INTRODUCTION TO CONTINUOUS CONTROL SYSTEMS
COLUMBIA UNIVERSITY MECHANICAL AND ELECTRICAL ENGINEERING DEPARTMENTS: E3601

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Homework 3

Problem 1 (Analyticity of the Trigonometric Functions)**.**
Prove thet the following functions are analytic everywhere in the $\mathbb{C}$ *plane, except at some special points, and that* $\sin(s)$ *and* $\cos(s)$ *are periodic with period* 2π*.*

Trigonometric Functions of complex variable s are defined in terms of the Exponential Function, e^s*, as follows,*

$$\sin(s) \triangleq \frac{e^{is}-e^{-is}}{2i} \tag{1}$$

$$\cos(s) \triangleq \frac{e^{is}+e^{-is}}{2} \tag{2}$$

$$\csc(s) \triangleq \frac{1}{\sin(s)} \tag{3}$$

$$\sec(s) \triangleq \frac{1}{\cos(s)} \tag{4}$$

$$\tan(s) \triangleq \frac{\sin(s)}{\cos(s)} \tag{5}$$

$$\cot(s) \triangleq \frac{\cos(s)}{\sin(s)} \tag{6}$$

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