

REVISED
EDITION
2020

QUANTUM

CAT

NEW CHAPTER
Number System

*Specially
Revised*

Profit, Loss
& Discount

Theory of
Equation

Permutations &
Combinations

Geometry

Mensuration

**2000
New MCQs**

Also Useful for
XAT | SNAP | CMAT | MAT

Sarvesh K Verma

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QUANTUM



Also Useful for
XAT | SNAP | CMAT | MAT

REVISED
EDITION
2020

QUANTUM CAT

Also Useful for
XAT | SNAP | CMAT | MAT

Sarvesh K Verma
(An IIM Alumnus)

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PREFACE

Let me take this opportunity to introduce you to the revised edition of the most sought after book for Quantitative Aptitude.

At the very outset it would be vital to let you know that this edition covers all the essential concepts, related examples, handcrafted problems with multiple solutions which are pretty useful in cracking major competitive exams across the globe.

This book is designed keeping certain key aspects in the mind –

- ✔ Every topic starts with very basic level concepts so that anyone can easily strengthen one's foundation of that topic and one does not have to struggle for basic understanding.
- ✔ Every concept is discussed in detail and supported by the variety of examples. Some topics are supported by 40-50 examples with all sorts of twists in the situations and wordings. And that's how your mind starts thinking differently.
- ✔ Shortcuts have been devised in order to reduce the time to answer the problem. Since time is the main factor in competitive exams, so it is vital to think of those solutions that are fast, accurate and reliable.
- ✔ Most of the problems are solved using alternative methods. It gives you a 360- degree approach to understand the problem and apply your lateral thinking.
- ✔ Special alternative solutions and examples are there to help the students who have left mathematics long back due to probably no or little interest in mathematics.
- ✔ Self-explanatory answers enable the students to prepare for the toughest exams without joining the coaching centers, especially the ones which charges you a hefty price.
- ✔ This edition has got thousands of new problems based on latest patterns and difficulty level to keep you updated with the new dynamics of the competitive exams.
- ✔ And, this edition has got scores of new concepts included to make you stay ahead of your competitors. I am sure that you will never find many of these new concepts in any other book or study material until they copy the same from the Quantum CAT. And, that is precisely the reason behind revising this mammoth book so that you have an extra edge over your peers.
- ✔ Every topic has 2-3 levels of questions making you prepare for the most difficult problems even beyond the CAT level. Practise these exercises based on the difficulty level of your exam.

From my experience most of the students believe in and require a through practice .That's why the number of problems based on any topic is way beyond your imagination. If you are serious about your exam, start early and refer this book smartly. There is no wonder why some 30-40 percentilers have become 99+ percentilers in 3-4 months after solving this book. This book has helped many students secure 100 percentile in CAT, All India Rank 1 in RBI Grade B exam, SBI PO and SSC CGL.

We all know that most of the students are scared about Quantitative Aptitude section. It happens because when they see a lot of graphs, functions and algebraic equations and try to solve these problems in a traditional manner they feel puzzled. However, I suggest them to look at these problems differently if they want their tension to disappear. Even I personally do not use too many formulae or equations while solving the problem. I try to use option-elimination method, or

substitution method or approximation method or diagrammatic approach or unit digit approach or some other method different from standard and traditional mathematics. After all, no one expects you to know all the theorems and formulae of mathematics. Examiners expect you to know that if you can look at the numbers, mathematical figures and graphs smartly and make some sense out of it. If you have a smart way to look at these things, you are naturally a problem solver. The idea is to test your common sense and not to identify a Ramanujan or Aryabhata in a future employee. There are numerous such solutions that have been offered in this book to develop your thought process as expected by the smart examiners.

For example, in the following figure (i), find the area of the smaller equilateral triangle if the area of the larger equilateral triangle is $60\sqrt{3}$ units. More often than not you might struggle for 2 to 3 minutes to find the relevant data, geometrical theorems and appropriate formulae. However, if you look at the figure (ii) in which I have just rotated the smaller equilateral triangle without changing its shape or size, you can easily tell me that the area of smaller triangle inside the circle is $1/4$ th of the larger triangle outside the circle, as the vertices of the smaller equilateral triangle meet the mid-points of the sides of the larger equilateral triangle and thereby creating 4 same size equilateral triangles. That means the area of the inscribed triangle is $15\sqrt{3}$ units.

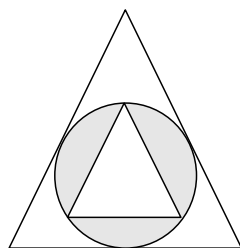


Fig (i)

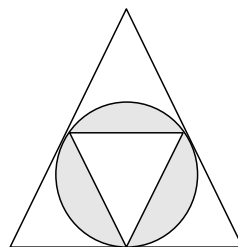


Fig (ii)

I request you to start and finish the chapter in the same order as they are given in the book. Also if you are not getting a desirable decent score, try solving each and every example and problem of the particular chapter. Mark and revise the difficult sums quite frequently and consistently to boost your confidence and score.

I have tried my best to keep it error free, however if any sort of error or discrepancy is found, it must be communicated to me if it is related to content, otherwise it must be communicated to Arihant publications and in case of any delivery issue it must be communicated to the delivery partner such as Amazon or Flipkart.

I am thankful to each one of you for letting me use this space to talk about the product – the reason for its creation, the reason for its success, the reason why one should trust it and the key features of the book, if one wants to understand the book before going through all the main pages. Now let me share some unusual yet important facts about this book with you.

- After regular appearance of its questions in CAT since its debut, from last couple of years its questions have started appearing in Bank PO and SSC CGL exams in conspicuously large numbers, as SBI PO and SSC CGL exams have increased their difficulty level a couple of notches up. Why do these exams choose to pick up the problems from Quantum CAT?

- It is because I strongly believe that only innovation driven products set the standards for others to take a leaf from. As far as competitive exams are concerned I try to design the problems that become unavoidable for a paper setter. And, that's why our students have always felt delighted to see the same or very similar questions of Quantum CAT appearing in their papers. Trust me no student want to face the uncertainty in the exam. And, that's why you could be able to experience the variety, depth and subtlety in the problems given in the Quantum CAT. Reinforcing the same philosophy, I have revamped the material of the book for the years to come.



Hello Sir,

8 November 2012 at 12:54 PM

*I appeared for my CAT exam on 5th Nov and I have to tell you that around 12 sums, from Number System, Time & Work and Geometry were directly picked up from your book. 2 sums were exactly word to word from the book and I marked the answer even before solving them (Although I didn't get overconfident and solved them before marking). Thanks a tonne! :) I can't tell u how happy I was to see questions that I had already done before!
A big thanks again!*

Swati Daga

- I am fortunate enough to be able to write a book that has been inspiring its readers and teachers alike. It does not help any of us to bask in the past glory, however, sharing my perspective with deep sense of humility will definitely help you understand the point that I am putting across. As an esteemed reader of this book you must be proud of the fact that whenever I design the problems or introduce a new concept it is well thought out from the perspective of what could be the future of the exam you are preparing for. If I am not able to visualize and foresee the trend, I won't be able to do justice to my work. Precisely, that has been the reason that I never felt to revise the Quantum CAT in last 15 years, while the other books on Quantitative Aptitude had to go through multiple revisions. In a hyper competitive environment as a student you deserve nothing but the best. When even 0.25 marks can jettison you out of the cut off list, my responsibility becomes huge in the sense that how could I develop a product which is as reliable as your parents. And, I am sure that when you see the new set of questions in Quantum CAT and you compare them with any other resource in the market, you will definitely find the positive difference with Quantum CAT.



Hello Sir,

May 24, 2015

My name is Raajit and I will be joining the PGP at IIM Ahmedabad this year. I had scored an 88 percentile in my first attempt. For the second attempt, I used your quantitative aptitude book (Quantum CAT) for preparation and through a lot of mock tests, I scored a 99+ percentile. I just personally wanted to thank you for this and when I came across your profile on Quora, I could not help but send a thank you message.

Raajit Kumar

IIM Ahmedabad, PGP 2015-17



Hi Sir,

9 July 2015 at 2:48 AM

My name is Krishnan, and I am currently based in London. I used your book – Quantum CAT extensively during 2012, and it helped me crack the CAT. I did not join the IIM at that point, but choose to continue working. Now, in 2015, I have decided to write the GMAT, and have ordered your book from India, and I am happy to return to this book after about 2 years. I would like to Congratulate you on coming up with such a wonderful book.

Thanks
Krishnan

In my opinion, a book is not just a collection of pages or random thoughts; rather it represents a personality, a belief and a purpose. I have tried my best to provide you a one-stop solution. However, I am always open to listen to your feedback and suggestions at my official email ID mentioned below. Please do get in touch with me with your valuable inputs. If you really love the book please do share your thoughts with your friends on social media. Also, you can use the hashtag #QuantumCAT while sharing your thoughts on social media.

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www.facebook.com/I.am.Sarvesh.Kumar.Verma

I wish you all the best for your bright career!

Author

ACKNOWLEDGEMENT

Let me take this opportunity to express my deepest gratitude to each and every person and institution that has been pivotal in my journey of writing the Quantum CAT and making it an enormously successful book.

First and foremost, I am thankful to maa Saraswati, the Goddess of Knowledge. I have no words to describe my feelings full of gratitude towards my beloved parents and venerable teachers. Parents have done everything for me – every support, every care and every sacrifice one can imagine. I wish every child could get the kind of support I always get from my parents. Likewise, my teachers have invariably influenced me, encouraged me, pushed me, and guided me to excel in my life, no matter what. I extend my deep gratitude to my sensei, Dr. Daisaku Ikeda for helping me realize my true potential by expanding my life state. Without him I would not have accomplished the revised edition of Quantum CAT.

I would like to thank my students spread around the globe who have encouraged, inspired and challenged me to come up with a world-class product from India. Without their ideas, questions, doubts and aspirations to succeed I would not have been able to understand the nuances of writing a book at such an early age in my life.

I deeply acknowledge the support of Jai Bhan Verma, who was instrumental in motivating me to accept this offer and embark on this promising journey of writing a book for millions of Indians. I am equally indebted to my friend and well wisher Praveen Verma, who eventually had to support me in the proof reading apart from being the most supportive person since the very first day.

I have a deep sense of gratitude towards Ms. Anjali Verma, my dearest sister, who has helped me immensely in developing the multiple choices for questions, proof reading the material, getting the courier ready to be shipped. She has worked equally hard since the very first day. If she would not have supported me, probably, I could not have done it alone.

I am highly indebted to Ms. Saundarya Katiyar, my cofounder, who has always been highly supportive, encouraging and of course she has been proof reading, contributing to a large pool of new questions and new ideas in the book for a couple of years. However, this year she had made immense contribution to the new edition of Quantum CAT, as preparing the revised edition has been a daunting and time consuming task. Without her support the revised edition would have never been with you at all.

My heartfelt gratitude to Mr. Abis Naqvi, Mr. S. A. Siddiqui, Mr. Bhanu Pratap Singh, Mr. Amza Rahman, Mr. Ravi Singh, Ms. Kiran Verma, Mr. (Col.) D. R. Patnaik, Pravesh K. Verma for their invaluable contribution in my book and in my life. Heartily, I have a deep regard for each one of you, as I owe my success to each one of you.

How can I forget to mention the contribution of all my friends, colleagues and well wishers who believed in my hard work and sincerity to accomplish a monumental book ! I'm grateful for having your back every moment of my life!

My deepest regard for those people who have shared their feedback and constructive criticism. Their efforts have paved the way and have helped me realize their invaluable contribution in enhancing the quality and accuracy of the content. You will always live in my heart. I request them if anyone wants to contribute to the Quantum CAT by sharing some ideas, questions, answers and doubts one can join the Quantum CAT Facebook Group: www.facebook.com/groups/117572951590692

The truth is how can a book see the light of the day without the strong commitment, patience and vision of Mr. Deepesh Jain and his dynamic team at Arihant publications. I would like to express my deep regard to Mr. Jain for bearing with me for so many years! And to my dearest friend Mr. Harsh Kumar, I am really touched by your patience, your interpersonal skills and for everything you have been doing for me since you joined the Publications. And, to all the teams from composing, typesetting, printing, logistics, marketing and accounting, all of you have been really a tremendous support to me. Without your skills and efforts this book would not have disrupted the market.



DEDICATION

To all the cosmic forces and deities who have blessed me with such a wonderful life in the human form with amazing parents, siblings extended family, friends, teachers, mentors, critics and well wishers!

To the ones who made me realize how much my life is beautiful! I wish I could dedicate not just my book, but also my whole being till eternity. The fun does not lie in writing a book rather it lies in the dynamics of expression of life that make us infinite. Let's continue to cherish and share a smile with the ones who need this book more than anything else at one point of time!

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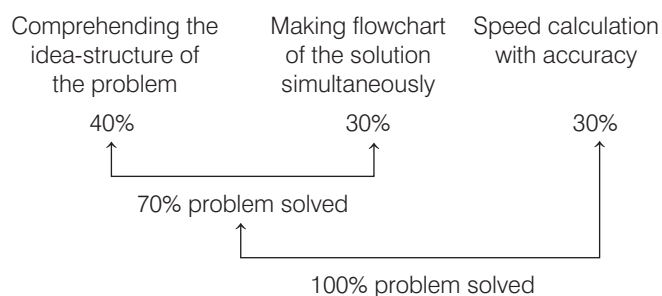
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CAT Test

CHAPTER 01

Number System

First of all I would like to mention that this chapter is deliberately written to promote your interest for CAT preparation in a systematic approach which will lay the strong foundation. Since most of the problems (*i.e.*, 20-40%) in CAT paper (*viz.*, QA section) belong to this chapter alone hence it becomes inevitable to discuss the nuances and subtleties of the various concepts objectively and comprehensively. That's why this chapter has become very bulky too. In your own interest it would be better to adhere to my advices and stipulations stated at appropriate places in this chapter even throughout the book. As per my own experiences there are basically three stages in the solution of a problem and each of the stage has its own contribution. The following diagrams illustrates the importance of each stage.



If any of the three above mentioned activities is botched up then you are prone to failure. Hence, it is obvious that speed calculation invariably enhances your rate of success in CAT and most importantly it eliminates the stress and anxiety to ensconce you in your own comfort zone. Remember that you can succeed only when you operate in your comfort zone else it turns out to be a challenge. Due to above mentioned reasons it is imperative to become handy in quick calculation which is required in DI too. The latter part, even major section, of the chapter is devoted to numbers, their kinds, operations and behaviour in mathematical milieu.

You are supposed to not to skip a bit of it since every bit and every concept is equally important. It is advised that you should make the flow chart of the solution while reading, since going back to the problem several times means irritation and wastage of invaluable time. Also, you should try to solve maximum no. of problems without pencil and paper since it makes you a quick respondent and saves a lot of time. Finally, avoid rote method of learning in maths instead be inquisitive and explorer to gain an advantage. The bottomline is that perceive logic and apply logic, since it is logical.

Chapter Checklist

- Calculation Techniques
- Square and Cubes
- Basic Numbers
- Integers
- Factors and Multiples
- HCF and LCM
- Fractions and Decimal Fractions
- Indices and Surds
- Factorial, Last Digits and Remainders
- Rational and Irrational Number
- Various Number Systems
- Patterns, Relations and Functions
- CAT Test

1.1 Calculation Techniques

Multiplication Rules

Multiplication is nothing but the shortest method of addition. In day-to-day life when we have to add up the same quantity many a time, so we multiply the given number by the number of times to which the given number is to be added repeatedly *e.g.*, $3 + 3 + 3 + 3 + 3 + 3 + 3 + 3$ can be added seven times but to get quick result we just multiply 3 by 7.

Hence, we get $3 \times 7 = 21$. Thus, I am sure that now you must have become aware of the fact that how much important is the multiplication table. So, I hope that you will learn the multiplication table.

Now, I would like to suggest you that in the competitive exams like CAT, where speed and accuracy are totally indispensable. So there are some novel and enticing techniques for multiplication given in this chapter. But in some techniques you have to know the square of some relevant numbers, normally the squares of 1 to 100. Last but not the least you don't need to bother about since I have some very innovative techniques to learn and calculate the squares very readily. Even most of the times you need not to use a pencil and paper for this work and that's my purpose.

Basically we divide these techniques into two categories. The first one which I have derived from some algebraic operations and the second one belongs to our ancient maths *i.e.*, Vedic maths. In this book I have laid emphasis on my own approach of multiplication, because years of research in numeric maths I have found that I myself and my students find it comfortable as it is very convenient and pragmatic to do within a few seconds.

Case 1. Multiplication by 5, 25 ($= 5^2$), 125 ($= 5^3$), 625 ($= 5^4$) etc.

Exp. 1) Multiply 187 by 5 *i.e.*, 187×5 .

Solution **Step 1:** Associate as many zeros at the end of the given number as there is the power to 5. Thus we write 187 as 1870. (because $5 = 5^1$)

Step 2: Divide the resultant number by 2 as many times as there is the power to 5. Thus, $1870 / 2 = 935$

Exp. 2) Multiply 369 by 125.

Solution **Step 1:** 369000 ($\because 125 = 5^3$)

hence 3 zeros will be associated.

$$\text{Step 2: } \frac{369000}{2 \times 2 \times 2} = \frac{369000}{8} = 46125$$

(Since, there is 5^3 hence it will be divided by 2^3 .)

In general, if you want to multiply any number K by 5^n , then you just need to divide that number by 2^n , after writing or (associating) ' n ' zeros at the end of the number *i.e.*,

$$K \times 5^n = \frac{K0000 \dots n \text{ zeros}}{2^n}$$

$$\text{e.g., } 732 \times 5^6 = \frac{732000000}{2 \times 2 \times 2 \times 2 \times 2 \times 2} = \frac{732000000}{64} = 11437500$$

Case 2. Multiplication of a number by 9^n

i.e., $9, 9^2 (= 81), 9^3 (= 729)$ etc.

Exp. 1) Multiply 238 by 9.

Solution Multiply the given number by 10 and then subtract the given number from the resultant number. So

$$238 \times 9 = 238 \times (10 - 1) = 2380 - 238 = 2142$$

[using distributive law as $a \times (b - c) = ab - ac$]

Exp. 2) Multiply 238 by 81.

Solution **Step 1:** $238 \times 81 = 238 \times 9 \times 9$

$$\text{Step 2: } 238 \times (10 - 1) \times (10 - 1)$$

$$\text{Step 3: } [2380 - 238] \times (10 - 1)$$

$$\text{Step 4: } 2142 \times (10 - 1)$$

$$\text{Step 5: } 21420 - 2142 = 19278$$

NOTE To multiply a number by the higher powers of 9 *i.e.*, 9 to power 3, 4, 5, ..., etc. It is better to do it conventionally.

Case 3. Multiplication of a number by the numbers whose unit (*i.e.*, last) digit is 9.

Exp. 1) Multiply 287 by 19.

Solution $287 \times (10 + 9) = 287 \times 10 + 287 \times 9$

$$= 2870 + 287 \times (10 - 1)$$

$$= 2870 + 2870 - 287 = 5740 - 287 = 5453$$

$$\text{or } 287 \times 19 = 287 \times (20 - 1) = 287 \times 20 - 287$$

$$= 287 \times 2 \times 10 - 287$$

$$= 574 \times 10 - 287 = 5740 - 287 = 5453$$

Case 4. Multiplication of a number by the number which contains all the digits as 9.

Exp. 1) Multiply 743 by 99.

Solution $743 \times 99 = 743 \times (100 - 1) = 74300 - 743 = 73557$

Exp. 2) Multiply 23857 by 9999.

Solution $23857 \times 9999 = 23857 \times (10000 - 1)$

$$= 238570000 - 23857 = 238546143$$

NOTE In general to multiply a number K by a number which is purely consists of n 9s, put n zeros at the end of the number K , then subtract the given number from the resultant number.

$$\text{e.g., } K \times 9999 \dots n \text{ times} = (K0000 \dots n \text{ zeros}) - K$$

Case 5. To multiply those two numbers, the sum of whose unit digit is 10 and the rest digits (left to the unit digit) are same in both the numbers.

Exp. 1) Multiply 38 by 32.

Solution

Step 1: Multiply the unit digits as $8 \times 2 = 16$

Step 2: Add 1 to the left digits, then multiply this increased number by the original digits *i.e.*, $3 \times (3 + 1) = 12$

$$\text{Thus, we get } 38 \times 32 = 1216$$

Exp. 2) Multiply 83 by 87.

Solution Step 1: $3 \times 7 = 21$
 Step 2: $8 \times (8 + 1) = 72$
 $\therefore 83 \times 87 = 7221$

Exp. 3) Multiply 71 by 79.

Solution Step 1: $9 \times 1 = 09$
 Step 2: $7 \times (7 + 1) = 56$
 $\therefore 71 \times 79 = 5609$

NOTE The last two places are reserved for the product of unit digits as in the above example $1 \times 9 = 9$ but we fill the last two places by writing "09". So don't write 569 since it is wrong.

Exp. 4) Multiply 125 by 125.

Solution Step 1: $5 \times 5 = 25$
 Step 2: $12 \times (12 + 1) = 156$
 So, $125 \times 125 = 15625$

Exp. 5) Multiply 431 by 439.

Solution Step 1: $1 \times 9 = 09$
 Step 2: $43 \times (43 + 1) = 1892$
 So, $431 \times 439 = 189209$

Exp. 6) Multiply 1203 by 1207.

Solution Step 1: $3 \times 7 = 21$
 Step 2: $120 \times (120 + 1) = 14520$
 So, $1203 \times 1207 = 1452021$

Case 6. To multiply the two numbers whose difference is always an even number.

Exp. 1) Multiply 38 by 52.

Solution Step 1: $52 - 38 = 14$ (here 14 is an even number)
 Step 2: $\frac{14}{2} = 7$
 Step 3: $38 + 7$ or $52 - 7 = 45$
 Step 4: $(45)^2 = 2025$
 Step 5: $7^2 = 49$
 Step 6: $2025 - 49 = 1976$

Exp. 2) Multiply 76 by 96.

Solution Step 1: $96 - 76 = 20$ (here 20 is an even number)
 Step 2: $\frac{20}{2} = 10$
 Step 3: $76 + 10 = 96 - 10 = 86$
 Step 4: $(86)^2 = 7396$
 Step 5: $(10)^2 = 100$
 Step 6: $7396 - 100 = 7296$

Exp. 3) Multiply 185 by 215.

Solution Step 1: $215 - 185 = 30$ (here 30 is an even number)
 Step 2: $\frac{30}{2} = 15$
 Step 3: $185 + 15 = 215 - 15 = 200$
 Step 4: $(200)^2 - (15)^2 = 40000 - 225 = 39775$

NOTE 1. In general, if there are any two numbers N_1 and N_2 and the difference between N_1 and N_2 is $2d$ then their product i.e.,

$$N_1 \times N_2 = (N_1 + d)^2 - d^2$$

$$\text{or } N_1 \times N_2 = (N_2 - d)^2 - d^2$$

2. Even this method can be applied for the two numbers whose difference is an odd number but since it contains decimal values so some students may feel it slightly difficult.

Exp. 1) Multiply 17 by 22.

Solution Step 1: $22 - 17 = 5$
 Step 2: $\frac{5}{2} = 2.5$
 Step 3: $17 + 2.5 = 22 - 2.5 = 19.5$
 Step 4: $(19.5)^2 - (2.5)^2 = 380.25 - 6.25 = 374.00$

Shortcut To multiply any two numbers say, N_1 and N_2 we apply the following methods.

$$N_1 \times N_2 = \left(\frac{N_1 + N_2}{2} \right)^2 - \left(\frac{N_2 - N_1}{2} \right)^2$$

So, if you know the square of the required numbers then using this method you can multiply any two numbers within

$$\begin{aligned} 2-3 \text{ seconds as } 56 \times 64 &= \left(\frac{56 + 64}{2} \right)^2 - \left(\frac{64 - 56}{2} \right)^2 \\ &= (60)^2 - (4)^2 \\ &= 3600 - 16 \\ &= 3584 \end{aligned}$$

Vedic Methods of Multiplication

Exp. 1) Solve 23×74 .

Solution

$$\begin{array}{r} 23 \\ 74 \\ \hline 2 \end{array}$$

$4 \times 3 = 12$

$$\begin{array}{r} 23 \\ 74 \\ 02 \end{array}$$

$(4 \times 2 + 7 \times 3) + 1 = 30$

$$\begin{array}{r} 23 \\ \times 74 \\ \hline 1702 \end{array}$$

$(7 \times 2) + 3 = 17$

$$\begin{array}{r} 23 \\ 74 \\ \hline 2 \end{array} \quad 12$$

$$\begin{array}{r} 23 \\ 74 \\ 02 \end{array} \quad 29 + 1 = 30$$

$$\begin{array}{r} 23 \\ 74 \\ \hline 1702 \end{array} \quad 14 + 3$$

Exp. 2) Solve 83×65 .

Solution

$$\begin{array}{r} 83 \\ 65 \\ \hline 5 \end{array} \quad \begin{array}{l} 5 \times 3 = 15 \\ (5 \times 8 + 6 \times 3) + 1 = 59 \\ (6 \times 8) + 5 = 53 \end{array}$$

$$\begin{array}{r} 83 \\ 65 \\ \hline 5395 \end{array}$$

Exp. 3) Solve 524×19 .

Solution

$$\begin{array}{r} 524 \\ 19 \\ \hline 6 \end{array} \quad \begin{array}{l} 9 \times 4 = 36 \\ (9 \times 2 + 1 \times 4) + 3 = 25 \\ (9 \times 5 + 1 \times 2) + 2 = 49 \\ (1 \times 5) + 4 = 9 \end{array}$$

$$\begin{array}{r} 524 \\ 19 \\ \hline 9956 \end{array}$$

Exp. 4) Evaluate 3456×89 .

Solution

$$\begin{array}{r} 3456 \\ 89 \\ \hline 4 \end{array} \quad \begin{array}{l} 9 \times 6 = 54 \\ (9 \times 5 + 8 \times 6) + 5 = 98 \\ (9 \times 4 + 8 \times 5) + 9 = 85 \\ (9 \times 3 + 8 \times 4) + 8 = 67 \\ 8 \times 3 + 6 = 30 \end{array}$$

$$\begin{array}{r} 3456 \\ 89 \\ \hline 307584 \end{array}$$

Exp. 5) Evaluate 245×367 .

Solution

$$\begin{array}{r} 245 \\ 367 \\ \hline 5 \end{array} \quad \begin{array}{l} 5 \times 7 = 35 \\ (7 \times 4 + 6 \times 5) + 3 = 61 \\ (7 \times 2 + 6 \times 4 + 3 \times 5) + 6 = 59 \\ (6 \times 2 + 3 \times 4) + 5 = 29 \\ (3 \times 2) + 2 = 8 \end{array}$$

$$\begin{array}{r} 245 \\ 367 \\ \hline 9915 \\ 89915 \end{array}$$

Exp. 6) Evaluate 123456×789 .

Solution

$$\begin{array}{r} 123456 \\ 789 \\ \hline 4 \end{array} \quad 9 \times 6 = 54$$

$$\begin{array}{r} 123456 \\ 789 \\ \hline 84 \end{array} \quad (9 \times 5 + 8 \times 6) + 5 = 98$$

$$\begin{array}{r} 123456 \\ 789 \\ \hline 784 \end{array} \quad (9 \times 4 + 8 \times 5 + 7 \times 6) + 9 = 127$$

$$\begin{array}{r} 123456 \\ 789 \\ \hline 6784 \end{array} \quad (9 \times 3 + 8 \times 4 + 7 \times 5) + 12 = 106$$

$$\begin{array}{r} 123456 \\ 789 \\ \hline 06784 \end{array} \quad (9 \times 2 + 8 \times 3 + 7 \times 4) + 10 = 80$$

$$\begin{array}{r} 123456 \\ 789 \\ \hline 406784 \end{array} \quad (9 \times 1 + 8 \times 2 + 7 \times 3) + 8 = 54$$

$$\begin{array}{r} 123456 \\ 789 \\ \hline 7406784 \end{array} \quad (8 \times 1 + 7 \times 2) + 5 = 27$$

$$\begin{array}{r} 123456 \\ 789 \\ \hline 97406784 \end{array} \quad (7 \times 1) + 2 = 9$$

Solution

$$\begin{array}{r}
 4567 \\
 \underline{1289} \\
 3
 \end{array}
 \quad 9 \times 7 = 63$$

$$\begin{array}{r}
 4567 \\
 \underline{1289} \\
 63
 \end{array}
 \quad (9 \times 6 + 8 \times 7) + 6 = 116$$

$$\begin{array}{r}
 4567 \\
 \underline{1289} \\
 863
 \end{array}
 \quad (9 \times 5 + 8 \times 6 + 2 \times 7) + 11 = 118$$

$$\begin{array}{r}
 4567 \\
 \underline{1289} \\
 6863
 \end{array}
 \quad (9 \times 4 + 8 \times 5 + 2 \times 6 + 1 \times 7) + 11 = 106$$

$$\begin{array}{r}
 4567 \\
 \underline{1289} \\
 6863 \\
 \underline{6863}
 \end{array}
 \quad (8 \times 4 + 2 \times 5 + 1 \times 6) + 10 = 58$$

$$\begin{array}{r}
 4567 \\
 \times 1289 \\
 \hline
 886863
 \end{array}
 \quad (2 \times 4 + 1 \times 5) + 5 = 18$$

$$\begin{array}{r}
 4567 \\
 \underline{1289} \\
 5886863
 \end{array}
 \quad (4 \times 1) + 1 = 5$$

Solution

325768	
<u>1234</u>	
<u>2</u>	$4 \times 8 = 32$
325768	
<u>1234</u>	
<u>12</u>	$(4 \times 6 + 3 \times 8) + 3 = 51$
325768	
<u>1234</u>	
<u>712</u>	$(4 \times 7 + 3 \times 6 + 2 \times 8) + 5 = 67$
325768	
<u>1234</u>	
<u>7712</u>	$(4 \times 5 + 3 \times 7 + 2 \times 6 + 1 \times 8) + 6 = 67$
325768	
<u>1234</u>	
<u>97712</u>	$(4 \times 2 + 3 \times 5 + 2 \times 7 + 1 \times 6) + 6 = 49$
325768	
<u>1234</u>	
<u>997772</u>	$(4 \times 3 + 3 \times 2 + 2 \times 5 + 1 \times 7) + 4 = 39$
325768	
<u>1234</u>	
<u>1997712</u>	$(3 \times 3 + 2 \times 2 + 1 \times 5) + 3 = 21$
325768	
<u>1234</u>	
<u>01997712</u>	$(2 \times 3 + 1 \times 2) + 2 = 10$
325768	
<u>1234</u>	
<u>401997712</u>	$(1 \times 3) + 1 = 4$

While solving the mathematical problems we need to divide one number by another number. In general we use four terms for this process as

Divisor

Dividend

Quotient

xyz

Remainder

for example $30 = (7 \times 4) + 2$

So, when the remainder is zero, we can say that the given number is divisible by the particular divisor number.

Now, $238 \div 6 = 39\frac{4}{6} = 39.66$ (decimal notation)

Further if we subtract the remainder from the dividend then the dividend (the resultant number) becomes divisible by particular divisor. *e.g.*,

$$238 - 4 = 234 \quad (\text{dividend} - \text{remainder})$$

$$\begin{array}{r} 6 \overline{)234} (39 \\ \underline{18} \\ 54 \\ \underline{54} \\ \times \end{array}$$

Another point is that if we add the difference of divisor and remainder to the given number (*i.e.*, dividend) the number becomes divisible by that particular divisor. *e.g.*,

$$6 - 4 = 2 \quad (\text{Divisor--Remainder})$$

$$238 + 2 = 240$$

$$\begin{array}{r} 6 \overline{) 240} (40 \\ \underline{240} \\ \times \end{array}$$

For negative integers

$$\text{e.g., } -37 \div 5 = \frac{-37 - 3 + 3}{5} = \frac{-40 + 3}{5} = \frac{-40}{5} + \frac{3}{5}$$

 NOTE The remainder is always a non-negative integer.

$$\begin{aligned} i.e., \quad & \frac{N}{D} \rightarrow \text{Remainder } 0 \\ \Rightarrow & \frac{NK}{D} \rightarrow \text{Remainder } 0 \end{aligned}$$

Rule 2. If N_1 and N_2 two different numbers are divisible by D individually, then their sum (i.e., $N_1 + N_2$) and their difference ($N_1 - N_2$) is also divisible by D .

Rule 3. If a number N_1 is divisible by N_2 and N_2 is divisible by N_3 then N_1 must be divisible by N_3 .

Rule 4. If two numbers N_1 and N_2 are such that they divide mutually each other it means they are same *i.e.*, $N_1 = N_2$ only.

How to Check the Divisibility

We have certain rules to check the divisibility by certain integral numbers. With the help of the following rules it has become easier to know whether a certain number is divisible or not by a particular number without actually dividing the number.

Divisibility by 2

Any integral number whose last digit (*i.e.*, unit digit) is even or in other words the unit digit is divisible by 2. It means any number whose last digit is either 0, 2, 4, 6 or 8, then this number must be divisible by 2.

e.g., 395672, 132, 790, 377754, etc.

Divisibility by 4

If the number formed by last two digits of the given number is divisible by 4, then the actual number must be divisible by 4. *i.e.*, the last two digits of a number can be 00, 04, 08, 12, 16, 20, 24, 28, 32, ..., 96.

e.g., 33932, 7958956, 2300, 1996, 3819764280 etc.

Divisibility by 8

If the number formed by last three digits of the given number is divisible by 8, then the actual number must be divisible by 8 *i.e.*, the last three digits of the divisible number can be 000, 008, 016, 024, 032, 040, ..., 096, 104, ..., 992.

e.g., 8537000, 9317640, 3945080, 23456008, 12345728, 3152408 etc.

Divisibility by 16

If the number formed by last four digits of the given number is divisible by 16, then the actual number must be divisible by 16 *i.e.*, the last 4 digits of the divisible number can be 0000, 0016, 0032, 0048, 0064, 0080, 0096, 0112, 0128, 0144, 0160, ..., 0960, 0976, ..., 0992, ..., 1600, ..., 9984.

Divisibility by 32, 64, 128, ... can be checked just by checking the last 5, 6, 7, ... digit number formed from the given number as in the above cases.

Divisibility by 5

A number is divisible by 5 if and only if the last (*i.e.*, unit) digit is either 0 or 5.

e.g., 5, 10, 15, 20, 25, 30, 35, ..., 275, 365, ..., 995, 70000, 438915 etc.

Divisibility by 3

If the sum of the digits of the given number is divisible by 3 then the actual number will also be divisible by 3.

e.g., 12375 is divisible by 3 since the sum of digits $1 + 2 + 3 + 7 + 5 = 18$ is divisible by 3.

Similarly 63089154 is also divisible by 3 since the sum of its digits $6 + 3 + 0 + 8 + 9 + 1 + 5 + 4 = 36$ is divisible by 3.

Divisibility by 9

If the sum of the digits of the given number is divisible by 9 then the actual number will also be divisible by 9.

e.g., 7329753 is divisible by 9, since

$$7 + 3 + 2 + 9 + 7 + 5 + 3 = 36 \text{ is divisible by } 9.$$

Similarly the divisibility by 27, 81, 243, ... can be checked.

Divisibility by 99

Consider any number. Starting from the right side split the number into the pairs like two digit numbers; if any digit is left unpaired in the left side of the original number take it as a single digit number.

If the sum of all these numbers is divisible by 99, then the given number will also be divisible by 99.

Exp. 1) Test whether 757845 is divisible by 99 or not.

Solution Split the given number starting from the right side as 75 78 45.

Now add all the numbers as $75 + 78 + 45 = 198$.

Since the sum of all these numbers (198) is divisible by 99, so the given number

757845 is also divisible by 99.

Exp. 2) Test whether 382546692 is divisible by 99 or not.

Solution Split the given number starting from the right side as 82 54 66 92

Now add all the numbers as $3 + 82 + 54 + 66 + 92 = 297$.

Since the sum of all these numbers (297) is divisible by 99, so the given number

362546692 is also divisible by 99.

Exp. 3) Test whether 967845 is divisible by 99 or not.

Solution Split the given number starting from the right side as 96 78 45.

Now add all the numbers as $96 + 78 + 45 = 219$.

Since the sum of all these numbers (219) is NOT divisible by 99, so the given number 967845 is NOT divisible by 99.

NOTE As you can see that when 219 is divided by 99, it leaves the remainder 21, so the same remainder 21 will be obtained when 967845 is divided by 99.

Divisibility by 999

Consider any number. Starting from the right side split the number into the triplets like three digit numbers; if any digits are left ungrouped in the left side of the original number take the remaining digit(s) as a single/double digit number. If the sum of all these numbers is divisible by 999, then the given number will also be divisible by 999.

Exp. 1) Test whether 579876543 is divisible by 999 or not.

Solution Split the given number starting from the right side as 579 876 543.

Now add all the numbers as $579 + 876 + 543 = 1998$.

Since the sum of all these numbers (1998) is divisible by 999, so the given number 579876543 is also divisible by 999.

Exp. 2) Test whether 34590900485988 is divisible by 999 or not.

Solution Split the given number starting from the right side as 34 590 900 485 988.

Now add all the numbers as $34 + 590 + 900 + 485 + 988 = 2997$.

Since the sum of all these numbers (2997) is divisible by 999, so the given number 34590900485988 is also divisible by 999.

Exp. 3) Test whether 6800900292 is divisible by 999 or not.

Solution Split the given number starting from the right side as 6 800 900 292.

Now add all the numbers as $6 + 800 + 900 + 292 = 1998$.

Since the sum of all these numbers (1998) is divisible by 999, so the given number

6800900292 is also divisible by 999.

Divisibility by 9999

Consider any number. Starting from the right side split the number into the 4-tuples like four digit numbers; if any digits are left untuppled in the left side of the original number take the remaining digit(s) as a single/double/triple digit number. If the sum of all these numbers is divisible by 9999, then the given number will also be divisible by 9999.

Exp. 1) Test whether 652587344739 is divisible by 9999 or not.

Solution Split the given number starting from the right side as 6525 8734 4739.

Now add all the numbers as $6525 + 8734 + 4739 = 19998$.

Since the sum of all these numbers (19998) is divisible by 9999, so the given number 652587344739 is also divisible by 9999.

Exp. 2) Test whether 239976 is divisible by 9999 or not.

Solution Split the given number starting from the right side as 23 9976.

Now add all the numbers as $23 + 9976 = 9999$.

Since the sum of all these numbers (9999) is divisible by 9999, so the given number 239976 is also divisible by 9999.

Divisibility by Some Special Numbers e.g., 7, 11, 13, 17, 19 etc.

Why are these numbers treated differently for checking the divisibility? Since, if we multiply these numbers by any other number (except 10 or multiples of 10) these numbers can never be divisible by 10. So to make the process easier we bring the given number (*i.e.*, divisor) closer to the multiples of 10 with the difference of 1 (*e.g.*, 9 or 11, 19 or 21, 29 or 31, 39 or 41 etc).

Now if the number is one less than the multiples of 10 (*i.e.*, 9, 19, 29, 39, ...) we need to increase it by 1 to make it further multiple of 10. Hence we call it “one more” osculator and the value of multiplier of 10 is called the value of “one more” osculator.

Similarly if the number is one more than the multiple of 10 (*i.e.*, 11, 21, 31, 41, ..., etc) we need to decrease it by 1 to make it the multiple of 10. So we call it ‘negative’ osculator and the value of multiplier of 10 is the value of ‘negative’ osculator.

e.g., The given number is 7 (divisor number) then we bring it closer to the multiple of 10 with the difference of 1, then

$$7 \times 3 = 21 = 20 + 1$$

$$\Rightarrow 20 = 21 - 1 = (2 \times 10) \quad \dots(i)$$

Now in the above expression 2 is the multiplier of 10 and 2 is known as the negative osculator for 7.

Now let us consider the next example of 13 and then bring it closer to the multiples of 10 with a difference of 1.

$$\text{Thus, } 13 \times 3 = 39 \text{ and } 39 + 1 = 40 = (4 \times 10) \quad \dots(ii)$$

In the above expression 4 is the multiplier of 10 and 4 is the value of ‘one more’ osculator.

Similarly for 17 we can find the osculator as

$$17 \times 3 = 51 = 50 + 1$$

$$\text{So, } 50 = 51 - 1 = (5 \times 10)$$

Here, 5 is the negative osculator for 17.

Similarly for 31 the osculator is negative and the value of negative osculator is 3.

$$\therefore 31 = 30 + 1 \Rightarrow 30 = 31 - 1 \Rightarrow (3 \times 10)$$

Now we will apply the osculator techniques to check the divisibility by 7, 11, 13, 17, 19, ... etc.

Divisibility by 7

To check the divisibility of a number by 7 we apply the following method. Let the number be 133.

Step 1. $133 \Rightarrow 13 - 3 \times 2 = 13 - 6 = 7$

Since 7 is divisible by 7, so the given number 133 will also be divisible by 7. In the above process 2 is multiplied with the last digit is the negative osculator for 7, which I have earlier discussed.

Exp. 1) Check whether 1071 is divisible by 7.

Solution **Step 1.** $1071 \Rightarrow 107 - 1 \times 2 = 105$

Step 2. $105 \Rightarrow 10 - 5 \times 2 = 0$

Since 0 is divisible by 7 hence the given number 1071 is also divisible by 7.

Exp. 2) Check whether 939715 is divisible by 7.

Solution **Step 1.** $939715 \Rightarrow 93971 - 5 \times 2 = 93961$

Step 2. $93961 \Rightarrow 9396 - 1 \times 2 = 9394$

Step 3. $9394 \Rightarrow 939 - 4 \times 2 = 931$

Step 4. $931 \Rightarrow 93 - 1 \times 2 = 91$

Step 5. $91 \Rightarrow 9 - 1 \times 2 = 7$

Hence, it is divisible by 7.

NOTE

1. In all the above examples we have to multiply the last digit by the appropriate osculator and then this value will be subtracted from the number formed by the rest digits of the number and this process is continued till you know that the resultant value is divisible by 7. Even you can stop the process in midway when you guess that the obtained value is divisible by 7. For example in the latest problem (example-2) we can stop at step 4 if we know that the 91 is divisible by 7 also we can stop even at step 3 if we have any idea that 931 is divisible by 7.
2. Remember that if the operating osculator is "one more" osculator then we add the product of last digit and one more osculator in the number formed by the rest digit else. We subtract if the operating osculator is 'negative' osculator.

Divisibility by 11

A number is divisible by 11 if the difference between the sum of the digits at odd places and sum of the digits at even places is equal to zero or multiple of 11 (i.e., 11, 22, 33 etc.)

For example : 57945822

Here sum of the digits at odd places = $2 + 8 + 4 + 7 = 21$

and sum of the digits at even places = $2 + 5 + 9 + 5 = 21$ and thus the difference = $0 (= 21 - 21)$

Hence 57945822 is divisible by 11.

Alternative Approach Consider any number. Starting from the right side split the number into the pairs like two digit numbers; if any digit is left unpaired in the left side of the original number take it as a single digit number.

If the sum of all these numbers is divisible by 11, then the given number will also be divisible by 11.

Exp. 1) Test whether 702845 is divisible by 11 or not.

Solution Split the given number starting from the right side as 70 28 45.

Now add all the numbers as $70 + 28 + 45 = 143$.

Since the sum of all these numbers (143) is divisible by 11, so the given number 70 28 45 is also divisible by 11.

Exp. 2) Test whether 382546692 is divisible by 11 or not

Solution Split the given number starting from the right side as 38 25 46 692

Now add all the numbers as $3 + 82 + 54 + 66 + 92 = 297$.

Since the sum of all these numbers (297) is divisible by 11, so the given number 382546692 is also divisible by 11.

Exp. 3) Test whether 967845 is divisible by 11 or not.

Solution Split the given number starting from the right side as 96 78 45.

Now add all the numbers as $96 + 78 + 45 = 219$.

Since the sum of all these numbers (219) is NOT divisible by 11, so the given number 967845 is NOT divisible by 11.

Exp. 4) Test whether 825466 is divisible by 11 or not, if not, what's remainder?

Solution Split the given number starting from the right side as 82 54 66.

Now add all the numbers as $82 + 54 + 66 = 202$.

Since the sum of all these numbers (202) is NOT divisible by 11, so the given number 825466 is also NOT divisible by 11.

Further, since when 202 is divided by 11 it leaves remainder 4, so when the original number 825466 is divided by 11 it will leave the same remainder 4.

Divisibility by 111

Consider any number. Starting from the right side split the number into the triplets like three digit numbers; if any digits are left unpaired in the left side of the original number take the remaining digit(s) as a single/double digit number.

If the sum of all these numbers is divisible by 111, then the given number will also be divisible by 111.

Exp. 1) Test whether 579876543 is divisible by 111 or not.

Solution Split the given number starting from the right side as 579 876 543.

Now add all the numbers as $579 + 876 + 543 = 1998$.

Since the sum of all these numbers (1998) is divisible by 111, so the given number 579876543 is also divisible by 111.

Exp. 2) Test whether 34590900485988 is divisible by 111 or not.

Solution Split the given number starting from the right side as 34 590 900 485 988.

Now add all the numbers as $34 + 590 + 900 + 485 + 988 = 2997$.

Since the sum of all these numbers (2997) is divisible by 111, so the given number 34590900485988 is also divisible by 111. To check that whether 2997 is divisible by 111, you can use the same technique as 2997 can be broken up as 2 997, then $2 + 997 = 999$. Since 999 is divisible by 111, so 2997 is also divisible by 111. As 2997 is divisible by 111, so 34590900485 is also divisible by 111.

Exp. 3) Test whether 6800900181 is divisible by 111 or not.

Solution Split the given number starting from the right side as 6 800 900 181.

Now add all the numbers as $6 + 800 + 900 + 181 = 1887$.

Since the sum of all these numbers (1887) is divisible by 111, so the given number 6800900181 is also divisible by 111.

Divisibility by 1111

Consider any number. Starting from the right side split the number into the 4-tuples like four digit numbers; if any digits are left unpaired in the left side of the original number take the remaining digit(s) as a single/ double/triple digit number. If the sum of all these numbers is divisible by 1111, then the given number will also be divisible by 1111.

Exp. 1) Test whether 652587344739 is divisible by 1111 or not

Solution Split the given number starting from the right side as 6525 8734 4739.

Now add all the numbers as $6525 + 8734 + 4739 = 19998$.

Since the sum of all these numbers (19998) is divisible by 1111, so the given number 652587344739 is also divisible by 1111.

NOTE To check that whether 19998 is divisible by 1111, you can use the same technique as 19998 can be broken up as 1 9998, then $1 + 9998 = 9999$. Since, 9999 is divisible by 1111, so 19998 is also divisible by 1111. As 19998 is divisible by 1111, so 652587344739 is also divisible by 1111.

Exp. 2) Test whether 217756 is divisible by 1111 or not.

Solution Split the given number starting from the right side as 21 7756.

Now add all the numbers as $21 + 7756 = 7777$.

Since the sum of all these numbers (7777) is divisible by 1111, so the given number 217756 is also divisible by 1111.

Divisibility by 13

Exp. 1) Check whether 2366 is divisible by 13.

Solution Step 1. 2366 $\Rightarrow 236 + 6 \times 4 = 260$

[Since, the osculator for 13 is 4 and it is 'one more' osculator. So we use addition]

Step 2. 260 $\Rightarrow 26 + 0 \times 4 = 26$

Since 26 (or 260) is divisible by 13 hence 2366 is also divisible by 13.

Exp. 2) Check whether 377910 is divisible by 13.

Solution Step 1. 377910 $\Rightarrow 37791 + 0 \times 4 = 37791$

Step 2. 37791 $\Rightarrow 3779 + 1 \times 4 = 3783$

Step 3. 3783 $\Rightarrow 378 + 3 \times 4 = 390$

Step 4. 390 $\Rightarrow 39 + 0 \times 4 = 39$

Since 39 is divisible by 13. So 377910 is also divisible by 13.

Divisibility by 17

Exp. 1) Find out whether 323 is divisible by 17.

Solution Step 1. 323 $\Rightarrow 32 - 3 \times 5 = 17$

[$\therefore 5$ is the negative osculator of 17]

Therefore 323 is divisible by 17.

Exp. 2) Checkout that 12716 is divisible by 17 or not.

Solution Step 1. 12716 $\Rightarrow 1271 - 6 \times 5 = 1241$

1241 $\Rightarrow 124 - 1 \times 5 = 119$

119 $\Rightarrow 11 - 9 \times 5 = -34$

so we can conclude that 12716 is divisible by 17. Since -34 and 119 both are simply visible that these two numbers are divisible by 17. As I have already mentioned that you can stop your checking process as soon as you can get a number which is easy to know that the particular number is divisible by the given divisor or not. Further you should know that every resultant value in the right hand side might be divisible by the divisor whether you readily recognise it or not.

Divisibility by 19

Exp. 1) Find out whether 21793 is divisible by 19.

Solution Step 1. 21793 $\Rightarrow 2179 + 3 \times 2 = 2185$

[$\therefore 2$ is the "one more" osculator of 19]

Step 2. 2185 $\Rightarrow 218 + 5 \times 2 = 228$

Step 3. 228 $\Rightarrow 22 + 8 \times 2 = 38$

Hence 21793 is divisible by 19.

Shortcut rule for the divisibility by 7, 11 and 13

A number can be divisible by 7, 11 or 13 if and only if the difference of the number formed by the last three digits and the number formed by the rest digits is divisible by 7, 11 or 13 respectively.

For example we have to check that 139125 is divisible by 7 or not.

So we take the difference as given below

$$139 - 125 = 14$$

Since, the difference is divisible by 7. Hence the given number is also divisible by 7.

Exp. 1) Check whether 12478375 is divisible by 13 or not.

Solution Step 1. $12478 - 375 = 12103$

$$\text{Step 2. } 12 - 103 = -91$$

Since 91 is divisible by 13 hence 12478375 is also divisible by 13.

Divisibility by Composite Numbers e.g., 4, 6, 8, 10, 12, 14, 15 etc.

Divisibility by 6

A number is divisible by 6 only when it is divisible by 2 and 3 both.

So first of all we see that the number is even or not then we check for the divisibility by 3.

Divisibility by 10

A number is divisible by 10 if and only if when it is divisible by both 2 and 5. So it can be easily observed that a number is divisible by 10 must end up with zero(s) at the right end (*i.e.*, last digits) of the given number itself.

NOTE Thus we can say that if there are '*n*' zeros at the end of the given number the number can be divided by 10^n means 10000... *n* zeros.

Divisibility by 12

A number is divisible by 12 only when it is divisible by 4 and 3 both at the same time. So first of all check the divisibility by 4 then by 3

Divisibility by 15

A number is divisible by 15 only when it is divisible by 3 and 5 both simultaneously. So first of all check the number by 5 then by 3. Thus we can conclude that any number which is divisible by a composite number, as mentioned above, must be divisible by all its factors whose L.C.M. is the given divisor.

Introductory Exercise 1.1

- The number 12375 is divisible by :
(a) 3, 11 and 9 (b) 3 and 11 only
(c) 11 and 9 only (d) 3 and 9 only
- The least number which must be subtracted from 6708 to make it exactly divisible by 9 is :
(a) 1 (b) 2
(c) 3 (d) 4
- The smallest number which must be added to 803642 in order to obtain a multiple of 9 is :
(a) 1 (b) 2 (c) 3 (d) 4
- The divisor when the quotient, dividend and the remainder are respectively 547, 171282 and 71 is equal to :
(a) 333 (b) 323 (c) 313 (d) 303
- In a problem involving division, the divisor is eight times the quotient and four times the remainder. If the remainder is 12, then the dividend is :
(a) 300 (b) 288
(c) 512 (d) 524
- 111111111111 is divisible by :
(a) 3 and 37 only
(b) 3, 37 and 11 only
(c) 3, 11, 37 and 111 only
(d) 3, 11, 37, 111 and 1001
- An integer is divisible by 16 if and only if its last *X* digits are divisible by 16. The value of *X* would be :
(a) three (b) four (c) five (d) six
- If the number 243×51 is divisible by 9, the value of the digit marked as *x* would be:
(a) 3 (b) 1 (c) 2 (d) 4
- Which of the following number is divisible by 999?
(a) 9999999 (b) 99999
(c) 987654321 (d) 145854
- Which of the following can divide 99999999 exactly?
(a) 9 (b) 9999
(c) 99 (d) Each of (a), (b), and (c)
- If $24AB4$ is divisible by 99, then $A \times B$ is:
(a) 25 (b) 30 (c) 20 (d) 15
- What is the largest possible two digit number by which 2179782 can be divided ?
(a) 88 (b) 50
(c) 66 (d) 99
- At least which number must be subtracted from 9999999 so that it will become the multiple of 125?
(a) 124 (b) 4
(c) 24 (d) none of these
- A number of the form $10^n - 1$ is always divisible by 11 for every *n* is a natural number, when :
(a) *n* is odd (b) *n* is prime
(c) *n* is even (d) can't say
- Out of the following numbers which is divisible by 132?
(a) 31218 (b) 78520
(c) 38148 (d) 52020

16. If $653xy$ is divisible by 80 then the value of $x + y$ is :
(a) 2 (b) 3
(c) 4 (d) 6
17. The value of k if $k35624$ is divisible by 11 :
(a) 2 (b) 5 (c) 7 (d) 6
18. If $42573k$ is divisible by 72 then the value of k is :
(a) 4 (b) 5
(c) 6 (d) 7
19. How many numbers between 1 and 1000 are divisible by 7?
(a) 777 (b) 142
(c) 143 (d) none of these
20. How many numbers between 55 and 555 including both the extreme values are divisible by 5?
(a) 100 (b) 111
(c) 101 (d) none of these
21. How many numbers are there from 100 to 200 ?
(a) 100 (b) 101
(c) 99 (d) none of these
22. How many numbers are divisible by 3 in the set of numbers 300, 301, 302, ..., 499, 500?
(a) 200 (b) 66
(c) 67 (d) none of these
23. How many numbers are there between 200 and 800 which are divisible by both 5 and 7?
(a) 35 (b) 16
(c) 17 (d) can't be determined
24. In the above question total numbers in the set of numbers $S = \{200, 201, \dots, 800\}$ which are either divisible by 5 or by 7 is :
(a) 210 (b) 190
(c) 199 (d) can't be determined
25. How many numbers are there in the set $S = \{200, 201, 202, \dots, 800\}$ which are divisible by neither of 5 or 7?
(a) 411 (b) 412
(c) 410 (d) none of these
26. Total number of numbers lying in the range of 1331 and 3113 which are neither divisible by 2, 3 or 5 is :
(a) 477 (b) 594
(c) 653 (d) none of these
27. Atleast what number must be subtracted from 434079 so that it becomes divisible by 137 ?
(a) 173 (b) 63
(c) 97 (d) can't be determined
28. In the above question, at least what number be added to 434079, so that it will become divisible by (or multiple of) 137 ?
(a) 97 (b) 74
(c) 75 (d) none of these
29. Which one number is closest to 193 which is divisible by 18 is :
(a) 180 (b) 195
(c) 198 (d) 108
30. The product of two numbers $ab7$ and $cd5$ could be, where $ab7$ and $cd5$ are individually three digit numbers :
(a) 8135 (b) 79236
(c) 8735255 (d) none of these
31. When a 3 digit number 984 is added to another 3 digit number $4p3$, we get a four digit number $13q7$, which is divisible by 11. The value of $p + q$ is :
(a) 10 (b) 11
(c) 12 (d) 13
32. When a number divided by 9235, we get the quotient 888 and the remainder 222, such a least possible number is :
(a) 820090 (b) 8200920
(c) 8200680 (d) none of these
33. The number which when divided by 33 is perfectly divisible and closer to 1000 is :
(a) 990
(b) 999
(c) 1023
(d) can't be determined
34. A number which when divided by 32 leaves a remainder of 29. If this number is divided by 8 the remainder will be :
(a) 0 (b) 1
(c) 5 (d) 3
35. A number when divided by 5 leaves a remainder of 4, when the double (i.e., twice) of that number is divided by 5 the remainder will be :
(a) 0
(b) 1
(c) 3
(d) can't be determined
36. When a number ' N ' is divided by a proper divisor ' D ' then it leaves a remainder of 14 and if the thrice of that number i.e., $3N$ is divided by the same divisor D , the remainder comes out to be 8. Again if the 4 times of the same number i.e., ' $4N$ ' is divided by D the remainder will be :
(a) 35 (b) 22
(c) 5 (d) can't be determined
37. A number when divided by 5 gives a number which is 8 more than the remainder obtained on dividing the same number by 34. Such a least possible number is:
(a) 175 (b) 75
(c) 680 (d) does not exist

38. When a natural number divided by a certain divisor, we get 15 as a remainder. But when the 10 times of the same number is divided by the same divisor we get 6 as a remainder. The maximum possible number of such divisors is :
- (a) 6 (b) 7
(c) 15 (d) can't be determined
39. A certain number 'C' when divided by N_1 it leaves a remainder of 13 and when it is divided by N_2 it leaves a remainder of 1, where N_1 and N_2 are the positive integers. Then the value of $N_1 + N_2$ is, if $\frac{N_1}{N_2} = \frac{5}{4}$:
- (a) 36
(b) 27
(c) 54
(d) can't be determined uniquely
40. In the above problem the value of c is :
- (a) $50 < c < 100$
(b) any multiple of 11
(c) $20 < c < 50$
(d) can't be determined
41. In how many parts a rod of length 19.5 m can be broken of equal length of 65 cm?
- (a) 20 (b) 30
(c) 3 (d) 130
42. A six digit number which is consisting of only one digits either 1, 2, 3, 4, 5, 6, 7, 8 or 9, e.g., 111111, 222222... etc. This number is always divisible by :
- (a) 7 (b) 11
(c) 13 (d) all of these
43. The maximum possible difference between the 4 digit numbers formed by using the 4 different digits 1, 2, 3, 5 is :
- (a) 4086 (b) 5076
(c) 4386 (d) 3242
44. The sum of all digits except the unity that can be substituted at the place of k in order to be divisible by 8 in the number $23487k2$:
- (a) 5 (b) 14
(c) 9 (d) none of these
45. A certain number N when multiplied by 13, the resultant values consists entirely of sevens. The value of N is :
- (a) 123459 (b) 58829
(c) 59829 (d) none of these
46. A teacher who teaches online at Lamamia wrote a 90 digit positive number 112222333333... in his laptop. Then he inserted a three-digit number between any two distinct digits. Now the new number cannot be perfectly divisible by 11. Find the number of possible values of three digit numbers.
- (a) 819 (b) 781
(c) 881 (d) none of these
47. A natural number is divisible by 1125, which consists of only 0s and 1s. Minimum how many 0s and 1s are there in such a number?
- (a) 5,9 (b) 7,5 (c) 3,3 (d) 3,9
48. A ten-digit number containing each distinct digit only once. The ten-digit number is divisible by 10. If the last digit is eliminated, the remaining number is divisible by 9. If the last 2 digits are eliminated, the remaining number is divisible by 8. If the last three digits are eliminated, the remaining number is divisible by 7. And so on. Find the number.
- (a) 1234567890 (b) 2436517890
(c) 1832547690 (d) 3816547290
49. What is the smallest 9 digit number containing all the non-zero digits 1, 2, ..., 9, which is divisible by 99?
- (a) 125364789 (b) 124365879
(c) 123475689 (d) none of these
50. An uninitiated student once visited a test prep portal www.Lamamia.in to assess her quantitative ability for SSC CGL. She took a test in which there were total 18 problems each with four options—A, B, C and D. After glancing through the whole test paper she realized that she was not able to crack even a single problem, so she marked the choice A in all the problems. Then after a while she changed the answers by marking B in each third problem, starting with the third problem. Unsatisfied with her answers, she again changed the answers by marking C in each second problem, starting with the second problem. In her final attempt to guess the answers, she changed the answers by marking D in each ninth problem, starting with the ninth problem. While changing the answers she moves from the first to the last problem in an orderly way. What are the numbers of final answers that she marked in terms of A, B, C and D, respectively?
- (a) 6, 2, 1, 9 (b) 6, 2, 8, 2
(c) 2, 4, 6, 6 (d) 4, 6, 2, 6

1.2 Square and Cubes

SQUARE TABLE

1	1	21	441	41	1681	61	3721	81	6561
2	4	22	484	42	1764	62	3844	82	6724
3	9	23	529	43	1849	63	3969	83	6889
4	16	24	576	44	1936	64	4096	84	7056
5	25	25	625	45	2025	65	4225	85	7225
6	36	26	676	46	2116	66	4356	86	7396
7	49	27	729	47	2209	67	4489	87	7569
8	64	28	784	48	2304	68	4624	88	7744
9	81	29	841	49	2401	69	4761	89	7921
10	100	30	900	50	2500	70	4900	90	8100
11	121	31	961	51	2601	71	5041	91	8281
12	144	32	1024	52	2704	72	5184	92	8464
13	169	33	1089	53	2809	73	5329	93	8649
14	196	34	1156	54	2916	74	5476	94	8836
15	225	35	1225	55	3025	75	5625	95	9025
16	256	36	1296	56	3136	76	5776	96	9216
17	289	37	1369	57	3249	77	5929	97	9409
18	324	38	1444	58	3364	78	6084	98	9604
19	361	39	1521	59	3481	79	6241	99	9801
20	400	40	1600	60	3600	80	6400	100	10,000

Squaring Techniques

Dear students let me tell you a simple secret of Number System. It is nothing but knowing the properties of numbers and so knowing the squares of numbers makes you smarter than the ones who don't know the squares or the ones who can't calculate the squares orally in a few seconds.

