shown in the figure below




From the data $DP : PQ : QC = 2 : 4 : 2$

Ar $DAP$ : Ar $PABQ$ : Ar $QBC = 1 : 4 : 1$

$\therefore$ Ar of $PABQ = \frac{4}{6}$ (Ar of trapezium) = $\frac{2}{3}\ (60) = 40$

Ar $\Delta PMQ = \frac{1}{2}\ (40) = 20$

$\Delta MSB \sim \Delta PSD$. The ratio of the sides is 1 : 1

$\therefore\ MS = SP = \frac{1}{2}\ MP$ (1)

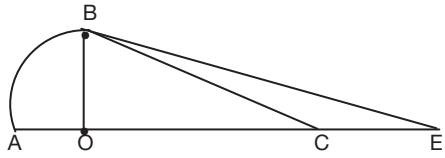
$\Delta MRB \sim \Delta QRD$. The ratio of the sides is 1 : 3

$\therefore\ MR = \frac{1}{4}\ MQ$ (2)

From (1), (2), Ar $\Delta MSR = \frac{1}{2}\ \left(\frac{1}{4}\right)$ Ar $\Delta MPQ = \frac{1}{8}\ (20)$
= 2.5

$\therefore$ Ar of $PQRS = 20 - 2.5 = 17.5$.

**25.**




Difficulty Level-4: Masters

Let the radius of the quadrant be $r \Rightarrow OA = OB = r$

$OC = 3(OA) = 3r \Rightarrow AC = 4r$ and $AE = (1.05)4r = 4.2r$.

Area of $ABCOA$ = Area of the quadrant $(AOB)$ + Area of the triangle $(BOC)$

$$= \frac{\pi r^2}{4} + \frac{1}{2}(OB)\ (OC)$$

$$= \frac{\pi r^2}{4} + \frac{1}{2}(r)\ (3r)$$

$$= 0.785\ r^2 + 1.5r^2 = 2.285\ r^2.$$

With 70% of the amount 70% of the above area can be tiled i.e., $0.7 \times 2.285\ r^2 = 1.5995r^2$.

But the actual area = Area of $(AOB)$ + Area of $(BOE)$

$$= 0.785r^2 + \frac{1}{2}r(OE)\ [OE = AE - AO = 4.2r - r = 3.2r]$$

$$= 0.785r^2 + 1.6r^2 = 2.385r^2$$

$\therefore$ The percentage of actual area tiled

$$= \frac{1.5995r^2}{2.385r^2} \times 100 \sim 67\%.$$

**26.** Let the radius of the circle be $r$.

The central angles subtended by each of the arcs $P_1P_2$, $P_2P_3$,..., $P_{23}P_{24}$, $P_{24}P_1$ are the same.

$\therefore$ Each of these is $\left(\frac{360^\circ}{24} \text{ i.e } 15^\circ\right)$

Central angle of the sector formed by the line segments $OP_1$, $OP_9$, and the arc $P_1P_9$ is $8(15^\circ)$ or $360^\circ - 8(15^\circ)$, i.e., $120^\circ$ or $240^\circ$

$$\frac{120}{360}\pi r^2 = 462 \text{ or } \frac{240}{360}\pi r^2 = 462$$

$\pi r^2 = 1386$ or $693$.

The sector formed by the line segment $OP_3$ the line segment $OP_{15}$ and the arc $P_3P_{15}$ is a semicircle.

$\therefore$ Its area $= \frac{\pi r^2}{2} = 693$ or $346.5$.

**27.** Area of $PQRS$ = Area of $\Delta PRS$ + Area of $\Delta PQR$

$PR = 24$ and $QS = 34$. Let $T$ be the point of intersection of $PR$ and $QS$.

Area of $\Delta PRS$ = Area of $\Delta PTS$ + Area of $\Delta STR$

$$= \frac{1}{2}(ST)\ (PT) \sin \angle PTS + \frac{1}{2}(ST)\ (RT) \sin \angle STR$$

$$= \frac{1}{2}(ST)\ (PT + RT) \sin \angle PTS\ (\sin\theta = \sin 180 - \theta)$$

$$= \frac{1}{2}(ST)\ (24) \sin \angle PTS$$

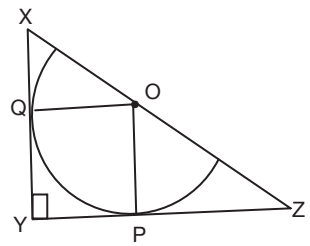
Area of $\Delta PQR = \frac{1}{2}(TQ)\ (24) \sin \angle PTQ$

Area of $PQRS = \frac{1}{2}(SQ)\ (24) \sin \angle PTS$

$$= \frac{1}{2}(34)\ (24) \sin \angle PTS$$

Area of $PQRS$ is maximum when $\sin \angle PTS$ is maximum i.e. when $\angle PTS$ is $90^\circ$. The area of the biggest such quadrilateral is 408.

**28.**




The triangle is right-angled at $Y$ ($\because 8^2 + 15^2 = 17^2$)

Let $O$ be the centre of the semicircle.

$\therefore \angle QOP = 90^\circ$. $QOPY$ is a rectangle. Also $OQ = OP$ ( = Radius). $\therefore$ $QOPY$ is a square whose side equals the radius.

Let the radius of the circle be $r$.

Triangles $XOQ$ and $XZY$ are similar.

$\therefore$ Let $XQ = 8K$, $QO = 15K$, $XO = 17K$

$XQ + QY = 8 \Rightarrow 8K + 15K = 8$ ($\because QY = QO$)

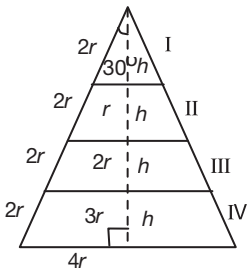
$K = \frac{8}{23}$. $\therefore$ Radius $= \frac{120}{23}$

Required area = Area of the square $QOPY$ – Area of the sector $OQP$.

$$= \left(\frac{120}{23}\right)^2 - \frac{90}{360}\pi\left(\frac{120}{23}\right)^2$$

$$= \left(\frac{120}{23}\right)^2 \left(1 - \frac{\pi}{4}\right)$$

**29.**




Let the radius of the original cone be $R = 4r$. Then, its slant height is $8r$. Let the four pieces be I, II, III and IV.

The *TSA* of the original cone = $\pi R L + \pi R^2$

$= \pi (4r)\ (8r) + \pi (4r)^2$

$= 48\pi r^2$

The sum of the *TSA*s of the 4 pieces

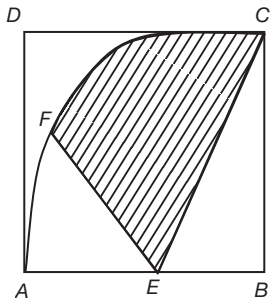
= *TSA* of the original cone + 2 [Base area of cone I + Base area of the cone formed by putting I and II together + Base area of the cone formed by putting the I, II and III together]

[It may be noted that each of the three cuts made generates two equal circular faces. So, the total of *TSA* of the four pieces is equal to the *TSA* of the original cone + the total area of the new faces generated]

$= 48\pi r^2 + 2\{\pi r^2 + 4\pi r^2 + 9\pi r^2\} = 76\pi r^2$

$\therefore$ The required ratio is 12 : 19

**30.**




Let $AB = a$ and $BE = h$

Area of the shaded region = Ar (sector $FBC$ + $\Delta FBE$ – $\Delta CBE$)

$$= \frac{1}{6}\pi a^2 + \frac{1}{2}(a)(h)\sin\angle FBE - \frac{1}{2}ah$$

So, $\frac{\pi a^2}{6} - \frac{1}{4}ah = \frac{\pi}{8}a^2$ ($\because \angle FBE = 30°$) $\Rightarrow h = \frac{\pi a}{6}$

But $a = 18$ cm $\Rightarrow h = 3\pi$ cm

So, $AE = AB - BE = (18 - 3\pi)$ cm.

## EXERCISE-5

***Solutions for questions 1 to 20:***

**1.** From Statement I, triangle $ABC$ is clearly right-angled. As we know the median, we can find all the sides as each of the sides has an integer value. $\therefore$ Statement I alone is sufficient.

From Statement II, as we know only the perimeter we cannot find the area of the triangle uniquely.

**2.** Let each of the equal sides be a cm and the unequal side be $b$ cm.

Then, the side of the equilateral triangle constructed is a cm. Now we want the value of $b$.

From Statement I alone,

$3a = 45 \Rightarrow a = 15$ (1)

But we want the value of $b$. Hence, I alone is not sufficient.

From Statement II alone, $2a + b = 40$ (2)

Since, we do not know the value of $a$, $b$ cannot be found. Hence, II alone is not sufficient.

From I and II together, we can find the value of $b$ as we know the value of $a$. ((2) – 2(1)).

**3.** Let $l$ be the length of the longer sides and $b$ be the length of the shorter side.

From Statement I alone, $2(l + b) = 3l \Rightarrow b = \frac{1}{2}$

Since, the lengths of the sides and the angle between any two adjacent sides is not known, area cannot be found.

From Statement II alone, given $l = 8$ cm

Since the shorter side and the angle between any two adjacent sides is not known, area cannot be found.

From I and II together, $l = 8$ cm and $b = \frac{1}{2} = 4$ cm

But the angle between the adjacent sides is still not known. Hence, I and II together are also not sufficient.

**4.** Let the side of the hexagon be a and length and breadth of the rectangle be $l$ and $b$.

$$6\left(\sqrt{\frac{3}{4}a^2}\right) = lb$$

let $l = a$, $b = 6\left(\sqrt{\frac{3}{4}\frac{a^2}{a}}\right) = \frac{3\sqrt{3}}{2}a$

As $b > l$ this is not possible. Therefore $b = a$.

$$l = \frac{6\left(\frac{\sqrt{3}}{4}a^2\right)}{a} = \frac{3\sqrt{3}}{2}a$$

Hence, the ratio of perimeters = $6a : 2(l + b)$ can be found.

Using Statement II, as the ratio of $l$, $b$ and a is known, let us say $l = 2k$, $b = k$ and $a = 3k$

The required ratio of perimeters = $6a : 2(l + b)$ can be found.

Hence, either statement can answer the question.

**5.** Let the length, breadth and height of the cuboid be $l$, $b$ and $h$.

Using Statement I, the surface areas of the three adjacent faces are $lb$, $lh$ and $bh$ respectively.

Their product = $(lbh)^2$ = (volume of cuboid)$^2$ = (8) (16) (8)

Hence, volume of the cuboid can be found.

From II alone, given that $l^2 + b^2 + h^2 = 36$ (1)

and $lh + bh + lb = 32$ (2)

$(l + b + h)^2 = (l^2 + b^2 + h^2) + 2(lh + bh + lb) = 36 + 64 = 100$

$\therefore l + b + h = 10$ (3)

(3) is dependant on (1) and (2).

So, we have only two independent equations with three unknowns. Hence, we have multiple solutions.

Hence, II alone is not sufficient.

**6.** $2\pi\ r = 44 \Rightarrow r = 7$

Total surface area = $2\pi\ r(r + h)$

Statement I: $r + h = 21$

Substituting $r$ and $r + h$ values in $2\pi\ r(r + h)$ we can find the total surface area.

Statement I is sufficient.

Statement II: Total surface area = curved surface area + $2\pi r^2$

As $r = 7$

Statement II is sufficient.

**7.** Statement I: We do not know the number of sides of the polygon, so, we can't find the area of the polygon. Not sufficient.

Statement II: Each exterior angle is $60^o$, so, the number of sides is $\frac{360}{60}$, viz 6.
But we do not know the side of the polygon.
Not sufficient.

Statements I and II. The length of longest diagonal in the hexagon is twice the length of side, so side of the hexagon is 10 cm. Area of the hexagon is $6\left(\frac{\sqrt{3}}{4}a^2\right)$.

Area of the hexagon = $6\left(\frac{\sqrt{3}}{4}\right)\left(10^2\right)$ cm$^2$.

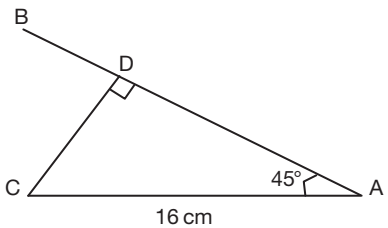
Sufficient.

**8.** Statement I alone is not sufficient, as it gives no numerical data.

Statement II alone is not sufficient, as we don't know the information about the other side or any of the angles of the triangle.

From I and II, we have

$\angle BAC = 45°$, $AC = 16$ cm, $BC = 12$ cm.




Let $D$ be the foot of the perpendicular from $C$ to $AB$.

$AB = AD + DB$

$= CD + \sqrt{BC^2 - CD^2} = 8\sqrt{2} + \sqrt{12^2 - \left(8\sqrt{2}\right)^2}$

$= 8\sqrt{2} + 4$

Area of $\Delta ABC = \frac{1}{2}(AB)(CD)$

$CD = AC \sin 45° = \frac{16}{\sqrt{2}} = 8\sqrt{2}\text{cm} \cong 11.3$ cm. Sufficient.

**9.** Let the length, breadth, and the height of the cuboid be $l$ cm, $b$ cm and $h$ cm respectively.

We need to find $2(lb + lh + bh)$.

Statement I: $l^2 + b^2 + h^2 = 36^2$

From this we can't find the total surface area as we have one equation with three unknowns. Not sufficient.

Statement II: $l + b + h = 38$

From this we can't find the total surface area as we have one equation with three unknowns. Not sufficient.

Statements I and II: $(l + b + h)^2 = 38^2$

$\Rightarrow l^2 + b^2 + h^2 + 2(lb + lh + bh) = 38^2 \Rightarrow 2(lb + lh + bh) = 38^2 - 36^2$. Sufficient.

**10.**




Statement I: $\angle BAC = 60°$.

Hence, the height of the cone $= \frac{3.5}{\tan 60°}$ cm

As we know the height and the radius of the base, we can find the volume of the cone. Sufficient.

Statement II: The ratio of the $TSA$ (in $\text{cm}^2$) to the volume (in $\text{cm}^3$) is 1.5 : 1

Let $l$ cm and $h$ cm be the slant height and the height of the cone respectively.

$\therefore l^2 - h^2 = (3.5)^2$ (1)

Given, $\frac{\pi r(r+\ell)}{\frac{1}{3}\pi r^2 h} = \frac{3}{2} =$

$2(r + l) = rh \Rightarrow 7 + 2l = 7/2\ h$ (2)

Solving (1) and (2), we get $l$ and $h$.

Sufficient.

**11.** Neither of the statements alone is sufficient as each has only partial information.

Combining Statements I and II,

$(1/2)\, d_1 d_2$ = area of rhombus. $1/2 \times 6 \times d_2 = 24 \Rightarrow d_2 = 8$

we have the side of the rhombus $= \sqrt{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2}$

So, $s = \sqrt{3^2 + 4^2} = 5$ ; $\therefore$ Perimeter $= 4s = 20$.

**12.** From Statement I, $6a^2 = 96$, so, $a$ can be found and the diagonal $a\sqrt{3}$ can be found.

From Statement II, $a^3 = 64$, so, $a$ can be found and the diagonal $a\sqrt{3}$ can be found.

**13.** From Statement I, $4\pi r^2 = 120$.

$\Rightarrow r = \sqrt{(120/4\pi)}$ ; $v = \frac{4}{3}\pi r^3$.

Hence, Statement I alone is sufficient.

Statement II alone cannot be sufficient since no dimensions of the parallelopiped are known.

**14.** From Statement I, $h = r/2$

From Statement II, $r = \sqrt{4} = 2$

Combining Statements I and II, $r = 2$ and $h = 1$.

Volume $= 1/3\pi r^2 h$.

So, we can find the volume of the cone.

**15.** Volume of a cylinder is $\pi r^2 h$. So, either of the statements alone is not sufficient as the relation between radii and heights is given in different statements. Combining Statements I and II, we get

$\frac{h_A}{h_B} = \frac{1}{2}; \frac{r_A}{r_B} = \frac{1}{4}$

$v = \pi r^2 h$

So,

$\frac{v_A}{v_B} = \frac{r^2_A h_A}{r^2_B h_B} = \frac{1}{32}$

So, both are required to answer the question.

**16.** Area of a rectangle $= l \times b$, where $l$ = length, $b$ = breadth.

Given that $lb = 77$

From Statement I, $b = 7$ cm, so, $l = 77/7 = 11$ cm

Perimeter of the rectangle is $2(l + b)$, so, we can find the perimeter of the rectangle as $l$ and $b$ are known.

From Statement II, $l = 11$ cm, so, $b = 77/11 = 7$ cm

So, Statement II alone is also sufficient.

**17.** Statement I alone is not sufficient, as the measure of the other side is not given.

Statement II alone is also not sufficient, as the measure of the sides is not given.

Combining both the statements, as one side is the square of the other side and the measure of the longer side is 16 cm, the measure of the shorter side is $\sqrt{16} = 4$ cm As the measure of both the sides is known, the area of the rectangle can be found.

**18.** Statement I alone is not sufficient, as the second side is not given.

Statement II alone gives the relationship between the two sides of the rectangle and no measurement of sides is given. Hence, it alone is also not sufficient.

Using both the statements also, we cannot find the area of the rectangle, as it is not known whether the given side refers to length or breadth.

**19.** Statement I alone is not sufficient as it gives only the relationship between the two sides.

Statement II alone is not sufficient as breadth is not known.

Using both the statements, as length is $p$ cm, the breadth will be $\sqrt{p}$ (if $p > 1$) or $p^2$ (if $p < 1$).

$\therefore$ The possible area of the rectangle is $p^3$ or $p\sqrt{p}$. As unique area of the rectangle cannot be found choice (D).

**20.** The side of the maximum square that can be cut of a rectangle is equal to the shorter side (width) of the rectangle.

From Statement I, since the area of maximum square that can be cut of the rectangle is 64 $cm^2$, the side of the square must be 8 cm. Hence, the shorter side of the rectangle is 8 cm.

$\therefore$ Statement I is sufficient.

If $l$ and $b$ are the length and width of the rectangle from Statement II, we can say that $(l - b)\ l = 32$, from which '$b$', the shorter side, cannot be calculated.

Hence, Statement II is not sufficient.

# 3 Coordinate Geometry

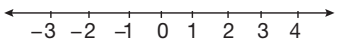
CHAPTER

## Learning Objectives

*In this chapter, you will*:

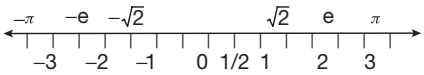
- Understand all about rectangular coordinate system
- Learn to find the distance between any two points in the coordinate plane
- Learn to calculate the area of a triangle
- Learn the Section formula to find the coordinates of any point which divides a line segment in a given ratio
- Learn about the straight line and it elements such as:
  - Slope of a line
  - Different ways of representing a line
  - Angle between two lines
  - Perpendicular distance of a point from a line
  - Perpendicular distance between two parallel lines
- Understand the equation of a circle
- Learn about Locus
- Learn about the pair of straight lines
- Learn about change of Axes such as
  - Translation of axes
  - Rotation of axes

NUMBER LINE is a geometrical representation of real numbers on a horizontal line. We begin by selecting an arbitrary point O, called origin, and associating it with the real number 0. By convention, we take all positive real numbers to the right of O and the negative real numbers to the left of O.



Integer line

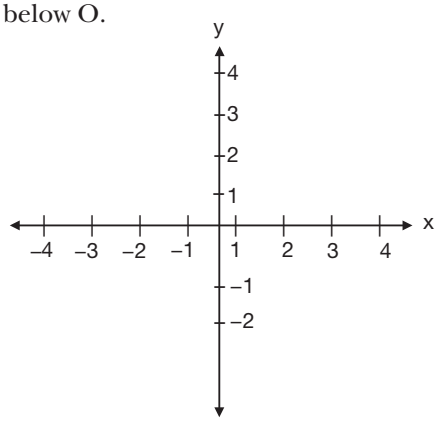
Primarily, we plot the integers. On subdividing of these segments, it is possible to locate rational and irrational numbers.




## ❒ RECTANGULAR COORDINATES

Consider two lines, one vertical and the other horizontal. Let the horizontal line be named as '*x*-axis' and the vertical line the '*y*-axis'.

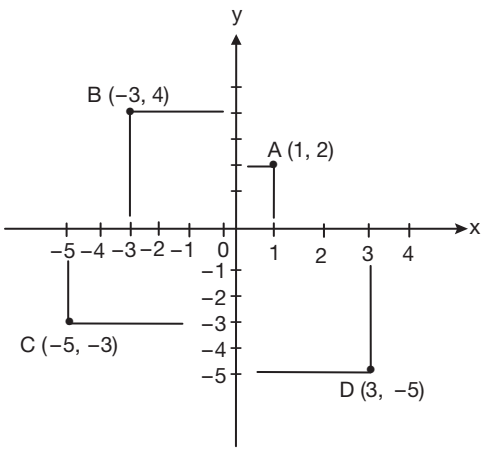
This time, we take the point of intersection of the axes as the origin (O). Once again on the *x*-axis, we follow the convention of associating the positive real numbers to the right of O and the negative real numbers to the left of O. On the *y*-axis, the positive real numbers are associated above O and the negative real numbers below O.



## ❒ ORDERED PAIR

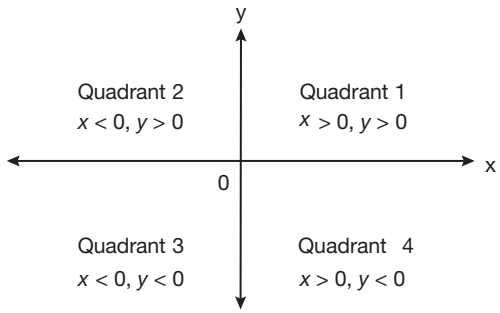
Any point $P$ in the plane formed by the $x$-axis and $y$-axis can be located by using an ordered pair of real numbers.

Let $x$ denote the signed distance of $P$ from the $y$-axis (by signed distance we mean, if $P$ is to the right of the $y$-axis, then $x > 0$ and if $P$ is to the left of the $y$-axis, then $x < 0$); and let $y$ denote the signed distance of $P$ from the $x$-axis. The ordered pair $(x, y)$ denotes the coordinates of $P$. This gives us the information to locate the point $P$. The points $A$, $B$, $C$, and $D$ located in the figure can be observed by the reader.




## ❒ X-COORDINATE AND Y-COORDINATE

If $(x, y)$ are the coordinates of a point $P$, then $x$ is called $x$-coordinate of $P$ and $y$ is called $y$-coordinate of $P$. For instance, the coordinates of the origin are $(0, 0)$. The $x$-coordinate of any point on the $y$-axis is 0, the $y$-coordinate of any point on the $x$-axis is 0. The coordinate system described here is also termed as the cartesian coordinate system. The plane is divided into 4 sections termed as quadrants.




**Examples:**

The point (3, 5) lies in the 1st quadrant.

The point (–4, 6) lies in the 2nd quadrant.

The point (–5, –6) lies in the 3rd quadrant.

The point (7, –2) lies in the 4th quadrant.

Some basic rules and formulae, which have to be remembered, are given below. Each formula is followed by one or more examples, which clearly explain its application.

**1. Distance formula**

(i) The distance between the two points $(x_1, y_1)$ and $(x_2, y_2)$ is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

(ii) The distance between the origin $(0, 0)$ and the point $(x, y)$ is $\sqrt{x^2 + y^2}$.

**2. Area of triangle**

(i) The area of the triangle formed by the vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is equal to value of the determinant

$$\frac{1}{2}\begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix} ; \text{ i.e.,}$$

$$\tfrac{1}{2}[(x_1 - x_2)(y_2 - y_3) - (y_1 - y_2)(x_2 - x_3)].$$

Alternately, the area can be found using

$$\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$$

$$= \frac{1}{2}\left|(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)\right|$$

(ii) The area of the triangle formed by the vertices $(0, 0)$, $(x_1, y_1)$ and $(x_2, y_2)$ is $1/2\,|x_1 y_2 - x_2 y_1|$.

**NOTE**

The area of a quadrilateral formed by the points $(x_1, y_1)$, $(x_2, y_2)$, $(x_3, y_3)$ and $(x_4, y_4)$ describing the consecutive vertices is given by $\frac{1}{2}\begin{vmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - x_4 & y_2 - y_4 \end{vmatrix}$.

Alternately, the area can be found using

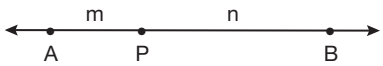
$$\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix} ; \text{ i.e.,}$$

$$\frac{1}{2}\left|\begin{matrix}(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) \\ + (x_4y_1 - x_1y_4)\end{matrix}\right|$$

**3. Section formulae**

**(i) Internal Division:** If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points given, then the coordinates of a point $P$, which divides the line joining $AB$ internally in the ratio $m : n$ is given by

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

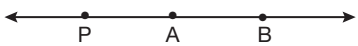



**NOTE**

The point $P$ is between $A$ and $B$ for internal division.

**(ii) External Division:** If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points, then the coordinates of a point $P$, which divides the line segment joining $AB$ in the ratio $m : n$ externally are given by

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$

Fig 1. 

(if $m < n$)

or

Fig 2. 

(if $m > n$)

**NOTE**

The point $P$ is beyond $A$ and $B$ for external division. It can be either beyond $A$ (Fig. 1) or beyond $B$ (Fig. 2).

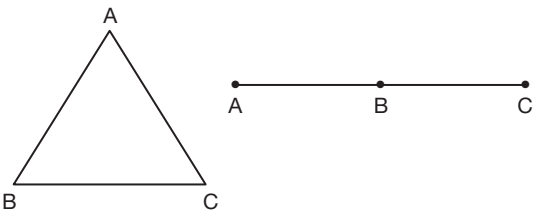
**Centroid:** If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle, then the centroid $G$ of the triangle $ABC$ is given by

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

**NOTE**

The centroid of a triangle is the point of concurrence of the medians of a triangle.

**Collinearity:** Given three distinct points in a plane $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, there are two possibilities. They may form a triangle or a straight line.



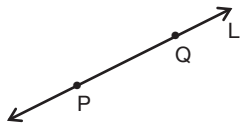

In case the three points A, B and C form a straight line, we say that they are 'collinear'.

Any of the following conditions are enough to show collinearity of the given three points.

1. $AB + BC = CA$ or $AC + CB = AB$ or $AB + AC = BC$.
2. The area of the triangle formed by $A$, $B$ and $C$ equals zero.
3. Slope of $AB$ = Slope of $BC$.

## ❒ THE STRAIGHT LINE

We now deal with a case where a specified relationship (equation) between $x$ and $y$ is given for various points $P(x, y)$. One such relationship is the linear equation. Its graph is called straight line. We know that there is one and only one line containing two distinct points $P$ and $Q$, from plane geometry.




If $P$ and $Q$ are each represented by ordered pairs of real numbers, the following definition can be given:

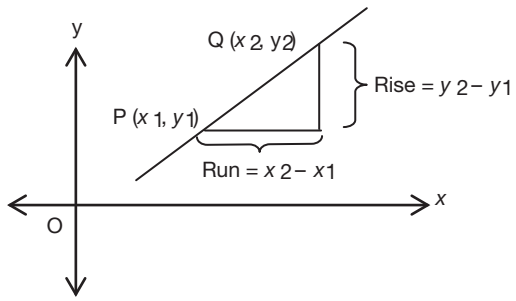
**Slope of a Line:** The slope of a line is a number that describes both the direction and steepness of a line.

Let $P$ and $Q$ be two distinct points with coordinates $(x_1, y_1)$ and $(x_2, y_2)$ respectively. The slope $m$ of the line $L$ containing $P$ and $Q$ is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ if } x_1 \neq x_2.$$

If $x_1 = x_2$, the slope $m$ of line $L$ is undefined (since this results in division by 0) and L in which case, is a vertical line.

For a non-vertical line, slope $= \dfrac{\text{Change in } y}{\text{Change in } x}$ or 'rise over run'




**NOTE**

$\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_1 - y_2}{x_1 - x_2}$

So, the slope is same whether changes are computed from $P$ to $Q$ or $Q$ to $P$.

Alternately, the slope of a line is the tangent value of the angle ($\theta$) made by the line with the positive direction of the $x$-axis in the anti-clockwise direction.

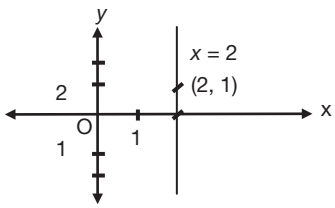
$m = \tan\theta$.

## ❒ EQUATIONS OF LINES

**Vertical Lines:** The equation of a vertical line passing through a point $(a, 0)$ is given by the equation $x = a$ where $a$ is a given real number.

**Example:**

**Non-vertical Lines:** Let $L$ be a non-vertical line with slope $m$ containing $(x_1, y_1)$. For any other point $(x, y)$ on $L$, we have




$m = \dfrac{y - y_1}{x - x_1}$ or $y - y_1 = m(x - x_1)$

**Point-slope Form:** The equation of a non-vertical line of slope $m$ and passing through the point $(x_1, y_1)$ is $y - y_1 = m(x - x_1)$

**Two-point Form:** The equation of a non-vertical line passing through $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

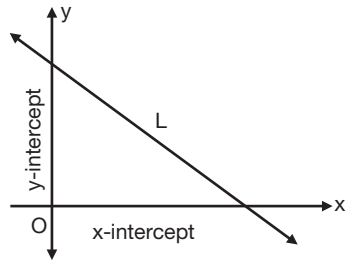
$y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

**NOTE**

(i) In algebra $ax + by + c = 0$ is termed as a first-degree equation in $x$ and $y$.
(ii) If $a = 0$ and $b \neq 0$, then $L$ will be a horizontal line.
(iii) If $b = 0$ and $a \neq 0$, then $L$ will be a vertical line.
(iv) If $c = 0$, then $L$ passes through the origin.

**Intercepts:** The portions cut off by a line on the co-ordinate axes are called intercepts.

The $x$-intercept is the portion on the $x$-axis and the $y$-intercept is the portion on the $y$-axis.




**Intercept Form:** $\dfrac{x}{a} + \dfrac{y}{b} = 1$, where

$x$-intercept is '$a$' and $y$-intercept is '$b$'.

**Slope Intercept Form:** The equation of a line with slope $m$ and $y$-intercept $b$ is $y = mx + b$

**NOTE**

(i) When the equation is written in this form, the coefficient of $x$ is the slope and the constant term gives the $y$-intercept of the line.
(ii) $y$ is explicitly written in terms of $x$. So, this form is also termed as the explicit form of the line.

**General Form:** The equation of a line L is in the general form when it is written as $ax + by + c = 0$ where $a$, $b$ and $c$ are real numbers with either $a \neq 0$ or $b \neq 0$.

**NOTE**

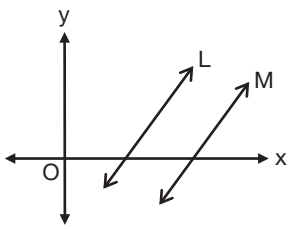
(i) When the equation is written in this form, the coefficient of $x$ is the slope and the constant term gives the $y$-intercept of the line.
(ii) $y$ is explicitly written in terms of $x$. So, this form is also termed as the explicit form of the line.

The following table summarises the various forms of equations of straight lines.

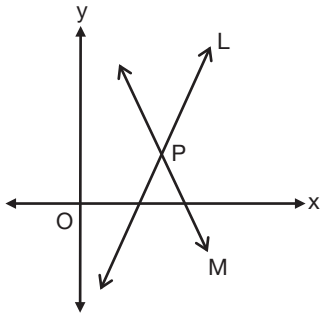
| You are given | You use | Equation |
|---|---|---|
| 1. Point $(x_1, y_1)$ and slope $m$ | Point-slope form | $y - y_1 = m(x - x_1)$ |
| 2. Two points $(x_1, y_1)$, $(x_2, y_2)$ | If $x_1 = x_2$, use vertical line equation | $x = x_1$ |
| | If $x_1 \neq x_2$, two point form | $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ |
| 3. $x$ and $y$ intercepts $a$ and $b$ | Intercept form | $x/a + y/b = 1$ |
| 4. Slope $m$, $y$-intercept $b$ | Slope-intercept form | $y = mx + b$ |

**Parallel and Intersecting Lines:** Let $L$ and $M$ be two lines. Exactly one of the following three relationships must hold for the lines $L$ and $M$.

1. All the points on $L$ are the same as the points on $M$. (Identical lines)
2. $L$ and $M$ have no points in common. (Parallel lines)
3. $L$ and $M$ have exactly one point in common. (Intersecting lines)




**Parallel lines**




**Intersecting lines**

**NOTE**

To find the coordinates of the point of intersection of the lines
$L : a_1x + b_1y + c_1 = 0$ and
$M : a_2x + b_2y + c_2 = 0$,
we solve the two equations to get the point of intersection as

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right)$$

## ❒ ANGLE BETWEEN TWO LINES

If $m_1$ and $m_2$ are the slopes of two lines, the angle '$\theta$' between them is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$$

**NOTE**

(i) Condition for parallel lines: $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ or $m_1 = m_2$

(ii) Condition for perpendicular lines:

$a_1a_2 + b_1b_2 = 0$ or $m_1m_2 = -1$.

The equation of a line through a point $P(x_1, y_1)$ and

(i) parallel to $ax + by + c = 0$ is given by $a(x - x_1) + b(y - y_1) = 0$.

(ii) perpendicular to $ax + by + c = 0$ is given by $b(x - x_1) - a(y - y_1) = 0$.

## ❒ SOME FORMULAE TO REMEMBER

**FORMULA**

1. The general form of the equation of a straight line is $ax + by + c = 0$. Here, the $y$-intercept is $-c/b$, the $x$-intercept is $-c/a$ and the slope is $-a/b$.
2. If $ax + by + c = 0$ is the equation of a line, the perpendicular distance from a point $(x_1, y_1)$ to this line is given by: $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$
3. The distance between two parallel straight lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is given by $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$
4. The equation of a circle centred at $(h, k)$ with radius $r$ units is $(x - h)^2 + (y - k)^2 = r^2$.
5. The equation of a circle centred at the origin with radius $r$ units is $x^2 + y^2 = r^2$.
6. The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents of a circle with centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$

**NOTE**

The general form of the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ with $g^2 + f^2 - c \geq 0$.

## ❒ LOCUS

Locus is a set of points that satisfy a geometrical condition. For example

1. If a point moves so that its distances from two fixed points are always equal, its locus is the perpendicular bisector of the line segment joining the two points.
2. If a point moves so that its distance from a fixed point is constant, then its locus is a circle with centre as the fixed point and the constant distance as radius. We can also say that the locus of a point equidistant from two fixed points, say $A$ and $B$ is the perpendicular bisector of the segment $AB$. Similarly, the locus of a point which is at a fixed distance (say $r$) from a fixed point, say $O$, is the circle with centre $O$ and radius $r$., i.e., the idea of motion is not a necessary part of the concept of locus. It is merely one way of expressing the idea.

## ❒ PAIR OF STRAIGHT LINES

Consider a general second degree equation of the form $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ (1)

If it can be split into two linear factors of the form $(a_1x + b_1y + c_1)(a_2x + b_2y + c_2)$,

i.e., if $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = (a_1x + b_1y + c_1)(a_2x + b_2y + c_2) = 0$

then the equation (1) represents a pair of straight lines represented by $a_1x + b_1y + c_1 = 0$ (2)

and $a_2x + b_2y + c_2 = 0$ (3)

## ❒ Change of Axes

Sometimes, to be able to express an equation in a simpler form, it may be required to change the original co-ordinate axes, either by way of shifting the origin or by changing the direction of the axes.

## ❒ Translation of Axes

In this case, the origin is shifted to a new point, keeping the direction of the axes intact. The new axes are parallel to the original axes.

Suppose the origin is shifted to $(h, k)$. If the original coordinates of a point $P$ are $(x, y)$ and $(X, Y)$ denote the coordinates of P with reference to the new axes, then we have $x = X + h$ and $y = Y + k$.

## ❒ Rotation of Axes

In this process, the origin is kept intact and the axes are rotated about the origin, through a required angle.

If $\theta$ is the angle of rotation and $(x, y)$ are the coordinates of a point $P$ with reference to the original axes and $(X, Y)$ with reference to the new axes, then the relation between them is given below:

| | |
|---|---|
| $x = X\cos\theta - Y\sin\theta$ | $y = X\sin\theta + Y\cos\theta$ |
| $X = x\cos\theta + y\sin\theta$ | $Y = -x\sin\theta + y\cos\theta$ |

These equations are called transformation equations.

**NOTE**

1. If the axes are rotated at an angle $\theta$ in the anti-clockwise direction, $\theta$ is considered positive.
2. Sometimes, we may have to translate and rotate or rotate and translate. The order is unimportant.

## SOLVED EXAMPLES

**3.01.** Find the distance between the points (3, 4) and (−2, 3).

**Sol:** Distance $= \sqrt{(-2-3)^2 + (3-4)^2} = \sqrt{25+1} = \sqrt{26}$ units.

**3.02.** What is the distance between the points (0, 0) and (24, 7)?

**Sol:** Distance $= \sqrt{24^2 + 7^2} = \sqrt{576+49} = \sqrt{625} = 25$ units.

**3.03.** Prove that the points (1, −1), (−1, 4), (4, 6) are the vertices of an isosceles right-angled triangle.

**Sol:** Let $A = (1, -1)$, $B = (-1, 4)$ and C = (4, 6).

Using the distance formula, we find $AB$, $BC$ and $CA$.

$AB = \sqrt{(-1-1)^2 + (4+1)^2} = \sqrt{4+25} = \sqrt{29}$ units

$\Rightarrow AB^2 = 29$.

$BC = \sqrt{(4+1)^2 + (6-4)^2} = \sqrt{25+4} = \sqrt{29}$ units

$\Rightarrow BC^2 = 29.$

$CA = \sqrt{(4-1)^2 + (6+1)^2} = \sqrt{9+49} = \sqrt{58}$ units

$\Rightarrow CA^2 = 58$

$AB^2 + BC^2 = 29 + 29 = 58$

Hence, $AB^2 + BC^2 = CA^2$ and $AB = BC$

$\therefore$ The given points form an isosceles right-angled triangle.

**3.04.** Find the area of the triangle formed by joining the points (1, 4), (–2, 5) and (4, –3).

**Sol:** Given: $(x_1, y_1) = (1, 4)$; $(x_2, y_2) = (-2, 5)$ and $(x_3, y_3) = (4, -3)$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1-(-2) & 4-5 \\ -2-4 & 5-(-3) \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 3 & -1 \\ -6 & 8 \end{vmatrix} = \frac{1}{2}[3 \times 8 - (-1) \times (-6)]$$

$$= \frac{1}{2}[24 - 6] = \frac{1}{2} \times (18) = 9 \text{ sq. units.}$$

**3.05.** Find the area of the triangle formed by joining the points (0, 0), (3, 0) and (3, 5).

**Sol:** Given: $(x_1, y_1) = (3, 0)$ and $(x_2, y_2) = (3, 5)$

$$\text{Area} = \frac{1}{2}|x_1y_2 - x_2y_1| \qquad = \frac{1}{2}\ 3 \times 5 - 3 \times 0\ = \frac{1}{2}$$

$|15| = 7.5$ sq. units.

**3.06.** Find the area of the triangle formed by joining the points (–3, –4), (–2, 5) and (–1, 14).

**Sol:** $$\text{Area} = \frac{1}{2}\begin{vmatrix} -3-(-2) & (-4)-5 \\ -2-(-1) & 5-14 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -1 & -9 \\ -1 & -9 \end{vmatrix} = \frac{1}{2}[(-1)\ (-9) - (-1)\ (-9)]$$

$$= \frac{1}{2}[9 - 9] = 0.$$

Since the area = 0, the points do not form a triangle but form a straight line.

**3.07.** Find the co-ordinates of the point $P$ which divides the line joining the points $A(2, 4)$ and $B(3, -3)$ in the ratio 3 : 4 internally.

**Sol:** From the section formula,

$$P = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Here, $m = 3$, $n = 4$; $(x_1, y_1) = (2, 4)$ and $(x_2, y_2) = (3, -3)$

The co-ordinates of P =

$$\left(\frac{3\times3 + 4\times2}{3+4}, \frac{3\times-3 + 4\times4}{3+4}\right)$$

$$= \left(\frac{17}{7}, \frac{7}{7}\right) = \left(\frac{17}{7}, 1\right)$$

**3.08.** Find the co-ordinates of the point $P$ which divides the line joining the points (3, –5) and (4, 6) in the ratio 2 : 1 externally.

**Sol:** $$\text{P} = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$

Here, $(x_1, y_1) = (3, -5)$; $(x_2, y_2) = (4, 6)$, $m = 2$ and $n = 1$.

$$\text{P} = \left(\frac{2(4) - 1(3)}{2-1}, \frac{2(6) - 1(-5)}{2-1}\right) = (5, 17)$$

**Note:** The midpoint of the line segment joining two points is a special case of section formula, when the ratio is 1 : 1.

**3.09.** Find the centre of the circle which has (5, 12) and (3, –8) as the extremities of its diameter.

**Sol:** The centre of any circle C is the midpoint of its diameter.

$$\therefore C = \left(\frac{5+3}{2}, \frac{12-8}{2}\right) = (4, 2).$$

**3.10.** Find the centroid of the triangle formed by the vertices $A(-1, 3)$, $B(5, -2)$ and $C(5, -4)$.

**Sol:** $G$ = Centroid of the triangle

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Here, $A(-1, 3) = (x_1, y_1)$, $B(5, -2) = (x_2, y_2)$, $C(5, -4) = (x_3, y_3)$.

$$G = \left(\frac{-1+5+5}{3}, \frac{3-2-4}{3}\right) = (3, -1)$$

**3.11.** Show that the points $A(3, -4)$, $B(7, -10)$, $C(5, -7)$ are collinear.

**Sol:** $AC = \sqrt{(5-3)^2 + (-7+4)^2} = \sqrt{4+9} = \sqrt{13}$

$BC = \sqrt{(5-7)^2 + (-7+10)^2} = \sqrt{4+9} = \sqrt{13}$

$AB = \sqrt{(7-3)^2 + (-10+4)^2}$

$= \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$

$\Rightarrow AC + BC = AB$

$\therefore A, B, C$ are collinear

(OR)

Applying the area of a triangle formula,

$$\frac{1}{2}\begin{vmatrix} 3-7 & -4+10 \\ 7-5 & -10+7 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -4 & 6 \\ 2 & -3 \end{vmatrix} = \frac{1}{2}[(-4)(-3) - (B)(6)] = 0$$

$\therefore A, B$ and $C$ are collinear.

**3.12.** Compute the slope of the line passing through the points (–1, 4) and (3, –1).

**Sol:** $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-4}{3+1} = \dfrac{-5}{4}$.

**3.13.** The lines $L_1$, $L_2$, $L_3$ and $L_4$ are given containing the following pairs of points, find their slopes and graph them.

$L_1$: (3, –4); (–1, 2) $L_2$: (3, –4); (3, 3)

$L_3$: (3, –4); (1, – 4) $L_4$: (3, –4); (5, 2)

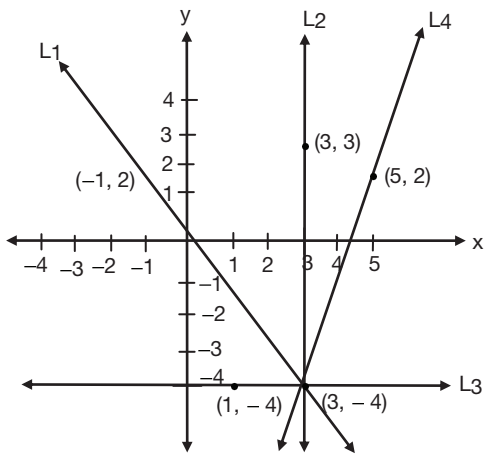
**Sol:** Let $m_1$, $m_2$, $m_3$, $m_4$ denote the slopes of the lines $L_1$, $L_2$, $L_3$ and $L_4$ respectively. Then

$$m_1 = \frac{2-(-4)}{-1-3} = \frac{6}{-4} = \frac{-3}{2}$$

$$m_2 = \frac{3-(-4)}{3-3} = \frac{7}{0}$$

$$m_3 = \frac{-4+4}{1-3} = 0$$

$$m_4 = \frac{2+4}{5-3} = 3$$




$m_1$ is negative, the line $L_1$ 'slants downwards' or decreasing trend; $m_2$ is not defined, the line $L_2$ is vertical; $m_3$ is 0, the line $L_3$ is horizontal; $m_4$ is positive, the line $L_4$ 'slants upward' or increasing trend.

**3.14.** Find the equation of the line which passes through (1, –1) and has a slope 2.

**Sol:** $y - y_1 = m(x - x_1)$

Here, $m = 2$, $(x_1, y_1) = (1, -1)$

$y + 1 = 2(x - 1)$

$y + 1 = 2x - 2 \Rightarrow y = 2x - 3$.

**3.15.** Find the equation of the line passing through the points (2, 1) and (4, 5).

**Sol:** Here, $(x_1, y_1) = (2, 1)$ and $(x_2, y_2) = (4, 5)$.

$$y - 1 = \frac{5-1}{4-2}(x-2)$$

$\Rightarrow y - 1 = 2(x - 2) \Rightarrow y = 2x - 3$.

**3.16.** Find the equation of the line joining the points (5, 3) and (2, 1) in the general form.

**Sol:** Using the two-point form, we get

$$y - 3 = \frac{1-3}{2-5}(x-5)$$

$\Rightarrow y - 3 = 2/3(x - 5)$

$\Rightarrow 3y - 9 = 2x - 10$

$\Rightarrow 2x - 3y - 1 = 0$ $(ax + by + c = 0)$

Here, $a = 2$, $b = -3$ and $c = -1$.

**3.17.** Find the $x$ and $y$ intercepts of the line $7x + 4y - 56 = 0$.

**Sol:** The given line cuts the $x$-axis when $y = 0$

$\Rightarrow 7x = 56$

$\Rightarrow x = 8$

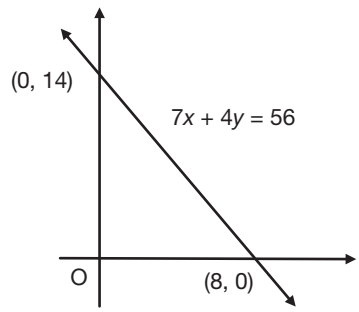
$\therefore$ $x$-intercept = 8

The given line cuts the $y$-axis when $x = 0$

$\Rightarrow 4y = 56$

$\Rightarrow y = 14$

$\therefore y$-intercept = 14.



**3.18.** Write the intercept form of the line whose general form is $3x + 4y - 12 = 0$.

**Sol:** Putting $x = 0$ in the equation, we get $y = 3$ and $y = 0$ in the equation, we get $x = 4$

$\therefore$ The intercept form of the line is $\frac{x}{4} + \frac{y}{3} = 1$.

**3.19.** Find the slope and the $y$-intercept of the line $y = 2x - 5$.

**Sol:** The given line is of the form $y = mx + b$ where $m = 2$, $b = -5$.
$\therefore$ slope $(m) = 2$
$y$-intercept $(b) = -5$

**3.20.** Find the point of intersection of the lines $3x - 2y - 12 = 0$ and $4x + y - 5 = 0$ and also find the number of regions into which the $xy$ plane is divided by these lines.

**Sol:** $L_1 = 3x - 2y - 12 = 0$ and $L_2 = 4x + y - 5 = 0$
On solving, we get $x = 2$ and $y = -3$.
So, $(2, -3)$ is the point of intersection.
As the lines intersect, the plane gets divided into 4 infinite regions.

**3.21.** Show that the lines $2x - 3y - 4 = 0$ and $-8x + 12y - 6 = 0$ are parallel.

**Sol:** $a_1 = 2$, $a_2 = -8$, $b_1 = -3$, $b_2 = 12$

$$\frac{a_1}{a_2} = \frac{2}{-8} = \frac{-1}{4}\ ;\ \frac{b_1}{b_2} = \frac{-3}{12} = \frac{-1}{4}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$\therefore$ The given lines are parallel.

**3.22.** Show that the lines $3x + 5y - 9 = 0$ and $10x - 6y + 7 = 0$ are perpendicular.

**Sol:** $a_1 = 3$, $b_1 = 5$, $a_2 = 10$, $b_2 = -6$
$a_1a_2 + b_1b_2 = (3)(10) + (5)(-6) = 30 - 30 = 0$
$\therefore$ The given lines are perpendicular.

**3.23.** Find the equation of the line through $(3, -3)$ and
(a) parallel to $3x + y + 1 = 0$
(b) perpendicular to $2x + 5y - 2 = 0$

**Sol:** (a) $3(x - 3) + 1(y + 3) = 0$
$\Rightarrow 3x - 9 + y + 3 = 0$
$\Rightarrow 3x + y - 6 = 0$
(b) $5(x - 3) - 2(y + 3) = 0$
$\Rightarrow 5x - 15 - 2y - 6 = 0$
$\Rightarrow 5x - 2y - 21 = 0$.

**3.24.** Find the equation of the line whose $x$ and $y$ intercepts are $2\frac{1}{3}$ and $-3$, respectively.

**Sol:** $\frac{b}{a} = 1$

$$\Rightarrow \frac{x}{7/3} + \frac{y}{-3} = 1 \Rightarrow \frac{-b}{a} = 1$$

$\Rightarrow 9x - 7y = 21$
$\Rightarrow 9x - 7y - 21 = 0$.

**3.25.** Find the equation of the line passing through $(2, -3)$ and parallel to the $x$-axis.

**Sol:** Equation of a line parallel to the $x$-axis is of the form $y = k$.
Substituting $(2, -3)$, we get $-3 = k$
$\therefore$ Equation of the required line is $y = -3$.

**3.26.** Find the perpendicular distance of the point $(2, 2)$ from the line $4x + 3y - 4 = 0$.

**Sol:** The length of the perpendicular from the point $P(x_1, y_1)$ to the line $ax + by + c = 0$ is given by

$$d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

$$\text{So, } d = \left| \frac{4(2) + 3(2) - 4}{\sqrt{4^2 + 3^2}} \right| = \left| \frac{8 + 6 - 4}{5} \right|$$

$$= \left| \frac{10}{5} \right| = 2 \text{ units.}$$

**3.27.** Find the equation of the circle with centre as $(3, 2)$ and radius 4 units.

**Sol:** Let $P(x_1, y_1)$ be any point on the circle.
The distance between $P$ and the centre of the circle equals the radius.

Accordingly, $\sqrt{(x_1 - 3)^2 + (y_1 - 2)^2} = 4$

On squaring, we get $(x_1 - 3)^2 + (y_1 - 2)^2 = 16$
$x_1^2 + y_1^2 - 6x_1 - 4y_1 - 3 = 0$
As $(x_1, y_1)$ was arbitrarily chosen, the equation of the circle is
$x^2 + y^2 - 6x - 4y - 3 = 0$

**3.28.** A line drawn through P$(2, -3)$ making an angle $45°$ with the $x$-axis cuts the $x$-axis at $Q$. Find $PQ$.

**Sol:** Equation of the line is $y + 3 = 1(x - 2)$
$\Rightarrow x - y - 5 = 0$.
This line cuts the $x$-axis, for $y = 0$
$\Rightarrow x = 5$
$\therefore$ $Q(5, 0)$

$PQ = \sqrt{(5-2)^2+(0+3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units.

**3.29.** Find the value of $k$, if $(x+y-1)-k(3x-7y+12) = 0$ is parallel to the $y$-axis.

**Sol:** $x(1-3k)+y(1+7k)-(1+12k) = 0$. (1)

$$\text{Slope} = \frac{-(1-3k)}{1+7k} = \frac{3k-1}{1+7k}$$

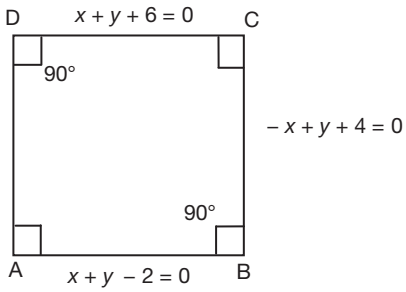
Since (1) is parallel to the $y$-axis, slope $= \infty$

$\Rightarrow 1+7k = 0$

$$\Rightarrow k = \frac{-1}{7}.$$

**3.30.** The lines $x+y-2=0$, $x+y+6=0$ and $-x+y+4=0$ form three sides of a square. Find the equation of the fourth side.

**Sol:**




Let the fourth line $by - x + y + k = 0$

Since $ABCD$ is a square, the distance between the lines $AB$ and $CD$ = the distance between the lines $AD$ and $BC$.

$$\frac{|6+2|}{\sqrt{1^2+1^2}} = \frac{|4-k|}{\sqrt{(-1)^2+1^2}}$$

$$\Rightarrow \frac{|8|}{\sqrt{2}} = \frac{|4-k|}{\sqrt{2}}$$

$\Rightarrow 4-k = \pm 8$.

$\Rightarrow k = -4$ or $k = 12$

$\therefore$ Equation of the fourth side is $-x+y-4=0$ or $-x+y+12=0$.

**3.31.** (a) Find the coordinates of the point (3, 5) when the origin is translated to (1, 3).

(b) The origin is translated to (5, –6) and the point $P$ is transformed to (1, 3). Find the original coordinates of $P$.

**Sol:** (a) Here, $x = X+h$, $y = Y+k$

Given: $(h, k) = (1, 3)$ and $(x, y) = (3, 5)$

$X = x - h = 3 - 1 = 2$

$Y = y - k = 5 - 3 = 2$

$\therefore (X, Y) = (2, 2)$

(b) Here, $(h, k) = (5, -6)$ and $(X, Y) = (1, 3)$

$x = X + h = 1 + 5 = 6$

$y = Y + k = 3 - 6 = -3$

$(x, y) = (6, -3)$

**3.32.** (a) If the axes are rotated through an angle 30°, then find the coordinates of the point (3, 5) in the new system.

(b) When the axes are rotated through an angle of 45°, the coordinates of a point $P$ in the new system are $(5\sqrt{3}, 6\sqrt{3})$. Find the coordinates of $P$ in the original system.

**Sol:** (a) Here, $X = x\cos\theta + y\sin\theta$, $Y = -x\sin\theta + y\cos\theta$

Where $\theta = 30^\circ$ and $(x, y) = (3, 5)$

$$X = 3\cdot\frac{\sqrt{3}}{2} + 5\cdot\frac{1}{2} = \frac{5+3\sqrt{3}}{2}$$

$$Y = -3\cdot\frac{1}{2} + 5\cdot\frac{\sqrt{3}}{2} = \frac{-3+5\sqrt{3}}{2}$$

The point in the new system is

$$\left(\frac{5+3\sqrt{3}}{2}, \frac{5\sqrt{3}-3}{2}\right)$$

(b) Here, $x = X\cos\theta - Y\sin\theta$, $y = X\sin\theta + Y\cos\theta$

Where $\theta = 45^\circ$ and $(X, Y) = (5\sqrt{3}, 6\sqrt{3})$

$$x = 5\sqrt{3}\cdot\frac{1}{\sqrt{2}} - 6\sqrt{3}\cdot\frac{1}{\sqrt{2}} = \frac{5\sqrt{3}-6\sqrt{3}}{\sqrt{2}} = -\sqrt{\frac{3}{2}}$$

$$y = 5\sqrt{3}\cdot\frac{1}{\sqrt{2}} + 6\sqrt{3}\cdot\frac{1}{\sqrt{2}} = \frac{11\sqrt{3}}{\sqrt{2}} = 11\sqrt{\frac{3}{2}}$$

Coordinates of $P$ are $\left(-\sqrt{\frac{3}{2}}, 11\sqrt{\frac{3}{2}}\right)$

**3.33.** Find the distance between the points (4, 3) and (3, –2).

**Sol:** Distance $= \sqrt{(4-3)^2+[3-(-2)]^2} = \sqrt{26}$.

**3.34.** Find the distance between the points (0, 0) and (9, 12).

**Sol:** Distance $= \sqrt{(9-0)^2+(12-0)^2} = 15$.

**3.35.** Prove that the lines formed by joining the points (2, –3), (–3, –7) and (–8, –11) do not form a triangle.

**Sol:** Let $X = (2, -3)$, $Y = (-3, -7)$ and $Z = (-8, -11)$

$$XY = \sqrt{(2-(-3))^2 + (-3-(-7))^2} = \sqrt{41} \text{ units.}$$

$$YZ = \sqrt{(-3-(-8))^2 + (-7-(-11))^2} = \sqrt{41} \text{ units.}$$

$$XZ = \sqrt{(2-(-8))^2 + (-3-(-11))^2} = 2\sqrt{41} \text{ units.}$$

$XY + YZ = XZ$

$\therefore$ $X$, $Y$ and $Z$ are three collinear points and do not form a triangle.

**3.36.** Find the area of the triangle formed by joining the points (–1, –5), (2, –2) and (4, –1).

**Sol:** Given: $(x_1, y_1) = (-1, -5)$, $(x_2, y_2) = (2, -2)$ and $(x_3, y_3) = (4, -1)$.

$$\text{Area} = \text{Mod of } \frac{1}{2}\begin{vmatrix} -1-2 & -5-(-2) \\ 2-4 & -2-(-1) \end{vmatrix}$$

$$= \frac{1}{2}\left|(-3)(-1) - (-3)(-2)\right| = \frac{3}{2} \text{ sq. units.}$$

**3.37.** Find the area of the triangle formed by joining the points (0, 0), (2, 0) and (2, 4).

**Sol:** Given: $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (2, 0)$ and $(x_3, y_3) = (2, 4)$.

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 0-2 & 0-0 \\ 2-2 & 0-4 \end{vmatrix}$$

$$\frac{1}{2}\left|(-2)(-4)\right| = 4 \text{ sq. units.}$$

**3.38.** Find the area of the triangle formed by joining the points (–1, –4), (–3, –5) and (–5, –6).

**Sol:** Given: $(x_1, y_1) = (-1, -4)$, $(x_2, y_2) = (-3, -5)$ and $(x_3, y_3) = (-5, -6)$.

$$\text{Area} = \text{Mod of } \frac{1}{2}\begin{vmatrix} -1-(-3) & -4-(-5) \\ -3-(-5) & -5-(-6) \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} = 0$$

Since the area is 0, the points do not form a triangle but form a straight line.

**3.39.** Find the co-ordinates of the point $P$ which divides the line segment joining the points $A(3, 5)$ and $B(1, -1)$ internally in the ratio 2 : 3.

**Sol:** From the section formula,

$$P = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$$= \left(\frac{2(1)+(3)(3)}{2+3}, \frac{2(-1)+(3)(5)}{2+3}\right) = \left(\frac{11}{5}, \frac{13}{5}\right)$$

**3.40.** Find the co-ordinates of the point $P$ which divides the line segment joining the points $A(3, 5)$ and $B(1, -1)$ in the ratio 2 : 3 externally.

**Sol:** From the section formula,

$$P = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$

$$= \left(\frac{(2)(1)-(3)(3)}{2-3}, \frac{2(-1)-(3)(5)}{2-3}\right) = (7, 17)$$

**NOTE**

The midpoint of the line segment joining the two points is a special case of section formula, in which the ratio is 1 : 1.

**3.41.** Find the centre of the circle which has (7, 20) and (1, –4) as the extremities of one of its diameters.

**Sol:** Centre = Midpoints of its diameter

$$= \left(\frac{7+1}{2}, \frac{20-4}{2}\right) = (4, 8)$$

**3.42.** Find the centroid of the triangle formed by the vertices $P(-3, 5)$, $Q(-5, 7)$ and $R(-1, 3)$.

**Sol:** $G$ = Centroid of the triangle

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$= \left(\frac{-3+-5+-1}{3}, \frac{5+7+3}{3}\right) = (-3, 5)$$

**3.43.** Show that the points $P(4, -5)$, $Q(6, -7)$ and $R(8, -9)$ are collinear.

**Sol:** Area of the triangle formed by the points $P$, $Q$ and $R = \frac{1}{2}\begin{vmatrix} 4-6 & -5-(-7) \\ 6-8 & -7-(-9) \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -2 & 2 \\ -2 & 2 \end{vmatrix}$

$$\frac{1}{2}\left[(-2)(B) - (B)(-2)\right] = 0$$

$\therefore$ $P$, $Q$ and $R$ are collinear.

**Alternate Method:**

If $PQ$, $QR$ and $PR$ are computed, they would be $2\sqrt{2}$, $2\sqrt{2}$ and $4\sqrt{2}$ respectively.

$PQ + QR = PR$. This is possible only when $P$, $Q$, $R$ are collinear and $Q$ lies between $P$ and $R$. Hence, $P$, $Q$, and $R$ do not form a triangle.

$\therefore$ P, Q and R are collinear.

**3.44.** Compute the slope of the line passing through the points $(-3, 5)$ and $(5, -1)$.

**Sol:** Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-(-1)}{-3-(5)} = \dfrac{-3}{4}$

**3.45.** The lines $L_1$, $L_2$, $L_3$ and $L_4$ pass through the following pairs of points. Find the slopes of the lines.

$L_1$: $(2, -3)$, $(-1, 2)$

$L_2$: $(3, -4)$, $(-4, 6)$

$L_3$ : $(-4, 5)$, $(-5, 8)$

$L_4$ : $(5, -6)$, $(3, -10)$

**Sol:** Let $m_1$, $m_2$, $m_3$ and $m_4$ denote the slopes of the lines $L_1$, $L_2$, $L_3$ and $L_4$ respectively.

$$m_1 = \frac{2-(-3)}{-1-2} = -\frac{5}{3}$$

$$m_2 = \frac{6-(-4)}{-4-3} = -\frac{10}{7}$$

$$m_3 = \frac{8-5}{-5-(-4)} = -3$$

$$m_4 = \frac{-10-(-6)}{3-5} = 2$$

**3.46.** Find the equation of the line which passes through $(2, -2)$ and has a slope 3.

**Sol:** $y - y_1 = m(x - x_1)$

$m = 3$, $(x_1, y_1) = (2, -2)$

$y - (-2) = 3(x - 2)$

$y = 3x - 8$

**3.47.** Find the equation of the line which passes through $(3, 2)$ and $(6, 7)$.

**Sol:** $(x_1, y_1) = (3, 2)$ and $(x_2, y_2) = (6, 7)$

$$y - 2 = \frac{7-2}{6-3}(x-3)$$

$3y - 6 = 5x - 15$

$5x - 3y = 9$

**3.48.** Find the $x$ and $y$ intercepts of the line $14x + 8y = 112$

**Sol:** $14x + 8y = 112$

$\therefore$ when $x = 0$, $y = 14$

and when $y = 0$, $x = 8$

$\therefore$ $x$-intercept $= 8$ and $y$-intercept $= 14$.

**3.49.** Write the intercept form of the line whose general form is $4x + 5y - 20 = 0$.

**Sol:** $4x + 5y = 20$

We get the intercept form by dividing by 20.

$\therefore$ The intercept form of the line is $\dfrac{x}{5} + \dfrac{y}{4} = 1$.

**3.50.** Find the slope and the $y$-intercept of the line $y = 3x + 4$.

**Sol:** The given line has the form $y = 3x + 4$.

Slope $= 3$ and $y$-intercept $= 4$.

**3.51.** Find the point of intersection of the lines $4x - 3y - 33 = 0$ and $3x - 2y - 25 = 0$ and also find the number of regions into which the $xy$ plane is divided by these lines.

**Sol:** $L_1$ is $4x - 3y - 33 = 0$

$L_2$ is $3x - 2y - 25 = 0$

On solving these equations,

$x = 9$ and $y = 1$

$\therefore$ $(9, 1)$ is the point of intersection.

As the lines intersect, the plane gets divided into 4 infinite regions.

**3.52.** Show that the lines $4x - 3y - 6 = 0$ and $-12x + 9y + 10 = 0$ are parallel.

**Sol:** $a_1 = 4$, $a_2 = -12$, $b_1 = -3$ and $b_2 = 9$

$$\frac{a_1}{a_2} = \frac{4}{-12} = -\frac{1}{3}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$\therefore$ the given lines are parallel.

**3.53.** Show that the lines $4x - 3y - 6 = 0$ and $3x + 4y + 10 = 0$ are perpendicular.

**Sol:** $a_1 = 4$, $b_1 = -3$, $a_2 = 3$ and $b_2 = 4$

$a_1a_2 + b_1b_2 = (4)(3) + (-3)(4) = 0$

$\therefore$ The given lines are perpendicular.

**3.54.** Find the equation of the line passing through $(4, -4)$ and

(i) parallel to $2x + 3y + 6 = 0$;

(ii) perpendicular to $2x + 3y + 6 = 0$

**Sol:** (i) As the required line is parallel to $2x + 3y = -6$, it is of the form $2x + 3y = k$. As it passes through $(4, -4)$, it is $2x + 3y = 2(4) + 3(-4) = -4$.

(ii) As the required line is perpendicular to $2x+3y=-6$, it is of the form $3x-2y=k$. As it passes through $(4,-4)$, it is $3x-2y=3(4)-2(-4)=20$

**3.55.** Find the equation of the line whose $x$ and $y$ intercepts are 4 and 5 respectively.

**Sol:** $\frac{x}{4}+\frac{y}{5}=1$

$\Rightarrow 5x+4y=20.$

**3.56.** Find the equation of the line passing through $(3,-4)$ and parallel to the $y$-axis.

**Sol:** The equation of a line parallel to the $y$-axis is of the form $x=a$ constant.
As the line required passes through $(3,-4)$ the required equation is $x=3$.

**3.57.** Find the perpendicular distance of the point $(1,1)$ from the line $12x+5y+8=0$.

**Sol:** The length of the perpendicular

$$=\left|\frac{12(1)+5(1)+8}{\sqrt{12^2+5^2}}\right|=\frac{25}{13}\text{ units.}$$

**3.58.** Find the equation of the circle with centre as $(2,3)$ and radius 5 units.

**Sol:** Let $(x_1, y_1)$ be any point on the circle.

$$\sqrt{(x_1-2)^2+(y_1-3)^2}=5$$

On squaring both sides, we get

$$x_1^2-4x_1+4+y_1^2-6y_1+9=25$$

$$x_1^2+y_1^2-4x_1-6y_1-12=0$$

$\therefore$ The required equation is $x^2+y^2-4x-6y-12=0$

**3.59.** A line drawn through $P(3,-4)$ makes an angle of $45°$ with the $x$-axis and cuts it at $Q$. Find $PQ$.

**Sol:** Given: $\theta=45$; Slope of the line is $\tan\theta=\tan 45°=1$.

Equation of the line is $\frac{y-(-4)}{x-3}=1$

$\Rightarrow y-x=-7$
At Q, $y=0$; $\therefore x=7$, $\therefore$ Q$(7,0)$.

$\therefore$ PQ $=\sqrt{(7-3)^2+(0-(-4))^2}=4\sqrt{2}$ units.

**3.60.** Find the value of $a$, if the line $(x+y+2)+a(4x+6y+18)=0$ is parallel to the $x$-axis.

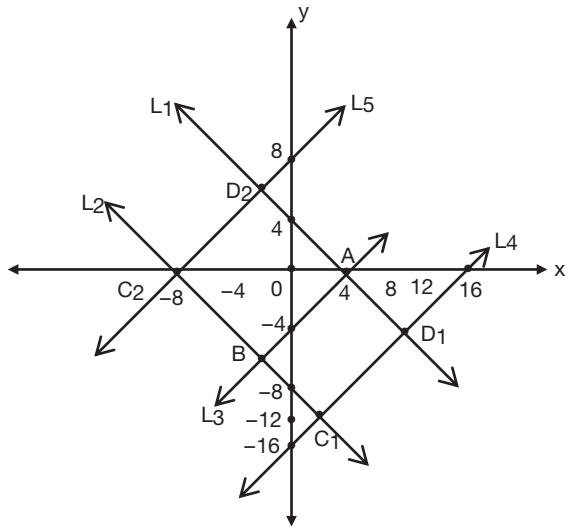
**Sol:** $x(1+4a)+y(1+6a)+2+18a=0$

$$\text{Slope}=\frac{-(1+4a)}{1+6a}$$

Since the given equation is parallel to the $x$-axis, slope = 0.

$$\therefore \frac{-(1+4a)}{1+6a}=0,\ \therefore\ a=\frac{-1}{4}.$$

**3.61.** The lines $x+y-4=0$, $x+y+8=0$ and $-x+y+4=0$ form three sides of a square. Find the equation of the fourth side.




$L_1=x+y-4=0$
$L_3=x-y-4=0$
$L_2=x+y+8=0$
Let the point of intersection of $L_1$ and $L_3$ be $A$ and the point of intersection of $L_2$ and $L_3$ be $B$.
There are two possible squares determined by these three lines. The fourth side of one square lies on $L_4$ which lies on one side of $L_3$ and the fourth side of the other lies on $L_5$ which lies on the opposite side of $L_3$.
The equation of $L_4$ and $L_5$ are of the form $x-y+k=0$
Since the lines form a square, the distance between $L_1$ and $L_2$ is equal to the distance between $L_3$ and $L_4$ or $L_3$ and $L_5$.

$$\therefore \frac{|8-(-4)|}{\sqrt{1^2+1^2}}=\frac{|k+4|}{\sqrt{1^2+1^2}}$$

i.e., $k+4=12$ or $k+4=-12$
i.e., $k=-16$ or $k=8$
$\therefore$ The required equation is $x-y-16=0$ or $x-y+8=0$.

**3.62.** Find the equation of locus of the point $P$ such that $PA^2+PB^2=36$, where $A=(2,3)$ and $B=(4,-1)$.

**Sol:** $P(x,y)$ is any point on the locus and $A=(2,3)$ and $B=(4,-1)$
Given: $PA^2+PB^2=36$
$\Rightarrow (x-2)^2+(y-3)^2+(x-4)^2+(y+1)^2=36$
$\Rightarrow 2(x^2+y^2-6x-2y-3)=0$
$\Rightarrow x^2+y^2-6x-2y-3=0$. This is the equation of the locus. It represents a circle.

**3.63.** Does the equation $x^2+2xy+y^2-5x-5y+6=0$ represent a pair of parallel lines?

**Sol:** $x^2+2xy+y^2-5x-5y+6=0$ can be expressed as $(x+y-2)(x+y-3)=0$,
i.e., $x^2+2xy+y^2-5x-5y+6=(x+y-2)(x+y-3)=0$
$\therefore$ The two straight lines represented by the given equations are $x+y-2=0$ and $x+y-3=0$ or $x+y=2$ and $x+y=3$. These are two parallel lines.

**3.64.** Find the coordinates of the point $(-3,4)$, when the origin is translated to $(1,-4)$.

**Sol:** The equations relating the coordinates are $x=X+h$,
$y=Y+k$
Given: $(x,y)=(-3,4)$ and $(h,k)=(1,-4)$
$X=x-h$ and $Y=y-k$
$X=-3-1$ and $Y=4-(-4)$
$\Rightarrow (X,Y)=(-4,8)$

**3.65.** The point to which the origin has to be shifted, so that the point $(-3,5)$ changes to $(1,-2)$ after translation, is

**Sol:** The equations relating the coordinates are $x=X+h$, $y=Y+k$
Here, $(x,y)=(-3,5)$ and $(X,Y)=(1,-2)$
$\therefore h=x-X$ and $k=y-Y$
$h=(-3,-1)$, $k=5-(-2)$
$\therefore (h,k)=(-4,7)$

**3.66.** The coordinates of the point $\left(-3\sqrt{2},5\sqrt{2}\right)$ in the new system, when the axes are rotated by an angle of $45°$ in the anti-clockwise direction, are

**Sol:** The transformation equations are
$X=x\cos\theta+y\sin\theta$, $Y=-x\sin\theta+y\cos\theta$
Here, $(x,y)=\left(-3\sqrt{2},\ 5\sqrt{2}\right)$ and $\theta=45°$
$\therefore X=-3\sqrt{2}\cos 45°+5\sqrt{2}\sin 45°$ and
$Y=+3\sqrt{2}\sin 45+5\sqrt{2}\cos 45°$
$X=-3\sqrt{2}.\frac{1}{\sqrt{2}}+5\sqrt{2}.\frac{1}{\sqrt{2}}$ and
$Y=3\sqrt{2}.\frac{1}{\sqrt{2}}+5\sqrt{2}.\frac{1}{\sqrt{2}}$
$X=-3+5$ and $Y=3+5$
$\therefore (X,Y)=(2,8)$

**3.67.** If the coordinates of a point $P$ change to $(4,0)$ when the axes are rotated by an angle $30°$ in the clockwise direction, then the coordinates of P are

**Sol:** Given: $(X,Y)=(4,0)$, let $P(x,y)$ and $\theta=-30°$
$\therefore x=X\cos\theta-Y\sin\theta$, $Y=x\sin\theta+y\cos\theta$
$x=4\cos(-30°)-0\sin(-30)°$, $y=4\sin(-30°)+0\cos(-30°)$
$x=4.\frac{\sqrt{3}}{2}$, $y=-4.\frac{1}{2}$
$\therefore (x,y)=(2\sqrt{3},-2)$

## EXERCISE-1

***Directions for questions 1 to 40*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** The point $(-4,3)$ lies in _____ quadrant
(A) I$^{st}$ (B) II$^{nd}$ (C) III$^{rd}$ (D) IV$^{th}$

**2.** What is the least distance (in units) between two non-parallel lines lying in the same plane, if the distance is measured along a direction which is perpendicular to one of the lines? _____

**3.** What is the maximum number of quadrants that a straight line can pass through? _____

**4.** Find the distance from the origin to (3, 4). _____

**5.** How many points are there which are at unit distance from the origin?
(A) 2 (B) 4 (C) 1 (D) infinite

**6.** Find the distance from the origin to the point (5, 12).
(A) 5 (B) 12 (C) 13 (D) 6

**7.** Find the perpendicular distance from (3, 5) to the $x$-axis.
(A) 3 (B) 5 (C) 8 (D) 4

**8.** Which of the following points is nearest to the origin?
(A) (1, 3) (B) (2, –3)
(C) (–3, 0) (D) (1, 2)

**9.** The point of intersection of the lines $x = 3$ and $y + 2 = 0$ is
(A) (–3, 2) (B) (–3, –2)
(C) (3, 2) (D) (3, –2)

**10.** In what ratio does the $x$-axis divide the line joining points (3, 5) and (4, –6)?
(A) 5 : 6 externally (B) 5 : 6 internally
(C) 3 : 4 internally (D) 3 : 4 externally

**11.** In triangle $ABC$, $D$ is the midpoint of $BC$. The centroid ($G$) divides $DA$ in the ratio of
(A) 1 : 2 (B) 2 : 1 (C) 3 : 1 (D) 1 : 3

**12.** A straight line with a slope of –1 passes through the I$^{st}$ quadrant. It can't pass through which quadrant?
(A) II$^{nd}$ (B) III$^{rd}$
(C) IV$^{th}$ (D) None of these

**13.** What is the distance of the straight line $y = mx + c$ from the origin?

(A) $\left|\frac{c}{\sqrt{1+m^2}}\right|$ (B) $\left|\frac{c}{m}\right|$

(C) $\left|\frac{c}{1+m}\right|$ (D) $\left|\frac{c}{1+m^2}\right|$

**14.** Distance from origin to the line $ax + by + c = 0$ is

(A) $\left|\frac{c}{a^2+b^2}\right|$ (B) $\left|\frac{c^2}{a^2+b^2}\right|$

(C) $\left|\frac{c^2}{\sqrt{a^2+b^2}}\right|$ (D) $\left|\frac{c}{\sqrt{a^2+b^2}}\right|$

**15.** What is the ratio in which the origin divides the line segment joining the points $(x_1, y_1)$ and $(-3x_1, -3y_1)$?
(A) 1 : 3 externally (B) 1 : 3 internally
(C) 1 : 2 internally (D) 2 : 1 internally

**16.** In a square, two adjacent vertices are (0, 0) and $(0, \alpha)$. What is the length of the diagonal of the square?
(A) $\sqrt{\alpha}$ (B) $2\sqrt{\alpha}$ (C) $\sqrt{2}\alpha$ (D) $\alpha^2$

**17.** If (1, 1) and (–1, –1) are the two vertices of an equilateral triangle, then the third vertex could be
(A) $\left(\sqrt{3}, -\sqrt{3}\right)$ (B) (–3, 3)
(C) (0, 0) (D) Both (A) and (C)

**18.** In triangle $ABC$, $D$, $E$, $F$ are the midpoints of $BC$, $CA$, $AB$ respectively. The area of the triangle $DEF$ is 5. The area of triangle $ABC$ is
(A) 30 (B) 10 (C) 20 (D) 5

**19.** The slope of a line perpendicular to $x = 3$ is
(A) 0 (B) 1 (C) –1 (D) undefined

**20.** If the slope of the line joining the points $(a, b)$ and $(c, d)$ is '0' then which of the following is true?
(A) $b + d > 0$ (B) $b = d$ (C) $a = c$ (D) $a + c = 0$

**21.** The slope of the line whose inclination is 135° is
(A) –1 (B) 1 (C) 2 (D) –2

**22.** The slope of the line $x + \sqrt{3}y + 8 = 0$ is
(A) 1 (B) $1/\sqrt{3}$ (C) –1 (D) $-1/\sqrt{3}$

**23.** What is the area of the circle whose largest chord is 2 units? (in sq. units)
(A) $\pi$ (B) $4\pi$ (C) $2\pi$ (D) None of these

**24.** If $(x_1, y_1)$ and $(-x_1, y_1)$ are two opposite vertices of a square and $y_1 > x_1$, the other two vertices are
(A) $(y_1 + x_1, 0)$ $(y_1 - x_1, 0)$ (B) $(0, y_1 + x_1)$ $(0, y_1 - x_1)$
(C) $(-x_1, -y_1)$, $(x_1, -y_1)$ (D) None of these

**25.** The area of the triangle formed by the points $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$ is zero. What can we say about the three points?
(A) They are collinear
(B) One of the points divides the join of the other two in some ratio
(C) Both (A) and (B)
(D) Neither (A) nor (B)

**26.** The angle made by the line $x - \sqrt{3}\,y + 4 = 0$ with $x$-axis is
(A) 30° (B) 45° (C) 60° (D) 90°

**27.** The angle between the lines $x = 5$ and $y = 7$ is
(A) 90° (B) 30° (C) 45° (D) 60°

**28.** Which of the following lines is perpendicular to the line $3x + 4y + 5 = 0$?
(A) $4x + 3y + 7 = 0$ (B) $4x - 3y + 5 = 0$
(C) $3x + 4y + 6 = 0$ (D) All the above

**29.** The equation of a line with slope $\frac{1}{m}$ and $y$ intercept $a$ is

(A) $y = mx + a$ (B) $my = x + a$

(C) $y = \frac{1}{m}x + am$ (D) $my = x + am$

**30.** The equation of the line parallel to the $x$-axis and passing through (3, 4) is

(A) $x = 4$ (B) $y = 3$ (C) $y = 4$ (D) $x = 3$

**31.** $(0, y_1)$ and $(0, -y_1)$ are the end points of a diagonal of a rhombus. The other diagonal is half the first diagonal. Which of the following is/are vertices of the rhombus?

(A) $\left(\frac{y_1}{2}, 0\right)$ (B) $\left(\frac{-y_1}{2}, 0\right)$

(C) Both (A) and (B) (D) Neither (A) nor (B)

**32.** The equation of a line perpendicular to $y = 3x + 1$ and passing through (1, 1) is

(A) $3y = -x + 12$ (B) $y = -3x + 4$

(C) $3y = -x + 4$ (D) $3y = x + 2$

**33.** The centroid of the triangle whose vertices are (0, 0), (5, 0) and (0, 12) is

(A) $\left(\frac{5}{2}, 6\right)$ (B) $\left(\frac{5}{4}, 3\right)$

(C) $\left(\frac{5}{3}, 4\right)$ (D) $\left(\frac{5}{6}, 2\right)$

**34.** Which of the following points lie on the line $2x - y + 7 = 0$?

(A) (–2, 3) (B) (0, 7)

(C) (1, 9) (D) All the above

**35.** The equation of the line passing through (2, 3) and sum of whose intercepts is zero is

(A) $x - y - 1 = 0$ (B) $x - y + 1 = 0$

(C) $x + y + 1 = 0$ (D) $x + y - 5 = 0$

**36.** The angle made (in degrees) by the line $x - y = 0$ with the positive direction of $x$-axis is _____.

**37.** The equation of a line parallel to $y = 2x$ and passing through (3, 4) is

(A) $y = 2x + 2$ (B) $y = \frac{1}{2}x + \frac{5}{2}$

(C) $y = \frac{-1}{2}x + \frac{11}{2}$ (D) $y = 2x - 2$

**38.** A circle is centered at the origin and passes through (3, 4). Find the circumference of the circle.

(A) $5\pi$ (B) $10\pi$ (C) $2.5\pi$ (D) $20\pi$

**39.** The midpoint of the line segment whose ends are (1, 6) and (9, 12) is

(A) (5, 9) (B) (7, 9) (C) (5, 8) (D) (6, 8)

**40.** A line passes through only two quadrants. Which of the following could be true?

(A) The slope of the line is 0.

(B) The line is parallel to the $y$-axis.

(C) The line passes through the origin.

(D) All the above

## EXERCISE-2

***Directions for questions 1 to 40*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** The distance between (6, 8) and the centre of the circle $x^2 + y^2 = a^2$ is

(A) 6 units (B) 8 units

(C) 10 units (D) 9 units

**2.** The ratio in which the $x$-axis divides the line joining (2, 3) and (–4, 1) is

(A) 1 : 2 internally (B) 3 : 1 externally

(C) 2 : 1 internally (D) 1 : 3 externally

**3.** If the points (2, 3), (3, 1) and (–1, $k$) are collinear, then $k$ =

(A) 3/2 (B) –1/2 (C) 9 (D) –2/3

**4.** If (1, 3), (–2, 1) and (4, –2) are the three consecutive vertices of a parallelogram, then the fourth vertex is

(A) (–1, 0) (B) (–1, 2) (C) (7, 0) (D) (0, 7)

**5.** The distance between (3, 7) and the centre of the circle $(x - 6)^2 + (y - 3)^2 = 25$ (in units) is _____.

**6.** The ratio in which $y$-axis divides the line joining points (4, 3) and (–6, 2) is

(A) 2 : 3 externally (B) 3 : 2 internally

(C) 2 : 3 internally (D) 3 : 2 externally

**7.** If the orthocentre and the centroid of a triangle are (4, 5) and (3, 3) respectively, then the circumcentre of the triangle is

(A) $\left(\frac{5}{2}, 2\right)$ (B) (5, 4) (C) (1, 6) (D) (6, 1)

**8.** The slope of the line joining the points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ is

(A) $\frac{1}{2(t_1+t_2)}$ (B) $\frac{1}{t_2+t_1}$

(C) $\frac{2}{t_2+t_1}$ (D) $\frac{1}{2a(t_1+t_2)}$

**9.** The line passing through (2, 3) and (3, 2) is
(A) $x+y-5=0$ (B) $x-y-1=0$
(C) $x+y+1=0$ (D) $x-y+1=0$

**10.** If the lines $x-2y-3=0$, $x+3y-3=0$ and $2x+y-1=0$ form a right angled triangle, then the vertex containing the right angle is
(A) (3, 0) (B) (0, 1) (C) (1, –1) (D) (1, 1)

**11.** The lines $2x+3y+5=0$ and $x+y+2=0$ intersect in the
(A) 1st quadrant. (B) 2nd quadrant.
(C) 3rd quadrant (D) 4th quadrant.

**12.** If the lines $\sqrt{3}x+\sqrt{2}y=6$ and $\sqrt{2}x+ky=2$ are parallel, then $k=$
(A) $\sqrt{6}$ (B) $2/\sqrt{3}$ (C) $\sqrt{3}/2$ (D) $\sqrt{3}$

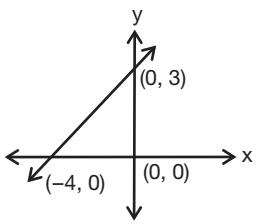
**13.** The angle made (in degrees) by the line $\sqrt{3}x-y+9=0$ with $y$-axis is _____.

**14.** The acute angle (in degrees) between the lines $2x+3y+7=0$ and $x-5y+3=0$ is _____.

**15.** If $3x+4y+7=0$ and $12x-9y+10=0$ are two adjacent sides of a rectangle and one of its vertex is (2, 3), then the area of the rectangle in sq. units is

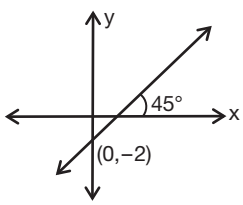
(A) 5 (B) 2 (C) $\frac{7}{3}$ (D) $\frac{7}{5}$

**16.** The equation of the line represented by the given graph is.




(A) $3x-2y+12=0$
(B) $3x+4y+12=0$
(C) $3x-4y+12=0$
(D) $3x-4y-12=0$

**17.** The equation of the line represented by the given graph is




(A) $x-2y+6=0$
(B) $x-y-2=0$
(C) $x+y-2=0$
(D) $x+y+2=0$

**18.** The area of the triangle formed by the line $3x+4y=24$ with co-ordinate axes in sq. units is
(A) 6 sq. units (B) 11 sq. units
(C) 48 sq. units (D) 24 sq. units

**19.** The distance of (1, 4) from the line $3x-4y+3=0$ is
(A) 1/5 units (B) 2/5 units
(C) 8/5 units (D) 2 units

**20.** If $x-2y+1=0$ and $x-2y+3=0$ are two opposite sides of a square, then area of the square in sq. units is
(A) 2/5 (B) 4/5
(C) 5 (D) 5/4

**21.** If $A(3, 5)$, $B(3, 9)$ and $C(6, 9)$ are the vertices of a right angled triangle $ABC$, right angled at $B$, then find the length of altitude drawn from $B$ to $AC$.
(A) 12/5 (B) 13/5
(C) 2 (D) 16/5

**22.** If $A(4, 5)$, $B(3, 6)$, $C(2, 1)$ are the three vertices of a triangle, then the equation of the altitude through $A$ is
(A) $3x-7y+23=0$ (B) $5x-y-15=0$
(C) $x-5y+4=0$ (D) $x+5y-29=0$

**23.** If the line $3x+4y+5+k(x-3y+2)=0$ is parallel to the $x$-axis, then the value of $k^2$ is _____.

**24.** If the lines $ax+3y+7=0$ and $4x+9y+15=0$ are perpendicular to each other, then the value of $a$ is

(A) $-\frac{27}{4}$ (B) $-\frac{3}{2}$ (C) $\frac{4}{27}$ (D) $\frac{2}{3}$

**25.** If the line $x+ky+3k+2=0$ passes through the point of intersection of the lines $4x+5y-23=0$ and $x+3y-11=0$, then $k=$

(A) $\frac{2}{3}$ (B) $-\frac{3}{2}$ (C) $\frac{3}{2}$ (D) $-\frac{2}{3}$

**26.** If the roots of the equation $x^2-5x-6=0$ represents the slope and $y$-intercept of a line, then the equation of the line can be
(A) $x+y+6=0$ (B) $6x-y-1=0$
(C) $6x+y+1=0$ (D) $x+y+1=0$

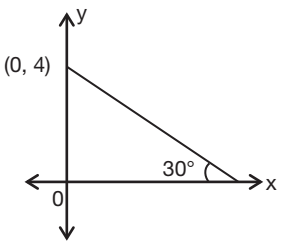
**27.** Two of the tangents to a circle are $x+y-7=0$ and $2x+2y+13=0$. Find the circumference of the circle.

(A) $\frac{27}{\sqrt{2}}\pi$ (B) $\frac{27}{2\sqrt{2}}\pi$

(C) $27\sqrt{2}\pi$ (D) $\frac{27}{4\sqrt{2}}\pi$

Difficulty Level-2: Moderate

**28.** What is the equation of the line shown?




(A) $x+\sqrt{3}\,y=4$

(B) $5x-3y+15=0$

(C) $5x-3y-15=0$

(D) $x+\sqrt{3}y\ -4\sqrt{3}=0$

**29.** A line passing through $P(3, 4)$ makes an angle of 30° with the $x$-axis and cuts the $y$-axis at $Q$. The length of $PQ$ is

(A) $5\sqrt{3}$ (B) $4\sqrt{3}$ (C) $\sqrt{3}$ (D) $2\sqrt{3}$

**30.** The perimeter of the triangle whose vertices are $A(1, 1)$, $B(5, 4)$ and $C(4, -3)$ is

(A) $10\left(2+\sqrt{2}\right)$ (B) $6\left(2+\sqrt{2}\right)$

(C) $5\left(2+\sqrt{2}\right)$ (D) $5\left(2-\sqrt{2}\right)$

**31.** From point $A(3, 2)$, line $l$ is drawn perpendicular to line $3x-2y=0$. If line $l$ intersects line $x+y=4$ at point $P(a, b)$, then $b^3-a^3$ is

(A) 64 (B) 56 (C) 37 (D) – 64

**32.** If the origin is shifted to point (3, 4), then find the transformed equation of $5x+6y+18=0$.

(A) $5x+6y+57=0$ (B) $5x+6y+21=0$

(C) $5x+6y-57=0$ (D) $5x+6y-21=0$

**33.** A circle touches the circle $x^2+y^2=4$ and the line $4x+3y+12=0$. Find the area of the smallest such circle.

(A) $\frac{\pi}{25}$ (B) $5\pi$ (C) $\frac{\pi}{16}$ (D) $\frac{\pi}{9}$

**34.** Find the equation of the line whose $x$ and $y$ intercepts are $2\frac{1}{3}$ and –3 respectively.

(A) $9x+7y-21\ 0$ (B) $9x-7y-21=0$

(C) $9x+7y+21=0$ (D) $7x+8y-21=0$

**35.** Find the number of points which have integer coordinates in the figure formed by the lines $x=0$, $y=0$ and $x+y=5$. _____

**36.** The transformed equation of a curve when the axes are translated to (1, 1) is $2x^2-3xy-y^2=5$. Its equation with respect to the original axes is

(A) $2x^2-3xy-y^2+x+5y+7=0$

(B) $2x^2-3xy+y^2-x-5y+7=0$

(C) $2x^2-3xy-y^2-x-5y-7=0$

(D) $2x^2-3xy-y^2-x+5y-7=0$

**37.** Let the values of $x$ that satisfy the quadratic equation $x^2+7x+12=0$ form an ordered pair $(a, b)$ such that $a>b$. Which of the following relations does the point $(a, b)$ satisfy?

(A) $a-3b=-6$ (B) $2a-3b=10$

(C) $2a-5b=14$ (D) $a-b=8$

**38.** If a line passing through $P(5, 3)$ makes an angle 60° with the $x$-axis and cuts the $y$-axis at $Q$, then the length of $PQ$ (in units) is _____.

**39.** In the $xy$-plane, the vertices of a triangle are (0, 20), (20, 0), (0, 0). The number of points with integer coordinates inside the triangle (excluding all the points on the boundary) is _____.

**40.** How many integral values of $k$ are possible, if the lines $3x+4ky+6=0$, and $kx-3y+9=0$ intersect in the 2nd quadrant? _____

## EXERCISE-3

***Directions for questions 1 to 40*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** The points (1, –1), (–1, 4), (4, 6) are the vertices of an

(A) isosceles right angled triangle

(B) right angled triangle

(C) equilateral triangle

(D) Scalene triangle

**2.** Find the equation of the line which passes through (1, –1) and has a slope 2.

(A) $y=2x+3$ (B) $y=2x-3$

(C) $3x-y+3=0$ (D) $y=3x+2$

**3.** Find the area of the polygon formed by joining the points (1, –1) (3, 2), (2, 5), (–1, 7) and (–4, 1) in the given order. _____

**4.** When the axes are rotated anti-clockwise through 45°, the coordinates of a point are $\left(-2\sqrt{2}, 3\sqrt{2}\right)$. Find the coordinates of the point in the original system.

(A) (5, –1) (B) (1, –5)

(C) (–5, 1) (D) (–5, –1)

**5.** Find the centre and radius of the circle $2(x^2+y^2)+12x-8y-46=0$.

(A) (–3, 2), 6 (B) (3, 2), 5

(C) (–3, –2), 4 (D) (3, 2), 5

**6.** Find the shortest distance between the curves $x^2 + y^2 + 6x - 10y + 9 = 0$ and $x^2 + y^2 + 22x + 20y + 140 = 0$. _____

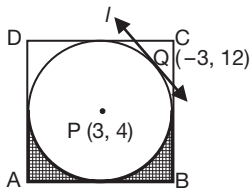
**7.** The distance between the parallel lines $3x + y - 5 = 0$ and $12x + 4y + 15 = 0$ is

(A) $\frac{35}{4\sqrt{10}}$ (B) $\frac{35}{\sqrt{10}}$ (C) $\frac{35}{2\sqrt{10}}$ (D) $\frac{7}{4\sqrt{10}}$

**8.** What ratio does line $4x + 5y = 6$ divide the line joining points $A(1, 2)$ and $B(3, -2)$?

(A) 1 : 2 externally (B) 1 : 2 internally
(C) 2 : 1 externally (D) 2 : 1 internally

**9.** $ABCD$ is square. A circle at centre $P(3, 4)$ is inscribed in the square $ABCD$. A tangent ( $\ell$ ) meets the circle at $Q(-3, 12)$. Find the area of the shaded region.

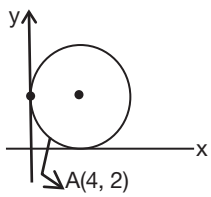



(A) $50(4 - \pi)$
(B) $100(4 - \pi)$
(C) $200(4 - \pi)$
(D) $75(4 - \pi)$

**10.** Line $x + y + 1 = 0$ cuts curve $x^2 - 4y - 16 = 0$ at two points $A$ and $B$. If $C$ (5, 8) is a point, then the triangle formed by points $A$, $B$, $C$ is

(A) right angled (B) acute angled
(C) isosceles (D) None of these

**11.** In the given figure, $A(4, 2)$ is a point on the circle. Find the area of the circle.




(A) 25 $\pi$ sq. units
(B) 100 $\pi$ sq. units
(C) 10 $\pi$ sq. units
(D) 75 $\pi$ sq. units

**12.** Find the shortest distance of point (6, –9) from curve $y = x^2 - 12x + 32$.

(A) 6 (B) 4 (C) 3 (D) 5

**13.** If the slope of the line joining the points (4, p) and $(p, 5)$ is –2, then the value of $p$ is _____.

**14.** The points $(p + 1, 1)$, $(2p + 1, 3)$ and $(2p + 2, 2p)$ lie on the same straight line only if

(A) $p = -1$ or 2 (B) $p = 2$ or 1
(C) $p = 2$ or $\frac{-1}{2}$ (D) $p = \frac{1}{2}$ or 2

**15.** Two points (1, 3) and (5, –5) are joined by a straight line. Which of the following points lies on this line?

(A) (1, –3) (B) (1, 2) (C) (4, –3) (D) (5, 2)

**16.** The equation of a line passing through the point (1, –6) and product of whose intercepts on the axes is 1, is

(A) $3x + y = -3$ (B) $9x + y = 3$
(C) $4x + y + 2 = 0$ (D) Either (B) or (C)

**17.** Find the slope of the chord of the circle $x^2 + y^2 + 4x + 6y - 12 = 0$, if the midpoint of the chord is (2, –1). _____

**18.** Find the number of common tangents that can be drawn to the two circles $(x - 2)^2 + (y - 3)^2 = 16$ and $(x + 1)^2 + (y - 2)^2 = 9$. _____

**19.** The orthocentre of the triangle formed by the lines $x + 2y = 3$, $x + y = 2$ and $4x - 2y + 5 = 0$ is $(p, q)$, then find $p + q$.

Enter your answer as a decimal value, rounded off to one decimal place. _____

**20.** If the lines $4x - 3y + 12 = 0$ and $8x - 6y + 15 = 0$ are two tangents of a circle, then find the area of the circle.

(A) $\frac{9}{20}\pi$ (B) $\frac{81}{400}\pi$
(C) $\frac{81}{100}\pi$ (D) $\frac{9}{20}\pi$

**21.** Two identical circles have a radius of 9 cm each. They touch each other externally at the origin. The line $y = 5x$ is a common tangent to the circles at the origin. Which of the following is the centre of one of these circles?

(A) $\left(\frac{9}{\sqrt{26}}, \frac{-45}{\sqrt{26}}\right)$ (B) $\left(\frac{-9}{26}, \frac{-45}{\sqrt{26}}\right)$
(C) $\left(\frac{-45}{\sqrt{26}}, \frac{9}{\sqrt{26}}\right)$ (D) $\left(\frac{-45}{\sqrt{23}}, \frac{-9}{\sqrt{26}}\right)$

**22.** If the circles $x^2 + y^2 - 2px - c = 0$ and $x^2 + y^2 - 2qy - c = 0$ touch each other externally, which of the following is true?

(A) $p^2 + q^2 = c^2$ (B) $p^2 q^2 = (p^2 + q^2)c^2$
(C) $p^2 q^2 = (p^2 - q^2)c$ (D) $p^2 q^2 + c(p^2 + q^2) = 0$

**23.** The length of the common chord of the circles $x^2 + y^2 + 3x = 0$ and $x^2 + y^2 + 6y = 0$ is

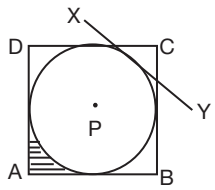
(A) $\frac{6}{\sqrt{5}}$ (B) $\frac{3}{\sqrt{5}}$ (C) $\frac{9}{\sqrt{5}}$ (D) $\frac{36}{\sqrt{5}}$

**24.** If the two circles $x^2 + y^2 - 4x - 6y + 9 = 0$ and $x^2 + y^2 - 10x - 14y + k = 0$ touch each other externally at $P$, then find the equation of the common tangent at $P$.

(A) $3x + 4y - 28 = 0$ (B) $4x - 3y + 28 = 0$
(C) $3x + 4y - 25 = 0$ (D) $4x - 3y - 18 = 0$

**25.** How many integral values of $k$ are possible, if the lines $4x + 5ky + 7 = 0$ and $kx - 6y + 12 = 0$ intersect in the 2nd quadrant?
(A) 4 (B) 5 (C) 6 (D) 7

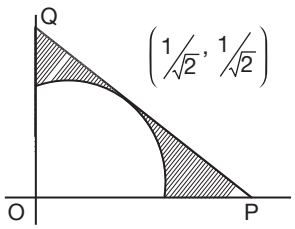
**26.** In the figure given below, ABCD is a square in which a circle is inscribed. The centre of the circle is $P$ (4, 7). The point of contact of the tangent $XY$ with the circle is (–1, –5). The area of the shaded region is




(A) $\frac{169}{4}[4-\pi]$
(B) $\frac{179}{4}[\pi-2]$
(C) $\frac{1}{4}[676-13\pi]$
(D) $\frac{13}{4}[4-\pi]$

**27.** In the figure given below, line PQ makes equal intercepts on the two axes. The area of the shaded region is

(A) $\left(\frac{\pi-1}{4}\right)$
(B) $\left(\frac{4-\pi}{4}\right)$
(C) $\left(\frac{2-\pi}{2}\right)$
(D) $\left(\frac{4-\pi}{2}\right)$




**28.** A line passing through the point $P(3, 2)$ is tangent to the circle $x^2 + y^2 + 2x + 4y - 2 = 0$ at $Q$. Find the distance $PQ$.
(A) $\frac{5}{2}$ (B) 3 (C) 10 (D) 5

**29.** From point $A(2, 1)$, a line $\ell$ is drawn to the line $m: 3x + y = 0$. The distance of $A$ from $m$ measured along $\ell$ is $d$. The distance $d$ is less than the distance of $A$ from $m$, measured along any other line (other than $\ell$). If line $\ell$ intersects line $5x - 7y = 11$ at point $(a, b)$, then $a^2 - b^2$ is
(A) 21 (B) 29 (C) 25 (D) 30

**30.** A line passing through the origin cuts two parallel lines $5x + 12y = 26$ and $10x + 24y = 39$ at the points $P$ and $Q$ respectively. Find the ratio in which the origin divides the line segment joining $P$ and Q.
(A) 4 : 3 externally (B) 4 : 3 internally
(C) 3 : 2 externally (D) 3 : 2 internally

**31.** The length of the intercept made by the line which passes through the points (3, 6) and (–3, 9) between the coordinate axes is
(A) $8\sqrt{5}$ (B) $\frac{7\sqrt{5}}{2}$ (C) $15\sqrt{5}$ (D) $\frac{15}{2}\sqrt{5}$

**32.** The points (0, 0), $(p, q)$, $(-p, -q)$ and $(pq, q^2)$ are
(A) vertices of a rectangle
(B) vertices of a rhombus
(C) Both A and B
(D) Neither A nor B

**33.** If $A(4, 9)$, $B(6, 5)$ and $C(7, 8)$ are the vertices of the triangle $ABC$, the slope of the line which bisects $\angle ACB$ is
(A) $\frac{-1}{2}$ (B) 2 (C) –2 (D) $\frac{1}{2}$

**34.** If (3,10) is a vertex and $5x - y + 12 = 0$ is the equation of a diagonal of a square, then find the equation of the other diagonal of the square.
(A) $x + 5y - 53 = 0$ (B) $x + 5y - 17 = 0$
(C) $x + 5y - 18 = 0$ (D) $x + 5y + 53 = 0$

**35.** Three lines $3x - y = 2$, $5x - ay = 3$ and $2x + y = 3$ are concurrent. $a =$ _____.

**36.** $PQR$ is an equilateral triangle $P = (-4, 0)$ and $Q = (4, 0)$. $R$ is a point on the $y$-axis. The equation of the circle circumscribing $PQR$ is
(A) $x^2 + \left(y \pm \frac{4\sqrt{3}}{3}\right)^2 = \frac{64}{3}$
(B) $x^2 + \left(y \mp \frac{2\sqrt{3}}{3}\right)^2 = \frac{16}{3}$
(C) $x^2 + \left(y \mp \frac{8\sqrt{3}}{3}\right)^2 = \frac{64}{3}$
(D) $x^2 + \left(y \mp \frac{2\sqrt{3}}{3}\right)^2 = \frac{8}{3}$

**37.** A line passes through $P$ 5, 3). The segment of this line intercepted by the $x$-axis and $y$-axis is divided by $P$ in the ratio 3 : 7. Find the equation of the line.
(A) $7x - 5y = 50$ (B) $7x - 4y = 28$
(C) $7x + 5y = 50$ (D) $4x - 7y = 13$

**38.** If two consecutive vertices of a square are (3, 2) and (5, 4), then find one of the other vertices.
(A) (6, 3) (B) (7, 2)
(C) (2, 7) (D) (5, 8)

**39.** The centroid of a triangle two of whose vertices are $A(1, 1)$ and $B(3, -1)$, is $(3, 1)$. Then the perimeter of the triangle is

(A) $2\left(\sqrt{2}+\sqrt{5}\right)$ units (B) $2\left(\sqrt{2}+2\sqrt{5}\right)$ units

(C) $2+2\sqrt{5}$ units (D) $2\sqrt{2}+\sqrt{5}$ units

**40.** The angle (in degrees) between the lines $(2-\sqrt{3})x + \sqrt{3}y = 1$ and $(2\sqrt{3}+3)x - y + 1 = 0$ is _____.

## EXERCISE-4

***Directions for questions 1 to 30***: For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** The circumcentre of the triangle formed by $A\left(\sqrt{3}, 0\right)$, $B(0, 3)$ and $C\left(-\sqrt{3}, 0\right)$ is

(A) $(0, 1)$ (B) $(0, 3)$ (C) $(-1, 0)$ (D) $(1, 1)$

**2.** If the axes are translated to the point $(7, -3)$, the new co-ordinates of the point $(1, 1)$ will be

(A) $(6, 4)$ (B) $(-6, -4)$
(C) $(-6, 4)$ (D) $(6, -4)$

**3.** If $A(5, 7)$, $B(2, 3)$ and $C(-2, 0)$ are the three consecutive vertices of a rhombus $ABCD$, find the area of the rhombus $ABCD$.

(A) 6 sq. units (B) 10 sq. units
(C) 7 sq. units (D) 21 sq. units

**4.** If $(4, 4)$ and $(-4, -4)$ are two vertices of an equilateral triangle, the third vertex can possibly be

(A) $(3, 1)$ (B) $\left(4\sqrt{3}, 4\sqrt{3}\right)$

(C) $\left(3\sqrt{3}, 3\sqrt{3}\right)$ (D) $(-4\sqrt{3}, 4\sqrt{3})$

**5.** Find the area of an isosceles right-angled triangle if $(0, 0)$ and $(3, 3)$ are the coordinates of the right-angled vertex and the midpoint of its hypotenuse respectively.

(A) 18 square units (B) 36 square units
(C) 9 square units (D) Cannot be determined

**6.** The vertices of a triangle are $(0, 0)$, $(0, 2)$ and $(2, 0)$. The distance between the orthocentre and the circumcentre is

(A) $5\sqrt{2}$ (B) $2\sqrt{2}$ (C) $\sqrt{2}$ (D) 2

**7.** If the lines $x + 2ay + a = 0$, $x + 3by + b = 0$ and $x + 4cy + c = 0$ are concurrent, then $a, b, c$ are in

(A) arithmetic Progression
(B) geometric Progression
(C) harmonic Progression
(D) None of these

**8.** The area of the triangle (in sq. units) formed by the $y$-axis, the straight line $L$ passing through $(3, 4)$ and $(6, 0)$ and a line perpendicular to $L$ and passing through $(2, 0)$, is closest to

(A) 17 (B) 24
(C) 18 (D) 21

**9.** The equations of the perpendicular bisectors of the sides $AB$ and $AC$ of $\Delta ABC$ are $x - y + 5 = 0$ and $x + 2y = 0$ respectively. If $A = (1, -2)$, the equation of the line $BC$ is

(A) $14x + 23y - 40 = 0$ (B) $3x - 4y + 7 = 0$
(C) $2x + 3y + 11 = 0$ (D) $7x + 8y - 12 = 0$

**10.** Find the distance of the point $(3, 5)$ from the line $2x - y + 2 = 0$, measured along a line parallel to $x - y + 6 = 0$.

(A) $\frac{3}{5}\sqrt{5}$ units (B) $\frac{3}{2}\sqrt{2}$ units

(C) $3\sqrt{2}$ units (D) $4\sqrt{2}$ units

**11.** A straight line through the point $A(1, 2)$ and making an angle of $135°$ with the negative direction of $x$-axis, meets the line $3x + 4y = 12$ at $P$. The distance $AP$ is

(A) $\frac{2}{7}$ (B) $\frac{2}{\sqrt{7}}$

(C) $\sqrt{\frac{2}{7}}$ (D) $\frac{\sqrt{2}}{7}$

**12.** The vertex of an equilateral triangle is $(2, 3)$ and the opposite side lies on $x + y = 2$. The equations of the other sides are

(A) $\left(2-\sqrt{3}\right)x - y - 2\sqrt{3} - 1 = 0$

$\left(2+\sqrt{3}\right)x - y - 2\sqrt{3} + 1 = 0$

(B) $\left(2+\sqrt{3}\right)x - y + 2\sqrt{3} - 1 = 0$

$\left(2+\sqrt{3}\right)x + y - 2\sqrt{3} + 1 = 0$

(C) $\left(2+\sqrt{3}\right)x-y-2\sqrt{3}-1=0$

$\left(2-\sqrt{3}\right)x-y+2\sqrt{3}-1=0$

(D) $\left(2+\sqrt{3}\right)x+y+2\sqrt{3}+1=0$

$\left(2-\sqrt{3}\right)x+y+2\sqrt{3}-1=0$

**13.** The points on the line $x+y=4$ that lie at a unit distance from the line $4x+3y-10=0$ are
(A) (3, 1), (1, 3) (B) (–7, 11) (11, – 7)
(C) (3, 1), (–7, 11) (D) (1, 3), (11, – 7)

**14.** If the lines $4x-y-1=0$ and $12x-3y+3=0$ represent a pair of opposite sides of a square, the area of the square is

(A) $\frac{2}{5}$ sq. units (B) $\frac{36}{\sqrt{153}}$ sq. units

(C) $\frac{18}{153}$ sq. units (D) $\frac{4}{17}$ sq. units

**15.** The acute angle between the lines $x+\sqrt{3}y+6\sqrt{3}=0$ and $y+\sqrt{3}x+2=0$ is
(A) 15° (B) 30°
(C) 60° (D) 75°

**16.** $A$ is (3, 5). The equation of the perpendicular bisector of $AB$ is $2y+x-23=0$. Find $B$.
(A) (7, 13) (B) (6, 11)
(C) (8, 15) (D) (9, 17)

**17.** Find the transformed equation of $2x-3y+7=0$, when the origin is translated to $(1,-1)$.
(A) $2X-3Y+12=0$ (B) $2X-3Y-12=0$
(C) $2X-3Y-12=0$ (D) $2X+3Y-12=0$

**18.** The transformed equation of $x-2y+5=0$, when the axes are rotated by an angle of 45° in the anti-clockwise direction is
(A) $3X+Y-5=0$ (B) $3X+Y+5\sqrt{2}=0$
(C) $X-3Y+5\sqrt{2}=0$ (D) $X+3Y-5\sqrt{2}=0$

**19.** A circle $(x-3)^2+(y-3)^2=9$ is drawn such that both the positive coordinate axes are tangents to it. Find the greatest possible distance from a point $P(2, 2)$ to a point on the circle.
(A) $3+\sqrt{2}$ (B) $3-\sqrt{2}$
(C) $\sqrt{3}-2$ (D) $\sqrt{3}+2$

**20.** The maximum distance between any point on $(x+8)^2+y^2=36$ and any point on $x^2+(y+15)^2=64$ is _____.

**21.** The number of points lying inside the circle $x^2+y^2=9$, which have integer coordinates is
(A) 30 (B) 25 (C) 13 (D) 15

**22.** The point of intersection of $4x+5y=26$ and $y=kx+2$ has integral coordinates. What is the number of integral values that $k$ can take? _____

**23.** If the area of the convex quadrilateral formed by the lines $x+3y=22$, $3x+y=22$ and the coordinate axes is $S$, find the value of $3S$. _____

**24.** The triangle formed by the points of intersection of the curve $|x|y=1$, the line $2y-1=0$ and the origin is
(A) equilateral (B) isosceles
(C) right angle (D) scalene

**25.** The lines $4x-3ky+4=0$ and $2x-5y+1=0$ intersect in the first quadrant. The $x$-coordinate of the point of intersection is twice the $y$-coordinate. Find the value of $k$. _____

**26.** The area of the polygon formed by joining the points $A(-4, 3)$, $B(2, 5)$, $C(5, -2)$, $D(6, -3)$ and $E(-3, -3)$ in order is _____.

**27.** Find the area of the quadrilateral formed by the lines $x+2y=8$ and $2x+y=8$ and the coordinate axes.
(A) $\frac{64}{3}$ (B) $\frac{32}{3}$ (C) $\frac{16}{3}$ (D) 64

**28.** A circle passing through the origin and (0, 5) touches the circle $x^2+y^2=169$ internally, at a point in the first quadrant. The centre of the smaller circle is
(A) (6, 5) (B) (5, 6)
(C) $\left(6, \frac{5}{2}\right)$ (D) $\left(3, \frac{5}{2}\right)$

**29.** A circle has the equation $x^2+y^2=r^2$. One of the lines $5x-4y-20=0$ and $5x-4y+40=0$ is a secant of the circle and the other is a tangent to the circle. Find the radius of the circle.
(A) $\frac{40}{\sqrt{41}}$ (B) $\frac{60}{\sqrt{41}}$ (C) $\frac{30}{\sqrt{41}}$ (D) $\frac{20}{\sqrt{41}}$

**30.** The minimum possible distance between the curves $x^2+y^2-8y+12=0$ and $x^2+y^2+6y=0$ is _____.

## EXERCISE-5 Based on Data Sufficiency

***Directions for questions 1 to 15:*** Each question is followed by two statements, I and II. Answer each question based on the following directions:

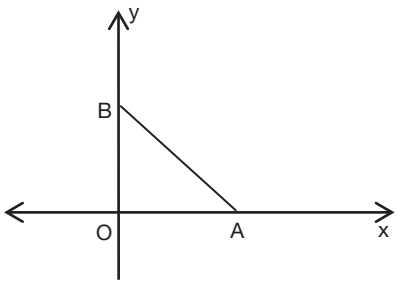
Choose (A) If the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.

Choose (B) If the question can be answered by using either statement alone.

Choose (C) If the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (D) If the question cannot be answered even by using both the statements together.

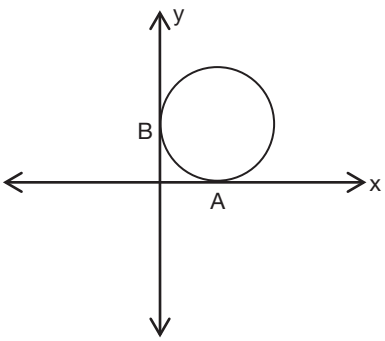
**1.** In the figure below, what is the area of triangle *AOB*?




I. The equation of the line *AB* is $4x + 3y = 12$.

II. The midpoint of the line segment *AB* is $\left(\frac{3}{2}, 2\right)$

**2.**




The circle given in the above figure touches the coordinate axes at the points *A* and *B*, respectively. What is the equation of the circle?

I. The coordinates of the point *A* are (4, 0)

II. The coordinates of the point *B* are (0, 4)

**3.** What is the area of triangle *ABC*?

I. *ABC* is an equilateral triangle.

II. The coordinates of the vertex *A* and the midpoint of the side *BC* are respectively (–3, 5) and (5, 2).

**4.** At which point does the line '$\ell$' intersect the *x*-axis?

I. The line $\ell$ is parallel to *y*-axis.

II. The line $\ell$ passes through the point (3, –4).

**5.** Find the equation of the line '*L*'.

I. *L* is parallel to *x*-axis.

II. The slope of *L* is 0.

**6.** Find the centroid of the triangle *ABC*.

I. The coordinates of *B* are (1, 2).

II. The midpoint of *AB* is (4, 7).

**7.** Does the point (7, 2) lie on the line *L*?

I. *L* passes through (5, 6).

II. The slope of the line *L* is 3/5.

**8.** Is point *P* in the first quadrant?

I. *P* lies within the circle with centre at the origin and radius 3.

II. *P* lies on the straight line $x + 2y = 4$.

**9.** What is the area of the triangle?

I. One of the sides of the triangle is on the *y*-axis.

II. Two of the sides lie on $x + y = 1$ and $x - y = 1$.

**10.** What is the area of the square *ABCD*?

I. The midpoints of *AB* and *CD* are (1, 4) and (1, 8) respectively.

II. The point of intersection of the diagonals is (2, 10) and one of the vertices of the square is (4, 6).

**11.** A circle touches the *x*-axis from above at *A* and its centre *C* is at a distance of *d* units from the *x*-axis. What is the equation of the circle?

I. The coordinates of *A* are (6, 0)

II. $d = 3$

**12.** What is the area of the triangle *AOB*?

I. *AOB* is a right angled isosceles triangle right angled at *O* the origin.

II. The midpoint of AB is (3, 4)

**13.** What is the value of $\theta$, where $\theta$ the angle of inclination of the line with the positive $x$-axis satisfies the condition $0 \le \theta < \pi$?

I. The equation of the line is $3x - \sqrt{3}\,y + k = 0$ ($k$ is real).

II. $\cos\theta = -\frac{1}{2}$

**14.** At how many points do the line and the circle intersect?

I. The equation of the circle is $x^2 + y^2 - 4x - 6y - 12 = 0$

II. The equation of the line is $3x + 4y + 7 = 0$.

**15.** Does the line ($l$) pass through the origin?

I. The line ($l$) passes through only two quadrants.

II. The line ($l$) passes through (5, 5).

## ANSWER KEYS

### Exercise-1

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (B) | 7. (B) | 13. (A) | 19. (A) | 25. (C) | 31. (C) | 37. (D) |
| 2. 0 | 8. (D) | 14. (D) | 20. (B) | 26. (A) | 32. (C) | 38. (B) |
| 3. 3 | 9. (D) | 15. (B) | 21. (A) | 27. (A) | 33. (C) | 39. (A) |
| 4. 5 | 10. (B) | 16. (C) | 22. (D) | 28. (B) | 34. (D) | 40. (D) |
| 5. (D) | 11. (A) | 17. (A) | 23. (A) | 29. (D) | 35. (B) | |
| 6. (C) | 12. (B) | 18. (C) | 24. (B) | 30. (C) | 36. 45 | |

### Exercise-2

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (C) | 7. (A) | 13. 30 | 19. (D) | 25. (D) | 31. (A) | 37. (C) |
| 2. (B) | 8. (C) | 14. 45 | 20. (B) | 26. (B) | 32. (A) | 38. 10 |
| 3. (C) | 9. (A) | 15. (C) | 21. (A) | 27. (B) | 33. (A) | 39. 171 |
| 4. (C) | 10. (C) | 16. (C) | 22. (D) | 28. (D) | 34. (B) | 40. 5 |
| 5. 5 | 11. (C) | 17. (B) | 23. 9 | 29. (D) | 35. 21 | |
| 6. (C) | 12. (B) | 18. (D) | 24. (A) | 30. (C) | 36. (D) | |

### Exercise-3

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (A) | 7. (A) | 13. 3 | 19. (1.3) | 25. (D) | 31. (D) | 37. (C) |
| 2. (B) | 8. (D) | 14. (C) | 20. (B) | 26. (A) | 32. (D) | 38. (B) |
| 3. (32.5) | 9. (A) | 15. (C) | 21. (C) | 27. (B) | 33. (D) | 39. (B) |
| 4. (C) | 10. (C) | 16. (D) | 22. (D) | 28. (D) | 34. (A) | 40. 90 |
| 5. (A) | 11. (B) | 17. (–2) | 23. (A) | 29. (A) | 35. 2 | |
| 6. 3 | 12. (D) | 18. 2 | 24. (A) | 30. (A) | 36. (A) | |

### Exercise-4

| | | | | | |
|---|---|---|---|---|---|
| 1. (A) | 6. (C) | 11. (D) | 16. (A) | 21. (B) | 26. 53 |
| 2. (C) | 7. (C) | 12. (C) | 17. (A) | 22. 1 | 27. (B) |
| 3. (C) | 8. (D) | 13. (C) | 18. (D) | 23. 121 | 28. (C) |
| 4. (D) | 9. (A) | 14. (D) | 19. (A) | 24. (B) | 29. (A) |
| 5. (A) | 10. (C) | 15. (B) | 20. 31 | 25. 4 | 30. 2 |

### Exercise-5

| | | | | |
|---|---|---|---|---|
| 1. (B) | 4. (C) | 7. (C) | 10. (B) | 13. (B) |
| 2. (B) | 5. (D) | 8. (D) | 11. (C) | 14. (C) |
| 3. (C) | 6. (D) | 9. (C) | 12. (C) | 15. (D) |

## SOLUTIONS

## EXERCISE-1

**1.** II$^{nd}$ quadrant.

**2.** Two non-parallel lines lying in the same plane have to intersect.

$\therefore$ The least distance is 0.

**3.** As seen in solution I above, a line can pass through at most 3 quadrants.

**4.** Distance from (0, 0) to (3, 4) is $\sqrt{3^2 + 4^2} = 5$ units.

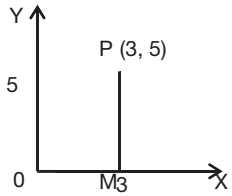
**5.** The points that are at unit distance from (0, 0) lie on a circle of radius 1 and centre at the origin. There are infinitely many such points on this circle.

**6.** The distance from the origin to the point $(x, y)$ is $\sqrt{x^2 + y^2}$

The distance from the origin to the point (5, 12) is $\sqrt{5^2 + 12^2} = \sqrt{169} = 13$.

**7.** The perpendicular distance from $P(3, 5)$ to the $x$-axis is the $y$-coordinate of the point $P(3, 5)$.

$\therefore$ Distance is 5 units.




**8.** The distance from (0, 0) to (1, 3) is $\sqrt{10}$

The distance from (0, 0) to (2, –3) is $\sqrt{13}$

The distance from (0, 0) to (–3, 0) is $\sqrt{9}$

The distance from (0, 0) to (1, 2) is $\sqrt{5}$

$\therefore$ The point nearest to the origin is (1, 2).

**9.** The given lines are $x - 3 = 0$ and $y + 2 = 0$

$\therefore$ Their point of intersection is (3, –2).

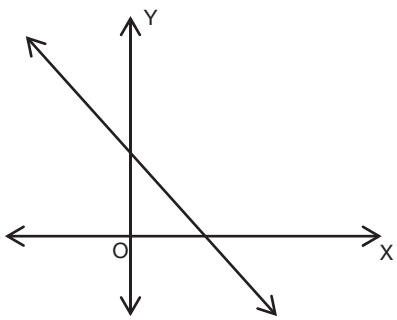
**10.** The $x$-axis divides the line joining the points $(x_1, y_1)$ and $(x_2, y_2)$ in the ratio $-y_1 : y_2$

The given points are (3, 5) and (4, –6)

The ratio is = –5 :–6 = 5 : 6. (internally)

**11.** $G$ divides $DA$ in the ratio 1 : 2.

**12.** As can be seen in the figure above, if a line with slope –1 passes through the I quadrant, it can't pass through the III quadrant




**13.** The distance of $(x_1, y_1)$ from $ax + by + c = 0$ is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$\therefore$ The distance of (0, 0) from $mx - y + c = 0$ is $\left|\frac{c}{\sqrt{1 + m^2}}\right|$.

**14.** The required distance is $\left|\frac{c}{\sqrt{a^2 + b^2}}\right|$

**15.** Let $A = (x_1, y_1)$ and $B = (-3x_1, -3y_1)$

The origin (0, 0) divides $AB$ in the ratio say $m : n$ ( say).

$\therefore 0 = \frac{m(-3x_1) + n(x_1)}{m + n}$

$\therefore -3m + n = 0 \Rightarrow n = 3m$

$\therefore m : n = m : 3m = 1 : 3$

As $m : n$ is positive, this is internal division.

**Alternative Solution:**

The slope of the line joining $(x_1, y_1)$ and $(-3x_1, -3y_1)$ is $\frac{y_1}{x_1}$.

$\therefore$ Its equation is $y = \frac{y_1}{x_1}x$.

The origin lies on the line.
Distance between the origin and $(x_1, y_1) = \sqrt{x^2{}_1 + y^2{}_1}$.

Distance between the origin and $(-3x_1, -3y_1)$

$= 3\sqrt{x_1{}^2 + y_1{}^2}$

The origin divides the line segment joining $(x_1, y_1)$ and $(-3x_1, -3y_1)$ in the ratio 1 : 3 internally. ($\because$ The origin lies on the line segment).

**16.** The side of the square is distance between (0, 0) and $(0, \alpha)$ i.e. $\alpha$ then its diagonal is $\sqrt{2}\,\alpha$.

**17.** Any equilateral triangle will contain at least one vertex which has irrational coordinates.

**18.** Area of $\Delta ABC = 4$ (area of $\Delta DEF$) = 4(5) = 20.

**19.** The line $x = 3$ is parallel to the $y$-axis.

A line perpendicular to the $y$-axis is parallel to the $x$-axis.

$\therefore$ The slope of $x$-axis (or a line parallel to $x$-axis) is 0.

**20.** Slope of the line joining the points $(a, b)$ and $(c, d)$ is $\frac{d - b}{c - a}$

Given slope is 0

$\frac{d - b}{c - a} = 0 \Rightarrow d - b = 0 \Rightarrow b = d$.

**21.** Given $\theta = 135^\circ \Rightarrow$ slope $(m) = \tan\theta = \tan 135^\circ = -1$.

**22.** Slope of the line $ax + by + c = 0$ is $-a/b$
$\therefore$ Slope of given line is $-1 / \sqrt{3}$.

**23.** The largest chord of a circle is its diameter. If the diameter is 2, then the radius is 1 and the area is $\pi(1^2) = \pi$

**24.** $(x_1, y_1)$ and $(-x_1, y_1)$ are two opposite vertices of the square.

$\therefore$ The midpoint of the line segment joining these vertices is the common midpoint of the diagonals, i.e., point of intersection of the diagonals is $(0, y_1)$ [$\because$ Midpoint of the line segment joining $(x_1, y_1)$ and $(x_2, y_2)$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$].

The diagonal joining the given opposite vertices lies on the $x$-axis.

$\therefore$ The other diagonal would lie on the $y$-axis ($\because$ The diagonals of the square are perpendicular). (1)

Distance from the intersection point of the diagonals to each of the given vertices is $|x_1|$ (2)

From (1) and (2), the other two vertices are $(0, y_1 + x_1)$ and $(0, y_1 - x_1)$.

**25.** The area of $\Delta ABC$ is zero means $A$, $B$, $C$, are collinear. If $A$ and $B$ are known points, we can say that $C$ lies on line $AB$.

**26.** The slope of the line is $x - \sqrt{3}\, y + 4 = 0$ is $\frac{1}{\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$

$\therefore$ The inclination of the line is $30^\circ$.

$\therefore$ The required angle is $30^\circ$.

**27.** We know that $x = 5$ is parallel to the $y$-axis and $y = 7$ is parallel to the $x$-axis.

$\therefore$ The angle between the two lines is $90^\circ$.

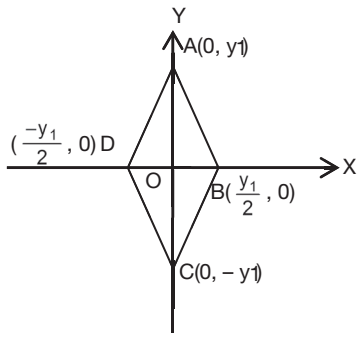
**28.** The equation of the line perpendicular to $3x + 4y + 5 = 0$ is of the form $4x - 3y + k = 0$.

**29.** We know that the equation of a line with slope $m$ and $y$-intercept $c$ is $y = mx + c$

Given slope is $\frac{1}{m}$; $y$-intercept is '$a$'.

$\therefore y = \frac{1}{m}x + a \Rightarrow my = x + am.$

**30.** The equation of a line parallel to the $x$-axis and passing through $(h, k)$ is $y = k$.




The equation of a line parallel to the $x$-axis and passing through $(3, 4)$ is $y = 4$.

**31.** The vertices $A$ and $C$ of the rhombus $ABCD$ are given

$\therefore$ The midpoint of $AC$, i.e., $(0, 0)$ is the point where BD intersects $AC$. Further $BD \perp AC$.

$\therefore BD$ lies on the $x$-axis

As $BD = (1/2)\ AC$, $B = \left(\frac{y_1}{2}, 0\right)$

and $D = \left(\frac{-y_1}{2}, 0\right)$

Both (A) and (B) are vertices.

**32.** The required line (ray $m$) is perpendicular to the given line (say $\ell$) $y = 3x + 1$

$\therefore$ The product of the slopes of $\ell$ and $m$ is $-1$.

The slope of $\ell$ is 3. $\therefore$ The slope of $m$ has to be $\frac{-1}{3}$

Equation of $m$ is $\frac{-1}{3} = \frac{y-1}{x-1}$ i.e. $3y = -x + 4$.

**33.** The centroid of the triangle whose vertices are $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is

$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

The centroid of the given triangle is

$\left(\frac{0+5+0}{3}, \frac{0+0+12}{3}\right)$, i.e., $\left(\frac{5}{3}, 4\right)$

**34.** The points in the choices $A$, $B$ and $C$ lie on the line.

**35.** Sum of intercepts $= 0 \Rightarrow a + b = 0$

$\Rightarrow a = -b\ \frac{x}{a} + \frac{y}{b} = 1 \Rightarrow x - y = -b$

It passes through $(2, 3)$

$2 - 3 = -b \Rightarrow b = +1$

The line is $x - y + 1 = 0$.

**36.** Given equation is $x - y = 0$

i.e., $y = x$

comparing with $y = mx + c$

We have, $m = 1$

But $m = \tan\theta$

$\therefore \tan\theta = 1 \Rightarrow \theta = 45°$

$\therefore$ The required angle is $45°$

**37.** The line is parallel to $y = 2x$.

$\therefore$ The line has the same slope as that of $y = 2x$

The slope of any line in the form $y = mx + c$ is $m$. Slope of the considered line = That of $y = 2x$, which is 2.

Equation of the line is $2 = \dfrac{y-4}{x-3}$, i.e., $y = 2x - 2$.

**38.** The circle is centred at the origin and pass through (3, 4).

$\therefore$ Radius of the circle = distance between the origin and

$(3, 4) = \sqrt{(3-0)^2 + (4-0)^2} = 5$.

Circumference of the circle = $2\pi$ (Radius) = $10\ \pi$.

**39.** The midpoint of the line segment whose end points are

$(x_1, y_1)$, $(x_2, y_2)$, is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Midpoint of the line segment whose ends are

(1, 6) and (9, 12) is $\left(\dfrac{1+9}{2}, \dfrac{6+12}{2}\right)$, i.e., (5, 9)

**40.** We know that a line passing through only two quadrants is parallel to the $x$-axis or the $y$-axis or passes through the origin. All options can be true.

## EXERCISE-2

**1.** Centre of the circle

$x^2 + y^2 = a^2$ is (0, 0)

Distance = $\sqrt{6^2 + 8^2} = 10$ units.

**2.** Let the ratio be $m : n$

$(x, 0) = \left(\dfrac{2n + (-4)m}{m+n}, \dfrac{3n+m}{m+n}\right)$

$\Rightarrow 3n + m = 0 \Rightarrow m/n = -3/1$

$\therefore$ 3 : 1 externally.

**3.** If 3 points are collinear then area of triangle formed = 0 or slopes are equal

$\Delta = \dfrac{1}{2}\begin{vmatrix} 2-3 & 3+1 \\ 3-1 & 1-k \end{vmatrix} = 0$

$\Rightarrow (-1)\ (1-k) - 8 = 0$

$\Rightarrow k - 1 - 8 = 0 \Rightarrow k = 9.$

**4.** Let the fourth vertex be $(x_4, y_4)$

$(x_4, y_4) = (x_1 - x_2 + x_3, y_1 - y_2 + y_3)$

$= (1 + 2 + 4, 3 - 1 - 2) = (7, 0)$.

**5.** The centre of the circle $(x-g)^2 + (y-f)^2 = r^2$ is $(g, f)$

$\therefore$ The centre of the circle $(x-6)^2 + (y-3)^2 = 25$ is (6, 3)

Distance between the points (3, 7), (6, 3) is

$\sqrt{(3-6)^2 + (7-3)^2} = \sqrt{9+16} = 5$ units

**6.** We know that the ratio in which the $y$-axis divides the line joining the points $(x_1, y_1)$ and $(x_2, y_2)$ is $-x_1 : x_2$. Here the points are (4, 3), (–6, 2).

$\therefore$ The required ratio is $-4 : -6 = 2 : 3$

**7.** In a triangle, the centroid divides the segment joining the orthocentre, 'O' and the circumcentre, $S$ in the ratio 2 : 1 internally.

Given O(4, 5), G(3, 3).

Let $S = (x, y)$

The centroid $G$ is

$\left(\dfrac{2x+4}{2+1}, \dfrac{2x+5}{2+1}\right) = (3,3)$

$2x + 4 = 9; \qquad 2y + 5 = 9$

$2x = 5 \qquad 2y = 4$

$x = \dfrac{5}{2} \qquad y = 2$

$\therefore$ Circumcentre $\left(\dfrac{5}{2}, 2\right)$

**8.** We know that, the slope of the line joining the points $(x_1, y_1)$ and $(x_2, y_2)$ is $\dfrac{y_2 - y_1}{x_2 - x_1}$.

Given, points are $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$

$\Rightarrow$ slope = $\dfrac{2at_2 - 2at_1}{at_2^2 - at_1^2} = \dfrac{2a(t_2 - t_1)}{a(t_2 + t_1)(t_2 - t_1)} = \dfrac{2}{t_2 + t_1}$

Difficulty Level-2: Moderate

**9.** Equation of the line passing through (2, 3) and (3, 2) is

$(y-3)=\left(\frac{2-3}{3-2}\right)(x-2) \Rightarrow y-3=-1(x-2)$

$\Rightarrow x+y-5=0.$

**10.** The vertex containing the right angle is the point of intersection of perpendicular lines, i.e., $x-2y-3=0$ and $2x+y-1=0$. Solving $x=1; y=-1$

**11.** $2x+3y+5=0$ (1)

$x+y+2=0$ (2)

Solving (1) and (2), we get $x=-1, y=-1$

$\therefore$ $(-1,-1)$ lies in the third quadrant.

**12.** $\frac{a_1}{a_2}=\frac{b_1}{b_2} \Rightarrow \frac{\sqrt{3}}{\sqrt{2}}=\frac{\sqrt{2}}{k} \Rightarrow k=\frac{2}{\sqrt{3}}.$

**13.** The given line is $\sqrt{3}x-y+9=0$.

Slope of the line is $\sqrt{3}$.

$\therefore$ The angle made by the line with $x$-axis is 60°.

Hence, the angle made by the line with $y$-axis is 90 – 60 = 30°.

**14.** If $m_1$ and $m_2$ are the slopes of two lines and $\theta$ is the acute angle between the lines, then $\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$.

Given lines are $2x+3y+7=0$ (1)

and $x-5y+3=0$ (2)

Slope of lines (1) $=m_1=-\frac{2}{3}$

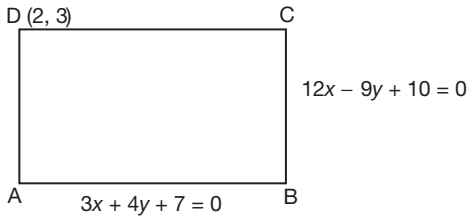
Slope of lines (2) $=m_2=\frac{1}{5}$

$\tan\theta=\left|\frac{-\frac{2}{3}-\frac{1}{5}}{1+\left(-\frac{2}{3}\right)\frac{1}{5}}\right|$

$\tan\theta=\frac{\frac{13}{15}}{\frac{13}{15}}$

$\tan\theta=1 \Rightarrow \theta=45°$

**15.** The given lines $3x+4y+7=0$ and $12x-9y+10=0$ represent the adjacent sides $AB$ and $BC$ of a rectangle $ABCD$, since the point (2, 3) does not lie on the two given lines. So, (2, 3) represents the vertex '$D$'.




Length/breadth of the rectangle is the perpendicular distance from (2, 3) to the line $12x-9y+10=0$

i.e., $\left|\frac{12(2)-9(3)+10}{\sqrt{12^2+(-9)^2}}\right|=\frac{7}{15}$ units.

Breadth of the rectangle is the perpendicular distance from (2, 3) to the line $3x+4y+7=0$

i.e.,

$\therefore$ Area of the rectangle = length × breadth

$=5\times\frac{7}{15}=\frac{7}{3}$ sq. units.

**16.** The given line passes through (0, 3) and (–4, 0)

$\therefore (y-3)=3/4(x-0) \Rightarrow 3x-4y+12=0.$

**17.** $m=\tan\theta=\tan45°=1$

The line passes through (0, –2) and makes an angle of 45° with $x$-axis. $\therefore y=mx+c$

$y=x+c$ (since $m=1$)

$-2=0+c \Rightarrow c=-2$

$\therefore$ Line is $x-y-2=0$.

**18.** Area formed by the line $ax+by+c=0$ and coordinate axes is $\frac{1}{2}\frac{c^2}{|ab|}$

$3x+4y=24$ $\therefore$Area $=\frac{1}{2}\frac{(24)^2}{12}=24$ sq. units.

**19.** $d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}=\frac{|3-16+3|}{\sqrt{3^2+4^2}}=\frac{10}{5}=2$ units.

**20.** Given lines are parallel

Distance between them is the length of the side

$d=\frac{|1-3|}{\sqrt{1+4}}=\frac{2}{\sqrt{5}}$, Area = $d^2=4/5$ sq. units.

**21.** The equation of the line joining $A(3,5)$, $C(6,9)$ is

$y-5=\frac{9-5}{6-3}(x-3)$

$y-5=\frac{4}{3}(x-3)$

$3y - 15 = 4x - 12 \Rightarrow 4x - 3y + 3 = 0$

The length of altitude drawn from $B$ to $AC$ = The perpendicular distance from $B(3, 9)$ to the line

$4x - 3y + 3 = 0$

$$\therefore \frac{|4(3) - 3(9) + 3|}{\sqrt{4^2 + (-3)^2}} = \frac{12}{5}$$

**22.** Given: $A(4, 5)$ $B(3, 6)$ and $C(2, 1)$ are the vertices of the triangle $ABC$.

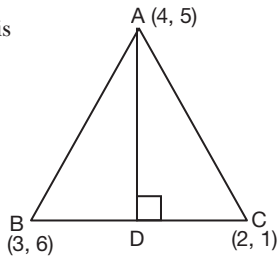
Let $AD$ be the altitude, then $AD \perp BC$.

Slope of $AD \times$ Slope of $BC = -1$

Slope of $AD \times 5 = -1$

Slope of $AD = -\frac{1}{5}$.

Equation of $AD$ having slope $= -\frac{1}{5}$ and passing through the point $A(4, 5)$ is




$y - y_1 = m(x - x_1)$

i.e., $y - 5 = -\frac{1}{5}(x - 4)$

$5y - 25 = -x + 4$

$x + 5y - 29 = 0$

**23.** The given line is $3x + 4y + 5 + k(x - 3y + 2) = 0$

i.e., $(3 + k)x + (4 - 3k)y + 5 + 2k = 0$

If a line is parallel to $x$-axis, then the coefficient of $x$ must be zero.

$\therefore 3 + k = 0 \Rightarrow k = -3 \Rightarrow k^2 = 9$

**24.** If two lines are perpendicular to each other, then $m_1 \times m_2 = -1$, where $m_1$ & $m_2$ are the slopes of the two lines.

The given lines are $ax + 3y + 7 = 0$, $4x + 9y + 15 = 0$.

The slopes of the lines are $-\frac{a}{3}, -\frac{4}{9}$ respectively.

$$\therefore -\frac{a}{3} \times -\frac{4}{9} = -1$$

$4a = -27 \Rightarrow a = -\frac{27}{4}$

**25.** Given: lines are $4x + 5y - 23 = 0$, $x + 3y - 11 = 0$

Solving these equations, we get, $x = 2$, $y = 3$

Since the line $x + ky + 3k + 2 = 0$ passes through the point $(2, 3)$,

$2 + 3k + 3k + 2 = 0$

$6k = -4$

$k = -\frac{4}{6} = -\frac{2}{3}$

**26.** On solving the equation $x^2 - 5x - 6 = 0$, we get the roots $6, -1$.

$\therefore$ The equation of the line can be

$y = 6x - 1$ (when slope = 6 and $y$-intercept is $-1$) or $y = -x + 6$ (when slope = $-1$ and $y$-intercept is 6).

**27.** Two of the tangents to the circle are $x + y = 7$ and $x + y = \frac{-13}{2}$. These are parallel tangents. The diameter of the circle is the distance between the tangents.

Distance between the tangents is $\frac{\left|-7 - \left(\frac{13}{2}\right)\right|}{\sqrt{2}}$, i.e., $\frac{27}{2\sqrt{2}}$

Circumference of the circle $= \frac{27}{2\sqrt{2}}\pi$.

**28.** The inclination of the line is $180° - 30° = 150°$

The slope of the line is $\tan 150° = \frac{-1}{\sqrt{3}}$

The equation of the line is $y = mx + c$; where $m$ is the slope and $c$ is the $y$-intercept. $\Rightarrow y = \frac{-1}{\sqrt{3}}x + 4$

$x + \sqrt{3}y = 4\sqrt{3}$.

**29.** The inclination of the line is $30°$

The slope of the line is $\tan 30° = \frac{1}{\sqrt{3}}$

The equation of the line is

$y - 4 = \frac{1}{\sqrt{3}}(x - 3)$

$x - 3 = \sqrt{3}y - 4\sqrt{3}$

$x - \sqrt{3}y + 4\sqrt{3} - 3 = 0$

This line cuts the $y$-axis at $Q(0, b)$

$0 - \sqrt{3} + 4\sqrt{3} - 3 = 0 \Rightarrow b = 4 - \sqrt{3}$

The distance between $P(3, 4)$ and $Q(0, 4 - \sqrt{3}$

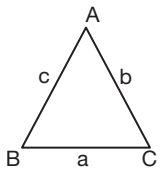
$= \sqrt{(3)^2 + \left(-\sqrt{3}\right)^2} = \sqrt{9 + 3} = \sqrt{12} = 2\sqrt{3}$.

**30.** Given: $A = (1, 1)$, $B = (5, 4)$, $C = (4, -3)$

$a = BC$

$= \sqrt{(4-5)^2 + (-3-4)^2}$

$= \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$




$b = CA$

$= \sqrt{(1-4)^2 + (1+3)^2} = \sqrt{9+16} = \sqrt{25} = 5$

$c = AB$

$= \sqrt{(5-1)^2 + (4-1)^2} = \sqrt{16+9} = \sqrt{25} = 5$

$\therefore$ The perimeter of the triangle $= a + b + c$

$= 5\sqrt{2} + 5 + 5 = 5\left(\sqrt{2} + 2\right)$.

**31.** Since $l$ is perpendicular to $3x - 2y = 0$, the equation of line $l$ is $2x + 3y + k = 0$.

The above line passes through $(3, 2)$.

$\therefore 2(3) + 3(2) + k = 0 \Rightarrow k = -12$

$\therefore$ The equation of the line $(l)$ in $2x + 3y - 12 = 0$

The point of intersection of $2x + 3y - 12 = 0$ and $x + y = 4$ is $(0, 4)$

$\therefore (a, b) = (0, 4) \Rightarrow a = 0; b = 4$

$b^3 - a^3 = 64 - 0 = 64$.

**32.** When the origin is shifted to point $(h, k)$, point $(x, y)$ is changed as $[(x + h), (y + k)]$. Since the origin is shifted to $(3, 4)$, point $(x, y)$ is changed as $(x + 3, y + 4)$

The equation $5x + 6y + 18 = 0$ changes as

$5(x + 3) + 6(y + 4) + 18 = 0$

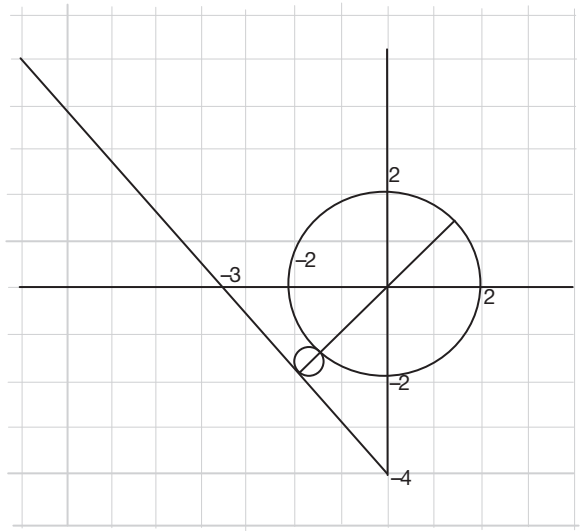
$5x + 15 + 6y + 24 + 18 = 0$

$5x + 6y + 57 = 0$.

**33.** The given circle is $x^2 + y^2 = 4^2$ (say $C$). The centre $(0, 0)$ radius $r = 2$. The given line is $4x + 3y + 12 = 0$ (say $L$). There are many circles which touch both $C$ and $L$. The smallest is shown in the figure.

The diameter of the required circle = perpendicular distance from origin to the line – radius of the given circle.

$$\left|\frac{12}{\sqrt{3^2+4^2}}\right| - 2 = \frac{2}{5} \Rightarrow \text{radius} = \frac{2}{5(2)} = \frac{1}{5}$$




Area of that circle $= \pi\left(\frac{1}{5}\right)^2 = \frac{\pi}{25}$.

**34.** $\frac{x}{a} + \frac{y}{b} = 1$

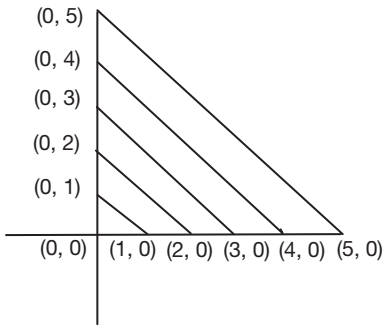
$\Rightarrow \frac{x}{7/3} + \frac{y}{-3} = 1$

$\Rightarrow \frac{3x}{7} - \frac{y}{3} = 1$

$\Rightarrow 9x - 7y = 21$

$\Rightarrow 9x - 7y - 21 = 0$.

**35.**




$P_1 = (0, 0)$

Line joining $(1, 0)$ $(0, 1)$ has 2 points

$P_2 = (1, 0)$, $P_3 = (0, 1)$

Line joining (2, 0) (0, 2) has 3 points

$P_4 = (2, 0), P_5 = (1, 1), P_6 = (0, 2)$

Line joining (3, 0) (0, 3) has 4 points

Line joining (4, 0) (0, 4) has 5 points

Line joining (5, 0) (0, 5) has 6 points

Totally 21 points.

**36.** Given: $(\alpha, \beta) = (1, 1)$ and $f(X, Y) = 2X^2 - 3XY - Y^2 - 5 = 0$

The original equation is $f(x - \infty, y - \beta) = f(x - 1, y - 1) = 0$

$2(x-1)^2 - 3(x-1)(y-1) - (y-1)^2 - 5 = 0$

$2x^2 - 3xy - y^2 - x + 5y - 7 = 0$

**37.** Given quadratic equation is $x^2 + 7x + 12 = 0$

$\Rightarrow (x + 3)(x + 4) = 0$

$\Rightarrow$ Roots are $-3, -4$

$\therefore$ The ordered pair $(x, y) = (-3, -4)$

Clearly it satisfies $2x - 5y = 14$

**38.** Let $L$ be the line,

Slope of the line $m = \tan 60^\circ = \sqrt{3}$.

Equation of the line having slope $\sqrt{3}$ and passing through the point (5, 3) is $y - 3 = \sqrt{3}(x - 5)$

$\sqrt{3}x - y - 5\sqrt{3} + 3 = 0$

Since this cuts the $y$-axis at $Q$, $x = 0$

$\therefore -y - 5\sqrt{3} + 3 = 0$

$\Rightarrow y = 3 - 5\sqrt{3}$,

$\therefore$ The point $Q(0, 3 - 5\sqrt{3})$.

$$PQ = \sqrt{(5-0)^2 + \left(3 - \left(3 - 5\sqrt{3}\right)\right)^2}$$

$$= \sqrt{25 + 75} = \sqrt{100} = 10 \text{ units}$$

**39.** The triangle formed by the given vertices is shown in the figure.

The triangle meets the coordinate axes at $A(20, 0)$ and $B(0, 20)$.

$\therefore$ Equation of line $AB$ is $\frac{x}{20} + \frac{y}{20} = 1$

$x + y = 20$

Now, we find the intercepts which satisfies the following conditions.

$x + y < 20$

$x > 0 ; y > 0$

When $x = 1$, the number of corresponding values are 18. Similarly, when $x = 2$, the number of corresponding values are 17.

$\therefore$ The total number of ordered pairs of $x$ and $y$ which satisfies the above conditions are

$$18 + 17 + \ldots + 1 = \frac{19 \times 18}{2} = 171$$

**40.** The given lines are

$3x + 4ky + 6 = 0$

$kx - 3y + 9 = 0$

Solving these two equations,

We get $\left(\frac{36k+18}{-(9+4k^2)}, \frac{6k-27}{-(9+4k^2)}\right)$

Since the point is in the second quadrant, $x < 0, y > 0$

$$\therefore \frac{36k+18}{-(9+4k^2)} < 0; \frac{6k-27}{-(9+4k^2)} > 0$$

$36k + 18 > 0$ and $6k - 27 < 0$

$k > -\frac{1}{2}$ $k < \frac{9}{2}$

$\therefore -\frac{1}{2} < k < \frac{9}{2}$

$\therefore$ The integral values that $k$ can take are 0, 1, 2, 3, 4.

Hence, 5 integral values of $k$ satisfy the equation.

## EXERCISE-3

**1.** Let $A = (1, -1)$, $B = (-1, 4)$ and $C = (4, 6)$.

Using the distance formula, we find $AB$, $BC$ and $CA$.

$$AB = \sqrt{(-1-1)^2 + (4+1)^2}$$

$$= \sqrt{4 + 25} = \sqrt{29} \text{ units}$$

$\Rightarrow AB^2 = 29.$

$$BC = \sqrt{(4+1)^2 + (6-4)^2}$$

$$= \sqrt{25 + 4} = \sqrt{29} \text{ units}$$

$\Rightarrow BC^2 = 29.$

$$CA = \sqrt{(4-1)^2 + (6+1)^2}$$

$= \sqrt{9+49} = \sqrt{58}$ units

$\Rightarrow CA^2 = 58$

$AB^2 + BC^2 = 29 + 29 = 58$

Hence, $AB^2 + BC^2 = CA^2$ and $AB = BC$

$\therefore$ The given points form an isosceles right-angled triangle.

**2.** $y - y_1 = m(x - x_1)$

Here, $m = 2$, $(x_1, y_1) = (1, -1)$

$y + 1 = 2(x - 1)$

$y + 1 = 2x - 2$

$\Rightarrow y = 2x - 3.$

**3.** The area of the polygon joining the points $(x_1, y_1)$, $(x_2, y_2)$, $(x_3, y_3)$, $(x_4, y_4)$ and $(x_5, y_5)$ is

$$\frac{1}{2}\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_5 & y_1 \end{matrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & 3 & 2 & -1 & -4 & 1 \\ -1 & 2 & 5 & 7 & 1 & -1 \end{vmatrix}$$

$$= \frac{1}{2}\,|2 + 3 + 15 - 4 + 14 + 5 + (-1 + 28) + 4 - 1|$$

$$= \frac{65}{2} = 32.5.$$

**4.** Let $(x, y)$ and $(X, Y)$ be the coordinates of the point in the original system and the new system respectively and the angle of rotation be '$\theta$'. Then

$x = X\cos\theta - Y\sin\theta$

$y = X\sin\theta + Y\cos\theta$

Here, $(X, Y) = \left(-2\sqrt{2}, 3\sqrt{2}\right)$ and $\theta = 45°$

$$\Rightarrow \quad x = -2\sqrt{2}\cos 45° - 3\sqrt{2}\sin 45° = \frac{-2\sqrt{2}}{\sqrt{2}} - \frac{3\sqrt{2}}{\sqrt{2}} = -5$$

and $y = -2\sqrt{2}\sin 45° + 3\sqrt{2}\cos 45°$

$$= \frac{-2\sqrt{2}}{\sqrt{2}} + \frac{3\sqrt{2}}{\sqrt{2}} = 1$$

$\therefore (x, y) = (-5, 1)$

**5.** $2(x^2 + y^2) + 12x - 8y - 46 = 0$

$\Rightarrow x^2 + y^2 + 6x - 4y - 23 = 0$

$\Rightarrow (x + 3)^2 + (y - 2)^2 - 9 - 4 - 23 = 0$

$\Rightarrow (x + 3)^2 + (y - 2)^2 = (6)^2$ which is in the form $(x - a)^2 + (y - b)^2 = r^2$ which represents a circle with centre $(a, b)$ and radius $r$, i.e., The centre is $(-3,2)$ and the radius is 6.

**Alternative Solution:**

The given equation can be expressed as $x^2 + y^2 + 6x - 4y - 23 = 0$

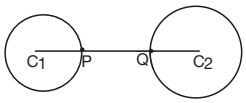
This is in the form of $x^2 + y^2 + 2gx + 2fy + c = 0$. The centre is $(-g, -f)$ and radius $r = \sqrt{g^2 + f^2 - c}$

$x^2 + y^2 + 6x - 4y - 23 = 0$

Centre is $\left(\frac{-6}{2}, \frac{4}{2}\right) = (-3, 2)$ and radius

$r = \sqrt{(-3)^2 + (2)^2 - (-23)} = 6$ units

**6.**




The centres and the radii of the two given circles are

$C_1 = (-3, 5)$, $r_1 = \sqrt{(-3)^2 + (5)^2 - 9} = 5$ and

$C_2 = (-11, -10)$, $r_2 = \sqrt{(11)^2 + (10)^2 - 140} = 9$

Let $PQ$ be the shortest distance between the circles

$\therefore PQ = C_1C_2 - C_1P - C_2Q = C_1C_2 - r_1 - r_2$

$C_1C_2 = \sqrt{(-11+3)^2 + (-10-5)^2} = 17$. Here, $C_1C_2 > r_1 + r_2$

$r_1 = 5$, $r_2 = 9$

$\therefore$ PQ = 17 – (5 + 9) = 3 units

**NOTE**

If $C_1C_2 < r_2 - r_1$ then the shortest distance between the circles would be $r_2 - r_1 - C_1C_2$.

**7.** The distance between parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}$

The distance between the parallel lines $3x + y - 5 = 0$ and $12x + 4y + 15 = 0$, i.e., $= 3x + y + \frac{15}{4} = 0$

$$\frac{\left|\frac{15}{4} + 5\right|}{\sqrt{9+1}} = \frac{35}{4\sqrt{10}}.$$

**8.** Let $P(a, b)$ be the point on the line $4x + 5y = 6$.

$\Rightarrow 4a + 5b = 6$ (1)

Let $P(a, b)$ divide $A(1, 2)$, $B(3, -2)$ in the ratio $k : 1$ then

$$P = \left(\frac{3k+1\times1}{k+1}, \frac{-2k+1\times2}{k+1}\right)$$

$$(a, b) = \left(\frac{3k+1}{k+1}, \frac{-2k+2}{k+1}\right)$$

$$a = \frac{3k+1}{k+1}, b = \frac{-2k+2}{k+1}$$

Substituting the values of $a$ and $b$ in (1)

$$4\frac{(3k+1)}{k+1} + 5\frac{(-2k+2)}{k+1} = 6$$

$12k + 4 - 10k + 10 = 6k + 6$

$10 + 4 - 6 = 6k - 2k \Rightarrow 4k = 8 \Rightarrow k = 2$

$\therefore$ The required ratio is 2 : 1 internally.

**Alternative Solution:**

If a line $ax + by + c = 0$ divides the join of $A(x_1, y_1)$ and $B(x_2, y_2)$ in $m : n$ ratio then $\frac{m}{n} = -\left(\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}\right)$, here the line is $4x + 5y - 6 = 0$,

A(1,2) and $B(3, -2)$

$$\therefore \frac{m}{n} = -\left(\frac{4+10-6}{12-10-6}\right) = \frac{2}{1}$$

$\therefore m : n = 2 : 1$ internally.

**9.** The radius of the circle = The distance between (3, 4) and (–3, 12)

i.e., $= \sqrt{6^2 + 8^2} = 10$

$\therefore$ The side of the square = diameter of the circle = 20

Area of the square = $(20)^2 = 400$

Area of the circle = $\pi r^2 = (10)^2 \pi = 100\pi$

Area of the shaded region

$$= \frac{\text{area of the square} - \text{area of the circle}}{2}$$

$$= \frac{400 - 100\pi}{2} = \frac{100(4-\pi)}{2} = 50(4-\pi)$$

**10.** $x + y + 1 = 0$ (1)

$x^2 - 4y - 16 = 0$ (2)

$y = -(x + 1)$

$x^2 - 4(-(x + 1)) - 16 = 0$

$x^2 + 4x - 12 = 0 = (x + 6)(x - 2) = 0$

$x = -6; 2$

$\therefore y = 5; -3$

The points of intersection are $A(-6, 5)$, $B(2, -3)$

Given: $C(5, 8)$

$$AB = \sqrt{8^2 + 8^2} = \sqrt{64 + 64} = \sqrt{128}$$

$$BC = \sqrt{3^2 + 11^2} = \sqrt{121 + 9} = \sqrt{130}$$

$$CA = \sqrt{11^2 + 3^2} = \sqrt{121 + 9} = \sqrt{130}$$

$\therefore ABC$ is an isosceles triangle.

**11.** Let the circle touch the $x$-axis and $y$-axis at $P$ and $Q$ respectively then $P(h, 0)$, $Q(0, h)$

$\therefore$ The centre of the circle is $C(h, h)$

Radius of the circle is $h$,

Given: $A(4, 2)$ is point on the circle

$AC = h$

$(h - 2)^2 + (h - 4)^2 = h^2$

$h^2 - 4h + 4 + h^2 - 8h + 16 = h^2$

$h^2 - 12h + 20 = 0$

$(h - 10)(h - 2) = 0$

$h = 10$ or $h = 2$

Since $A(4, 2)$ is left side to $P$, therefore, $h$ must be 10

$\therefore$ Radius of the circle is 10

$\therefore$ The area of the circle is $\pi r^2 = 2\pi(10)^2$

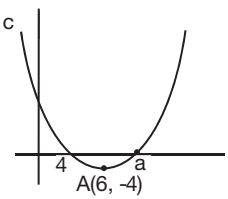
$= 100\pi$ sq. units.

**12.** Given curve is $x^2 - 12x + 32$

$= x^2 - 12x + 36 - 4$

$= (x - 6)^2 - 4$

The minimum value of the expression is –4 and is attained at $x = 6$




The shortest distance from the curve $x^2 - 12x + 32$ to the point (6, –9) is equal to the distance between (6, –4) and (6, –9), i.e., the required distance is = 5.

**13.** We have slope of the line joining the points

$A(x_1, y_1)$ and $B(x_2, y_2)$ is,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Given: $(x_1, y_1) = (4, p)$

$(x_2, y_2) = (p, 5)$ and

$m = -2$

$\therefore \dfrac{5-p}{p-4} = -2$

$\Rightarrow 5 - p = -2\,(p-4) \Rightarrow 5 - p = -2p + 8$

$\Rightarrow p = 3$

**14.** The points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are said to be collinear if slope of $AB$ = slope of $BC$

Given: $A\,(x_1, y_1) = (p+1, 1)$

$B\,(x_2, y_2) = (2p+1, 3)$

$C\,(x_3, y_3) = (2p+2, 2p)$

We have,

Slope, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Slope of AB = Slope of BC

$\Rightarrow \dfrac{3-1}{(2p+1)-(p+1)} = \dfrac{2p-3}{(2p+2)-(2p+1)}$

$\Rightarrow \dfrac{2}{p} = \dfrac{2p-3}{1} \Rightarrow p(2p-3) = 2$

$\Rightarrow 2p^2 - 3p - 2 = 0$

$\Rightarrow (2p+1)(p-2) = 0$

$\Rightarrow p = 2, \dfrac{-1}{2}$

**15.** Equation of the line passing through the points (1, 3) and (5, –5) is $y - 3 = \dfrac{-5-3}{5-1}(x-1)$

$\Rightarrow y - 3 = \dfrac{-8}{4}(x-1)$

$\Rightarrow y - 3 = -2x + 2$

$\Rightarrow 2x + y - 5 = 0$

Of the given choices, only the point (4, –3) satisfies this equation.

**16.** Clearly for the lines $9x + y = 3$ and $4x + y + 2 = 0$, the product of the intercepts is 1

Also, the point (1, –6) satisfies both these equations.

Hence, the required line is $9x + y = 3$ or $4x + y + 2 = 0$

**17.** The given circle is

$x^2 + y^2 + 4x + 6y - 12 = 0$ or

$x^2 + 4x + 4 + y^2 + 6y + 9 = 4 + 9 + 12$ or

$(x+2)^2 + (y+3)^2 = 25$

$\therefore$ The centre is (–2, –3) and the radius is 5

(2, –1) is the midpoint of the chord.

Let us denote this midpoint by $A$ and the centre by $C$.

The line joining $C$ and $A$ is perpendicular to the chord (1)

Slope of $CA$ = (2, –1) (–2, –3)

$= \dfrac{-3-(-1)}{-2-2} = \dfrac{-2}{-4} = \dfrac{1}{2}$

$\therefore$ The slope of chord is –2.

**18.** The given circles are

$(x-2)^2 + (y-3)^2 = 16$ (1)

$(x+1)^2 + (y-2)^2 = 9$ (2)

$\therefore$ The centre and radius of (1) are (2, 3) and 4, respectively

The centre and radius of (2) are (–1, 2) and 3, respectively.

The distance between the two centres is

$\sqrt{(2+1)^2 + (3-2)^2} = \sqrt{9+1} = \sqrt{10}$

$\therefore C_1C_2 < r_1 + r_2$

Hence, we can draw only two common tangents to the two circles.

**19.** Given lines: $x + 2y = 3$ (1)

$4x - 2y + 5 = 0$ (2)

$x + y = 2$ (3)

Clearly (1) and (2) are perpendicular

$\therefore$ Orthocentre = point of intersection of (1) and (2)

Solving (1) and (2), we get the orthocentre

$= \left(-\dfrac{2}{5}, \dfrac{17}{10}\right)$

$\therefore p = -\dfrac{2}{5}, q = \dfrac{17}{10}$

$p + q = -\dfrac{2}{5} + \dfrac{17}{10}$

$= \dfrac{-4+17}{10} = \dfrac{13}{10} = 1.3$

**20.** By observation, the given lines are parallel

$\therefore$ The distance between these lines = diameter of the circle = $d$ (say), Distance between these lines

$$= \frac{12-\frac{15}{2}}{\sqrt{4^2+3^2}} = \frac{9}{2(5)} = \frac{9}{10} \therefore d = \frac{9}{10}.$$

$$\therefore r = \frac{d}{2} = \frac{9}{10(2)} = \frac{9}{20}$$

$$\therefore \text{Area of the circle} = \pi r^2 = \pi\left(\frac{9}{20}\right)^2 = \frac{81}{400}\pi$$

**21.** Let us call the origin $O$.

Let $A$ and $B$ be the centres of the first and the second circles respectively.

Slope of the common tangent = 5

$AB$ must be perpendicular to the common tangent

$$\therefore \text{Slope of } AB = \frac{-1}{5}$$

$AB$ passes through the origin and has a slope of $\frac{-1}{5}$.

$$\therefore \text{Equation of } AB \text{ is } y = \frac{-1}{5}x$$

$$\text{Let } A = \left(p, \frac{-1}{5}p\right). \text{ Let } B = \left(q, \frac{-1}{5}q\right)$$

$OA$ = Radius = $OB$

$$\sqrt{(p-0)^2+\left(\frac{-1}{5}p-0\right)^2} = 9 = \sqrt{(q-0)^2+\left(\frac{-1}{5}q-0\right)^2}$$

Squaring, we have $\frac{26}{25}p^2 = 81 = \frac{26}{25}q^2$

One of $p$ and $q$ is $\frac{45}{\sqrt{26}}$ and the other is $\frac{-45}{\sqrt{26}}$

One of $A$ and $B$ is $\left(\frac{45}{\sqrt{26}}, \frac{-9}{\sqrt{26}}\right)$ and the other is

$\left(\frac{-45}{\sqrt{26}}, \frac{9}{\sqrt{26}}\right)$.

**22.** If $C$, $D$ are centres and $r$, $s$ are radii of the given circles then $CD = r + s$ ($\because$ the circles touch each other externally)

Here, $C = (p, 0)$, $r = \sqrt{p^2+c}$ and $D = (0, q)$, $s = \sqrt{q^2+c}$

$$CD = \sqrt{p^2+q^2}$$

$$CD = r+s \Rightarrow \sqrt{p^2+q^2} = \sqrt{p^2+c} + \sqrt{q^2+c}$$

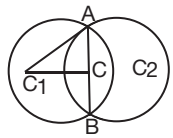
Squaring both the sides

$$p^2 + q^2 = p^2 + c + q^2 + c + 2\sqrt{p^2+c}\,.\sqrt{q^2+c}$$

$$-c = \sqrt{(p^2+c)(q^2+c)}$$

$$\Rightarrow c^2 = p^2q^2 + c(p^2+q^2) + c^2 \Rightarrow p^2q^2 + c(p^2+q^2) = 0.$$

**23.** Let the centre and radius of the circle $x^2 + y^2 + 6y = 0$ be $c_1(0, -3)$, $r_1 = 3$.




Let $AB$ be the common chord of the two circles.

The equation of the common chord is obtained as follows. $x^2 + y^2 + 6y = x^2 + y^2 + 3x$

$$\Rightarrow x - 2y = 0$$

$$AB \equiv x - 2y = 0 \qquad (1)$$

$C_1A = r_1 = 3$

Distance from $C_1$ to $AB$ is $\left|\frac{6}{\sqrt{1^2+2^2}}\right| = \frac{6}{\sqrt{5}}$

$$AC = \sqrt{C_1A^2 - C_1C^2} = \sqrt{9 - \frac{36}{5}} = \frac{3}{\sqrt{5}}$$

$$\therefore \text{Length of common chord } AB = 2AC = \frac{6}{\sqrt{5}}.$$

**24.** $x^2 + y^2 - 4x - 6y + 9 = 0 \qquad (1)$

$x^2 + y^2 - 10x - 14y + k = 0 \qquad (2)$

Centre of the first circle is $C(2, 3)$ and the radius is 2.

Centre of the second circle is $D(5, 7)$.

Let $r$ be radius of the second circle.

The distance between the two centres is

$$\sqrt{(5-2)^2+(7-3)^2} = 5$$

Since the two circles touch each other externally, the distance between two centres = sum of the two radii

$5 = 2 + r \Rightarrow r = 3$

$\therefore$ The point $P$ divides $CD$ in the ratio 2 : 3 internally.

$$\therefore P = \left(\frac{2(5)+3(2)}{2+3}, \frac{2(7)+3(3)}{2+3}\right) = \left(\frac{16}{5}, \frac{23}{5}\right)$$

The slope of $CD$ is $\frac{4}{3}$.

The tangent is perpendicular to $CD$, so slope of tangent is $\frac{-3}{4}$.

$$\therefore \text{Equation of tangent at } P \text{ is } y - \frac{23}{5} = \frac{-3}{5}\left(x - \frac{16}{5}\right)$$

$\Rightarrow 5y - 23 = \frac{-3}{4}(5x - 16) \Rightarrow 20y - 92 = -15x + 48$

$15x + 20y - 140 = 0 \Rightarrow 3x + 4y - 28 = 0.$

**25.** The point of intersection of the lines $4x + 5ky + 7 = 0$

$kx - 6y + 12 = 0$ is $\left(\frac{60k + 42}{5k^2 + 24}, \frac{48 - 7k}{5k^2 + 24}\right)$

It is in the 2nd quadrant, the $y$-coordinate is positive.

$48 - 7k > 0 \Rightarrow 7k < 48 \Rightarrow k < 48/7$ (1)

Also, the $x$ coordinate is positive.

$60k + 42 > 0 \Rightarrow k > \frac{-7}{10}$ (2)

from (1) and (2) $k \in \left(\frac{-7}{10}, \frac{48}{7}\right)$

Integral values of $k$ are 0, 1, 2, 3, 4, 5, 6, i.e., there are 7 such values.

**26.** The radius of the circle is the distance between centre $P$ and point of contact $(-1,-7)$ which is $\sqrt{(4+1)^2 + (7+5)^2} = 13$

Area of the circle = $\pi r^2 = \pi(13)^2 = 169\pi$

Side of the square = diameter of the circle = $2(13) = 26$

Area of square = $26^2 = 676$

Area of the shaded region = $\frac{1}{4}$ [Area of square – Area of circle]

$= \frac{1}{4}[676 - 169\pi] = \frac{169}{4}[4 - \pi]$

**27.** $OARB$ is a sector whose central angle is $90\sigma$

$OR = \sqrt{\frac{1}{2} + \frac{1}{2}} = 1$

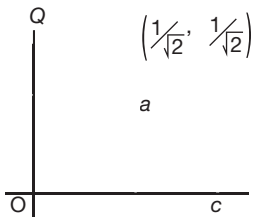
Area of the sector = $\frac{1}{4}\pi(1)^2 = \pi/4$

Let $P = (a, 0)$ and $Q = (0, b)$

$R$ is the midpoint of $PQ$.

$\therefore \left(\frac{a}{2}, \frac{b}{2}\right) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\Rightarrow a = \sqrt{2}, b = \sqrt{2}$




$\therefore$ Area of $\Delta OPQ = \frac{1}{2} OP \cdot OQ = \frac{1}{2}\sqrt{2}\sqrt{2} = 1$

$\therefore$ Area of the shaded region = Area of $\Delta OPQ$ – Area of the sector $= 1 - \frac{\pi}{4} = \left(\frac{4 - \pi}{4}\right)$

**28.** The centre of the circle $x^2 + y^2 + 2x + 4y - 2 = 0$ is $C$ $(-1, -2)$. The radius is $\sqrt{1 + 4 - (-2)} = \sqrt{7}$

The points $P$, $Q$ and C form a right angled triangle right angled at $Q$.

$\therefore PC^2 = PQ^2 + CQ^2$

$PC = \sqrt{(3+1)^2 + (2+2)^2} = \sqrt{32} \Rightarrow PC^2 = 32$

$CQ$ = radius of the circle = $\sqrt{7}$

$\therefore PQ^2 = PC^2 - CQ^2 = 32 - 7 = 25$

$PQ = 5$.

**29.** Since the distance from $3x + y = 0$ to the line $(l)$ is minimum, line $(l)$ is perpendicular to $3x + y = 0$

$\therefore$ The equation of line $(l)$ is of the form $x - 3y + k = 0$

Given it is passing through (2, 1)

$\therefore 2 - 3 + k = 0 \Rightarrow k = 1$

The equation of $(l)$ is $x - 3y + 1 = 0$

The point of intersection of $x - 3y + 1 = 0$

and $5x - 7y - 11 = 0$ is $(a, b)$

$\therefore (a, b) = (5, 2)$

$\therefore a^2 - b^2 = 25 - 4 = 21.$

**30.** The distance from origin to the line $5x + 12y = 26$ is

$OP \frac{|26|}{\sqrt{5^2 + 12^2}} = 2$

The distance from origin to the line $10x + 24y = 39$ is

$OQ = \frac{|-39|}{\sqrt{10^2 + 24^2}} = \frac{39}{26} = \frac{3}{2}$

$\therefore OP : OQ = 2 : \frac{3}{2} = 4 : 3$

As the origin lies on the same side of both the lines, origin divides the given lines externally in the ratio 4 : 3.

**31.** The equation of a straight line passing through the points (3,6) and (–3, 9) is $\frac{y-6}{x-3} = \frac{9-6}{-3-3}$

$= \frac{y-6}{x-3} = \frac{-1}{2}$

$2y - 12 = -x + 3$

$= 1x + 2y - 15 = 0$

Let the line $x + 2y - 15 = 0$

Intersect the $x$-axis at $A$, $y$-axis at $B$

$\therefore A = (15, 0)$ and $B = \left(0, \frac{15}{2}\right)$

$\therefore$ Length of intercept between the axes = distance between $AB$

$$= \sqrt{(15)^2 + \frac{(15)^2}{(2)^2}} = 15\sqrt{\frac{5}{4}}$$

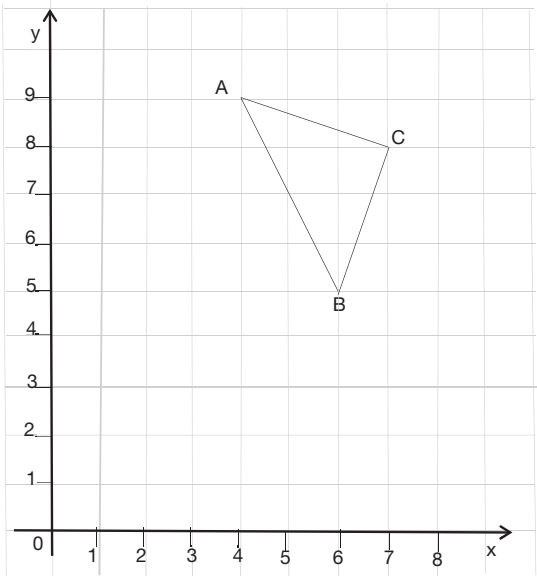
$$= \frac{15}{2}\sqrt{5}$$

**32.** Slope of the line joining (0, 0) and (p, q) is $\frac{q-0}{p-0}$

Slope of the line joining (0, 0) and $(p, q)$, the slope of the line joining $(p, q)$ and $(-p, -q)$, the slope of the line joining $(-p, -q)$ and $(pq, q^2)$ are all the same.

$\therefore$ The given points are collinear.

**33.**




We note that $CA = CB = \sqrt{10}$, i.e., $\Delta CAB$ is isosceles. The angle bisector of $\angle C$ is perpendicular to $AB$. Slope of $AB$ is $-2$.

$\therefore$ The slope of the line which bisects $\angle ACB$ is $\frac{1}{2}$.

**34.** Let $ABCD$ be the square one of whose vertices is $A(3, 10)$. Since (3, 10) is not on the line $5x - y + 12 = 0$, the equation represents diagonal $BD$. Let $AC$ be the required diagonal. Since $AC$ is perpendicular to $BD$, the slope of $AC \times$ slope of $BD = -1$

slope of $AC \times 5 = -1$.

slope of $AC = \frac{-1}{5}$

The equation of $AC$ passing through the point $A(3, 10)$ and having slope $\frac{-1}{5}$ is

$$y - 10 = \frac{-1}{5}(x - 3)$$

$$5y - 50 = -x + 3$$

$$x + 5y - 53 = 0$$

**35.** Three lines are said to be concurrent, if the point of intersection of any two lines lies on the third line.

Given lines are

$3x - y = 2$ (1)

$2x + y = 3$ (2)

and $5x - ay = 3$ (3)

Solving (1) and (2), we get $(x, y) = (1, 1)$

Substituting (1, 1) in (3), we have

$5(1) - a(1) = 3$

$\Rightarrow 5 - a = 3$

$\Rightarrow a = 5 - 3 = 2.$

**36.** $PQ = 8$

The circumradius of an equilateral triangle of side a is $\frac{a}{\sqrt{3}}$

The circumradius of $PQR$ is $\frac{8}{\sqrt{3}}$

As $PQR$ is equilateral, the circumcentre of $PQR$ is the same as its centroid.

$R$ is a point on the $y$-axis. Let $R = (0, a)$

$PR = \sqrt{4^2 + a^2} = PQ = 8$

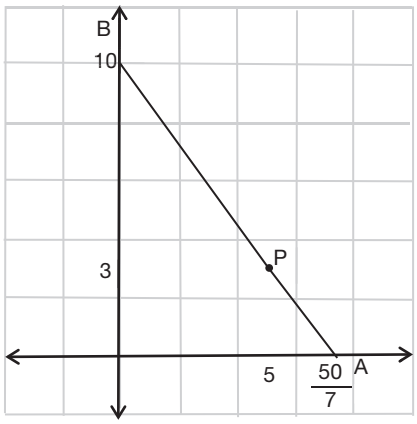
$a = \pm 4\sqrt{3}$

Centroid of $PQR = \left(\frac{-4+4+0}{3}, \frac{0+0+a}{3}\right) = \left(0, \pm\frac{4\sqrt{3}}{3}\right)$

Equation of the circle circumscribing $PQR$ is $(x - 0)^2 + \left(y - \frac{4\sqrt{3}}{3}\right) = \left(\frac{8}{\sqrt{3}}\right)^2$, i.e.,

$$x^2 + \left(y \pm \frac{4\sqrt{3}}{3}\right)^2 = \frac{64}{3}.$$

**37.**




Let the equation of the straight line in the intercept form be

$\frac{x}{a}+\frac{y}{b}=1$

i.e., the line cuts the $x$-axis at $A(a, 0)$ and the $y$-axis at $B(0, b)$

Given: $(5, 3)$ divides $AB$ in the ratio $3:7$

$\therefore \left(\frac{3(0)+7(a)}{3+7}, \frac{3(b)+7(0)}{3+7}\right)=(5, 3)$

$7a=50 \Rightarrow a=\frac{50}{7}$ and $3b=30 \Rightarrow b=10$

$\therefore$ Required line is $\frac{x}{\frac{50}{7}}+\frac{y}{10}=1$ or $7x+5y=50$.

**38.** Say $A=(3, 2)$, $B=(5, 4)$

From $A$ to $B$ the $x$ coordinate increases by 2 (i.e., $\Delta x = 2$) and $\Delta y = 2$. From $B$ to the point $C$, which we want to determine,

we have two options.

$\Delta x=2, \Delta y=-2 \Rightarrow C=(7, 2)$

$\Delta x=-2, \Delta y=2 \Rightarrow C=(3, 6)$

For each of these possible locations of $C$, there is a unique location for $D$.

$C=(7, 2) \Rightarrow D=(5, 0)$ while $C=(3, 6) \Rightarrow D=(1, 4)$

Among these 4 points—$(7, 2)$, $(3, 6)$, $(5, 0)$ and $(1, 4)$—only 1 appears among the options.

**39.** Given: $A(1, 1)$; $B(3, -1)$; $G(3, 1)$

Let the third vertex be $(x_3, y_3)$

Centroid $=\left(\frac{1+3+x_3}{3}, \frac{1-1+y_3}{3}\right)$

$(3, 1)=\left(\frac{4+x_3}{3}, \frac{y_3}{3}\right) \Rightarrow (x_3, y_3)=(5, 3)$

Now, $AB=\sqrt{4+4}=2\sqrt{2}$ $BC=\sqrt{4+16}=2\sqrt{5}$

$AC=\sqrt{16+4}=2\sqrt{5}$

$\therefore$ Perimeter $=AB+BC+CA=2\left(\sqrt{2}+2\sqrt{5}\right)$ units

**40.** Given lines: $(2-\sqrt{3})x+\sqrt{3}y=1$ (1)

$(2\sqrt{3}+3)x-y+1=0$ (2)

slope of (1) is: $m_1=\frac{-(2-\sqrt{3})}{\sqrt{3}}$

slope of (2) is: $m_2=\sqrt{3}(2+\sqrt{3})$

clearly $m_1 m_2=-1$

$\therefore$ Angle between the given lines $=\frac{\pi}{2}=90°$.

## EXERCISE-4

**1.** $AB=\sqrt{12}, BC=\sqrt{12}, CA=2\sqrt{3}=\sqrt{12}$

$\Delta ABC$ is an equilateral triangle.

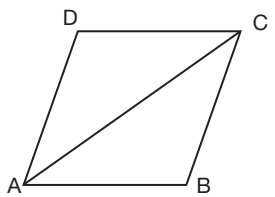
$\therefore$ its circumcentre coincides with the centroid.

$\therefore$ Circumcentre $\equiv\left(\frac{\sqrt{3}+0+-\sqrt{3}}{3}, \frac{0+3+0}{3}\right)=(0, 1)$

**2.** Translation equations are $X=x-7$ & $Y=y+3$

$\therefore$ New co-ordinates of $(1, 1)$ are $(1-7, 1+3)=(-6, 4)$

**3.** The diagonal $AC$ divides the rhombus into two equal parts.




$\therefore$ Area of $ABCD=2$ Area of $\Delta ABC$

$=2\times\frac{1}{2}\begin{vmatrix}5-2 & 2-(-2)\\ 7-3 & 3-0\end{vmatrix}=\begin{vmatrix}3 & 4\\ 4 & 3\end{vmatrix}=7$ sq. units

**4.** Side of the equilateral triangle is $8\sqrt{2}$.

The given points lie on the line $y = x$.

$\therefore$ The third vertex lies on the line $y = -x$.

$\therefore$ Its coordinates have the form $(a, -a)$

$\left(\sqrt{2}a\right)^2 = \left(4\sqrt{6}\right)^2$

$a = \pm 4\sqrt{3}$

$\therefore$ The third vertex can be $\left(-4\sqrt{3}, 4\sqrt{3}\right)$

$\therefore$ Third vertex can be $\left(-4\sqrt{3}, 4\sqrt{3}\right)$

**5.** Let $ABC$ be an isosceles right angled triangle right angled at $A$. Then, $BC$ is hypotenuse.

Let equal sides have lengths of $a$ each. Let $D$ be midpoint of $BC$.

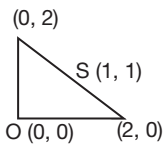
Then from geometry, $BC = \sqrt{2}a$, $AD = \dfrac{1}{\sqrt{2}}$

Now, $\dfrac{a}{\sqrt{2}} = \sqrt{(0-3)^2 + (0-3)^2} = 3\sqrt{2}$

$\therefore$ $a = 6$ units.

$\therefore$ Area $= \dfrac{1}{2} \times a^2 = 18$ sq. units.

**6.** For a right triangle, orthocentre is the vertex containing right angle and circumcenter is the midpoint of hypotenuse.




$\therefore$ Orthocentre = (0, 0)

Circumcentre = (1, 1)

$\therefore$ distance $= \sqrt{1^2 + 1^2} = \sqrt{2}$

**7.** $x + 2ay + a = 0$ (1)

$x + 3by + b = 0$ (2)

$x + 4cy + c = 0$ (3)

Since the three lines are concurrent, the point of intersection of (1) and (2) must be lie on (3).

The point of intersection of (1) and (2) is

$\left(\dfrac{-ab}{3b-2a}, \dfrac{a-b}{3b-2a}\right)$

Substitute this point in (3)

$\dfrac{-ab}{3b-2a} + \dfrac{4c(a-b)}{3b-2a} + c = 0$

$-ab + 4ac - 4bc + 3bc - 2ac = 0$

$-ab + 2ac - bc = 0 \Rightarrow 2ac = ab + bc \Rightarrow \dfrac{2ac}{a+c} = b$

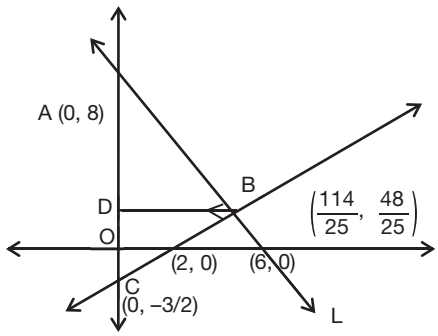
Hence, $a$, $b$, $c$ are in H.P.

**8.** Equation of the line through (3, 4) and (6, 0) is

$\equiv \dfrac{y-4}{x-3} = \dfrac{-4}{3} \Rightarrow 4x + 3y = 24.$

Equation of the line perpendicular to $4x + 3y = 24$ and passing through (2, 0) is $3x - 4y = 6$

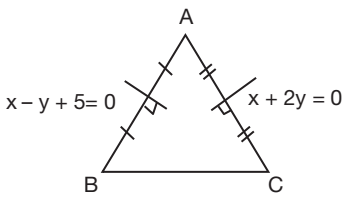
Plotting the lines on a graph, we see that the point of intersection of the lines is $\left(\dfrac{114}{25}, \dfrac{48}{25}\right)$




$\therefore$ Area of $\Delta ABC = \dfrac{1}{2} AC \times BD$

$= \dfrac{1}{2} \times 9.5 \times \dfrac{114}{25} = 21.66 = 21$ sq. units

**9.**




$B$ is the image of $A(1, -2)$ in $x - y + 5 = 0$ and

$C$ is the image of $A(1, -2)$ in $x + 2y = 0$

$\dfrac{h-1}{1} = \dfrac{k+2}{-1} = \dfrac{-2(1+2+5)}{2}$

$\Rightarrow h = -8 + 1 = -7$

$k = 8 - 2 = 6$

$(h, k) = (-7, 6) = B$

$\dfrac{h-1}{1} = \dfrac{k+2}{2} = \dfrac{-2(1-4)}{1+4} = \dfrac{6}{5}$

$\Rightarrow h = \dfrac{6}{5} + 1 = \dfrac{11}{5}$, $k = \dfrac{12}{5} - 2 = \dfrac{2}{5}$

$(h, k) = \left(\frac{11}{5}, \frac{2}{5}\right) = c$

Equation of $BC$ is $y - 6 = \frac{\frac{2}{5} - 6}{\frac{11}{5} + 7}(x + 7)$

i.e., $y - 6 = \frac{-28}{46}(x + 7)$

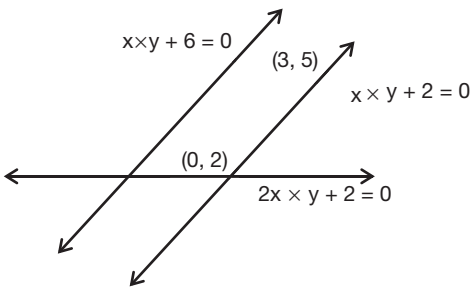
$23(y - 6) = -14(x + 7)$

$14x + 23y - 138 + 98 = 0$

$14x + 23y - 40 = 0$

**10.** The line passing through (3, 5) and parallel to $l: x - y + 6 = 0$ is given by $m: x - y + 2 = 0$.

Now, we find the point of intersection of $x - y + 2 = 0$ and $2x - y + 2 = 0$, as (0, 2)




The distance between (0, 2) and (3, 5) gives the required distance.

$\therefore$ The required distance $= \sqrt{3^2 + 3^2} = 3\sqrt{2}$ units.

**11.** Clearly the slope of the line through $A(1, 2)$ is

$\tan 45 = 1$

$\therefore$ The equation of the line is $y - 2 = x - 1$

i.e., $x - y + 1 = 0$ (1)

Given line: $3x + 4y = 12$ (2)

Solving (i) and (ii)

$\frac{x}{12 - 4} = \frac{y}{3 + 12} = \frac{1}{4 + 3}$

$\therefore x = \frac{8}{7}, y = \frac{15}{7}$

$\therefore P = \left(\frac{8}{7}, \frac{15}{7}\right)$

$AP = \sqrt{\left(\frac{8}{7} - 1\right)^2 + \left(\frac{15}{7} - 2\right)^2} = \sqrt{\frac{1}{49} + \frac{1}{49}} = \frac{\sqrt{2}}{7}$

**12.** Let the slope of a line through (2, 3) and making an angle of 60° with $x + y = 2$ be $m$

Then $\tan 60 = \left|\frac{m + 1}{1 - m}\right| = \frac{1 + m}{1 - m} = \pm\sqrt{3}$

Case (i): $\frac{1 + m}{1 - m} = -\sqrt{3}$

$1 + m = \sqrt{3}(1 - m)$

$\Rightarrow = 1 + m = \sqrt{3} - \sqrt{3}\, m$

$\Rightarrow = (1 + \sqrt{3})\, m = (\sqrt{3} - 1)$

$m = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$ Or $m = \frac{3 + 1 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$

Case (ii) if $\frac{1 + m}{1 - m} = -\sqrt{3}$,

$(1 + m) = -\sqrt{3}(1 - m)$

$1 + m = -\sqrt{3} + \sqrt{3}\, m$

$m(1 - \sqrt{3}) = -\left(1 + \sqrt{3}\right)$

$m = \frac{1 + \sqrt{3}}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{3 + 1 + 2\sqrt{3}}{2} = 2 + \sqrt{3}$

The equation of the lines passing through the point (2, 3) and having slopes $2 - \sqrt{3}$ $2 + \sqrt{3}$ are

(i) $y - 3 = (2 - \sqrt{3})(x - 2)$

$(2 - \sqrt{3})x - y + 2\sqrt{3} - 1 = 0$

(ii) $y - 3 = (2 + \sqrt{3})(x - 2)$

$(2 + \sqrt{3})x - y - 1 - 2\sqrt{3} = 0$

**13.** Any point on $x + y = 4$ is in the form $(t, 4 - t)$

$\frac{|4t + 3(4 - t) - 10|}{\sqrt{16 + 9}}$

$= 1 = |t + 2| = 5 \Rightarrow t = \pm 5 - 2 \Rightarrow t = 3, -7$

$\therefore$ Required points are (3, 1), (–7, 11)

**14.** The given parallel lines represent the parallel sides of the squares, hence, the distance between them gives the length of the side.

$d = \frac{|3 + 3|}{\sqrt{12^2 + 3^2}} = \frac{6}{\sqrt{153}}$

$\therefore$ Area $= d^2 = \frac{36}{153} = \frac{4}{17}$ sq. units.

**15.** $\cos\theta = \dfrac{a_1 a_2 + b_1 b_2}{\sqrt{a_1^2 + b_1^2}\sqrt{a_2^2 + b_2^2}} = \dfrac{+\sqrt{3} + \sqrt{3}}{\sqrt{1+3}\sqrt{1+3}}$

$= \dfrac{+2\sqrt{3}}{4} = \dfrac{+\sqrt{3}}{2}$

$\therefore \theta = \cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = 30°$

**Alternative Method:**

Slope of $x + \sqrt{3}y + 6\sqrt{3} = 0$,

$m_1 = \dfrac{-1}{\sqrt{3}}$ or $\theta_1 = 150°$ and slope of $y + \sqrt{3}\,x + 2 = 0$,

$m_2 = -\sqrt{3}$ or $\theta_2 = 120°$ .

$\therefore$ The angle between the lines = 150 – 120 = 30°

**16.** The slope of $2y + x - 23 = 0$ is $\dfrac{-1}{2}$

$\therefore$ Slope of $XY = \dfrac{-1}{\frac{-1}{2}} = 2$

The equation $XY$ is $y - 5 = 2(x - 3)$

$y = 2x - 1$

midpoint $XY$ is the point of intersection of $y = 2x - 1$ and $2y + x - 23 = 0$

$\therefore$ midpoint of $XY$ is (5, 9).

Let $Y$ be $(a, b)$, $\left(\dfrac{3+a}{2}, \dfrac{5+b}{2}\right) = (5, 9)$

$(a, b) = (7, 13)$

**17.** The equations relating the coordinates are $x = X + 1$,

$y = Y - 1$

Let $f(x, y) = 2x - 3y + 7$, then the transformed equation is given by $f(X, Y)$

$= 2(X + 1) - 3(Y - 1) + 7 = 0$

$\Rightarrow 2X - 3Y + 12 = 0$

**18.** When the axes are rotated through an angle of 45° in anti-clockwise direction, then the equations relating the coordinates are

$x = X\cos 45° - Y\sin 45°$ and $y = X\sin 45° + Y\cos 45°$

$x = \dfrac{X - Y}{\sqrt{2}}$, and $y = \dfrac{X + Y}{\sqrt{2}}$

$\therefore$ The transformed equation of $f(x, y) = 0$, is

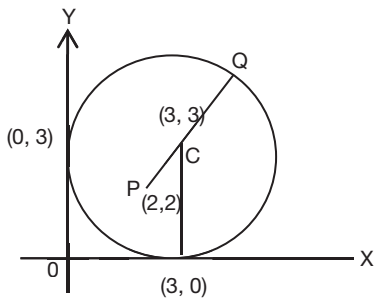
$f\left(\dfrac{x-y}{\sqrt{2}}, \dfrac{x+y}{\sqrt{2}}\right) = 0$

i.e., $\dfrac{X - Y}{\sqrt{2}} - 2\left(\dfrac{X + Y}{\sqrt{2}}\right) + 5 = 0$

$\Rightarrow X - Y - 2(X + Y) + 5\sqrt{2} = 0$

$\Rightarrow -X - 3Y + 5\sqrt{2} = 0 \Rightarrow X + 3Y - 5\sqrt{2} = 0$

**19.**




The equation of the circle is $(x - 3)^2 + (y - 3)^2 = 9$ the centre is (3, 3) and radius = 3.

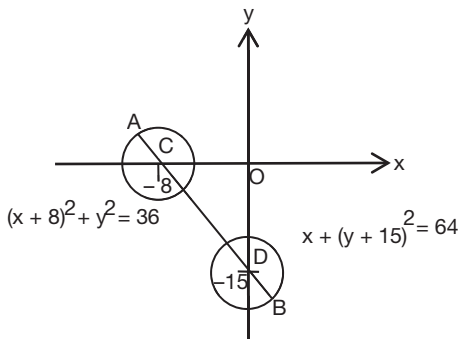
P(2, 2) is inside the circle as $(2 - 3)^2 + (2 - 3)^2 < 9$.

$\therefore$ The largest possible distance from $P$ to any point on the circle is $PQ$, where $Q$ is the end of the diameter passing through $P$

$\therefore PQ = PC + CQ = \sqrt{(3-2)^2 + (3-2)^2} + 3$ ($\because CQ = r = 3$)

$\therefore PQ = 3 + \sqrt{2}$

**20.** The maximum distance between any point on one circle $C_1$ and any point on another circle $C_2$ is that between the points which are on the line joining the centres of the circles (between $A$ and $B$ in the figure below).




$AB = AC + CD + BD$, i.e., $6 + CD + 8$

$CD = \sqrt{CO^2 + OB^2} = 17$

$\therefore$ Maximum distance is 31.

**21.** The points we have to consider must satisfy the condition $x^2 + y^2 < 9$.

If $x^2 = 0$, $x = 0$ and $y^2 < 9$. $y = 0, \pm 1, \pm 2$

If $x^2 = 1$, $x = \pm 1$ and $y^2 < 8$. $y = 0, \pm 1, \pm 2$

If $x^2 = 4$, $x = \pm 2$ and $y^2 < 5$. $y = 0, \pm 1, \pm 2$

A total of 25 points satisfy the given condition.

**22.** The point of intersection has integral coordinates. Let this point be $(x_0, y_0)$

$4x_0 + 5y_0 = 26$ and $y_0 = kx_0 + 2$

$$4x_0 + (kx_0 + 2) = 26 \Rightarrow x_0 = \frac{16}{4 + 5k}$$

$x_0$ is an integer

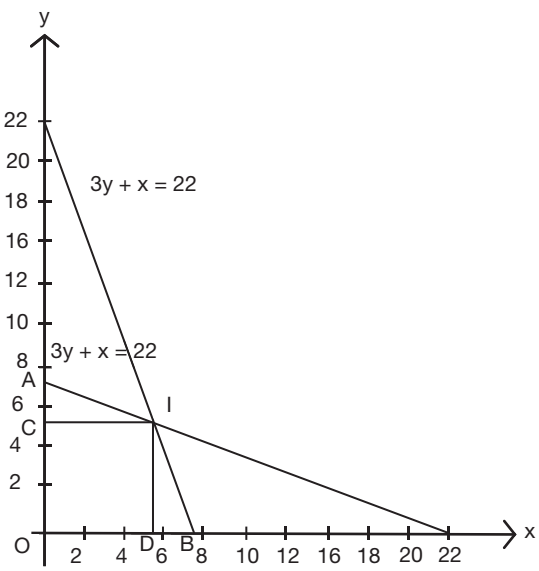
$\therefore$ $5 + 5k$ is a factor (positive or negative) of 16. Also, it is odd.

$4 + 5k = \pm 1$

$$k = \frac{-3}{5} \text{ or } -1$$

$k$ has only one integer value. It can be verified that when $k = -1$, $y_0$ is an integer.

**23.**




At $I$, $x + 3y = 22$ and $3x + y = 22$

Let I be $(x_0, y_0)$

$x_0 + 3y_0 = 22$ and $3x_0 + y_0 = 22$

Solving these $(x_0, y_0) = \left(\frac{11}{2}, \frac{11}{2}\right)$

The convex quadrilateral formed by the given lines and the coordinate axes is $OAIB = S$

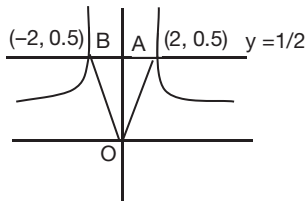
$S$ = Area of $AIO$ + Area of $BIO$

$$= 2\text{Area of } AIO = (AO)(IC) = \frac{22}{3}\left(\frac{11}{2}\right) = \frac{121}{3}$$

$\therefore$ $3S = 121$

**24.** The given curve and the line are $|x|y = 1$ and $2y - 1 = 0$.

$y = \frac{1}{|x|}$ and $y = \frac{1}{2}$ intersect at $(-2, 0.5)$ and $(2, 0.5)$




Clearly $OA = OB$. $\therefore \Delta OAB$ is isosceles.

**25.** Given the lines $4x - 3ky + 4 = 0$ (1)

and $2x - 5y + 1 = 0$ (2)

which intersect at a point whose $x$-coordinate is twice its $y$-coordinate.

So, let the point of intersection be $(2p, p)$.

As $(2p, p)$ lies on (2), $4p - 5p + 1 = 0 \Rightarrow p = 1$

$\therefore$ The point of intersection is $(2, 1)$.

Now, this point $(2, 1)$ also lies on (1).

$\Rightarrow 4(2) - 3k(1) + 4 = 0 \Rightarrow k = 4$

**26.** The polygon is a pentagon. Its area is given by

$$\frac{1}{2}\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_5 & y_1 \end{matrix}$$

i.e., $\frac{1}{2}\begin{vmatrix} -4 & 2 & 5 & 6 & -3 & -4 \\ 3 & 5 & -2 & -3 & -3 & 3 \end{vmatrix}$

$$= \frac{1}{2}(-20 - 6) + (-4 - 25) + (-15 + 12) + (-18 - 9) + (-9 - 12)$$

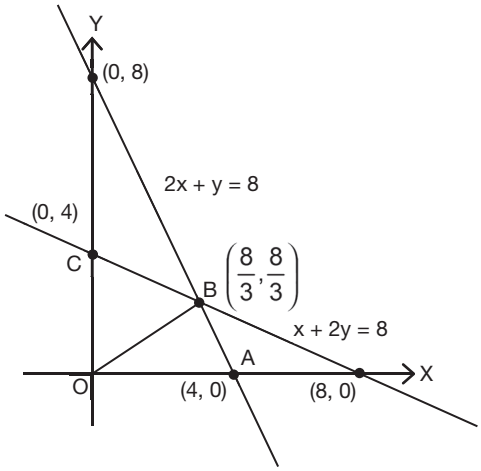
$= 53$.

**27.** The point of intersection of the lines $x + 2y = 8$ and $2x + y = 8$ is $\left(\frac{8}{3}, \frac{8}{3}\right)$

The quadrilateral formed by these lines and axes is shown in the figure

The area of the quadrilateral formed by the points $(x_1, y_1)$, $(x_2, y_2)$, $(x_3, y_3)$, $(x_4, y_4)$ is given by

$$\frac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_3 & y_2 - y \; 4 \end{vmatrix}$$




$\because$ Area of $OABC = \frac{1}{2}\begin{vmatrix} 4 & \frac{8}{3} \\ -4 & \frac{8}{3} \end{vmatrix}$

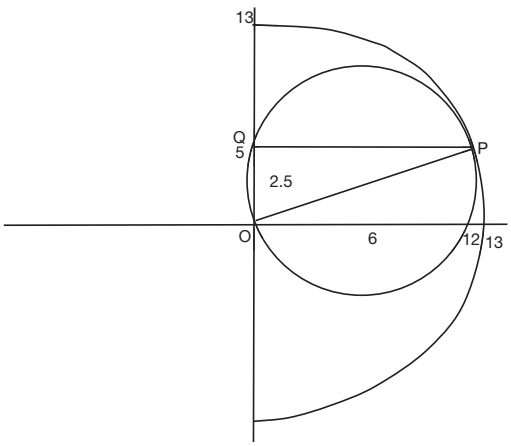
$\frac{1}{2}\left[4\times\frac{8}{3}-\left(-4\times\frac{8}{3}\right)\right] = \frac{64}{2\times 3} = \frac{32}{3}$ sq. units.

**Alternate Solution:**

Area of $OABC$

$= \frac{1}{2}\begin{vmatrix} 0 & 4 & \frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{8}{3} & 4 & 0 \end{vmatrix} = \frac{1}{2}\left|4\left(\frac{8}{3}\right)+\frac{8}{3}(4)\right| = \frac{32}{3}.$

**28.**




Given $x^2 + y^2 = 169$ which is a circle with centre at origin and radius = 13, i.e., $OP = 13$ ($P$ is the point of contact). A part of this circle is shown in the figure above. The 2 circles touch at $P$ and $OP$ is the radius of the larger circle, so the centre of the smaller circle also lies on $OP$. $\therefore$ $OP$ is the diameter of the smaller circle.

Let $PQ$ be joined where $Q = (0, 5)$

$\Delta OPQ$ is a right-angle triangle, in which OP is the hypotenuse ($\because$ $OP$ is a diameter and $\angle OQP$, the angle in a semi circle will be a right angle).

$\therefore$ $OP^2 = OQ^2 + PQ^2$

$169 = 5^2 + PQ^2 \Rightarrow PQ = 12 \Rightarrow P = (12, 5)$

$\therefore$ The centre of the smaller circle is $(6, \frac{5}{2})$

(It is the midpoint of $PO$).

**29.** Any secant of a circle must be closer to the circle's centre than any tangent to the circle. In the given problem, the circle is centred at the origin. The origin must be closer to the secant than the tangent.

The distance between the origin and the line

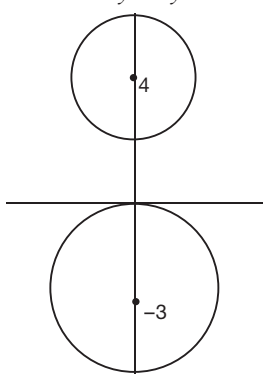
$5x - 4y - 20 = 0$ is $\frac{|5(0)-4(0)-20|}{\sqrt{5^2+(-4)^2}}$

i.e., $\frac{20}{\sqrt{41}}$. The distance between the origin and the line

$5x - 4y + 40 = 0$ is $\frac{5(0)-4(0)+40}{\sqrt{5^2+(-4)^2}}$, i.e., $\frac{40}{\sqrt{41}}$

The line $5x - 4y - 20 = 0$ is closer to the origin than the other line. $\therefore$ This must be secant and the other line must be tangent. Radius = Distance from the centre to the line $5x - 4y + 40 = 0$. $\therefore$ Radius = $\frac{40}{\sqrt{41}}$

**30.** The given circles are $x^2 + y^2 - 8y + 12 = 0$ and $x^2 + y^2 + 6y = 0$




i.e., $x^2 + (y-4)^2 = 2^2$ and $x^2 + (y+3)^2 = 3^2$

centre $C_1(0, 4)$ and $r_1 = 2$ and $C_2(0, -3)$ and $r_2 = 3$

As the centres lie on the $y$-axis, PQ is the shortest distance between the circles.

$C_1C_2 = 4 - (-3) = 7$

$PQ = C_1C_2 - (r_1 + r_2) = 7 - (2 + 3) = 2$

## EXERCISE-5

**1.** From Statement I, the equation of the line $AB$ is $4x + 3y = 12$.

When $y = 0$, $x = 3$

When $x = 0$, $y = 4$

So, $OA = 3$ and $OB = 4$

$\therefore$ The area of $\Delta AOB = 1/2 \times OA \times OB$

$= 1/2 \times 3 \times 4 = 6$ sq. units

So, Statement I alone is sufficient.

From Statement II, the midpoint of the line segment $AB$ is $\left(\frac{3}{2}, 2\right)$

Let $A(a, 0)$ and $B(o, b)$

$\left(\frac{a+o}{2}, \frac{o+b}{2}\right) = \left(\frac{3}{2}, 2\right)$

$\Rightarrow a = 3$ and $b = 4$

$\therefore$ The area of $\Delta AOB = 1/2|a\ b|$

$= 1/2 \times 3 \times 4 = 6$ sq. units.

So, Statement II alone is sufficient.

Hence, either of the statements alone is sufficient to answer the question.

**2.** Let $C$ be the centre of the circle.

From Statement I, $A(4, 0) \Rightarrow CA = CB = 4$

So, the centre of the circle is $(4, 4)$ and its radius is 4 units.

$\therefore$ The equation of the circle is $(x-4)^2 + (y-4)^2 = 16$

So, Statement I alone is sufficient.

From Statement II, $B(0, 4) \Rightarrow CB = CA$.

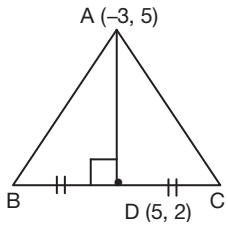
So, the centre of the circle is $(4, 4)$ and its radius is 4 units.

So, the equation of the circle is $(x-4)^2 + (y-4)^2 = 16$.

$\therefore$ Statement II alone is sufficient. Hence, either of the statements alone is sufficient to answer the question.

**3.** Clearly, either of the statements alone is not sufficient to answer the question. Combining both the statements, $\Delta ABC$ is an equilateral triangle and the vertex is $A(-3, 5)$, the midpoint of $BC$ is $D(5, 2)$.

So, the height of the $\Delta ABC$ corresponding to the base $BC$ is $AD = \sqrt{64 + 9} = \sqrt{73}$.




$\Rightarrow$ The side a of the $\Delta ABC = \frac{2}{\sqrt{3}} AD$ [$\because h = \sqrt{3}/2\ a$]

$a = \frac{2}{\sqrt{3}} \sqrt{73}$ units

$\therefore$ The area of $\Delta ABC = \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times \frac{4}{3} \times 73 = \frac{73}{\sqrt{3}}$ sq. units

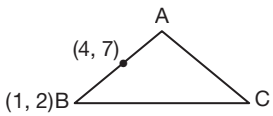
Hence, both I and II together are sufficient to answer the question.

**4.** Clearly, either of the statements alone is not sufficient to answer the question. Combining both I and II, the equation of the line '$\ell$' is, $x = 3$. The line $x = 3$ meets the $x$-axis at the point $(3, 0)$. Hence, both I and II together are sufficient to answer the question.

**5.** The line equation can be $y = 2$ or $y = 7$. Hence, we cannot determine the equation uniquely.

So, both data is insufficient.

**6.**



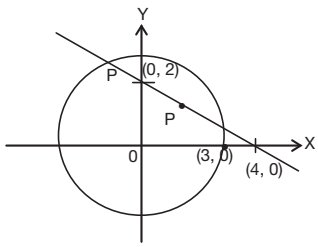

Unless we know the coordinates of C, it is not possible to find the centroid.

$\therefore$ Using both the statements also, we cannot solve.

**7.** Using both the statements together, we can find the equation of the line using point slope form and then we can check whether $(7, 2)$ lies on $L$ or not.

Hence, both statements together are sufficient.

**8.** Even after using both the statements, $P$ can lie in the 1st or in the 2nd quadrant.

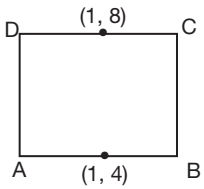


**9.** From Statement I alone, the area cannot be found as the equation of only one side is known.

From Statement II alone, the area cannot be found as the equations of only two sides are known.

From I and II together, the area can be found as all the three equations are known.

**10.**




From Statement I,

The distance between the midpoints of the sides is length of the side of square

the length of the square = 4.

∴ Area = 16 sq units. Statement I alone is sufficient.

From Statement II, we can find the length of diagonal.

From that we can find area also.

∴ Statement II alone is also sufficient.

Hence, either of the statements is sufficient to answer the question.

**11.** From Statement I: The circle touches the $x$-axis at A(6, 0). With this alone, we cannot find the equation of the circle

∴ Statement I alone is not sufficient

Statement II: The distance from centre of the circle to the $x$-axis is 3 units. We don't know the $x$-coordinate. We cannot find the equation of the circle.

Combining I and II, $d$ = radius = 3 units and circle touches $x$-axis at (6, 0)

The $x$-coordinate of the centre is 6 and the $y$-coordinate is radius, which is 3

∴ centre is (6, 3) and the radius $r = 3$ units

∴ combining statements I and II, we can find the equation of the circle.

**12.** From Statement I, we get the location of one vertex. This is not enough to find the area of the triangle

∴ Statement I alone is not sufficient.

From Statement II, we know only the midpoint of AB. We cannot find the area of the triangle.

But combining the two statements, the triangle is right angled and isosceles. The length of the median to the hypotenuse is 5. Each leg in $5\sqrt{2}$ and the area is 25. We can get the area by combining the two statements.

**13.** From Statement I, the slope of the line $m = \tan\theta$

$$= -\frac{3}{-\sqrt{3}} = \sqrt{3}$$

$\Rightarrow \theta = 60°$

∴ Statement I alone is sufficient

From statement II, given $\cos\theta = -\frac{1}{2}$

$$\therefore \theta = \frac{2\pi}{3}$$

∴Statement II alone is also sufficient

We can answer the question from either of the statement.

**14.** In Statement I, there is no information about the line.

In Statement II, there is no information about the circle.

But combing I and II, the centre of the circle is (2, 3) and radius = 5

Perpendicular distance from (2, 3) to the line $3x + 4y + 7 = 0$ is

$$\left|\frac{3(2)+4(3)+7}{\sqrt{3^2+4^2}}\right| = 5$$

∴ radius = perpendicular distance

∴ there is only one common point

When combing both statements, we can answer the question.

**15.** Statement I: If a line passes through only two quadrants, then it is either parallel to the $X$-axis or parallel to the $Y$-axis or passes through the origin I is not sufficient.

Statement II: Only one point is given. II is not sufficient.

Statements I, II together are also not sufficient.

# 4 Trigonometry

CHAPTER

## Learning Objectives

*In this chapter, you will*:

- Learn about the measurement systems of angles such as:
  - Sexagesimal system
  - Circular system
- Gain understanding of trigonometric concepts such as:
  - Basic trigonometric ratios
  - Signs of trigonometric ratios
  - Maximum and minimum values of trigonometric ratios
  - Graphs of sine, cosine, tangent functions
- Discover various trigonometric identities
- Learn about compound angles formulae such as:
  - Addition and subtraction formulae
  - Double angle formulae
  - Half angle formulae
  - Triple angle formulae
  - Formulae for changing the product into sum or difference
  - Formulae for changing the sum or difference into product
- Learn about triangle and its properties
  - Sine rule
  - Cosine rule
  - Area of a triangle
- Learn how to calculate heights and distances based on angle of elevation/depression

TRIGONOMETRY is a branch of Mathematics that deals with the relation between the sides and angles in any triangle. Trigonometry literally means 'three-angle measure'.

## ❒ MEASUREMENT OF ANGLES

The angle between two lines can be measured in various systems. We consider two of these systems,

1. Sexagesimal system
2. Circular system

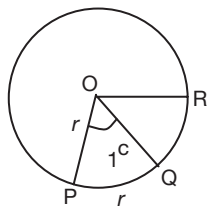
**Sexagesimal system:** A right angle is taken as the unit of measurement.

$^1/_{90}$ of a right angle is termed as a degree ($1^o$).
$^1/_{60}$ of a degree is termed as a minute (1').
$^1/_{60}$ of a minute is termed as a second (1").

## ❒ BASIC CONVERSIONS

60" = 1', 60' = 1°, and
90° = 1 right angle.

**Circular system:** A radian ($1^c$) is taken as the basic unit of this measure.




A radian ($1^c$) is defined as the angle subtended at the centre of a circle by an arc whose length ($l$) equals the radius ($r$).

i.e., $l = r$

$\therefore$ If arc $PQ = r$, then $\angle POQ = 1$ radian.

**NOTE**

(i) Radian is a real constant.

(ii) The circular and sexagesimal measures are related by $\pi^c$ or $\pi$ radians = 180°.

$2\pi$ radians = 360°.

(iii) 1 revolution = $2\pi$ radians.

## ❒ IMPORTANT CONVERSIONS

| Sexagesimal measure | Circular measure |
|---|---|
| 30° | $\pi/6$ |
| 45° | $\pi/4$ |
| 60° | $\pi/3$ |
| 90° | $\pi/2$ |
| 270° | $3\pi/2$ |

Given a right-angled triangle $ABC$, and $\angle ACB = \theta$ the trigonometric ratios are defined as follows:

$$\sin\theta = \frac{opp}{hyp} = \frac{AB}{AC}$$

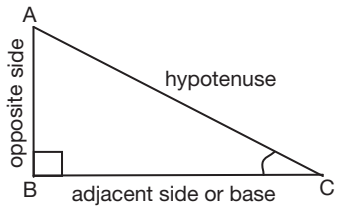
$$\cos\theta = \frac{adj}{hyp} = \frac{BC}{AC}$$

$$\tan\theta = \frac{opp}{adj} = \frac{AB}{BC}$$

$$\cot\theta = \frac{adj}{opp} = \frac{BC}{AB}$$

$$\sec\theta = \frac{hyp}{adj} = \frac{AC}{BC}$$

$$\text{cosec}\,\theta = \frac{hyp}{opp} = \frac{AC}{AB}$$




From the ratios, we can easily observe the following relations:

1. **Reciprocal relations:**

$$\text{cosec}\,\theta = \frac{1}{\sin\theta}; \qquad \sin\theta = \frac{1}{\text{cosec}\,\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}; \qquad \cos\theta = \frac{1}{\sec\theta}$$

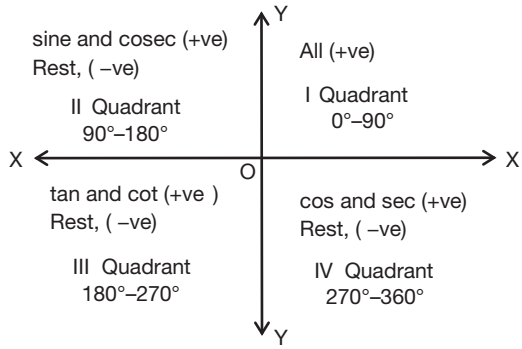
$$\tan\theta = \frac{1}{\cot\theta}; \qquad \cot\theta = \frac{1}{\tan\theta}$$

2. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}; \qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}.$

## ❒ Signs of Trigonometric Ratios

1. If $\theta$ lies in the first quadrant $(0 < \theta < \pi/2)$, then all the trigonometric ratios are positive.
2. If $\theta$ lies in the second quadrant $(\pi/2 < \theta < \pi)$, then only sin $\theta$ and cosec $\theta$ are positive and the rest of the ratios are negative.
3. In the third quadrant $(\pi < \theta < 3\pi/2)$, only tan $\theta$ and cot $\theta$ are positive and the rest of the ratios are negative.
4. In the fourth quadrant $(3\pi/2 < \theta < 2\pi)$, only cos $\theta$ and sec $\theta$ are positive and the rest of the ratios are negative.

The signs of the trigonometric ratios can be remembered with the help of the diagram given below.




1. The absolute value of the ratio is to be arrived at as per the following table.

| Ratio | When $n$ is even | When $n$ is odd |
|---|---|---|
| sin $(n\pi / 2 \pm \theta)$ | sin $\theta$ | cos $\theta$ |
| cos $(n\pi / 2 \pm \theta)$ | cos $\theta$ | sin $\theta$ |
| tan $(n\pi / 2 \pm \theta)$ | tan $\theta$ | cot $\theta$ |

2. Values of trigonometric functions of some standard angles.

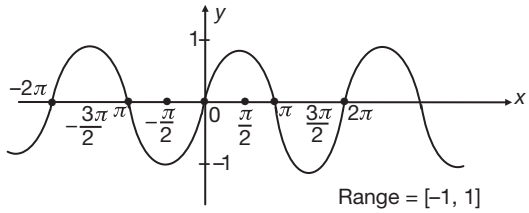
| $\theta$ | 0 | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ | $\pi$ | $3\pi/2$ | $2\pi$ |
|---|---|---|---|---|---|---|---|---|
| $\sin\theta$ | 0 | 1/2 | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 | 0 | –1 | 0 |
| $\cos\theta$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | ½ | 0 | –1 | 0 | 1 |
| $\tan\theta$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | ∞ | 0 | ∞ | 0 |

## ❒ Maximum and Minimum Values of Trigonometric Ratios

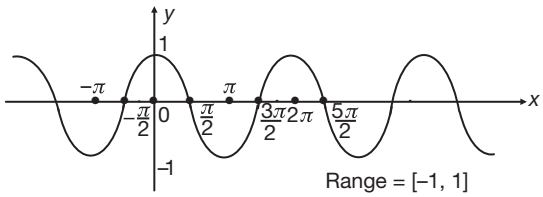
1. The sine and cosine of an angle can never be less than –1 and cannot be greater than +1.
2. The secant and cosecant of an angle will not have any value between –1 and +1.
3. The tangent and co-tangent of an angle can take any real value.

## ❒ Graphs of Sine, Cosine and Tangent Functions

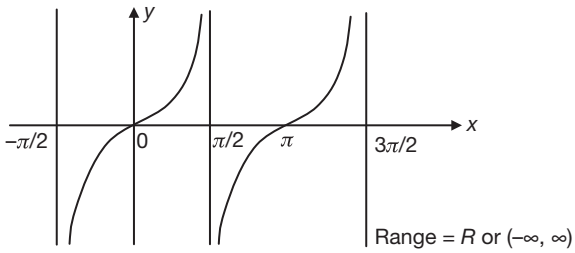
**Sine function:** $y = \sin x$




**Cosine function:** $y = \cos x$




**Tangent function:** $y = \tan x$




## ❒ Some Important Trigonometric Identities

**FORMULA**

(i) (a) $\sin^2\theta + \cos^2\theta = 1$
(b) $1 + \tan^2\theta = \sec^2\theta$
(c) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
(ii) (a) $\sin(-\theta) = -\sin\theta$
(b) $\cos(-\theta) = \cos\theta$
(c) $\tan(-\theta) = -\tan\theta$

**NOTE**

$(\sin\theta)^2$ is represented as $\sin^2\theta$ and so on.

**Compound Angle:** An angle made by the sum or difference of two or more angles is called a compound angle.

## ❒ Addition and Subtraction Formulae

(i) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

(ii) $\sin(A - B) = \sin A \cos B - \cos A \sin B$

(iii) $\cos(A + B) = \cos A \cos B - \sin A \sin B$

(iv) $\cos(A - B) = \cos A \cos B + \sin A \sin B$

(v) $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(vi) $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

(vii) $\cot(A + B) = \dfrac{\cot A \cot B - 1}{\cot A + \cot B}$

(viii) $\cot(A - B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

(ix) $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

(x) $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

(xi) $\tan(A + B + C) = \dfrac{\sum \tan A - \pi \tan A}{1 - \sum \tan A \tan B}$

## ❒ Double Angle Formulae

(i) $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1 + \tan^2\theta}$

(ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$

(iii) From (ii) above,

$$\cos\theta = \sqrt{\frac{1 + \cos 2\theta}{2}}; \sin\theta = \sqrt{\frac{1 - \cos 2\theta}{2}}$$

(iv) $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

(v) $\cot 2\theta = \dfrac{\cot^2\theta - 1}{2\cot\theta}$

## ❒ Half-angle Formulae

Replacing $\theta$ by $\theta/2$ in the formulae given in (2), we get the following formulae.

(i) $\sin\theta = 2\sin\theta/2\cos\theta/2$

$$= \frac{2\tan\theta/2}{1+\tan^2\theta/2}$$

(ii) $\cos\theta = \cos^2\theta/2 - \sin^2\theta/2$

$= 2\cos^2\theta/2 - 1$

$$= 1 - 2\sin^2\theta/2 = \frac{1-\tan^2\theta/2}{1+\tan^2\theta/2}$$

(iii) $$\tan\theta = \frac{2\tan\theta/2}{1-\tan^2\theta/2}$$

(iv) $$\cot\theta = \frac{\cot^2\theta/2 - 1}{2\cot\theta/2}$$

## ❒ Triple Angle Formulae

(i) $\sin3\theta = 3\sin\theta - 4\sin^3\theta$

(ii) $\cos3\theta = 4\cos^3\theta - 3\cos\theta$

(iii) $$\tan3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$$

(iv) $$\cot3\theta = \frac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta - 1} = \frac{3\cot\theta - \cot^3\theta}{1-3\cot^2\theta}$$

## ❒ Formulae for Changing the Product into Sum or Difference

(i) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

(ii) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

(iii) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

(iv) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

## ❒ Formulae for Changing the Sum or Difference into Products

(i) $$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

(ii) $$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

(iii) $$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

(iv) $$\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

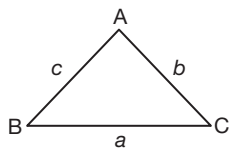
## ❒ OTHER STANDARD RESULTS

1. **Sine rule:**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

($R$ is the circumradius.)

2. **Area of a triangle:**
$ABC$, $(\Delta) = (1/2)\ ab\sin C$ (where $C$ is the angle included between the sides of the lengths $a$ and $b$) Alternatively, $\Delta = 1/2\ bc\sin A$ and $1/2\ ca\sin B$ can also be taken.



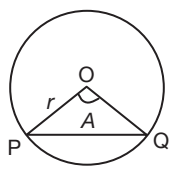

3. **Cosine rule:**

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

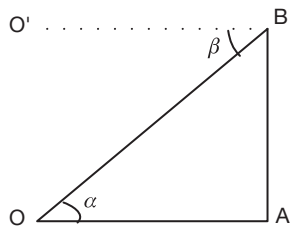
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

4. O is the centre.
$PQ$ is the chord.
Chord $PQ = 2r\sin\frac{A}{2}$, $r$ = radius of the circle.



## ❒ HEIGHTS AND DISTANCES




Let $AB$ be a vertical line drawn from $B$ to $A$ to meet the horizontal line drawn from $O$ (which is at a lower level than $B$) and let $BO'$ be the horizontal drawn through $B$ (i.e., $BO'$ is parallel to $OA$)

Then, $\angle AOB$ is called the angle of elevation of point $B$ as seen from point $O$ and $\angle O'BO$ is the angle of depression of point $O$ as seen from point $B$. It can be readily seen that $\alpha = \beta$

i.e., Angle of elevation = Angle of depression (since $AO$ is parallel to $BO'$).

Using the trigonometric ratios on angles of elevation and depression, we can find out heights and distances as seen in the examples given in the later part of this chapter.

## SOLVED EXAMPLES

**4.01.** Convert $\frac{7\pi}{12}$ radians into degrees.

**Sol:** $\pi$ radians $= 180°$

$\therefore \frac{7\pi}{12} = \frac{7\pi}{12} \cdot \frac{180°}{\pi} = 105°$

**4.02.** If an athlete runs 1800 m around a circular path of circumference 250 m, then find the angle covered by the athlete in radians.

**Sol:** The athlete makes $\frac{1800}{250} = 7\frac{1}{5}$ rounds of the circular path.

1 round $= 2\pi$ radians

$7\frac{1}{5}$ rounds $= 7\frac{1}{5} \times 2\pi$ radians

$= \frac{36}{5} \times 2\pi$ radians $= \frac{72\pi}{5}$ radians.

**4.03.** If the angles of a triangle are in arithmetic progression with a common difference of 20°, then express the angles in radian measure.

**Sol:** Let $A, B, C$ denote the angles of the triangle.

So, $A + B + C = 180°$

The angles being in A.P., with a common difference of 20°, $A = B - 20°$ and $C = B + 20°$

$\therefore B - 20° + B + B + 20° = 180°$

$\Rightarrow 3B = 180°$

$\Rightarrow B = 60°, A = 40°$ and $C = 80°$

$\therefore$ Angles in circular measure are $\frac{2\pi}{9}, \frac{\pi}{3}, \frac{4\pi}{9}$.

**4.04.** Find the angle covered by a minute hand in 54 minutes.

**Sol:** The angle covered by a minute hand in 60 minutes $= 360°$

$\therefore$ Angle covered in 54 minutes $= \frac{360° \times 54}{60}$

$= 324° = \frac{9\pi}{5}$ radians.

**4.05.** If $\tan^2\theta - 3 = 0$ and $\theta$ is in the III quadrant, then find the values of $\sin\theta$ and $\sec\theta$.

**Sol:** $\tan^2\theta - 3 = 0$

$\Rightarrow \tan^2\theta = 3$

$\Rightarrow \tan\theta = \pm\sqrt{3}$

Since $\theta$ lies in the III quadrant, $\tan\theta$ is positive, whereas, $\sin\theta$ and $\sec\theta$ are negative.

$\sec\theta = \sqrt{1 + \tan^2} = \sqrt{4} = -2$

$\Rightarrow \cos\theta = -\frac{1}{2}$

$\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \frac{1}{4}} = -\frac{\sqrt{3}}{2}$.

**4.06.** Show that $\frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\text{cosec}\theta$.

**Sol:** $\frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta}$

$= \frac{\sin\theta[1 + \cos\theta + 1 - \cos\theta]}{(1 - \cos^2\theta)}$

$= \frac{2\sin\theta}{\sin^2\theta} = 2\text{cosec}\theta$

[Using the identity $\cos^2\theta + \sin^2\theta = 1$]

**4.07.** Prove that $(\text{cosec}\theta - \cot\theta)(\text{cosec}\theta + \cot\theta) - \sin 2\theta + (\sin\theta + \cos\theta)^2 = 2$.

**Sol:** $(\operatorname{cosec}\theta - \cot\theta)\ (\operatorname{cosec}\theta + \cot\theta) - \sin 2\theta + (\sin\theta + \cos\theta)^2$
$= (\operatorname{cosec}^2\theta - \cot^2\theta) - \sin 2\theta + \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2.$
$[\because \operatorname{cosec}^2 - \cot^2\theta = 1;\ \sin^2\theta + \cos^2\theta = 1$ and $2\sin\theta\cos\theta = \sin 2\theta]$

**4.08.** If 6, 7, 8 represent the lengths of the sides of a triangle, then find the cosine of the greatest angle in the triangle.

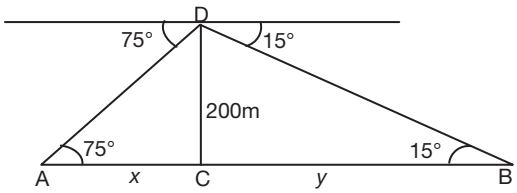
**Sol:** Let $a = 6$, $b = 7$ and $c = 8$
The greatest angle of a triangle will be opposite to the greatest side of the triangle.
Here, $c$ is the greatest side
$\therefore \angle C$ is the greatest angle.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{36 + 49 - 64}{2(6)\ (7)}$$

$$= \frac{21}{2(6)\ (7)} = \frac{1}{4}$$

**4.09.** From the top of a tower of height 200 m, the angle of depression of two points on opposite sides of a tower are observed to be 15° and 75°. Find the distance between the two points on the surface in a straight line with the foot of the tower.
(Hint: $\tan 15° = 2 - \sqrt{3}$ and $\tan 75° = 2 + \sqrt{3}$ )

**Sol:**




Let $CD$ be the height of the tower and $A$ and $B$ be two points on the ground. From $\Delta ACD$,

$$\tan 75° = \frac{200}{x}$$

$$\Rightarrow x = \frac{200}{2 + \sqrt{3}} \Rightarrow x = 200\left(2 - \sqrt{3}\right) \text{m}$$

From $\Delta BCD$,

$$\tan 15° = \frac{200}{y}$$

$$\Rightarrow y = \frac{200}{2 - \sqrt{3}}$$

$$\Rightarrow y = 200\left(2 + \sqrt{3}\right) \text{m}$$

Distance $AB$ is $x + y = 200\left[2 - \sqrt{3} + 2 + \sqrt{3}\right]$
$= 800$ m.

**4.10.** Find $\sin^8\theta + \cos^8\theta$, when $\theta = \frac{\pi}{3}$.

**Sol:** $\sin^8\theta + \cos^8\theta = \sin^8\frac{\pi}{3} + \cos^8\frac{\pi}{3}$

$$= \left(\frac{\sqrt{3}}{2}\right)^8 + \left(\frac{1}{2}\right)^8 = \frac{1}{2^8}[3^4 + 1]$$

$$= \frac{82}{256} = \frac{41}{128}.$$

**4.11.** Find the value of $\cos(-1200°)$.

**Sol:** We know that $\cos(-\theta) = \cos\theta$.
$\Rightarrow \cos(-1200°) = \cos 1200°$
$= \cos(3 \times 360° + 120°)$
$= \cos 120°$
$= \cos(180° - 60°)$
$= -\cos 60° = -1/2$

**4.12.** Find the value of $\cos^2 15° - \cos^2 75°$.

**Sol:** $\cos^2 15 - \cos^2 75 = \sin(75° + 15°)\ \sin(75° - 15°)$
$= \sin 90°.\sin 60° = \frac{\sqrt{3}}{2}.$

**4.13.** Find the value of $\frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°}$.

**Sol:** $$\frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20°\cos 20°} = \frac{2\left[\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right]}{\frac{1}{2}(2\sin 20°\cos 20°)}$$

$$= \frac{4(\sin 60°\cos 20° - \cos 60°\sin 20°)}{\sin 40°}$$

$$= 4\left(\frac{\sin 40°}{\sin 40°}\right) = 4$$

**4.14.** If $\sin\alpha = \frac{4}{5}$ and $\cos\beta = \frac{5}{13}$, then find the value of $\sin\frac{\alpha - \beta}{2}$.

**Sol:** Consider $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$= \frac{3}{5}.\frac{5}{13} + \frac{4}{5}.\frac{12}{13} \Rightarrow \cos(\alpha - \beta) = \frac{63}{65}$$

We know that $\sin\left(\frac{\alpha - \beta}{2}\right)$

$$= \sqrt{\frac{1 - \cos(\alpha - \beta)}{2}} = \sqrt{\frac{1 - \frac{63}{65}}{2}} = \sqrt{\frac{2}{2(65)}} = \frac{1}{\sqrt{65}}$$

**4.15.** Convert $\frac{5\pi}{12}$ radians into degrees.

**Sol:** $\pi$ radians $= 180°$

$\frac{5\pi}{12}$ radians $= \frac{5}{12}(180°) = 75°$

**4.16.** An athlete ran 2000 m around a circular path of circumference 200 m. Find the angle covered by him (in radians).

**Sol:** The athlete makes $\frac{2000}{200} = 10$ rounds of the circular path.

1 round $= 2\pi$ radians.

10 rounds $= (10)\ (2\pi$ radians$)$

$= 20\pi$ radians.

**4.17.** The angles of a triangle are in arithmetic progression with a common difference of 20°. Find the measures of the angles in radians.

**Sol:** Let the measures of the angles be $P°$, $Q°$ and $R°$

$P + Q + R = 180°$

$P = Q - 20$ and $R = Q + 20$

$\therefore (Q - 20) + Q + (Q + 20) = 180$

$\Rightarrow Q = 60$

$P = 40$ and $R = 80$

The measures of the angles (in radians) are $\frac{2\pi}{9}, \frac{\pi}{3}$ and $\frac{4\pi}{9}$.

**4.18.** Find the angle covered by the minutes hand in 36 minutes.

**Sol:** The angle covered by the minute hand in 60 minutes $= 360°$

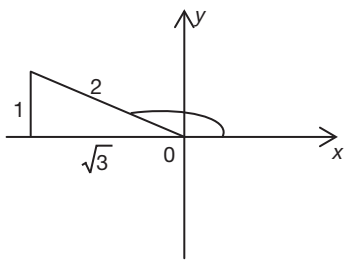
Angle covered by it in 36 minutes

$= \frac{36}{60}(360)° = 216° = \frac{6\pi}{5}$ radians.

**4.19.** If $\tan^2\alpha = \frac{1}{3}$ and $\alpha$ lies in the II quadrant, find the values of $\sin\alpha$ and $\cos\alpha$.

**Sol:** $\tan^2\alpha = \frac{1}{3} \Rightarrow \tan\alpha = \pm\frac{1}{\sqrt{3}}$

Since $\alpha$ lies in the II quadrant, $\sin\alpha > 0$, $\cos\alpha < 0$ and $\tan\alpha < 0$.




$\therefore \tan\alpha = -\frac{1}{\sqrt{3}}$

$\sec\alpha = \sqrt{1+\tan^2\theta} = -\frac{2}{\sqrt{3}}$

$\therefore \cos\alpha = -\sqrt{3}/2$ and $\sin\alpha = \tan\alpha\cos\alpha$

$= \frac{-1}{\sqrt{3}}\left(\frac{-\sqrt{3}}{2}\right) = \frac{1}{2}$.

**4.20.** Show that $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$

**Sol:** $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta}$

$= \frac{\cos\theta(1+\sin\theta+1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{2\cos\theta}{1-\sin^2\theta} = \frac{2\cos\theta}{\cos^2\theta} = 2\sec\theta$

**4.21.** Show that $(\sec\theta - \tan\theta)\ (\sec\theta + \tan\theta) + \sin2\theta - (\sin\theta + \cos\theta)^2 = 0$

**Sol:** $(\sec\theta - \tan\theta)\ (\sec\theta + \tan\theta) = \sec^2\theta - \tan^2\theta = 1$

$(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$= 1 + \sin2\theta$

$\therefore \sin2\theta - (\sin\theta + \cos\theta)^2 = -1$

$\therefore (\sec\theta - \tan\theta)\ (\sec\theta + \tan\theta) + \sin2\theta - (\sin\theta + \cos\theta)^2 = 1 + (-1) = 0$

**4.22.** If $\cos\theta = -\frac{\sqrt{7}}{2\sqrt{2}}$ and $\theta$ is not in second quadrant, then find the value of $\frac{\text{cosec}^2\theta + \sec^2\theta}{\text{cosec}^2\theta - \sec^2\theta}$.

**Sol:** Given $\cos\theta = -\frac{\sqrt{7}}{2\sqrt{2}}$ and $\theta \notin Q_2 \Rightarrow \theta \in Q_3$

$\therefore \text{cosec}\ \theta = -2\sqrt{2}$

and $\sec\theta = \frac{-2\sqrt{2}}{\sqrt{7}}$

$\therefore \frac{\text{cosec}^2\theta + \sec^2\theta}{\text{cosec}^2\theta - \sec^2\theta} = \frac{8 + \frac{8}{7}}{8 - 8/7} = \frac{8\left(1+\frac{1}{7}\right)}{8\left(1-\frac{1}{7}\right)}$

$= \frac{8}{7} \times \frac{7}{6} = \frac{4}{3}$

**4.23.** If $\theta = \pi/6$ then find the value of $\sin^6\theta + \cos^6\theta$.

**Sol:** $\left(\sin\frac{\pi}{6}\right)^6 + \left(\cos\frac{\pi}{6}\right)^6 = \left(\frac{1}{2}\right)^6 + \left(\frac{\sqrt{3}}{2}\right)^6$

$= \frac{1+27}{64} = \frac{7}{16}$

**4.24.** Find the value of cos (–1200°).

**Sol:** We know that cos $(-\theta)$ = cos $\theta$.

$\Rightarrow$ cos (–1200°) = cos 1200°

= cos $(3 \times 360° + 120°)$

= cos 120°

= cos $(180° - 60°)$

= – cos 60° = –1/2

**4.25.** If $\sin\alpha = \frac{4}{5}$ and $\cos\beta = \frac{5}{13}$, then find the value of $\sin\frac{\alpha-\beta}{2}$.

**Sol:** Consider $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \frac{3}{5}.\frac{5}{13} + \frac{4}{5}.\frac{12}{13}$

$\Rightarrow \cos(\alpha-\beta) = \frac{63}{65}$

We know that

$\sin\left(\frac{\alpha-\beta}{2}\right) = \sqrt{\frac{1-\cos(\alpha-\beta)}{2}}$

$= \sqrt{\frac{1-\frac{63}{65}}{2}} = \sqrt{\frac{2}{2(65)}} = \frac{1}{\sqrt{65}}$

**4.26.** Find the value of cos 20° cos 40° – sin 5° sin 25°.

**Sol:** Required value = $\frac{1}{2}$ [2cos 20° cos 40° – 2 sin 5° sin 25°]

$= \frac{1}{2}$ [cos 60° + cos 20° + cos 30° – cos 20°]

$= \frac{1}{2}\left[\frac{1}{2} + \frac{\sqrt{3}}{2}\right] = \frac{\sqrt{3}+1}{4}$

**4.27.** Find the value of $\frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°}$.

**Sol:** $\frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20°\cos 20°} = \frac{2\left[\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right]}{\frac{1}{2}(2\sin 20°\cos 20°)}$

$= \frac{4(\sin 60°\cos 20° - \cos 60°\sin 20°)}{\sin 40°}$

$= 4\left(\frac{\sin 40°}{\sin 40°}\right) = 4$

**4.28.** If the lengths of the sides of a triangle are 8, 9 and 10, find the cosine of the least angle in it.

**Sol:** Let $a = 8$, $b = 9$ and $c = 10$.

The least angle of the triangle will be opposite to the least side in it.

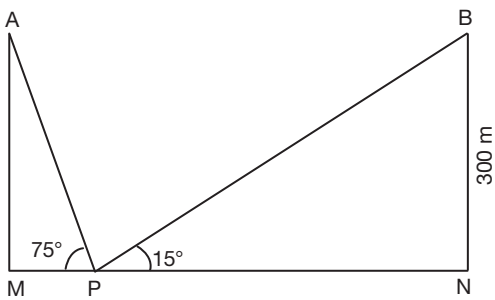
$\therefore \angle A$ is the least angle.

$\cos\angle A = \frac{b^2+c^2-a^2}{2bc} = \frac{9^2+10^2-8^2}{(2)(9)(10)} = 0.65$

$\angle A = \cos^{-1}(0.65)$

**4.29.** From a point $P$ on the ground the angles of elevations of the tops of two buildings ($B$ and $A$) are 15° and 75°, respectively. Each building is 300 m high. Find the distance between the tops of both the buildings. (tan 15° = 2 – $\sqrt{3}$ and tan 75° = 2 + $\sqrt{3}$ )

**Sol:**




Distance between the tops of the two buildings

$= AB = MP + PN$

= 300 (cot 75° + cot 15°)

= 300 (tan 15° + tan 75°)

$= 300\left(2 - \sqrt{3} + 2 + \sqrt{3}\right)$

= 1200 m.

## EXERCISE-1

***Directions for questions 1–40*:** For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** If the angle between two rays $OX$ and $OA$ is 1420°, then $OA$ lies in (here, $OX$ denotes positive $X$-axis) the
(A) first quadrant (B) second quadrant
(C) third quadrant (D) fourth quadrant

**2.** The value of $\frac{6\pi}{5}$ in sexagesimal measure is
(A) 144° (B) 216° (C) 240° (D) 120

**3.** The value of 72° in circular measure is
(A) $\frac{2\pi}{5}$ (B) $\frac{4\pi}{5}$ (C) $\frac{3\pi}{5}$ (D) $\frac{6\pi}{5}$

**4.** $\sin(270° - A) =$
(A) $\cos A$ (B) $\sin A$ (C) $-\cos A$ (D) $-\sin A$

**5.** $\sin 750° =$
(A) 1 (B) –1 (C) 1/2 (D) –1/2

**6.** $\sin 25° + \cos 115° =$ _____.

**7.** The angle subtended by a ray OA when OA completes three revolutions about origin 'O' is
(A) 1020° (B) 1040° (C) 1080° (D) 1440°

**8.** If the radius of a circle is 14 cm, then the angle subtended by the arc of length 110 cm at the centre of the circle in radians is
(A) $\frac{7\pi}{2}$ (B) $\frac{9\pi}{2}$ (C) $\frac{3\pi}{2}$ (D) $\frac{5\pi}{2}$

**9.** Given that $\sin\theta$ is positive and $\cos\theta$ is negative, then $\theta$ is in _____ quadrant.
(A) 1st (B) 2nd (C) 3rd (D) 4th

**10.** If $\theta$ is acute and $\operatorname{cosec}\theta = \frac{17}{8}$ then $\cot\theta$ is
(A) $\frac{8}{15}$ (B) $\frac{15}{17}$ (C) $\frac{15}{8}$ (D) $\frac{17}{15}$

**11.** $\sin\theta \times \operatorname{cosec}\theta =$ _____.

**12.** If $\tan\theta = \frac{3}{4}$, then $\theta$ lies in
(A) $Q_2$ (B) $Q_3$ (C) $Q_1$ (D) $Q_1$ or $Q_3$

**13.** If $\tan\theta = \frac{1}{2}$ and $\cos\theta = -\frac{2}{\sqrt{5}}$, then $\theta$ lies in _____ quadrant.
(A) $Q_1$ (B) $Q_2$ (C) $Q_3$ (D) $Q_4$

**14.** If the lengths of the sides of a triangle are 1, $\sqrt{3}$ and 2, then the angles opposite to these sides are _____, respectively.
(A) 45°, 60°, 75° (B) 30°, 60°, 90°
(C) 30°, 45°, 105° (D) None of these

**15.** If '$\theta$' is an acute angle, and $\sin\theta = \cos\theta$, then $\theta =$
(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

**16.** For $0 < \theta < \frac{\pi}{4}$, $\frac{\tan\theta}{\cot\theta}$ is
(A) less than 1 (B) greater than 1
(C) equal to 1 (D) $\infty$

**17.** If $\theta \in Q_2$ and $\sin\theta = \frac{5}{13}$, then $\cot\theta =$
(A) $\frac{12}{5}$ (B) $-\frac{12}{5}$ (C) $\frac{5}{12}$ (D) $-\frac{5}{12}$

**18.** If $3\tan\theta - 4 = 0$ and $180° < \theta < 270°$ then $\operatorname{cosec}\theta =$
(A) $\frac{5}{4}$ (B) $\frac{4}{5}$ (C) $-\frac{5}{4}$ (D) $-\frac{4}{5}$

**19.** $\sec^4\theta + \tan^4\theta - 2\sec^2\theta\tan^2\theta =$ _____.

**20.** If $\tan\alpha = \cot\beta = 1$ and $\alpha, \beta$ are both acute angles, then find the value of $2\alpha + \beta$.
(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{2\pi}{3}$ (D) $\frac{4\pi}{3}$

**21.** $\sin^2 45° + \cos^2 45° =$ _____.

**22.** $\sin^2 225° + \cos^2 225° =$ _____.

**23.** If $p = \operatorname{cosec}\theta + \cot\theta$, then the value of $p + \frac{1}{p} =$
(A) $\frac{2}{\sin\theta}$ (B) $2\sin\theta$ (C) $\frac{2}{\cos\theta}$ (D) $2\cot\theta$

**24.** If $\sec\theta - \tan\theta = \frac{1}{5}$, then $\sec\theta + \tan\theta =$ _____.

**25.** The value of $(1 + \sin\theta)(1 - \sin\theta)\sec^2\theta =$ _____.

**26.** $\sec\theta\cos\theta = 1$ is not true for $\theta =$
(A) 30° (B) 0° (C) 60° (D) 90°

**27.** If $\cot\theta = \dfrac{1+\cos 2\theta}{\sin 2\theta}$, then the value of $\cot 22^{1}/_{2}^{\circ}$ is

(A) $2\left(\sqrt{2}-1\right)$ (B) $2\left(\sqrt{2}+1\right)$

(C) $\sqrt{2}+1$ (D) $\sqrt{2}-1$

**28.** $\sin 30^\circ + \sqrt{3}\ \tan 60^\circ - \sec 0^\circ =$ _____.

**29.** Which of the following can be true?

(A) $\sin\theta = \pi$ (B) $\cos\theta = -\pi$

(C) $\sec\theta = \dfrac{1}{2}$ (D) $\operatorname{cosec}\theta = -3$

**30.** $\sec^2 60^\circ - \tan^2 60^\circ =$ _____.

**31.** If $x = \sin\theta$ and $y = \cos\theta$ and $0 < \theta < 45^\circ$, then

(A) $x = y$ (B) $x > y$

(C) $x < y$ (D) Cannot say

**32.** If $\operatorname{cosec}\theta = -\sqrt{2}$ and $\tan\theta = -1$, then $\cos\theta =$

(A) $\sqrt{2}$ (B) $\dfrac{1}{\sqrt{2}}$ (C) $-\sqrt{2}$ (D) $\dfrac{-1}{\sqrt{2}}$

**33.** If $3\sin\theta = 2$ and '$\theta$' is in II quadrant, then $\tan\theta =$

(A) $\dfrac{-2}{\sqrt{5}}$ (B) $\dfrac{2}{\sqrt{5}}$ (C) $\dfrac{2}{5}$ (D) $\dfrac{-2}{5}$

**34.** The value of $\sec^4\theta - \tan^4\theta$, when $\theta = \dfrac{\pi}{3}$ is _____.

**35.** The range of $2\cos\theta$ is

(A) $[-2, 2]$ (B) $[-1, 1]$ (C) $[0, 1]$ (D) $[-2, 1]$

**36.** In $\Delta ABC$, if $AB = 6$ units, $BC = 4$ units and $\angle A = 30^\circ$, then the value of sinC is

(A) $\dfrac{3}{4}$ (B) $\dfrac{4}{3}$ (C) $\dfrac{5}{6}$ (D) $\dfrac{2}{3}$

**37.** The maximum value of $3\sin x + 4\cos x$ is _____.

**38.** At how many points do the graphs of sinx and cosx intersect when $0 < x < \pi/2$? _____

**39.** If $\theta$ is acute and $\sec\theta = \dfrac{13}{5}$, then tan θ can be

(A) $\dfrac{12}{5}$ but not $-\dfrac{12}{5}$ (B) $-\dfrac{12}{5}$ but not $\dfrac{12}{5}$

(C) either $\dfrac{12}{5}$ or $-\dfrac{12}{5}$ (D) neither $\dfrac{12}{5}$ nor $-\dfrac{12}{5}$

**40.** If $\cot\theta + \tan\theta = 2$, then $\cos\theta =$

(A) $\pm\dfrac{1}{\sqrt{3}}$ (B) $\pm\dfrac{1}{2}$ (C) $\pm\dfrac{1}{\sqrt{2}}$ (D) $\pm\dfrac{\sqrt{3}}{2}$

## EXERCISE-2

***Directions for questions 1–40***: For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple-choice questions, write your answer in the space provided.

**1.** The angles of a triangle are in *A.P.* If the least angle is $\dfrac{1}{3}$ of the greatest, then the angles of the triangle in circular measure are

(A) $\dfrac{\pi}{6}, \dfrac{\pi}{3}, \dfrac{\pi}{2}$ (B) $\dfrac{\pi}{4}, \dfrac{\pi}{4}, \dfrac{\pi}{2}$

(C) $\dfrac{3\pi}{4}, \dfrac{\pi}{8}, \dfrac{\pi}{8}$ (D) $\dfrac{2\pi}{3}, \dfrac{\pi}{6}, \dfrac{\pi}{6}$

**2.** If $\tan\theta = \dfrac{15}{8}$ $(\theta \in Q_1)$, then find the values of $\sin\theta$, $\cos\theta$, $\sec\theta$, $\operatorname{cosec}\theta$ and $\cot\theta$.

(A) $\dfrac{3}{8}$ (B) $\dfrac{8}{15}$ (C) $\dfrac{7}{15}$ (D) $\dfrac{17}{15}$

**3.** The value of $(1+\tan^2\theta)\ (1+\cot^2\theta)\ (\sin^2\theta - 1)\ (\cos\theta^2 - 1)\left(\dfrac{1}{\tan^2\theta} - \dfrac{1}{\sin^2\theta}\right)$ is .

**4.** Find the value of $\cos A\left(\dfrac{\cot A}{\operatorname{cosec} A + 1} + \dfrac{\cot A}{\operatorname{cosec} A - 1}\right)$.

**5.** Find the value of $\operatorname{cosec}(-1560^\circ)$.

(A) $\dfrac{1}{\sqrt{3}}$ (B) $\dfrac{\sqrt{3}}{2}$ (C) $\dfrac{-2}{\sqrt{3}}$ (D) $-\sqrt{3}$

**6.** If $\cos\theta = -\dfrac{15}{7}$ and $\operatorname{cosec}\theta = \dfrac{17}{8}$, then the value of $\dfrac{\sin\theta + \cot\theta}{\sin\theta - \cot\theta}$ is

(A) $\dfrac{319}{191}$ (B) $\dfrac{19}{39}$ (C) $\dfrac{-319}{191}$ (D) $\dfrac{-191}{319}$

**7.** Simplify $\dfrac{\sin 4A(1-\cos 2A)\cos A}{(1+\cos 2A)\left(\cos^2 A - \sin^2 A\right)\sin^2 A}$.

(A) $\sin A$ (B) $4\operatorname{cosec} A$

(C) $4\sin A$ (D) $4\cos A$

Difficulty Level-2: Moderate

**8.** Find the value of $\cos\frac{\pi}{12}+\cos\frac{2\pi}{12}+\cos\frac{3\pi}{12}+\cos\frac{4\pi}{12}+\cos\frac{5\pi}{12}+\cos\frac{7\pi}{12}+\cos\frac{9\pi}{12}+\cos\frac{10\pi}{12}+\cos\frac{11\pi}{12}$.

**9.** Find the range of the function $f(\theta)=\sin^4\theta+\cos^2\theta$.

(A) $\left[\frac{-3}{4},1\right]$ (B) $\left[\frac{1}{2},1\right]$

(C) $\left[\frac{1}{2},\frac{1}{2}\right]$ (D) $\left[\frac{3}{4},1\right]$

**10.** The difference between the maximum and minimum values of $12\sin x+5\cos x+11$ is _____.

**11.** The value of $\cos 28^\circ+\cos 65^\circ+\cos 115^\circ+\cos 208^\circ+\cos 240^\circ+\cos 300^\circ$ is _____.

**12.** If $13\sin\theta=12$ and $\theta$ is acute, then $\frac{\operatorname{cosec}\theta+\cot\theta}{\tan\theta+\sec\theta}=$ .

(A) $\frac{3}{8}$ (B) $\frac{3}{10}$ (C) $\frac{10}{3}$ (D) $\frac{8}{3}$

**13.** $(1+\tan\theta+\sec\theta)(1+\sec\theta-\tan\theta)-2\sec\theta=$ _____.

**14.** If $\cos\theta+\cos^2\theta=1$, then $\sin^2\theta+\sin^4\theta=$ _____.

**15.** If $\sec\theta+\tan\theta=$ m, then $\cos\theta=$

(A) $\frac{m}{m^2+1}$ (B) $\frac{m}{m^2-1}$ (C) $\frac{m^2+1}{m^2-1}$ (D) $\frac{2m}{m^2+1}$

**16.** If $\operatorname{cosec}\theta-\cot\theta=p$, then $\operatorname{cosec}\theta+\cot\theta=$

(A) $p$ (B) $-p$ (C) $\frac{1}{p^2}$ (D) $\frac{1}{p}$

**17.** If $5\cos\theta=3$ and $\theta$ is acute, then $\frac{\tan\theta+\cot\theta}{\sec\theta+\operatorname{cosec}\theta}$ is

(A) 7/5 (B) 5/7 (C) 1 (D) 3/5

**18.** If $a=b\sin\theta+c$ and $b=a\cos\theta+c$, then

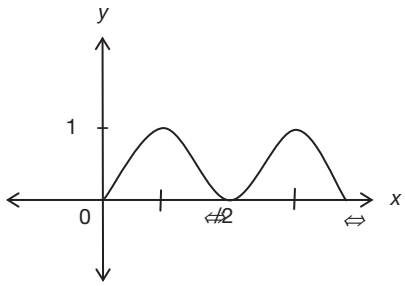
(A) $\frac{(a-b)^2}{c^2}+\frac{(b-a)^2}{c^2}=1$ (B) $\left(\frac{a-c}{a}\right)^2+\left(\frac{b-c}{b}\right)^2=1$

(C) $\left(\frac{a-c}{c}\right)^2+\left(\frac{b-c}{c}\right)^2=1$ (D) $\frac{(a-c)^2}{b^2}+\frac{(b-c)^2}{a^2}=1$

**19.** The minimum value of $\sin^6x+\cos^6x$ is

(A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $\frac{1}{4}$ (D) $\sqrt{2}$

**20.** Which of the following relations best describes the grah?




(A) $y=|\sin 2x|$ (B) $y=|\sin x/2|$

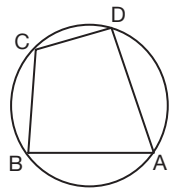
(C) $y=|\cos x|$ (D) $y=\sin|x|$

**21.** Find the number of solutions of the equation $\sin^8x+\cos^8x=1$ in the interval $[-2\pi, 2\pi]$.

(A) 12 (B) 4 (C) 5 (D) 9

**22.** If $3\cos^2A=\cos 60^\circ+\sin^2 45^\circ$, then $\sec^2A=$ _____.

**23.** In the figure shown below, if $\tan A+\tan B+\tan C=5$, then $\tan D=$




(A) 2 (B) 0 (C) 5 (D) –5

**24.** If the lengths of the sides of triangle are 2, $3\sqrt{3}$ and 7, then the greatest angle (in degrees) of the triangle is _____.

**25.** If $\sin\theta+\cos\theta=\sqrt{2}$, then find $\tan^n\theta+\cot^n\theta$. _____

**26.** If $(1-\cot 4^\circ)(1-\cot 5^\circ)(1-\cot 6^\circ)$ _____ $(1-\cot 41^\circ)=2^P$, then the value of $P$ is .

**27.** The maximum value of $\sin^2x-\cos 2x$ is

(A) 1 (B) 2 (C) 3 (D) –2

**28.** If $f(x)=3\sin x-4\cos x$, the minimum value of $f(x)$ is

(A) –5 (B) –3

(C) 0 (D) None of these

**29.** The value of $\sin 12^\circ\sin 48^\circ\sin 54^\circ$ is equal to

[Given $4\sin A\sin(60+A)\sin(60-A)=\sin 3A$)]

(A) 1/16 (B) 1/8 (C) 1/4 (D) 1/2

**30.** $3(\cos^2 45^\circ+\sin^2 225^\circ)-3(\sin^2 225^\circ+\cos^4 225^\circ)$

(A) 4/3 (B) 1/9 (C) 3/4 (D) 1/2

**31.** If $0 < \theta < 45°$, then find the value of $\sqrt{2-\sqrt{2-\sqrt{2-\cos 2\theta}}}$

(A) $\sqrt{2+\sqrt{2-2\cos\theta}}$ (B) $\sqrt{2-\sqrt{2-2\cos\theta}}$

(C) $\sqrt{2-\sqrt{2-2\sin\theta}}$ (D) $\sqrt{2+\sqrt{2-\sin\theta}}$

**32.** Malini observed the top of a tower of height 60 m at an angle of elevation of 15° from a certain point on the ground. How far was she away from the foot of the tower? (Given $\tan 15° = 2 - \sqrt{3}$ )

(A) $60\left(2-\sqrt{3}\right)$ m (B) $\left(2+\sqrt{3}\right)$ m

(C) $60\left(2+\sqrt{3}\right)$ m (D) $30\left(2+\sqrt{3}\right)$ m

**33.** A man moving at a speed of 36 kmph covered the shadow of a tower of height 75 m in 3 seconds. If the height of the person is 1.75 m, the length of the shadow cast by the man in meters is

(A) 1.5 (B) 0.7 (C) 1 (D) 0.5

**34.** The angle of elevation of the top of a hill from a point $A$ is $30^o$. After walking a distance of 1.5 km on a slope inclined at $15^o$ to the horizontal, the angle of elevation of the top is found to be 45°. The height of the hill in kilometres is

(A) $\left(\sqrt{3}+1\right)(1.5)$ (B) $\dfrac{\left(\sqrt{3}+1\right)(1.5)}{2}$

(C) $\dfrac{\sqrt{2}\left(\sqrt{3}+1\right)(1.5)}{2}$ (D) $\dfrac{\sqrt{2}\left(\sqrt{3}+1\right)(1.5)}{4}$

**35.** $PQ$ is a tower whose base is $P$. $R$ is the midpoint of $PQ$. $A$ is a point on the ground. The portion $RQ$ subtends an angle $\alpha$. at $A$. If $AP = 3\ PQ$, then $\cot\alpha =$

(A) $\dfrac{16}{19}$ (B) $\dfrac{3}{19}$ (C) $\dfrac{19}{3}$ (D) $\dfrac{19}{16}$

**36.** A man was travelling towards a lighthouse in a boat. At a certain instant, the man observed the top of the lighthouse at an angle of elevation of $45^0$. Five minutes later, he observed the top of the lighthouse at an angle of elevation of $60^0$. If the boat travelled at an uniform speed of 5 m/min, the height of the lighthouse (in metres) is

(A) $\dfrac{25\left(3+\sqrt{3}\right)}{2}$ (B) $\dfrac{25\left(\sqrt{3}+1\right)}{2}$

(C) $25\left(3+\sqrt{3}\right)$ (D) $25\left(\sqrt{3}+1\right)$

**37.** A man, standing at the foot of a hillock, observed the top of the hillock at an angle of elevation of 45°. There is a straight path towards the top. The path makes an angle of 30° with the horizontal. After covering a distance '$d$' along this path, the man sees the top of the hillock at an angle of elevation of 60°. If the height of the hillock is 40 m, find the distance the man covers along the path.

(A) $20\left(\sqrt{3}+1\right)$m (B) $20\left(\sqrt{3}-1\right)$m

(C) $40\left(\sqrt{3}-1\right)$m (D) $40\left(\sqrt{3}+1\right)$m

**38.** Ranvir observes from a point on the ground with the angles of elevation of the top and bottom of a flag post standing on the top of a building 300 m high to be 45° and 60°, respectively. The height (in metres) of the flag post is _____.

**39.** $AB$ is a vertical pole. The end $A$ is on the level ground. $C$ is the midpoint of $AB$. $P$ is a point on the level ground such that the portion $BC$ subtends an angle $\theta$ at $P$. If $AP = nAB$, then the value of $\cot\theta$ is

(A) $\dfrac{2n^2+1}{n}$ (B) $\dfrac{n}{2n^2+1}$

(C) $\dfrac{2n^2+1}{2n}$ (D) $\dfrac{2n}{2n^2+1}$

**40.** From the top of a light house 30 m high, with its base at the sea level, the angle of depression of a boat is 75°. Find the distance of the boat from the foot of the light house. (in m) _____

## EXERCISE-3

***Directions for questions 1–40***: For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

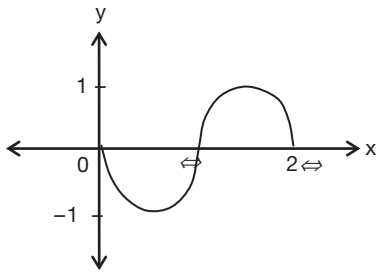
**1.** If $\cot\theta = \sqrt{\dfrac{1+\cos 2\theta}{1-\cos 2\theta}}$, then $\cot(22\frac{1}{2}°)$ is

(A) $2-\sqrt{3}$ (B) $2+\sqrt{3}$ (C) $\sqrt{2}-1$ (D) $\sqrt{2}+1$

Difficulty Level-3: Advanced

**2.** If $(\cot 10°)(\cot 11°)(\cot 12°)\ldots\ldots(\cot 80°) = x$, then the value of $x$ is _____.

**3.** Which of the following relations best describes the graph?




(A) $y = \sin x$ (B) $y = -\cos x$
(C) $y = -\sin x$ (D) $x = \sin y$

**4.** If $\operatorname{cosec}\theta$ and $\cot\theta$ are the roots of the equation $cx^2 + bx + a = 0$, then which of the following is true?
(A) $b^4 = 4ab^2c + c^4$ (B) $c^4 = 4ab^2c + b^4$
(C) $b^4 = 4ab^2c - c^4$ (D) $b^4 + c^4 = 4ab^2c$

**5.** If $3\tan\theta - 4 = 0$, and $\theta$ is acute, then the value of $\dfrac{3\sec\theta + 2\operatorname{cosec}\theta}{\cot\theta - 5\sin\theta}$ is

(A) $\dfrac{15}{13}$ (B) $\dfrac{13}{30}$ (C) $\dfrac{-30}{13}$ (D) $\dfrac{-15}{13}$

**6.** If $\dfrac{\sin^2 x}{1+\cot x} + \dfrac{\cos^2 x}{1+\tan x} = k - \sin x\cos x$, then $k =$ _____.

**7.** $\tan 17° + \tan 28° + \tan 17°\tan 28° =$ _____.

**8.** $\sqrt{\dfrac{1-\sin x}{1+\sin x}} =$

(A) $\sec x - \tan x$ (B) $\sec x + \tan x$
(C) 1 (D) $\sec^2 x + \tan^2 x$

**9.** $\dfrac{\cos x}{1-\tan x} + \dfrac{\sin x}{1-\cot x} =$

(A) $\sin x + \cos x$ (B) $\sin^2 x - \cos^2 x$
(C) $(\sin x + \cos x)^2$ (D) None of these

**10.** If $\sin A = 1/3$; $\tan B = 3/4$; $0° < A < 90°$ and $180° < B < 270°$, the value of $9\cos^2 A + 20\sec B$ is _____.

**11.** $\log\cot 30° + \log\cot 45° + \log\cot 60° =$

(A) $\log 3$ (B) $1/\sqrt{3}$ (C) 1 (D) 0

**12.** If $2\sqrt{3} - 2$, $2\sqrt{3} + 2$, $\sqrt{24}$ are the lengths of the sides of a triangle, then the greatest angle of the triangle is

(A) $\dfrac{\pi}{6}$ (B) $\dfrac{5\pi}{12}$ (C) $\dfrac{7\pi}{12}$ (D) $\dfrac{5\pi}{9}$

**13.** $a = b\sec\theta + c$ and $b = a\tan\theta + c$, then which of the following is true?

(A) $\dfrac{(a-c)^2}{b^2} + \dfrac{(b-c)^2}{a^2} = 1$ (B) $\dfrac{(a-c)^2}{b^2} - \dfrac{(b-c)^2}{a^2} = 1$

(C) $-\dfrac{(a-c)^2}{b^2} + \dfrac{(b-c)^2}{a^2} = 2$ (D) $\dfrac{(b-c)^2}{a^2} - \dfrac{(a-c)^2}{b^2} = 1$

**14.** If $g(x) = 3(|\sin x| - |\cos x|)$, where $x \in \left[\dfrac{3\pi}{4}, \dfrac{5\pi}{4}\right]$ then which of the following represents the range of $g(x)$?
(A) $[-1, 0]$ (B) $[-3, 0]$ (C) $[0, 1]$ (D) $[0, 3]$

**15.** If $\sin\theta - \cos\theta = \dfrac{1}{5}$, find the value of $\sin^3\theta - \cos^3\theta$.

(A) $\dfrac{5}{12}$ (B) $\dfrac{7}{25}$ (C) $\dfrac{63}{125}$ (D) $\dfrac{37}{125}$

**16.** If $A + B = 45°$, then the value of $(1 + \tan A)(1 + \tan B)$ is _____.

**17.** The range of $f(x) = \dfrac{1}{5 - \cos 3x}$ is

(A) $\left[\dfrac{1}{3}, \dfrac{2}{3}\right]$ (B) $\left[\dfrac{1}{6}, \dfrac{1}{4}\right]$

(C) $\left[-1, \dfrac{1}{4}\right]$ (D) $\left[-\dfrac{1}{3}, \dfrac{1}{4}\right]$

**18.** Find the range of the function $f(\theta) = \sin^2\theta - \cos^2\theta$.
(A) $(0, \infty)$ (B) $[-1, 1]$ (C) $(-1, 0)$ (D) $(1, \infty)$

**19.** The range of $8\sin x + 15\cos x + 13$ is
(A) $[-2, 10]$ (B) $[-4, 30]$
(C) $(0, \infty)$ (D) $[-10, 20]$

**20.** If $13\sin\theta = 5$ and $\theta$ is obtuse, then $\dfrac{\cot\theta + \sec\theta}{\tan\theta + \cos\theta} =$

(A) 5/12 (B) 13/12
(C) 13/5 (D) 5/13

**21.** In a triangle $ABC$, if $a = 3$ cm, $b = 4$ cm and $\angle C = 60°$, then the length of the third side is

(A) 13 (B) $\sqrt{3}$ (C) $\sqrt{14}$ (D) $\sqrt{15}$

**22.** The expression

$$9\left[\sin^4\left(\frac{5\pi}{2}-\alpha\right)+\sin^4(6\pi+\alpha)\right]$$
$$-6\left[\sin^6\left(\frac{3\pi}{2}+\alpha\right)+\sin^6(7\pi-\alpha)\right] \text{ is equal to}$$

(A) 0 (B) 1 (C) 2 (D) 3

**23.** If two sides of a triangle are 8 units and 6 units and the included angle is 75°, then the area of the triangle is

(A) $6\left(\sqrt{6}-\sqrt{2}\right)$ (B) $6\sqrt{2}$

(C) $\sqrt{6}+\sqrt{2}$ (D) $6\left(\sqrt{6}+\sqrt{2}\right)$

**24.** $a\cos\alpha + b\sin\alpha = c$, where $\alpha = \theta_1$ or $\theta_2$ find $\sin\theta_1 + \sin\theta_2 =$

(A) $\frac{2bc}{a^2+b^2}$ (B) $\frac{c^2-a^2}{a^2+b^2}$

(C) $\frac{2ac}{a^2+b^2}$ (D) None of these

**25.** $\log_3(\sin^2 15° + \cos^2 75°) \times \log_5(\sin^2 30° + \cos^2 60°) \times \log_3(\sin^2 45° + \cos^2 45°) \times \log_5(\sin^2 60° + \cos^2 30°) \times \log_3(\sin^2 60° + \cos^2 30°) - \log_5(\sin^2 60° + \cos^2 60°)$

= _____.

**26.** The minimum value of $81^{\cos^2 x} + 81^{\sin^2 x}$ is _____.

**27.** $\sin^2(\theta-45)° + \sin^2(\theta+15)° - \sin^2(\theta-15)° =$ .

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 0

**28.** The value of $\frac{\cos 20° + \sin 50°}{\sin 20° + \cos 50°}$ is

(A) 3 (B) $\sqrt{3}$ (C) 1/3 (D) $1/\sqrt{3}$

**29.** For all $\theta$, if $\tan\theta = \frac{\sin 2\theta}{1+\cos 2\theta}$, then $\tan 22\frac{1}{2} =$

(A) $\sqrt{2}+1$ (B) $\sqrt{2}-1$

(C) $\frac{\sqrt{2}-1}{\sqrt{2}}$ (D) $\frac{\sqrt{2}-1}{2\sqrt{2}}$

**30.** $\frac{\sin 2A\,(1-\cos 2A)}{(1+\cos 2A)\sin A}(\cos A) =$

(A) $\cos A$ (B) $2\sin A$
(C) $\sin 2A$ (D) $2\sin^2 A$

**31.** There is a plot in the shape of an equilateral triangle. A flagstaff is erected at the midpoint of one of its sides. The angles of elevation of the top of the flagstaff from two of the vertices of the plot are 45° each. Find the height of the flagstaff (in meters) if the length of one of the medians of the plot is $60\sqrt{3}$ m.

(A) $50\sqrt{3}$ (B) $45\sqrt{3}$ (C) $60\sqrt{3}$ (D) 60

**32.** Raghuram, a Maths historian observed the tip of the hour hand and minute hand of an ancient clock tower at 6 o' clock from a point on the ground that was 1m away from the foot of the tower. Using the sextant, he found that the angles of elevation were 30° and 45° in that order of observation. Raghuram could measure the diameter of the dial approximately. The approximate value of the diameter in centimetres is
(A) 40.36 (B) 23.85 (C) 42.26 (D) 45.25

**33.** A man moving towards a lighthouse at a speed of 54 kmph observes the top of the lighthouse at an angle of elevation of 30°. After three minutes, he finds the angle of elevation of the top of the lighthouse to be 45°. The height of the lighthouse in metres is

(A) $1250\left(\sqrt{3}-1\right)$ (B) $1350\left(\sqrt{3}-1\right)$

(C) $1350\left(\sqrt{3}+1\right)$ (D) $1250\left(\sqrt{3}+1\right)$

**34.** Find the angle of elevation of the sun, if the length of the shadow of a pole is $1/\sqrt{3}$ times its height?

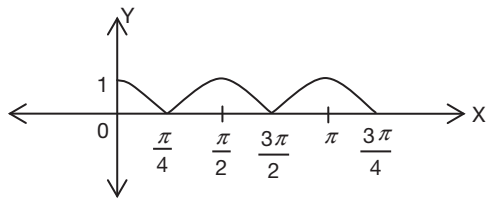
(A) $\pi/6$ (B) $\pi/5$ (C) $\pi/9$ (D) $\pi/3$

**35.** Govind observed an aeroplane flying exactly 1 km above the ground at an angle of elevation of 60°. If after 15 seconds, the plane moved away from him and made an angle of elevation of 30° at the same height, then the uniform speed of the plane was

(A) $240\sqrt{3}$ kmph (B) $160\sqrt{3}$ kmph

(C) $120\sqrt{3}$ kmph (D) $144\sqrt{3}$ kmph

**36.** Which of the following relations best describes the graph?



(A) $x = \cos 2y$ (B) $y = \cos 2x$
(C) $x = |\cos 2y|$ (D) $y = |\cos 2x|$

**37.** The angles of depression of two ships due east of a lighthouse from the top of the light house are 37° and 45°. If the ships are 80 m apart, then the height of the lighthouse (in m) is (Given sin 37° = 0×6) _____.

**38.** A plot was in the shape of an equilateral triangle. A flagstaff was erected at its centroid. The top of the flagstaff was connected to each of the vertices of the plot by a rope. The angle of elevation of the top of the flagstaff at each vertex of the plot was $30^0$. If the height of the flagstaff was 24 m, the side of the triangle (in m) is _____.

**39.** A flagpost stands on the top of a tower. The angles of elevation of the top of the tower and the flag post from a point 180 m from the foot of the tower are 30° and 60°, respectively. What is the height of the flagpost? (in m)

(A) $120\sqrt{3}$ (B) $40\sqrt{3}$ (C) $60\sqrt{3}$ (D) 60

**40.** The upper part of a pole, broken over by the wind, makes an angle of 45° with the ground. The distance from the foot of the pole to the point where the top of the pole touches the ground is $30\left(\sqrt{2}-1\right)$m . What was the original height of thepole? (in m)

## EXERCISE-4

***Directions for questions 1–30***: For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple-choice questions, write your answer in the space provided.

**1.** If $\cot\theta = \frac{b}{a}$ where $a, b > 0$ and $\theta \in Q_1$, the value of $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$ is

(A) $\frac{b-a}{b+a}$ (B) $\frac{b^2-a^2}{b^2+a^2}$
(C) $\frac{b+a}{b-a}$ (D) $\frac{a^2-b^2}{a^2+b^2}$

**2.** The value of $\left(\frac{\cot A}{\text{cosec}\, A+1} + \frac{\cot A}{\text{cosec}\, A-1}\right)\frac{1}{\sec A}$ is

(A) –2 (B) –1 (C) 2 (D) 3

**3.** If $x = a\cot\theta + b\,\text{cosec}\,\theta$ and $y = a\cot\theta - b\,\text{cosec}\,\theta$, which of the following is true?

