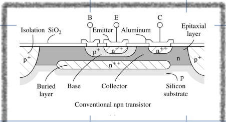
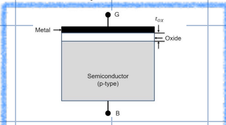
# BJT



11/7 1. Basic Theory / Band Diagram / Carrier Distribution / Current / Different Operation Regime

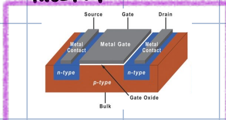
11/14 2. Capacitance / Different Model / Nonideal Effect

# MOS Cap



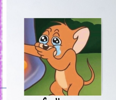
12/5 1. Basic Theory / Band Diagram / Carrier Distribution / Current / Different Operation Regime

# MOSFET 12/12 final review



1. Basic Operation / long vs short channel / Secondary Effect

Final Exam 12/18

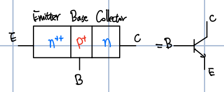


finally...

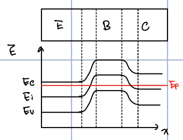
11/28 

## Bipolar Junction Transistor

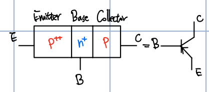
N-type (npn structure)



Band Diagram at Equilibrium

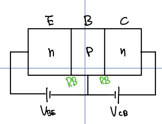


Similarly, P-type (pnp structure)

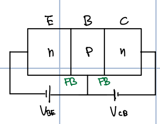


BJT Operation Modes

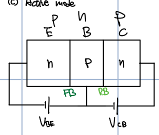
(a) Cutoff mode



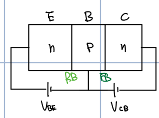
(b) Saturation mode



(c) Active mode

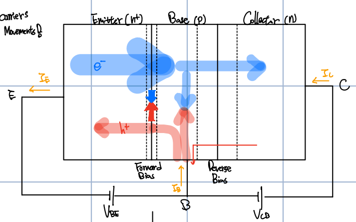


(d) Reverse Active

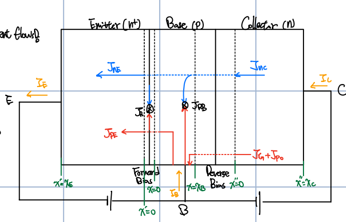


(c) Active mode (Amplifier!)

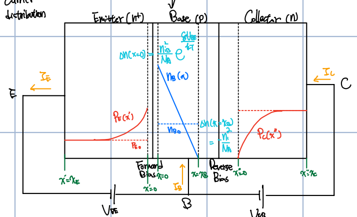
carriers Movements!



current flow!



carrier distribution



for Currents:

$J_{nE}$ : Diffusion of minority carrier electrons in the base at $x=0$

$J_{nC}$ : Diffusion of minority carrier electrons in the base at $x=x_B$

$J_{RB}$ : Difference between $J_{nE}$ and $J_{nC}$ ⇒ Recombination in Base (also the $h^+$ flow into base to replace the $h^+$ lost by recombination)

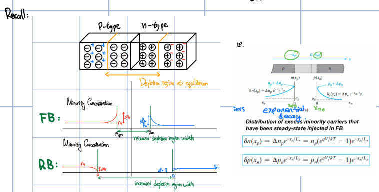
$J_{pE}$ : Diffusion of minority carrier holes in the emitter at $x'=0$

$J_R$ : (Nonideal) Recombination in FB B-E junction

$J_G$ : (Nonideal) Generation in RB B-C junction

$J_{p0}$ : Reverse-saturation current in B-C junction

Recall:



KVL & KCL:

$$V_{CE} = V_{BE} + V_{CB}$$

$$I_E = I_B + I_C$$

Gain:

$$I_C = \beta I_B$$

$$I_E = I_C + I_B = I_C + \frac{I_C}{\beta} = \left(\frac{1+\beta}{\beta}\right) I_C = \frac{1}{\alpha} I_C$$

$$\Rightarrow I_C = \alpha I_E \quad \text{where } \alpha = \frac{\beta}{\beta+1}$$

$$\because \alpha = \frac{I_C}{I_E} = \frac{J_{nC}}{J_{nE}+J_{pE}+J_R} = \frac{J_{nE}+J_{pE}}{J_{nE}+J_{pE}+J_R} \times \frac{J_{nE}}{J_{nE}+J_{pE}} \times \frac{J_{nC}}{J_{nE}}$$