A couple of techniques are illustrated in this book that will definitely help you calculate the squares faster than many of your competitors. My only request is that do not undermine the role of squares due to your ignorance or impatience.

- 1. Square of the numbers whose unit (i.e., last) digit is zero (0) :** Any number whose last digit is zero, we double the number of zeros at the right side of the number and in the left of the zeros we write the square of the non-zero numbers (or rest digits) which is in the left of the zero(s)

e.g., square of 40 :

$$(40)^2 \Rightarrow \begin{array}{ll} \text{Step 1.} & 00 \\ \text{Step 2.} & 4^2 = 16 \\ \text{Step 3.} & 1600 \end{array}$$

Similarly, square of 700 :

$$(700)^2 \Rightarrow \begin{array}{ll} \text{Step 1.} & 0000 \\ \text{Step 2.} & (7)^2 = 49 \\ \text{Step 3.} & 490000 \end{array}$$

Square of 13000 :

$$(13000)^2 \Rightarrow \begin{array}{ll} \text{Step 1.} & 000000 \\ \text{Step 2.} & (13)^2 = 169 \\ \text{Step 3.} & 169000000 \end{array}$$

Square of 21000 :

$$(21000)^2 \Rightarrow \begin{array}{ll} \text{Step 1.} & 000000 \\ \text{Step 2.} & (21)^2 = 441 \\ \text{Step 3.} & 441000000 \end{array}$$

- 2. Square of the numbers whose unit digit is 5 :**

To calculate the square of the number whose unit digit is 5, first of all, we write 25 as the last two digits of the square of the given number. For remaining digits which has to be written to the left of 25 we write the product of the number formed by remaining digits of the given number and the its successive number.

For example :

(i) The square of 35 :

$$(35)^2 \Rightarrow \text{Step 1. } 25$$

$$\text{Step 2. } 3 \times (3 + 1) = 12$$

$$\text{Step 3. } 1225$$

(ii) The square of 65 :

$$(65)^2 \Rightarrow \text{Step 1. } 25$$

$$\text{Step 2. } 6 \times (6 + 1) = 42$$

$$\text{Step 3. } 4225$$

(iii) The square of 75 :

$$(75)^2 \Rightarrow \text{Step 1. } 25$$

$$\text{Step 2. } 7 \times (7 + 1) = 56$$

$$\text{Step 3. } 5625$$

(iv) The square of 125 :

$$(125)^2 \Rightarrow \text{Step 1. } 25$$

$$\text{Step 2. } 12 \times (12 + 1) = 156$$

$$\text{Step 3. } 15625$$

(v) The square of 1405

$$\text{Step 1. } 25$$

$$\text{Step 2. } 140 \times (140 + 1) = 19740$$

$$\text{Step 3. } 1974025$$

(vi) The square of 139005

$$(139005)^2 \Rightarrow \text{Step 1. } 25$$

$$\text{Step 2. } 13900 \times (13900 + 1) = 193223900$$

$$\text{Step 3. } 19322390025$$

(vii) The square of 155 :

$$(155)^2 \Rightarrow \text{Step 1. } 25$$

$$\text{Step 2. } 15 \times (15 + 1) = 240$$

$$\text{Step 3. } 24025$$

3. (i) Square of the numbers ending with the digit 1 :

These numbers are very easy to solve. To get the square of any such number just write the square of the previous number and then add the previous number and the number whose square is being asked.

For example the square of 21

$$(21)^2 = (20)^2 + (20 + 21) = 441$$

$$\text{Similarly, } (31)^2 = (30)^2 + (30 + 31) = 961$$

$$\text{and } (41)^2 = (40)^2 + (40 + 41) = 1681$$

$$\text{and } (81)^2 = (80)^2 + (80 + 81) = 6561$$

$$\begin{aligned} \text{and } (131)^2 &= (130)^2 + (130 + 131) \\ &= 16900 + 261 = 17161 \end{aligned}$$

$$\begin{aligned} \text{and } (151)^2 &= (150)^2 + (150 + 151) \\ &= 22500 + 301 = 22801 \end{aligned}$$

$$\begin{aligned} \text{and } (1111)^2 &= (1110)^2 + (1110 + 1111) \\ &= (1110)^2 + 2221 \\ &= 1232100 + 2221 = 1234321 \end{aligned}$$

(ii) **Square of the numbers whose unit digit is 9 :** It is very similar to the previous case. The difference is that here we have to subtract instead of addition. For example the square of 39

$$\begin{aligned} (39)^2 &= (40)^2 - (39 + 40) \\ &= 1600 - 79 = 1521 \end{aligned}$$

$$\begin{aligned} \text{Similarly, } (49)^2 &= (50)^2 - (49 + 50) \\ &= 2500 - 99 = 2401 \end{aligned}$$

$$\begin{aligned} \text{and } (89)^2 &= (90)^2 - (89 + 90) \\ &= 8100 - 179 = 7921 \end{aligned}$$

$$\begin{aligned} \text{and } (139)^2 &= (140)^2 - (139 + 140) \\ &= 19600 - 279 = 19321 \end{aligned}$$

$$\begin{aligned} \text{and } (249)^2 &= (250)^2 - (249 + 250) \\ &= 62500 - 499 = 62001 \end{aligned}$$

 **NOTE** In the case of unit digit of 9, we subtract from the base square because base is higher than the number whose square is to be calculated.

(iii) Square of the numbers whose last digit is 2 or 8 :

$$\begin{aligned} \text{For example } (52)^2 &= (50)^2 + 2(50 + 52) \\ &= 2500 + 204 = 2704 \end{aligned}$$

$$\begin{aligned} \text{Similarly, } (112)^2 &= (110)^2 + 2(110 + 112) \\ &= 12100 + 444 = 12544 \end{aligned}$$

$$\begin{aligned} \text{and } (38)^2 &= (40)^2 - 2(38 + 40) \\ &= 1600 - 156 = 1444 \end{aligned}$$

$$\begin{aligned} \text{and } (88)^2 &= (90)^2 - 2(88 + 90) \\ &= 8100 - 356 = 7744 \end{aligned}$$

(iv) Square of the numbers whose unit digit is 3 or 7 :

$$\begin{aligned} \text{For example : } (23)^2 &= (20)^2 + 3(20 + 23) \\ &= 400 + 129 = 529 \end{aligned}$$

$$\begin{aligned} \text{Similarly, } (53)^2 &= (50)^2 + 3(50 + 53) \\ &= 2500 + 309 = 2809 \end{aligned}$$

$$\begin{aligned} \text{and } (123)^2 &= (120)^2 + 3(120 + 123) \\ &= 14400 + 729 = 15129 \end{aligned}$$

$$\begin{aligned} \text{and } (47)^2 &= (50)^2 - 3(47 + 50) \\ &= 2500 - 291 = 2209 \end{aligned}$$

$$\text{and} \quad (137)^2 = (140)^2 - 3(137 + 140) \\ = 19600 - 831 = 18769$$

$$\text{and} \quad (2347)^2 = (2350)^2 - 3(2347 + 2350) \\ = 5522500 - 4691 = 5517809$$

(v) **Square of the numbers which ends with 4 or 6 :**

$$\text{For example : } (34)^2 = (30)^2 + 4(30 + 34) \\ = 900 + 256 = 1156$$

$$\text{or} \quad (34)^2 = (35)^2 - (34 + 35) \\ = 1225 - 69 = 1156$$

$$\text{Similarly, } (36)^2 = (35)^2 + (35 + 36) \\ = 1225 + 71 = 1296$$

$$\text{and} \quad (126)^2 = (125)^2 + (125 + 126) \\ = 15625 + 251 = 15876$$

NOTE In general square of any number can be found by using the square of any convenient number as base square *e.g.*, the numbers whose unit digit is either 0 or 5, since the square of these numbers is easy to find and learn. However we can calculate the square by considering any base square. For example, if

$$(32)^2 = (30)^2 + 2(30 + 32) = 1024$$

$$\text{or} \quad (32)^2 = (31)^2 + (31 + 32) \\ = 961 + 63 = 1024$$

$$\text{or} \quad (32)^2 = (33)^2 - (32 + 33) \\ = 1089 - 65 = 1024$$

$$\text{or} \quad (32)^2 = (35)^2 - 3(32 + 35) \\ = 1225 - 201 = 1024$$

Now, it can be generalized as :

If N is the number whose square is to be calculated and B is the base and d is difference between N and B then $(N)^2 = (B)^2 + d(B + N)$ when $B < N$

$$\text{or} \quad (N)^2 = (B)^2 - d(B + N) \text{ when } B > N$$

Properties of Squares

- If the unit digit of the number is zero, then the unit digit of the square of this number will also be zero and the number of zeros will be double in the square than that of its root. *e.g.*, $(60)^2 = 3600$, $(130)^2 = 16900$
- If the unit digit of the number is 5, then the unit digit of its square is also 5 and the number formed by last two digits is 25. *e.g.*, $(35)^2 = 1225$, $(45)^2 = 2025$, $(55)^2 = 3025$ etc.

- If the unit digit of any number is 1 or 9, then the unit digit of the square of its number is always 1.

$$\text{e.g., } (71)^2 = 5041, (31)^2 = 961, (19)^2 = 361$$

- If the unit digit of any number is 2 or 8, then the unit digit of the square of its number is always 4.

- If the unit digit of any number is 3 or 7, then the unit digit of its square is always 9.

$$\text{e.g., } (23)^2 = 529,$$

$$(27)^2 = 729$$

- If the unit digit of any number is 4 or 6, then the unit digit of its square is always 6.

$$\text{e.g., } (26)^2 = 676,$$

$$(24)^2 = 576,$$

$$(14)^2 = 196,$$

$$(16)^2 = 256 \text{ etc.}$$

- The square of any number is always positive irrespective of the nature of the given number.

- The numbers with unit digit 0, 1, 5 and 6 always give the same unit digits respectively, on squaring.

- 2, 3, 7 and 8 never appear as unit digit in the square of a number.

Perfect Square : The square of any natural number is known as perfect square *e.g.*, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, ... etc. are the squares of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, ... etc.

Square root

A number n is called the square root of a number $n^2 = (n \times n)$ where n^2 is obtained by multiplying the number n with itself only once.

The symbol for square root is ' $\sqrt{\quad}$ '.

For example

the square root of 4 is $\sqrt{4} = 2$,

the square root of 9 is $\sqrt{9} = 3$, and

the square root of 25 is $\sqrt{25} = 5$, etc.

Generally there are two methods for finding the square root :

- Prime Factorisation method
- Division method

(i) Factorisation Method

In this method first we find out the prime factors and then we pair them as given below.

Exp. 1) Find the square root of 3600.

Solution The factors of 3600
 $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$
 So the square root of 3600 i.e.,
 $\sqrt{3600} = 2 \times 2 \times 3 \times 5$
 $= 60$

Exp. 2) Find the square root of 144.

Solution Factors of 144 = $2 \times 2 \times 2 \times 2 \times 3 \times 3$
 So $\sqrt{144} = 2 \times 2 \times 3$
 $\sqrt{144} = 12$

Exp. 3) Find the square root of 10404.

Solution The factors of 10404
 $= 2 \times 2 \times 3 \times 3 \times 17 \times 17$
 $\therefore \sqrt{10404} = 2 \times 3 \times 17$
 or $\sqrt{10404} = 102$

(ii) **Division Method**

In this method first of all we make pairs from the right side towards left and then solve as given below.

Exp. 1) Find the square root of 3600.

Solution

	60
6	<u>36</u> 00
6	<u>36</u>
120	00 00
0	00 00

$\therefore \sqrt{3600} = 60$

Exp. 2) Find the square root of 10404.

Solution

	102
1	<u>1</u> <u>04</u> <u>04</u>
1	<u>1</u>
202	0 <u>04</u> <u>04</u>
02	<u>4</u> <u>04</u>
	<u>4</u> <u>04</u>

$\therefore \sqrt{10404} = 102$

Exp. 3) Find the square root of 15876.

Solution

	126
1	<u>1</u> <u>58</u> <u>76</u>
1	<u>1</u>
22	<u>58</u>
2	<u>44</u>
246	<u>1476</u>
6	<u>1476</u>
	<u>1476</u>

$\therefore \sqrt{15876} = 126$

2	<u>3600</u>
2	<u>1800</u>
2	<u>900</u>
2	<u>450</u>
5	<u>225</u>
5	<u>45</u>
3	<u>9</u>
3	<u>3</u>

2	<u>144</u>
2	<u>72</u>
2	<u>36</u>
2	<u>18</u>
3	<u>9</u>
3	<u>3</u>

2	<u>10404</u>
2	<u>5202</u>
3	<u>2601</u>
3	<u>867</u>
17	<u>289</u>
17	<u>17</u>

Exp. 4) Find the square root of 120409.

Solution

	347
3	<u>12</u> <u>04</u> <u>09</u>
3	<u>9</u>
64	<u>304</u>
4	<u>256</u>
687	<u>4809</u>
7	<u>4809</u>
	<u>4809</u>

$\therefore \sqrt{120409} = 347$

Exp. 5) Find the square root of 5793649.

Solution

	2407
2	<u>5</u> <u>79</u> <u>36</u> <u>49</u>
2	<u>4</u>
44	<u>179</u>
4	<u>176</u>
4807	<u>33649</u>
07	<u>33649</u>

$\therefore \sqrt{5793649} = 2407$

NOTE

If $(11)^2 = 121$, then $(101)^2 = 10201$
 If $(21)^2 = 441$, then
 $(201)^2 = 40401$ and so on...
 If $(11)^2 = 121$, then $(1001)^2 = 1002001$
 If $(31)^2 = 961$, then $(3001)^2 = 9006001$
 If $(12)^2 = 144$, then $(1002)^2 = 1004004$ and so on

Go in depth and find the limitation of this logic ...

NOTE

$1^2 = 1$
 $11^2 = 121$
 $111^2 = 12321$
 $1111^2 = 1234321$
 $11111^2 = 123454321$
 $111111^2 = 12345654321$

Cubing Techniques

If a number is multiplied by itself twice, then the resultant value is called as the cube of that number. For example $n \times n \times n = n^3$, read as n cube.

If we say that what is the cube of 4, then it means

$$4^3 = 4 \times 4 \times 4 = 64$$

Similarly, $7^3 = 7 \times 7 \times 7 = 343$ and $8^3 = 8 \times 8 \times 8 = 512$

But, for the greater (or larger) number sometimes it becomes tedious to calculate the cube very quickly. So use a new approach to calculate the cube of larger numbers from 11 to 99.

Since you can easily calculate the cubes of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 60, 70, 80, 90 and 100 etc. So you can leverage it for further calculation.

Method to find cube of two digit numbers

Step 1. Write down the cube of tens digit.

Step 2. Write down the resultant value in the same ratio as the ratio of tens and unit digit in four terms as a G.P.

Step 3. Write down twice the value obtained in the second and third term below the second and third term respectively.

Step 4. Now you can add all the numbers in the prescribed manner as given in the following examples :

 **NOTE** While doing the addition (in step 4) subscripts should be treated as a "carryover" for the next term.

Exp. 1) $(11)^3 \Rightarrow$ Step 1. $1^3 = 1$
 Step 2. 1 1 1 1
 Step 3. 1 1 1 1
 + 2 2
 Step 4. 1 3 3 1
 $\therefore (11)^3 = 1331$

Exp. 2) $(12)^3 \Rightarrow$ Step 1. $1^3 = 1$
 Step 2. 1 2 4 8
 Step 3. 1 2 4 8
 + 4 8
 Step 4. 1 7 2 8
 $\therefore (12)^3 = 1728$

Exp. 3) $(13)^3 \Rightarrow$ Step 1. $1^3 = 1$
 Step 2. 1 3 9 27
 Step 3. 1 3 9 27
 + 6 18
 Step 4. 2 1 2⁹ 2⁷
 $\therefore (13)^3 = 2197$

Exp. 4) $(25)^3 \Rightarrow$ Step 1. $2^3 = 8$
 Step 2. 8 20 50 125
 Step 3. 8 20 50 125
 + 40 100
 Step 4. 15 7⁶ 16² 12⁵
 $\therefore (25)^3 = 15625$

Exp. 5) $(26)^3 \Rightarrow$ Step 1. $2^3 = 8$
 Step 2. 8 24 72 216
 Step 3. 8 24 72 216
 + 48 144
 Step 4. 17 9⁵ 23⁷ 21⁶
 $\therefore (26)^3 = 17576$

Exp. 6) $(27)^3 \Rightarrow$ Step 1. $2^3 = 8$
 Step 2. 8 28 98 343
 Step 3. 8 28 98 343
 + 56 196
 Step 4. 19 11⁶ 32⁸ 34³
 $\therefore (27)^3 = 19683$

Exp. 7) $(53)^3 \Rightarrow$ Step 1. $5^3 = 125$
 Step 2. 125 75 45 27
 Step 3. 125 75 45 27
 + 150 90
 Step 4. 148 23⁸ 13⁷ 2⁷
 $\therefore (53)^3 = 148877$

Exp. 8) $(67)^3 \Rightarrow$ Step 1. $(6)^3 = 216$
 Step 2. 216 252 294 343
 Step 3. 216 252 294 343
 + 504 588
 Step 4. 300 84⁷ 91⁶ 34³
 $\therefore (67)^3 = 300763$

Exp. 9) $(94)^3 \Rightarrow$ Step 1. $(9)^3 = 729$
 Step 2. 729 324 144 64
 Step 3. 729 324 144 64
 + 648 288
 Step 4. 830 101⁵ 43⁸ 6⁴
 $\therefore (94)^3 = 830584$

Exp. 10) $(88)^3 \Rightarrow$ Step 1. $(8)^3 = 512$
 Step 2. 512 512 512 512
 Step 3. 512 512 512 512
 + 1024 1024
 Step 4. 681 169⁴ 158⁷ 51²
 $\therefore (88)^3 = 681472$

Exp. 11) $(55)^3 \Rightarrow$ Step 1. $(5)^3 = 125$
 Step 2. 125 125 125 125
 Step 3. 125 125 125 125
 + 250 250
 Step 4. 166 41³ 38⁷ 12⁵
 $\therefore (55)^3 = 166375$

Exp. 12) $(72)^3 \Rightarrow$ Step 1. $(7)^3 = 343$
 Step 2. 343 98 28 8
 Step 3. 343 98 28 8
 + 196 56
 Step 4. 373 30² 8⁴ 8
 $\therefore (72)^3 = 373248$

Cube Roots

The cube root of a number n is expressed as $\sqrt[3]{n}$ or $(n)^{1/3}$ or in other words $n = (n)^{1/3} \times (n)^{1/3} \times (n)^{1/3}$

For example, cube root of 27 i.e., $\sqrt[3]{27} = 3$

and $\sqrt[3]{343} = 7$ and $\sqrt[3]{64} = 4$

Exp. 1) Find the cube root of 125.

Solution $\therefore 125 = 5 \times 5 \times 5$

$\therefore \sqrt[3]{125} = 5$

$$\begin{array}{r|l} 5 & 125 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

Exp. 2) Find the cube root of 1728.

Solution $1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$\sqrt[3]{1728} = 2 \times 2 \times 3$

$\sqrt[3]{1728} = 12$

$$\begin{array}{r|l} 2 & 1728 \\ 2 & 864 \\ 2 & 432 \\ 2 & 216 \\ 2 & 108 \\ 2 & 54 \\ 3 & 27 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

NOTE In this method first we factorize the given numbers then we make the triplet of the factors and thus we multiply these triplets to get the cube root of the given number.

Exp. 3) On 14th November, in my school, each child received as many packs of chocolates as there were total number of the students in the school. Further, each pack of chocolates contains as many chocolates as there were the total number of packs which a child had. Total how many chocolates have been distributed among all the children of school :

- (a) 729 (b) 196
(c) 961 (d) can't be determined

Solution (d) is the appropriate answer since we don't have sufficient data to calculate.

Exp. 4) In the above question if there were 8 packs of chocolates with every child. Then the total number of chocolates distributed among them was :

- (a) 512 (b) 64
(c) 256 (d) can't be determined

Solution Let there are n children it means each child has n packs of chocolates and since every pack contains n chocolates. Therefore total number of chocolates

$$= n \times n \times n = n^3$$

Hence total number of chocolates $= (8)^3 = 512$

Thus option (a) is correct.

Introductory Exercise 1.2

- The square root of 6280036 is :
(a) 1308 (b) 2903
(c) 2506 (d) none of these
- The square root of 1296 is :
(a) 33 (b) 44 (c) 34 (d) 36
- The square root of 7744 is :
(a) 94 (b) 88
(c) 77 (d) none of these
- The square root of 56169 is :
(a) 359 (b) 323
(c) 227 (d) none of these
- The square root of 1238578 is :
(a) 3254 (b) 3724
(c) 3258 (d) None of these
- The least number by which we multiply to the 11760, so that we can get a perfect square number :
(a) 2
(b) 3
(c) 5
(d) none of the above
- In the above question, by which least possible number we divide to the 11760 so the resultant number becomes a perfect square :
(a) 3
(b) 15
(c) 7
(d) can't be determined
- The least possible positive number which should be added to 575 to make a perfect square number is :
(a) 0 (b) 1
(c) 4 (d) none of these
- The least possible number which must be subtracted from 575 to make a perfect square number is :
(a) 5 (b) 50
(c) 46 (d) 37
- The square of a number 'A' is the sum of the square of other two numbers 'B' and 'C'. Where $5B = 12C$ and B, C are positive numbers. The least possible positive value of A is :
(a) 10 (b) 12
(c) 13 (d) 16

11. Lieutenant Kalia when arranged all his 1500 soldiers in such a way that the number of soldiers in a line were the same as there were the number of lines. So he was left with 56 soldiers, who were not a part of this arrangement. The number of lines in this arrangement is :
 (a) 44 (b) 36
 (c) 38 (d) none of these
12. $\sqrt{289} \div \sqrt{x} = \frac{1}{5}$, then the value of x is :
 (a) $\frac{17}{25}$ (b) $\frac{34}{35}$
 (c) 235 (d) 7225
13. Out of the following statements which one is incorrect?
 (a) $\sqrt{5184} = 72$ (b) $\sqrt{15625} = 125$
 (c) $\sqrt{1444} = 38$ (d) $\sqrt{1296} = 34$
14. If $2 * 3 = \sqrt{13}$ and $3 * 4 = 5$, then the value of $5 * 12$ is
 (a) 17 (b) $\sqrt{29}$
 (c) 21 (d) 13
15. If $a * b * c = \sqrt{\frac{(a+2)(b+3)}{(c+1)}}$, then the value of $(6 * 15 * 3)$ is :
 (a) 6 (b) 3
 (c) 4 (d) can't be determined
16. The smallest number that must be added to 1780 to make it a perfect square is :
 (a) 69 (b) 156
 (c) 49 (d) 59
17. If $\frac{100\sqrt{25}}{\sqrt{25} + x} = 50$ then the value of x is :
 (a) 25 (b) $\frac{1}{\sqrt{25}}$ (c) $\sqrt{25}$ (d) $\frac{1}{25}$
18. The least possible number which we should add to 1720 to make a perfect cube number is :
 (a) 0 (b) 1
 (c) 8 (d) 7
19. The least positive number which is subtracted from 1369 to make it a perfect cube is :
 (a) 17 (b) 38
 (c) 34 (d) none of these
20. The least possible natural number by which if we multiply to the 1372, we get a perfect cube number is :
 (a) 2 (b) 3
 (c) 5 (d) can't be determined
21. The least possible number by which if we divide 1372, it will become a perfect cube number is :
 (a) 2 (b) 7 (c) 3 (d) 4

1.3 Basic Numbers

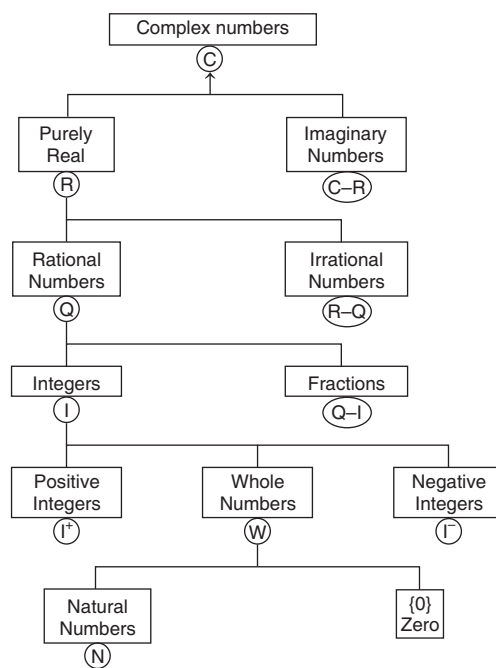
It has been said by a great mathematician that “Mathematics is the queen of Science and Arithmetic is the queen of Mathematics.” Arithmetic is the maths of numerals and digits and so the other branches of Maths are dependent upon Arithmetic.

Either the counting of the currency or counting of the stars in all the ways it has been a very popular subject especially among Indians.

One thing which is very well known that the great Indian Mathematician Aryabhatta has introduced ‘0’ (zero) in the counting numbers and the whole world has adopted it. In fact, a computer is dead if this figure does not exist. So one can say zero has revolutionised the whole world.

Here we will discuss all sorts of numbers, their properties, applications besides the different number systems used widely.

The following chart briefly illustrates the different kinds of numbers with their relationship.



Natural Numbers

The counting numbers 1, 2, 3, 4, 5, ... are called the natural numbers and are denoted by N . As you may like to recall that a child when he/she is very young he/she starts counting his/her toys, sweets, fruits, etc. as 1, 2, 3, 4, 5, ..., as he/she has no idea about 0 (zero) and decimal numbers such as 1.5, 2.75 or 3.33 or any other type of numbers. That's why probably the numbers 1, 2, 3, 4, 5, ... are called natural numbers.

Natural numbers are represented by N , where $N = \{1, 2, 3, \dots\}$.

All natural numbers are the positive integers.

Properties of Natural Numbers

- Successor** : The next natural number just after any natural number n is called its successor ' n^+ ', where $n^+ = n + 1$ for example the successor of 2 is 3, successor of 6 is 7 etc.
- Closure law** : For any two natural numbers a and b

$$(a + b) \in N \quad \text{and} \quad (a \times b) \in N$$

e.g., $3 + 4 = 7 \in N$ and $3 \times 4 = 12 \in N$
- Commutative law** : For any two natural numbers a and b

$$a + b = b + a \quad \text{and} \quad a \times b = b \times a$$

e.g., $5 + 6 = 6 + 5 = 11$
and $5 \times 6 = 6 \times 5 = 30$
- Associative law** : For any three natural numbers
$$(a + b) + c = a + (b + c)$$

$$(a \times b) \times c = a \times (b \times c)$$

e.g., $(7 + 8) + 9 = 7 + (8 + 9) = 24$
 $(7 \times 8) \times 9 = 7 \times (8 \times 9) = 504$
- Multiplicative Identity** : 1 is the multiplicative identity of every natural number as
$$4 \times 1 = 4$$

$$5 \times 1 = 5$$

$$10 \times 1 = 10$$

$$17 \times 1 = 17$$
- Cancellation law** : For any three natural numbers a, b, c

$$a + b = c + b \Rightarrow a = c$$

and $a \times b = c \times b \Rightarrow a = c$
- Distributive law** : For any three natural numbers a, b, c

$$a \times (b + c) = a \times b + a \times c$$

but $a + (b \times c) \neq (a + b) \times (a + c)$
e.g., $3 \times (4 + 5) = 3 \times 4 + 3 \times 5 = 27$
- Trichotomy law** If there are any two natural numbers a and b then there exists one and only one relation necessarily is
 - $a > b$
 - $a = b$
 - $a < b$

Even Numbers

All the natural numbers which are divisible by 2 are known as even numbers e.g., 2, 4, 6, 8, 10, ...

Odd Numbers

All the natural numbers which are not divisible by 2 are known as 'Odd numbers' e.g., 1, 3, 5, 7, ...

Important Note

Even + Even = Even	Even $\times$ Odd = Even
Even - Even = Even	Even + Odd = Even
Even $\times$ Even = Even	Odd + Even = Odd
Even + Even = Even of odd	Odd - Even = Odd
	Odd $\times$ Even = Even
	Odd + Even = (never divisible)
Odd + Odd = Even	
Odd - Odd = Even	
Odd $\times$ Odd = Odd	(even) ^{even/odd} = Even
Odd + Odd = Even	(odd) ^{odd/even} = Odd
Even + Odd = Odd	
Even - Odd = Odd	

Prime Numbers

Except 1 each natural number which is divisible by only 1 and itself is called as prime number e.g., 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, ... etc.

- There are total 25 prime numbers upto 100
- There are total 46 prime numbers upto 200
- 2 is the only even prime number and the least prime number.
- 1 is neither prime nor composite number.
- There are infinite prime numbers.
- A list of all the prime numbers upto 100 is given below.

Table of Prime Numbers (1 - 100) :

2	11	23	31	41	53	61	71	83	97
3	13	29	37	43	59	67	73	89	
5	17			47			79		
7	19								

How to test whether a number is prime or not : To test a number n take the square root of n and consider as it is, if it is a natural number otherwise just increase the square root of it to the next natural number. Then divide the given number by all the prime numbers below the square root obtained. If the number is divisible by any of these prime numbers then it is not a prime number else it is a prime number.

Exp.) Check that whether 241 is prime.

Solution When we take the square root of 241 it is approximate 15, so we consider it 16. Now we divide 241 by all the prime numbers below 16 viz., 2, 3, 5, 7, 11, 13

Since 241 is not divisible by any one of the prime numbers below 16. So it is a **prime number**.

NOTE Any digit if it is written continuously 3 times, 6 times, 9 times ... etc. then it is divisible by 3
e.g., 111; 555555; 777777; 22222222; 888, 222 etc.

Co-prime Numbers

Two natural numbers are called co-prime (or relatively prime) numbers if they have no common factor other than 1 or in other words. The highest common factor *i.e.*, H.C.F. between co-prime numbers is 1. *e.g.*, (15, 16), (14, 25), (8, 9), (13, 15) etc.

NOTE It is not necessary that the numbers involved in the pair of co-primes will be prime even they can be composite numbers as seen in the above examples.

Twin Primes

When the difference between any two prime number is 2, these prime numbers are called twin primes.

e.g., (3, 5), (5, 7), (11, 13), (17, 19), (29, 31), (41, 43), (59, 61), (71, 73), (101, 103), (107, 109), (137, 139), etc.

Composite Numbers

A number other than one which is not a prime number is called a composite number. It means it is divisible by some other number(s) other than 1 and the number itself. *e.g.*, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26...

Every natural number except 1 is either prime or composite.

Whole Numbers

The extended set of natural numbers in which '0' is also included is called as the set of whole numbers and is denoted by

$$W = \{0, 1, 2, 3, 4, 5, \dots\}$$

NOTE '0' (zero) is an even number.

Consecutive Numbers

A series of numbers in which the next number is 1 more than the previous number or the predecessor number is 1 less than the successor or just they can be differed by 1

e.g., 10, 11, 12, or 17, 18, 19, 20, 21 or 717, 718, 719, 720, 721... etc.

Perfect Number

When the sum of all the factors (including 1 but excluding the number itself) of the given number is the same number then this number is called as Perfect Number.

For example 6, 28, 496, 8128, ...etc.

So far only 27 perfect numbers are known. The factors of 28 are 1, 2, 4, 7, 14, 28

Now, $1 + 2 + 4 + 7 + 14 = 28$

Hence 28 is a perfect Number.

Triangular Number

A triangular number is obtained by adding the previous number to the n th position in the sequence of triangular numbers, where the first triangular number is 1. The sequence of triangular numbers is given as follows

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, ...etc.

Introductory Exercise 1.3

- The set of natural numbers is closed under the binary operations of :
 - addition, subtraction, multiplication and division
 - addition, subtraction, multiplication but not division
 - addition and multiplication but not subtraction and division
 - addition and subtraction but not multiplication and division
- If p be a prime, $p > 3$ and let x be the product of natural numbers 1, 2, 3, ..., $(p - 1)$, then consider the following statements :
 - x is a composite number divisible by p .
 - x is a composite number not divisible by p , but some prime greater than p may divide x .
 - x is not divisible by any prime $(p - 2)$.
 - All primes less than $(p - 1)$ divide x .

Of these statements :

- 1 and 2 are correct
- 2 and 3 are correct
- 3 and 4 are correct
- 4 alone is correct

- Consider the following statements :

- If $p > 2$ is a prime, then it can be written as $4n + 1$ or $4n + 3$ for suitable n .
- If $p > 2$ is a prime, then $(p - 1)(p + 1)$ is always divisible by 4.

Of these statements :

- 1 and 2 are false
- 1 and 2 are true
- 1 is true but 2 is false
- 1 is false but 2 is true

- A number n is said to be 'perfect' if the sum of all its divisors excluding n 'itself' is equal to n . An example of perfect number is :

- 9
- 15
- 21
- 6

5. If $2^p + 1$ is a prime number, then p must be power of
 (a) 2 (b) 3
 (c) 5 (d) 6
6. The number of composite numbers between 101 and 120 is :
 (a) 11 (b) 12
 (c) 13 (d) 14
7. The total number of prime numbers between 120 and 140 is :
 (a) 7 (b) 6 (c) 5 (d) 4
8. The unit digit of every prime number (other than 2 and 5) must be necessarily :
 (a) 1, 3 or 5 only (b) 1, 3, 7 or 9
 (c) 7 or 9 only (d) 1 or 7 only

1.4 Integers

The extended set of whole numbers in which negative integers are also included is known as the set of integers and is denoted by

$$\mathbb{Z} \text{ or } I = \{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, \dots\}$$

(a) **Positive integers** : The set of integers $\{1, 2, 3, \dots\}$ is known as positive integers.

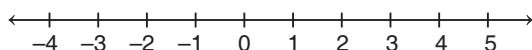
(b) **Negative integers** : The set of integers $\{-1, -2, -3, -4, \dots\}$ is known as negative integers.

(c) **Non-negative integers** : The set of integers $\{0, 1, 2, 3, \dots\}$ is called as non-negative integers.

(d) **Non-positive integers** : The set of integers $\{0, -1, -2, -3, \dots\}$ is called as non-positive integers.

- '0' is neither positive nor negative integer.

Representation of the Integers on a Number Line



All the integers (whole number, natural number etc.) can be shown on this number line, where every integer is represented by some point on the line.

It can be said that :

- There is no any largest or smallest integer.
- Every integer has a predecessor and a successor.
- An integer is smaller than all those integers which are on the right side of it and is greater than all those integers which are on the left side of it on the number line.
 e.g., $-4 > -5, 3 > 1, 10 > -4$ etc.
 or $-6 < 0, -2 < 1$ etc.

Exp. 1) Arrange the following integers in ascending order $-3, -7, 8, 5, 0, 3, 17, -23$.

Solution $-23, -7, -3, 0, 3, 5, 8, 17$

Exp. 2) Arrange the following integers in descending order $-17, 18, 15, 32, 81, -5, 87$.

Solution $87, 81, 32, 18, 15, -5, -17$

Properties of Integers

- Closure law is followed by all the integers
- Commutative law and Associative law is not followed by all the integers for the subtraction.

e.g., $4 - 6 \neq 6 - 4$

and $(4 - 6) - 2 \neq 4 - (6 - 2)$

but it is valid for the addition and multiplication as

$$4 + 6 = 6 + 4$$

or $(-3) + (-2) = (-2) + (-3)$

and $4 \times 6 = 6 \times 4$

or $-3 \times -2 = -2 \times -3$

and $(-2) + [(-3) + (-7)] = [(-2) + (-3)] + (-7)$ etc.

- Additive identity of all the integers is zero (0) and Multiplicative identity of all the integers is 1.

e.g., $-3 + 0 = -3, 8 + 0 = 8$

and $-5 \times 1 = -5, 7 \times 1 = 7$ etc.

- Additive inverse of an integer a is $-a$.

e.g. the additive inverse of 7, 8, 9, 15, $-3, -5, -6$

etc. are $-7, -8, -9, -15, 3, 5$ and 6 respectively.

- Distributive law of multiplication over addition or subtraction

i.e., $a \times (b \pm c) = a \times b \pm a \times c$

For example: $3 \times (4 \pm 6) = 3 \times 4 \pm 3 \times 6$

NOTE

- Division by zero is not defined in Mathematics.
- Division by 1 is actually unification (not division) so it is an improper divisor.

Some important rules regarding the sign convention in mathematical operations

(i) $(a) + (b) = + (a + b)$

$$(-a) + (b) = b - a$$

$$(a) + (-b) = a - b$$

$$(-a) + (-b) = - (a + b)$$

i.e., $(+) + (+) = +$

$(-) + (+) = +$ if the numerical value of $+$ is greater
 $(-) + (+) = -$ if the numerical value of $-$ is greater
 $(-) + (-) = -$

For example $(4) + (7) = 11$, $(-3) + (8) = 5$

$$(-8) + (3) = -5$$

$$(-5) + (-3) = -8 \text{ etc.}$$

Similarly, $(5) - (2) = 3$, $(2) - (5) = -3$

$$(-5) - (2) = -7$$
, $(5) - (-2) = 7$

$$(-5) - (-2) = -3$$
, $(-2) - (-5) = 3$

- (ii) $a \times b = ab$ *i.e.*, $(+) \times (+) = +$
 $-a \times b = -ab$ *i.e.*, $(-) \times (+) = -$
 $a \times -b = -ab$ *i.e.*, $(+) \times (-) = -$
 $-a \times -b = ab$ *i.e.*, $(-) \times (-) = +$

For example, $(2) \times (3) = 6$, $(-2) \times (3) = -6$,
 $2 \times (-3) = -6$ and $(-2) \times (-3) = 6$

$$\text{Similarly, } \frac{8}{2} = 4, \frac{-8}{2} = -4,$$

$$\frac{8}{-2} = -4, \frac{-8}{-2} = 4$$

$$\text{i.e., } \frac{(+)}{(+)} = (+), \frac{(-)}{(+)} = (-),$$

$$\frac{(+)}{(-)} = (-), \frac{(-)}{(-)} = (+)$$

Practice Exercise

Evaluate the following

1. $4 + (-5)$	2. $-4 - (-2)$
3. $8 - (-5)$	4. $-13 + (56)$
5. $(-94) + (-239)$	6. $(-526) - (-217)$
7. $7 + (-5) + (-2)$	8. $-6 + (-2) - (-3) + 1$
9. 6×9	10. $(-9) \times (-13)$
11. -7×8	12. $13 \times (-15)$
13. $(-3) \times (8) \times (-5)$	14. $(-8) \times (-19) \times (-15)$
15. $[(-6) \times (-8)] \times 5$	16. $(-18) \div 6$
17. $126 \div (-14)$	18. $-13 \times (7 - 8)$
19. $(-32) \div (-4)$	20. $(-31) + (31)$

Answers

1. -1	2. -2	3. 13	4. 43	5. -333
6. -309	7. 0	8. -4	9. 54	10. 117
11. -56	12. -195	13. 120	14. -2280	15. 240
16. -3	17. -9	18. 13	19. 8	20. 0

Numerical Expression

Collection of numbers connected by one or more operations of addition, subtraction, multiplication and division is called a numerical expression.

It can also involve some brackets.

e.g., $7 + 18 \div 3 \div 1 \times 6$

and $84 - 6 \div 2 - 3 \times (-7)$

and $9 + (-2) \times \{72 \div 8\} + 21$, etc.

Simplification Rules

The order in which various mathematical operations must be done can be remembered with the word 'BODMAS'

Where B $\rightarrow$ Brackets

O $\rightarrow$ Off

D $\rightarrow$ Division

M $\rightarrow$ Multiplication

A $\rightarrow$ Addition

S $\rightarrow$ Subtraction

So first of all we solve the innermost brackets moving outwards. Then we perform 'of' which means multiplication then, Division, Addition and Subtraction.

- Addition and Subtraction can be done together or separately as required.
- Between any two brackets if there is no any sign of '+' or '-' it means we have to do multiplication *e.g.*,

$$(7)(2) = 7 \times 2 = 14$$

$$[3(5) + 7] = 15 + 7 = 22$$

Brackets : They are used for the grouping of things or entities. The various kind of brackets are :

- '-' is known as line (or bar) bracket or vinculum
- () is known as parenthesis or common bracket
- { } is known as curly bracket or brace.
- [] is known as rectangular (or big) bracket.

The order of eliminating brackets is :

- line bracket
- common bracket
- curly bracket
- rectangular bracket

The significance of various brackets is as follows:

- { } contains only the particular values that are explicitly mentioned.
- () contains all the values of the defined range, except the extreme values of the given range.

- [] contains all the values of the defined range including both the extreme values of the given range.
- () contains all the values of the defined range, but does not include the lowest extreme value of the given range.
-] contains all the values of the defined range, but does not include the highest extreme value of the given range.

Exp. 1) Solve the following expressions :

- (a) $3 + 2 - 1 \times 4 \div 2$
- (b) $7 \times 3 + 8 - 2$
- (c) $53 \times 2 - 1 \times 6$
- (d) $12 + (-3) + 5 - (-2)$

Solution (a) $3 + 2 - 1 \times 4 \div 2 = 3 + 2 - 1 \times 2 = 3 + 2 - 2 = 3$
 (b) $7 \times 3 + 8 - 2 = 21 + 8 - 2 = 27$
 (c) $53 \times 2 - 1 \times 6 = 106 - 6 = 100$
 (d) $12 + (-3) + 5 - (-2) = 12 - 3 + 5 + 2 = 9 + 7 = 16$

Exp. 2) Evaluate or simplify the following expression

- (a) $9 - [7 - 24 \div (8 + 6 \times 2 - 16)]$
- (b) $(-3) \times (-12) \div (-4) + 3 \times 6$
- (c) $17 - \{8 \div (2 \times 3 - 4)\}$
- (d) $5 \times 2 - [3 - \{5 - (7 + 2 \div 4 - 19)\}]$

Solution (a) $9 - [7 - 24 \div (8 + 6 \times 2 - 16)]$
 $= 9 - [7 - 24 \div (8 + 12 - 16)] = 9 - [7 - 24 \div 4]$
 $= 9 - [7 - 6] = 9 - [1] = 8$
 (b) $(-3) \times (-12) \div (-4) + 3 \times 6 = 36 \div (-4) + 18$
 $= -9 + 18 = 9$
 (c) $17 - \{8 \div (2 \times 3 - 4)\} = 17 - \{8 \div (6 - 4)\}$
 $= 17 - \{8 \div 2\} = 17 - \{4\} = 13$
 (d) $5 \times 2 - [3 - \{5 - (7 + 2 \div 4 - 19)\}]$
 $= 5 \times 2 - [3 - \{5 - (7 + 0.5 - 19)\}]$
 $= 10 - [3 - \{5 - (-11.5)\}] = 10 - [3 - \{9\}]$
 $= 10 - [-6] = 10 + 6 = 16$

Exp. 3) Which one of the following options is correct for each of the given expressions?

- (i) $45 - [28 - \{37 - (15 - k)\}] = 58$, the value of k is :
 (a) 19 (b) -19
 (c) -39 (d) none of these
- (ii) $1 + [1 \div \{5 \div 4 - 1 \div (13 \div 3 - 1 \div 3)\}]$ is equal to :
 (a) $2/5$ (b) 2
 (c) $3/2$ (d) none of these
- (iii) $2 - [3 - \{6 - (5 \div 4 - 3)\}]$ is equal to :
 (a) 0 (b) 1
 (c) -3 (d) none of these

Ans. (i) (a), (ii) (b), (iii) (a)

Exp. 4) A student gets 4 marks for a correct answer and 1 mark is deducted for a wrong answer. If she has attempted 80 questions at all and she has got only 240 marks, the number of correct answers she has attempted is.

- (a) 40 (b) 60
- (c) 64 (d) can't be determined

Solution If she has attempted all the 80 questions correctly she must have got 320 ($= 80 \times 4$) marks, but if she attempts one wrong answer it means she is liable to lose 5 marks ($4 + 1$). Thus for every wrong answer she loses 5 marks.

Now since she has lost total 80 marks ($320 - 240 = 80$).

This implies that she has attempted 16 questions wrong ($\therefore \frac{80}{5} = 16$). It means she has done only 64 questions

($80 - 16 = 64$) correctly.

Hence (c) is the correct option.

Alternatively Since she has obtained less marks than the maximum possible marks it means she must have attempted some questions incorrectly.

Now, we can check through options. Let us consider option (c).

Correct answer	Wrong answers	Total questions
64	16	80
Marks for correct answers	Marks for wrong answers	Net marks
$64 \times 4 = 256$	$16 \times -1 = -16$	$256 - 16 = 240$

Hence the assumed option (c) is correct.

Alternatively If she has attempted x question correctly it means she has attempted $(80 - x)$ questions incorrectly.

$$\begin{aligned} \text{So} \quad & 4 \times (x) - 1 \times (80 - x) = 240 \\ \Rightarrow & 5x - 80 = 240 \\ \Rightarrow & 5x = 320 \\ \Rightarrow & x = 64 \end{aligned}$$

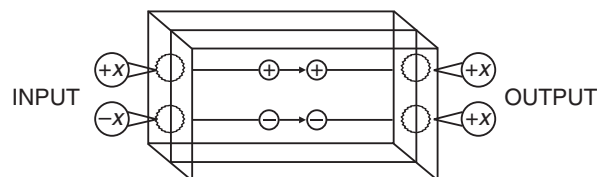
Hence she has attempted 64 answers correctly.

Absolute value of an integer or Modulus

The absolute value of an integer is its numerical value irrespective of its sign (or nature). The absolute value of an integer x is written as $|x|$ and is defined as

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Modulus (Machine Model)



It means any integer whether it is positive or negative if it is operated upon modulus, it always gives a positive integer.

for example $|-7| = 7$, $|-4| = 4$, $|3| = 3$ etc.

Also, $\text{Max } \{x, -x\} = |x|$

and $-\text{Min } \{x, -x\} = |x|$

and $\sqrt{(x)^2} = |x|$

Properties of a Modulus or Mod

1. $|a| = |-a|$
2. $|ab| = |a| |b|$
3. $\left| \frac{a}{b} \right| = \frac{|a|}{|b|}$
4. $|a + b| \leq |a| + |b|$

(The sign of equality holds only when the sign of a and b are same)

5. If $|a| \leq k \Rightarrow -k \leq a \leq k$
6. If $|a - b| \leq k \Rightarrow -k \leq a - b \leq k$
 $\Rightarrow b - k \leq a \leq b + k$

Exp. 1) Solution set of the equation $|x - 2| = 5$ is :

- (a) $\{3, -7\}$
- (b) $\{-3, 7\}$
- (c) $\{3, 6\}$
- (d) none of these

Solution $|x - 2| = 5 \Rightarrow x - 2 = 5$

or $x - 2 = -5$

$\Rightarrow x = 7$ or $x = -3$

Hence $x = \{-3, 7\}$, (b) is the correct option.

Exp. 2) The maximum value of the expression $27 - |9x - 8|$ is

- (a) 27
- (b) 17
- (c) 44
- (d) 26

Solution The maximum value of the expression $27 - |9x - 8|$ will be maximum only when the value of $|9x - 8|$ is minimum, but the minimum possible value of any $|k|$ is zero. Hence the maximum value of $27 - |9x - 8| = 27 - (0) = 27$

Hence option (a) is correct.

Exp. 3) The minimum value of the expression $|17x - 8| - 9$ is

- (a) 0
- (b) -9
- (c) $\frac{8}{17}$
- (d) none of the above

Solution The minimum value of expression $|17x - 8| - 9$ is minimum only when $|17x - 8|$ is minimum. But the minimum value of $|k|$ is zero.

Hence minimum value of $|17x - 8| - 9 = 0 - 9 = -9$

Exp. 4) If $2a - 9 = b + a$, then the value of

$(|a - b| + |b - a|)$ is :

- (a) 18
- (b) 11
- (c) 1
- (d) 0

Solution $\therefore 2a - 9 = b + a$

$\Rightarrow 2a - a = b + 9$

$\Rightarrow a = b + 9$

$\Rightarrow a - b = 9$ or $b - a = -9$

Hence $|a - b| + |b - a| = |9| + |-9|$
 $= 9 + 9 = 18$

Thus (a) is the correct option.

Exp. 5) The value of x for which the value of $|3x + 15|$ is minimum :

- (a) 3
- (b) 5
- (c) -5
- (d) none of the above

Solution The minimum value of $|3x + 15| = 0$

$\Rightarrow 3x + 15 = 0 \Rightarrow x = -5$

Hence (c) is correct option.

Greatest Integer Less Than Or Equal To A Number

This number is represented as $\lfloor N \rfloor$. The resultant number is always an integral number.

1. when the given number is not an integer, the resultant number is the greatest integer less than the given number. For example, $\lfloor 3.1 \rfloor = 3$, $\lfloor 2.5 \rfloor = 2$, $\lfloor 1.99 \rfloor = 1$, $\lfloor 0.75 \rfloor = 0$, $\lfloor -0.45 \rfloor = -1$, $\lfloor -1.45 \rfloor = -2$, etc.
2. When the given number is integer, the resultant number is the same as the given Number. For example, $\lfloor 3 \rfloor = 3$, $\lfloor 2 \rfloor = 2$, $\lfloor 0 \rfloor = 0$, $\lfloor -1 \rfloor = -1$, $\lfloor -2 \rfloor = -2$, etc

Least Integer Greater Than Or Equal To A Number

This number is represented as $\lceil N \rceil$. The resultant number is always an integral number.

1. When the given number is not a integer, the resultant number is the least integer less than the given number. For example, $\lceil 3.1 \rceil = 4$, $\lceil 2.5 \rceil = 3$, $\lceil 1.99 \rceil = 2$, $\lceil 0.75 \rceil = 1$, $\lceil -0.45 \rceil = 0$, $\lceil -1.45 \rceil = -1$, etc.
2. When the given number is integer, the resultant number is the same as the given Number. For example, $\lceil 3 \rceil = 3$, $\lceil 2 \rceil = 2$, $\lceil 0 \rceil = 0$, $\lceil -1 \rceil = -1$, $\lceil -2 \rceil = -2$, etc.

Introductory Exercise 1.4

- $\frac{3}{4} \times \left(\frac{-2}{3} + \frac{3}{5} \right)$ is equal to :
 (a) $\frac{-3}{20}$ (b) $\frac{-19}{20}$
 (c) $\frac{-1}{20}$ (d) $\frac{1}{20}$
 - The value of $4 \times 100 + 3 \times 10 + \frac{9}{1000}$ is :
 (a) 430.09 (b) 430.0009
 (c) 430.009 (d) 430.900
 - If x and y are positive real numbers, then :
 (a) $x > y \Rightarrow -x > -y$ (b) $x > y \Rightarrow -x < -y$
 (c) $x > y \Rightarrow \frac{1}{x} > \frac{1}{y}$ (d) $x > y \Rightarrow \frac{-1}{x} < \frac{-1}{y}$
 - If $x < 0 < y$, then :
 (a) $\frac{1}{x^2} < \frac{1}{xy} < \frac{1}{y^2}$ (b) $\frac{1}{x^2} > \frac{1}{xy} > \frac{1}{y^2}$
 (c) $\frac{1}{x} < \frac{1}{y}$ (d) $\frac{1}{x} > 1$
 - On the set of integers I , if a binary operation 'o' be defined as $aob = a - b$ for every $a, b \in I$, then :
 (a) association law holds
 (b) commutative law holds
 (c) I is not closed under this operation
 (d) I is closed under this operation
 - If $|x - 2| < 3$ then :
 (a) $1 < x < 5$
 (b) $-1 < x < 2$
 (c) $-1 < x < 5$
 (d) $0 < x < 6$
 - How many of the following relations is/are always true for any real values of p and q ?
 (i) $|p + q| = |p| + |q|$ (ii) $|p - q| = |p| - |q|$
 (iii) $\left| \frac{p}{q} \right| = \frac{|p|}{|q|}$ (iv) $|pq| = |p| |q|$
 (v) $|p^q| = |p|^{|q|}$ (vi) $\sqrt[q]{p} = \sqrt[q]{|p|}$
 (a) None (b) one
 (c) two (d) three
 - Which of the following relations is NOT always true?
 (a) $|p + q| \leq |p| + |q|$
 (b) $|p - q| \leq |p| - |q|$
 (c) $|p| - |q| > 0$, if $p^2 - q^2 > 0$
 (d) $|p - q|^2 = (p - q)^2$
 - The total number of integral values of x for which $||x - 2| - 4| - 6| < 10$
 (a) 39 (b) 42
 (c) 43 (d) 38
 - If x satisfies $|x - 1| + |x - 2| + |x - 3| \geq 6$, then :
 (a) $0 \leq x \leq 4$
 (b) $x \leq 0$ or $x \geq 4$
 (c) $x \leq -2$ or $x \geq 4$
 (d) $x \geq -2$ or $x \leq 4$
 - For the positive integers a, b, c, d find the minimum value of $(-1)^a + (-1)^b + (-1)^c + (-1)^d$, If $a + b + c + d = 1947$.
 (a) -1 (b) -2
 (c) -3 (d) -4
- Directions** (for Q. Nos. 12 and 13) *In a test, conducted by Lamamia, a student attempted n problems out of total 30 problems and she scored net 17 marks. In this test each correct response awards 3 marks and each wrong response penalizes the test taker by deducting 1 mark from the total score.*
- At most how many problems are attempted by the student?
 (a) 7
 (b) 10
 (c) 17
 (d) none of the above
 - If there are total m students who took the same test in which all the students scored net 17 marks and none of these students attempted same number of problems, what is the maximum value of m ?
 (a) 17 (b) 10
 (c) 7
 (d) none of the above

1.5 Factors and Multiples

Product

When two or more number are multiplied together then the resultant value is called the product of these numbers.

For example

$$3 \times 4 = 12, \quad 2 \times 7 \times 5 = 70, \quad 3 \times 5 \times 11 = 165$$

where 12, 70 and 165 are called the products.

But we see that $12 = 4 \times 3$ it means 4 and 3 are the **factors** of 12.

and $70 = 2 \times 5 \times 7$

where 2, 5, 7 are known as **factors** of 70.

Again 12 is called as the **multiple** of 3 or **multiple** of 4.

Similarly 70 is called as the **multiple** of 2 or 5 or 7 or 10 or 35 or 14.

Thus a number which divides a given number exactly is called factor (or divisor) of that given number and the given number is called a multiple of that number.

Now, 15 is exactly divisible by 1, 3, 5, 15 so 1, 3, 5, 15 are called as the factors of 15 while 15 is called as the multiple of these factors. Where 1, 15 are **improper factors** and 3, 5 are called **proper factors** of 15.

Therefore 1 and itself (the number) are called the improper factors of the given number.

So the factors of $24 = 1, 2, 3, 4, 6, 8, 12, 24$

Factors of $35 = 1, 5, 7, 35$

Factors of $36 = 1, 2, 3, 4, 6, 9, 12, 18, 36$

Similarly the multiples of 2 are 2, 4, 6, 8, 10, 12, ...

multiples of 7 are 7, 14, 21, 28, 35, 42, 49, ...

multiples of 10 are 10, 20, 30, 40, 50, ...

- 1 is a factor of every number.
- Every number is a factor of itself.
- Every number, except 1 has atleast two factors viz., 1 and itself.
- Every factor of a number is less than or equal to that number.
- Every multiple of a number is greater than or equal to itself.
- Every number has infinite number of its multiples.
- Every number is a multiple of itself.

Prime Factorisation

If a natural number is expressed as the product of prime numbers (factors) then the factorisation of the number is called its prime factorisation.

For example :

(i) $72 = 2 \times 2 \times 2 \times 3 \times 3$

(ii) $420 = 2 \times 2 \times 3 \times 5 \times 7$

(iii) $3432 = 2 \times 2 \times 2 \times 3 \times 11 \times 13$

2	72
2	36
2	18
3	9
3	3
	1
2	420
2	210
3	105
5	35
7	7
	1
2	3432
2	1716
2	858
3	429
11	143
13	13
	1

Practice Exercise

Direction (for Q. Nos 1-6) factorize the following numbers into the prime factors.

1. 210 2. 120 3. 3315
4. 3465 5. 1197 6. 157573

7. Find the least number by which 22932 must be multiplied or divided so as to make it a perfect square:
(a) 10 (b) 4
(c) 11 (d) 13
8. How many natural numbers upto 150 are divisible by 7?
(a) 21 (b) 14
(c) 22 (d) 17
9. How many numbers between 333 and 666 are divisible by 5?
(a) 67 (b) 70
(c) 75 (d) 55
10. How many numbers between 11 and 111 are the multiples of both 2 and 5?
(a) 10 (b) 12
(c) 11 (d) 70

Answers & Solutions

1. $2 \times 3 \times 5 \times 7$ 2. $2 \times 2 \times 2 \times 3 \times 5$
3. $3 \times 5 \times 13 \times 17$ 4. $3 \times 3 \times 5 \times 7 \times 11$
5. $3 \times 3 \times 7 \times 19$ 6. $13 \times 17 \times 23 \times 31$
7. $22932 = 2^2 \times 3^2 \times 7^2 \times 13$. Therefore in order to make 22932 a perfect square you can either divide or multiply by 13.

8. 7, 14, 21, 28, 35, ..., 140, 147.
7 (1, 2, 3, ..., 20, 21)
Therefore we have 21 natural numbers between 1 and 150 which are divisible by 7.
9. 335, 340, ..., 660, 665
5(67, 68, ..., 132, 133)
Therefore the required numbers = $133 - 66 = 67$.
10. 20, 30, 100, 110.
10(2, 3, ..., 11)
Therefore the required number = $11 - 1 = 10$

Number of Prime Factors of a Composite Number

Let us assume a composite number say 24, then find the number of prime factors of 24.

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3^1$$

We can see that there are two unique prime factors 2 and 3, however, the power of 2 is 3 and the power of 3 is 1, so the total number of prime factors is $4 (= 3 + 1)$.

Let there be a composite number N and its prime factors be $a, b, c, d, \dots$ etc. and $p, q, r, s, \dots$ be the indices (or powers) of $a, b, c, d, \dots$ then the number of total prime factors (or divisors) of N

$$= p + q + r + s + \dots$$

Exp. 1) Find the number of unique Prime factors of 210.

Solution $210 = 2 \times 3 \times 5 \times 7$

Therefore 210 has 4 unique prime factors, namely, 2, 3, 5 and 7.