(A) $\frac{(x-y)^2}{4a^2} + \frac{(x+y)^2}{4b^2} = 1$ (B) $\frac{(x-y)^2}{4b^2} + \frac{(x+y)^2}{4a^2} = 1$
(C) $\frac{(x-y)^2}{4a^2} - \frac{(x+y)^2}{4b^2} = 1$ (D) $\frac{(x-y)^2}{4b^2} - \frac{(x+y)^2}{4a^2} = 1$

**4.** If $\sin\theta + \cos\theta = \sqrt{2}$, find $\tan^n\theta + \cot^n\theta$.

(A) 0 (B) 1 (C) –1 (D) 2

**5.** If $\sin 18° = \frac{\sqrt{5}-1}{4}$, and $\cos 36° = \frac{\sqrt{5}+1}{4}$, the value of tan 9° – tan 27° – tan 63° + tan 81° is

(A) 1 (B) 2 (C) 3 (D) 4

**6.** The range of $\cos^2\theta + \sin^4\theta$ is

(A) $\left[\frac{3}{4}, 1\right]$ (B) $\left[\frac{1}{4}, \frac{3}{4}\right]$
(C) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (D) $\left[-\frac{1}{2}, 1\right]$

**7.** The minimum value of $1 + 8\sin^2 x^2\cos^2 x^2$ is

(A) 1 (B) 2 (C) 3 (D) 0

**8.** $\sin^2(\theta - 45)° + \sin^2(\theta + 15)° - \sin^2(\theta - 15)° =$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 0

**9.** The minimum value of $4\tan^2 x + 9\cot^2 x$ is

(A) 9 (B) 4 (C) 6 (D) 12

**10.** In $\Delta ABC$, if $\frac{4\sin A + 3\sin B}{4\sin A - 3\sin B} = \frac{7}{3}$, and $AC = 16$, $BC =$

(A) 18 (B) 20 (C) 30 (D) 24

**11.** $P = 1 - \cos^2\theta + \cos^4\theta$. For all real $\theta$,

(A) $\frac{1}{2} < P \le 1$ (B) $\frac{3}{4} \le P \le 1$
(C) $\frac{13}{16} \le P \le 1$ (D) $\frac{1}{4} \le P \le 1$

**12.** The number of values of $x$ in the interval $(0, 2\pi)$, that satisfy the equation $27^{1+|\sin x|+|\sin^2 x|+\ldots} = 3^6$ is _____.

**13.** Find the value of $(1 + \tan 15°)(1 + \tan 16°)(1 + \tan 17°) \ldots (1 + \tan 30°)$.

(A) 0 (B) 1 (C) 512 (D) 256

**14.** $\sin(\alpha+\beta)=\frac{4}{5}$ and $\cos(\alpha-\beta)=\frac{12}{13}$, where $0<\beta<\alpha<\pi/4$. Find the value of $\cot 2\alpha$.

(A) $\frac{16}{63}$ (B) $\frac{25}{63}$ (C) $\frac{63}{16}$ (D) $\frac{63}{25}$

**15.** $\left(1+\cos 22\frac{1}{2}\right)\left(1+\cos 67\frac{1}{2}\right)\left(1+\cos 112\frac{1}{2}\right)\left(1+\cos 157\frac{1}{2}\right)=$ _____.

**16.** If $6\cos A+8\sin B=12$ and $8\cos B+6\sin A=2\sqrt{13}$ and $A, B, C$ are the angles of $\Delta ABC$, find $\angle C$ in degrees. ____

**17.** If $x=\cos 50^\circ+\cos 55^\circ+\cos 60^\circ$ and $y=\sin 20^\circ+\sin 25^\circ+\sin 30^\circ$, then

(A) $\frac{x}{y}>1$ (B) $\frac{x}{y}<1$ (C) $\frac{x}{y}=1$ (D) $x+y=0$

**18.** If $h(y)=3\left[|\sin y|+|\cos y|\right]$, which of the following represents the range of h(y)?

(A) $3\le h(y)\le 3\sqrt{2}$ (B) $0\le h(y)\le 3\sqrt{2}$

(C) $3\le h(y)\le 4$ (D) None of these

**19.** If $x^2+y^2\le 9$ and $|\tan^2\pi x|+|\cos^2\pi y|=0$, the number of possible values of $(x, y)$ is ____.

**20.** If $\theta<45^\circ$, find the value of $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 2\theta}}}$.

(A) $4\cos^2\theta/4$ (B) $4\sin^2\theta/4$
(C) $2\cos\theta/4$ (D) $2\sin\theta/4$

**21.** If $f(x)=\frac{\sin^3 x}{\cos^2 x}-\frac{\cos^3 x}{\sin^2 x}$ and $g(x)=\frac{\cos^2 x}{\sin^3 x}-\frac{\sin^2 x}{\cos^3 x}$, where $\frac{\pi}{4}<x<\frac{\pi}{2}$, which of the following best describes the relation between $f(x)$ and $g(x)$?

(A) $f(x)\ge g(x)$ (B) $f(x)\le g(x)$
(C) $f(x)>g(x)$ (D) $f(x)<g(x)$

**22.** In $\Delta PQR$, if $p=8$ cm, $q=10$ cm and $\cos R=\frac{\sqrt{5}+1}{4}$, find $r$.

(A) $2\sqrt{41-10\sqrt{5}}$ (B) $2\sqrt{36-10\sqrt{5}}$

(C) $2\sqrt{31-10\sqrt{5}}$ (D) $2\sqrt{26-10\sqrt{5}}$

**23.** An aeroplane flying horizontally 1 km above the ground with a uniform speed is observed at an elevation of 53° from a point on the ground. The plane is moving away from the observer. After 12 seconds, the angle of elevation of the plane is observed to be 45° from the same point. Find the speed of the aeroplane. (Given, $\sin 37^\circ=0\times 6$)

(A) 48 km/sec (B) $\frac{1}{48}$ km/sec

(C) 24 km/sec (D) $\frac{1}{24}$ km/sec

**24.** The angles of elevation of the tops of two towers, standing on level ground, from a point on the ground mid-way between the towers are 60° and 45°, respectively. The height of the first tower is $112\sqrt{3}$ m. The height of the second tower is

(A) 56 m (B) 224 m
(C) $112\sqrt{3}$ m (D) 112 m

**25.** A man at the top of a 24-m tall lighthouse observes a motorboat approaching the foot of the lighthouse in a straight line at a uniform speed. He recorded the angle of depression to be 30° at 10:06 a.m. After 16 minutes, he observed the angle of depression to be 60°. Find the time at which the boat reaches the foot of the lighthouse.

(A) 10:30 a.m. (B) 10:22 a.m.
(C) 10:28 a.m. (D) 10:45 a.m.

**26.** The angle of depression of the water image of a 35-m tall tree which is on one side of a river as observed by a man standing on a building of height 5 m on the other side of the river is $\sin^{-1} 40/41$. The cosine of the angle of elevation of the top of the tree when observed from the same point is

(A) $\frac{9}{\sqrt{109}}$ (B) $\frac{1}{\sqrt{109}}$ (C) $\frac{3}{\sqrt{109}}$ (D) $\frac{30}{\sqrt{109}}$

**27.** From a point on the ground, a person observed the top of a hill at an angle of elevation of 30°. Starting from the point where he stood, the ground sloped upwards at an angle of 15° with the horizontal. He walked towards the hill for 300 m along the slope. He then found the angle of elevation of the top of the hill to be 45°. Find the height of the hill (in metres). $\left(\sin 15^\circ=\frac{\sqrt{6}-\sqrt{2}}{4}\right)$

(A) $75\left(\sqrt{3}+1\right)$ (B) $75\left(\sqrt{2}-\sqrt{2}\right)$

(C) $75\left(\sqrt{3}-1\right)$ (D) $75\left(\sqrt{6}+\sqrt{2}\right)$

**28.** A person standing at a point on the circumference of a circular park observes the top of a flag staff situated at the centre of the park at an angle of elevation of 60°. The circumference of the circle is 440m. Find the height of the flag staff in meters. (Ignore the height of the person).

(A) $\frac{70}{2}\sqrt{3}$ (B) $140\sqrt{3}$

(C) $70\sqrt{3}$ (D) $\frac{140}{\sqrt{3}}$

**29.** A pole $EP$ is erected vertically at the midpoint of the diagonal $E$ of a square $ABCD$ in such a way that the side $AB$ subtends an angle of 60° at $P$. If the side of the square is 5 m, find the height of the tower in meters._____ $\sqrt{2}$

**30.** The angles of elevation of the tip '$P$' of a pole $PQ$ from two points, $A$ and $B$, lying on either side of the pole are found to be 30° and 60°, respectively.$A$ point '$C$' lying on the line segment joining $AB$ is such that $2\angle APC = \angle APB$. Find the length of $AC$ if $BC = 30$ m.

(A) $30\sqrt{3}$m (B) 60 m (C) $10\sqrt{3}$ m (D) 45 m

## EXERCISE-5 Based on Data Sufficiency

***Directions for questions 1–10*:** Each question is followed by two statements, I and II. Answer each question based on the following directions:

Choose (A) if the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.

Choose (B) if the question can be answered by using either statement alone.

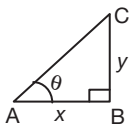
Choose (C) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (D) if the question cannot be answered even by using both the statements together.

**1.** Find the value of $y$ in the following figure.

I. $\theta = 45°$

II. $x = 100$ m




**2.** Find the value of $\cos\theta$.

I. $\tan\theta = \frac{3}{4}$ II. $\sec\theta = \frac{5}{4}$

**3.** Is $\text{cosec}\,\theta < \sec\theta$?

I. $\theta$ is in the first quadrant.

II. $\theta$ is in the second quadrant.

**4.** Find the value of $\tan\theta$.

I. $\cos^2\theta = 1$

II. $0° \le \theta < 90°$

**5.** From the top of a building of height '$h$' m, two points $A$ and $B$ are observed. What is the distance between $A$ and $B$?

I. The angles of depression of $A$ and $B$ are 30° and 45°, respectively.

II. $A$ is at a distance of 60 m from the building.

**6.** Does $\theta$ lie in the third quadrant?

I. $\cos\theta > 0$.

II. $\sin\theta < 0$.

**7.** Find the value of $(\cos A + \sin A)^2 - (\cos A - \sin A)^2$.

I. $A = 0°$.

II. $\cos^2 A + \sin^2 A = 1$.

**8.** Find the value of $\tan A + \tan B + \tan C$.

I. $A, B, C$ are angles of a triangle.

II. $\Delta ABC$ is equilateral.

**9.** Which is taller, The Refill Tower or The Statue of Freedom?

I. The angle of elevation of the head of The Statue of Freedom, from a distance of 15 m is 60°.

II. The angle of elevation of the middle of The Refill Tower from a distance of 15 m is 45°.

**10.** What is the height of the tower $PQ$?

I. A man standing on the ground at a point $A$, 30 m from the foot of the tower observes the top of the tower at an angle of elevation of 60°.

II. A man standing at a point $B$, 20 m from the foott of the tower observes the midpoint of the tower at an angle of elevation of 30°.

## ANSWER KEYS

### Exercise-1

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (D) | 7. (C) | 13. (C) | 19. (1) | 25. (1) | 31. (C) | 37. (5) |
| 2. (B) | 8. (D) | 14. (B) | 20. (A) | 26. (D) | 32. (B) | 38. (1) |
| 3. (A) | 9. (B) | 15. (C) | 21. (1) | 27. (C) | 33. (A) | 39. (A) |
| 4. (C) | 10. (C) | 16. (A) | 22. (1) | 28. (2.5) | 34. (7) | 40. (C) |
| 5. (C) | 11. (1) | 17. (B) | 23. (A) | 29. (D) | 35. (A) | |
| 6. (0) | 12. (D) | 18. (C) | 24. (5) | 30. (1) | 36. (A) | |

### Exercise-2

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (A) | 7. (C) | 13. (2) | 19. (C) | 25. (2) | 31. (C) | 37. (C) |
| 2. (B) | 8. (0.5) | 14. (1) | 20. (A) | 26. (19) | 32. (C) | 38. (219.6) |
| 3. (1) | 9. (D) | 15. (D) | 21. (D) | 27. (B) | 33. (B) | 39. (A) |
| 4. (2) | 10. (26) | 16. (D) | 22. (3) | 28. (A) | 34. (D) | 40. (8).04 |
| 5. (C) | 11. (0) | 17. (B) | 23. (D) | 29. (B) | 35. (C) | |
| 6. (D) | 12. (B) | 18. (D) | 24. (150) | 30. (C) | 36. (A) | |

### Exercise-3

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (D) | 7. (1) | 13. (B) | 19. (B) | 25. (0) | 31. (D) | 37. (240) |
| 2. (1) | 8. (A) | 14. (B) | 20. (C) | 26. (18) | 32. (C) | 38. (72) |
| 3. (C) | 9. (A) | 15. (D) | 21. (B) | 27. (B) | 33. (C) | 39. (A) |
| 4. (A) | 10. (–17) | 16. (2) | 22. (D) | 28. (B) | 34. (D) | 40. (30) |
| 5. (C) | 11. (D) | 17. (B) | 23. (D) | 29. (B) | 35. (B) | |
| 6. (1) | 12. (C) | 18. (B) | 24. (A) | 30. (D) | 36. (D) | |

### Exercise-4

| | | | | | |
|---|---|---|---|---|---|
| 1. (D) | 6. (A) | 11. (B) | 16. (90) | 21. (C) | 26. (C) |
| 2. (C) | 7. (A) | 12. (4) | 17. (A) | 22. (C) | 27. (D) |
| 3. (D) | 8. (B) | 13. (D) | 18. (A) | 23. (B) | 28. (C) |
| 4. (D) | 9. (D) | 14. (A) | 19. (26) | 24. (D) | 29. (2.5) |
| 5. (D) | 10. (C) | 15. (0.125) | 20. (C) | 25. (A) | 30. (A) |

### Exercise-5

| | | | | |
|---|---|---|---|---|
| 1. (C) | 3. (A) | 5. (D) | 7. (A) | 9. (C) |
| 2. (A) | 4. (A) | 6. (A) | 8. (A) | 10. (B) |

## SOLUTIONS

## EXERCISE-1

***Solutions for questions 1–40:***

**1.** $1420° = (4 \times 360 - 20°)$ lies in the fourth quadrant.

**2.** $\frac{6\pi^c}{5} = \frac{6}{5} \times 180 = 216°$

**3.** $72° = 72 \times \frac{\pi}{180} = \frac{2\pi^c}{5}$.

**4.** $\sin(270° - A) = -\cos A$

**5.** $\sin(750°) = \sin[2(360°) + 30°] = \sin 30° = \frac{1}{2}$

**6.** $\sin 25° + \cos 115° = \sin 25° + \cos(90° + 25°)$

$= \sin 25° - \sin 25° = 0$

**7.** Each revolution makes an angle of 360°. In three revolutions, the angle made by $\overrightarrow{OA}$ is $3 \times 360° = 1080°$.

**8.** Arc of the sector $l = r\theta$ when $\theta$ is in radians,

$$\theta = \frac{l}{r} = \frac{110}{14} = \frac{55}{7}, \frac{5}{2} \times \frac{22}{7} = \frac{5\pi^c}{2}$$

**9.** $\sin\theta$ is positive $\Rightarrow$ $\theta$ belongs to I quadrant or II quadrant
$\cos\theta$ is negative $\Rightarrow$ $\theta$ belongs to II quadrant or III quadrant
Hence, $\theta$ belongs to II quadrant

**10.** 8, 15 and 17 are pythagorean numbers. So, $\text{cosec}\theta = \frac{17}{8}$;

$\sin\theta = \frac{8}{17}$; $\cos\theta = \frac{15}{17}$

$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{15}{8}$.

**11.** $\sin\theta \,\text{cosec}\theta = \sin\theta \,\frac{1}{\sin\theta} = 1$

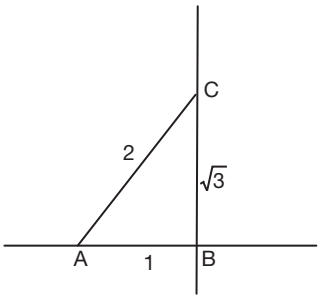
**12.** We know that $\tan\theta$ is positive in the first and third quadrants.

$\therefore\ \theta$ lies in $Q_1$ or $Q_3$.

**13.** Given $\tan\theta = \frac{1}{2}$ and $\cos\theta = \frac{-2}{\sqrt{5}}$

$\Rightarrow \theta \in Q_3$ as $\tan\theta$ positive and $\cos\theta$ is negative in third quadrant $(Q_3)$.

**14.**




Let $\Delta ABC$ be the given triangle and $AB = 1$ unit,

$BC = \sqrt{3}$ units and $AC = 2$ units.

Since, $(1)^2 + \left(\sqrt{3}\right)^2 = 4 = (2)^2$ i.e., $AB^2 + BC^2 = AC^2$,

$\Delta ABC$ is a right-angled triangle.

$\therefore \sin A = \frac{BC}{AC} = \frac{\sqrt{3}}{2}$

$\Rightarrow A = 60°$

$\therefore\ C = 90° - 60° = 30°$

Hence, the angles of the triangle are 30°, 60° and 90°.

**15.** $\sin\theta = \cos\theta \Rightarrow \theta = 45°$ ($\because \theta$ is acute)

Hence, the curves $y = \sin\theta$ and $y = \cos\theta$ meet at $\theta = \frac{\pi}{4}$

**16.** In the interval $0 < \theta < \frac{\pi}{4}$,

$\tan\theta < \cot\theta. \Rightarrow \frac{\tan\theta}{\cot\theta} < 1.$

**17.** Given $\sin\theta = \frac{5}{13} \Rightarrow \text{cosec}\ \theta = \frac{13}{5}$

$\cot^2\theta = \text{cosec}^2\theta - 1 = \left(\frac{13}{5}\right)^2 - 1 = \cot\theta = \frac{12}{5}$

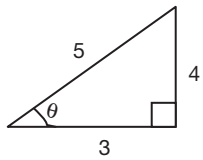
$\therefore\ \theta \in Q_2 \Rightarrow \cot\theta = \frac{-12}{5}$

**18.** $3\tan\theta - 4 = 0$

$\Rightarrow \tan\theta = \frac{4}{3}$

$\therefore\ \text{cosec}\theta = \frac{-5}{4}$

$(\therefore 180° < \theta < 270°)$.




**19.** $\sec^4\theta + \tan^4\theta - 2\sec^2\theta\tan^2\theta$

$= (\sec^2\theta)^2 + (\tan^2\theta)^2 - 2\sec^2\theta\tan^2\theta$

$= (\sec^2\theta - \tan^2\theta)^2 = (1)^2 = 1$

**20.** Given, $\tan\alpha = \cot\beta = 1$

$\Rightarrow \tan\alpha = 1; \cot\beta = 1$

$\alpha = \frac{\pi}{4}$ and $\beta = \frac{\pi}{4}$

$2\alpha + \beta = 2\left(\frac{\pi}{4}\right) + \frac{\pi}{4}$

$= \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

**21.** We know that, $\sin^2\theta + \cos^2\theta = 1$

$\therefore \sin^2 45° + \cos^2 45° = 1$

**22.** For every value of $\theta$, we know that $\sin^2\theta + \cos^2\theta = 1$

$\therefore \sin^2 225° + \cos^2 225° = 1.$

**23.** Given $p = \text{cosec}\ \theta + \cot\theta$

$\Rightarrow \frac{1}{P} = \text{cosec}\theta - \cot\theta$

$p + \frac{1}{P} = \text{cosec}\theta + \cot\theta + \text{cosec}\theta - \cot\theta = 2\text{cosec}\theta$

$= \frac{2}{\sin\theta}$.

**24.** $\sec^2\theta - \tan^2\theta = 1$

$(\sec\theta - \tan\theta)\ (\sec\theta + \tan\theta) = 1$

$\sec\theta + \tan\theta = \frac{1}{\sec\theta - an\theta} = \frac{1}{1/5} = 5.$

**25.** $(1+\sin\theta)(1-\sin\theta)\sec^2\theta = (1-\sin^2\theta)\sec^2\theta$

$= \cos^2\theta \cdot \sec^2\theta = 1.$

**26.** When $\theta = 90°$, $\sec\theta$ is not defined.

$\therefore$ The given statement is not true for $\theta = 90°$.

**27.** Given $\cot\theta = \dfrac{1+\cos 2\theta}{\sin 2\theta}$

Put $\theta = 22^1/_2° \Rightarrow 2\theta = 45°$

$$\cot 22^1/_2° = \frac{1+\cos 45°}{\sin 45°} = \frac{1+\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \sqrt{2}+1.$$

**28.** $\sin 30° + \sqrt{3}\ \tan 60° - \sec 0°$

$$= \frac{1}{2} + \sqrt{3}\,\sqrt{3} - 1 = \frac{1}{2} + 2 = \frac{5}{2}$$

**29.** Options A, B and C can't be true ($\because$ $\sin\theta$ and $\cos\theta$ lie in the range $[-1, 1]$). follows.

**30.** $\sec^2\theta - \tan^2\theta = 1$ for all $\theta$.

**31.** For $0 < \theta < \dfrac{\pi}{4}$, $\sin\theta < \cos\theta$

**32.** Given, $\operatorname{cosec}\theta = -\sqrt{2}$ and $\tan\theta = -1$

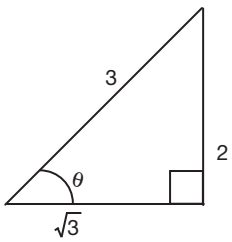
$\Rightarrow \theta$ is in IV quadrant and $\theta = 360° - 45° = 315°$

$\therefore \cos\theta = \cos 315° = \cos(360° - 45°) = \cos 45° = \dfrac{1}{\sqrt{2}}$

**33.** Given, $3\sin\theta = 2 \Rightarrow \sin\theta = \dfrac{2}{3}$

Also $\theta$ is in II quadrant.

Hence, $\tan\theta = \dfrac{-2}{\sqrt{5}}$




**34.** The given expression E

$= \sec^4\theta - \tan^4\theta = (\sec^2\theta)^2 - (\tan^2\theta)^2$

$= (\sec^2\theta - \tan^2\theta)(\sec^2\theta + \tan^2\theta)$

$= 1\left(\sec^2\dfrac{\pi}{3} + \tan^2\dfrac{\pi}{3}\right)$ when $\theta = \dfrac{\pi}{3}$, $\sec\theta = 2$, $\tan\theta = \sqrt{3}$

$E = (2)^2 + \left(\sqrt{3}\right)^2 = 7.$

**35.** The range of $\cos\theta$ is $[-1, 1]$

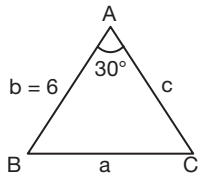
$\therefore$ The range of $2\cos\theta$ is $[-2, 2]$.

**36.** By sine rule

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{4}{\sin 30°} = \frac{6}{\sin C}$$

$$\sin C = \frac{6\sin 30°}{4} = \frac{6}{4}\frac{1}{2} = \frac{3}{4}.$$




**37.** The maximum value of $a\cos x + b\sin x + c$ is $c + \sqrt{a^2+b^2}$

$\therefore$ The maximum value of $3\cos x + 4\sin x$ is $0 + \sqrt{3^2+4^2}$

$= 5.$

**38.** When $0 \le x \le \pi/2$, the graphs of $\sin x$ and $\cos x$ intersect only at $x = \pi/4$, lie at only one point.

**39.** $\sec\theta = \dfrac{13}{5} \therefore \cos\theta = \dfrac{5}{13}$

$\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta = 1 - \left(\frac{5}{13}\right)^2 = \frac{144}{169}$$

As $\theta$ is acute, $\sin\theta$, $\cos\theta$, $\tan\theta$ are all positive.

$$\sin\theta = +\sqrt{\frac{144}{169}} = \frac{12}{13}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5}.$$

Alternately, $\sec^2\theta - \tan^2\theta = 1$ for all $\theta$

$$\tan^2\theta = \left(\frac{13}{5}\right)^2 - 1 = \frac{144}{25} = \left(\frac{12}{5}\right)^2.$$

As $\theta$ is acute, $\tan\theta$ is positive

$$\tan\theta = +\sqrt{\left(\frac{12}{5}\right)^2} = \frac{12}{5}.$$

**40.** $\cot\theta + \tan\theta = 2$

$\dfrac{1}{\tan\theta}$ $\tan\theta$ 2

$\tan^2\theta - 2\tan\theta + 1 = 0$

$(\tan\theta - 1)^2 = 0$, i.e., $\tan\theta = 1$

$\sec^2\theta = 1 + \tan^2\theta = 1 + 1^2 = 2$

$\sec\theta = \pm\sqrt{2}$

$$\cos\theta = \frac{1}{\sec\theta} = \pm\frac{1}{\sqrt{2}}$$

## EXERCISE-2

***Solutions for questions 1–40***:

**1.** Let the angles of the triangle (in degrees) be $a-d$, $a$, $a+d$

$(a-d)+a+(a+d)=180 \Rightarrow a=60$

Also, $a-d=\frac{1}{3}(a+d)$

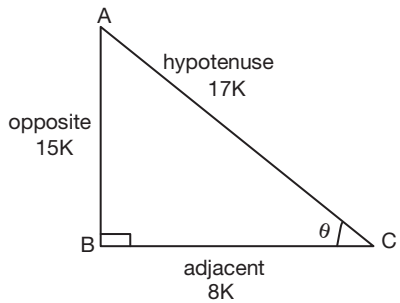
$\Rightarrow a=2d$

$\Rightarrow d=30$

$\therefore$ The angles of the triangle are 30°, 60° and 90°

i.e., $30(\frac{\pi}{180}), 60(\frac{\pi}{180}), 90(\frac{\pi}{180})$, i.e., $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$.

**2.** Given $\tan\theta=\frac{15}{8}=\frac{opp}{adj}$




$\therefore \sin\theta=\frac{opp}{hypo}=\frac{15}{17}$

$\cos\theta=\frac{adj}{hypo}=\frac{8}{17}$

$\text{cosec}\,\theta=\frac{1}{\sin\theta}=\frac{17}{15}$, $\sec\theta=\frac{1}{\cos\theta}=\frac{17}{8}$

$\cot\theta=\frac{1}{\tan\theta}=\frac{8}{15}$

**3.** Given $(1+\tan^2\theta)(1+\cot^2\theta)(\sin^2\theta-1)(\cos^2-1)\left(\frac{1}{\cot^2\theta}-\frac{1}{\cos^2\theta}\right)\left(\frac{1}{\tan^2\theta}-\frac{1}{\sin^2\theta}\right)$

We know $1+\tan^2\theta=\sec^2\theta$, $1+\cot^2\theta=\text{cosec}^2\theta$, $\sin^2\theta+\cos^2\theta=1$

Using the above results, we have

$\sec^2\theta\,\text{cosec}^2\theta\,(-\cos^2\theta)\,(-\sin^2\theta)\,(\tan^2\theta-\sec^2\theta)$

$(\cot^2\theta-\text{cosec}^2\theta)$

$=\sec^2\theta\,\text{cosec}^2\theta\,(\cos^2\theta)\,(\sin^2\theta)\,(-1)\,(-1)=1.$

**4.** $\cot A\left(\frac{1}{\text{cosec}\,A+1}+\frac{1}{\text{cosec}\,A-1}\right)\cos A=$

$\cot A\cos A\left(\frac{2\,\text{cosec}\,A}{\text{cosec}^2A-1}\right)=\cot A\cos A\left(\frac{2\,\text{cosec}\,A}{\text{cosec}^2A}\right)=2$

**5.** $\text{cosec}(-1560°)=-\text{cosec}\,1560°$

Here, $\theta>360°$. We express the given angle as $(360n\pm\theta)°$

$1560°=[4(360)+120]$

$\therefore \text{cosec}\,1560°=\text{cosec}\,[4(360)+120]=\text{cosec}120°$

$=\text{cosec}\,(180-60)°=\text{cosec}\,60°$ ($\because 120°\in Q_2$ in which $\text{cosec}\theta$ is positive) $=\frac{2}{\sqrt{3}}$

$\therefore \text{cosec}\,1560°=\frac{2}{\sqrt{3}}$

$\therefore \text{cosec}\,(-1560)=-\text{cosec}\,1560°=-2/\sqrt{3}$

**6.** Given $\cos\theta=\frac{-15}{17}$ and $\text{cosec}\theta=\frac{17}{8}$. $\cos\theta$ is negative and $\text{cosec}\theta$ is positive.

$\therefore \theta\in Q_2$. $\therefore \sin\theta$ is positive and $\cot\theta$ is negative.

$\Rightarrow \sin\theta=\frac{8}{17}$ and $\cot\theta=-\sqrt{\text{cosec}^2\theta-1}$

$=-\sqrt{\left(\frac{17}{8}\right)^2-1}=\frac{-15}{8}$

$\therefore \frac{\sin\theta+\cot\theta}{\sin\theta-\cot\theta}=\frac{\frac{8}{17}+\left(\frac{-15}{8}\right)}{\frac{8}{17}-\left(\frac{-15}{8}\right)}=\frac{64-255}{64+255}=\frac{-191}{319}$

**7.** We arrange the factors so that the angles are in increasing order.

$\frac{\cos A}{\sin A}\frac{1-\cos 2A}{1+\cos 2}\frac{2\sin 2A\cos 2A}{\cos 2A}$

$=\frac{\cos A}{\sin^2 A}\frac{2\sin^2 A}{2\cos^2 A}4\sin A\cos A=4\sin A$

**8.** Let $\frac{\pi}{12}=\alpha$, then $\pi=12\alpha$. Using this, the given expression can be written as

$\cos\alpha+\cos 2\alpha+\cos 3\alpha+\cos 4\alpha+\cos 5\alpha+\cos 7\alpha+\cos 9\alpha+\cos 10\alpha+\cos 11\alpha$

There are 9 cos ratios. Eight of them get cancelled and we are left with $\cos 4\alpha = \cos 60^\circ = \frac{1}{2} = 0.5$

**9.** Given $f(\theta) = \sin^4\theta + \cos^2\theta = \left(\frac{1-\cos 2\theta}{2}\right)^2 + \frac{1+\cos 2\theta}{2}$

$(\because 2\cos^2\theta = 1 + \cos 2\theta$ and $2\sin^2\theta = 1 - \cos 2\theta)$

$\Rightarrow \frac{1}{4} - \frac{2\cos 2\theta}{4} + \frac{\cos^2 2\theta}{4} + \frac{1}{2} + \frac{\cos 2\theta}{2} = \frac{3}{4} + \frac{1}{4}\left(\frac{1+\cos 4\theta}{2}\right)$

$f(\theta) = \frac{3}{4} + \frac{1}{8} + \frac{\cos 4\theta}{8} = \frac{7}{8} + \frac{\cos 4\theta}{8}$

Maximum value of $f(\theta) = \frac{7}{8} + \frac{1}{8} = 1$

Minimum value of $f(\theta) = \frac{7}{8} - \frac{1}{8} = \frac{3}{4}$

$\therefore$ Range of $f(x) = \left[\frac{3}{4}, 1\right]$

**10.** The range of $f(x) = a\cos x + b\sin x + c$ is

$(c - \sqrt{a^2+b^2}\,,\ c + \sqrt{a^2+b^2}\,)$

Here, $c = 11$, $a = 12$, $b = 5$

$\therefore c \pm \sqrt{a^2+b^2} = 11 \pm \sqrt{12^2+5^2} = 24$

and $C - \sqrt{a^2+b^2} = 11 - 13 = -2$

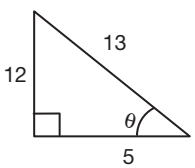
$\therefore$ Range of the given function is $(-2, 24)$

The required difference is $24 + 2 = 26$

**11.** $\cos 28^\circ + \cos 65^\circ + \cos 115^\circ + \cos 240^\circ + \cos 208^\circ + \cos 300^\circ$

$\Rightarrow \cos 28^\circ + \cos 65^\circ - \cos 65^\circ - \cos 60^\circ - \cos 28^\circ + \cos 60^\circ = 0.$

**12.**




$13\sin\theta = 12$

$\Rightarrow \sin\theta = \frac{12}{13}$; $\cos\theta = \frac{5}{13}$; $\sec\theta = \frac{13}{5}$;

$\operatorname{cosec}\theta = \frac{13}{12}$; $\cot\theta = \frac{5}{12}$ and $\tan\theta = \frac{12}{5}$

$\therefore \frac{\operatorname{cosec}\theta + \cot\theta}{\tan\theta + \sec\theta} = \frac{\frac{13}{12} + \frac{5}{12}}{\frac{12}{5} + \frac{13}{5}} = \frac{18}{25} \times \frac{5}{12} = \frac{3}{10}$

**13.** $(1 + \tan\theta + \sec\theta)\ (1 + \sec\theta - \tan\theta) - 2\sec\theta$

$= [(1 + \sec\theta) + \tan\theta]\ [(1 + \sec\theta) - \tan\theta] - 2\sec\theta$

$= (1 + \sec\theta)^2 - \tan^2\theta - 2\sec\theta$

$= 1 + \sec^2\theta + 2\sec\theta - \tan^2\theta - 2\sec\theta$

$= 1 + (\sec^2\theta - \tan^2\theta)$

$= 1 + 1 = 2$

**14.** $\cos\theta + \cos^2\theta = 1 \Rightarrow \cos\theta = 1 - \cos^2\theta = \sin^2\theta$ and

$\cos^2\theta = (\sin^2\theta)^2 = \sin^4\theta$

$\therefore \sin^2\theta + \sin^4\theta = \cos\theta + \cos^2\theta = 1.$

**15.** Given, $\sec\theta + \tan\theta = m$

$\Rightarrow \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = m$

Squaring on both the sides, we get

$\Rightarrow \frac{(1+\sin\theta)^2}{\cos^2\theta} = m^2$

$\Rightarrow \frac{(1+\sin\theta)^2}{(1-\sin^2\theta)} = m^2$

$\Rightarrow \frac{1+\sin\theta}{1-\sin\theta} = m^2 \Rightarrow \sin\theta = \frac{m^2-1}{m^2+1}$

$\therefore \cos\theta = \sqrt{1 - \left(\frac{m^2-1}{m^2+1}\right)^2} = \frac{2m}{m^2+1}$

**Alternate Solution:**

Given, $\sec\theta + \tan\theta = m$ (1)

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{m}$ (2)

Adding (1) and (2), we get

$2\sec\theta = m + \frac{1}{m} \Rightarrow \sec\theta = \frac{m^2+1}{2m}$

$\therefore \cos\theta = \frac{2m}{m^2+1}$

**16.** Given, $\operatorname{cosec}\theta - \cot\theta = p$

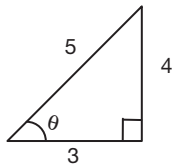
$\therefore \operatorname{cosec}\theta + \cot\theta = \frac{1}{\operatorname{cosec}\theta - \cot\theta} = \frac{1}{p}$

**17.** $\cos\theta = 3/5$

$\tan\theta = 4/3$, $\sec\theta = 5/3$

$\cot\theta = 3/4$, $\operatorname{cosec}\theta = 5/4$

$$\frac{\tan\theta + \cot\theta}{\sec\theta + \operatorname{cosec}\theta} = \frac{\frac{4}{3} + \frac{3}{4}}{\frac{5}{3} + \frac{5}{4}}$$




$$= \frac{\frac{16+9}{12}}{\frac{20+15}{12}} = \frac{25}{35} = \frac{5}{7}.$$

**18.** $a = b\sin\theta + c \Rightarrow \dfrac{a-c}{b} = \sin\theta$ (1)

$b = a\cos\theta + c \Rightarrow \dfrac{b-c}{a} = \cos\theta$ (2)

Squaring and adding (1) and (2)

$$\sin^2\theta + \cos^2\theta = \frac{(a-c)^2}{b^2} + \frac{(b-c)^2}{a^2} = 1.$$

**19.** $\sin^6 x + \cos^6 x = (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x \cos^2 x (\sin^2 x + \cos^2 x)$

$= 1 - \dfrac{3}{4}(2\sin x \cos x)^2$, $(\sin^2 x + \cos^2 x = 1)$

$= 1 - \dfrac{3}{4}(\sin 2x)^2$

Minimum value $= 1 - \dfrac{3}{4}(1) = \dfrac{1}{4}$ ($\because$ Minimum value is obtained when $\sin 2x$ is maximum i.e., $\sin 2x = 1$)

**20.** We note that this graph is above the $x$-axis. So, it takes the form $y = |f(x)|$. It resembles $\sin x$ as it passes through the origin (from the choices). We see that $y = \left|\sin\dfrac{x}{2}\right|$ can be ruled out as $(0, \pi)$ is not satisfied.

So, $y = |\sin 2x|$ is appropriate.

**21.** $\sin^8 x + \cos^8 x = 1$

The above equation is true only when $\sin x = 1$ or $-1$ ($\therefore \cos x = 0$) or $\cos x = 1$ or $-1$ ($\therefore \sin x = 0$)

The number of solutions for $\sin x = 1$ or $-1$ in $[-2\pi, 2\pi]$ is 4 i.e., $\left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{-\pi}{2}, \dfrac{-3\pi}{2},\right)$ Similarly, the number of solutions for $\cos x = 1$ or $-1$ is 5 $(0, \pi, 2\pi, -\pi, -2\pi)$

Hence, 9 solutions exist for $\sin^8 x + \cos^8 x = 1$.

**22.** $3\cos^2 A = \dfrac{1}{2} + \left(\dfrac{1}{\sqrt{2}}\right)^2$

$3\cos^2 A = \dfrac{1}{2} + \dfrac{1}{2}$

$\cos^2 A = \dfrac{1}{3}$

$\sec^2 A = 3$.

**23.** In the figure given, $ABCD$ is a cyclic quadrilateral

$\Rightarrow A + C = 180° = B + D$

$\Rightarrow \tan A = \tan(180° - C) = -\tan C$

$\therefore \tan B = \tan(180° - D) = -\tan D$

$\Rightarrow \tan A + \tan B = -(\tan C + \tan D)$

$\Rightarrow \tan A + \tan B + \tan C = -\tan D$

Given, $\tan A + \tan B + \tan C = 5$

$\Rightarrow -\tan D = 5$

$\therefore \tan D = -5$

**24.** Given, $a = 2$, $b = 3\sqrt{3}$ and $c = 7$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{4 + 27 - 49}{2 \times 2 \times 3\sqrt{3}}$$

$$= \frac{-18}{12\sqrt{3}} = \frac{-\sqrt{3}}{2}.$$

$\therefore \angle C = 150°$.

**25.** $\sin\theta + \cos\theta = \sqrt{2}$

It is possible only when $\theta = 45°$

$\tan^n\theta + \cot^n\theta = \tan^n 45 + \cot^n 45 = 1 + 1 = 2$

**26.** If $A + B = 45°$ then $\cot(A + B) = \cot 45$

$$\frac{\cot A \cot B - 1}{\cot B + \cot A} = 1$$

$\Rightarrow \cot A \cot B - 1 = \cot B + \cot A$

$\Rightarrow -1 = \cot A + \cot B - \cot A \cot B$

$-2 = \cot A(1 - \cot B) - 1(1 - \cot B)$

$-2 = (\cot A - 1)(1 - \cot B)$

Or $(1 - \cot A)(1 - \cot B) = 2$

$\therefore (1 - \cot 4°)(1 - \cot 41°)(1 - \cot 5°)(1 - \cot 40°)$ - - - - - - - - $(1 - \cot 22)(1 - \cot 23) = 2^P$

i.e., $2^{19} = 2^P$

$\Rightarrow P = 19$

**27.** $\sin^2 x - \cos 2x = \dfrac{1-\cos 2x}{2} - \cos 2x$

$= \dfrac{1-3\cos 2x}{2}$

The above expression will take the maximum value if $\cos 2x$ is $-1$ $\dfrac{1-3\cos 2x}{2} = \dfrac{1+3}{2} = 2$

**28.** Min $= -\sqrt{(3)^2 + (-4)^2} = -5$

**29.** Given: $\sin 12^\circ \sin 48^\circ \sin 54^\circ$

$= \dfrac{\sin 12^\circ \sin(60^\circ - 12^\circ)\sin(60^\circ + 12^\circ)\sin 54^\circ}{\sin 72^\circ}$

(We know that $\sin A \cdot \sin(60^\circ + A)\sin(60^\circ - A)$

$= \dfrac{1}{4}\sin 3A$)

$= \left(\dfrac{1}{4}\right)\dfrac{\sin 3(12^\circ)\sin 54^\circ}{\sin 72^\circ}$

$= \left(\dfrac{1}{8}\right)\dfrac{2\sin 36^\circ \cos 36^\circ}{\sin 72^\circ} = \dfrac{\sin 72^\circ}{8.\sin 72} = \dfrac{1}{8}$

**30.** $3\left(\dfrac{1}{2}+\dfrac{1}{2}\right) - 3\left(\dfrac{1}{2}+\dfrac{1}{4}\right) = 3 - 3.\dfrac{3}{4}$

$= 3\left[1-\dfrac{3}{4}\right] = 3.\dfrac{1}{4} = \dfrac{3}{4}$.

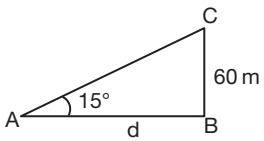
**31.** $\sqrt{2-\sqrt{2-\sqrt{2-2\cos 2\theta}}} = \sqrt{2-\sqrt{2-\sqrt{2(2\sin^2\theta)}}}$

$= \sqrt{2-\sqrt{2-2\sin\theta}}$

**32.** $\tan 15^\circ = \dfrac{60}{d}$

$2-\sqrt{3} = \dfrac{60}{d}$

$\therefore d = \dfrac{60}{2-\sqrt{3}}$ or $d = 60\left(2+\sqrt{3}\right)$ m.




**33.** Distance travelled in 3 seconds with a speed of 36 kmph is $36\left(\dfrac{5}{18}\right)(3)$ or 30m

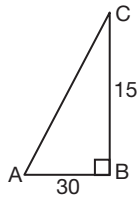
From the figure $\tan\theta = \dfrac{BC}{AB} = \dfrac{75}{30} = \dfrac{5}{2}$

The height of the person is 1.75 m

Let the length of the shadow $x$ m

$\text{Tan}\,\theta = \dfrac{1.75}{x} \Rightarrow \dfrac{5}{2} = \dfrac{1.75}{x}$

$\Rightarrow x = \dfrac{1.75}{5}(2) = 0.7$




**34.** Let $BC$ be the height of the hill.

We are given $AH$. We can find $AC$ and then $BC$.

Consider $\Delta AHC$, $\angle AHC = 360^o - (45 + 90 + 75)^\circ = 150^o$

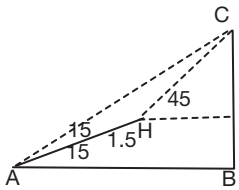
$\dfrac{AC}{\text{Sin}\angle AHC} = \dfrac{AH}{\text{Sin}\angle ACH}$

$\Rightarrow AC = \dfrac{\sin 150^\circ}{\sin 15^\circ}(1.5)\text{ km} = \dfrac{\sqrt{2}}{\sqrt{3}-1}(1.5)\text{ km}$

$= \dfrac{\sqrt{2}\left(\sqrt{3}+1\right)(1.5)}{2}$

In $\Delta ABC$, $\dfrac{BC}{AC} = \sin 30^\circ = 1/2$

$BC = \dfrac{AC}{2} = \dfrac{\sqrt{2}\left(\sqrt{3}+1\right)(1.5)}{4}$ km




**35.** Let $\angle RAP = \beta$

Given $AP = 3\,PQ$

$\dfrac{PQ}{AP} = \dfrac{1}{3}$ and $PR = RQ = \dfrac{PQ}{2}$

In $\Delta APR$, $\tan\beta = \dfrac{PR}{AP}$

$= \dfrac{PQ}{2(3PQ)} = \dfrac{1}{6}$

In $\Delta APQ$, $\tan(\alpha+\beta) = \dfrac{PQ}{AP} = \dfrac{1}{3}$

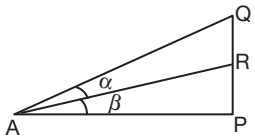
$\dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \dfrac{1}{3}$

$\Rightarrow 3\left(\dfrac{1}{6}+\tan\alpha\right) = \left(1-\dfrac{1}{6}\tan\alpha\right)$

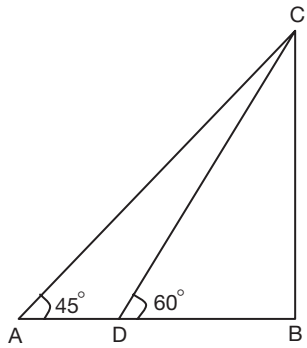
$\Rightarrow \dfrac{1}{2} + 3\tan\alpha = 1 - \dfrac{1}{6}\tan\alpha \Rightarrow 3\tan\alpha + \dfrac{1}{6}\tan\alpha = 1 - \dfrac{1}{2}$

$\Rightarrow \dfrac{19}{6}\tan\alpha = \dfrac{1}{2}$

$\Rightarrow \tan\alpha = \dfrac{3}{19} \Rightarrow \cot\alpha = \dfrac{19}{3}$



**36.**




Let the height of the lighthouse be '$h$'.

Distance covered by the man in 5 minutes

= 25 m.

$AD = 25$ m

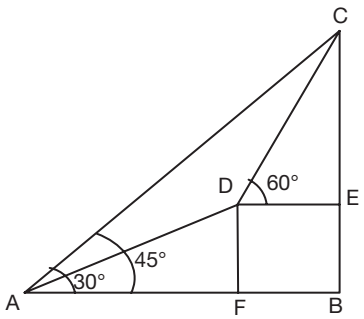
$AB - DB = 25$ m

$$\frac{BC}{\tan 45^\circ} - \frac{BC}{\tan 60^\circ} = 25 \text{ m i.e., } \frac{h}{1} - \frac{h}{\sqrt{3}} = 25$$

$$\Rightarrow h\left(\sqrt{3}-1\right) = 25\sqrt{3}$$

$$\Rightarrow \quad h = \frac{25\sqrt{3}}{\sqrt{3}-1} = \frac{25\sqrt{3}\left(\sqrt{3}+1\right)}{2} = \frac{25\left(3+\sqrt{3}\right)}{2}.$$

**37.**




Let $D$ be the point up to which the man goes.

Let $F$ be the point vertically below $D$ which is in line with $A$, the initial position of the man.

The man covers $AD$.

Let $AD = x$.

$AB = AF + FB = x \cos 30^\circ + DE = x \cos 30^\circ + CE \cot 60^\circ$

$$= \frac{x\sqrt{3}}{2} + \frac{CE}{\sqrt{3}} =$$

$$\frac{x\sqrt{3}}{2} + \frac{(CB - EB)}{\sqrt{3}} = \frac{x\sqrt{3}}{2} + \frac{(40 - DF)}{\sqrt{3}}$$

$$= \frac{x\sqrt{3}}{2} + \frac{40 - x\sin 30^\circ}{\sqrt{3}}$$

Also, $AB = BC \tan 45^\circ$, $BC = 40$.

$$\therefore BC = \frac{x\sqrt{3}}{2} + \frac{40 - \frac{x}{2}}{\sqrt{3}} = 40$$

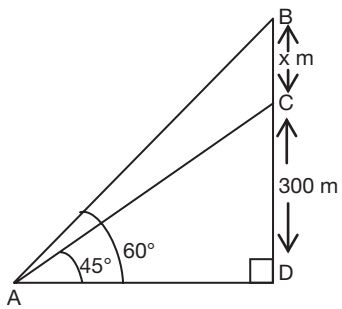
$$3x + 80 - x = 80\sqrt{3}$$

$$2x = 80\ (\sqrt{3} - 1)$$

$$x = 40\ (\sqrt{3} - 1)$$

**38.** Let the height of the flag post be $x$ metres

In $\Delta ACD$, $\tan 45^\circ = 1 = \frac{300}{AD}$



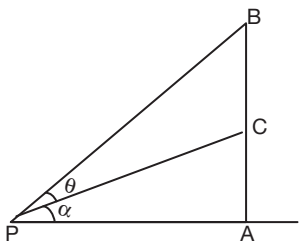

$\Rightarrow AD = 300$ m

In $\Delta DAB$, $\tan 60^\circ = \sqrt{3} = \frac{(300 + x)}{300}$

$$\Rightarrow x = 300\left(\sqrt{3} - 1\right) \text{ m} = 219.6 \text{ m}$$

$\therefore$ The height of the flag post is 219.6 m

**39.**




$AC = CB$

In $\Delta PAC$, $\tan\alpha = \frac{AC}{AP} = \frac{AB}{2AP} = \frac{1}{2n}$

In $\Delta PAB$, $\tan(\alpha + \theta) = \frac{AB}{AP} = \frac{1}{n}$

$$\Rightarrow \frac{\tan\alpha+\tan\theta}{1-\tan\alpha\tan\theta}=\frac{1}{n} \Rightarrow \frac{\frac{1}{2n}+\tan\theta}{1-\frac{1}{2n}\tan\theta}=\frac{1}{n}$$

$$\Rightarrow \frac{1}{2n}+\tan\theta=\frac{1}{n}-\frac{1}{2n^2\tan\theta} \Rightarrow \frac{1}{2n}=\left(1+\frac{1}{2n^2}\right)\tan\theta$$

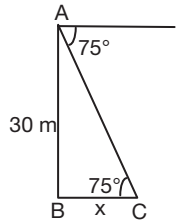
$$\therefore \cot\theta=\frac{2n^2+1}{n}$$

**40.** $\tan 75^\circ = \frac{30}{x}$

$\Rightarrow x = 30\cot 75^\circ = 30\left(2-\sqrt{3}\right)$

$= 30\ (2 - 1.732) = 30\ (0\times 268)$

$x = 8.04$ m




## EXERCISE-3

***Solutions for questions 1–40:***

**1.** $\cot 22\frac{1}{2}^\circ = \sqrt{\frac{1+\cos 45^\circ}{1-\cos 45^\circ}} = \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}} = \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}} = \sqrt{2}+1.$

**2.** Given $x = \cot 10^\circ \cot 11^\circ \cot 12^\circ \ldots\ldots \cot 79^\circ \cot 80^\circ$

$x = \cot 10^\circ \cot 11^\circ \cot 12^\circ \ldots \cot(90^\circ - 12^\circ)$

$\cot(90^\circ - 11^\circ)\ \cot(90^\circ - 10^\circ)$

$= \cot 10^\circ \cot 11^\circ \ldots \cot 45^\circ \ldots \tan 11^\circ \tan 10^\circ$

$= (\cot 10^\circ \tan 10^\circ)\ (\cot 11^\circ \tan 11^\circ)(\cot 12^\circ \tan 12^\circ)\ldots.$

$(\cot 44^\circ \tan 44^\circ)\cot 45^\circ$

$= 1(1) \ldots (1) = 1.$

**3.** The graph is the reflection of $y = \sin x$ in the $x$-axis so, its equation is $y = -\sin x$.

**4.** Given, $\operatorname{cosec}\theta$ and $\cot\theta$ are the roots of $cx^2 + bx + a = 0$.

$\therefore \operatorname{cosec}\theta + \cot\theta = \frac{-b}{c}$

$\Rightarrow \operatorname{cosec}\theta - \cot\theta = \frac{-c}{b}$

$\operatorname{cosec}\theta \,.\, \cot\theta = \frac{a}{c}$

$(a+b)^2 - (a-b)^2 = 4ab$

$(\operatorname{cosec}\theta + \cot\theta)^2 - (\operatorname{cosec}\theta - \cot\theta)^2 = 4\operatorname{cosec}\theta \,.\, \cot\theta$

$\frac{b^2}{c^2} - \frac{c^2}{b^2} = \frac{4a}{c}$

$\Rightarrow \frac{b^4 - c^4}{b^2c^2} = \frac{4a}{c}$

$\Rightarrow b^4 - c^4 = 4ab^2c$

$\Rightarrow b^4 = 4ab^2c + c^4.$

**5.** Given, $3\tan\theta - 4 = 0,\ 3\tan\theta = 4 \Rightarrow \tan\theta = \frac{4}{3}$

$\sin\theta = \frac{4}{5},\ \operatorname{cosec}\theta = \frac{5}{4},$

$\cot\theta = \frac{3}{4},\ \sec\theta = \frac{5}{3}.$

$\therefore \frac{3\sec\theta + 2\operatorname{cosec}\theta}{\cot\theta - 5\sin\theta}$

$= \frac{3\left(\frac{5}{3}\right)+2\left(\frac{5}{4}\right)}{\frac{3}{4}-5\left(\frac{4}{5}\right)}$

$= \frac{5+\frac{5}{2}}{\frac{3}{4}-4} = \frac{\frac{15}{2}}{\frac{-13}{4}} = \frac{-30}{13}.$

**6.** Given, $\frac{\sin^2 x}{1+\cot x} + \frac{\cos^2 x}{1+\tan x} = k - \sin x\cos x$

$\Rightarrow \frac{\sin^2 x}{1+\frac{\cos x}{\sin x}} + \frac{\cos^2 x}{1+\frac{\sin x}{\cos x}} = k - \sin x\cos x$

$\Rightarrow \frac{\sin^3 x}{\sin x+\cos x} + \frac{\cos^3 x}{\cos x+\sin x} = k - \sin x\cos x$

$\Rightarrow \frac{\sin^3 x + \cos^3 x}{\sin x+\cos x} = k - \sin x\cos x$

$\Rightarrow \frac{(\sin+\cos x)\left(\sin^2 x - \sin x\cos x + \cos^2 x\right)}{(\sin x+\cos x)}$

$= k - \sin x\cos x$

$[\because\ a^3 + b^3 = (a+b)\ (a^2 - ab + b^2]$

$\Rightarrow \sin^2 x + \cos^2 x - \sin x\cos x = k - \sin x\cos x$

$\Rightarrow 1 - \sin x\cos x = k - \sin x\cos x \Rightarrow k = 1$

**7.** We know that $17° + 28° = 45°$

$\tan(17° + 28°) = \tan 45°$

$\frac{\tan 17° + \tan 28°}{1 - \tan 17° \tan 28°} = 1 \Rightarrow \tan 17° + \tan 28° = 1 - \tan 17°$

$\tan 28° \tan 17° + \tan 28° + \tan 17°. \tan 28° = 1$

**8.** $\sqrt{\frac{1-\sin x}{1+\sin x}} + \sqrt{\frac{(1-\sin x)^2}{1-\sin^2 x}}$

$= \frac{1-\sin x}{\cos x}$

$= \sec x - \tan x$

**9.** $\frac{\cos x}{\frac{\cos x - \sin x}{\cos x}} + \frac{\sin x}{\frac{\sin x - \cos x}{\sin x}}$

$= \frac{\cos^2 x - \sin^2 x}{(-\sin x + \cos x)}$

$= \frac{(\cos x + )(\cos x - \sin x)}{(\cos x - \sin x)}$

$= \sin x + \cos x$

**10.** $\sin A = 1/3$

$\tan B = 3/4$

$9 \cos^2 A + 20 \sec B$

$= 9. \frac{8}{9} + 20 . (-5/4)$

$= 8 - 25 = -17$

**11.** $\log a + \log b + \log c = \log abc$

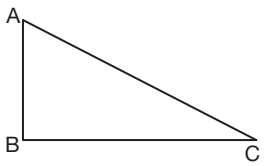
(standard result)

Now, the expression becomes

$\log (\cot 30° \times \cot 45° \times \cot 60°)$

$= \log \left(\frac{1}{\sqrt{3}}.1.\sqrt{3}\right) = \log 1 = 0.$

**12.** Let $a = 2\sqrt{3} - 2$, $b = 2\sqrt{3} + 2$ and $c = \sqrt{24}$ be the sides of the triangle $ABC$.




Since $b$ is the largest side, $(b > 5, c < 5)$ $\angle B$ is the greatest angle. We know that, $b^2 = a^2 + c^2 - 2ac \cos B$

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

We can cancel the common factor of 2 and take $a = \sqrt{3} - 1$, $b = \sqrt{3} + 1$, $c = \sqrt{6}$

$= \frac{\left(\sqrt{3}-1\right)^2 + \left(\sqrt{6}\right)^2 - \left(\sqrt{3}+1\right)^2}{2\sqrt{6}\left(\sqrt{3}-1\right)}$

$= \frac{6 - 4\sqrt{3}}{2\left(\sqrt{18} - \sqrt{6}\right)} = \frac{3 - 2\sqrt{3}}{\sqrt{18} - \sqrt{6}} = \frac{\left(3 - 2\sqrt{3}\right)\left(\sqrt{18} + \sqrt{6}\right)}{12}$

$= \frac{\sqrt{2}\left(3 - 2\sqrt{3}\right)\left(3 + \sqrt{3}\right)}{12} = \left(3 - 3\sqrt{3}\right) = \frac{\sqrt{2} - \sqrt{6}}{4}$

$= \sin(-15°) = \cos(90° + 15°) = \cos(105°) = \cos\frac{7\pi}{12}$

$\therefore$ The greatest angle in $B = \frac{7\pi}{12}$

**13.** Given $a = b \sec\theta + c$; $b = a \tan\theta + c$

$\frac{a-c}{b} = \sec\theta$ ; $\frac{b-c}{a} = \tan\theta$

We know that $\sec^2\theta - \tan^2\theta = 1$

$\left(\frac{a-c}{b}\right)^2 - \left(\frac{b-c}{a}\right)^2 = 1$

**14.** When $x = \frac{3\pi}{4}$ or $\frac{5\pi}{4}$, $|\text{Sin}\, x| - |\text{Cos}\, x| = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$ (1)

and when $x = \pi$, $|\text{Sin}\ x| = 0$ and $|\text{Cos}\ x| = 1$

$|\text{Sin}\ x| - |\text{Cos}\ x| = -1$ (2)

From (1) and (2), the range of $f(x)$ is $3[-1, 0]$, i.e., $[-3, 0]$.

**15.** $\sin\theta - \cos\theta = 1/5. \Rightarrow \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = 1/25$

$\Rightarrow \sin\theta\cos\theta = 12/25$

$\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)^3 + 3\sin\theta\cos\theta\,(\sin\theta - \cos\theta)$

$= \frac{1}{125} + 3.\frac{12}{25}\left(\frac{1}{5}\right) = \frac{37}{125}.$

**16.** Given $A + B = 45° \Rightarrow B = 45° A$

$(1 + \tan A)\ (1 + \tan B) = (1 + \tan A)\ [1 + \tan\ (45° - A)]$

$= (1 + \tan A)\left(1 + \frac{\tan 45 - \tan A}{1 + \tan 45 \tan A}\right),$

$\left(\because\ \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right)$

$(1 + \tan A)\left(1 + \frac{1 - \tan A}{1 + \tan A}\right)$, ($\because \tan 45° = 1$)

$(1 + \tan A)\ \frac{(1 + \tan A + 1 - \tan A)}{1 + \tan A} = 2$

**17.** Given $f(x) = \frac{1}{5 - \cos 3x}$

We know that $-1 \le \cos 3x \le 1$

$-1 \le -\cos 3x \le 1$

$5 - 1 \le 5 - \cos 3x \le 5 + 1$

$4 \le 5 - \cos 3x \le 6$

$\frac{1}{6} \le \frac{1}{5 - \cos 3x} \le \frac{1}{4}$

$\therefore$ range of $f(x)$ is $\left[\frac{1}{6}, \frac{1}{4}\right]$

**18.** Given $f(\theta) = \sin^2\theta - \cos^2\theta = -(\cos^2\theta - \sin^2\theta)$

$= -\cos 2\theta$

Range of $\cos 2\theta$ is $[-1, 1]$

$\therefore$ Range of $f(\theta)$ is $[-1, 1]$.

**19.** The range of $f(x) = a\cos x + b\sin x + c$ is

$(c - \sqrt{a^2 + b^2}\ , c + \sqrt{a^2 + b^2}\ )$

Here, $c = 13$, $a = 8$, $b = 15$

$\therefore c \pm \sqrt{a^2 + b^2} = 13 \pm \sqrt{8 - 15}$

and $C - \sqrt{a^2 + b^2} = 13 - 17 = -4$

$C + \sqrt{a^2 + b^2} = 13 + 17 = 30$

$\therefore$ Range of the given function is $[-4, 30]$

**20.** $13\sin\theta = 5 \Rightarrow \sin\theta = \frac{5}{13}$

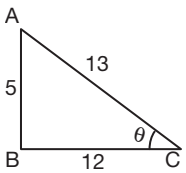
Since $\theta$ is obtuse, $\theta$ lies in the second quadrant.

$\cos\theta = \frac{-12}{13}$

$\sec\theta = \frac{-13}{12}$

$\tan\theta = \frac{-5}{12}$

$\cot\theta = \frac{-12}{5}$




$$\frac{\cot\theta + \sec\theta}{\tan\theta + \cos\theta} = \frac{\frac{-12}{5} \frac{-13}{12}}{\frac{-5}{12} \frac{12}{13}}$$

$$= \left(\frac{-144 - 65}{60}\right)\left(\frac{156}{-65 - 144}\right) = \frac{156}{60} = \frac{13}{5}.$$

**21.** Given $a = 3$ cm, $b = 4$ cm and $\angle C = 60°$

By the cosine rule, we have $c^2 = a^2 + b^2 - 2ab\cos c$

$= 9 + 16 - 2(3)(4)\cos 60° = 25 - 12 = 13$

$\therefore c = \sqrt{13}$

**22.** $9\left[\sin^4\left(\frac{5\pi}{2} - \alpha\right) + \sin^4(6\pi + \alpha)\right]$

$-6\left[\sin^6\left(\frac{3\pi}{2} + \alpha\right) + \sin^6(7\pi - \alpha)\right]$

$= 9[\cos^4\alpha + \sin^4\alpha] - 6[\cos^6\alpha + \sin^6\alpha]$

$= 9[(\cos^2\alpha + \sin^2\alpha)^2 - 2\sin^2\alpha\cos^2\alpha] - 6[(\cos^2\alpha + \sin^2\alpha)(\cos^4\alpha + \sin^4\alpha - \cos^2\alpha\sin^2\alpha]$

$= 9(1 - 2\sin^2\alpha\cos^2\alpha) - 6[(\cos^2\alpha + \sin^2\alpha) - 2\sin^2\alpha\cos^2\alpha - \cos^2\alpha\sin^2\alpha]$

$= 9 - 18\sin^2\alpha\cos^2\alpha - 6[1 - 3\sin^2\alpha\cos^2\alpha]$

$= 3.$

**23.** Let $AB = c$ and $AC = b$ be two sides of the triangle, then the area of the triangle is $\frac{1}{2}cb\sin A$.

The area of the triangle whose sides are 8 cm, 6 cm, and the included angle is 75° is

$\frac{1}{2}\ 8(6)\ \sin 75° = 24\ \frac{\sqrt{3} + 1}{2\sqrt{2}}$

$= 6\sqrt{2}\left(\sqrt{3} + 1\right) = 6\left(\sqrt{6} + \sqrt{2}\right)$

**24.** $a\cos\alpha + b\sin\alpha = c$

$\Rightarrow a\cos\alpha = c - b\sin\alpha$

$\Rightarrow a^2\cos^2\alpha = c^2 + b^2\sin^2\alpha - 2bc\sin\alpha$

$\Rightarrow a^2 - a^2\sin^2\alpha = c^2 + b^2\sin^2\alpha - 2bc\sin\alpha$

$\Rightarrow (b^2 + a^2)\sin^2\alpha - 2bc\sin\alpha + c^2 - a^2 = 0$

If $\sin\theta_1$ and $\sin\theta_2$ are the roots of this equation, then

The sum of the roots $= \sin\theta_1 + \sin\theta_2 = \frac{2bc}{a^2 + b^2}$

**25.** $\log_3(\text{Sin}^2\theta + \text{Cos}^2\theta) = \log_3 1 = 0.$

**Similarly,**

$\log_5(\text{Sin}^2\theta + \text{Cos}^2\theta) = 0$

$\therefore$ The required value is 0.

**26.** We know AM $(a, b) \geq$ GM $(a, b)$

AM $\left(81^{\cos^2 x}, 81^{\sin^2 x}\right) \geq$ GM $\left(81^{\cos^2 x}, 81^{\sin^2 x}\right)$

$$\frac{81^{\cos^2 x} + 81^{\sin^2 x}}{2} \geq \sqrt{81^{\cos^2 x} \times 81^{\sin^2 x}}$$

$$81^{\cos^2 x} + 81^{\sin^2 x} \geq 2\sqrt{81^{\cos^2 x + \sin^2 x}} = 18.$$

$\therefore$ The minimum value of $81^{\cos^2 x} + 81^{\sin^2 x}$ is 18. It occurs when $\cos^2 x = \sin^2 x = 1/2$.

**27.** $\sin^2(\theta - 45°) + \sin^2(\theta + 15°) - \sin^2(\theta - 15°)$

$= \sin^2(\theta - 45)° + [\sin(\theta + 15°) + \sin(\theta - 15°)]$
$(\sin(\theta + 15°) - \sin(\theta - 15°)$

$= \sin^2(\theta - 45°) + (\sin 2\theta . \sin 30°)$

$$= \frac{1 - \cos(90° - 2\theta)}{2} + \frac{1}{2}\sin 2\theta$$

$$= \frac{1}{2} - \frac{1}{2}\sin 2\theta + \frac{1}{2}\sin 2\theta = \frac{1}{2}$$

**28.** $$\frac{\cos(90° - 70°) + \sin 50°}{\sin 20° + \cos(90° - 40°)} = \frac{\sin 70° + \sin 50°}{\sin 20° + \sin 40°}$$

$$= \frac{2\sin 60° \cos 10°}{2\sin 30° \cos 10°}$$

$$\left(\because \sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}\right)$$

$$= \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

**29.** Given, $\tan\theta = \dfrac{\sin 2\theta}{1 + \cos 2\theta}$

substitute $\theta = 22\frac{1}{2}$

Then, $\tan 22\frac{1}{2} = \dfrac{\sin 45°}{1 + \cos 45°}$

$$= \frac{\left(\frac{1}{\sqrt{2}}\right)}{1 + \left(\frac{1}{\sqrt{2}}\right)} = \frac{1}{\sqrt{2} + 1}$$

$$= \frac{\left(\sqrt{2} - 1\right)}{\left(\sqrt{2} + 1\right)\left(\sqrt{2} - 1\right)} = \sqrt{2} - 1$$

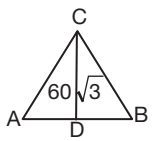
Hence, $\tan 22\frac{1}{2} = \sqrt{2} - 1$

**30.** $$\frac{\sin 2A}{1 + \cos 2A}\cdot\frac{(1 - \cos 2A)}{\sin A}\cos A$$

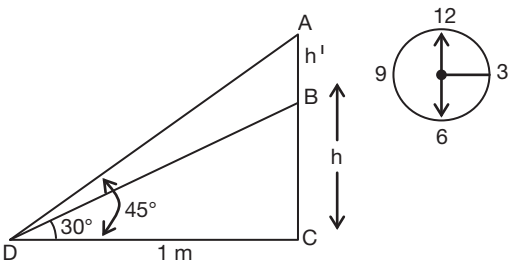
$$= \frac{(2\sin A \cos A)}{(2\cos^2 A)} \cdot \frac{(2\sin^2 A).\cos A}{\sin A}$$

$= 2\sin^2 A$

**31.** Let the flagstaff be erected at $I$, the midpoint of $AB$. Let $DP$ be the flagstaff. The angle of elevation of the top of the flagstaff from $A$ and $B$ (say $\alpha$ and $\beta$, respectively) are equal, while the angle of elevation from $C$ (Say $\gamma$) would be less. (Two of these angles are given to be 45°. Therefore, $\alpha = \beta = 45°$ and then $\gamma = 30°$) As the median $CD$ (the longer leg in the 30°, 60° and 90° triangle $DBC$) $= 60\sqrt{3}$ m, it follows that $DB$ (the shorter leg) = 60 m and hence $DP = 60$ m




**32.**




In the diagram, $AB$ represents the diameter of the clock

In $\Delta ADC$, $\tan 45° = \dfrac{h + h^1}{1} \Rightarrow h + h^1 = 100$ cm (1)

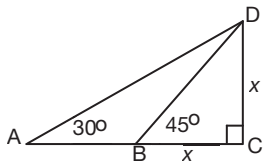
In $\Delta DBC$, $\tan 30° = \dfrac{h}{1} \Rightarrow h = \dfrac{1}{\sqrt{3}}$ m $= 57.73$ cm (2)

using (1) and (2)

$h^1 = 100 - 57.73 = 42.26$ cm. (app)

**33.** Let $CD$ be the tower. $A$, $B$ are the first and second points of observation. Let $BC = x$ (All lengths are in metres)

It took 3 minutes to cover $AB$ at 54 kmph




$$\Rightarrow AB = \frac{54(5)(3)(60)}{18} = 2700 \text{ m}$$

From $\Delta ACD$, $\tan 30° =$

$$\frac{CD}{AC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{CD}{AB+BC} = \frac{CD}{2700+x}$$

$$\Rightarrow CD = \frac{2700+x}{\sqrt{3}} \qquad (1)$$

From $\Delta BCD$, $\tan 45° = \frac{CD}{BC} \Rightarrow CD = BC = x$

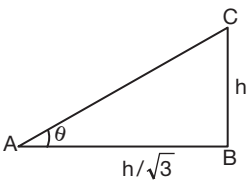
From (1) and (2), $\frac{2700+x}{\sqrt{3}} = x$

$x\left(\sqrt{3}-1\right) = 2700$

$x = \frac{2700}{\sqrt{3}-1}\frac{\sqrt{3}+1}{\sqrt{3}+1} = 1350\left(\sqrt{3}+1\right)$

Height of the tower $CD = 1350\left(\sqrt{3}+1\right)$ m.

**34.**

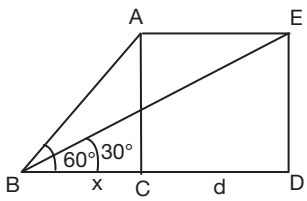



$\tan\theta = h/\left(h/\sqrt{3}\right) = \sqrt{3} \Rightarrow \theta = 60° = \pi/3.$

**35.** Given $\angle ABC = 60°$

$AC = ED = 1$ km

$\angle EBD = 30°$




$\tan 60° = \frac{1}{x}$ and

$\tan 30° = \frac{1}{x+d}$

so $x = \frac{1}{\sqrt{3}}$ and $x + d = \sqrt{3}$

$\therefore d = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$ km

So speed of the plane $= \frac{3600}{15} \times \frac{2}{\sqrt{3}} = 160\sqrt{3}$ kmph.

**36.** When $x = \frac{\pi}{4}, \frac{3\pi}{4}, \ldots$

$y$ is zero.

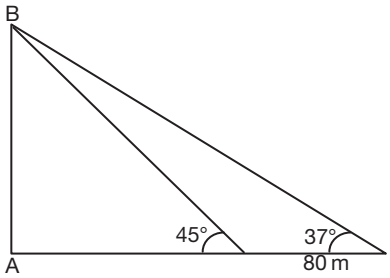
when $x = 0, \frac{\pi}{2}, \pi, \ldots\ldots$

$y$ is one.

In given choices,

only $y = |\cos 2x|$ satisfies both conditions.

**37.**




Let $AB$ be the lighthouse and $C$ and $D$ denote the positions of the two ships.

$CD = 80$ m

Since, $\angle ADB = 45°$, $AB = AD$

$\therefore AC = AD + 80 = AB + 80$

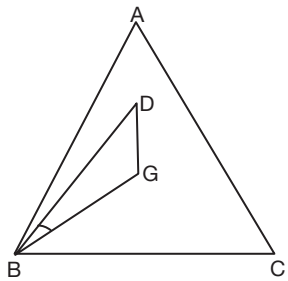
Now, $\tan 37° = \frac{AB}{AC}$

$\Rightarrow \frac{3}{4} = \frac{AB}{AB+80}$ $\left(\therefore \sin 37° = 0 \times 6 = \frac{3}{5}\right)$

$\Rightarrow 3AB + 3 \times 80 = 4AB$

$\therefore AB = 240$ m.

**38.**




Let $G$ be the centroid and $DG$ be the flagstaff.

$\tan \angle DBG = \frac{DG}{BG}$ i.e., $\tan 30° = \frac{24}{BG}$

$BG = 24\sqrt{3}$

$BG$ = Distance between vertex $B$ of the equilateral triangle $AND$ and the centroid $G$

The distance between any vertex of an equilateral triangle and the centroid of the triangle is $\frac{a}{\sqrt{3}}$

$BG = \frac{a}{\sqrt{3}} = 24\sqrt{3}$,

$a = 72$.

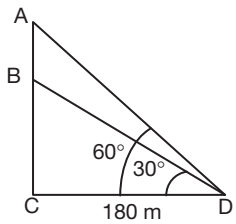
**39.** Let $BC$ represent the tower, $AB$ the flag post and $D$ be the point of observation.

Given $CD = 180$m

From triangle $BCD$, $\tan 30° = BC/CD$

$\Rightarrow BC$

$= 180\left(\frac{1}{\sqrt{3}}\right) = 60\sqrt{3}$m




from $\Delta ACD$, $\tan 60° = AC/CD$

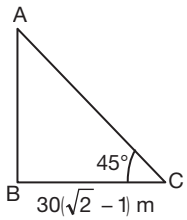
$= (AB + BC)/CD$

$\Rightarrow AB + BC = 180\left(\sqrt{3}\right)$

$\Rightarrow AB = 180\sqrt{3} - 60\sqrt{3}$

$= 60\sqrt{3}\,(3-1) = 120\sqrt{3}$m

**40.**




From the figure, $AB + AC$ represents the height of the pole and $AC$ is the broken part of the pole.

$\tan 45° = AB/BC = AB/30\left(\sqrt{2}-1\right)$

$\Rightarrow AB = 30$ m and $\cos 45° = BC/AC$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{30\left(\sqrt{2}-1\right)}{AC} \Rightarrow AC = 30\sqrt{2}\left(\sqrt{2}-1\right)$

$\therefore$ Height of the pole $= AB + AC = 30$ m

## EXERCISE-4

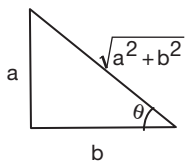
***Solutions for questions 1–30***:

**1.** Given: $\cot\theta = \frac{b}{a}$ and $\theta \in Q_1$

$\Rightarrow \theta \in Q_3$

$\therefore \sin\theta = \frac{-a}{\sqrt{a^2+b^2}}$, $\cos\theta = \frac{-b}{\sqrt{a^2+b^2}}$

$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{\frac{-a^2+b^2}{\sqrt{a^2+b^2}}}{\frac{-\left(a^2+b^2\right)}{\left(\sqrt{a^2+b^2}\right)}}$$




$= \frac{a^2-b^2}{a^2+b^2}$

**Alternate Method:**

$\cot\theta = \frac{b}{a}$, $\Rightarrow \frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a - \frac{b}{a}}{a + \frac{b}{a}} = \frac{a^2-b^2}{a^2+b^2}$

**2.** $\left(\frac{\cot A}{\operatorname{cosec} A + 1} + \frac{\cot A}{\operatorname{cosec} A - 1}\right)\frac{1}{\sec A}$

$= \left(\frac{\frac{\cos A}{\sin A}}{\frac{1}{\sin A}+1} + \frac{\frac{\cos A}{\sin A}}{\frac{1}{\sin A}-1}\right)$

$= \left(\frac{\cos A}{1+\sin A} + \frac{\cos A}{1-\sin A}\right)\frac{1}{\sec A}$

$= \cos A\left(\frac{2}{\cos^2 A}\right)\cos A = 2 = 2$

**3.** $x = a\cot\theta + b\operatorname{cosec}\theta$ (1)

$y = a\cot\theta - b\operatorname{cosec}\theta$ (2)

Solving (1) and (2) for $\cot\theta$ and $\operatorname{cosec}\theta$,

$\cot\theta = \frac{(x+y)}{2a}$ and $\operatorname{cos\,ec}\,\theta = \frac{(x-y)}{2b}$

Since $\operatorname{cosec}^2\theta - \cot^2\theta = 1$,

$\frac{(x-y)^2}{4b^2} - \frac{(x+y)^2}{4a^2} = 1$

**4.** $\sin\theta + \cos\theta = \sqrt{2}$

It is possible only when $\theta = 45°$

$\tan^n\theta + \cot^n\theta = \tan^n 45 + \cot^n 45 = 1 + 1 = 2$

**5.** $\tan 9° + \tan(90° - 9) - [\tan 27° + \tan(90° - 27°)]$

$\Rightarrow \tan 9° + \cot 9° - (\tan 27° + \cot 27°)$

$\Rightarrow \frac{\sin 9°}{\cos 9°} + \frac{\cos 9°}{\sin 9°} - \left(\frac{\sin 27°}{\cos 27°} + \frac{\cos 27°}{\sin 27°}\right)$

$\Rightarrow \frac{2\left(\sin^2 9° + \cos^2 9°\right)}{2\sin 9°\cos 9°} - \frac{2\left(\sin^2 27° + \cos^2 27°\right)}{2\sin 27°\cos 27°}$

$\Rightarrow \frac{2}{\sin 18°} - \frac{2}{\sin 54°} = 2\left[\frac{1}{\sin 18} - \frac{1}{\cos 36}\right]$

$$\Rightarrow 2\left[\frac{4}{\sqrt{5}-1}-\frac{4}{\sqrt{5}+1}\right]=8\left[\frac{\sqrt{5}+1-\sqrt{5}+1}{4}\right]=4$$

**6.** $\cos^2\theta+\sin^4\theta=\sin^4\theta+1-\sin^2\theta$

$$=\left\{\left(\sin^2\theta\right)^2-2.\sin^2\theta\times\frac{1}{2}+\left(\frac{1}{2}\right)^2\right\}+1-\left(\frac{1}{2}\right)^2$$

$$=\left(\sin^2\theta-\frac{1}{2}\right)^2+\frac{3}{4}$$

When $\sin^2\theta=0$, the given expression is maximum

$$\text{Maximum value}=\left(\frac{-1}{2}\right)^2+\frac{3}{4}=\frac{1}{4}+\frac{3}{4}=1$$

$$\text{Minimum value}=0+\frac{3}{4}=\frac{3}{4}$$

$$\therefore\text{The range}=\left[\frac{3}{4},1\right]$$

**7.** $1+8\sin^2x^2\cos^2x^2=1+2\ (2\sin x^2\cos x^2)^2$

$=1+2\sin^2 2x^2$

$=1+(1-\cos 4x^2)=2-\cos 4x^2$

Minimum value is $c-\sqrt{a^2+b^2}$ here, $c=2$, $a=1$, $b=0$

$\therefore$ The required minimum value is $2-1=1$.

**8.** $\sin^2(\theta-45°)+\sin^2(\theta+15°)-\sin^2(\theta-15°)$

$=\sin^2(\theta-45°)+(\sin 2\theta.\sin 30°)$

$(\sin^2A-\sin^2B)=\sin(A+B).\sin(A-B)$

$$=\frac{1-\cos(90°-2\theta)}{2}+\frac{1}{2}\sin 2\theta$$

$\because\cos\theta$ is an even function.

$$=\frac{1}{2}-\frac{1}{2}\sin 2\theta+\frac{1}{2}\sin 2\theta=\frac{1}{2}$$

**9.** We know that AM $\geq$ GM

$$\Rightarrow\frac{4\tan^2x+9\cot^2x}{2}\geq\sqrt{4\tan^2x.9\cot^2x}$$

$\therefore 4\tan^2x+9\cot^2x\geq 12$

$\therefore$ The minimum value of the function is 12.

**10.** $\dfrac{4\text{SinA}+3\text{SinB}}{4\text{SinA}+3\text{SinB}}=\dfrac{7}{3}$

$\Rightarrow 12\sin A+9\sin B=28\sin A-21\sin B$

$\Rightarrow 16\sin A=30\sin B$

$$\frac{15}{Sin\,A}=\frac{8}{Sin\,B}\Rightarrow \text{Sin}A:\text{Sin}B$$

$$=15:8=\text{a}:b\left[\because\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}\right]$$

But given, $AC=16\Rightarrow b=16$

$\Rightarrow BC=a=15\times 2=30$.

**11.** Given $P=1-\cos^2\theta+\cos^4\theta$

$$=1-\frac{1+\cos 2\theta}{2}+\left(\frac{1+\cos 2\theta}{2}\right)^2$$

$$=1-\frac{1}{2}-\frac{\cos 2\theta}{2}+\frac{1}{4}+\frac{2\cos 2\theta}{4}+\frac{\cos^2 2\theta}{4}$$

$$=\frac{3}{4}+\frac{1}{4}\frac{(1+\cos 4\theta)}{2}$$

$$=\frac{3}{4}+\frac{1}{8}+\frac{\cos 4\theta}{8}=\frac{7}{8}+\frac{1}{8}\cos 4\theta$$

P is maximum when $\cos 4\theta=1$ and minimum when $\cos 4\theta=-1$

$$\therefore\ \frac{7}{8}-\frac{1}{8}\leq P\leq\frac{7}{8}+\frac{1}{8}\text{ or }\frac{3}{4}\leq P\leq 1.$$

**12.** Given $27^{1+|\sin x|+|\sin^2 x|+\ldots}=3^6$

$$\Rightarrow 3^{3\left(1+|\sin x|+|\sin^2 x|+\ldots\right)}=3^6$$

$$\Rightarrow 1+|\sin x|+|\sin^2 x|+\ldots=2$$

$$\Rightarrow\frac{1}{1-|\sin x|}=2\left(\because\text{The sum of the infinite GP }a,ar,ar^2,\ldots=\frac{a}{1-r}\right)$$

$$1-|\sin x|=\frac{1}{2}\Rightarrow|\sin x|=\frac{1}{2}$$

$$\sin x=\pm\frac{1}{2}\Rightarrow x=30°,150°,210°,330°.$$ There are 4 values of $x$ that satisfy the given equation.

**13.** If $A+B=45°$, then $\tan A+\tan B=1-\tan A\tan B$

or $(1+\tan A)\ (1+\tan B)=2$

$\therefore\ (1+\tan 15°)\ (1+\tan 30°)=2$

Similarly, $(1+\tan 16°)\ (1+\tan 29°)=2$ ($\because$ $16+29=45$)

$(1+\tan 17)\ (1+\tan 28)=2\ldots$

$\therefore$ The given expression consists of 8 pairs of terms. In each pair, the sum of the angles is 45°

$\therefore$ The value of the given expression is $2^8$.

**14.** $\cot 2\alpha = \cot\left(\overline{\alpha+\beta}+\overline{\alpha-\beta}\right) = \dfrac{\cot(\alpha+\beta)\cot(\alpha-\beta)-1}{\cot(\alpha+\beta)+\cot(\alpha-\beta)}$

$\sin(\alpha+\beta) = \dfrac{4}{5} \Rightarrow \tan(\alpha+\beta) = 4/3$

$(\because\ 0 < \alpha+\beta < \pi/2)$ and $\cot(\alpha+\beta) = \dfrac{3}{4}$

And $\cos(\alpha-\beta) = \dfrac{12}{13} \Rightarrow \cot(\alpha-\beta) = \dfrac{12}{5}$ (As $\alpha-\beta > 0$, cot $(\alpha-\beta) = \dfrac{12}{5}$ rather than $\dfrac{-12}{5}$)

$\therefore \cot 2\alpha = \dfrac{\frac{3}{4}\left(\frac{12}{5}\right)-1}{\frac{3}{4}+\frac{12}{5}} = \dfrac{16}{63}$

**15.** $\left(1+\cos 22\dfrac{1^\circ}{2}\right)\left(1+\cos 67\dfrac{1^\circ}{2}\right)\left(1+\cos 112\dfrac{1^\circ}{2}\right)\left(1+\cos 157\dfrac{1^\circ}{2}\right)$

$= \left(1+\cos 22\dfrac{1^\circ}{2}\right)\left(1+\cos 67\dfrac{1^\circ}{2}\right)\left(1+\cos\left(180^\circ - 67\dfrac{1^\circ}{2}\right)\right)$

$\left(1+\cos\left(180-22\dfrac{1^\circ}{2}\right)\right)$

$\left(1+\cos 22\dfrac{1^\circ}{2}\right)\left(1+\cos 67\dfrac{1^\circ}{2}\right)\left(1-\cos 67\dfrac{1^\circ}{2}\right)\left(1-\cos 22\dfrac{1^\circ}{2}\right)$

$= \left(1-\cos^2 22\dfrac{1^\circ}{2}\right)\left(1-\cos^2 67\dfrac{1^\circ}{2}\right)$

$= \sin^2 22\dfrac{1}{2}\sin^2 67\dfrac{1^\circ}{2}$

$= \sin^2 22\dfrac{1}{2}\sin^2\left(90-22\dfrac{1^\circ}{2}\right)$

$= \sin^2 22\dfrac{1^\circ}{2}\cos^2 22\dfrac{1^\circ}{2} = \dfrac{1}{4}\left(2\sin 22\dfrac{1^\circ}{2}\cos 22\dfrac{1^\circ}{2}\right)^2$

$= \dfrac{1}{4}(\sin 45^\circ)^2 = \dfrac{1}{4}\left(\dfrac{1}{\sqrt{2}}\right)^2 = \dfrac{1}{8} = 0.125.$

**16.** $6\ \text{Cos}\ A + 8\ \text{Sin}\ B = 12$ (1)

$8\ \text{Cos}\ B + 6\ \text{Sin}\ A = 2\sqrt{13}$ (2)

Squaring and adding (1) and (2), we get

$36(\text{Cos}^2A + \text{Sin}^2A) + 64\ (\text{Cos}^2B + \text{Sin}^2B)$
$+96(\text{Sin}B\ \text{Cos}A + \text{Cos}B\ \text{Sin}A) = 144 + 52$

$\Rightarrow 36 + 64 + 96\ [\text{Sin}\ (A+B)] = 196$

$\Rightarrow \text{Sin}\ (A+B) = 1$

$\Rightarrow \text{Sin}(180 - C) = \text{Sin}\ \pi/2$

$\Rightarrow \text{Sin}\ C = \text{Sin}\ \pi/2$

$\Rightarrow \angle C = 90^\circ$

**17.** $x = \cos 50^\circ + \cos 55^\circ + \cos 60^\circ$

$y = \sin 20^\circ + \sin 25^\circ + \sin 30^\circ$

Since $\cos 60^\circ = \sin 30^\circ$, we compare the remaining terms. The cosine function is a decreasing function from $0^\circ$ to $90^\circ$, and since $\cos 60^\circ = \dfrac{1}{2}$; $\cos 50^\circ, \cos 55^\circ > \dfrac{1}{2}$;

$\Rightarrow \cos 50^\circ + \cos 55^\circ > 1.$

The sine function is a increasing function from $0^\circ$ to $90^\circ$, and since $\sin 30^\circ = \dfrac{1}{2}$; , $\sin 20^\circ + \sin 25^\circ < 1$.

$\therefore x > y \Rightarrow \dfrac{x}{y} > 1.$

**18.** $h(y) = 3\left[|\sin y| + |\cos y|\right]$

$\sin^2 x \le |\sin x|$

$\cos^2 x \le |\cos x|$

$\therefore \sin^2 x + \cos^2 x \le |\sin x| + |\cos x|$

i.e $1 \le |\sin x| + |\cos x| \Rightarrow |\sin x| + |\cos x| \ge 1$

Also, $\sin x + \cos x = \sqrt{2}\left(\dfrac{\sin x}{\sqrt{2}} + \dfrac{\cos x}{\sqrt{2}}\right)$

$= \sqrt{2}\ (\sin x \cos \pi/4 + \cos x \sin \pi/4) = \sqrt{2}\sin\left(x + \dfrac{\pi}{4}\right)$

$\therefore$ The maximum sum is $\sqrt{2}$ ; $|\sin x| + |\cos x| \le \sqrt{2}$

Given $h(y) = 3\left[|\sin y| + |\cos y|\right]$

$\therefore$ minimum value of $h(y)$ is = 3 and maximum value is $3\sqrt{2}$

$\therefore 3 \le h(y) \le 3\sqrt{2}$

**19.** $|\tan^2 \pi x| + |\cos^2 \pi y| = 0 \Rightarrow |\tan^2 \pi x| = |\cos^2 \pi y| = 0$

$\Rightarrow \tan \pi x = \cos \pi y = 0$

$\tan \pi x = 0$ implies $\pi x = 0, \pm\pi, \pm 2\pi \ldots$ and

$\cos \pi y = 0$ implies $\pi y = \pm\dfrac{\pi}{2}, \pm\dfrac{3\pi}{2}, \pm\dfrac{5\pi}{2}\ldots$

$\therefore x = 0, \pm 1, \pm 2, \ldots$ and $y = \pm\dfrac{1}{2}, \pm\dfrac{3}{2}, \pm\dfrac{5}{2}\ldots$

The points which satisfy $x^2 + y^2 \le 9$ must be such that $x^2 \le 9$ and $y^2 \le 9$, i.e., $|x| \le 3$ and $|y| \le 3$

The possible values of $(x, y)$ are

$\left(0, \pm\frac{1}{2}\right)\left(0, \pm\frac{3}{2}\right)\left(0, \pm\frac{5}{2}\right)\left(\pm 1, \pm\frac{1}{2}\right)$

$\left(\pm 1, \pm\frac{3}{2}\right)\left(\pm 1, \pm\frac{5}{2}\right)\left(\pm 2, \pm\frac{1}{2}\right)\left(\pm 2, \pm\frac{3}{2}\right)$

$\therefore$ $(x, y)$ has 26 possible values.

**20.** $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 2\theta}}} = \sqrt{2+\sqrt{2+\sqrt{2\left(2\cos^2\theta\right)}}}$

$= \sqrt{2+\sqrt{2.2\cos^2\theta/2}} = \sqrt{2+2\cos\theta/2}$

$= \sqrt{2.2\cos^2\theta/4} = 2\cos\theta/4$

**21.** $f(x) = \dfrac{\sin^5 x - \cos^5 x}{\cos^2 x \sin^2 x}$ and

$g(x) = \dfrac{\cos^5 x - \sin^5 x}{\sin^3 x \cos^3 x}$

$\frac{\pi}{4} < x < \frac{\pi}{2}$ (1)

As $\theta$ Increases from 0 to $\frac{\pi}{2}$, $\sin\theta$ increases from 0 to 1

and $\cos\theta$ decreases from 1 to 0. Also,

$\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$.

$\therefore$ $\sin\theta < \cos\theta$ when $0 \le \theta \le \frac{\pi}{4}$ and $\sin\theta > \cos\theta$ when

$\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$

(1) implies $\sin x > \cos x$

$\therefore$ $\sin^5 x > \cos^5 x$

$f(x) = \dfrac{\text{A positive value}}{\text{A positive value}}$ = a positive value

Also, $g(x)$ is a negative value.

$\therefore$ $f(x) > g(x)$

**22.** $r^2 = p^2 + q^2 - 2pq\cos\angle R$

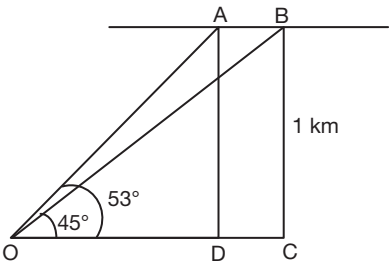
$= 8^2 + 10^2 - 2(8)(10)\left(\dfrac{\sqrt{5}+1}{4}\right)$

$= 164 - 40\left(\sqrt{5}+1\right)$

$= 124 - 40\sqrt{5}$

$r = 2\sqrt{31 - 10\sqrt{5}}$

**23.**




Let $A$ be the position of the aeroplane at the given instant of time.

Let $v$ km/sec be the speed of the aeroplane.

Then $DC = AB = 12v$ km

$\angle BOC = 45°$,

$OC = BC = 1$km

$\Rightarrow OD + DC = 1$

$OD = 1 - 12v$

Now, $\tan 53° = \dfrac{AD}{OD} = \dfrac{1}{1-12v}$

$\Rightarrow \tan(90° - 37°) = \dfrac{1}{1-12v}$

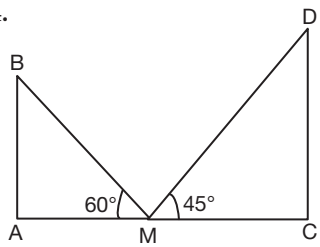
$\Rightarrow \dfrac{1}{\tan 37°} = \dfrac{1}{1-12v}$

Given: $\sin 37° = 0 \times 6 = \frac{3}{5}$ $\therefore$ $\tan 37° = \frac{3}{4}$

$\Rightarrow \frac{4}{3} = \frac{1}{1-12v} \Rightarrow 4 - 48v = 3$

$\therefore$ $v = \frac{1}{48}$ km/sec.

**24.**

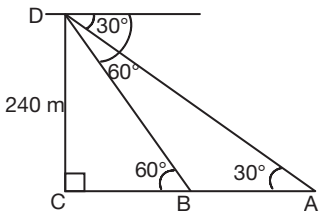



Let $M$ be the midpoint of $AC$ and $AB = 112\sqrt{3}$m

In $\Delta MAB$, $\tan 60° = \dfrac{AB}{AM}$ $\therefore$ $AM = \dfrac{112\sqrt{3}}{\sqrt{3}} = 112$m

Since $\angle DMC = 45°$, $MC = CD = 112$ m. ($\because$ $MC = AM$)

**25.**

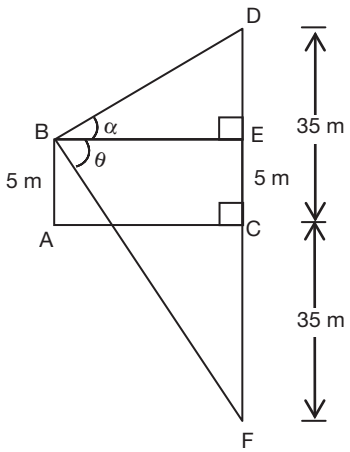



Let $CD$ be the lighthouse and $A$, $B$ the two positions when the boat was observed. The time when the boat was at $A$ was 10.06 a.m.

$DC = \sqrt{3}\ BC$ and $CA = \sqrt{3}\ DC = 3(BC)$. $\therefore\ BA = 2CB$.

The boat takes 16 min to cover $AB$. $\therefore$ It takes 8 min to cover $BC$ or a total of 24 min to cover $AC$. $\therefore$ It reaches the lighthouse at 10:06 + 24 min = 10:30 a.m.

**26.**




The height of the building $AB = 5$ m

$CD$ is the tree, $CF$ its image, $AC$ is the width of the river, $\theta$ and $\alpha$ are the angles of depression and the angle of elevation, respectively.

In $\Delta BEF$, $\sin\theta = \dfrac{EF}{BF} = \dfrac{40}{41}$ (given).

The height of the tree is 35 m and $EF = 40$ m,

$\therefore\ BF = 41$ m;

From right angled triangle $BEF$, $BE^2 = BF^2 - EF^2$

$= 41^2 - 40^2\ m^2$

$BE = \sqrt{81}$ m $= 9$ m

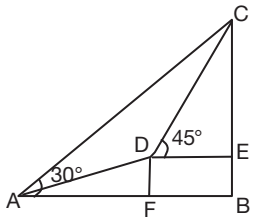
From $\Delta BED$, $BD^2 = BE^2 + DE^2$

$= 9^2 + 30^2\ m^2 = 981\ m^2$

$BD = \sqrt{981}$ m

$\therefore \cos\alpha = \dfrac{BE}{BD} = \dfrac{9}{\sqrt{981}} = \dfrac{3}{\sqrt{109}}$.

**27.** In the figure, $BC$ is the hill $\angle DAB = 15°$, $\angle CDE = 45°$ and $AD = 300$m




In $\Delta ADF$, $DF = AD \sin 15° = 300\left(\dfrac{\sqrt{6}-\sqrt{2}}{4}\right)$ m

$= 75\left(\sqrt{6}-\sqrt{2}\right)$ m

From $\Delta ADC$

$\angle ADC = 360° - \angle ADF - \angle FDE - \angle CDE$

$= (360 - 75 - 90 - 45)° = 150°$

$\angle CAF = 30°$ and $\angle DAF = 15° \Rightarrow \angle DAC = 15°$

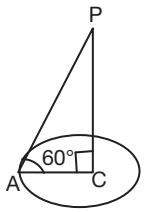
$\therefore\ \angle ACD = 180° - (150 + 15)° = 15°$

$\therefore\ ADC$ is isosceles with $AD = DC = 300$ m. In $\Delta DCE$,

$= DC/\sqrt{2} = 150\sqrt{2}$.

$\therefore$ Height of the hill $= CE + BE = 75\left(\sqrt{6}+\sqrt{2}\right)$ m.

**28.** Let $C$ be the centre of the circle, $CP$ be the flagstaff and $A$ be the observer.




Given that $2\pi r = 440$

$r = \dfrac{440}{2\pi}$ m $= 70$

In right-angled triangle $ACP$, (in which $\angle A = 60°$) $CP = \sqrt{3}\ AC = 70\ \sqrt{3}$ m.

**29.** Given side $AB$ subtends and angle of 60° at the top of the pole $P$.

$\therefore\ APB$ is an equilateral triangle.

$\therefore\ AP = 5$ m

$E$ is mid point of diagonal

$\Rightarrow AE = \dfrac{1}{2}\,5\sqrt{2} = \dfrac{5}{\sqrt{2}}$ m

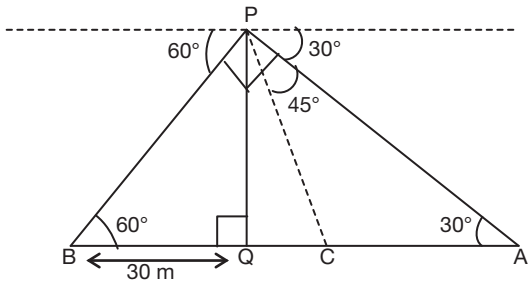
In right triangle $APE$

$PE^2 = AP^2 - AE^2 = 5^2 - \left(\dfrac{5}{\sqrt{2}}\right)^2$

$\therefore\ PE = \dfrac{5}{\sqrt{2}}$ m or $2.5\sqrt{2}$ m.

**30.** Let $PQ$ be the tower and $A$, $B$ be the points of observation.

Now, consider the figure below;




Given $\angle APB = 2\angle APC$.

$\therefore$ $PC$ is the angle bisector of $\angle APB$.

$\Rightarrow AP : BP = AC : BC$

In $\Delta BPQ$, $\sin 60° = \frac{PQ}{BP}$ and in $\Delta APQ$, $\sin 30° = \frac{PQ}{AP}$

$\therefore AP : BP = \sin 60° : \sin 30° = \sqrt{3} : 1$

$\Rightarrow AC : BC = \sqrt{3} : 1$

But given, $BC = 30$ m, hence $AC = 30\sqrt{3}$m .

## EXERCISE-5

***Solutions for questions 1–10:***

**1.** Neither statement is sufficient, as only one value is known.

Statements I, II. $\theta = 45°$, $x = 100$ and $ABC$ is a right-angled isosceles triangle, $\therefore$ $x = y = 100$. I, II are sufficient.

**2.** Statement I. $\tan\theta = \frac{3}{4} \Rightarrow \cos\theta = \pm\frac{4}{5}$

$\therefore$ I alone is not sufficient.

Statement II $\sec\theta = \frac{5}{4} \Rightarrow \cos\theta = \frac{4}{5}$

II is sufficient.

**3.** Statement I. If $\theta = 45°$, $\operatorname{cosec}\theta = \sec\theta$

If $\theta = 30°$, $\operatorname{cosec}\theta > \sec\theta$

If $\theta = 60°$, $\operatorname{cosec}\theta < \sec\theta$

I is not sufficient.

Statement II $\theta \in Q_2$

In the second quadrant, $\sec\theta$ is negative and $\operatorname{cosec}\theta$ is positive.

$\therefore \operatorname{cosec}\theta > \sec\theta$. II is sufficient.

**4.** Statement I, $\cos^2\theta = 1$ or $\cos\theta = 1, -1$ $\therefore \theta = 0$ or $\pi$

In either case, $\tan\theta = 0$.

I is sufficient.

Statement II alone is not sufficient as we do not know the value of $\theta$.

**5.** From Statement I, it is not possible to find $AB$, we can conclude only that $A$ is further away from the building.

Statement I is not sufficient

From Statement II: Even though the distance of one point is known without knowing their positions we cannot find $AB$. Statement II is not sufficient

Even if we combine I and II, we cannot find $AB$.

**6.** Using Statement I alone, $\cos\theta > 0$

$\Rightarrow \theta$ is in $1^{st}$ or $4^{th}$ quadrant.

Hence, it is not in the third quadrant.

**7.** Using Statement II alone, we can not answer as $\sin\theta < 0$

$\Rightarrow \theta$ is in

The expression equals $4\cos A \sin A$.

When $A = 0°$, $4\cos A \sin A = 0$.

$\therefore$ Statement I alone is sufficient to answer $Q_3$ or $Q_4$; we cannot answer Statement II.

**8.** Using Statement II alone,

$A = B = C = 60°$.

$\tan A + \tan B + \tan C = 3\sqrt{3}$ .

With Statement I alone, we cannot answer

**9.** From Statement I, we know that the angle of elevation and the distance. So, we can find the height of the Statue of Freedom.

Similarly from Statement II, we can find the height of Refill tower.

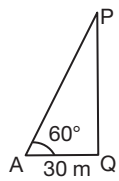
$\therefore$ Using both the Statements, we can find which is taller.

**10.** From Statement I,

In $\Delta AQP$, $BQ = 30$m, $\angle A = 60°$

$\therefore$ $PQ = 30\sqrt{3}$ m

$\therefore$ Statement I is sufficient.



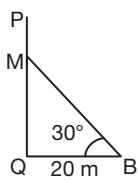

From Statement II,

In $\Delta BQM$, $BQ = 20$m, $\angle B = 30°$

$\therefore$ $QM = 20/\sqrt{3}$ and $PQ = 40/\sqrt{3}$

$\therefore$ Statement II alone is also sufficient.

Hence, either of the statements is sufficient to answer the question.



# Unit 5

# Algebra


Chapter 1 Sequences and Series
Chapter 2 Indices and Surds
Chapter 3 Logarithms
Chapter 4 Quadratic Equations
Chapter 5 Inequalities and Modulus
Chapter 6 Functions
Chapter 7 Graphs
Chapter 8 Operator-Based Questions

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# 1 Sequences and Series

CHAPTER

**LEARNING OBJECTIVES**

*In this chapter, you will*:

- Learn about progressions
- Understand the basic concept of Arithmetic Progression (A.P.), including
  - Finding the nth term of an A.P.
  - Finding the sum of $n$ terms of an A.P.
  - Finding the arithmetic mean of an A.P.
- Understand basic concept of Geometric Progression (G.P.), including
  - Finding the nth term of a G.P.
  - Finding the sum of $n$ terms of a G.P.
  - Finding the geometric mean of a G.P.
  - Finding the sum of terms of an infinite geometric progression
- Gain knowledge of the applications of an infinite geometric G.P.
- Understand basic concept of Harmonic Progression (H.P.)
- Gain introduction to standard results to calculate $\Sigma n$, $\Sigma n^2$, $\Sigma n^3$
- Learn about sequences and series–convergent series and divergent series

## ❒ PROGRESSIONS

PROGRESSION is a series in which each term is related to its previous term in some logical pattern. In this chapter, we will look at the problems on sequences or progressions of numbers, where the terms of the sequence follow a particular pattern—either addition of a constant (arithmetic sequence or arithmetic progression) or multiplication by a constant (geometric sequence or geometric progression). A third type of progression—harmonic progression—has also been defined later.

### ❒ Arithmetic Progression

An Arithmetic Progression (A.P.) is a sequence of numbers in which any number (other than the first) is more (or less) than the immediately preceding number by a constant value. This constant value is called the common difference. In other words, any term of an arithmetic progression can be obtained by adding the common difference to the preceding term.

Let $a$ be the first term of an arithmetic progression, $d$ the common difference and $n$ the number of terms in the progression.

The $n$th term is normally represented by $T_n$ and the sum to $n$ terms is denoted by $S_n$.

$$T_n = n\text{th term} = a + (n-1)\ d$$

**FORMULA**

$$S_n = \text{Sum of } n \text{ terms} = \frac{n}{2} \times [2a + (n-1)\,d]$$

The progression can be represented as $a$, $a + d$, $a + 2d$, …, $[a + (n-1)\,d]$. Here, quantity $d$ is to be added to any chosen term to get the next term of the progression.

The sum to $n$ terms of an arithmetic progression can also be written in a different manner.

$$\text{Sum of first } n \text{ terms} = \frac{n}{2} \times [2a + (n-1)d]$$

$$= \frac{n}{2} \times \{a + [a + (n-1)d]\}$$

However, when there are $n$ terms in an arithmetic progression, $a$ is the first term and $[a + (n-1)d]$ is the last term. Hence, $S_n = \frac{n}{2} \times [\text{first term} + \text{last term}]$

Arithmetic Mean (A.M.) is the average of a set of numerical values. Since the average is equal to the sum of all the quantities/number of quantities, arithmetic mean of an A.P. is equal to the sum of the terms of the arithmetic progression divided by the number of terms in the arithmetic progression.

**FORMULA**

Arithmetic mean of $n$ terms in an arithmetic progression

$$= \frac{Sn}{n} = \frac{1}{2}[2a + (n-1)d]$$

$$= \frac{1}{2} \times (\text{first term} + \text{last term})$$

$$= \frac{(\text{First Term} + \text{Last Term})}{2}$$

i.e., A.M. of an A.P. is the average of the first and the last terms of the A.P.

Arithmetic mean can also be obtained by taking the average of any two terms which are equidistant from the two ends of the A.P., i.e.,

1. The average of the second term from the beginning and the second term from the end will be equal to the A.M.
2. The average of the third term from the beginning and the third term from the end will also be equal to the A.M. and so on.

In general, **the average of the *k*th term from the beginning and the *k*th term from the end will be equal to the A.M.**

**Conversely, if the A.M. of an A.P. is known, the sum to n terms of the series ($S_n$) can be expressed as**

$$S_n = n \times \text{A.M.}$$

**For two numbers $a$ and $b$, their A.M. = $\frac{(a+b)}{2}$.**

If three numbers are in arithmetic progression, the middle number is called the arithmetic mean, i.e., if $a$, $b$, $c$ are in A.P., then $b$ is the A.M. of the three terms and $b = \frac{a+c}{2}$.

**If three numbers are in A.P., we can represent the three numbers as $(a-d)$, $a$ and $(a+d)$.**

**If four numbers are in A.P., we can represent the four numbers as $(a-3d)$, $(a-d)$, $(a+d)$ and $(a+3d)$; (in this case, $2d$ is the common difference).**

**If five numbers are in A.P., we can represent the five numbers as $(a-2d)$, $(a-d)$, $a$, $(a+d)$ and $(a+2d)$.**

## ❒ Geometric Progression

Numbers taken in a certain order are said to be in geometric progression if the ratio of any term (other than the first) to the preceding one is the same. This ratio is called the common ratio. In other words, any term of a geometric progression can be obtained by multiplying the preceding number by the common ratio.

The common ratio is normally represented by $r$. The first term of a geometric progression is denoted by $a$.

A geometric progression can be represented as $a$, $ar$, $ar^2$, ... where $a$ is the first term and $r$ is the common ratio of the geometric progression.

$n$th term of the geometric progression is $ar^n - 1$.

Sum to $n$ terms:

$$\frac{a\left(1-r^n\right)}{1-r} \text{ or } \frac{a\left(r^n-1\right)}{r-1}$$

$$= \frac{rar^{n-1} - a}{r-1} = \frac{r \times \text{Last term} - \text{First term}}{r-1}$$

Thus, the sum to $n$ terms of a geometric progression can also be written as

$$S_n = \frac{r \times \text{Last term} - \text{First term}}{r-1}$$

If $n$ terms $a_1$, $a_2$, $a_3$, ... $a_n$ are in G.P., then the geometric mean (G.M.) of these $n$ terms is given by $\sqrt[n]{a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_n}$.

If three terms are in geometric progression, then the middle term is a geometric mean of the other two terms, i.e., if $a$, $b$ and $c$ are in G.P., then $b$ is the geometric mean of the three terms and $b^2 = ac$.

If there are two terms $a$ and b, their geometric mean (G.M.) is given by $\sqrt{ab}$.

When there are three terms in geometric progression, we can represent the three terms to be $a/r$, $a$ and $ar$.

When there are four terms in a geometric progression, we can represent the four terms as $\frac{a}{r^3}$, $\frac{a}{r}$, $ar$ and $ar^3$.

(In this case, $r^2$ is the common ratio)

## ❒ Infinite Geometric Progression

If $-1 < r < +1$ or $|r| < 1$, then the sum of the terms of a geometric progression does not increase infinitely; it 'converges' to a particular value. Such a G.P. is referred to as an infinite geometric progression. The sum of an infinite geometric progression is represented by $S_\infty$ and is given by the formula

**FORMULA**

$$S_\infty = \frac{a}{1-r}$$

## ❒ Harmonic Progression

If the reciprocals of the terms of a sequence are in arithmetic progression, the sequence is said to be a harmonic progression, For example, $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ is a harmonic progression. In general, the sequence $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots$ is a harmonic progression.

For two numbers $a$ and b, their harmonic mean is given by $\frac{2ab}{a+b}$.

For any two positive numbers $a$ and $b$,

$$\text{A.M.} \geq \text{G.M.} \geq \text{H.M.}$$

If $a$, $b$, $c$ are in harmonic progression, $b$ is said to be the harmonic mean of $a$ and $c$. In general, if $x_1, x_2, \ldots x_n$ are in harmonic progression, $x_2, x_3, \ldots x_{n-1}$ are the $n-2$ harmonic means between $x_1$ and $x_n$.

## ❒ SOME IMPORTANT RESULTS

The results of the sums to $n$ terms of the following series are quite useful, and hence should be remembered by students.

Sum of the first $n$ natural numbers

$$= \sum n = \frac{n(n+1)}{2}$$

Sum of squares of the first $n$ natural numbers

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

Sum of cubes of first $n$ natural numbers

$$\sum n^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2(n+1)^2}{4} = \left[\sum n\right]^2$$

## ❒ SEQUENCES AND SERIES

Any function for which the domain is the set {1, 2, …$n$} is called a **Sequence** (or a finite sequence). If the domain is $N = \{1, 2, 3, \ldots\}$, it is an infinite sequence.

Thus, $1, \frac{1}{2}, \frac{1}{3}, \ldots \frac{1}{1000}$ is a finite sequence, whereas 1, $\frac{1}{2}, \frac{1}{3}, \ldots$ or $1, \frac{1}{2^2}, \frac{1}{3^2}, \ldots$ are infinite sequences.

An expression of the kind $x_1 + x_2 + \ldots + x_n$ is a series (or a finite series) whereas expressions of the kind $x_1 + x_2 + \ldots$ are infinite series.

Consider the infinite sequences $A$ and $B$ and infinite series $C$ and $D$ given below.

$A: 1, 2, 3, \ldots$

$B: 1 + \frac{1}{1}, 1 + \frac{1}{2}, 1 + \frac{1}{3}, \ldots$

$A$ is said to be a divergent sequence whereas $B$ is a convergent sequence. This is because there is no finite number to which the terms of $A$ tend to, whereas in the case of $B$, the terms tend to 1, i.e., if $t_n$ denotes the $n$th term, we can make sure that $t_n$ is as close to 1 as we please, by taking any value of $n$, which is sufficiently large. (The condition is stated quantitatively as follows. For any value of $\delta$, no matter how small, we can find a number N, such that if $n \geq N$, then $|t_n - 1| < \delta$.)

$C: 1 + \frac{1}{2} + \frac{1}{3} + \ldots$

$D: 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

Similarly $C$ is a divergent series while $D$ is a convergent series. This is because there is no finite value to which $C$ tends while $D$ tends to 2, i.e., we can ensure that the sum up to the $n$th term of $D$, say $S_n$, is as close to 2 as we please, by taking any value of $n$ which is sufficiently large. (This condition is stated quantitatively as follows. For any value of $\delta$, no matter has small, we can find a number $N$ such that if $n \geq N$, then $|s_n - 2| < \delta$.)

We shall look at some useful models on series/sequence which have appeared in management entrances. The series could include A.P., G.P. or other patterns of summations which involve concepts of progressions. There could also be other series which appear to be related to progressions, but actually involve techniques of mathematical manipulation. These techniques are best illustrated or learned using examples. However, for most questions, observing the pattern proved to be a useful method for arriving at the answer.

**NOTE**

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

## SOLVED EXAMPLES

**1.01.** Find the 10th term of an arithmetic progression whose first term is 2 and the common difference is 3.

**Sol:** The $n$th term of an arithmetic progression is given by $a + (n-1)d$, where $a$ and $d$ are the first term and the common difference of the arithmetic progression, respectively.

As $n = 10$, $a = 2$ and $d = 3$, the 10th term $= 2 + (10 - 1)\ 3 = 29$.

**1.02.** Find the number of terms in an arithmetic progression with the first term being 3 and the last term being 67, given that the common difference is 4.

**Sol:** The $n^{th}$ term $= a + (n-1)\ d$

$67 = 3 + (n-1)\ 4$

$\Rightarrow n - 1 = \dfrac{67 - 3}{4} = 16$

$\Rightarrow n = 17$.

**1.03.** Find the first term and common difference of an A.P., if the fourth term is 14 and the eleventh term is 42.

**Sol:** Fourth term $= a + 3d = 14$

Eleventh term $= a + 10d = 42$

Subtracting the first equation from the second, $7d = 28 \Rightarrow d = 4$.

Substituting $d$ in any one of the two equations, we get $a = 2$.

Hence, the first term and the common difference are 2 and 4, respectively.

**1.04.** Find the number of terms in an A.P. whose sixth term is 19 and the twelfth term is 37 and the last term is 67.

**Sol:** Sixth term $= a + 5d = 19$ (1)

Twelfth term $= a + 11d = 37$ (2)

Subtracting equation (1) from equation (2),

$6d = 18 \Rightarrow d = 3$.

Substituting $d = 3$ in equation (1) or (2), $a = 4$

The last term, if the A.P. has $n$ terms, is

$a + (n-1)\ d$

$= 4 + (n-1)\ 3 = 67$

$\Rightarrow n - 1 = 21$

$\Rightarrow n = 22$.

Hence, there are 22 terms in the A.P.

**1.05.** Find the sum of the first 24 terms of the A.P. given that the first term is 3 and the common difference is 5.

**Sol:** The sum of the first $n$ terms of an arithmetic progression is given by $\dfrac{n}{2}\left[2a + (n-1)d\right]$

$\therefore$ Sum of the first 24 terms $= \dfrac{24}{2}\left[(2\times 3) + (23 \times 5)\right]$

$= 1452$.

**1.06.** Find the arithmetic mean of an A.P. with 33 terms if its first term is 1 and common difference is 2.

**Sol.** Arithmetic mean $= \dfrac{1}{2}\left[2a + (n-1)d\right]$

$= \dfrac{1}{2}\left[2(1) + 32(2)\right] = 33$.

**1.07.** There are three numbers in A.P. whose sum is 24 and product is 480. Find the numbers.

**Sol:** Let the three numbers be $a-d$, $a$ and $a+d$.
$a-d+a+a+d=24$
$\Rightarrow 3a=24$
$\Rightarrow a=8$
$(8-d)\ (8)\ (8+d)=480$
$\Rightarrow 64-d^2=60$
$\Rightarrow d^2=4$
$\Rightarrow d=\pm 2$.
Hence, the three numbers are 6, 8 and 10 when $d=2$ and the same numbers in reverse order, when $d=-2$.

**1.08.** 148 is split into four parts which are in arithmetic progression such that the product of the second and third parts is 8 more than the product of the first and last part. What are the four parts?

**Sol:** Let the four parts be $a-3d$, $a-d$, $a+d$ and $a+3d$.
Sum of the four parts
$=a-3d+a-d+a+d+a+3d=148$
$\Rightarrow 4a=148$
$\Rightarrow a=37$.
$(a-d)\ (a+d)=(a-3d)\ (a+3d)+8$
$a^2-d^2=a^2-9d^2+8$
$\Rightarrow d^2=1$
$\Rightarrow d=\pm 1$.
Hence, the four parts are 34, 36, 38 and 40 when $d=1$. When $d=-1$, we get the same numbers in descending order.

**1.09.** The first term and the last term of an A.P. are 7 and 51, respectively. If the sum of the terms of the A.P. is 348, find the common difference.

**Sol:** Let the number of terms be $n$.

$$\frac{n}{2}[7+51]=29n=348$$

$$n=12.$$

Let common difference be $d$.
$7+(12-1)\ d=51$
$\Rightarrow 11d=44$
$\Rightarrow d=4$.

**1.10.** The 5th term and the 21st term of a series in an A.P. are 10 and 42, respectively. Find the 31st term.

**Sol:** $a+4d=10$ (1)
$a+20d=42$ (2)
By subtracting equation (1) from equation (2),
$16d=32$
$\Rightarrow d=2$.
Substituting $d=2$ in either (1) or (2), $a=2$.
31st term $=a+30d=2+30(2)=62$.

**1.11.** Find the values of three numbers in arithmetic progression such that their sum is 30 and the sum of their squares is 308.

**Sol:** Let the three numbers in A.P. be $a-d$, $a$ and $a+d$.
$a-d+a+a+d=30$
$\Rightarrow 3a=30$
$\Rightarrow a=10$.
$(a-d)^2+a^2+(a+d)^2$
$=a^2-2ad+d^2+a^2+a^2+2ad+d^2$
$3a^2+2d^2=308$
$2d^2=308-3a^2=8$, as $a=10$.
$\Rightarrow d^2=4\Rightarrow d=\pm 2$.
Hence, the numbers are 8, 10 and 12 when $d=2$ and the same numbers in reverse order when $d=-2$.

**1.12.** The first term and the common difference of an arithmetic progression are 4 and 3, respectively. Find the $11^{th}$ term.

**Sol:** The $n^{th}$ term of A.P. $=a+(n-1)\ d$
$11^{th}$ term $=4+10\ (3)=34$

**1.13.** Find the number of terms in an arithmetic progression in which the first and last terms are 7 and 45, respectively, and the common difference is 2.

**Sol:** Let the number of terms be $n$.
Given $a=7$, $d=2$ and $a+(n-1)\ d=45$
$\Rightarrow 45=7+(n-1)\ 2$
$\therefore n=20$

**1.14.** The sixth and the tenth terms of an arithmetic progression are 22 and 38, respectively. Find the first term and the common difference.

**Sol:** Let the first term and the common difference be $a$ and $d$, respectively.
$a+5d=22$ (1)
$a+9d=38$ (2)
Subtracting (1) from (2),
$4d=16$, $d=4$
Substituting $d$ in (1) or (2),
we get $a=2$

**1.15.** The 12th term, the 14th term and the last term of an arithmetic progression are 25, 31 and 37, respectively. Find the first term, common difference and the number of terms.

**Sol:** Let the first term, the common difference and the number of terms be $a$, $d$ and $n$, respectively.
Given that
$a+11d=25$ (1)
$a+13d=31$ (2)

Subtracting (1) from (2),
$2d = 6$
$d = 3$
Substituting $d = 3$ in (1) or (2),
$a = -8$
Given, $t_n = -8 + (n-1)\ 3 = 37$
$n = 16$

**1.16.** Find the sum of the first 20 terms of an arithmetic progression in which the first term is 6 and the common difference is 2.

**Sol:** The sum of the first $n$ terms of an A.P.

$$= \frac{n}{2}\ [2a + (n-1)\ d]$$

Sum of the first 20 terms

$$= \frac{20}{2}[2(6) + 19(2)] = 500$$

**1.17.** Find the arithmetic mean of the first 31 terms of an arithmetic progression in which the first term is 3 and the common difference is 5.

**Sol:** Arithmetic mean of the terms of an A.P.

$$= \frac{\frac{n}{2}[2a + (n-1)d]}{n} = \frac{2a + (n-1)d}{2}$$

$$\therefore \text{Arithmetic mean} = \frac{2(3) + (30)(5)}{2} = 78$$

**1.18.** Divide 100 into four parts which are in arithmetic progression such that the product of the second part and third part is 72 more than the product of the other two parts.

**Sol:** Let the four parts be $a-3d$, $a-d$, $a+d$ and $a+3d$
Given, $a-3d+a-d+a+d+a+3d = 100$
$\therefore a = 25$
Given, $(a-d)\ (a+d) = (a-3d)\ (a+3d) + 72$
$a^2 - d^2 = a^2 - 9d^2 + 72$
$d = \pm 3$
If $d = 3$, the numbers are 16, 22, 28 and 34. If $d = -3$, the numbers are same but in the descending order.

**1.19.** Three terms in arithmetic progression have a sum of 45 and a product of 3240. Find them.

**Sol:** Let the terms be $a-d$, $a$ and $a+d$.
$a-d+a+a+d = 45$
$a = 15$
$(a-d)\ a\ (a+d) = 3240$
$15^2 - d^2 = 216$
$d = \pm 3$
If $d = 3$, the terms are 12, 15 and 18. If $d = -3$, the terms are same but in the descending order.

**1.20.** The first term and the last term of an arithmetic progression are 9 and 69, respectively. If the sum of all the terms is 468, find the number of terms and the common difference.

**Sol:** Let the number of terms and the common difference be $n$ and $d$, respectively,

$$S_n = \frac{n}{2}[9 + 69] = 468 \Rightarrow 39n = 468$$

$n = 12$
$t_n = 9 + 11d \Rightarrow 11d = 60$

$$\Rightarrow d = \frac{60}{11}$$

**1.21.** The 17th term and the 28th term of an arithmetic progression are 66 and 110, respectively. Find the 33rd term.

**Sol:** Let the first term and the common difference be $a$ and $d$, respectively.
$a + 16d = 66$ (1)
$a + 27d = 110$ (2)
Subtracting (1) from (2), we get
$11d = 44$
$d = 4$
Substituting $d = 4$ in (1) or (2), we get
$a = 2$
33rd term = 2 + 32(4) = 130.

**1.22.** The sum of three numbers which are in arithmetic progression is 24. The sum of their squares is 200. Find the numbers.

**Sol:** Let the numbers be $a-d$, $a$ and $a+d$.
Given, $a-d+a+a+d = 24$
$\therefore a = 8$
$(a-d)^2 + a^2 + (a+d)^2 = 200$
$3a^2 + 2d^2 = 200$
$\Rightarrow d^2 = 4$
$\therefore d = \pm 2$
If $d = 2$, the numbers are 6, 8 and 10. If $d = -2$, the numbers are same, but in the descending order.

**1.23.** If the 7th term of a G.P., having all positive terms, is (729/1024) and the first term is 4, find the common ratio.

**Sol:** The $n$th term of a geometric progression is $ar^{n-1}$ where $a$ is the first term, $r$ is the common ratio and $n$ is the number of terms.

$$\Rightarrow 4r^6 = \frac{729}{1024}$$

$$\Rightarrow r^6 = \frac{729}{4096}$$

$\Rightarrow r^6 = \left(\frac{3}{4}\right)^6$

$\Rightarrow r = \frac{3}{4}$.

(positive value only is considered because all the terms are positive)

Hence, the common ratio is 3/4.

**1.24.** The sum of the first five terms of a G.P. is 363. If the common ratio is 1/3 find the first term.

**Sol:** Sum of the first $n$ terms of a G.P. $= \frac{a(1-r^n)}{1-r}$, where $a$ is the first term, $r$ is the common ratio and $n$ is the number of terms.

$$363 = \frac{a\left(1-\left(\frac{1}{3}\right)^5\right)}{1-\frac{1}{3}}$$

$$a = \frac{363\times\left(1-\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^5} = 243.$$

**1.25.** Find the sum of the terms of a G.P. if the first term is 4 and the last term is (1/64) and the common ratio is (1/2).

**Sol:** The sum of the terms of a G.P. is

$$\frac{r\times\text{last term}-\text{first term}}{r-1}$$

Since the last term is less than the first term, for the sake of convenience, we rewrite the above expression as

$$\frac{\text{first term}-r\times\text{last term}}{1-r} = \frac{4-\frac{1}{2}\left(\frac{1}{64}\right)}{1-\frac{1}{2}}$$

$$= \frac{\frac{511}{128}}{\frac{1}{2}} = \frac{511}{64}.$$

**1.26.** Find the last term of a G.P. whose first term is 9 and common ratio is (1/3) if the sum of the terms of the G.P. is (40/3).

**Sol:** Sum of the G.P. $= \frac{\text{first term} - r\times\text{last term}}{1-r}$

$$\frac{40}{3} = \frac{9-\frac{1}{3}(\text{last term})}{\frac{2}{3}}$$

$$\Rightarrow \text{last term} = \left(\frac{-40}{3}\times\frac{2}{3}+9\right)\times 3$$

$$= \frac{-80}{3}+27 = \frac{1}{3}$$

**1.27.** Find three numbers in G.P. having a sum of 21 and a product of 216.

**Sol:** Let the three numbers be $\frac{a}{r}$, $a$ and $ar$.

Given that $\frac{a}{r}\times a\times ar = 216$

$\Rightarrow a^3 = 216 \Rightarrow a = 6$

Sum of the numbers $= \left(\frac{a}{r}\right) + a + ar = 21$

$a + ar + ar^2 = 21r$

but $a = 6$

$6 + 6r + 6r^2 = 21r$

$6r^2 - 15r + 6 = 0$

$2r^2 - 5r + 2 = 0$

$\Rightarrow (2r-1)(r-2) = 0$

$\Rightarrow r = (1/2)$ or 2

Hence, the numbers are 3, 6 and 12 when $r = 2$ and the same numbers in reverse order when $r = (1/2)$.

**1.28.** The sum to infinity of a G.P. is 27 and the sum of the squares of the terms is 243. Find the common ratio.

**Sol:** Let the first term be $a$ and the common ratio be $r$. Given that,

$$\frac{a}{1-r} = 27 \qquad (1)$$

and

$$\frac{a^2}{1-r^2} = 243 \qquad (2)$$

Dividing (2) by (1), we get

$$\frac{a}{1+r} = 9 \qquad (3)$$

Dividing (3) by (1), we get

$$\frac{1-r}{1+r} = \frac{1}{3}$$

$\Rightarrow 3 - 3r = 1 + r$

$\Rightarrow 4r = 2 \Rightarrow r = \frac{1}{2}$

**1.29.** Given that $|y| < 1$, find the value of $3 + 6y + 9y^2 + 12y^3 + \ldots$

**Sol:** Let $S = 3 + 6y + 9y^2 + 12y^3 + \ldots$

Multiplying the equation by $y$, we get $yS = 3y + 6y^2 + 9y^3 + 12y^4 + \ldots$

Subtracting second equation from the first,

$S(1 - y) = 3 + 3y + 3y^2 + \ldots \infty$

$$S = \frac{(3 + 3y + 3y^2 + \ldots \infty)}{1 - y}$$

Again $3 + 3y + 3y^2 \ldots \infty$ is a geometric progression of infinite terms

$$\therefore S = \frac{\frac{3}{1 - y}}{1 - y} = \frac{3}{(1 - y)^2}$$

**1.30.** Find the sum of the terms of the series

$$1, \frac{4}{5}, \frac{16}{25}, \frac{64}{125}, \ldots \infty$$

**Sol:** It can be noticed that that the given series is a G.P. with infinite terms. The common ratio is (4/5) and this is < 1.

The sum to infinity of a series with first term $a$ and the common ratio

$r = [a / (1 - r)]$

As $a = 1$ and $r = 4/5$,

the sum to infinity $= \frac{1}{1 - (4/5)} = 5$

**1.31.** Find the sum of the following series, $2\sqrt{2}, \frac{4}{\sqrt{3}}, \frac{4\sqrt{2}}{3}, \ldots \infty$

**Sol:** The ratio of any term to its previous term starting from the second term is $\sqrt{\frac{2}{3}}$. Hence, the series is in G.P. The sum to infinity of the series $= \frac{2\sqrt{2}}{1 - \sqrt{\frac{2}{3}}}$

$$= \frac{2\sqrt{6}}{\sqrt{3} - \sqrt{2}}$$

$$= \frac{2\sqrt{6}\left(\sqrt{3} + \sqrt{2}\right)}{\left(\sqrt{3} - \sqrt{2}\right)\left(\sqrt{3} + \sqrt{2}\right)}$$

(multiplying both numerator and denominator by $\sqrt{3} + \sqrt{2}$ )

$$= \frac{2\sqrt{6}\left(\sqrt{3} + \sqrt{2}\right)}{3 - 2} = 2\sqrt{6}\left(\sqrt{3} + \sqrt{2}\right)$$

**1.32.** If $(2^3 - t_1) + (4^3 - t_2) + (6^3 - t_3) + (8^3 - t_4) + (10^3 - t_5) + \ldots \{(2n)^3 - t_n)\}$

$= \frac{n^2 (n + 1)^2}{4}$, find the general expression for $t_n$.

**Sol:** $2^3 + 4^3 + 6^3 + \ldots (2n)^3$

$= 8\,(1^3 + 2^3 + 3^3 + \ldots + n^3)$

$$= 8\left(\frac{n(n+1)}{2}\right)^2 = \frac{8n^2 (n+1)^2}{4}$$

$= 2n^2 (n + 1)^2$ (1)

Substituting (1) in the given equation and transposing the terms, the equation becomes

$t_1 + t_2 + t_3 + t_4 + \ldots + t_n$

$$= 2n^2 (n + 1)^2 - \frac{n^2 (n + 1)^2}{4}$$

$$= \frac{7}{4} n^2 (n+1)^2 \quad (2)$$

If this sum is denoted by $S_n$, then

$$S_{n-1} = \frac{7}{4}(n-1)^2 (n)^2 \quad (3)$$

But, $t_n = S_n - S_{n-1}$

$$= \frac{7}{4} n^2 (n+1)^2 - \frac{7}{4}(n-1)^2 n^2$$

$$= \frac{7}{4} n^2 \left[(n+1)^2 - (n-1)^2\right]$$

$$= \frac{7}{4} n^2 (4n)$$

$= 7n^3$

**1.33.** Find the 7th term of the geometric progression whose first term is 6 and common ratio is 2.

**Sol:** $n^{\text{th}}$ term of a G.P. $= ar^{n-1}$

$7^{\text{th}}$ term $= 6(2^6) = 384$

**1.34.** A geometric progression has its first term as 64 and its common ratio as $\frac{1}{2}$. Find the sum of its first five terms.

**Sol:** Sum of the first $n$ terms of a G.P. $= \frac{a(1-r^n)}{1-r}$

Sum of its first five terms $= \frac{64\left(1-\left(\frac{1}{2}\right)^5\right)}{1-\frac{1}{2}} = 124$

**1.35.** Find the common ratio of the geometric progression whose first and last terms are 5 and $\frac{1}{25}$, respectively, and the sum of its terms is $\frac{624}{100}$.

**Sol:** Sum of the terms of a geometric progression whose common ratio is $r$, is given by

$$\frac{r(\text{last term}) - (\text{first term})}{r-1}$$

$$\frac{r\left(\frac{1}{25}\right)-5}{r-1} = \frac{624}{100}$$

$$4r - 500 = 624r - 624$$

$$\Rightarrow r = \frac{1}{5}.$$

**1.36.** In the previous example, find the number of terms.

**Sol:** Let the number of terms be $n$.

$$\frac{1}{25} = 5\left(\frac{1}{5}\right)^{n-1}$$

$$\left(\frac{1}{5}\right)^3 = \left(\frac{1}{5}\right)^{n-1}$$

Comparing both sides, $n - 1 = 3$

$\therefore n = 4$

**1.37.** Three numbers in geometric progression have a sum of 42 and a product of 512. Find the numbers.

**Sol:** Let the numbers be $\frac{a}{r}$, $a$ and $ar$.

$$\frac{a}{r} + a + ar = 42$$

$$\left(\frac{a}{r}\right)(a)(ar) = 512$$

$$a = 8$$

$$\frac{8}{r} + 8 + 8r = 42$$

$$8r^2 - 34r + 8 = 0$$

$$8r^2 - 32r - 2r + 8 = 0$$

$$(r-4)(4r-1) = 0$$

$$r = 4 \text{ or } \frac{1}{4}$$

If $r = 4$, the numbers are 2, 8 and 32. If $r = \frac{1}{4}$, the numbers are same, but in the descending order.

**1.38.** The sum of the terms of an infinite geometric progression is 27. The sum of their squares is 364.5. Find the common ratio.

**Sol:** Let the first term and the common ratio be $a$ and $r$, respectively.

Given that

$$\frac{a}{1-r} = 27 \Rightarrow \left(\frac{a}{1-r}\right)^2 = 729$$

and $\frac{a^2}{1-r^2} = 364.5$

$$\Rightarrow a^2 = 729\ (1-r)^2 = 364.5\ (1-r^2)$$

$$729\ (1-r)^2 - \frac{729}{2}(1-r)(1+r) = 0$$

$$\frac{729}{2}(1-r)[2(1-r)-(1+r)] = 0$$

$$\Rightarrow (1-r)\ (1-3r) = 0$$

$$r \neq 1\ (\because |r| < 1)$$

$$\therefore\ r = \frac{1}{3}$$

**1.39.** If $|x| < 1$, find the value of $3 + 6x + 9x^2 + 12x^3 + \ldots$

**Sol:** Let $S = 3 + 6x + 9x^2 + 12x^3 + \ldots$ (1)

$xS = 3x + 6x^2 + 9x^3 + \ldots$ (2)

Subtracting (2) from (1), we get

$$S(1-x) = 3\ (1 + x + x^2 + \ldots)$$

As $|x| < 1$, $S = \frac{3\left(\frac{1}{1-x}\right)}{1-x} = \frac{3}{(1-x)^2}$

**1.40.** Find the sum of the series $1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \ldots \infty$

**Sol:** The series is an infinite geometric progression with first term as 1 and common ratio as $\frac{2}{3}$.

Sum to infinity of an infinite geometric progression $= \frac{a}{1-r}$

$\therefore$ Sum of the series $= \frac{1}{1-\frac{2}{3}} = 3$

**1.41.** If $(1^3 + t_1) + (2^3 + t_2) + (3^3 + t_3) + (4^3 + t_4) + \ldots + (n^3 + t_n) = \dfrac{n^2(n+6)}{4}$, find $t_{10}$.

**Sol:** $(1^3 + 2^3 + 3^3 + \ldots n^3) + (t_1 + t_2 + \ldots t_n)$

$= \dfrac{n^2(n+6)}{4}$

$\Rightarrow t_1 + t_2 + \ldots t_n$

$= \dfrac{n^2(n+6)}{4} - (1^3 + 2^3 + 3^3 + \ldots n^3)$

$= \dfrac{n^2(n+6)}{4} - \dfrac{1}{4}(n(n+1)^2) = \dfrac{n^2[-n^2 - n + 5]}{4}$

Let $S_n = t_1 + t_2 + \ldots t_n$

$t_{10} = S_{10} - S_9$

$S_{10} = \dfrac{10^2[-100 - 10 + 5]}{4} = -2625$

$S_9 = \dfrac{81[-81 - 9 + 5]}{4} = \dfrac{-6885}{4}$

$\therefore\ t_{10} = -2625 - \left(\dfrac{-6885}{4}\right) = \dfrac{-3615}{4}$

**1.42.** In a certain sequence, $t_1 = 5$ and $t_{n+1} = 2t_n - 3n + 3$. Find $t_{100}$

**Sol:** To get any term, we need to multiply the preceding one by 2.

$\therefore$ We should try to relate the $n$th term $t_n$ to $2^n$

$t_1 = 5 = 2 + 3$

$t_2 = 2(5) - 3(1) + 3 = 10 = 2^2 + 3(2)$

$t_3 = 2(10) - 3(2) + 3 = 17 = 2^3 + 3(3)$

$t_4 = 2(17) - 3(3) + 3 = 28 = 2^4 + 3(4)$

We can guess that $t_n = 2^n + 3n$.

$\therefore\ t_{100} = 2^{100} + 3(100)$

We shall consider this general problem in reverse, i.e., we start with the answer ($t_n = r^n + bn$) and obtain the question (the way $t_{n+1}$ is related to $t_n$)

Let $t_n = ar^n + bn$ (1)

$\Rightarrow rt_n = ar^{n+1} + brn$

$\therefore\ t_{n+1} = a \,.\, r^{n+1} + bn + b = rt_n - brn + bn + b$

$= rt_n + b(1 - r)n + b$ (2)

Let us compare this with the relation given in the example above

$t_{n+1} = 2t_n - 3n + 3$

i.e., $r = 2$, $b = 3$ and $b(1 - r) = 3(1 - 2) = -3$

We see that the relation between $t_{n+1}$ and $t_n$ is of the kind (2) where $r = 2$, $b = 3$

$\therefore\ t_n = 2^n + 3n$

In general, if a relation of the type $t_{n+1} = rt_n + cn + b$ is given, where $c = b(1 - a)$, then $t_n$ is given by (1).

**1.43.** Find the value of the expression

$$\frac{1(2)(3)+2(3)(4)+3(4)(5)+\ldots+8(9)(10)}{1^2(2)+2^2(3)+3^2(4)+\ldots+8^2(9)}$$

**Sol:** $\dfrac{1(2)(3)+2(3)(4)+3(4)(5)+\ldots+8(9)(10)}{1^2(2)+2^2(3)+3^2(4)+\ldots+8^2(9)}$

$$= \frac{\sum_{n=1}^{8} n(n+1)(n+2)}{\sum_{n=1}^{8} n^2 \cdot (n+1)}$$

$$= \frac{\sum_{n=1}^{8} n^3 + 3\sum_{n=1}^{8} n^2 + 2\sum_{n=1}^{8} n}{\sum_{n=1}^{8} n^3 + \sum_{n=1}^{8} n^2}$$

$$= \frac{36^2 + \frac{3(8)(9)(17)}{6} + \frac{2(8)(9)}{2}}{36^2 + \frac{8(9)(17)}{6}} = \frac{33}{25}$$

**1.44.** If $A = \dfrac{1}{35(68)} + \dfrac{1}{36(67)} + \dfrac{1}{37(66)} + \ldots + \dfrac{1}{68(35)}$ and $B = 1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \ldots + \dfrac{1}{67} - \dfrac{1}{68}$ then the value of $\dfrac{A}{B}$ is

**Sol:** $A = \dfrac{1}{35(68)} + \dfrac{1}{36(67)} + \ldots + \dfrac{1}{68(35)}$

$= \dfrac{1}{103}\left[\dfrac{103}{35(68)} + \dfrac{103}{36(67)} + \ldots + \dfrac{103}{68(35)}\right]$

$= \dfrac{1}{103}\left[\dfrac{1}{35} + \dfrac{1}{68} + \dfrac{1}{36} + \dfrac{1}{67} + \ldots + \dfrac{1}{68} + \dfrac{1}{35}\right]$

$= \dfrac{2}{103}\left[\dfrac{1}{35} + \dfrac{1}{36} + \ldots + \dfrac{1}{68}\right]$

$B = 1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \ldots + \dfrac{1}{67} - \dfrac{1}{68}$

$$=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{67}+\frac{1}{68}-2\left[\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\ldots\frac{1}{68}\right]$$

$$=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{67}+\frac{1}{68}-\left[1+\frac{1}{2}+\frac{1}{3}+\ldots\frac{1}{34}\right]$$

$$=\frac{1}{35}+\frac{1}{36}+\frac{1}{37}+\ldots+\frac{1}{68}$$

$$\frac{A}{B}=\frac{2}{103}$$

**1.45.** Find the sum of the terms

$$\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\ldots+\sqrt{1+\frac{1}{14^2}+\frac{1}{15^2}}$$

**Sol:** $1+\frac{1}{n^2}+\frac{1}{(n+1)^2}=\frac{(n+1)^2n^2+n^2+(n+1)^2}{n^2(n+1)^2}$

$$=\frac{n^4+2n^3+3n^2+2n+1}{[n(n+1)]^2}=\frac{(n^2+n+1)^2}{[n(n+1)]^2}$$

$$\therefore\sqrt{1+\frac{1}{n^2}+\frac{1}{(n+1)^2}}=\frac{n^2+n+1}{n(n+1)}=1+\frac{1}{n(n+1)}$$

$$=1+\frac{1}{n}-\frac{1}{n+1}$$

$$\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\ldots\sqrt{1+\frac{1}{14^2}+\frac{1}{15^2}}$$

$$=1+\frac{1}{1}-\frac{1}{2}+1+\frac{1}{2}-\frac{1}{3}+\ldots+1+\frac{1}{14}-\frac{1}{15}$$

$$=14+1-\frac{1}{15}=14\frac{14}{15}$$

**1.46.** In a certain series, for $n\geq 2$, $T_n$ equals $5T_{n-1}-3$. If $T_1=5$, then $T_{100}=$

**Sol:** It is given

$$T_n=5T_{n-1}-3$$
$$=5(5T_{n-2}-3)-3$$
$$=5^2T_{n-2}-5(3)-3$$
$$=5^2(5T_{n-3}-3)-5(3)-3$$
$$=5^3\ T_{n-3}-3(5^2)-3(5)-3(1)$$

So, we can get

$$T_n=5^{n-1}T_1-3(5^{n-2})-3(5^{n-1})-\ldots\ 3(1)$$
$$=5^n-3(1+5^1+\ldots+5^{n-2})$$
$$=5^n-3\frac{\left(5^{n-1}-1\right)}{5-1}$$
$$=\frac{4(5^n)-3(5^{n-1})+3}{4}=\frac{17(5^{n-1})+3}{4}$$

$$T_{100}=\frac{17(5^{99})+3}{4}$$

**1.47.** Evaluate $\frac{1}{1(2)}+\frac{1}{2(3)}+\frac{1}{3(4)}+\ldots+\frac{1}{99(100)}$.

**Sol:** $\frac{1}{1(2)}=\frac{1}{1}-\frac{1}{2}$

$$\frac{1}{2(3)}=\frac{1}{2}-\frac{1}{3}$$

Finally $\frac{1}{99(100)}=\frac{1}{99}-\frac{1}{100}$

The given expression is $1-\frac{1}{100}=\frac{99}{100}$

**1.48.** Find the sum of

$$\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}},\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}},\ldots,\sqrt{1+\frac{1}{20^2}+\frac{1}{21^2}}$$

**Sol:** The general term is given by

$$t_{n^2}=1+\frac{1}{n^2}+\frac{1}{(n+1)^2}$$
$$=\frac{n^2(n+1)^2+(n+1)^2+n^2}{n^2(n+1)^2}$$
$$=\frac{[n(n+1)]^2+2n(n+1)+1}{[n(n+1)]^2}=\frac{(n^2+n+1)^2}{n^2(n+1)^2}$$

$$\therefore\ t_n=\frac{n^2+n+1}{n(n+1)}=1+\frac{1}{n(n+1)}$$

$$t_1=1+\frac{1}{1(2)}$$

$$t_2=1+\frac{1}{2(3)}$$

And finally, $t_{20}=1+\frac{1}{20(21)}$

$$\therefore\ S=20+1-\frac{1}{21}=20\frac{20}{21}$$

## EXERCISE-1

***Directions for questions 1–45:*** For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple-choice questions, write your answer in the space provided.

**1.** What is the sum to 7 terms of the arithmetic progression in which the first term is 2 and the common difference is 4?
(A) 49 (B) 98
(C) 126 (D) 77

**2.** What is the arithmetic mean of the arithmetic progression 6, 8, 10, 12, 14, 16?
(A) 22 (B) 11
(C) 24 (D) 12

**3.** What is the fourth term of the arithmetic progression in which the first term is 4 and the seventh term is 28?
(A) 16 (B) 8
(C) 12 (D) 2

**4.** What is the arithmetic mean of an arithmetic progression with 13 terms, in which the seventh term is 9?
(A) 9 (B) $\frac{91}{7}$
(C) $\frac{95}{7}$ (D) Cannot be determined

**5.** What is the sum to 8 terms of an arithmetic progression in which the first term is 3 and the last term is 31?
(A) 136 (B) 58.5
(C) 132 (D) Cannot be determined

**6.** Find the eighth term of an arithmetic progression whose first term is 7 and the common difference is 5.
(A) 24 (B) 47
(C) 42 (D) Cannot be determined

**7.** Find the sum of the first 9 terms of an arithmetic progression whose first term is 5 and the last term is 33.
(A) 85.5 (B) 171
(C) 342 (D) Cannot be determined

**8.** Which term of the series 2, 5, 8, 11, … is 266?
(A) 88th (B) 89th
(C) 90th (D) 91st

**9.** Find the fourth term of the arithmetic progression whose first term is 6 and the seventh term is 24.
(A) 15 (B) 7.5
(C) 9 (D) Cannot be determined

**10.** Find the sum of the first 25 terms of an arithmetic progression whose 13th term is 20.
(A) 250 (B) 400
(C) 500 (D) Cannot be determined

**11.** In an arithmetic progression having 100 terms, the $m$th term from the beginning and the $m$th term from the end are 10 and 20, respectively. What is the sum of all the terms?
(A) 3000 (B) 1500
(C) 3200 (D) Cannot be determined

**12.** What is the sum to 15 terms of an arithmetic progression whose 8th term is 4?
(A) 30 (B) 60
(C) 40 (D) Cannot be determined

**13.** What is the seventh term of an arithmetic progression whose first term is 9 and the common difference is 3?
(A) 27 (B) 36
(C) 33 (D) 30

**14.** What is the sum of all the terms in an arithmetic progression in which the first term is 5, the last term is 15 and the number of terms is 11?
(A) 55 (B) 110
(C) 115 (D) Cannot be determined

**15.** If sum of first 51 terms of an arithmetic progression is zero, then which of the following terms is zero?
(A) 13th (B) 26th
(C) 17th (D) Cannot be determined

**16.** Find the geometric mean of 4, 8, 16 and 32.
(A) 128 (B) $\sqrt{128}$
(C) 15 (D) 16

**17.** Find the fourth term of a geometric progression whose second term is 8 and sixth term is 32.
(A) 16 (B) –16
(C) Either (A) or (B) (D) Neither (A) or (B)

**18.** If $a$, $b$ and $c$ are distinct positive numbers in geometric progression, log $a$, log $b$ and log $c$ will be in
(A) arithmetic progression
(B) geometric progression
(C) harmonic progression
(D) None of these

**19.** Find the sum of the first four terms of a geometric progression whose first term is 4 and the common ratio is 3.
(A) 320 (B) 160
(C) 80 (D) Cannot be determined

**20.** Two distinct positive numbers have their geometric mean equal to 9. Their arithmetic mean will be
(A) $< 9$ (B) $= 9$
(C) $> 9$ (D) Cannot be determined

**21.** What is the geometric mean of the geometric progression 2, 4, 8, 16?
(A) 32 (B) $\sqrt{32}$
(C) 64 (D) 8

**22.** What is the seventh term of a geometric progression whose first term is 3 and the common ratio is 2?

(A) 96 (B) 384
(C) 192 (D) 288

**23.** What is the sum to 4 terms of a geometric progression whose first term is 6 and the common ratio is 3?
(A) 300 (B) 360
(C) 270 (D) 240

**24.** If the geometric mean of two distinct positive numbers is 4, then the arithmetic mean of these two numbers is
(A) $< 4$ (B) $= 4$
(C) $> 4$ (D) Cannot be determined

**25.** What is the fourth term of a geometric progression in which the second term is 4 and the sixth term is 64?
(A) 8 (B) 32
(C) 64 (D) 16

**26.** The sum of first 20 terms in an arithmetic progression is 210. Find the sum of the 10th and the 11th terms.
(A) 21 (B) 10.5
(C) 42 (D) Cannot be determined.

**27.** Find the sum of the cubes of the first ten natural numbers _____.

**28.** Find the sum to infinity of 1, 1/4, 1/16,…
(A) $\frac{3}{4}$ (B) $\frac{5}{4}$
(C) $\frac{4}{3}$ (D) $\frac{4}{5}$

**29.** Find the sum of the first 7 terms of a geometric progression whose first term is 1 and the fourth term is 8 _____.

**30.** The 30th term from the beginning in a 100-term series is the _____ term from the end.
(A) 69th (B) 70th
(C) 71st (D) 72nd

**31.** What is the geometric mean of the geometric progression having a total of 13 terms, given the 7th term is 4?
(A) 2 (B) 4
(C) 16 (D) Cannot be determined

**32.** What is the sum of the cubes of first 9 natural numbers?
(A) 45 (B) 2025
(C) 91125 (D) Cannot be determined

**33.** A geometric progression has its seventh term equal to 2. Find the product of its first 13 terms.
(A) 4 (B) 16
(C) 2048 (D) 8192

**34.** If $m$, $n$ and $p$ are in arithmetic progression, then the $m$th term, $n$th term and $p$th term of a geometric progression are in
(A) arithmetic progression
(B) geometric progression
(C) harmonic progression
(D) not necessarily in arithmetic progression or geometric progression.

**35.** What is the product of the first 9 terms of a geometric progression having a total of 13 terms given that the 5th term is 2?
(A) 512 (B) 32
(C) 16 (D) Cannot be determined

**36.** The sum of the first 30 terms of an arithmetic progression is 40. The sum of its first 80 terms is also 40. Find the sum of its 31st and 80th terms _____.

**37.** An arithmetic progression has 200 terms. The $p$th term from the beginning is 15. The $p$th term from the end is 45. Find the sum of all the terms.
(A) 12000 (B) 6000
(C) 9000 (D) Cannot be determined

**38.** Find the sum of the first seven terms of an arithmetic progression whose first term is one and the common difference is 3 _____.

**39.** If $p$, $q$ and $r$ are in arithmetic progression, then the $p$th term, $q$th term and the $r$th term of a geometric progression are in
(A) arithmetic progression
(B) geometric progression
(C) harmonic progression
(D) Both (A) and (B)

**40.** If $p$, $q$ and $r$ are in arithmetic progression, the $p$th term, $q$th term and the $r$th term of an arithmetic progression are in
(A) arithmetic progression
(B) geometric progression
(C) harmonic progression
(D) None of these

**41.** If the sum of first 20 terms of an arithmetic progression is 30 and the sum of first 50 terms is also 30, then what is the sum of 21st term and the 50th term?
(A) 0 (B) 30
(C) 15 (D) Cannot be determined

**42.** What is the sum of infinite geometric series 1, 1/2, 1/4, 1/8, 1/16, … ?
(A) $1\frac{255}{256}$ (B) 2
(C) 3 (D) 4

**43.** Find the eighth term of a geometric progression whose first term as well as common ratio is 2 _____.

**44.** What is the sum to 7 terms of a geometric progression whose first term is 1 and the 4th term is 27?
(A) 1093 (B) 2186
(C) 3279 (D) 4372

**45.** Find the arithmetic mean of an arithmetic progression with 17 terms whose ninth term is 11.

(A) 22 (B) 11
(C) 33 (D) Cannot be determined

## EXERCISE-2

***Directions for questions 1–45:*** For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple-choice questions, write your answer in the space provided.

**1.** In an A.P., the first term is 7 and the common difference is 2. Find the fifteenth term _____.

**2.** Find the sum up to 25 terms of the A.P. whose first term is $\frac{3}{4}$ and 25th term is $\frac{77}{4}$ _____.

**3.** The sum of the first three terms of an A.P. is equal to the sum of the first four terms of the same A.P. What is the sum of the first seven terms? _____

**4.** Find the sum up to 50 terms of an A.P. whose first term is 5 and common difference is 2 _____.

**5.** If the fifth term of an A.P. is 10, what is the sum of the first nine terms? _____

**6.** In an arithmetic progression, the fifth term is 11 and its 15th term is 31, the 25th term of the arithmetic progression is _____.

**7.** The 41st term of the series $8, 10\frac{1}{2}, 13, 15\frac{1}{2}, \ldots$ is_____.

**8.** The $n$th term of the arithmetic progression 11, 16, 21, 26, … is 66. then $n =$ _____.

**9.** The sum of the first $n$ terms of the arithmetic progression 21, 28, 35, 42, … is 175. Find $n$.
(A) 7 (B) 8
(C) 5 (D) 9

**10.** The sum of the terms of the series 49, 42, 35, …7 is _____.

**11.** The sum of the first three terms of a G.P. is 7 and sum of their squares is 21. The 4th, 5th and 6th terms of this series are
(I) 8, 16, 32 (II) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$
(III) 4, 2, 1
(A) either (I) or (III) (B) only (II)
(C) either (I) or (II) (D) only (I)

**12.** If $T_N = (N-1)\ T_{N-1}$ where $N \geq 2$ and $T_1 = 2000!$, and $T_{2001} = b$, find $\sum_{i=2}^{2000} \log_b i$
(A) 2 (B) $\frac{1}{2}$
(C) $\frac{1}{4}$ (D) None of these

**13.** If the sum of the first $p$, first $q$ and first $r$ terms of an A.P. are $a$, $b$ and $c$, respectively, then
(A) $\frac{c}{p}(q-r)+\frac{a}{q}(r-p)+\frac{b}{r}(p-q)=0$
(B) $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$
(C) $\frac{a}{p}(r-p)+\frac{b}{q}(q-r)+\frac{c}{r}(p-q)=0$
(D) None of these

**14.** The sum of the first eight terms of a geometric series is 10001 times the sum of its first four terms. Find the common ratio of the series.
(A) 10 (B) –10
(C) Either (A) or (B) (D) Neither (A) nor (B)

**15.** Find the first term of a geometric series whose 7th term is 448 and 11th term is 7168 (common ratio > 0)_____.

**16.** The sum of four numbers in an arithmetic progression is 56. The difference between the product of the two numbers in the middle and that of the two numbers in the extreme ends is 8. Find the smallest of the four numbers _____.

**17.** The product of the first three terms of a geometric progression is 216 and its 5th term is 162. Find the first term of the geometric progression _____.

**18.** Find the 35th term of an arithmetic progression whose first and the last terms are, respectively, 4 and 241 and the sum of all the terms is 9800.
(A) 101 (B) 104
(C) 106 (D) 112

**19.** The 69th term of an arithmetic progression is 16 times the fourth term of that arithmetic progression. Find the 20th term of the arithmetic progression, if its 7th term is 22 _____.

**20.** The sum of three positive integers is 84 and their product is 13,824. If the three numbers are in geometric progression, then find the largest of the three numbers.
(A) 36 (B) 48
(C) 60 (D) 72

**21.** In an arithmetic progression, the third term is 28 and the sum of the first 21 terms is 1932. Find the sum of the first 11 terms.
(A) 572 (B) 580
(C) 593 (D) 600

Difficulty Level-2: Moderate

**22.** The maximum value of the sum of the series 50, 46, 42, … is
(A) 356 (B) 492
(C) 368 (D) 338

**23.** The ratio of the sum of the first $n$ terms of two arithmetic progressions is given by $5n+4 : 8n-15$. Find the ratio of the 12th terms of the two arithmetic progressions.
(A) 125 : 161 (B) 119 : 169
(C) 123 : 166 (D) 115 : 169

**24.** Four times the 91st term of an arithmetic progression is equal to five times its 81st term. If the 31st term of the progression is 630, find its 51st term.
(A) 810 (B) 729
(C) 630 (D) 567

**25.** The sum of the first, fifth, sixth, eighth, ninth and thirteenth terms of an arithmetic progression is 243. Find the sum of the first 13 terms of the arithmetic progression.
(A) 1134 (B) 1296
(C) 1458 (D) 526.5

**26.** If $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\infty = s$,
find $\frac{1}{1^2}-\frac{1}{2^2}-\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}-\frac{1}{6^2}\ldots\infty$.
(A) $2s$ (B) $s/2$
(C) $s^2$ (D) $s^2/2$

**27.** Find the value of $8+88+888+\ldots n$ terms.
(A) $\frac{10(10^n-1)}{9}-n$ (B) $\frac{8}{9}\frac{10(10^n-1)}{9}+1$
(C) $\frac{8}{9}\frac{10(10^n-1)}{9}-n$ (D) $\frac{10}{9}\frac{8(8^n-1)}{9}-n$

**28.** Find the value of $1+2x+3x^2+4x^3+\ldots\infty$ where $x=\frac{1}{2}$.
_____.

**29.** Find the sum of $(1+2+3+\ldots n)+(2+3+\ldots n)+(3+4+\ldots n)+\ldots+(n)$ _____.

**30.** Find the value of $1+3x+6x^2+10x^3+15x^4+\ldots\infty$ where $x=\frac{2}{3}$ _____.

**31.** Which term of the sequence $-3/5, -13/30, -8/30, \ldots$ is $37/30$? _____.

**32.** Find the sum to $n$ terms of the series $8+88+888+\ldots$
(A) $\frac{80}{81}(10^n-1)-n$ (B) $\frac{80}{81}(10^n-1)-\frac{8n}{9}$
(C) $\frac{80}{81}(10^n-1)-\frac{80n}{81}$ (D) $\frac{80}{81}(10^n-1)-\frac{8n}{81}$

**33.** A man employed in a company is promised a salary of ₹3000 every month for the first year and an increment of ₹1000 in his monthly salary every succeeding year. How much does the man earn from the company in 20 years? _____

**34.** In an A.P. consisting of even number of terms, the last term is less than the first by 76. The sum of the even terms is −250 and this is less than the sum of the odd terms by 40. What is the number of terms of the progression?
(A) 25 (B) 50
(C) 10 (D) 20

**35.** Find the value of $n$, if the sum to $n$ terms of a geometric series is $4488\frac{3}{4}$. The first term and the common ratio of the series are 216 and 3/2, respectively_____.

**36.** A group of 810 children arranged themselves in $N$ rows for a group photograph. In each row, there were three more children than the row behind. Which of the following can be the value of $N$?
(A) 6 (B) 8
(C) 5 (D) 10

**37.** Find the sum up to 25 terms of the series $1/2+3/4+7/8+15/16+\ldots$
(A) $\frac{25.2^{25}-1}{2^{25}}$ (B) $\frac{24.2^{25}+1}{2^{25}}$
(C) $\frac{25.225+1}{2^{25}}$ (D) $\frac{2.2^{24}-1}{2^{25}}$

**38.** Find the value of $(100\times1)+(99\times2)+(98\times3)+\ldots+(2\times99)+(1\times100)$
(A) 12850 (B) 171700
(C) 23250 (D) 50500

**39.** Find the number of terms which are common to the progressions 2, 5, 8, 11, … 434 and 3, 7, 11, 15, … 579.
_____.

**40.** Find the sum of the series $p(p+q)+p^2(p^2+q^2)+p^3(p^3+q^3)+\ldots\ldots$up to infinite terms if $-1<p<1$ and $-1<q<1$.
(A) $\frac{2+p^2+q^2}{(1-p^2)(1-q^2)}$ (B) $\frac{p^2-q^2-2}{(1-p^2)(1-q^2)}$
(C) $\frac{p^2-q^2+2}{(1-p^2)(1-q^2)}$ (D) None of these

**41.** Find the sum $\frac{1}{2}+\frac{2^2}{2^2}+\frac{3^2}{2^3}+\frac{4^2}{2^4}+\frac{5^2}{2^5}+\frac{6^2}{2^6}+\ldots$
(A) 4 (B) 5
(C) 6 (D) 7

**42.** Find the sum of the first 20 terms of the following series $1^2(3)+2^2(4)+3^2(5)+4^2(6)+\ldots$
(A) 46970 (B) 49670
(C) 46790 (D) 49840

**43.** Find the value of
$\frac{1}{3^2-2^2}+\frac{1}{7^2-2^2}+\frac{1}{11^2-2^2}+\ldots+\frac{1}{39^2-2^2}$.

(A) 5/41 (B) 31/41
(C) 18/37 (D) 10/41

**44.** If $S_1 = 1(30) + 2(29) + 3(28) + ... + 30(1)$
$S_2 = 1(60) + 2(59) + 3(58) + 4(57) ... + 60(1)$ then $2\frac{S_2}{S_1}$ is

(A) $\frac{31}{8}$ (B) $\frac{31}{4}$
(C) $\frac{61}{8}$ (D) $\frac{61}{4}$

**45.** If $S = \frac{1}{7} + \frac{3}{7^2} + \frac{7}{7^3} + \frac{13}{7^4} + ...$, then $S =$

(A) $\frac{25}{111}$ (B) $\frac{25}{108}$
(C) $\frac{25}{216}$ (D) $\frac{1}{4}$

## EXERCISE-3

***Directions for questions 1–45*:** For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple-choice questions, write your answer in the space provided.

**1.** The third term of an A.P. is 15 and the 23rd term is 55. Find the 40th term _____.

**2.** The sum of 3 terms in an A.P. is 18 and the product of the extreme terms is 20. Find the terms.
(A) 2, 8, 12 (B) −4, −8, −12
(C) 2, 6, 10 (D) −2, −6, −10

**3.** Consider the two sequences, 3, 6, 9, 12, 15, … 102 and 4, 8, 12, 16, … 124. Find the number of terms common to both the sequences _____.

**4.** In a G.P., the first term is 11 and the last term is 5632. The sum of all the terms is 11253. Find the common ratio _____.

**5.** In a G.P., $T_5 = 3$. What is the product of the first nine terms? _____

**6.** The sum of 3 numbers in a G.P. is 13 and their product is 27. Find the numbers.
(A) 2, −6, 17 (B) 1, 3, 9
(C) −1, −4, 18 (D) −18, 13, 18

**7.** In a G.P., the first term is 2 and the common ratio is 3. What is the sum of the first 6 terms? _____

**8.** The fourth term of an A.P. is 24 and the 44th term is 124. Find $t_{24}$. _____.

**9.** In an A.P., the sum of the first six terms is equal to the sum of the first eight terms. What is the sum of the first 14 terms?
(A) 0 (B) 56
(C) 28 (D) 14

**10.** In an A.P., the sum of the first four terms is 6 and the sum of the first six terms is 4. Find the sum of the first 10 terms.
(A) 10 (B) 0
(C) −20 (D) −10

**11.** In a geometric progression, the first three terms are $10a + 36$, $6a + 12$ and $4a$, respectively. Find the sixth term of the geometric progression, if $a > 0$.
(A) 5 (B) 2
(C) 3 (D) 9

**12.** A square $S_1$ has a side of 20 m. By joining the midpoints of its sides, another square $S_2$ is formed. By joining the midpoints of the sides of $S_2$, another square $S_3$ is formed. This process is continued infinitely. Find the sum of the perimeters of all the squares formed.
(A) 36 *m* (B) 45 *m*
(C) $\frac{80\sqrt{2}}{\left(\sqrt{2}+1\right)}m$ (D) $80\sqrt{2}\left(\sqrt{2}+1\right)m$

**13.** If $5x + y$, $3x + 2y$ and $3x + y$ are in arithmetic progression and $5x + 1$, $3(x + 1)$ and $3x$ are in geometric progression, find $(x, y)$.
(A) (4, 3) (B) (5, 2)
(C) $\left(\frac{3}{2}, \frac{3}{2}\right)$ (D) (3, 3)

**14.** A ball dropped from 36 m above the ground rebounds to 1/3 of the height it falls from. If it continues to rebound in this manner, find the total distance the ball can cover.
(A) 96 m (B) 72 m
(C) 54 m (D) 76 m

**15.** Three distinct numbers in geometric progression have a product of 1728. The sum of the products taking two numbers at a time is 456. Find the least number.
(A) 8 (B) 9
(C) 12 (D) 16

**16.** The sum to first 20 terms of an arithmetic progression is 420. If the numbers are arranged in ascending order, then the sum of the first 5 terms is 30. Find the largest number among all the numbers _____.

**17.** Find the sum of the squares of the first 10 terms of the series whose sum up to $n$ terms is given by $2n^2 + 4n$ _____.

**18.** Five heavy stones are placed on the road connecting P and Q at intervals of 4m, with the first stone at P itself. Ajay started from P and began to move all the stones to

Q by carrying one stone at a time. If the distance PQ is 200 m, find the minimum distance he has to travel (in m) _____.

**19.** If $a = b - b^2 + b^3 - b^4 + \ldots$, express $b$ in terms of $a$ if it is given that $|b| < 1$.

(A) $\frac{a}{1+a}$ (B) $\frac{a}{1-a}$

(C) $\frac{a^2}{1-a}$ (D) $\frac{a^2}{1+a}$

**20.** The sum of the first 13 terms of an arithmetic progression equals the sum of its first 27 terms. Find the sum of its first 40 terms.

(A) 0 (B) −1

(C) 1 (D) Cannot be determined

**21.** $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \ldots + \frac{10}{1+10^2+10^4} =$

(A) $\frac{55}{111}$ (B) $\frac{5}{11}$

(C) $\frac{25}{111}$ (D) $\frac{35}{111}$

**22.** $1 + (1 + 2) + (1 + 2 + 3) + \ldots + (1 + 2 + 3 + \ldots n) =$

(A) $\frac{n(n+1)}{4}$ (B) $\frac{n(n+1)(n+4)}{6}$

(C) $\frac{n^2(n+1)^2}{6}$ (D) $\frac{n(n+1)(n+2)}{6}$

**23.** $\left(1+\frac{1}{3}\right)\left(1+\frac{1}{9}\right)\left(1+\frac{1}{81}\right)\left(1+\frac{1}{6561}\right) =$

(A) $\frac{(3281)(6560)}{(2187)(6561)}$ (B) 1

(C) $\frac{6560}{6531}$ (D) $\frac{3281}{6561}$

**24.** Find the value of $1 + 4/6 + 9/36 + 16/216 + 25/1296 + \ldots\infty$.

(A) $\frac{52}{125}$ (B) $\frac{152}{251}$

(C) $\frac{252}{125}$ (D) 1

**25.** $\frac{1}{5\times7} + \frac{1}{7\times9} + \frac{1}{9\times11} \ldots \infty =$ _____.

**26.** If $S_n = \sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}} + \sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}} + \sqrt{1+\frac{1}{3^2}+\frac{1}{4^2}} + \ldots + \sqrt{1+\frac{1}{n^2}+\frac{1}{(n+1)^2}}$, then find $S_4 + S_5 + S_6 + S_7$.

(A) $17\frac{55}{84}$ (B) $16\frac{9}{14}$

(C) $15\frac{55}{84}$ (D) $25\frac{307}{840}$

**27.** $\frac{1}{(2)(5)(8)} + \frac{1}{(5)(8)(11)} + \frac{1}{(8)(11)(14)} + \ldots$ up to 15 terms =

(A) $\frac{39}{2350}$ (B) $\frac{35}{2151}$

(C) $\frac{351}{25010}$ (D) None of these

**28.** Find the sum of the given terms in the following series

$\frac{1}{1+2^{\frac{1}{3}}+2^{\frac{2}{3}}} + \frac{1}{2^{\frac{2}{3}}+6^{\frac{1}{3}}+3^{\frac{2}{3}}} + \frac{1}{3^{\frac{2}{3}}+12^{\frac{1}{3}}+4^{\frac{2}{3}}} + \ldots + \frac{1}{(63)^{\frac{2}{3}}+(4032)^{\frac{1}{3}}+(64)^{\frac{2}{3}}}$

(A) 4 (B) 3

(C) 5 (D) 2

**29.** If $X = \frac{1}{(90)(46)} + \frac{1}{(89)(47)} + \ldots + \frac{1}{(46)90}$

$Y = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots - \frac{1}{90}$, then $X : Y$ is

(A) $\frac{3}{68}$ (B) $\frac{1}{34}$

(C) $\frac{1}{68}$ (D) $\frac{1}{16}$

**30.** The value of sum of first 75 terms of the sequence $150 \times 2 + 148 \times 4 + 146 \times 6 + \ldots$ is

(A) 252850 (B) 292600

(C) 273950 (D) 284050

**31.** There are three numbers $a$, $b$ and $c$ such that $a$, $b$ and $c$ are in arithmetic progression and $a$, $c$ and $b$ are in geometric progression. If the common ratio of the progression $a$, $c$ and $b$ is not equal to 1, find the value of $a : b$ _____.

**32.** Find the sum to $n$ terms of the series $\log_2 2, \log_2 4, \log_2 8, \log_2 16 \ldots$

(A) $\frac{n(n+1)}{2}$ (B) $\frac{n(2n+1)(n+1)}{6}$

(C) $\left[\frac{n(n+1)}{2}\right]^2$ (D) $\frac{n^2(n+1)^2}{2}$

**33.** Find the sum to 15 terms of the series $3.4^2 + 4.5^2 + 5.6^2 + \ldots$ _____.

**34.** Find the series in geometric progression for which the sum to infinity is 1 and $|r| < 1$.

(A) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \ldots$ (B) $\frac{1}{4}, \frac{3}{16}, \frac{9}{64} \ldots$

(C) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \ldots$ (D) Both (A) and (B)

**35.** There are three numbers in geometric progression. Their product is 343 and the sum of the products of the numbers taken in pairs is 171.5. Find the largest number among the three numbers _____.

**36.** In an arithmetic progression, the sum of the squares of the seventh term and the eleventh term is 482. The product of its second term and the sixteenth term is 29. Find the product of its first and the seventeenth terms.
(A) 31 (B) –31
(C) –33 (D) 33

**37.** The first, second and third terms of a geometric progression with distinct terms, are equal to the first, ninth and thirty first terms, respectively, of an arithmetic progression. If the first term of either progression is 11, find the common ratio of the geometric progression.
(A) 4/11 (B) 11/4
(C) 9/31 (D) 31/9

**38.** A number of saplings were lying at a point P, by the side of a straight road. Raju planted these saplings in a straight line with a distance of 20 metres between consecutive saplings. The first sapling was planted 20 m from P and all the saplings were planted on the same side of P. Raju carried only one sapling at a time and returned to P after planting each sapling, including the last one. If he travelled a total distance of 2.2 km, find the number of saplings that he planted.
(A) 10 (B) 12
(C) 14 (D) 16

**39.** The first term of an arithmetic progression consisting of integers, is the common ratio of a geometric progression. The first term of the geometric progression is the common difference of the arithmetic progression. The sum of the first 11 terms of the arithmetic progression is 341 and the sum of the first three terms of the geometric progression is 18. Find the sum of the common difference and the common ratio.
(A) 8 (B) 7
(C) 12 (D) 14

**40.** In an arithmetic progression, the 10th term is 11 and the 11th term is 10. How many consecutive terms (starting from the first term) of the arithmetic progression should be considered so as to make their sum equal to zero?
(A) 33 (B) 41
(C) 37 (D) 39

**41.** Two new charity organizations $C_1$ and $C_2$ were formed, with $x$ members each, on January 1, 2003. On the first day of each subsequent month in $C_1$, the number of members increases by a certain number $a$, while in $C_2$, the number of members increases in such a way that the ratio of the number of members in a month to the preceding month bear a ratio equal to $b$. On May 1, 2003, both organizations had the same number of members. If $a = 20x$, find $b$ _____.

**42.** Find the sum of the first 50 terms of the series $1 - 3 - 5 - 7, 3 - 5 - 7 - 9, 5 - 7 - 9 - 11 \ldots$ _____.

**43.** In a certain series, the nth term $T_n$ equals $3T_{n-1} + n - 1$. If $T_1 = 3$, then the value of $T_{100}$ is
(A) $\frac{3^{100} - 201}{2}$ (B) $3^{100} - 201$
(C) $\frac{5\left(3^{100}\right) - 201}{2}$ (D) $\frac{3^{101} - 201}{2}$

**44.** Find the sum of the first 20 terms of the series $2 + 13 + 28 + 47 + 70 + \ldots$ _____.

**45.** 3080 balls, each having a radius of 2 cm, are stocked in a pile. There are 2 balls in the top most layer, 6 balls in the second, 12 balls in the third, 20 in the fourth and so on. Find the number of layers in the pile _____.

## EXERCISE-4

***Directions for questions 1–30:*** For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple-choice questions, write your answer in the space provided.

**1.** $\frac{1}{5.10} + \frac{1}{10.15} + \frac{1}{15.20} + \ldots\infty =$
(A) 1/10 (B) 1/80
(C) 1/5 (D) 1/25

**2.** The sum of the infinite series $1 + (1 + b)r + (1 + b + b^2)r^2 + \ldots$ where $r$ and $b$ are proper fractions is
(A) $\frac{1}{1 - rb}$ (B) $\frac{1}{(1 - r)(1 - b)}$
(C) $\frac{1}{(1 - r)(1 - br)}$ (D) None of these

**3.** $1 + 4/5 + 7/5^2 + 10/5^3 + \ldots\infty =$
(A) 35/16 (B) 37/17
(C) 16/35 (D) 17/37

**4.** $\frac{1^3}{2} + \frac{1^3 + 2^3}{2 + 4} + \frac{1^3 + 2^3 + 3^3}{2 + 4 + 6} + \ldots$ up to $n$ terms =
(A) $\frac{n(n+1)^2(n+2)}{12}$ (B) $\frac{n(n+1)(n+3)}{12}$
(C) $\frac{n(n+1)(n+2)}{12}$ (D) $\frac{n^2(n+1)}{4n+2}$

**5.** If $7^{1+x} + 7^{1-x}$, $\frac{p}{2}$ and $10^x + 10^{-x}$ are three consecutive terms of an arithmetic progression, find the range of $p$.
(A) $[14, \infty)$ (B) $[16, \infty)$
(C) $[10, \infty)$ (D) $[12, \infty)$

**6.** If $a_1, a_2, a_3, \ldots, a_{39}$ are in arithmetic progression and $a_1 + a_{10} + a_{15} + a_{25} + a_{30} + a_{39} = 270$, find the value of $a_1 + a_{19} + a_{21} + a_{39}$ _____.

**7.** Find the sum of the series $0.7 + 0.77 + 0.777 + \ldots$ up to $n$ terms.

(A) $\frac{7}{9}\left(9n + \frac{1}{10^n} - 1\right)$ (B) $\frac{7}{81}\left(9n + \frac{1}{10^n} - 1\right)$

(C) $\frac{7}{81}\left(9n - \frac{1}{10^n} + 1\right)$ (D) $\frac{7}{9}\left(9n - \frac{1}{10^n} + 1\right)$

**8.** Find the sum of the series $1 + 5x + 9x^2 + 13x^3 + \ldots$

(A) $\frac{1-3x}{(1+x)^2}$ (B) $\frac{1+3x}{(1-x)^2}$

(C) $\frac{1+3x}{(1+x)^2}$ (D) $\frac{1-3x}{(1-x)^2}$

**9.** Find the sum of the first $n$ terms of $\frac{3}{4} + \frac{5}{36} + \frac{7}{144} + \frac{9}{400} + \ldots$

(A) $\frac{n^2+n}{n^2+2n+1}$ (B) $\frac{n^2}{n^2+2n+1}$

(C) $\frac{2n}{n^2+2n+1}$ (D) None of these

**10.** A real estate dealer tried to make a deal with the owner of a plot of land. The plot owner did not agree to the terms and conditions of the dealer. Mr. Madhya Varthi tried to settle their deal and he was successful. The dealer was given two options. He could pay the entire amount of ₹1,80,000 at once or he could pay an interest of 10% on the outstanding amount in addition to a fixed amount of ₹10,000 every year, starting from the end of the first year. The businessman agreed to pay the annual instalments. By the end of the instalments, what additional amount would the businessman have paid to the owner of the plot? (in ₹) _____

**11.** Garibilal borrowed some money from Amirilal to admit his son in a reputed engineering college. He agreed to pay the interest-free loan of ₹60,000 in monthly instalments which increased by a constant amount. After the 20th instalment, he found that he had paid 3/4 of the loan. If the entire loan was cleared this way in exactly 25 instalments, find the value of the first instalment.

(A) ₹1,200 (B) ₹1,320

(C) ₹1,440 (D) ₹1,680

**12.** When a rubber ball is dropped from a certain height on to a granite floor, the ball rebounds to 3/4 of the height from which it has fallen. A boy drops the ball from a height of 25 m and the ball rebounds several times and finally comes to rest. Out of curiosity, the boy asks his uncle to tell him the total distance travelled by the ball from the moment it had been dropped, till it came to rest. What should be the uncle's correct reply? (in m) _____

**13.** A clerk in a bank was to count ₹2500 in ten-rupee notes. He could count ₹220 in the first minute. As his fingers began to pain, he counted ₹10 less in every subsequent minute. In this way, he counted all the notes. How much time did he take?

(A) 20 minutes (B) 25 minutes

(C) Either (A) or (B) (D) 45 minutes

**14.** A man took an interest-free loan of ₹25,000 with the promise that he would pay back ₹500 at the end of the first month and from the second month onwards, he would pay ₹50 more than what he paid in the previous month. The instalment that would be less than or equal to the preceding instalment would thus be the final instalment. What was the final instalment? (in ₹) _____

**15.** My son participated in the solo event 'biscuit race' conducted on the Annual Sports day of his school. There were 10 biscuits hung, each separated by 25 m. The first biscuit was hung at a distance of 100 m from the starting point. Each participant is supposed to reach the first biscuit, eat it and come back to the starting point and without wasting any time turn back for the second biscuit and this process of eating and coming back to the starting point continued for all the 10 biscuits. How many metres in all did my son run?

(A) 1500 m (B) 2125 m

(C) 3400 m (D) 4250 m

**16.** I went to my sister's house and returned in the evening. On my return journey, I felt very weak. Though I covered 10 km during the first hour of my return journey, for every subsequent hour, my speed reduced to a half of what it was in the previous hour. If I had travelled this way, in how many hours would I have been 25 kilometres away from my sister's house?

(A) 10 hours (B) 16 hours

(C) 20 hours (D) None of these

**17.** Mr. Singh has five children—Dolly, Polly, Molly, Solly and Lolly—named in the decreasing order of their ages. The age difference between any two consecutive children is the same (an integral number of years). If Dolly is 14 years old, which among the following could be the age of Molly?

I. 13 years

II. 12 years

III. 11 years

(A) Only II (B) Only III

(C) Only II or III (D) Only I or II

**18.** In a simulated gravity experiment, a ball is thrown vertically downwards from an altitude of 3000 m. If the distances through which the ball falls in successive seconds is 6 m, 16 m, 26 m and so on, what is the distance (in m) through which the ball falls in the 20th second? _____

**19.** In a one-day series, Kumble, Harbhajan and Zaheer took a total of 21 wickets between the three of them. Kumble took more wickets than Harbhajan who took more wickets than Zaheer. Incidentally, the number of wickets taken by them forms an A.P. Had Zaheer taken one more wicket,

Harbhajan two more and Kumble 15 more wickets, the number of wickets taken by them would have been in G.P. Find the number of wickets taken by Kumble _____.

**20.** For an interest-free loan of ₹32,400 that I gave to my friend Rajesh, he promised to pay the amount in monthly instalments starting with a certain amount and increasing the instalments every month by ₹100. In this way, he could clear off the loan in 24 months. What was the instalment for the first month? (in ₹) _____

**21.** A group of friends have some money with each of them, and the amounts were in an increasing G.P. The total amount with the first and the last friend was ₹66 and the product of the amounts (in ₹) with the second friend and the second last friend was 128. If the total amount with all the friends was ₹126, how many friends were there?

(A) 6 (B) 5
(C) 3 (D) Cannot be determined

**22.** There are some numbers in a series of brackets as given below.

{1}, {2, 3, 4}, {5, 6, 7, 8, 9} and so on. Find the sum of all the numbers in the 25th bracket _____.

**23.** My son who studies in V standard gets pocket money of ₹500 every month. One day he listened to a lecture on the importance of thrift and decided to save more and more money every month. He could save ₹120 in the first month. In the second month, he saved five more rupees than he had saved in the first month. In the third month, he saved five rupees more than he had saved in the second month. If he continued saving in this way, how much would he be saving in two years? (in ₹) _____

**24.** Find the sum of the coefficients of $x^{150}$ and $x^{151}$ in the expansion of $(1+x)^{350} + (1+x)^{349}\,x + (1+x)^{348}\,x^2 + (1+x)^{347}\,x^3 + \ldots + x^{350}$.

(A) ${}^{351}C_{151}$ (B) ${}^{351}C_{150}$
(C) ${}^{352}C_{151}$ (D) ${}^{352}C_{150}$

**25.** $S = \frac{1}{126} + \frac{1}{127} + \frac{1}{128} + \frac{1}{129} + \ldots + \frac{1}{250}$

Which of the following statements is/are true?

I. $S = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots - \frac{1}{250}$

II. $S = 188\left[\frac{1}{(126)(250)} + \frac{1}{(127)(249)} + \frac{1}{(128)(248)} + \ldots + \frac{1}{(250)(126)}\right]$

III. $S = \frac{1}{(126)(250)} + \frac{1}{(127)(249)} + \frac{1}{(128)(248)} + \ldots + \frac{1}{(250)(126)}$

(A) Only I (B) I and II
(C) I and III (D) Only III

**26.** Find the sum to 20 terms of the series $3(4)^2 + 5(5)^2 + 7(6)^2 + 9(7)^2 + \ldots$

(A) 103703 (B) 130730
(C) 170330 (D) 130370

**27.** $T_n = \left(1 + \frac{4}{n^2 - 4}\right) T_{n-1}$, for $n \geq 3$ and $T_2 = \frac{3}{4}$. Find $T_{20}$.

(A) $\frac{285}{77}$ (B) $\frac{513}{140}$
(C) $\frac{693}{184}$ (D) $\frac{77}{155}$

**28.** In a sequence of 130 terms, the sum of any three consecutive terms is 150. The 60th term and the 49th term of the sequence are 96 and 40, respectively. Find the 80th term of the sequence _____.

**29.** In a certain sequence, the $N$th term $T_N$ equals the $(N-1)$th term minus the $(N-2)$th term for all $N \geq 3$. If $T_1 = 2$ and $T_2 = 5$, find the sum of the first 20 terms of the sequence _____.

**30.** $\frac{1}{6+12+18} + \frac{1}{6+12+18+24} + \frac{1}{6+12+18+24+30} + \ldots + \frac{1}{6+12+18+24+\ldots+1200} =$

(A) $\frac{22}{201}$ (B) $\frac{34}{603}$
(C) $\frac{41}{603}$ (D) $\frac{17}{201}$

## EXERCISE-5 Based on Data Sufficiency

***Directions for questions 1–20***: Each question is followed by two statements, I and II. Answer each question based on the following directions:

Choose (A): If the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.

Choose (B): If the question can be answered by using either statement alone.

Choose (C): If the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (D): If the question cannot be answered even by using both the statements together.

Data Sufficiency

**1.** What is the sum of the first 25 terms of an arithmetic progression?
I. The first term of the progression is 3.
II. The sum of the first 10 terms of the progression is equal to the sum of the first 15 terms of the progression.

**2.** Three numbers are in geometric progression and the least of them is 1. Find the middle number.
I. The sum of the numbers is 21.
II. The product of the numbers is 64.

**3.** Are $x$, $y$ and $z$ in geometric progression?
I. $x + y = 20$ and $y + z = 60$
II. $y/x = -2$

**4.** What is the fourth term of the series?
I. The first term of the series is 1.
II. Starting from the second term, each term in the series is obtained by squaring the number obtained by adding one to the previous term.

**5.** What is the arithmetic mean of $x$ and $y$?
I. The geometric mean of $x$ and $y$ is 4.
II. The arithmetic mean of $x$, $y$, 4 and 8 is 5.

**6.** $G$ is a geometric progression. The sum of all the terms of $G$ is $\frac{3^8 - 1}{2}$ and the product of all the terms is $3^{28}$. Find its common ratio.
I. If each term in $G$ is multiplied by its common ratio, the sum of all the resulting terms would be $\frac{3^8 - 1}{2}$.
II. If each term in $G$ is multiplied by its common ratio, the product of the resulting terms would be $3^{36}$. Also, G has 8 terms.

**7.** The sum of all the terms of an infinite geometric progression which has all positive terms is 8. Find its first term.
I. The sum of the squares of all the terms of the progression is $\frac{64}{3}$.
II. Each term equals the sum of all the terms following it.

**8.** The sum of the 12 terms of an arithmetic progression is 100. Find its common difference.
I. The ratio of the sum of its odd numbered terms and even numbered terms is 11 : 14.
II. The sum of its even numbered terms is 12 more than that of the odd numbered terms.

**9.** What is the middle term of an arithmetic progression of 11 numbers?
I. The average of 11 numbers is 63.
II. The average of the first six numbers is 60 and that of the last six numbers is 66.

**10.** If $a$, $b$ and $c$ are in geometric progression, is the common ratio negative?
I. $b$ is less than $a$.
II. $(a)(b)(c)$ is greater than $(a)(c)$.

**11.** Find the 17th term of the arithmetic progression A.
I. The sum of the first 33 terms of the progression is 1089.
II. The first term and the common difference are both 1.

**12.** A geometric progression has 9 terms. Find its fifth term.
I. The sum of its terms is 511.
II. The product of its terms is $2^{36}$.

**13.** An arithmetic progression has 81 terms. Find the sum of its terms.
I. Twice its twenty first term exceeds its first term by 63.
II. Its second term is 5.

**14.** The sum of all the terms in a geometric progression is 2047. Find its common ratio.
I. Its last term is 1024 and the first term is 1.
II. Its last but one term is 512.

**15.** Four integers are in arithmetic progression. The product of the first and the last numbers is 512 less than the product of the middle numbers. Find the first number.
I. The sum of the numbers is 104.
II. The first and second numbers when halved would be single digit natural numbers. The first is less than the second.

**16.** There are $n$ natural numbers in an arithmetic progression. Find the common difference of the progression if $n > 1$.
I. If each term is increased by the common difference, the sum of the terms would increase by 17.
II. The common difference is also a natural number less than 3.

**17.** Find the arithmetic mean of the terms of an arithmetic progression having an even number of terms.
I. The sum of the lesser of the middle terms and the last term of the progression is 24.
II. The sum of the larger of the middle terms and the first term of the progression is 12.

**18.** What is the third number in an arithmetic progression consisting of nine numbers?
I. The sum of the first and the last numbers is 24.
II. The first number is 4 and the last number is 20.

**19.** Four non-zero integers are in geometric progression. Find the square of the common ratio.
I. The sum of the middle numbers is more than the first number by five times the first number.
II. The product of the second and the last numbers exceeds the product of the first and the third numbers by 12 times the square of the first number.

**20.** If $a$, $b$ and $c$ are all distinct positive integers, are $a$, $b$ and $c$ in ascending order?
I. $b$ is the arithmetic mean of $a$ and $c$.
II. $c + 2$ is greater than $a + 1$.

## ANSWER KEYS

### Exercise-1

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (B) | 8. (B) | 15. (B) | 22. (C) | 29. 127 | 36. 0 | 43. 256 |
| 2. (B) | 9. (A) | 16. (B) | 23. (D) | 30. (C) | 37. (B) | 44. (A) |
| 3. (A) | 10. (C) | 17. (A) | 24. (C) | 31. (B) | 38. 70 | 45. (B) |
| 4. (A) | 11. (B) | 18. (A) | 25. (D) | 32. (B) | 39. (B) | |
| 5. (D) | 12. (B) | 19. (B) | 26. (A) | 33. (D) | 40. (A) | |
| 6. (C) | 13. (A) | 20. (C) | 27. 3025 | 34. (B) | 41. (A) | |
| 7. (D) | 14. (B) | 21. (B) | 28. (C) | 35. (A) | 42. (B) | |

### Exercise-2

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. 35 | 8. 12 | 15. 7 | 22. (D) | 29. (D) | 36. (C) | 43. (D) |
| 2. 250 | 9. (C) | 16. 11 | 23. (B) | 30. 27 | 37. (B) | 44. (D) |
| 3. 0 | 10. 196 | 17. 2 | 24. (C) | 31. 12 | 38. (B) | 45. (B) |
| 4. 2700 | 11. (C) | 18. (C) | 25. (D) | 32. (B) | 39. 36 | |
| 5. 90 | 12. (B) | 19. 61 | 26. (B) | 33. 30,00,000 | 40. (D) | |
| 6. 51 | 13. (B) | 20. (B) | 27. (C) | 34. (D) | 41. (C) | |
| 7. 108 | 14. (C) | 21. (A) | 28. 4 | 35. 6 | 42. (D) | |

### Exercise-3

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. 89 | 8. 74 | 15. (A) | 22. (D) | 29. (C) | 36. (B) | 43. (C) |
| 2. (C) | 9. (A) | 16. 40 | 23. (A) | 30. (B) | 37. (B) | 44. 6690 |
| 3. 8 | 10. (D) | 17. 7080 | 24. (C) | 31. 4 | 38. (A) | 45. 20 |
| 4. 2 | 11. (C) | 18. 1720 | 25. 0.1 | 32. (A) | 39. (B) | |
| 5. 19683 | 12. (D) | 19. (B) | 26. (D) | 33. 27110 | 40. (B) | |
| 6. (B) | 13. (D) | 20. (A) | 27. (A) | 34. (D) | 41. 3 | |
| 7. 728 | 14. (B) | 21. (A) | 28. (B) | 35. 14 | 42. –5600 | |

### Exercise-4

| | | | | | |
|---|---|---|---|---|---|
| 1. (D) | 6. 180 | 11. (D) | 16. (D) | 21. (A) | 26. (B) |
| 2. (C) | 7. (B) | 12. 175 | 17. (A) | 22. 29449 | 27. (A) |
| 3. (A) | 8. (B) | 13. (A) | 18. 196 | 23. 4260 | 28. 14 |
| 4. (C) | 9. (D) | 14. 850 | 19. 12 | 24. (C) | 29. 7 |
| 5. (B) | 10. 1,71,000 | 15. (D) | 20. 200 | 25. (B) | 30. (A) |

### Exercise-5

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (A) | 4. (C) | 7. (B) | 10. (C) | 13. (A) | 16. (A) | 19. (A) |
| 2. (B) | 5. (A) | 8. (B) | 11. (B) | 14. (A) | 17. (C) | 20. (C) |
| 3. (C) | 6. (B) | 9. (B) | 12. (A) | 15. (A) | 18. (A) | |

## SOLUTIONS

## EXERCISE-1

**1.** Sum of the first $n$ terms of an A.P.

$$S_n = \left(\frac{n}{2}\right)[2a + (n-1)d]$$

$$S_7 = \left(\frac{7}{2}\right)[2(2) + (7-1)4]$$

$$= \left(\frac{7}{2}\right)[4 + 24] = \frac{7}{2}(28) = 98$$

**2.** Arithmetic mean $= \dfrac{\text{Sum of the terms}}{\text{Number of terms}}$

$$= \frac{6+8+10+12+14+16}{6} = \frac{66}{6} = 11$$

**Alternative Solution:**

Arithmetic mean of an A.P. $= \dfrac{\text{First term} + \text{Last term}}{2}$

$$= \frac{6+16}{2} = \frac{22}{2} = 11$$

**3.** The fourth term is equidistant (3 terms away) from the first and the seventh term in the arithmetic progression.
∴ The fourth term is the arithmetic mean of the first term and the seventh term in an A.P.
∴ Fourth term $= \frac{4+28}{2} = \frac{32}{2} = 16$

**4.** Given total number of terms is 13, i.e., $n = 13$
In an A.P., if $n$ is odd, then $\frac{(n+1)^{th}}{2}$ term is the arithmetic mean of that A.P.
As $\frac{(13+1)}{2} = 7$ th, the 7th term is the arithmetic mean.
Given, 7th term is 9
∴ 9 is the arithmetic mean.

**5.** Given first term = 3 and the last term = 31
As we do not know the total number of terms, we cannot find the common ratio.
∴ We cannot find the sum of first 8 terms of the progression.

**6.** 8th term $= 7 + 7(5) = 42$

**7.** As we do not know the total number of terms nor the common difference, nor the ninth term, we cannot answer the question.

**8.** Given that $a = 2$ and $d = 5 - 2 = 3$
$t_n = a + (n-1)d = 266$
$\Rightarrow 2 + 3(n-1) = 266$
$\Rightarrow n - 1 = \frac{266-2}{3} = 88$
$\Rightarrow n = 89.$

**9.** The fourth term is equidistant from the first term and the seventh term.
∴ Fourth term = arithmetic mean of 6 and 24 = 15

**10.** Sum of the first 25 terms = 25 (arithmetic mean)
= 25 (middle term) = 25 (20) = 500

**11.** As the average of the $k$th term from the beginning and the $k$th term from the end is equal to the arithmetic mean, the average of 10 and 20 is the arithmetic mean
i.e., the mean is $\frac{10+20}{2} = 15$
In an A.P. as $S_n = n$ (A.M.)
$S_{100} = 100(15) = 1500$

**12.** The 8th term is equidistant from the first term and the 15th term.
∴ 8th term is the arithmetic mean of first 15 terms.
Sum of first 15 terms = 15 × (eight term)
= 15(4) = 60

**13.** In an A.P., $n$th term $(t_n) = a + (n-1)\,d$
$\therefore t_7 = 9 + (7-1)\,3 = 9 + 6\,(3) = 27$

**14.** In A.P.,
$S_n = \left(\frac{n}{2}\right)$[First term + Last term]
$S_{11} = \left(\frac{11}{2}\right)[5+15] = 110$

**15.** In an A.P., if the sum of the first $n$ terms is zero, and $n$ is odd, the $\frac{(n+1)^{th}}{2}$ term is zero.
$\therefore \left(\frac{51+1}{2}\right)^{th}$, i.e., 26th term is zero.

**16.** Geometric mean $= \sqrt[4]{(4)(8)(16)(32)} = 2\left(\sqrt{32}\right) = \sqrt{128}$

**17.** $4^{th}$ term $= ar^3$
$2^{nd}$ term $= ar$
$6^{th}$ term $= ar^5$
$(2^{nd}$ term$)\,(6^{th}$ term$) = (ar^3)^2$
$(ar^3)^2 = (8)\,(32)$
$ar^3 = \pm 16$
But as $ar = 8$, $ar^3$ must be positive, i.e., 16.

**18.** The quantities log $a$, log $b$ and log $c$ are in arithmetic progression if $a$, $b$ and $c$ are in geometric progression.

**19.** Sum of the first 4 terms $= 4\frac{(3^4-1)}{3-1} = 160$

**20.** Two distinct positive numbers have their arithmetic mean greater than their geometric mean. In the given problem, the arithmetic mean will exceed 9.

**21.** Geometric mean $= \left[(2)(4)(8)(16)\right]^{1/4}$
$= \left[(2^1)(2^2)(2^3)(2^4)\right]^{1/4} = 2^{10/4} = \sqrt{32}$

**Alternative Solution:**
Geometric mean of a geometric progression
$= \sqrt{\text{First term} \times \text{Last term}} = \sqrt{(2)(16)} = \sqrt{32}$

**22.** In a G.P.,
$T_n = ar^{n-1}$
$T_7 = 3(2)^{7-1} = 3(64) = 192$

**23.** In a G.P.,
$S_n = \frac{a\left(1-r^n\right)}{1-r}$, where $r$ is the common ration, $a$ the first term and $S_n$ in the sum to $n$ terms.
$\therefore S_4 = \frac{6(1-3^4)}{1-3} = \frac{6(1-81)}{-2} = (3)(80) = 240$

**24.** For any two unequal positive numbers $a$ and $b$,
$\frac{a+b}{2} > \sqrt{ab}$
Given $\sqrt{ab} = 4$
∴ Arithmetic mean, $\frac{a+b}{2} > 4$

**25.** The fourth term is equidistant from the second term and the sixth term.
∴ In a G.P., the fourth term is the geometric mean of the second term and the sixth terms.

$\therefore$ Fourth term $= \sqrt{(\text{Second term})(\text{Sixth term})}$

$= \sqrt{(4)(64)} = 16$

**NOTE**

It cannot be –16. The common ratio would be complex and we are considering only progressions with real terms.

**26.** $\frac{20}{2}[2a + 19d] = 210$

$\Rightarrow 2a + 19d = 21$

$\Rightarrow a + 9d + a + 10d = 21$

$10^{\text{th}}$ term + $11^{\text{th}}$ term = 21

**NOTE**

In an arithmetic progression with $N$ terms, the sum of all the terms is equal to

(a) $N$ (middle term), if $N$ is odd.

(b) $N$ (average of the 2 middle terms), if $N$ is even.

**27.** The sum of the cubes of the first 10 natural numbers

$= \left(\frac{(10)(11)}{2}\right)^2 = 3025$

**28.** The series is a geometric progression

Sum to infinity $= \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$.

**29.** $a = 1$

$ar^3 = r^3 = 8$

$r = 2$

Sum of the first 7 terms

$= 1\frac{(2^7 - 1)}{2 - 1} = 127$

**30.** The 30th term from the beginning is the 71st term from the end.

**31.** In a G.P., if the number of terms is $n$ which is odd, then the middle term, i.e., $\left(\frac{n+1}{2}\right)^{\text{th}}$ term is the geometric mean.

$\therefore \left(\frac{13+1}{2}\right)^{\text{th}}$ term, i.e., the 7th term is the geometric mean, i.e., the geometric mean is 4.

**32.** Sum of the first 9 natural numbers $= \frac{9(9+1)}{2} = 45$

Sum of the cubes of first $n$ natural numbers

= (Sum of first $n$ natural numbers)$^2$

$\therefore$ Sum of the cubes of first 9 natural numbers $= 45^2$

$= 2025$

**33.** In a geometric progression, the product of any odd number of consecutive terms is (middle term)$^{\text{number of terms}}$

$\therefore$ Required product $= 2^{13} = 8192$

**34.** Given $m$, $n$ and $p$ are in A.P.

Let $n - m = p - n = k$

$\therefore n = m + k$, $p = m + 2k$

Consider a G.P.

$g_1, g_2, g_3, \ldots$ whose common ratio is $r$.

$g_m(r) = g_{m+1}$ [$g_m$ is $m^{\text{th}}$ term]

$g_m(r^k) = g_{m+k} = g_n$ [$\because m + k = n$]

$g_m(r^{2k}) = g_{m+2k} = g_p$ [$\because m + 2k = p$]

$$\frac{g_p}{g_n} = \frac{g_m r^{2k}}{g_m r^k} = r^k$$

$$\therefore \frac{g_n}{g_m} = \frac{g_p}{g_n} = r^k$$

$\therefore g_m$, $g_n$ and $g_p$ are in G.P.

$\therefore m$th term, $n$th term and $p$th term are in G.P.

**35.** The 5th term is equidistant from the first term and the 9th term

$\therefore$ 5th term is the geometric mean of first and ninth term and moreover, it is the geometric mean of first 9 terms.

The product of the first $n$ terms in a G.P. = (Geometric mean of first $n$ terms)$^n$

$\therefore$ Product $= 2^9 = 512$

**36.** If the sum of the first $p$ terms of an arithmetic progression is $q$ and the sum of its first $r$ terms is also $q$ (where $r > p$), the sum of its $(p + 1)$st term and the $r$th term is always 0. Applying this concept for the given problem, the required sum = 0.

**37.** Sum of all the terms $= \frac{200}{2}(15 + 45) = 6000$

($\because$ Sum of the $p$th term from the beginning and the $p$th term from the end = sum of the first and the last terms).

**38.** Sum of the first 7 terms $= \frac{7}{2}[2\ (1) + 6(3)] = 70$

**39.** If $p$, $q$, $r$ are in arithmetic progression, the $p$th term, $q$th term and the $r$th term of a geometric progression will be in geometric progression.

**40.** If $p$, $q$, $r$ are in arithmetic progression, the $p$th term, $q$th term and $r$th term of an arithmetic progression will be in arithmetic progression.

**41.** Given $S_{20} = 30$ and $S_{50} = 30$

$S_{50} = S_{20} + t_{21} + \ldots + t_{50}$

$\therefore 30 = 30 + t_{21} + \ldots t_{50}$

$\therefore t_{21} + t_{22} + \ldots + t_{50} = 0$

The mean of $t_{21}$ to $t_{50}$ is 0.

$\therefore$ Sum of $t_{21}$ and $t_{50}$ is zero.

**42.** Common ratio $= \frac{1/2}{1} = \frac{1}{2}$

$\therefore S_a = \frac{a}{1-r} = \frac{1}{1-1/2} = 2$

**43.** Eighth term $= ar^7 = 2\ (2)^7 = 256$

**44.** Given $a = 1,\ t_4 = 27$

$t_4 = ar^3$

$27 = (1)\ r^3 \Rightarrow r = 3$

$S_7 = 1\frac{\left(1-3^7\right)}{1-3} = \frac{1-2187}{-2} = \frac{-2186}{-2} = 1093$

**45.** Arithmetic mean = middle term, i.e., 9th term = 11

## EXERCISE-2

**1.** $T_{15} = a + 14d$

$a = 7$

$d = 2$

$T_{15} = 7 + 14(2) = 35$

**2.** $S_{25} = \frac{25}{2}\{T_1 + T_{25}\} = \frac{25}{2}\left\{\frac{3}{4} + \frac{77}{4}\right\}$

$= \frac{25}{2} \times \frac{80}{4} = 250$

**3.** $S_3 = t_1 + t_2 + t_3$

$S_4 = t_1 + t_2 + t_3 + t_4$

As $S_3 = S_4,\ t_4 = 0$

$S_7 = 7 \times t_4 = 7 \times (0) = 7\ (0) = 0$

**4.** $S_{50} = \frac{50}{2}[2(5) + 49(2)] = 25\ \{10 + 98\} = 108 \times 25 = 2700$

**5.** $S_9 = \frac{9}{2}\{2a + 8d\}$

$\frac{9}{2}[2(a + 4d)] = 9\ (a + 4d) = 9 \times 10 = 90$

**6.** Let the first term and the common difference of the A.P. be $a$ and $d$, respectively.

$t_5 = a + 4d = 11$ (1)

$t_{15} = a + 14d = 31$ (2)

Subtracting equation (1) from equation (2), we have

$10d = 20 \Rightarrow d = 2$

25th term of the A.P. $= a + 24d$

$= a + 4d + 20d = 11 + 20\ (2) = 51$

**7.** The series is an arithmetic progression with first term 8, and common difference $2\frac{1}{2}$. So, $a = 8,\ d = 2\frac{1}{2}$

$t_n = a + (n-1)d$

$t_{41} = 8 + (40 \times 2^1/_2) = 108$

**8.** The A.P. has a first term of 11 and a common difference of 5. The $n$th term of the A.P. is 66.

i.e., $11 + (n-1)5 = 66$

$n = 12$.

**9.** Let the number of terms of the A.P. to be added be $n$. The A.P. has a first term of 21 and common difference of 7.

Sum of the first $n$ terms of the A.P.

$= \frac{n}{2}\ [2\ (21) + (n-1)7]$

$\Rightarrow \frac{n}{2}\ [35 + 7n] = 175 \Rightarrow 7n\ [5 + n] = 350$

Substituting each of the choices for $n$ in the above equation only $n = 5$, satisfies it.

**10.** The series has a first term as 49, common difference –7 and the last term is 7.

Let the number of terms in the series be $n$.

$\therefore 7 = 49 - 7\ (n-1)$

$42 = 7\ (n-1)$

$6 = n - 1$

$7 = n$

Sum to $n$ terms of the series $= \frac{n}{2}$ (First term + Last term)

$= \frac{7}{2}(49 + 7) = 196$

**11.** Let the G.P. be $a,\ ar,\ ar^2$

$a + ar + ar^2 = 7$

$a^2 + a^2r^2 + a^2r^4 = 21$

$\therefore \frac{a^2(1 + r^2 + r^4)}{\left[a\left(1 + r + r^2\right)^2\right]} = \frac{21}{(7)^2}$

$\Rightarrow \frac{\left(1 + 2r^2 + r^4\right) - r^2}{\left(1 + r + r^2\right)^2} = \frac{3}{7}$

$\Rightarrow \frac{\left(1 + r^2 + r\right)\left(1 + r^2 - r\right)}{\left(1 + r + r^2\right)^2} = \frac{3}{7} \Rightarrow \frac{1 + r^2 - r}{1 + r + r^2} = \frac{3}{7}$

$\Rightarrow 2r^2 - 5r + 2 = 0 \Rightarrow r = 2,\ \frac{1}{2}$

When $r = 2,\ a\ (1 + 2 + 4) = 7 \Rightarrow a = 1$

$\therefore$ The 4th, 5th, 6th terms are 8, 16, 32

When $r = 1/2,\ a\left(1 + \frac{1}{2} + \frac{1}{4}\right) = 7$

$\therefore a = 4$ and the 4th, 5th, 6th, terms are $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$.

Either I or II could be the specified terms.

**12.** $T_2 = 1\ T_1 = 2000! = 2a$ (say)
$T_3 = 2T_2 = 2!a$
$T_4 = 3T_3 = 3!$ A and so on.
$T_N = (N-1)\,!(a)$ for all $N \geq 2$
$T_{2001} = [(2000)!]\,a = a^2 = b$
$\sum_{1=2}^{2000} \log_b{}^i = \log_b{}^a = \frac{1}{2}$.

**13.** Let the 1st term and the common difference be $A$ and $d$, respectively.

$\therefore a = \frac{p}{2}[2A + (p-1)d]$

$\Rightarrow \frac{a}{p} = A + (p-1)\frac{d}{2}$ (1)

Similarly $\frac{b}{q} = A + (q-1)\frac{d}{2}$ (2)

$\frac{c}{r} = A + (r-1)\frac{d}{2}$ (3)

(1) $(q-r)$ + (2) $(r-p)$ + (3) $(p-q)$

$\Rightarrow \frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q)$

$\Rightarrow = a(q-r+r-p+p-q) + \frac{d}{2}[(p-1)(q-r) + (q-1)(r-p)$

$+(r-1)(p-q)] = 0$

**14.** For a G.P., whose common ratio is $r$, the sum of the first $m$ terms (say $Sm$) multiplied by $r^m$ gives the sum of the next $m$ terms ($S_{2m} - S_m$). For the given G.P., $S_8 = 10001\ S_4$, i.e., $S_8 - S_4 = 10^4 S_4$.
$= (\pm 10)^4 S_4$
$r^4 = (\pm 10)^4$
$r = \pm 10$

**15.** In the G.P., $t_7 = 448$, $t_{11} = 7168$ and the common ratio $r > 0$.

$r^4 = \frac{t_{11}}{t_7} = \frac{7168}{448} = \frac{1792}{112} = \frac{448}{28} = 16 \Rightarrow r = 2$

$\therefore\ t_1 = \frac{t_7}{r^6} = \frac{448}{68} = \frac{7(64)}{64} = 7$

**16.** Let the numbers be $a-3d$, $a-d$, $a+d$ and $a+3d$.
$(a-3d) + (a-d) + (a+d) + (a+3d) = 56 \Rightarrow a = 14$
Also, $(a-d)\ (a+d) - (a-3d)\ (a+3d) = 8$
$\Rightarrow 9d^2 - d^2 = 8\ \therefore\ d = 1$
$\therefore$ The smallest number is $14 - 3(1) = 11$.

**17.** Let the terms be $a$, $ar$ and $ar^2$
$a \times ar \times ar^2 = 216 \Rightarrow a^3 r^3 = 216 \Rightarrow ar = 6$

Now, $ar^4 = 162 \Rightarrow \frac{ar^4}{ar} = \frac{162}{6} \Rightarrow r^3 = 27$

$\therefore r = 3\ \therefore a = 6/3 = 2$

**18.** $S_n = \frac{n}{2}(a+l)$

$\Rightarrow \frac{n}{2}(4 + 241) = 9800$

$\Rightarrow n = \frac{2 \times 9800}{245} = 80$

$\therefore 4 + (80-1)\ d = 241$

$\Rightarrow d = \frac{237}{79} = 3$

$\therefore t_{35} = a + 34d = 4 + 34 \times 3 = 106$

**19.** $t_{69} = 16 \times t_4 \Rightarrow a + 68d = 16\ (a + 3d)$
$\Rightarrow 15a = 20d \Rightarrow a = 4/3d$
Also, $a + 6d = 22$
$\Rightarrow 4/3d + 6d = 22 \Rightarrow d = 3$
$\therefore a = 4\ \therefore t_{20} = 4 + 19 \times 3 = 61$

**20.** Let the numbers be $a/r$, $a$ and $ar$
$a/r \times a \times ar = 13824 \Rightarrow a^3 = 13824 \Rightarrow a = 24$
Now $24/r + 24 + 24r = 84$
$\Rightarrow 24\ (r + 1/r) = 60 \Rightarrow r + 1/r = 5/2\ \therefore r = 2$ or $1/2$
$\therefore$ The numbers are 12, 24 and 48.

**21.** Let $a$ and $d$ be the first term and common diference of the progression. Given $a + 2d = 28$ (1)

$S_{21} = \frac{21}{2}[2a + 20d] = 1932$

$a + 10d = 92$ (2)

Solving (1) and (2) $a = 12$; $d = 8$
$\therefore$ The sum of first 11 terms

$S_{11} = \frac{11}{2}[2a + 10d]$

$= \frac{11}{2}(24 + 80) = 572.$

**22.** Let the number of terms of the series which give the maximum sum be $n$.

Sum $= \frac{n}{2}[2(50) + (n-1)(-4)]$

$= \frac{n}{2}[104 - 4n] = n\,[52 - 2n] = 2n\,[26 - n]$

When the sum of two quantities is constant, their product is maximum when the quantities are equal. As $n$ and $26 - n$ have a constant sum of 26, their product is maximum when $n = 26 - n \Rightarrow \frac{26}{2} = 13 = n$

Maximum sum = (2) (13) (13) = 338.

**23.** Let the two arithmetic progressions have first terms and common differences as $a_1$, $d_1$ and $a_2$, $d_2$, respectively.

Ratio of the sum of their first $n$ terms $= \dfrac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]}$

$$\frac{a_1\frac{n-1}{2}d_1}{a_2\frac{n-1}{2}d_2} = \frac{5n+4}{8n-15} \quad (1)$$

Ratio of their $12^{th}$ terms $= \dfrac{a_1+11d_1}{a_2+11d_2}$

Substituting $\dfrac{n-1}{2}=11$ in the L.H.S., we get ratio of their 12th terms.

$n-1=22$, $n=23$. Ratio of their $12^{th}$ terms

$$= \frac{5(23)+4}{8(23)-15} = \frac{119}{169}$$

**24.** Let the first term and the common difference of the arithmetic progression be $a$ and $d$.

$4\ (a+90d) = 5\ (a+80d); \Rightarrow -40\ d = a$

Thirty first term of that arithmetic progression

$= a+30d = -630d = +63.$

Fifty first term of that arithmetic progression

$= a+50d = -40d+50d = +10d = 630.$

**25.** Let the first term and the common difference of the arithmetic progression be $a$ and $d$, respectively.

$a+(a+4d)+(a+5d)+(a+7d)+(a+8d)+(a+12d)$

$= 243$

$6a+36d = 243 \quad (1)$

Sum of the first 13 terms $= \dfrac{13}{2}[2a+12d] = 13[a+6d]$

$= \dfrac{(13)(243)}{6}$, (from (1)) $= 526.5$

**26.** Given $\dfrac{1}{1^2}+\dfrac{1}{2^2}+\dfrac{1}{3^2}+\dfrac{1}{4^2}+\ldots\infty = s$

$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\frac{1}{5^2}\ldots$$

$$= \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots-2\left[\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+\ldots\right]$$

$$= s-2.\frac{1}{2^2}\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\right]$$

$$= s-\frac{s}{2} = \frac{s}{2}$$

**27.** $8+88+888+\ldots$

$$= \frac{8}{9}\ (9+99+999+\ldots)$$

$$= \frac{8}{9}\ [(10-1)+(10^2-1)+\ldots+(10^n-1)]$$

$$= \frac{8}{9}\ [(10+10^2+\ldots+10^n)-n]$$

$$= \frac{8}{9}\left[\frac{10(10^n-1)}{10-1}-n\right]$$

$$= \frac{8}{9}\left[\frac{10(10^n-1)}{9}-n\right]$$

**28.** The first look reveals that it is neither in A.P. nor in G.P. but a closer examination reveals that coefficients are in A.P. and that $x$ is in G.P. Such a series is called A.G.P.

$S = 1+2x+3x^2+4x^3+\ldots\infty \quad (1)$

$Sx = \quad x+2x^2+3x^3+\ldots\infty \quad (2)$

from (1) – (2)

$(1-x)\ S = 1+x+x^2+x^3+\ldots\infty \quad (\because\ |x|<1)$

$(1-x)\ S = \dfrac{1}{(1-x)}$

$$S = \frac{1}{(1-x)^2} = \frac{1}{\left(1-\frac{1}{2}\right)^2} = \frac{1}{\left(\frac{1}{2}\right)^2} = 4$$

**29.** The terms can be visualized as follows:

$1+2+3+4\ \ldots\ n$

$2+3+4\ \ldots\ n$

$3+4\ \ldots\ n$

$4\ \ldots\ n$

$\therefore$ Sum $= 1(1)+2(2)+3(3)+\ldots\ n(n)$

$= 1^2+2^2+3^2+\ldots\ n^2$

$$= \frac{n(n+1)(2n+1)}{6}.$$

**30.** $S = 1+3x+6x^2+10x^3+15x^4+\ldots\infty$

$Sx = \quad x+3x^2+6x^3+10x^4+\ldots\infty$

$(1-x)\ S = 1+2x+3x^2+4x^3+5x^4+\ldots\infty$

$(1-x)\ S = \dfrac{1}{(1-x)^2}$ (from previous solution)

$$S = \frac{1}{(1-x)^3}$$

$$= \frac{1}{\left(1-\frac{2}{3}\right)^3} = \frac{1}{\left(\frac{1}{3}\right)^3} = 3^3 = 27$$

**31.** The given sequence is $-\dfrac{3}{5}, -\dfrac{13}{30}, -\dfrac{8}{30}, \ldots$

or $-\dfrac{18}{30}, -\dfrac{13}{30}, -\dfrac{8}{30}, \ldots$

The negative terms in the numerator end in 8 and 3 alternately.

The positive terms would end in 2 and 7 alternately.

Let the $n^{th}$ term be $\dfrac{37}{30}$

$$-\frac{18}{30}+(n-1)\frac{5}{30} = \frac{37}{30}$$

$-18+5(n-1) = 37 \Rightarrow n = 12.$

**32.** $8 + 88 + \ldots 88 \ldots 8$

$= 8(1 + 11 + \ldots + 11 \ldots 1)$

$= \frac{8}{9}(9 + 99 + \ldots + 99 \ldots 9)$

$= \frac{8}{9}(10 + 10^2 + \ldots + 10^n - n)$

$= \frac{8}{9}\left[(10)\frac{(10^n - 1)}{9} - n\right] = \frac{80}{81}(10^n - 1) - \frac{8n}{9}$

**33.** In the first year, the man's salary was ₹3000 p.m.
In the second year, it was ₹4000 p.m.
In the twentieth year, it was ₹22000 p.m.
The total amount paid over the 20 years is

$\left(\frac{3000 + 22000}{2}\right)(20)(12)$ or $(25{,}000)(120) = 30{,}00{,}000.$

**34.** The given A.P. has even number of terms; it can be represented as '$2n$'
The last term is less than first term by 76.
Hence, $T_{2n} = a - 76$; but $T_{2n} = a + (2n - 1)\ d$
Hence, $(2n - 1) = -76$ (1)
Given sum of even terms = –250
If $a,\ a + d,\ a + 2d,\ a + 3d,\ \ldots + (a + 2n - 1\ d)$ is the progression, then sum of even terms (number of terms $= 2n/2 = n$)
$= (a + d) + (a + 3d) + (a + 5d) + \ldots + (a + 2n - 1\ d)$
$= n/2\ [2(a + d) + (n - 1)2d]$
Hence, $n/2 \cdot 2[a + d + nd - d] = -250$
$\Rightarrow n\ (a + nd) = -250$ (2)
The sum of even terms is less than the sum of odd terms by 40
Hence, –250 = (sum of odd term) –40
Sum of odd terms = –250 + 40 = –210
$\Rightarrow n/2\ [2a + n - 1\ 2d] = -210$
$\Rightarrow (a + nd - d) = -210$ (3)
Subtracting (2) from (3), we get
$n\ (a + nd - d) - n\ (a + nd) = -210 - (-250)$
$-nd = +40$ (4)
$\Rightarrow$ From (1) and (4)
$2nd - d = -76,\ 2x - 40 - d = -76$
$-d = -76 + 80 = +4;\ d = -4$ (5)
Substituting in (1)
$(2n - 1)d = -76,\ (2n - 1)\ (-4) = -76$
$2n - 1 = 19,\ 2n = 20$
Number of terms = 20

**35.** $a = 216,\ r = 3/2$

$S_n = \frac{17955}{4}$

$\Rightarrow \frac{216\left[(3/2)^n - 1\right]}{(3/2) - 1} = \frac{17955}{4} \Rightarrow n = 6$

**36.** If there are an odd number of rows, the average number of children per row is an integer. (This is equal to the number of children in the middle row)
If there are an even number of rows, the average number of children per row is the average for the middle two rows, which is an integer + 1/2.
We consider the choices.
(A) 810/6 = 135 ≠ integer + 1/2
(B) 810/8 = 101.25 ≠ integer + 1/2
(C) 810/5 = 162 = integer
(D) 810/10 = 81 ≠ integer + 1/2
∴ Only 5 is a possible value for the number of rows.

**37.** $1/2 + 3/4 + 7/8 + 15/16 + \ldots$

$= (1 - 1/2) + (1 - 1/2^2) + (1 - 1/2^3) + (1 - 1/2^4) + \ldots$ Upto 25 terms

$= 25 - (1/2 + 1/2^2 + 1/2^3 + \ldots + 1/2^{25})$

$= 25 - 1/2\ .\ \frac{1 - (1/2)^{25}}{1 - 1/2} = 25 - \frac{(2^{25} - 1)}{2^{25}}$

$= \frac{24.2^{25} + 1}{2^{25}}$

**38.** The general term of the series $t_n = (101 - n).n$

$= 101n - n^2$

$\therefore S_n = 101\ \Sigma n - \Sigma n^2$

$= 101.\ \frac{n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$

But, there are 100 terms in the series.

$\therefore$ Required sum $= 101 \times \frac{100 \times 101}{2} - \frac{100(101)(21)}{6}$

$= 510050 - 338350 = 171700$

**39.** 2, 5, 8, 11, … 434
Let $434 = t_n \Rightarrow 2 + (n - 1)3 = 434$
$\Rightarrow n = 145$
3, 7, 11, 15, … 579
Let $579 = t_n$
$\Rightarrow 3 + (m - 1)4 = 579 \Rightarrow m = 145$
∴ each of the progressions has 145 terms
Let $p^{\text{th}}$ term of the first A.P.
$= q^{\text{th}}$ term of the second A.P.
$\Rightarrow 2 + (p - 1)3 = 3 + (q - 1) \Rightarrow 3p - 1 = 4q - 1$
$\Rightarrow 3p = 4q \Rightarrow p/q = 4/3 = k$ (say)
$\therefore p = 4k,\ q = 3k$
Now, $p \le 145$ and $q < 145$
$\Rightarrow 4k \le 145$ and $3k < 145$
$\Rightarrow k \le 36\ 1/4$ and $k < 48\ 1/3$
∴ From the two, we can say that $k \le 36$
∴ There would be 36 common terms in the two A.P.'s.

**40.** $p\ (p + q) + p^2\ (p^2 + q^2) + p^3(p^3 + q^3) + \ldots$

$= (p^2 + p^4 + p^6 + \ldots) + (pq + p^2q^2 + \ldots)$

$= \frac{p^2}{1 - p^2} + \frac{pq}{1 - pq}$

$[-1< p<1$ and $-1<q<1$
$\Rightarrow -1<pq<1$ and $0<p^2<1]$

$$=\frac{p^2-2p^3q+pq}{(1-p^2)(1-pq)}$$

**41.** Let $S=\frac{1^2}{2^2}+\frac{2^2}{2^2}+\frac{3^2}{2^3}+\frac{4^2}{2^4}+\frac{5^2}{2^5}+\frac{6^2}{2^6}+\ldots$

$$\frac{S}{2}=\frac{1}{2^2}+\frac{2^2}{2^3}+\frac{3^2}{2^4}+\frac{4^2}{2^5}+\frac{5^2}{2^6}+\ldots$$

$$S-\frac{S}{2}=\frac{S}{2}=\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\frac{7}{2^4}+\frac{9}{2^5}+\frac{11}{2^6}+\ldots$$

$$\frac{S}{4}=\frac{1}{2^2}+\frac{3}{2^3}+\frac{5}{2^4}+\frac{7}{2^5}+\frac{9}{2^6}+\ldots$$

$$\frac{S}{2}=\frac{S}{4}=\frac{S}{4}=\frac{1}{2}+\frac{2}{2^2}+\frac{2}{2^3}+\frac{2}{2^4}+\frac{2}{2^5}+\ldots$$

$$=\frac{1}{2}+\left[\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\ldots\right]$$

$$=\frac{1}{2}+\frac{1/2}{1-1/2}=\frac{1}{2}+1=\frac{3}{2}\Rightarrow S=\frac{3}{2}\times 4=6$$

**42.** $1^2(3)+2^2(4)+3^2(5)+4^2(6)+\ldots 20^2(22)$

$=1^2(1+2)+2^2(2+2)+3^2(3+2)+\ldots+20^2(20+2)$

$=1^3+1^2(2)+2^3+2(2)^2+3^3+2(3)^2+\ldots+20^3+2(20)^2$

$=1^3+2^3+3^3+4^3+\ldots+20^3+2(1^2+2^2+\ldots 20^2)$

$$=\left(\frac{20(21)}{2}\right)^2+\frac{2(20)(21)(41)}{6}$$

$$=(20)(21)\left[\frac{(20)(21)}{4}+\frac{82}{6}\right]$$

$$=(20)(21)\left(\frac{1260+164}{12}\right)$$

$$=\frac{(20)(21)(1424)}{12}=49840$$

**43.** $\frac{1}{3^2-2^2}=\frac{1}{(3-2)(3+2)}=\frac{1}{4}\left[1-\frac{1}{5}\right]$

$\frac{1}{7^2-2^2}=\frac{1}{(7-2)(7+2)}=\frac{1}{4}\left[\frac{1}{5}-\frac{1}{9}\right]$

$\frac{1}{39^2-2^2}=\frac{1}{(39-2)(39+2)}=\frac{1}{4}\left[\frac{1}{37}-\frac{1}{41}\right]$

$\therefore$ The required sum is $1/4[1-1/41]=10/41$

**44.** $S_1=1(30)+2(29)+3(28)+\ldots+30(1)$

$$S_1=\sum_{n=1}^{30}n(31-n)=\sum_{n=1}^{30}n(31n-n^2)=31\sum_{n=1}^{30}n-\sum_{n=1}^{30}n^2$$

$$=(31)\frac{(30)(31)}{2}-\frac{30(31)(61)}{6}=\frac{(30(31))}{6}[93-61]$$

$$=\frac{(30)(31)(32)}{6}$$

$S_2=1(60)+2(59)+3(58)+\ldots+60(1)$

$$=\sum_{n=1}^{60}n(61-n)=\sum_{n=1}^{60}(61n-n)=61\sum_{n=1}^{60}n-\sum n^2$$

$$=(61)\frac{(60)(61)}{2}-\frac{60(61)(121)}{6}=\frac{60(61)}{6}(183-121)$$

$$=\frac{60(61)(62)}{6}$$

$$\therefore\frac{2S_2}{S_1}=(2)\frac{60(61)(62)}{30(31)(32)}=\frac{61}{4}$$

**45.** $S=\frac{1}{7}+\frac{3}{7^2}+\frac{7}{7^3}+\frac{13}{7^4}+\ldots$ (1)

Multiplying both sides by $\frac{1}{7}$,

$$\frac{1}{7}S=\frac{1}{7^2}+\frac{3}{7^3}+\frac{7}{7^4}+\frac{13}{7^5}+\ldots \quad (2)$$

(1) − (2) gives

$$\frac{6}{7}S=\frac{1}{7}+\left[\frac{2}{7^2}+\frac{4}{7^3}+\frac{6}{7^4}+\frac{8}{7^5}+\ldots\right] \quad (3)$$

Again multiplying by $\frac{1}{7}$,

$$\frac{6}{7^2}S=\frac{1}{7^2}+\frac{2}{7^3}+\frac{4}{7^4}+\frac{6}{7^4}+\frac{8}{7^6}+\ldots \quad (4)$$

(3) − (4) gives

$$\frac{36}{49}S=\frac{1}{7}+\frac{1}{7^2}+\frac{2}{7^3}+\frac{2}{7^4}+\frac{2}{7^5}+\frac{2}{7^6}+\ldots$$

$$\frac{36S}{49}=\frac{7+1}{49}+2\left(\frac{1}{7^3}+\frac{1}{7^4}+\frac{1}{7^5}+\frac{1}{7^6}+\ldots\right)$$

$$\frac{36S}{49}=\frac{8}{49}+\frac{2}{7^3}\left(1+\frac{1}{7}+\frac{1}{7^2}+\frac{1}{7^3}+\ldots\right)$$

$$=\frac{8}{49}+\frac{2}{343}\frac{1}{1-\frac{1}{7}}=\frac{8}{49}+\frac{2}{343}\left(\frac{7}{6}\right)=\frac{25}{147}$$

$$\therefore S=\frac{49(25)}{14(36)}=\frac{25}{108}$$

## EXERCISE-3

**1.** $t_3=a+2d=15$ (1)

$t_{23}=a+22d=55$ (2)

(2) − (1)

$\Rightarrow 20d=40$

$d=2$

$a=11$

$t_{40}=a+39d=11+78=89$

**2.** Let the terms be $a-d$, $a$, $a+d$.
$3a = 18 \Rightarrow a = 6$
$(6-d)\ (6+d) = 20$
$36 - d^2 = 20$
$d^2 = 16 \Rightarrow d = \pm 4$
The terms are 2, 6, 10 or 10, 6, 2.

**3.** First term that is common is 12. Next term will be 24. [L.C.M of (3, 4) = 12].
$12 + (n-1)\ 12 \le 102$
$(n-1)\ 12 \le 90$
$(n-1) \le 7$
$n = 8$

**4.** $11 + 11r + 11r^2 + \ldots 11r^{n-1} = 11253$
$11\ (1 + r + r^2 + \ldots r^{n-1}) = 11253$
$11\dfrac{\left(r^{n-1}\right)}{r-1} = 11253$
$\dfrac{r\left(11r^{n-1}\right)-11}{r-1} = 11253$
$\dfrac{r(5632)-11}{r-1} = 11253$
$11253r - 11253 = r\,(5632) - 11$
$5621r = 11242$
$r = 2$

**5.** Required value $= t_1 \times t_2 \times t_3 \times \ldots \times t_9$
$= a \times ar \times ar^2 \times \ldots ar^8$
$= a^9.r^{36} = (ar^4)^9 = 3^9 = 19683$

**6.** $\left(\dfrac{a}{r}\right) \times (a) \times (ar) = 27$
$a^3 = 27$
$a = 3$
$\dfrac{3}{r} + 3 + 3r = 13$
$3r^2 + 3r + 3 = 13r$
$3r^2 - 10r + 3 = 0$
$3r^2 - 9r - rr + 3 = 0$
$3r - (rr - 3)\ (rr - 3) = 0$
$(3r-1)\ (r-3) = 0$
$r = \dfrac{1}{3}, 3.$
The numbers are 1, 3, 9 or 9, 3, 1

**7.** $S_6 = a\,.\,\dfrac{r^6-1}{r-1} = 2\,.\,\dfrac{3^6-1}{3-1} = 728$

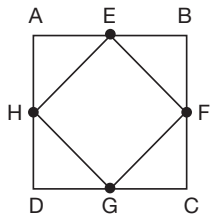
**8.** $t_4 = a + 3d = 24$
$t_{44} = a + 43d = 124$
$t_4 + t_{44} = 2a + 46d = 148$
$t_{24} = a + 23d = 74$

**9.** $S_6 = t_1 + t_2 + t_3 + t_4 + t_5 + t_6$
$S_8 = t_1 + t_2 + t_3 + t_4 + \ldots + t_7 + t_8$
$t_7 + t_8 = 0$
$S_{14} = \dfrac{14}{2}\{T_1 + T_{14}\} = \dfrac{14}{2}\{T_7 + T_8\} = 0$

**10.** $S_4 = t_1 + t_2 + t_3 + t_4 = 6$ (1)
$S_6 = t_1 + t_2 + t_3 + t_4 + t_5 + t_6 = 4$ (2)
From (2) – (1),
$t_5 + t_6 = -2$
$\Rightarrow a + 4d + a + 5d = -2$
$2a + 9d = -2$
$S_{10} = \dfrac{10}{2}\{2a + 9d\}$
$= \dfrac{10}{2}\ (-2) = -10$

**11.** $(6a+12)^2 = (10a+36)\ 4a$
$36a^2 + 144a + 144 = 40a^2 + 144a$
$144 = 4a^2$, $a = 6$ as $a > 0$.
Sixth term of the G.P. $= (10a+36)\left(\dfrac{6a+12}{10a+36}\right)^5$
$= 96\left(\dfrac{48}{96}\right)^5 = 3$

**12.**




Let $S_1$ be ABCD shown above. Let the side of the square $S_1$ be $a$.
Let the midpoints of AB, BC, CD and AD be E, F, G and H, respectively.
$FE = \sqrt{BE^2 + BF^2} = \sqrt{\left(\dfrac{a}{2}\right)^2 + \left(\dfrac{a}{2}\right)^2} = \dfrac{a}{\sqrt{2}}$. The side of the square formed by joining the midpoints of ABCD, i.e., EF is $\dfrac{1}{\sqrt{2}}$ times the length of the side of the square ABCD. Similarly, each of the squares formed by joining midpoints of its outer square will have a side equal to $\dfrac{1}{\sqrt{2}}$ times the length of the side of its outer square. Sum of the perimeters of all the squares formed is:

$$= 4a + \frac{4a}{\sqrt{2}} + \frac{4a}{\left(\sqrt{2}\right)^2} + \ldots\infty = \frac{4a}{1 - \frac{1}{\sqrt{2}}} = \frac{4a\sqrt{2}}{\sqrt{2}-1}$$

As $a = 20$, sum of the perimeters of all the squares formed

$$= \frac{80\sqrt{2}}{\sqrt{2}-1} = 80\sqrt{2}\left(\sqrt{2}+1\right)$$

**13.** $2(3x+2y) = 5x+y+3x+y$

$6x+4y = 8x+2y \Rightarrow x = y$

$[3(x+1)]^2 = (5x+1)\,3x$

$9x^2+18x+9 = 15x^2+3x$

$6x^2-15x-9 = 0$

Dividing both sides by 3.

$2x^2-5x-3 = 0$

$(2x+1)(x-3) = 0$

$x = \dfrac{-1}{2}$ or 3; going by the choices, $x = 3$.

Choice (D) follows.

**14.** The height to which the ball rebounds after the first fall $= 36\left(\dfrac{1}{3}\right) = 12$m, the 2nd time it falls from 12 m height.

The distance the ball rebounds after the second time it falls $= 36\left(\dfrac{1}{3}\right)\left(\dfrac{1}{3}\right) = 4$ m, it falls from 4 m height

Similarly, the distance rebounded by the ball after every time can be found to be $\dfrac{1}{3}$ of distance rebounded after the previous time it fell.

Total distance it covers. $36+12+12+4+4+\ldots\infty$

$= (36+12+4+\ldots\infty)+(12+4+\ldots\infty)$

$= \left(\dfrac{36}{1-1/3}\right)+\left(\dfrac{12}{1-1/3}\right) = 54+18 = 72$ m.

**15.** Let the numbers in geometric progression be $\dfrac{a}{r}$, $a$ and $ar$.

$\left(\dfrac{a}{r}\right)(a)(ar) = 1728$

$a^3 = 1728$

$a = \sqrt[3]{1728} = 12$

$\dfrac{a}{r}(ar)+a(ar)+\dfrac{a}{r}(a) = 456$

$a^2\left(1+r+\dfrac{1}{r}\right) = 456$

$\dfrac{r+r^2+1}{r} = \dfrac{456}{12^2} = \dfrac{19}{6}$

$6r+6r^2+6 = 19r$

$6r^2-13r+6 = 0$

$6r^2-9r-4r+6 = 0$

$3r(2r-3)-2(2r-3) = 0$

$(3r-2)(2r-3) = 0$, $rr = 2/3$ or $3/2$

The least number $= \dfrac{12}{3/2} = 8$.

**16.** $S_{20} = \dfrac{20}{2}[2a+19d] = 420$

$2a+19d = 42$ (1)

$S_5 = \dfrac{5}{2}[2a+4d] = 30$

$2a+4d = 12$ (2)

Solving (1) and (2), we get

$d = 2$ and $a = 2$

The largest term will be the 20th term

$T_{20} = a+19d = 2+19\times 2 = 40$

**17.** $t_n = S_n - S_{n-1}$

$= 2n^2+4n-[2(n-1)^2+4(n-1)] = 4n+2$

$\therefore$ Sum of the squares of the terms $= S(4n+2)^2$

$= 16\Sigma n^2+16\Sigma n+4n$

$= \dfrac{16n(n+1)(2n+1)}{6}+\dfrac{16n(n+1)}{2}+4n$

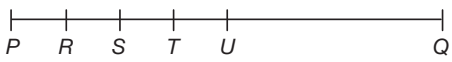
When $n = 10$,

Required sum

$= \dfrac{16\times 10\times 11\times 21}{6}+\dfrac{16\times 10\times 11}{2}+4\times 10$

$= 6160+880+40 = 7080$

**18.**




Let $P$, $R$, $S$, $T$ and $U$ be the locations of the stones.

Let $PR = RS = ST = TU = 4$ m.

The distance covered when the first, second, third, fourth and fifth stones are moved to $Q$ is

$= 200+2(196+192+188+184) = 1720$ m.

**19.** $r = -b$

$\therefore a = \dfrac{b}{1-(-b)}$

$\Rightarrow a+ab = b$

$\Rightarrow b(1-a) = a \Rightarrow b = a/(1-a)$

**20.** Let the first term and the common difference of the progression be $a$ and $d$, respectively.

$\dfrac{13}{2}[2a+12d] = \dfrac{27}{2}[2a+26d]$

$13a+78d = 27a+351d$

$0 = 2a+39d$

$\therefore$ The sum of the first 40 terms $= \dfrac{40}{2}[2a+39d] = 0$

**21.** Let $n$th term be $\dfrac{n}{1+n^2+n^4} = \dfrac{n}{(1+n+n^2)(1-n+n^2)}$

$t_n = \dfrac{1}{2}\left[\dfrac{1}{1-n+n^2}-\dfrac{1}{1+n+n^2}\right]$

$t_1 = \dfrac{1}{2}\left[\dfrac{1}{1-1+1}-\dfrac{1}{1+1+1}\right] = \left[1-\dfrac{1}{3}\right]\times\dfrac{1}{2}$

$$t_2 = \frac{1}{2}\left[\frac{1}{1-2+2^2} - \frac{1}{1+2+2^2}\right] = \left[\frac{1}{3} - \frac{1}{7}\right] \times \frac{1}{2}$$

$$t_3 = \frac{1}{2}\left[\frac{1}{1-3+3^2} - \frac{1}{1+3+3^2}\right] = \left[\frac{1}{7} - \frac{1}{13}\right] \times \frac{1}{2}$$

$$t_{10} = \frac{1}{2}\left[\frac{1}{1-10+10^2} - \frac{1}{1+10+10^2}\right] = \left[\frac{1}{91} - \frac{1}{111}\right] \times \frac{1}{2}$$

$$t_1 + t_2 + t_3 + \ldots + t_{10} = \frac{1}{2}\left[1 - \frac{1}{111}\right] = \frac{55}{111}$$

**22.** The $i^{\text{th}}$ term is $t_i = 1 + 2 + \ldots + i = \dfrac{i(i+1)}{2}$

$$\therefore S_n = \sum_{i=1}^{n} t_i = \sum \frac{i^2 + i}{2} = \frac{1}{2}(\Sigma i^2 + \Sigma i) = \frac{n(n+1)(2n+1)}{12}$$

$$+ \frac{n(n+1)}{4} = \frac{n(n+1)}{12}(2n+1+3) = \frac{n(n+1)(n+2)}{6}$$

(For $n = 1, 2, 3$, $S_n = 1, 4, 10$, respectively).

**23.** $\left(1+\frac{1}{3}\right)\left(1+\frac{1}{9}\right)\left(1+\frac{1}{81}\right)\left(1+\frac{1}{6561}\right)$

$$= \frac{\left(1-\frac{1}{3}\right)}{\left(1-\frac{1}{3}\right)}\left(1+\frac{1}{3}\right)\left(1+\frac{1}{9}\right)\left(1+\frac{1}{81}\right)\left(1+\frac{1}{6561}\right)$$

$$= \frac{1}{\frac{2}{3}}\left(1-\frac{1}{9}\right)\left(1+\frac{1}{9}\right)\left(1+\frac{1}{81}\right)\left(1+\frac{1}{6561}\right)$$

$$= \frac{3}{2}\left(1-\frac{1}{(6561)^2}\right) = \frac{3}{2}\left(\frac{6562}{6561}\right)\left(\frac{6560}{6561}\right)$$

$$= \frac{(3281)(6560)}{(2187)(6561)}$$

**24.** Taking $x = 1/6$, the expression becomes

$$\begin{aligned} S &= 1 + 4x + 9x^2 + 16x^3 + 25x^4 + \ldots \infty \\ Sx &= \quad x + 4x^2 + 9x^3 + 16x^4 + \ldots \infty \end{aligned}$$

$$\begin{aligned} (1-x)S &= 1 + 3x + 5x^2 + 7x^3 + 9x^4 + \ldots \infty \\ x(1-x)S &= \quad x + 3x^2 + 5x^3 + 7x^4 + \ldots \infty \\ (1-x)^2 S &= 1 + 2x + 2x^2 + 2x^3 + 2x^4 + \ldots \infty \\ (1-x)^2 S &= 1 + 2x(1 + x + x^2 + x^3 + \ldots \infty) \\ (1-x)^2 S &= 1 + 2x/(1-x) \\ S &= (1+x) \;/\; (1-x)^3 \\ S &= (1+1/6) \;/\; (1-1/6)^3 = 252/125 \end{aligned}$$

**25.** $\dfrac{1}{5\times 7} = \dfrac{1}{2}\left[\dfrac{1}{5} - \dfrac{1}{7}\right]$

$\dfrac{1}{7\times 9} = \dfrac{1}{2}\left[\dfrac{1}{7} - \dfrac{1}{9}\right]$, etc

So, the sum is

$$\frac{1}{2}\left[\frac{1}{5} - \frac{1}{7}\right] + \frac{1}{2}\left[\frac{1}{7} - \frac{1}{9}\right] + \ldots\infty$$

$$= \frac{1}{2}\left[\frac{1}{5} - \frac{1}{7} + \frac{1}{7} - \frac{1}{9} + \ldots\infty\right]$$

$$= \frac{1}{2} \times \frac{1}{5} \text{ (All other terms cancel out)} = \frac{1}{10}$$

**26.** $S_1 = \sqrt{1 + \frac{1}{1^2} + \frac{1}{2^2}} = \sqrt{\frac{4+4+1}{4}} = \frac{3}{2} = 2 - \frac{1}{2}$

$$S_2 = \sqrt{1 + \frac{1}{1^2} + \frac{1}{2^2}} + \sqrt{1 + \frac{1}{2^2} + \frac{1}{3^2}} = \frac{3}{2} + \frac{7}{6} = 3 - \frac{1}{3}$$

$$S_3 = \sqrt{1 + \frac{1}{1^2} + \frac{1}{2^2}} + \sqrt{1 + \frac{1}{2^2} + \frac{1}{3^2}} + \sqrt{1 + \frac{1}{3^2} + \frac{1}{4^2}}$$

$$= \frac{3}{2} + \frac{7}{6} + \frac{13}{2} = 4 - \frac{1}{4}$$

Similarly

$$S_4 = 5 - \frac{1}{5};\ S_5 = 6 - \frac{1}{6};\ S_6 = 7 - \frac{1}{7}$$

$$S_7 = 8 - \frac{1}{8}$$

$$\therefore S_4 + S_5 + S_6 + S_7 = 5 - \frac{1}{5} + 6 - \frac{1}{6} + 7 - \frac{1}{7} + 8 - \frac{1}{8}$$

$$= 26 - \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right)$$

$$= 26 - \left(\frac{1}{5} + \frac{1}{7} + \frac{1}{6} + \frac{1}{8}\right) = 26 - \left(\frac{12}{35} + \frac{7}{24}\right) = 26 - \frac{533}{840}$$

$$= 25 + \frac{307}{840}$$

**NOTE**

The algebraic proof of the result (required to simplify each term) is given below

$$1 + \frac{1}{n^2} + \frac{1}{(n+1)^2} = \frac{n^2(n+1)^2 + (n+1)^2 + n^2}{n^2(n+1)^2}$$

$$= \frac{n^2(n+1)^2 + 2n^2 + 2n + 1}{n^2(n+1)^2} = \left[\frac{n(n+1)+1}{n(n+1)}\right]^2$$

$$\therefore \sqrt{1 + \frac{1}{n^2} + \frac{1}{(n+1)^2}} = \frac{n(n+1)+1}{n(n+1)}$$

$$= 1 + \frac{1}{n(n+1)} = 1 + \frac{1}{n} - \frac{1}{n+1}$$

**27.** $\frac{1}{a(a+a)(a+2d)} = \frac{1}{2d^2}\left[\frac{1}{a} - \frac{2}{a+d} + \frac{1}{a+2d}\right]$

$\frac{1}{2(5)(8)} = \frac{1}{2(3)^2}\left[\frac{1}{2} - \frac{2}{5} + \frac{1}{8}\right]$

$\frac{1}{5(8)(11)} = \frac{1}{2(3)^2}\left[\frac{1}{5} - \frac{2}{8} + \frac{1}{11}\right]$

$\vdots$

$\frac{1}{41(44)(47)} = \frac{2}{2(3)^2}\left[\frac{1}{41} - \frac{2}{44} + \frac{1}{47}\right]$

$\frac{1}{41(47)(50)} = \frac{2}{2(3)^2}\left[\frac{1}{44} - \frac{2}{47} + \frac{1}{50}\right]$

When all these brackets are added only the first two terms from the first bracket, the first term from the second, the third term of the 14th bracket and the second and third terms of the 15th bracket remain.

$\therefore \frac{1}{2(5)(8)} + \frac{1}{5(8)(11)} + \frac{1}{8(11)(14)} + \ldots + \frac{1}{44(47)(50)}$

$= \frac{1}{18}\left[\frac{1}{2} - \frac{2}{5} + \frac{1}{5} + \frac{1}{47} - \frac{2}{47} + \frac{1}{50}\right]$

$= \frac{1}{18}\left[\frac{1}{2} - \frac{1}{5} - \frac{1}{47} + \frac{1}{50}\right] = \frac{351}{21150} = \frac{39}{2350}$

**28.** $\frac{1}{1+2^{\frac{1}{3}}+2^{\frac{2}{3}}} = \frac{2^{\frac{1}{3}}-1}{\left(1+2^{\frac{2}{3}}+2^{\frac{2}{3}}\right)\left(2^{\frac{1}{3}}-1\right)} = \frac{2^{\frac{1}{3}}-1}{2-1} = 2^{\frac{1}{3}}-1$

$\frac{1}{2^{\frac{2}{3}}+6^{\frac{1}{3}}+3^{\frac{2}{3}}} = \frac{3^{\frac{1}{3}}-2^{\frac{1}{3}}}{\left(2^{\frac{2}{3}}+6^{\frac{1}{3}}+3^{\frac{2}{3}}\right)\left(3^{\frac{1}{3}}-2^{\frac{1}{3}}\right)}$

$= \frac{3^{\frac{1}{3}}-2^{\frac{1}{3}}}{3-2} = 3^{\frac{1}{3}}-2^{\frac{1}{3}}$

$\frac{1}{3^{\frac{2}{3}}+12^{\frac{1}{3}}+4^{\frac{2}{3}}} = \frac{4^{\frac{1}{3}}-3^{\frac{1}{3}}}{\left(3^{\frac{2}{3}}+12^{\frac{1}{3}}+4^{\frac{2}{3}}\right)\left(4^{\frac{1}{3}}-3^{\frac{1}{3}}\right)}$

$= \frac{4^{\frac{1}{3}}-3^{\frac{1}{3}}}{4-3} = 4^{\frac{1}{3}}-3^{\frac{1}{3}}$

Similarly,

$\frac{1}{(63)^{\frac{2}{3}}+(4032)^{\frac{1}{3}}+(64)^{\frac{2}{3}}} = \frac{(64)^{\frac{1}{3}}-(63)^{\frac{1}{3}}}{64-63}$

$= (64)^{\frac{1}{3}}-(63)^{\frac{1}{3}}$

$\therefore$ Required sum

$= 2^{\frac{1}{3}}-1+3^{\frac{1}{3}}-2^{\frac{1}{3}}+4^{\frac{1}{3}}-3^{\frac{1}{3}}+\ldots+(64)^{\frac{1}{3}}-(63)^{\frac{1}{3}}$

$= -1+(64)^{\frac{1}{3}} = 4-1 = 3$

**29.** $X = \frac{1}{(90)(46)} + \frac{1}{(89)(47)} + \ldots + \frac{1}{(46)(90)}$

$= \frac{1}{136}\left[\frac{136}{(90)(46)} + \frac{136}{(89)(47)} + \ldots + \frac{136}{(46)(90)}\right]$

$= \frac{1}{136}\left[\frac{1}{46} + \frac{1}{90} + \frac{1}{47} + \frac{1}{89} + \ldots + \frac{1}{90} + \frac{1}{46}\right]$

$X = \frac{2}{136}\left[\frac{1}{46} + \frac{1}{47} + \frac{1}{48} + \cdots \frac{1}{90}\right]$

$Y = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots - \frac{1}{90}$

$= \left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \frac{1}{90}\right] - 2\left[\frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{90}\right]$

$= \left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{90}\right] - \left[\frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{45}\right]$

$= \frac{1}{46} + \frac{1}{47} + \frac{1}{48} + \cdots + \frac{1}{90}$

(i.e., the sum of the reciprocals of all the odd numbers upto $2n$ minus the sum of the reciprocals of all the even numbers upto $2n$ is equal to the sum of the reciprocals of numbers from $n+1$ to $2n$.)

$\frac{X}{Y} = \frac{\frac{2}{136}\left[\frac{1}{46} + \frac{1}{47} + \frac{1}{48} + \cdots + \frac{1}{90}\right]}{\frac{1}{46} + \frac{1}{47} + \frac{1}{48} + \cdots \frac{1}{90}} = \frac{1}{68}$

**30.** Given series is $150 \times 2 + 148 \times 4 + 146 \times 6 + \ldots$ 75 terms.

The numbers 150, 148, 146 … from an arithmetic progression, with $a = 150$, $d = -2$.

The numbers 2, 4, 6 … from an arithmetic progression with $a = 2$, $d = 2$.

Hence, $t_n$, the $n$th term of the series is:

$[150 + (n-1)(-2)][2 + (n-1)2] = (152 - 2n)2n$.

$t_n = 304n - 4n^2$.

Hence, sum to 75 terms is:

$s_{75} = 304(1 + 2 + 3 \ldots + 75) - 4(1^2 + 2^2 + 3^2 + \ldots 75^2)$.

$= 304 \times \frac{75 \times 76}{2} - 4 \times \frac{75 \times 76 \times 151}{6} = 292600$

**31.** Let $b = a + d$,

$c = a + 2d$

As $a$, $a + 2d$, $a + d$ are in G.P.,

$(a + 2d)^2 = a(a + d)$

$\Rightarrow a^2 + 4ad + 4d^2 = a^2 + ad$

$\Rightarrow 4d^2 = -3ad$

$\Rightarrow d(4d + 3a) = 0$

$\Rightarrow d = 0$ or $d = -\frac{3}{4}a$.

If $d = 0$, $\frac{a}{c} = 1$, which is violating the condition that $r \neq 1$.

$\therefore d = -\frac{3}{4}a$.

$\frac{a}{b} = \frac{a}{a - \frac{3}{4}a} = \frac{a}{a/4} = 4.$

**32.** $\log_2 2 + \log_2 2^2 + \log_2 2^3 + \ldots$up to $n$ terms

$(\log_2 2)\ [1 + 2 + 3 + 4 + \ldots$up to $n$ terms$] = \left[\frac{n(n+1)}{2}\right]$

**33.** Given series is $3.4^2 + 4.5^2 + 5.6^2 + \ldots$

In series $1 \times 2^2 + 2 \times 3^2 + 3 \times 4^2 + 4 \times 5^2 + 5 \times 6^2$

The $n^{th}$ term will be of the form $T_n = n(n+1)^2$

$S_n = \sum T_n = \sum n(n^2 + 1 + 2n)$

$= \sum n^3 + \sum n + 2\sum n^2$

We are asked to find the sum of 15 terms, and first term is starting with '3'

$\therefore$ We have to take $n = 17$

$\left[\frac{17(18)}{2}\right]^2 + \left[\frac{17 \times 18}{2}\right] + 2\left[\frac{17(18)(35)}{3 \times 2}\right]$

$= (17 \times 9)^2 + (17 \times 9) + (17 \times 6 \times 35)$

$= 23409 + 153 + 3570 = 27132$

This includes the sum of 17 terms of the series which has 2 extra terms. They are $1.2^2$ and $2.3^2$. So, we have to subtract this total from the above result, i.e., subtract 1.4 + 2.9 = 22

$\therefore 27132 - 22 = 27110$

**34.** Option (A):

$r = \frac{1}{2}$, $a = \frac{1}{2}$

$S_\infty = \frac{a}{1-r} = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$

Option (B):

$r = \frac{3}{4}$, $a = \frac{1}{4}$

$S_a = \frac{\frac{1}{4}}{1 - \frac{3}{4}} = \frac{\frac{1}{4}}{\frac{1}{4}} = 1$

Series is in the option C is not in G.P.

$\therefore$ Both (A) & (B) satisfy.

**35.** Let the terms be $\frac{a}{r}$, $a$, $ar$

Given, $\frac{a}{r} \times a \times ar = 343$

$a^3 = 343, \Rightarrow a = 7$

But given $\frac{a}{r} \times a + a \times ar + ar \times \frac{a}{r} = 171.5$

$a^2\left(\frac{1}{r} + r + 1\right) = 171.5$

$7^2(1 + r^2 + r) = 171.5 \times r$

$1 + r^2 + r = \frac{7}{2}r$

$2 + 2r^2 + 2r - 7r = 0$

$2r^2 - 5r + 2 = 0$

$2r(r-2) - 1(r-2) = 0$

$(r-2)(2r-1) = 0$

$r = 2$, $r = \frac{1}{2}$

If we substitute $r = 2$

The series is $\frac{7}{2}$, 7, 14

If $r = \frac{1}{2}$, then the series is

14, 7, $\frac{7}{2}$

Anyhow, the largest number is 14

**36.** Let the first term and common difference of the arithmetic progression be $a$ and $d$, respectively.

Given that $t_7^2 + t_{11}^2 = 482$

$\Rightarrow (a + 6d)^2 + (a + 10d)^2 = 482$

$\Rightarrow 2a^2 + 32ad + 136d^2 = 482$ (1)

and $(a + d)(a + 15d) = 29$

$\Rightarrow a^2 + 16ad + 15d^2 = 29$ (2)

By (1) $- 2 \times$ (2), we get

$106d^2 = 482 - 58 = 424$

$\Rightarrow d^2 = 4$ (3)

Consider, $t_1 \times t_{17} = a(a + 16d) = a^2 + 16ad$

$= (2) - 15 \times (3) = 29 - 15 \times 4 = -31$

**37.** Let the G.P. be 11, $11r$, $11r^2$. $r \neq 1$.

The 1st, 9th and 31st terms of the arithmetic progression are 11, 11 + 8$d$, 11 + 30$d$.

$\therefore 11r = 11 + 8d \Rightarrow 11(r-1) = 8d$ (1)

$11r^2 = 11 + 30d \Rightarrow 11(r^2 - 1) = 30d$ (2)

(2) $\div$ (1) $\Rightarrow r + 1 = \frac{15}{4} \Rightarrow r = \frac{11}{4}$.

**38.** To carry the first sapling to plant it and return, Raju has to travel 40 m. To carry the second one to plant it and return, Raju has to travel 80 m. Hence, until he plants the final sapling ($n^{th}$ sapling), he will have to travel

$40(1 + 2 + \ldots n) = 2200$ (given)

$\Rightarrow \frac{n(n+1)}{2} = 55 \Rightarrow n_2 + n - 110 = 0 \Rightarrow n = 10$ or $-11$

Hence, $n = 10$, as $n > 0$.

**39.** If $r$ is the common ratio of the geometric progression and $d$ is the common difference of arithmetic progression, first term of arithmetic progression = $r$ and that of geometric progression = $d$.

$\frac{11}{2}[2r + (11-1)d] = 341$

$\Rightarrow r + 5d = 31$ (1)

$\Rightarrow d + dr + dr^2 = 18$ (2)

As arithmetic progression consists of only integers, both $r$ and $d$ are integers. So, geometric progression also consists of only integers. By trial and error we get $r = 1$ and $d = 6$. $\therefore\ r + d = 7$

**40.** $T_n = a + (n-1)\ d$

($a$ = first term, $d$ = common difference, $T_n = n^{\text{th}}$ term)

$T_{10} = a + 9d = 11$ (1)

$T_{11} = a + 10d = 10$ (2)

Solving (1) and (2)

$d = -1,\ a = 20$

Say '$n$' terms are needed so as to make sum = 0

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 0 \Rightarrow \frac{n}{2}[40 + (n-1)(-1)] = 0$

$\Rightarrow 40 - n + 1 = 0 \Rightarrow n = 41$

First 41 terms of the arithmetic progression when added give the sum as zero

**41.** Number of members in $C_1$ on May 1, 2003 = $x + 4a$

Number of members in $C_2$ on May 1, 2003 = $xb^4$

$x + 4a = xb^4$ and $a = 20x$

$\therefore\ x(b^4 - 81) = 0$

As $x \neq 0$, $b^4 - 81 = 0$

$\therefore\ b = 3$

**42.** The first 50 terms of the given series are $-14, -18, \ldots, -210$. These are in A.P. whose first term is $-14$ and common difference is $-4$

Sum of the first 50 terms $= \frac{50}{2}[-14 + (-210)] = -5600$

**43.** $T_n = (n-1) + 3\ T_{n-1}$

$= (n-1) + 3^1(n-2) + 3^2\ T_{n-2}$

$= (n-1) + 3^1(n-2) + 3^2\ (n-3) + 3^3\ T_{n-3}$

$= (n-1) + 3^1(n-2) + \ldots + 3^{n-2}(1) + 3^{n-1}T_1$

(The sum of the index of 3 and the coefficient of the power of 3 is $n-1$ for each term except the last)

Let

$S_n = \quad (n-1) + 3\ (n-2) + 3^2(n-3) + \ldots + 3^{n-2}(1)$ (1)

$3S_n = +3(n-1) + 3^2(n-2) + \ldots + 3^{n-2}(2) + 3^{n-1}(1)$ (2)

(2) − (1) $2S_n = -(n-1) + 3 + 3^2 + \ldots 3^{n-2} + 3^{n-1}$

$= -(n-1) + 3\frac{3^{n-1} - 1}{2}$

$S_n = -\frac{(n-1)}{2} + 3\ \frac{(3^{n-1} - 1)}{4}.$

$\therefore T_n = \frac{3}{4}(3^{n-1} - 1) - \left(\frac{n-1}{2}\right) + 3^{\ n-1}T_1$

$= \frac{3}{4}\ (3^{n-1} - 1) - \left(\frac{n-1}{2}\right) + 3^n \qquad (\because T_1 = 3)$

$T_{100} = \frac{3}{4}(3^{99} - 1) - \frac{99}{2} + 3^{100}$

$= \frac{3^{100}}{4} + 3^{100} - \frac{3}{4} - \frac{99}{2}$

$= \frac{5.3^{100} - 201}{4}$

**44.** $2 + 13 + 28 + 47 + 70 + \ldots$

$= 4 - 2 + 15 - 2 + 30 - 2 + 49 - 2 + 72 - 2 + \ldots$

$= 1(4) - 2 + 3(5) - 2 + 5(6) - 2 + 7(7) - 2 + \ldots$

$\sum_{n=1}^{20}[(2n-1)(n+3) - 2]$

$= \sum_{n=1}^{20}(2n^2 + 5n - 5)$

$= 2\sum_{n=1}^{20} n^2 + 5\sum_{n=1}^{20} n - 5\sum_{n=1}^{20} 1$

$= \frac{2(20)(21)(41)}{6} + \frac{5(20)(21)}{2} - 5(20) = 6690$

**45.** Let the number of layers in the pile be $N$.

Number of balls in the Kth layer where $K \geq 1 = K(K+1)$

$\sum_{K=1}^{N} K(K+1) = 3080$

$\sum_{K=1}^{N} K^2 + K = 3080$

$\frac{1}{6}N(N+1)(2N+1) + \frac{N(N+1)}{2} = 3080$

$N(N+1)(N+2) = 9240 = (20)\ (21)\ (22)$

Comparing the two sides, $N = 20$

## EXERCISE-4

**1.** $\frac{1}{5.10}+\frac{1}{10.15}+\frac{1}{15.20}\ldots\infty$ terms

$=\left(\frac{1}{5}-\frac{1}{10}\right)\frac{1}{5}+\left(\frac{1}{10}-\frac{1}{5}\right)\frac{1}{5}+\left(\frac{1}{15}-\frac{1}{20}\right)\frac{1}{5}+\infty$

$=\frac{1}{5}\left(\frac{1}{5}-\frac{1}{10}+\frac{1}{10}-\frac{1}{15}+\frac{1}{15}-\frac{1}{20}+\frac{1}{20}\ldots\infty\right)$

$=\frac{1}{5}\times\frac{1}{5}=\frac{1}{25}$

**2.** $S=1+(1+b)r+(1+b+b^2)r^2+(1+b+b^2+b^3)r^3+\ldots$
multiplying both sides by $r$

$rS=r+(1+b)r^2+(1+b+b^2)r^3+(1+b+b^2+b^3)r^4+\ldots$

Subtracting the second equation from the first equation, we get $(1-r)S=1+br+b^2r^2+\ldots$ The R.H.S is a G.P. with first term as 1 and common ratio as $br$.

The sum of this series on RHS $= a/(1-br)$

i.e., $(1-r)S=1/(1-br)$; So, $S=1/[(1-r)\ (1-br)]$

**3.** $S=1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}\ldots$

$1/5\ .\ S=1/5+4/5^2+7/5^3+10/5^4+\ldots$

(Multiplying both sides by 1/5)

Subtracting the second equation from the first equation,

$(1-1/5)S=1+3/5+3/5^2+3/5^3+\ldots$

$=1+3[(1/5)/1-1/5]=1+3(1/4)=1+3/4=7/4$

$S=7/4\times5/4=35/16$

**4.** $S_n=\frac{1^3}{2}+\frac{1^3+2^3}{2+4}+\frac{1^3+2^3+3^3}{2+4+6}+\ldots+\frac{1^3+2^3+\ldots n^3}{2+4+6+\ldots+2n}$

$\therefore t_n=\frac{1^3+2^3+\ldots+n^3}{2+4+6+\ldots+2n}$

$=\frac{\sum n^3}{2(1+2+\ldots+n)}=\frac{\sum n^3}{2\sum n}$

$=\frac{1}{2}\frac{n^2(n+1)^2}{4}\frac{n(n+1)}{2}=\frac{n(n+1)}{4}$

$\therefore S_n=\sum t_n$

$=\sum\frac{n(n+1)}{4}=\frac{1}{4}\left[\sum n^2+\sum n\right]$

$=\frac{1}{4}\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}$

$=\frac{n(n+1)(n+2)}{12}$

**5.** Given $7^{1+x}+7^{1-x}$, $\frac{p}{2}$ and $10^x+10^{-x}$ are in A.P.

$\therefore p=7^{1+x}+7^{1-x}+10^x+10^{-x}$

$=7\cdot7^x+\frac{7}{7^x}+10^x+\frac{1}{10^x}$

$=7\left(7^x+\frac{1}{7^x}\right)+\left(10^x+\frac{1}{10^x}\right)$

We know that the sum of any positive number and its reciprocal is always greater than or equal to 2

$\therefore p\geq14+2$ ($\because 7^x$ and $10^x$ are positive).

$p\geq16$

$\therefore$ The range of $p$ is $[16,\infty)$

**6.** Given $a_1+a_{10}+a_{15}+a_{25}+a_{30}+a_{39}=270$

$(a_1+a_{39})+(a_{10}+a_{30})+(a_{15}+a_{25})=270$

We know that, in an finite arithmetic progression the sum of the terms equidistant from the beginning and end is equal to the sum of first and last term

$\therefore a_1+a_{39}=a_{10}+a_{30}=a_{15}+a_{25}$

$\therefore 3(a_1+a_{39})=270$

$a_1+a_{39}=90$ (1)

Now $a_1+a_{19}+a_{21}+a_{39}=(a_1+a_{39})+(a_{19}+a_{21})$

$=2(a_1+a_{39})$

$=2\ (90)=180$

**7.** $0.7+0.77+0.777+\ldots$

$=7\ (0.1+0.11+0.111+\ldots)$

$=\frac{7}{9}\ (0.9+0.99+0.999+\ldots\ n\text{ terms})$

$=\frac{7}{9}\left(1-\frac{1}{10}+1-\frac{1}{10^2}+1-\frac{1}{10^3}+\ldots+n\text{ terms}\right)$

$=\frac{7}{9}\left[1+1+1+\ldots n\text{ terms}-\left(\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+\ldots n\text{ terms}\right)\right]$

$=\frac{7}{9}\left\{n-\left[\frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}\right]\right\}$

$=\frac{7}{9}\left[n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]=\frac{7}{81}\left(9n+\frac{1}{10^n}-1\right)$

**8.** Let

$S=1+5x+9x^2+13x^3+\ldots$

$Sx=\quad x+5x^2+\ 9x^3+13x^4+\ldots$

$(S-Sx)=1+4x+4x^2+4x^3+\ldots$

$=1+4x(1+x+x^2+\ldots)$

$S(1-x)=1+\frac{4x}{1-x}$

$$S(1-x)=\frac{1-x+4x}{1-x}$$

$$S=\frac{1+3x}{(1-x)^2}$$

**Alternative Solution:**

Given $1+5x+9x^2+13x^3+\ldots$

The $n$th term of 1, 5, 9, 13,…

$1+(n-1)\ (4)=4n-3$

$\therefore$ The $n$th term of the series $T_n=(4n-3)x^{n-1}$

$S_n=\Sigma T_n=\Sigma[4nx^{n-1}-3x^{n-1}]$

$=4[1+2x+3x^2+4x^3+\ldots]-3[1+x+x^2+\ldots]$

$$=4\frac{1}{(1-x^2)}-\frac{3}{1-x}$$

$$S=\frac{4-3(1-x)}{(1-x^2)}=\frac{1+3x}{(1-x^2)}$$

**9.** The given series can be written as

$$\frac{3}{1^2\cdot 2^2}+\frac{5}{2^2\cdot 3^2}+\frac{7}{3^2\cdot 4^2}+\ldots$$

$$\therefore \text{ The } n^{\text{th}} \text{ term} = T_n=\frac{(2n+1)}{n^2(n+1)^2}=\frac{1}{n^2}-\frac{1}{(n+1)^2}$$

Putting $n=1, 2, 3\ \ldots n$ and adding, we get

$$S_n=1-\frac{1}{(n+1)^2}=\frac{n^2+2n}{n^2+2n+1}$$

**10.** Interest on ₹1,80,000 $=\frac{10}{100}\times 1,80,000=18,000$

$\therefore$ First instalment $=10,000+18,000=$ ₹28,000

Amount left = ₹1,70,000

Interest of ₹1,70,000 $=\frac{10}{100}\times 1,70,000=17,000$

$\therefore$Second instalment $=10,000+17,000=$ ₹27,000

Total amount paid is in 18 months.

$=\frac{18}{2}\ [2\times 28,000+17\ (-1,000)]=$ ₹3,51,000

$\therefore$ Additional amount paid $=3,51,000-1,80,000$

= ₹1,71,000

**11.** Let the first instalment be $a$ and the increment be $d$.

$(20/2)\ \{2a+19d\}=(3/4)\ 60,000$

$\Rightarrow 20a+190d=45,000$

$\Rightarrow 2a+19d=4,500$ (1)

Also, $(25/2)\ [2a+24d]=60,000$

$\Rightarrow 2a+24d=4,800$ (2)

(2) − (1) $\Rightarrow 5d=300$

$d=60$

$$\therefore a=\frac{4,500-19\times 60}{2}=\frac{3,360}{2}=1,680$$

**12.** Total distance travelled by the ball =

$25+2\ \{3/4\times 25+(3/4)^2\times 25+(3/4)^3\times 25+\ldots\text{to}\ \infty\}$

$$=25+2\times\frac{(3/4)25}{1-(3/4)}=25+150=175\text{ m}$$

**13.** Let the time taken be $n$ minutes.

$$\frac{n}{2}\ [2\times 220+(n-1)\ (-10)]=2,500$$

$5n^2-225n+2,500=0$

$\Rightarrow n^2-45n+500=0$

$\Rightarrow (n-20)\ (n-25)=0$

$\Rightarrow n=20$ or 25

Since, 20 < 25, he would have finished counting in 20 minutes.

**14.** Here, $a=500$; $d=50$

Let the loan be cleared in $n$ months

$$\frac{n}{2}\ [2\times 500+(n-1)\ 50]=25,000$$

$\Rightarrow 50n^2+950n-50,000=0$

$\Rightarrow n^2+19n-1,000=0$

Solving this equation, we find that $n\approx 23.5$

$\therefore\ n=24$

$\therefore$ The amount paid in 23 months

$$=\frac{23}{2}\ 2\times 500+22\times 50=₹24,150$$

$\therefore$ last instalment $=25,000-24,150=$ ₹850

**15.** The distances to be run form an A.P. where

$a=100$, $d=25$, $n=10$

$$\text{Total distance run}=2\times=2\times\frac{10}{2}\{200+9\times 25\}\text{m}=4,250\text{ m}$$

**16.** The maximum distance that I can cover would tend to $\frac{10}{1-1/2}=20$ km, and hence, I can never be 25 km away from the starting points.

**17.**

| If common difference | Age of Molly |
|---|---|
| = 1 | $14-2\times 1=12$ |
| = 2 | $14-2\times 2=10$ |
| = 3 | $14-2\times 3=8$ |

We cannot take c.$d$ as 4, since then the age of Lolly would be $14-4\times 4$ which is a negative value.

So, only II is acceptable.

**18.** The distance that the ball falls through in successive seconds is in A.P. with a common difference of 10 m. The distance that it falls through in the 20th second (in m) is $6+19\ (10)=196$.

**19.** Let the number of wickets be $a+d$, $a$ and $a-d$.

$(a+d)+a+(a-d)=21\Rightarrow a=7$

Now, $7+d+15$, $7+2$ and $7-d+1$

i.e., $22 + d$, 9 and $8 - d$ are in G.P.

$(22 + d)\ (8 - d) = 9^2$

$\Rightarrow 176 - 14d - d^2 = 81 \Rightarrow d^2 + 14d - 95 = 0$

$\Rightarrow (d + 19)\ (d - 5) = 0$

$\therefore d = 5$

(Since if $d = -19$; $7 + (-19) = -12$ which cannot be the number of wickets taken)

$\therefore$ The number of wickets taken by Kumble $= 7 + 5 = 12$

**20.** Let the first instalment be $a$

Then, $\frac{24}{2}\ [2a + 23\ (100)] = 32{,}400$

$\Rightarrow 24a + 27{,}600 = 32{,}400$

$\Rightarrow 24a = 4{,}800 \Rightarrow a = 200$

**21.** Let there be $n$ friends

$a + arn^{-1} = 66$

$\Rightarrow ar^{n-1} = 66 - a$ (1)

and, $ar \cdot ar^{n-2} = 128 \Rightarrow a^2 r^{n-1} = 128$

$\Rightarrow a\ [66 - a] = 128$

$\Rightarrow a^2 - 66a + 128 = 0$

$\Rightarrow (a - 64)\ (a - 2) = 0$

$a = 2$ or 64.

As the amounts are in an increasing G.P., $a = 2$.

The sum of the amounts with all the friends is 126.

i.e., $2\ \frac{r^n - 1}{r - 1} = 126 \Rightarrow \frac{r^n - 1}{r - 1} = 63$ (2)

Also, from (1);

$2\ (r^{n-1}) = 66 - 2 \Rightarrow r^{n-1} = 32 \Rightarrow r^n = 32r$

From (2);

$\frac{32r - 1}{r - 1} = 63$

$\Rightarrow 32r - 1 = 63r - 63$

$\Rightarrow 31r = 62 \Rightarrow r = 2$

Now, $2^{n-1} = 32 = 2^5$

$\Rightarrow n - 1 = 5 \Rightarrow n = 6$

**22.** The numbers in the 25th bracket are {577, 576,…, 625}

The sum of all these numbers $n\left(\frac{577 + 625}{2}\right)(49)$

$= (601)\ (49) = 29{,}449.$

**23.** Total savings in rupees $= \frac{24}{2}\ [2\ (120) + 23\ (5)] = 4{,}260$

**24.** Consider

$(1 + x)^2\ x^0 + (1 + x)\ x + (1 + x)^0\ x^2$

$= 1 + 2x + x^2 + x + x^2 + x^2$

$= 1 + 3x + 3x^2 = (1 + x)^3 - x^3$

Similarly $(1 + x)^3 \cdot x^0 + (1 + x)^2\ x + (1 + x)^1\ (x)^2 + (1 + x)^0\ x^3$

$= (1 + x)^4 - x^4$

$\therefore (1 + x)^{350}\ x^0 + (1 + x)^{349}\ x + (1 + x)^{348}\ x^2 + (1 + x)^{347}\ x^3 + \ldots + (1 + x)^0\ x^{350} = (1 + x)^{351} - x^{351}.$

The coefficients of $x^{150}$ and $x^{151}$ in the above expansion are

$^{351}C_{150}$ and $^{351}C_{151}$

$\therefore$ sum of the coefficients $=$

$^{351}C_{150} + {}^{351}C_{151} = {}^{352}C_{151}$

**Alternative Solution:**

$(1 + x)^n\ x^m$ can be represented as a series of terms. In each term, there are two factors $(1 + x)^r$ and $x^s$ and the coefficients of all terms (unlike in the binomial theorem) are all 1. We can let the indices of $1 + x$ decrease and those of $x$ increase. Thus,

$(1 + x)^n\ x^m = (1 + x)^{n-1}\ x^m\ (1 + x)$

$= (1 + x)^{n-1} x^m + (1 + x)^{n-1} x^{m+1}$

and so on.