$\delta$ (Recombination Factor)

$\gamma$ (Emitter Injection Efficiency) = what enters and exists Emitter

$\alpha_T$ (Base Transport Factor) [B] = How much is left after passing through Base

$$\beta = \frac{\alpha_T \gamma}{1-\alpha_T \gamma}$$

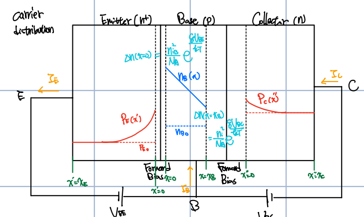
$$I_C = A_E q \frac{D_B}{W_B} \frac{n_{iB}^2}{N_B} \left(e^{\frac{qV_{BE}}{kT}} - 1\right) = A_E \frac{q n_i^2}{G_B} \left(e^{\frac{qV_{BE}}{kT}} - 1\right) = I_S \left(e^{\frac{qV_{BE}}{kT}} - 1\right)$$

$$G_B = \frac{n_i^2}{n_{iB}^2} \frac{N_B}{D_B} W_B = \frac{n_i^2}{n_{iB}^2} \cdot \frac{P}{D_B} W_B \quad \text{for npn}$$

$$I_B = A_E q \frac{D_E}{W_E} \frac{n_{iE}^2}{N_E} \left(e^{\frac{qV_{BE}}{kT}} - 1\right) = A_E \cdot q \cdot \frac{n_i^2}{G_E} \left(e^{\frac{qV_{BE}}{kT}} - 1\right)$$

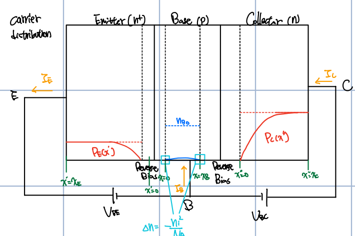
(b) Saturation

carrier distribution



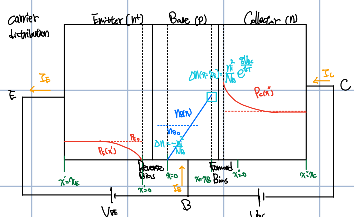
(c) Cutoff

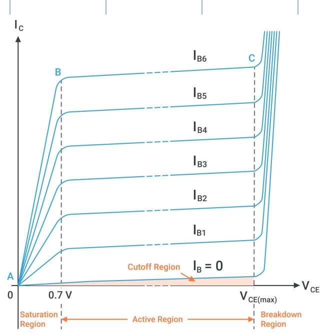
carrier distribution



(d) Inverse Active

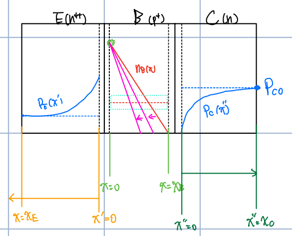
carrier distribution





## Nonideal Effect for BJT

① Base Width Modulation




when $V_C$ ↑ → RB ↑ → Depletion Region of Base-Collector Junction ↑ (ie. $x_B$ ↓)
→ become steeper (slope ↑) ⇒ $I_C$ ↑ (from the formula: slope decides current)

ie. →

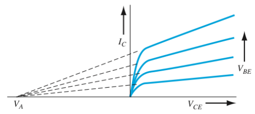
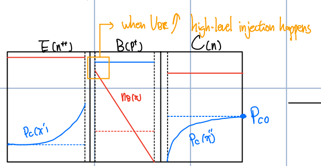


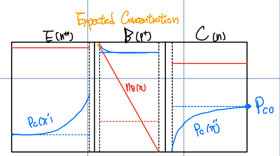

**Figure 12.22** | The collector current versus collector–emitter voltage showing the Early effect and Early voltage.

aka. Early Effect $\left[ r_0 = \left(\frac{\partial I_C}{\partial V_A}\right)^{-1} = \frac{V_A}{I_C} \right]$

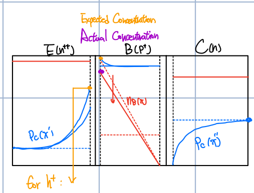
② High-level Injection




→




→ but that also means EHPs recombination ↑
can't reach the expected conc.