Exp. 2) Find the total number of Prime factors of 210

Solution $210 = 2 \times 3 \times 5 \times 7 = 2^1 \times 3^1 \times 5^1 \times 7^1$

Therefore the total number of prime factors of
 $210 = 1 + 1 + 1 + 1 = 4$

Exp. 3) Find the number of unique Prime factors of 120.

Solution $120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3^1 \times 5^1$

Therefore 120 has 3 unique prime factors, namely 2, 3 and 5.

Exp. 4) Find the total number of Prime factors of 120

Solution $120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3^1 \times 5^1$

Therefore the total number of prime factors of
 $120 = 3 + 1 + 1 = 5$

Exp. 5) Find the number of unique Prime factors of 600.

Solution $600 = 2 \times 2 \times 2 \times 3 \times 5 \times 5 = 2^3 \times 3^1 \times 5^2$

Therefore 600 has 3 unique prime factors, namely, 2, 3 and 5.

Exp. 6) Find the total number of Prime factors of 600

Solution $600 = 2 \times 2 \times 2 \times 3 \times 5 \times 5 = 2^3 \times 3^1 \times 5^2$

Therefore the total number of prime factors of $600 = 3 + 1 + 2 = 6$

Number of All the Factors of a Composite Number

Number of Factors (or Divisors) of A Given Number (composite number)

Let us assume a composite number say 24 then find the no. of factors.

$$\begin{aligned} 24 &= 1 \times 24 \\ &2 \times 12 \\ &3 \times 8 \\ &4 \times 6 \end{aligned}$$

We see that there are total 8 factors namely,

1, 2, 3, 4, 6, 8, 12 and 24. But for the larger numbers it becomes difficult to find the total number of factors. So we use the following formula which can be derived with the help of theory of permutation and combination. Let there be a composite number N and its prime factors be $a, b, c, d, \dots$ etc. and $p, q, r, s, \dots$ etc. be the indices (or powers) of the $a, b, c, d, \dots$ etc. respectively. That is if N can be expressed as $N = a^p \cdot b^q \cdot c^r \cdot d^s \dots$, then the number of total divisors or factors of N is $(p+1)(q+1)(r+1)(s+1)\dots$

Exp. 1) Find the total number of factors of 24.

Solution $24 = 2^3 \times 3^1$ [$\because 24 = 2 \times 2 \times 2 \times 3$]

$\therefore$ Number of factors = $(3+1)(1+1) = 8$

Exp. 2) Find the total number of factors of 540.

(a) 24 (b) 20 (c) 30 (d) none

Solution $540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$ or $540 = 2^2 \times 3^3 \times 5^1$

Therefore total number of factors of 540 is

$$(2+1)(3+1)(1+1) = 24$$

Exp. 3) The total number of divisors of 10500 except 1 and itself is :

(a) 48 (b) 50 (c) 46 (d) 56

Solution $10500 = 2 \times 2 \times 3 \times 5 \times 5 \times 5 \times 7$

$$10500 = 2^2 \times 3^1 \times 5^3 \times 7^1$$

$\therefore$ Total number of divisors of 10500 is

$$(2+1)(1+1)(3+1)(1+1) = 48$$

But we have to exclude 1 and 10500

So there are only $48 - 2 = 46$ factors of 10500 except 1 and 10500.

Hence, (c) is the correct option.

Exp. 4) Total number of factors of 36 is :

(a) 4 (b) 9 (c) 6 (d) none

Solution $36 = 2^2 \times 3^2$

$\therefore$ Number of factors = $(2+1)(2+1) = 3 \times 3 = 9$

Hence (b) is the correct option.

 **NOTE** A perfect square number always contains odd no. of factors.

Practice Exercise

Find the number of factors of the following numbers :

1. 1008 2. 101 3. 111 4. 7056 5. 18522
6. 7744 7. 3875 8. 1458 9. 1339 10. 512

Answers

1. 30 2. 2 3. 4 4. 45 5. 32
6. 21 7. 8 8. 14 9. 4 10. 10

Number of Odd Factors of a Composite Number

Let us assume as small number, e.g. 24, which can be expressed as $24 = 2^3 \times 3^1$

Now, we have $24 = (1 \times 24) = (2 \times 12) = (3 \times 8)$

Now, we can see that there are total 2 odd factors, namely, 1 and 3.

Further, assume another number, say, 36 which can be expressed as $36 = 2^2 \times 3^2$

Now, we have $36 = (1 \times 36) = (2 \times 18) = (3 \times 12)$
 $= (4 \times 9) = (6 \times 6)$

So, we can see that there are only 3 odd factors, namely, 1, 3 and 9.

Once again we assume another number, say, 90, which can be expressed as $90 = 2 \times 3^2 \times 5$.

Now, we have $90 = (1 \times 90) = (2 \times 45) = (3 \times 30)$
 $= (5 \times 18) = (6 \times 15) = (9 \times 10)$

Thus, there are only 6 odd factors, namely, 1, 3, 5, 9, 15, 45.

To get the number of odd factors of a number N first of all express the number N as $N = (p_1^a \times p_2^b \times p_3^c \times \dots) \times (e^x)$

(where, $p_1, p_2, p_3, \dots$ are the odd prime factors and e is the even prime factor.)

Then the total number of odd factors

$$= (a+1)(b+1)(c+1)\dots$$

Exp. 1) The number of odd factors (or divisors) of 24 is:

Solution $\therefore 24 = 2^3 \times 3^1$

Here 3 is the odd prime factor

So, total number of odd factors $= (1+1) = 2$

Exp. 2) The number of odd factors of 36 is :

Solution $\therefore 36 = 2^2 \times 3^2$

$\therefore$ Number of odd factors $= (2+1) = 3$

Exp. 3) The number of odd factors of 90 is

Solution $\therefore 90 = 2^1 \times 3^2 \times 5^1$

$\therefore$ Total number of odd factors of
 $90 = (2+1)(1+1)$
 $= 6$

Number of Even Factors of a Composite Number

Number of even factors of a number

$= (\text{Total number of factors of the given number} - \text{Total number of odd factors})$

Thus in the above

Example (1), number of even factors $= 8 - 2 = 6$

Example (2), number of even factors $= 9 - 3 = 6$

Example (3), number of even factors $= 12 - 6 = 6$

Sum of Factors of a Composite Number

Once again if you want to find the sum of a small composite number, then you can do it manually, but for larger number it is a problem.

e.g., Sum of factors of $24 = 1 + 2 + 3 + 4 + 6 + 8 + 12 + 24 = 60$

Let N be the composite number and $a, b, c, d, \dots$ be its prime factors and p, q, r, s be the indices (or powers) of $a, b, c, d, \dots$. That is if N can be expressed as

$$N = a^p \cdot b^q \cdot c^r \cdot d^s \dots$$

then the sum of all the divisors (or factors) of N

$$= \frac{(a^{p+1} - 1)(b^{q+1} - 1)(c^{r+1} - 1)(d^{s+1} - 1)}{(a-1)(b-1)(c-1)(d-1)} \dots$$

Exp. 1) Find the sum of factors of 24.

Solution $\therefore 24 = 2^3 \times 3^1$

$$\therefore \text{Sum of factors of } 24 = \frac{(2^4 - 1)(3^2 - 1)}{(2-1)(3-1)} \\ = \frac{15 \times 8}{1 \times 2} = 60$$

Exp. 2) Find the sum of factors of 270.

Solution $\therefore 270 = 2^1 \times 3^3 \times 5^1$

$$\therefore \text{Sum of factors of } 270 = \frac{(2^{1+1} - 1)(3^{3+1} - 1)(5^{1+1} - 1)}{(2-1)(3-1)(5-1)} \\ = \frac{3 \times 80 \times 24}{1 \times 2 \times 4} = 720$$

Exp. 3) The sum of factors of 1520 except the unity is :

- (a) 3720 (b) 2730
(c) 2370 (d) none of these

Solution Since $1520 = 2^4 \times 5^1 \times 19^1$

$$\begin{aligned}\therefore \text{Sum of all the factors of } 1520 &= \frac{(2^5 - 1)(5^2 - 1)(19^2 - 1)}{(2 - 1)(5 - 1)(19 - 1)} \\ &= \frac{31 \times 24 \times 360}{1 \times 4 \times 18} = 3720\end{aligned}$$

But since unity is to be excluded.

$\therefore$ The net sum of the factors = 3719

$\therefore$ (d) is the correct option.

Exp. 4) The sum of factors of 19600 is :

- (a) 54777 (b) 33667
(c) 5428 (d) none of these

Solution $19600 = 2^4 \times 5^2 \times 7^2$

$$\begin{aligned}\therefore \text{Sum of factors of } 19600 &= \frac{(2^{4+1} - 1)(5^{2+1} - 1)(7^{2+1} - 1)}{(2 - 1)(5 - 1)(7 - 1)} \\ &= \frac{31 \times 124 \times 342}{1 \times 4 \times 6} = 54777\end{aligned}$$

Hence (a) is the correct option.

Product of Factors of a Composite number

Let us assume a very small number 24 and see the factors

$$24 = 1 \times 24 = 2 \times 12 = 3 \times 8 = 4 \times 6$$

Now, it is obvious from the above explanation that the product of factors of 24

$$\begin{aligned}&= (1 \times 24) \times (2 \times 12) \times (3 \times 8) \times (4 \times 6) \\ &= 24 \times 24 \times 24 \times 24 = (24)^4\end{aligned}$$

Thus, the product of factors of composite number $N = N^{n/2}$, where n is the total number of factors of N .

Exp. 1) Product of divisors of 7056 is :

- (a) $(84)^{48}$ (b) $(84)^{44}$
(c) $(84)^{45}$ (d) none of these

Solution $\therefore 7056 = 2^4 \times 3^2 \times 7^2$

$\therefore$ Number of factors/divisors of 7056 = $(4 + 1)(2 + 1)(2 + 1) = 45$

$\therefore$ Product of factors = $(7056)^{45/2} = (84)^{45}$

Hence (c) is the correct option.

Exp. 2) Product of factors of 360 is :

- (a) $(360)^{12}$ (b) $(36)^{120}$
(c) $(360)^{22}$ (d) $6^{24} \times 10^{10}$

Solution $\therefore 360 = 2^3 \times 3^2 \times 5^1$

$\therefore$ Number of factors of 360 = $(3 + 1)(2 + 1)(1 + 1) = 24$

Thus the product of factors = $(360)^{24/2} = (360)^{12}$

Hence (a) is the correct option.

NOTE As we know the number of divisors of any perfect number is an odd number so we can express the given number in the form of the perfect square to eliminate the $\frac{1}{2}$ from the power of the number as it can be seen in the above example no. 1.

Number of Ways of Expressing a Composite Number as a Product of two Factors

It has long been discussed in the previous examples of composite numbers but you might have not noticed it.

Let us consider an example of small composite number say, 24

$$\begin{aligned}\text{Then } 24 &= 1 \times 24 \\ &= 2 \times 12 \\ &= 3 \times 8 \\ &= 4 \times 6\end{aligned}$$

So it is clear that the number of ways of expressing a composite no. as a product of two factors

$$= \frac{1}{2} \times \text{the no. of total factors}$$

Exp. 1) Find the number of ways of expressing 180 as a product of two factors.

Solution $180 = 2^2 \times 3^2 \times 5^1$

$$\begin{aligned}\text{Number of factors} &= (2 + 1)(2 + 1)(1 + 1) \\ &= 18\end{aligned}$$

Hence, there are total $\frac{18}{2} = 9$ ways in which 180 can be expressed as a product of two factors.

NOTE As you know when you express any perfect square number ' N ' as a product of two factors namely $\sqrt{N}$ and $\sqrt{N}$, and you also know that since in this case $\sqrt{N}$ appears two times but it is considered only once while calculating the no. of factors so we get always an odd number as number of factors so we can not divide the odd number exactly by 2 as in the above formula. So if we have to consider these two same factors then we find the number of ways of expressing N as a product of two factors = $\frac{(\text{Number of factors} + 1)}{2}$.

Again if it is asked that find the no. of ways of expressing N as a product of Two distinct factors then we do not consider 1 way (i.e., $N = \sqrt{N} \times \sqrt{N}$) then no. of ways = $\frac{(\text{Number of factors} - 1)}{2}$.

Exp. 2) Find the number of ways expressing 36 as a product of two factors.

Solution $36 = 2^2 \times 3^2$

$$\begin{aligned}\text{Number of factors} &= (2 + 1)(2 + 1) = 9\end{aligned}$$

Hence the no. of ways of expressing 36 as a product of two factors = $\frac{(9 + 1)}{2} = 5$.

as $36 = (1 \times 36) = (2 \times 18) = (3 \times 12) = (4 \times 9) = (6 \times 6)$

Exp. 3) In how many ways can 576 be expressed as the product of two distinct factors?

Solution $\therefore 576 = 2^6 \times 3^2$

$\therefore$ Total number of factors

$$= (6 + 1)(2 + 1) = 21$$

So the number of ways of expressing 576 as a product of two distinct factors

$$= \frac{(21 - 1)}{2} = 10$$

NOTE Since the word 'distinct' has been used therefore we do not include 26 two times.

Number of Ways of Expressing a Composite Number as a Product of Three Factors

Exp. 1) Find the number of ways of expressing 2310 as product of 3 distinct factors.

Solution $a \times b \times c = 2310 = 2^1 \times 3^1 \times 5^1 \times 7^1 \times 11^1$

There are total 5 prime numbers. Since we know that 1 can also be a factor of 2310, so we have to consider 3 different cases.

Case I When out of three factors two factors are 1 and 1. That is $1 \times 1 \times 2310$

Thus we can express 2310 as a product of 3 factors in 1 way.

Case II When out of three factors one factor is 1. That is $(1 \times 2 \times 1150), (1 \times 3 \times 770), \dots, (1 \times 15 \times 77)$.

There are two possibilities to group the prime numbers: $(1 \times p \times pppp)$ or $(1 \times pp \times pp)$

This can be done in ${}^5C_1 + {}^5C_2 = 10 + 15 = 25$ ways.

Case III When none of the three factors is 1. That is $(2 \times 3 \times 385), (2 \times 5 \times 231), \dots, (2 \times 15 \times 77)$

There are two possibilities to group the prime numbers: $(p \times pp \times pp)$ or $(ppp \times p \times p)$. This can be done in

$${}^5C_1 \left(\frac{{}^4C_2 \times {}^2C_2}{2} \right) + {}^5C_3 \left(\frac{{}^2C_1 \times {}^1C_1}{2} \right) = 15 + 10 = 25 \text{ ways.}$$

Therefore the number of ways of expressing 2310 as a product of 3 factors

$$= 1 + 15 + 25 = 41$$

NOTE Refer the Permutation and Combination chapter to understand this topic better.

Alternatively If a number can be expressed as a product of n distinct prime numbers, it can be expressed as a product of 3 numbers in $\frac{3^{n-1} + 1}{2}$ ways.

Now $2310 = 2 \times 3 \times 5 \times 7 \times 11$. That is 2310 can be expressed as a product of 5 distinct prime numbers.

Therefore 2310 can be expressed as a product of 3 factors in

$$\frac{3^{n-1} + 1}{2} = \frac{3^4 + 1}{2} = 41 \text{ ways}$$

When the third factor is	The other two factors have prime numbers	The number of ways of expressing the other prime numbers into two factors	The number of ways of expressing 2310 as a product of 3 factors	The number of ways of expressing 2310 as a product of 3 Distinct factors
1	2, 3, 5, 7, 11	16	16	$16 - 1 = 15$
2	3, 5, 7, 11	8	8	$8 - 1 = 7$
3	2, 5, 7, 11	8	8	$8 - 2 = 6$
5	2, 3, 7, 11	8	8	$8 - 3 = 5$
7	2, 3, 5, 11	8	8	$8 - 4 = 4$
11	2, 3, 5, 7	8	8	$8 - 5 = 3$

Number of Co-Primes of A Composite Number N

For a composite number $N = a^p \times b^q \times c^r \dots$, the number of co-primes of N is given by

$$\phi(N) = N \left(1 - \frac{1}{a} \right) \left(1 - \frac{1}{b} \right) \left(1 - \frac{1}{c} \right) \dots,$$

where a, b, c are prime numbers.

Exp. 1) Find the number of co-prime numbers of 10.

Solution $10 = 2 \times 5$ Number of co-prime numbers of 10 is given

by $\phi(10) = \left(1 - \frac{1}{2} \right) \left(1 - \frac{1}{5} \right) = 4$. These numbers are 1, 3, 7, 9.

Exp. 2) Find the number of co-prime numbers of 336.

Solution $10 = 2^4 \times 3 \times 7$ Number of co-prime numbers of 336 is

given by $\phi(336) = 336 \left(1 - \frac{1}{2} \right) \left(1 - \frac{1}{3} \right) \left(1 - \frac{1}{7} \right) = 96$

Exp. 3) Find the number of co-prime numbers of 49, which are composite numbers.

Solution $49 = 7^2$ Number of co-prime numbers of 49 is given by $\phi(49) = (1 - 1/7) = 42$

These numbers are 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48. Since, there are 15 numbers, 1, 2, 3, 5, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47 which are not the composite numbers, so the required number = $42 - 15 = 27$.

Sum of All The Co-Primes of a Composite Number N

For a composite number $N = a^p \times b^q \times c^r \dots$, the sum of all the co-primes of N is given by

$$\frac{N}{2} [\phi(N)] = \frac{N}{2} \left[N \left(1 - \frac{1}{a} \right) \left(1 - \frac{1}{b} \right) \left(1 - \frac{1}{c} \right) \dots \right],$$

where a, b, c are prime numbers.

Exp. 1) Find the sum of all the co-prime numbers of 10.

Solution $10 = 2 \times 5$

Sum of all the co-prime numbers of 10 is given by

$$\frac{10}{2} [\phi(10)] = \frac{10}{2} \left(1 - \frac{1}{2} \right) \left(1 - \frac{1}{5} \right) = 20$$

The required sum is $1 + 3 + 7 + 9 = 20$

Exp. 2) Find the sum of all the co-prime numbers of 336.

Solution $336 = 2^4 \times 3 \times 7$

Sum of all the co-prime numbers of 336 is given by

$$\frac{336}{2} [\phi(336)] = \frac{336}{2} \left[336 \left(1 - \frac{1}{2} \right) \left(1 - \frac{1}{3} \right) \left(1 - \frac{1}{7} \right) \right] = 16128$$

Exp. 3) Find the sum of all the co-prime numbers of 49, which are composite number.

Solution $49 = 7^2$

Sum of all the co-prime numbers of 49 is given by

$$\frac{49}{2} [\phi(49)] = \frac{49}{2} \left[49 \left(1 - \frac{1}{7} \right) \right] = 1029$$

The required sum

$$= 1029 - (1 + 2 + 3 + 5 + 11 + 13 + 17 + 19 + 23 + 29 + 31 + 37 + 41 + 43 + 47) = 707.$$

Number of Ways of Expressing a Composite Number As A Product of Co-Prime Factors

For a composite number N , the number of ways of expressing a composite number as product of co-prime factors is 2^{n-1} , where n is the number of unique prime factors of N .

Exp. 1) Find the number of ways of expressing 10 as a product of co-prime factors.

Solution $10 = 2 \times 5$

The required number of ways $= 2^{(2-1)} = 2$

$(1 \times 10), (2 \times 5)$ are two different ways in which 10 can be expressed as a product of co-prime factors.

Exp. 2) Find the number of ways of expressing 336 as a product of co-prime factors.

Solution $336 = 2^4 \times 3 \times 7$

The required number of ways $= 2^{(3-1)} = 4$

$(1 \times 336), (16 \times 21), (48 \times 7), (112 \times 3)$ are four different ways of expressing 336 as a product of co-prime factors.

Exp. 3) Find the number of ways of expressing 49 as a product of co-prime factors.

Solution $49 = 7^2$

The required number of ways $= 2^{(1-1)} = 1$,

(1×49) is the only way to express 49 as a product of co-prime factors.

Introductory Exercise 1.5

- The number of prime factors in the expression $6^4 \times 8^6 \times 10^8 \times 12^{10}$ is :
(a) 48 (b) 64 (c) 72 (d) 80
- A number is expressed as $2^m \times 3^n$ and the sum of all its factors is 124, find m and n .
(a) 2, 3 (b) 3, 1 (c) 4, 1 (d) 3, 2
- The sum of all possible factors of 500 (including 1 and 500 themselves) equals :
(a) 784 (b) 980 (c) 1092 (d) 1350
- The perimeter of a rectangle is 72 cm. If the sides are positive integers, maximum how many distinct areas can it have?
(a) 18 (b) 9 (c) 36 (d) 35
- The area of a rectangle is 72 sq cm. If the sides are positive integers, maximum how many distinct perimeters can it have?
(a) 12 (b) 8 (c) 9 (d) 6
- Among the first 100 even natural numbers how many numbers have even number of factors?
(a) 37 (b) 50 (c) 73 (d) 93
- In a set of first 350 natural numbers find the number of integers, which are not divisible by 5.
(a) 280 (b) 75 (c) 150 (d) 240
- In a set of first 350 natural numbers find the number of integers, which are not divisible by 7.
(a) 120 (b) 280 (c) 300 (d) 240
- In a set of first 350 natural numbers find the number of integers, which are not divisible by any of the numbers 5 or 7.
(a) 240 (b) 120
(c) 150 (d) 300
- In a set of first 420 natural numbers find the number of integers that are divisible by 7 but not by any of 2, 3 or 5.
(a) 16 (b) 24
(c) 96 (d) 42
- In a set of first 210 natural numbers find the number of integers which are not divisible by any of 2, 3, 4, ..., 10.
(a) 42 (b) 52
(c) 48 (d) 63

12. In a set of first 61 natural numbers find the number of integers, which are divisible by neither 2 nor 3 nor 5.
 (a) 0
 (b) 1
 (c) 16
 (d) 17
13. In a set of first 123 natural numbers find the number of integers, which are divisible by either 2 or 3 or 5.
 (a) 90
 (b) 31
 (c) 33
 (d) 11

14. In a set of first 180 natural numbers find the number of integers, which are divisible by neither 2 nor 3 nor 5 nor 7.
 (a) 14
 (b) 15
 (c) 41
 (d) 51
15. In a set of first 180 natural numbers find the number of prime numbers.
 (a) 48
 (b) 24
 (c) 41
 (d) 38
16. In a set of first 1000 natural numbers find the number of prime numbers.
 (a) 181
 (b) 168
 (c) 200
 (d) 224

1.6 HCF and LCM

In the previous articles I have discussed in detail about the factors. Now I will move towards common factor, Highest common factors (HCF) and least common multiple (LCM).

Common Factors

When any factor which is the factor of two or more given numbers then it is said that this particular factor is common.

$$\begin{aligned}\text{For example} \quad 6 &= 2 \times 3 \\ 15 &= 3 \times 5\end{aligned}$$

We see that 3 is a common factor in both 6 and 15.

$$\begin{aligned}\text{Similarly,} \quad 6 &= 2 \times 3 \\ 8 &= 2 \times 2 \times 2 \\ 30 &= 2 \times 3 \times 5\end{aligned}$$

So, we can see that only 2 is common in all the three numbers 6, 8 and 30.

No other factor is common in all the three numbers.

Highest Common Factor (HCF) or Greatest Common Divisor (GCD)

HCF of two or more than two numbers is the greatest possible number that can divide all these numbers exactly, without leaving any remainder. For example, find the HCF of 84 and 126.

$$\begin{aligned}\therefore \quad 84 &= 2 \times 3 \times 7 \times 2 = 42 \times 2 \\ 126 &= 2 \times 3 \times 7 \times 3 = 42 \times 3\end{aligned}$$

$$\text{So, the HCF of 84 and 126} = 2 \times 3 \times 7 = 42$$

Thus we can see that 42 is the greatest common divisor since it can divide exactly both 84 and 126.

Basically there are two methods of finding the HCF.

- (i) Factor Method
- (ii) Division Method

(i) Factor Method

In this method first we break (or resolve) the numbers into prime factors then take the product of all the common factors. This resultant product is known as the HCF of the given numbers.

Exp. 1) Find the HCF of 1680 and 3600.

$$\begin{aligned}\text{Solution} \quad \therefore 1680 &= 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 7 \\ 3600 &= 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 3 \times 5\end{aligned}$$

$$\begin{aligned}\text{So the product of common factors} &= 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 240 \\ \text{Hence, the HCF of 1680 and 3600 is 240.}\end{aligned}$$

Exp. 2) Find the HCF of 750, 6300, 18900.

$$\begin{aligned}\text{Solution} \quad 750 &= 2 \times 3 \times 5 \times 5 \times 5 \\ 6300 &= 2 \times 3 \times 5 \times 5 \times 3 \times 2 \times 7 \\ 18900 &= 2 \times 3 \times 5 \times 5 \times 3 \times 2 \times 3 \times 7 \\ \text{So the product of common factors} &= 2 \times 3 \times 5 \times 5 = 150 \\ \text{Hence 150 is the HCF of 750, 6300, 18900.}\end{aligned}$$

(ii) HCF by Division Method

Consider two smallest numbers, then divide the larger one of them by the smaller one and then divide this divisor by the remainder and again divide this remainder by the next remainder and so on until the remainder is zero. If there are more than two numbers of which HCF is to be found, we continue this process as we divide the third lowest number by the last divisor obtained in the above process.

Exp. 1) Find the HCF of 120 and 180.

$$\begin{array}{r} 120 \overline{)180} \quad (1 \\ \underline{120} \\ 60 \overline{)120} \quad (2 \\ \underline{120} \\ \times \end{array}$$

Hence, 60 is the HCF of 120 and 180.

Exp. 2) Find the HCF of 420 and 1782.

Solution

$$\begin{array}{r}
 420 \overline{)1782} \begin{array}{l} 4 \\ 1680 \\ \hline 102 \end{array} \\
 102 \overline{)420} \begin{array}{l} 4 \\ 408 \\ \hline 12 \end{array} \\
 12 \overline{)102} \begin{array}{l} 8 \\ 96 \\ \hline 6 \end{array} \\
 6 \overline{)12} \begin{array}{l} 2 \\ 12 \\ \hline \times \end{array}
 \end{array}$$

Hence 6 is the HCF of 420 and 1782.

Exp. 3) Find the HCF of 210, 495 and 980.

Solution First we consider the two smallest numbers *i.e.*, 210 and 495.

$$\begin{array}{r}
 210 \overline{)495} \begin{array}{l} 2 \\ 420 \\ \hline 75 \end{array} \\
 75 \overline{)210} \begin{array}{l} 2 \\ 150 \\ \hline 60 \end{array} \\
 60 \overline{)75} \begin{array}{l} 1 \\ 60 \\ \hline 15 \end{array} \\
 15 \overline{)60} \begin{array}{l} 4 \\ 60 \\ \hline \times \end{array} \\
 15 \overline{)980} \begin{array}{l} 65 \\ 975 \\ \hline 5 \end{array} \\
 5 \overline{)15} \begin{array}{l} 3 \\ 15 \\ \hline \times \end{array}
 \end{array}$$

Hence the HCF is 5.

Euclidean Algorithm

If you watch carefully the above discussion you will find that the HCF is the factor of difference of the given numbers. So there is short cut for you to find the HCF.

You can divide the given numbers by their lowest possible difference if these numbers are divisible by this difference then this difference itself is the HCF of the given numbers. If this difference is not the HCF then any factor of this difference must be the HCF of the given numbers.

Be'zout's Identity

If 'H' be the HCF of any two positive integers *a* and *b* then there exists unique integers *p* and *q* such that $H = ap + bq$.

Exp. 1) Find the HCF of 63 and 84.

Solution The difference = $84 - 63 = 21$

Now divide 63 and 84 by 21, since the given numbers 63 and 84 are divisible by their difference 21, thus 21 itself is the HCF of 63 and 84.

Exp. 2) Find the HCF of 42 and 105.

Solution The difference = $105 - 42 = 63$

Now divide 42 and 105 by 63 but none of these is divisible by their difference 63.

So we factorize 63 and then divide 42 and 105 by the factor of 63.

The greatest factor of 63 which can divide 42 and 105 that will be the HCF of the given numbers.

So $63 = 1 \times 63, 3 \times 21, 7 \times 9$

$\therefore$ The factor of 63 are 1, 3, 7, 9, 21 and 63.

Obviously we divide 42 and 105 by 21 (since the division by 63 is not possible) and we see that the given numbers are divisible by 21, hence 21 is the HCF of 42 and 105.

Exp. 3) Find the HCF of 30, 42 and 135.

Solution The difference between 30 and 42 = 12

The difference between 42 and 135 = 93

Now divide, 30, 42 and 135 by 12 but it can not divide all the numbers. So take the factors of 12.

The factors of 12 are 1, 2, 3, 4, 6, 12

Now we divide 30, 42 and 135 by 6 but it can not divide all these numbers.

So we divide now 30, 42 and 135 by 4, but 4 too can not divide all the numbers, Now we divide the given numbers by 3 and we will find that all the numbers are divisible by 3 hence 3 is the HCF of the given numbers.

HCF with Remainders

Case 1. Find the greatest possible number with which when we divide 37 and 58, it leaves the respective remainder of 2 and 3.

Solution Since when we divide 37 and 58 by the same number then we get remainders 2 and 3 respectively.

So $(37 - 2)$ and $(58 - 3)$ must be divisible, hence leaving the remainders zero. It means 35 and 55 both are divisible by that number so the HCF of 35 and 55 is 5.

Hence the greatest possible number is 5.

Case 2. Find the largest possible number with which when 60 and 98 are divided it leaves the remainders 3 in each case.

Solution Since 60 and 98 both leave the remainders 3 when divided by such a number.

Therefore $57 = (60 - 3)$ and $95 = (98 - 3)$ will be divisible by the same number without leaving any remainder. That means the HCF of 57 and 95 is 19.

Hence 19 is the highest possible number.

Case 3. Find the largest possible number with which when 38, 66 and 80 are divided the remainder remains the same.

Solution In this case (since we do not know the value of remainder) we take the HCF of the differences of the given numbers.

So the HCF of $(66 - 38)$, $(80 - 66)$, $(80 - 38) =$ HCF of 28, 14, 42 = 14. Hence 14 is the largest possible number which leaves same remainder (= 10) when it divides either 38, 66 or 80.

NOTE In case 1, the remainders are different
 In case 2, the remainders are same in each case.
 In case 3, the remainders are same in each case,
 but the value of remainder is unknown.

Least Common Multiple (LCM)

In this chapter I have already discussed the multiples of a number for example integral multiples of 7 are 7, 14, 21, 28, 35, 42, ... and the integral multiples of 8 are 8, 16, 24, 32, 40, 48, 56, ...etc. In short we can write the positive integral multiples of any number N as NK ,

where $K = 1, 2, 3, 4, 5, 6, \dots$

Now if we consider at least two numbers say 2 and 3 then we write the multiples of each as 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30... and 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, ...

Thus it is clear from the above illustrations that some of the multiples of 2 and 3 are common viz., 6, 12, 18, 24, 30...

Again if we consider any 3 numbers say, 3, 5 and 6 then the multiples of each of the 3, 5, 6 are as follows:

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, ...

5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, ...

6, 12, 18, 24, 30, 36, 42, 48, 54, 60, ...

So, we can see that the common multiples of 3, 5 and 6 are 30, 60, 90, 120, ... etc.

But, out of these common multiples, 30 is the least common multiple. Similarly in the previous example, 6 is the least common multiple.

NOTE Generally there is no any greatest common multiple unless otherwise the condition is stated.

Basically, there are two methods to find the LCM.

- (i) Factor Method
- (ii) Division Method

(i) Factor Method

Resolve the given numbers into their prime factors, then take the product of all the prime factors of the first number with those prime factors of second number which are not common to the prime factors of the first number.

Now this resultant product can be multiplied with those prime factors of the third number which are not common to the factors of the previous product and this process can be continued for further numbers if any.

Exp. 1) Find the LCM of 48, 72, 140.

Solution We can write the given numbers as

$$\begin{aligned} 48 &= 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3 \\ 72 &= 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2 \\ 140 &= 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7 \end{aligned}$$

So the LCM of 48, 72 and 140 = $2 \times 2 \times 2 \times 3 \times 2 \times 3 \times 5 \times 7 = 5040$

Exp. 2) Find the LCM of 42, 63 and 231.

$$\begin{aligned} \text{Solution } 42 &= 2 \times 3 \times 7 = 2 \times 3 \times 7 \\ 63 &= 3 \times 3 \times 7 = 3^2 \times 7 \\ 231 &= 3 \times 7 \times 11 = 3 \times 7 \times 11 \end{aligned}$$

$\therefore$ The LCM of 42, 63 and 231 = $3 \times 7 \times 2 \times 3 \times 11 = 1386$

(ii) Division Method

First of all write down all the given numbers in a line separated by the comma (,) then divide these numbers by the least common prime factors say 2, 3, 5, 7, 11... to the given numbers, then write the quotients just below the actual numbers separated by comma (,).

If any number is not divisible by such a prime factor then write this number as it is just below itself, then continue this process of division by considering higher prime factors, if the division is complete by lower prime factor, till the quotient in the last line is 1.

Then take the product of all the prime factors by which you have divided the numbers (or quotient) in different lines (or steps). This product will be the LCM of the given numbers.

Exp. 3) Find the LCM of 108, 135 and 162.

Solution	2	108, 135, 162
	2	54, 135, 81
	3	27, 135, 81
	3	9, 45, 27
	3	3, 15, 9
	3	1, 5, 3
	5	1, 5, 1
	1	1, 1, 1

Thus the required LCM

$$\begin{aligned} &= 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \\ &= 1620 \end{aligned}$$

Exp. 4) Find the LCM of 420, 9009, 6270.

Solution	2	420, 9009, 6270
	2	210, 9009, 3135
	3	105, 9009, 3135
	3	35, 3003, 1045
	5	35, 1001, 1045
	7	7, 1001, 209
	11	1, 143, 209
	13	1, 13, 19
	19	1, 1, 19
		1, 1, 1

Thus the required LCM = $2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 11 \times 13 \times 19$
 $= 3423420$

Exp. 5) Find the least possible number which can be divided by 32, 36 and 40.

Solution The number which is divisible by 32, 36 and 40, it must be the common multiple of all the given numbers. Since we need such a least number then we have to find out just the LCM of 32, 36 and 40.

The LCM of 32, 36 and 40 = 1440

Hence, the least possible no. is 1440, which is divisible by all the given numbers.

Exp. 6) What is the least possible number of 5 digits which is divisible by all the numbers 32, 36 and 40.

Solution Since the least possible no. is 1440, but it is a four digit number.

So we can take the integral multiples of 1440 which must be divisible by the given numbers.

Now since the least possible 5 digit number is 10000, so the required number must be equal to or greater than 10,000. So when we multiply 1440 by 7. We get the required result i.e., 10,080. Thus 10080 is the least possible 5 digit number which is divisible by 32, 36 and 40.

Alternatively Divide the least possible 5 digit no. by 1440 and then add the difference of the divisor and remainder to the least possible 5 digit number. This will be the required number.

So,

$$\begin{array}{r} 1440 \overline{)10000} (6 \\ \underline{8640} \\ 1360 \end{array}$$

Now, $1440 - 1360 = 80$

Thus the required number = $10000 + 80 = 10080$

Exp. 7) Find the largest possible number of 4 digits which is exactly divisible by 32, 36 and 40.

Solution Since the largest 4 digit number is 9999, so the required number can not exceed 9999 any how. Now we take the appropriate multiple of 1440. Since 1440 is divisible by the given numbers, so the multiples of 1440 must be divisible by the given numbers.

Hence $1440 \times 6 = 8640$ is the largest possible number since $1440 \times 7 = 10080$ is greater than 9999 which is not admissible. Hence, 8640 is the largest possible 4 digit no. which is divisible by all the given numbers.

Alternatively Divide the greatest 4 digit number by 1440 (the LCM of the given no.) and then subtract the remainder from the greatest 4 digit number. So,

$$\begin{array}{r} 1440 \overline{)9999} (6 \\ \underline{8640} \\ 1359 \end{array}$$

Hence, the required number = $9999 - 1359 = 8640$

Exp. 8) Find the number of numbers lying between 1 and 1000 which are divisible by each of 6, 7 and 15.

Solution The least possible number which is divisible by 6, 7, and 15 = LCM of 6, 7, 15 = 210

So, the first such number is 210 and the other numbers are the multiples of 210 i.e., 210, 420, 630, 840.

Thus there are total 4 numbers lying between 1 and 1000 which are divisible by 6, 7 and 15.

Exp. 9) Find the number of numbers lying between 1000 and 1,00,000 which are divisible by 15, 35 and 77 and are even also.

Solution The required numbers must be the multiples of the LCM of 15, 35 and 77. Now the LCM of 15, 35 and 77 = 1155.

So the other numbers are 1155, 2310, 3465, 4620, 5775, ..., 99330. Thus there are total 86 numbers which are divisible by 15, 35 and 77 but only 43 numbers are even i.e., every alternate number is even. Thus there are total 43 required numbers.

Exp. 10) Find the least possible perfect square number which is exactly divisible by 6, 40, 49 and 75.

Solution The required number must be divisible by the given numbers so it can be the LCM or its multiple number.

Now the LCM of 6, 40, 49 and 75 = $2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 7 \times 7$

But the required number is a perfect square

Thus the LCM must be multiplied by $2 \times 3 = 6$.

Thus the required number

$$\begin{aligned} &= (2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 7 \times 7) \times (2 \times 3) \\ &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 \times 7 = 176400 \end{aligned}$$

Exp. 11) Three bells in the bhootnath temple toll at the interval of 48, 72 and 108 second individually. If they have tolled all together at 6 : 00 AM then at what time will they toll together after 6 : 00 AM?

Solution The three bells toll together only at the LCM of the times they toll individually.

Thus the LCM of 48, 72 and 108 is 432 seconds.

Therefore all the bells will toll together at 6 : 07 : 12 AM

($\because$ 432 seconds = 7 minuts 12 seconds)

Exp. 12) In the above problem how many times these bells will toll together till the 6 : 00 PM on the same day.

Solution The total time since 6 : 00 AM till 6 : 00 PM
 $= 12 \times 60 \times 60$ seconds.

Now since all these bells toll together at the interval of 432 seconds. So the number of times when they will toll together

$$= \frac{\text{Total time}}{\text{least duration of tolling together}} \\ = \frac{12 \times 60 \times 60}{432} = 100$$

but since the bells toll together at 6 : 00 PM also. Hence total 101 (= 100 + 1) times these bells will toll together in the given duration of time.

LCM with Remainders

Case 1. When the remainders are same for all the divisors.

Case 2. When the remainders are different for different divisors, but the respective difference between the divisors and the remainders remains constant.

Case 3. When neither the divisors are same nor the respective differences between divisors and the remainders remain constant.

Case 1

Exp. 13) What is the least possible number which when divided by 24, 32 or 42 in each case it leaves the remainder 5?

Solution Since we know that the LCM of 24, 32 and 42 is divisible by the given numbers. So the required number

$$= (\text{LCM of } 24, 32, 42) + (5) = 672 + 5 = 677$$

Hence such a least possible number is 677.

Exp. 14) In the above question how many numbers are possible between 666 and 8888?

Solution Since the form of such a number is $672m + 5$, where $m = 1, 2, 3, \dots$

So, the first no. = $672 \times 1 + 5 = 677$ and the highest possible number in the given range = $672 \times 13 + 5 = 8736 + 5 = 8741$

Thus the total numbers between 666 and 8888 are 13.

Exp. 15) What is the least possible number which when divided by 21, 25, 27 and 35 it leaves the remainder 2 in each case?

Solution The least possible number

$$= (\text{LCM of } 21, 25, 27 \text{ and } 35) m + 2$$

$$= 4725m + 2$$

$$= 4727 \quad (\text{for the least possible value we take } m = 1)$$

Exp. 16) What is the least possible number which must be added to 4722 so that it becomes divisible by 21, 25, 27 and 35?

Solution The number which is divisible by 21, 25, 27 and 35 is the LCM of 21, 25, 27 and 35 = 4725

So the required number = $4725 - 4722 = 3$

Exp. 17) What is the least possible number which when divided by 8, 12 and 16 leaves 3 as the remainder in each case, but when divided by 7 leaves no any remainder?

Solution The possible value = $m (\text{LCM of } 8, 12 \text{ and } 16) + 3$
 $= m (48) + 3$

Now put the value of m such that " $m (48) + 3$ " becomes divisible by 7. So $3 \times 48 + 3 = 147$ which is the least possible required number.

Case 2

Exp. 18) What is the least possible number which when divided by 18, 35 or 42 it leaves. 2, 19, 26 as the remainders, respectively?

Solution Since the difference between the divisors and the respective remainders is same.

Therefore, the least possible number

$$= (\text{LCM of } 18, 35 \text{ and } 42) - 16$$

$$= 630 - 16 \quad [\because (18 - 2) = (35 - 19) = (42 - 26) = 16]$$

$$= 614$$

Exp. 19) What is the least possible number which when divided by 2, 3, 4, 5, 6 it leaves the remainders 1, 2, 3, 4, 5 respectively?

Solution Since the difference is same as

$$(2 - 1) = (3 - 2) = (4 - 3) = (5 - 4) = (6 - 5) = 1$$

Hence the required number = $(\text{LCM of } 2, 3, 4, 5, 6) - 1$

$$= 60 - 1 = 59$$

Exp. 20) In the above problem what is the least possible 3 digit number which is divisible by 11?

Solution Since the form of the number is $60m - 1$, where $m = 1, 2, 3, \dots$

but the number $(60m - 1)$ should also be divisible by 11 hence at $m = 9$ the number becomes 539 which is also divisible by 11.

Thus the required number = 539.

Exp. 21) Find the least possible 5 digit number which when divided by 2, 4, 6 and 8, it leaves the remainders 1, 3, 5 and 7, respectively.

Solution The possible value = $(\text{LCM of } 2, 4, 6, 8)m - 1 = 24m - 1$
 Since, the least 5 digit number is 10000. So the required number must be atleast 10000. So putting the value of $m = 417$, we get $(10008 - 1) = 10007$, which is the required number.

Case 3

Exp. 22) What is the least possible number which when divided by 13 it leaves the remainder 3 and when it is divided by 5 it leaves the remainder 2.

Solution Let the required number be N then it can be expressed as follows

$$N = 13k + 3 \quad \dots(i)$$

$$\text{and } N = 5l + 2 \quad \dots(ii)$$

where k and l are the quotients belong to the set of integers.

$$\text{Thus } 5l + 2 = 13k + 3 \Rightarrow 5l - 13k = 1 \Rightarrow 5l = 13k + 1$$

$$\Rightarrow l = \frac{13k + 1}{5}$$

Now, we put the value of k such that numerator will be divisible by 5 or l must be integer so considering $k = 1, 2, 3, \dots$ we find that $k = 3$, l becomes 8. So the number $N = 5 \times 8 + 2 = 42$

Thus the least possible number = 42

To get the higher numbers which satisfy the given conditions in the above problem we just add the multiples of the given divisors (*i.e.*, 13 and 5) to the least possible number (*i.e.*, 42)

NOTE The next higher number = (Multiple of LCM of 13 and 5) + 42 = $65m + 42$

Exp. 23) In the above problem what is the greatest possible number of 4 digits?

Solution Since the general form of this number is $65m + 42$. So by putting $m = 153$ we get $(9945 + 42) = 9987$, which is required number.

Hint To get the value of 9945 we can simply divide 9999 (which is the greatest 4 digit number) by 65 and then subtract the remainder from 9999.

Exp. 24) How many numbers lie between 11 and 1111 which when divided by 9 leave a remainder of 6 and when divide by 21 leave a remainder of 12?

Solution Let the possible number be N then it can be expressed as

$$N = 9k + 6 \quad \text{and } N = 21l + 12$$

$$\therefore 9k + 6 = 21l + 12 \Rightarrow 9k - 21l = 6$$

$$\text{or } 3(3k - 7l) = 6 \text{ or } 3k = 7l + 2 \quad \text{or } k = \frac{7l + 2}{3}$$

So put the min. possible value of l such that the value of k is an integer or in other words numerator (*i.e.*, $7l + 2$) will be divisible by 3.

Thus at $l = 1$, we get $k = 3$ (an integer). So the least possible number $N = 9 \times 3 + 6 = 21 \times 1 + 12 = 33$.

Now the higher possible values can be obtained by adding 33 in the multiples of LCM of 9 and 21. *i.e.*, The general form of the number is $63m + 33$. So the other number in the given range including 33 are 96, 159, 222, 285, 348, ..., 1104. Hence there are total 18 numbers which satisfy the given condition.

Important Formula Product of two numbers
 = Product of their HCF and their LCM.

Exp. 25) The HCF and LCM of the two numbers is 12 and 600 respectively. If one of the numbers is 24, then the other number will be

- (a) 300 (b) 400
 (c) 1500 (d) none of these

Solution The answer is (a)

since product of numbers = HCF $\times$ LCM

$$\therefore N \times 24 = 12 \times 600 \Rightarrow N = 300$$

Successive Division

If the quotient in a division is further used as a dividend for the next divisor and again the latest obtained divisor is used as a dividend for another divisor and so on, then it is called the "successive division" *i.e.*, if we divide 625 by 3, we get 208 as quotient and 1 as a remainder then if 208 is divided by another divisor say 4 then we get 52 as a quotient and '0' (zero) as remainder and again, if we divide 52 by another divisor say 6 we get 8 as quotient and 4 as a remainder *i.e.*, we can represent it as following

$$\begin{array}{r|l} 3 & 625 \\ \hline 4 & 208 \rightarrow 1 (\text{Remainder}) \\ 6 & 52 \rightarrow 0 (\text{Remainder}) \\ & 8 \rightarrow 4 (\text{Remainder}) \end{array}$$

Now you can see that the quotient obtained in the first division behaves as a dividend for another divisor 4. Once again the quotient 52 is treated as a dividend for the next divisor 4. Thus it is clear from the above discussion as

Dividend	Divisor	Quotient	Remainder
625	3	208	1
208	4	52	0
52	6	8	4

So the 625 is successively divided by 3, 4 and 6 and the corresponding remainders are 1, 0 and 4.

Exp. 1) The least possible number of 3 digits when successively divided by 2, 5, 4, 3 gives respective remainders of 1, 1, 3, 1 is :

- (a) 372 (b) 275
 (c) 273 (d) 193

Solution The problem can be expressed as

$$\begin{array}{rcl} 2 & | & A \\ 5 & | & B \rightarrow 1 \\ 4 & | & C \rightarrow 1 \\ 3 & | & D \rightarrow 3 \\ & & E \rightarrow 1 \end{array} \left. \vphantom{\begin{array}{rcl} 2 & | & A \\ 5 & | & B \rightarrow 1 \\ 4 & | & C \rightarrow 1 \\ 3 & | & D \rightarrow 3 \\ & & E \rightarrow 1 \end{array}} \right\} \text{Remainder}$$

So it can be solved as

$$((((E \times 3) + 1) \times 4 + 3) \times 5 + 1) \times 2 + 1 = A$$

(where A is the required number)

So for the least possible number E

$$= 1 (\text{the least positive integer})$$

$$\text{then } A = (((((1 \times 3) + 1) \times 4 + 3) \times 5 + 1) \times 2 + 1)$$

[Since at $E = 0$, we get a two digit number]

Alternatively We use the following convention while solving this type of problem.

First we write all the divisors as given below then their respective remainders just below them.

$$\begin{array}{ccccccc} & 2 & & 5 & & 4 & & 3 \\ \swarrow & \downarrow & \swarrow & \downarrow & \swarrow & \downarrow & \swarrow & \downarrow \\ 1 & & 1 & & 3 & & 1 & \end{array} \begin{array}{l} \text{Divisors} \\ \text{Remainders} \end{array}$$

Where the arrow downwards means to add up and the arrow slightly upward (at an angle of 45°) means multiplication.

So we start from the right side remainder and move towards left since while writing down the divisors and remainders we write the first divisor first (*i.e.*, leftmost), second divisor at second position (from the left) and so on.

Now solve it as : **Step 1.** $(1 \times 4) + 3 = 7$

$$\text{Step 2. } 7 \times 5 + 1 = 36$$

$$\text{Step 3. } 36 \times 2 + 1 = 73$$

$$\text{or } (((((1 \times 4) + 3) \times 5 + 1) \times 2 + 1) = 73$$

but we are required to find a three digit number so the next higher numbers can be obtained just by taking the multiples of the product of the divisors and then adding to it the least such number.

The next higher number

$$\begin{aligned} &= (2 \times 5 \times 4 \times 3)m + 73 \\ &= 120m + 73 \end{aligned}$$

So by putting $m = 1, 2, 3, \dots$ we can get the higher possible numbers.

Now we just need a least possible 3 digit number so we can get it by putting $m = 1$

Hence the required number

$$= 120 \times 1 + 73 = 193$$

Hence (d) is the correct answer.

Exp. 2) A number when successively divided by 2, 3 and 5 it leaves the respective remainders 1, 2 and 3. What will be the remainder if this number is divided by 7?

Solution Write the divisors and remainders as given below

$$\begin{array}{ccc} & 2 & & 3 & & 5 \\ \swarrow & \downarrow & \swarrow & \downarrow & \swarrow & \downarrow \\ 1 & & 2 & & 3 & \end{array}$$

then solve it as follows $((((3 \times 3) + 2) \times 2 + 1) = 23$

$$\text{or } 3 \times 3 = 9$$

$$9 + 2 = 11$$

$$11 \times 2 = 22$$

$$22 + 1 = 23$$

So, the least possible number is 23 and the higher numbers can be obtained as $(2 \times 3 \times 5)m + 23 = 30m + 23$.

So the higher numbers are 53, 83, 113, 143, 173, 213, 333, ...etc.

But when we divide all these possible numbers by 7 we get the different remainders. So we can not conclude the single figure as a remainder.

Exp. 3) A number when divided successively by 6, 7 and 8, it leaves the respective remainders of 3, 5 and 4. What will be the last remainder when such a least possible number is divided successively by 8, 7 and 6?

Solution First we find the actual least possible no. then we move further

$$\begin{array}{ccc} & 6 & & 7 & & 8 \\ \swarrow & \downarrow & \swarrow & \downarrow & \swarrow & \downarrow \\ 3 & & 5 & & 4 & \end{array}$$

$$\begin{aligned} (((4 \times 7) + 5) \times 6) + 3 &= (((28 + 5) \times 6) + 3) \\ &= 33 \times 6 + 3 = 198 + 3 = 201 \end{aligned}$$

Now we divide 201 successively by 8, 7 and 6.

$$\begin{array}{rcl} 8 & | & 201 \\ 7 & | & 25 \rightarrow 1 \text{ (Remainder)} \\ 6 & | & 3 \rightarrow 4 \text{ (Remainder)} \\ & & 0 \rightarrow 3 \text{ (Remainder)} \end{array}$$

So 3 is the last remainder.

Exp. 4) How many numbers lie between 100 and 10000 which when successively divided by 7, 11 and 13 leaves the respective remainders as 5, 6 and 7?

Solution The least possible number can be obtained as

$$\begin{array}{ccc} & 7 & & 11 & & 13 \\ \swarrow & \downarrow & \swarrow & \downarrow & \swarrow & \downarrow \\ 5 & & 6 & & 7 & \end{array}$$

$$\begin{aligned} (((7 \times 11) + 6) \times 7 + 5) &= ((77 + 6) \times 7 + 5) \\ &= (83 \times 7 + 5) = (581 + 5) = 586 \end{aligned}$$

The general form for the higher numbers is

$$(7 \times 11 \times 13)m + 586 = (1001)m + 586$$

So, the numbers can be obtained by considering $m = 0, 1, 2, 3, \dots$ so the first number is 586 and the last number is 9595 which can be attained at $m = 9$. So there are total 10 such numbers lying between 100 and 10000.

Introductory Exercise 1.6

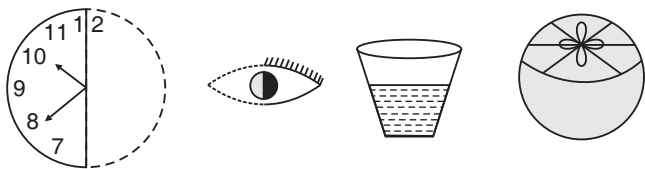
- HCF of 1007 and 1273 is :
(a) 1 (b) 17 (c) 23 (d) 19
- The GCD of two whole numbers is 5 and their LCM is 60. If one of the numbers is 20, then other number would be :
(a) 25 (b) 13 (c) 16 (d) 15
- The number of possible pairs of numbers, whose product is 5400 and HCF is 30 :
(a) 1 (b) 2 (c) 3 (d) 4
- The number of pairs lying between 40 and 100, such that HCF is 15, is :
(a) 3 (b) 4 (c) 5 (d) 6
- A merchant has 140 litres, 260 litres and 320 litres of three kinds of oil. He wants to sell the oil by filling the three kinds of oil separately in tins of equal volume. The volume of such a tin is :
(a) 20 litres (b) 13 litres
(c) 16 litres (d) 70 litres
- If x and y be integer such that $3x + 2y = 1$ then consider the following statements regarding x and y :
1. x and y can be found using euclidean algorithm.
2. x and y are positive.
3. x and y are uniquely determined.
Of these statements :
(a) 1 and 2 are correct (b) 1 alone is correct
(c) 1 and 3 are correct (d) 1, 2 and 3 are correct
- If d is the HCF of a and b , then $d = \lambda a + \mu b$ where :
(a) λ and μ are uniquely determined
(b) λ and μ are both positive
(c) λ and μ are both negative
(d) one of the λ and μ is negative and the other is positive
- The product of two numbers is 84 and their HCF is 2. Find the numbers of such pairs.
(a) 8 (b) 5 (c) 4 (d) 2
- The product of two numbers is 15120 and their HCF is 6, find the number of such pairs.
(a) 5 (b) 6 (c) 3 (d) 8
- The number of pairs of two numbers whose product is 300 and their HCF is 5 :
(a) 2 (b) 3
(c) 4 (d) can't be determined
- The largest possible number by which when 76, 132 and 160 are divided the remainders obtained are the same is :
(a) 6 (b) 14
(c) 18 (d) none of these
- Find the number of pairs of two numbers whose HCF is 5 and their sum is 50.
(a) 4 (b) 3
(c) 2 (d) none of these
- The largest possible length of a tape which can measure 525 cm, 1050 cm and 1155 cm length of cloths in a minimum number of attempts without measuring the length of a cloth in a fraction of the tape's length
(a) 25 (b) 105
(c) 75 (d) none of these
- In the above question minimum how many attempts are required to measure whole length of cloths?
(a) 16 (b) 26
(c) 24 (d) 30
- Minimum how many similar tiles of square shape are required to furnish the floor of a room with the length of 462 cm and breadth of 360 cm?
(a) 4420 (b) 4220
(c) 4120 (d) 4620
- The ratio of two numbers is 15 : 11. If their HCF be 13 then these numbers will be :
(a) 15 : 11 (b) 75 : 55
(c) 105 : 77 (d) 195 : 143
- The three numbers are in the ratio 1 : 2 : 3 and their HCF is 12. These numbers are :
(a) 4, 8, 12 (b) 5, 10, 15
(c) 24, 48, 72 (d) 12, 24, 36
- There are three drums with 1653 litre, 2261 litre and 2527 litre of petrol. The greatest possible size of the measuring vessel with which we can measure up the petrol of any drum, while every the vessel must be completely filled:
(a) 31 (b) 27
(c) 19 (d) 41
- Two pencils are of 24 cm and 42 cm. If we want to make them of equal size then minimum no. of similar pencils is
(a) 6 (b) 11
(c) 12 (d) none of these
- The HCF of two numbers is 27 and their sum is 216. These numbers are :
(a) 27, 189 (b) 81, 189
(c) 108, 108 (d) 154, 162
- Mr. Baagwan wants to plant 36 mango trees, 144 orange trees and 234 apple trees in his garden. If he wants to plant the equal no. of trees in every row, but the rows of mango, orange and apple trees will be separate, then the minimum number of rows in his garden is :
(a) 18 (b) 23
(c) 36 (d) can't be determined
- If the product of the HCF and the LCM of 3 natural numbers p, q, r equals pqr , then p, q, r must be :
(a) such that $(p, q, r) = 1$
(b) prime number
(c) odd number
(d) such that $(p, q) = (p, r) = (q, r) = 1$

1.7 Fractions And Decimal Fractions

Suppose you have borrowed Rs. 20 from your friend in the last month. Now he has asked you to return his money. But you are paying him only Rs. 10 and the rest amount you want to return in the next month.

It means you are paying the total amount not in a one lot but in a fraction that means “in parts”.

From figures you can see that these things are in fraction.



So, we can say that when any unit of a thing is divided into equal parts and some parts are considered, then it is called a fraction. For example $\frac{1}{2}, \frac{3}{4}, \frac{2}{5}, \frac{3}{7}$ etc.

$\frac{1}{2}$ is read as one half



$\frac{3}{4}$ is read as three fourth



Denominator : The lower value indicates the number of parts into which the whole thing or quantity is being equally divided and it is known as Denominator.

Numerator : The upper value indicates the number of parts taken into consideration (or for use) out of the total parts in known as Numerator.

So in the fractions $\frac{2}{3}, \frac{5}{7}, \frac{1}{4}$ etc. 2, 5, 1 are the numerators and 3, 7, 4 are the denominators. The numerators and denominators of a fraction are also called the ‘terms’ of a fraction.

Proper Fraction : A fraction whose numerator is less than its denominator, but not equal to zero is called a proper fraction.

For example $\frac{1}{2}, \frac{3}{4}, \frac{2}{7}, \frac{11}{20}$, etc.

Improper Fraction : A fraction whose numerator is equal to or greater than its denominator is called an improper fraction. For example $\frac{7}{2}, \frac{8}{5}, \frac{215}{15}, \frac{63}{15}$ etc.

NOTE 1. Every natural number can be expressed as a fraction, for example $7 = \frac{7}{1}, 8 = \frac{8}{1}, 11 = \frac{11}{1}$ etc. which are improper fraction.

2. When the numerator and denominator of a fraction are equal, then it is equal to unity.

3. The denominator of a fraction can never be equal to zero.

Mixed Fraction : A number which consists of two parts

(i) a natural number (ii) a proper fraction

is called a mixed fraction. For example $2\frac{5}{13}, 1\frac{7}{8}, 18\frac{9}{17}$ etc.

where 2, 1, 18 are natural numbers and $\frac{5}{13}, \frac{7}{8}$ and $\frac{9}{17}$ are

proper fractions. and $2\frac{5}{13} = 2 + \frac{5}{13}, 1\frac{7}{8} = 1 + \frac{7}{8}$

and $18\frac{9}{17} = 18 + \frac{9}{17}$

NOTE 1. Every mixed fraction can be written as an improper fraction and every improper fraction can be written as a mixed fraction as $\frac{45}{7} = 6\frac{3}{7}$

2. The numbers for example $-\frac{2}{3}, -\frac{7}{8}, -\frac{9}{5}$ etc are not

the fraction numbers, but these are called as fraction like numbers.

Like Fractions : The fractions whose denominators are same are called like fractions, for example

$\frac{3}{11}, \frac{4}{11}, \frac{9}{11}, \frac{8}{11}, \frac{17}{11}$ etc.

Equivalent Fractions : The fractions whose values are same i.e., the ratio is same are called equivalent fractions

for example, $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{12}{18} = \frac{20}{30} = \frac{26}{39}$

It implies that :

1. If we multiply the numerator and the denominator by the same non-zero number, the value of the fraction remains unchanged.
2. If we divide the numerator and the denominator by the same non zero number, the value of the fractions remains unchanged.

Cancellation : Division of the numerator and denominator by the same (non-zero) number is called the cancellation when the numerator and denominator of a fraction have no any common factors between them, then it is said the Reduced or Simplest form of fraction or in lowest term. For example $\frac{2}{3}, \frac{3}{4}, \frac{7}{8}$ etc are the reduced fractions in their lowest term.