Thus, $(1 + x)^{351} = (1 + x)^{350}\ (1 + x) = (1 + x)^{350} + (1 + x)^{350} x$ (1)

$(1 + x)^{350}\ x = (1 + x)^{349} x(1 + x) = (1 + x)^{349} x + (1 + x)^{349} x^2$ (2)

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$(1 + x)\ x^{350} = (1 + x)^0 x^{350} + x^{351}$ … (351)

Adding these 351 equations, we get

$(1 + x)^{351} = (1 + x)^{350} + (1 + x)^{349}\ x + \ldots + (1 + x)^0\ x^{350} + x^{351}$

$\therefore$ The given series is equal to $(1 + x)^{351} - x^{351}$

$T_{150} = {}^{351}C_{150}\ x^{150}$ and $T_{151} = {}^{351}C_{151}\ x^{151}$

The sum of these coefficients (i.e., $^{351}C_{150} + {}^{351}C_{151}$) is $^{352}C_{151}$

**25.** Consider I

$$1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots - \frac{1}{250}$$

$$= \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots + \frac{1}{250}\right)$$

$$-2\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \ldots + \frac{1}{250}\right)$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots + \frac{1}{250} - \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{125}\right)$$

$$= \frac{1}{126} + \frac{1}{127} + \frac{1}{128} + \ldots + \frac{1}{250}$$

Consider II

$$(188)\left[\frac{1}{(126)(250)} + \frac{1}{(127)(249)} + \frac{1}{(128)(248)} + \cdots + \frac{1}{(250)(126)}\right]$$

$$\frac{1}{2}\left[\frac{376}{(126)(250)} + \frac{376}{(127)(249)} + \ldots + \frac{376}{(250)(126)}\right]$$

$$\frac{1}{2}\left[\frac{126+250}{(126)(250)}+\frac{127+249}{(127)(249)}+\ldots+\frac{250+126}{(250)(126)}\right]$$

$$=\frac{1}{2}\left[\frac{1}{250}+\frac{1}{126}+\frac{1}{249}+\frac{1}{127}+\frac{1}{248}+\frac{1}{128}\ldots+\frac{1}{126}+\frac{1}{250}\right]$$

$$=\frac{2}{2}\left[\frac{1}{126}+\frac{1}{127}+\frac{1}{128}+\frac{1}{129}+\frac{1}{130}+\ldots+\frac{1}{250}\right]$$

$$=\frac{1}{126}+\frac{1}{127}+\frac{1}{128}+\ldots+\frac{1}{250}$$

$\therefore$ Both I and II are true.

**26.** $3(4)^2+5(5^2)+7(6^2)+9(7^2)+\ldots$

$\therefore$ the $n$th term in the above series is $t_n=(2n+1)\ (n+3)^2$

$\therefore\ t_n=(2n+1)\ (n^2+6n+9)$

$=2n^3+13n^2+24n+9$

$S_n=\Sigma t_n=2\Sigma n^3+13\Sigma n^2+24\Sigma n+9\Sigma 1$

$$=\frac{n^2(n+1)^2}{2}+13\frac{n(n+1)(2n+1)}{6}+12n(n+1)+9n$$

Put $n=20$.

$$\therefore\ S_{20}=\frac{(400)(441)}{2}+13\frac{(20)(21)(41)}{6}+12(20)(21)+9(20)$$

$=88200+37310+5040+180=130730.$



**NOTE**

We can consider the $n$th term (i.e., $t_n$) to be $(2n-5)n^2$. We would then be dealing with only two summations.

**27.** $T_n=\left(1+\frac{4}{n^2-4}\right)T_{n-1}=\left(\frac{n^2-4+4}{n^2-4}\right)T_{n-1}$

$$=\left(\frac{n^2}{n^2-4}\right)T_{n-1}=\left(\frac{n}{n-2}\cdot\frac{n}{n+2}\right)T_{n-1}$$

$$\therefore T_3=\left(\frac{3}{1}\frac{3}{5}\right)T_2=\left(\frac{3}{1}\right)\left(\frac{3}{5}\right)T_2$$

$$T_4=\left(\frac{4}{2}\frac{4}{6}\right)T_3=\left(\frac{3}{1}\frac{4}{2}\right)\left(\frac{3}{5}\frac{4}{6}\right)T_2$$

$$T_5=\left(\frac{5}{3}\frac{5}{7}\right)T_4=\left(\frac{3}{1}\frac{4}{2}\frac{5}{3}\right)\left(\frac{3}{5}\frac{4}{6}\frac{5}{7}\right)T_2$$

Similarly,

$$T_{20}=\left(\frac{3}{1}\frac{4}{2}\frac{5}{3}\ldots\frac{18}{16}\frac{19}{17}\frac{20}{18}\right)\left(\frac{3}{5}\frac{4}{6}\frac{5}{7}\ldots\frac{18}{20}\frac{19}{21}\frac{20}{22}\right)T_{20}$$

$$=\frac{3^2(4)^2 19^2(20)^2 T_2}{1(2)(3)(4)19(20)(21)(22)}=\frac{3(4)19(20)}{2(21)(22)}\left(\frac{3}{4}\right)=\frac{285}{77}$$

**28.** The sum of any three consecutive terms of the sequence is150.

$T_1+T_2+T_3=T_2+T_3+T_4=T_3+T_4+T_5=\ldots=150$

$\therefore\ T_1=T_4,\ T_2=T_5,\ T_3=T_6$, and so on.

$T_N=T_{N+3}$. This is a periodic sequence with period 3.

$T_{60}=T_3$ and $T_{49}=T_1$

$T_{80}=T_2=150-(T_3+T_1)$

$=150-(T_{60}+T_{49})=14$

**29.** $T_1=2$

$T_2=5$

$T_3=T_2-T_1=3$

$T_4=-2$

$T_5=-5$

$T_6=-3$

The sum of the terms $T_1$ to $T_6$ is 0.

$T_7=2$

$T_8=5$

$T_9=3$ ($\because$ we see that $T_{n+6}=T_n$ for all $n$)

$\vdots$

$T_{12}=-3$

The sum of the terms $T_7$ to $T_{12}$ is 0.

The sum of the terms $T_{13}$ to $T_{18}$ will be 0

$T_{19}$ (and $T_{13}$, $T_7$) $=T_1=2$ and $T_{20}=T_2=5$.

Sum of the first 20 terms of the sequence $=T_{19}+T_{20}=7$

**30.** $\frac{1}{6+12+18}+\frac{1}{6+12+18+24}+\frac{1}{6+12+18+24+30}+\ldots$

$$+\frac{1}{6+12+18+24+\ldots+1200}=S(\text{say})$$

$$S=\frac{1}{6(1+2+3)}+\frac{1}{6(1+2+3+4)}=\frac{1}{6(1+2+3+4+5)}+\ldots$$

$$+\frac{1}{6(1+2+3+4+\ldots+200)}$$

$$=\frac{1}{6}\left[\frac{2}{3(4)}+\frac{2}{4(5)}+\frac{2}{5(6)}+\ldots+\frac{2}{200(201)}\right]$$

$$=\frac{1}{6}\left[\frac{2}{3}-\frac{2}{201}\right]=\frac{66}{603}=\frac{22}{201}$$

## EXERCISE-5

**1.** From Statement I, nothing can be concluded.

From Statement II, sum of the first $n$ terms of the A.P. $=n/2(2a+(n-1)d)$ where $a$ is the first term and $d$ is the common difference.

$10/2(2a+9d) = 15/2(2a+14d) \Rightarrow 2a = -24d$.
Sum of first 25 terms of A.P. $= 25/2(2a+24d)$
$\Rightarrow 25/2(-24d+24d) = 0$
So, Statement II alone is sufficient.

**2.** Let the common ratio be $r$.
If the first is the least, the terms are 1, $r$ and $r^2$.
If the last is the least, the terms are $\frac{1}{r^2}, \frac{1}{r}$ and 1
Using Statement I,
$1 + r + r^2 = 21$
$r^2 + r - 20 = 0$
$(r+5)\ (r-4) = 0$
$\Rightarrow r = -5$ or 4. If $r = -5$, the numbers 1, –5 and 25. But the least here is –5. $\therefore r \neq -5$. Hence, $r = 4$ (we may check for the consistency here also)
I is sufficient.
Using Statement II,
$(1)\ (r)\ (r^2) = 64$
$\Rightarrow r^3 = 64 \Rightarrow r = 4$
$\therefore$ The middle term is 4
$\therefore$ II is sufficient.

**3.** From Statement I,
If $x = 5$, $y = 15$ and $z = 45$
then $x$, $y$ and $z$ are in G.P.
If $x = 4$, $y = 16$ and $z = 44$,
then $x$, $y$ and $z$ are not in G.P.
So, Statement I alone is not sufficient.
From Statement II, we do not know about $z$ so the second statement alone is not sufficient.
Using both the statements, $y/x = -2 \Rightarrow y = -2x$
$x + y = 20 \Rightarrow x = -20$ so $y = 40$
$y + z = 60 \Rightarrow z = 20$
So, $x$, $y$ and $z$ are not in geometric progression.

**4.** From Statement I, $a_1 = 1$
From Statement II, $a_{n+1} = (a_n + 1)^2$
Combining Statements I and II, we can answer the question.

**5.** From Statement I, $\sqrt{xy} = 4 \Rightarrow xy = 16$
but we can't find the arithmetic mean of $x$ and $y$ as we do not know the values of $x$ and $y$.
Statement I alone is not sufficient.
From Statement II, $\frac{x+y+4+8}{4} = 5$
$\Rightarrow x + y = 20 - 12 = 8$ $\frac{x+y}{2} = 4$.
Statement II alone is sufficient.

**6.** Let the first term of G and its common ratio be $a$ and $r$, respectively. Let the number of its terms be $n$.

$$\frac{a(r^n - 1)}{r-1} = \frac{3^8 - 1}{2} \quad (1)$$

$(a)\ (ar)\ (ar^2)\ \ldots\ (ar^{n-1}) = 3^{28}$

Using Statement I, $r\ \frac{a(r^n-1)}{r-1} = \frac{3(3^8-1)}{2}$ (2)

Dividing (2) by (1), $r = 3$. Hence, I is sufficient.
Using Statement II, $n = 8$
$r^8\ [(a)\ (ar)\ (ar^2)\ \ldots\ (ar^7)] = 3^{36}$ (3)
Dividing (3) by (2),
$r^8 = 3^8 \Rightarrow r = \pm 3$
When $r = 3$, sum of the terms of G $= \frac{a(3^8-1)}{3-1} = \frac{3^8-1}{2}$
(given) $\Rightarrow a = 1$ and the product of the terms of G $= 3^{28}$.
$\therefore r = 3$
When $r = -3$, sum $= \frac{a((-3)^8-1)}{-3-1} = \frac{3^8-1}{2} \Rightarrow a = -2$
But the product of the terms $= 2^8 \times 3^{28}$ ; $\therefore r \neq -3 \Rightarrow r = 3$
II is also sufficient

**7.** Let the first term and the common ratio of the progression be $a$ and $r$, respectively.

$$\frac{a}{1-r} = 8 \quad (1)$$

Using statement I, $a^2 + (a\,r^2) + (ar^2)^2 + \ldots = \frac{64}{3}$
$a^2(1 + r^2 + (r^2)^2 + \ldots) = \frac{64}{3}$
$\frac{a^2}{1-r^2} = \frac{64}{3}$ (2)

(1) $\Rightarrow \frac{a^2}{(1-r)^2} = 64$ (3)

Dividing (3) by (2), $\frac{(1-r)(1+r)}{(1-r)(1-r)} = 3$
I is sufficient.
Using Statement II, first term = sum of all terms following it.
$\therefore a$ = sum to infinity of the progression $- a$
$2a = \frac{a}{1-r}$
$a\ (2\ (1-r) - 1) = 0\ a > 0)$
$\therefore 2\ (1-r) - 1 = 0$ ; $r = \frac{1}{2}$
II is sufficient.
Either of the statements is sufficient.

**8.** Let the first term and the common difference of the arithmetic progression be $a$ and $d$, respectively.
Using statement I, sum of the odd numbered terms
$= a + a + 2d + a + 4d + a + 6d + a + 8d + a + 10d = 6a + 30d$
Let $6a + 30d = 11x$
$6a + 36d = 14x$
$11x + 14x = 100$

$x = 4$

$14x - 11x = 6d$

$\therefore 2 = d$

I is sufficient.

Using Statement II, $6a + 36d - (6a + 30d) = 12$

$d = 2$

II is sufficient.

Either of the statements is sufficient.

**9.** Let the middle term of the A.P. be $a$ and the common difference be $d$. So, the terms are $a - 5d$, $a - 4d$, $a - 3d$, $a - 2d$, $a - d$, $a$, $a + d$, $a + 2d$, $a + 3d$, $a + 4d$, $a + 5d$.

From statement I, $\frac{11a + 0}{11} = 63 \Rightarrow a = 63$

So, the middle term of the A.P. is 63

From statement II,

$$\frac{6a - 15d}{6} = 60 \Rightarrow 6a - 15d = 360 \quad (1)$$

$$\frac{6a + 15d}{6} = 66 \Rightarrow 6a + 15d = 396 \quad (2)$$

Solving the equations (1) and (2), we can get the value of $a$ which is the middle term of the A.P.

**10.** From Statement I,

If $a = 4$ and the common ratio is $1/2$, then $b < a$.

If $a = 6$ and the common ratio is $-2$, then $b <$ .

So, Statement I alone is not sufficient.

From Statement II,

$abc > ac$, so $b > 1$ as $a$ and $c$ must be of the same sign.

But using this alone, we cannot say whether the common ratio is negative or not. (e.g., $a$, $b$, $c$ can be 8, 4 and 2 or $-8$, 4 and $-2$)

Using both the statements, $b > 1$ and $b < a$, so $a$ must be positive.

$a$ and $b$ are positive; so the common ratio is also positive.

**11.** Let the first term and the common difference of the arithmetic progression be $a$ and $d$, respectively.

$17^{th}$ term $= a + 16\ d$.

Using statement I, $\frac{33}{2}(2a + 32d) = 1089$

$a + 16d = 33$

I alone is sufficient.

Using Statement II, $a = d = 1$

$a + 16d$ can be found

II alone is also sufficient

Either of the statements is sufficient.

**12.** Let the first term and the common ratio of the geometric progression be $a$ and $r$, respectively.

Fifth term $= ar^4$

Using Statement I, $\frac{a(r^9 - 1)}{r - 1} = 511$

We have only one equation but two unknown.

$\therefore$ $a$ and $r$ cannot be found.

$\therefore$ $ar^4$ cannot be found. I is not sufficient.

Using Statement II, $(a)\ (ar)\ (ar^2)\ \ldots\ ar^8 = 2^{36}$

$a^9\ r^{1+2+3+\ldots\ 8} = 2^{36}$

$a^9\ r^{(36)} = 2^{36}$

$(a\ r^4)^9 = (2^4)^9$

$\therefore$ $ar^4$ can be found. II is sufficient.

**13.** Let the first term and the common difference of the arithmetic progression be $a$ and $d$, respectively.

Required sum $= \frac{80}{2}(2a + 80d) = 80\ (a + 40\ d)$

Using Statement I, $2\ (a + 20\ d) - a = 63$

$a + 40\ d = 63$

Required sum can be found

I is sufficient

Using Statement II, $a + d = 5$.

We have only one equation but two unknowns.

$\therefore$ $a$ and $d$ cannot be found.

$\therefore$ The required sum cannot be found. II is not sufficient.

**14.** Let the first term and the common ratio of the geometric progression be $a$ and $r$, respectively.

$$\frac{ar^n - 1}{r - 1} = 2047$$

$$\frac{r(ar^{n-1}) - a}{r - 1} = 2047 \quad (1)$$

Using Statement I, $ar^{n-1} = 1024$ and $a = 1$.

From (1), $r$ can be found

I is sufficient

Using Statement II, $ar^{n-2} = 512$

But as we do not know $a$, we cannot answer the question.

II is not sufficient.

**15.** Let the first, second, third and fourth numbers be $a - 3d$, $a - d$, $a + d$ and $a + 3d$, respectively.

$(a - 3d)\ (a + 3d) = (a - d)\ (a + d) - 512$

$d^2 = 64$

$d = \pm 8$

Using Statement I, $4a = 104$

$a = 26$. $\Rightarrow$ The first number can be $26 \pm (3 \times 8)$

First number is not unique.

I is not sufficient.

Using Statement II, $\frac{a - 3d}{2}$ and $\frac{a - d}{2}$

are single digit natural numbers and $\frac{a-3d}{2} < \frac{a-d}{2}$
$\Rightarrow d > 0 \therefore d = 8.$

As two single – digit natural numbers differ by 8, they must be 1 and 9.

$\therefore \frac{a-3d}{2} = 2 \Rightarrow a - 3d = 4$

II alone is sufficient.

**16.** Let the first term and the common difference be $a$ and $d$, respectively.

Using Statement I, increase in the sum of the terms = $n.d$.

$n.d = 17$ = a prime number.

Both $n$ and $d$ must be natural numbers with $n > 1$

$\therefore n = 17$ and $d = 1$.

I is sufficient.

Using Statement II, $d = 1$ or 2. Hence, II is not sufficient.

**17.** Let the first term, the lower of the middle terms, higher of the middle terms and the last term of the arithmetic progression be $a$, $b$, $c$ and $d$, respectively.

Arithmetic mean of the progression = Average of its first and last term = Average of its middle terms

$\therefore$ Arithmetic mean $= \frac{a+d}{2} = \frac{b+c}{2}$ (1)

Using Statement I, $b + d = 24$

More information is required. I is not sufficient.

Using Statement II, $a + c = 12$

More information is required II is not sufficient.

Using both statements,

$b + d = 24$ and $a + c = 12$

$\therefore a + b + c + d = 36$

From (1), $a + d = b + c$

$\therefore a + d = \frac{1}{2}(a + b + c + d) = 18$

Both statements taken together are sufficient to answer the question.

**18.** Let the first number of the A.P. be $a$ and the common difference be $d$

$n$th term of A.P. $= a + (n - 1)d$.

so we have to find $a + 2d$ which is the third term of the A.P.

From Statement I,

$a + a + 9d = 24$

We do not know either $a$ or $d$; so we cannot find $a + 2d$

From Statement II,

$a + a + 9d = 20 \Rightarrow 4 + 4 + 9d = 20$

So, we can find the value of $d$ and thereby, $a + 2d$.

So, Statement II alone is sufficient.

**19.** Let the first, second, third and fourth numbers be $a$, $ar$, $ar^2$ and $ar^3$, respectively.

Using Statement I,

$ar + ar^2 - a = 5a$

$a(r^2 + r - 6) = 0$

$\Rightarrow r^2 + r - 6 = 0 \ (a \neq 0)$

$(r + 3)(r - 2) = 0$

$r = -3$ or 2

$r^2$ is not unique I is not sufficient.

Using Statement II,

$(ar)(ar^3) - (a)(ar^2) = 12\ a^2$

$a^2 (r^2)^2 - r^2 - 12 = 0$

$\Rightarrow (r^2)^2 - r^2 - 12 = 0 \ (a^2 \neq 0)$

$(r^2)^2 - 4r^2 + 3r^2 - 12 = 0$

$r^2 (r^2 - 4) + 3(r^2 - 4) = 0$

$(r^2 - 4)(r^2 + 3) = 0$

As $r^2 > 0$, $r^2 = 4$

$\therefore$ II is sufficient.

**20.** From Statement I, $b = \frac{a+c}{2}$

$\Rightarrow a < b < c$ or $c < b < a$

But from Statement II, $c > a$

Using both the statements, $c < b < a$.

We can answer the questions using both I and II, statements.

# 2 Indices and Surds

CHAPTER

**LEARNING OBJECTIVES**

*In this chapter, you will:*

- Learn all about Indices, such as:
  - Laws of Indices
  - Simplification
  - Solving for a value of an unknown
  - Comparison of numbers
- Learn all about surds, such as:
  - Rationalization of a surd
  - Square root of a surd
  - Comparison of surds

## ❒ INDICES

INDICES (Plural for Index) is a simplified way of representing large numbers in the form of a base and an exponent. For e.g., 1048576 is $2 \times 2 \times 2 \ldots$ (20 times) which can be expressed as $2^{20}$.

In general, we multiply $a \times a \times a \ldots$ ($n$ times), we write it as an. Here, '$a$' is called 'base' and $n$ is called 'power' or 'index' or 'exponent'. Also, an is read as '$a$ to the power $n$' or '$a$ raised to the power $n$.'

**FORMULA**

$a^n = a \times a \times a \times a \times \ldots$ $n$ times.

For example,

$$2^3 = 2 \times 2 \times 2 = 8 \text{ and } 3^4 = 3 \times 3 \times 3 \times 3 = 81$$

While the examples taken are for positive integer values of $n$, the powers can also be negative integers or positive or negative fractions or irrational numbers. In the sections that follow, we will also see how to deal with numbers where the powers are fractions or negative integers.

**NOTE**

If a number raised to a certain power is inside brackets and quantity is then raised to a power again {i.e., *a* number of the type $(a^m)^n$ – read as '*a* raised to the power *m* whole raised to the power *n*' or '*a* raised to power *m* whole to the power *n*'}, then the number inside the brackets is evaluated first and then this number is raised to the power which is outside the brackets.

For example, to evaluate $(2^3)^2$, we first find out the value of the number inside the bracket $(2^3)$ as 8 and now raise this to the power 2. This gives $8^2$ which is equal to 64. Thus, $(2^3)^2$ is equal to 64.

If we have powers in the manner of 'steps', then such a number is evaluated by starting at the topmost of the 'steps' and coming down one 'step' in each operation.

For example, $2^{4^3}$ is evaluated by starting at the topmost level '3'. Thus, we first calculate $4^3$ as equal to 64. Since 2 is raised to the power $4^3$, we now have $2^{64}$.

Similarly, $2^{3^2}$ is equal to '2 raised to the power $3^2$' or '2 raised to the power 9' or $2^9$ which is equal to 512.

There are certain basic rules/laws for dealing with numbers having powers. These are called **Laws of Indices**. The important ones are listed down and the students have to know these rules and be able to apply any of them in solving problems. Most of the problems in indices will require one or more of these rules. These rules should be internalized by the students to the extent that after some practice application of these rules should come naturally and the student should not feel that he is applying some specific formula.

**Table of Rules/Laws of Indices**

| Rule/Law | Example |
|---|---|
| 1. $a^m \times a^n = a^{m+n}$ | $5^2 \times 5^7 = 5^9$ |
| 2. $\frac{a^m}{a^n} = a^{m-n}$ | $\frac{7^5}{7^3} = 7^2 = 49$ |
| 3. $(a^m)^n = a^{mn}$ | $(4^2)^3 = 4^6$ |
| 4. $a^{-m} = \frac{1}{a^m}$ | $2^{-3} = \frac{1}{2^3} = \frac{1}{8} = 0.125$ |
| 5. $\sqrt[m]{a} = a^{1/m}$ | $\sqrt[3]{64} = 64^{1/3} = 4$ |
| 6. $(ab)^m = a^m \cdot b^m$ | $(2 \times 3)^4 = 2^4 \cdot 3^4$ |
| 7. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ | $\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$ |
| 8. $a^0 = 1$ (where $a \neq 0$) | $3^0 = 1$ |
| 9. $a^1 = a$ | $4^1 = 4$ |

These rules/laws will help you in solving a number of problems. In addition to the above, the student should also remember the following rules:

**Rule 1**: When the bases of two equal numbers are equal, then their powers also will be equal. (If the bases are neither zero nor ± 1.)

For example, if $2^n = 2^3$, then it means $n = 3$

**Rule 2**: When the powers of two equal numbers are equal (integer and not equal to zero), two cases arise:

1. if the power is an odd number, then the bases are equal. For example, if $a^3 = 4^3$, then $a = 4$
2. if the powers are even numbers, then the bases are numerically equal but can have different signs. For example, if $a^4 = 3^4$, then $a = +3$ or $-3$.

The problems associated with indices are normally of Three types:

1. **Simplification:** Here, the problem involves terms with same or different bases and powers which have to be simplified using the rules discussed in the table above.
2. **Solving for the value of an unknown:** Here, the problem will have an equation where an unknown (like $x$ or $y$) will appear in the base or in the power and using Rule 1 and Rule 2 discussed above, values of unknown are to be determined.
3. **Comparison of numbers:** Here, two or more quantities will be given—each being a number raised to a certain power. These numbers have to be compared in magnitude—either to find the largest or the smallest of the quantities or to arrange the given quantities in ascending or descending order.

The questions in the solved examples will give clarity about the different types of problems pertaining to the above-mentioned types

## ❒ SURDS

Any number of the form $p/q$, where $p$ and $q$ are integers and $q \neq 0$ is called rational number. Any real number which is not a rational number is an irrational number. Amongst irrational numbers, of particular interest to us are SURDS. Surds are irrational roots of positive integers whose values cannot be exactly determined. Amongst surds, we will specifically be looking at 'quadratic surds' – surds of the type $a+\sqrt{b}$ and $a+\sqrt{b}+\sqrt{c}$, etc., where the terms involve only square roots and not any higher roots. We do not need to go very deep into the area of surds – what is required is a basic understanding of some of the operations on surds.

If there is a surd of the form $(a+\sqrt{b})$, then a surd of the form $\pm(a-\sqrt{b})$ is called conjugate of the surd $(a+\sqrt{b})$. The product of a surd and its conjugate will always be a rational number.

## ❒ Rationalization of a Surd

When there is a surd of the form $\frac{1}{a+\sqrt{b}}$, it is difficult to perform arithmetic operations on it. Hence, the denominator is converted into a rational number thereby facilitating ease of handling the surd. This process of converting the denominator into a rational number without changing the value of the surd is called rationalization.

To convert the denominator of a surd into a rational number, multiply the denominator and the numerator with the conjugate of the surd in the denominator so that the denominator gets converted to a rational number without changing the value of the surd. That is,

if there is a surd of the type $a+\sqrt{b}$ in the denominator, both the numerator and the denominator have to be multiplied with a surd of the form I – $\sqrt{b}$ or a surd of the form $(-a+\sqrt{b})$ to convert the denominator into a rational number.

If there is a surd of the form $(a+\sqrt{b}+\sqrt{c})$ in the denominator, then the process of multiplying the denominator with its conjugate surd has to be carried out **twice** to rationalize the denominator.

## ❒ Square Root of a Surd

If there exists a square root of a surd of the type $a+\sqrt{b}$, then it will be of the form $\sqrt{x}+\sqrt{y}$. We can equate the square of $\sqrt{x}+\sqrt{y}$ to $a+\sqrt{b}$, and thus solve for $x$ and $y$. Here, one point should be noted—when there is an equation with rational and irrational terms, the rational part on the left-hand side is equal to the rational part on the right-hand side and, the irrational part on the left-hand side is equal to the irrational part on the right-hand side of the equation.

However, for the problems which are expected in the entrance exams, there is no need of solving for the square root in such an elaborate manner. We will look at finding the square root of the surd in a much simpler manner. Here, first the given surd is written in the form of $(\sqrt{x}+\sqrt{y})^2$ or $(\sqrt{x}-\sqrt{y})^2$. Then the square root of the surd will be $(\sqrt{x}+\sqrt{y})$ or $(\sqrt{x}-\sqrt{y})$.

## ❒ Comparison of Surds

Sometimes we will need to compare two or more surds either to identify the largest/smallest one or to arrange the given surds in ascending/descending order. The surds given in such cases will be such that they will be close to each other and hence we will not be able to identify the largest one by taking the approximate square root of each of the terms. In such a case, the surds can both be squared and the common rational part be subtracted. At this stage, normally one will be able to make out the order of the surds. If, even at this stage, it is not possible to identify the larger of the two, then the numbers should be squared once more.

## SOLVED EXAMPLES

**2.01.** Simplify: $\left(\frac{256}{576}\right)^{1/4}\times\left(\frac{64}{27}\right)^{-1/3}\times\left(\frac{216}{8}\right)^{-1}$.

**Sol:** As $256=2^8$; $576=24^2$; $64=2^6$; $27=3^3$; $216=6^3$ and $8=2^3$, $\left(\frac{256}{576}\right)^{1/4}\times\left(\frac{64}{27}\right)^{-1/3}\times\left(\frac{216}{8}\right)^{-1}$

$$=\left(\frac{2^8}{24^2}\right)^{1/4}\times\left(\frac{3^3}{2^6}\right)^{1/3}\times\frac{8}{216}$$

$$=\frac{2^2}{\sqrt{24}}\times\frac{3}{4}\times\frac{1}{27}=\frac{1}{2\sqrt{6}\times 9}=\frac{1}{18\sqrt{6}}.$$

**2.02.** Simplify the following:

$$\left(\frac{l^4m^6}{n^8}\right)^3\times\left(\frac{m^8n^4}{l^{-6}}\right)^{-2}\times\left(\frac{n^6l^6}{m^4}\right)^2.$$

**Sol:** $\frac{l^{4\times3}\ m^{6\times3}}{n^{8\times3}}\times\frac{m^{8\times-2}\ n^{4\times-2}}{l^{-6\times-2}}\times\frac{n^{6\times2}\ l^{6\times2}}{m^{4\times2}}$

$$=l^{12-12+12}\ m^{18-16-8}\ n^{-8+12-24}=\frac{l^{12}}{m^6\cdot n^{20}}.$$

**2.03.** Solve the given equation for the value of $x$.

$$\sqrt[3]{\left(\frac{5}{4}\right)^{x+2}}=\frac{4096}{15625}.$$

**Sol:** The given equation can be written as

$$\left(\frac{5}{4}\right)^{\frac{x+2}{3}}=\left(\frac{15625}{4096}\right)^{-1}$$

$$\Rightarrow\left(\frac{5}{4}\right)^{\frac{x+2}{3}}=\left(\frac{5^6}{4^6}\right)^{-1}\Rightarrow\left(\frac{5}{4}\right)^{\frac{x+2}{3}}=\left(\frac{5}{4}\right)^{-6}$$

$$\Rightarrow\frac{x+2}{3}=-6\ \Rightarrow x+2=-18$$

$\therefore x=-20.$

**2.04.** If $(32768)^{x-2}=(32)^x$, then find the value of $x$.

**Sol:** The given equation can be rewritten as

$(2^{15})^{x-2}=(2^5)^x$

$\Rightarrow 15x-30=5x\Rightarrow 10x=30$

$x=3$

**2.05.** If $\left(\frac{1}{16\times81}\right)^{3-\frac{x}{2}}=(48)^{2x-12}$, then find the value of $x$

**Sol:** The given equation can be rewritten as

$$\left(3^{-4}\times2^{-4}\right)^{3-\frac{x}{2}}=(2^4)^{2x-12}\times3^{2x-12}$$

$$\Rightarrow 3^{2x-12}\times2^{2x-12}=2^{8x-48}\times3^{2x-12}$$

$$\Rightarrow 2^{2x-12}=2^{8x-48}$$

$$\Rightarrow 2x-12=8x-48\Rightarrow x=6$$

**2.06.** Arrange the following number in ascending order $(125)^{10}$, $(625)^{9}$, and $(25)^{16}$

**Sol:** Each of the given numbers can be expressed with 5 as the base.

Hence, the given numbers can be written as $(5^3)^{10}$, $(5^4)^9$, and $(5^2)^{16}$

$\Rightarrow 5^{30}$, $5^{36}$, and $5^{32}$

As the bases are equal, the values can be compared based on the powers.

Hence, $5^{30} < 5^{32} < 5^{36}$

i.e., $(125)^{10} < (25)^{16} < (625)^9$

**2.07.** Arrange the following numbers in descending order $(144)^3$, $(256)^2$, and $(36)^6$.

**Sol:** Each of the given bases can be expressed in exponential form.

Hence, the given numbers are

$12^6$, $16^4$, and $6^{12}$; i.e., $(12^{1/2})^{12}$, $(16^{1/3})^{12}$, and $(6)^{12}$

All these numbers have the same power. Hence, they can be compared based on the bases of the numbers.

As $12^{1/2}$ lies between 3 and 4 and $16^{1/3}$ lies between 2 and 3, $16^{1/3} < 12^{1/2} < 6$.

Thus, $6^{12} > 12^6 > 16^4$

$\Rightarrow 36^6 > 144^3 > 256^2$ is the descending order.

**2.08.** Simplify: $\dfrac{1}{4+\sqrt{2}} - \dfrac{1}{4-\sqrt{2}}$.

**Sol:** $\dfrac{1}{4+\sqrt{2}} - \dfrac{1}{4-\sqrt{2}}$

$= \dfrac{(4-\sqrt{2})-(4+\sqrt{2})}{(4+\sqrt{2})\ (4-\sqrt{2})} = \dfrac{-2\sqrt{2}}{14} = \dfrac{-\sqrt{2}}{7}$.

**2.09.** Rationalize the denominator of the surd $\dfrac{1}{4-\sqrt{13}}$.

**Sol:** Since the denominator of the surd is $4-\sqrt{13}$, to rationalize, we multiply both the numerator and the denominator with $4+\sqrt{13}$

$\dfrac{1}{4-\sqrt{13}} \times \dfrac{4+\sqrt{13}}{4+\sqrt{13}} = \dfrac{4+\sqrt{13}}{16-13} = \dfrac{4+\sqrt{13}}{3}$.

**2.10.** Rationalize the denominator of the surd $\dfrac{1}{4+\sqrt{6}-\sqrt{10}}$.

**Sol:** Here, first take $4+\sqrt{6}$ as one term and $\sqrt{10}$ as the second term and carry out the rationalization by multiplying the numerator and denominator with $\left(4+\sqrt{6}-\sqrt{10}\right)$

$\dfrac{1}{(4+\sqrt{6}-\sqrt{10})}$

$= \dfrac{(4+\sqrt{6}+\sqrt{10})}{(4+\sqrt{6}-\sqrt{10})(4+\sqrt{6}-\sqrt{10})}$

$= \dfrac{(4+\sqrt{6}+\sqrt{10})}{(4+\sqrt{6})^2-(\sqrt{10})^2}$

$= \dfrac{4+\sqrt{6}+\sqrt{10}}{16+6+8\sqrt{6}-10} = \dfrac{4+\sqrt{6}+\sqrt{10}}{12+8\sqrt{6}}$.

As the denominator has still irrational part, it should be rationalized one more time by multiplying the numerator and denominator with its conjugate surd.

$= \dfrac{(4+\sqrt{6}+\sqrt{10})\ (3-2\sqrt{6})}{4(3+2\sqrt{6})\ (3-2\sqrt{6})}$

$= \dfrac{12+3\sqrt{6}+3\sqrt{10}-8\sqrt{6}-12-2\sqrt{60}}{4(9-24)}$

$= \dfrac{-5\sqrt{6}+3\sqrt{10}-2\sqrt{60}}{-60} = \dfrac{5\sqrt{6}-3\sqrt{10}+4\sqrt{15}}{60}$.

**2.11.** Find the positive square root of the surd $32+4\sqrt{15}$.

**Sol:** The given surd is to be written as $\left(a+\sqrt{b}\right)^2$, as the irrational part is positive.

(if the irrational part is negative, we would have written it as $\left(a-\sqrt{b}\right)^2$)

In the expansion of $\left(a+\sqrt{b}\right)^2$, we get the terms $a^2$, $b$, and $2a\sqrt{b}$.

Since the coefficient of irrational term is 2, we will keep the co-efficient of irrational term of the given surd as 2.

Consider the term $4\sqrt{15}$. Here, the coefficient of the irrational term is 4, we will retain only 2 and take the remaining 2 under the square root. Hence, $4\sqrt{15}$ will become $2\sqrt{60}$. Thus, the given surd is $32+2\sqrt{60}$.

Now, resolve 60 into a pair of factors such that their sum is 32. The pair of factors 30 and 2 satisfies the condition.

$$32 + 2\sqrt{60} = (30+2) + 2\sqrt{30\times 2}\ .$$

Hence, the positive square root $= \sqrt{30}+\sqrt{2}$

$= \sqrt{2}\left(\sqrt{15}+1\right)$.

(Please note that although the square root can be with a positive or a negative sign, when written in the form $\sqrt{32+2\sqrt{60}}$, positive root is implied.)

**2.12.** Find which of the following two surds is greater: $\sqrt{6}+\sqrt{26}$ and $\sqrt{3}+\sqrt{31}$.

**Sol:** If we try to take approximate values of both the surds, we find that both are more than 7; and we will not able to judge which surd is greater. The comparison can be done by squaring the surds and then comparing them. Now, we get

$(\sqrt{6}+\sqrt{26})^2 = 32+2\sqrt{156}$ ; $(\sqrt{3}+\sqrt{31})^2$

$= 34+2\sqrt{93}$.

The square of the second surd can be written as $32+2+2\sqrt{93}$. Since 32 is the common rational part for both the surds, we need to compare $2\sqrt{156}$ and $2+2\sqrt{93}$ only.

We know that $\sqrt{156}$ lies between 12 and 13 (since $12^2=144$ and $13^2=169$)

$\therefore 2\sqrt{156}$ lies between 24 and 26.

Also $\sqrt{93}$ lies between 9 and 10, so $2+2\sqrt{93}$ lies between 2 + 2 (9 to 10) i.e., 20 to 22.

As 24 to 26 > 20 to 22, $32+2\sqrt{156} > 34+2\sqrt{93}$ i.e., $\sqrt{6}+\sqrt{26} > \sqrt{3}+\sqrt{31}$.

**2.13.** Simplify:

$$\left(\frac{729}{1728}\right)^{\frac{-2}{3}} \times \left(\frac{1024}{9}\right)^{\frac{1}{2}} \div \left(\frac{24}{324}\right)$$

**Sol:**

$$\left(\frac{729}{1728}\right)^{\frac{-2}{3}} \times \left(\frac{1024}{9}\right)^{\frac{1}{2}} \div \left(\frac{24}{324}\right)$$

$$= \left(\frac{9^2}{12^2}\right)^{-1} \times \left(\frac{32^2}{3^2}\right)^{1/2} \times \left(\frac{324}{24}\right)$$

$$= \frac{144}{81} \times \frac{32}{3} \times \frac{324}{24} = 256$$

**2.14.** Simplify: $\left(\frac{a^4 b^3}{c^2}\right)^2 \times \left(\frac{b^4 c^3}{a^2}\right)^3 \times \left(\frac{c^4 a^3}{b^2}\right)^4$

**Sol:**

$$\left(\frac{a^4 b^3}{c^2}\right)^2 \times \left(\frac{b^4 c^3}{a^2}\right)^3 \times \left(\frac{c^4 a^3}{b^2}\right)^4$$

$$= \frac{a^8 b^6}{c^4} \times \frac{b^{12} c^9}{a^6} \times \frac{c^{16} a^{12}}{b^8}$$

$$= a^{8-6+12}\, b^{6+12-8}\, c^{-4+9+16} = a^{14}\, b^{10}\, c^{21}$$

**2.15.** In the equation given below, solve for $x$

$$\sqrt[3]{\left(\frac{5}{7}\right)^{x+1}} = \frac{125}{343}$$

**Sol:** Given:

$$\left(\frac{5}{7}\right)^{x+1} = \left[\left(\frac{5}{7}\right)^3\right]^3 = \left(\frac{5}{7}\right)^9.$$

By equating their indices,

$x+1=9$

$x=8$.

**2.16.** If $625^{x-2} = 25^{x+2}$, find the value of $x$.

**Sol:** Given, $(5^4)^{x-2} = (5^2)^{(x+2)}$, by equating their Indices,

$4(x-2) = 2(x+2)$

$x=6$.

**2.17.** If $\left(\frac{49}{2401}\right)^{4-x} = 49^{2x-6}$, find the value of $x$.

**Sol:** $\left(\frac{49}{2401}\right)^{4-x} = \left(49^{-1}\right)^{4-x} = 49^{x-4}$

Given, $49^{x-4} = 49^{2x-6}$

$x-4 = 2x-6$

$x=2$

**2.18.** Arrange the following in ascending order $625^6$, $125^7$ and $25^{10}$

**Sol:** $625^6 = (5^4)^6 = 5^{24}$

$125^7 = (5^3)^7 = 5^{21}$

$25^{10} = (5^2)^{10} = 5^{20}$

$25^{10} < 125^7 < 625^6$

**2.19.** Simplify: $\frac{1}{4-\sqrt{5}} - \frac{1}{4+\sqrt{5}}$

**Sol:** $\frac{1}{4-\sqrt{5}} - \frac{1}{4+\sqrt{5}} = \frac{4+\sqrt{5}-(4-\sqrt{5})}{(4-\sqrt{5})(4+\sqrt{5})} = \frac{2\sqrt{5}}{11}$

**2.20.** Rationalize the denominator: $\frac{1}{1+\sqrt{6}-\sqrt{7}}$

**Sol:** A rationalizing factor of $1+\sqrt{6}-\sqrt{7}$ is $1+\sqrt{6}+\sqrt{7}$

$$\frac{1}{1+\sqrt{6}-\sqrt{7}} = \frac{(1+\sqrt{6}+\sqrt{7})}{(1+\sqrt{6}-\sqrt{7})(1+\sqrt{6}+\sqrt{7})}$$

$$= \frac{1+\sqrt{6}+\sqrt{7}}{(1+\sqrt{6})^2-(\sqrt{7})^2} = \frac{1+\sqrt{6}+\sqrt{7}}{2\sqrt{6}}$$

The rationalizing factor of $\sqrt{6}$ is $\sqrt{6}$

$$= \frac{\sqrt{6}+6+\sqrt{42}}{12}$$

**2.21.** Find the value of $\sqrt{62+\sqrt{480}}$.

**Sol:** Let $\sqrt{62+\sqrt{480}} = \sqrt{a}+\sqrt{b}$

Squaring both sides, we get

$62+\sqrt{480}=a+b+2\sqrt{ab}$

$62+\sqrt{480}=a+b+\sqrt{4ab}$

Equating the corresponding rational and irrational parts on both sides, $a+b=62$ and $4ab=480 \Rightarrow ab=120$

As $a+b=60+2$ and $ab=(60)(2)$, it follows that $a=60$ and $b=2$ or vice versa.

$\therefore \sqrt{a}+\sqrt{b}=\sqrt{60}+\sqrt{2}$

**2.22.** Which of the surds given below is greater?

$\sqrt{3}+\sqrt{23}$ and $\sqrt{6}+\sqrt{19}$

**Sol:** $(\sqrt{3}+\sqrt{23})^2=26+2\sqrt{69}$

$\sqrt{69}$ lies between $\sqrt{64}$ and $\sqrt{81}$

$\therefore 26+2\sqrt{69}$ lies between $26+2(8)$ and $26+2(9)$, i.e., 42 and 44.

Similarly, $(\sqrt{6}+\sqrt{19})^2$ lies between 45 and 47.

$\therefore (\sqrt{3}+\sqrt{23})^2<(\sqrt{6}+\sqrt{19})^2$

$\therefore \sqrt{6}+\sqrt{19}>\sqrt{3}+\sqrt{23}$

## EXERCISE-1

***Directions for questions 1 to 45:*** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** If $3^{2x}=81$, what is the value of $x$?
(A) 1 (B) 2 (C) 3 (D) 4

**2.** Simplify: $x^{\frac{a}{a+b+c}}\cdot x^{\frac{b}{a+b+c}}\cdot x^{\frac{c}{a+b+c}}$
(A) 1 (B) $x$ (C) 0 (D) $x^{abc}$

**3.** Which is greater between $2^{2^3}$, $3^{2^2}$ ?
(A) $2^{2^3}$ (B) $3^{2^2}$ (C) Cannot be determined

**4.** What is the value of $x$ if $2^{x^x}=16$ ?
(A) 0 (B) 1 (C) 3 (D) 2

**5.** If $(2^x)(3^y)=216$ and $x$ and $y$ are integers, then what are the values of $x$ and $y$?
(A) 3, 2 (B) 2, 2 (C) 3, 4 (D) 3, 3

**6.** If $3^{1/2}\cdot 3^{3/2}\cdot 3^{5/2}\cdot 3^{7/2}=9^x$, what is the value of $x$?
(A) 4 (B) 3 (C) 2 (D) 1

**7.** If $4^{2x}=256$, find $x$ _____.

**8.** Simplify : $x^{\frac{b+c-a}{a+b+c}}\cdot x^{\frac{a+c-b}{a+b+c}}\cdot x^{\frac{a+b-c}{a+b+c}}$.
(A) 1 (B) $x$ (C) $x^2$ (D) $x^3$

**9.** Find the rationalizing factor of $4^{\frac{1}{3}}+3^{\frac{1}{3}}$.
(A) $4^{\frac{2}{3}}-2^{\frac{1}{3}}$ (B) $4^{\frac{2}{3}}+2^{\frac{1}{3}}$
(C) $4^{\frac{2}{3}}+12^{\frac{1}{3}}+3^{\frac{2}{3}}$ (D) $4^{\frac{2}{3}}-12^{\frac{1}{3}}+3^{\frac{2}{3}}$

**10.** Which is greater: $2^{3^3}$ or $3^{3^2}$ ?
(A) $2^{3^3}$ (B) $3^{3^2}$ (C) Both are equal

**11.** If $3^{x^x}=81$, find $x$ _____.

**12.** If $2^{\frac{1}{2}}2^{\frac{3}{2}}2^{\frac{5}{2}}2^{\frac{7}{2}}2^{\frac{9}{2}}2^{\frac{11}{2}}=8^x$, find $x$.
(A) 6 (B) 4 (C) 8 (D) 10

**13.** If $2^p3^q=432$, and $p$ and $q$ are integers, find $p+q$ _____.

**14.** Simplify: $32^5\times 16^{-4}\times 8^8\div 2^6$
(A) $2^{18}$ (B) $2^{27}$ (C) $2^{32}$ (D) $2^{24}$

**15.** Simplify: $\left(\frac{729}{343}\right)^{\frac{1}{6}}\left(\frac{625}{49}\right)^{-\frac{1}{4}}\left(\frac{125}{729}\right)^{\frac{1}{3}}$
(A) $1/3$ (B) $2/3$ (C) $1/9$ (D) $1/3^3$

**16.** What are the values of $x$ and $y$ that satisfy both the equations?
$2^x 9^y=648$
$4^x 27^y=46656$ $x=$ _____, $y=$ _____.

**17.** Solve for $x$, if $3^{3x-4}=3^{2^{1^{2^{3^4}}}}$ _____.

**18.** If $(a\sqrt[3]{a})^{\sqrt{a}}=\left(a^2\sqrt{a}\right)^{a\sqrt{a}}$, what is the value of $a$?
(A) $\frac{8}{15}$ (B) 2 (C) 1 (D) Either A or C

**19.** If $(32)^{2x+5}=(128)^{2x-1}$, then find the value of $x$ _____.

**20.** Compare the following numbers and identify the greater one in each case.
(A) $3^{200}$ and $4^{150}$ (B) $12^{75}$ and $8^{100}$
(C) $3^{1/4}$ and $2^{1/3}$ (D) $64^{325}$ and $32^{400}$

**21.** Simplify: $\left(1-\frac{1}{1+x^{-y}}\right)+\left(1-\frac{1}{1+x^{y}}\right)$ _____.

**22.** Arrange the numbers $3^{1/5}$, $4^{1/6}$, $5^{1/3}$, and $2^{1/2}$ in ascending order.
(A) $3^{1/5}, 2^{1/2}, 5^{1/3}, 4^{1/6}$ (B) $3^{1/5}, 4^{1/6}, 2^{1/2}, 5^{1/3}$,
(C) $5^{1/3}, 2^{1/2}, 3^{1/5}, 4^{1/6}$, (D) $2^{1/2}, 3^{1/5}, 4^{1/6}, 5^{1/3}$

**23.** Arrange the numbers 22222, $222^{22}$, $22^{22^2}$, and $2^{2222}$ in descending order.

(A) $2^{2222}$, $22^{22^2}$, $222^{22}$, 22222

(B) 22222, $222^{22}$, $22^{22^2}$, $2^{2222}$

(C) $22^{22^2}$ 22222, $222^{22}$, $2^{2222}$

(D) $2^{2222}$,22222, $22^{22^2}$ $222^{22}$

**24.** If $3^{3x-4} = 3^{2^{-1^{3^{4^5}}}}$, then find the value of $x$ _____.

**25.** If $3^{2x+3} + 3^{2x-3} = 2190$, find $x$ _____.

**26.** What is the conjugate of the surd $\sqrt{5} - 2$ ?

(A) $\sqrt{5} + 2$ (B) $-\sqrt{5} + 2$

(C) Either (A) or (B) (D) None of these

**27.** What is the positive square root of $\sqrt{48} - \sqrt{27}$ ?

(A) $3^{1/2}$ (B) $3^{1/8}$

(C) $3^{1/4}$ (D) None of these

**28.** What is the positive square root of $7 + 4\sqrt{3}$ ?

(A) $\sqrt{3} + 1$ (B) $2 + 2\sqrt{3}$

(C) $\sqrt{2} + \sqrt{3}$ (D) $2 + \sqrt{3}$

**29.** Simplify: $\frac{1}{\sqrt{3} + \sqrt{2}} + \frac{1}{2 + \sqrt{3}}$.

(A) $2 - \sqrt{2}$ (B) $2 - \sqrt{3}$

(C) $\sqrt{3} - \sqrt{2}$ (D) $2 - 2\sqrt{2}$

**30.** What is the positive square root of $\sqrt{72} - \sqrt{40}$ ?

(A) $2^{1/2}\left(\sqrt{5} - 1\right)$ (B) $2^{1/4}\left(\sqrt{5} - 1\right)$

(C) $2^{1/4}\left(\sqrt{3} - \sqrt{2}\right)$ (D) $2^{1/4}\left(\sqrt{5} - \sqrt{3}\right)$

**31.** Find the positive square root of $\sqrt{108} - \sqrt{75}$.

(A) $\sqrt{6}$ (B) 3 (C) $\sqrt{3}$ (D) $\sqrt[4]{3}$

**32.** Find the positive square root of $14 + 6\sqrt{5}$.

(A) $3 + \sqrt{5}$ (B) $3 - \sqrt{5}$

(C) $3 + 2\sqrt{5}$ (D) $3 - 2\sqrt{5}$

**33.** Find the conjugate of the surd $\sqrt{13} - 2$.

(A) $\sqrt{13} + 2$ (B) $\sqrt{13} - 2$

(C) $-\left(2 + \sqrt{13}\right)$ (D) $2 - \sqrt{13}$

**34.** Simplify: $\frac{3}{\sqrt{5} + \sqrt{2}} + \frac{1}{3 + \sqrt{8}}$.

(A) $3 + \sqrt{5} + 3\sqrt{2}$ (B) $3 + \sqrt{5} - 3\sqrt{2}$

(C) $3 - \sqrt{5} - 3\sqrt{2}$ (D) $-3 + \sqrt{5} + 3\sqrt{2}$

**35.** Find the positive square root of $\sqrt{128} - \sqrt{56}$.

(A) $\sqrt[4]{2}\left(\sqrt{7} - 1\right)$ (B) $\sqrt[4]{2}\left(\sqrt{7} + 1\right)$

(C) $\sqrt{2}\left(\sqrt{7} - 1\right)$ (D) $\sqrt{2}\left(\sqrt{7} + 1\right)$

**36.** Which is the rationalizing factor of $6 + 3\sqrt{5}$ ?

(A) $-6 - 3\sqrt{5}$ (B) $\sqrt{5} - 2$

(C) $-\sqrt{5} - 2$ (D) None of these

**37.** Which of the following is greater? $\frac{1}{\sqrt{8} + \sqrt{5}}, \frac{1}{\sqrt{5} + \sqrt{2}}$

(A) $\frac{1}{\sqrt{8} + \sqrt{5}}$ (B) $\frac{1}{\sqrt{5} + \sqrt{2}}$

(C) Cannot be determined

**38.** What is the rationalizing factor of $4^{1/3} - 3^{1/3}$?

(A) $4^{2/3} + 12 + 3^{2/3}$ (B) $4^{2/3} - 12 + 3^{2/3}$

(C) $4^{2/3} + 12^{1/3} + 3^{2/3}$ (D) $4^{4/3} + 12^{2/3} + 3^{4/3}$

**39.** $\frac{5}{3^{2/3} - 6^{1/3} + 2^{2/3}}$ is equal to

(A) $3^{1/3} + 2^{1/3}$ (B) $3^{1/3} - 2^{1/3}$

(C) $2^{1/3} - 3^{1/3}$ (D) $3^{2/3} + 2^{2/3}$

**40.** Simplify: $\left[(64)^{-1/6} \times (81)^{3/4} \times (64)^{2/3}\right]^{1/3}$ _____.

**41.** If $x$ and $y$ are integers and $2^x \cdot 3^{2y} = 144$, then what is the value of $(x + y)$? _____

**42.** Simplify: $\frac{6}{2\sqrt{3} + \sqrt{6}} - \frac{1}{\sqrt{3} - \sqrt{2}} + \frac{4}{\sqrt{6} - \sqrt{2}}$.

(A) 1 (B) $\sqrt{3}$ (C) $2\sqrt{6}$ (D) $\sqrt{2} - \sqrt{3} + \sqrt{6}$

**43.** Find the ascending order of $3^{\frac{1}{2}}, 4^{\frac{1}{3}}$ and $5^{\frac{1}{4}}$.

(A) $5^{\frac{1}{4}}, 3^{\frac{1}{2}}, 4^{\frac{1}{3}}$ (B) $4^{\frac{1}{3}}, 3^{\frac{1}{2}}, 5^{\frac{1}{4}}$

(C) $3^{\frac{1}{2}}, 4^{\frac{1}{3}}, 5^{\frac{1}{4}}$ (D) $5^{\frac{1}{4}}, 4^{\frac{1}{3}}, 3^{\frac{1}{2}}$

**44.** Solve for $x$: $\left(2\sqrt{3}\right)^{3x-4} = 20736$ _____.

**45.** Simplify: $\left(\frac{x^4 y^{-5} z^2}{x^{-3} y z^4}\right)^{11} \div \left(\frac{x^2 y z^3}{x^{-8} y^2 z^5}\right)^7$.

(A) $\frac{x^7}{y^{59} z^8}$ (B) $x^7 y^{59} z^8$ (C) $\frac{y^{59}}{x^7 z^8}$ (D) $\frac{x^7 y^{59}}{z^8}$

## EXERCISE-2

***Directions for questions 1 to 45***: For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** If $5^{1/a} = 7^{1/b} = 35^{1/c}$, find $c$ in terms of $a$ and $b$.

(A) $\frac{ab}{a+b}$ (B) $a+b$ (C) $\frac{ab}{a-b}$ (D) $a-b$

**2.** Simplify: $\left(\frac{a^3 b^6 c^9}{a^4 b^8 c^{12}}\right)^9 \div \left(\frac{a^2 b^4 c^6}{a^3 b^7 c^5}\right)^8$.

(A) $a\,b^6\,c^{35}$ (B) $\frac{b^6}{ac^{35}}$ (C) $\frac{ac^{35}}{b^6}$ (D) $\frac{c^{35}}{ab^6}$

**3.** Simplify: $\left((216)^{4/3}(384)^{1/7}(729)^{1/7}\right)^{\frac{1}{2}}$.

(A) $36\sqrt{3}$ (B) $54\sqrt{3}$ (C) $72\sqrt{6}$ (D) $36\sqrt{6}$

**4.** Simplify: $\frac{2^{m+2}6^{m+2}10^{2m+2}15^{m+1}25^{m-n}}{3^{2m-3}4^{2m}6^6 5^{5m+2n}25^{1-2n}}$.

(A) 1 (B) $2^m\,3^{m-1}\,5$
(C) $5^{2n}$ (D) 5

**5.** Find the value of the expression $y^3 - 15y^2 + 60y + 25$, if $y = 5 + 5^{\frac{1}{3}} + 5^{\frac{2}{3}}$.

(A) 105 (B) 145 (C) 150 (D) 115

**6.** If $x - 4 = \sqrt{11} + \sqrt{7}$, then find the value of $x^4 - 16x^3 + 60x^2 + 32x$.

(A) 244 (B) 264 (C) 280 (D) 304

**7.** If $p+q+r=0$, then find the value of $\sqrt[qr]{a^{p^2}} \cdot \sqrt[rp]{a^{q^2}} \cdot \sqrt[pq]{a^{r^2}}$.

(A) 1 (B) $a^3$
(C) $a^{pqr}$ (D) None of these

**8.** Solve for $x$ and $y$: $x^y = y^x$ and $y = x^2$ where $x > 1$.

(A) $x=0, y=0$ (B) $x=0, y=2$
(C) $x=1, y=2$ (D) $x=2, y=4$

**9.** Write the following surds in ascending order: $\sqrt{5}, \sqrt[4]{7}, \sqrt[3]{11}$.

(A) $\sqrt[4]{7}, \sqrt[3]{11}, \sqrt{5}$ (B) $\sqrt[4]{7}, \sqrt{5}, \sqrt[3]{11}$
(C) $\sqrt[3]{11}, \sqrt[4]{7}, \sqrt{5}$ (D) $\sqrt{5}, \sqrt[3]{11}, \sqrt[4]{7}$

**10.** Simplify: $\sqrt[648]{\sqrt[4]{\sqrt[3]{\sqrt{512^{216}}}}}$.

(A) $2^{\frac{1}{3}}$ (B) $2^{\frac{1}{4}}$ (C) $2^{\frac{1}{8}}$ (D) $2^{\frac{1}{7}}$

**11.** Simplify: $\frac{2}{\sqrt{5}+\sqrt{3}} - \frac{1}{\sqrt{2}-\sqrt{3}} + \frac{3}{\sqrt{5}+\sqrt{2}}$.

(A) $2\sqrt{5}$ (B) $2\sqrt{5} - 2\sqrt{2}$
(C) $-2\sqrt{3}$ (D) $\sqrt{2} - \sqrt{3} + \sqrt{5}$

**12.** Find the values of $a$ and $b$ that satisfy both the equations given below:

$5^{0.75b}.7^{-0.5a} = \frac{625\sqrt{35}}{343}$, $25^{0.1a}.49^{0.3b} = 245.(2401)^{\frac{2}{5}}$.

(A) $a=5, b=4$ (B) $a=5, b=8$
(C) $a=7, b=6$ (D) $a=5, b=6$

**13.** One of the values of $\sqrt{\sqrt{75} - \sqrt{72}}$ is

(A) $3^{1/4}\left(\sqrt{3}-1\right)$ (B) $3\left(\sqrt{3}-\sqrt{2}\right)$
(C) $\sqrt{3}\left(\sqrt{3}-1\right)$ (D) $3^{1/4}\left(\sqrt{3}-\sqrt{2}\right)$

**14.** Arrange the following surds in ascending order: $\sqrt{2}, \sqrt[4]{3}, \sqrt[8]{6}$

(A) $\sqrt[8]{6}, \sqrt[4]{2}, \sqrt[4]{3}$ (B) $\sqrt[8]{6}, \sqrt[4]{3}, \sqrt{2}$
(C) $\sqrt[4]{3}, \sqrt[8]{6}, \sqrt{2}$ (D) $\sqrt{2}, \sqrt[8]{6}, \sqrt[4]{3}$

**15.** Simplify: $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{8-2\sqrt{15}}} + \frac{\sqrt{11+2\sqrt{30}}}{\sqrt{6}-\sqrt{5}}$

(A) $25+\sqrt{10}+2\sqrt{30}$ (B) $40+\sqrt{10}+2\sqrt{30}$
(C) $30+\sqrt{20}+2\sqrt{30}$ (D) $15+\sqrt{15}+2\sqrt{30}$

**16.** Find the value of

$$\frac{\sqrt{9+\sqrt{24}+\sqrt{48}+\sqrt{32}} + \sqrt{9-\sqrt{24}+\sqrt{48}-\sqrt{32}}}{\sqrt{9+\sqrt{24}+\sqrt{48}+\sqrt{32}} - \sqrt{9-\sqrt{24}+\sqrt{48}-\sqrt{32}}}.$$

(A) $\sqrt{\frac{3}{2}}+2$ (B) $\frac{\sqrt{3}}{2}+2$
(C) $\sqrt{\frac{3}{2}}+\frac{1}{\sqrt{2}}$ (D) $\sqrt{\frac{3}{2}}+\sqrt{2}$

**17.** If $x = \frac{1}{2}$, then find the value of $\sqrt{\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}}$.

(A) $\frac{1}{\sqrt{2}}\left(2+\sqrt{6}\right)$ (B) $\frac{1}{2}\left(\sqrt{2}+\sqrt{6}\right)$
(C) $\sqrt{2}+1$ (D) $\sqrt{3}+1$

**18.** If $x = \sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}}$, then find the value of $7x^2 - 7x$ _____.

Difficulty Level-2: Moderate

**19.** If $\frac{12}{3+\sqrt{5}-2\sqrt{2}} = x + a\sqrt{2} + b\sqrt{5} + c\sqrt{10}$ and $x$, $a$, $b$ and $c$ are rational, then find the value of $x + a + b + c$ _____.

**20.** If $x = \frac{\left(\sqrt{\left(\sqrt{3}+\sqrt{2}\right)} + \sqrt{\left(\sqrt{3}-\sqrt{2}\right)}\right)^2}{\sqrt{3}-1}$, then find the value of $\sqrt{x}$.

(A) $\left(\sqrt{3}+1\right)$ (B) $\left(\sqrt{3}-1\right)$

(C) $\left(2+\sqrt{3}\right)$ (D) $\left(2-\sqrt{3}\right)$

**21.** If $\frac{P}{p}=\frac{Q}{q}=\frac{R}{r}=\frac{S}{s}$, what is the value of $\sqrt{Pp}+\sqrt{Qq}+\sqrt{Rr}+\sqrt{Ss}$, given that $(p+q+r+s) \neq 0$?

(A) $\sqrt{(p+q+r+s)(P+Q+R+S)}$

(B) $(p+q+r+s)\left(\sqrt{P}+\sqrt{Q}+\sqrt{R}+\sqrt{S}\right)$

(C) $\left[\frac{\sqrt{P+Q+R+S}}{p+q+r+s}\right]$

(D) $\left[\frac{P+Q+R+S}{\sqrt{p+q+r+s}}\right]$

**22.** If $y = \frac{32-\sqrt{600}-\sqrt{54}}{8+\sqrt{150}-\sqrt{24}}$, then find the value of $\sqrt{y}$.

(A) $\sqrt{6}+10$ (B) $\sqrt{24}-5$

(C) $5-2\sqrt{6}$ (D) $\sqrt{24}+5$

**23.** Given: $\frac{\sqrt{9+4\sqrt{5}}-\sqrt{9-4\sqrt{5}}}{\sqrt{9+4\sqrt{5}}+\sqrt{9-4\sqrt{5}}} = \frac{a}{\sqrt{5}}$. Find the value of $a$.

(A) $\frac{5}{2}$ (B) $\frac{2}{\sqrt{5}}$ (C) $\frac{1}{\sqrt{5}}$ (D) 2

**24.** Simplify: $\left(3^{\frac{1}{3}} 9^{\frac{2}{3}} 27^{\frac{4}{3}} 81^{\frac{8}{3}} 243^{\frac{10}{3}} 729^{\frac{12}{3}}\right)^{\frac{2}{19}}$.

(A) 27 (B) 81 (C) 243 (D) 729

**25.** Find the value of the expression

$$\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{7}+\sqrt{5}}+\frac{1}{\sqrt{9}+\sqrt{7}}+\ldots+\frac{1}{\sqrt{49}+\sqrt{47}}.$$

(A) 3 (B) $\sqrt{49}$ (C) $\sqrt{47}$ (D) 4

**26.** Arrange the given surds in ascending order:

$\sqrt{7}+\sqrt{17}+\sqrt{8}+\sqrt{14}, \sqrt{10}+\sqrt{14}$ and $\sqrt{3}+\sqrt{22}$.

(A) $\sqrt{8}+\sqrt{14}, \sqrt{7}+\sqrt{17}, \sqrt{10}+\sqrt{14}, \sqrt{3}+\sqrt{72}$

(B) $\sqrt{8}+\sqrt{14}, \sqrt{3}+\sqrt{22}, \sqrt{10}+\sqrt{14}, \sqrt{7}+\sqrt{17}$

(C) $\sqrt{8}+\sqrt{14}, \sqrt{3}+\sqrt{22}, \sqrt{7}+\sqrt{17}, \sqrt{10}+\sqrt{14}$

(D) $\sqrt{3}+\sqrt{22}, \sqrt{8}+\sqrt{14}, \sqrt{7}+\sqrt{17}, \sqrt{10}+\sqrt{14}$

**27.** If $\sqrt{11\sqrt{11\sqrt{11\ldots\infty}}} = 14641^x$, then find the value of $x$.

(A) 11 (B) 4

(C) 1/4 (D) None of these

**28.** Find the value of

$$\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\ldots+\frac{1}{\sqrt{100}+\sqrt{99}}$$ _____.

**29.** Find the square root of $\frac{3}{2}(x+4)+\sqrt{2x^2+19x+35}$.

(A) $\frac{1}{2}\left(\sqrt{x+7}+\sqrt{2x+5}\right)$ (B) $\frac{1}{2}\left(\sqrt{x-7}+\sqrt{2x+5}\right)$

(C) $\frac{1}{\sqrt{2}}\left(\sqrt{x+7}+\sqrt{2x+5}\right)$ (D) $\frac{1}{\sqrt{2}}\left(\sqrt{x+7}+\sqrt{2x-5}\right)$

**30.** If $xyz = 1$, $\frac{1}{1+y^2+x^{-2}}+\frac{1}{1+z^2+y^{-2}}+\frac{1}{1+x^2+z^{-2}} =$

(A) 1 (B) 3

(C) $(x+y+z)^2$ (D) $x^{-2}+y^{-2}+z^{-2}$

**31.** If $\left(\sqrt[3]{\frac{3}{5}}\right)^{3x+12} = \left(\sqrt[4]{\frac{25}{9}}\right)^{3x+1}$, then find the value of $x$?

(A) $-\frac{17}{7}$ (B) $-\frac{9}{5}$ (C) 7 (D) –15

**32.** Simplify: $\frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2+\sqrt{3}}}-\frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2-\sqrt{3}}}$.

(A) 1 (B) $\frac{1}{\sqrt{2}}$ (C) $-\frac{\sqrt{2}}{\sqrt{3}}$ (D) $\frac{\sqrt{2}\left(3\sqrt{3}-1\right)}{\sqrt{3}\left(\sqrt{3}+1\right)}$

**33.** If $a$, $b$ and $c$ are three real numbers lying between 0 and 1 and $\sqrt{a}+\sqrt{1-a} = \sqrt{b}+\sqrt{1-b} = \sqrt{c}+\sqrt{1-c}$, which of the following may be true?

(A) $a = b = c$ (B) $a = b = 1 - c$

(C) $b = c = 1 - a$ (D) All the above

**34.** Simplify: $P+\sqrt{P^2+\sqrt{P^4+\sqrt{P^8+\sqrt{P^{16}\ldots\infty}}}}$.

(A) $P\left(\frac{\sqrt{5}+2}{2}\right)$ (B) $P\left(\frac{3+\sqrt{5}}{2}\right)$

(C) $\frac{P}{1+\sqrt{P}}$ (D) $P\left(\frac{\sqrt{5}+1}{2}\right)$

**35.** Given: $\sqrt{7\sqrt{7\sqrt{7\sqrt{7...\infty}}}} = 16807^{y}$. Find the value of $y$.

(A) 1/4 (B) 1/6 (C) 1/5 (D) 1/8

**36.** If $\frac{(x-\sqrt{3})(\sqrt{12}+\sqrt{8})}{(\sqrt{12}-\sqrt{8})} = 1$, then find the value of $x$.

(A) $5-2\sqrt{3}+\sqrt{6}$ (B) $5+2\sqrt{6}+\sqrt{3}$

(C) $5+\sqrt{3}-2\sqrt{6}$ (D) $5-\sqrt{3}+2\sqrt{6}$

**37.** Solve for $x$ if $\left(\sqrt[4]{\frac{2}{7}}\right)^{(3x+1)} = \left(\sqrt[3]{\frac{49}{4}}\right)^{\frac{(5x-37)}{2}}$ _____.

**38.** One of the values of $\sqrt{x^2+2y\sqrt{x^2-y^2}}+\sqrt{x^2-2y\sqrt{x^2-y^2}}$ is

(A) $y^2+x^2$ (B) $x^2-y^2$ (C) $2y$ (D) $2x$

**39.** If $\frac{x-y}{\sqrt{x}+\sqrt{y}}+\frac{x-y}{\sqrt{x}-\sqrt{y}} = 4$, then find the value of $x$ _____.

**40.** If $3^{2x+2} = 6561$ and $2^{3y-1} = 2048$, then find the value of $\frac{2x+3y}{y-x}$ _____.

**41.** If $S = \frac{3}{9+\sqrt{90}} + \frac{\sqrt{10}}{10+\sqrt{110}} + \frac{\sqrt{11}}{11+\sqrt{132}} + \frac{\sqrt{12}}{12+\sqrt{156}} + \ldots + \frac{\sqrt{399}}{399+\sqrt{159600}}$, then find the value of $S$.

(A) 24 (B) $\sqrt{399}-3$

(C) $\frac{50}{\sqrt{399}}$ (D) 17

**42.** Simplify: $\frac{\sqrt{15}+\sqrt{10}+\sqrt{5}}{\sqrt{6+2\sqrt{2}+2\sqrt{6}+2\sqrt{3}}}$.

(A) $\sqrt{5}$ (B) $\sqrt{10}$ (C) $\sqrt{15}$ (D) 5

**43.** Find the square root of $x+2+\frac{\sqrt{4x^2+16x+15}}{2}$.

(A) $\frac{1}{\sqrt{2}}\left(\sqrt{2x+5}+\sqrt{2x+3}\right)$

(B) $\frac{1}{2}\left(\sqrt{2x+5}+\sqrt{2x+3}\right)$

(C) $\frac{1}{2}\left(\sqrt{2x+5}+\sqrt{2x-3}\right)$

(D) $\frac{1}{\sqrt{2}}\left(\sqrt{2x-5}+\sqrt{2x+3}\right)$

**44.** If $S = \left(\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{6}}\right)^2+\left(\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}-\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{6}}\right)^2+\left(\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}-\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{6}}\right)^2$, then find the value of $S$.

(A) $3\frac{1}{2}$ (B) $4\frac{1}{5}$ (C) 4 (D) None of these

**45.** Find the value of $\sqrt{11+\sqrt{48}+\sqrt{24}+\sqrt{72}}$.

(A) $2+2\sqrt{3}+\sqrt{6}$ (B) $2+\sqrt{3}+\sqrt{6}$

(C) $\sqrt{2}+\sqrt{3}+\sqrt{6}$ (D) $3+\sqrt{2}+\sqrt{6}$

## EXERCISE-3

***Directions for questions 1 to 45:*** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Find the value of $\frac{1+4x}{1+\sqrt{1+4x}}+\frac{1+4x}{1-\sqrt{1+4x}}$, when $x = 1/4$ _____.

**2.** Simplify: $\sqrt{\sqrt[3]{\sqrt[4]{\sqrt[6]{25^{72}}}}}$ _____.

**3.** If $32^{a} = 25b$ and $2^{a} = 50b$, then find the value of $a$.

(A) –1 (B) –1/2 (C) –1/4 (D) 2

**4.** If $2^{x} = 7^{y} = 14^{z}$, then find the value of $z$ in terms of $x$ and $y$.

(A) $\frac{x+y}{x-y}$ (B) $\frac{xy}{x+y}$

(C) $x+y+xy$ (D) $xy-(x+y)$

**5.** Simplify: $\frac{2^{m+2}\,6^{m-2}\,10^{2m}\,15^{m-1}\,25^{m-n}}{4^{2m}\,3^{2m-3}\,5^{5m-2n+5}\,5^{-8}}$.

(A) 1 (B) $2^{m}\,3^{m-1}\,5$

(C) $2\,3^{m}\,5^{2m-2n}$ (D) 25

**6.** Which is the greatest among $7^{\frac{1}{6}}, 4^{\frac{1}{3}}, 3^{\frac{1}{2}}, 10^{\frac{1}{9}}$ and $19^{\frac{1}{18}}$?

(A) $7^{\frac{1}{6}}$ (B) $4^{\frac{1}{3}}$ (C) $3^{\frac{1}{2}}$ (D) $10^{\frac{1}{9}}$

**7.** Given $3^{4x+2} = 729$ and $2^{3y+1} = 8192$, find $\frac{2y+x}{y-x}$.

(A) 2 (B) 1 (C) 4 (D) 3

**8.** Simplify: $\dfrac{\left(x^{2^{2n-1}}+y^{2^{2n-1}}\right)\left(x^{2^{2n-1}}-y^{2^{2n-1}}\right)}{x^{2^{2n}}-y^{2^{2n}}}$.

(A) 1 (B) $x^n+y^n$
(C) $x^{2n}-y^{2n}$ (D) $x^{2n}+y^{2n}$

**9.** Given: $3^x = 115 + 2^y$ and $3^{x-4} + 2^{y-5} = 7$ Find $x$ and $y$ satisfying the above equations.

(A) $x=6, y=5$ (B) $x=5, y=7$
(C) $x=5, y=6$ (D) $x=y=6$

**10.** Simplify: $\left(729b^{14} \div 27b^8\right)^{\frac{1}{3}} \div \left(\dfrac{243b^{10}}{9b^4}\right)^{\frac{2}{3}}$.

(A) $\dfrac{1}{9b^2}$ (B) $\dfrac{1}{3b^2}$ (C) $3b^2$ (D) $\dfrac{1}{3}b^2$

**11.** $\left[(512)^{\frac{2}{9}} \times (2187)^{\frac{3}{7}} \times (3375)^{\frac{1}{3}} \times 5\right]^{\frac{1}{2}} =$

(A) 30 (B) 90 (C) $30\sqrt{5}$ (D) $30\sqrt{3}$

**12.** If $(2.56)^a = (0.00256)^b = 10^c$, then find the relationship between $a$, $b$ and $c$.

(A) $\dfrac{2}{c}=\dfrac{1}{a}-\dfrac{1}{b}$ (B) $\dfrac{3}{c}=\dfrac{1}{b}-\dfrac{1}{a}$
(C) $\dfrac{1}{a}=\dfrac{3}{c}+\dfrac{1}{b}$ (D) $\dfrac{2}{b}=\dfrac{1}{a}+\dfrac{1}{c}$

**13.** Write the following terms in descending order: $\sqrt[3]{4}, \sqrt[4]{7}$ and $\sqrt[6]{17}$.

(A) $\sqrt[3]{4}, \sqrt[6]{17}$ and $\sqrt[4]{7}$ (B) $\sqrt[6]{17}, \sqrt[3]{4}$ and $\sqrt[4]{7}$
(C) $\sqrt[4]{7}, \sqrt[6]{17}$ and $\sqrt[3]{4}$ (D) $\sqrt[4]{7}, \sqrt[3]{4}$ and $\sqrt[6]{17}$

**14.** Simplify: $\dfrac{2^a \cdot 2^{a+2} \cdot 3^{a+2} \cdot 2^{6a} \cdot 3^{4a+4b}}{3^{a-b} \cdot 3^{2a+b-2} \cdot 2^{4a+2b-4} \cdot 2^6 \cdot 3^2}$.

(A) $\dfrac{(4)^{2a-b}}{(9)^{(a+2b+1)}}$ (B) $\dfrac{4}{9}$ (C) $\dfrac{2^{4a-2b}}{9^{a+b}}$ (D) $4^{2a-b} \times 9^{a+2b+1}$

**15.** If $\dfrac{5^x}{5^y} = 125$ and $\dfrac{2^x}{4^y} = 2$, then find the value of $\left(\dfrac{3^{2x}}{27^y}+4\right)$.

**16.** Simplify: $\dfrac{\left(\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}\right)^2}{\left(\sqrt{3}+\sqrt{2}\right)^2\left(5-2\sqrt{6}\right)}$

(A) $4\sqrt{15}+4\sqrt{10}$ (B) $2\sqrt{10}+2\sqrt{15}$
(C) $4\sqrt{15}-2\sqrt{10}$ (D) $2\sqrt{5}+2$

**17.** If $\sqrt{\left(10+2\sqrt{21}\right)^3}+\sqrt{\left(10-2\sqrt{21}\right)^3} = p\sqrt{7}+q\sqrt{3}$, then find the value of $p+q$ where $p$ and $q$ are rational numbers.

(A) 20 (B) 24 (C) 28 (D) 32

**18.** Given: $\left(a-\sqrt{2}\right)\dfrac{\left(\sqrt{24}-\sqrt{18}\right)}{\sqrt{24}+\sqrt{18}} = 1$. Find $a$.

(A) $7-3\sqrt{3}+2\sqrt{2}$ (B) $7+4\sqrt{3}+\sqrt{2}$
(C) $7+3\sqrt{3}+2\sqrt{2}$ (D) $7+6\sqrt{3}+\sqrt{2}$

**19.** Simplify: $\dfrac{6\sqrt{2}}{2\sqrt{2}-\sqrt{\left(\sqrt{3}+2\right)}}+2\sqrt{12-6\sqrt{3}}$.

(A) 6 (B) 9 (C) 12 (D) 18

**20.** If $a$ and $b$ are real numbers satisfying $\dfrac{\sqrt{a}+\sqrt{b}}{a-b}-\dfrac{\sqrt{b}-\sqrt{a}}{a+b}$ $= 4$, then find the value of $x$ if $\dfrac{a^x+b^x}{a^2-b^2} = 2$.

(A) 1/2 (B) 2 (C) 3/2 (D) 1

**21.** Simplify: $\dfrac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2+\sqrt{3}}}+\dfrac{2-\sqrt{3}}{\sqrt{2}+\sqrt{2-\sqrt{3}}}$.

(A) 1 (B) $\dfrac{1}{\sqrt{2}}$ (C) $\sqrt{2}$ (D) $\dfrac{\sqrt{2}\left(3\sqrt{3}-1\right)}{\sqrt{3}\left(\sqrt{3}+1\right)}$

**22.** Simplify: $\dfrac{2+\sqrt{2}+\sqrt{6}}{\sqrt{3}+\sqrt{2}+\sqrt{3}+\sqrt{6}}$.

(A) 1 (B) 2 (C) $\dfrac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

**23.** If $\dfrac{\sqrt{3+2\sqrt{2}}-\sqrt{3-2\sqrt{2}}}{\sqrt{3+2\sqrt{2}}+\sqrt{3-2\sqrt{2}}} = \sqrt{\dfrac{x}{32}}$, then find the value of $x$.

(A) 2 (B) 4 (C) 8 (D) 16

**24.** If $x = \dfrac{1}{2}\left(\sqrt{5}+\dfrac{1}{\sqrt{5}}\right)$, then find the value of $\dfrac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}}$.

(A) –1 (B) 1 (C) –2 (D) 2

**25.** $\left(24-16\sqrt{2}\right)^{\frac{1}{6}}\left(32+16\sqrt{2}\right)^{\frac{1}{3}} =$

(A) 2 (B) 4 (C) $(2)^{\frac{1}{3}}$ (D) $(4)^{\frac{1}{3}}$

**26.** If $n = 8(m-2)$, $m \neq 4$ and $m > n > 1$, then simplify $\sqrt{\dfrac{m-\sqrt{n}}{m+\sqrt{n}}}+\sqrt{\dfrac{m+\sqrt{n}}{m-\sqrt{n}}}-\dfrac{8}{\sqrt{(m+\sqrt{n})(m-\sqrt{n})}}$

(A) 1 (B) $\dfrac{m}{m-4}$ (C) 2 (D) $(m-4)^{1/2}$

**27.** Which of the following is the mean proportional of $8-3\sqrt{7}$ and $16-5\sqrt{7}$ ?

(A) $7-4\sqrt{7}$ (B) $11-4\sqrt{7}$
(C) $12-4\sqrt{7}$ (D) $4-4\sqrt{7}$

**28.** If $\frac{1}{\sqrt{a}+\sqrt{a+1}}+\frac{1}{\sqrt{a+1}+\sqrt{a+2}}+\frac{1}{\sqrt{a+2}+\sqrt{a+3}}+\ldots$

$+\frac{1}{\sqrt{a+63}+\sqrt{a+64}}=4$, which of the following is a possible value of $a$?

(A) 16 (B) 64 (C) 36 (D) 196

**29.** Given: $b=\frac{1}{2}\left(7\sqrt{7}+\frac{1}{\sqrt{7}}\right)$. Find the value of $\frac{\sqrt{b^2-7}}{b-\sqrt{b^2-7}}$.

(A) 7 (B) 24/7 (C) 24 (D) 168

**30.** Simplify: $\frac{1}{\sqrt{2\left(\sqrt{6+2\sqrt{5}}\right)+4\left(4+\sqrt{5}\right)}}$.

(A) $\frac{3-\sqrt{5}}{36}$ (B) $\frac{3-\sqrt{5}}{18}$ (C) $\frac{3-\sqrt{5}}{72}$ (D) $\frac{\sqrt{3}\left(\sqrt{5}-1\right)}{12}$

**31.** Simplify: $\frac{\sqrt{8}+\sqrt{6}-\sqrt{10}}{\sqrt{6}+2\sqrt{3}-2\sqrt{5}-\sqrt{15}}$.

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

**32.** Simplify: $\frac{\left(17+2\sqrt{72}\right)^{\frac{3}{2}}+\left(17-2\sqrt{72}\right)^{\frac{3}{2}}}{\left(6+4\sqrt{2}\right)^{\frac{3}{2}}+\left(6-4\sqrt{2}\right)^{\frac{3}{2}}}$

(A) $\frac{99}{20}$ (B) $\frac{5}{4}$ (C) $\frac{35}{23}$ (D) $35\sqrt{8}$

**33.** $\sqrt{15+2\sqrt{35}-2\sqrt{15}-2\sqrt{21}}=$

(A) $\sqrt{7}-\sqrt{5}-\sqrt{3}$ (B) $\sqrt{7}-\sqrt{5}+\sqrt{3}$

(C) $\sqrt{7}+\sqrt{5}-\sqrt{3}$ (D) $\sqrt{5}+\sqrt{3}-\sqrt{7}$

**34.** If $x=\sqrt[3]{4+\sqrt{8}}+\sqrt[3]{4-\sqrt{8}}$, then find the value of $x^3-6x$ = _____.

**35.** If $\frac{8}{2+2\sqrt{3}+2\sqrt{3}}=x+y\sqrt{2}+z\sqrt{6}$, then find the value of $x+y+z$ _____.

**36.** Find the greater of $\sqrt{8}+\sqrt{19}$, $\sqrt{12}+\sqrt{13}$ and $\sqrt{14}+\sqrt{12}$.

(A) $\sqrt{8}+\sqrt{19}$ (B) $\sqrt{14}+\sqrt{12}$

(C) $\sqrt{12}+\sqrt{13}$ (D) With A and B are equal

**37.** Rationalize the denominator of $\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$.

(A) $\frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{6}$ (B) $\frac{\sqrt{2}-\sqrt{3}-\sqrt{5}}{6}$

(C) $\frac{\left(\sqrt{2}+\sqrt{3}-\sqrt{5}\right)\sqrt{6}}{6}$ (D) $\frac{\left(\sqrt{2}+\sqrt{3}+\sqrt{5}\right)\sqrt{6}}{12}$

**38.** $\frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}}-\frac{18\sqrt{3}}{\sqrt{5}-\sqrt{2}}+\frac{8}{\sqrt{5}+\sqrt{3}}=$

(A) $4\sqrt{5}-5\sqrt{15}-6\sqrt{6}-4\sqrt{3}-\sqrt{10}$

(B) $4\sqrt{5}+5\sqrt{15}+6\sqrt{6}+4\sqrt{3}+\sqrt{10}$

(C) $4\sqrt{5}-5\sqrt{15}+6\sqrt{6}+\sqrt{10}$

(D) $4\sqrt{5}-5\sqrt{15}+6\sqrt{6}+4\sqrt{3}-\sqrt{10}$

**39.** The value of $\frac{13}{\sqrt{19-8\sqrt{3}}}-\frac{23}{\sqrt{31+2\sqrt{3}}}-\frac{24}{\sqrt{48+24\sqrt{3}}}$ is

(A) $12-4\sqrt{3}$ (B) 0

(C) $8+2\sqrt{3}$ (D) 8

**40.** If $\left(6+\sqrt{35}\right)^{x/2}+\left(6-\sqrt{35}\right)^{x/2}=12$, then number of solutions for $x$ is

(A) 0 (B) 2 (C) 1 (D) 4

**41.** What is the cube root of $(3\sqrt{3})(5\sqrt{5})+11\sqrt{11}+(3\sqrt{3})(\sqrt{5})$ $(\sqrt{11})\left[(\sqrt{3})(\sqrt{5})+\sqrt{11}\right]$?

(A) $\sqrt{5}+\sqrt{11}$ (B) $\sqrt{15}+\sqrt{11}$

(C) $\sqrt{15}+\sqrt{33}$ (D) $15+\sqrt{11}$

**42.** If $a=\frac{\left(\sqrt{5}-\sqrt{3}\right)^3}{8}$, $b=\left(\sqrt{6}+\sqrt{5}\right)^3$, $c=\frac{\left(\sqrt{3}-\sqrt{6}\right)^3}{27}$, what is the value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$?

(A) $3\sqrt{3}-9\sqrt{5}$ (B) $3\left(2\sqrt{6}-\sqrt{5}-\sqrt{3}\right)$

(C) $9\sqrt{5}+3\sqrt{3}$ (D) $3\left(2\sqrt{6}+\sqrt{5}+\sqrt{3}\right)$

**43.** If $\frac{\left(4^x\right)^x\cdot\left(4^y\right)^y\cdot\left(4^z\right)^z}{\left(16^x\right)^{-y}\cdot\left(16^y\right)^{-z}\cdot\left(16^z\right)^{-x}}=64$, which of the following is a possible value of $(x+y+z)$?

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 2 (D) $2\sqrt{2}$

**44.** If $x=\frac{1}{3-2\sqrt{2}}, y=\frac{1}{3+2\sqrt{2}}$, what is the value of $3x^2+6xy-3y^2$?

(A) $6+24\sqrt{2}$ (B) $6+72\sqrt{2}$

(C) $6+48\sqrt{2}$ (D) $6+96\sqrt{2}$

**45.** Simplify: $\frac{x+y}{\sqrt[3]{x^2}-\sqrt[3]{xy}+\sqrt[3]{y^2}}-\frac{x-y}{\sqrt[3]{x^2}+\sqrt[3]{xy}+\sqrt[3]{y^2}}$.

(A) $2x^{1/3}$ (B) $2y^{1/3}$ (C) $2(xy)^{1/3}$ (D) $x^{1/3}-y^{1/3}$

Difficulty Level-3: Advanced

## EXERCISE-4

***Directions for questions 1 to 25***: For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** If $5^{\log_5 x^{\log_x y \log_z z^2}} = 54$ and $x, y, z > 0$, then find the value of $y$.

(A) 27 (B) 2916 (C) $3\sqrt{6}$ (D) 108

**2.** If $a^{5-x} b^{3x} = a^{5x} b^{x+5}$ and $a^3 = b^3 \neq -1, 0$ or 1, then find the value of $x$.

(A) 0 (B) 3/2
(C) $-5/2$ (D) Cannot be determined

**3.** If $450^x = 570^y = 722^z$, then find the value of $\frac{1}{x} - \frac{1}{y}$.

(A) $\frac{1}{y} + \frac{1}{z}$ (B) $\frac{1}{z} - \frac{1}{y}$

(C) $\frac{1}{y} - \frac{1}{z}$ (D) None of these

**4.** The greatest of $\sqrt{19} + \sqrt{6}$, $\sqrt{23} + \sqrt{3}$, $\sqrt{18} + \sqrt{7}$, $\sqrt{24} + \sqrt{2}$ is

(A) $\sqrt{19} + \sqrt{6}$ (B) $\sqrt{23} + \sqrt{3}$

(C) $\sqrt{18} + \sqrt{7}$ (D) $\sqrt{24} + \sqrt{2}$

**5.** The greatest of $3^{\frac{1}{4}}, 4^{\frac{1}{5}}, 5^{\frac{1}{6}}, 6^{\frac{1}{7}}, 7^{\frac{1}{8}}$ is

(A) $3^{\frac{1}{4}}$ (B) $4^{\frac{1}{5}}$ (C) $6^{\frac{1}{7}}$ (D) $7^{\frac{1}{8}}$

**6.** If $\frac{3 - \sqrt{a+5}}{a-4} = \frac{-1}{6}$, how many values can $a$ take? _____

**7.** $\sum_{x=2}^{225} \frac{1}{\sqrt{x} + \sqrt{x-1}} =$

(A) 16 (B) 15 (C) 14 (D) 13

**8.** $\left(29 + 12\sqrt{5}\right)^{1/2} - \left(24 - 8\sqrt{5}\right)^{1/2} =$

(A) 1 (B) 3 (C) 5 (D) 7

**9.** The greatest of $31^{15}$, $17^{19}$, $7^{25}$, $63^{12}$ is

(A) $31^{15}$ (B) $17^{19}$ (C) $7^{25}$ (D) $63^{12}$

**10.** $X = \frac{1}{16\sqrt{17} + 17\sqrt{16}} + \frac{1}{17\sqrt{18} + 18\sqrt{17}} + \ldots + \frac{1}{144\sqrt{145} + 145\sqrt{144}}$. Find $X$.

(A) $\frac{1}{\sqrt{17}} - \frac{1}{\sqrt{145}}$ (B) $\frac{1}{\sqrt{16}} - \frac{1}{\sqrt{145}}$

(C) $\frac{1}{\sqrt{16}} - \frac{1}{\sqrt{144}}$ (D) $\frac{1}{\sqrt{17}} - \frac{1}{\sqrt{144}}$

**11.** If $N = \sqrt{17 + \sqrt{17 - \sqrt{17 + \sqrt{17 - \ldots\infty}}}}$, then find the value of $N$.

(A) $\frac{\sqrt{69} - 3}{2}$ (B) $\frac{\sqrt{65} + 1}{2}$

(C) $\frac{\sqrt{65} - 3}{2}$ (D) $\frac{\sqrt{69} - 1}{2}$

**12.** If $a > 1$, $b > 1$, $a^{1/a} = b^{1/b}$ and $a^8 = b^6$, $b$ equals $\frac{}{81}$.

**13.** If $x^{\sqrt{x}^{\sqrt{x}}} = \left(x^{\sqrt{x}}\right)^{\sqrt{x}}$ and $x > 0$, $x$ can be

(A) $\sqrt{2}$ (B) 8 (C) $2\sqrt{2}$ (D) 4

**14.** $X = \frac{\sqrt{5}}{\sqrt{30} + \sqrt{35}} + \frac{\sqrt{6}}{\sqrt{42} + \sqrt{48}} + \frac{\sqrt{7}}{\sqrt{56} + \sqrt{63}} + \ldots + \frac{\sqrt{19}}{\sqrt{380} + \sqrt{399}}$. $X$ lies between

(A) 1 and 1.2 (B) 1.4 and 1.6
(C) 1.8 and 2 (D) 2 and 2.2

**15.** If $7x^{7a} < 8x^{8b}$ for all positive values of $x$, then

(A) $a < b$ (B) $a > b$
(C) $a : b = 7 : 8$ (D) $a : b = 8 : 7$

**16.** If $A^{\frac{1}{B}} = B^{\frac{1}{C}} = C^{\frac{1}{A}}$, each of these is equal to

(A) $(ABC)^{\frac{ABC}{AB+BC+CA}}$ (B) $(ABC)^{\frac{AB+BC+CA}{ABC}}$

(C) $(ABC)^{\frac{ABC}{A+B+C}}$ (D) $(ABC)^{\frac{1}{A+B+C}}$

**17.** Solve for $x$: $\sqrt[3]{12\sqrt[3]{12\sqrt[3]{12\ldots\infty}}} = 144^x$

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$

**18.** If $a = e^{\frac{1}{e}}$, find the value of $a^{a^{a^{a^{\ldots}}}}$, where the exponential expression extends indefinitely.

(A) $\sqrt{e}$ (B) $e$ (C) $e^2$ (D) 3

**19.** $\sqrt{\sqrt{192} - \sqrt{180}} =$

(A) $\sqrt{5\sqrt{3}} - \sqrt{3\sqrt{3}}$ (B) $3^{1/2}\left(\sqrt{5} - \sqrt{3}\right)$

(C) $3^{1/4}\left(\sqrt{5} + \sqrt{3}\right)$ (D) $3^{1/4}\left(\sqrt{5} - \sqrt{2}\right)$

**20.** If $x = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$, find the value of $x^4 + \frac{1}{x^4}$.

(A) $-194$ (B) $112\sqrt{3}$
(C) 194 (D) $-112\sqrt{3}$

**21.** $\sqrt{\dfrac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}}$ =

(A) $(3\sqrt{2}+2\sqrt{3})/6$ (B) $(3\sqrt{2}-2\sqrt{3})/\sqrt{6}$

(C) $\sqrt{3}+\sqrt{2}$ (D) $\sqrt{3}-\sqrt{2}$

**22.** $\sqrt[4]{49-20\sqrt{6}}$ =

(A) $\sqrt{2}-1$ (B) $\sqrt{2}+\sqrt{3}$

(C) $\sqrt{5}-\sqrt{3}$ (D) $\sqrt{3}-\sqrt{2}$

**23.** If $(9\sqrt{3}+11\sqrt{2})^{1/3}=\sqrt{a}+\sqrt{b}$ and $a$ and $b$ are integers, where $a>b$, which of the following is true?

(A) $a=\sqrt{3}$ (B) $a=2$ (C) $b=\sqrt{3}$ (D) $b=2$

**24.** If $\left(8-3\sqrt{7}\right)^{x^2-3}+\left(8-3\sqrt{7}\right)^{x^2-3}=16$, the values of $x$ are

(A) 2, – 2 (B) $\sqrt{2},-\sqrt{2}$

(C) 2, $\sqrt{2}$ (D) 2, – 2, $\sqrt{2}$, $-\sqrt{2}$

**25.** $\left(5\sqrt{2}+7\right)^{\frac{1}{3}}+\left(5\sqrt{2}-7\right)^{\frac{1}{3}}$ is

(A) an integer.
(B) a rational number but not an integer.
(C) an irrational number.
(D) None of these

## EXERCISE-5 Based on Data Sufficiency

***Directions for questions 1 to 20:*** Each question is followed by two Statements, I and II. Answer each question based on the following directions:

Choose (A) if the question can be answered by using one of the Statements alone, but cannot be answered by using the other Statement alone.

Choose (B) if the question can be answered by using either Statement alone.

Choose (C) if the question can be answered by using both the Statements together, but cannot be answered by using either Statement alone.

Choose (D) if the question cannot be answered even by using both the Statements together.

**1.** If $x$ and $y$ are positive integers, what is the value of $(2x)^y$?

I. $x^y=16$. II. $2x=8$.

**2.** If $p$, $q$ and $r$ are non-zero real numbers, find the value of $\left(4^{\frac{1}{qr}}\right)^{p^2}\times\left(4^{\frac{1}{pq}}\right)^{r^2}\times\left(4^{\frac{1}{pq}}\right)^{q^2}$.

I. $p^3+q^3+r^3=3pqr$. II. $p=1$, $q=2$ and $r=-3$.

**3.** If $b$ is a natural number, which is greater among $a^{3b}$ and $(a+1)^{2b}$?

I. $a$ is a whole number.
II. $a$ is a natural number.

**4.** Find $a+b$ where $a$ and $b$ are non-negative integers.

I. $2^a+3^b=17$. II. $9^a+8^b=145$.

**5.** If $a$ and $b$ are positive, which is greater among $P$ and $Q$?

I. $P=\sqrt[3]{a^3+b^3}$ II. $Q=\sqrt{a^2+b^2}$

**6.** If $54q=2^p$, what is the value of $p$?

I. $27q=64^p$ II. $108q=16^{P+1}$

**7.** What is the value of $\dfrac{2y+x}{2y-x}$?

I. $3^{4x+y}=243$ II. $2^{8y+x}=512$

**8.** If $a$ and $b$ are positive integers, what is the value of $a+b$?

I. $5^a+2^{b+2}=253$ II. $5^{a+1}+2^{b-2}=633$

**9.** If $x$ and $y$ are positive, is $x+2\sqrt{y}>5+\sqrt{108}$?

I. $x<5$ II. $y=99$

**10.** If $a$, $b$, $c$ and $d$ are all positive, is $a^d>b^c$?

I. $a>b$
II. $d$ is geometric mean of $a$ and $b$ while $c$ is arithmetic mean of $a$ and $b$.

**11.** If $a$, $b$ and $c$ are real numbers, is $c=a+b$?

I. $5^{\frac{1}{a}}=7^{\frac{1}{b}}$ II. $5^{\frac{1}{c}}=7^{\frac{c-b}{bc}}$

**12.** It is given that $A$, $B$ are surds and $8\left(5+\sqrt{18}\right)$ $AB=\left(6+2\sqrt{3}\right)$. What is the value of $A$?

I. $28AB=15-3\sqrt{6}-9\sqrt{2}+5\sqrt{3}$

II. $\dfrac{1}{A}+\dfrac{1}{B}=7+3\sqrt{2}-\dfrac{2}{\sqrt{3}}$

**13.** If $x$ is positive number, what is the value of $x$?

I $\dfrac{1}{x}\left\{\dfrac{1}{\sqrt{3}+1}+\dfrac{1}{\sqrt{5}+\sqrt{3}}+\dfrac{1}{\sqrt{7}+\sqrt{5}}+\ldots\dfrac{1}{\sqrt{49}+\sqrt{47}}\right\}$

$=p$ and $p$ is a positive integer.

II. $x=3^n$ where $n$ is an integer.

**14.** If $p$, $q$ and $s$ are integers, what is the numerical value of $s$?

I. $\dfrac{p^s}{q^{2s}}=\dfrac{27}{64}$ II. $q+s=5$

**15.** What is the value of $y-2x$?

I. $(2^{6x-3y})(3^{4x-2y})=2^{18}\times 6^{36}$
II. $x$ and $y$ are integers.

**16.** If $a$, $b$ and $c$ are all positive integers and $a$ is a factor of $c$, is $x$ an integer?

I. $x = \left(\frac{a^3b^6c^9}{a^4b^8c^{12}}\right)^9 \div \left(\frac{a^2b^4c^6}{a^3b^7c^5}\right)^8$

II. $x = \left(\frac{a^5b^8c^{10}}{a^2b^6c^4}\right)^5 \div \left(\frac{ab^2c^2}{a^5b^7c^5}\right)^9$

**17.** If $n \neq 0$, what is the value of $\left(\frac{W^n + X^n}{Y^n + Z^n}\right)^{\frac{2}{n}}$ ?

I. $\frac{x}{y} = \frac{w}{z}$ II. $\frac{w+x}{y+z} = \frac{1}{2}$

**18.** If $x$ is a positive integer and $x < 100$, is $\sqrt[x]{\sqrt[4]{\sqrt[3]{512^{216}}}}$ an integer?

I. $x$ is a multiple of 10

II. $x = 3^n$ where $n$ is an integer.

**19.** If $x$, $y$ are positive integers, what is the value of $x + y$?

I. $(4)^{x+2} + (729)^{y-4} = 1025$

II. $3^{2x}\,2^y = 324$

**20.** If $ab \neq 0$, what is the value of $\frac{\left(a^3\right)^2 b^3}{a^2}$?

I. $(ab)^3 = \frac{3}{4a}$ II. $b = \frac{1}{2a^{\frac{4}{3}}}$

## ANSWER KEYS

### Exercise-1

1. (B) 2. (B) 3. (A) 4. (D) 5. (D) 6. (A) 7. 2 8. (B) 9. (D) 10. (A) 11. 2 12. (A) 13. 7 14. (B) 15. (A) 16. $x = 3$, $y = 2$ 17. 2 18. (D) 19. 8 20. 3200, 8100 $3^{\frac{1}{4}}$, 32400 21. 1 22. (B) 23. (A) 24. 1.5 25. 2 26. (D) 27. (C) 28. (D) 29. (A) 30. (B) 31. (D) 32. (A) 33. (C) 34. (B) 35. (A) 36. (B) 37. (B) 38. (C) 39. (A) 40. 6 41. 5 42. (B) 43. (D) 44. 4 45. (A)

### Exercise-2

1. (B) 2. (B) 3. (D) 4. (D) 5. (A) 6. (D) 7. (B) 8. (D) 9. (A) 10. (C) 11. (A) 12. (D) 13. (D) 14. (B) 15. (D) 16. (D) 17. (B) 18. 7 19. 2 20. (A) 21. (A) 22. (C) 23. (D) 24. (D) 25. (A) 26. (D) 27. (C) 28. 9 29. (C) 30. (A) 31. (B) 32. (C) 33. (D) 34. (B) 35. (C) 36. (C) 37. 5 38. (C) 39. 4 40. 18 41. (D) 42. (A) 43. (B) 44. (D) 45. (C)

### Exercise-3

1. −4 2. 5 3. (C) 4. (B) 5. (D) 6. (C) 7. (D) 8. (A) 9. (B) 10. (B) 11. (B) 12. (C) 13. (C) 14. (D) 15. 85 16. (D) 17. (D) 18. (B) 19. (C) 20. (C) 21. (D) 22. (B) 23. (D) 24. (D) 25. (B) 26. (C) 27. (B) 28. (C) 29. (C) 30. (D) 31. (C) 32. (A) 33. (C) 34. 8 35. 2 36. (B) 37. (C) 38. (A) 39. (B) 40. (B) 41. (B) 42. (B) 43. (B) 44. (B) 45. (B)

### Exercise-4

1. (C) 2. (A) 3. (C) 4. (C) 5. (B) 6. 0 7. (C) 8. (C) 9. (B) 10. (B) 11. (B) 12. 256 13. (D) 14. (D) 15. (D) 16. (D) 17. (B) 18. (B) 19. (A) 20. (C) 21. (C) 22. (D) 23. (D) 24. (D) 25. (C)

### Exercise-5

1. (C) 2. (B) 3. (D) 4. (A) 5. (C) 6. (B) 7. (C) 8. (B) 9. (A) 10. (D) 11. (C) 12. (D) 13. (D) 14. (C) 15. (A) 16. (B) 17. (C) 18. (B) 19. (B) 20. (B)

## SOLUTIONS

## EXERCISE-1

**1.** $3^{2x} = 81 \Rightarrow 3^{2x} = 3^4$

$\Rightarrow 2x = 4$ [$\because$ If $a^m = a^n$, then $m = n$]

$\Rightarrow x = 2$

**2.** $x^{\frac{a}{a+b+c}} \cdot x^{\frac{b}{a+b+c}} \cdot x^{\frac{c}{a+b+c}}$

$= x^{\frac{a}{a+b+c}+\frac{b}{a+b+c}+\frac{c}{a+b+c}}$ [$\because a^m . a^n = a^{m+n}$]

$= x^{\frac{a+b+c}{a+b+c}} = x^1 = x$

**3.** $2^{2^3} = 2^8 = 2^{2\times 4} = (2^2)^4 = 4^4$

$3^{2^2} = 3^4$

$4^4 > 3^4 \therefore 2^{2^3} > 3^{2^2}$

**4.** $2^{x^x} = 16 \Rightarrow 2^{x^x} = 2^4$

$\Rightarrow x^x = 4$ [$\because a^m = a^n$, then $m = n$]

$\Rightarrow x^x = x^2 \Rightarrow x = 2$

**5.** $(2^x)(3^y) = 216$

$(2^x)(3^y) = 6^3$

$(2^x)(3^y) = (2.3)^3 \Rightarrow x = 3$ and $y = 3$

**6.** $3^{1/2} \cdot 3^{3/2} \cdot 3^{5/2} \cdot 3^{7/2} = 9^x$

$\Rightarrow 3^{\frac{1}{2}+\frac{3}{2}+\frac{5}{2}+\frac{7}{2}} = \left(3^2\right)^x$

$\Rightarrow 3^8 = 3^{2x} \Rightarrow 8 = 2x$

$\Rightarrow x = 4$

**7.** $4^{2x} = 4^4$

Equating powers of 4 on both sides, $2x = 4$.

$x = 2$

**8.** The given expression is

$x^{\frac{(b+c-a)+(c+a-b)+(a+b-c)}{a+b+c}} = x^{\frac{a+b+c}{a+b+c}} = x$

**9.** Rationalizing factor of $a^{\frac{1}{3}} + b^{\frac{1}{3}}$ is $a^{\frac{2}{3}} - (ab)^{\frac{1}{3}} + b^{\frac{2}{3}}$

Rationalizing factor of $4^{\frac{1}{3}} + 3^{\frac{1}{3}}$ is $4^{\frac{2}{3}} - 12^{\frac{1}{3}} + 3^{\frac{2}{3}}$

**10.** $2^{3^3} = 2^{27}$

$3^{3^2} = 3^9$

$2^{27} = (2^3)^9$

$2^3 > 3$

$\therefore (2^3)^9 > 3^9$

$\therefore 2^{3^3}$ is greater.

**11.** $3^{x^x} = 3^4 = 3^{2^2}$ comparing both sides, $x = 2$.

**12.** $2^{\frac{1+3+5+7+9+11}{2}} = (2^3)^x$

$2^{18} = 2^{3x}$

$\Rightarrow x = 6$

**13.** $2^p 3^q = 2(216) = 2(6^3) = 2\ (2\times 3)^3$

$2^p 3^q = (2^4)\ (3^3)$

comparing both sides, $p = 4$ and $q = 3$

$p + q = 7$

**14.** As all the given powers have the base as 2, change all of them into base 2.

$\frac{(2^5)^5 \times (2^4)^{-4} \times (2^3)^8}{2^6} = \frac{2^{25} \times 2^{-16} \times 2^{24}}{2^6}$

$= 2^{25-16+24-6} = 2^{27}$.

**15.** Change each of the bases into respective powers of 3, 5, or 7 and simplify

$\left[\frac{(3^6)}{(7^3)}\right]^{\frac{1}{6}} \left[\frac{(5^4)}{(7^2)}\right]^{-\frac{1}{4}} \left[\frac{(5^3)}{(3^6)}\right]^{\frac{1}{3}}$

$\Rightarrow \frac{(3^6)^{\frac{1}{6}}}{(7^3)^{\frac{1}{6}}} \times \frac{(5^4)^{-\frac{1}{4}}}{(7^2)^{-\frac{1}{4}}} \times \frac{(5^3)^{\frac{1}{3}}}{(3^6)^{\frac{1}{3}}}$

$\Rightarrow \frac{3}{7^{\frac{1}{2}}} \times \frac{5^{-1}}{7^{-\frac{1}{2}}} \times \frac{5}{3^2}$

$\Rightarrow \frac{3^{1-2} \times 5^{-1+1}}{7^{+\frac{1}{2}-\frac{1}{2}}} = 3^{-1}$, i.e., $\frac{1}{3}$

**16.** As $x$ and $y$ are not necessarily integers, it does not necessarily follow that the powers of 2 and 9 on both sides are equal.

$4^x = \frac{46656}{27^y}$. This is also equal to $(2^x)^2 = \left(\frac{648}{9^y}\right)^2$.

$\frac{46656}{27^y} = \left(\frac{648}{9^y}\right)^2 \Rightarrow y = 2$

From any of the equations given, $x = 3$.

**17.** Given, $3^{3x-4} = 3^{2^{1^{2^{3^4}}}}$

$\Rightarrow 3^{3x-4} = 3^{2^{1^{2^{8^1}}}}$

Though $2^{81}$ is a very big number, 1 raised to any number will be 1.

$\therefore 3^{3x-4} = 3^{2^1}$

$\Rightarrow 3^{3x-4} = 3^2$

$\Rightarrow 3x-4=2$

$\Rightarrow x=2$

**18.** Given: $\left(a\sqrt[3]{a}\right)^{\sqrt{a}}=\left(a^2\times\sqrt{a}\right)^{a\sqrt{a}}$

$\Rightarrow\left(a\times a^{\frac{1}{3}}\right)^{\sqrt{a}}=\left(a^2\times a^{\frac{1}{2}}\right)^{a\sqrt{a}}$

$\Rightarrow\left(a^{\frac{4}{3}}\right)^{\sqrt{a}}=\left(a^{\frac{5}{2}}\right)^{a\sqrt{a}}$

The two expressions are in the form of $(a^m)^n$ which can be written as $a^{mn}$.

$\therefore a^{\frac{4}{3}(\sqrt{a})}=a^{\frac{5}{2}(a\sqrt{a})}$

$\therefore a$ could be 1, 0, –1 or the indices have to be equal. We have to reject –1 and 0, as $\sqrt{-1}$ would be imaginary and $0^0$ is undefined. But $a$ could be 1.

Equating the indices, we have

$\frac{4}{3}\sqrt{a}=\frac{5}{2}\times a\sqrt{a}$

$\Rightarrow a=\frac{4}{3}\times\frac{2}{5}$

$\Rightarrow a=\frac{8}{15}$

**19.** $(32)^{2x+5}=(128)^{2x-1}$ can be written as $2^{10x+25}=2^{14x-7}$

As the bases are equal, equating the powers, we have $10x+25=14x-7$

$\Rightarrow 4x=32$

$\Rightarrow x=8$.

**20.** (i) $3^{200}$ and $4^{150}$

Both these numbers can be written with power as 50.

i.e., $(3^4)^{50}$ and $(4^3)^{50}$

i.e., $81^{50}$ and $64^{50}$

(ii) $12^{75}$ and $8^{100}$

In this case also, we can go for making the powers equal, as making both the bases equal leads to complex calculations.

$12^{75}=(12^3)^{25}=1728^{25}$

and $8^{100}=(8^4)^{25}=(4096)^{25}$

(iii) $3^{1/4}$ and $2^{1/3}$

In this case, we have rational powers. To compare them, take the L.C.M. of denominators, i.e., 4 and 3 which is 12.

Rewrite the given numbers with $\frac{1}{12}$ as the power.

$\therefore 3^{1/4}=(3^3)^{1/12}=27^{1/12}$

and $2^{1/3}=(2^4)^{1/12}=16^{1/12}$

As the powers are equal, by looking at the bases, we have $3^{1/4}>2^{1/3}$.

(iv) $64^{325}$ and $32^{400}$

In this case, making the powers equal will not serve the purposes. Hence, it is better we go for making the bases equal.

$64^{325}=(2^6)^{325}=2^{1950}$

$32^{400}=(2^5)^{400}=2^{2000}$

$\therefore 32^{400}>64^{325}$

**21.** Given: $\left(1-\frac{1}{1+x^{-y}}\right)+\left(1-\frac{1}{1+x^y}\right)$

$\Rightarrow\left(1-\frac{1}{1+\frac{1}{x^y}}\right)+\left(\frac{1+x^y-1}{1+x^y}\right)$

$\Rightarrow\left(1-\frac{x^y}{x^y+1}\right)+\left(\frac{x^y}{1+x^y}\right)$

$\Rightarrow\left(\frac{1+x^y-x^y}{1+x^y}\right)+\left(\frac{x^y}{1+x^y}\right)$

$\Rightarrow\frac{1}{1+x^y}+\frac{x^y}{1+x^y}$

$\Rightarrow\frac{1+x^y}{1+x^y}=1$

**22.** The L.C.M. of denominators of 2, 3, 5 and 6 is 30

$\therefore 3^{1/5}=(3^6)^{1/30}$

$4^{1/6}=(4^5)^{1/30}$

$5^{1/3}=(5^{10})^{1/30}$ and $2^{1/2}=(2^{15})^{1/30}$

As the common power is 1/30, now compare $3^6$; $4^5$; $5^{10}$ and $2^{15}$.

**23.** As $2^{12}=4096$; $2^{13}=8192$; $2^{14}=16384$ and $2^{15}>22222$.

$\therefore$ 22222 lies between $2^{14}$ and $2^{15}$

$222^{22}=(2^{7\text{ to }8})^{22}=2^{154\text{ to }176}$

$22^{222}=22^{484}\approx(2^{4.5})^{484}\approx 2^{2178}$

$2^{2222}$ is the last number.

$\therefore 2^{2222}>22^{222}>222^{22}>22222$.

**24.** $3^{3x-4}=3^{2-131024}$

$\Rightarrow 3^{3x-4}=3^{2-1}$

$\Rightarrow 3x-4=\frac{1}{2}$

$\Rightarrow 3x=\frac{9}{2}$

$\Rightarrow x=\frac{3}{2}$

**25.** Given expression can be written as

$3^{2x}\times 3^3+\frac{3^{2x}}{3^3}=2190$

$\Rightarrow 3^{2x}\left(3^3+\frac{1}{3^3}\right)=2190$

$\Rightarrow 3^{2x}\left(27+\frac{1}{27}\right)=2190$

$\Rightarrow 3^{2x}\left(\frac{729+1}{27}\right)=2190$

$\Rightarrow 3^{2x}\left(\frac{730}{27}\right)=2190$

$\Rightarrow 3^{2x}=3^4$

$\Rightarrow 2x=4 \Rightarrow x=2$

**26.** Conjugate of a surd of the form $a+\sqrt{b}$ is $\left(a-\sqrt{b}\right)$

$\therefore$ The conjugate of $\sqrt{5}-2$, i.e., $-2+\sqrt{5}$ is $-2-\sqrt{5}$

**27.** $\sqrt{\sqrt{48}-\sqrt{27}}=\sqrt{\sqrt{16(3)}-\sqrt{9(3)}}$

$=\sqrt{4\sqrt{3}-3\sqrt{3}}=\sqrt{\sqrt{3}}=3^{1/4}$

**28.** $\sqrt{7+4\sqrt{3}}=\sqrt{7+2\sqrt{3(4)}}$

$=\sqrt{3+4+2\sqrt{3(4)}}=\sqrt{3}+\sqrt{4}=2+\sqrt{3}$

**29.** $\frac{1}{\sqrt{3}+\sqrt{2}}$

$=\frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)}=\frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}=\frac{\sqrt{3}-\sqrt{2}}{1}$

Similarly,

$\frac{1}{2+\sqrt{3}}=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=\frac{2-\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2}=\frac{2-\sqrt{3}}{1}$

$\therefore \frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{2+\sqrt{3}}$

$=\left(\sqrt{3}-\sqrt{2}\right)+\left(2-\sqrt{3}\right)=2-\sqrt{2}$

**30.** $\sqrt{\sqrt{72}-\sqrt{40}}=\sqrt{\sqrt{36(2)}-\sqrt{20(2)}}$

$=\sqrt{\sqrt{2}}\left[\sqrt{36}-\sqrt{20}\right]=2^{1/4}\sqrt{6-\sqrt{4(5)}}$

$=2^{1/4}\sqrt{(5+1)-2\sqrt{5(1)}}=2^{1/4}\left(\sqrt{5}-1\right)$

**31.** $\sqrt{108}=6\sqrt{3}$

$\sqrt{75}=5\sqrt{3}$

$\sqrt{108}-\sqrt{75}=\sqrt{3}$

Its positive square root $=\sqrt[4]{3}$

**32.** Let $\sqrt{14+6\sqrt{5}}=\sqrt{a}+\sqrt{b}$, squaring on both sides,

$14+6\sqrt{5}=a+b+2\sqrt{ab}$

$a+b=14$

$ab=45$

Solving for $a$ and $b$, $a=9$ and $b=5$ or vice versa positive square root $=\sqrt{9}+\sqrt{5}=3+\sqrt{5}$

**33.** Conjugate of the surd $\sqrt{13}-2=-(\sqrt{13}+2)$

**34.** $\frac{3}{\sqrt{5}+\sqrt{2}}=\frac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}=\sqrt{5}-\sqrt{2}$

$\frac{1}{3+\sqrt{8}}=\frac{3-\sqrt{8}}{(3+\sqrt{8})(3-\sqrt{8})}=3-2\sqrt{2}$

$\frac{3}{\sqrt{5}+\sqrt{2}}+\frac{1}{3+\sqrt{8}}=3+\sqrt{5}-3\sqrt{2}$

**35.** $\sqrt{128}-\sqrt{56}=8\sqrt{2}-2\sqrt{14}=\sqrt{2}(8-2\sqrt{7})$

Its positive square root $=\sqrt[4]{2}(\sqrt{7+1-2\sqrt{7\times1}})$

$\therefore$ Positive square root $=\sqrt[4]{2}(\sqrt{7}-1)$

**36.** $6+3\sqrt{5}=(3)(2)+3\sqrt{5}=3\left(2+\sqrt{5}\right)$

To rationalize $3\left(\sqrt{5}+2\right)$, we need to multiply by $\sqrt{5}-2$.

$\therefore \sqrt{5}-2$ is the rationalizing factor of $6+3\sqrt{5}$

**37.** $(\sqrt{5}+\sqrt{2})<(\sqrt{8}+\sqrt{2})$ *as* $\sqrt{5}<\sqrt{8}$.

$\therefore \frac{1}{\sqrt{5}+\sqrt{2}}$ is greater than $\frac{1}{\sqrt{8}+\sqrt{5}}$

**38.** $4^{1/3}-3^{1/3}$ is of the form $a-b$. If we multiply $(a-b)$ with $(a^2+ab+b^2)$, we get $a^3-b^3$. If we multiply $4^{1/3}-3^{1/3}$ with $\left(4^{1/3}\right)+4^{1/3}\cdot3^{1/3}+\left(3^{1/3}\right)^2$, we get $\left(4^{1/3}\right)^3-\left(3^{1/3}\right)^3=4-3=1$, which is national

$\therefore 4^{2/3}+12^{1/3}+3^{2/3}$ is the nationalizing factor of $4^{1/3}-3^{1/3}$

**39.** $\frac{5}{3^{2/3}-6^{1/3}+2^{2/3}}$

$=\frac{5}{3^{2/3}-[(3)(2)]^{1/3}+2^{2/3}}$

$=\frac{5}{\left(3^{1/3}\right)^2-3^{1/3}\cdot2^{1/3}+\left(2^{1/3}\right)^2}$

The denominator is of the form $a^2-ab+b^2$, the rationalizing factor for which is $(a+b)$.

$\therefore \frac{5}{3^{2/3}-6^{1/3}+2^{2/3}}$

$=\frac{5}{(3^{1/3})^2\,3^{1/3}\cdot2^{1/3}+(2^{1/3})^2}\times\frac{\left(3^{1/3}+2^{1/3}\right)}{\left(3^{1/3}+2^{1/3}\right)}$

$=\frac{5\left(3^{1/3}+2^{1/3}\right)}{\left(3^{1/3}\right)^3+\left(2^{1/3}\right)^3}=\frac{5\left(3^{1/3}+2^{1/3}\right)}{3+2}=3^{1/3}+2^{1/3}$

**40.** $\left[(64)^{-1/6}\times(81)^{3/4}\times(64)^{2/3}\right]^{1/3}$

$=\left[\left(2^6\right)^{-1/6}\times\left(3^4\right)^{3/4}\times\left(2^6\right)^{2/3}\right]^{1/3}$

$=[2^{-1}\times 3^3\times 2^4]^{1/3}$

$=\left[(2\times 3)^3\right]^{1/3}=6$

**41.** $2^x\cdot 3^{2y}=144\Rightarrow 2^x\cdot 3^{2y}=2^4\cdot 3^2$

$\therefore\ x=4$ and $2y=2$ or $y=1$

$\therefore\ x+y=4+1=5$

**42.** $\dfrac{6}{2\sqrt{3}+\sqrt{6}}-\dfrac{1}{\sqrt{3}-\sqrt{2}}+\dfrac{4}{\sqrt{6}-\sqrt{2}}$

$=\dfrac{6(2\sqrt{3}-\sqrt{6})}{(2\sqrt{3})^2-(\sqrt{6})^2}-\dfrac{1(\sqrt{3}+\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2}+\dfrac{4(\sqrt{6}+\sqrt{2})}{(\sqrt{6})^2-(\sqrt{2})^2}$

$=2\sqrt{3}-\sqrt{6}-\sqrt{3}-\sqrt{2}+\sqrt{6}+\sqrt{2}=\sqrt{3}$

**43.** Raising each number to the power 12, we get

$\left(3^{\frac{1}{2}}\right)^{12},\left(4^{\frac{1}{3}}\right)^{12}$ and $\left(5^{\frac{1}{4}}\right)^{12}$ i.e., $3^6$, $4^4$, and $5^3$

i.e., 729, 256, and 125.

$125<256<729$

$\therefore\ 5^{\frac{1}{4}}<4^{\frac{1}{3}}<3^{\frac{1}{2}}$

**44.** $\left(2\sqrt{3}\right)^{3x-4}=2\sqrt{3}^{8}\Rightarrow 3x-4=8\Rightarrow 3x=12$

$\therefore\ x=4$

**45.** $\left[\dfrac{x^4y^{-5}z^2}{x^{-3}yz^4}\right]\div\left[\dfrac{x^2yz^3}{x^{-8}y^2z^5}\right]^7$

$=[x^7\ y^{-6}\ z^{-2}]^{11}\div[x^{10}\ y^{-1}\ z^{-2}]^7=[x^{77}\ y^{-66}\ z^{-22}]\times[x^{-70}\ y^7\ z^{14}]$

$=x^7\ y^{-59}\ z^{-8}=\dfrac{x^7}{y^{59}z^8}$

## EXERCISE-2

**1.** Let $5^{\frac{1}{a}}=7^{\frac{1}{b}}=35^{\frac{1}{c}}=k$

$5=k^a$

$7=k^b$

$35\ (=(5)\ (7))=k^c$

$k^a\ k^b=k^c$

$k^{a+b}=k^c$

$c=a+b.$

**2.** $\left(\dfrac{a^3b^6c^9}{a^4b^8c^{12}}\right)=\left(a^{3-4}b^{6-8}c^{9-12}\right)^9$

$=\left(a^{-1}b^{-2},c^{-3}\right)^9=a^{-9}b^{-18}c^{-27}$

$\left(\dfrac{a^2b^4c^6}{a^3b^7c^5}\right)^8=\left(a^{2-3}b^{4-7}c^{6-5}\right)^8$

$=(a^{-1}\ b^{-3}\ c^1)^8=a^{-8}\ b^{-24}\ c^8$

$\left(\dfrac{a^3b^6c^9}{a^4b^8c^{12}}\right)^9\div\left(\dfrac{a^2b^4c^6}{a^3b^7c^5}\right)^8=a^{-9-(-8)}\ b^{-18-(-24)}\ c^{-27-8}$

$=a^{-1}\ b^6\ c^{-35}=\dfrac{b^6}{ac^{35}}.$

**3.** $(216)^{\frac{4}{3}}=\left(6^3\right)^{\frac{4}{3}}=6^4=3^4\left(2^4\right)$

$(384)^{\frac{1}{7}}=\left[\left(2^7\right)(3)\right]^{\frac{1}{7}}=(2)\left(3^{\frac{1}{7}}\right)$

$(729)^{\frac{1}{7}}=\left(3^6\right)^{\frac{1}{7}}=3^{\frac{6}{7}}$

$\left[(216)^{\frac{4}{3}}(384)^{\frac{1}{7}}(729)^{\frac{1}{7}}\right]^{\frac{1}{2}}$

$=\left[3^5\times 2^5\right]^{\frac{1}{2}}=36\sqrt{6}.$

**4.** Numerator

$=2^{(m+2+m+2+2m+2)}\cdot 3^{(m+2+m+1)}\cdot 5^{(2m+2+m+1+2m-2n)}$

$=2^{4m+6}\cdot 3^{2m+3}\cdot 5^{5m-2n+3}$

Denominator $=2^{(4m+6)}\cdot 3^{(2m-3+6)}\cdot 5^{(5m+2n+2-4n)}$

$\dfrac{\text{Numerator}}{\text{Denominator}}=5$

**5.** $y-5=5^{\frac{1}{3}}+5^{\frac{2}{3}}$

$(y-5)^3=\left(5^{\frac{1}{3}}+5^{\frac{2}{3}}\right)^3\Rightarrow y^3+75y-5y^2-125$

$=\left(5^{\frac{1}{3}}\right)^3+\left(5^{\frac{2}{3}}\right)^3+3\left(5^{\frac{1}{3}}\right)\left(5^{\frac{2}{3}}\right)\left[\left(5^{\frac{1}{3}}\right)+\left(5^{\frac{2}{3}}\right)\right]$

$\Rightarrow y^3-5y^2+75y-125=5+5^2+15\ (y-5)$

$\Rightarrow y^3-5y^2+60y=80$

Adding 25 to both sides $y^3-5y^2+60y+25=105$.

**6.** $x-4=\sqrt{11}+\sqrt{7}$

$(x-4)^2=x^2-8x+16$

$=18+2\quad\Rightarrow 2\sqrt{77}=x^2-8x-2$ (1)

Squaring (1), we get

$x^4+64x^2+4-16x^3+32x-4x^2=308$

$\Rightarrow x^4-16x^3+60x^2+32x+4=308$

$\therefore x^4-16x^3+60x^2+32x=304$

**7.** $\sqrt[qr]{p^2}\cdot\sqrt[rp]{q^2}\cdot\sqrt[pq]{r^2}$

$= a^{\frac{p^2}{qr}}\cdot a^{\frac{q^2}{rp}}\cdot a^{\frac{r^2}{pq}}$

$= a^{\frac{p^2}{qr}+\frac{q^2}{rp}+\frac{r^2}{pq}} = a^{\frac{p^2+q^2+r^2}{pqr}} = a^{\frac{3pqr}{pqr}} = a^3$

(since $p^3+q^3+r^3 = 3pqr$, when $p+q+r=0$)

**8.** $x^y = y^x$ (1)

$y = x^2$ (2)

Substituting the value of $y$ from (2) to (1)

$x^{x^2} = (x^2)^x \Rightarrow x^{x^2} = x^{2x} \Rightarrow x^2 = 2x$

Either $x=0$ or $x=2$

If $x=0$, $y=0$, which is not possible, since it won't satisfy (1). When $x=2$, $y=x^2=2^2=4$.

**9.** L.C.M. of 2, 3, and 4 is 12.

$\sqrt{5} = (5^6)^{1/12} = (15625)^{1/12}$

$\sqrt[4]{7} = (7^3)^{1/12} = (343)^{1/12}$

$\sqrt[3]{11} = (11^4)^{1/12} = (14641)^{1/12}$

Since $343 < 14641 < 15625$

$\sqrt[4]{7} < \sqrt[3]{11} < \sqrt{5}$

**10.** $\left\{\left\{\left[\left(512^{216}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}}\right\}^{\frac{1}{4}}\right\}^{\frac{1}{648}}$

$= \left\{\left\{\left[\left(2^9\right)^{108}\right]^{1/3}\right\}^{1/4}\right\}^{1/648} = \left[\left(2^{324}\right)^{1/4}\right]^{1/648}$

$= \left(2^{81}\right)^{1/648} = 2^{\frac{1}{8}}$

**11.** $\frac{2}{\sqrt{5}+\sqrt{3}} = \frac{2\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2} = \frac{2\sqrt{5}-\sqrt{3}}{2} = \sqrt{5}-\sqrt{3}$

(by multiplying numerator and denominator with $\sqrt{5}-\sqrt{3}$ )

Similarly, $-\frac{1}{\sqrt{2}-\sqrt{3}} = \frac{1}{\sqrt{3}-\sqrt{2}} = \sqrt{3}+\sqrt{2}$,

$\frac{3}{\sqrt{5}+\sqrt{2}} = \frac{3\left(\sqrt{5}-\sqrt{2}\right)}{5-2} = \sqrt{5}-\sqrt{2}$

$\therefore$ Answer $= \sqrt{5}-\sqrt{3}+\sqrt{3}+\sqrt{2}+\sqrt{5}-\sqrt{2} = 2\sqrt{5}$

**12.** Consider the first equation

$5^{0.75b}\cdot 7^{-0.5a} = \frac{625\sqrt{35}}{343}$

$\Rightarrow 5^{0\cdot75b}.7^{-0/5a} = 5^4\cdot 5^{\frac{1}{2}}\cdot 7^{\frac{1}{2}}\cdot 7^{-3}$

$\Rightarrow 5^{0\cdot75b}.\ 7^{-0\cdot5a} = 5^{4\cdot5}.\ 7^{-2\cdot5}$

Comparing the two sides, $0\cdot75b = 4\cdot5$ and $-0\cdot5a = -2\cdot5$

$\Rightarrow b=6$ and $a=5$

Consider the second equation

$25^{0.1a}.49^{0.3b} = 245\cdot(2401)^{\frac{2}{5}}$

$\Rightarrow (5^2)^{0.1a}.\ (7^2)^{0.3b} = 7^2\cdot 5\ \left(7^4\right)^{\frac{2}{5}}$

$\Rightarrow 5^{0.2a}\cdot 7^{0.6b} = 7^{3.6}\cdot 5$

Comparing the two sides, $0.2a=1$ and $0.6b=3.6$

$\Rightarrow a=5$ and $b=6$.

Since $a=5$ and $b=6$ satisfy both the equation, that can be taken as the solution for the given pair of equations.

**13.** $\sqrt{\sqrt{75}-\sqrt{72}} = \sqrt{\sqrt{3}\left(5-2\sqrt{6}\right)} = 3^{\frac{1}{4}}\sqrt{5-2\sqrt{6}}$

We find the square root of $5-2\sqrt{6}$ using the procedure below.

Let $\sqrt{5-2\sqrt{6}} = \sqrt{a}-\sqrt{b}$, where $a$, $b$ are rational.

Squaring both sides, $5-2\sqrt{6} = a+b-2\sqrt{ab}$

$a+b=5$ (1)

$ab=6$ (2)

The product of $a$ and $b$, i.e., 6 can be written as (3) (2)

As $3+2=5$, $a=3$ and $b=2$ satisfy both (1) and (2)

$\sqrt{\sqrt{75}-\sqrt{18}} = 3^{\frac{1}{4}}\left(\sqrt{3}-\sqrt{2}\right)$.

**14.** The given surds are $\sqrt{2} = 2^{\frac{1}{2}}, \sqrt[4]{3} = 3^{\frac{1}{4}}, \sqrt[8]{6} = 6^{\frac{1}{8}}$

By making all the indices equal, we can compare the three numbers

$2^{\frac{1}{2}} = \left(2^4\right)^{\frac{1}{8}} = 16^{\frac{1}{8}}$

$3^{\frac{1}{4}} = \left(3^2\right)^{\frac{1}{8}} = 9^{\frac{1}{8}}$

$6^{\frac{1}{8}} = 6^{\frac{1}{8}}$, since $6<9<16$

$\therefore 6^{\frac{1}{8}} < 3^{\frac{1}{4}} < 2^{\frac{1}{2}}$

$\therefore 6^{\frac{1}{8}} < 3^{\frac{1}{4}} < 2^{\frac{1}{2}}$

**15.** The denominator of the expression on the left of the + sign can be found to be $\sqrt{5}-\sqrt{3}$ using the method in solution 18. Shown below is the method to find the square root of numerator of the expression to the right of the + sign.

Let $\sqrt{11+2\sqrt{30}} = \sqrt{a}+\sqrt{b}$, squaring both sides

$11+2\sqrt{30} = a+b+2\sqrt{ab}$ $a+b=11$ (1)

$ab=30$ (2)

Hence, $\sqrt{11+2\sqrt{30}} = \sqrt{6}+\sqrt{5}$

$$\frac{\sqrt{11+2\sqrt{30}}}{\sqrt{6}-\sqrt{5}}=\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}=\frac{\left(\sqrt{6}+\sqrt{5}\right)\left(\sqrt{6}+\sqrt{5}\right)}{\left(\sqrt{6}-\sqrt{5}\right)\left(\sqrt{6}-\sqrt{5}\right)}$$

$=11+2\sqrt{30}$

The expression to be simplified becomes

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+11+2\sqrt{30}$$

$$=\frac{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}+11+2\sqrt{30}$$

$$=\frac{8+2\sqrt{15}}{2}+11+2\sqrt{30}=15+\sqrt{15}+2\sqrt{30}.$$

**16.** We note that $2+3+4=9$, $2(3)=6$, $2(4)=8$, and $3(4)=12$.

$\therefore\ \sqrt{9+2\sqrt{6}+2\sqrt{8}+2\sqrt{12}}=\sqrt{2}+\sqrt{3}+\sqrt{4}.$

$$\frac{\left(\sqrt{2}+\sqrt{3}+\sqrt{4}\right)+\left(-\sqrt{2}+\sqrt{3}+\sqrt{4}\right)}{\left(\sqrt{2}+\sqrt{3}+\sqrt{4}\right)-\left(-\sqrt{2}+\sqrt{3}+\sqrt{4}\right)}$$

$$=\frac{2\left(\sqrt{3}+\sqrt{4}\right)}{2\sqrt{2}}=\sqrt{\frac{3}{2}+\sqrt{2}}.$$

**17.** $\dfrac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}$

$$=\frac{\left(\sqrt{1+x}+\sqrt{1+x}\right)^2}{\left(\sqrt{1+x}+\sqrt{1-x}\right)\left(\sqrt{1+x}-\sqrt{1-x}\right)}$$

$$=\frac{1+x+1-x+2\sqrt{(1+x)(1-x)}}{1+x-1+x}=\frac{1+\sqrt{1-x^2}}{x}$$

$$\therefore \text{ Required value }=\sqrt{\frac{1+\sqrt{1-\frac{1}{4}}}{\frac{1}{2}}}$$

$$=\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{\frac{1}{2}}}=\sqrt{2+\sqrt{3}}$$

$$=\sqrt{\frac{1}{2}\left(4+2\sqrt{3}\right)}=\frac{\sqrt{3}+1}{\sqrt{2}}=\frac{1}{2}\left(\sqrt{2}+\sqrt{6}\right)$$

**18.** $\sqrt{\dfrac{\sqrt{5}+1}{\sqrt{5}-1}}$

$$=\sqrt{\frac{\left(\sqrt{5}+1\right)^2}{\left(\sqrt{5}\right)^2-1^2}}=\sqrt{\frac{\left(\sqrt{5}+1\right)^2}{4}}=\frac{\sqrt{5}+1}{2}$$

$$x^2=\frac{\left(\sqrt{5}\right)^2+2\sqrt{5}+1}{4}$$

$$=\frac{6+2\sqrt{5}}{4}=\frac{3+\sqrt{5}}{2}$$

$$\therefore\ 7x^2-7x=\frac{21+7\sqrt{5}}{2}-\frac{7\sqrt{5}+7}{2}=14/2=7.$$

**19.** $\dfrac{12}{3+\sqrt{5}-2\sqrt{2}}$

$$=\frac{12\left(3+\sqrt{5}+2\sqrt{2}\right)}{\left(3+\sqrt{5}\right)^2-\left(2\sqrt{2}\right)^2}=\frac{36+12\sqrt{5}+24\sqrt{2}}{14+6\sqrt{5}-8}$$

$$=\frac{6\left(6+2\sqrt{5}+4\sqrt{2}\right)}{6\left(\sqrt{5}+1\right)}=\frac{\left(6+2\sqrt{5}+4\sqrt{2}\left(\sqrt{5}-1\right)\right)}{\left(\sqrt{5}\right)^2-1^2}$$

$$=\frac{4\left(1-\sqrt{2}+\sqrt{5}+\sqrt{10}\right)}{4}=1-\sqrt{2}+\sqrt{5}+\sqrt{10}$$

$\Rightarrow x=1,\ a=-1,\ b=1,\ c=1$

$x+a+b+c=1-1+1+1=2$

**20.** $\dfrac{\left[\sqrt{\left(\sqrt{3}+\sqrt{2}\right)}+\sqrt{\left(\sqrt{3}-\sqrt{2}\right)}\right]^2}{\sqrt{3}-1}$

$$=\frac{\left[\sqrt{\sqrt{3}+\sqrt{2}}\right]^2+\left[\sqrt{(\sqrt{3}-\sqrt{2}}\right]^2+\frac{2(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})}}{\sqrt{3}-1}$$

$$==\frac{2\sqrt{3}+2}{\sqrt{3}-1}=\frac{2\left(\sqrt{3}+1\right)}{\sqrt{3}-1}=\frac{2\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}\right)^2-1}=\left(\sqrt{3}+1\right)^2$$

$\therefore \sqrt{x}=\left(\sqrt{3}+1\right)$

**21.** Let $\dfrac{P}{p}=\dfrac{Q}{q}=\dfrac{R}{r}=\dfrac{S}{s}=k$

$P=pk,\ Q=qk,\ R=rk,\ S=sk$

$\therefore\ \sqrt{Pp}+\sqrt{Qq}+\sqrt{Rr}+\sqrt{Ss}=$

$$=\sqrt{(pk)p}+\sqrt{(qk)q}+\sqrt{(rk)r}+\sqrt{(sk)s}$$

$$=p\sqrt{k}+q\sqrt{k}+r\sqrt{k}+s\sqrt{k}=(p+q+r+s)\sqrt{k}$$

$\Rightarrow \sqrt{Pp}+\sqrt{Qq}+\sqrt{Rr}+\sqrt{Ss}=$

$$=\left(\sqrt{p+q+r+s}\right)\left(\sqrt{p+q+r+s}\right)\sqrt{k}$$

$$=\left(\sqrt{p+q+r+s}\right)\left(\sqrt{pk+qk+rk+sk}\right)$$

$$=\left(\sqrt{p+q+r+s}\right)\left(\sqrt{P+Q+R+S}\right).$$

**22.** $y=\dfrac{32-10\sqrt{6}-3\sqrt{6}}{8+5\sqrt{6}-2\sqrt{6}}=\dfrac{32-13\sqrt{6}}{8+3\sqrt{6}}$

$$=\frac{\left(32-13\sqrt{6}\right)\left(8-3\sqrt{6}\right)}{10}$$

$$= \frac{(256+234)-(96+104)\sqrt{6}}{10}$$

$$= 49 - 20\sqrt{6} = 49 - 2\sqrt{600}$$

$$\therefore \sqrt{y} = 5 - \sqrt{24} = 5 - 2\sqrt{6}.$$

**23.** Given: $\frac{\sqrt{9+4\sqrt{5}}-\sqrt{9-4\sqrt{5}}}{\sqrt{9+4\sqrt{5}}+\sqrt{9-4\sqrt{5}}} = \frac{a}{\sqrt{5}}$

$$= \frac{\left(\sqrt{9+4\sqrt{5}}-\sqrt{9-4\sqrt{5}}\right)^2}{\left(\sqrt{9+4\sqrt{5}}\right)-\left(\sqrt{9-4\sqrt{5}}\right)} = \text{(by rationalization)}$$

$$= \frac{9+4\sqrt{5}+9-4\sqrt{5}-2\sqrt{\left(9+4\sqrt{5}\right)\left(9-4\sqrt{5}\right)}}{8\sqrt{5}}$$

$$= \frac{18-2\sqrt{81-80}}{8\sqrt{5}} = \frac{18-2}{8\sqrt{5}}$$

$$= \frac{16}{8\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{a}{\sqrt{5}}$$

$$\therefore a = 2$$

**24.** $\left(3^{\frac{1}{3}}9^{\frac{1}{3}}27^{\frac{1}{3}}81^{\frac{1}{3}}243^{\frac{1}{3}}729^{\frac{1}{3}}\right)^{\frac{2}{19}}$

$$= \left(3^{\frac{1}{3}}3^{\frac{4}{3}}3^{\frac{12}{3}}3^{\frac{32}{3}}3^{\frac{50}{3}}3^{\frac{72}{3}}\right)^{\frac{2}{19}}$$

$$= \left(3^{\frac{1}{3}+\frac{4}{3}+\frac{12}{3}+\frac{32}{3}+\frac{50}{3}+\frac{72}{3}}\right)^{\frac{2}{19}}$$

$$= \left(3^{57}\right)^{\frac{2}{19}} = 729$$

**25.** $\frac{1}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)} = \frac{\sqrt{3}-1}{2}$

(multiplying both numerator and denominator by $\sqrt{3}-1$)

Similarly, $\frac{1}{\sqrt{5}+\sqrt{3}}, \frac{1}{\sqrt{7}+\sqrt{5}}, \frac{1}{\sqrt{9}+\sqrt{7}}, \ldots \frac{1}{\sqrt{49}+\sqrt{47}}$

would be $\frac{\sqrt{5}-\sqrt{3}}{2}, \frac{\sqrt{7}-\sqrt{5}}{2}, \frac{\sqrt{9}-\sqrt{7}}{2} \quad \frac{\sqrt{49}-\sqrt{47}}{2}$

respectively.

The value of the given expression is

$$\frac{\sqrt{3}-1+\sqrt{5}-\sqrt{3}+\sqrt{7}-\sqrt{5}+\sqrt{9}-\sqrt{7}+\ldots\sqrt{49}-\sqrt{47}}{2}$$

$$= \frac{(7-1)}{2}, \text{ i.e., } 3$$

**26.** Let $a = \sqrt{7}+\sqrt{17} \Rightarrow a^2 = 24+2\sqrt{119}$

$b = \sqrt{8}+\sqrt{14} \Rightarrow b^2 = 22+2\sqrt{112}$

$c = \sqrt{10}+\sqrt{14} \Rightarrow c^2 = 24+2\sqrt{140}$

$d = \sqrt{3}+\sqrt{22} \Rightarrow d^2 = 25+2\sqrt{66}$

Taking ranges

$a^2 = 24 + (21 \text{ to } 22) = 45 \text{ to } 46$

$b^2 = 22 + (21 \text{ to } 22) = 43 \text{ to } 44$

$c^2 = 24 + (23 \text{ to } 24) = 47 \text{ to } 48$

$d^2 = 25 + (16 \text{ to } 17) = 41 \text{ to } 42$

Thus, we get $d < b < a < c$.

**27.** Let $\sqrt{11\sqrt{11\sqrt{11\sqrt{11\ldots\infty}}}} = x$

Squaring both sides, we get

$= 11\sqrt{11\sqrt{11\sqrt{11\ldots\infty}}} = x^2 \Rightarrow 11x = x^2 \Rightarrow x(x-11) = 0$

Either, $x = 0$ (which is not possible) or $x = 11$

Now, $11 = 14641^x \Rightarrow 11 = (11^4)^x \Rightarrow 11^{4x} = 11$

$\Rightarrow 4x = 1$ or $x = ¼$.

**28.** $\frac{1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{\left(\sqrt{2}\right)^2-1^2} = \sqrt{2}-1$

Similarly, $\frac{1}{\sqrt{2}+1} = \sqrt{3}-\sqrt{2}$

$\frac{1}{\sqrt{4}+\sqrt{3}} = \sqrt{4}-\sqrt{3}$

...

...

...

$\frac{1}{\sqrt{100}+\sqrt{99}} = \sqrt{100}-\sqrt{99}$

$\therefore$ The given expression is equal to

$$\left(\sqrt{2}-1\right)+\left(\sqrt{3}-\sqrt{2}\right)+\left(\sqrt{4}-\sqrt{3}\right)+\ldots\left(\sqrt{100}-\sqrt{99}\right)$$

$$= \sqrt{100} - 1 = 10 - 1 = 9$$

**29.** $\frac{3}{2}(x+4)+\sqrt{2x^2+19x+35}$

$$= \frac{1}{2}\left[3x+12+2\sqrt{(x+7(2x-5))}\right]$$

$$= \frac{1}{2}\left[(x+7)+(2x+5)+2\sqrt{(x+7(2x+5))}\right]$$

$$= \frac{1}{2}\left[\left(\sqrt{x+7}\right)^2+\left(\sqrt{2x+5}\right)^2+2\sqrt{(x+7(2x+5))}\right]$$

$$= \frac{1}{2}\left[\sqrt{x+7}+\sqrt{2x+5}\right]^2$$

$$\therefore \sqrt{\frac{3}{2}(x+4)+\sqrt{2x^2+19x+35}}$$

$$= \frac{1}{\sqrt{2}}\left[\sqrt{x+7}+\sqrt{2x+5}\right].$$

**30.** $xyz = 1 \Rightarrow x^2 y^2 z^2 = 1$

Multiplying the numerator and the denominator of $\frac{1}{1+y^2+x^2}$ by $x^2$, it becomes $\frac{x^2}{x^2+x^2y^2+1}$.

Multiplying the numerator and the denominator of $\frac{1}{1+z^2+y^2}$ by $x^2y^2$, it becomes $\frac{x^2y^2}{x^2y^2+1+x^2}$

$$\frac{1}{1+x^2+z^2} = \frac{1}{x^2+x^2y^2+1}$$

$\therefore$ The given expression equals $\frac{x^2+x^2y^2+1}{x^2+x^2y^2+1} = 1$.

**31.** $\left(\frac{3}{5}\right)^{\frac{3x+12}{3}} = \left[\left(\frac{5}{3}\right)^{\frac{2}{4}}\right]^{3x+1} = \left(\frac{3}{5}\right)^{\frac{-(3x+1)}{2}}$

Equating the powers of $\frac{3}{5}$ both sides,

$2(x+4) = -3x - 1 \Rightarrow 5x = -9$

$\Rightarrow x = -9/5.$

**32.** $\frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2}+\sqrt{3}} - \frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2}-\sqrt{3}}$

$$\sqrt{2+\sqrt{3}} = \sqrt{\frac{4+2\sqrt{3}}{2}} = \frac{\sqrt{3}+1}{\sqrt{2}}$$

Similarly, $= \sqrt{2-\sqrt{3}} = \frac{\sqrt{3}-1}{\sqrt{2}}$

$$\therefore \frac{2+\sqrt{3}}{\sqrt{2}+\frac{\sqrt{3}+1}{\sqrt{2}}} - \frac{2+\sqrt{3}}{\sqrt{2}+\frac{\sqrt{3}-1}{\sqrt{2}}} = \sqrt{2}\left[\frac{2+\sqrt{3}}{3+\sqrt{3}} - \frac{2+\sqrt{3}}{2+\sqrt{3}-1}\right]$$

$$= \sqrt{2}\left[\frac{2+\sqrt{3}}{\sqrt{3}\left(\sqrt{3}+1\right)} - \frac{2+\sqrt{3}}{\sqrt{3}+1}\right]$$

$$= \sqrt{2}\left[\frac{2+\sqrt{3}-2\sqrt{3}-3}{\sqrt{3}\left(\sqrt{3}+1\right)}\right] = \frac{\left(-\sqrt{3}-1\right)\sqrt{2}}{\sqrt{3}\left(\sqrt{3}+1\right)} = -\frac{\sqrt{2}}{\sqrt{3}}.$$

**33.** $\sqrt{a}+\sqrt{1-a} = \sqrt{b}+\sqrt{1-b} = \sqrt{c}+\sqrt{1-c}$

Squaring, we get

$1+2\sqrt{a(1-a)} = 1+2\sqrt{b(1-b)} = 1+2\sqrt{c(1-c)}$

$\therefore 2\sqrt{a(1-a)} = 2\sqrt{b(1-b)} = 2\sqrt{c(1-c)}$

$\Rightarrow a(1-a) = b(1-b) = c(1-c)$

Lets take $a(1-a) = b(1-b)$

$a - a^2 = b - b^2$

$a - b = a^2 - b^2$

$(a-b)\ (a+b-1) = 0$

$\Rightarrow a = b$ or $a + b = 1$

Similarly, $a = c$ or $a + c = 1$

Thus, we have the following possibilities for $b$, $c$ in terms of $a$

| $a$ | $b$ | $c$ |
|---|---|---|
| $a$ | $a$ | $a$ |
| $a$ | $a$ | $1-a$ |
| $a$ | $1-a$ | $A$ |
| $a$ | $1-a$ | $1-a$ |

We see that while choice (B) or (C) could be true, they are not definitely (always) true.

**34.** Let $P + \sqrt{P^2+\sqrt{P^4+\sqrt{P^8+\sqrt{P^{16}+\ldots\infty}}}} = x$

$$= P + \sqrt{P^2+\sqrt{P^4+\sqrt{P^8+P^8\sqrt{1+\ldots\infty}}}}$$

$$= P + \sqrt{P^2+\sqrt{P^4+P^4\sqrt{1+\sqrt{1+\ldots\infty}}}}$$

$$= P + \sqrt{P^2+P^2\sqrt{1+\sqrt{1+\sqrt{1+\ldots\infty}}}}$$

$$= P + P\sqrt{1+\sqrt{1+\sqrt{1+\ldots\infty}}}$$

Let $\sqrt{1+\sqrt{1+\sqrt{1+\ldots\infty}}} = q$

$\Rightarrow \sqrt{1+q} = q$

Squaring on both sides

$1 + q = q^2 \Rightarrow q^2 - q - 1 = 0$

$$\therefore q = \frac{1 \pm \sqrt{1-4(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$\therefore q = \frac{\sqrt{5}+1}{2} \text{ or } \frac{1-\sqrt{5}}{2}$$

As $q > 0$, $q = \frac{\sqrt{5}+1}{2}$

$\therefore x = P + Pq = P(1+q)$

$$= P\left(1+\frac{\sqrt{5}+1}{2}\right) = P\left(\frac{3+\sqrt{5}}{2}\right).$$

**35.** $\sqrt{7\sqrt{7\sqrt{7\sqrt{7}\ldots\infty}}}$

$$= 7^{\frac{1}{2}}7^{\frac{1}{4}}7^{\frac{1}{8}}\ldots = 7^{\frac{1}{2}}7^{\frac{1}{4}}7^{\frac{1}{8}}\ldots = 7^{\frac{\frac{1}{2}}{1-\frac{1}{2}}} = 7$$

$(16807)^y = \left(7^5\right)^y$

$= 5y$ (equating the powers of 7 both sides).

$y = \frac{1}{5}$.

**36.** $\dfrac{\left(x-\sqrt{3}\right)\left(\sqrt{12}+\sqrt{8}\right)}{\left(\sqrt{12}-\sqrt{8}\right)}=1$

$\Rightarrow \dfrac{(x-\sqrt{3})\,(12+8+2\sqrt{12\times 8})}{12-8}=1$

$\dfrac{(x-\sqrt{3})\,(12+8+2\sqrt{12\times 8})}{12-8}=1$

$\Rightarrow (x-\sqrt{3})\left(5+2\sqrt{6}\right)=1$

$\Rightarrow x-\sqrt{3}=\dfrac{1}{5+2\sqrt{6}}$

$\Rightarrow x-\sqrt{3}=\dfrac{5-2\sqrt{6}}{(5)^2-\left(2\sqrt{6}\right)^2}=5-2\sqrt{6}$

$\Rightarrow x=5+\sqrt{3}-2\sqrt{6}$

**37.** $\left(\dfrac{2}{7}\right)^{\frac{3x+1}{4}}=\left[\left(\left(\dfrac{2}{7}\right)^{-2}\right)^{1/3}\right]^{\frac{5x-37}{2}}$

$\Rightarrow \dfrac{3x+1}{4}=\dfrac{37-5x}{3}$

$\Rightarrow 9x+3=148-20x \Rightarrow 29x=145 \Rightarrow x=5$

**38.** Let $z=\left[\sqrt{x^2+2y\sqrt{x^2-y^2}}+\sqrt{x^2-2y\sqrt{x^2-y^2}}\right]$

$\Rightarrow z^2=\left[\sqrt{x^2+2y\sqrt{x^2-y^2}}+\sqrt{x^2-2y\sqrt{x^2-y^2}}\right]$

$+2\sqrt{\left\{x^2+2y\sqrt{x^2-y^2}\right\}\times\left\{\sqrt{x^2-2y\sqrt{x^2-y^2}}\right\}}$

$\Rightarrow z^2=x+2y\sqrt{x^2-y^2}+x^2-2y\sqrt{x^2-y^2}$

$+2\sqrt{x^4-4y^2\left(x^2-y^2\right)}$

$=2x^2+2\sqrt{\left(2y^2\right)^2-2\left(x^2\right)\left(2y^2\right)+\left(x^2\right)^2}$

$=2x^2+2(2y^2-x^2)=4y^2=(2y)^2$

$\therefore z=2y$

**39.** $\dfrac{x-y}{\sqrt{x}+\sqrt{y}}+\dfrac{x-y}{\sqrt{x}-\sqrt{y}}=4$

$(x-y)\left\{\dfrac{\left(\sqrt{x}-\sqrt{y}\right)\left(\sqrt{x}+\sqrt{y}\right)}{\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)}\right\}=4$

$\Rightarrow \sqrt{x}-\sqrt{y}+\sqrt{x}+\sqrt{y}=4 \Rightarrow 2\sqrt{x}=4$

$\Rightarrow \sqrt{x}=2 \Rightarrow x=4$

**40.** $3^{2x+2}=3^8 \Rightarrow 2x+2=8$

$x=3$

Also, $2^{3y-1}=2^{11}$

$3y-1=11 \Rightarrow y=4$

$\therefore \dfrac{2x+3y}{y-x}=\dfrac{6+12}{4-3}=18$

**41.** $S=\dfrac{3}{3\left(3+\left(\sqrt{10}\right)\right)}+\dfrac{\sqrt{10}}{\sqrt{10}\left(\sqrt{10}+\sqrt{11}\right)}+\dfrac{\sqrt{11}}{\sqrt{11}\left(\sqrt{11}+\sqrt{12}\right)}$

$+\ldots+\dfrac{\sqrt{399}}{\sqrt{399}\left(\sqrt{399}+\sqrt{400}\right)}$

$=\dfrac{1}{\sqrt{10}+\sqrt{9}}+\dfrac{1}{\sqrt{11}+\sqrt{10}}+\dfrac{1}{\sqrt{12}+\sqrt{11}}+\ldots+\dfrac{1}{\sqrt{400}+\sqrt{399}}$

$=\dfrac{\sqrt{10}-\sqrt{9}}{\left(\sqrt{10}\right)^2-\left(\sqrt{9}\right)^2}+\dfrac{\sqrt{11}-\sqrt{10}}{\left(\sqrt{11}\right)^2-\left(\sqrt{10}\right)^2}+\dfrac{\sqrt{12}-\sqrt{11}}{\left(\sqrt{12}\right)^2-\left(\sqrt{11}\right)^2}$

$+\ldots+\dfrac{\sqrt{400}-\sqrt{399}}{\left(\sqrt{400}\right)^2-\left(\sqrt{399}\right)^2}$

$=\sqrt{10}-\sqrt{9}+\sqrt{11}-\sqrt{10}+\sqrt{12}-\sqrt{11}+\ldots+\sqrt{400}-\sqrt{399}$

$=\sqrt{400}-\sqrt{9}$

$=20-3=17.$

**42.** Let $\sqrt{6+2\sqrt{2}+2\sqrt{6}+2\sqrt{3}}$

$=\sqrt{a}+\sqrt{b}+\sqrt{c}$

Squaring both sides, $6+2\sqrt{2}+2\sqrt{6}+2\sqrt{3}$

$=a+b+c+2\sqrt{ab}+2\sqrt{bc}+2\sqrt{ca}$

Equating the rational and irrational parts on both sides,

$6=a+b+c$

$2=ab \quad (1)$

$6=bc \quad (2)$

$3=ac \quad (3)$

$(ab)\,(bc)\,(ac)=(2)\,(6)\,(3)$

$a^2\,b^2\,c^2=36$

$abc=6$ (as $a$, $b$, $c$ are all positive)

From (1), $c=\dfrac{6}{ab}=3$

From (2), $a=\dfrac{6}{bc}=1$

From (3), $b=\dfrac{6}{ac}=2$

Hence, $\sqrt{6+2\sqrt{2}+2\sqrt{6}+2\sqrt{3}}=1+\sqrt{2}+\sqrt{3}$

$\dfrac{\sqrt{15}+\sqrt{10}+\sqrt{5}}{\sqrt{6}+2\sqrt{5}+2\sqrt{6}+2\sqrt{3}}$

$\dfrac{\sqrt{15}+\sqrt{10}+\sqrt{5}}{1+\sqrt{2}+\sqrt{3}}=\sqrt{5}$

**43.** Squaring each of the choices, we have only choice (B)'s square resulting, i.e., $\frac{x+2+\sqrt{4x^2+16x+15}}{4}$.

**44.** The given expression is of the form

$E=(a+b+c+d)^2+(a-b+c-d)^2+(a-b-c+d)^2+(a+b-c-d)^2$

Now, $[(a+b)+(c+d)]^2+[(a+b)-(c+d)]^2=2[(a+b)^2+(c+d)^2]$

And $[(a-b)+(c-d)]^2+[(a-b)-(c-d)]^2=2[(a-b)^2+(c-d)^2]$

$\therefore E=2[(a+b)^2+(a-b)^2+(c+d)^2+(c-d)^2]=4[(a^2+b^2)+(c^2+d^2)]$

$\therefore$ The given expression is $4\left[\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\right]$

$=4\left[\frac{20}{60}+\frac{15}{60}+\frac{12}{60}+\frac{10}{60}\right]=\frac{19}{5}$.

**45.** Let $\sqrt{11+\sqrt{48}+\sqrt{24}+\sqrt{72}}=\sqrt{a}+\sqrt{b}+\sqrt{c}$

Squaring both sides,

$11+\sqrt{48}+\sqrt{24}+\sqrt{72}=a+b+c+2\sqrt{ab}+\sqrt{bc}+\sqrt{ca}$

$\therefore 11+2\sqrt{6}+2\sqrt{12}+2\sqrt{18}$

$=(a+b+c)+2\sqrt{ab}+2\sqrt{bc}+2\sqrt{ca}$

By inspection $a=2$, $b=3$, $c=6$ and the required square root is $\sqrt{2}+\sqrt{3}+\sqrt{6}$.

## EXERCISE-3

**1.** $\frac{1+4x}{1+\sqrt{1+4x}}+\frac{1+4x}{1-\sqrt{1+4x}}$

$=(1+4x)\left[\frac{1}{1+\sqrt{1+4x}}+\frac{1}{1-\sqrt{1+4x}}\right]$

$=(1+4x)\left[\frac{2}{1-(1+4x)}\right]=-\frac{1+4x}{2x}$

when $x=1/4$, the given expression becomes

$-\frac{1+4(1/4)}{2(1/4)}=-4$

**2.** $\sqrt{\sqrt[3]{\sqrt[4]{\sqrt[6]{25^{72}}}}}=\sqrt{\sqrt[3]{\sqrt[4]{25^{72/6}}}}=\sqrt{\sqrt[3]{25^{12/4}}}$

$=\sqrt{25^{3/3}}=\sqrt{25}=5$

**3.** $\frac{2^a}{32^a}=\frac{50b}{25b}\Rightarrow 2^{a-5a}=2\Rightarrow 2^{-4a}=2$

$\Rightarrow -4a=1\Rightarrow a=-1/4$

**4.** Let $2^x=7^y=14^z=k$

$\Rightarrow 2=k^{1/x}$

$7=k^{1/y}$

and $14=k^{1/z}$

But $14=2\times 7$

$\Rightarrow k^{1/z}=k^{1/x}\cdot k^{1/y}\Rightarrow k^{1/z}=k^{1/x+1/y}$

$\Rightarrow \frac{1}{z}=\frac{1}{x}+\frac{1}{y}=\frac{x+y}{xy}$

$\therefore\ z=\frac{xy}{x+y}$

**5.** The given expression

$=\frac{2^{(m+2)}\cdot 2^{(m-2)}\cdot 3^{(m-2)}\cdot 2^{2m}\cdot 5^{2m}\cdot 3^{(m-1)}\cdot 5^{(m-1)}\cdot 5^{(2m-2n)}}{\left(2^2\right)^{2m}\cdot 3^{2m-3}\cdot 5^{5m-2n+5}\cdot 5^{-3}}$

$=2^{(m+2)+(m-2)+2m-4m}\cdot 3^{(m-2)+(m-1)-2m+3}\cdot 5^{2m+(m-1)+(2m-2n)-(5m-2n+5)+8}$

$=2^0\cdot 3^0\cdot 5^2=25$

**6.** Let us find the H.C.F. of the indices.

H.C.F. of the indices $=\frac{\text{H.C.F.(their numerators)}}{\text{L.C.M.(their denominators)}}=\frac{1}{18}$

We can express the 5 powers as different numbers to the power of 1/18.

$7^{\frac{1}{6}}=7^{\frac{3}{18}}=\left(7^3\right)^{\frac{1}{18}}$.

Similarly, we see that $4^{\frac{1}{3}}=\left(4^6\right)^{\frac{1}{18}}$, $3^{\frac{1}{2}}=\left(3^9\right)^{\frac{1}{18}}$ and $10^{\frac{1}{9}}=\left(10^2\right)^{\frac{1}{18}}$.

$\therefore$ The numbers to be compared are $\left(7^3\right)^{\frac{1}{18}},\left(4^6\right)^{\frac{1}{18}},\left(3^9\right)^{\frac{1}{18}},\left(10^2\right)^{\frac{1}{18}}$ and $\left(19^1\right)^{\frac{1}{18}}$.

$\therefore$ The comparison of the given powers is equivalent to comparison of the bases.

$7^3=343$

$4^6=4096$

$3^9=19683$

$10^2=100$

$19^1=19$

$\therefore 3^9$ is the greatest of these bases or $3^{\frac{1}{2}}$ is the greatest of the given numbers.

**7.** $3^{4x+2}=729=3^6$

$\Rightarrow 4x+2=6$

(equating the powers of 3)

$\Rightarrow x=1$

$2^{3y+1}=8192=2^{13}$

$\Rightarrow 3y+1=13$

$\Rightarrow y=4$

$$\frac{2y+x}{y-x}=\frac{2(4)+1}{4-1}=3$$

**8.** $\left(x^{2^{2n-1}}+y^{2^{2n-1}}\right)\left(x^{2^{2n-1}}-y^{2^{2n-1}}\right)$

$=\left(x^{2^{2n-1}}\right)^2-\left(y^{2^{2n-1}}\right)^2=x^{2^{2n}}-y^{2^{2n}}$

The given expression has a value of

$=\dfrac{x^{2^{2n}}-y^{2^{2n}}}{x^{2^{2n}}-y^{2^{2n}}}=1.$

**NOTE**

$\left(x^{2^{2n-1}}\right)^2=\left(x^{2^{2n-1}}\right)\left(x^{2^{2n-1}}\right)$

$=x^{2^{2n-1}+2^{2n-1}}=x^{2\left(2^{2n-1}\right)}=x^{2^{2n}}$

**9.** $3^x=115+2^y$ (1)

$3^{x-4}+2^{y-5}=7$ (2)

In equation (1), $3^x$ and $2^y$ are both positive.

Hence, $3x>115$.

As $x$ and $y$ are integers in all the options, $x$ must be a minimum of 5 in order to satisfy equation (1).

Hence, $x$ can be 5 or 6.

When $x=5$, $y=7$ from (1) and when $x=6$, '$y$' is not an integer (since all answer options are integers).

Hence, choice (B) follows.

**10.** $\left(\dfrac{729\,b^{14}}{27b^8}\right)^{\frac{1}{3}}\div\left(\dfrac{243b^{10}}{9b^4}\right)^{\frac{2}{3}}$

$=\left(27b^6\right)^{\frac{1}{3}}\div\left(27b^6\right)^{\frac{2}{3}}=(3b^2)\div(3b^2)^2=\dfrac{1}{3b^2}$

**11.** $\left[2^{9\times\frac{2}{9}}\times3^{7\times\frac{3}{7}}\times3^{3\times\frac{1}{3}}\times5\right]^{\frac{1}{2}}$

$=\left[2^2\times15\times3^3\times5\right]^{\frac{1}{2}}=2\times15\times3=90$

**12.** Given: $(2.56)^a=(0.00256)^b=10^c$

$2.56=10^{\frac{c}{a}}$ (1)

$0.00256=10^{\frac{c}{b}}$ (2)

$\dfrac{(1)}{(2)}=\dfrac{2.56}{0.00256}=\dfrac{10^{\frac{c}{a}}}{10^{\frac{c}{b}}}$

$10^3=10^{\frac{c}{a}-\frac{c}{b}}\Rightarrow3=\dfrac{c}{a}-\dfrac{c}{b}$

$\dfrac{1}{a}-\dfrac{1}{b}=\dfrac{3}{c}\Rightarrow\dfrac{1}{a}=\dfrac{3}{c}+\dfrac{1}{b}$

**13.** Given terms are $\sqrt[3]{4},\sqrt[4]{7}$ and $\sqrt[6]{17}$, i.e., $\left(4^4\right)^{\frac{1}{12}},\left(7^3\right)^{\frac{1}{12}},\left(17^2\right)^{\frac{1}{12}}$

$(256)^{\frac{1}{12}},(343)^{\frac{1}{12}}$ and $(289)^{\frac{1}{12}}$

$\therefore$ The descending order is $\sqrt[4]{7},\sqrt[6]{17}$ and $\sqrt[3]{4}$,

**14.** $\dfrac{2^{a+a+2+6a}\times3^{a+2+4a+4b}}{2^{4a+2b-4+6}\times3^{a-b+2a+b-2+2}}$

$=2^{8a+2-4a-2b-2}\times3^{5a+4b+2-3a}$

$=2^{4a-2b}\times3^{2a+4b+2}=4^{2a-b}\times9^{a+2b+1}$

**15.** $\dfrac{5^x}{5^y}=125\Rightarrow5^{x-y}=5^3\Rightarrow x-y=3$ (1)

$\dfrac{2^x}{4^y}=2\Rightarrow\dfrac{2^x}{2^{2y}}=2\Rightarrow2^{x-2y}=2$

$\Rightarrow x-2y=1$ (2)

Solving (1) and (2), we get

$x=5,\ y=2$

$\therefore\ \dfrac{3^{2x}}{27^y}+4=\dfrac{3^{10}}{3^{3\times2}}+4=3^4+4=85$

**16.** $\dfrac{\left[\left(\sqrt{\sqrt5+2}\right)^2+\left(\sqrt{\sqrt5-2}\right)^2+2\sqrt{\left(\sqrt5+2\right)\left(\sqrt5-2\right)}\right]}{\left(5-2\sqrt6\right)\left(5+2\sqrt6\right)}$

$=\dfrac{\sqrt5+2+\sqrt5-2+2\sqrt{5-4}}{25-24}=2\sqrt5+2$

**17.** $\left(\sqrt{10+2\sqrt{21}}\right)^3+\left(\sqrt{10-2\sqrt{21}}\right)^3$ is the expression.

Consider $\sqrt{10+2\sqrt{21}}=\sqrt{(\sqrt7)^2+(\sqrt3)^2+2(\sqrt7)(\sqrt3)}$

$=\sqrt{(\sqrt7+\sqrt3)^2}=\sqrt7+\sqrt3$

Similarly, $\sqrt{10-2\sqrt{21}}=\sqrt7-\sqrt3$

So, $\sqrt{10+2\sqrt{21}}$ and $\sqrt{10-2\sqrt{21}}$ have values of $\sqrt7+\sqrt3$ and $\sqrt7-\sqrt3$ respectively.

$\therefore\left(\sqrt7+\sqrt3\right)^3+\left(\sqrt7-\sqrt3\right)^3=2[(\sqrt7)^3+3(\sqrt7)(\sqrt3)^2]$

$=14\sqrt7+18\sqrt7$ (using the identity $(a+b)^3+(a-b)^3$)

$=2a^3+6ab^2$ where in this case $a=\sqrt7$ and $b=\sqrt3$.

As $p=32$ and $q=0$, $p+q=32$.

**18.** $a-\sqrt2=\dfrac{\sqrt{24}+\sqrt{18}}{\sqrt{24}-\sqrt{18}}$

$a-\sqrt2=\dfrac{\left(\sqrt{24}+\sqrt{18}\right)\left(\sqrt{24}+\sqrt{18}\right)}{\left(\sqrt{24}-\sqrt{18}\right)\left(\sqrt{24}+\sqrt{18}\right)}$

$$= \frac{24+18+2\sqrt{(24)(18)}}{24-18} = \frac{42+24\sqrt{3}}{6} = 7+4\sqrt{3}$$

$\therefore a = 7+4\sqrt{3}+\sqrt{2}$

**19.** $\sqrt{\sqrt{3}+2} = \frac{1}{\sqrt{2}}\left(\sqrt{2\sqrt{3}+4}\right)$

$$= \frac{1}{\sqrt{2}}\left(\sqrt{(\sqrt{3})^2+1^2+2\sqrt{3}}\right)$$

$$= \frac{1}{\sqrt{2}}\sqrt{(\sqrt{3}+1)^2} = \frac{\sqrt{3}+1}{\sqrt{2}}$$

By substituting the value of $\sqrt{\sqrt{3}+2}$ in the given expression, it becomes

$$\frac{6\sqrt{2}}{2\sqrt{2}-\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right)}+6-2\sqrt{3}$$

$$= \frac{6\sqrt{2}\left(\sqrt{2}\right)}{2\left(\sqrt{2}\right)\left(\sqrt{2}\right)-\left(\sqrt{3}+1\right)}+6-2\sqrt{3}$$

$$= \frac{12}{3-\sqrt{3}}+6-2\sqrt{3} = \frac{12\left(3+\sqrt{3}\right)}{\left(3-\sqrt{3}\right)\left(3+\sqrt{3}\right)}+6-2\sqrt{3}$$

(Multiplying the numerator and denominator of $\frac{12}{3-\sqrt{3}}$ by $3+\sqrt{3}$ )

$$= 2\left(3+\sqrt{3}\right)+6-2\sqrt{3} = 12$$

**20.** $\frac{\sqrt{a}+\sqrt{b}}{a-b} - \frac{\sqrt{b}-\sqrt{a}}{a+b}$

$$= \frac{(a+b)\left(\sqrt{a}+\sqrt{b}\right)-\left(\sqrt{b}-\sqrt{a}\right)(a-b)}{(a-b)(a+b)}$$

$$= \frac{a\sqrt{a}+a\sqrt{b}+b\sqrt{a}+b\sqrt{b}-a\sqrt{b}+a\sqrt{a}+b\sqrt{b}-b\sqrt{a}}{(a-b)(a+b)}$$

$$= \frac{2a\sqrt{a}+2b\sqrt{b}}{(a-b)(a+b)} = 4 \text{ (Given)}$$

$$\Rightarrow \frac{a\sqrt{a}+b\sqrt{b}}{a^2-b^2} = 2.$$

Comparing with the given relation, $x = \frac{3}{2}$

**21.** Let $x = \frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2}+\sqrt{3}} + \frac{2-\sqrt{3}}{\sqrt{2}+\sqrt{2}-\sqrt{3}}$

$$= \sqrt{2}+\sqrt{3} = \sqrt{1/2\left(4+2\sqrt{3}\right)} = \frac{\sqrt{3}+1}{\sqrt{2}}$$

and $\sqrt{2-\sqrt{3}} = \frac{\sqrt{3}-1}{\sqrt{2}}$

$$\therefore x = \frac{2+\sqrt{3}}{\sqrt{2}+\frac{\sqrt{3}+1}{\sqrt{2}}} + \frac{2-\sqrt{3}}{\sqrt{2}+\frac{\sqrt{3}-1}{\sqrt{2}}}$$

$$= \frac{\frac{2+\sqrt{3}}{3+\sqrt{3}}}{\sqrt{2}} + \frac{\frac{2-\sqrt{3}}{\sqrt{3}+1}}{\sqrt{2}} = \frac{\sqrt{2}\left(2+\sqrt{3}\right)}{\sqrt{3}\left(\sqrt{3}+1\right)} + \frac{\left(2-\sqrt{3}\right)\sqrt{2}}{\sqrt{3}+1}$$

$$= \frac{\sqrt{2}}{\left(\sqrt{3}+1\right)}\left\{\frac{2+\sqrt{3}+\sqrt{3}\left(2-\sqrt{3}\right)}{\sqrt{3}}\right\}$$

$$= \frac{\sqrt{2}}{\sqrt{3}\left(\sqrt{3}+1\right)}\left(3\sqrt{3}-1\right)$$

**22.** $2(3+\sqrt{2}+\sqrt{3}+\sqrt{6}) = 6+2\sqrt{2}+2\sqrt{3}+2\sqrt{6}$

$$= 1^2+\left(\sqrt{2}\right)^2+\left(\sqrt{3}\right)^2+2.1.\sqrt{2}+2.1.\sqrt{3}+2.\sqrt{2}.\sqrt{3}$$

$$= \left(1+\sqrt{2}+\sqrt{3}\right)^2$$

$$\therefore \sqrt{3+\sqrt{2+\sqrt{3+\sqrt{6}}}}$$

$$= \sqrt{1/2\left(6+2\sqrt{2}+2\sqrt{3}+2\sqrt{6}\right)} = \frac{1}{2}\left(1+\sqrt{2}+\sqrt{3}\right)$$

$\therefore$ The given expression becomes

$$\frac{\sqrt{2}\left(1+\sqrt{2}+\sqrt{3}\right)}{\frac{1}{\sqrt{2}}\left(1+\sqrt{2}+\sqrt{3}\right)} = 2$$

**23.** $\frac{\sqrt{3+2\sqrt{2}}-\sqrt{3-2\sqrt{2}}}{\sqrt{3+2\sqrt{2}}+\sqrt{3-2\sqrt{2}}} = \frac{x}{32}$

$$= \frac{\left(\sqrt{2}+1\right)-\left(\sqrt{2}-1\right)}{\left(\sqrt{2}+1\right)+\left(\sqrt{2}-1\right)} = \sqrt{\frac{x}{32}} = \frac{1}{\sqrt{2}} = \sqrt{\frac{x}{32}}$$

Squaring both sides, we get

$\frac{x}{32} = \frac{1}{2} \Rightarrow x = 16$

**24.** $\frac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}} = \frac{\sqrt{x^2-1}\left(x+\sqrt{x^2-1}\right)}{x^2-\left(\sqrt{x^2-1}\right)}$

$$= x\sqrt{x^2-1}+x^2-1$$

Now, $x^2 = \frac{1}{2}\left[\sqrt{5}+\frac{1}{\sqrt{5}}\right] = \frac{3}{\sqrt{5}}$ $x^2 = 9/5$

$$\Rightarrow x^2-1 = \frac{9}{5}-1 = \frac{4}{5} = \sqrt{x^2-1} = \frac{2}{\sqrt{5}}$$

$$\therefore x\sqrt{x^2-1}+(x^2-1)$$

$$= \frac{3}{\sqrt{5}}\times\frac{2}{\sqrt{5}}\times\frac{4}{5} = \frac{10}{5} = 2$$

**25.** $\left(24-16\sqrt{2}\right)^{\frac{1}{6}}\times\left(32+16\sqrt{2}\right)^{\frac{1}{3}}$

$=\left[8\left(3-2\sqrt{2}\right)^{\frac{1}{6}}\right]\times\left[16\left(2+\sqrt{2}\right)\right]^{\frac{1}{3}}$

$=8^{\frac{1}{6}}\left[\left(\sqrt{2}-1\right)^{2}\right]^{\frac{1}{6}}\times\left[16\left(2+\sqrt{2}\right)\right]^{\frac{1}{3}}$

$=8^{\frac{1}{6}}16^{\frac{1}{3}}\left[\left(\sqrt{2}-1\right)\left(2+\sqrt{2}\right)\right]^{\frac{1}{3}}$

$=2^{\frac{1}{2}}\cdot 2^{\frac{4}{3}}\cdot 2^{\frac{1}{6}}=2^{\frac{1}{2}+\frac{4}{3}+\frac{1}{6}}=2^{2}=4$

**26.** $\sqrt{\frac{m-\sqrt{n}}{m+\sqrt{n}}}+\sqrt{\frac{m+\sqrt{n}}{m-\sqrt{n}}}-\frac{8}{\sqrt{\left(m+\sqrt{n}\right)\left(m-\sqrt{n}\right)}}$

$=\frac{\sqrt{\left(m-\sqrt{n}\right)^{2}}+\sqrt{\left(m+\sqrt{n}\right)^{2}}-8}{\sqrt{\left(m+\sqrt{n}\right)\left(m-\sqrt{n}\right)}}$

$=\frac{m-\sqrt{n}+m+\sqrt{n}-8}{\sqrt{m^{2}-n}}=\frac{2m-8}{\sqrt{m^{2}-n}}$

$=\frac{2\left(m-4\right)}{\sqrt{m^{2}-8\left(m-2\right)}}\quad\left[\because n=8\left(m-2\right)\right]$

$=\frac{2\left(m-4\right)}{\sqrt{m^{2}-8m+16}}=\frac{2\left(m-4\right)}{\sqrt{\left(m-4\right)^{2}}}=2$

**27.** Mean proportional of $(8-3\sqrt{7})$ and $(16-5\sqrt{7})$ is

$\sqrt{\left(8-3\sqrt{7}\right)\left(16-5\sqrt{7}\right)}$, let it be $x$

$\therefore x=\sqrt{128-40\sqrt{7}-48\sqrt{7}+105}=\sqrt{233-88\sqrt{7}}$

$=\sqrt{233-2\sqrt{44\times44\times7}}=\sqrt{232-2\sqrt{11\times11\times16\times7}}$

$=\sqrt{(121+112)-2\sqrt{121\times112}}$

$=\sqrt{\left(\sqrt{121}-\sqrt{112}\right)^{2}}=11-4\sqrt{7}$

**28.** Let $\frac{1}{\sqrt{a}+\sqrt{a+1}}+\frac{1}{\sqrt{a+1}+\sqrt{a+2}}+\ldots$

$+\frac{1}{\sqrt{a+63}+\sqrt{a+64}}=x$

By rationalizing the denominators of every term in $x$, we get

$x=\frac{\sqrt{a+1}-\sqrt{a}}{(a+1)-a}+\frac{\sqrt{a+2}-\sqrt{a+1}}{(a+2)-(a+1)}+\ldots$

$+\frac{\sqrt{a+64}-\sqrt{a+63}}{(a-64)-(a+63)}$

$=\sqrt{a+1}-\sqrt{a}+\sqrt{a+2}-\sqrt{a+1}+\ldots$

$+\sqrt{a+64}-\sqrt{a+63}$

$x=\sqrt{a+64}-\sqrt{a}$

Given: $x=4$

$\Rightarrow\sqrt{a+64}-\sqrt{a}=4$

Of the given choices, $a=36$ satisfies the above equation i.e., $\sqrt{36+64}-\sqrt{36}=4$

$10-6=4$

$\therefore a=36$

**29.** Given: $b=\frac{1}{2}\left(\frac{7(7)+1}{\sqrt{7}}\right)=\frac{25}{\sqrt{7}}$

$\sqrt{b^{2}-7}=\sqrt{\frac{625-49}{7}}$

$=\frac{24}{\sqrt{7}}$

$\frac{\sqrt{b^{2}-7}}{b-\sqrt{b^{2}-7}}=\frac{\frac{24}{\sqrt{7}}}{\frac{25}{\sqrt{7}}-\frac{24}{\sqrt{7}}}=24$

**30.** The square root of $6+2\sqrt{5}$ can be found to be $\sqrt{5}+1$.

Consider $\sqrt{2\left(\sqrt{6+2\sqrt{5}}\right)+4\left(4+\sqrt{5}\right)}$

$=\sqrt{18+6\sqrt{5}}=\sqrt{18+2\sqrt{45}}=\sqrt{15}+\sqrt{3}$

$\frac{1}{\sqrt{\sqrt{6+2\sqrt{5}}+3\left(4+\sqrt{5}\right)}}=\frac{1}{\sqrt{15}+\sqrt{3}}$

$=\frac{\sqrt{15}-\sqrt{3}}{15-3}=\frac{\sqrt{3}\left(\sqrt{5}-1\right)}{12}$

(Multiplying both the numerator and denominator by $\sqrt{15}-\sqrt{3}$)

**31.** $\frac{\sqrt{8}+\sqrt{6}-\sqrt{10}}{\sqrt{6}+2\sqrt{3}-2\sqrt{5}-\sqrt{15}}$

$=\frac{\sqrt{8}+\sqrt{6}-\sqrt{10}}{\sqrt{6}+\sqrt{12}-\sqrt{20}-\sqrt{15}}$

$=\frac{\sqrt{2}\left(2\sqrt{2}+\sqrt{6}-\sqrt{10}\right)}{\sqrt{12+2\sqrt{12}-2\sqrt{20}}-2\sqrt{15}}$

$=\frac{4+2\sqrt{3}-2\sqrt{5}}{\sqrt{4}+\sqrt{3}-\sqrt{5}}=2$

**32.** Consider $\sqrt{17+2\sqrt{72}}=\sqrt{17+2\sqrt{9\times8}}$

$= \sqrt{9} + \sqrt{8} = 3 + \sqrt{8}$

$\sqrt{6+4\sqrt{2}} = \sqrt{6+2\sqrt{4\times 2}} = \sqrt{2}+2$

$\therefore$ The given expression $= \dfrac{\left(3+\sqrt{8}\right)^3 + \left(3-\sqrt{8}\right)^3}{\left(2+\sqrt{2}\right)^3 + \left(2-\sqrt{2}\right)^3}$

$= \dfrac{2\left[(3)^3 + 3(3)(8)\right]}{2\left[(2)^3 + 3(2)(2)\right]} = \dfrac{99}{20}$

**33.** $\sqrt{15+2\sqrt{35}-2\sqrt{21}-2\sqrt{15}}$

$= \sqrt{15+2\sqrt{7}\sqrt{5}-2\sqrt{7}\sqrt{3}-2\sqrt{5}\sqrt{3}}$

$= \sqrt{\begin{matrix}(\sqrt{7})^2+(\sqrt{5})^2+(-\sqrt{3})^2+2(\sqrt{7})(\sqrt{5}) \\ +2(\sqrt{7})(-\sqrt{3})+2(\sqrt{5})(-\sqrt{3})\end{matrix}}$

$= \sqrt{(\sqrt{7}+\sqrt{5}-\sqrt{3})^2}$

$(\because\ a^2+b^2+c^2+2ab+2bc+2ca = (a+b+c)^2)$

$= \sqrt{7}+\sqrt{5}-\sqrt{3}$ .

**34.** Let $a = 4+\sqrt{8}$ and $b = 4-\sqrt{8}$

$x = a^{\frac{1}{3}} + b^{\frac{1}{3}}$

Cubing on both sides, $x^3 = a+b+3(ab)^{1/3}\left(a^{\frac{1}{3}}+a^{\frac{1}{3}}\right)$

$\Rightarrow x^3 = 8 + 3(8)^{\frac{1}{3}}(x)$

$\Rightarrow x^3 - 6x = 8$

**35.** $\dfrac{8}{2+2\sqrt{2}+2\sqrt{3}} = \dfrac{4}{1+\sqrt{2}+\sqrt{3}}$

$= \dfrac{4\sqrt{2}}{2+\sqrt{2}+\sqrt{6}} = \dfrac{4\sqrt{2}\left(2+\sqrt{2}-\sqrt{6}\right)}{\left(2+\sqrt{2}\right)^2-\left(\sqrt{6}\right)^2}$

$= \dfrac{4\sqrt{2}\left(2+\sqrt{2}-\sqrt{6}\right)}{6+4\sqrt{2}-6} = 2+\sqrt{2}-\sqrt{6}$

$\therefore\ 2+\sqrt{2}-\sqrt{6} = x+y\sqrt{2}+z\sqrt{6}$

$x+y+z = 2+1-1 = 2$

**36.** Squaring the given surds, we have

$\left(\sqrt{8}+\sqrt{19}\right)^2 = 27+2\sqrt{152}$

$\left(\sqrt{12}+\sqrt{13}\right)^2 = 25+2\sqrt{156}$ and

$\left(\sqrt{14}+\sqrt{12}\right)^2 = 26+2\sqrt{168}$

By observation, eliminate $\sqrt{12}+\sqrt{13}$ ,

as $\sqrt{168}$ is close to 13 and $\sqrt{152}$ is close to 12.3

Hence, $\left(\sqrt{8}+\sqrt{19}\right)^2 = 51.6$

and $\left(\sqrt{14}+\sqrt{12}\right)^2 = 52$

$\therefore\ \sqrt{14}+\sqrt{12}$ is the greatest.

**37.** Here, we take $\sqrt{2}+\sqrt{3}$ as one part in $\sqrt{2}+\sqrt{3}+\sqrt{5}$

$\dfrac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}} \times \dfrac{(\sqrt{2}+\sqrt{3})-(\sqrt{5})}{(\sqrt{2}+\sqrt{3})-(\sqrt{5})} = \dfrac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{5})^2}$

$= \dfrac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{5+2\sqrt{6}-5} = \dfrac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2\sqrt{6}}$

But still $\sqrt{6}$ is irrational multiply both numerator and denominator with $\sqrt{6}$.

Hence, it becomes $\dfrac{(\sqrt{2}+\sqrt{3}-\sqrt{5})\sqrt{6}}{6}$

**38.** Rationalizing the denominators of given surds, we get

$\dfrac{\sqrt{5}}{\sqrt{3}+\sqrt{2}} = \sqrt{5}(\sqrt{3}-\sqrt{2})$

$\dfrac{18\sqrt{3}}{\sqrt{5}-\sqrt{2}} = 6\sqrt{3}(\sqrt{5}+\sqrt{2})$

$\dfrac{8}{\sqrt{5}+\sqrt{3}} = 4(\sqrt{5}-\sqrt{3})$

$\therefore$ The required answer will be

$\sqrt{5}\left(\sqrt{3}-\sqrt{2}\right)-6\sqrt{3}\left(\sqrt{5}+\sqrt{2}\right)+4\left(\sqrt{5}-\sqrt{3}\right)$

$= \sqrt{15}-\sqrt{10}-6\sqrt{15}-6\sqrt{6}+4\sqrt{5}-4\sqrt{3}$

$= 4\sqrt{5}-5\sqrt{15}-6\sqrt{6}-4\sqrt{3}-\sqrt{10}$

**39.** Consider $\dfrac{13}{\sqrt{19-8\sqrt{3}}} = \dfrac{13}{\sqrt{19-2\sqrt{16\times 3}}}$

$= \dfrac{13}{4-\sqrt{3}} = 4+\sqrt{3}$ (1)

Consider $\dfrac{23}{\sqrt{31+12\sqrt{3}}} = \dfrac{23}{\sqrt{31+2\sqrt{36\times 3}}}$

$= \dfrac{23}{\sqrt{31+2\sqrt{27\times 4}}}$

$= \dfrac{23}{\sqrt{27}+2} = \sqrt{27}-2 = 3\sqrt{3}-2$ (2)

Consider $\dfrac{24}{\sqrt{48+24\sqrt{3}}} = \dfrac{24}{\sqrt{48+2\sqrt{144\times 3}}}$

$= \dfrac{24}{\sqrt{48+2\sqrt{36\times 2}}} = \dfrac{24}{6+\sqrt{12}} = 6-2\sqrt{3}$ (3)

$\therefore$ (1) – (2) – (3)

$= 4+\sqrt{3}-3\sqrt{3}+2-6+2\sqrt{3} = 0$

**40.** $\left(6+\sqrt{35}\right)^{x/2}=a \Rightarrow \left(6-\sqrt{35}\right)^{x/2}=\frac{1}{a}$

$\Rightarrow a+\frac{1}{a}=12 \Rightarrow a^2-12a+1=0$

$\Rightarrow a=\frac{12\pm\sqrt{144-4}}{2}=\frac{12\pm\sqrt{140}}{2}=6\pm\sqrt{35}$

$(6+\sqrt{35})^{x/2}=6\pm\sqrt{35}=(6+\sqrt{35})^{\pm 1}$

$x/2=\pm 1 \therefore$ Two solutions for $x$.

**41.** Let $\left(3\sqrt{3}\right)\left(5\sqrt{5}\right)+11\sqrt{11}$

$+\left(3\sqrt{3}\right)\left(\sqrt{5}\right)\left(\sqrt{11}\right)\left(\sqrt{3}\right)\left(\sqrt{5}\right)+\sqrt{11}=x$

$x=\left(\sqrt{3}\right)^3\left(\sqrt{5}\right)^3+\left(\sqrt{11}\right)^3$

$+3\left(\sqrt{3}\right)\left(\sqrt{5}\right)\left(\sqrt{11}\right)\left(\sqrt{3}\right)\left(\sqrt{5}\right)+\sqrt{11}$

$=\left(\sqrt{15}\right)^3+\left(\sqrt{11}\right)^3+3\left(\sqrt{15}\right)\left(\sqrt{11}\right)\left[\left(\sqrt{15}+\sqrt{11}\right)\right]$

$=\left(\sqrt{15}+\sqrt{11}\right)^3$

$\therefore$ The cube root of $x$ is

$x^{1/3}=\left[\left(\sqrt{15}+\sqrt{11}\right)^3\right]^{1/3}=\sqrt{15}+\sqrt{11}$

**42.** $a=\frac{\left(\sqrt{5}-\sqrt{3}\right)^3}{8},\ b=\left(\sqrt{6}+\sqrt{5}\right)^3,\ c=\frac{\left(\sqrt{3}-\sqrt{6}\right)^3}{27}$

$\frac{1}{a}=\left[\frac{2}{\sqrt{5}-\sqrt{3}}\right]^3=\left[\frac{2\times\left(\sqrt{5}+\sqrt{3}\right)}{5-3}\right]^3$

$\left[\because \text{To rationalize multiply and divide with } \left(\sqrt{5}+\sqrt{3}\right)\right]$

$\frac{1}{a}=\left(\sqrt{5}+\sqrt{3}\right)^3$

$\frac{1}{b}=\left[\frac{1}{\sqrt{6}+\sqrt{5}}\right]^3=\left[\frac{\sqrt{6}-\sqrt{5}}{6-5}\right]^3=\left(\sqrt{6}-\sqrt{5}\right)^3$

$\frac{1}{c}=\left[\frac{3}{\sqrt{3}-\sqrt{6}}\right]^3=\left[\frac{3\times\left(\sqrt{3}+\sqrt{6}\right)}{3-6}\right]^3=-\left(\sqrt{6}+\sqrt{3}\right)^3$

$\therefore \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\left(\sqrt{5}+\sqrt{3}\right)^3+\left(\sqrt{6}-\sqrt{5}\right)^3-\left(\sqrt{6}+\sqrt{3}\right)^3$

$=5\sqrt{5}+15\sqrt{3}+9\sqrt{5}+3\sqrt{3}+6\sqrt{6}-18\sqrt{5}$

$+15\sqrt{6}-5\sqrt{5}-6\sqrt{6}-18\sqrt{3}-9\sqrt{6}-3\sqrt{3}$

$=-3\sqrt{3}-9\sqrt{5}+6\sqrt{6}=3$

**43.** $\frac{\left(4^x\right)^x.\left(4^y\right)^y.\left(4^z\right)^z}{\left(16^x\right)^{-y}.\left(16^y\right)^{-z}.\left(16^z\right)^{-z}}=64$

$=\frac{4^{\left(x^2+y^2+z^2\right)}}{16^{(-xy-yz-zx)}}=64=\frac{4^{\left(x^2+y^2+z^2\right)}}{4^{2(-xy-yz-zx)}}=4^3$

$=4^{\left[x^2+y^2+z^2-2(-xy-yz-zx)\right]}=4^3$

$\Rightarrow (x^2+y^2+z^2+2xy+2yz+2zx)=3 \Rightarrow (x+y+z)^2=3$

$\therefore x+y+z=\sqrt{3}$

**44.** $x=\frac{1}{3-2\sqrt{2}}=\frac{1}{3-2\sqrt{2}}\times\frac{3+2\sqrt{2}}{3+2\sqrt{2}}$

$=\frac{3+2\sqrt{5}}{9-8}=3-2\sqrt{2}$

Similarly, $y=\frac{1}{3+2\sqrt{2}}=3-2\sqrt{2}$

$3x^2+6xy-3y^2=3(x^2-y^2)+6xy$

$=3\left[\left(3+2\sqrt{2}\right)^2-\left(3-2\sqrt{2}\right)^2\right]+6\left(3+2\sqrt{2}\right)\left(3-2\sqrt{2}\right)$

$=3\left[4(3)(2\sqrt{2})\right]+6[9-8]=3\left[24\sqrt{2}\right]+6=72\sqrt{2}+6$

**45.** Consider $\frac{x+y}{x^{2/3}-(xy)^{1/3}+y^{2/3}}$

Multiply numerator and denominator with $x^{1/3}+y^{1/3}$

$\Rightarrow \frac{(x+y)\left(x^{1/3}+y^{1/3}\right)}{\left(x^{1/3}+y^{1/3}\right)\left(x^{2/3}-\left(x^{1/3}\right)\left(y^{1/3}\right)+y^{2/3}\right)}$

$=\frac{(x+y)\left(x^{1/3}+y^{1/3}\right)}{\left(x^{1/3}\right)^3+\left(y^{1/3}\right)^3}$

$[\because (a+b)(a^2-ab+b^2)=a^3+b^3]$

$=\frac{(x+y)\left(x^{1/3}+y^{1/3}\right)}{(x+y)}=x^{1/3}+y^{1/3}$

similarly, $\frac{x-y}{x^{2/3}+(xy)^{1/3}+y^{2/3}}=x^{1/3}-y^{1/3}$

$\therefore \frac{x+y}{\sqrt[3]{x^2}-\sqrt[3]{2y}+\sqrt[3]{y^2}}-\frac{x-y}{\sqrt[3]{x^2}+\sqrt[3]{xy}+\sqrt[3]{y^2}}$

$=x^{1/3}+y^{1/3}-\left(x^{1/3}-y^{1/3}\right)=2y^{1/3}$

## EXERCISE-4

**1.** $5^{\log_5 x^{\log_x y \log_z z^2}} = 54$

$5^{\log_5 x^{\log_x y^2}} = 54$

$5^{\log_5 y^2} = 54$

$\Rightarrow y^2 = 54$

$\Rightarrow y = \sqrt{54} = 3\sqrt{6}$

**2.** As $a^3 = b^3 \Rightarrow a = b$

$a^{5-x} b^{3x} = a^{5x} b^{x+5}$

$\Rightarrow a^{2x+5} = a^{6x+5}$

$\Rightarrow x = 0$ (as $a \neq -1, 0$ or $1$)

**3.** $450^x = 570^y = 722^z = K$ (say)

$450 = k^{\frac{1}{x}}$

$570 = k^{\frac{1}{y}}$

$722 = k^{\frac{1}{z}}$

we note that $a = 450 = 2(9)(25)$, $b = 570 = 2(3)(5)(19)$ and $c = 722 = 2(19)(19)$. Therefore, $ac = b^2$, or $\frac{a}{b} = \frac{b}{c}$

$\therefore k^{\frac{1}{x} - \frac{1}{y}} = k^{\frac{1}{y} - \frac{1}{z}}$

$\frac{1}{x} - \frac{1}{y} = \frac{1}{y} - \frac{1}{z}$

**4.** Squaring each of the given terms, we have

| Choice | $x$ | $x^2$ |
|---|---|---|
| A | $\sqrt{19} + \sqrt{6}$ | $25 + 2\sqrt{114}$ |
| B | $\sqrt{23} + \sqrt{3}$ | $26 + 2\sqrt{69}$ |
| C | $\sqrt{18} + \sqrt{7}$ | $25 + 2\sqrt{126}$ |
| D | $\sqrt{24} + \sqrt{2}$ | $26 + 2\sqrt{48}$ |

$\therefore$ C > A and B > D

We have to compare only B and C

$C^2 = 25 + 2\sqrt{126}$ as $11 < \sqrt{126} < 12$,

it follows that $47 < C^2 < 49$

$B^2 = 26 + 2\sqrt{69}$ as $8 < \sqrt{69} < 9$,

it follows that $42 < B^2 < 44$

$\therefore$ C = $\sqrt{18} + \sqrt{7}$ is the greatest.

**5.**

| $x$ | $3^{1/4}\ 4^{1/5}\ 5^{1/6}\ 6^{1/7}$ |
|---|---|
| $x^{20}$ | $3^5 \quad 4^4$ |
| $x^{30}$ | $4^6 \quad 5^5$ |
| $x^{42}$ | $5^7 \quad 6^6$ |

We see that $3^5 < 4^4$, $4^6 > 5^5$, $5^7 > 6^6$ and so on

$\therefore 4^{1/5}$ in the greatest.

**6.** Given $\frac{3 - \sqrt{a+5}}{a-4} = \frac{-1}{6}$

Rationalizing the numerator on the left-hand side, we have $\frac{(3 - \sqrt{a+5})(3 + \sqrt{a+5})}{(a-4)(3 + \sqrt{a+5})} = \frac{-1}{6}$

$\Rightarrow \frac{(9 - a - 5)}{(a-4)(3 + \sqrt{a+5})} = \frac{-1}{6}$

$\Rightarrow \frac{4 - a}{(a-4)(3 + \sqrt{a+5})} = \frac{-1}{6}$

$\Rightarrow \frac{1}{(a-4)(3 + \sqrt{a+5})} = \frac{-1}{6}$

$\Rightarrow 3 + \sqrt{a+5} = 6$

$\Rightarrow a + 5 = 9$

$\Rightarrow a = 4$. But if $a = 4$, the initial expression will be undefined as $a - 4 = 0$. Hence, $a$ can't take any value.

**7.** $\sum_{x=2}^{225} \frac{1}{\sqrt{x} + \sqrt{x-1}}$

$= \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \ldots \frac{1}{\sqrt{225}+\sqrt{224}}$

(Rationalizing the denominators of all the fractions, we get)

$= \sqrt{225} - 1 = 15 - 1 = 14$

**8.** $\sqrt{29 + 2\sqrt{180}} - \sqrt{24 - 2\sqrt{80}}$

$= \sqrt{\left(\sqrt{20}\right)^2 + \left(\sqrt{9}\right)^2 + 2\sqrt{20(9)}}$

$- \sqrt{\left(\sqrt{20}\right)^2 + \left(\sqrt{4}\right)^2 - 2\sqrt{20(4)}}$

$= \sqrt{\left(\sqrt{20} + \sqrt{9}\right)^2} - \sqrt{\left(\sqrt{20} + \sqrt{4}\right)^2}$

$= \left(\sqrt{20} + \sqrt{9}\right) - \left(\sqrt{20} + \sqrt{4}\right)$

$= \left(\sqrt{20} + 3\right) - \left(\sqrt{20} - 2\right) = 3 + 2 = 5$

**9.** The base in the 4 expressions are of the form $2^k \pm 1$.
$31^{15} < 32^{15}$ which is $(2^5)^{15}$, i.e., $2^{75}$
$17^{19} > 16^{19}$ which is $2^{76}$
$7^{25} < 2^{75}$ and $63^{12} < 2^{72}$
$17^{19}$ is the greatest.

**10.** Each term in $X$ has the form $\dfrac{1}{N\sqrt{N+1}+(N+1)\sqrt{N}}$

$$\frac{1}{N\sqrt{N+1}+(N+1)\sqrt{N}} = \frac{(N+1)-N}{\sqrt{N}\sqrt{N+1}\left(\sqrt{N}+\sqrt{N+1}\right)}$$

$$= \frac{\sqrt{N+1}-\sqrt{N}}{\sqrt{N}\sqrt{N+1}}$$

$$\left(\because \frac{a^2-b^2}{a+b} = a-b\right) = \frac{1}{\sqrt{N}} - \frac{1}{\sqrt{N+1}}$$

$$X = \frac{1}{\sqrt{16}} - \frac{1}{\sqrt{17}} + \frac{1}{\sqrt{17}} - \frac{1}{\sqrt{8}} + \ldots + \frac{1}{\sqrt{144}} - \frac{1}{\sqrt{145}}$$

$$= \frac{1}{\sqrt{16}} - \frac{1}{\sqrt{145}}.$$

**11.** $N = \sqrt{17+\sqrt{17-N}}$

Squaring both sides, $N^2 = 17 + \sqrt{17-N}$

$N^2 - 17 = \sqrt{17-N}$

Squaring both sides, $(N^2-17)^2 = 17 - N$ (1)

$\Rightarrow N^4 - 34N^2 + 289 = 17 - N$

$\Rightarrow N^4 - 34N^2 + N + 272 = 0$ (2)

This is a fourth degree equation. It will be difficult to solve. Let us go by the choices.

Choices A and C: In choice A, $\sqrt{69}$ is less than 9.

$\therefore$ Choice A is less than 3. But $N$ is more than $\sqrt{17}$ (which is a value more than 4) $\therefore$ $N$ is more than 4.

$\therefore$ Choice A is ruled out. Also choice C (which is less than Choice A) is ruled out. Choice (D) lies between 3.5 and 4.

Choices (B) is more than $\sqrt{17}$.

$\therefore$ Choice B follows.

**NOTE**

$N = \dfrac{\sqrt{65}+1}{2}$, $\Rightarrow (2N-1)^2 = 65$

$\Rightarrow 4N^2 - 4N + 1 = 65 \Rightarrow N^2 - N - 16 = 0$

We can verify that $N^2 - N - 16$ is a factor of the L.H.S of (2)

**12.** $a^{\frac{1}{a}} = b^{\frac{1}{b}}$ and $a^8 = b^6$

$a = b^{\frac{a}{b}}$ and $a = \left(b^6\right)^{\frac{1}{8}} = b^{\frac{3}{4}}$

$b^{\frac{a}{b}} = b^{\frac{3}{4}} \Rightarrow \dfrac{a}{b} = \dfrac{3}{4}$

As $a = b^{\frac{3}{4}}, \dfrac{b^{\frac{3}{4}}}{b} = \dfrac{3}{4}$

$\Rightarrow b^{\frac{1}{4}} = \dfrac{4}{3}$ or $b = \dfrac{256}{81}$.

**Alternative Solution:**

$a^{1/a} = b^{1/b} \Rightarrow \dfrac{\log a}{a} = \dfrac{\log b}{b}$ (1)

$a^8 = b^6 \Rightarrow 8 \log a = 6 \log b$ (2)

$\therefore \dfrac{\log a}{\log b} = \dfrac{3}{4}$ and $\dfrac{a}{b} = \dfrac{3}{4}$ (from 1)

Let $a = 3k$. $\therefore$ $b = 4k$.

$$\frac{\log a}{\log b} = \frac{\log 3 + \log k}{\log 4 + \log k} = \frac{3}{4} \Rightarrow \log k = 3 \log 4 - 4 \log 3$$

$$= \log \frac{64}{81}$$

$$\therefore k = \frac{64}{81} \text{ and } b = 4k = \frac{256}{81}$$

**13.** Given: $x^{\sqrt{x}^{\sqrt{x}}} = \left(x^{\sqrt{x}}\right)^{\sqrt{x}}$

$\Rightarrow x^{\sqrt{x}^{\sqrt{x}}} = x^x$ $\therefore$ either $x = 1$ or

$x^{\sqrt{x}} = x^1$, say $x \neq 1$

$\Rightarrow x^{\sqrt{x}/2} = x^1$, either $x = 1$ or

$\Rightarrow \dfrac{\sqrt{x}}{2} = 1 \Rightarrow x = 4$

**14.** $$X = \frac{1}{\sqrt{6}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{9}} + \ldots + \frac{1}{\sqrt{20}+\sqrt{21}}$$

$$= \frac{7-6}{\sqrt{7}+\sqrt{6}} + \frac{8-7}{\sqrt{8}+\sqrt{7}} + \frac{9-8}{\sqrt{9}+\sqrt{8}} + \ldots + \frac{21-20}{\sqrt{21}+\sqrt{20}}$$

$$= \sqrt{7} - \sqrt{6} + \sqrt{8} - \sqrt{7} + \sqrt{9} - \sqrt{8} + \ldots + \sqrt{21} - \sqrt{20}$$

$= \sqrt{21} - \sqrt{6}$ = (A value between 4.5 and 4.6) – (A value between 2.4 and 2.5) = A value between (4.5 – 2.5) and (4.6 – 2.4) = A value between 2 and 2.2.

**15.** $7x^{7a} < 8x^{8b}$

$$\frac{x^{7a}}{x^{8b}} < \frac{8}{7}$$

$x^{7a-8b} < \dfrac{8}{7}$. This holds true for all positive $x$.

If $7a - 8b$ is positive, there is at least one value of $x$ violating the inequation ($\because$ $x = 2$, $7a - 8b = 1$).

If $7a - 8b$ is negative, there is at least one value of $x$ violating the inequation ( $\because x = \frac{1}{2}$, $7a - 8b = -1$).

$7a - 8b$ can be neither positive nor negative.

$7a - 8b = 0$

$\frac{a}{b} = \frac{8}{7}$.

**16.** $A^{\frac{1}{B}} = B^{\frac{1}{C}} = C^{\frac{1}{A}}$. Let each of these be $= K$.

Then $A = K^B$, $B = K^C$ and $C = K^A$.

$\therefore A.B.C. = K^{B+C+A} = K^{A+B+C}$

$\therefore K = (A.B.C)^{\frac{1}{A+B+C}}$

**17.** Let $\sqrt[3]{12\sqrt[3]{12\sqrt[3]{12...\infty}}} = y$ (1)

$\sqrt[3]{12y} = y$

Cubing both sides, $12y = y^3$

$\therefore y = \sqrt{12} = 12^{\frac{1}{2}}$ ( $\because y$ is positive).

$\therefore 12^{\frac{1}{2}} = 144^x = (12^2)^x \quad 12^{\frac{1}{2}} = 12^{2x}$

Equating the powers of 12 on both sides,

$\frac{1}{2} = 2x \Rightarrow \frac{1}{4} = x$

**18.** Let the value of the expression $a^{a^{a^{a....}}}$ be $b$.

$\therefore a^b = b \Rightarrow b \log_e a = \log_e b$, as $a = e^{\frac{1}{e}}$, $b(1/e) = \log_e b$

By inspection, $b = e$ satisfies this equation.

**19.** $\sqrt{\sqrt{192} - \sqrt{180}} = \sqrt{\sqrt{3}\left(\sqrt{64} - \sqrt{60}\right)} = 3^{1/4}\sqrt{8 - 2\sqrt{15}}$

$= 3^{1/4}\sqrt{5 + 3 - 2\sqrt{5 \times 3}} = 3^{1/4}\left(\sqrt{5} - \sqrt{3}\right)$

$\sqrt{5\sqrt{3}} - \sqrt{3\sqrt{3}}$

**20.** $x = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$

$= \frac{3 + 1 - 2\sqrt{3}}{3 - 1}$

$x = 2 - \sqrt{3}$

$x^4 = (2 - \sqrt{3})^4 = (4 + 3 - 4\sqrt{3})^2 = (7 - 4\sqrt{3})^2$

$= 49 + 48 - 56\sqrt{3} = 97 - 56\sqrt{3}$

Similarly, $\frac{1}{x} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = \frac{\left(\sqrt{3} + 1\right)^2}{2} = 2 + \sqrt{3}$

$\frac{1}{x^4} = (2 + \sqrt{3})^4 = 97 + 56\sqrt{3}$

$x^4 + \frac{1}{x^4} = 97 - 56\sqrt{3} + 97 + 56\sqrt{3} = 194$

**21.** $\sqrt{\frac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{2} - 2\sqrt{3}}} = \sqrt{\frac{(3\sqrt{2} + 2\sqrt{3}) \times (3\sqrt{2} + 2\sqrt{3})}{(3\sqrt{2} - 2\sqrt{3}) \times (3\sqrt{2} + 2\sqrt{3})}}$

$= \sqrt{\frac{(3\sqrt{2} + 2\sqrt{3})^2}{(3\sqrt{2})^2 - (2\sqrt{3})^2}} = \frac{3\sqrt{2} + 2\sqrt{3}}{\sqrt{18 - 12}}$

$= \frac{3\sqrt{2} + 2\sqrt{3}}{\sqrt{6}} = \sqrt{3} + \sqrt{2}$

**22.** $\sqrt[4]{49 - 20\sqrt{6}} = \sqrt[4]{49 - 2\sqrt{600}} = \sqrt[4]{25 + 24 - 2\sqrt{25.24}}$

$= \sqrt[4]{(5 - 2\sqrt{6})^2} = \sqrt{5 - 2\sqrt{6}}$

$= \sqrt{(\sqrt{3} - \sqrt{2})^2} = \sqrt{3} - \sqrt{2}$

**23.** Let $(9\sqrt{3} + 11\sqrt{2})^{1/3} = \sqrt{x} + \sqrt{y}$

Cubing on both sides, we get

$9\sqrt{3} + 11\sqrt{2} = x\sqrt{x} + y\sqrt{y} + 3x\sqrt{y} + 3y\sqrt{x}$

$= \sqrt{x}\ (x + 3y) + \sqrt{y}\ (3x + y)$

Let $x + 3y = 9$ (1)

$3x + y = 11$ (2)

Solving (1) and (2), we get $x = 3$, $y = 2$

$\sqrt[3]{9\sqrt{3} + 11\sqrt{2}} = \sqrt{3} + \sqrt{2} = \sqrt{a} + \sqrt{b}$

Given that $a > b$

$\therefore a = 3$ and $b = 2$

**24.** Let $\left(8 + 3\sqrt{7}\right)^{x^2 - 3} = a$, then

$\left(8 - 3\sqrt{7}\right)^{x^2 - 3} = \frac{1}{a} \Rightarrow a + \frac{1}{a} = 16 \Rightarrow a^2 - 16a + 1 = 0$

$\Rightarrow a = \frac{16 \pm \sqrt{256 - 4}}{2} = 8 \pm 3\sqrt{7}$

$\left(8 + 3\sqrt{7}\right)^{x^2 - 3} = \left(8 \pm 3\sqrt{7}\right) = \left(8 + 3\sqrt{7}\right)^{\pm 1}$

$x^2 - 3 = \pm 1 \Rightarrow x = \pm 2, \pm\sqrt{2}$

**25.** Let $(5\sqrt{2} + 7)^{\frac{1}{3}} = a$ and $(5\sqrt{2} - 7)^{\frac{1}{3}} = b$

$5\sqrt{2} + 7 = a^3$ and $5\sqrt{2} - 7 = b^3$

$ab = ((5\sqrt{2} + 7)(5\sqrt{2} - 7))^{\frac{1}{3}} = ((5\sqrt{2})^2 - 7^2)^{\frac{1}{3}}$

$(50 - 49)^{\frac{1}{3}} = 1$

$a^3 + b^3 = 10\sqrt{2}$.

$10\sqrt{2} = (a + b)^3 - 3ab\,(a + b)$

$\therefore 10\sqrt{2} = (a + b)^3 - 3(a + b)$

If $a + b$ is rational, R.H.S would be rational. But L.H.S. is irrational.

$\therefore a + b$ cannot be rational.

$\therefore a + b$ must be irrational.

## EXERCISE-5

**1.** From Statement I, $x^y = 16$.
So if $x = 2$, then $y = 4$ and if $x = 4$, then $y = 2$.
If $x = 2$, then $(2x)^y = 4^4 = 256$
If $x = 4$, then $(2x)^y = 8^2 = 64$
So, Statement I alone is not sufficient.
From Statement II, $2x = 8 \Rightarrow x = 4$.
Combining both the Statements, $y = 2$.
$(2x)^y = (8)^2 = 64$.

**2.** $\left(4^{\frac{1}{gr}}\right)^{p^2} \times \left(4^{\frac{1}{pq}}\right)^{r^2} \times \left(4^{\frac{1}{pq}}\right)^{q^2}$

$= 4^{\frac{p^2}{qr}} \times 4^{\frac{r^2}{pq}} \times 4^{\frac{q^2}{pq}} = 4^{\frac{p^2}{qr}+\frac{r^2}{pq}+\frac{q^2}{pq}} = 4^{\frac{p^3+r^3+q^3}{pqr}}$

From Statement I, we know $\frac{p^3+r^3+p^3}{pqr} = 3$
I is sufficient.
Using Statement II, we get $p + q + r = 0$
When $p + q + r = 0$, it follows that

$$\frac{p^3+r^3+p^3}{pqr} = 3$$

II is sufficient.
Either of the Statements is sufficient.

**3.** $a^{3b} = (a^3)^b$
$(a+1)^{2b} = ((a+1)^2)^b$
The greater of $a^{3b}$ and $(a+1)^{2b}$ is the greater of $a^3$ and $(a+1)^2$
The first few values of $a$, $a^3$, $(a+1)^2$ are tabulated below.

| $a$ | $a^3$ | $(a+1)^2$ |
|---|---|---|
| 1 | 1 | 4 |
| 2 | 8 | 9 |
| 3 | 27 | 16 |

We see that $a^3$ may be less or more than $(a+1)^2$.
Statement I and II are not significantly different. Combining them results in Statement II itself.
$\therefore$ Even the combination is not sufficient.

**4.** Using Statement I, we know that
If $b = 0$, $a = 4$ and if $b = 2$, $a = 3$.
$\therefore$ $a + b$ is not unique
I is not sufficient.
Using Statement II, if $a \geq 3$, $8^b$ would be negative which is not possible. $\therefore$ $a < 3$.
If $a = 0$, or 1, $b$ would not be an integer.
$\therefore$ $a = 2$, $\therefore$ $b = 2$
II is sufficient.

**5.** As $P$ and $Q$ are given in different Statements, we cannot answer the question using either Statement.
Using both Statements, we get

$P^6 = \left(\sqrt[3]{a^3+b^3}\right)^6 = (a^3+b^3)^2 = a^6 + b^6 + 2a^3 b^3$

$Q^6 = \left(\sqrt{a^2+b^2}\right)^6 = (a^2+b^2)^3 = a^6 + b^6 + 3\ a^2 b^2(a^2+b^2)$

$P^6 - Q^6 = 2a^3 b^3 - 3a^2 b^2(a^2+b^2)$
$= a^2 b^2(2ab - 3(a^2+b^2)) = a^2b^2(-3(a-b)^2 - 4\ ab)$
$= a^2 b^2$ (A number which is negative)
$\therefore$ $P^6 - Q^6$ is negative.
Both statements together are required to answer the question.

**6.** From Statement I, we have $27q = 64P$. Multiplying it by 2, we get $54q = 2^{6p}$ (2), i.e., $54q = 2^{6p+1}$.
$\Rightarrow 2^{6p+1} = 2^p$
$\therefore$ $p = -1/5$
Hence, Statement I alone is sufficient
From Statement II, we have $108q = 2^{4(p+1)}$
We know then $54q = 2^p \Rightarrow 108 = 2^{p+1}$
$\therefore 4(p+1) = p+1 \Rightarrow p = -1$
$\therefore$ Hence, Statement (II) alone is sufficient.

**7.** From Statement I, we have $3^{4x+y} = 3^5$
i.e., $4x + y = 5$
We cannot find $x$ and $y$.
From Statement II, we have
$2^{8y+x} = 2^9$
i.e., $8y + x = 9$
We can not find $x$ and $y$
Using both the Statements, we can find $x$ and $y$

**8.** We know that $a$ and $b$ are positive integers.
From Statement I,
we have $5^a + 2^{b+2} = 253$
This is possible only when $a = 3$ and $b = 5$.
$\therefore$ $a + b$ can be found uniquely.
From Statement II, we have
$5^{a+1} + 2^{b-2} = 633$
This is possible only for $a = 3$ and $b = 5$.

**9.** From Statement I, $x < 5$.
But, we do not know $y$, so I alone is not sufficient.
From Statement II, $y = 99$.
The lower limiting value of $x + 2\sqrt{y}$ is

$0 + 2\sqrt{y} = 2\sqrt{299} \approx 20$, while

$5 + \sqrt{108} < 5 + 11 = 16$.

Hence, Statement II alone is sufficient to say that
$x + 2\sqrt{y} > 5 + \sqrt{108}$.

**10.** Using Statement I alone, we know that $a > b$. But we do not know about the indices. So, we cannot say $a^d > b^c$.
Statement I alone is insufficient.

From Statement II, $c$ is the AM of $(a, b)$ while $d$ is a GM of $(a, b)$. $\therefore c > d$

But as we do not know anything about $a$ and $b$, we cannot compare $a^d$ and $b^c$.

We now consider both Statements together.

$a > b$, $d =$ GM, $c =$ AM

We have to compare the greater raised to GM and the smaller raised to AM.

Let $b$, $br$ and $br^2 = a$ be in GP, where $r > 1$.

We have to compare $(br^2)^{br}$ and $b^{\frac{b(1+r^2)}{2}}$

or $(br^2)^{2r}$ and $b^{1+r^2}$

Lets consider only $r = 2$.

We need to compare $(4b)^4$ and $b^5$ or $4^4$ and $b$.

Clearly, one quantity is the greater if $b < 4^4$ and the other if $b > 4^4$.

**11.** Statement I gives, $5^{\frac{1}{a}} = 7^{\frac{1}{b}}$

But as there is no information about $c$, Statement I alone is insufficient.

From Statement II alone, we have $5^{\frac{1}{c}} = 7^{\frac{c-b}{b-c}}$. This alone is insufficient as there is no information about $a$. Combining the two Statements, we get the following result.

I $\Rightarrow 5^b = 7^a$ and II $\Rightarrow 5^b = 7^{c-b}$

$\therefore a = c - b \Rightarrow a + b = c$.

**12.** Statement I alone doesn't give any new information as the value of $AB$ which is given in I can be calculated from the information given in the question. Therefore, we cannot find the value of $A$.

From Statement II alone,

$\frac{1}{A} + \frac{1}{B} = K$ (say the given number)

$A + B = KAB$

($AB$ can be calculated from the data given in the questions)

So, we have the sum and product of the two numbers.

We will get a quadratic equation for $a$ (or $b$) and normally two possible values for each of $a$ and $b$. If the sum of two number $(a, b)$ is $s$ and the product is $P$,

$a = \frac{s + \sqrt{s^2 - 4p}}{2}, b = \frac{s - \sqrt{s^2 - 4p}}{2}$ or vice versa. We would expect that $a$ is not uniquely determined. But, we have to make sure that $s^2 - 4p \neq 0$. Otherwise, $a$ would be uniquely determined. Therefore, we need to compute $s^2 - 4p$

$s = A + B = 7 + 3\sqrt{2} - \frac{2}{3}\sqrt{2}$

and $4p = 4AB = \left(\frac{1}{7}\right)\left[15 - 3\sqrt{6} - 9\sqrt{2} + 5\sqrt{3}\right]$

The rational part of $(A + B)^2 = 49 + 18 + \frac{4}{3}$. As this is not equal to the rational part of $4AB$, $(A + B)^2 \neq 4AB$. And consequently $A$ is not uniquely determined.

Therefore, II alone is not sufficient. As I is redundant and II is insufficient, I and II together also are not sufficient.

**13.** From Statement I, we have,

$$\frac{1}{x}\left\{\frac{1}{\sqrt{3}+1} + \frac{1}{\sqrt{5}+\sqrt{3}} + \ldots \frac{1}{\sqrt{49}+\sqrt{47}}\right\} = p$$

$$= \frac{1}{x}\left\{\sqrt{3} - 1 + \sqrt{5} - \sqrt{3} + \sqrt{7} - \sqrt{5} + \ldots\right\}(1/2)$$

$$= \frac{1}{x}\{7 - 1\}(1/2) = \frac{1}{x}(3) = p$$

As $p$ is an integer,

$\therefore x = 1, 3$ are possible values of $x$

$\therefore$ Statement I alone is insufficient

From the Statement II, $x = 3^n$.

$x = 1, 3, 9 \ldots$, etc., are the possibilities. Hence, Statement II alone is insufficient.

Combining the two Statements, $\frac{3}{x} = p$. As $p$ is a positive integer, both 1 and 3 satisfy the given conditions.

Hence, both I and II together also are not sufficient.

**14.** From Statement I,

$$\frac{p^s}{q^{2s}} = \frac{3^3}{2^6} = \frac{(27)^1}{8^2}$$

i.e., $$\frac{p^s}{q^{2s}} = \frac{3^3}{2^6} = \frac{(27)^1}{8^2} = \frac{\left(3x^2\right)^3}{(2x)^6} = \frac{\left(27a^2\right)^1}{(8a)^2}$$

$\therefore p = 3x^2$ or $27a^2$, where $x$ and $a$ can be any numbers, i.e., $p$ can't be determined. Similarly, $q$ can't be determined. But as we want the index in the denominetor to be twice the index in the numberator, these indices can only be 6, 3 or 2, 1 respectively, i.e., the index $s$ can be determined to this extent (For example, we can't make these indices 8 and 4. To make the index in the denominator 8, we can multiply $2^6$ with $2^2x^4$. At the same time, as we want to make the index in the numerator 4, we can multiply $3^3$ with $3x^4$. But $2^2x^4$ can never be equal to $3x^4$)

From Statement II alone, we cannot find the value of $s$ as $p$ is not known.

Using both the Statements, it follows that $s = 3$ and $q = 2$.

$\therefore$ we can find the value of $s$, by using both Statements but not by using either one alone.

**15.** From Statement I alone, $2^{6x-3y} \times 3^{4x-2y} = 2^{18} \times 2^{36} \times 3^{36}$

$\Rightarrow (2^3)^{2x-y} \times (3^2)^{2x-y} = 2^{54} \times 3^{36}$

$\Rightarrow (72)^{2x-y} = [2^3 3^2]^{18} = 72^{18}$

$2x - y = 18$

Hence, I alone is sufficient.
From Statement II alone, we cannot find the value of $y - 2x$.

**16.** $x$ is an integer if the denominator is a factor of the numerator.

Using Statement I, simplifying the expression, we get $x = \frac{b^6}{a^1 \cdot c^{35}}$. Since we do not know the value of $c$ or its relationship with $b$, we cannot find whether $x$ is an integer or not.
Using Statement II and simplifying the expression, we get
$x = (a^{51})(b^{55})(c^{57})$, which is always an integer.
Hence, Statement II alone is sufficient.

**17.** From Statement I, we have

$$\frac{x}{y} = \frac{w}{z} = \frac{x+w}{y+z}$$

But since we do not now the value of $x + w/y + z$, Statement I alone is insufficient

From Statement II $\frac{w+x}{y+z} = \frac{1}{2}$

This alone is insufficient.
Combining both the Statements, we get

$$\frac{x}{y} = \frac{w}{z} = \frac{x+w}{y+z} = \frac{1}{2}$$

Similarly, $\frac{x^n}{y^n} = \frac{w^n}{z^n} = \frac{x^n + w^n}{y^z = z^n} = \frac{1}{2^n}$

$$\therefore \left(\frac{w^n + x^n}{y^n + z^n}\right)^{\frac{2}{n}} = \left(\frac{1}{2^n}\right)^{\frac{2}{n}} = \frac{1}{2^2} = \frac{1}{4}$$

**18.** $\sqrt[x]{\sqrt[4]{\sqrt[3]{\sqrt[x]{2^{9(216)}}}}} = \left(2^{9\times 216}\right)^{\frac{1}{2}\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)\left(\frac{1}{x}\right)} = 2^{\frac{81}{x}}$

If $x$ is a factor of 81, then $2^{\frac{81}{x}}$ is an integer.
From Statement I, $x$ is a multiple of 10.
Hence, $2^{\frac{81}{x}}$ is not an integer.
Hence, Statement I alone is sufficient.
From Statement II, $x = 3^n$. Since, it is known that $x < 100$, the possible values of $n$ are 1, 2, 3 and 4.
$\therefore$ $x = 3, 9, 27, 81$ only.
Hence, $2^{\frac{81}{x}}$ is definitely an integer.
Hence, Statement II alone is also sufficient.

**19.** From Statement I, we have $4^{x+2} + (729)^{y-4} = 1025$
This is possible only for $2^{10} + 3^0$, i.e., $4^5 + 729^0$
$\therefore$ $y - 4 = 0$
$\Rightarrow$ $y = 4$ and $x = 3$
$\therefore$ Statement I alone is sufficient.
From Statement II, we have $3^{2x}2^y = 324 = 2^2 3^4$
$y = 2$, $x = 2$
Hence, Statement II alone is sufficient.

**20.** Simplifying $\frac{\left(a^3\right)^2 b}{a^2}$, we get $\frac{a^6 b^3}{a^2} = a^4 b^3$
From Statement I, we have $(ab)^3 a = 3/4a$
$\Rightarrow (ab)^3 = 3/4a \Rightarrow a^4 b^3 = 3/4$
$\Rightarrow a^4 b^3 = 3/4$
Hence, Statement I alone is sufficient.
From Statement II, we have

$$b = \frac{1}{2a^{\frac{4}{3}}} \Rightarrow a^{4/3} = 1/2b \Rightarrow a^4 b^3 = 1/8$$

$\therefore$ Statement II alone is also sufficient.

# 3 Logarithms

CHAPTER

## Learning Objectives

*In this chapter, you will*:

- Learn all about logarithms
- Learn about characteristic and mantissa of logarithm
- Learn about various rules of logarithms

In the equation $a^x = N$, we are expressing $N$ in terms of $a$ and $x$. The same equation can be re-written as, $a = N^{1/x}$. Here, we are expressing $a$ in terms of $N$ and $x$. But, among $a$, $x$ and $N$, by normal algebraic methods known to us, we cannot express $x$ in terms of the other two parameters $a$ and $N$. This is where logarithms come into the picture. When $a^x = N$, then we say $x$ = logarithm of $N$ to the base $a$ and write it as $x = \log_a N$. The definition of logarithm is given as: '**The logarithm of any number to a given base is the index or the power to which the base must be raised in order to equal the given number**'.

Therefore the formula can be written as:

**FORMULA**

If $a^x = N$, then $x = \log_a N$

This is read as 'log $N$ to the base $a$'.

In the above equation, $N$ is a **positive number** and $a$ is a **positive number other than 1**.

This basic definition of logarithm is very useful in solving a number of problems on logarithms.

Example of a logarithm: $216 = 6^3$ can be expressed as $\log_6 216 = 3$.

Since logarithm of a number is a value, it will have an 'integral' part and a 'decimal' part. The integral part of the logarithm of a number is called **Characteristic** and the decimal part of the logarithm is called **Mantissa**.

Logarithms can be expressed to any base (positive number other than 1). Logarithms in one base can be converted to logarithms in any other base. (One of the formulae given below will help do this conversion.) However, there are two types of logarithms that are commonly used.

1. **Natural logarithms or Napierian logarithms:** These are logarithms expressed to the base of a number called '$e$'.
2. **Common logarithms:** These are logarithms expressed to the base 10. For most of the problems under logarithms, it is common logarithms that we deal with. In examinations also, if logarithms are given without mentioning any base, then it can be taken to be logarithms to the base 10.

Students should remember the following points regarding common logarithms:

1. The characteristic of the common logarithm of a number greater than unity is positive and is one less than the number of digits in its integral part. For example, the characteristic of log 245 will be 2 (because the number has 3 digits and the characteristic should be one less than the number of digits in the number).

   Similarly, the characteristic of log 4758 will be 3.
2. The characteristic of the common logarithm of a number between 0 and 1 is negative and its magnitude is one more than the number of zeroes immediately after the decimal point.

   For example, the characteristic of log 0.0034 will be –3 or $\bar{3}$.
3. The mantissas are the same for the logarithms of all numbers which have the same significant digits in the same order. The values of mantissas, which are necessary to solve a problem, are usually given in the problem itself, as a part of the data.

   Let us look at the value of log 0.02 given that the value of log 2 is 0.3010.

   The characteristic of log 0.02 will be $\bar{2}$. The mantissa will be the same as that for log 2. Hence, the value of log 0.02 is $\bar{2}$.3010. Here, the mantissa 0.3010 is positive while the characteristic is negative. But, the same can be written with a negative mantissa, in which case, the characteristic will be –1. Let us see how to do this conversion.

   $\bar{2}.3010 = -2 + 0.3010$
   $= -2 + 1 - 1 + 0.3010$ (by adding and subtracting 1)
   $= \{-2 + 1\} + \{-1 + 0.3010\}$
   $= -1 + (-0.6990) = -1.6990$

So, the value of log 0.02 can be written as $\bar{2}$.3010 or as –1.6990 and both are the same.

Similarly, given that the value of log 3 is 0.4771, we can find out the value of log 0.003. Since there are two zeroes in this number immediately after the decimal point, the characteristic is $\bar{3}$ and the mantissa is positive and the same as that for log 3. Hence, the value of log 0.003 is $\bar{3}$.4771. This can also be written as –2.5229 (you should do this conversion of $\bar{3}$.4771 into –2.5229 in the same way as we did for log 0.02).

Given below are some important rules in logarithms:

**NOTE**

(i) $\log_a a = 1$ (logarithm of any number to the same base is 1)
(ii) $\log_a 1 = 0$ (log of 1 to any base other than 1 is 0)
(iii) $\log_a (mn) = \log_a m + \log_a n$
(iv) $\log_a (m/n) = \log_a m - \log_a n$
(v) $\log_a mp = p \times \log_a m$
(vi) $\log_a b = \frac{1}{\log_b a}$
(vii) $\log_a m = \frac{\log_b m}{\log_b a}$
(viii) $\log_{a^q} m^p = \frac{p}{q} \log_a m$
(ix) $a^{\log_a N} = N$
(x) $a^{\log b} = b^{\log a}$

You should memorise these rules because they are very helpful in solving the problems.

As in the chapter on Indices, in logarithms also there will be problems on:

1. Simplification using the rules listed above
2. Solving for the value of an unknown given in an equation:

The following rules also should be remembered while solving problems on logarithms:

Given an equation $\log_a M = \log_b N$,

**NOTE**

(i) If $M = N$, then $a$ will be equal to $b$; if $M \neq 1$ and $N \neq 1$.
(ii) If $a = b$, then $M$ will be equal to $N$.

Please note that unless otherwise specified, all the logarithms are taken to the base 10.

## SOLVED EXAMPLES

**3.01.** Evaluate $\log_{10} 200 + \log_{10} 40 + 2 \log_{10} 25$.

**Sol:** $\log_{10} 200 + \log_{10} 40 + 2\log_{10} 25$
$= \log_{10} (2 \times 100) + \log_{10} (4 \times 10) + 2 \log_{10} 5^2$
$= \log_{10} 2 + \log_{10} 10^2 + \log_{10} 2^2 + \log_{10} 10 + 4 \log_{10} 5 = 3 \log_{10} 2 + 3 + 4 \log_{10} 5$
$= 3(\log_{10} 2 + \log_{10} 5) + 3 + \log_{10} 5$
$= 3(\log_{10} 10) + 3 + \log_{10} 5 = 6 + \log_{10} 5$

**Alternate method:**

As $\log_{10} 200 + \log_{10} 40 + \log_{10} 25^2$ is in the form of $\log m + \log n + \log p$, it can be written as $\log (m \times n \times p)$.

$\log_{10} (200 \times 40 \times 25 \times 25) = \log_{10} (5 \times 10^6)$
$= \log_{10} 10^6 + \log_{10} 5 = 6 \log_{10} 10 + \log_{10} 5$
$= 6 + \log_{10} 5$

**3.02.** Simplify the following expression and express the answer in terms of log 5: $\log_{10}300 + 2\log_{10}243 - 3\log_{10}81 + \log_{10}48 - \log_{10}800$.

**Sol:** As $\log m + \log n = \log mn$ and $\log m - \log n = \log (m/n)$, we have $\log_{10}300 + \log_{10}243^2 - \log_{10}81^3 + \log_{10}(2^4 \times 3) - \log_{10}800$.

$$= \log_{10}\left[300 \times \left(3^5\right)^2 \times 2^4 \times 3\right] - \log_{10}[81^3 \times 800]$$

$$= \log_{10}\left[\frac{3 \times 100 \times 3^{10} \times 2^4 \times 3}{3^{12} \times 8 \times 100}\right] = \log 10^2$$

$$= \log_{10}{}^{(10/5)} = \log_{10}10 - \log_{10}5 = 1 - \log_{10}5$$

**3.03.** Solve for $x$: $\log_{10} 80x = 6$.

**Sol.** $\log_{10}80x = 6$

By the definition of logarithm,

$80x = 10^6 = 10 \times 10^5$

$\Rightarrow 8x = 10^5 = 1000 \times 10^2$

$\Rightarrow x = 125 \times 10^2 = 12500$

**3.04.** Solve for $x$: $\log 4x - \log 8 = \log 12$.

**Sol:** Since no base is given, we take the base as 10.

$\therefore \log_{10} 4x - \log_{10} 8 = \log_{10}12$

$\Rightarrow \log_{10} 4x = \log_{10} 8 + \log_{10} 12 = \log_{10} 96$

As the logarithms on both sides are to the same base and are equal, we have $4x = 96$, Hence, $x = 24$.

**3.05.** Solve for $x$: $\log (x + 1) + \log (x - 1) = \log 224$.

**Sol.** Given $\log_{10}(x + 1) + \log_{10}(x - 1) = \log_{10}224$

As the logarithms on both sides are to the same base and they are equal, we have $(x + 1)\ (x - 1) = 224$.

$\Rightarrow x^2 - 1 = 224$

$\Rightarrow x^2 = 225$

$\Rightarrow x = \pm 15$

As logarithms are not defined for negative values, the value of $x$ is 15.

**3.06.** Express $\log \left(\frac{\sqrt[3]{a^4}}{c^2b^6}\right)$ in terms of $\log a$, $\log b$ and $\log c$.

**Sol.** $\log \frac{\sqrt[3]{a^4}}{c^2b^6} = \log a^{4/3} - \log c^2b^6$

$= 4/3 \log a - (\log c^2 + \log b^6)$

$= 4/3 \log a - 2 \log c - 6 \log b$

**3.07.** Find the number of digits in $(14175)^{11}$ given that $\log 2 = 0.3010$, $\log 3 = 0.477$ and $\log 7 = 0.845$.

**Sol.** This is an example where common logarithms can be put to practical use.

Values of logarithms to the base 10 of 2, 3 and 7 are given.

Hence, the number 14175 needs to be expressed in terms of 2, 3, 7 and 10.

$14175 = 25 \times 567 = 25 \times 9 \times 63$

$= 5^2 \times 3^2 \times 3^2 \times 7 = 3^4 \times 5^2 \times 7$

$$= 3^4 \times \left(\frac{10}{2}\right)^2 \times 7$$

$$= \frac{3^4 \times 7^1 \times 10^2}{2^2}$$

Hence, $\log_{10}(14175)$

$= 4\log_{10}3 + \log_{10}7 + 2\log_{10}10 - 2\log_{10}2$

$= 4(0.477) + (0.845) + 2 - 2(0.3010) = 4.151$

Hence, $\log_{10}(14175)^{11}$

$= 11.\ \log_{10}(14175)$

$= 11 \times 4.151 = 45.661$

As the characteristic is 45, the number $(14175)^{11}$ will have $45 + 1 = 46$ digits.

**3.08.** Obtain an equation between $x$ and $y$ from the following equation without involving logarithms, $3\log x = 4 \log y + \log 5$.

**Sol.** $3\log x = 4 \log y + \log 5$

$\Rightarrow \log x^3 = \log y^4 + \log 5$

$\Rightarrow \log x^3 = \log 5y^4$

$\therefore x^3 = 5y^4$

**3.09.** Find the logarithm of $3125\ x\ \sqrt[4]{125}$ to the base of $\sqrt[4]{5}$.

**Sol.** Let $x$ be the required logarithm.

Hence, $x = \log_{\sqrt[4]{5}}\ 3125\ \sqrt[4]{125}$

By replacing the logarithm, we get $\left(\sqrt[4]{5}\right)^x = 3125\ \sqrt[4]{125}$.

$\Rightarrow 5^{x/4} = 5^5\ 5^{3/4} \Rightarrow 5^{x/4} = 5^{23/4}$

$\Rightarrow x/4 = 23/4 \Rightarrow x = 23$

**3.10.** Find the number of zeroes between the decimal point and the first non-zero digit in $(4/9)216$, given $\log 2 = 0.301$ and $\log 3 = 0.477$.

**Sol.** In common logarithms of a number less than 1, if the mantissa is expressed as a positive figure, then the magnitude of the characteristic is one more than the number of zeros after the decimal point. But, if the mantissa is also taken as negative, then the magnitude of the characteristic is equal to the number of zeroes immediately after the decimal point.

$\therefore \log (4/9)^{216} = 216 \log 4/9 = 216 \log (2/3)^2$

$= 432(\log 2 - \log 3) = 432(0.301 - 0.477)$

$= 432(-0.176) = -76.032 = \overline{77}.968$

The characteristic of the above log is $\overline{77}$.

Therefore, the number of zeroes after the decimal point but before the first non-zero digit is 76.

**3.11.** If log 2 = 0.301, find the values of log 1250, log 0.001250 and log 125000.

**Sol.** Let us write down 1250 in terms of its prime factors.

$1250 = 2 \times 625 = 2 \times 5^4$

$\log 1250 = \log (2 \times 5^4)$

$= \log 2 + \log 5^4 = \log 2 + 4 \log 5$

log 5 can be written as $\log (10/2) = 1 - \log 2$

So $\log 1250 = \log 2 + 4(1 - \log 2)$

$= 4 - 3 \log 2 = 4 - 3(0.301) = 3.097$

log 0.00125 and log 125000 will have the same mantissa as log 1250 but will have different characteristics.

log 0.00125 has a characteristic of –3 (as there are 2 zeroes after the decimal point) and log 125000 will have a characteristic of 5 (as there are six digits in 125000).

Therefore, $\log 0.00125 = \bar{3}.097$ and log 12500 = 5.097.

**3.12.** Simplify: log 315 + 4 log 25 – 6 log 9 – 3 log 49.

**Sol:** log 315 + 4 log 25 – 6 log 9 – 3 log 49

$= \log ((9)\ (7)\ (5)) + 4 \log 5^2 - 6 \log 9 - 3 \log 7^2$

$= \log 9 + \log 7 + \log 5 + 8 \log 5 - 6 \log 9 - 6 \log 7$

$= 9 \log 5 - 5 \log 7 - 5 \log 3^2$

$= 9 \log 5 - 5 \log 7 - 10 \log 3$

**3.13.** Simplify: log 700 + log 1280 + 3 log 25.

**Sol:** log 700 + log 1280 + 3 log 25

$= \log (7)(10)^2 + \log (2^7)(10) + 3 \log 5^2$

$= \log 7 + 2 + 7 \log 2 + 1 + 6 \log 5$

$= 3 + \log 7 + \log 2 + 6 (\log 2 + \log 5)$

$= 9 + \log 14$

**3.14.** Solve for $x$: $\log_{10} 20x = 4$.

**Sol:** Given that $\log_{10} 20x = 4$

$\Rightarrow 20x = 10^4 = 10000$

$\therefore x = 500$

**3.15.** Solve for $x$: $\log 3x - \log 6 = \log 12$

**Sol:** $\log 3x - \log 6 = \log 12$

$\log 3x = \log 12 + \log 6 = \log 72$

$3x = 72$

$x = 24$

**3.16.** Solve for $x$: $\log (x+3) + \log (x-3) = \log 72$

**Sol:** $\log (x+3) + \log (x-3) = \log 72$

$\log (x+3)\ (x-3) = \log 72$

$(x+3)\ (x-3) = 72$

$x^2 = 81$

$x = 9$ (If $x = -9$, then $\log (x-3)$ would be undefined.)

**3.17.** Express $\log \dfrac{\sqrt{a^3}}{b^6\ c^4}$ in terms of log $a$, log $b$ and log $c$.

**Sol:** $\log \dfrac{\sqrt{a^3}}{b^6\ c^4}$

$= \log \sqrt{a^3} - \log b^6 - \log c^4$

$= \dfrac{3}{2} \log a - 6 \log b - 4 \log c$

**3.18.** Find the number of digits in $294^{20}$ given that log 6 = 0.778 and log 7 = 0.845.

**Sol:** $\log 294^{20} = 20 \log (7^2\ 6)$

$= 20\ (2 \log 7 + \log 6)$

$= 20\ (2\ (0.845) + 0.778) = 20\ (1.69 + 0.778)$

$= 49.36$

Characteristic = 49.

$\therefore 294^{20}$ has 50 digits.

**3.19.** Obtain an equation between $x$ and $y$, without involving logarithms, if $3 \log x = 4 \log y + 5$.

**Sol:** $3 \log x = 4 \log y + 5$

$\log x^3 = \log y^4 + \log 10^5$

$\log x^3 = \log y^4 10^5$

$\therefore x^3 = 10^5\ y^4$

**3.20.** Find the value of $\log_{\sqrt[3]{2}} 32 \sqrt[3]{16}$.

**Sol:** $\log_{\sqrt[3]{2}} 32 \sqrt[3]{16} = \log_{2^{1/3}} 2^5 \left(2^4\right)^{\frac{1}{3}}$

$= \log_{2^{1/3}} \left(2^{\frac{1}{3}}\right)^{19} = 19$

**3.21.** Find the number of zeros after the decimal point in $\left(\dfrac{3}{4}\right)^{500}$, given that log 3 = 0.4771 and log 2 = 0.3010.

**Sol:** $\log \left(\dfrac{3}{4}\right)^{500} = 500 \left(\log \dfrac{3}{4}\right)$

$= 500\ (\log 3 - 2 \log 2)$

$= 500\ (0.4771 - 2\ (0.3010)) = -62.4500.$

$\therefore$ Number of zeros after the decimal point is 62.

**3.22.** If log 2 = 0.301, then find the values of log 1250, log 0.001250 and log 125000.

**Sol:** $\log 1250 = \log \dfrac{10000}{8}$

$= 4 \log 10 - 3 \log 2 = 4 - 3\ (0.3010) = 3.097$

$\log 0.001250 = \log \dfrac{1250}{10^6} = 3.097 - 6 = -2.903$

$\log 125000 = \log (1250)\ (100)$

$= \log 1250 + 2 = 5.097$

## EXERCISE-1

***Directions for questions 1 to 45:*** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Find the value of $\log_5 m^0$, where $m \neq 0$, _____.

**2.** Simplify: $\log_{(81)\,(343)}\,(189)\,(147)$.
(A) 0.5 (B) 1
(C) 1.5 (D) 2

**3.** Simplify: $\dfrac{\log_5 27}{\log_5 64}$.
(A) $\log_4 3$ (B) $\log_3 4$
(C) $\log_{64} 27$ (D) $\log_{27} 64$

**4.** If $5 = \log_3 p$, find $p$ _____.

**5.** If $\log_3 4 + \log_3 24 = \log_3 m$, find $m$ _____.

**6.** Simplify: $\log_2 48 - \log_2 6$.
(A) $\log_2 42$ (B) $\log_2 54$
(C) $\log_2 288$ (D) $\log_2 8$

**7.** Simplify: $\log_{(27)\,(16)}\,(24)\,(18)$.
(A) 4 (B) 2
(C) 1 (D) 3

**8.** What is the value of $\log_4 m^0$ where $m \neq 0$.
(A) 1 (B) 2
(C) 3 (D) 0

**9.** If $\log_3 6 + \log_3 12 = \log_3 m$, then what is the value of $m$?
(A) 9 (B) 18
(C) 3 (D) 72

**10.** $\log\left(\dfrac{x^2}{yz}\right) + \log\left(\dfrac{y^2}{zx}\right) \log\left(\dfrac{z^2}{xy}\right) =$ _____.

**11.** N is a 15-digit number. Find the integral part of $\log_{10}$N.
(A) 14 (B) 15
(C) 16 (D) 17

**12.** If $\log_{36} 49 = \dfrac{\log_{36} 7}{\log_x 6}$, then find $x$.
(A) 6 (B) $\dfrac{1}{6}$
(C) $\dfrac{1}{36}$ (D) 36

**13.** Simplify: $\log_{16} 8^4$.
(A) 4 (B) 3
(C) 2 (D) 5

**14.** Find the integral part of $\log_2 10000$ _____.

**15.** log2 + log4 + log6 =
(A) log 12 (B) log 48
(C) log 24 (D) log 36

**16.** If $4 = \log_3 p$, then what is the value of $p$?
(A) 81 (B) 27
(C) 9 (D) 243

**17.** If $\log_4 9 \cdot \log_x 2 = 1$, then what is the value of $x$?
(A) 9 (B) 1/9
(C) 1/3 (D) 3

**18.** If $2^{\log_2 3^2} = k$, then what is the value of $k$?
(A) 81 (B) 27
(C) 9 (D) 243

**19.** If $\log_9 16 = \dfrac{\log_9 4}{\log_x 3}$, then what is the value of $x$?
(A) 3 (B) 9
(C) 27 (D) 81

**20.** If $\log_x a = \log_y a$ where $a$ is a natural number and both $x$ and $y$ are greater than $a$, then which of the following is necessarily true?
(A) $x$ is always equal to $y$.
(B) $x$ is never equal to $y$.
(C) $x$ need not be equal to $y$.
(D) for any values of $x$ and $y$.

**21.** If $\log_z x = \log_z y$, where $z \neq 1$ and $x$ and $y$ are positive, then which of the following is necessarily true?
(A) $x = y$
(B) $x \neq y$
(C) $x$ need not be equal to $y$
(D) $x \geq y$

**22.** Simplify: $\log_2 96 - \log_2 3$ _____.

**23.** If $\log_p r = \log_q r$, which of the following holds true?
(A) $p = q$
(B) $p \neq q$
(C) $p$ need not be equal to $q$.
(D) $p \geq q$

**24.** If $\log_{81} 25 = k \log_3 5$, then find $k$.
(A) 2 (B) 4
(C) $\dfrac{1}{2}$ (D) $\dfrac{1}{4}$

**25.** If $5^{\log_5 7^2} = k$, find $k$ _____.

**26.** If N is a 20-digit number, then what is the integral part of $\log_{10}$N?
(A) 118 (B) 19
(C) 120 (D) 21

**27.** If $\log_a x = \log_a y$, where $a \neq 1$, $x$ and $y$ are positive, then which of the following is necessarily true?
(A) $x$ is always equal to $y$.
(B) $x$ is never equal to $y$.
(C) $x$ need not be equal to $y$.

**28.** Simplify: $\dfrac{\log_7 9}{\log_7 16}$.
(A) $\log_3 2$ (B) $\log_2 3$
(C) $\log_4 3$ (D) $\log_3 4$

**29.** What is the integral part of $\log_2 10000$?
(A) 11 (B) 12
(C) 13 (D) 14

**30.** If $\log_{16} 9 = k\log_2 3$, then what is the value of $k$?
(A) 1 (B) 2
(C) 1/2 (D) 1/4

**31.** If $\dfrac{\log x}{3} = \dfrac{\log y}{4} = \dfrac{\log z}{5}$, then $xz =$ _____ (in terms of $y$).
(A) $y^2$ (B) $2y$
(C) $y + 2$ (D) $y^2 + 1$

**32.** If $\log 171 = 2.2196$ and $\log 2 = 0.3010$, then find the value of $\log 8550$ _____.

**33.** $3 + \log_4 x = 3 \log_4 y$, then express $y$ in terms of $x$.
(A) $y = \sqrt[3]{x}$ (B) $y = 4\sqrt{x}$
(C) $y = 2\sqrt[3]{x}$ (D) $y = 4\sqrt[3]{x}$

**34.** If $\dfrac{\log a}{b-c} = \dfrac{\log b}{c-a} = \dfrac{\log c}{a-b}$, then $a^a b^b c^c$ is _____.

**35.** Find the value of:
(A) $\log_{1/9} 243 =$ _____.
(B) $\log_{2\sqrt{6}} 24 =$ _____.
(C) $\log_{0.25} 16 =$ _____.

**36.** If $\log_x 243 = 10$, then find the value of $x =$
(A) $\sqrt{3}$ (B) $\sqrt[3]{3}$
(C) $\sqrt[3]{3^2}$ (D) $\sqrt{3^3}$

**37.** Simplify $\log 108 - \log 343 + \log\left(\dfrac{49}{8}\right) + \log\left(\dfrac{7}{12}\right)$.
(A) $\log 2$ (B) $2 \log 2$
(C) $3 \log 2$ (D) 1

**38.** Simplify $(\log_{y^2} x)(\log_{z^3} y)(\log_{x^4} z)$.
(A) $\dfrac{1}{8}$ (B) $\dfrac{1}{24}$
(C) $\dfrac{1}{6}$ (D) $\dfrac{2}{3}$

**39.** The value $\log_{27} \log_4 64$ is
(A) $\dfrac{1}{2}$ (B) $\dfrac{2}{3}$
(C) $\dfrac{1}{5}$ (D) $\dfrac{1}{3}$

**40.** Solve for $x$, if $\log_9 x + \log_9 x^2 + \log_{1/9} x = \dfrac{1}{9}$.
(A) 33 (B) $3^{1/9}$
(C) $3^{2/3}$ (D) $3^{1/3}$

**41.** Simplify the following logarithm.
$\log 125 + \log 8 - \log 10 =$ _____.

**42.** **Solve** $\log_{81}\left(\dfrac{1}{81}\right)$
(A) 1 (B) −1
(C) 0 (D) 2

**43.** $\log_5 40 + \log_5 150 - \log_5 48 - 2 =$ _____.

**44.** Solve the following for $x$.
$\dfrac{\log 2401}{\log 343} = \log x$
(A) 1 (B) 10
(C) 100 (D) $\sqrt[3]{10000}$

**45.** Solve the following for $x$.
$\log x + \log 3 = \log 15$

## EXERCISE-2

***Directions for questions 1 to 45*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** $\log_2 \log_2 \log_2 \log_2 \log_2 (65536)$ equals
(A) 0 (B) 2
(C) $4 \log 2$ (D) $8 \log 2$

**2.** $\dfrac{1}{\log_2 a} + \dfrac{1}{\log_3 a} + \dfrac{1}{\log_4 a} + \ldots + \dfrac{1}{\log_{100} a}$ equals

Difficulty Level-2: Moderate

(A) $\log_{5050} a$ (B) $\log_a 5050$

(C) $\log_a(100!)$ (D) $\dfrac{1}{\log_a(100)!}$

**3.** If $\dfrac{\log_p q}{s}=\dfrac{\log_q r}{t}=\dfrac{\log_r p}{st}=1$, then which of the following cannot be a possible value of $(s, t)$?

(A) $(1, -1)$ (B) $\left(2, \dfrac{1}{2}\right)$

(C) $(0.25, -4)$ (D) $(\pi, -\pi)$

**4.** Find the value of $\dfrac{1}{1+\log_a bc}+\dfrac{1}{1+\log_b ca}+\dfrac{1}{1+\log_c ab}$.

(A) 4 (B) 1

(C) 3 (D) 0

**5.** If $\dfrac{\log a}{y-z}=\dfrac{\log b}{z-x}=\dfrac{\log c}{x-y}$, compute the value of $(a^{y+z}b^{z+x}c^{x+y})$.

(A) 0 (B) 1

(C) 2 (D) 4

**6.** Find $x$, if $\log_3 x+\log_9 x+\log_{27} x+\log_{81} x=\dfrac{25}{4}$ _____.

**7.** Find the value of $\sqrt{\dfrac{a}{b}}$, if

$\log_2(\log_2 2^{(a-b)})=2\log_2(\sqrt{a}-\sqrt{b})+1$.

**8.** Which of the following is true if $\dfrac{1}{xy}+\dfrac{1}{yz}+\dfrac{1}{xz}=\dfrac{1}{xyz}$?

(A) $\log(xy+yz+xz)=xyz$

(B) $\log\left(\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}\right)=xyz$

(C) $\log(x+y+z)=0$

(D) $\log\left(\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}\right)=\log 1$

**9.** The logarithm of a number to a certain base is 9. The logarithm of 64 times the number to a base which is 11 times the original base is 6. Find the original base.

(A) 11/6 (B) 33/7

(C) $11^1/_4$ (D) 121/4

**10.** Find $\dfrac{x}{y}+\dfrac{y}{x}$, if $\log(x+y)+\log(x-y)=2\log(x+y)-\log x-\log y-\log 2$.

(A) $x^2-y^2-1$ (B) $\dfrac{x^2-y^2-1}{2}$

(C) $2(x^2-y^2-1)$ (D) $x^2+y^2+1$

**11.** Simplify: $\log_8 6^{\log_5[\log_9{}^{10}\times\log_{10} 9]}$.

(A) 1/2 (B) 1

(C) 0 (D) 3/4

**12.** If $\log(a+b)+\log(a-b)=2\log b\sqrt{3}$, which of the following is true about $a/b$?

(A) $a/b=2$ or $1/2$ (B) $a/b=2$ or $-2$

(C) $a/b=-2$ or $1/2$ (D) $a/b=-2$ or $-1/2$

**13.** Given $p=\log_9 3$, $q=\log_{18} 9$ and $r=\log_{27} 18$, find $pqr$.

(A) 1/2 (B) 1/3

(C) 1 (D) 1/4

**14.** If $\dfrac{1}{4}\log_2 P+4\log_2 Q=1+\log_{0.00165}$, then

(A) $P^{16}=\dfrac{4}{Q}$ (B) $Q=\dfrac{16}{P^{16}}$

(C) $P=\dfrac{16}{Q^{16}}$ (D) $Q^{16}=\dfrac{8}{P}$

**15.** Given that $4\log a=\dfrac{24}{\left(\dfrac{\log 125}{\log 16}\right)\left(\dfrac{\log 10}{\log 25}\right)}$,

(all log expressions are to the base 10). Find $a$.

(A) $2^8$ (B) $2^{12}$

(C) $2^{16}$ (D) $2^{14}$

**16.** If $(abc)^x=a+b+c$ and $abc\neq 0$, then $\dfrac{1}{x}$ is

(A) $\log_{a+b+c}(abc)$

(B) $\log_{a+b+c}(ab+bc+ca)$

(C) $\log_{abc}(a+b+c)$

(D) $\log_{ab+bc+ca}(a+b+c)$

**17.** Which of the following is true, if $\dfrac{a}{\log_a abc}+\dfrac{b}{\log_b abc}+\dfrac{c}{\log_c abc}=1$?

(A) $a^a+b^b+c^c=1$

(B) $a^a\,b^b\,c^c=1$

(C) $a^{a-1}\,b^{b-1}\,c^{c-1}=abc$

(D) $a^{a-1}\,b^{b-1}\,c^{c-1}=1$

**18.** If $2^x\,(4^{y/3})\,(8^{z/4})=1$, then which of the following expressions is always equal to $a^{x^3}\sqrt[27]{(a^8)^{y^3}}\sqrt[64]{(a^{27})^{z^3}}$?

(A) $\left(\sqrt{a}\right)^{xyz}$ (B) $\left(\sqrt[3]{a}\right)^{3xyz}$

(C) $\left(\sqrt{a}\right)^{3xyz}$ (D) $(a)^{9xyz}$

**19.** Find the value of $b$, if $\log_{\sqrt[q]{y}} x^a=1$, $\log_{\sqrt[q]{y}} x^{a^{b-1}}=1$ and $a^2\neq 1$.

(A) 2 (B) $b-a=1$

(C) $b=1-a$ (D) $b=a$

**20.** If $2\log_4 y+2\log_4 z+2\log_4 x=1$, then find the value of $xyz$ _____.

**21.** If $\log_{54} 72=b$, then the value of $\log_{96} 128$ in terms of $b$ is

(A) $\dfrac{7(3b-2)}{7-16b}$ (B) $\dfrac{3b-2}{2b-1}$

(C) $\dfrac{7-16b}{7(3b-2)}$ (D) $\dfrac{2b-1}{3b-2}$

**22.** If $\log_4 a - 2\log_{64} a = 3\log_a 4$, then $a$ possible value of a is
(A) 4 (B) 1/4
(C) 1/64 (D) 16

**23.** If $\log_4(64x^2 + 640x + 1600) - \log_4(16x^2 + 64x + 64) = 2$, what could be the value of $x$?
(A) –3 (B) 2
(C) 3 (D) –1

**24.** If $16^{\log 2^x} + 4^{\log x^{16}} = 65540$, then $x$ could be
I. $\sqrt{2}$ II. $2\sqrt{2}$
III. $4\sqrt{2}$ IV. 16
(A) II or IV (B) III or IV
(C) I or II (D) I or IV

**25.** Find the value of $\sqrt[6]{0.002048}$, given that $\log 2048 = 3.3113$ and $\log 35636 = 4.55188$.
(A) 0.85634 (B) 0.64192
(C) 0.35636 (D) 0.44562

**26.** Given $\log 2 = 0.3010$ and $\log 3 = 0.4771$, solve the equation $(4^x)\,(81^{1-x}) = 50$ and find an approximate value of $x$.
(A) 0.16 (B) 0.24
(C) 0.44 (D) 0.52

**27.** How many digits are there in $(2205)^{25}$, given that $\log 2 = 0.3010$, $\log 3 = 0.4771$ and $\log 7 = 0.8450$? _____

**28.** If $\log_{(x+y)}(x-y) = a$, $\dfrac{1+a}{a}$ is equal to
(A) $\log_{(x-y)}(x^2 - y^2)$ (B) $\log_{(x+y)}(x^2 - y^2)$
(C) $\log_{(x+y)}\left(\dfrac{x^2}{y^2}\right)$ (D) $\log_{(x+y)}(x^2 + y^2)$

**29.** If $\log_p 8 = b$ and $\log_p 12 = a$, express $\log_p\left(\dfrac{3}{p}\right)^3$ in terms of $a$ and $b$.
(A) $a - b - 1$ (B) $3a - 2b - 3$
(C) $a - 2b - 3$ (D) $2a - 2b - 3$

**30.** If $(x+1)\log y + \log x + \log z = 2\log z + \log y + \log x$, which of the following is true?
(A) $x^{y^z} = xyz$ (B) $(x^y)^z = xyz$
(C) $y = z^x$ (D) $y^x = z$

**31.** Express $\log\left\{\dfrac{(x^3y^2)^{1/5}}{(z^3y^4)^{1/5}} \Big/ \dfrac{(xy^3)^{2/3}}{(x^3yz)^2}\right\}$ in terms of $\log x$, $\log y$ and $\log z$.
(A) $\dfrac{1}{15}[89\log x - 34\log y - 55\log z]$
(B) $\dfrac{1}{15}[55\log x - 89\log y + 55\log z]$
(C) $\dfrac{1}{15}[55\log x - 89\log y - 55\log z]$
(D) $\dfrac{1}{15}[89\log x - 34\log y]$

**32.** $(x^{\log y - \log z})\,(y^{\log z - \log x})\,(z^{\log x - \log y}) =$
(A) 0 (B) 4
(C) 1 (D) 3

**33.** If $a^x = b^y = c^z$ and $y = \sqrt{xz}$, find the value of $\dfrac{\log a \log c}{(\log b)^2}$.
(A) 3 (B) 1
(C) 4 (D) 5

**34.** The value of $\dfrac{1}{\log_n m} + \dfrac{1}{\log_{\sqrt{n}} m^{1/4}} + \dfrac{1}{\log_{\sqrt[3]{n}} m^{1/9}} + \cdots + \dfrac{1}{\log_{(10\sqrt{n})} m^{(1/10)}}$ is
(A) $\log_m n^{45}$ (B) $\log_n m^{1/55}$
(C) $\log_m n^{55}$ (D) $\log_m n^{1/55}$

**35.** Simplify the expression: $\dfrac{\log_a x \log_b x[1 - 2\log_a b]}{\log_a x - \log_b x}$.
(A) $\dfrac{\log a/b^2}{\log b/a}$ (B) $\dfrac{\log a(\log a - \log b^2)}{\log x(\log b - \log a)}$
(C) $\dfrac{\log b^2/a}{\log a/b}$ (D) $\log_a x \dfrac{(\log a - \log b^2)}{(\log b - \log a)}$

**36.** If $\log_{(x-y)}(x + y) = \dfrac{1}{2}$, then find the value of $\log_{(x^2-y^2)}(x^2 - 2xy + y^2)$.
(A) $\dfrac{1}{2}$ (B) 1
(C) $\dfrac{4}{3}$ (D) $\dfrac{1}{3}$

**37.** If $\dfrac{1}{3}\log[(a^3 + b^3)(a+b)] = \log(a+b)$ and $a = 2b$, then find the respective values of $a$ and $b$.
(A) 2, 1 (B) 1, 2
(C) 4, 3 (D) 4, 2

**38.** If $1/2\log_2 P + 2\log_2 Q = 1 + \log_{0.0625} 2$, then which of the following is always true?
(A) $\dfrac{P^2}{Q^8} = 8$ (B) $\dfrac{P^8}{Q^2} = 8$
(C) $P^2 Q^8 = 8$ (D) $P^8 Q^2 = 8$

**39.** If $0 < \log_{x+3}(3x-1) + \log_{3x-1}(x+3) \le 2$, then the number of possible values of $x$ is _____.

**40.** If $\log 8246 = 3.9162$, then find the number whose logarithm is –3.0838.
(A) 0.08246 (B) 0.008246
(C) 0.0008246 (D) 0.8246

**41.** What is the value of $\dfrac{\log\sqrt{p\sqrt{p\sqrt{p\sqrt{p\ldots\infty}}}}}{\log\sqrt{q\sqrt{q\sqrt{q\sqrt{q\ldots\infty}}}}}$?

(A) $\frac{1}{2}\log_q P$ (B) $\frac{1}{4}\log_q p$

(C) $\infty$ (D) $\log_q p$

**42.** If the mantissas of the common logarithm of 2548 and 6732 are 4062 and 8282, respectively, what is the common logarithm of 9th root of (0.002548/0.6732)?

(A) $\bar{1}\cdot 3976$ (B) $\bar{1}\cdot 7309$

(C) 0·3976 (D) 0·7309

**43.** If $pqr = 1$, then which of the following expressions is equal to $\log p \cdot \log q \cdot \log r$?

(A) $3\log(p+q+r)$

(B) $(1/3)[(\log p)^3 + (\log q)^3 + (\log r)^3]$

(C) $\log(p+q+r)$

(D) $\frac{1}{3}\log(p+q+r)$

**44.** If $a, b, c$ in arithmetic progression, then $\log_a x$, $\log_b x$ and $\log_c x$ will be in

(A) arithmetic progression.

(B) geometric progression.

(C) harmonic progression.

(D) None of these

**45.** If $\log_{1727}\left\{-\left[1-\left(1+\frac{1}{x^3-1}\right)\right]\right\}^{-1} = 1$, then which of the following represents the value of $x$?

(A) 10 (B) 11

(C) 12 (D) 9

## EXERCISE-3

***Directions for questions 1 to 45:*** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** If $\log_3 \log_2 \log_a (2)^{2048} = 1$, then $a$ is

(A) 64 (B) 128

(C) 32 (D) 256

**2.** If $\log(10^x+2) + \log(10^x-8) = 2\log(10^x-4)$, then find $x$.

(A) 4 log 2 (B) 3 log 2

(C) 2 log 2 (D) log 2

**3.** Find the value of $\log_{a/b}\dfrac{\left(b-\frac{1}{a}\right)}{\left(a-\frac{1}{b}\right)}$.

(A) $-1$ (B) $-3$

(C) 2 (D) $-2$

**4.** Given $k = \log_3 2$, find the value of $\frac{1}{\log_3 4} + \frac{1}{\log_{3^2} 4} + \frac{1}{\log_{3^3} 4} + \ldots + \frac{1}{\log_{3^{15}} 4}$.

(A) $60k$ (B) $\frac{60}{k}$

(C) $120k$ (D) $\frac{120}{k}$

**5.** If $\log_a 2 \log_{a/16} 2 = \log_{a/64} 2$, then find '$a$'.

I. 2 II. 4

III. 8 IV. 16

(A) I or II (B) I or III

(C) II or III (D) I or III or IV

**6.** If $1/4 \log_{10} a + 4\log_{10} b = 1 + \log_{0.0016} 5$, then which of the following holds true?

(A) $a = 10b^4$ (B) $b = 10a^4$

(C) $b = 1000a^{16}$ (D) $1000 = ab^{16}$

**7.** Given that $a = \sqrt{b} = \sqrt[3]{c} = \sqrt[4]{d} = \sqrt[5]{e}$, find $\log_a abcde$.

(A) 15 (B) $2\frac{23}{60}$

(C) $2\frac{19}{20}$ (D) $2\frac{29}{60}$

**8.** Solve for $x$, if $2\log x + \log(x^4 + 1 + 2x^2) = \log(x^2+1) + \log x^2 + 1$ (All the log expressions are to the base 10).

**9.** Given that $4 + \log_2 a = 4\log_2 b$, find $a$ in terms of $b$.

(A) $a = b^4$ (B) $a = (b/2)^4$

(C) $a = 2b^4$ (D) $a = (2b)^4$

**10.** Find the sum of the first $n$ terms of the series: $\log(a/b^2) + \log(a^3/b^4) + \log(a^5/b^6) + \log(a^7/b^8) + \ldots + \log(a^{2n-1}/b^{2n})$.

(A) $\log\left(\frac{a^n}{b^{n+1}}\right)^n$ (B) $\log\left(\frac{a^{n+1}}{b^n}\right)^n$

(C) $\log\left(\frac{a^{n-1}}{b^{n+1}}\right)^n$ (D) $\log\left(\frac{a^{n+1}}{b^{n-1}}\right)^n$

**11.** If $\log_b a + 2\log_{\sqrt{b}} a^2 + 3\log_{\sqrt[3]{b}} a^3 + \ldots + n\log_{\sqrt[n]{b}} a^n = 6084 \log_b a$, then find $n$ _____.

**12.** $P$, $Q$ and $R$ are positive numbers. $1 < P \le Q \le R$
$E_1 = \log_P Q + \log_Q R + \log_R P$
$E_2 = \log_Q P + \log_R Q + \log_P R$
Consider the following statements:
I. $E_1 - E_2$ must be non-negative.
II. $E_1 - E_2$ must be non-positive.
Which of the following is/are true?
(A) Only I (B) Only II
(C) Either I nor II (D) Neither I nor II

**13.** If $\log_b(11x - y) - \log_b(x - 11y) = \log_b\left(\frac{x}{y}\right)$, then find $\left(\frac{x}{y}\right)^3 + \left(\frac{y}{x}\right)^3$ _____.

**14.** If $p$, $q$, $r$ are positive numbers, then the value of $\left[\log pqr - \left(\log\frac{pq}{r}\right)^2 - \left(\log\frac{qr}{p}\right)^2 - \left(\log\frac{rp}{q}\right)^2\right]$ cannot be
(A) $\frac{1}{8}$ (B) $\frac{1}{2}$
(C) $\frac{3}{4}$ (D) $\frac{7}{8}$

**15.** $N_1, N_2, \ldots, N_{15}$ are positive integers such that $\log N_1, \log N_2, \ldots, \log N_{15}$ are in arithmetic progression. If $N_8 - N_4 = 3600$ and $N_7 - N_5 = 1440$, the sum of $N_1, N_2, \ldots, N_{15}$ is
(A) $15(4^{15} - 1)$ (B) $60(4^{15} - 1)$
(C) $30(2^{15} - 1)$ (D) $60(2^{15} - 1)$

**16.** $\log_2 \frac{5^{2x-1} + 3}{5^x + 7} = 2$ Find the sum of possible values of $x$.
(A) 2 (B) 15
(C) 1 (D) 20

**17.** Given $b = \frac{1}{2}\left(a^x - a^{-x}\right)$ and $c = \frac{1}{2}\left(a^x + a^{-x}\right)$ where $a > 0$. Find $c$ in terms of $b$.
(A) $\sqrt{b^2 + 1}$ (B) $\sqrt{b^2 - 1}$
(C) $-\sqrt{b^2 + 1}$ (D) $-\sqrt{b^2 - 1}$

**18.** (log tan 1°) (log tan 2°) (log tan 3°) ... (log tan 88°) (log tan 89°) = $k$, what is the value of $k$?
(A) log (cos75°)88 (B) log (sin75°)88
(C) 1 (D) 0

**19.** $N_1, N_2, \ldots N_{15}$ are positive integers such that $\log N_1, \log N_2, \ldots, \log N_{15}$ are in arithmetic progression. If $N_8 - N_4 = 3600$ and $N_7 - N_5 = 1440$, the sum of $N_1, N_2, \ldots N_{15}$ is
(A) $15(4^{15} - 1)$ (B) $60(4^{15} - 1)$
(C) $30(2^{15} - 1)$ (D) $60(2^{15} - 1)$

**20.** If $\log_{10} 252 = a$, $\log_{10} 35 = b$ and $\log_{10} 5 = c$, then express $\log_{10} 36$ in terms of $a$, $b$ and $c$.
(A) $\frac{ac}{b}$ (B) $\frac{a}{bc}$
(C) $a - b + c$ (D) $a - b - c$

**21.** If $3\log x = \frac{\log_2 8}{\log_9 16.\log_4 10}$, then find the value of $x$.

**22.** Arrange in ascending order:
$A = \log_7 2401$, $B = \log_{7\sqrt{7}} 343$, $C = \log_{\sqrt{6}} 216$,
$D = \log_2 32$.
(A) ACBD (B) BDCA
(C) DCAB (D) BADC

**23.** If $\log_y x = 8$ and $\log_{10y} 16x = 4$, then find the value of $y$.
(A) 1 (B) 2
(C) 3 (D) 5

**24.** If $\log_{10}\{1 - [1 - (1 - x^2)^{-1}]^{-1}\} - 1/2 = 1$, then which of the following is the value of $x$?
(A) 1 (B) 2
(C) 10 (D) 0

**25.** If $(\log x^3)(\log_{x/81} 3) = \log_{x/729} 3$, then the value of $x$ is
(a) 3 (b) 9
(c) 27 (d) 81
(A) $a$ or $b$ (B) $a$ or $c$
(C) $b$ or $c$ (D) $a$, $c$ or $d$

**26.** Solve for $x$: $4^{\log_2 8} + 27^{\log_{27} 81} = 144 + \log_{10} x$.

**27.** Find $m$, if $\log_2 [3\log_2 2^{2m} + 2\log_2 4] = 4$.

**28.** If $(\log 144)(\log_{12} 1/18 + \log_a 9) = \log(1/4)$, find $a$.
(A) 144 (B) $\frac{1}{12}$
(C) $\frac{1}{144}$ (D) 12

**29.** Simplify: $\log_{17} 6^{\log_7[\log_{20} 25 \times \log_{25} 20]}$.
(A) 0 (B) 1/2
(C) 3/4 (D) 1

**30.** Find the value of $x$, if
$2^{\frac{1}{\log_x 4}} \times 2^{\frac{1}{\log_x 16}} \times 2^{\frac{1}{\log_x 256}} \ldots \infty = 2$.