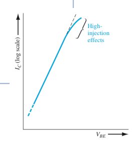


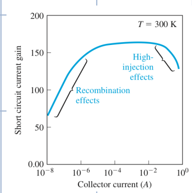

for $h^+$:
not reaching high-level injection yet
∴ Expected conc. = Actual conc.

∴ For these two points: Expected $J_{nE}$ ↑ $J_{pE}$ ↑ Actual $J_{nE}$ ✗ $J_{pE}$ ↑ ⇒ $\gamma = \frac{1}{1+\frac{J_{pE}}{J_{nE}}}$ ↑
(decide two current)

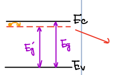
∴ $\gamma$ ↓ ⇒ $\alpha$ & $\beta$ ↓

→ ie.




③ Emitter Bandgap narrowing



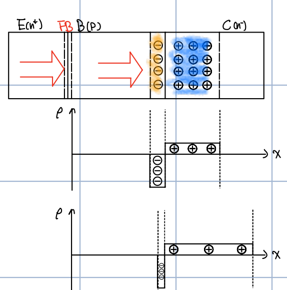

→ discrete donor energy level splits into a band of energies

⇒ ∵ $E_g \to E_g'$ ⇒ $n_i$ also changes ⇒ $\gamma = \frac{1}{1+\frac{G_B}{G_E}} \simeq \frac{1}{1+\frac{x_B D_E N_B n_{iE}^2}{x_E D_B N_E n_{iB}^2}}$

$E_g$ ↓ ⇒ $n_{iE}^2$ ⇒ $\gamma$ ↓ ⇒ $\alpha$ & $\beta$ ↓

How to resolve? Use wide-bandgap material for Emitter ⇒ HBT ⇒ increase $\alpha$
Heterojunction Bipolar Transistor

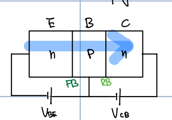
④ Kirk Effect



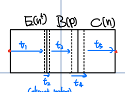

Under FB, large $e^-$ current flows from E to C
So inside BC junction, negative charged carrier ↑ → DR ↓
→ Base quasi-neutral region width ↑ (ie. $x_B$ ↑) ⇒ $\alpha$ & $\beta$ ↓

Same, positive charged carriers recombines more → DR ↑
equivalent charge ↓ to compensate

⑤ Nonuniform Base Doping (we want this!)

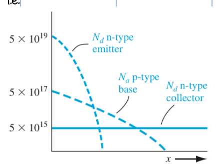



we want shorter transit time
(E ⟶ C)

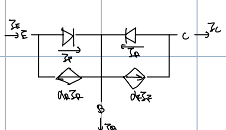



→ to make it faster:
F=ma: make a E-field!
a E-field with ← direction
but which side higher? Emitter or Collector?
∵ p-type, if distribution is like ╲ ⇒ ⊖ ← $h^+$ ∴ Emitter higher
$h^+$ ⤳ diffuse

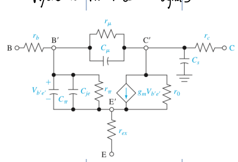
ie.




Ebers-Moll Model for DC (large-signal)




Hybrid-π model (small-signal)




$I_B = I_F + I_R - \alpha_F I_F - \alpha_R I_R$
$= (I_F - \alpha_R I_R) - (\alpha_F I_F - I_R)$
$= I_E - I_C$

$I_E = I_F - \alpha_R I_R = I_{E0}(e^{\frac{V_{EB}}{V_T}}-1) - \alpha_R I_{C0}(e^{\frac{V_{CB}}{V_T}}-1)$

$I_C = \alpha_F I_F - I_R = \alpha_F I_{E0}(e^{\frac{V_{EB}}{V_T}}-1) - I_{C0}(e^{\frac{V_{CB}}{V_T}}-1)$

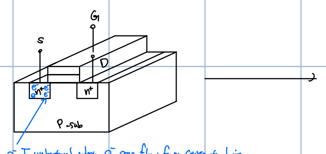
Active Mode:

$I_E = I_{E0}(e^{\frac{V_{EB}}{V_T}}-1) + \alpha_R I_{C0}$

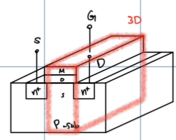
$I_C = \alpha_F I_{E0}(e^{\frac{V_{EB}}{V_T}}-1) + I_{C0}$