Reduction of a Fraction to its Lowest Terms or Simplest Form

1. HCF Method

Divide the numerator and denominator both by their HCF.

e.g., to reduce $\frac{21}{35}$ to its lowest term just divide 21 and 35 by

their HCF. So, $\frac{21 \div 7}{35 \div 7} = \frac{3}{5}$

2. Prime Factorisation Method

Just cancel out the common prime factors of both the numerator and denominator. For example to reduce 42 and 140 we cancel out the common prime factors as

$$\frac{42}{140} = \frac{2 \times 3 \times 7}{2 \times 2 \times 5 \times 7} = \frac{3}{10}$$

Reduction of the given fractions into like fractions :

Obtain the LCM of the denominators of the given fractions and then make all the denominators equal to the LCM obtained, in such a way that the value of every fraction remains unchanged. For example $\frac{6}{35}, \frac{10}{21}$.

Since the LCM of the 35 and 21 is 105, so the new fraction will be as $\frac{6 \times 3}{35 \times 3}, \frac{10 \times 5}{21 \times 5}$

$$\Rightarrow \frac{18}{105}, \frac{50}{105}$$

Relationship among fractions : If the denominators of all the fractions are same then the fraction with the greater numerator will be greater.

For example $\frac{8}{19} > \frac{7}{19} > \frac{5}{19} > \frac{4}{19}$

NOTE

1. To equalize the denominator of the fractions, take the LCM of the denominators.

2. (A) If $\frac{x}{y}$ be a proper positive fraction and 'a' be a positive integers when $y \neq 0$

$$(i) \frac{x+a}{y+a} > \frac{x}{y} \quad (ii) \frac{x-a}{y-a} < \frac{x}{y}$$

- (B) If 'b' being a positive integer such that $a > b$, then

$$(iii) \frac{x}{y} \times \frac{a}{b} > \frac{x}{y} \quad (iv) \frac{x}{y} \times \frac{b}{a} < \frac{x}{y}$$

3. If $\frac{x}{y}$ be an improper positive fraction, then

$$(v) \frac{x+a}{y+a} < \frac{x}{y} \quad (vi) \frac{x-a}{y-a} > \frac{x}{y}$$

$$(vii) \frac{x}{y} \times \frac{a}{b} > \frac{x}{y} \quad (viii) \frac{x}{y} \times \frac{b}{a} < \frac{x}{y}$$

4. For any fraction $\frac{x}{y}$

$$(ix) \frac{x+nx}{y+ny} = \frac{x}{y} \quad (x) \frac{x-nx}{y-ny} = \frac{x}{y}$$

$$(xi) \frac{x \pm m}{y \pm n} \neq \frac{x}{y} \text{ if } \frac{m}{n} \neq \frac{x}{y}$$

Practice Exercise

1. State (a) if the fraction is proper, state (b) if the fraction is improper state (c) if the fraction is mixed, state if (d) none of these

$$(i) \frac{3}{7} \quad (ii) \frac{7}{3} \\ (iii) \frac{35}{12} \quad (iv) 4\frac{3}{11}$$

2. State (a) if the fractions are equivalent and state (b) if the fractions are like fractions else state (c)

$$(i) \frac{7}{16}, \frac{8}{16}, \frac{9}{16}, \frac{31}{16} \quad (ii) \frac{-5}{-6}, \frac{15}{18}, \frac{-45}{-54}$$

$$(iii) \frac{3}{5}, \frac{7}{10}, \frac{14}{20} \quad (iv) \frac{9}{25}, \frac{4}{16}, \frac{13}{29}$$

3. State (a) if the fractions are in ascending order state (b) if the fractions are in descending order, else state (c)

$$(i) \frac{3}{5}, \frac{7}{9}, \frac{13}{8} \quad (ii) \frac{9}{25}, \frac{11}{26}, \frac{13}{28} \\ (iii) \frac{17}{16}, \frac{35}{52}, \frac{65}{75} \quad (iv) \frac{30}{45}, \frac{30}{50}, \frac{38}{80}$$

Answers

1. (i) a (ii) b (iii) b (iv) c
2. (i) b (ii) a (iii) c (iv) c
3. (i) a (ii) a (iii) a (iv) b

Type of Fraction

Reciprocal Fraction : The reciprocal fraction of a number k be $1/k$ i.e., the product of a reciprocal number with the number itself is unity (i.e., 1). Hence the reciprocal of 6 is $\frac{1}{6}$,

the reciprocal of $\frac{1}{8}$ is 8, the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.

Compound Fraction : The fraction of a fraction is called its compound fraction. For example $\frac{1}{3}$ of $\frac{1}{2}$ (i.e., $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$) is the compound fraction.

Complex Fraction : If the numerator or denominator or both of a fraction are fraction then it is called as complex fraction. For example $\frac{2/3}{7/8}, \frac{7/5}{2}, \frac{3/7}{8/9}, \frac{2}{3/7}$ etc.

NOTE

$$\frac{a/b}{c/d} = \frac{a}{b} \times \frac{d}{c} \\ \frac{a/b}{c} = \frac{a}{b \times c} \\ \frac{a}{b/c} = \frac{a \times c}{b}$$

Continued Fraction : A continued fraction consists of the fractional denominators. For example,

$$(i) 1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4 + \frac{1}{5}}}} \quad (ii) 2 + \frac{3}{5 + \frac{2}{7 + \frac{6}{5 + \frac{2}{3}}}}$$

 **NOTE** These fractions are solved starting from the bottom towards upside.

Simplification of Fractions

Exp. 1) Solve the following expressions :

$$(i) \frac{3}{5} + \frac{2}{7} \quad (ii) \frac{7}{3} + \frac{9}{2} \quad (iii) 8\frac{3}{7} + 6\frac{1}{5} \quad (iv) \frac{8}{3} - \frac{13}{6}$$

$$(v) \frac{6}{55} - \frac{7}{88} \quad (vi) 9\frac{3}{5} - 5\frac{3}{9} \quad (vii) \frac{16}{5} - \frac{3}{8} + \frac{11}{10}$$

$$(viii) \frac{193}{75} + \frac{37}{81} - \frac{18}{65}$$

Solution (i) $\frac{3}{5} + \frac{2}{7} = \frac{3 \times 7}{5 \times 7} + \frac{2 \times 5}{7 \times 5} = \frac{21 + 10}{35} = \frac{31}{35}$

(ii) $\frac{7}{3} + \frac{9}{2} = \frac{7 \times 2}{3 \times 2} + \frac{9 \times 3}{2 \times 3} = \frac{14 + 27}{6} = \frac{41}{6} = 6\frac{5}{6}$

(iii) $8\frac{3}{7} + 6\frac{1}{5} = \frac{59}{7} + \frac{31}{5} = \frac{59 \times 5}{7 \times 5} + \frac{31 \times 7}{5 \times 7}$
 $= \frac{295 + 217}{35} = \frac{512}{35} = 14\frac{22}{35}$

(iv) $\frac{8}{3} - \frac{13}{6} = \frac{8 \times 2}{3 \times 2} - \frac{13}{6} = \frac{16}{6} - \frac{13}{6} = \frac{3}{6} = \frac{1}{2}$

(v) $\frac{6}{55} - \frac{7}{88} = \frac{6 \times 8}{55 \times 8} - \frac{7 \times 5}{88 \times 5} = \frac{48}{440} - \frac{35}{440} = \frac{13}{440}$

(vi) $9\frac{3}{5} - 5\frac{3}{9} = \frac{48}{5} - \frac{48}{9} = \frac{48 \times 9}{5 \times 9} - \frac{48 \times 5}{9 \times 5}$
 $= \frac{432}{45} - \frac{240}{45} = \frac{192}{45} = \frac{64}{15} = 4\frac{4}{15}$

(vii) $\frac{16}{5} - \frac{3}{8} + \frac{11}{10} = \frac{16 \times 8}{5 \times 8} - \frac{3 \times 5}{8 \times 5} + \frac{11 \times 4}{10 \times 4}$
 $= \frac{128 - 15 + 44}{40} = \frac{157}{40} = 3\frac{37}{40}$

(viii) $\frac{193}{75} + \frac{37}{81} + \frac{18}{65} = \frac{193 \times 27 \times 13}{75 \times 27 \times 13} + \frac{37 \times 25 \times 13}{81 \times 25 \times 13}$
 $+ \frac{18 \times 81 \times 5}{65 \times 81 \times 5}$
 $= \frac{67743 + 12025 + 7290}{26325} = \frac{87058}{26325} = 3\frac{8083}{26325}$

Exp. 2) Simplify the following expressions :

$$(i) \frac{3}{7} \times \frac{5}{4} \quad (ii) \frac{3}{8} \times \frac{9}{16} \times \frac{7}{5}$$

$$(iii) \frac{9}{7} \times \frac{21}{6} \times \frac{5}{8} \times \frac{5}{6} \quad (iv) \frac{4}{9} \text{ of } \frac{729}{48}$$

(v) $\frac{1}{2} \text{ of } \left(\frac{16}{7} + \frac{5}{3} \right)$

(vii) $\frac{6}{15} \div \frac{3}{2}$

(ix) $\frac{3}{7} \div \frac{2}{7} \times \frac{21}{8}$

(xi) $5\frac{1}{3} \div \frac{4}{15}$

(vi) $\frac{2}{3} \text{ of } 21\frac{4}{9} - 9\frac{4}{11}$

(viii) $\frac{14}{9} \div \frac{3}{7}$

(x) $\frac{9}{5} \text{ of } \frac{37}{8} \times \frac{36}{74}$

(xii) $36 \div 4 \text{ of } \frac{1}{2} + \frac{3}{4} \times \frac{3}{2}$

Solution (i) $\frac{3}{7} \times \frac{5}{4} = \frac{15}{28}$

(ii) $\frac{3}{8} \times \frac{9}{16} \times \frac{7}{5} = \frac{189}{640}$

(iii) $\frac{9}{7} \times \frac{21}{6} \times \frac{5}{8} \times 3\frac{5}{6} = \frac{9}{7} \times \frac{21}{6} \times \frac{5}{8} \times \frac{23}{6} = \frac{345}{32} = 10\frac{25}{32}$

(iv) $\frac{4}{9} \text{ of } \frac{729}{48} = \frac{4}{9} \times \frac{729}{48} = \frac{1}{9} \times \frac{729}{12}$
 $= \frac{1}{1} \times \frac{81}{12} = \frac{1}{1} \times \frac{27}{4} = \frac{27}{4} = 6\frac{3}{4}$

(v) $\frac{1}{2} \text{ of } \left(\frac{16}{7} + \frac{5}{3} \right) = \frac{1}{2} \times \left(\frac{16}{7} + \frac{5}{3} \right)$
 $= \frac{1}{2} \times \left(\frac{48}{21} + \frac{35}{21} \right) = \frac{1}{2} \times \left(\frac{83}{21} \right) = \frac{83}{42} = 1\frac{41}{42}$

(vi) $\frac{2}{3} \text{ of } 21\frac{4}{9} - 9\frac{4}{11} = \frac{2}{3} \times 21\frac{4}{9} - 9\frac{4}{11}$
 $= \frac{2}{3} \times \frac{193}{9} - \frac{103}{11} = \frac{386}{27} - \frac{103}{11}$
 $= \frac{386 \times 11}{27 \times 11} - \frac{103 \times 27}{11 \times 27} = \frac{4246 - 2781}{297} = \frac{1465}{297} = 4\frac{277}{297}$

(vii) $\frac{6}{15} \div \frac{3}{2} = \frac{6/15}{3/2} = \frac{6}{15} \times \frac{2}{3} = \frac{4}{15}$

(viii) $\frac{14}{9} \div \frac{3}{7} = \frac{14/9}{3/7} = \frac{14}{9} \times \frac{7}{3} = \frac{98}{27} = 3\frac{17}{27}$

(ix) $\frac{3}{7} \div \frac{2}{7} \times \frac{21}{8} = \frac{3}{7} \times \frac{7}{2} \times \frac{21}{8} = \frac{63}{16} = 3\frac{15}{16}$

(x) $\frac{9}{5} \text{ of } \frac{37}{8} \times \frac{36}{74} = \frac{9}{5} \times \frac{37}{8} \times \frac{36}{74} = \frac{81}{20} = 4\frac{1}{20}$

(xi) $5\frac{1}{3} \div \frac{4}{15} = \frac{16}{3} \div \frac{4}{15} = \frac{16}{3} \times \frac{15}{4} = \frac{20}{1} = 20$

(xii) $36 \div 4 \text{ of } \frac{1}{2} + \frac{3}{4} \times \frac{3}{2} = 36 \div 4 \times \frac{1}{2} + \frac{3}{4} \times \frac{3}{2}$
 $= 36 \div 2 + \frac{9}{8} = 18 + \frac{9}{8} = \frac{153}{8} = 19\frac{1}{8}$

Exp. 3) Simplify the following expressions :

(a) $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{6}}}$

(b) $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}}}$

$$(c) 2 - \frac{1}{3 + \frac{1}{4 - \frac{1}{5 + \frac{1}{6 - \frac{1}{7}}}}}$$

Solution (a) $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{6}}} = 1 + \frac{1}{1 + \frac{1}{7/6}}$

$$= 1 + \frac{1}{1 + \frac{6}{7}} = 1 + \frac{1}{13/7}$$

$$= 1 + \frac{7}{13} = \frac{20}{13} = 1\frac{7}{13}$$

(b) $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}}} = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}}$

$$= 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{2}{3}}}} = 1 + \frac{1}{1 + \frac{1}{1 + \frac{3}{5}}}$$

$$= 1 + \frac{1}{1 + \frac{1}{1 + \frac{5}{8}}} = 1 + \frac{1}{1 + \frac{8}{13}} = 1 + \frac{13}{21}$$

$$= 1 + \frac{13}{21} = \frac{34}{21} = 1\frac{13}{21}$$

(c) $2 - \frac{1}{3 + \frac{1}{4 - \frac{1}{5 + \frac{1}{6 - \frac{1}{7}}}}} = 2 - \frac{1}{3 + \frac{1}{4 - \frac{1}{5 + \frac{1}{41/7}}}}$

$$= 2 - \frac{1}{3 + \frac{1}{4 - \frac{1}{5 + \frac{7}{41}}}} = 2 - \frac{1}{3 + \frac{1}{4 - \frac{1}{212/41}}}$$

$$= 2 - \frac{1}{3 + \frac{1}{4 - \frac{41}{212}}} = 2 - \frac{1}{3 + \frac{1}{807/212}}$$

$$= 2 - \frac{1}{3 + \frac{212}{807}} = 2 - \frac{1}{\frac{2633}{807}} = 2 - \frac{807}{2633}$$

$$= \frac{4459}{2633} = 1\frac{1826}{2633}$$

HCF and LCM of Fractions

HCF of fractions : The greatest common fraction is called the HCF of the given fractions.

$$\text{HCF of fractions} = \frac{\text{HCF of Numerator}}{\text{LCM of denominator}}$$

For example, The HCF of $\frac{4}{3}, \frac{4}{9}, \frac{2}{15}, \frac{36}{21}$

$$= \frac{\text{HCF of } 4, 4, 2, 36}{\text{LCM of } 3, 9, 15, 21} = \frac{2}{315}$$

LCM of fractions : The least possible number of fraction which is exactly divisible by all the given fractions is called the LCM of the fractions.

$$\text{LCM of fractions} = \frac{\text{LCM of Numerator}}{\text{HCF of Denominator}}$$

For example The LCM of $\frac{4}{3}, \frac{4}{9}, \frac{2}{15}, \frac{36}{21}$

$$= \frac{\text{LCM of } 4, 4, 2, 36}{\text{HCF of } 3, 9, 15, 21}$$

$$= \frac{36}{3} = 12$$

Exp. 1) Find the smallest positive number which is exactly divisible by $\frac{1}{3}, \frac{1}{2}, \frac{3}{7}$ and $\frac{4}{11}$.

Solution The LCM of the given numbers will be the required number. So, the LCM of $\frac{1}{3}, \frac{1}{2}, \frac{3}{7}, \frac{4}{11}$

$$= \frac{\text{LCM of } 1, 1, 3, 4}{\text{HCF of } 3, 2, 7, 11} = \frac{12}{1} = 12$$

Thus 12 is the smallest positive numbers, which is required.

Exp. 2) Four runners started running the race in the same direction around a circular path of 7 km. Their speeds are 4, 3, 9 and 3.5 km/hr individually. If they have started their race at 6 o'clock in the morning, then at what time will they be at the starting point?

Solution Time required by everyone to complete one revolution individually is $\frac{7}{4}, \frac{7}{3}, \frac{7}{9}, \frac{7}{3.5}$ hours.

Therefore everyone must reach at the starting point after the time of the LCM of the individual time period for one revolution.

$$\text{So the LCM of } \frac{7}{4}, \frac{7}{3}, \frac{7}{9}, \frac{7}{3.5} = \frac{7}{4}, \frac{7}{3}, \frac{7}{9}, \frac{2}{1}$$

$$= \frac{\text{LCM of } 7, 7, 7, 2}{\text{HCF of } 4, 3, 9, 1} = \frac{14}{1} = 14 \text{ hours.}$$

Hence after 14 hours i.e., at 8 o'clock in the evening of the same day they will meet at the starting point.

Square, square root, cube and cube root of fractions

Square of a fraction = $\frac{\text{Square of numerators}}{\text{Square of denominators}}$

Similarly, Cube of a fraction = $\frac{\text{Cube of numerator}}{\text{Cube of denominator}}$

NOTE $\sqrt{ab} = \sqrt{a} \sqrt{b}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$$(ab)^2 = a^2 b^2 \quad \text{and} \quad \left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$$

Exp. 3) Evaluate the following :

(a) $\left(\frac{3}{4}\right)^2$ (b) $\left(\frac{17}{25}\right)^2$ (c) $\left(\frac{19}{-31}\right)^2$ (d) $\left(\frac{78}{36}\right)^2$
 (e) $\left(\frac{9}{7}\right)^3$ (f) $\left(-\frac{13}{12}\right)^3$ (g) $\left(\frac{20}{7}\right)^3$ (h) $\left(6\frac{1}{2}\right)^3$

Solution (a) $\frac{9}{16}$ (b) $\frac{289}{625}$ (c) $\frac{361}{961}$ (d) $\frac{1521}{324}$
 (e) $\frac{729}{343}$ (f) $-\frac{2197}{1728}$ (g) $\frac{8000}{343}$ (h) $\frac{2197}{8}$

Exp. 4) Evaluate the following :

(a) $\sqrt{\frac{961}{625}}$ (b) $\frac{\sqrt{7744}}{20}$ (c) $\frac{31}{\sqrt{1681}}$ (d) $\sqrt{\frac{1156}{529}}$
 (e) $\sqrt{50} \times \sqrt{18}$ (f) $\sqrt{39} \times \sqrt{\frac{75}{2197}}$

Solution (a) $\sqrt{\frac{961}{625}} = \frac{31}{25}$ (b) $\frac{\sqrt{7744}}{20} = \frac{88}{20} = 4\frac{8}{20} = 4\frac{2}{5}$
 (c) $\frac{31}{\sqrt{1681}} = \frac{31}{41}$ (d) $\frac{\sqrt{1156}}{\sqrt{529}} = \frac{34}{23}$
 (e) $\sqrt{50} \times \sqrt{18} = \sqrt{50 \times 18} = \sqrt{900} = 30$
 (f) $\sqrt{39} \times \sqrt{\frac{75}{2197}} = \sqrt{\frac{39 \times 75}{2197}} = \sqrt{\frac{13 \times 3 \times 3 \times 25}{13 \times 13 \times 13}} = \frac{15}{13} = 1\frac{2}{13}$

Exp. 5) The value of $\left\{ \frac{1}{(\sqrt{2}-2)} + \frac{1}{(\sqrt{2}+2)} + (\sqrt{2}+2) \right\}$ is :

(a) 2 (b) 4 (c) $2\sqrt{2}$ (d) none

Solution $\left\{ \frac{(\sqrt{2}+2) + (\sqrt{2}-2)}{(\sqrt{2}-2)(\sqrt{2}+2)} + (\sqrt{2}+2) \right\} = 2$

Exp. 6) The value of $\left\{ \frac{1}{(\sqrt{6}-\sqrt{5})} - \frac{1}{(\sqrt{5}-\sqrt{4})} + \frac{1}{(\sqrt{4}-\sqrt{3})} - \frac{1}{(\sqrt{3}-\sqrt{2})} + \frac{1}{(\sqrt{2}-1)} \right\}$ is

(a) 5 (b) 7 (c) $\sqrt{12}$ (d) none

Solution $\frac{1}{(\sqrt{6}-\sqrt{5})} - \frac{1}{(\sqrt{5}-\sqrt{4})} + \frac{1}{(\sqrt{4}-\sqrt{3})} - \frac{1}{(\sqrt{3}-\sqrt{2})} + \frac{1}{(\sqrt{2}-1)}$

$$= (\sqrt{6} + \sqrt{5}) - (\sqrt{5} + \sqrt{4}) + (\sqrt{4} + \sqrt{3}) - (\sqrt{3} + \sqrt{2}) + (\sqrt{2} + 1)$$

$$= \sqrt{6} + 1$$

$$\left[\because \frac{1}{\sqrt{a}-\sqrt{b}} = \frac{(\sqrt{a}+\sqrt{b})}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})} = \frac{\sqrt{a}+\sqrt{b}}{(a-b)} \right]$$

Hence (d) is the correct option.

Practice Exercise

- Find the value of $\frac{1}{\sqrt{9}-\sqrt{4}}$.
- Find the value of x if $\sqrt{1+\frac{x}{169}} = \frac{14}{13}$.
- Find the value of x if $140\sqrt{x} + 315 = 1015$.
- If $\sqrt{1+\frac{27}{169}} = \left(1+\frac{x}{13}\right)$ then find the value of x.
- Find the value of $\left[6\frac{1}{4} \div \left\{2\frac{1}{4} - \frac{1}{2} \left(2\frac{1}{2} - \frac{1}{4} - \frac{1}{6}\right)\right\}\right]$.
- Find the value of $\frac{3}{2}$ of $\left(\frac{4}{3} \div \frac{5}{7}\right) + \frac{1}{2} \div \left[3\frac{4}{5} - \left\{\frac{2}{5} \div \left(\frac{1}{3} + \frac{1}{2} + \frac{1}{5} - \frac{1}{6}\right)\right\}\right]$.
- Find the value of $1 + \frac{1}{2 + \frac{1}{4 + \frac{1}{8 + \frac{1}{16}}}}$.
- Find the value of $1 - \frac{1}{2 + \frac{1}{2 - \frac{1}{2 + \frac{1}{2}}}}$.
- Calculate the value of $\frac{2}{5} + 2\frac{4}{9} \div \left[\left(7\frac{5}{12} - 5\frac{3}{4}\right) \div 22\frac{1}{2} + 10 \times \frac{5}{18}\right] - \frac{4}{5}$.
- Calculate the value of $\frac{1}{2 + \frac{1}{2 - \frac{1}{2}}}$.
- Calculate the value of $2 + \frac{1}{2 - \frac{1}{2}}$.
- Calculate the value of $\left(1\frac{29}{36} + 1\frac{7}{11} \times 4\frac{1}{8}\right) \div \left(5\frac{1}{9} - \frac{63}{8} \div 9\frac{9}{20}\right)$.
- Calculate the value of $\frac{4}{2\frac{1}{3}} + \frac{3}{1\frac{3}{4}} - \frac{5}{3\frac{1}{2}}$.

Answers

- 1
- 27
- 25
- 1
- 6
- 2 $\frac{2061}{2170}$
- 1 $\frac{532}{1193}$
- $\frac{13}{21}$
- $\frac{16}{35}$
- $\frac{8}{19}$
- 2
- 2

Exp. 1) From a rope of length $38\frac{3}{5}m$, a piece of length $5\frac{3}{38}m$ is cut off. The length of the remaining rope is :

- (a) $190\frac{99}{33}$ (b) $33\frac{90}{190}$
 (c) $33\frac{99}{190}$ (d) none of these

Solution Total length of the rope = $38\frac{3}{5} = \frac{193}{5}$

The length of the rope which has been removed = $5\frac{3}{38} = \frac{193}{38}$

∴ The length of the remaining part of the rope
 $= \frac{193}{5} - \frac{193}{38} = 33\frac{99}{190}$

Hence (c) is the correct option.

Exp. 2) The cost of 1 metre cloth is ₹ $21\frac{1}{2}$, then the cost of $\frac{42}{43}$ metre

- (a) ₹ 21 (b) ₹ 42
 (c) ₹ $\frac{43}{2}$ (d) none of these

Solution ∴ The price of 1m cloth = $21\frac{1}{2} = \frac{43}{2}$

∴ The price of $\frac{42}{43}$ m cloth = $\frac{42}{43} \times \frac{43}{2} = 21$

Hence option (a) is correct.

Exp. 3) A drum of kerosene oil is $\frac{3}{4}$ full. When 15 litres of oil is drawn from it, it is $\frac{7}{12}$ full. The capacity of the drum is :

- (a) 45 (b) 90
 (c) 60 (d) can't be determined

Solution Let the capacity of drum be x

Then $\frac{3}{4}x - 15 = \frac{7}{12}x \Rightarrow \frac{9x - 7x}{12} = 15 \Rightarrow x = 90$

Thus (b) is the correct option.

☞ **Alternatively** If you consider option (b) then

$$90 \times \frac{3}{4} - 15 = 90 \times \frac{7}{12}$$

☞ **Alternatively** The decrease in amount = $\frac{x}{6} = 15 \Rightarrow x = 90$ litre.

Exp. 4) A sum of ₹ 11200 is shared among Mr. Khare, Mr. Patel and Mr. Verma. Mr. Khare gets $\frac{1}{4}$ th of it while

Mr. Patel gets $\frac{1}{5}$ th of it. The amount of Mr. Verma is :

- (a) 6006 (b) 6160
 (c) 3080 (d) data insufficient

Solution The share of Mr. Khare = $\frac{1}{4}$

The share of Mr. Patel = $\frac{1}{5}$

∴ The share of Mr. Verma = $1 - \left(\frac{1}{4} + \frac{1}{5}\right) = \frac{11}{20}$

So, the amount of Mr. Verma = $\frac{11}{20} \times 11200 = 6160$

Thus option (b) is correct.

Exp. 5) When Sarvesh travelled 33 km, he found that $\frac{2}{3}$ rd of the entire journey was still left. The length of the total journey is :

- (a) 66 (b) 132
 (c) 99 (d) 100

Solution Since $\frac{2}{3}$ rd journey is left, it means Sarvesh has travelled only $\frac{1}{3}$ rd of his journey which is equal to 33. Thus *i.e.*,

$$\frac{1}{3}x = 33 \Rightarrow x = 99$$

Hence (b) is the right choice.

Exp. 6) Mrs. Verma earns Rs. 18000 per month. She spends $\frac{7}{12}$ on house hold items and $\frac{1}{8}$ on rest of the things. The amount she saves is :

- (a) 7120 (b) 5250
 (c) 5520 (d) none of these

Solution Her total expenditure = $\frac{7}{12} + \frac{1}{8} = \frac{17}{24}$

∴ Her savings = $1 - \frac{17}{24} = \frac{7}{24}$

∴ Her saving in ₹ = $\frac{7}{24} \times 18000 = 5250$

Hence (b) is the correct option.

Exp. 7) Neha, a working lady, earns ₹ x per month. If she spends $\frac{2}{5}$ th of her earning for personal uses and $\frac{3}{4}$ th of the personal uses, she spends in entertainment while $\frac{7}{20}$ th of

the expenditure in entertainment she spends in movies only. Her salary would be, if her expenditure in movies is in integers.

- (a) 4225 (b) 2175 (c) 200 (d) 3465

Solution Since her salary is ₹ x then her expenditure in movies is

$$\frac{7}{20} \text{ of } \frac{3}{4} \text{ of } \frac{2}{5} \text{ of } x = \frac{7}{20} \times \frac{3}{4} \times \frac{2}{5} \times x = \frac{21x}{200}$$

Now, In order to $\frac{21x}{200}$ be an integer so x must be equal to 200.

Thus (c) is the correct option.

Decimal Fractions

In the previous topics I have discussed the integral and fractional numbers.

Now in the ongoing discussion, I am extending the theory of numbers as “decimal numbers”.

In the number 8637, the place value of 7 is $7 \times 1 = 7$, the place value of 3 is $10 \times 3 = 30$, the place value of 6 is $6 \times 100 = 600$ and the place value of 8 is $8 \times 1000 = 8000$.

Thus, as we move from right to left each next digit is multiplied by 10, 100, 1000, 10000, 100000 etc.

It means moving from left to right the place value of each digit is divided by 10, 100, 1000, etc.....

In the above number 7 is known as unit digit (the right most digit), 3 is known as tens digit, 6 as hundreds digit and 8 as thousands digit.

Now if we move towards the right of unit place, the place value of the digit next to the right of unit digit (is the tenths digit) is being divided by 10, while the place value of the next digit is the hundredth digit is being divided by 100 and so on.

So, for our convenience we put a small dot just right to the unit digit and it is known as decimal point.

Hence, the place value of a digit in a number depends on the place or position with respect to the unit digit position while the **face value** of a digit is always fixed as the face value of 1 is 1, 2 is 2, 3 is 3, 4 is 4 and so on.

Place Value Chart

Places	Ten thousands	Thousands	Hundreds	Tens	Unit	Decimal point	Tenths	Hundredths	Thousandths
Value	10000	1000	100	10	1	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Thus a decimal number say 4637.938 can be expressed as

$$4 \times 1000 + 6 \times 100 + 3 \times 10 + 7 \times 1 + \frac{9}{10} + \frac{3}{100} + \frac{8}{1000}.$$

NOTE The digits written in the right side of decimal point read separately as we read 2.359 as two point three five nine.

Thus the fraction whose denominators are 10, 100, 1000 etc. are called decimal fractions.

The number left to the decimal point is a whole or integral number, while the number to right to the decimal point is a **decimal number**. In the number 735.82, 735 is a whole number and .82 is the **decimal number**.

Conversion of Decimal Numbers into Decimal Fractions

Write the number of zeros as the number of digits in the decimal number preceded by 1, below the actual number without decimal point as

$$23.6 = \frac{236}{10}, \quad 4.579 = \frac{4579}{1000}$$

$$75.89 = \frac{7589}{100}, \quad 0.18870 = \frac{18870}{100000} = \frac{1887}{10000}$$

Conversion of Decimal Fractions into Decimal Numbers

Count the number of zeros in the denominator and then count the same number of digits in the numerator starting from the unit digit of the numerator moving to the left and then place the decimal point as :

$$\frac{2375}{10} = 237.5, \quad \frac{2375}{100} = 23.75, \quad \frac{2375}{1000} = 2.375,$$

$$\frac{2375}{100000} = 0.02375, \quad \frac{2375}{10000000} = 0.0002375 \text{ etc.}$$

NOTE 0.35 = 0.350 = 0.3500 = 0.35000 etc. because increasing the number of zeros in the rightmost of a decimal number is inconsiderable.

For example $0.35 = \frac{35}{100}$; $0.350 = \frac{350}{1000} = \frac{35}{100}$

and $0.3500 = \frac{3500}{10000} = \frac{35}{1000}$

Some Mathematical Operations on Decimal Numbers

- (i) **Addition and Subtraction** : The decimal numbers are written in such a manner that decimal points of all the numbers fall in one column or fall below one another.

For example

$$\begin{array}{r} 2.358 \\ 709.21 \\ 0.35 \\ + 2.0067 \\ \hline 713.9247 \end{array} \quad \text{and} \quad \begin{array}{r} 635.888 \\ - 28.0125 \\ \hline 607.8755 \end{array}$$

- (ii) **Multiplication** : Suppose there is no decimal point as natural numbers and then multiply them. Then put the decimal point in the product as the sum of the decimal places in the multiplicands. For example

Decimal Places

(i) $23 \times 1.1 = 25.3$	$0 + 1 = 1$
(ii) $3.8 \times 4.6 = 17.48$	$1 + 1 = 2$
(iii) $2.456 \times 7.8 = 19.1568$	$3 + 1 = 4$
(iv) $8.125 \times 100 = 812.500$	$(3 + 0) = 3$
(v) $75 \times 0.3 = 22.5$	$(0 + 1) = 1$

- (iii) **Division** : Make the divisor as a natural number by shifting the decimal in the right hand side equally in dividend and divisor. Now divide the resultant dividend by this resultant divisor as usual.

For Example :

$$65 \div 1.3 = \frac{65}{1.3} = \frac{650}{13} = 50$$

$$235 \div 4.7 = \frac{235}{4.7} = \frac{2350}{47} = 50$$

$$1.17 \div 13 = \frac{1.17}{13} = 0.09$$

$$\left[\because \frac{1.17}{13} = \frac{117}{13} \times \frac{1}{100} = 9 \times \frac{1}{100} \right]$$

$$172.8 \div 0.12 = \frac{172.8}{0.12} = \frac{17280}{12} = 1440$$

$$1.068 \div 8.9 = \frac{1.068}{8.9} = \frac{10.68}{89} = 0.12$$

$$78 \div 0.0039 = \frac{78}{0.0039} = \frac{780000}{0039} = \frac{780000}{39} = 20000$$

Recurring Decimal A decimal number in which a digit or a set of digits repeats regularly, over a constant period, is called a recurring decimal or periodic decimal. For example

2.33333..., 7.5555, ..., 108.232323...

9.142857142857142857.....

where $2.3333..... = 2.\bar{3}$ or $2.\dot{3}$

$7.5555... = 7.\bar{5}$ or $7.\dot{5}$

$108.232323... = 108.\bar{23}$

and $9.142857142857142857...$

$= 9.\overline{142857} = 9.\dot{1}4285\dot{7}$

So the recurring of the decimal number is expressed by putting the bar or dot over the period of a recurring decimal.

Pure Recurring decimal

A decimal fraction in which all the figures (or digits) occur repeatedly, is called a pure recurring decimal as 7.4444, ..., 2.666, ..., 9.454545, ... etc.

Mixed Recurring decimal : A decimal, number in which some of the digits do not recur is called a mixed recurring decimal for example 327.63454545...

Non-recurring decimals : A decimal number in which there is no any regular pattern of repetition of digits after decimal point is called a non-recurring decimal. e.g., 3.2466267628...

Conversion of Recurring Decimal into Vulgar Fraction

(a) Pure Recurring Decimal

Write down as many 9's in the denominator as the number of digits in the period of decimal number, below the given decimal number, for example,

$$(i) 0.77777... = 0.\bar{7} = \frac{7}{9}$$

$$(ii) 0.545454... = 0.\bar{54} = \frac{54}{99}$$

$$(iii) 2.357357357357... = 2.\overline{357} = 2 + \frac{357}{999}$$

$$= 2 + \frac{357}{999} = \frac{2355}{999}$$

$$(iv) 0.768397683976839... = 0.\overline{76839} = \frac{76839}{99999}$$

(b) Mixed Recurring Decimal

Step 1. Write down the number and then subtract from it the number formed by the non recurring digits after decimal point.

Step 2. Write down as many 9s as there are digits in the period of the recurring decimal followed by as many zeros as there is number of digits which are not recurring after the decimal point.

Step 3. Divide the value of step 1 by the value obtained in step 2.

For example :

$$1. 0.17777... = 0.1\bar{7} = \frac{17-1}{90} = \frac{16}{90} = \frac{8}{45}$$

$$2. 0.8737373... = 0.8\bar{73} = \frac{873-8}{990} = \frac{865}{990} = \frac{173}{198}$$

$$3. 0.78943943943... = 0.789\bar{43} = \frac{78943-78}{99900}$$

$$= \frac{78865}{99900} = \frac{15773}{19980}$$

$$4. 0.250121212... = 0.2501\bar{2} = \frac{25012-250}{99000}$$

$$= \frac{24762}{99000} = \frac{12381}{49500}$$

$$5. 5.00\bar{72} = 5 + 0.00\bar{72} = 5 + \left(\frac{72-00}{9900} \right)$$

$$= 5 + \frac{72}{9900} = 5 + \frac{2}{275} = 5\frac{2}{275}$$

Addition and Subtraction of Recurring Decimals

Exp. 1) $5.\overline{732} + 8.\overline{613}$

Solution $5.\overline{732} + 8.\overline{613} = 5 + 0.\overline{732} + 8 + 0.\overline{613}$

$$= 5 + \frac{732-7}{990} + 8 + \frac{613}{999} = 5 + \frac{725}{990} + 8 + \frac{613}{999}$$

$$= 13 + \frac{725}{990} + \frac{613}{999} = 13 + \frac{145}{198} + \frac{613}{999}$$

$$= 13 + \frac{145 \times 3 \times 37 + 613 \times 2 \times 11}{2 \times 3 \times 9 \times 11 \times 37}$$

$$= \frac{13 \times 2 \times 3 \times 9 \times 11 \times 37 + 145 \times 3 \times 37 + 613 \times 2 \times 11}{2 \times 3 \times 9 \times 11 \times 37}$$

$$= \frac{285714 + 16095 + 13486}{21978} = \frac{315295}{21978} = 14.3459368$$

☞ **Alternatively**

Step 1. Express the numbers without bar as

$$5.732323232 + 8.613613613$$

Step 2. Write the numbers as one above other *i.e.*,

$$5.732323232$$

$$8.613613613$$

Step 3. Divide this number into two parts. In the first part *i.e.*, left side write as many digits as there will be integral value with non recurring decimal. In the right side write as many digits as the LCM of the number of recurring digits in the given decimal number *e.g.*,

5.7	323232	(Since 5.7 is the integral + non-recurring part]
8.6	136136	(The LCM of 2 and 3 is 6]

Step 4. Now add or subtract as usual.

$$\begin{array}{r} 5.7 \quad 323232 \\ 8.6 \quad 136136 \\ \hline 14.3 \quad 459368 \end{array}$$

Step 5. Put the bar over the digits which are on the right side in the resultant value.

$$\begin{array}{r} 14.3459368 \\ \hline \text{Thus } 5.\overline{732} + 8.\overline{613} = 14.\overline{3459368} \end{array}$$

Exp. 2) Solve the following : $19.\overline{368421} - 16.\overline{2053}$

Solution $19.\overline{368421} - 16.\overline{2053} = 19.36842168421 - 16.205353535353...$

$$= 19.3684216842168421$$

$$= 16.205353535353535353$$

$$\begin{array}{r} 19.36 \quad 8421684216 \\ - 16.20 \quad 5353535353 \\ \hline 3.16 \quad 3068148863 \end{array}$$

(Since 16.20 is integral part with non recurring digits]
[The LCM of 5 and 2 is 10]

In the first number there are 5 recurring digits and in the second number there are 2 recurring digits]

$$19.\overline{368421} - 16.\overline{2053} = 3.\overline{163068148863}$$

Multiplication and Division of Recurring Decimals

It can be done as usual. Just convert the decimals into vulgar fractions and then operate as required.

Exp. 1) $97.\overline{281} \times 100 = 97.2818181 \times 100$

$$= 9728.181818 = 9728.\overline{181}$$

(Remember that the set of recurring digits is not altered as in the above problem the set of recurring digits will remain $\overline{81}$ but not $\overline{18}$ since initially it was $\overline{81}$.)

Exp. 2) $25.\overline{632} \times 55 = (25 + 0.\overline{632}) \times 55$

$$= 25 \frac{632-6}{990} \times 55 = 25 \frac{626}{990} \times 55$$

$$= 25 \frac{313}{495} \times 55$$

$$= \frac{12688}{495} \times 55$$

$$= \frac{12688}{9} = 1409.7777$$

$$= 1409.\overline{7}$$

Exp. 3) $13.00\overline{5} \times 20 \times 8.\overline{423} = 13 \frac{5}{900} \times 20 \times 8 \frac{423-4}{990}$

$$= 13 \frac{1}{180} \times 20 \times 8 \frac{419}{990}$$

$$= \frac{2341}{180} \times 20 \times \frac{8339}{990} = \frac{2341 \times 8339}{9 \times 990}$$

$$= \frac{19521599}{9 \times 990} = 2190.9763187$$

Exp. 4) $0.08\overline{9} \div 100 = 0.089999... \div 100$

$$= 0.00089999... = 0.0008\overline{9}$$

Exp. 5) $53.08\overline{53} \div 6 = \frac{53 + 0.08\overline{53}}{6} = \frac{53 + \frac{0853-08}{990}}{6}$

$$= \frac{53 + \frac{845}{990}}{6} = \frac{53 + \frac{169}{1980}}{6}$$

$$= \frac{\left(\frac{105109}{1980}\right)}{6} = \frac{105109}{1980 \times 6} = \frac{105109}{11880}$$

$$= 8.847558922558922558922$$

$$= 8.847\overline{558922}$$

☞ Alternatively $53.0853 \div 6 = 53.08535353... \div 6$

$$\begin{array}{r}
 6 \overline{) 53.08535353...} \quad (8.847558922558922 \\
 \underline{48} \\
 50 \\
 \underline{48} \\
 28 \\
 \underline{24} \\
 45 \\
 \underline{42} \\
 33 \\
 \underline{30} \\
 35 \\
 \underline{30} \\
 53 \\
 \underline{48} \\
 55 \\
 \underline{54} \\
 13 \\
 \underline{12} \\
 15 \\
 \underline{12} \\
 33 \\
 \underline{30} \\
 35 \\
 \underline{30} \\
 53 \\
 \underline{48} \\
 55 \\
 \underline{54} \\
 1 \text{ etc.}
 \end{array}$$

Square of Decimals

First we assume that there is no decimal point and then square up the given decimal number. But at last we put the dot (as decimal point), counting the digits from the rightmost digit. The no. of places will be double as that of the given number. **For example :**

1. $(2.3)^2 = 5.29$	$[(23)^2 = 529]$
2. $(1.07)^2 = 1.1449$	$[(107)^2 = 11449]$
3. $(0.11)^2 = 0.0121$	$[(11)^2 = 121]$
4. $(1.352)^2 = 1.827904$	$[\therefore (1352)^2 = 1827904]$
5. $0.1^2 = 0.01$	$[\therefore (1)^2 = 1]$
6. $(9.99)^2 = 99.8001$	$[(999)^2 = 998001]$

Square root of Decimals

The process of finding the square root is same as that of integers. Here we put the decimal point as in the division of decimal numbers.

For example :

1. Find the square root of 5.29

$$\begin{array}{r}
 2.3 \\
 2 \overline{) 5.29} \\
 \underline{24} \\
 43 \quad 129 \\
 \underline{3 \quad 129} \\
 \times
 \end{array}
 \quad \therefore \sqrt{5.29} = 2.3$$

2. Find the square root of 0.0121

$$\begin{array}{r}
 0.11 \\
 0 \overline{) 0.0121} \\
 \underline{00} \\
 01 \quad 01 \\
 \underline{1 \quad 01} \\
 21 \quad 21 \\
 \underline{1 \quad 21} \\
 \times
 \end{array}
 \quad \therefore \sqrt{0.0121} = 0.11$$

3. Find the square root of 536.85374

$$\begin{array}{r}
 23.1701 \\
 2 \overline{) 536.85374} \\
 \underline{24} \\
 43 \quad 136 \\
 \underline{3 \quad 129} \\
 461 \quad 785 \\
 \underline{1 \quad 461} \\
 4627 \quad 32437 \\
 \underline{7 \quad 32389} \\
 463401 \quad 484000 \\
 \underline{01 \quad 463401}
 \end{array}$$

The process can be continued if required. But generally we stop our calculation after 2-3 places of decimal point.

☞ **NOTE** To find the n th power of a decimal number in which the decimal point is placed at m places before the rightmost digit, we simply solve it considering as an integer then we put the decimal point in the resultant value before the $m.n$ places from the right most digit, where m, n are positive integers.

For example : $(2.13)^4 = 20.58346161$

[The number of places of decimal point = $2 \times 4 = 8$]

HCF and LCM of Decimals

HCF

Step 1. First of all equate the number of places in all the numbers by using zeros, wherever required.

Step 2. Then considering these numbers as integers find the HCF of these numbers.

Step 3. Put the decimal point in the resultant value as many places before the right most digit as that of in the every equated number.

Exp. 1) Find the HCF of 0.0005, 0.005, 0.15, 0.175, 0.5 and 3.5.

Solution	0.0005	$\Rightarrow$ 5
	0.0050	$\Rightarrow$ 50
	0.1500	$\Rightarrow$ 1500
	0.1750	$\Rightarrow$ 1750
	0.5000	$\Rightarrow$ 5000
	3.5000	$\Rightarrow$ 35000

Then the HCF of 5, 50, 1500, 1750, 5000 and 35000 is 5.
So the HCF of the given numbers is 0.0005 (since there are four digits in all the adjusted (or equated) decimal places).

Exp. 2) Find the HCF of 0.9, 0.36 and 1.08.

Solution
$$\begin{array}{l} 0.9 \} \rightarrow 0.90 \} \rightarrow 90 \\ 0.36 \} \rightarrow 0.36 \} \rightarrow 36 \\ 1.08 \} \rightarrow 1.08 \} \rightarrow 108 \end{array}$$

Now the HCF of 90, 36 and 108 is 18. So the required HCF of the given numbers is 0.18. (Since there are two digits after decimal places in every number in second step)

LCM

Step 1. First of all equate the no. of places in all the given numbers by putting the minimum possible number of zeros at the end of the decimal numbers, wherever required.

Step 2. Now consider the equated numbers as integers and then find the LCM of these numbers.

Step 3. Put the decimal point in the LCM of the numbers as many places as that of in the equated numbers.

Exp. 1) Find the LCM of 1.8, 0.54 and 7.2

Solution
$$\begin{array}{l} 1.8 \} \rightarrow 1.80 \} \rightarrow 180 \\ 0.54 \} \rightarrow 0.54 \} \rightarrow 54 \\ 7.2 \} \rightarrow 7.20 \} \rightarrow 720 \end{array}$$

Now the LCM of 180, 54 and 720 is 2160
Therefore the required LCM is 21.60.

Exp. 2) Find the LCM of 4.44, 37 and 55.5

Solution
$$\begin{array}{l} 4.44 \} \rightarrow 4.44 \} \rightarrow 444 \\ 37.0 \} \rightarrow 37.00 \} \rightarrow 3700 \\ 55.5 \} \rightarrow 55.50 \} \rightarrow 5550 \end{array}$$

Now the LCM of 444, 3700, 5550 is 11100.
Hence the required LCM is 111.00 = 111.

Introductory Exercise 1.7

1. $-\frac{2}{3}, \frac{5}{2}, -\frac{1}{6}$ in descending order can be arranged as :

- (a) $\frac{5}{2}, -\frac{1}{6}, -\frac{2}{3}$ (b) $-\frac{2}{3}, \frac{5}{2}, -\frac{1}{6}$
(c) $-\frac{1}{6}, \frac{5}{2}, -\frac{2}{3}$ (d) $\frac{5}{2}, -\frac{2}{3}, -\frac{1}{6}$

2. Which one of the following represents the numbers $-\frac{3}{7}, \frac{2}{3}$ and $-\frac{1}{3}$ in descending order?

- (a) $-\frac{1}{3}, -\frac{3}{7}, \frac{2}{3}$ (b) $\frac{2}{3}, -\frac{1}{3}, -\frac{3}{7}$
(c) $-\frac{3}{7}, \frac{2}{3}, -\frac{1}{3}$ (d) $\frac{2}{3}, -\frac{3}{7}, -\frac{1}{3}$

3. Find the greatest among 1029/1025, 1030/1026, 256/255, 1023/1019

- (a) 1029/1025
(b) 1030/1026
(c) 256/255
(d) 1023/1019

4. The fundamental arithmetical operations on 2 recurring decimals can be performed directly without converting them to vulgar fractions :

- (a) only in addition and subtraction
(b) only in addition and multiplication
(c) only in addition, subtraction and multiplication
(d) in all the four arithmetical operations

5. Find the value of x and y in the given equation

$$5\frac{7}{x} \times y\frac{1}{13} = 12 :$$

- (a) -2, 9 (b) 9, 2
(c) 4, 3 (d) 9, 4

6. The square root of $32\frac{137}{484}$ is :

- (a) $5\frac{15}{22}$
(b) $15\frac{5}{32}$
(c) $5\frac{15}{28}$
(d) none of the above

7. $1 \div \frac{1}{1 \div \frac{1}{1 \div \frac{1}{3}}}$ is equal to :

- (a) $\frac{1}{3}$ (b) 1
(c) 3 (d) $1\frac{1}{3}$

8. The value of $\frac{4\frac{1}{7} - 2\frac{1}{4}}{3\frac{1}{2} + 1\frac{1}{7}} \div \frac{1}{2 + \frac{1}{2 + \frac{1}{5 - \frac{1}{5}}}}$:

- (a) 7/29 (b) 5/6
(c) 1 (d) 9/13

9. Mallika travelled $\frac{5}{16}$ of his journey by coach and $\frac{7}{20}$ by rail and then she walked the remaining 10 km. How far did she go altogether?

- (a) $27\frac{17}{29}$ (b) $33\frac{7}{27}$
(c) $19\frac{7}{27}$ (d) $29\frac{17}{27}$

10. For every positive integer x_n find the value of $\frac{x_1}{x_2}$

$$\text{if } x_1 + \frac{1}{x_2 + \frac{1}{x_3 + \frac{1}{x_4 + \dots}}} = \sqrt[3]{10}$$

- (a) $1/3$ (b) $3/10$
 (c) $2/5$ (d) none of these
11. Vishvamitra has some guavas of same shape, size and colour. He wants to distribute them equally among some of his chosen disciples. In order to do this he can cut a guava into uniform pieces only. Which of the following statements is/are false?
- (i) He can divide 7 guavas equally among his 12 disciples, such that no any guava is cut into more than 6 pieces.
 (ii) He can divide 6 guavas equally among his 12 disciples, such that no any guava is cut into more than 3 pieces.
 (iii) He can divide 11 guavas equally among his 15 disciples, such that no any guava is cut into more than 6 pieces.
 (iv) He can divide 7 guavas equally among his 15 disciples, such that no any guava is cut into more than 6 pieces.
- (a) (i) (b) (ii)
 (c) (iii) (d) (iv)

1.8 Indices and Surds

In the previous articles we have studied the multiplication, as $2 \times 2 = 4$, $2 \times 2 \times 2 = 8$, $2 \times 2 \times 2 \times 2 = 16$, ...

Now, here we extend our discussion to the higher power or index of a number, as following

$$\begin{aligned} \text{as } 2 \times 2 &= 2^2 = 4 \\ 2 \times 2 \times 2 &= 2^3 = 8 \\ 2 \times 2 \times 2 \times 2 &= 2^4 = 16 \\ 2 \times 2 \times 2 \times 2 \times 2 &= 2^5 = 32 \end{aligned}$$

Similarly, $2 \times 2 \times 2 \times \dots$ upto n times $= 2^n$

Thus $\underbrace{a \times a \times a \times a \times \dots a}_{n \text{ factors}} = a^n$

Where a is any number and n is a natural number. i.e., a^n is continued product of n equal quantities, each equal to a and is called the n th power of a . ' n ' is called the **index** or **exponent** and ' a ' is called the **base** of a^n .

Therefore a^n is the exponential expression. a^n is read as ' a raised to the power n ' or ' a to the power n '

12. The expression $33.33 \div 1.1$ simplifies as :

- (a) 33.3 (b) 303
 (c) 33.0 (d) 30.3

13. Which one of the following is not correct?

- (a) $\sqrt{0.4096} = 0.64$ (b) $\sqrt{40.96} = 6.4$
 (c) $\sqrt{0.04096} = 0.064$ (d) $\sqrt{4096} = 64$

14. The value of $\sqrt{900} + \sqrt{0.09} - \sqrt{0.000009}$ is :

- (a) 30.297 (b) 30.197
 (c) 30.097 (d) 30.397

15. The value of $\frac{10\sqrt{6.25}}{\sqrt{6.25} - 0.5}$ is :

- (a) 125 (b) 0.125
 (c) 1.25 (d) 12.5

16. Simplify $\frac{\sqrt{25.4016} - \sqrt{1.0609}}{\sqrt{25.4016} + \sqrt{1.0609}}$:

- (a) $\frac{401}{607}$ (b) $\frac{104}{706}$
 (c) $41/76$ (d) none of these

17. Let n be a positive integer. If $\frac{1}{n}$ has a terminating decimal expansion, then which one of the following is true?

- (a) n is of the form 5^x , where x is a positive integer
 (b) n is of the form 2^y , where y is non-positive integer
 (c) n is of the form $2^x \cdot 5^y$ for some non-negative integers x and y
 (d) n is of the form 10^z for some positive integer z

Laws of Indices

If m and n are positive integers, then

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$; ($a \neq 0, m \geq n$)
- $(ab)^n = a^n \cdot b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$; ($b \neq 0$)
- $(a^m)^n = a^{m \cdot n}$
- $(a^0) = 1$; ($a \neq 0$)

Some Important Results

- If $a^x = k$, then $a = (k)^{1/x}$
- If $a^{1/x} = k$, then $a = k^x$
- If $a^x = b^y$, then $a = (b)^{y/x}$ and $b = (a)^{x/y}$
- If $a^x = a^y$, then $x = y$, where $a \neq 0, 1$
- $a^{-n} = 1/a^n$ and $a^n = 1/a^{-n}$
- $a^{b^c} \neq (a^b)^c$; $b \neq c$
- HCF : The HCF of $(a^m - 1)$ and $(a^n - 1)$ is equal to the $(a^{\text{HCF of } m, n} - 1)$

Exp. 1) Solve the following :

(i) $(5)^3$ (ii) $(-6)^4$ (iii) $(-2)^5$ (iv) $(4)^3$ (v) $(-4)^3$

Solution (i) $5^3 = 5 \times 5 \times 5 = 125$

(ii) $(-6)^4 = (-6) \times (-6) \times (-6) \times (-6) = 1296$

(iii) $(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) = -32$

(iv) $(4)^3 = 4 \times 4 \times 4 = 64$

(v) $(-4)^3 = (-4) \times (-4) \times (-4) = -64$

Exp. 2) Solve the following expressions.

(i) $(-2)^3 \times 5^2$

(ii) $(-4)^3 \times 5^2 \times 7^0$

(iii) $(32)^{-1/5}$

(iv) $(243)^{4/5}$

(v) $(36)^{1/6}$

(vi) $\left(-\frac{1}{343}\right)^{-2/3}$

(vii) $3^{-3} + (-3)^3$

(viii) $(2^2 + 2^3 + 2^{-2} + 2^{-3})$

(ix) $2^{2x-1} = \frac{1}{8^{(x-3)}}$, then $x = ?$

(x) $4^{2x+1} = 8^{x+3}$ then $x = ?$

(xi) $3^{x-1} + 3^{x+1} = 90$, then $x = ?$

Solution (i) $(-2)^3 \times 5^2 = -8 \times 25 = -200$

(ii) $(-4)^3 \times 5^2 \times 7^0 = -64 \times 25 \times 1 = -1600$

(iii) $(32)^{-1/5} = (2^5)^{-1/5} = 2^{5 \times -\frac{1}{5}} = 2^{-1} = \frac{1}{2}$

(iv) $(243)^{4/5} = (3^5)^{4/5} = 3^{5 \times \frac{4}{5}} = 3^4 = 81$

(v) $(36)^{1/6} = (6^2)^{1/6} = 6^{2 \times \frac{1}{6}} = 6^{\frac{1}{3}} = \sqrt[3]{6}$

(vi) $\left(-\frac{1}{343}\right)^{-2/3} = \left(-\frac{1}{7^3}\right)^{-2/3} = (-7^{-3})^{-2/3} = (-7)^{-3 \times -2/3} = (-7)^2 = 49$

(vii) $3^{-3} + (-3)^3 = \frac{1}{3^3} + (-3)^3 = \frac{1}{27} - 27 = \frac{1-729}{27} = -\frac{728}{27}$

(viii) $(2^2 + 2^3 + 2^{-2} + 2^{-3}) = \left(4 + 8 + \frac{1}{4} + \frac{1}{8}\right) = 12 + \frac{3}{8} = \frac{99}{8}$

(ix) $2^{2x-1} = \frac{1}{8^{x-3}} \Rightarrow 2^{2x-1} = \frac{1}{2^{3(x-3)}}$

$\Rightarrow 2^{2x-1} = \frac{1}{2^{3x-9}}$

$\Rightarrow (2^{2x-1})(2^{3x-9}) = 1$

$\Rightarrow 2^{(2x-1)+(3x-9)} = 1$

$\Rightarrow 2^{5x-10} = 1 \Rightarrow 2^{5(x-2)} = 1$

$\Rightarrow 2^{5(x-2)} = 2^0$

$\Rightarrow 5(x-2) = 0$

$\Rightarrow x-2 = 0 \Rightarrow x = 2$

(x) $4^{2x+1} = 8^{x+3}$

$\Rightarrow (2^2)^{(2x+1)} = (2^3)^{(x+3)}$

$\Rightarrow 2^{4x+2} = 2^{3x+9}$

$\Rightarrow 4x+2 = 3x+9$ ($\because$ base in both side is same)

$\Rightarrow x = 7$

(xi) $3^{x-1} + 3^{x+1} = 90$

$\Rightarrow \frac{3^x}{3} + 3 \cdot 3^x = 90$

$\Rightarrow 3^x + 3^2 \cdot 3^x = 90 \times 3$

$\Rightarrow 3^x(1+3^2) = 270$

$\Rightarrow 3^x(10) = 270 \Rightarrow 3^x = 27$

$\Rightarrow 3^x = 3^3 \Rightarrow x = 3$

Exp. 3) Solve the followings :

(i) $\left[\left(x + \frac{1}{y}\right)^a \left(x - \frac{1}{y}\right)^b\right] \div \left[\left(y + \frac{1}{x}\right)^a \left(y - \frac{1}{x}\right)^b\right]$ is equal to

(a) $\left(\frac{x}{y}\right)^{a+b}$ (b) $\left(\frac{y}{x}\right)^{(a+b)}$ (c) $\frac{x^a}{y^b}$ (d) $(xy)^{a+b}$

(ii) $\left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b \times \left(\frac{x^a}{x^b}\right)^c$ is equal to :

(a) 0 (b) 1
(c) abc (d) none of these

(iii) If $x^{3/2} = (x^{3/2})^x$, then the value of x is :

(a) $\frac{3}{2}$ (b) $\frac{9}{4}$ (c) $\frac{16}{25}$ (d) $\frac{8}{27}$

(iv) If $x^a = y^b = z^c$ and $y^2 = zx$ then the value of $\frac{1}{a} + \frac{1}{c}$ is :

(a) $\frac{b}{2}$ (b) $\frac{c}{2}$ (c) $\frac{2}{b}$ (d) $2a$

(v) $(a^{m-n})^l \times (a^{n-l})^m \times (a^{l-m})^n$

(a) 1 (b) 0 (c) 2 (d) a^{lmn}

(vi) If $a^x = b$, $b^y = c$ and $c^z = a$, then the value of xyz is

(a) 0 (b) 1 (c) $x+y+z$ (d) abc

(vii) $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$ is equal to :

(a) 1 (b) 0
(c) 2 (d) none of these

(viii) The value of

$\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{c-b}+x^{a-b}} + \frac{1}{1+x^{a-c}+x^{b-c}}$ is :

(a) 0 (b) x^{abc} (c) 1 (d) $x^{(a+b+c)}$

(ix) $2^{2^x} = 16^{2^3 x}$, then x is equal to

(a) -1 (b) 0
(c) 1 (d) none of these

(x) The value of the expression

$$\frac{4^n \times 20^{m-1} \times 12^{m-n} \times 15^{m+n-2}}{16^m \times 5^{2m+n} \times 9^{m-1}} \text{ is :}$$

- (a) 500 (b) 1 (c) 200 (d) $\frac{1}{500}$

(xi) The value of the expression

$$\frac{(0.3)^{1/3} \cdot \left(\frac{1}{27}\right)^{1/4} \cdot (9)^{1/6} \cdot (0.81)^{2/3}}{(0.9)^{2/3} \cdot (3)^{-1/2} \cdot \left(\frac{1}{3}\right)^{-2} \cdot (243)^{-1/4}} \text{ is :}$$

- (a) 0.3 (b) 0.9 (c) 1.27 (d) 0.09

(xii) The value of expression $\frac{(0.6)^0 - (0.1)^{-1}}{\left(\frac{3}{2^3}\right)^{-1} \cdot \left(\frac{3}{2}\right)^3 + \left(-\frac{1}{3}\right)^{-1}}$ is :

- (a) $-\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{9}{4}$

(xiii) If $\frac{9^n \times 3^2 \times (3^{-n/2})^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$

then the value of $(m-n)$ is :

- (a) -1 (b) 1 (c) 2 (d) -2

Solution (i) $\left[\left(x + \frac{1}{y} \right)^a \left(x - \frac{1}{y} \right)^b \right] \div \left[\left(y + \frac{1}{x} \right)^a \left(y - \frac{1}{x} \right)^b \right]$

$$= \frac{\left(x + \frac{1}{y} \right)^a \left(x - \frac{1}{y} \right)^b}{\left(y + \frac{1}{x} \right)^a \left(y - \frac{1}{x} \right)^b} = \frac{\left(\frac{xy+1}{y} \right)^a \left(\frac{xy-1}{y} \right)^b}{\left(\frac{xy+1}{x} \right)^a \left(\frac{xy-1}{x} \right)^b}$$

$$= \frac{(xy+1)^a \cdot (xy-1)^b}{y^a \cdot y^b} \cdot \frac{x^a \cdot x^b}{(xy+1)^a \cdot (xy-1)^b}$$

$$= \frac{(xy+1)^a (xy-1)^b}{y^{a+b}} \times \frac{x^{a+b}}{(xy+1)^a (xy-1)^b}$$

$$= \frac{x^{a+b}}{y^{a+b}} = \left(\frac{x}{y} \right)^{a+b}$$

Hence (a) is the correct option.

$$\begin{aligned} \text{(ii)} \quad \left(\frac{x^b}{x^c} \right)^a \times \left(\frac{x^c}{x^a} \right)^b \times \left(\frac{x^a}{x^b} \right)^c &= (x^{b-c})^a \times (x^{c-a})^b \times (x^{a-b})^c \\ &= x^{(ab-ac)} \times x^{(bc-ab)} \times x^{(ac-bc)} \\ &= x^{(ab-ac) + (bc-ab) + (ac-bc)} = x^0 = 1 \end{aligned}$$

Hence (b) is the correct option.

$$\begin{aligned} \text{(iii)} \quad x^{x^{3/2}} &= (x^{3/2})^x \Rightarrow x^{x^{3/2}} = x^{(3/2)x} \\ \Rightarrow x^{3/2} &= \frac{3}{2}x \Rightarrow x^{1/2} = \frac{3}{2} \Rightarrow x = \frac{9}{4} \end{aligned}$$

Hence (b) is the correct option.

