

**INTRODUCTION TO CONTINUOUS CONTROL SYSTEMS**  
**COLUMBIA UNIVERSITY MECHANICAL AND ELECTRICAL ENGINEERING**  
**DEPARTMENTS: E3601**

*Homayoon Beigi<sup>†</sup>*

1340 Mudd Building  
Columbia University, New York City, NY 10027  
hb87@columbia.edu

## Homework 3

**Problem 1** (Analyticity of the Trigonometric Functions).

*Prove that the following functions are analytic everywhere in the  $\mathbb{C}$  plane, except at some special points, and that  $\sin(s)$  and  $\cos(s)$  are periodic with period  $2\pi$ .*

*Trigonometric Functions of complex variable  $s$  are defined in terms of the Exponential Function,  $e^s$ , as follows,*

$$\sin(s) \triangleq \frac{e^{is} - e^{-is}}{2i} \quad (1)$$

$$\cos(s) \triangleq \frac{e^{is} + e^{-is}}{2} \quad (2)$$

$$\csc(s) \triangleq \frac{1}{\sin(s)} \quad (3)$$

$$\sec(s) \triangleq \frac{1}{\cos(s)} \quad (4)$$

$$\tan(s) \triangleq \frac{\sin(s)}{\cos(s)} \quad (5)$$

$$\cot(s) \triangleq \frac{\cos(s)}{\sin(s)} \quad (6)$$

---

COPYRIGHT HOMAYOON BEIGI, 2025 THIS DOCUMENT IS COPYRIGHTED BY HOMAYOON BEIGI AND MAY NOT BE SHARED WITH ANYONE OTHER THAN THE STUDENTS REGISTERED IN THE COLUMBIA UNIVERSITY EEME-E3601 COURSE.

<sup>†</sup>Homayoon Beigi is Professor of Professional Practice in the department of mechanical engineering and in the department of electrical engineering at Columbia University