**31.** If $3(\log_{10} y - \log_{10} \sqrt[3]{y}) = 8\log_y 10$, find $y$.
(A) 100 (B) 1/100
(C) 10 (D) Either (A) or (B)

**32.** If $p \ge q$ and $q > 1$, the value of $\log p \frac{p^3}{q^2} + \log q \frac{q^3}{p^2}$ cannot be
(A) 1 (B) 2
(C) 2.5 (D) 1.5

**33.** Find $\sqrt[8]{0.004225}$, given $\log 4225 = 3.6258$ and $\log 50492 = 4.70322$.

(A) 0.46755 (B) 0.50492
(C) 0.70322 (D) 0.40592

**34.** If $\log_{10}24 = x$, $\log_{10}80 = y$ and $\log_{10}25 = z$, then find $\log$ 1048 in terms of $x$, $y$ and $z$.
(A) $x+y+z+3$ (B) $x+y+z-3$
(C) $x^2+y^2+z^2+4$ (D) $x^2+y^2+z^2-4$

**35.** If $\log_2\log_2\log_2\left(\sqrt{x-13}+\sqrt{x-45}\right)=1$ how many values can $x$ take?

**36.** Find the value of $x$ satisfying $x^{\log_4 9} = 1458 - 9^{\log_4 x}$
(A) 3 (B) 256
(C) 729 (D) 64

**37.** Find the number of zeroes immediately after the decimal point in $(3/4)^{500}$ given $\log 2 = 0.3010$ and $\log 3 = 0.4771$.
(A) 61 (B) 62
(C) 63 (D) 67

**38.** If $\log_4 15$, $\log_4(3x-7)$ and $\log_4\left(3^x-\frac{1}{3}\right)$ are in arithmetic progression, then the value of $x$ is equal to
(A) 2 (B) 3
(C) $\log_3 2$ (D) Either (B) or (C)

**39.** The logarithm of a number to a certain base is 8. The logarithm of 9 times the number to a base which is thrice the original base is 5. Find the original base.
(A) 3/4 (B) 3/2
(C) 6 (D) 3

**40.** If $\log_4 a+\log_{16}a+\log_{64}a+\log_{256}a = 25/3$, the find $a$.
(A) 128 (B) 16
(C) 64 (D) 256

**41.** If $\log_4(x+6) = \log_4(x-6) + 2\log_4 2$, then find $x$.
(A) 8 (B) 9
(C) 10 (D) 12

**42.** Simplify the expression: $\dfrac{\log_a x \log_b x[1-2\log_a b]}{\log_a x - \log_b x}$.
(A) $\dfrac{\log a/b^2}{\log b/a}$ (B) $\dfrac{\log a}{\log x}\dfrac{(\log a-\log b^2)}{(\log b-\log a)}$
(C) $\dfrac{\log b^2/a}{\log a/b}$ (D) $\log_a x\dfrac{(\log a-\log b^2)}{(\log b-\log a)}$

**43.** Find the value of $\log\sqrt{512}$ to the base 0.3125.
(A) $\dfrac{2+\log_8 2}{\log_8 5^{-1}}$ (B) $\dfrac{\log_2 2^{9/2}}{\log_2\frac{5}{16}}$
(C) $\dfrac{\log_2 2^{7/2}}{\log_2\frac{7}{16}}$ (D) $\dfrac{\log_2 2^{11/2}}{\log\frac{11}{16}}$

**44.** If $\dfrac{\log_3 a\log_3 2\log_{11}9\log_b 13}{\log_b(21-a)\log_3 13\log_{11}a} = 1$, then $a$ is
(A) 18 (B) 17
(C) 12 (D) 19

**45.** If $\log_b{}^a + 2\log_{\sqrt{b}} a^2 + 3\log_{\sqrt[3]{b}} a^3 + \ldots + n\log_{\sqrt[n]{b}} a^n = 3025\log_b a$, then find $n$.
(A) 9 (B) 11
(C) 10 (D) 12

## EXERCISE-4

***Directions for questions 1 to 20***: For the multiple choice questions, select the correct alternative from the given choices. for the non-multiple choice questions, write your answer in the space provided.

**1.** If $\log_b a + \log_b(1+a) = 0$, find the value of $a^2 - 1$.
(A) $a$ (B) $ab$
(C) $-a$ (D) $-ab$

**2.** Simplify: $x^{\log y - \log z}\times y^{\log z - \log x}\times z^{\log x - \log y}$.
(A) 0 (B) 1
(C) $xyz$ (D) $x+y+z$

**3.** If $a = b^2 = c^4 = d^6 = e^8$, then find the value of $\log_a abcde$.
(A) 21 (B) $\dfrac{49}{24}$
(C) 384 (D) $\dfrac{53}{24}$

**4.** How many digits are there in the number $1125^{225}$ given that $\log 2 = 0.3010$ and $\log 3 = 0.4771$?

**5.** If $\log_8(\log_6 x) < 0$, then which of the following is true?
(A) $1 < x < 6$ (B) $(-\infty, 6)$
(C) $(6, \infty)$ (D) None of these

**6.** If $\log_{0.9}(\log_5 x) < 0$, which of the following is true?
(A) $1 < x < 5$ (B) $(5, \infty)$
(C) $(-\infty, 5)$ (D) None of these

**7.** $\dfrac{1}{\log_2 n}+\dfrac{1}{\log_3 n}+\ldots+\dfrac{1}{\log_{100} n} =$
(A) 1 (B) $\log_{100!} n$
(C) $\log_n 100!$ (D) None of these

**8.** Find the solution set of the inequation $\log_{1/2}(x^2-6x+3) \geq 2$.

(A) $\left[\frac{1}{2}, 3-\sqrt{6}\right)\cup\left(3+\sqrt{6}, \frac{11}{2}\right]$

(B) $\left[\frac{1}{2}, 3-\sqrt{6}\right]\cup\left[3+\sqrt{6}, \frac{11}{2}\right]$

(C) $\left[\frac{1}{2}, 3-\sqrt{6}\right]\cup\left(3+\sqrt{6}, \frac{11}{2}\right]$

(D) $\left(-\infty, \frac{1}{2}\right]\cup\left[\frac{1}{2}, \infty\right)$

**9.** If $p=\log_e \frac{(fg)^2}{e^3}$, $q=\log_f \frac{(eg)^2}{f^3}$ and $r=\log_g \frac{(ef)^2}{g^3}$ where $e$, $f$ and $g$ are pairwise co-primes and $p$, $q$ and $r$ are rational numbers, then find $\frac{p}{p+2}+\frac{q}{q+2}+\frac{r}{r+2}$.

(A) 2 (B) 1

(C) 5/3 (D) 7/3

**10.** The value of $\log\left(\sqrt{6}-\sqrt{3}-2+\sqrt{2}\right)+\log\left(\sqrt{6}+\sqrt{3}+2+\sqrt{2}\right)$ is

(A) log 2 (B) 2 log 3

(C) $\frac{1}{2}\log 6$ (D) None of these

**11.** If $\log_{|p-4|}|p-3|+\log_{|p-3|}|p-4|\le 2$ and $\log_{|p-4|}|p-3|>0$, then the value of $p$ is

(A) $\frac{1}{7}$ (B) $\frac{5}{3}$

(C) $\frac{7}{2}$ (D) $\frac{3}{4}$

**12.** Let $x, y, z$ and $w$ be positive real numbers. The maximum value of $x^{\log y(\log z-\log w)}+y^{\log z(\log w+\log x)}+z^{\log w(\log x-\log y)}+w^{\log x(\log y-\log z)}$ is _____.

**13.** If $\log_3 (9x+8y)+\log_3 (9x-8y)=5$, where $x$ and $y$ are integers, find $|x|+|y|$.

(A) 8 (B) 9

(C) 10 (D) No possible value exists

**14.** If $P \ge Q \ge R \ge 1$ and $K=\log_P(\log_P Q)+\log Q(\log_Q R)+\log_R(\log_R P)$, then consider the following statements:

I. K cannot be 0.

II. K cannot be negative.

Which of the statements given above is/are true?

(A) Only I (B) Only II

(C) Both I and II (D) Neither I nor II

**15.** If $\log_{0.01}(x^2-2^x)^4 \ge \log_{0.1} 9(x-2)$, then the range of $x$ is

(A) $(2, \infty)$ (B) $(3, \infty)$

(C) $(2, 3]$ (D) None of these

**16.** $Z=\sqrt[3]{2\sqrt{13}+5}+\sqrt[3]{2\sqrt{13}-5}$ $Z$ lies between

(A) 3.4 and 3.6 (B) 3.8 and 4

(C) 3.6 and 3.8 (D) 3.2 and 3.4

**17.** If $0<A<B<1$ and $E=\log_A \frac{B}{A}+\log_B \frac{A}{B}$, which of the following is/are true?

I. E cannot be zero.

II. E cannot be negative.

(A) Only I (B) Only II

(C) Both I and II (D) Neither I nor II

**18.** If $2\log_5 (x+11)<2\log_5 x+\log_5 1.33^2$, then it follows that

(A) $x>\frac{100}{3}$ (B) $x<\frac{100}{3}$

(C) $x>\frac{50}{3}$ (D) $x<\frac{50}{3}$

**19.** If $\log_2 (5x+3y)+\log_2 (5x-3y)=10$, where $x$ and y are integers, then the number of possible values of $(x, y)$ is _____.

**20.** If $\log_{2\sqrt{3}}\left(\frac{75}{13}\right)+4\log 144\left(\frac{13}{4x}\right)\ge 2$, then it follows that $x$ is at most $\frac{\quad}{16}$.

## ANSWER KEYS

### EXERCISE-1

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. 0 | 8. (D) | 15. (B) | 22. 5 | 29. (C) | (b) 2 | 41. 2 |
| 2. (B) | 9. (D) | 16. (A) | 23. (C) | 30. (C) | (c) –2 | 42. (B) |
| 3. (A) | 10. 0 | 17. (D) | 24. (C) | 31. (A) | 36. (A) | 43. 1 |
| 4. 243 | 11. (A) | 18. (C) | 25. 49 | 32. 3.915 | 37. (C) | 44. (D) |
| 5. 96 | 12. (D) | 19. (B) | 26. (B) | 33. (D) | 38. (B) | 45. 5 |
| 6. (D) | 13. (B) | 20. (C) | 27. (A) | 34. 1 | 39. (D) | |
| 7. (C) | 14. 13 | 21. (A) | 28. (C) | 35. (a)–2.5 | 40. (B) | |

## EXERCISE-2

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (A) | 8. (C) | 15. (C) | 22. (C) | 29. (B) | 36. (C) | 43. (B) |
| 2. (C) | 9. (D) | 16. (A) | 23. (A) | 30. (D) | 37. (A) | 44. (D) |
| 3. (D) | 10. (C) | 17. (D) | 24. (D) | 31. (D) | 38. (C) | 45. (C) |
| 4. (B) | 11. (C) | 18. (C) | 25. (C) | 32. (C) | 39. 1 | |
| 5. (B) | 12. (B) | 19. (A) | 26. (A) | 33. (B) | 40. (C) | |
| 6. 27 | 13. (B) | 20. 2 | 27. 84 | 34. (C) | 41. (D) | |
| 7. 3 | 14. (D) | 21. (B) | 28. (A) | 35. (D) | 42. (B) | |

## EXERCISE-3

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (D) | 8. (3) | 15. (C) | 22. (D) | 29. (A) | 36. (D) | 43. (B) |
| 2. (A) | 9. (B) | 16. (A) | 23. (D) | 30. 2 | 37. (B) | 44. (B) |
| 3. (A) | 10. (A) | 17. (A) | 24. (C) | 31. (D) | 38. (B) | 45. (C) |
| 4. (B) | 11. (B) | 18. (D) | 25. (C) | 32. (C) | 39. (D) | |
| 5. (C) | 12. (A) | 19. (C) | 26. 10 | 33. (B) | 40. (D) | |
| 6. (D) | 13. 10582 | 20. (C) | 27. 2 | 34. (B) | 41. (C) | |
| 7. (A) | 14. (D) | 21. 3 | 28. (D) | 35. 1 | 42. (D) | |

## EXERCISE-4

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (C) | 4. 687 | 7. (C) | 10. (D) | 13. (D) | 16. (C) | 19. (4) |
| 2. (B) | 5. (A) | 8. (A) | 11. (C) | 14. (B) | 17. (C) | 20. (25) |
| 3. (B) | 6. (B) | 9. (B) | 12. (4) | 15. (C) | 18. (A) | |

## SOLUTIONS

## EXERCISE-1

***Solutions for questions 1 to 45:***

**1.** As, $m^0 = 1$, $\log_5 m^0 = 0$ ($\because \log\ 1$ to any base is zero.)

**2.** $\log_{(81)\,(343)}(189)\,(147) = \log\,(3^4)\,(7^3)\,(3^3)\,(7)\,(7^2)\,(3)$

$= \log_{(3^4.7^3)}(3^4 \cdot 7^3) = 1$

**3.** $\dfrac{\log_5 27}{\log_5 64} = \dfrac{\log_5 3^3}{\log_5 4^3} = \dfrac{3\log_5 3}{3\log_5 4}\log_4 3$

**4.** $p = 3^5 = 243$

**5.** $\log_3 4 + \log_3 24 = \log_3 96 = \log_3 m \Rightarrow m = 96$

($\because \log_m a + \log_m b = \log_m ab$)

**6.** $\log_2 48 - \log_2 6 = \log_2(48/6)$

[$\because \log_x m - \log_x n = \log_x(m/n)$]

$= \log_2 8 = \log_2 2^3 = 3\log_2 2 = 3$

**7.** $\log_{27(16)}24(18)$

$(24)\,(18) = (3)\,(8)\,(2)\,(9) = 3^3 \times 2^4$

$(27)\,(16) = (3^3)\,(2^4)$

$\therefore\ (24)\,(18) = (27)\,(16)$

$\therefore \log_{27(16)}24(18) = 1$

[$\therefore \log_a a = 1$]

**8.** $\log_4 m^0 = \log_4 1 = 0$ [$\because m^0 = 1$ and $\log 1 = 0$]

**9.** $\log_3 6 + \log_3 12 = \log_3 m \Rightarrow \log_3 6 \times 12 = \log_3 m$

[$\because \log_a x + \log_a y = \log_a x.y$]

$\Rightarrow m = (6)\,(12) = 72$

**10.** $\log\left(\dfrac{x^2}{yz}\right) + \log\left(\dfrac{y^2}{zx}\right) + \log\left(\dfrac{z^2}{xy}\right)$

$= \log\ \dfrac{x^2y^2z^2}{x^2y^2z^2} = \log\ 1 = 0$

**11.** A 15-digit number is of the form $k.1014$ where $1 \le k < 10$.

$\log_{10}(k.10^{14}) = \log_{10} k + \log_{10} 10^{14}$ As $0 \le \log_{10} k.10^{14} < 1$, $14 \le \log_{10}(k.10^{14}) < 15$

$\therefore$ integral part of $\log_{10} N$ is 14.

**Alternative Method:**

Integral part of the logarithm of any number greater than 1 to the base 10 is always 1 less than the number of digits in the number. In the given problem, as $N$ has 15 digits, integral part of $\log_{10}$: $N$ is 14.

**12.** $\log_{36}49 = \dfrac{\log 7^2}{\log 6^2} = \dfrac{2\log 7}{2\log 6} = \dfrac{\log_{36} 7}{\log_{36} 6} = \dfrac{\log_{36} 7}{\log_x 6}$ (given)

Comparing the two sides, $x = 36$.

**13.** $\log_{16} 8^4 = \log_{2^4}^{8^4} = \log_2 4\ (2^4)^3$

$= 3\ \log_{2^4} 2^4 = 3$

**14.** $2^{13} = 8192$ and $2^{14} = 16384$

$\therefore$ Integral part of $\log_2 10000 = 13$

**15.** $\log 2 + \log 4 + \log 6$

$\therefore \log 2 + \log 4 + \log 6 = \log 8 + \log 6$

$= \log 8.6 = \log 48.$

**16.** $4 = \log_3^p \Rightarrow 3^4 = p$

[$\because$ If $N = \log_a x$, then $x = a^N$.]

$\therefore p = 81$

**17.** $\log_4 9 \,.\, \log x_2 = 1 \Rightarrow \log_{2^2} 3^2 . \log_x 2 = 1$

$\Rightarrow \dfrac{2}{2}\log_2 3 \,.\, \log_x 2 = 1$

$\Rightarrow \dfrac{2}{2}\log_2 3.\log_x 2 = 1 \Rightarrow \dfrac{\log 3}{\log 2}.\dfrac{\log 2}{\log x} = 1 \Rightarrow \log 3 = \log x$

$\Rightarrow x = 3$

**18.** $2^{\log_2 3^2} = k \Rightarrow 3^2 = k$ [$\because a \log_a^N = N$] $\Rightarrow k = 9$

**19.** $\log_9 16 = \dfrac{\log_9 4}{\log_x 3} \Rightarrow \log_{3^2} 4^2 = \dfrac{\log_9 4}{\log_x 3}$

$\Rightarrow \dfrac{2}{2}\log_3 4 = \dfrac{\dfrac{\log 4}{\log 9}}{\dfrac{\log 3}{\log x}}$

$\Rightarrow \dfrac{\log 4}{\log 3} = \dfrac{\log 4.\log x}{\log 3.\log 9} \Rightarrow \log x = \log 9$

$\Rightarrow x = 9$

**20.** Given $\log_x a = \log_y a$,

If $a = 1$, $\log_x 1 = 0$ and $\log_y 1 = 0$.

$\therefore$ Irrespective of values of $x$ and $y$, $\log_x a$ is equal to $\log_y a$.

If $a = 2$, then $\log_x 2 = \log_y 2$.

$\Rightarrow \dfrac{\log 2}{\log x} = \dfrac{\log 2}{\log y}$ $\left[\because \log_b a = \dfrac{\log a}{\log b}\right]$

$\Rightarrow \log y = \log x$

$\Rightarrow y = x$, when $a \neq 1$, $x = y$.

In general, $x$ need not be equal to $y$.

**21.** Only if $x = y$, $\log_z x = \log_z$.

**22.** $\log_2 96 - \log_2 3 = \log_2 96/3$

$= \log_2 32 = 5$

**23.** If $r = 1$, $\log_p r = \log_q r = 0$, then values of $p$ and $q$ need not be equal.

**24.** $\log_{3^4} 5^2 = \dfrac{2\log 5}{4\log 3}$ $(1/2)\ \log_3 5$

$\therefore (1/2)\ \log_3 5 = k \log_3 5$ or $k = 1/2$

**25.** $5^{\log_5 7^2} = 7^2$

$\therefore k = 7^2 = 49$

**26.** Any 20-digit number lies between $10^{19}$ and $10^{20}$.

$\therefore$ The logarithm of a 20-digit number to the base 10 lies between 19 and 20.

$\therefore$ The integral part is 19.

**27.** If $\log_a x = \log_a y$, $a \neq 1$, then $x$ is always equal to $y$.

**28.** $\dfrac{\log_7 9}{\log_7 16} = \dfrac{\dfrac{\log 9}{\log 7}}{\dfrac{\log 16}{\log 7}}$ $\left[\because \log_b a = \dfrac{\log a}{\log b}\right]$

$= \dfrac{\log 9}{\log 6} = \dfrac{\log 3^2}{\log 4^2} = \dfrac{2}{2}\dfrac{\log 3}{\log 4} = \log_4 3$

**29.** 10,000 lies between $2^{13}$ and $2^{14}$.

$\therefore$ The logarithm of 10,000 to the base 2 lies between 13 and 14.

$\therefore$ The integral part is 13.

**30.** $\log_{16} 9 = k \log_2 3$

$\Rightarrow \log_{2^4} 3^2 = k \log_2^3$

$\Rightarrow \dfrac{2}{4}\log_2 3 = k \log_2 3$

$\Rightarrow \dfrac{2}{4} = k \Rightarrow k = \dfrac{1}{2}$

**31.** Let $\dfrac{\log x}{3} = \dfrac{\log y}{4} = \dfrac{\log z}{5} = k$

$\therefore \log x = 3k$, $\log y = 4k$ and $\log z = 5k$.

$\log x + \log z = 3k + 5k = 8k$

$\log (xz) = 8k = 2 \times 4k = 2\log y$

$\Rightarrow \log (xz) = \log y^2$

$\Rightarrow xz = y^2$

**32.** (a) $\log 8550 = \log 171 \times 5 \times 10 = \log 171 \times 25 \times 2$

$= \log 171 + \log 25 + \log 2$

$= 2.2196 + \log\left(\dfrac{100}{4}\right) + \log 2$

$= 2.2196 + 2 - \log 2$

$= 2.2196 + 2 - 0.3010 = 3.915$

**33.** $3 + \log_4 x = 3\log_4 y$

$\Rightarrow 3\log_4 4 + \log_4 x = 3\log_4 y$

$\Rightarrow \log_4 4^3 + \log_4 x = \log_4 y^3$

$\Rightarrow \log_4 (4^3 \times x) = \log_4 y^3$

$\Rightarrow 64x = y^3$

$\Rightarrow y = 4\sqrt[3]{x}$

**34.** Let $\frac{\log a}{b-c}=\frac{\log b}{c-a}=\frac{\log c}{a-b}=k$

$\log a=k(b-c)$, $\log b=k(c-a)$, $\log c=k(a-b)$

$a^a\,b^b\,c^c=x$

$\Rightarrow \log x=a\log a+b\log b+c\log c$

$=ak(b-c)+kb(c-a)+kc(a-b)$

$=k[ab-ac+bc-ab+ac-bc]$

$\log x=0$

$\Rightarrow x=1 \Rightarrow a^a\,b^b\,c^c=1$

**35.** (a) $\log_{1/9}243=\log_{3^{-2}}3^5=-\frac{5}{2}$

(b) $\log_{\sqrt[3]{6}}24=\log_{\sqrt[3]{2}\sqrt{3}}2^3\times3^1$

$=\log_{\sqrt{2}^3\sqrt{3}}\sqrt{2}^6\times\sqrt{3}^2$

$\Rightarrow \log_{\sqrt{2}^3\sqrt{3}}\left(\sqrt{2}^3\times\sqrt{3}^1\right)^2=2$

(or) $\log_{\sqrt[3]{6}}24=\log_{\sqrt[3]{6}}(2\sqrt{6})^2=2$

(c) $\log_{0.25}16=\log_{1/4}2^4=\log_{2^{-2}}2^4$

$=\frac{4}{-2}=-2$

**36.** Given $\log_x 243=10$

$\Rightarrow 243=x^{10}$

$\Rightarrow 3^5=x^{10}$

$\Rightarrow \sqrt{3}^{10}=x^{10} \Rightarrow x=\sqrt{3}$

**37.** $\log\left(\frac{108\times49\times7}{343\times8\times12}\right)=\log\left(\frac{9}{8}\right)=2\log3-3\log2$

**38.** $\frac{\log x}{2\log y}\times\frac{\log y}{3\log z}\times\frac{\log z}{4\log x}$

$\Rightarrow \frac{1}{2}\times\frac{1}{3}\times\frac{1}{4}=\frac{1}{24}$

**39.** $\log_{27}(\log_4 64)=\log_{27}(\log_4 4^3)=\log 3^3\,(3\log_4 4)$

$=\log_{3^3}3=\frac{1}{3}\log_3 3=\frac{1}{3}$

**40.** $\log_9 x+\log_9 x^2+\log_{1/9}x=\frac{1}{9}$

$\Rightarrow \frac{1}{2}\log_3 x+\frac{2}{2}\log_3 x+(-\frac{1}{2})\log_3 x=\frac{1}{9}$

$\Rightarrow \frac{1}{2}[\log_3 x+2\log_3 x-\log_3 x]=\frac{1}{9}$

$\Rightarrow \frac{1}{2}\log_3\left(\frac{x\times x^2}{x}\right)=\frac{1}{9}$

$\Rightarrow \log_3 x^2=\frac{2}{9} \Rightarrow x^2=3^{2/9} \Rightarrow x=3^{1/9}$

**41.** $\log 125+\log 8-\log 10$

$=\log\left(\frac{125\times8}{10}\right)=\log 100$

As the base is not mentioned, the value of the given expression is $\log_{10}100=\log_{10}10^2=2$.

**42.** $\log_{81}\left(\frac{1}{81}\right)=\log_{81}81^{-1}=-1$

**43.** $\log_5 40+\log_5 150-\log_5 48-2$

$=\log_5\left[\frac{(40)(150)}{48}\right]-2=\log_5(5^3)-2=1.$

**44.** $\frac{\log 2401}{\log 343}=\frac{\log 7^4}{\log 7^3}=\frac{4\log 7}{3\log 7}$

$\frac{4}{3}=\log_{10}x$ (since the base is not mentioned)

$x=10^{\frac{4}{3}}=\sqrt[3]{10000}$

**45.** $\log x+\log 3=\log 15$

$\log(3x)=\log 15$

Equating arguments of logarithms on both sides,

$\Rightarrow 3x=15 \Rightarrow x=5$

## EXERCISE-2

***Solutions for questions 1 to 45*:**

**1.** $\log_2\log_2\log_2\log_2\log_2 2^{16}$

$=\log_2\log_2\log_2\log_2 16$

$=\log_2\log_2\log 24$

$=\log_2\log_2 2=\log_2 1=0$

**2.** $\frac{1}{\log_2 a}+\frac{1}{\log_3 a}+\frac{1}{\log_4 a}+\ldots+\frac{1}{\log_{100} a}$

$=\log_a 2+\log_a 3\,\log_a 4+\ldots\log_a 100$

$=\log_a[(2)\,(3)\,(4)\,\ldots\,(100)]=\log_a 100!$

**3.** $\frac{\log_P q}{3}+\frac{\log_q r}{s}+\frac{\log_r p}{st}=1$

$\Rightarrow \log_p q=S$, $\log_q r=t$ and $\log_n p=st$

$\therefore (\log_p q)\,.\,(\log_q r)\,.\,(\log_n p)=(s)\,(t)\,(st)$

$\Rightarrow \frac{\log q}{\log p}\cdot\frac{\log r}{\log q}\cdot\frac{\log p}{\log r}=(st)^2 \Rightarrow 1=(st)^2 \Rightarrow st=\pm1$

Only choice (D) violates this condition.

**4.** $\frac{1}{1+\log_a bc}+\frac{1}{1+\log_a ca}+\frac{1}{1+\log_c ab}$

$=\frac{1}{1+\frac{\log bc}{\log a}}+\frac{1}{1+\frac{\log ca}{\log b}}+\frac{1}{1+\frac{\log ab}{\log c}}$

$$= \frac{\log a}{\log a + \log bc} + \frac{\log b}{\log b + \log ca} + \frac{\log c}{\log c + \log ab}$$

$$= \frac{\log a}{\log abc} + \frac{\log b}{\log abc} + \frac{\log c}{\log abc}$$

$$= \frac{\log a + \log b + \log c}{\log abc} = \frac{\log abc}{\log abc} = 1.$$

**5.** Let $\frac{\log a}{y-z} = \frac{\log b}{z-x} = \frac{\log c}{x-y} = k$

$\log a = k\,(y-z)$ (1)

$\log b = k\,(z-x)$ (2)

$\log c = k\,(x-y)$ (3)

Multiplying (1), (2) and (3) by $(y+z)$, $(z+x)$ and $(x+y)$, respectively, and adding them, we get

$(y+z)\log a + (z+x)\log b + (x+y)\log c = 0.$

$\log a^{y+z} + \log b^{z+x} + \log a^{x+y} = 0$

$\log a^{y+z} . b^{z+x} . a^{x+y} = 0$

$\Rightarrow a^{y+z} . b^{z+x} . a^{x+y} = 1.$

**6.** $\log_9 x = \log_{3^2} x = 1/2 \log_3 x$

Similarly, $\log_{27} x = 1/3 \log_3 x$

And $\log_{81} x = 1/4 \log_3 x$

$(1 + 1/2 + 1/3 + 1/4) \log_3 x = \frac{25}{4} \Rightarrow \frac{25}{12} \log_3 x = \frac{25}{4}$

$\Rightarrow \log_3 x = 3 \Rightarrow x = 3^3 = 27$

**7.** Given $\log_2\left(\log_2 2^{(a-b)}\right) = 2\log_2\left(\sqrt{a} - \sqrt{b}\right) + 1$

$\Rightarrow \log_2(a-b) = \log_2\left(\sqrt{a} - \sqrt{b}\right)^2 + 1$

$\Rightarrow \log_2(a-b) - \log_2\left(\sqrt{a} - \sqrt{b}\right)^2 = 1$

$\Rightarrow \log_2\left|\frac{a-b}{\left(\sqrt{a} - \sqrt{b}\right)^2}\right| = 1 \Rightarrow 2 = \frac{a-b}{\left(\sqrt{a} - \sqrt{b}\right)^2}$

$\Rightarrow 2 = \frac{\left(\sqrt{a} - \sqrt{b}\right)\left(\sqrt{a} + \sqrt{b}\right)}{\left(\sqrt{a} - \sqrt{b}\right)^2} \Rightarrow \frac{2}{1} = \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}}$

$\Rightarrow 2\left(\sqrt{a} - \sqrt{b}\right) = \sqrt{a} + \sqrt{b} \Rightarrow \sqrt{a} = 3\sqrt{b} \Rightarrow \frac{\sqrt{a}}{\sqrt{b}} = 3$

$\Rightarrow \sqrt{\frac{a}{b}} = 3$

**8.** Given $\frac{1}{xy} + \frac{1}{yz} + \frac{1}{xz} = \frac{1}{xyz}$

$\Rightarrow \frac{x+y+z}{xyz} = \frac{1}{xyz}$

$\therefore x + y + z = 1$

Taking log on both sides

$\log (x+y+z) = \log 1$

$\log (x+y+z) = 0$

**9.** Let the number be $x$ and the base be $y$.

$\log_y x = 9 \Rightarrow x = y^9$ (1)

$\log_{11y} 64x = 6 \Rightarrow 64x = 11^6 . y^6$ (2)

Dividing (2) ÷ (1) $\Rightarrow 64 = \frac{11^6}{y^3}$

$\Rightarrow y^3 = \frac{\left(11^2\right)^3}{(4)^3} = (121/4)^3$

$\therefore y = \frac{121}{4}$

**10.** Given $\log (x+y) + \log (x-y) = 2\log (x+y) - \log x - \log y - \log 2$

$\Rightarrow \log (x+y) + \log (x-y) = \log (x+y)^2 - [\log x + \log y + \log 2]$

$\Rightarrow \log (x^2 - y^2) = \log\left|\frac{(x+y)^2}{2xy}\right|$

$\therefore x^2 - y^2 = \frac{(x+y)^2}{2xy}$

$\Rightarrow x^2 - y^2 = \frac{x^2 + y^2 + 2xy}{2xy}$

$\Rightarrow x^2 - y^2 = \frac{x^2}{2xy} + \frac{y^2}{2xy} + \frac{2xy}{2xy}$

$\Rightarrow x^2 - y^2 - 1 = \frac{x}{2y} + \frac{y}{2x} \Rightarrow x^2 - y^2 - 1 = \frac{1}{2}\left[\frac{x}{y} + \frac{y}{x}\right]$

$\Rightarrow 2\,(x^2 - y^2 - 1) = \frac{x}{y} + \frac{y}{x}$

**11.** $\log_8 6^{\log_5\left[\frac{\log 10}{\log 9} \times \frac{\log 9}{\log 10}\right]} = \log_8 6^0 = \log_8 1 = 0$

**12.** $\log (a+b) + \log (a-b) = \log (a^2 - b^2)$

$2\log b\sqrt{3} = \log 3b^2$

$\log (a^2 - b^2) = \log (3b^2)$

$\Rightarrow a^2 - b^2 = 3b^2 \Rightarrow a^2 = 4b^2 \Rightarrow a = \pm 2b$

**13.** $pqr = \log_9 3 \log_{18} 9 \log_{27} 18$

$= (\log 3/\log 9)\ (\log 9/\log 18)\ (\log 18/\log 27)$

$= \frac{\log 3}{3\log 3} = \frac{1}{3}$

**14.** $\log_2 P^{1/4} + \log_2 Q^4 = 1 + \log_{5^{-4}} 5 = 1 - \frac{1}{4} = \frac{3}{4}$

$\log_2 P^{1/4} Q^4 = \frac{3}{4}$

$\Rightarrow P^{1/4} Q^4 = 2^{3/4} \Rightarrow PQ^{16} = 2^3.$

**15.** $4\log a = \frac{24}{\left(\frac{\log 5^3}{\log 2^4}\right)\left(\frac{\log 10}{\log 5^2}\right)} = \frac{24}{\frac{3\log 5}{4\log 2}\frac{\log 10}{2\log 5}}$

$= 64\log_{10} 2 \Rightarrow \log a = 16\log_{10} 2 = \log_{10} 2^{16}$

$\Rightarrow a = 2^{16}$

**16.** $(abc)^x = a + b + c$

Taking log on both sides, we get

$x \log (abc) = \log (a+b+c)$

$$\Rightarrow \frac{1}{x}=\frac{\log abc}{\log a+b+c}=\log_{a+b+c}(abc)$$

**17.** Given $\frac{a}{\log_a abc}+\frac{b}{\log_b abc}+\frac{c}{\log_c abc}+1=1$

$\Rightarrow a\log_{abc}a+b\log_{abc}b+c\log_{abc}c=1$
$\Rightarrow \log_{abc}a^a+\log_{abc}b^b+\log_{abc}c^c=1$
$\Rightarrow \log_{abc}a^a\,b^b\,c^c=1$
$\Rightarrow (abc)^1=a^a\,b^b\,c^c$

$\Rightarrow \frac{a^a b^b c^c}{abc}=1 \Rightarrow a^{a-1}\,b^{b-1}\,c^{c-1}=1$

**18.** Given $2^x\,2^{2y/3}\,2^{3z/4}=1 \Rightarrow 2^{\left(x+\frac{2y}{3}+\frac{3z}{4}\right)}+2^0$.

$\Rightarrow x+\frac{2y}{3}+\frac{3z}{4}=0$

$\Rightarrow x^3+\frac{8y^3}{27}+\frac{27z^3}{64}=3(x)\left(\frac{2y}{3}\right)\left(\frac{3z}{4}\right)=\frac{3}{2}xyz$

($\therefore$ if $a+b+c=0$, then $a^3+b^3+c^3=3abc$.)

$a^{\left(x^3+\frac{8y^3}{27}+\frac{27z^3}{64}\right)}=a^{\frac{3}{2}xyz}$

$\therefore a^{x^3}\,a^{\frac{8y^3}{27}}\,a^{\frac{27z^3}{64}}=a^{\frac{3xyz}{2}}=\sqrt{a^{3xyz}}$

$=\left(\sqrt{a}\right)^{3xyz}$

**19.** Given $\log_{\sqrt[q]{y}}x^a=1$

$\Rightarrow x^a=y^{1/a}$ (1)

$\log_{\sqrt[q]{y}}x^{a^{b-1}}=1$

$\Rightarrow x^{a^{b-1}}=y^{1/a}$ (2)

From (1) and (2), we get

$\therefore x^a=x^{a^{b-1}}$

As $x$ is not $-1$, 0 or 1 $a=a^{b-1}$ (3)

As the expression $\sqrt[q]{y}$ occurs in the question, $a\neq 0$, and as $a^2\neq 1$, $a$ is not 1 or $-1$ $\therefore$equating the indices on the two sides of Eq. (3), $b=2$.

**20.** Given $2\log_4(y)+2\log_4(z)+2\log_4(x)=1$

$\log_4(xyz)^2=1$
$\Rightarrow (xyz)^2=4$
$\therefore xyz=2$
($\therefore$ $xyz$ is positive.)

**21.** $\log_{54}72=b$

$\frac{\log_2 72}{\log_2 54}=b$

$\frac{\log_2 9+\log_2 8}{\log_2 27+\log_2 2}=b$

$\frac{2\log_2 3+3}{3\log_2 3+1}=b$

Let $\log_2 3=a$

$\frac{2a+3}{3a+1}=b$

$a=\frac{b-3}{2+3b}$

$\log_{96}128=\frac{\log_2 7}{\log_2 3+\log_2 2^5}=\frac{7}{b+5}=\frac{-2-3b}{1-2b}$

$=\frac{3b-2}{2b-1}$.

**22.** $\log_4 a-2\log_{64}a=3\log_a 4$

$\Rightarrow \log_4 a-2\log_{4^3}a=3\log_a 4$

$\Rightarrow \log_4 a-\frac{2}{3}\log_4 a=3\log_a 4$

$\Rightarrow \frac{1}{3}\log_4 a=3\log_a 4$

$\Rightarrow \log_4 a=\frac{9}{\log_4 a}\quad\left[\therefore \log_b a=\right]\frac{1}{\log_b a}$

$\Rightarrow (\log_4 a)^2=9=\log_4 a+\pm 3 \Rightarrow a=4^3$ or $a=4^{-3}$

$\therefore a=64$ or $1/64$

**23.** $\log_4(8x+40)^2-\log_4(4x+8)^2$

$\Rightarrow 4^2=\left(\frac{8x+40}{4x+8}\right)^2 \Rightarrow \frac{8x+40}{4x+8}=\pm 4$

Case (1):
$8x+40=-4(4x+8)$
$x+5=-(2x+4)$
$3x=-9$, $x=-3$.
Case (2):
$8x+40=4(4x+8)$
$x+5=2x+4$, $x=1$
$\therefore$The two solutions are $x=-3$, $x=1$.
Out of which, we have got $x=-3$ in the options.

**24.** $16^{\log 2^x}+4^{\log x^{16}}=65540$,

$16^{\log 2^x}+16^{\log x^4}=65536+4=16^4+16^{\frac{1}{2}}$ (1)

If $\log_2{}^x=4, \log_x{}^4=\frac{1}{2}$. If $\log_2{}^x=\frac{1}{2}$, $\log_x{}^4=4$.

In either case, (1) is satisfied.

$\therefore$ $\log_2{}^x$ can be 4 or $\frac{1}{2}$. $\therefore$ $x$ can be 16 or $\sqrt{2}$.

**25.** $\log\sqrt[6]{0.002048}=\log\left[2048(10^{-6})\right]^{1/6}$
$=(1/6)\ (\log 2048+\log 10^{-6})=(1/6)\ (3.3113-6)$
$=(1/6)\ (-6+3.3113)=\bar{1}.55188\approx\bar{1}.5519$
Given, $\log 35636=4.5519$
Hence, 0.35636 is the value of $\sqrt[6]{0.002048}$

**26.** $4^x\,.\,81^{1-x}=50$
$\Rightarrow 2^{2x}\,.\,3^{4-4x}=50$
Taking log on both sides, we get
$\log 2^{2x}+\log 3^{4-4x}=\log 50$

$\Rightarrow 2x(\log 2) + (4-4x)\log 3 = \log(100/2)$

$\Rightarrow (2x0.3010)x + (4-4x)(0.4771) = 2 - 0.3010$

$\Rightarrow 0.6020x + 1.9084 - 1.9084x = 1.699$

$\Rightarrow (1.9084 - 0.602\,0)x = 1.9084 - 1.699$

$\Rightarrow 1.3064x = 0.2094$

$\Rightarrow x = \dfrac{0.2094}{1.3064} = 0.16$

**27.** Let $N = (2205)^{25} = [(5)(441)]^{25} = [(3)^2(5)(7^2)]^{25}$
$= 3^{50}5^{25}7^{50}$
$\therefore \log N = 50\log 3 + 25\log 5 + 50\log 7$
$= 50(0.4771) + 25(1 - 0.301) + 50(0.845) = 83.58$
The characteristic is 83
$\therefore$ The given number has (83 + 1) or 84 digits.

**28.** Given $\log_{x+y} x - y = a$.

Now, $\dfrac{1+a}{a} = \dfrac{1}{a} + 1 = \dfrac{1}{\log_{x+y} x - y} + 1 = \log_{(x-y)}(x+y) + 1$

$= \log_{(x-y)}(x+y) + \log_{(x-y)}(x-y) = \log_{(x-y)}[(x+y)(x-y)]$
$= \log_{(x-y)}(x^2 - y^2)$

**29.** Given $\log_p 8 = b \Rightarrow \log_p 2 = b/3$. (1)
$\log_p 12 = a$
$\log_p 3 + 2\log_p 2 = a$
$\log_p 3 = a - 2\left(\dfrac{b}{3}\right)$ ($\because$ from Eq. (1))

$\Rightarrow \log_p 3 - 1 = a - \dfrac{2b}{3} - 1$

$\Rightarrow \log_p 3 - \log_p p = a - \dfrac{2b}{3} - 1$

$\Rightarrow \log_p\left(\dfrac{3}{p}\right) = \dfrac{(3a - 2b - 3)}{3}$

$\Rightarrow 3\log_p\left(\dfrac{3}{p}\right) = 3a - 2b - 3$

$\Rightarrow \log_p\left(\dfrac{3}{p}\right)^3 = 3a - 2b - 3$

**30.** Given $(x+1)\log y + \log x + \log z = 2\log z + \log y + \log x$.
$\Rightarrow x\log y = \log z \Rightarrow y^x = z$

**31.** Consider $\left\{\dfrac{\left(x^3y^2\right)^{1/5}}{\left(z^3y^4\right)^{2/3}} \Big/ \dfrac{\left(xh^3\right)^{2/3}}{\left(x^3yz\right)^2}\right\}$

$= \dfrac{\left(x^3y^2\right)^{1/5} \cdot \left(x^3yz\right)^2}{\left(z^3y^4\right)^{2/3}\left(xy^3\right)^{2/3}} \left[\because \dfrac{a/b}{c/d} = \dfrac{ad}{bc}\right]$

$= \dfrac{x^{3/5} \cdot y^{2/5} \cdot x^6 \cdot y^2 \cdot z^2}{z^2 \cdot y^{8/3} \cdot x^{2/3} \cdot y^2}$

$= (x)^{\frac{3}{5}+6-\frac{2}{3}} \cdot (y)^{\frac{2}{5}+2-\frac{8}{3}-2} \cdot (z)^{2-2}$

$= (x)^{\frac{89}{15}} \cdot (y)^{\frac{-34}{15}} \cdot (z)^0 = \left(x^{89} \cdot y^{-34}\right)^{1/15}$

$\therefore \log \left\{\dfrac{\left(x^3y^2\right)^{1/5}}{\left(z^3y^4\right)^{2/3}} \Big/ \dfrac{\left(xh^3\right)^{2/3}}{\left(x^3yz\right)^2}\right\}$

$= \log\left(x^{89} \cdot y^{-34}\right)^{1/15} = \dfrac{1}{15}\log\left(x^{89} \cdot y^{-34}\right)$

$= \dfrac{1}{15}\left[\log x^{89} \cdot \log y^{-34}\right]$

$= \dfrac{1}{15}\left[89\log x - 34\log y\right]$

**32.** Let, $(x^{\log y - \log z})(y^{\log z - \log x})(z^{\log x - \log y}) = k$
$\Rightarrow \log k = (\log y - \log z)\log x + (\log z - \log x)\log y$
$+ (\log x - \log y)\log z = 0$,
$\Rightarrow k = 10^0 \Rightarrow k = 1$

**33.** $a^x = b^y = c^z$
$a = b^{\frac{y}{x}}$, $c = b^{\frac{y}{z}}$

$\log a = \dfrac{y}{x}\log b$, $\log c = \dfrac{y}{z}\log b$

$\log a \log c = \dfrac{y^2}{xy}(\log b)^2$

$\dfrac{\log a \log c}{(\log b)^2} = \dfrac{y^2}{xz} = 1$ (as $y = \sqrt{xz}$).

**34.** $\dfrac{1}{\log_n m} + \dfrac{1}{\frac{1}{2}\log_n m} + \dfrac{1}{\frac{1}{3}\log_n m} + \ldots + \dfrac{1}{\frac{1}{100}\log_n m}$

$= \log_m n + \dfrac{\log_m n}{1/2} + \dfrac{\log_m n}{1/3} + \ldots + \dfrac{\log_m n}{1/10}$

$= \log_m n + 2\log_m n + 3\log_m n + \ldots + 10\log_m n$
$= (1 + 2 + 3 + \ldots + 10)\log_m n$
$= 55\log_m n = \log_m n^{55}$

**35.** $\dfrac{\log_a \log_b [1 - 2\log_a b]}{\log_a x - \log_b x}$

$= \dfrac{\log_a a - \log_a b^2}{\log_x b - \log_x a}$

(dividing numerator and denominator by $\log_a x \log_b x$)

$= \dfrac{\log x}{\log a}\dfrac{\log a - \log b}{\log b - \log a} = \log_a x\left(\dfrac{\log a - \log b^2}{\log b - \log a}\right)$

**36.** Given $\log_{(x-y)}(x+y) = \dfrac{1}{2}$

Consider $\log_{x^2-y^2} x^2 - 2xy + y^2 = \log_{(x-y)(x+y)}(x-y)^2$

$= 2\log_{(x-y)(x+y)}(x-y) = \dfrac{2}{\log_{(x-y)}(x-y)(x+y)}$

$= \dfrac{2}{\log_{(x-y)}(x-y)+\log_{(x-y)}(x+y)} = \dfrac{2}{1+\frac{1}{2}} = \dfrac{4}{3}$

**37.** $\dfrac{1}{3}\log\left[\left(a^3+b^3\right)(a+b)\right] = \log(a+b)$

$\Rightarrow \log[(a^3+b^3)(a+b)] = 3\log(a+b)$

$\Rightarrow (a^3+b^3)(a+b) = (a+b)^3 \Rightarrow a^3+b^3 = (a+b)^2$

But given $a = 2b$.

$\therefore (2b)^3 + b^3 = (2b+b)^2 \Rightarrow 8b^3 + b^3 = (3b)^2$

$\Rightarrow 9b^3 = 9b^2 \Rightarrow b = 1$ and $a = 2$

**38.** $\dfrac{1}{2}\log_2 P + 2\log_2 Q = 1 + \log_{\left(\frac{5}{10}\right)^4}\dfrac{10}{5}$

The second term on the RHS is $\log_{(5/10)^4}(5/10)^{-1} = \dfrac{-1}{4}$

$\therefore \log_2 P^{\frac{1}{2}} + \log_2 Q^2 = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

$\Rightarrow \log_2 P^{\frac{1}{2}} Q^2 = \dfrac{3}{4}$

$\Rightarrow P^{\frac{1}{2}} Q^2 = 2^{\frac{3}{4}}$

$\Rightarrow \left(P^{\frac{1}{2}}Q^2\right)^4 = \left(2^{\frac{3}{4}}\right)^4$

$\Rightarrow P^2 Q^8 = 8$

**39.** $0 < \log_{x+3}(3x-1) + \log_{3x-1}(x+3) \le 2$

Let $\log_{x+3}(3x-1)$ be $a$.

$0 < a + \dfrac{1}{a} \le 2$

$a + \dfrac{1}{a}$ is positive and at most 2.

$\therefore a$ must be positive.

$\dfrac{a^2+1}{a} \le 2$

$a^2 + 1 \le 2a$

$a^2 - 2a + 1 \le 0$, i.e., $(a-1)^2 \le 0$

But, $(a-1)^2$ cannot be negative.

$\therefore (a-1)^2 = 0$

$a = 1$

$\therefore 3x - 1 = x + 3$

$x = 2$. There is only 1 possible value for $x$.

**40.** $\bar{3}.0838 = -3 - 0.0838 = -4 + 1 - 0.0838 = \bar{4}.9162$

$\therefore$ Antilog $(\bar{3}.0838)$ = Antilog $(\bar{4}.9162)$

$= 0.0008246$

**41.** Let $\sqrt{p\sqrt{p\sqrt{p\sqrt{p...\infty}}}} = K$

$\Rightarrow \sqrt{pk} = k$

Squaring both sides, $pk = k^2$

$\Rightarrow k^2 - pk = 0$

$\therefore k = 0$ or $k = p$

If then $k = 0$, $\log k$ is not defined.

$\therefore k = p$

Similarly, $\sqrt{q\sqrt{q\sqrt{q\sqrt{q...\infty}}}} = q$

$\therefore \dfrac{\log\sqrt{p\sqrt{p\sqrt{p\sqrt{p...\infty}}}}}{\log\sqrt{q\sqrt{q\sqrt{q\sqrt{q...\infty}}}}} = \log_q p.$

**42.** Mantissa of log 2548 is 4062.

Hence, $\log(0.002548) = \bar{3}.4062$ (1)

Similarly, $\log(0.6732) = \bar{1}.8282$ (2)

$\log\left(\dfrac{0.002548}{0.6732}\right) = \bar{3}.4062 - \bar{1}.8282 = \bar{3}.5780$

$\log$ of $\left(9\text{th root of } \dfrac{0.002548}{0.6732}\right) = \dfrac{1}{2}(\bar{3}.5780)$

$= \dfrac{1}{9}\left[\bar{9} + 6.5780\right] = \bar{1}.73088 = \bar{1}.7309$ (rounded of to $4^{th}$ decimal).

**43.** $pqr = 1$.

$\therefore \log p + \log q + \log r = 0$

$\Rightarrow (\log p)^3 + (\log q)^3 + (\log r)^3$

$= 3\log p \log q \log r$

$\therefore \log p \log q \log r = (1/3)\left[(\log p)^3 + (\log q)^3 + (\log r)^3\right]$.

**44.** If 3 numbers are in GP, their logs to the same base are in AP. If the three bases are in AP, then we cannot say anything about the logs of a certain number to the 3 bases. e.g., $\log_{10}100$, $\log_{100}100$ and $\log_{190}100$, i.e., 2, 1 and $\log_{190}100$ are neither in AP, not in GP, not in HP, though the bases 10, 100, 190 are in A.P.

**45.** $\left\{-\left[1-\left(1+\dfrac{1}{x^3-1}\right)\right]\right\} = \dfrac{1}{1727} \Rightarrow \dfrac{1}{x^3-1} = \dfrac{1}{1727}$

$\therefore x^3 - 1 = 1727 \Rightarrow x^3 = 1728 \Rightarrow x = 12$

## EXERCISE-3

***Solutions for questions 1 to 45:***

**1.** $\log_3\log_2\log_2\log_a 2^{2048} = 1$

$\Rightarrow \log_2\log_2\log_a 2^{2048} = 3$

$\Rightarrow \log_2\log_a 2^{2048} = 8$

$\Rightarrow \log_a 2^{2048} = 2^8$

$\Rightarrow a^{2^8} = 2^{2048} = 2^{8(256)} = \left(2^8\right)^{2^8}$

$\therefore a = 2^8 = 256$.

**2.** $\log(10^x+2)+\log(10^x-8)=\log(10^x-4)^2$
$\Rightarrow \log[10^{2x}-(6)(10^x-16)]=\log[10^{2x}-8.10^x+16]$
Equating the arguments of logarithms on both sides, we get $10^x=16$.
Taking log to the base 10 on both sides, we get
$\log 16=x \Rightarrow x=4\log 2$

**3.** $\log_{a/b}\dfrac{\left(b-\dfrac{1}{a}\right)}{a-\dfrac{1}{b}}=\log_{a/b}\dfrac{\left(\dfrac{ba-1}{a}\right)}{\dfrac{ab-1}{b}}$
$=\log_{a/b}(a/b)^{-1}=-1$

**4.** Given $k=\log_3{}^2$.
$\therefore \dfrac{1}{\log_3 4}+\dfrac{1}{\log_{3^2}4}+\ldots+\dfrac{1}{\log_{3^{15}}4}$
$=\log_4 3+2\log_4 3+\ldots+15\log_4 3$
$=\dfrac{1}{2}\log_2 3[1+2+\ldots+15]$
$=\dfrac{1}{2}\dfrac{1}{\log_3 2}\left[\dfrac{(15)(16)}{2}\right]$
$=\dfrac{1}{2}\left(\dfrac{1}{k}\right)\dfrac{(15)(16)}{2}=\dfrac{60}{k}$

**5.** Given $\log_a 2\log_{a/16}2=\log_{a/64}2$.
$\Rightarrow \dfrac{1}{\log_2 a}\times\dfrac{1}{\log_2 a-\log_2 16}=\dfrac{1}{\log_2 a-\log_2 64}$
Take $\log_2 a=x$, then $\dfrac{1}{x}\dfrac{1}{(x-4)}=\dfrac{1}{(x-6)}$
$x-6=x^2-4x$
$x^2-5x+6=0=x=3,\ x=2$
$\therefore \log_2 a=3$
$\Rightarrow a=8=\log_2 a=2$
$\Rightarrow a=4$ $\therefore$ options (II) or (III)

**NOTE**

The choices (A), (B), and (D) contain I. By substituting $a = 2$, the equation is not satisfied. Hence, the answer is (C).

**6.** $1/4\log_{10}a+4\log_{10}b=1+\log_{(0.2)^4}5$
$=1-\dfrac{1}{4}\log_5 5=\dfrac{3}{4}=1-1/4=3/4$
$a^{1/4}b^4=10^{3/4}\Rightarrow ab^{16}=10^3=1000.$

**7.** Let $a=\sqrt{b}=\sqrt[3]{c}=\sqrt[4]{d}=\sqrt[5]{e}=k$
$\sqrt{b}=k\Rightarrow b=k^2$
$\sqrt[3]{c}=k\Rightarrow c=k^3$
$\sqrt[4]{d}=k\Rightarrow d=k^4$
$\sqrt[5]{e}=k\Rightarrow e=k^5$
$\log_a abcde=\log_k kk^2k^3k^4k^5$
$=\log_k k^{1+2+3+4+5}=\log_k k^{15}$
$=15\log_k k=15(1)=15$

**8.** $2\log x+\log(x^4+1+2x^2)=\log(x^2+1)+\log x^2+1$
$\Rightarrow \log(x^2+1)^2-\log(x^2+1)=1$
$\Rightarrow \log\left[\dfrac{(x^2+1)^2}{(x^2+1)}\right]=1\Rightarrow \log(x^2+1)]=1$
$\Rightarrow x^2+1=10$
$\Rightarrow x^2=9\Rightarrow x=3$
We have to reject the negative value as log $x$ appears in the equation.

**9.** $4=4\log_2 b-\log_2 a=\log_2 b^4-\log_2 a=\log_2 b^4/a$
$2^4=b^4/a\Rightarrow a=\left(\dfrac{b}{2}\right)^4$

**10.** $\log a/b^2+\log a^3/b^4+\log a^5/b^6+\log a^7/b^8+\ldots$
$=\log a-\log b^2+\log a^3-\log b^4+\log a^5-\log b^6+\ldots$
$=(\log a)\ [1+3+\ldots+(2n-1)]-$
$(\log b)[2+4+6+\ldots+2n]$
$=(\log a)\ (n^2)-(\log b)[n\ (n+1)]$
[The sum of the first $n$ odd numbers, i.e., from 1 to $2n-1$ is $n^2$ and the sum of the first $n$ even numbers, i.e., from 2 to $2n$ is $2(1+2+3+\ldots n)$, i.e., $\dfrac{2(n(n+1))}{2}$
$\Rightarrow n(n+1)$.
$=\log\dfrac{a^{n^2}}{b^{n(n+1)}}=\log\left(\dfrac{a^n}{b^{n+1}}\right)^n$

**11.** $\log_{x^2-y^2}\left(x^2+2xy+y^2\right)$
$\dfrac{2\log(x+y)}{\log(x+y)+\log(x-y)}$ $[(\therefore \log_b a=(\log a)/(\log b)]$
Considering the logarithm to the base $(x+y)$, the given function is $\dfrac{2}{1+\log_{(x+y)}x-y}=\dfrac{2}{1+3}=\dfrac{2}{4}=\dfrac{1}{2}$

**12.** Given expression equals
$\log_a(x)+\dfrac{1}{\dfrac{1/4}{1/2}\log_a x}+\ldots+\dfrac{1}{\dfrac{1/400}{1/20}\log_a x}.$
$=\log_x a+2\log_x a+\ldots+20\log_x a$
$=\log_x a^{1+2+\ldots+20}=\log_x a^{210}$

**13.** $\log_b(11x-y)-\log_b(x-11y)=\log_b\left(\dfrac{x}{y}\right)$
$\Rightarrow \log_b\dfrac{11x-y}{x-11y}=\log_b\left(\dfrac{x}{y}\right)\Rightarrow\dfrac{11x-y}{x-11y}=\dfrac{x}{y}$

Cross multiplying and simplifying, $x^2 - 22xy + y^2 = 0$

$x = \frac{22y \pm \sqrt{(22y)^2 - 4(1)(y^2)}}{2} = 11y \pm \sqrt{120}y$

$\left(\frac{x}{y}, \frac{y}{x}\right) = \left(11 + \sqrt{120}, 11 - \sqrt{120}\right)$ or $\left(11 - \sqrt{120}, 11 - \sqrt{120}\right)$

$\left(\frac{x}{y}\right)^3 + \left(\frac{y}{x}\right)^3 = 2(11)^3 + 6(11)\left(\sqrt{120}\right)^2$ $((a+b)^3 + (a-b)^3$

$= 2a^3 + 6ab^2) = 10582.$

**14.** Let $\log\frac{pq}{r} = x, \log\frac{qr}{p} = y, \log\frac{rp}{q} = z$

$\log pqr = x + y + z$

Given expression say $E = x + y + z - (x^2 + y^2 + z^2)$

$= -[(x^2 - x) + (y^2 - y) + (z^2 - z)]$

Say $F$ = the minimum value of each of the 3 brackets is $-\frac{1}{4}$. Therefore, the minimum value of $F$ is $-\frac{3}{4}$ or the maximum value of $E$ is $\frac{3}{4}$.

$\therefore$ It cannot be $\frac{7}{8}$.

**15.** $\log N_1, \log N_2, \ldots \log N_{15}$ are in arithmetic progression. This is only possible if $N_1, N_2 \ldots N_{15}$ are in geometric progression.

Let $N_1 = a$ and $\frac{N_2}{N_1} = r$.

$N_2 = ar, N_3 = ar^2 \ldots N_{15} = ar^{14}$

$N_8 - N_4 = ar^3(r^4 - 1) = 3600$ and $N_7 - N_5 = ar^4(r^2 - 1)$

$= 1440$

$\frac{ar^3(r^4-1)}{ar^4(r^2-1)} = \frac{3600}{1440}$, i.e., $\frac{r^2+1}{r} = \frac{5}{2}$

$2r^2 - 5r + 2 = 0$

$r = 2$ or $\frac{1}{2}$

If $r$ is $\frac{1}{2}$ $a$ will be negative. This is not possible.

$\therefore$ $r$ is 2

$(r, a) = (2, 30)$.

$N_1 + N_2 + \ldots N_{15} = \frac{a\left(r^{15}-1\right)}{r-1} = 30(2^{15} - 1).$

**16.** $\frac{5^{2x-1}+3}{5^x+7} = 2^2$

$\Rightarrow (5^x)^2 - 20(5^x) - 125 = 0$

$(5^x + 5)(5^x - 25) = 0$

$5^x = -5$ or $5^2$

$\Rightarrow x = \log_5 - 5$ or $\log_5 5^2$

As logarithms are not defined for negative numbers, $x = 2$.

**17.** $4b^2 = a^{2x} - 4 + a^{-2x}$ and

$4c^2 = a^{2x} + 2 + a^{-2x}$

$\therefore 4c^2 = 4b^2 + 4$ or $c^2$

$= b^2 + 1$ or $c = \sqrt{b^2+1}$, as $c$ cannot be negative.

**18.** $\tan 45° = 1$, and $\log 1 = 0$

In the given product, $\log \tan 45°$ is present, which is equal to 0.

$\therefore$ The product will be 0.

$\therefore$ $k = 0$

**19.** $\log N_1, \log N_2, \ldots \log N_{15}$ are in arithmetic progression. This is only possible if $N_1, N_2 \ldots N_{15}$ are in geometric progression.

Let $N_1 = a$ and $\frac{N_2}{N_1} = r$.

$N_2 = ar, N_3 = ar^2 \ldots N_{15} = ar^{14}$

$N_8 - N_4 = ar^3(r^4 - 1) = 3600$ and $N_7 - N_5 = ar^4(r^2 - 1)$

$= 1440$

$\frac{ar^3(r^4-1)}{ar^4(r^2-1)} = \frac{3600}{1400}$, i.e., $\frac{r^2+1}{r} = \frac{5}{2}$

$2r^2 - 5r + 2 = 0$

$r = 2$ or $\frac{1}{2}$

If $r$ is $\frac{1}{2}$ $a$ will be negative. This is not possible.

$\therefore$ $r$ is 2.

$(r, a) = (2, 30)$

$N_1 + N_2 + \ldots N_{15} = \frac{a\left(r^{15}-1\right)}{r-1} = 30(2^{15} - 1)$

**20.** $b = \log_{10}35$ and $c = \log_{10}5$

$a = \log_{10}252 = \log_{10}(36)(7) = \log_{10}36 + \log_{10}7$

$= \log_{10}36 + \log_{10}35 - \log_{10}5$

$= \log_{10}36 + b - c$

$\therefore \log_{10}36 = a - b + c$

**21.** $\log_9 16 \times \log_4 10 = \frac{\log 16}{\log 9} \times \frac{\log 10}{\log 4}$

$= \frac{2\log 4}{2\log 3} \times \frac{1}{\log 4} = \frac{1}{\log 3}$

Also, $\log_2 8 = 3 \log_2 2 = 3$

$\therefore 3 \log x = \frac{3}{\frac{1}{\log 3}} \Rightarrow 3 \log x = 3 \log 3 \Rightarrow \log x = \log 3$

$\therefore x = 3$

**22.** $A = \log_7 2401 = \log_7 7^4 = 4$

$B = \log_{7\sqrt{7}} 343 = \log_{7\sqrt{7}}\left(7\sqrt{7}\right)^2 = 2$

$C = \log_{\sqrt{6}} 216 = \log_{\sqrt{6}}\left(\sqrt{6}\right)^6 = 6$

$D = \log_2 32 = \log_2 2^5 = 5$

$\therefore$ The arrangement in ascending order would be BADC.

**23.** $\log_y x = 8 \Rightarrow x = y^8$ (1)

and, $\log_{10y} 16x = 4 \quad \Rightarrow 16x = (10y)^4$

$\Rightarrow 16x = 2^4 \times 5^4 \times y^4 \Rightarrow x = 5^4\, x\, y^4$ (2)

Combining (1) and (2), we get

$y^8 = 5^4 \times y^4$

$\Rightarrow y^4\,(y^4 - 5^4) = 0$

$y^4 = 0 \Rightarrow y = 0$, which is not possible.

or, $y^4 - 5^4 = 0 \Rightarrow y = 5$

**24.** $\{1-[1-(1-x^2)^{-1}]^{-1}\}^{-1/2} = \{1-[1-1/1-x^2]^{-1}\}^{-1/2}$

$= \{1-[-x^2/1-x^2]^{-1}\}^{-1/2} = [1+1-x^2/x^2]^{-1/2} = (1/x^2)^{-1/2}$

$= x$

$\therefore$ LHS $= \log x \Rightarrow \log x = 1 \ \therefore x = 10$

**25.** Let $\log_3 x = a$.

$\therefore \log_x 3 = 1/\log_3 x = 1/a$

$= \log_{x/81} 3 = \dfrac{1}{\log_3 x81}$

$= \dfrac{1}{\log_3 x - \log_3 81} = \dfrac{1}{\log_3 x - 4} = \dfrac{1}{a-4}$

and $\log_{x/729} 3 = \dfrac{1}{\log_3 x/729}$

$= \dfrac{1}{\log_3 x - \log_3 729} = \dfrac{1}{\log_3 x - 6} = \dfrac{1}{a-6}$

Now, $= \dfrac{1}{a} \times \dfrac{1}{a-4} = \dfrac{1}{a-6}$

$\Rightarrow a^2 - 4a = a - 6$

$\Rightarrow a^2 - 5a + 6 = 0 \Rightarrow (a-2)\,(a-3) = 0$

either $a = 2$

$\Rightarrow \log_3 x = 2 \Rightarrow x = 3^2 = 9$

or, $a = 3 \Rightarrow \log_3 x = 3 \Rightarrow x = 3^3 = 27$

**26.** $\log_2 8 = 3; \Rightarrow {}_4\log_2 8 = 4^3 = 64$

Also, ${}_{27}\log_{27} 81 = 81$

$\therefore 144 + \log_{10} x = 64 + 81$

$\Rightarrow \log_{10} x = 1 \Rightarrow x = 10$

**27.** $\log_2 [3\log_2 2^{2m} + 2\log_2 4] = 4$

$\Rightarrow 3(2\ m) + 4 = 2^4$

$\Rightarrow 6\ m + 4 = 16$

$\Rightarrow 6\ m = 12 \Rightarrow m = 2$

**28.** $\log_{12} 1/18 + \log_a 9 = \log_{144} 1/4$

$\therefore \log_a 9 = \log_{12} 1/2 - \log_{12} 1/18 = \log_{12} 9$

$\therefore\ a = 12$

**29.** $\log_{20} 25 \times \log_{25} 20 = \dfrac{\log 25}{\log 20} \times \dfrac{\log 20}{\log 25} = 1$

$\therefore \log_7 1 = 0$

$\therefore$ The given expression becomes $\log_{17} 6^0$.

$= \log_{17} 1 = 0$

**30.** $2^{\frac{1}{\log_x 4}} \times 2^{\frac{1}{\log_x 16}} \times 2^{\frac{1}{\log_x 256}} \ldots \infty = 2$

Let $\log_x 4 = a$

$2^{\frac{1}{a}+\frac{1}{2a}+\frac{1}{4a}} + \ldots = 2^1$

Equating the indices on the two sides, we get

$\dfrac{1}{a} + \dfrac{1}{2a} + \cdots \infty = 1$

$\dfrac{1}{a}\left[1 + \dfrac{1}{2} + \dfrac{1}{4} + \ldots\infty\right] = 1$

$\Rightarrow \dfrac{1}{a}(2) = 1$ ($\because$ sum of infinite terms)

$\Rightarrow a = 2$

$\therefore \log_x 4 = 2 \Rightarrow 4 = x^2$

$\Rightarrow x = 2$

**31.** $3\ (\log_{10} y - \log_{10} \sqrt[3]{y}\ ) = 8 \log_y 10$

$\Rightarrow 3\left(\log_{10} y - \dfrac{1}{3}\log_{10} y\right) = \dfrac{8}{\log_{10} y}$

$\Rightarrow 2\ (\log_{10} y)^2 = 8$

$\Rightarrow \log_{10} y = \pm 2$

$\therefore\ y = 100$ or $\dfrac{1}{100}$

**32.** $\log_p \dfrac{p^3}{q^2} + \log_q \dfrac{q^3}{p^2} = \log_p p^3 - \log_p q^2 + \log_q q^3 - \log_q p^2$

$= 3 - 2\log_p q + 3 - 2\log_q p$

$= 6 - 2\,(\log_q p + \log_p q)$

For any two positive numbers $x$ and $y$ where $x \geq y > 1$,

$\log_y x + \log_x y \geq 2$.

$\therefore \log_q p + \log_p q \geq 2$

$\therefore$ The given expression cannot exceed 2. Only 2.5 violates this condition.

**33.** $\log\ (0.004225)^{1/8} = \dfrac{1}{8}\ (3.6258) = \dfrac{1}{8}\ (-2.3742)$

$= -0.296775 = \bar{1}\ .703225 = \bar{1}\ .70322$

Since, $\log 50492 = 4.70322$

$\therefore$Antilog $\bar{1}\ .70322 = 0.50492$

$\therefore 8\sqrt{0.004225} = 0.50492$

**34.** $\log_{10} 24 + \log_{10} 80 + \log_{10} 25 = x + y + z$

$\Rightarrow \log_{10}(24 + 80 + 25) = x + y + z$

$\Rightarrow \log_{10} 48000 = x + y + z$

$\Rightarrow \log_{10} 48 + \log_{10} 1000 = x + y + z$

$\therefore \log_{10} 48 = x + y + z - 3$

**35.** $\log_2 \log_2 \log_2 (\sqrt{x-13} + \sqrt{x-45}) = 1$

$\Rightarrow \log_2 \log_2 \left(\sqrt{x-13} + \sqrt{x-45}\right) = 2$

$\Rightarrow \log_2 \left(\sqrt{x-13} + \sqrt{x-45}\right) = 4$

$\Rightarrow \left(\sqrt{x-14} + \sqrt{x-45}\right) = 16$

Squaring both sides, $2x - 58 + 2\sqrt{x^2 - 58x + 585} = 256$

$\Rightarrow 2\sqrt{x^2 - 58x + 585} = 314 - 2x$

$\Rightarrow 4(x^2 - 58x + 585) = 98596 - 1256x + 4x^2$

$\Rightarrow 1024x = 96256 \Rightarrow x = 94.$

$\sqrt{x-13} + \sqrt{x-45}$ is defined when $x$ is 94.

$\therefore$ $x$ can be 94

$x$ has only one possible value.

**36.** $x^{\log_4 9} + 9^{\log_4 x} = 1458$

Let $x = 4^t$.

$\therefore\ 4^{t\log_4 9} + 9^{\log_4 4^t} = 1458$

$\Rightarrow\ 4^{\log_4 9^t} + 9^{\log_4} = 1458$

$\Rightarrow 9^t + 9^t = 1458 \Rightarrow 9^t = 729 \Rightarrow \therefore\ t = 3$

$\therefore\ x = 4^t = 4^3 = 64$

**Alternative Method:**

$x^{\log_4 9} = x^{\log_{2^2} 3^2}$

$= x^{\log_2 3}$ and $9^{\log_4 x} = x^{\log_4 9} = x^{\log_2 3}$ $\therefore$ given equation is $2\left(x^{\log_2 3}\right) = 1458.$

$x^{\log_2 3} = 729.$

$x = 729^{\frac{1}{\log_2 3}} = 729^{\log_3 2}$

$= (3^6)^{\log_3 2} = 3^{(6\log_3 2)} = 3^{\log_3 2^6} = 2^6 = 64.$

**37.** Let $x = \left(\frac{3}{4}\right)^{500}$.

Taking logarithm to base 10, we have $\log x = 500(\log 3 - 2\log 2) = 500\ (0.4771 - 0.6020)$

$= 500\ (-0.1249) = -62.45 = \overline{63}\ .\ 55$

The number of zeros after the decimal point equals one less than the numerical value of the characteristic. Therefore, there are 62 zeroes.

**38.** Given $\log_4 15$, $\log_4(3^x - 7)$ and $\log_4(3^x - 1/3)$ are in arithmetic progression.

$\Rightarrow$ 2 $\log_4(3^x - 7)$ and $\log_4(3^x - 1/3)$

$\Rightarrow (3^x - 7)^2 = 15\ \left(3^x - 1/3\right)$

$\Rightarrow (3^x)^2 - 14(3^x) + 49 = 15(3^x) - 5$

$\Rightarrow (3^x)^2 - 29(3^x) + 54 = 0$

$\Rightarrow (3^x - 27)\ (3^x - 2) = 0$

$\therefore\ 3^x = 27 \Rightarrow x = 3$ or $3^x = 2 \Rightarrow x = \log_3 2$

But, if $x = \log_3 2$, i.e., $3^x = 2$,

$3^x - 7 = 2 - 7 = -5$, which is negative quantity.

$\therefore\ \log_4\ (3x - 7)$ cannot be defined.

$\therefore\ x \neq \log_4(3x - 7)\ \ \therefore\ x = 3$

**39.** Let the number be N and the base be $b$.

$\log_b N = 8\ \Rightarrow N = b^8$

$N = b^8$ (1)

$\log_{3b} 9N = 5$

$9N = (3b)^5 = 243b^5$ (2)

Substituting N's value in (1) and in (2), we get

$9b^8 = 243b^5 \Rightarrow b^3 = 27 \Rightarrow b = 3$

**40.** $\log_4 a \times \frac{\log_4 a}{\log_4 16} + \frac{\log_4 a}{\log_4 64} + \frac{\log_4 a}{\log_4 256}$

$= \log_4 a\left(1 + \frac{1}{\log_4 4^2} + \frac{1}{\log_4 4^3} + \frac{1}{\log_4 4^4}\right)$

$= \log_4 a\ (1 + 1/2 + 1/3 + 1/4)$

$= (25/12)\ \log_4 a = 25/3 \Rightarrow \log_4 a = 4$

$a = 4^4 = 256$

**41.** $\log_4(x+6) - \log_4(x-6) = \log_4 2^2 = \log_4 4$

$\log_4(x + 6/x - 6) = \log_4 4$

$x + 6/x - 6 = 4$

$\Rightarrow x = 10$

**42.** $\frac{\log_a \log_b[1 - 2\log_a b]}{\log_a x - \log_b x}$

$= \frac{\log_a a - \log_a b^2}{\log_x b - \log_x a}$

(dividing numerator and denominator by $\log_a x \log_b x$)

$= \frac{\log x}{\log a}\frac{\log a - \log b^2}{\log b - \log a} = \log_a x\left(\frac{\log a - \log b^2}{\log b - \log a}\right)$

**43.** $0.3125 = \frac{3125}{10000} = \frac{5}{16}$ and $\log_{0.3125} \sqrt{512}$

$= \frac{\log_2 2^{9/2}}{\log_2 5/16}$

**44.** $\frac{\log_3 a \log_3 2 \log_{11} 9 \log_b 13}{\log_b(21-a)\log_3^{13}\log_{11}^{a}}$

$\frac{\left(\frac{\log a}{\log 3}\right)\left(\frac{\log 2}{\log 3}\right)\left(\frac{\log 9}{\log 11}\right)\left(\frac{\log 13}{\log b}\right)}{\left(\frac{\log(21-a)}{\log b}\right)\left(\frac{\log 13}{\log 3}\right)\left(\frac{\log a}{\log 11}\right)} = \frac{(\log 2)(\log 9)}{(\log 3)\log(21-a)}$

$= \frac{(\log 2)(2\log 3)}{(\log 3)(\log(21-a))} = \frac{\log 4}{\log(21-a)}$. This is 1.

$\therefore\ 4 = 21 - a$ or $a = 17$

**45.** $\log_b{}^a + 2\log_b{}^{\frac{1}{2}} a^2 + 3\log_{b^{\frac{1}{3}}} a^3 + \cdots + n\log_b{}^{\frac{1}{n}} a^n$

$= 3025\ \log_b{}^a$

$\log_b{}^a + 2\left(\frac{2}{\frac{1}{2}}\right)\log_b{}^a + 3\left(\frac{3}{\frac{1}{3}}\right)\log_b{}^a + \ldots + n\left(\frac{n}{n}\right)\log_b{}^a$

$= 3025\ \log_b{}^a$

$\Rightarrow (1^3 + 2^3 + \ldots\ n^3)\ \log_b^a = 3025\ \log_b^a$

$\Rightarrow \left(\frac{n(n+1)}{2}\right)^2 = 3025 \Rightarrow \frac{n(n+1)}{2} = 55$

$\Rightarrow n = 10$

## EXERCISE-4

***Solutions for questions 1 to 20:***

**1.** $\log_b a(1+a) = 0$
$a(1+a) = b^0$
$a + a^2 = 1 \Rightarrow a^2 - 1 = -a$

**2.** Let $k = x^{\log y - \log z} \cdot y^{\log z - \log x} \cdot z^{\log x - \log y}$.
Taking logarithms on both sides, we have
$\log k = (\log y - \log z)\log x + (\log z - \log x)\log y + (\log x - \log y)\log z = 0$.
Therefore, $k = 1$.

**3.** Given $a = b^2 = c^4 = d^6 = e^8$
$\therefore b = a^{\frac{1}{2}}$, $c = a^{\frac{1}{4}}$, $d = a^{\frac{1}{6}}$ and $e = a^{\frac{1}{8}}$
$\log_a abcde = \log_a a^1 \cdot a^{\frac{1}{2}} \cdot a^{\frac{1}{4}} \cdot a^{\frac{1}{6}} \cdot a^{\frac{1}{8}}$
$= \log_a a^{\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+}$
$\log_a a^{\frac{24+12+6+4+3}{24}} = \log_a a^{\frac{49}{24}} = \frac{49}{24}$

**4.** $x = (1125)^{225} \Rightarrow \log x = 225 \log (1125)$
$\log x = 225 \log (9000/8)$
$\log x = 225 (\log 9000 - \log 8)$
$\log x = 225 [\log (3^2 \times 10^3) - \log (2^3)]$
$\log x = 225 [\log 3^2 + \log 10^3 - 3 \log 2]$
$\log x = 225 [2 \log 3 + 3 - 3 \log 2]$
$\log x = 225 [0.9542 + 3 - 0.9030]$
$\log x = 225 [3.9542 - 0.903] = 225(3.0512) = 686.52$
Hence, 687 digits

**5.** $\log_6 x < 1 \Rightarrow x < 6$ and $x > 1$
$\therefore 1 < x < 6$

**6.** $\log_5 x > (0.9)^\circ \Rightarrow \log_5 x > 1$ $x > 5$
$\therefore x \in (5, \infty)$

**7.** $\log_n 2 + \log_n 3 + \ldots + \log_n 100 = \log_n 100!$

**8.** $\log_{\frac{1}{2}} x^2 - 6x + 3 \geq \log_{\frac{1}{2}} \frac{1}{4}$
$x^2 - 6x + 3 \leq \frac{1}{4}$ $(\therefore \log_a x > \log_a y \Rightarrow x < y$ when $0 < a < 1)$
$x^2 - 6x + 3 - \frac{1}{4} \leq 0$
$\Rightarrow 4x^2 - 24x + 11 \leq 0$
$4x^2 - 2x - 22x + 11 \leq 0$
$2x(2x-1) - 11(2x-1) \leq 0$
$(2x-1)(2x-11) \leq 0$
$x \in \left[\frac{1}{2}, \frac{11}{2}\right]$ (1)
Since log is defined only for positive numbers, $x^2 - 6x + 3 > 0$.
$x > 3 + \sqrt{6}$ or $x < 3 - \sqrt{6}$ (2)
From (1) and (2) $\left[\frac{1}{2}, 3-\sqrt{6}\right) \cup \left(3+\sqrt{6}, \frac{11}{2}\right]$

**9.** $p = \log_e \frac{(fg)^2}{e^3}$, $q = \log_f \frac{(eg)^2}{f^3}$ and $r = \log_g \frac{(ef)^2}{g^3}$
$e^p = \frac{(fg)^2}{e^3}$, $f^q = \frac{(eg)^2}{f^3}$ and $g^r = \frac{(ef)^2}{g^3}$
$e^p . f^q . g^r = \frac{(e^2 f^2 g^2)^2}{(efg)^3} = \frac{(efg)^4}{(efg)^3} = efg$
Equating the powers of $p$, $q$ and $r$ on both sides, $p = q = r = 1$.
$\therefore$ Required value $= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$.

**10.** $\log\left(\sqrt{6} - \sqrt{3} - 2 + \sqrt{2}\right) + \log\left(\sqrt{6} + \sqrt{3} + 2 + \sqrt{2}\right)$
$= \log\left[\left(\sqrt{6} + \sqrt{2}\right) - \left(\sqrt{3} + \sqrt{2}\right)\right]\left[\left(\sqrt{6} + \sqrt{2}\right) + \left(\sqrt{3} + \sqrt{2}\right)\right]$
$= \log\left[\left(\sqrt{6} + \sqrt{2}\right)^2 - \left(\sqrt{3} + \sqrt{2}\right)^2\right]$
$= \log\left(6 + 2 + 4\sqrt{3} - 3 - 4 - 4\sqrt{3}\right) = \log 1 = 0$.

**11.** The first inequality states that the sum of a number and its reciprocal is less than or equal to 2.
The second states that the number is positive.
$\therefore$ The reciprocal is also positive. But, the sum of a positive number and its reciprocal is greater than or equal to 2.
$\therefore$ This sum is exactly equal to 2, i.e., both the number and the reciprocal are 1.
$\log_{|p-4|} |p-3| = 1 \Rightarrow |p-3| = |p-4|$
$\Rightarrow p - 4 = \frac{-1}{2}$ and $p - 3 = \frac{1}{2}$, i.e., $p = 3\frac{1}{2} = \frac{7}{2}$.

**12.** Let $\log x = a$, $\log y = b$, $\log z = c$ and $\log w = d$
$\Rightarrow x = e^a$, $y = e^b$, $z = e^c$ and $w = e^d$
Now,
$x^{\log y(\log z - \log w)} + y^{\log z(\log w - \log x)} + z^{\log w(\log x - \log y)}$
$+ w^{\log x(\log y - \log z)}$
$= e^{ab(c-d)} + e^{bc(d-a)} + e^{cd(a-b)} + e^{da(b-c)}$
$= e^{abc-abd} + e^{bcd-abc} + e^{acd-bcd} + e^{abd-acd}$
Now, $= e^{abc-abd} + e^{bcd-abc} + e^{acd-bcd} + e^{abd-acd}$
$\geq 4\sqrt[4]{e^{abc-abd} e^{bcd-abc} e^{acd-bcd} e^{abd-acd}}$
$= 4$

**13.** $\log_3 (9x+8y)(9x-8y) = 5$
$(9x+8y)(9x-8y) = 3^5 = 243$
$x$ and $y$ are integers.
$\therefore 9x + 8y$ and $9x - 8y$ must divide 243 exactly. Also, their sum must be divisible by 18.

$(9x+8y, 9x-8y) = (9, 27)$ or $(27, 9)$ or $(-9, -27)$ or $(-27, -9)$. But, in none of these cases, $x$ and $y$ are integers.

$\therefore |x| + |y|$ has no possible value.

**14.** The four numbers, 1, $R$, $Q$, $P$ and their logs to base $Q$ are listed below.

| **Numbers** | 1 | $R$ | $Q$ | $P$ |
|---|---|---|---|---|
| **log to base $Q$** | 0 | $b$ | 1 | $a$ |

$K = \log_P \frac{1}{a} + \log_Q b + \log_R \frac{a}{b}$

$L = \log_Q \frac{1}{a} + \log_Q b + \log_Q \frac{a}{b}$

$= \log_Q \frac{1}{a} b \frac{a}{b} = 0.$

Compare $K$ and $L$, term wise.

$\log_P \frac{1}{a} = \log_P Q \log_Q \frac{1}{a} = \left(\frac{1}{a}\right) \log_Q \frac{1}{a}$

= (a positive number less than or equal to 1) (the negative number $\log \frac{1}{a}$)

$\geq$ the negative number $\log \frac{1}{a}$

Similarly, $\log_R \frac{a}{b} = \log_{R^Q} \log_Q \frac{a}{b} = \frac{1}{b} \log_Q \frac{a}{b}$

= (a positive number greater than or equal to 1) (the positive quantity $\log_Q \frac{a}{b}$)

$\geq \log_Q \frac{a}{b}$

$\therefore K \geq L$. As $L = 0$, $K \geq 0$. It cannot be negative.

**15.** $\frac{\log_{0.1}\left(x^2 - 2x\right)^4}{\log_{0.1} 0.01} \geq \log_{0.1} 9(x-2)$

$\log_{0.1} (x^2 - 2x)^2 \geq \log_{0.1} 9(x-2)$

On the LHS, the 2 in the denominator cancels with a 2 in the index.

When $\log_b a \geq \log_b c$ and $0 < b < 1$, it follows that $a \leq c$

$(x^2 - 2x)^2 \leq 9(x-2)$

$(x^2 - 2x)^2 - 9(x-2) \leq 0$

$(x-2)\ (x^2\ (x-2) - 9) \leq 0$ (1)

For $\log_{0.1} (x-2)$ to be defined, $x$ must be more than 2.

$\therefore$ (1) implies $x > 2$ and $x^2\ (x-2) - 9 \leq 0$

$x^3 - 2x^2 - 9 \leq 0$

$x^3 - 3x^2 + x^2 - 9 \leq 0$

$(x^2 + x + 3)\ (x-3) \leq 0$

$\Rightarrow \left(\left(x + \frac{1}{2}\right)^2 + \frac{11}{4}\right)(x-3) \leq 0$

$\Rightarrow \left(x + \frac{1}{2}\right)^2 + \frac{11}{4}$ is always positive.

$\Rightarrow \therefore x - 3 \leq 0$, i.e., $x \leq 3$

Range of $x$ is $(2, 3)$.

**16.** $= \sqrt[3]{\frac{16\sqrt{13} + 40}{8}} = \sqrt[3]{\frac{13\sqrt{13} + 3\sqrt{13} + 3(13) + 1...}{8}}$

$= \sqrt[3]{\left(\frac{\sqrt{13}+1}{2}\right)^3} = \frac{\sqrt{13}+1}{2}$

$Z = \frac{\sqrt{13}+1}{2} + \frac{\sqrt{13}-1}{2} = \sqrt{13}$, which is slightly more than 3.6.

$3.6 < Z < 3.8$.

**17.** Given $0 < A < B < 1$.

$\therefore \log_B A > 0$. Logs of big numbers (>1) to big bases (>1) and of small numbers (<1) to small bases (<1) are positive.

$E = \log_A \frac{B}{A} + \log_B \frac{A}{B} = (\log_A B + \log_B A - 2)$

Since $A < B$

$\log_A B + \log_B A > 2$

$\therefore \log_A B + \log_B A - 2 > 0$

($\because \log_B A$ is positive and $\log_A B$ is its reciprocal. The sum of a positive number and its reciprocal is greater than equal to 2. As $A \neq B$, it cannot be 2.)

$\therefore E$ must be positive.

Both I and II are true.

**18.** $2\log_5 (x+11) < 2\log_5 x + \log_5 1.33^2$

$\log_5^{x+11} < \log_5 1.33x$

$x + 11 < 1.33x$

$\frac{11}{0.33} < x$

or $x > \frac{100}{3}$

**19.** $\log_2 (5x+3y)\ (5x-3y) = 10$

$(5x+3y)\ (5x-3y) = 1024$. Also, $5x+3y$ and $5x-3y$ must be positive integers such that their sum is divisible by 10. The only possible values for $(5x+3y, 5x-3y)$ are $(256, 4)$, $(4, 256)$, $(64, 16)$ and $(16, 64)$.

$\therefore (x, y) = (26, 42)$, $(26, -42)$ $(8, 8)$ or $(8, -8)$

**20.** $\log_{2\sqrt{3}} \frac{75}{13} + 4\log_{144} \frac{13}{4x} \geq 2$

$\log_{2\sqrt{3}} \frac{75}{13} + 4\log_{\left(2\sqrt{3}\right)^4} \frac{13}{4x} \geq 2$

$\log_{2\sqrt{3}} \frac{75}{13} + 4\left[\frac{1}{4}\log_{2\sqrt{3}} \frac{13}{4x}\right] \geq 2$

$\log_{2\sqrt{3}} \frac{75}{4x} \geq 2$

$\frac{75}{4x} \geq \left(2\sqrt{3}\right)^2$, i.e., 12

$\frac{75}{48} \geq x$, i.e., $x \leq \frac{25}{16}$

# 4 Quadratic Equations

CHAPTER

## Learning Objectives

*In this chapter, you will*:

- Understand about quadratic equations and their standard forms
- Learn to find the roots of a quadratic equation—factorization and formula approach
- Grasp knowledge of discriminant of a quadratic equation and the nature of the roots
- Learn how to find the sum and product of the roots and their applications, including
  - Relation between the roots and the coefficients
  - Relation between signs of the roots and the coefficients
- Construct a new quadratic equation using given roots
- Construct a quadratic equation by changing the given roots
- Understand quadratic and higher degree expressions, such as:
  - Finding the minima and maxima of quadratic expressions
  - Polynomial division
- Understand and apply remainder and factor theorems
- Learn to apply Descartes' rule of signs to find out the possible number of positive, negative and non-real roots

## ❒ QUADRATIC EQUATIONS

'If a variable occurs in an equation with all positive integer powers and the highest power is two, then it is called quadratic equation (in that variable)'.

In other words, a second-degree polynomial in $x$ equated to zero will be a quadratic equation. For such an equation to be a quadratic equation, the co-efficient of $x^2$ should not be zero.

The most general form of a quadratic equation is $ax^2 + bx + c = 0$, where $a \neq 0$ (and $a$, $b$, $c$ are real).

Some examples of quadratic equations are as follows:

$x^2 - 5x + 6 = 0$ (1)

$x^2 - x - 6 = 0$ (2)

$2x^2 + 3x - 2 = 0$ (3)

$2x^2 + x - 3 = 0$ (4)

Like a first-degree equation in $x$ has one value of $x$ satisfying the equation, a quadratic equation in $x$ will have two values of $x$ that satisfy the equation. The values of $x$ that satisfy the equation are called roots of the equation. These roots may be real or imaginary.

For the four quadratic equations given above, the roots are as given below:

Equation (1) : $x = 2$ and $x = 3$

Equation (2) : $x = -2$ and $x = 3$

Equation (3) : $x = 1/2$ and $x = -2$

Equation (4) : $x = 1$ and $x = -3/2$

In general, the roots of a quadratic equation can be found out in two ways.

1. by factorising the expression on the left-hand side of the quadratic equation
2. by using the standard formula

All the expressions may not be easy to factorize, whereas applying the formula is simple and straightforward.

## ❒ Finding the Roots by Factorization

If the quadratic equation $ax^2 + bx + c = 0$ can be written in the form $(x - \alpha)(x - \beta) = 0$, then the roots of the equation are $\alpha$ and $\beta$.

To find the roots of a quadratic equation, we should first write it in the form of $(x - \alpha)(x - \beta) = 0$, i.e., the left-hand side $ax^2 + bx + c$ of the quadratic equation $ax^2 + bx + c = 0$ should be factorized into two factors.

For this purpose, we should go through the following steps. We will understand these steps with the help of the equation $x^2 - 5x + 6 = 0$ which is the first of the four quadratic equations we looked at as examples above.

1. First write down $b$ (the co-efficient of $x$) as the sum of two quantities whose product is equal to ac.
   In this case –5 has to be written as the sum of two quantities whose product is 6. We can write –5 as (–3) + (–2) so that the product of (–3) and (–2) is equal to 6.
2. Now, rewrite the equation with the '$bx$' term split in the above manner.
   In this case, the given equation can be written as $x^2 - 3x - 2x + 6 = 0$
3. Take the first two terms and rewrite them together after taking out the common factor between the two of them. Similarly, the third and fourth terms should be rewritten after taking out the common factor between the two of them. In other words, you should ensure that what is left from the first and the second terms (after removing the common factor) is the same as that left from the third and the fourth terms (after removing their common factor).
   In this case, the equation can be rewritten as $x(x - 3) - 2(x - 3) = 0$; Between the first and second terms as well as the third and fourth terms, we are left with $(x - 3)$ as a common factor.
4. Rewrite the entire left-hand side to get the form $(x - \alpha)\ (x - \beta)$.
   In this case, if we take out $(x - 3)$ as the common factor, we can rewrite the given equation as $(x - 3)\ (x - 2) = 0$
5. Now, $\alpha$ and $\beta$ are the roots of the given quadratic equation.

∴ For $x^2 - 5x + 6 = 0$, the roots of the equation are 3 and 2.

For the other three quadratic equations given above as examples, let us see how to factorize the expressions and get the roots.

For equation (2), i.e., $x^2 - x - 6 = 0$, the co-efficient of $x$ which is –1 can be rewritten as (–3) + (+2) so that their product is –6 which is equal to $ac$(1 multiplied by –6). Then we can rewrite the equation as $(x - 3)\ (x + 2) = 0$ giving us the roots as 3 and –2.

For equation (3), i.e., $2x^2 + 3x - 2 = 0$, the co-efficient of $x$ which is 3 can be rewritten as (+4) + (–1) so that their product is –4 which is the value of $ac$(–2 multiplied by 2). Then we can rewrite the equation as $(2x - 1)(x + 2) = 0$ giving the roots as 1/2 and –2.For equation (4), i.e., $2x^2 + x - 3 = 0$, the co-efficient of $x$ which is 1 can be rewritten as (+3) + (–2) so that their product is –6 which is equal to $ac$(2 multiplied by –3). Then we can rewrite the given equation as $(x - 1)\ (2x + 3) = 0$ giving us the roots as 1 and –3/2.

## ❒ Finding the Roots by Using the Formula

If the quadratic equation is $ax^2 + bx + c = 0$, then we can use the standard formula given below to find out the roots of the equation.

**FORMULA**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The four quadratic equations we took as examples above can be taken and their roots found out by using the formula. The student is advised to check it out for himself/herself that the roots can be obtained by using this formula also.

## ❒ SUM AND PRODUCT OF ROOTS OF A QUADRATIC EQUATION

For the quadratic equation $ax^2 + bx + c = 0$, the sum of the roots and the product of the roots are given by the following:

**FORMULA**

Sum of the roots = $-b/a$
Product of the roots = $c/a$

These two rules will be very helpful in solving problems on quadratic equations.

## ❒ NATURE OF THE ROOTS

We mentioned already that the roots of a quadratic equation with real co-efficients can be real or complex. When the roots are real, they can be equal or unequal. All this will depend on the expression $b^2 - 4ac$. $b^2 - 4ac$ determines the nature of the roots of the quadratic equation and it is called 'discriminant' of the quadratic equation.

1. If $b^2 - 4ac > 0$, then the roots of the quadratic equation will be real and distinct.
2. If $b^2 - 4ac = 0$, the roots are real and equal.
3. If $b^2 - 4ac < 0$, then the roots of the quadratic equation will be complex conjugates.

Thus, we can write down the following about the nature of the roots of a quadratic equation when $a$, $b$ and $c$ are all rational.

| | |
|---|---|
| when $b^2 - 4ac < 0$ | the roots are complex and unequal |
| when $b^2 - 4ac = 0$ | the roots are rational and equal |
| when $b^2 - 4ac > 0$ and a perfect square | the roots are rational and unequal |
| when $b^2 - 4ac > 0$ but not a perfect square | the roots are irrational and unequal |

Whenever the roots of the quadratic equation are irrational, ($a$, $b$, $c$ being rational) they will be of the form $a + \sqrt{b}$ and $a - \sqrt{b}$, i.e., whenever $a + \sqrt{b}$ is one root of a quadratic equation, then $a - \sqrt{b}$ will be the second root of the quadratic equation and vice versa.

## ❒ SIGNS OF THE ROOTS

We can comment on the signs of the roots, i.e., whether the roots are positive or negative, based on the signs of the sum of the roots and the product of the roots of the quadratic equation. The following table will make clear the relationship between the signs of the sum and the product of the roots and the signs of the roots themselves.

| Sign of product of the roots | Signs of sum of the roots | Sign of the roots |
|---|---|---|
| + ve | + ve | Both the roots are positive |
| + ve | – ve | Both the roots are negative |
| – ve | + ve | The numerically larger root is positive and the other root is negative. |
| – ve | – ve | The numerically larger root is negative and the other root is positive. |

## ❒ CONSTRUCTING A QUADRATIC EQUATION

We can build a quadratic equation in the following three cases:

1. when the roots of the quadratic equation are given
2. when the sum of the roots and the product of the roots of the quadratic equation are given.
3. when the relation between the roots of the equation to be framed and the roots of another equation is given.

If the roots of the quadratic equation are given as $\alpha$ and $\beta$, the equation can be written as

$(x - \alpha)\ (x - \beta) = 0$, i.e., $x^2 - x\ (\alpha + \beta) + \alpha\beta = 0$

If $p$ is the sum of the roots of the quadratic equation and $q$ is the product of the roots of the quadratic equation, then the equation can be written as $x^2 - px + q = 0$.

### ❒ Constructing a New Quadratic Equation by Changing the Roots of a Given Quadratic Equation

If we are given a quadratic equation, we can build a new quadratic equation by changing the roots of this equation in the manner specified to us.

For example, let us take a quadratic equation $ax^2 + bx + c = 0$ and let its roots be $\alpha$ and $\beta$ respectively. Then we can build new quadratic equations as per the following patterns:

1. A quadratic equation whose roots are the reciprocals of the roots of the given equation $ax^2 + bx + c = 0$, i.e., the roots are $1/\alpha$ and $1/\beta$:
   This can be obtained by substituting $1/x$ in place of $x$ in the given equation giving us $cx^2 + bx + a = 0$, i.e., we get the equation required

by interchanging the co-efficient of $x^2$ and the constant term.

2. A quadratic equation whose roots are $k$ more than the roots of the equation $ax^2 + bx + c = 0$, i.e., the roots are $(\alpha + k)$ and $(\beta + k)$
   This can be obtained by substituting $(x - k)$ in place of $x$ in the given equation.
3. A quadratic equation whose roots are $k$ less than the roots of the equation $ax^2 + bx + c = 0$, i.e., the roots are $(\alpha - k)$ and $(\beta - k)$
   This can be obtained by substituting $(x + k)$ in place of $x$ in the given equation.
4. A quadratic equation whose roots are $k$ times the roots of the equation $ax^2 + bx + c = 0$, i.e., the roots are $k\alpha$ and $k\beta$.
   This can be obtained by substituting $x/k$ in place of $x$ in the given equation.
5. A quadratic equation whose roots are $1/k$ times the roots of the equation $ax^2 + bx + c = 0$, i.e., the roots are $\alpha/k$ and $\beta/k$
   This can be obtained by substituting $kx$ in place of $x$ in the given equation.
   An equation whose degree is '$n$' will have $n$ roots.

## ❒ MAXIMUM OR MINIMUM VALUE OF A QUADRATIC EXPRESSION

$ax^2 + bx + c = 0$ is called a quadratic equation, where as a quadratic expression is an expression of the form $f(x) = ax^2 + bx + c$.

As $x$ varies from $-\infty$ to $+\infty$, (i.e., when $x$ is real) the quadratic expression $ax^2 + bx + c$

1. has a minimum value whenever $a > 0$ (i.e., $a$ is positive). The minimum value of the quadratic expression is $(4ac - b^2) / 4a$ and it occurs at $x = -b/2a$.
2. has a maximum value whenever $a < 0$ (i.e., $a$ is negative). The maximum value of the quadratic expression is $(4ac - b^2) / 4a$ and it occurs at $x = -b/2a$.

## ❒ Polynomials and Polynomial Equations

**An expression of the form $a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ is called polynomial.** If we denote it as $f(x)$, $f(x) = 0$ is a polynomial equation.

**Remainder Theorem:** When a polynomial $p(x)$ of degree $n$ is divided by $x - a$ (a linear polynomial), there results a quotient polynomial $q(x)$ (of degree $(n - 1)$) and a remainder (of degree 0), i.e., a constant.

i.e., $p(x) = (x - a)\ q(x) + R$

This relation is true for all values of $x$. In particular, for $x = a$, we get, $p(a) = R$. This result is the Remainder Theorem.

**NOTE**

1. If $p(a) = 0$, we say that '$a$' is a zero of the polynomial $p(x)$.
2. If $p(x)$ is a polynomial and '$a$' is a zero of $p(x)$, then $p(x) = (x - a)\ q(x)$.
3. If $p(x)$ is divided by $ax + b$, then the remainder is given by $p\left(\frac{-b}{a}\right)$.
4. If $p(x)$ is divided by $ax - b$, then the remainder is given by $p\left(\frac{b}{a}\right)$.
5. The degree of remainder is always less than the degree of divisor.

**Factor theorem:** If $R = 0$, i.e., $p(a) = 0$, then $x - a$ is $a$ factor of $p(x)$ and conversely, if $x - a$ is a factor of $p(x)$, then $p(a) = 0$. **This immediate consequence of the Remainder Theorem is called Factor Theorem.** This can be restated as follows: The number $a$ is a root of $p(x) = 0$, if and only if $(x - a)$ is a factor of $p(x)$.

## ❒ Division of a Polynomial by a Polynomial

**Long Division Method**

Step 1: First arrange the terms of the dividend and the divisor in the descending order of their degrees.

Step 2: Now the first term of the quotient is obtained by dividing the first term of the dividend by the first term of the divisor.

Step 3: Then multiply all the terms of the divisor by the first term of the quotients and subtract the result from the dividend.

Step 4: Consider the remainder as new dividend and proceed as before.

Step 5: Repeat this process till we obtain a remainder which is either 0 or a polynomial of degree less than that of the divisor.

**Example:** Divide $2x^3 + 9x^2 + 4x - 15$ by $2x + 5$.

**Solution:**

$$2x+5)\ 2x^3+9x^2+4x-15\ (x^2+2x-3$$
$$2x^3+5x^2$$
$$(-)\ \ (-)$$
$$4x^2+4x$$
$$4x^2+10x$$
$$(-)\ \ (-)$$
$$-6x-15$$
$$-6x-15$$
$$(+)\ \ (+)$$
$$0$$

$\therefore (2x^3 + 9x^2 + 4x - 15) \div (2x + 5) = x^2 + 2x - 3$

**Example:** Find the quotient and the remainder when $x^4 + 4x^3 - 31x^2 - 94x + 120$ is divided by $x^2 + 3x - 4$.

**Solution:**

$$x^2+3x-4)\ x^4+4x^3-31x^2-94x+120\ (x^2+x-30$$
$$x^4+3x^3-4x^2$$
$$-\ \ -\ \ +$$
$$x^3-27x^2-94x+120$$
$$x^3+3x^2-4x$$
$$-\ \ -\ \ +$$
$$-30x^2-90x+120$$
$$-30x^2-90x+120$$
$$+\ \ +\ \ -$$
$$0$$

$\therefore$ the quotient is $x^2 + x - 30$ and the remainder is '0'

## ❒ Relations between Roots and Coefficients

An $n^{th}$ order equation has $n$ roots. Corresponding to every root, there is a factor. If $\alpha$ is a root of $f(x) = 0$, then $x - \alpha$ is a factor of $f(x)$. Sometimes $(x - \alpha)^2$ may also be a factor. In such a case, $\alpha$ is said to be a double root. Similarly, equations can have triple roots, quadruple roots and roots of multiplicity $m$. If $m$ is the greatest value of $k$, for which $(x - \alpha)^k$ is a factor of $f(a)$, then $\alpha$ is said to be a root of multiplicity $m$. If all the roots are counted by taking their multiplicity into account, the number of roots is equal to $n$, the degree of the equation.

If $\alpha_1, \alpha_2, \ldots, \alpha_n$ (not necessarily distinct) are the roots of $f(x) = 0$, then

$f(x) = a_n(x - \alpha_1)\ (x - \alpha_2)\ \ldots.(x - \alpha_n)$

$= a_n[x^n - S_1\ x^{n-1} + S_2 x^{n-2} \ldots.. + (-1)^n\ S_n]$

where $S_1$ = the sum of the roots

$S_2$ = the sum of the products of the roots taken 2 at a time

$S_3$ = the sum of the product of the roots taken 3 at a time and so on.

$S_n$ = the 'sum' of the product of the roots taken $n$(or all) at a time. Thus, $S_n$ is a single term.

$S_n = \alpha_1\ \alpha_2\ \ldots..\alpha_n$

Let us write down the polynomial $f(x)$ in two forms:

The standard form

$f(x) = a_n\ x^n + a_{n-1}\ x^{n-1} + a_{n-2}\ x^{n-2} \ldots + a_1 x + a_0$

In terms of the roots of the corresponding equation.

$f(x) = a_n\ [x_n - S_1 x^{n-1} + S_2 x^{n-2} + \ldots. + (-1)^{x-1}\ S_{n-1} x + (-1)^n S_n]$

These polynomials are identically equal, i.e., equal for all values of $x$. Therefore the corresponding coefficients are equal. The sum of the roots $S_1 = -a_{n-1}/a_n$

The sum of the products of the roots, taken two at a time, $S_2 = a_{n-2}/a_n$

The sum of the products of the roots, taken three at a time, $S_3 = -a_{n-3}/a_n$ and so on.

The 'sum' of the 'products' of the roots taken $m$ ($m \le n$) at a time $S_m = \Sigma\alpha_1\ \alpha_2\ \alpha_3 \ldots..\alpha_m = (-1)^m\ \dfrac{a_{n-m}}{a_n}$

$\therefore S_n = \alpha_1\ \alpha_2\ \alpha_3 \ldots..\alpha_n = (-1)^n\ \dfrac{a_0}{a_n}$

For example, consider the polynomial equation

$(x - 1)(x - 2)(x - 3) = x^3 - 6x^2 + 11x - 6 = 0$

(We can see immediately that the roots are 1, 2, 3)

The sum of roots = $(1 + 2 + 3) = -(-6)/1$

The sum of the products of the roots, taken two at a time $S_2 = 1(2) + 1(3) + 2(3) = 11 = 11/1$

We can drop the word 'sum' and 'products' for the last relation, because there is only one term (only one product). The product = $1(2)(3) = 6 = -(-6)/1$.

## ❒ Roots of Equations and Descartes' Rule

If the coefficients are all real and the complex number $z_1$, is a root of $f(x) = 0$, then the conjugate of $z_1$, viz, $\overline{z_1}$ is also a root of $f(x) = 0$. Thus, for equations with real, coefficients, complex roots occur in pairs.

A consequence of this is that any equation of an odd degree must have at least one real root.

The number of roots is related to very simple properties of the equation as illustrated below.

Let $\alpha_1$ be a positive root, i.e., $x - \alpha_1$, is a factor.

Let $\alpha_2$ be another positive root, i.e., $x^2 - (\alpha_1+\alpha_2)x + \alpha_1\alpha_2$ is a factor.

Let $\alpha_3$ be another positive root, i.e., $x^3 - (\alpha_1+\alpha_2+\alpha_3)x^2 + (\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_1)x - \alpha_1\alpha_2\alpha_3$ is a factor. We note that every positive root introduces a sign change in the polynomial. For 1 root, there is 1 sign change (the coefficient of $x$ is positive and $-\alpha_1$ is negative).

The second root results in a second sign change [$x^2 - (\alpha_1 + \alpha_2)x + \alpha_1\alpha_2$ has 2 sign changes] and so on.

But every sign change need not correspond to a real positive root. (For example, $x^2 - 2x + 4$ has two sign changes but the corresponding equation $x^2 - 2x + 4 = 0$ has no real roots.)

**The number of positive roots of $f(x) = 0$ is at the most equal to the number of sign changes in $f(x)$. It could also be less than that by 2, 4…, i.e., if there are $k$ sign changes in $f(x)$, the number of positive roots could be $k$, $k$–2, $k$–4, …**

**This is called Descartes' Rule of Signs. This rule can be extended to negative roots as follows. The number of negative roots of $f(x) = 0$ is at the most equal to the number of sign charges in $f(-x)$.**

For example, consider

$f(x) = x^5 - 3x^3 + 6x^2 - 28x + 24$. There are 4 sign changes in $f(x)$

$\therefore$ The number of positive roots could be 4, 2 or 0,

Consider $g(x) = f(-x)$

$(-x)^5 - 3(-x)^3 + 6(-x)^2 - 28(-x) + 24$

$= -x^5 + 3x^3 + 6x^2 + 28x + 24$

There is only one sign change in $f(-x)$. $\therefore$ The number of negative roots of $f(x) = 0$ is 1. (It cannot be –1, –3, …)

The following table shows the various possibilities for the five roots.

| Negative | Positive | Complex |
|---|---|---|
| 1 | 4 | 0 |
| 1 | 2 | 2 |
| 1 | 0 | 4 |

We can use more advanced techniques to find the actual roots. But even without that, using only Descartes' Rule, we expect exactly one of the 3 possibilities shown in the table above.

## SOLVED EXAMPLES

**4.01.** $f(x) = x^2 + 12x + 20$. Find the remainder when $f(x)$ is divided by $x + 14$.

**Sol:** $f(x) = x^2 + 12x + 20$

Remainder theorem: If $f(x)$ (i.e., a function of $x$) is divided by $x - a$, the remainder of the division is $f(a)$.

The remainder of the division of $f(x)$ by $x + 14$ i.e., $x - (-14)$ is $f(-14)$.

$f(-14) = (-14)^2 + 12(-14) + 20 = 196 - 168 + 20 = 48$

**4.02.** $g(x) = 4x^2 + 12x + 20$. Find the remainder when $g(x)$ is divided by $2x - 1$.

**Sol:** If $f(x)$ is divided by $ax \pm b$, the remainder of the division is $f\left(\mp\frac{b}{a}\right)$.

$g(x) = 4x^2 + 12x + 20$

If $g(x)$ is divided by $2x - 1$, the remainder of the division is $g\left(\frac{1}{2}\right)$.

$$g\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^2 + 12\left(\frac{1}{2}\right) + 20 = 4\frac{1}{4} + 6 + 20 = 27$$

**4.03.** A linear function of $x$ leaves a remainder of 20 when divided by $x - 5$. The function is divisible by $x + 5$. Find the linear function.

**Sol:** The linear function is divisible by $x + 5$.

$\therefore$ It must be $k(x + 5)$, where $k$ is a constant.

Let us denote the linear function by $g(x)$.

$g(x) = k(x + 5)$

Remainder theorem: If $f(x)$ is divided by $x - a$, then the remainder of the division is $f(a)$.

The remainder of the division of $g(x)$ by $x - 5$ is $g(5)$. This is also equal to 20.

$g(5) = 20 \Rightarrow k(5 + 5) = 20$

$\Rightarrow k = 2$

$g(x) = k(x + 5) = 2x + 10$.

**4.04.** If $(a - 2)x^3 + (a + 1)x^2 + x - 2a$ is divisible by $x + 3$, find the value of $a$.

**Sol:** Let $g(x) - (a - 2)x^3 + (a + 1)x^2 + x - 2a$

Factor theorem: $x - a$ is a factor of $f(x)$ if and only if $f(a) = 0$.

$g(x)$ is divisible by $x + 3$ (i.e., $x - (-3)$) is a factor of $g(x)$.

$\therefore g(-3) = 0$

$g(-3) = (a-2)(-27) + (a+1)(9) + (-3) - 2a$
$= -20a + 60$
$g(-3) = 0$
$\Rightarrow -20a + 60 = 0$
$\Rightarrow a = 3$

**4.05.** If $ax^2 + bx + c$ is divisible by $3x - 4$, the value of $16a + 12b + 9c$ is _____.

**Sol:** Factor theorem: $x - a$ is a factor of $f(x)$ if and only if $f(a) = 0$.
If $ax \pm b$ is a factor of $f(x)$, $f\left(\mp\frac{b}{a}\right) = 0$ ($\because$ This follows from the Factor theorem)
$ax^2 + bx + c$ is divisible by $3x - 4$. $\therefore$ $3x - 4$ is a factor of $ax^2 + bx + c$.
Let $g(x) = ax^2 + bx + c$
As $3x - 4$ is a factor of $g(x)$, $g\left(\frac{4}{3}\right) = 0$

$$a\left(\frac{4}{3}\right)^2 + b\left(\frac{4}{3}\right) + c = 0$$

$16a + 12b + 9c = 0$

**4.06.** $h(x) = 4x^2 + 12x + 20$. Find the remainder when $h(x)$ is divided by $2x + 1$.

**Sol:** If $f(x)$ is divided by $ax \pm b$, the remainder of the division is $f\left(\mp\frac{b}{a}\right)$.
$h(x) = 4x^2 + 12x + 20$
if $h(x)$ is divided by $2x + 1$, the remainder of the division is $h\left(\frac{-1}{2}\right)$.

$$h\left(\frac{-1}{2}\right) = 4\left(\frac{-1}{2}\right)^2 + 12\left(\frac{-1}{2}\right) + 20 = 15$$

**4.07.** $Q(x)$ is a quadratic expression and $Q(0) = 12$. The remainder of $Q(x)$ divided by $x - 1$ is 5. The remainder of $Q(x)$ divided by $x + 2$ is 38. Find $Q(x)$.

**Sol:** Let $Q(x) = ax^2 + bx + c$. $Q(0) = 12$. $\therefore$ $c = 12$
$Q(x) = ax^2 + bx + 12$
Remainder theorem: If $f(x)$ is divided by $x - a$, the remainder of the division is $f(a)$.
The remainder of $Q(x)$ divided by $x - 1$ is $Q(1)$. This is also equal to 5.
$Q(1) = 5$ Also $Q(-2) = 38$
$a + b + 12 = 5$ and $4a - 2b + 12 = 38$
$\therefore$ $a = 2$ and $b = -9$
$Q(x) = ax^2 + bx + 12 = 2x^2 - 9x + 12$

**4.08.** $f(x) = ax^3 - 2x^2 - x + b$. If $f(x)$ is divisible by $x^2 - 1$, the value of $(a, b)$ is _____.

**Sol:** $f(x)$ is divisible by $x^2 - 1$, i.e., $(x+1)(x-1)$
$\therefore$ $(x+1)(x-1)$ is a factor of $f(x)$.
$x + 1$ and $x - 1$ are both factors of $f(x)$.
Factor theorem: $x - a$ is a factor of $f(x)$ if and only if $f(a) = 0$
$f(-1) = a(-1)^3 - 2(-1)2 - (-1) + b = 0$ (1)
and $f(1) = a - 2 - 1 + b = 0$ (2)
Solving (1) and (2), $a = 1$ and $b = 2$

**4.09.** A quadratic expression in $x$ leaves remainders of 9 and 4 when divided by $x + 2$ and $x - 3$, respectively. The expression is divisible by $x + 1$. Find the quadratic expression.

**Sol:** Let the quadratic expression in $x$ be $Q(x)$
Let $Q(x) = ax^2 + bx + c$
Remainder theorem: If $f(x)$ is divided by $x - a$, then the remainder of the division is $f(a)$.
The respective remainders when $Q(x)$ is divided by $x + 2$, $x - 3$, $x + 1$ are 9, 4, 0.
$Q(-2) = 4a - 2b + c = 9$ (1)
$Q(3) = 9a + 3b + c = 4$ (2)
$Q(-1) = a - b + c = 0$ (3)
(1)–(2): $-5a - 5b = 5 \Rightarrow a + b = -1$
(1)– (3): $3a - b = 9$
Solving these, $a = 2$. $\therefore$ $b = -3$. $\therefore$ $c = -5$
$Q(x) = 2x^2 - 3x - 5$

**4.10.** If $3x - 1$ is a common factor of $6x^2 + (6b + 1)x - (a + 1)$ and $3ax^2 - (b - 1)x - 1$, then $(a, b) =$

**Sol:** Let $g(x) = 6x^2 + (6b + 1)x - (a + 1)$ and
$h(x) = 3ax^2 - (b - 1)x - 1$
Factor theorem: $x - a$ is a factor of $f(x)$ if and only if $f(a) = 0$
If $ax + b$ is a factor of $f(x)$, $f\left(\frac{-b}{a}\right) = 0$ ($\because$ This follows from the Factor theorem)
$3x - 1$ is a common factor of $g(x)$ and $h(x)$.

$$\therefore g\left(\frac{-1}{3}\right) = h\left(\frac{1}{3}\right) = 0$$

$$g\left(\frac{1}{3}\right) = 6\left(\frac{1}{3}\right)^2 + (6b+1)\left(\frac{1}{3}\right) - (a+1) = 0 \quad (1)$$

$$h\left(\frac{1}{3}\right) = 3a\left(\frac{1}{3}\right)^2 - (b-1)\frac{1}{3} - 1 = 0 \quad (2)$$

$\Rightarrow a = 2b$

$$\Rightarrow \frac{a}{3} - \frac{(b-1)}{3} - 1 = 0$$

(1) $\Rightarrow 2 + (6.6 + 1) - 3(a+1) = 0 \Rightarrow a = 26$
(2) $\Rightarrow a - (b - 1) = 3 \Rightarrow a - b = 2$
$\therefore$ $(a, b) = (4, 2)$

**4.11.** Find the remainder of $x^{999}$ divided by $x^2 - 6x + 5$.

**Sol:** Let the quotient and the remainder of $x^{999}$ divided by $x^2 - 6x + 5$, i.e., $(x-1)(x-5)$ be $q(x)$ and $r(x)$, respectively.

$x^{999} = (x-1)(x-5)\,q(x) + r(x)$

As the divisor $x^2 - 6x + 5$ is a quadratic expression, the remainder $r(x)$ must be a linear expression.

Let $r(x) = ax + b$, where a and $b$ are constants.

$x^{999} = (x-1)(x-5)\,q(x) + ax + b$ (1)

Setting $x = 1$ in (1): $1 = a + b$ (2)

Setting $x = 5$ in (1): $5^{999} = 5a + b$ (3)

(3) − (2): $5^{999} - 1 = 4a$

$$a = \frac{5^{999} - 1}{4}$$

$$\therefore\ b = \frac{-5^{999} + 5}{4} \quad (\because \text{From (2) or (3)})$$

Remainder is $ax + b$, i.e., $\left(\frac{5^{999} - 1}{4}\right)x + \frac{-5^{999} + 5}{4}$

**4.12.** Find the remainder when $x^5$ is divided by $x^3 - 9x$.

**Sol:** Let the quotient and the remainder of $x^5$ divided by $x^3 - 9x$, i.e., $x(x+3)(x-3)$ be $q(x)$ and $r(x)$, respectively.

$x^5 = x(x+3)(x-3)\,q(x) + r(x)$

As the divisor $x^3 - 9x$ is a cubic expression, the remainder $r(x)$ must be an expression whose degree is at most 2, i.e., it must be a quadratic expression or a linear expression.

Let $r(x) = ax^2 + bx + c$ (if $a = 0$, $r(x)$ is a linear expression. Otherwise, it is a quadratic expression)

$x^5 = x(x+3)(x-3)\,q(x) + ax^2 + bx + c$ (1)

Setting $x = 0$ in (1): $0 = c$

Setting $x = -3$ in (1): $-243 = 9a - 3b + c$

Setting $x = 3$ in (1): $243 = 9a + 3b + c$

$-243 = 9a - 3b$ and $243 = 9a + 3b$

$-81 = 3a - b$ and $81 = 3a + b$

Solving these, $a = 0$ and $b = 81$

$r(x) = ax^2 + bx + c = 0\ x^2 + 81x + 0 = 81x$

**4.13.** If $f(x) = x^2 + 14x + 30$, find the remainder when $f(x)$ is divided by $x + 4$.

**Sol:** We know that when $f(x)$ is divided by $x + a$, the remainder is $f(-a)$,

Given, $f(x) = x^2 + 14x + 30$. The required remainder is $f(-4) = (-4)^2 + 14(-4) + 30 = -10$

$\therefore$ Remainder is $-10$,

**4.14.** If $g(x) = 3x^2 + 7x + 15$, find the remainder when $g(x)$ is divided by $3x - 2$.

**Sol:** We know that when $f(x)$ is divided by $ax - b$, the remainder is $f(b/a)$,

$g(x) = 3x^2 + 7x + 15$

$$\therefore\ \text{Rem}\ \frac{g(x)}{3x-2} = g\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + 15$$

$$= \frac{4}{3} + \frac{14}{3} + 15$$

$= 6 + 15 = 21$

$\therefore$ Remainder is 21.

**4.15.** A linear function in $x$ leaves a remainder of 32 when divided by $x - 4$. If the function is divisible by $x + 4$, find the linear function.

**Sol:** Since $x + 4$ is a factor of the linear function, let the linear function be $k(x + 4)$.

$f(x) = k(x + 4)$

Given $f(4) = 32 \Rightarrow k(4 + 4) = 32 \Rightarrow k = 4$

$\therefore$ The linear function $= 4(x + 4) = 4x + 16$.

**4.16.** If $(a-3)x^3 + (a+2)x^2 + x - 2a$ is divisible by $x + 2$, find the value of $a$.

**Sol:** Let $f(x) = (a-3)x^3 + (a+2)x^2 + x - 2a$

Given $x + 2$ is a factor of $f(x)$

$\therefore\ f(-2) = 0$

$(a-3)(-8) + (a+2)(4) - 2 - 2a = 0$

$\Rightarrow -8a + 24 + 4a + 8 - 2 - 2a = 0$

$\Rightarrow -6a + 30 = 0 \Rightarrow a = 5.$

**4.17.** If $ax^2 + bx + c$ is divisible by $5x - 3$, find the value of $9a + 15b + 25i$.

**Sol:** Let $f(x) = ax^2 + bx + c$

Given $f(x)$ is divisible by $5x - 3$

$$\therefore\ f\left(\frac{3}{5}\right) = 0$$

$$a\left(\frac{9}{25}\right) + b\left(\frac{3}{5}\right) + c = 0 \Rightarrow\ 9a + 15b + 25c = 0.$$

**4.18.** If $f(x) = 3x^2 + 9x + 15$, find the remainder when $f(x)$ is divided by $3x + 1$.

**Sol:** We know that when $f(x)$ is divided by $ax + b$, the remainder is $f\left(\frac{-b}{a}\right)$

$f(x) = 3x^2 + 9x + 15$ and the divisor is $3x + 1$

required remainder is $f\left(\frac{-1}{3}\right)$

$$= 3\left(\frac{-1}{3}\right)^2 + 9\left(\frac{-1}{3}\right) + 15 = \frac{1}{3} - 3 + 15 = \frac{37}{3}$$

**4.19.** $Q(x)$ is a quadratic expression and $Q(0) = 14$. The remainder of $Q(x)$ when divided by $x + 1$ is

25. The remainder of $Q(x)$ when divided by $x-2$ is 10. Find $Q(x)$.

**Sol:** Let $Q(x) = ax^2 + bx + c$

Given $Q(0) = 14 \Rightarrow c = 14$

$Q(-1) = 25$

$\therefore\ a - b + c = 25$

$\Rightarrow a - b = 25 - 14 \Rightarrow a - b = 11$ (1)

given $Q(+2) = 10$

$4a + 2b + c = 10 \Rightarrow 4a + 2b = 10 - 14$

$\Rightarrow 4a + 2b = -4$

$\Rightarrow 2a + b = -2$ (2)

Solving (1) and (2) $\Rightarrow\ a = 3;\ b = -8$

$\therefore\ Q(x) = 3x^2 - 8x + 14.$

**4.20.** $f(x) = ax^3 + 3x^2 - 4x + b$. If $f(x)$ is divisible by $x^2 - 4$, find $a + b$.

**Sol:** Given $f(x)$ is divisible by $x^2 - 4$.

$\therefore\ f(x)$ is also divisible by $x-2$ and $x+2$.

$f(2) = 0;\ f(-2) = 0$

$f(x) = ax^3 + 3x^2 - 4x + b$

$f(2) = 0 \Rightarrow 8a + 12 - 8 + b = 0 \Rightarrow 8a + b = -4$ (1)

$f(-2) = 0 \Rightarrow -8a + 12 + 8 + b = 0$

$\Rightarrow -8a + b = -20$ (2)

Solving (1) and (2) $a = 1;\ b = -12$

$\therefore\ a + b = 1 - 12 = -11.$

**4.21.** A quadratic expression in $x$ leaves remainders 13 and 8 when divided by $x-2$ and $x+3$, respectively. If the expression is divisible by $x-1$, find the quadratic expression.

**Sol:** Let the quadratic expression be $Q(x)$

$= ax^2 + bx + c$

Given $Q(1) = 0$; and $Q(2) = 13$, $Q(-3) = 8$.

$\Rightarrow a + b + c = 0$ (1)

$\Rightarrow 4a + 2b + c = 13$ (2)

$\Rightarrow 9a - 3b + c = 8$ (3)

Solving (1), (2) and (3) we will get $a = 3$, $b = 4$, $c = -7$

$\therefore\ Q(x) = 3x^2 + 4x - 7.$

**4.22.** If $3x-2$ is a common factor of $12x^2 + (5b+2)x - (3a+1)$ and $6ax^2 + (4b+5)x - 14$, find $(a, b)$.

**Sol:** Let $f(x) = 12x^2 + (5b+2)x - (3a+1)$

$g(x) = 6ax^2 + (4b+5)x - 14$

given $3x-2$ is a factor of both $f(x)$ and $g(x)$.

$\therefore\ f(2/3) = 0$ and $g(2/3) = 0$

$$12.\frac{4}{9} + (5b+2)\left(\frac{2}{3}\right) - (3a+1) = 0$$

$$\frac{16}{3} + \frac{10b+4}{3} - 3a - 1 = 0$$

$10b - 9a = -17$ (1)

$$g(2/3) = 0 \Rightarrow 6a\frac{4}{9} + (4b+5)\frac{2}{3} - 14 = 0$$

$$\frac{8a}{3} + \frac{8b}{3} + \frac{10}{3} - 14 = 0$$

$8a + 8b = 32$

$a + b = 4$ (2)

solving (1) and (2) $a = 3$, $b = 1$, i.e., $(a, b) = (3, 1)$.

**4.23.** Find the remainder when $x^{888}$ is divided by $x^2 - 5x + 6$.

**Sol:** Let $f(x) = x^{888}$

Let $f(x)$ be divided by $x^2 - 5x + 6$.

The remainder is $ax + b$ and $Q(x)$ is the quotient.

By division algorithm

$f(x) = (x^2 - 5x + 6)\ Q(x) + R(x)$

$x^{888} = (x-3)\ (x-2)\ Q(x) + ax + b$

put $x = 2$

$2^{888} = 2a + b$ (1)

Put $x = 3$

$3^{888} = 3a + b$ (2)

Solving (1) and (2)

$a = 3^{888} - 2^{888}$

$b = 3 \cdot 2^{888} - 2 \cdot 3^{888}$

$\therefore$ Required remainder is $(3^{888} - 2^{888})x + 3\ (2^{888}) - 2\ (3^{888})$.

**4.24.** Find the remainder when $x^5$ is divided by $x^3 - 4x$.

**Sol:** Let $f(x) = x^5$

Since divisor $x^3 - 4x$ is a cubic expression, the remainder must be a quadratic expression.

If $f(x)$ is divided by $x^3 - 4x$, the remainder is $ax^2 + bx + c$ and quotient is, say $q(x)$

By division algorithm, $f(x) = (x^3 - 4x)\ q(x) + r(x)$

$x^5 = x(x-2)\ (x+2)\ q(x) + ax^2 + bx + c.$

Set $x = 0$ we get $c = 0$

Set $x = 2$ we get $2^5 = 4a + 2b \Rightarrow 16 = 2a + b$ (1)

Set $x = -2$

$(-2)^5 = 4a - 2b$

$-16 = 2a - b$ (2)

Solving (1) and (2) $a = 0$, $b = 16$

$\therefore$ Required remainder is $16x$.

Alternately, by dividing $x^5$ by $x^3 - 4x$ using long division, we will get the same remainder.

**4.25.** Find the roots of the equation $2x^2 + 13x + 18 = 0$.

**Sol:** To find the roots of a quadratic equation, the following steps are required. First, write the coefficient of $x$, i.e., 13 as the sum (or difference) of

two parts such that the product of these two parts is equal to the coefficient of $x^2$ term and constant term, i.e., product of 2 and 18 which is 36.
We see that 13 can be written as the sum of 4 and 9 and the product of these two numbers is 36.
$2x^2 + 13x + 18 = 0 \Rightarrow 2x^2 + 4x + 9x + 18 = 0$
Taking $2x$ common from the first two terms and taking 9 common from the last two terms, we have:
$\Rightarrow 2x\,(x+2) + 9\,(x+2) = 0$
$\Rightarrow (x+2)\,(2x+9) = 0,\ x = -2 \text{ or } -\frac{9}{2}$.

**4.26.** Find the roots of the equation $x^2 + x - 12 = 0$.

**Sol:** Given equation is $x^2 + x - 12 = 0$
Applying the procedure described above, we have
$\Rightarrow x^2 + 4x - 3x - 12 = 0$
$\Rightarrow x\,(x+4) - 3\,(x+4) = 0$
$\Rightarrow (x-3)\,(x+4) = 0 \Rightarrow x = 3 \text{ or } x = -4$.

**4.27.** Find the roots of the equation $11x^2 - 37x + 30 = 0$.

**Sol:** We have to write $-37$ as the sum of two parts whose product should be equal to $(11) \times (30)$
$(-22) + (-15) = -37$ and $(-22)\,(-15) = 11 \times 30$
Therefore, $11x^2 - 37x + 30 = 0$
$\Rightarrow 11x^2 - 22x - 15x + 30 = 0$
$\Rightarrow 11x\,(x-2) - 15\,(x-2) = 0$
$\Rightarrow (11x - 15)\,(x-2) = 0\ x = \frac{15}{11}$ or 2.

**4.28.** Discuss the nature of the roots of the equation $8x^2 - 2x - 4 = 0$.

**Sol:** For the quadratic equation $ax^2 + bx + c = 0$ the nature of the roots is given by the discriminant $b^2 - 4ac$.
Discriminant of $8x^2 - 2x - 4 = 0$ is $(-2)^2 - 4(8)(-4) = 132$.
Since the discriminant is positive but not a perfect square, the roots of the equation are irrational and unequal.

**4.29.** Comment on the nature of the roots of $3x^2 - x - 4 = 0$.

**Sol:** Discriminant of $3x^2 - x - 4 = 0$ is $(-1)^2 - 4\,(3)\,(-4) = 1 + 48 = 49$. Since the discriminant is positive and a perfect square, the roots of the equation are rational and unequal.

**4.30.** If the sum of the roots of the equation $Rx^2 + 5x - 24 = 0$ is $5/11$, then find the product of the roots of that equation.

**Sol:** For a quadratic equation $ax^2 + bx + c = 0$, the sum of the roots is $(-b/a)$ and the product of the roots is $(c/a)$.
Sum of the roots of the equation
$Rx^2 + 5x - 24 = 0$ is $\left(\frac{-5}{R}\right)$ which is given as $\frac{5}{11}$
$\therefore R = -11$
In the given equation, product of the roots $= \frac{-24}{R} = \frac{-24}{-11} = +\frac{24}{11}$.

**4.31.** Find the value of $k$, so that the roots of $6x^2 - 12x - k = 0$ are reciprocals of each other.

**Sol:** If the roots of the equation are reciprocals of each other, then the product of the roots should be equal to 1.
$\Rightarrow \frac{-k}{6} = 1$. Therefore $k = -6$.

**4.32.** If $4 + \sqrt{7}$ is one root of a quadratic equation with rational coefficients, then find the other root of the equation.

**Sol:** When the coefficients of a quadratic equation are rational and the roots are irrational, they occur only in pairs like $p \pm \sqrt{q}$, i.e., if $p + \sqrt{q}$ is one root, then the other root of the equation will be $p - \sqrt{q}$. So, in this case, the other root of the equation will be $4 - \sqrt{7}$.

**4.33.** Find the positive value of $k$ if one root of the equation $x^2 - kx + 243 = 0$ is thrice the other root.

**Sol:** If one root of the equation is $\alpha$, then the other root will be $3\alpha$.
We have $(\alpha)\,(3\alpha) = 3\alpha^2 = 243$
$\Rightarrow \alpha^2 = 81$
$\Rightarrow \alpha = \pm 9$. Hence, $3\alpha = \pm 27$.
Sum of the roots $= -\left(\frac{-k}{1}\right) = k = 4\alpha = \pm 36$.
Since we need the positive value of $k$, so $k = 36$.

**4.34.** Form a quadratic equation whose roots are 4 and 21.

**Sol:** Sum of the roots $= 4 + 21 = 25$.
Product of the roots $= 4 \times 21 = 84$.
We know that if $p$ is the sum of the roots and $q$ is the product of the roots of a quadratic equation, the equation will be $x^2 - px + q = 0$. Hence, the required equation will be $x^2 - 25x + 84 = 0$.

**4.35.** Form a quadratic equation with rational coefficients, one of whose roots is $5 + \sqrt{6}$.

**Sol:** If $5+\sqrt{6}$ is one root, then the other root is $5-\sqrt{6}$ (because the coefficients are rational).

Sum of the roots $= 5+\sqrt{6}+5-\sqrt{6} = 10$.

Product of the roots $= (5+\sqrt{6})(5-\sqrt{6}) = 25-6 = 19$.

Thus, the required equation is $x^2 - 10x + 19 = 0$.

**4.36.** If the price of each book goes up by ₹5, then Atul can buy 20 books less for ₹1200. Find the original price and the number of books Atul could buy at the original price.

**Sol:** Let the original price of each book be $x$.

Then the new price of each book will be $x+5$.

Number of books that can be bought at the original price $= \frac{1200}{x}$

Number of books that can be bought at the new price $= \frac{1200}{x+5}$

Given that Atul gets 20 books less at new price

i.e., $\frac{1200}{x} - \frac{1200}{x+5} = 20$

$\Rightarrow \frac{60}{x} - \frac{60}{x+5} = 1 \quad \Rightarrow \frac{60(x+5-x)}{x^2+5x} = 1$

$\Rightarrow 300 = x^2 + 5x \Rightarrow x^2 + 5x - 300 = 0$

$\Rightarrow (x+20)(x-15) = 0 \Rightarrow x = -20$ or $15$

As the price cannot be negative, the original price is ₹15.

**4.37.** If $\alpha$ and $\beta$ are the roots of the equation $x^2 - 3x - 180 = 0$ such that $\alpha < \beta$, then find the values of

(i) $\alpha^2 + \beta^2$ (ii) $\frac{1}{\alpha} + \frac{1}{\beta}$ (iii) $\alpha - \beta$

**Sol:** From the given equation, we get $\alpha + \beta = 3$ and $\alpha\beta = -180$

(i) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (3)^2 - 2(-180) = 369$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{+3}{-180} = \frac{-1}{60}$

(iii) $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$\Rightarrow \alpha - \beta = \pm\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= \pm\sqrt{(+3)^2 - 4(-180)} = \pm\sqrt{9+720}$

$= \pm\sqrt{729} = \pm 27$ ; as $\alpha < \beta$, $\alpha - \beta = -27$

**4.38.** If $\sqrt{x+4} + \sqrt{x+8} = 7$, then find the value of $x$.

**Sol:** Given $\sqrt{x+4} + \sqrt{x+8} = 7$

Squaring on both sides, we get

$x+4+x+8+2\left(\sqrt{x+4}\sqrt{x+8}\right) = 49$

$\Rightarrow 2x + 12 + 2\sqrt{x^2+12x+32} = 49$

$\Rightarrow 2x - 37 = -2\sqrt{x^2+12x+32}$

Squaring again on both sides, we have

$(2x-37)^2 = 4(x^2+12x+32)$

$\Rightarrow 4x^2 - 148x + 1369 = 4x^2 + 48x + 128$

$\Rightarrow 1241 = 196x$

$\Rightarrow x = \frac{1241}{196}$

**4.39.** If $4^{2x+1} + 4^{x+1} = 80$, then find the value of $x$.

**Sol:** Given $4^{2x+1} + 4^{x+1} = 80$

$\Rightarrow 4^{2x} \times 4 + 4^x \times 4 = 80$

$4^{2x} + 4^x = 20$

Substituting $4^x = a$,

we get $a^2 + a = 20$

$\Rightarrow a^2 + a - 20 = 0$

$\Rightarrow (a+5)(a-4) = 0$

$\Rightarrow a = -5$ or $4$

If $4^x = -5$, there is no possible value for $x$ as no power of 4 gives negative value.

If $4^x = 4$, then $x = 1$.

**4.40.** Find the nature of roots of the equation, $f(x) = x^3 + x - 2 = 0$.

**Sol:** There is only 1 change of sign in $f(x)$.

We know that when $f(x)$ has $r$ changes of sign then $f(x)$ has $r$, $r-2$, $r-4$, …. positive roots.

$\therefore f(x) = 0$ has one positive root.

Now $f(-x) \equiv -x^3 - x - 2 = 0$. $q = 0$

Since there is no change of sign in $f(-x)$, $f(x)$ has no negative roots. The number of complex roots is even.

$\therefore$ The equation has one positive root, and two complex roots.

Hence, $f(x) = 0$ has 1 real root and two complex roots.

**4.41.** How many non-real roots does the equation $x^4 - 2x^2 + 3x - 2 = 0$ have?

**Sol:** Let $f(x) = x^4 - 2x^2 + 3x - 2$

$f(x)$ has 3 sign changes

$\therefore f(x)$ has 3 or 1 positive roots.

$f(-x) = x^4 - 2x^2 - 3x - 2$

$\therefore f(-x)$ has one sign change

$\therefore f(x)$ has exactly one negative root.

As the sum of the co-efficient of $f(x)$ is zero, $x = 1$ is a root of $f(x) = 0$

$\therefore f(x) = (x-1)\ (x^3 + x^2 - x + 2) = (x-1)\ f_1(x)$

By trial, $f_1(-2) = 0$

$\therefore f_1(x) = (x+2)\ (x^2 - x + 1)$

We can see that $x^2 - x + 1 = 0$ has two non-real roots.

$\therefore f(x)$ has one positive, one negative and two non-real roots.

**4.42.** If $p-q,\ p,\ p+q$ are the roots of the equation $x^3 - 18x^2 + 99x - 162 = 0$, then find the values of $p$ and $q$.

**Sol:** Given $p-q,\ p,\ p+q$ are the roots of the equation.

$\therefore$ The sum of the roots is $(p-q) + p + (p+q) = 18$

$\Rightarrow 3p = 18 \Rightarrow p = 6$

and the product of the roots is $(p-q)\ p\ (p+q) = 162$

$p^2 - q^2 = \dfrac{162}{6} = 27 \Rightarrow 36 - q^2 = 27$

$\Rightarrow q = \pm 3 \ \therefore\ p = 6$ and $q = \pm 3$

**4.43.** Find the range of the expression $\dfrac{x-2}{x^2+x+3}$ where $x$ is real.

**Sol:** Let $f(x) = \dfrac{x-2}{x^2+x+3} = y$

$\Rightarrow x^2y + xy + 3y = x - 2$

i.e., $x^2y + x\ (y-1) + 3y + 2 = 0$

$f(x)$ can have any value $y$, provided the above equation in $x$ has real roots

$\therefore b^2 - 4ac \geq 0$

$\Rightarrow (y-1)^2 - 4y\ (3y+2) \geq 0$

i.e., $11y^2 + 10y - 1 \leq 0$

$(11y-1)\ (y+1) \leq 0 \Rightarrow -1 \leq y \leq \dfrac{1}{11}$

$\therefore$ The range of $y$ is $\left[-1, \dfrac{1}{11}\right]$

**4.44.** Solve the equation $x^4 - 2x^3 - 19x^2 + 8x + 60 = 0$, given that two of the roots are $\alpha$ and $-\alpha$, where $\alpha > 0$.

**Sol:** Let the roots of the equation be $\alpha, -\alpha, \beta, \gamma$. Let $\beta > \gamma$

The sum of the roots $= \alpha - \alpha + \beta + \gamma = 2$

$\Rightarrow \beta + \gamma = 2$

The sum of the products of the roots taken three at a time $=$

$-\alpha^2\beta - \alpha^2\gamma + \alpha\beta\gamma - \alpha\beta\gamma = -8$

$\alpha^2\ (\beta + \gamma) = 8. \ \therefore\ \alpha^2 = 4$

Product of the roots $= -\alpha^2\beta\gamma = 60.\ \beta\gamma = -15$

$\beta + \gamma = 2$ and $\beta\gamma = -15,\ \alpha^2 = 4$

$\beta = 5$ and $\gamma = -3,\ \alpha = \pm 2$

$\Rightarrow$ The roots are $-2, 2, -3$ and $5$.

**4.45.** Find the sum of the squares of the roots of the equation $x^3 - 4x^2 + x + 6 = 0$.

**Sol:** Let $\alpha, \beta, \gamma$ be the roots of the given equation.

$\alpha + \beta + \gamma = 4,\ \alpha\beta + \beta\gamma + \alpha\gamma = 1$ (and $\alpha\beta\gamma = -6$).

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$

$= (4)^2 - 2(1) = 14$

**4.46.** Find the roots of the equation $2x^2 + 13x + 18 = 0$.

**Sol:** To find the roots of a quadratic equation, the following steps are required. First write the coefficient of $x$, i.e., 13 as the sum (or difference) of two parts such that the product of these two parts is equal to the coefficient of $x^2$ term and constant term, i.e., product of 2 and 18 which is 36.

We see that 13 can be written as the sum of 4 and 9 and the product of these two numbers is 36.

$2x^2 + 13x + 18 = 0 \Rightarrow 2x^2 + 4x + 9x + 18 = 0$

Taking $2x$ common from the first two terms and taking 9 common from the last two terms, we have:

$\Rightarrow 2x\ (x+2) + 9\ (x+2) = 0$

$\Rightarrow (x+2)\ (2x+9) = 0,\ x = -2$ or $-\dfrac{9}{2}$.

**4.47.** Find the roots of the equation $x^2 + x - 12 = 0$.

**Sol:** Given equation is $x^2 + x - 12 = 0$

Applying the procedure described above, we have

$\Rightarrow x^2 + 4x - 3x - 12 = 0$

$\Rightarrow x\ (x+4) - 3\ (x+4) = 0$

$\Rightarrow (x-3)\ (x+4) = 0 \Rightarrow x = 3$ or $x = -4$.

**4.48.** Find the roots of the equation $11x^2 - 37x + 30 = 0$.

**Sol:** We have to write $-37$ as the sum of two parts whose product should be equal to $(11) \times (30)$

$(-22) + (-15) = -37$ and $(-22)\ (-15) = 11 \times 30$

Therefore, $11x^2 - 37x + 30 = 0$

$\Rightarrow 11x^2 - 22x - 15x + 30 = 0$

$\Rightarrow 11x\ (x-2) - 15\ (x-2) = 0$

$\Rightarrow (11x - 15)\ (x-2) = 0\ x = \dfrac{15}{11}$ or 2.

**4.49.** Discuss the nature of the roots of the equation $8x^2 - 2x - 4 = 0$.

**Sol:** For the quadratic equation $ax^2 + bx + c = 0$ the nature of the roots is given by the discriminant $b^2 - 4ac$.

Discriminant of $8x^2 - 2x - 4 = 0$ is

$(-2)^2 - 4(8)\ (-4) = 132$.

Since the discriminant is positive but not a perfect square, the roots of the equation are irrational and unequal.

**4.50.** Comment on the nature of the roots of $3x^2 - x - 4 = 0$.

**Sol:** Discriminant of $3x^2 - x - 4 = 0$ is $(-1)^2 - 4\,(3)\,(-4) = 1 + 48 = 49$. Since the discriminant is positive and a perfect square, the roots of the equation are rational and unequal.

**4.51.** If the sum of the roots of the equation $Rx^2 + 5x - 24 = 0$ is $5/11$, then find the product of the roots of that equation.

**Sol:** For a quadratic equation $ax^2 + bx + c = 0$, the sum of the roots is $(-b/a)$ and the product of the roots is $(c/a)$.

The sum of the roots of the equation

$Rx^2 + 5x - 24 = 0$ is $\left(\frac{-5}{R}\right)$ which is given as $\frac{5}{11}$

$\therefore R = -11$

In the given equation, product of the roots $= \frac{-24}{R} = \frac{-24}{-11} = +\frac{24}{11}$.

**4.52.** Find the value of $k$, so that the roots of $6x^2 - 12x - k = 0$ are reciprocals of each other.

**Sol:** If the roots of the equation are reciprocals of each other, then the product of the roots should be equal to 1.

$\Rightarrow \frac{-k}{6} = 1$. Therefore $k = -6$.

**4.53.** If $4 + \sqrt{7}$ is one root of a quadratic equation with rational co-efficients, then find the other root of the equation.

**Sol:** When the coefficients of a quadratic equation are rational and the roots are irrational, they occur only in pairs like $p \pm \sqrt{q}$ i.e., if $p + \sqrt{q}$ is one root, then the other root of the equation will be $p - \sqrt{q}$. So, in this case, the other root of the equation will be $4 - \sqrt{7}$.

**4.54.** Find the positive value of $k$ if one root of the equation $x^2 - kx + 243 = 0$ is three times the other root.

**Sol:** If one root of the equation is $\alpha$, then the other root will be $3\alpha$.

We have $(\alpha)\,(3\alpha) = 3\alpha^2 = 243$

$\Rightarrow \alpha^2 = 81$

$\Rightarrow \alpha = \pm 9$. Hence, $3\alpha = \pm 27$.

Sum of the roots $= -\left(\frac{-k}{1}\right) = k = 4\alpha = \pm 36$.

Since we need the positive value of $k$, so $k = 36$.

**4.55.** Form a quadratic equation whose roots are 4 and 21.

**Sol:** The sum of the roots $= 4 + 21 = 25$.

The product of the roots $= 4 \times 21 = 84$.

We know that if $p$ is the sum of the roots and $q$ is the product of the roots of a quadratic equation, the equation will be $x^2 - px + q = 0$. Hence, the required equation will be $x^2 - 25x + 84 = 0$.

**4.56.** Form a quadratic equation with rational coefficients, one of whose roots is $5 + \sqrt{6}$.

**Sol:** If $5 + \sqrt{6}$ is one root, then the other root is $5 - \sqrt{6}$ (because the coefficients are rational).

The sum of the roots $= 5 + \sqrt{6} + 5 - \sqrt{6} = 10$.

The product of the roots $= (5 + \sqrt{6})\,(5 - \sqrt{6})$

$= 25 - 6 = 19$.

Thus the required equation is

$x^2 - 10x + 19 = 0$.

**4.57.** If the price of each book goes up by ₹5, then Atul can buy 20 books less for ₹1200. Find the original price and the number of books Atul could buy at the original price.

**Sol:** Let the original price of each book be $x$.

Then the new price of each book will be $x + 5$.

The number of books that can be bought at the original price $= \frac{1200}{x}$

The number of books that can be bought at the new price $= \frac{1200}{x+5}$

Given that Atul gets 20 books less at new price,

i.e., $\frac{1200}{x} - \frac{1200}{x+5} = 20$

$\Rightarrow \frac{60}{x} - \frac{60}{x+5} = 1 \Rightarrow \frac{60(x+5-x)}{x^2+5x} = 1$

$\Rightarrow 300 = x^2 + 5x \Rightarrow x^2 + 5x - 300 = 0$

$\Rightarrow (x + 20)\,(x - 15) = 0 \Rightarrow x = -20$ or $15$

As the price cannot be negative, the original price is ₹15.

**4.58.** If $\alpha$ and $\beta$ are the roots of the equation $x^2 - 3x - 180 = 0$ such that $\alpha < \beta$, then find the values of

(i) $\alpha^2 + \beta^2$ (ii) $\frac{1}{\alpha} + \frac{1}{\beta}$ (iii) $\alpha - \beta$

**Sol:** From the given equation, we get $\alpha + \beta = 3$ and $\alpha\beta = -180$

(i) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (3)^2 - 2(-180) = 369$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{+3}{-180} = \frac{-1}{60}$

(iii) $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$\Rightarrow \alpha - \beta = \pm\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= \pm\sqrt{(+3)^2 - 4(-180)} = \pm\sqrt{9+720}$

$= \pm\sqrt{729} = \pm 27$ ; as $\alpha < \beta$, $\alpha - \beta = -27$

**4.59.** If $\sqrt{x+4} + \sqrt{x+8} = 7$, then find the value of $x$.

**Sol:** Given $\sqrt{x+4} + \sqrt{x+8} = 7$

Squaring on both sides, we get

$x + 4 + x + 8 + 2\left(\sqrt{x+4}\sqrt{x+8}\right) = 49$

$\Rightarrow 2x + 12 + 2\sqrt{x^2+12x+32} = 49$

$\Rightarrow 2x - 37 = -2\sqrt{x^2+12x+32}$

Squaring again on both sides, we have

$(2x-37)^2 = 4(x^2+12x+32)$

$\Rightarrow 4x^2 - 148x + 1369 = 4x^2 + 48x + 128$

$\Rightarrow 1241 = 196x$

$\Rightarrow x = \frac{1241}{196}$

**4.60.** If $4^{2x+1} + 4^{x+1} = 80$, then find the value of $x$.

**Sol:** Given $4^{2x+1} + 4^{x+1} = 80$

$\Rightarrow 4^{2x} \times 4 + 4^x \times 4 = 80$

$4^{2x} + 4^x = 20$

Substituting $4^x = a$,

we get $a^2 + a = 20$

$\Rightarrow a^2 + a - 20 = 0$

$\Rightarrow (a+5)(a-4) = 0$

$\Rightarrow a = -5$ or $4$

If $4^x = -5$, there is no possible value for $x$ as no power of 4 gives negative value.

If $4^x = 4$, then $x = 1$.

**4.61.** Find the nature of roots of the equation, $f(x) = x^3 + x - 2 = 0$.

**Sol:** There is only 1 change of sign in $f(x)$.

We know that when $f(x)$ has $r$ changes of sign then $f(x)$ has $r$, $r-2$, $r-4$, … positive roots.

$\therefore f(x) = 0$ has one positive root.

Now, $f(-x) \equiv -x^3 - x - 2 = 0$. $q = 0$

Since there is no change of sign in $f(-x)$, $f(x)$ has no negative roots. The number of complex roots is even.

$\therefore$ The equation has one positive root, and two complex roots.

Hence, $f(x) = 0$ has 1 real root and two complex roots.

**4.62.** How many non-real-roots does the equation $x^4 - 2x^2 + 3x - 2 = 0$ have?

**Sol:** Let $f(x) = x^4 - 2x^2 + 3x - 2$

$f(x)$ has 3 sign changes

$\therefore f(x)$ has 3 or 1 positive roots.

$f(-x) = x^4 - 2x^2 - 3x - 2$

$\therefore f(-x)$ has one sign change

$\therefore f(x)$ has exactly one negative root.

As the sum of the co-efficient of $f(x)$ is zero,

$x = 1$ is a root $ff(x) = 0$

$\therefore f(x) = (x-1)(x^3 + x^2 - x + 2) = (x-1) f_1(x)$

By trial, $f_1(-2) = 0$

$\therefore f_1(x) = (x+2)(x^2 - x + 1)$

We can see that $x^2 - x + 1 = 0$ has two non-real roots.

$\therefore f(x)$ has one positive, one negative and two non-real roots.

**4.63.** If $p-q$, $p$, $p+q$ are the roots of the equation $x^3 - 18x^2 + 99x - 162 = 0$, then find the values of $p$ and $q$.

**Sol:** Given $p-q$, $p$, $p+q$ are the roots of the equation.

$\therefore$ The sum of the roots is $(p-q) + p + (p+q) = 18$

$\Rightarrow 3p = 18 \Rightarrow p = 6$

and the product of the roots is $(p-q)\,p\,(p+q) = 162$

$p^2 - q^2 = \frac{162}{6} = 27 \Rightarrow 36 - q^2 = 27$

$\Rightarrow q = \pm 3$ $\therefore p = 6$ and $q = \pm 3$

**4.64.** Find the range of the expression $\frac{x-2}{x^2+x+3}$ where $x$ is real.

**Sol:** Let $f(x) = \frac{x-2}{x^2+x+3} = y$

$\Rightarrow x^2y + xy + 3y = x - 2$

i.e, $x^2y + x(y-1) + 3y + 2 = 0$

$f(x)$ can have any value $y$, provided the above equation in $x$ has real roots

$\therefore b^2 - 4ac \geq 0$

$\Rightarrow (y-1)^2 - 4y(3y+2) \geq 0$

i.e, $11y^2 + 10y - 1 \leq 0$

$(11y-1)(y+1) \leq 0 \Rightarrow -1 \leq y \leq \frac{1}{11}$

$\therefore$ The range of $y$ is $\left[-1, \frac{1}{11}\right]$

**4.65.** Solve the equation $x^4 - 2x^3 - 19x^2 + 8x + 60 = 0$, given that two of the roots are $\alpha$ and $-\alpha$, where $\alpha > 0$.

**Sol:** Let the roots of the equation be $\alpha, -\alpha, \beta, \gamma$. Let $\beta > \gamma$
The sum of the roots $= \alpha - \alpha + \beta + \gamma = 2$
$\Rightarrow \beta + \gamma = 2$
The sum of the products of the roots taken three at a time =
$-\alpha^2\beta - \alpha^2\gamma + \alpha\beta\gamma - \alpha\beta\gamma = -8$
$\alpha^2(\beta + \gamma) = 8$. $\therefore \alpha^2 = 4$
The product of the roots $= -\alpha^2\beta\gamma = 60$. $\beta\gamma = -15$
$\beta + \gamma = 2$ and $\beta\gamma = -15$, $\alpha^2 = 4$
$\beta = 5$ and $\gamma = -3$, $\alpha = \pm 2$
$\Rightarrow$ The roots are $-2, 2, -3$ and $5$.

**4.66.** Find the sum of the squares of the roots of the equation $x^3 - 4x^2 + x + 6 = 0$.

**Sol:** Let $\alpha, \beta, \gamma$ be the roots of the given equation.
$\alpha + \beta + \gamma = 4$, $\alpha\beta + \beta\gamma + \alpha\gamma = 1$ (and $\alpha\beta\gamma = -6$).
$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$
$= (4)^2 - 2(1) = 14$

**4.67.** Find the roots of the equation $x^2 + 9x - 10 = 0$.

**Sol:** $x^2 + 9x - 10 = 0$
$\Rightarrow x^2 + 10x - x - 10 = 0$
$\Rightarrow x(x + 10) - 1(x + 10) = 0$
$\Rightarrow (x + 10)(x - 1) = 0$ $\therefore x = -10$ or $1$

**4.68.** Find the roots of the equation $4x^2 - 17x + 4 = 0$.

**Sol:** $4x^2 - 17x + 4 = 0$
$\Rightarrow 4x^2 - 16x - x + 4 = 0$
$\Rightarrow (4x - 1)(x - 4) = 0$
$\Rightarrow x = \frac{1}{4}$ or $4$

**4.69.** Find the nature of the roots of the equation $9x^2 - 3x + 1 = 0$.

**Sol:** Discriminant $= (-3)^2 - 4(9)(1) = -27$
($\because$ discriminant $= b^2 - 4ac$)
Since the discriminant is negative, the roots are complex conjugates.

**4.70.** Find the nature of the roots of the equation $5x^2 - x - 4 = 0$.

**Sol:** Discriminant $= (-1)^2 - 4(5)(-4) = 81$. As the coefficients are rational and the discriminant is positive and a perfect square, the roots are rational and unequal.

**4.71.** If the sum of the roots of the equation $kx^2 - 52x + 24 = 0$ is $\frac{13}{6}$, find the product of its roots.

**Sol:** $\frac{13}{6} = \frac{52}{k}$
($\because$ sum of the roots of $ax^2 + bx + c = 0$ is $-\frac{b}{a}$)
$k = 24$
The product of its roots $= \frac{24}{k} = 1$
($\because$ product of the roots of $ax^2 + bx + c = 0$ is $\frac{c}{a}$).

**4.72.** If the roots of the equation $6x^2 - 7x + b = 0$ are reciprocals of each other, find $b$.

Sol: Let the roots be $\alpha$ and $\frac{1}{\alpha}$
The product of the roots $= \frac{b}{6}$
$= (\alpha)\left(\frac{1}{\alpha}\right) = 1$
$\therefore b = 6$

**4.73.** The roots of a quadratic equation are a and $-a$. The product of its roots is $-9$. Form the equation in variable $x$.

**Sol:** The sum of the roots $= 0$ and the product of the roots $= -9$
The quadratic equation whose roots are $\alpha$ and $\beta$ is $x^2 - x(\alpha + \beta) + \alpha\beta = 0$ i.e.,
$x^2 - x$ (sum of the roots) + product $= 0$
Hence, the quadratic equation is $x^2 - 9 = 0$

**4.74.** The roots of the equation $x^2 - 12x + k = 0$ are in the ratio $1 : 2$. Find $k$.

**Sol:** Let the roots be $m$ and $2m$.
$m + 2m = -(-12) = 12$
$\Rightarrow m = 4$
$k$ = the product $(m)(2m) = 32$

**4.75.** A quadratic equation has rational coefficients. One of its roots is $2 + \sqrt{2}$. Find its other root.

**Sol:** If a quadratic equation has rational coefficients, and one of its roots is $p + \sqrt{q}$ (where $p$, $q$ are rational), the other root is $p - \sqrt{q}$.
$\therefore$ The other root of the given equation is $2 - \sqrt{2}$.

**4.76.** I can buy a certain number of books for ₹1050. If the price of the book increases by ₹15, the number of books I can buy for the same amount decreases by 9. Find the original price and the number of books I could buy at that price.

**Sol:** Let the original price be ₹ $x$ per book
The number of books I could buy at that price
$= \frac{1050}{x}, \frac{1050}{x + 15}$
$= \frac{1050}{x} - 9$
$\frac{1050}{x} - \frac{1050}{x + 15} = 9$

$(1050)\ 15 = 9\ (x+15)\,x$

$x^2 + 15x - 1750 = 0$

$$x = \frac{-15 \pm \sqrt{15^2 - 4(1)(-1750)}}{2} = 35 \text{ or } -50$$

as $x > 0$, $x = 35$

**4.77.** $P$ and $Q$ are the roots of the equation $x^2 - 22x + 120 = 0$.
Find the value of

(i) $P^2 + Q^2$

(ii) $\frac{1}{P} + \frac{1}{Q}$

(iii) difference of $P$ and $Q$

**Sol:** $P + Q = 22$ and $PQ = 120$

(i) $P^2 + Q^2 = (P+Q)^2 - 2PQ$
$= 22^2 - 2\ (120) = 244$

(ii) $\frac{1}{P} + \frac{1}{Q} = \frac{Q+P}{PQ} = \frac{22}{120} = \frac{11}{60}$

(iii) (Difference of $P$ and $Q)^2$
$= (P+Q)^2 - 4PQ = 22^2 - 4\ (120) = 4$
$\therefore$ Difference of $P$ and $Q = 2$

**4.78.** Find the remainder when $f(x) = x^2 + 6x + 8$ is divided by $2x + 1$.

**Sol:** Given polynomial is $f(x) = x^2 + 6x + 8$ and divisor is $2x + 1$.

$\therefore$ Remainder $= f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 + 6\left(-\frac{1}{2}\right) + 8$

$= \frac{1}{4} - 3 + 8 = 5\frac{1}{4}$

**4.79.** If $4^{x+2} + 4^{2x+1} = 1280$, find $x$.

**Sol:** $4^{x+2} + 4^{2x+1} = 1280$
$(4^x)\ (4^2) + (4^x)^2\ (4) = 1280$
$4\ (4^x)^2 + 16\ (4^x) = 1280$
Dividing both sides by 4
$(4^x)^2 + 4\ (4^x) = 320$
let $4^x = y$
$\Rightarrow y^2 + 4y - 32 = 0$
$\Rightarrow (y + 20)\ (y - 16) = 0$
$\Rightarrow y = -20$ or $16$
$\therefore 4^x = 16$ or $-20$
as $4^x > 0$, $4^x = 16 \Rightarrow x = 2$

**4.80.** The minimum value of $2x^2 + bx + c$ is known to be $\frac{15}{2}$ and occurs at $x = -\frac{5}{2}$. Find the values of $b$ and $c$.

**Sol:** The minimum value of $ax^2 + bx + c$ (where $a > 0$) occurs at $x = -b/2a$ and it equals $\frac{4ac - b^2}{4a}$.

$\Rightarrow \frac{-b}{2(2)} = -\frac{5}{2}$

$b = 10$

minimum value $= \frac{4(2)c - 10^2}{4(2)} = \frac{15}{2}$

$\Rightarrow 8c - 100 = 60$

$\Rightarrow c = \frac{100 + 60}{8} = 20$

$\therefore$ The values of $b$ and $c$ are 10 and 20, respectively.

**4.81.** Find the number of positive and negative roots of the equation
$x^3 - ax + b = 0$ where $a > 0$ and $b > 0$.

**Sol:** Let $f(x) = x^3 - ax + b$

In $f(x)$ there are two changes of sign $(+ - +)$

$\therefore$ By Descartes' rule there exist at most two positive roots. The number of positive roots could be 2 or 0.

$f(-x) = (-x)^3 - a\ (-x) + b$

$f(-x) = -x^3 + ax + b$

There is only one sign change in $f(-x)$. Therefore there exists at most one negative root. As the number of negative roots cannot be negative, it is exactly equal to 1. Therefore, $f(x) = 0$ has 2 or 0 positive roots and exactly 1 negative root.

**4.82.** If $-1$ and 2 are two of the roots of the equation $x^4 - 3x^3 + 2x^2 + 2x - 4 = 0$, then find the other two roots.

**Sol:** Given equation is $x^4 - 3x^3 + 2x^2 + 2x - 4 = 0$ and $-1$, 2 are the roots of it.

We can find the Quotient as follows

$$\begin{array}{r|l} & x^3 - 4x^2 + 6x - 4 \\ \hline x+1 & x^4 - 3x^3 + 2x^2 + 2x - 4 \\ & x^4 + x^3 \\ \hline & -4x^3 + 2x^2 \\ & -4x^3 - 4x^2 \\ \hline & 6x^2 + 2x \\ & 6x^2 + 6x \\ \hline & -4x - 4 \\ & -4x - 4 \\ \hline & 0 \end{array}$$

$$x-2\overline{)\,x^3-4x^2+6x-4\;\;\;x^2-2x+2}$$
$$x^3-2x^2$$
$$-2x^2+6x$$
$$-2x^2+4x$$
$$2x-4$$
$$2x-4$$
$$0$$

∴ The quotient when the given expression is divided by $(x+1)\,(x-2)$ is $x^2-2x+2$ which is a quadratic equation whose discriminant is negative.

The roots of $x^2-2x+2=0$ are $\frac{2+\sqrt{4-8}}{2}$ i.e., $1\pm i$.

The division shown above can be performed in very compact way, using Horner's Method of Synthetic Division.

| | | | | | |
|---|---|---|---|---|---|
| $x=-1$ | 1 | –3 | 2 | 2 | –4 |
| | 0 | –1 | 4 | –6 | 4 |
| $x=2$ | 1 | –4 | 6 | –4 | 0 |
| | 0 | 2 | –4 | –4 | 0 |
| | 1 | –2 | 2 | 0 | |

**4.83.** If one root of the cubic equation $ax^3+bx^2+cx+d=0$ is the negative of another, what is the condition that the coefficients satisfy?

**Sol:** Let the roots be $\alpha, -\alpha, \beta$

$\therefore \alpha-\alpha+\beta=\frac{-b}{a}$, i.e., $\frac{-b}{a}$ is a root of the equation

$$\therefore\; a\left(\frac{-b^3}{a^3}\right)+b\left(\frac{b^2}{a^2}\right)+c\left(\frac{-b}{a}\right)+d=0$$

$\Rightarrow ad=bc$

**4.84.** If $\alpha, \beta, \gamma$ are the roots of $x^3+10x^2+x+7=0$, find the value of $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

**Sol:** Let $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=E$

$$\therefore\; \frac{a\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}=E$$

$\alpha\beta+\beta\gamma+\gamma\alpha=1$

And $\alpha\beta\gamma=-7\;\therefore E=-1/7$

**4.85.** Solve the following equation:

$F(x)=x^4-5x^3+10x^2-20x+24=0$

**Sol:** By trial

$F(1)=1-5+10-20+24=10\neq 0$

$F(-1)=1+5+10+20+24\neq 0$

$F(2)=16-40+40-40+24=0$

∴ One of the roots is 2 and $x-2$ is a factor of $f(x)$. dividing $f(x)$ by $x-2$, we get,

$f(x)=(x-2)(x^3-3x^2+4x-12)$

$=(x-2)g(x)$ say

By trial $g(3)$ [no need to check for $g(1)$ or $g(-1)$]

$g(3)=27-27+12-12=0$

Dividing by $x-3$, we get

$F(x)=(x-2)(x-3)(x^2+4)$

∴ The roots are 2, 3, ±2i

**4.86.** For the cubic equation $ax^3+bx^2+bx+a=0$, find the roots.

**Sol:** Let $\alpha$ be a root ie

$a\alpha^3+b\alpha^2+b\alpha+a=0$ (1)

consider $x=1/\alpha$

$$\frac{a}{\alpha^3}+\frac{b}{\alpha^2}+\frac{b}{\alpha}+a$$

$$=\frac{1}{\alpha^3}\left(a+b\alpha+b\alpha^2+a\alpha^3\right)=0 \text{ (from 1)}$$

∴ $1/\alpha$ is also a root.

All together there are 3 roots. Also, if $\alpha$ is a root $1/\alpha$ is also a root. ∴ one of the roots has to be its own reciprocal (1, or –1)

If it is 1, $a+b=0$ (1)

If it is –1, no additional condition has to be satisfied. Consider A. The equation would be

$ax^3-ax^2-ax+a=0$

$\Rightarrow (x^2-1)(x-1)=0$

The roots are 1, –1, 1.

Consider the other possibility.

One of the roots is –1. The three roots are $\alpha, 1/\alpha$, –1.

The equation would be $(x^2+px+1)\,(x+1)=0$.

Or $x^3+(p+1)x^2+(p+1)x+1=0$ (2)

The roots of (2) are –1, $\frac{-p\pm\sqrt{p^2-4}}{2}$

∴The roots of (1) are –1,

$$\frac{\left(\frac{-b}{a}+1\right)\pm\sqrt{\left(\frac{-b}{a}+1\right)^2-4}}{2}$$

$\left(\because\; p+1=\frac{b}{a}\right)$

## EXERCISE-1

***Directions for questions 1–45:*** For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple-choice questions, write your answer in the box provided.

**1.** Find the quadratic equation in $x$ whose roots are 2 and 3.
(A) $x^2 + 5x + 6 = 0$ (B) $x^2 - 5x + 6 = 0$
(C) $x^2 - 5x - 6 = 0$ (D) $x^2 + 5x - 6 = 0$

**2.** What are the signs of the roots of the equation $x^2 + 6x + 9 = 0$?
(A) Both are positive
(B) Both are negative
(C) One is negative, and the other is positive
(D) None of these

**3.** If the sum of the roots of a quadratic equation in $x$ is 4 and the constant term of the equation is 15, then find the quadratic equation.
(A) $x^2 - 4x + 15 = 0$ (B) $x^2 + 4x + 15 = 0$
(C) $x^2 - 4x - 15 = 0$ (D) $x^2 + 4x - 15 = 0$

**4.** Find the roots of the quadratic equation $x^2 - 8x + 9 = 0$.
(A) 1, 9 (B) –1, –9
(C) $4 + \sqrt{7}, 4 - \sqrt{7}$ (D) $3 + \sqrt{6}, 3 - \sqrt{6}$

**5.** If the sum of the roots of a quadratic equation is 7 and the product of the roots is 12, then find the equation.
(A) $x^2 + 7x + 12 = 0$ (B) $x^2 + 7x - 12 = 0$
(C) $x^2 - 7x + 12 = 0$ (D) $x^2 - 7x - 12 = 0$

**6.** Find a quadratic equation in $x$ whose roots are 5 and 6.
(A) $x^2 + 11x + 30 = 0$ (B) $x^2 - 11x - 30 = 0$
(C) $x^2 - 11x + 30 = 0$ (D) $x^2 + 11x - 30 = 0$

**7.** Find the sum of the roots of $9x^2 - 144x + 92 = 0$. _____

**8.** Find the roots of the quadratic equation
$x^2 - 9x + \frac{41}{4} = 0$.
(A) –1, –10 (B) 1, 10
(C) $\frac{9+5\sqrt{2}}{2}, \frac{9-5\sqrt{2}}{2}$ (D) None of these

**9.** The sum of the roots of a quadratic equation is 8 and the constant term of the equation is 15. Find the quadratic equation.
(A) $x^2 - 8x - 15 = 0$
(B) $x^2 - 8x + 15 = 0$
(C) $x^2 + 8x + 15 = 0$
(D) Cannot be determined uniquely.

**10.** If the sum of the roots of a quadratic equation is 20 and the product of its roots is 36, find the equation.
(A) $x^2 + 20x + 36 = 0$ (B) $x^2 - 20x + 36 = 0$
(C) $x^2 - 20x - 36 = 0$ (D) $x^2 + 20x - 36 = 0$

**11.** If the sum of the roots of a quadratic equation is 11 and product of the roots is 24, find the roots of the equation.
(A) 6, 4 (B) 12, 2 (C) 8, 3 (D) 1, 24

**12.** Construct a quadratic equation whose roots are reciprocals of the roots of the equation $3x^2 + 5x + 7 = 0$
(A) $7x^2 + 5x + 3 = 0$ (B) $5x^2 + 7x + 3 = 0$
(C) $5x^2 + 3x + 7 = 0$ (D) $7x^2 + 3x + 5 = 0$

**13.** Find the discriminant of the equation $2x^2 + 3x + 4 = 0$.
(A) –23 (B) –19 (C) –25 (D) –27

**14.** What is the nature of the roots of the quadratic equation $x^2 + kx + 7 = 0$?
(A) Complex conjugates (B) Real and equal
(C) Conjugate surds (D) Cannot be determined

**15.** What is the sum of the roots of the equation $9x^2 - 36x + 35 = 0$?
(A) 36 (B) –36 (C) 4 (D) –4

**16.** Find the number of roots of $(x^n - b)^2 = 0$.
(A) $n$ (B) $n^2$
(C) $2n$ (D) Infinitly many

**17.** Find the nature of the roots of $4x^2 + 8x - 11 = 0$.
(A) Real and equal
(B) Complex conjugates
(C) Conjugate surds
(D) Rational and unequal

**18.** Find the quadratic equation whose roots are 3 more than the roots of $x^2 - 4x + 10 = 0$.
(A) $x^2 - 10x - 23 = 0$ (B) $x^2 + 10x - 31 = 0$
(C) $x^2 + 10x + 23 = 0$ (D) $x^2 - 10x + 31 = 0$

**19.** What can be said about the roots of $ax^2 + bx + a = 0$?
(A) Roots are equal in magnitudes but opposite in sign.
(B) The roots are reciprocals to each other.
(C) One root is the square of the other.
(D) Both (A) and (B)

**20.** For what value of $p$ does the equation $x^2 + px + 81 = 0$ have equal roots?
(A) 18 (B) 9 (C) ± 9 (D) ± 18

**21.** The expression $\frac{4ac - b^2}{4a}$ gives what value of the quadratic expression $ax^2 + bx + c$?

(I) maximum value when $a > 0$

(II) minimum value when $a < 0$

(III) minimum value when $a > 0$

(IV) maximum value when $a < 0$

(A) Only I (B) Only III
(C) Both I and III (D) Both III and IV

**22.** What is the degree of the equation $(x^2 - 3)^3 - 4x^4 = 0$?
(A) 3 (B) 4 (C) 6 (D) 7

**23.** Construct a quadratic equation whose roots are 4 more than the roots of $x^2 + 7x + 16 = 0$
(A) $x^2 + 15x + 60 = 0$ (B) $x^2 + 7x + 60 = 0$
(C) $x^2 - x + 4 = 0$ (D) $x^2 + 15x + 16 = 0$

**24.** What is the minimum value of the quadratic expression $2x^2 + 5x + 7$?

(A) $\frac{23}{8}$ (B) $\frac{27}{8}$ (C) $\frac{35}{8}$ (D) $\frac{31}{8}$

**25.** How many distinct roots does the quadratic equation $(x - 4)^2 = 0$ have?
(A) 4 (B) 2 (C) 3 (D) 1

**26.** If the sum of the roots of a quadratic equation is 22 and the product of its roots is 72, find the roots of the equation.
(A) 1, 72 (B) 2, 36 (C) 3, 24 (D) 4, 18

**27.** If one of the roots of a quadratic equation with rational coefficients is $3 + 2\sqrt{2}$, what is the sum of the roots of the equation?

(A) $-4\sqrt{2}$ (B) $-4\sqrt{2}$ (C) 6 (D) – 6

**28.** If _____ is added to the cubic expression $5x^3 - 2x^2 - 3x - 2$, then the resulting expression will be exactly divisible by $x - 1$.
(A) 4 (B) –2 (C) –4 (D) 2

**29.** If the roots of a quadratic equation are reciprocal of one another and the constant term is 3, what is the coefficient of the second-degree term?
(A) 1 (B) 3
(C) –3 (D) Cannot be determined

**30.** Find the quadratic equation whose roots are the reciprocals of the roots of $3x^2 + 4x + 2 = 0$.
(A) $2x^2 + 4x + 3 = 0$ (B) $4x^2 + 2x + 3 = 0$
(C) $2x^2 + 3x + 4 = 0$ (D) $4x^2 + 3x + 2 = 0$

**31.** If the expression $6x^2 + 13x + k$ is divisible by $2x + 3$, then which of the following is the factor of the expression?
(A) $3x + 1$ (B) $3x + 4$
(C) $3x + 2$ (D) $3x + 5$

**32.** If _____ is added to $3x^3 + 5x^2 - 6x + 3$, then it is exactly divisible by $x - 1$.
(A) 4 (B) –3 (C) 5 (D) –5

**33.** The remainder when $f(x) = x^3 + 5x^2 + 2x + 3$ is divided by $x$, is
(A) 3 (B) 4 (C) 2 (D) 5

**34.** If $px^2 + qx + r$ is exactly divisible by $x + 1$, then which of the following is necessarily true?
(A) $p + q = r$ (B) $p + r = q$
(C) $q + r = p$ (D) None of these

**35.** The remainder when $2x^6 - 5x^3 - 3$ is divided by $x^3 + 1$, is
(A) 3 (B) 2 (C) 4 (D) 7

**36.** Find the quadratic equation whose roots are the reciprocals of the roots of $2x^2 + 5x + 3 = 0$.
(A) $3x^2 + 5x - 2 = 0$ (B) $3x^2 + 5x + 2 = 0$
(C) $3x^2 - 5x + 2 = 0$ (D) $3x^2 - 5x - 2 = 0$

**37.** Find the signs of the roots of $x^2 - 14x + p^2 = 0$, where $p$ is a non-zero-real number.
(A) Both positive
(B) Both negative
(C) One positive, one negative
(D) None of these

**38.** The quadratic expression $px^2 + qx + r$ has its maximum or minimum value at

(A) $\frac{-q}{2p}$ (B) $\frac{q}{2p}$ (C) $\frac{-2q}{p}$ (D) $\frac{2q}{p}$

**39.** The remainder of $3x^2 - 5x - 2$ divided by $x - 2$ is _____.

**40.** If $9x^2 - 13x + c$ is divided by $x + 1$, then the remainder is 12. Find the value of $c$. _____

**41.** The quadratic expression $ax^2 + bx + c$ has its maximum or minimum value at $x =$
(A) $-b/2a$ (B) $b/2a$ (C) $-2b/a$ (D) $2b/a$

**42.** What is the maximum value of the quadratic expression $-7x^2 + 2x + 5$?

(A) $\frac{37}{7}$ (B) $\frac{39}{7}$ (C) $\frac{35}{7}$ (D) $\frac{36}{7}$

**43.** Construct a quadratic equation, whose roots are half of the roots of the equation $x^2 + 5x + 9 = 0$.
(A) $4x^2 + 10x + 9 = 0$ (B) $x^2 + 10x + 36 = 0$
(C) $4x^2 + 5x + 36 = 0$ (D) $4x^2 + 5x + 9 = 0$

**44.** Find the maximum or minimum value of the expression $x^2 + x + 5$.

(A) Minimum value of $\frac{19}{4}$

(B) Minimum value of $\frac{19}{2}$

(C) Maximum value of $\frac{19}{4}$

(D) Maximum value of $\frac{19}{2}$

**45.** If $ax^2 - bx - c$ is exactly divisible by $x + 1$, then it follows that

(A) $b + c = a$ (B) $a + c = b$

(C) $a + b = c$ (D) None of these

## EXERCISE-2

***Directions for questions 1–45*:** For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple-choice questions, write your answer in the box provided.

**1.** Solve: $x^2 - 7x + 12 = 0$.

(A) 3, 4 (B) –3, 4

(C) 3, –4 (D) –3, –4

**2.** If the roots of $x^2 - kx + 36 = 0$ are not real, what values can $k$ take?

(A) $(-\infty, \infty)$ (B) $(-\infty, -12)$

(C) (–12, 12) (D) $(12, \infty)$

**3.** Form the equation, whose roots are one less than the roots of the equation $x^2 + 7x + 12 = 0$?

(A) $x^2 + 9x - 20 = 0$ (B) $x^2 + 9x + 20 = 0$

(C) $x^2 - 9x - 20 = 0$ (D) $x^2 - 9x + 20 = 0$

**4.** Find the quadratic equation whose roots are 2 and 7.

(A) $x^2 + 9x - 14 = 0$ (B) $x^2 - 9x - 14 = 0$

(C) $x^2 + 9x + 14 = 0$ (D) $x^2 - 9x + 14 = 0$

**5.** If the roots of $x^2 - kx + 49 = 0$ are real, what is the least positive integer value that $k$ can take? _____

**6.** If the roots of the equation $y^2 - 7y + k = 0$ are $y_1$ and $y_2$ and $y_1 - y_2 = 3$, then find $k$. _____

**7.** If $4 + \sqrt{5}$ is a root of the equation $y^2 - 8y + m = 0$, where $m$ is rational, then find the value of $m$.

(A) 11 (B) 13 (C) 9 (D) 15

**8.** If $\alpha$ and $\beta$ are the roots of the equation $x^2 - 8x + 9 = 0$, then $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} =$

(A) $\frac{56}{9}$ (B) $\frac{-46}{9}$ (C) $\frac{46}{9}$ (D) $\frac{-56}{9}$

**9.** If the roots of the equation $py^2 + qy + r = 0$ are in the ratio $m : n$, then $\frac{mn}{(m+n)^2} =$

(A) $pr - q^2$ (B) $\frac{pr}{q^2}$ (C) $\frac{q^2}{pr}$ (D) $q^2 - pr$

**10.** If the sum and the product of the roots of a quadratic equation in $x$ are –4 and 9, respectively, which of the following represents the equation?

(A) $x^2 - 4x + 9 = 0$ (B) $x^2 + 4x - 9 = 0$

(C) $x^2 - 4x - 9 = 0$ (D) $x^2 + 4x + 9 = 0$

**11.** Identify the nature of roots of the following equations.

(a) $9x^2 - 10x + 3 = 0$

(A) Real and equal (B) Rational and unequal

(C) Complex (D) Irrational

(b) $x^2 - 2x - 3 = 0$

(A) Real and equal (B) Rational and unequal

(C) Complex (D) Irrational

(c) $x^2 - 12x + 36 = 0$

(A) Real and equal (B) Rational and unequal

(C) Complex (D) Irrational

**12.** In the quadratic equation $16x^2 - qx + 81 = 0$, one root is the cube of the other. Find the value of $q$.

(A) ±78 (B) 39 (C) –39 (D) 56

**13.** Find the possible values of $x$ of the equation $\sqrt{3 - 2x} + \sqrt{7 + 2x} = 4$.

(A) –3, 1 (B) 3, –1

(C) 3, –2 (D) 3, 2

**14.** If $x$ is an integer and $\frac{6}{x+2} + \frac{8}{x+3} = \frac{10}{x+4} + \frac{4}{x+1}$ the value of $x$ equals to _____.

**15.** The discriminant of the quadratic equation $(a - b)x^2 + 2ax + (a + b) = 0$, where a and b are real numbers is

(A) non-positive. (B) non-negative.

(C) zero. (D) None of these

**16.** Find the value of $k$, if the product of the roots of the equation $x^2 - \left(\sqrt{2} + \sqrt{3}k\right)x + 4y^{2\log_y k} = 0$ is 256. _____

Difficulty Level-2: Moderate

**17.** If a positive number is decreased by 2 and then squared the resultant number is 2 less than the original number. Find a possible value of the number.
(A) 1 (B) 4 (C) 3 (D) 7

**18.** Find the larger of the two positive numbers whose sum is 24 and twice the square of the smaller number is 4 more than the square of the larger number. _____

**19.** The roots of the equation $2x^2 + 5x + 2 = 0$ are $k$ less than those of the equation $px^2 + qx + r = 0$. Which of the following could be true?
(A) $k(pk + 4p + q) + 4p + 2q + r = 0$
(B) $k(pk - 4p + q) + 4p - 2q + r = 0$
(C) $k(pk + 4p + q) + 4p - 2q + r = 0$
(D) $k(pk - 4p + q) + 4p + 2q + r = 0$

**20.** A journey of 1600 km by an express train takes 24 hours less than that by an ordinary train. If the ordinary train is 15 km/hr slower than the express train, find the speed of the ordinary train. (in km/hr). _____

**21.** Find the value of

$$\cfrac{1}{1+\cfrac{1}{-4+\cfrac{1}{1+\cfrac{1}{-4+..}}}}$$

(A) 2 (B) –3 (C) –2 (D) 3

**22.** For the quadratic equation $ax^2 + bx + c = 0$, where both the roots are negative, the sum of the cubes of the roots is $-7/2$ and the product of the roots is $3/4$, then find $a/b$.
(A) 2 (B) 3/2 (C) 2/3 (D) 1/2

**23.** $(3 + n^2)x^2 - 5nx + n^2 - 14 = 0$ is a quadratic equation in $x$. $n_1$ and $n_2$ are two values of $n$ such that the sum of the roots of the equation is equal to the product of the roots. Find $(n_1^2 + n_2^2)^2$.
(A) 2401 (B) 2704 (C) 2089 (D) 2809

**24.** If $a$ and $c$ are integers and $3 \le a \le 7$ and $0 \le c$, how many equations of the form $ax^2 + 12x + c = 0$ have real roots?
(A) 44 (B) 48 (C) 36 (D) 40

**25.** In the quadratic equation $ax^2 + bx + c = 0$, $a = 1$, $b = c$ and both the roots are integers. The difference between the roots is
(A) 0 (B) 1 (C) 8 (D) 4

**26.** $K$ is a positive integer satisfying $K^2 \le 36$. How many equations of the form $x^2 + Kx + 4 = 0$ exist such that the roots are real and unequal? _____.

**27.** A and B had a total of 50 chocolates. The sum of the squares of the numbers of chocolates with them is 1300. A sells each chocolate at ₹5 and B sells each chocolate at ₹6. Find the total amount they get, if each had as many chocolates as the other had. How much more/ less would they realize totally in selling them?
(A) Amount = 270/-, ₹10 more
(B) Amount = 270/-, ₹10 less
(C) Amount = 280/-, ₹10 less
(D) Either (A) or (C)

**28.** If one root of the equation $x^2 - 2Rx + 6 = 0$ is twice one root of the equation $x^2 - 5x + 6 = 0$, then R can be
(A) 7/2 (B) 11/4
(C) Either (A) or (B) (D) –7/2

**29.** For the equation $x^2 - 2px + 2p = 0$ where $p$ is real, find the nature of the roots.
(A) Real and unequal
(B) Real and equal
(C) Imaginary
(D) Cannot be determined

**30.** How many roots of the equation $x^3 - 3x^2 + 4 = 0$ are also roots of the equation $x^2 + x - 6 = 0$? _____

**31.** Ashok bought some pencils for ₹180. He sold all but 2 pencils for ₹160, making a profit of ₹2 per pencil. Find the number of pencils he bought. _____

**32.** If a quadratic equation $ax^2 + bx + c = 0$ has the sum of the roots equal to the product of the roots, which of the following holds true?
(A) $b = c$ (B) $b = -c$ (C) $a = b$ (D) $a = -c$

**33.** If $b^2 - 4ac$ has a value of 49 where $a$, $b$ and $c$ are rational numbers satisfying $ax^2 + bx + c = 0$, the roots are
(A) irrational. (B) rational and equal.
(C) complex. (D) rational and unequal

**34.** A natural number is tripled and then increased by 2. The result when squared equals 32 times the original number. Find the number. _____

**35.** Sum of squares of two consecutive natural numbers is 841. Find the least among the two numbers. _____

**36.** Find the maximum value of $12 + 22x - 11x^2$.
(A) 30 (B) 28 (C) 23 (D) 21

**37.** If the equation $x^2 + 2(m + 3)\ x + 81 = 0$ has equal roots, find the value of $m$.

(A) 6 (B) –12
(C) Either (A) or (B) (D) Neither (A) nor (B)

**38.** If $(x+2)^2 + (x-2)^2 = 40$, find $x$.
(A) $\pm 4$ (B) $\pm 5$ (C) $\pm 6$ (D) $\pm 7$

**39.** For what value of $K$ are the roots of the quadratic equation $(K-4)\ x^2 + (3K-12)\ x + 9 = 0$ real and equal?
(A) 8 (B) 5 (C) 6 (D) 4

**40.** Find the equation which has each of its roots one more than the roots of the equation $x^2 - 13x + 21 = 0$
(A) $x^2 + 15x + 35 = 0$ (B) $x^2 + 15x - 35 = 0$
(C) $x^2 - 15x - 35 = 0$ (D) $x^2 - 15x + 35 = 0$

**41.** $x^4 - 4x^3 - x^2 - 6x + 4 = 0$, Using Descartes' rule the maximum number of negative roots for the given equation is _____.

**42.** The equation $x^2 + 4x + 3 = 0$ has _____ number of positive roots _____.

**43.** If $p$ is real, the nature of roots of $x^2 + 2(p+1)\ x + 2p = 0$, will be?
(A) real and equal (B) real and unequal
(C) not real (D) Cannot say anything

**44.** Two of the roots of the equation $x^4 - px^3 + 2x^2 + 2x + q = 0$ are –1, 2 then find the other two roots.
(A) 1, –2 (B) –1, –2 (C) 1, 2 (D) not real

**45.** '$A$' is a single digit prime number and '$B$' is a natural number. How many equations of the form $x^2 - 4\sqrt{A}\ x + 3B = 0$ will have real roots?

_____.

## EXERCISE-3

***Directions for questions 1–45*:** For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple-choice questions, write your answer in the box provided.

**1.** If one root of the quadratic equation $25x^2 - 45x + k = 0$, is twice the other root, find the value of $k$.
(A) 9 (B) 27 (C) 18 (D) 12

**2.** Find the quadratic equation which has each of its roots being thrice the roots of $2x^2 - 11x + 12 = 0$
(A) $2x^2 + 33x + 108 = 0$ (B) $2x^2 + 33x - 108 = 0$
(C) $2x^2 - 33x + 108 = 0$ (D) $2x^2 - 33x - 108 = 0$

**3.** If $K(K+1)\ (K+2)\ (K+3) = 120$, then find $K^2 + 3K$ given that it is positive. _____

**4.** One of the roots of the quadratic equation $ax^2 + bx + c = 0$ is thrice the other root. Which of the following always holds true?
(A) $b^2 = 2ac$ (B) $b^2 = 5ac$
(C) $3b^2 = 10ac$ (D) $3b^2 = 16ac$

**5.** Find the quadratic equation in $x$ whose roots are twice the roots of $x^2 - 9x + 18 = 0$.
(A) $x^2 + 18x - 72 = 0$ (B) $x^2 - 18x + 72 = 0$
(C) $x^2 - 18x + 36 = 0$ (D) $x^2 - 18x - 36 = 0$

**6.** If $\alpha, \beta, \gamma$ are the roots of the equation $x^3 - 4x^2 + 6x - 4 = 0$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ is _____.

**7.** The equation $x^2 + 4x + 3 = 0$ has roots $\alpha, \beta$. Find the equation with roots $(1+\frac{\alpha}{\beta})$ and $(1+\frac{\beta}{\alpha})$.
(A) $3x^2 + 16x - 16 = 0$ (B) $3x^2 - 16x + 16$
(C) $x^2 - 16x + 16 = 0$ (D) $3x^2 - 16x - 16 = 0$

**8.** If the roots of $2x^2 + (4m+1)\ x + 2(2m-1) = 0$ are reciprocals of one another; find '$m$'. _____

**9.** Given $\left(x^2 + \frac{1}{x^2}\right) - 4\left(x + \frac{1}{x}\right) + \frac{23}{4} = 0$

Find $\left(x + \frac{1}{x}\right)$. _____

**10.** The equation $x^2 - 4x + 3 = 0$ has _____ number of positive roots _____.

**11.** If $\alpha$ and $\beta$ are the roots of the equation $3x^2 + 5x + k = 0$ and $\alpha^2 - \beta^2 = 25$, then find the value of $k$.
(A) $-\frac{500}{9}$ (B) $\frac{400}{3}$ (C) $-\frac{200}{3}$ (D) $-\frac{500}{3}$

**12.** Find the range of p such that one of the roots of the equation $x^2 - 8x + p = 0$ lies between 0 and 1.
(A) (–7, 0) (B) (2, 3)
(C) (1, 8) (D) (0, 7)

**13.** If the quadratic equations, $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$, where $a, b, c$ are distinct, have one common root, then the common root is

(A) –1 (B) 1 (C) 2 (D) –2

**14.** Find a possible value of the sum of the squares of the roots of the quadratic equation

$x^2 - (a-3)\,x + 5 - a = 0$.

(A) – 3 (B) – 8 (C) – 7 (D) – 9

**15.** In the quadratic equation $ax^2 + bx + c = 0$ if the sum of the roots is equal to twice the product of the roots, then which of the following is true?

(A) $b = c$ (B) $b = 2c$ (C) $b = -c$ (D) $b = -2c$

**16.** Find the real roots of the equation $(x+5)\,(x+6)\,(x+7)\,(x+8) = 840$.

(A) $-1, -12, \dfrac{3 \pm \sqrt{5}}{2}$ (B) –1, 12

(C) –1, –12 (D) $\dfrac{3 \pm \sqrt{5}}{2}$

**17.** If I had walked 2 km/hr faster, I would have taken 60 minutes less to walk 12 km. What is my normal speed?

(A) 1 km/hr (B) 2 km/hr
(C) 3 km/hr (D) 4 km/hr

**18.** In a joint family there are 3 couples. The ages (all in years) of the husbands are $4x$, $3x$ and $2x$. The age of each women is 4 years less than her husband's age. The product of the ages of the first couple is more than the sum of the products of the ages of the other two couples by 832. Find $x$. _____

**19.** A man is moving from A to B. CD is a pole between A and B which is of height 6 metres, He found the distance from the top of the pole C to A and B as 10 metres and 2 $\sqrt{34}$ metres, respectively. Find AB. (in m) _____

**20.** If a number is increased by 4, then result is equal to 5 more than the twice the reciprocal of the number. Find the original number.

(A) 1 (B) 2
(C) –1 (D) Either (B) or (C)

**21.** There are $m$ ways of going from A to B, $m + 3$ ways of going from B to C and $m + 8$ ways of going from C to D. If the number of ways of going from A to D (via B and C) is 336. Find the number of ways of going from A to C.

(A) 28 (B) 84 (C) 48 (D) 56

**22.** If $x^2 + y^2 = 1.0$ and $|x - y| = 1.4$, then $|x + y|$ is

(A) 0.3 (B) 0.5 (C) 0.2 (D) 1.4

**23.** If $65\left(\dfrac{x+5}{x-2}\right)^4 + 24\left(\dfrac{x+5}{x-2}\right)^2 = 432$, then $x =$

(A) $\dfrac{-36}{13}$ or $\dfrac{12}{5}$ (B) $\pm\dfrac{12}{5}$

(C) $\dfrac{36}{13}$ or $\dfrac{-12}{5}$ (D) None of these

**24.** Find the roots of the equation $x^3 + 3x^2 - 6x - 8 = 0$

(A) 1, 2, 4 (B) –1, 2, –4
(C) 1, 0, –3 (D) 1, 3, 9

**25.** A student was asked to write down a $5^{th}$ degree equation with real coefficients that had 1 negative, 2 positive and two non-real roots. If the coefficients of $x^5$, $x^4$, $x^3$, $x^2$, $x$ and the constant terms he got were $a$, $b$, $c$, $d$, $e$, and f, respectively, which of the following can be true?

(i) $a > 0, b > 0, c < 0, d > 0, e < 0, f > 0$

(ii) $a > 0, b > 0, c > 0, d < 0, e < 0, f < 0$

(iii) $a < 0, b > 0, c > 0, d > 0, e < 0, f > 0$

(A) Only (i) (B) Both (i) and (ii)
(C) Both (ii) and (iii) (D) Only (ii)

**26.** If $2x + \dfrac{1}{x} = -\sqrt{6}$, then the value of $x^{12} - x^6 =$

(A) $\dfrac{9}{64}$ (B) $\dfrac{7}{64}$ (C) $\dfrac{11}{64}$ (D) $\dfrac{5}{64}$

**27.** If $x^{10} + x^{11} + \ldots + x^{20}$ is divided by $x^3 + x$, then what is the remainder?

(A) $x$ (B) $-x$ (C) $x^2$ (D) $-x^2$

**28.** Two of the roots of the equation $x^3 - 7x^2 + 36 = 0$ are such that one is thrice the other. These roots have opposite signs. Find the difference of the greatest two roots.

_____

**29.** Three of the roots of the equation

$x^4 + lx^3 + mx^2 + nx + 24 = 0$ are 3, 1 and –2. Which of the following could be the value of $l + m - n$?

(A) 0 (B) 1 (C) 2 (D) 3

**30.** If the equation $3x^4 - 13x^3 + 7x^2 + 17x + a - 10 = 0$ has exactly three positive roots, then a can be

(A) 11 (B) 4 (C) 13 (D) 12

**31.** If $x + \dfrac{1}{x} = -\sqrt{3}$, the value of $x^{42} + x^{48} + x^{54} + x^{60} + x^{66} + x^{72}$ is

(A) (B) 1 (C) 0 (D) –1

**32.** What should be added to $2x^3 + 4x^2 - 7x + 4$ to make it exactly divisible by $x - 1$?

(A) –3 (B) 3 (C) 2 (D) –2

**33.** If $px^3 + qx^2 + rx + s$ is exactly divisible by $x^2 - 1$, then which of the following is / are necessarily true?

(a) $p = r$ (b) $q = s$ (c) $p = -r$ (d) $q = -s$

(A) Both (a) and (b) (B) Both (c) and (d)
(C) Both (a) and (d) (D) Both (b) and (c)

**34.** The remainders of a polynomial $f(x)$ in $x$ are 10 and 15, respectively, when $f(x)$ is divided by $(x-3)$ and $(x-4)$. Find the remainder when f$(x)$ is divided by $(x-3)(x-4)$.
(A) $x-1$ (B) $x+1$ (C) $5x-5$ (D) $5x+5$

**35.** Let $f(x) = a_{12}x^{12} + a_{10}x^{10} + a_8 x^8 + \ldots\ldots + a_2x^2 + a_0$
The coefficients $a_{12}$, $a_{10}$, $a_2$, $a_0$ are real. There are three sign changes of $f(x)$ and $f(x) = 0$ has four non-real roots. Which of the following is true?
(A) $a_{10} = 0$ (B) $a_{10} > 0$ (C) $a_{10} < 0$ (D) $a_0 = 0$

**36.** F$(x)$ is a polynomial whose degree is at least 3. When it is divided by $x-5$, the remainder is 17. When F$(x)$ is divided by $x-6$, the remainder is 19. Find the remainder when F$(x)$ is divided by $(x-5)(x-6)$.
(A) $3x+2$ (B) $4x+3$ (C) $4x-5$ (D) $2x+7$

**37.** The remainder of $x^{72} + x^{60} + x^{48} + x^{36} + x^{24}$ divided by $x^3 - x$ is
(A) $2x^2$ (B) $5x^2$ (C) $3x^2$ (D) $x^2$

**38.** If $x + \frac{6}{x} = 2\sqrt{3}$, $x^8 + x^{12} =$
(A) $36^2(35)$ (B) $36^2(37)$
(C) $-36^2(37)$ (D) $-36^2(35)$

**39.** All the roots of $x^4 - 12x^3 + ax^2 + bx + 81 = 0$ are non-negative. The ordered pair $(a, b)$ can be
(A) (9, 36) (B) (27, –108)
(C) (54, –108) (D) (36, 108)

**40.** The roots of the equation $x^3 + lx^2 + mx + n = 0$ are the real numbers $\alpha$, $\beta$ and $\gamma$, where $\alpha < \beta < \gamma$. Also, one of these roots is 2. If $l - 3 = m$, $m + 8 = n$, then the ratio $\beta : \gamma$ is
(A) $\frac{1}{2}$ (B) 1 (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

**41.** Find the value of $r$ for which the following pair of equations yields a unique solution for $p$, which is positive $p^2 - q^2 = 0$ $(p+r)^2 + q^2 = 1$
(A) –2 (B) 2 (C) $-\sqrt{2}$ (D) $\sqrt{2}$

**42.** The denominator of a fraction is one more than the square of the numerator. When the numerator is increased by 2 and the denominator is increased by 9, the new fraction formed is equal to $\frac{1}{5}$. If the numerator is positive, find the original fraction.
(A) $\frac{1}{2}$ (B) $\frac{2}{5}$ (C) $\frac{3}{10}$ (D) $\frac{5}{26}$

**43.** A positive number exceeds its reciprocal by 168/13. Find the number.
(A) 12 (B) 13 (C) 14 (D) 15

**44.** If $6(x^2 + 1/x^2) - 5(x + 1/x) = 38$ and $x$ is positive and rational, then $x$ is
(A) 3 or 1/3 (B) –3 or –1/3
(C) 2 or 3/2 (D) 3 or 2/3

**45.** If $x + 2y = 5$, then find the maximum (or) minimum possible value of $x^2 + y^2$.
(A) Maximum value, 8 (B) Minimum value, 8
(C) Minimum value, 5 (D) Maximum value, 5

## EXERCISE-4

***Directions for questions 1–30*: For** the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple-choice questions, write your answer in the box provided.

**1.** If $x$ and $y$ are real and $x^2 - 8x + y^2 - 4y + 20 = 0$, find the relation between $x$ and $y$.
(A) $\frac{x}{2} > y$ (B) $x \le y$ (C) $2x = y$ (D) $x = 2y$

**2.** If $E = 3x^2 + 8x + 5$, which of the following is true?
(A) E has a maximum value of $\frac{-1}{3}$
(B) E has a minimum value of $\frac{-2}{3}$
(C) E has a minimum value of $\frac{-1}{3}$
(D) E has a maximum value of $\frac{-2}{3}$

**3.** If the quadratic equation $bx + (a + b\sqrt{2}) + (a - b\sqrt{2})x^2 = 0$ has equal roots, which of the following can be true?
(A) $2b = 3a$ (B) $b\sqrt{2} = 3a$
(C) $3b = 2a$ (D) $3a + b\sqrt{2} = 0$

**4.** Two quadratic equations have a common positive root. The equation satisfied by the other two roots is $x^2 - 6x + 8 = 0$. The sum of all possible products of the four roots, taken two at a time is 116. Find the product of all possible sums of the four roots, taken two at a time. _____

**5.** If $(y+1)^2 - 2y = \dfrac{2}{2-\sqrt{3}}$, the value of $y$ is

(A) $\dfrac{3^{1/4}\left(\sqrt{3}+1\right)}{\sqrt{2}}$ (B) $\dfrac{\sqrt{3}+1}{2^{1/4}}$

(C) $\dfrac{\sqrt{2}\left(\sqrt{3}+1\right)}{3^{1/4}}$ (D) $\dfrac{3^{1/4}\left(\sqrt{3}+1\right)}{2^{1/4}}$

**6.** If a is real and $\sqrt{12+\sqrt{12\sqrt{12+\ldots\infty}}} = a^2$, find $a^4$.

(A) 16 (B) 4 (C) 3 (D) 9

**7.** Both the roots of $x^2 - 70x + k = 0$ are prime numbers. Find the number of possible values of $k$.

(A) 3 (B) 6 (C) 4 (D) 5

**8.** The equation $x^6 + 3x^5 - x^4 - 7x^3 - 14x^2 - 10x - 12 = 0$ has exactly one negative root. How many imaginary roots does it have?

(A) 0 (B) 2 (C) 3 (D) 4

**9.** If the roots of $x + \dfrac{4}{x} = m$ are real, the least positive value of $m$ is _____.

**10.** The expression $ax^2 + bx + c$ takes a maximum value of 5 at $x = 1$ and takes the value of $-1$ at $x = 0$. Find the value of the expression at $x = -5$.

(A) –89 (B) –91 (C) –210 (D) –211

**11.** If $x + \dfrac{1}{x} = \sqrt{2}$, find the value of

$x^{52} + x^{48} + x^{44} + x^{40} + x^{36} + x^{32} + x^{28} + 7$ _____.

**12.** What is the remainder when $z^{1990} - z^3 + z^2 - 1$ is divided by $z + 1$? _____.

**13.** The value of $\cfrac{2}{2+\cfrac{2}{2+\cfrac{2}{2+\ldots\infty}}}$ is

(A) –2.732 (B) 1.732

(C) 0.268 (D) None of these

**14.** If $p$, $q$, $r$ and s are distinct non-zero real numbers such that $(p^2 + q^2 + r^2)\, x^2 - 2\,(pq + qr + rs)\, x + (q^2 + r^2 + s^2) \le 0$, which of the following is true?

(A) $p$, $q$, $r$ and s are in AP

(B) $p$, $q$, $r$ and s are in HP

(C) $p$, $q$, $r$ and s are in GP

(D) $pq = rs$

**15.** Find the quadratic equation whose roots are

$$2 + \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ldots \text{upto } \infty}}}$$

and $$2 + \cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+\ldots \text{upto } \infty}}}.$$

(A) $x^2 + x\left(\sqrt{5}+\sqrt{2}-1\right) + \sqrt{10} - \sqrt{5} = 0$

(B) $x^2 - x\left(1-\sqrt{2+\sqrt{5}}\right) - \sqrt{10} + \sqrt{5} = 0$

(C) $x^2 - x\left(1+\sqrt{2}+\sqrt{5}\right) + \sqrt{10} + \sqrt{5} = 0$

(D) $x^2 + x\left(\sqrt{5}-\sqrt{2}-1\right) - \sqrt{10} - \sqrt{5} = 0$

**16.** If the roots of the equation

$l(m-n)x^2 + m(n-l)x + n(l-m) = 0$ are equal, which of the following is true?

(A) $l + m + n = 0$ (B) $\dfrac{1}{l} + \dfrac{1}{n} = \dfrac{2}{m}$

(C) $l + m + n = 1$ (D) $\dfrac{1}{m} + \dfrac{1}{l} + \dfrac{2}{n}$

**17.** The length of a rectangle R, is reduced by 3 cm and its breadth is increased so that the area remains constant to produce a rectangle Q. The other side of Q is 12 cm. If the length of R is increased by 3 cm, again keeping the area constant to produce a rectangle S, the breadth decreases by 10/7 cm. Find the area of R (in sq. cm).

(A) 180

(B) 72/5

(C) 36

(D) Can be more than one of the above

**18.** If $\alpha$ and $\beta$ are the roots of the equation $x^2 - (p+1)^2\,(p^2 + 4pq + 4q^2 + 2)\, x + (p+1) = 0$, what is the minimum value of $(\alpha+\beta)\,/\,\alpha\beta$, given that $p$ is a particular positive number while $q$ is any real number?

(A) $p + 1$ (B) $2(p+1)$

(C) $p + 2$ (D) $2(p+2)$

**19.** What are the roots of the following equation?

$4x^4 - 4x^3 - 71x^2 + 4x + 4 = 0$

(A) $\dfrac{-7 \pm \sqrt{65}}{4}, \dfrac{9 \pm \sqrt{97}}{4}$

(B) $\frac{-7 \pm \sqrt{97}}{4}, \frac{9 \pm \sqrt{65}}{4}$

(C) $\frac{-7 \pm \sqrt{65}}{4}, \frac{9 \pm \sqrt{47}}{2}$

(D) $\frac{-7 \pm \sqrt{97}}{2}, \frac{9 \pm \sqrt{65}}{2}$

**20.** If $x^4 + (a+b)x^2 + ab = 0$ has exactly two real roots, which of the following is true?

(A) $(a+b)\ ab < 0$ (B) $(a+b)\ ab > 0$

(C) $ab < 0$ (D) $ab > 0$

**21.** If $x^2 - (2a+3)x + 4a^2 - 6a + 9 = 0$, what is the value of $x^3 - 18ax + 8a^3$? _____.

**22.** If $x$ is a real number, what are the values that $y = \frac{x^2 + \left(3+2\sqrt{3}\right)x + 3\sqrt{3}}{x^2 + 3x + 2}$ can take?

(A) $y \le 3$ or $7 \le y$ (B) $3 \le y \le 7$

(C) $y \le -7$ or $-3 \le y$ (D) $-7 \le y \le -3$

**23.** $F(x)$ is a cubic polynomial. The remainders of $F(x)$ when divided by $x-1$, $x-2$ and $x-3$ are 28, 64 and 124, respectively. Find the remainder when $F(x)$ is divided by $x^3 - 6x^2 + 11x - 6$.

(A) $8x^2 + 2x + 12$ (B) $16x^2 - 4x + 8$

(C) $12x^2 + 16$ (D) $20x^2 - 32$

**24.** If $(3x)^2 + (27.3^{1/p} - 15)x + 4 = 0$ has real and equal roots, what is the value of $p$? $(3^{1/p} \neq 1)$

Enter your answer as a decimal value, rounded off to one decimal place. _____.

**25.** If $\alpha$ and $\beta$ are the roots of $x^2 - x + l = 0$ while $\gamma$ and $\delta$ are the roots of $x^2 - 4x + m = 0$, where $\alpha, \beta, \gamma$ and $\delta$ form a geometric progression, what is the sum of the integer values of $l$ and $m$?

**26.** What is the value of $p$ in the equation $3x^2 + 17x + 18 - 3p = 0$ such that $\alpha/\beta + \beta/\alpha \ge 2$, where $\alpha$ and $\beta$ are the roots of the equation? $(\alpha \neq \beta$ and $\alpha, \beta \neq 0)$

(A) $p < 6$ (B) $p > 6$

(C) $p \ge 6$ (D) $p \le 6$

**27.** $f(x) = ax^2 + bx + c,\ a < 0$

$f(x) = 0$ has real and equal roots and $p_1$, $p_2$, and $p_3$ are real numbers such that $p_1 < p_2 < p_3 < \frac{-b}{2a}$. Which of the following is true?

(A) $f(p_1) < f(p_2)$ and $f(p_2) > f(p_3)$

(B) $f(p_1) < f(p_2) < f(p_3)$

(C) $f(p_1) > f(p_2) > f(p_3)$

(D) $f(p_1) > f(p_2)$ and $f(p_2) < f(p_3)$

**28.** $f(x) = ax^2 + bx + c,\ a < 0$

If $\alpha, \beta$ are the distinct real roots of the equation $f(x) = 0$ and $p_1$ $p_2$ and $p_3$ are real numbers such that $p_1 < \alpha < p_2 < \beta < p_3$, which of the following is true?

(A) $f(p_1) < f(p_2) < f(p_3)$

(B) $f(p_1) > f(p_2)$ and $f(p_2) < f(p_3)$

(C) $f(p_1) > f(p_2) > f(p_3)$

(D) $f(p_1) < f(p_2)$ and $f(p_2) > f(p_3)$

**29.** If $p$ is the greatest negative integer for which $p^2 \ge 20p + 3500$, which of the following is true?

(A) $-50 < p \le -29$ (B) $-29 < p \le -1$

(C) $-73 < p \le -50$ (D) $-102 \le p \le -73$

**30.** In the cubic equation $x^3 + p_1x^2 + p_2x + p_3 = 0$, $p_1$, $p_2$ and $p_3$ are in GP and a, b and c are the roots of the equation. If $a + b + c = -1$, then $(ab + bc + ca)^2 =$

(A) $2\,abc$ (B) $3\,abc$ (C) $\frac{1}{2}abc$ (D) $-abc$

## EXERCISE-5 Based on Data Sufficiency

***Directions for questions 1–25:*** Each question is followed by two statements, I and II. Answer each question based on the following directions:

Choose (A) If the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.

Choose (B) If the question can be answered by using either statement alone.

Choose (C) If the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (D) If the question cannot be answered even by using both the statements together.

**1.** Is $x = 2$?

I. $x^2 - 4x + 3 = 0$.

II. $x^2 - x + 2 = 0$.

**2.** What is the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$?

I. The product of the roots is the same as the sum of the roots.

II. The sum of the roots of the quadratic equation $cx^2 + ax - b = 0$ is 2.

**3.** If $ax^2 + bx + c = 0$, does $x$ have a real value?

I. $0 < a < c < 1$.

II. $b > 3$.

**4.** Are the roots of the quadratic equation real?

I. The square of the sum of its roots is at most equal to 4 times the product of its roots.

II. The square of the sum of its roots is at least equal to 4 times the product of its roots.

**5.** In $Ax^2 + Bx + C$, one of $A$, $B$ and $C$ is negative. Which of them is negative?

I. $Ax^2 + Bx + C$ is positive when $x = 1$ and negative when $x = -1$.

II. $C$ is not negative.

**6.** $A$ and $B$ are natural numbers. The equation $Ax^2 + Bx + 4A = 0$ has real roots, neither of which exceed 10. Find $A$.

I. $B < 5$

II. $A \le 2$

**7.** $P$ and $Q$ are natural numbers. Find $\frac{P}{Q}$.

I. $Px^2 + Qx + Q = 0$ has real roots.

II. $Qx^2 + 4Px + P = 0$ has real roots.

**8.** Q is a quadratic expression in $x$. Find its maximum value.

I. The discriminant of Q = 0 is –12.

II. The coefficient of $x^2$ in Q is –1.

**9.** Are the roots of $x^2 + ax + b = 0$ equal given that they are integers?

I. $a + b = 0$.

II. The roots are reciprocal to each other.

**10.** The roots of $(a^2 + b^2)\ x^2 + 2(ab + bc)\ x + b^2 + c^2 = 0$, where $a$, $b$, $c$ are real numbers, are $p$ and $q$. Find $\frac{b^2}{ac}$.

I. $p$ and $q$ are real.

II. $b = -2$, $a = c = 2$

**11.** $ax^2 + 3x + 2 = 0$ has roots $\alpha$ and $\beta$. Is $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ greater than zero?

I. $a < 0$ II. $2 > a > 0$

**12.** What is the sum of the roots of $ax^2 + bx + c = 0$?

I. Product of the roots of $cx^2 - ax + b = 0$ is 3.

II. Sum of the roots of $cx^2 + ax + b = 0$ is 4.

**13.** If $\alpha, \beta$ are the roots of the equation $x^2 + \gamma x + \delta = 0$, and $\gamma > 0$, what is the value of $\gamma$?

I. $\alpha^2 + \beta^2 = 1300$ II. $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{1}{12}$

**14.** Is $2x^2 - ax + 20$ greater than 0?

I. $10 > a > 0$ II. $a < 10$

**15.** If the roots of the equation $ax^2 + bx + c = 0$ are real, is the sum of the roots of the equation greater than 0?

I. $abc > 0$ II. $a + b + c < 0$

**16.** What are the values of $b$ in the equation $x^2 - bx + c = 0$?

I. The roots are reciprocal of each other.

II. The sum of the roots is twice the product of the roots.

**17.** What is the value of $x$ if

$$x^2 - (\log_b a^3)x + 9\log_b\left(\frac{a}{b}\right) = 0?$$

I. $x$ is an integer.

II. $a$, $b$ are relative primes and neither $a$ nor $b$ is 1.

**18.** In the quadratic equation $ax^2 - (a + 1)x + a = 0$, find the value of $a$.

I. The roots of the equation are reciprocals of each other.

II. The roots of the equation are equal.

**19.** Does $x^2 + y^2 + 4x - 6y + b = 0$ have a unique solution?

I. $b = 13$

II. $b = 26$

**20.** If the quadratic equations $2x^2 + kx - 5 = 0$ and $x^2 - 3x - 4 = 0$ have a root in common, what is the value of $k$?

I. $k$ is an integer

II. $k + 5$ is a natural number.

**21.** What is the value of $k$ given $ax^2 + kx + 72 = 0$, where $a$, $k$ are positive integers?

I. The roots of the equation are real and equal.

II. $a = 8$

**22.** Is there a real solution for $ax^2 + bx + c = 0$?

I. $a$, $b$ and $c$ are in GP.

II. $b$ is the arithmetic mean of $2a$ and $2c$

**23.** If $ax^2 + bx + c = 0$, find the value of $b + c$.

I. $a$, $b$ and $c$ are successive terms of a geometric progression

II. The roots of the equation are real and equal.

**24.** What is the minimum value of the quadratic expression $3x^2 + bx + c$?

I. The two positive roots of the equation $3x^2 + bx + c = 0$ are reciprocal to each other.

II. The sum of the roots of $3x^2 + bx + c = 0$ is 2.

**25.** The roots of the equation $x^2 - 5x + a = 0$ are $p$ and $q$ and $a$ is an integer. Is $\left(\frac{p^3 + q^3}{pq}\right)$ an integer?

I. $1 < a < 5$ II. $0 < a < 6$

## ANSWER KEYS

### Exercise - 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (B) | 8. (D) | 15. (C) | 22. (C) | 29. (B) | 36. (B) | 43. (A) |
| 2. (B) | 9. (D) | 16. (C) | 23. (C) | 30. (A) | 37. (A) | 44. (A) |
| 3. (A) | 10. (B) | 17. (C) | 24. (D) | 31. (C) | 38. (A) | 45. (C) |
| 4. (C) | 11. (C) | 18. (D) | 25. (D) | 32. (D) | 39. 0 | |
| 5. (C) | 12. (A) | 19. (B) | 26. (D) | 33. (A) | 40. –10 | |
| 6. (C) | 13. (A) | 20. (D) | 27. (C) | 34. (B) | 41. (A) | |
| 7. 16 | 14. (D) | 21. (D) | 28. (D) | 35. (C) | 42. (D) | |

### Exercise - 2

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (A) | 9. (B) | 12. (A) | 20. 25 | 28. (C) | 36. (C) | 44. (D) |
| 2. (C) | 10. (D) | 13. (A) | 21. (A) | 29. (D) | 37. (C) | 45. 21 |
| 3. (B) | 11. (a) | 14. 1 | 22. (D) | 30. 2 | 38. (A) | |
| 4. (D) | (C) | 15. (B) | 23. (D) | 31. 10 | 39. (A) | |
| 5. 15 | (b) | 16. 8 | 24. (A) | 32. (B) | 40. (D) | |
| 6. 10 | (B) | 17. (C) | 25. (A) | 33. (D) | 41. 2 | |
| 7. (A) | (c) | 18. 14 | 26. 2 | 34. 2 | 42. 0 | |
| 8. (C) | (A) | 19. (B) | 27. (D) | 35. 20 | 43. (B) | |

### Exercise - 3

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (C) | 8. 1 | 15. (D) | 22. (C) | 29. (D) | 36. (D) | 43. (B) |
| 2. (C) | 9. 2.5 | 16. (C) | 23. (D) | 30. (B) | 37. (B) | 44. (A) |
| 3. 10 | 10. 2 | 17. (D) | 24. (B) | 31. (C) | 38. (D) | 45. (C) |
| 4. (D) | 11. (D) | 18. 16 | 25. (A) | 32. (A) | 39. (C) | |
| 5. (B) | 12. (D) | 19. 18 | 26. (A) | 33. (B) | 40. (A) | |
| 6. 1.5 | 13. (B) | 20. (D) | 27. (B) | 34. (C) | 41. (C) | |
| 7. (B) | 14. (A) | 21. (A) | 28. 3 | 35. (D) | 42. (D) | |

### Exercise - 4

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (D) | 6. (A) | 11. 6 | 16. (B) | 21. –27 | 26. (A) | |
| 2. (C) | 7. (D) | 12. 2 | 17. (D) | 22. (A) | 27. (B) | |
| 3. (C) | 8. (D) | 13. (D) | 18. (B) | 23. (C) | 28. (D) | |
| 4. 460800 | 9. 4 | 14. (C) | 19. (A) | 24. –0.5 | 29. (C) | |
| 5. (A) | 10. (D) | 15. (C) | 20. (C) | 25. –34 | 30. (D) | |

### Exercise - 5

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (B) | 5. (C) | 9. (B) | 13. (C) | 17. (C) | 21. (C) | 25. A |
| 2. (C) | 6. (A) | 10. (B) | 14. (A) | 18. (D) | 22. (A) | |
| 3. (C) | 7. (C) | 11. (A) | 15. (D) | 19. (B) | 23. (C) | |
| 4. (A) | 8. (C) | 12. (C) | 16. (C) | 20. (B) | 24. (C) | |

## SOLUTIONS

## EXERCISE-1

**1.** The quadratic equation is $(x-2)\ (x-3) = 0$

or $x^2 - 2x - 3x + 6 = 0$ or $x^2 - 5x + 6 = 0$.

**2.** $x^2 + 6x + 9 = 0 \Rightarrow x^2 - (-6)\ x + 9 = 0$

The sum of the roots is negative and the product of the roots is positive.

$\therefore$ Both the roots are negative.

**3.** We know that, if $s$ is the sum and $p$ is the product of the roots of a quadratic equation then the equation is $x^2 - 5x + p = 0$

Note that the constant term in the equation is nothing but the product of the roots.

Given, $s = 4$, $p = 15$.

$\therefore$ The quadratic equation is $x^2 - 4x + 15 = 0$.

**4.** $x^2 - 8x + 9 = 0$. The roots are $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(9)}}{2(1)}$$

$$= \frac{8 + \sqrt{64 - 36}}{2} = \frac{8 \pm \sqrt{28}}{2} = \frac{8 \pm 2\sqrt{7}}{2} = 4 \pm \sqrt{7}$$

$\therefore$ The roots are $4 + \sqrt{7}$ and $4 - \sqrt{7}$.

**5.** Given the sum of the roots is 7 and product of the roots is 12.

$\therefore$ The equation is

$x^2 -$ (sum of the roots) $x +$ (product of the roots) $= 0$

i.e., $x^2 - 7x + 12 = 0$.

**6.** Any quadratic equation in $x$,

can be expressed as $x^2 -$ (sum of the roots) $x +$ (product of the roots) $= 0$.

In the given problem, as the roots are 5 and 6, the quadratic equation in $x$ is $x^2 - 11x + 30 = 0$

**7.** $9x^2 - 144x + 92 = 0$ will have the sum of its roots equal to $-\left(\dfrac{-144}{9}\right) = 16$

**8.** $x^2 - 9x + \dfrac{41}{4} = 0$ $x = \dfrac{9 \pm \sqrt{(-9)^2 - 4(1)\left(\frac{41}{4}\right)}}{2}$

$$= \frac{9 \pm 2\sqrt{10}}{2}$$

**9.** As the product of the roots is unknown, the quadratic equation cannot be uniquely determined.

**10.** The equation is $x^2 - 20x + 36 = 0$

**11.** Let $\alpha, \beta$ be the roots of the equation. Given, $\alpha + \beta = 11$ and $\alpha\beta = 24$.

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = 11^2 - 4(24) = 121 - 96 = 25$

$\alpha - \beta = 5$ (1)

$\alpha + \beta = 11$ (2)

(1) + (2) $\Rightarrow 22 = 16 \Rightarrow \alpha = 8$

$\beta = 11 - 8 = 3$.

$\therefore$ The roots are 8, 3.

**12.** To find a new quadratic equation, whose roots are reciprocals of the roots of $3x^2 + 5x + 7 = 0$, we have replace $x$ by $1/x$.

$\therefore$ The equation is $3\left(\dfrac{1}{x}\right)^2 + 5\left(\dfrac{1}{x}\right) + 7 = 0$

$$\Rightarrow \frac{3}{x^2} + \frac{5}{x} + 7 = 0 \Rightarrow 3 + 5x + 7x^2 = 0$$

$\therefore 7x^2 + 5x + 3 = 0$.

**13.** Discriminant of the quadratic equation $ax^2 + bx + c = 0$ is $b^2 - 4ac$.

$\therefore$ The discriminant of $2x^2 + 3x + 4 = 0$ is

$3^2 - 4(2)(4) = 9 - 32 = -23$.

**14.** $x^2 + kx + 7 = 0$

Discriminant $= k^2 - (4)\ (1)\ (7) = k^2 - 28$.

As we do not know whether $k^2 > 28$ or $k^2 = 28$ or $k^2 < 28$, we cannot say anything about the nature of the roots.

**15.** The sum of the roots of the equation $ax^2 + bx + c = 0$ is $-b/a$.

$\therefore$ The sum of the roots of $9x^2 - 36x + 35 = 0$ is $\dfrac{-(-36)}{9} = 4$

**16.** $(x^n - b)^2 = 0$ can also be written as $x^{2n} - 2x^n + b^2 = 0$ as the degree of this equation is $2n$, it has $2n$ roots.

**17.** Discriminant $= 8^2 - (4)\ (4)\ (-11) = 240$ which is positive but not a perfect square

$\therefore$ The roots of $4x^2 + 8x - 11 = 0$ will be conjugate surds.

**18.** Let the roots of $x^2 - 4x + 10 = 0$

be $\alpha$ and $\beta$. $\alpha + \beta = 4$ and $\alpha\beta = 10$

The required equation will have roots of $\alpha + 3$ and $\beta + 3$

$\alpha + 3 + \beta + 3 = \alpha + \beta + 6 = 10$

$(\alpha + 3)\ (\beta + 3) = \alpha\beta + 3\ (\alpha + \beta) + 9 = 31$

$\therefore$ Required equation is $x^2 - 10x + 31 = 0$.

**19.** Product of the roots is $\frac{a}{a} = 1$.

As the product of the roots is 1, the roots are reciprocals of each other.

Hence, choice $(B)$ follows.

**20.** As the equation $x^2 + px + 81 = 0$ has equal roots,

$p^2 - 4\ (1)\ (81) = 0$

$\Rightarrow p^2 = 324$

$\Rightarrow p = \sqrt{324} = \pm 18$

**21.** Quadratic expression is $ax^2 + bx + c$

where $a > 0$, $\frac{4ac - b^2}{4a}$ given the minimum value of the expression.

When $a < 0$, $\frac{4ac - b^2}{4a}$ gives the maximum value of the expression.

**22.** $(x^2 - 3)^3 - 4x^4 = 0$

$(x^2)^3 + 3x^2\ (-3)\ (x^2 - 3) + (-3)^3 - 4x^4 = 0$

$x^6 - 9x^4 + 27x^2 - 27 - 4x^4 = 0$

$\Rightarrow x^6 - 13x^4 + 27x^2 - 27 = 0.$

The highest power of $x$ in the equation is called the degree of the equation.

$\therefore$ The degree of the equation is 6.

**23.** To find a new quadratic equation, whose roots are 4 more than the roots of $x^2 + 7x + 16 = 0$, we have to replace $x$ by $(x - 4)$.

$\therefore$ The equation is $(x - 4)^2 + 7\ (x - 4) + 16 = 0$

$\Rightarrow x^2 - 8x + 16 + 7x - 28 + 16 = 0$

$\Rightarrow x^2 - x + 4 = 0.$

**24.** The minimum value for the quadratic expression $ax^2 + bx + c$, where $a$ is positive is given by $\frac{4ac - b^2}{4a}$.

$\therefore$ The minimum value of $2x^2 + 5x + 7$ is

$= \frac{4(2)(7) - 5^2}{4(2)} = \frac{56 - 25}{8} = \frac{31}{8} = 3\frac{7}{8}.$

**25.** Here the equation $(x - 4)^2 = 0$ is given.

$\therefore$ Number of distinct roots = 1.

**26.** Going by the choices, only choice $(D)$ satisfies the given conditions.

**27.** If the root of a quadratic equation with rational coefficient is $3 + 2\sqrt{2}$, the other root is $3 - 2\sqrt{2}$.

Therefore, their sum is 6.

**28.** $5x^3 - 2x^2 - 3x - 2 = x(5x^2 - 2x - 3) - 2)$

$= x(x - 1)\ (5x + 3) - 2$

If 2 is added to $5x^3 - 2x^2 - 3x - 2$, then the resulting expression is exactly divisible by $x - 1$.

**29.** The product of the reciprocal roots $= \frac{c}{a} = 1$

$c = 3,\ a = 3.$

**30.** The equation whose roots are the reciprocals of the roots of $ax^2 + bx + c = 0$ is $cx^2 + bx + a = 0$

$\therefore$ The required equation is $2x^2 + 4x + 3 = 0$

**31.** Let $f(x) = 6x^2 + 13x + k$.

Given $2x + 3$ is a factor of $f(x)$.

By factor theorem,

$f\left(-\frac{3}{2}\right) = 0$

$\Rightarrow 6\left(-\frac{3}{2}\right)^2 + 13\left(-\frac{3}{2}\right) + k = 0$

$\Rightarrow \frac{27}{2} - \frac{39}{2} + k = 0$

$\Rightarrow k = 6$

$\therefore f(x) = 6x^2 + 13x + 6$

$= (2x + 3)\ (3x + 2)$

$\therefore$ The other factor is $3x + 2$.

**32.** We know that the remainder when $f(x)$ is divided by $x - a$ is $f(a)$.

$\therefore$ The remainder when $3x^3 + 5x^2 - 6x + 3$ is divided by $x - 1$ is $3(1)^3 + 5(1)^2 - 6(1) + 3 = 5$.

$\therefore$ When $-5$ is added to $f(x)$ it is exactly divisible by $x - 1$.

**33.** Given, $f(x) = x^3 + 5x^2 + 2x + 3$.

Remainder $= f(0) = (0)^3 + 5(0)^2 + 2(0) + 3 = 3.$

**34.** Given, $px^2 + qx + r$ is divisible by $x + 1$.

By factor theorem,

$p(-1)^2 + q(-1) + r = 0$

$\Rightarrow p - q + r = 0 \Rightarrow p + r = q.$

**35.** $f(x) = 2x^6 - 5x^3 - 3 = 2(x^3)^2 - 5(x^3) - 3.$

Let $x^3 = y$

$f(y) = 2y^2 - 5y - 3.$

$\therefore$ The remainder when $f(y)$ is divided by $y + 1$ is $f(-1) = 2 - 5(-1) - 3.$

$= 4$

**36.** The quadratic equation whose roots are reciprocal of $2x^2 + 5x + 3 = 0$ can be obtained by replacing $x$ by $\frac{1}{x}$ Hence, the required equation is $2\left(\frac{1}{x}\right)^2 + 5\left(\frac{1}{x}\right) + 3 = 0$

$\Rightarrow 3x^2 + 5x + 2 = 0$

**37.** As the sum of the roots as well as the product of the roots is positive, both roots are positive.

**38.** Maximum/Minimum value of the quadratic expression $px^2 + qx + r$ occurs at $x = \frac{-q}{2p}$

**39.** $3x^2 - 5x - 2 = (3x + 1)(x - 2)$

$3x^2 - 5x - 2$ is divisible by $x - 2$

$\therefore$ Remainder is 0.

**40.** Remainder theorem: If $f(x)$ is divided by $x - a$, then the remainder of the division is $f(a)$.

Let $f(x) = 9x^2 - 13x + c$

The remainder of the division of $f(x)$ by $x + 1$

i.e., $x - (-1)$ is $f(-1)$.

This is also equal to 12.

$9(-1)^2 - 13(-1) + c = 12 \Rightarrow c = -10.$

**41.** If $a > 0$, the quadratic expression $ax^2 + bx + c$, has its minimum value at $-b/2a$ and if $a < 0$ it has its maximum value at $-b/2i$.

$\therefore$ Any quadratic expression of the form $ax^2 + bx + c$ has its maximum or minimum value at $-b/2a$.

**42.** The maximum value for quadratic expression $ax^2 + bx + c$, where $a$ is negative is given by $\frac{4ac - b^2}{4a}$.

$\therefore$ The maximum value of $-7x^2 + 2x + 5$ is

$= \frac{4(-7)(5) - 2^2}{4(-7)} = \frac{-140 - 4}{-28} = \frac{-144}{-28} = \frac{36}{7} = 5\frac{1}{7}.$

**43.** To find $a$ new quadratic equation, whose roots are half of the roots of the equation, $x^2 + 5x + 9 = 0$, we have to replace $x$ by $2x$.

$\therefore$ The equation is $(2x)^2 + 5(2x) + 9 = 0$

$\Rightarrow 4x^2 + 10x + 9 = 0.$

**44.** A quadratic expression of the form of $ax^2 + bx + c = 0$, where $b$ and $c$ are real numbers always has a minimum value when $a$ is positive and has a maximum value when $a$ is negative. Its maximum/minimum value is given by

$= \frac{4ac - b^2}{4a}$

For the given expression $a = 1$, $b = 1$ and $c = 5$. Hence, it has a minimum value, The minimum value is given by

$\frac{4(1)(5) - 1^2}{4(1)} = \frac{19}{4}$

**45.** Let $g(x) = ax^2 - bx - c$

$x + 1$ i.e., $x - (-1)$ is a factor of $g(x)$ (1)

Factor Theorem: $x - a$ is a factor of $f(x)$ if and only if $f(a) = 0$.

$g(-1) = 0$ ($\because$ From (1))

$g(-1) = a + b - c.$ $\therefore a + b - c = 0$

$a + b = c$

## EXERCISE-2

**1.** $x^2 - 7x + 12 = 0$

$x^2 - 3x - 4x + 12 = 0$

$x(x - 3) - 4(x - 3) = 0$

$(x - 3)(x - 4) = 0$

$\therefore x = 3$ or $4$

**2.** $(k^2 - 144) < 0$

$k^2 < 144$

$-12 < k < 12$

**3.** The roots of the given equation is $-3, -4$. To form an equation whose roots are $-4, -5$. The required equation is $x^2 + 9x + 20 = 0$.

In some cases the roots may not be integers

Substitute '$(x + 1)$' for '$x$' in the original equation.

$(x + 1)^2 + 7(x + 1) + 12 = 0$ and you get the equation $x^2 + 9x + 20 = 0$.

**4.** If $\alpha$ and $\beta$ are the root of the QE, then the QE will be $x^2 - (\alpha+\beta)x + \alpha x \beta = 0$

$\Rightarrow x^2 - (2+7)x + 2 \; x \; 7 = 0$

$\Rightarrow x^2 - 9x + 14 = 0$

**5.** $(k^2 - 196) > 0$

$k^2 > 196$

$k > 14$ or $k < -14$

**6.** Given $y_1$ and $y_2$ are the roots of $y^2 - 7y + k = 0$

$$y_1 + y_2 = -\left(\frac{-7}{1}\right) = 7 \qquad (1)$$

given

$$y_1 - y_2 = 3 \qquad (2)$$

the product of roots = solving (1) and (2), $y_1 = 5$ and $y_2 = 2$

$y_1 y_2 = k$

$\therefore k = 10.$

**7.** Let the other root be $\alpha$

$$4 + \sqrt{5} + \alpha = \frac{-(-8)}{1} = 8$$

$\alpha = 4 - \sqrt{5}$

The other root is $4 - \sqrt{5}$. The product of the roots = $m$

$m = \left(4+\sqrt{5}\right)\left(4-\sqrt{5}\right) = 4^2 - \left(\sqrt{5}\right)^2$

$= 16 - 5 = 11.$

**8.** Given that $\alpha$ and $\beta$ are the roots of $x^2 - 8x + 9 = 0$.

$\alpha + \beta = 8$ and $\alpha.\beta = 9$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{8^2 - 2(9)}{9}$$

$$= \frac{46}{9}$$

**9.** Let the roots of the equation $py^2 + qy + r = 0$ be $mk$ and $nk$.

$$mk + nk = \frac{-q}{p} \text{ and } (mk)\,(nk) = \frac{r}{p}$$

$$m + n = \frac{-q}{kp} \text{ and } mn = \frac{r}{pk^2}$$

$$\frac{mn}{(m+n)^2} = \frac{\frac{r}{pk^2}}{\left(\frac{-q}{kp}\right)^2} = \left(\frac{r}{pk^2}\right)\left(\frac{k^2p^2}{q^2}\right) = \frac{pr}{q^2}.$$

**10.** A quadratic equation in $x$ whose sum of the roots is $s$ and product of the roots is $p$ is $x^2 - sx + p = 0$

The equation whose sum of the roots is $-4$ and the product of the roots is 9 is $x^2 - (-4)x + 9 = 0$

$\Rightarrow x^2 + 4x + 9 = 0.$

**11.** For a quadratic equation $ax^2 + bx + c = 0$

If $b^2 - 4ac > 0$ but not a perfect square, the roots are real and unequal and irrational.

If $b^2 - 4ac > 0$ and a perfect square, the roots are real and unequal and rational.

If $b^2 - 4ac = 0$, the roots are rational and equal.

If $b^2 - 4ac < 0$, the roots are complex.

(a) As $b^2 - 4ac = -8$ which is $< 0$, the roots are complex.

(b) As $b^2 - 4ac > 0$ and a perfect square, the roots are rational and unequal.

(c) As $b^2 - 4ac = 0$, the roots are real and equal.

**12.** Let the roots be $\alpha$ and $\alpha^3$

Product of the roots = $\alpha^4 = \frac{81}{16} \Rightarrow \alpha = \pm\frac{3}{2}$

$$\frac{q}{16} = \alpha + \alpha^3 = \pm\frac{3}{2} \pm \frac{27}{8}$$

$\Rightarrow q = \pm 78$

**13.** Squaring both sides,

$$3 - 2x + 7 + 2x + 2\sqrt{(3-2x)(7+2x)} = 16$$

$$\Rightarrow \sqrt{21 - 8x - 4x^2} = 3$$

Squaring on both sides,

$21 - 8x - 4x^2 = 9$

$\Rightarrow 4(x^2 + 2x - 3) = 0$

$\Rightarrow 4(x^2 + 3x - x - 3) = 0$

$\Rightarrow 4(x\,(x+3) - 1(x+3)) = 0$

$\Rightarrow 4((x-1)\,(x+3) = 0$

$\Rightarrow x - 1 = 0$ or $x + 3 = 0$

$\Rightarrow x = 1$ or $x = -3.$

both these values satisfy the original equation.

**14.** $$\frac{6(x+3)+8(x+2)}{(x+2)(x+3)} = \frac{10(x+1)+4(x+4)}{(x+4)(x+1)}$$

$$\frac{14x+34}{x^2+5x+26} = \frac{14x+26}{x^2+5x+4}$$

$(x^2+5x)14x+(x^2+5x)34+4(14x+34)$

$=(x^2+5x)(14x)+(x^2+5x)26+6(14x+26)$

$\Rightarrow 8x^2+12x-20=0$

$\Rightarrow 8x^2+20x-8x-20=0$

$\Rightarrow x(8x+20)-1(8x+20)=0$

$(8x+20)(x-1)=0$

$8x+20=0$ (or) $x-1=0$

$x=\frac{-20}{8}$ or 1

Going by the choices, $x=1$

**NOTE**

By directly substituting the choices in the given equation L.H.S = R.H.S

Only when $x=1$.

**15.** The discriminant of the equation is

$(2a)^2-4(a-b)(a+b)$

$=4a^2-4(a^2-b^2)=4b^2$ which is 0 when $b$ is 0 and positive otherwise.

**16.** Product of the roots $=4y^{\log_y k^2}=4k^2$

$\therefore 4k^2=256\Rightarrow k^2=64$

$\Rightarrow k=\pm 8$

But $k$ cannot be negative $\therefore k=8$

**17.** Let $x$ be the positive number. From the given condition we get $(x-2)^2=x-2$

$\Rightarrow x^2-4x+4=x-2$

$\Rightarrow x^2-5x+6=0$

$\Rightarrow x^2-3x-2x+6=0$

$\Rightarrow x(x-3)-2(x-3)=0$

$\Rightarrow (x-3)(x-2)=0\Rightarrow x=3$ or 2

**18.** Let the smaller number be $x$, then the other number will be $(24-x)$

$2x^2-(24-x)^2=4$

$\Rightarrow x^2+48x-580=0$ $x=10,-58$

$\therefore$ The larger of the two numbers is 24 – 10 i.e. 14

**19.** $2x^2+5x+2=0$

$\Rightarrow 2x^2+4x+x+2=0$

$2x(x+2)+1(x+2)=0$

$\therefore x=-2$ (or) $x=\frac{-1}{2}$

The roots of $2x^2+5x+2=0$ are $k$ less than the roots of $px^2+qx+r=0$ $\therefore$ The roots of $px^2+qx+r=0$ are $-2+k$ and $-\frac{1}{2}+k$ $\therefore -2+k$ satisfies

$p(-2+k)^2+q(-2+k)+r=0$

$\therefore p(k-2)^2+q(k-2)+2=0$

$p(k^2+4-4k)+q(k-2)+r=0$

$pk^2+4p-4pk+qk-2q+r=0$

$k(pk-4p+q)+4p-2q+r=0$

$\therefore$ Choice $(B)$ could be true

**20.** Let the speed of the ordinary train be $x$ km/hr

$\frac{1600}{x}-\frac{1600}{x+15}=24$

Solving we get; $x=25$

**21.** If the expression is $x$,

$x=\frac{1}{1+\frac{1}{-4+x}}\Rightarrow x=\frac{-4+x}{-3+x}$

$x^2-3x=x-4$

$x^2-4x+4=0$

$(x-2)^2=0\Rightarrow x=2.$

**22.** Let $\alpha,\beta$ be the roots of $ax^2+bx+c=0$

$\alpha+\beta=\frac{-b}{a}$ ; $\alpha\beta=\frac{c}{a}$.

Given $\alpha^3+\beta^3=\frac{-7}{2}$, $\alpha\beta=\frac{3}{4}$

$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

$\frac{-7}{2}=\left(-\frac{b}{a}\right)^3-3\left(\frac{3}{4}\left(\frac{-b}{a}\right)\right)$

Let $\frac{-b}{a} = p$

$-\frac{7}{2} = p^3 - \frac{9}{4}p$

$\Rightarrow 4p^3 - 9p + 14 = 0.$

$p = -2$ satisfies the above equation.

$\therefore p + 2$ is a factor of $4p^3 - 9p + 14$.

The other factor must have the form $4p^2 + kp + 7$.

Where $k$ is a constant ($\because \frac{4p^3}{p} = 4p^2$ and $\frac{14}{2} = 7$).

$(p+2)(4p^2 + kp + 7) = 4p^3 - 9p + 14.$

Coefficient of $p^2$ on the $L.H.S = 2(4) + 1(k)$

$\therefore 2(4) + 1(k) = 0$

$k = -8.$

$\therefore p + 2 = 0$ or $4p^2 - 8p + 7 = 0.$

The second equation has a – ve discriminant.

$\therefore$ Only possible value of $p = -2$.

$\frac{-b}{a} = -2 \Rightarrow \frac{a}{b} = \frac{1}{2}$ .

**23.** Let $\alpha$ and $\beta$ be the roots of the equation

$(3 + n^2)\ x^2 - 5nx + n^2 - 14 = 0$ for some values of $n_1$ and $n_2$ $\alpha + \beta = \alpha\beta$

$\frac{5n}{n^2+3} = \frac{n^2-14}{n^2+3}$

$\Rightarrow (n^2 - 5n - 14)\ (n^2 + 3) = 0$

$\Rightarrow (n - 7)\ (n + 2) = 0\ (\because\ n^2 + 3 \neq 0)$

$n_1 = 7\ ,\ n_2 = -2$

$\therefore \left(n_1^2 + n_2^2\right)^2 = [(7)^2 + (-2)^2]^2 = (53)^2 = 2809.$

**24.** The given equation is $ax^2 + 12x + c = 0,$ (1)

where $3 \le a \le 7$ and $0 \le c$.

Equation (1) has real roots if and if only $144 - 4ac \ge 0$ or $ac \le 36$.

if $a = 3$, $c = 0, 1, 2, 3, \ldots 12 \rightarrow$ i.e., 13,

if $a = 4$, $c = 0, 1, 2, \ldots 9 \rightarrow$ i.e., 10,

if $a = 5$, $c = 0, 1, \ldots 7 \rightarrow$ i.e., 8,

if $a = 6$, $c = 0, 1, \ldots 6 \rightarrow$ i.e., 7,

if $a = 7$, $c = 0, 1, \ldots 5 \rightarrow$ i.e., 6.

There are a total of 44 equations with real roots.

**25.** Let $\alpha, \beta$ be the roots of the quadratic equation

$ax^2 + bx + c = 0.$

Given $a = 1$, $b = c$. Let $b = c = k$.

The given quadratic equation is $x^2 + kx + k = 0$

$\alpha + \beta = -k\ ;\ \alpha\beta = k$

$\alpha + \beta + \alpha\beta = 0.$

**Method** 1

$\alpha = -\beta\ (1 + \alpha)$

$\therefore \beta = \frac{-\alpha}{\alpha + 1}$

$\alpha$ and $\beta$ both are integers. Since $\beta$ is an integer, $\frac{-\alpha}{\alpha+1}$ is also an integer. This is possible only when $\alpha = 0$ or $-2$. In either case, $\alpha = \beta$. $\therefore$ Difference = 0.

**Method** 2

Adding 1 on both sides, $\alpha\beta + \alpha + \beta + 1 = 1$

$(\alpha + 1)\ (\beta + 1) = 1$

$\alpha, \beta$ are integers.

$\therefore \alpha + 1$ and $\beta + 1$ are integers.

$\therefore \alpha + 1, \beta + 1 = 1$ or $-1$.

In either case, $\alpha = \beta$.

$\therefore$ Difference = 0.

**26.** As the roots of $x^2 + kx + 4 = 0$ are real and unequal,

$(k)^2 - 4(1)\ (4) > 0$. Also, $k^2 \le 36$

$k^2 > 16$ and $k^2 \le 36$ As $k > 0$, $k = 5$ or 6

$\therefore$ There are 2 equations

**27.** Let the number of chocolates with $A$ and $B$ be $x$ and $y$ Given $x + y = 50$ and $x^2 + y^2 = 1300$.

By trial $x = 20$, $y = 30$ or $x = 30$, $y = 20$

The possible number of chocolates with $A$ and $B$, the rates at which they sell their chocolates and the possible amounts they get are tabulated below.

| | **A** | **B** | **A** | **B** |
|---|---|---|---|---|
| Number | 20 | 30 | 30 | 20 |
| Rate | 5 | 6 | 5 | 6 |
| Amount | 100 | 180 | 150 | 120 |
| Total Amount for *A*,B | 280 | | 270 | |

∴ The total amount with $A$ and $B$ could be ₹280. If each had as many chocolates as the other, they would get ₹10 less.

Alternately, the total amount they get could be ₹270. If each had as many as the other had, they would get ₹10 more.

**28.** $x^2 - 5x + 6 = 0$

The second equation say $(B)$ is $x^2 - 3x - 2x + 6 = 0$

$x(x-3) - 2(x-3) = 0$ $x = 3$ (or) $x = 2$

Twice of one root of $x^2 - 5x + 6 = 0$ is equal to one root of the first equation, say $A$, viz

$x^2 - 2Rx + 6 = 0$ If $2(3) = 6$ is one root, of $A$

$(6)^2 - 2R(6) + 6 = 0 \Rightarrow R = 7/2$

If $2(2) = 4$ is one root of $A$, then $(4)^2 - 2R(4) + 6 = 0 \Rightarrow R = 11/4$ $\therefore R = \frac{7}{2}$ or $\frac{11}{4}$

**29.** $b^2 - 4ac = (-2p)^2 - 4\ (1)\ (2p) = 4p^2 - 8p = 4p\ (p-2)$

If $p > 2$, $b2 - 4ac$ is positive.

In this case the roots will be real and unequal.

If $p = 2$, $b - 4ac = 0$.

In this case the roots will be real and equal.

If $0 < p < 2$, $b^2 - 4ac < 0$.

In this case the roots will be complex conjugates

∴ the nature of the roots cannot be determined.

**30.** The first equation $(A)$ is $x^3 - 3x^2 + 4 = 0$.

We note that $(-1)^3 - 3(-1)^2 + 4 = 0$.

∴ By the factor theorem, $x + 1$ is a factor of $x^3 - 3x^2 + 4$. Dividing $x^3 - 3x^2 + 4$ by $x + 1$, we get $x^2 - 4x + 4$ in the quotient i.e., $x^3 - 3x^2 + 4 = (x+1)\ (x-2)^2$

The second equation $(B)$ is $x^2 + x - 6 = 0$

$\Rightarrow (x+3)\ (x-2) = 0$. The roots of $A$ are $-1, 2, 2$

The roots of $B$ are $-3$ and $2$.

We see that 2 roots of $A$ are also roots of $B$, while only one root of $B$ is also a root of $A$.

**31.** Let the number of pencils bought by Ashok be $x$ and cost of each pencil be $y$

$\therefore xy = 180$

$(x-2)\ (y+2) = 160$

$xy - 2y + 2x - 4 = 160$

$180 - 2\left(\frac{180}{x}\right) + 2x - 164 = 0$

Multiplying both sides by $-x$ and simplifying,

$x^2 + 8x - 180 = 0$

$\Rightarrow x^2 + 18x - 10x - 180 = 0$

$\Rightarrow x\ (x+18) - 10\ (x+18) = 0$

$\Rightarrow (x-10)\ (x+18) = 0$

$\Rightarrow x - 10 = 0$ or $x + 18 = 0$

$\Rightarrow x = 10$ or $-18$

As $x > 0$, $x = 10$.

**32.** Sum of the roots $= -\frac{b}{a}$

Product of the roots $= \frac{c}{a}$

$-\frac{b}{a} = \frac{c}{a}$

$\Rightarrow$ cross multiplying, $-ab = ac$

$a\ (c + b) = 0$

$a = 0$ or $c + b = 0$, i.e., $b = -c$

As a cannot be 0, for any quadratic equation,

$\therefore b = -c$.

**33.** As $b^2 - 4ac$ is positive and a perfect square, the roots are rational and unequal.

**34.** Let the natural number be $x$

$(3x+2)^2 = 32x$

$9x^2 + 12x + 4 = 32x$

$9x^2 - 20x + 4 = 0$

$9x^2 - 18x - 2x + 4 = 0$

$9x\ (x-2) - 2\ (x-2) = 0$

$(9x-2)\ (x-2) = 0$

$9x - 2 = 0$ or $x - 2 = 0$

$\Rightarrow x = \frac{2}{9}$ or $2$

As $x$ is a natural number $x = 2$

**35.** Let the consecutive numbers be $x$ and $x + 1$.

$x^2 + (x+1)^2 = 841$

$\Rightarrow x^2 + x^2 + 2x + 1 - 841 = 0$

$\Rightarrow 2(x^2 + x - 420) = 0$

$x^2 + 21x - 20x - 420 = 0$

$x(x+21) - 20(x+21) = 0$

$x - 20 = 0$ or $x + 21 = 0$

$x = 20$ or $x = -21$

As $x$ is a natural number $x > 0$. Hence, the smaller natural number is 20.

**36.** Maximum value of $ax^2 + bx + c$ is $\frac{4ac - b^2}{4a}$

$\therefore$ Maximum value of $12 + 22x - 11x^2 = \frac{4(-11)(12) - 22^2}{4(-11)}$

$= \frac{-44(12+11)}{-44} = 12 + 11 = 23.$

**37.** We know that equation if $ax^2 + bx + c = 0$ have equal roots then $b^2 - 4ac = 0$

given $x^2 + 2(m+3)x + 81 = 0$ has equal roots

$\therefore$ Its discriminant $= [2(m+3)]^2 - 4(1)(81) = 0$

$[2(m+3)]^2 = (4)(81) \Rightarrow (m+3)^2 = 81$

$\Rightarrow m + 3 = \pm 9$

$\Rightarrow m = 6$ or $-12$.

**38.** $(x+2)^2 = x^2 + 4x + 4$

$(x-2)^2 = x^2 - 4x + 4$

$\therefore (x+2)^2 + (x-2)^2 = 2x^2 + 8 = 40$

$x^2 = 16 \Rightarrow x = \pm 4$

**39.** As the roots are real and equal, the discriminate equals 0

$\therefore (3k-12)^2 - 4(k-4)(9) = 0$

$9(k-4)[(k-4) - 4] = 0$

$\Rightarrow 9(k-4)(k-8) = 0$

$k - 4 = 0$ or $k - 8 = 0$

If $k - 4 = 0$, the equation will not be quadratic.

$\therefore k - 4 \neq 0$. $\therefore k - 8 = 0$

$\therefore k = 8$

**40.** The required equation can be obtained by replacing $x$ by $x - 1$ in the given equation as $x^2 - 15x + 35 = 0$.

**41.** Let $f(x) = x^4 - 4x^3 - x^2 - 6x + 4$

$f(x)$ has two sign changes and hence $f(x) = 0$ can have 2 or 0 positive roots.

$f(-x) = x^4 + 4x^3 - x^2 + 6x + 4$

Number of sign changes is 2. Hence, $f(x) = 0$ has 2 or 0 negative roots.

| | Positive roots | Negative roots | Complex roots |
|---|---|---|---|
| 1 | 2 | 2 | 0 |
| 2 | 2 | 0 | 2 |
| 3 | 0 | 2 | 2 |
| 4 | 0 | 0 | 4 |

From: Descartes Rule, we conclude that these are the four possibilities. But the equation is completely specified, i.e. all the coefficients are given. Actually, only one of the 4 possibilities given above is true. The others are not true.

To determine which of the four possibilities is true, we need to use other techniques. But quite often, when we seek some limited information, the results that we get by applying Descartes rule are very useful.

**42.** The roots are $-1$ & $-3$. Both roots are negative.

**43.** $b^2 - 4ac = 4(p+1)^2 - 8p$

$= 4(p^2 + 1)$

Thus, roots will be real and unequal.

**44.** Let $f(x) = x^4 - px^3 + 2x^2 + 2x + q$, given $-1$, and 2 are roots of $f(x)$

$\Rightarrow f(-1) = 0$ and $f(2) = 0$.

$f(-1) = 0 \Rightarrow (-1)^4 - p(-1)^3 + 2(-1)^2 + 2(-1) + q = 0$

$p + q = -1$ (1)

$f(2) = 0 \Rightarrow (2)^4 - p(2)^3 + 2(2)^2 + 2(2) + q = 0$

$\Rightarrow 8p - q = 28$ – (2) solving (1) and (2) we get $p = 3q = -4$

$\therefore f(x) = x^4 - 3x^3 + 2x^2 + 2x - 4$ by dividing $f(x)$ by $(x+1)(x-2)$ we get quotient as $x^2 - 2x + 2 = 0$ wich does not have any real roots as $b^2 - 4ac = 4 - 8 < 0$

**45.** $b^2 - 4ac \geq 0$

$\Rightarrow 16A \geq 12B$

$\Rightarrow 4A \geq 3B$

$A$ can take values:

$A = 2$ $\quad 3B \leq 4A$; Thus, $B = 1, 2$

$A = 3$ $\quad B = 1, 2, 3, 4$

$A = 5$ $\quad B = 1, 2, 3, 4, 5, 6$

$A = 7$ $\quad B = 1, 2, 3, 4, 5, 6, 7, 8, 9$

Thus, we have total 21 equations.

## EXERCISE-3

**1.** Let the roots of the Q.E be $\alpha, 2\alpha$

Sum $= 3\alpha = \frac{45}{25} = \frac{9}{5}$

$\Rightarrow \alpha = \frac{3}{5}, 2\alpha = \frac{6}{5}$

Product $= \frac{3}{5} \times \frac{6}{5} = \frac{k}{25} \Rightarrow k = 18$

**2.** The equation each of whose roots are $k$ times the roots of the equation $ax^2 + bx + c = 0$ is given by $ax^2 + kbx + k^2 c = 0$

As $K = 3$, the required equation is

$2x^2 + 3(-11)x + 9(12) = 0$

i.e., $2x^2 - 33x + 108 = 0$

**3.** $K(K+1)(K+2)(K+3) = 120$

$K(K+3)(K+1)(K+2) = 120$

$(K^2 + 3K)(K^2 + 3K + 2) = 120$ (1)

Let $K^2 + 3K = x$

$x^2 + 2x = 120$

$\Rightarrow x^2 + 2x - 120 = 0$

$(x+12)(x-10) = 0$

$x + 12 = 0$ or $x - 10 = 0$

$x = -12$ or $x = 10$

As $K^2 + 3K$ is positive, $x$ is positive.

$\therefore x = 10$

**4.** Let the roots of the equation be $k$ and $3k$.

The sum of its roots $= \frac{-b}{a} = 4k$

The product of its roots $= \frac{c}{a} = 3k^2$

The ratio of the square of the sum of its roots and the product of its roots $= \frac{\left(\frac{b}{a}\right)^2}{\frac{c}{a}} = \frac{(4k)^2}{3k^2}$

$\frac{b^2}{ac} = \frac{16}{3}$

$\Rightarrow 3b^2 = 16ac$

**5.** The quadratic equation whose roots are twice the roots of a given equation is obtained by substituting $\frac{x}{2}$ in place of $x$.

$\therefore$ The required equation is $\left(\frac{x}{2}\right)^2 - 9\left(\frac{x}{2}\right) + 18 = 0$

$\Rightarrow x^2 - 18x + 72 = 0$

**6.** Given $\alpha, \beta, \gamma$ are the roots of the equation

$x^3 - 4x^2 + 6x - 4 = 0$

$\alpha + \beta + \gamma = 4$

$\alpha\beta + \beta\gamma + \alpha\gamma = 6$

$\alpha\beta\gamma = 4$

$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \alpha\gamma}{\alpha\beta\gamma} = \frac{6}{4} = \frac{3}{2}$

**7.** The new roots are

$1 + \frac{\alpha}{\beta} = \frac{\alpha+\beta}{\beta}$ and $1 + \frac{\beta}{\alpha} = \frac{\alpha+\beta}{\alpha}$

Sum of roots $= \frac{\alpha+\beta}{\alpha} + \frac{\alpha+\beta}{\beta} = \frac{16}{3}$

Product of roots $= \left(\frac{\alpha+\beta}{\alpha}\right)\left(\frac{\alpha+\beta}{\beta}\right) = \frac{16}{3}$

The required equation is

$x^2 - \frac{16x}{3} + \frac{16}{3} = 0$

$3x^2 - 16x + 16 = 0$

**8.** Product of roots = 1

$\Rightarrow 2\frac{(2m-1)}{2} = 1 \Rightarrow m = 1$

**9.** $\left(x^2 + \frac{1}{x^2}\right) = \left(x + \frac{1}{x}\right)^2 - 2$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 - 4\left(x + \frac{1}{x}\right) + \frac{15}{4} = 0$

Let $y = \left(x + \frac{1}{x}\right)$

$y^2 - 4y + 15/4 = 0$

$4y^2 - 16y + 15 = 0$

$4y^2 - 10y - 6y + 15 = 0$

$2y(2y-5) - 3(2y-5) = 0$

$(2y-3)\ (2y-5) = 0$

$y = \left(x + \frac{1}{x}\right) = \frac{3}{2} = \frac{5}{2}$

**10.** The roots are 1 and 3. Both roots are positive.

**11.** Given $\alpha, \beta$ are the roots of the equation $3x^2 + 5x + k = 0$

$\alpha + \beta = \frac{-5}{3}$ (1);

$\alpha\beta = \frac{k}{3}$

$\alpha^2 - \beta^2 = 25$

$(\alpha + \beta)\ (\alpha - \beta) = 25$

$\frac{-5}{3}\ (\alpha - \beta) = 25$

$\Rightarrow \alpha - \beta = -15$ (2).

Solving (1) and (2) $\alpha = \frac{-25}{3}$ and $\beta = \frac{20}{3}$

$k = 3\alpha\beta$

$= 3\left(\frac{-25}{3}\right)\left(\frac{20}{3}\right) = -\frac{500}{3}$.

**12.** The given equation is $x^2 - 8x + p = 0$.

The roots of the above equation are

$\frac{8 \pm \sqrt{64 - 4p}}{2} = 4 \pm \sqrt{16 - p}$

$= 4 + \sqrt{16 - p}$ or $4 - \sqrt{16 - p}$.

Since one root lies between 0 and 1,

$4 - \sqrt{16 - p}$ must lie between 0 and 1

$\Rightarrow 0 < 4 - \sqrt{16 - p} < 1$

$\Rightarrow -4 < -\sqrt{16 - p} < -3$

$\Rightarrow 3 < \sqrt{16 - p} < 4$

$\Rightarrow 9 < 16 - p < 16$

$\Rightarrow -7 < -p < 0$

$\Rightarrow 0 < p < 7$.

$\therefore$ The range of $p$ is $(0, 7)$.

**13.** Let $\alpha$ be the common root of the equation $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$

$\Rightarrow a\alpha^2 + b\alpha + c = 0$ $b\alpha^2 + c\alpha + a = 0$.

$\frac{\alpha^2}{ab - c^2} = \frac{-\alpha}{a^2 - cb} = \frac{1}{ac - b^2}$

$\alpha^2 = \frac{ab - c^2}{ac - b^2}$ (1)

$\alpha = \frac{cb - a^2}{ac - b^2}$ (2)

From (1) and (2)

$\left(\frac{cb - a^2}{ac - b^2}\right)^2 = \frac{ab - c^2}{ac - b^2}$

$(cb - a^2)^2 = (ac - b^2)\ (ab - c^2)$

$\Rightarrow c^2 b^2 + a^4 - 2a^2 bc = a^2 bc - ac^3 - ab^3 + b^2 c^2$

$\Rightarrow a^4 + ac^3 + ab^3 = 3a^2 bc$.

Since $a \neq 0$ and $a, b, c$ are distinct, $a^3 + b^3 + c^3 = 3abc$

$\Rightarrow a + b + c = 0$.

$\therefore$ Sum of the coefficients is zero. Hence, '1' is the root of both the equations.

**NOTE**

It can be observed that $x = 1$ is a common root of the given equations.

**14.** Let $\alpha, \beta$ be the roots of the equation.

$x^2 - (a-3)x + 5 - a = 0$

$\alpha + \beta = a - 3$; $\alpha\beta = 5 - a$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (a-3)^2 - 2\ (5 - a)$

$= a^2 - 6a + 9 - 10 + 2a$

$= a^2 - 4a - 1$

$= (a-2)^2 - 5$.

$\therefore$ The minimum value of $\alpha^2 + \beta^2 = -5$. From the options only choice $A$ satisfies this condition.

**15.** Let $\alpha$ and $\beta$ be the roots of the equation $ax^2 + bx + c = 0$

$\alpha + \beta = \frac{-b}{a}$ , $\alpha\beta = \frac{c}{a}$ .

Given $\alpha + \beta = 2\ \alpha\beta$

$\frac{-b}{a} = \frac{2c}{a} \Rightarrow a(-b-2c) = 0$

$a \neq 0\ \therefore -b-2c = 0$

$\therefore\ b = -2c.$

**16.** Given the equation,

$(x+5)\ (x+6)\ (x+7)\ (x+8) = 840$ (1)

$[(x+5)\ (x+8)]\ [(x+6)\ (x+7)] = 840$

$\Rightarrow (x^2 + 13x + 40)\ (x^2 + 13x + 42) = 840.$

Let $y = x^2 + 13x + 40 \Rightarrow x^2 + 13x + 42 = y + 2$

$\therefore y(y+2) = 840$

$\Rightarrow y^2 + 2y - 840 = 0 \Rightarrow (y+30)\ (y-28) = 0$

$\Rightarrow y = -30$ or $28$

$\Rightarrow x^2 + 13x + 40 = -30$ or $x^2 + 13x + 40 = 28$

$\Rightarrow x^2 + 13x + 70 = 0$ or $x^2 + 13x + 12 = 0$

$\Rightarrow$ Complex roots or $x = -1$ or $-12$.

$\therefore$ The given equation has only two real roots and they are $-1$ and $-12$.

**17.** Let my usual speed be $x$ km/hr

$(12/x) - (12/x+2) = 1 \Rightarrow x = 4$

**18.** Given:

$4x(4x-4) = 3x(3x-4) + 2x(2x-4) + 832$

$\Rightarrow 16x^2 - 16x = 9x^2 - 12x + 4x^2 - 8x + 832$

$\Rightarrow 3x^2 + 4x - 832 = 0$

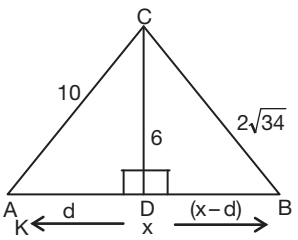
$\Rightarrow 3x^2 - 48x + 52x - 832 = 0$

$\Rightarrow 3x\ (x-16) + 52\ (x-16) = 0$

$\Rightarrow x = 16$ (or) $x = -52/3$

As $x > 0$, $x = 16$

**19.**




In triangle $ACD$, $AD^2 = AC^2 - CD^2$

i.e., $d^2 = 100 - 36 = 64$

$\Rightarrow d = 8$

In triangle $CDB$, $DB^2 = CB^2 - CD^2$

I.e. $(x-d)^2 = \left(\sqrt{136}\right)^2 - 36$

$\Rightarrow (x-8)^2 = 100 \Rightarrow x = 18$

**20.** Let the number be $x$. Given that

$\Rightarrow x + 4 = \frac{2}{x} + 5 \Rightarrow x + 4 = \frac{2+5x}{x}$

$\Rightarrow x^2 + 4x - 5x - 2 = 0 \Rightarrow x^2 - x - 2 = 0$

$\Rightarrow x^2 - 2x + x - 2 = 0 \Rightarrow x\ (x-2) + 1(x-2) = 0$

$\Rightarrow (x-2)\ (x+1) = 0 \Rightarrow x = 2$ (or) $-1$

**21.** The number of ways of going from $A$ to $D$ (via $B$ and $C$) is equal to the product of the number of ways of going from $A$ to $B$, $B$ to $C$ and $C$ to $D$.

i.e., $m(m+3)\ (m+8) = 336 = 3(7)\ (16)$

$= 4(7)\ (12)\ \therefore\ m = 4$

The number of ways of going form $A$ to $C$ is $m(m + 3) = 4(7) = 28$.

**22.** Given, $x^2 + y^2 = 1.0$ and $|x - y| = 1.4$.

Now, $(x-y)^2 = x^2 + y^2 - 2xy$

$(1.4)^2 = 1.0 - 2xy$

$2xy = -0.96.$

Now, $(x+y)^2 = x^2 + y^2 + 2xy$

$\Rightarrow 1.0 + (-0.96) = 0.04 \Rightarrow |x + y| = 0.2.$

**23.** Let $(x + 5/x - 2)^2 = y.$

$\therefore 65y^2 + 24y - 432 = 0$

$\Rightarrow (13y + 36)\ (5y - 12) = 0$

$\Rightarrow y = -36/13$ or $12/5$

$y \geq 0$, $y = 12/5$

$\frac{x+5}{x-2} = \pm\sqrt{\frac{12}{5}}$ , $x$ takes two irrational values other than $\pm\sqrt{\frac{12}{5}}$ . So, we can eliminate choices $(A)$, $(B)$, and $(C)$

**24.** Let the roots be $\alpha, \beta, \gamma$ then $\alpha + \beta + \gamma = -3$

$\alpha\beta + \beta\gamma + \alpha\gamma = -6$

$\alpha\beta\gamma = 8$

By back substitution, we find that only choice (*B*) satisfies all the three equations above.

**25.** The equation has one negative and two positive roots.

$\therefore$ There should be 2 (or 4) sign changes in $f(x)$ and 1 (or 3) sign changes in $f(-x)$.

The three options and the corresponding signs of the terms in $f(x)$ and $f(-x)$ are tabulated below.

| Option | Function | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | Number of sign changes in $f(x)$ | Number of sign changes in $f(-x)$ |
|---|---|---|---|---|---|---|---|---|---|
| (i) | $f(x)$ | + | + | − | + | − | + | 4 | 1 |
| | $f(-x)$ | − | + | + | + | + | + | | |
| (ii) | $f(x)$ | + | + | + | − | − | + | 3 | 2 |
| | $f(-x)$ | + | + | − | − | + | + | | |
| (iii) | $f(x)$ | − | + | + | + | − | + | 3 | 2 |
| | $f(-x)$ | + | + | − | + | + | + | | |

As there should be 1 or 3 sign changes in $f(-x)$, only option *A* is possible.

**26.** $2x + \frac{1}{x} = -\sqrt{6}$

Cubing both sides, we get $8x^3 + \frac{1}{x^3} + 3(2)\left(2x + \frac{1}{x}\right)$

$= -6\sqrt{6}$

$$8x^3 + \frac{1}{x^3} + 3(2)\left(-\sqrt{6}\right) = -6\sqrt{6}$$

$$8x^3 + \frac{1}{x^3} = 0$$

$$8x^6 = -1$$

$$x^6 = -\frac{1}{8}$$

$$x^{12} - x^6 = \frac{-1}{8}\left(\frac{-1}{8} - 1\right) = \frac{9}{64}$$

**27.** The dividend is a polynomial of degree 20 and it has 11 terms. The divisor is $x^3 + x$, a binomial of degree 3. The degree of the remainder of degree 0, 1 or 2. The dividend can be split into the 11 terms and the remainder of each term can be obtained. The required remainder is the sum of these 11 remainders. The various terms, including the monomials of lower degree and the corresponding remainders are shown below. The divisor is $x^3 + x$.

| Monomial | Remainder |
|---|---|
| 1 | 1 |
| $x$ | $x$ |
| $x^2$ | $x^2$ |
| $x^3$ | $-x$ |
| $x^4$ | $-x^2$ |
| $x^5$ | $-x^3 \equiv x$ |
| $x^6$ | $x^2$ |

We see that the remainder can only be $-x$, $-6 - x^2$, $x$ or $x^2$. These 4 possible remainders, and the corresponding terms (from among the given 11 terms) are tabulated below.

| Monomial | Remainder |
|---|---|
| $-x$ | $x^{11}, x^{15}, x^{19}$ |
| $-x^2$ | $x^{12}, x^{16}, x^{20}$ |
| $x$ | $x^{13}, x^{17}$ |
| $x^2$ | $x^{10}, x^{14}, x^{18}$ |

$\therefore$ The net remainder is $-x$.

**28.** The given equation is $x^3 - 7x^2 + 36 = 0$ (1)

Let $\alpha, -3\alpha, \beta$ be the roots of the equation

$\alpha - 3\alpha + \beta = 7 \Rightarrow \beta - 2\alpha = 7$ (2)

$\alpha(3\alpha) + (-3\alpha)\beta + \beta(\alpha) = 0$

$\Rightarrow -3\alpha^2 - 2\alpha\beta = 0$ (3)

(2), (3) $\Rightarrow -3\alpha^2 - 2\alpha(7 + 2\alpha) = 0 \Rightarrow -3\alpha^2 - 14\alpha - 4\alpha^2 = 0$

$\Rightarrow 7\alpha^2 + 14\alpha = 0 \Rightarrow 7\alpha(\alpha + 2) = 0$

$\alpha = 0$, or $\alpha = -2$

$\alpha = -2$ satisfies equation (1) while $\alpha = 0$ does not

$\beta = 7 + 2\alpha = 7 - 4 = 3$

$\therefore$ The roots of the equation are $-2, 6, 3$

The difference of the greatest two roots $= 6 - 3 = 3$

**29.** The given equation is $x^4 + lx^3 + mx^2 + nx + 24 = 0$. Let the roots 3, 1, –2 and the fourth root be denoted by $\beta, \gamma, \delta$ and $\alpha$, respectively.

The product of the roots = 24

If the fourth root is $\alpha$,

$3(1)(-2)\alpha = 24$

$\Rightarrow \alpha = -4$

$\therefore$ The roots of the equation are $\alpha = -4$, $\beta = 3$, $\gamma = 1$ and $\delta = -2$

$-l$ = (Sum of roots) = $(-4+3+1-2) = -2 \therefore l = 2$.

$m = (\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta) = -13$

$-n = (\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta) = 14$

$\therefore n = -14$

$l + m - n = 2 - 13 + 14 = 3$

**30.** $f(x) = 3x^4 - 13x^3 + 7x^2 + 17x + a - 10 = 0$ has 3 positive roots. The number of sign changes in $f(x)$ have to be 3, 5, ….. as $f(x)$ is a $4^{th}$ degree polynomial there have to be exactly 3 sign changes $a - 10$ must be negative, i.e., $a < 10$. a can be 4.

**31.** $x + \frac{1}{x} = -\sqrt{3}$

Cubing both sides $x^3 + \frac{1}{x^3} + 3(x)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right) = -3\sqrt{3}$

$x^3 + \frac{1}{x^3} + 3\left(-\sqrt{3}\right) = -3\sqrt{3} \Rightarrow x^3 + \frac{1}{x^3} = 0.$

Multiplying both sides by $x^3$, $x^6 = -1$

$x^{42} + x^{48} + x^{54} + x^{60} + x^{66} + x^{72} = (x^6)^7 + (x^6)^8 + (x^6)^9 + (x^6)^{10} + (x^6)^{11} + (x^5)^{12}$

$= (-1)^7 + (-1)^8 + (-1)^9 + (-1)^{10} + (-1)^{11} + (-1)^{12} = 0.$

**32.** If $f(x)$ is exactly divisible by $x - 1$, sum of coefficients must be zero. Here sum of coefficients is $2 + 4 - 7 + 4 = 3$.

$\therefore$ If 3 is subtracted or $-3$ is added the given expression is exactly divisible by $x - 1$.

**33.** Let $f(x) = px^3 + qx^2 + rx + s$

Given $x^2 - 1$ is a factor of $f(x)$.

$\therefore (x+1)$ and $(x-1)$ are factors of $f(x)$

$\therefore f(-1) = 0$ and $f(1) = 0$

$-p + q - r + s = 0$

$\Rightarrow p + r = q + s$ (1)

$p + q + r + s = 0$ (2)

From (1) and (2), we have

$p + r = 0$ and $q + s = 0$

$\Rightarrow p = -r$ and $q = -s$

**34.** Given: $f(3) = 10$ and $f(4) = 15$

$f(x) = (x-3)(x-4)q(x) + mx + n$, where $q(x)$ and $mx + n$ are the quotient and the remainder, respectively.