# MOS Capacitor

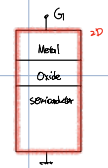
For MOSFET:

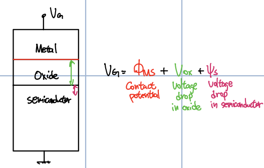


$e^-$: To understand when $e^-$ can flow from source to drain
we first need to understand when "the channel" turn on



we focus on MOS structure first
(metal-oxide-semiconductor)
Aka two-terminal device

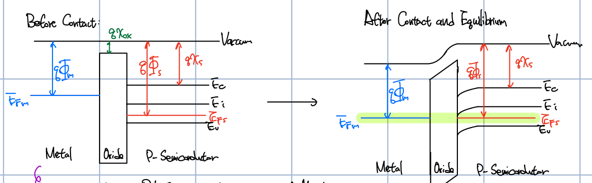




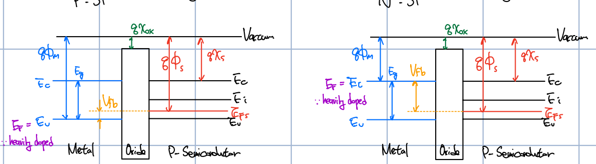
$V_G = \Phi_{MS} + V_{ox} + V_s$

Contact potential; Voltage drop in oxide; Voltage drop in semiconductor

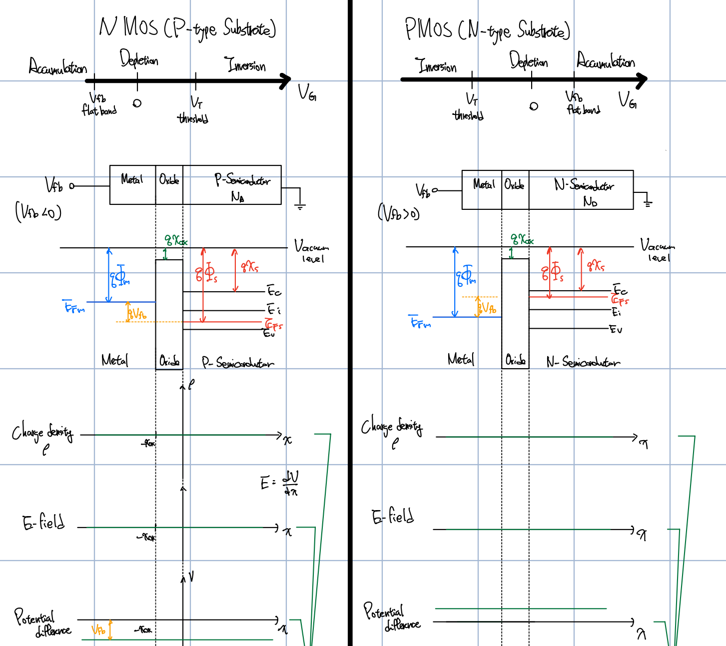
Band Diagrams & Big Three Plots for Different Operating Regimes ⟷ Physic Implication: How much the Energy bands must bend to balance the applied voltage