(iv) If $x^a = y^b = z^c$ and $y^2 = zx$

$$\text{Let } x^a = y^b = z^c = k$$

$$\Rightarrow x = k^{1/a}, y = k^{1/b}, z = k^{1/c}$$

$$\text{Now, } \therefore y^2 = zx$$

$$\therefore (k^{1/b})^2 = (k^{1/c}) \cdot (k^{1/a})$$

$$\Rightarrow k^{2/b} = k^{\frac{1}{c} + \frac{1}{a}}$$

$$\Rightarrow \frac{1}{a} + \frac{1}{c} = \frac{2}{b}$$

Hence (c) is the correct option.

$$\begin{aligned} \text{(v)} \quad (a^{m-n})^l \times (a^{n-l})^m \times (a^{l-m})^n &= a^{ml-nl} \times a^{nm-lm} \times a^{ln-mn} \\ &= a^{ml-nl+nm-lm+ln-mn} \\ &= a^0 = 1 \end{aligned}$$

Thus (a) is correct option.

(vi) If $a^x = b$, $b^y = c$, $c^z = a$

$$\therefore a^x = b \therefore (c^z)^x = b$$

$$\Rightarrow c^{zx} = b \Rightarrow (b^y)^{zx} = b$$

$$\Rightarrow b^{xyz} = b \Rightarrow xyz = 1$$

Hence (b) is the correct option.

$$\begin{aligned} \text{(vii)} \quad \left(\frac{x^a}{x^b} \right)^{a+b} \times \left(\frac{x^b}{x^c} \right)^{b+c} \times \left(\frac{x^c}{x^a} \right)^{c+a} \\ &= (x^{a-b})^{(a+b)} \times (x^{b-c})^{(b+c)} \times (x^{c-a})^{(c+a)} \\ &= x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)} \\ &= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2} \\ &= x^{a^2-b^2+b^2-c^2+c^2-a^2} = x^0 = 1 \end{aligned}$$

Thus (a) is the correct option.

$$\begin{aligned} \text{(viii)} \quad \frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{c-b}+x^{a-b}} + \frac{1}{1+x^{a-c}+x^{b-c}} \\ &= \frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \frac{1}{1+\frac{x^c}{x^b}+\frac{x^a}{x^b}} + \frac{1}{1+\frac{x^a}{x^c}+\frac{x^b}{x^c}} \\ &= \frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^b+x^c+x^a} + \frac{x^c}{x^c+x^a+x^b} \\ &= \frac{x^a+x^b+x^c}{x^a+x^b+x^c} = 1 \end{aligned}$$

Hence (c) is correct option.

$$\begin{aligned} \text{(ix)} \quad 2^{2^x} &= 16^{2^{3x}} \\ \Rightarrow 2^{2^x} &= (2^4)^{2^{3x}} \\ \Rightarrow 2^{2^x} &= 2^4 \cdot 2^{3x} = 2^{3x+2} \\ \Rightarrow 2^{2^x} &= 2^{3x+2} \quad [\text{Since base in both sides is equal}] \\ \Rightarrow 2^x &= 2^{3x+2} \\ \Rightarrow x &= 3x+2 \\ \Rightarrow 2x &= -2 \Rightarrow x = -1 \end{aligned}$$

Hence (a) is correct option.

$$\begin{aligned}
 \text{(x)} \quad & \frac{4^n \times 20^{m-1} \times 12^{m-n} \times 15^{m+n-2}}{16^m \times 5^{2m+n} \times 9^{m-1}} \\
 &= \frac{2^{2n} \times 2^{2m-2} \times 5^{m-1} \times 2^{2m-2n} \times 3^{m-n}}{2^{4m} \times 5^{2m+n} \times 3^{2m-2}} \\
 &= \frac{2^{2n+2m-2+2m-2n-4m} \times 3^{m-n+m+n-2-2m+2}}{5^{m-1+m+n-2-2m-n}} \\
 &= 2^{-2} \times 3^0 \times 5^{-3} = \frac{1}{4} \times \frac{1}{125} = \frac{1}{500}
 \end{aligned}$$

Hence (d) is the correct option.

$$\begin{aligned}
 \text{(xi)} \quad & \frac{(0.3)^{1/3} \cdot \left(\frac{1}{27}\right)^{1/4} \cdot (9)^{1/6} \cdot (0.81)^{2/3}}{(0.9)^{2/3} \cdot (3)^{-1/2} \cdot \left(\frac{1}{3}\right)^{-2} \cdot (243)^{-1/4}} \\
 &= \frac{\left(\frac{3}{10}\right)^{1/3} \left(\frac{1}{3}\right)^{3/4} (3)^{1/3} \cdot \left(\frac{81}{100}\right)^{2/3}}{\left(\frac{9}{10}\right)^{2/3} \left(\frac{1}{3}\right)^{1/2} \cdot (3)^2 (3)^{-5/4}} \\
 &= \frac{3^{\frac{1}{3} + \frac{1}{3} - \frac{3}{4} + \frac{8}{3}} \times 10^{-\frac{1}{3} - \frac{4}{3}}}{3^{\frac{4}{3} + 2 - \frac{1}{2} - \frac{5}{4}} \times 10^{-\frac{2}{3}}} \\
 &= \frac{3^{31/12} \times 10^{-5/3}}{3^{19/12} \times 10^{-2/3}} = 3 \times 10^{-1} = \frac{3}{10} = 0.3
 \end{aligned}$$

$$\begin{aligned}
 \text{(xii)} \quad & \frac{(0.6)^0 - (0.1)^{-1}}{\left(\frac{3}{2^3}\right)^{-1} \cdot \left(\frac{3}{2}\right)^3 + \left(-\frac{1}{3}\right)^{-1}} \\
 &= \frac{1 - \left(\frac{1}{10}\right)^{-1}}{(3^{-1})(2^3) \times 3^3 \times 2^{-3} + (-3)^1} \\
 &= \frac{1 - 10}{3^{-3-1} \cdot 2^0 - 3} = \frac{-9}{9 - 3} = \frac{-9}{6} = -\frac{3}{2}
 \end{aligned}$$

Hence (a) is the correct option.

$$\begin{aligned}
 \text{(xiii)} \quad & \frac{9^n \times 3^2 \times (3^{-n/2})^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27} \\
 \Rightarrow & \frac{3^{2n} \times 3^2 \times 3^n - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{3^3} \\
 \Rightarrow & \frac{3^{2n+2+n} - 3^{3n}}{8 \times 3^{3m}} = \frac{1}{3^3} \\
 \Rightarrow & \frac{3^{3n+2} - 3^{3n}}{3^{3m} \times 8} = 3^{-3} \\
 \Rightarrow & \frac{3^{3n}(3^2 - 1)}{3^{3m} \times 8} = 3^{-3} \\
 \Rightarrow & 3^{3n-3m} = 3^{-3} \Rightarrow 3n - 3m = -3 \Rightarrow m - n = 1
 \end{aligned}$$

Hence (b) is the correct option.

Exp. 4) Find the HCF of $(2^{315} - 1)$ and $(2^{25} - 1)$ is

- (a) 5
- (b) 2
- (c) 31
- (d) none of the above

Solution The required HCF = $(2)^{\text{HCF of } 315, 25} - 1 = 2^5 - 1 = 31$

Hence (c) is the correct option.

Surds

In the previous articles we have studied about the square roots and cube roots etc. There we have studied those numbers whose roots can be found in the form of an integer, but here we study only those numbers whose roots are not the integers.

Surds : When a root of a rational number (*i.e.*, quantities of the type $\sqrt[n]{a}$, a being a rational number) can not be exactly obtained, then this root is called a **surd**.

For Example $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{10}$, etc.

$$\sqrt[3]{2}, \sqrt[3]{3}, \sqrt[3]{4}, \sqrt[3]{5}, \sqrt[3]{6}, \sqrt[3]{7}, \sqrt[3]{9}, \sqrt[3]{10}, \dots \text{etc}$$

$$\sqrt[4]{2}, \sqrt[4]{3}, \sqrt[4]{4}, \sqrt[4]{5}, \sqrt[4]{6}, \sqrt[4]{7}, \dots \text{etc}$$

But $\sqrt{4} = 2, \sqrt[3]{27} = 3, \sqrt[4]{16} = 2, \sqrt[5]{32} = 2, \sqrt[6]{729} = 3, \dots$ etc are not the surds.

Thus it can be said that “all surds are irrational numbers, but all irrational numbers are not the surds”.

For example π and e are irrational, but not surds.

Order of Surds : In a surd $\sqrt[n]{a} [= (a)^{1/n}]$, the value of n is called the order of the surd and a is called the **radicand**.

The surds of second order are called quadratic surds *e.g.*, $\sqrt{5}, \sqrt{6}, (7)^{3/2}$. The surds of third order are called cubic surds *e.g.*, $\sqrt[3]{3}, \sqrt[3]{7}, (6)^{5/3}, (9)^{2/3}$. The surds of fourth order are called biquadratic surds *e.g.*, $\sqrt[4]{2}, \sqrt[4]{3}, \sqrt[4]{7}, (5)^{7/4}$.

Mixed Surds : The product of a rational number with a surd is known as a **mixed surd**.

For example, $3 \cdot \sqrt{2}, 4\sqrt{7}, 5 \cdot \sqrt[3]{6}, \sqrt{18} (= 3\sqrt{2})$

Pure Surd : A surd having no rational factor is known as pure surd.

e.g., $\sqrt[4]{2}, \sqrt[3]{7}, \sqrt[5]{8}$ etc.

Similar Surds : If the radicands of two or more rationalised surds are same, then these surds are called the **similar surds**. For example, $\sqrt{5}, 3\sqrt{5}, \sqrt{20} (= 2\sqrt{5})$ and $\sqrt{80} (= 4\sqrt{5})$ etc.

Conjugate or Complementary Surds : For every surd of the form $\sqrt{a} + \sqrt{b}$ there exists another surd $\sqrt{a} - \sqrt{b}$, which is called the conjugate surd of previous surd and *vice-versa*.

For example,

$\{(2\sqrt{3} + 5\sqrt{3}) \text{ and } (2\sqrt{3} - 5\sqrt{3})\}$, $\{(\sqrt{7} + \sqrt{8}) \text{ and } (\sqrt{7} - \sqrt{8})\}$, ... etc are the pairs of conjugate surds.

Properties of Surds

1. A quadratic surd cannot be equal to the sum or difference of a rational number and a quadratic surd e.g., $\sqrt{a} + b \neq \sqrt{c}$ or $\sqrt{a} - b \neq \sqrt{c}$, where $\sqrt{a}$, $\sqrt{c}$ are quadratic surds and b is a rational number.
2. The product and quotient of two dissimilar quadratic surds can not be rational e.g., $\sqrt{2} \cdot \sqrt{5} = \sqrt{10}$ or $\frac{\sqrt{6}}{\sqrt{2}} = \sqrt{3}$
3. If $a + \sqrt{b} = c + \sqrt{d}$ or $a - \sqrt{b} = c - \sqrt{d}$, then $a = c$ and $b = d$.
4. If $a \pm \sqrt{b} = 0$ then $a = b = 0$
5. If $a + \sqrt{b} = c + \sqrt{d}$, then $a - \sqrt{b} = c - \sqrt{d}$ and its vice-versa.
6. If $\sqrt{a + \sqrt{b}} = \sqrt{c} + \sqrt{d}$, then $\sqrt{a - \sqrt{b}} = \sqrt{c} - \sqrt{d}$ and its vice-versa.

Addition : Only similar surds can be simplified

$$\text{e.g., } 2\sqrt{3} + 7\sqrt{3} = 9\sqrt{3}$$

$$\text{and } 8\sqrt{5} + 3\sqrt{5} + \sqrt{20} = 8\sqrt{5} + 3\sqrt{5} + 2\sqrt{5} = 13\sqrt{5}$$

Subtraction : Only similar surds can be simplified

$$\text{e.g., } 7\sqrt{3} - 2\sqrt{3} = 5\sqrt{3}$$

$$\text{and } 8\sqrt{5} - 3\sqrt{5} = 5\sqrt{5} \text{ etc.}$$

NOTE $8\sqrt{5} \pm 2\sqrt{6}$ can not be simplified, because radicands are different.

Multiplication : $3^{1/4} \times 6^{1/2} \times 9^{1/3}$

$$= 3^{3/12} \times 6^{6/12} \times 9^{4/12}$$

$$= (3^3 \times 6^6 \times 9^4)^{1/12}$$

$$= (3^3 \times 2^6 \times 3^6 \times 3^8)^{1/12}$$

$$= (3^{17} \times 2^6)^{1/12}$$

$$\text{Division : } 25 \div \sqrt{5} = \frac{25}{\sqrt{5}} = \frac{(5)^2}{(5)^{1/2}} = 5^{2-\frac{1}{2}} = 5^{3/2} = \sqrt{125}$$

Rationalisation : Making two or more surds a rational number by multiplication, is called the rationalisation

$$\text{e.g., } 3\sqrt{2} \times 4\sqrt{2} = 24$$

$$7\sqrt{5} \times \sqrt{5} = 35$$

$$(9\sqrt{3} - 5\sqrt{3})(9\sqrt{3} + 5\sqrt{3}) = (9\sqrt{3})^2 - (5\sqrt{3})^2 = 168$$

Laws of Surds

$$(i) \sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b} \quad (ii) \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$(iii) (\sqrt[n]{a})^m = \sqrt[n]{a^m} \quad (iv) (\sqrt[n]{a})^n = (a^{1/n})^n = a$$

$$(v) \sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}$$

Exp. 1) Which one is smaller out of $\sqrt[3]{2}$ and $\sqrt[4]{3}$?

$$\text{Solution } \sqrt[3]{2} = (2)^{1/3} = 2^{4/12} = (16)^{1/12}$$

$$\text{and } \sqrt[4]{3} = (3)^{1/4} = 3^{3/12} = (27)^{1/12}$$

(Taking the LCM of surd)

$$\text{Hence } \sqrt[3]{2} < \sqrt[4]{3}$$

Exp. 2) Which one is greatest out of $\sqrt[3]{5}$, $\sqrt[4]{3}$, $\sqrt[5]{4}$?

$$\text{Solution } \sqrt[3]{5} = (5)^{1/3} = (5)^{20/60} = (5^{20})^{1/60} = \sqrt[60]{5^{20}}$$

$$\sqrt[4]{3} = (3)^{1/4} = (3)^{15/60} = (3^{15})^{1/60} = \sqrt[60]{3^{15}}$$

$$\sqrt[5]{4} = (4)^{1/5} = (4)^{12/60} = (4^{12})^{1/60} = \sqrt[60]{4^{12}}$$

Obviously $\sqrt[3]{5}$ is the greatest.

Exp. 3) If $p = \sqrt[3]{5}$, $q = \sqrt[4]{3}$, $r = \sqrt[5]{4}$ then the correct relationship is :

$$(a) p > q > r$$

$$(b) q < r < p$$

$$(c) r > p > q$$

$$(d) \text{ can't be determined}$$

Solution In the previous question we have found that p is the greatest.

$$\text{Again } q = \sqrt[60]{3^{15}} \text{ and } r = \sqrt[60]{4^{12}}$$

$$\text{Now } 3^{15} \geq 4^{12}$$

[Here the sign $\geq$ signifies the trichotomy- relation i.e., if $x \geq y$, then either $x < y$ or $x = y$ or $x > y$]

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

Thus

Hence $p > r > q$, so (b) is the correct option.

Exp. 4) If $5\sqrt{5} \times 5^3 \div 5^{-3/2} = 5^{a+2}$ then the value of a is :

$$(a) 4$$

$$(b) 5$$

$$(c) 9$$

$$(d) 16$$

$$\text{Solution } \frac{5\sqrt{5} \times 5^3}{5^{-3/2}} = 5^{a+2}$$

$$\Rightarrow \frac{5^{4+\frac{1}{2}}}{5^{-\frac{3}{2}}} = 5^{a+2} \Rightarrow 5^{4+\frac{1}{2}+\frac{3}{2}} = 5^{a+2}$$

$$\Rightarrow 5^6 = 5^{a+2}$$

$$\Rightarrow 6 = a + 2 \Rightarrow a = 4$$

Hence (a) is the correct option.

Exp. 5) Find the value of $\frac{2 + \sqrt{3}}{2 - \sqrt{3}}$.

Solution $\frac{2 + \sqrt{3}}{2 - \sqrt{3}} = \frac{2 + \sqrt{3}}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$
 (Multiplying numerator and denominator by conjugate)

$$= \frac{(2 + \sqrt{3})^2}{4 - 3} = \frac{4 + 3 + 4\sqrt{3}}{1}$$

$$= 7 + 4\sqrt{3}$$

Exp. 6) The value of $\frac{\sqrt{5} - 2}{\sqrt{5} + 2}$ is :

- (a) $4 - 9\sqrt{5}$ (b) $9 + 4\sqrt{5}$
 (c) $9 - 4\sqrt{5}$ (d) $7 + 4\sqrt{5}$

Solution $\frac{\sqrt{5} - 2}{\sqrt{5} + 2} = \frac{(\sqrt{5} - 2)}{(\sqrt{5} + 2)} \times \frac{(\sqrt{5} - 2)}{(\sqrt{5} - 2)}$

$$= \frac{(\sqrt{5} - 2)^2}{5 - 4} = \frac{5 + 4 - 4\sqrt{5}}{1} = 9 - 4\sqrt{5}$$

Thus (c) is the correct option.

Exp. 7) Find the value of $\frac{5}{6\sqrt{6}} \times \frac{12\sqrt{30}}{25\sqrt{5}}$.

Solution $\frac{5}{6\sqrt{6}} \times \frac{12\sqrt{30}}{25\sqrt{5}} = \frac{5}{6\sqrt{6}} \times \frac{12\sqrt{6} \cdot \sqrt{5}}{25\sqrt{5}} = \frac{2}{5}$

Exp. 8) Arrange the following in descending order

$$\sqrt{3} - \sqrt{2}, \sqrt{4} - \sqrt{3}, \sqrt{5} - \sqrt{4}, \sqrt{2} - 1.$$

Solution $\sqrt{3} - \sqrt{2} = \frac{\sqrt{3} - \sqrt{2}}{1} \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}}$

$$= \frac{3 - 2}{\sqrt{3} + \sqrt{2}} = \frac{1}{\sqrt{3} + \sqrt{2}}$$

Similarly, $\sqrt{4} - \sqrt{3} = \frac{1}{\sqrt{4} + \sqrt{3}}, \sqrt{5} - \sqrt{4} = \frac{1}{\sqrt{5} + \sqrt{4}}$

and $\sqrt{2} - 1 = \frac{1}{\sqrt{2} + 1}$

As we know, if the numerator is same then the fraction whose denominator is larger the fraction will be lower.

Hence the correct order of descending number is

$$(\sqrt{2} - 1) > (\sqrt{3} - \sqrt{2}) > (\sqrt{4} - \sqrt{3}) > (\sqrt{5} - \sqrt{4})$$

Exp. 9) $\sqrt[r]{\frac{9^{\left(r + \frac{1}{4}\right)} \sqrt{3 \cdot 3^{-r}}}{3 \cdot \sqrt{3^{-r}}}} = k$, then the value of k is

- (a) 3 (b) 3^2 (c) 3^3 (d) $\sqrt[3]{3}$

Solution $\sqrt[r]{\frac{3^{2r + \frac{1}{2}} \cdot 3^{\left(\frac{1-r}{2}\right)}}{3^{1 - \frac{r}{2}}}} = \sqrt[r]{\frac{3^{\frac{4r + 1 + 1 - r}{2}}}{3^{\frac{2-r}{2}}}} = \sqrt[r]{3^{\left(\frac{3r+2}{2}\right) - \left(\frac{2-r}{2}\right)}}$

$$= \sqrt[r]{3^{2r}} = 3^{\frac{2r}{r}} = 3^2$$

Hence (b) is the correct option.

Exp. 10) If m, n are the positive integers ($n > 1$) such that $m^n = 121$, then value of $(m - 1)^{n+1}$ is :

- (a) 12321 (b) 1
 (c) 1000 (d) 11

Solution $\because m^n = 121$ and $n > 1$

$$\therefore (11)^2 = 121$$

$$\Rightarrow m = 11 \text{ and } n = 2$$

$$\text{Hence } (m - 1)^{n+1} = (10)^3 = 1000$$

Thus (c) is the correct option.

Exp. 11) Simplify : $\sqrt{192} - \frac{1}{2}\sqrt{48} - \sqrt{75}$

Solution $\sqrt{192} - \frac{1}{2}\sqrt{48} - \sqrt{75} = 8\sqrt{3} - \frac{4}{2}\sqrt{3} - 5\sqrt{3}$

$$= 8\sqrt{3} - 2\sqrt{3} - 5\sqrt{3} = \sqrt{3}$$

Exp. 12) Find the square root of $7 - 2\sqrt{10}$.

Solution $7 - 2\sqrt{10} = 5 + 2 - 2\sqrt{5} \times 2$

$$\Rightarrow 7 - 2\sqrt{10} = (\sqrt{5})^2 + (\sqrt{2})^2 - 2\sqrt{5} \cdot \sqrt{2}$$

$$\Rightarrow 7 - 2\sqrt{10} = (\sqrt{5} - \sqrt{2})^2$$

$$\text{Thus the } \sqrt{7 - 2\sqrt{10}} = \pm (\sqrt{5} - \sqrt{2})$$

Exp. 13) Find the value of $\left(\frac{1}{\sqrt{5} - 2} + \frac{1}{\sqrt{5} + 2}\right)^2$.

Solution $\left(\frac{1}{\sqrt{5} - 2} + \frac{1}{\sqrt{5} + 2}\right)^2 = \left[\frac{\sqrt{5} + 2 + \sqrt{5} - 2}{(\sqrt{5} - 2)(\sqrt{5} + 2)}\right]^2$

$$= \left[\frac{2\sqrt{5}}{(\sqrt{5})^2 - (2)^2}\right]^2 = \frac{4 \times 5}{5 - 4} = \frac{20}{1} = 20$$

Introductory Exercise 1.8

- In the equation $4^{x+2} = 2^{x+3} + 48$, the value of x will be
 (a) $-\frac{3}{2}$ (b) -2
 (c) -3 (d) 1
- Consider the following statements :
Assertion (A) $a^0 = 1, a \neq 0$
Reason (R) $a^m \div a^n = a^{m-n}$, m, n being integers.
 Of these statements :
 (a) both A and R are true and R is the correct explanation of A
 (b) both A and R are true and R is not the correct explanation of A
 (c) A is true, but R is false
 (d) A is false, but R is true
- If $3^n = 27$ then 3^{n-2} is :
 (a) 3 (b) $1/3$
 (c) $1/9$ (d) 9
- If $4^{x+3} \times 2^{x-3} - 128 = 0$ then the value of x is :
 (a) $\frac{1}{3}$ (b) $\frac{2}{3}$
 (c) $\frac{3}{3}$ (d) $\frac{4}{3}$
- If $a^m \cdot a^n = a^{mn}$, then $m(n-2) + n(m-2)$ is :
 (a) 0 (b) 1
 (c) -1 (d) $\frac{1}{2}$
- $\frac{6^6 + 6^6 + 6^6 + 6^6 + 6^6 + 6^6}{3^6 + 3^6 + 3^6} \div \frac{4^6 + 4^6 + 4^6 + 4^6}{2^6 + 2^6} = 2^n$,
 then the value of n is :
 (a) -1 (b) 0
 (c) $\frac{1}{2}$ (d) 1
- If $x^y = y^x$ and $y = 2x$, then x is equal to :
 (a) 2 (b) -2
 (c) 1 (d) -1
- If $2^{x+3} \cdot 4^{2x-5} = 2^{3x+7}$, then the value of x is :
 (a) 3 (b) 4
 (c) 6 (d) 7
- $9^{3/2} + (243)^{-2/3}$ simplifies to :
 (a) $3^{10/3}$ (b) $3^{19/3}$
 (c) $3^{1/3}$ (d) 3^{19}
- The solution of $(25)^{x-2} = (125)^{2x-4}$:
 (a) $3/4$ (b) 0
 (c) 2 (d) -2
- The value of
 $\left(\frac{x^a}{x^b}\right)^{(a^2+ab+b^2)} \left(\frac{x^b}{x^c}\right)^{(b^2+bc+c^2)} \left(\frac{x^c}{x^a}\right)^{(c^2+ca+a^2)}$ is :
 (a) -1 (b) x^{abc}
 (c) 1 (d) $x^{(a+b+c)}$
- If $a^x = b^y = c^z$ and $\frac{b}{a} = \frac{c}{b}$, then $\frac{2z}{x+z}$ is equal to :
 (a) $\frac{y}{x}$ (b) $\frac{x}{y}$
 (c) $\frac{x}{2}$ (d) $\frac{z}{x}$
- If $a^{1/m} = b^{1/n} = c^{1/p}$ and $abc = 1$, then $m+n+p$ is equal to :
 (a) 0 (b) 2
 (c) 1 (d) -2
- Value of $[(x^l)^{1-\frac{1}{l}}]^{l-1}$ is :
 (a) x (b) 1
 (c) x^{-1} (d) x^{-l}
- Which one of the following sets of surds is in correct sequence of ascending order of their values?
 (a) $\sqrt[4]{10}, \sqrt[3]{6}, \sqrt{3}$ (b) $\sqrt{3}, \sqrt[4]{10}, \sqrt[3]{6}$
 (c) $\sqrt{3}, \sqrt[3]{6}, \sqrt[4]{10}$ (d) $\sqrt[4]{10}, \sqrt{3}, \sqrt[3]{6}$
- If $x = \frac{1}{\sqrt{2}-1}$, then the value of $x^2 - 6 + \frac{1}{x^2}$ is :
 (a) 0 (b) 1
 (c) $\sqrt{2}$ (d) can't be determined
- The value of $\frac{3}{5\sqrt[4]{27}}$, when its denominator is a rational number :
 (a) $\frac{\sqrt[3]{3}}{6}$ (b) $\frac{\sqrt[4]{3}}{5}$
 (c) $\frac{\sqrt[3]{4}}{5}$ (d) none of these
- The value of denominator when $\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{18}+\sqrt{48}}$ is rationalised :
 (a) 15 (b) 25 (c) 30 (d) 50
- If $\sqrt{5+\sqrt[3]{x}} = 3$, then the value of x is :
 (a) 9 (b) 27
 (c) 64 (d) 343
- If $A = \sqrt{2}$, $B = \sqrt[3]{3}$ and $C = \sqrt[4]{4}$ then which of the relation is correct?
 (a) $A > B > C$ (b) $A > B = C$
 (c) $A = C < B$ (d) none of these

1.9 Factorials, Last Digits and Remainders

Factorial

The product of n consecutive natural numbers (or positive integers) starting from 1 to n is called as the factorial ' n '.

$$\text{i.e., } n! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \dots (n-2)(n-1)n$$

$$\text{So, the } 4! = 1 \times 2 \times 3 \times 4 = 4 \times 3 \times 2 \times 1$$

$$5! = 1 \times 2 \times 3 \times 4 \times 5 = 5 \times 4 \times 3 \times 2 \times 1$$

$$6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

NOTE 1. Factorial n is written as " $n!$ ". 2. $0! = 1$ and $1! = 1$

Properties

1. $n!$ is always an even number if $n \geq 2$

2. $n!$ always ends with zero if $n \geq 5$

Exp. 1) If $m^n - m = (m-n)!$ where $m > n > 1$ and $m = n^2$, then the value of $m^2 + n^2$ is :

- (a) 272 (b) 90
(c) 20 (d) none of (a), (b), (c)

Solution Let us consider $n = 2$ ($\because n > 1$)

$$\text{Then } m^n - m = 4^2 - 4 = 12 \quad (\because m = n^2)$$

$$\text{and } (m-n)! = (4-2)! = 2! = 2$$

Hence it is impossible.

Again consider $n = 3$, then

$$m^n - m = 9^3 - 9 = 720$$

$$\text{and } (m-n)! = (9-3)! = 6! = 720$$

Thus we get $n = 3$ and $m = 9$ the probable values

$$\text{Now } m^2 + n^2 = 9^2 + 3^2 = 81 + 9 = 90$$

Hence (b) is the correct option.

Exp. 2) If $P + P! = P^3$, then the value of P is :

- (a) 4 (b) 6 (c) 0 (d) 5

Solution Consider $P = 5$, then

$$5 + 5! = 5^3$$

$$5 + 120 = 125$$

$$125 = 125$$

Thus (d) is correct option.

Exp. 3) If $P + P! = P^2$, then the value of P is :

- (a) 5 (b) 7 (c) 3 (d) 0

Solution Consider $P = 3$ then

$$3 + 3! = 3^2;$$

$$\Rightarrow 3 + 6 = 9 \Rightarrow 9 = 9$$

Hence (c) is the correct option.

Exp. 4) If $n! - n = n$, then the value of n is :

- (a) 4 (b) 5 (c) 6 (d) 3

Solution Consider $n = 3$ then

$$3! - 3 = 3$$

$$\Rightarrow 6 - 3 = 3$$

$$\Rightarrow 3 = 3$$

Hence (d) is the correct answer.

Exp. 5) The appropriate value of P for the relation $(P! + 1) = (P + 1)^2$ is :

- (a) 3 (b) 4
(c) 5 (d) none of these

Solution Let us consider $P = 4$ then

$$4! + 1 = (4 + 1)^2$$

$$\Rightarrow 24 + 1 = 5^2$$

$$\Rightarrow 25 = 25$$

Hence (b) is the correct option.

Exp. 6) The value of $8! \div 4!$ is :

- (a) 4 (b) 16 (c) 2 (d) 1680

$$\text{Solution } \frac{8!}{4!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1}$$

$$= 8 \cdot 7 \cdot 6 \cdot 5 = 1680$$

Hence (d) is the correct option.

Exp. 7) If $n! = \frac{(n+4)!}{(n+1)!}$, then the value of n is :

- (a) 5 (b) 18 (c) 6 (d) 9

$$\text{Solution } n! = \frac{(n+4)!}{(n+1)!}$$

$$\Rightarrow n! = \frac{1 \cdot 2 \cdot 3 \dots n \cdot (n+1) \cdot (n+2) \cdot (n+3) \cdot (n+4)}{1 \cdot 2 \cdot 3 \dots n \cdot (n+1)}$$

$$\Rightarrow n! = (n+2)(n+3)(n+4)$$

$$n! = (n+2)(n+3)(n+4)$$

$$\text{now } 5! \neq 7 \times 8 \times 9$$

$$\text{and } 18! \neq 20 \times 21 \times 22$$

$$\text{and } 9! \neq 11 \times 12 \times 13$$

$$\text{but } 6! = 8 \times 9 \times 10$$

$$720 = 720$$

Hence $n = 6$ is the correct answer, thus (c) is the right choice.

Exp. 8) The value of

$$(1.2.3\dots 9) \cdot (11.12.13\dots 19) \cdot (21.22.23\dots 29) \cdot (31.32.33\dots 39) \cdot$$

$$(41.42.43\dots 49) \cdot (51.52.53\dots 59) \dots (91.92\dots 99)$$

$$(a) \frac{100!}{36288 \times 10^{11}} \quad (b) \frac{99!}{388 \times 10^{11}}$$

$$(c) \frac{99!}{36288 \times 10^{10}} \quad (d) \text{ can't be determined}$$

Solution $(1.2.3\dots 9) \cdot (11.12.13\dots 19) \cdot (21.22.23\dots 29) \dots$

$$\dots (91.92.93\dots 99)$$

$$= (1 \cdot 2 \cdot 3 \dots 9) \frac{10}{10} (11 \cdot 12 \cdot 13 \dots 19) \frac{20}{20}$$

$$\begin{aligned}
 & (21 \cdot 23 \cdot 23 \dots 29) \frac{30}{30} \dots (91 \cdot 92 \cdot 93 \dots 99) \\
 &= \frac{1 \cdot 2 \cdot 3 \dots 99}{10 \cdot 20 \cdot 30 \dots 90} = \frac{99!}{362880 \times 10^9} = \frac{99!}{36288 \times 10^{10}}
 \end{aligned}$$

Hence (c) is the correct option.

Exp. 9) The expression $1! + 2! + 3! + 4! + \dots + n!$ (where $n \geq 5$) is not a/an :

- (a) composite number (b) odd number
(c) perfect square (d) multiple of 3

Solution $1! + 2! + 3! + 4! + 5! = 1 + 2 + 6 + 24 + 120 = 153$

Since, the options (a), (b) and (d) are ruled out so the remaining option (c) is the correct one.

Exp. 10) The HCF and LCM of $13!$ and $31!$ are respectively :

- (a) $12!$ and $32!$ (b) $13!$ and $31!$
(c) 26 and 403 (d) can't be determined

Solution Since $13!$ is contained in $31!$ so the LCM is $31!$ and $13!$ is common in $13!$ and $31!$, so the HCF is $13!$

Number of zeros at the end of the product

We know that $10 = 5 \times 2$, $100 = 5^2 \times 2^2$, $1000 = 5^3 \times 2^3$ etc. So we can say that for n number of zeros at the end of the product we need exactly n combinations of " 5×2 ". For example,

$2 \times 3 \times 5 \times 7 = 210$. There is only one zero at the end of the product (or resultant value)

$$\begin{aligned}
 \text{Again, } 2 \times 3 \times 5 \times 6 \times 7 \times 15 &= 2 \times 3 \times 5 \times 2 \times 3 \times 7 \times 3 \times 5 \\
 &= \underline{2 \times 2} \times \underline{5 \times 5} \times 3 \times 3 \times 3 \times 7 \\
 &= 100 \times 189 = 18900
 \end{aligned}$$

Thus there are two zeros because there are two combinations of " 5×2 ".

Now, $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9$

$$\begin{aligned}
 &= 1 \times 2 \times 3 \times 2 \times 2 \times 5 \times 2 \times 3 \times 7 \times 2 \times 2 \times 2 \times 3 \times 3 \\
 &= 1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 3 \times 3 \times 3 \times 3 \times 7 \\
 &= 1 \times 2^7 \times 5 \times 3^4 \times 7 \\
 &= 1 \times 2^6 \times 2 \times 5 \times 3^4 \times 7 \\
 &= 10 \times 2^6 \times 3^4 \times 7 \\
 &= 10 \times 64 \times 81 \times 7 \\
 &= 10 \times 36288 = 362880
 \end{aligned}$$

Thus there is only one zero at the end of the product since there is only one combination of 5×2 .

$$\begin{aligned}
 \text{Again } 4 \times 125 \times 3 &= 2 \times 2 \times 5 \times 5 \times 5 \times 3 \\
 &= 2^2 \times 5^3 \times 3 = 2^2 \times 5^2 \times 5 \times 3
 \end{aligned}$$

$$\begin{aligned}
 &= (2 \times 5)^2 \times 15 \\
 &= 100 \times 15 = 1500
 \end{aligned}$$

Thus there are only two zeros at the end of the product since there are only two combinations of " 5×2 ".

NOTE The number of zeros at the end of the product depends upon 2×5 , but the condition is that

- (i) $2^k \times 5^l$ gives k number of zeros if $k < l$
(ii) $2^k \times 5^l$ gives l number of zeros if $l < k$

Exp. 1) Find the number of zeros in the product of $10!$.

Solution $10! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$

It is obvious from the above expression that there are only two 5s and eight 2s.

Since the number of 5s are less (than the number of 2's) so the number of 5s will be effective to form the combination of " 5×2 ". Thus there are only 2 zeros at the end of the product of $10!$.

Exp. 2) Find the number of zeros at the end of the product of $2^{222} \times 5^{555}$.

Solution Since the number of 2s are less than the number of 5s hence the restriction is imposed by the number of 2s.

Thus there can be only 222 pairs of (5×2) . Hence the number of zeros at the end of the product of the given expression will be 222.

Exp. 3) Find the number of zeros at the end of the product of the expression

$$2^1 \times 5^2 \times 2^3 \times 5^4 \times 2^5 \times 5^6 \times 2^7 \times 5^8 \times 2^9 \times 5^{10}$$

Solution Since the number of 2's are less than the number of 5's hence number of 2s will be effective. Thus

$$\begin{aligned}
 &2^1 \times 2^3 \times 2^5 \times 2^7 \times 2^9 \times 2^{11} \times 2^{13} \times 2^{15} \times 2^{17} \times 2^{19} \\
 &= 2^{25} \times 5^{30} = 2^{25} \times 5^{25} \times 5^5 = (2 \times 5)^{25} \times 5^5 \\
 &= 10^{25} \times 5^5 = 5^5 \times 10^{25}
 \end{aligned}$$

Thus there will be 25 zeros at the end of the product of the given expression.

Exp. 4) The number of zeros at the end of the expression $10 + 100 + 1000 + \dots + 10000000000$ is :

- (a) 1 (b) 10
(c) 55 (d) none of these

Solution

$$\begin{array}{r}
 10 \\
 100 \\
 1000 \\
 \dots\dots\dots \\
 10000000000 \\
 \hline
 11111111110
 \end{array}$$

Thus there is only one zero at the end of resut.

Hence (a) is the correct option.

Exp. 5) The number of zeros at the end of the product of the expression $10 \times 100 \times 1000 \times 10000 \times \dots 10000000000$ is :

- (a) 10 (b) 100
(c) 50 (d) 55

Solution $10 \times 100 \times 1000 \times \dots 10000000000$
 $= 10^1 \times 10^2 \times 10^3 \times \dots \times 10^{10}$
 $= 10^{(1+2+3+\dots+10)} = 10^{55}$

Hence (d) is the correct option.

Exp. 6) Number of zeros at the end of the following expression $(5!)^{5!} + (10!)^{10!} + (50!)^{50!} + (100!)^{100!}$ is :

- (a) 165 (b) 120
(c) 125 (d) none of these

Solution The number of zeros at the end of $(5!)^{5!} = 120$
 $[\because 5! = 120 \text{ and thus } (120)^{120} \text{ will give 120 zeros}]$
 and the number of zeros at the end of the $(10!)^{10!}$, $(50!)^{50!}$ and $(100!)^{100!}$ will be greater than 120. Now since the number of zeros at the end of the whole expression will depend on the number which has the least number of zeros at the end of the number among other given numbers. So, the number of zeros at the end of the given expression is 120.

Divisibility of a Factorial Number by the Largest Power of Any Number

Let us start with very simple example. As if we consider $1!$, $2!$, $3!$ or $4!$, none of them is divisible by 5, because 5 is not the factor, involved in the given factorial number.

Now, if we consider any factorial number greater than $4!$, every number consists of 5 or the higher powers of 5 in the factorial numbers. For example starting from $5!$ every number has 5 as its factor. That is

$$5! = 1 \times 2 \times 3 \times 4 \times 5$$

$$6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6$$

$$7! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7$$

$$\dots \dots \dots$$

$$10! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \text{ etc.}$$

Thus it is obvious that every number greater than $4!$ is divisible by 5. But, as we move towards higher number we find that the frequency of the occurrence of 5 (or any particular number) increases. As $5!$ contains only one 5, $10!$ contains two 5s, $15!$ contains three 5s, $20!$ contains four 5s but $25!$ contains suddenly six 5s instead of five 5s. So it becomes a very tedious work to calculate the occurrence of any particular number involved as a factor of any higher factorial number.

Similarly, we can find that how many times 3 is contained in the $30!$ or in other words what is the largest power of 3 that can divide $30!$

Now, if we try to do it manually, we see that there are 10 multiples of 3 viz., 3, 6, 9, 12, 15, 18, 21, 24, 27, 30. But, there are some multiples which contain 3 at least two times for Example 9), 18 and 27.

Again there are some multiples which contain 3 three times as in 27. Thus the factor 3 totally occurs $14 (= 10 + 3 + 1)$ times in $30!$.

Hence you can understand that how tedious the calculation could be if we have to find the highest power of 3 that can exactly divide $9235!$

Therefore to make the calculation easier we can use the following formula.

Suppose we have to find the highest power of k that can exactly divide $n!$, we divide n by k , n by k^2 , n by k^3 ... and so

on till we get $\left\lfloor \frac{n}{k^x} \right\rfloor$ equal to 1 (where, $[P]$ means the greatest

integer less than or equal to P) and then add up as

$$\left\lfloor \frac{n}{k} \right\rfloor + \left\lfloor \frac{n}{k^2} \right\rfloor + \left\lfloor \frac{n}{k^3} \right\rfloor + \left\lfloor \frac{n}{k^4} \right\rfloor + \dots + \left\lfloor \frac{n}{k^x} \right\rfloor$$

NOTE For more information about greatest integer number (or function) i.e., GIF refer the chapter "FUNCTIONS".

Exp. 1) Find the largest power of 5 contained in $124!$

Solution $\left\lfloor \frac{124}{5} \right\rfloor + \left\lfloor \frac{124}{5^2} \right\rfloor = 24 + 4 = 28$

[We cannot do it further since 124 is not divisible by 5^3]
 Hence, there are 28 times 5 involved as a factor in $124!$

Exp. 2) Find the largest power of 2 that can divide $268!$

Solution $\left\lfloor \frac{268}{2} \right\rfloor + \left\lfloor \frac{268}{2^2} \right\rfloor + \left\lfloor \frac{268}{2^3} \right\rfloor + \left\lfloor \frac{268}{2^4} \right\rfloor$
 $+ \left\lfloor \frac{268}{2^5} \right\rfloor + \left\lfloor \frac{268}{2^6} \right\rfloor + \left\lfloor \frac{268}{2^7} \right\rfloor + \left\lfloor \frac{268}{2^8} \right\rfloor$
 $= 134 + 67 + 33 + 16 + 8 + 4 + 2 + 1 = 265$

Thus the greatest power of 2 is 265 that can divide exactly $268!$

Dear students you might have noticed that if we further increase the value of $n!$ and consider a slightly higher prime. Number say 7, 13, 17, 19 etc. for the divisibility, then we feel the real pain in solving the problem, since we don't know the values of higher powers of these divisors say 17^4 , 17^5 , 19^8 , 19^9 , ... etc. So this formula in this present form is not appropriate.

Now you must have noticed that we consider only integral values of the quotient q as $[q]$ which gives an integer just less than or equal to q . So we can do this calculation slightly in a easier manner.

Here we just divide the given number and then the succeeding quotients will be divided by the same remainder as in the case of successive division. So, for better understanding we can solve the previous problem in this manner as follows.

$$\begin{array}{r}
 2 \overline{) 268} \\
 2 \overline{) 134} \rightarrow \\
 2 \overline{) 67} \rightarrow \\
 2 \overline{) 33} \rightarrow \\
 2 \overline{) 16} \rightarrow \\
 2 \overline{) 8} \rightarrow \\
 2 \overline{) 4} \rightarrow \\
 2 \overline{) 2} \rightarrow \\
 2 \overline{) 1} \rightarrow \\
 0
 \end{array}
 \quad \left. \begin{array}{l} \text{[Divide successive quotients till} \\ \text{you get 0 as the last quotient]} \end{array} \right\} 265 \text{ (Add up all the quotients)}$$

So we can frankly say that this method of calculation is easy since we need not to know the values of $2^1, 2^2, 2^3, 2^4, \dots, 2^8$ etc. Also the division by 2 is easier than the division by 2^x , where x is any large integer.

Exp. 3) Find the largest power of 7 that can exactly divide 777!

Solution

$$\begin{array}{r}
 7 \overline{) 777} \\
 7 \overline{) 111} \rightarrow \\
 7 \overline{) 15} \rightarrow \\
 7 \overline{) 2} \rightarrow \\
 0
 \end{array}
 \left. \begin{array}{l} \rightarrow \\ \rightarrow \\ \rightarrow \end{array} \right\} 128$$

Thus the highest power of 7 is 128 by which 777! can be completely divided.

Exp. 4) Find the largest value of n in the 10^n which can exactly divide 1000!

Solution Since 10 is made up of 5 and 2 i.e., $10 = 2 \times 5$.

So you can see that before 10! there is the presence of 10 in 5!, 6!, 7!, 8! or 9!.

$$\text{So } 10^n = (5 \times 2)^n = 5^n \times 2^n$$

Thus we find the powers of 2 and 5 individually

$$\begin{array}{r}
 2 \overline{) 1000} \\
 2 \overline{) 500} \rightarrow \\
 2 \overline{) 250} \rightarrow \\
 2 \overline{) 125} \rightarrow \\
 2 \overline{) 62} \rightarrow \\
 2 \overline{) 31} \rightarrow \\
 2 \overline{) 15} \rightarrow \\
 2 \overline{) 7} \rightarrow \\
 2 \overline{) 3} \rightarrow \\
 1
 \end{array}
 \quad \text{and} \quad
 \begin{array}{r}
 5 \overline{) 1000} \\
 5 \overline{) 200} \rightarrow \\
 5 \overline{) 40} \rightarrow \\
 5 \overline{) 8} \rightarrow \\
 1
 \end{array}
 \left. \begin{array}{l} \rightarrow \\ \rightarrow \\ \rightarrow \end{array} \right\} 249$$

So there is 2^{994} and 5^{249} but we can make only 249 combinations due to the restriction imposed by the lower power of 5!.

Thus the highest power of 10 is 249 which can divide 1000!

$$\left(\begin{array}{l} \therefore 2^{994} \times 5^{249} \Rightarrow 2^{745} \times 2^{249} \times 5^{249} \\ \Rightarrow 2^{745} \times (10)^{249} \end{array} \right)$$

Exp. 5) Find the number of zeros at the end of 1000!

Solution Since we know that the zeros at the end of any product are due to the presence of 10 as the factor of the product and the number of zeros depends upon the number of times 10 is involved. For example if there are seven 10s (i.e., 7 combinations of 5×2) in the $n!$, then the number of zeros at the end of $n!$ will be 7. But, since we have solved the same problem previous to this problem so we can conclude that the number of 10s in 1000! is 249. Hence the number of zeros at the end of the 1000! is 249.

Exp. 6) The number of zeros at the end of 100! is :

- (a) 25 (b) 50
(c) 24 (d) 100

Solution

$$\begin{array}{r}
 2 \overline{) 100} \\
 2 \overline{) 50} \rightarrow \\
 2 \overline{) 25} \rightarrow \\
 2 \overline{) 12} \rightarrow \\
 2 \overline{) 6} \rightarrow \\
 2 \overline{) 3} \rightarrow \\
 1
 \end{array}
 \left. \begin{array}{l} \rightarrow \\ \rightarrow \\ \rightarrow \end{array} \right\} 97$$

and

$$\begin{array}{r}
 5 \overline{) 100} \\
 5 \overline{) 20} \rightarrow \\
 4
 \end{array}
 \left. \begin{array}{l} \rightarrow \\ \rightarrow \end{array} \right\} 24$$

So, there will be only 24 combinations of 5×2 , it means there will be 24 zeros at the end of 100!.

Hence (c) is the correct answer.

Exp. 7) Find the highest power of 63 which can completely divide 6336!.

Solution $63 = 3^2 \times 7$

Since this technique is applicable only for the prime factors. So we solve it by breaking 63 in its prime factors.

$$\begin{array}{r}
 3 \overline{) 6336} \\
 3 \overline{) 2112} \rightarrow \\
 3 \overline{) 704} \rightarrow \\
 3 \overline{) 234} \rightarrow \\
 3 \overline{) 78} \rightarrow \\
 3 \overline{) 26} \rightarrow \\
 3 \overline{) 8} \rightarrow \\
 2
 \end{array}
 \left. \begin{array}{l} \rightarrow \\ \rightarrow \\ \rightarrow \end{array} \right\} 3164$$

and

$$\begin{array}{r|l} 7 & 6336 \\ 7 & 905 \rightarrow \\ 7 & 129 \rightarrow \\ 7 & 18 \rightarrow \\ 2 & \rightarrow \end{array} \left. \vphantom{\begin{array}{r|l} 7 & 6336 \\ 7 & 905 \rightarrow \\ 7 & 129 \rightarrow \\ 7 & 18 \rightarrow \\ 2 & \rightarrow \end{array}} \right\} 1054$$

$$\therefore 63 = 9 \times 7 = 3^2 \times 7$$

$\therefore$ Since to make a 9 we need 3×3 i.e., two times. So we divide 3164 by 2 and get 1582.

$$\text{So } 3^{3164} \times 7^{1054}$$

$$\Rightarrow 9^{1582} \times 7^{1054}$$

Thus the largest power of 63 is 1054. That can completely divide 6336!

Exp. 8) Find the highest power of 40 which can completely divide 4000!

$$\text{Solution } 40 = 8 \times 5 = 2^3 \times 5$$

So,

$$\begin{array}{r|l} 2 & 4000 \\ 2 & 2000 \rightarrow \\ 2 & 1000 \rightarrow \\ 2 & 500 \rightarrow \\ 2 & 250 \rightarrow \\ 2 & 125 \rightarrow \\ 2 & 62 \rightarrow \\ 2 & 31 \rightarrow \\ 2 & 15 \rightarrow \\ 2 & 7 \rightarrow \\ 2 & 3 \rightarrow \\ 1 & \rightarrow \end{array} \left. \vphantom{\begin{array}{r|l} 2 & 4000 \\ 2 & 2000 \rightarrow \\ 2 & 1000 \rightarrow \\ 2 & 500 \rightarrow \\ 2 & 250 \rightarrow \\ 2 & 125 \rightarrow \\ 2 & 62 \rightarrow \\ 2 & 31 \rightarrow \\ 2 & 15 \rightarrow \\ 2 & 7 \rightarrow \\ 2 & 3 \rightarrow \\ 1 & \rightarrow \end{array}} \right\} 3994$$

and

$$\begin{array}{r|l} 5 & 4000 \\ 5 & 800 \rightarrow \\ 5 & 160 \rightarrow \\ 5 & 32 \rightarrow \\ 5 & 6 \rightarrow \\ 1 & \rightarrow \end{array} \left. \vphantom{\begin{array}{r|l} 5 & 4000 \\ 5 & 800 \rightarrow \\ 5 & 160 \rightarrow \\ 5 & 32 \rightarrow \\ 5 & 6 \rightarrow \\ 1 & \rightarrow \end{array}} \right\} 999$$

$$\text{Now since } 2^{3m} \times 5^n = 8^m \times 5^n$$

$$\therefore 2^{3994} \times 5^{999} = (2^3)^{1331} \times 2 \times 5^{999}$$

$$\Rightarrow 2 \times (8^{1331} \times 5^{999})$$

$$\Rightarrow 2 \times 8^{332} \times (8 \times 5)^{999}$$

Thus the highest power of 40 is 999 that can completely divide 4000!.

Exp. 9) Find the highest power of 81 that can completely divide 1800!.

$$\text{Solution } 81 = 3^4$$

So,

$$\begin{array}{r|l} 3 & 1800 \\ 3 & 600 \rightarrow \\ 3 & 200 \rightarrow \\ 3 & 66 \rightarrow \\ 3 & 22 \rightarrow \\ 3 & 7 \rightarrow \\ 2 & \rightarrow \end{array} \left. \vphantom{\begin{array}{r|l} 3 & 1800 \\ 3 & 600 \rightarrow \\ 3 & 200 \rightarrow \\ 3 & 66 \rightarrow \\ 3 & 22 \rightarrow \\ 3 & 7 \rightarrow \\ 2 & \rightarrow \end{array}} \right\} 897$$

Thus we get 3^{897} . But we need 81.

$$\text{Now since } 81 = 3^4$$

$$\text{Therefore } 3^{897} = (3^4)^{224} \times 3^1 = (81)^{224} \times 3.$$

Therefore the highest power of 81 is 224 which can divide completely 1800! so $(81)^{224}$ can divide 1800!

NOTE Sometimes for your convenience you can skip some unnecessary calculation. As if you are asked to find the highest power of 30 which can completely divide 1357!

Since $30 = 2 \times 3 \times 5$. Now if you have been serious while doing the previous examples so you can conclude that you need not to solve to find the highest powers of 2 and 3. What you need is just to find the highest power of 5 (which is the greatest factor therefore it occurs least frequently) and as you know that the least frequent factor impose the restriction and hence it is the only effective value. Hence we just calculate the highest power of 5 and get the required result. Still there are some special points which are really difficult to explain on the paper.

These points can be effectively explained only in the classroom by teacher and last but not the least if you are intelligent enough so you can pick up these subtle but the important points by yourself as you are learning the concept religiously. Please note that there can be some shortcuts which I have not mentioned because it is difficult to explain exactly on paper and even some students don't fathom the logic of the shortcut. So it becomes dangerous and hence it is better to find the shortcuts by themselves just by intensive practice or seeking the help of an expert.

Concept of Unit Digit

Look at the following :

$$1 \times 5 = 5$$

$$3 \times 5 = 15$$

$$5 \times 5 = 25$$

$$7 \times 5 = 35$$

$$9 \times 5 = 45$$

$$11 \times 5 = 55$$

... ..
... ..

i.e., if the number whose last digit is 5, is multiplied by any odd number, the unit digit of the product will always be 5.

For example

$$13 \times 15 = 195, 19 \times 35 = 665 \text{ etc.}$$

Now, $2 \times 5 = 10$

$$4 \times 5 = 20$$

$$6 \times 5 = 30$$

$$8 \times 5 = 40$$

$$10 \times 5 = 50$$

$$12 \times 5 = 60$$

$$\dots \dots \dots$$

i.e., if the number whose last is 5, is multiplied by any even number (including zero), the unit digit of the product is always zero. For example

$$82 \times 15 = 1230,$$

and $156 \times 45 = 7020$

and $62 \times 13 \times 65 = 52390 \text{ etc.}$

Now we will discuss that the unit digit of the resultant value depends upon the unit digits of all the participating numbers *i.e.*,

$$12 + 17 + 13 + 47 = 89$$

Thus it is clear that the unit digit of the resultant value 89 depends upon the unit digits 2, 7, 3, 7.

Similarly, $6 \times 7 \times 9 = 378$

$$3 \times 7 \times 8 = 168$$

So we can find out the unit digit of the resultant value only by solving the unit digits of the given expression.

Exp. 1) Find the unit digit of $123 + 345 + 780 + 65 + 44$.

Solution We can find the unit digit just by adding the unit digits 3, 5, 0, 5, 4 as

$$3 + 5 + 0 + 5 + 4 = 17$$

So the unit digit (or the last digit) of the resultant value of the expression $123 + 345 + 780 + 65 + 44$ will be 7. (you can verify it by doing the whole sum)

Exp. 2) Find the unit digit of $676 \times 543 \times 19$.

Solution We can find the unit digit of the product of the given expression just by multiplying the unit digits (6, 3, 9) instead of doing the whole sum.

Thus $6 \times 3 \times 9 = 162$

Hence, the unit digit of the product of the given expression will be 2. (you can verify it by doing the complete sum)

Exp. 3) Find the unit digit of $135 \times 361 \times 970$.

Solution The unit digit can be obtained by multiplying the unit digits 5, 1, 0. then $5 \times 1 \times 0 = 0$ thus the unit digit will be zero.

Exp. 4) Find the unit digit of the product of all the odd prime numbers.

Solution The odd prime numbers are 3, 5, 7, 11, 13, 17, 19...etc.

Now we know that if 5 is multiplied by any odd number it always gives the last digit 5. So the required unit digit will be 5.

Cyclicity of the Unit Digit

Now, look at the following

$$\begin{aligned} 1 \times 1 &= 1 \\ 1 \times 1 \times 1 &= 1 \\ 1 \times 1 \times 1 \times 1 &= 1 \\ 1 \times 1 \times 1 \times 1 \times 1 &= 1 \text{ etc.} \end{aligned}$$

$$2^1 = 2$$

$$2^2 = 2 \times 2 = 4$$

$$2^3 = 2 \times 2 \times 2 = 8$$

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

$$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$$

$$2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$$

$$2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$$

$$2^9 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 512$$

$$2^{10} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1024$$

$$2^{11} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2048$$

$$2^{12} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 4096$$

Similarly $3^1 = 3$

$$3^2 = 9$$

$$3^3 = 27$$

$$3^4 = 81$$

$$3^5 = 243$$

$$3^6 = 729$$

$$3^7 = 2187$$

$$3^8 = 6561$$

$$3^9 = 19683 \text{ etc.}$$

Similarly, $4^1 = 4$

$$4^2 = 16$$

$$4^3 = 64$$

$$4^4 = 256$$

$$4^5 = 1024$$

$$4^6 = 4096$$

Thus we can say that the unit digit follows a periodic pattern that is after a particular period it repeats in a cyclic form.

The unit digit of $2^1, 2^5, 2^9, 2^{13}, \dots$ is the same which is 2.

Similarly $2^2, 2^6, 2^{10}, 2^{14}, \dots$ etc. has the same unit digit, which is 4.

Again the last digit of 3^1 is 3.

and the last digit of 3^2 is 9.

and the last digit of 3^3 is 7.

and the last digit of 3^4 is 1.

and the last digit of 3^5 is 3.

and the last digit of 3^6 is 9.

and the last digit of 3^7 is 7.

and the last digit of 3^8 is 1.

Thus the last digit must follow a pattern. It can be seen that the last digit of 2 repeats after every four steps and the last digit of 4 repeats after every 2 steps.

NOTE

1. The last digit (or unit digit) of 0, 1, 5 and 6 is always the same irrespective of their powers raised to them.
2. The last digit of 4 and 9 follows the pattern of odd-even i.e., their period is 2.
3. The last digit of 2, 3, 7, 8 repeats after every 4 steps i.e., their cyclic period is 4.

Exp. 1) Find the last digit of 2^{35} .