$f(3) = 0 \times q(x) + 3m + n \Rightarrow 3m + n = 10$ (1)

$f(4) = 0 \times q(x) + 4m + n \Rightarrow 4m + n = 15$ (2)

Solving (1) and (2), we have, $m = 5$ and $n = -5$.

Hence, the required remainder is $5x - 5$ i.e., $5(x-1)$.

**35.** As $f(x)$ has terms of only even degree, $f(-x)$ is identical to $f(x)$. The number of sign changes in $f(x)$ is 3.

The number of positive roots is 3 or 1. Similarly the number of negative roots is 3 or 1.

Also there are 4 non-real roots. This accounts for 1 + 1 + 4 or 1 + 3 + 4 or 3 + 1 + 4 or 3 + 3 + 4 roots;

i.e., 6 or 8 or 10 roots. The remaining roots have to be 0. i.e., the multiplicity of the root 0 is at least 2 (it could be 4 or 6 as well), i.e., $a_0 = 0$.

**36.** Let $q(x)$ and $r(x)$ be the quotient and the remainder, respectively when $F(x)$ is divided by $(x-4)(x-6)$

$F(x) = (x-5)(x-6)q(x) + r(x)$

As the divisor $(x-5)(x-6)$ is a quadratic expression, the remainder $r(x)$ must be a linear expression.

Let $r(x) = ax + b$, where $a$ and $b$ are constants.

$F(x) = (x-5)(x-6)q(x) + ax + b$

Remainder theorem: If $f(x)$ is divided by $x - a$, the remainder of the division is $f(a)$

The remainders of the divisions of $F(x)$ by $x - 5$ and $x - 6$ are $F(5)$ and $F(6)$, respectively.

$F(5) = 17$ and $F(6) = 19$

$F(5) = a(5) + b$ and $F(6) = a(6) + b$

$a(5) + b = 17$ and $a(6) + b = 19$

$5a + b = 17$ and $6a + b = 19$

Solving these, $a = 2$ and $b = 7$

The remainder when $F(x)$ is divided by $(x-5)(x-6)$ is $2x + 7$.

**37.** Let $f(x) = x^{72} + x^{60} + x^{48} + x^{36} + x^{24}$

If $A$ and $B$ are functions of $x$, then the remainder of A$x$ divided by B$x$ is $x$ times the remainder of $A$ divided by $B$.

The remainder of the given expression $E(x)$ (say) divided by $x^3 - x$ is $x$ times the remainder of $\frac{E(x)}{x}$ g=F(x) (say) divided by $x^2 - 1$.

$F(x) = x^{71} + x^{59} + x^{47} + x^{35} + x^{23}$ (1)

As $x^2 - 1$ is a quadratic expression, the remainder of $F(x)$ divided by $x^2 - 1$ is a linear expression.

Let this remainder be $ax + b$.

$F(x) = (x+1)(x-1)Q(x) + ax + b$ (2)

From (1), $F(-1) = -5$ and $F(1) = 5$

From (2), $F(-1) = a(-1) + b$ and $F(1) = a(1) + b$

$\therefore a(-1) + b = -5$ and $a(1) + b = 5$

$a = 5$ and $b = 0$

$ax + b = 5x$

Required remainder = $\text{Rem}\dfrac{E(x)}{x^3 - x} = x\,\text{Rem}\left(\dfrac{F(x)}{x^2 - 1}\right)$

$= x(5x) = 5x^2$

**38.** $x + \dfrac{6}{x} = 2\sqrt{3}$

Squaring both sides, $x^2 + \dfrac{36}{x^2} + 12 = 12$

$x^2 + \dfrac{36}{x^2} = 0$

$x^2 = \dfrac{-36}{x^2}$

$x^4 = -36$

$x^8 + x^{12} = x^8(1 + x^4) = (x^4)^2\,(1 + x^4) = (-36)^2\,(1 + (-36))$

$= 36^2(-35) = -36^2(35)$

**39.** Let $\alpha, \beta, \gamma, \delta$ be the roots of the given equation

$\alpha + \beta + \gamma + \delta = 12$

and $\alpha\beta\gamma\delta = 81$

$AM$ of roots $= \dfrac{12}{4} = 3$ and $GM = \quad = 3$

$AM = GM \Rightarrow \alpha = \beta = \gamma = \delta = 3$

$\therefore a = \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$

$= 6(9) = 54$

$-b = \alpha\beta\gamma + \alpha\gamma\delta + \beta\gamma\delta + \alpha\beta\delta$

$= 4\,(27) = 108$

$\therefore (a, b) = (54, -108)$

**40.** Given 2 is a root of $x^3 + lx^2 + mx + n = 0$

$\Rightarrow 2^3 + l(2)^2 + m(2) + n = 0$

$\Rightarrow 8 + 4l + 2m + n = 0$

Also $l - 3 = m$ and $m + 8 = n$

$\Rightarrow 8 + 4l + 2(l - 3) + (m + 8) = 0$

or

$8 + 4l + 2l - 6 + l - 3 + 8 = 0$

$\Rightarrow 7l = -7 \Rightarrow \quad l = -1$

$m = l - 3 = -4$ and $n = m + 8 = 4$

$\therefore$ The given equation is $x^3 - x^2 - 4x + 4 = 0$

$x = 2$ is a root.

When $x = -2$ and $x = 1$, the given equation is satisfied.

$\therefore -2, 1$ and $2$ are the roots of the given equation.

As $\alpha < \beta < \gamma$, $\alpha = -2$, $\beta = 1$ and $\gamma = 2$.

$\therefore \dfrac{\beta}{\gamma} = \dfrac{1}{2}$

**41.** $p^2 - q^2 = 0$

$\Rightarrow p^2 = q^2\ (p + r)^2 + q^2 = 1$

$\Rightarrow 2p^2 + 2pr + r^2 - 1 = 0$

$p = \dfrac{-2r \pm \sqrt{4\left(r^2 - 2\left(r^2 - 1\right)\right)}}{2}$

$= \dfrac{-2r \pm 2\sqrt{2 - r^2}}{2} = -r \pm \sqrt{2 - r^2}$

As the solution for $p$ must be unique, $2 - r^2 = 0$.

As it must be positive, $r$ must be negative.

$\therefore r = -\sqrt{2}$

**42.** Let the numerator and denominator be $n$ and $d$, respectively.

$d = n^2 + 1$ (1)

$\dfrac{n + 2}{d + 9} = \dfrac{1}{5}$

$\Rightarrow 5n + 10 = d + 9 = n^2 + 10$

$\Rightarrow n^2 - 5n = 0$

$\Rightarrow n\,(n - 5) = 0$

$\Rightarrow n = 0$ or $n - 5 = 0$

$\Rightarrow n = 0$ or $5$

As the fraction has a positive numerator, $n$ is not 0. Hence, $n$ is 5.

From (1), $d = 26$

Hence, $\dfrac{n}{d} = \dfrac{5}{26}$

**43.** Let the positive number be $x$.

$x - \dfrac{1}{x} = \dfrac{168}{13}$

$13x^2 - 13 = 168x$

$\Rightarrow 13x^2 - 168x - 13 = 0$

$13x^2 - 169x + x - 13 = 0$

$13x\,(x - 13) + 1\,(x - 13) = 0$

$\Rightarrow (x-13)\ (13x+1) = 0$

$x - 13 = 0$ or $13x + 1 = 0 \Rightarrow x = 13$

**44.** $6(x^2 + 1/x^2 + 2) - 5(x + 1/x) - 50 = 0$ getting $x + 1/x = y$

$\Rightarrow 6y^2 - 5y - 50 = 0$

$\Rightarrow (2y + 5)\ (3y - 10) = 0$

$\Rightarrow y = -5/2$ or $10/3$

As $x$ is positive $x + 1/x = 10/3$, as $x = 3$ or $1/3$.

**45.** $x = 5 - 2y$, $x^2+y^2 = (5 - 2y)^2+y^2 = 5y^2 - 20y + 25$ which is a quadratic expression in $y^2$.

It has a minimum value since the coefficient of $y^2$ is positive. Its minimum value $= \dfrac{(4)(5)(25)-20^2}{4(5)} = 5.$

## EXERCISE-4

**1.** $x^2 - 8x + 16 + y^2 - 4y + 4 = 0$

$(x - 4)^2 + (y - 2)^2 = 0$

(As $x - 4$, $y - 2$ are real, their squares are non-negative)

$\Rightarrow x = 4$ and $y = 2$.

Hence, $x = 2y$.

**2.** The maximum/minimum value of $ax^2 + bx + c$ is

$$\frac{-(\text{discriminant})}{4(\text{coefficient of } x^2)} = \frac{-(b^2 - 4ac)}{4a}.$$

Given that $(a, b, c) = (3, 8, 5)$

$\therefore$ Maximum/Minimum value $=$

$$\frac{-\left[-8^2 - (4)(3)(5)\right]}{4(3)} = -\frac{1}{3}.$$

As a $> 0$ $(a = 3)$, minimum value is $-\dfrac{1}{3}$.

**3.** $(a - b\sqrt{2})\ x^2 + bx + (a + b\sqrt{2}) = 0$

Roots are equal.

Hence, the discriminant $= 0$

$\Rightarrow (b)^2 - 4\ (a - b\sqrt{2})\ (a + b\sqrt{2}) = 0$

$\Rightarrow b^2 - 4a^2 + 8b^2 = 0 \Rightarrow 9b^2 = 4a^2$

$\therefore 3b = +2a$ or $-2a$.

**4.** Let the common root be $a$.

The other roots satisfy $x^2 - 6x + 8 = 0$.

$\therefore$ Other roots are 2 and 4.

$\therefore$ The four roots are $a$, 2, $a$, 4.

The sum of all possible products of the 4 roots taken two at a time is $(2a + a^2 + 4a) + (2a + 8) + 4a = a^2 + 12a + 8 = 116$ given $\Rightarrow (a + 18)\ (a - 6) = 0$

As $a > 0$, $a = 6$.

The product of all possible sums of the 4 roots (i.e., 6, 2, 6, 4) taken two at a time is $(6 + 2)(6 + 6)(6 + 4)(2 + 6)(2 + 4)(6 + 4) = 8(12)(10)(8)(6)(10) = 460800$

**5.** Given $(y + 1)^2 - 2y = \dfrac{2}{2 - \sqrt{3}}$

$$\Rightarrow (y + 1)^2 - 2y = \frac{2\left(2 + \sqrt{3}\right)}{\left(2 + \sqrt{3}\right)\left(2 - \sqrt{3}\right)}$$

$$\Rightarrow (y)^2 + (1)^2 = \frac{2\left(2 + \sqrt{3}\right)}{4 - 3}$$

$$\Rightarrow y^2 = \sqrt{3}\left(\sqrt{3} + 2\right)$$

$$\Rightarrow y = \sqrt{\sqrt{3}\left(2 + \sqrt{3}\right)}$$

$$= \sqrt{\frac{\sqrt{3}\left(4 + 2\sqrt{3}\right)}{2}} = \sqrt{\frac{\sqrt{3}\left(\sqrt{3} + 1\right)^2}{2}} = 3^{1/4}\left[\frac{\sqrt{3} + 1}{\sqrt{2}}\right]$$

**6.** Given, $\sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots\infty}}} = a^2$

$$\Rightarrow \sqrt{12 + a^2} = a^2$$

$\Rightarrow a^4 - a^2 - 12 = 0$

$\Rightarrow (a^2 - 4)\ (a^2 + 3) = 0$

$\Rightarrow$ As $a^2$ cannot be negative, $a^2 = 4$

Hence, $a^4 = 16$

**7.** $x^2 - 70x + k = 0$

Let $\alpha, \beta$ be the prime roots of the equation.

$\Rightarrow$ $\alpha + \beta$ and $k$ values are listed below.

| $\alpha+\beta=70$ | | $K=(a\beta)$ |
|---|---|---|
| $a$ | $\beta$ | |
| 3 | 67 | 201 |
| 11 | 59 | 649 |
| 17 | 53 | 901 |
| 23 | 47 | 1081 |
| 29 | 41 | 1189 |

$\therefore$ There exist five values of $k$ such that both roots are prime numbers.

**8.** Let $f(x) = x^6 + 3x^5 - x^4 - 7x^3 - 14x^2 - 10x - 12$

The number of sign changes in $f(x)$ is one. (1)

$f(-x) = x^6 - 3x^5 - x^4 + 7x^3 - 14x^2 + 10x - 12$

The number of sign changes in $f(-x)$ is five. (2)

From (1), it can be concluded that $f(x) = 0$ has at most one positive root and hence exactly one positive root. From (2), it can be concluded that $f(x) = 0$ has at most five negative roots. It is also given that $f(x) = 0$ has exactly one negative root

$\Rightarrow f(x) = 0$ has exactly 4 imaginary roots

**9.** $x + \frac{4}{x} = m$ has real roots.

$\therefore\ x^2 - mx + 4 = 0$ has real roots

$\Rightarrow\ m^2 - 16 \geq 0$ (discriminant $\geq 0$)

$(m+4)\ (m-4) \geq 0$

$\Rightarrow\ m \leq -4$ or $m \geq 4$

$\therefore$ The minimum positive value of $m$ is 4

**10.** The maximum value of the expression will occur at $x = -\frac{b}{2a}$.

It is given that $\frac{-b}{2a} = 1 \Rightarrow b = -2a$

The value of the expression at $x = 0$ is $c$.

It is given that $c = -1$

So, the expression is $ax^2 - 2ax - 1$

It takes a value of 5 at $x = 1$

$a(1)^2 - 2a(1) - 1 = 5$

$\Rightarrow -a = 6$

$\Rightarrow a = -6$

The expression is $-6x^2 + 12x - 1$

The value of the expression at $x = -5$ is $-6\ (-5)^2 + 12\ (-5) - 1 = -211$

**11.** Given $x + \frac{1}{x} = \sqrt{2}$

Squaring on both sides,

$x^2 + \frac{1}{x^2} + 2 = 2$

$\Rightarrow\ x^2 + \frac{1}{x^2} = 0$

$\Rightarrow\ x^4 + 1 = 0$

$\Rightarrow\ x^4 = -1$

$x^{52} + x^{48} + x^{44} + x^{40} + x^{36} + x^{32} + x^{28} + 7$

$= (x^4)^{13} + (x^4)^{12} + (x^4)^{11} + (x^4)^{10} + (x^4)^9 + (x^4)^8 + (x^4)^7 + 7$

$= (-1)^{13} + (-1)^{12} + (-1)^{11} + (-1)^{10} + (-1)^9 + (-1)^8 + (-1)^7 + 7$

$= -1 + 1 - 1 + 1 - 1 + 1 - 1 + 7 = 6$

**12.** Let $f(z) = z^{1990} - z^3 + z^2 - 1$. When $f(z)$ is divided by $z + 1$, the remainder is $f(-1)$.

$f(-1) = (-1)^{1990} - (-1)^3 + (-1)^2 - 1$

$= 1 + 1 + 1 - 1 = 2$

**13.** Let $x = \cfrac{2}{2 + \cfrac{2}{2 + \cfrac{2}{2 + \ldots\infty}}}$

Then we have $x = \frac{2}{2+x}$

$\Rightarrow x^2 + 2x - 2 = 0$

$\Rightarrow x = \frac{-2 \pm \sqrt{4+8}}{2} = -1 \pm \sqrt{3}$

$\therefore$ the possible values are $-2.732$ and $0.732$

But the expression cannot be negative.

Hence, the value is 0.732

**14.** $\left(p^2x^2 - 2pqx + q^2\right) + \left(q^2x^2 - 2qrx + r^2\right)$

$+ (r^2x^2 - 2rsx + s^2) \leq 0$

$\Rightarrow (px - q)^2 + (qx - r)^2 + (rx - s)^2 \leq 0$

$\Rightarrow x = \frac{q}{p} = \frac{r}{q} = \frac{s}{r} \therefore p, q, r, s$ are in $GP$.

**15.** Let $P = 2 + \cfrac{1}{2 + \cfrac{1}{2 + \ldots}} \Rightarrow P - 2 = \frac{1}{P}$

$\Rightarrow P^2 - 2P - 1 = 0 \quad \therefore P = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$

but, $P$ cannot be –ve $\therefore P = 1 + \sqrt{2}$

Let $q = \cfrac{2 + \cfrac{1}{4 + \cfrac{1}{4 + \ldots}}} \Rightarrow q - 2 = \frac{1}{q+2}$

$\Rightarrow q^2 - 4 = 1 \Rightarrow q = \pm\sqrt{5}$. But, $q$ cannot be –ve

$\therefore q = \sqrt{5}$. $\therefore$ The required quadratic equation is

$x^2 - x\left(1 + \sqrt{2} + \sqrt{5}\right) + \sqrt{5}\left(1 + \sqrt{2}\right) = 0$

**16.** The discriminant of the given equation is zero, as the roots are equal, i.e., $m^2 (n-l)^2 = 4ln (m-n) (l-m)$

$\Rightarrow m^2n^2 + m^2 l^2 - 2lnm^2 = 4ln (ml - m^2 - nl + mn)$

$\Rightarrow m^2n^2 + m^2l^2 - 2 lnm^2 = 4nm l^2 - 4lnm^2 - 4n^2l^2 + 4 lmn^2$

$\Rightarrow m^2n^2 + m^2 l^2 + 4n^2l^2 - 4nml^2 + 2l nm^2 - 4n^2lm = 0$

$\Rightarrow (lm + mn - 2nl)^2 = 0$

$lm + mn - 2nl = 0$ dividing throughout by $mnl$, we get, $\frac{1}{n} + \frac{1}{\ell} = \frac{2}{m}$

**17.** The data is tabulated below

| | Q | R | S |
|---|---|---|---|
| Breadth | 12 | $y$ | $y - 10/7$ |
| Length | $x - 3x$ | $x$ | $x + 3$ |
| Area | $A$ | $A$ | $A$ |

We have $yx = 12(x-3)$ (1)

And $yx = (x - 10/7)(y + 3)$ (2)

(1) $\Rightarrow yx = 12x - 36 \quad \Rightarrow y = 12 - 36/x$

(2) $\Rightarrow -10x/7 + 3y = 30/7$

i.e., $-10x/7 + 3(12 - 36/x) = 30/7$

$\Rightarrow -10x/7 + 36 - 30/7 - 108/x = 0$

$\Rightarrow 10x^2 - 222x + 7(108) = 0$

$\Rightarrow 5x^2 - 111x + 7(9)(2)(3) = 0 \Rightarrow (x - 18)$

$\Rightarrow (5x - 21) = 0$ i.e., $x = 18$ or $x = 21/5$

The dimensions of $Q$, $R$, $S$ are as follows.

| | Q | R | S |
|---|---|---|---|
| Breadth | 12 | 10 | $10 - 10/7 = 60/7$ |
| Length | 15 | 18 | 21 |
| Area | 180 | 180 | 180 |
| Or | | | |
| One dimension | 12 | $72/21 = 24/7$ | $24/7 - 10/7 = 2$ |
| Other dimension | 6/5 | 21/5 | 36/5 |
| Area | 72/5 | 72/5 | 72/5 |

$\therefore$ The area of $R$ could be 180 $cm^2$ or 72/5 $cm^2$

**18.** Let the roots of the equation

$x^2 - (p+1)^2 (p^2 + 4pq + 4q^2 + 2)x + (p+1)c = 0$

be, $\alpha, \beta$

$\therefore \alpha + \beta = (p+1)^2 (p^2 + 4pq + 4q^2 + 2)$ and

$\alpha\beta = (p+1)$

$\therefore \frac{\alpha + \beta}{\alpha\beta} = (p+1)[(p+2q)^2 + 2]$

$p$ is a particular positive constant, while $q$ is any real number. Hence, the minimum value of $\frac{\alpha + \beta}{\alpha\beta}$ is 2 $(p+1)$

**19.** Dividing by $x^2$, we get $4x^2 - 4x - 71 + 4/x + 4/x^2 = 0$

We notice that the coefficients of $x$ and $1/x$ have opposite signs. Therefore, instead of trying to express the equation as a quadratic in $x + 1/x$, we try to express it as a quadratic in $x - 1/x$.

i.e., $4(x^2 - 2 + 1/x^2) - 4(x - 1/x) - 63 = 0$

$\Rightarrow [2(x - 1/x) - 9][2(x - 1/x) + 7] = 0$

$\therefore x - 1/x = 9/2$ or $x - 1/x = -7/2$

i.e., $2x^2 - 9x - 2 = 0$ or $2x^2 + 7x - 2 = 0$

i.e. $x = \frac{9 \pm \sqrt{81+16}}{4} = \left(9 \pm \sqrt{97}\right)/4$

or $x = \frac{-7 \pm \sqrt{49+16}}{4} = \frac{-7 \pm 5}{4}$

**20.** The equation $x^4 + (a+b)x^2 + ab = 0$

i.e., $(x^2 + a)(x^2 + b)$ has exactly two real roots. Hence, of $a$ and $b$, one is negative, the other is positive i.e., $ab < 0$.

**21.** We have $x^2 - (2a+3)x + (4a^2 - 6a + 9) = 0$ (1)

We need to evaluate $E = x^3 - 18ax + 8a^3$ (2)

Solving (1), we can get $x$ in terms of $a$. We can substitute in the expression $E$ and try to simplify it. But a little reflection would show that, though this is the most direct method, it may be very complicated.

We recognize some of the terms of the expression

$A^3 + B^3 + C^3 - 3ABC$ in (2)

and it strikes us that the identity $A^3 + B^3 + C^3 - 3ABC$

$= (A + B + C)\ (A^2 + B^2 + C^2 - AB - BC - CA)$ is likely to be useful.

$E = x^3 - 18ax + 8a^3 = x^3 + (2a)^3 - 3(x)\ (2a)$ (3)

$= x^3 + (2a)^3 + 3^3 - 3(x)\ (2a)\ (3) - 3^3$, which is of the form $A^3 + B^3 + C^3 - 3ABC - 3^3$

The expression corresponding to

$A^2 + B^2 + C^2 - AB - BC - CA$ would be

$x^2 + 4a^2 + 9 - 2ax - 6a - 3x$

$= x^2 - (2a+3)x + (4a^2 - 6a + 9)$ which is equal to zero (1).

Hence, $A^3 + B^3 + C^3 - 3ABC = 0$

and $E = -27$

**22.** The expression $\dfrac{x^2 + \left(3 + 2\sqrt{3}\right)x + 3\sqrt{3}}{x^2 + 3x + 2}$ can take a particular value $y$ for some real $x$, provided the value of $x$ satisfies the equation

$x^2(y-1) + (3y - 3 - 2\sqrt{3})x + (2y - 3\sqrt{3}) = 0$

This equation has real roots, provided

$$\Delta = \left(3y - 3 - 2\sqrt{3}\right)^2 - 8(y-1)\left(y - \frac{3}{2}\sqrt{3}\right) \geq 0$$

i.e., $9y^2 - 6\left(3 + 2\sqrt{3}\right)y + 9 + 12\sqrt{3} + 12$

$$-8y^2 + 8\left(1 + \frac{3\sqrt{3}}{2}\right)y - 12\sqrt{3} \geq 0$$

i.e., $y^2 - 10y + 21 \geq 0$ i.e., $y \leq 3$ or $7 \leq y$

**23.** Let $F(x) = k(x-1)\ (x-2)\ (x-3) + G(x)$ where $k$ is a constant and $G(x)$ is a quadratic expression.

The remainder theorem: The remainder of $f(x)$ divided by $x - a$ is $f(a)$, where $a$ is a constant and $f(x)$ is a function of $x$.

Remainders of $F(x)$ divided by $x-1$, $x-2$, $x-3$ are $F(1)$, $F(2)$, $F(3)$, respectively

Let $G(x) = ax^2 + bx + c$

$F(1) = G(1) = a + b + c = 28$

$F(2) = G(2) = 4a + 2b + c = 64$

$F(3) = G(3) = 9a + 3b + c = 124$

Solving for $a$, $b$, $c$ we have $a = 12$, $b = 0$ and $c = 16$.

We have to find the remainder when $F(x)$ is divided by $(x-1)\ (x-2)\ (x-3)$. This remainder is $G(x)$.

Required remainder $= 12x^2 + 16$.

**24.** $(3x)^2 + \left[3^{3+\frac{1}{p}} - 15\right]x + 4 = 0$

Given $\Delta = 0$

$$\left(3^{3+\frac{1}{p}} - 15\right)^2 = 4 \times 4 \times 9$$

$$3^{6+\frac{2}{p}} + 225 - 30.3^{3+\frac{1}{p}} = 4 \times 4 \times 9$$

$$3^4 \times 3^{\frac{2}{p}} + 25 - 90.3^{\frac{1}{p}} = 4 \times 4$$

$$81.3^{\frac{2}{p}} - 90 \times 3^{\frac{1}{p}} + 9 = 0$$

$\Rightarrow 9\ x\ 3^{2/p} - 10\ x\ 3^{1/p} + 1 = 0$

$\Rightarrow (9\ x\ 3^{1/p} - 1)\ (3^{1/p} - 1)$

$\Rightarrow 3^{1/p} = 1/9 \Rightarrow p = -1/2$

**25.** $\alpha, \beta$ are roots of $x^2 - x + l = 0$ (1)

Hence, $\alpha + \beta = -(-1) = 1$

and $\alpha\beta = l$ (2)

$\gamma, \delta$ are roots of $x^2 - 4x + m = 0$ (3)

Hence, $\gamma + \delta = -(-4) = 4$ and

$\gamma\delta = m$ (4)

$\alpha, \beta, \gamma, \delta$ form a G.P. If $r$, is the common ratio.

$\beta = r\,.\,\alpha$, $\gamma = r^2.\ \alpha$, $\delta = r^3\alpha$ (5)

From (4) and (2)

$$\frac{\gamma + \delta}{\alpha + \beta} = \frac{\alpha.r^2 + \alpha.r^3}{\alpha + \alpha.r} = \frac{4}{1} \Rightarrow \frac{\alpha r^2(1+r)}{\alpha(1+r)} = r^2 = 4$$

$\Rightarrow r = \pm 2$ (6)

From (4) and (2)

$\alpha + \beta + \gamma + \delta = 5 \Rightarrow S_4 = 5$

$$\Rightarrow \frac{\alpha.\left(r^4 - 1\right)}{r - 1} = 5 \Rightarrow \alpha\left[(\pm 2)^4 - 1\right] = 5(\pm 2 - 1)$$

$\Rightarrow \alpha . 15 = 5$ or $-15$

$\Rightarrow \alpha = \frac{1}{3}$ or $-1$ (7)

From (4) and (2)

$m = \gamma\delta = \alpha r^2 . \alpha r^3$

$= \alpha^2 r^5 = \left(\frac{1}{3}\right)^2 (+2)^5$ or $(-1)^2 (-2)^5$

$m = \frac{32}{9}$ or $-32$ (8)

$l = \alpha\beta = \alpha.\alpha r = \alpha^2 r = \left(\frac{1}{3}\right)^2 (+2)$ or $(-1)^2 (-2)$

$= \frac{2}{9}$ — or $-2$ (9)

From (8) or (9), the integer values of $l$ and $m$ are $-2$ and $-32$

$\therefore l + m = -2 - 32 = -34$

**26.** For satisfying the condition $\mu/\beta + \beta/\mu \geq 2$

The sign of both $\mu$ and $\beta$ are either positive or negative (i.e., same)

Hence, product of roots is positive.

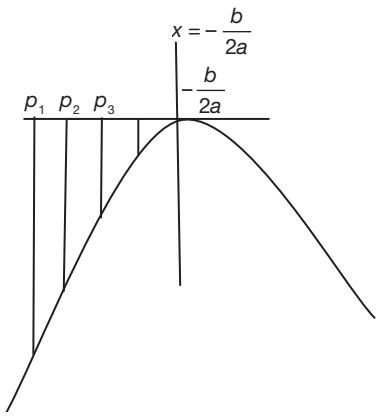
$\Rightarrow \frac{18 - 3p}{3} > 0 \Rightarrow p < 6$

**27.** Given,

$f(x) = ax^2 + bx + c,\ a < 0$ (1)

Since $a < 0$, the parabola represented, by (1), downwards is concave, as shown in the figure.

Further, the curve that touches the $x$-axis, since the roots of $f(x) = 0$ are equal and $\frac{-b}{2a}$ is the double root of the equation $f(x) = 0$




The curve is symmetrical about $x = \frac{-b}{2a}$

$\therefore p_1 < p_2 < p_3 < \frac{-b}{2a}$

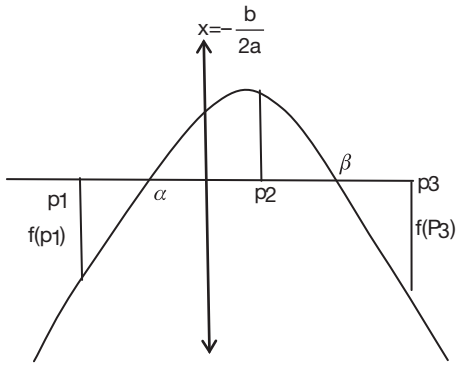
$\Rightarrow f(p_1) < f(p_2) < f(p_3) < f\left(\frac{-b}{2a}\right) = 0.$

**28.** Given, $S$

$F(x) = ax^2 + bx + c,\ a < 0$

$\alpha, \beta$ are the distinct real roots of the equation $f(x) = 0$

$\therefore$ The parabola would be concave downwards, with the curve intersecting the $x$-axis at two distinct points.




Given,

$p_1 < \alpha < p_2 < \beta < p_3$

From the figure,

$f(p_1) < 0, f(\alpha) = 0, f(p_2) > 0, f(\beta) = 0, f(p_3) < 0$

$\therefore f(p_1) < f(p_2) > f(p_3).$

**29.** $P$ is the greatest negative integer such that

$P^2 \geq 20P + 3500,$

i.e., $P^2 \geq 20P + 3500$

$\Rightarrow P^2 - 20P - 3500 \geq 0$

$\Rightarrow (P + 50)\ (P - 70) \geq 0$

$\Rightarrow P \leq -50$ or $P \geq 70$

i.e., the greatest negative integral value of $P$ is $-50$.

**30.** $a$, $b$ and $c$ are the roots of the equation

$x^3 + p_1x^2 + p_2x + p_3 = 0$ (1)

such that $a + b + c = -1$

$\Rightarrow -p_1 = -1 \Rightarrow p_1 = 1$

Since, coefficients $p_1$, $p_2$ and $p_3$ are in $GP$, co-efficients

$p_1$, $p_2$ and $p_3$ would be in the form of 1, $r$, $r^2$, respectively.

Equation (1) reduces to

$x^3 + x^2 + rx + r^2 = 0$

Now,

$ab + bc + ca = r$ (2)

and

$abc = -r^2$ (3)

From (2) and (3)

$(ab + bc + ca)^2 = r^2 = -abc$

$(ab + bc + ca)^2 = -abc.$

## EXERCISE-5

**1.** From Statement I,

$x^2 - 4x + 3 = 0, \Rightarrow (x-3)\ (x-1) = 0.$

So, $x$ is either 3 or 1. (i.e., $x$ is not equal to 2)

So, Statement I alone is sufficient.

From Statement II,

$x^2 - x + 2 = 0$, which has no real solution.

So, we can answer the question with Statement II alone also.

**2.** For $ax^2 + bx + c = 0$, sum of the roots is $\frac{-b}{a}$ and product is $\frac{c}{a}$.

From Statement I, $\frac{c}{a} = \frac{-b}{a}$

From Statement II, $\frac{-a}{c} = 2\ , \Rightarrow \frac{a}{c} = -2$

Combining both the statements, we can say that sum of the roots is $\frac{c}{a} = \frac{1}{2}$

**3.** $ax^2 + bx + c = 0$ has a real solution if $b^2 - 4ac \geq 0$.

Either of the statements alone is not sufficient as the information about $a$, $c$ and $b$ is given in different statements.

Combining statements I and II,

$0 < a < c < 1 \Rightarrow 4ac < 4.$

$b > 3 \Rightarrow b^2 > 9.$, So, $b^2 - 4ac > 5$

$\therefore$ It has a real solution.

**4.** Let the equation be $ax^2 + bx + c = 0$

Using Statement I, we get, $\left(\frac{-b}{a}\right)^2 \leq 4\frac{c}{a}$

$b^2 - 4ac \leq 0$

If $b^2 - 4ac = 0$, the roots are real.

If $b^2 - 4ac < 0$, the roots are not real.

I is not sufficient.

Using Statement II, we get, $\left(-\frac{b}{a}\right)^2 \geq 4\frac{c}{a}$, $b^2 - 4ac \geq 0.$

$\therefore$ The roots are always real. II is sufficient.

**5.** Using Statement I, we get

$A\ (1)^2 + B\ (1) + C > 0$ and $A\ (-1)^2 + B\ (-1) + c < 0$

i.e. $A + B + C > 0$ and $A - B + C < 0$

$\therefore A + B + C - (A - B + C) > 0, \therefore B > 0.$

$\therefore A$ or $C$ is negative.

I is not sufficient.

Using Statement II, we know that $A$ or $B$ is negative. II is not sufficient.

Using both statements, we know that $A$ is negative. Both statements taken together are required to answer the question.

**6.** Discriminant $= B^2 - 4(A)\ (4A) = B^2 - 16\ A^2$

As the roots are real, $B^2 - 16\ A^2 \geq 0$

$\therefore (B - 4A)\ (B + 4A) \geq 0$

$A$ and $B$ are natural numbers

$\therefore B + 4\ A \geq 0$

$\therefore B - 4A \geq 0$ i.e. $B \geq 4A$ (1)

From statement $A$, $B < 5$ (2)

$A \geq 1$

$\therefore$ (1) $\Rightarrow B \geq 4$ (3)

From (2) and (3), $B = 4$

From (1), $A \leq 1, \therefore A = 1$

I is sufficient.

From Statement II, $1 \leq A \leq 2$

$\therefore A = 1$ or 2

II is not sufficient.

**7.** From Statement I, discriminate = $Q^2 - 4PQ \geq 0$

$Q(Q - 4P) \geq 0$

$Q \geq 0$

$\therefore\ Q - 4P \geq 0$ i.e. $Q \geq 4P$ (1)

$\frac{P}{Q}$ is not unique.

From Statement II, discriminate = $(4P)^2 - 4(Q)\ (P) \geq 0$

$4P(4P - Q) \geq 0$

$4P \geq 0$

$\therefore\ 4P - Q \geq 0$ i.e. $Q \leq 4P$ (2)

$\frac{P}{Q}$ is not unique.

II is not sufficient

Using both statements, from (1) and (2), we get

$Q = 4P$

$\therefore \frac{P}{Q} = \frac{1}{4}$

The two statements together are required to answer the question.

**8.** Let $Q$ be $ax^2 + bx + c$

Maximum value of $Q$

$$= \frac{4ac - b^2}{4a} = -\frac{(\text{Discriminant of } Q = 0)}{4(\text{Coeffecient of } x^2 \text{ in } Q)}$$

As the discriminant of $Q = 0$ and the coefficient of $x^2$ are given in different statements, the question can be answered only by combining the two statements.

**9.** Let the roots be $p$ and $q$.

$p + q = -a$ and $pq = b$

Using Statement I, we know $a + b = 0 \Rightarrow -a = b$

$\therefore\ p + q = pq$

$pq - p - q = 0$

$pq - p - q + 1 = 1$

$p(q - 1) - 1(q - 1) = 1$

$(p - 1)\ (q - 1) = 1$

The roots are integers. $\therefore\ p - 1$ and $q - 1$ are factors of 1.

$\therefore\ p - 1 = q - 1 = \pm 1$

$\therefore\ p = q = 2$ or 0

I is sufficient.

Using Statement II, $p$ and $q$ are reciprocal to each other.

$\therefore\ p = \frac{1}{q}$ i.e. $pq = 1$.

Also $p$ and $q$ are integers.

$\therefore\ p$ and $q$ are factors of 1.

$\therefore\ p = q = 1$ or $-1$.

II is sufficient.

Either of the statements is sufficient.

**10.** Discriminant = $[2(ab + bc)]^2 - 4(a^2 + b^2)\ (b^2 + c^2)$

$= 4[a^2 b^2 + 2ab^2 c + b^2 c^2 - (a^2 b^2 + a^2 c^2 + b^4 + b^2 c^2)]$

$= -4(b^2 - ac)^2$

$\Rightarrow$ The discriminant is (0) or negative (1)

Using Statement I, we know that $p$ and $q$ are real, i.e. the discriminant is non-negative. (2).

From (1) and (2)

discriminant is 0.

$\therefore\ b^2 - ac = 0$,

$\therefore \frac{b^2}{ac} = 1$

I is sufficient.

Using Statement II, $\frac{b^2}{ac} = 1$

II is sufficient.

Either of the statements is sufficient.

**11.** For the equation $ax^2 + 3x + 2 = 0$, the roots are $\mu$ and $\beta$

$\therefore\ \alpha + \beta = \frac{-3}{a}$, $\alpha\beta = \frac{2}{a}$

$\therefore \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$

$$= \frac{(\alpha + \beta)^3 - 3(\alpha + \beta)\alpha\beta}{\alpha\beta} = \frac{\left(\frac{-3}{a}\right)^3 - 3\left(\frac{-3}{a}\right)\left(\frac{2}{a}\right)}{\frac{2}{a}}$$

$$= \frac{\left(\frac{-27}{a^3} + \frac{18}{a^2}\right)}{\frac{2}{a}} = \frac{18a - 27}{2a^2} = \frac{9}{2a^2}(2a - 3)$$, which is negative if $a < 3/2$, zero if $a = 3/2$ and positive if $a > 3/2$.

From Statement I, if $a$ is –ve, $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}<0$.

Hence, I alone is sufficient.

If $a>0$ and lesser than $\frac{3}{2}$ then $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}<0$

But if $a>\frac{3}{2}$, then $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}>0$

Hence, Statement II is not sufficient

**12.** Sum of the roots of $ax^2+bx+c=0$ is $\frac{-b}{a}$

From Statement I, $-b/c=3$

This statement alone is not sufficient.

From Statement II, sum of the roots of $cx^2+ax+b=0$ is 4, i.e., $-a/c=4$. This statement alone is not sufficient.

But if we use both statements, we have $\frac{-a}{c}=3,\ \frac{-a}{c}=4$

Hence, $\frac{-a}{c}=\frac{-a}{c}$

$\therefore$ I and II together are sufficient.

**13.** $\alpha+\beta=-r$ and $\alpha\ \beta=\delta$ From (1) $\alpha^2+\beta^2=1300$

$\Rightarrow (\alpha+\beta)^2-2\alpha\beta=1300$

$\gamma^2-2\ \delta=1300$

Since there are 2 unknowns, Statement I is insufficient.

From II : $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{1}{12}$ i.e. $\frac{\alpha+\beta}{\alpha\beta}=\frac{1}{12}$

$\frac{-\gamma}{\delta}=\frac{1}{12}$

But this statement alone is insufficient.

But if we consider the data in both Statement I and II we get $\delta=-12\gamma$

$\therefore \gamma^2+24\gamma=1300$

$\Rightarrow \gamma^2+24\ Y-1300=0 \Rightarrow (\gamma+50)\ (\gamma-26)=0$

$\therefore \gamma=26$

**14.** The minimum value of the given expression is $\frac{4(2)(20)-a^2}{4(2)} \Rightarrow 20-\frac{a^2}{8}>0$

From Statement I, for any value of $a$ (*i.e*, $0<a<10$),

$20-\frac{a^2}{8}>0$ $\therefore$ Statement I, alone is sufficient.

From Statement II, we cannot say anything as far some values (say $a=-20$) of '$a$' the expression can be positive or negative.

$\therefore$ Statement II is not sufficient

**15.** The sum of the roots is $-b/a$ It is greater than 0 if $a$, $b$ are of opposite signs.

From I $a\ b\ c>0$. Hence, $a$, $b$, $c$ are either all +ve or any 2 are –ve. We cannot determine if $a$, $b$ are of opposite sign. Hence, Statement I alone is insufficient.

From Statement II, $a+b+c<0$

We cannot determine if $a$, $b$ are of opposite sign.

Hence, Statement II is insufficient.

If we consider both I and II we can determine that out of $a$, $b$, $c$ two are definitely negative. But we still cannot determine if $a$ and $b$ are of opposite sign.

**16.** From Statement I, product of the roots $=c=1$

But we cannot find the value of $b$

$\therefore$ Statement I alone is not sufficient

Form Statement II, $b=2c$

As we do not know the value of $c$, we cannot find $b$

$\therefore$ Statement II is not sufficient

By combining I and II, we get $b=2$

**17.** $x^2-(\log_b a^3)x+9\log_b\frac{a}{b}=0$

To make it a quadratic we write the equation as

$x^2-\left(\log_b\frac{a^3b^3}{b^3}\right)x+9\log_b\frac{a}{b}=0$

$\Rightarrow x^2-3\ \log_b\left(\frac{a}{b}\right)x-3x+3\left(3\log_b\left(\frac{a}{b}\right)\right)=0$

$\Rightarrow (x-3)\left(x-3\log_b\left(\frac{a}{b}\right)\right)=0$ i.e. $x=3$ or

$x=3\log_b\left(\frac{a}{b}\right)$

From I, $x$ is an integer i.e. $x$ could be 3 or same other integer 3. as we do not know anything about $a$, $b$. Hence, Statement I alone is insufficient.

From Statement II, $a$, $b$ are relatively prime. Hence, $\log_b\left(\frac{a}{b}\right)$ is not an integer.

$\therefore$ $x$ can be 3 or $\log_b(a/b)$

Statement II alone is not sufficient. If we consider both statements I and II we get $x = 3$

**18.** From Statement I, no additional data is given.

It is redundant.

The product of the roots = $a/a = 1$

$\therefore$ Statement I is not sufficient

From Statement II, $(a+1)^2 = 4a^2$

$\Rightarrow a^2 + 2a + 1 = 4a^2 \Rightarrow 3a^2 - 2a - 1 = 0$

$\Rightarrow (a-1)(3a+1) = 0$

$\Rightarrow a = 1$ or $a = -1/3$.

As $a$ has two possible values, Statement II is not sufficient. As Statement I is redundant, i.e., gives only data which can be inferred from the problem, even the combination of I and II is not sufficient.

**19.** $x^2 + y^2 + 4x - 6y + b = 0$

From statement (1) we get

$x^2 + y^2 + 4x - 6y + 13 = 0$

$(x+2)^2 + (y-3)^2 = 0$

Since it is expressed as a sum of two squares, both terms must be 0.

$\therefore$ $x = -2$, $y = 3$,i.e., there is a unique solution for $(x, y)$

Hence, Statement I alone is sufficient.

From Statement II we get

$x^2 + y^2 + 4x + 6y + 26 = 0$

$\Rightarrow (x+2)^2 + (y-3)^2 = 13$ i.e., $(x, y)$ can have many values, i.e., the equation does not have a unique solution.

$\therefore$ Statement II alone is also sufficient

**20.** The roots of $x^2 - 3x - 4 = 0$ i.e.,$(x-4)(x+1) = 0$ are

$x = 4$ or $x = -1$

if $x = 4$ is the common root and then the other root is $P_2$, then $4P_2 = -\frac{5}{2}$ (product of the roots) or $P_2 = -\frac{5}{8}$

$\therefore -\frac{k}{2} = 4 + -\left(\frac{5}{8}\right) = \frac{27}{8}$

$\therefore k = -\frac{5}{2}$

if $x = -1$ is the common root then $-P_2 = -\frac{5}{2}$

$P_2 = \frac{5}{2}$ i.e. $-\frac{5}{2} = -1 + \frac{5}{2} = \frac{5}{2}$

$k = -3$

From Statement I, $k$ is an integer. Hence, $k = -3$.

Hence, Statement I alone is sufficient.

From Statement II, $k + 5$ is a natural number.

This is possible only when $k = -3$.

Hence, Statement II is also sufficient.

**21.** From Statement I, $k^2 - 4a(72) = 0 \Rightarrow k^2 = 288a$

But as we do not know the value of $a$, we cannot find $k$.

$\therefore$Statement I alone is not sufficient

From Statement II, $a = 8$

But no further information is given.

$\therefore$ Statement II alone is not sufficient.

By combining I and II,

$k^2 = 288(8)$

$\Rightarrow k = \pm 48$

But given $k$ is positive

$\Rightarrow k = 48$

**22.** $ax^2 + bx + c = 0$ has real roots if $b^2 - 4ac \geq 0$

From Statement I, $a$, $b$, $c$ are in $G.P.$ Hence, $b^2 = ac$.

$\therefore$ $b^2 - 4ac = b^2 - 4b^2 = -3b^2 \leq 0$

The equation does not have real roots. If $-3b^2 < 0$ and has real roots if $-3b^2 = 0$, i.e., $b = 0$.

Hence, Statement I alone is not sufficient.

From Statement II,

$b$ is the $AM$ of $2a$ and $2c$

$\therefore$ $b = \frac{2a+2c}{2} \Rightarrow b = a + c$

$\Rightarrow a - b + c = 0 \Rightarrow x = -1$ is one of the roots.

$\therefore$ $b^2 \geq 4ac$

$\therefore$ $b^2 - 4ac \geq 0$

Hence, Statement II is sufficient.

**23.** From Statement I, $b^2 = ac$

But as we do not know the value of at least one variable, we cannot find the values. ∴Statement I is not sufficient

From Statement II, $b^2 = 4ac$

For the same reason as above, we cannot find any value. ∴Statement II is not sufficient

By combining I and II,

$b^2 = 4b^2 \Rightarrow b = 0 \Rightarrow ac = 0$

As $a$ cannot be 'zero', $c = 0$

$\therefore b + c = 0$

**24.** The minimum value of the expression $3x^2 + bx + c$ is

$$\frac{4(3)(c) - b^2}{4(3)} = c - \frac{b^2}{12}$$

From Statement I , the product of the roots = $c/3 = 1$

$\Rightarrow c = 3$. But we do not know the value of $b$

∴Statement I is not sufficient.

From Statement II, $-b/3 = 2 \Rightarrow b = -6$

But we do't know the value of $c$

∴Statement II is not sufficient.

By combining I and II,

the minimum value = $3 - 36/12 = 0$

**25.** $x^2 - 5x + a = 0$

$p$ and $q$ are the roots.

$p + q = 5 \qquad pq = a$

$$\frac{\left(p^3 + q^3\right)}{pq} = \frac{(p+q)^3 - 3pq(p+q)}{pq}$$

$$= \frac{5^3 - 3a(5)}{a} = \frac{125 - 15a}{a} = \frac{125}{a} - 15$$

From Statement I, $a = 2$, 3 or 4 At any of these values,

$\frac{125}{a} - 15$ is not on integer

∴ Statement I alone is sufficient

From Statement II, when $a = 1$ or 5, $125/a - 15$ is an integer and in the other cases it is not.

∴Statement II is not sufficient.

# 5 Inequalities and Modulus

CHAPTER

**LEARNING OBJECTIVES**

*In this chapter, you will*:

- Learn about different types of inequalities and their properties such as:
  - The rules of inequalities
  - Linear inequalities
  - Quadratic inequalities
  - Higher degree inequalities
- Gain understanding about modulus
  - Definition of modulus
  - Properties of modulus
- Learn all about all about standard maxima/minima models

An inequality is a mathematical expression where the Left-Hand Side (LHS) is not equal to the Right-Hand Side (RHS). Hence, if LHS ≠ RHS, then
LHS < RHS, which means LHS is less than RHS.
LHS > RHS, which means LHS is greater than RHS.
LHS ≤ RHS, which means LHS is less than or equal to RHS.
LHS ≥ RHS, which means LHS is greater than or equal to RHS.
If '$a$' is any real number, then '$a$' is either positive or negative or zero. When '$a$' is positive, then we write $a > 0$, which is read as '$a$ is greater than zero'. When '$a$' is negative, then we write $a < 0$, which is read as '$a$ is less than zero'. If '$a$' is zero, then we write $a = 0$, and in this case, '$a$' is neither positive nor negative.

## ❒ SYMBOLS AND NOTATIONS

For any two non-zero real numbers $a$ and $b$,

1. $a$ is said to be greater than $b$ when $a - b$ is positive.
2. $a$ is said to be less than $b$ when $a - b$ is negative.

These two statements are written as:

1. $a > b$ when $a - b > 0$
2. $a < b$ when $a - b < 0$.

For example, 3 is greater than 2 because $3 - 2 = 1$ and 1 is greater than zero. $-3$ is less than $-2$ because $-3 - (-2) = -1$ and $-1$ is less than zero.

An example of an inequality is $3x + 4 < 5x - 3$

Certain properties and useful results pertaining to inequalities are given below. A thorough understanding of these properties/results is very essential for being able to solve the problems pertaining to inequalities. (In the following list of properties and results, numbers such as $a$, $b$, $c$, $d$, etc., are real numbers.)

**IMPORTANT FACTS**

(i) For any two real numbers $a$ and $b$, either $a > b$ or $a < b$ or $a = b$.
(ii) If $a \not< b$, then $a \geq b$ and if $a \not> b$, then $a \leq b$.
(iii) If $a > b$ and $b > c$, then $a > c$.
(iv) If $a < b$ and $b < c$, then $a < c$.
(v) If $a > b$, then $a \pm c > b \pm c$.
(vi) If $a > b$ and $c > 0$, then $ac > bc$.
(vii) If $a < b$ and $c > 0$, then $ac < bc$.
(viii) If $a > b$ and $c < 0$, then $ac < bc$.
(ix) If $a < b$ and $c < 0$, then $ac > bc$.

(x) If $a > b$ and $c > d$, then $a + c > b + d$.

(xi) If $a < b$ and $c < d$, then $a + c < b + d$.

(xii) The square of any real number is always greater than or equal to 0.

(xiii) The square of any non-zero real number is always greater than 0.

(xiv) If $a > 0$, then $-a < 0$ and if $a > b$, then $-a < -b$.

(xv) If $a$ and $b$ are positive numbers and $a > b$, then

(a) $1/a < 1/b$

(b) $a/c > b/c$ if $c > 0$ and

(c) $a/c < b/c$ if $c < 0$

(xvi) For any two positive numbers $a$ and $b$,

(a) If $a > b$, then $a^2 > b^2$.

(b) If $a^2 > b^2$, then $a > b$.

(c) If $a > b$, then for any positive value of $n$, $a^n > b^n$.

(xvii) Let, $A$, $G$ and $H$ be the arithmetic mean, geometric mean and harmonic mean, respectively, of $n$ positive real numbers. Then $A \geq G \geq H$, the equality occurring only when the numbers are all equal.

(xviii) If the sum of two positive quantities is given, then their product is the greatest when they are equal, and if the product of two positive quantities is given, then their sum is the least when they are equal.

(xxi) If $a > b$ and $c > d$, then we cannot say anything conclusively about the relationship between $(a - b)$ and $(c - d)$; depending on the values of $a$, $b$, $c$ and $d$, it is possible to have $(a - b) > (c - d)$, $(a - b) = (c - d)$ or $(a - b) < (c - d)$.

(xx) When two numbers $a$ and $b$ have to be compared, we can use one of the following two methods:

(a) If both $a$ and $b$ are positive, then we can take the ratio $a/b$ and depending on whether $a/b$ is less than, equal to or greater than 1, we can conclude that $a$ is less than, equal to or greater than $b$. In other words, for two positive numbers $a$ and $b$,

If $a/b < 1$ then $a < b$.

If $a/b = 1$ then $a = b$.

If $a/b > 1$ then $a > b$.

(b) If one or both of $a$ and $b$ are not positive or we do not know whether they are positive, negative or zero, then we can take the difference of $a$ and $b$ and depending on whether $(a - b)$ is less than, equal to or greater than zero, we can conclude that $a$ is less than, equal to or greater than $b$. In other words, for any two real numbers $a$ and $b$,

If $a - b < 0$, then $a < b$.

If $a - b = 0$, then $a = b$.

If $a - b > 0$, then $a > b$.

(xxi) For any positive number $x \geq 1$,

$$2 \leq \left(1 + \frac{1}{x}\right)^x < 2.8.$$

The equality in the first part will occur only if $x = 1$.

(xxii) For any positive number, the sum of the number and its reciprocal is always greater than or equal to 2, i.e., $x + \frac{1}{x} \geq 2$ where $x > 0$.

The equality in this relationship will occur only when $x = 1$.

## ❒ ABSOLUTE VALUE

Absolute value is written as $|x|$ and read as 'modulus of $x$'. For any real number $x$, the absolute value is defined as follows:

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \text{ and} \\ -x, & \text{if } x < 0 \end{cases}$$

## ❒ Properties of Modulus

For any two real numbers $x$ and $y$,

1. $x = 0 \Leftrightarrow |x| = 0$
2. $|x| \geq 0$ and $-|x| \leq 0$
3. $|x + y| \leq |x| + |y|$
4. $\big||x| - |y|\big| \leq |x - y|$
5. $-|x| \leq x \leq |x|$
6. $|x \cdot y| = |x||y|$
7. $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$; $(y \neq 0)$
8. $|x|^2 = x^2$

In inequalities, the variables generally take a range of values unlike in the case of equations where the variables in general, take one value or a finite set of values.

## ❒ INTERVAL NOTATION

Generally, the solution set or the range of values satisfied by inequalities are not discrete. Hence, it is important to understand the 'interval notation'. $(a, b)$ read as 'open interval $a, b$' means all real numbers between $a$ and $b$ excluding $a$ and $b$ $(a < b)$. $[a, b]$ read as 'closed interval $a, b$' means all real numbers between $a$ and $b$ including $a$ and $b$ $(a < b)$. $[a, b)$ means all numbers between $a$ and $b$, with $a$ being included and $b$ excluded $(a < b)$.

The problems on inequalities are normally of three categories:

1. Consisting of first degree expressions in $x$.
2. Consisting of second degree expressions in $x$, directly in the problem or consisting single expression which reduces to quadratic expression.
3. Consisting of expressions including 'modulus'.

## ❒ SOME USEFUL MODELS

Quite often, when dealing with positive real numbers, we come across situations where the sum (or product) of certain variables is given and we are required to maximise (or minimise) the product (or sum) of the same. We illustrate the technique involved with a couple of examples.

### ❒ Model 1

If $ax + by = k$, where $a, b, x, y$ are all positive, maximise $x^m y^n$ where $m$ and $n$ are positive integers.

### ❒ Model 2

If $x^m y^n = k$ where $x > 0$, $y > 0$ and $m$ and $n$ are positive integers, minimise $ax + by$ where $a > 0$, $b > 0$.

### ❒ Model 3

The greatest value of $(a - x)^m (b + x)^n$, for any real value of $x$ numerically less than $a$, $b$ and $m, n \in Z^+$, occurs when $\frac{a-x}{m} = \frac{b+x}{n}$ or at $x = \frac{an-bm}{m+n}$.

### ❒ Model 4

If $a$ and $b$ are two positive numbers, then the mean of their $m^{th}$ powers (say $M$) and the $m^{th}$ power of their mean (say $P$) are related as follows, depending on the value of $M$.

1. If $m < 0$ or $m > 1$, then $P$ is less than or equal to $M$.

   i.e., $\left(\frac{a+b}{2}\right)^m \le \left(\frac{a^m + b^m}{2}\right)$

   $\left(\text{For example, } \left(\frac{a+b}{2}\right)^2 \le \frac{a^2+b^2}{2}\right)$

   The equality holds if $a = b$.

2. If $m = 0$ or $1$, then $P = M$

   i.e., $\left(\frac{a+b}{2}\right)^0 = \frac{a^0 + b^0}{2} = 1$ and $\left(\frac{a+b}{2}\right)^1 = \frac{a^1 + b^1}{2}$

   $= \frac{a+b}{2}$

3. If $0 < m < 1$, then $M$ is less than or equal to $P$.

   i.e., $\frac{a^m + b^m}{2} \le \left(\frac{a+b}{2}\right)^m$

   (For example, $\frac{\sqrt{a} + \sqrt{b}}{2} < \sqrt{\frac{a+b}{2}}$ ) The equality holds if $a = b$.

This rule can be extended for 3 or more quantities.

**NOTE**

When $a > 0$, $b > 0$, $c > 0$, then

1. If $m < 0$ or $m > 1$, then $\left(\frac{a+b+c}{3}\right)^m \le \frac{a^m + b^m + c^m}{3}$.

   The equality holds if $a = b = c$.

2. If $m = 0$ or $1$, then $\left(\frac{a+b+c}{3}\right)^m = \frac{a^m + b^m + c^m}{3}$

3. If $0 < m < 1$, then $\frac{a^m + b^m + c^m}{3} \le \left(\frac{a+b+c}{3}\right)^m$.

The equality holds if $a = b = c$.

The generalisation of this is given below.

### ❒ Model 5

The minimum value of $\frac{(x+a)(x+b)}{x+c}$, where $a > c$, $b > c$ and $x + c > 0$ is given by $a - c + b - c + 2\sqrt{(a-c)(b-c)}$ and the corresponding value of $x$ is $\sqrt{(a-c)(b-c)} - c$.

## SOLVED EXAMPLES

**5.01.** Express all real numbers between 3 and 8 in the interval notation from where
(a) 3 and 8 are excluded.
(b) 3 and 8 are included.
(c) 3 is included and 8 is excluded.
(d) 3 is excluded and 8 is included.

**Sol:** (a) $(3, 8)$, i.e., $3 < x < 8$ and $x$ is a real number.
(b) $[3, 8]$, i.e., $3 \le x \le 8$ and $x$ is a real number.
(c) $[3, 8)$, i.e., $3 \le x < 8$ and $x$ is a real number.
(d) $(3, 8]$, i.e., $3 < x \le 8$ and $x$ is a real number.

**5.02.** Express the following in the interval notation:
(a) All positive real numbers
(b) All negative real numbers
(c) All non-zero real numbers
(d) All real numbers.

**Sol:** (a) $(0, +\infty)$ or simply $(0, \infty)$, i.e., $0 < x < \infty$
As we cannot write the greatest positive real number, we use the symbol $\infty$ (infinity).
(b) $(-\infty, 0)$, i.e., $-\infty < x < 0$.
(c) $(-\infty, 0) \cup (0, \infty)$
Here, zero is excluded and the symbol '$\cup$' stands for the union of two sets.
(d) $(-\infty, \infty)$ is the notation and this would include all real numbers.

**5.03.** Express the following in the interval notation:
(a) All real numbers greater than 3
(b) All real numbers less than or equal to $-4$

**Sol:** (a) $(3, +\infty)$ or simply $(3, \infty)$, i.e., $3 < x < \infty$.
As we cannot write the greatest positive real number, we use the symbol $\infty$ (infinity).
(b) $(-\infty, -4]$, i.e., $-\infty < x \le -4$.
Here, we use $-\infty$ to represent the least negative real number and we use the '$]$' bracket to denote '$\le$' relation.

**5.04.** If $11x - 21 \ge 2x + 15$, then what is the range of values that $x$ can take?

**Sol:** $11x - 21 \ge 2x + 15$
$11x - 2x \ge 15 + 21 \Rightarrow 9x \ge 36 \Rightarrow x \ge 4$
Interval notation is $[4, \infty)$.

**5.05.** Solve the inequalities which hold simultaneously $4x + 17 < 33$ and $8x - 12 < 36$.

**Sol:** $4x + 17 < 33$ (1)
$8x - 12 < 36$ (2)
$(1) \Rightarrow 4x < 33 - 17 \Rightarrow 4x < 16 \Rightarrow x < 4$ (3)
$(2) \Rightarrow 8x < 36 + 12 \Rightarrow 8x < 48 \Rightarrow x < 6$ (4)
We have to take the intersection of the range of values represented by the inequalities (3) and (4), i.e., $(x < 4) \cap (x < 6)$, which is $x < 4$.
Interval notation is $(-\infty, 4)$.

**5.06.** Which of these two numbers, $20^{21}$ and $21^{20}$ is greater?

**Sol:** Let $a = 21^{20}$ and $b = 20^{21}$.

Consider $\frac{a}{b} = \frac{21^{20}}{20^{21}} = \left(\frac{21}{20}\right)^{20} \times \frac{1}{20}$

$\Rightarrow \frac{a}{b} = \frac{(1+1/20)^{20}}{20}$

As $(1+1/x)^x$ lies between 2 and 2.8, for all positive values of $x$, $(1+1/20)^{20}$, cannot exceed 2.8.

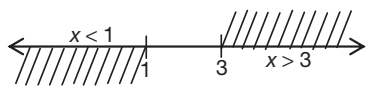
So, $(1+1/20)^{20} < 2 \cdot 8$, i.e., $\frac{(1+1/20)^{20}}{20} < \frac{2 \cdot 8}{20} < 1$

As $\frac{a}{b} < 1$, $a < b$, i.e., $21^{20} < 20^{21}$.

**5.07.** Solve the simultaneous inequalities:
$9x + 8 > 2x + 29$ and $7x - 12 < 5x - 10$.

**Sol:** $9x + 8 > 2x + 29$ (1)
$7x - 12 < 5x - 10$ (2)
From (1), $9x - 2x > 29 - 8 \Rightarrow 7x > 21$
$\Rightarrow x > 3$ (3)
From (2), $7x - 5x < -10 + 12$
$\Rightarrow 2x < 2 \Rightarrow x < 1$ (4)
The intersection of (3) and (4), i.e., $(x > 3) \cap (x < 1)$ is empty.

This is because we cannot find $a$ number less than 1 and at the same time greater than 3. The student can understand this on the number line.




Hence, the given inequalities have no solutions.

**5.08.** Solve for $x$ if $3x^2 - 14x + 15 > 0$.

**Sol:** $3x^2 - 14x + 15 > 0$
$\Rightarrow 3x^2 - 9x - 5x + 15 > 0$
$\Rightarrow (3x - 5)(x - 3) > 0$
$\Rightarrow (3x - 5) > 0$ and $(x - 3) > 0$
(or) $(3x - 5) < 0$ and $(x - 3) < 0$
i.e., $x > \frac{5}{3}$ and $x > 3$
(or) $x < \frac{5}{3}$ and $x < 3$
i.e., $x > 3$ or $x < \frac{5}{3}$

i.e., all values of $x < 5/3$ or all values of $x > 3$ satisfy the inequality.
The interval notation is $(-\infty, 5/3) \cup (3, \infty)$.

**5.09.** Solve for $x$: $4x^2 - 12x + 17 > 0$.

**Sol:** We try to resolve this expression into factors of first degree. As such, it may not work out this way. So, we identify the 'hidden perfect square' in it. We rewrite the expression as $((2x)^2 - 2 \cdot (2x) \cdot (3) + 3^2) + 8 \geq 0$.
$\Rightarrow (2x-3)^2 + 8 \geq 0$
As $(2x-3)^2 \geq 0$ for all $x \in R$, $(2x-3)^2 + 8 \geq 8 + 0$
So, $(2x-3)^2 + 8 > 0$ for all $x \in R$
Hence, all real numbers would satisfy this inequality. The interval notation is $(-\infty, \infty)$.

**5.10.** Solve for $x$: $\dfrac{x^2+5x-14}{2x^2+x-6} < 0$.

**Sol:** Let $f(x) = \dfrac{x^2+5x-14}{2x^2+x-6} < 0$
We resolve the numerator and denominator into linear factors.
Now, $f(x) = \dfrac{(x+7)(x-2)}{(2x-3)(x+2)} < 0$.
We multiply and divide the expression with $(2x-3)\ (x+2)$.

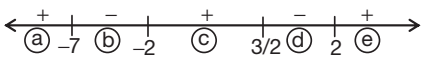


**NOTE**

Cross multiplying should not be done as we are not sure of the sign of the expression in the denominator.

i.e., $\dfrac{(x+7)(x-2)(2x-3)(x+2)}{(2x-3)^2(x+2)^2} < 0$

$(x+7)\ (x-2)\ (2x-3)\ (x+2) < 0$
(as the denominator is positive).
We find the zeros (or roots) of the expression and place them on the number line in order.




We see that there are 5 sections on the number line indicated as ⓐ, ⓑ, ⓒ, ⓓ and ⓔ.

In region ⓐ, the expression is positive since each of $(x+7)$, $(x+2)$, $(2x-3)$ and $(x-2)$ is negative while in ⓑ, the expression is negative as $(x+7)$ is positive while all the other 3 terms take *a* negative value.

Similarly, the sign of expression can be evaluated in each region, and we conclude that in ⓒ it is positive, in ⓓ it is negative and in ⓔ it is positive.

As we want the expression to take *a* 'negative sign', the solution set is the union of sections ⓑ and ⓓ.

The interval notation is $(-7, -2) \cup (3/2, 2)$.

**5.11.** Solve the inequality: $|2x+7| > -5$.

**Sol:** As the modulus of any quantity is non-negative and any non-negative quantity is greater than $-5$, every real number $x$ would satisfy the inequality.

Symbolically, $|2x+7| \geq 0 > -5$. So $|2x+7| > -5$ for all $x \in R$. So, all real numbers ($R$) form the solution set.

The interval notation is $(-\infty, \infty)$.

**5.12.** Solve for $x$: $|x^2 + x + 1| < -1$.

**Sol:** Any quantity less than $-1$ has to be negative. This means $|x^2 + x + 1|$ should be negative. But, the modulus of *a* quantity cannot be negative. So, no solution exists for this inequality. The notation for this is that of empty set $\phi$.

**5.13.** Solve for $x$: $|x-3| = 7$.

**Sol:** $x - 3 = 7$ or $-(x-3) = 7$
$\Rightarrow x = 10$ or $x = -4$.
If $p > 0$, then $|x-a| = p \Rightarrow x = a + p$ or $a - p$.

**5.14.** Solve for $x$: $|2x+5| < 13$.

**Sol:** By definition of modulus,

$$|2x+5| = \begin{cases} 2x+5; \text{ if } 2x+5 \geq 0 \text{ or } x \geq -5/2 \\ -(2x+5); \text{ if } 2x+5 < 0 \text{ or } x < -5/2 \end{cases}$$

Hence, we have to consider two cases $x \geq -5/2$ and $x < -5/2$.

Case (i): $x \geq -5/2$.
$|2x+5| < 13$
$\Rightarrow 2x + 5 < 13 \Rightarrow x < 4$
Hence, the admissible range of values of $x$ is $x \geq -5/2$ and $x < 4$, i.e., $x \in [-5/2, 4)$.

Case (ii): $x < -5/2$
$|2x+5| < 13 \Rightarrow -(2x+5) < 13 \Rightarrow x > -9$
Hence, the admissible range of values of $x$ is $x > -9$ and $x < -5/2$, i.e., $x \in (-9, -5/2)$.

Hence, all the values of $x$, for which $x \in (-9, -5/2) \cup [-5/2, 4) = (-9, 4)$, satisfy the given inequality.

**5.15.** Express all real numbers between 2 and 10 in the interval form where
(a) 2 and 10 are excluded.
(b) 2 and 10 are included.
(c) 2 is included and 10 is excluded.
(d) 2 is excluded and 10 is included.

**Sol:** (a) $(2, 10)$, i.e., $2 < x < 10$ and $x$ is *a* real number.
(b) $[2, 10]$, i.e., $2 \leq x \leq 10$ and $x$ is *a* real number.
(c) $[2, 10)$, i.e., $2 \leq x < 10$ and $x$ is *a* real number.
(d) $(2, 10]$, i.e., $2 < x \leq 10$ and $x$ is *a* real number.

**5.16.** Express the following in the interval notation:
(a) All non-negative real numbers
(b) All non-positive real numbers
(c) All non-zero real numbers
(d) All real numbers

**Sol:** (a) $[0, \infty)$, i.e., $0 \le x < \infty$
(b) $(-\infty, 0]$, i.e., $-\infty < x \le 0$
(c) $(-\infty, 0) \cup (0, \infty)$
(d) $(-\infty, \infty)$

**5.17.** Express the following in the interval notation:
(a) All real numbers less than $-2$
(b) All real numbers greater than or equal to 3

**Sol:** (a) $(-\infty, -2)$, i.e., $-\infty < x < -2$
(b) $[3, \infty)$, i.e., $3 \le x < \infty$

**5.18.** If $13x - 19 \le 4x + 26$, then find the range of $x$.

**Sol:** $13x - 4x \le 26 + 19$
$x \le 5$
$(-\infty, 5]$ in the interval notation

**5.19.** Solve the following inequalities:
$5x + 21 < 46$ and $4x + 18 < 54$.

**Sol:** $5x + 21 < 46 \Rightarrow x < 5$ (1)
$4x + 18 < 54 \Rightarrow x < 9$ (2)
The common inequality satisfying (1) and (2) is $x < 5$ or $(-\infty, 5)$ in the interval notation.

**5.20.** Which of the numbers $50^{51}$ and $51^{50}$ is greater?

**Sol:** Let $a = 50^{51}$ and $b = 51^{50}$.

$$\frac{b}{a} = \frac{51^{50}}{50^{51}} = \left(\frac{51}{50}\right)^{50}\left(\frac{1}{50}\right) = \left(1+\frac{1}{50}\right)^{50}\left(\frac{1}{50}\right)$$

$\left(1+\frac{1}{x}\right)^x$ where $x > 0$ always lies between 2 and 2.8.

$\therefore \frac{b}{a}$ lies between $\frac{2}{50} = 0.04$ and $\frac{2.8}{50} = 0.056$

$\therefore \frac{b}{a} < 1$

$\therefore a > b$

**5.21.** Solve the simultaneous inequations:
$10x + 7 > 2x + 39$ and $9x - 14 > 11x - 28$.

**Sol:** $10x + 7 > 2x + 39 \Rightarrow x > 4$ (1)
$9x - 14 > 11x - 28 \Rightarrow x < 7$ (2)
From Eq. (1) and Eq. (2), $4 < x < 7$

**5.22.** Solve for $x$ if $4x^2 - 21x + 20 > 0$

**Sol:** $4x^2 - 21x + 20 > 0 \Rightarrow (4x - 5)\ (x - 4) > 0$
Both factors are positive (i.e., the smaller is positive) or both are negative (i.e., the greater is negative), i.e., $x > 4$ or $x < \frac{5}{4}$ or it can be expressed in the interval notation as $(4, \infty) \cup (-\infty, \frac{5}{4})$.

**5.23.** Solve for $x$, if $\frac{x^2 + 5x - 24}{2x^2 - 5x - 3} < 0$.

**Sol:** $x^2 + 5x - 24 = (x + 8)\ (x - 3)$
Similarly, $2x^2 - 5x - 3 = (2x + 1)\ (x - 3)$

Given: $\frac{x^2 + 5x - 24}{2x^2 - 5x - 3} < 0 \Rightarrow \frac{(x+8)(x-3)}{(2x+1)(x-3)} < 0$

$$\Rightarrow \frac{x+8}{2x+1} < 0$$

$$\frac{(x+8)(2x+1)}{(2x+1)^2} < 0$$

$$\Rightarrow (x+8)\left(x+\frac{1}{2}\right) < 0$$

$$\therefore -8 < x < -\frac{1}{2}.$$

**5.24.** Solve the inequality $|3x + 6| > -12$.

**Sol:** The modulus of any number is always non-negative.
$\therefore |3x + 6| \ge 0$
$\therefore$ The given inequality is always satisfied.
$\therefore -\infty < x < \infty$

**5.25.** Solve the inequality $|2x + 4| < -6$.

**Sol:** The modulus of any number is always non-negative.
$\therefore |2x + 4| \ge 0$
$\therefore$ The given inequality will never hold.
There is no solution.

**5.26.** Solve for $x$: $|2x - 3| = 5$

**Sol:** $2x - 3 = 5$ or $2x - 3 = -5$
(If $|y| = a$, then $y = \pm a$) $\Rightarrow x = 4$ or $x = -1$

**5.27.** Solve the inequality $|4x - 5| > 3$.

**Sol:** From the definition of modulus,
$|4x - 5| = 4x - 5$ if $4x - 5 \ge 0$, i.e., $x \ge \frac{5}{4}$
$= -(4x - 5)$ if $x < \frac{5}{4}$
$\therefore$ If $x \ge \frac{5}{4}$, $4x - 5 > 3 \Rightarrow x > 2$
and If $x < \frac{5}{4}$, $-4x + 5 > 3 \Rightarrow x < \frac{1}{2}$

**5.28.** Find the maximum value of $g(x) = 16 - |- x - 6|$; $x \in R$.

**Sol:** $g(x)$ is maximum when $|- x - 6|$ is minimum.
The minimum value of the modulus of all numbers is 0.
$\therefore$ The maximum value of $g(x) = 16 - 0 = 16$.

**5.29.** Find the maximum value of $f(x) = 8 - |-4 - x|$; $x \in R$.

**Sol:** $f(x) = 8 - |-4 - x|$ is the maximum when $|-4 - x|$ is the minimum. As the minimum value of $|-4 - x|$ is 0, the maximum value of $f(x)$ is 8.

**5.30.** If $4x + 7y = 18$, then find the maximum value of $x^4y^5$.

**Sol:** Consider the expression $\left(\frac{4x}{4}\right)^4 \left(\frac{7y}{5}\right)^5$.

Now, the sum of all the factors of the above expression is $4 \cdot \frac{4x}{4} + 5 \cdot \frac{7y}{5} = 4x + 7y = 18$ (constant).

$\therefore$ Sum of the factors is constant.

$\left(\frac{4x}{4}\right)^4 \left(\frac{7y}{5}\right)^5$ is maximum when $\frac{4x}{4} = \frac{7y}{5}$.

$\therefore \quad \frac{4x}{4} = \frac{7y}{5} = \frac{4x + 7y}{9}$

$\Rightarrow \frac{4x}{4} = \frac{7y}{5} = 2; \Rightarrow x = 2$ and $y = \frac{10}{7}$

$\therefore$ The maximum value of $x^4y^5 = 24\left(\frac{10}{7}\right)^5$.

**5.31.** If $x^2y^3 = 2^2 3^6 5^2$, then find the minimum value of $3x + 5y$.

**Sol:** Given $x^2y^3 = 2^2 3^6 5^2$

$\Rightarrow 3^2x^2\, 5^3y^3 = 2^2 3^8 5^5$

$\Rightarrow \left(\frac{3x}{2}\right)^2 \left(\frac{5y}{3}\right)^3 = 3^5 5^5$

The LHS is the product of 5 factors, and the sum of these factors, i.e., $3x + 5y$, will have its minimum value, when all these factors are equal.

i.e., when $\frac{3x}{2} = \frac{5y}{2}$

$\Rightarrow y = \frac{9x}{10}$

Given $x^2y^3 = 2^2 3^6 5^2$

$\Rightarrow x^2\left(\frac{9x}{10}\right)^3 = 2^2 3^6 5^2$

$\Rightarrow x^5 = 2^5 5^5$ or $x = 10$

$\therefore y^3 = \frac{2^2 3^6 5^2}{2^5 5^5} = 3^6$

$\Rightarrow y = 9$

$\therefore 3x + 5y = 3(10) + 5(9) = 75$

**5.32.** Find the maximum value of $(x-6)^2 (11 - x)^3$ for $6 \le x \le 11$.

**Sol:** If $E = (x - a)^m (b - x)^n$ and $a \le x \le b$, then the maximum value of $E$ occurs when $\frac{x-a}{m} = \frac{b-x}{n}$.

Here, $a = 6$, $b = 11$, $m = 2$ and $n = 3$.

The given expression is maximum when $\frac{x-6}{2} = \frac{11-x}{3} \Rightarrow x = 8$.

$\therefore$ The maximum value is $2^2 3^3 = 108$.

**5.33.** Find the minimum value of $\frac{(6+x)(12+x)}{(4+x)}$ $(x > -4)$.

**Sol:** $\frac{(6+x)(12+x)}{4+x} = \frac{(4+x+2)(4+x+8)}{4+x}$

$= \frac{(4+x)^2 + 10(4+x) + 16}{4+x}$

$= 4 + x + 10 + \frac{16}{4+x} = 10 + (4 + x) + \left(\frac{16}{4+x}\right)$

As $x > -4$, $4 + x > 0$. $\therefore \frac{16}{4+x} > 0$ and hence, both $4 + x$ and $\frac{16}{4+x}$ are greater than 0.

$\therefore$ A.M. $\left(4 + x, \frac{16}{4+x}\right) \ge$ GM $\left(4 + x, \frac{16}{4+x}\right)$

$\therefore \left(4 + x + \frac{16}{4+x}\right) \ge 2\sqrt{(4+x)\left(\frac{16}{4+x}\right)}$

i.e., $\left(4 + x + \frac{16}{4+x}\right) \ge 8$

$\therefore 10 + (4 + x) + \left(\frac{16}{4+x}\right) \ge 18$, i.e., the minimum value of the given expression is 18.

**Alternative Method:**

Let $E_m$ be the required minimum value.

For $x > -4$, $E_m = a - c + b - c + 2\sqrt{(a-c)(b-c)}$

Here $a = 6$, $b = 12$ and $c = 4$

$\therefore E_m = 6 - 4 + 12 - 4 + 2\sqrt{(6-4)(12-4)}$

$= 2 + 8 + 2\sqrt{2(8)} = 18$

**5.34.** If $x > -6$, then find the minimum value of the expression $\frac{(9+x)(18+x)}{6+x}$.

**Sol:** The minimum value of the expression for $x > -6$ is $a - c + b - c + 2\sqrt{(a-c)(b-c)}$.

Here, $a = 9$, $b = 18$, and $c = 6$

$= 9 - 6 + 18 - 6 + 2\sqrt{(9-6)(18-6)} = 27$

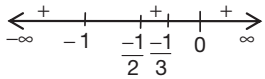
**5.35.** If $\frac{2x}{3x+1} + \frac{1}{x} > 0$, then find the range of real values of $x$.

**Sol:** $\frac{2x^2+3x+1}{x(3x+1)} > 0$

$\Rightarrow \frac{(2x+1)(x+1)}{x(3x+1)} > 0$

$\Rightarrow x(2x+1)\ (3x+1)(x+1) > 0$

$\therefore$ The critical points, (points on the number line for which the expression is 0) are $\frac{-1}{2}, \frac{-1}{3}$, $-1, 0$




The expression is positive in the ranges marked with $a$ positive sign.

$\therefore$ Solution is $(-\infty, -1) \cup \left(\frac{-1}{2}, \frac{-1}{3}\right) \cup (0, \infty)$.

**5.36.** If $2x + 3y = 30$, where $x, y > 0$, then find the maximum value of $x^2y^3$.

**Sol:** $x^2y^3 = \left(\frac{2x}{2}\right)^2 \left(\frac{3y}{3}\right)^3$

Sum of all the factors of the above expression $= 2\left(\frac{2x}{2}\right) + 3\left(\frac{3y}{3}\right) = 2x + 3y = 30$ ($a$ constant).

When the sum of two or more positive quantities is constant, their product is maximum when all the quantities are equal.

$\therefore \left(\frac{2x}{2}\right)^2 \left(\frac{3y}{3}\right)^3$ is maximum when $\frac{2x}{2} = \frac{3y}{3}$, i.e., $x = y$.

When $x = y$, $2x + 3y = 5y = 30$.

$\therefore y = 6$

$\therefore$ Maximum value $= 6^2 . 6^3 = 7776$

**NOTE**

When the expression $ax + by$ is constant, the maximum value of $x^m y^n$ is realized when $\frac{ax}{m} = \frac{by}{n}$.

**5.37.** If $x, y$ are positive and $xy^2 = 216$, then find the minimum value of $32x + y$.

**Sol:** The given condition ($C$) is $xy^2 = 216$. We need the minimum value of the expression $E$: $32x + y$. If the product of several factors is constant, we can obtain the minimum value of the sum of all the factors. We should modify $C$, so that the sum of all the factors is $E$. After modifying, $8(xy^2) = 8\ (216)$. We should think of the factors on the LHS as $32x$, $\frac{y}{2}$ and $\frac{y}{2}$. The sum of these factors is $E$.

$C \Rightarrow (32x)\left(\frac{y}{2}\right)\left(\frac{y}{2}\right) = 8\ (216)$

The sum of the 3 factors, i.e., $32x + y$ is minimum, when $32x = \frac{y}{2} = 2\ (6) = 12$.

Thus, minimum value is $24 + 12 = 36$.

**NOTE**

When the expression $x^m y^n$ is constant, the minimum value of $ax + by$ is realized when $\frac{ax}{m} = \frac{by}{n}$.

**5.38.** Find the maximum value of $(9 - x)^2(-4 + x)^3$.

**Sol:** $m = 2$, $n = 3$ are positive integers. The given expression is maximum when $\frac{9-x}{2} = \frac{x-4}{3}$.

$\Rightarrow x = 7$

$\therefore$ The maximum value is $(9 - 7)^2\ (-4 + 7)^3$.

$= 2^2\ 3^3 = 108$

**5.39.** Find the minimum value of $\frac{(3+x)(6+x)}{2+x}$.

**Sol:** Let $E = \frac{(3+x)(6+x)}{2+x} = \frac{(2+x+1)(2+x+4)}{2+x}$

$= \frac{(2+x)^2 + 5(2+x) + 4}{2+x}$

$= (2+x) + 5 + \left(\frac{4}{2+x}\right) = F + 5.$

The minimum value of $F$ is obtained as follows:

Since $x > -2 \Rightarrow 2 + x > 0$

$\therefore A.M\left(2+x, \frac{4}{2+x}\right) \geq G.M\left(2+x, \frac{4}{2+x}\right) = 2$

$\therefore 2 + x + \frac{4}{2+x} \geq 4$ (The equality holds if and only if $2 + x = \frac{4}{2+x}$. As $x + 2 > 0$, this means $2 + x = 2$.)

$\therefore E_{min} = \text{Min}\ \frac{(3+x)(6+x)}{2+x} = 9$

## EXERCISE-1

***Directions for questions 1 to 45:*** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** All the real numbers lying between –2 and 3 excluding –2 but including 3 is
(A) $[-2, 3]$ (B) $(-2, 3]$
(C) $[-2, 3)$ (D) $(-2, 3)$

**2.** If $x \in [1, 7]$, then which of the following is true?
(A) $1 < x < 7$ (B) $1 \le x \le 7$
(C) $1 \le x < 7$ (D) $1 < x \le 7$

**3.** $x < 3$ represents which of the following intervals?
(A) $(3, \infty)$ (B) $(3, \infty)$
(C) $(-\infty, 3]$ (D) $(-\infty, 3)$

**4.** $2 < x \le 5$ when represented in interval form is
(A) $(2, 5)$ (B) $[2, 5]$
(C) $[2, 5)$ (D) $(2, 5]$

**5.** $x \le -2$ or $x \ge 5$ can also be written as
(A) $[-2, 5]$ (B) $(-2, 5)$
(C) $(-\infty, -2] \cup [5, \infty)$ (D) $(-\infty, -2) \cup (5, \infty)$

**6.** For real numbers $a$, $b$, $c$ and $d$, which of the following is always true?
(A) If $a > b$ and $b > c$, then $a > c$.
(B) If $a > b$ and $c > d$, then $ac > bd$.
(C) If $a > b$ and $c > d$, then $a - c > b - d$.
(D) All the above

**7.** The set consisting of all real numbers lying between –5 and 2 including 2 but excluding –5 is
(A) $[-5, 2]$ (B) $(-5, 2)$
(C) $(-5, 2]$ (D) $[-5, 2)$

**8.** If $a$ and $b$ are two positive real numbers, then which of the following is not always positive?
(A) $a + b$ (B) $a - b$
(C) $ab$ (D) $\frac{a}{b}$

**9.** Which of the following sets represents the set of all real numbers lying between 2 and 5 excluding 2 and 5?
(A) $[2, 5]$ (B) $[2, 5)$
(C) $(-\infty, 2) \cup (5, \infty)$ (D) $(2, 5)$

**10.** If '$a$' and '$b$' are two real numbers, then which of the following is always true?
(A) If $\frac{a}{b} > 1$, then $a > b$.
(B) If $\frac{a}{b} < 1$, then $a < b$.
(C) If $a > 0$, $b > 0$ and $\frac{a}{b} > 1$, then $a > b$.
(D) All the above

**11.** The set of real values of $x$ satisfying the inequation $x + 3 \ge 7$ is
(A) $[4, \infty)$ (B) $(4, \infty)$
(C) $(-8, 4)$ (D) $(-\infty, 4]$

**12.** Solution set of $-2x - 6 \ge 0$ is
(A) $x \ge 3$ (B) $x \ge -3$
(C) $x \le 3$ (D) $x \le -3$

**13.** If $5 - 2x \le 6$, then $x$ belongs to
(A) $\left[\frac{3}{4}, \infty\right]$ (B) $\left[\frac{-1}{2}, \infty\right)$
(C) $\left(\frac{1}{2}, \infty\right)$ (D) $\left[\frac{3}{2}, \infty\right)$

**14.** Which of the following is always true?
(A) If $a < b$, then $a^2 > b^2$.
(B) If $a > b$, then $a^2 > b^2$
(C) If $a < 0$, $b < 0$ and $a < b$, then $a^2 < b^2$.
(D) None of these

**15.** The solution set of $(x - 3)(x + 5) \le 0$ is
(A) $[-3, 5]$ (B) $[-5, 3]$
(C) $(-\infty, -3] \cup [5, \infty)$ (D) $(-\infty, -5) \cup (3, \infty)$

**16.** The number of values of $x$ satisfying the equation $|x + 2| = 2x - 1$ is _____.

**17.** The value of $x$ at which the expression $5 - |2 - x|$ has the maximum value is _____.

**18.** The number of integer values of $x$ satisfying the inequation $|x + 3| \le 5$ is _____.

**19.** Which of the following expressions is always positive?
(A) $x^2 - 3x + 2$, $x \in R$. (B) $x^2 + 2x - 35$, $x \in R$.
(C) $x^2 + 4x + 5$, $x \in R$. (D) $x^2 - x - 6$, $x \in R$.

**20.** The minimum value of $10 + |x|$ is _____.

**21.** The solution of the inequation $|2x - 3| > -5$ is
(A) $\{\ \}$ (B) $R$
(C) $\left[\frac{5}{2}, \infty\right]$ (D) $[5, \infty)$

**22.** Which of the following is positive for all $x \in R$?
(A) $x^2 - 6x - 7$ (B) $x^2 - 5x + 6$
(C) $x^2 - 4x + 8$ (D) $x^2 - 7x + 12$

**23.** The number of integral values of $x$ satisfying the inequation $|x| < 4$ is
(A) 7 (B) 6
(C) 9 (D) 8

**24.** The number of integral values of $x$ satisfying the inequation $x^2 - 7x - 18 \le 0$ is

(A) 11 (B) 14
(C) 13 (D) 12

**25.** If $x$ is a negative real number, then the maximum value of $x+\frac{1}{x}$ is
(A) $-1$ (B) $-5$
(C) $-3$ (D) $-2$

**26.** The solution set of the in equations in $-5 \le x < 7$ represented as
(A) $(-5, 7)$ (B) $[-5, 7]$
(C) $[-5, 7)$ (D) $(-5, 7]$

**27.** Which of the following is/are always true?
(A) If $a > b$, then $a^2 < b^2$.
(B) If $a > b$, then $a^2 > b^2$.
(C) If $a > 0$, $b > 0$ and $a > b$, then $a^2 > b^2$.
(D) All the above

**28.** The solution set of $2x-5 \ge 7x+10$ is
(A) $(-3, \infty)$ (B) $(-\infty, 3)$
(C) $(-\infty, -3]$ (D) $R$

**29.** The solution set of the inequation $|x+2| < -3$ is
(A) $\phi$ (B) $[0, \infty)$
(C) $(-\infty, \infty)$ (D) $(-\infty, 0]$

**30.** The solution set of the inequation $|x-1| > -2$ is
(A) $\phi$ (B) $[0, \infty)$
(C) $(-\infty, \infty)$ (D) $(-\infty, 0]$

**31.** For what value of $x$ does the expression $6 + |3 - 2x|$ have its minimum value?
(A) 6 (B) $\frac{3}{2}$
(C) $\frac{2}{3}$ (D) 0

**32.** The solution set of the inequation $|3x+5| < -7$ is
(A) $\{\ \}$ (B) R
(C) $\left[\frac{7}{3}, \infty\right)$ (D) $\left[\frac{-7}{3}, \infty\right)$

**33.** The maximum value of $20 - |x|$ is
(A) 19 (B) 18
(C) 20 (D) 25

**34.** The number of distinct values of $x$ satisfying the equation $|x+3| + |x-5| = 0$ is
(A) 1 (B) 0
(C) 2 (D) 3

**35.** If $4x + 7 > 9$, then
(A) $x > 1$ (B) $x > \frac{5}{6}$
(C) $x > \frac{1}{2}$ (D) $x > \frac{3}{4}$

**36.** If $3 - x \le 4$, then
(A) $x \le -1$ (B) $x \ge 1$
(C) $x \ge -1$ (D) $x \le 1$

**37.** The number of integer values of $x$ satisfying the inequation $x^2 - 5x - 14 \le 0$ is
(A) 10 (B) 9
(C) 8 (D) 6

**38.** The real values of $x$ satisfying the inequation $x + 2 \ge 5$ are
(A) $[3, \infty)$ (B) $(3, \infty)$
(C) $(-\infty, -3]$ (D) $(-\infty, -3)$

**39.** The solution set of the inequation $|x-2| \le 3$ is
(A) $(-\infty, -1] \cup [5, \infty)$ (B) $[-5, 1]$
(C) $[-1, 5]$ (D) $(-\infty, -5] \cup [1, \infty)$

**40.** The solution set of the inequation $2x + 3 > 4$ is
(A) $[1/2, \infty)$ (B) $(-\infty, 1/2)$
(C) $(-\infty, 1/2]$ (D) $(1/2, \infty)$

**41.** If '$a$' is a positive real number, then the minimum value of $a+\frac{1}{a}$ is _____.

**42.** The number of integral solutions of $|x+3| < 5$ is
(A) 9 (B) 10
(C) 11 (D) 8

**43.** The equation in which has a solution as $x \le -3$ or $x \ge 5$ is
(A) $(x+3)(x-5) \ge 0$ (B) $(x-3)(x+5) \le 0$
(C) $(x+3)(x-5) \le 0$ (D) $(x-3)(x+5) \ge 0$

**44.** The solution set of $(x+13)(x-15) \ge 0$ is
(A) $[-13, 15]$ (B) $[-15, 13]$
(C) $x \le -13$ or $x \ge 15$ (D) None of these

**45.** If $3x + 4 \le -5x + 12$, then which of the following is true?
(A) $x \ge 1$ (B) $x \le 1$
(C) $x \ge -1$ (D) $x \le -1$

## EXERCISE-2

***Directions for questions 1 to 45*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Solve the system of inequalities $3x + 4 \le -8$ and $2x + 7 \ge 5x - 8$.
(A) $(-\infty, -4)$ (B) $(-\infty, -4)$
(C) $[-4, \infty)$ (D) $[4, \infty)$

**2.** Solve the system of inequations $2x-5>3$ and $3x+7<4$.
(A) $\phi$ (B) $R$
(C) $(4, \infty)$ (D) $(-1, 4)$

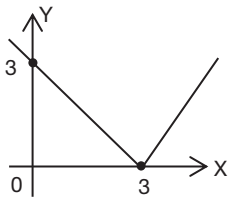
**3.** Find the solution of $2x+7 \geq -5$ and $x+12 \geq 3x+8$.
(A) $[2, 6]$ (B) $[-6, 2]$
(C) $[-6, 2]$ (D) $(-\infty, -2)$

**4.** What values of $x$ satisfy $x^2 - x - 6 > 0$?
(A) $(-\infty, 2)$ (B) $(-3, \infty]$
(C) $[-2, 3]$ (D) $(-\infty, -2) \cup (3, \infty)$

**5.** The number of positive integer solutions of $(x-5)(x+9)(x-4) \leq 0$ is _____.

**6.** $|2x-1| > 3-x, x>0 \Rightarrow x >$
(A) 3/4 (B) 4/3
(C) 2/3 (D) 5/3

**7.** The graph given below is best represented by which of the following?




(A) $y=|x|-3$ (B) $y=|x-3|$
(C) $y=|x+3|$ (D) $y=|x|+3$

**8.** If $\left|\frac{3x-6}{2}\right| \leq 12$, then
(A) $-2 \leq x \leq 6$ (B) $-1 \leq x \leq 6$
(C) $-3 \leq x \leq 5$ (D) $-6 \leq x \leq 10$

**9.** Which of the following set of values of $x$ satisfies $|8x-5| = 3$?
(A) $\{1, 4\}$ (B) $\{1, 2\}$
(C) $\left\{\frac{1}{4}, 1\right\}$ (D) $\left\{\frac{1}{4}, \frac{1}{2}\right\}$

**10.** Solve: $x^2 - 5x + 6 < 0$.
(A) $-3 < x < -2$ (B) $-3 < x < 2$
(C) $2 < x < 3$ (D) $-2 < x < 3$

**11.** If $x^2 - x - 6 < 0$, then which of the following always holds true?
(A) $x \in (2, 3)$ (B) $x \in (-3, 2)$
(C) $x \in (-2, 3)$ (D) $x \in (-3, -2)$

**12.** If $x^2 - 4x + 10 > -4$, then which of the following represent the solution set for $x$?
(A) $\{\}$ (B) $R$
(C) $(1, \infty)$ (D) $(-\infty, 1]$

**13.** Solve for real $x$: $6x^2 + x - 12 \geq 0$.
(A) $R$ (B) $R - (-3/2, 4/3)$
(C) $R - (-2/3, 3/4)$ (D) $(-3/2, 4/3)$

**14.** If $3x-8 \leq 5x-2$ and $6x-4 \leq 4x+1$, then
(A) $x \in [5/2, 3]$ (B) $x \in (-3, 5/2)$
(C) $x \in [-3, 5/2]$ (D) $x \in (5/2, 3)$

**15.** If $3x+14 > 5x+24$, then
(A) $x>5$ (B) $x<5$
(C) $x>-5$ (D) $x<-5$

**16.** Solve: $\frac{x-3}{x+5} \leq 4$.
(A) $(5, \infty)$ (B) $\left(\frac{-23}{3}, \infty\right)$
(C) $\left(-\infty, -\frac{23}{3}\right] \cup (-5, \infty)$ (D) $\left(-\infty, \frac{-23}{3}\right)$

**17.** What is the difference between the largest and the smallest integer that satisfies the inequality $\left|\frac{12}{x-15}\right| > 5; x \neq 15$ ?
_____

**18.** If $\frac{x}{x-2} - \frac{1}{2x} > 0$, then the range of '$x$' is
(A) $(0, 2)$ (B) $(-\infty, 0) \cup (2, \infty)$
(C) $(0, -2)$ (D) $(-2, 2)$

**19.** (i) What is the maximum value of $14 - |2x-5|$?
(ii) At what value of $x$ does the expression $13 + |3x-7|$ have its minimum value?
(A) 3/7 (B) 7/3
(C) 13 (D) 1

**20.** The solution of the inequality $\frac{5x^2+8x-21}{x^2-8x+15} < 0$ is
(A) $\left(\frac{7}{5}, 5\right)$ (B) $\left(-\infty, \frac{7}{5}\right)$
(C) $\left(-3, \frac{7}{5}\right) \cup (3, 5)$ (D) $(3, 5)$

**21.** The range of $f(x) = |x| + |x-5| + |x+7|$ is
(A) $[-5, 7]$ (B) $[-7, 5]$
(C) $[-7, \infty)$ (D) $[12, \infty)$

**22.** If $|x| \geq |6 - x^2|$, then find the range of $x$ that satisfies the given equations.
(A) $[-3, 2] \cup [3, \infty)$ (B) $(-\infty, -3] \cup [3, \infty)$
(C) $(-\infty, -3] \cup [2, 3]$ (D) $[-3, -2] \cup [2, 3]$

**23.** If $|x-3| \leq 9$ and $|4-x| < 5$, then
(A) $-6 < x < 9$ (B) $-1 < x < 12$
(C) $-1 < x < 9$ (D) $-1 \leq x < 10$

**24.** For two real numbers $a$ and $b$, let $p = |a| + |b|$, $q = |a| - |b|$, and $r = |a-b|$. Which of the following is true?
(A) $r \leq p \leq q$ (B) $p \leq q \leq r$
(C) $q \leq r \leq p$ (D) $q \leq p \leq r$

**25.** What are the real values of $x$ that satisfy the inequation $2x^2 + 10x + 17 < 0$?

(A) $(-4, -1)$ (B) $(4, \infty)$
(C) $\{\}$ (D) $(-\infty, 0)$

**26.** If $\frac{x^2-2x-8}{3x^2+7x+2} < 0$, then the solution set for the inequality is
(A) $R$ (B) $R - [-1/3, 4]$
(C) $(-1/3, 4)$ (D) None of these

**27.** If $\frac{x-3}{x+3} > 3$ with $x \neq -3$, then which of the following always holds true?
(A) $x \in (-6, -3)$ (B) $x \in (-3, 3)$
(C) $x \in R - [-3, 3]$ (D) $x \in [-6, -2]$

**28.** Solve: $|x^2 - 9| = 0$.
(A) $[-3, 3]$ (B) $(-3, 3)$
(C) $\{-3, 3\}$ (D) None of these

**29.** Solve: $|2x + 3| > 2$.
(A) $(-5/2, -1/2)$
(B) $(-\infty, -5/2)$
(C) $(-\infty, -5/2) \cup (-1/2, \infty)$
(D) $(-1, \infty)$

**30.** If $|x + 5| < 7$, then solution set for the inequality is
(A) $(-12, 2)$ (B) $(-5, 2)$
(C) $(-12, -5)$ (D) $(-12, -5) \cup (-5, 2)$

**31.** The solution of inequation $|x| - 4 < 2 + x$ is
(A) $[-1, 3]$ (B) $(-\infty, -1]$
(C) $(-\infty, \infty)$ (D) $(-3, \infty)$

**32.** For what value of $x (x \in R)$ is the minimum value of the function $|x + 2| + |x - 1| + |x + 5|$ obtained?_____

**33.** For three positive numbers $p, q, r$,
$\frac{(4p^2 + p + 4)(11q^2 + q + 11)(13r^2 + r + 13)}{27\,pqr}$ can be
(A) 207 (B) 253
(C) 256.5 (D) All the above

**34.** Max min $(x - 2, x + 2)$, min $(x - 1, x - 3)$ is equal to
(A) $x + 2$ (B) $x - 1$
(C) $x - 3$ (D) $x - 2$

**35.** If $3x + 5y = 18$ and the expression $x^4y^2$ has its maximum value, then the value of $y$ is $(x, y > 0)$.
(A) 3 (B) 1/4
(C) 6/5 (D) 5/6

**36.** For which of the following range of values of $x$ is $x^3 + 1$ greater than $x^2 + x$?
(A) $(-1, 1) \cup (1, \infty)$ (B) $(-2, \infty)$
(C) $(-\infty, 1)$ (D) $(-2, 1)$

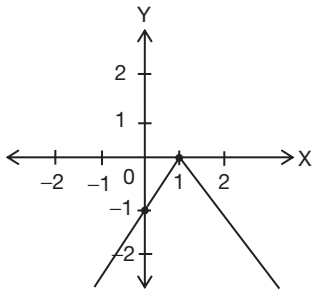
**37.** If $\frac{5x}{x+2} - \frac{1}{2x} < 0$, then the range of $x$ is
(A) $(-\infty, -2) \cup (1/2, \infty)$
(B) $(-2, -2/5) \cup (1/2, \infty)$
(C) $(-2, -2/5) \cup (0, 1/2)$
(D) $(-2/5, 0) \cup (1/2, \infty)$

**38.** Find the number of integers satisfying the inequality $(x^2 + 5x - 6)(x^2 - 6x - 7) < 0$_____.

**39.** For a positive integer '$n$', $2 - \frac{1}{n} < x \leq 4 + \frac{1}{n}$. The range of $x$ is
(A) $(1, 5]$ (B) $[3/2, 9/2]$
(C) $[0, 4]$ (D) $(2, 4)$

**40.** If $\left(\frac{1}{2x-1}\right) > \frac{3}{x}$ where $x > 0$, then find the range of $x$.
(A) $(0, 5)$ (B) $(1, 3)$
(C) $(2, 5)$ (D) $(1/2, 3/5)$

**41.** Which of the following equations best describes the graph given below?




(A) $y = |x - 1|$ (B) $y = -|x - 1|$
(C) $x = |y + 1|$ (D) $x = -|y + 1|$

**42.** (i) The number of distinct solutions of the equation $x^2 - 13|x| + 36 = 0$ is _____.
(ii) The number of distinct solutions of the equation $||x + 5| - 9| = 7$ is _____.

**43.** Solve for $x$: $|x + 1| + |x - 3| = 6$.
(A) $\{-2\}$ (B) $\{4, 3\}$
(C) $\{-2, 4\}$ (D) $\{-1, -2, 4\}$

**44.** If $x^2 - 12x + 42 > -5$, then the solution set for $x$ is
(A) $(11, \infty)$ (B) R
(C) $[6, \infty)$ (D) None of these

**45.** If $\frac{x^2-5x+10}{3x^2-5x-22} \leq 1$, then the solution set for the inequality is
(A) $(4, \infty)$
(B) $(-\infty, -4) \cup (2, \infty)$
(C) $(-\infty, -4) \cup \left(-2, \frac{11}{3}\right) \cup (4, \infty)$
(D) $(-\infty, -4) \cup (4, \infty)$.

## EXERCISE-3

***Directions for questions 1 to 45*:** For the multiple choice questions, select the correct alternative from the given choices. For the non–multiple choice questions, write your answer in the space provided.

**1.** If $A = \{x : x|x-6| = 7\}$ and $x \in N$, then A =
(A) { } (B) {1, 7}
(C) {1} (D) {7}

**2.** The number of distinct solutions of the equation $|x - |4x + 1|| = 7$ is _____.

**3.** The number of distinct solutions of the equation $x^2 - 9|x| + 18 = 0$ is _____.

**4.** The number of distinct solutions of the equation $f(x) = 0$, where $f(x) = |x+2| + |x-2|$; $x \in R$ is _____.

**5.** $|3 - 5x| > 5 - x$, $x > 0$, $\Rightarrow x >$
(A) 3/4 (B) 5/3
(C) 2/7 (D) 4/3

**6.** Find the range of the real values of $x$ satisfying the inequalities $3x + 4 \le 6$ and $4x + 3 \ge 6$.
(A) $\left[\frac{3}{4}, \frac{3}{2}\right]$ (B) $(-\infty, 2/3]$
(C) $[3/4, \infty)$ (D) $\phi$

**7.** What are the values of '$x$' that satisfy the inequality $4x^2 + 5x - 9 \le 0$?
(A) $[-9/4, -1]$ (B) $[-5/2, 1]$
(C) $[-1, 9/4]$ (D) $[-9/4, 1]$

**8.** What are the real values of '$x$' that satisfy the inequality $x^2 - 9x - 10 > 0$?
(A) $(-1, 9)$ (B) $(-1, 10)$
(C) $(-\infty, -1) \cup (10, \infty)$ (D) $(-\infty, 1) \cup (9, \infty)$

**9.** If $\frac{x-7}{x+5} > 3$ and $x \ne -5$, then $x \in$
(A) $(-\infty, -5)$ (B) $(-11, -5)$
(C) $(-11, -8)$ (D) $(5, 11)$

**10.** The solution set for the inequality $\frac{2x^2+5x-3}{x^2-3x+2} \le 0$ is
(A) $(-4, -3) \cup (2, \infty)$ (B) $R - [-1/2, 1]$
(C) $[-3, 1/2] \cup (1, 2)$ (D) $(-3, 2)$

**11.** If $\frac{3x}{x-3} - \frac{1}{x} < 0$, then the range of '$x$' is
(A) $(3, \infty)$ (B) $(-\infty, 0)$
(C) $(0, 3)$ (D) $(-\infty, 3)$

**12.** Let $g(y) = \min(-y + 3, 2 + 7y)$. Find the maximum possible value of $g(y)$.
(A) $\frac{21}{8}$ (B) $\frac{25}{8}$
(C) $\frac{23}{8}$ (D) $\frac{5}{2}$

**13.** The minimum value of the function $|x-2| + |x-1| + |x+11|$ where $x$ is a real number is _____.

**14.** If $f(x) = \min(x-3, x+2)$ and $g(x) = \max(x-2, x-3)$ and if $h(x) = f(x) + g(x)$, then find the range of $x$ for which $h(x) \le 7$.
(A) $(-\infty, 0]$ (B) $(-\infty, 6]$
(C) $[0, 6]$ (D) $[6, \infty)$

**15.** Find the complete range of values of $x$ for which, $(x^2 - x + 1)^x < 1$.
(A) $(-\infty, 0)$ (B) $(0, 1)$
(C) $(-\infty, 0) \cup (0, 1)$ (D) $(0, 1) \cup (1, \infty)$

**16.** For what values of $x$, does the inequality $|x-5| > x^2 - 4x + 1$ hold true?
(A) (2, 3) (B) (1, 4)
(C) (–1, 4) (D) (0, 3)

**17.** Which of the following is true for $a, b, c > 0$?
(A) $a + b + c \le \frac{bc}{a} + \frac{ca}{b} + \frac{ab}{c}$, where $a, b, c > 0$
(B) $\left(\frac{a}{bc}\right)^{\frac{1}{2}} + \left(\frac{b}{ca}\right)^{\frac{1}{2}} + \left(\frac{c}{ab}\right)^{\frac{1}{2}} \ge a^{-\frac{1}{2}} + b^{-\frac{1}{2}} + c^{-\frac{1}{2}}$,
(C) $a^4 + b^4 + c^4 \ge a^2.b^2 + b^2.c^2 + c^2.a^2$
(D) All the above

**18.** For which of the following range of the values of $x$ is $x^3 - 8 < x^2 - 2x$?
(A) $(-2, 2)$ (B) $(-2, \infty)$
(C) $(2, \infty)$ (D) $(-\infty, 2)$

**19.** If $x$, $y$ and $z$ are positive real numbers such that $x^2 + y^2 + z^2 = 3$, then which of the following is always true?
(A) $xy + yz + zx \le 3$ (B) $xy + yz + zx \ge 1$
(C) $xy + yz + zx \le 2$ (C) $xy + yz + zx \ge 3$

**20.** If $a$, $b$ and $c$ are the lengths of the sides of $a$ triangle then the range of the values of $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$ is
(A) [1, 2) (B) [3/2, 3]
(C) [3/2, 2) (D) [1, 3)

**21.** For what values of '$k$' is the expression $x^2 - 3kx + 2k^2 + 4$ greater than zero for all $x \in R$?
(A) $(-\infty, -4)$ (B) $(4, \infty)$
(C) $(-4, 4)$ (D) $(-\infty, -4) \cup (4, \infty)$

**22.** Find the values of $y$ satisfying the inequation $y^{\frac{2}{3}} - y^{\frac{1}{3}} - 72 \le 0$.

Difficulty Level-3: Advanced

(A) $512 \le y \le 729$ (B) $-512 \le y \le 729$
(C) $-729 \le y \le 512$ (D) $-512 \le y \le 512$

**23.** Find the number of integers satisfying the inequality $(x^2 + x - 6)\ (x^2 - x - 20) < 0$_____.

**24.** If $\frac{x-5}{x+2} < 2$, $(x \ne -2)$, then $x \in$
(A) $(-9, -2)$ (B) $(2, 9)$
(C) $(-\infty, -9) \cup (-2, \infty)$ (D) $(-\infty, 2) \cup (9, \infty)$

**25.** For which of the following range of values of $x$ is $x^3 - 1$ greater than $x^2 - x$?
(A) $(-3, \infty)$ (B) $(0, \infty)$
(C) $(-\infty, 1)$ (D) $(1, \infty)$

**26.** The number of distinct solutions of the equation $|x - |6x + 1|| = 9$ is_____.

**27.** The number of integral values that satisfy the inequation $\frac{7x}{5x+4} - \frac{2}{x} < 0$ is _____.

**28.** Solve $(x^2 + x + 1)^x > 1$.
(A) $(-\infty, 1)$ (B) $(-1, \infty)$
(C) $(0, \infty)$ (D) $(1, \infty)$

**29.** For the positive numbers, $p, q, r$,
$\frac{(2p^2 + p + 2)(7q^2 + q + 7)(6r^2 + r + 6)}{5pqr}$ can be
(A) 225 (B) 200
(C) 215 (D) All of the above

**30.** If $x$ and $y$ are positive real numbers such that $x + y = 5$, then the maximum value of $x^3y^2$ is _____.

**31.** If $a, b, c$ are positive real numbers, then which of the following is true?
(A) $a + b + c \le ab/c + bc/a + ac/b$
(B) If $a > b$, then $a^b . b^a > a^a . b^b$
(C) $a^2 + b^2 + c^2 > 2(ab + bc + ca)$
(D) Both (A) and (B)

**32.** How many solutions are possible for the inequality, $|x - 3| + |x - 4| < 1$?
(A) 3 (B) 4
(C) 0 (D) Infinitely many

**33.** If $x, y, z$ are positive real numbers, then which one of the following holds?
(A) $x^2 + y^2 + z^2 \ge xy + yz + zx$
(B) $(x + y)\ (y + z)\ (z + x) \ge 8xyz$
(C) $1/x + 1/y > 2/(x + y)$
(D) (A), (B) and (C)

**34.** The range of values of x such that $|2x - 1| - |x + 5| + 3 < 0$ is
(A) $\left(-\frac{1}{3}, \frac{1}{2}\right) \cup \left(\frac{1}{2}, 3\right)$ (B) $\left(-\infty, \frac{-1}{3}\right)$
(C) $(3, \infty)$ (D) $\left(-\frac{1}{3}, 3\right)$

**35.** If $a$, $b$ and $c$ are positive real numbers, then the minimum value of $\frac{a^4 + b^4}{a^2b^2} + \frac{b^4 + c^4}{b^2c^2} + \frac{c^4 + a^4}{c^2a^2}$ is _____.

**36.** If $|b| \le 1$ and $x = -|a|b$, then which of the following is/are always true?
(A) $a - xb \ge 0$ (B) $a + xb \ge 0$
(C) Both (A) and (B) (D) Neither (A) nor (B)

**37.** Find the values of $y$ satisfying the inequation
$$\left(y^{\frac{1}{3}} - 1\right)^2 + y^{\frac{1}{3}} - 13 \le 0.$$
(A) $-3 \le y \le 4$ (B) $27 \le y \le 64$
(C) $-27 \le y \le 64$ (D) $-64 \le y \le -27$

**38.** If $x$ is $a$ positive integer satisfying $\frac{5x + 124}{5(5x^2 + 1)} < \frac{1}{50}$, then the least value of x is _____.

**39.** The function $f(x) = |x - 3| + |3.5 - x| + |4.6 - x|$, where $x$ is $a$ real number, has a minimum value of M. When $f(x) = M$, $x =$ _____.

**40.** If $2 \le x \le 8$ and $4 \le y \le 12$, then the minimum value of $\frac{x + y}{x}$ is
(A) 4 (B) 5
(C) $\frac{1}{2}$ (D) None of these

**41.** If $x$ and $y$ are positive real numbers $3x + 4y = 20$, then maximum value of $x^3y^2$ is
(A) 326 (B) 280
(C) 512 (D) 256

**42.** If $x$ and $y$ are positive real numbers and $x^4y^2 = 1024$, then the least value of $x + y$ is
(A) 6 (B) 12
(C) 9 (D) 5

**43.** If $2x^3y^2 = (15)^3$, $y > 0$ and $2x + 5y$ has its minimum value, then the value of x is
(A) 2/15 (B) 15
(C) 2 (D) 15/2

**44.** Max [min $(x - 1, x + 2)$, max $(x + 3, x + 5)$] is equal to
(A) $x + 3$ (B) $x + 5$
(C) $x + 2$ (D) $x - 1$

**45.** The minimum value of the expression
$\left(\frac{a}{b} - \frac{b}{c}\right)^2 + \left(\frac{c}{a} - \frac{b}{a}\right)^2 + \left(\frac{c}{b} - \frac{a}{c}\right)^2 + 2\left(\frac{a}{c} + \frac{bc}{a^2} + \frac{a}{b}\right)$, where $a, b, c$ are positive real numbers is _____.

## EXERCISE-4

***Directions for questions 1 to 25:*** For the multiple choice questions, select the correct alternative from the given choices. For the non–multiple choice questions, write your answer in the space provided.

**1.** If $x \in z^+$ and $(x+1)^x < x^x + 1$, then which of the following best describes $x$?
(A) No such $x$ exists (B) $x \geq 1$
(C) $x \geq 3$ (D) $x \geq 4$

**2.** If $x = 10/(a+b)$ and $y = 10(a+b)$ and if $a, b > 0$, then $x + y$ is
(A) always less than 10.
(B) always greater than or equal to 20.
(C) always between 10 and 20.
(D) always greater than or equal to 30.

**3.** If $a$, $b$ and $c$ are three distinct positive real numbers, then $\frac{a^2(b+c)+b^2(c+a)+c^2(a+b)}{abc}$ is always
(A) greater than 9. (B) greater than 7.
(C) greater than 6. (D) greater than 8.

**4.** Find the set of all values of $x$ for which $\frac{2x}{2x^2+5x+2} > \frac{1}{x+1}$.
(A) $(-2, -1/2)$
(B) $(-2, -2/3)$
(C) $(-2, -1) \cup (-2/3, -1/2)$
(D) None of these

**5.** If $x \in R$, the greatest value that $\frac{x^4}{1+x^8}$ can take is
(A) 2/5 (B) 1/3
(C) 1/2 (D) 3/4

**6.** If $p^4 q^4 + q^4 r^4 + r^4 p^4 \geq k$, then $k =$
(A) $4(p^2 + q^2 + r^2)$ (B) $2 p^2 q^2 r^2$
(C) $pqr(p^2 + q^2 + r^2)$ (D) $p^2 q^2 r^2 (p^2 + q^2 + r^2)$

**7.** If $x < -2$ and $3^{|x+2|} - 3^x = |3^x - 1| + 2$, the value of $x$ is _____.

**8.** The minimum value of the expression $\frac{(a+b+c)^3}{(a+b+c)(a+c-b)(b+c-a)}$, where $a$, $b$ and $c$ are the sides of a triangle is _____.

**9.** If $p+q+r+s = 10$, where $p$, $q$, $r$ and $s$ are positive numbers and $x = (p+q)(r+s)$, which of the following is true?
(A) $0 \leq x \leq 5$ (B) $5 \leq x \leq 25$
(C) $0 < x \leq 25$ (D) $3 \leq x \leq 25$

**10.** If $p_1, p_2, p_3, \ldots, p_n$ are positive real numbers and $(p_1 + p_1^2 + p_1^3)(p_2 + p_2^2 + p_2^3) \ldots (p_n + p_n^2 + p_n^3) \geq k (p_1 p_2 \ldots p_n)^2$, then the least value of $k$ is
(A) 3 (B) $3^n$
(C) $\frac{1}{3}$ (D) $\frac{1}{3^n}$

**11.** If $|p| < 1$ and $|q| < 1$, then which of the following is/are possible?
(A) $|1 + pq| = |p + q|$
(B) $|1 + pq| > |p + q|$
(C) $|1 - pq| > |p + q|$
(D) More than one of the above

**12.** How many negative integers satisfy $|x + 4| + |x - 7| < 13$? _____

**13.** If $p > 0$, $q > 0$ and $pq = 1$, then find the minimum value of $(p+2)(q+2)$ _____.

**14.** $\text{Max}[\min(x+6, x-3), \min(x+5, x-7)] =$
(A) $x+6$ (B) $x+5$
(C) $x-3$ (D) $x-7$

**15.** If $a^2 + b^2 + c^2 = l^2 + m^2 + n^2 = 3$, then find the maximum value of $al + bm + cn$.
(A) 3 (B) 3.5
(C) 4 (D) 9

**16.** How many integral values of $x$ satisfy the inequality $14 < |x-5| + |x+3| + |x+7| < 25$? _____

**17.** Find the greatest value of $abc$, given that $ab + bc + ca = 15$, where, $a$, $b$ and $c$ are positive.
(A) 125 (B) $5\sqrt{5}$
(C) 625 (D) $25\sqrt{5}$

**18.** If $10 \leq x \leq 20$ and $2y - x = 2$, then the maximum value of $\frac{x}{x+y}$ is
(A) $\frac{20}{31}$ (B) $\frac{5}{8}$
(C) $\frac{5}{6}$ (D) $\frac{20}{11}$

**19.** Find the minimum value of $\left(\frac{a^2}{l^2} + \frac{b^2}{m^2} + \frac{c^2}{n^2}\right)\left(\frac{m^2}{b^2} + \frac{n^2}{c^2} + \frac{l^2}{a^2}\right)$ where $a, b, c, l, m, n$ are real numbers _____.

**20.** Three distinct positive numbers $p$, $q$ and $r$ are such that the sum of any two is greater than the third. If $p + q + r = 2s$ and $E = \frac{1}{-p+q+r} + \frac{1}{p-q+r} + \frac{1}{p+q-r}$, which of the following is true?
(A) $E > 9/s$ (B) $E < 9/s$
(C) $E > 9/(2s)$ (D) $E < 9/(2s)$

**21.** How many solutions does the inequality $7|x| \leq 6.93x$ have? _____

**22.** If $3x + 4y + 5z = 27$ and $x$, $y$, $z$ are positive real numbers, then the maximum value of $x^2 y^3 z^4$ is

(A) $\frac{(2^8)3^{10}}{5^4}$ (B) $\frac{(2^4)3^8}{5^4}$

(C) $\frac{(2^4)3^{10}}{5^4}$ (D) $\frac{(2^8)\,3^8}{5^4}$

**23.** If $x$ is real, $(3x^2 - x - 24)(3x^2 - 14x - 24) \geq 0$ and $3x^2 - x - 24 \leq 0$, then what is the maximum value that $x$ can take? _____

**24.** The range of values of $x$ such that $|3x - 1| - |x + 2| + 5 < 0$ is

(A) $\left(-1, \frac{1}{3}\right)$ (B) $\left(-2, \frac{1}{3}\right)$

(C) $(\infty, 2)$ (D) None of these

**25.** If the roots of $x + \frac{4}{x} = m$ are real, then the least positive value of $m$ is _____.

## EXERCISE-5 Based on Data Sufficiency

***Directions for questions 1 to 20*:** Each question is followed by two Statements I and II. Answer each question based on the following directions:

Choose (A) If the question can be answered by using one of the statements alone but cannot be answered by using the other statement alone.
Choose (B) If the question can be answered by using either statement alone.
Choose (C) If the question can be answered by using both the statements together but cannot be answered by using either statement alone.
Choose (D) If the question cannot be answered even by using both the statements together.

**1.** Is $x < y$?
I. $x^2 > y^3$
II. $y^2 > x^3$

**2.** Is $x > 1$?
I. $x^2 > x^3$
II. $x^3 > x$

**3.** What is the maximum value of $|x+6z|$?
I. $|x-2y| \leq 5$
II. $|y+3z| \leq 9$

**4.** Is $|3x+5| < 5x+1$?
I. $(x+5)(x+10)(x-6) > 0$
II. $8x^2 - 10x - 12 > 0$

**5.** What is the value of $\left|\frac{x+5}{x+7}\right|$?
I. $x^2 + 7x - 18 = 0$
II. $3x^2 + 46x + 171 = 0$

**6.** Is $|x+2y-2z| > 9$?
I. $|x+4y| < 3$
II. $|y+z| > 6$

**7.** Is $x < 0$?
I. $x > x^2$
II. $x^2 > x^4$

**8.** If $x > 0$, is $x < 1$?
I. $x > \sqrt{x}$
II. $x > \sqrt[3]{x}$

**9.** If $x > 0$, is $x^2 > 1$?
I. $x^4 > 1$
II. $\sqrt{x} > \sqrt[4]{x}$

**10.** Is $13 - |x+4| + |x+1| > 14$?
I. $x$ is any integer.
II. $-100 \leq x \leq -4$

**11.** Is $(a^2 + b^2 + c^2)\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right) \geq 9$?
I. $a$, $b$ and $c$ are non–zero integers.
II. $a$, $b$ and $c$ are positive real numbers.

**12.** What is the minimum value of $\frac{|y|}{|x| + |y|}$?
I. $-4 \leq x \leq 2$
II. $2 \leq y \leq 5$

**13.** What is the value of $k$?
I. The number of integer values of $x$ which satisfy the inequation $|x - 8| \leq k$ is 7.
II. $k$ is an integer.

**14.** What is the value of $|x + 2|$?
I. $x^2 - 3x + 2 = 0$
II. $x^2 + 4x - 5 = 0$

**15.** Is $|x - 2y| < 4$?
I. $|x| < 2$
II. $|y| > 3$

**16.** What is the minimum value of $|x + 2| + |x - 3|$?
I. $-2 \leq x \leq 3$
II. $x$ is a real number.

**17.** What is the value of $|x - y|$?
I. $(x-3)^2 + (y-4)^2 = 0$
II. $|x - 3| + |y - 4| = 0$

**18.** What is the minimum value of $x + \frac{1}{x}$?
I. $x$ is a positive integer.
II. $x$ is a positive real number.

**19.** Is $|x-1|<4$?
I. $x^2-2x-15<0$
II. $x(x-4)>0$

**20.** Is $a>b$?
I. $|a|>|b|$
II. $a/b<1$

## ANSWER KEYS

### Exercise-1

1. (B) 2. (B) 3. (D) 4. (D) 5. (C) 6. (A) 7. (C) 8. (B) 9. (D) 10. (C) 11. (A) 12. (D) 13. (B) 14. (D) 15. (B) 16. 1 17. 2 18. 11 19. (C) 20. 10 21. (B) 22. (C) 23. (A) 24. (D) 25. (D) 26. (C) 27. (C) 28. (C) 29. (A) 30. (C) 31. (B) 32. (A) 33. (C) 34. (B) 35. (C) 36. (C) 37. (A) 38. (A) 39. (C) 40. (D) 41. 2 42. (A) 43. (A) 44. (C) 45. (B)

### Exercise-2

1. (B) 2. (A) 3. (C) 4. (D) 5. 2 6. (B) 7. (B) 8. (D) 9. (C) 10. (C) 11. (C) 12. (B) 13. (B) 14. (C) 15. (D) 16. (C) 17. 4 18. (B) 19. (i) 14 (ii) (B) 20. (C) 21. (D) 22. (D) 23. (C) 24. (C) 25. (C) 26. (C) 27. (A) 28. (C) 29. (C) 30. (A) 31. (D) 32. –2 33. (D) 34. (D) 35. (C) 36. (A) 37. (C) 38. 9 39. (A) 40. (D) 41. (B) 42. (i) 4 (ii) 4 43. (C) 44. (B) 45. (C)

### Exercise-3

1. (D) 2. 2 3. 4 4. 0 5. (D) 6. (D) 7. (D) 8. (C) 9. (B) 10. (C) 11. (C) 12. (C) 13. 13 14. (B) 15. (C) 16. (C) 17. (D) 18. (D) 19. (A) 20. (C) 21. (C) 22. (B) 23. 2 24. (C) 25. (D) 26. 2 27. 1 28. (B) 29. (D) 30. 108 31. (A) 32. (C) 33. (D) 34. (D) 35. 6 36. (D) 37. (C) 38. 22 39. 3.5 40. (D) 41. (D) 42. (A) 43. (D) 44. (B) 45. 6

### Exercise-4

1. (C) 2. (B) 3. (C) 4. (C) 5. (C) 6. (D) 7. –3 8. 27 9. (C) 10. (B) 11. (D) 12. 4 13. (9) 14. (C) 15. (A) 16. 11 17. (B) 18. (A) 19. 9 20. (C) 21. 1 22. (C) 23. 3 24. (D) 25. 4

### Exercise-5

1. (D) 2. (A) 3. (C) 4. (D) 5. (A) 6. (C) 7. (A) 8. (B) 9. (B) 10. (A) 11. (B) 12. (C) 13. (C) 14. (A) 15. (C) 16. (B) 17. (B) 18. (B) 19. (A) 20. (D)

## SOLUTIONS

## EXERCISE-1

**1.** The set of all numbers excluding $-2$ and including 3 is denoted by $(-2, 3]$.

**2.** $x \in [1, 7] \Rightarrow 1 \le x \le 7$

**3.** $x < 3 \Rightarrow x < (-\infty, 3)$

**4.** $2 < x \le 5$ can also be represented as $(2, 5]$.

**5.** $x \le -2 \Rightarrow x \in (-\infty, -2]$
$x \ge 5 \Rightarrow x \in [5, \infty)$
$\therefore x \le -2$ or $x \ge 5 \Rightarrow (-\infty, -2] \cup [5, \infty)$

**6.** Choice (A) is a standard result.
Consider choice (B), $4 > 2$ and $-3 > -4$
But $4(-3) < 2(-4)$
Consider choice (C), $4 > 2$ and $-2 > -5$
But $4 + 2 < 2 + 5$

**7.** $(-5, 2]$ (By definition)

**8.** When $a = 2$ and $b = 3$, $a - b = 2 - 3 = -1$
$\therefore a - b$ need not always be positive.

**9.** $(2, 5)$ (By definition)

**10.** We know that when $a > 0$, $b > 0$ and $a > b$, then $\frac{a}{b} > 1$.
$\therefore$ Choice (C) is always true.
When $a = -4$ and $b = -2$, $\frac{a}{b} = \frac{-4}{-2} = 2 > 1$. But $-4 < -2$
$\therefore$ Choice (A) is not always true.
When $a = -2$ and $b = -4$, $\frac{a}{b} = \frac{-2}{-4} = \frac{1}{2} < 1$
But $-2 < -4$
$\therefore$ Choice (B) is not always true.

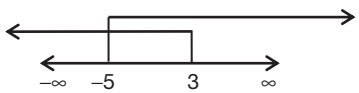
**11.** $x + 3 \ge 7 \Rightarrow x \ge 4 \Rightarrow x \in [4, \infty)$

**12.** $-2x - 6 \ge 0$
$-2x \ge 6$
$x \le -3$

**13.** $5 - 2x \le 6 \Rightarrow -2x \le 6 - 5 \Rightarrow -2x \le 1 \Rightarrow x \ge \frac{-1}{2}$
$\therefore x \in \left[\frac{-1}{2}, \infty\right)$

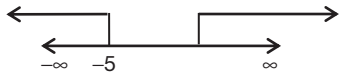
**14.** If $a < b$, then $a^2 < b^2$ or $a^2 > b^2$.
Choice (A) is not always true.
Similarly, choice (B) and choice (C) are not always true.

**15.** $(x - 3)(x + 5) \le 0$
$\Rightarrow x - 3 \le 0$ and $x + 5 \ge 0$




$\Rightarrow x \in [-5, 3]$

or $x - 3 \ge 0$ and $x + 5 \le 0$




$\Rightarrow$ No solution
$\therefore$ The solutions set of $(x - 3)(x + 5) \le 0$ if $x \in [-5, 3]$.

**16.** Given, $|x + 2| = 2x - 1$
**Case (i):** Let $x < -2$
Then $|x + 2| = -(x + 2)$
$\therefore |x + 2| = 2x - 1$
$\Rightarrow -(x + 2) = 2x - 1$
$\Rightarrow -x - 2 = 2x - 1$
$\Rightarrow 3x = -1 \Rightarrow x = \frac{-1}{3}$
But, $x = \frac{-1}{3}$ does not satisfy the given equation.
**Case (ii):** Let $x \ge -2$
Then, $|x + 2| = x + 2$
$\therefore |x + 2| = 2x - 1 \Rightarrow x + 2 = 2x - 1$
$\Rightarrow x = 3$
Clearly, $x = 3$ satisfies the given equation.
Hence, the required number of solutions is 1.
**Alternative Method:**
If $x \le -2$, $|x + 2| \ge 0$, while $2x - 1 < 0$, then there are no solutions.
If $x > -2$, then $|x + 2| = x + 2$.
$\therefore x + 2 = 2x - 1 \Rightarrow x = 3$

**17.** The expression $5 - |2 - x|$ will have a maximum value when $|2 - x| = 0$, i.e., when $x = 2$.

**18.** Given, $|x + 3| \le 5$
$\Rightarrow -5 \le x + 3 \le 5 \Rightarrow -5 - 3 \le x \le 5 - 3$
$\Rightarrow -8 \le x \le 2$
$\therefore$ The integers which satisfy the given inequation are $-8$, $-7$, $-6$, $-5$, $-4$, $-3$, $-2$, $-1$, 0, 1 and 2.
Hence, the required number of integers is 11.

**19.** We have $x^2 - 3x + 2 = (x - 1)(x - 2)$, which is not always positive.
$x^2 + 2x - 35 = (x + 7)(x - 5)$, which is not always positive.
$x^2 + 4x + 5 = (x + 2)^2 + 1$, which is always positive.
$x^2 - x - 6 = (x - 3)(x + 2)$, which is not always positive.

**20.** The minimum value of $|x|$ is 0.
$\therefore$ The minimum value of $10 + |x|$ is $10 + 0 = 10$.

**21.** Since the modulus of a number is always non-negative, the given inequality is true for all values of $x$. $x \in R$.

**22.** $x^2 - 4x + 8 = x^2 - 4x + 4 + 4 = (x - 2)^2 + 4$, which is always positive.

**23.** $|x| < a \Rightarrow -a < x < a$
$|x| < 4 \Rightarrow -4 < x < 4$
The integral values of $x$ are $\{-3, -2, -1, 0, 1, 2, 3\}$, i.e., there are 7 integral values.

**24.** $x^2 - 7x - 18 \le 0 \Rightarrow (x-9)\ (x+2) \le 0$
$\therefore -2 \le x \le 9$
The number of integral values in the interval $[-2, 9]$ is 12.

**25.** Given $x$ is negative.
Let $x = -a$ when $a$ is positive.
$x + \frac{1}{x} = -a + \frac{1}{-a} \Rightarrow -\left(a + \frac{1}{a}\right)$
We know that the minimum value of $a + \frac{1}{a}$ when $a > 0$ is 2.
$\therefore$ The maximum value of $-\left(a + \frac{1}{a}\right)$ is $-2$.
Hence, the maximum value of $x + \frac{1}{x}$ is $-2$.

**26.** $-5 \le x < 7$ means $x$ can be any real number lying between $-5$ and 7 including $-5$ but not 7. One way to represent the solution set is $[-5, 7)$.

**27.** We know that $a > b \Rightarrow a^2 > b^2$ if $a$ and $b$ are positive real numbers.

**28.** $2x - 5 \ge 7x + 10$
$\Rightarrow 2x - 7x \ge 10 + 5$
$\Rightarrow -5x \ge 15$
$x \le -3$ The solution set is $(-\infty, -3]$.

**29.** For every real value of $x$, $|x+2| \ge 0$.
So, no real value of $x$ satisfies the inequation $|x+2| < -3$.
$\therefore$ There is no solution for the given inequation.

**30.** For every real value of $x$, $|x-1| \ge 0 > -2$.
$\therefore$ The required solution set is $(-\infty, \infty)$.

**31.** Since $|3 - 2x|$ is always positive, $6 + |3 - 2x| \ge 6$.
The expression is equal to 6 when $|3 - 2x| = 0$.
$3 = 2x \Rightarrow x = \frac{3}{2}$

**32.** $|3x - 5| < -7$
Since the modulus of a real number is always non-negative, it cannot be less than $-7$.
The solution set is $\{\ \}$.

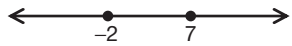
**33.** Since $|x|$ is always positive, $20 - |x| \le 20$.
$\therefore$ The maximum values is 20.

**34.** Since the modulus of a number is always greater than or equal to 0 and $|x+3|$ and $|x-5|$ simultaneously cannot be zero, for any real value of $x$, the number of solutions = 0.

**35.** $4x + 7 > 9 \Rightarrow 4x > 2 \Rightarrow x > \frac{2}{4} \Rightarrow x > \frac{1}{2}$

**36.** Given $3 - x \le 4$
$\Rightarrow -x \le 4 - 3 \Rightarrow -x \le 1$
$\Rightarrow x \ge -1$

**37.** Given $x^2 - 5x - 14 \le 0$
$\Rightarrow x^2 - 7x + 2x - 14 \le 0$
$\Rightarrow x\,(x-7) + 2\,(x-7) \le 0$
$\Rightarrow (x-7)\ (x+2) \le 0$
The critical values are $-2$, 7.




Clearly, '0' satisfies the given inequation.
So, the solution set is $[-2, 7]$.
The integers in the solution set are $-2, -1, 0, 1, 2, 3, 4, 5, 6$ and 7.

**38.** Given, $x + 2 \ge 5$
$\Rightarrow x \ge 5 - 2$
$\Rightarrow x \ge 3$
$\therefore x \in [3, \infty)$

**39.** Given, $|x-2| \le 3$
$\Leftrightarrow -3 \le x - 2 \le 3$
$\Leftrightarrow -3 + 2 \le x \le 3 + 2$
$\Leftrightarrow -1 \le x \le 5$
$\therefore$ The required solution set is $[-1, 5]$.

**40.** Given $2x + 3 > 4$
$\Rightarrow 2x > 4 - 3$
$\Rightarrow 2x > 1 \Rightarrow x > \frac{1}{2}$
$\therefore$ The required solution set is $\left(\frac{1}{2}, \infty\right)$.

**41.** Since '$a$' is a positive real number.
A.M. $\left(a, \frac{1}{a}\right) \ge$ G.M. $\left(a, \frac{1}{a}\right)$
$\Rightarrow \frac{\left(a + \frac{1}{a}\right)}{2} \ge \sqrt{a \cdot \frac{1}{a}} \Rightarrow a + \frac{1}{a} \ge 2$
$\therefore$ The minimum value of $a + \frac{1}{a}$ is 2.

**42.** $|x+3| < 5 \Rightarrow -5 < x + 3 < 5$
$\Rightarrow -5 - 3 < x + 3 - 3 < 5 - 3 - 8 < x < 2$
The number of integer solutions are $-7, -6, -5, -4, -3, -2, -1, 0, 1$ which are 9.

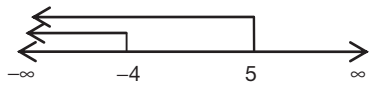
**43.** $\alpha, \beta\ (\alpha < \beta)$ are the roots of the equation $(x-\alpha)\ (x-\beta) = 0$ and the solution sets for the inequality $(x-\alpha)\ (x-\beta) \ge 0$
$(-\alpha, \alpha]\ \cup\ [\beta, \alpha)$
$\Rightarrow$ Here, $\alpha = -3$ and $\beta = 5$, $\alpha$
$\therefore$ The required equation is $(x+3)\ (x-5) \ge 0$.

**44.** If $(x-\alpha)\ (x-\beta) \ge 0$ then
$x \le \alpha$, or $x \ge \beta\ (\alpha < \beta)$
Here, $\alpha = -13$ and $\beta = 15$
$x \le -13$ or $x \ge 15$

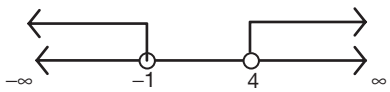
**45.** $\Rightarrow 3x + 4 \le -5x + 12 \Rightarrow 3x + 5x \le 12 - 4$
$\Rightarrow 8x \le 8 \Rightarrow x \le 1$

## EXERCISE-2

**1.** $3x+4 \le -8$ and $2x+7 \ge 5x-8$
$3x \le -12$ and $2x-5x \ge -8-7$
$x \le -4$ and $-3x \ge -15 \Rightarrow x \le 5$
$\therefore$ The required solution is $x \le -4$, i.e., $x \in (-\infty, -4)$.

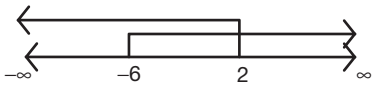



**2.** $2x-5>3$ and $3x+7<4$
$\Rightarrow 2x>8$ and $3x<-3$
$x>4$ and $x<-1$




There is no common solution.
$\therefore$ Solution is $\phi$.

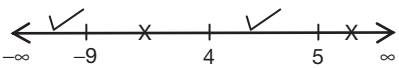
**3.** $2x+7 \ge -5$ and $x+12 \ge 3x+8$
$2x \ge -12$ and $x-3x \ge -4$
$x \ge -6$ and $2x \ge -4$ $x \le 2$




$-6 \le x \le 2$ $\quad x \in [-6, 2]$

**4.** $x^2-x-6>0$
$(x-3)\ (x+2)>0$
$\Rightarrow x-3>0$ and $x+2>0$ (1)
or
$\Rightarrow x-3<0$ and $x+2<0$ (2)
From Eq. (1): $x>3$ and $x>-2$
$\Rightarrow x>3$ (combining)
From Eq. (2): $x<3$ and $x<-2$
$\Rightarrow x<-2$
Thus, $x>3$ or $x<-2$
$x \in (-\infty, -2) \cup (3, \infty)$

**5.**




$(x-5)\ (x+9)\ (x-4)=0$ at $x=-9, 4, 5$ when $x=0$; the given inequation is false then the neighbouring intervals $(-\infty, -9]$ and $[4, 5]$ are the required solutions.
$\therefore$ Only 4, 5 are the positive integral solutions.

**6.** When $x>\frac{1}{2}$, $|2x-1| = (2x-1)$, then $|2x-1|>3-x$ becomes $2x-1>3-x \Rightarrow 3x>4$.
$\Rightarrow x>\frac{4}{3}$
When $x<1/2$, $|2x-1| = -(2x-1)$, then $|2x-1|>3-x$ becomes $\Rightarrow -(2x-1)>3-x \Rightarrow 1-2x>3-x$.
$\Rightarrow 1-3>2x-x \Rightarrow -2>x \Rightarrow x<-2$
Since $x>0$, $x>\frac{4}{3}$

**7.** When $x=3$ and $y=0$, only first and second choices satisfy the graph. In the first choice, when $x=-3$ also $y=0$, but the graph cuts $x$-axis at only one point.
$\therefore$ The best represented equation is $y=|x-3|$.

**8.** $\left|\frac{3x-6}{2}\right| \le 12$
$\frac{|x-2|}{2} \le 4$
$|x-2| \le 8$
$-8 \le x-2 \le 8$
$-6 \le x \le 10$

**9.** $|8x-5|=3$
$\Rightarrow 8x-5=3$ or $8x-5=-3$
$8x=8$ or $8x=2$
$\Rightarrow x=1$ or $x=\frac{2}{8}=\frac{1}{4}$
$\therefore$ Solution set $=\left\{\frac{1}{4}, 1\right\}$

**10.** $x^2-5x+6<0$
$\Rightarrow (x-2)\ (x-3)<0$




The critical points are 2, 3. When $x=0$, the inequation is not satisfied.
$\Rightarrow$ When $x \in (2, 3)$, the inequation is satisfied.
$\therefore$ Solution set is $2<x<3$.

**11.** $x^2-x-6<0 \Rightarrow (x-3)\ (x+2)<0$, which is of the form $(x-\alpha)\ (x-\beta)<0$, whose solution is given by $x \in (\alpha, \beta)$.
$\therefore$ Required solution is $x \in (-2, 3)$.

**12.** $x^2-4x+10>4 \Rightarrow (x-2)^2+10>0$, as $(x-2)^2 \ge 0$
$\Rightarrow (x-2)^2+10>0$ holds for all real numbers $(x-2)^2+10 \ge 10>0$.
$\therefore$ Solution set is $R$.

**13.** $6x^2+x-12 \ge 0 \Rightarrow (2x+3)\ (3x-4) \ge 0$
This is of the form $(x-\alpha)\ (x-\beta) \ge 0$ $(\alpha<\beta)$, whose solution is $x \in R-(\alpha, \beta)$.
$\therefore$ Solution is $x \in R-\left(\frac{-3}{2}, \frac{4}{3}\right)$.

**14.** $3x-8 \le 5x-2 \Rightarrow 3x-5x \le -2+8$
$\Rightarrow -2x \le 6 \Rightarrow x \ge -3$ (1)
and $6x-4 \le 4x+1 \Rightarrow 6x-4x \le 1+4$
$\Rightarrow 2x \le 5 \Rightarrow x \le \frac{5}{2}$ (2)
The region that satisfies both Eq. (1) and Eq. (2), i.e., $x \ge -3$ and $x \le \frac{5}{2}$ is $x \in \left[-3, \frac{5}{2}\right]$.

**15.** Given that $3x+14>5x+24$.
$\Rightarrow 3x-5x>24-14 \Rightarrow -2x>10 \Rightarrow x<-5$

**NOTE**

When an inequality is multiplied by a negative number, the inequality is reversed.

**16.** $\frac{x-3}{x-5} \le 4 \Rightarrow \frac{x-3}{x+5} - 4 \le 0 \Rightarrow \frac{x-3-4x-20}{x+5} \le 0$

$\Rightarrow \frac{-3x-23}{x+5} \le 0 \Rightarrow \frac{-(3x+23)(x+5)}{(x+5)^2} \le 0$

$\Rightarrow (3x+23)\ (x+5) > 0 \Rightarrow \left(x+\frac{23}{3}\right)(x+5) > 0$

$\Rightarrow x + \frac{23}{3} < 0$ or $x+5 \ge 0 \Rightarrow x \le \frac{-23}{3}$ or $x \ge -5$

$\therefore$ The solution set is $\left(-\infty, \frac{-23}{3}\right] \cup (-5, \infty)$.

**17.** Given $\left|\frac{1}{x-15}\right| > \frac{5}{12}$

$|x-15| < \frac{12}{5}$

$\frac{-12}{5} < x-15 < \frac{12}{5}$

$-\frac{12}{5} + 15 < x < \frac{12}{5} + 15$

$\frac{63}{5} < x < \frac{87}{5}$

$12{\cdot}6 < x < 17{\cdot}4$

The minimum integer is 13 and the maximum integer is 17.

$\therefore$ Their difference is 4.

**18.** Given $\frac{x}{x-2} - \frac{1}{2x} > 0$

$\Rightarrow \frac{2x^2 - x + 2}{2x(x-2)} > 0 \Rightarrow x(x-2)(2x^2 - x + 2) > 0$ (1)

The expression $2x^2 - x + 2$ is always positive.

($\because$ The discriminant of the equation $2x^2 - x + 2 = 0$ is negative.)

Eq. (1) $\Rightarrow x(x-2) > 0 \Rightarrow x \notin [0, 2]$ or $x \in (-\infty, 0) \cup (2, \infty)$

**19.** (i) Let $f(x) = 14 - |2x-5|$.

Since $|2x-5|$ is always positive, $14 - |2x-5| \le 14$.

$\therefore f(x) \le 14$

$\therefore$ The maximum value of $f(x)$ is 14.

(ii) $f(x) = 13 + |3x-7|$

Since $|3x-7|$ is positive. $\therefore$ $13 + |3x-7|$ is greater than 13.

$\therefore f(x) \ge 13$ and $f(x)$ is equal to 13 when $|3x-7|$ is zero.

$\therefore 3x - 7 = 0 \Rightarrow x = \frac{7}{3}$

**20.** $\frac{5x^2 + 8x - 21}{x^2 - 8x + 15} < 0$

$5x^2 + 8x - 21 = 5x^2 + 15x - 7x - 21 = (x+3)\ (5x-7)$

Also, $x^2 - 8x + 15 = (x-5)\ (x-3)$

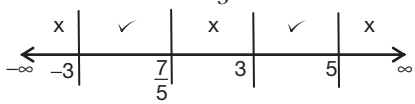
$\frac{5x^2 + 8x - 21}{x^2 - 8x + 15} < 0$

$\frac{(x+3)(5x-7)}{(x-5)(x-3)} < 0$

$\frac{(x+3)(5x-7)(x-3)(x-5)}{(x-5)^2(x-3)^2} < 0$

$(x+3)\ (x-3)\ (5x-7)\ (x-5) < 0$

The critical points are $-3, \frac{7}{5}, 3, 5$




$x = 0$ satisfies the inequality.

$\therefore$ The solution set is $\left(-3, \frac{7}{5}\right) \cup (3, 5)$.

**21.** $f(x) = |x| + |x-5| + |x+7|$

$\forall\ x \in R$, $f(x)$ has no maximum value.

The minimum value of $f(x)$ is possible when $x = 0, 5$ or $-7$.

$f(0) = 0 + 5 + 7 = 12$

$f(5) = 5 + 12 = 17$ and $f(-7) = 7 + 12 + 0 = 19$

Of these, the minimum value of $f(x)$ is 12 is at $x = 0$.

$\therefore$ Range of $f(x)$ is $[12, \infty]$.

**22.** Given $|x| \ge |6 - x^2|$

**Case (i):** When $x \ge 0$ and $x^2 \le 6$, then

$x \ge 6 - x^2$

$x^2 + x - 6 \ge 0$

$(x+3)\ (x-2) \ge 0$

$x \notin (-3, 2)$

$\Rightarrow x \in [2, \sqrt{6}\ ]$ (1)

**Case (ii):** When $x \ge 0$, and $x^2 > 6$, then

$x \ge x^2 - 6$

$-x^2 + x - 6 \ge 0$ or $x^2 - x + 6 \le 0$

$\Rightarrow x \in [-2, 3]$

$x \in (\sqrt{6}, 3]$

**Case (iii):** When $x < 0$ and $x^2 > 6$, then

$-x \ge x^2 - 6$

$-x^2 - x + 6 \ge 0$

$x^2 + x - 6 \le 0$

$(x+3)(x-2) \le 0$

$x \in [-3, 2]$

$x \in [-3, -\sqrt{6}\ )$

**Case (iv):** $x < 0$, $x^2 < 6$

Then $-x \ge 6 - x^2$

$x^2 - x - 6 \ge 0$

$(x-3)\ (x+2) \ge 0$

$x \notin (-2, 3)$

$\therefore x \in [-\sqrt{6}, -2]$

From the four cases, $x \in [-3, -2] \cup [2, 3]$.

**23.** $|x-3| \le 9$

$\Rightarrow -9 \le x-3 \le 9$

$-6 \le x \le 12$ (1)

$|4-x| < 5$

$\Rightarrow -5 < 4-x < 5$

$-9 < -x < 1$

$-1 < x < 9$ (2)

From (1) & (2) $-1 < x < 9$

**24.** We know that $|a|-|b| \le |a-b|$ and $|a-b| \le |a|+|b|$

$\therefore |a|-|b| \le |a-b| \le |a|+|b|$

i.e., $q \le r \le p$

**25.** Given $2x^2+10x+17<0$

$\Rightarrow x^2+5x+\frac{17}{2}<0$

$\Rightarrow x^2+5x+\frac{25}{4}-\frac{25}{4}+\frac{17}{2}<0$

$\Rightarrow \left(x+\frac{5}{2}\right)^2+\frac{9}{4}<0$

Since $\left(x+\frac{5}{2}\right)^2$ is always greater than or equal to 0, for any value of $x$.

There is no real value of $x$ such that $2x^2+10x+17<0$.

**26.** $\frac{x^2-2x-8}{3x^2+7x+2}<0$

$\Rightarrow \frac{(x+2)(x-4)}{(x+2)(3x+1)}<0 \Rightarrow \frac{x-4}{3x+1}<0 \; ; (x \ne -2)$

$\Rightarrow \frac{(x-4)(3x+1)}{(3x+1)^2}<0 \Rightarrow (x-4)(3x+1)<0$

$\Rightarrow x \in \left(-\frac{1}{3}, 4\right)$

**27.** $\frac{(x-3)}{(x+3)} \times \frac{(x+3)}{(x+3)} > 3 \Rightarrow x^2-9 > 3(x+3)^2$

$\Rightarrow -2x^2-18x-36>0 \Rightarrow x^2+9x+18<0$

$\Rightarrow (x+3)(x+6)<0 \Rightarrow x \in (-6, -3)$

$\therefore$ Solution set is $(-6, -3)$.

**28.** $|x^2-9|=0$

$\Rightarrow (x+3)(x-3)=0$

$\Rightarrow x=3$ or $-3$

$\therefore$ Solution set is $\{-3, 3\}$.

**29.** We know that $|x| > a$.

$\Rightarrow x < -a$ or $x > a$

$|2x+3| > 2 \Rightarrow 2x+3 < -2$ or $2x+3 > 2$

$x < -\frac{5}{2}$ or $x > -\frac{1}{2}$

$\Rightarrow \left(-\infty, -\frac{5}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$

**30.** Given $|x+5| < 7, -7 < x+5 < 7$

$\Rightarrow -12 < x < 2$

$\therefore$ Solution is $(-12, 2)$.

**31.** Given $|x|-4 < 2+x$

When $x \ge 0$, $|x| = x$

$\Rightarrow x-4 < 2+x \Rightarrow -4 < 2$ true for all $x > 0$.

$x \in [0, \infty)$ (1)

When $x < 0$, $|x| = -x \Rightarrow -x-4 < 2+x$.

$-2x < 6$

$x > -3 \Rightarrow (-3, 0)$ (2)

From Eqs (1) and (2), solution is $(-3, \infty)$.

**32.** Let $f(x) = |x+2|+|x-1|+|x+5|$.

When $x = -5$, $f(x) = 3+6+0 = 9$.

When $x = -2$, $f(x) = 0+3+3 = 6$.

When $x = 1$, $f(x) = 3+0+6 = 9$.

The minimum value of $f(x)$ is 6 and it occurs at $x = -2$.

**33.** Consider $4p^2+p+4 = p\left[4\left(p+\frac{1}{p}\right)+1\right]$, as

$p+\frac{1}{p} \ge 2 \Rightarrow p(4(\ge 2)+1) \ge 9p$.

Similarly, $11q^2+q+11 \ge 23q$ and $13r^2+r+13 \ge 27r$

$\therefore \frac{(4p^2+p+4)(11q^2+q+11)(13r^2+r+13)}{27pqr}$

$\ge \frac{9p \times 23q \times 27r}{27pqr} \ge 207$

**34.** Min $(x-2, x+2) = x-2$

Min $(x-1, x-3) = x-3$

Max $(x-2, x-3) = x-2$

**35.** Given $3x+5y=18$

Consider the product $\left(\frac{3x}{4}\right)^4\left(\frac{5y}{2}\right)^2$.

The sum of these six factors is $4\left(\frac{3x}{4}\right)+2\left(\frac{5y}{2}\right) = 3x+5y$ $= 18 =$ constant.

The sum is constant. $\therefore$ The product is maximum when they are equal.

$\frac{3x}{4}=\frac{5y}{2}=\frac{3x+5y}{6}=\frac{18}{6}=3$

$\therefore x=4$ and $y=\frac{6}{5}$

**36.** Let $x^3+1 > x^2+x$.

$\Rightarrow x^3-x^2-x+1>0$

$\Rightarrow x^2(x-1)-1(x-1)>0$

$\Rightarrow (x-1)(x^2-1)>0$

$\Rightarrow (x-1)^2(x+1)>0$

$\Rightarrow x+1>0$ and $x \ne 1$

$[\because (x-1)^2 \ge 0 \; x \in R]$

$\Rightarrow x>1$ and $x \ne 1$

$\Rightarrow x \in (-1, 1) \cup (1, \infty)$

$\therefore$ The solution set is $(-1, 1) \cup (1, \infty)$.

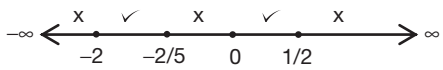
**37.** Given $\frac{5x}{x+2}-\frac{1}{2x}<0$ $\frac{10x^2-x-2}{2x(x+2)}<0$

$\frac{10x^2-5x+4x-2}{x(x+2)}<0 \Rightarrow \frac{5x(2x-1)+2(2x-1)}{x(x+2)}<0$

$\Rightarrow x(x+2)(5x+2)(2x-1)<0$

The critical points are $x=0$, $x=-2$, $x=-\frac{2}{5}$, $x=\frac{1}{2}$.

Representing the above points on the number line, we have $\frac{x(x+2)(5x+2)(2x-1)}{x^2(x+2)^2}<0$.




$x=-1$ satisfies the given inequality.

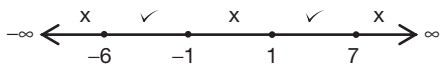
∴ The union of the intervals marked with '√' is the required solution, i.e., $x \in (-2, -2/5) \cup (0, 1/2)$.

**38.** Given, $(x^2+5x-6)(x^2-6x-7)<0$

$(x^2+6x-x-6)(x^2-7x+x-7)<0$

$(x+6)(x-1)(x+1)(x-7)<0$

The critical points are –6, –1, 1 and 7.




When $x=0$, the inequality is not true.

∴The solution set is $x \in (-6, -1) \cup (1, 7)$.

The integral values of $x$ in the above solution set are –5, –4, –8, –3, –2, 2, 3, 4, 5 and 6.

∴ The required number of integral values is 9.

**39.** Given $2-\frac{1}{n}<x\le 4+\frac{1}{n}$

$n=1 \Rightarrow 2-1<x\le 4+1$

$1<x\le 5$

$x=2 \Rightarrow \frac{3}{2}<x<\frac{9}{2}$ and so on

The range of $x$ is $(1, 5]$.

**40.** $\frac{1}{2x-1}>\frac{3}{x}$

$\frac{2x-1}{(2x-1)^2}>\frac{3x}{x^2}$

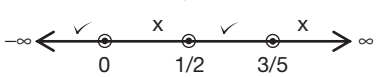
$x^2(2x-1)>3x(2x-1)^2$

$x(2x-1)(x-6x+3)>0$

$x(2x-1)(3-5x)>0$

$x(2x-1)(5x-3)<0$

Critical values are $x=0, \frac{1}{2}, \frac{3}{5}$




When $x=-1$, the inequality is true.

∴ Solution is $(-\infty, 0) \cup \left(\frac{1}{2}, \frac{3}{5}\right)$.

Given $x>0$, solution is $\left(\frac{1}{2}, \frac{3}{5}\right)$

**41.** According to the graph, $x$ takes both negative and positive values.

∴ Choices (C) and (D) are not correct.

The values of $y$ are only negative so choice (A) is ruled out.

**42.** (a) Given $x^2-13|x|+36=0$

$|x|^2-13|x|+36=0$

$|x|^2-9|x|-4|x|+36=0$

$(|x|-9)(|x|-4)=0$

$|x|-9=0$ or $|x|-4=0$

$|x|=9$ or $|x|=4 \Rightarrow x=\pm 9;\ x=\pm 4$

∴ The number of solutions is 4.

(b) Given $||x+5|-9|=7$

**Case (i):** $x>-5 \Rightarrow x+5>0 \Rightarrow |x+5|=x+5$

So, $||x+5|-9|=7 \Rightarrow |x+5-9|=7 \Rightarrow |x-4|=7$

$x-4=\pm 7$

$x=7+4;\ x=-7+4$

$\Rightarrow x=11$ or $x=-3$ which agree with $x>-5$.

**Case (ii):** $x<-5$

$|x+5|=-(x+5)$

∴ $||x+5|-9|=7 \Rightarrow |-x-5-9|=7$

$\Rightarrow |x+14|=7 \Rightarrow x+14=\pm 7$

$x=-7$ or $x=-21$ which agree with $x<-5$

∴ The number of solutions is 4.

**43.** Given $|x+1|+|x-3|=6$

**Case (i):** When $x>3$,

$|x+1|=x+1, |x-3|=x-3$

∴ $|x+1|+|x-3|=6 \Rightarrow x+1+x-3=6$

$\Rightarrow 2x-2=6 \Rightarrow 2x=8 \Rightarrow x=4$

**Case (ii):** When $x<-1$,

$|x+1|=-(x+1); |x-3|=-(x-3)$

∴ $|x+1|+|x-3|=6 \Rightarrow -(x+1)-(x-3)=6$

$\Rightarrow -2x+2=6 \Rightarrow 2x=-4 \Rightarrow x=-2$

**Case (iii):** When $-1<x<3$, $\Rightarrow |x+1|=x+1$ ; $|x-3|=3-x$

∴ $|x+1|+|x-3|=6 \Rightarrow x+1+3-x=6 \Rightarrow 4=6$, which is not possible.

∴ The solution set is {–2, 4}.

**44.** $x^2-12x+42>-5$

$\Rightarrow x^2-12x+42+5>0$

$\Rightarrow x^2-12x+47>0$

$\Rightarrow x^2-12x+36+11>0$

$\Rightarrow (x-6)^2+11>0$

$(x-6)^2$ is always positive.

∴ For all values of $x$, $(x-6)^2+11$ is positive.

∴ Solution is $x \in R$.

**45.** $\frac{x^2-5x+10}{3x^2-5x-22} \le 1$

$\Rightarrow \frac{x^2-5x+10}{3x^2-5x-22} - 1 \le 0$

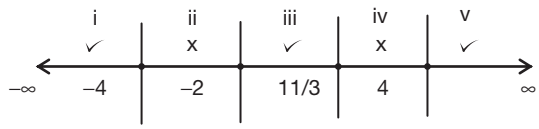
$\Rightarrow \frac{x^2-5x+10-3x^2+5x+22}{3x^2-5x-22} \le 0$

$\Rightarrow \frac{-2x^2+32}{3x^2-5x-22} \le 0 \Rightarrow \frac{-2(x^2-16)}{(3x-11)(x+2)} \le 0$

$\Rightarrow \frac{(x^2-16)(3x-11)(x+2)}{(3x-11)^2(x+2)^2} \ge 0$

$\Rightarrow (x^2-16)(3x-11)(x+2) \ge 0$

Critical points are $-4, -2, \frac{11}{3}, 4$




When $x = 0$, the above inequation is satisfied.

$\therefore$ i, iii, v regions together form the solution.

Solution set is $(-\infty, -4) \cup \left(-2, \frac{11}{3}\right) \cup (4, \infty)$.

## EXERCISE-3

**1.** $x|x-6| = 7$

If $x \ge 6$, then $|x-6| = x-6$ $\therefore x|x-6| = x(x-6) = 7$.

$x^2 - 6x - 7 = 0$

$\therefore x = 7$ or $-1$

But, $x > 6 \Rightarrow x = 7$

$x < 6$, we get non-natural number.

**Alternative Method:**

Proceeding from the choices only $x = 7$ satisfies $x|x-6| = 7$.

**2.** **Case (i):** Let $x \ge -\frac{1}{4}$.

Then, $|4x+1| = 4x+1$

So, $|x-|4x+1|| = 7$

$\Rightarrow |x-4x-1| = 7$

$\Rightarrow |3x+1| = 7 \Rightarrow x = 2$ or $x = \frac{-8}{3}$

Since $x \ge \frac{-1}{4}$ $x = 2$ is the only possibility.

**Case (ii):** Let $x < -\frac{1}{4}$.

Then, $|4x+1| = -(4x+1)$

So, $|x-|4x+1|| = 7$

$\Rightarrow |x+4x+1| = 7$

$\Rightarrow |5x+1| = 7 \Rightarrow x = \frac{6}{5}$ or $x = \frac{-8}{5}$

Since $x < \frac{-1}{4}$, so $x = \frac{-8}{5}$ is the only possibility.

Thus, we get two solutions for this equation.

**3.** **Case (i):** When $x \ge 0$, $|x| = x$.

$\therefore x^2 - 9x + 18 = 0$

$\Rightarrow (x-3)(x-6) = 0 \Rightarrow x = 6$ or $x = 3$

**Case (ii):** When $x < 0$, $|x| = -x$

$\therefore x^2 + 9x + 18 = 0$

$\Rightarrow (x+3)(x+6) = 0$

$\Rightarrow x = -6$ or $x = -3$

Thus, there are 4 distinct solutions of the given equation.

**Alternative Method:**

$x^2 = |x|^2$

$\therefore$ The given equation becomes $|x|^2 - 9|x| + 18 = 0$.

$|x| = 6$ or $3$

$\therefore x = \pm 6$ or $\pm 3$

$\therefore x$ has 4 distinct solutions.

**4.** $|x+2| + |x-2|$ is a positive quantity, and so, it cannot be zero. Hence, no solution is possible for $f(x) = 0$.

**5.** Given $|3-5x| > 5-x$, if $x < \frac{3}{5}$, $|3-5x| = 3-5x$

$\therefore 3-5x > 5-x$

$-2 > 4x$

$x < -\frac{1}{2}$

Given $x > 0$, there is no solution.

$x > 3/5$ $|3-5x| = 5x-3$

Inequation becomes $5x-3 > 5-x$.

$6x > 8$

$x > \frac{8}{6} \Rightarrow x > \frac{4}{3}$

**6.** Given $3x+4 \le 6$ and $4x+3 \ge 6$

$3x \le 2$ and $4x \ge 3$

$x \le \frac{2}{3}$ and $x \ge \frac{3}{4}$

There is no real value of $x$ satisfying the above inequations simultaneously.

$\therefore$ The solution set is empty set.

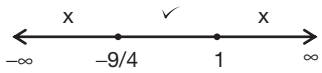
**7.** Given $4x^2 + 5x - 9 \le 0$

$\Rightarrow 4x^2 + 9x - 4x - 9 \le 0$

$x(4x+9) - 1(4x+9) \le 0$

$(4x+9)(x-1) \le 0$

$\therefore$ The critical points are $-\frac{9}{4}$, 1.



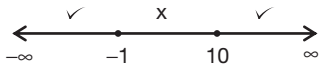

When $x = 0$, the given inequation is true.

Hence, the solution set is $x \in \left[-\frac{9}{4}, 1\right]$.

**8.** Given $x^2 - 9x - 10 > 0 \Rightarrow (x - 10)\ (x + 1) > 0$

The critical points of the above inequality are –1 and 10.

When $x = 0$, the inequality is not true.




So, when $x < -1$ or $x > 10$, the inequation will be true.

$\therefore$ The solution set is $(-\infty, -1) \cup (10, \infty)$.

**9.** Given $\frac{x-7}{x+5} > 3$

$$\frac{x-7}{x+5} - 3 > 0$$

$$\Rightarrow \frac{x-7-3(x+5)}{x+5} > 0$$

$$\Rightarrow \frac{x-7-3x-15}{x+5} > 0$$

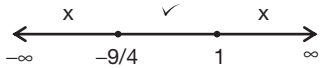
$$\Rightarrow \frac{-2x-22}{x+5} > 0$$

$$\Rightarrow \frac{x+11}{x+5} < 0$$

$$\Rightarrow \frac{(x+11)(x+5)}{(x+5)^2} < 0$$

$(x + 11)\ (x + 5) < 0$

The critical points are –11 and –5.




When $x = 0$, the inequality is not true.

$\therefore$ The solution set of the inequality is $(-11, -5)$.

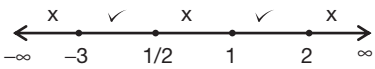
**10.** Given $\frac{2x^2+5x-3}{x^2-3x+2} \le 0$

$$\Rightarrow \frac{(x+3)(2x-1)}{(x-2)(x-1)} \le 0$$

$$\Rightarrow \frac{(x+3)(2x-1)(x-2)(x-1)}{(x-2)^2(x-1)^2} \le 0$$

$\Rightarrow (x + 3)\ (2x - 1)\ (x - 1)\ (x - 2) \le 0$

$\Rightarrow$ The critical points are –3, 1/2, 1 and 2.




When $x = 0$, the inequality is true.

The solution set is $[-3, 1/2] \cup (1, 2)$.

**11.** Given $\frac{3x}{x-3} - \frac{1}{x} < 0 \Rightarrow \frac{3x^2 - x + 3}{x(x-3)} < 0$

$$\Rightarrow \frac{x(x-3)\left(3x^2 - x + 3\right)}{x^2(x-3)^2} < 0 \Rightarrow x(x-3)\left(3x^2 - x + 3\right) < 0$$

Consider the equation $3x^2 - x + 3 = 0$. The discriminant $b^2 - 4ac = 1 - 36 < 0$.

$\therefore$ The expression $3x^2 - x + 3$ is positive for all real values of $x$.

$\Rightarrow x(x - 3) < 0 \Rightarrow x \in (0, 3)$

**12.** $g(y) = \min\ (-y + 3, 2 + 7y)$

If $-y + 3 = 2 + 7y$, then $y = \frac{1}{8}$.

$g(y) = \min\ (2 + 7y, 2 + 7y) = 2 + 7y = \frac{23}{8}$

Suppose $-y + 3 > 2 + 7y \Rightarrow 1 > 8y$

$\Rightarrow y < 1/8$

$\therefore y < 1/8 \Rightarrow 2 + 7y < 3 - y$ so $g(y) = 2 + 7y$

$\therefore$ When $y < \frac{1}{8} \Rightarrow g(y) < \frac{23}{8}$.

Similarly, if $-y + 3 < 2 + 7y \Rightarrow y > \frac{1}{8}$

When $y > 1/8$, then $g(y) = 3 - y$.

$\therefore$ When $y > 1/8$, then $g(y) < \frac{23}{8}$.

Maximum value of $g(y)$ is $\frac{23}{8}$.

**13.** Let $f(x) = |x - 2| + |x - 1| + |x + 11|$.

When $x \neq 2, 1, -11$.

$|x - 2|, |x + 1|, |x + 11|$ are none zero values.

$\therefore$ When $x = 1$ or 2 or –11 one of $|x - 2|, |x - 1|, |x + 11|$ is zero.

$\therefore f(x)$ is minimum when one of $x = 2$, or 1, –11.

$f(2) = 0 + 1 + 13 = 14$

$f(1) = 1 + 0 + 12 = 13$

$f(-11) = 13 + 12 = 25$

Least among these values is 13 at $x = 1$.

**14.** $f(x) = \min(x - 3, x + 2) = x - 3$

$g(x) = \max(x - 2, x - 3) = x - 2$

$h(x) = f(x) + g(x) = x - 3 + x - 2 = 2x - 5$

$h(x) \le 7 \Rightarrow 2x - 5 \le 7$ or $x \le 6$

**15.** Given $(x^2 - x + 1)^x < 1$

Taking log on both sides, we get $x \log\ (x^2 - x + 1) < 0$.

Now, two cases arise:

**Case (i):** $x < 0$, $\log\ (x^2 - x + 1) > 0 \Rightarrow x^2 - x + 1 > 1$

$\Rightarrow x(x - 1) > 0 \Rightarrow x \in (-\infty, 0) \cup (1, \infty)$

as $x < 0$

The solution in this case is $(-\infty, 0)$.

**Case (ii):** $x > 0$ and $\log(x^2 - x + 1) < 0 \Rightarrow x^2 - x + 1 < 1$

$\Rightarrow x(x - 1) < 0 \Rightarrow 0 < x < 1$

So, $0 < x < 1$ in this range.

Combining both cases, we get the solution set as $(-\infty, 0) \cup (0, 1)$.

**16.** $|x-5| > x^2 - 4x + 1$

**Case (i):** If $x > 5$, then $|x-5| = x-5$.

So, $x - 5 > x^2 - 4x + 1$

$x^2 - 4x + 1 < x - 5$

$x^2 - 5x + 6 < 0$

$(x-3)\ (x-2) < 0$

$x \in (2, 3)$ which does not agree with $x < 5$.

**Case (ii):** If $x < 5$, then $|x-5| = 5 - x$.

So, $5 - x > x^2 - 4x + 1$

$x^2 - 4x + 1 < 5 - x$

$x^2 - 3x - 4 < 0$

$x^2 - 4x + x - 4 < 0$

$(x-4)\ (x+1) < 0$

$x \in (-1, 4)$

$\therefore$ The solution set is $(-1, 4)$.

**17.** Given $a, b, c$ are positive real numbers.

We know that A.M. $\left(\frac{bc}{a}, \frac{ca}{b}\right) \geq$ G.M. $\left(\frac{bc}{a}, \frac{ca}{b}\right)$.

$$\geq \frac{\frac{bc}{a} + \frac{ca}{b}}{2} \geq \sqrt{\frac{bc}{a} \cdot \frac{ca}{b}}$$

$$\frac{bc}{a} + \frac{ca}{b} \geq 2c$$

Similarly, we can show that $\frac{ca}{b} + \frac{ab}{c} \geq 2a$ and $\frac{ab}{c} + \frac{bc}{a} \geq 2b$.

Adding these three inequalities, we get

$$\frac{bc}{a} + \frac{ca}{b} + \frac{ab}{c} \geq a + b + c.$$

$$a + b + c \leq \frac{bc}{a} + \frac{ca}{b} + \frac{ab}{c}$$

$\therefore$ Choice (A) is true.

We know that $\frac{\left(\frac{a}{bc}\right)^{\frac{1}{2}} + \left(\frac{b}{ca}\right)^{\frac{1}{2}}}{2} \geq \sqrt{\left(\frac{a}{bc}\right)^{\frac{1}{2}} \cdot \left(\frac{b}{ca}\right)^{\frac{1}{2}}}$.

$$\left(\frac{a}{bc}\right)^{\frac{1}{2}} + \left(\frac{b}{ca}\right)^{\frac{1}{2}} \geq 2\frac{1}{\sqrt{c}}$$

Similarly, we can show that $\left(\frac{b}{ca}\right)^{\frac{1}{2}} + \left(\frac{c}{ab}\right)^{\frac{1}{2}} \geq \frac{2}{\sqrt{c}}$ and $\left(\frac{a}{bc}\right)^{\frac{1}{2}} + \left(\frac{c}{ab}\right)^{\frac{1}{2}} \geq \frac{2}{\sqrt{b}}$.

Adding these inequalities, we get

$$\left(\frac{a}{bc}\right)^{\frac{1}{2}} + \left(\frac{b}{ca}\right)^{\frac{1}{2}} + \left(\frac{c}{ab}\right)^{\frac{1}{2}} \geq \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}.$$

$\therefore$ Choice (B) is true.

Now, $\frac{(a^2)^2 + (b^2)^2}{2} \geq \sqrt{a^4 . b^4}$

$$\Rightarrow \frac{a^4 + b^4}{2} \geq a^2 b^2$$

Similarly, we can show that $\frac{b^4 + c^4}{2} \geq b^2 c^2$ and $\frac{c^4 + a^4}{2} \geq c^2 a^2$.

Adding these inequalities, we get $a^4 + b^4 + c^4 \geq a^2 b^2 + b^2 c^2 + c^2 a^2$.

$\therefore$ Choice (C) is also true.

**18.** Given $x^3 - 8 < x^2 - 2x$

$\Rightarrow (x-2)\ (x^2 + 2x + 4) < x(x-2)$

$\Rightarrow (x-2\ (x^2 + 2x + 4 - x) < 0$

$\Rightarrow (x-2)\ (x^2 + x + 4) < 0$

$$\Rightarrow (x-2)\left[\left(x + \frac{1}{2}\right)^2 + \frac{15}{4} < 0\right]$$

$$\Rightarrow x - 2 < 0 \left[\because \left(x + \frac{1}{2}\right)^2 + \frac{15}{4} > 0\right]$$

$\Rightarrow x - 2 < 0$

$\Rightarrow x < 2$

$\therefore$ The solution set is $(-\infty, 2)$.

**19.** Given $x^2 + y^2 + z^2 = 3$

We know that $x^2 + y^2 + z^2 \geq xy + yz + zx$.

$\therefore\ xy + yz + zx \leq x^2 + y^2 + z^2$

i.e., $xy + yz + zx \leq 3$

**20.** We know that A.M. $(b+c, c+a, a+b) \geq$ H.M. $(b+c, c+a, a+b)$.

$$\Rightarrow \frac{b + c + c + a + a + b}{3} \geq \frac{3}{\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}}$$

$$\Rightarrow \frac{2}{9}(a + b + c) \geq \frac{1}{\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}}$$

$$\Rightarrow \frac{9}{2(a+b+c)} \leq \frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}$$

$$\Rightarrow \frac{9}{2} \leq \frac{a+b+c}{b+c} + \frac{a+b+c}{c+a} + \frac{a+b+c}{a+b}$$

$$\Rightarrow \frac{9}{2} \leq \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} + 3$$

$$\Rightarrow \frac{3}{2} \leq \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \quad (1)$$

Now, since $a, b$ and $c$ are the lengths of a triangle, $b + c > a$.

$2b + 2c > a + b + c$

$$\frac{1}{2(b+c)} < \frac{1}{a+b+c}$$

$$\frac{a}{b+c} < \frac{2a}{a+b+c}$$

Similarly, we have $\frac{b}{c+a} < \frac{2b}{a+b+c}$ and $\frac{c}{a+b} < \frac{2c}{a+b+c}$.

Adding these inequalities,

we get $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}<2$ (2)

$\therefore$ From (1) and (2), we have $\frac{3}{2}\le\frac{a}{b+c}+\frac{b}{b+c}+\frac{c}{a+b}<2$.

**21.** Given $x^2-3kx+2k^2+4>0$

If $b^2-4ac<0$, then the given expression is always positive.

$\Rightarrow 9k^2-4(2k^2+4)<0$

$\Rightarrow 9k^2-8k^2-16<0$

$k^2-16<0$

$\Rightarrow k\in(-4,4)$

**22.** Given $y^{\frac{2}{3}}-y^{\frac{1}{3}}-72\le 0$

$\Rightarrow \left(y^{\frac{1}{3}}\right)^2-y^{\frac{1}{3}}-72\le 0$

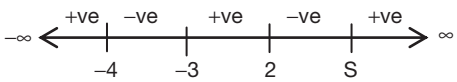
$\Rightarrow \left(y^{\frac{1}{3}}-9\right)\left(y^{\frac{1}{3}}+8\right)\le 0$

$\Rightarrow -8\le y^{\frac{1}{3}}\le 9\Rightarrow -512\le y\le 729$

**23.** $(x^2+x-6)(x^2-x-20)<0$

$\Rightarrow (x+3)(x-2)(x-5)(x+4)<0$

Putting the roots on the number line in order




Hence, the inequality holds for all real values in $(-4,-3)\cup(2,5)$.

$\therefore$ The integer values that satisfy the inequality are 3, 4.

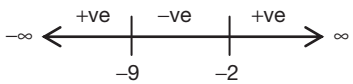
Hence, the number of integers that satisfy the inequality is 2.

**24.** $\frac{x-5}{x+2}<2$, i.e., $\frac{x-5}{x+2}-2<0\Rightarrow\frac{-x-9}{x+2}<0$

or $\frac{x+9}{x+2}>0$

$\Rightarrow \frac{x+9}{x+2}\cdot\frac{x+2}{x+2}>0$

$\Rightarrow (x+9)(x+2)>0$




Hence, solution is $(-\infty,-9)\cup(-2,\infty)$.

**25.** Let $a=x^3-1$ and $b=x^2-x$.

We need to find a range where $a-b>0$.

$\Rightarrow (x^3-1)-(x^2-x)>0\Rightarrow x^2(x-1)+1(x-1)>0$

$\Rightarrow (x^2+1)(x-1)>0$

As $(x^2+1)>0$ for any $x$.

$\Rightarrow (x-1)>0\Rightarrow x>1$

Hence, the range is $(1,\infty)$.

**26. Case (i):** Let $x\ge -\frac{1}{6}$.

Then $|6x+1|=6x+1$

So, $|x-|6x+1||=9$

$\Rightarrow |x-6x-1|$

$\Rightarrow |5x+1|=9$

$\Rightarrow x=\frac{8}{5}$ or $-2$

As $x\ge\frac{-1}{6}$, $x=\frac{8}{5}$ but not $-2$

**Case (ii):** Let $x<\frac{-1}{6}$.

$|x-[-(6x+1)]|=9$

$|7x+1|=9$

$x=\frac{8}{7}$ or $\frac{-10}{7}$

As $x\le\frac{-1}{6}$, $x=\frac{-10}{7}$ but not $\frac{8}{7}$.

Two solutions exist.

**27.** Given $\frac{7x}{5x+4}-\frac{2}{x}<0$

$\frac{7x^2-10x-8}{x(5x+4)}<0$

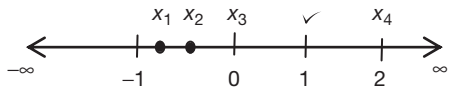
$\Rightarrow \frac{7x^2-14x+4x-8}{x(5x+4)}<0$ (or) $\frac{(7x+4)(x-2)}{x(5x+4)}<0$

$\Rightarrow \frac{x(5x+4)(7x+4)(x-2)}{x^2(5x+4)^2}<0$

$\Rightarrow x(5x+4)(7x+4)(x-2)<0$

The critical points are $x_1=\frac{-4}{5}$, $x_2=\frac{-4}{7}$, $x_3=0$, $x_4=2$.

Representing the above points on the number line, we have




$x=-1$ does not satisfy the given inequality.

$\therefore$ The region containing $-1$ is not part of the solution. In the regions marked with '✓', the inequality (1) is satisfied.

$\therefore x\in(-4/5,-4/7)\cup(0,2)$

$\therefore$ The only integral root is $x=1$.

$\therefore$ The number of integral roots is one.

**28.** Given $(x^2+x+1)^x>1$

Applying logarithm on both sides, we get

$\log(x^2+x+1)^x>\log 1$

$x\log(x^2+x+1)>0$

$\Rightarrow x>0$ and $\log(x^2+x+1)>0$ (or) $x<0$ and $\log(x^2+x+1)<0$

**Case (i):** Let $x>0$ and $\log(x^2+x+1)>0$.

$\Rightarrow x^2+x+1>1$ and $x>0$

$\Rightarrow x^2+x>0$ and $x>0$

$\Rightarrow x(x+1) > 0$ and $x > 0$
$\Rightarrow x \in (-\infty, -1) \cup (0, \infty)$ and
$\Rightarrow x > 0$ (1)

**Case (ii):** Let $x < 0$ and $\log(x^2 + x + 1) < 0$.
$\Rightarrow x^2 + x + 1 < 1$ and $x < 0$
$\Rightarrow x^2 + x < 0$ and $x < 0$
$\Rightarrow x(x+1) < 0$ and $x < 0$
$\Rightarrow x \in (-1, 0)$ (2)
$\therefore$ From (1) and (2)
$\therefore$ The solution set is $(-1, \infty)$.

**29.** $2p^2 + p + 2 = p\left[2\left(p + \frac{1}{p}\right) + 1\right] \geq p(2 \times 2 + 1) \geq 5p$

Similarly, $7q^2 + q + 7 \geq 15q$ and $6r^2 + r + 6 \geq 13r$

$\therefore \frac{(2p^2+p+2)(7q^2+q+7)(6r^2+r+6)}{5pqr} \geq \frac{5p \times (15q) \times (13r)}{5pqr}$

$\geq 195$

**30.** Given $x + y = 5$

Consider $\left(\frac{x}{3}\right)^3 + \left(\frac{y}{2}\right)^2$

$\frac{x}{3} + \frac{x}{3} + \frac{x}{3} + \frac{y}{2} + \frac{y}{2} = x + y = 5$ (constant)

$\therefore$ The product $\left(\frac{x}{3}\right)^3 \left(\frac{y}{2}\right)^2$ is maximum,

when $\frac{x}{3} = \frac{y}{2}$

$\Rightarrow \frac{x}{3} = \frac{y}{2} = \frac{x+y}{3+2}$

$\frac{x}{3} = \frac{y}{2} = 1$

$\Rightarrow x = 3; y = 2$

$\therefore$ The maximum value of $x^3y^2$ is $(3)^3 (2)^2 = 108$.

**31.** A.M. $\geq$ G.M. $\therefore \frac{\frac{a}{b} + \frac{b}{a}}{2} \geq \sqrt{\frac{a}{b}\frac{b}{a}}$

$\Rightarrow \frac{a}{b} + \frac{b}{a} \geq 2$,

Multiplying both sides with $c$, we get

$\frac{ac}{b} + \frac{bc}{a} \geq 2c$ (1)

Similarly $\frac{b}{c} + \frac{c}{b} \geq 2$

Multiplying both sides with $a$, we get

$\frac{ab}{c} + \frac{ca}{b} \geq 2a$ (2)

also $\frac{c}{a} + \frac{a}{c} \geq 2$

Multiplying both sides with $b$, we get

$\frac{bc}{a} + \frac{ac}{c} \quad \frac{bc}{a} + \frac{ac}{c} \quad 2b$ (3)

Adding Eqs (1), (2) and (3), we get $2\left(\frac{ab}{c} + \frac{bc}{a} + \frac{ca}{b}\right) \geq 2(c + a + b)$.

$\Rightarrow \frac{ab}{c} + \frac{bc}{a} + \frac{ca}{b} \geq a + b + c$

$\therefore$ Choice (A) is correct.

Given $a > b$, raising both the sides to the power $a - b$,
$a^{a-b} > b^{a-b}$ (as $a - b > 0$, the inequality does not change)

$\frac{a^a}{a^b} > \frac{b^a}{b^b} \Rightarrow a^a b^b > b^a a^b$

Choice (B) is not correct.

We know that if $a$, $b$, $c$ are positive, then the range of $\frac{a^2 + b^2 + c^2}{ab + bc + ca}$ is [1, 2); hence, $a^2 + b^2 + c^2$ is never greater than $2(ab + bc + ca)$. Choice (C) is not correct.

**32.** We now consider 3 cases:

**Case (i):** $x \leq 3$
$|x - 3| + |x - 4| = (3 - x) + (4 - x) = 7 - 2x$
So, the expression becomes $7 - 2x < 1 \Rightarrow -2x < -6 \Rightarrow x > 3$.
This is inconsistent, so no solution in this domain.

**Case (ii):** $3 < x < 4$
$|x - 3| + |x - 4| = (x - 3) + (4 - x) = 1$
Now, the expression becomes $1 < 1$.
This is inconsistent, so no solution in this domain.

**Case (iii):** $x \geq 4$
$|x - 3| + |x - 4| = (x - 3) + (x - 4) = 2x - 7$
Now, the expression becomes $(2x - 7) < 1$
$\Rightarrow 2x < 8 \Rightarrow x < 4$.
This is inconsistent, so no solutions in this domain.
Hence, no solution at all.

**33.** We make use of the fact that for any two positive numbers, their arithmetic mean $\geq$ geometric mean $\geq$ harmonic mean.

**Consider choice (A):**

$\frac{x^2 + y^2}{2} \geq \sqrt{x^2 y^2}$ (since A.M. of $x^2$, $y^2 \geq$ G.M. of $x^2$, $y^2$)

$\therefore x^2 + y^2 \geq 2xy$.

Similarly, we have $x^2 + z^2 \geq 2zx$

$y^2 + z^2 \geq 2yz$. Adding these three inequalities, we get

$2(x^2 + y^2 + z^2) \geq 2(xy + yz + zx)$

$\therefore x^2 + y^2 + z^2 \geq xy + yz + zx$

$\therefore$ Choice (A) is true.

**Consider choice (B):**

$\frac{x + y}{2} \geq \sqrt{xy}$ (A.M. $\geq$ G.M.)

$x+y \geq 2\sqrt{xy} \Rightarrow y+x \geq 2\sqrt{yz} \Rightarrow z+x \geq 2\sqrt{zx}$

Multiplying the three equations, we get

$(x+y)\ (y+z)\ (z+x) \geq 8\sqrt{x^2y^2z^2}$

i.e. $(x+y)\ (y+z)\ (z+x) \geq 8xyz$

$\therefore$ Choice (B) is correct.

**Consider choice (C):**

We have $x^2+y^2>0$, i.e., $(x+y)^2>2xy$.

$\frac{x+y}{xy} > \frac{2}{x+y}$

i.e., $\frac{1}{x}+\frac{1}{y} > \frac{2}{x+y}$

Choice (C) is also correct.

$\therefore$ Choice (D) is the correct answer.

**34.** The equivalent expressions for $|2x-1|$ and $|x+5|$ are tabulated below for different values of $x$.

| Case | Values of $x$ | $\|2x-1\|$ | $\|x+5\|$ |
|---|---|---|---|
| (i) | $x<-5$ | $-2x+1$ | $-x-5$ |
| (ii) | $-5 \leq x \leq \frac{1}{2}$ | $-2x+1$ | $x+5$ |
| (iii) | $\frac{1}{2} < x$ | $2x-1$ | $x+5$ |

**Case (i):** $-2x+1-(-x-5)+3<0$

$\Rightarrow -x+9<0 \Rightarrow x>9$

$x$ cannot be less than $-5$ and greater than 9 at the same time.

**Case (ii):** $-2x+1-(x+5)+3<0$

$\Rightarrow -3x-1<0 \Rightarrow x>-1/3$

$\therefore x \in \left(\frac{-1}{3}, \frac{1}{2}\right]$

**Case (iii):** $2x-1-(x+5)+3<0$

$\Rightarrow x-3<0 \Rightarrow x<3$

$\therefore x \in \left[\frac{1}{2}, 3\right)$

$\therefore$ The range of values is $\left(\frac{-1}{3}, 3\right)$.

**35.** $\frac{a^4+b^4}{a^2b^2}+\frac{b^4+c^4}{b^2c^2}+\frac{c^4+a^4}{c^2a^2}$

$=\left(\frac{a^2}{b^2}+\frac{b^2}{a^2}\right)+\left(\frac{b^2}{c^2}+\frac{c^2}{b^2}\right)\left(\frac{c^2}{a^2}+\frac{a^2}{c^2}\right)$

We know that the minimum value of $\frac{x}{y}+\frac{y}{x}$ is 2.

The required minimum value is $2+2+2=6$.

**36.** Given $|b| \leq 1$ and $x=-|a|\ b$.

$\therefore a-xb=a+|a|\ b^2$

When $a<0$, $a-xb=a-ab^2=a(1-b^2)<0$

When $a \geq 0$, $a-xb=a+ab^2=a(1+b^2) \geq 0$

$a-xb$ can be less than zero or greater than zero.

Now, $a+xb=a-|a|b^2$.

When $a<0$, $a+xb=a+ab^2=a\ (1+b^2)<0$

When $a \geq 0$, $a+xb=a-ab^2=a\ (1-b^2) \geq 0$

$(\because |b|<1)$

$\therefore a+xb$ can be less than zero or greater than zero.

**37.** $\left(y^{\frac{1}{3}}-1\right)^2+y^{\frac{1}{3}}-13 \leq 0 \Rightarrow y^{\frac{2}{3}}-2y^{\frac{1}{3}}+1y^{\frac{1}{3}}-13 \leq 0$

$\Rightarrow y^{\frac{2}{3}}-2y^{\frac{1}{3}}-12 \leq 0$

Let $y^{\frac{1}{3}}=a^2-a-12\ a \leq 0$

$(a-4)\ (a+3) \leq 0 \Rightarrow -3 \leq a \leq 4$

$\Rightarrow -3 \leq y^{\frac{1}{3}} \leq 4 \Rightarrow -27 \leq y \leq 64$

**38.** $x$ is a positive integer.

$\therefore \left|\frac{5x+124}{5\left(5x^2+1\right)}\right| = \frac{5x+124}{5\left(5x^2+1\right)}$

$\frac{5x+124}{5\left(5x^2+1\right)} < \frac{1}{50}$

$\Rightarrow 10(5x+124) < 5x^2+1$

$\Rightarrow 0 < 5x^2-50x-1239$

$\Rightarrow 0 < x^2-10x-247.8$

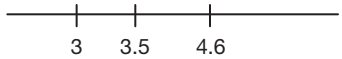
$\Rightarrow 247.8 < x^2-10x$

Adding 5 on both sides, $272.8 < (x-5)^2$

$\therefore x-5>16$ or $x>21$.

$\therefore$ The least value of $x$ satisfying the given inequality is 22.

**39.** $|x-a|$ is the distance from $x$ to $a$ on the number line.




$f(x)=|x-3|+|3.5-x|+14.6-x|$

$f(x)$ is the sum of the distances from $x$ to 3, 3.5 and 4.6.

$f(3)=2.1$

$f(4.6)=2.7$

$f(3-k)$, where $k$ is any positive number, is more than 2.1

$f(4.6+k)$, where $k$ is any positive number, is more than 2.7.

If $3<x<4.6$, $|x-3|+|4.6-x|=1.6$.

$\therefore f(x)=1.6+|3.5-x|$. This is minimum when $x$ is 3.5.

$\therefore f(x)=1.6\ (\because |3.5-x|=x-3.5)$.

Minimum value of $(x)$, i.e., $M$ is 1.6. The value of $x$ when $f(x)=M$ is 3.5.

**40.** $2 \leq x \leq 8$ and $4 \leq y \leq 12$

$\frac{x+y}{x}=1+\frac{y}{x}$. This is minimum when $\frac{y}{x}$ has its minimum value. This is when $y$ is minimum and $x$ is maximum.

$1+\frac{4}{8}=\frac{3}{2}$

**41.** Given $3x+4y=20$

Consider $\left(3\frac{x}{3}\right)^3\cdot\left(\frac{4y}{2}\right)^2$

Sum of the factor $=\frac{3x}{3}+\frac{3x}{3}+\frac{3x}{3}+\frac{4y}{2}+\frac{4y}{2}$

$=3x+4y=20$

$\therefore$ If sum of the factors is constant, then their product is maximum when the factors are equal i.e., $\frac{3x}{3}=\frac{4y}{2}=\frac{3x+4y}{5}$.

$x=2y=\frac{20}{5}$

$x=4;\ y=2.$

$\therefore$ The maximum value of $x^3y^2$ is $4^3\,2^2=256$.

**42.** Given $x^4y^2=1024$.

Consider $\left(\frac{x}{4}\right)^4\left(\frac{y}{2}\right)^2$

$=\frac{x}{4}\cdot\frac{x}{4}\cdot\frac{x}{4}\cdot\frac{x}{4}\cdot\frac{y}{2}\cdot\frac{y}{2}=\frac{1}{4^4}\cdot\frac{1}{2^2}\cdot x^4y^2=\frac{1024}{4^4 2^2}=1$

$\therefore$ The product is given constant.

$\therefore$ The sum of the factors is minimum when $\frac{x}{4}=\frac{y}{2}=1$.

$\Rightarrow x=4$ and $y=2$

$\frac{x}{4}+\frac{x}{4}+\frac{x}{4}+\frac{x}{4}+\frac{y}{2}+\frac{y}{2}=x+y$

$\therefore$ The minimum value of $x+y$ is $4+2=6$.

**43.** Given $2x^3y^2=(15)^3$, $y>0$.

$\therefore$ $x$ has to be positive.

Consider the product $\left(\frac{2x}{3}\right)^3\left(\frac{5y}{2}\right)^2$. The A.M. of these 5 positive numbers is greater than the G.M.

$\therefore\ \frac{\frac{2x}{3}+\frac{2x}{3}+\frac{2x}{3}+\frac{5y}{2}+\frac{3y}{2}}{5}\geq\sqrt[5]{\left(\frac{2x}{3}\right)^3\left(\frac{5y}{2}\right)^2}$

$\frac{2x+5y}{5}\geq\sqrt[5]{\frac{8}{27}\cdot\frac{25}{4}x^3y^2}\geq\sqrt[5]{\frac{25}{27}(15)^3}\geq\sqrt[5]{5^5}$

$\frac{2x+5y}{5}\geq 5$

$2x+5y\geq 25$

The minimum value of $2x+5y$ is 25.

It occurs when all the factors are equal, i.e., when $\frac{2x}{3}=\frac{5y}{2}=5$.

$\therefore\ x=\frac{15}{2}$ and $y=2$

$\therefore$ The value of $x$ is $\frac{15}{2}$.

**44.** Min $(x-1,\ x+2)=x-1$

Max $(x+3,\ x+5)=x+5$

Max $(x-1,\ x+5)=x+5$

**45.** $\left(\frac{a}{b}-\frac{b}{c}\right)^2+\left(\frac{c}{c^2}-\frac{b}{a}\right)^2+\left(\frac{c}{b}-\frac{a}{c}\right)^2+2\left(\frac{a}{c}+\frac{bc}{a^2}+\frac{a}{b}\right)$

$\Rightarrow\frac{a^2}{b^2}+\frac{b^2}{c^2}-\frac{2a}{c}+\frac{c^2}{a^2}+\frac{b^2}{a^2}-\frac{2bc}{a^2}+\frac{c^2}{b^2}$

$+\frac{a^2}{c^2}-\frac{2a}{b}+\frac{2a}{c}+\frac{2bc}{a^2}+\frac{2a}{b}$

$\Rightarrow\left(\frac{a}{b}+\frac{b}{a}\right)^2-2+\left(\frac{b}{c}+\frac{c}{b}\right)^2-2+\left(\frac{a}{c}+\frac{c}{a}\right)^2-2$

$\Rightarrow\geq(2)^2+(2)^2+(2)^2-6\left(\therefore x+\frac{1}{x}\geq 2\right)$

$\therefore\ \geq 6$; the minimum value of the given expression is 6.

## EXERCISE-4

**1.** Since $x\geq 1$, $(x+1)^x$ and $x^x+1$ are positive.

Now, $(x+1)^x<x^x\cdot x$

or $(x+1)^x/x^x<x\Rightarrow(1+1/x)^x<x$

We know that for $x\geq 1$; $2\leq(1+1/x)^x<2.8$

$\therefore$ For $x\geq 3$; $(1+1/x)^x<x$

**2.** Given $x=10/(a+b)$ and $y=10(a+b)$

$\therefore\ x+y=10[1/(a+b)+(a+b)]\geq 10(2)$

As $k+\frac{1}{k}\geq 2$, for $k>0$

$\therefore\ x+y\geq 20$

**3.** Say $k=\frac{a^2(b+c)+b^2(c+a)+c^2(a+b)}{abc}$

$\Rightarrow k=\frac{a^2b+a^2c+b^2c+b^2a+c^2a+c^2b}{abc}$

Now, $a^2b+a^2c+b^2c+b^2a+c^2a+c^2b\geq 6\sqrt[6]{a^6b^6c^6}$

(since A.M. $\geq$ G.M.)

$\Rightarrow a^2b+a^2c+b^2c+b^2a+c^2a+c^2b\geq 6abc$

$\Rightarrow k\geq 6abc/abc$, i.e., $k\geq 6$

Since $a$, $b$, $c$ are distinct, $k>6$.

**4.** Given $\frac{2x}{2x^2+5x+2}>\frac{1}{x+1}$

$\Rightarrow\frac{2x}{2x^2+5x+2}-\frac{1}{x+1}>0$

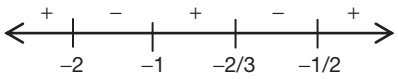
$$\Rightarrow \frac{2x(x+1)-\left(2x^2+5x+2\right)}{\left(2x^2+5x-2\right)(x+1)}>0$$

$$\text{or } \frac{2x^2+2x-2x^2-5x-2}{\left(2x^2+5x+2\right)(x+1)}>0$$

$$\Rightarrow \frac{(-3x-2)}{\left(2x^2+5x+2\right)(x+1)}>0$$

$\Rightarrow (x+1)\ (3x+2)\ (2x^2+5x+2)<0$

or $(x+1)(3x+2)\ (x+2)\ (2x+1)<0$




$\therefore x \in (-2,-1) \cup (-2/3,-1/2)$

**5.** For $x=0$; $\frac{x^4}{1+x^8}=0$

for $x\neq 0$; $x^4$ is positive and $1/x^4$ is positive.

As A.M ≥ G.M.

$$\frac{x^4+\frac{1}{x^4}}{2}=\sqrt{x^4\times\frac{1}{x^4}} \Rightarrow \frac{x^8+1}{x^4}\geq 2 \text{ or } \frac{x^4}{1+x^8}\leq\frac{1}{2}$$

**6.** We know that A.M $(p^4\ q^4, q^4\ r^4) \geq$ G.M $(p^4\ q^4, q^4\ r^4)$

$$\frac{p^4q^4+q^4r^4}{2}\geq\left(p^4q^4q^4r^4\right)^{\frac{1}{2}}$$

$p^4q^4+q^4r^4\geq 2\ p^2q^4r^2$ (1)

Similarly $q^4r^4+r^4\ p^4\geq 2\ p^2q^2r^4$ (2)

and

$r^4p^4+p^4q^4\geq 2p^4q^2\ r^2$ (3)

Adding (1), (2) and (3), we get

$2\ (p^4\ q^4+q^4\ r^4+r^4\ p^4)\geq 2[p^2q^2\ r^2\ (p^2+q^2+r^2)]$

$p^4\ q^4+q^4\ r^4+r^4\ p^4\geq p^2q^2\ r^2\ (p^2+q^2+r^2)$

**7.** Given: $3^{|x+2|}-3^x=(|3^x-1|+2)$

If $x<-2$, then $|x+2|=-(x+2)$ and $|3^x-1|$

$=-(3^x-1)$

∴The given equation can be written as

$3^{-(x+2)}-3^x=-3^x+1+2$

$3^{-(x+2)}=3$

$-x-2=1 \Rightarrow x=-3$

**8.** Given $a, b, c$ are the sides of the triangle.

The sum of two sides is greater than the third side

i.e., $a+b>c$; $b+c>a$; $c+a>b$

let $p=a+b-c$, $q=a+c-b$ and $r=b+c-a$

$a+b+c=p+q+r$

We know that A .M $(p, q, r) \geq$ G.M $(p, q, r)$

$$\therefore \frac{p+q+r}{3}\geq(pqr)^{\frac{1}{3}}$$

$(p+q+r)^3\geq 27\ pqr$

$(a+b+c)^3\geq 27\ (a+b-c)\ (a+c-b)\ (b+c-a)$

$$\frac{(a+b+c)^3}{(a+b-c)(a+c-b)(b+c-a)}\geq 27$$

∴ The minimum value is 27.

**9.** Let $m=p+q$ and $n=r+s$

$m+n=p+q+r+s=10$

$mn=(p+q)\ (r+s)$

We know that when the sum of two positive quantities is constant the product of these quantities is maximum when they are equal.

∴ $mn$ is maximum when $m=n=5$.

∴ Maximum $mn=25$

∴ $0<x\leq 25$

**10.** We know that A.M. $(p_1+p_1^2+p_1^3)\geq$ G.M $(p_1+p_1^2+p_1^3)$

$$=\frac{p_1+p_1^2+p_1^3}{3}\geq\sqrt[3]{p_1\ p_1^2\ p_1^3}$$

$p_1+p_1^2+p_1^3\geq 3(p_1p_1^2p_1^3)^3$

$p_1+p_1^2+p_1^3\geq 3\ p_1^2$

Similarly $p_2+p_2^2+p_2^3\geq 3\ p_2^2$

$p_n+p_n^2+p_n^3\geq 3\ p_n^2$

$(p_1+p_1^2+p_1^3)\ (p_2+p_2^2+p_2^3)\ \ldots..\ (p_n+p_n^2+p_n^3)$

$\geq 3^n(p_1p_2\ldots p_n)^2$

**11.** B: $p=0.3$, $q=0.2$ satisfies B.

C: $p=0.3$, $q=-0.2$ satisfies C.

Only B and C are possible.

**12.** As we are looking at only negative values of $x$, we take $x<0$

**Case 1:** $x\leq -4$:

$|x+4|+|x-7|<13$

$-x-4+7-x<13$; $-2x<10 \Rightarrow x>-5$

As $x\leq -4$, $x=-4$

**Case 2:** $-4<x<0$

$|x+4|+|x-7|<13$

$x+4+7-x<13$

$\Rightarrow 11<13$, this being always true, the inequality is true for $-4<x<0$, hence $x=-1, -2, -3$,

∴ $x$ can take four negative integral values

i.e., $-1, -2, -3, -4$.

**13.** $(p+2)\ (q+2)=pq+2p+2q+4$

$=2\ (p+q)+5$

We know that, A.M $(p, q) \geq$ G.M $(p, q)$

$$\frac{p+q}{2}\geq(pq)^{\frac{1}{2}}$$

$p+q\geq 2$

$2\ (p+q)\geq 4$

$2\ (p+q)+5\geq 4+5$

$2\ (p+q)+5\geq 9$

∴ $(p+2)\ (q+2)\geq 9$.

∴ the minimum value = 9.

**14.** Min $(x+6, x-3) = x-3$, for all values of $x$.
Min $(x+5, x-7) = x-7$, for all values of $x$.
$\therefore$ Required value $= \max(x-3, x-7)$
$= x-3$ for all values of $x$.

**15.** As Arithmetic mean $\geq$ Geometric mean

$$\frac{a^2+l^2}{2} \geq \sqrt{a^2 \times l^2} \Rightarrow a^2+l^2 \geq 2al$$

Similarly, $b^2+m^2 \geq 2bm$ and $c^2+n^2 \geq 2cn$
$\therefore a^2+l^2+b^2+m^2+c^2+n^2 \geq 2al+2bm+2cn$
$\Rightarrow (a^2+b^2+c^2)+(l^2+m^2+n^2) \geq 2al+2bm+2cn.$
Using the given condition, $3+3 \geq 2\ (al+bm+cn)$
$\Rightarrow al+bm+cn \leq 3$

**16.** $|x-a|$ is the distance of $x$ from $a$ on the number line
Let $|x-5|+|x+3|+|x+7| = 25$.
If $x$ is 7, $|x-5|+|x+3|+|x+7| = 26$.
At $x=-7$, E = 16
At $x=5$, E = 20
For $x>5$, if $x$ is increased by 1, E increased by 3.
For $x<-7$, if $x$ is decreased by 1, E increased by 3.
$\therefore$ At $x=-10$, E $= 16+|-10-(-7)| = 3 = 16+3(3) = 25$
At $x=6$, E $= 20+|7-6|3 = 23$
At $x=7$, E $= 10+2\ (3) = 26$.
If $x$ lies between $-7$ and 5, $|x+7|+|x-5|$ is 12.
For $|x-5|+|x+3|+|x+7|$ to be greater than 14,
$|x+3|$ must be greater than 2.
Thus, $x$ cannot lie in the range $-5 \leq x \leq -1$
Therefore, $x$ can take the values $-9, -8, -7, -6$ and 0, 1, 2, 3, 4, 5 and 6, i.e., $x$ can take 11 integral values.

**17.** For $ab+bc+ca = 15$, $ab \times bc \times ca$ is maximum.
When $ab = bc = ca = 15/3 = 5$
$\therefore$ Maximum value of $ab \times bc \times ca = 125$
$\Rightarrow$ Max $(a^2\ b^2\ c^2) = 125$
Max $(abc) = \sqrt{125} = 5\sqrt{5}$

**18.** $10 \leq x \leq 20$ and $2y-x=2$

Let $E = \dfrac{x}{x+y} = \dfrac{x}{x+(x+2)/2} = \dfrac{2x}{3x+2} = \dfrac{2}{3+2/x}$

Here, $E$ is maximum when $\left(3+\dfrac{2}{x}\right)$ is minimum, $\left(3+\dfrac{2}{x}\right)$ is minimum when $x$ is maximum, i.e., $x=20$.

$\therefore E_{\text{Max}} = \dfrac{2}{3+0.1} = \dfrac{20}{31}$

**19.** The given expression is of the form
$(x+y+z)\ (1/x+1/y+1/z)$

Now $(x+y+z)\left(\dfrac{yz+zx+xy}{xyz}\right)$

$$= \frac{(x+y+z)(yz+zx+xy)}{(xyz)}$$

since A.M $\geq$ G.M, $x+y+z \geq 3\sqrt[3]{xyz}$

and $yz+zx+xy \geq 3\sqrt[3]{x^2y^2z^2}$

$$\therefore \frac{(x+y+z)(yz+zx+xy)}{(xyz)} \geq \frac{9\sqrt[3]{(xyz)^3}}{xyz} \geq 9$$

**20.** Let $a = p+q-r$, $b = p+r-q$ and $c = r+q-p$
From the given data $a$, $b$, $c$ are distinct and positive
Applying AM > HM ($\because$ $a$, $b$, $c$ are distinct and therefore AM $\neq$ HM

$$\therefore \frac{a^{-1}+b^{-1}+c^{-1}}{3} > \left(\frac{a+b+c}{3}\right)^{-1}$$

$$\Rightarrow a^{-1}+b^{-1}+c^{-1} > 3 \cdot \frac{3}{a+b+c}$$

$$\Rightarrow a^{-1}+b^{-1}+c^{-1} > \frac{9}{p+q+r} (\because a+b+c = p+q+r)$$

$$\therefore \frac{1}{p+q-r}+\frac{1}{p+r-q}+\frac{1}{r+q-p} > \frac{9}{p+q+r}$$

i.e., $E > 9/(2s)$.

**21.** $7|x| \leq 6.93x$
If $x \leq 0$,
$-7x \leq 6.93x$
There is only one solution ie, $x=0$
If $x \geq 0$,
$7x \leq 6.93\ x$ here too, there is only one solution ie, $x=0$.
$\therefore$ The given inequation has only one solution, $x=0$

**22.** Given $3x+4y+5z = 27$

Consider the expression $\left(\dfrac{3x}{2}\right)^2\left(\dfrac{4y}{3}\right)^3\left(\dfrac{5z}{4}\right)^4$ (1)

The sum of the all factors of the above expression is

$$\frac{3x}{2}+\frac{3x}{2}+\frac{4y}{3}+\frac{4y}{3}+\frac{4y}{3}+\frac{5z}{4}+\frac{5z}{4}+\frac{5z}{4}+\frac{5z}{4}$$

$= 3x+4y+5z = 27$ which is constant
$\therefore$ The expression (1) is maximum when factors are equal

$$\therefore \frac{3x}{2} = \frac{4y}{3} = \frac{5z}{4} = \frac{27}{9} = 3$$

$\Rightarrow \dfrac{3x}{2} = 3 \ \therefore x = 2, \ \dfrac{4y}{3} = 3 \ \therefore y = \dfrac{9}{4}$ and $\dfrac{5z}{4} = 3$

$\therefore z = \dfrac{12}{5}$

$\therefore$ When $x=2$; $y = \dfrac{9}{4}$, $z = \dfrac{12}{5}$, $x^2y^3z^4$ is maximum.

The maximum value of $x^2\ y^3\ z^4$ is $= 2^2\left(\dfrac{9}{4}\right)^3\left(\dfrac{12}{5}\right)^4$

$$= \frac{\left(2^4\right)3^{10}}{5^4}$$

**23.** $3x^2-x-24 \leq 0$
$\Rightarrow (3x+8)\ (x-3) \leq 0$
$\Rightarrow -8/3 \leq x \leq 3$ (1)
$(3x^2-14x-24)\ (3x^2-x-24) \geq 0$

Since $(3x^2 - x - 24) \le 0$ it follows that
$3x^2 - 14x - 24 \le 0$
$\Rightarrow (3x + 4)\ (x - 6) \le 0$
$\Rightarrow -4/3 \le x \le 6$ (2)
$x$ has to satisfy both (1) and (2)
$\Rightarrow -4/3 \le x \le 3$
Hence, the maximum value $x$ can take is 3.

**24.** Given $|3x-1| - |x+2| + 5 < 0$ (1)

When $x > \frac{1}{3}$, $(3x-1) > 0 \Rightarrow |3x-1| = 3x-1$ and $x+2 > 0$
$\Rightarrow |x+2| = x+2$ and A becomes
$3x - 1 - x - 2 + 5 < 0$
$\Rightarrow 2x + 2 < 0 \Rightarrow x < -1$
As $x > \frac{1}{3}$ (assumption) there is no solution.
When $x < -2$, $3x - 1 < 0 \Rightarrow |3x-1| = -(3x-1)$ and $x + 2 < 0 \Rightarrow |x+2| = -(x+2)$
$\therefore$ A becomes $-(3x-1) + (x+2) + 5 < 0$
$-3x + 1 + x + 2 + 5 < 0 \Rightarrow -2x + 8 < 0 \Rightarrow x > 4$
As $x < -2$, there is no solution
When $-2 < x < \frac{1}{3} \Rightarrow 3x - 1 < 0 \Rightarrow |3x-1|$
$= -(3x-1)$ and $(x+2) > 0 \Rightarrow |x+2| = x+2$
$\therefore$ A becomes $-(3x-1) - (x+2) + 5 < 0$
$\Rightarrow -3x + 1 - x - 2 + 5 < 0 \Rightarrow -4x + 4 < 0 \Rightarrow x > 1$
As $-2 < x < \frac{1}{3}$, there is no solution
$\therefore$ There exists no solution for A.

**25.** $x + \frac{4}{x} = m$ has real roots.
$\therefore x^2 - mx + 4 = 0$ has real roots.
$\Rightarrow m^2 - 16 \ge 0$ (discriminant $\ge 0$)
$(m+4)\ (m-4) \ge 0$
$\Rightarrow m \le -4$ or $m \ge 4$
$\therefore$ The minimum positive value of $m$ is 4.

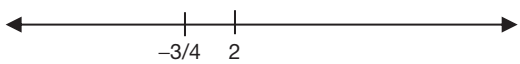
## EXERCISE-5

**1.** Using Statement I,
Suppose $y = 4$, $x^2 > 64$, $x > 8$ or $x < -8$
If $x > 8$, then $x > y$.
If $x < -8$, then $x < y$.
Statement I is not sufficient.
Using Statement II,
Suppose $x = 9$, $y^2 > 729$, $y > 27$ or $y < -27$
If $y > 27$, then $x < y$.
If $y < -27$, then $x > y$.
Statement II is not sufficient.
Using both statements,
Suppose $x = y = \frac{1}{2}$, $x^2 > y^3$ and $y^2 > x^3$ would hold true.
Suppose $x = -\frac{1}{2}$ and $y = \frac{1}{2}$, $x^2 > y^3$ and $y^2 > x^3$ would hold true. In this case, $x < y$. We cannot answer the question. Both statements even when taken together are not sufficient to answer the question.

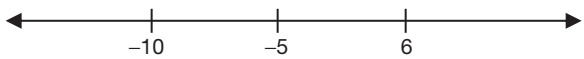
**2.** Using Statement I, $x^2 - x^3 > 0$ and $x^2(1 - x) > 0$.
As $x^2 > 0$, $1 - x > 0$
$x < 1$
Statement I is sufficient.
Using Statement II, $x^3 - x > 0$, $x(x^2 - 1) > 0$, $x > 0$ and $x^2 - 1 > 0$, (i.e., $x > 1$) or $x < 0$ and $x^2 - 1 < 0$ (i.e., $-1 < x < 0$)
We cannot answer the question.
Statement II is not sufficient.

**3.** Neither of the statements alone is sufficient. Combining the two statements, we get
$|x-2y| \le 5$ and $|y+3z| \le 9$ $|x-2y| \le 5$
$\Rightarrow -5 \le x - 2y \le 5$ (1)
$|y+3z| \le 9 \Rightarrow -9 \le y + 3z \le 9$ (2)
$(2) \times 2 \Rightarrow -18 \le 2y + 6z \le 18$ (3)
Adding Eq. (1) and Eq. (3), we get
$-5 - 18 \le x + 6z \le 23$
$-23 \le x + 6z \le 23$
$\therefore$ The maximum value of $|x+6z|$ is 23.
By combining both statements, we can answer the question.

**4.** $|3x+5| < 5x+1$
$-(5x+1) < 3x+5 < 5x+1$
$-5x - 1 < 3x + 5 < 5x + 1$
$-5x - 1 < 3x + 5$ and $3x + 5 < 5x + 1$
$-8x < 6$ and $3x - 5x < -4$
$x > -\frac{6}{8}$ and $-2x < -4$
$x > -\frac{3}{4}$ and $x > 2$




When $x > 2$ $|3x+5| < 5x+1$ and $x < 2$ $|3x+5| > (5n+1)$
Now from Statement I, $(x+5)\ (x+10)\ (x-6) > 0$.




$x \in (-10, -5) \cup (6, \infty)$
$\therefore$ Statement I is not sufficient to answer the question.

From Statement II, $8x^2 - 10x - 12 > 0$.

$$4x^2 - 5x - 6 > 0$$
$$4x^2 - 8x + 3x - 6 > 0$$
$$4x(x-2) + 3(x-2) > 0$$
$$(4x+3)(x-2) > 0$$
$$x \in \left(-\infty, \frac{-3}{4}\right) \cup (2, \infty)$$

$\therefore$ Statement II is also not sufficient to answer the question. By combining both the statements, we get $x \in (-10, -5) \cup (6, \infty)$.

$\therefore$ The question cannot be answered even by combining both the statements also.

**5.** From Statement I, $x = 2$ or $-9$.

When $x = 2$, $\left|\frac{x+5}{x+7}\right| = 7/9$ and when $x = -9$, $\left|\frac{x+5}{x+7}\right| = 2$.

$\therefore$ Statement I alone is not sufficient.

From Statement II, $x = \frac{-19}{3}$ or $-9$. In either case, $\left|\frac{x+5}{x+7}\right| = 2$.

$\therefore$ Statement II alone is sufficient to answer the question.

**6.** Either of the statements alone is not sufficient to answer the question.

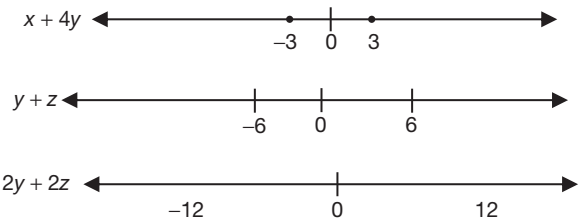
Now, from Statement I, $|x+4y| < 3$.

$\Rightarrow -3 < x + 4y < 3 \Rightarrow x + 4y > -3$ and $x + 4y < 3$

From Statement II, $|y+z| > 6$.

$\Rightarrow y + z < -6$ or $y + z > 6 \Rightarrow 2y + 2z < -12$ or $2y + 2z > 12$

We can represent the possible values $x + 4y$, $y + z$ and $2y + 2z$ on the number line as shown below.




The minimum value of $|(x+4y)-(2y+2z)|$ is 9, and it occurs when $x + 4y = -3$ and $2y + 2z = -12$ or $x + 4y = 3$ and $2y + 2z = 12$.

$\therefore |x+2y-2z|$ is always greater than 9.

By combining the two statements, we can answer the question.

**7.** Using Statement I, $x - x^2 > 0 \Rightarrow x(1 - x) > 0$

$x > 0$ and $1 - x > 0$ or $x < 0$ and $1 - x < 0$

If $x > 0$ and $1 - x > 0$, then $0 < x < 1$.

If $x < 0$ and $1 - x < 0$, then $x$ has no possible value.

$\therefore 0 < x < 1$

Statement I is sufficient.

Using statement II, $x^2 - x^4 > 0$

$x^2(1 - x^2) > 0$

As $x^2 > 0$, $1 - x^2, > 0$

$-1 < x < 1$

We cannot say whether $x < 0$ or not. Statement II is not sufficient.

**8.** Using Statement I, $x > \sqrt{x} \geq 0$ (1)

$\therefore 1 < x$. $\therefore$ Statement I is sufficient.

Using Statement II, $x > 1$. $\therefore$ Statement II is sufficient.

Either of the statements alone is sufficient.

**9.** Using statement I, $(x^2)^2 - 1 > 0$

$(x^2 - 1)(x^2 + 1) > 0$

$\therefore x^2 - 1 > 0$; $\therefore x^2 > 1$

Statement I is sufficient.

Using statement II, $\sqrt[4]{x}\,(\sqrt[4]{x} - 1) > 0$

$\sqrt[4]{x}, > 0$ and $\sqrt[4]{x} - 1 > 0$

$\therefore x > 1$ and $x^2 > 1$

$\therefore$ Statement II is sufficient.

**10.** $|x+1|$ represents the distance of $x$ from $-1$ on the number line.

$|x + 4|$ is the distance of $x$ from $-4$. If $x \geq -1$, then the expression $E = |x + 1| - |x + 4|$ is $-3$. If $x \leq -4$, then $E = 3$. If $-4 < x < -1$, then $x + 1 < 0$ and $|x + 1| = -x - 1$, whereas $x + 4 > 0$ and $|x + 4| = x + 4$.

$\therefore E = (-x-1) - (x+4) = -2x - 5 = -(2x + 5)$

$\therefore -3 \leq E \leq$ and hence $10 \leq 13 + E \leq 16$.

From Statement I, $13 + E$ could be 16 or 10 (or some intermediate value). We cannot answer the question.

From Statement II, $E_{\min} = 3$ and $13 + E_{\min} = 16$. We can answer the question.

**11.** We know that for any three positive real numbers $x$, $y$ and $z$, A.M. $(x, y, z) \geq$ H.M. $(x, y, z)$. As for any three non-zero real numbers $a$, $b$ and $c$, the numbers $a^2$, $b^2$ and $c^2$ are positive, we have A.M. $(a^2, b^2, c^2) \geq$ H.M. $(a^2, b^2, c^2)$.

$$\Rightarrow \frac{a^2+b^2+c^2}{3} \geq \frac{3}{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}$$

$$\Rightarrow (a^2 + b^2 + c^2)\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right) \geq 9$$

Hence, either of the statements alone is sufficient to answer the question.

**12.** Clearly, either of the statements alone is not sufficient.

We have $\frac{|y|}{|x|+|y|} = \frac{1}{1+\frac{|x|}{|y|}}$.

So, $\frac{|y|}{|x|+|y|}$ will be minimum.

The maximum value of $\frac{|x|}{|y|}$ is $\frac{4}{2} = 2$.

So, the minimum value of $\frac{|y|}{|x|+|y|}$ is $\frac{1}{1+2}=\frac{1}{3}$.

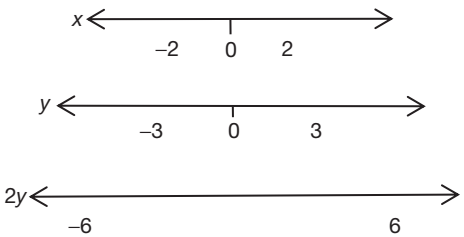
Hence, both Statements I and II together are sufficient to answer the question.

**13.** $|x-8| \le k \Rightarrow -k \le x-8 \le k \Rightarrow -k+8 \le x \le k+8$
When $k=3$, we have $5 \le x \le 11$. There are 7 integer values of $x$ satisfying the above inequation. Also, for any value of $k$ such that $3 \le k < 4$, the inequation $|x-8| \le k$ has 7 integer solutions.

So, Statement I alone is not sufficient. Also, Statement II alone is also not sufficient. But, by combining both I and II, we get $k=3$. Hence, both Statements I and II together are sufficient to answer the question.

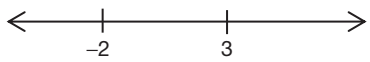
**14.** From Statement I, $x^2-3x+2=0$.
$\Rightarrow (x-1)\ (x-2)=0$
$\Rightarrow x=1$ or 2
$\Rightarrow |x+2|=3$ or 4
$\therefore$ Statement I alone is not sufficient.
From statement II, $x^2+4x-5=0$.
$\Rightarrow (x+5)\ (x-1)=0 \Rightarrow x=1$ or $-5$
When $x=1$, $|x+2|=3$ and when $x=-5$, $|x+2|=3$
$\therefore$ Statement II alone is sufficient.

**15.** Clearly, either of the statements alone is not sufficient to answer the question.
Now, $|x|<2 \Rightarrow -2<x<2$
$\Rightarrow x>-2$ and $x<2$
and $|y|>3 \Rightarrow y<-3$ or $y>3$
$\Rightarrow 2y<-6$ or $2y>6$
We can represent the possible values of $x$, $y$ and $2y$ on three number lines as shown below.




We can see that the minimum value of $|x-2y|$ occurs when $x=2$ and $2y=6$ or when $x=-2$ when $2y=-6$ and this minimum value is 4.
$\therefore |x-2y|$ is always greater than 4. By combining the two statements, we can answer the question.

**16.**




When $x<-2$, $|x-3|>5$
$\Rightarrow |x+2|+|x-3|>5$
When $x>3$, $|x+2|>5$
$\Rightarrow |x+2|+|x-3|>5$
When $-2 \le x \le 3$, $|x+2|+|x-3|=5$
So, the minimum value of $|x+2|+|x-3|$ is 5.
Clearly, either of the statements alone is sufficient.

**17.** From statement I, $(x-3)^2+(y-4)^2=0$.
$\Rightarrow x=3$ and $y=4 \Rightarrow |x-y|=1$
From statement II, $|x-3|+|3+4|=0$.
$\Rightarrow x=3;\ y=4 \Rightarrow |x-y|=1$.
$\therefore$ Either of the statements alone is sufficient to answer the question.

**18.** We know that, for any positive real number $x$, A.M. $\left(x, \frac{1}{x}\right)$ $\ge$ G.M. $\left(x, \frac{1}{x}\right)$.

$$\frac{x+\frac{1}{x}}{2} \ge \sqrt{x \cdot \frac{1}{x}} \Rightarrow x+\frac{1}{x} \ge 2$$

So, the minimum value of $x+\frac{1}{x}$ is 2.
Clearly, either of the statements alone is sufficient to answer the question.

**19.** From statement I, $x^2-2x-15<0$.
$\Rightarrow (x-5)\ (x+3)<0 \Rightarrow x \in (-3, 5)$
$\Rightarrow x-1<(-4, 4) \Rightarrow |x-1|<4$
$\therefore$ Statement I alone is sufficient.
From statement II, $x(x-4)>0$.
$\Rightarrow x<0$ or $x>4$
$\Rightarrow x-1<-1$ or $x-1>3$
$\Rightarrow |x-1|>1$ or $|x-1|>3$
i.e., $|x-1|>1$
$\therefore$ Statement II alone is not sufficient.

**20.** $|a|>|b| \Rightarrow a>b$ or $a<b$
$\therefore$ Statement I alone is not sufficient.
$\frac{1}{3}>1 \Rightarrow a>b$ or $a<b$
$\therefore$ Statement II alone is not sufficient.
When $a=-2$ and $b=-1$, $|a|>|b|$ and $\frac{a}{b}>1$.
In this case, $a<b$.
$\therefore$ Both Statements I and II together are also not sufficient and some additional information is required to answer the question.

# 6 Functions

CHAPTER

## Learning Objectives

*In this chapter, you will*:

- Learn all about Sets, including
  - Defining different types of sets
  - Basic set operations
  - Standard Rules
- Understand all about Relations, such as:
  - Domain and range of a relation
  - Inverse of a relation
- Gain complete knowledge about Functions, including
  - Types of functions
    - One–one, Many–one functions
    - Onto, Into functions
  - Operations on functions
    - Inverse of a function
    - Composition of functions
  - Properties of functions
    - Even and odd functions
    - Periodic functions
    - Increasing and decreasing functions
  - Standard functions
    - Polynomial functions
    - Modulus functions
    - Signum functions
    - Trigonometric functions
  - Functional equations

## ❒ SETS

A SET is a well defined collection of objects. The objects of the set are called its elements. Sets are usually denoted by capital letters and the elements of the set are denoted by lower case letters. If an element $x$ belongs to set $A$, it is denoted by $x \in A$. If $x$ is not an element of $A$, it is denoted by $x \notin A$.

$A$ set, in general, is represented in two forms:

1. **Roster Form:** In this form, a set is described by actually listing out the elements. For example, the set of all even natural numbers less than 12 is represented by {2, 4, 6, 8, 10}
2. **Set Builder Form:** In this form, a set is described by a characterizing property. For example, the set of all even natural numbers less than 12 is represented by $\{x \in N \mid x < 12$ and $x$ is even$\}$. The symbol $\mid$ is read as 'such that.'

## ❒ Some Definitions

**Null Set:** $A$ set is said to be a null set if it has no elements. It is also called an empty set or a void set and is denoted by $\phi$.

**Examples:**

(i) $\{x \mid x$ is an integer, $1 < x < 2\}$

(ii) $\{x \mid x$ is a real number and $x^2 < 0\}$

**Finite and Infinite sets:** $A$ set '$A$' is said to be finite if it is either an empty set or contains a finite number of elements. Otherwise, it is said to be infinite.

**Examples:**

(i) The set of vowels in the English alphabet is finite.

(ii) The set of natural numbers is infinite.

**Cardinality of a Set:** The number of elements in a set is called cardinality (or order) of the set. If a finite set $A$ has $n$ elements, the cardinality of the set is $n$ and is denoted by $O(A)$ or $n(A)$. The cardinality of the empty set is 0.

**Example:**
The cardinality of $A = \{x, y, z, t\}$ is 4.

**Singleton Set:** A set consisting of a single element is called singleton set, i.e., a set whose cardinality is 1 is a singleton set.

**Examples:**
(i) {3}
(ii) {a}

**Equal Sets:** Two sets $A$ and $B$ are said to be equal if they have the same elements, i.e., if every element of $A$ is an element of $B$ and every element of $B$ is an element of $A$.

**Subsets and Supersets:** Let $A$ and $B$ be two sets. If every element of $A$ belongs to $B$, then $A$ is said to be a subset of $B$. This is written as $A \subseteq B$.

$\therefore A \not\subset B$ means $A$ is not a subset of $B$. If $A$ is a subset of $B$ and there is at least one element in $B$ that is not there in $A$, $A$ is said to be a proper subset of $B$. This is written as $A \subset B$.

$\therefore A \not\subset B$ means $A$ is a not a proper subset of $B$.

**NOTE**

$A \subset B \Rightarrow A \subseteq B$. But the converse is not true.

If $A$ is a subset of $B$ we say that $B$ contains $A$ or $B$ is a superset of $A$. This is written as $B \supseteq A$

**NOTE**

1. Every set is a subset of itself.
2. The empty set is a subset of every set.
3. If $A$ is a finite set of cardinality $n$, then the total number of subsets of $A$ is $2^n$.

**Power Set:** If $A$ is any set, then the set of all subsets of $A$ is called power set of $A$ and is denoted by $P(A)$, i.e., $P(A) = \{S \mid S \subseteq A\}$

**Example:** If $A = \{1, 2, 3\}$, then $P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

**NOTE**

1. $A, \phi \in P(A)$.
2. If $A$ is a finite set having $n$ elements, then the cardinality of $P(A)$ is $2^n$.

**Universal Set:** The set that contains all the elements in a given context is called Universal Set. It is denoted by $\mu$ or $U$.

**Examples:**

(i) In Plane Geometry, the set of all points in the plane is the universal set.

(ii) In the context of divisibility tests, the set of natural numbers is the universal set.

## ❒ Basic Set Operations

**Union of Sets:** If $A$ and $B$ are two sets, the union of $A$ and $B$ is the set of all those elements which belong to either $A$ or $B$ or both $A$ and $B$. This is denoted by $A \cup B$.

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

**NOTE**

1. If $A \subseteq B$, then $A \cup B = B$
2. $A \cup \phi = A$
3. $A \cup \mu = \mu$

**Intersection of Sets:** Let $A$ and $B$ be two sets. The intersection of $A$ and $B$ is the set of all those elements that belong to both $A$ and $B$. It is denoted by $A \cup B$.

$A \cup B = \{x \mid x \in A \text{ and } x \in B\}$

**NOTE**

1. $A \subseteq B$, then $A \cap B = A$.
2. $A \cap \phi = \phi$.
3. $A \cap \mu = A$.

**Disjoint Sets:** Two sets $A$ and $B$ are said to be disjoint if they have no element in common.

$\therefore$ If $A$ and $B$ are disjoint, then $A \cup B = \phi$.

**Difference of Sets:** The difference of two sets is the set of all elements which are there in one set but not in the other.

Let $A$ and $B$ be two sets. $A - B$ is the set of all those elements of $A$ which do not belong to $B$.

$A - B = \{x \mid x \in A \text{ and } x \notin B\}$

Similarly, $B - A = \{x / x \in B \text{ and } x \notin A\}$.

**Example:** $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 8, 10\}$

$A - B = \{1, 2\}$
$B - A = \{8, 10\}$

**Complement of a Set:** The complement of set $A$ is the set of all those elements that do not belong to set $A$, i.e., the complement of a set $A$ is the difference of the universal set and set $A$ and is denoted by $A'$ or $Ac$.

**Example:** If $\mu$ is the set of natural numbers, $A$ is the set of even natural numbers, then $A'$ is the set of odd natural numbers.

**Symmetric Difference of Two Sets:** Let $A$ and $B$ be two sets. The symmetric difference of the sets $A$ and $B$ is the set $(A - B) \cup (B - A)$ and is denoted by $A \Delta B$.

**Example:** $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, then $A \Delta B = \{1, 2, 5, 6\}$

**Some Results:**

1. $A \cup A = A$; $A \cup A = A$
2. $A \cup B = B \cup A$
3. $A \cup B = B \cup A$
4. $A \cup (B \cup C) = (A \cup B) \cup C$
5. $A \cup (B \cup C) = (A \cup B) \cup C$
6. $A \cup (B \cup C) = (A \cup B) \cup (A \cup C)$
7. $A \cup (B \cup C) = (A \cup B) \cup (A \cup C)$
8. $C - (C - A) = C \cup A$
9. $C - (A \cup B) = (C - A) \cup (C - B)$
10. $C - (A \cup B) = (C - A) \cup (C - B)$
11. $(Ac)c = A$
12. $(A \cup B)c = Ac \cup Bc$
13. $(A \cup B)c = Ac \cup Bc$

**Cartesian Product of Two Sets:** Let $A$ and $B$ be any two sets. Then, the Cartesian Product of $A$ and $B$ is the set of all ordered pairs $(a, b)$ where $a \in A$ and $b \in B$. The product is denoted by $A \times B$

$A \times B = \{(a, b) \mid a \in A, b \in B\}$

**Example:** $A = \{1, 2, 3\}$ $B = \{a, b\}$ then

$A \times B = \{(1, a), (2, a), (3, a), (1, b), (2, b), (3, b)\}$
$B \times A = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

**NOTE**

1. If $A$ and $B$ are two sets such that $n(A) = x$ and $n(B) = y$, then the number of ordered pairs in $A \times B$ is $xy$
2. $A \times B$ is not necessarily equal to $B \times A$
3. $n(A \times B) = n(B \times A)$

## RELATIONS

Any subset of $A \times B$ is a relation from $A$ to $B$. A relation pairs up elements of $A$ with elements of $B$. If $a$ from $A$ is paired with $b$ from $B$ in a relation $R$, then we write $(a, b) \in R$ or $aRb$.

$A$ subset of $A \times A$ is a relation in the set $A$.

**Example:** $A = \{1, 2, 3\}$, $B = \{a, b\}$, then $R = \{(1, a) (3, b)\}$ is a possible relation.

**Domain and Range of a Relation:** The set of all first co-ordinates of the ordered pairs of a relation $R$ is called Domain of $R$ and the set of all second coordinates of the ordered pairs of $R$ is called range of $R$, i.e., if $R$ is a relation from $A$ to $B$, then Domain of $R = \{x / (x, y) \in R$ for some $y\}$ and Range of $R = \{y / (x, y) \in R$ for some $x\}$. The domain and range of $R$ are denoted as Dom $R$ and Range $R$.

**Inverse of a Relation:** Let $A$ and $B$ be two sets and let $R$ be a relation from $A$ to $B$. Then, the inverse of $R$ denoted by $R^{-1}$ is a relation from $B$ to $A$ and is defined by

$R^{-1} = \{(b, a) \mid (a, b) \in R\}$

We note that Domain of $R$ = Range of $R^{-1}$ and Range of $R$ = Domain of $R^{-1}$

## FUNCTIONS

A relation $f$, which associates to each element of a set $A$ exactly one element of set $B$ is called function from $A$ to $B$ and is denoted as $f: A \to B$ (read as $f$ maps $A$ to $B$). If $B = A$, then $f$ is said to be a function from $A$ to $A$ (or in $A$)

If $(a, b) \in f$, then '$b$' is called image of '$a$' under $f$. This is written as $b = f(a)$ and '$a$' is called pre-image of '$b$'.

If $f: A \to B$, then $A$ is called domain of $f$ and $B$ is called co-domain of $f$ and the set $R = \{f(a) / a \in A\}$ is called range of $f$. The range of $f$ is also called image of $f$ and is denoted by Im $(f)$ or $f(A)$

**NOTE**

1. Range $\subseteq$ Co-domain.
2. $f \subseteq A \times B$.
3. Every element of $A$ has a unique $f$-image in $B$.
4. Two or more elements of $A$ can have the same $f$-image in $B$.
5. There may be elements in $B$ which are not $f$-images of any element of $A$.
6. The number of functions from a set $A$ containing $m$ elements to another set $B$ containing $n$ elements is $n^m$.

**One–One Function (Injection):** $A$ function $f: A \to B$ is called one–one function if distinct elements of $A$ have distinct images in $B$. One–One functions are also called injective functions or injections.

i.e., $f: A \to B$ is one–one if $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$ equivalently $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

**NOTE**

1. $A$ one–one function is possible from $A$ to $B$ if $n(A) \leq n(B)$.
2. If $n(A) = m$, $n(B) = n$ and $m \leq n$, then the number of one to one functions is ${}^nP_m$.
3. If $n(A) = m$ and $n(B) = 2$, there are $2^m - 2$ onto functions from $A$ to $B$.

**Many–One Function:** A function which is not one–one is called many–one function.

**Onto Function (Surjection):** $A$ function $f: A \to B$ is called an onto function if every element of $B$ is an image of at least one element of $A$, i.e., $f: A \to B$ is onto, if for each $y \in B$, there exists $x \in A$ such that $f(x) = y$. Onto functions are also called surjective functions or surjections.

**NOTE**

1. If $f$ is onto, Range of $f$ = co-domain of $f$.
2. An onto function is possible from $A$ to $B$ if $n(A) \geq n(B)$.
3. If $n(A) = m$, $n(B) = n$, then the number of onto functions is $n^m - {}^nC_1 (n-1)^m + {}^nC_2 (n-2)^m ....+...(n \leq m)$

**Into Function:** $A$ function which is not onto is called into function, i.e., $f: A \to B$ is into if Range of $f$ is a proper subset of $B$.

**Bijection:** If a function is both one–one and onto, then it is called bijective function or bijection.

**NOTE**

1. $A$ bijection from $A$ to $B$ is possible if $n(A) = n(B)$.
2. If $A$ is a set with $n$ elements, the number of bijections from $A$ to $A$ (or in $A$) is $n!$.

**Constant Function:** $A$ function $f: A \to B$ is said to be a constant function if $f(x) = k$, for all $x \in A$, where $k$ is a fixed element of $B$.

**NOTE**

Range of $f = \{k\}$

**Identity Function:** The function $f: A \to A$ defined by $f(x) = x$ is called identity function, denoted by $IA$.

**NOTE**

1. For the identity function, Range = Domain. Symbolically, $I_A(x) = x$ for $x \in A$.
2. The identity function is a bijection.

**Inverse Function:** If $f: A \to B$ is a bijective function, then the function $f^{-1}: B \to A$, where $f^{-1} = \{(b, a)/(a, b) \in f\}$ is called the inverse of the function $f$.

**NOTE**

1. If $f: A \to B$ is bijective, $f^{-1}: B \to A$ is also bijective
2. If $f(a) = b$, then $a = f^{-1}(b)$

**Composition of functions:** If $f: A \to B$ and $g: B \to C$ are two functions, $g(f(a))$ for every $a \in A$ and is called composite function of $f$ and $g$. It is denoted as $gof$, which is read as $g$ circle $f$.

**NOTE**

1. If $f: A \to A$, then $(fof^{-1})(x) = x$ or $fof^{-1} = I$.
2. If $A$ is a finite set $f: A \to A$ is injective, then $f$ is bijective. But if $A$ is infinite this need not be true.
3. If $f: A \to B$ is bijective, then $f^{-1}of = I_A$ and $fof^{-1} = I_B$.
4. If $f: A \to B$ and $g: B \to A$ are two functions such that $gof = I_A$ and $fog = I_B$, then $g = f^{-1}$.
5. If $f: A \to B$, $g: B \to C$ are two bijective functions, then $(gof)^{-1} = f^{-1}og^{-1}$.
6. If $f: A \to B$ and $g: B \to C$ are two functions then,
   (i) $f$ and $g$ are injective $\Rightarrow$ $gof$ is injective.
   (ii) $f$ and $g$ are surjective $\Rightarrow$ $gof$ is surjective.
   (iii) $f$ and $g$ are bijective $\Rightarrow$ $gof$ is bijective.
   (iv) If $gof$ is injective, then '$f$' must be injective but '$g$' need not be.
   (v) If $gof$ is surjective, then '$g$' must be surjective but '$f$' need not be.
   (vi) If $gof$ is bijective, then '$f$' must be injective and g' must be surjective.
7. If $h: A \to B$, $g: B \to C$ and $f: C \to D$ be any three functions, then $fo(goh) = (fog)oh$.

**Real Function:** If $A$ is a non-empty subset of $\boldsymbol{R}$, then a function $f: A \to R$ is called real valued function of a real number (or a real function)

**Operations on Real Functions:** If $f: D_1 \to \boldsymbol{R}$ and $g: D_2 \to \boldsymbol{R}$ and $D = D_1 \cup D_2$, then we can define

1. $f \pm g: D \to \boldsymbol{R}$ such that for all $x \in D$, $(f \pm g)(x) = f(x) \pm g(x)$
2. $fg: D \to \boldsymbol{R}$ such that for all $x \in D$, $(fg)(x) = f(x) \cdot g(x)$
3. $f/g: D \to \boldsymbol{R}$ such that for all $x \in D$, $f/g(x) = f(x)/g(x)$ provided $g(x) \neq 0$
4. for some constant $c$, $(cf): D_1 \to \boldsymbol{R}$ such that for all $x \in D_1$, $(cf)(x) = cf(x)$.

**NOTE**

Domain of $f \pm g$, $fg$, $f/g$, is $D_1 \cap D_2$ where $D_1$ is the domain of $f$ and $D_2$ is domain of $g$.

## ❒ Ways of expressing Functions

**Explicit and Implicit Functions:** If the relation between the variables is of the form $y = f(x)$, then $y$ is an explicit function of $x$. Similarly $x$ is an explicit function of $y$ if $x = f(y)$.

**Examples:**

(i) $x = y^2 + 3$

(ii) $y = \sin x + x^3 + 2$

*A* function which is expressed implicitly is called an implicit function. *A* function in two variables $x$, $y$ is of the form $f(x, y) = 0$.

**Examples:** (i) $x^2 + exy \log y = \sin x$; (ii) $x^2 + y^2 = a^2$. Here, we can think of $y$ as an implicit function of $x$ or $x$ as an implicit function of $y$.

## ❒ Some Properties of Functions

1. **Even and Odd Functions:** A function $f(x)$ is said to be an even function if for any number $x$ in its domain, $-x$ is also there in its domain and $f(-x) = f(x)$.

**Examples :** (i) $x^2 + 5$ (ii) $\cos x$

*A* function $f$ is said to be odd if for any number $x$ in its domain, $-x$ is also in its domain and $f(-x) = -f(x)$.

**Examples :** (i) $x^3$ (ii) $\sin x$

**NOTE**

Only certain functions are either even or odd. The others are neither even nor odd. This property (of being even or odd) is called parity of functions which has to be carefully distinguished from the property of parity of numbers. If *f*, *g* are two functions, the following table shows how the parity of $f \pm g$, *fg*, *f*/*g* *gof* and *fog* depends on the parity of *f* and *g*. (*e* denotes even and *o* denotes odd)

| *f* | *g* | $f+g$ | $f-g$ | *fg* | *f*/*g* | *gof* | *fog* |
|---|---|---|---|---|---|---|---|
| *e* | *e* | *e* | *e* | *e* | *e* | *e* | *e* |
| *e* | *o* | – | – | *o* | *o* | *e* | *e* |
| *o* | *e* | – | – | *o* | *o* | *e* | *e* |
| *o* | *o* | *o* | *o* | *e* | *e* | *o* | *o* |

The student should construct the corresponding table for $x \pm y$, $xy$, $x/y$ where $x$ and $y$ are integers and carefully note the points at which the two tables differ.

2. **Periodic Functions:** A function $f(x)$ is said to be periodic if there exists a non-negative real number $T$ such that $f(x + T) = f(x)\ \forall\ x \in R$. If $T$ is the smallest positive real number such that $f(x + T) = f(x)$ for all $x \in R$, then $T$ is called fundamental period of $f(x)$. Any integral multiple of $T$ is also a period. But when we speak of the period, without further description, we refer to the fundamental period.

**NOTE**

(i) If $f(x)$ is a periodic function with period $T$, then the function $f(ax + b)$, where $a > 0$ is periodic with period $T/a$. The period of both $\sin x$ and $\cos x$ is $2\pi$ and the period of $\tan x$ is $\pi$

(ii) The constant function is a periodic function without a fundamental period.

3. **Increasing and Decreasing Functions:** Let $f(x)$ be a function defined in a certain domain $D$. If for all $a$, $b$ in $D$, where $a < b$, $f(a) \le f(b)$, then $f(x)$ is said to be an increasing function, and if $f(a) < f(b)$, then $f(x)$ is said to be strictly increasing.

Consider the following two functions defined on $[0, 5]$

$f(x) = 2x$ for $0 \le x \le 5$

$g(x) = 2x$ for $0 \le x \le 3$

$= 6$ for $3 \le x \le 5$

We see that $f(x)$ is strictly increasing in $[0, 5]$, while $g(x)$ is an increasing function in $[0, 5]$. (It is strictly increasing in $[0, 3]$)

Let $f(x)$ be a function defined on a certain domain $D$. If for all $a$, $b$ in $D$, where $a < b$, $f(a) \ge f(b)$, then $f(x)$ is said to be a decreasing function and if $f(a) > f(b)$, then $f(x)$ is said to be a strictly decreasing function.

Let $F(x) = -f(x)$, $G(x) = -g(x)$, where $f(x)$ and $g(x)$ are the functions defined above. $F(x)$ is a strictly decreasing function in $[0, 5]$ while $G(x)$ is a decreasing function. (It is strictly decreasing in $[0, 3]$)

For most functions that we need to deal with, it will be possible to break up the domain into different parts so that throughout each part the function is either increasing or decreasing.

## ❒ Some Types of Functions

1. **Polynomial Functions:** A function of the form $f(x) = a_0x^n + a_1x^{n-2} + a_2x^{n-2} + \ldots + a_n$ where $x$, $a_0$, $a_1$, $a_2$ ... $an$ are real numbers, $a_0 \neq 0$ and $n \in N$ is called polynomial function of degree $n$.

The domain and the range of $f$ is $R$.

2. **Modulus Functions:** The function defined by $f(x) = x$ if $x \ge 0$ and $-x$ if $x < 0$ is called modulus function. This function is denoted as $|x|$. Similarly, we can define $|g(x)|$, where $g(x)$ is an arbitrary function.

Here, the domain of $f = R$, range of $f$ = set of all non-negative real numbers

**Example:**

$|7| = 7$; $|-7| = 7$

3. **Signum Functions:** The function $f$ defined by $sg(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is called signum function.

   Domain of $f = R$, Range of $f = \{-1, 0, 1\}$.

   Similarly, we can define $sg(g(x))$, where $g(x)$ is an arbitrary function.

4. **Trigonometric Functions:** We give the domain and range of various trigonometric functions.

| Function | Domain | Range |
|---|---|---|
| sin $x$ | $R$ | $[-1, 1]$ |
| cos $x$ | $R$ | $[-1, 1]$ |
| tan $x$ | $R-\{(2n+1)\ \pi/_2 \mid n \in Z\}$ | $R$ |
| cot $x$ | $R - \{n\pi \mid n \in Z\}$ | $R$ |
| sec $x$ | $R-\{(2n+1)\pi/_2 \mid n \in Z\}$ | $(-\infty,-1]\cup[1,\infty)$ |
| cosec $x$ | $R-\{n\pi \mid n \in Z\}$ | $(-\infty,-1]\cup[1, \infty)$ |

## ❒ Functional Equations

Equations involving unknown functions are called functional equations. Some basic functional equations and their solutions, i.e., the functions which satisfy these equations are given below.

| | Equation | Solution |
|---|---|---|
| 1. | $f(x + y) = f(x) + f(y)$ | $f(x) = kx,\ k \in R$ |
| 2. | $f(x + y) = f(x)f(y)$ | $f(x) = 0$ or $f(x) = ax,\ a > 0$ |
| 3. | $f(xy) = f(x) + f(y)$ | $f(x) = k \ln x,\ k \in R$ |
| 4. | $f(xy) = f(x)f(y)$ | $f(x) = 0$ or $f(x) = xn,\ n \in R$ |

There are some unusual functions (sometimes called pathological functions) which may also satisfy the above conditions in addition to the solutions given but they are way beyond the scope of every MBA entrance exam in our country.

## SOLVED EXAMPLES

**6.01.** The sets $A$ and $B$ are given as $A = \{a, b, c, d\}$ and $B = \{a, d, i\}$ Find $A \cup B$, $A \cup B$, $A - B$, $B - A$, $A \Delta B$, $A \times B$ and $B \times A$.

**Sol:** (i) $A \cup B$ is the set of all elements which are in set $A$ OR set $B$ or in both
$\therefore A \cup B = \{a, b, c, d, i\}$

(ii) $A \cup B$ is the set of elements which are in both set $A$ AND set $B$ (common elements)
$\therefore A \cup B = \{a, d\}$.

(iii) $A - B$ is the set of elements that are present in set $A$ but NOT in set $B$
$\therefore A - B = \{b, c\}$.

(iv) $B - A$ is the set of elements that are in set $B$ but NOT in set $A$
$\therefore B - A = \{i\}$.

(v) $A \Delta B = (A \cup B) - (A \cup B) = \{a, b, c, d, i\} - \{a, d\} = \{b, c, i\}$.

(vi) $A \times B$ = set of all ordered pairs with first element from set $A$ and second element from set $B$
$A \times B = \{(a, a), (a, d), (a, i), (b, a), (b, d), (b, i), (c, a), (c, d), (c, i), (d, a), (d, d), (d, i)\}$.

(vii) $B \times A$ = Set of ordered pairs with first element from set $B$ and second element from set $A$
$\therefore B \times A = \{(a, a), (a, b), (a, c), (a, d), (d, a), (d, b), (d, c), (d, d), (i, a), (i, b), (i, c), (i, d)\}$.

**6.02.** The sets $A$ and $B$ are given as $A = \{a, b, c, d, e\}$ and $B = \{a, c, e f\}$ and $\mu = \{a, b, c, d, e, f, g, h\}$ ($\mu$ is the universal set), then verify the following
(i) $(A \cup B)c = Ac \cup Bc$
(ii) $(A \cup B)c = Ac \cup Bc$

**Sol:** (i) $A \cup B = \{a, b, c, d, e, f\} \therefore (A \cup B)c = \{g, h\}$ (1)
$Ac = \{f, g, h\}$, and $Bc = \{b, d, g, h\}$
$\therefore Ac \cup Bc = \{g, h\}$ (2)
From (1) and (2) $(A \cup B)\ c = A^c \cup B^C$.

(ii) $A \cup B = \{a, c, e\} \therefore (A \cup B)\ c = \{c, d, f, g, h\}$ (1)
$Ac \cup Bc = \{b, d, f, g, h\}$ (2)
From (1) and (2), $(A \cup B)\ c = Ac \cup Bc$.

**6.03.** If $A = \{a, b, c, d, e\}$, then find the number of subsets of $A$ that contain '$a$' but not '$e$'.

**Sol:** In the formation of any subset, each element of $A$ either gets included or excluded. Thus, each element has two ways to be dealt with. Now that $a$ is included and $e$ is excluded, each of the remaining elements $b$, $c$, $d$ can be dealt with in 2 ways. $\therefore$ The number of subsets containing '$a$' but not 'e' is $2^3$ or 8.

**6.04.** If $A = \{a, d, f, h\}$, $R_1 = \{(a, f), (a, h), (f, h)\}$ and $R_2 = \{(a, d), (d, f), (f, h), (a, h)\}$, then find $R_2^{-1}$, $R_1 \cup R_2$ and $R_1 \cup R_2$.

**Sol:** (i) $R_2^{-1} = \{(d, a), (f, d), (h, f), (h, a)\}$ (Reverse the order).

(ii) $R_1 \cup R_2 = \{(a, f), (a, h), (f, h), (a, d), (d, f)\}$.
(iii) $R_1 \cup R_2 = \{(a, h), (f, h)\}$.

**6.05.** Which of the following relations is a function on $A = \{1, 3, 5, 7\}$?
(i) $f_1 = \{(1, 3), (1, 5), (1, 7)\}$
(ii) $f_2 = \{(1, 3), (3, 5), (5, 7)\}$
(iii) $f_3 = \{(1, 1), (3, 3), (5, 5)\}$
(iv) $f_4 = \{(1, 3), (3, 5), (5, 7), (7, 1)\}$

**Sol:** $f_4$ is a function on $A$ $f_1$ is not a function on $A$ as 1 has more than one image. Further 3, 5, 7 have no images $f_2, f_3$ are not functions on $A$ as 7 has no image

**6.06.** Compute the inverse of $f(x) = \frac{2x+1}{x-5}$; $(x \neq 5)$, $x \in R\, x \neq 2$

**Sol:** Let $y = f(x)$ (given $y$ in terms of $x$)

$$y = \frac{2x+1}{x-5} \Rightarrow xy - 5y = 2x + 1$$

$$\Rightarrow x(y-2) = 5y + 1$$

$$\Rightarrow x = \frac{5y+1}{y-2} \text{ (expressing } x \text{ in terms of } y)$$

$$\therefore f^{-1}(y) = \frac{5y+1}{y-2} \Rightarrow f^{-1}(x) = \frac{5x+1}{x-2} \text{ (put } y = x)$$

**6.07.** If $f(x) = 2x + 3$, $g(x) = \log \cos x$, $0 < x < \frac{\pi}{2}$, then find $fog(x)$ and $gof(x)$.

**Sol:** $(fog)(x) = f[g(x)] = f(\log \cos x) = 2(\log \cos x) + 3$
$(gof)(x) = g[f(x)] = g(2x + 3) = \log \cos(2x + 3)$

**NOTE**

$fog \neq gof$

**6.08.** If $f(x) = e^2 x + 3x^2$, $g(x) = \frac{f(x)+f(-x)}{2}$ and $h(x) = \frac{f(x)-f(-x)}{2}$, show that

(i) $g(x)$ is even and (ii) $h(x)$ is odd

**Sol:** (i) If $g(x) = g(-x)$, then $g(x)$ is said to be an even function.

$$g(x) = \frac{f(x)+f(-x)}{2} = \text{while } g(-x)$$

$$= \frac{f(-x)+f(x)}{2}$$

As $g(x) = g(-x)$, $g(x)$ is even

(ii) If $h(-x) = -h(x)$, then $h(x)$ is said to be an odd function.

$$h(x) = \frac{f(x)-f(-x)}{2}, \text{ while } h(-x)$$

$$\frac{f(-x)-f(x)}{2}$$

As $h(-x) = -h(x)$, $h(x)$ is odd.

**6.09.** If $f(x) = \frac{4x-5}{6}$ for all $x \in R$, then show that $f$ is bijection.

**Sol:** We first show that $f$ is one–one

$$f(x_1) = f(x_2) \Rightarrow \frac{4x_1-5}{6} = \frac{4x_2-5}{6}$$

$\Rightarrow x_1 = x_2$
$\therefore f$ is one–one.
Then we show that $f$ is onto

$$y = \frac{4x-5}{6} \Rightarrow x = \frac{6y+5}{4}$$

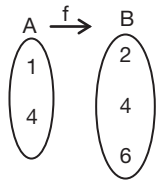
$$\text{Consider } f\left(\frac{6y+5}{4}\right) = \frac{4\left(\frac{6y+5}{4}\right)-5}{6} = y$$

So, in order to get any real number $y$ as an $f$ image, the pre-image should be $\frac{6y+5}{4}$

Thus, every real number gets included in the range and hence $f$ is onto.
$\therefore$ As $f$ is one - one and onto, it is a bijection.

**6.10.** If $A = \{1, 4, 6\}$, $B = \{2, 4, 6, 8, 10\}$. How many one–one functions are possible from $A$ to $B$?

**Sol:** $f: A \to B$ is any one–one function
Now 1 of $A$ can be assigned to any one of the five elements of $B$.
Then 4 can be assigned to any one of the four remaining elements of $B$. Similarly, 6 can be assigned to any three of the remaining elements.




Thus $5(4)(3)$, i.e., ${}^5P_3$ one–one functions are possible from $A$ to $B$.

**6.11.** $A = \{1, 3, 5, 7, 9, 11, 13\}$ and $B = \{2, 4\}$. How many onto functions are possible from $A$ to $B$?

**Sol:** The total number of functions from $A$ to $B$ is $2^7$.
The constant functions, $f(x) = 2$ and $f(x) = 4$ are not onto.
$\therefore$ The number of onto functions is $2^7 - 2 = 126$.

**6.12.** If $f(x) = x + \frac{1}{x^2}$ $x \neq 0$; $x \in R$, then compute

$f\left(\frac{1}{x}\right)$, $(fof)\left(\frac{1}{x}\right)$.

**Sol:** Given $f(x) = x + \frac{1}{x^2}$

$$f\left(\frac{1}{x}\right) = \frac{1}{x} + \frac{1}{\left(\frac{1}{x}\right)^2} = x^2 + \frac{1}{x}$$

$$(fof)\left(\frac{1}{x}\right) = f\left[f\left(\frac{1}{x}\right)\right] = f\left(x^2 + \frac{1}{x}\right)$$

$$= \left(x^2 + \frac{1}{x}\right) + \frac{1}{\left(x^2 + \frac{1}{x}\right)^2}$$

**6.13.** If $f(x) = \sin \log(x)$ where $x > 0$, $x \in R$, then find

$f\left(\frac{x}{y}\right)$ and $f\left(\frac{y}{x}\right)$.

Also, show that $f\left(\frac{y}{x}\right) = -f\left(\frac{x}{y}\right)$.

**Sol:** $f\left(\frac{x}{y}\right) = \sin \log\left(\frac{x}{y}\right) = \sin\ (\log x - \log y)$

$f\left(\frac{y}{x}\right) = \sin \log\left(\frac{y}{x}\right) = \sin\ (\log y - \log x)$

$= \sin\ (-\ (\log x - \log y)) = -\sin\ (\log x - \log y)$

$= -f\left(\frac{x}{y}\right)$

$\therefore f\left(\frac{y}{x}\right) = -f\left(\frac{x}{y}\right)$

**6.14.** If $A = \{1, 2, 3, 4, 6, 8\}$ and $B = \{1, 2, 3, 5, 7\}$, then find $A \cup B$, $A \cup B$, $A - B$, $B - A$, $A \Delta B$, $A \times B$ and $B \times A$

**Sol:**
- $A \cup B$ is the set having the elements that are in $A$ or $B$ or in both
  $\therefore A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- $A \cup B = \{1, 2, 3\}$. The elements that are present in both $A$ and $B$
- $A - B = \{4, 6, 8\}$. The elements present in $A$ but not in $B$.
- $B - A = \{5, 7\}$. The elements present in $B$ but not in $A$.
- $A \Delta B = (A \cup B) - (A \cup B) = \{4, 5, 6, 7, 8\}$
- $A \times B = \{(1, 1), (1, 2), (1, 3), (1, 5), (1, 7),$
  $(2, 1), (2, 2), (2, 3), (2, 5), (2, 7)$
  $(3, 1), (3, 2), (3, 3), (3, 5), (3, 7),$
  $(4, 1), (4, 2), (4, 3), (4, 5), (4, 7),$
  $(6, 1), (6, 2), (6, 3), (6, 5), (6, 7),$
  $(8, 1), (8, 2), (8, 3), (8, 5), (8, 7)\}$
  We have to list all the possible ordered pairs taking the first element from set $A$ and second element from set $B$.
  $B \times A =$
- $\{(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (1, 8),$
  $(2, 1), (2, 2), (2, 3), (2, 4), (2, 6), (2, 8)$
  $(3, 1), (3, 2), (3, 3), (3, 4), (3, 6), (3, 8),$
  $(5, 1), (5, 2), (5, 3), (5, 4), (5, 6), (5, 8)$
  $(7, 1), (7, 2), (7, 3), (7, 4), (7, 6), (7, 8)\}$
  We have to listed all the possible ordered pairs taking the first element from set $B$ and the second element from set $A$.
  Note: In general $A \times B \neq B \times A$ but
  $n(A \times B) = n(B \times A)$

**6.15.** If $A = \{1, 2, 3, 4\}$ and $B = \{1, 3, 5, 7\}$, then verify the following:
(i) $(A \cup B)c = Ac \cup Bc$
(ii) $(A \cup B)c = Ac \cup Bc$ when $\mu = \{1, 2, 3 \ldots 10\}$.

**Sol:** (i) $A \cup B = \{1, 2, 3, 4, 5, 7\}$
$(A \cup B)^c = \mu - (A \cup B)$
$= \{6, 8, 9, 10\}$ (1)
$Ac = \mu - A = \{5, 6, 7, 8, 9, 10\}$
$Bc = \mu - B = \{2, 4, 6, 8, 9, 10\}$
$Ac \cup Bc = \{6, 8, 9, 10\}$ (2)
From (1) and (2), we notice that
$(A \cup B)^c = Ac \cup Bc$.

(ii) $A \cup B = \{1, 3\}$
$(A \cup B)^c = \{2, 4, 5, 6, 7, 8, 9, 10\}$ (3)
$Ac \cup Bc = \{2, 4, 5, 6, 7, 8, 9, 10\}$ (4)
From (3) and (4), we notice that $(A \cup B)c$ $= Ac \cup Bc$.

**6.16.** If $A = \{1, 2, 3, 4, 5\}$, then find the number of subsets of $A$ which do not contain 3.

**Sol:** The number of subsets of $A$ which do not contain 3 is $2^4$.

**6.17.** If $A = \{a, b, c\}$; $R = \{(a, b), (b, c), (c, a)\}$ and $R_1 = \{(a, c), (b, c), (a, a)\}$, then find $R^1$, $R \cup R_1$ and $R \cup R_1$.

**Sol:** (i) $R^1 = \{(b, a), (c, b), (a, c)\}$
(ii) $R \cup R_1 = \{(a, a), (a, b), (b, c), (c, a), (a, c)\}$
(iii) $R \cup R_1 = \{(b, c)\}$

**6.18.** Which of the following relations is a function on $\{1, 2, 3\}$?
(i) $f_1 = \{(2, 1), (2, 2), (3, 3)\}$

(ii) $f_2 = \{(1, 2), (2, 1), (3, 3)\}$
(iii) $f_3 = \{(1, 1), (1, 3), (2, 2)\}$
(iv) $f_4 = \{(3, 1), (3, 2)\}$

**Sol:** Clearly $f_2$ is a function while in $f_1$, '2' has more than one image. In $f_3$, 1 has more than one image and in $f_4$, 3 has more than one image.
$\therefore f_1, f_3$, and $f_4$ are not functions.

**6.19.** Let $f$ and $g$ be two real valued functions defined on the domain $\{-2, -1, 0, 1\}$. If $f = \{(-2, -1), (-1, 0), (0, 1), (1, 2)\}$ and $g = \{(-2, 0), (-1, 1), (0, 2), (1, 3)\}$ then, find $2f + 3g$, $f^2$ and $fg$.

**Sol:** (i) $2f(-2) + 3g(-2) = 2(-1) + 3(0) = -2$
$2f(-1) + 3g(-1) = 2(0) + 3(1) = 3$
$2f(0) + 3g(0) = 2(1) + 3(2) = 8$
$2f(1) + 3g(1) = 2(2) + 3(3) = 13$
$\therefore 2f + 3g = \{(-2, -2), (-1, 3), (0, 8), (1, 13)\}$

(ii) $f^2(x)$
$= \{f(x)\}^2 = \{(-2, 1), (-1, 0), (0, 1), (1, 4)\}$

(iii) $fg(x) = f(x) \cdot g(x)$
$f(-2) \cdot g(-2) = (-1)(0) = 0$, $f(-1)g(-1) = 0$,
$f(0)g(0) = 1 \cdot 2 = 2$ and
$f(1)g(1) = (2)(3) = 6$
$\therefore fg = \{(-2, 0), (-1, 0), (0, 2), (1, 6)\}$

**6.20.** If $f(x) = 2x^2 - 1$, $g(x) = 3x + 1$, then find the domain of $\sqrt{\frac{f}{g}}$.

**Sol:** $\sqrt{\frac{f(x)}{g(x)}} = \sqrt{\frac{2x^2 - 1}{3x + 1}}$

$\frac{2x^2 - 1}{3x + 1} \geq 0$

$(2x^2 - 1)(3x + 1) \geq 0$

Critical points are $\frac{-1}{\sqrt{2}}, \frac{-1}{3}, \frac{1}{\sqrt{2}}$

$\left(\sqrt{2}x - 1\right)\left(\sqrt{2}x + 1\right)(3x + 1) \geq 0$

When $x = 0$, the above equation is not satisfied.

$\therefore$ The domain is $\left[\frac{-1}{\sqrt{2}}, \frac{-1}{3}\right) \cup \left[\frac{1}{\sqrt{2}}, \infty\right)$

**6.21.** If $f(x) = \frac{3x + 2}{5x - 3}$ and $\neq \frac{3}{5}$, then find the relation between $f$ and $f^{-1}$.

**Sol:** Let $\frac{3x + 2}{5x - 3} = y \Rightarrow 3x + 2 = 5xy - 3y$

$\Rightarrow x(5y - 3) = 3y + 2$
$\Rightarrow x = \frac{3y + 2}{5y - 3}$

$f^{-1}(y)\ \frac{3y + 2}{5y - 3} = f(y)$

$\therefore f^{-1}(x)\ \frac{3x + 2}{5x - 3} = f(x) \Rightarrow \therefore f^{-1}(x) = f(x)$

**6.22.** If $f(x) = \frac{\operatorname{cosec} x \sec x}{\tan x + \cot x}$, then verify $f(x)$ is even function or not.

**Sol:** $f(-x)$

$= \frac{\operatorname{cosec}(-x)\sec(-x)}{\tan(-x) + \cot(-x)} = \frac{-\operatorname{cosec} x \sec x}{-(\tan x + \cot x)} = f(x)$

$\therefore f(x)$ is even.

**6.23.** Find the domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$

**Sol:** $f(x) = \frac{1}{\sqrt{|x| - x}}$ is defined only if $|x| - x > 0$

i.e., $|x| > x$, i.e., $x < 0$
$\therefore$ Domain is $(-\infty, 0)$.

**6.24.** Find the domain of the following functions.

(i) $f(x) = \frac{1}{x}\sqrt{x + 6}$

(ii) $f(x) = \log_e(3x - 1)$

**Sol:** (i) $f(x)\ \frac{1}{x}\sqrt{x + 6}$

Division by 0 is not allowed.
so, $x \neq 0$.
Square root is defined only for non-negative quantities.
$\therefore\ x + 6 \geq 0$ or $x \geq -6$.
$\therefore$ domain is $[-6, 0) \cup (0, \infty)$

(ii) $f(x) = \log_e(3x - 1)$.
As the logarithm is defined for positive numbers only, we must have $3x - 1 > 0$ or $x\ \frac{1}{3}$.
Domain is $\frac{1}{3}, \infty)$.

**6.25.** Find the range of the following functions
(i) $f(x) = x^2 + 4x + 10;\ x \in R$

(ii) $f(x) = x - [x]$ $x \in R$, where $[x]$ represents the greatest integer less than or equal to $x$.

**Sol:** (i) $f(x) = x^2 + 4x + 10$
$= (x+2)^2 + 6$
Since $(x+2)^2 \geq 0$
$\therefore (x+2)^2 + 6 \geq 6$.
$\therefore f(x) \geq 6$.
The range of $f(x)$ is $[6, \infty)$

(ii) $f(x) = x - [x]$ or $x = [x] + f(x)$
Any real number $x$ can be expressed as the sum of its integer part and the fractional part.
The fractional part lies between 0 and 1 where 1 is excluded.
In the above relation $f(x)$ is the fractional part.
$\therefore$ The range of $f(x) = [0, 1)$

**6.26.** Find the inverse of the function $f(x) = \dfrac{3x+2}{4x-1}$ where $x \neq \dfrac{1}{4}$

**Sol:** Let $y = f(x) = \dfrac{3x+2}{4x-1}$, $x \neq \dfrac{1}{4}$

The inverse of a function is found by expressing $x$ in terms of $y$. We think of $y$ as the argument of the inverse function and $x$ as the value of the inverse function.
$4xy - y = 3x + 2$
$\Rightarrow x(4y-3) = y + 2$
$\Rightarrow x = \dfrac{y+2}{4y-3}$
$\therefore$ The inverse function $f^{-1}(y) = \dfrac{y+2}{4y-3}$, where $y \neq \dfrac{3}{4}$ or $f^{-1}(x) = \dfrac{x+2}{4x-3}$

**6.27.** Find the zeros of the function $f(x) = e^x(x-1)(x+2)$, $x \in R$.

**Sol:** To find the zeros of $f(x)$, we equate $f(x)$ to 0 and find the values of $x$ satisfying
$e^x(x-1)(x+2) = 0$.
For all $x$, $e^x > 0$
$\therefore (x-1)(x+2) = 0$
$\therefore x = 1$ or $-2$.

**6.28.** If $f(x) = \dfrac{1}{x^3}$ and $g(x)$ $\sqrt{x-6}$, find $f(g(x))$ and also find its domain, given that it is a real function.

**Sol:** $f(g(x)) = \left(\sqrt{x-6}\right) = \dfrac{1}{\left(\sqrt{x-2}\right)^3} = \dfrac{1}{\sqrt{(x-6)^3}}$

Domain is $x > 6$ since $\dfrac{1}{\sqrt{x-6}}$ would not be defined if $x = 6$ and would be complex if $x < 6$.
Domain $(6, \infty)$.

**6.29.** Find the range of the function $f(x) = |x-6| - |x-7|$.

**Sol:** Let us divide the possible values of $x$ into three.
Case (i) $x \leq 6$
If $x \leq 6$, $|x-6| = 6 - x$ and
$|x-7| = 7 - x$
$\therefore f(x) = -1$
Case (ii) $6 \leq x \leq 7$.
If $6 \leq x \leq 7$,
$|x-6| = x - 6$ and $|x-7| = 7 - x$
$\therefore f(x) = 2x - 13$
$\therefore$ range of $f(x)$ is $-1 \leq f(x) \leq 1$
Case (iii) $x \geq 7$ : If $x \geq 7$, $|x-6| = x - 6$ and
$|x-7| = x - 7$
$\therefore f(x) = 1$.
$\therefore$ range of $f(x)$ is $-1 \leq f(x) \leq 1$

## EXERCISE-1

***Directions for questions 1 to 40:*** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** How many elements are there in the set of letters occurring in the word Mathematics?
(A) 11 (B) 9
(C) 8 (D) None of these

**2.** $X = \{x / x$ is an odd prime number less then 18$\}$. Which of the following set represents $X$?
(A) {2, 3, 5, 7 , 11, 13, 17}
(B) {3, 5, 7, 11, 13, 17}
(C) {2, 3, 5, 7, 11, 13}
(D) {2, 3, 5, 7, 9, 11, 13, 17}

**3.** Which of the following is an empty set?
(A) { } (B) {$\phi$} (C) {0} (D) None of these

**4.** Which of the following represents the set $A = \{1, 3, 5, \ldots, 15\}$
(A) $A = \{x/x \in N \text{ and } x \leq 15\}$
(B) $A = \{x/x = 2n,\ x \in N \text{ and } x \leq 15\}$
(C) $A = \{x/x = 2y - 1,\ y \in N \text{ and } y \leq 8\}$
(D) $A = \{x/x = 2y - 1,\ y \in W \text{ and } y \leq 8\}$

**5.** Which of the following is a null set?
(A) $\{x/x \neq x\}$
(B) $\{x/x = 2y,\ y \in N \text{ and } y \text{ is an odd number}\}$
(C) $\{x/x = 2n - 1,\ n \in N,\ n < 5 \text{ and } n \text{ is an even number}\}$
(D) All the above

**6.** The cardinal number of the set formed by the letters of the word 'MISSISSIPPI' _____.

**7.** $A = \{1, 2, 3, 4\}$, $B = \{x/x \in N,\ x \leq 4\}$. Which of the following is/ are true?
I. $A \subseteq B$
II. $B \subseteq A$
III. $A = B$
(A) only I (B) Only II
(C) Only III (D) I, II, III

**8.** The number of elements of the set $\{2, \{2, 3\}, 3, \{1, 2, 3\}\}$ is _____.

**9.** The minimum number of subsets that are possible for any non-empty set is _____.

**10.** Which of the following sets is/are subset/s of $\{\{1, 2\}, 2, 3, 4\}$?
(A) $\{1, 2\}$ (B) $\{2, 3\}$
(C) $\{1, 2, 3\}$ (D) All of the above

**11.** If Set $A = \{a, e, b, d, i, o, u\}$ and set $B = \{a, c, e, i, r, o, g, u\}$, then $A \cap B =$
(A) $\{a, e, i, o, u\}$ (B) $\{a, b, e, i, d, o\}$
(C) $\{b, d, f, g\}$ (D) $\{a, b, c, d, e, f, i, o, u\}$

**12.** Which of the following is a subset of the set $\{a, \{a, b\}, b, c\}$?
(A) $\{a, b\}$ (B) $\{\{a, b\}\}$
(C) Both (A) and (B) (D) None of these

**13.** If $A = \{a, b, c, d\}$ and $B = \{b, c, e, f\}$, then $A \cup B =$
(A) $\{a, d, e, f\}$ (B) $\{a, e, f, b, c\}$
(C) $\{a, d, e, f, c\}$ (D) $\{a, b, c, d, e, f\}$

**14.** The set which is equal to the set $\{1, 2, \ldots, 10\}$ is
(A) $\{x/x \in N \text{ and } x \leq 10\}$
(B) $\{1, 4, 6, 8, 3, 5, 2, 9, 10, 7\}$
(C) $\{x/x \in N, \text{ and } x^2 < 110\}$
(D) All the above

**15.** The number of subsets of the set $A = \{1, 2, 4, 6, 8\}$ is
(A) 32 (B) 5 (C) 16 (D) 24

**16.** If $A = \{1, 2, 3, 4, 5, 6\}$, then the number of subsets of $A$ is _____.

**17.** If set $A = \{2, 3, 5, 7\}$ and set $B = \{1, 2, 3, 4\}$, then $A \cap B =$
(A) $\{2, 3\}$. (B) $\{1, 2, 3\}$.
(C) $\{2, 3, 4\}$. (D) $\{1, 2, 3, 4, 5, 7\}$.