since people like to use Poly-Si as gate electrodes instead of Metal




## A. Flat-Band




Formula

$V_{fb} = \Phi_{MS} + V_{ox} + \psi_s$

$= \Phi_m - \Phi_s + 0 + 0$

$= \Phi_m - (q\chi_s + \frac{E_g}{2} + \phi_B)$

$= \Phi_m - (q\chi_s + \frac{E_g}{2} + \frac{kT}{q}\ln\frac{N_a}{n_i})$

☆ Assumption: there's no charge in oxide

Formula

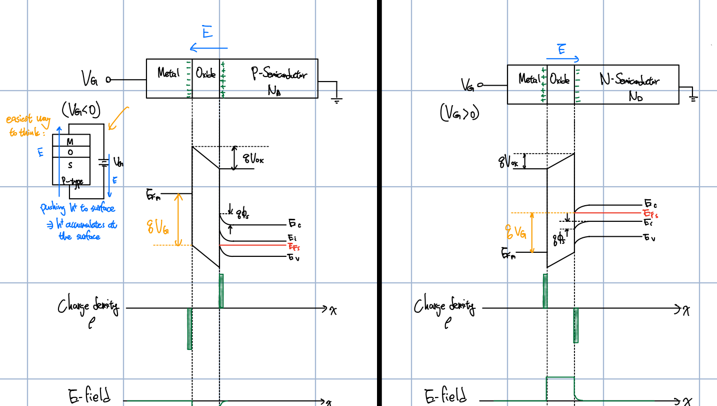
$V_{fb} = \Phi_m - \Phi_s$

$= \Phi_m - (q\chi_s + \frac{E_g}{2} - \phi_B)$

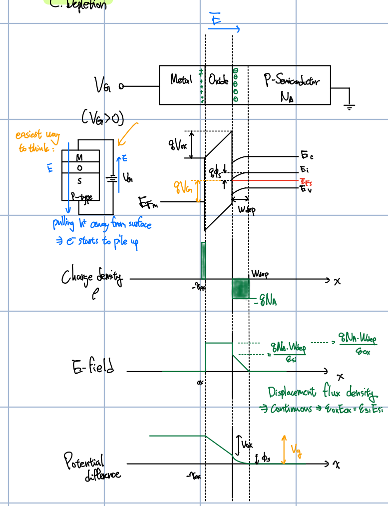
$= \Phi_m - (q\chi_s + \frac{E_g}{2} - \frac{kT}{q}\ln\frac{N_D}{n_i})$

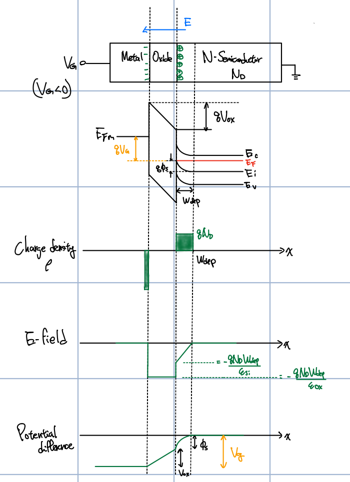
※ Assume: there's no charge in oxide

## B. Accumulation




## C. Depletion



Formula

$V_g = V_{fb} + \phi_s + V_{ox}$

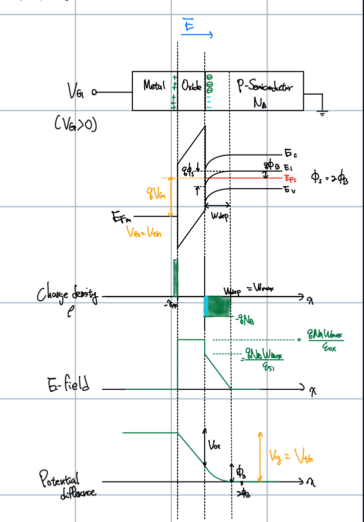
$V_{ox} = -\frac{Q_{sub}}{C_{ox}} = \frac{-Q_{dep}}{C_{ox}} = \frac{qN_aW_{dep}}{C_{ox}}$

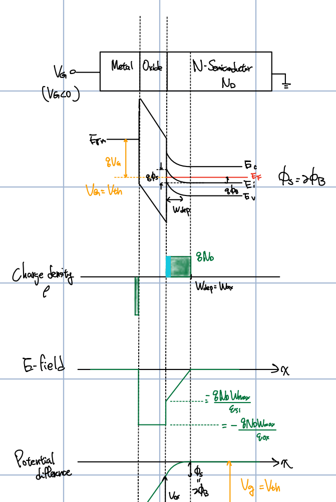
$\left(W = \sqrt{\frac{2\varepsilon}{q}V_{bi}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}\right)$ ($V_{bi} = \phi_s$)

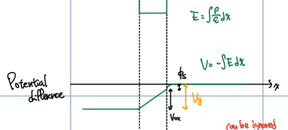
$\because W_{dep} = \sqrt{\frac{2\varepsilon_s\phi_s}{q}\frac{1}{N_a}}$ $\therefore \phi_s = \frac{qN_aW_{dep}^2}{2\varepsilon_s}$

$\Rightarrow V_g = V_{fb} + \frac{qN_aW_{dep}^2}{2\varepsilon_s} + \frac{qN_aW_{dep}}{C_{ox}}$ #

## D. Threshold






Formula

$V_g = V_{fb} + V_{ox} + \phi_s$ (can be ignored)