Solution The last digit of $2^1 \Rightarrow 2$

$$2^2 \Rightarrow 4$$

$$2^3 \Rightarrow 8$$

$$2^4 \Rightarrow 6$$

$$2^5 \Rightarrow 2$$

Since its cyclic period is four, it means the unit digit of 2 will repeat after every 4 steps. Hence we can say that the last digit of $2^1, 2^5, 2^9, 2^{13}, 2^{17}, 2^{21}, 2^{25}, 2^{29}, 2^{33}, 2^{37}$ is the same and the last digit of $2^2, 2^6, 2^{10}, 2^{14}, 2^{18}, \dots, 2^{30}, 2^{34}, 2^{38}$ is same and the last digit of $2^3, 2^7, 2^{11}, \dots, 2^{31}, 2^{35}, 2^{39}$ is same and the last digit of $2^4, 2^8, 2^{12}, 2^{16}, \dots, 2^{32}, 2^{36}, 2^{40}$ etc. is same.

Hence the last digit of 2^{35} is 8.

Alternatively You can see that the powers which are divisible by 4 (i.e., cyclic period) give the same unit digit as 2^4 . As $2^8, 2^{12}, 2^{16}, \dots$ etc.

Again the powers which leave the remainder 1 when divided by 4 (which is the cyclic period) give the same unit digit as 2^1 . As $2^5, 2^9, 2^{13}, \dots$ etc.

Similarly the powers which leave the remainder 2 when divided by 4, give the same unit digit as 2^2 . As $2^6, 2^{10}, 2^{14}, 2^{18}, \dots$ etc.

Similarly the powers which leave the remainder 3 when divided by 4 give the same unit digit as 2^3 . As $2^7, 2^{11}, 2^{15}, \dots$ etc.

Since when 35 is divided by 4, the remainder is 3. Hence, the unit digit of 2^{35} is equal to the unit digit of 2^3 , which is equal to 8.

Exp. 2) Find the unit digit of $(12)^{78}$.

Solution The unit digit of $(12)^{78}$ will be same as $(2)^{78}$. Now since we know that the cyclic period of unit digit of 2 is 4. The remainder when 78 is divided by 4 is 2. Hence the unit digit of 2^{78} will be same as 2^2 which is 4. Thus the unit digit of 12^{78} is 4.

Exp. 3) Find the unit digit of $(33)^{123}$.

Solution Since we know that the unit digit of $(33)^{123}$ will be same as $(3)^{123}$. Now the unit-digit of 3^{123} will be 7 since it will be equal to the unit digit of 3^3 .

Thus the unit digit of $(3)^{123}$ is 7.

Exp. 4) Find the unit digit of $3^{47} + 7^{52}$.

Solution The unit digit of the given expression will be equal to the unit digit of the sum of the unit digits of both the terms individually.

Now, unit digit of 3^{47} is 7 (since it will be equal to 3^3) and the unit digit of 7^{52} is 1 (since it will be equal to 7^4)

Thus the unit digit of $3^{47} + 7^{52}$ is $7 + 1 = 8$.

Exp. 5) Find the unit digit of $3^6 \times 4^7 \times 6^3 \times 7^4 \times 8^2 \times 9^5$.

Solution The unit digit of 3^6 is 9

The unit digit of 4^7 is 4

The unit digit of 6^3 is 6

The unit digit of 7^4 is 1

The unit digit of 8^2 is 4

The unit digit of 9^5 is 9

Therefore the unit digit of the given expression is 6, (since $9 \times 4 \times 6 \times 1 \times 4 \times 9 = 7776$).

Exp. 6) Find the unit digit of $111!$ (factorial 111).

Solution $111! = 1 \times 2 \times 3 \times 4 \times 5 \times \dots \times 110 \times 111$

Since there is a product of 5 and 2 hence it will give zero as the unit digit.

Hence the unit digit of $111!$ is 0 (zero).

Exp. 7) Find the unit digit of the product of all the prime number between 1 and $(11)^{11}$.

Solution The set of prime number $S = \{2, 3, 5, 7, 11, 13, \dots\}$

Since there is one 5 and one 2 which gives 10 after multiplying mutually, it means the unit digit will be zero.

Exp. 8) Find the unit digit of the product of all the elements of the set which consists all the prime numbers greater than 2 but less than 222.

Solution The set of required prime numbers = $\{3, 5, 7, 11, \dots\}$

Since there is no any even number in the set so when 5 is multiplied with any odd number, it always gives 5 as the last digit.

Hence the unit digit will be 5.

Exp. 9) Find the last digit of $222^{888} + 888^{222}$.

Solution The last digit of the expression will be same as the last digit of $2^{888} + 8^{222}$.

Now the last digit of 2^{888} is 6 and the last digit of the 8^{222} is 4. Thus the last digit of $2^{888} + 8^{222}$ is 0 (zero), since $6 + 4 = 10$.

Exp. 10) Find the last digit of $32^{32^{32}}$.

Solution The last digit of $32^{32^{32}}$ is same as $2^{32^{32}}$.

$$\text{But } 2^{32^{32}} = 2^{32 \times 32 \times 32 \times \dots \times 32 \text{ times}}$$

$$\Rightarrow 2^{32^{32}} = 2^{4 \times 8 \times (32 \times 32 \times \dots \times 31 \text{ times})}$$

$$\Rightarrow 2^{32^{32}} = 2^{4n},$$

where $n = 8 \times (32 \times 32 \times \dots \times 31 \text{ times})$

Again $2^{4n} = (16)^n \Rightarrow$ unit digit is 6, for every $n \in \mathbb{N}$

Hence, the required unit digit = 6.

Exp. 11) Find the last digit of the expression :

$$1^2 + 2^2 + 3^2 + 4^2 + \dots + 100^2.$$

Solution The unit digit of the whole expression will be equal to the unit digit of the sum of the unit digits of the expression.

Now adding the unit digits of $1^2 + 2^2 + 3^2 + \dots + 10^2$ we get

$$1 + 4 + 9 + 6 + 5 + 6 + 9 + 4 + 1 + 0 = 45$$

Hence, the unit digit of $1^2 + 2^2 + 3^2 + \dots + 10^2$ is 5.

Now since there are 10 similar columns of numbers which will yield the same unit digit 5. Hence the sum of unit digits of all the 10 columns is 50 ($= 5 + 5 + 5 + \dots + 5$).

Hence, the unit digit of the given expression is 0 (zero).

Exp. 12) Find the unit digit of $1^1 + 2^2 + 3^3 + \dots + 10^{10}$.

Solution The unit digit of $1^1 = 1$

The unit digit of $2^2 = 4$

The unit digit of $3^3 = 7$

The unit digit of $4^4 = 6$

The unit digit of $5^5 = 5$

The unit digit of $6^6 = 6$

The unit digit of $7^7 = 3$

The unit digit of $8^8 = 6$

The unit digit of $9^9 = 9$

The unit digit of $10^{10} = 0$

Thus the unit digit of the given expression will be 7. ($\because 1 + 4 + 7 + 6 + 5 + 6 + 9 + 4 + 1 + 0 = 47$)

Exp. 13) Find the unit digit of

$$13^{24} \times 68^{57} + 24^{13} \times 57^{68} + 1234 + 5678.$$

Solution The unit digit of 3^{24} is 1

The unit digit of 8^{57} is 8

The unit digit of 4^{13} is 4

The unit digit of 7^{68} is 1

Therefore

$$= 1 \times 8 + 4 \times 1 + 4 + 8$$

$$= 8 + 4 + 4 + 8 = 24$$

Thus the unit digit of the whole expression is 4.

Exp. 14) The unit digit of the expression $\frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}{100}$ is :

(a) 7

(b) 9

(c) 8

(d) none of these

Solution Since in the numerator of the product of the expression there will be 2 zeros at the end and these two zeros will be cancelled by 2 zeros of the denominator. Hence finally we get a non-zero unit digit in the expression.

$$\begin{aligned} \text{Now, } \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}{100} &= \frac{1 \times 2^8 \times 3^4 \times 5^2 \times 7^1}{5^2 \times 2^2} \\ &= 1 \times 2^6 \times 3^4 \times 7 \end{aligned}$$

Therefore, the unit digit of the given expression will be same as that of $1 \times 2^6 \times 3^4 \times 7$.

Now, the unit digit of $1 \times 2^6 \times 3^4 \times 7$ is 8.

($\because$ the product of unit digits of 1, 2^6 , 3^4 , 7 is $1 \times 4 \times 1 \times 7 = 28$)

Hence, the unit digit of $\frac{10!}{100}$ is 8.

Exp. 15) Find the unit digit of the expression

$$888^{9235!} + 222^{9235!} + 666^{2359!} + 999^{9999!}.$$

Solution First of all we individually need to find the unit digit individually of all the four terms

So, the unit digit of $888^{9235!}$ is equal to the unit digit of $8^{9235!}$

Now, the unit digit of $8^{9235!}$ is equal to the unit of 8^4 (since 9235! is divisible by 4), which is 6.

Again the unit digit of $222^{9235!}$ is equal to the unit digit of $2^{9235!}$ and the unit digit of $2^{9235!}$ is same as that of 2^4 which is 6, since 9235! is divisible by 4. Further the unit digit of $666^{2359!}$ is 6, which is always constant for all the powers except zero and the unit digit of $999^{9999!}$ is 1 since the value of 9999! is even.

Thus the unit digit of the expression is 9. ($\because 6 + 6 + 6 + 1 = 19$)

Exp. 16) The last digit of the following expression is :

$$(1!)^1 + (2!)^2 + (3!)^3 + (4!)^4 + \dots + (10!)^{10}$$

(a) 4

(b) 5

(c) 6

(d) 7

Solution The unit digit of the given expression will be equal to the unit digit of the sum of the unit digits of every term of the expression.

Now, The unit digit of $(1!)^2 = 1$

The unit digit of $(2!)^2 = 4$

The unit digit of $(3!)^3 = 6$

The unit digit of $(4!)^4 = 6$

The unit digit of $(5!)^5 = 0$

The unit digit of $(6!)^6 = 0$

Thus the unit digit of the $(7!)^7, (8!)^8, (9!)^9, (10!)^{10}$ will be zero.

So, the unit digit of the given expression = 7

($\because 1 + 4 + 6 + 0 + 0 + 0 + 0 + 0 + 0 = 17$)

Exp. 17) The last 5 digits of the following expression will be $(1!)^5 + (2!)^4 + (3!)^3 + (4!)^2 + (5!)^1 + (10!)^5 + (100!)^4 + (1000!)^3 + (10000!)^2 + (100000!)^1$

(a) 45939

(b) 00929

(c) 20929

(d) can't be determined

Solution The last digit of $(1!)^5 = 1$

The last digit of $(2!)^4 = 16$

The last digit of $(3!)^3 = 216$

The last digit of $(4!)^2 = 576$

The last digit of $(5!)^1 = 120$

The last 5 digit of $(10!)^5 = 00000$

The last 5 digit of $(100!)^4 = 00000$

The last 5 digit of $(1000!)^3 = 00000$

The last 5 digit of $(10000!)^2 = 00000$

The last 5 digit of $(100000!)^1 = 00000$

Thus the last 5 digits of the given expression = 00929

[$\because 1 + 16 + 216 + 576 + 120 + 00000 + 00000 + 00000 + 00000 + 00000 = 00929$]

Involution

As in the previous articles we have studied that $a \cdot a \cdot a \dots (n \text{ times}) = a^n$. So the process of multiplication of a number several times by itself is known as INVOLUTION.

Similarly, we use the same method in some algebraic expressions as follows :

$$1. (a+b)^2 = a^2 + b^2 + 2ab = (a+b)(a+b)$$

$$2. (a-b)^2 = a^2 + b^2 - 2ab = (a-b)(a-b)$$

$$3. (a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca) \\ = (a+b+c)(a+b+c)$$

$$4. (a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$5. (a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

$$6. (a+b+c)^3 = a^3 + b^3 + c^3 + 3(a+b)(b+c)(c+a)$$

$$7. (a+b)^2 = (a-b)^2 + 4ab$$

$$8. (a-b)^2 = (a+b)^2 - 4ab$$

$$9. a^2 - b^2 = (a+b)(a-b)$$

$$10. a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$$

$$11. a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$$

$$12. a^3 + b^3 + c^3 - 3abc = (a+b+c)$$

$$(a^2 + b^2 + c^2 - ab - bc - ac)$$

The above rules are used widely. But these are important in finding the square, cube, etc. of a number besides helping as find digits the remainder or divisibility of a number.

1. $a^n + b^n$ is divisible by $(a+b)$, when n is odd.

2. $a^n + b^n$ is never divisible by $(a-b)$

3. $a^n + b^n$ is not divisible by $(a+b)$, when n is even.

4. $a^n - b^n$ is always divisible by $(a-b)$

5. $a^n - b^n$ is divisible by $(a+b)$ only, when n is even.

6. $a^n - b^n$ is not divisible by $(a+b)$, when n is odd.

Exp. 1) Find the value of $107 \times 107 - 93 \times 93$

Solution $(107)^2 - (93)^2 \quad [\because a^2 - b^2 = (a+b)(a-b)]$
 $= (107 + 93)(107 - 93) = 200 \times 14 = 2800$

Exp. 2) Find the value of 734856×9999 .

Solution $734856 \times 9999 = 734856 \times (10000 - 1)$
 $= 7348560000 - 734856 = 7347825144$

Exp. 3) $6798 \times 223 + 6798 \times 77 = k$, then the value of k will be

(a) 2034900 (b) 3029400 (c) 2039400 (d) none

Solution $6798 \times 223 + 6798 \times 77 = 6798(223 + 77)$
 $= 6798(300) = 2039400$

Exp. 4) $123 \times 123 + 77 \times 77 + 2 \times 123 \times 77 = ?$

Solution $123 \times 123 + 77 \times 77 + 2 \times 123 \times 77$
 $= (123)^2 + (77)^2 + 2 \times 123 \times 77$
 $= (123 + 77)^2 \quad [(a+b)^2 = a^2 + b^2 + 2ab]$
 $= (200)^2 = 40000$

Exp. 5) $\frac{(941 + 149)^2 + (941 - 149)^2}{(941 \times 941 + 149 \times 149)} = ?$

Solution $\frac{(941 + 149)^2 + (941 - 149)^2}{(941 \times 941 + 149 \times 149)}$
 $\therefore \frac{(a+b)^2 + (a-b)^2}{a^2 + b^2} = \frac{(a^2 + b^2 + 2ab) + (a^2 + b^2 - 2ab)}{(a^2 + b^2)}$
 $= \frac{2(a^2 + b^2)}{(a^2 + b^2)} = 2$

Hence, the value of the given expression is 2. So you need not calculate the given numbers in the expression, where $a = 941, b = 149$.

Exp. 6) $\frac{888 \times 888 \times 888 - 222 \times 222 \times 222}{888 \times 888 + 888 \times 222 + 222 \times 222} = ?$

Solution
$$\frac{888 \times 888 \times 888 - 222 \times 222 \times 222}{888 \times 888 + 888 \times 222 + 222 \times 222}$$

$$= \frac{(888)^3 - (222)^3}{(888)^2 + 888 \times 222 + (222)^2}$$

$$= \frac{(888)^3 - (222)^3}{(888)^2 + 888 \times 222 + (222)^2}$$

$$= 888 - 222$$

$$\left[\because \frac{a^3 - b^3}{(a^2 + b^2 + ab)} = \frac{(a-b)(a^2 + b^2 + ab)}{(a^2 + b^2 + ab)} \right]$$

$$= 666$$

Exp. 7) If $(64)^2 - (36)^2 = 20k$ then the value of k is :

- (a) 140 (b) 120
(c) 80 (d) none of these

Solution $(64)^2 - (36)^2 = 20k$

$$\Rightarrow (64 + 36)(64 - 36) = 20k$$

$$\Rightarrow 100 \cdot 28 = 20k$$

$$\Rightarrow 20 \times 5 \times 28 = 20k$$

$$\Rightarrow k = 28 \times 5 = 140$$

Hence (a) is the correct answer.

Exp. 8) If $a + \frac{1}{a} = 3$, then the value of $a^2 + \frac{1}{a^2}$ is :

- (a) 6 (b) 7
(c) 9 (d) can't be determined

Solution $\because \left(a + \frac{1}{a}\right) = 3$

$$\therefore \left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2a \times \frac{1}{a} = 9$$

$$\Rightarrow a^2 + \frac{1}{a^2} + 2 = 9 \Rightarrow a^2 + \frac{1}{a^2} = 7$$

Hence (b) is the correct option.

Exp. 9) If $a + \frac{1}{a} = 3$, then the value of $a^3 + \frac{1}{a^3}$ is :

- (a) 15 (b) 18
(c) 27 (d) none of these

Solution $\left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3a \cdot \frac{1}{a} \left(a + \frac{1}{a}\right)$

$$\Rightarrow (3)^3 = a^3 + \frac{1}{a^3} + 3(3)$$

$$\left[\because a \times \frac{1}{a} = 1 \right]$$

$$\left[\text{and } (a+b)^3 = a^3 + b^3 + 3ab(a+b) \right]$$

$$\Rightarrow 27 = a^3 + \frac{1}{a^3} + 9 \Rightarrow a^3 + \frac{1}{a^3} = 18$$

Hence (b) is the correct option.

Exp. 10) What is the value of

$$\frac{2.75 \times 2.75 \times 2.75 - 2.25 \times 2.25 \times 2.25}{2.75 \times 2.75 + 2.75 \times 2.25 + 2.25 \times 2.25} ?$$

Solution $\because a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$

$$\therefore \frac{a^3 - b^3}{(a^2 + b^2 + ab)} = \frac{(a-b)(a^2 + b^2 + ab)}{(a^2 + b^2 + ab)} = (a-b)$$

$$\therefore \text{The value of the expression} = 2.75 - 2.25 = 0.50$$

Exp. 11) If $(\sqrt{a} + \sqrt{b}) = 17$ and $(\sqrt{a} - \sqrt{b}) = 1$, then the value of $\sqrt{ab}$ is :

- (a) 72 (b) 27
(c) 35 (d) none of these

Solution $(x+y)^2 - (x-y)^2 = (x^2 + y^2 + 2xy) - (x^2 + y^2 - 2xy)$

$$\Rightarrow (x+y)^2 - (x-y)^2 = 4xy$$

$$\Rightarrow xy = \frac{(x+y)^2 - (x-y)^2}{4}$$

Thus
$$\sqrt{ab} = \frac{(\sqrt{a} + \sqrt{b})^2 - (\sqrt{a} - \sqrt{b})^2}{4}$$

$$= \frac{(17)^2 - (1)^2}{4} = \frac{288}{4} = 72$$

Hence (a) is the correct option.

Exp. 12) Find the value of $a^3 + b^3 + c^3 - 3abc$ if $a + b + c = 12$ and $ab + bc + ca = 47$.

Solution $\because a + b + c = 12$

$$\therefore (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) = 144$$

$$\Rightarrow a^2 + b^2 + c^2 + 2 \times 47 = 144$$

$$\Rightarrow a^2 + b^2 + c^2 = 50$$

Now, since $a^3 + b^3 + c^3 - 3abc$

$$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$$

$$\text{or } a^3 + b^3 + c^3 - 3abc = 12(50 - 47) = 12 \times 3 = 36$$

Exp. 13) If $a + b + c = 0$, then the value of $a^3 + b^3 + c^3$ is :

- (a) 0 (b) abc
(c) $3abc$ (d) none of these

Solution $\because a^3 + b^3 + c^3 - 3abc$

$$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$$

$$\text{or } a^3 + b^3 + c^3 - 3abc = 0$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

Hence (c) is the correct option.

Exp. 14) If $x + y + z = 0$, then the value of

$$\frac{x^2y^2 + y^2z^2 + z^2x^2}{x^4 + y^4 + z^4} \text{ is :}$$

- (a) 0 (b) $1/2$ (c) 1 (d) 2

Solution $\because (x + y + z) = 0$

$$\Rightarrow x^2 + y^2 + z^2 + 2(xy + yz + zx) = 0$$

$$\begin{aligned} \Rightarrow x^2 + y^2 + z^2 &= -2(xy + yz + zx) \\ \Rightarrow (x^2 + y^2 + z^2)^2 &= 4(xy + yz + zx)^2 \\ \Rightarrow x^4 + y^4 + z^4 + 2(x^2y^2 + y^2z^2 + z^2x^2) \\ &= 4[x^2y^2 + y^2z^2 + z^2x^2 + 2xyz(x + y + z)] \\ \therefore x^4 + y^4 + z^4 &= 2(x^2y^2 + y^2z^2 + z^2x^2) \quad [\because (x + y + z) = 0] \\ \therefore \frac{x^2y^2 + y^2z^2 + z^2x^2}{x^4 + y^4 + z^4} &= \frac{1}{2} \end{aligned}$$

Hence (b) is the correct option.

Exp. 15) The value of $a^3 + b^3 + c^3 - 3abc$, when $a = 87$, $b = -126$ and $c = 39$ is :

- (a) 0 (b) 1259
(c) -48 (d) none of these

Solution $\because a + b + c = 0$

$$\therefore a^3 + b^3 + c^3 - 3abc = 0$$

Thus (a) is the correct option.

Exp. 16) The greatest divisor of

$$(a-b)(a+b)(a^2+b^2)(a^4+b^4)(a^8+b^8)(a^{16}+b^{16}) \text{ is :}$$

(a) $a^{16} - b^{16}$ (b) $a^{16} + b^{16}$ (c) $a^{31} - b^{31}$ (d) $a^{32} - b^{32}$

Solution $(a-b)(a+b)(a^2+b^2)(a^4+b^4)(a^8+b^8)(a^{16}+b^{16})$
 $= a^{32} - b^{32}$

Hence, the greatest divisor is $a^{32} - b^{32}$. Hence (d) is the correct option.

Exp. 17) In the above example which of the following can not divide the given expression :

- (a) $a^4 - b^4$ (b) $a^{12} - b^{12}$
(c) $a^{16} - b^{16}$ (d) can't be determined

Solution Since $a^{12} - b^{12}$ is not the factor of the given expression. Hence, it can not divide the given expression. Thus (b) is the appropriate option.

Exp. 18) The expression $31^n + 17^n$ is divisible by, for every odd positive integer n :

- (a) 48 (b) 14
(c) 16 (d) both (a) and (c)

Solution $a^n + b^n$ is divisible by $(a + b)$ for every odd n , $n \in N$

Hence $31^n + 17^n$ is divisible by $48 (= 31 + 17)$ and since 16 is the factor of 48. So the given expression is divisible by both (a) and (c). Hence (d) is the most appropriate answer.

Exp. 19) Which is not the factor of $4^{6n} - 6^{4n}$ for any positive integer n ?

- (a) 5 (b) 25
(c) 7 (d) none of these

Solution $\because 4^{6n} - 6^{4n} = (64)^{2n} - (36)^{2n} = (64^n + 36^n)(64^n - 36^n)$

For $n = 1, 3, 5, \dots$ etc. $(64^n + 36^n)$ is divisible by 100 and all its factors. Also $(64^n - 36^n)$ is divisible by 28 and all its factors.

Again for $n = 2, 4, 6, \dots$ etc. $(64^n - 36^n)$ is always divisible by 100 and all its factors. Also it is divisible by 28 and all its factors. Hence (d).

Exp. 20) $19^n - 1$ is :

- (a) always divisible by 9 (b) always divisible by 20
(c) is never divisible by 19 (d) only (a) and (c) are true

Solution $19^n - 1$ is divisible by $18 (= 19 - 1)$ when n is even or odd. So (a) is correct.

$19^n - 1$ is divisible by 20 only when n is even so (b) is wrong.

$19^n - 1$ is never divisible by 19 which is correct. Thus (d) is the most appropriate answer.

Concepts of Remainder

Some Important Results

- Let us assume that when $a_1, a_2, a_3, \dots, a_n$ are individually divided by d , the respective remainders are $R_1, R_2, R_3, \dots, R_n$. Now, if we divide $(a_1 + a_2 + a_3 + \dots + a_n)$ by d , we get the same remainder as when we get by dividing $(R_1 + R_2 + R_3 + \dots + R_n)$ by d .

Exp. 1) Find the remainder when $38 + 71 + 85$ is divided by 16.

Solution When we divide 38, 71 and 85 we obtain the respective remainders as 6, 7 and 5. Now we can obtain the required remainder by dividing the sum of remainders (i.e., $6 + 7 + 5 = 18$) by 16, which is 2 instead of dividing the sum of actual given numbers (i.e., $38 + 71 + 85 = 194$) by 16, which also gives us the same remainder 2.

Exp. 2) Find the remainder when

$1661 + 1551 + 1441 + 1331 + 1221$ is divided by 20.

Solution The remainders when 1661, 1551, 1441, 1331, 1221 are divided by 20 are 1, 11, 1, 11, 1. So the required remainder can be obtained just by dividing $1 + 11 + 1 + 11 + 1 (= 25)$ by 20. Hence the required remainder is 5.

Exp. 3) What is the remainder when

$678 + 687 + 6879 + 6890$ is divided by 17?

Solution The individual remainders when 678, 687, 6879, 6890 are divided by 17 are 15, 7, 11, 5, respectively. Hence the required remainder can be obtained by dividing $15 + 7 + 11 + 5 (= 38)$ by 17. Thus the required remainder is 4.

Exp. 4) What is the remainder when

$(10 + 10^2 + 10^3 + 10^4 + 10^5)$ is divided by 6?

Solution The remainder when 10 is divided by 6 is 4

The remainder when 100 is divided by 6 is 4

The remainder when 1000 is divided by 6 is 4

The remainder when 10000 is divided by 6 is 4

The remainder when 100000 is divided by 6 is 4

So the required remainder can be obtained by dividing

$4 + 4 + 4 + 4 + 4 (= 20)$ by 6. Thus the required remainder is 2.

2. When 'a' is divided by 'd' the remainder is R and when 'a₁' and 'a₂' are divided by 'd' the remainders are R₁ and R₂, then the remainder R will be equal to the difference of remainders R₁ and R₂ if a₁ - a₂ = a.

For example the remainder when 63 is divided by 35 is 28 and this can be obtained by taking the 63 as difference of any two numbers say 100 and 37 then the remainders when 100 and 37 are divided by 35 are 30 and 2 then the required remainder is 28 (= 30 - 2) [i.e., Remainder of $\frac{63}{35}$

$$= \text{Remainder of } \frac{100}{35} - \text{Remainder of } \frac{37}{35} = 30 - 2 = 28]$$

NOTE If the obtained remainder in a division has negative sign, then the actual remainder can be obtained by adding the divisor with initially obtained remainder containing negative sign.

For example, the remainder of $\frac{63}{35}$

$$= \text{Remainder of } \frac{120}{35} - \text{Remainder of } \frac{57}{35} = 15 - 22 = -7$$

So, the required remainder = 35 - 7 = 28

3. When a₁, a₂, a₃, ... are divided by a divisor d the respective remainders obtained are R₁, R₂, R₃, ..., then the remainder when (a₁ × a₂ × a₃ × ...) is divided by 'd' can be obtained by dividing (R₁ × R₂ × R₃ × ...) by d. For example when 365 × 375 × 389 is divided by 35 the remainders can be obtained by dividing the product of the remainders which are obtained by dividing 365, 375, 389 individually. Now since the remainders are 15, 25, 4 so the required remainder = Remainder of (15 × 25 × 4) when divided by 35.

$$\begin{aligned} \text{Thus the required remainder} &= \frac{15 \times 25 \times 4}{35} = \frac{60 \times 25}{35} \\ &= \frac{25 \times 25}{35} = \frac{625}{35} = 30 \end{aligned}$$

Thus 30 is the remainder.

NOTE The required remainder can be obtained by considering the remainders as the dividends.

$$\begin{aligned} \text{Thus the required remainder} &= R_1 \times R_2 \times R_3 \times R_4 \\ &= R_5 \times R_3 \times R_4 = R_6 \times R_4 = R_7 \end{aligned}$$

where R₇ is the required remainder and R₁, R₂, ..., R₆ are the remainders which are obtained by dividing the remainders perse.

Exp. 1) Find the remainder when 123 × 1234 is divided by 15.

Solution The remainder of $\frac{123 \times 1234}{15} = \text{Remainder of } \frac{3 \times 4}{15}$

(3 and 4 are the remainders of 123 and 1234 when divided by 15)

$$= \text{Remainder of } \frac{12}{15} = 12$$

Exp. 2) Find the remainder when

1719 × 1721 × 1723 × 1725 × 1727 is divided by 18.

Solution Since you can understand that applying the traditional method of getting the remainder is very difficult since first of all you have to take the product of all the numbers as given in the expression, then this a very large product that has to be divided by 18, which is very lengthy and tedious process.

Thus we find the remainder of the remainders.

Hence, the remainder of

$$\begin{aligned} &\frac{1719 \times 1721 \times 1723 \times 1725 \times 1727}{18} \\ &= \frac{9 \times 11 \times 13 \times 15 \times 17}{18} \end{aligned}$$

(9, 11, 13, 15, 17 are the remainders when the given numbers are divided by 18.)

$$\begin{aligned} &= \frac{99 \times 13 \times 15 \times 17}{18} \\ &= \frac{9 \times 13 \times 255}{18} = \frac{117 \times 255}{18} \\ &= \frac{9 \times 3}{18} = \frac{27}{18} = 9 \end{aligned}$$

Exp. 3) Find the remainder when 7⁷ is divided by 4.

$$\begin{aligned} \text{Solution } \frac{7^7}{4} &= \frac{7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7}{4} \\ &= \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{4} = \frac{9 \times 9 \times 9 \times 3}{4} \\ &= \frac{1 \times 1 \times 1 \times 3}{4} = \frac{3}{4} \end{aligned}$$

Thus the required remainder is 3.

Exp. 4) Find the remainder when 11⁸ is divided by 7.

$$\begin{aligned} \text{Solution } \frac{11^8}{7} &= \frac{11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11}{7} \\ &= \frac{4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4}{7} \\ &= \frac{16 \times 16 \times 16 \times 16}{7} \\ &= \frac{2 \times 2 \times 2 \times 2}{7} = \frac{16}{7} \end{aligned}$$

Thus the required remainder = 2

Now you can see that, if in the above problem it is to be asked that find the remainder when 11⁷⁵ is divided by 7, then it would be very difficult to solve it within a short span of time because it would involve a very large calculation if done in the above shown manner.

So follow the steps in which I will show you the appropriate process, which is generally known as "pattern method to find the remainder."

The remainder when 11^1 is divided by 7 is 4

The remainder when 11^2 is divided by 7 is 2

The remainder when 11^3 is divided by 7 is 1

The remainder when 11^4 is divided by 7 is 4

The remainder when 11^5 is divided by 7 is 2

The remainder when 11^6 is divided by 7 is 1

The remainder when 11^7 is divided by 7 is 4

So you can see that you are getting a pattern, if you follow it you can get the remainder for higher powers of 11.

Now you are seeing that after every 3 steps the cycle of remainders is repeating its course as after every 7 days a particular day repeats itself.

Now if we assume that the month of March starts with Monday, then which day will fall on 24th March of the same year? So you will quickly respond that it would be Wednesday.

Let us see how?

M	T	W	Th	F	Sat	Sun
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

As we can see that after every 7 days Monday comes on 1st, 8th, 15th, 22nd etc. So what we do is just divide the given date by 7. If it is divisible by 7 then the last day of the week (*i.e.*, last day of the cycle) will be the required day. If it is not divisible by 7 then it must leave some remainder then we need to calculate the day according to the remainder.

For instance in the above example when we divide 24 by 7, then we get the remainder 3.

So we calculate the 3rd day starting from the first day of the week (or cycle). So the required day will be Wednesday.

Now, If I ask that which day will fall on 7th, 14th, 21st or 28th of this month then your response will be Sunday since all the dates given above are divisible by 7 which is the period of the cycle.

If the dates were 1, 8, 15, ... etc then the required day would have been Monday since when we divide these given dates by 7 the remainder is 1 so the dates which are corresponding to 1 *i.e.*, 8, 15, 22, 29 etc. will fall on Monday.

So I hope this discussion will be very helpful to you while solving the concerned problems. Thus we can say that the remainder when 11^{75} is divided by 7 is 1, since 75 is divisible by 7.

Exp. 5) Find the remainder when 5^{123} is divided by 7.

Solution The remainder when 5^1 is divided by 7 is 5

The remainder when 5^2 is divided by 7 is 4

The remainder when 5^3 is divided by 7 is 6

The remainder when 5^4 is divided by 7 is 2

The remainder when 5^5 is divided by 7 is 3

The remainder when 5^6 is divided by 7 is 1

The remainder when 5^7 is divided by 7 is 5

The remainder when 5^8 is divided by 7 is 4 etc.

So we see that the cyclic period of remainder is 6, since after 6 steps the remainders start repeating.

Now we divide the power of 5 *i.e.*, 123 by 6, then it leaves the remainder 3.

It means the required remainder will be equal to the corresponding remainder when 5^3 is divided by 7, which is 6.

So the remainder when 5^{123} is divided by 7 is 6.

Explanation : Remainder $\frac{5^{123}}{7} = \text{Remainder } \frac{5^{120} \times 5^3}{7}$
 $= \text{Remainder } \frac{(5^6)^{20} \times 5^3}{7}$
 $= \text{Remainder } \frac{5^3}{7}$
 $= \text{Remainder is 6.}$

NOTE Remember that when you get the remainder 1 while obtaining the cyclic period, you can stop the further calculation because at this step your cyclic period ends and in the next step the remainder starts repeating as in the above example at 5^6 we get the remainder 1.

So we need not calculate further for $5^7, 5^8, 5^9$ etc., because all these numbers will give the same remainders as $5^1, 5^2, 5^3, \dots$, etc. respectively.

Exp. 6) Find the remainder when 123^{321} is divided by 5.

Solution The remainder when 123 is divided by 5 is 3

So, the remainder when 123^{321} is divided by 5 is same as when 3^{321} is divided by 5.

Now, the remainder when 3^1 is divided by 5 is 3

The remainder when 3^2 is divided by 5 is 4

The remainder when 3^3 is divided by 5 is 2

The remainder when 3^4 is divided by 5 is 1

The remainder when 3^5 is divided by 5 is 3

So the cyclic period is 4 since at 3^4 we get the remainder 1 (after which the cycle starts repeating).

Thus the remainder when 3^{321} is divided by 5 is 3 since we get the remainder 1 when 321 is divided by 4 (the cyclic period).

or Rem. $\frac{123^{321}}{5} = \text{Rem. } \frac{3^{321}}{5} = \text{Rem. } \frac{3^{320} \times 3^1}{5}$
 $= \text{Rem. } \frac{(3^4)^{80} \times 3^1}{5} = \text{Rem. } \frac{3^1}{5}$
 $= \text{Remainder is 3.}$

Exp. 7) Find the remainder when $923^{888} + 235^{222}$ is divided by 4.

Solution Rem $\frac{923^{888} + 235^{222}}{4}$
 $= \text{Rem. } \frac{3^{888} + 3^{222}}{4}$
 $= \text{Rem. } \frac{1 + 1}{4} = \frac{2}{4}$

Thus the remainder is 2.

Exp. 8) Find the remainder of $\frac{3^{9415}}{80}$.

Solution $\text{Rem } \frac{3^{9415}}{80} = \text{Rem } \frac{3^{9412} \times 3^3}{80}$
 $= \text{Rem } \frac{(3^4)^{2353} \times 3^3}{80}$ [Since the cyclic period is 4]
 $= \text{Rem } \frac{(81)^{2353} \times 3^3}{80} = \text{Rem } \frac{1 \times 27}{80} = \text{Rem } \frac{27}{80}$

Thus the remainder is 27.

Exp. 9) Find the remainder when

$10^1 + 10^2 + 10^3 + 10^4 + 10^5 + \dots + 10^{99}$ is divided by 6.

Solution The remainder when 10^1 is divided by 6 is 4

The remainder when 10^2 is divided by 6 is 4

The remainder when 10^3 is divided by 6 is 4

The remainder when 10^4 is divided by 6 is 4

The remainder when 10^5 is divided by 6 is 4

Thus the remainder is always 4.

So, the required remainder = $\frac{4 + 4 + 4 + \dots \text{99 times}}{6} = \frac{396}{6}$

Thus the remainder is zero.

Exp. 10) The remainder of $\frac{888^{222} + 222^{888}}{3}$ is :

- (a) 0 (b) 1 (c) 2 (d) 3

Solution Remainder $\frac{888^{222} + 222^{888}}{3} = \text{Remainder is zero,}$
 since 888 and 222 both (bases) are divisible by 3.

Exp. 11) The remainder of $\frac{888^{222} + 222^{888}}{5}$ is :

- (a) 0 (b) 1 (c) 3 (d) 4

Solution $\text{Rem. } \frac{888^{222} + 222^{888}}{5}$
 $= \text{Rem. } \frac{888^{222}}{5} + \text{Rem. } \frac{222^{888}}{5}$
 $= \text{Rem. } \frac{3^{222}}{5} + \text{Rem. } \frac{2^{888}}{5}$
 $= \text{Rem. } \frac{(3^4)^{55} \times 3^2}{5} + \text{Rem. } \frac{(2^4)^{222}}{5}$
 $= \text{Rem. } \frac{1 \times 9}{5} + \text{Rem. } 1$
 $= \text{Rem. } \frac{4}{5} + \text{Rem. } \frac{1}{5} = \text{Rem. } \frac{4+1}{5} \Rightarrow \frac{5}{5}$

Thus the remainder is zero.

☞ Alternatively [To check the divisibility by 5 just see the sum of the unit digits which is $10 (= 4 + 6)$

$\therefore 8^{222} \rightarrow 4$ (unit digit)

and $2^{888} \rightarrow 6$ (unit digit)

Hence it is divisible. So there is no remainder]

Exp. 12) The remainder of $\frac{32^{32^{32}}}{7}$:

- (a) 2 (b) 1 (c) 4 (d) none

Solution $32^{32^{32}}$ means $32^{(32 \cdot 32 \cdot 32 \dots 32 \text{ times})}$

Now the remainder of $\frac{32}{7}$ is 4.

Again $\frac{4^{32 \cdot 32 \cdot 32 \dots 32 \text{ times}}}{7} \Rightarrow \frac{4^{2 \cdot 2 \cdot 2 \dots 32 \text{ times}}}{7}$

$$\left[\because \frac{4^1}{7} \rightarrow 4, \frac{4^2}{7} \rightarrow 2, \frac{4^3}{7} \rightarrow 1 \right]$$

Remainder = 4

Since remainder of $\frac{4^2}{7} = \frac{4^8}{7} = \frac{4^{32}}{7} = \frac{4^{128}}{7} = \dots$ is 2

and remainder of $\frac{4^4}{7} = \frac{4^{16}}{7} = \frac{4^{64}}{7} = \frac{4^{256}}{7} = \dots$ is 4

NOTE 1. $\frac{(a+1)^n}{a}$ gives always remainder 1.

2. $\frac{a^n}{(a+1)}$ gives remainder 1 when n is even and gives the remainder a itself when n is odd, where a is any integer and n being the positive integer.

Exp. 13) The remainder when 8^{1785} is divided by 7 is :

- (a) 5 (b) 1
(c) 6 (d) can't be determined

Solution $\therefore \frac{(a+1)^n}{a}$ leaves always remainder 1.

So $\frac{8^{1785}}{7} = \frac{(7+1)^{1785}}{7}$ gives the remainder 1.

Exp. 14) The remainder when $(16)^{3500}$ is divided by 17 is

- (a) 1 (b) 0 (c) 16 (d) none

Solution $\therefore \frac{a^n}{a+1}$ gives remainder 1 when n is even

So the remainder of $\frac{16^{3500}}{17} = \frac{16^{3500}}{(16+1)}$ is 1.

Hence (a) is the correct option.

Exp. 15) The remainder when $(3)^{81}$ is divided by 28 is :

- (a) 1 (b) 8 (c) 18 (d) 26

Solution The remainder of $\frac{3^{81}}{28} \left[= \frac{(3^3)^{27}}{28} = \frac{(27)^{27}}{28} \right]$ is 27.

Since $\frac{a^n}{(a+1)}$ gives remainder a when n is odd.

Exp. 16) The remainder of $\frac{2^{243}}{3^2}$ is :

- (a) 8 (b) 10 (c) 4 (d) none

Solution $\frac{2^{243}}{3^2} = \frac{(2^3)^{81}}{9} = \frac{8^{81}}{9} = \frac{8^{81}}{(8+1)}$

Hence the remainder is 8 since the power of 8 is odd.

Exp. 17) The remainder when $(3)^{67!}$ is divided by 80 :

- (a) 0 (b) 1
(c) 2 (d) can't be determined

Solution $\because 3^4 = 81$, so $\frac{3^{4n}}{80}$ gives the remainder 1.

Thus $\frac{3^{67!}}{80}$ will also give the remainder 1.

Since $67! = 4n$ for a positive integer n .

Exp. 18) The remainder of $\frac{39^{93!}}{40}$ is :

- (a) 0 (b) 1 (c) 39 (d) 13

Solution $\because \frac{a^n}{a+1}$ gives remainder 1, when n is even, Now since

$93!$ is an even number, hence the remainder is 1. Thus (b) is the correct choice.

Exp. 19) The remainder of $\frac{2^{59!}}{255}$ is :

- (a) 0 (b) 1 (c) 55 (d) 56

Solution $2^{59!}$ can be expressed as 2^{8n}

$$\text{So, Rem. } \frac{2^{59!}}{255} = \text{Rem. } \frac{2^{8n}}{255} = \text{Rem. } \frac{(256)^n}{255}$$

Hence the remainder is 1.

Introductory Exercise 1.9

1. Find the last two digits of 64^{81}

- (a) 84 (b) 24
(c) 64 (d) 44

2. If $x + \frac{1}{x} = 2$, then the value of $x^2 + \frac{1}{x^2}$ is :

- (a) 6 (b) 4
(c) 2 (d) 0

3. If $a = 24, b = 26, c = 28$, then the value of $a^2 + b^2 + c^2 - ab - bc - ac$ will be :

- (a) 0 (b) 4
(c) 8 (d) 12

4. If $(a^2 + b^2)^3 = (a^3 + b^3)^2$ and $ab \neq 0$ then $\left(\frac{a}{b} + \frac{b}{a}\right)^6$ is equal to :

- (a) $\frac{a^6 + b^6}{a^3b^3}$ (b) $\frac{64}{729}$
(c) 1 (d) $\frac{a^6 + a^3b^3 + b^6}{a^2b^4 + a^4b^2}$

5. If $x - y = 1$, then the value of $x^3 - y^3 - 3xy$ will be :

- (a) 1 (b) -1
(c) 3 (d) -3

6. If $x = 1234, y = 4321, z = -5555$, what is the value of $\frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy}$ is:

- (a) 0 (b) 1
(c) 3 (d) $9/4$

7. Find the remainder when $2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^{255}$ is divided by 255.

- (a) 0 (b) 1
(c) 127 (d) none of these

8. Among the expression $(1 - 3p), [1 - (3p)^2], [1 - (3p)^3]$ and $[1 - (3p)^4]$ the number of factors of $(1 - 81p^4)$ is :

- (a) one (b) two (c) four (d) three

9. The remainder when $x^4 - y^4$ is divided by $x - y$ is :

- (a) 0 (b) $x + y$
(c) $x^2 - y^2$ (d) $2y^4$

10. If $x - \frac{1}{x} = 2$, then the value of $x^4 + \frac{1}{x^4}$ is :

- (a) 4 (b) 8
(c) 12 (d) 34

11. If the sum and the product of two numbers are 25 and 144; respectively, then the difference of the numbers must be :

- (a) 3 (b) 5
(c) 7 (d) 11

12. The sum of two numbers is 9 and the sum of their squares is 41. The numbers are :

- (a) 4 & 5 (b) 1 & 8
(c) 3 & 6 (d) 2 & 7

13. The square root of $\frac{\left(3\frac{1}{4}\right)^4 - \left(4\frac{1}{3}\right)^4}{\left(3\frac{1}{4}\right)^2 - \left(4\frac{1}{3}\right)^2}$:

- (a) $5\frac{7}{12}$ (b) $5\frac{5}{12}$
(c) $5\frac{12}{13}$ (d) $5\frac{1}{6}$

14. The value of $\frac{(119)^2 + (119)(111) + (111)^2}{(119)^3 - (111)^3}$ is :

- (a) 8 (b) $\frac{1}{8}$ (c) 230 (d) $\frac{1}{230}$

15. The value of $\frac{(1.5)^3 + (4.7)^3 + (3.8)^3 - 3 \times 1.5 \times 4.7 \times 3.8}{(1.5)^2 + (4.7)^2 + (3.8)^2 - (1.5 \times 4.7) - (4.7 \times 3.8) - (1.5 \times 3.8)}$ is :

- (a) 8 (b) 9
(c) 10 (d) 11

16. Simplified value of $\frac{8.73 \times 8.73 \times 8.73 + 4.27 \times 4.27 \times 4.27}{8.73 \times 8.73 - 8.73 \times 4.27 + 4.27 \times 4.27}$ is :
 (a) 11 (b) 12 (c) 13 (d) 14
17. The expression $(a - b)^3 + (b - c)^3 + (c - a)^3 = 0$ if :
 (a) $a < b < c$ (b) $a > b > c$
 (c) $a = b = c$ (d) $a \neq b = c$
18. If $a + b + c = 11$, $a^2 + b^2 + c^2 = 51$, what is the value of $ab + bc + ac$?
 (a) 24 (b) 28 (c) 32 (d) 35
19. If $x + y + z = 0$, then the value of $\frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy}$ is :
 (a) 3 (b) -3 (c) 0 (d) $3xyz$
20. If $\left(x - \frac{1}{x}\right) = 5$, then $x^3 - \frac{1}{x^3}$ equals :
 (a) 125 (b) 130
 (c) 135 (d) 140
21. The value of $\left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$ is :
 (a) 0 (b) 1 (c) 4 (d) 16
22. The continued product of $(1 + x)$, $(1 + x^2)$, $(1 + x^4)$, $(1 + x^8)$ and $(1 - x)$ is :
 (a) $(1 - x^8 + x^{16})$
 (b) $(x^8 + x^{16})$
 (c) $(1 - x^{16})$
 (d) $(x^{16} - 1)$

1.10 Rational and Irrational Numbers

Rational Numbers

The numbers which can be expressed as the ratio of any two integers *i.e.*, in the form of $\frac{p}{q}$, where p, q are integers, prime

to each other (*i.e.*, p and q must be coprime) and $q \neq 0$ are called the rational numbers. These numbers are denoted by ' Q '. Thus $3, -4, \frac{1}{7}, -\frac{3}{8}, \sqrt{16}, \sqrt{25}$ etc. are rational numbers.

All integers and all fractions are rational numbers, which are also called commensurable quantities.

Facts About Rational Numbers

- The denominator of any rational number can never be zero since the division by zero is undefined.
- Every integer *e.g.*, $0, \pm 1, \pm 2, \pm 3$ etc. is a rational number since it can be expressed as
 $7 = \frac{7}{1}, -3 = -\frac{3}{1}, 0 = \frac{0}{1}$ etc.
- All the decimal numbers which are terminating are rational numbers *e.g.*,
 $0.14 = \frac{14}{100}, -8.5 = -\frac{85}{10}, 0.006 = \frac{6}{1000}$ etc.
- All the recurring decimals, which are non terminating, are the rational numbers.
e.g., $0.4444\dots, 1.4141\dots, 6.785785\dots, 1.42857142857\dots$
 $\dots \frac{22}{7}$ etc.

 **NOTE** π is not a rational number, since the exact value of π is not $\frac{22}{7}$.

Properties of Rational Numbers

- Closure law** Addition and multiplication of any two rational number is also a rational number.
- Commutative law** For any two rational numbers $\frac{a}{b}, \frac{c}{d}$, $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$ and $\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$
- Associative law** : For any three rational numbers $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$.

$$\left(\frac{a}{b} + \frac{c}{d}\right) + \frac{e}{f} = \frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right)$$

and $\left(\frac{a}{b} \cdot \frac{c}{d}\right) \cdot \frac{e}{f} = \frac{a}{b} \cdot \left(\frac{c}{d} \cdot \frac{e}{f}\right)$

- Additive and Multiplicative Identity :**

$$\frac{a}{b} + 0 = \frac{a}{b} \quad (\because 0 = \frac{0}{1} \text{ is an additive identity})$$

and $\frac{a}{b} \times 1 = \frac{a}{b} \quad (\because 1 = \frac{1}{1} \text{ is a multiplicative identity})$

- Additive and Multiplicative Inverse :**

$$\frac{a}{b} + \left(-\frac{a}{b}\right) = \frac{0}{1} \quad \left(-\frac{a}{b} \text{ is additive inverse of } \frac{a}{b}\right)$$

$$\frac{a}{b} \times \left(\frac{b}{a}\right) = \frac{1}{1} \quad \left(\frac{b}{a} \text{ is multiplicative inverse of } \frac{a}{b}\right)$$

where $a \neq 0, b \neq 0$

- Rational Numbers** Do not follow the Commutative law and Associative law for the subtraction and division.

7. **Distribution law :** For any three rational numbers a, b, c $a(b+c) = a \cdot b + a \cdot c$ and $a(b-c) = a \cdot b - a \cdot c$
8. If x and y are any two rational numbers such that $x < y$ then $\frac{1}{2}(x+y)$ is also a rational number lying between x and y which shows that **there are infinite number of rational numbers between any two rational numbers.**

Irrational Numbers

The numbers which can't be expressed in the form $\frac{p}{q}$, where

p, q are two integers prime to each other and $q \neq 0$ are called irrational numbers. Thus $\sqrt{2}, \sqrt{3}, -\sqrt{3}, \sqrt[3]{4}, \sqrt[3]{6}, \sqrt{5}, \pi$, etc. are irrational numbers. The decimal representation of these numbers is non-repeating and non-terminating.

e.g., $7.2030030003 \dots, \sqrt{2} = 1.41421\dots, \sqrt{3} = 1.732\dots$

Properties of Irrational Numbers

- (i) The set 'P' of all irrational numbers is not closed for addition since the sum of two irrationals need not be irrational e.g., $(3+\sqrt{5}) \in P, (3-\sqrt{5}) \in P$ but $(3+\sqrt{5}) + (3-\sqrt{5}) = 6 \notin P$
- (ii) The set P of all irrational number is not closed for multiplication since the product of two irrationals need not be irrational.
e.g., $\sqrt{3} \in P, -\sqrt{3} \in P$ but $\sqrt{3} \times -\sqrt{3} = -3 \notin P$

Real Numbers

All the rational and all irrational numbers are called as real numbers i.e., the set of real numbers is the union of entire rational and irrational numbers.

e.g., $-3, 2, 0, \frac{5}{7}, \sqrt{4}, \sqrt{2}, \sqrt{3}, \pi, e, \frac{22}{7}, 0$, etc.

NOTE For some real numbers a and b

1. $a > b \Rightarrow -a < -b$
2. $a > b \Rightarrow \frac{1}{a} < \frac{1}{b}$
3. $a > b \Rightarrow a + k > b + k$
4. $a > b \Rightarrow ak > bk$, when $k > 0$
5. $a \cdot b = 0 \Rightarrow$ either a is zero or b is zero or both a and b are zero.

Imaginary Numbers

If the square of a number is negative then this number is called as an imaginary number. e.g., $\sqrt{-1}, \sqrt{-2}, \sqrt{-3}$ etc.

An imaginary number is denoted by 'i', where $i = \sqrt{-1}$

Facts About Imaginary Numbers

1. $i^0 = 1, i^1 = i, i^2 = -1, i^3 = -i, i^4 = 1$, etc.
2. $i^n = i^{4p+r} = (i^4)^p \times i^r = 1 \times i^r = i^r$
3. $\sqrt{a} \times \sqrt{b} \neq \sqrt{ab}$, if both a, b are negative i.e., imaginary otherwise at least one of a, b must be whole number e.g.,

$$\sqrt{-5} \times \sqrt{-7} = \sqrt{5i} \times \sqrt{7i} = \sqrt{35i^2} = -\sqrt{35}$$

$$\text{but } \sqrt{-5} \times \sqrt{-7} = \sqrt{-5 \times -7} \neq \sqrt{35}$$

Exp. 1) Find the value of i^{63} .

Solution $i^{63} = (i^4)^{15} \times i^3 = i^3 = -i$

Exp. 2) Find the value of i^{170} .

Solution $i^{170} = (i^4)^{42} \times i^2 = i^2 = -1$

Exp. 3) Find the value of $\frac{1}{i^{123}}$

Solution $\frac{1}{i^{123}} = \frac{1}{i^{120} \times i^3} = \frac{1}{1 \times i^3} = \frac{1}{i^3} = \frac{1}{i^3} \times \frac{i}{i} = \frac{i}{i^4} = i$

Exp. 4) Find the value of $\sqrt{-16} \times \sqrt{-25}$.

Solution $\sqrt{-16} \times \sqrt{-25} = 4i \times 5i = 20i^2 = -20$

Exp. 5) Find the value of $(\sqrt{-1})^{4n+1}$.

Solution $(\sqrt{-1})^{4n+1} = i^{4n+1} = i^{4n} \times i = i$

Exp. 6) Find the value of i^{-343} .

Solution $i^{-343} = \frac{1}{i^{343}} = \frac{1}{(i^4)^{85} \times i^3} = \frac{1}{1 \times i^3} = \frac{1 \times i}{i^3 \times i} = \frac{i}{i^4} = i$

Exp. 7) Find the value of $i^{248} + i^{341} + i^{442} + i^{543}$.

Solution $i^{248} + i^{341} + i^{442} + i^{543} = 1 + i + (-1) + (-i) = 0$

Exp. 8) The value of $\frac{1}{i^n} + \frac{1}{i^{(n+3)}} + \frac{1}{i^{(n+2)}} + \frac{1}{i^{(n+1)}}$ is :

- (a) -1 (b) 1
(c) 0 (d) can't be determined

Solution $\frac{1}{i^n} + \frac{1}{i^{(n+3)}} + \frac{1}{i^{(n+2)}} + \frac{1}{i^{(n+1)}}$ is :

$$\begin{aligned} &= \frac{1}{i^n} + \frac{1}{i^n \times i^3} + \frac{1}{i^n \times i^2} + \frac{1}{i^n \times i} \\ &= \frac{1}{i^n} + \frac{1}{i^n} \times \frac{i}{i^4} + \frac{1}{i^n} \times \frac{i^2}{i^4} + \frac{1}{i^n} \times \frac{i^3}{i^4} \\ &= \frac{1}{i^n} + \frac{i}{i^n} + \frac{i^2}{i^n} + \frac{i^3}{i^n} = \frac{1}{i^n} + \frac{i}{i^n} + \frac{-1}{i^n} - \frac{i}{i^n} = 0 \end{aligned}$$

Hence (c) is the correct option.

Complex Numbers

The combination of real number and imaginary number is known as complex number *i.e.*, the number of the form $a + ib$ where a and b are purely real numbers and $i = \sqrt{-1}$, is called as a complex number. It is denoted by $z = a + ib$ where real $z = a$ and imaginary $z = b$.

- Property of order :** For any two complex numbers $(a + ib)$ and $(c + id)$ $(a + ib) < (\text{or } >) c + id$ is not defined *e.g.*, $9 + 4i < 3 - i$ makes no sense.
- A complex number is said to be purely real if $\text{Im}(z) = 0$ and it is said to be purely imaginary if $\text{Re}(z) = 0$. The complex number $0 = 0 + i \cdot 0$ is both purely real and purely imaginary.
- The sum of four consecutive powers of i is zero.

$$\text{i.e., } i^k + i^{k+1} + i^{k+2} + i^{k+3} = 0$$

Operations on Complex Numbers

Let z_1, z_2 be two complex numbers such that $z_1 = (a_1 + ib_1)$ and $z_2 = (a_2 + ib_2)$ then :

- $z_1 \pm z_2 = (a_1 \pm a_2) + i(b_1 \pm b_2)$
- $z_1 \cdot z_2 = (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + b_1 a_2)$
- $z_1 = z_2 \Leftrightarrow a_1 = a_2 \text{ and } b_1 = b_2$

Conjugate Complex Number

The complex number $z = a + ib$ and $\bar{z} = (a - ib)$ are called the complex conjugate of each other, where $i = \sqrt{-1}$, $b \neq 0$ and a, b are real numbers.

Properties of Conjugate

For any z there exists the mirror image of z along the real axis denoted as $\bar{z}$, then

- $z = \bar{z} \Leftrightarrow z$ is purely real
- $z = -\bar{z} \Leftrightarrow z$ is purely imaginary
- $\overline{(\bar{z})} = z$
- $\text{Re}(z) = \text{Re}(\bar{z}) = \frac{z + \bar{z}}{2}$
- $\text{Im}(z) = \frac{z - \bar{z}}{2i}$
- $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$
- $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$
- $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$
- $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$

$$(x) \quad z_1 \bar{z}_2 + \bar{z}_1 z_2 = 2 \text{Re}(\bar{z}_1 z_2) = 2 \text{Re}(z_1 \bar{z}_2)$$

$$(xi) \quad \overline{z^n} = (\bar{z})^n$$

$$(xii) \quad \text{If } z = f(z_1), \text{ then } \bar{z} = f(\bar{z}_1)$$

Modulus of a Complex Number

The modulus of a complex number

$$z = (a + ib) \text{ is represented as } |z| = \sqrt{a^2 + b^2}.$$

Properties of Modulus

- $|z| \geq 0 \Rightarrow |z| = 0$ iff $z = 0$ and $|z| > 0$ iff $z \neq 0$
- $-|z| \leq \text{Re}(z) \leq |z|$ and $-|z| \leq \text{Im}(z) \leq |z|$
- $|z| = |\bar{z}| = |-z| = |-\bar{z}|$
- $z \bar{z} = |z|^2$
- $|z_1 \pm z_2| \leq |z_1| + |z_2|$
- $|z_1 \pm z_2 \pm z_3 \pm \dots \pm z_n| \leq |z_1| \pm |z_2| \pm |z_3| \pm \dots \pm |z_n|$
- $|z_1 \pm z_2| \geq ||z_1| - |z_2||$
- $|z_1 z_2| = |z_1| |z_2|$
- $\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$
- $|z^n| = |z|^n$
- $||z_1| - |z_2|| \leq |z_1 + z_2| \leq |z_1| + |z_2|$
- $|z_1 \pm z_2|^2 = (z_1 \pm z_2)(\bar{z}_1 \pm \bar{z}_2) = |z_1|^2 + |z_2|^2 \pm (z_1 \bar{z}_2 + \bar{z}_1 z_2)$
- $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

n th root of Unity

- Unity has n roots namely $1, \omega, \omega^2, \omega^3, \dots, \omega^{n-1}$ which are in geometric progression and the sum of these n roots is zero.
- Product of n roots is $(-1)^{n-1}$.

Square Root

The square root of $z = a + ib$ are

$$\pm \left(\sqrt{\frac{|z| + a}{2}} + i \sqrt{\frac{|z| - a}{2}} \right) \text{ for } b > 0$$

$$\text{and } \pm \left(\sqrt{\frac{|z| + a}{2}} - i \sqrt{\frac{|z| - a}{2}} \right) \text{ for } b < 0$$

Remember

1. The square root of i are $\pm \left(\frac{1+i}{\sqrt{2}}\right)$ when $b=1$
2. The square root of $-i$ are $\pm \left(\frac{1-i}{\sqrt{2}}\right)$ when $b=-1$
or $\sqrt{a+ib} = \pm(x+iy)$ and $\sqrt{a-ib} = \pm(x-iy)$
where $x = \left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{1/2}$ and $y = \left(\frac{\sqrt{a^2+b^2}-a}{2}\right)^{1/2}$

Cube root of unity

Cube root of unity are $1, \omega, \omega^2$ where

- (a) $1 + \omega + \omega^2 = 0$
- (b) $\omega^3 = 1$
- (c) $\bar{\omega} = \omega^2$ and $(\bar{\omega})^2 = \omega$
- (d) $\omega^{3n} = 1, \omega^{3n+1} = \omega, \omega^{3n+2} = \omega^2$

NOTE Iota (i) is neither 0, nor greater than 0, or nor less than 0.

