INTRODUCTION TO CONTINUOUS CONTROL SYSTEMS

COLUMBIA UNIVERSITY MECHANICAL AND ELECTRICAL ENGINEERING DEPARTMENTS: E3601

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Homework 10

Problem 1 (Routh)**.**
Use the Routh criterion on the following polynomials to determine the number of roots in the right-half complex plane.
A.

$$P(s) = s^7 + 3s^6 + 11s^5 + 19s^4 + 36s^3 + 38s^2 + 36s + 24 \tag{1}$$

B.

$$P(s) = s^6 + 2s^5 + 4s^4 + 8s^3 + 6s^2 + 8s + 4 \tag{2}$$

Problem 2 (Routh-Hurwitz)**.**
A system has the following characteristic polynomial,

$$s^5 + 8s^4 + 24s^3 + 32s^2 + as + ab = 0 \tag{3}$$

where a and b are unspecified parameters. Use the Routh criterion to determind the constraints that must be imposed on the values of a and b to make the above system stable.

Problem 3 (Lyapunov Stability)**.**
A. *Does the choice of* $\mathbf{Q} = \mathbf{I}$ *provide a Lyapunov function for the following system,*

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -1 & 1 \\ -4 & -1 \end{bmatrix} \mathbf{x}(t) \tag{4}$$

B. Determine the stability of the above system using Routh-Hurwitz.

C. Find a Lyapunov function for the above system.

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Problem 4 (Root Locus).
Consider the closed-loop system of Figure 1 where the transfer functions for the corresponding blocks are given by the following,

$$C(s) = s+4 \tag{5}$$

$$G(s) = \frac{1}{s(s+2)(s+3)(s+6)} \tag{6}$$

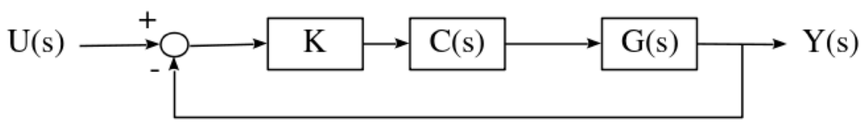

Fig. 1: Closed-Loop Block Diagram

A. *Find all the pertinent information according to rules 1 through 11 and plot the root locus according to the outcome of these rules.*

B. *Using the above application of rules, dettermine at what K the system will go unstable.*

C. *Plot the root locus using Matlab and compare to your hand-drawn plot.*

Problem 5 (Amplitude and Phase).
Express the following solutions by combining cosine and sine pairs to be represented in terms of sine only, with an amplitude and a phase.

A.

$$y(t) = \frac{10}{13}e^{-2t} - \frac{17}{10}e^{-t} - \frac{9}{130}\cos(3t) - \frac{7}{130}\sin(3t) \tag{7}$$

B.

$$y(t) = \frac{103}{100}e^{-2t} + \frac{29}{10}te^{-2t} - \frac{3}{100}\cos(4t) + \frac{1}{25}\sin(4t) \tag{8}$$

C.

$$y(t) = 1 - \frac{35}{1261}\cos(6t) + \frac{6}{1261}\sin(6t) - \frac{2}{1261}e^{-\frac{t}{2}}\left(613\cos(\frac{\sqrt{3}}{2}t) + \frac{1262}{\sqrt{3}}\sin(\frac{\sqrt{3}}{2}t))\right) \tag{9}$$

D.

$$y(t) = 1 + \frac{1}{5}e^{-2t} - \frac{6}{5}\cos(t) + \frac{2}{5}\sin(t) \tag{10}$$