**18.** The number of proper subsets of the set $A = \{a, b, c, d\}$ is _____.

**19.** Which of the following can be the number of proper subsets of a given non-empty set?
(A) 24 (B) 31 (C) 20 (D) 120

**20.** If set $A = \{1, 3, 4, 5\}$ and $B = \{1, 2, 4, 6\}$, then $A \cup B =$
(A) $\{1, 2, 4, 5, 6\}$. (B) $\{1, 2, 3, 4, 5, 6\}$.
(C) $\{1, 3, 4, 6\}$. (D) $\{1, 2, 3, 5, 6\}$.

**21.** For any sets $A$ and $B$, if $n(A) = n(B)$, then $A$, $B$ are called
(A) equal sets (B) equivalent sets
(C) disjoint sets (D) None of these

**22.** If $A$, $B$ are any two sets, then $A \Delta B =$
(A) $(A - B) \cup (B - A)$ (B) $(A \cup B) - (A \cap B)$
(C) Both (A) and (B) (D) None of these

**23.** If $A \subseteq B$, then $A \cup B =$
(A) $A$ (B) $B$
(C) $\phi$ (D) None of these

**24.** Which of the following can be the number of subsets of a given set? _____

**25.** For any two sets $A$ and $B$, if $n(A \cap B) = 0$, then $A$, $B$ are called
(A) equal sets (B) equivalent sets
(C) disjoint sets (D) None of these

**26.** If $\mu = \{1, 2, 3, 4, \ldots\ldots, 10\}$, $A = \{1, 3, 5, 7\}$, then $Ac =$
(A) $\{1, 3, 5, 7\}$. (B) $\{2, 4, 6, 8, 10\}$.
(C) $\{2, 4, 6, 8, 9, 10\}$. (D) $\{1, 2, 3, 4, 5, 6\}$.

**27.** Set $A$ has 5 elements and set $B$ has 8 elements. Then, the maximum number of elements that $A \cup B$ can have is _____.

**28.** If $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{2, 4, 6, 8, 10\}$, then $B - A =$
(A) $\{8, 10\}$. (B) $\{1, 3, 5, 7\}$.
(C) $\{2, 4, 6\}$. (D) $\{2, 4, 6, 8\}$.

**29.** If $\mu$ is universal set and $A$ is any non-empty set, then $A \cup A' =$
(A) $\mu$ (B) $A$ (C) $A'$ (D) $\phi$

**30.** Set $A$ has 6 elements and set $B$ has 9 elements. Then, the minimum number of elements $A \cup B$ can have is
(A) 6. (B) 9. (C) 15. (D) 3.

**31.** If $f(x) = \dfrac{2x-1}{x+2}$ $x \neq -2$, then $f^{-1}(3) =$
(A) 5 (B) –7 (C) 7 (D) –5

**32.** Which of the following is a relation from set $A$ to set $B$ when $A = \{1, 2, 3\}$, $B = \{a, b\}$
(A) $\{(1, a), (2, a), (3, b), (a, 1)\}$
(B) $\{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2)\}$
(C) $\{(1, a), (1, b), (2, a), (3, a), (3, b)\}$
(D) $\{(1, a), (b, 2), (3, a)\}$

**33.** If a relation $R = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$, then the inverse relation $R^1 =$
(A) $\{(2, 1), (3, 2), (4, 3), (1, 4)\}$
(B) $\{(2, 1), (2, 3), (3, 4), (1, 4)\}$
(C) $\{(1, 3), (4, 3), (1, 4), 2, 3\}$
(D) None of these

**34.** If $(a + b, 2) = (6, a - b)$, then $2a - 3b =$
(A) 1 (B) 2 (C) 3 (D) 4

**35.** If $A = \{1, 2, 3\}$ and $B = \{4, 5\}$, then $A \times B =$
(A) $\{(1,4),(1,5),(2,5),(3,5)\}$
(B) $\{(4,1),(4,2),(4,3),(5,1)\}$
(C) $\{(1,4),(1,5),(4,1),(4,2),(4,3)\}$
(D) $\{(1,4),(1,5),(2,4),(2,5),(3,4),(3,5)\}$

**36.** A relation $R = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$. $R^{-1}$, the inverse relation of $R =$
(A) $\{(1, 4), (3, 4), (3, 2), (2, 1)\}$
(B) $\{(2, 1), (3, 2), (4, 3), (1, 4)\}$
(C) $\{(4, 1), (4, 3), (2, 3), (2, 1)\}$
(D) $\{(4, 1), (4, 3), (2, 1), (2, 3), (3, 4)\}$

**37.** Which of the following is $A \times B$, where $A$, $B$, are any two non-empty sets?
(A) $\{(x, y)/, x \in B, y \in A\}$ (B) $\{(x, y)/x, y \in A \cap B\}$
(C) $\{(x, y)/x \in A, y \in B\}$ (D) $\{(y, x)/ y \in B, x \in A\}$

**38.** Which of the following is a relation from set $A$ to set $B$, where $A = \{1, 2, 3\}$, $B = \{4, 5\}$?
(A) $\{(1, 4), (2, 4), (3, 5)\}$
(B) $\{(1, 4), (2, 4), (4, 5)\}$
(C) $\{(1, 5), (4, 5), (3, 4)\}$
(D) $\{(1, 4), (4, 2), (3, 4), (5, 4)\}$

**39.** Set $A$ has 4 elements and set $B$ has 7 elements. Then, the maximum number of elements that $A \cap B$ can have is _____.

**40.** If $n(A) = 3$ and $n(B) = 4$, then $n(A \times B)$ equals
(A) 3. (B) 4. (C) 12. (D) 7.

## EXERCISE-2

***Directions for questions 1 to 40***: For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** The number of proper subsets of the set {1, 3, 5, 7, 9, 11, 13} is
(A) 128 (B) 127 (C) 64 (D) 63

**2.** If the number of proper subsets other than the empty set of a set $P$ is 126, then the number of elements of $P$ is _____.

**3.** The set $A$ contains $a$, $b$, $c$ and some other elements. The difference between the number of proper subsets of $A$ and the number of subsets of $A$ that contain $a$, b, c is 55. The cardinal number of set $A$ is _____.

**4.** The number of subsets of the set {1, 2, 3, 4, 5, 6, 7} that contain 1, 3, 7 and do not contain 2, 4 is
(A) 2 (B) 8 (C) 4 (D) 14

**5.** If the number of subsets of $A$ that contain exactly 3 elements is 56, then the number of elements of set $A$ is
(A) 9 (B) 7 (C) 6 (D) 8

**6.** If the number of subsets of a set $A$ that contain $a$ but not $b$ of a set $A$ is 16, then the number of elements in set $A$ is _____.

**7.** The number of elements in the power set of $A$ is denoted by $n(P(A))$. Find $n(P(A))$, where $A = \{a, b, c, d, e\}$.
(A) 25 (B) 16 (C) 32 (D) 31

**8.** The number of functions that can be defined from set $A$ to set $B$ is 625 and $n(A) = 4$. Then, $n(B) =$ _____.

**9.** The number of one-one functions that can be defined from set $A$ to set $B$ where $n(B) = 16$ is 3360. Then, $n(A) =$ _____.

**10.** The number of onto functions possible from set $A$ to set $B$ when $n(A) = 6$ and $n(B) = 4$ is
(A) 1560. (B) 1260. (C) 1460. (D) 1660.

Difficulty Level-2: Moderate

**11.** Which of the following represents the set builder form of $A = \{1, 2, 3, 4, \ldots..12\}$?
(A) $\{x / x$ is a whole number $\leq 12\}$
(B) $\{x / x$ is a natural number$\}$
(C) $\{x / x$ is a natural number $\leq 12\}$
(D) $\{x / x$ is a natural number $< 12\}$

**12.** Which of the following laws gives the relation $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$?
(A) De Morgan's law.
(B) Associative law.
(C) Distributive law.
(D) Commutative law

**13.** If $A = \{2, 4, 5, 7, 9\}$, $B = \{1, 3, 5, 7, 8\}$ and C = $\{1, 2, 5, 8\}$, then $(A \cup B) \cap C =$
(A) $C$ (B) $A \cap B$ (C) $B$ (D) $A$

**14.** If $A = \{0, 2, 4, 6, 9\}$ and $B = \{1, 3, 5, 7\}$, then $A \Delta B =$
(A) $A \cap B$ (B) $A$ (C) $A \cup B$ (D) $B$

**15.** Which of the following cannot be the cardinal number of power set of any finite set?
(A) 1056 (B) 256 (C) 1 (D) 64

**16.** If $f(x - 1/x) = x^2 + 1/x^2$, $x \neq 0$, then $f(x^2 + 1/x^2) =$
(A) $x^4 + 1/x^4 + 4$ (B) $x - 1/x$
(C) $x^2 + 2$ (D) $x^4 - 1/x^4$

**17.** If $f(x) = x^2 + 35$ and $g(x) = x^2 - 10x + 12$, then the value of $x$ such that $f(2x) = g(x - 1)$ is
(A) $-2$ (B) 2 (C) 1 (D) 0

**18.** The number of bijections that can be defined from set $A$ to set $B$ is 120. The number of elements in set $B$ is _____.

**19.** Of all the functions that can be defined from set $A$ to set $B$, where $n(A) = 4$ and $n(B) = 6$, the number of functions which are not one-one, is
(A) 900. (B) 1296. (C) 936. (D) 1200.

**20.** Let $f(x) = \begin{cases} 5, \text{ when } x \text{ is a rational number.} \\ -5, \text{ when } x \text{ is nota rational number} \end{cases}$
Find the value of the expression
$f(\sqrt{5}) + f(\sqrt{3}) + |f(5)| + \sqrt{|f(3)|} + |\sqrt{f(2)}| + |f(\sqrt{5})|$.
(A) $\sqrt{5}$ (B) $-\sqrt{5}$ (C) $2\sqrt{5}$ (D) $-2\sqrt{5}$

***Directions for questions 21 to 24*:** These questions are based on the following data.

If $A = \{1, 2, 3, 4, 6, 12\}$, then how many of the subsets of $A$

**21.** contain 6 but not 12 is _____.

**22.** have atmost one element is _____.

**23.** contain 3 and 4 is _____.

**24.** contain exactly 4 elements is _____.

**25.** If $A = \{2, 3, 5, 7\}$, then the maximum number of elements in a relation defined on $A$ is _____..

**26.** Which of the following is a function on $\{1, 2\}$?
(A) $\{(1, 1), (1, 2), (2, 2)\}$
(B) $\{(1, 1), (2, 2)\}$
(C) $\{(1, 2), (1, 1)\}$
(D) $\{(2, 1), (2, 2), (1, 1), (1, 2)\}$.

**27.** Set $A$ and set $B$ are two equivalent sets and $n(A) = 6$. How many bijections can be defined from $B$ to $A$? _____

**28.** Find the domain and range of the function
$f = \{(1, a), (2, b), (3, c), (4, c)\}$
(A) $\{1, 2, 3, 4\}$, $\{a, b, c\}$
(B) $\{1, 3, 4\}$, $\{a, b\}$
(C) $\{1, 2, 4\}$, $\{b, c\}$
(D) $\{1, 2, 3, 4, 5\}$, $\{a, b, c, d\}$

**29.** Find the domain of the function
$f(x) = \log(2x - 3) + \sqrt{x^2 - 9} + \dfrac{1}{\log(x - 4)}$.
(A) $(-5, 5)$ (B) $(-4, 5)$
(C) $(-5, \infty)$ (D) $(4, 5) \cup (5, \infty)$

**30.** Find the range of the function $f(x) = \dfrac{3x}{x^2 - x + 1}$.
(A) $[-3, 1]$ (B) $[-1, 3]$
(C) $(-3, 3)$ (D) $(3, \infty)$

**31.** If $h(x) = K^x$, then which of the following is/are true?
(A) $h(x+2) - 2h(x+2)h(-1) + h(x)h(0) = (K-1)^2 h(x)$
(B) $h(x + 1)\ h(x - 1) = (h(x))^2$
(C) $h(x)\ h(-x) = 1$
(D) All the above.

**32.** If $h(x + y) = h(x)h(y)$ for all real values of $x$ and $y$ and $h(6) = 15625$, then $h(3) =$
(A) 216 (B) 243 (C) 125 (D) 625

**33.** If $f(x + 1) = 3[f(x)]^2 - 2$ for all numbers $x$, then $f(x + 2) =$
(A) $9[f(x)]^4 - 36 f(x) + 10$
(B) $9[f(x)]^4 - 36[f(x)]^2 - 10$
(C) $3[f(x)]^4 - 12[f(x)]^2 - 10$
(D) None of these

**34.** Let $g(x)$ be a function satisfying $\dfrac{g(a)}{g(b)} = g\left(\dfrac{a}{b}\right)$ for all real values of $a$ and $b$. If $g(3) = \dfrac{1}{9}$, then find $g\left(\dfrac{1}{3}\right)$.
(A) $\dfrac{1}{3}$ (B) 3 (C) $\dfrac{1}{9}$ (D) 9

**35.** If $f_1(x) = 2x^2 - 5$ and $f_2(x) = 2x^2 - 5x - 36$, the value of $x$ such that $f_1(x-1) = f_2(x+1)$ is
(A) 10 (B) 12 (C) 8 (D) 9

**36.** If $f(x+y) = f(x) + f(y)$, $\forall\ x, y \in R$ and $f(3) = 18$, then find the value of $f\left(\frac{1}{3}\right)$. _____

**37.** If $g(x) = 3x^2 + 8$; $x \in R$ and $h(x) = 3x^2 + 4x - 56$; $x \in R$, The value of $x$ such that $g(x-2) = h(x+2)$ is _____.

**38.** $H(x)$ is a function such that $H(1) = 2460$ and $H(1) + H(2) + H(3) + \ldots + H(N) = \frac{2N[H(N)]}{N-1}$, where $N$ is any natural number greater than 1. Find $H(4)$.
(A) 2952 (B) 3640 (C) 2840 (D) 4280

**39.** If $f(x) = \frac{4^x}{4^x + 2}$, then find $f\left(\frac{1}{2}\right) + f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right)$.
(A) $\frac{3}{2}$ (B) 2
(C) 4 (D) None of these

**40.** If $h(x)$ is a real function such that $h(0) = 1$, $h(1) = 2$ and $h(x+1) = 3h(x) - 2h(x-1)$ for all $x \geq 1$, then $h(6)$ is equal to _____.

## EXERCISE-3

***Directions for questions 1 to 40*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Which of the following represents the roster form of the set $A = \{x / x$ is an odd natural number less than 15$\}$?
(A) {1, 3, 5, 7, 9, 11, 13, 15}
(B) (1, 3, 5, 7, 9, 11, 13)
(C) {0, 1, 3, 5, 7, 9, 11, 13}
(D) {1, 3, 5, 7, 9, 11, 13}

**2.** If $A = \{2, 3, 5, 7\}$ and $B = \{2, 4, 6, 8\}$, then which of the following is true for the sets $A$ and $B$?
(A) $A$ and $B$ are equal.
(B) $A$ and $B$ are disjoint.
(C) $A$ and $B$ are equivalent.
(D) None of these

**3.** Which of the following is/are a subset of {2, 4, 6, 8}?
(A) { } (B) {2, 8}
(C) Both (A) and (B) (D) None of these

**4.** If $A$ denotes the set of all factors of 10 and $B$ denotes the set of all factors of 12, then $A \cap B$ is
(A) {1, 3, 4} (B) {1, 2} (C) {1, 3} (D) {1, 2, 5}

**5.** If $f(x) = \frac{3x-5}{2x+1}$ for $x \neq -1/2$, then $f^{-1}(x) =$
(A) $\frac{3x+5}{2x-1}$ (B) $\frac{3x-5}{2x+1}$ (C) $\frac{x+5}{3-2x}$ (D) $\frac{x+5}{2x-3}$

**6.** Suppose $X_1, X_2, X_3, \ldots X_{15}$ are 15 sets, each with six elements and $Y_1, Y_2 \ldots Y_n$ are $n$ sets, each with five elements such that $\bigcup_{i=1}^{15} X_i = \bigcup_{i=1}^{n} Y_i = S$. If each element of $S$ belongs to exactly 10 of $X$'s and exactly 5 of the $Y$'s, then the value of '$n$' is
(A) 12 (B) 18 (C) 15 (D) 9

**7.** Let set $S = \{x / x$ is a prime number$\}$ and for all natural numbers $n > 3$ $An = \{x / x$ is a multiple of $n\}$. Then, $S - UAn$, (where $UAn$ is the union of the sets $A_4, A_5, \ldots$) is
(A) {1,2,3} (B) {2,3}
(C) {2,3,5,7} (D) {1,2,3,4}

**8.** If $n(A) = 5$, find the number of functions that can be defined, that are neither bijections nor constant functions from set $A$ to $A$.
(A) 3000 (B) 3125 (C) 3025 (D) 3005

**9.** The number of onto functions from set $A$ to set $B$ if $n(A) = 5$ and $n(B) = 3$ is
(A) 140 (B) 150 (C) 120 (D) 130

**10.** If $h(x, y, z, w) = xz - yw$ and $h(p-1, p, p+1, 3) = h(4, 3, 5, 1)$, then the possible values of $p$ are
(A) –3, 6 (B) 3, 6
(C) 3, –6 (D) –3, –6

**11.** If $f(x) = 3x - 2$ and $g(x) = 2x + 5$, then $fog(x) =$
(A) $6x + 13$ (B) $6x - 13$
(C) $6x - 1$ (D) $6x + 1$

**12.** If $f(2x-3) = 4x^2 - 12x + 4$, then $f(3x-2) =$
(A) $9x^2 + 12x + 1$ (B) $9x^2 - 12x - 1$
(C) $x^2 - 12x - 1$ (D) $8x^2 - 12x - 1$

**13.** If $f(x) = x^2 + 3$ and $g(x) = x + 5$, then $f[g(x)] =$
(A) $x^2 + 8$ (B) $x^2 + 10x + 28$
(C) $x^3 + 5x^2 + 3x + 15$ (D) $x^3 + 15$

**14.** If $f = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$, then $fof =$
(A) $\{(1, 4), (2, 4), (3, 1)\}$
(B) $\{(1, 3), (2, 4), (3, 1)\}$
(C) $\{(2, 4), (3, 1), (4, 2), (2, 1)\}$
(D) $\{(1, 3), (2, 4), (3, 1), (4, 2)\}$

**15.** If $A = \{1, 2, 3\}$ and $B = \{4, 5\}$, then which of the following set of ordered pairs can represent a function from $A$ to $B$?
(A) $\{(1, 4), (2, 5), (1, 5)\}$
(B) $\{(1, 4), (2, 5)\}$
(C) $\{(1, 4), (2, 4), (3, 5)\}$
(D) $\{(1, 4), (1, 5), (2, 4), (3, 5)\}$

**16.** If $f(x)$ is a fourth degree polynomial function such that $f(x) + f\left(\frac{1}{x}\right) = -f(x)\, f\left(\frac{-4}{3}, 0\right)$ and $f(5) = 624$, then find $f(3)$.
(A) 81 (B) 80 (C) 82 (D) 96

**17.** If $f(x) + f(1 - x) = 6$, then find the value of $f\left(\frac{5}{30}\right) + f\left(\frac{6}{30}\right) + f\left(\frac{7}{30}\right) + \ldots + f\left(\frac{25}{30}\right)$.
(A) 63 (B) 73 (C) 61 (D) 65

**18.** $F(x)$ is a function. It satisfies the conditions below.
$F(1) = 3465$ and $F(1) + F(2) + \ldots + F(N) = 2NF(N)$ where $N$ is any natural number greater than 1. Find $F(6)$.
(A) 620 (B) 640 (C) 660 (D) 680

**19.** Given: $f(x) + 2f\left(\frac{1}{x}\right) = 2x - 1$. Find the value of $f\left(\frac{1}{3}\right)$.
(A) $\frac{20}{9}$ (B) $\frac{4}{3}$ (C) $\frac{31}{9}$ (D) $\frac{-17}{9}$

**20.** If $f(x) = \frac{x}{\sqrt{1+x^2}}$, then $(fofofof)\,(4) =$
(A) $\frac{4}{\sqrt{65}}$ (B) $\frac{1}{\sqrt{17}}$
(C) 3 (D) $\frac{4}{9}$

**21.** If $f(x) = 2x - 1$, $x \in R$ and $g(x) = 3x + 4$, $x \in R$, then $(gof)^{-1}(0) =$
(A) $\frac{1}{6}$ (B) $-\frac{1}{6}$ (C) $\frac{-7}{6}$ (D) $\frac{7}{6}$

**22.** Find the domain of the function
$f(x) = \frac{1}{\log|x-1|} + \frac{1}{x+1}$.
(A) $R - \{-1, 0, 2\}$ (B) $R - \{-1, 0, 1\}$
(C) $R - \{-1, 0, 1, 2\}$ (D) $R - \{0, 1, 2\}$

**23.** The domain of the real function
$f(x) = \frac{2x}{\sqrt{x^2 - 2x - 15}}$ is
(A) $[-3, 5]$. (B) $[-3, \infty)$.
(C) $(-\infty, -3) \cup (5, \infty)$. (D) $(-5, 3)$.

**24.** Find the range of the function $f(x) = x - [x]$, where $[x]$ is the greatest integer less than or equal to $x$.
(A) $[0, 1]$ (B) $(0, 1]$
(C) $[0, 1)$ (D) $(0, 1)$

**25.** Domain of the function $f(x) = \frac{2}{1-[x]}$ is
(A) $(-\infty, 1) \cup [2, \infty)$ (B) $(-\infty, -1] \cup [2, \infty)$
(C) $(-\infty, -1) \cup (2, \infty)$ (D) $[-4, -1] \cup [2, 4]$

**26.** Find the domain of the function $f(x) = \log(2x - 1) + \frac{1}{\sqrt{x^2 - 9}}$.
(A) $(-3, 3)$ (B) $(-\infty, -3) \cup (3, \infty)$
(C) $(3, \infty)$ (D) $\left(\frac{1}{2}, \infty\right)$

**27.** If $f(xy) = f(x)f(y)$ for all $x, y \in R$ and $f(8) = 64$, then find $\frac{|x|}{x} f(n)$
(A) 124 (B) 1640
(C) 1240 (D) 2480

**28.** If $2f(x) - 3f\left(\frac{1}{x}\right) = 5x^2$ for all $x \in R$, then find the value of $f(0.01)$.
(A) –30,000.0002 (B) –30002
(C) –0.3002 (D) –20.000.0002

**29.** If $h(x) + h(2 - x) = 7$, then find the value of
$h\left(\frac{3}{99}\right) + h\left(\frac{4}{99}\right) + h\left(\frac{5}{99}\right) + \ldots + h\left(\frac{195}{99}\right)$.
(A) 575.5 (B) 775 (C) 695.5 (D) 675.5

**30.** Given $f(x+1) = 2x^2 + 4x - 9$ for all $x \in R$, then the value of $f(-2)$ is
(A) 3 (B) –3 (C) –9 (D) –11

**31.** If $f(x) = \sqrt{x(x+1)(x+2)(x-3)}$ is a real valued function, find the domain of $f(x)$.
(A) $(-\infty, -2] \cup [3, \infty)$ (B) $(-\infty, 3]$
(C) $(-\infty, -2] \cup [-1, 0] \cup [3, \infty)$ (D) $[-1, \infty)$

**32.** $f_1(x) = 5x^2 + 7$; $x \in R$, and $f_2(x) = 3x^2 - 3x + 12$; $x \in R$. The positive value of $x$ such that $f_1(x) = f_2(x-1)$ is _____.

**33.** Given $f(x) + f(2-x) = 6$. The value of
$f\left(\frac{1}{150}\right) + f\left(\frac{2}{150}\right) + \ldots\ldots + f\left(\frac{299}{150}\right)$ is _____.

**34.** The range of the function $f(x) = \frac{1}{4 - \sin 2x}$; $x \in R$ is
(A) $\left(\frac{1}{4}, \frac{1}{2}\right)$. (B) $\left(\frac{-1}{2}, \frac{1}{4}\right)$.
(C) $\left[\frac{-1}{4}, \frac{1}{2}\right]$. (D) $\left[\frac{1}{5}, \frac{1}{3}\right]$.

**35.** For all $x \in R$, $f(x) = \min(x-4, x+2)$ and $g(x) = \max(x-2, x+3)$. If $h(x) = f(x) + g(x)$, then the set of values of $x$ for which $h(x) \le 5$ is
(A) $(-\infty, 3]$ (B) $[3, \infty)$ (C) $[-2, 4]$ (D) $[-3, 4]$

**36.** The function $f(x)$ is defined such that $f(2x) + f(3x) + f(x+2) + f(3-x) = x$, for all real values of $x$. Find $f(0)$.
(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

**37.** If $f(x) = px + q$ and $f\{f[f(x)]\} = 8x - 56$, then find the value of $f^{-1}(-6)$.
(A) 0 (B) 1 (C) –2 (D) 2

**38.** If $f(x\,y) = f(x)f(y)$, $\forall\ x, y \in R$ and $f(3) = 27$, then find $\sum_{n=1}^{12} f(n)$. _____

**39.** The greatest integer less than the least value of the function $f(x) = \frac{-x^2}{x^2 + x + 1}$ is
(A) 2 (B) –1 (C) 1 (D) –2

**40.** For $|x| < 1$, $g(x) = \log\left(\frac{1-x}{1+x}\right)$ and $h(x) = \frac{x^3 + 3x}{3x^2 + 1}$, then $(goh)(x)$ is
(A) $3g(x)$ (B) $3h(x)$
(C) $(g(x))^3$ (D) $(h(x))^3$

## EXERCISE-4

***Directions for questions 1 to 25*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** If $f(p)$ is a polynomial satisfying $f(p)\,f\left(\frac{1}{p}\right) = f(p) + f\left(\frac{1}{p}\right)$ and $f(5) = -124$, then $f(9) =$ _____.

**2.** If $f(x)$ and $f(y)$ satisfy the relation $f(x+y) = f(x) + f(y)$ where $x, y \in R$ and $f(1) = 2$, then $\sum_{i=1}^{10} f(i) =$ _____.

**3.** $f: R \to R$ is defined as $f(x) = x - 1$ for $x < -2$
$= x$ for $-2 \le x \le 2$
$= 1 - 2x$ for $x > 2$
The value of $f(-2.5) + f(-1.5) + f(-3.5) =$ _____.

**4.** If $f(x) = 2^x + 2^{-x}$ and $g(x) = 2^x - 2^{-x}$, the value of $f(x)\,g(y) + f(y)\,g(x) =$
(A) $g(x+y)$ (B) $f(x+y)$
(C) $2g(x+y)$ (D) $2f(x+y)$

**5.** The domain of the function $f(x) = \sqrt{\frac{5-|x|}{7-|x|}}$ is
(A) $[-5, 5]$ (B) $[-5, 5] \cup (-\infty, -7) \cup (7, \infty)$
(C) $(-\infty, -7)$ (D) $(7, \infty)$

***Directions for questions* 6 *to* 8:** These questions are based on the following data.

For real numbers $x$ and $y$, consider the following definitions. $f(x+y) = f(x)\,f(y)$; $g(x) + g(y) = g(x \times y)$ and $fn(x) = f(x) \times (x) \times \ldots\ n$ times. Given that $f(x) \ne 0$, for $x \ne 0$, answer the following questions.

**6.** If $f(1) = 4$, then find $f^6(4)$.
(A) $2^{34}$ (B) $2^{24}$ (C) $2^{48}$ (D) $2^{44}$

**7.** If $g(2) = 3$ and $f(3) = 2$, then find $f(g(8))$. _____

**8.** Which of the following is incorrect?
(A) $fn(x) = f(nx)$
(B) $g(1) = 0$
(C) If $g(2) = 3$ and $f(3) = 4$, then $f(g(4)) = 16$
(D) $f(0) = 0$

**9.** If $(x) = x^2 + 3x + 6$, $g(x) = \frac{|x|}{x}$ $h(x) = 1 - 2x^5$ and $i(x) = x^2 - 4$, find the value of $fig[h(i(1.5))]\}$.
(A) 4 (B) 10
(C) 16.5 (D) None of these

**10.** $f(x)f(y) = f(x) + f(y) + f(xy) - 2$ where $x$ and $y$ are positive real numbers. If $f(3) = 10$ and $f(2) = 5$, find $f(12)$. _____

**11.** If $f(x-y) = \frac{f(x)}{f(y)}$, find $f(50x)$.
(A) $f(x)$ (B) $50f(x)$
(C) $[f(x)]^{50}$ (D) $100f(x)$

**12.** $f(t) = \frac{1-t}{1+t}$, $(t \neq -1)$, $f^2(t) = f[f(t)]$ and for $n > 2$, $f^n(t) = f(f^{n-1}(t))$. Find the value of $f^{132}(0)$.
(A) 1/2 (B) 1 (C) 0 (D) 2

**13.** If $f(xy) = f(x)\ f(y)$ and $f(2) = 1/8$, find $f(1) + f(1/2) + f(1/3)$. _____

**14.** Let $A = \{1, 2, 3.....100\}$ and $B$ be a non-empty subset of $A$. How many such sets $B$ can be formed such that the product of all the elements of $B$ is even?
(A) $2^{50} - 1$ (B) $2^{100} - 2^{49}$
(C) $2^{50}\ (2^{50} - 1)$ (D) $(2^{50} - 1)\ (2^{50} - 1)$

**15.** If $f(x) = \frac{5^x + 5^{-x}}{2}$, $2f(x+y)f(x-y) =$
(A) $f(2x) + f(2y)$ (B) $f(2x)\ f(2y)$
(C) $\frac{f(2x)}{f(2y)}$ (D) $f(2x) - f(2y)$

**16.** If $-1 < x < 1$, find the inverse of the function $f(x) = \frac{3^x - 3^{-x}}{3^x + 3^{-x}}$.
(A) $\frac{1}{2}\log_3\left(\frac{1+x}{1-x}\right)$ (B) $\frac{1}{2}\log_3\left(\frac{1-x}{1+x}\right)$
(C) $\log_3\left(\frac{1+x}{1-x}\right)$ (D) $\log_3\left(\frac{1-x}{1+x}\right)$

**17.** If $f(4x+3) = 32x^2 + 68x + 40$, $f(3x+4) =$
(A) $40x^2 + 68x + 32$ (B) $18x^2 + 63x + 59$
(C) $32x^2 + 68x + 40$ (D) $12x^2 + 30x + 42$

**18.** The domain of the real function
$f(x) = \frac{1}{\sqrt{x^2 - 9}} + \log(x-2) + \frac{1}{x+2}$ is
(A) $(2,\infty)$ (B) $(-3, 3)$
(C) $(3, \infty)$ (D) $(-\infty, -3)$

**19.** The range of the function $\frac{x^2 + x + 1}{x^2 - x - 1}$ is
(A) $(-\infty, 1]$ (B) $[1, \infty)$
(C) $(-\infty, -1] \cup \left[\frac{3}{5}, \infty\right)$ (D) None of these

**20.** For two sets $A$, $B$, $n(A) = 6$ and $n(B) = 5$. Find the number of functions which are not onto that can be defined from $A$ to $B$. _____

**21.** If $f(x) = \frac{x+1}{x-1}$ for $x \neq 1$, find $f^{-1}of^{-1}of^{-1}of^{-1}of^{-1}(5)$.
(A) $\frac{1}{5}$ (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) 5

**22.** For a real number $x$, $f(x) = 1/(1+x)$.
Also, $f^1(x) = f(x)$ and $f^n(x) = f(f^{n-1}(x))$, for $n = 2, 3, \ldots$
What is the value of $f(2)f^2(2)f^3(2)f^4(2)f^5(2)$?
(A) 1/3 (B) 3 (C) 1/18 (D) 18

**23.** If $f(x) = px + q$ and $f\{f[f(x)]\} = 64x + 147$, the value of $2p + 3q$ is _____.

**24.** If $f(x) = 27x^4$ and $g(x) = 3\sqrt{f(x)}$, the value of $log_3\ [fog(729)]$ is _____.

**25.** If $f(x) = px^5 + qx^3 - rx + 3$ and $f(4) = 10$, $f(-4) =$
(A) $-3$ (B) 2 (C) $-4$ (D) $-7$

## ANSWER KEYS

### Exercise-1

| | | | | | |
|---|---|---|---|---|---|
| 1. (C) | 7. (D) | 13. (D) | 19. (B) | 25. (C) | 31. (B) |
| 2. (B) | 8. 4 | 14. (D) | 20. (B) | 26. (C) | 32. (C) |
| 3. (A) | 9. 2 | 15. (A) | 21. (B) | 27. 13 | 33. (A) |
| 4. (C) | 10. (B) | 16. 64 | 22. (C) | 28. (A) | 34. (B) |
| 5. (A) | 11. (A) | 17. (A) | 23. (B) | 29. (A) | 35. (D) |
| 6. 4 | 12. (C) | 18. 15 | 24. 128 | 30. (B) | 36. (B) |

### Exercise-2

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (B) | 7. (C) | 13. (A) | 19. (C) | 25. 16 | 31. (D) | 37. 2 |
| 2. 7 | 8. 5 | 14. (C) | 20. (C) | 26. (B) | 32. (C) | 38. (A) |
| 3. 6 | 9. 3 | 15. (A) | 21. 16 | 27. 720 | 33. (D) | 39. (A) |
| 4. (C) | 10. (A) | 16. (A) | 22. 7 | 28. (A) | 34. (D) | 40. 64 |
| 5. (D) | 11. (C) | 17. (A) | 23. 16 | 29. (D) | 35. (B) | |
| 6. 6 | 12. (C) | 18. 5 | 24. 15 | 30. (B) | 36. 2 | |

### Exercise-3

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (D) | 7. (B) | 13. (B) | 19. (C) | 25. (A) | 31. (C) | 37. (B) |
| 2. (C) | 8. (A) | 14. (D) | 20. (A) | 26. (C) | 32. 1 | 38. 6084 |
| 3. (C) | 9. (B) | 15. (C) | 21. (B) | 27. (C) | 33. 897 | 39. (D) |
| 4. (B) | 10. (A) | 16. (B) | 22. (C) | 28. (A) | 34. (D) | 40. (A) |
| 5. (C) | 11. (A) | 17. (A) | 23. (C) | 29. (D) | 35. (A) | |
| 6. (D) | 12. (B) | 18. (B) | 24. (C) | 30. (B) | 36. (D) | |

### Exercise-4

| | | | | | |
|---|---|---|---|---|---|
| 1. –728 | 6. (C) | 11. (C) | 16. (A) | 21. (D) | 37. (C) |
| 2. 110 | 7. 8 | 12. (C) | 17. (B) | 22. (C) | 38. (A) |
| 3. –9.5 | 8. (D) | 13. 36 | 18. (C) | 23. 29 | 39. 4 |
| 4. (C) | 9. (B) | 14. (C) | 19. (C) | 24. 39 | 40. (C) |
| 5. (B) | 10. 145 | 15. (A) | 20. 13825 | 25. (C) | |

## EXERCISE-1

**1.** In sets, every element occurs only once. The given set can be written as {M, *A*, *T*, *H*, *E*, *I*, *C*, *S*}.

∴ The given set has 8 elements.

**2.** Given $x = \{x / x$ is an odd prime numbers upto 18$\}$.

$= \{3, 5, 7, 11, 13, 17\}$.

**3.** The empty set is a set with no elements.

**4.** The given set of odd natural numbers upto 15 clearly from option (C), $x = 2y - 1$, $y \in N \leq 8$ gives odd numbers upto 15.

**5.** Option (A) $\{x / x \neq x\}$ as any one is equal to itself. ∴ it is an empty set. Option (B) $\{x / x = 2y$, $y$ is an odd number$\}$ is a set containing 2, 6, 10, … ∴not an empty set

Option (C) $\{x / x = 2n - 1$, $n \in N$ $n$ is an even number$\}$

∴ $n \in \{2,4\}$ is not an empty set.

**6.** There are 4 distinct letters *M*, *I*, *S*, *P*. We do not repeat elements in a set. Thus, the cardinality of the set is 4.

**7.** As sets *A* and *B* have the same elements, the two sets are equal.

∴ $A \subseteq B$ and $B \subseteq A$ are also true.

**8.** The elements of the set are 2, {2, 3}, 3, {1, 2, 3} which are 4 in number.

**9.** For any non-empty set, there exists a minimum of two subsets, One is the null set and the other is the set itself.

**10.** Only the set mentioned in choice (B) is the subset of the given set. Choice A is an element of the given set. {2, 3} would be a subset.

**11.** $A \cap B$ is the set containing the elements belonging to both set *A* and *B*, i.e., $A \cap B = \{a, e, i, o, u\}$

**12.** In the given set *a*, *b* are elements ∴ {*a*, *b*} is a subset also the set contains {*a*, *b*} as an element, hence, {*a*, *b*} is also a subset.

**13.** $A \cup B$ is the set containing the elements of *A* or *B* or both

∴ $A \cup B = \{a, b, c, d. e, f\}$.

**14.** Choice (A) $x/x \in N \leq 10 \Rightarrow \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Choice (B), in set as the order of writing the elements has no importance it is also same as {1, 2, ..., 10}

Choice (C), as each element in the set is counted once and repetition of elements has no value the given set is {1, 2, ..., 10}.

**15.** The number of subsets of a set containing $n$ elements is $2n$. Here, $n = 5$ $\therefore$ The number of subsets = $2^5 = 32$.

**16.** The number of subsets of a set with $n$ elements is $2n$. For the given set, $n = 6$. $\therefore$ Number of sub sets $2^6 = 64$.

**17.** $A \cap B$ is the set containing the elements that are present in both $A$ and $B$. (common elements make the intersection). i.e., $A \cap B = \{2, 3\}$

**18.** For a set with $n$ elements, the number of proper subsets is $2n - 1$. For the given set, this is $2^4 - 1$ or 15.

**19.** The number of proper subsets of a set containing $n$ elements is given by $2n - 1$. Among the choices, only 31 can be expressed as $2^5 - 1$.

**20.** $A \cup B$ is the set containing the elements of $A$ or $B$ or both.

$\therefore A \cup B = \{1, 2, 3, 4, 5, 6\}$

**21.** If $n(A) = n(B)$, then $A$, $B$ are called equivalent sets, i.e., equivalent sets have the same number of elements.

**22.** $A \Delta B = (A - B) \cup (B - A)$ or $(A \cup B) - (A \cap B)$.

**23.** If $A \subseteq B$, then $A \cup B = B$.

**24.** Out of the options only Choice (D), i.e., 128 can be expressed as $2n$, i.e., $2^7$.

**25.** If $n(A \cap B) = 0$, then sets $A$ and $B$ are called disjoint sets.

**26.** $A^c = \mu - A = \{2, 4, 6, 8, 9, 10\}$

**27.** $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

If $A$ and $B$ are disjoint, then $A \cup B$ will have the maximum number of elements, which is $n(A) + n(B) = 5 + 8 = 13$.

**28.** $B - A$ is the set containing the elements that are there in set $B$ but not in $A$.

$\therefore B - A = \{8, 10\}$

**29.** $A \cup A' = \mu$ (Standard result)

**30.** $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. If $A \cap B$ has the maximum number of elements, i.e., if $A \subseteq B$, then $A \cup B$ will have the minimum number of elements which is the number of elements of $B$ itself, which is 9.

**31.** Given $y = f(x) = \dfrac{2x-1}{x+2}$ $x \neq 2$

$xy + 2y = 2x - 1 \Rightarrow x(y - 2) = -(2y + 1)$

$x = -\dfrac{2x+1}{y-2}$

$\therefore f^{-1}(x) = \dfrac{2x+1}{x-2} \Rightarrow f^{-1}(3) = \dfrac{6+1}{2-3} = -7.$

**32.** Only in option (C), all the first coordinates are from set $A$ and all the second coordinates from set $B$.

**33.** We know if $(a, b) \in R$, then $(b, a) \in R^1$

$\Rightarrow R^{-1} = \{(2, 1), (3, 2), (4, 3), (1, 4)\}$.

**34.** Given $(a + b, 2) = (6, a - b) \Rightarrow a + b = 6$ and $a - b = 2$

$\therefore a = 4$ and $b = 2$ and $2a - 3b = 2(4) - 3(2) = 2$.

**35.** $A \times B = \{(x, y)/\ x \in A$ and $y \in B\}$; here, $A = \{1, 2, 3\}$, $B = \{4, 5\}$ is $A \times B = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$

**36.** $R^{-1}$ is a the set of reverse pairs corresponding to all the pairs in $R$.

$R^{-1} = \{(x, y)/(y, x) \in R\} = \{(2, 1), (3, 2), (4, 3), (1, 4)\}$

**37.** $A \times B = \{(x, y)/x \in A, y \in B)$

**38.** The first coordinates should be from $A$ and the second from $B$. Only choice $A$ satisfies these conditions.

**39.** If $A \subseteq B$, then $A \cap B$ will have the maximum number of elements. If this is the case, $n(A \cap B) = n(A) = 4$.

**40.** $n(A \times B) = n(A)\ n(B) = 3(4) = 12$ (Standard Result)

## EXERCISE-2

**1.** The number of proper subsets of $A$ is $2^7 - 1 = 127$.

**2.** If set $P$ contains $n$ elements, then the number of proper subsets other than empty set = $2^n - 2 = 126$

$\Rightarrow 2^n = 128 = 2^7$ $\therefore$ $n = 7$

**3.** Let $n(A) = n$

$(2^n - 1) - 2^{n-3} = 55 \Rightarrow 2^n - 2^{n-3} \Rightarrow 2^{n-3}(7) = 56$

$\Rightarrow n = 6$, $\therefore$ $n(A) = 6$

**4.** The required number of subsets

$= 2^{n-p-q}$ where $n$ is the number of elements in the set, $p$ is the number of elements which have to be included and $q$ is the number of elements that have to be excluded.

$= 2^{7-(3+2)} = 2^2 = 4$

**5.** Let $n(A) = P$. Then, the number of subsets that contain exactly 3 elements is given by

$PC_3 = 56 \Rightarrow \dfrac{P(P-1)(P-2)}{1(2)(3)} = 7(8)$

$P(P-1)(P-2) = 8\ (7)\ (6) \Rightarrow P = 8$

**6.** The number of subsets containing a but not $b$ of a set $A$ having $n$ elements is given by $2^{n-2}$

$\therefore 2^{n-2} = 16$

$\Rightarrow 2^{n-2} = 2^4$

$\Rightarrow n - 2 = 4$

$\Rightarrow n = 6.$

**7.** $n(P(A)) = 2^n$. Here, $n(A) = 5 \therefore n(P(A)) = 2^5 = 32$

**8.** $(n(B))^{n(A)} = 625 \Rightarrow n(B)^4 = 5^4$

$\Rightarrow n(B) = 5$

**9.** The number of one-one functions that can be defined from set $A$ with $m$ elements to set $B$ with $n$ elements = ${}^nP_m$

${}^nP_m = 3360 = 16\ (210) = 16\ (15)\ (14)$

$= {}^{16}P_3 \therefore n(A) = 3,$

**10.** The number of onto functions can be defined from set $A$ to set $B$, where

$n(A) = m$ and $n(B) = n$ is $X = n^m - {}^nC_1\ (n-1)^m + {}^nC_2\ (n-2)^m + ... + [(-1)^{n-1}]\ {}^nC_{n-1}(1)^m$

Here, $n = 4,\ m = 6$

i.e., $X = 4^6 - {}^4C_1\ (4-1)^6 + {}^4C_2\ (4-2)^6 - {}^4C_3(4-3)^6$

$= 4096 - 4(729) + 6(64) - 4(1) = 1560$

**11.** Given: $A = \{1, 2, 3, 4, ..... 12\}$

Set builder form of $A$ is $\{x/x$ is a natural number $\leq 12\}$

**12.** $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ is distributive law.

**13.** Given, $A = \{2, 4, 5, 7, 9\}$, $B = \{1, 3, 5, 7, 8\}$ and $C = \{1, 2, 5, 8\}$

$\therefore (A \cup B) = \{1, 2, 3, 4, 5, 7, 8, 9\}$

$\Rightarrow (A \cup B) \cap C = \{1, 2, 5, 8\} = C$

**14.** Given: $A = \{0, 2, 4, 6, 9\}$ and $B = \{1, 3, 5, 7\}$

We know that, $A \Delta B = (A \cup B) - (A \cap B)$

Since $A \cap B = \phi$, $A \Delta B = A \cup B$.

**15.** If a set has '$n$' elements, then the cardinal number of its power set is $2^n$.

Options (B), (C) and (D) can be expressed as $2^8$, $2^0$ and $2^6$ respectively, but option (A) cannot be expressed in the form $2^n$.

$\therefore$ cardinal number of power set of any finite set cannot be 1056.

**16.** $f(x - 1/x) = x^2 + 1/x^2$

$= x^2 + 1/x^2 - 2 + 2 = (x - 1/x)^2 + 2$

i.e., $f(x) = x^2 + 2$

$f(x^2 + 1/x^2) = (x^2 + 1/x^2)^2 + 2$

$= x^4 + 1/x^4 + 2 + 2 = x^4 + 1/x^4 + 4$

**17.** Given: $f(x) = x^2 + 35$ and $g(x) = x^2 - 10x + 12$

$f(2x) = g(x-1) \Rightarrow (2x)^2 + 35 = (x-1)^2 - 10(x-1) + 12$

$\Rightarrow 4x^2 + 35 = x^2 - 12x + 23$

$\Rightarrow 3x^2 + 12x + 12 = 0$ or $x^2 + 4x + 4 = 0$ or $(x+2)^2 = 0$

$\Rightarrow x = -2$

**18.** Given the number of bijections = $n! = 120 \therefore n = 5$

As bijections are possible from $A$ to $B$, $n(A) = n(B) = 5$

**19.** The total number of functions that can be defined from set $A$ to set $B$ is $(n(B))^{n(A)} = 6^4 = 1296$

The number of one-one functions from set $A$ to set $B$ is ${}^{[n(B)]}P_{n(A)} = {}^6P_4 = 360$

$\therefore$ The number of functions which are not one-one

$= 1296 - 360 = 936$

**20.** Given: $f(x) = 5$ if $x$ is rational and $f(x) = -5$ if $x$ is irrational.

$\therefore\ f(\sqrt{5}) = -5,\quad |f(5)| = 5\quad \sqrt{f(2)} = \sqrt{5}$

$f(\sqrt{3}) = -5\quad \sqrt{\|f(3)\|} = \sqrt{5}\quad |f|\sqrt{5}\|| = 5$

$\therefore f(\sqrt{5}) + f(\sqrt{3}) + |f(5)| + \sqrt{|f(3)|} + \sqrt{|f(2)|} + |f(\sqrt{5})|$

$= -5 - 5 + 5 + \sqrt{5} + \sqrt{5} + 5 = 2\sqrt{5}$

**21.** The number of subsets which contain 6 but not 12 is $2^4$, i.e., 16

**22.** The number of subsets which have atmost one element is ${}^6C_0 + {}^6C_1$, i.e., $1 + 6 = 7$

**23.** Given, $A = \{1, 2, 3, 4, 6, 12\}$

The number of subsets which contain 3 and 4 is $2^4$, i.e., 16.

**24.** The number of subsets which contain exactly 4 elements is ${}^6C_4$, i.e., 15 .

**25.** Given, $A = \{2, 3, 5, 7\}$

$\Rightarrow n(A) = 4$

$\therefore$ maximum number of elements in any relation is $4 \times 4 = 16$.

**26.** $\{(1, 1), (1, 2), (2, 2)\}$ is not a function, as the element 1 has as two images.

$\{(1, 1), (2, 2)\}$ alone is a function

**27.** $A$ and $B$ are equivalent sets $\therefore n(A) = n(B) = 6$

$\therefore$ The number of bijections from $A$ to $B$ (or the number of bijections from $B$ to $A$) = 6! = 720

**28.** The domain of $f$ is the set of all the first coordinates, i.e., $\{1, 2, 3, 4\}$

The range of $f$ is the set of all the second coordinates, i.e., $\{a, b, c\}$

**29.** Given: $f(x) = \log(2x-3) + \sqrt{x^2-9} + \dfrac{1}{\log(x-4)}$

$\log(2x-3)$ is defined only when $2x-3>0$

or when $x > 3/2$ (1)

$\sqrt{x^2-9}$ is defined only when $x^2-9 \geq 0$

$\Rightarrow (x-3)(x+3) \geq 0$

$\Rightarrow x \notin [-3, 3]$ (2)

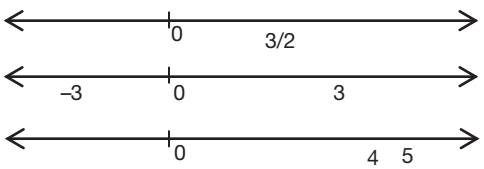
$\dfrac{1}{\log(x-4)}$ is defined only when $x-4>0$ and

$x-4 \neq 1$

$x>4$ but $x \neq 5$

$\Rightarrow x \in (4, 5) \cup (5, \infty)$ (3)

$\therefore$ The domain of $f(x)$ is the common region for (1), (2), (3), which is (3) itself.




$\therefore$ Domain $= (4, 5) \cup (5, \infty)$

**30.** Let $\dfrac{3x}{x^2-x+1} = m$

$\Rightarrow mx^2 - mx + m = 3x$

Or $mx^2 - x(m+3) + m = 0$ …

The given expression can assume only these values $m$ for which the roots of (1) are real.

$\Rightarrow b^2 - 4ac \geq 0$

$(m+3)^2 - 4m^2 \geq 0$

$\Rightarrow 3m^2 - 6m - 9 \leq 0$

$\Rightarrow m^2 - 2m - 3 \leq 0$

$\Rightarrow (m-3)(m+1) \leq 0$

$\Rightarrow m \in [-1, 3]$

$\therefore$ range of $f$ is $[-1, 3]$

**31.** Given $h(x) = K^x$

Consider option (A)

$h(x+2) - 2h(x+2)\ h(-1) + h(x)\ h(0)$

$= k^{x+2} - 2\ k^{x+2} \cdot k^{-1} + k^x \cdot k^o = k^x\ (k^2 - 2k + 1)$

$= k^{x+2} - 2k^{x+1} + k^x$

$= (k-1)^2\ k^x = (k-1)^2\ h(x)$ True.

Option (B) $h(x+1)\ h(x-1) = k^{x+1} \cdot k^{x-1} = k^{2x} = (kx)^2$ $= (h(x))^2$ True

Option (C) $h(x)\ h(-x) = k^x \cdot k^{-x} = 1$ True.

**32.** Let $h(x) = k^x$

$h(x+y) = k^{x+y} = k^x \cdot k^y = h(x) \cdot h(y)$

Given $h(5) = 15625 \Rightarrow k^6 = 15625 = 5^6 \Rightarrow k = 5$

$\therefore\ h(3) = 5^3 = 125$

**33.** Let $f(x) = y$

$f(x+1) = 3y^2 - 2$

$f(x+2) = 3(3y^2-2)^2 - 2 = 3[9y^4 - 12y^2 + 4] - 2$

$= 27y^4 - 36y^2 + 10$

$= 27[f(x)]^4 - 36[f(x)]^2 + 10$

**34.** Given $g\left(\dfrac{a}{b}\right) = \dfrac{g(a)}{g(b)}$ and $g(3) = \dfrac{1}{9}$

$\therefore\ 9\left(\dfrac{3}{1}\right) = \dfrac{g(3)}{g(1)}$

$\therefore\ g(1) = 1$

$g\left(\dfrac{1}{3}\right) = \dfrac{g(1)}{g(3)} = \dfrac{1}{\frac{1}{9}} = 9$.

**35.** Given $f_1(x) = 2x^2 - 5$ and $f_2(x) = 2x^2 - 5x - 36$

$f_1(x-1) = f_2(x+1)$

$\Rightarrow 2(x-1)^2 - 5 = 2(x+1)^2 - 5(x+1) - 36$

$\Rightarrow 2x^2 - 4x - 3 = 2x^2 - x - 39$

$-3x = -36 \Rightarrow x = 12$

**36.** $f(x) = kx$

given $f(3) = 18 = k(3)$

$\Rightarrow\ k = 6$

$\therefore\ f(x) = 6x$

$f\left(\dfrac{1}{3}\right) = 6 \times \dfrac{1}{3} = 2$

**37.** Given $g(x) = 3x^2 + 8$ and $h(x) = 3x^2 + 4x - 56$ and

$g(x-2) = h(x+2) \Rightarrow 3(x-2)^2 + 8 = 3(x+2)^2 + 4(x+2) - 56$

$3x^2 - 12x + 12 + 8 = 3x^2 + 12x + 12 + 4x + 8 - 56 - 28$

$x = -56$

$\Rightarrow x = \dfrac{56}{28} = 2$

**38.** Given: $H(1) + H(2) + H(3) + ... + H(N) = \frac{2NH(N)}{N-1}$ (1)

Given: $H(1) = 2460$

Setting $N = 2$, $H(1) + H(2) = \frac{2(2)H(2)}{1}$

$\Rightarrow 3\,H(2) = 2460$

$\Rightarrow H(2) = 820$

Setting $N = 3$, $H(1) + H(2) + H(3) = \frac{2(3)H(3)}{2}$

$\Rightarrow 2460 + 820 = 2\,H(3)$

$\Rightarrow H(3) = 1640$

Setting $N = 4$, $H(1) + H(2) + H(3) + H(4) = \frac{2(4)H(4)}{3}$

$2460 + 820 + 1640 = \frac{5}{3}H(4)$

$\therefore H(4) = 2952$

**39.** Given: $f(x) = \frac{4x}{4^x + 2}$

$$f\left(\tfrac{1}{2}\right) = \frac{4^{1/2}}{4^{1/2}+2} = \frac{2}{2+2} = \frac{1}{2}$$

$$f\left(\tfrac{1}{4}\right) = \frac{4^{1/4}}{4^{1/4}+2}$$

$$f\left(\tfrac{3}{4}\right) = \frac{4^{3/4}}{4^{3/4}+2} = \frac{4^{1-\frac{1}{4}}}{4^{1-\frac{1}{4}}+2}$$

$$= \frac{4}{4+2.4^{1/4}}$$

$$= \frac{2}{2+4^{1/4}}$$

$$\therefore f\left(\frac{1}{2}\right) + f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right)$$

$$= \frac{1}{2} + \frac{4^{1/4}}{4^{1/4}+2} + \frac{2}{2+4^{1/4}}.$$

$$= \frac{1}{2} + \frac{4^{1/4}+2}{4^{1/4}+2}$$

$$= \frac{1}{2} + 1 = \frac{3}{2}$$

**40.** Given: $h(x+1) = 3\,h(x) - 2h(x-1)$ and $h(0) = 1$, $h(1) = 2$

Setting $x = 1$, $h(2) = 3\,h(1) - 2\,h(0) = 3(2) - 2(1) = 4$

Setting $x = 2$, $h(3) = 3\,h(2) - 2\,h(1) = 3(4) - 2(2) = 8$

Setting $x = 3$, $h(4) = 3\,h(3) - 2\,h(2) = 3(8) - 2(4) = 16$

Setting $x = 4$, $h(5) = 3\,h(4) - 2\,h(3) = 3(16) - 2(8) = 32$

Setting $x = 5$, $h(6) = 3\,h(5) - 2\,h(4) = 3(32) - 2(16) = 64$

## EXERCISE-3

**1.** Given: $A = \{x / x$ is an odd natural number less than 15$\}$

Roster form of the set $A$ is $\{1, 3, 5, 7, 9, 11, 13\}$

**2.** Given $A = \{2, 3, 5, 7\}$ and $B = \{2, 4, 6, 8\}$

$\therefore n(A) = 4$ and $n(B) = 4$

$\therefore A$ and $B$ are equivalent

**3.** Let $A = \{2, 4, 6, 8\}$

$\{\}$ and $\{2, 8\}$ are the subsets of $A$

[since empty set is a subset of every set]

**4.** Given, $A = \{1, 2, 5, 10\}$

$B = \{1, 2, 3, 4, 6, 12\}$

$\Rightarrow A \cap B = \{1, 2\}$

**5.** Let $y = \frac{3x-5}{2x+1}$

$\Rightarrow 2xy + y = 3x - 5$

$\Rightarrow x\,(2y - 3) = -(y + 5)$

$\Rightarrow x = \frac{-(y+5)}{2y-3}$ or $x = \frac{y+5}{3-2y}$

$\therefore f^{-1}(x) = \frac{x+5}{3-2x}$.

**6.** Each of the 15 sets $X_1, X_2, \ldots X_{15}$ has 6 elements.

These are not all distinct. Let the number of distinct elements be $p$. Therefore, $S$, the union of the 15 sets, has $p$ elements. The total number of occurrences of the $p$ elements in $S$ is 6(15) or 90. As each element occurs in 10 of the $X$'s the number of elements, $p$ has to be 9.

i.e., No of occurrences = (No of sets) (No of elements in each set)

= (No of elements) (No of occurrences of each element)

or

$15(6) = p(10) \Rightarrow p = 9$.

Among the $Y$'s, each of the 9 elements occurs 5 times while each of the $n$ sets has 5 elements.

$\therefore n(5) = 9(5) \Rightarrow n = 9$

$\therefore$ The value of $n$ is 9.

**7.** $S = \{2, 3, 5, 7, 11,\ldots\}$

$A_4 = \{4, 8, 12, 16 \ldots\}$,

$A_5 = \{5, 10, 15,\ldots\}$,

$A_6 = \{6, 12, 18,\ldots\}$, ...

$UA_n = A_4 \cup A_5 \cup A_6 \ldots \{4, 5, 6, \ldots\}$

$\therefore S - UA_n = \{2,3\}$

**8.** Total number of functions from set $A$ to $A$ is $n(A)^{n(A)}$.

Here, $n(A) = 5$, i.e., the number of functions is $5^5$ or 3125.

The number of bijections from set $A$ to $A$ is 5!, i.e.,120.

The number of constant functions from set $A$ to $B$ is $n(B)$, which is 5.

$\therefore$ The number of functions which are neither bijections nor constant is $3125 - 120 - 5 = 3{,}000$

**9.** The number of onto functions when $n(A) = p$ and $n(B) = q$, where $q < p$ is $q^p - {}^qC_1 (q-1)^p + {}^qC_2 (q-2)^p \ldots$

Here, $p = 5$ and $q = 3$

$\Rightarrow 3^5 - {}^3C_1(3-1)^5 + {}^3C_2 (3-2)^5 - {}^3C_3(3-3)^5$

$= 243 - 3(32) + 3(1) = 150$

**10.** Given $h(x, y, z, w) = xz - yw$ also

$\Rightarrow h(p-1, p, p+1, 3) = h(4, 3, 5, 1)$

$\Rightarrow (p-1)(p+1) - 3p = 20 - 3$, i.e., $p^2 - 3p - 18 = 0$

$\Rightarrow (p-6)(p+3) = 0$

$\Rightarrow p = -3$ or 6

**11.** $fog(x) = f[g(x)] = f[2x+5] = 3(2x+5) - 2$

$= 6x + 13$.

**12.** Given: $f(2x-3) = 4x^2 - 12 + 4 \Rightarrow f(2x-3) = (2x-3)^2 - 5$

$\therefore f(x) = x^2 - 5 \Rightarrow f(3x-2) = (3x-2)^2 - 5$

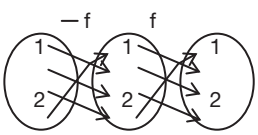
$= 9x^2 - 12x - 1$.

**13.** $f(g(x)) = f(x+5)$

$= (x+5)^2 + 3$

$= x^2 + 10x + 28$

**14.**




$\therefore fof = \{(1, 3), (2, 4), (3, 1), (4, 2)\}$

**15.** In options (A) and (D), multiple images exist for 1. So, these are not functions.

In option (B), 3 is not having any image. So, it is not a function.

**16.** Given $f(x) + f\left(\frac{1}{x}\right) = -f(x)\, f\left(\frac{1}{x}\right)$

When $f(x)$ is an nth degree polynomial satisfying the above condition, then $f(x)$ must be $x^n - 1$

Given $f(5) = 624 \Rightarrow f(5) = 5^n - 1 = 624 \Rightarrow n = 4$

$\therefore f(x) = x^4 - 1$

$f(3) = (3)^4 - 1 = 80$

**17.** Given: $f(x) + f(1-x) = 6$

$$f\left(\frac{5}{30}\right) + f\left(\frac{6}{30}\right) + f\left(\frac{7}{30}\right) + \ldots + f\left(\frac{25}{30}\right)$$

$$= \left[f\left(\frac{5}{30}\right) + f\left(\frac{25}{30}\right)\right] + \left[f\left(\frac{6}{30}\right) + f\left(\frac{24}{30}\right)\right]$$

$$+ \left[f\left(\frac{7}{30}\right) + \left(\frac{23}{30}\right)\right]$$

$$+ \ldots \left[f\left(\frac{14}{30}\right) + f\left(\frac{16}{30}\right)\right] + f\left(\frac{15}{30}\right)$$

If $a + b = 1$, then $f(a) + f(b) = 6$.

Setting $a = b = \frac{1}{2}$, we get $f\left(\frac{1}{2}\right) = 3$.

Required value $= 6 + 6 + \ldots + 6$

10 times $+ f\left(\frac{1}{2}\right) = 10(6) + 3 = 63$

**18.** Given $F(1) + F(2) + \ldots + F(N) = 2NF(N)$

When $N = 2$, $F(1) + F(2) = 4F(2)$

$\therefore F(2) = \frac{F(1)}{3} = \frac{3465}{3} = 1155$

When $N = 3$, $F(1) + F(2) + F(3) = 6F(3)$

$\therefore F(3) = \frac{F(1) + F(2)}{5} = \frac{3465 + 1155}{3} = 924$

When $N = 4$, $F(1) + F(2) + F(3) + F(4) = 8F(4)$

$\therefore F(4) = \frac{F(1) + F(2) + F(3)}{7}$

$= \frac{3465 + 1155 + 924}{7} = 792$

When $N = 5$, $F(1) + F(2) + F(3) + F(4) + F(5) = 10F(5)$

$\therefore F(5) = \frac{F(1) + F(2) + F(3) + F(4)}{9} = \frac{6336}{9} = 704$

When $N = 6$, $F(1) + F(2) + F(3) + F(4) + F(5) + F(6) = 12F(6)$

$$\therefore F(6) = \frac{F(1)+F(2)+F(3)+F(4)+F(5)}{11}$$

$$= \frac{7040}{11}$$

$$= 640.$$

**19.** $f(x) + 2f\left(\frac{1}{x}\right) = 2x - 1$ (1)

Put $x = \frac{1}{x}$ in (1) $\Rightarrow$ $2(x) + f\left(\frac{1}{x}\right) = \frac{2}{x} - 1$ (2)

$2(2) - (1) \Rightarrow 3f(x) = \frac{4}{x} - 2 - 2x + 1$

$\Rightarrow f(x) = \frac{1}{3}\left[\frac{4}{x} - 2x - 1\right]$

$f\left(\frac{1}{3}\right) = \frac{1}{3}\left[12 - \frac{2}{3} - 1\right] = \frac{31}{9}$

**20.** If $f(x) = \frac{x}{\sqrt{1+x^2}}$, then $fofo\ldots k$ times $= \frac{x}{\sqrt{1+kx^2}}$

$\therefore (fofofof)(x) = \frac{x}{\sqrt{1+4x^2}}$

$\therefore (fofofof)(4) = \frac{4}{\sqrt{65}}$

**21.** $(gof)[x] = g[f(x)] = g(2x-1) = 3(2x-1) + 4 = 6x + 1$

Let $y = 6x + 1 \Rightarrow x = \frac{y-1}{6}$

$\therefore (gof)^{-1}(x) = \frac{x-1}{6} \Rightarrow (gof)^{-1}(0) = \frac{-1}{6}$

**22.** The function $\frac{1}{\log|x-1|}$ is not defined if $|x-1| = 0$ or $|x-1| = 1$

i.e., if $x = 1$ or $x = 0, 2$

i.e., if $x = 0, 1$ or $2$

The function $\frac{1}{x+1}$ is not defined if $x + 1 = 0$, i.e., $x = -1$.

Hence, the domain of $f(x)$ is $R - \{-1, 0, 1, 2\}$

**23.** $f(x) = \frac{2x}{\sqrt{x^2 - 2x - 15}}$

$f(x)$ is real $\Rightarrow x^2 - 2x - 15 > 0$

$(x-5)(x+3) > 0$

$\Rightarrow x \in R - [-3, 5]$

$\therefore$ The domain of $f(x)$ is $(-\infty, -3) \cup (5, \infty)$

**24.** Every real number '$x$' lies between two integers.

Given: $[x]$ is the greatest integer less than or equal to $x$.

So, $0 \le x - [x] < 1$

**25.** $f(x) = \frac{2}{1-[x]}$

For the function $f(x)$ to be defined, the denominator should not be zero.

$1 - [x] \neq 0 \Rightarrow [x] \neq 1$

From the definition of $[x]$, $[x] = 1$, for $x \in [1, 2)$

Therefore, the domain of $f(x)$ is $(-\infty, 1) \cup [2, \infty)$

**26.** (i) $\log(2x-1)$ is defined only when $2x - 1 > 0$

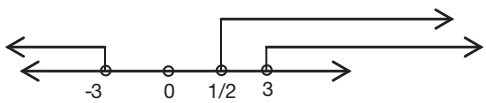
$\Rightarrow x > \frac{1}{2} \Rightarrow x \in \left(\frac{1}{2}, \infty\right)$ (1)

(ii) $\sqrt{x^2 - 9}$ is real only when $x^2 - 9 \ge 0$. Also, $x^2 - 9$ cannot be 0.

$\therefore (x+3)(x-3) > 0$

$\Rightarrow x \notin [-3, 3]$ (2)

Representing (1) and (2) on the number line we have




From the above, $x > 3$ is the common region for (1) and (2).

$\therefore$ The required domain of $f(x)$ is $(3, \infty)$.

**27.** Given $f(xy) = f(x)\,f(y)$. This relation is satisfied only when $f(x) = x^k$

Given: $f(8) = 64$

$\Rightarrow f(8) = 8^k = 8^2 \Rightarrow k = 2$

$\therefore \sum_{k=1}^{15} f(k) = 1^2 + 2^2 + 3^2 + 4^2 \ldots + 15^2$

$= \frac{15(15+1)(30+1)}{6} = 1240$

**28.** $2f(x) - 3f\left(\frac{1}{x}\right) = 5x^2$ (1)

Let $f(100) = a$, $f(0.01) = b$

Setting $x = 100$, in (1), $2a - 3b = 50{,}000$ (2)

Setting $x = 0.01$, in (1), $-3a + 2b = 0.0005$ (3)

$\therefore f(0.01) = b = \frac{2(0.0005) + 3(50000)}{2(3) - 3(3)}$

$$= \frac{-150000.001}{5}$$

$= -30000.0002$

**29.** Let $E = h\left(\frac{3}{99}\right) + h\left(\frac{4}{99}\right) + h\left(\frac{5}{99}\right) + \ldots + h\left(\frac{193}{99}\right) + h\left(\frac{194}{99}\right) + h\left(\frac{195}{99}\right)$

$= \left[h\left(\frac{3}{99}\right) + h\left(\frac{195}{99}\right)\right] + \left[h\left(\frac{4}{99}\right) + h\left(\frac{194}{99}\right)\right] + \left[h\left(\frac{5}{99}\right) + h\left(\frac{193}{99}\right)\right] + \ldots + \left[h\left(\frac{98}{99}\right) + h\left(\frac{100}{99}\right)\right] + h\left(\frac{99}{99}\right)$

If $a + b = 2$, then $f(a) + f(b) = 7$.

There are $98 - 2$ or 96 brackets in the expression above.

Also, $f(1) = \frac{7}{2}$

$\therefore E = 96(7) + \frac{7}{2} = 675.5$

**30.** $f(x+1) = 2x^2 + 4x - 9$

$f(x+1) = 2(x^2 + 2x + 1) - 11$

$= 2(x+1)^2 - 11$

$\therefore f(x) = 2x^2 - 11$

$f(-2) = 2(-2)^2 - 11 = -3$

**Alternate Solution**:

Put $x = -3$, we get

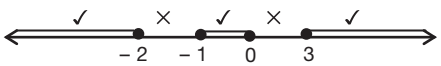
$f(-3+1) = 2(-3)^2 + 4(-3) - 9$

$f(-2) = 18 - 12 - 9 = -3$.

**31.** $f(x) = \sqrt{x(x+1)(x+2)(x-3)}$

$f(x)$ is real if $x(x+1)(x+2)(x-3) \geq 0$

$\therefore$ Critical values of $x$ are $0, -1, -2, 3$




When $x = 1$, the equation is not satisfied

$\Rightarrow$ The interval $(0, 3)$ is not a solution

By observation, we notice that the interval with '✓' marks represents the solution regions

$\Rightarrow$ Domain of $f$ is $(-\infty, -2] \cup [-1, 0] \cup [3, \infty)$

**32.** Given: $f_1(x) = f_2(x-1)$

$\Rightarrow 5x^2 + 7 = 3(x-1)^2 - 3(x-1) + 12$

$\Rightarrow 5x^2 + 7 = 3x^2 - 6x + 3 - 3x + 3 + 12$

$\Rightarrow 2x^2 + 9x - 11 = 0 \Leftrightarrow (2x+11)(x-1) = 0$

$\Rightarrow x = \frac{-11}{2}, x = 1 \; [x > 0 \Rightarrow x = 1]$

**33.** Consider $f$

$\left(\frac{1}{150}\right) + f\left(\frac{2}{150}\right) + f\left(\frac{3}{150}\right) + \ldots + f\left(\frac{297}{150}\right) + f\left(\frac{298}{150}\right) + f\left(\frac{299}{150}\right)$

$\left[f\left(\frac{1}{150}\right) + f\left(\frac{299}{150}\right)\right] + \left[f\left(\frac{2}{150}\right) + f\left(\frac{298}{150}\right)\right] + \left[f\left(\frac{3}{150}\right) + f\left(\frac{297}{150}\right)\right] + \ldots 149 \text{ pairs of terms} + f\left(\frac{150}{150}\right)\Big]$

The pairs are in the form of $f(x) + f(2-x)$

$\therefore$ The value of each of the above pairs of terms is 6. The required value is $149(6) + f(1)$

Given: $f(x) + f(2-x) = 6$

setting $x = 1$, $f(1) + f(1) = 6$

$\Rightarrow f(1) = 3$

$\therefore$ The required value is $894 + 3 = 897$

**34.** $f(x) = \frac{1}{4 - \sin 2x}$

We know $-1 \leq \sin 2x \leq 1$

$-1 \leq -\sin 2x \leq 1$

$\Leftrightarrow 3 \leq 4 - \sin 2x \leq 5$

$\Rightarrow \frac{1}{5} \leq \frac{1}{4 - \sin 2x} \leq \frac{1}{3}$

$\therefore$ Range of $f(x)$ is $\left[\frac{1}{5}, \frac{1}{3}\right]$

**35.** $f(x) = \min(x-4, x+2) = x - 4$

$g(x) = \max(x-2, x+3) = x + 3$

$h(x) = f(x) + g(x) = x - 4 + x + 3 = 2x - 1$

$h(x) \leq 5$

$\Rightarrow 2x - 1 \leq 5$

$\Rightarrow x \leq 3$

$\Rightarrow x \in (-\infty, 3]$

**36.** $f(2x) + f(3x) + f(x+2) + f(3-x) = x$

Let $x = 0$ $\therefore f(0) + f(0) + f(2) + f(3) = 0$ (1)

Let $x = 1$ $\therefore f(2) + f(3) + f(3) + f(2) = 1$

$\Rightarrow f(2) + f(3) = \frac{1}{2}$ (2)

(1), (2) $\Rightarrow 2f(0) + \frac{1}{2} = 0 \Rightarrow f(0) = -\frac{1}{4}$

**37.** $f(x) = px + q$

$F[f(x)] = p(px+ q) + q = p^2x + pq + q$

$F[f[f(x)]] = p^2(px + q) + pq + q$

$= p^3x + p^2q + pq + q$

Given: $f[f[f(x)]] = 8x - 56 \Rightarrow p^3 x + p^2q + pq + q$

$= 8x - 56 \Rightarrow p^3 = 8$

$\Rightarrow p = 2$

$p^2q + pq + q = -56$

$4q + 2q + q = -56$

$q = -8$

$\therefore f(x) = 2x - 8$

$f^{-1}(x) = \dfrac{x+8}{2}$

$\Rightarrow f^{-1}(-6) = \dfrac{-6+8}{2} = 1$

**38.** Given: $f(xy) = f(x)f(y)$ which is true only when

$f(x) = x^k$

so $f(3) = 27$

$\Rightarrow 3k = 27 = 3^3 \Rightarrow k = 3$

$\therefore f(x) = x^3$

$$\sum_{n=1}^{12} f(x) = 1^3 + 2^3 + 3^3 + \ldots + 12^3 = \left(\frac{12(12+1)}{2}\right)$$

$= 6084$

**39.** Let $f(x) = k = \dfrac{-x^2}{x^2 + x + 1}$

$\Rightarrow kx^2 + kx + k = -x^2$ or $x^2(1 + k) + k(x) + k = 0$

Since it is a quadratic equation with real roots, the discriminant $b^2 - 4ac \geq 0$.

$\Rightarrow k^2 - 4(1 + k)(k) \geq 0$

$\Rightarrow -(3k^2) - 4k \geq 0$

$k(3k+4) \leq 0$

$\Rightarrow$ Range of $(x)$ is $\left(\dfrac{-4}{3}, 0\right)$, i.e., $(-1.33, 0)$

$\therefore$ The greatest integer value less than $-1.33$ is $-2$.

**40.** $g(x) = \log\left(\dfrac{1-x}{1+x}\right)$, $h(x) = \dfrac{x^3 + 3x}{3x^2 + 1}$

$$(goh)(x) = g[h(x)] = g\left[\frac{x^3 + 3x}{3x^2 + 1}\right]$$

$$= \log\left(\frac{1 - \dfrac{x^3 + 3x}{3x^2 + 1}}{1 + \dfrac{x^3 + 3x}{3x^2 + 1}}\right) = \log\left(\frac{3x^2 + 1 - x^3 - 3x}{3x^2 + 1 + x^3 + 3x}\right)$$

$$= \log\frac{(1-x)^3}{(1+x)^3} = 3\log\left(\frac{1-x}{1+x}\right) = 3g(x)$$

## EXERCISE-4

**1.** If $f(p)\cdot f\left(\dfrac{1}{p}\right) = f(p) + \left(\dfrac{1}{p}\right) f(p) = 1 \pm pn$

Here, given $f(5) = -124 = 1 - 5^3 \Rightarrow f(x) = 1 - xn$

$\therefore f(9) = -9^3 + 1 = -728$.

**2.** Given: $f(x + y) = f(x) + f(y)$

$$\sum_{i=1}^{10} f(i) = f(1) + f(2) + f(3) + f(4) + \ldots + f(10)$$

Given: $f(1) = 2$

$f(2) = f(1 + 1) = f(1) + f(1) = 2 + 2 = 2.2 = 4$

$f(3) = f(1 + 2) = f(1) + f(2) = 2 + 4 = 2.3 = 6$

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$f(10) = f(1 + 9) = f(1) + f(9) = 2 + 18 = 2.10$

$$\therefore \sum_{i=1}^{10} f(i) = 2(1 + 2 + 3 + \ldots + 10)$$

$$= \frac{2(10)(10+1)}{2} = 110$$

**3.** $f(-2.5) + f(-1.5) + f(-3.5) = (-2.5 - 1) - 1.5 + (-3.5 - 1)$

$= -9.5$.

**4.** Given: $f(x) = 2^x + 2^{-x}$ and $g(x) = 2^x - 2^{-x}$

$f(x)\cdot g(y) + f(y)\ g(x) = (2^x + 2^{-x})(2^y - 2^{-y}) + (2^y + 2^{-y})(2^x - 2^{-x})$

$\Rightarrow 2^{x+y} - 2^{x-y} + 2^{-x+y} - 2^{-x-y} + 2^{x+y} - 2^{-x+y} - 2^{-x-y} + 2^{-y+x}$

$\Rightarrow 2(2^{x+y} - 2^{-(x+y)}) = 2g(x + y)$

**5.** $\frac{5-|x|}{7-|x|} \geq 0$

$\therefore 5-|x|$ and $7-|x| \geq 0$ or $5-|x| \leq 0$ and $7-|x| \leq 0$

$5-|x|$ and $7-|x|$ are positive in the interval $(-5, 5)$ and both are negative in the intervals $(-\infty, -7)$ and in $(7, \infty)$.

Also, $5-|x|$ can be 0 but not $7-|x|$

$\therefore$ The domain is $[-5, 5] \cup (-\infty, -7) \cup (7, \infty)$

**6.** Given $f(1) = 4$ and $f(x+y) = f(x)f(y)$ for all real values of $x$ and $y$.

$\Rightarrow f(2) = f(1+1) = f(1)\, f(1) = 4^2$

$\therefore f(4) = f(2+2) = f(2)\, f(2) = (4^2)\,(4^2) = 4^4$

$\therefore f^6(4) = f(4)f(4) \ldots 6$ times

$= 4^4 \times 4^4 \times \ldots 6$ times $= 4^{24} = 2^{48}$

**7.** Given: $g(2) = 3$ and $f(3) = 2$.

$f(g(8)) = f[g(2 \times 2 \times 2)] = f[g(2) + g(2) + g(2)]$

$= f(3+3+3)$

$= f(3)\, f(3)\, f(3) = 2 \times 2 \times 2 = 8$

**8.** Given $f\,n(x) = f(x)f(x)f(x) \ldots n$ times

$\Rightarrow fn(x) = f(x + x + x + \ldots. n \text{ times}) = f(nx)$.

Consider, $g(1) = g(1 \times 1) = g(1) + g(1)$

$\Rightarrow g(1) = 2g(1)$

$\Rightarrow g(1) = 0$

Consider, $f(0) = f(0+0) = f(0) \cdot f(0)$

$\Rightarrow f(0) = f(0) \cdot f(0)$

$\therefore f(0) = 0$ or 1.

If $f(0) = 0$,

$f(5) = f(5+0) = f(5) \cdot f(0) = f(5) \times 0 = 0$

$\Rightarrow f(5) = 0$, but given that for $x \neq 0$, $f(x) \neq 0$.

$\therefore f(0) = 1$.

$g(4) = g(2\times2) = g(2) + g(2) = 3 + 3 = 6$

$f(g(4)) = f(6) = f(3+3) = f(3) \cdot f(3) = 4 \cdot 4 = 16$ true

**9.** Since $x = 1.5$, $i(x) = (1.5)^2 - 4 < 0$,

hence, $h(i(x)) = 1 - 2\,(i(x))^5$, as $i(x) < 0$, $h(x) > 0$.

$\Rightarrow g(h(x)) = \frac{|h(x)|}{h(x)} = 1$, as $h(x) > 0$.

$\therefore f(g(x)) = f(1) = 1^2 + 3 + 6 = 10$

**10.** $f(x)f(y) = f(x) + f(y) + f(xy) - 2$

$f(3)f(2) = f(3) + f(2) + f(6) - 2$

$10(5) = 10 + 5 + f(6) - 2$

$\Rightarrow f(6) = 50 - 15 + 2 = 37$

put $x = 6$, $y = 2$

$f(6)f(2) = f(6) + f(2) + f(12) - 2$

$f(12) = 37(5) - 37 - 5 + 2 = 145$

**11.** $f(x-y) = \frac{f(x)}{f(y)}$

The function which satisfies the above condition is $akx$, i.e, $f(x) = akx$

$f(50x) = ak^{50}x = (axk)^{50} = [f(x)]^{50}$

**12.** Given: $f(t) = \frac{1-t}{1+t}$; $f(f(t)) = \frac{1-\frac{1-t}{1+t}}{1+\frac{1-t}{1+t}} = \frac{2t}{2} = t$

$\therefore f^2(t) = f(f(t)) = t$; $f^3(t) = f(f^2(t)) = f(t)$; ...

In general, $fn(t) = t$ when $n$ is even and $fn(t) = f(t)$ when $n$ is odd.

$f^1(0) = 1$, $f^2(0) = 0$, $f^3(0) = 1$, $f^4 = 0$, ...

$\therefore f^{132}(0) = 0$.

**13.** $f(xy) = f(x)f(y) \Rightarrow f(x) = x^n$

$f(2) = \frac{1}{8} \Rightarrow n = -3$

$\therefore f(x) = x^{-3}$

$f(1) = 1$, $f(1/2) = 8$, $f(1/3) = 27$

and $f(1) + f(1/2) + f(1/3) = 36$

**14.** The product of all the elements in the set $B$ is even when at least one of the elements is even.

The total number of non-empty subsets formed is $2^{100} - 1$.

The number of subsets that contain only odd numbers is

${}^{50}C_1 + {}^{50}C_2 + \ldots.. + {}^{50}C_{50} = 2^{50} - 1$

Hence, required number of subsets

$= 2^{100} - 1 - (2^{50} - 1) = 2^{100} - 2^{50}$

$= 2^{50}\,(2^{50} - 1)$.

**15.** Given $f(x) = \frac{5^x + 5^{-x}}{2}$

$f(x+y) = \frac{5^{x+y} + 5^{-(x+y)}}{2}$

$$f(x-y) = \frac{5^{x-y} + 5^{-(x-y)}}{2}$$

$$2f(x+y)\,f(x-y) = 2\left(\frac{5^{x+y} + 5^{-(x+y)}}{2}\right)\left(\frac{5^{x-y} + 5^{-(x-y)}}{2}\right)$$

$$= \frac{5^{2x} + 5^{2y} + 5^{-2x} + 5^{-2y}}{2} = \frac{5^{2x} + 5^{-2x}}{2} + \frac{5^{2y} + 5^{-2y}}{2}$$

$$= f(2x) + f(2y)$$

**16.** $f(x) = \dfrac{3^x - 3^{-x}}{3^x + 3^{-x}}$

let $f^{-1}(x) = y \Rightarrow x = f(y)$

$$\Rightarrow x = \frac{3^y - 3^{-y}}{3^y + 3^{-y}} = \frac{3^{2y} - 1}{3^{2y} + 1}$$

$$x(3^2y) + x = 3^2y - 1$$

$$3^2y(1 - x) = x + 1$$

$$3^2y = \frac{x+1}{1-x}$$

$$\log_3 3^2y = \log_3\left(\frac{1+x}{1-x}\right)$$

$$2y = \log_3\left(\frac{1+x}{1-x}\right)$$

$$\therefore y = \frac{1}{2}\log_3\left(\frac{1+x}{1-x}\right)$$

**17.** $f(4x+3) = 32x^2 + 68x + 40$

Let $4x + 3 = t \Rightarrow x = \dfrac{t-3}{4}$

$$f(t) = 32\left(\frac{t-3}{4}\right)^2 + 68\left(\frac{t-3}{4}\right) + 40$$

$$= 2(t^2 - 6t + 9) + 17t - 51 + 40$$

$$= 2t^2 - 12t + 18 + 17t - 11$$

$$f(t) = 2t^2 + 5t + 7$$

$$\therefore f(3x+4) = 2(3x+4)^2 + 5(3x+4) + 7$$

$$= 2(9x^2 + 24x + 16) + 15x + 20 + 7$$

$$= 18x^2 + 48x + 32 + 15x + 20 + 7$$

$$f(3x+4) = 18x^2 + 63x + 59$$

**18.** Let I $= \dfrac{1}{\sqrt{x^2 - 9}}$, II $= \log(x-2)$, III $= \dfrac{1}{x+2}$

$\sqrt{x^2 - 9}$ is defined only when $x^2 - 9 > 0$

$\Rightarrow (x+3)\,(x-3) > 0\ x \in R - [-3, 3]$ (1)

$\therefore$ Domain of I is $x \in R - [-3, 3]$

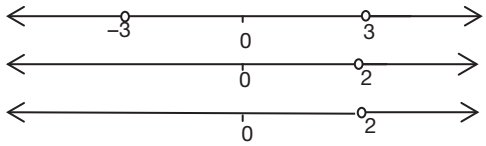
II. $\log(x-2)$ is defined when $x - 2 > 0 \Rightarrow x > 2$ (2)

III. $\dfrac{1}{x+2}$ is defined for all $x \neq -2$

$\therefore$ Domain of the given function

$R - [-3, 3] \cap (2, \infty) \cap R - \{-2\}$

i.e.,




$\therefore$ The required domain is $(3, \infty)$

**19.** Let $\dfrac{x^2 + x + 1}{x^2 - x - 1} = k$ (1)

$$\Rightarrow kx^2 - kx - k - x^2 - x - 1 = 0$$

$$\Rightarrow x^2(k-1) - x(k+1) - k - 1 = 0 \quad (2)$$

The roots are $\dfrac{(k+1) \pm \sqrt{\Delta}}{k-1}$, where $\Delta \geq 0$ (3)

$$\Rightarrow (k+1)^2 + 4(k-1)\,(k+1) \geq 0$$

$$(k+1)\,(k+1+4k-4) \geq 0\ (k+1)\,(5k-3) \geq 0$$

$\Rightarrow k \in R - (-1, \frac{3}{5})$

$\therefore$ The range of the given function is $(-\infty, -1] \cup \left[\frac{3}{5}, \infty\right]$

**20.** When $n(A) = 6$, and $n(B) = 5$

the total number of functions from $A$ to $B$ is given by $n(B)^{n(A)} = 5^6$

The number of onto functions from $A$ to $B$ is given by

$n^m - {}^nC_1\,(n-1)^m + {}^nC_2\,(n-2)^m - {}^nC_3\,(n-3)^m + \ldots$

Here, $n = 5$, $m = 6$

$$= 5^6 - {}^5C_1\,(4)^6 + {}^5C_2\,(3)^6 - {}^5C_3\,(2)^6 + {}^5C_4\,(1)^6$$

$$= 5^6 - 5\,(4096) + 10(729) - 10(64) + 5 = 5^6 - 13825$$

$\therefore$ The required number of functions

$= 5^6 - \{5^6 - 13825\} = 13825$

**21.** Given: $f(x) = \dfrac{x+1}{x-1} = y$ (say)

$$\Rightarrow xy - y = x + 1 \Rightarrow x = \frac{y+1}{y-1} \text{ or } f^{-1}(x) = \frac{x+1}{x-1} = f(x)$$

$(f^{-1} o f^{-1})(x) = (f^{-1} o f)\,x = x$

As $f^{-1}$ occurs 6 times (an even number of times the given function is $x$ itself).

$\therefore (f^{-1} o f^{-1} o f^{-1} o f^{-1} o f^{-1} o f^{-1})(5) = 5$

**22.** $f(2) = \frac{1}{1+2} = \frac{1}{3}$

$f^2(2) = f(f(2)) = f(1/3) = \frac{1}{1+1/3} = \frac{3}{4}$

$f^3(2) = f(f^2(2)) = f(3/4) = \frac{1}{1+3/4} = \frac{4}{7}$

$f^4(2) = f(f^3(2) = f(4/7) = \frac{1}{1+4/7} = \frac{7}{11}$

$f^5(2) = f(f^4(2) = f(7/11) = \frac{1}{1+7/11} = \frac{11}{18}$

$\therefore f(2)\, f^2(2)\, f^3(2)\, f^4(2)\, f^5(2)$

$= \frac{1}{3}\left(\frac{3}{4}\right)\left(\frac{4}{7}\right)\left(\frac{7}{11}\right)\left(\frac{11}{18}\right) = \frac{1}{18}$

**23.** $f(x) = px + q$

$f[f(x)] = f[px + q]$

$= p(px + q) + q$

$= p^2x + pq + q$

$f[f(f(x)] = f[p^2x + pq + q]$

$= p[p^2x + pq + q] + q$

$= p^3x + p^2q + pq + q$

Given, this is equal to $64x + 147$

$\Rightarrow p^3 = 64$ and $p^2q + pq + q = 147$

$p = 4$ and $q(p^2 + p + 1) = 147$

$\Rightarrow q = \frac{147}{21} = 7$

$\therefore 2p + 3q = 2(4) + 3(7)$

$= 8 + 21 = 29.$

**24.** $f(x) = 27x^4$ $g(x) = 3\sqrt{f(x)}$

$= 3\sqrt{27x^4} = 3x^{\frac{4}{3}}$

$fog(x) = f[g(x)]$

$= f\left[3x^{\frac{4}{3}}\right]$

$= 27.\left(3x^{\frac{4}{3}}\right)^4$

$= 27.3^4\ x^{\frac{16}{3}}$

$fog(x) = 3^7\ x^{\frac{16}{3}}$

$fog(729) = 3^7\ (729)^{\frac{16}{3}}$

$= 3^7\ \left(3^6\right)^{\frac{16}{3}}$

$= 3^7.3^{32} = 3^{39}$

$\therefore \log_3 fog(729) = \log_3 3^{39}$

$= 39.$

**25.** $f(x) = px^5 + qx^3 - rx + 3$

Given: $f(4) = 10$

$4^5p + 4^3q - 4r + 3 = 10$

$4^5p + 4^3q - 4r = 7$

$f(-4) = (-4^5)p - (4^3)q + 4r + 3$

$= -(4^5p + 4^3q - 4r) + 3$

$= -7 + 3 = -4.$

# 7 Graphs

CHAPTER

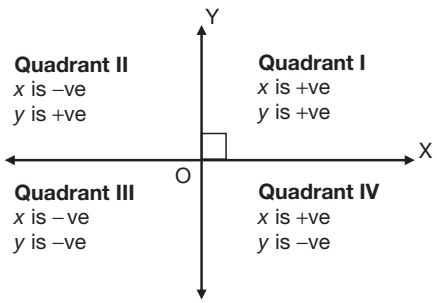
## Learning Objectives

*In this chapter, you will:*

- Grasp the basic concept of coordinate system and learn how to plot graphs on the *xy*-plane
- Understand graphs for the following standard functions/equations:
  - Line graph
  - Modulus functions
  - Circle
  - Logarithmic functions
  - Trigonometric functions
- Learn about shifting graphs horizontally and vertically, as well as stretching and compressing graphs
- Understand reflection of graphs in the *x*-axis and *y*-axis
- Learn graphs of some commonly used functions
- Learn and practice floor and ceiling functions, Exponential functions
- Learn to calculate area enclosed by graphs and coordinate axes

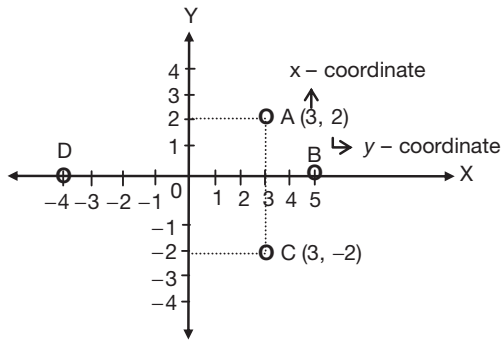
By a GRAPH, we mean the set of points $(x, y)$ that satisfy a relation of the form $f(x, y) = k$ or $f(x, y) \leq k$ or $f(x, y) \geq k$. In this chapter, we will be studying curves that can be drawn in the xy-plane

The rectangular coordinate system is shown below:



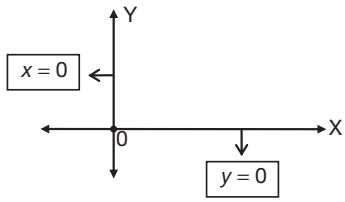

The rectangular coordinate system is divided into four quadrants by two mutually perpendicular lines called coordinate-axes. The horizontal line is called *x*-axis while the vertical line is called *y*-axis. The point O, where the two lines meet is called origin.

Any point on the *x*-axis can be represented as $(a, 0)$, whereas any point on the *y*-axis can be represented as $(0, b)$.



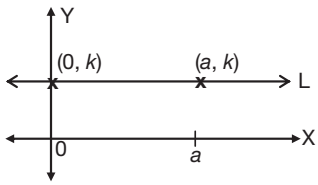

The co-ordinates of the point A are (3, 2). If we want to specify the coordinates of the points, we can write A = (3, 2), B = (5, 0), etc., and if we want to refer to the points we can write A (3, 2), B (5, 0), etc.

## ❒ LINE GRAPHS



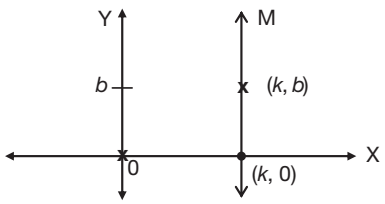

1. We note that any point on the $x$-axis has the $y$-coordinate as 0.
   So, $y = 0$ is the condition satisfied by any point on the line.
   $\therefore$ The equation of the $x$-axis is $y = 0$
2. The equation of the $y$-axis is $x = 0$.
3. The equation of a horizontal line or any line parallel to the $x$-axis is $y = k$ (constant).

   **Reason:** We note that every point on the line L has the $y$-coordinate as $k$.

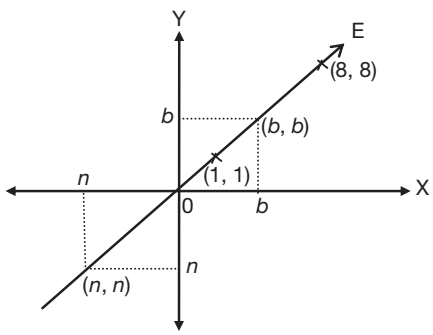



4. The equation of a vertical line or any line parallel to the $y$-axis is $x = k$ (constant).

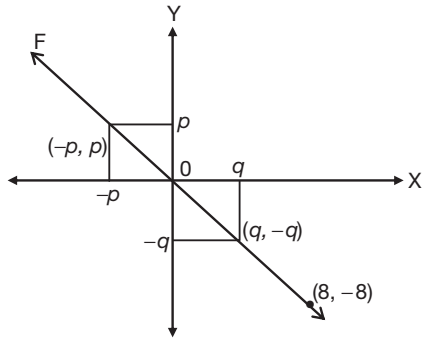
   **Reason:** We note that every point on the line M has the $x$-coordinate as $k$.



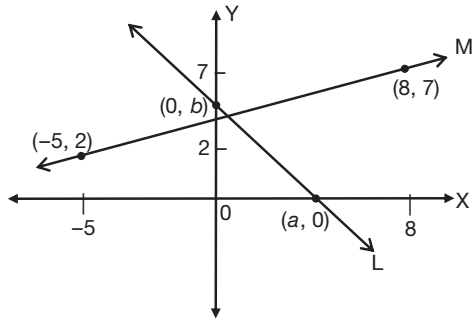

5. On the line E shown in the figure, we find that $x$ and $y$-co-ordinates are equal.
   So, $y = x$ is the equation.




6. On the line F, as shown in the figure, we find that $x$ and $y$ co-ordinates are equal in magnitude but opposite in sign.
   So, $y = -x$ is the equation.




7. Obtaining the equation of a line if two points on it are known.




   We apply the two-point formula for line equation

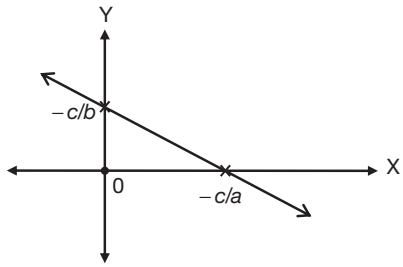
$$y - y_1 = -\frac{y_2 - y_1}{x_{2-}x_1}(x - x_1)$$

   (Refer to the coordinate geometry chapter).
   Accordingly, the equation of line L is

$$y - 0 = \frac{b - 0}{0 - a}(x - a) \text{ or } \frac{x}{a} + \frac{y}{b} = 1$$

   The equation of M, is $y - 7 = \dfrac{2 - 7}{-5 - 8}(x - 8)$

   or $5x - 13y + 51 = 0$.
8. Plotting the graph of an inclined line:
   For a line given as $ax + by + c = 0$; $ab \neq 0$
   Step (i): Get the $x$-intercept as $x = -c/a$, by plugging $y = 0$.
   Step (ii): Get the $y$-intercept as $y = -c/b$, by plugging $x = 0$.
   Step (iii): Plot the points $(0, -c/b)$ and $(-c/a, 0)$ on the graph sheet.
   Step (iv): Join these points by a line.

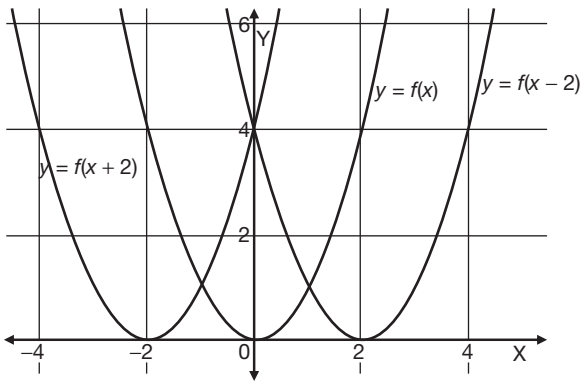



9. Graphing the regions bounded by lines
   (i) $x \geq 0$: The right half-plane (including the $y$-axis)
   (ii) $y \geq 0$: The upper half-plane (including the $x$-axis)
   (iii) $x < 0$: The left half-plane (excluding the $y$-axis)
   (iv) $y < 0$: The lower half-plane (excluding the $x$-axis)
   (v) $x \geq 0$ and $y \geq 0$: The first quadrant (with the boundaries)
   (vi) $xy \leq 0$: The second and fourth quadrants together (with the boundaries)
   (vii) $xy > 0$: The first and third quadrants together (without the boundaries)
   (viii) $ax + by + c \geq 0$ or $\leq 0$: represents the half-planes demarcated by the line $ax + by + c = 0$ (with the line).

## ❒ Shifting of Graphs

Consider the graph of $f(x)$. To get the graph of $f(x + a)$ $(a > 0)$ move the graph $a$ units to the left. Similarly to get the graph of $f(x - a)$ (where $a > 0$) move the graph $a$ units to the right.

One such example of shifting a graph horizontally is illustrated below, using the function $y = x^2$.




## ❒ GRAPHS OF COMMONLY USED FUNCTIONS

The table below shows how to get the graphs of some commonly used functions from the graph of a given function, say $y = f(x)$.

Table showing the changes made to $y = f(x)$ and the corresponding effect in the graph.

| Function | Obtaining the graph from the graph of $y = f(x)$ |
|---|---|
| (1) $y = \lvert f(x) \rvert$. | (1) unchanged when $f(x) \geq 0$ and reflected in the $x$-axis when $f(x) < 0$. |
| (2) $y = -\lvert f(x) \rvert$. | (2) unchanged when $f(x) \leq 0$ and reflected in the $x$-axis when $f(x) > 0$. |
| (3) $x = f(y)$. | (3) reflected in the line $y = x$. |
| (4) (i) $y = cf(x)$, $c > 1$, (ii) $y = cf(x)$, $0 < c < 1$. | (4) (i) vertical stretch, (ii) vertical shrink. |
| (5) (i) $f(cx)$; $c > 1$, (ii) $f(cx)$; $0 < c < 1$. | (5) (i) horizontal shrink, (ii) horizontal stretch. |
| (6) $y - k = f(x - h)$. | (6) translates '$h$' units horizontally to the right and '$k$' units vertically down. |

## ❒ Floor and Ceiling (or Ceil) Functions

For any real number $x$, the greatest integer less than or equal to $x$ is called the floor function. It is denoted as $\lfloor x \rfloor$. The least integer greater than or equal to $x$ is called ceiling (or ceil) function. It is denoted as $\lceil x \rceil$.

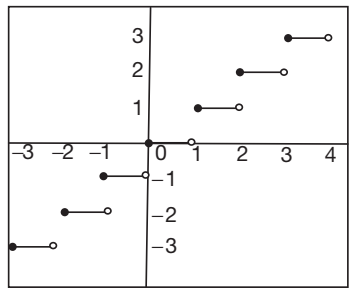
Thus, $\lfloor 4.1 \rfloor = 4, \lfloor -1.2 \rfloor = -2$ and $\lfloor -7 \rfloor = -7$

while $\lceil 4.1 \rceil = 5, \lceil -1.2 \rceil = -1$ and $\lceil -7 \rceil = -7$

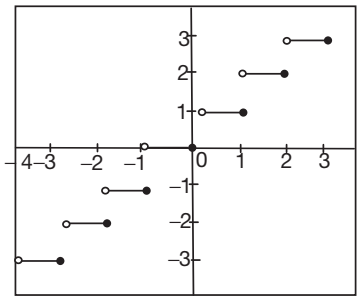
When the symbol $[x]$ is used without any further explanation, it normally means $\lfloor x \rfloor$.

The graphs of these two functions are shown below

Floor function




Ceil function

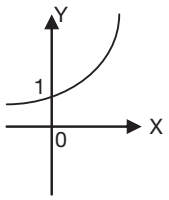


We note that for any integer $n$, $\lfloor n \rfloor = n = \lceil n \rceil$ and for any non-integer $x$, $\lfloor x \rfloor + 1 = \lceil x \rceil$

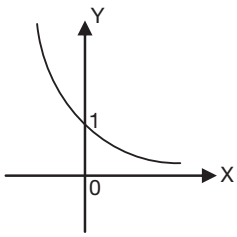
We also note that for the floor function, the endpoint on the left of each segment or 'step' is included, while the endpoint on the right is excluded. For the ceil function, it is vice-versa.

## ❒ Exponential Function ($e^{ax}$)

**Case 1:** $a > 0$



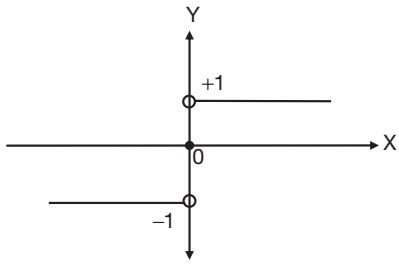

**Case 2:** $a < 0$




**NOTE**

$e = 2.718$ (app)

## ❒ Signum Function




**Definition**

$$sg(x) = \begin{cases} \dfrac{|x|}{x}; & x \neq 0 \\ 0; & x = 0 \end{cases}$$

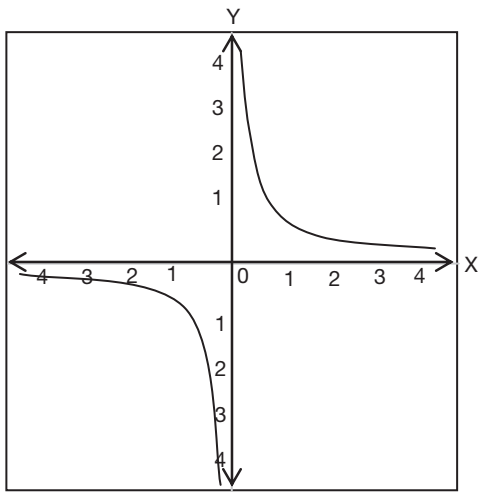
**Examples**

$sg(7) = +1$, $sg(0) = 0$
$sg(-7) = -1$.

## SOLVED EXAMPLES

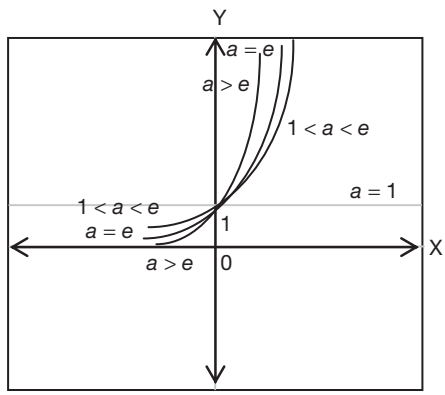
**7.01.** Draw the graph of $y = \dfrac{1}{x}$, $(x \neq 0)$.

**Sol:** $y = \dfrac{1}{x}$



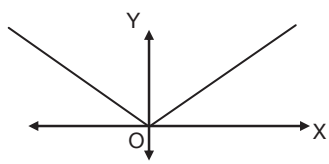

**7.02.** Draw the graph of $y = a^x$ for
(a) $1 < a < e$
(b) $a > e$
(c) $a = e$

**Sol:** $y = a^x$

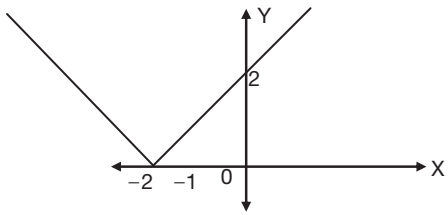



**7.03.** Draw the graph of $y = |x + 2|$.

**Sol:** The graph of $f(x) = |x|$ is shown below:



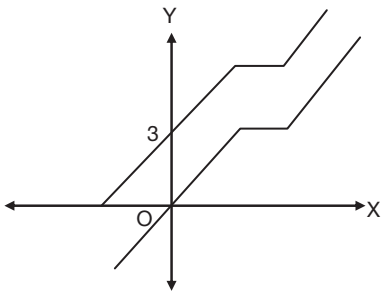
To get the graph of $|x+2|$, move the above graph 2 units to the left. Then, the graph is as follows:




**7.04.** The graph of the following function is shown below.

$$f(x) = x \text{ for } x \le 2$$
$$= 2 \text{ for } 2 < x \le 3$$
$$= x - 1 \text{ for } 3 < x$$

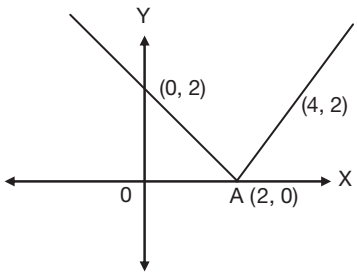
**Sol:**



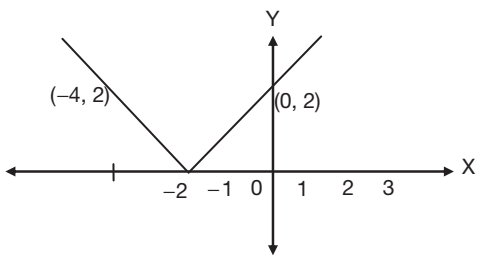

To get the graph of $f(x) + 3$, shift the graph of $f(x)$, 3 units up.

**7.05.** Get the graph of $|x+2|$ from the graph of $|x-2|$

**Sol:** The graph of $f(x) = |x-2|$ is as follows:

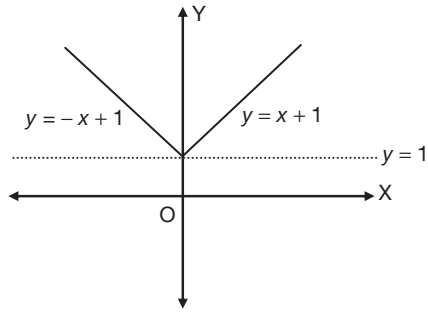



To get the graph of $|x+2|$, shift this graph 4 units to the left. Then the graph is as follows:




**7.06.** Draw the graph of $y = |x| + 1$.

**Sol:**




$$y = |x| + 1$$

Clearly, the least value taken by $y$ is 1, as no portion of the line is below $y = 1$.

The equation $y = |x| + 1$, can be written as

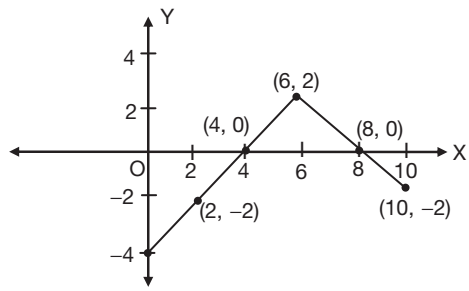
$$y = \begin{cases} x+1; & x \ge 0 \\ -x+1; & x < 0 \end{cases}$$

**7.07.** Sketch the graph of the function

$$y = \begin{cases} x-4; & 0 \le x < 6 \\ 8-x; & 6 \le x \le 10 \end{cases}$$

Also, determine the maximum] and the minimum values of $y$ in the given domain.

**Sol:**




The graph is made up of 2-line segments.

For $0 \le x < 6$, we take the points $(0, -4)$, $(2, -2)$, $(4, 0)$.

For $6 \le x \le 10$, we take the points $(6, 2)$, $(8, 0)$, $(10, -2)$.

The maximum value of $y$ is 2 and the minimum value is $-4$.

**7.08.** Sketch the graph of $y = \dfrac{|x-7| - |x-3|}{2}$ and hence $x = \dfrac{|y-7| - |y-3|}{2}$.

**Sol:** We first write the function $y$ in terms of $x$, more explicitly.

Case (i): $x \le 3$:

$$y = \frac{[-(x-7)]-[-(x-3)]}{2} = \frac{7-x+x-3}{2} = 2.$$

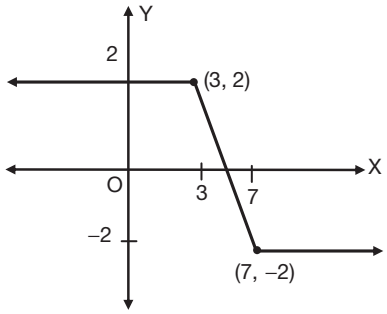
Case (ii): $3 \le x \le 7$:

$$y = \frac{[-(x-7)-(x-3)]}{2} = \frac{-2x+10}{2} = -x+5.$$

Case (iii): $x \ge 7$:

$$y = \frac{(x-7)-(x-3)}{2} = \frac{-4}{2} = -2.$$

The graph can be represented as follows:

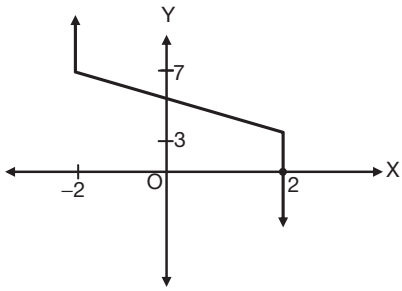



Thus, $y = \begin{cases} 2; & \text{for } x \le 3 \\ -x+5; & \text{for } 3 \le x < 7 \\ -2; & \text{for } x \ge 7 \end{cases}$

To get the graph of

$x = \dfrac{|y-7|-|y-3|}{2}$, we need to reflect the graph discussed above in the line $y = x$.

The graph is as follows:

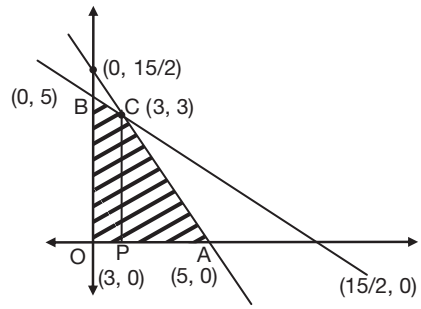



The expression being

$$x = \begin{cases} 2; & \text{for } y \le 3 \\ -y+5; & \text{for } 3 \le y \le 7 \\ -2; & \text{for } y > 2 \end{cases}.$$

**7.09.** Find the area enclosed by the region bounded by the relations $2x+3y \le 15$, $3x+2y \le 15$, $x \ge 0$, $y \ge 0$.

**Sol:** The demarcating lines for the region are $2x+3y = 15$, $3x+2y = 15$, $x = 0$, $y = 0$. The area of the region required is that of the quadrilateral OACB. We break it as trapezium OPCB and triangle PAC.



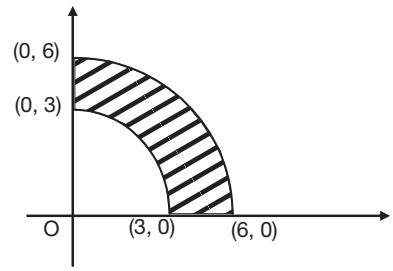

The sum of the areas

$$= \frac{1}{2}[(5+3)3] + \frac{1}{2} \times 2 \times 3 = 12+3$$

$= 15$ sq. units.

**7.10.** Two points P and Q move in such a way that the distance of P from the origin is 6 units and that of Q is 3 units. Find the area enclosed between the truncated annular region in the first quadrant.

**Sol:**




The set of points traversed by P will be the points on the arc of the circle with radius 6 centred at the origin and that of Q will be on the arc of the circle with radius 3 centred at the origin. The area enclosed equals

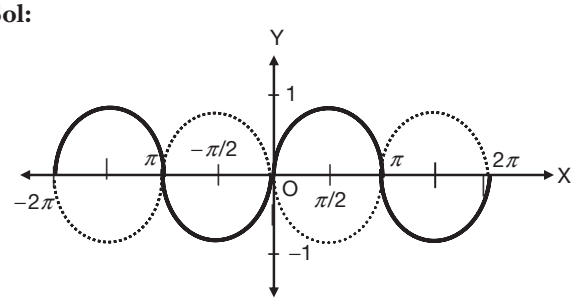
$$= \frac{1}{4}\left(\pi(6)^2 - \pi(3)^2\right) = \frac{27\pi}{4} \text{ sq. units.}$$

**7.11.** Sketch the graph of

(a) $y = |\sin x|$

(b) $y = -|\sin x|$ on $[-2\pi, 2\pi]$

**Sol:**

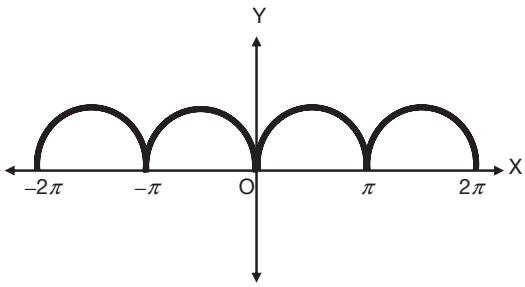


We first recall the graph of $y = \sin x$ on $[-2\pi, 2\pi]$

(a) For the graph of $y = |\sin x|$, we need to reflect in the $x$-axis, the portion of the graph defined between $(-\pi, 0)$ and $(\pi, 2\pi)$.

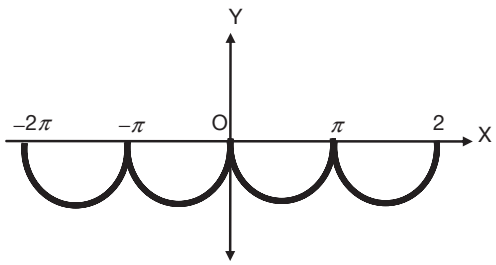
The dotted portion above $x$-axis is to be traced back. Analytically, $y = |\sin x| \geq 0$
$\Rightarrow y \geq 0$ (no portion is below the $x$-axis)

Accordingly, we get the graph as




(b) $y = -|\sin x|$. The graph can be obtained by reflecting in the $x$-axis, the portion of the graph defined between $(-2\pi, -\pi)$ and $(0, \pi)$.

Accordingly, we get the graph as




The portion that is above in $y = \sin x$ moves down. Analytically, $y = -|\sin x| \leq 0 \Rightarrow y \leq 0$ (no portion is above $x$ – axis)

**7.12.** Sketch the graph of

(a) $y = \log_e x$,

(b) $y = \log(-x)$,

(c) $y = |\log x|$ and

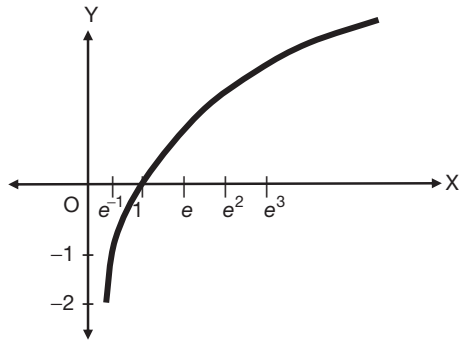
(d) $y = -|\log x|$.

(taking the natural base as '$e$')

**Sol:** (a) $y = \log x$ (given)

We know that $y$ is defined only for $x > 0$.

| **X** | 1 | $e$ | $e^2$ | $e^3$ | $e^{-1}$ | $e^{-2}$ | $e^{-3}$ |
|---|---|---|---|---|---|---|---|
| **Y** | 0 | 1 | 2 | 3 | –1 | –2 | –3 |




**NOTE**

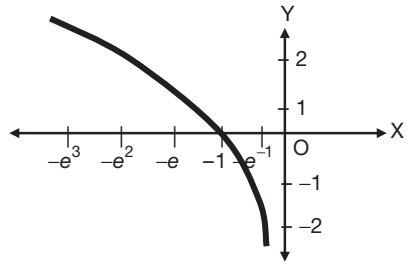
(a) No portion of the graph lies in the second and the third quadrants as $y$ is defined only for positive values of $x$.

(b) As $x \to 0$ we note that $y \to -\infty$.

(b) We note that $x$ is replaced with $(-x)$ in $\log x$.

Therefore, it just means reflection in $y$-axis; here, we note that $\log(-x)$ is defined only for $x < 0$.

Accordingly, the graph becomes,



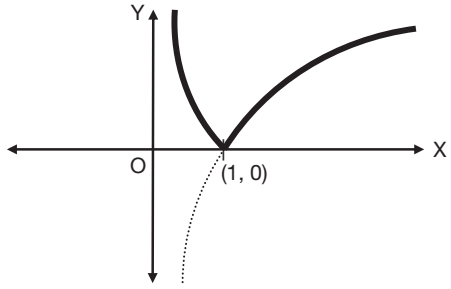

**NOTE**

No portion of the graph is in the first and the fourth quadrants as $x < 0$.

(c) $y = |\log x|$

We know that $y$ is defined for $x > 0$. Also, $y$ is non-negative. The portion in $\log x$, $x > 0$ that is below the $x$-axis is reflected in $x$-axis.

Accordingly, the graph becomes



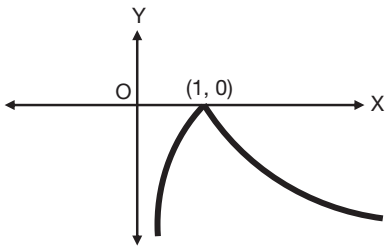
**NOTE**

The dotted line indicates the original form $y = \log x$. The introduction of modulus into $\log x$ makes the portion below to move above.

(d) $y = -|\log x|$;

We know that the graph is defined for $x > 0$.

As it is of the form $y = -|f(x)|$, it first means reflection of $|f(x)|$ in the $x$-axis. Here $f(x) = \log x$. Accordingly, the graph becomes




**NOTE**

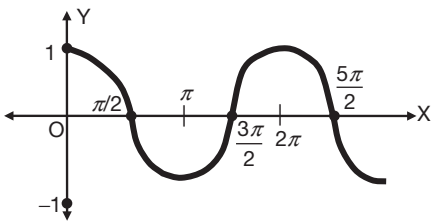
No portion of the graph is above the $x$-axis.

**7.13.** Sketch the graph of $y = \cos x$ and hence sketch the graph of

(a) $y = \cos 2x$ (b) $y = \cos x/2$

(d) $y = 2 \cos x$ (d) $y = 1/2 \cos x$, on $[0, 2\pi]$

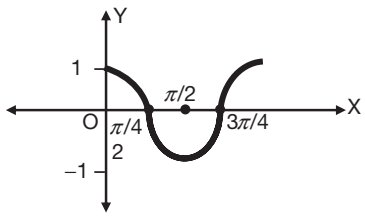
**Sol:** $y = \cos x$




(a) $y = \cos 2x$

$-1 \le y \le 1$

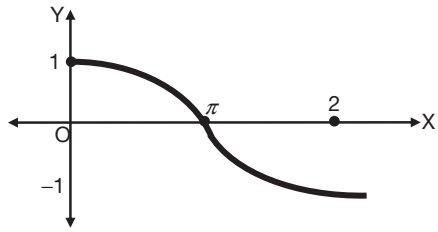
($f(x)$ is shrunk horizontally)




(b) $y = \cos(x/2)$

$-1 \le y \le 1$

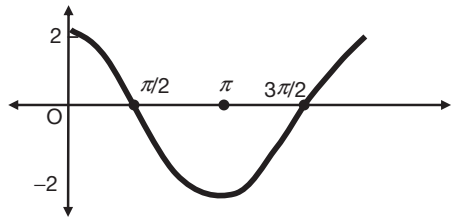
($f(x)$ is stretched horizontally)




(c) $y = 2\cos x$

$-2 \le y \le 2$

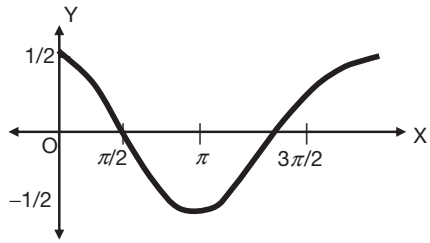
($f(x)$ is stretched vertically)




(d) $y = 1/2 \cos x$

$-1/2 \le y \le 1/2$

($f(x)$ is shrunk vertically)



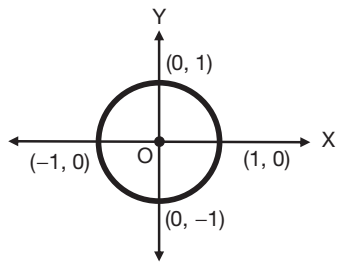

**7.14.** The graph of $x^2 + y^2 = 1$ is a circle centred at the origin whose radius = 1. Sketch the graph of

(a) $(x-2)^2 + y^2 = 1$

(b) $x^2 + (y-2)^2 = 1$

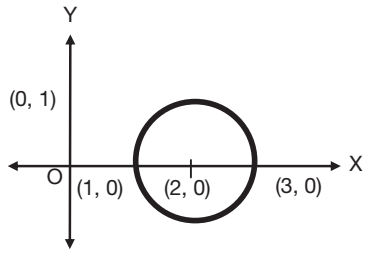
(c) $(x-2)^2 + (y-2)^2 = 1$

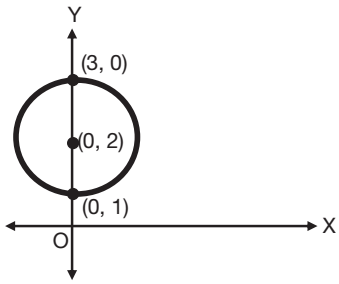
**Sol:**




The graph of $x^2 + y^2 = 1$ is as shown above.

(a) The graph of $(x-2)^2 + y^2 = 1$ is obtained by translating the graph of $x^2 + y^2 = 1$, horizontally 2 units to the right

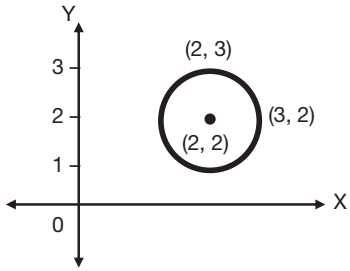



(b) The graph of $x^2 + (y-2)^2 = 1$ is obtained by translating the graph of $x^2 + y^2 = 1$, vertically upwards by 2 units.

(c) The graph of $(x-2)^2 + (y-2)^2 = 1$ is obtained by translating the graph of $x^2 + y^2 = 1$ vertically upwards by 2 units and 2 units to the right horizontally.

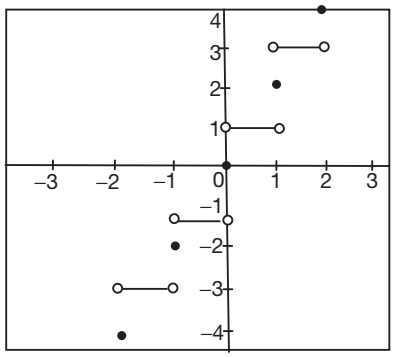
The graph is as shown below:




**7.15.** Sketch the graph of $y = \lceil x \rceil + \lfloor x \rfloor$.

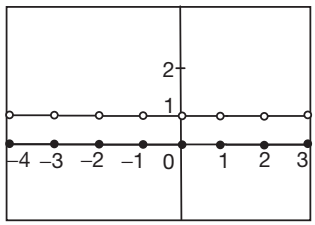
For integral values of $x$, $\lfloor x \rfloor = \lceil x \rceil = x$ and $y = 2x$, an even number. For other values of $x$, $y$ is odd. Endpoints on both ends of the steps are excluded.

**Sol:** $y = \lceil x \rceil + \lfloor x \rfloor$.




**7.16.** Sketch the graph of $y = \lceil x \rceil - \lfloor x \rfloor$.

**Sol:** $y = \lceil x \rceil - \lfloor x \rfloor$.



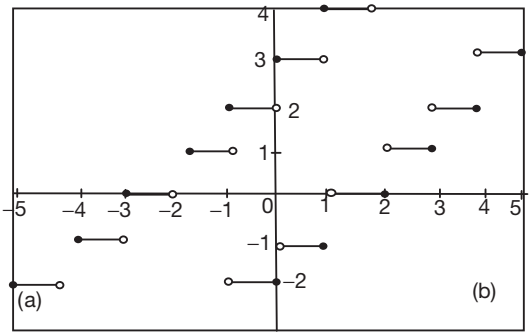

The graph is the line $y = 1$ (with missing points) for non-integral values of $x$ and the isolated points given by $y = 0$ for integral values of $x$.

**7.17.** Sketch the graph of

(a) $\lfloor x \rfloor + 3$ or $\lfloor x + 3 \rfloor$

(b) $\lceil x \rceil - 2$ or $\lceil x - 2 \rceil$

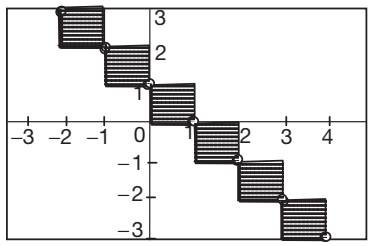
**Sol:**




We see that for all $x$, $\lfloor x + 3 \rfloor = \lfloor x \rfloor + 3$ and for all $x$, $\lceil x - 2 \rceil = \lceil x \rceil - 2$

**7.18.** Sketch the graph of $\lfloor x \rfloor + \lfloor y \rfloor = 0$.

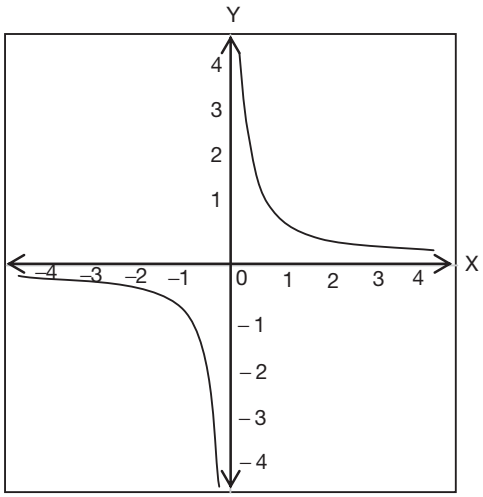
**Sol:**




We get a sequence of squares, in each of which the left and lower sides are included, which is indicated by the solid line while the right and upper sides are excluded, which is indicated by the broken line.

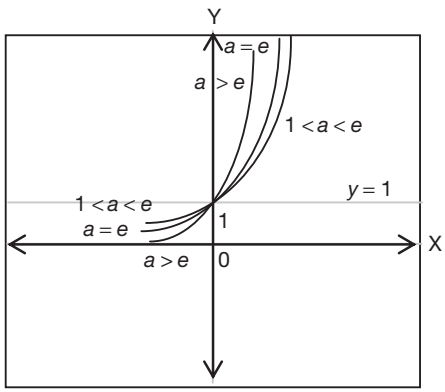
**7.19.** Draw the graph of $y = \frac{1}{x}$, $(x \neq 0)$

**Sol:** $y = \frac{1}{x}$



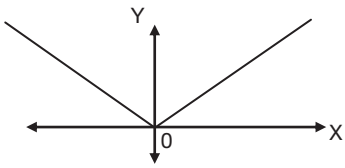

**7.20.** Draw the graph of $y = a^x$ for
(a) $1 < a < e$ (b) $a > e$ (c) $a = e$

**Sol:** $y = a^x$

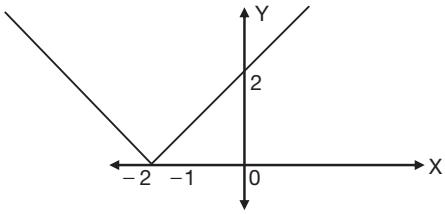



**7.21.** Draw the graph of $y = |x + 2|$.

**Sol:** The graph of $f(x) = |x|$ is shown below:




To get the graph of $|x+2|$, move the above graph 2 units to the left. Then graph is as follows:



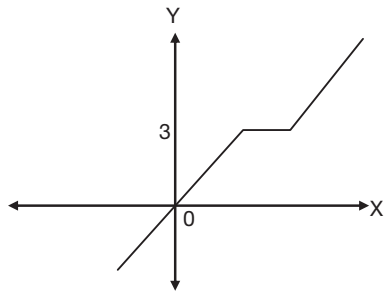

**NOTE**

Similarly, the graph of $|x - 2|$ is obtained by shifting the graph of $|x|$, 2 units to the right.

To get the graph of $f(x) + k$, (where $k>0$) shift the graph of $f(x)$, $k$ units up and similarly to get the graph of $f(x) - k$, shift the graph of $f(x)$ $k$ units down.

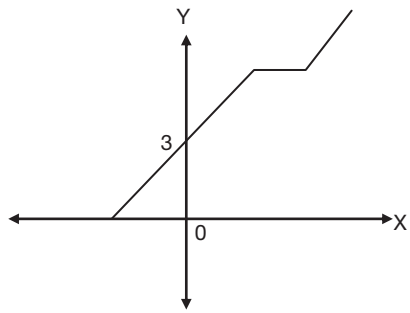
**7.22.** The graph of the following function is shown below.

$$f(x) = x \text{ for } x \le 2$$
$$= 2 \text{ for } 2 < x \le 3$$
$$= x - 1 \text{ for } 3 < x$$




Draw the graph of $y = f(x) + 3$.

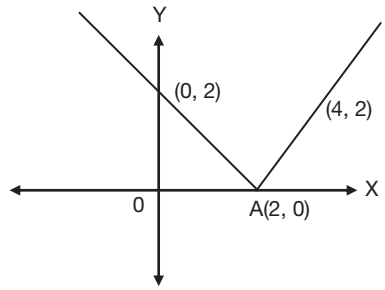
**Sol:**



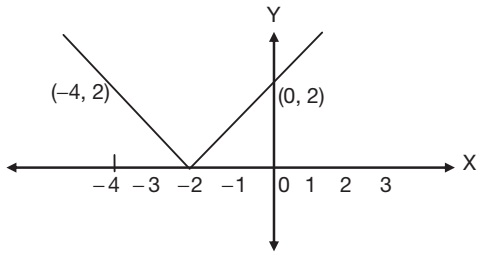

To get the graph of $f(x) + 3$, shift the graph of $f(x)$, 3 units up.

**7.23.** Draw the graph of $y = |x + 2|$ from the graph of $y = |x - 2|$

**Sol:** The graph of $f(x) = |x - 2|$ is as follows:

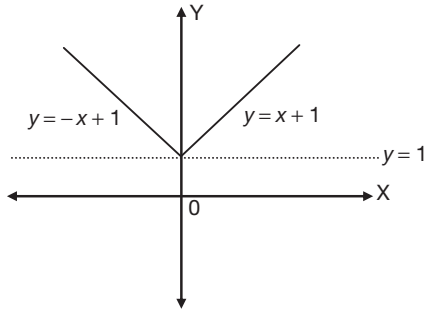



To get the graph of $|x+2|$, shift this graph 4 units to the left. Then the graph is as follows:




**7.24.** Draw the graph of $y = |x| + 1$

**Sol:**




$$y = |x| + 1$$

The least value taken by $y$ is 1, as no portion of the graph $y = |x| + 1$ is below $y = 1$.

The equation $y = |x| + 1$ can be written as

$$y = \begin{cases} x+1; & x \geq 0 \\ -x+1; & x < 0 \end{cases}$$

**7.25.** Sketch the graph of the function

$$y = x - 4;\ 0 \leq x < 5$$

$$9 - \frac{3}{2}x,\ 5 \leq x \leq 10$$

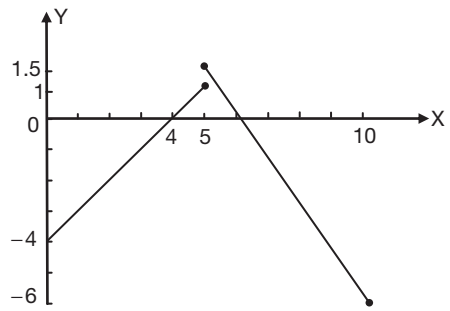
Also, determine the maximum and minimum values of $y$ in the given domain.

**Sol:** The maximum and minimum values of $y$ can be determined from the graph.

The max value occurs at $x = 5$.

$$y = 9 - \frac{3}{2}x = 9 - \frac{3}{2}(5) = 1.5$$

Minimum value of $y$ occurs at $x = 10$ and it is $-6$.




**7.26.** Sketch the graph of $y = \dfrac{|x-9| - |x-5|}{2}$ and $x = \dfrac{|y-9| - |y-5|}{2}$.

**Sol:** Let us divide the possible values of $x$ into three cases.

Case (i) $x \leq 5$: If $x \leq 5$, $|x - 5| = 5 - x$ and $|x - 9| = 9 - x$

$$\therefore y = \frac{9 - x - (5 - x)}{2} = 2$$

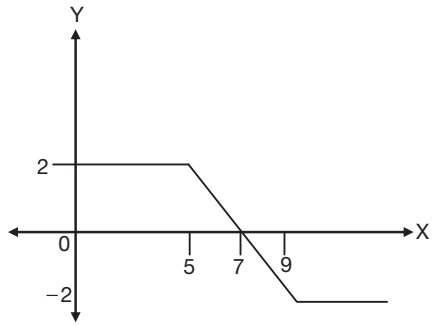
Case (ii) $5 \leq x \leq 9$: If $5 \leq x \leq 9$, $|x - 5| = x - 5$ and $|x - 9| = 9 - x$

$$\therefore y = \frac{9 - x - (x - 5)}{2} = 7 - x$$

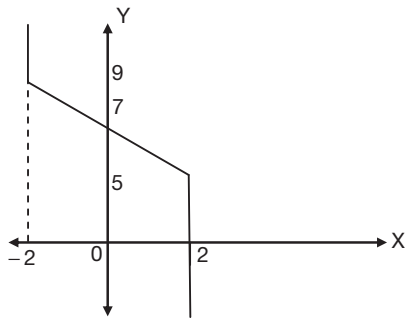
Case (iii) $x \geq 9$: If $x \geq 9$, $|x - 5| = x - 5$ and $|x - 9| = x - 9$

$$\therefore y = \frac{x - 9 - (x - 5)}{2} = -2$$

The graph can be represented as follows:

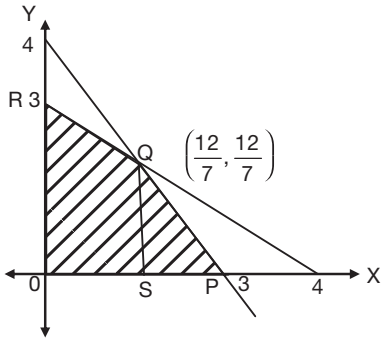



To get the graph of $x = \dfrac{|y-9| - |y-5|}{2}$, we need to reflect the graph discussed above in the line $y = x$. The graph is as follows.




**7.27.** Find the area enclosed by the region bounded by the graphs of $4x + 3y \leq 12$, $3x + 4y \leq 12$, $x \geq 0$ and $y \geq 0$.

**Sol:**




Sum of the areas
= Area of trapezium RQSO + Area of ΔQSP

$$= \frac{1}{2}\text{OS (OR + QS)} + \frac{1}{2}\text{SP} \times \text{QS}$$

$$= \frac{1}{2}\left(\frac{12}{7}+3\right)\left(\frac{12}{7}\right) + \frac{1}{2}\left(\frac{12}{7}\right)\left(3-\frac{12}{7}\right)$$

$$= \frac{36}{7} \text{ sq. units}$$

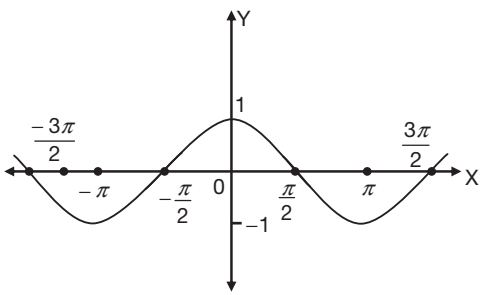
**7.28.** Two points A and B move in such a way that A is 8 units from the origin and B is 4 units from the origin. Find the area of the annular region in the first quadrant.

**Sol:** The set of points transversed by A will be the points on the area of the circle with radius 8 centred at the origin and that of B will be on the area of the circle with radius 4 centred at the origin and are in the first quadrant.

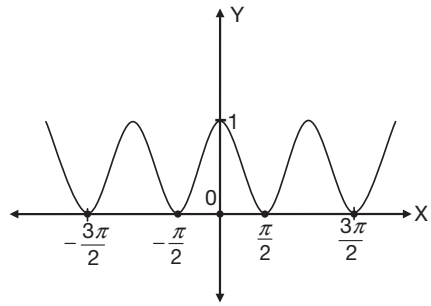
$$\text{Area enclosed} = \frac{1}{4}\left[\pi(8)^2 - \pi(4)^2\right] = 12\pi$$

**7.29.** Sketch the graph of
(a) $y = |\cos x|$
(b) $y = -|\cos x|$

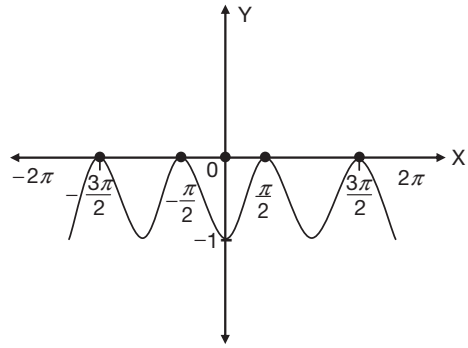
**Sol:** The graph of $y = \cos x$ is




(a) $y = |\cos x|$



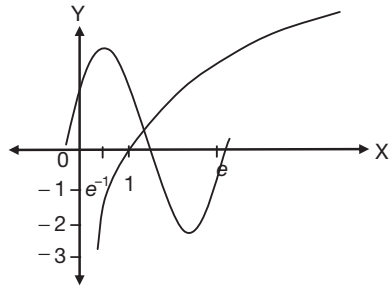

(b) $y = -|\cos x|$



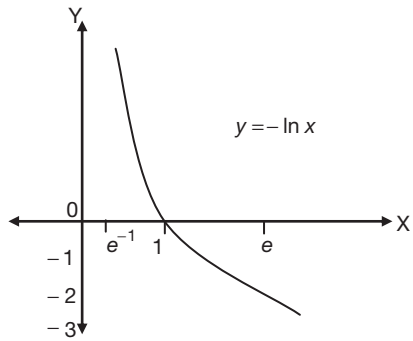

**7.30.** Sketch the graph of
(a) $y = \ln x$
(b) $y = -\ln x$
(c) $y = |\ln x|$
(d) $y = \ln(-x)$

**Sol:** (a) $y = \ln x$
Only when $x > 0$, $y$ is defined.

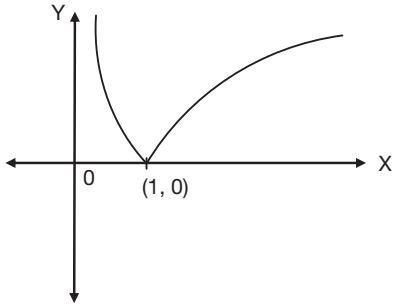
| $x$ | 1 | $e$ | $e^2$ | $e^3$ | $e^{-1}$ | $e^{-2}$ | $e^{-3}$ |
|---|---|---|---|---|---|---|---|
| $y$ | 0 | 1 | 2 | 3 | −1 | −2 | −3 |



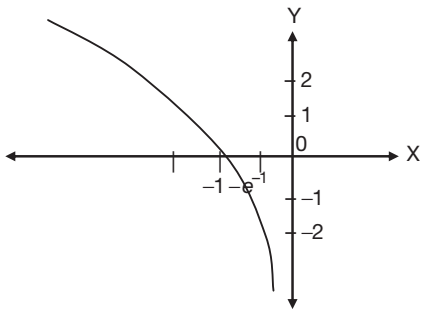

(b)



(c)




(d)




**7.31.** Sketch the graph of $y = \sin x$ and hence sketch the graph of

(a) $y = \sin 2x$

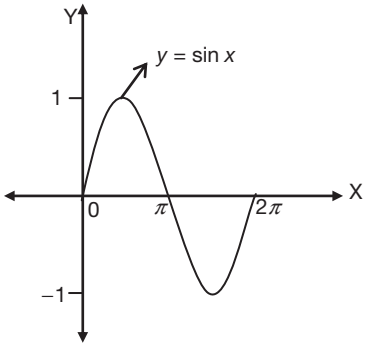
(b) $y = \sin \frac{x}{2}$

(c) $y = 2 \sin x$

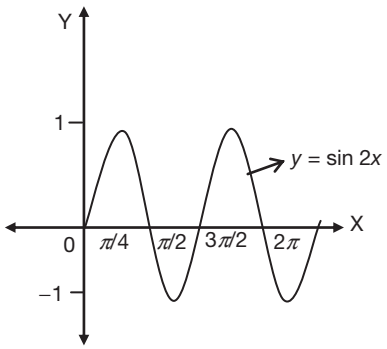
(d) $y = \frac{1}{2} \sin x$

in the interval $[0, 2\pi]$.

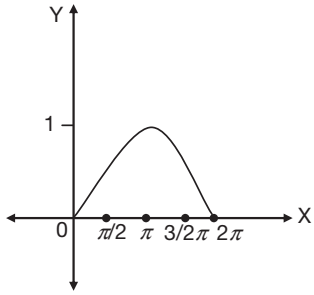
**Sol:**



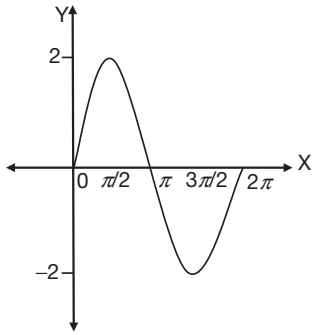

(a)



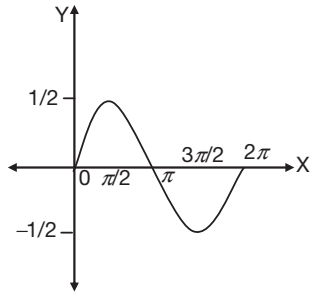

(b) $y = \sin x/2$




(c) $y = 2 \sin x$




(d) $y = \frac{1}{2} \sin x$



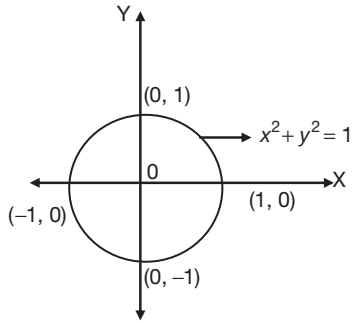

**7.32.** Sketch the graph of

(a) $(x+1)^2 + y^2 = 1$

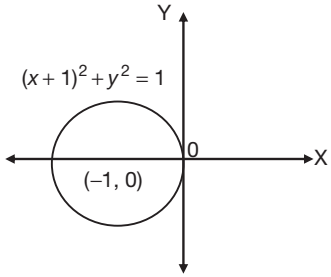
(b) $x^2 + (y-1)^2 = 1$

(c) $(x+1)^2 + (y+1)^2 = 1$.

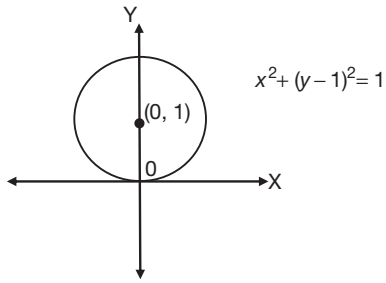
**Sol:** The graph of $x^2 + y^2 = 1$ is a circle of radius 1, centred at the origin.



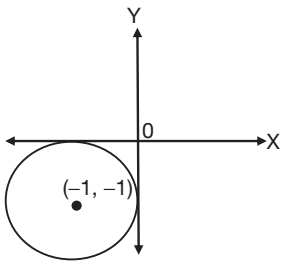
(a) The graph of $(x+1)^2 + y^2 = 1$ is obtained by translating the graph of $x^2 + y^2 = 1$ horizontally 1 unit to the left.




(b) The graph of $x^2 + (y-1)^2 = 1$ is obtained by translating the graph of $x^2 + y^2 = 1$ vertically by 1 unit upwards.



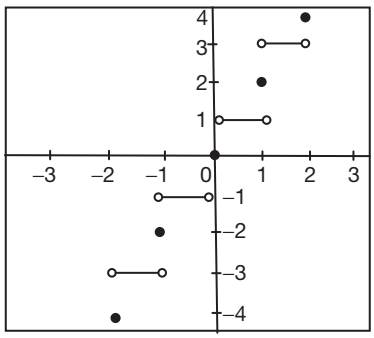

(c) The graph of $(x+1)^2 + (y+1)^2 = 1$ is obtained by translating the graph of $x^2 + y^2 = 1$ vertically downwards by 1 unit and horizontally by 1 unit to the left.




**7.33.** Sketch the graph of $y = \lceil x \rceil + \lfloor x \rfloor$

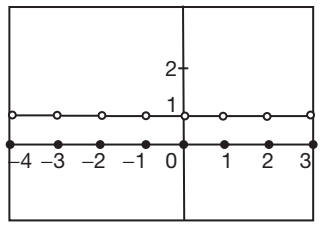
For integral values of $x$, $\lfloor x \rfloor = \lceil x \rceil = x$ and $y = 2x$, an even number. For other values of $x$, $y$ is odd. End-points on both ends of the steps are excluded.

**Sol:** $y = \lceil x \rceil + \lfloor x \rfloor$




**7.34.** Sketch the graph of $y = \lceil x \rceil - \lfloor x \rfloor$.

**Sol:** $y = \lceil x \rceil - \lfloor x \rfloor$



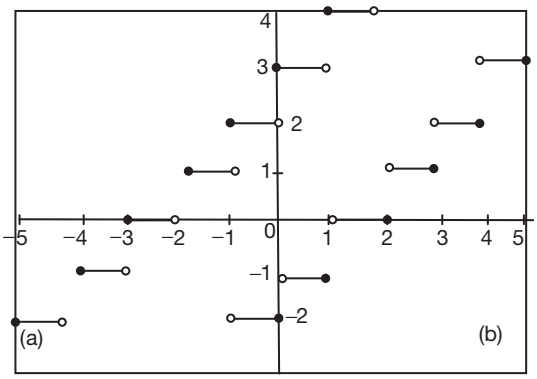

The graph is the line $y = 1$ (with excluded points represented by) for non-integral values of $x$ and the isolated points given by $y = 0$ for integral values of $x$.

**7.35.** Sketch the graph of

(a) $y = \lfloor x \rfloor + 3$ or $y = \lfloor x + 3 \rfloor$

(b) $y = \lceil x \rceil - 2$ or $y = \lceil x - 2 \rceil$

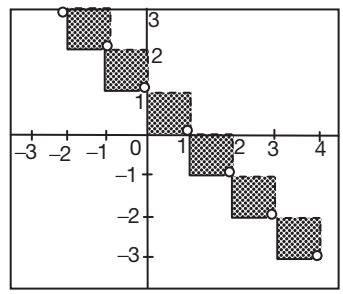
**Sol:**




We see that for all $x$, $\lfloor x + 3 \rfloor = \lfloor x \rfloor + 3$ and for all $x$, $\lceil x - 2 \rceil = \lceil x \rceil - 2$

**7.36.** Sketch the graph of $\lfloor x \rfloor + \lfloor y \rfloor = 0$

**Sol:**




We get a sequence of squares, in each of which the left and lower sides are included, which is indicated by the solid line while the right and upper sides are excluded, which is indicated by the broken line.

## EXERCISE-1

***Directions for questions 1–40:*** For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple-choice questions, write your answer in the space provided.

**1.** The graph of the function $y = x$ lies in the quadrants
(A) I, II (B) II, III
(C) I, III (D) II, IV

**2.** The graph of the equation $x + y - 1 = 0$ meets the $x$-axis at the point
(A) (0, 1) (B) (1, 0)
(C) (−1, 0) (D) (1, 1)

**3.** To which quadrants does the graph of the curve $y = x^2$ belong?
(A) I, II (B) II, III
(C) III, IV (D) I, IV

**4.** At what point(s) does the graph of the equation $x^2 + y^2 = 1$ meet the $y$-axis?
(A) (0, 1), (0, −1) (B) (0, 1), (0, 2)
(C) (0, 2), (0, −2) (D) None of these

**5.** At what points do the graphs of the equations $y = 2x + 1$ and $x^2 - 2 = y$ intersect?
(A) (3, 7), (1, 1) (B) (3, 7), (−1, −1)
(C) (−3, 7), (1, −1) (D) (3, −7), (−1, 1)

**6.** The reflection of the point (−2, 4) in the line $y = 3$ is
(A) (−2, −3) (B) (−2, 2)
(C) (2, 4) (D) (2, −4)

**7.** The reflection of the point $(x_1, y_1)$ in the origin is
(A) $(x_1, y_1)$ (B) $(-x_1, -y_1)$
(C) $(-x_1, y_1)$ (D) $(x_1, -y_1)$

**8.** The reflection of the point $(x_1, y_1)$ in $y$-axis is
(A) $(x_1, -y_1)$ (B) $(-x_1, -y_1)$
(C) $(-x_1, y_1)$ (D) $(x_1, y_1)$

**9.** The reflection of the point (2, −1) in the origin is
(A) (2, 1) (B) (−2, 1)
(C) (2, −1) (D) (−2, −1)

**10.** The reflection of the point $(-a, 0)$ in the $x$-axis is
(A) $(0, a)$ (B) $(0, -a)$
(C) $(a, 0)$ (D) $(-a, 0)$

**11.** The curve $x^2 = y$ is symmetric about
(A) $x$-axis
(B) $y$-axis
(C) a line parallel to $x$-axis
(D) a line parallel to $y$-axis

**12.** When $ac = 0$ and $b \neq 0$, the equation $ax + by + c = 0$ always represents
(A) a horizontal line (B) a vertical line
(C) an inclined line (D) Either (A) or (C)

**13.** If $a^2 + b = 0$, where $a$ and $b$ are real, and $a \neq 0$, then the equation $ax + by + c = 0$ always represents
(A) a horizontal line (B) a vertical line
(C) an inclined line (D) not a straight line

**14.** At how many points do the curves $y = x - 1$ and $x^2 = 2y$ intersect? _____

**15.** The curve $x^2 + y^2 - 2x + 2y + 1 = 0$ passes through
(A) (0, 0) (B) (0, 1)
(C) (1, 0) (D) None of these

**16.** The graph $y = -|x|$ is symmetric with respect to
(A) the $x$-axis (B) the $y$-axis
(C) both $x$ and $y$-axes (D) None of these

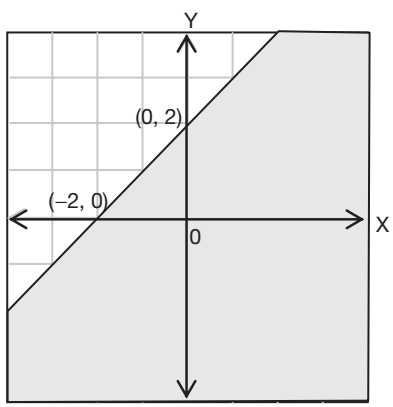
**17.** The graph $2x - 3y = 0$ is a line
(A) parallel to $x$-axis
(B) parallel to $y$-axis
(C) passing through origin
(D) None of these

**18.** The graph of $y - 7 = 0$ is a line
(A) parallel to $x$-axis
(B) parallel to $y$-axis
(C) passing through origin
(D) None of these

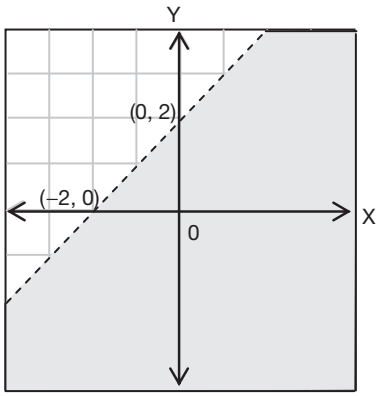
**19.** The graph $y + x = 0$ lies entirely in which of the following quadrants?
(A) $Q_1$, $Q_2$ (B) $Q_2$, $Q_3$
(C) $Q_1$, $Q_3$ (D) $Q_2$, $Q_4$

**20.** The graph of $y = -|x| + c$ is symmetric, w.r.t.,
(A) the $x$-axis (B) the $y$-axis
(C) both $x$ and $y$-axes (D) None of these

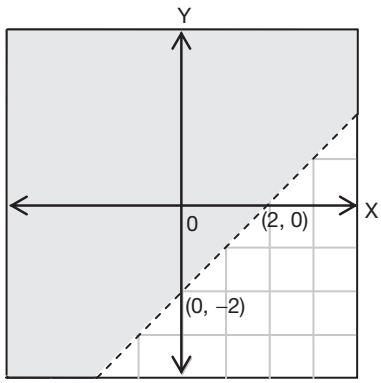
**21.** The graph of $y < x + 2$ is
(A)



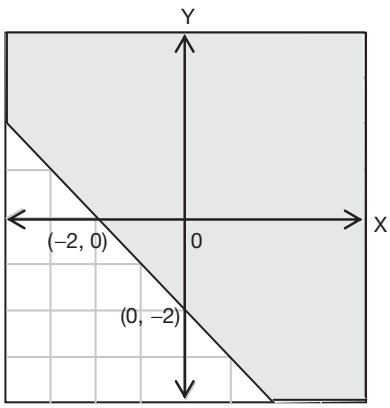
(B)




(C)




(D)




**22.** The graph of $2x - 3 = 0$ is a line
(A) parallel to the $x$-axis
(B) parallel to the $y$-axis
(C) passes through the origin
(D) lies along the $y$-axis.

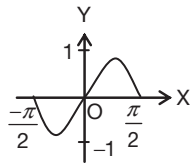
**23.** The graph of the line $2x + 3y = 0$ is
(A) parallel to the $x$-axis
(B) parallel to the $y$-axis
(C) passes through the origin
(D) lies along the $x$-axis.

**24.** The curve $x = y^2 - 2$ is symmetric about
(A) $x$-axis
(B) $y$-axis
(C) a line parallel to $x$-axis
(D) a line parallel to $y$-axis

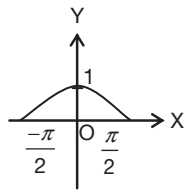
**25.** A line drawn parallel to $y$-axis meets the graph of the curve $y = x^2$ at _____ point(s)?

**26.** The graph of $f(x) = \sin x$ in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is

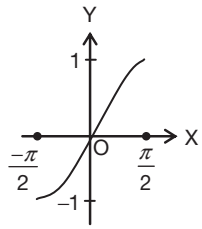
(A)



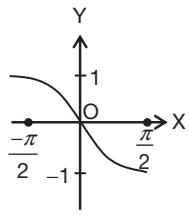

(B)



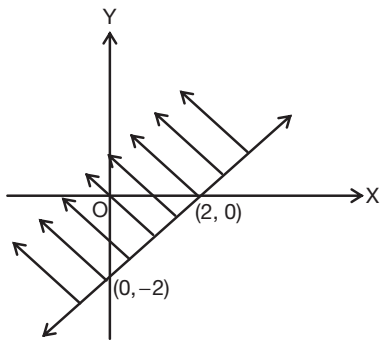

(C)




(D)



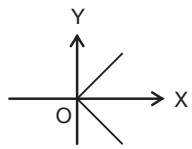

**27.** Which is the inequality that represents the region shown in the graph below?



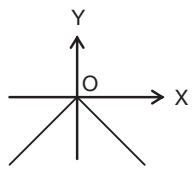

(A) $x + y \le 2$ (B) $x - y \ge 2$
(C) $x + y \ge -2$ (D) $x - y \le 2$

**28.** Which of the following represents the graph of the function $y = |x|$?

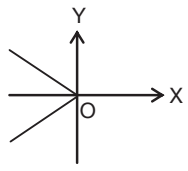
(A)



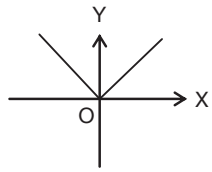

(B)



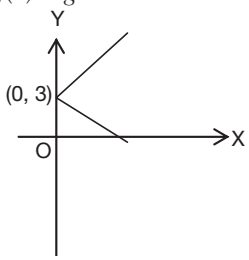

(C)



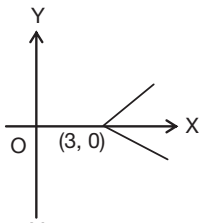

(D)

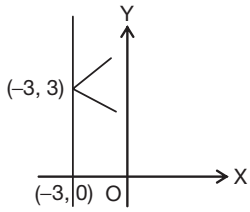


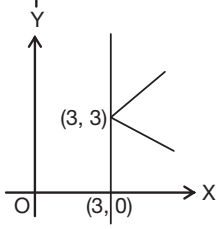
**29.** The graph of $f(x)$ is given below.

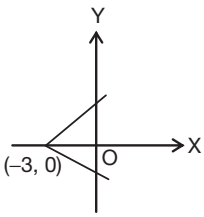



What is the graph of $f(x-3)$?

(A)


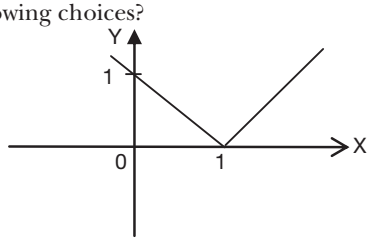

(B)



(C)



(D)



**30.** If $y = f(x)$ is the sine function, then $y = f(x) + c$ $(c > 0)$ is obtained by moving $f(x)$
(A) towards right for $c$ units
(B) towards left for $c$ units
(C) vertically above for $c$ units
(D) vertically below for $c$ units

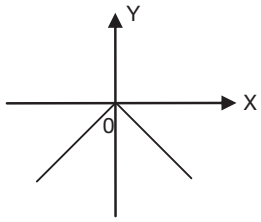
**31.** The graph given below is best represented by which of the following choices?




(A) $y = |x| - 1$ (B) $y = |x - 1|$
(C) $y = |x + 1|$ (D) $y = |x| + 1$

**32.** The graph of $y = mx + c$ meets the $y$-axis at the point
(A) $(c, 0)$ (B) $(0, c)$
(C) $(0, m)$ (D) $(0, -c)$

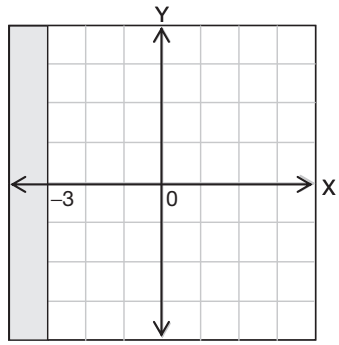
**33.** The graph given below is best represented by which of the following choices?

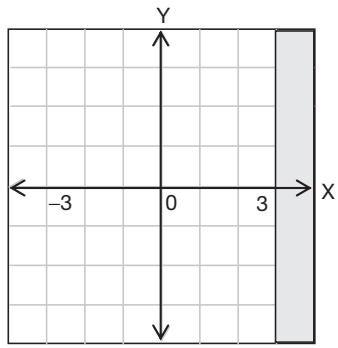


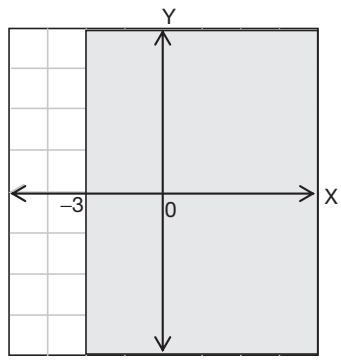

(A) $y = -|x|$ (B) $y = |x|$
(C) $x = |y|$ (D) None of these

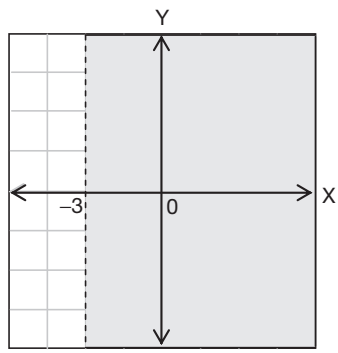
**34.** A line drawn parallel to $x$-axis meets the graph of the curve $x^2 + y^2 = 1$ at _____ point(s)?
(A) only one (B) only two
(C) more than two (D) None of these

**35.** The graph of $x \geq -3$ is

(A)



(B)


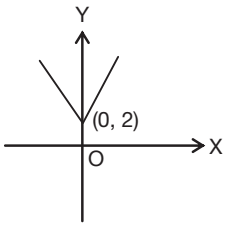

(C)



(D)


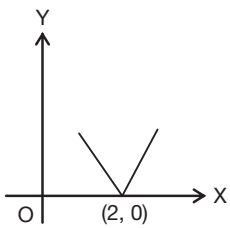
**36.** If $y = |x|$ is symmetric with respect to the $y$-axis, then $x = |y|$ is symmetric with respect to
(A) $x$-axis (B) $y$-axis
(C) both $x$ and $y$-axes (D) None of these

**37.** If $g(x) = f(-x)$, then $g(x)$ is obtained from $f(x)$ by
(A) reflecting $f(x)$ in $y$-axis
(B) reflecting $f(x)$ in $x$-axis
(C) reflecting $f(x)$ in both axes
(D) None of these

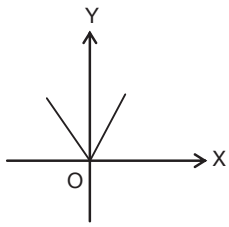
**38.** The graph of $f(x)$ is given below.



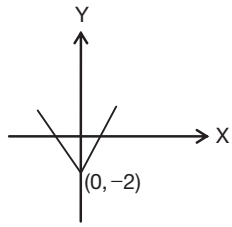

The graph of $f(x) - 2$ is
(A)



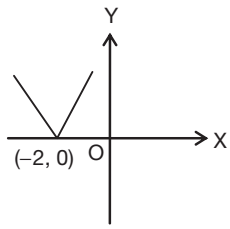

(B)



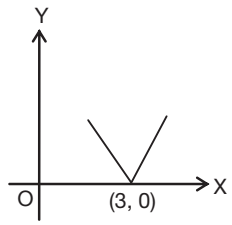

(C)



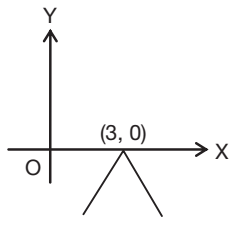

(D)



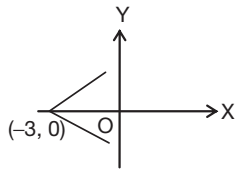

**39.** Which of the following is the graph of $y = |x - 3|$?
(A)



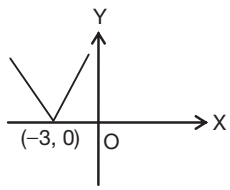

(B)




(C)




(D)




**40.** The graph $y = \log x$ crosses the $x$-axis at the point
(A) (0, 1) (B) (1, 0)
(C) (0, 0) (D) (2, 0)

## EXERCISE-2

***Directions for questions 1–40:*** For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple-choice questions, write your answer in the space provided.

**1.** (a) Draw the graph of $y = |x|$.

(b) Draw the graph of $y = -|x|$

**2.** Draw the graphs of

(a) $y = |x - 3|$, (b) $y = |x + 4|$

**3.** Draw the graphs of

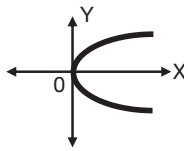
(a) $y = |x| + 4$, (b) $y = |x| - 3$

**4.** Draw the graph of $f(x) = [x]$, $[x]$ represents greatest integer $\le x$.

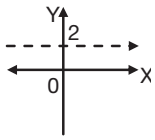
**5.** Draw the graph of $f(x) = \dfrac{|x|}{x}$, $x \neq 0, = 0$ for $x = 0$.

***Directions for questions 6–8:*** Match the equations with their graphs.

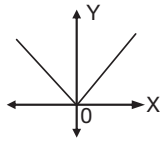
**6.** The graph of the equation $y = |x|$.
(A)



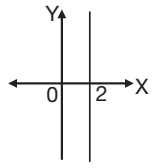

**7.** The graph of the equation $y^2 = x$.
(B)




**8.** The graph of the equation $y = 2$.
(C)

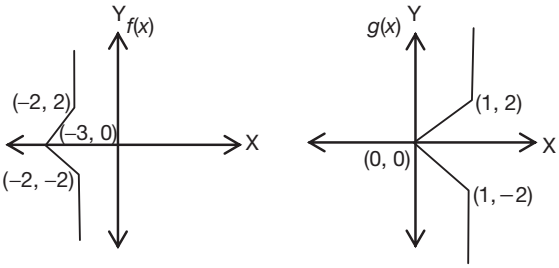



(D)



***Directions for questions 9–11*:** Select the correct alternative from the given choices.

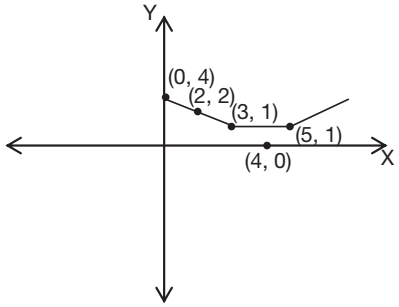
**9.**




(A) $g(x) = f(x-3)$ (B) $g(x) = f(x+3)$

(C) $g(x-3) = f(x)$ (D) $g(x) + 3 = f(x)$

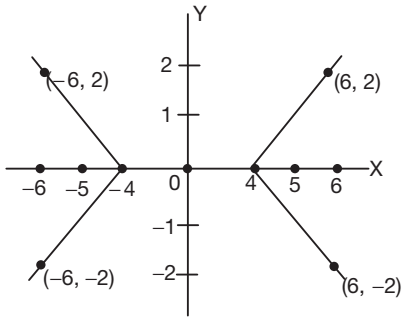
**10.**




(A) $y = \dfrac{|x-3|+|5-x|}{3}$ (B) $y = \dfrac{|x-1|+|x-5|}{2}$

(C) $y = \dfrac{|x-3|+|x-5|}{2}$ (D) $y = \dfrac{|x-1|-|x-5|}{2}$

**11.**

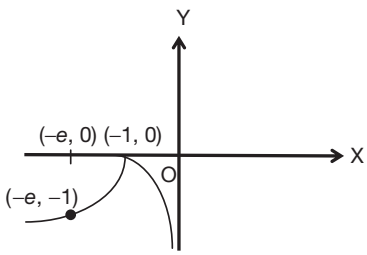



(A) $|x+y|+|x-y| = 8$ (B) $|x+y|-|x-y| = 0$

(C) $|x|-|y| = 4$ (D) $|x+y|-|x-y| = 4$

***Directions for questions 12 and 13*:** In these questions, a graph is given. The relation that best describes the graph is to be identified

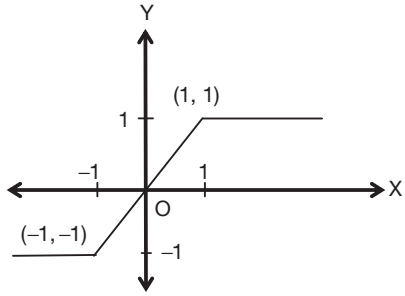
**12.**




(A) $y = -|\log x|;\ x > 0$ (B) $y = \log(-x);\ x < 0$

(C) $y = -|\log(-x)|;\ x < 0$ (D) $y = |-\log(-x)|,\ x < 0$

**13.**

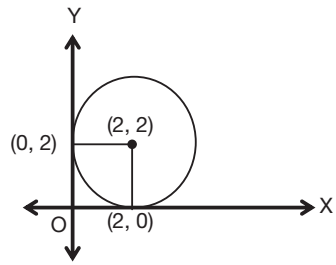



(A) $x = \dfrac{|y+1|-|y-1|}{2}$ (B) $y = \dfrac{|x|-|x+1|}{2}$

(C) $y = \dfrac{|x+1|-|x-1|}{2}$ (D) $y = \dfrac{|x-1|-|x+1|}{2}$

***Direction for question 14*:** Select the correct alternative from the given choices.

**14.** Find the coordinates of the point on the circle that is farthest from origin. The *x*-axis and the *y*-axis are tangents to the circle.

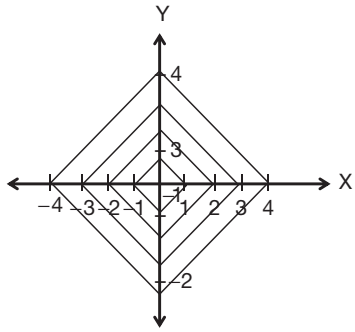



(A) $(2-\sqrt{2}, 2-\sqrt{2})$ (B) $(2+\sqrt{2}, 2+\sqrt{2})$

(C) $(\sqrt{2}, \sqrt{2})$ (D) $(3, 3)$

***Direction for question 15*:** In these questions a graph is given. The relation that best describes the graph is to be identified

**15.**




(A) $|x| + |y| = k$; $k$ is an integer

(B) $|x| - |y| = k$; $k$ is a non-zero integer

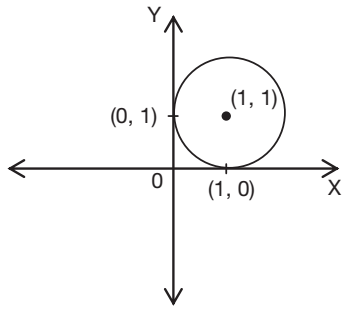
(C) $|x| + |y| = k$; $k$ is a positive integer

(D) $|x| + |y| = k$, $k$ is not an integer

Difficulty Level-2: Moderate

**16.** Find the area enclosed by the region described by $1 \le x \le 4$ and $0 \le |y-1| \le 2$ in square units _____.

**17.** On the given circle, find the coordinates of the point, that is farthest from the origin.




(A) $\left(1+\frac{1}{\sqrt{2}}, 1+\frac{1}{\sqrt{2}}\right)$ (B) $\left(1+\frac{1}{\sqrt{2}}, 1\right)$

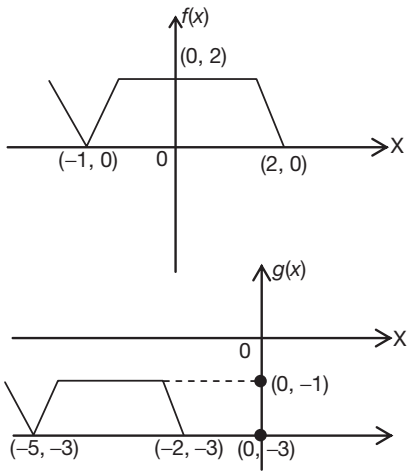
(C) $\left(1+\frac{1}{\sqrt{2}}, 1-\frac{1}{\sqrt{2}}\right)$ (D) $(1, 1)$

**18.** Find the area enclosed by the region described by $2 \le |x-3| \le 3$ and $1 \le |y-2| \le 3$ in square units.

(A) 8 (B) 10

(C) 12 (D) 16

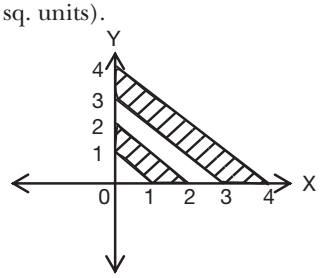
**19.** Which of the following statement best describes the relation between the two graphs?




(A) $g(x) = f(x-4) + 3$ (B) $g(x) = f(x+4) - 3$

(C) $g(x) = f(x+3) - 4$ (D) $g(x) = f(x-3) + 4$

**20.** In the figure given below, find the area of the shaded region (in sq. units).




***Directions for questions 21–23*:** These questions are based on the following data.

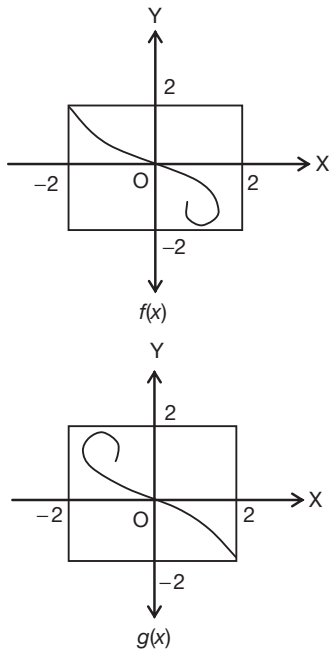
In each of these questions a pair of graphs $f(x)$ and $g(x)$ is given. The graphs are shown as thick curves in the interval $[-2, 2]$. The following relations may hold between the graphs.

(A) $f(x) = g(-x)$ (B) $f(x) = -g(x)$

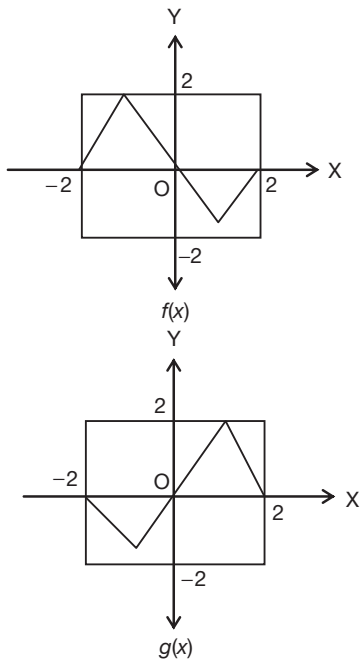
(C) $f(x) = -g(-x)$ (D) $g(x) = -|f(x)|$

Mark your options A, B, C, D if the corresponding relation holds true.

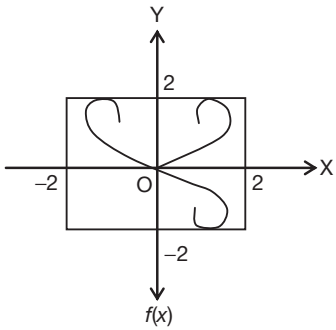
**21.**

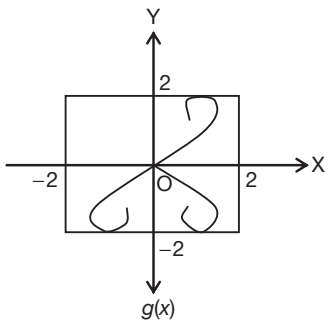



**22.**



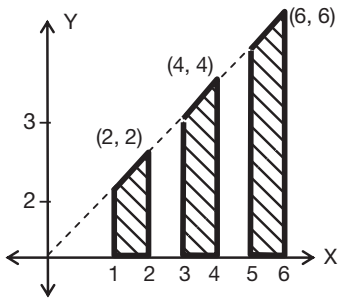
**23.**




***Directions for question 24*:** Select the correct alternative from the given choices.

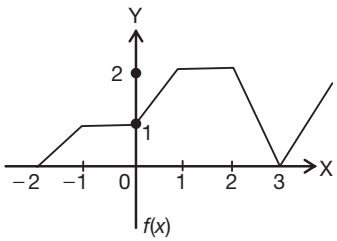
**24.** Find the least number of trapeziums that are required to make the sum of the area exceed 100 sq.units?

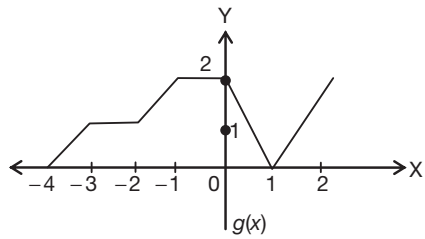



(A) 8 (B) 7
(C) 9 (D) 10

***Directions for question 25*:** In the following question, the graph of $f(x)$ and $g(x)$ are given. Select the choice which describes the relation between the given graph.

**25.**

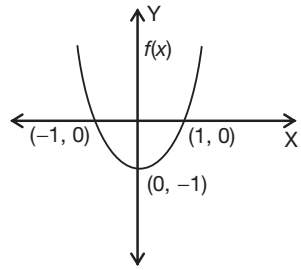


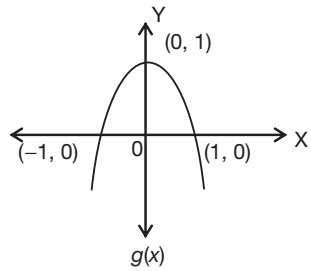




(A) $g(x) = f(x+2)$ (B) $g(x) = f(x-2)$
(C) $g(x) = f(x) + 2$ (D) $g(x) = f(x-2) + 2$

***Directions for question 26*:** In the following question the graphs of $f(x)$ and $g(x)$ are given. Select the choice which describes the relation between the given graphs.

**26.**

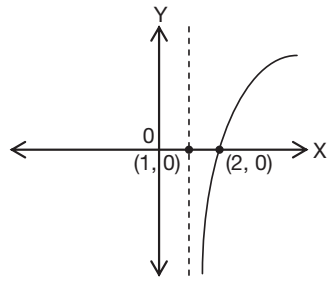



(A) $f(x) = g(x)$ (B) $f(x) = g(-x)$, for $x > 0$
(C) $f(x) = g(-x)$ for $x < 0$ (D) $f(x) = -g(-x)$

***Directions for questions 27–29*:** In these questions a graph is given. Select the choice which best describes the given graph.

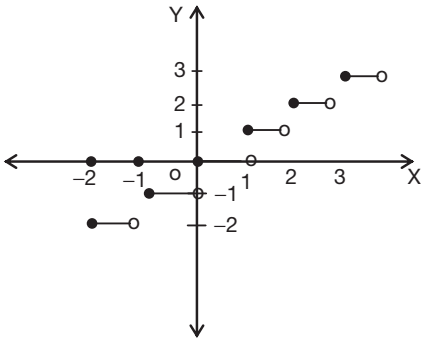
**27.**




(A) $y = \log_e x$ (B) $y = \log_e(x-1)$
(C) $y = \log_e\left(\frac{x}{2}\right)$ (D) $y = \log_e 2x$

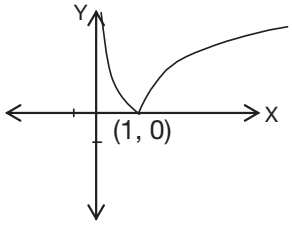
**28.** The reflection of the point (–1, 4) in the origin is
(A) (–1, 4) (B) (1, 4)
(C) (–4, 1) (D) (1, –4)

**29.**




(A) $y = [x]$ (B) $y = 1 - [x]$
(C) $y = 1 + [x]$ (D) $y = [2x]$

**30.**




(A) $y = \log x$ (B) $y = -\log x$
(C) $y = |\log x|$ (D) $y = -|\log x|$

**31.** The equation $x^2 - 4x + 3 = 0$ has _____ number of positive roots.

**32.** Given $\left(x^2 + \frac{1}{x^2}\right) - 4\left(x + \frac{1}{x}\right) + \frac{23}{4} = 0$. Find $\left(x + \frac{1}{x}\right)$.

**33.** The equation $x^2 + 4x + 3 = 0$ has roots $\alpha, \beta$. Find the equation with roots $\left(1 + \frac{\alpha}{\beta}\right)$ and $\left(1 + \frac{\beta}{\alpha}\right)$.

(A) $3x^2 + 16x + 16 = 0$ (B) $3x^2 - 16x + 16 = 0$
(C) $3x^2 - 16x - 16 = 0$ (D) $x^2 - 16x + 16 = 0$

**34.** If $\alpha, \beta, \gamma$ are the roots of the equation $x^3 - 4x^2 + 6x - 4 = 0$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ is _____.

**35.** When a polynomial $f(x)$ is divided by $2x + 3$, the quotient is $x^2 + 3x - 2$ and the remainder is 4. Find $f(x)$.
(A) $x^3 - 9x^2 - 2$ (B) $2x^3 + 9x^2 - 5x + 2$
(C) $2x^3 + 9x^2 + 5x - 2$ (D) $x^3 - 9x^2 - 5x - 2$

**36.** Two of the roots of the equation $x^4 - px^3 + 2x^2 + 2x + q = 0$ are $-1, 2$ then find the other two roots.
(A) $2 \pm i$ (B) $\pm i$
(C) $1 \pm 2i$ (D) $1 \pm i$

***Directions for questions 37–40*:** In each of these questions the relation satisfied by the points on a graph is given. Give your response as

(A) if a horizontal line can intersect the graph more than once.
(B) if a vertical line can intersect the graph more than once.
(C) if both (A) and (B) are possible.
(D) otherwise.

**37.** $y = \tan x$ where $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

**38.** The graph is a set of points $P(x, y)$ such that $xy \geq 0$ and P is at a distance of $k$ units from the origin.

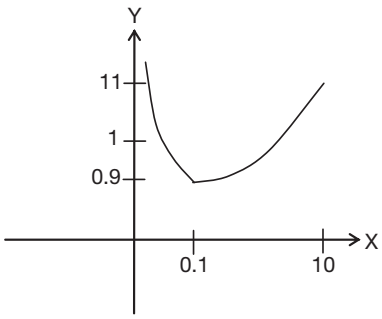
**39.** $x = |y - 3| + 2$

**40.** $y = -|\log(-x)|, x < 0$

## EXERCISE-3

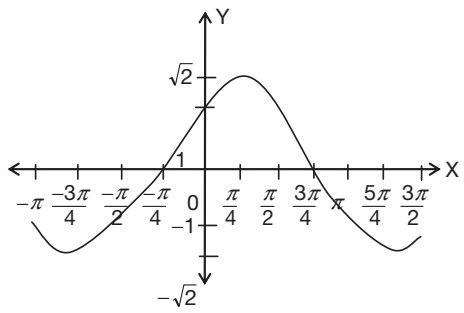
***Directions for questions 1–40*:** For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple-choice questions, write your answer in the space provided.

**1.** For the graph given below, find the function that best describes the graph.




(A) $|\log_{10} x|$ (B) $|\log_{10} x + x|$
(C) $|\log_{10} x| + x$ (D) None of the above

**2.** For the graph given below, the function that corresponds to the given graph is




(A) $|\sin 2x|$ (B) $|\cos 2x|$
(C) $\sin x + \cos x$ (D) $\sqrt{2}[\sin x + \cos x]$

***Direction for questions 3 and 4*:** These questions are based on the following data:

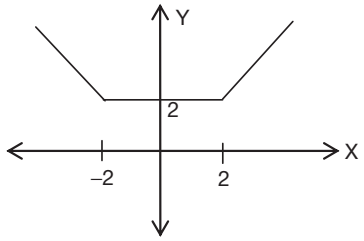
In each of these questions, the relation satisfied by the points on the graph is given. Use this relation and decide your response as

(A) if a vertical line intersecting the graph more than once exists.
(B) if a horizontal line intersecting the graph more than once exists.
(C) if a vertical and a horizontal line intersecting the graph more than once exists.
(D) None of the above

**3.** $xy = 1$, when $x, y > 0$.

**4.** $\sin |x|$

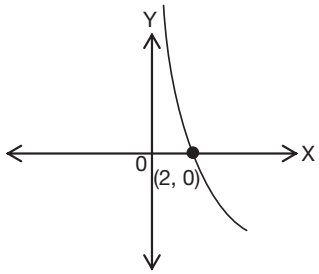
**5.** For the graph given below, find the function that best describes the graph.




(A) $\frac{|x+2|+|x-2|}{2}$ (B) $\frac{|x+2|-|x+2|}{2}$

(C) $\frac{|x+1|+|x-1|}{2}$ (D) $\frac{|x+1|-|x-1|}{2}$

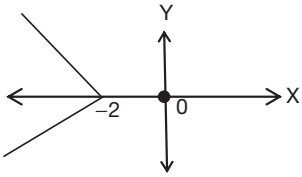
**6.** Which of the following equations best describes the graph given below?




(A) $y = \log_{0.5} x$ (B) $y = \log_{0.5}(x-1)$

(C) $y = \log_{0.5}\left(\frac{x}{2}\right)$ (D) $y = \log_{0.5} 2x$

**7.** Which of the following equations best describes the graph given below?

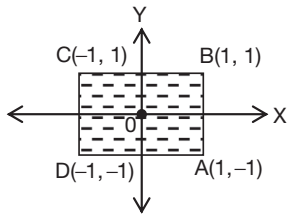



(A) $y = -|x+2|$ (B) $x = |y+2|$

(C) $x = -(|y|+2)$ (D) $x = 2 - |y|$

***Directions for questions 8–10*:** In each of these questions, a graph is given. Choose the relation that best describes the graph from the choices given.

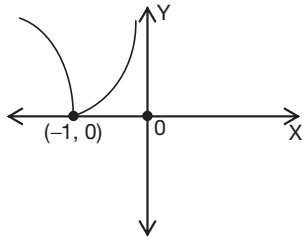
**8.**




(A) $0 \le x \le 1$ and $0 \le y \le 1$ (B) $|x| \le 1$ and $|y| \ge 1$

(C) $|x| \ge 1$ and $|y| \ge 1$ (D) $|x| \le 1$ and $|y| \le 1$

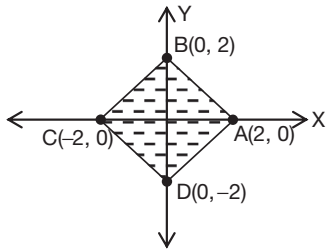
**9.**




(A) $y = |\log x|, x > 0$ (B) $y = -|\log x|, x < 0$

(C) $y = |\log(-x)|, x > 0$ (D) $y = |\log(-x)|, x < 0$

**10.**



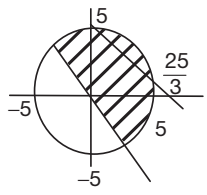

(A) $|x| + |y| \le 1$ (B) $|x| + |y| \ge 1$

(C) $|x| + |y| \le 3$ (D) $|x| + |y| \le 2$

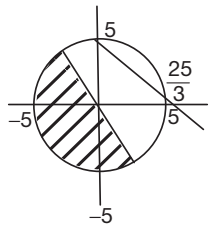
**11.** Find the region in which $x$ and $y$ satisfy the following conditions

$x^2 + y^2 \le 25$,
$x \ge -y$ and $3x + 5y \ge 25$

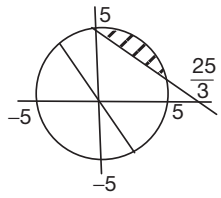
(A)



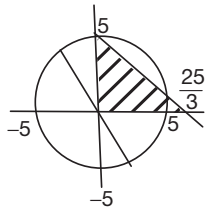

(B)




(C)



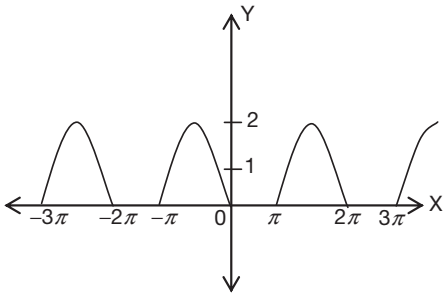

(D)



**12.** The area of the region described by the inequality $|x| + |y| \le 3$ is _____.

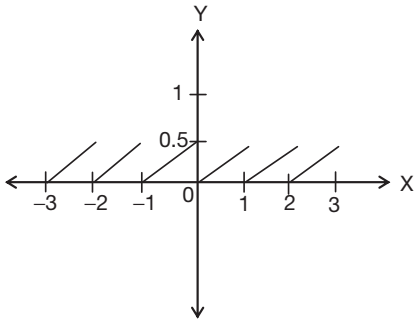
**13.** Find the area (in sq. units) of the figure formed by the lines $x$-axis, $x = 2$, $x = 5$ and $4x + 3y = 23$ _____.

**14.** For the graph given below, find the function that best describes the graph.



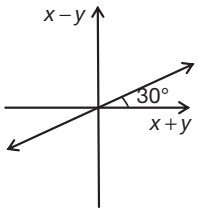

(A) $|\sin x - x|$ (B) $|\cos x + \sin x|$
(C) $|\sin x| - \sin x$ (D) None of the above

**15.** For the graph given below, find the relation that best describes the graph.
$\lceil x \rceil$ = Least integer greater than or equal to $x$.
$\lfloor x \rfloor$ = Greatest integer less than or equal to $x$.

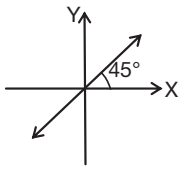


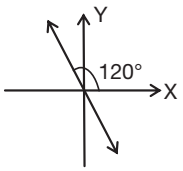

(A) $\dfrac{\lceil x \rceil + x}{2}$ (B) $\dfrac{\lfloor x \rfloor + x}{2}$
(C) $\dfrac{\lceil x \rceil - x}{2}$ (D) $\dfrac{x - \lfloor x \rfloor}{2}$

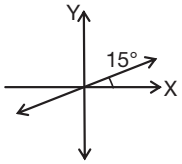
**16.** The graph of $x - y$ against $x + y$ is shown below.

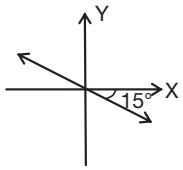



Which of the following represents the above line, w.r.t., $y$ – axis and $x$ – axis?

(A)



(B)


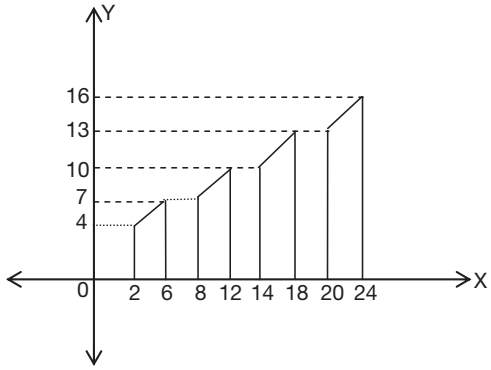

(C)



(D)



**17.** How many points with integral coordinates lie inside the region bounded by the lines $|x| = 2$, $x + y = 12$ and $x - y = 5$?

**18.** Find the area of the region described by the relations $|x| + |y| \ge 2$ and $x^2 + y^2 \le 4$ in square units.
(A) $4(\pi - 1)$ (B) $4(\pi - 2)$
(C) $4(4 - \pi)$ (D) $2(4 - \pi)$

**19.** Find the sum of the areas of the trapeziums that are shown in the figure below. (in sq. units)




(A) 80 (B) 160
(C) 120 (D) 320

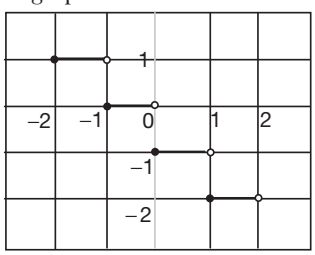
**20.** How many points in the region enclosed by $x \ge 0$, $y \ge 0$ and $4x + 7y \le 35$ have both the coordinates as positive integers _____?

**21.** Find the area of the region described by the relations $1 \le |x - 3| \le 5$ and $1 \le |y - 3| \le 5$ in square units _____.

***Directions for questions 22–25*:** These questions are based on the following data.

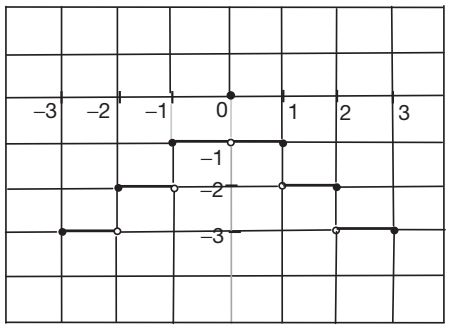
The functions $f(x)$, $g(x)$, $h(x)$, $i(x)$ are defined below.
$f(x)$ is the greatest integer less than $x$.
$g(x)$ is the greatest integer less than or equal to $x$, i.e., floor function.
$h(x)$ is the least integer greater than or equal to $x$, i.e., ceil function.
$i(x)$ is the least integer greater than $x$.

**22.** How many of the following functions (for $-2 \le x \le 2$) represent the graph below?



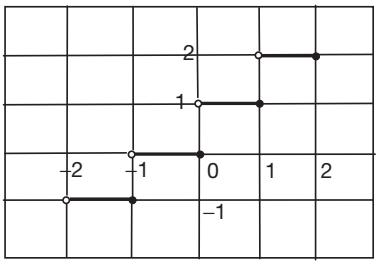
(i) $f(-x)$ (ii) $-f(x)$
(iii) $g(-x)$ (iv) $-g(x)$
(A) 4 (B) 1
(C) 2 (D) 0

**23.** Which of the following functions represents the graph below?



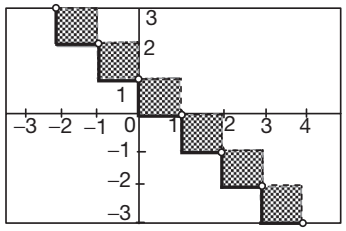

(A) $-g(|x|)$ (B) $-h(|x|)$
(C) $-f(|x|)$ (D) $-i(|x|)$

**24.** Which of the following functions (for $-2 \le x \le 2$) represents the graph below?




(A) $y = f(x)$ (B) $y = g(x)$
(C) $y = i(x)$ (D) $y = h(x)$

**25.** Which of the following equations represents the graph shown below?

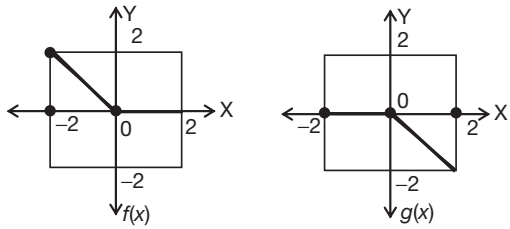



(A) $g(x) + g(y) = 1$ (B) $f(x) + f(y) = 1$
(C) $g(x) + g(y) = 0$ (D) $f(x) + f(y) = 0$

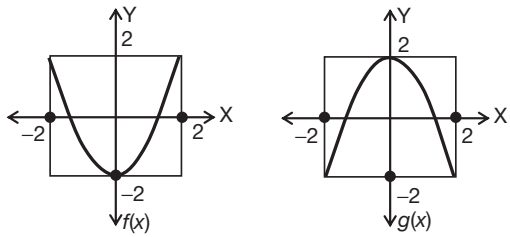
***Directions for questions 26–30*:** In each of these questions, a pair of graphs f$(x)$ and g$(x)$ is given. The graphs are shown as thick curves in the interval $[-2, 2]$. The following relations may hold between the graphs. Mark your answer as

(A) if $f(x) = \pm g(-x)$
(B) if $f(x) = -g(x)$
(C) if $f(x) = |g(x)|$
(D) None of the above relation holds.

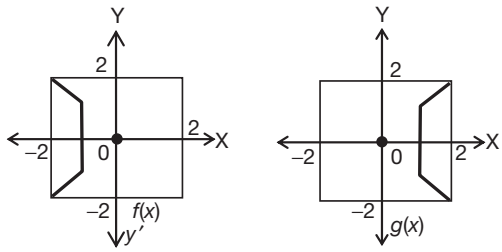
**26.**



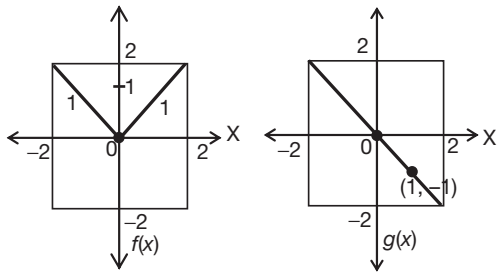

**27.**



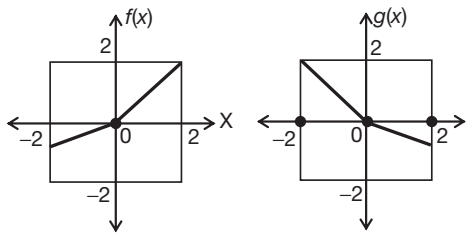

**28.**




**29.**

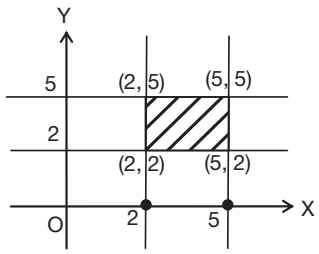



**30.**




***Direction for question 31*:** In each of these questions, a graph is given. Choose the relation that best describes the shaded region.

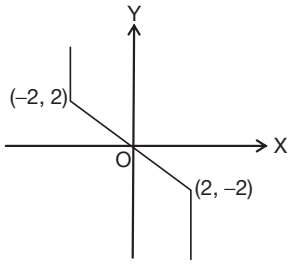
**31.**




(A) $2 \le |x| \le 5$ and $2 \le |y| \le 5$
(B) $2 \le x \le 5$ and $2 \le y \le 5$
(C) $0 \le |x| \le 5$ and $0 \le |y| \le 5$
(D) $2 \le x \le 5$ or $2 \le |y| \le 5$

***Directions for questions 32*:** In each of these questions, a graph is given. Choose the relation that best describes the graph.

**32.**

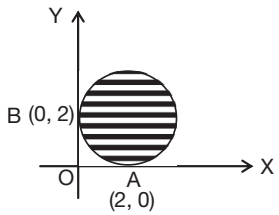



(A) $y = \dfrac{|x-2|+|x+2|}{2}$ (B) $x = \dfrac{|y-2|+|y+2|}{2}$

(C) $y = \dfrac{|x-2|-|x+2|}{2}$ (D) $x = \dfrac{|y-2|-|y+2|}{2}$

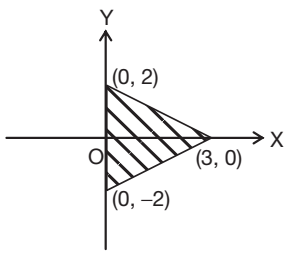
***Direction for question 33*:** In each of these questions, a graph is given. Choose the relation that best describes the shaded region.

**33.**




(A) $x^2 + y^2 \le 4$ (B) $x^2 + y^2 - 4(x+y) \le 0$
(C) $x^2 + y^2 \le 4(x+y-1)$ (D) $(x-2)^2 + (y-2)^2 = 4$

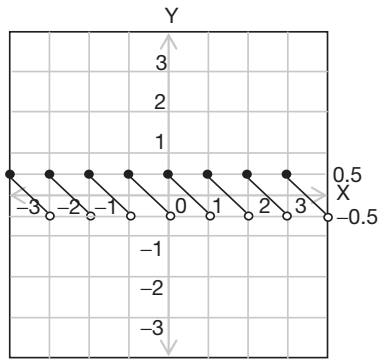
**34.**




(A) $|2x| + |3y| \le 6,\ x \ge 0$ (B) $x \le 3$ and $y \le 2$
(C) $|2x| + 3y \le 6$ (D) $|2x| + |3y| \le 6$

**35.** The number of common points of the graphs $3x + 4y - 7 = 0$ and $x^2 + y^2 - 4 = 0$ is
(A) 1 (B) 2
(C) 0 (D) Cannot be determined

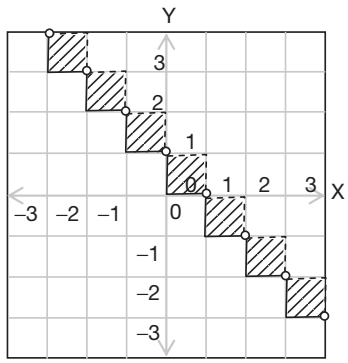
**36.** Which of the following relations represents the graph shown below?




(A) $y = [x] - x$ (B) $y = x - [x] - \dfrac{1}{2}$

(C) $y = [x] - x + \dfrac{1}{2}$ (D) $y = \dfrac{x - [x]}{2}$

**37.** What is the equation of the graph shown below?




(A) $[x] = [y]$ (B) $[x] + [y] = 1$
(C) $[x] - 1 = [y]$ (D) $[x] + [y] = 0$

***Directions for questions 38 and 39*:** These questions are based on the following data.

The functions $f(x)$, $g(x)$, $h(x)$, $i(x)$ are defined below.
$f(x)$ is the greatest integer less than $x$.
$g(x)$ is the greatest integer less than or equal to $x$, i.e., floor function.
$h(x)$ is the least integer greater than or equal to $x$, i.e, ceil function.
$i(x)$ is the least integer greater than $x$.

**38.** If $p(x) = sg(x) + g(x)$, and $sg(x) = -1, 0$ or $1$ when $x < 0$, $x = 0$ or $x > 0$, respectively, what is the maximum value of $p(x + 1) - p(x)$?

(A) 1 (B) 2

(C) 3 (D) None of these

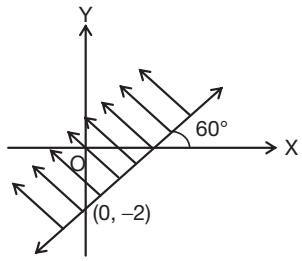
**39.** The graph of the four functions above consists of a series of line segments. In each, the end point on one side is omitted and the end point on the other side is included. For which functions are the right end points included?

(A) $f(x), g(x)$ (B) $f(x), h(x)$

(C) $f(x), i(x)$ (D) $g(x), i(x)$

***Direction for question 40*:** In each of these questions, a graph is given. Choose the relation that best describes the shaded region.

**40.**




(A) $x - \sqrt{3}y - 2 \le 0$ (B) $\sqrt{3}x - y - 2 \le 0$

(C) $\sqrt{3}x - y - 2 \ge 0$ (D) $x - \sqrt{3}y - 2 \ge 0$

## EXERCISE-4

***Directions for questions 1–15*:** For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple-choice questions, write your answer in the space provided

**1.** If the graph of $y = x^2 + 4x + 9$ is symmetric about $x = k$, the value of $k$ is _____.

**2.** What is the maximum distance between any two points on the graph $4|x| + 5|y| = 40$?

(A) 24 (B) 16

(C) 10 (D) 20

**3.** Find the area of the region bounded by the straight lines $4|x| + 5|y| = 20$ _____.

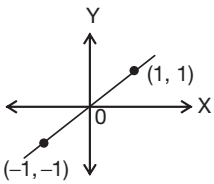
***Directions for questions 4 and 5*:** These questions are based on the information given below.

The functions $f(x)$ and $g(x)$ are defined as

$f(x) = \dfrac{x - |x|}{2}$ and $g(x) = \dfrac{x + |x|}{2}$.

In these questions, choose the alternative that best describes the graph given.

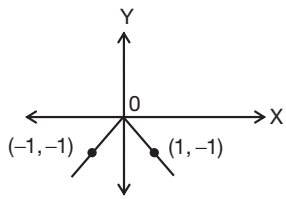
**4.**




(A) $y = g(x) - f(x)$ (B) $y = f(x) - g(x)$

(C) $y = \dfrac{f(x) + g(x)}{2}$ (D) $y = f(x) + g(x)$

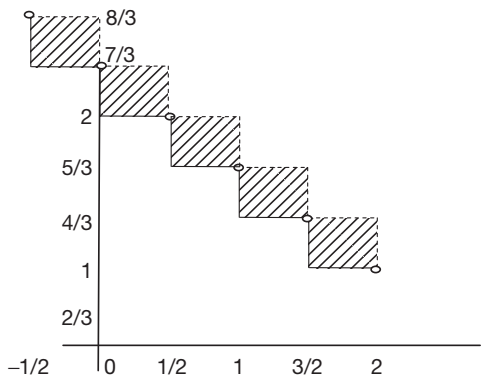
**5.**




(A) $y = \dfrac{f(x) + g(x)}{2}$ (B) $y = f(x) - g(x)$

(C) $y = f(x) + g(x)$ (D) $y = g(x) - f(x)$

**6.**




Which of the following best describes the graph above for $-\dfrac{1}{2} \le x \le 2$?

(A) $\left[\dfrac{x}{2}\right] + \left[\dfrac{y}{3}\right] = 6$ (B) $\left[\dfrac{x}{2}\right] - \left[\dfrac{y}{3}\right] = 6$

(C) $[2x] + [3y] = 6$ (D) $[2x] - [3y] = 6$

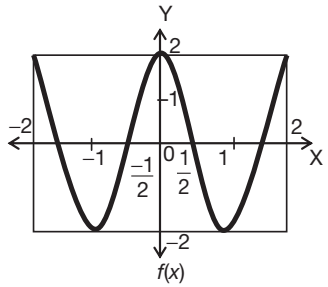
**7.** The equations of lines $l$ and $m$ are $x = 7 + |a|$ and $y = 6 + |b|$, respectively. In how many points do $l$ and $m$, respectively meet the graph of $5|x| - 6|y| = 30$?

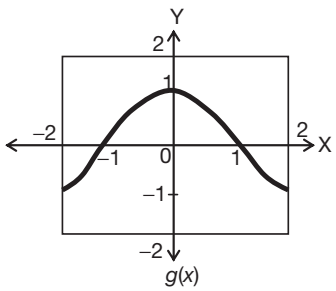
(A) 1, 2
(B) 2, 1
(C) 2, 2
(D) depends on the values of $a$, $b$

***Directions for questions 8 and 9*:** These questions are based on the information given below:

In each question a pair of graphs, $y = f(x)$ and $y = g(x)$ are provided. From the given options, find the relation that best describes the relation between $f(x)$ and $g(x)$.

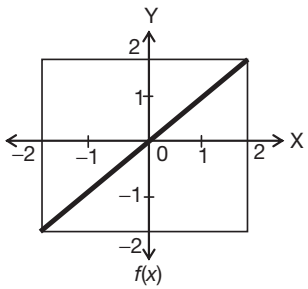
**8.**

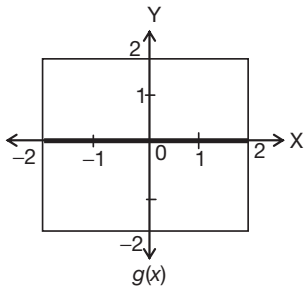



(A) $f(x) = 2g(x)$ (B) $f(2x) = 2g(x)$
(C) $f(x) = g(2x)$ (D) $f(x) = 2g(2x)$

**9.**




(A) $f(x) = g(x) + g(-x)$ (B) $g(x) = f(x) + f(-x)$
(C) $g(x) = f(x) - f(-x)$ (D) $g(x) = f(x) + |f(-x)|$

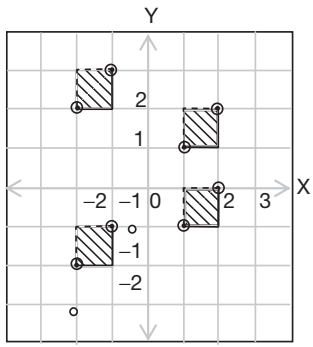
***Directions for questions 10–15*:** These questions are based on the information given below:

**NOTE**

(a) In questions 10–15, $\lfloor x \rfloor$ or the floor function, denotes the greatest integer less than or equal to $x$ and $\lceil x \rceil$ or the ceil function denotes the least integer greater than or equal to $x$. In the graphs, the dark line along the edge of the shaded region indicates that the points on that edge are included along with the region, the broken line indicates their exclusion from the region. Where the broken line meets the dark line, the dark dot indicates inclusion and the hollow dot indicates exclusion.

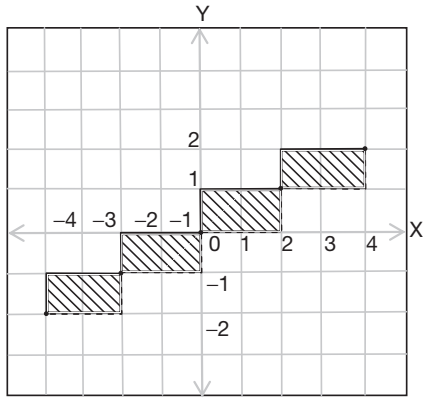
(b) The corners along the line $x + y = 0$ are included, though they have not been marked with dark dots. [It may be presumed that as there are two dark lines meeting at these corners (in addition to the two dotted lines), these points are included in the graph].

**10.** Which of the following relations between $x$ and $y$, for $1 \le |x| \le 2$, represents the given graph?



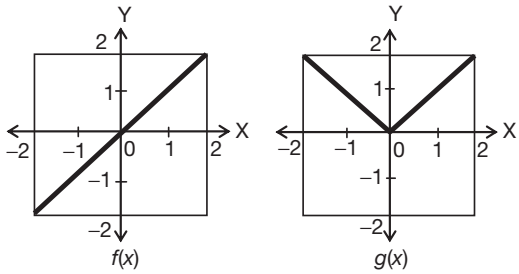

(A) $\|\lfloor x \rfloor\| + \|\lceil y \rceil\| = 3$ (B) $\|\lceil x \rceil\| + \|\lfloor y \rfloor\| = 3$
(C) $\lceil |x| \rceil + \lfloor |y| \rfloor = 3$ (D) $\lfloor |x| \rfloor + \lceil |y| \rceil = 3$

**11.** Which of the following relations between $x$ and $y$, for $-4 \le x \le 4$, represents the graph shown?



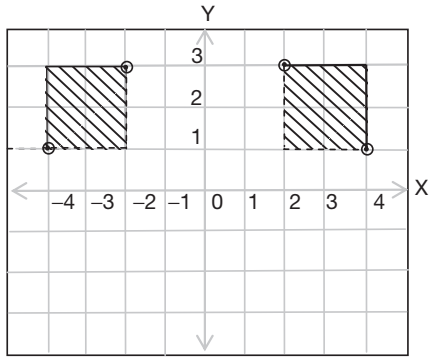
(A) $\lceil x/2 \rceil - \lceil y \rceil = 0$ (B) $\lceil x/2 \rceil - \lfloor y \rfloor = 1$
(C) $\lfloor x/2 \rfloor - \lfloor y \rfloor = 0$ (D) $\lfloor x/2 \rfloor - \lceil y \rceil = -1$

**12.** From the given options, find the relation that best describes the relation between $f(x)$ and $g(x)$.



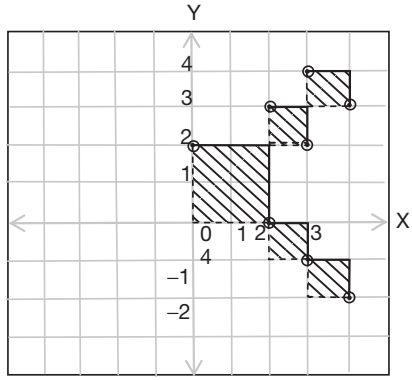

(A) $f(x) = |g(-x)|$ (B) $g(x) = |f(x)|$
(C) $f(x) = -(g(-x))$ (D) $g(x) = |f(-x)|$

**13.** Which of the following relations between $x$ and $y$, for $2 < |x| \le 4$, represents the given graph?




(A) $\left\lfloor \left| \frac{x}{2} \right| \right\rfloor + \left\lceil \frac{y-1}{2} \right\rceil = 3$ (B) $\left\lceil \left| \frac{x}{2} \right| \right\rceil + \left\lceil \frac{|y-1|}{2} \right\rceil = 3$
(C) $\left\lfloor \left| \frac{x}{2} \right| \right\rfloor + \left\lceil \frac{y-1}{2} \right\rceil = 3$ (D) $\left\lceil \left\| \frac{x}{2} \right\| \right\rceil + \left\lceil \left\| \frac{y-1}{2} \right\| \right\rceil = 3$

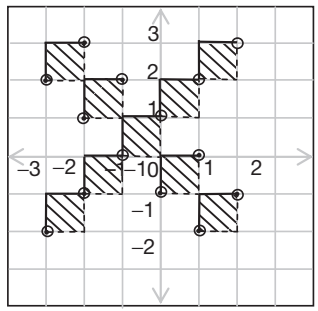
**14.** If $\lceil x \rceil = a$ and $\lceil y \rceil = b$, which of the following relations between $x$ and $y$, for $0 \le x \le 4$, represents the graph shown below?




(A) $a(a+2) = b(b+2)$ (B) $a(a+3) = b(b+3)$
(C) $a(a-2) = b(b-2)$ (D) $a(a-3) = b(b-3)$

**15.** Which of the following relations between $x$ and $y$, for $-3 \le x \le 2$, represents the graph shown below?

$p = \lfloor x \rfloor$, $q = \lceil y \rceil$



(A) $p^2 - 2p = q^2 + 2q$ (B) $p^2 + 2p = q^2 - 2q$
(C) $p^2 + 2p = q^2 + 2q$ (D) $p^2 - 2p = q^2 - 2q$

## ANSWER KEYS

### Exercise-1

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (C) | 7. (B) | 13. (C) | 19. (D) | 25. 1 | 31. (B) | 37. (A) |
| 2. (B) | 8. (C) | 14. 0 | 20. (B) | 26. (C) | 32. (B) | 38. (B) |
| 3. (A) | 9. (B) | 15. (C) | 21. (B) | 27. (D) | 33. (A) | 39. (A) |
| 4. (A) | 10. (D) | 16. (B) | 22. (B) | 28. (D) | 34. (D) | 40. (B) |
| 5. (B) | 11. (B) | 17. (C) | 23. (C) | 29. (C) | 35. (C) | |
| 6. (B) | 12. (D) | 18. (A) | 24. (A) | 30. (C) | 36. (A) | |

### Exercise-2

1–5. For answers to questions 1–5 please refer the solutions of exercise 2 on page 4.253 & 4.254.

| | | | | | | |
|---|---|---|---|---|---|---|
| 6. (C) | 11. (C) | 16. 12 | 21. (C) | 26. (D) | 31. 2 | 36. (D) |
| 7. (A) | 12. (C) | 17. (A) | 22. (A) | 27. (B) | 32. 2.5, 1.5 | 37. (D) |
| 8. (B) | 13. (C) | 18. (A) | 23. (B) | 28. (D) | 33. (B) | 38. (C) |
| 9. (A) | 14. (B) | 19. (B) | 24. (D) | 29. (A) | 34. 1.5 | 39. (B) |
| 10. (C) | 15. (C) | 20. 5 | 25. (A) | 30. (C) | 35. (C) | 40. (A) |

## Exercise-3

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (B) | 7. (C) | 13. 9 | 19. (B) | 25. (C) | 31. (B) | 37. (D) |
| 2. (C) | 8. (D) | 14. (C) | 20. 16 | 26. (A) | 32. (D) | 38. (C) |
| 3. (D) | 9. (D) | 15. (D) | 21. 64 | 27. (D) | 33. (C) | 39. (B) |
| 4. (B) | 10. (D) | 16. (C) | 22. (B) | 28. (D) | 34. (A) | 40. (B) |
| 5. (A) | 11. (C) | 17. 48 | 23. (B) | 29. (C) | 35. (B) | |
| 6. (C) | 12. 18 | 18. (B) | 24. (D) | 30. (A) | 36. (C) | |

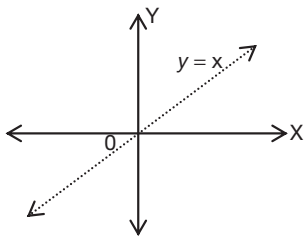
## Exercise-4

| | | | | |
|---|---|---|---|---|
| 1. –2 | 4. (D) | 7. (C) | 10. (B) | 13. (C) |
| 2. (D) | 5. (B) | 8. (D) | 11. (D) | 14. (D) |
| 3. 40 | 6. (C) | 9. (B) | 12. (B) | 15. (B) |

## SOLUTIONS

## EXERCISE-1

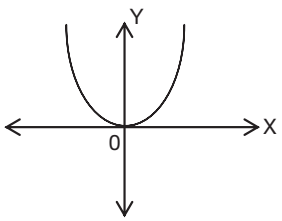
**1.** The graph of the function $y = x$ is:




Hence, the graph of $y = x$ lies in the I and III quadrants.

**2.** Put $y = 0$ in the equation $x + y - 1 = 0$.
Then, $x + 0 - 1 = 0$
$\Rightarrow x = 1$
$\therefore$ The curve $x + y - 1 = 0$ meets the $x$-axis at the point $(1, 0)$.

**3.** The graph of $y = x^2$ is :

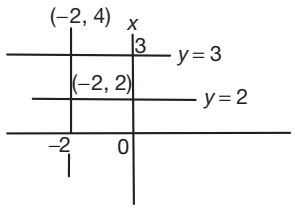



Hence, the graph of $y = x^2$ belongs to I and II quadrants.

**4.** Put $x = 0$ in the equation $x^2 + y^2 = 1$.
Then, $0 + y^2 = 1 \Rightarrow y^2 = 1$
$\Rightarrow y = \pm 1$
$\therefore$ The graph of the equation $x^2 + y^2 = 1$ meets the $y$-axis at the points $(0, -1)$ and $(0, 1)$.

**5.** The given curves are:
$y = 2x + 1$ (1)
and $y = x^2 - 2$ (2)
Solving (1) and (2), we get
$x^2 - 2 = 2x + 1$
$\Rightarrow x^2 - 2x - 3 = 0$
$\Rightarrow (x - 3)(x + 1) = 0$
$\Rightarrow x = 3$ or $x = -1$
$\therefore y = 7$ or $y = -1$
Hence, the required points of intersection are $(3, 7)$ and $(-1, -1)$

**6.** The reflection of the point $(-2, 4)$ in the line $y = 3$ is obtained as follows.




The required point is 1 unit vertically below $y = 3$, i.e., $y = 2$
$\therefore$ The required point is $(-2, 2)$.

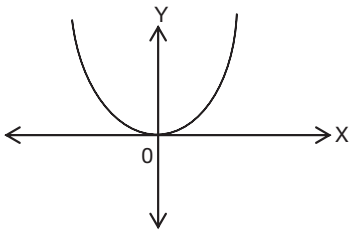
**7.** When a point is reflected in the origin, then the sign of both the coordinates of the point changes.
$\therefore$ Reflection of $(x_1, y_1)$ in the $y$-axis is $(-x_1, -y_1)$.

**8.** When a point is reflected in the $y$-axis the sign of the $x$-coordinate changes.
$\therefore$ Reflection of $(x_1, y_1)$ in $y$-axis is $(-x_1, y_1)$.

**9.** If any point is reflected in the origin, the signs of both the coordinates change.
$\therefore$ Reflection of $(2, -1)$ in origin becomes $(-2, 1)$.

**10.** $(-a, 0)$ lies on the $x$-axis. Every point on the $x$-axis reflects into itself, when reflected in $x$-axis.

**11.** Graph of the curve $x^2 = y$ is:




Hence, the curve $x^2 = y$ is symmetric about the $y$-axis.

**12.** $ac = 0 \Rightarrow a = 0$ or $c = 0$

Case (i): If $a = 0$ and $c \neq 0$, then $ax + by + c = 0$

$\Rightarrow by + c = 0$ which represents a horizontal line.

Case (ii): If $a \neq 0$ and $c = 0$, then $ax + by + c = 0$

$\Rightarrow ax + by = 0$

which represents a line passing through the origin

Case (iii): If $a = 0$ and $c = 0$, then $ax + by + c = 0$

$\Rightarrow by = 0$

$\Rightarrow y = 0$ ($\because b \neq 0$), which represents the $x$-axis.

Hence, $ax + by + c = 0$, $ac = 0$ represents either a horizontal line or an inclined line.

**13.** Given, $a^2 + b = 0$ and $a \neq 0$

$\Rightarrow a \neq 0$ and $b \neq 0$

Hence, the equation $ax + by + c = 0$ represents an inclined line. (i.e., a line that in not parallel to either axis)

**14.** Given curves are:

$y = x - 1$ (1)

and $x^2 = 2y$ (2)

Solving (1) and (2), we get

$x^2 = 2(x - 1)$

$\Rightarrow x^2 - 2x + 2 = 0$

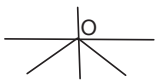
$\Rightarrow (x - 1)^2 + 1 = 0$

For no real value of $x$, is $(x - 1)^2 + 1 = 0$

Hence, the curves $y = x - 1$ and $x^2 = 2y$ do not intersect.

**15.** The point (1, 0) alone satisfies the equation $x^2 + y^2 - 2x + 2y + 1 = 0$

**16.** The graph of $y = -|x|$ is as follows.

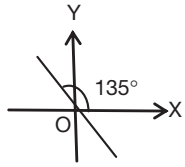



This is symmetric w.r.t. the $y$-axis.

**17.** The line $2x - 3y = 0$ is of the form $y = mx$, which passes through the origin.

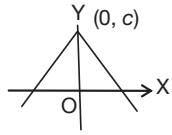
**18.** $y - 7 = 0$ represents a line parallel to the $x$-axis

**19.** $y + x = 0 \Rightarrow y = -x$. Here, $m = -1 \Rightarrow \tan\theta = -1$




$\Rightarrow \theta = 135°$. The given line lies in $Q_2$ and $Q_4$.

**20.** $y = -|x| + c$. The graph is obtained by making a vertical shift of the graph $y = -|x|$ by $c$ units.




This is symmetric, w.r.t., the $y$-axis.

**21.** Given $y < x + 2$ (1)

Consider the graph $y = x + 2$

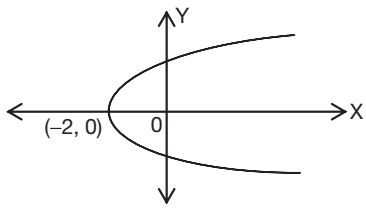
| $x$ | 0 | –2 | 1 | –1 |
|---|---|---|---|---|
| $y$ | 2 | 0 | 3 | 1 |

Plotting these points, we get the line shown in the graph. As (0, 0) Satisfies the inequality (1), (1) represents the shaded region in the graph.

**22.** The given equation is in the form of $x = k$ which represents a line parallel to the $y$-axis.

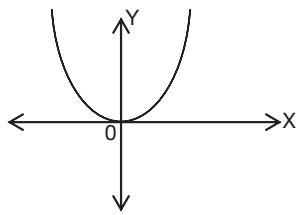
**23.** The equation $2x + 3y = 0$ is satisfied by (0, 0). This represents a line that passes through the origin.

**24.** Graph of the curve $x = y^2 - 2$ is:



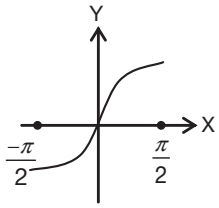

Hence, the curve $x = y^2 - 2$ is symmetric about the $x$-axis.

**25.** Graph of the curve $y = x^2$ is:




Hence, a line parallel to $y$-axis meets the graph at exactly one point.

**26.** $y = \sin x$ graph in the interval $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is as follows.

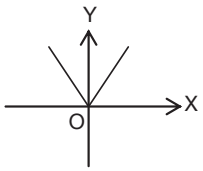



**27.** (2, 0) and (0, –2) lie on the graph.

$\therefore$ Equation of the line joining them is given by $\frac{x}{2} + \frac{y}{-2} = 1$

$\Rightarrow x - y = 2$. Since origin side is shaded when the coordinates of the origin are substituted, it must result in a true statement. The required relation is $x - y \le 2$.

**28.** $y = |x|$ is given by




**29.** The graph of $y = f(x - 3)$ is obtained by shifting the graph of $y = f(x)$ to the right (horizontally) to 3 units.

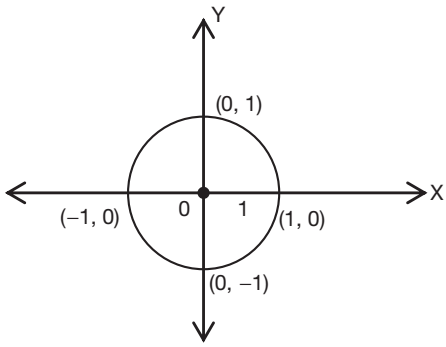
**30.** The given function is $y = f(x)$. The graph of $y = f(x) + c$, $c > 0$ is obtained by moving the graph $y = f(x)$ vertically above for $c$ units

**31.** The graph represents the equation $y = |x - 1|$

**32.** The graph of $y = mx + c$ meets the $y$-axis at (0, c)

**33.** $y = -|x|$

**34.** Graph of the curve $x^2 + y^2 = 1$ is:




Hence, a line parallel to the $x$-axis meets the graph at exactly either one or two points.

**35.** Choice (C) is the graph of $x \ge -3$.

**36.** Interchange of $x$ and $y$ in $y = f(x)$ changes the symmetry of the curve. As the given curve $y = |x|$ is symmetric, w.r.t., the $y$-axis $x = |y|$ is symmetric, w.r.t., the $x$-axis.

**37.** A change of sign of $x$ reflects the graph in the $y$-axis.

**38.** If $f(x)$ is the given graph then the graph $f(x) - 2$ is obtained by moving it down (vertically) by 2 units.

**39.** If $f(x)$ is a given graph, then $f(x - a)$ is obtained by shifting the graph of $f(x)$ to the right (horizontally) by a units.

**40.** $y = -\log x$ when $x = 1$, $y = 0$

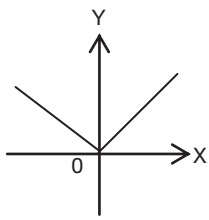
$\therefore$ The graph crosses the $x$-axis at (1, 0).

## EXERCISE-2

**1. (a)** $y = |x|$. This graph entirely lies in the first and second quadrants. If $\neq 0$, $y$ is positive. When $x = 0$, $y = 0$.

$\therefore$ The graph passes through the origin.

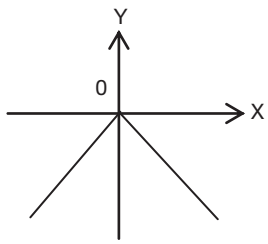
$\therefore$ The required graph is




**(b)** $y = -|x|$. This graph is symmetric with respect to the $y$-axis, it passes through the origin and $y$ is always negative.

**NOTE**

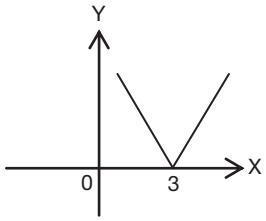
It is obtained by reflecting the graph of $y = |x|$ in the $x$-axis.




**2. (a)** $y = |x - 3|$ which is in the form of $y = f(x - a)$ where $f(x) = |x|$

$\therefore$ The graph of $f(x - 3)$ is obtained by shifting the graph of $y = |x|$ horizontally 3 units towards the right.

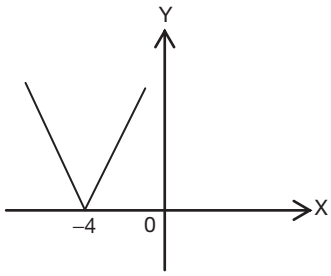
∴ The required graph is




**(b)** $y = |x + 4|$ which is in the form of $y = f(x + a)$ where $f(x) = |x|$

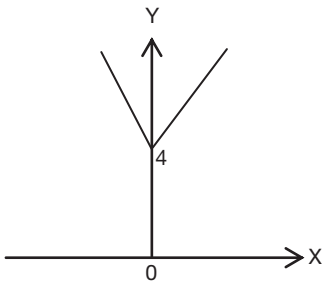
∴ The graph of $f(x + 4)$ is obtained by shifting the graph of $f(x)$ horizontally 4 units towards left.

∴ The required graph is

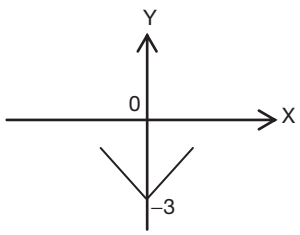



**3. (a)** $y = |x| + 4$ is in the form of $y = f(x) + c$. The graph of $y$ is $y = |x| + 4$ is obtained by shifting the graph $y = f(x)$ vertically above by $c$ units

∴ The required graph of $y = |x| + 4$ is



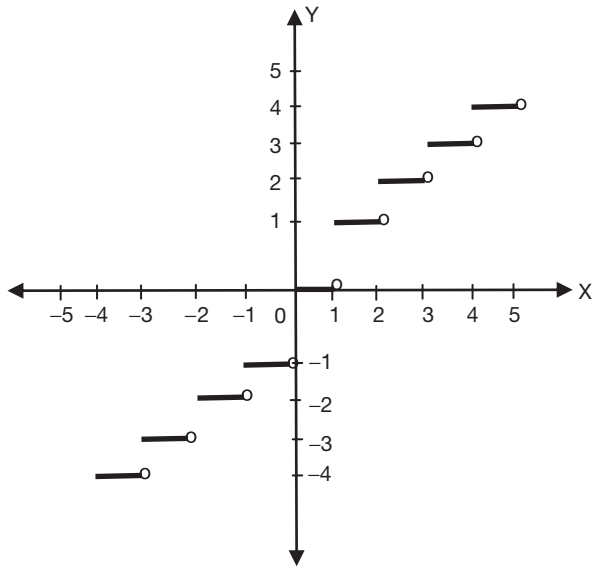

**(b)** The graph of $y = |x|. -3$ is obtained by shifting the graph of $y = |x|$ vertically down by 3 units. i.e.,




**4.** $f(x) = [x]$ is called the greatest integer function (Step Function):

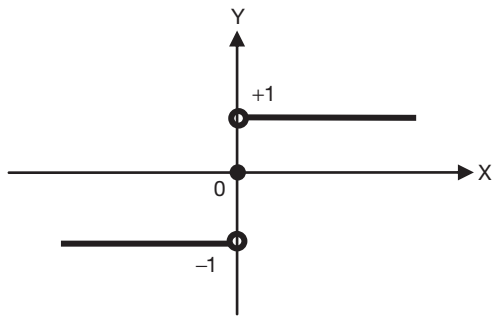
$f(x) = [x]$ is the greatest integer less than or equal to $x$.

$f(x) = x \forall x \in z$ and $f(x) =$ the greatest integer $\leq x$ for all non-integers. The graph of $f(x) = [x]$ is as follows.

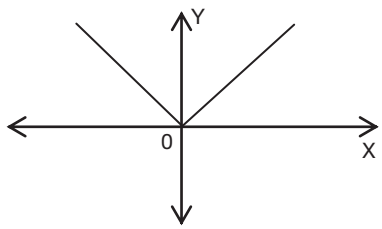



**5.** $f(x) = \frac{|x|}{x}, \forall x \neq 0, = 0$ for $x = 0$ is called signum function:

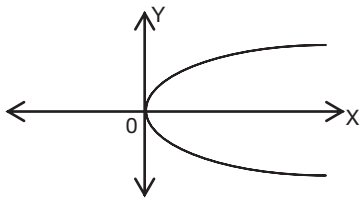
The function $f$ defined by $sg(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$. Domain of $f$ = R, Range = {–1, 0, 1}. The graph of $f(x)$ is as follows.



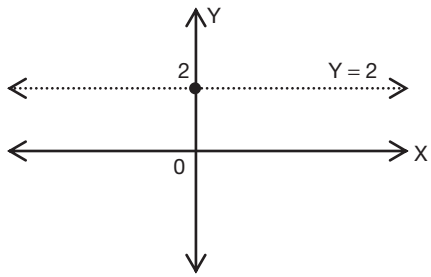

**6.** Graph of the equation $y = |x|$ is:



**7.** The graph of the equation $y^2 = x$ is:




**8.** Graph of the equation $y = 2$ is:




**9.** The graph $f(x)$ is shifted towards right by 3 units and the graph $g(x)$ is obtained
$\therefore\ g(x) = f(x-3)$

**10.** The given points satisfy the relation $y = \dfrac{|x-3|+|x-5|}{2}$

Therefore, the function which represents the figure given is $y = \dfrac{|x-3|+|x-5|}{2}$

**11.** Substituting the points in the given graph in all the options. We notice only choice C satisfies.

**12.** The graph is in third quadrant, so $x < 0$ and $y < 0$. This is possible with option (C) alone.
Option (A) admits $x > 0$, option (B) admits $y > 0$
Option (D) admits $y > 0$

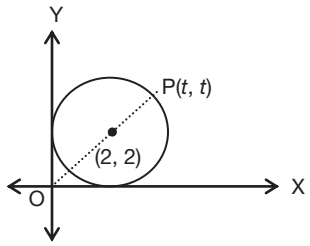
**13.** The graph can be understood in 3 cases
Case (i) $x \geq 1$, $y = 1$
Case (ii) $-1 \leq x \leq 1$, $y = x$
Case (iii) $x \leq -1$ $y = -1$
consider Option (C),

for $x \geq 1$; $y = \dfrac{(x+1)-(x-1)}{2} = 1$;

$-1 \leq x \leq 1$; $y = \dfrac{(x+1)+(x-1)}{2} = x$; and

$x \leq -1$; $y = \dfrac{(-1-x)-(1-x)}{2} = -1$;

**14.** The graph represents a circle centred at (2, 2) with radius as 2 units. The farthest point on the circle should lie on the line $y = x$. Let P$(t, t)$, be that point.
$(t-2)^2 + (t-2)^2 = 4$ (using distance formula)

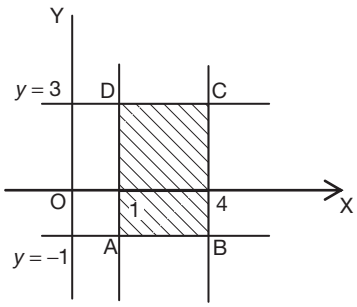



$\Rightarrow (t-2)^2 = 2 \Rightarrow t = 2 \pm \sqrt{2}$ as we want the farthest point on the circle it is $\left(2+\sqrt{2}, 2+\sqrt{2}\right)$

**15.** Basically, we have the set of rhombuses
$|x| + |y| = 1$,
$|x| + |y| = 2$ and so on.
Thus, $|x| + |y| = k$, where $k$ is a positive integer.

**16.** $1 \leq x \leq 4$ would represent an infinite vertical band in the first quadrant and similarly $0 \leq |y-1| \leq 2$.
or $-2 \leq y - 1 \leq 2$
$-1 \leq y \leq 3$ represents an infinite horizontal band.

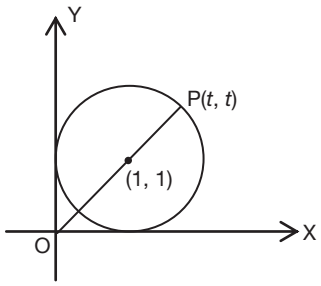
The intersection is shown as the shaded region in the figure below.




The area of the shaded region is = AB × BC = 3 × 4 = 12 units.

**17.** The graph represents a circle centred at (1, 1) with radius as 1 unit. The farthest point on the circle should lie on the line $y = x$.

Let P$(t, t)$ be that point.



$(t-1)^2+(t-1)^2=1$ (Using distance formula).

$(t-1)^2=\frac{1}{2}$

$t=\left(1+\frac{1}{\sqrt{2}}\right)\text{or}\left(1-\frac{1}{\sqrt{2}}\right)$

As we want the farthest point on the circle, it is

$\left(1+\frac{1}{\sqrt{2}},1+\frac{1}{\sqrt{2}}\right)$

**18.** First, we will simplify $2\le|x-3|\le 3$

Case (i) $(x>3)$

$2\le x-3\le 3$

$5\le x\le 6$

Case (ii) $(x<3)$

$2\le 3-x\le 3$

$-2\ge x-3\ge -3$

$\Rightarrow 1\ge x\ge 0$

$\Rightarrow 0\le x\le 1$

Similarly, we simplify $1\le|y-2|\le 3$

Case (i) $(y>2)$

$1\le y-2\le 3$

$3\le y\le 5$

Case (ii) $(y<2)$

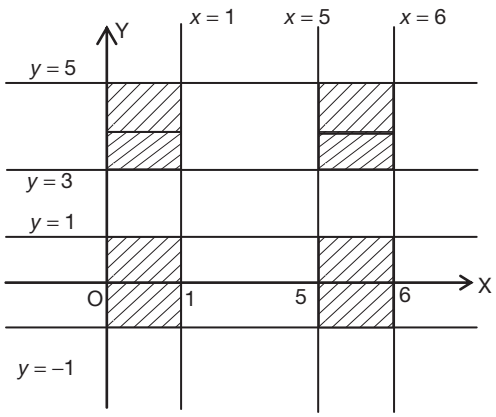
$1\le 2-y\le 3$

$-1\ge y-2\ge -3$

$1\ge y\ge -1$

$\Rightarrow -1\le y\le 1.$

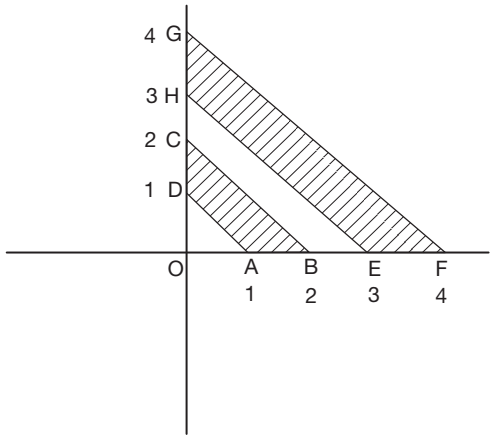
Now, observing the graph obtained




Area enclosed, is equal to the area of the shaded regions, i.e., $2+2+2+2=8$

**19.** $g(x)$ is obtained from $f(x)$ by translating the graph horizontally left by 4 units and vertically (down) by 3 units.

**20.**




Area of the shaded region = Area of ABCD + Area of EFGH.

Area of ABCD = Area of $\Delta OBC$ – Area of $\Delta OAD$.

$=\frac{(2\times 2)}{2}-\frac{(1\times 1)}{2}=\frac{3}{2}$

Similarly, Area of EFGH $=8-\frac{9}{2}=\frac{7}{2}$

Total area of shaded region = 5 sq. units.

**21.** The graph $g(x)$ is obtained from that of $f(x)$ by two successive reflections in $x$ and $y$-axis in either order.

$\therefore f(x)=-g(-x)$

**22.** The graph of $f(x)$ can be obtained by reflecting the graph of $g(x)$ in the $y$-axis.

$\therefore f(x)=g(-x)$

**23.** The graph of $f(x)$ can be obtained by reflecting the graph of $g(x)$ in the $x$-axis.

$\therefore f(x)=-g(x)$

**24.** Area of a trapezium

$=\frac{1}{2}$ (sum of parallel sides) × distance between them

Let us say we require $n$ such trapeziums are required.

Accordingly,

$\frac{1}{2}[(2+1)1]+\frac{1}{2}[(3+4)1]+\frac{1}{2}[2n+(2n-1)1]\ge 100$

$\Rightarrow \frac{1}{2}[3+7+11+\ldots+(4n-1)]\ge 100$

$\Rightarrow \frac{1}{2}\left\{\frac{n}{2}[3+(4n-1)]\right\}\ge 100$

$\Rightarrow n[2n+1]\ge 200$

We guess that $n=10$.

So, a minimum of 10 such trapeziums are required.

**25.** The graph of $g(x)$ can be obtained by shifting the graph of $f(x)$ to the left by 2 units.

$\Rightarrow g(x)=f(x+2)$

**26.** We can notice that the graph $g(x)$ can be obtained from that of $f(x)$ by reflecting it in the in $y$-axis and then in the $x$-axis.
$\therefore$ It is a double reflection
i.e., $f(x) = -g(-x)$

**27.** Both options (B) and (C) are satisfied by the point (2, 0)
But, the graph is not defined for $x \le 1$.
So, $y = \log_e (x-1)$ is the required relation.

**28.** The reflection of the point $(x, y)$ in the origin is $(-x_1, -y_1)$
$\therefore$ The reflection of $(-1, 4)$ in the origin is $(1, -4)$.

**29.** The figure given is a graph of discontinuous function.
According to the figure,
$f(x) = 0 \ \forall \ x \in [0, 1)$
$f(x) = 1 \ \forall \ x \in [1, 2)$ and so on.
Clearly, this represents the function $f(x) = [x]$.

**30.** The graph represents the function $y = |\log x|$

**31.** The roots are 1 and 3. Both roots are positive.

**32.** $\left(x^2 + \frac{1}{x^2}\right) = \left(x + \frac{1}{x}\right)^2 - 2$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 - 4\left(x + \frac{1}{x}\right) + \frac{15}{4} = 0$

Let $y = \left(x + \frac{1}{x}\right)$

$y^2 - 4y + \frac{15}{4} = 0$

$4y^2 - 16y + 15 = 0$
$4y^2 - 10y - 6y + 15 = 0$
$2y\,(2y-5) - 3(2y-5) = 0$
$(2y-3)\,(2y-5) = 0$

$y = \left(x + \frac{1}{x}\right) = \frac{3}{2} = \frac{5}{2}$

**33.** The new roots are

$1 + \frac{\alpha}{\beta} = \frac{\alpha+\beta}{\beta}$ and $1 + \frac{\beta}{\alpha} = \frac{\alpha+\beta}{\alpha}$

Sum of roots $= \frac{\alpha+\beta}{\alpha} + \frac{\alpha+\beta}{\beta} = \frac{16}{3}$

Product of roots $= \left(\frac{\alpha+\beta}{\alpha}\right)\left(\frac{\alpha+\beta}{\beta}\right) = \frac{16}{3}$

The required equation is

$x^2 - \frac{16x}{3} + \frac{16}{3} = 0$

$3x^2 - 16x + 16 = 0$

**34.** Given $\alpha, \beta, \gamma$ are the roots of the equation

$x^3 - 4x^2 + 6x - 4 = 0$
$\alpha + \beta + \gamma = 4$
$\alpha\beta + \beta\gamma + \alpha\gamma = 6$
$\alpha\beta\gamma = 4$

$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \alpha\gamma}{\alpha\beta\gamma} = \frac{6}{4} = \frac{3}{2}$

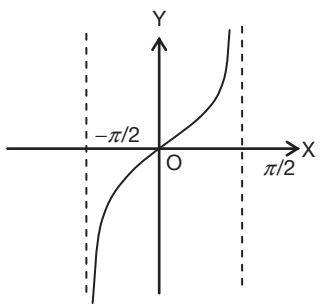
**35.** We know that dividend = [(Divisor × Quotient)] + Remainder
$f(x) = (x^2 + 3x - 2)\ (2x + 3) + 4$
$= 2x^3 + 9x^2 + 5x - 2.$

**36.** Let $f(x) = x^4 - px^3 + 2x^2 + 2x + q$, given $-1$, and 2 are roots of $f(x)$
$\Rightarrow f(-1) = 0$ and $f(2) = 0$.
$f(-1) = 0 \Rightarrow (-1)^4 - p(-1)^3 + 2(-1)^2 + 2(-1) + q = 0$
$p + q = -1 \quad (1)$
$f(2) = 0 \Rightarrow (2)^4 - p(2)^3 + 2(2)^2$
$-8p + q = -28 \quad (2)$
Solving (1) and (2), we get $p = 3$, $q = -4$
$\therefore$ The given equation is $x^4 - 3x^3 + 2x^2 + 2x - 4 = 0$

| | | | | | |
|---|---|---|---|---|---|
| $-1$ | 1 | $-3$ | 2 | $-2$ | $-4$ |
| | 0 | $-1$ | 4 | $-6$ | 4 |
| 2 | 1 | $-4$ | 6 | $-4$ | |
| | 0 | $-2$ | $-4$ | 4 | |
| | 1 | $-2$ | 2 | 0 | |

$\therefore$ The quotient is $x^2 - 2x + 2 = 0$
$b^2 - 4ac < 0$
$\therefore$ The other two roots are imaginary and they are
$\frac{2 \pm \sqrt{4-8}}{2} = 1 \pm i$

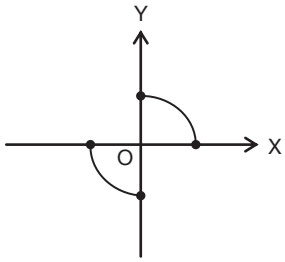
**37.** The graph of $y = \tan x$ in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ is shown below.




No horizontal line and no vertical line intersecting the graph more than once can be drawn.

**38.** A set of points equidistant from the origin represents a circle but given $xy \ge 0 \Rightarrow$ both $x$, $y$ should have the same sign.

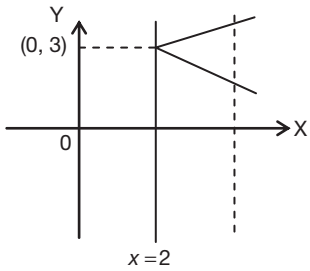
∴ The graph as per the given conditions is shown below. Both the axes intersect the graph at more than one point.




Both horizontal and vertical lines intersect the graph more than once.

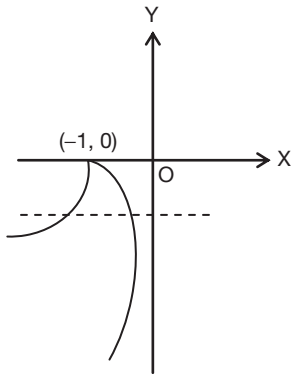
**39.** Given: $x = |y - 3| + 2$

The graph of the above equation is shown in the figure below.




A vertical line intersects the graph more than once.

**40.** The graph of $y = -|\log(-x)|$ is shown in figure.




The horizontal line intersecting the graph more than once exists.

## EXERCISE-3

**1.** Let us substitute the value of $x = 0.1$ in all the choices.

Choice (A) $\Rightarrow |\log_{10} 0.1| = |-1| = 1$

∴Choice (A) is rejected as we need 0.9 as the answer.

Choice (C) $\Rightarrow |\log_{10} 0.1| + 0.1 = |-1| + 0.1$

$= 1.1$. ∴ Choice (C) is also rejected.

Choice (B) $\Rightarrow |(\log_{10} 0.1) + 0.1| = |-1 + 0.1| = |-0.9| = 0.9$.

For all other values of $x$ also, it satisfies the graph.

**2.** Let us consider each choice

Choice (A) $\Rightarrow |\sin 2x|$ For $x = 0 \Rightarrow |\sin 0| = 0$

Hence, Choice (A) is rejected.

Choice (B) $\Rightarrow |\cos 2x|$

For $x = \frac{\pi}{4} \Rightarrow \left|\cos \frac{\pi}{2}\right| = 0$

Hence, Choice (B) is rejected.

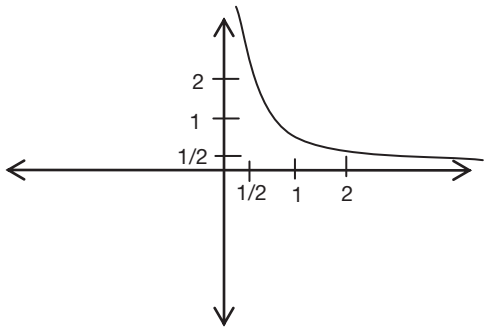
Choice (D) $\Rightarrow \sqrt{2}(\sin x + \cos x)$

For $x = \frac{\pi}{4} \Rightarrow \sqrt{2} \cdot \sin \frac{\pi}{4} + \cos \frac{\pi}{4}$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \sqrt{2}\left(\frac{2}{\sqrt{2}}\right) = 2.$

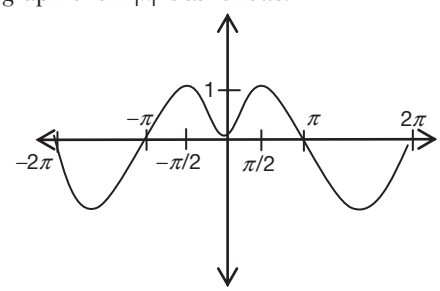
Hence, Choice (D) is rejected.

**3.** The graph of $xy = 1$, where $x > 0$, $y > 0$ looks as follows.




It is obvious that no horizontal or vertical line can cut the graph more than once.

**4.** The graph of $\sin|x|$ is as follows:



It is obvious here, that there exist horizontal lines that cut the graph more than once.

**5.** Let us check each choice one by one.

Choice (A) $\Rightarrow \dfrac{|x+2|+|x-2|}{2}$.

If $x \le -2$, $\dfrac{(-2+x)-(x-2)}{2} = \dfrac{-2-x-x+2}{2} = -x$.

If $-2 < x < 2$, $\dfrac{x+2-(x-2)}{2} = 2$

If $x \ge 2$, $\dfrac{x+2-(x-2)}{2} = x$.

The set of equations is

$$y = \begin{cases} -x \text{ if } & x \le -2 \\ 2 \text{ if } & -2 < x < 2 \\ x \text{ if } & x \ge 2 \end{cases}$$

This satisfies the graph. Hence, Choice (A) required answer.

**6.** Both options (2) and (3) satisfy the point (2, 0).
But $\log_e(x-1)$ is not defined for $x < 1$.

So, $y = \log_{0.5}\left(\dfrac{x}{2}\right)$ is the required relation.

**7.** The graph represents $x = -(|y| + 2)$

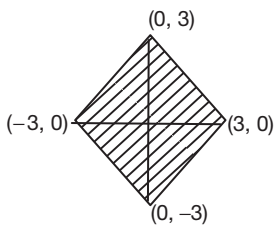
**8.** The shaded region lies between the lines $x = \pm 1$ and $y = \pm 1$.
Hence, $-1 \le x \le 1$ and $-1 \le y \le 1$ is the required relation.

**9.** From the given choices, the relation $y = |\log(-x)|$, $x < 0$ best describes the given graph.

**10.** The relation $|x|+|y|=2$ represents four lines $x+y=2$, $-x+y=2$, $x-y=2$ and $-x-y=2$
These four lines form a square with vertices (2, 0), (0, 2), (–2, 0) and (0, –2).
Hence, the shaded region is described by the relation $|x|+|y| \le 2$.

**11.** Only choice (C) satisfies all the three given conditions.

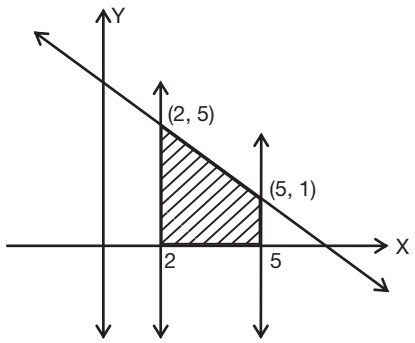
**12.**




The graph of $|x| + |y| \le 3$ is shown above.

$\therefore$ Area of the region is $\dfrac{1}{2}(6)(6) = 18$

**13.**




The figure formed is a trapezium whose area is $\dfrac{1}{2}h(a+b)$, where $h$ is the height between parallel sides and $a$ and $b$ are lengths of the parallel sides.
$h$ = Distance between parallel sides = 3
[$\because$ Distance between $x = 2$ and $x = 5$ is 3 units]
To find $a$, $b$, we find where $4x + 3y = 23$ meets with $x = 2$, $x = 5$
It meets $x = 2$ at $3y = 23 - 8 = 15 \Rightarrow y = 5$
$\therefore$ At (2, 5)
It meets $x = 5$ at $3y = 23 - 20 = 3$
$\Rightarrow y = 1$
$\therefore$ At (5, 1) Hence, $a = 5$, $b = 1$

$\therefore$ Area $= \dfrac{1}{2} \times 3(5+1) = 3 \times 3 = 9$ square units.

**14.** The graph is such that $y = 0$ at places where $\sin x$ is positive and is double the height of the $-\sin x$ curve in places where $\sin x$ is negative.
i.e., $y = 0$ if $x \in [2n\pi, (2n+1)\pi]$
if $x \in [(2n-1)\pi, 2n\pi]$
$y = -2\sin x$
If $x \in [2n\pi, (2n+1)\pi]$, $|\sin x| = \sin x$
and if $x \in [(2n-1)\pi, 2n\pi]$, $|\sin x| = -\sin x$.
$\therefore$ The function is $|\sin x| - \sin x$.
So that, if $x \in [2n\pi, (2n+1)\pi]$,
$|\sin x| - \sin x = \sin x - \sin x = 0$
$x \in [(2n-1)\pi, 2n\pi]$, $|\sin x| - \sin x = -2\sin x$.

**15.** Given graph is such that the value of $y$ increases from 0 to 0.5 when the value of $x$ increases from $n$ to $n+1$, where $n \in z$. Let us substitute $x = -0.5$ in all the choices.
Choice (A) $\Rightarrow s = -0{\cdot}25$

Choice (B) $\Rightarrow \dfrac{\lfloor -0.5 \rfloor - 0.5}{2} = \dfrac{-1-0.5}{2} = -0{\cdot}75$

Choice (C) $\Rightarrow \dfrac{\lceil -0.5 \rceil - (0.5)}{2} = \dfrac{0+0.5}{2} = 0.25$

Choice (D) $\Rightarrow \dfrac{-0.5 - \lfloor -0.5 \rfloor}{2} = \dfrac{-0.5+1}{2} = 0.25$

To decide between Choice (C) and (D) let us take $x = 0{\cdot}9$.

Choice (C) $\Rightarrow \dfrac{\lceil 0.9 \rceil - 0.9}{2} = \dfrac{1 - 0.9}{2} = 0.05.$

Choice (C) is rejected.

**16.** $\tan\theta = \dfrac{x-y}{x+y}$

$\tan 30° = \dfrac{x-y}{x+y} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{x-y}{x+y}$

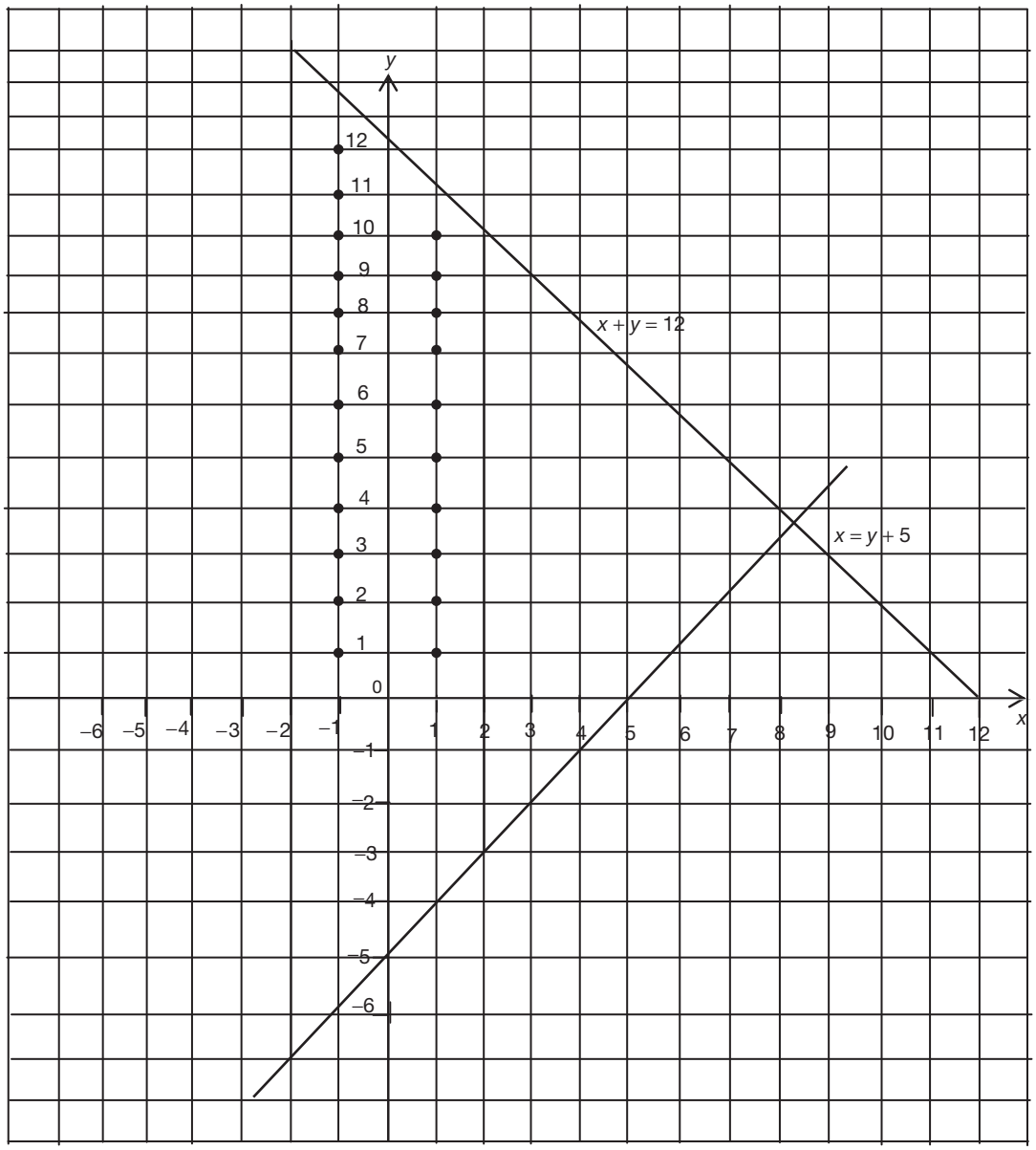
$\Rightarrow \sqrt{3} = \dfrac{x+y}{x-y}$

$\Rightarrow \dfrac{\sqrt{3}+1}{\sqrt{3}-1} = \dfrac{x}{y} \Rightarrow \dfrac{y}{x} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

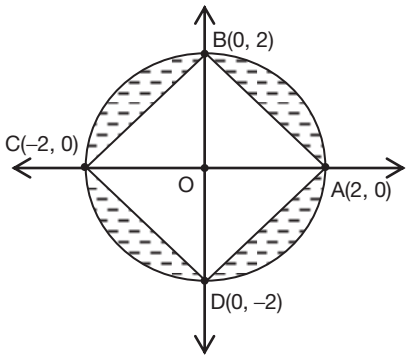
$\dfrac{y}{x} = \tan 15°$

$\therefore$ The given line appears as shown in choice C, when viewed wrt $x$, $y$ axes.

**17.** The given lines are $x = +2$, $x = -2$, $x + y = 12$ and $x - y = 5$. We see that we have a total of 48 points, lying inside the region bounded by the given lines, which have integral coordinates.

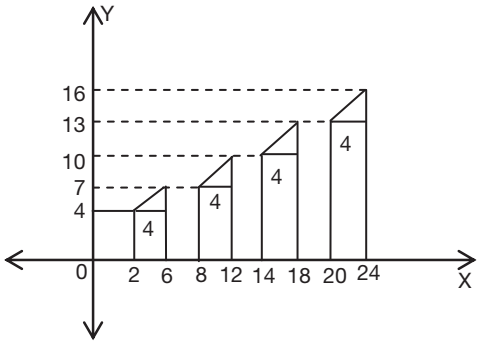


**18.** The relation $|x| + |y| = 2$ represents a rhombus and the relation $x^2 + y^2 = 4$ represents a circle.
The region described by the relations $|x| + |y| \geq 2$ and $x^2 + y^2 \leq 4$ is shaded



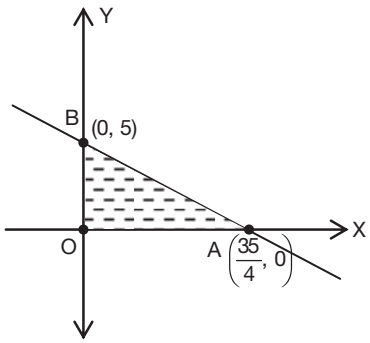

The shaded region is the required region.
$\therefore$ Required area = Area of the circle – Area of rhombus ABCD $= \pi(2)^2 - \frac{1}{2} 4 \times 4 = 4\pi - 8 = 4(\pi - 2)$ sq. units.

**19.**



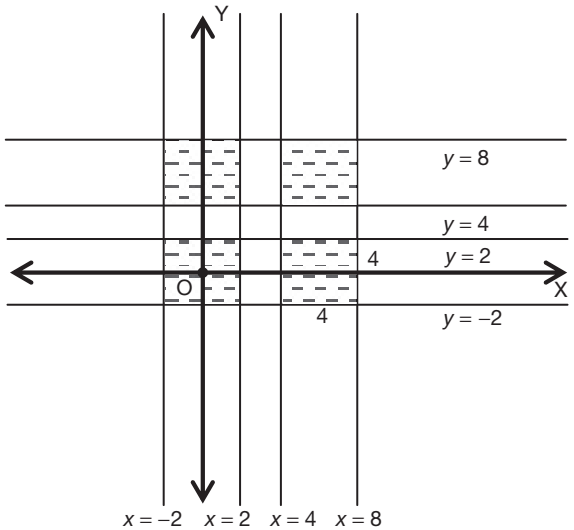

We know that, the area of trapezium = $\frac{1}{2} \times$ (sum of the lengths of parallel sides) $\times$ distance between the parallel sides.
$\therefore$ The required area = $\frac{1}{2} \times (4 + 7) \times 4 + \frac{1}{2} \times (7 + 10) \times 4 + \frac{1}{2} \times (10 + 13) \times 4 + \frac{1}{2} \times (13 + 16) \times 4$
$= 22 + 34 + 46 + 58 = 160$ sq. units.

**20.** The relations $x \geq 0$, $y \geq 0$ and $4x + 7y \leq 35$
Describe the required region OAB.




The possible values of $y$ satisfying the relation $4x + 7y \leq 35$ are:
$y = 1, 2, 3$ and $4$ ($\because$ for $y \geq 5$, $7y \geq 35$)
When $y = 1$, $\quad 4x \leq 28$
$\Rightarrow x \leq 7$
$\Rightarrow x = 1, 2, 3, \ldots 7$
When $y = 2$, $4x \leq 21$
$\Rightarrow x \leq \frac{21}{4}$
$\Rightarrow x = 1, 2, 3, 4, 5$
When $y = 3$, $4x \leq 14$
$\Rightarrow x \leq \frac{14}{4}$
$\Rightarrow x = 1, 2, 3$
When $y = 4$, $4x \leq 7$
$\Rightarrow x \leq \frac{7}{4}$
$\Rightarrow x = 1$
Hence, the required number of points is $7 + 5 + 3 + 1 = 16$.

**21.** $|x - 3| = 1 \Rightarrow x = 2, 4$
$|x - 3| = 5 \Rightarrow x = -2, 8$
$|y - 3| = 1 \Rightarrow y = 2, 4$
$|y - 3| = 5 \Rightarrow y = -2, 8$
$\therefore$ The region described by the relations $1 \leq |x - 3| \leq 5$ and $1 \leq |y - 3| \leq 5$ is



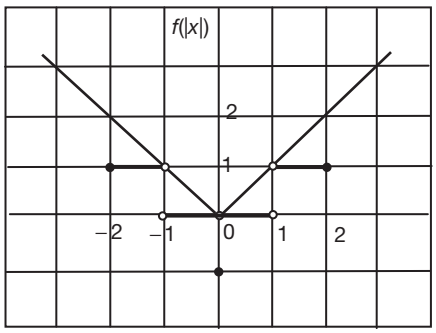

The shaded region is the required region. The region consists of 4 squares each of side 4 units.
Hence, the required area = $4(4 \times 4) = 64$ sq. units.

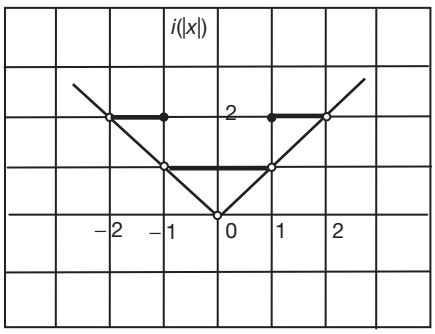
**22.** We note that in the graphs of $f(x)$ as well as $-f(x)$, the right end points are included. If we consider $f(-x)$ (or $-f(-x)$), we can verify that the left end points would be included.

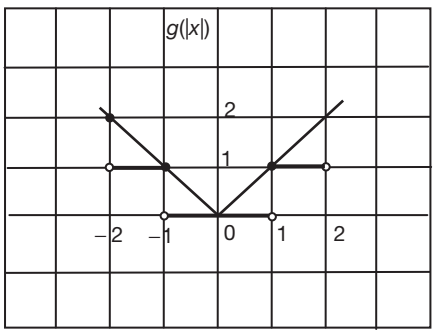
Similarly, in the graphs of $g(x)$ and $-g(x)$, the left end points are included, while in the graph of $g(-x)$ and $-g(-x)$, the right end points are included.

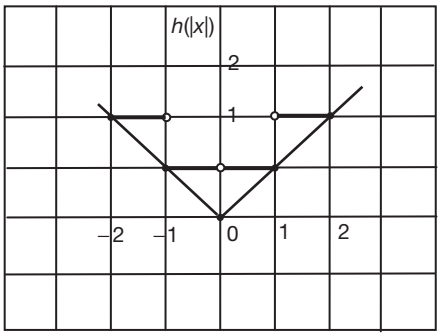
In the given graph, the left end points are included. Therefore, we can consider $f(-x)$ and $g(-x)$. The others do not describe the graph. While $f(-x)$ does describe the graph, $g(-x)$ does not. Therefore, only 1 function describes the graph.

**23.** From the options, we see that we need to have an idea of how the graphs of $f(|x|)$, $g(|x|)$, $h(|x|)$ and $i(|x|)$ look like. These are given below.

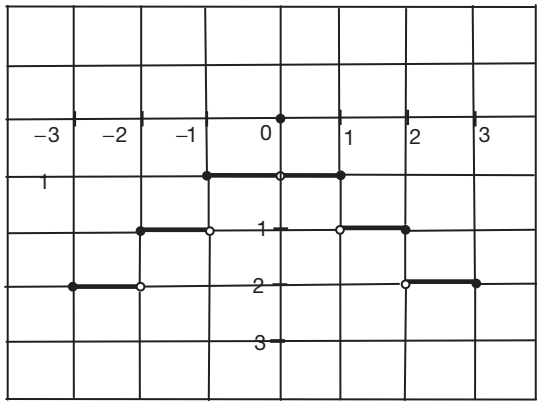



We can compare the graphs given in the question with that of $h(|x|)$. We can see that the given graph is that of $-h(|x|)$.




**24.** The points for which $y = 1$ and $0 < x \le 1$ are there on the graph. This is the graph of the least integer greater than or equal to $x$, i.e., of $h(x)$.

**25.** We can take 3 points—one in the interior, one on an included edge and one on an excluded edge. The first two should satisfy the relation, whereas the third one should not. These points. The corresponding values of the function that occur in the options are calculated below.

| | Point | $f(x)$ | $f(y)$ | $g(x)$ | $g(y)$ |
|---|---|---|---|---|---|
| **Internal** | (0.5, 0.5) | 0 | 0 | 0 | 0 |
| **Included edge** | (0.5, 0) | 0 | –1 | 0 | 0 |
| **Excluded edge** | (1, 0.5) | 0 | 0 | 1 | 0 |

We see that for (0.5, 0.5) $g(x) + g(y) = 0$
and for (0.5, 0) $g(x) + g(y) = 0$
While for (1, 0.5) $g(x) + g(y) \neq 0$
i.e., $g(x) + g(y) = 0$ describes the given graph and $f(x) + f(y) = 0$ does not describe because $f(1) + f(0.5) = 0 + 0 = 0$.

**26.** Clearly, we can observe that $f(x) = -g(-x)$

**27.** $g(x)$ can be obtained by reflecting $f(x)$ only in $x$-axis OR by reflecting $f(x)$, first in $y$-axis and then in $x$-axis.

**28.** $g(x)$ can be obtained from $f(x)$ in the following ways.
(i) Reflect the graph $f(x)$ in $x$-axis then in $y$-axis.
i.e., $f(x) = -g(-x)$
(ii) Reflect the graph $f(x)$ in $y$-axis.
$f(x) = g(-x)$.

**29.** $g(x)$ is a straight line passing through the origin and lying in the II$^{nd}$ and IV$^{th}$ quadrants.
$\therefore g(x) \equiv x + y = 0$.
The graph $f(x)$ is always positive.
$\therefore f(x) = |g(x)|$.

**30.** The graph $g(x)$ can be obtained only by reflecting the graph $f(x)$ in the $y$-axis.

$$f(x) = g(-x).$$

**31.** In the figure, in the shaded region $x$ ranges from 2 to 5 and $y$ is from 2 to 5.

$\therefore$ i.e., $\{2 \le x \le 5\}$ and $\{2 \le y \le 5\}$

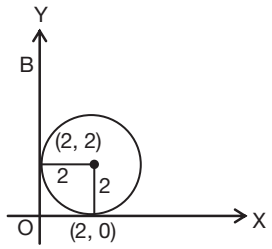
**32.** From the given graph, $x$ is dependent on $y$.

$\therefore$ The equation of the graph must be in the form of $x = f(y)$. Only options (B), (D) and in this form. Option (D) alone satisfies all the points given in the graph.

**33.** The shaded region is the interior of the circle with centre (2, 2) and radius 2.

$\therefore$ The equation of the circle is $(x-2)^2 + (y-2)^2 = 2^2$.

i.e., $x^2 + y^2 - 4x - 4y + 4 = 0$. The interior of the circle is represented by $x^2+y^2-4x-4y+4 \le 0$ or $x^2+y^2-4(x+y-1) \le 0$ or $x^2 + y^2 \le 4(x + y - 1)$.




**34.** In the entire shaded region, $x \ge 0$.

Equation of the line joining the points (3, 0) and (0, 2) is $\frac{x}{3} + \frac{y}{2} = 1$ or $2x + 3y = 6$.

The shaded region is bounded by lines $2x \pm 3y = 6$ or $2x + |3y| = 6$.

$\therefore$ The shaded region is represented by the inequality $|2x| + |3y| \le 6$ where $x \ge 0$.

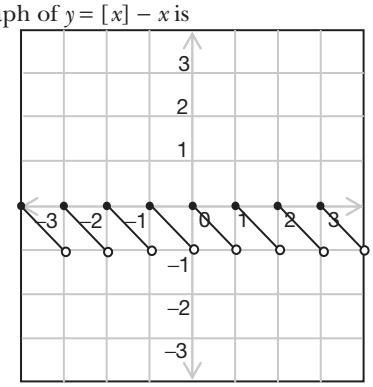
**35.** $x^2 + y^2 = 4$ represents a circle with centre origin and $3x + 4y - 7 = 0$ represents a line distance of the line from the centre of the circle is $= \left|\frac{3(0)+4(0)-7}{\sqrt{3^2+4^2}}\right| = \frac{7}{5} = 1.4$ units

but radius of the circle is 2 units

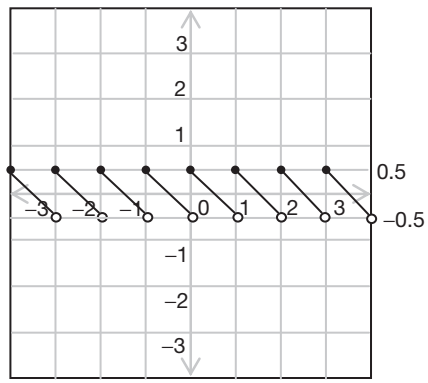
$\therefore$ $r > d$

$\therefore$ the line cuts the circle at two points.