Exp. 1) Find the modulus of $5 + \sqrt{-11}$.

Solution $\because z = 5 + \sqrt{-11} = 5 + \sqrt{11}i$
 $\therefore |z| = \sqrt{25 + 11} = \sqrt{36} = 6$

Exp. 2) If the multiplicative inverse of a complex number is $\frac{\sqrt{5} + 6i}{41}$ then the complex number itself is:

- (a) $\sqrt{5} - 6i$
- (b) $\sqrt{5} + 6i$
- (c) $6 + \sqrt{7}i$
- (d) none of these

Solution Since $z \cdot \frac{(\sqrt{5} + 6i)}{41} = 1$

$$\Rightarrow z = \frac{41}{(\sqrt{5} + 6i)} \times \frac{\sqrt{5} - 6i}{\sqrt{5} - 6i} = \sqrt{5} - 6i$$

Hence (a) is the correct option.

Exp. 3) Which of the following is correct?

- (a) $7 + i > 5 - i$
- (b) $8 + i > 9 - i$
- (c) $13 + i < 25 - 3i$
- (d) none of these

Solution Since the relation is not defined i.e., $(a + ib) \gtrless (c + id)$ is indeterminable. Hence (d) is the correct option.

Exp. 4) The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = -1$ is:

- (a) 5
- (b) 1
- (c) 2
- (d) none of these

Solution Given that $\left(\frac{1+i}{1-i}\right)^n = -1$

$$\Rightarrow \left[\frac{(1+i)(1+i)}{(1-i)(1+i)}\right]^n = -1 \Rightarrow i^n = -1$$

$$\Rightarrow (-1)^{\frac{n}{2}} = (-1) \Rightarrow \frac{n}{2} = 1$$

$$\therefore n = 2$$

Hence (c) is the correct option.

Exp. 5) The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$ is:

- (a) 2
- (b) 4
- (c) 6
- (d) 8

Solution $\left(\frac{1+i}{1-i}\right)^n = \left(\frac{(i+i)(1+i)}{(1-i)(1+i)}\right)^n = \left(\frac{(1+i)^2}{2}\right)^n = i^n = 1$

$$\Rightarrow i^n = 1 = i^4 \Rightarrow n = 4$$

Hence (b) is the correct option.

Exp. 6) $(3 + 2i)^3 = ?$

- (a) $9 - 46i$
- (b) $9 + 46i$
- (c) $-9 + 46i$
- (d) none of these

Solution $(3 + 2i)^3 = 3^3 + (2i)^3 + 3 \cdot 3 \cdot 2i(3 + 2i)$
 $= 27 + 8i^3 + 18i(3 + 2i) = 27 + 8i^3 + 54i + 36i^2$
 $= 27 - 8i + 54i - 36 = (-9 + 46i)$

Hence (c) is the correct option.

Exp. 7) The multiplicative inverse of $(3 + 2i)^2$ is:

- (a) $\frac{12}{169} - \frac{5i}{169}$
- (b) $\frac{5}{169} - \frac{12i}{169}$
- (c) $\frac{5}{13} - \frac{12i}{13}$
- (d) none of these

Solution $Z = (3 + 2i)^2 = 9 + 4i^2 + 12i = 9 - 4 + 12i = 5 + 12i$

$$\text{then } z^{-1} = \frac{1}{5 + 12i} = \frac{1}{5 + 12i} \times \frac{5 - 12i}{5 - 12i}$$

$$= \frac{5 - 12i}{25 - 144i^2} = \frac{5 - 12i}{169} = \left(\frac{5}{169} - \frac{12i}{169}\right)$$

Thus (b) is the correct option.

Exp. 8) If $|z| = 1$ then $\left(\frac{1+z}{1+\bar{z}}\right)$ equals:

- (a) $\bar{z}$
- (b) z
- (c) z^{-1}
- (d) none of these

Solution $\frac{1+z}{1+\bar{z}} = \left(\frac{1+z}{1+\bar{z}}\right) \frac{z}{z} = \frac{z(1+z)}{(z+1)} = z$ ($\because |z| = 1$)

Hence (b) is the correct option.

Exp. 9) The value of $\sqrt{i} + \sqrt{-i}$ is:

- (a) 1
- (b) $\sqrt{2}$
- (c) $-i$
- (d) none of these

Solution $\sqrt{i} + \sqrt{-i} = \sqrt{(\sqrt{i} + \sqrt{-i})^2}$
 $= \sqrt{i - i + 2\sqrt{i}\sqrt{-i}} = \sqrt{2\sqrt{-i^2}} = \sqrt{2}$

Hence (b) is the correct option.

Exp. 10) The value of $\log_e(-1)$ is :

- (a) $i\pi$ (b) 0
(c) i^{-1} (d) does not exist

Solution $\log_e(-1) = \log_e(e^{i\pi}) = i\pi$

Hence (a) is the correct option.

Exp. 11) The number of solutions of the equation $z^2 + |z|^2 = 0$, where z is a complex number is :

- (a) 1 (b) 2
(c) 4 (d) infinitely many

Solution $z^2 + |z|^2 = 0 \Rightarrow z^2 + z\bar{z} = 0$

$$\Rightarrow z(z + \bar{z}) = 0$$

$$\text{or } z \cdot 2\operatorname{Re}(z) = 0$$

$$\therefore z = 0 \text{ and } \operatorname{Re}(z) = 0$$

$$\text{If } z = x + iy \Rightarrow 0 = x + iy \Rightarrow (x, y) = (0, 0)$$

$$\text{and if } z = 0 + ib \Rightarrow (a, b) = (0, b) \text{ but } b \in \mathbb{R}$$

So, there are infinitely many solutions. Since $b \in \mathbb{R}$ Hence (d) is the correct option.

Exp. 12) The value of $\left(\frac{i+\sqrt{3}}{2}\right)^{100} + \left(\frac{i-\sqrt{3}}{2}\right)^{100}$ is :

- (a) -1 (b) 1 (c) 0 (d) i

$$\text{Solution } \frac{i+\sqrt{3}}{2} = \left(\frac{i+\sqrt{3}}{2}\right) \cdot \frac{i}{i} = \left(\frac{-1+\sqrt{3}i}{2i}\right) = \frac{\omega}{i}$$

($\because$ A complex number $a + ib$ for which $|a : b| = 1 : \sqrt{3}$ or $\sqrt{3} : 1$ can always be expressed in terms of i, ω or ω^2)

$$\text{Similarly, } \frac{i-\sqrt{3}}{2} = \frac{\omega^2}{i}$$

$$\therefore \left(\frac{i+\sqrt{3}}{2}\right)^{100} + \left(\frac{i-\sqrt{3}}{2}\right)^{100} = \left(\frac{\omega}{i}\right)^{100} + \left(\frac{\omega^2}{i}\right)^{100}$$

$$= \omega^{99} \cdot \omega + \omega^{198} \cdot \omega^2 \quad [\because \omega^{99} = \omega^{198} = 1]$$

$$= \omega + \omega^2 = -1$$

Hence (a) is the correct option.

Exp. 13) If $1, \omega, \omega^2$ be the cube roots of unity, then the value of $(1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5$ is :

- (a) 0 (b) 16 (c) 32 (d) 64

Solution $\because 1 + \omega + \omega^2 = 0$

$$\therefore 1 + \omega = -\omega^2 \text{ and } 1 + \omega^2 = -\omega$$

$$\therefore (1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5 = (-2\omega)^5 + (-2\omega^2)^5$$

$$= -32(\omega^5 + \omega^{10}) \quad [\because \omega^5 = \omega^2 \text{ and } \omega^{10} = \omega \text{ and } \omega + \omega^2 = -1]$$

$$= 32$$

Hence (c) is the correct option.

Exp. 14) If $1, \omega, \omega^2, \omega^3, \dots, \omega^{n-1}$ be the n th roots of unity, the value of $(9 - \omega)(9 - \omega^2) \dots (9 - \omega^{n-1})$ will be :

- (a) 0 (b) $\frac{9^n - 1}{8}$
(c) $\frac{8^n - 1}{3}$ (d) none of these

Solution Let $x^n = 1 \Rightarrow x = (1)^{1/n}$

$$\Rightarrow x^n - 1 = 0$$

$$\text{or } x^n - 1 = (x - 1)(x - \omega)(x - \omega^2) \dots (x - \omega^{n-1})$$

$$\Rightarrow \frac{x^n - 1}{x - 1} = (x - \omega)(x - \omega^2) \dots (x - \omega^{n-1})$$

Putting $x = 9$ in both sides, we have

$$(9 - \omega)(9 - \omega^2)(9 - \omega^3) \dots (9 - \omega^{n-1}) = \frac{9^n - 1}{8}$$

Hence (b) is the correct choice.

Exp. 15) If $\left(\frac{1-i}{1+i}\right)^{100} = x + iy$ then the value of (x, y) is :

- (a) (0, 1) (b) (0, 0)
(c) (1, 0) (d) (-1, 0)

$$\text{Solution } \frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = \frac{-2i}{2} = -i$$

$$\text{Hence } \left(\frac{1-i}{1+i}\right)^{100} = (-i)^{100} = 1$$

$$\text{Thus } x + iy = 1 \Rightarrow x = 1, y = 0$$

So the value of $(x, y) = (1, 0)$

Hence (c) is the correct option.

Exp. 16) The inequality $a + ib < c + id$ holds if

- (a) $a < c, b < d$ (b) $a < c, b > d$
(c) $a > c, b < d$ (d) none of these

Solution Option (d) is correct.

Exp. 17) The points z_1, z_2, z_3, z_4 in the complex plane form the vertices of a parallelogram iff :

- (a) $z_1 + z_2 = z_3 + z_4$ (b) $z_1 + z_3 = z_2 + z_4$
(c) $z_1 + z_3 > z_2 + z_4$ (d) none of these

Solution If the mid point of $z_1 z_3$ is the same as that of $z_2 z_4$.

$$\text{i.e., } \frac{1}{2}(z_1 + z_3) = \frac{1}{2}(z_2 + z_4)$$

$\Rightarrow z_1 + z_3 = z_2 + z_4$, only then these points can form the parallelogram.

Exp. 18) If $a + ib = \sqrt{\frac{(u+iv)}{(x+iy)}}$ then the value of $a^2 + b^2$ is :

- (a) $\sqrt{\frac{u^2 + v^2}{x^2 + y^2}}$ (b) $\frac{u^2 - v^2}{x^2 - y^2}$
(c) can't be determined (d) none of these

Solution $a^2 + b^2 = |a + ib|^2$

$$= \left| \frac{u + iv}{x + iy} \right|^2$$

(Putting the value of $a + ib$)

$$= \frac{\sqrt{u^2 + v^2}}{\sqrt{x^2 + y^2}}$$

Hence (a) is the correct option.

Exp. 19) The maximum value of $|z|$ when z satisfies the condition $\left|z + \frac{2}{z}\right| = 2$ is :

- (a) $\sqrt{3} - 1$ (b) $\sqrt{3}$ (c) $\sqrt{3} + 1$ (d) 1

Solution $\therefore \left|z + \frac{2}{z} - \frac{2}{z}\right|$
 $|z| \leq \left|z + \frac{2}{z}\right| + \left|\frac{2}{z}\right|$

It means $|z| \leq 2 + \frac{2}{|z|}$

$\Rightarrow |z|^2 \leq 2|z| + 2$

$\Rightarrow |z|^2 - 2|z| + 1 \leq 2 + 1$

$\Rightarrow (|z| - 1)^2 \leq 3$

$\Rightarrow -\sqrt{3} \leq |z| - 1 \leq \sqrt{3}$

$\Rightarrow 1 - \sqrt{3} \leq |z| \leq 1 + \sqrt{3}$

Therefore the maximum value of $|z|$ is $1 + \sqrt{3}$

Exp. 20) The modulus of the complex number $\frac{(2 - i\sqrt{5})}{(1 + 2\sqrt{2}i)}$

is :

- (a) -1 (b) 1 (c) $\sqrt{3}$ (d) $-\sqrt{3}$

Solution The modulus of $\left(\frac{2 - i\sqrt{5}}{1 + 2\sqrt{2}i}\right) = \left|\frac{2 - i\sqrt{5}}{1 + 2\sqrt{2}i}\right|$
 $= \frac{\sqrt{4 + 5}}{\sqrt{1 + 8}} = \frac{3}{3} = 1$

Hence (b) is the correct answer.

Introductory Exercise 1.10

1. Which one of the following is a rational number?

- (a) $(\sqrt{2})^2$ (b) $2\sqrt{2}$ (c) $2 + \sqrt{2}$ (d) $\frac{\sqrt{2}}{2}$

2. Rational number $\frac{-18}{5}$ lies between consecutive integers :

- (a) -2 and -3 (b) -3 and -4
 (c) -4 and -5 (d) -5 and -6

3. Which one of the following statements is correct?

- (a) There can be a real number which is both rational and irrational
 (b) The sum of two irrational number is always irrational
 (c) For any real numbers x and y , $x < y \Rightarrow x^2 < y^2$
 (d) Every integer is a rational number

4. Which one of the following statements is not correct?

- (a) If 'a' is a rational number and b is irrational, then $a + b$ is irrational
 (b) The product of a non-zero rational number with an irrational number is always irrational
 (c) Addition of any two irrational numbers can be an integer
 (d) Division of any two integers is an integer

5. If x be a rational number and y be an irrational number, then :

- (a) both $x + y$ and xy are necessarily irrational
 (b) both $x + y$ and xy are necessarily rational
 (c) xy is necessarily irrational, but $x + y$ can be either rational or irrational
 (d) $x + y$ necessarily irrational, but xy can be either rational or irrational

6. A rational equivalent to $\frac{-24}{20}$ with denominator 25 is :

- (a) $\frac{-30}{25}$ (b) $\frac{30}{25}$ (c) $\frac{-29}{25}$ (d) $\frac{-19}{25}$

7. Let $\frac{a}{b} = \frac{c}{d}$, (where a and b are odd prime numbers). If $c > a$ and $d > b$, then :

- (a) c is not a multiple of a (b) d is not a multiple of b
 (c) $c = ka$, $d = kb$ with $k > 1$
 (d) $c = ka$, $d = lb$ with $k \neq l$

Directions (for Q. Nos. 8 to 10) Answer the following questions based on the information given below. From the set of first 81 natural numbers two numbers p and q are

chosen in order to form the rational number $\frac{p}{q}$ such that

p and q are co-prime.

8. Find the number of values of q so that $\frac{p}{q}$ is always a recurring decimal number.

- (a) 9 (b) 14 (c) 67 (d) 72

9. Find the total number of rational numbers such that $\frac{p}{q}$ is always a terminating decimal number.

- (a) 622 (b) 441 (c) 134 (d) 1134

10. Find the number of non-integral rational numbers $\frac{p}{q}$ between 0 and 1, which can always be expressed as the terminating decimals?

- (a) 167 (b) 144
 (c) 214 (d) 377

11. In the online game of Angry Birds, each bird destroys on an average A number of pigs. If $A = 3$. $abbcabbcabbc \dots$ and out of a , b and c not more than one digit is 0, which of the following could be the total number of birds?

- (a) 1991 (b) 1998
 (c) 29992 (d) 49995

1.11 Various Number Systems

Decimal Number System

Generally we use decimal system in our day to day mathematical applications *e.g.*, counting of the things, accounting in offices etc. Why we call it decimal system?

Since there we use 10 symbols *viz.*, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 to represent the data, that is why we say that the representation is in decimal system.

The sequence of decimal numbers goes on as 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21.. etc. So after 9, each successive number is a combination of 2 (or more) unique symbols of this system.

The decimal system is a positional-value system in which the value of a digit in a number depends on its position. For example 879. Here 8 represents 800, 7 represents 70 and 9 represents 9 units. In essence, 8 carries the most weight of three digits, it is referred as the most significant digit (MSD). The 9 carries the least weight and is called as the least significant digit (LSD)

$$\text{i.e., } 879 = 8 \times 10^2 + 7 \times 10^1 + 9 \times 10^0$$

$$\text{Similarly } 63.78 = 6 \times 10^1 + 3 \times 10^0 + 7 \times 10^{-1} + 8 \times 10^{-2}$$

So, this number is actually equal to 6 tens plus 3 units plus 7 tenths plus 8 hundredths.

$$\begin{aligned} \text{So } 3859276 &= 3 \times 10^6 + 8 \times 10^5 + 5 \times 10^4 \\ &\quad + 9 \times 10^3 + 2 \times 10^2 + 7 \times 10^1 + 6 \times 10^0 \\ \text{and } 9253.827 &= 9 \times 10^3 + 2 \times 10^2 + 5 \times 10^1 + 3 \times 10^0 \\ &\quad + 8 \times 10^{-1} + 2 \times 10^{-2} + 7 \times 10^{-3} \end{aligned}$$

In general, any number is simply the sum of the products of each digit value and its positional (or place) value.

Positional values (weights)	→	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
		↓	↓	↓	↓	↓	↓	↓
		9	2	5	3	8	2	7
		↑			↑			↑
		MSD			Decimal point			LSD

Thus a two digit number can be expressed as $x \times 10^1 + y \times 10^0$, where x is the tens and y is the unit digit.

Thus the two digit number xy is expressed as $10x + y$
Similarly a three digit number xyz is expressed as $10^2x + 10^1y + 10^0z$

or xyz is expressed as $100x + 10y + z$ where x, y, z are the hundreds tens and unit digit respectively.

In general an n digit number in decimal representation, can be expressed as

$$k_1 \times 10^{n-1} + k_2 \times 10^{n-2} + k_3 \times 10^{n-3} + \dots + k_n \times 10^0$$

Important Facts about Decimal Numbers

1. The digits which are always same *i.e.*, when they are seen upside down, they appear the same. *e.g.*, 0, 1 and 8.
2. The digits which create confusion when they are written upside down they change their face values. *e.g.*, 6 and 9.
3. When the digits of a two digit number are reversed *i.e.*, when their position is interchanged, then the difference of the original number and this resultant number is always divisible by 9. Also the difference of these two numbers is exactly the product of the difference of the two digits (*i.e.*, difference of their face values) with 9.

$$\begin{array}{r} xy \\ - yx \\ \hline 9(x - y) \end{array}$$

$$[\because (10x + y) - (10y + x) = 9(x - y)]$$

$$\text{e.g., } 53 - 35 = 9 \times 2 = 18 \quad (\because 5 - 3 = 2)$$

$$\text{e.g., } 82 - 28 = 9 \times 6 = 54 \quad (\because 8 - 2 = 6)$$

NOTE The sum of these two numbers is always divisible by 11.

4. When the digits of a three digit number are reversed *i.e.*, the unit digit becomes hundreds digit and *vice versa*, then the difference between these two numbers is always divisible by 99. Also the difference of these two numbers (*i.e.*, original and resultant number) is exactly equal to the product of the difference of unit and hundreds digit with 99, *i.e.*, $xyz - zyx = 99(x - z)$
 $[\because (100x + 10y + z) - (100z + 10y + x) = 99(x - z)]$

$$\text{For example (i) } 852 - 258 = 99 \times 6 \quad [\because 8 - 2 = 6]$$

$$= 594$$

$$\text{(ii) } 703 - 307 = 99 \times 4 \quad [\because 7 - 3 = 4]$$

$$= 396$$

5. When the digits of a four digit number are reversed *i.e.*, unit digit with thousands digit and *vice versa* and tens digit with hundreds digit and *vice versa*, then the difference between the original number and the resultant number is always divisible by 9.

Exp. 1) The difference between the highest and lowest two digit numbers is :

- (a) 88 (b) 89
(c) 22 (d) 99

Solution Simply $99 - 10 = 89$

Hence (b) is the correct option.

Exp. 2) When a two digit number is reversed, then the new number becomes $\frac{5}{6}$ th of the original number. The

original number is :

- (a) 56 (b) 45
(c) 48 (d) 54

Solution The best way is to go through options.

Now, we consider those options which are divisible by 6, as, the number is an integer.

Again we pick up only those values which on being reversed (their digits) decreases. So only option (d) is suitable. Check it out.

☞ Alternatively Let the original number be $(10x + y)$ then the new number will be $(10y + x)$, then $(10y + x) = \frac{5}{6}(10x + y)$

$$\Rightarrow 55y = 44x$$

$$\Rightarrow \frac{x}{y} = \frac{5}{4}$$

Thus the only possible values of x, y are 5, 4 respectively.

Hence the original number is 54.

☞ Alternatively When you multiply the given (original) number by $\frac{5}{6}$ then the digit of the resultant value gets interchanged.

Exp. 3) The number of two digit numbers which on being reversed (*i.e.*, their digits exchanged the position) gives out perfect square two digit numbers :

- (a) 1 (b) 4
(c) 6 (d) 10

Solution These numbers are 61, 52, 63, 94, 46 and 18. Since these numbers on being reversed give out 16, 25, 36, 49, 64, 81 the two digit perfect square numbers.

Hence (c) is the correct answer.

Exp. 4) The number of two digit numbers which are prime :

- (a) 25 (b) 23
(c) 21 (d) can't be determined

Solution There are 25 prime numbers between 1 and 100. There are 4 prime numbers of 1 digit (*viz.* 2, 3, 5, 7)

So, the number of two digit prime numbers = $25 - 4 = 21$

Hence (c) is the correct option.

Exp. 5) The number of two digit prime numbers, with distinct digits, on being reversed they again give the prime numbers is :

- (a) 3 (b) 5
(c) 8 (d) 11

Solution See, the total number of prime numbers in this case (when there are distinct digits *i.e.*, there are no same digits) the number of prime numbers must be an even number.

Hence (c) is the correct option.

☞ Alternatively These numbers are 13, 31, 17, 71, 37, 73 and 79, 97.

Thus there are total 8 such numbers.

Exp. 6) The total number of two digit numbers whose unit digit is either, same, double, triple or quadruple of the tens digit :

- (a) 13 (b) 9
(c) 18 (d) 90

Solution 11, 22, 33, 44, 55, 66, 77, 88, 99
12, 24, 36, 48
13, 26, 39
14, 28

Thus there are total 18 such numbers. Hence (c) is the correct answer.

Exp. 7) The number of two digit numbers which are perfect square and perfect cube both is :

- (a) 0 (b) 2
(c) can't be determined (d) none of these

Solution Two digit perfect cubes are 27, 64

Two digit perfect squares are 16, 25, 36, 49, 64, 81

Thus there is only one value *i.e.*, 64 which is both perfect square and perfect cube.

Thus (d) is the most appropriate choice.

Exp. 8) Aprajita multiplies a number by 72 instead of 27, then her new answer will increase by, than the actual result :

- (a) $\frac{7}{2}$ (b) $\frac{8}{3}$
(c) $\frac{5}{3}$ (d) none of these

Solution Let the number be k then the original product will be $k \times 27$ and the new product will be $k \times 72$.

So, the difference in the product value

$$= 72k - 27k = 45k$$

Thus, new product will increase by $\frac{45k}{27k} = \frac{5}{3}$

Hence (c) is the correct answer.

☞ Alternatively $\frac{72 - 27}{27} = \frac{5}{3}$

Exp. 9) If the numerator and denominator of a fraction are exchanged then the product of these two fractions becomes equal to 1. The total number of such fractions is

- (a) 1 (b) 31
(c) 13 (d) infinitely many

Solution Let $\frac{x}{y}$ be the fraction where $x, y \neq 0$, then

$$\frac{x}{y} \times \frac{y}{x} = 1 \quad (\text{always})$$

Now since x and y can assume infinite values so (d) is the best answer.

Exp. 10) $\frac{3}{4}$ th of a number is 20 more than half of the same number. The required number is :

- (a) 50 (b) 180 (c) 90 (d) 80

Solution Since the number is an integer so it must be divisible by 4. Hence option (a) and (c) are ruled out.

Now, if we check option (b) we find it is wrong so (d) is the correct.

$$\text{As } 80 \times \frac{3}{4} = 80 \times \frac{1}{2} + 20 \Rightarrow 60 = 60$$

$$\text{Alternatively } x \times \frac{3}{4} = x \times \frac{1}{2} + 20 \Rightarrow x = 80$$

Exp. 11) When 50 is added to the 50% of a number, then the number becomes itself. The required number is :

- (a) 375 (b) 100 (c) 150 (d) 500

$$\text{Solution } 50 + \frac{x}{2} = x \Rightarrow \frac{x}{2} = 50 \Rightarrow x = 100$$

Hence (b) is the correct option.

Exp. 12) If we reverse the digits of a two digit number then the difference between the original number and new number is 27, the difference between the digits is :

- (a) 9 (b) 3
(c) can't be determined (d) none of these

Solution Since $xy - yx = 9(x - y)$

$$\text{Here } 9(x - y) = 27$$

$$\text{Thus } x - y = 3$$

So (b) is the correct answer.

Exp. 13) A two digit number is 4 times the sum of its digits and the unit digit is 3 more than the tens digit. The number is :

- (a) 52 (b) 61 (c) 63 (d) 36

Solution Go through option 36 = 4(3 + 6) and 6 - 3 = 3

Hence (d) is the correct answer.

Alternatively Let x, y be the tens and unit digits respectively, then

$$(10x + y) = 4(x + y) \quad \dots(i)$$

$$\Rightarrow 6x = 3y$$

$$\Rightarrow \frac{x}{y} = \frac{1}{2} \quad \dots(ii)$$

and $y - x = 3 \quad \dots(iii)$
from equation (ii) and (iii)

$$x = 3 \quad \text{and} \quad y = 6$$

Thus the required number is 36.

Exp. 14) When a two digit number is subtracted from the other two digit number which consists of the same digits but in reverse order, then the difference comes out to be a two digit perfect square. The number is :

- (a) 59 (b) 73
(c) 36 (d) not unique

Solution Since the difference between number is a perfect square. So this difference can be only 36, because 36 is the only two digit perfect square contains 9 as a factor.

But there are total 5 numbers possible viz., 15, 26, 37, 48, 59. Since the only condition is that

$$(10x + y) - (10y + x) = 9(x - y) = 36$$

$$\Rightarrow (x - y) = 4$$

Thus (d) is the most appropriate answer.

Exp. 15) There is a three digit number such that the sum of its end digits (unit digit and hundredth place digit) is always a single digit number. Another three digit number is obtained by reversing the position of end digits of the original number. Then what can be the possible sum of the tens digits of both these numbers?

- (a) 5 (b) 12 (c) 15 (d) 26

Solution Since in both the numbers the middle digits i.e., the tens digits are same, it means the sum of these tens digits is always an even number ($\because x + x = 2x$)

So the option (a) and (c) are ruled out.

Now, since the largest possible digit is 9, so the maximum possible sum of these two digits can be 18 (= 9 + 9). Therefore option (d) is also ruled out.

Thus the possible answer is (b).

Exp. 16) A three digit number which on being subtracted from another three digit number consisting of the same digits in reverse order gives 594. The minimum possible sum of all the three digits of this number is :

- (a) 8 (b) 12
(c) 6 (d) can't be determined

Solution Let x, y, z be the hundred, tens and unit digits of the original number then

$$(100z + 10y + x) - (100x + 10y + z) = 594$$

$$\Rightarrow 99(z - x) = 594$$

$$\Rightarrow (z - x) = 6$$

So the possible values of (x, z) are (1, 7), (2, 8) and (3, 9). Again the tens digit can have the values viz., 0, 1, 2, 3, ... 9.

So the minimum possible value of $x + y + z = 1 + 0 + 7 = 8$.

Hence (a) is the correct option.

NOTE x and z can never be zero since if the left most digit becomes zero, then it means this number is only two digit number.

Digital Sum

The digital sum of a number is a single digit number obtained by an iterative (or repeated) process of summing the digits. In this process all the digits are added to form a new number, and then again all the digits of new number are added to form another number. The process continues until a single digit number is obtained.

Exp. 1) Find the digital sum of 7586902.

Solution $7 + 5 + 8 + 6 + 9 + 0 + 2 = 37$

$$3 + 7 = 10$$

$$1 + 0 = 1$$

Therefore the digital sum of 7586902 is 1.

Additive Persistence The number of times the digits must be summed to reach the digital sum is called a number's additive persistence. In the above example, the additive persistence of 7586902 is 3.

Properties of Digital Roots

1. The digital sum of any number in the decimal system is the same as the remainder obtained when that number is divided by 9. In terms of digital sum the remainder 0 is equivalent to the digit 9.
2. Adding 9 or its multiples to any number does not change the digital sum of that number.
3. Removing the digit 9 or removing the cluster of digits those add up to 9 would not affect the digital sum of the given number.
4. A number is divisible by 3, if its digital sum is 0, 3, 6 or 9.
5. A number is divisible by 9, if its digital sum is 0 or 9.
6. The digital root of a square is 1, 4, 7, or 9. Digital roots of square numbers progress in the sequence 1, 4, 9, 7, 7, 9, 4, 1, 9.
7. The digital root of a perfect cube is 1, 8 or 9, and digital roots of perfect cubes progress in that exact sequence.
8. The digital root of a prime number (except 3) is 1, 2, 4, 5, 7, or 8.
9. The digital root of a power of 2 is 1, 2, 4, 5, 7, or 8. Digital roots of the powers of 2 progress in the sequence 1, 2, 4, 8, 7 and 5. This even applies to negative powers of 2; for example, 2 to the power of 0 is 1; 2 to the power of -1 (minus one) is .5, with a digital root of 5; 2 to the power of -2 is .25, with a digital root of 7; and so on, ad infinitum in both directions.

This is because negative powers of 2 share the same digits (after removing leading zeroes) as corresponding positive powers of 5, whose digital roots progress in the sequence 1, 5, 7, 8, 4, 2.

10. The digital root of an even perfect number (except 6) is 1.
11. The digital root of a star number is 1 or 4. Digital roots of star numbers progress in the sequence 1, 4, 1.
12. The digital root of a nonzero multiple of 9 is 9.
13. The digital root of a nonzero multiple of 3 is 3, 6 or 9.
14. The digital root of a triangular number is 1, 3, 6 or 9. Digital roots of triangular numbers progress in the sequence 1, 3, 6, 1, 6, 3, 1, 9, 9.
15. The digital root of a factorial $\geq 6!$ is 9.
16. The digital root of Fibonacci numbers is a repeating pattern of 1, 1, 2, 3, 5, 8, 4, 3, 7, 1, 8, 9, 8, 8, 7, 6, 4, 1, 5, 6, 2, 8, 1, 9.
17. The digital root of Lucas numbers is a repeating pattern of 2, 1, 3, 4, 7, 2, 9, 2, 2, 4, 6, 1, 7, 8, 6, 5, 2, 7, 9, 7, 7, 5, 3, 8.
18. The digital root of the product of twin primes, other than 3 and 5, is 8. The digital root of the product of 3 and 5 (twin primes) is 6.
19. The digital root of a non-zero number is 9 if and only if the number is itself a multiple of 9.

Digital Sum Rule of Multiplication

The digital sum of the product of two numbers is equal to the digital sum of the product of the digital sums of the two numbers.

Exp. 2) The product of 174 and 26 is 4524.

The digital sum of 174 is 3 and that of 26 is 8.

The product of digital sums 3 and 8 is 6.

The digital sum of 4524 is 6.

Therefore the digital sum of $174 \times 26 =$ Digital sum of 4524.

Determining Whether a Number is a Perfect Square or Not

The digital sum of the numbers that are perfect squares will always be 1, 4, 7, or 9.

However, a number will NOT be a perfect square if its digital sum is NOT 1, 4, 7, or 9, but it may or may not be a perfect square if its digital sum is 1, 4, 7, or 9.

The numbers 1, 81, 1458 and 1729 are each the product of their own digit sum and its reversal, for example $1 + 7 + 2 + 9 = 19$, and $19 \times 91 = 1729$.

Exp. 3) Find the product of 102030x405060x708090

- (a) 39064716832962000
- (b) 281635785138962000
- (c) 29264135978862000
- (d) 291635781359782000

Solution Getting the exact product of the 3 numbers through direct multiplication is indubitably a cumbersome and time-consuming process. So it's better to use the following approach.

The digital root of 102030 is 6, and that of 405060 is 6 and that of 708090 is 6.

So the digital root of $6 \times 6 \times 6$ is 9.

It means the digital root of $102030 \times 405060 \times 708090$ is 9.

Again, the digital root of the number in choice (a) is 3

and the digital root of the number in choice (b) is 2

Similarly the digital root of the number in choice (c) is 9

Finally the digital root of the number in choice (d) is 4

Since the digital root of the numbers in choices (a), (b) and (d) is not the same as the digital root of the product of the 3 numbers, so these choices are wrong.

Hence choice (c) is the correct one.

Hint Since $6 \times 6 \times 6 = 216$, which is divisible by 9, therefore the digital root of $6 \times 6 \times 6$ is 9.

Exp. 4) For any natural number n , if $D(n)$ is the sum of all the digits of $D(n)$, find the remainder when $\sum_{n=1}^{99} D(n)$ is

divided by 99.

- (a) 9 (b) 0
(c) 1 (d) 89

Solution Consider the following expansion.

$$\sum_{n=1}^{99} D(n) = D(1) + D(2) + D(3) + D(4) + \dots + D(99)$$

$$\sum_{n=1}^{99} D(n) = 11(1 + 2 + 3 + 4 + \dots + 9) = 11 \times 45$$

$$\sum_{n=1}^{99} D(n) = 99 \times 5$$

Therefore the required remainder is zero, as the given expression is divisible by 99.

Hence choice (b) is the correct one.

Exp. 5) Which of the following is not a valid Pythagorean relation, in which $x^2 + y^2 = z^2$?

- (a) $576^2 + 768^2 = 960^2$ (c) $576^2 + 756^2 = 945^2$
(b) $675^2 + 816^3 = 1059^2$ (d) $756^2 + 1205^2 = 1431^2$

Solution As it is apparent that squaring the both sides for all the choices would be an irritating task, so let's have another approach, as follows.

Choice (a): The digital sum of 576 is 9, so the digital sum of 576^2 is also 9.

The digital sum of 768 is 3, so the digital sum of 768^2 is 9.

Thus the digital sum of $576^2 + 768^2$ is 9.

And the digital sum of 960 is 6, so the digital sum of 960^2 is 9.

As the digital sum of both the sides is same, so this is a valid relation.

You can prove other choices (b) and (c) too as valid ones, in the similar fashion.

Choice (d): The digital sum of 756^2 is 9 and the digital sum of 1205^2 is 1. So the digital sum of $756^2 + 1205^2$ is 1.

And the digital sum of 1431^2 is 9.

Since the digital roots (or sums) of the both the sides are not the same, so it's an invalid relation.

Hence choice (d) is the correct one.

Exp. 6) Two friends Anjaniputra and Bajrangbali who earned some money by teaching at Lamamia mobile app, and now they have some Indian currency in their wallets. Each one has some notes of denominations of ₹ 1000, ₹ 100, ₹ 10 and ₹ 1. The total number of notes with each one is same but the total amount with each one is distinct. None of them has 10 or more notes of any denomination.

What could be the possible difference of amounts with Anjaniputra and Bajrangbali?

- (a) 2365 (b) 1991 (c) 1957 (d) 2025

Solution Let's assume that they have a , b , c and d number of notes of various denominations in any random order. Then there would be total 24 different combinations of amounts such as

$$1000a + 100b + 10c + d, 1000a + 100b + 10d + c,$$

$$1000a + 100c + 10b + d, 1000a + 100c + 10d + b, \text{ etc.}$$

However, the difference of any two combinations would always be divisible by 9, which means the difference in the two amounts has the digital root equal to 9. Since digital roots of 2365, 1991 and 1957 are 7, 2 and 4, respectively, so choices (a), (b) and (c) cannot be the correct ones. Now, the digital root of 2025 is 9, so 2025 can be the difference in the amounts with Anjaniputra and Bajrangbali. Hence, choice (d) is the correct one.

Hint For example, consider any two combinations and take the difference and you will see that this difference is always divisible by 9.

$$(1000a + 100b + 10c + d) -$$

$$(1000c + 100b + 10d + a) = 9(111a - 110c - d)$$

Digital Number Systems

There are various number systems used in digital representation. The most common number systems are binary, octal, decimal, hexadecimal systems.

Base The number that determines the positional value of every digit in a number is called the base of that number. For example the base of binary system is 2, octal is 8, decimal is 10 and hexadecimal is 16. It is also known as **Radix**. Also the base tells us that the number of distinct (or unique) symbols used in that system.

For example we use only 2 distinct symbols viz., 0 and 1 in binary system. Also in decimal system there are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 unique symbols. In the binary system (base is 2) there are only two symbols or possible digits viz., 0 and 1. Even it can represent a very-very large number that can be represented in any other number system.

Conversion of Decimal Numbers into Another Number System

To convert any decimal number in any base ' n '. We divide the number successively by ' n ' i.e., first of all divide the given number, then divide the quotients obtained one by one successively, till the last quotient becomes zero, then write down all the remainders in proper order. As the last remainder is MSD (most significant digit) and the first remainder is LSD (least significant digit). It means the first remainder will be the right most digit and the last remainder will be left most digit.

Decimal to Binary Numbers

Exp. 1) Convert $(17)_{10}$ into binary system (or in base 2)

Solution

$$\begin{array}{r|l}
 2 & 17 \\
 2 & 8 \rightarrow 1 \\
 2 & 4 \rightarrow 0 \\
 2 & 2 \rightarrow 0 \\
 2 & 1 \rightarrow 0 \\
 & 0 \rightarrow 1
 \end{array}
 \rightarrow \text{Remainders}$$

$\therefore (17)_{10} = (10001)_2$

Exp. 2) Convert $(35)_{10}$ into base 3.

Solution

$$\begin{array}{r|l}
 3 & 35 \\
 3 & 11 \rightarrow 2 \\
 3 & 3 \rightarrow 2 \\
 3 & 1 \rightarrow 0 \\
 & 0 \rightarrow 1
 \end{array}
 \rightarrow \text{Remainders}$$

$\therefore (35)_{10} = (1022)_3$

Exp. 3) Convert $(169)_{10}$ into base 7.

Solution

$$\begin{array}{r|l}
 7 & 169 \\
 7 & 24 \rightarrow 1 \\
 7 & 3 \rightarrow 3 \\
 & 0 \rightarrow 3
 \end{array}
 \rightarrow \text{Remainders}$$

$\therefore (169)_{10} = (331)_7$

Exp. 4) Convert $(8976)_{10}$ into base 9.

Solution

$$\begin{array}{r|l}
 9 & 8976 \\
 9 & 997 \rightarrow 3 \\
 9 & 110 \rightarrow 7 \\
 9 & 12 \rightarrow 2 \\
 9 & 1 \rightarrow 3 \\
 & 0 \rightarrow 1
 \end{array}
 \rightarrow \text{Remainders}$$

So $(8976)_{10} = (13273)_9$

Exp. 5) Convert $(128)_{10}$ into binary system.

Solution

$$\begin{array}{r|l}
 2 & 128 \\
 2 & 64 \rightarrow 0 \\
 2 & 32 \rightarrow 0 \\
 2 & 16 \rightarrow 0 \\
 2 & 8 \rightarrow 0 \\
 2 & 4 \rightarrow 0 \\
 2 & 2 \rightarrow 0 \\
 2 & 1 \rightarrow 0 \\
 & 0 \rightarrow 1
 \end{array}$$

$\therefore (128)_{10} = (10000000)_2$

Exp. 6) Convert $(127)_{10}$ into binary system.

Solution

$$\begin{array}{r|l}
 2 & 127 \\
 2 & 63 \rightarrow 1 \\
 2 & 31 \rightarrow 1 \\
 2 & 15 \rightarrow 1 \\
 2 & 7 \rightarrow 1 \\
 2 & 3 \rightarrow 1 \\
 2 & 1 \rightarrow 1 \\
 & 0 \rightarrow 1
 \end{array}$$

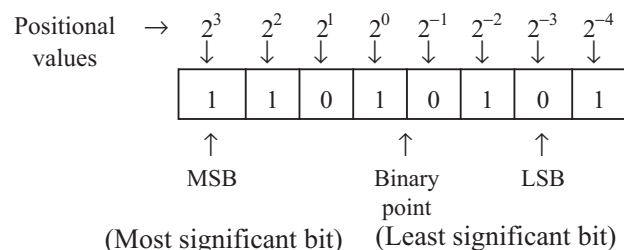
$\therefore (127)_{10} = (1111111)_2$

Representation of Binary Numbers

The binary system is also a positional value system, wherein each binary digit has its own value or weight expressed as a power of 2.

NOTE We have studied the representation of the Decimal number system in the previous articles.

The number 1101.0101 is shown represented in the figure



Here, places to the left of the binary point (i.e., counter part of the decimal point) are positive powers of 2 and places to the right are negative powers of 2.

Binary into Decimal.

To find the decimal equivalent of binary number, we take the sum of the products of each digit value (0 or 1) with its positional value.

Exp. 1) Convert $(100101)_2$ into decimal representation.

Solution $1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
 $= 32 + 0 + 0 + 4 + 0 + 1 = 37$

Exp. 2) Convert $(1101.0101)_2$ in base 10 (i.e., decimal system)

Solution $(1101.0101) = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1$
 $+ 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4}$
 $= 8 + 4 + 0 + 1 + 0 + 0.25 + 0 + 0.0625 = (13.3125)_{10}$

Exp. 3) The decimal number corresponding to the binary number $(111000.0101)_2$ is :

(a) $(5.6312)_{10}$ (b) $(56.3125)_{10}$ (c) $(563.125)_{10}$ (d) $(5631.2)_{10}$

Solution $1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2$
 $+ 0 \times 2^1 + 0 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4}$
 $= 32 + 16 + 8 + 0 + 0 + 0 + 0 + 0.25 + 0 + 0.0625$
 $= (56.3125)_{10}$

NOTE We use the subscripts to denote the base number so as to avoid confusion. For example $(10010)_2$ means the base of the number is 2 and $(20102)_3$ means the base of the number is 3 etc. In the binary system the term "binary digit" is often abbreviated as "bit". The leftmost bit carries the largest weight and hence is called the most significant bit (MSB). The rightmost bit carries the smallest weight and hence it is called the least significant bit (LSB). So the sequence of binary numbers goes as 00, 01, 10, 11, 100, 101, 110, 111, 1000, 1001, ... etc. See the peculiarity in the pattern of binary numbers.

00
01
10
11
100
101
110
111
1000
1001
1010
1011
1100
1101
1110
1111
10000
10001

Now, you can see that the unit digit goes as 0, 1, 0, 1, 0, 1... and tens digit goes as 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1 and Hundreds digit goes as 0000, 1111, 0000, 1111, 0000, 1111, ... etc. and thousands digit goes as 00000000, 11111111, 00000000, 11111111, ... etc.

Representation of Octal Numbers

This system has base of 8. So there are 8 unique symbols viz., 0, 1, 2, 3, 4, 5, 6, 7 are used for representation. For example consider a number 5324.01246.

Then

8^3 ↓	8^2 ↓	8^1 ↓	8^0 ↓	8^{-1} ↓	8^{-2} ↓	8^{-3} ↓	8^{-4} ↓	8^{-5} ↓
5	3	2	4	0	1	2	4	6
↑				↑				↑
MSD				Octal point				LSD

The decimal equivalent of $(5324.012)_8$ is given below :

$$(5324.012)_8 = 5 \times 8^3 + 3 \times 8^2 + 2 \times 8^1$$

$$+ 4 \times 8^0 + 0 \times 8^{-1} + 1 \times 8^{-2} + 2 \times 8^{-3}$$

$$= 2560 + 192 + 16 + 4 + 0.015625 + 2 \times 0.001953125$$

$$= (2772.01953125)_{10}$$

The sequence of octal number goes as :

0, 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 30, 31, ...

Hexadecimal Number System

It has base 16. So it uses 16 unique symbols viz., 1, 2, 3, ...9 and A, B, C, D, E, and F.

Thus $(1)_{10} = (1)_{16}$, $(2)_{10} = (2)_{16}$,

$(3)_{10} = (3)_{16}$ $(9)_{10} = (9)_{16}$

and $(10)_{10} = (A)_{16}$,

$(11)_{10} = (B)_{16}$, ... $(15)_{10} = (F)_{16}$.

Conversion of Decimal Fractions into Binary

To convert a decimal fraction into binary, we multiply successively the decimal fraction by the radix i.e., base 2 (in case of binary) and write down all the integral values (i.e., the figures left of the decimal point) in that order.

As the first integral figure (0 or 1) will be just right to the binary point and the last integral will be the right most figure in the binary representation.

Exp. 1) Convert $(0.875)_{10}$ to binary.

Solution	Integer part
$2 \times 0.875 = 1.750$	1
$2 \times 0.75 = 1.50$	1
$2 \times 0.50 = 1.00$	1
$\therefore (0.875)_{10} = (0.111)_2$	

Exp. 2) Convert $(0.3125)_{10}$ to binary equivalent.

Solution	Integer part
$2 \times 0.3125 = 0.625$	0
$2 \times 0.625 = 1.25$	1
$2 \times 0.25 = 0.50$	0
$2 \times 0.50 = 1.00$	1
Thus $(0.3125)_{10} = (0.1010)_2$	

Exp. 3) Convert 0.454545 to binary equivalent.

Solution	Integer part
$2 \times 0.454545 = 0.909090$	0
$2 \times 0.90909 = 1.81816$	1
$2 \times 0.81818 = 1.63638$	1
$2 \times 0.63636 = 1.27272$	1
$2 \times 0.27272 = 0.54544$	0
$2 \times 0.54544 = 1.09088$	1
$2 \times 0.09088 = 0.18176$	0
$2 \times 0.18176 = 0.36352$	0
$2 \times 0.36352 = 0.72704$	0
$2 \times 0.72704 = 1.45408$	1

Since, the sequence is continuous so can never be expressed as exactly in binary. Therefore writing down the integral parts.
 $(0.454545)_{10} = (0.1000101110)_2$

Exp. 4) Convert $(38.21)_{10}$ to its binary equivalent.

Solution First we convert the integer part *i.e.*, 38.

2	38	
2	19	→ 0 Remainder
2	9	→ 1
2	4	→ 1
2	2	→ 0
2	1	→ 0
	0	→ 1

$$(38)_{10} = (100110)_2$$

Now we convert the fractional part *i.e.*, 0.21

	Integer part
$2 \times 0.21 = 0.42$	0
$2 \times 0.42 = 0.84$	0
$2 \times 0.84 = 1.68$	1
$2 \times 0.68 = 1.36$	1
$2 \times 0.36 = 0.72$	0
$2 \times 0.72 = 1.44$	1
$2 \times 0.44 = 0.88$	0
$2 \times 0.88 = 1.76$	1
$2 \times 0.76 = 1.52$	1
$2 \times 0.52 = 1.04$	1

Thus $(0.21)_{10} = (0.0011010111)_2$

$\therefore (38.21)_{10} = (100110.0011010111)_2$

Exp. 5) Convert $(0.825)_{10}$ to octal.

Solution	Integer part
$8 \times 0.825 = 6.6006$	
$8 \times 0.6 = 4.8$	4
$8 \times 0.8 = 6.4$	6
$8 \times 0.4 = 3.2$	3
$8 \times 0.2 = 1.6$	1
$8 \times 0.6 = 4.8$	4

Since it is a recurring number so it will not be stop.
 Thus we write only 5 figures.

Hence, $(0.825)_{10} = (.64631)_8$

Octal to Binary Conversion

The following table helps in quick conversion.

Octal Digit	0	1	2	3	4	5	6	7
Binary Equivalent	000	001	010	011	100	101	110	111

Exp. 6) Convert $(273)_8$ to binary.

Solution	2	7	3
	↓	↓	↓
	010	111	011

Hence $(273)_8 = (010111011)_2$

Exp. 7) Convert $(16504)_8$ to binary.

Solution	1	6	5	0	4
	↓	↓	↓	↓	↓
	001	110	101	000	100

Hence $(16504)_8 = (001110101000100)_2$

Exp. 8) Convert $(37.42)_8$ to binary.

Solution	3	7	4	2
	↓	↓	↓	↓
	011	111	100	010

$\therefore (37.42)_8 = (0111111.100010)_2$

Binary to Octal Conversion

This is the inverse process of the previous conversion. In this process we make the triplets starting from the right (*i.e.*, LSB). If the number of digits in the binary representation are not in the group of 3 (*i.e.*, 3, 6, 9, 12, ... etc) then we associate either 1 or 2 zeros at the left of the MSB, then we write the octal equivalent of each triplet.

Exp. 9) Convert $(100011010110)_2$ to octal.

Solution	100	011	010	110	(see the table)
	↓	↓	↓	↓	
	4	3	2	6	

Hence $(100011010110)_2 = (4326)_8$

Exp. 10) Convert $(1001011001)_2$ to octal.

Solution $\begin{array}{cccc} 001 & 001 & 011 & 001 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 1 & 3 & 1 \end{array}$ [two more zeros are added in the left to make the triplet]
Thus $(1001011001)_2 = (1131)_8$

Exp. 11) Convert $(10110.0101)_2$ to octal.

Solution $\begin{array}{cccc} 010 & 110 & 010 & 100 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 4 & 6 & 2 & 4 \end{array}$
Thus $(10110.0101)_2 = (46.24)_8$

Note that, after binary point, the groups of 3 bits are made starting from left to right. That's why we added two zeros to make a group of three bits as the last group had only 1.

Decimal to Hex Conversion

Exp. 12) Convert $(725)_{10}$ to hexadecimal.

Solution $\begin{array}{r|l} 16 & 725 \\ 16 & 45 \rightarrow 5 \\ 16 & 2 \rightarrow D \\ & 0 \rightarrow 0 \end{array}$ [$\therefore (13)_{10} = (D)_{16}$]
 $\therefore (725)_{10} = (2D5)_{16}$

Exp. 13) Convert $(0.03125)_{10}$ to base 16.

Solution $\begin{array}{rcl} 16 \times 0.03125 & = & 0.5 \\ 16 \times 0.5 & = & 8.0 \end{array}$ Integer part
So $(0.03125)_{10} = (0.08)_{16}$

Hex to Decimal Numbers

Exp. 14) Convert $(765)_{16}$ to decimal.

Solution $(765)_{16} = 7 \times 16^2 + 6 \times 16^1 + 5 \times 16^0$
 $\Rightarrow (765)_{16} = 1792 + 96 + 5 = 1893$
Thus $(765)_{16} = (1893)_{10}$

Exp. 15) Convert $(3C8.08)_{16}$ to decimal.

Solution $(3C8.08)_{16} = 3 \times 16^2 + C \times 16^1$
 $\quad \quad \quad + 8 \times 16^0 + 0 \times 16^{-1} + 8 \times 16^{-2}$
 $= 3 \times 16^2 + 12 \times 16^1 + 8 \times 16^0 + 0 \times 16^{-1} + 8 \times 16^{-2}$
 $= 768 + 192 + 8 + 0 + 0.03125 = (968.03125)_{10}$
So $(3C8.08)_{16} = (968.03125)_{10}$

Roman Number System

In this system there are basically seven symbols used to represent the whole number system. The symbols and their respective values are given below.

I = 1, V = 5, X = 10, L = 50,
C = 100, D = 500 and M = 1000

In general, the symbols in the numeral system are read from left to right, starting with the symbol representing the largest value; the same symbol cannot occur contiguously more than three times; the value of the numeral is the sum of the values of the symbols.

For example LX VII = 50 + 10 + 5 + 1 + 1 = 67.

An exception to the left to the right reading occurs when a symbol is followed immediately by a symbol of greater value, then the smaller value is subtracted from the larger. For example,

CDXLVIII = (500 - 100) + (50 - 10) + 5 + 1 + 1 + 1 = 448.

Important Conversions

1 trillion = 10^{12} = 1000000000000

1 billion = 10^9 = 1000000000

1 million = 10^6 = 1000000

1 crore = 10^7 = 100 lakh

10 lakh = 10^6 = 1 million

1 lakh = 10^5 = 100000 = 100 thousand

1 thousand = 10^3 = 1000

Exp. 1) The value of the numeral MCDXLIV is :

- (a) 1666 (b) 664
(c) 1464 (d) 656

Solution MCDXLIV = 1000 + (500 - 100) + 50 + 10 + (5 - 1) = 1464

Exp. 2) Which of the following can represent the numeral for 2005 ?

- (a) DDDDV (b) MMV
(c) MDDXV (d) MDVX

Solution 2005 = 2000 + 5 = 1000 + 1000 + 5 = MMV

Exp. 3) The value of the numeral MDCCLXXXIX is :

- (a) 1789 (b) 1987
(c) 2989 (d) 2311

Solution MDCCLXXXIX = 1000 + 500 + 100 + 100 + 50 + 10 + 10 + 10 + (10 - 1) = 1987

Exp. 4) Which of the following represents the numeral for 2949?

- (a) MMMIXL (b) MMXMIX
(c) MMCMIL (d) MMCMXLIX

Solution 2949 = 2000 + 900 + 40 + 9 = (1000 + 1000) + (1000 - 100) + (50 - 10) + (10 - 1) = MMCMXLIX

Introductory Exercise 1.11

- The sum of the digits of a two digit number is 8. The number obtained by reversing the digits is 18 less than the original number. The original number is :
(a) 62 (b) 17
(c) 53 (d) 35
- A number consists of two digits. The sum of the digits is 11, reversing the digits, the number decreases by 45, the number is :
(a) 38 (b) 65
(c) 74 (d) 83
- If a number of two digit is ' k ' times the sum of its digits then the number formed by interchanging the digits is the sum of the digits multiplied by :
(a) $(9 + k)$ (b) $(10 - k)$
(c) $(11 - k)$ (d) $(k - 1)$
- The value of x when $(25)_x = (85)_{10}$ is :
(a) 2 (b) 8
(c) 40 (d) can't be determined
- Which one is an invalid number?
(a) $(325)_7$ (b) $(345)_5$
(c) $(543)_6$ (d) none of these
- $(35)_n, (37)_n, (45)_n$ and $(51)_n$ all are written in base n then the value of n , such that all these four numbers when written in decimal system, they must be the consecutive prime numbers :
(a) $n = 8$ (b) $n = 2$ (c) $n = 12$ (d) $n = 6$
- The last three digits in the binary representation of $(365247728)_{10}$ is
(a) 000 (b) 100
(c) 110 (d) 010
- The last three digits, if $(12345956)_{10}$ is expressed in binary system
(a) 210 (b) 100
(c) 110 (d) 010
- The last four digits in the binary representation of $(8009)_{10}$ are
(a) 1001 (b) 0001
(c) 0101 (d) none of these
- A number of decimal system when represented in binary system then its first and last digits are same and the rest digits are also of another kind. Further this number appears to be palindrome of numerals. The number is :
(a) 57 (b) 19
(c) 8 (d) 9
- When a two digit number is reversed, the new number is reduced by 63 and the sum of digits of these individual numbers is 9. Now, if these same numbers are converted into base ' x ' the larger number becomes 5 times that of smaller one the value of x is :
(a) 36 (b) 13 (c) 15 (d) 18
- If $(25)_n \times (31)_n = (1015)_n$, then the value of $(13)_n \times (52)_n$ is, when $n > 0$:
(a) $(626)_4$ (b) $(462)_n$
(c) $(716)_n$ (d) $(676)_n$
- What is the decimal equivalent of the hexadecimal number $(100 \dots 001)_{16}$?

23 zeros

(a) $2^{23} + 1$ (b) $2^{24} + 1$
(c) $2^{92} + 1$ (d) $2^{96} + 1$
- If the decimal number 2^{111} is written in the octal system, then what is its unit place digit?
(a) 0 (b) 1 (c) 2 (d) 3
- Match list I (Binary) with list II (Octal) and select the correct answer using the codes given below the lists :

List I (Binary)	List II (Octal)
I · 101110	(A) 135
II · 1101110	(B) 56
III · 1011101	(C) 176
IV · 1111110	(D) 156

Codes

- I-A, II-C, III-B, IV-D
- I-B, II-D, III-A, IV-C
- I-A, II-D, III-B, IV-C
- I-B, II-C, III-A, IV-D

1.12 Patterns, Relations and Functions

Progressions

This topic is directly related to our real life. For example whenever the things (or activities) occur in a definite pattern, we say there is a progression.

Arithmetic Progressions

Suppose A has 2 books, B has 4 books, C has 6 books, D has 8 books etc., then we can say that E must have 10 books, if all these form a progression.

So if the same quantity is added in the next term or value, then it is said to be in Arithmetic Progression e.g., $a, (a + d), (a + 2d), (a + 3d), (a + 4d), \dots$

Here you can see that each next term is 'd' more than its previous term.

Some more examples of **Arithmetic Progression** are as follows

$$1, 2, 3, 4, 5, 6, 7, \dots$$

$$3, 5, 7, 9, 11, \dots$$

$$7, 11, 15, 19, 23, \dots$$

$$-9, -4, 1, 6, 11, 16, \dots$$

$$13, 9, 5, 1, -3, -7, -11, \dots$$

The first term of the series is denoted by 'a' and the common difference by 'd' and n th term by t_n and sum of the n terms of the series is denoted by ' S_n '.

Also the last term is denoted by 'l'

The common difference is the difference between any two consecutive terms of the series.

Let a_1 be the first term and a_2 be the second term and a_3 be the third term then $d = a_3 - a_2 = a_2 - a_1$ etc.

The n^{th} term of a series is $T_n = a + (n - 1)d$

and the sum of the n terms of a series is

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

and the arithmetic mean A.M. = $\left(\frac{a_1 + a_2 + a_3 + \dots + a_n}{n} \right)$

and the last term $= l = a + (n - 1)d$.

Exp. 1) Find the 10th term of the following series 7, 10, 13, 16, ... etc.

Solution $T_{10} = 7 + (10 - 1) \cdot 3$ [Here $a = 7, d = 3$]
 $T_{10} = 34$

Exp. 2) Find the sum of the 10 terms of the following series $-11, -8, -5, -2, \dots$

Solution $S_n = \frac{n}{2} [2a + (n - 1)d]$
 $S_{10} = \frac{10}{2} [2 \times (-11) + (9) \cdot 3]$
 [Here $a = -11, d = 3$]
 $= 5 \times (-22 + 27) = 25$

Exp. 3) Find the arithmetic mean of the following series $-17, -13, -9, -5, -1, \dots$

Solution A.M. = $\frac{(-17) + (-13) + (-9) + (-5) + (-1)}{5}$
 A.M. = $-\frac{45}{5} = -9$

NOTE 1. The arithmetic mean (A.M.) of odd numbers of consecutive terms is the middle most term itself.

2. If a, b, c be three consecutive terms of an A.P. then the

$$\text{A.M.} = b = \frac{a + c}{2}$$

3. The consecutive odd number of terms of an A.P. are written as $(a - 2d), (a - d), a, (a + d), (a + 2d), \dots$ etc.

4. The consecutive even number of terms of an A.P. are written as $(a - 5d), (a - 3d), (a - d), (a + d), (a + 3d), (a + 5d), \dots$ etc.

Geometric Progression

In the series each successive (or next) term is multiplied by a fix number.

For example: 1, 3, 9, 27, 81, ...

$$2, 8, 32, 128, 512, \dots$$

$$-3, 6, -12, 24, -48, \dots$$

If a is the first term, l is the last term, r is the common ratio, T_n is the n th and S_n is the sum of n term.

then, $l = ar^{(n-1)}$

$$T_n = ar^{(n-1)}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \text{ if } r > 1$$

and $S_n = \frac{a(1 - r^n)}{1 - r} \text{ if } r < 1$

Also the geometric mean G.M.

$$= (a_1 \cdot a_2 \cdot a_3 \cdot a_4 \dots a_n)^{1/n}$$

Exp. 1) Find the 6th term of 2, 4, 8, ...

Solution $T_6 = 2(2)^5 = 2^6 = 64$ [$\because T_n = a(r)^{n-1}$]

Exp. 2) Find the sum of 7 terms of the following series
1, 3, 9, 27, 81, ...

Solution $\therefore S_n = \frac{a(r^n - 1)}{r - 1}$

$\therefore S_7 = \frac{1(3^7 - 1)}{3 - 1} = \frac{(2187 - 1)}{2} = 1093$

Exp. 3) Find the Geometric mean of, 3, 9, 27.