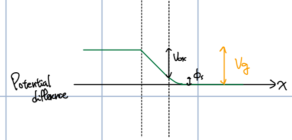
$= V_{fb} + V_{ox}$

* $P_s = N_A e^{-q\phi_s/kT}$ surface concentration

* $V_{ox}$ ⇒ $V = \frac{Q}{C}$ ⇒ $V_{ox} = -\frac{Q_{acc}}{C_{ox}}$

$Q_{acc} = -C_{ox}V_{ox}$

$= -C_{ox}(V_G - V_{fb})$




Formula

$V_g = V_{fb} + V_{ox} + \phi_s$

* $n_s = N_D e^{q\phi_s/kT}$

* $V_{ox} = -\frac{Q_{acc}}{C_{ox}} > 0$

Formula

$q\phi_B = \frac{E_g}{2} - (E_F - E_v)_{bulk} = kT \ln\frac{N_A}{n_i}$

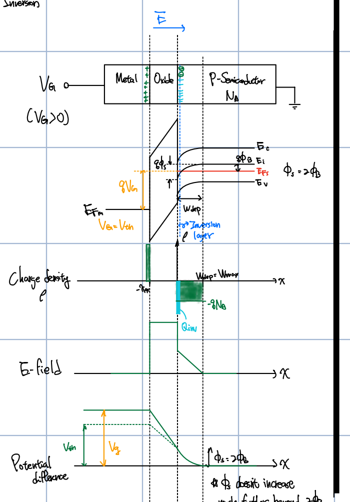
$\phi_s = 2\phi_B = 2\frac{kT}{q}\ln\frac{N_A}{n_i}$

Inversion Condition

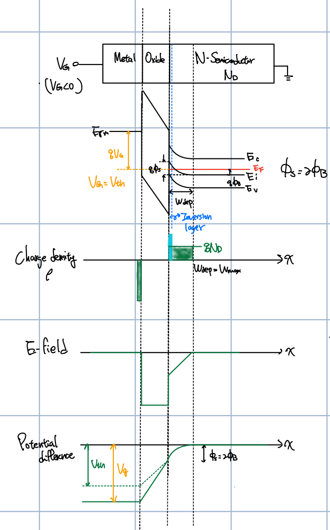
∴ $V_t = V_{fb} + \phi_s + V_{ox} = V_{fb} + 2\frac{kT}{q}\ln\frac{N_A}{n_i} + \frac{\sqrt{qN_A 2\epsilon_s 2\phi_B}}{C_{ox}}$

Threshold Voltage in Device Physics

E. Inversion




* $\phi_s$ doesn't increase much further beyond $2\phi_B$,
So, $W_{dep}$ still $= W_{max} = \sqrt{\frac{2\epsilon_s 2\phi_B}{qN_A}}$




Formula

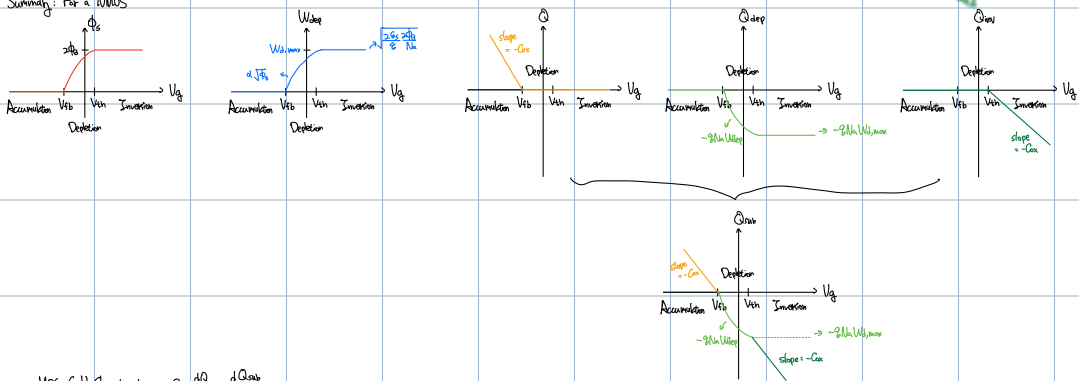
$V_g = V_{fb} + \phi_s - \frac{Q_{dep}}{C_{ox}} - \frac{Q_{inv}}{C_{ox}}$

$= V_{fb} + 2\phi_B + \frac{\sqrt{2qN_A\epsilon_s 2\phi_B}}{C_{ox}} - \frac{Q_{inv}}{C_{ox}}$