**36.** The graph of $y = [x] - x$ is




By adding $\frac{1}{2}$ to $y$, we get




That is

$y = [x] - x + \frac{1}{2}$

**37.** From the graph, we can observe the following:

For $0 \le x < 1$, the range of $y$ is $0 \le y < 1$

For $1 \le x < 2$, the range of $y$ is $-1 \le y < 0$

For $-1 \le x < 0$, the range of $y$ is $1 \le y < 2$

The equation of the graph is $[x] + [y] = 0$

**38.** The function $sg(x)$ shows a jump of 2 from $a$ to $a+1$, where $-1 < a < 0$. The function $g(x)$ shows a jump of 1 from $a$ to $a+1$ (for all values of $a$).

$\therefore$ $p(x) = sg(x) + g(x)$ shows a jump of 3 from say $-\frac{1}{2}$ to $\frac{1}{2}$

$p\left(-\frac{1}{2}\right) = -1 + -1 = -2$ while $p\left(\frac{1}{2}\right) = 1 + 0 = 1$.

**39.** We want the right end point to be included. Consider $0 < x \le 1$. The image of all these values of $x$ should be the same. We should consider the greatest integer that is less than $x$ $\left(\text{so that } p\left(\frac{1}{2}\right) \text{ is 0 and } p(1) \text{ is also 0. The graph would be the segment described by } y = 0,\ 0 < x \le 1\right)$. We can also consider the least integer that is greater than or equal to $x$ $\left(\text{so that } p\left(\frac{1}{2}\right) = 1 \text{ and } p(1) \text{ is also 1. The graph would be the segment described by } y = 1,\ 0 < x \le 1\right)$.

$\therefore$ $f(x)$ and $h(x)$ are the required functions.

**40.** In the graph, the inclination of the line is 60° and an intercept is –2.

$\therefore$ Equation of the line is $y = \tan 60°\ x - 2$.

$\Rightarrow y = \sqrt{3}\ x - 2 \Rightarrow \sqrt{3}\ x - y - 2 = 0$

Since the origin belongs to the shaded region, the half plane containing the origin is the required region.

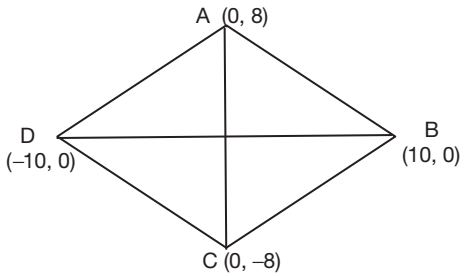
i.e., $\sqrt{3}\ (0) - (0) - 2 = -2 \le 0$

$\therefore$ The required inequality for the graph is $\sqrt{3}\ x - y - 2 \le 0$.

## EXERCISE-4

**1.** $y = x^2 + 4x + 9$
$= x^2 + 4x + 4 + 5$
$y = (x+2)^2 + 5$
The minimum value of the above function is 5. The graph is symmetric about $x + 2 = 0$, i.e., $x = -2$
$\therefore k = -2$.

**2.** The graph of $4|x| + 5|y| = 40$ is as follows:




The maximum distance between any two points on the graph is same as the distance between the points B and D
$\therefore$ BD = 20.

**3.** The region bounded by the straight lines $4|x| + 5|y| = 20$ represents a rhombus whose diagonals are 8 and 10. Its area $= \frac{1}{2} \cdot 8 \cdot 10 = 40$.

**4.** The given graph is represented by the line $y = x$.

Consider Choice (D): $y = f(x) + g(x) = \frac{x - |x|}{2} + \frac{x + |x|}{2} = x$ or $y = x$.

**5.** The given graph is obtained by reflecting the graph of $y = |x|$ in the $x$-axis, hence the graph is represented by $y = -|x|$.

Consider Choice (B): $f(x) - g(x) = \left(\frac{x - |x|}{2}\right) - \left(\frac{x + |x|}{2}\right)$

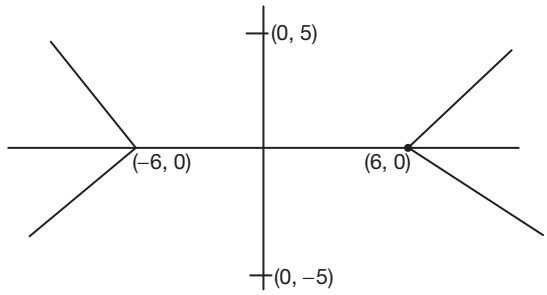
$= \frac{-2|x|}{2} = -|x|$

**6.** From the graph, we observe that when $x \in \left[0, \frac{1}{2}\right)$, $y \in [2, 7/3)$ and when $x \in \left[\frac{1}{2}, 1\right)$; $y \in \left[\frac{5}{2}, 2\right)$

Option C satisfies this condition.

**7.** The graph of $5|x| - 6|y| = 30$ is as follows.

The vertical line $x = 7 + |a|$, (where $a$ is any real number) intersects the graph in 2 points.




$\therefore$ Every horizontal line intersects the graph at two points.

***Solutions for questions 8 and 9:***
To stretch the graph of $f(x)$ vertically, define $g(x) = k\,f(x)$ where $k > 1$.
To compress the graph of $f(x)$ vertically, define $g(x) = \frac{1}{k} f(x)$ where $k > 1$.
To stretch the graph of $f(x)$ horizontally, define $g(x) = f\left(\frac{x}{k}\right)$ where $k > 1$.
To compress the graph of $f(x)$ horizontally, define $g(x) = f(kx)$, where $k > 1$.

**8.** The graph $g(x)$ can be obtained by stretching the graph of $f(x)$, horizontally and compressing it vertically. Therefore,
$\frac{1}{2} f\left(\frac{x}{2}\right) = g(x)$ or $\left(\text{setting } \frac{x}{2} = y\right)$; $f(y) = 2g(2y)$

$\therefore f(x) = 2g(2x)$.

**9.** From the graph of $f(x)$ it can be observed that
$f(x) = -f(-x) \Rightarrow f(x) + f(-x) = 0$.
Since $g(x) = 0$, $f(x) + f(-x) = 0$

**10.** In the given domain, $-2 \le x \le -1$ and $1 \le x \le 2$, four unit squares are included and for each of them the right and bottom edges are included.
Let us first consider the square in the first quadrant (say $S_1$) we may as well drop the mod function. As the right edge is included, we need $\lceil x \rceil$ (rather than $\lfloor x \rfloor$). As the bottom edge is included, we need $\lfloor y \rfloor$ (rather than $\lceil y \rceil$). To describe just this square (in terms of the floor/ceil functions), we can write down $\lceil x \rceil + \lfloor y \rfloor = 3$ ($\therefore$ We can consider only choices (B) and (C).

We need to consider the points (both internal and on the boundary, if necessary) in the other three squares (which we can denote as $S_2$, $S_3$, $S_4$ lying in $Q_2$, $Q_3$, $Q_4$, respectively) and see which of these two choices are satisfied by all of them.

| Point | $\lceil x \rceil$ | $\lfloor y \rfloor$ | $\lceil x \rceil + \lfloor y \rfloor$ | $\lvert x \rvert$ | $\lvert y \rvert$ | $\lceil \lvert x \rvert \rceil + \lfloor \lvert y \rvert \rfloor$ |
|---|---|---|---|---|---|---|
| (–1.5, 2.5) | –1 | 2 | 1 + 2 = 3 | 1.5 | 2.5 | 2 + 2 = 4 |
| (–1.5, –1.5) | –1 | –2 | 1 + 2 = 3 | 1.5 | 1.5 | 2 + 1 = 3 |
| (1.5, –0.5) | 2 | –1 | 2 + 1 = 3 | 1.5 | 0.5 | 2 + 0 = 2 |

We see that it is not necessary now to consider the boundary points. Only choice (B) covers all these internal points.

We can confirm that for each of the squares $S_2$, $S_3$, $S_4$, the boundaries are covered by choice (B).

**NOTE**

All the corners, where the dotted line meets the dark line are excluded.

**11.** The graph shows an ascending series of rectangles.

∴We should expect minus signs, if all the terms are on one side. ($mx - y = 0$ when $m > 0$ represents an ascending line while $mx + y = 0$ $(m > 0)$ represents a descending line). The length is along $x$-axis and it is 2 units. We should expect the term $\frac{x}{2}$ (rather than say $2x$). Also, the left boundaries are included. (We should expect $\lfloor x/2 \rfloor$ rather than $\lceil x/2 \rceil$).

The breadth is along the $y$-axis and it is 1 unit. The upper boundaries are included. We should expect $\lceil y \rceil$ rather than $\lfloor y \rfloor$.

∴We select choice (D).

**NOTE**

The points (–4, –2); (–2, –1); (0, 0); (2, 1) and (4, 2) are included.

***Solution for question 12*:**

To stretch the graph of $f(x)$ vertically, define $g(x) = k\,f(x)$ where $k > 1$.

To compress the graph of $f(x)$ vertically, define $g(x) = \frac{1}{k}\,f(x)$ where $k > 1$.

To stretch the graph of $f(x)$ horizontally, define $g(x) = f\left(\frac{x}{k}\right)$ where $k > 1$.

To compress the graph of $f(x)$ horizontally, define $g(x) = f(kx)$, where $k > 1$.

**12.** The graph $g(x)$ can be obtained by reflecting the negative points of $f(x)$ in the $x$-axis.

$\therefore\ g(x) = |f(x)|$

**13.** There are only two squares, one in $Q_1$ (say $S_1$), the other in $Q_2$ (say $S_2$). For $S_1$, the right and upper edges are included. The mod of the $x$ coordinates have the same range, viz., $2 < |x| \le 4$ for both $S_1$ and $S_2$ and $y$ coordinates have the same range, viz, $1 < y \le 3$ for both $S_1$ and $S_2$. We should expect mod signs with the $x$-coordinates and no mod signs with the $y$-coordinates. (If the mod sign appears with the $y$-coordinates, there should be two more squares in $Q_3$ and $Q_4$, respectively) (We should reject B and even D). As the right edge is included, we should expect $\lceil x \rceil$ (rather than $\lfloor x \rfloor$), and therefore, we should reject A also. We can verify that choice (C) describes the given squares.

**NOTE**

All the corners, where the dotted line meets the dark line are excluded.

**14.** There are two series of squares: $\lceil x \rceil + \lceil y \rceil = 3$ and $\lceil x \rceil = \lceil y \rceil$, i.e., $(a + b - 3)(a - b) = 0 \Rightarrow a^2 - b^2 = 3(a - b)$ $\Rightarrow a^2 - 3a = b^2 - 3b$. Where the two series cross each other, a bigger $(2 \times 2)$ square is formed.

**NOTE**

All the corners, where the dotted line meets the dark line are excluded.

**15.** There are 2 series of squares: $\lfloor x \rfloor + \lceil y \rceil = 0$ and $\lfloor x \rfloor - \lceil y \rceil = -2$, i.e., $(p + q)(p - q + 2) = 0$

$p^2 - q^2 + 2(p + q) = 0$

$p^2 + 2p = q^2 - 2q$

For the square, which is common to the series, two of the vertices are included.

**NOTE**

All the corners, where the dotted line meets the dark line are excluded.

# 8 Operator-Based Questions

CHAPTER

## Learning Objectives

*In this chapter, you will:*

- Learn about defining new mathematical operators using the existing ones
- Learn about performing operations using the newly defined operators

QUESTIONS BASED ON OPERATORS or definition–based questions have been given in various entrance exams in the past.

The basic idea deals with giving a set of operators/definitions followed by questions which require the use of these operators/definitions.

## SOLVED EXAMPLES

***Directions for questions 8.01–8.03*:** These questions are based on the following data.
Consider the following operators:

$$a\ \alpha\ b = a^3 + b^3;\ a \cup b = a^3 - b^3;\ a\ \beta\ b = \frac{a^2 - b^2}{a^2 + b^2};$$

$$a \subset b = \frac{a^2 + b^2}{a^2 - b^2};\ a \to b = ab.$$

**8.01.** What is the value of $(2\ \alpha\ 1)\ \beta\ (2 \cup 1)$?

**Sol:** $2\ \alpha\ 1 = 2^3 + 1^3 = 8 + 1 = 9;$
$2 \cup 1 = 2^3 - 1^3 = 8 - 1 = 7$

$$\therefore (2\ \alpha\ 1)\ \beta\ (2 \cup 1) = \frac{9^2 - 7^2}{9^2 + 7^2}$$

$$= \frac{32}{130} = \frac{16}{65}.$$

**8.02.** What is the value of $(37\ \beta\ 4) \to (37 \subset 4)$

**Sol:** Since $a\ \beta\ b = \frac{a^2 - b^2}{a^2 + b^2}$ and $a \subset b = \frac{a^2 + b^2}{a^2 - b^2}$.
The given expression is of the form

$$= (a\ \beta\ b) \to (a \subset b) = \left(\frac{a^2 - b^2}{a^2 + b^2}\right) \to \left(\frac{a^2 + b^2}{a^2 - b^2}\right)$$

$$= \left(\frac{a^2 - b^2}{a^2 + b^2}\right)\left(\frac{a^2 + b^2}{a^2 - b^2}\right) = 1,$$ for all values of $a$ and $b$,

where $a^2 + b^2,\ a^2 - b^2 \neq 0$.

**8.03.** Find the value of the expression:
$((a\ \alpha\ b)\ \alpha(a \cup b)) \to ((a\ \alpha\ b) \cup (a \cup b))$

**Sol:** Considering the first term:
$(a\ \alpha\ b)\ \alpha(a \cup b) = (a^3 + b^3)\ \alpha(a^3 - b^3)$
$= (a^3 + b^3)^3 + (a^3 - b^3)^3 = 2a^9 + 6a^3\ b^6$
$= 2a^3(a^6 + 3b^6)$

The second term is $(a\ \alpha\ b) \cup (a \cup b)$
$= (a^3 + b^3) \cup (a^3 - b^3) = (a^3 + b^3)^3 - (a^3 - b^3)^3$
$= 6a^6\ b^3 + 2b^9 = 2b^3\ (3a^6 + b^6)$
$\therefore ((a\ \alpha\ b)\ \alpha(a \cup b) \rightarrow (a\ \alpha\ b) \cup (a \cup b))$
$= 4a^3\ b^3\ (a^6 + 3b^6)\ (b^6 + 3a^6)$

***Directions for questions 8.04–8.08*:** These questions are based on the following operators:

$a(x, y) = |(x^2 + y^3) - (x^3 + y^2)|$
$b(x, y) = (x^3 - y^2) + (y^3 - x^2)$
$c(x, y)$ = L.C.M. $(a(x, y), b(x, y))$
$d(x, y)$ = H.C.F. $(a(x, y), b(x, y))$

**8.04.** Find the value of $a(4, 7)$.
**Sol:** $a(4, 7) = |(4^2 + 7^3) - (4^3 + 7^2)|$
$= |359 - 113| = 359 - 113 = 246.$

**8.05.** Find the value of $b(3, 6)$.
**Sol:** $b(3, 6) = (3^3 - 6^2) + (6^3 - 3^2)$
$= -9 + 207 = 198$

**8.06.** Find the value of $c(3, 5)$.
**Sol:** $c(3, 5)$ = L.C.M. $(a(3, 5), b(3, 5))$
$a(3, 5) = |(3^2 + 5^3) - (3^3 + 5^2)|$
$= |134 - 52| = 82$
$b(3, 5) = (3^3 - 5^2) + (5^3 - 3^2) = 2 + 116 = 118$
$\therefore c(3, 5)$ = L.C.M. $(82, 118) = 4838.$

**8.07.** Find the value of $d(4, 5)$.
**Sol:** $d(4, 5)$ = H.C.F. $(a(4, 5), b(4, 5))$
$a(4, 5) = |(4^2 + 5^3) - (4^3 + 5^2)|$
$= |141 - 89| = 52$
$b(4,5) = (4^3 - 5^2) + (5^3 - 4^2)$
$= 39 + 109 = 148.$
$\therefore$ H.C.F. $(a(4, 5), b(4, 5))$ = H.C.F. $(52, 148) = 4.$

**8.08.** Find the value of $c(1, 2) - d(2, 1)$.
**Sol:** $c(1, 2)$ = L.C.M. $(a(1, 2), b(1, 2))$
$a(1, 2) = |(1^2 + 2^3) - (1^3 + 2^2)| = 4$
$b(1, 2) = (1^3 - 2^2) + (2^3 - 1^2) = 4$
$\therefore c(1, 2)$ = L.C.M. $(4, 4) = 4$
$d(2, 1)$ = H.C.F. $(a(2, 1), b(2, 1))$
$a(2, 1) = |(2^2 + 1^3) - (2^3 + 1^2)| = 4$
$b(2, 1) = (2^3 - 1^2) + (1^3 - 2^2) = 4$
$\therefore d(2, 1)$ = H.C.F. $(4, 4) = 4$
$\therefore c(1, 2) - d(2, 1) = 4 - 4 = 0.$

***Directions for questions 8.09–8.11*:** These questions are based on the following data.
Consider the following operations:

$p\ \alpha\ q = p^2 + q^2$
$p\ \beta\ q = p^2 - q^2$
$p \cup q = \dfrac{p^3 + q^3}{p^3 - q^3}$
$p \cap q = \dfrac{p^3 - q^3}{p^3 + q^3}$
$p/q = pq$

**8.09.** Find the value of $(3\ \alpha\ 2) \cup (3\ \beta\ 2)$.
**Sol:** $3\ \alpha\ 2 = 3^2 + 2^2 = 13$
$3\ \beta\ 2 = 3^2 - 2^2 = 5$
$(3\ \alpha\ 2) \cup (3\ \beta\ 2)$
$= \dfrac{13^3 + 5^3}{13^3 - 5^3} = \dfrac{2322}{2072} = \dfrac{1161}{1036}$

**8.10.** Find the value of $(5 \cup 4)/(5 \cap 4)$.
**Sol:** $5 \cup 4 = \dfrac{5^3 + 4^3}{5^3 - 4^3} = \dfrac{189}{61}$

$5 \cap 4 = \dfrac{5^3 - 4^3}{5^3 + 4^3} = \dfrac{61}{189}$

$\therefore (5 \cup 4)/(5 \cap 4) = \dfrac{189}{61}\left(\dfrac{61}{189}\right) = 1$

**8.11.** Simplify:

$$\left(p^{\frac{3}{2}}\alpha q^{\frac{3}{2}}\right)\alpha\left(p^{\frac{3}{2}}\beta q^{\frac{3}{2}}\right) \Big/ \left(p^{\frac{3}{2}}\alpha q^{\frac{3}{2}}\right)\beta\left(p^{\frac{3}{2}}\beta q^{\frac{3}{2}}\right)$$

**Sol:** Let $A = p^{\frac{3}{2}}\ \alpha q^{\frac{3}{2}}$ and $B = p^{\frac{3}{2}}\ \beta q^{\frac{3}{2}}$
$\therefore A = p^3 + q^3$ and $B = p^3 - q^3$
The required expression is $(A\ \alpha\ B)/(A\ \beta\ B)$
$= [(p^3 + q^3)^2 + (p^3 - q^3)^2]/[(p^3 + q^3)^2 - (p^3 - q^3)^2]$
$= 2(p^6 + q^6)/(4\ p^3\ q^3) = 8\ p^3\ q^3\ (p^6 + q^6)$

***Directions for questions 8.12–8.15*:** These questions are based on the following data.
Consider the following definitions.

$m(x, y) = |(x^3 + y^2) - (x^2 + y^3)|$
$n(x, y) = x^2 - y^3 + y^2 - x^3$
$o(x, y)$ = L.C.M. $(x, y)$
$p(x, y)$ = H.C.F. $(m(x, y), n(x, y))$

**8.12.** Find the value of $m(6, 4)$.
**Sol:** $m(6, 4) = |(6^3 + 4^2) - (6^2 + 4^3)| = 132$

**8.13.** Find the value of $n(6, 4)$.
**Sol:** $n(6, 4) = 6^2 - 4^3 + 4^2 - 6^3 = -228$

**8.14.** Find the value of $o(m(6, 4), -n(6, 4))$
**Sol:** $o(m(6, 4), -n(6, 4)) = 0\ (132, 228)$
= L.C.M. $[12\ (11), 12(19)] = 12(11)(19) = 2508$

**8.15.** Find the value of $p(m(6, 4), -n(6, 4))$.
**Sol:** The given expression $= p(132, 228)$
= H.C.F. $(132, 228) = 12.$

## EXERCISE-1

***Directions for questions 1–40***: For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

***Directions for questions 1–4***: These questions are based on the following data.

For real numbers $a$, $b$, $c$ and $d$, an operator $\oplus$ is defined as follows:

| $\oplus$ | $a\ b\ c\ d$ |
|---|---|
| $a$ | $b\ d\ a\ c$ |
| $b$ | $d\ c\ b\ a$ |
| $c$ | $a\ b\ c\ d$ |
| $d$ | $c\ a\ d\ b$ |

i.e., $a \oplus a = b$, $a \oplus b = d$, $a \oplus c = a$ and so on.
Further, for $x = a, b, c$ and $d$,
$x^2 = x \oplus x$, $x^3 = x^2 \oplus x$, etc.

**1.** $c^{100} \oplus a^{100} =$
(A) $a$ (B) $b$ (C) $c$ (D) $d$

**2.** $(a \oplus b) \oplus (d \oplus c) =$
(A) $a$ (B) $c$ (C) $b$ (D) $d$

**3.** $a \oplus b \oplus (c \oplus d) =$
(A) $a$ (B) $b$
(C) $c$ (D) Cannot be determined

**4.** The least positive integer $n$ such that $d^n = c$ is
(A) 1 (B) 2 (C) 3 (D) 4

**5.** For two positive real numbers '$a$' and '$b$', let $a \$ b = \log_{10}(ab)$

$$a \,\%\, b = \left(\frac{1}{2}\right)^{ab}$$

$$a \,\Delta\, b = a - b$$

$$a \setminus b = a + b - ab$$

Which of the following is always positive?
(A) $a \$ b$ (B) $a \,\Delta\, b$ (C) $a \,\%\, b$ (D) $a \setminus b$

**6.** For two real numbers $a \rightarrow b = 2ab$, $a \leftarrow b = \dfrac{3a}{2b}$, which of the following is an integer?

(A) $\dfrac{1}{2} \rightarrow \dfrac{1}{6}$ (B) $\dfrac{1}{3} \leftarrow \dfrac{3}{5}$

(C) $\dfrac{1}{8} \leftarrow \dfrac{9}{16}$ (D) $\dfrac{3}{5} \rightarrow \dfrac{5}{6}$

**7.** If $x \oplus y$ = product of $x$ and $2y$, then $4 \oplus 5 =$
(A) 20 (B) 40 (C) 90 (D) 60

**8.** $a \rightarrow b = a + 2b$; $a \leftarrow b = 2a - b$. Which of the following is not negative?

(A) $\dfrac{1}{3} \rightarrow \left(-\dfrac{1}{2}\right)$ (B) $\dfrac{1}{5} \leftarrow \left(-\dfrac{1}{3}\right)$

(C) $-\dfrac{1}{6} \leftarrow \dfrac{1}{5}$ (D) $-5 \rightarrow 2$

**9.** $a \nabla b = a^2 - b^2 + ab$, then $2 \nabla 3 =$
(A) 5 (B) 3 (C) 1 (D) 2

**10.** For two positive real numbers $a$ and $b$, let $a \# b = \log_{10}\left(\dfrac{a}{b}\right)$; $a \varepsilon b = \left(\dfrac{1}{3}\right)^{ab}$

$a \$ b = a \times b - a^2$ and $a £ b = a + b - ab$
Which of the following is never negative?
(A) $a \# b$ (B) $a \varepsilon b$ (C) $a \$ b$ (D) $a £ b$

***Directions for questions 11–13***: These questions are based on the following data.

Let L $(a, b)$ = L.C.M $(a, b)$ and H $(a, b)$ = H.C.F $(a, b)$

**11.** L[L[L[L(L(1, 3), 6), 12], 24], 48] = _____.

**12.** H [H[H[ H(H(64,32), 16), 8], 4], 2] = _____.

**13.** H [L(4,18), L(12,18)] = _____.

**14.** For two real numbers '$a$' and '$b$', $a \downarrow b = a - b$, $a \uparrow b = a + b$, $a \rightarrow b = a \times b$ and $a \leftarrow b = \dfrac{a}{b}$, which of the following is an integer?

(A) $\dfrac{1}{2} \downarrow \dfrac{1}{4}$ (B) $(2 \leftarrow 3) \uparrow (4 \leftarrow 3)$

(C) $(2 \rightarrow 3) \leftarrow 4$ (D) None of these

**15.** If $x - y$ = sum of $x$ and $y$, then $3 - 2 =$ _____.

**16.** If $a \rightarrow b = 2a + 3b - ab$ and $c \leftarrow d = cd - (c + d)$, then the value of $(2 \rightarrow 3) \leftarrow (4 \leftarrow 3)$ is _____.

**17.** If $x \,\Delta\, y = x^2 + y^2 - xy$, then $(3 \,\Delta\, 4) \,\Delta 5 =$ _____.

**18.** If $M$ = average of first 10 natural numbers, $N$ = average of squares of first six natural numbers and $M \downarrow N = M + N - MN$, then $M \downarrow N =$
(A) $\dfrac{257}{4}$ (B) $\dfrac{-251}{4}$ (C) $\dfrac{250}{7}$ (D) $\dfrac{-351}{7}$

**19.** If $f(x, y, z) = \max(x, y, z)$, $g(p, q, r, s) = \text{L.C.M.}(p, q, r, s)$ and $h(a, b) = \text{H.C.F.}(a, b)$, then the value of $h(f(8, 32, 14), g(2, 8, 12, 16))$ is _____.

**20.** If $a * b = \dfrac{a+b}{a-b}$, then $(5 * 6) * -3 =$ _____.

***Directions for questions 21–24:*** For $s = \{a, b, c, d\}$, the operation $\otimes$ in $s$ is defined as follows:

Also, for $x \in A$,
$x^2 = x \otimes x$, $x^3 = x \otimes x^2$
$x^4 = x \otimes x^3$, etc.

| * | a | b | c | d |
|---|---|---|---|---|
| a | b | c | a | d |
| b | a | d | b | c |
| c | c | a | d | b |
| d | d | b | c | a |

**21.** $(b * c) * (d * a) =$
(A) $c$ (B) $a$ (C) $b$ (D) $d$

**22.** The least positive integer $n$ greater than 1 such that $c^n = c$, is
(A) 6 (B) 7 (C) 3 (D) 5

**23.** $d^6 * a^5 =$
(A) $a$ (B) $b$ (C) $c$ (D) $d$

**24.** $a * b * c =$
(A) $a$ (B) $b$
(C) $c$ (D) Cannot be determined

***Directions for questions 25–27:*** These questions are based on the following data.

For real numbers $x$ and $y$, let $\alpha(x, y) = x^3 + y^3$, $\beta(x, y) = x^3 - y^3$, $\gamma(x, y) = x + y$, $\delta(x, y) = x - y$

**25.** Find $\alpha[\beta(1, 1), \gamma(2, 1)]$.
(A) 125 (B) 51 (C) 64 (D) 27

**26.** Which of the following is always equal to 1?

(A) $\dfrac{\alpha(x,y)-\beta(x,y)}{\gamma(x,y)-\delta(x,y)}$ (B) $\dfrac{\alpha(x,y)-\beta(x,y)}{\gamma(x,y)+\delta(x,y)}$

(C) $\dfrac{\alpha(x,y)\beta(x,y)}{\gamma(x,y)\delta(x,y)}$ (D) None of these

**27.** If $1 < y < x$, which of the following is true?
(A) $\alpha(x, y) < \beta(x, y) < \gamma(x, y)$
(B) $\beta(x, y) < \gamma(x, y) < \alpha(x, y)$
(C) $\delta(x, y) < \beta(x, y) < \alpha(x, y)$
(D) $\delta(x, y) < \gamma(x, y) < \beta(x, y)$

***Directions for questions 28–33:*** These questions are based on the following data.

For two real numbers $a$ and $b$, let $a \odot b = a + b + ab$.

**28.** If $a \neq -1$, find the number $e$ such that $a \odot e = a$.
(A) 0 (B) 1 (C) –1 (D) None of these

**29.** Find the number $a$ such that $a \odot 2 = 0$.

(A) $\dfrac{3}{2}$ (B) $\dfrac{2}{3}$ (C) $\dfrac{-3}{2}$ (D) $\dfrac{-2}{3}$

**30.** Find $a$ such that $a \odot 1 = a$.
(A) 1 (B) –1 (C) 0 (D) 2

***Directions for questions 31–8:*** For two real numbers $a$ and $b$, $a \oplus b = a + b - ba$.

**31.** If $a \neq 1$, find the number $e$, such that $a \oplus e = a$.

(A) 1 (B) 0 (C) 5 (D) $-\dfrac{1}{3}$

**32.** Find the number $a$, such that $a \oplus 3 = 0$.

(A) $\dfrac{3}{2}$ (B) $\dfrac{2}{3}$ (C) –3 (D) $-\dfrac{3}{2}$

**33.** Find the number $a$, such that $a \oplus 1 = a$.
(A) 0 (B) 1 (C) 2 (D) 3

***Directions for question 34:*** These questions are based on the following data.

If $t(a, b) = \text{L.C.M.}(a, b)$ and $H(a, b) = \text{H.C.F.}(a, b)$

**34.** Find the value of $H\{t(3, 4), t(5, 7)\}$.
(A) 4 (B) 3 (C) 1 (D) 2

**35.** If $a \Delta b = a^2 + b^2 - ab$, then $1 \Delta 2 =$ _____.

**36.** If $a \Delta b = a - b$ and $a \nabla b = a + b$, then which of the following is always non-negative?
(A) $(a \nabla b) - (a \Delta b)$ (B) $(a \nabla b)^2 - (a \Delta b)^2$
(C) $(a \Delta b) + (a \nabla b)$ (D) $(a \Delta b)^2 + (a \nabla b)^2$

***Directions for questions 37–39:*** For real numbers $a$ and $b$, $f(a, b) = a - b$, $g(a, b) = (a - b)^2$ and $h(a, b) = (a - b)^3$.

**37.** If $a \neq b$, which of the following is true?
(A) $f(a, b) < g(a, b) < h(a, b)$
(B) $f(a, b) > g(a, b) > h(a, b)$
(C) $g(a, b) > h(a, b) > f(a, b)$
(D) None of the above

**38.** $\dfrac{h(3,4)-f(3,6)}{g(1,3)} =$

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{3}$ (C) 2 (D) 3

**39.** $\dfrac{h(5,7)+g(8,6)}{f(1,3)-h(1,5)} =$

(A) $\dfrac{2}{31}$ (B) $-\dfrac{2}{31}$ (C) $-\dfrac{1}{31}$ (D) $\dfrac{1}{31}$

**40.** If $a \# b = a^2 + b^2$, $a \rightarrow b = (a + b)(ab)$, $a \leftarrow b = (a - b)\ ab$ and $a \nabla b = 2a + 3b$, then which of the following is / are divisible by 5?

(A) $3 \# 4$ (B) $5 \rightarrow 7$

(C) $7 \leftarrow 8$ (D) both A and B

## EXERCISE-2

***Directions for questions 1–40*:** For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** If $a \oplus b = a^3 + b^3$, $a \otimes b = a^2 + b^2 - ab$, then $\dfrac{a \oplus b}{a \otimes b} =$

(A) $a - b$ (B) $ab$ (C) $a + b$ (D) $\dfrac{a}{b}$

**2.** If $x * y = x^3 + y^3$, $x \Delta y = x^3 - y^3$, $x \nabla y = x + y$ and $x \rightarrow y = x - y$, then the value of $\{[(2 * 3) \rightarrow (2 \Delta 3)] \nabla 4\}$ is _____.

**3.** If $A(x, y) = (x + y)^2$, $B(x, y) = (x - y)^2$ and $C(x, y) = 4xy$, then the value of $\dfrac{A(a,b) - B(a,b)}{C(a,b)}$ is _____.

**4.** If $a \uparrow b = a^2 + b^2 - 3ab$, $a \downarrow b = a^2 - b^2 + 3ab$, $a * b = \sqrt{\dfrac{a+b}{2}}$, then $(a \uparrow b) * (a \downarrow b) =$

(A) $a$ (B) $b$ (C) $a + b$ (D) $a - b$

***Directions for questions 5–7*:** These questions are based on the following data.

$$f(x, y) = \frac{a^x + b^y}{2},\ g(x, y) = \frac{a^x - a^{-y}}{2}$$

$$p(x, y) = \log_a\left(\frac{x+y}{x-y}\right),$$

$$q(x, y) = \log_a\left(\frac{x-y}{x+y}\right)$$

**5.** The value of $q(f(x, x), g(x,-x))$ is

(A) $\dfrac{a^x - a^{-x}}{a^x + a^{-x}}$ (B) $\dfrac{a^x + a^{-x}}{a^x - a^{-x}}$

(C) 1 (D) 0

**6.** $\dfrac{f\left(p(x,y), q(x,y)\right)}{g\left(q(x-y), p(x,y)\right)} =$

(A) $\dfrac{(x+y)^2}{2xy}$ (B) $\dfrac{x+y}{2xy}$ (C) $\dfrac{2xy}{x^2 - y^2}$ (D) $\dfrac{2xy}{(x+y)^2}$

**7.** The value of $q(p(f(6, 4), g(6, 4)), 4) =$

(A) $\log_a 21$ (B) $\log_a\left(\dfrac{3}{7}\right)$

(C) $\log_a \dfrac{7}{3}$ (D) $\log_a 343$

***Directions for questions 8–11*:** These questions are based on the following information.

$(a \cup b) = \dfrac{a+b}{a-b}$ $(a \cap b) = \dfrac{2ab}{a+b}$ $a \ominus b = a + b - ab$, $a \oplus b = a - b + ab$, $a \Delta b = (a \cup b) - (a \cap b)$, $a \nabla b = (a \cup b)(a \cap b)$

**8.** $(8 \cup 12) \cap (15 \ominus 1) =$

(A) $\dfrac{5}{2}$ (B) $-\dfrac{5}{2}$ (C) 2 (D) –1

**9.** $(21 \oplus 7) \cup (12 \ominus 8) =$

(A) $\dfrac{76}{161}$ (B) $\dfrac{85}{237}$ (C) $-\dfrac{85}{237}$ (D) $\dfrac{76}{161}$

**10.** $(5 \nabla 8) \cup (6 \Delta 3) =$

(A) 1 (B) $\dfrac{77}{83}$ (C) $\dfrac{83}{77}$ (D) –1

**11.** $[(6 \oplus 4) \Delta (6 \ominus 4)] \cap (8 \cup 7) =$

(A) $\dfrac{54870}{2279}$ (B) $\dfrac{18290}{2279}$ (C) $\dfrac{2279}{1829}$ (D) 1

***Directions for questions 12–15*:** These questions are based on the following data.

Consider the following operators and answer the questions.

Sweet $(a, b) = e^a + e^b$

Sour $(a, b) = e^a - e^b$

Mint $(a, b) = e^{a+b}$

Hint $(a, b) = e^{a-b}$

Difficulty Level-2: Moderate

**12.** Which of the following statements is always true?
(A) Sweet $(a, b) = 2$ for $a = 1$ and $b = 1$
(B) Sour $(a, b)$ is always positive
(C) Sweet $(a, b) = 2$, for $a = 0$ and $b = 0$
(D) Mint $(a, b) = 2$; for $a = 1$, $b = 1$

**13.** Which of the following statements is true?
(A) Mint $(a, b)$ = Sweet $(a, b)$
(B) Hint $(a, b)$ = Sour $(a, b)$
(C) Sweet $(a, b)$ Sour $(a, b)$ = Mint $(a, b)$ Hint $(a, b)$
(D) Mint $(a, b)$ = Hint $(a, b)$ [sweet $(a, b)$ − sour $(a, b)$]

**14.** Which of the following is not true?
(A) Mint $(a, b) \times$ Hint $(a, b)$ = Sweet $(a, b)$ + Sour $(a, b)$
(B) Sweet $(a, b)$ is less than 1 when $a = b = -2$
(C) Sour $(a, b)$ is negative when $a = 1/2$ and $b = 1$
(D) Arithmetic mean of Sweet $(a, b)$ and Sour $(a, b)$ is equal to 1 for $a = 0$

**15.** Which of the following statements will enable us to find $a$ and $b$?
(A) Mint $(a, b) = e^8$, Hint $(a, b) = e^4$
(B) Sweet $(a, b) = e^6$
(C) Sour $(a, b) = e^{10}$
(D) Hint $(a, b) = e^6$

***Directions for questions 16–18*:** These questions are based on the following data.
The following functions are defined for $x$, $y$ and $z$ which are distinct and positive real numbers.
$f(x, y, z) = \min(\max(x, y), \min(y, z)\ \min(z, x))$
$g(x, y, z) = \max(\min(x, y), \min(y, z)\ \min(z, x))$
$h(x, y, z) = \max(\max(x, y), \max(y, z), \max(z, x))$
$i(x, y, z) = \min(\min(x, y), \min(y, z), \min(z, x))$
$j(x, y, z) = \max(\min(x, y), \max(y, z), \max(z, x)$
$k(x, y, z) = \min(\max(x, y), \max(y, z), \max(z, x))$

**16.** Which of the following is less than 1?
(A) $f(x, y, z) + k(x, y, z)/g(x, y, z) - h(x, y, z)$
(B) $i(x, y, z) + h(x, y, z)/f(x, y, z) - g(x, y, z)$
(C) $h(x, y, z) + g(x, y, z)/i(x, y, z) - f(x, y, z)$
(D) More than one of the above

**17.** Which of the following is positive?

(A) $\dfrac{f(x,y,z)-h(x,y,z)}{i(x,y,z)-k(x,y,z)}$ (B) $\dfrac{j(x,y,z)-h(x,y,z)}{g(x,y,z)-h(x,y,z)}$

(C) $\dfrac{g(x,y,z)-f(x,y,z)}{k(x,y,z)-h(x,y,z)}$ (D) $\dfrac{i(x,y,z)+h(x,y,z)}{i(x,y,z)-f(x,y,z)}$

**18.** Which of the following is undefined?

(A) $\dfrac{g(x,y,z)-h(x,y,z)}{f(x,y,z)-j(x,y,z)}$ (B) $\dfrac{h(x,y,z)-j(x,y,z)}{k(x,y,z)-i(x,y,z)}$

(C) $\dfrac{k(x,y,z)-i(x,y,z)}{f(x,y,z)-k(x,y,z)}$ (D) None of these

***Directions for question 19–20*:** These questions are based on the following information that $x$ and $y$ are real numbers, such that
$f(x, y) = -|x + y|$, $F(f(x, y)) = -f(x, y)$
$G(f(x, y)) = -F(f(x, y))$

**19.** Find the value of $f\left[G\left(f(2,3)\right), f\left(G(f(4,3)F(f(4,3))\right)\right]$.

(A) −5 (B) −3 (C) 0 (D) −4

**20.** Which of the following expressions yields $x^2$ as a result?
(A) $F(f(x, -x))\ G(f(x, -x))$
(B) $F(f(x, x))\ G(f(x, x)) \cdot 4$
(C) $-F(f(x, x))\ G(f(x, x))/4$
(D) $f(x, x) \cdot f(x, -x)$

***Directions for questions 21–23*:** These questions are based on the following data.
For positive real numbers $x$, $y$, $z$,
$f(x, y, z) = \min(\max(x, y), \max(y, z), \max(z, x))$
$g(x, y, z) = \max(\min(x, y), \min(y, z), \min(z, x))$
$h(x, y, z) = \max(\min(x, y), \min(y, z), \max(z, x))$
$k(x, y, z) = \min(\min(x, y), \min(y, z), \max(z, x))$
$j(x, y, z) = \min(\min(x, y), \min(y, z), \min(z, x))$
$i(x, y, z) = \max(\max(x, y), \max(y, z), \max(z, x))$

**21.** Which of the following is greater than 1?

(A) $\dfrac{f(x,y,z)-g(x,y,z)}{h(x,y,z)+j(x,y,z)}$ (B) $\dfrac{f(x,y,z)+k(x,y,z)}{g(x,y,z)+i(x,y,z)}$

(C) $\dfrac{h(x,y,z)-g(x,y,z)}{k(x,y,z)-i(x,y,z)}$ (D) $\dfrac{i(x,y,z)-k(x,y,z)}{h(x,y,z)-g(x,y,z)}$

**22.** Which of the following is negative?

(A) $\dfrac{-k(x,y,z)+f(x,y,z)}{h(x,y,z)}$ (B) $\dfrac{h(x,y,z)+g(x,y,z)}{k(x,y,z)+j(x,y,z)}$

(C) $\dfrac{k(x,y,z)+g(x,y,z)}{j(x,y,z)-f(x,y,z)}$ (D) $\dfrac{g(x,y,z)-i(x,y,z)}{j(x,y,z)-f(x,y,z)}$

**23.** Which of the following expressions is undefined?

(A) $\dfrac{f(x,y,z)-h(x,y,z)}{-k(x,y,z)-j(x,y,z)}$ (B) $\dfrac{g(x,y,z)}{k(x,y,z)-i(x,y,z)}$

(C) $\dfrac{h(x,y,z)+j(x,y,z)}{j(x,y,z)-k(x,y,z)}$ (D) $\dfrac{g(x,y,z)+f(x,y,z)}{h(x,y,z)-j(x,y,z)}$

***Directions for questions 24–26*:** These questions are based on the following data:

$$a \uparrow b = \frac{3ab}{2}$$

$$a \downarrow b = \frac{4ab}{2}$$

$$a \rightarrow b = 2a + 3b$$

$$a \leftarrow b = 4a - 5b$$

**24.** Which of the following is an integer?
(A) $((3 \downarrow 7) \rightarrow 9) \uparrow 5$ (B) $((3 \rightarrow 7) \downarrow 9) \uparrow 5$
(C) $((3 \uparrow 7) \downarrow 9) \rightarrow 5$ (D) $((3 \uparrow 7) \rightarrow 9) \downarrow 5$

**25.** Which of the following is always true?
(A) $\{[(a \uparrow b) \rightarrow b] \downarrow ab\} \leftarrow b = 3a - 7b$
(B) $\{[(a \leftarrow b) \uparrow b] \rightarrow ab\} \downarrow b = 6a + 5b$
(C) $\{[(a \rightarrow b) \uparrow b] \leftarrow ab\} \downarrow b = 28a + 72b$
(D) $\{[(a \downarrow b) \rightarrow b] \uparrow ab\} \leftarrow b = \frac{3a - 7b}{5}$

**26.** A perfect square among the following is
(A) $\{\{[(4 \downarrow 5) \rightarrow 7] \uparrow 9\} \leftarrow 8\}$
(B) $\{\{[(4 \uparrow 5) \downarrow 7] \rightarrow 9\} \leftarrow 8\}$
(C) $\{\{[(4 \leftarrow 5) \rightarrow 7] \downarrow 9\} \uparrow 8\}$
(D) $\{\{[(4 \rightarrow 5) \leftarrow 7] \downarrow 9\} \uparrow 8\}$

***Directions for questions 27–29*:** These questions are based on the following data.

$$a \rightarrow b = a \times 3b$$

$$a \leftarrow b = \frac{2a}{b}$$

$$a \uparrow b = 3a + b$$

$$a \downarrow b = 2a - 3b$$

**27.** Which of the following is a perfect cube?
(A) $[(9 \uparrow 7) \rightarrow 4] \downarrow 29$ (B) $[(9 \rightarrow 7) \downarrow 4] \uparrow 29$
(C) $[(9 \downarrow 7)\ 4] \rightarrow 29$ (D) $[(9 \uparrow 7) \downarrow 4] \rightarrow 29$

**28.** Which of the following is a multiple of 13?
(A) $[(15 \uparrow 6) \rightarrow 9] \leftarrow 2$ (B) $[(15 \rightarrow 6) \uparrow 9] \leftarrow 2$
(C) $[(15 \downarrow 6) \leftarrow 9] \rightarrow 2$ (D) $[(15 \leftarrow 6) \downarrow 9] \uparrow 2$

**29.** The value of the expression $\sqrt[3]{30 \uparrow 35} - \sqrt{14 \downarrow 4}$
(A) 6 (B) 5 (C) 1 (D) –1

**30.** If $\$(x, y)$ = H.C.F. $(x, y)$, $\Delta(x, y)$ = AM $(x, y)$, $\nabla(x, y)$ = L.C.M. $(x, y)$, $\sigma(x, y)$ = quotient when $x$ is divided by $y$, then find the value of
$\sigma(\Delta(\nabla(\$(\Delta(240, 180), 70), 50), 90), 10)$. _____

***Directions for questions 31 and 32*:** These questions are based on the following data.

$a$ and $b$ are two non-zero real numbers and $*$, $\oplus$ are defined as $a * b = \frac{ab}{3}$; $a \oplus b = a + b - ab$

**31.** If $a * b = a \oplus b$, then which of the following is true?

(A) $\frac{1}{a} + \frac{1}{b} = 2$ (B) $\frac{1}{a} + \frac{1}{b} = \frac{4}{3}$

(C) $\frac{a+b}{ab} = 5$ (D) $\frac{ab}{a+b} = \frac{4}{3}$

**32.** $\{[(3 * 5) \oplus 7] * 9\} \oplus 4 =$
(A) 211 (B) –69 (C) –211 (D) 63

***Directions for questions 33–35*:** These questions are based on the following information.

$$a * b = (a+b)^{a^3 - b^3}$$

$$a \$ b = a^{a^3 - b^3} + b^{a^3 - b^3}$$

$$a \vee b = a^{a^3 - b^3}$$

$$a \wedge b = b^{a^3 - b^3}$$

**33.** The value of $(a \$ b) - (a * b)$ when $a = 2$ and $b = 1$ is
(A) $3^7 + 1 - 2^7$ (B) $2^7 - 3^7 + 1$
(C) $3^7 - 1 - 2^7$ (D) $2^7 - 3^7 - 1$

**34.** Which of the following is/are true for all values $a, b > 0$?
(A) $a \vee b \geq a \wedge b$ (B) $a \wedge b > 1$
(C) $a * b = a \$ b$ (D) Both (A) and (B)

**35.** Which of the following is/are false when $a = 1$, $b = 2$?
(A) $a * b > 0$ (B) $a \vee b = 1$
(C) $a \wedge b = 1 - a \$ b$ (D) Both (A) and (B)

## EXERCISE-3

***Directions for questions 1–40*:** For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple-choice questions, write your answer in the space provided.

**1.** To simplify the algebraic expression, we follow the order BOADSM where B stands for Bracket, O stands for of, A stands for addition, D stands for division, S stands for subtraction, and M stands for Multiplication. Using the above rule what is the value of $(3 \times 9) + 28 \div 7 \times 24 - 10$?

(A) 113 (B) $\frac{1250}{7}$ (C) 110 (D) $\frac{55}{158}$

***Directions for questions 2 and 3*:** These questions are based on the following data.

Let $f(x, y) = a^{x+y}$, $g(x, y) = a^{x-y}$, $h(x, y) = f(x, y) \times g(x, y)$, $I(x, y) = \dfrac{f(x,y)}{g(x,y)}$

**2.** $\dfrac{h(x,y)}{I(x,y)} =$

(A) $(g(x, y))^2$ (B) $(f(x, y))^2$
(C) $x + y$ (D) $x - y$

**3.** $h(x, y) \times I(x, y) =$
(A) 1 (B) $f(x, y)$
(C) $g(x, y)$ (D) $(f(x, y))^2$

***Directions for questions 4–6*:** These questions are based on the following data:

$a \sim b$ is H.C.F. of $a^2, b^2$
$a \% b$ is L.C.M. of $a^2, b^2$
$a \Delta b$ is $(a + b)^2 - 4ab$
$a \nabla b$ is $(a - b)^2 + 4ab$

**4.** The value of the expression $(((4 \sim 5) \Delta 6) \nabla 3) =$ _____.

**5.** If $a = 5$, $b = 6$, then which of the following is true?
(A) $a \% b = a \Delta b$ (B) $a \sim b = a \Delta b$
(C) $a \% b = a \nabla b - a \Delta b$ (D) $a \nabla b = a \Delta b + 1$

**6.** For distinct integers $a$ and $b$, which of the following is/are always false?
(A) $a \sim b > a \% b$ (B) $a \Delta b > a \nabla b$
(C) $a \% b < a \nabla b$ (D) Both (A) and (B)

***Directions for questions 7–11*:** Two operations $*$ and $\oplus$ are defined as follows.

| $*$ | $a$ | $B$ | $c$ | $d$ |
|---|---|---|---|---|
| $a$ | $d$ | $C$ | $b$ | $a$ |
| $b$ | $c$ | $D$ | $a$ | $b$ |
| $c$ | $b$ | $A$ | $d$ | $c$ |
| $d$ | $a$ | $B$ | $c$ | $d$ |

| $\oplus$ | $a$ | $B$ | $c$ | $d$ |
|---|---|---|---|---|
| $a$ | $b$ | $d$ | $a$ | $c$ |
| $b$ | $d$ | $a$ | $c$ | $b$ |
| $c$ | $c$ | $b$ | $d$ | $a$ |
| $d$ | $a$ | $c$ | $b$ | $d$ |

and $a^2 = a * a$, $a^3 = a^2 * a$, etc.
$2a = a \oplus a$, $3a = 2a \oplus a$, etc.

**7.** If $cn = c$, then the least positive integer value of $n$, that is greater than 1 is _____.

**8.** The expression $(b^4 * 5c) \oplus d^5] * 4a$ is
(A) $b$ (B) $c$ (C) $a$ (D) $d$

**9.** Which of the following is always true?
(A) $*$ satisfies the commutative property.
(B) $\oplus$ satisfies the associative property.
(C) $\oplus$ satisfies the commutative property.
(D) (B) and (C)

**10.** The value of $\{[(a * b) \oplus c] * d\} \oplus a$
(A) $b$ (B) $c$
(C) $d$ (D) $a$

**11.** If $a^n = na = a$, then the least value of $n$ is _____.

***Directions for questions 12–15*:** These questions are based on the following data.

$a + b = 12ab$
$a - b = \dfrac{2a}{5b}$
$a \times b = 3a + 7b$
$\dfrac{a}{b} = 4a - 3b$

**12.** The expression $\left[\left(\dfrac{9}{2} - 6\right) \times 4 + 3\right] =$ _____

**13.** Which of the following is multiple of 23?
(A) $\dfrac{[(5+6)-2]}{4}$ (B) $[5 \times 6] - 2 + 4$
(C) $\dfrac{[(5-6)+2]}{4}$ (D) $[(5 + 2) - 6) \times 4$

**14.** The value of the expression $\dfrac{(7 \times 9)}{11} - 13$ is
(A) $\dfrac{606}{5}$ (B) $\dfrac{606}{13}$ (C) $\dfrac{606}{65}$ (D) $\dfrac{609}{56}$

**15.** Which of following is a perfect square?
(A) $\dfrac{(10 - 8 \times 16)}{4}$ (B) $\dfrac{(10 - 16) \times 8}{4}$
(C) $\dfrac{(10 + 8) - 4}{16}$ (D) $[(10 - 4) + 8] \times 16$

**16.** The value of the expression $\sqrt{17 \times 10} + \sqrt[3]{\dfrac{25}{12}}$ is _____.

***Directions for questions 17–20*:** These questions are based on the following data.
Consider the following operators
$a \uparrow b$ is L.C.M. of $a$ and $b$
$a \downarrow b$ is H.C.F. of $a$ and $b$

$a \to b$ is $(a+b)^3 - (a-b)^3$
$a \leftarrow b$ is $(a+b)^3 + (a-b)^3$
$a \,\Delta\, b$ is $(a-b)(a^2+ab+b^2)$

**17.** The value of $(93)\ \Delta\ (16 \downarrow 24)$ is _____.

**18.** Which of the following statements is true if $a = 5$, $b = 25$?
(A) $(a \uparrow b) \div (a \downarrow b) + (a \,\Delta\, b)$ is 7750
(B) $\sqrt[3]{(a \uparrow b) x (a \downarrow b)} = b$
(C) $a \to b > a \leftarrow b$
(D) (A) and (B)

**19.** Which of the following statements is true?
(A) $(a \uparrow b)(a \downarrow b)$ is divisible by $a$ but not $b$
(B) $(a \uparrow b)(a \downarrow b)$ is divisible by $b$ but not $a$
(C) $(a \uparrow b)(a \downarrow b)$ is divisible by neither $a$ nor $b$.
(D) $(a \uparrow b)(a \downarrow b)$ is divisible by both $a$ and $b$.

**20.** Which of the following is true?
(A) when $a = 7$, $b = 2$; $a \,\Delta\, b = 300$
(B) when $a = 5$, $b = 2$, $a \to b = 318$
(C) when $a = 9$, $b = 2$, $(a \to b) + (a \leftarrow b) = 2662$
(D) when $a = 2$, $b = 3$, $(a \downarrow b)\ \Delta\ (a \uparrow b) = 215$

***Directions for questions 21–23*:** These questions are based on the following data:

$f(x, y) = \lceil 2x \rceil + \lceil 2y \rceil + \lceil x + y \rceil$
$g(x, y) = \lceil 3x \rceil + \lceil 3y \rceil$
$h(x, y) = \lfloor 3x \rfloor + \lfloor 3y \rfloor$

where $\lceil x \rceil$ denotes the least integral value greater than or equal to $x$.
$\lfloor x \rfloor$ denotes the greatest integral value less than or equal to $x$.

**21.** For real numbers $x$ and $y$, which of the following is/are true?
(A) $f(x, y) = g(x, y)$ (B) $g(x, y) \geq h(x, y)$
(C) $f(x, y) < h(x, y)$ (D) Both (A) and (B)

**22.** Under which of the following conditions is $g(x, y) = h(x, y)$?
(A) Neither $x$ nor $y$ is an integer.
(B) $x$ is an integer but $y$ is not an integer.
(C) $x$ is not an integer but $y$ is an integer.
(D) None of these

**23.** The value of $h(g(f(3.5, 7.9), 8.2), 7) =$ _____.

***Directions for questions 24 and 25*:** For the equation $ax^2 + by^2 + 2gx + 2fy + c = 0$ the operation $\Delta$ and $\nabla$ are defined as follows:

$$\Delta = \frac{2}{b}\sqrt{g^2 - ac}$$

$$\nabla = \frac{2}{a}\sqrt{f^2 - bc}$$

**24.** For the equation $3x^2 + 5y^2 + 7x + 11y + 12 = 0$ which of the following is true?
(A) $\Delta^2 > 0$
(B) $\nabla^2 < 0$
(C) $\Delta^2 > \nabla^2$
(D) More than one of the above

**25.** For the equation $5x^2 + 7y^2 + x - 3y - 15 = 0$ The value of $\Delta^2$ is approximately
(A) 25 (B) 35
(C) $\sqrt{6}$ (D) 6

***Directions for questions 26–29*:** These questions are based on the following data.

$a \to b$ = L.C.M. of $2a$ and $b$
$a \leftarrow b$ = H.C.F. of $2a$ and $b$
$a \uparrow b = (2a+b)^2 - (2a-b)^2$
$a \downarrow b = (2a+b)^2 + (2a-b)^2$
$a \leftrightarrow b = (2a+b)\ (2a-b)$

**26.** Which of the following is true?
(A) $a = 3$, $b = 5$ then $a \to b$ is 15
(B) $a = 6$, $b = 4$ then $a \leftarrow b$ is 2
(C) $a = 1$, $b = 0$ then $a \leftrightarrow b = 22$
(D) $a = 7$, $b = 6$ then $a \uparrow b = 336$

**27.** The value of the expression $\{[(4 \to 6) \leftarrow 3] \uparrow 2\} \leftrightarrow 1$ is
(A) 52 (B) 48 (C) 9152 (D) 9215

**28.** Which of the following statements is true for all positive values of $a$ and $b$?
(A) $a \to b$ is always greater than $a \leftarrow b$.
(B) $(a \uparrow b)$ is always less than $a \leftrightarrow b$.
(C) $a \downarrow b$ is always greater than $a \leftrightarrow b$.
(D) Both (A) and (C)

**29.** Which of the following is always true?
(A) $(a \to b)$ is divisible by $a$ but not $b$.
(B) $(a \leftarrow b)$ is divisibly by $b$ but not $a$.
(C) $(a \uparrow b)\ (a \downarrow b)$ is divisible by $a$ but not $b$.
(D) $(a \uparrow b)\ (a \downarrow b)$ divisible by both $a$ and $b$.

***Directions for questions 30–33*:** These questions are based on the following data.

$a + b = a \times 2b$
$a - b = 3a/b$
$a \times b = a + 4b$
$a/b = 6a - b$

**30.** The expression $(((14/3) - 27) \times 6) + 13 =$ _____.

**31.** Which of the following is not a multiple of 7?
(A) $((21 \times 7) - 9) + 6$ (B) $((21 - 7) + 9) \times 6$
(C) $((21/7) \times 9) - 6$ (D) Both (B) and (C)

**32.** The expression $(6 \times 4) + (9/2) =$ _____.

**33.** Which of the following is a perfect square?
(A) $((10 + 5) - 3) \times 24$
(B) $((10 - 5) + 3) \times 24$
(C) $((10 \times 5) - 3) + 24$
(D) (A) and (B)

***Directions for questions 34–36*:** These questions are based on the following data.

$$f(x, y) = 2^{x+y},\ g(x, y) = 2^{x-y}$$

$$p(x, y) = \log_2 xy,\ q(x, y) = \log_2\left(\frac{x}{y}\right)$$

**34.** The value of $q(f(x, -x), g(x, x))$ is _____.

**35.** The value of $p(f(3, 4), g(4, 5))$ is _____.

**36.** $\dfrac{f\left[p(5,6), g(6,5)\right]}{g\left[p(4,5), g(5,6)\right]} =$ _____.

***Directions for questions 37–40*:** These questions are based on the following data.
For the equation, $ax^2 + 2hxy + by^2 = 0$, the operators $\Delta$ and $\nabla$ are defined as follows:

$$\Delta = \frac{2}{b}\sqrt{h^2 - ab}$$

$$\nabla = \frac{2}{b}\sqrt{ab - h^2}$$

**37.** For the equation $2x^2 + 7xy + 7y^2 = 0$,
Which of the following statements is true?
(A) $\Delta^2$ is positive
(B) $\nabla^2$ is negative
(C) $\nabla^2 > \Delta^2$
(D) $\Delta^2 > \nabla^2$

**38.** If a new equation is formed by interchanging the coefficients of $x^2$ and $y^2$ in the equation $7x^2 + 10xy - 8y^2 = 0$, then which of the following is true for the new equation?
(A) $\Delta^2 = \nabla^2$ (B) $\Delta^2 > \nabla^2$
(C) $\Delta^2 < \nabla^2$ (D) $\Delta^2 = 2\nabla^2$

**39.** If $D = \dfrac{\sqrt{h^2 - ab}}{ab}$, then which of the following is true?

(A) $D = a\Delta$ (B) $\Delta = 2aD$
(C) $2\Delta = aD$ (D) $\Delta = aD$

**40.** Which of the following is true for $px^2 + 2hxy + qy^2 = 0$ if $pq < h^2$?
(A) $\nabla = \Delta$ (B) $\nabla > \Delta$
(C) $\nabla^2 > \Delta^2$ (D) $\nabla^2 < \Delta^2$

## EXERCISE-4

***Directions for questions 1–20*:** For the multiple-choice questions, select the correct alternative from the given choices. For the non-multiple-choice questions, write your answer in the space provided.

***Directions for questions 1 and 2*:** These questions are based on the information given below.

$$f(a,b) = \left|\sqrt{a^2 + b^2}\right|;\quad g\left[(a,b),(c,d)\right] = \frac{ac + bd}{c^2 + d^2};$$

$$h[(a, b), (c, d)] = \frac{bc - ad}{c^2 + d^2};\quad i\left[(a,b),(c,d)\right] = ac - bd;$$

$$j[(a, b), (c, d)] = ad + bc.$$

**1.** Find $f\{g[(3, 4), (12, 16)], h[(3, 4), (12, 16)]\}$.

(A) $\frac{1}{2}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{1}{4}$ (D) $\frac{1}{8}$

**2.** Find $f\{i[(3, 4), (12, 16)], j[(3, 4), (12, 16)]\}$
(A) 54 (B) 100
(C) 125 (D) None of these

***Directions for questions 3 and 4*:** These questions are based on the information given below.

$x$ and $y$ are real numbers and $f(x, y) = |x + y|$, $F(z) = -z$, $G(z) = -F(z)$ where $z = f(x, y)$

**3.** What is the value of $f\{G(f(1, 0)), f[F(f(1, 2)), G(f(1, 2))]\}$? _____

**4.** Which of the following expressions is equal to $x^2$?
(A) $F[f(x, -x)]\ G[f(x, -x)]$
(B) $F[f(x, x)]\ G[f(x, x)]$
(C) $-F[f(x, x)]\ G[f(x, x)] \div 4$
(D) $f(x, x)\ f(x, x)$

**5.** If $(a, b) * (c, d) = (ad - bc, ab - cd)$ and $(x, y) = [(1, 3) * (2, 4)] * [(5, 6) * (7, 8)]$ then $(x+y, x-y) * (x-y, x+y)$ equals
(A) $(42^2, 0)$ (B) $(-84^2, 0)$ (C) $(0, -84^2)$ (D) $(-42^2, 0)$

**6.** $a \odot b = \dfrac{1}{(a+1)(b+1)} - \dfrac{1}{ab}$.

$(1 \odot 2) + (2 \odot 3) + (3 \odot 4) + \ldots + (9 \odot 10) =$

(A) $\frac{-54}{55}$ (B) $\frac{-27}{55}$ (C) $\frac{-54}{101}$ (D) $\frac{27}{55}$

**7.** $a, b, c, d$ are four positive real numbers. Consider the following definitions.

(1) $f(a, b, c, d) = \min [\max (a, b, c), \max (a, b, d) \max (a, c, d), \max (b, c, d)]$

(2) $g(a, b, c, d) = \max [\max (a, b, c), \max (a, b, d) \max (a, c, d), \max (b, c, d)]$

(3) $h(a, b, c, d) = \max [\max (a, b, c), \min (a, b, d) \min (a, c, d) \max (b, c, d)]$

(4) $i(a, b, c, d) = \min [\max (a, b, c) \max (a, b, d) \max (a , c, d), \min (b, c, d)]$

Which of the following is undefined?

(A) $\dfrac{f(a,b,c,d)+g(a,b,c,d)}{h(a,b,c,d)-i(a,b,c,d)}$

(B) $\dfrac{g(a,b,c,d)-h(a,b,c,d)}{i(a,b,c,d)-f(a,b,c,d)}$

(C) $\dfrac{i(a,b,c,d)+f(a,b,c,d)}{h(a,b,c,d)-g(a,b,c,d)}$

(D) $\dfrac{g(a,b,c,d)-i(a,b,c,d)}{f(a,b,c,d)-h(a,b,c,d)}$

***Directions for question 8*:** The question is based on the information given below.

For the equation $ax^2 + 2hxy + by^2 = 0$, the operators $\Delta$ and $\nabla$ are defined as follows

$\Delta = \dfrac{1}{ab}\sqrt{h^2 + ab};$

$\nabla = \dfrac{1}{ab}\sqrt{h^2 - ab};$ and $\nabla^2 = \nabla \times, \nabla;\ \Delta^2 = \Delta \times \Delta.$

**8.** If $D = \dfrac{2}{b}\sqrt{h^2 - ab}$, then which of the following is true?

(A) $D = 2a\nabla$

(B) $\nabla = 2aD$

(C) $2\nabla = aD$

(D) $D = a\nabla$

***Directions for questions 9 and 10*:** These questions are based on the information given below.

For real numbers $x$ and $y$,

$L(x, y) = \lfloor x \rfloor + \lfloor y \rfloor + \lfloor x + y \rfloor;$

$R(x, y) = \lfloor 2x \rfloor + \lfloor 2y \rfloor;$

$T(x, y) = \lceil 2x \rceil + \lceil 2y \rceil.$

Where, $\lfloor x \rfloor$ denotes the greatest integer lesser or equal to $x$; and $\lceil x \rceil$ denotes the least integer greater or equal to $x$.

**9.** Which of the following is never true for positive real numbers $x$ and $y$?

(A) $L(x, y) \neq R(x, y)$

(B) $L(x, y) = R(x, y)$

(C) $L(x, y) < R(x, y)$

(D) $L(x, y) > R(x, y)$

**10.** For $x, y \in [0, 5]$, the number of pairs for which $R(x, y) = T(x, y)$, is

(A) 25 (B) 12 (C) 36 (D) 20

**11.** The natives of Bannanna island, use the 'BOSAMD' rule, which gives the order in which the mathematical operations Brackets (B), Of (O), Subtraction (S), Addition (A), Multiplication (M) and Division (D), should be applied.

Find the value of $13 \times 5 + 35 \div 8 - (2 \times 5)$, using the 'BOSAMD' rule. _____

***Directions for questions 12 and 13*:** These questions are based on the information given below.

$f(x, y) = e^{x+y}$; $g(x, y) = e^{x-y}$; $p(x, y) = \log_e(xy)$ and $q(x, y) = \log_e(x/y)$.

**12.** Find the value of $p(f(x, x), g(x, -x))$.

(A) $f(2x, 2x)$ (B) $2f(x, x)$

(C) $2g(x, x)$ (D) $q(e^6x, e^2x)$

**13.** The value of $f(p(x, y), q(x, y))$; for $x = 5$ and $y = 13$ is _____

***Directions for questions 14 and 15*:** These questions are based on the information given below.

For the equation $ax^2 + 2hxy + by^2 = 0$, the operators $\Delta$ and $\nabla$ are defined as follows

$$\Delta = \frac{1}{ab}\sqrt{h^2 + ab};$$

$$\nabla = \frac{1}{ab}\sqrt{h^2 - ab};$$

and $\nabla^2 = \nabla \times, \nabla;\ \Delta^2 = \Delta \times \Delta.$

**14.** Which of the following statements is true for the equation $3x^2 + 6xy + 4y^2 = 0$?

(A) $\nabla^2$ is positive. (B) $\Delta^2$ is negative.

(C) $\Delta^2 > \nabla^2$ (D) $\Delta^2 < \nabla^2$

**15.** If a new equation is formed by interchangingthe coefficients of $x^2$ and $y^2$ in the equation $4x^2 + 8xy - 7y^2 = 0$, then which of the following is/are true for the new equation?

(A) $\Delta^2 = \nabla^2$ (B) $\Delta^2 > \nabla^2$

(C) $\Delta^2 < \nabla^2$ (D) Both (B) and (C)

***Directions for questions 16 and 17*:** These questions are based on the information given below.

Let $[x]$ be the greatest integer less than or equal to $x$ and $(x)$ be the least integer greater than $x$.

**16.** Let $[A+B+C+D] = [A] + [B] + [C] + [D] + k$. Which of the following is not a possible value of $k$?
(A) 0 (B) 1 (C) 2 (D) 4

**17.** Let $(A+B+C) = (A) + (B) + (C) + i$. Which of the following is a possible value of $i$?
(A) 1 (B) 2 (C) 3 (D) –1

**18.** If $p \Delta q = p + q$, $p \nabla q = pq$ and $p \updownarrow q = p - q$, find the value of $x \nabla y$, where $x = \{(a \nabla a) \Delta (b \nabla b) \Delta (c \nabla c) \updownarrow (a \nabla b) \updownarrow (b \nabla c) \updownarrow (a \nabla c)\}$ and $y = (a \Delta b) \Delta c$. (Where there are no brackets, the operations have to be performed from left to right).
(A) $a^3 + b^3 + c^3$ (B) $a^2 + b^2 + c^2$
(C) $(a+b+c)^2$ (D) $a^3 + b^3 + c^3 - 3abc$

**19.** If $\$(x, y)$ = L.C.M. $(x, y)$, $\Delta(x, y)$ = H.C.F. $(x, y)$, $\nabla(x, y)$ = A.M. $(x, y)$ and $\pi(x, y)$ = remainder when $x$ is divided by $y$, then $\pi[\$\{\Delta\{\nabla\{\$(240, 180), 360\}, 420\}, 600\}, 2] =$ _____

**20.** If GOS $(a, b, c)$ is the geometric mean of $a$, $b$ and $c$, GOSS $(a, b, c)$ is the geometric mean of squares of $a$, $b$ and $c$, GOC $(a, b, c)$ is the geometric mean of cubes of $a$, $b$, and $c$, which of the following statements is not true for $a$, $b$, $c \geq 1$?
(A) GOC $(a, b, c) \geq$ GOSS $(a, b, c)$
(B) GOSS $(a, b, c) \geq$ GOS $(a, b, c)$
(C) GOC $(a, b, c) \geq$ GOS $(a, b, c)$
(D) GOSS $(a, b, c) \geq$ GOC $(a, b, c)$

## ANSWER KEYS

### Exercise-1

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (C) | 7. (B) | 13. 36 | 19. 16 | 25. (D) | 31. (B) | 37. (D) |
| 2. (C) | 8. (B) | 14. (B) | 20. 1.75 | 26. (D) | 32. (A) | 38. (A) |
| 3. (B) | 9. (C) | 15. 5 | 21. (A) | 27. (C) | 33. (B) | 39. (B) |
| 4. (D) | 10. (B) | 16. 23 | 22. (C) | 28. (A) | 34. (C) | 40. (D) |
| 5. (C) | 11. 48 | 17. 129 | 23. (B) | 29. (D) | 35. 3 | |
| 6. (D) | 12. 2 | 18. (B) | 24. (D) | 30. (B) | 36. (D) | |

### Exercise-2

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (C) | 6. (A) | 11. (A) | 16. (D) | 21. (D) | 26. (C) | 31. (B) |
| 2. 58 | 7. (B) | 12. (C) | 17. (A) | 22. (C) | 27. (A) | 32. (A) |
| 3. 1 | 8. (A) | 13. (D) | 18. (D) | 23. (C) | 28. (B) | 33. (B) |
| 4. (A) | 9. (B) | 14. (A) | 19. (A) | 24. (B) | 29. (C) | 34. (A) |
| 5. (D) | 10. (C) | 15. (A) | 20. (C) | 25. (C) | 30. 22 | 35. (C) |

### Exercise-3

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (C) | 7. 3 | 13. (A) | 19. (D) | 25. (D) | 31. (D) | 37. (C) |
| 2. (A) | 8. (C) | 14. (C) | 20. (C) | 26. (D) | 32. 2288 | 38. (B) |
| 3. (D) | 9. (A) | 15. (D) | 21. (B) | 27. (D) | 33. (A) | 39. (B) |
| 4. 784 | 10. (D) | 16. 528 | 22. (D) | 28. (C) | 34. 0 | 40. (D) |
| 5. (B) | 11. 7 | 17. 217 | 23. 411 | 29. (D) | 35. 6 | |
| 6. (A) | 12. 1224 | 18. (C) | 24. (D) | 30. 858 | 36. 1.5 | |

### Exercise-4

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (C) | 4. (C) | 7. (C) | 10. (C) | 13. 25 | 16. (D) | 19. 0 |
| 2. (B) | 5. (B) | 8. (A) | 11. –260 | 14. (C) | 17. (D) | 20. (D) |
| 3. 1 | 6. (B) | 9. (D) | 12. (D) | 15. (C) | 18. (D) | |

## SOLUTIONS

## EXERCISE-1

***Solutions for questions 1–20:***

**1.** $c^2 = c \oplus c = c$
$\therefore\ c^{100} = c$
$a = a$
$a^2 = a \oplus a = b$
$a^3 = a^2 \oplus a = b \oplus a = d$
$a^4 = a^3 \oplus a = d \oplus a = c$
Similarly, $a^5$, $a^6$, $a^7$, $a^8$ are $a$, $b$, $d$, and $c$, respectively.
$\therefore\ a^{100} = c$
$\therefore\ c^{100} \oplus a^{100} = c \oplus c = c$

**2.** $(a \oplus b) \oplus (d \oplus c)$
$= d \oplus d = b$

**3.** $a \oplus b \oplus (c \oplus d) = a \oplus b \oplus d$
$= (a \oplus b) \oplus d = d \oplus d = b$
or $a \oplus (b \oplus d) = a \oplus a = b$

**4.** $d^2 = d \oplus d = b$
$d^3 = b \oplus d = a$
$d^4 = a \oplus d = c$
$\therefore\ n = 4$

**5.** $\left(\frac{1}{2}\right)^{ab}$ is positive for any two real numbers $a$ and $b$.

**6.** Option (A)

$\frac{1}{2} \rightarrow \frac{1}{6} = 2\left(\frac{1}{2}\right)\frac{1}{6} = \frac{1}{6}$ not an integer.

Option (B)

$\frac{1}{3} \leftarrow \frac{3}{5} = \frac{3\frac{1}{3}}{2\frac{3}{5}} = \frac{5}{6}$ not an integer.

Option (C)

$\frac{1}{8} \leftarrow \frac{9}{16} = \frac{3\frac{1}{8}}{2\frac{9}{16}} = \frac{1}{3}$ not an integer.

Option (D)

$\frac{3}{5} \rightarrow \frac{5}{6} = 2\frac{3}{5}\frac{5}{6} = 1$ is an integer.

**7.** $x \oplus y$ = product of $x$ and $2y = 2xy$
$4 \oplus 5 = 2(4)\ (5) = 40$.

**8.** We consider the options,
$a \rightarrow b = a + 2b$.
$a \leftarrow b = 2a - b$.

Option (A)

$\frac{1}{3} \rightarrow \left(-\frac{1}{2}\right) = \frac{1}{3} + 2\left(-\frac{1}{2}\right) = -\frac{2}{3} < 0$

Option (B)

$\frac{1}{5} \leftarrow \left(-\frac{1}{3}\right) = \frac{2}{5} + \frac{1}{3} = \frac{6+5}{15} > 0$

Option (C)

$\left(\frac{-1}{6}\right) \leftarrow \frac{1}{5} = -\frac{2}{6} - \frac{1}{5} = \frac{10-6}{30} = < 0$

Option (D)

$-5 \rightarrow 2 = -5 + 2\ (2) = -1 < 0$.

**9.** Given
$a \nabla b = a^2 - b^2 + ab$
$2 \nabla 3 = 2^3 - 3^2 + 2(3)$
$4 - 9 + 6 = 1$.

**10.** For any values of $a$ and $b$,

$a \in b = \left(\frac{1}{3}\right)^{ab}$ is always positive.

**11.** L {L[L[L(L(1, 3), 6), 12], 24], 48}
= L {L[L[6, 12], 24], 48} = 48

**12.** H {H[H[H(H(64, 32), 16), 8], 4], 2}
= H {H[H[64, 8], 4], 2} = 2

**13.** H [L(4,18), L(12,18)] = H (36, 36) = 36

**14.** $(2 \leftarrow 3)\ (4 \leftarrow 3) = \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2$

$\therefore$ Choice (B) gives an integer value.

**15.** 3 – 2 = sum of 3 and 2 = 5

**16.** $2 \rightarrow 3 = 2 \times 2 + 3 \times 3 - 2 \times 3 = 7$
$4 \leftarrow 3 = 4 \times 3 - (4 + 3) = 5$
$(2 \rightarrow 3) \leftarrow (4 \leftarrow 3) = 7 \leftarrow 5 = 7 \times 5 - (7 + 5)$
$= 23$

**17.** Given $x \Delta y = x^2 + y^2 - xy$
$3 \Delta 4 = 3^2 + 4^2 - 3 \times 4 = 13$
$(3 \Delta 4) \Delta 5 = 13 \Delta 5 = 13^2 + 5^2 - 13 \times 5 = 129$

**18.** $M = \frac{1 + 2 + \ldots + 10}{10} = \frac{10(11)}{2 \times 10}$ $\left(\therefore \Sigma n = \frac{n(n+1)}{2}\right)$

$M = \frac{11}{2}$

$N = \frac{1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2}{6} = \frac{6(7)(13)}{6 \times 6} = \frac{91}{6}$

$M \downarrow N = \frac{11}{2} \downarrow \frac{91}{6} = \frac{11}{2} + \frac{91}{6} - \frac{11}{2}\left(\frac{91}{6}\right)$

$= \frac{33+91}{6} - \frac{11}{2}\left(\frac{91}{6}\right)$

$= \frac{248-1001}{12} = \frac{-753}{12} = -\frac{251}{4}$

**19.** $f(8, 32, 14) = \max(8, 32, 14) = 32$
$g(2, 8, 12, 16) = \text{L.C.M.}(2, 8, 12, 16) = 48$
$h[f(8, 32, 14), g(2, 8, 12, 16)] = h(32, 48) = \text{H.C.F.}(32, 48) = 16.$

**20.** $(5 * 6) = \frac{5+6}{5-6} = -11$

$(5 * 6) * -3 = -11 * -3 = \frac{-11-3}{-11+3} = \frac{7}{4}$

$= 1.75$

***Solutions for questions 21–24:***

**21.** $(b * c) * (d * a) = b * d = c$

**22.** $c^2 = c * c = d,\ c^3 = d * c = c$
$\therefore n = 3.$

**23.** $d^2 = d * d = a,\ \ d^3 = a * d = d,\ d^4 = a,$ etc.
$a^2 = a * a = b,\ a^3 = b * a = a,\ a^4 = b,$ etc.
$\therefore d^6 * a^5 = a * a = b.$

**24.** $(a * b) * c = c * c = d$
While $a * (b * c) = a * (b * c) = a * b = c$
Unless the order in which the operations are performed are indicated, we cannot determine the value of $a * b * c$.

***Solutions for questions 25–40:***

**25.** $\gamma(x, y) = x + y \quad \alpha(x, y) = x^3 + y^3$
$\delta(x, y) = x - y \quad \beta(x, y) = x^3 - y^3$
$\therefore \beta(1, 1) = 0$ and $\gamma(2, 1) = 3$
$\therefore \alpha(\beta(1, 1), (2, 1)) = 0^3 + 3^3 = 27.$

**26.** Option (A)

$\frac{\alpha(x, y) - \beta(x, y)}{\gamma(x, y) - \delta(x, y)} = \frac{2y^3}{2y} = y^2$

Option (B)

$\frac{\alpha(x, y) - \beta(x, y)}{\gamma(x, y) + \delta(x, y)} = \frac{\left(x^3 + y^3\right) - \left(x^3 - y^3\right)}{(x + y) + (x - y)} = \frac{y^3}{x}$

Option (C)

$\frac{\alpha(x, y)\beta(x, y)}{\gamma(x, y)\delta(x, y)} = \frac{\left(x^3 + y^3\right)\left(x^3 - y^3\right)}{(x + y)(x - y)}.$

**27.** $\delta = x - y \qquad \beta = x^3 - y^3$
$\gamma = x + y \qquad \alpha = x^3 + y^3$
As $1 < y < x$, $\delta < \gamma$ and $\beta < \alpha$. Also, $\delta < \beta$ and $\gamma < \alpha$ but $\gamma$ may be less than, equal to or greater than $\beta$.
Among the options, only $\delta < \beta < \alpha$ is true.

**28.** $a \odot e = a$
$\Rightarrow a + e + ae = a$
$\Rightarrow e(1 + a) = 0$
$\Rightarrow e = 0\ (\because a \neq -1)$

**29.** $a \odot 2 = 0$
$\Rightarrow a + 2 + 2a = 0$
$\Rightarrow 3a = -2$
$\Rightarrow a = \frac{-2}{3}$

**30.** $a \odot 1 = a$
$\Rightarrow a + 1 + a = a$
$\Rightarrow a + 1 = 0$
$\Rightarrow a = -1$

**31.** For all $a, b$,
$a \oplus b = a + b - ab$
$\therefore a \oplus e = a + e - ae$
$a = a + e - ae$ (given)
$\Rightarrow e(1 - a) = 0 \Rightarrow e = 0\ \therefore a \neq 1.$

**32.** $a \oplus 3 = 0 \Rightarrow a + 3 - 3a = 0.$

$\Rightarrow 3 - 2a = 0 \Rightarrow a = \frac{3}{2}.$

**33.** $a \oplus 1 = a$
$\Rightarrow a + 1 - a = a \Rightarrow a = 1.$

**34.** Given $t(a, b) = \text{L.C.M. of } a, b$
$H(a, b) = \text{H.C.F. of } a, b$
$L(3, 4) = \text{L.C.M. of } 3, 4 = 12$
$L(5, 7) = \text{L.C.M. of } 5, 7 = 35$
$H(L(3, 4), (5, 7) = H(12, 35) = 1.$

**35.** $1 \Delta 2 = (1)^2 + (2)^2 - 1\ (2) = 3$

**36.** Choice (D) is the sum of two squares. It has to be non-negative.

**37.** Given $f(a, b) = a - b$
$g(a, b) = (a - b)^2$
$h(a, b) = (a - b)^3$
since $a$ and $b$ distinct,
when $a < b\ h(a, b) < f(a, b) < g(a, b)$
$a > b\ h(a, b) > g(a, b) \geq f(a, b).$

**38.** $h(3, 4) = (3 - 4)^3 = -1$
$f(3, 6) = 3 - 6 = -3$
$g(1, 3) = (1 - 3)^2 = 4$

$\frac{h(3,4) - f(3,6)}{g(13)} = \frac{-1+3}{4} = \frac{2}{4} = \frac{1}{2}.$

**39.** $\frac{h(5,7) + g(8,6)}{f(1,3) - h(1,5)} = \frac{(-2)^3 + \left(2^2\right)}{(-2) - (-4)^3}$

$= \frac{-8+4}{-2+64} = \frac{-4}{62} = \frac{-2}{31}.$

**40.** (A) $3 \# 4 = 3^2 + 4^2 = 25$ is divisible by 5
(B) $5 \rightarrow 7 = (5 + 7)\ (5 \times 7)$ is divisible by 5
(C) $7 \leftarrow 8 = (7 - 8)\ (7 \times 8) = -56$ is not divisible by 5

## EXERCISE-2

***Solutions for questions 1–20:***

**1.** $\frac{a \oplus b}{a \otimes b} = \frac{a^3 + b^3}{a^2 + b^2 - ab} = \frac{(a+b)\left(a^2 + b^2 - ab\right)}{a^2 + b^2 - ab} = a + b$

**2.** $2 * 3 = 2^3 + 3^3 = 35$

$2 \Delta 3 = 2^3 - 3^3 = -19$

$(2 * 3) \rightarrow (2 \Delta 3) = (35) \rightarrow (-19) = 35 - (-19) = 54$

$\{(2 * 3) \rightarrow (2 \Delta 3)\} \nabla 4 = 54 \nabla 4 = 54 + 4 = 58$

**3.** $\frac{A(a,b) - B(a,b)}{C(a,b)} = \frac{(a+b)^2 - (a-b)^2}{4ab} = \frac{4ab}{4ab} = 1$

**4.** $a \uparrow b = a^2 + b^2 - 3ab$

$a \downarrow b = a^2 - b^2 + 3ab$

$(a \uparrow b) * (a \downarrow b) = (a^2 + b^2 - 3ab) * (a^2 - b^2 + 3ab)$

$= \sqrt{\frac{a^2 + b^2 - 3ab + a^2 - b^2 + 3ab}{2}} = a$

***Solutions for questions 5–7:***

**5.** $f(x, x) = \frac{a^x + a^{-x}}{2}$

$g(x, -x) = \frac{a^x - a^{-(-x)}}{2} = 0$

$\therefore\ q(f(x, x), g(x,-x)) = q\left(\frac{a^x + a^{-x}}{2}, 0\right)$

$= \log_a\left(\frac{a^x + a^{-x} - 0}{a^x + a^{-x} + 0}\right) = \log_a 1 = 0$

**6.** $f(p(x, y), q(x, y)) = \frac{a^{\log_a \frac{x+y}{x-y}} + a^{-\left(\log_a \frac{x-y}{x+y}\right)}}{2}$

$= \frac{\frac{x+y}{x-y} + \frac{x+y}{x-y}}{2} = \frac{(x+y)}{(x-y)} \rightarrow$ (1)

$g(q(x, -y), p(x, y)) = \frac{a^{\log_a \frac{x+y}{x-y}} - a^{-\log_a \left(\frac{x+y}{x-y}\right)}}{2}$

$= \frac{\frac{x+y}{x-y} - \frac{x-y}{x+y}}{2} = \frac{(x+y)^2 - (x-y)^2}{2(x^2 - y^2)} = \frac{2xy}{x^2 - y^2}$

$\therefore\ \frac{f(p(x,y), q(x,y))}{g(q(x-y), p(x,y))} = \frac{\frac{(x+y)}{(x-y)}}{\frac{2xy}{x^2 - y^2}}$

$= \frac{(x+y)}{(x-y)} \frac{(x^2 - y^2)}{2xy} = \frac{(x+y)^2}{2xy}$

**7.** $f(6, 4) = \frac{a^6 + a^{-4}}{2}$

$g(6, 4) = \frac{a^6 - a^{-4}}{2}$

$p(f(6, 4), g(6, 4)) = \log_a \left( \frac{\frac{a^6 + a^{-4} + a^6 - a^{-4}}{2}}{\frac{a^6 + a^{-4} - (a^6 - a^{-4})}{2}} \right)$

$= \log_a \left(\frac{a^6}{a^{-4}}\right) = \log_a a^{10} = 10$

$q(10, 4) = \log_a \left(\frac{10 - 4}{10 + 4}\right) = \log_a \frac{6}{14} = \log_a \left(\frac{3}{7}\right)$

***Solutions for questions 8–11:***

**8.** Let $A = (8 \cup 12) = \frac{8+12}{8-12} = \frac{20}{-4} = -5$

Let $B = (15 \ominus 1) = 15 + 1 - 15(1) = 1$

$A \cap B = -5 \cap 1 = \frac{2(-5)(1)}{-5 + 1} = \frac{5}{2}$

**9.** Let $A = 21 \oplus 7 = 21 - 7 + 21(7) = 161$

Let $B = 12 \ominus 8 = 12 + 8 - 12(8) = -76$

$A \cup B = (161 \cup -76) = \frac{161 - 76}{161 - (-76)} = \frac{85}{237}$

**10.** $(5 \nabla 8) = (5 \cup 8)\ (5 \cap 8)$

$= \left(\frac{5+8}{5-8}\right)\left(\frac{2 \times 5 \times 8}{5+8}\right) = -\frac{80}{3}$

$(6 \Delta 3) = (6 \cup 3) - (6 \cap 3)$

$= \frac{6+3}{6-3} - \frac{2 \times 6 \times 3}{6+3} = -1$

$(5 \nabla 8) \cup (6 \Delta 3) = \left(-\frac{80}{3}\right) \cup (-1) = \frac{-\frac{80}{3} - 1}{-\frac{80}{3} + 1} = \frac{83}{77}$

**11.** Let $A = 6 \oplus 4 = 6 - 4 + 6(4) = 26$

Let $B = 6 \ominus 4 = 6 + 4 - 6(4) = -14$

$A \Delta B = (26) \Delta (-14)$

$= (26 \cup -14) - (26 \cap -14)$

$= \left(\frac{26 - 14}{26 + 14}\right) - \frac{2(26) - 14}{26 - 14}$

Difficulty Level-2: Moderate

$$= \frac{12}{40} + \frac{182}{3} = \frac{1829}{30}$$

Let $C = 8 \cup 7 = \dfrac{8+7}{8-7} = 15$

$\therefore (A \Delta B) \cap C$

$$= \left(\frac{1829}{30}\right) \cap (15) = \frac{2\left(\frac{1829}{30}\right)(15)}{\frac{1829}{30} + 15}$$

$$= \frac{54870}{2279}$$

***Solutions for questions 12–15:***

**12.** Consider choice (A)
Sweet $(a, b)$ = Sweet $(1, 1) = e^1 + e^1 = 2e \neq 2$
Consider choice (B)
Sour $(a, b) = e^a - e^b$ need not be always positive as Sour $(a, b)$ is negative when $b > a$
Consider choice (C)
Sweet $(a, b) = e^a + e^b$
Sweet $(0, 0) = e^0 + e^0 = 2$.

**13.** $e^{a+b} \neq e^a + e^b$
$\therefore$ Choice (A) is not correct
$e^{a-b} \neq e^a - e^b$
$\therefore$ Choice (B) is not correct
Sweet $(a, b)$ Sour $(a, b) = (e^a + e^b)(e^a - e^b) = e^{2a} - e^{2b}$
while Mint $(a, b)$ Hint $(a, b) = e^{2a}$
$\therefore$ Choice (C) is not correct.

**14.** Consider choice (A)
Mint$(a, b) \times$ Hint $(a, b) = e^{2a}$ while Sweet $(a, b)$ + Sour $(a, b) = 2e^a$
$\therefore$ Choice (A) is not true.
Consider (B) Sweet $(a, b) = e^a + e^b$
$= e^{-2} + e^{-2} = 1/e^2 + 1/e^2 = 2/e^2 < 1$ (since $e^2 > 2$)
$\therefore$ Consider (B) is true
Consider (3) Sour $(a, b) = e^a - e^b$
$a = 1/2$, $b = 1$ Sour $(a, b) = e^{1/2} - e < 0$
$\therefore$ Choice (C) is true
Consider choice (D)
Arithmetic Mean of Sweet $(a, b)$

and Sour $(a, b) = \dfrac{e^a + e^b - e^a + e^b}{2} = \dfrac{2e^a}{2} = e^a$

$= e^0$ (since $a = 0$) $= 1$
$\therefore$ choice (D) is true.

**15.** Consider Choice (A)
Mint $(a, b) = e^8 \Rightarrow e^{a+b} = e^8$
$\Rightarrow a + b = 8$ (1)
Hint $(a, b) = e^4$
$\Rightarrow e^{a-b} = e^4$
$\Rightarrow a - b = 4$ (2)
Solving (1) and (2) we can get $a$ and $b$.

***Solutions for questions 16–18:***

Without loss of generality, we may assume
$x < y < z$
$f(x, y, z) = \min(\text{ma}(x, y), \min(y, x), \min(x, z))$
$= \min(y, x, x) = x$
$g(x, y, z) = \max(\min(x, y), \min(y, z), \min(z, x)$
$= \max(x, y, x) = y$
$h(x, y, z) = \max(\max(x, y), \max(x, z), \max(z, x)$
$= \max(x, z, z) = z$
$i(x, y, z) = \min(\min(x, y) \min(y, z) \min(z, x)$
$= \min(x, y, x) = x$
$j(x, y, z) = \max(\min(x, y), \max(y, z), \max(z, x)$
$= \max(x, y, z) = z$
$k(x, y, z) = \min(\max(x, y), \max(y, z) \max(z, x)$
$= \min(y, z, z) = y$
These results are tabulated below:

| | | |
|---|---|---|
| $x$ | $y$ | $z$ |
| $F$ | $g$ | $h$ |
| $I$ | $k$ | $j$ |

**16.** Option A $= \dfrac{x+y}{y-z} < 1$

Option B $= \dfrac{x+z}{x-y} < 1$

Option C $= \dfrac{z+y}{x-x}$ = undefined

Option D $= \dfrac{x+y}{x+y} = 1$

Both (A) and (B) are less than 1.

**17.** Option (A) $= \dfrac{x-y}{x-y} > 0$

Option (B) $= \dfrac{z-z}{y-z} = 0$

Option (C) $= \dfrac{y-x}{y-z} < 0$

Option (D) $= \dfrac{x+z}{x-x}$ is undefined.

**18.** Option (A) $= \dfrac{y+z}{x-z} < 0$

Options (B) $= \dfrac{z-z}{y-x} = 0$

Option (C) $= \dfrac{y+x}{x-y} < 0.$

***Solutions for questions 19 and 20:***

**19.** $f(x, y) = -|x+y|$

$F(f(x, y)) = -f(x, y) = |x+y|$

$G\big(F(x,y)\big) = -F\big(f(x,y)\big) = -|x+y| = f(x, y)$

$G\big(f(2,3)\big) = -|2+3| = -5$

$G\big(f(4,3)\big) = -|4+3| = -7$

$F\big(f(4,3)\big) = (4+3) = 7$

$f(G\big(f(4,3)\big), F(f(4,3)) = -|-7+7| = 0$

$\therefore\ f\Big[\big(G\big(f(2,3)\big)\big), f\big(G\big(f(4,3)\big)\big), F\big(f(4,3)\big)\Big]$

$= f(-5, 0) = -|5+0| = -5.$

**20.** $F\big(f(x,y)\big) = 0$

$G(f(x, -x)) = 0$

$\therefore F(f(x, -x))\ Gf(x, -x) = 0$

Option (A) is not true

$F(f(x, x)) = 2x$

$G(f(x, x) = -2x.$

$F(f(x, x)\ G(f(x, x)) = -4x^2$. option (B) is not true

$$\frac{-F\big(f(x,x)\big)G\big(f(x,x)\big)}{4} = \frac{-\big(-4x^2\big)}{4} = x^2.$$

***Solutions for questions 21–23:***

Without loss of generality, let $x < y < z$.

$f(x, y, z) = \min(y, z, z) = y$

$g(x, y, z) = \max(x, y, x) = y$

$h(x, y, z) = \max(x, y, z) = z$

$k(x, y, z) = \min(x, y, z) = x$

$j(x, y, z) = \min(x, y, x) = x$

$i(x, y, z) = \max(y, z, z) = z$

As $x < y < z$,

$k = j < f = g < h = i$

**21.** (i) $\dfrac{f(x,y,z)-g(x,y,z)}{h(x,y,z)+j(x,y,z)} = \dfrac{y-y}{z+x} < 1$

(ii) $\dfrac{f(x,y,z)+k(x,y,z)}{g(x,y,z)+i(x,y,z)} = \dfrac{y+x}{y+z} < 1$

(iii) $\dfrac{h(x,y,z)-g(x,y,z)}{k(x,y,z)-i(x,y,z)} = \dfrac{z-y}{x-z} < 1$

(iv) $\dfrac{i-k}{h-g} = \dfrac{z-x}{z-y} > 1.$

**22.** (i) $\dfrac{-k(x,y,z)+f(x,y,z)}{h(x,y,z)} = \dfrac{-x+y}{z} \not< 0$

(ii) $\dfrac{h(x,y,z)+g(x,y,z)}{k(x,y,z)+j(x,y,z)} = \dfrac{z+y}{x+x} \not< 0$

(iii) $\dfrac{k(x,y,z)+g(x,y,z)}{j(x,y,z)-f(x,y,z)} = \dfrac{x+y}{x-y} < 0$

(iv) $\dfrac{g-i}{j-f} = \dfrac{y-z}{x-y} \not< 0$

**23.** If the function is indeterminate, then the denominator will be = 0. Among the options, only option C equals 0.

***Solutions for questions 24–26:***

**24.** Considering option (B), we get

$3 \to 7 = 2(3) + 3(7) = 27$

$((3 \to 7) \downarrow 9) \uparrow 5 = (27 \downarrow 9) \uparrow 5$

$= \left(\dfrac{4(27)}{9}\right) \uparrow 5 = \dfrac{3(12)(5)}{2} = 90$

**25.** Considering option (C), we get

$((a \to b) \uparrow b) \leftarrow ab) \downarrow b$

$((2a+3b) \uparrow b) = \dfrac{3(2a+3b)b}{2}$

$\left(\dfrac{6ab+9b^2}{2}\right) \leftarrow ab = \dfrac{4(6ab+9b^2)}{2} - 5ab = 7ab + 18b^2$

$(7ab+18b^2) \downarrow b = \dfrac{4(7ab+18b^2)}{b} = 28a + 72b$

**26.** Considering option (A), we get

$((4 \downarrow 5) \to 7) \uparrow 9) \leftarrow 8) = \dfrac{7198}{5}$

$((4 \uparrow 5) \downarrow 7) \to 9) \leftarrow 8) = \dfrac{1436}{7}$

$((4 \leftarrow 5) \to 7) \downarrow 9) \uparrow 8) = 16$

$((4 \to 5) \leftarrow 7) \downarrow 9) \uparrow 8) = 304$

***Solutions for questions 27–30:***

**27.** $((9 \uparrow 7) \to 4) \downarrow 29 = (34 \to 4 \downarrow 29)$

$= (34 \times 12) \downarrow 29$

$= (34 \times 24) - 3 \times 29$

$= 729$

which is a perfect cube.

The other three expressions are not perfect cubes.

$((9 \to 7) \downarrow 4) \uparrow 29$
$= (189 \downarrow 4) \uparrow 29$
$= 366 \uparrow 29$
$= 3 \times 366 + 29 = 1127$
$((9 \downarrow 7) \uparrow 4) \to 29 = -435$
$((9 \uparrow 7) \downarrow 4) \to 29 = 4872$

**28.** Consider
$(15 \uparrow 6 \to 9) \leftarrow 2 = 1377$
$(15 \to 6) \uparrow 9) \leftarrow 2 = 819$
$(15 \downarrow 6) \leftarrow 9) \to 2 = 16$
$(15 \leftarrow 6) \downarrow 9) \uparrow 2 = -49$
Among these, only 819 is divisible by 13.

**29.** $\sqrt[3]{30 \uparrow 35} - \sqrt{14 \downarrow 4}$

$= \sqrt[3]{90 + 35} - \sqrt{28 - 12}$

$= \sqrt[3]{125} - \sqrt{16} = 5 - 4 = 1$

**30.** Given $\$(x, y)$ = H.C.F. $(x, y)$, $\Delta(x, y)$ = AM $(x, y)$, $\nabla$ $(x, y)$ = L.C.M. $(x, y)$, $\sigma(x, y)$ = quotient when $x$ is divided by $y$

$\Delta(240, 180) = \text{AM}(240, 180) = \dfrac{240 + 180}{2} = 210$

$\$(\Delta(240,180), 70) = \$(210, 70) = \text{H.C.F.}(210, 70) = 70$
$\nabla(70, 50) = \text{L.C.M. of } (70, 50) = 350$
$\Delta(350, 90) = \text{AM}(350, 90)$

$= \dfrac{350 + 90}{2} = 220$

$\sigma(220, 10)$ = Quotient when 220 is divided by 10 = 22

**31.** Given $a * b = a \oplus b$

$\dfrac{ab}{3} = a + b - ab$

$\dfrac{4ab}{3} = a + b$

$\dfrac{4}{3} = \dfrac{1}{b} + \dfrac{1}{a}$

**32.** $3 * 5 = \dfrac{(3)(5)}{3} = 5$

$((3 * 5) \oplus 7) = (5 \oplus 7) = 5 + 7 - 35 = -23$

$(-23 * 9) = \dfrac{-23\,(9)}{3} = -69$

$-69 \oplus 4 = -69 + 4 + 69\ (4) = 211$

***Solutions for questions 33–35:***

**33.** $a \$ b = a^{a^3 - b^3} + b.$

$a * b = (a + b)^{a^3 - b^3}$

$\therefore 2 \$ 1 = 2^{2^3 - 1} + 1^{2^3 - 1} = 2^7 + 1$

and $2 * 1 = (2 + 1)^{2^3 - 1} = 3^7$

$(2 \$ 1) - (2 * 1) = 2^7 + 1 - 3^7$

**34.** Consider option (B)

$a \wedge b > b^{a^3 - b^3}$

$a = 1;\ b = 2$ we get $2^{1^3 - 2^3} = 2^{-7} = \dfrac{1}{2^7} < 1$

$\therefore a \wedge b > 1$ is false

Consider option (C)
Given $a * b = a \$ b$
Let $a = b$

$a * b = (a + b)^{a^3 - b^3} = (2a)^{a^3 - a^3} = 1$

$a \$ b = a^{a^3 - b^3} + b^{a^3 - b^3}$

$= a^{a^3 - a^3} + a^{a^3 - a^3} = 1 + 1 = 2$

$a * b \neq a \$ b$

Consider option (A)

$a \vee b = a^{a^3 - b^3}$

$a \wedge b = b^{a^3 - b^3}$

$\dfrac{a \vee b}{a \wedge b} = \dfrac{a^{a^3 - b^3}}{b^{a^3 - b^3}} = \left(\dfrac{a}{b}\right)^{a^3 - b^3}$

If $a < b$, $a^3 - b^3 \leq 0$ and $= \left(\dfrac{a}{b}\right)^{a^3 - b^3} \geq 1$

Similarly if $a > b$, $\left(\dfrac{a}{b}\right)^{a^3 - b^3} \geq 1$

In both cases $\left(\dfrac{a}{b}\right)^{a^3 - b^3} \geq 1$

$\therefore \dfrac{a \vee b}{a \wedge b} > 1 \Rightarrow a \vee b \geq a \wedge b$

**35.** Given $a = 1$, $b = 2$

$a * b = (a + b)^{a^3 - b^3}$

$= (1 + 2)^{1 - 2^3} = 3^{-7} = \dfrac{1}{3^7} > 0$

which is true.

$a \vee b = a^{a^3 - b^3} = 1^{1 - 8} = 1$

It is also true.

$a \wedge b = b^{a^3 - b^3} = 2^{1 - 8} = \dfrac{1}{2^7}$

$a \$ b = a^{a^3 - b^3} + b^{a^3 - b^3} = 1 + \dfrac{1}{2^7}$

$a \$ b = 1 + a \wedge b$

$\therefore$ Choice (C) is not true.

## EXERCISE-3

***Solutions for questions 1–40:***

**1.** $(3 \times 9) + 28 \div 7 \times 24 - 10$
$= (27 + 28) \div 7 \times 24 - 10 = 55 \div 7 \times 24 - 10$
$= \frac{55}{7} \times 24 - 10 = \frac{55}{7} \times 14 = 110$

**2.** $h(x, y) = f(x, y) \times g(x, y) = a^{x+y} \times a^{x-y} = a^{2x}$

$I(x, y) = \frac{f(x,y)}{g(x,y)} = \frac{a^{x+y}}{a^{x-y}} = a^{2y}$

$\frac{h(x,y)}{I(x,y)} = \frac{a^{2x}}{a^{2y}} = a^{2(x-y)} = (a^{x-y})^2 = (g(x, y))^2$

**3.** $h(x, y) \times I(x, y) = a^{2x} \times a^{2y} = a^{2(x+y)} = (a^{x+y})^2$
$= (f(x, y))^2$.

**4.** $4 \sim 5$ = H.C.F. of $4^2$ and $5^2 = 1$
$(4 \sim 5) \Delta 6 = 1 \Delta 6 = (1 + 6)^2 - 4\ (1)\ (6) = 25$
$((4 \sim 5) \Delta 6) \nabla 3 = 25 \nabla 3$
$= (25 - 3)^2 + 4(25)\ 3 = 784$

**5.** $a \% b$ = L.C.M. of $a^2$ and $b^2$
= L.C.M. of $5^2$ and $6^2 = 900$
$a \Delta b = (a + b)^2 - 4ab = (5 + 6)^2 - 4(5)6 = 1$
$a \sim b$ = H.C.F. of $a^2$ and $b^2$ = H.C.F. of $5^2$ and $6^2 = 1$
$a \nabla b = (a - b)^2 + 4ab = (5 - 6)^2 + 4(5)\ (6) = 121$
$\therefore$ From the above, $a \Delta b = a \sim b$

**6.** $a$ and $b$ are distinct integers.
$a \Delta b = (a + b)^2 - 4ab = (a - b)^2$
$a \nabla b = (a - b)^2 + 4ab = (a + b)^2$
When $a > 0$; $b < 0$ $a \Delta b > a \nabla b$
$\therefore$ Choice (B) is not false.
$a \% b$ = L.C.M. of $a^2$, $b^2$
$a \nabla b = (a + b)^2$
Let $a = 1$; $b = 2$
$\therefore$ $a \% b = 4$; $a \nabla b = 9$
$a \% b < a \nabla b$ Choice (C) is also not false.
Since $a$ and $b$ are distinct integers H.C.F. of $a^2$, $b^2$ is always less than L.C.M. of $a^2$, $b^2$
$a \sim b < a \% b$
Choice (A) is false.

**7.** If $c^n = c$
$c^3 = (c * c) * c = c$
$\therefore n = 3$.

**8.** To evaluate the expression, we need the 'powers' of $b$, $d$ and the 'multiplies' of $a$ and $c$.
These are tabulated below:

| $x$ | $a$ | $c$ |
|---|---|---|
| $x$ | $a$ | $c$ |
| $2x$ | $b$ | $d$ |
| $3x$ | $d$ | $b$ |
| $4x$ | $a$ | $c$ |
| $5x$ | $b$ | $d$ |

| $x$ | $b$ | $d$ |
|---|---|---|
| $x$ | $b$ | $d$ |
| $x^2$ | $d$ | $d$ |
| $x^3$ | $b$ | $d$ |
| $x^4$ | $d$ | $d$ |
| $x^5$ | $b$ | $d$ |

The given expression is
$[(b^5 * 5c) \oplus d^5] * 4a = [(d * d) \oplus d] * a$
$= [d \oplus d] * a = d * a = a$.

**9.** The table for $*$ is symmetric about the main diagonal, i.e., for all $x$, $y$.
$x * y = y * x$
$\therefore$ $*$ is commutative, while $\oplus$ is not commutative.
$\oplus$ does not satisfy the associative property. Consider
$(a \oplus b) \oplus c = d \oplus c = b$
while $a \oplus (b \oplus c) = a \oplus c = a$
$\therefore$ $\oplus$ is not associative.

**10.** $(a * b) = c$
$c \oplus c = d$
$d * d = d$
$d \oplus a = a$.

**11.** If $a^n = a$, $n = 3, 5, 7$, etc.
If $na = a$, $n = 4, 7, 10$, etc.
$\therefore$ The minimum value of $n$ such that $a^n = na = a$ is 7.

**12.** $\left[\left(\frac{9}{2} - 6\right) \times 4 + 3\right]$

$\frac{9}{2} = 4(9) - 3(2) = 30$

$(30 - 6) = \frac{30(2)}{5(6)} = 2$

$2 \times 4 = 3(2) + 7(4) = 34$
$34 + 3 = 12\ (34)\ (3) = 1224$.

**13.** Consider (A) option

$\frac{[(5+6) - 2]}{4}$

$5 + 6 = 12(5)\ (6) = 360$

$(360 - 2) = \frac{2(360)}{5(2)} = 72$

$\frac{72}{4} = 4(72) - 3(4) = 4(69) = 12(23)$.

**14.** $7 \times 9 = 3(7) + 7(9) = 84$

$\frac{84}{11} = 4(84) - 3(11) = 303$

$303 - 13 = \frac{2(103)}{5(13)} = \frac{606}{65}$.

**15.** Option (A)

$10 - 8 = \frac{2(10)}{5(8)} = \frac{1}{2}$

$\frac{1}{2} \times 16 = 3\left(\frac{1}{2}\right) + 7(16) = \frac{227}{2}$

$\frac{227}{2}/4 = 4\ \frac{227}{2} - 3(4) = 442$

Option (B)

$10 - 16 = \frac{2(10)}{5(16)} = \frac{1}{4}$

$\frac{1}{4} \times 8 = 3\left(\frac{1}{4}\right) + 7(8) = 56\frac{3}{4}$

$\left(56\frac{3}{4}\right)/4 = 4\left(\frac{227}{4}\right) - 3(4) = 215$

Option (C)

$(10 + 8) = 12(10)\ (8) = 960$

$960 - 4 = \frac{2(960)}{5(4)} = 96$

$\frac{96}{16} = 4(96) - 3(16) = 336$

Option (D)

$10 - 4 = \frac{2(10)}{5(4)} = 1$

$1 + 8 = 12(1)\ (8) = 96$

$96 \times 16 = 3(96) + 16(7) = 400 = 20^2$

**16.** $17 \times 10 = 3(17) + 7(10) = 121$ and $\sqrt{121} = 11$

$\frac{25}{12} = 4(25) - 3(12) = 64$ and $\sqrt[3]{64} = 4$

$11 + 4 = 12(11)\ (4) = 528.$

**17.** $(9 \uparrow 3)$ = L.C.M. of 9 and 3 = 9

$(16 \downarrow 24)$ = H.C.F. of 16 and 24 = 8

$(9 \uparrow 3)\ \Delta\ (16 \downarrow 24) \quad = 9\ \Delta\ 8 = (9 - 8)\ (9^2 + 9 \times 8 + 8^2)$

$= 9^3 - 8^3 = 729 - 512 = 217$

**18.** Here, $a = 5,\ b = 25$

$a \uparrow b$ is 25 while $a \downarrow b$ is 5.

$a\ \Delta\ b = (-20)\ (25 + 125 + 625) = -15500$

Hence, $(a \uparrow b) + (a \downarrow b) + (a\ \Delta\ b) = -15495$

Consider choice (B).

$(a \uparrow b) \times (a \downarrow b)$ = L.C.M. $(a, b) \times$ H.C.F.$c(a, b)$

$= ab$ and $\sqrt[3]{ab} = \sqrt[3]{125} = 5 = a$

$a \rightarrow b = (5 + 25)^3 - (5 - 25)^3 = 30^3 + 20^3$

$a \leftarrow b = (5 + 25)^3 + (5 - 25)^3 = 30^3 - 20^3$

$\therefore\ a \rightarrow b > a \leftarrow b.$

**19.** H.C.F. × L.C.M. = product of two numbers

$(a \uparrow b)\ (a \downarrow b) = ab$

$\Rightarrow (a \uparrow b)\ (a \downarrow b)$ is divisible by both $a$ and $b$.

**20.** $a\ \Delta\ b = 335$

$\Rightarrow (a - b)\ (a^2 + ab + b^2) = 335 \Rightarrow a^3 - b^3 = 335$

Choices (A) and (C) are ruled out as the values of $a$ and $b$ do not satisfy the above equation.

Choice (B) is ruled out as there may be some other possible values for $a - b$ and $a^2 + ab + b^2$.

With $a = 7,\ b = 2,\ a\ \Delta\ b = 335.$

Choice (A) is not correct.

With $a = 5,\ b = 2,\ a \rightarrow b = 7^3 - 3^3 = 316.$

Choice (B) is not correct.

While in , $(a \rightarrow b) + (a \leftarrow b)$

$= 2(a + b)^3 = 2(11)^3 = 2662.$

**21.** When, $x = 2.5;\ y = 1.5$

$f(x, y) = 12,\ g(x, y) = 13$

$\therefore f(x, y) < g(x, y)$

Option (A) is false.

When, $x = 2.5;\ y = 1.5$

$f(x, y) = 12$

$h(x, y) = 11$

$f(x, y) > h(x, y)$

Option (C) is false.

When $x$ and $y$ are integers $g(x, y) = h(x, y)$

When $x$ and $y$ are not integers

$g(x, y) > h(x, y)$

$\therefore$ Option (B) is always true.

**22.** When $x = \frac{1}{3}$ and $y = \frac{1}{3}$

then $g(x, y) = h(x, y)$

So for option (A), the statement is not true.

When $x = 2$ and $y = \frac{1}{3}$

then $g(x, y) = h(x, y)$

So for option (B), the statement is not true.

When $x = \frac{1}{3}$ and $y = 2$

then $g(x, y) = h(x, y)$

So for option (C), the statement is not true.

**23.** $f(3.5, 7.9) = 7 + 16 + 12 = 35$

$g(35, 8.2) = 105 + 25 = 130$

$h(130, 7) = 390 + 21 = 411$

**24.** $3x^2 + 5y^2 + 7x + 11y + 12 = 0$

$a = 3;\ b = 5,\ 2g = 7,\ 2f = 11$

$c = 12$

$\Delta = \frac{2}{b}\sqrt{g^2 - ac}$

$= \frac{2}{5}\sqrt{\frac{49}{4} - 36} = \frac{2}{5}\sqrt{\frac{-95}{4}} \quad \therefore \Delta^2 = \frac{-19}{5}$

$\nabla = \frac{2}{3}\sqrt{\frac{-119}{4}} \Rightarrow \nabla^2 = \frac{-119}{9}$

As $\frac{19}{5} < \frac{119}{9}, \frac{-19}{5} > \frac{-119}{9}$

i.e., $\Delta^2 > \nabla^2$

Choices (B) and (C) are true.

**25.** Given $5x^2 + 7y^2 + x - 3y - 15 = 0$

$$\Delta = \frac{2}{b}\sqrt{g^2 - ac} = \frac{2}{7}\sqrt{\left(\frac{1}{2}\right)^2 - (-15)5}$$

$$= \frac{2}{7}\sqrt{\frac{1}{4} + 75} = \frac{2}{7}\sqrt{\frac{301}{4}}$$

$$\therefore \Delta^2 = \frac{301}{49} \approx 6.$$

***Solutions for questions 26–29:***

$a \uparrow b = (2a + b)^2 - (2a - b)^2 = (4a)\ (2b) = 8ab$
$a \downarrow b = (2a + b)^2 + (2a - b)^2 = 2(4a^2 + b^2) = 8a^2 + 2b^2$
$a \leftrightarrow b - (2a + b)\ (2a - b) = 4a^2 - b^2$

**26.** We consider the choices,
(A) $a = 3, b = 5\ a \rightarrow b =$ L.C.M. of 6 and$15 = 15$. False
(B) $a = 6, b = 4, a \leftarrow b =$ H.C.F. $(2(6), 4) = 2$. False
(C) $a = 1, b = 8, a \leftrightarrow b - 4(1) - 64 = 22$. False
(D) $a = 7, b = 6$ then $a \uparrow b = 8(7)\ (6) = 336$. True

**27.** The given expression is $[\{(4 \rightarrow 6) \leftarrow 3\}\ 2] \leftrightarrow 1$.
$4 \rightarrow 6 =$ L.C.M. $(8, 6) = 24$
$24 \leftarrow 3 =$ H.C.F. $(48, 3) = 3$
$3\ 2 = 8(3)\ (2) = 48$
$48 \leftrightarrow 1 = 96^2 - 1 = 9215$

**28.** We consider the options,
(A) the H.C.F. is always less than L.C.M. of $2a$ and $b$.
Not necessarily true. When $2a = b$
H.C.F. = L.C.M.
(B) $a \uparrow b = (2a + b)^2 - (2a - b)^2 = 8ab$
$a \leftrightarrow b = (2a + b)\ (2a - b) = 4a^2 - b^2$
take $a = 1$: and $b = 2$
Statement is false
(C) $8a^2 + 2b^2 > 4a^2 - b^2$ true.

**29.** $(a \rightarrow b) =$ L.C.M. of $2a$ and $b$ it is always divisible by both $a$ and $b$. choice (A) is false.
$(a \leftarrow b)$ is H.C.F. of $a$ and $b$.
It is always divisible by both $a$ and $b$.
choice (B) is false.
$a \uparrow b = (2a + b)^2 - (2a - b)^2 = 4.2ab$ which always divisible by both $a$ and $b$
$a \downarrow b = (2a + b)^2 + (2a - b)^2 = 2(4\ a^2 + b^2)$
$(a \uparrow b)\ (a \downarrow b) = 16ab\ (4a^2 + b^2)$ it is always divisible by both $a$ and $b$. choice 4 is true.

**30.** $14/3 = 6(14) - 3 = 84 - 3 = 81$
$87 - 27 = 3(81)/27 = 9$
$9 \times 6 = 9 + 4(6) = 33$
$33 + 13 = 33 \times 26 = 858$.

**31.** Consider choice (A)
$21 \times 7 = 21 + 4(7) = 49$
$49 - 9 = 3(49)/9 = 49/3$
$49/3 + 6 = 49/3 \times 2(6) = 49 \times 4$
which is $a$ multiple of 7
Consider choice (B)
$21 - 7 = 3(21)/7 = 9$
$9 + 9 = 9 \times 2(9) = 9 \times 18 = 162$
$162 \times 6 = 162 + 4(6) = 162 + 24 = 186$
which is not $a$ multiple of 7
Consider choice (C)
$21/\ 7 = 6(21) - 7 = 119$
$119 \times 9 = 119 + 4(9) = 155$
$155 - 6 = 3(155)/6 = 155/2$
which is not $a$ multiple of 7.

**32.** $6 \times 4 = 6 + 4(4) = 6 + 16 = 22$
$9/2 = 6(9) - 2 = 52$
$(6 \times 4) + (9/2) = 22 + 52 = 22 \times 2 \times 52 = 2288$.

**33.** Consider choice (A)
$10 + 5 = 10 \times 2\ (5) = 10 \times 10 = 100$
$100 - 3 = 3(100)/3 = 100$
$100 \times 24 = 100 + 4(24) = 196$
which is a perfect square.

**34.** Given $f(x, y) = 2^{x+y}$, $g(x, y) = 2^{x-y}$

$$p(x, y) = \log_2 xy,\ q(x, y) = \log_2\left(\frac{x}{y}\right)$$

$f(x, -x) = 2^{x-x} = 2^0 = 1$
$g(x, x) = 2^{x-x} = 2^0 = 1$
$q(f(x, -x), g(x, x)) = q(1, 1)$

$$= \log_2\left(\frac{1}{1}\right) = 0$$

**35.** $f(3, 4) = 2^{3+4} = 2^7$
$g(4, 5) = 2^{4-5} = 2^{-1} = 1/2$
$p(f(3, 4), g(4, 5)) = p(2^7, 1/2)$

$$= \log_2 2^7.\ \frac{1}{2} = \log_2 2^6 = 6$$

**36.** $p(5, 6) = \log_2 5(6) = \log_2 30$

$$q(6, 5) = \log_2\left(\frac{6}{5}\right)$$

and $p(5, 6) + q\ (6,5)$

$$= \log_2(30) + \log\frac{6}{5} = \log_2 36$$

$\therefore f[g(5, 6), q(6, 5)]$
$= 2\log_2 36 = 36$
$p(4, 5) = \log_2(4)\ (5) = \log_2 20$
$q(5, 6) = \log_2 5/6$
$\therefore g[p(4, 5), q(5, 6)]$
$\therefore p(4,5) - q(5, 6) = \log_2 20 - \log_2 5/6$

$$= \log_2 \frac{20(6)}{5} = \log_2 24 = 2\log_2 24 = 24$$

$$\therefore \frac{f(p(5,6), q(6,5))}{g(p(4,5), q(5,6)} = \frac{36}{24} = \frac{3}{2}$$

**37.** Here, $a=2,\ b=7,\ h=7/2$

$$\Delta^2=\frac{4}{b^2}\ (h^2-ab)$$

$$\Delta^2=\frac{4}{49}\ (\frac{49}{4}-14)<0$$

$$\nabla^2=\frac{4}{49}\ (14-\frac{49}{4})>0$$

$\nabla^2>\Delta^2$, i.e., $\Delta^2<\nabla^2$

**38.** With the coefficients of $x^2$ and $y^2$ being interchanged, the new equation will have $a=-8,\ b=7$ and $h=5$.

$$\Delta^2=\frac{4}{49}(25+56)=\frac{4}{49}(81)$$

$$\nabla^2=\frac{4}{49}(-56-25)=\frac{4}{49}(-81)$$

Hence, $\Delta^2>\nabla^2$.

**39.** $D=\dfrac{\sqrt{h^2-ab}}{ab}\Rightarrow aD=\dfrac{\sqrt{h^2-ab}}{b}$

$$\Rightarrow 2aD=\frac{2}{b}\sqrt{h^2-ab}\ \Rightarrow 2aD=\Delta.$$

**40.** $a=p,\ b=q$

$pq<h^2\Rightarrow pq-h^2<0$

$$\Delta=\frac{2}{b}\sqrt{h^2-ab}\ =\frac{2}{q}\sqrt{h^2-pq}$$

$$\Rightarrow\Delta^2=\frac{4}{q^2}\sqrt{h^2-pq}$$

$$\nabla=\frac{2}{b}\sqrt{ab-h^2}\ =\frac{2}{q}\sqrt{pq-h^2}$$

$$\nabla^2=\frac{4}{q^2}\sqrt{pq-h^2}$$

Given $pq<h^2\Rightarrow pq-h^2<0$

$\Rightarrow\nabla^2$ is negative while $\Delta^2$ is positive.

$\therefore\nabla^2<\Delta^2$.

## EXERCISE-4

***Solutions for questions 1 and 2:***

**1.** $f\{g[(3,4),(12,16)],h[(3,4),(12,16)]\}$

$$=\sqrt{\frac{3^2+4^2}{12^2+16^2}}=\sqrt{\frac{25}{400}}=\sqrt{\frac{1}{16}}=\frac{1}{4}.$$

**2.** $f\{i[(3,4),(12,6)],j[(3,4),(12,16)]\}$.

$$=\sqrt{\left(3^2+4^2\right)\times\left(12^2+16^2\right)}$$

$$=\sqrt{25\times400}=100$$

***Solutions for questions 3 and 4:***

We are given

$f(x,y)=|x+y|$. This is always positive.

$F(f(x,y))=-f(x,y)=-|x+y|$. This is always negative.

$G(f(x,y))=-F(f(x,y))=f(x,y)=|x+y|$

This is always positive.

**3.** $f(x,y)=|x+y|$, $F(z)=-z$ and $G(z)=-f(z)=z$ ($G$ is the identity function). We need to evaluate

$f\{G(f(1,0)),f[F(f(1,2)),G(f(1,2))]\}$

$f(1,0)=1.\ \therefore\ G(f(1,0))=1$

$f(1,2)=3.\ \therefore\ F(f(1,2))=-3$

$f(1,2)=3.\ \therefore\ G(f(1,2))=3$

$f(-3,3)=0$

$f(1,0)=1$

**4.** We have to check each choice. However, when we look at the choices, we find that each of them deals with $f(x,x)$ or $f(x,-x)$ or the product of these two terms. $f(x,x)$ will be $2x$ and $f(x,-x)$ will be 0.

In Choice (A), the product of the two terms equals zero.

In Choice (B), the product of the three terms is $(-2x)(2x)(4)$ and this is not equal to $x^2$.

Choice (C). $-F(f(x,x))\ G(f(x,x))\div4$

$$=\frac{|2x||2x|}{4}$$

Choice (D) is $(2x)(2x)$ which is equal to $4x^2$.

**5.** $(x+y,x-y)*(x-y,x+y)=[(x+y)^2-(x-y)^2,(x^2-y^2)-(x^2-y^2)]=(4xy,0)$ (1)

$(1,3)*(2,4)=(4-6,3-8)=(-2,-5)$

$(5,6)*(7,8)=(40-42,30-56)=(-2,-26)$

$\therefore\ (x,y)=(-2,-5)*(-2,-26)=(52-10,10-52)$

$=(42,-42)$

$\therefore(x,y)=(42,-42)$ and $(x+y,x-y)*(x-y,x+y)$

$=(4xy,0)=(-84^2,0)$

**6.** $a\odot b=\dfrac{1}{(a+1)(b+1)}-\dfrac{1}{ab}$

$$\therefore\ a\odot(a+1)=\frac{1}{(a+1)(a+2)}-\frac{1}{a(a+1)}$$

$$=\frac{1}{a+1}-\frac{1}{a+2}-\frac{1}{a}+\frac{1}{a+1}$$

$$=\frac{-1}{a}+\frac{2}{a+1}-\frac{1}{a+2}$$

$$\therefore\ 1\odot2=\frac{-1}{1}+\frac{2}{2}-\frac{1}{3}\ \rightarrow$$ (1)

$$2 \odot 3 = \frac{-1}{2} + \frac{2}{3} - \frac{1}{4} \quad \rightarrow \quad (2)$$

$$3 \odot 4 = \frac{-1}{3} + \frac{2}{4} - \frac{1}{5} \quad \rightarrow \quad (3)$$

$$7 \odot 8 = \frac{-1}{7} + \frac{2}{8} - \frac{1}{9} \quad \rightarrow \quad (7)$$

$$8 \odot 9 = \frac{-1}{8} + \frac{2}{9} - \frac{1}{10} \quad \rightarrow \quad (8)$$

$$9 \odot 10 = \frac{-1}{9} + \frac{2}{10} - \frac{1}{11} \quad \rightarrow \quad (9)$$

When these 9 equations are added, only the first two terms from (1), the first from (2), the third from (8) and the second and third terms from (9) remain.

$\therefore$ The given expression is $\left(-1+1-\frac{1}{2}\right)+\left(\frac{-1}{10}+\frac{2}{10}-\frac{1}{11}\right)$

$= \frac{-1}{2} + \frac{1}{110} = \frac{-54}{110} = \frac{-27}{55}$

**7.** Without loss of generality, let
$a \le b \le c \le d$
$f(a, b, c, d) = \min(c, d, d, d) = c$
$g(a, b, c, d) = \max(c, d, d, d) = d$
$h(a, b, c, d) = \max(c, a, a, d) = d$
$i(a, b, c, d) = \min(c, d, d, b) = b$
$\therefore g(a, b, c, d) = h(a, b, c, d)$
$\therefore$ Option $c$ is not defined.

***Solutions for questions 8:***

**8.** $D = \frac{2}{b}\sqrt{h^2 - ab}$

$\Rightarrow D/a = \frac{2}{ab}\sqrt{h^2 - ab}$

$\Rightarrow D/2a = \frac{2}{ab}\sqrt{h^2 - ab} = \nabla$

$\Rightarrow D/2a = \nabla \Rightarrow D = 2a\nabla$

***Solutions for questions 9 and 10:***

**9.** For a positive number $\lfloor 2x \rfloor \ge \lfloor x \rfloor + \lfloor x \rfloor$,
for example, $\lfloor 1.2 \rfloor + \lfloor 1.2 \rfloor = \lfloor 2 \times 1.2 \rfloor = \lfloor 2.4 \rfloor = 2$
whereas $\lfloor 1.8 \rfloor + \lfloor 1.8 \rfloor = 2 < \lfloor 2 \times 1.8 \rfloor = \lfloor 3.6 \rfloor = 3$
$\Rightarrow \lfloor 2x \rfloor + \lfloor 2y \rfloor \ge \lfloor x \rfloor + \lfloor x \rfloor + \lfloor y \rfloor + \lfloor y \rfloor$
$\ge \lfloor x \rfloor + \lfloor y \rfloor + \lfloor x \rfloor + \lfloor y \rfloor$
$\ge \lfloor x \rfloor + \lfloor y \rfloor + \lfloor x + y \rfloor$ as $\lfloor x + y \rfloor \ge \lfloor x \rfloor + \lfloor y \rfloor$
Hence $R(x, y) \ge L(x, y)$

**10.** For all integral values of $x$ and $y$; $R(x, y) = T(x, y)$. In the interval $(0, 5)$ there are 6 integers.
So, the number of such pairs $6 \times 6 = 36$.

**11.** The order in which operations should be performed: BOSAMD.
$\therefore 13 \times 5 + 35 \div 8 - (2 \times 5)$
$= 13 \times 5 + 35 \div 8 - 10 = 13 \times 5 + 35 \div (-2)$
$= 13 \times 40 \div (-2) = 520 \div (-2) = -260$

***Solutions for questions 12 and 13:***

**12.** $p(f(x, x), g(x, -x)) = p(e^{2x}, e^{2x})$
$= \log_e(e^{2x}, e^{2x})$
$= \log_e e^{4x} = 4x$
$= q(e^{6x}, e^{2x})$

**13.** $f(p(x, y), q(x, y)) = f(\log xy, \log x/y)$
$= e^{\log xy + \log x/y} = e^{\log x^2} = x^2 = 25$

***Solutions for questions 14 and 15:***

**14.** Here $a = 3$, $b = 4$, $h = 3$

$\Delta^2 = \frac{1}{a^2b^2}\left(h^2 + ab\right)$

$= 1/144\ (9 + 12) = 21/144 > 0$

and $\nabla^2 = \frac{1}{a^2b^2}\left(h^2 - ab\right)$

$= 1/144\ (9 - 12) = -3/144 < 0$

Choices (A) and (B) are false.
Also $21/144 > -3/144$
$\therefore \Delta^2 > \nabla^2$

**15.** With the coefficients of $x^2$ and $y^2$ being interchanged, the new equation will have $a = -7$, $h = 4$, $b = 4$

$\Delta^2 = \frac{1}{a^2b^2}\left(h^2 + ab\right)$

$= \frac{1}{49 \times 16}\ (16 - 7 \times 4) = \frac{-12}{49 \times 16}$

$\nabla^2 = \frac{1}{a^2b^2}\left(h^2 + ab\right)$

$= \frac{1}{49 \times 16}\ (16 - (-7) \times 4) = \frac{44}{49 \times 16}$

$\therefore \Delta^2 < \nabla^2$

***Solutions for questions 16 and 17:***

Let $i$ be an integer, $f$ be a proper fraction (i.e., $0 \le f < 1$) and $r$ be any real number. We have the following basic results, following from the definition of $[x]$ and $(x)$.

(I) $[i] = i$
(II) $[i + f] = i$
(III) $[i + r] = i + [r]$
(IV) $(i) = i + 1$
(V) $(i + f)] = i + 1$
(VI) $[i + r] = i + (r)$
(VII) $[r] + 1 = (r)$

**16.** Let $A = [A] + a$
$B = [B] + b$
$C = [C] + c$
$D = [D] + d$

$0 \le a+b+c+d = s < 4$ (1)
and A + B + C + D = [A] + [B] +[C] + [D] +$s$
$\therefore$ [A] + [B] + [C] + $k$
Where $k = 0, 1, 2$ or 3 (from (1))
$\therefore$ 4 is not a possible value for $k$.

**17.** Let A = [A] +$a$
B = [B] + $b$
C = [C] + $c$
$\therefore 0 \le a+b+c = s < 3 \rightarrow$ (1)
(A + B + C) = ([A] + [B] +[C] + $s$)
= [A] + [B] + [C] + ($s$)
= [A] + [B] + [C] + $k \rightarrow$ (2)
Where $k = 1, 2$ or 3 (from (1))
Also (A) + (B) + (C)
= [A] + [B] + [C] + 3 $\rightarrow$ (3)
Comparing (2) and (3), l can be 0, –1 or –2. Among the options, only –1 is there.

**18.** $a \nabla a = a^2$, $b \nabla b = b^2$, $c \nabla c = c^2$, and
$a \nabla b = ab$, $b \nabla c = bc$, $a \nabla c = ac$
$(a \nabla a) \Delta (b \nabla b) \Delta (c \nabla c)$
$\updownarrow (a \nabla b) \updownarrow (b \nabla c) \updownarrow (a \nabla c)$
$= a^2 + b^2 + c^2 - ab - bc - ac$ (1)
$y = (a \Delta b) \Delta c = a + b + c$ (2)
$\therefore x \nabla y = (a^2 + b^2 + c^2 - ab - bc - ac)(a + b + c)$
$= a^3 + b^3 + c^3 - 3abc$

**19.** Since all the numbers provided are even and L.C.M., H.C.F. or the Arithmetic mean of two even numbers is even, the given question reduces to $\pi$(even, 2) = 0

**20.** GOS $(a, b, c) = \sqrt[3]{a.b.c}$

GOSS $(a, b, c) = \sqrt[3]{a^2.b^2.c^2}$

GOC $(a, b, c) = \sqrt[3]{a^3.b^3.c^3}$

For $a, b, c > 1$; $a^3b^3c^3 \ge a^2b^2c^2 \ge$ abc

$\Rightarrow \sqrt[3]{a^3b^3c^3} \ge \sqrt[3]{a^2b^2c^2} \ge \sqrt[3]{abc}$

i.e., GOC $(a, b, c) \ge$ GOSS $(a, b, c) \ge$ GOS $(a, b, c)$.
Choices (A), (B) and (C) are clearly true.

While Choice (D), i.e., $\sqrt[3]{a^2.b^2.c^2} \ge abc$ is not true.

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# Unit 6

# Modern Maths


Chapter 1 Permutation and Combinations

Chapter 2 Probability

Chapter 3 Statistics

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# 1 Permutations and Combinations

CHAPTER

**Learning Objectives**

*In this chapter, you will*:

- Learn about principle of counting
- Understand what are permutation and its properties, such as:
  - ${}^nP_r$– what is it and its interpretation and calculation
  - Learn about permutations with repeated objects
  - Learn about circular permutations
- Learn all about combinations and its properties, such as:
  - ${}^nC_r$–what is it, its interpretation and calculation
  - Learn about selections from dissimilar objects
  - Learn about selections from similar objects
- Learn how to divide items into similar/dissimilar groups
- Learn how to find the number of positive/non-negative integral solutions for an equation
- Learn how to solve other P&C models like grids, networks, etc.

PERMUTATIONS and COMBINATIONS is one of the important areas in many exams because of two reasons. First, solving questions in this area is a measure of students' reasoning ability. Secondly, solving problems in areas like probability requires thorough knowledge of permutations and combinations.

Before discussing permutations and combinations, let us look at what is called fundamental rule.

'If one operation can be performed in '$m$' ways and (when, it has been performed in any one of these ways), a second operation then can be performed in '$n$' ways, the number of ways of performing the two operations will be $m \times n$'.

This can be extended to any number of operations.

If there are three cities A, B and C such that there are 3 roads connecting A and B and 4 roads connecting B and C, then the number of ways one can travel from A to C is $3 \times 4$, i.e., 12.

This is a very important principle and we will be using it extensively in permutations and combinations. As we use it very extensively, we do not explicitly state every time that the result is obtained by the fundamental rule but directly write down the result.

## ❒ PERMUTATIONS

Each of the arrangements which can be made by taking some or all of a number of items is called permutation. In permutations, 'order of the items' is important.

The permutations of three items $a$, $b$ and $c$ taken two at a time are $ab$, $ba$, $ac$, $ca$, $cb$ and $bc$. Since the order in which the items are taken is important, $ab$ and $ba$ are counted as two different permutations. The words 'permutation' and 'arrangement' are synonymous and can be used interchangeably.

The number of permutations of $n$ things taking $r$ at a time is denoted by ${}^nP_r$ (and read as '$nPr$')

## ❒ COMBINATIONS

Each of the groups or selections which can be made by taking some or all of a number of items is called combination. In combinations, the order in which the

items are taken is not considered as long as the specific things are included.

The combination of three items $a$, $b$ and $c$ taken two at a time are $ab$, $bc$ and $ca$. Here, $ab$ and $ba$ are not considered separately because the order in which $a$ and $b$ are taken is not important but it is only required that a combination including $a$ and $b$ is what is to be counted. The words 'combination' and 'selection' are synonymous.

The number of combinations of $n$ things taking $r$ at time is denoted by ${}^nC_r$ (and read as '$nCr$')

When a problem is read, it should first be clear to you as to whether it is a permutation or combination that is being discussed. Sometimes, the problem specifically states whether it is the number of permutations (or arrangements) or the number of combinations (or selections) that you should find out. The questions can be as follows:

For permutations, 'Find the number of permutations that can be made ...' OR 'Find the number of arrangements that can be made...' OR 'Find the number of ways in which you can arrange...'

For combinations,

'Find the number of combinations that can be made ...' OR 'Find the number of selections that can be made...' OR 'Find the number of ways in which you can select...'

Sometimes, the problem may not explicitly state whether what you have to find out is a permutation or a combination but the nature of what is to be found out will dictate whether it is the number of permutations or the number of combinations that you have to find out. Let us look at the following two examples to clarify this.

'How many four digit numbers can be made from the digits 1, 2, 3 and 4 using each digit once?'

Here, since we are talking of numbers, the order of the digits matters, and hence, what we have to find out is permutations.

'Out of a group of five friends that I have, I have to invite two for dinner. In how many different ways can I do this?'

Here, if the five friends are $A$, $B$, $C$, $D$ and $E$, whether the two friends that I call for dinner on a particular day are $A$ and $B$ or $B$ and A, it does not make any difference, i.e., here the order of the 'items' does not play any role, and hence, it is the number of combinations that we have to find out.

Now we will find out the number of permutations and combinations that can be made from a group of given items.

Initially, we impose two constraints (conditions) while looking at the number of permutations. They are:

1. All the $n$ items are distinct or dissimilar (or no two items are of the same type)
2. Each item is used at most once (i.e., no item is repeated in any arrangement)

**Number of linear permutations of '$n$' dissimilar items taken '$r$' at a time without repetition (${}^nP_r$)**

Consider $r$ boxes each of which can hold one item. When all the $r$ boxes are filled, what we have is an arrangement of $r$ items taken from the given $n$ items. So, each time we fill up the $r$ boxes with items taken from the given $n$ items, we have an arrangement of $r$ items taken from the given $n$ items without repetition. Hence, the number of ways in which we can fill up the $r$ boxes by taking things from the given $n$ things is equal to the number of permutations of $n$ things taking $r$ at a time.

Boxes $\square\square\square\square\ldots\square$

1 2 3 4 $r$

The first box can be filled in $n$ ways (because any one of the $n$ items can be used to fill this box). Having filled the first box, to fill the second box we now have only $(n-1)$ items; any one of these items can be used to fill the second box, and hence, the second box can be filled in $(n-1)$ ways; similarly, the third box in $(n-2)$ ways and so on the $r^{th}$ box can be filled in $\{n-(r-1)\}$ ways, i.e. $[n-r+1]$ ways. Hence, from the Fundamental Rule, all the $r$ boxes together can be filled in

$n \cdot (n-1) \cdot (n-2)\ldots(n-r+1)$ ways

So, ${}^nP_r = n \cdot (n-1) \cdot (n-2)\ldots(n-r+1)$

This can be simplified by multiplying and dividing the right hand side by $(n-r)\ (n-r-1)\ldots 2.1$ giving us

$${}^nP_r = n(n-1)\ (n-2)\ \ldots\ [n-(r-1)]$$

$$= \frac{(n-1)(n-2)\ldots\left[n-(r-1)\right].(n-r)\ldots 3.2.1}{(n-r)\ldots 3.2.1}$$

$$= \frac{n!}{(n-r)!}$$

**FORMULA**

The number of permutations of $n$ distinct items taking $r$ items at a time is

$$^nP_r = \frac{n!}{(n-r)!}$$

If we take $n$ items at a time, then we get ${}^nP_n$. From a discussion similar to that we had for filling the $r$ boxes above, we can find that ${}^nP_n$ is equal to $n!$

The first box can be filled in $n$ ways, the second one in $(n-1)$ ways, the third one in $(n-2)$ ways and so on, then the $n^{th}$ box in 1 way; hence, all the $n$ boxes can be filled in $n(n-1)\ (n-2)\ \ldots\ 3.2.1$ ways, i.e., $n!$ ways. Hence,

$$^nP_n = n!$$

But if we substitute $r = n$ in the formula for ${}^nP_r$, then we get ${}^nP_n = \frac{n!}{0!}$; since we already found that ${}^nP_n = n!$, we can conclude that $0! = 1$

**Number of combinations of *n* dissimilar things taken *r* at a time**

Let the number of combinations ${}^nC_r$ be $x$, consider one of these $x$ combinations. Since this is a combination, the order of the $r$ items is not important. If we now impose the condition that order is required for these $r$ items, we can get $r!$ arrangements from this one combination. So, each combination can give rise to $r!$ arrangements. $x$ combinations will thus give rise to $x \cdot r!$ arrangements. But since these are all permutations of $n$ things taken $r$ at a time, this must be equal to ${}^nP_r$. So,

$$x.r! = {}^nP_r \frac{n!}{(n-r)!} \Rightarrow {}^nC_r = \frac{n!}{r!(n-r)!}$$

The number of combinations of $n$ dissimilar things taken all at a time is 1.

Out of $n$ things lying on a table, if we select $r$ things and remove them from the table, we are left with $(n - r)$ things on the table—that is, whenever $r$ things are selected out of $n$ things, we automatically have another selection of the $(n - r)$ things. Hence, the number of ways of selecting $r$ out of $n$ things is the same as selecting $(n - r)$ things out of $n$ given things, i.e.,

$$\boldsymbol{{}^nC_r = {}^nC_{n-r}}$$

When we looked at ${}^nP_r$, we imposed two constraints which we will now release one by one and see how to find out the number of permutations.

**Number of arrangements of *n* items of which *p* are of one type, *q* are of a second type and the rest are distinct**

When the items are all not distinct, then we **cannot** talk of a general formula for $\boldsymbol{{}^nP_r}$ for any $r$ but we can talk of only ${}^nP_n$ (which is given below). If we want to find out ${}^nP_r$ for a specific value of $r$ in a given problem, we have to work on a case to case basis (this has been explained in one of the solved examples).

The number of ways in which $n$ things may be arranged taking them all at a time, when $p$ of the things are exactly alike of one kind, $q$ of them exactly alike of another kind, $r$ of them exactly alike of a third kind, and the rest all distinct is $\frac{n!}{p!\,q!\,r!}$.

**Number of arrangements of *n* distinct items where each item can be used any number of times (i.e., repetition allowed)**

You are advised to apply the basic reasoning given while deriving the formula for ${}^nP_r$ to arrive at this result also. The first box can be filled up in $n$ ways; the second box can be filled again in $n$ ways (even though the first box is filled with one item, the same item can be used for filling the second box also because repetition is allowed); the third box can also be filled in $n$ ways and so on... the $r^{\text{th}}$ box can be filled in $n$ ways. Now all the $r$ boxes together can be filled in {$n.n.n.n$ ... $r$ times} ways, i.e., $\boldsymbol{n^r}$ ways.

The number of permutations of $n$ things, taken $r$ at a time when each item may be repeated once, twice, ... up to $r$ times in any arrangement is $\boldsymbol{n^r}$

What is important is not this formula by itself but the reasoning involved. So, even while solving problems of this type, you will be better off if you go from the basic reasoning and not just apply this formula.

## ❒ Total Number of Combinations

Out of $n$ given things, the number of ways of selecting **one or more** things is where we can select 1 or 2 or 3 ... and so on up to $n$ things at a time; hence, the number of ways is ${}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots {}^nC_n$

This is called total number of combinations and is equal to $2^n - 1$ where $n$ is the number of things.

The same can be reasoned out in the following manner also.

There are $n$ items to select from. Let each of these be represented by a box.

| | 1 | 2 | 3 | 4 | | $n$ |
|---|---|---|---|---|---|---|
| No. of ways of dealing | ☐ | ☐ | ☐ | ☐ | ... | ☐ |
| with the boxes | 2 | 2 | 2 | 2 | | 2 |

The first box can be dealt with in two ways. In any combination that we consider, this box is **either** included **or** not included. These are the two ways of dealing with the first box. Similarly, the second box can be dealt with in two ways, the third one in two ways and so on, the $n^{\text{th}}$ box in two ways. By the Fundamental Rule, the number of ways of dealing with all the boxes together in $2 \cdot 2 \cdot 2 \cdot \ldots$ $n$ times ways, i.e., in $2^n$ ways. But out of these, there is one combination where we 'do not include the first box, do not include the second box, do not include the third box and so on, do not include the $n$ th box.' That means, no box is included. But this is not allowed because we have to select one or more of the items (i.e., at least one item). Hence, this combination of no box being included is to be subtracted from the $2^n$ ways to give the result of

**FORMULA**

Number of ways of selecting one or more items from $n$ given items is $2^n - 1$

## ❒ Dividing Given Items into Groups

**Dividing $(p + q)$ items into two groups of $p$ and q items**

Out of $(p + q)$ items, if we select $p$ items (which can be done in ${}^{p+q}C_p$ ways), then we will be left with $q$ items, i.e., we have two groups of $p$ and $q$ items, respectively. So, the number of ways of dividing $(p + q)$ items into two groups of $p$ and $q$ items, respectively is equal to ${}^{p+q}C_p$ which is equal to $\frac{(p+q)!}{p!\,q!}$

**FORMULA**

The number of ways of dividing $(p + q)$ items into two groups of $p$ and $q$ items, respectively is $\frac{(p+q)!}{p!\,.q!}$

If $p = q$, i.e., if we have to divide the given items into two EQUAL groups, then two cases arise

(i) when the two groups have distinct identity and
(ii) when the two groups do not have distinct identity.

In the first case, we just have to substitute $p = q$ in the above formula which then becomes

**FORMULA**

The number of ways of dividing $2p$ items into two equal groups of $p$ each is $\frac{(2p)!}{(p!)^2}$ where the two groups have distinct identity.

In the second case, where the two groups do not have distinct identity, we have to divide the above result by 2!, i.e., it then becomes

**FORMULA**

The number of ways of dividing $2p$ items into two equal groups of $p$ each is $\frac{(2p)!}{2!(p!)^2}$ where the two groups do not have distinct identity.

**Dividing $(p + q + r)$ items into three groups consisting of $p$, $q$ and $r$ items, respectively**

**FORMULA**

The number of ways in which $(p + q + r)$ things can be divided into three groups containing $p$, $q$ and $r$ things, respectively is $\frac{(p+q+r)!}{p!q!r!}$

If $p = q = r$, i.e., if we have to divide the given items into three equal groups, then we have two cases where no two groups are identical and where the groups are identical.

When no two groups are identical, the number of ways is $\frac{(3p)!}{(p!)^3}$

When the three groups are identical, then the number of ways is $\frac{(3p)!}{3!(p!)^3}$

## ❒ CIRCULAR PERMUTATIONS

When $n$ distinct things are arranged in a straight line taking all the $n$ items, we get $n!$ permutations. However, if these $n$ items are arranged in a circular manner, then the number of arrangements will not be $n!$ but it will be less than that. This is because in a straight line manner, if we have an arrangement $ABCDE$ and if we move every item one place to the right (in cyclic order), the new arrangement that we get $EABCD$ is not the same as $ABCDE$ and this also is counted in the $n!$ permutations that we talked of. However, if we have an arrangement $ABCDE$ in a circular fashion, by shifting every item by one place in the clockwise direction, we still get the same arrangement $ABCDE$. So, if we now take $n!$ as the number of permutations, we will be counting the same arrangement more than once.

The number of arrangements in circular fashion can be found out by first fixing the position of one item. Then, the remaining $(n - 1)$ items can be arranged in $(n - 1)!$ ways. Now, even if we move these $(n - 1)$ items by one place in the clockwise direction, then the arrangement that we get will not be the same as the initial arrangement because one item is fixed and it does not move.

Hence, the number of ways in which $n$ distinct things can be arranged in a circular arrangement is $(n - 1)!$

If we take the case of five persons *A, B, C, D* and *E* sitting around a table, then the two arrangements ABCDE (in clockwise direction) and *AEDCB* (the same order but in anticlockwise direction) will be different. Here we say that the clockwise and anticlockwise arrangements are different. However, if we consider the circular arrangement of a necklace made of five precious stones *A, B, C, D* and *E*, the two arrangements talked of above will be the same because we take one arrangement and turn the necklace around (front to back), then we get the other arrangement. Here, we say that there is no difference between the clockwise and anticlockwise arrangements. In this case the number of arrangements will be half of what it is in the case where the clockwise and anticlockwise arrangements are different.

The number of **circular arrangements** of ***n* distinct items** is

**$(n-1)!$** if there is difference between clockwise and anticlockwise arrangements and

**$(n-1)!/2$** if there is no difference between clockwise and anticlockwise arrangements

## ❒ Sum of All Numbers Formed from Given Digits

If $n$ distinct digits are used to make all the possible $n$-digit numbers, we get $n!$ numbers. We now want to find out the sum if all these $n!$ numbers are added. Let us take an example and understand how it is to be done and then look at it as a formula.

To find the sum of all the four digit numbers formed using the digits 2, 3, 4 and 5 without repetition:

We can form a total of 4! or 24 numbers. When we add all these numbers, let us look at the contribution of the digit 2 to the sum.

When 2 occurs in the thousands place in a particular number, its contribution to the total will be 2000. The number of numbers that can be formed with 2 in the thousands place is 3!, i.e., 6 numbers. Hence, when 2 is in the thousands place, its contribution to the sum is $3! \times 2000$.

Similarly, when 2 occurs in the hundreds place in a particular number, its contribution to the total will be 200 and since there are 3! numbers with 2 in the hundreds place, the contribution 2 makes to the sum when it comes in the hundreds place is $3! \times 200$.

Similarly, when 2 occurs in the tens and units place respectively, its contribution to the sum is $3! \times 20$ and $3! \times 2$, respectively. Thus the total contribution of 2 to the sum is $3! \times 2000 + 3! \times 200 + 3! \times 20 + 3! \times 2$, i.e., $3! \times 2222$. This takes care of the digit 2 completely.

In a similar manner, the contribution of 3, 4 and 5 to the sum will respectively be $3! \times 3333$, $3! \times 4444$ and $3! \times 5555$, respectively.

The sum can now be obtained by adding the contributions of these four digits. Hence, the sum of the numbers formed by using the four digits is $3! \times (2222 + + 3333 + 4444 + 5555)$, i.e.,

$$3! \times (2 + 3 + 4 + 5)\ x\ 1111$$

We can now generalize the above as

If all the possible $n$-digit numbers using $n$ distinct digits are formed, the sum of all the numbers so formed is equal to $(n-1)! \times \{\text{sum of the } n \text{ digits}\} \times \{11111 \ldots\}$ $n$ times

## ❒ Rank of a Word

Finding the rank of a given word is basically finding out the position of the word when all possible words have been formed using all the letters of this word exactly once and arranged in alphabetical order as in the case of dictionary. Let us understand this by taking an example.

Let us look at the word 'POINT'. The letters involved here when taken in alphabetical order are *I, N, O, P, T*.

To arrive at the word 'POINT', initially we have to go through the words that begin with *I*, then all those that begin with *N*, those that begin with *O* which are 4! in each case. Then we arrive at words that begin with *PI, PN* which are 3! in each case. Then we arrive at the word POINT.

There are $3 \times 4! + 2 \times 3! = 84$ words that precede the word POINT, i.e., POINT is the $85^{th}$ word. Hence, rank of 'POINT' is 85.

**The number of diagonals in a *n*-sided convex polygon**

An $n$-sided convex polygon has $n$ vertices. Joining any two vertices we get a line of the polygon which are ${}^nC_2$ in number. Of these ${}^nC_2$ lines, $n$ of them are sides. Hence, number of diagonals is

$$ {}^nC_2 - n = \frac{n(n-3)}{2} $$

**Number of integral solutions of the equation**

$$x_1 + x_2 + \ldots + x_n = s$$

Consider the equation

$$x_1 + x_2 + x_3 = 10$$

If we consider all possible integral solutions of this equation, there are infinitely many. But the number of positive (or non-negative) integral solutions is finite.

We would like the number of positive integral solutions of this equation, i.e., values of $(x_1, x_2, x_3)$ such that each $x_i > 0$.

We imagine 10 identical objects arranged on a line. There are 9 gaps between these 10 objects. If we choose any two of these gaps, we are effectively splitting the 10 identical objects into 3 parts of distinct identity ($x_1$ is the first part, $x_2$ is the second part and $x_3$ is the third part). Conversely, every split of these 10 objects corresponds to a selection of 2 gaps out of the 9 gaps.

Therefore, the number of positive integral solutions is ${}^{9}C_{2}$. In general, if $x_1 + x_2 + ... + x_n = s$ where $s \geq n$, the number of positive integral solutions is ${}^{s-1}C_{n-1}$.

If we need the number of non negative integral solutions, we proceed as follows. Let $a_1$, $a_2$, ... be a non-negative integral solution. Than $a_1 + 1$, $a_2 + 1$, ... $a_n + 1$ is a positive integral solution of the equation $x_1 + x_2 + ... + x_n = s + n$. Therefore, the number of non-negative integral solutions of the given equation is equal to the number of positive integral solutions of $x_1 + x_2 + ... + x_n = s + n$, which is ${}^{s+n-1}C_{n-1}$.

For $x_1 + x_2 + x_3 + \ldots + x_n = s$ where $s \geq 0$, the number of positive integral solutions (when s ≥ n) is ${}^{s-1}C_{n-1}$ and the number of non-negative integral solutions is ${}^{n+s-1}C_{n-1}$

## ❒ SOME ADDITIONAL POINTS

1. Suppose there are $n$ letters and $n$ corresponding addressed envelopes. The numbers of ways of placing these letters into the envelopes such that no letter is placed in its corresponding envelope is often referred as derangements. The number of derangements of $n$ objects is given by

$$D(n) = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + ... + (-1)^n \frac{1}{n!}\right]$$

For example, when $n = 3$, the number of derangements is

$$D(3) = 3!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right] = 2 \text{ and when } n = 4, D(4) = 4!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right] = 9$$

2. The total number of ways in which a selection can be made by taking some or all out of $p + q + r + ...$ things where $p$ are alike of one kind, $q$ alike of a second kind, $r$ alike of a third kind and so on is $[\{(p+1)(q+1)(r+1) ...\} - 1]$.
3. ${}^{n+1}C_r = {}^{n}C_r + {}^{n}C_{r-1}$ and ${}^{n}P_r = r.{}^{n-1}P_{r-1} + {}^{n-1}P_r$

### Example

Consider the word PRECIPITATION. Find the number of ways in which

(i) a selection
(ii) an arrangement of 4 letters can be made from the letters of this word.

**Solution** The word PRECIPITATION has 13 letters *I, I, I, P, P, T, T, E, R, C,* A, *O, N* of 9 different sorts. In taking 4 letters, the following are the possibilities to be considered.

(a) all 4 distinct.
(b) 3 alike, 1 distinct.
(c) 2 alike of one kind, 2 alike of other kind.
(d) 2 alike, 2 other distinct.

## ❒ SELECTIONS

1. 4 distinct letters can be selected from 9 distinct letters (I, *P, T, E, R, C,* A, *O, N*) in ${}^{9}C_{4} = 126$ ways.
2. As 3 letters have to be alike, the only possibility is selecting all the I's. Now the 4th letter can be selected from any of the remaining 8 distinct letters in ${}^{8}C_{1} = 8$ ways.
3. Two pairs of two alike letters can be selected from I's, P's and T's in ${}^{3}C_{2} = 3$ ways.
4. The two alike letters can be selected in ${}^{3}C_{1} = 3$ ways and the two distinct letters can now be selected from the 8 distinct letters in ${}^{8}C_{2} = 28$ ways. Hence required number of ways are $3 \times 28 = 84$ Hence, the total selections are $126 + 8 + 3 + 84 = 221$.

## ❒ ARRANGEMENTS

For arrangements, we find the arrangements for each of the above selections and add them up.

1. As the 4 letters are distinct, there are 4! arrangements for each selection. Hence, required arrangements are $126 \times 4! = 3024$
2. Since 3 of the 4 letters are alike, there are $\frac{4!}{3!}$ arrangements for each selection. Hence, required arrangements are $8 \times \frac{4!}{3!} = 32$.

3. The required arrangements here are $3 \times \frac{4!}{2!\,2!} = 18$

4. The required arrangements are $84 \times \frac{4!}{2!} = 1008$.

Total number of arrangements are 3024 + 32 + 18 + 1008 = 4082.

## SOLVED EXAMPLES

**1.01.** Out of 8 persons in a group, find the number of ways of selecting 3 persons and the number of ways of arranging these 3 selected persons in a row.

**Sol:** The number of ways of selecting 3 persons from 8 persons is

$$^8C_3 = \frac{8!}{3!\,5!} = 56 \left[ {}^nC_r = \frac{n!}{(n-r)!\,r!} \right]$$

The number of ways of arranging 3 persons taken from 8 persons is

$$^8P_3 = \frac{8!}{5!} = 336 \left[ {}^nP_r = \frac{n!}{(n-r)!} \right]$$

**1.02.** There are 6 distinct letters of English alphabet and 4 distinct digits. All possible 6-character alphanumeric codes are generated using any 4 letters of the alphabet and any 2 available digits. If in any given code, the characters are all distinct, then what is the maximum number of such codes that can be generated?

**Sol:** We are looking for all possible codes that can be generated using 4 of the 6 letters of the given alphabet and 2 of the 4 given digits. Initially we need to look at the total number of such choices that can be made which are $^6C_4 \times {}^4C_2$. Having selected the 6 characters, they can be arranged among themselves in 6! ways. Hence, the total number of codes that can be generated is $^6C_4 \times {}^4C_2 \times 6! = 64800$

**1.03.** In a cricket tournament, each participating team plays once against every other team and in all 36 matches are played. Find the number of teams that participated in the tournament.

**Sol:** Let us assume that '$n$' teams participated in the tournament. As exactly 2 teams are required to play a match, the total number of matches that are played in the tournament are $^nC_2$.

Given that $^nC_2 = 36$ i.e., $\frac{n(n-1)}{2} = 36$

We get $n(n-1) = 9 \times 8$ which gives $n = 9$.

Hence, 9 teams participated in the tournament.

***Directions for examples 1.04 to 1.09*:** The examples are based on the information given below.

The letters of the word ADROIT are permuted in all possible ways to form 6! different words. In each of the examples below, certain conditions are given. Find the number of permutations subject to these conditions.

**1.04.** How many of these words begin with *T*?

**Sol:** The word ADROIT has got 6 letters. As we want to consider words that begin with *T*, we fix *T* in the first position. Now the other 5 positions can be filled with the remaining 5 letters in 5! = 120 ways. Hence, 120 words begin with *T*.

**1.05.** How many words begin with *A*, but do not end in *T*?

**Sol:** The first position can be filled in only one way (must be *A*). As the words cannot end in *T*, the last position can be filled with any of the other 4 letters, i.e., *D, R, O, I*. Having taken care of the first and the last positions, the other 4 positions can be filled with the remaining 4 letters in 4! ways. Hence total possibilities are $4 \times 4! = 96$

**1.06.** How many words can be formed which either begin with *T* or end in *A*?

**Sol:** There are 5! words that begin with *T* and 5! words that end in *A*, while 4! words begin with *T* and end in *A*. Hence of the 5! + 5! words that either begin with *T* or end in *A*, we exclude the 4! words, which begin with *T* and end in *A*. Hence the required number of words = 5! + 5! – 4! = $2 \times 5! - 4! = 240 - 24 = 216$.

**1.07.** How many words can be formed which neither begin with *T* nor end in *A*?

**Sol:** We will consider the following two cases

Case1: Words that begin with *A*.

Since *A* is in the first place, it takes care of both the conditions given (that *T* should not be in the

first place and *A* should not be in the last place). Now, there are 5! words that begin with *A*.

Case 2: Words that do not begin with *A*. Here we must ensure that *T* does not come in the first place. So, the first place can be filled up in 4 ways (using any letter except *A* and *T*). Out of the remaining 5 letters *A* cannot go into the last place. Hence the last place can be filled in 4 ways. The other 4 places ($2^{nd}$ to $5^{th}$) can now be filled in 4! ways. Hence, the number of words is $4 \times 4 \times 4!$

Combining both the cases, the total number of words are $5! + 16 \times 4! = 504$

**Alternative method:**

From example 1.06 we know that 216 words either begin with *T* or end in *A*. If we exclude these from the total number of words which are 6! = 720, then we get the words that neither begin with *T* nor end in *A*. Hence, 720 –216 = 504 words neither begin with *T* nor end in *A*.

**1.08.** How many words can be formed so that the vowels occupy the even places?

**Sol:** Out of a total of 6 places, there are 3 even places and the word ADROIT has 3 vowels. The 3 vowels can be arranged in the 3 even places in 3! ways and the consonants which are 3 can be arranged in the remaining 3 places in 3! ways. Hence, the total number of words in which vowels occupy even places is $3! \times 3! = 36$.

**1.09.** How many words can be formed such that the vowels are always together?

**Sol:** As the vowels must be together, the 3 vowels can be treated as one unit. Now the 3 consonants and this unit, can be arranged in 4! ways. Further the three vowels can be arranged among themselves in 3! ways. Hence, the total number of words in which the vowels are together is $4! \times 3! = 144$.

**1.10.** If the letters of the word 'NUMBERS' are permuted in all possible ways, then in how many of these permutations, are the vowels never together, i.e., vowels are separated?

**Sol:** The word NUMBERS have 7 distinct letters of which 2 are vowels. The word can be permuted in 7! ways, of which $6! \times 2!$ permutations have the vowels together. Hence $7! - 2!\ 6! = 7.6! - 2.6! = 5.6! = 3600$ permutations will not have the vowels together. (Indirect counting is done here).

**1.11.** In how many ways can the letters of the word MANIFOLD be arranged so that the vowels are separated?

**Sol:** The word MANIFOLD has 8 distinct letters of which 3 are vowels and 5 are consonants. The vowels have to be separated is same as saying no two vowels are together. Hence between any two vowels there is at least one consonant present which acts as a separator. We first arrange the 5 consonants in 5! ways and then the 3 vowels can be arranged in the 6 possible places as shown below

('C' represents a consonant and - represents a possible position for vowels)

– *C* – *C* – *C* – *C* – *C* –

The vowels can be arranged in ${}^6P_3$ ways. Hence, the total number of words is $5! \times {}^6P_3 = 14400$.

***Directions for examples 1.12 to 1.17*:** The examples are based on the information given below.

Anusha has a collection of 7 CDs with her, viz. Ben Hur, The Sound of Music, Titanic, Small Wonder, Stuart Little, Chariots of Fire and Toy Story. Nikhil wanted to borrow four of these CDs to watch during vacation. Anusha has no reservation against lending any of the CDs.

**1.12.** In how many ways can Nikhil borrow the CDs?

**Sol:** Nikhil can borrow any 4 of the 7 CD's, i.e., he can select any 4 of the 7 CDs in ${}^7C_4 = 35$ ways.

**1.13.** In how many ways can Nikhil borrow the CDs, if he wants to include Stuart Little in his selection?

**Sol:** Since Stuart Little is included in the selection, Nikhil can choose 3 of the remaining 6 CDs in ${}^6C_3 = 20$ ways.

**1.14.** If Nikhil wants to exclude Titanic, in how many ways can the selection be made?

**Sol:** If Titanic must be excluded, Nikhil must select his 4 CDs from the remaining 6 CDs in ${}^6C_4 = 15$ ways.

**1.15.** In how many ways can Nikhil borrow the CDs, if he is particular that his selection should include both Small Wonder and Toy Story if at all one of them is included?

**Sol:** Here we need to consider two cases –

**Case(i):** Both Small Wonders and Toy Story are included.

**Case(ii):** Neither Small Wonders nor Toy Story is included.

In Case(i), Nikhil needs to select 2 more CDs which can be done in ${}^5C_2$ ways.

In Case (ii), Nikhil needs to select 4 CDs from the remaining 5 CDs which can be done in ${}^5C_4$

ways. Hence, the required number of ways= ${}^5C_2 + {}^5C_4 = 15$.

**1.16.** How many selections can be made if Nikhil does not want to include Small Wonder and Toy Story together?

**Sol:** The total number of selections that can be made are ${}^7C_4 = 35$. Of these, we want to exclude the selections where Toy Story and Small Wonder are together, which are ${}^5C_2 = 10$ in number. Hence, the required number of ways are 35 – 10 = 25.

**1.17.** In how many of the selections would exactly one of Ben Hur or Chariots of Fire be included?

**Sol:** The given problem can be broken into two cases
(i) Ben Hur is included but not Chariots of Fire
(ii) Chariots of Fire is included but not Ben Hur.
**Case (i):** When Ben Hur is included, and Chariots of Fire is excluded, the other three CDs can be borrowed from the remaining 5 CDs in ${}^5C_3$ =10 ways.
Similarly, in Case (ii) there are 10 ways of selecting. Hence, the total number of ways are 20.

**1.18.** Consider the word PRECIPITATION. Find the number of ways in which
(i) a selection
(ii) an arrangement of 4 letters can be made from the letters of this word.

**Sol:** The word PRECIPITATION has 13 letters *I, I, I, P, P, T, T, E, R, C,* A, *O, N* of 9 different sorts.
In taking 4 letters, the following are the possibilities to be considered.
(a) all 4 distinct.
(b) 3 alike, 1 distinct.
(c) 2 alike of one kind, 2 alike of other kind.
(d) 2 alike, 2 other distinct.

**1.19.** If all possible four-digit numbers are formed using the digits 5, 6, 7, 8 without repetition and arranged in ascending order of magnitude then find the position of the number 7685.

**Sol:** To find the position of the number 7685, we first count all four-digit numbers that precede 7685 when arranged in ascending order of magnitude.
The number of four-digit numbers that
begin with 5 is 3!
begin with 6 is 3!
begin with 75 is 2!
begin with 765 is 1.
The next number in the order is 7685. Hence 3! + 3! + 2! + 1 = 15 numbers precede 7685. Hence, the position of 7685 is 16.

**1.20.** A committee of 7 is to be formed from 6 ladies and 8 gentlemen. In how many ways can it be done when the committee consists of
(i) exactly 4 ladies.
(ii) at least 4 ladies.

**Sol:** (i) The committee must have 4 ladies and 3 gentlemen. The selection of 4 ladies and 3 gentlemen from 6 ladies and 8 gentlemen can be done in ${}^6C_4 \times {}^8C_3 = 15 \times 56 = 840$ ways.
(ii) Here the committee can consist of 4 or 5 or 6 ladies and accordingly 3 or 2 or 1 gentleman, respectively. This can be done in ${}^6C_4 \times {}^8C_3 + {}^6C_5 \times {}^8C_2 + {}^6C_6 \times {}^8C_1 = 840 + 168 + 8 = 1016$ ways.

**1.21.** (i) How many four letter words can be formed using the letters of the word 'ROAMING'?
(ii) How many three letter words can be formed using the letters of the word 'PRACTICES'?

**Sol:** (i) None of the letters in the word are repeated.
∴The number of four letter words that can be formed $= {}^7P_4 = \frac{7!}{3!} = (7)\ (6)\ (5)\ (4) = 840$.

**Sol:** (ii) PRACTICES
The combinations, the number of combinations and the number of permutations (for each combination) as well as the total are tabulated below.

| Combinations | Number of combinations | Number of permutations for each combination | Total number of permutations |
|---|---|---|---|
| *x, y, z* | ${}^8C_3 = 56$ | 6 | 336 |
| *x, x, y* | ${}^7C_1 = 7$ | 3 | 21 |
| i.e., *c, c, y* | | | 357 |

**1.22.** In a party, each person shook hands with every other person present. The total number of hand shakes was 28. Find the number of people present in the party.

**Sol:** Let the number of people present in the party be *n*.
**Method 1**
The first person shakes hands with a total of $(n - 1)$ persons, the second with $(n - 2)$ other people and so on.

The total number of hand shakes is
$(n-1) + (n-2) + \ldots + 2 + 1$

$$\frac{n(n-1)}{2} = 28 \text{ (given)} \Rightarrow n = 8$$

**Method 2**
Number of hand shakes = Number of ways of selecting 2 people out of $n = {}^{n}C_{2}$.
${}^{n}C_{2} = 28$

$$\frac{n(n-1)}{2!} = 28 \Rightarrow n = 8$$

***Directions for examples 1.23 to 1.27*:** The following examples are based on the data below.

The letters of NESTLE are permuted in all possible ways.

**1.23.** How many of these words begin with T?
**Sol:** NESTLE has 6 letters of which the letter *E* occurs two times. Therefore the required number of words = Number of ways of filling *N*, *E*, *S*, *E* and *L* in the second to sixth positions $= \frac{5!}{2!} = 60$.

**1.24.** How many of these words begin and end with *E*?
**Sol:** The required number of words = The number of ways of filling *N*, *S*, *T* and *L* in the second to fifth positions = 4! = 24.

**1.25.** How many of these words begin with *S* and end with *L*?
**Sol:** The required number of words = The number of ways of filling *N*, *E*, *T* and *E* in the second to fifth positions $= \frac{4!}{2!} = 12$.

**1.26.** How many of these words neither begin with *S* nor end with *L*?
**Sol:** The required number of words = The total number of words which can be formed using the letters *N*, *E*, *S*, *T*, *L* and *E* – (Number of words which begin with *S* or end with *L*)

$= \frac{6!}{2!}$ – (Number of words beginning with *S* + Number of words ending with *L* – Number of words beginning with *S* and ending with *L*)

$= \frac{6!}{2!} - \left(\frac{5!}{2!} + \frac{5!}{2!} - \frac{4!}{2!}\right)$

$= 360 - (60 + 60 - 12) = 252.$

**1.27.** How many of these words begin with *T* and do not end with N?
**Sol:** The required number of words = The number of words beginning with *T* – The number of words beginning with *T* and ending with *N*

$= \frac{5!}{2!} - \frac{4!}{2!} = 48.$

***Directions for examples 1.08 to 1.11*:** The following examples are based on the data below.

The letters of FAMINE are permuted in all possible ways.

**1.28.** How many of these words have all the vowels occupying odd places?
**Sol:** FAMINE has 3 vowels and 3 consonants.
The vowels can be arranged in the odd places in 3! or 6 ways.
The consonants would have to be arranged in even places. This is possible in 3! or 6 ways as well.
∴ The required number of words = $6^2 = 36$.

**1.29.** How many of these words have all the vowels together?
**Sol:** If all the vowels are together, the vowels can be arranged in 3! ways among themselves.
Considering the vowels as a separate unit and each of the other letters as a unit, we have a total of 4 units which can be arranged in 4! ways.
∴ The required number of words = 4! 3! = 144

**1.30.** How many of these words have at least two of the vowels separated?
**Sol:** The required number of words = The total number of words which can be formed using the letters *F*, *A*, *M*, *I*, *N* and *E* – The number of words with all the vowels together = 6! – 4! 3! = 576.

**1.31.** How many of these words have no two vowels next to each other?
**Sol:** To ensure that no two vowels are together, we first arrange the 3 consonants say – $c_1$ – $c_2$ – $c_3$ – and place the vowels in the gaps between the consonants or the initial or final position. For each arrangement of the consonants, there are 4 places where the vowels can go. The vowels can be dealt with in 4 (3) (2) ways.
∴ The total number of words is 3! 4! = 144.

***Directions for examples 1.32 to 1.34*:** The following examples are based on the data below.

Raju wrote 7 letters *A*, *B*, *C*, *D*, *E*, *F* and *G* on a black board.

**1.32.** How many 4-letter words can be formed using these letters such that at least one letter of the word is a vowel?

**Sol:** The vowels, the number of ways in which the consonants can be selected and the number of words are tabulated below:

| Vowels | No. of ways of selecting consonants | Number of words |
|---|---|---|
| $A$ | ${}^5C_3 = 10$ | 10 (24) |
| $E$ | ${}^5C_3 = 10$ | 10 (24) |
| $AE$ | ${}^5C_2 = 10$ | 10 (24) |
| | | 10 (72) = 720 |

**1.33.** How many 7 letter words can be formed using these letters such that the last two letters of the word are adjacent consonants?

**Sol:** $B$ and $C$, $C$ and $D$ as well as $F$ and $G$ are adjacent consonants. The letters in each of these pairs can be arranged in 2! = 2 ways.
The remaining letters can be arranged in 5! = 120 ways.
$\therefore$ The required number of words = (3) (2) (120) = 720.

**1.34.** How many 7 letter words can be formed using these letters such that the letter at one of its ends is a vowel and that at the other end is a consonant?

**Sol:** If the first letter is a vowel, it has 2 possibilities.
In this case, the last letter has 5 possibilities.
The remaining 5 letters can be arranged in 5! = 120 ways.
$\therefore$ The total number of words = (2) (5) (120)
If the last letter is a vowel, it similarly follows that the number of words = (2) (5) (120)
So the total number of words = (2) (2) (5) (120) = 2400.

***Directions for examples 1.35 and 1.36*:** The following examples are based on the data below.

All possible four-digit numbers are formed using the digits 1, 2, 3 and 4 without repeating any digit.

**1.35.** How many of these numbers have the even digits in even places?

**Sol:** There are two even places.
$\therefore$ The even digits can be arranged in 2! = 2 ways.
$\therefore$ The required number = (2) (2) = 4.

**1.36.** If all the numbers are arranged in an ascending order of magnitude, find the position of the number 3241.

**Sol:** If the first digit is 1 or 2, in each case, the remaining digits can be arranged in 3! or 6 ways.
$\therefore$ a total of (2) (6) = 12 numbers have their first digit as 1 or 2.
If the first digit is 3, the possible numbers in ascending order are 3124, 3142, 3214, 3241,
$\therefore$ The position of 3241 = 16.

***Directions for examples 1.37 and 1.38*:** The following examples are based on the data below.

A committee of 5 is to be formed from 4 women and 6 men.

**1.37.** In how many ways can it be formed if it consists of exactly 2 women?

**Sol:** The committee must have 2 women and 3 men.
$\therefore$ The required number of ways = ${}^4C_2\ {}^6C_3 = 120$

**1.38.** In how many ways can it be formed if it consists of more women than men?

**Sol:** The committee must have either 4 women and 1 man or 3 women and 2 men.
$\therefore$ The required number of ways
$= {}^4C_4\ {}^6C_1 + {}^4C_3\ {}^6C_2 = 6 + 60 = 66.$

**1.39.** Find the number of four-digit numbers which can be formed using four of the digits 0, 1, 2, 3 and 4 without repetition.

**Sol:** The first digit has 4 possibilities (1, 2, 3 and 4).
The second digit has 4 possibilities (0 and any of the three digits not used as the first digit).
The third digit has 3 possibilities.
The last digit has 2 possibilities.
$\therefore$ The required number of numbers = (4) (4) 3 (2) = 96.

**1.40.** The number of diagonals of a regular polygon is four times the number of its sides. How many sides does it have?

**Sol:** Let the number of sides in the polygon be $n$.

$$\frac{n(n-3)}{2} = 4n$$

$$n(n-11) = 0$$
$$n > 0$$
$$\therefore\ n - 11 = 0$$
$$\therefore\ n = 11$$

## EXERCISE-1

***Directions for questions 1 to 50*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Find the value of ${}^{8}P_{3}$.
(A) 166 (B) 336 (C) 672 (D) 536

**2.** Find the value of ${}^{10}C_{7}$.
(A) 60 (B) 360 (C) 120 (D) 720

**3.** Find the value of ${}^{12}C_{2}$.
(A) 55 (B) 66 (C) 132 (D) 110

**4.** If, ${}^{n}C_{12} = {}^{n}C_{18}$, then find ${}^{n}C_{3}$.
(A) 4060 (B) 240 (C) 30 (D) 1140

**5.** If ${}^{n}P_{4} = {}^{n}P_{5}$, then $n =$ _____.

**6.** If ${}^{n}C_{r} = {}^{n}P_{r}$, then the value of $r$ can be
(A) 0
(B) 1
(C) 2
(D) More than one of the above

**7.** If ${}^{n}C_{5} = {}^{n}C_{7}$, then ${}^{2n+1}C_{2}$ is
(A) 250 (B) 300 (C) 240 (D) 280

**8.** Find the value of ${}^{120}C_{119}$.
(A) 119! (B) 120! (C) 120 (D) 119

**9.** Find the value of ${}^{35}P_{34}$.
(A) 35! (B) 35 (C) 34! (D) 34

**10.** When a dice is rolled for $n$ times, then the number of total outcomes is
(A) 6 (B) $6^{n-1}$ (C) $6^{n}$ (D) $6^{n+1}$

**11.** If ${}^{n}P_{r} = 5040$ and ${}^{n}C_{r} = 210$, then find the value of $r$.
(A) 3 (B) 6 (C) 4 (D) 5

**12.** Relation between ${}^{n}C_{r}$ and ${}^{n}P_{r}$ is
(A) ${}^{n}P_{r} = {}^{n}C_{r}$ (B) ${}^{n}P_{r} = {}^{n}C_{n-r}$
(C) ${}^{n}P_{r} = r!{}^{n}C_{r}$ (D) ${}^{n}C_{r} = r!{}^{n}P_{r}$

**13.** ${}^{8}C_{7} + {}^{8}C_{6} =$
(A) ${}^{9}C_{6}$ (B) ${}^{8}C_{7}$ (C) ${}^{9}C_{7}$ (D) 8

**14.** If ${}^{n}C_{7} = {}^{n}C_{3}$, then $n =$
(A) 10 (B) 4 (C) 7 (D) 3

**15.** Number of ways of arranging 3 persons in 8 distinct chairs is
(A) ${}^{8}P_{3}$ (B) ${}^{8}C_{3}$ (C) 8! (D) 3!

**16.** In how many ways can 4 letters be posted into 3 letters boxes?
(A) 32 (B) 64 (C) 27 (D) 81

**17.** When a coin is tossed for $(n-1)$ times, then the number of total outcomes is
(A) 2 (B) $2^{n}$ (C) $2^{n+1}$ (D) $2^{n-1}$

**18.** In how many ways can 8 letters be posted into 5 letter boxes?
(A) ${}^{8}P_{5}$ (B) $5^{8}$ (C) $8^{5}$ (D) ${}^{8}C_{5}$

**19.** When a coin is tossed for $n$ times, then the number of ways of getting exactly '$r$' heads is
(A) $2^{r}$ (B) ${}^{n}C_{r}$ (C) ${}^{n}P_{r}$ (D) $2^{n}$

**20.** How many words can be formed using all the letters of the word GINGER?
(A) 720 (B) 240 (C) 380 (D) 360

**21.** Number of even numbers that can be formed using all the digits 1, 3, 5, 6, 7 is
(A) 5! (B) 5 (C) 4! (D) 0

**22.** Find the number of ways of arranging all the letters of the word ARRANGE.
(A) 7! (B) $\frac{7!}{(2!)^{2}}$ (C) $\frac{7!}{2!}$ (D) 4!7!

**23.** Number of odd numbers that can be formed using the digits 2, 4, 6, 8 is _____.

**24.** The number of different words that can be formed using all the letters of the word SPECIAL that begin with $S$ and end with $L$ is
(A) 5! (B) 7! (C) 6! (D) 4!

**25.** Number of ways of selecting 5 out of 7 persons is
(A) ${}^{7}P_{5}$ (B) ${}^{7}C_{5}$ (C) 7! (D) 5!

**26.** How many different words can be formed using all the letters of the word TRINETRA?
(A) 20160 (B) 40320
(C) 5040 (D) 10080

**27.** In how many ways can 4 boys be seated in 6 chairs?
(A) 180 (B) 720 (C) 360 (D) 240

**28.** How many odd numbers can be formed using the digits {0, 2, 4, 6}?
(A) 0 (B) 192 (C) 18 (D) 20

**29.** How many different words can be formed using the letters of the word MARKET so that they begin with *K* and end with *R*?
(A) 16 (B) 24 (C) 12 (D) 64

**30.** All possible four-digit numbers, with distinct digits are formed, using the digits {1, 3, 4, 5, 6}. How many of them are divisible by 5?
(A) 8 (B) 12 (C) 24 (D) 20

**31.** How many arrangements can be made by using all the letters of the word 'COMPLEX'?_____

**32.** The number of diagonals in a regular decagon is _____.

**33.** In a library there are 12 research scholars. In how many ways can we select 4 of them?
(A) 400 (B) 495
(C) 320 (D) 240

**34.** Neha has 10 different chocolates. In how many ways can she give them to two of her friends?
(A) 10! (B) $2^{10}$ (C) 10 (D) 9

**35.** In how many ways can 6 persons be seated around a circular table?_____

**36.** In how many ways can three blue balls be selected from a bag which contains four white balls six blue balls?
(A) 20 (B) 10 (C) 120 (D) 210

**37.** If the number of diagonals of a polygon is five times the number of sides, the polygon is
(A) 13 (B) 20 (C) 15 (D) 17

**38.** In how many ways can three boys and two girls be seated in a row, so that all girls sit together?
(A) 12 (B) 24 (C) 84 (D) 48

**39.** In how many ways can four letters be selected from the word EDUCATION?
(A) ${}^9C_8$ (B) ${}^9C_7$ (C) ${}^9C_4$ (D) ${}^9C_6$

**40.** In how many ways can five men and three women be seated around a circular table?
(A) 720 (B) 5040 (C) 4020 (D) 2520

**41.** Number of ways of arranging 5 persons in a row is
(A) 5 (B) 10 (C) 5! (D) 20

**42.** In how many ways can we select two vertices in a octagon?_____

**43.** A set has 8 elements. How many non-empty subsets are there for the set?_____

**44.** Find the number of diagonals in a regular duodecagon.
_____

**45.** In how many ways can 6 letters be posted into 4 letterboxes?
(A) $6^4$ (B) ${}^6P_4$ (C) $4^6$ (D) ${}^6C_4$

**46.** Ten points are selected on a plane, such that no three of them are collinear. How many different straight lines can be formed by joining these points?
(A) 54 (B) 45 (C) 90 (D) 108

**47.** If *n* books can be arranged on an ordinary shelf in 720 ways, then in how many ways can these books be arranged on a circular shelf?
(A) 120 (B) 720 (C) 360 (D) 60

**48.** In how many ways can two consonants be selected from the English alphabet?
(A) 420 (B) 105 (C) 210 (D) 300

**49.** Find the number of ways in which 11 delegates can be arranged around a table for a meeting. _____

**50.** Rahul has six friends. In how many ways can he invite five or more friends for dinner?
(A) 1 (B) 6 (C) 7 (D) 8

## EXERCISE-2

***Directions for questions 1 to 50*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** In how many ways can the letters of the word VICTORY be arranged so that consonants appear only in the even places?
(A) 180 (B) 360
(C) 240 (D) 0

**2.** In how many ways can the letters of the word TROUBLE be arranged such that no two vowels are together?
(A) 24 (B) 1440 (C) 60 (D) 720

**3.** In how many ways can the letters of the word MALBORNE be arranged so that the vowels are always together?
(A) 4320 (B) 1440 (C) 2160 (D) 5040

**4.** How many words can be formed using all the letters of the word PROBLEM without repetition such that the vowels occupy the even places?

(A) 1440 (B) 360 (C) 720 (D) 120

**5.** The number of arrangements that can be made using all the letters of the word QUARTZ which begin with A but do not end with *R* is
(A) 36 (B) 96 (C) 48 (D) 24

**6.** How many 5-digit numbers can be formed using the digits 0, 2, 3, 4, 5, 8 and 9, if repetition of the digits is not allowed?_____

**7.** The number of words that can be formed using all the letters of the word LIMPET, so that the vowels are always together is _____.

**8.** Find the number of ways of arranging all the letters of the word THUNDER, so that the vowels appear in the odd places. _____

**9.** Twelve varieties of ice-cream are available at an ice-cream parlour. The number of ways of selecting
(i) at least one of them is
(A) 4096 (B) 4095
(C) 12! (D) 12! – 1

**10.** Twelve varieties of ice-cream are available at an ice-cream parlour. The number of ways of selecting
(ii) at least 2 of them is
(A) 4084 (B) 13
(C) 66 (D) 4083

**11.** In how many ways can 21 differently coloured beads be strung on a necklace?
(A) (21)! (B) (20)! (C) $\frac{(21)!}{2}$ (D) $\frac{(20)!}{2}$

**12.** In how many ways can the crew of a ten oared boat be arranged, when of the 10 persons available, two of whom can row only on the bow side and three of whom can row only on the stroke side?
(A) $\frac{10!}{2!\,3!}$ (B) $\frac{10!}{8!\,7!}$ (C) $\frac{5!}{3!\,2!}$ (D) $\frac{(5!)^3}{3!2!}$

**13.** How many numbers can be formed using all the digits 5, 4, 7, 6, 1, 4, 5, 4, 1 such that the even digits always occupy the even places?
(A) $\frac{9!}{2!\,2!\,2!}$ (B) $\frac{9!}{4!\,4!}$
(C) $\frac{4!\,5!}{3!\,2!\,2!}$ (D) $\frac{4!\,5!}{2!\,2!}$

**14.** How may five-digit numbers can be formed using the digits 0 to 8 if no digit is to occurmore than once in any number?
(A) 1680 (B) 6720
(C) 13440 (D) 1344

**15.** How many of the numbers in the above problem are divisible by 5?
(A) 1680 (B) 1470 (C) 3150 (D) 3360

**16.** If all the letters of the word RATE are taken and permuted and arranged in alphabetical order as in a dictionary, then what is the rank of the word TEAR?
(A) 20 (B) 23 (C) 22 (D) 21

**17.** There are 5 different Maths books, 4 different Physics books and 3 different Chemistry books. The number of ways in which at least one book can be given away is
(A) $(2^5 - 1)\ (2^4 - 1)\ (2^3 - 1)$
(B) $2^{12}$
(C) $2^{12} - 1$
(D) $(2^5 - 1)\ (2^4 - 1)\ (2^3 - 1) - 1$

**18.** Find the sum of all numbers that can be formed by taking all the digits at a time from 4, 5, 6, 7, 8 without repetition.
(A) 133320 (B) 244420
(C) 7999920 (D) 2233440

**19.** In how many ways can a representation of 12 students consisting of 8 boys and 4 girls be selected from 15 boys and 10 girls, if a particular boy A and a particular girl *B* are never together in the representation?
(A) ${}^{14}C_8 \times {}^{9}C_4$
(B) ${}^{15}C_8 \times {}^{10}C_4 - {}^{14}C_8 \times {}^{9}C_4$
(C) ${}^{15}C_8 \times {}^{10}C_4 - {}^{14}C_7 \times {}^{9}C_3$
(D) ${}^{14}C_7 \times {}^{9}C_3$

**20.** Rohit attempts a multiple choice question paper consisting of 10 questions and each question having 4 choices. The number of ways in which he can attempt the entire paper if he is marking the answers at random is
(A) ${}^{10}P_4$ (B) $4^{10}$ (C) ${}^{10}C_4$ (D) $10^4$

**21.** Two red pencils, three black pencils and two white pencils are to be arranged in a row such that:
(a) no two adjacent pencils are of the same colour and
(b) the pencils at the two ends of the row are of same colour.
In how many ways can the pencils be arranged?
(A) 12 (B) 8 (C) 9 (D) 10

**22.** Fifteen lines are drawn in a plane such that four of them are parallel. What is the maximum number of regions into which the plane is divided?
(A) 121 (B) 116
(C) 115 (D) 114

**23.** A certain number of students of a school participated in the chess tournament of their Annual Sports Meet. Each

player played 1 game against each of the other player. It was found that in 66 games both the players were girls, and in 240 games one was a girl and the other was a boy. The number of games in which both the players were boys is _____.

**24.** There is an unlimited supply of identical red, blue and green coloured balls. In how many ways can 12 balls be selected from the supply? _____

**25.** Mangalam has forgotten his friend's 8-digit telephone number but remembers the following:
(i) The first 3 digits are either 270 or 279.
(ii) The digit 0 occurs exactly three times and the digit 9 occurs exactly once.
(iii) The number is an even number.
If Mangalam were to use a trial and error method to reach his friend, what is the maximum number of trials he has to make to be sure to succeed?
(A) 832 (B) 1664 (C) 1280 (D) 2000

**26.** A man has 7 friends whom he wants to invite for dinner. The number of ways in which he can invite
(i) at least one of them is
(A) 127 (B) 128 (C) 126 (D) 100
(ii) at least 3 of them is
(A) 98 (B) 100 (C) 99 (D) 92

**27.** Find the number of selections that can be made by taking four letters from the word SENTENCE.
(A) 21 (B) 22 (C) 17 (D) 10

**28.** Seven boxes numbered 1 to 7 are arranged in a row. Each is to be filled by either a black or blue coloured balls such that no two adjacent boxes contain blue coloured balls. In how many ways can the boxes be filled with the balls?
(A) 23 (B) 33 (C) 34 (D) 32

**29.** A question paper consists of 10 sections with each section having two questions. In how many ways can a candidate attempt one or more questions choosing not more than one question per section?
(A) $3^{10}$ (B) $3^{10}-1$ (C) $2^{10}-1$ (D) $2^{10}$

**30.** If all possible four-digit numbers are formed using the digits 3, 5, 6, 9 without repetition and arranged in ascending order of magnitude, then the position of the number 6953 is
(A) 20 (B) 16 (C) 18 (D) 15

**31.** From numbers 8, 9, 10, 11, … 25. Eight numbers are selected. such that the greatest is 19. In how many ways can these 8 numbers be permuted?
(A) 11! (B) (330)8!
(C) $\frac{(11)!}{3}$ (D) $\frac{(11)!}{6}$

**32.** In how many ways five letters are posted into 3 post boxes such that at least one letter is posted in each box?_____

**33.** There are 3 groups. A, *B*, *C*—with 8, 6 and $n$ persons, respectively. Person in each group shakes hands with every person in the other two within any groups exactly once and no two persons shakes hands. The total number of handshakes among them is 104. Find the value of $n$. _____

**34.** Four atheletesPravin, Visharath, Bhushan and Durandhar participate in 6 athletic events. There is only one prize for winning in each event and each of them won in at least one event. In how many ways could they have won the six prizes?_____

**35.** There are 6 letters and corresponding 6 addressed envelops. If the letters are placed into the envelopes randomly (each letter is placed only in one envelope), in how many ways can exactly two letters be placed into their corresponding envelopes?_____

**36.** There are 6 copies of a biography, 5 copies of an autobiography and 4 copies of a novel. The number of ways in which one or more books can be given away is_____.

**37.** In a regular polygon with 15 sides, find the number of triangles that can be formed with the vertices of the polygon, such that none of the sides of the triangles are taken from the sides of the polygon. _____

**38.** Find the sum of all numbers that can be formed using all the digits 1, 2, 8, 9 and 5 without repetition.
(A) 5555500 (B) 666600
(C) 4444400 (D) 6666600

**39.** In how many ways can 6 girls and 6 boys sit around a circular table so that no two boys sit together?
(A) $(5!)^2$ (B) $(6!)^2$ (C) 5! 6! (D) 11!

**40.** Find the number of ways of arranging the letters of the word INSTALLATION in all possible ways so that the Ns come together?
(A) 11!2! (B) $\frac{11!}{(2!)^4}$ (C) $\frac{12!}{(2!)^5}$ (D) $\frac{11!}{(2!)^3}$

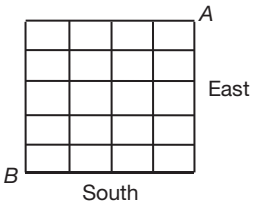
**41.** In how many ways can 12 books be divided equally
(i) among 3 boys?
(A) $\frac{12!}{(4!)^3}$ (B) $\frac{12!}{3!(4!)^3}$
(C) $\frac{12!}{4!}$ (D) 4! 4! 4!

**42.** In how many ways can 12 books be divided equally
(ii) into 3 parcels?

(A) $\frac{12!}{4!}$ (B) $\frac{12!}{4!3!}$ (C) $\frac{12!}{(4!)^3}$ (D) $\frac{12!}{3!(4!)^3}$

**43.** There are 5 copies of a Maths book, 4 copies of a Physics book and 3 copies of a Chemistry book. The number of ways in which one or more books can be given away is
(A) 89 (B) 119 (C) 60 (D) 59

**44.** In the figure below, the lines represent the one way roads allowing cars to travel only west or south. In how many ways can a car travel from the point $A$ to point $B$?




(A) 160 (B) 128 (C) 150 (D) 126

**45.** The number of terms in the expansion $(a+b+c+d)^{20}$ is
(A) 1671 (B) 1717 (C) 1771 (D) 7171

**46.** The number of positive integer solutions for the equation $x+y+z=20$ where $x \geq 2$, $y \geq 3$, $z \geq 4$ is _____.

**47.** The number of positive integral solutions of the equation $a+b+c=15$ is
(A) 76 (B) 105
(C) 91 (D) 86

**48.** The number of non-negative integral solutions of the equation $p+q+r+s=30$ is
(A) 4598 (B) 5324
(C) 5546 (D) 5456

**49.** What is the number of non-negative integer solutions of $A+B+C+D \leq 15$?
(A) ${}^{18}C_3$ (B) ${}^{19}C_3$
(C) ${}^{18}C_4$ (D) ${}^{19}C_4$

**50.** How many positive integers less than 100,000 and divisible by 125 can be formed using the digits 0, 1, 2, 5 and 8, if repetition is allowed?
(A) 90 (B) 300
(C) 99 (D) 129

## EXERCISE-3

***Directions for questions 1 to 50*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** If ${}^nP_r = 120$ and ${}^nC_r = 20$, then $n =$
(A) 20 (B) 12 (C) 6 (D) 4

**2.** ${}^{25}C_{12} + {}^{25}C_{19} - {}^{25}C_6 - {}^{25}C_{13} =$
(A) 25 (B) 0 (C) 13 (D) 2

**3.** How many different signals can be formed by arranging 3 out of 6 different coloured flags in a row?
(A) 120 (B) 20 (C) 216 (D) 729

**4.** City $A$ is connected to City $B$ by four highways and City $B$ is connected to City $C$ by three highways. In how many ways can one travel from $A$ to $C$ via $B$?
(A) 64 (B) 4 (C) 7 (D) 12

**5.** Using all the letters of the word 'NOKIA', how many words can be formed, which begin with $N$ and end with $A$?
(A) 3 (B) 6 (C) 24 (D) 120

**6.** A boy has nine trousers and 12 shirts. In how many different ways can he select a trouser and a shirt?
(A) 21 (B) 12 (C) 9 (D) 108

**7.** Using all the letters of the word 'THURSDAY', how many different words can be formed?
(A) 8 (B) 8! (C) 7! (D) 7

**8.** If there are 3 routes from a city $A$ to city $B$ and 5 routes from city $B$ to city $C$, then the number of distinct routes form $A$ to $C$ through $B$ are _____.

**9.** How many three letter words can be formed using the letters of the word TIME?
(A) 12 (B) 20 (C) 16 (D) 24

***Directions for question 10*:** These questions are based on the following data.

The number of words which can be formed using the letters of the word 'MEADOWS'.

**10.** have no two vowels together is
(A) 240 (B) 720 (C) 1440 (D) 120

**11.** Find the number of ways in which the letters of the word 'ELECTIONS' can be arranged so that the vowels occupy the odd places.
(A) 3600 (B) 7200
(C) 1440 (D) 2880

***Directions for question 12*:** These questions are based on the following data.

The number of words which can be formed using the letters of the word 'MEADOWS'.

**12.** Starting with *M* and not ending with *W* is
(A) 480 (B) 120 (C) 240 (D) 600

**13.** In how many ways can 5 boys and 7 girls be arranged in a row so that no two boys are together?
(A) $5! \times 7!$ (B) ${}^{7}P_5 \times 5!$
(C) $7! \times {}^{8}P_5$ (D) ${}^{7}P_5 \times 7!$

***Directions for questions 14 and 15*:** These questions are based on the following data.

The number of words which can be formed using the letters of the word 'MEADOWS'.

**14.** Starting with A is
(A) 120 (B) 720 (C) 5040 (D) 24

**15.** Having all the vowels together is
(A) 240 (B) 720 (C) 480 (D) 360

**16.** The number of 5 distinct digit numbers that can be formed using all the digits 1, 2, 3, 4, 5, which are divisible by 6 is _____.

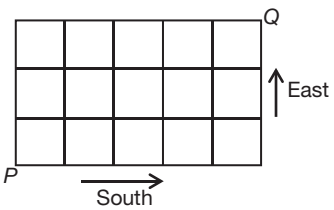
**17.** The number of ways 4 teachers and a principal can be seated in 5 chairs such that the principal always sits in the middle is
(A) 5! (B) 4! (C) $3! \times 2!$ (D) 3!

**18.** How many 7 digit numbers can be formed using the digits 0, 1, 1, 2, 2, 3, 3?
(A) 198 (B) 200 (C) 240 (D) 540

**19.** In how many ways can 12 prizes be distributed among 3 boys?
(A) $3^3$ (B) $12^3$ (C) $3^{12}$ (D) $3^{15}$

**20.** In how many ways can 6 letters be posted when 4 mail boxes are available for posting the letters?
(A) ${}^{6}C_4$ (B) $4^6$ (C) ${}^{6}P_4$ (D) $6^4$

**21.** In the figure below, the lines represent the one-way roads allowing cars to travel only eastwards or southwards. In how many ways can a car travel from the point *P* to the point *Q*?




(A) 15 (B) 66 (C) 56 (D) 76

**22.** Nine points are marked on a straight line and 10 points are marked on another line which is parallel to the first line. How many triangles can be formed with these points as vertices?_____

**23.** Let *k* be an integer such that the sum of the digits of *k* is 2 and $10^9 < k < 10^{10}$. How many values can *k* have?
(A) 10 (B) 7 (C) 9 (D) 45

**24.** In an 'akhada', *m* wrestlers stand around a circle. Each possible pair of persons not standing next to each other play a match with in the middle of the circle for 1minute and 30 seconds. If the total time taken for all the matches is 30 minutes, then *m* is equal to _____.

**25.** Ten points are plotted in a plane such that no three of them lie on a straight line. Four of these points are joined to each of the remaining six points and each of the remaining six points is joined to exactly five points. How many line segments are formed?
(A) 27 (B) 25 (C) 29 (D) 24

**26.** How many non-negative integral solutions does the equation $x_1 + x_2 + x_3 + x_4 = 15$ have?
(A) 815 (B) 816 (C) 618 (D) 680

**27.** How many different words can be formed using all the letters of the word 'COMBINATION', such that the vowels as well as consonants appear in alphabetical order?
(A) 426 (B) 462
(C) 624 (D) None of these

**28.** Group *A* contains 12 persons, group *B* contains 15 persons and group *C* contains *n* persons. Every person in one group shakes hands with every person in the other groups exactly once. The total number of handshakes is 666. Then find handshakes the value of *n*. (There is no handshake between two persons in the same group)
(A) 16 (B) 25 (C) 18 (D) 20

**29.** From the numbers 4,5,6,…20, nine numbers are selected such that the greatest is 15. In how many ways can these 9 numbers be permuted?
(A) (33)10! (B) 165(9!)
(C) (33)9! (D) 145 (10!)

**30.** In how many ways can 7 identical balls be placed into four boxes *P*, *Q*, *R*, *S* such that the two boxes *P* and *Q* have at least one ball each?
(A) 84 (B) 70 (C) 120 (D) 56

**31.** The number of ways of arranging the letters of the word 'MATERIAL' such that all the vowels are always together is_____.

**32.** A question paper consists of five problems, each problem having three internal choices. The number of ways a candidate can attempt one or more problems is _____.

**33.** In how many ways can three consonants and two vowels be selected from the letters of the word 'TRIANGLE'?
(A) 25 (B) 30 (C) 40 (D) 20

**34.** The number of ways in which five boys and three girls can be made to sit in a row such that all boys sit together is _____.

**35.** The number of four digit even numbers that can be formed using the digits {2, 3, 5, 1, 7, 9} (repetition of digits is not allowed) is_____.

**36.** There are 12 points in a plane of which 4 are ona straight line and no three of the other points lieona straight line.

(i) How many straight lines can be formed by joining these points?
(A) 60 (B) 61 (C) 59 (D) 66

(ii) How many triangles can be formed by joining these points?
(A) 220 (B) 216 (C) 224 (D) 66

**37.** Twelve villages in a district are divided into 3 zones with four villages per zone. The telephone department of the district intends to connect the villages with telephone lines such that every two villages in the same zone are connected with three direct lines and every two villages belonging to different zones are connected with two direct lines. How many direct lines are required?
(A) 210 (B) 96 (C) 54 (D) 150

**38.** There is a grid of 6 uniformly spaced horizontal lines interacting 6 uniformly spaced vertical lines. The distance between any two adjacent horizontal lines or any two adjacent vertical lines is 1 unit. An ant has to go from $A(0,0)$ to $B(5,5)$. If it can move only along the lines, find the numbers of shortest paths it can take. _____

**39.** A certain number of students of a school have participated in the chess tournament of their Annual Sports Meet. It was found that in 105 games both the players were girls and in 300 games both the players were boys. The number of games in which one was a girl and another a boy is
(A) 500 (B) 600 (C) 375 (D) 210

**40.** There are 8 lines, such that no two lines are parallel and no three lines are concurrent. Find the number of regions that are formed with these 8 lines. _____

**41.** Find the sum of all the numbers that can be formed by taking all the digits at a time from 3, 4, 6, 7 and 9 without repetition.
(A) 7733652 (B) 7733256
(C) 7373256 (D) 7373652

**42.** Nine villages in a district are divided into three zones with three villages per zone. The telephone department of the district intends to connect the villages with telephone lines such that every two villages in the same zone are connected with four direct lines and every two villages belonging to different zones are connected with two direct lines. How many direct lines are required?
(A) 85 (B) 90 (C) 57 (D) 100

**43.** If all words that can be formed from the letters of the word GARNE are taken and arranged in alphabetical order, then the rank of the word RANGE is
(A) 101 (B) 100 (C) 102 (D) 78

**44.** There are 7 copies of an R.K. Narayan novel, 6 copies of Sidney Sheldon novel and 5 copies of the complete works of Milton. The number of ways in which one or more books can be given away is
(A) 300 (B) 288 (C) 336 (D) 335

**45.** If all the letters of the word 'BANANA' are taken and rearranged in all possible way, in alphabetical order as in a dictionary, then what is the rank of the arrangement 'NANAAB'?
(A) 46 (B) 50 (C) 49 (D) 48

**46.** Find the sum of all possible 5-digit numbers that can be formed using the digits 1, 3, 4, 6 and 8 without repetition.
(A) 4880008 (B) 5040068
(C) 5866608 (D) 5866068

**47.** From a group of 10 professors and 6 assistant professors, a management institute desires to send a delegation of 8 persons consisting of 5, professors and 3 assistant professors to the IIMs annual meet. If Prof.Balamurali, a science Professor refuses to be in the delegation if Assistant Prof.Sheshadri, an arts professor is included in the delegation, then in how many ways can the delegation be formed?

(A) ${}^9C_4\,{}^4C_3$ (B) ${}^9C_5\,{}^4C_2$

(C) ${}^{10}C_5\,{}^6C_3 - {}^9C_4\,{}^5C_2$ (D) ${}^9C_4\,{}^4C_3 + {}^9C_5\,{}^4C_2$

**48.** Sixteen villages in a district are divided into 4 zones with four villages per zone. The telephone department of the district intends to connect the villages with telephone lines such that every two villages in the same zone are connected with two direct lines and every two villages belonging to different zones are connected with one

direct line. How many direct telephone lines are required?_____

**49.** Among all convex polygons with $n$ sides, the maximum number of points of intersection of the diagonals inside the polygon is 495. The value of $n$ is _____.

**50.** A question paper consists of 15 'true or false' questions. In how many ways can a candidate answer the entire paper?
(A) $3^{15}$ (B) $3^{15}-1$
(C) $2^{15}$ (D) $2^{15}-1$

## EXERCISE-4

***Directions for questions 1 to 30*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** If all the letters of the word 'COCOCOLA' are permuted in all possible ways and are arranged in a dictionary order, the $561^{th}$ word is
(A) LOOOACCC (B) ACCCOOOL
(C) LACCCOOO (D) CCCOOOAL

**2.** How many multiples of 4 greater than 40,000 but less than 70,000 can be formed using the digits 0, 1, 3, 4, 6, 7, 8, if repetition of digits is allowed?_____

**3.** In how many ways can the letters of the word ROBUST be reordered so that no letter is in itsoriginal place?
(A) 265 (B) 720 (C) 719 (D) 120

**4.** From a collection of umbrellas hung in a row, one or more can be selected in 511 ways. In how many ways can 4 umbrellas be selected such that the selection contains no two consecutive umbrellas?
(A) 45 (B) 28 (C) 36 (D) 15

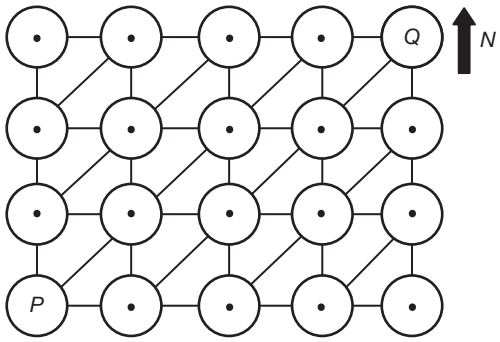
**5.** In how many ways can 20 identical coins be placed in 3 different boxes such that no box contains less than 3 or more than 8 coins?
(A) 35 (B) 21 (C) 20 (D) 15

**6.** How many triangles can be formed using the eight vertices and the center of the regular octagon ABCDEFGH, such that at least one of A and $E$ is a vertex of the triangle?
(A) 64 (B) 52 (C) 34 (D) 48

**7.** In how many ways can 5 boys and 18 girls be seated in a row such that each arrangement begins and ends with a boy and in between any two successive boys an odd number of girls sit?
(A) (14400) (18!) (B) (12600) (18!)
(C) (22500) (18!) (D) (13600) (18!)

**8.** Find the number of positive integral solutions of $x_1\ x_2\ x_3\ x_4 = 600$. _____

***Directions for questions 9 and 10*:** The figure given below depicts a network of routes connecting the circular cells of an ant's nest.




**9.** If an ant can travel only in the North or in the East direction, in how many possible ways can an ant travel from $P$ to $Q$?
(A) 35 (B) 21 (C) 105 (D) 45

**10.** If an ant can travel only in the North, the East or the North-East direction, in how many possible ways can an ant travel from $P$ to $Q$?
(A) 143 (B) 129 (C) 126 (D) 152

**11.** Jim, Joe and Julian decide to divide 20 chocolates among themselves as follows. Jim first picks a card from a set of 21 cards, numbered 0 through 20.Jim gets as many chocolates as the number on the card, and then puts the card back in the set after shuffling. After that, Joe picks up a card and replaces it as often as necessary, till he gets a card numbered greater than or equal to that picked by Jim. Joe gets as many chocolates as the difference between the numbers on the cards picked by him and Jim while Julian gets the remaining chocolates. In how many ways can the chocolates be divided? _____

**12.** Miss Susie, a famous psephologist, constructs a Venn diagram to study the voting pattern among a sample population of 5000 voters. The survey requires the voter to identify election issues from among a list of five issues. A voter may identify none, one, or more than one of the issues listed as an election issue. How many distinct regions should the Venn diagram contain? _____

**13.** There are 15 points in space, of which 5 points lie on the same plane and of the other 10, no four points are in the same plane. Find the number of distinct planes which are determined by the 15 points.
(A) 432 (B) 200 (C) 120 (D) 446

**14.** From the first 20 natural numbers, a combination of 3 distinct numbers is selected and then arranged in ascending order. How many such combinations can be picked so that the average of the extreme numbers gives the middle number?
(A) 102 (B) 90 (C) 94 (D) 86

**15.** Five points $A$, $B$, $C$, $D$ and $E$ lie on a line $l_1$ and points $P$, $Q$, $R$, and $S$ lie on another line $l_2$. Each of the five points on $l_1$ is connected to each of the points on $l_2$, by means of linesegments terminated by the points. Excluding the nine points, the maximum number of points at which the lines can intersect is _____.

**16.** Ten pencils are to be distributed among Alice, Bryan and Crane, such that Alice receives more pencils than Bryan, who in turn receives more pencils than Crane. Find the number of ways in which the distribution can be done if each of them has to receive at least one pencil.
(A) 4 (B) 8
(C) 12 (D) None of these

**17.** All possible six-digit numbers are formed using the digits 1, 2, 3, 4, 5 and 6 without repetition. How many of the numbers thus formed are divisible by every factor of the digit present in its units place?_____

**18.** Fifty chocolates are to be completely distributed among 15 children such that each receives an odd number of chocolates. Find the number of ways in which the distribution can be carried out.
(A) 50!/35! 15! (B) 35!/15!
(C) 50!/2!35!15! (D) None of these

**19.** In how many ways can one choose 4 letters from the word EFFERVESCENT?_____

**20.** A coaching institute was to send 9 parcels to its correspondence students, four of whom were from Bihar and the remaining five were from Delhi. The dispatch clerk made a mistake in addressing the parcels. He addressed them in such a way that no student received the correct parcel although all the parcels for Bihar were sent to Bihar and all the parcels for Delhi were sent to Delhi. Find the number of ways in which he could have addressed those 9 parcels. _____

**21.** From a box containing $n$ items, if at most $(n-2)$ items can be selected in 247 ways, find the number of ways of selecting at least 3 items. _____

**22.** In a convex polygon with $n$ sides, the number of points of intersection of the diagonals in the interior of the polygon is 70. Find the minimum value of $n$.
(A) 9 (B) 8 (C) 10 (D) 12

**23.** Find the sum of all possible 4-digit numbers that can be formed using the digits 1, 2, 3 and 4. Given that each digit can be used more than once in any number.
(A) 2844160 (B) 66160
(C) 66660 (D) 711040

**24.** A bag contains 5 identical white balls, 6 identical green balls and 4 identical red balls. In how many ways can these balls be arranged in a row such that all the red balls are together and no two green balls are together?_____

**25.** The letters of the word BANANA are permuted in all possible ways and listed in alphabetical order as in a dictionary. What is the rank of the word NANAAB?
(A) 50 (B) 39 (C) 51 (D) 49

**26.** Bag A contains 7 maths books, bag $B$ contains 5 physics books and bag $C$ contains $x$ chemistry books. The number of ways of selecting three books such that two books are from one bag and the third book is from a different bag is 541. The value of $x$ is_____.

**27.** My mother gave me 5 rasgullas and went out at 7:00 P.M. She is expected to return at 8:00 P.M. She knew that I would eat them all and so she told me to give a gap of at least 5 minutes between any two rasgullas. If I can instantaneously eat a rasagulla and I eat them only at the middle of a minute, what is the number of combinations of times at which I could have eaten my rasgullas (of course all) during that one hour?
(A) ${}^{39}C_5$ (B) ${}^{40}C_5$ (C) ${}^{41}C_5$ (D) ${}^{44}C_5$

**28.** Five boys including Suresh and five girls including Suneetaare to be seated around a circular table such that no two boys are adjacent and Suneeta is not adjacent to Suresh. In how many ways can this be done?
(A) $4! \times 5!$ (B) $(4!)^2$
(C) $3 \times (4!)^2$ (D) $3 \times (5!)^2$

**29.** Find the number of ways in which 14 boys and 10 girls can be seated in a row such that no two girls sit adjacent to each other and a particular boy and a particular girl sit adjacent to each other.
(A) $\frac{(14!)^2}{60}$ (B) $\frac{14! \times 13!}{120}$
(C) $\frac{(14!)^2}{120}$ (D) $\frac{14! \times 13!}{60}$

**30.** How many 9-digit multiples of 6 can be formed using only the digits 8 or 9? _____

## ANSWER KEYS

### Exercise-1

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (B) | 9. (A) | 17. (D) | 25. (B) | 33. (B) | 41. (C) | 49. 3628800 |
| 2. (C) | 10. (C) | 18. (B) | 26. (D) | 34. (B) | 42. 28 | 50. (C) |
| 3. (B) | 11. (C) | 19. (B) | 27. (C) | 35. 120 | 43. 255 | |
| 4. (A) | 12. (C) | 20. (D) | 28. (A) | 36. (A) | 44. 54 | |
| 5. 5 | 13. (C) | 21. (C) | 29. (B) | 37. (A) | 45. (C) | |
| 6. (D) | 14. (A) | 22. (B) | 30. (C) | 38. (D) | 46. (B) | |
| 7. (B) | 15. (A) | 23. 0 | 31. 5040 | 39. (C) | 47. (A) | |
| 8. (C) | 16. (D) | 24. (A) | 32. 35 | 40. (B) | 48. (C) | |

### Exercise-2

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (D) | 9. (B) | 17. (C) | 25. (B) | 32. 150 | 40. (B) | 48. (D) |
| 2. (B) | 10. (D) | 18. (C) | 26. (i) (A) | 33. 4 | 41. (A) | 49. (D) |
| 3. (A) | 11. (D) | 19. (C) | (ii) (C) | 34. 1560 | 42. (D) | 50. (C) |
| 4. (C) | 12. (D) | 20. (B) | 27. (B) | 35. 135 | 43. (B) | |
| 5. (B) | 13. (C) | 21. (D) | 28. (C) | 36. 209 | 44. (D) | |
| 6. 2160 | 14. (C) | 22. (C) | 29. (B) | 37. 275 | 45. (C) | |
| 7. 240 | 15. (C) | 23. 190 | 30. (C) | 38. (D) | 46. 78 | |
| 8. 1440 | 16. (D) | 24. 91 | 31. (B) | 39. (C) | 47. (C) | |

### Exercise-3

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (C) | 9. (D) | 17. (B) | 25. (A) | 33. (B) | 40. 37 | 48. 144 |
| 2. (B) | 10. (C) | 18. (D) | 26. (B) | 34. 2880 | 41. (B) | 49. 12 |
| 3. (A) | 11. (B) | 19. (C) | 27. (B) | 35. 60 | 42. (B) | 50. (C) |
| 4. (D) | 12. (D) | 20. (B) | 28. (C) | 36. (i) (B) | 43. (C) | |
| 5. (B) | 13. (C) | 21. (C) | 29. (B) | (ii) (B) | 44. (D) | |
| 6. (D) | 14. (B) | 22. 765 | 30. (D) | 37. (D) | 45. (B) | |
| 7. (B) | 15. (B) | 23. (A) | 31. 1440 | 38. 252 | 46. (C) | |
| 8. 15 | 16. 48 | 24. 8 | 32. 1023 | 39. (C) | 47. (C) | |

### Exercise-4

| | | | | | |
|---|---|---|---|---|---|
| 1. (C) | 6. (D) | 11. 231 | 16. (A) | 21. 219 | 26. 6 |
| 2. 1470 | 7. (A) | 12. 32 | 17. 648 | 22. (B) | 27. (D) |
| 3. (A) | 8. 800 | 13. (D) | 18. (D) | 23. (D) | 28. (C) |
| 4. (D) | 9. (A) | 14. (B) | 19. 121 | 24. 42 | 29. (A) |
| 5. (D) | 10. (B) | 15. 60 | 20. 396 | 25. (A) | 30. 85 |

## SOLUTIONS

## EXERCISE-1

**1.** ${}^8P_3 = \frac{8!}{5!} = 8\ (7)\ (6) = 336.$ $\left(\because {}^nP_r = \frac{n!}{(n-r)!}\right)$

**2.** ${}^{10}C_7 = \frac{10!}{3!\,7!} = \frac{10\,(9)\,(8)}{3\,(2)\,(1)} = 120$ $\left(\because {}^nC_r = \frac{n!}{(n-r)!}\right)$

**3.** ${}^{12}C_2 = \frac{12!}{10!\,2!} = \frac{12(11)}{2(1)} = 66$

**4.** Given, ${}^nC_{12} = {}^nC_{18}$

$\Rightarrow n = 12 + 18 = 30$ $\left(\because n_{c_r} = n_{c_s} \Rightarrow n = r + s\right)$

$\therefore {}^{30}C_3 = \frac{30!}{27!\,3!} = \frac{30\,(29)\,(28)}{3\,(2)\,(1)} = 4060$

**5.** Given, ${}^nP_4 = {}^nP_5 \Rightarrow \frac{n!}{(n-4)!} = \frac{n!}{(n-5)!} \Rightarrow \frac{1}{n-4} = 1$

$\Rightarrow n - 4 = 1 \Rightarrow n = 5$

**6.** Given ${}^nC_r = {}^nP_r$

We know that ${}^nP_r = {}^nC_r.r! \Rightarrow {}^nC_r = {}^nC_r.r!$

$\Rightarrow r! = 1 \quad \therefore r = 0$ or 1

**7.** Given ${}^nC_5 = {}^nC_7$

We know that if ${}^nC_r = {}^nC_s$, then $r + s = n$

${}^nC_5 = {}^nC_7 \Rightarrow n = 5 + 7 = 12$

${}^{2n+1}C_2 = {}^{2(12)+1}C_2 = {}^{25}C_2 = 300$

**8.** We know that

${}^{120}C_{119} = {}^{120}C_{120-119} = {}^{120}C_1 = 120$

**9.** We know that ${}^nP_{n-1} = n!$

${}^{35}P_{34} = 35!$

**10.** When a dice is rolled $n$ times the number of outcomes is $6^n$.

**11.** ${}^nP_r = 5040 = 10 \times 98 \times 7 = {}^{10}P_4$

$\Rightarrow n = 10, r = 4$ also ${}^{10}C_4 = 210 \quad \therefore r = 4$

**12.** ${}^nP_r = r!\ {}^nC_r$

**13.** ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r \Rightarrow {}^8C_7 + {}^8C_6 = {}^9C_7$.

**14.** We know if ${}^nC_r = {}^nC_s \Rightarrow r = s$ or $n = r + s$

$\therefore {}^nC_7 = {}^nC_3 \Rightarrow n = 7 + 3 = 10$.

**15.** ${}^8P_3$.

**16.** Each letter can be posted in 3 ways.

Four letters can be posted in 3 (3) (3) (3) or 81 ways.

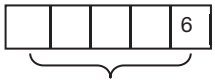
**17.** When a coin is tossed for $(n-1)$ times the number of out comes are $2^{n-1}$.

**18.** Each letter can be dealt in 5 ways, i.e., post it into any of the 5 boxes. Hence, total ways are $5^8$. This can be remembered as $(\text{boxes})^{\text{letters}}$

**19.** When a coin is tossed $n$ times the number of ways of getting exactly $r$ heads is ${}^nc_r$.

**20.** In the given word GINGER there are six letters of thereG occurring two times $\therefore$ the number of 6 letter words are $\frac{6!}{2!} = 360$.

**21.** There is only one even digit, i.e., 6 fixing it at the units place and arrange the remaining four digits.



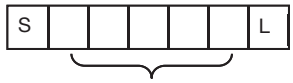

This can be done in 4! ways.

**22.** In the given word letter $A$ is repeated for two times and $R$ also repeated for two times.

Hence, the required number of arrangements possible are $\frac{7!}{2!2!}$.

**23.** All the given digits are even. Hence, no odd number can be formed.

**24.** We can arrange the letters of the given word as shown below.




$\therefore$ The required number of ways = 5!.

**25.** ${}^7C_5$.

**26.** The number of different words that can be formed using all the letters of word TRINETRA is $\frac{8!}{2!2!}$ or 10080.

**27.** 4 boys can be seated in 6 chairs in ${}^6P_4 = 360$ ways.

**28.** Since all the digits are even, no odd numbers can be formed. Number of odd numbers formed = 0.

**29.** Consider 6 blanks

_ _ _ _ _ _

Since the first and the last place are to be filled with $K$ and 'R', the remaining four letters can be arranged in 4! or24 ways.

**30.** Consider four blanks

_ _ _ _

The units place is filled with 5. The remaining three blanks can be filled with 4 digits in ${}^4P_3$ ways.

$\therefore$ The number of four-digit numbers required is 4 (3) (2) or 24.

**31.** The required number of arrangements is 7! = 5040

**32.** A decagon has 10 vertices.

$\therefore$ The number of diagonals in a regular decagon is

${}^{10}C_2 - 10 = 35$

**33.** Out of 12 scholars, we can select 4 of them in ${}^{12}C_4$ ways.

${}^{10}C_4 = \frac{12 \times 11 \times 10 \times 9}{1 \times 2 \times 3 \times 4} = 495$ ways.

**34.** Every chocolate can be dealt with in 2 ways (Neha may give away the chocolate to one friend or the other).

$\therefore$ The required number of ways is $2^{10}$.

**35.** We know that, $n$ persons can be seated around a circular table in $(n-1)$ !ways.

$\therefore$ 6 persons can be seated around a circular table in $(6-1)$ ! = 5 ! = 120 ways.

**36.** The number of blue balls is 6. 3 balls can be selected from 6 blue balls in ${}^6C_3$ ways = 20.

**37.** Let the number of sides be $n$.
Then, the number of diagonals is $\frac{n(n-3)}{2}$

Given $\frac{n(n-3)}{2} = 5n$

$\Rightarrow n - 3 = 10$

$\Rightarrow n = 13$

**38.** Treat all the girls as one unit. Now there are 3 boys and 1 unit of girls. They can be arranged in 4! ways.

The two girls can be arranged among themselves in 2! ways.

Total number of arrangements 4!2! = 24(2) = 48.

**39.** The number of letters in the word EDUCATION is 9.

$\therefore$ 4 letters can be selected from these 9 letters in ${}^9C_4$ ways.

**40.** We know that $n$ persons can be positioned around a circle in $(n-1)!$ ways.

$\therefore$ 8 persons can be arranged in 7! or 5040 ways.

**41.** $n$ persons can be arranged in a row in n! ways.

$\therefore$ 5 persons can be arranged in 5! Ways.

**42.** Anoctagon has 8 vertices. Any 2 vertices can be selected in ${}^8C_2$ ways.

$= \frac{8\times 7}{1\times 2} = 28$ ways

**43.** Every element can be dealt with in 2 ways. It may or may not be included in a subset.

$\therefore$ The required number of non-empty subsets of A is $2^8 - 1 = 255$.

**44.** The number of diagonals in a regular polygon of $n$ sides is $\frac{n(n-3)}{2}$.
Anduo decagon has 12 sides.

Hence, number of diagonals is

$= \frac{12(12-3)}{2} = 54$

**45.** Each letter can be posted in 4 ways.

$\therefore$ The required number of ways is $4^6$.

**46.** A straight line is formed by joining any two points.

Two points can be selected from 10 points in ${}^{10}C_2$ or $\frac{10\times 9}{2\times 1} = 45$ ways

**47.** $n$ books can be arranged in $n!$ ways

Given $n! = 720 = 6!$

$\Rightarrow n = 6$

6 books can be arranged on a circular shelf in $(6-1)!$ ways = 5! = 120

**48.** We know that there are 21 consonants.

Two consonants can be selected from 21 consonants in ${}^{21}C_2$ or $= \frac{21(20)}{2} = 210$ ways.

**49.** Number of ways of arranging '11' delegates around a table.

$= (11-1)! = 10! = 3628800$

**50.** Rahul can invite 5 friends or 6 friends. 5 friends can be invited in ${}^6C_5$ ways and 6 friends can be invited in ${}^6C_6$ ways

$\therefore$ The total number of required ways = ${}^6C_5 + {}^6C_6$

= 6 + 1 = 7.

## EXERCISE-2

**1.** In VICTORY, the consonants are *V, C, T, R, Y* but there are only 3 even places. So, all the consonants cannot be arranged in even places.

$\therefore$ The number of words is 0.

**2.** In TROUBLE, there are three vowels and 4 consonants. The four consonants can be arranged in 4! ways.

_____

Now there are 5 gaps and the three vowels can be arranged in these gaps in ${}^5P_3$ ways.

Total number of words is $4! \times {}^5P_3 = 1440$.

**3.** In MALBORNE, we can treat all vowels as one unit.

Now there are 5 consonants and the 1 unit of vowels can be arranged in 6! ways.

The 3 vowels can be arranged among themselves in 3! ways.

$\therefore$ The number of words that can be formed, so that the vowels are together is 6!3!=4320

**4.** In the word PROBLEM, the vowels are *O, E,*

There are three even places.



These two vowels can be arranged in these three places in ${}^3P_2$ ways. The remaining 5 letters can be arranged in 5! ways.

Total number of different words that can be formed is ${}^3P_2 .5! = 720$.

**5.** The given word is QUARTZ

Consider 6 blanks __ __ __ __ __ __

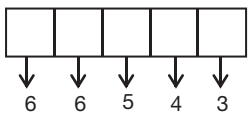
The first blank is filled by A.

Since the last blank should not be filled with *R*, it can be filled with 4 letters in 4 ways.

Now 4 letters are left and the remaining four blanks can be filled with these four letters in 4! ways.

∴ The total number of words = 4 (4!) = 96

**6.** The first place cannot be filled with 0




So, the first place can be filled in 6 ways.

Now, the remaining 4 places can be filled in 6, 5, 4 and 3 ways, respectively.

Hence, the number of five-digit numbers

= (6) ( 6) (5) (4) (3) = 2160

**7.** In the word LIMPET, the vowels are *I*, *E* and the consonants are *L*, *M*, *P*, *T*.

Assume the two vowels *I* and *E* as a single unit.

This unit together with the 4 consonants can be arranged in 5! ways.

The two vowels can be arranged internally in 2! ways.

Hence, the required number of ways.

= 5! (2!) = 120 (2) = 240

**8.** In the word THUNDER the vowels are *E*, *U* and the consonants are *T*, *H*, *N*, *D*, *R*. Of the seven places available, we have 4 odd places.

∴ The two vowels *E* and *U* can be arranged in these 4 places in ${}^4P_2 = 12$ ways.

Now, in the remaining 5 places the consonants *T*, *H*, *N*, *D* and *R* can be arranged in 5 !or 120 ways.

Hence, the required number of ways = 12 (120) = 1440

**9.** Number of ways of selecting at least one variety of ice-cream = $2^{12} - 1$

= 4096 – 1 = 4095

**10.** Number of ways of selecting at least 2 of them

= Total ways – number of ways of selecting at most one variety of ice-cream.

= $2^{12} - ({}^{12}C_0 + {}^{12}C_1) = 4096 - 13 = 4083$

**11.** The number of circular arrangements of *n* distinct items is $\frac{(n-1)!}{2}$, if there is no difference between the clockwise and anticlockwise direction. Hence, the 21 beads can be strung on a necklace in $\frac{(21-1)!}{2}$ ways, i.e., $\frac{20!}{2}$ ways.

**12.** Since it is a 10 oared boat, we need to have 5 on each of bow side and stroke side

The situation can be depicted as follows

| | **Available bow side** | **Total 10 stroke side** |
|---|---|---|
| Require | 5 | 5 |
| Fix | 2 | 3 |
| Left with | | 5 |
| Require | 3 | 2 |

The 2 people who can row only on bow side are sent there and the other 3 people who can row only on stroke side are fixed there. Having taken care of them, we are left with 5 people of whom we want to send 3 onto the bow side and 2 on to the stroke side and this can be done in 5!/3! 2! ways.

Further, the group of 5 people on each side can be arranged among themselves in 5! ways. Hence, total arrangements are $\frac{5!}{3!\,2!} \times 5! \times 5!$

**13.** The digits are 5, 4, 7, 6, 1, 4, 5, 4, 1.

There are 4 even slots and 4 even digits of which 3 are alike. Hence, the number of arrangements are 4!/3!

There are 5 odd slots and 5 odd digits of which 2 are alike of one kind and 2 of another kind. Hence, total arrangements are 5!/2!2!

∴ Required ways are 4!/3! × 5!/2! 2!

**14.** Given digits are 0, 1, 2, 3, 4, 5, 6, 7, 8. Total number of 5 digits numbers with repetition not allowed that can be formed are

– – – – –
↓ ↓ ↓ ↓ ↓

$8 \times 8 \times 7 \times 6 \times 5 = 13440$

**15.** Number that are divisible by 5 have to end in either 0 or 5 and the number of ways in each case are

– – – – 0
↓ ↓ ↓ ↓

$8 \times 7 \times 6 \times 5 = 1680$

– – – – 5
↓ ↓ ↓ ↓

$7 \times 7 \times 6 \times 5 = 1470$

Hence, total ways are 3150.

**16.** RATE is the given word. When the alphabets are arranged in alphabetical order, we have *A*, *E*, *R*, *T*.

| | | |
|---|---|---|
| words beginning with A | → | 3! = 6 |
| words beginning with E | → | 3! = 6 |
| words beginning with *R* | → | 3! = 6 |
| words beginning with TA | → | 2! = 2 |
| Required word is TEAR | | 1 |
| | | 21 |

Hence, the rank of the word TEAR is 21.

**17.** Since the books are all different, we have 12 different books and we can give one or more of them in $2^{12} - 1$ ways.

**18.** We know that the sum of all numbers that are formed by the digits *a*, *b*, *c*, *d*, *e* is $4!(11111)\ (a + b + c + d + e)$.

we get $4!(11111) \times 30 = 7999920$

**19.** Total number of ways of selecting 8 boys and 4 girls from 15 boys and 10 girls is ${}^{15}C_8 \times {}^{10}C_4$.

*A*, *B* are together in the representation in

${}^{14}C_7 \times {}^{9}C_3$ ways. Hence, they are never together in

${}^{15}C_8 \times {}^{10}C_4 - {}^{14}C_7 \times {}^{9}C_3$ ways.

**20.** Each question can be attempted in 4 ways. Hence, 10 questions can be attempted in $4^{10}$ ways.

**21.** The possible ways of arranging the pencils are: RBWBWBR, WBRBRBW, BWBRWRB, BRBWRWB, BWRBWRB, BWRRBRWB, BRWBRWB, BRWBWRB, BWRWBRB and BRWBBWB

i.e., a total of 10 possible arrangements.

**22.** The lines, the number of bounded and unbounded regions they create are tabulated below.

| Lines | B | U |
|---|---|---|
| 1 to 4 | 0 | 5 |
| $5^{th}$ | 0 | 5 |
| $6^{th}$ | 4 | 2 |
| – | – | – |
| 15 | 13 | 2 |

The 8 rows for lines 7 to 14 have not been shown explicitly.

$\therefore$ The total number of bounded regions is $4 + 5 + \ldots + 13 = 91 - 6 = 85$

The total number of unbounded regions is $5 + 5 + 2(10) = 30$

The total number of regions is 85 + 30 or 115

**23.** Let *m*, *n* be the number of girls and the number of boys, respectively.

Then, ${}^{m}C_2 = 66$

$\Rightarrow \dfrac{m(m\,1)}{2!} = 66\ . \Rightarrow m = 12$

Also, ${}^{m}C_1 \times {}^{n}C_1 = 240$

$\Rightarrow n = 20.$

Hence, the number of games in which both the players were boys is ${}^{20}C_2 = \dfrac{20(19)}{2} = 190.$

**24.** Let the number of red, blue and green balls selected from the supply be $x_1$, $x_2$ and $x_3$, respectively.

Then, $x_1 + x_2 + x_3 = 12$. The number of non-negative integral solutions of the above equation is

$(n + k - 1)_{c_{k-1}} = 14_{c_2} = 91.$

**25.** As the number is an even number, the last digit can be 0, 2, 4, 6 or 8.

*The number of 8-digit telephone numbers which begin with 270 and end with 0:*

In this case, 270 ____ 0 of the remaining four places, two places must be occupied by the digits 0 and 9 and the remaining places can be occupied by any of the digits other than 0 and 9. This can be done in $({}^{4}C_2 \times 2!) \times 8 \times 8 = 768$ ways.

*The number of 8-digit telephone numbers which begin with 270 and end with 2, 4, 6 or 8:*

In this case, of the remaining four places, 2 places must be 270 _ _ _ _ _ occupied by '0', one place by '9' and the remaining place by any one of the remaining 8 digits.

This can be done in $4 \times \left( {}^4C_3 \times \frac{3!}{2!} \right) \times 8 = 384$ ways.

Similarly, the number of 8-digit telephone numbers which begin with 279 and end with 0 is 279 _ _ _ _ 0. ${}^4C_2 \times 8 \times 8 = 384$.

The number of 8-digit telephone numbers which begin with 279 and end with 2, 4, 6 or 8 is $4 \times {}^4C_3 \times 8 = 128$.

Hence, the required number of telephone numbers is $768 + 384 + 384 + 128 = 1664$.

**26.** (i) The number of ways in which at least one friend is invited for dinner = total number of ways of inviting the 7 friends – number of ways in which no friend is invited $= 2^7 - 1 = 127$.

(ii) At least three of them invited are = 3 or more members for the dinner

$= {}^7C_3 + {}^7C_4 + {}^7C_5 + {}^7C_6 + {}^7C_7$

$= 35 + 35 + 21 + 7 + 1 = 99$

***Solutions for question 27*:**

We have to select 4 letters from SENTENCE, i.e., 3*Er*, 2*Ns*, 1*C*,1*S*,1*T*. The type of letters and the number of ways of selecting them are tabulated below. The number of words that can be formed is in the fourth column:

| Type | No. of selections | No. of arrangements | No. of words |
|---|---|---|---|
| 3, 1 | 4 | 4! / 3! = 4 | 4(4) = 16 |
| 2, 2 | 1 | 4! / 2!2! = 6 | 1(6) = 6 |
| 2, 1, 1 | 12 | 4!/2! = 12 | 12(12) = 144 |
| 1, 1, 1, 1 | 5 | 4! = 24 | 5(24) = 120 |
| | 22 | | 286 |

**27.** The total number of selections is 22.

**28.** Suppose only 1 box is filled with a blue ball, any one of the boxes can be filled with a blue ball.

This can be done in 7 ways.

Suppose two boxes are filled with blue balls. The remaining 5 boxes are filled with black balls.

We first fill up 5 boxes with black balls. In the 6 gaps that are created (one in front of each box and the $6^{th}$ after the $5^{th}$ box), we can place 2 boxes with blue balls. This can be done in ${}^6C_2$ ways. Similarly, we can work out the number of ways of filling up the boxes for 3 and 4 boxes with blue balls. We get the following table.

| No of boxes with blue balls | No of boxes with black balls | No of ways of filling up |
|---|---|---|
| 0 | 7 | ${}^8C_0 = 1$ |
| 1 | 6 | ${}^7C_1 = 7$ |
| 2 | 5 | ${}^6C_2 = 15$ |
| 3 | 4 | ${}^5C_3 = 10$ |
| 4 | 3 | ${}^4C_4 = 1$ |
| | | Total = 34 |

∴ The total number of ways of filling up the boxes so that no two adjacent boxes have blue balls is 34.

**29.** A candidate can deal with each section in 3 ways i.e., he may attempt the 1st question or the 2nd question or he may leave the section.

10 questions can be attempted in $3^{10}$ ways. Out of these, there is one way in which no question is attempted.

∴ The number of ways = $3^{10} - 1$

**30.** The number of four-digit numbers begin with 3 or 5 is 3! each.

The number of four-digit numbers that begin with 63, 65 is 2! each.

After that, the numbers are 6935, 6953.

The rank of 6953 is 2(3!) + 2(2!) + 2 = 18.

**31.** The greatest number is 19. We select the remaining seven numbers from 8 to 18. This can be done in ${}^{11}C_7$ ways. These 8 numbers can be permuted in 8! ways

∴ Total number of arrangements is

$${}^{11}C_7 \, 8! = {}^{11}C_4 \, 8! = (330) \, 8!$$

**32.** Let $B_1$, $B_2$, $B_3$ be the three boxes 5 letters can be posted in these boxes in the following way.

| $B_1$ | $B_2$ | $B_3$ | No. ways |
|---|---|---|---|
| 1 | 1 | 3 | ${}^5C_1 \, {}^4C_1 \, {}^3C_3 = 20$ |
| 1 | 3 | 1 | ${}^5C_1 \, {}^4C_3 \, {}^1C_1 = 20$ |
| 3 | 1 | 1 | ${}^5C_3 \, {}^2C_1 \, {}^1C_1 = 20$ |
| 1 | 2 | 2 | ${}^5C_1 \, {}^4C_2 \, {}^2C_2 = 30$ |
| 2 | 1 | 2 | ${}^5C_2 \, {}^3C_1 \, {}^2C_2 = 30$ |
| 2 | 2 | 1 | ${}^5C_2 \, {}^1C_1 \, {}^1C_1 = 30$ |

Hence, the total number of ways = 3(20) + 3(30) = 150

**33.** The number of handshakes between group $A$ and group $B$ is $= 8(6) = 48$

Similarly the number of handshakes between group $B$ and group $C$ is $6n$

The number of handshakes between group $C$ and group $A$ is $8n$

Given total number of handshakes = 104

$\therefore 48 + 6n + 8n = 104 \Rightarrow 14n = 56 \Rightarrow n = 4$

**34.** 6 prizes have to be distributed among 4 athletes, such that each gets at least one. Either the split (of the prizes) is 3, 1, 1, 1 or 2, 2, 1, 1. There are 4 cases corresponding to the first split (Any one of the 4 athletes could get 3 prizes) and 6 cases corresponding to the second split (Any two of the 4 athletes could get 2 prizes each) consider the split 3, 1, 1, 1. The number of ways of selecting the prizes to be given to one athlete is ${}^6C_3$. The other 3 prizes can be distributed among the other 3 athletes in ${}^3P_3$ or 3! ways.

$\therefore$ There are ${}^6C_3 {}^3P_3$ or $20(6)$ or 120 ways.

Consider the second split 2, 2, 1, 1. The number of ways of selecting the prizes for each of this splits is ${}^6C_2 {}^4C_2 {}^2C_1$ (or ${}^6C_2 {}^4C_2 {}^2P_2$) which is $15(6)(2)$ or 180.

$\therefore$ The total number of ways in which the prizes might get distributed is $4(120) + 6(180) = 1560$.

**35.** Out of 6 letters any two letters can be placed into their corresponding envelopes in ${}^6C_2$ ways.

Of the remaining 4 letters, no letter should be placed into its corresponding envelope. This can be done in

$4!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right]$ viz 9 ways

$\therefore$ The total number of ways that 6 letters are placed into envelopes such that exactly two letters are placed into their corresponding envelopes is ${}^9C_2 = 9\ (15) = 135$

**36.** Thenumber of ways of dealing with the 6 copies of a biography = 7 (one way of not giving away and 6 ways of giving away). Similarly, the number of ways of dealing with the 5 copies of the autobiography and the 4 copies of the novel are 6 and 5, respectively. Hence, the number of ways of giving away one or more books = $(6 + 1)\ (5 + 1)\ (4 + 1) - 1 = 210 - 1 = 209$.

**37.** The required number of triangles = $T - N_1 - N_2$

Where

$T$ = Total number of triangles.

$N_1$ = Number of triangles with 1 side in common with the polygon.

$N_2$ = Number of triangles with 2 sides in common with the polygon.

$= {}^{15}C_3 - ({}^{15}C_1 {}^{11}C_1) - 15 = 275.$

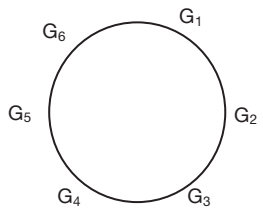
**38.** Sum of all the numbers

$= (n - 1)!$ (Sum of the digits) (111 ... 1) ($n$ times)

$= (5 - 1)!\ (1 + 2 + 8 + 9 + 5)(11111)$

$= 4!\ (25)(11111) = 6666600$

**39.**




Six girls can be arranged around a circle in 5! ways. Now, the six boys can be arranged in the six places in between the girls in 6! ways.

Hence, the required number of ways = 5!6!

**40.** Assume the two Ns form a single unit

This unit together with the remaining 10 letters can be arranged in $\frac{11!}{2!\,2!\,2!\,2!}$ ways

**41.** 12 books can be divided equally among 3 boys in $\frac{12!}{(4!)^3}$ ways.

**42.** 12 books can be divided equally into 3 parcels in $\frac{12!}{(4!)^3 3!}$ ways ($\because$ the three parcels are not distinct)

**43.** The 5 copies of Maths books can be dealt in 6 ways – do not give at all or give 1 or 2 or 3 or 4 or all 5 copies. Similarly, 4 Physics books can be dealt in 5 ways and 3 Chemistry books in 4 ways, i.e., in $6 \times 5 \times 4$ of which we remove the case of not giving any of the copies. Thus required ways are $(6 \times 5 \times 4) - 1 = 119$

**44.** We know that if there are $m$ horizontal steps and $n$ vertical steps, then the number of ways of travelling from one corner to another opposite corner is ${}^{(m+n)}C_n$.

Here $m = 4$, $n = 5$; the number of ways a car can travel from A to $B$ is ${}^{(4+5)}C_5 = {}^9C_5 = 126$.

**45.** Given $(a + b + c + d)^{20}$

Any term in the expansion of $(a + b + c + d)^{20}$ is of the form $a^p b^q c^r d^s$ such that $p + q + r + s = 20$ where $p$, $q$, $r$ and $s$ are whole numbers.

The number of solutions for $x + y + z + ___ k$ terms $= n$.

where $x, y, z ___ W$ is given by ${}^{n+k-1}C_{k-1}$

$\therefore$ The required number of solutions for $p + q + r + s = 20$ is

${}^{20+4-1}C_{4-1} = {}^{23}C_3 = 1771$

**46.** Given that $x + y + z = 20$ where $x \geq 2$, $y \geq 3$, $z \geq 4$.

Let $x = x + 2$, $y = y + 3$, $z = z + 4$

$\Rightarrow x + y + z = 20 - (2 + 3 + 4) = 11$ (1)

and $x \geq 0$, $y \geq 0$, $z \geq 0$. We know that the number of non-negative integral solutions of the equation of the form $x_1 + x_2 + \ldots + x_n = n$ is ${}^{n+k-1}C_{k-1}$.

The number of non-negative integral solutions of (1) is ${}^{11+3-1}C_{3-1} = {}^{13}C_2 = 78$.

**47.** The number of positive integral solutions of the equation $x_1 + x_2 + \ldots + x_n = n$ is ${}^{n+k-1}C_{k-1}$.

$\therefore$ The number of positive integral solutions of the equation $a + b + c = 15$ is ${}^{15-1}C_{3-1}$

$= {}^{14}C_2 = 91$.

**48.** The number of non-negative integral solutions of the equation $p + q + r + s = 30$ is ${}^{30+4-1}C_{4-1} = {}^{33}C_3$

$= \dfrac{33 \times 32 \times 31}{3 \times 2 \times 1} = 5456.$

**49.** Given that $A + B + C + D \leq 15$ (1)
and $A, B, C, D$ are non negative integers.

Let $A + B + C + D = 15 - E$ where $E$ is an integer satisfying $0 \leq E \leq 15$

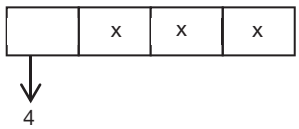
$A + B + C + D + E = 15$ (2)

Here $A, B, C, D, E$ arenon negative integers.

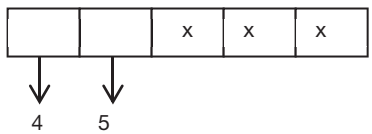
The solutions of (2) is ${}^{(15+5-1)}C_{(5-1)} = {}^{19}C_4$

**50.** A number is divisible by 125 if and only if the number formed by its last three digits is divisible by 125. So, the possible combinations for the last three digits are: 000, 125, 250 and 500. Now, the number of three-digit numbers divisible by 125 is 3.

The number of four-digit numbers divisible by 125 is 4 (4) = 16.




the number of five-digit numbers divisible by 125 is 4 (4) (5) = 80.




Hence, the required number of numbers is 3 + 16 + 80 = 99.

## EXERCISE-3

**1.** ${}^nP_r = 120$, ${}^nC_r = 20$

$({}^nC_r)$ $r! = {}^nP_r$; $r! = 120/20 = 6 \Rightarrow r = 3$

${}^nC_3 = 20 \Rightarrow \dfrac{n(n-1)(n-2)}{3.2.1} = 20$ ; $n(n-1)\ (n-2) = 120$

From choices, $n = 6$ satisfies the above equation.

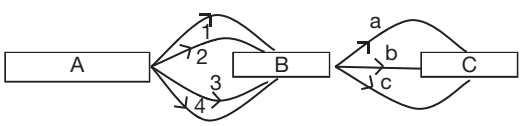
**2.** We know that ${}^nC_r = {}^nC_{n-r}$,

$\therefore {}^{25}C_{12} = {}^{25}C_{13}$, and ${}^{25}C_{19} = {}^{25}C_6$

$\therefore$ value of the required expression = 0

**3.** We want to consider 3 flags out of 6 different coloured flags. As the order of the colours play a role in forming the signals, $\therefore {}^6P_3 = 6 \times 5 \times 4 = 120$ different signals can be generated.

**4.**




A person can travel from $A$ to $B$ using any of the four highways 1, 2, 3, 4. Having reached $B$, he can travel to $C$ using any of the three highways $a$, $b$, $c$, i.e., a person can reach $C$ from $A$ via $B$ using the highways in any of the following possible ways.

1, $a$ ; 1, $b$ ; 1, $c$ ; 2, a ; 2, $b$ ; 2, $c$ ;

3, $a$ ; 3, $b$ ; 3, $c$ ; 4, a ; 4, $b$ ; 4, $c$ ;

Hence, there are a total of 12 ways.

**5.** There are five letters in the given word.

Consider 5 blanks . . . .

The first blank and last blank must be filled with $N$ and $A$ and the remaining three blanks can be filled with the remaining 3 letters in 3! ways.

$\therefore$ The number of words = 3! = 6.

**6.** The boy can select one trouser in nine ways.

The boy can select one shirt in 12 ways.

∴ The number of ways in which he can select one trouser and one shirt is $9 \times 12 = 108$ ways.

**7.** Total number of letters = 8

Using these letters the number of 8 letter words formed is ${}^{8}P_{8} = 8!$.

**8.** One can travel from city $A$ to city $B$ in 3 ways and from city $B$ to city $C$ in 5 ways. Since the two operations are independent, by fundamental theorem, number of distinct routes from $A$ to $C$ are $3 \times 5 = 15$

**9.** The number of letters in the given word is four.

The number of three letter words that can be formed using these four letters is ${}^{4}P_{3} = 4 \times 3 \times 2 = 24$.

***Solutions for question 10*:**

The given word is MEADOWS and it contains 4 consonants and 3 vowels.

**10.** As no two vowels are together, we need to have a consonant present in between two vowels which act as a separator for vowels. As there is no condition on consonants, we first arrange them and this can be done in 4! ways. Now, there are 5 possible positions for the3 vowels as indicated below.

$V\,C\,V\,C\,V\,C\,V$

In these 5 positions 3 vowels can be arranged in ${}^{5}P_{3}$ ways.

∴ The required number of words = $4! \times {}^{5}P_{3}$

$= 24 \times 120/2 = 1440$

**11.** The total number of letters = 9 and two of them are $E$, one I and one $O$. There are 5 odd places. The vowels can be arranged in these 5 places in ${}^{5}C_{4} \times \frac{4!}{2!} = \frac{5 \times 4 \times 3 \times 2}{2}$ = 60 ways. Now the consonants can be arranged in 5! = 120 ways. Total number of arrangements = 60 $x$ 120 = 7200.

***Solutions for question 12*:**

The given word is MEADOWS and it contains 4 consonants and 3 vowels.

**12.** The number of words when the word starts with $M = 6!$

The number of words when the word starts with $M$ and ends with $W = 5!$

∴ The required number of words = $6! - 5! = 5 \times 5! = 600$

**13.** As no two boys are together we need the presence of a girl between two boys

The seven girls can be arranged in 7! ways.

_G_G_G_G_G_G_G_. The 5 boys can now be arranged in 8 available gaps in ${}^{8}P_{5}$ ways, therefore, the total number of arrangements are $7! \times {}^{8}P_{5}$.

***Solutions for questions 14and 15*:**

The given word is MEADOWS and it contains 4 consonants and 3 vowels.

**14.** If the word starts with letter 'A', then remaining 6 letters can be arranged in the remaining 6 positions in 6! = 720 ways.

∴ The required number of words = 720

**15.** $\boxed{A\,E\,O}$ + _ _ _ _

As all the vowels are to be together, we treat the three vowels as one unit. Now, there are 4 other letters, these 4 letters and the unit of vowels can be permuted in 5! ways, while the vowels can be permuted among themselves in 3! ways.

The required number of words = $5! \times 3! = 120 \times 6 = 720$

**16.** As the sum of the digits is 15 the number formed will always be divisible by 3 for the number to be divisible by 6 it should also be divisible by 2

i.e., The number should end with even digit

4! numbers end with 2 and 4! numbers end with 4

∴ The required number of numbers = 4! + 4!

= 24 + 24 = 48

**17.** $\underline{\quad} \times \underline{\quad} \times \underline{P} \times \underline{\quad} \times \underline{\quad}$
$4 \quad 3 \quad 1 \quad 2 \quad 1$

The number of ways the 4 teachers can be seated in 4 chairs is 4! ways

**18.** Number of 7 digit numbers formed using the digits 0, 1, 1, 2, 2, 3, 3 is = $\frac{7!}{2!2!2!}$

| | | | | | | |
|---|---|---|---|---|---|---|
| X | X | X | X | X | X | X |

Of which some of the numbers start with zero,

Now, numbers (7 digit) starting with zero are to be excluded

Number of 7 digit numbers starting with zero

$= \frac{6!}{2!2!2!}$

Required number of numbers = $\frac{7!}{2!2!2!} - \frac{6!}{2!2!2!}$

$= \frac{6!}{(2!)^3}(7-1) = \frac{720}{8} \times 6 = 540$

**19.** Each prize can be given to any of the 3 boys in 3 ways

$\Rightarrow$ Total number of ways

$= 3 \times 3 \times 3 \times \ldots$ for 12 times $= 3^{12}$

**20.** Each letter can be dealt in 4 ways–post it into any of the 4 available boxes.

Hence, 6 letters can be dealt in $4^6$ ways.

**21.** To travel from the point $P$ to the $Q$, one must use 5 horizontal roads and 3 vertical roads. This can be done in $\frac{8!}{5!3!} = 56$ ways.

**22.** We know that the points lying on a straight line do not form triangles.

Hence, the number of triangles that can be formed by using the given points is

${}^{19}C_3 - ({}^{9}C_3 + {}^{10}C_3) = 765$.

**23.** The possibilities for $k$ are: 1100000000, 1010000000, 1001000000, 100100000, 1000010000, 1000001000, 1000000100, 1000000010, 1000000001 and 2000000000 i.e., there are 10 possible values of $k$.

**24.** The total number of matches played is $\frac{m(m-3)}{2}$

Given, the total time taken for all the matches is 30 minutes.

$\therefore \frac{m(m-3)}{2} \times 1\frac{1}{2} = 30$

$\Rightarrow \frac{3(3-3)}{2} \times \frac{3}{2} = 30$

$\Rightarrow m\,(m-3) = 40 = 8 \times 5$

$\therefore\ m = 8.$

**25.** The number of line segments that can be drawn by joining 4 points with each of the remaining 6 points is $4 \times 6 = 24$.

Since each of the remaining 6 points are joined to exactly 5 points, the 6 points must form 3 pairs and the points in each pair must be joined by a line.

Hence, the total number of lines drawn in the plane were $24 + 3 = 27$.

**26.** The number of non-negative integral solutions of the equation $x_1 + x_2 + \ldots\ldots\ldots + x_s = n$ is ${}^{n+s-1}C_{s-1}$.

The given equation is $x_1 + x_2 + x_3 + x_4 = 15$. The number of solutions is ${}^{15+4-1}C_{4-1} = {}^{18}C_3 = 816$.

**27.** The word 'COMBINATION' consists of 11 letters, of which 5 are vowels and the rest are consonants. Consider 11 blanks

— — — — — — — — — — —

Since vowels appear in alphabetical order, the five vowels can be placed in these 11 places in ${}^{11}C_5$ ways. The remaining 6 places are occupied by consonants in exactly one way (since they also appear in alphabetical order)

Hence, required number of words $= {}^{11}C_5 = 462$.

**28.** The number of handshakes exchanged between a person from group A and a person from group $B$ is ${}^{12}C_1 . {}^{15}C_1 = 180$

Similarly, the number of handshakes between a person from group and one person from group $C = {}^{15}C_1 . {}^{n}C_1$
The number of handshakes between person from group A and a person from group $C$ is ${}^{n}C_1 . {}^{12}C_1$

Hence, the total number of handshakes $= 180 + 15.n + 12.n = 27n + 180 = 666$

$\Rightarrow 27n = 486$

$\Rightarrow n = \frac{486}{27} = 18$

**29.** The greatest number is 15

So, 15 is always selected. The remaining 8 numbers have to be selected from 4 to 14. This can be done in ${}^{11}C_8$ ways. Now, these 9 numbers can be arranged in 9! ways .

Total number of arrangements

$= {}^{11}C_8 .9!$

$= \frac{11(10)(9)}{3(2)(1)} .9!$

$= 165(9!).$

**30.** We distribute 7 balls into 4 boxes such that $P$ and $Q$ containsatleast one ball. This is represented in the following equation

$p + q + r + s = 7$ such that (1)

$p \geq 1,\ q \geq 1,\ r \geq 0,\ s \geq 0.$

Where $p$, $q$, $r$, $s$ are number of balls in the respective boxes.

Let $p = p_1 + 1 \Rightarrow 0 \leq p_1 \leq 5$

$q = q_1 + 1 \Rightarrow 0 \leq q_1 \leq 5$

(1) equation becomes

$p_1 + q_1 + r + s = 5$ where $p_1$, $q_1$, $r$, $s$ arenon negative integers

The number of solutions for (2) is

$= {}^{5+4-1}C_{4-1} = {}^{8}C_3 = 56.$

**31.** In the word, 'MATERIAL' there are three vowels *A, I, E.*

If all the vowels are together, the arrangement is MTRL AAEI

Consider AAEI as one unit. The arrangement is as follows.

M T R L AA E I

The above 5 items can be arranged in 5! ways and AAEI can be arranged among themselves in $\frac{4!}{2!}$ ways.

$\therefore$ Number of required ways of arranging the above letters $= 5! \times \frac{4!}{2!} = \frac{120 \times 24}{2} = 1440$ ways

**32.** Given that, the question paper consists of five problems. For each problem, one or two or three or none of the choices can be attempted.

$\therefore$ Hence, the required number of ways $= 4^5 - 1.$

$= 2^{10} - 1 = 1024 - 1 = 1023$

**33.** The word contains five consonants,

Three vowels, three consonants can be selected from five consonants in ${}^5C_3$ ways, two vowels can be selected from three vowels in ${}^3C_2$ ways.

$\therefore$ 3 consonants and 2 vowels can be selected in ${}^5C_2 \times {}^3C_2$ ways, i.e., $10 \times 3 = 30$ ways.

**34.** Treat all boys as one unit. Now there are four students and they can be arranged in 4! ways Again the five boys can be arranged among themselves in 5! ways. $\therefore$ required number of arrangements $4! \times 5! = 24 \times 120 = 2880$

**35.** The given digits are 1, 2, 3, 5, 7, 9

A number is even when its units digit is even. Of the given digits, two is the only even digit.

Units place is filled with only '2' and the remaining three places can be filled in ${}^5P_3$ ways.

$\therefore$ Number of even numbers $= {}^5P_3 = 60.$

**36.** (i) 2 points determine a line.

$\therefore$ 12 points can give ${}^{12}C_2$ straight lines.

But 4 points are collinear. They give only one line.

$\therefore$ The number of straight lines that can be formed

$= {}^{12}C_2 - {}^4C_2 + 1$

$= 66 - 6 + 1 = 61$

(ii) Three points determine a triangle.

$\therefore$ 12 non-collinear points can give ${}^{12}C_3$ triangles.

But 4 points are collinear. So, these 4 points do not give any triangles.

Hence, the number of triangles.

$= {}^{12}C_3 - {}^4C_3 = 216$

**37.** The number of direct lines required between any two villages belonging to the same zone is

$3 \times \left({}^4C_2 \times 3\right) = 54$

The number of direct lines required between any two villages belonging to the different zones is

${}^3C_2\left({}^4C_2 \times {}^4C_1 \times 2\right) = 96$

Hence, the required number of direct lines is 54 + 96 = 150.

**38.** We know that if there are $m$ horizontal lines and $n$ vertical lines the number of ways of travelling from one corner to another corner is ${}^{m+n}C_n$

Here $m = 5;\ n = 5$

Required number of paths ${}^{5+5}C_5 = {}^{10}C_5 = 252$

**39.** Let $m$ and $n$ be the number of girls and the number of boys respectively, then

${}^mC_2 = 300 \Rightarrow \frac{m(m-1)}{2} = 300 \Rightarrow m(m-1) = 600$

$\Rightarrow m = 25.$

Also ${}^nC_2 = 105$

$\frac{n(n-1)}{2} = 105 \Rightarrow n(n-1) = 210 \Rightarrow n = 15$

Hence, the number of games in which one player is a boy and the other is a girl is

${}^{25}C_1 \times {}^{15}C_1 = 25\,(15) = 375$

**40.** Let there are be $n$ lines in a plane of which no two lines are parallel and no three lines are concurrent. The number of regions that these $x$ lines divide the plane is $\left(\sum_1^n i\right) + 1$

$= \frac{8.(9)}{2} + 1 = 37$

**41.** The sum of all the numbers that can be formed by $n$ digits is $(n-1)!$ (sum of digits) (111 … 1 for $n$ times).

The given digits are 3, 4, 6, 7, 9.

$\therefore$ Required sum $= (4!)\ (11111)(3 + 4 + 6 + 7 + 9)$ $= 7733256$

**42.** Let the three zones be $z_1, z_2, z_3$.

Each zone contains 3 villages.

We select any two villages of the same zone in ${}^3C_2$ ways and they are connected with 4 direct lines.

$\therefore$ Total number of lines that are used in any one zone is ${}^3C_2$ (4) (3) = 36.

Now two zones can be selected from 3 zones in ${}^3C_2$ ways. Any two villages can be selected in ${}^3C_1.{}^3C_1$ ways.

These villages are connected by two direct lines.

Total number of lines = ${}^3C_2{}^3C_1{}^3C_1$ 2 = 54

$\therefore$ The total number of lines used = 36 + 54 = 90

**43.** The letters of the word 'RANGE' when arranged in alphabetical order we get *A, E, G, N, R.*

The number of words that begin with *A, E, G* and *N* are 4! each.

The number of words begin with RAE, RAG is 2! each

The next word is RANEG and

The next word is RANGE.

The rank of the word range is = 4(4!) + 2(2!) + 1 + 1

= 96 + 4 + 1 + 1 = 102

**44.** The *R.*K. Narayan novels can be given in 8 ways, Sidney Sheldon novels in 7 ways and Milton in 6 ways.

The total number of ways that the books can be given is 8(7)(6) = 336.

The total number of ways that one or more books can be given is 336 – 1 = 335.

**45.** The alphabetical order of the letters in the given word is *A, A, A, B, N, N*

The number of words that start with $A = \frac{5!}{2!2!} = 30$

The number of words that start with $B = \frac{5!}{3!2!} = 10$

The number of words that start with *NAA* = 6

The number of words that begin with *NAB*. $= \frac{3!}{2!} = 3$

The next word is 'NANAAB'.

$\therefore$ The rank of the word 'NANAAB' is

= 30 + 10 + 6 + 3 + 1 = 50.

**46.** We know that, the sum of all *n*-digit numbers that can be formed using *n*-digits is

$(n-1)!$ (sum of the digits) $\times \underbrace{11....1}_{n \text{ times}}$

$\therefore$ The required sum is (1 + 3 + 4 + 6 + 8) × 11111

= 5866608

**47.** Prof. Balamurali and Asst. *B* of Sheshadri cannot be included together in the delegation.

Hence, the number of ways of forming the delegation is ${}^{10}c_5 \cdot {}^6c_3 - {}^9c_4 \cdot {}^5c_2$

**48.** The number of direct lines required between any two villages belonging to the same zone is

$4 \times ({}^4C_2 \times 2) = 48$.

The number of direct lines required between any two villages belonging to different zones is

${}^4C_2 \times ({}^4C_1 \times {}^4C_1) \times 1 = 96$.

Hence, the required number of direct lines is 48 + 96 = 144.

**49.** The maximum possible number of points of intersection of diagonals inside the polygon is 495

$\therefore {}^nC_4 = 495 \Rightarrow \frac{n(n-1)(n-2)(n-3)}{2(3)(4)} = 495$

= 5 (9) (11)

$\Rightarrow$n $(n-1)$ $(n-2)$ $(n-3)$ = 12 (11) (10) (9)

$\Rightarrow n = 12$.

**50.** Every question can be attempted in 2 ways.

So, the entire paper can be attempted in $2^{15}$ ways.

## EXERCISE-4

**1.** Arranging the letters of the word in the alphabetical order we get *A, C, L, O.*

The number of words starting with *A* are $\frac{7!}{3! \times 3!}$

$= \frac{7 \times 6 \times 5 \times 4}{3 \times 2 \times 1} = 140$

The number of words starting with *C* are $\frac{7!}{3! \times 2!}$

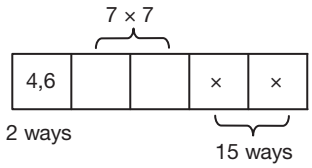
$= \frac{7 \times 6 \times 5 \times 4}{2} = 420$

$\therefore$ The 561$^{st}$ word is LACCCOOO

**2.** As the numbers lie between 40,000 and 70,000, the first digit could be 4 or 6.

Numbers whose last two digits are divisible by 4, could be:

00, 04, 40, 60, 08, 80, 16, 36, 64, 48, 84, 76, 68, 44, 88 (15 possibilities).




The second and third digits can be selected in $7 \times 7$ ways.

$\therefore$ The total required numbers are:

$= 2(7^2)\ (15) = 1470$

**3.** We need the number of derangements of the word ROBUST. This is

$$= 6!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\frac{1}{6!}\right)$$

$$= 720\left(\frac{360-120+30-6+1}{720}\right)$$

$= 391 - 126 = 265.$

**4.** Let the number of umbrellas hung in a row be $n$

We have,

${}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n = 511$

$\Rightarrow 2^n - 1 = 511 \Rightarrow 2^n = 512 = 2^9 \Rightarrow n = 9$

The number of ways of selection four umbrellas such that no two are consecutive

$${}^{n-r+1}C_r = {}^{9-4+1}C_4$$

$$= {}^{n-r+1}C_r = {}^{9-4+1}C_4$$

$$= {}^6C_4 = {}^6C_2 = \frac{6(5)}{2} = 15.$$

**5.** We need the number of positive integral solutions of $x_1 + x_2 + x_3 = 20$ (1)

where $3 \le x \le 8$ for $l = 1, 2, 3.$ or of $y_1 + y_2 + y_3 = 14$ (2)

where $1 \le y_i \le 6$ for $l = 1, 2, 3.$ All the solutions of (2) can be easily listed as shown below.

$y_1 + y_2 + y_3 = 14$

| | | |
|---|---|---|
| 2 | 6 | 6 |
| 3 | 5 | 6 |
| | 6 | 5 |
| 4 | 4 | 6 |
| | 5 | 5 |
| | 6 | 4 |
| 5 | 3 | 6 |
| | 4 | 5 |
| | 5 | 4 |
| | 6 | 3 |
| 6 | 2 | 6 |
| | 3 | 5 |
| | 4 | 4 |
| | 5 | 3 |
| | 6 | 2 |

Thus, there are 15 ways of placing 20 identical coins into 3 different boxes.

**Alternate Method:**

The required number of ways (say $N$) is equal to the number of positive integral solutions of

$x_1 + x_2 + x_3 = 20$, where $3 \le x_1, x_2, x_3 \le 8$

$\therefore N =$ coefficient of $x^{20}$ in $(x^3 + x^4 + \ldots + x^8)^3$

$=$ coefficient of $x^{20}$ in $x^9\ (1 + x + \ldots + x^5)^3$

$=$ coefficient of $x^{11}$ in $\left(\frac{1-x^6}{1-x}\right)^3$

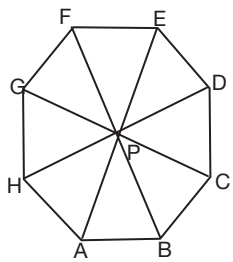
$(1 - x^6)^3 = (1 - 3x^6 + 3x^{12} - x^{18})$

$(1 - x)^{-3} =$

$$(1+3x+\frac{3(4)}{2}x^2+\ldots+\frac{3(4)(5)(6)(7)}{1(2)(3)(4)(5)}x^5 \quad +\ldots+\frac{3(4)\ldots(13)}{1(2)\ldots(11)}x^{11}+\ldots)$$

$$\therefore N = \frac{(-3)3(4)(5)(6)(7)}{1(2)(3)(4)(5)} + \frac{12(13)}{1(2)} = -63 + 78 = 15.$$

**6.**




We have a total of 9 points (8 vertices and the centre $P$). The three points (two vertices and the centre $P$), on each of the diagonals $AE$, $BF$, $CG$ and $DH$ correspond to a straight line, and not a triangle.

$\therefore$ The total number of triangles is ${}^9C_3 - 4$. The number of triangles that do not include $A$ or $E$ is ${}^7C_3 - 3$

$$\left({}^9C_3 - 4\right) - \left({}^7C_3 - 3\right) = {}^9C_3 - {}^7C_3 - 1$$

$$= \frac{9(8)(7)}{6} - \frac{7(6)(5)}{6} - 1 = 48.$$

**7.** At first 5 boys can be arranged in 5! ways as shown,

$B \times B \times B \times B \times B$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$x_1 \quad x_2 \quad x_3 \quad x_4$

Let $x_1$, $x_2$, $x_3$ and $x_4$ be the number of girls in between successive boys

i.e., $x_1 + x_2 + x_3 + x_4 = 18$ (1)

where $x_1 = 2n_1 - 1$, $x_2 = 2n_2 - 1$, $x_3 = 2n_3 - 1$ and $x_4\, 2n_4 - 1$ and $n_1, n_2, n_3, n_4 \in N$

Eq (1) reduces to $2(n_1 + n_2 + n_3 + n_4) = 18 + 4$

$\Rightarrow n_1 + n_2 + n_3 + n_4 = 11$

The number of positive solutions is ${}^{n-1}C_{4-1}$ or ${}^{10}C_3$

Number of ways in which the 18 girls and 5 boys can be arranged = 18! 5! ${}^{10}C_3$ = 18! (120) $\frac{10(9)(8)}{6}$ = $120^2$ (18!)

$\therefore$ Required number of ways = $(120)^2$ 18! = (14400) (18!)

**8.** We have $x_1\, x_2\, x_3\, x_4 = 600 = 2^3\, 3^1\, 5^2$ (1)

The number of positive integral solutions of (1)

$= {}^6C_3\, {}^4C_3\, {}^5C_3 = 20\ (4)\ (10) = 800$

**NOTE**

The 3 identical 2's have to be divided into 4 distinct groups we need the number of nonnegative integral solutions of $a + b + c + d = 3$, which is equal to the number of positive integral solutions of $a_1 + b_1 + c_1 + d_1 = 7$, which is ${}^6C_3$. Similarly, the one 3 can be divided into 4 distinct groups in ${}^4C_3$ ways and the two 5's can be divided into 4 distinct groups in ${}^5C_3$ ways. The number of integral solutions of (1) is = $2^{4-1}$ (800) = 6400

**9.** In order to reach the cell $Q$ from the cell $P$, the ant will have to take 4 horizontal steps and 3 vertical steps. Now, this implies that ant will have to take in all 7 steps of which 4 are horizontal ($H$) and 3 vertical ($V$), i.e., HHHHVVV in any possible order. Now HHHHVVV can be arranged in 7!/4!3! = 35 ways.

$\therefore$ The number of such routes is 35.

**10.** Let $H$ indicate each horizontal step, $V$ a vertical step and $D$ a diagonal step.

Initially we note that with each diagonal step the number of horizontal and vertical steps will come down by 1 each.

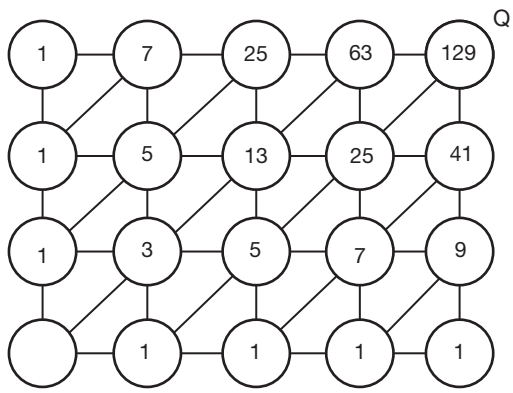
Now the different combinations, that can lead an ant from $P$ to $Q$ are.

1. HHHHVVV: Number of routes = 7!/4!3! = 35
2. HHHVVD: Number of routes = 6!/3!2! = 60
3. HHVDD: Number of routes = 5!/2!2! = 30
4. HDDD: Number of routes = 4!/3! = 4

$\therefore$ Total number of routes = 35 + 60 + 30 + 4 = 129

**Alternative method:**

In the diagram below, we write down the number of paths to each cell from $P$. The number of paths to any cell is equal to the sum of the numbers appearing in the cells that lead to the given cell.




**11.** Let the number of chocolates received by Jim, Joe and Julian be $x$, $y$ and $z$, respectively. We note that $y$ is not the number of the card selected by Joe.

As per the given conditions $x + y + z = 20$; $x \geq 0$, $y \geq 0$, $z \geq 0$.

The number of solutions to $x + y + z + \ldots\ldots r$ terms $= n$, where $x, y, z, \ldots \geq 0$ and $n \in N$ is given by ${}^{n+r-1}C_{r-1}$

$\therefore$ Number of ways = ${}^{20+3-1}C_{3-1} = {}^{22}C_2 = 231$

**Alternate Method:**

If Jim selected 0, Joe could select any number from 0 to 20, i.e., there are 21 way of distributing the chocolates. If Jim selects 1, there are 20 ways and so on. The total number of ways of distributing the chocolates is 21 + 20 + 19 + … + 1 = (21) × (11) = 231.

**12.** The Venn diagram has to describe 5 sets, say *A, B, C, D, E.*

Now, a point in a Venn diagram may belong to none, one or more of the five sets.

Number of regions belonging to all 5 sets = ${}^5C_5$

Number of regions common to only 4 sets = ${}^5C_4$

Similarly, the number of regions common to 3, 2 and 1 sets is given by ${}^5C_3$, ${}^5C_2$ and ${}^5C_1$.

Also ${}^5C_0$ gives the number of regions belonging to none of the sets.

$\therefore$ Total number of regions = ${}^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 2^5 = 32$.

**13.** To determine a plane uniquely, we require a set of 3 points. Hence, the number of planes that are determined by 15 points is ${}^{15}C_3$, if no 4 of them are coplanar.

Now, since 5 of the points are coplanar, instead of getting ${}^5C_3$or 10 planes, these 5 points determine only 1 plane..

$\therefore$ Total number of such planes = ${}^{15}C_3 - {}^5C_3 + 1$

$= 455 - 10 + 1 = 446$

**14.** From the first 20 natural numbers a combination is to be selected such that when arranged in ascending order, we get an A.P.

Now, the middle element of any such combination has to be greater than 1.

For 2 as the middle element, we have only one A.P, i.e., (1, 2, 3).

For 3 as the middle element, we have two A.P's possible, i.e., (2, 3, 4) and (1, 3, 5). It may be noted that the number of A.P.'s with 2 as middle term is same as those with 19 as middle term and those with 3 as middle term is same as those with 18 as middle term and so on. We list below the number of A.P.'s possible in each case. For the middle element; 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18 and 19, the number of A.P.'s.is 1, 2, 3, 4, 5, 6, 7, 8, 9, 9, 8, 7, 6, 5, 4, 3, 2 and 1, respectively.

$\therefore$ Total number of such combinations = 90.

**15.** In order to get a point of intersection we need to select two line segments, i.e., we need to select two points from the first line and two from the second.

$\therefore$ The number of points of intersection = ${}^5C_2 \times {}^4C_2$

$= 10 \times 6 = 60$

**16.** The various ways of distributing the pencils are listed below.

| *C* | *B* | *A* |
|---|---|---|
| 1 | 2 | 7 |
| 1 | 3 | 6 |
| 1 | 4 | 5 |
| 2 | 3 | 5 |

$\therefore$ Only 4 such ways exist.