$= V_{th} - \frac{Q_{inv}}{C_{ox}}$

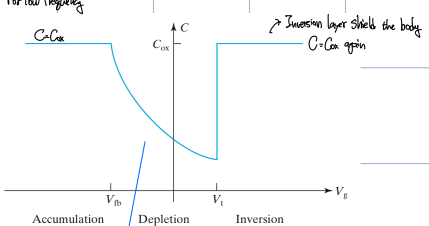
∴ $Q_{inv} = -C_{ox}(V_g - V_{th})$

Summary : For a NMOS



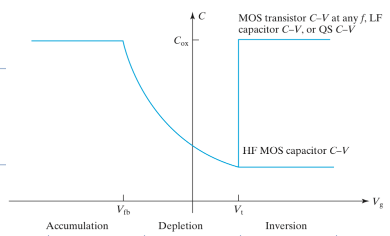

MOS C-V Characteristic ⇒ $C = \frac{dQ}{dV_g} = \frac{dQ_{sub}}{dV_g}$

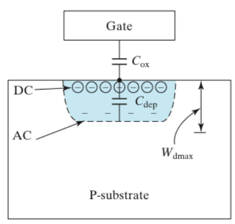
For low frequency

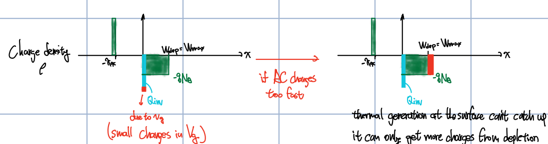



$\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_{dep}}$ where $C_{dep} = \frac{\epsilon_s}{W_{dep}}$

$= \sqrt{\frac{1}{C_{ox}^2} + \frac{2(V_g - V_{fb})}{qN_A\epsilon_s}}$

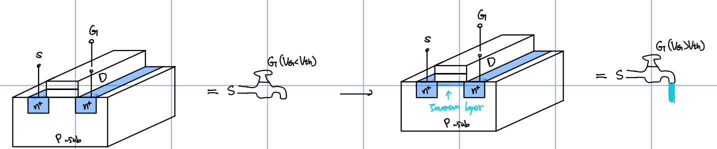


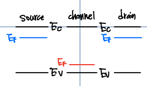




$C = C_{min}$

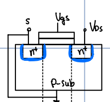
MOSFET  now, we know how to turn on the device!



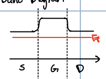




Different Operation Regions

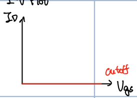
① Cutoff : $V_{gs} < V_{th}$ (no channel)
$V_{ds} = 0$



Band Diagram



I-V Plot

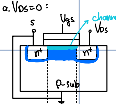


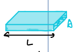

Current Calculation

$I_D = W \cdot Q_{inv}(x) \cdot v$
$= 0$

① Triode (Linear) : $V_{gs} \geq V_{th}$
$V_{DS} < V_{GS} - V_{th}$

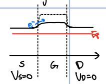
a. $V_{DS} = 0$ :



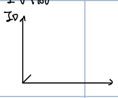




$R = \rho \frac{L}{A}$ just like a resistor

Band Diagram

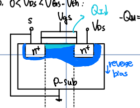


I-V Plot

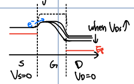


$I_D = W \cdot Q_{inv}(x) \cdot v$
$= W \cdot Q_{inv} \, \mu_n \, E$
$= W \, C_{ox}[V_{gs} - V_t - V(x)] \, \mu_n \frac{dV(x)}{dx}$
$\Rightarrow \int_0^L I_D = W C_{ox} \mu_n \int_0^{V_{DS}} V_{gs} - V_t - V \, dV$
$I_D = \frac{W C_{ox} \mu_n}{L} (V_{gs} - V_t - \frac{1}{2} V_{ds}) V_{ds}$
when $V_{ds}$ small $\Rightarrow I_D = \frac{W \cdot C_{ox} \mu_n}{L} \cdot V_{DS}(V_{GS} - V_{th})$

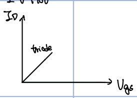
b. $0 < V_{DS} < V_{GS} - V_{th}$ : $Q_{inv}\downarrow$ $-Q_M = Q_{dep} + Q_{inv}$




Band Diagram

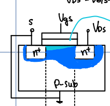



I-V Plot

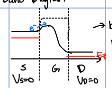



when $V_{DS}\uparrow$
$I_D = \frac{W C_{ox} \mu_n}{L} (V_{gs} - V_t - \frac{1}{2} V_{ds}) V_{ds}$

① Saturation : $V_{gs} \geq V_{th}$
$V_{DS} = V_{GS} - V_{th}$

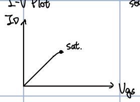



Band Diagram



→ barrier : from source
further decrease drain side doesn't help

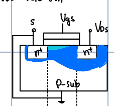

I-V Plot




when $V_{DS} = V_{GS} - V_{th}$
$I_D = \frac{W C_{ox} \mu_n}{L} (V_{gs} - V_t - \frac{1}{2} V_{ds}) V_{ds}$
$= \frac{\mu W C_{ox}}{L} [(V_{GS} - V_t) - \frac{1}{2}(V_{GS} - V_t)](V_{GS} - V_t)$
$= \frac{\mu W C_{ox}}{2L} (V_{GS} - V_t)^2$

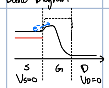
keep increasing $V_{ds}$ →

$V_{gs} \geq V_{th}$
$V_{DS} > V_{GS} - V_{th}$

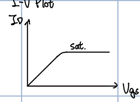


$\Rightarrow E_{pinch\text{-}off} = \frac{V_{ds} - (V_{gs} - V_t)}{L - L_1}$

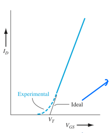
Band Diagram



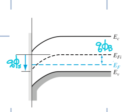
I-V Plot



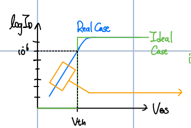

Subthreshold Conduction




→ expected $V_{gs} < V_{th}$
~~$I_D$ should~~ $= 0$
Subthreshold current: the drain currents exists for $V_{GS} \leq V_{th}$



∵ weak inversion ($\phi_B < \phi_s < 2\phi_B$)
Inverted charges $\propto \frac{q\phi_s}{kT} \Rightarrow I_D \propto \frac{qV_{GS}}{kT}$




ie. If slope↑ ⇒ closer to Ideal case ——→ SS↓

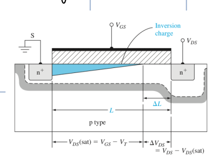
$SS = slope^{-1} = \left(\frac{\Delta \log I_D}{\Delta V_{GS}}\right)^{-1} = \frac{\partial V_{GS}}{\partial \log I_D}$ [=] $\frac{mV}{dec}$

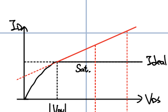
dec: how many voltage change when you change currents by 10 times

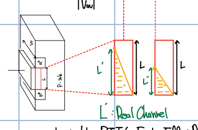
∵ SS need to be small → small leakage current is desired → less charge conc. at the interface

8

Channel length modulation



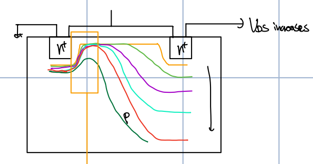



just like BJT's Early Effect!

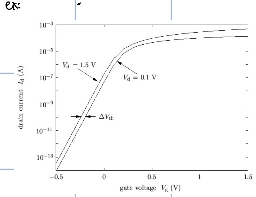
In Sat. Region, $V_{DS}\uparrow$ Pinch-off↑
∴ $L\downarrow \Rightarrow I_D\uparrow$ → need a correction
$I_D = K(V_{GS} - V_t)^2 \times (1 + \frac{V_{DS}}{V_A})$ #
$K = \frac{C_{ox} \mu W}{2L}$

DIBL (Drain-induced Barrier lowering)



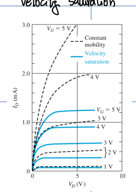

carriers can tunnel through

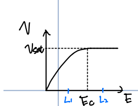
ex:



$DIBL = \frac{\Delta V_{th}}{\Delta V_d}$ [=] $\frac{mV}{V}$

Velocity saturation






$E = \frac{V_{ds}}{L}$ , $L_2 < L_1$

⇒

Mobility correction $\mu = \frac{\mu_{eff}}{\left[1 + \left(\frac{\mu_{eff} E}{v_{sat}}\right)^2\right]^{\frac{1}{2}}}$