**17.** Number of 6 digit numbers that can be formed using 1, 2, 3, 4, 5 and 6 without repetition = 6! = 720.We are interested in finding out those numbers which are divisible by every factor of the number present in its units place. This implies that the number should be divisible by the number in the units place. Now any number having a 1, 2 and 5 in the units place is divisible by 1, 2 or 5 respectively, $3 \times 5!$, i.e., 360 such numbers exist. Also since 1 + 2 + 3 + 4 + 5 + 6 = 21, is divisible by 3 any number with 3 in its units place is divisible by 3, 120 more such numbers exist. Numbers ending in 6 are also divisible by 6, as the number is divisible by 2 and also divisible by 3 as the sum of digits is 21. Thus, 120 more such numbers exist. Now if a number ending in 4 has to be divisible by 4, the last two digits should be 24 or 64. 48 such numbers exist.

$\therefore$ A total of 648 such numbers exist.

**18.** Assuming that each child gets an odd number of chocolates, sum of 15 odd numbers should be odd. 50 being an even number, such a distribution is not possible.

**19.** The composition of the given word: 4*E*s, 2*I*s and a letter each of *R, S, C, T, N, V*; i.e., a set of 8 distinct letters.

Any four letters when selected can be of the following combinations:

(i) All four alike: Number of ways = 1 (EEEE)

(ii) 3 Alike, 1 Different: Number of ways = $1 \times {}^7C_1 = 7$

(iii) 2 Alike, 2 Different: Number of ways = ${}^2C_1 \times {}^7C_2 = 42$

(iv) 2 Alike of first kind, 2 Alike of second kind:

Number of ways = ${}^2C_2 = 1$

(v) All different : Number of ways = ${}^8C_4 = 70$

$\therefore$ the total number of combinations = 1 + 7 + 42 + 1 + 70 = 121.

**20.** The total number of ways in which the parcels for Bihar can be addressed

$= 4! \left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right)$

$= 24 \left(\frac{1}{2} - \frac{1}{6} + \frac{1}{20}\right)$

$= 24\left(\frac{12-4+1}{24}\right) = 9$

The total number of ways in which the parcels for Delhi can be addressed

$= 5!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5}\right)$

$= 120\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120}\right)$

$= 120\left(\frac{60-20+5-1}{120}\right) = 44$

Therefore, the total number of ways in which all the parcels can be addressed = 9 (44) = 396.

**21.** The number of ways of selecting any number of items from the box containing $n$ items

$= {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots. {}^nC_{n-1} + {}^nC_n = 2^n$

No. of ways of selecting at most $(n-2)$ items

$= 2^n$ – (no. of ways of selecting at least $(n-1)$ items)

$= 2^n - ({}^nC_{n-1} + {}^nC_n) = 2^n - n - 1$

But $2^n - n - 1 = 247$ (given)

Only $n = 8$ satisfies the above equation $\therefore\ n = 8$

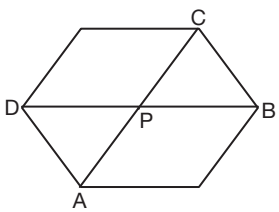
Required no. of ways of selecting at least 3 items

= total no of ways – (no. of ways of selecting at most 2)

$= 2^n - ({}^nC_0 + {}^nC_1 + {}^nC_2) = 2^8 - ({}^8C_0 + {}^8C_1 + {}^8C_2)$

$= 256 - (1 + 8 + 28) = 219$

**22.**




In the polygon shown in the figure above, the two diagonals AC and BD intersect in the interior of the region. i.e., every selection of 4 distinct vertices corresponds to one point of intersection of the diagonals in the interior of the region. Different sets of 4 vertices may produce the same point. If they all produce different points, we would get the maximum number of points of intersection (in the interior). This maximum number is ${}^nC_4$ [where $n$ is the number of vertices (or sides)].

From the given data ${}^nC_4 = 70$.

$\Rightarrow n(n-1)\ (n-2)\ (n-3) = 70\ (24) = 8(1)(6)(5)$

$\Rightarrow n \geq 8$ If $n = 8$, the maximum value of $p$ (the number of points of intersection of the diagonals lying in the interior) is 70.

$\therefore$ If $p = 70$, the minimum value of n is 8.

**23.** Each of the digits, 1, 2, 3 and 4 occur (4) (4) (4) = 64 times in the units place. The contribution to the sum from the units place is (1 + 2 + 3 + 4) (64) $10^0$. Like wise for tens place it is (1 + 2 + 3 + 4) (64) $10^1$ and so on.

Thus we get the sum to be

$(1 + 2 + 3 + 4)\ (64)\ (10^0 + 10^1 + 10^2 + 10^3)$

$= (10)\ (64)\ (1111) = 711040.$

**24.** First arrange the 5 white balls. There are 6 slots. Treat all the 4 red balls as one unit. This unit of red balls can be arranged in 6 ways.

Now there are 7 slots. We can arrange 6 green balls in ${}^7C_6$, i.e., 7 ways.

$\therefore$ Total required number of arrangements = 7(6) = 42.

**25.** Given word is 'BANANA'.

The number of letters are

| A | B | N |
|---|---|---|
| 3 | 1 | 2 |

The number of words that begin with A is $\frac{5!}{2!\,2!} = 30.$

The number of words that begin with $B$ is $\frac{5!}{3!\,2!} = 10.$

The number of words that begin with NAA is 3! = 6.

The number of words that begin with NAB is $\frac{3!}{2} = 3.$

The next word is NANAAB.

$\therefore$ The rank of the word = 30 + 10 + 6 + 3 + 1 = 50.

**26.** Two books from A and one book from remaining bags can be selected in ${}^7C_2(5 + x)$. Similarly two books from $B$ and one book from the remaining bags can be selected in ${}^5C_2(7 + x)$ ways. Two books from bag $C$ and one book from remaining bag can be selected in

${}^xC_2$ 12 ways.

$\therefore$ Total number of ways = ${}^7C_2\ (5 + x) + {}^5C_2\ (7 + x) + {}^xC_2$ $(12) = 541$

$\therefore\ 21(5 + x) + 10(7 + x) + x(x-1)\ 6 = 541$

$\Rightarrow 6x^2 + 25x - 366 = 0$

$\Rightarrow x = 6$

**27.** Following any minute in which I gulp a rasgolla, there have to be at least 4 minutes of rest. The earliest I can gulp another one is in the $5^{th}$ minute after the minute in which I consume one. Thus, there is a total period of 16 minutes in which I can't gulp any rasgolla (mother's instructions). I need to select 5 of the remaining 44 minutes. This can be done in ${}^{44}C_5$ ways. (For each of these ways, I can give a gap of 4 minutes, after each of the first 4 rasgollas. We can see that the conditions of minimum gap is satisfied.)

The number of combinations of times at which I could have eaten the rasgollas is ${}^{44}C_5$.

**Alternate Solution:** Let the first Rasgulla be consumed 'a' minutes after 7.00 p.m. Let the last Rasgulla be consumed '*b*' minutes before 8.00 p.m.

Let the time gap between the $1^{st}$ and $2^{nd}$, $2^{nd}$ and $3^{rd}$, $3^{rd}$ and $4^{th}$, $4^{th}$ and $5^{th}$ Rasgullas be $c$, $d$, $e$ and $f$, respectively.

$a + b + c + d + e + f = 55$

Where $c, d, e, f \geq 4$ and a, $b \geq 0$.

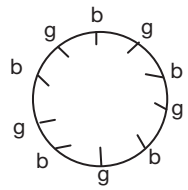
Let $c = c_1 + 4$, $d = d_1 + 4$, $e = e_1 + 4$, $f = f_1 + 4$.

Where $c_1, d_1, e_1, f_1 \geq 0$

$\therefore a + b + c_1 + d_1 + e_1 + f_1 = 39$

Number of non-negative integral solutions of $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = n$ is ${}^{n+5}C_5$. Applying the same, the Rasgullas, can be consumed at ${}^{44}C_5$ different time combinations.

**28.** As no two boys can sit next to each other, we should start with the girls and then make sure that at the most 1 boy goes into the slots that are created. As Suneeta is mentioned by name, we can use her position as a reference. The other 4 girls can be seated in 4! ways. Suresh cannot sit on either side of Suneeta. He can be seated in any of the other 3 slots. The 4 remaining boys can be seated in 4 !ways.




$\therefore$ Total number of ways = $4!\ {}^{3}C_1\ 4! = 3(4!)^2$

**29.** 14 boys can be seated in a row in 14! ways. Since no two girls can be seated adjacent to each other, the girls can be seated in the 15 gaps between the 14 boys. The particular girl who is to be seated beside a particular boy can be seated in 2 different ways. After this the remaining 9 girls can be seated in the remaining 14 places in ${}^{14}P_9$ ways.

Hence, the total number of ways = $14!(2)\ {}^{14}P_9$

$= \dfrac{(14!)^2 \times 2}{5!} = \dfrac{(14!)^2}{60}$

**30.** The 9-digit numbers must have either no 8 *s*, three 8 *s*, six 8 *s* or all 8 *s*.

But, as the numbers are even, each of them must end in 8. Therefore, we can't have zero 8's. Number of possible values with nine 8's is 1. Number of possible values with six 8 *s* is ${}^{8}C_3$

[the three 9 *s* can occupy any 3 of the remaining 8 positions.] Number of possible values with three 8's is ${}^{8}C_6$ [The six 9*s* can occupy any 6 of the remaining 8 positions] Number of possible values with zero 8 *s* is 0.

Total number of such 9 digit even numbers
$= 1 + {}^{8}C_3 + {}^{8}C_2 + 0 = 1 + 56 + 28 = 85.$

# 2 Probability

CHAPTER

**Learning Objectives**

*In this chapter, you will*:

- Learn about random events and probability
- Learn about the various types of events, such as:
  - Equally likely outcomes
  - Mutually exhaustive events
  - Collectively exhaustive events
- Learn all about addition theorem of probability
- Learn all about conditional probability and its main elements
  - Multiplication theorem
  - Bayes' theorem
- Understand Expected Value and learn how to solve problems based on it

## ❒ PROBABILITY

This is an important topic for MBA entrance tests. Besides the entrance tests, the topic is an important part of the management courses themselves. Therefore, students aspiring to be future managers need a sound understanding of the basics.

Natural phenomena are of two types—**deterministic** and **probabilistic**. For example, the direction in which or the time at which the sun rises everyday is a deterministic phenomenon, while, where or when it may rain is a probabilistic phenomenon. Similarly, **experiments** (the operations of doing or observing something resulting in some final outcomes) are of two types—those in which the outcome is definite and others in which the actual outcome may be any one of all the possible outcomes. For example, hydrogen is allowed to react with oxygen. They react in a certain ratio and produce water. The outcome is definite-while if a coin is tossed, it may turn up showing either heads or tails, the outcome may be one of the two possible outcomes. **Experiments of the second kind are called random experiments (RE)**. One instance of such an experiment is a trial. The set of all possible outcomes for a particular random experiment is its **sample space** *S*. This corresponds to the concept of the universal set in **set theory**. Each outcome is said to be a point in *S*. Thus for the experiment of tossing a coin, the sample space is the set of the two outcomes—heads and tails. For the experiment of rolling a die, it is {1, 2, 3, 4, 5, 6}. For drawing a card from a deck of cards it is the set of all the 52 possible outcomes—corresponding to the 52 cards. Any subset of the sample space is a simple event. Two (or more) events which occur for two (or more) different experiments—or for two (or more) trials of the same experiment are called **compound events**. Thus if a coin is tossed and a die is rolled, the event of getting a head (in the case of the coin) and say 5 (in the case of the die) is a compound event. Similarly, if a die is rolled twice, the event of getting an even number on the first roll and an odd on the second is a compound event.

### ❒ Equally Likely Outcomes

If a normal coin is tossed, it may come up either heads or tails. Both the outcomes are equally likely. We can accept

this intuitively, even though at the moment, we do not know how to compute (or measure) the probability of either outcome. For the purpose of this discussion, we start with this assumption—that we can recognise intuitively whether all the possible outcomes are equally likely or not. We know from experience that all coins are not unbiased. Sometimes, the mass in the coin is so distributed that it shows up one side more than the other. Such coins (or dice) are said to be biased.

We can now consider one definition of probability. This is the only one that we need for the questions that we shall face. For a random experiment with $n$ 'equally likely' outcomes, if $E$ is an event which can be considered to have occurred for $m$ of the outcomes, the probability (mathematical probability or a priori probability) of $E$ **is** $\frac{m}{n}$**, i.e.,** $P(E) = \frac{m}{n}$

The complement of an event is the event of the non-occurrence of $E$. It is denoted by $\bar{E}$ and $P(\bar{E}) = 1 - \frac{m}{n}$

For example, if the RE is tossing a coin and $E$ is the event of getting heads, $P(E) = \frac{1}{2}$. Also $P(\bar{E}) = 1 - \frac{1}{2} = \frac{1}{2}$. The complement of getting a head is getting a tail. If the RE is rolling a die and $E = \{2, 3\}$, $\bar{E} = \{1, 4, 5, 6\}$. In this case, $P(E) = \frac{1}{3}$, $P(\bar{E}) = \frac{2}{3}$

With this definition, we can consider the two extreme cases. An event is any subset of $S$. If $E$ is the null set $P(E) = 0$ (an impossible event) and if $E = S$, $P(E) = 1$ (a certain event)

For example, let the RE be rolling a die and consider the 'event' of getting a 0. In our notation, $E$ would be the null set. For no element of $S$, can it be said that $E$ has occurred. $\bar{E}$ would be the event of not getting a 0 and $P(\bar{E}) = 1$

Instead of $E$ saying that the probability of an event is $m/n$, we can also say that the odds in favour of the event are $m$ to $n - m$, i.e., $P(E)/P(\bar{E})$. Similarly, the odds against the event are $n - m$ to $m$, i.e., $P(\bar{E})/P(E)$.

## ❒ Mutually Exclusive Events

If there is a set of events, such that if any one of them occurs, none of the others can occur, the events are said to be mutually exclusive. Consider the RE of rolling a die and the following events.

$E_1 = \{1\}$

$E_2 = \{2, 3\}$

$E_3 = \{4, 5\}$

The three events are mutually exclusive.

## ❒ Collectively Exhaustive Events

If there is a set of events such that at least one of them is bound to occur, the events are said to be collectively exhaustive. For the RE considered above, if $E_4 = \{1, 2, 3, 4\}$ and $E_5 = \{3, 4, 5, 6\}$, $E_4$ and $E_5$ are collectively exhaustive.

If a set of events are both mutually exclusive and collectively exhaustive, the sum of their probabilities is 1.

## ❒ ADDITION THEOREM OF PROBABILITY

If $A$ and $B$ are two events, then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

This result follows from the corresponding result in set theory. If $n(X)$ represents the number of elements in set $X$, $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

**Example:** If a die is rolled, what is the probability that the number that comes up is either even or prime?

$A$ = The event of getting an even number = $\{2, 4, 6\}$

$B$ = The event of getting a prime = $\{2, 3, 5\}$

$A \cup B = \{2, 3, 4, 5, 6\}$

$A \cap B = \{2\}$

$P(A) = \frac{3}{6}$, $P(B) = \frac{3}{6}$, $P(A \cup B) = \frac{5}{6}$ and $P(A \cap B) = \frac{1}{6}$. We can verify that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

## ❒ Conditional Probability and the Multiplication Theorem of Probability

The events that we have considered so far are without reference to other events (or conditions). But, very often, we need to consider events, in relation to other events (or conditions). We can continue with the same RE. Let $A$ be the event of getting a prime and $B$ be the event of getting an even number, i.e., $A = \{2, 3, 5\}$ and $B = \{2, 4, 6\}$, $P(A) = \frac{3}{6}$

But if $B$ is known to have occurred, then $P(A) = \frac{1}{3}$. We write this as follows:

$P\left(\frac{A}{B}\right) = \frac{1}{3}$, This is read as follows:

The probability of $A$, given $B$ is $\frac{1}{3}$.

In general,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \text{ or } P(A \cap B) = P(B).P\left(\frac{A}{B}\right) \quad (1)$$

Also, as $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$, it follows that

$$P(B \cap A) = P(A).P\left(\frac{B}{A}\right). \quad (2)$$

This result (1) or (2) is known as the multiplication theorem of probability.

If $P\left(\frac{A}{B}\right) = P(A)$, then $A$ and $B$ are said to be independent. $A$ is independent of $B$, because whether $B$ occurs or does not occur, the probability of $A$ does not change.

**Example:** A number is selected at random from the integers 1 to 50. $A$ is the event of getting a multiple of 5 and $B$ is the event of getting an even number.

Then, $A = \{5, 10, 15...45, 50\}$, $B = \{2, 4, ..., 48, 50\}$

$$P(A) = \frac{10}{50}$$

$$P\left(\frac{A}{B}\right) = \frac{5}{25}, \text{ also } P\left(\frac{A}{\overline{B}}\right) = \frac{5}{25}$$

$\therefore$ $A$ and $B$ are independent events.

If $A$ and $B$ are independent, $P\left(\frac{A}{B}\right) = P(A)$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = P(A) \text{ or } P(A \cap B) = P(A).P(B)$$

If $A$ and $B$ are independent, so are $A$, $\overline{B}$; $\overline{A}$, $B$ and $\overline{A}, \overline{B}$

i.e., if $P(A) = P\left(\frac{A}{B}\right)$, then each of the following is true

$$(A) = P\left(\frac{A}{B}\right)$$

$$P(B) = P\left(\frac{B}{A}\right) = P\left(\frac{B}{A}\right)$$

We can show the following results

If $P(A) = P\left(\frac{A}{B}\right)$, then $P(A) = \left(\frac{A}{\overline{B}}\right)$.

Also $P\left(\frac{B}{A}\right) = P(B) = P\left(\frac{B}{\overline{A}}\right)$

## ❒ Pairwise Independence and Mutual Independence

If $A$, $B$, $C$ are three events such that each of the 3 pairs $A$, $B$; $B$, $C$ and $C$, $A$ are independent, $A$, $B$, $C$ are said to be pairwise independent. Let $P(A)$, $P(B)$, $P(C)$ be $a$, $b$, $c$ respectively.

As $A$, $B$ are independent, $P(A \cap B) = ab$,
As $B$, $C$ are independent, $P(B \cap C) = bc$,
As $A$, $C$ are independent, $P(C \cap A) = ca$

Even if these three conditions are true, $P(A \cap B \cap C)$ is not necessarily equal to $abc$. In case it is, the events are said to be mutually independent. We note that mutual independence is a stronger condition. It implies pairwise independence (while pairwise independence does not necessarily mean mutual independence)

For more events, we can generalise the concept, we can talk of pairwise (or 2-wise), tripletwise (or 3-wise), quadrapletwise (or 4-wise) independence and so on. Mutual independence of $n$ events would mean $i$-wise independence for $i = 2, 3, 4...n$

**Example:**

$A$ positive integer from 1 to 60 is selected at random.
$A$ is the event of selecting a multiple of 3.
$B$ is the event of selecting a multiple of 4.
$C$ is the event of selecting a multiple of 5.
The following results are true.

| | | |
|---|---|---|
| $a = P(A) = \frac{1}{3}$ | $P(A \cap B) = \frac{5}{60} = \frac{1}{12}$ | $P(A \cap B \cap C) = \frac{1}{60}$ |
| $b = P(B) = \frac{1}{4}$ | $P(A \cap C) = \frac{4}{60} = \frac{1}{15}$ | |
| $c = P(C) = \frac{1}{5}$ | $P(B \cap C) = \frac{3}{60} = \frac{1}{20}$ | |

Thus, $P(A \cap B) = ab$, $P(A \cap C) = ac$, $P(B \cap C) = bc$ and $P(A \cap B \cap C) = abc$,

$\therefore$ $A$, $B$, $C$ are not merely pairwise independent but also mutually independent

**Example:**

There are four three-digit numbers;
1 1 2; 1 2 1; 2 1 2; 2 2 1
One of these is selected at random.

$A_1$ is the event that the first digit is 1
$A_2$ is the event that the second digit is 1
$A_3$ is the event that the third digit is 1
What can be said about the 3 pairs, and the triplets?

$P(A_1) = \frac{1}{2}$, $P(A_2) = \frac{1}{2}$, $P(A_3) = \frac{1}{2}$, $P(A_1 \cap A_2) = \frac{1}{4}$,

$P(A_2 \cap A_3) = 0$, $P(A_3 \cap A_1) = \frac{1}{4}$

$P(A_1 \cap A_2 \cap A_3) = 0$

$\therefore$ $A_1$, $A_2$ are independent;

$A_2$, $A_3$ are not independent;

$A_3$, $A_1$ are independent;

The question of mutual independence need not be considered as $A_2$, $A_3$ are not independent, i.e., $A_1$, $A_2$, $A_3$ are not mutually independent because they are not pairwise independent.

## ❒ Bayes' Theorem

Let $A_1, A_2 \ldots A_n$ be $n$ mutually exclusive and collectively exhaustive events with respective probabilities of $p_1$, $p_2 \ldots p_n$. Let $B$ be an event such that $P(B) \neq 0$ and $P\left(\frac{B}{A_i}\right)$ for $i = 1$ to $n$ be $q_1, q_2 \ldots q_n$. Then, the conditional probability of $A_i$ given $B$ is $\frac{p_i q_i}{p_1 q_1 + p_2 q_2 + \ldots + p_n q_n}$

**Example:**

Box 1 contains 3 white and 2 black balls. Box 2 contains 1 white and 4 black balls. A ball is picked from one of the two boxes. It turns out to be black. Find the probability that it was drawn from box 1.

The data is tabulated below

| | Box 1 | Box 2 |
|---|---|---|
| White | 3 | 1 |
| Black | 2 | 4 |

The event that box 1 is selected is say $A_1$.
The event that box 2 is selected is $A_2$

$\therefore$ $p_1 = P(A_1) = \frac{1}{2}$ and $p_2 = P(A_2) = \frac{1}{2}$

Let $B$ be the event that a black ball is selected

$\therefore$ $P\left(\frac{B}{A_1}\right) = \frac{2}{5}$ and $P\left(\frac{B}{A_2}\right) = \frac{4}{5}$

Now it is given that a black ball has been drawn and we need to find $P(A_1)$, i.e., we need $P\left(\frac{A_1}{B}\right)$. We use the result above

$$P\left(\frac{A_1}{B}\right) = \frac{P(A_1)P\left(\frac{B}{A_1}\right)}{P(A_1)P\left(\frac{B}{A_1}\right) + P(A_2)P\left(\frac{B}{A_2}\right)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{2}{5}\right)}{\left(\frac{1}{2}\right)\left(\frac{2}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{4}{5}\right)} = \frac{2}{2+4} = \frac{1}{3}$$

## ❒ EXPECTED VALUE

The concept of 'expected value' is very important in the theory of probability. This concept is very useful in managerial decision-making.

The theory of probability has its origin in gambling. When people went to gambling houses or casinos, they used to get certain amounts of money if they achieved a certain result in the game. Mathematicians wanted to find out as to how much a person will earn if the game is played a large number of times.

Let us say that a man is playing a game of 'throwing a die'. He is given ₹6 if he throws a 'four' and ₹9 if he throws a 'six' on the die and not paid anything if he throws any other number (of course, he will have to pay some amount to the gambling house owner each time he wants to throw the die and this aspect will be considered later). Suppose he throws the die a large number of times—say 6,00,000 times. As the number of times the experiment is repeated becomes very large, we know that the number of times each event will occur is given by probability.

A 'four' will appear with a probability of 1/6, i.e., it is expected to appear 1,00,000 times out of a total of 6,00,000 times the die is thrown. Similarly, a 'six' will appear 1,00,000 times (because the probability is 1/6). Hence, the amount he will get in the long run will be $1,00,000 \times 6 + 1,00,000 \times 9 = 15,00,000$. The amount he gets per throw will be 1500000/600000 = ₹2.5.

We say that the person's expected value for this game per throw in the long run is ₹2.5.

This can be calculated without the number of throws coming into the picture. Once the events are defined, we should have the probabilities of all the events and the monetary value associated with each event (i.e., how much money is earned or given away if that particular event occurs). Then,

**FORMULA**

Expected Value = $\sum_i$ [Probability $(E_i)$ × [Monetary value associated with event $E_i$]

**Example:**

*A* person tosses a coin. If it comes up heads, he gets ₹10. If it comes up tails, he has to pay ₹5. What is his expected value?

| Event | Heads | Tails |
|---|---|---|
| Probability | $\frac{1}{2}$ | $\frac{1}{2}$ |
| Expectation | 10 | –5 |

$$\therefore E = \frac{\left(\frac{1}{2}\right)10 + \left(\frac{1}{2}\right)(-5)}{\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)} = 2.5$$

## SOLVED EXAMPLES

**2.01.** Two fair dice are thrown simultaneously. What is the probability that one dice shows a number greater than '4' and the other shows a number less than '3'?

**Sol:** When two dice are thrown simultaneously, the total number of outcomes is $6 \times 6 = 36$.

Out of these, favourable cases are 8, i.e., (5, 1), (5, 2), (6, 1), (6, 2), (1, 5), (2, 5), (1, 6) and (2, 6).

Hence, the required probability $= \frac{8}{36} = \frac{2}{9}$.

**2.02.** When two fair dice are thrown simultaneously, what is the probability that the sum obtained is not equal to 7?

**Sol:** We will have more cases to deal for the sum not being 7.

Rather than looking at finding the probability of this event directly, we will find the probability of non-occurrence of this event and subtract it from 1 to get the required probability.

So, we deal with the case where the sum is equal to 7.

The sum being 7: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1).

There are 6 favourable occurrences.

$\therefore P$ (sum is 7) $= \frac{6}{36} = \frac{1}{6}$.

Hence, the required probability $= 1 - \frac{1}{6} = \frac{5}{6}$.

**2.03.** When three fair dice are thrown simultaneously, what is the probability that the first dice shows an even number, the second dice shows an even prime number and the third dice shows a composite number?

**Sol:** Let *A*: the event of the first dice showing up an even number, i.e,. 2, 4, or 6.

*B*: the event of the second dice showing up an even prime number, i.e., 2.

*C*: the event of the third dice showing up a composite number, i.e., 4 or 6.

Their respective probabilities being,

$P(A) = \frac{3}{6} = \frac{1}{2}$, $P(B) = \frac{1}{6}$ and $P(C) = \frac{2}{6} = \frac{1}{3}$.

The required probability is obtained by compounding these events, i.e., by multiplying individual probabilities. Hence, the required probability is $\frac{1}{2} \times \frac{1}{6} \times \frac{1}{3} = \frac{1}{36}$.

**2.04.** If four fair dice are thrown simultaneously, what is the probability that the sum of the numbers is more than 21?

**Sol:** Total number of cases $= 6^4 = 1296$.

The various combinations for the sum being greater than 21 and the corresponding number of arrangements in each case are as follows:

Sum being 22:

(6, 6, 6, 4) $\frac{4!}{3!} = 4$

(6, 6, 5, 5) $\frac{4!}{2!2!} = 6$

Sum being 23: (6, 6, 6, 5) $\frac{4!}{3!} = 4$

Sum being 24: (6, 6, 6, 6) = 1

The number of favourable cases is 15.

Hence, the required probability $= \frac{15}{1296} = \frac{5}{432}$.

**2.05.** If 4 fair coins are tossed together, what is the probability of getting exactly 3 heads?

**Sol:** The event of getting exactly 3 heads will be the combination of 3 heads and 1 tail. The number of arrangements with this combination is $\frac{4!}{3!1!}$ = 4. Hence, the probability is $\frac{4}{2^4}=\frac{1}{4}$.

**2.06.** If five fair coins are tossed together, what is the probability of getting at the most 3 tails?

**Sol:** $P$(getting at most 3 tails)
= 1 – $P$(getting at least 4 tails)
= 1 – $P$(getting 4 tails or 5 tails)
The event of getting exactly 4 tails is same as obtaining 4 tails and 1 head in any order and the total occurrences in which case are $\frac{5!}{4!}=5$.

Hence, $P$(getting exactly 4 tails) = $\frac{5}{32}$.

$P$(getting exactly 5 tails) = $\frac{1}{32}$.

$\therefore$ $P$(getting at most 3 tails) = $1-\left[\frac{5}{32}+\frac{1}{32}\right]=\frac{13}{16}$.

**2.07.** A card is drawn from a well-shuffled pack of cards. Find the probability that it is
(a) an ace.
(b) a red numbered card.
(c) a spade.
(d) a black king.

**Sol:** The number of ways of selecting one card out of 52 cards is ${}^{52}C_1 = 52$.

(a) One card can be drawn from 4 aces in ${}^4C_1$ = 4 ways.
Hence, the required probability is
$$\frac{{}^4C_1}{{}^{52}C_1}=\frac{4}{52}=\frac{1}{13}$$

(b) A pack of cards has 18 red numbered cards. A red numbered card can be drawn in ${}^{18}C_1$ = 18 ways.
Hence, the probability of drawing a red numbered card is $\frac{18}{52}=\frac{9}{26}$.

(c) A pack of cards has 13 spades. Hence, the probability of drawing a spade is
$$\frac{{}^{13}C_1}{{}^{52}C_1}=\frac{13}{52}=\frac{1}{4}.$$

(d) A pack of cards has 2 black kings. Hence, the probability of drawing a black king is
$$\frac{{}^2C_1}{{}^{52}C_1}=\frac{2}{52}=\frac{1}{26}.$$

**2.08.** If two cards are drawn simultaneously from a well shuffled pack of cards, then find the probability of both being
(a) queens.
(b) honours.
(c) red honours.

**Sol:** Two cards can be dawn from a pack of cards in ${}^{52}C_2$ ways.

(a) There are 4 queens in a pack of cards. Two queens can be drawn in ${}^4C_2$ ways. Probability of both being queens is $\frac{{}^4C_2}{{}^{52}C_2}$.

(b) There are 16 honours in a pack of cards. Probability of both being honours is $\frac{{}^{16}C_2}{{}^{52}C_2}$.

(c) There are 8 red honours in a pack of cards. Probability of both the cards drawn being red honours is $\frac{{}^8C_2}{{}^{52}C_2}$.

**2.09.** If two cards are drawn at random from a pack of cards, what is the probability that one of them is a numbered card and the other is a king?

**Sol:** Two cards can be drawn from a pack of cards in ${}^{52}C_2$ ways.
A numbered card can be drawn in ${}^{36}C_1$ ways and a king can be drawn in ${}^4C_1$ ways.

Hence, the required probability is $\frac{{}^{36}C_1\times{}^4C_1}{{}^{52}C_2}$.

**2.10.** If two cards are drawn simultaneously from a pack of cards, what is the probability that both are spades or both are clubs?

**Sol:** Here, both the cards should be spades or both should be clubs. These are two mutually exclusive events. Let $A$ denote the event of getting both spades and $B$ denote the event of getting both clubs.
$n(A) = {}^{13}C_2$, $n(B) = {}^{13}C_2$ and $n(A \cap B) = 0$.

We need to compute $P(A \cup B)$.

$$P(A\cup B)=P(A)+P(\mathrm{B})=\frac{{}^{13}C_2}{{}^{52}C_2}+\frac{{}^{13}C_2}{{}^{52}C_2}.$$

**2.11.** When two cards are drawn simultaneously from a pack of cards, what is the probability that both are red, or both are kings?

**Sol:** Here, we have two events which are not mutually exclusive. While the two cards are red they can simultaneously be kings also. Let $A$ and $B$ be the events of selecting both red or both kings respectively. Then, we need to find out $P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{{}^{26}C_2}{{}^{52}C_2} + \frac{{}^{4}C_2}{{}^{52}C_2} - \frac{{}^{2}C_2}{{}^{52}C_2}$$

**2.12.** When three cards are drawn in succession from a pack of cards with replacement, what is the probability that the first drawn card is a heart, the second is a red card and the third is an honour?

**Sol:** Let $E_1$, $E_2$ and $E_3$ denote the events of drawing a heart, a red card and a honour card in the first, second and third draws, with replacement in that order. As the cards drawn are being replaced, $E_1$, $E_2$, $E_3$ are independent.

Hence, the required probability

$= P(E_1 \cap E_2 \cap E_3) = P(E_1)\ P(E_2)\ P(E_3)$

$$= \frac{{}^{13}C_1}{{}^{52}C_1} \times \frac{{}^{26}C_1}{{}^{52}C_1} \times \frac{{}^{16}C_1}{{}^{52}C_1} = 1/4 \times 1/2 \times 4/13$$

$= 1/26.$

**2.13.** When three cards are drawn simultaneously from a pack of cards, what is the probability that one of them is a black card, another a heart, and the third a diamond?

**Sol:** There are 26 black cards, 13 hearts and 13 diamonds in a pack of cards. The required probability is $\frac{{}^{26}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_3}$.

**2.14.** When two balls are drawn in succession without replacement from a box containing 4 red balls and 7 green balls, find the probability that

(a) the first one is red and the second one is green.

(b) both are red.

**Sol:** The first ball can be drawn in ${}^{11}C_1 = 11$ ways and as the ball is not being replaced, the second ball can be drawn in ${}^{10}C_1 = 10$ ways.

(a) $A$ red ball can be drawn in ${}^{4}C_1 = 4$ ways in the first draw and a green ball can be drawn in the second draw in ${}^{7}C_1 = 7$ ways.

$\therefore$ The required probability $= \frac{4}{11} \times \frac{7}{10} = \frac{14}{55}$

(b) $A$ red ball can be drawn in ${}^{4}C_1 = 4$ ways in the first draw. Since this ball is not replaced, a red ball in the second draw can be drawn in ${}^{3}C_1 = 3$ ways.

$\therefore$ The required probability $= \frac{4}{11} \times \frac{3}{10} = \frac{6}{55}$.

**2.15.** $A$ bag contains 5 black balls, 6 red balls and 3 white balls. If a ball is drawn at random, what is the probability that it is

(a) not a white ball?

(b) a red ball?

**Sol:** One ball can be drawn out of the 14 balls present in the bag in ${}^{14}C_1$ ways.

(a) $A$ ball other than a white ball $(5 + 6 = 11)$ can be drawn in ${}^{11}C_1$ ways. Hence, the probability of not drawing a white ball is $\frac{{}^{11}C_1}{{}^{14}C_1} = \frac{11}{14}$.

(b) One red ball out of the six red balls present in the bag can be drawn in ${}^{6}C_1$ ways.

Hence, the probability of drawing a red ball is $\frac{{}^{6}C_1}{{}^{14}C_1} = \frac{3}{7}$.

**2.16.** $A$ bag contains 7 blue, 5 red and 4 green balls. If three balls are drawn simultaneously at random, what is the probability that

(i) one is blue, one is red and the other is green?

(ii) two are red and one is green?

(iii) all three are of the same colour?

**Sol:** Three balls can be drawn from 16 balls in ${}^{16}C_3$ ways.

(i) We can draw one blue ball out of 7 blue balls in ${}^{7}C_1$ ways, one red ball out of 5 red balls in ${}^{5}C_1$ ways and 1 green ball out of 4 green balls in ${}^{4}C_1$ ways.

Required Probability $= \frac{{}^{7}C_1 \times {}^{5}C_1 \times {}^{4}C_1}{{}^{16}C_3}$.

(ii) We can select two red out of 5 red balls in ${}^{5}C_2$ ways and one green out of 4 green balls in ${}^{4}C_1$ ways.

The required probability $= \frac{{}^{5}C_2 \times {}^{4}C_1}{{}^{16}C_3}$.

(iii) To select three balls of the same colour, we can choose either 3 blue balls from 7 blue balls or 3 red balls from 5 red balls or 3 green balls from 4 green balls in ${}^{7}C_3$, ${}^{5}C_3$ and ${}^{4}C_3$ ways, respectively.

The required probability $= \frac{{}^{7}C_3 + {}^{5}C_3 + {}^{4}C_3}{{}^{16}C_3}$.

**2.17.** Nakul Kumar has 3 fifty-rupee notes, 4 hundred-rupee notes and 6 five hundred-rupee notes in his pocket. If 2 notes are taken at random, what are the odds in favour of both notes being of hundred-rupee denomination?

**Sol:** Total number of ways in which 2 notes can be taken from the pocket containing 13 notes is ${}^{13}C_2$ and the number of ways in which 2 hundred-rupee notes can be taken is ${}^{4}C_2$.

$\therefore$ The probability of choosing 2 hundred-rupee notes is $\frac{{}^{4}C_2}{{}^{13}C_2} = \frac{4\times3}{13\times12} = \frac{1}{13}$.

Odds in favour of the event = favourable ways: unfavourable ways = 1: 12

**2.18.** Shivcharan, who is interested in Philately, saw 4 Indonesian, 5 Mexican and 6 Egyptian stamps in a box. He drew 2 stamps from the box one after the other. What is the probability that the stamp drawn second is Mexican when the stamp drawn first is

(i) replaced?

(ii) not replaced?

**Sol:** (i) If the stamp drawn first is replaced, the box will contain all the stamps it originally had. The probability of drawing a Mexican stamp is $5/15 = 1/3$

(ii) If the stamp is not replaced, there are 2 cases to deal with.

(a) The first draw yielding a Mexican stamp.

(Or)

(b) The first draw not yielding a Mexican stamp.

The corresponding probabilities being

Case (a) $5/15 \times 4/14 = 2/21$

Case (b) $10/15 \times 5/14 = 5/21$

**2.19.** The probability of a snake being venomous is 0.5, the probability of a snake not being venomous or not an instant killer is 0.8. If it is known that a snake is venomous, find the probability that it is an instant killer.

**Sol:** Let $A$ be the event that the snake is venomous, and $B$ be the event that the snake is an instant killer.

It is given that

$P(A) = 0.5$

$P\left(\overline{A} \cup \overline{B}\right) = 0.8$

$\Rightarrow P\left(\overline{A \cap B}\right) = 0.8$

$\Rightarrow P(A \cap B) = 1 - 0.8 = 0.2$

$\therefore P\left(B/A\right) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.5} = \frac{2}{5}$

**2.20.** One number is selected at random from the set.

$S$ = {2222, 4422, 6622, 2244, 4444, 6644, 2266, 4466, 6666}

$A$ is the event that the number start with 22.

B is the event that the number end with 44.

C is the event that the units digit and thousands digit are equal.

Which of the following is true?

(A) $A$ and $B$ are dependent.

(B) B and $C$ are dependent.

(C) $A$, $B$, $C$ are mutually independent.

(D) $A$, $B$, $C$ are pair wise independent.

**Sol:** $P(A) = \frac{3}{9} = \frac{1}{3}$ $\quad P(A \cap B) = \frac{1}{9}$

$P(B) = \frac{3}{9} = \frac{1}{3}$ $\quad P(A \cap C) = \frac{1}{9}$

$P(C) = \frac{3}{9} = \frac{1}{3}$ $\quad P(B \cap C) = \frac{1}{9}$

$P(A \cap B) = P(A)\ P(B)$, $P(B \cap C) = P(B)P(C)$, $P(C \cap A) = P(C)P(A)$

but $P(A)P(B)P(C) \neq P(ABC)$

They are pairwise independent but not mutually independent.

**2.21.** During the next one year, the probability that Apple releases a product is 0.6. The probability that a product is a success, given that it is released by Apple is 0.7. The probability that a product is a success and released by Google is 0.4.$A$ product released by either Apple or Google during the next one year is a success. Find the probability that it is released by Apple.

**Sol:** Let $A$ be the event of a product being released by Apple.

Let $B$ be the event of a product being released by Google.

Let $E$ be the event of a product being a success.

$P(A) = 0.6$

$P\left(E/A\right) = 0.7$

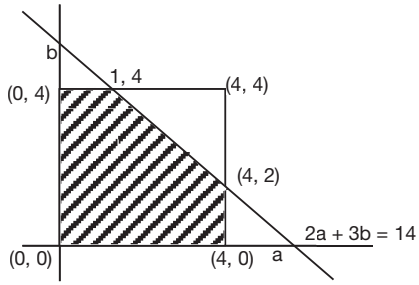
$P(E \cap B) = 0.4$

$$P(A/E)=\frac{P(A).P(E/A)}{P(A).P(E/A)+P(B).P(E/B)}$$

$$=\frac{0.6(0.7)}{0.6(0.7)+0.4}=\frac{0.42}{0.82}=\frac{21}{41}$$

**2.22.** *a*, *b* are two positive numbers such that $a < 4$, $b < 4$. Find the probability that $2a + 3b < 14$.

**Sol:**




If we add *a*-on *x*-axis, and *b*-on *y*-axis,
$2a + 3b = 14$ is a line as shown above,
In the shaded region $2a + 3b < 14$
Required probability

$$=\frac{4(4)-\frac{1}{2}(3)(2)}{4(4)}=\frac{13}{16}$$

**2.23.** Subash participates in a game that involves throwing an unbiased dice, where a participant is given twice as many rupees as the number that turns up on the dice if it is prime and thrice as many rupees as the number that turns up on the dice, if it is composite. What will be the expected value per throw in the long run, if the participant must pay 42, if one turns upon the dice?

**Sol:** When an unbiased dice is rolled, the probability of getting a 1 is 1/6. Similar is a case for the outcomes 2, 3, 4, 5 and 6, i.e., $P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6$.

Monetary value when a prime number (i.e., 2, 3 or 5) turns up = 2 times the number on dice.

Monetary value when a composite number (i.e., 4 or 6) turns up = 3 times the number on the dice.

Monetary value when 1 turns up = –42.

(As the participant must pay in this case).

∴ Expected value

$$=\frac{1}{6}\times 2\times(2)+\frac{1}{6}\times 2\times(3)+\frac{1}{6}\times 3\times(4)$$

$$+\frac{1}{6}\times 2\times(5)+\frac{1}{6}\times 3\times(6)-\frac{1}{6}\times 42$$

$$=\frac{8}{6}=1.33 \text{ (approx)}$$

**2.24.** *A* game involving a biased dice is such that ₹5 is paid each time the dice shows up a score of 3, while ₹8 is paid for every other score on the dice. The dice is such that the score of 3 occurs 4 times as frequently as any other score. How much would a person be willing to pay as entry fee each time, if in the long run, there has to be neither a profit nor a loss for taking part in this game?

**Sol:** As the game should result neither in profit nor in loss, the entry fee should be equal to the expected value of the game.

To calculate the expected value, we need the probability of the events involved.

Let us assume that the probability of getting any number other than three as *p*. The probability of getting a three is 4*p*.

Since all the events are mutually exclusive and collectively exhaustive, the sum of their probabilities should be equal to 1.

Hence, $p + p + 4p + p + p + p = 1 \Rightarrow 9p = 1$
or $p = 1/9$

∴ The probability that the number three appears is 4/9 and the probability that any other number appears is 1/9.

Hence, the expected value is

$$=\frac{4}{9}(5)+\frac{1}{9}(8+8+8+8+8)$$

$$=\frac{20}{9}+\frac{40}{9}=\frac{20}{3}=6.66$$

Hence, the person should be willing to pay ₹6.66, as entry fee.

**2.25.** Two dice are rolled simultaneously. Find the probability that one of them shows a number which is at least 4 and the other shows a number which is at most 3.

**Sol:** When two dice are rolled simultaneously, total number of outcomes = (6) (6) = 36.

Out of these, favourable cases are (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2) and (6, 3). i.e., 9

∴ Required probability $=9\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\times 2=\frac{1}{2}$

**2.26.** Two dice are rolled simultaneously. Find the probability that the sum of the numbers on them is not 8.

**Sol:** Probability of sum of the numbers is not 8 = 1 – (Probability of sum of the numbers on them is 8)

Favourable cases for sum of the numbers being8 are (2, 6), (3, 5), (4, 4), (5, 3) and (6, 2)

Probability (sum of the numbers on them is 8)

$$= 5\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{5}{36}$$

$$\text{Required probability} = 1 - \frac{5}{36} = \frac{31}{36}$$

**2.27.** Three fair dice are rolled simultaneously. Find the probability that the first die shows a composite number, second die shows an odd number and third die shows an odd prime number.

**Sol:** Required probability = probability of first die shows a composite number $x$ probability of second die shows an odd number $x$ probability of third die shows an odd prime number

$$= \frac{2}{6}\left(\frac{3}{6}\right)\left(\frac{2}{6}\right) = \frac{1}{18}.$$

**2.28.** Five fair coins are tossed together. Find the probability of getting exactly 4 heads.

**Sol:** The event of getting exactly 4 heads will be the combination of 4 heads and 1 tail.

Number of arrangements possible with this combination

$${}^5C_4 = \frac{5!}{4!\,1!} = 5$$

$$\therefore \text{Required probability} = \frac{5}{2^5} = \frac{5}{32}.$$

**2.29.** Five fair coins are tossed together. Find the probability of getting at most 3 heads.

**Sol:** Probability of getting at most 3 heads

= 1 – probability of getting at least 4 heads

= 1 – [(probability of getting 4 heads) + (probability of getting 5 heads)]

$$= 1 - \left[{}^5C_4\left(\frac{1}{32}\right) + \frac{1}{32}\right] = 1 - \frac{6}{32} = \frac{13}{16}.$$

**2.30.** A card is drawn from a well shuffled pack of cards. Find the probability of it being

(a) a jack.

(b) a black numbered card.

(c) a diamond.

(d) a red king.

**Sol:** Number of ways of selecting one card out of 52 cards = ${}^{52}C_1 = 52$.

(a) One jack can be drawn from 4 jacks in ${}^4C_1 = 4$ ways.

$$\therefore \text{The required probability} = \frac{{}^4C_1}{{}^{52}C_1} = \frac{4}{52} = \frac{1}{13}$$

(b) A pack of cards has 26 black cards of which 18 are numbered. A black numbered card can be drawn in ${}^{18}C_1 = 18$ ways.

$\therefore$ The required probability

$$= \frac{{}^{18}C_1}{{}^{52}C_1} = \frac{18}{52} = \frac{9}{26}$$

(c) A pack of cards has 13 diamonds.

A diamond can be drawn in ${}^{13}C_1 = 13$ ways.

$$\therefore \text{The required probability} = \frac{{}^{13}C_1}{{}^{52}C_1} = \frac{13}{52} = \frac{1}{4}$$

(d) A pack of cards has 2 red kings.

A red king can be drawn in ${}^2C_1 = 2$ ways.

$\therefore$ The required probability

$$= \frac{{}^2C_1}{{}^{52}C_1} = \frac{2}{52} = \frac{1}{26}$$

**2.31.** Two cards are drawn simultaneously from a well shuffled pack of cards. Find the probability of both being

(a) kings

(b) honours

(c) black honours

**Sol:** Two cards can be drawn from a pack of cards in ${}^{52}C_2$ ways.

(a) There are 4 kings in a pack of cards.

Two kings can be drawn in ${}^4C_2$ ways.

$$\text{The required probability} = \frac{{}^4C_2}{{}^{52}C_2}$$

(b) There are 16 honours in a pack of cards

$$\text{The required probability} = \frac{{}^{16}C_2}{{}^{52}C_2}.$$

(c) There are 8 black honours in a pack of cards.

$$\therefore \text{The required probability} = \frac{{}^8C_2}{{}^{52}C_2}$$

**2.32.** Two cards are drawn simultaneously from a pack of cards. Find the probability that both are hearts or both are diamonds.

**Sol:** Both cards should be hearts or diamonds.
These are mutually exclusive events.
Let $X$ denote the event of getting both hearts and $Y$ denote the event of getting both diamonds.

$$P(X) = \frac{^{13}C_2}{^{52}C_2} \text{ and } P(Y) = \frac{^{13}C_2}{^{52}C_2}$$

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

$$2\left(\frac{^{13}C_2}{^{52}C_2}\right) \quad [\because P(X \cap Y) = 0]$$

**2.33.** Two cards are drawn simultaneously from a pack of cards. Find the probability that both are black or both are queens.

**Sol:** Here, there are two events which are not mutually exclusive.
If both cards are black it is possible for them to be queens also.
Let $P$ and $Q$ be the events of selecting both black and both queens respectively.
The required probability
$= P(P \cup Q) = P(P) + P(Q) - P(P \cap Q)$

$$= \frac{^{26}C_2}{^{52}C_2} + \frac{^{4}C_2}{^{52}C_2} - \frac{^{2}C_2}{^{52}C_2}$$

**2.34.** Three cards are drawn in succession from a pack of cards with replacement. Find the probability of the first card drawn being a club, the second card drawn being black and the third card being a non-honour card.

**Sol:** Let $X$, $Y$ and $Z$ be the events of drawing a club, drawing a black card and drawing a non honoured cards respectively.
The cards are replaced.
$\therefore$ $X$, $Y$ and $Z$ are independent.
$\therefore$ required probability $= P(X \cap Y \cap Z)$

$$= P(X).\ P(Y).\ P(Z) = \frac{^{13}C_1}{^{52}C_1}\frac{^{26}C_1}{^{52}C_1}\frac{^{36}C_1}{^{52}C_1}$$

$$= \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{9}{13}\right) = \frac{9}{104}.$$

**2.35.** Three cards are drawn simultaneously from a pack of cards. Find the probability that one of them is red, another is a spade and the third is a club.

**Sol:** There are 26 red cards, 13 spades and 13 clubs in a pack of cards.

$$\therefore \text{ The required probability} = \frac{^{26}C_1\ ^{13}C_1\ ^{13}C_1}{^{52}C_3}.$$

**2.36.** Two balls are drawn in succession without replacement from a box containing 3 black balls and 6 blue balls. Find the probability that
(a) the first ball is black and the second ball is blue.
(b) both are blue.

**Sol:** The first ball can be drawn in $^{9}C_1 = 9$ ways and as it is not replaced, the second ball can be drawn in $^{8}C_1 = 8$ ways.
(a) $A$ black ball can be drawn in $^{3}C_1 = 3$ ways in the first draw. $A$ blue ball can be drawn in $^{6}C_1 = 6$ ways in the second draw.

$$\therefore \text{ The required probability} = \left(\frac{3}{9}\right)\left(\frac{6}{8}\right) = \frac{1}{4}$$

(b) $A$ blue ball can be drawn in $^{6}C_1 = 6$ ways in the first draw.
Since this ball is not replaced, a blue ball can be drawn in $^{5}C_1 = 5$ ways in the second draw.

$$\therefore \text{ The required probability} = \left(\frac{6}{9}\right)\left(\frac{5}{8}\right) = \frac{5}{12}$$

**2.37.** A bag contains 3 blue balls, 4 green balls and 5 red balls. A ball is drawn at random. Find the probability that it is
(a) not a green ball.
(b) a red ball.

**Sol:** $A$ ball can be drawn from 12 balls in $^{12}C_1 = 12$ ways.
(a) $A$ ball other than a green ball $(3 + 5 = 8)$ can be drawn in $^{8}C_1$ ways.
Hence, the probability of not drawing a green ball $= \frac{^{8}C_1}{^{12}C_1} = \frac{2}{3}$.

(b) $A$ red ball out of the 5 red balls present in the bag can be drawn in $^{5}C_1 = 5$ ways.
Hence, the probability of drawing a red ball

$$= \frac{^{5}C_1}{^{12}C_1} = \frac{5}{12}.$$

**2.38.** $A$ bag contains 3 blue balls, 4 green balls and 5 red balls. Three balls are drawn simultaneously at random. Find the probability that
(a) the balls are of different colour.
(b) two are blue and one is green.

(b) the balls are of the same colour.

**Sol:** Three balls can be drawn from 12 balls in ${}^{12}C_3$ ways.

(a) We can draw a blue ball out of 3 blue balls in ${}^3C_1$ ways, a green ball out of 4 green balls in ${}^4C_1$ ways and a red ball out of 5 red balls in ${}^5C_1$ ways.

$$\text{The required probability} = \frac{{}^3C_1\ {}^4C_1\ {}^5C_1}{{}^{12}C_3}$$

$$= \frac{3}{11}$$

(b) Two blue balls and one green ball can be drawn in ${}^3C_2.{}^4C_1$ ways

$$\therefore \text{ The required probability} = \frac{{}^3C_2\ {}^4C_1}{{}^{12}C_3} = \frac{3}{55}$$

(c) 3 blue balls or 3 green balls or 3 red balls can be drawn in ${}^3C_3$ or ${}^4C_3$ or ${}^5C_3$ ways Required

$$\text{probability} = \frac{{}^3C_3 + {}^4C_3 + {}^5C_3}{{}^{12}C_3} = \frac{3}{44}$$

**2.39.** Mohan had 5 fifty rupee notes, 3 hundred rupee notes and 4 five hundred rupee notes in his pocket. Two notes were drawn at random. Find the odds in favour of both being fifty rupee notes.

**Sol:** Total number of ways of drawing two notes from the pocket containing 12 notes is ${}^{12}C_2$ ways. The number of ways in which two fifty rupee notes can be drawn = ${}^5C_2$

The probability of choosing two fifty rupee notes

$$= \frac{{}^5C_2}{{}^{12}C_2} = \frac{5}{33}$$

$\therefore$ Odds in favour of the event = favourable ways : unfavourable ways = 5 : 28

**2.40.** There are 4 red, 6 blue and 8 white balls in a bag. Raju drew 2 balls from it one after another. Find the probability of the second ball drawn being blue if the first ball is

(a) replaced.

(b) not replaced.

**Sol:** (a) The required probability = (Probability of drawing any ball from the bag as the first ball). (Probability of drawing a blue ball from it as the second ball) $= 1\left(\frac{6}{18}\right) = \frac{1}{3}$

(b) There are two cases. The first ball is blue or it is not blue.

$\therefore$ The probability that the second ball is blue is

$$\frac{6}{18}\left(\frac{5}{17}\right) + \frac{12}{18}\left(\frac{6}{17}\right) = \frac{6}{18}\left(\frac{5}{17} + \frac{12}{17}\right) = \frac{6}{18} = \frac{1}{3}$$

**2.41.** Rohit participated in a game involving rolling an unbiased die. Each participant was given thrice as many rupees as the number which came up if it was prime and twice as many rupees as the number which came up, if it was composite. If 1 turned up, instead of getting money, the participant would have to pay ₹48. Find the expected value per roll in the long run for Rohit.

**Sol:** The outcomes and the corresponding money that Rohit would get are tabulated below.

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Money | –48 | 6 | 9 | 8 | 15 | |

$\therefore$ The expected value = $\Sigma P_i \times$ Monetary value

$$= \frac{1}{6}(-48 + 6 + 9 + 8 + 15 + 12) = \frac{1}{3}$$

**2.42.** *A* game involved a biased die. Each time the die shows up a score of 2, the participant was paid ₹4.50. For every other score he was paid ₹8. The dice was such that the score of 2 occurred twice as frequently as any other score. Find the amount that a person should be willing to pay as entry fee for each time he plays, if he neither wishes to gain nor lose in the long run.

**Sol:** In order for him to neither gain nor lose in the long run, the entry fee should be equal to the expected value of the game.

Let the probability of getting any number other than 2 be $P$.

Total probability $= P + 2P + P + P + P + P = 1$

$\Rightarrow 7P = 1$

$$P = \frac{1}{7},\ 2P = \frac{2}{7}$$

expected value $= \frac{2}{7}(4.50) + \frac{1}{7}(8 + 8 + 8 + 8 + 8) = ₹7.$

**2.43.** Sneha is known to speak the truth in 75% of the cases. She selects a natural number from 1 to 10 at random and says that it is a prime number. What is the probability that the number selected is actually a prime number?

**Sol:** Let $E$ denote the event that the number selected is a prime number and $A$ denote the event that Sneha says that the number is a prime number.

Then, $P(E) = \frac{4}{10}$ and $P(\overline{E}) = \frac{6}{10}$

$P\left(\frac{A}{E}\right) = \frac{3}{4}$ and $P\left(\frac{A}{\overline{E}}\right) = \frac{1}{4}$

From Bayes' rule,

$$P\left(\frac{E}{A}\right) = \frac{P(E)\,P\left(\frac{A}{E}\right)}{P(E)P\left(\frac{A}{E}\right) + P(\overline{E})\,P\left(\frac{A}{\overline{E}}\right)}$$

$$= \frac{\frac{4}{10}\left(\frac{3}{4}\right)}{\frac{4}{10}\left(\frac{3}{4}\right) + \frac{6}{10}\left(\frac{1}{4}\right)} = \frac{12}{18} = \frac{2}{3}$$

**2.44.** Akshay speaks the truth in 45% of the cases. In the rainy season, on each day there is a 75% chance of raining. On a certain day in the rainy season, Akshay tells his mother that it is raining outside. What is the probability that it is actually raining?

**Sol:** Let $E$ denote the event that it is raining and $A$ denote the event that Akshay tells his mother that it is raining outside.

Then, $P(E) = \frac{3}{4}$,

$P(\overline{E}) = \frac{1}{4}$

$P\left(\frac{A}{E}\right) = \frac{45}{100} = \frac{9}{20}$ and $P\left(\frac{A}{\overline{E}}\right) = \frac{11}{20}$

From Bayes' Rule,

$$P\left(\frac{E}{A}\right) = \frac{P(E)\,P\left(\frac{A}{E}\right)}{P(E)\,P\left(\frac{A}{E}\right) + P\left(\overline{E}\right)P\left(\frac{A}{\overline{E}}\right)}$$

$$= \frac{\frac{3}{4}\left(\frac{9}{20}\right)}{\frac{3}{4}\left(\frac{9}{20}\right) + \frac{1}{4}\left(\frac{11}{20}\right)} = \frac{27}{38} =$$

**2.45.** One number is selected at random from the set.
$S = \{2222, 4422, 6622, 2244, 4444, 6644, 2266, 4466, 6666\}$
$A$ is the event that the number starts with 22.
$B$ is the event that the number ends with 44.
$C$ is the event that the units digit and thousands digit are equal.
Which of the following is true?
(A) $A$ and $B$ are dependent.
(B) $B$ and $C$ are dependent.
(C) $A, B, C$ are mutually independent.
(D) $A, B, C$ are pair-wise independent.

**Sol:** $P(A) = \frac{3}{9} = \frac{1}{3}$ $\quad P(A \cap B) = \frac{1}{9}$

$P(B) = \frac{3}{9} = \frac{1}{3}$ $\quad P(A \cap C) = \frac{1}{9}$

$P(C) = \frac{3}{9} = \frac{1}{3}$ $\quad P(B \cap C) = \frac{1}{9}$

$P(A \cap B) = P(A)P(B)$

$P(B \cap C) = P(B)P(C)$

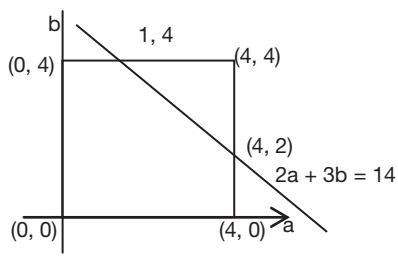
$P(C \cap A) = P(C)P(A)$

but $P(A)P(B)P(C) \neq P(ABC)$

They are pairwise independent but not mutually independent.

**2.46.** If $a$, $b$ are two positive numbers such that $a < 4$ and $b < 4$, find the probability that $2a + 3b < 14$.

**Sol:**




If we have a on the $x$-axis, and $b$-on the $y$-axis, $2a + 3b = 14$ is a line as shown above.
In the shaded region $2a + 3b < 14$
Required probability

$$= \frac{4(4) - \frac{1}{2}(3)(2)}{4(4)} = \frac{13}{16}$$

## EXERCISE-1

***Directions for questions 1 to 40*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** If $A$ and $B$ are two independent events such that $P(A) = 1/3$ and $P(B) = 2/7$, then the probability that both the events occur is

(A) $\frac{13}{21}$ (B) $\frac{5}{24}$ (C) $\frac{1}{7}$ (D) $\frac{2}{21}$

**2.** When '$n$' dice are rolled, the probability that all the dice show '1' is

(A) $\frac{1}{6^{n-1}}$ (B) $\frac{2}{6^n}$ (C) $\frac{1}{6^n}$ (D) $\frac{10}{6^n}$

**3.** When three dice are rolled, the number of possible cases that the total score is greater than 16 is _____.

**4.** When five coins are tossed, the number of possible outcomes are _____.

**5.** When four dice are rolled, the probability that the total score on the four dice is maximum is

(A) $\frac{1}{1296}$ (B) $\frac{1}{216}$ (C) $\frac{1}{432}$ (D) $\frac{5}{1296}$

**6.** The range of $P(A)$ is

(A) [0, 1] (B) (0, 1]
(C) [0, 1) (D) (0, 1)

**7.** The probability of an impossible event is

(A) 1 (B) 0 (C) $\frac{2}{3}$ (D) $\frac{1}{3}$

**8.** When 7 coins are tossed, the number of total outcomes is

(A) 67 (B) 64 (C) 128 (D) 256

**9.** The probability of a sure event is

(A) $\frac{6}{7}$ (B) $\frac{1}{7}$ (C) 0 (D) 1

**10.** The Sum of probability of all simple events in the Random experiment is

(A) < 1 (B) = 1 (C) >1 (D) None of these

**11.** When 7 coins are tossed the probability that all of them show up either heads or tails is

(A) $\frac{1}{2^7}$ (B) $\frac{5}{2^7}$ (C) $\frac{3}{2^7}$ (D) $\frac{1}{2^6}$

**12.** A bag contains 5 red balls, 3 green balls and 2 white balls. If one ball is drawn from the bag, then the probability that it is a green ball is _____.

**13.** If $P(A) = \frac{3}{4}$, $P(B) = \frac{7}{10}$ and $P(A \cap B) = \frac{3}{5}$, then $P(A \cup B)$ is

(A) $\frac{17}{20}$ (B) $\frac{13}{20}$ (C) $\frac{3}{7}$ (D) $\frac{7}{10}$

**14.** When a dice is thrown, the probability that the number on the dice is even or odd is _____.

**15.** A card is drawn from a pack of cards the probability that it is a red card is _____.

**16.** When two dice are rolled together, the probability that both the dice show odd number is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) $\frac{5}{7}$

**17.** If five unbiased coins are tossed together, then the probability of getting at most two heads is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$

**18.** When a dice is rolled, the probability that a prime number turns up is

(A) $\frac{5}{6}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) $\frac{1}{2}$

**19.** When a dice is rolled, the probability of getting a number greater than 6 is

(A) 0 (B) 1 (C) $\frac{1}{9}$ (D) $\frac{2}{9}$

**20.** If an unbiased coin is tossed four times, what is the probability that there would be exactly three heads?

(A) $\frac{3}{4}$ (B) $\frac{1}{4}$ (C) $\frac{3}{16}$ (D) $\frac{1}{16}$

**21.** A number is chosen at random from {1, 2, 3, 4, 5, 6, 7, 8, 9}. The probability that it will be a multiple of 3 is

(A) 1/3 (B) 2/3 (C) 3/4 (D) 5/9

**22.** If four cards are drawn at random from a well shuffled pack of cards, what is the probability that each card is an ace?

(A) $\frac{6}{{}^{52}C_4}$ (B) $\frac{4}{{}^{52}C_4}$ (C) $\frac{1}{{}^{52}C_4}$ (D) None of these

**23.** A card is selected at random from a pack of cards. What is the probability that it is a black king?
(A) 2/13 (B) 1/13 (C) 12/13 (D) 1/26

**24.** When a letter is selected at random from the English alphabet, the probability that it is a consonant is
(A) 5/26 (B) 10/13 (C) 21/26 (D) 7/13

**25.** In the above problem the probability that exactly one letter goes to a wrong addressed envelope is _____.

**26.** A letter is selected at random from the English alphabet. The probability that the letter is a vowel is

(A) $\frac{5}{26}$ (B) $\frac{3}{26}$ (C) $\frac{21}{26}$ (D) $\frac{23}{26}$

**27.** A card is selected at random from a pack of cards. What is the probability that it is a red queen?

(A) $\frac{1}{26}$ (B) $\frac{1}{13}$ (C) $\frac{3}{26}$ (D) $\frac{5}{26}$

**28.** A fair dice is rolled twice, the probability that the number obtained in the first trial is a multiple of the number obtained in the second trial is

(A) $\frac{4}{9}$ (B) $\frac{9}{10}$ (C) $\frac{7}{18}$ (D) $\frac{1}{2}$

**29.** When two distinct numbers are selected from the set {1, 3, 5, 7, 9}, the probability that the sum of the numbers is even is

(A) $\frac{1}{2}$ (B) 0 (C) 1 (D) $\frac{3}{4}$

**30.** When four dice are rolled, the probability that the total score on them is the minimum or the maximum is

(A) $\frac{1}{216}$ (B) $\frac{3}{245}$ (C) $\frac{1}{648}$ (D) $\frac{1}{324}$

**31.** If $A$ and $B$ are two mutually exclusive and collectively exhaustive events and the probability that the non-occurrence of $A$ is 3/4, then the probability of occurrence of $B$ is _____.

**32.** Four letters and four addressed envelopes are given to a dispatching clerk to dispatch. If he inserts the letters into the envelopes randomly, find the probability that all the letters are inserted into corresponding addressed envelopes.
(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{4!}$ (D) 0

**33.** If $A_1$, $A_2$, … $A_n$ are mutually exclusive and collectively exhaustive events of an experiment, then the sum of the probabilities of all the events is _____.

**34.** When two fair coins are tossed together, what is the probability that they both show the same face? _____

**35.** If the probability of occurrence of an event $A$ is $\frac{3}{7}$, then the probability of non-occurrence of $A$ is

(A) $\frac{4}{7}$ (B) $\frac{5}{7}$ (C) $\frac{6}{7}$ (D) $\frac{3}{7}$

**36.** $A$ and $B$ are two events. If $P(A) = \frac{3}{4}$, $P(B) = \frac{4}{7}$ and $P(A \cup B) = \frac{5}{6}$, then $P(A \cap B) =$

(A) $\frac{41}{84}$ (B) $\frac{10}{21}$ (C) $\frac{43}{84}$ (D) $\frac{1}{42}$

**37.** Three letters are selected at random from the English alphabet. What is the probability that all are vowels?

(A) $\frac{3}{260}$ (B) $\frac{1}{130}$ (C) $\frac{1}{260}$ (D) $\frac{1}{65}$

**38.** When 6 letters are placed into the corresponding 6 addressed envelopes, the probability that exactly five letters are placed into the corresponding envelopes is
(A) 3 (B) 1 (C) 0 (D) 2/5

**39.** A bag contains 5 red balls, 3 green balls and 2 white balls. If one ball is selected from the bag, the probability that it is not white is

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{3}{5}$ (D) $\frac{4}{5}$

**40.** If three dice are rolled together, the probability of the same number appearing on them is

(A) $\frac{1}{18}$ (B) $\frac{1}{36}$ (C) $\frac{5}{36}$ (D) $\frac{1}{9}$

## EXERCISE-2

***Directions for questions 1 to 40*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** Three fair dice are rolled simultaneously. What is the probability that at least one dice shows an even number?

(A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) $\frac{7}{8}$ (D) $\frac{3}{4}$

**2.** When 10 fair coins are tossed together, what is the probability of getting at most 3 heads?

(A) $\frac{11}{64}$ (B) $\frac{175}{1024}$ (C) $\frac{89}{256}$ (D) $\frac{15}{128}$

**3.** When two fair dice are rolled together, what is the probability that the product of the numbers obtained on the dice is a perfect cube?

(A) $\frac{1}{18}$ (B) $\frac{1}{12}$ (C) $\frac{1}{36}$ (D) $\frac{1}{9}$

**4.** Three natural numbers are selected simultaneously at random from the first five natural numbers. What is the probability that the sum of the numbers is 9?

(A) $\frac{1}{10}$ (B) $\frac{1}{5}$ (C) $\frac{3}{10}$ (D) $\frac{2}{5}$

**5.** When a fair dice is rolled once, what is the probability of getting an even number or a prime number?

(A) $\frac{2}{3}$ (B) $\frac{1}{4}$ (C) $\frac{5}{6}$ (D) $\frac{1}{2}$

***Directions for questions* 6 *and* 7:** These questions are based on the data given below.

Three six faced dice are rolled together. The probability that

**6.** No two dice show the same number on them is
(A) 7/12 (B) 5/9 (C) 1/36 (D) 5/12

**7.** Exactly two dice show the same number on them is
(A) 5/9 (B) 5/12 (C) 1/36 (D) 7/12

**8.** In rolling an unbiased dice, the probability of getting an even number or a number less than 4 is
(A) 2/3 (B) 2/5 (C) 1/3 (D) 5/6

**9.** The probability of getting no head when five fair coins are tossed, is _____.

**10.** The probability of getting at least three heads when six coins are tossed, is _____.

**11.** A five digit number is formed at random using the digits 1, 2, 3, 4, 7, without repetition. The probability that the number is even is
(A) 2/5 (B) 1/5 (C) 3/5 (D) 4/5

**12.** A fair dice is rolled twice. What is the probability that the number obtained in the first trial is less than that in the second trial?

(A) $\frac{7}{12}$ (B) $\frac{1}{6}$ (C) $\frac{5}{12}$ (D) $\frac{5}{6}$

**13.** When three fair dice are rolled together, what is the probability that exactly two of them will show number 6?

(A) $\frac{1}{36}$ (B) $\frac{5}{12}$ (C) $\frac{5}{72}$ (D) $\frac{5}{216}$

**14.** A person gets ₹50 when a coin shows heads, he loses ₹25 when the coin shows tail. If four coins are tossed, what is the probability that the person gets ₹50?
(A) 3/4 (B) 5/8 (C) 1/2 (D) 3/8

**15.** A park has 10 gates. A person can enter or leave the park through any gate. What is the probability that a person will enter the park through a gate and will leave the park through another gate?

(A) $\frac{9}{10}$ (B) $\frac{1}{10}$ (C) $\frac{1}{9}$ (D) None of these

***Directions for questions 16 to 20*:** These questions are based on the following information.

A box contains twelve bulbs of which two are defective. If three bulbs are chosen at random, find the probability that

**16.** All the bulbs are good.
(A) 9/22 (B) 5/11 (C) 6/11 (D) 7/22

**17.** Two bulbs are good and one is defective .
(A) 9/22 (B) 1/22 (C) 1/5 (D) 7/11

**18.** At least one bulb is good is _____.

**19.** At least one bulb is defective.
(A) 9/11 (B) 8/11 (C) 7/22 (D) 5/11

**20.** One bulb is good and the other two are defective.
(A) 7/22 (B) 1/22 (C) 1/11 (D) 2/5

**21.** Five digit numbers are formed using the digits 0 to 5 without repetition. The probability that the number so formed is divisible by 5 is _____.

**22.** If three cards are drawn at random, from a well shuffled pack of cards, then what is the probability that all of them are from the same suit?

(A) $\frac{4\left({}^{13}C_3\right)}{{}^{52}C_3}$ (B) $\frac{\left({}^{13}C_3\right)^3}{{}^{52}C_3}$

(C) $\frac{4\left({}^{13}C_3\right)^3}{{}^{52}C_3}$ (D) $\frac{{}^{13}C_3}{{}^{52}C_3}$

**23.** If three cards are drawn at random, from a well shuffled pack of cards, then what is the probability that all of them are from different suits?

(A) $\frac{13}{{}^{52}C_3}$ (B) $\frac{4(13)^3}{{}^{52}C_3}$

(C) $\frac{{}^{39}C_3}{{}^{52}C_3}$ (D) $\frac{(13)^4}{{}^{52}C_3}$

**24.** If three cards are drawn at random, from a well shuffled pack of cards, then what is the probability that two of them are numbered cards from the same suit and the 3rd number card is from a different suit?

(A) $\frac{{}^{9}C_2\ {}^{9}C_1}{{}^{52}C_3}$ (B) $\frac{{}^{9}C_2\ {}^{27}C_1}{{}^{52}C_3}$

(C) $\frac{4\left({}^{9}C_2\ {}^{27}C_1\right)}{{}^{52}C_3}$ (D) $\frac{4\left({}^{9}C_2\ {}^{9}C_1\right)}{{}^{52}C_3}$

**25.** If four dice are thrown together, then the probability that the product of the numbers showing up on them is a prime number is

(A) $\frac{1}{54}$ (B) $\frac{1}{108}$ (C) $\frac{1}{432}$ (D) 0

**26.** If four fair dice are thrown together, then the probability that the sum on them together is either 19 or 23.

(A) $\frac{4}{108}$ (B) $\frac{12}{108}$ (C) $\frac{9}{108}$ (D) $\frac{5}{108}$

**27.** If an unbiased coin is flipped 5 times, the probability that the same face does not show up in any three consecutive flips is

(A) $\frac{1}{2}$ (B) $\frac{5}{8}$ (C) $\frac{3}{8}$ (D) $\frac{7}{8}$

**28.** Two cards are drawn at random from a well-shuffled pack of cards. What is the probability that

(i) both are kings or both are queens?

(A) $\frac{14}{221}$ (B) $\frac{55}{221}$ (C) $\frac{2}{221}$ (D) $\frac{66}{221}$

(ii) both are jacks or both are blacks?

(A) $\frac{44}{221}$ (B) $\frac{14}{221}$ (C) $\frac{2}{221}$ (D) $\frac{55}{221}$

(iii) both are kings or both are spades?

(A) $\frac{14}{221}$ (B) $\frac{2}{221}$ (C) $\frac{55}{221}$ (D) $\frac{16}{221}$

**29.** Two fair dice are thrown one after the other. What is the probability that the first die shows up a number greater than that on the second die?

(A) $\frac{30}{36}$ (B) $\frac{6}{36}$ (C) $\frac{15}{36}$ (D) $\frac{3}{36}$

**30.** Six unbiased coins are tossed together. The probability that the number of heads exceeds the number of tails is

(A) $\frac{21}{64}$ (B) $\frac{11}{32}$ (C) $\frac{1}{6}$ (D) $\frac{5}{16}$

**31.** A basket contains 8 green balls, 11 white balls and 12 black balls. If 20 balls are picked at random, then the probability that they do not contain a green ball is

(A) $\frac{{}^{5}C_4}{{}^{15}C_4}$ (B) $\frac{{}^{23}C_{11}}{{}^{31}C_{20}}$ (C) $\frac{{}^{11}C_8}{{}^{31}C_8}$ (D) $\frac{{}^{11}P_8}{{}^{31}P_8}$

**32.** If $P(A) = 1/4$, $P(B) = 2/5$ and $P(A \cup B) = 1/2$, then $P(A \cap B)$ is _____.

**33.** A bag contains 9 white and 5 yellow balls, and another bag contains 6 white and 8 yellow balls. If one of the bags is selected at random and two balls are drawn at random from the bag, then the probability that both the balls are white is

(A) $\frac{15}{26}$ (B) $\frac{51}{182}$ (C) $\frac{40}{91}$ (D) $\frac{131}{182}$

**34.** Two independent witnesses $A$ and $B$ whose chances of speaking truth are 3 out of 4 and 7 out of 10 respectively agree in making a certain statement, then the probability that the statement is true is _____.

**35.** On a biased dice, any even number appears four times as frequently as any odd number. If the dice is rolled thrice what is the probability that the sum of the scores on them is more than 16?

(A) $\frac{26}{375}$ (B) $\frac{112}{375}$ (C) $\frac{26}{3375}$ (D) $\frac{112}{3375}$

**36.** Arun speaks truth in 70% of cases while Bhargav speaks truth in 65% of cases. The probability that they will contradict each other while stating the same fact is _____.

**37.** Ten people are to be seated in a row. The probability that two particular persons never sit together is _____.

**38.** Let $X$ and $Y$ be two events such that $P(X \cup Y) = \frac{3}{5}$; $P(\overline{X}) = \frac{2}{3}$; $P(\overline{Y}) = \frac{3}{5}$; then events $x$ and $y$ are

(A) independent and equally likely
(B) independent but not equally likely
(C) mutually exclusive and independent
(D) None of these

**39.** Rahul picks up a card at random from a set of cards numbered from 201 to 350. If the number on the card that he picks up is a multiple of 8, he wins ₹15. If it is a multiple of 13, he wins ₹40, and if it is a multiple of both 8 and 13, he wins ₹80. In the long run, what is the approximate amount in ₹ that Rahul will gain on an average if he has to pay ₹2 as a participation fee for each draw? _____

**40.** The odds against an event $A$ are 3 to 2 while the odds in favour of another independent event $B$ are 2 to 5. The probability that at least one of them would happen is

(A) 5/7 (B) 2/7
(C) 4/7 (D) 3/7

## EXERCISE-3

***Directions for questions 1 to 40*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** When two fair dice are rolled together, the probability that the sum of the numbers that appear on the dice is at least 9 is

(A) $\frac{1}{9}$ (B) $\frac{5}{18}$ (C) $\frac{1}{4}$ (D) $\frac{2}{9}$

**2.** When four cards are drawn simultaneously at random from a pack of cards, what is the probability of getting two diamonds and two spades?

(A) $\frac{{}^{26}C_4}{{}^{52}C_4}$ (B) $\frac{{}^{13}C_2}{{}^{52}C_4}$

(C) $\frac{{}^{13}C_2 \times {}^{13}C_2}{{}^{52}C_4}$ (D) $\frac{{}^{26}C_2 \times {}^{26}C_2}{{}^{52}C_4}$

**3.** A number is selected from the first 100 natural numbers. What is the probability that it is divisible by 8 or 12?

(A) 21/100 (B) 3/25
(C) 17/100 (D) 4/25

**4.** Ten books are arranged in a shelf in a row. If two books are selected at random from the shelf, the probability that the books are adjacent to each other is

(A) $\frac{4}{5}$ (B) $\frac{1}{5}$ (C) $\frac{2}{5}$ (D) $\frac{3}{5}$

**5.** In a leap year, what is the probability that the year has 53 Mondays and 53 Tuesdays?

(A) 2/7 (B) 1/7 (C) 3/7 (D) 4/7

**6.** If a number is chosen at random from the set {1, 2, 3,…, 100}, then the probability that the chosen number is a perfect cube is _____.

**7.** A two-digit number is formed by using the digits 0, 2, 3, 5, 8 without repetition. If a two digit number is selected at random out of these numbers, then the probability that it is a multiple of '2' is _____.

**8.** If ten students are to be seated in a row, then the probability that two particular students never sit together is _____.

**9.** A bag contains five red balls, three black balls and a white ball. If three balls are drawn from the bag, the probability that the three balls are of different colours is

(A) 23/28 (B) 5/28
(C) 3/28 (D) None of these

**10.** The probability of getting at least one 4 when two unbiased dice are thrown together is

(A) 5/18 (B) 11/36 (C) 1/3 (D) 13/36

**11.** The mean and variance of five observations are 10 and 11.6 respectively. If three of the observations are 8, 12 and 15, then the remaining two observations are

(A) 3, 12 (B) 8, 7 (C) 5, 10 (D) 4, 11

**12.** When three fair dice are rolled together, what is the probability that the three dice show the same number?

(A) $\frac{1}{6}$ (B) $\frac{1}{36}$ (C) $\frac{1}{216}$ (D) $\frac{5}{36}$

**13.** Two cards are drawn in succession without replacement from a pack of cards. What is the probability that the first card is a diamond and the second card is a spade?

(A) $\frac{13}{102}$ (B) $\frac{1}{12}$ (C) $\frac{1}{4}$ (D) $\frac{13}{204}$

**14.** A number is chosen at random from the natural numbers 1 to 100. What is the probability that the number has an odd number of factors?

(A) $\frac{2}{25}$ (B) $\frac{1}{100}$ (C) $\frac{9}{100}$ (D) $\frac{1}{10}$

**15.** If the average of the squares of 20 observations is 289 and the average of the observations is 15, then the standard deviation is

(A) 2 (B) 8 (C) 4 (D) $\sqrt{15}$

**16.** A bag contains 6 five rupee coins, 5 two rupee coins and 4 one rupee coins. If 5 coins are selected at random from the bag, then find the odds in favour of the draw yielding the minimum possible amount.

(A) 1 : 3002 (B) 5 : 2998
(C) 2 : 3003 (D) 5 : 3003

**17.** If A and $B$ are two events of an experiment such that $P(A \cup B) = \frac{3}{4}$, $P(A) = \frac{7}{20}$, then find $P(B)$ given that

(i) $A$ and $B$ are mutually exclusive. _____.

(ii) $A$ and $B$ are equally likely. _____.

(iii) $A$ and $B$ are independent events.

(A) $\frac{7}{13}$ (B) $\frac{8}{13}$ (C) $\frac{6}{13}$ (D) $\frac{2}{5}$

**18.** If a square is selected at random from a $8 \times 8$ chess board, what is the probability that the square is of dimension $1 \times 1$?

(A) 1 (B) 0 (C) $\frac{16}{51}$ (D) $\frac{17}{108}$

**19.** $A$ and $B$ pick up a card at random from a well shuffled pack of cards one after the other, replacing it every time till one of them gets a queen. If $A$ starts the game, then the probability that $B$ wins the game is _____.

**20.** In a biased coin, head occurs three times as frequently as tail occurs. If the coin is tossed 3 times, what is the probability of getting two heads?

(A) $\frac{3}{32}$ (B) $\frac{3}{64}$ (C) $\frac{9}{64}$ (D) $\frac{27}{64}$

**21.** From a bag containing 7 red and 9 green balls, 6 balls are drawn simultaneously at random. The probability that 4 of them are red and the rest are green is

(A) $\frac{300}{1001}$ (B) $\frac{45}{286}$ (C) $\frac{23}{143}$ (D) $\frac{150}{2002}$

**22.** From a box containing 20 bulbs, of which two-fifths are defective, three bulbs are chosen at random to fit into the three bulb holders in a room. The probability that the room gets lighted is

(A) $\frac{126}{285}$ (B) $\frac{217}{285}$ (C) $\frac{216}{285}$ (D) $\frac{271}{285}$

**23.** The odds against an event are 3 to 4 and the odds in favour of another independent event are 2 to 5. The probability that

(i) exactly one of them occurs is

(A) $\frac{23}{49}$ (B) $\frac{26}{49}$ (C) $\frac{6}{49}$ (D) $\frac{8}{49}$

(ii) at least one of them occurs is

(A) $\frac{34}{49}$ (B) $\frac{15}{49}$ (C) $\frac{23}{49}$ (D) $\frac{26}{49}$

**24.** The odds against an event are 4 to 5 and the odds in favour of another independent event are 3 to 7. The probability that

(i) both of them occur is

(A) 2/18 (B) 5/18
(C) 9/18 (D) 3/18

(ii) exactly one of them occurs is

(A) $\frac{43}{90}$ (B) $\frac{47}{90}$ (C) $\frac{53}{90}$ (D) $\frac{57}{90}$

**25.** If a number is selected randomly from the natural numbers 1 to 35. The probability that the number is divisible by 3 or 8 is

(A) $\frac{3}{7}$ (B) $\frac{1}{5}$ (C) $\frac{2}{5}$ (D) $\frac{3}{5}$

**26.** The probability that two particular persons do not have their birthdays on the same day (where they were born in a non-leap year) is

(A) $\frac{2}{365}$ (B) $\frac{363}{365}$ (C) $\frac{1}{365}$ (D) $\frac{364}{365}$

**27.** A frog fell into a box, which had four holes. Through one hole it can come out of the box in 4 minutes, through another hole it takes 3 minutes, through a third hole, it takes 2 minutes. But through the fourth hole, after travelling for 3 minutes, the frog falls back into the box. Every time it falls back into the box, the properties of the 4 holes get jumbled and the frog is equally likely to try

out any one of the 4 holes. Find the expected value for the time the frog takes to come out (in minutes). _____

**28.** An unbiased coin is tossed a fixed number of times. If the probability of getting 4 heads equals the probability of getting 7 heads, then what is the probability of getting 2 heads?

(A) $\frac{45}{1024}$ (B) $\frac{55}{1024}$ (C) $\frac{55}{2048}$ (D) $\frac{27}{1024}$

**29.** If $P(A \cap B) = 0.2$, $P(A) = 0.5$ and $P(A \cup B) = 0.7$, then $P(B) =$ _____.

**30.** If a line segment joining any two vertices of a regular hexagon is chosen at random, what is the probability of the line segment to be a diagonal? _____

**31.** On a biased dice every odd number appears four times as frequently as every even number. If the dice is rolled three times, what is the probability that the sum is 17 or more?

(A) $\frac{1}{3375}$ (B) $\frac{12}{625}$ (C) $\frac{13}{625}$ (D) $\frac{13}{3375}$

**32.** Jennifer picked a letter from the set of English alphabet and found it to be a vowel. What is the probability that it is E?

(A) $\frac{25}{26}$ (B) $\frac{3}{5}$ (C) $\frac{5}{26}$ (D) $\frac{1}{5}$

**33.** There are two boxes—one containing three white and three black balls and the other containing four white and three black balls. One box is picked at random and then one ball is picked from that, again at random. It turns out to be white. What is the probability that it is from the first box?

(A) $\frac{7}{15}$ (B) $\frac{4}{7}$ (C) $\frac{3}{7}$ (D) $\frac{8}{15}$

**34.** *A* and *B* pick up a card at random from a well-shuffled pack of cards, one after the other, replacing it every time till one of them gets an honour. If *A* begins the game, then the probability that the game ends with *B* is

(A) $\frac{9}{22}$ (B) $\frac{5}{22}$ (C) $\frac{17}{25}$ (D) $\frac{13}{22}$

**35.** Arjun picked a number from the first 30 natural numbers and found it to be prime. The probability that the number is 7 is

(A) $\frac{9}{22}$ (B) $\frac{1}{10}$ (C) $\frac{2}{11}$ (D) $\frac{9}{23}$

**36.** Three mountaineers Akil, Dikil, and Sunil are climbing up a mountain with their respective probability of reaching the summit being $\frac{2}{3}$, $\frac{5}{8}$ and $\frac{4}{7}$ respectively. What is the probability that

(i) none of them reach the summit?

(A) $\frac{1}{14}$ (B) $\frac{3}{56}$ (C) $\frac{5}{56}$ (D) $\frac{3}{14}$

(ii) exactly two of them reaches the summit?

(A) $\frac{37}{84}$ (B) $\frac{5}{12}$ (C) $\frac{19}{28}$ (D) $\frac{6}{17}$

(iii) at least two of them reaches the summit?

(A) $\frac{5}{21}$ (B) $\frac{3}{56}$ (C) $\frac{37}{84}$ (D) $\frac{19}{28}$

**37.** Two cards are drawn at random from a well shuffled pack of cards. Given that both are black, what is the probability that the cards have

(i) the same honour on them?

(A) $\frac{8}{325}$ (B) $\frac{4}{325}$

(C) $\frac{1}{65}$ (D) $\frac{2}{65}$

(ii) different honours and belong to different suits?

(A) $\frac{9}{325}$ (B) $\frac{18}{325}$

(C) $\frac{36}{325}$ (D) $\frac{12}{325}$

**38.** Five unbiased coins are tossed together. The probability that the number of heads exceeds the number of tails is _____.

**39.** The odds against an event *A* are 3 to 2 while the odds in favour of another independent event *B* are 2 to 5. The probability that at least one of them would happen is

(A) 5/7 (B) 2/7
(C) 4/7 (D) 3/7

**40.** Arun speaks truth in 70% of cases while Bhargav speaks truth in 65% of cases. The probability that they will contradict each other while stating the same fact is _____.

## EXERCISE-4

***Directions for questions 1 to 30*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the space provided.

**1.** In a purse are 30 coins, of which twenty were one-rupee coins and the rest were 50-paise coins. Twelve coins are picked simultaneously and are placed in a box. If a coin is now picked from the box, find the probability that it is a one-rupee coin.
(A) 4/7 (B) 1/2 (C) 2/3 (D) 5/6

**2.** *A*, *B* and *C* are three students who attend the same tutorial classes. The probability that on a particular day exactly one of *A* and *B* attends the class is 7/10; exactly one of *B* and *C* attends is 4/10; exactly one of *A* and *C* attends is 7/10. If the probability that all the three attend the class is 9/100, find the probability that at least one attends the class.
(A) 46/100 (B) 63/100
(C) 74/100 (D) 99/100

**3.** A box contains 3 oranges and 2 apples. In each draw a fruit is drawn at random. If it is an orange, an additional orange is removed (if present) from the box and in case it is an apple, the fruit is placed in the box and an additional apple is added to the box. Find the probability of getting an apple in the third such draw.
(A) 107/140 (B) 15/31
(C) 14/29 (D) 105/139

**4.** A box contains 10 balls numbered 1 through 10. Anju, Anshu and Ankitha pick a ball each, one after the other, each time replacing the ball. What is the probability that Anju picks a ball numbered less than that picked by Anshu, who in turn picks a lesser numbered ball than Ankitha?
(A) 3/25 (B) 1/6 (C) 4/25 (D) 81/400

**5.** Ten students stand in a row. If all possible arrangements of these ten students are equally likely, what is the probability there are exactly 3 students between two particular students Ram and Shyam?
(A) 1/10 (B) 2/5 (C) 2/15 (D) 1/15

**6.** A biased die has a probability of 1/4 of showing a 5, while the probability of showing any of 1, 2, 3, 4 or 6 is the same. If three such dice are rolled, what is the probability of getting a sum of at least 14 without getting a 6 on any die?
(A) 5/124 (B) 9/160 (C) 1/30 (D) 7/160

**7.** 6 boys and 2 girls stand in a circle. If all arrangements are equally likely, what is the probability that the two girls are separated by at least 2 boys in both directions?
(A) 1/3 (B) 3/7 (C) 3/8 (D) 1/4

**8.** Three persons A, B and C are to address a gathering along with 7 other persons. If the order in which the speakers address the gathering is selected at random, what is the probability that *A* delivers the speech before *B*, who in turn delivers his speech before *C*?
(A) 5/6 (B) 1/6 (C) 1/8 (D) 2/5

**9.** 12 marbles are selected at random from a large collection of white, red, green and yellow marbles. The number of marbles of each colour is unlimited. Find the probability that the selection contains at least one marble of each colour and a distinct number of marbles of each colour.

(A) $\frac{48}{455}$ (B) $\frac{72}{455}$ (C) $\frac{36}{455}$ (D) None of these

**10.** One number is selected at random from the following list of three-digit numbers: 235, 257, 337, 355, 435, 457, 537 and 555. With regards to the selected number, *A* is the event that the first digit is 2, *B* is the event that the second digit is 3 while *C* is the event that the third digit is 5. Which of the following is/are true?

I. *A*, *B* are independent

II. *A*, *C* are independent

III. *B*, *C* are independent

(A) I only (B) I, III only
(C) I, II, III (D) II, III only

**11.** A gambler throws a die. If 1 or 2 turns up, he is given ₹10. If 3 or 4 turns up, he is given ₹20 and if 5 or 6 turns up, he gets a chance to throw the dice once again. What is his expected gain in the long run? (in ₹) _____

**12.** A person is enticed into an enchanted garden, which has four gates that appear identical and are therefore impossible to differentiate. Through the first 3 gates it takes 1 min, 2 min and 3 min respectively to return to the normal world. The fourth is not an exit. A person walking through this takes 4 min to go through a maze that leads him back to the garden, where he started. Find the expected time (in minutes) the person takes to come out.

(A) 3 (B) 4 (C) $3\frac{1}{3}$ (D) $4\frac{1}{3}$

**13.** Three vertices of a regular hexagon are chosen at random. Find the probability that the triangle formed is equilateral. _____

**14.** In a game of dice, $A$, $B$ and $C$ roll an unbiased die one after the other in that order till one of them rolls a composite number. The person who rolls a composite number first is the winner. If $A$ begins the game, find the probability that $B$ wins.

(A) $\frac{9}{19}$ (B) $\frac{5}{19}$ (C) $\frac{4}{19}$ (D) $\frac{6}{19}$

**15.** A cubical die is marked '$x$' on three of its faces, '$y$' on two of its faces and '$z$' on the remaining face. If this die is rolled twice, what is the probability that the outcome of the first roll is the same as that of the second roll?

(A) $\frac{13}{36}$ (B) $\frac{1}{6}$ (C) $\frac{7}{18}$ (D) $\frac{11}{18}$

**16.** The letters of the word SERENDIPITY are arranged in all possible ways. What is the probability that an arrangement chosen at random ends in 'PITY'?

(A) $\frac{1}{2005}$ (B) $\frac{1}{3960}$ (C) $\frac{1}{7920}$ (D) $\frac{1}{7}$

**17.** Anil speaks the truth 3 out of 5 times. A coin is tossed, and Anil said that it showed heads. What is the probability that it actually showed heads? _____

**18.** Seven letters are to be placed into their corresponding addressed envelopes. If the task is done randomly what is the probability that exactly 4 letters are placed into their corresponding envelopes?

(A) $\frac{1}{144}$ (B) $\frac{1}{72}$ (C) $\frac{1}{36}$ (D) $\frac{1}{84}$

***Directions for questions* 19 *and* 20:** These questions are based on the information given below.

In a game involving tossing a coin, a player tosses a coin 3 times. A player is paid ₹30 if 3 tails show up, ₹20 if 2 tails show up and he pays ₹40 if no tails show up. In case of any other event no amount is paid to the player nor is he required to pay any amount.

**19.** If the coin is biased such that $P(H) = 1/4$ and $P(T) = 3/4$, in the long run, per game a player makes an average (approximately)

(A) profit of ₹21. (B) profit of ₹20.45.
(C) loss of ₹21. (D) loss of ₹20.45.

**20.** What is the expected gain to the player in the long run, if the coin used is a fair coin?

(A) ₹6.25 (B) ₹5.25
(C) ₹4.50 (D) ₹7.50

**21.** In an 8 cm × 8 cm chessboard, what is the probability that the length of the diagonal of a square (of any possible size) chosen at random, is $\sqrt{32}$ cm?

(A) $\frac{1}{8}$ (B) $\frac{25}{204}$ (C) $\frac{4}{51}$ (D) $\frac{8}{51}$

**22.** In an aptitude test, 10 multiple-choice questions were given. Each question has 4 choices, of which one is correct. Johnson, a candidate appearing for the test, attempted all the 10 questions by guessing randomly. What is the probability that exactly 3 questions are answered correctly?

(A) ${}^{10}C_3\left(\frac{3^7}{4^{10}}\right)$ (B) $\frac{3^7}{4^{10}}$

(C) $\frac{3}{10}$ (D) $\frac{1}{4}$

**23.** If a natural number is selected from the first 40 natural numbers, what is the probability that it satisfies the inequation $\frac{x}{5} + \frac{42.3}{x} > 6.5$?

(A) $\frac{4}{5}$ (B) $\frac{1}{5}$ (C) $\frac{5}{8}$ (D) $\frac{3}{5}$

**24.** If the integers $m$ and $n$ are chosen at random between 1 and 50, the probability that a number of the form $4^m + 4^n$ is divisible by 5 is

(A) $\frac{2}{3}$ (B) $\frac{5}{26}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

**25.** The odds in favour of a player being selected for the national team with three independent selectors are 4:3, 2:1 and 1:4 respectively. What is the probability that of the three selectors a majority will favour the player?

(A) $\frac{2}{5}$ (B) $\frac{10}{21}$ (C) $\frac{6}{35}$ (D) $\frac{2}{21}$

**26.** Sadikh, Akhil and Afroz start a game of dice. They throw the die by turns, first Sadikh, followed by Akhil, then Afroz, then once again Sadikh, Akhil and Afroz and so on till one of them wins. Sadikh will won if he throws an odd number, Akhil will win if he throws a prime and Afroz will win if he throws an even number. What is the probability that Sadikh wins?_____

**27.** $A$ and $B$ are two events of an experiment such that $P(\overline{A}) = 0.7$ and $P(\overline{A} \cap \overline{B}) = 0.2$. Find $P(B)$ given that

(i) $A$ and $B$ are mutually exclusive. _____

(ii) $A$ and $B$ are independent.

(A) $\frac{5}{7}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{8}$

**28.** In a non-leap year, what is the probability that in a group of 10 people, (none of whom was born on 29th February) at least two have the same birthday?

(A) $\frac{^{365}P_{10}}{365^{10}}$ (B) $\frac{^{364}P_{9}}{365^{10}}$

(C) $\frac{365^{9} - {}^{364}P_{9}}{365^{9}}$ (D) $\frac{^{365}P_{3}}{365^{10}}$

**29.** Two letters arrive—one from Hyderabad and the other from Adilabad. The name of the originating post office is marked on the envelopes with a rubber stamp. One of the envelopes is put into a paper shredder. On one of the shreds, just two consecutive letters are visible. Assume that every pair of consecutive letters has an equal chance of appearing on such a shred. What is the probability that the envelope that is put into the shredder is the one from Adilabad given that the consecutive letters visible are '*AD*'?

(A) $\frac{7}{23}$ (B) $\frac{2}{3}$ (C) $\frac{16}{23}$ (D) $\frac{3}{4}$

**30.** Ameer tossed a biased coin in which heads turn up on 80% of the occasions. In a game involving this coin, if Ameer is paid ₹ 25 per head and he has to pay ₹ 30 for a tail, in the long run, per game, Ameer makes an average

(A) loss of ₹ 8 (B) profit of ₹ 12

(C) loss of ₹ 20 (D) profit of ₹ 14

## ANSWER KEYS

### Exercise-1

| | | | | | |
|---|---|---|---|---|---|
| 1. (D) | 7. (B) | 13. (A) | 19. (A) | 25. 0 | 31. 0.75 |
| 2. (C) | 8. (C) | 14. 1 | 20. (B) | 26. (A) | 32. (C) |
| 3. 4 | 9. (D) | 15. 0.5 | 21. (A) | 27. (A) | 33. 1 |
| 4. 32 | 10. (B) | 16. (B) | 22. (C) | 28. (C) | 34. 0.5 |
| 5. (A) | 11. (D) | 17. (A) | 23. (D) | 29. (C) | 35. (A) |
| 6. (A) | 12. 0.3 | 18. (D) | 24. (C) | 30. (C) | 36. (A) |

### Exercise-2

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (C) | 7. (B) | 13. (C) | 19. (D) | 25. (B) | 29. (C) | 35. (D) |
| 2. (A) | 8. (D) | 14. (D) | 20. (B) | 26. (D) | 30. (B) | 36. 0.44 |
| 3. (B) | 9. 0.03125 | 15. (A) | 21. 0.36 | 27. (A) | 31. (D) | 37. 0.8 |
| 4. (B) | 10. 0.65625 | 16. (C) | 22. (A) | 28. (C), (D), (A) | 32. 0.1 | 38. (B) |
| 5. (C) | 11. (A) | 17. (A) | 23. (B) | | 33. (B) | 39. 3.06 |
| 6. (B) | 12. (C) | 18. 1 | 24. (C) | | 34. 0.875 | 40. (C) |

### Exercise-3

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (B) | 8. 0.8 | 15. (B) | 20. (D) | 27. 4 | 34. (A) | 38. 0.5 |
| 2. (C) | 9. (B) | 16. (B) | 21. (B) | 28. (C) | 35. (B) | 39. (C) |
| 3. (D) | 10. (B) | 17. (i) 0.4 (ii) 0.35 (iii) (B) | 22. (D) | 29. 0.6 | 36. (B), (A), (D) | 40. 0.44 |
| 4. (B) | 11. (C) | | 23. (B), (A) | 30. 0.6 | | |
| 5. (B) | 12. (B) | | 24. (D), (B) | 31. (D) | | |
| 6. 0.04 | 13. (D) | 18. (C) | 25. (C) | 32. (D) | 37. (B), (D) | |
| 7. 0.625 | 14. (D) | 19. 0.48 | 26. (D) | 33. (A) | | |

### Exercise-4

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (C) | 6. (D) | 11. 15 | 16. (B) | 21. (B) | 26. 4 | 30. (D) |
| 2. (D) | 7. (B) | 12. (C) | 17. 0.6 | 22. (A) | 27. (i) 0.5 (ii) (A) | 37. (C) |
| 3. (A) | 8. (B) | 13. 0.1 | 18. (B) | 23. (C) | | 38. (C) |
| 4. (A) | 9. (A) | 14. (D) | 19. (B) | 24. (C) | 28. (C) | 39. (D) |
| 5. (C) | 10. (C) | 15. (C) | 20. (A) | 25. (B) | 29. (C) | 40. (B) |

## SOLUTIONS

## EXERCISE-1

**1.** If $A$ and $B$ are independent events, then $P(A \cap B) = P(A)$.

$P(B) = \frac{1}{3} \times \frac{2}{7} = \frac{2}{21}$.

**2.** When '$n$' dice are rolled total possible outcomes are $6^n$.

There is only one favourable case that all dice shows the number '1'.

$\therefore$ Required probability is $\frac{1}{6^n}$

**3.** When three dice are rolled the possibilities that the total score is greater than 16 are as follows.

(5, 6, 6), (6, 6, 5) (6, 5, 6) and (6, 6, 6)

Hence, total number of possible cases are 4.

**4.** When five coins are tossed the possible outcomes are $2^5 = 32$.

**5.** When four dice are rolled, out comes are $6^4 = 1296$.

If total score on the dice is maximum, then all the dice shows 6 only.

This is occur only once.

$\therefore$ Required probability is $= \frac{1}{1296}$.

**6.** We know that $P(A)$ always lies in [0, 1].

**7.** Probability of an impossible event is 0.

**8.** When one coin is tossed, there outcomes, are 2.

When seven coins are tossed, there are $2^7$ or 128 outcomes.

**9.** The probability of a sure event is 1.

**10.** The sum of probability of all events = 1.

**11.** When 7 coins are tossed the total out comes are $2^7$.

There are only two favourable out comes that all are shows head or all shows tail.

$\therefore$ Required probability $= \frac{2}{2^7} = \frac{1}{2^6}$

**12.** Total number of balls is 5 + 2 + 3 = 10.

One ball can be drawn from 10 balls in ${}^{10}C_1$ ways.

One green ball can be drawn from 3 green balls in ${}^{3}C_1$ ways.

$\therefore$ The required probability is $= \frac{3}{10}$.

**13.** We know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{4} + \frac{7}{10}\frac{3}{5} = \frac{15+14-12}{20} = \frac{17}{20}$.

**14.** When a dice is rolled, the $n$ dice shows either an even or an odd number. It is a sure event

$\therefore$ Required probability = 1.

**15.** One card can be drawn from 52 cards in ${}^{52}C_1$ ways.

One red card can be drawn from 26 red cards in ${}^{26}C_1$ ways.

$\therefore$ The required probability $= \frac{26}{52} = \frac{1}{2}$.

**16.** The probability that one dice shows an odd number is $\frac{1}{2}$.

The probability that both the dice show an odd number is $\frac{1}{2}.\frac{1}{2} = \frac{1}{4}$.

**17.** Probability of getting at most two heads = probability of getting no head + probability of getting one head + probability of getting two heads.

$= \frac{{}^5C_0}{2^5} + \frac{{}^5C_1}{2^5} + \frac{{}^5C_2}{2^5} = \frac{1+5+10}{32} = \frac{16}{32} = \frac{1}{2}$

**18.** When a dice is rolled, there are 6 possible outcomes.

The prime numbers are 2, 3, 5, i.e., there are 3 primes.

$\therefore$ The required probability $= \frac{3}{6} = \frac{1}{2}$

**19.** When a dice is rolled the outcomes are {1, 2, 3, 4, 5, 6} with this dice getting a number greater than 6 is not possible.

$\therefore$ The required probability is 0.

**20.** As there should be exactly 3 heads, we have 3 heads and one tail HHHT which can appear in any of $\frac{4!}{3!} = 4$ ways.

Hence, favourable ways are 4 while total ways are $2 \times 2 \times 2 \times 2 = 16$.

Hence, the required probability is $\frac{4}{16} = \frac{1}{4}$.

**21.** Given set = {1, 2, 3, 4, 5, 6, 7, 8, 9}

From this set multiples of 3 are {3, 6, 9}

$n(E) = 3$

$n(S) = 9$

Hence, probability = $\frac{3}{9} = \frac{1}{3}$

**22.** Four cards can be drawn from a pack in ${}^{52}C_4$ ways.

Let $E$ be the event of each card being an ace.

This can be done in ${}^{4}C_4$, i.e., 1 way.

So $P(E) = \frac{1}{{}^{52}C_4}$

**23.** There are 2 black kings in a pack of cards.

$\therefore$ The required probability is $\frac{2}{52} = \frac{1}{26}$

**24.** There are 21 consonants in the English alphabet.

$\therefore$ The required probability is $\frac{21}{26}$

**25.** In the above problem if one letters goes to a wrong addressed envelop then at least some other letter also goes to a wrongly addressed envelop.

$\therefore$ It is an impossible event, hence, its probability = 0

**26.** In the English alphabet, there are five vowels.

The probability of selecting a vowel is $\frac{5}{26}$.

**27.** In a pack of cards, we have two red queens.

Probability of drawing a red queen is $\frac{2}{52} = \frac{1}{26}$.

**28.** When a fair dice is rolled twice, there are 36 possible outcomes

The favourable cases are

(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (2, 2), (4, 2), (6, 2), (3, 3), (6, 3), (4, 4), (5, 5), (6, 6)

$\therefore$ i.e., there are 14 cases.

Required probability = $\frac{14}{36} = \frac{7}{18}$

**29.** In the given set all the numbers are odd, any two numbers are selected their sum is always even.

$\therefore$ The required probability is 1.

**30.** When four dice are rolled, there are $6^4$ or 1296 outcomes.

When the outcomes are (1, 1, 1, 1) and (6, 6, 6, 6) the sum is the minimum and maximum respectively.

$\therefore$ There are 2 possibilities.

$\therefore$ The required probability is $\frac{2}{6^4} = \frac{1}{648}$.

**31.** Since $A$ and $B$ are mutual exclusive and exhaustive events, the non-occurrence of A is equal to the occurrence of $B$.

$P(B) = P(\overline{A}) = \frac{3}{4}$.

**32.** Letters can arranged in 4 envelops in 4! Ways but all the letters can be inserted into their respective envelops in only one way.

$\therefore$ The required probability = $\frac{1}{4!}$

**33.** The sum of all probabilities in an experiment is 1.

**34.** The possible outcomes are HH, HT, TH and TT of which the outcomes HT and TH are favourable.

$\therefore$ The required probability is $\frac{2}{4} = \frac{1}{2}$

**35.** The probability of occurrence of A is $\frac{3}{7}$, i.e., $P(A) = \frac{3}{7}$

$\therefore P(\overline{A}) = 1 - P(A) = 1 - \frac{3}{7} = \frac{4}{7}$.

**36.** We know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{3}{4} + \frac{4}{7} - \frac{5}{6} = \frac{63 + 48 - 70}{84} = \frac{41}{84}$.

**37.** Three letters can be selected from 26 letters in ${}^{26}C_3$ ways.

Three vowels can be selected from 5 vowels in ${}^{5}C_3$ ways

The probability of drawing three vowels is $\frac{{}^{5}C_3}{{}^{26}C_3} = \frac{1}{260}$

**38.** If five letters are placed into 5 corresponding envelopes, then the $6^{th}$ letter is also placed into the corresponding envelopes.

Exactly five letters cannot be placed into corresponding envelopes.

Probability = 0

**39.** Total number of balls = 5 + 3 + 2 = 10

The bag contains 2 white balls. The rest are not white.

$\therefore$ The probability of drawing a ball that is not white is

$= \frac{8}{10} = \frac{4}{5}$.

**40.** When three dice are rolled, there are 6(6) (6) or 216 outcomes.

The number of ways in which all the dices show the same number is 6.

$\therefore$ The required probability $= \frac{6}{216} = \frac{1}{36}$

## EXERCISE-2

**1.** We know that, the probability of a dice shows up an even number is $\frac{1}{2}$.

So, when three dice are rolled together, $P$(at least one dice shows up an even number)

$= 1 - P$(None of the dice shows an even number)

$= 1 - \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{7}{8}$

**2.** $P$(getting at most 3 heads)

$$= \frac{{}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3}{2^{10}}$$

$$= \frac{1+10+45+120}{2^{10}}$$

$$= \frac{176}{1024} = \frac{11}{64}$$

**3.** The total number of outcomes when two dice are rolled together is $6 \times 6 = 36$

The favourable combinations are:

(1, 1), (2, 4) and (4, 2)

Hence, the required probability

$= \frac{3}{36} = \frac{1}{12}$

**4.** The first 5 natural numbers are 1, 2, 3, 4 and 5.

Out of these, 3 numbers can be selected simultaneously in ${}^5C_3 = 10$ ways

The combinations which give a sum of 9 are: (2, 3, 4) and (1, 3, 5).

Hence, the required probability $= \frac{2}{10} = \frac{1}{5}$

**5.** When a fair dice is rolled, the possible outcomes are 1, 2, 3, 4, 5 and 6.

Out of these, the outcomes 2, 3, 4, 5 and 6 are favourable outcomes.

Hence, the required probability $= \frac{5}{6}$

***Solutions for questions 6 and 7:***

**6.** No two dice show same number would mean all the three should show different numbers. The first can fall in any one of the 6 ways. The second die can show a different number in 5 ways. The third should show a number that is different from the first and the second. This can happen in 4 ways.

Thus, $6 \times 5 \times 4 = 120$ favourable cases.

The total number of cases is $6 \times 6 \times 6 = 216$.

Hence, the required probability $= \frac{120}{216} = \frac{5}{9}$.

**7.** Total number of cases is $6 \times 6 \times 6 = 216$.

Out of three any two dice show same number. This can be happen in ${}^3C_2$ ways. Let first two dice show the same number. This can be happen in 6 ways. The third die shows a different number. This can be happen in 5 ways

$\therefore$ Total favourable cases $= {}^3C_2 . 6.5 = 90$

$\therefore$ Required probability $= \frac{90}{216} = 5/12$

**8.** The favourable cases are 1, 2, 3, 4 and 6. These are 5 in number. Hence, the required probability is $\frac{5}{6}$

**9.** The number of exhaustive events $= 2^5 = 32$.

Let $E$ be the event of getting zero heads. Then, $n(E) = 1$. So $P(E) = 1/32 = 0.03125$.

**10.** The number of exhaustive outcomes is $2^6 = 64$

Let $E$ be the event of getting at least 3 heads

$\therefore$ Number of favourable cases are ${}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$

$= 20 + 15 + 6 + 1 = 42$

$P(E) = \frac{42}{64} = \frac{21}{32} = 0.65625$

**11.** The number of five digits numbers formed using the digits 1, 2, 3, 4, 7 is 5! =120. If unit place is even then the numbers formed are even Consider five blanks —— Unit place can be filled in 2 ways (2 or 4). Remaining four places can be filled with four digits in 4! ways. Total number of five digit even numbers are = $2 \times 4!$

$\therefore$ Required probability = $\frac{2 \times 4!}{5!} = 2/5$

**12.** The total number of outcomes when two dice are rolled together = $6 \times 6 = 36$.

The favourable combinations are:

(1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 3), (2, 4), (2, 5), (2, 6)

(3, 4), (3, 5), (3, 6)

(4, 5), (4, 6)

(5, 6)

Hence, the required probability = $\frac{15}{36}$

$= \frac{15}{12}$

**13.** The total number of possible outcomes when three fair dice are rolled together = $6 \times 6 \times 6$

The favourable combinations in which exactly two dice show the number 6 are listed below:

| I | II | III |
|---|---|---|
| 6 | 6 | 1, 2, 3, 4, 5 |
| 6 | 1, 2, 3, 4, 5 | 6 |
| 1, 2, 3, 4, 5 | 6 | 6 |

So, there are a total of 15 favourable cases.

Hence, the required probability = $\frac{15}{6 \times 6 \times 6} = \frac{5}{72}$

**14.** When four coins are tossed, only if two coins show heads, and two coins show tails, the person gets ₹50. $\therefore$When four coins are tossed the probability of getting two heads is $\frac{{}^4c_2}{2^4} = \frac{6}{16} = \frac{3}{8}$

**15.** The total number of ways of entering and leaving the park = $10 \times 10 = 100$.

The favourable number of ways in which a person enters the park through a gate and leaves the park through another gate = $10 \times 9$

Hence, the required probability = $\frac{10 \times 9}{10 \times 10} = \frac{9}{10}$

***Solutions for questions 16 to 20*:**

As two bulbs are defective out of 12, there are 10 good bulbs. The total number of ways of choosing 3 bulbs out of 12 bulbs is ${}^{12}C_3 = \frac{12.11.10}{1.2.3} = 220$ ways.

**16.** Three good bulbs can be chosen in ${}^{10}C_3 = 120$ ways

Hence, the probability is $\frac{120}{220} = \frac{6}{11}$

**17.** Two defective and one good can be chosen in ${}^{10}C_2 \times {}^2C_1$ $= 45 \times 2 = 90$ ways.

Hence, the probability = $\frac{90}{220} = \frac{9}{22}$

**18.** There can be at the most two defective bulbs when we choose 3 bulbs. Thus it is certain that we get at least one good bulb. Hence, the probability is 1.

**19.** The possibilities are 1 defective and 2 good or 2 defective and 1 good. Number of ways = $({}^2C_1 \times {}^{10}C_2) + ({}^2C_2 \times {}^{10}C_1)$ $= 90 + 10 = 100$

Hence, the probability = $\frac{100}{220} = \frac{5}{11}$

**Alternate Method:**

$P$(atleast one bulb is defective) = 1 – $P$(no bulb is defective)

$1 - \frac{120}{220} = 1 - \frac{6}{11} = \frac{5}{11}$ (indirect way)

**20.** One good and two defective bulbs can be chosen in ${}^{10}C_1$ $\times {}^2C_2 = 10$ ways.

Hence, the probability is $\frac{10}{220} = 1/22$

**21.** The number of 5 digit numbers that can be formed using the digits 0, 1, 2, 3, 4, 5 is

$5 \times {}^5P_4 = 5 \times 120 = 600$

We know that, if the unit place in a number is either '0' or 5 the number is divisible by 5.

The number of numbers such that the unit place contain '0' is ${}^5P_4 = 120$.

The number of numbers such that the unit place contain '5' is $4.{}^4P_3 = 4 \times 24 = 96$

Favourable 5 digit numbers which are divisible by 5 is $120 + 96 = 216$

$\therefore$ Required probability $= \dfrac{216}{600} = \dfrac{9}{25}$

**22.** Three cards can be drawn from 52 cards in ${}^{52}C_3$ ways $n(S) = {}^{52}C_3$

Since we have 4 suits, i.e., Diamonds, Spades, Clubs and Hearts and each suit contains 13 cards. Three cards can be selected from 13 cards in ${}^{13}C_3$ ways.

$\therefore$ Number of favourable outcomes $= 4 \cdot {}^{13}C_3$

Required probability $= 4. \dfrac{{}^{13}C_3}{{}^{52}C_3}$

**23.** Three cards can be drawn from 52 cards in ${}^{52}C_3$ ways $n(S) = {}^{52}C_3$

From the four suits we can select 3 suits in ${}^4C_3$ ways.One card can be selected from each suit in ${}^{13}C_1 \cdot {}^{13}C_1 \cdot {}^{13}C_1$

$\therefore$ Number of favourable outcomes

$= {}^4C_3 \cdot {}^{13}C_1 \cdot {}^{13}C_1 \cdot {}^{13}C_1$

$\therefore$ Required probability $= \dfrac{4(13)^3}{{}^{52}C_3}$

**24.** Three cards can be drawn from 52 cards in ${}^{52}C_3$ ways $n(S) = {}^{52}C_3$

We know that, each suit contains 9 number cards. 2 number cards can be drawn from 9 cards in ${}^9C_2$ ways.

Third card can be selected from the remaining 27 number cards in ${}^{27}C_1$ ways.

$\therefore$ Total number of favourable outcomes is $4({}^9C_2)\ ({}^{27}C_1)$

Required probability $= \dfrac{4\left({}^9C_2\right)\left({}^{27}C_1\right)}{{}^{52}C_3}$

**25.** When four dice are thrown, the total number of outcomes is $n(s) = 6 \times 6 \times 6 \times 6 = 6^4$.

Let $E$ be the event, that the product of the numbers showing up is a prime number.

$\therefore$ The possible events are (1, 1, 1, 2) and (1, 1, 1, 3), (1, 1, 1, 5)

(1, 1, 1, 2) can be permuted in $\dfrac{4!}{3!} = 4$ ways

(1, 1, 1, 3) can be permuted in $\dfrac{4!}{3!} = 4$ ways

(1, 1, 1, 5) can be permuted in $\dfrac{4!}{3!} = 4$ ways

$\therefore$ Total number of favourable outcomes is 12.

$\therefore$ Required probability $= \dfrac{12}{6^4} = \dfrac{1}{108}$

**26.** With 4 dice thrown together, the sum has to be either 19 or 23.

The following are the possibilities for the sum to be 19.

6661; 6652; 6643; 6553; 6544; 5,5,5,4

Each of these combinations give rise to further possibilities of having these numbers distributed on the four dices. Each of 6661 and 5554 give rise to $\dfrac{4!}{3!} = 4$ different rearrangements, while each of 6652, 6643, 6553, 6544 give rise to $\dfrac{4!}{2!} = 12$ rearrangements. Hence, totally $4 \times 2 + 12 \times 4 = 56$ ways of having 19. For 23 the possibilities are $23 \rightarrow 6665$, i.e., $\dfrac{4!}{3!} = 4$.

Hence, the number of ways of having the sum to be 19 or 23 is 60.

Required probability $= \dfrac{60}{6 \times 6 \times 6 \times 6} = \dfrac{5}{108}$

**27.** When a coin is flipped 5 times, we have $2^5 = 32$, possible outcomes in all. The favourable cases would arise when there are 4 heads, 1 tail; 3 heads, 2 tails; 2 heads, 3 tails and 1 head, 4 tails.

In 4 heads 1 tail case, we have HHTHH as the only favourable sequence (1).

In 3 heads, 2 tails case, we have 10 cases in all of which HHHTT, TTHHH, THHHT are unfavourable sequences. Thus we get 7 cases from this. (7)

Similarly, we can count $7 + 1 = 8$ cases for 2 heads, 3 tails and 1 head, 4 tails.

Thus 16 cases are favourable out of 32.

Thus the probability $= 16/32 = 1/2$.

**28.** (i) $P$(both are kings or both are queens)

$$\frac{{}^4C_2 + {}^4C_2}{{}^{52}C_2} = \frac{6+6}{26 \times 51} = \frac{2}{221}$$

(ii) $P$(Both are Jacks or both are blacks)

$$\frac{{}^4C_2 + {}^{26}C_2\ {}^2C_2}{{}^{52}C_2} = \frac{6+325-1}{26 \times 51} = \frac{55}{221}$$

**NOTE**

There are exactly 2 cards which are common to Jacks and Blacks

(iii) $P$(Both are Kings or both are spades)

$$\frac{{}^4C_2 + {}^{13}C_2}{{}^{52}C_2} = \frac{6+78}{26\times 51} = \frac{14}{221}$$

**NOTE**

It may be noted that there is exactly one spade which is a king. Hence, if both cards have to be either both kings or both spades, the favourable ways are ${}^4C_2 + {}^{13}C_2$

**29.** When the two dice are thrown the possibilities are both the dice show the same number or one of them shows a number greater than the other.

Of the 36 ways that we have, there are 6 ways in which equal number appear on both the dice and in half, the remaining situations we have first die showing a large number then the second, i.e., in 15 ways.

Hence, the required probability is $\frac{15}{36}$

**30.** With six coins tossed, having more heads than tails happens with 6 heads, 0 tails; 5 heads, 1 tail; 4 heads, 2 tails. The number of ways being

HHHHHH 1 way

HHHHHT $\frac{6!}{5!} = 6$ ways

HHHHTT $\frac{6!}{4!\,2!} = 15$ ways

Hence, the favourable ways are 22, while total ways are $2^6 = 64$.

Hence, the required probability is $\frac{22}{64} = \frac{11}{32}$

**31.** Total number of balls = 8 + 11 + 12 = 31

20 balls can be selected from 31 balls in $= {}^{31}C_{20}$

There are 23 non green balls

20 balls can be selected from 23 non green balls in

$= {}^{23}C_{20}$ ways

$\therefore$ Required probability $= \frac{{}^{23}C_{20}}{{}^{31}C_{20}}$

$$= \frac{23\times 22\times 21\times 11\times 10\times 9\times 8\times 7\times 6\times 5\times 4\times 3\times 2\times 1}{31\times 30\times 29\times 28\times 27\times 26\times 25\times 24\times 23\times 22\times 21\times 3\times 2\times 1}$$

$$= \frac{11_{P_8}}{31_{P_8}}$$

**32.** $P(A \cap B)$

$= P(A \cup B) - P(B)$

$= \frac{1}{2} - \frac{2}{5} = 0.1$

**33.** Let $A$ and $B$ be the two bags.

The number of balls in the bag $A = 9 + 5 = 14$

The number of balls in the bag $B = 6 + 8 = 14$

The probability of selecting a bag is $\frac{1}{2}$

Case 1

Suppose bag $A$ is selected.

2 balls can be drawn from the 14 balls in ${}^{14}C_2$ ways.

2 white balls can be drawn from 9 white balls in ${}^9C_2$ ways.

$\therefore$ The probability of drawing two white balls from bag $A$ is $\frac{{}^9C_2}{{}^{14}C_2}$

Hence, the probability of selecting the bag $A$ and then drawing two white balls from it $= \frac{1}{2}\left(\frac{{}^9C_2}{{}^{14}C_2}\right)$

Case 2

Suppose bag $B$ is selected

Probability of drawing two white balls from bag $B$ is $\frac{{}^6C_2}{{}^{14}C_2}$

So, the probability of selecting the bag $B$ and then drawing two white balls from it $= \frac{1}{2}\left(\frac{{}^6C_2}{{}^{14}C_2}\right)$

$\therefore$ The required probability $= \frac{1}{2}\left(\frac{{}^9C_2}{{}^{14}C_2}\right) + \frac{1}{2}\left(\frac{{}^6C_2}{{}^{14}C_2}\right)$

$= \frac{36}{182} + \frac{15}{182} = \frac{51}{182}$

**34.** Let $X$ be the event that $A$ and $B$ agree and $T$ be the event of the statement is true.

We require $P(T/X) = \dfrac{P(X \cap T)}{P(X)}$

$(X \cap T)$ is the event that the statement is true and they agree, i.e., they both speak truth.

$\therefore P(X \cap T) = \dfrac{2}{4} \times \dfrac{7}{10} = \dfrac{21}{40}$

$X$ is the event that they agree. They agree either when both speak truth or both lie.

$\therefore P(X) = \dfrac{3}{4} \times \dfrac{7}{10} + \dfrac{1}{4} \times \dfrac{3}{10} = \dfrac{24}{40}$

$\therefore P(T/X) = \dfrac{21}{24} = \dfrac{7}{8}$

**35.** Let $p$ be the probability of getting an even number and $q$ be the probability of getting an odd number. Then, $p + q = 1$

Given, $p = 4q$

$\therefore 4q + q = 1 \Rightarrow q = \dfrac{1}{5} \Rightarrow p = \dfrac{4}{5}$

$\therefore$ For each even number, the probability of getting it is $\dfrac{4}{15}$ and for each odd number, probability is $\dfrac{1}{15}$

The favourable cases for getting the total score more than 16 are: 566, 656, 665 and 666.

Probability of the combination 566, 656 or 665 is

$\dfrac{1}{15}\left(\dfrac{4}{15}\right)\left(\dfrac{4}{15}\right) = \dfrac{16}{3375}$

$\therefore$ Probability of the combination 666 $= \dfrac{4}{12}\left(\dfrac{4}{15}\right)\left(\dfrac{4}{15}\right)$

$= \dfrac{64}{3375}$

$\therefore$ Required probability $= 3\left(\dfrac{16}{3375}\right) + \dfrac{64}{3375} = \dfrac{112}{3375}$

**36.** Probability of Arun speaking truth $= P(A) = \dfrac{70}{100} = \dfrac{7}{10}$

and that of Bhargav speaking truth $= P(B) = \dfrac{65}{100} = \dfrac{13}{20}$

They contradict when one speaks truth and the other lies, i.e., $P(\bar{A}\,B \text{ or } A\,\bar{B}) = \dfrac{7}{10} \times \dfrac{7}{20} + \dfrac{3}{10} \times \dfrac{3}{20}$

$= \dfrac{88}{200} = 0.44$

**37.** Ten people can be seated in a row in 10! ways Let $E$ be event that two boys are never together, $\bar{E}$ be the event that the two boys, are always together.

Let the two particular persons be always together. Considering the two people as a unit, the total number of persons = 9

9 persons can be arranged in 9! ways and the two people can be arranged among themselves in 2! ways

$n(\bar{E}) = 2! \times 9!$

$P(\bar{E}) = \dfrac{n(\bar{E})}{n(s)} = \dfrac{9! \times 2!}{10!} = \dfrac{1}{5}$

$\therefore$ Required probability $P(E) = 1 - P(\bar{E})$

$= 1 - \dfrac{1}{5} = \dfrac{4}{5}$

**38.** $P(X \cap Y) = P(X) + P(Y) - P(X \cup Y)$

$= -\dfrac{3}{5} + \dfrac{1}{3} + \dfrac{2}{5} = \dfrac{-9 + 5 + 6}{15} = \dfrac{12}{15}$

$= \dfrac{1}{3} \times \dfrac{2}{5} = P(X).\,P(Y)$

$\Rightarrow X, Y$ are independent but not equally likely.

**39.** The multiples of 8 in between 201 to 350 is 18.

The multiples of 13 in between 201 to 350 is 11.

The multiple of 8 and 13 in between 201 and 350 is 2.

$\therefore$ Probability of getting a number which is a multiple of 8 (and not 13) is $\dfrac{16}{150}$

Probability of getting a number which is a multiple of 13 (and not 8) is $\dfrac{9}{150}$

Probability of getting a number which is a multiple of both 8 and 13 is $\dfrac{2}{150}$

$\therefore$ Expected amount

$= \dfrac{16}{150} \times 15 + \dfrac{9}{150} \times 40 + \dfrac{2}{150} \times 80 = \dfrac{240 + 360 + 160}{150}$

$= \dfrac{760}{150} = 5.06$

Since he pays each time ₹2, he will make a profit of ₹5.06 – 2 = ₹3.06

**40.** $P(A) = \frac{2}{3+2} = \frac{2}{5}$

$P(\overline{A}) = 1 - \frac{2}{5} = \frac{3}{5}$

$P(B) = \frac{2}{2+5} = \frac{2}{7}$

$P(\overline{B}) = 1 - \frac{2}{7} = \frac{5}{7}$

$P$(atleast one of $A$, $B$ occur)

$= 1 - P$(none occurs)

$1 - P(\overline{A} \cap \overline{B})$

$= 1 - \frac{3}{5}, \frac{5}{7} = \frac{4}{7}$

## EXERCISE-3

**1.** The total number of outcomes when two dice are rolled together is $6 \times 6 = 36$.

The favourable combinations for getting a sum 9 are :

(3, 6), (4, 5), (5, 4) and (6, 3).

The favourable combinations for getting a sum 10 are :

(4, 6), (5, 5) and (6, 4)

The favourable combinations for getting a sum 11 are :

(5, 6) and (6, 5)

The favourable combinations for getting a sum 12 is:

(6, 6) only

Hence, $P$(getting a sum of at least 9 on the dice)

$= \frac{4+3+2+1}{6 \times 6} = \frac{10}{36} = \frac{5}{18}$

**2.** The total number of ways of drawing 4 cards from a pack $= {}^{52}C_4$.

The number of ways of selecting 2 diamonds and 2 spades $= {}^{13}C_2 \times {}^{13}C_2$

Hence, the required probability $= \frac{{}^{13}C_2 \times {}^{13}C_2}{{}^{52}C_4}$

**3.** Let $A$ be the event that the number is divisible by 8.

$\therefore n(A) = 12$ and $P(A) = \frac{12}{100}$

Let $B$ be the event that the number is divisible by 12

$\therefore n(B) = 8$ and $P(B) = \frac{8}{100}$.

$A \cap B$ is the event that the number is divisible by both 8 and 12, i.e., 24 .

$\therefore n(A \cap B) = 4$ and $P(A \cap B) = \frac{4}{100}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow = \frac{12}{100} + \frac{8}{100} - \frac{4}{100} = \frac{16}{100} = \frac{4}{25}$

**4.** The total number of ways of selecting two books from the shelf $= {}^{10}C_2 = 45$.

The favourable combinations of the positions of the books selected are : (1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (8, 9) and (9, 10).

Hence, the required probability

$= \frac{9}{45} = \frac{1}{5}$

**5.** In a leap year there are 366 days = 52 weeks + 2days. The is two days may be any one of the following possibilities {(sun, mon), (mon, tue), (tue, wed), (wed, thu), (thu, fri), (fri, sat) (sat, sun), i.e., = 7. If the two days are (mon, tue) then the year had 53 Mondays and 53 Tuesdays.

$\therefore$ the required probability $= \frac{1}{7}$

**6.** We have 1, 8, 27 and 64 as perfect cubes from 1 to 100. Thus, the probability of picking a perfect cube is

$\frac{4}{100} = \frac{1}{25} = 0.04$

**7.** The number of exhaustive cases $= 4 \times 4 = 16$.

The number of favourable cases to the given event are 10.

$\therefore$ Required probability $= 10/16 = 5/8 = 0.625$

**8.** Ten students can be seated in a row in 10! ways. Let $E$ be the event that the students do not sit together.

Then, $\overline{E}$ is the event of two students being seated together, then $P(\overline{E}) = 2(9!)/10! = 2/10 = 1/5$

$\therefore P(E) = 1 - 1/5 = 4/5 = 0.8$

**9.** Three balls can be drawn from a bag containing 9 balls in ${}^9C_3$ ways.

Let $E$ be the event of drawing different coloured balls.

$\therefore P(E) = \dfrac{{}^5C_1 \times {}^3C_1 \times {}^1C_1}{{}^9C_3} = \dfrac{15}{84} = \dfrac{5}{28}$

**10.** When two dice are rolled the total outcomes $= 6 \times 6 = 36$.

Let $E$ be the event that atleast one dice shows 4.

$\overline{E}$ is the event no dice shows 4

$\therefore n(\overline{E}) = 5 \times 5 = 25.$

$P(\overline{E}) = \dfrac{25}{36}$.

$P(E) = 1 - P(\overline{E}) = 1 - \dfrac{25}{36} = \dfrac{11}{36}$.

**11.** Let the two observations be $a$, $b$. The AM of $a$, $b$, 8, 12, 15 is $\dfrac{a+b+8+12+15}{5}$ $10\ a + b + 35 = 50 \Rightarrow a + b$

$= 15$ (1).

The variance is 11.6

$$\frac{(10-2)^2 + (10-b)^2 + (10-8)^2 + (12-10)^2 + (15-10)^2}{5}$$

$= 11.6$

$(10 - a)^2 + (10 - b)^2 + 4 + 4 + 25 = 58$ $(10 - a)^2 + (10 - b)^2 = 25$ (2)

Solving (1) and (2) we get $a = 5$; $b = 10$.

$\therefore$ The two observations are 5, 10

**12.** The total number of possible outcomes $= 6 \times 6 \times 6$

The favourable combinations of getting a triplet are:

(1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5) and (6, 6, 6).

Hence, the required probability $= \dfrac{6}{6 \times 6 \times 6} = \dfrac{1}{36}$

**13.** The total number of ways of drawing two cards in succession without replacement $= 52 \times 51$.

$\therefore P$ (selecting the diamond first and then secondly a spade)

$= \dfrac{13 \times 13}{52 \times 51} = \dfrac{13}{204}$

**14.** A number which is a perfect square will have an odd number of factors. Hence, the required probability

$= \dfrac{10}{100} = \dfrac{1}{10}$

**15.** Given, $\dfrac{\sum x_i^2}{20} = 289$ and $\dfrac{\sum x_i}{20} = 15$

$\therefore$ S.D $= \sqrt{\dfrac{\sum x_i^2}{n} - \left(\dfrac{\sum x_i}{n}\right)}$

$= \sqrt{\dfrac{\sum x_i^2}{20} - \left(\dfrac{\sum x_i}{20}\right)^2}$

$= \sqrt{289 - (15)^2}$

$= \sqrt{289 - 225}$

$= \sqrt{64} = 8$

**16.** Total number of coins in the bag $= 6 + 5 + 4 = 15$.

5 coins can be drawn from 15 coins in ${}^{15}C_5$ ways.

When 4 coins are one rupee coins and fifth coin is a two-rupee coin the amount will be minimum.

$\therefore$ Number of favourable cases $= {}^4C_4 \cdot {}^5C_1 = 5$

The probability that the amount will be minimum is $\dfrac{5}{{}^{15}C_5}$ $P(E) = \dfrac{5}{3003}$ $\therefore P(\overline{E}) = 1 - P(E)$

$= 1 - \dfrac{5}{3003} = \dfrac{2998}{3003}$

$\therefore$ Odds in favour of the required event $= 5 : 2998$

**17.** Given, $P(A \cup B) = \dfrac{3}{4}$; $P(A) = \dfrac{7}{20}$.

(i) Given: $A$ and $B$ are mutually exclusive, then $A \cap B = \phi$

$P(A \cup B) = P(A) + P(B)$ $P(B) = P(A \cup B) - P(A)$

$= \dfrac{3}{4} - \dfrac{7}{40} = \dfrac{15-7}{20} = \dfrac{8}{20} = \dfrac{2}{5}$ =.

(ii) If $A$ and $B$ are equally likely, then $P(A) = P(B)$

$\therefore P(B) = \dfrac{7}{20}$.

(iii) If $A$ and $B$ are independent events, then

$P(A \cap B) = P(A) \cdot P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(A).P(B)$

$P(A \cup B) = P(A) + P(B)\ (1 - P(A))$

$\dfrac{3}{4} = \dfrac{7}{20} + P(B)\left(1 - \dfrac{7}{20}\right)$ $\dfrac{3}{4} - \dfrac{7}{20} = P(B)\left(\dfrac{13}{20}\right)$

$\dfrac{8}{20} = p(B) = \dfrac{8}{20} \times \dfrac{20}{13} = \dfrac{8}{13}$

**18.** A square can be selected from a chess board in

$1^2 + 2^2 + \ldots + 8^2 = 204$ ways.

We have 64 squares of size $1 \times 1$.

$\therefore$ Required probability $= \frac{64}{204} = \frac{16}{51}$

**19.** Probability of drawing a queen card is $\frac{1}{13}$. Probability of drawing a card which is not a queen is. $\frac{12}{13}$

The winning sequence of $A$ can be $A$, $\bar{A}\bar{B}A$, $\bar{A}\bar{B}\bar{A}\bar{B}A$, … where $\bar{A}$ is the event of $A$ not drawing a queen card, $\bar{B}$ is the event of $B$ not drawing a queen card. According the above sequence $A$ may pick the card in 1st, 3rd, 5th … trials. The probability of $A$ winning the game is

$P(A) + P(\bar{A}).P(\bar{B}).P(A) + P(\bar{A}).P(\bar{B}).P(\bar{A}).P(\bar{B}).P(A) + \ldots$

$= P(A)(1 + P(\bar{A}).P(\bar{B}) + P(\bar{A}).P(\bar{B}).P(A) + \ldots)$

$= \frac{1}{13}\left(1 + \frac{12}{13}.\frac{12}{13} + \frac{12}{13}.\frac{12}{13}.\frac{12}{13}.\frac{12}{13} + \ldots\right)$

$= \frac{1}{13}\left(1\left(\frac{12}{13}\right)^2 + \left(\frac{12}{13}\right)^4 + \ldots\right)$

$= \frac{1}{13}\left(\frac{1}{1-\left(\frac{12}{13}\right)^2}\right)\left(\because s = \frac{a}{1-r}\right)$

$= \frac{1}{13} \times \frac{169}{169-144} = \frac{13}{25}$

$\therefore$ Probability that $B$ wins the game is $P(B) = 1 - P(A)$

$= 1\frac{13}{25} = \frac{12}{25}$

**20.** Given $P(H) = 3P(T)$

We know that $P(H) + P(T) = 1$ $3P(T) + P(T) = 1$ $P(T)$

$= \frac{1}{4}$

$P(H) = \frac{3}{4}$

Since the coin is tossed 3 times, two heads may occur in 1st and 2nd trial, 2nd and 3rd trial or 1st and 3rd trial.

$\therefore$ The required probability

$= \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} \times \frac{3}{4}$

$= \frac{9}{64} + \frac{9}{64} + \frac{9}{64} = \frac{27}{64}$.

**21.** Total number of balls = 7 + 9 = 16.

6 balls can be drawn from 16 balls in ${}^{16}C_6$ ways.

4 red balls can be drawn from 7 red balls in ${}^{7}C_4$ ways.

2 green balls can be drawn from 9 green balls in ${}^{9}C_2$ ways.

6 balls can be drawn of which 4 are red and 2 are green in ${}^{7}C_4.{}^{9}C_2$ ways.

Required probability $= \frac{{}^{7}C_4 {}^{9}C_2}{{}^{16}C_6} = \frac{45}{286}$

**22.** Total number of bulbs = 20

Number of defective bulbs $= \frac{2}{5}$ (20) = 8

Number of good bulbs = 12

Three bulbs can be selected from 20 bulbs in ${}^{20}C_3$ ways.

If at least one bulb is good, then the room is lighted.

Number of ways of drawing 3 defective bulbs from 8 bulbs is ${}^{8}C_3$

The probability that all the three bulbs are defective is

$\frac{{}^{8}C_3}{{}^{20}C_3} = \frac{14}{285}$

The probability that at least one bulb is good

= 1 – probability that no bulb is good $= 1 - \frac{14}{285} = \frac{271}{285}$

**23.** Let the events be $E_1$ and $E_2$

Odds against $E_1$ are 3 to 4

$\therefore P(E_1) = \frac{4}{7}$

Odd in favour of $E_2$ are 2 to 5

$\therefore P(E_2) = \frac{2}{7}$

(i) The probability that exactly one occurs

$= P(\bar{E}_1 E_2 \text{ or } E_1 \bar{E}_2) = \frac{4}{7} \times \frac{5}{7} + \frac{3}{7} \times \frac{2}{7} = \frac{26}{49}$

(ii) $P$(at least one occurs) = 1 – $P$(none occurs)

$= 1 - \frac{3}{7} \times \frac{5}{7} = \frac{34}{49}$

**24.** Let $A$ be the event

The data is tabulated below

| | Event | Complement | Total |
|---|---|---|---|
| A | 5 | 4 | 9 |
| B | 3 | 7 | 10 |

(i) both $A$, $B$ occur

$$P(A \cap B) = P(A)\ .P(B) = \frac{5}{9}.\frac{3}{10} = \frac{3}{18}$$

(ii) exactly one of them occurs is

$$P(\overline{A} \cap B) + P(A \cap \overline{B}) = P(\overline{A})\ P(B) + P(A)\ .P(\overline{B})$$

$$= \frac{4}{9}.\frac{3}{10} + \frac{5}{9}.\frac{7}{10} = \frac{12+35}{90} = \frac{47}{90}$$

**25.** One number can be selected from 1 to 35 in 35 ways.

There are 11 multiples of 3 and 4 multiples of 8 and 1 multiple of both 3 and 8, i.e., 24.

The number of favourable cases is 11 + 4 – 1 = 14

The probability that the number is a multiple of 3 or 8 is $= \frac{14}{35} = \frac{2}{5}$

**26.** The total outcomes = 365 × 365

Favourable cases that the two persons are not born on the same day= 365 × 364

∴ Required probability

$$= \frac{365 \times 364}{365 \times 365} = \frac{364}{365}$$

**27.** The possible time ($M$ minutes) and the corresponding probabilities are tabulated below.

| Time | 4 | 3 | 2 |
|---|---|---|---|
| Prob | 1/4 | 1/4 | 1/4 |
| Time | 7 | 6 | 5 |
| Prob | $1/4^2$ | $1/4^2$ | $1/4^2$ |

etc.

∴ The expected value of the time that the frog takes (in minutes) to come out is $E = E_1 + E_2 + E_3$ where

$$E_1 = 4\left(\frac{1}{4}\right) + 7\left(\frac{1}{4^2}\right) + 10\left(\frac{1}{4^3}\right)$$

$$E_2 = 3\left(\frac{1}{4}\right) + 6\left(\frac{1}{4^2}\right) + 9\left(\frac{1}{4^3}\right) + \ldots$$

$$E_3 = 2\left(\frac{1}{4}\right) + 5\left(\frac{1}{4^2}\right) + 8\left(\frac{1}{4^3}\right) + \ldots.$$

This is the sum of 3 AGP, in which $|r| < 1$.

Consider the $AP$ $a,\ a+d,\ a+2d,\ldots$

the $GP$ $b,\ br,\ br^2,\ldots$

and the AGP, $ab,\ (a+d)\ br,\ (a+2d)\ br^2,\ldots$

The sum to infinity of the AGP, is

$$\frac{ab}{1-r} + \frac{dbr}{(1-r)^2} = \frac{\text{First term of AGP}}{1-r} + \frac{d(\text{second term of GP})}{(1-r)^2}$$

$$E_1 = \frac{1}{3/4} + \frac{3/4^2}{3^2/4^2} = \frac{4}{3} + \frac{1}{3}$$

$$E_2 = \frac{3/4}{3/4} + \frac{3/4^2}{3^2/4^2} = \frac{3}{3} + \frac{1}{3}$$

$$E_3 = \frac{2/4}{3/4} + \frac{3/4^2}{3^2/4^2} = \frac{2}{3} + \frac{1}{3}$$

$$\therefore E = \frac{12}{3} = 4$$

**28.** $P(4 \text{ heads}) = P(7 \text{ heads})$

$${}^nC_4 (½)^4\ (½)^{n-4} = {}^nC_7\ (½)^7\ (½)^{n-7}$$

$${}^nC_4 = {}^nC_7 \qquad \Rightarrow n = 11$$

$$P(2 \text{ heads}) = {}^{11}C_2\ (½)^{11} = \frac{55}{2048}$$

**29.** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$0.7 = 0.5 + P(B) - 0.2$$

$$P(B) = 0.4$$

$$P(\overline{B}) = 1 - 0.4 = 0.6$$

**30.** A regular hexagon has 6 sides and $\frac{6(6-3)}{2} = 9$

Diagonals, i.e., totally 15 line segments. Hence, the probability of a line segment a diagonal $= \frac{9}{15} = \frac{3}{5}$

**31.** Given: $P(o) = 4P(e)$

We know that $P(o) + P(e) = 1$

$$\Rightarrow 5P(e) = 1;\ P(e) = \frac{1}{5},\ P(o) = \frac{4}{5}$$

When three dice are rolled the favourable cases for getting 17 or more is

(5, 6, 6) (6, 6, 5) (6, 5, 6) (6, 6, 6).

Probability getting $5 = \frac{4}{15}$

Probability of getting $6 = \frac{1}{15}$

$\therefore$ Probability is $3\left(\frac{4}{15}.\frac{1}{15}.\frac{1}{15}\right)+\frac{1}{5}.\frac{1}{15}.\frac{1}{15}$

$= \frac{12+1}{(15)^3} = \frac{13}{3375}$

**32.** Given that a vowel has been selected. Our focus is on the 5 vowels. Now the probability of this to be $E$ is $1/5$.

**33.** The data is tabulated below

| | Box 1 | Box 2 |
|---|---|---|
| White | 3 | 4 |
| Black | 3 | 3 |

The probability that the ball is white is $\frac{1}{2}\left(\frac{3}{6}\right)+\frac{1}{2}\left(\frac{4}{7}\right)$

The probability that the ball is from the first box is

$$\frac{\frac{1}{2}\left(\frac{3}{6}\right)}{\frac{1}{2}\left(\frac{3}{6}\right)+\frac{1}{2}\left(\frac{4}{7}\right)} = \frac{3(7)}{3(7)+6(4)} = \frac{7}{7+8} = \frac{7}{15}$$

**34.** The number of honours in a pack is 16.
The probability of drawing a honour card is $\frac{16}{52} = \frac{4}{13}$.

The probability of drawing a number card is

$1-\frac{4}{13} = \frac{9}{13}$.

Let $p$ be the probability of success, $q$ be the probability of failure

$p = \frac{4}{13}$, $q = \frac{9}{13}$

$B$ wins the game in the following cases.

(i) $A$ loses the game then $B$ wins the game.

(ii) $A$, $B$ lose the game and again $A$ loses the game then $B$ wins the game.

(iii) $A$, $B$ each lose the game for $n$ times. Then, $A$ loses again and then $B$ wins the game.

The probability is

$qp + qqqp + qqqqqp + \ldots$

$qp[1 + q^2 + q^4 + q^6 + \ldots]$

$$qp\left(\frac{1}{1-q^2}\right) = \frac{\frac{9}{13}.\frac{4}{13}}{1-\left(\frac{9}{13}\right)^2} = \frac{36}{88} = \frac{9}{22}$$

**35.** The prime numbers between 1 and 30 are

{2, 3, 5, 7, 11, 13, 17, 19, 23, 29}, i.e., there are 10 primes.

$\therefore$ The probability that the number is 7 is $= \frac{1}{10}$.

**36.** The probability that Akil reach the summit $= P(A) = 2/3$

$\therefore P\overline{A} = 1/3$

The probability that Nikil reach the summit is $P(N) = 5/8$

$\therefore P\overline{N} = 3/8$

The probability that Sunil reach the summit is $P(S) = \frac{4}{7}$

$\therefore P\overline{S} = \frac{3}{7}$

(i) Probability that none of them reaches the summit

$= P\left(\overline{A} \cap \overline{N} \cap \overline{S}\right) = P\left(\overline{A}\right) P\left(\overline{N}\right) P\left(\overline{S}\right)$

$= \frac{1}{3} \times \frac{3}{8} \times \frac{3}{7} = \frac{3}{56}$.

(ii) Probability that exactly two reaches the summit

$= P(A \cap S \cap \overline{N}) + P(A \cap N \cap \overline{S}) + P(N \cap S \cap \overline{A})$

$= P(A)\ P(S)\ P(\overline{N}) + P(A)\ P(N)\ P(\overline{S}) + P(N)\ P(S)$

$P(\overline{A}) = \frac{2}{3}.\frac{4}{7}.\frac{3}{8}.\frac{2}{3}.\frac{5}{8}.\frac{3}{7} + \frac{1}{3}.\frac{5}{8}.\frac{4}{7} = \frac{37}{84}$

(iii) Probability that at least two of them reaches the summit = Exactly two of them reach the summit + three of them reach the summit

$= \frac{37}{84} + \frac{5}{21} = \frac{57}{84} = \frac{19}{28}$

**37.** (i) Two black cards can be selected in ${}^{26}C_2$ ways.

A pack contain 8 black honours.

$\therefore$ The number ways of selecting two same honours is 4.

$\therefore$ Required probability $= \frac{4}{{}^{26}C_2} = \frac{4}{325}$

(ii) Suppose one card is king, then the second card can be any of the other 3 cards from the other suit and it can be selected in $4 \times 3 = 12$ ways. Since there are four honour cards.

$\therefore$ Required probability $= \frac{12}{325}$.

**38.** When five coins are tossed, the total possible out comes are $2^5 = 32$.

When head occurs for three are more times, then the number of heads exceeds the number of tails.

$\therefore$ Required probability = $P$(getting 3 heads) + $P$(getting 4 heads + $P$(getting 5 heads)

$$= \frac{{}^5C_3}{2^5} + \frac{{}^5C_4}{2^5} + \frac{{}^5C_5}{2^5} = \frac{10+5+1}{2^5} = \frac{16}{32} = \frac{1}{2}$$

$$= \frac{10+5+1}{2^5} = \frac{16}{32} = \frac{1}{2} =$$

**39.** $P(A) = \frac{2}{3+2} = \frac{2}{5}$

$P(\overline{A}) = 1 - \frac{2}{5} = \frac{3}{5}$

$P(B) = \frac{2}{2+5} = \frac{2}{7}$

$P(\overline{B}) = 1 - \frac{2}{7} = \frac{5}{7}$

$P$(atleast one of $A$, $B$ occur)

= 1 – $P$(none occurs)

$1 - P(\overline{A} \cap \overline{B})$

$= 1 - \frac{3}{5}.\frac{5}{7} = \frac{4}{7}$

**40.** Probability of Arun speaking truth = $P(A) = \frac{70}{100} = \frac{7}{10}$

and that of Bhargav speaking truth = $P(B) = \frac{65}{100} = \frac{13}{20}$

They contradict when one speaks truth and the other lies, i.e., $P(\overline{A}\,B$ or $A\,\overline{B}) = \frac{7}{10} \times \frac{7}{20} + \frac{3}{10} \times \frac{13}{20}$

$= \frac{88}{200} = 0.44$

## EXERCISE-4

**1.** The purse contains 50 paise and one-rupee coins in the ratio 1 : 2.

If some coins are selected at random from the box, the expected value of the ratio of the 50 paise coins and the 1 rupee coins is 1 : 2.

$\therefore$ If a coin is picked from the selection the probability of getting a one-rupee coin is 2/3.

**2.** Let $P(A)$, $P(B)$ and $P(C)$ represent the probability of $A$, $B$ and $C$ attending the class respectively.

Given: $P(A) + P(B) - 2P(A \cap B) = 7/10$

$P(B) + P(C) - 2P(B \cap C) = 4/10$

$P(C) + P(A) - P(A \cap C) = 7/10$

$P(A \cap B \cap C) = 9/100$

Now, $P$(at least one attending the class) = $P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= \frac{1}{2}[P(A) + P(B) - 2P(A \cap B) + P(B) + P(C) - 2P(B \cap C) + P(C) + P(A) - 2P(A \cap A)] + P(A \cap B \cap C)$

$P(A \cup B \cup C) = 1/2\ [7/10 + 4/10 + 7/10] + 9/100$

$= 9/10 + 9/100 = 99/100$

**3.** Listed below are the combinations and their respective probabilities, in which the third draw can yield an apple.

Case 1: $P(OOA) = 3/5 \times 1/3 \times 1$

Case 2: $P(AAA) = 2/5 \times 3/6 \times 4/7$

Case 3: $P(OAA) = 3/5 \times 2/3 \times 3/4$

Case 4: $P(AOA) = 2/5 \times 3/6 \times 3/4$

$\therefore$ Required probability = $1/5 + 4/35 + 3/10 + 3/20$

$= \frac{28+16+42+21}{140} = \frac{107}{140}$

**4.** Since Anju should pick a ball numbered lesser than that picked by Anshu and in turn Anshu should pick a ball numbered lesser than Anitha, the number of ways is given by the combinations of three distinct numbers that we can select from the numbers 1 to 10.

$\therefore$ Favourable ways = ${}^{10}C_3$

Total number of ways = $10 \times 10 \times 10 = 1000$

Required probability = ${}^{10}C_3/1000 = 3/25$

**5.** Number of ways of arranging Ram and Shyam in any two of the ten positions available is ${}^{10}P_2$.

Now, if three students are required to be in between Ram and Shyam, the positions that Ram and Shyam can occupy are given by (1, 5) (2, 6) (3, 7) (4, 8) (5, 9) or (6, 10) in each of these positions they can arrange themselves in 2 ways.

$\therefore$ Required probability $= 12/{}^{10}P_2 = 12/10 \times 2 = 2/15$

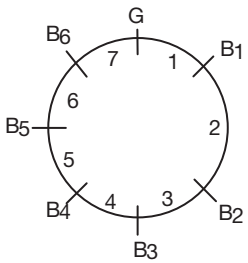
**6.** Without getting a 6 on any die the maximum sum that can be obtained is 15. Hence, we find the probability of getting the sum as 14 or 15. Given $P(5) = 1/4$ and $P(1) = P(2) = P(3) = P(4) = P(6) = x$.

$\therefore 5x + 1/4 = 1$ or $x = 3/20$

| Combinations | Arrangements | Probability |
|---|---|---|
| (5, 5, 5) | 1 | $1/4 \times 1/4 \times 1/4 = 1/64$ |
| (5, 5, 4) | 3 | $3(1/4 \times 1/4 \times 3/20)$ |
| | | $= 9/320$ |

$\therefore$ Required probability $= 1/64 + 9/320 = 14/320 = 7/160$

**7.** Let the 6 boys and a girl be positioned as shown in the figure.




Now the second girl can occupy one of the seven places, numbered 1, 2, …, 7.

To have at least 2 boys in between the 2 girls in both directions, the second girl can take the position 3, 4 or 5

$\therefore$ Required probability $= 3/7$

**8.** Irrespective of when $A$, $B$ and $C$ deliver their speech with respect to the other speakers, we look at the order only in terms of $A$, $B$ and $C$. The number of ways $A$, $B$ and $C$ can be arranged $= 3! = 6$. Out of these 6 ways there exists only one order which is favourable.

$\therefore$ Required probability $= 1/6$

**9.** Let $x_1$, $x_2$, $x_3$ and $x_4$ represent number of white, red green and yellow coloured marbles contained in the selection of 12 marbles

The number of ways of selecting 12 marbles is equal to the number of non–negative integral solutions of $x_1 + x_2 + x_3 + x_4 = 12$

i.e., total number of ways $= {}^{12+4-1}C_{4-1} = {}^{15}C_3 = 5(7)(13)$

The number of ways of selecting at least one of each colour and a distinct number of each colour is equal to the number of distinct positive integral solutions of $x_1 + x_2 + x_3 + x_4 = 12$

Without loss of generality, let $x_1 > x_2 > x_3 > x_4$

Let $x_2 = x_1 + \alpha$

$x_3 = x_2 + \beta$

$x_4 = x_3 + \gamma$ where $\alpha$, $\beta$ and $\gamma$ are positive integers

Now, we have, $4x_1 + 3\alpha + 2\beta + \gamma = 12$

Where $x_1 = 1$, $\alpha = 1$, $1 \le \beta \le 2$, $1 \le \gamma \le 3$

number of ways

$= 4!$ [coefficient of $x^{12}$ in $x^4 \cdot (x^3)\ (x^2 + x^4)\ (x + x^2 + x^3)$]

$= 4!$ [coefficient of $x^5$ in $(x^2 + x^4)\ (x + x^2 + x^3)$]

$= 4!$ [coefficient of $x^5$ in $(x^3 + x^4 + x^5 + x^5 + x^6 + x^7)$]

$= 4!$ [coefficient of $x^5$ in $(x^3 + x^4 + 2x^5 + x^6 + x^7$]

$= 4!\ (2) = 48$

Required probability $= \dfrac{48}{5(7)(13)} = \dfrac{48}{455}$

**10.** The 8 numbers are

235 435

257 457

337 537

355 555

$P(A) = \dfrac{2}{8} = \dfrac{1}{4}$ $\qquad$ $P(A \cap B) = \dfrac{1}{8}$

$P(B) = \dfrac{4}{8} = \dfrac{1}{2}$ $\qquad$ $P(A \cap C) = \dfrac{1}{8}$

$P(C) = \dfrac{4}{8} = \dfrac{1}{2}$ $\qquad$ $P(B \cap C) = \dfrac{2}{8} = \dfrac{1}{4}$

As $P(A \cap B) = P(A)\ P(B)$, $A, B$ are independent

As $P(A \cap C) = P(A)\ P(C)$, $A, C$ are independent

As $P(B \cap C) = P(B)\ P(C)$, $B, C$ are independent

**11.** The expected gain in the long run is

$$\frac{1}{3}(10) + \frac{1}{3}(20) + \frac{1}{3}\left[\frac{1}{3}(10) + \frac{1}{3}(20) + \frac{1}{3}(....)\right]$$

Regrouping the coefficients of 10 and 20, we get

$$10\left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + ...\right) + 20\left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27}\right)$$

$= 10(1/2) + 20(1/2) = 15$

**12.** The possible time (minutes) and the corresponding probabilities are tabulated below

| Time | 1 | 2 | 3 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| Probability | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\left(\frac{1}{4}\right)^2$ | $\left(\frac{1}{4}\right)^2$ | $\left(\frac{1}{4}\right)^2$ |

The expected value for the time that the person takes to come out (in minutes) is

$E = E_1 + E_2 + E_3$, where

$$E_1 = 1\left(\frac{1}{4}\right) + 5\left(\frac{1}{4^2}\right) + 9\left(\frac{1}{4^3}\right) + \ldots$$

$$E_2 = 2\left(\frac{1}{4}\right) + 6\left(\frac{1}{4^2}\right) + 10\left(\frac{1}{4^3}\right) + \ldots$$

$$E_3 = 3\left(\frac{1}{4}\right) + 7\left(\frac{1}{4^2}\right) + 11\left(\frac{1}{4^3}\right) + \ldots$$

$E_1$, $E_2$ and $E_3$ are 3 AGP's in which $|r| < 1$

$\therefore$ Sum of infinite terms of AGP $= \frac{ab}{1-r} + \frac{dbr}{(1-r)^2}$ where the AP is $a, a+d, a+2d \ldots$ and the GP is $b, br, br^2 \ldots$

For $E_1$, $a = 1$, $d = 4$, $b = \frac{1}{4}$, $r = \frac{1}{4}$

$$\therefore E_1 = \frac{1\left(\frac{1}{4}\right)}{1-\frac{1}{4}} + \frac{4\frac{1}{4^2}}{\left(1-\frac{1}{4}\right)^2} = \frac{1}{3} + \frac{4}{9} = \frac{7}{9}$$

$$E_2 = \frac{1\left(\frac{2}{4}\right)}{1-\frac{1}{4}} + \frac{4\frac{1}{4^2}}{\left(1-\frac{1}{4}\right)^2} = \frac{10}{9}$$

$$\text{And } E_3 = \frac{1\left(\frac{3}{4}\right)}{1-\frac{1}{4}} + \frac{4\left(\frac{1}{4}\right)^2}{\left(1-\frac{1}{4}\right)^2} = \frac{13}{9}$$

$$\therefore E = \frac{30}{9} = \frac{10}{3} = 3\frac{1}{3}$$

Alternately,

$$E = \frac{1}{4}(3) + \frac{1}{4}(2) + \frac{1}{4}(1) + \frac{1}{4}(4 + E)$$

$$\Rightarrow E = \frac{10}{3} \text{ min}$$

**13.** Let $(s)$ be the sample space associated with trial of forming triangles from the six vertices of regular hexagon $ABCDE$ by randomly selecting three vertices (fig)

The number of out comes:

$n(s) = {}^6C_3$

Let $\beta$ be the event that the selected triangle is equilateral.

From the fig; the possible equilateral triangles are $ACE$ and $BDF$.

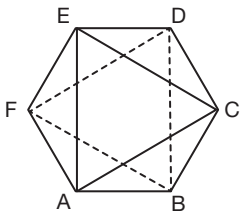
i.e., no. of favourable out comes

$n(\beta) = 2$

$\therefore$ Required probability:

$$P(\beta) = \frac{n(\beta)}{n(s)} = \frac{2}{{}^6C_3}$$

$$= \frac{2(6)}{6(5)(4)} = \frac{1}{10}$$




**14.** From the numbers, 1 to 6, 4 and 6 are composite. The other four are not.

Let $E$ be the event of a player winning the game in a particular roll.

$$\therefore P(E) = \frac{2}{6} = \frac{1}{3} \text{ and } P(\overline{E}) = \frac{2}{3}$$

The probability of $B$ winning, $P(B)$

$$= \frac{2}{3}\left(\frac{1}{3}\right) + \left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right) + \left(\frac{2}{3}\right)^7\left(\frac{1}{3}\right) + \left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right) + \ldots$$

$$= \frac{2}{3}\left(\frac{1}{3}\right)\left[1 + \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^6 + \left(\frac{2}{3}\right)^9 + \ldots\right]$$

$$= \frac{2}{9}\left[\frac{1}{1-\left(\frac{2}{3}\right)^3}\right] = \frac{2}{9}\left(\frac{27}{19}\right) = \frac{6}{19}$$

**Alternative Solution:**

First $A$ rolls, then $B$ and then $C$. The probability that $A$, $B$ and $C$ win in the first round is $\frac{1}{3}$,

$\frac{2}{3}\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2\frac{1}{3}$ respectively, i.e., $\frac{1}{3}, \frac{2}{9}, \frac{4}{27}$ or $\left(\frac{9}{27}, \frac{6}{27}, \frac{4}{27}\right)$

The probabilities that $A$, $B$, $C$ win in any round are given by 3 GPs, for which the first terms are 9/27, 6/27, 4/27 respectively and the common ratio is the same $(2/3)^3$ or

8/27 ($\therefore$ The probability that one of them wins (in some round) is $\frac{\frac{19}{27}}{1-\frac{8}{27}} = 1$)

$\therefore$ The probability that $B$ wins is $\frac{6}{9+6+4} = \frac{6}{19}$

**15.** Case (i): The probability that $x$ appears on both occasions is $3/6 \times 3/6 = 9/36$.

Case (ii): The probability that $y$ appears on both occasions is $2/6 \times 2/6 = 4/36$

Case (iii): The probability that $z$ appears on both occasions is $1/6 \times 1/6 = 1/36$.

Thus the required probability $= \frac{9}{36} + \frac{4}{36} + \frac{1}{36}$

$= 14/36 = 7/18$

**16.** As the last 4 places are fixed, the first seven places can be filled by $S, E, R, E, N, D, I$ in $\frac{7!}{2!}$ ways.

The total number of ways of arranging the letters $S, E, R, E, N, D, I, P, I, T, Y = \frac{11!}{2!\,2!}$

$\therefore$ The required probability $= \frac{7!\,2!}{11!/2!\,2!} = \frac{7!\,2!}{11}!$

$= \frac{2}{11(10)(9)(8)} = \frac{1}{3960}$

**17.** The outcomes of the toss, the way they are reported and the corresponding probability is tabulated below.

| Outcomes | $H\left(\frac{1}{2}\right)$ | $T\left(\frac{1}{2}\right)$ |
|---|---|---|
| Reported as | $H\left(\frac{3}{5}\right)$ | $H\left(\frac{2}{5}\right)$ |
| Probability | $\frac{3}{10}$ | $\frac{2}{10}$ |

$\therefore$ The required probability is $\frac{\frac{3}{10}}{\frac{3}{10}+\frac{2}{10}} = \frac{3}{5}$

**18.** 7 letters can be placed into 7 envelopes in 7! ways $n(S) = 7!$. Out of these 7, four letters are placed into corresponding envelopes, this can be done in ${}^7C_4$ ways. Then, the remaining three letters must be placed into wrong envelopes it can happen in 2 ways.

$\therefore$ The number of ways that four letters can be placed into exactly four corresponding envelopes is

$n(E) = {}^7C_4.2 = 35.\ 2. = 70.$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{70}{7!} = \frac{1}{72}$.

**19.** The expected value is given by

$E.V. = \Sigma$ (probability $\times$ monetary value)

$= (30)\ (3/4)\ (3/4)\ (3/4) + 20\ (3)\ (3/4)\ (3/4)\ (1/4) + (-40)\ (1/4)\ (1/4)\ (1/4)$

$= \frac{810+540-40}{64} = \frac{1310}{64} = 20.45$

$A$ gain of ₹ 20.45 on an average.

**20.** Since, the coin is fair, $P(H) = P(T) = 1/2$

$E.V. = 30\left(\frac{1}{2}\right)^3 + 20\left(3\left(\frac{1}{2}\right)^3\right) + (-40)\left(\left(\frac{1}{2}\right)^3\right)$

$= \frac{30+60-60}{8} = \frac{50}{8} = ₹\,6.25$

$\therefore$ It is a gain of ₹ 6.25.

**21.** The total number of squares in a $8 \times 8$ chess board is $1^2 + 2^2 + ... + 8^2 = 204$. For the diagonal to be of $\sqrt{32}$ cm we must chose the squares of dimensions $(4 \times 4)$, which are $5^2 = 25$ in number. Thus the probability is $\frac{25}{204}$

**NOTE**

In a $8 \times 8$ chess board the number of $n \times n$ squares is given by $(9-n)^2$; $1 \le n \le 8$.

**22.** The probability of guessing a correct answer is 1/4 and 3/4 for an incorrect answer. As exactly 3 questions are answered correctly, we may choose these 3 questions in ${}^{10}C_3$ ways.

Thus the required probability is

${}^{10}C_3\ (1/4)^3\ (3/4)^7 = {}^{10}C_3\left(\frac{3^7}{4^{10}}\right)$

**23.** One number can be selected from 40 numbers in ${}^{40}C_1$ ways. $n(S) = {}^{40}C_1$

Given $\frac{x}{5}+\frac{42.3}{x}>6.5$

$\frac{x}{5}+\frac{42.3}{x}>\frac{65}{10}$

$2x^2+423>65x$

$2x^2-65x+423>0$

$2x^2-47x-18x+423>0$

$x(2x-47)-9(2x-47)>0$

$(2x-47)\ (x-9)>0$

$x<9$ or $x>\frac{47}{2}$.

The number of natural numbers less than 9 is 8 and number of natural numbers greater than 23 and not exceeding 40 is 17.

$\therefore$ The favourable '$x$' values are $8+17=25$

$\therefore$ Required probability $=\frac{25}{40}=\frac{5}{8}$

**24.** Rem $(4^m/5)=1$ if $m$ is even

$=4$ if $m$ is odd

$\therefore$ Rem $\left(\frac{4^m+4^n}{5}\right)=2$ if both $m$, $n$ are even

$=0$ if one of $m$, $n$ is even and the other is odd $=3$ if both are odd.

As there are 24 even numbers and 24 odd numbers between 1 and 50 we get the following results for the number of ways in which the different remainders can be obtained.

| $R(4^m+4^n/5)$ | Number of ways |
|---|---|
| 2 | 24(24) |
| 0 | 2(24)(24) |
| 3 | (24)(24) |

$\therefore$ Probability that $R(4^m+4^n/5)=0$

is $\frac{2(24)(24)}{4(24)(24)}=\frac{1}{2}$

**25.** Given,

$P(A)$: $P(\overline{A})=4:3$

$P(A)=\frac{4}{7}$ and $P(\overline{A})=\frac{3}{7}$

$P(B)$ : $P(\overline{B})=2:1$

$P(B)=\frac{2}{3}$ and $P(\overline{B})=\frac{1}{3}$

$P(C)$ : $P(\overline{C})=1:4$

$P(C)=\frac{1}{5}$ and $P(\overline{C})=\frac{4}{5}$

Now, the majority of the selectors are favourable if any two are favourable and the third is unfavourable or all the three are favourable.

Hence, required probability $=\frac{4}{7}\left(\frac{2}{3}\right)\left(\frac{4}{5}\right)+\frac{3}{7}\left(\frac{1}{5}\right)\left(\frac{2}{3}\right)+\left(\frac{4}{7}\right)\left(\frac{1}{5}\right)\left(\frac{1}{3}\right)+\left(\frac{4}{7}\right)\left(\frac{2}{3}\right)\left(\frac{1}{5}\right)$

$=\frac{32+6+4+8}{105}=\frac{50}{105}=\frac{10}{21}$

**26.** Probability of Sadikh winning the game is $P(S)=\frac{1}{2}$

Probability of Akheel winning the game is $P(A)=\frac{3}{6}=\frac{1}{2}$

Probability of Afroz winning the game is $P(Z)=\frac{1}{2}$

And also

$P(\overline{S})=\frac{1}{2}$, $P(\overline{A})=\frac{1}{2}$, $P(\overline{Z})=\frac{1}{2}$

Required probability $=P(S)+P(\overline{S}).P(\overline{A}).P(\overline{Z}).P(S)+P(\overline{S}).P(\overline{A}).P(\overline{Z}).P(\overline{S}).P(\overline{A}).P(\overline{Z}).P(S)+\ldots$

$=\frac{1}{2}+\left(\frac{1}{2}\right)^4+\left(\frac{1}{2}\right)^7+\left(\frac{1}{2}\right)^{10}+\ldots$

$=\frac{\frac{1}{2}}{1-\frac{1}{8}}=\frac{\frac{1}{2}}{\frac{7}{8}}=\frac{1}{2}\left(\frac{8}{7}\right)=\frac{4}{7}$

**27.** (i) Given that

$P(\overline{A})=0.7$

$P(A)=0.3$

$P\left(\overline{A}\cap\overline{B}\right)=0.2$

$P(A\cup B)=1-P\left(\overline{A}\cap\overline{B}\right)=1-0.2=0.8$

Given:

$A$ and $B$ are mutually exclusive events

$\Rightarrow P(A\cup B)=P(A)+P(B)$

$0.8=0.3+P(B)$

$P(B)=0.5$

(ii) Given: $A$ and $B$ are independent

$P(A \cup B) = P(A) + P(B) - P(A)P(B)$

$0.8 = 0.3 + P(B) - 0.3P(B)$

$0.8 - 0.3 = P(B)\ (0.7)$

$\frac{5}{7} = P(B)$

**28.** We first find the probability that no two persons have the same birthday and subtract the result from 1. As leap years are excluded, there can be 365 different birthdays in a year. Second person also can have 365 birthdays and so on for the remaining persons also.

Hence, the total number of cases = $365^{10}$.

And the number of possible ways for none of the 10 birthdays to coincide is ${}^{365}P_{10}$.

$\therefore$ Required probability $= 1 - \frac{{}^{365}P_{10}}{365^{10}} = 1 - \frac{{}^{364}P_{9}}{365^{9}}$

$= \frac{365^{9-364}\,P_9}{365^9}$

**29.** The probability that the envelope is from Hyderabad ($H$) or Ahmedabad ($A$) is ½ in each case. But we need the probability of $A$, given a particular condition. Let $E$ be the event that the two consecutive letters are $AD$.

$P\left(\frac{E}{A}\right) = \frac{1}{8}, P\left(\frac{E}{B}\right) = \frac{2}{7}$

$\therefore P(E) = P(A)\ P\left(\frac{E}{A}\right) + P(B)\ P\left(\frac{E}{B}\right)$

$= \frac{1}{2}\left(\frac{1}{8}\right) + \frac{1}{2}\left(\frac{2}{7}\right) = \frac{1}{2}\left(\frac{23}{56}\right)$

By Bayes' Theorem

$P\left(\frac{B}{E}\right) = \frac{P(B)\,P\left(\frac{E}{B}\right)}{P(E)}$

$= \frac{\frac{1}{2}\left(\frac{2}{7}\right)}{\frac{1}{2}\left(\frac{23}{56}\right)} = \frac{16}{23}$

**30.** The probability of getting heads $= \frac{80}{100} = \frac{4}{5}$

The probability of getting tails $= \frac{20}{100} = \frac{1}{5}$

Expected amount $= \frac{4}{5}\ (25) - \frac{1}{5} \times 30$

$= 20 - 6 = ₹\,14$

# 3 Statistics

CHAPTER

## Learning Objectives

*In this chapter, you will*:

- Learn about various measures of central tendencies, such as:
  - Arithmetic mean
  - Geometric mean
  - Harmonic mean
  - Median
  - Mode
- Learn about various measures of dispersion, such as:
  - Range
  - Quartile deviation
  - Mean deviation
  - Standard deviation

STATISTICS deals with the collection, classification, presentation, analysis and interpretation of numeric data (quantitative data).

The simplest form of numerical data is a series of values of a certain variable. Such values can be given as individual numbers. Therefore, such data is called an individual series of raw data.

In such a series, some numbers could occur repeatedly. In such cases, it would be convenient to group these numbers. This grouping can be done in two ways. The numbers can be arranged in ascending (or descending) order and the number of times each number occurs (its frequency) can be specified. This results in a discrete series. Alternately, we may choose to ignore the actual number and record only the fact that it lies between certain limits. Such grouping results in a continuous series. The values between the limits are called intervals. For example, consider the marks obtained by the students of a class. This data has been represented in each of the ways below.

**Individual series**

60, 55, 70, 60, 70
65, 75, 65, 70, 55
58, 74, 67, 68, 73

**Discrete series**

| **Frequency** | 2 | 1 | 2 | 2 | 1 | 1 | 3 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Number** | 55 | 58 | 60 | 65 | 67 | 68 | 70 | 73 | 74 | 75 |

**Continuous series**

We denote the marks obtained by the students as

| **Intervals** | **Frequency** |
|---|---|
| $x \leq 55$ | 2 |
| $55 < x \leq 60$ | 3 |
| $60 < x \leq 65$ | 2 |
| $65 < x \leq 70$ | 5 |
| $70 < x \leq 75$ | 3 |

Collection of data, especially on a large scale, is typically an expensive process and requires big teams of field workers. Therefore, it needs careful planning. The primary consideration is the purpose for which the collected data will be used.

After data has been collected, classified and presented, it has to be analysed and interpreted. To analyse the data, the important features of the data have to be identified and relations between quantities (variables) have to be studied. Two important features of any data are the central tendency and dispersion. Instead of

thinking of all the numbers in a set, we would like to think of a typical number, the 'central' value. Two sets of numbers, which may have the same 'central' value, may differ in terms of variation of all the numbers in the set, i.e., the extent to which the numbers 'disperse' from this central value.

Various quantitative measures can be defined to measure these two properties, i.e., central tendency and dispersion.

In this chapter, we shall limit ourselves to an individual series and study such properties.

## ❒ MEASURES OF CENTRAL TENDENCIES

The measures we discuss here are as follows:

(A) Arithmetic mean
(B) Geometric mean
(C) Harmonic mean
(D) Median
(E) Mode

## ❒ Arithmetic Mean (A.M.)

Given $x_1, x_2, \ldots, x_n$ ($n$ individual items)

**FORMULA**

$$\text{A.M.} = \bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

or $$\bar{x} = \frac{\text{Sum of the observations}}{\text{The number of observations}}$$

**Examples**

(i) The arithmetic mean of (5, 8, 9, 14, –16) is
$$= \frac{5 + 8 + 9 + 14 - 16}{5} = \frac{20}{5} = 4$$

(ii) The arithmetic mean of (3, 4, 5, 6, 7, 8, 9, 10) is
$$= \frac{3+4+5+6+7+8+9+10}{8} = \frac{52}{10} = 5.2$$

**NOTE**

(a) The algebraic sum of deviations about the mean is 0 or $\Sigma(x - \bar{x}) = 0$.
(b) The arithmetic mean of two numbers $a$, $b$ is $\frac{a+b}{2}$.
(c) If $b$ = A.M. of ($a$, $c$), then $a$, $b$ and $c$ are in arithmetic progression.

## ❒ Geometric Mean (G.M.)

Given $x_1, x_2, \ldots, x_n$ ($n$ individual items such that all being positive.)

**FORMULA**

$$\text{G.M.} = (x_1 \times x_2 \times \ldots x_n)^{1/n}$$

or G.M. = $n^{\text{th}}$ root of the product of the numbers.

**Examples**

(i) The geometric mean of (10, 30, 90) is
$$= (10 \times 30 \times 90)^{1/3}$$
$$= (10 \times 30 \times 3 \times 30)^{1/3}$$
$$= (30 \times 30 \times 30)^{1/3} = 30$$

(ii) The geometric mean of (24, 20, 15, 45, 75) is
$$= (8 \times 3 \times 5 \times 4 \times 3 \times 5 \times 3^2 \times 5 \times 3 \times 5^2)^{1/5}$$
$$= (2^5 \times 3^5 \times 5^5)^{1/5}$$
$$= 2 \times 3 \times 5 = 30$$

**NOTE**

(a) Geometric mean is not very commonly used as it involves finding the $n^{\text{th}}$ root, and hence, it requires complex calculations for higher values of $n$.
(b) The geometric mean of two positive numbers $a$, $b$ is $\sqrt{ab}$.
(c) If $b$ = G.M. of ($a$, $c$), then $a$, $b$ and $c$ are in geometric progression.

## ❒ Harmonic Mean (H.M.)

Given $x_1, x_2, \ldots, x_n$ ($n$ individual observations such that none of them is equal to 0.)

**FORMULA**

$$\text{H.M.} = \frac{n}{1/x_1 + 1/x_2 + \ldots + 1/x_n}$$

**Examples**

(i) The harmonic mean of (1, 2, 3, 4, 5) is
$$= \frac{5}{\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}} = \frac{5 \times 60}{60 + 30 + 20 + 15 + 12}$$
$$= \frac{300}{137}$$

(ii) The harmonic mean of $\left(\frac{1}{1}, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}\right)$
$$= \frac{4}{1 + 3 + 9 + 27} = \frac{4}{40} = \frac{1}{10}$$

**NOTE**

(a) H.M. of two numbers $a$, $b$ is $\frac{2ab}{a+b}$.
(b) If $b$ = H.M. of ($a$, $c$), then $a$, $b$ and $c$ are in harmonic progression.
(c) For any two positive numbers $a$, $b$
(i) A.M. $\geq$ G.M. $\geq$ H.M.
(ii) $(\text{G.M.})^2 = (\text{A.M.})(\text{H.M.})$

## ❒ Median

Median is the magnitude of the 'middle-most' item in a series of value of variables, when the values have been arranged in order of their magnitude. When there are odd number of observations, the middle number, when the values are arranged in ascending or descending order, is the median. When there are even number of observations, the average of the two numbers, at the middle, when the values are arranged in ascending or in descending order, is the median.

**Examples**

(i) The median of 3, 7, 12, 16, 22 is 12.

(ii) The median of 4, 5, 9, 11, 14, 15 is $\frac{9+11}{2} = 10$.

**NOTE**

(a) The median divides the distribution into two equal parts.
(b) Median is suitable for qualitative data as well.

## ❒ Mode

It is the item which is 'most often' found in the given set of observations, i.e., the value occurring the highest number of times.

**Examples**

(i) For the observations, 2, 1, 4, 2, 6, 4, 3, 2, 8, 2, 2, 1, 4, 6, 7
Mode = 2

(ii) For the observations, 1, 2, 2, 3, 3, 3, 3 4, 4, 4, 4, 5, 5, 6
Modes are 3, 4.

(iii) Consider the observations 4, 7, 11, 35, 36, 26.
Here, no item occurs more than once. So, mode is ill-defined.

**FORMULA**

**Empirical Formula:**

Mode = 3 median – 2 mean

This formula is valid for the distributions that are moderately symmetric (symmetry being coincidence of mean, median and mode).

## ❒ MEASURES OF DISPERSION

The measures we discuss here are as follows:

(A) Range
(B) Quartile deviation
(C) Mean deviation
(D) Standard deviation/variance

## ❒ Range

Given $x_1, x_2, \ldots, x_n$ ($n$ individual observations)

**FORMULA**

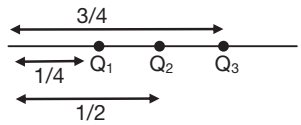
Range = maximum value – minimum value

**Examples**

(i) Range (2, 4, 5, 7, 8, 12, 13) = 13 – 2 = 11
(ii) Range (5, 4, 9, 1, 6, 12, 15) = 15 – 1 = 14

## ❒ Quartile Deviation (Q.D.) or Semi-interquartile Range

Quartiles are those values that divide the distribution into four equal parts when the values are arranged in ascending or descending order of magnitude.




$Q_1$ is called the first quartile, $Q_2$ is the middle quartile and $Q_3$ is the third quartile. The second quartile is also referred to as the median.

As the name semi-interquartile range itself suggests:

**FORMULA**

$\text{Q.D.} = \frac{Q_3 - Q_1}{2}$ (one-half the range of quartiles)

**For Calculation**

$Q_1 = \text{size of } \left(\frac{n+1}{4}\right)^{\text{th}} \text{ item}$

$Q_3 = \text{size of } 3\left(\frac{n+1}{4}\right)^{\text{th}} \text{ item}$

**Examples**

(i) Find the Q.D. of the observations 4, 9, 12, 15, 20, 23, 25.

**Solution** Since there are 7 terms, $n = 7$

$$Q_1 = \left(\frac{7+1}{4}\right)^{\text{th}} \text{ item} = 2^{\text{nd}} \text{ item} = 9$$

$$Q_3 = 3\left(\frac{7+1}{4}\right)^{\text{th}} \text{ item} = 6^{\text{th}} \text{ item} = 23$$

$$\therefore \text{Q.D} = \frac{Q_3 - Q_1}{2}$$

$$= \frac{23-9}{2} = \frac{14}{2} = 7$$

(ii) Find the Q.D. of 3, 7, 18, 24, 27, 31.

**Solution** The observations are 3, 7, 18, 24, 27, 31.

$$Q_1 = \left(\frac{6+1}{4}\right)^{\text{th}} \text{ item} = 1^3/_4{}^{\text{th}} \text{ item}$$

$$= 1^{\text{st}} \text{ item} + \frac{3}{4}(2^{\text{nd}} \text{ item} - 1^{\text{st}} \text{ item})$$

$$Q_1 = 3 + \frac{3}{4}(7-3) = 6$$

$$Q_3 = 3\left(\frac{6+1}{4}\right)^{\text{th}} \text{ item} = 5^1/_4{}^{\text{th}} \text{ item}$$

$$= 5^{\text{th}} \text{ item} + \frac{1}{4}(6^{\text{th}} \text{ item} - 5^{\text{th}} \text{ item})$$

$$Q_3 = 27 + \frac{1}{4}(31-27) = 28$$

$$\therefore \text{Q.D} = \frac{Q_3 - Q_1}{2} = \frac{28-6}{2} = 11$$

## ❒ Mean Deviation (M.D.)

The mean deviation is calculated about mean or median or mode. But, by default, mean deviation is about mean. Mean deviation is the average of deviations of each item in the data set from the mean.

**FORMULA**

$$\text{M.D.} = \frac{\sum_{i=1}^{n} |x_i - A|}{n}$$

A = mean/median/mode; $n$ = number of items

**Example**

Find the mean deviation of 2, 3, 9, 11, 15.

**Solution** Mean of the observations $\bar{x} = 40/5 = 8$

So, the mean deviation

$$\text{M.D.} = \frac{|2-8| + |3-8| + |9-8| + |11-8| + |15-8|}{5}$$

$$= \frac{6+5+1+3+7}{5} = \frac{22}{5} = 4.4$$

**NOTE**

(a) Mean Deviation of two numbers $a$, $b = \frac{|a-b|}{2}$.

(b) Mean deviation is based on each and every observation.

## ❒ Standard Deviation (S.D.)

It is the root mean squared deviation taken about the mean.

**FORMULA**

S.D. $= \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}}$, where $x_1, x_2, x_3, \ldots, x_n$ are the $n$ items given.

The expression $\sqrt{\frac{\sum(x_i - \bar{x})^2}{n}}$ also equals to $\sqrt{\frac{\sum x_i^2}{n} - (\bar{x})^2}$.

**Example**

Find the S.D. of (2, 5, 7, 10, 13, 17).

**Solution** Mean of the observations = 54/6 = 9

$$\text{S.D.} = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}}$$

$$= \sqrt{\frac{(-7)^2 + (-4)^2 + (-2)^2 + (1)^2 + (4)^2 + (8)^2}{6}}$$

$$= \sqrt{\frac{49+16+4+1+16+64}{6}} = \sqrt{\frac{150}{6}} = \sqrt{25} = 5$$

**NOTE**

(a) The square of the standard deviation is variance.

(b) The standard deviation is non-negative.

## SOLVED EXAMPLES

**3.01.** Find the mean, median and mode of the individual series 4, 6, 10, 12, 14, 14, 20 and 22.

**Sol:** Mean $= \dfrac{4+6+10+12+2(14)+20+22}{8} = 12.75$

**Median:** The given numbers are in a non-decreasing order.
The two middle numbers are 12 and 14.
$\therefore$ Median $= \dfrac{12+14}{2} = 13$

**Mode:** The observation 14 has the greatest frequency.
$\therefore$ 14 is the mode.

**3.02.** If the arithmetic mean of 25, 27, 31, 35, 39 and $x$ is 38, then find $x$.

**Sol:** $\bar{x} = \dfrac{25+27+31+35+39+x}{6} = 38$

$\Rightarrow x = 71$

**3.03.** Find the arithmetic mean and the median of the first 9 consecutive natural numbers.

**Sol:** The numbers are 1, 2, 3, 4, 5, 6, 7, 8 and 9.
The numbers are in A.P.
$\therefore$ Their mean is the mean of the first and the last terms, i.e., $\dfrac{1+9}{2} = 5$, which is the middle term.
Since the middle term is 5, the median is also 5.

**3.04.** Find the geometric mean of 2, 6, 18, 54 and 162.

**Sol:** Geometric mean $= \left[(2)\,(6)\,(18)\,(54)\,(162)\right]^{\frac{1}{5}}$

$= \left[(2)\,(2)\,(3)\,(2)(3)^2\,(2)\,(3)^3\,(2)\left(3^4\right)\right]^{\frac{1}{5}}$

$= \left[(2)^5\,\left(3^{10}\right)\right]^{\frac{1}{5}} = (2)\,(3^2) = 18$

**3.05.** Find the geometric mean of the observations 8, 12, 18 and 27.

**Sol:** Geometric mean $= \left[(8)\,(12)\,(18)\,(27)\right]^{\frac{1}{4}}$

$= \left[(8)\,(8)\left(\dfrac{3}{2}\right)(8)\left(\dfrac{3}{2}\right)^2\,(8)\left(\dfrac{3}{2}\right)^3\right]^{\frac{1}{4}}$

$= \left[(8)^4\left(\dfrac{3}{2}\right)^6\right]^{\frac{1}{4}} = 12\sqrt{\dfrac{3}{2}} = 6\sqrt{6}$

**3.06.** Find the harmonic mean of the observations 4, 8, 12, 16 and 20.

**Sol:** H.M. (4, 8, 12, 16, 20)

$= \dfrac{5}{\frac{1}{4}+\frac{1}{8}+\frac{1}{12}+\frac{1}{16}+\frac{1}{20}} = \dfrac{1200}{137}$

**3.07.** Show that the H.M., G.M. and A.M. of the positive numbers $p$ and $q$ must be in _____ (A.P./G.P./H.P.)

**Sol:** H.M. $(p, q) = \dfrac{2pq}{p+q}$

G.M. $(p, q) = \sqrt{pq}$

A.M. $(p, q) = \dfrac{p+q}{2}$

$[\text{G.M. } (p, q)]^2 = pq$
$[\text{A.M. } (p, q)]\,[\text{H.M. } (p, q)]\,pq = [\text{G.M. } (p, q)]^2$
$\therefore$ H.M. $(p, q)$, G.M. $(p, q)$ and A.M. $(p, q)$ are in geometric progression.

**3.08.** Find the range, quartile deviation and mean deviation for the data 5, 7, 12, 14, 20, 26 and 28.

**Sol:** Range = maximum value – minimum value
$= 28 - 5 = 23$
Quartile deviation:

$Q_1 = \left(\dfrac{n+1}{4}\right)^{\text{th}}$ item $= 2^{\text{nd}}$ item $= 7$

$Q_3 = \left(\dfrac{n+1}{4}\right)^{\text{th}}$ item $= 6^{\text{th}}$ item $= 26$

$\therefore$ Q.D. $= \dfrac{Q_3 - Q_1}{2} = 9.5$

Mean deviation:
Mean $(\bar{x})$ of the observations $= \dfrac{112}{7} = 16$

M.D. $= \dfrac{\sum |xi - \bar{x}|}{n}$

$= \dfrac{1}{7}[|5-16| + |7-16| + |12-16| + |14-16| + |20-16| + |26-16| + |28-16|] = \dfrac{52}{7}$

**3.09.** Find the standard deviation and variance of the observations 6, 10, 16, 20 and 24.

**Sol:** Mean of the observations

$= \dfrac{\sum x_i}{n} = \dfrac{6+10+16+20+24}{5} = 15.2$

$\dfrac{\sum x_i^2}{n} = \dfrac{36+100+256+400+576}{5} = \dfrac{1368}{5}$

$$\text{S.D.} = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2} = \sqrt{\frac{1368}{5} - 15.2^2}$$

$$= \sqrt{42.56} \Rightarrow \text{Variance} = (\text{S.D.})^2 = 42.56$$

**3.10.** Find the range and the quartile deviation of the following prices of 15 items in a general store: 7, 9, 15, 21, 27, 17, 33, 34, 39, 25, 11, 29, 23, 43 and 41.

**Sol:** Arranging the prices in ascending order, we have 7, 9, 11, 15, 17, 21, 23, 25, 27, 29, 33, 34, 39, 41 and 43.

Range = maximum price – minimum price
= 43 – 7 = 36

$$Q_1 = \left(\frac{n+1}{4}\right)^{\text{th}} \text{item} = \left(\frac{15+1}{4}\right)^{\text{th}} \text{item} = 15$$

$$Q_3 = 3\left(\frac{n+1}{4}\right)^{\text{th}} \text{item} = 12^{\text{th}} \text{item} = 34$$

$$\text{Q.D.} = \frac{34-15}{2} = 9.5$$

**3.11.** The median of a set of observations is 12. Their mean is 13. Find their mode.

**Sol:** Mode = 3 median – 2 mean
= 3(12) – 2(13) = 10.

**3.12.** Find the mean, median and mode of the individual series 2, 5, 9, 11, 13, 13, 17, 18.

**Sol:** $\text{Mean} = \bar{x} = \frac{\sum x_i}{n} = \frac{2+5+\ldots+18}{8}$

$= 88/8 = 11$

*Median:* The given numbers are in ascending order, with 11 and 13 as the middle terms.

$\therefore \text{Median} = \frac{11+13}{2} = \frac{24}{2} = 12$

*Mode:* As the observation 13 has the greatest frequency, 13 is the mode.

**3.13.** If the arithmetic mean of 28, 31, 36, 43, 30 and $K$ is 35, then find the value of $K$.

**Sol:** $\bar{x} = \frac{28+30+31+36+43+K}{6} = 35$

$\Rightarrow 168 + K = 210$

or $K = 210 - 168 = 4$

**3.14.** Find the arithmetic mean and the median of the first seven natural numbers.

**Sol:** The numbers are 1, 2, 3, 4, 5, 6, 7. As the numbers are in an A.P., the arithmetic mean is the average of first and last terms, i.e., 4.

Since, the middle term is 4, the value of the median is also equal to 4.

**3.15.** Find the geometric mean of 3, 6, 24, 48.

**Sol:** G.M. = (3, 6, 24, 48)
$= (3 \times 6 \times 24 \times 48)^{1/4}$
$= (3 \times 2 \times 3 \times 2^3 \times 3 \times 2^4 \times 3)^{1/4}$
$= (2^8 \times 3^4)^{1/4} = 2^2 \times 3 = 12$

**3.16.** Find the geometric mean of the observations 1, 2, 4, 8, …, 256.

**Sol:** G.M. $= (2^0 \cdot 2^1 \cdot 2^2 \ldots 2^8)^{1/9}$
$= (2^{0+1+2+\ldots 8})^{1/9} = (2^{36})^{1/9} = 2^4 = 16$

**3.17.** Find the harmonic mean of the observations 10, 20, 30, 40, 50.

**Sol:** H.M. (10, 20, 30, 40, 50)

$$= \frac{5}{\frac{1}{10}+\frac{1}{20}+\frac{1}{30}+\frac{1}{40}+\frac{1}{50}}$$

$$= \frac{5 \times 600}{60+30+20+15+12} = \frac{3000}{137}$$

**3.18.** Show that A.M., G.M. and H.M. of two positive numbers $a$ and $b$ are in geometric progression.

**Sol:** For two positive numbers $a$ and $b$,

$$\text{A.M.} = \frac{a+b}{2},\ \text{G.M.} = \sqrt{ab},\ \text{H.M.} = \frac{2ab}{a+b}$$

$$\text{Now, } (\text{G.M.})^2 = ab = \frac{a+b}{2} \times \frac{2ab}{a+b}$$

$\therefore (\text{G.M.})^2 = \text{A.M.} \times \text{H.M.}$

Since $(\text{G.M.})^2$ = A.M. × H.M., A.M., G.M. and H.M. of two positive numbers form a geometric progression.

**3.19.** Find the range, quartile deviation and mean deviation for the data: 7, 11, 12, 14, 18, 24, 26.

**Sol:** Range = maximum value – minimum value
= 26 – 7 = 19

Quartile deviation:

$$Q_1 = \left(\frac{n+1}{4}\right)^{\text{th}} \text{item} = 2^{\text{nd}} \text{item} = 11$$

$$Q_3 = 3\left(\frac{n+1}{4}\right)^{\text{th}} \text{item} = 6^{\text{th}} \text{item} = 24$$

$$\therefore \text{Q.D.} = \frac{Q_3 - Q_1}{2} = \frac{24-11}{2} = 6.5$$

Mean deviation:

Mean of the observations = 112/7 = 16

$$\text{M.D.} = \frac{\sum |x_i - \bar{x}|}{n}$$

$$= \frac{|7-16| + |11-16| + |12-16| + |14-16| + |18-16| + |24-16| + |26-16|}{7}$$

$$= \frac{9+5+4+2+2+8+10}{7} = \frac{40}{7} \simeq 5.71$$

**3.20.** Find the standard deviation and variance of the observations: 8, 12, 20, 24, 36.

**Sol:** Mean of the observations

$$\bar{x} = \frac{\sum x}{n} = \frac{8+12+20+24+36}{5} = 20$$

$$\frac{\sum x^2}{n} = \frac{64+144+400+576+1296}{5} = \frac{2480}{5} = 496$$

$$\therefore (\text{S.D.}) = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$= \sqrt{496-400} = \sqrt{96} = 4\sqrt{6}$$

and variance $= (\text{S.D.})^2 = (4\sqrt{6})^2 = 96$

**3.21.** Find the range and quartile deviation from the following prices of 11 items in a super market: 13, 21, 42, 19, 23, 64, 56, 71, 85, 25, 9.

**Sol:** Arranging the prices in ascending order, we get 9, 13, 19, 21, 23, 25, 42, 56, 64, 71, 85

Range = maximum price – minimum price

$= 85 - 9 = 76$

$$Q_1 = \left(\frac{n+1}{4}\right)^{\text{th}} \text{item} = \left(\frac{11+1}{4}\right)^{\text{th}} \text{item}$$

$= 3^{\text{rd}}$ item $= 19$

$$Q_3 = 3\left(\frac{n+1}{4}\right)^{\text{th}} \text{item} = 3\left(\frac{11+1}{4}\right)^{\text{th}} \text{item}$$

$= 9^{\text{th}}$ item $= 64$

$$\text{Q.D.} = \frac{64-19}{2} = \frac{45}{2} = 22.5$$

## EXERCISE-1

***Directions for questions 1 to 40*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the blank space provided.

**1.** The difference between the lower (or upper) limits of two successive classes is the
(A) lower bound.
(B) upper bound.
(C) mid-value of the class.
(D) size of the class, for a continuous distribution.

**2.** The mid-value of the class 27.5–37.5 is
(A) 32 (B) 32.5
(C) 33 (D) 33.5

**3.** If the mid-value of an inclusive class of size 7 is 9, then the class interval is
(A) 5–13 (B) 6–12
(C) 8–10 (D) None of these

**4.** The size of the exclusive class interval 24–34 is
(A) 9 (B) 11
(C) 10 (D) 24

**5.** The upper boundary of an inclusive type class 10–14 is
(A) 14 (B) 10
(C) 14.5 (D) 9.5

**6.** The mid-value of the class 45–65 is _____.

**7.** The arithmetic mean of $3a$, $3b$, $3c$ is
(A) $3(a+b+c)$ (B) $a+b+c$
(C) $\frac{a+b+c}{3}$ (D) $a^3+b^3+c^3$

**8.** The empirical relation between mean, median and mode is
(A) mean + median = mode
(B) mean = 3 median – 2 mode
(C) mode = 3 median – 2 mean
(D) None of these

**9.** The size of the class 12–22 is (the previous class is 2 – 12) _____.

**10.** For a symmetric distribution, which of the following is true?
(A) mode = 2 median – 3 mean
(B) mode = 3 median + 2 mean
(C) mean = median = mode
(D) mean = mode + median

**11.** For a moderately symmetric distribution, mode-median is equal to
(A) median – mean (B) mode – mean
(C) 3(median – mean) (D) 2(median – mean)

**12.** For a symmetric distribution, the mode is 24. The arithmetic mean of the distribution is
(A) 22 (B) 26
(C) 24 (D) Cannot be determined

**13.** The arithmetic mean and the sum of observations of individual data is 9 and 108, respectively. The number of observations of the individual data is
(A) 12 (B) 10
(C) 11 (D) 5

**14.** The sum of the deviations about mean of an individual data is equal to
(A) 0 (B) its arithmetic mean
(C) its mean deviation (D) its range

**15.** The median of the data 8, 12, 13, 17 and 19 is
(A) 12.5 (B) 13
(C) 13.5 (D) 6.5

**16.** The geometric mean of $a$, $b$ and $c$ is
(A) $(abc)^3$ (B) $abc$
(C) $\sqrt[3]{abc}$ (D) None of these

**17.** The mode of the data 1, 2, 1, 5, 6, 7, 8, 7, 1 is _____.

**18.** The range of the data 10, 8, 12, 23, 18, 35, 56, 82, 49, 76 is _____.

**19.** Find the geometric mean of the data 5, 75 and 9 _____.

**20.** The mean deviation of 30 and 40 is _____.

**21.** The geometric mean of the data 1, 3, 12 is
(A) $\sqrt{6}$ (B) 6
(C) $\sqrt[3]{36}$ (D) 3

**22.** If the range and the minimum value of the observations are 17 and 88, respectively, then the maximum value of the data is
(A) 100 (B) 105
(C) 71 (D) 110

**23.** The third quartile ($Q_3$) of the data 16, 21, 23, 25, 29, 32, 46, 48, 51, 53 and 54 is
(A) 51 (B) 48
(C) 29 (D) 53

**24.** The median of the first five prime numbers is
(A) 11 (B) 5
(C) 7 (D) 2

**25.** If the variance of the series $x_1, x_2, \ldots, x_n$ is $p$, then the standard deviation of the series $2x_1 + 3, 2x_2 + 3, 2x_n + 3$ is
(A) $\sqrt{p}$ (B) $2\sqrt{p} + 3$
(C) $2p + 3$ (D) $2\sqrt{p}$

**26.** The second quartile of a distribution is equal to its
(A) mean (B) median
(C) mode (D) range

**27.** If the arithmetic mean of a set of 15 observations is 12, then the sum of the observations is
(A) 180 (B) 150
(C) 240 (D) 160

**28.** The arithmetic mean of the first '$n$' natural numbers is 8 $n$ = _____.

**29.** If the standard deviation of $x_1, x_2, \ldots, x_n$ is $S$, then the variance of $x_1 + c, x_2 + c, \ldots, x_n + c$ is
(A) $S$ (B) $S^2$
(C) $2S$ (D) $\sqrt{S}$

**30.** If the arithmetic mean of $x_1, x_2, \ldots, x_n$ is $A$, then the arithmetic mean of $x_1 + a, x_2 + a, \ldots, x_n + a$ is
(A) A + $a$ (B) A – $a$
(C) $a$ . A (D) A

**31.** Mode of the data 3, 1, 2, 3, 2, 1, $x$, 3, 4, 3, 6 is
(A) 3 (B) 2
(C) $x$ (D) Cannot be determined

**32.** The range of the values 7, 8, 12, 9, 6, 13, 15, 21, 19, 5 is
(A) 15 (B) 13
(C) 14 (D) 16

**33.** The arithmetic mean of the series 2, 5, 8, 11, 14 is
(A) 8 (B) 6
(C) 9 (D) 7

**34.** Mean deviation of 8 and 17 is
(A) 4 (B) 3.5
(C) 4.5 (D) 5.5

**35.** When $10 < x < 15$, then the median of the data 6, 18, 21, 9, 23, 5 and $x$ is
(A) 9 (B) 21
(C) $x$ (D) Cannot be determined

**36.** The mode of the data 2, 5, 11, 12, 15 and 32 is
(A) 2 (B) 32
(C) 30 (D) ill-defined

**37.** If the standard deviation of 10, 20, 30, 40 and 50 is $S$, then the standard deviation of 20, 30, 40, 50 and 60 is
(A) $S$ (B) $S + 10$
(C) $S - 10$ (D) 10 $S$

**38.** If the range and the maximum value of a data are, respectively, 15 and 101, then the minimum value of the data is _____.

**39.** If $A$, $G$ and $H$ are the arithmetic mean, geometric mean and harmonic mean of two positive numbers $a$ and $b$, then which of the following is true?
(A) $A.G = H^2$ (B) $A^2 H^2 = G$
(C) $G.H = A^2$ (D) $A.H = G^2$

**40.** The first quartile ($Q_1$) of the observations 4, 8, 10, 15, 17, 29 and 32 is
(A) 8 (B) 16
(C) 29 (D) 53

## EXERCISE-2

***Directions for questions 1 to 40*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the blank space provided.

**1.** The arithmetic mean of the data 24, 38, 14, 8, 12, 18 is _____.

**2.** The median of the observations 21, 37, 25, 32, 19, 24, 29 is _____.

**3.** The mode of the data 24, 26, 32, 26, 28, 26, 24, 31, 24 is _____.

**4.** The mode of the data 22, 15, 17, 22, 16, 15, 21, 16, 22, 33, 22, 18 is _____.

**5.** If the arithmetic mean of $n$ observations $x_1, x_2, x_3 \ldots, x_n$ is $\bar{x}$, then the arithmetic mean of $kx_1 + c$, $kx_2 + c$, $kx_3 + c, \ldots, kx_n + c$ is

(A) $\bar{x}$ (B) $\bar{x} + c$

(C) $k\bar{x} + c$ (D) $k + c$

**6.** The arithmetic mean of 24, 4, 12, 8, 36, 48, 20, 16, 28, 32, 40 and 44 is

(A) 52 (B) 26

(C) 28 (D) 56

**7.** If the arithmetic mean of the series $a_1, a_2, \ldots, a_n$ is $A$ and the arithmetic mean of the series $b_1, b_2, \ldots, b_n$ is $B$, then the arithmetic mean of the series $a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n$ is

(A) $\frac{A+B}{2}$ (B) $A + B$

(C) $2(A + B)$ (D) $\frac{A+B}{n}$

**8.** In calculating the arithmetic mean of a set of 15 observations as 25, four observations 4, 12, 19 and 35 were misread as 1, 3, 8 and 13, respectively. Find the correct mean _____.

**9.** The arithmetic mean of the multiples of 7 between the numbers 100 and 200 is _____.

**10.** The average of a set of 120 observations is 20. If the average of 80 of these observations is also 20, then the average of the remaining 40 observations is _____.

**11.** The median of the observations $k+3$, $k-\frac{5}{2}$, $k-1$, $k-4$, $k+1$, $k+\frac{1}{2}$, $k+5$ $(k>0)$ is

(A) $\frac{k+1}{2}$ (B) $k-\frac{1}{2}$

(C) $k+\frac{1}{2}$ (D) $k-1$

**12.** If the median of the data $m_1, m_2, m_3, \ldots, m_n$ is $M$, then the median of the data $m_1 - 10, m_2 - 10, \ldots, m_n - 10$ is

(A) M (B) M + 10

(C) M – 10 (D) 10M

**13.** A denotes the arithmetic mean of two positive integers. G denotes their geometric mean and $H$ denotes their harmonic mean. $2G$ must be

(A) at most equal to (A + H).

(B) at least equal to (A + H).

(C) equal to (A + H).

(D) equal to $\frac{(A+H)}{2}$.

**14.** The quartile deviation of 12, 18, 20, 15, 19, 23, 25 is _____.

**15.** The mean deviation of 113, 117, 120, 122 and 128 is _____.

**16.** The range of 13, 38, 51, 56, 70, 72, 63, 65, 93 is _____.

**17.** The arithmetic mean of the range and the quartile deviation of the observations 1, 3, 5, 7, 9, 11, 13 is _____.

**18.** The standard deviation of the data $x_1, x_2, \ldots x_n$ is $p$, then the standard deviation of $2x_1 + 1$, $2x_2 + 1$, $2x_3 + 1$, …, $2x_n + 1$ is

(A) $4p + 1$ (B) $2p$

(C) $4p$ (D) $2p + 1$

**19.** The mean of the first eleven consecutive positive integral multiples of '4' is _____.

**20.** The arithmetic mean of $3^3$, $4^3$, $5^3$, $6^3$ ..., $20^3$ is

(A) 4899.5 (B) 2449.5

(C) 4012.5 (D) 1750

**21.** The geometric mean of 1, 4, 9, 27, 256 is

(A) 6 (B) 9

(C) 24 (D) 12

**22.** If mean and mode of a distribution are 6 and 8, respectively, then the median is

(A) 6.67 (B) 7

(C) 5 (D) 6.5

**23.** If the arithmetic mean of a set of 8 observations is 42 and another set of 12 observations is 38, then the mean for the combined set is
(A) 40 (B) 40.5
(C) 80 (D) 39.6

**24.** The sum of squares of first $m$ natural numbers is 385, then the arithmetic mean of squares of numbers is
(A) 38.5 (B) 47
(C) 35 (D) 13

**25.** The median of 10.2781, 10.8253, 10.3671, 10.2252, 10.8412, 10.5324 and 10.2225 is
(A) 10.2252 (B) 10.2781
(C) 10.8412 (D) 10.3671

**26.** The arithmetic mean of the first 100 natural numbers is _____.

**27.** If the arithmetic mean of the series $x_1, x_2, x_3, \ldots, x_n$ is $M$, then the arithmetic mean of the series $\frac{2x_1-3}{5}, \frac{2x_2-3}{5}, \ldots, \frac{2x_n-3}{5}$ is
(A) $\frac{2M+3}{5}$ (B) $\frac{2M-3}{5}$
(C) $M-\frac{3}{5}$ (D) $\frac{2M}{5}$

**28.** The arithmetic mean of a set of 22 observations is 25. If two observations 23 and 47 are discarded, then the arithmetic mean of the remaining observations is
(A) 24 (B) 24.5
(C) 23.5 (D) 25

**29.** Which of the following can be the range of the values of the median for the series 14, 12, 23, $x$, 15, 29, 5?
(A) [12, 14] (B) [14, 15]
(C) [15, 29] (D) [13, 18]

**30.** The median of the series 1, $\frac{2}{3}, \frac{12}{5}, \frac{13}{6}, \frac{1}{2}$ and 2 is _____.

**31.** The geometric mean of $\sqrt{27}$, 8, $\sqrt{3}$, 18 is
(A) 9 (B) 36
(C) 6 (D) 12

**32.** The geometric mean of 1, 4, $4^2$, …, $4^{101}$ is
(A) $2^{101}$ (B) $4^{50}$
(C) $2^{101}\sqrt{2}$ (D) $4^{51}$

**33.** The arithmetic mean of the first '$n$' even natural numbers is
(A) $n(n+1)$ (B) $n+1$
(C) $\frac{n+1}{2}$ (D) $n$

**34.** Find the mean deviation of the observations 8, 19, 21, 6, 16, 22 and 29 from median.
(A) 8 (B) 6
(C) 7 (D) 9

**35.** If the mean of 12 observations is 8 and their standard deviation is 2, then find the sum of the squares of the observations.
(A) 824 (B) 724
(C) 916 (D) 816

**36.** The geometric mean of 45, 245, 21 and 525 is
(A) 135 (B) 105
(C) 75 (D) 95

**37.** The standard deviation of a series of '$n$' observations is $\sigma$. If each observation is multiplied by 3, then the standard deviation of the new series is
(A) $\sigma$ (B) $9\sigma$
(C) $\frac{\sigma}{3}$ (D) $3\sigma$

**38.** The median of the values 12.12346, 12.12355, 12.12382, 12.13245, 12.15632, 12.12345, 12.12432, 12.18932, 12.12344, 12.12346 and 12.12349 is _____.

**39.** The mode of the data 2, 5, 3, 2, 6, 2, 7, 8, 2, 7, 7, 3, 7, 8, 7 and 10 is _____.

**40.** The algebraic sum of the squares of the deviations of 11 observations about their arithmetic mean is 275. The standard deviation of the 11 observations is _____.

## EXERCISE-3

***Directions for questions 1 to 40:*** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the blank space provided.

**1.** If the mean of 10 observations is 12 and that of another 12 observations is 10, then the mean of the total observations is
(A) 10 (B) 120/11
(C) 31/25 (D) 27/20

**2.** The geometric mean of 3, 5, 15, 45, 75 is _____.

**3.** The mean of 100 observations was found to be 39. Three observations were incorrectly copied as 13, 19, and 26 instead of 18, 12, and 28, respectively. The correct value of mean is _____.

**4.** The mean of squares of first 10 natural numbers is
(A) 5.5 (B) 38.5
(C) 35.8 (D) 53.8

**5.** The A.M. of two numbers is 10 and their G.M. is 8. Then their H.M. is _____.

**6.** If the mean of the following distribution is 3, then the value of '$a$' is _____.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f$ | a | 2a | 8 | 4 | 2 |

**7.** The mode of the following distribution is _____.

| $x$ | 3 | 7 | 9 | 11 | 12 |
|---|---|---|---|---|---|
| $f$ | 6 | 10 | 8 | 3 | 2 |

**8.** The median of the data 28, 31, 42, 37, 26, 34, 18, 23 is _____.

**9.** Find the harmonic mean of $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$.

**10.** Find the quartile deviation of the data 17, 15, 9, 6, 21, 32, 38, 30, 23, 25 and 27.

**11.** If the variance of the data $\frac{x_1+a}{3}, \frac{x_2+a}{3}, \frac{x_3+a}{3}, \ldots, \frac{x_n+a}{3}$ is $n$, then the standard deviation of $x_1, x_2, x_3, \ldots, x_n$ is
(A) $\sqrt{n}$ (B) $\sqrt{n}/3$
(C) $\sqrt[3]{n}$ (D) $n/3$

**12.** The standard deviation of the data 23, 15, 12, 21, 19 is _____.

**13.** Find the mean, median and mode of the frequency distribution table given below.

| CI | 0–5 | 5–10 | 10–15 | 15–20 | 20–25 |
|---|---|---|---|---|---|
| F | 4 | 8 | 12 | 5 | 1 |

**14.** The mean deviation of $(a+b)$ and $(a-b)$ is
(A) $a$ (B) $b$
(C) $a+b$ (D) $a-b$

**15.** The standard deviation of 5, 5, 5, 5, 5, 5, 5, 13 is
(A) 1.5 (B) 7
(C) $2\sqrt{2}$ (D) $\sqrt{7}$

**16.** If mean of 10 items is 25 and their standard deviation is 3, then the sum of squares of items is
(A) 2590 (B) 6430
(C) 6340 (D) 63400

**17.** Which of the following is true?
I : Range (2, 3, 5, 7, 9, 11, 6) = 4
II : Range (10, 20, 25, 30, 45, 80) = 70
(A) I alone (B) II alone
(C) Both I and II (D) Neither I nor II

**18.** Which of the following numbers cannot be the median of the series $x$, 12, 18, 13, 15, 40, 47?
(A) 17 (B) 19
(C) 15 (D) 16

**19.** The standard deviation of eleven consecutive integral multiples of 3 is
(A) $\sqrt{90}$ (B) $\sqrt{30}$
(C) $\sqrt{10}$ (D) 3

**20.** The harmonic mean of 1, 1/3, 1/6, 1/10, 1/12 is
(A) 1/6 (B) 160
(C) 5/32 (D) 300/101

**21.** The median of the series $\sin 1^\circ, \sin 2^\circ, \sin 3^\circ, \ldots, \sin 179^\circ$ is
(A) 1 (B) $\frac{1}{\sqrt{2}}$
(C) $\sin 46^\circ$ (D) $\sin 89^\circ$

**22.** The algebraic sum of the deviations of 12 numbers about 9 is 60. Find the arithmetic mean of the numbers.
(A) 14 (B) 12
(C) 13 (D) 15

**23.** The harmonic mean of 1, 2, 4, 7, 14 and 28 is _____.

**24.** The mean deviation of the series 12, 5, 9, 15, 31, 20, 4, 17 and 22 about the mean is
(A) $\frac{58}{9}$ (B) $\frac{56}{9}$
(C) $\frac{20}{3}$ (D) 6

**25.** The quartile deviation of 2, 12, 19, 17, 52, 11, 32, 23, 25, 12 and 39 is _____.

**26.** Find the standard deviation of the observations 8, 12, 4, 16, 20, 28, 32, 24, 36, 44 and 40.
(A) $\sqrt{80}$ (B) $\sqrt{40}$
(C) $\sqrt{160}$ (D) $\sqrt{360}$

**27.** Which of the following can be the range of the values of the median for the series 36, 8, 12, 42, 16, 21, 32, 23, 33, $x$, 9, 6, 25, 40, 37, 45 and 19?
(A) [25, 32] (B) [23, 25]
(C) [23, 32] (D) [21, 23]

**28.** If the third term and the tenth term of an arithmetic progression are 14 and 56, respectively, then the arithmetic mean of the first 15 terms of the arithmetic progression is
(A) 44 (B) 84
(C) 35 (D) 40

**29.** The median of the series 18, 21, 18, 12, 40, 38, 32, 40, 36, 12, 32, 38 and 36 is
(A) 36 (B) 38
(C) 21 (D) 32

**30.** The standard deviation of $p+q$ and $p-q$ is
(A) $p$ (B) $p+q$
(C) $q$ (D) $p-q$

**31.** The arithmetic mean (A.M.) of the series $x_1, x_2, \ldots, x_n$ is $M$. Let $y_i$ = A.M. $(x_i, x_{i+1}, x_{i+2})$ for $1 \le i \le n-2$ and $y_{n-1}$ = A.M. $(x_{n-1}, x_n, x_1)$, $y_n$ = A.M. $(x_n, x_1, x_2)$.
The arithmetic mean of $y_1, y_2, \ldots, y_n$ is
(A) $\frac{2}{3}$M (B) M
(C) $\frac{3}{2}$M (D) 3M

**32.** If 3, 2 and 9 occur with frequencies 2, 5 and 3, respectively, then their arithmetic mean is _____.

**33.** If the standard deviation of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 is M, then the standard deviation of 101, 102, 103, 104, …, and 111 is
(A) M (B) 100 + M
(C) 100 – M (D) M – 100

**34.** Which of the following can be the range of the values of the median for the values 5, 11, 8, 3, 25, 39, $x$, 14, 23, 4, 25, 6, 9, 15 and 29?
(A) [14, 15] (B) [9, 11]
(C) [11, 14] (D) [23, 25]

**35.** The arithmetic mean of the cubes of the first $n$ natural numbers is
(A) $\frac{(n+1)(2n+1)}{6}$ (B) $\frac{(n+1)^2}{4}$
(C) $\frac{n(n+1)^2}{4}$ (D) $\frac{n+1}{2}$

**36.** A man travels a distance of 200 km on a motorcycle and cover the first 100 km at an average speed of $V_1$ km/h and the second 100 km at an average speed of $V_2$ km/h $(V_1 \ne V_2)$. The average speed of the bike for the entire journey is equal to
(A) arithmetic mean of $V_1$ and $V_2$.
(B) geometric mean of $V_1$ and $V_2$.
(C) harmonic mean of $V_1$ and $V_2$.
(D) arithmetic mean of $\frac{1}{V_1}$ and $\frac{1}{V_2}$.

**37.** The standard deviation of 7, 7, 7, 7, 7, 7, 7, 7, 7, 7 and 18 is
(A) 10 (B) $2\sqrt{2}$
(C) $\sqrt{10}$ (D) $2\sqrt{5}$

**38.** If the standard deviation of the observations 3, 12, 6, 10, 14, 21 and 24 is $p$, then the variance of the observations 6, 33, 15, 27, 39, 60 and 69 is
(A) $p$ (B) $3p$
(C) $9p^2$ (D) $3p^2$

**39.** The harmonic mean of $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \ldots, \frac{1}{20}$ is
(A) $\frac{1}{10}$ (B) $\frac{1}{11}$
(C) $\frac{2}{5}$ (D) $\frac{3}{5}$

**40.** Find the quartile deviation of 12, 6, 10, 14, 18, 8, 15 19, 4, 21 and 13.
(A) 4.5 (B) 5
(C) 4 (D) 3

## EXERCISE-4

***Directions for questions 1 to 25*:** For the multiple choice questions, select the correct alternative from the given choices. For the non-multiple choice questions, write your answer in the blank space provided.

**1.** If the sum of deviations of a set of 10 observations about 25 is –2, then find the sum of the observations.
(A) 252 (B) 230
(C) 248 (D) 270

**2.** Find the mean of the series containing $n$ elements, such that the $j^{th}$ element $x_j = (j+1)\,j;\ 1 \le j \le n$.
(A) $\frac{(2n+1)(n+1)}{6}$ (B) $\frac{(2n+5)(n+1)}{6}$
(C) $\frac{(2n+4)(n+1)}{6}$ (D) $\frac{(n+4)(2n+1)}{6}$

**3.** If the variance of the sequence of terms represented by $5x_i + 7$ where $i = 1$ to 11 is 225, then the standard deviation of the sequence $7x_i + 53$ $\sqrt{35}$ is
(A) 21 (B) 441
(C) $3\sqrt{35}$ (D) 315

**4.** If the mean of 15 observations is 9 and the standard deviation is 2, then the sum of the squares of the observations is
(A) 1275 (B) 1572
(C) 1362 (D) 16

**5.** Find the mode of the following distribution _____.

| $x$ | 6 | 5 | 4 | 3 | 2 | 1 |
|---|---|---|---|---|---|---|
| $f$ | 3 | 5 | 8 | 7 | 12 | 15 |

**6.** Find the arithmetic mean of the cubes of the first $n$ even natural numbers.

(A) $2n^2(n+1)^2$ (B) $2n(n+1)^2$
(C) $2n^2(n+1)^2$ (D) $n^2(n+1)^2$

**7.** Find the arithmetic mean of $4^2$, $5^2$, $6^2$, $7^2$, $8^2$ and $9^2$.
(A) 31.66 (B) 35.6
(C) 44.5 (D 45.16

**8.** Find the arithmetic mean of 4, 16, 64,…, 4096 _____.

**9.** The arithmetic mean of a set of 50 numbers is 42. If two numbers 75 and 105 are discarded, then the arithmetic mean of the remaining set is
(A) 40 (B) 52
(C) 48 (D) 38.4

**10.** The arithmetic mean of a set of 7 observations is 36 and that of another set of 13 observations is 46. The mean of the combined set is
(A) 41 (B) 42.5
(C) 41.5 (D) 43.5

**11.** The harmonic mean of $1, \frac{1}{4}, \frac{1}{7}, \frac{1}{8}, \frac{1}{10}$ is
(A) $\frac{5}{29}$ (B) $\frac{1}{5}$
(C) $\frac{1}{6}$ (D) $\frac{1}{8}$

**12.** The geometric mean of $3^1$, $3^2$, $3^3$, ..., $3^{99}$ is
(A) $27^{10}$ (B) $3^{40}$
(C) $9^{25}$ (D) $9^{15}$

**13.** The median of 15.5213, 15.3125, 15.4132, 15.3215, 15.3142, 15.2413 and 15.3143 is _____.

**14.** The median of a series of 17 numbers is 23. If two more numbers 20 and 27 are added to this set, then find the median of the new set so formed.
(A) 27 (B) 23
(C) 23.5 (D) Cannot be determined

**15.** Find the standard deviation of $a$, $b$ where $a > b$.
(A) $a-b$ (B) $\frac{a^2+b^2}{2}$
(C) $\sqrt{\frac{a^2-b^2}{2}}$ (D) $\frac{a-b}{2}$

**16.** The algebraic sum of the deviations of 10 numbers taken about 7 is 40. The arithmetic mean of these numbers is _____.

**17.** The mean of 11 observations is calculated as 36. It was later discovered that 3 observations 23, 52 and 61 were mistakenly taken as 32, 25 and 16, respectively. The error is corrected and two more observations 45 and 42 are added. Find the mean of the 13 observations.

**18.** Find the range of values that the median of the following series can take 3, $x$, 10, 18, 20, 23, 14.
(A) [13, 15] (B) [15, 20]
(C) [14, 18] (D) Cannot be determined

**19.** The arithmetic mean of the first $x$ odd natural numbers is $y$. Find the arithmetic mean of the first $x$ even natural numbers.
(A) $y$ (B) $y+1$
(C) $y+2$ (D) $y+3$

**20.** The arithmetic mean of the series $y_1$, $y_2$, $y_3$, ..., $y_n$ is $A$. Find the arithmetic mean of the series $\frac{3y_1+2}{4}, \frac{3y_2+2}{4}, \ldots, \frac{3y_n+2}{4}$.
(A) $\frac{3A+2}{4}$ (B) $\frac{3A-2}{4}$
(C) $A-\frac{1}{2}$ (D) $A+\frac{1}{2}$

**21.** The arithmetic mean of the series $Y_1$, $Y_2$, ..., $Y_n$ is $M$. If $Y_n - 1$ is replaced by $Yr$, then the arithmetic mean of the new series will be
(A) $\frac{Mn - Y_r + Y_n - 1}{n}$ (B) $\frac{Mn - Y_r - Y_n - 1}{n}$
(C) $\frac{nM + Y_r + Y_{n-1}}{n}$ (D) $\frac{Mn + Y_r + Y_{n-1}}{n}$

**22.** The arithmetic mean of the series $x_1$, $x_2$, ..., $x_n$ is $x$. The arithmetic mean of the series $y_1$, $y_2$, ..., $y_n$ is $y$. Find the arithmetic mean of the series $x_1 - y_1$, $x_2 - y_2$, ..., $x_n - y_n$.
(A) $x-y$ (B) $\frac{x-y}{n}$
(C) $n(x-y)$ (D) $\frac{x-y}{2}$

**23.** If 5, 4 and 6 occur with frequencies of 2, 3 and 4, respectively, then find their arithmetic mean.
(A) $\frac{44}{9}$ (B) $\frac{50}{9}$
(C) $\frac{52}{9}$ (D) $\frac{46}{9}$

**24.** Raju ran once around a square plot ABCD. He ran along AB at an average speed of $S_1$ kmph and ran along BC at an average speed of $S_2$ kmph. His average speed in running from $A$ to $C$ is
(A) the arithmetic mean of $S_1$ and $S_2$.
(B) the geometric mean of $S_1$ and $S_2$.
(C) the harmonic mean of $S_1$ and $S_2$.
(D) the arithmetic mean of $\frac{1}{S_1}$ and $\frac{1}{S_2}$.

**25.** The terms $Y_1$, $Y_2$, ..., $Y_n$ have an arithmetic mean of $A$. Let $Z_i$ = A.M. $(Y_i, Y_{i+1}, Y_{i+2}, Y_{i+3})$ for $1 \le i \le n-3$.

$Z_{n-2}$ = A.M. ($Y_{n-2}$ $Y_{n-1}$, $Y_n$, $Y_1$)
$Z_{n-1}$ = A.M. ($Y_{n-1}$ $Y_n$, $Y_1$, $Y_2$)
$Z_n$ = A.M. ($Y_n$, $Y_1$, $Y_2$, $Y_3$).

Find the arithmetic mean of $Z_1$, $Z_2$, ..., $Z_n$.
(Assume $n > 10$) _____ $A$

## ANSWER KEYS

### Exercise-1

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (D) | 7. (B) | 13. (A) | 19. 15 | 25. (D) | 31. (A) | 37. (A) |
| 2. (B) | 8. (C) | 14. (A) | 20. 5 | 26. (B) | 32. (D) | 38. 86 |
| 3. (B) | 9. 10 | 15. (B) | 21. (C) | 27. (A) | 33. (A) | 39. (D) |
| 4. (C) | 10. (C) | 16. (C) | 22. (B) | 28. 15 | 34. (C) | 40. (A) |
| 5. (C) | 11. (D) | 17. 1 | 23. (A) | 29. (B) | 35. (C) | |
| 6. 55 | 12. (C) | 18. 74 | 24. (B) | 30. (A) | 36. (D) | |

### Exercise-2

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. 19 | 7. (B) | 13. (A) | 19. 24 | 25. (D) | 31. (C) | 37. (D) |
| 2. 25 | 8. 28 | 14. 4 | 20. (B) | 26. 50.5 | 32. (A) | 38. 12.12355 |
| 3. 24, 26 | 9. 150.5 | 15. 4 | 21. (D) | 27. (B) | 33. (B) | 39. 7 |
| 4. 22 | 10. 20 | 16. 80 | 22. (A) | 28. (A) | 34. (B) | 40. 5 |
| 5. (C) | 11. (C) | 17. 8 | 23. (D) | 29. (B) | 35. (D) | |
| 6. (B) | 12. (C) | 18. (B) | 24. (A) | 30. 1.5 | 36. (B) | |

### Exercise-3

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (B) | 7. 7 | 13. 11.82 | 19. (A) | 25. 10 | 31. (B) | 37. (C) |
| 2. 15 | 8. 29.5 | 14. (B) | 20. (C) | 26. (C) | 32. 4.3 | 38. (C) |
| 3. 39 | 9. 0.2 | 15. (D) | 21. (B) | 27. (B) | 33. (A) | 39. (B) |
| 4. (B) | 10. 7.5 | 16. (C) | 22. (A) | 28. (A) | 34. (C) | 40. (B) |
| 5. 6.4 | 11. (C) | 17. (B) | 23. 3 | 29. (D) | 35. (C) | |
| 6. 2 | 12. 4 | 18. (B) | 24. (C) | 30. (C) | 36. (C) | |

### Exercise-4

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (C) | 5. 1 | 9. (A) | 13. 15.3143 | 17. 42 | 21. (D) | 25. 1 |
| 2. (C) | 6. (B) | 10. (B) | 14. (B) | 18. (C) | 22. (A) | |
| 3. (A) | 7. (D) | 11. (C) | 15. (D) | 19. (B) | 23. (D) | |
| 4. (A) | 8. 910 | 12. (C) | 16. 11 | 20. (A) | 24. (C) | |

## SOLUTIONS

## EXERCISE-1

***Solutions for questions 1 to 40:***

**1.** Length or size of the class (standard definition).

**2.** Mid-value $= \frac{\text{upper limit} + \text{lower limit}}{2} = \frac{37.5 + 27.5}{2} = 32.5$

**3.** Given mid-value of a class is 9 whose size of the class is 7.
$\Rightarrow$ Lower limit of the class is $9 - 3 = 6$.
Upper limit of the class is $9 + 3 = 12$.
$\therefore$ Class interval is 6–12.

**4.** Size or length of the class is the difference between the lower (or upper) limits of two successive classes, i.e., $34 - 24 = 10$.

**5.** The boundaries of a class are obtained by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit.
$\therefore$ The upper boundary of the given class is 14.5.

**6.** The mid-value of the class 45 – 65 is $\frac{45+65}{2} = \frac{110}{2} = 55$.

**7.** A.M. $(3a, 3b, 3c) = \frac{3a+3b+3c}{3} = \frac{3(a+b+c)}{3} = a+b+c$

**8.** Mode = 3 median – 2 mean

**9.** The size of the class 12 – 22 is 22 – 12 = 10.

**10.** For a symmetric distribution, mean = median = mode.

**11.** We know that for a moderately symmetric distribution
mode = 3 median – 2 mean and
mode – median = 2(median – mean)

**12.** We know that for a symmetric distribution, arithmetic mean, median and mode coincides.
∴ Arithmetic mean = mode = 24

**13.** We know that arithmetic mean $= \dfrac{\text{sum of observations}}{\text{number of observations}}$

$$\text{Number of observations} = \frac{\text{sum of observations}}{\text{arithmetic mean}}$$

$$= \frac{108}{9} = 12$$

**14.** The sum of deviations about mean is zero. (standard result)

**15.** When the number of observations is odd, the median of observations, when arranged in order, is the exact middle most observation. Here, middle observation is 13.
∴ Median = 13

**16.** G.M. $(a, b, c) = \sqrt[3]{abc}$

**17.** 1 occurs more frequently in the given data. So, the mode is 1.

**18.** Range = maximum value – minimum value = 82 – 8 = 74

**19.** G.M. $(5, 75, 9) = \sqrt[3]{5\times 75\times 9} = \sqrt[3]{5^3\times 3^3} = 5\times 3 = 15$

**20.** We know that mean deviation $(a, b) = \dfrac{|a-b|}{2}$.

$$\therefore \text{M.D } (30, 40) = \frac{|30-40|}{2} = 5$$

**21.** Geometric mean of $x_1$, $x_2$, $x_3$ is $\sqrt[3]{x_1.x_2.x_3}$, i.e., $\sqrt[3]{1\times 3\times 12} = \sqrt[3]{36}$.

**22.** Range = maximum value – minimum value
17 = Maximum value – 88
∴ Maximum value = 88 + 17 = 105

**23.** The third quartile $Q_3$ is the $3\left(\dfrac{n+1}{4}\right)^{\text{th}}$ observation.

$3\left(\dfrac{11+1}{4}\right) = 9^{\text{th}}$ observation is $Q_3$.

$\therefore Q_3 = 51$

**24.** The median of five observations is the third observation. The third prime number = 5.

**25.** The variance of the series $x_1$, $x_2$, $x_3$, ..., $x_n$, is $p$.
∴ The standard deviation of the series $x_1$, $x_2$, ..., $x_n$ is $\sqrt{p}$. The standard deviation of the series $2x_1 + 3$, $2x_2 + 3$, ..., $2x_n + 3$ is $2\sqrt{p}$.

**26.** $Q_2$ is equal to the median.

**27.** We know that, A.M. $= \dfrac{\text{sum of the observations}}{\text{number of the observations}}$

So, $12 = \dfrac{\text{sum}}{15}$

∴ The sum of the observations = 12 × 15 = 180

**28.** Given, the A.M. of the first '$n$' natural numbers is 8.

$$\therefore \frac{n+1}{2} = 8 \Rightarrow n = 15$$

**29.** S.D. $(x_1 + c, x_2 + c, \ldots, x_3 + c)$
= S.D. $(x_1, x_2, \ldots, x_3) = S$
∴ Variance $(x_1 + c, x_2 + c, \ldots, x_n + c) = S^2$

**30.** A.M. $((x_1 + a, x_2 + a, \ldots, x_n + a)$
= A.M. $(x_1, x_2, \ldots, x_n) + a = A + a$

**31.** Of all the given observations, 3 is occurring for 4 times even if the value of $x$ is either 2 or 1 still 3 is the most occurring value.
∴ Mode = 3

**32.** Range = maximum value – minimum value
= 21 – 5 = 16

**33.** If the given values are in A.P., then the arithmetic mean

$$= \frac{\text{first term} + \text{last term}}{2} = \frac{2+14}{2} = 8.$$

**34.** Mean deviation of 8 and 17 is $\dfrac{|17-8|}{2} = 4.5$.

**35.** Since given $10 < x < 15$ and when arranged the given observations in order the value that lies between 9 and 21 is the median, i.e., $x$.
∴ Median of the data is $x$.

**36.** Since no observation occurs more than once, the mode is ill-defined.

**37.** We know that on adding a constant value to each of the given observation the standard deviation remains unchanged.
∴ S.D. (10, 20, 30, 40, 50)
= S.D. (20, 30, 40, 50, 60) = $S$

**38.** Range = maximum value – minimum value
∴ 15 = 101 – minimum value
So, minimum value = 101 – 15 = 86

**39.** Given, $A = \dfrac{a+b}{2}$, $G = \sqrt{ab}$ and $H = \dfrac{2ab}{a+b}$

$$\therefore \text{A.H.} = \left(\frac{a+b}{2}\right)\left(\frac{2ab}{a+b}\right) = ab = G^2$$

**40.** When the observations are in order (ascending), then the $\left(\frac{n+1}{4}\right)^{th}$ observation is the first quartile ($Q_1$).

Here, $n = 7 \therefore \left(\frac{7+1}{4}\right)$ i.e. 2nd observation.

$\therefore Q_1 = 8$

## EXERCISE-2

***Solutions for questions 1 to 40:***

**1.** Arithmetic mean (A.M.) $= \frac{\text{sum of observations}}{\text{number of observations}}$

A.M. (24, 38, 14, 8, 12, 18) $= \frac{24+38+14+8+12+18}{6}$

$= \frac{114}{6} = 19$

**2.** Arranging the given observations in ascending order, we have 19, 21, 24, 25, 29, 32, 37.

The middle value is 25.

$\therefore$ Median = 25

**3.** In the given data, the values 24 and 26 occur 3 times each. Hence, both of them are modes.

$\therefore$ The modes of the data are 24 and 26.

**4.** The most frequently occurring value of the data is called the mode. In the given data, 22 appears 4 times

$\therefore$ Mode = 22

**5.** Arithmetic mean of the new series is $k\bar{x} + c$.

**6.** Arranging the given numbers in an increasing order, we get 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44 and 48, which is an arithmetic progression.

$\therefore$ The required arithmetic mean $= \frac{a+l}{2} = \frac{4+48}{2} = 26$

**7.** A.M.$(a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n)$

$= \frac{(a_1+b_1)+(a_2+b_2)+\ldots+(a_n+b_n)}{n}$

$= \frac{(a_1+a_2+\ldots a_n)}{n} + \frac{(b_1+b_2+\ldots b_n)}{n} = A + B$

**8.** Given, the arithmetic mean of a set of 15 observations is 25.

So, the sum of the 15 observations is $15 \times 25 = 375$.

But, four observations 4, 12, 19 and 35 were misread as 1, 3, 8 and 13, respectively.

So, the actual sum of the observations is $375 + (4 - 1) + (12 - 3) + (19 - 8) + (35 - 13) = 420$.

Hence, the correct mean is $\frac{420}{15} = 28$.

**9.** The multiples of 7 between 100 and 200 are 105, 112, 119, …, 196.

The above numbers are in arithmetic progression.

$\therefore$ The required arithmetic mean is $\frac{a+l}{2} = \frac{105+196}{2}$

$= \frac{301}{2} = 105.5$.

**10.** Let the average of the remaining 40 observations be $\bar{x}$.

Then, $120 \times 20 = 80 \times 20 + 40\bar{x}$.

$\Rightarrow 40\bar{x} = 2400 - 1600 \Rightarrow 40\bar{x} = 800$

$\therefore \bar{x} = 20$

**11.** By arranging the observations in ascending order, we get

$k - 4, k - \frac{5}{2}, k - 1, k + \frac{1}{2}, k + 1, k + 3, k + 5.$

Median = middle most term $= k + \frac{1}{2}$

**12.** When 10 is subtracted from each observation, the median also reduces by 10.

So, the required median is M – 10.

**13.** Let the two positive integers be $x$ and $y$.

$A = \frac{x+y}{2}$, $G = \sqrt{xy}$ and $H = \frac{2xy}{x+y}$

$A - G = \frac{x+y-2\sqrt{xy}}{2} = \frac{\left(\sqrt{x}-\sqrt{y}\right)^2}{2}$

$G - H = \frac{\sqrt{xy}\left(x+y-2\sqrt{xy}\right)}{x+y} = \frac{\sqrt{xy}\left(\sqrt{x}-\sqrt{y}\right)^2}{x+y}$

$A - G - (G - H) = \frac{\left(\sqrt{x}-\sqrt{y}\right)^2\left(x+y-2\sqrt{xy}\right)}{2(x+y)}$

$\therefore A + H - 2G = \frac{\left(\sqrt{x}-\sqrt{y}\right)^4}{2(x+y)}$, which is 0 when $x = y$ and is positive otherwise.

$\therefore A + H - 2G \geq 0$, i.e., $A + H \geq 2G$.

$\therefore 2G$ will be at most equal to $A + H$.

**Alternative Solution:**

As A.M. $\geq$ G.M., $\frac{A+H}{2} \geq \sqrt{AH}$

$\Rightarrow \frac{A+H}{2} \geq \sqrt{\left(\frac{x+y}{2}\right)\left(\frac{2xy}{x+y}\right)}$

$\Rightarrow \frac{A+H}{2} \geq \sqrt{xy} \Rightarrow A+H \geq 2G.$

$\therefore$ $2G$ is less than or equal to $A+H$.

**14.** The ascending order of the observation is 12, 15, 18, 19, 20, 23, 25.

$Q_1 = \left(\frac{7+1}{4}\right)^{th}$, i.e., $2^{nd}$ observation = 15

$Q_3 = \left[3\frac{(7+1)}{4}\right]^{th}$, i.e., $6^{th}$ observation = 23

$\therefore$ Q.D. $= \frac{Q_3 - Q_1}{2} = \frac{23-15}{2} = \frac{8}{2} = 4$

**15.** Mean of (113, 117, 120, 122, 128) is

$\frac{113+117+120+122+128}{5} = 120.$

Mean deviation $= \frac{\Sigma|x_i - A|}{n}$

$= \frac{|113-120|+|117-120|+|120-120|+|122-120|+|128-120|}{5}$

$= \frac{7+3+2+8}{5} = 4$

**16.** Range = maximum value − minimum value

= 93 − 13 = 80

**17.** Range = maximum value − minimum value = 12

Quartile deviation $= \frac{Q_3 - Q_1}{2}$

Where $Q_1 = \left(\frac{n+1}{4}\right)^{th}$ term $= 2^{nd}$ term = 3 and

$Q_2 = \left(3\frac{n+1}{4}\right)^{th}$ term $= 6^{th}$ term = 11

$\therefore$ Q.D. $= \frac{11-3}{2} = 4.$

$\therefore$ Arithmetic mean of the range and the quartile deviation is $\frac{12+4}{2} = 8$

**18.** We know S.D. $(Ax+b) = A \times$ [S.D. $(x)$]

Now, S.D. $\times (2x+1) = 2 \times$ [S.D.$(x)$] $= 2p$.

**19.** Given observations: {4, 8, …, 44}

$\therefore$ Mean $= \frac{4+8+....+44}{11} = \frac{11}{2}\frac{(48)}{11} = 24$

**20.** A.M. $(3^3, 4^3, \ldots, 20^3)$

$= \frac{3^3+4^3+...+20^3}{18}$

$= \frac{1^3+2^3+3^3+4^3+...+20^3-1^3-2^3}{18}$

$= \frac{(210)^2-9}{18} = \frac{9\times(4900-1)}{18}$

$= \frac{4899}{2} = 2449.5$

**21.** G.M. (1, 4, 9, 27, 256)

$(1 \cdot 4 \cdot 9 \cdot 27 \cdot 256)^{1/5}$

$(1 \cdot 2^2 \cdot 3^2 \cdot 3^3 \cdot 2^8)^{1/5} = (3^5 \cdot 2^{10})^{1/5} = (12^5)^{1/5} = 12.$

**22.** Mean = 6, mode = 8

Mode = 3 median − 2 mean

$\Rightarrow$ Median $= \frac{\text{mode}+2\,\text{mean}}{3} = \frac{8+12}{3} = 6.67$

**23.** $n_1 = 8,\ \bar{x}_1 = 42$

$n_2 = 12,\ \bar{x}_2 = 38$

Combined mean $= \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2}$

$= \frac{336+456}{20} = \frac{792}{20} = 39.6$

**24.** The squares of m natural numbers

$= \frac{m(m+1)(2m+1)}{6} = 385$

$\Rightarrow m(m+1)\ (2m+1) = 10.11.21$

$\therefore\ m = 10$

Mean of squares of 10 natural numbers

$= \frac{\text{sum of squares of 10 natural numbers}}{10}$

$= 385/10 = 38.5$

**25.** The data in ascending order is 10.2225, 10.2252, 10.2781, 10.3671, 10.5324, 10.8253 and 10.8412.

Median is $\left(\frac{7+1}{2}\right)^{th}$ item $= 4^{th}$ item = 10.3671.

**26.** We know that, the arithmetic mean of first '$n$' natural numbers is $\frac{n+1}{2}$.

$\therefore$ The arithmetic mean of the first 100 natural numbers is $\frac{100+1}{2} = 50.5.$

**27.** Given, the A.M.$(x_1, x_2, \ldots, x_n) = M$

We know that if the A.M. $(x_1, x_2, \ldots, x_n) = A$, then the A.M. $(ax_1+b, ax_2+b, \ldots, ax_n+b) = aA+b$.

$\therefore$ The A.M. $\left(\frac{2x_1-3}{5}, \frac{2x_2-3}{5}, \ldots \frac{2x_n-3}{5}\right)$

$= \frac{2}{3}M - \frac{3}{5} = \frac{2M-3}{5}$

**28.** Given, the A.M. of 22 observations is 25.

So, the sum of the 22 observations = $22 \times 25 = 550$.

After discarding the observations 23 and 47, the new sum is 550 – (23 + 47) = 480.

Hence, the required mean = $\frac{480}{20} = 24$.

**29.** Arranging the given values other than '$x$' in increasing order, we have 5, 12, 14, 15, 29, 23.
If $x \le 14$, then median is 14.
If $x \ge 15$, then median is 15.
If $14 < x < 15$, the median is $x$.
So, the range of the values of the median is [14, 15].

**30.** Increasing order of the given numbers is $\frac{1}{2}, \frac{2}{3}, 1, 2, \frac{13}{6}$ and $\frac{12}{5}$.

$\therefore$ The median is $\frac{1+2}{2} = 1.5$.

**31.** Geometric mean of $\sqrt{27}, 8, \sqrt{3}, 18$ is $\left[\left(\sqrt{27}\right)(8)\left(\sqrt{3}\right)(18)\right]^{\frac{1}{4}}$.

$= \left[\left(\sqrt{81}\right)\left(2^3\right)(2)\left(3^2\right)\right]^{\frac{1}{4}} = \left(3^4 2^4\right)^{\frac{1}{4}} = 3(2) = 6$

**32.** We know that, the G.M.$(x_1, x_2, \ldots, x_n)$
$\therefore$ The G.M.$(1, 4, 4^2, \ldots, 4^{101})$

$= \left(4^{1+2+\ldots+101}\right)^{\frac{1}{102}} = \left(4^{\frac{101\times 102}{2}}\right)^{\frac{1}{102}} = (4)^{\frac{101}{2}} = 2^{101}$

**33.** We know that the sum of the first '$n$' even natural numbers is $n(n+1)$.

Hence, the required arithmetic mean is $\frac{n(n+1)}{n} = n+1$.

**34.** The given observations when arranged in order, we get 6, 8, 16, 19, 21, 22, 29.
Median of the observations is the exact middle observation which is 19.
$\bar{x} = 19$($\because$ here $\bar{x}$ is median.)

Mean deviation = $\Sigma\frac{|x_i - \bar{x}|}{n}$

$$\frac{|6-19|+|8-19|+|16-19|+|19-19|+|21-19|+|22-19|+|29-19|}{7}$$

$$\frac{13+11+3+0+2+3+10}{7} = \frac{42}{7} = 6$$

**35.** The standard deviation $(\sigma) = 2$, arithmetic mean $(\bar{x}) = 8$ and number of observations $(x) = 12$

$\therefore\ \sigma = \sqrt{\frac{\Sigma x_i^2}{n} - (\bar{x})^2}$

$\Rightarrow 2 = \sqrt{\frac{\Sigma x_i^2}{12} - 8^2}$

$4 = \frac{\Sigma x_i^2}{12} - 64 \Rightarrow \Sigma x_i^2 = (4+64)(12) = 816$

**36.** We have $45 = 3^2 \times 5$, $245 = 5 \times 7^2$, $21 = 3 \times 7$ and $525 = 3 \times 5^2 \times 7$.
We know that, the G.M. $(x_1, x_2, \ldots, x_n)$.

$= (x_1 \times x_2 \times x_3 \ldots x_n)^{\frac{1}{n}}$

$\therefore$ The G.M. of 45, 245, 21, 525

$= (45 \times 245 \times 21 \times 525)^{\frac{1}{4}}$

$= (3^5 \times 5^4 \times 7^4)^{\frac{1}{4}} = 3 \times 5 \times 7 = 105$

**37.** The standard deviation of the series will be $3\sigma$. (Standard result)

**38.** Arranging the given numbers in increasing order, we get 12.12344, 12.12345, 12.12346, 12.12346, 12.12349, 12.12355, 12.12382, 12, 12432, 12.13245, 12.15632 and 12.18932.
There are 11 values. So, the median is the 6th observation which is 12.12355.

**39.** 7 is the most often occurring observation in the given data. So, the mode of the data is 7.

**40.** Let $x_1, x_2, \ldots, x_{11}$ be the 11 observations and $\bar{x}$ be their arithmetic mean.

$\sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}} = \sqrt{\frac{275}{11}} = \sqrt{25} = 5$

## EXERCISE-3

***Solutions for questions 1 to 40:***

**1.** The combined mean is $\frac{n_1 x_1 + n_2 x_2}{n_1 + n_2}$.

where $n_1 = 10$, $n_2 = 12$, $x_1 = 12$; $x_2 = 10$

Combined mean = $\frac{10\times 12 + 12 \times 10}{12+10} = \frac{240}{22} = \frac{120}{11}$

**2.** G.M. $(3, 5, 15, 45, 75) = \sqrt[5]{3\times 5\times 15\times 45\times 75}$

$= \sqrt[5]{15\times 15\times 15\times 15\times 15} = \sqrt[5]{15^5} = 15$

**3.** Since sum of correct observations is equal to sum of wrong observations, there is no change in the mean.

**4.** Mean of squares of first '$n$' natural numbers

$$= \frac{(n+1)(2n+1)}{6} = \frac{(10+1)(2(10)+1)}{6}$$

$$= \frac{(11)(21)}{6} = \frac{77}{2} = 38.5$$

**5.** We know that $(\text{G.M.})^2 = (\text{A.M.})\ (\text{H.M.})$

$(8)^2 = 10\ (\text{H.M.})$

$$\text{H.M.} = \frac{64}{10} = 6.4$$

**6.** $\text{Mean} = \dfrac{\Sigma fx}{\Sigma f}$

$$\frac{a+4a+24+16+10}{a+2a+8+4+2} = 3$$

$\Rightarrow 5a + 50 = 3(3a + 14) \Rightarrow 5a + 50 = 9a + 42$

$\Rightarrow 4a = 8$

$a = 2$

**7.** Since frequency is maximum for $x = 7$, mode = 7.

**8.** By arranging the given data in ascending order, we have 18, 23, 26, 28, 31, 34, 37, 42.

The two middle values in the above series are 28 and 31.

$$\therefore \text{Median} = \frac{28+31}{2} = \frac{59}{2} = 29.5$$

**9.** Harmonic mean of $(x_1, x_2, \ldots, x_n) = \dfrac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}}$

$$\text{H.M.}\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}\right) = \frac{4}{2+4+6+8} = \frac{4}{20} = \frac{1}{5} = 0.2$$

**10.** Arranging the given values in ascending order, we have 6, 9, 15, 17, 21, 23, 25, 27, 30, 32, and 38.

Here $n = 11$

$$Q_1 = \left(\frac{n+1}{4}\right)^{\text{th}} \text{value } \frac{11+1}{4} = 3^{\text{rd}} \text{ which is 15.}$$

$\therefore Q_1 = 15$

$$Q_3 = 3\left(\frac{n+1}{4}\right)^{\text{th}} \text{ value} = 3 \times 3 = 9^{\text{th}} \text{ which is 30.}$$

$$\therefore \text{Q.D.} = \frac{Q_3 - Q_1}{2} = \frac{30-15}{2} = 7.5$$

**11.** The variance of the given data is $n$.

$\therefore$ Standard deviation is $\sqrt{n}$ ($\because$ S.D. = $\sqrt{\text{Variance}}$ )

Standard deviation of $\frac{x_1}{3} + \frac{a}{3}, \frac{x_2}{3} + \frac{a}{3}, \ldots, \frac{x_n}{3} + \frac{a}{3}$ is $\sqrt{n}$.

$\therefore$ S.D. of $\frac{x_1}{3}, \frac{x_2}{3}, \ldots, \frac{x_n}{3}$ is $\sqrt{n}$ ($\because$ If a constant is added to each observation, then the standard deviation does not change.)

S.D. of $x_1, x_2, \ldots, x_n$ is $3\sqrt{n}$.

**12.** Standard deviation (S.D.) = $\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}}$

$$\bar{x}\ (\text{A.M.}) = \frac{23+15+12+21+19}{5} = 18$$

$$\text{S.D.} = \left[\frac{(23-18)^2 + (15-18)^2 + (12-18)^2 + (21-18)^2 + (19-18)^2}{5}\right]^{1/2}$$

$$= \left(\frac{25+9+36+9+1}{5}\right)^{1/2} \Rightarrow \text{S.D.} = \sqrt{16} = 4$$

**13.**

| CI | $F$ | Mid value | $fx$ | Cum $f$ |
|---|---|---|---|---|
| 0 – 5 | 4 | 2.5 | 10.0 | 4 |
| 5 – 10 | $f_1 = 8$ | 7.5 | 60.0 | (12) $m$ |
| $L$ 10 – 15 | $f = 12$ | 12.5 | 150.0 | 24 |
| 15 – 20 | $f_2 = 5$ | 17.5 | 87.5 | 29 |
| 20 – 25 | 1 | 22.5 | 22.5 | 30 |
| | $\Sigma f = 30$ | | $\Sigma fx = 330.0$ | |

$$\text{Arithmetic mean } (\bar{x}) = \frac{\Sigma fx}{\Sigma f} = \frac{330}{30} = 11$$

$$\text{Median} = \ell + \left[\frac{\left(\frac{N}{2} - M\right)}{f} \times C\right] \text{ here } \frac{N}{2} = \frac{30}{2} = 15,\ L = 10,$$

$f = 12,\ C = 5,\ m = 12$

$$= 10 + \left(\frac{15-12}{12}\right)(5) = 11.25$$

$$\text{Mode} = \ell + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times C, \text{ here } \Delta_1 = f - f_1 = 12 - 8 = 4$$

$$D_2 = f - f_2 = 12 - 5 = 7 = 10 + \frac{4(5)}{4+7} = 11.82$$

**14.** M.D. $((a+b), (a-b))$

$$= \left|\frac{(a+b)-(a-b)}{2}\right| = \left|\frac{a+b-a+b}{2}\right| = 2b/2 = b.$$

**15.** The S.D. does not differ if a constant is added or subtracted from all data items.

S.D. $(5, 5, 5, 5, 5, 5, 5, 13)$ = S.D. $(0, 0, 0, 0, 0, 0, 0, 8)$

Mean $(0, 0, 0, 0, 0, 0, 0, 8) = 1$

$$\text{S.D.} = \sqrt{\frac{7(0-1)^2 + (8-1)^2}{8}} = \sqrt{\frac{7+49}{8}} = \sqrt{7}$$

**16.** S.D. = $\sqrt{\dfrac{\Sigma x_i^2}{n} - \left(\dfrac{\Sigma x_i^2}{n}\right)}$

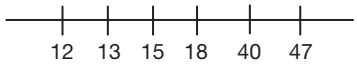
$$3 = \sqrt{\frac{\Sigma x_i^2}{10} - (25)^2}$$

$$9 = \frac{\Sigma x_i^2}{10} - 625$$

$$\Sigma x_i^2 = 6340$$

**17.** I : Range (2, 3, 5, 7, 9, 11, 6)
= maximum value – minimum value = 11 – 2 = 9 ≠ 4
∴ I is false.
II : Range (10, 20, 25, 30, 45, 80) = 80 – 10 = 70
∴ II is true.

**18.** As $x$ is unknown, three cases arise after arranging the numbers on the number line in order.




(i) $x \le 15$; 15 is the median.
(ii) $x \ge 18$; 18 is the median.
(iii) $15 < x < 18$; x is the median.
Hence, 19 cannot be the median.

**19.** The observations are $k$, $k+3$, $k+6$, …, $k+30$ where $k$ is a multiple of 3.

$$\text{S.D.} = |d|\sqrt{\frac{n^2-1}{12}} = 3\sqrt{\frac{121-1}{12}} = 3\sqrt{10} = \sqrt{90}$$

**20.** H.M. = (1, 1/3, 1/6, 1/10, 1/12)

$$= \frac{5}{1+3+6+10+12} = \frac{5}{32}$$

**21.** We have, sin179° = sin1°, sin178° = sin2°, …, sin91° = sin89°
∴ The increasing order of the values sin1° . sin2°, sin3°, … sin89°, sin90°, … sin179° is sin1°. sin179°. sin2°, sin178°… sin90°.
There are 179 observations. So, the middle observation is the 90th observation which is sin45°.
Hence, the median of the series is sin45°, i.e., $\frac{1}{\sqrt{2}}$.

**22.** Let $x_1, x_2, \ldots, x_{12}$ be the twelve numbers.
Then, $(x_1 - 9) + (x_2 - 9) + \ldots (x_{12} - 9) = 60$
Given,
$\Rightarrow x_1 + x_2 + \ldots + x_{12} = 60 + 108 = 168$
Hence, the A.M. $(x_1 . x_2 \ldots x_{12}) = \frac{168}{12} = 14$

**23.** We know that, the H.M. $(x_1, x_2, \ldots x_n)$

$$= \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}}$$

∴ The H.M.(1, 2, 4, 7, 14, 28)

$$= \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}}$$

$$= \frac{6}{\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{7}+\frac{1}{14}+\frac{1}{28}} = \frac{6}{\left(\frac{56}{28}\right)} = 3$$

**24.** A.M. (12, 5, 9, 15, 31, 20, 4, 17, 22)

$$= \frac{12+5+9+15+31+20+4+17+22}{9} = \frac{135}{9} = 15$$

$$\therefore \text{The mean deviation} = \frac{\Sigma|x_i - M|}{n}$$

$$= \frac{3+10+6+0+16+5+11+2+7}{9} = \frac{60}{9} = \frac{20}{3}$$

**25.** Arranging the given numbers in increasing order, we get 2, 11, 12, 12, 17, 19, 23, 25, 32, 39, 52.

$$\therefore Q_1 = \text{Size of } \left(\frac{n+1}{4}\right)^{\text{th}} \text{ observation}$$

$$= \text{size of } \frac{11+1}{4} = 3^{\text{rd}} \text{ observation} = 12$$

$$\therefore Q_3 = \text{Size of } 3\left(\frac{n+1}{4}\right)^{\text{th}} \text{ observation}$$

$$= \text{size of } 3\left(\frac{11+1}{4}\right) = 9^{\text{th}} \text{ observation} = 32$$

$$\text{Hence, quartile deviation} = \frac{Q_3 - Q_1}{2} = \frac{32-12}{2} = 10$$

**26.** When the given observations are written in order, we get 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, which are in arithmetic progression with common difference $d = 4$. The number of observations = 11.

$$\therefore \text{S.D.} = \text{c.d}\sqrt{\frac{n^2-1}{12}} = 4\sqrt{\frac{11^2-1}{12}} = 4\sqrt{10} = \sqrt{160}$$

**27.** When the given series (except $x$) is arranged in order, we get 6, 8, 9, 12, 16, 19, 21, 23, 25, 32, 33, 36, 37, 40, 42, 45, including $x$ there are 17 observations.
∴ The middle most observation is the median.
If $x$ is the middle most observation, then median is $x$.
In this case, $x > 23$ and $x < 25$.
If $x < 23$, then median is 23, and if $x > 25$, then median is 25.
∴ Median can be any value in the interval [23, 25].
∴ Range values of median is [23, 25].

**28.** Let $a$ and $d$ be the first term and common difference of the arithmetic progression.

Given $\quad t_3 = a + 2d = 14 \quad (1)$

and $\quad t_{10} = a + 9d = 56 \quad (2)$

By solving (1) and (2), we get $a = 2$ and $d = 6$

$t_{15} = a + (15 - 1)d = 2 + 14 \times 6 = 86$

$t_{15} = 86$

A.M. of the first 15 terms of A.P. is $\frac{t_1 + t_{15}}{2} = \frac{2 + 86}{2} = 44$

**29.** The given series when arranged in the ascending order, we have 12, 12, 18, 18, 21, 32, 32, 36, 36, 38, 38, 40, 40.

There are 13 values.

$\therefore$ Median is $\frac{n+1}{2} = \frac{13+1}{2} = 7^{\text{th}}$ observation.

$\therefore$ Median = 32

**30.** The standard deviation of two numbers $x$ and $y$ is given by $\left|\frac{x-y}{2}\right|$.

$\therefore$ The standard deviation of $p + q$ and $p - q$ is $\left|\frac{p+q-p+q}{2}\right| = q$.

**31.** Given $y_i = \text{A.M.}(x_i, x_{i+1}, x_{i+2})$ for $1 \le i \le n-2$.

So, $y_1 = \frac{x_1 + x_2 + x_3}{3}$

$\Rightarrow 3y_1 = x_1 + x_2 + x_3$

$y_{n-2} = \frac{x_{n-2} + x_{n-1} + x_n}{3}$

$\Rightarrow 3y_{n-2} = x_{n-2} + x_{n-1} + x_n$

Also, $y_{n-1} = \text{A.M.}(x_{n-1}, x_n, x_1)$

$\Rightarrow 3y_{n-1} = x_{n-1} + x_n + x_1$

and $y_n = \text{A.M.}(x_n, x_1, x_2)$

$\Rightarrow 3y_n = x_n + x_1 + x_2$

$\therefore 3y_1 + 3y_2 + \ldots + 3y_n = (x_1 + x_2 + x_3) + (x_2 + x_3 + x_4) + \ldots + (x_n + x_1 + x_2)$

$\Rightarrow 3(y_1 + y_2 + \ldots + y_n) = 3(x_1 + x_2 + \ldots + x_n)$

$\Rightarrow y_1 + y_2 + \ldots + y_n = x_1 + x_2 + \ldots + x_n$

Hence, $\text{A.M.}(y_1, y_2, \ldots, y_n) = \text{A.M.}(x_1, x_2, \ldots, x_n) = M$

**32.** We have A.M. $= \frac{\Sigma f_i x_i}{\Sigma f}$.

$= \frac{6 + 10 + 27}{2 + 5 + 2} = \frac{43}{10} = 4.3$

**33.** We know that when a constant term is subtracted from each of the given observations, the standard deviation remains unchanged.

$\therefore$ S.D.(101, 102, 103,..., 111)

= S.D.(1, 2, 3, ..., 11)

(Subtracting 100 from each of the observations) = $M$

**34.** Arranging the given numbers other than '$x$' in increasing order, we get, 3, 4, 5, 6, 8, 9, 11, 14, 15, 23, 25, 25, 29 and 39.

If $x \le 11$, then the median is 11.

If $x \ge 14$, then the median is 14.

If $11 < x < 14$, then the median is $x$.

Hence, the range of the values of the median is [11, 14]

**35.** We know that the sum of the cubes of the first '$n$' natural numbers is $\frac{n^2(n+1)^2}{4}$.

So, the required arithmetic mean is $= \frac{\frac{n^2(n+1)^2}{4}}{n} = \frac{n(n+1)^2}{4}$.

**36.** Let $t_1$, $t_2$ be the time taken by the man to cover the first 100 km and the second 100 km, respectively.

Then, $V_1 = \frac{100}{t_1} \Rightarrow t_1 = \frac{100}{V_1}$

and $V_2 = \frac{100}{t_2} \Rightarrow t_2 = \frac{100}{V_2}$

Now, let $V$ be the average speed of the motor cycle for the entire journey.

Then, $V = \frac{200}{t_1 + t_2}$

$\Rightarrow V = \frac{200}{\frac{100}{V_1} + \frac{100}{V_2}}$

$\therefore V = \frac{2}{\frac{1}{V_1} + \frac{1}{V_2}}$

i.e., $V$ is the harmonic mean of $V_1$ and $V_2$.

**37.** S.D. (7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 18)

= S.D. (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 11)

Now, A.M. (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 11)

$= \frac{11}{11} = 1$

$\therefore$ S.D. (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 11)

$= \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$, where $\bar{x}$ is the arithmetic mean.

$= \sqrt{\frac{1+1+1+1+1+1+1+1+1+1+(10)^2}{11}}$

$= \sqrt{\frac{110}{11}}$

$= \sqrt{10}$

**38.** The standard deviation of the observations 3, 12, 6, 10, 14, 21 and 24 is $p$.

The observations 6, 33, 15, 27, 39, 60 and 69 can be written as 3 (3) – 3,12 (3) – 3,6 (3) – 3, 10 (3) – 3, 14 (3) – 3 and 21 (3) – 3.

$\therefore$ The standard deviation of the new series is $3p$.

Variance for the new series is $(\text{S.D.})^2 = 9p^2$.

**39.** The harmonic mean of $x_1, x_2, x_3, ..., x_n$ is

$$\frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+.....+\frac{1}{x_n}}.$$

$\therefore$ The harmonic mean of $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, ..., \frac{1}{20}$ is

$$\frac{10}{2+4+6+...+20}=\frac{10}{110}=\frac{1}{11}.$$

**40.** When the given data is arranged in order, we get 4, 6, 8, 10, 12, 13, 14, 15, 18, 19, 21.

$Q_1$ is the $\left(\frac{n+1}{4}\right)^{th}$ observation, i.e., $\frac{11+1}{4}=$ 3rd observation.

$Q_1 = 8$

$Q_3 = 3\left(\frac{n+1}{4}\right)^{th}$ observation

$= 3\left(\frac{11+1}{4}\right) = 9^{th}$ observation

$= 18$

$\therefore$ Q.D. $= \frac{Q_3 - Q_1}{2} = \frac{18-8}{2} = 5$

## EXERCISE-4

***Solutions for questions 1 to 25:***

**1.** Given that the sum of the deviations of the elements $x_1, x_2, x_3, ..., x_i$ about 25 is – 2.

i.e., $(x_1 - 25) + (x_2 - 25) + (x_3 - 25) + ... + (x_{10} - 25)$

$= -2 \therefore x_1 + x_2 + x_3 + ... + x_{10} = 248$

**2.** A.M. $(1, 2, 3, 4, ..., n)$

Now, A.M. $(2 \times 1, 3 \times 2, 4 \times 3, 5 \times 4, ..., (n+1)\ n)$

= A.M. $(1^2 + 1, 2^2 + 2, 3^2 + 3, 4^2 + 4, ..., n^2 + n)$

$$=\frac{\left(1^2+2^2+...+n^2\right)+(1+2+3+...+n)}{n}$$

$$=\frac{\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}}{n}$$

$$=\frac{(n+1)(2n+1)}{6}+\frac{(n+1)}{2}=\frac{(2n+4)(n+1)}{6}$$

**3.** Var $(5x_i + 7) = 225$

S.D. $(5x_i + 7) = 15$

S.D. $(5x_i) = 15$

S.D. $(x_i) = 3$

S.D. $(7x_i + 5) = 7 \times$ S.D. $(x_i) = 7 \times 3 = 21$

**4.** Given S.D. = 2; $\bar{x} = 9$, $n = 15$

$$\text{S.D.} = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$2 = \sqrt{\frac{\Sigma x_i^2}{15} - (9)^2}$$

$$4 = \frac{\Sigma x_i^2}{15} - 81$$

$$85 = \frac{\Sigma x_i^2}{15}$$

$Sx_i^{2-} = 85 \times 15 = 1275$

**5.** Given distribution:

| $x$ | 6 | 5 | 4 | 3 | 2 | 1 |
|---|---|---|---|---|---|---|
| $f$ | 3 | 5 | 8 | 7 | 12 | 15 |

Mode = data item with maximum occurrence = 1 (since frequency is 15).

**6.** The sum of the cubes of the first $n$ even natural numbers

$$= 8\left(\frac{n(n+1)^2}{2}\right).$$

Their arithmetic mean $= 8\frac{\left(\frac{n(n+1)}{2}\right)^2}{n} = 2n(n+1)^2.$

**7.** The sum of the squares of first '$n$' natural numbers is $n(n+1)\ (2n+1)/6$.

If $n = 9$, we get the sum as $9(9+1)\ (2(9)+1)/6 = 285$.

We deduct $1^2 + 2^2 + 3^2$ from 285 to get 271.

We are given 6 numbers. The mean of the required numbers is $271/6 = 45.16$.

**8.** The numbers are in geometric progression. The sum of these numbers is $4.\frac{4^6-1}{4-1} = 4.\frac{4095}{3} = 5460$.

The arithmetic mean is $5460/6 = 910$.

**9.** $\bar{x} = \Sigma x/n$

So, $\Sigma x = n\bar{x}$

The sum of the numbers is $50 \times 42 = 2100$.

On subtracting 75 and 105 from 2100, we get 2100 – (105 + 75) = 1920.

So, the sum of the remaining set of 48 observations is 1920. Hence, the mean of the remaining set is 1920/48 = 40.

**10.** Given $n_1 = 7$, $\bar{x}_1 = 36$; $n_2 = 13$, $\bar{x}_2 = 46$.

Combined mean $\bar{x}_c = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \dfrac{7\times 36 + 13\times 46}{7+13}$

$= \dfrac{850}{20} = 42.5$

**11.** H.M. $(x_1, x_2, x_3, \ldots, x_n) = \dfrac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}}$

H.M. $(1, \frac{1}{4}, \frac{1}{7}, \frac{1}{8}, \frac{1}{10}) = \dfrac{5}{1+4+7+8+10} = \dfrac{5}{30} = \dfrac{1}{6}$

**12.** G.M. (G.P.) $= \sqrt{\text{first term} \times \text{last term}}$

G.M. $(3^1, 3^2, 3^3, \ldots, 3^{99})$

$= \sqrt{3\times 3^{99}} = 3^{50} = 9^{25}$

**13.** Arranging the numbers in ascending order, we have 15.2413, 15.3125, 15.3142, 15.3143, 15.3215, 15.4132, 15.5213. Since there are 7 terms, the median is $\left(\dfrac{7+1}{2}\right)^{\text{th}}$ term, i.e., $4^{\text{th}}$ term.

$\therefore$ median = 15.3143

**14.** As the two numbers included into the series flank the median (M), the position of the median is not affected. Hence, the median remains the same.

**15.** The arithmetic mean of $a$ and $b$ is $\dfrac{a+b}{2}$.

S.D. $= \sqrt{\dfrac{(a-(a+b)/2)^2 + (b-(a+b)/2)^2}{2}}$

S.D. $= \sqrt{(a-b)^2/4} = \left|\dfrac{a-b}{2}\right|$. As $a > b$, $|a-b| = a-b$.

Thus, S.D. $= (a-b)/2$

**16.** $\Sigma(x-7) = 40$

$\Rightarrow \Sigma x - \Sigma 7 = 40$

Dividing throughout by 10, we get $\Sigma x/10 - \Sigma 7/10 = 40/10$.

$\bar{x} - 7 = 4;\ \bar{x} = 11$

**17.** Mean (calculated) of 11 observations = 36

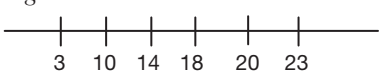
Total of 11 observations = 36 (11)

Correct mean = Total − (sum of wrong observations) + sum of corrected values + new observations/(11 + 2)

$= \dfrac{396 - (32+25+16) + (23+52+61) + 45 + 42}{13}$

$= \dfrac{396 - 73 + 223}{13} = 42$

**18.** On placing the numbers on the number line, we get




If $x < 14$, then the median = 14, if $x > 18$ then, median = 18 and if $14 < x < 18$, then the median is $x$ itself.

Thus, $14 \le$ median $\le 18$.

**19.** The sum of the first $x$ odd natural numbers = $xy$. The $p$th even natural number is 1 more than the $p$th odd natural number.

$\therefore$ The sum of the first $x$ even natural numbers = $xy + x(1)$ = $x(y+1)$.

$\therefore$ The arithmetic mean of the first $x$ even natural numbers $= \dfrac{x(y+1)}{x} = y+1$.

**20.** A.M. $(ax_1 + b_1, ax_2 + b_2 \ldots ax_n + b) = a$(A.M. $(x_1, x_2, \ldots, x_n) + b$.

Given A.M. $(y_1, y_2, y_3, \ldots y_n) = A$

$\therefore$ A.M. $\left(\dfrac{3y_1+2}{4}, \dfrac{3y_2+2}{4}, \ldots, \dfrac{3y_n+2}{4}\right) = \dfrac{3A+2}{4}$

**21.** $\dfrac{Y_1 + Y_2 + \ldots + Y_n}{n} = M$

$Y_1 + Y_2 + \ldots + Y_{n-2} + Y_{n-1}\ \ Y_n = M_n$

$Y_1 + Y_2 + \ldots + Y_{n-2} + Y_n = M_n - Y_{n-1}$

$Y_1 + Y_2 + \ldots + Y_{n-2} + Y_r + Y_n = M_n - Y_{n-1} + Y_r$

$\therefore$ The arithmetic mean of the new series will be $\dfrac{M_n - Y_{n-1} + Y_r}{n}$.

**22.** $\dfrac{x_1 + x_2 + \ldots + x_n}{n} = x$

$\Rightarrow x_1 + x_2 + \ldots + x_n = nx$ (1)

$\dfrac{y_1 + y_2 + \ldots + y_n}{n} = y$

$\Rightarrow y_1 + y_2 + \ldots + y_n = ny$ (2)

Subtracting (2) from (1),

$(x_1 - y_1) + (x_2 - y_2) + \ldots + (x_n - y_n) = n(x-y)$

Required mean $= \dfrac{x_1 - y_1 + x_2 - y_2 + \ldots + x_n - y_n}{n} = \dfrac{n(x-y)}{n} = x - y$

**23.** Arithmetic mean $= \dfrac{2(5) + 3(4) + 4(6)}{2+3+4} = \dfrac{46}{9}$

**24.** Let the side of ABCD be $x$ pm.

Time taken by Raju to travel AB $= \dfrac{x}{S_1}$ hours.

Time taken by Raju to travel BC $= \dfrac{x}{S_2}$ hours.

Average speed $= \dfrac{2x}{\frac{x}{S_1} + \frac{x}{S_2}} = \dfrac{2S_1S_2}{S_1 + S_2}$, which is the harmonic mean of $S_1$ and $S_2$

**25.** $Z_1 = \dfrac{Y_1 + Y_2 + Y_3 + Y_4}{4}$

$Z_2 = \dfrac{Y_2 + Y_3 + Y_4 + Y_5}{4}$

$Z_3 = \dfrac{Y_3 + Y_4 + Y_5 + Y_6}{4} \ldots$

$Z_{n-2} = \dfrac{Y_{n-2} + Y_{n-1} + Y_n + Y_1}{4}$

$Z_{n-1} = \dfrac{Y_{n-1} + Y_n + Y_1 + Y_2}{4}$

$Z_n = \dfrac{Y_n + Y_1 + Y_2 + Y_3}{4}$

$\therefore$ It would be seen that each $Y_i$, where $1 \le i \le n$ occurs exactly in four of the $Z_i$ s.

$\therefore$ Arithmetic mean of $Z_1, Z_2, \ldots, Z_n$

$= \dfrac{4(Y_1 + Y_2 + \ldots + Y_n)}{4n} = A$