Solution G.M. = $(a_1 \cdot a_2 \cdot a_3)^{1/3}$

G.M. = $(3 \cdot 9 \cdot 27)^{1/3} = (3^6)^{1/3} = 3^2 = 9$

NOTE The G.M. of odd number of consecutive terms of a G.P. is the middlemost term itself.

2. If a, b, c be three consecutive terms of a G.P. then
G.M. = $b = \sqrt{ac} = (a \cdot c)^{1/2}$

3. The consecutive odd number of terms of a G.P. are written as $\dots, \frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2, \dots$ etc.

4. The consecutive even number of terms of a G.P. are written as $\dots, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, \dots$ etc.

Important Series

1. The sum of first n terms of natural numbers

$$= \frac{n(n+1)}{2}$$

2. The sum of first n even natural numbers = $n(n+1)$

3. The sum of first n odd natural numbers = n^2

4. The sum of first n perfect squares (numbers)

$$= \frac{n(n+1)(2n+1)}{6}$$

5. The sum of first n perfect cubes (numbers)

$$= \left[\frac{n(n+1)}{2} \right]^2$$

6. The sum of the first n terms of the following series :

1, 3, 6, 10, 15, 21, ... is $\frac{n(n+1)(n+2)}{6}$.

Functions/Operations/ Coding/Decoding

In this article we will learn to operate the numbers with specially defined *i.e.*, user defined operations. Basically this is a kind of coding-decoding.

Directions (for Q. Nos.1 to 3) : On the basis of information given below :

If $a \oplus b = a^3 + b^2 + 1$

$a \odot b = (a - b)^2 + 1$

$a * b = a^3 - b^2 + 1$

Exp. 1) If $P = (2 \oplus 3) \odot (3 * 5)$, then the value of P is :

- (a) 225 (b) 226
(c) 256 (d) none of these

Solution $P = (2 \oplus 3) \odot (3 * 5)$

$\Rightarrow P = (2^3 + 3^2 + 1) \odot (3^3 - 5^2 + 1)$

$\Rightarrow P = 18 \odot 3 = (18 - 3)^2 + 1$

$\Rightarrow P = 226$

Exp. 2) The value of $P = (((1 \oplus 2) * 2) \odot 1)$ is :

- (a) 40001 (b) 10804
(c) 40401 (d) none of these

Solution $(((1 \oplus 2) * 2) \odot 1) = ((6 * 2) \odot 1)$

$= (213 \odot 1)$

$= (212)^2 + 1 = 44945$

Exp. 3) The value of $P = (((2 \oplus 2) \odot 2) \odot 2)$ is :

- (a) 14441 (b) 14401
(c) 144001 (d) none of these

Solution $P = (((2 \oplus 2) \odot 2) \odot 2)$

$= (((13) \odot 2) \odot 2)$

$= (122 \odot 2) = 14401$

Directions (for Q. Nos. 4 to 6) : In the following division problem some digits are replaced by letters $a, b, c, \dots$ etc. Every missing digit is uniquely determinable by $a, b, c, \dots$ etc.

$$\begin{array}{r} ab3 \overline{) c61de1} \quad 4cfd \\ \underline{eg2} \\ 6gd \\ \underline{fa5} \\ 8be \\ \underline{d3j} \\ 8f1 \\ \underline{8f1} \\ \times \end{array}$$

Exp. 4) The value of J is :

- (a) 1 (b) 6
(c) 8 (d) none of these

Exp. 5) The value of f is :

- (a) 4 (b) 6
(c) 0 (d) can't be determined

Exp. 6) The value of $a + g$ is :

- (a) 7 (b) 8
(c) 10 (d) none of these

Solution (for Q. Nos. 4 to 6) From the second step, $g = 9$, since $11 - 2 = 9$ again from the second step, $c = 5$, since $ab3 \times c = fa5$ (using unit digit property)
Now, from the last step $ab3 \times d = 8f1$

$\Rightarrow d = 7$ (only)
 so, the value of $a = 1$ (only)
 therefore $f = 6$ (only) $\Rightarrow b = 2$
 Therefore, $e = 4$, from first step (since $a = 1, b = 2$)
 Thus $a = 1, b = 2, c = 5, d = 7, e = 4, f = 6, g = 9, j = 8$

Exp. 7) If $F = a \cdot (|b|) \cdot c \cdot (|d|) \cdot e \cdot (|f|) \cdot g \cdot (|h|) \cdot i \cdot (|j|)$ and
 $G = a \cdot b \cdot c \cdot d \cdot e \cdot f \cdot g \cdot h \cdot i \cdot j$
 Again, if $a = (-1)^1, b = (2)^{-2}, c = (-3)^3, d = (4)^{-4}, \dots$
 $j = (10)^{-10}$ then the correct relation is :

- (a) $F = -G$
 (b) $F = G$
 (c) $F \cdot G = 1$
 (d) none of the above

Solution For F , the modulus of b, d, f, h, j is
 ineffective since for the given values of b, d, f, h, j the
 value of $(|b|) = b, (|d|) = d, \dots$ etc.
 Therefore $F = G$

Exp. 8) If $2f(x) = f(2x), 3f(x) = f(3x), 4f(x) = f(4x) \dots$ etc,
 then $f(1) + f(2) + f(3) + \dots + f(n)$ equals to where $f(1) = 1$:

- (a) $n(n+1)$ (b) 0
 (c) $\frac{n(n+1)}{2}$ (d) $n!$

Solution $f(1) = 1$
 $f(2) = 2$
 $f(3) = 3$
 $f(4) = 4$
 $\dots \dots \dots$
 $f(n) = n$
 $\therefore f(1) + f(2) + f(3) + \dots + f(n) = \frac{n(n+1)}{2}$

Exp. 9) If $f\left(x - \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$, then $f(t)$ equals to :

- (a) $t^2 - \frac{1}{t^2}$ (b) $t^2 + \frac{1}{t^2}$
 (c) $t^2 + 2$ (d) $(t-1)^2$

Solution Since $f\left(x - \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$
 $\Rightarrow f\left(x - \frac{1}{x}\right) = \left(x - \frac{1}{x}\right)^2 + 2$
 Hence $f(t) = t^2 + 2$

Directions (for Q. Nos. 10 to 12)

$$p * q = p^2 - q^2$$

$$p \$ q = p^2 + q^2$$

$$p @ q = pq + p + q$$

$$p \Delta q = \text{Remainder of } \frac{p}{q}$$

$$p \odot q = \text{greatest integer less than or equal to } \frac{p}{q}$$

Exp. 10) If $p = 11$ and $q = 7$, then the value of
 $(p * q) @ (p \$ q)$ is :

- (a) 14641 (b) 12482
 (c) 12243 (d) none of these

Solution $(11 * 7) @ (11 \$ 7) = 72 @ 170$
 $= 72 \times 170 + 72 + 170 = 12482$

Exp. 11) If $p = 8$ and $q = 10$, then the value of
 $[(p \$ q) \Delta (p @ q)] * [(q * p) @ (q \odot p)]$ is :

- (a) 5329 (b) 5239
 (c) 12100 (d) -973

Solution $[(8 \$ 10) \Delta (8 @ 10)] * [(10 * 8) @ (10 \odot 8)]$
 $= (164 \Delta 98) * (36 @ 1)$
 $= 66 * 73 = -973$

Exp. 12) If $p = 15$ and $q = 25$, then the value of the
 expression $[(q \odot p) @ (p \$ q)] \odot [(q * p) \Delta (p \$ q)]$ is :

- (a) 4 (b) 5
 (c) 101 (d) 6

Solution $[(25 \odot 15) @ (15 \$ 25)] \odot [(25 * 15) \Delta (15 \$ 25)]$
 $= (1 @ 850) \odot (400 \Delta 850)$
 $= (1701) \odot (400) = 4$

Direction (for Q. No. 13) Follow the steps to solve the
 given problem :

Step 1. Put $x = 2$ and $y = 3$

Step 2. Multiply x by 2

Step 3. Multiply y by 3

Step 4. If $y =$ greatest possible 3 digit perfect square, then
 stop, else follow step 5.

Step 5. Go to step 2.

Exp. 13) What is the maximum possible value of x ?

- (a) 32 (b) 64
 (c) less than 50 (d) none of these

Solution $x = 2, 4, 8, 16, 32, 64$
 $y = 3, 9, 27, 81, 243, 729$
 Hence, the required maximum value of $x = 64$

Exp. 14) $p @ q = p^2 - 20$ if $p > q$, $p @ q = p^2 + 20$ if $p \leq q$,
 then for $a = 5$ and $b = 4$ the value of
 $[(a @ b) @ (a @ b)] - [(b @ a) @ (b @ a)]$ is :

- (a) 416
 (b) -461
 (c) -1271
 (d) none of these

Solution $[(5 @ 4) @ (5 @ 4)] - [(4 @ 5) @ (4 @ 5)]$
 $= [5 @ 5] - [36 @ 36]$
 $= 45 - 1316 = -1271$

Exp. 15) If $x \# y = \frac{x+y}{xy}$, then the value of $p \# (q \# r)$ for every $p, q, r \in \mathbb{N}$:

- (a) $\frac{pq + qr + r^2}{p(p+q+r)}$ (b) $\frac{pqr + q + r}{p(q+r)}$
 (c) $\frac{p(qr + q + r)}{pqr}$ (d) none of these

Solution $p \# (q \# r) = p \# \left(\frac{q+r}{qr} \right)$

$$= \frac{p + \left(\frac{q+r}{qr} \right)}{p \left(\frac{q+r}{qr} \right)} = \frac{\frac{pqr + q + r}{qr}}{\frac{p(q+r)}{qr}} = \frac{pqr + q + r}{p(q+r)}$$

Exp. 16) If $f(x, y) = \frac{(x+y)^2}{xy}$, for every $x, y > 0$, then $f(x, y)$ must be:

- (a) ≥ 2 (b) < 5
 (c) ≥ 4 (d) none of these

Solution $f(x, y) = \frac{x^2 + y^2 + 2xy}{xy}$

$$= \frac{x}{y} + \frac{y}{x} + 2$$

The minimum value of $\frac{x}{y} + \frac{y}{x} = 2$ at $x = y$

Therefore $f(x, y) = \left(\frac{x+y}{xy} \right)^2 \geq 4$

Exp. 17) The function f_1 and f_2 are defined as follows : $f_1 = a! + b!$ and $f_2 = a^a + b^b$, for $a, b > 1$ and $a, b \in \mathbb{N}$, then the correct relation is :

- (a) $f_1 < f_2$ (b) $f_1 = f_2$ (c) $f_1 > f_2$ (d) can't say

Solution Check for any value of $(a, b) > 1$ and $a, b \in \mathbb{N}$

since $a! = a \cdot (a-1) \cdot (a-2) \dots 1$ but $a^a = a \cdot a \cdot a \dots a$ times

Therefore, $f_1 < f_2$

Directions : For any natural number p and q

(i) $p \# q = p^3 + q^3 + 3$ and $p * q = p^2 + q^2 + 2$

and $p \$ q = |p - q|$

(ii) $\text{Max}(p, q) = \text{Maximum of } (p, q) \text{ and } \text{Min}(p \text{ and } q) = \text{Minimum of } (p, q)$

Exp. 18) The value of $[(4 \# 5) \$ (14 * 15)]$ is :

- (a) -196 (b) 231 (c) -225 (d) 229

Solution $[(4 \# 5) \$ (14 * 15)] = (64 + 125 + 3) \$ (196 + 225 + 2)$
 $= (192 \$ 423) = 231$

Exp. 19) The value of $[(1 \$ 2) \# (3 \$ 4)] * [(5 \$ 6) \# (7 \$ 8)]$ is

- (a) 25 (b) 52 (c) 36 (d) none

Solution $[(1 \$ 2) \# (3 \$ 4)] * [(5 \$ 6) \# (7 \$ 8)]$
 $= (1 \# 1) * (1 \# 1) = 5 * 5 = 25$

Exp. 20) The value of :

$[\text{Max}(2, 4) \$ \text{Min}(6, 8)] \# [\text{Min}(10, 12) \$ \text{Max}(14, 16)]$

- (a) 227 (b) 225 (c) 224 (d) none

Solution $[\text{Max}(2, 4) \$ \text{Min}(6, 8)] \# [\text{Min}(10, 12) \$ \text{Max}(14, 16)] = (4 \$ 6) \# (10 \$ 16) = 2 \# 6 = 227$

Introductory Exercise 1.12

1. $\left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{5}\right) \dots \left(1 - \frac{1}{n}\right)$ equals :

- (a) $\frac{1}{n}$ (b) $\frac{2}{n}$ (c) $\frac{3}{n}$ (d) $\frac{4}{n}$

2. If the sum of three consecutive integers is 21, then the sum of the two smaller integers is :

- (a) 11 (b) 5 (c) 12 (d) 13

3. The sum of squares of first ten natural numbers is :

- (a) 375 (b) 385 (c) 475 (d) 485

4. The arithmetic mean of first 50 odd natural numbers is

- (a) 50 (b) 625 (c) 175 (d) none

5. A boy draws n squares with sides 1, 2, 3, 4, 5, ... in inches. The average area covered by these n squares will be :

- (a) $\left(\frac{n+1}{2}\right)$ (b) $\left(\frac{n+1}{2}\right) \left(\frac{2n+1}{3}\right)$
 (c) $\left(\frac{n+1}{2}\right) \left(\frac{2n+1}{3}\right)^{-1}$ (d) $\left(\frac{n+1}{2}\right) - 1 \left(\frac{2n+1}{3}\right)$

6. The first and the last term of an arithmetic progression (AP) are 6 and 66, respectively. If it has at least 3 terms and all the terms are integers, how many different values of common difference of the AP can be possible?

- (a) 7 (b) 8 (c) 9 (d) 12

7. If the 9th and 19th terms of a G.P. are 12 and 18, what is the G.M. of the first 27 terms of this G.P.?

- (a) 9 (b) 14 (c) 15 (d) none

8. If the arithmetic mean of the numbers $x_1, x_2, x_3, \dots, x_n$ is $\bar{x}$, then the arithmetic mean of the numbers $ax_1 + b, ax_2 + b, ax_3 + b, \dots, ax_n + b$, where a and b are two constants, would be :

- (a) $\bar{x}$ (b) $n\bar{x} + nb$ (c) $a\bar{x}$ (d) $a\bar{x} + b$

9. Which one of the following numbers belongs to the given series 18, 26, 34, 42, 50?

- (a) 438 (b) 338
 (c) 232 (d) 132

CAT-Test

Questions Helping you bell the CAT

LEVEL 01 > BASIC LEVEL EXERCISE

- 1 When the natural numbers 1, 2, 3, ..., 500 are written, then the digit 3 is used n times in this way. The value of n is :
 (a) 100 (b) 200
 (c) 300 (d) 280
- 2 In how many ways can 1146600 be written as the product of two factors?
 (a) 100 (b) 108
 (c) 216 (d) 273
- 3 How many natural numbers upto 990 are divisible by 5 & 9 both, but not by 7?
 (a) 18 (b) 19
 (c) 22 (d) none of a, b, c
- 4 The number of solution of $|x| + |y| \leq 0$, for $(x, y) \in R$, is :
 (a) 0 (b) 1
 (c) 2 (d) infinitely many
- 5 The remainder when 2^{39} is divided by 39 is :
 (a) 0 (b) 2
 (c) 8 (d) 1
- 6 The unit digit of the following expression
 $(1!)^{99} + (2!)^{98} + (3!)^{97} + (4!)^{96} + \dots (99!)^1$ is :
 (a) 1 (b) 3
 (c) 7 (d) 6
- 7 The sum of all four digit numbers which are divisible by 7 is :
 (a) 7071071 (b) 7^7
 (c) 7107073 (d) 10019996
- 8 When the numerator of a positive fraction is increased by 2 and the denominator of the same fraction is multiplied by 2, the new fraction can be reduced to $\frac{1}{2}$ to its lowest term. The sum of the numerator and denominator of the original fraction can be :
 (a) 13
 (b) 45
 (c) 16
 (d) any even integer greater than 3
- 9 In the given expression $pq = p - q + 9$; q is a fraction and p is any positive integer. The value of p that is inadmissible is :
 (a) 5 (b) 4
 (c) 8 (d) 2
- 10 The digits of a three digit number are in G.P. When the digits of this number are reversed and this resultant number is subtracted from the original number the difference comes out to be 792. The actual number is :
 (a) 842 (b) 961
 (c) 421 (d) 931
- 11 How many even integers n ; $13 \leq n \leq 313$ are of the form $3k + 4$, where k is any natural number?
 (a) 101 (b) 51
 (c) 50 (d) none of these
- 12 In the above question the number of values of n which are odd :
 (a) 10 (b) 51
 (c) 32 (d) none of a, b, c
- 13 If a and b are two odd distinct prime numbers and if $a > b$ then $a^2 - b^2$ can never be divided by :
 (a) 13 (b) 11
 (c) 17 (d) all of these
- 14 If $P = (101)^{100}$ and $Q = (100)^{101}$, then the correct relation is
 (a) $P > Q$ (b) $P < Q$
 (c) $P = Q$ (d) $P = \frac{11}{10} Q$
- 15 If $k^2 - 25$ is an odd integer then which one of the following values gives the even number?
 (i) $k^{(2k-1)}$ (ii) $(3k-3)(2k-2)$
 (iii) $(k+5)(k-5)$
 (a) None (b) exactly one
 (c) all three (d) exactly two
- 16 $(a+1)(b-1) = 625$; $(a \neq b) \in I^+$, then the value of $(a+b)$ is :
 (a) $a+b \geq 25$ (b) $a+b \geq 50$
 (c) $a+b = 24$ (d) $a+b = 26$

- 17** If $p + \frac{1}{p} = q$, then for $p > 0$:
 (a) $q = 0$ (b) $-2 < q < 2$
 (c) $q \geq 2$ (d) none of a, b, c
- 18** If $a^b = b^a$; $(a \neq b) > 1$, then the value of $(a + b)$ is :
 (a) 5 (b) 6
 (c) 7 (d) does not exist
- 19** If $m^n - n^m = (m + n)$; $(m, n) \in$ prime numbers, then what can be said about m and n :
 (a) m, n are only even integers
 (b) m, n are only odd integers
 (c) m is even and n is odd
 (d) none of the above
- 20** There is a unique 3 digit number which is cube of a natural number. If we shift the position of the digits of this number, the new number also becomes the cube of another number. The original number is :
 (a) 343 (b) 729
 (c) 125 (d) does not exist
- 21** For $n > 2$ such that $n \in I^+$, the given expression $n^4 - n^2$ is divisible is
 (a) 4 (b) 8
 (c) 12 (d) all of these
- 22** If a, b represents two distinct positive integers such that $(aa)^b = abba$ is a valid relation. Then the value of $(a^b \cdot b^a + b^a \cdot a^b)$ is :
 (a) 4 (b) 6
 (c) 13 (d) none of these
- 23** At our training institute we have $p-1, p-2, p-3$ and $p-4$ processors in the ratio of $\frac{1}{6}, \frac{1}{5}, \frac{1}{3}$ and $\frac{1}{2}$, respectively. Minimum number of processors in our institute is :
 (a) 16 (b) 30
 (c) 32 (d) 36
- 24** $(392)^n - (392)^{n-1}$ is not divisible by :
 (a) 56 (b) 23 (c) 13 (d) 17
- 25** Mr. Chaalu while travelling by Ferry Queen has travelled the distance one kilometre more than the fare he paid per km. Initially, he had total amount of ₹ 350 in his wallet. Now he is only left with the minimum sum of (if all the distance travelled by him is in integers) :
 (a) ₹ 26 (b) ₹ 8
 (c) ₹ 19 (d) can't be determined
- 26** A person starts typing the numbers from 1 to 1999. He press the keys total 'n' number of times. The value of n is
 (a) 6889 (b) 1000
 (c) 2888 (d) none of these
- 27** The remainder when $(20)^{23}$ is divided by 17 is :
 (a) 11 (b) 3
 (c) 6 (d) can't be determined
- 28** Let p be a prime number such that $3 < p < 50$, then $p^2 - 1$ is :
 (a) always divisible by 8 (b) always divisible by 24
 (c) always divisible by 12 (d) all of a, b, c
- 29** If p be a prime number, then $p^2 + 1$ can not have its unit digit equal to
 (a) 3 (b) 9
 (c) 7 (d) all of these
- 30** The number of numbers from 1 to 200 which are divisible by neither 3 nor 7 is :
 (a) 115 (b) 106
 (c) 103 (d) less than 100
- 31** p is a prime number and $(p^2 + 3)$ is also a prime number. Find the number of values of p satisfying the given criterias.
 (a) 3 (b) 2
 (c) 1 (d) can't say
- 32** At a bangle shop, when the shopkeeper tries to displays all the bangles in the form of a square, he is left with 38 bangles that cannot be accommodated in the square arrangement. If he wants to increase the size of the square by one unit, he need to by 25 more bangles to complete the square arrangement. However, when the arranges the bangles he keep all the bangles adjacent to each other without any overlapping. What is the actual number of bangles in his shop?
 (a) 1690 (b) 999
 (c) 538 (d) can't be determined
- 33** If a and b are two integers which are multiples of 5, which of the following is not necessarily true ?
 (a) $a - b$ is divisible by 5 (b) $a + b$ is divisible by 10
 (c) $a^2 - b^2$ is divisible by 5 (d) none of the above
- 34** What are the values of the digits a and b , respectively, in the number $a5523879b$, if it is divisible by both a and b :
 (a) (8, 6) (b) (7, 2) (c) (8, 1) (d) not unique
- 35** A six digit number $abcabc$ such that $a, b, c \in N$, then which is the most correct statement :
 (a) It is divisible by 91 (b) It can be divided by 143
 (c) It is divisible by 6 (d) Only a and b are correct
- 36** Two numbers a and b are such that one is odd and the other is even. Which statement is necessarily true?
 (a) a^{2b} is even
 (b) $(a + b)$ is even
 (c) $a^b \times b^a$ is even
 (d) $a^2 - b^2$ is even

- 37** A gardener plants his garden with 5550 trees and arranges them so that there is one plant more per row as there is the number of rows, then number of trees in a row is :
 (a) 56 (b) 74 (c) 76 (d) 75
- 38** The value of 'a' when $3^a = 9^b$ and $4^{(a+b+2)b} = 16^{ab}$ is :
 (a) 2 (b) 1 (c) 4 (d) none of these
- 39** The number of two digit prime numbers which remain prime even on inverting the position of its digits is :
 (a) 4 (b) 5 (c) 9 (d) 10
- 40** Halfway through the journey from Delhi to Lahore Atal Bihari begins to look out of the window of the Samjhauta Express and continues looking out until the distance which is yet to be covered becomes half of the distance that he has already covered. At this point of time how much distance is he yet to cover?
 (a) $\frac{2}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$
- 41** At the end of 1996, I found that the height of my son was 90 cm. But, at the end of 2003, I found that the height of my son was $\frac{1}{9}$ th more than it was at the end of 2002. However, over the years I have observed that every year his height is increasing by equal amount and it is expected to increase at the same way. Can you find the height of my son at the end of 2008?
 (a) 360 cm (b) 450 cm (c) 250 cm (d) 270 cm
- 42** Chris widener had a servant who was determined to be paid \$ 250, a wrist watch and a ration for whole year? But after 9 months Widener migrated to India and he had just paid him \$ 270 and the ration for the 9 months. What is the cost of the wrist watch?
 (a) \$ 20 (b) \$ 120
 (c) \$ 110 (d) data insufficient
- 43** The sum of the squares of a two digit number is 10. If we add 18 to this number we get another number consisting of the same digits written in reverse order. The original number is :
 (a) 10 (b) 46
 (c) 13 (d) none of (a), (b), (c)
- 44** A two digit number ab is added to another number ba , which is obtained by reversing the digits then we get a three digit number. Thus $a + b$ equals to :
 (a) at least 18 (b) $2ab$
 (c) $2(a + b)$ (d) $(a + b) \geq 10$
- 45** At Wharton School every student is awarded with the grades A, B or C only. 57.1428571428 ...% students obtained 'A' grade while 26.4444...% students obtained 'B' grade. If there are less than 3500 students then the maximum number of students obtained the 'C' grade while no one is declared fail.
 (a) 517 (b) 533
 (c) 428 (d) can't be determined
- 46** Which one of the following is wrong?
 (a) The sum of two even numbers, each raised to an odd power is even
 (b) The sum of two odd numbers, each raised to an odd power is even
 (c) The remainder when dividing an even number by an odd number is even or zero
 (d) The remainder when dividing an odd number by an even number is always odd
- 47** The sum of the expression $551 + 552 + 553 + \dots + 560$ is :
 (a) 3450 (b) 5555 (c) 555 (d) 6060
- 48** Find x, if $x^2 + x = x^3 - x$:
 (a) 1 (b) 2 (c) 3 (d) 4
- 49** In the above problem the number of values of x is :
 (a) 1 (b) 2 (c) 3 (d) 4
- 50** How many 3s you have to write down while writing the numbers from 3301 to 3401?
 (a) 220 (b) 218
 (c) 198 (d) none of these
- 51** $(2^{19} + 1)$ is divisible by :
 (a) 3 (b) 4 (c) 6 (d) 3 & 6 both
- 52** Which one of the following is a prime number : 3123, 219, 573, 467 ?
 (a) 219 (b) 467 (c) 573 (d) 3123
- 53** Which is not a prime number?
 (a) 97 (b) 1001 (c) 127 (d) 101
- 54** What is the least number which must be multiplied to 5400 to get a perfect square?
 (a) 2 (b) 3 (c) 6 (d) 10
- 55** For every p, q positive integers at $x = 0$ or $x = 1$, the valid relation can be :
 (a) $p^x q^{(1-x)} = qx + p(1-x)$
 (b) $p^x q^{(1-x)} = px + q(1-x)$
 (c) $p^x q^{(1-x)} = p^{(1-x)}qx$
 (d) either (b) or (c)
- 56** How many times does the sum of 3780 and 2835 contains their difference?
 (a) 4 (b) 5 (c) 6 (d) 7
- 57** The expression $(x + y)^{-1} \cdot (x^{-1} + y^{-1})$ is equivalent to :
 (a) 1 (b) $(xy)^{-1}$
 (c) x^{-y} (d) $xy^{-1} + x^{-1}y$
- 58** For any odd prime number p there exists a positive integer k where $1 < k < p$, such that the remainder of $\frac{k^2}{p}$ is 1. Then the number of positive integers k is :
 (a) 0 (b) 1
 (c) $p - 1$ (d) can't be determined

- 59** At Lucknow Public School $\frac{1}{9}$ students were absent in an exam and only $\frac{19}{24}$ of those who appeared for the exam passed it. Now we know that 500 students failed in the exam. Total number of students registered for the exam is :
 (a) 2000 (b) 2400
 (c) 2700 (d) 3000
- 60** If, $0 < m < n < 1$. Then the expression $km < kn$ is true if :
 (a) $k < 0$ (b) $k > 0$
 (c) $k = 1$ only (d) all of a, b, c
- 61** If $a = 0.1818181818 \dots$ and $b = 0.3030030003 \dots$ then $(a + b)$ is :
 (a) a rational no. (b) a perfect number
 (c) an irrational no. (d) both (b) and (c)
- 62** Half life of a substance is defined as the time period in which a substance becomes just half of it. If it is known that the half life of a substance "DECAY" is 1122 years, then after 4488 years, 80 gm of "DECAY" becomes.
 (a) 4 gm (b) 20 gm
 (c) 5 gm (d) none of these
- 63** In the examination of CBSE, a candidate must get $\frac{2}{5}$ th marks to pass, out of total marks. Vinod appeared in the same exam and got 210 marks and still failed it by 40 marks. The maximum marks which a candidate can get is :
 (a) 500 (b) 625
 (c) 390 (d) can't be determined
- 64** Sunny gets $3\frac{1}{2}$ times as many marks in 'QA' as he gets in 'English'. If his total combined marks in both the papers is 90. His marks in 'QA' is :
 (a) 50 (b) 60
 (c) 70 (d) none of these
- 65** I know a two digit number, but when its digits swap their places we get another two digit number. But, when these two digit numbers are added, it amounts to 99. Further if I just consider the difference between these numbers, it comes out to be 45. What is the number which I know?
 (a) 27 (b) 38
 (c) 72 (d) data insufficient
- 66** Which of the following is/are correct?
 (i) $a^{x+y} = ax + ay$ (ii) $(a^x)^y = ya^x$
 (iii) $a(x \cdot y) = ax \cdot ay$ (iv) $\frac{a^x}{a^y} = a^{x-y}$
 (a) (i) only
 (b) (ii) & (iii) only
 (c) (iv) only
 (d) none of the above
- 67** In a Mock CAT at Lamamia Mobile App, a student was asked to multiply a two digit number with another two digit number. However, while doing the multiplication he reversed the digits (i.e., tens to unit and unit to tens) of each number, still his answer was correct. Such a pair of numbers is :
 (a) 16, 32 (b) 28, 42
 (c) 31, 23 (d) 12, 63
- 68** The relation $\frac{p}{(p+1)} > 1$ is valid when :
 (a) $p \geq -1$ (b) $p > -1$
 (c) $p < -1$ (d) $-1 < p < 0$
- 69** A number when divided by 14 leaves a remainder of 8, but when the same number is divided by 7, it will leave the remainder :
 (a) 3 (b) 2
 (c) 1 (d) can't be determined
- 70** The unit digit of $(316)^{3^{4n}} + 1$ is :
 (a) 4 (b) 5 (c) 1 (d) 7
- 71** The sum of two numbers is 18. The greatest product of these two numbers can be :
 (a) 17 (b) 81
 (c) 80 (d) can't be determined
- 72** Largest four digit number which when divided by 15 leaves a remainder of 12 and if the same number is divided by 8 it leaves the remainder 5. Such a greatest possible number is :
 (a) 9963 (b) 9957
 (c) 9945 (d) 9999
- 73** In a mobiles shop $\frac{7}{12}$ mobiles are imported and rest are manufactured in India. Further $\frac{1}{5}$ th Indian mobiles are coloured while $\frac{5}{7}$ th imported mobiles are black and white. If there are total 150 coloured mobiles in his shop, then total number of mobile phones in his shop is :
 (a) 500 (b) 600
 (c) 800 (d) data insufficient
- 74** In a call centre at New Delhi, it is observed that it gets a call at an interval of every 10 minutes from California, at every 12 minutes from Texas, at the interval of 20 minutes from Washington DC and after every 25 minutes it gets the call from London. If in the early morning at 5 : 00 a.m. it has recieved the calls simultaneously from all the four destinations, then at what time will it receive the calls simultaneously from all the places on the same day?
 (a) 10 : 00 a.m. (b) 3 : 00 a.m.
 (c) 5 : 00 p.m. (d) both (a) and (b)
- 75** The H.C.F. and L.C.M. of $2^4, 8^2, 16^2, 20^3$ are :
 (a) $2^3; 32000$ (b) $2^4; 32000$
 (c) $2^4; 25600$ (d) $2^2; 3200$

- 76** When we divide 15192327 by 99 the remainder will be :
 (a) 98 (b) 84
 (c) 30 (d) none of these
- 77** The number of numbers lying between 1 and 200 which are divisible by either of 2, 3 or 5 is :
 (a) 146 (b) 145
 (c) 158 (d) none of these
- 78** If xAy means $x + y$, xSy means $x - y$, xMy means $x \times y$ and xDy means $x \div y$. Then the value of $4D2S3M6A12$ is :
 (a) -4 (b) 18
 (c) $-\frac{47}{4}$ (d) none of these
- 79** The L.C.M. of two numbers is 1020 and their H.C.F. is 34, the possible pair of numbers is :
 (a) 255, 34 (b) 102, 204
 (c) 204, 170 (d) none of these
- 80** The sum of 100 terms of the series
 $1 - 3 + 5 - 7 + 9 - 11 + 13 - 15 + \dots$ is :
 (a) 100 (b) 50
 (c) 200 (d) none of these
- 81** The value of $\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) \dots \left(1 - \frac{1}{n}\right)$ is :
 (a) 1 (b) $\left(1 - \frac{1}{n}\right)^n$
 (c) $\frac{1}{n}$ (d) can't be determined
- 82** The minimum and maximum possible values of $\frac{x}{y}$, where $2 \leq x \leq 8$ and $16 \leq y \leq 32$, respectively, are :
 (a) $\frac{1}{8}, \frac{1}{4}$ (b) $\frac{1}{16}, \frac{1}{2}$ (c) 2, 16 (d) not unique
- 83** A rectangular floor in my office has its area equal to 56 m^2 . The minimum number of tiles required, if all the tiles are in square shape is :
 (a) 15
 (b) 9
 (c) 14
 (d) can't be determined
- 84** A string of length 221 metre is cut into two parts such that one part is $\frac{9}{4}$ th as long as the rest of the string, then the difference between the larger piece and the shorter piece is
 (a) 58 m (b) 53 m
 (c) 85 m (d) none of these
- 85** Total number of prime numbers between 1 and 200 is :
 (a) 34 (b) 46
 (c) 56 (d) 71
- 86** What is the remainder of $\frac{6^{36}}{215}$?
 (a) 0 (b) 1
 (c) 2 (d) none of these
- 87** The remainder when $(12^{13} + 23^{13})$ is divided by 11 :
 (a) 0 (b) 1
 (c) 2 (d) none of these
- 88** The four digit smallest positive number which when divided by 4, 5, 6 or 7, it always leaves the remainder as 3 :
 (a) 1000 (b) 1257
 (c) 1263 (d) 1683
- 89** Which one of the following is correct?
 (i) $13^{31} > 31^{13}$ (ii) $10^{100} < 100^{10}$
 (iii) $2^{32} < 32^2$
 (a) (i) and (ii) (b) (i) and (iii)
 (c) (ii) and (iii) (d) (i) only
- 90** If $n_1, n_2, n_3, \dots, n_k$ are such that out of these k elements $\frac{k}{2}$ elements are even and rest are odd numbers. Which is necessarily even?
 (a) $(n_1 + 2n_2 + 3n_3 + 4n_4 + \dots + k \cdot n_k)$
 (b) $n_1 \cdot n_2 \cdot n_3 \dots n_{\left(\frac{k}{2}+1\right)}$
 (c) $(n_1 + n_3 + n_5 + n_7 + \dots + n_{k-1})$
 (d) $n_2 + n_4 + n_6 + \dots + n_k$
- 91** The H.C.F. of two numbers is 43 and their sum is 430. Total number of distinct pairs of two such numbers is :
 (a) 5 (b) 2
 (c) 6 (d) data insufficient
- 92** $2^{73} - 2^{72} - 2^{71}$ is same as :
 (a) 2^{72} (b) 2^{71}
 (c) 2^{70} (d) none of these
- 93** $N = 55^3 + 17^3 - 72^3$, then N is divisible by :
 (a) 3 & 17 (b) 40 & 11
 (c) 11 & 15 (d) all of these
- 94** $abcde$ is a five digit number when multiplied by 13 it gives a number, which is purely formed by the digit 9. Then the value of $a + b + c + d + e$ is :
 (a) divisible by 8 (b) equal to 27
 (c) divisible by 11 (d) all of these
- 95** The sum of 3 consecutive even numbers is always divisible by :
 (a) 24 (b) 48
 (c) 10 (d) none of these
- 96** The remainder obtained when $23^3 + 31^3$ is divided by 54
 (a) 0 (b) 1
 (c) 3 (d) can't be determined

- 97** The largest possible number by which the product of any five consecutive natural numbers can be divided :
 (a) 120 (b) 160
 (c) 100 (d) none of these
- 98** If $x^2 + y^2 = 25$ and $xy = 12$, then the value of $x^{-1} + y^{-1}$ is :
 (a) $\frac{12}{5}$ (b) $\frac{7}{12}$
 (c) $-\frac{7}{12}$ (d) both (b) and (c)
- 99** The remainder when 75^{75} is divided by 37 :
 (a) 0 (b) 1
 (c) 5 (d) 7
- 100** Let p be a prime number strictly greater than 3. Then $p^2 + 17$ will leave a remainder k , when divided by 12, the value of k is :
 (a) 1 (b) 5
 (c) 6 (d) none of these
- 101** The unit digit of $(12345k)^{72}$ is 6. The value of k is :
 (a) 6 (b) 8
 (c) 2 (d) all of these
- 102** What is the least number which must be subtracted from 369 to make it a perfect cube?
 (a) 8 (b) 26
 (c) 2 (d) 25
- 103** The product of two consecutive even numbers is 624, then one of the numbers is :
 (a) 13 (b) 25
 (c) 26 (d) 28
- 104** What is the remainder when $1719 \times 1715 \times 1713$ is divided by 17?
 (a) 1 (b) 16
 (c) 3 (d) none of these
- 105** The quotient when 12.5 is divided by $\frac{1}{6}$ is :
 (a) 75 (b) 25
 (c) 2.48 (d) none of these
- 106** Kavita was supposed to multiply a natural number by 23. Instead, she multiplied that natural number by 32 and thus her result got increased by 315. The original product was :
 (a) 135 (b) 805
 (c) 775 (d) data insufficient
- 107** A fraction in its lowest form is such that when it is squared and then its numerator is reduced by $\frac{1}{3}$ rd and denominator is reduced to $\frac{1}{5}$ th, the resultant fraction becomes twice of the original fraction. The sum of numerator and denominator could be :
 (a) 7 (b) 8
 (c) 9 (d) 17
- 108** When the three numbers 104, 221 and 377 are divided by a number we get the same remainder in each case. Such a largest possible number is :
 (a) 56 (b) 13
 (c) 39 (d) it does not exist
- 109** If $(ab^2)^{1/3} = 125$ where $a > b > 1$ and $(a, b^2) \in N$, then the correct relation is :
 (a) $a = b$
 (b) $a = b^4$
 (c) $a^{3/2} = b^{2/3}$
 (d) none of these
- 110** If a five digit number 'm21n2' is divisible by 24 then the maximum number of possible combinations of m and n :
 (a) 4 (b) 9 (c) 16 (d) 10
- 111** If a fraction is divided by its reciprocal and then multiplied by itself then the fraction becomes $\frac{512}{125}$. The difference between the actual fraction and its reciprocal fraction is :
 (a) $\frac{29}{30}$ (b) $\frac{39}{40}$
 (c) data insufficient (d) none of these
- 112** A perfect square less than 500 is such that when a prime number p_1 is subtracted or another prime number p_2 is added to it in both cases it becomes the perfect square when $p_2 = 2 + p_1$. Then the $p_1 + p_2$ is necessarily divisible by :
 (a) 8 (b) 11
 (c) 4 (d) all of these
- 113** If $(11)^a = (19)^b = (209)^c$ then the correct relation is :
 (a) $ab = (c)^{ab}$ (b) $a^b = c$
 (c) $ab = c(a + b)$ (d) none of these
- 114** The remainder when $(2^{54} - 1)$ is divided by 9 is :
 (a) 0 (b) 8
 (c) 7 (d) none of these
- 115** A number $1 < N < 100$ is such that it is a perfect square and perfect cube both, then the sum of digits of N is :
 (a) 1
 (b) 10
 (c) 20
 (d) can't be determined
- 116** The product of the digits of a three digit number which is a perfect square as well as a perfect cube is :
 (a) 126 (b) 256
 (c) 18 (d) none of these
- 117** A natural number N is such that $N = a^2 = b^4 = c^8$, where a, b, c are distinct positive integers, then the least possible value of N is :
 (a) 729 (b) 1000
 (c) 256 (d) none of these

118 The possible value of $|x| + |x - 1| = 2$ is :

- (a) $\left\{\frac{3}{2}, -\frac{1}{2}\right\}$ (b) $\left\{\frac{3}{2}, \frac{1}{2}\right\}$
 (c) $\left\{\frac{2}{3}, -\frac{1}{2}\right\}$ (d) none of these

119 If $(pqr)^2 = (ijkpqr)$, where $i, j, k, p, q, r \in W$, and pqr and $ijkpqr$ are three digits and 6 digits numbers respectively. Then the value of $i \times j \times k \times p \times q \times r$ is :

- (a) 0 (b) 720
 (c) can't say (d) none of these

120 $3^{6n} + 4^{6n}$ is necessarily divisible by 25 when :

- (a) n is an even integer (b) n is an odd integer
 (c) only n is a prime (d) $n \geq 6; n \in I^+$

121 The number of zeros at the end of $100!$ is :

- (a) $(100)^{100}$ (b) 100×10
 (c) data insufficient (d) none of these

122 The greatest power of 3, when $41!$ is expressed in prime factors :

- (a) 13 (b) 18
 (c) can't be determined (d) none of these

123 A two digit number is such that it is the product of the two distinct perfect squares. The tens digit, unit digit and the sum of these two digits are in A.P. Further if we reverse the digits mutually, the new number increases by 27. The original number is :

- (a) 72 (b) 19
 (c) 36 (d) none of these

124 The value of the expression

$$7777 + 7777 \times 7777 \times (5 + 77) \times (11 \div 35) :$$

- (a) 1234321 (b) 12344321
 (c) 7^{7777} (d) none of these

125 The sum and difference of a number with its reciprocal are $\frac{113}{56}$ and $\frac{15}{56}$, respectively, the number is :

- (a) $\frac{11}{4}$ (b) $\frac{13}{6}$ (c) $\frac{14}{8}$ (d) $\frac{7}{8}$

126 If 4 is added to the numerator of a fraction, it becomes $\frac{1}{3}$ and if 3 is added to the denominator of the same fraction it becomes $\frac{1}{6}$ then the sum of the numerator and denominator is :

- (a) 32 (b) 7 (c) 4 (d) 3

127 In an opera house, there are 7777 chairs to be placed, but the organiser of the event arranged all the chairs in such a way that there were as many columns as there were rows. So he had to remove minimum ' n ' chairs from the total 7777 chairs. The minimum value of n is :

- (a) 121 (b) 44 (c) 33 (d) 25

128 If $p^r \cdot p^{-1} \cdot p^s = (\sqrt{p^3})^2$ and $p^{3/2} \cdot p^r = p^s \cdot p^{-1/2}$ then the value of $(r + s)^{(r + s)}$ is :

- (a) 27 (b) 64
 (c) 4 (d) none of these

129 If $5^{-k} = \frac{1}{l}$, then the value of 5^{3k} is equal to :

- (a) $\frac{1}{3l}$
 (b) $3l$
 (c) l^3
 (d) data insufficient

130 $5^{x-1} + 5^x + 5^{x+1} = 775$ then the value of x for every positive integer x , is :

- (a) 1 (b) 3
 (c) 2 (d) can't be determined

131 If a, b are two perfect square digits and ab is a two digit perfect square number, such that $(a \times b) + (a + b) = ab$, then the value of $[ba - (b + a)]$ is :

- (a) 27 (b) b^2
 (c) 81 (d) either of b and c

132 The greatest possible number which can always divide the sum of the cubes of any three consecutive integers is :

- (a) 3 (b) 4
 (c) 9 (d) 11

133 The least possible divisor of 25930800, by which we divide this number, we get the quotient as a perfect square :

- (a) 2 (b) 3
 (c) 5 (d) can't be determined

134 Arun and Prabhat have some books with them. Once Prabhat said to Arun that if Arun gives 3 books to Prabhat then Arun will have only $\frac{1}{2}$ of the books that Prabhat will

have with him. Then Arun asked Prabhat that if Prabhat gives only two books to Arun, then Prabhat will have as many books as Arun will have. The total number of books that Arun and Prabhat have with them is :

- (a) 25 (b) 56
 (c) 30 (d) can't be determined

135 The number of co-primes of 200 lying between 1 to 100 is :

- (a) 100 (b) 40
 (c) 50 (d) none of these

136 A soda water bottle is exactly filled with coke weighs 1600 gm but when it is $\frac{1}{3}$ rd filled it weighs only 900 gm.

The weight of the empty bottle is :

- (a) 650
 (b) 1100
 (c) 550
 (d) data insufficient

- 137** If ab , cd , ba and dc are two digit numbers then the maximum value of $(ab \times cd) - (ba \times dc)$ is, where a , b , c , d are distinct non-zero integers :
 (a) 7938 (b) 7128
 (c) 6930 (d) none of these
- 138** In how many ways 12600 can be expressed as a product of two factors which are relatively prime :
 (a) 12 (b) 4
 (c) 8 (d) 72
- 139** The solution set of the expression $\frac{1}{(1+p)^p} = 1$ is :
 (a) $[-1, 2]$ (b) $(0, 1)$
 (c) $\{-2, 0\}$ (d) $\{-1, 1\}$
- 140** The number of zeros at the end of $(2^{123} - 2^{122} - 2^{121}) \times (3^{234} - 3^{233} - 3^{232})$:
 (a) 0
 (b) 1
 (c) 121
 (d) none of the above
- 141** The sum of a number and its reciprocal is thrice the difference of the number and its reciprocal. The number is :
 (a) $\pm \sqrt{2}$ (b) $\pm \frac{1}{\sqrt{2}}$
 (c) $\pm \sqrt{3}$ (d) $\pm \frac{1}{\sqrt{3}}$
- 142** A lad was asked his age by his friend. The lad said, "The number you get when you subtract 25 times my age from twice the square of my age will be thrice your age". If the friend's age is 14, then the age of the lad is :
 (a) 21 (b) 28
 (c) 14 (d) 25
- 143** If the sum of two numbers added to the sum of their squares is 42 and the product of these numbers is 15, then the numbers are :
 (a) 15, 1 (b) $\frac{15}{6}$, 6
 (c) $2\frac{1}{2}$, 6 (d) 5, 3
- 144** If the equality $\frac{1}{x-1} = \frac{2}{x-2}$ is satisfied by x , then the value of x will be :
 (a) 2 (b) 1
 (c) $1/2$ (d) 0
- 145** The product $\frac{a}{b^2} \times \frac{b}{a^2}$ expressed as the sum of two identical terms, is :
 (a) $\frac{1}{a+b} + \frac{1}{a+b}$ (b) $\frac{a}{b} + \frac{a}{b}$
 (c) $\frac{b}{a} + \frac{b}{a}$ (d) $\frac{1}{2ab} + \frac{1}{2ab}$
- 146** If $x = 6$ and $y = 3$ then the value of $[x + y]^{x/y}$ is :
 (a) 30 (b) 36 (c) 81 (d) 18
- 147** Which one of the following statements is not correct ?
 (a) every integer is a rational number
 (b) every natural number is an integer
 (c) every natural number is a real number
 (d) every real number is a rational number
- 148** A page contains 60 lines. A chapter contains 125 pages. A book contains 5 chapters. 20 such books form a bound. If there are total 30 lakh lines in an x number of bounds then the value of x is :
 (a) 4 (b) 2 (c) 5 (d) 6
- 149** If x is a natural number, which is a perfect square, then the number $x + \sqrt{x}$ must end in :
 (a) 0 or 5 (b) 0 or 1 or 9
 (c) 0 or 2 or 6 (d) 0 or 4 or 8
- 150** The number $10^N - 1$ is divisible by 11 for :
 (a) even values of N
 (b) all values of N
 (c) odd values of N
 (d) N must be a multiple of 11
- 151** The expression $\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{5}\right) \dots \left(1 + \frac{1}{n}\right)$ simplifies to :
 (a) $\frac{n+1}{3}$ (b) $\frac{n}{n+1}$
 (c) $\frac{3}{n}$ (d) $1 + \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{5} \dots \frac{1}{n}$
- 152** If $x = 2^{1/3} + 2^{-1/3}$ then the value of $2x^3 - 6x$ will be :
 (a) 5 (b) -5 (c) 1 (d) 0
- 153** If $2s = a + b + c$, then the value of $(s-a)^2 + (s-b)^2 + (s-c)^2 + s^2 - a^2 - b^2 - c^2$ will be :
 (a) -1 (b) 1 (c) 2 (d) 0
- 154** If $ab + bc + ca = 0$, then the value of $\frac{1}{a^2 - bc} + \frac{1}{b^2 - ca} + \frac{1}{c^2 - ab}$ will be :
 (a) -1 (b) $a + b + c$
 (c) abc (d) 0
- 155** x is a five digit number. The digit in ten thousands place is 1. The number formed by its digits in units and tens places is divisible by 4. The sum of all the digits is divisible by 3. If 5 and 7 also divide x , then x will be :
 (a) 14020 (b) 12060
 (c) 10020 (d) 10080
- 156** If $2s = 9$, then the value of $s^2 + (s-1)^2 + (s-3)^2 + (s-5)^2$ is :
 (a) 9 (b) 25
 (c) 45 (d) 35

- 157** If $x^a \times x^b \times x^c = 1$, then $a^3 + b^3 + c^3$ is equal to :
 (a) 9 (b) abc (c) $a + b + c$ (d) $3abc$
- 158** The greatest integer that divides 358, 376, 232 leaving the same remainder in each case is :
 (a) 6 (b) 7
 (c) 8 (d) 9
- 159** The least number which when divided by 2, 3, 4, 5 and 6 leaves the remainder 1 in each case. If the same number is divided by 7 it leaves no remainder. The number is :
 (a) 231 (b) 301
 (c) 371 (d) 441
- 160** Three bells, toll at interval of 36 sec, 40 sec and 48 sec respectively. They start ringing together at particular time. They will toll together next time after :
 (a) 6 minutes (b) 12 minutes
 (c) 18 minutes (d) 24 minutes
- 161** A has certain amount in his account. He gives half of this to his eldest son and one third of the remaining to his youngest son. The amount with him now is :
 (a) $\frac{1}{3}$ of the original (b) $\frac{2}{3}$ of the original
 (c) $\frac{3}{4}$ of the original (d) $\frac{1}{6}$ of the original
- 162** If $x + y + z = 0$, then $x^3 + y^3 + z^3$ is equal to :
 (a) 0 (b) $3xyz$
 (c) $\frac{xy + yz + zx}{xyz}$ (d) $xyz(xy + yz + zx)$
- 163** If $a = b^x$, $b = c^y$, $c = a^z$, then xyz is :
 (a) -1 (b) 0
 (c) 1 (d) abc
- 164** If $p = x^{1/3} + x^{-1/3}$, then $p^3 - 3p$ is equal to :
 (a) 3 (b) $\frac{1}{2}(x + x^{-1})$
 (c) $x + x^{-1}$ (d) $2(x + x^{-1})$
- 165** If a language of natural numbers has binary vocabulary of 0 and 1, then which one of the following strings does represent the natural number 7?
 (a) 11 (b) 101
 (c) 110 (d) 111
- 166** The greatest number that will divide 398, 436 and 542 leaving 7, 11 and 15 as remainders, respectively, is :
 (a) 16 (b) 17 (c) 18 (d) 19
- 167** Mohan gets 3 marks for each correct sum and loses 2 marks for each wrong sum. He attempts 30 sums and obtains 40 marks. The number of sums solved correctly is :
 (a) 10 (b) 15
 (c) 20 (d) 25
- 168** It costs ₹ 10 a kilometer to fly and ₹ 2 a km to drive. If one travels 200 km covering x km of the distance by flying and the rest by driving, then the cost of the trip is :
 (a) ₹ 2000 (b) ₹ 24000
 (c) ₹ $(8x + 400)$ (d) ₹ $(12x + 400)$
- 169** The least number which is a perfect square and has 540 as a factor is :
 (a) 8100 (b) 6400 (c) 4900 (d) 3600
- 170** If $\frac{x}{2} = \frac{y}{3}$, then $\left[\frac{4}{5} + \frac{y-x}{y+x} \right]$ equals
 (a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) 1 (d) $\frac{6}{5}$
- 171** For a journey the cost of a child ticket is $\frac{1}{3}$ rd of the cost of an adult ticket. If the cost of the tickets for 4 adults and 5 children is ₹ 85, the cost of a child ticket is :
 (a) ₹ 5 (b) ₹ 6 (c) ₹ 10 (d) ₹ 15
- 172** A fraction becomes 4 when 1 is added to both the numerator and denominator and it becomes 7 when 1 is subtracted from both the numerator and denominator. The numerator of the given fraction is :
 (a) 2 (b) 3 (c) 7 (d) 15
- 173** Two numbers 34041 and 32506 when divided by a certain number of three digits, leave the same remainder. The number is
 (a) 307 (b) 211 (c) 247 (d) 299
- 174** Mr. Black has three kinds of wine. Of the first kind 403 litres, of the second kind 434 litres of the third kind 465 litres. What is the least number of full casks of equal size in which this can be stored without mixing?
 (a) 31 (b) 39 (c) 42 (d) 51
- 175** The largest sum of money which is contained in both ₹ 49.56 and ₹ 38.94 an exact number of times is :
 (a) 2.7 (b) 3.54 (c) 4.28 (d) none
- 176** Tanya gives away to each of four girls $\frac{1}{12}$, $\frac{5}{18}$, $\frac{7}{30}$, $\frac{7}{48}$ of the apples in a basket and has only just enough apples to be able to do so without dividing an apple. The minimum number of apples she has in her basket :
 (a) 250 (b) 720 (c) 750 (d) none
- 177** Abhishek, Bobby and Charlie start from the same point and travel in the same direction around an island 6 km in circumference. Abhishek travels at the rate of 3, Bobby at the rate of $2\frac{1}{2}$ and Charlie at the rate of $1\frac{1}{4}$ km/hr. In how many hours will they come together again?
 (a) 6 hrs (b) 12 hrs (c) 24 hrs (d) 15 hrs

- 178** $2^{\sqrt{3}}$ is :
 (a) a natural number (b) an integer
 (c) a rational number (d) an irrational number
- 179** If $p > 0, q < 0$, then which of the following is correct?
 (a) $p + q > 0$ (b) $p - q > 0$
 (c) $p + q < 0$ (d) $p - q < 0$
- 180** Simplify : $(x^{2^{n-1}} + y^{2^{n-1}})(x^{2^{n-1}} - y^{2^{n-1}})$
 (a) $x^{2^n} - y^{2^n}$ (b) $x^{2^n} + y^{2^n}$
 (c) $(x^n - y^n)^2$ (d) $(x^2 - y^2)^{n-1}$
- 181** If $x = 2^{1/3} + 2^{-1/3}$, then the value of $2x^3$ is :
 (a) $6x + 5$ (b) $5x + 6$
 (c) $6x - 5$ (d) $5x - 6$
- 182** If $x = 3 + 3^{2/3} + 3^{1/3}$, then the value of $x^3 - 9x^2 + 18x - 12$ is :
 (a) 1 (b) 0 (c) -1 (d) $\sqrt{2}$
- 183** If $x^{1/3} + y^{1/3} + z^{1/3} = 0$, then the value of $(x + y + z)^3$ is :
 (a) 27 (b) $27xyz$
 (c) 81 (d) $(xyz)^3$
- 184** If $x^{x(\sqrt{x})} = (x\sqrt{x})^x$, then the value of x is :
 (a) $\frac{9}{4}$ (b) $\frac{4}{9}$
 (c) 8 (d) 4
- 185** The value of (x, y) if $5^x + 3^y = 8, 5^{x-1} + 3^{y-1} = 2$ is :
 (a) 2, 3 (b) 3, 2
 (c) 1, 1 (d) 0, 1
- 186** The value of (x, y) if $x^y = y^x$ and $x^2 = y^3$ is :
 (a) $\frac{27}{8}, \frac{9}{4}$ (b) $\frac{9}{8}, \frac{27}{4}$
 (c) $\frac{8}{27}, \frac{4}{9}$ (d) $\frac{8}{9}, \frac{4}{27}$
- 187** The value of $(ax^{1/4} + 3a^{1/2}x^{1/2} + 4x^{3/4})(a - 3a^{1/2}x^{1/4} + 4x^{1/2})$ is :
 (a) $a^2x^{1/4} - ax^{3/4} + 16x^{5/4}$
 (b) $ax^2 + x^{3/4} + 9x^{1/4}$
 (c) 0
 (d) 1
- 188** The value of $\frac{1}{x^b + x^{-c} + 1} + \frac{1}{x^c + x^{-a} + 1} + \frac{1}{x^a + x^{-b} + 1}$ given that $a + b + c = 0$ is :
 (a) 1 (b) 0 (c) abc (d) x
- 189** The value of $\frac{1}{1 + p + q^{-1}} + \frac{1}{1 + q + r^{-1}} + \frac{1}{1 + r + p^{-1}}$; given that $pqr = 1$ is
 (a) 1 (b) 0
 (c) $(p + q + r) - 1$ (d) $p + q + r$
- 190** In a nationwide Smart City Business Idea hunt the jury selected total 100 proposals from six cities Bengaluru, Mumbai, Delhi, Hyderabad, Chennai and Pune. Bengaluru outsmarted all other cities by getting accepted 50 proposals alone from it. Pune stood last as only 5 proposals were selected from this city.
 The second highest number of proposals was selected from Mumbai. What is the minimum number of proposals that would have been selected from Mumbai if distinct number of proposals were selected from each of the six cities?
 (a) 9 (b) 10
 (c) 12 (d) 13
- 191** Government has arrested 10 suspects who are believed to have more than the authorized number of gas connections (LPG). One of the arrested suspects Halwaai Babu has maximum number of 7 gas connections. Which of the following statements is true about these suspects?
 (a) Exactly two suspects have the same number of gas connections
 (b) Exactly three suspects have the same number of gas connections
 (c) At least three suspects have the same number of gas connections
 (d) At least two suspects have the same number of gas connections
- 192** Consider the following series -

$$S(n) = \frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{n} + \sqrt{n+1}}$$
 For which value of n is $S(n)$ a rational number?
 (i) 5! (ii) $30 + 30 \times 30 + 30$
 (iii) $\log 10^8$ (iv) 2^{2^2} (v) 2^3
 (a) only (iv) and (v)
 (b) only (i), (iii) and (iv)
 (c) only (ii) and (iii)
 (d) only (i), (ii), (iii) and (v)
- 193** For any natural number n , if $D(n)$ is the sum of all the digits of $D(n)$, find the remainder when $\sum_{i=1}^{99} D(n)$ is divided by 99.
 (a) 9 (b) 0
 (c) 1 (d) 90
- 194** The digit 6 is written contiguously 225 times to form a natural number N . What's the remainder when N is divided by 455?
 (a) 230
 (b) 5
 (c) 211
 (d) none of the above

- 195** If n is always a natural number, how many terms in the following sequence are integers?

$$210, 84, \frac{105}{2}, \dots, \frac{420}{3n-1}$$

- (a) 3
(b) 5
(c) 6
(d) none of the above

Directions (for Q. Nos. 196 and 197) *Answer the following questions based on the information given below.*

In a medical college examination, where Munnabhai is pursuing an MBBS degree, a group of students are found cheating in the exam hall. The ticket numbers of the students who are found cheating are the consecutive positive integers and their sum is 600.

- 196** Find the number of students who are found cheating in the exam hall.

- (a) 24 (b) 120
(c) 200 (d) cannot be determined

- 197** Out of the students who are found cheating in the exam, if Munnabhai and his best friend Curkit are sitting closely such that their ticket numbers are consecutive numbers and half of the remaining students' ticket numbers are below the ticket numbers of Munnabhai and Curkit and half of that are above the ticket numbers of Munnabhai and Curkit, what could be the possible ticket number of Munnabhai?

- (a) 37 (b) 39
(c) 36 (d) none of these

- 198** If p, q are the positive integers and r, s, t are prime numbers such that the L.C.M. of p, q is $r^4 s^7 t^2$, then the number of ordered pairs (p, q) is

- (a) 567 (b) 765
(c) 120 (d) 180

- 199** How many factors of the number $16!$ don't have their unit digit 5?

- (a) 252 (b) 7201
(c) 5124 (d) 5040

LEVEL 02 > HIGHER LEVEL EXERCISE

- 1** In Mahabalipuram Temple there are some magical bells which toll 18 times in a day, simultaneously. But every bell tolls at a different interval of time, but not in fraction of minutes. The maximum number of bells in the temple can be :
- (a) 18 (b) 10
(c) 24 (d) 6
- 2** Three numbers p, q, r are such that $p^q = q^r$, where $p, q, r > 1$ then the correct relation between q and r is :
- (a) $\frac{q}{r} = 1$ (b) $q < r$
(c) $q > r$ (d) indeterminable
- 3** $p^q - q^r = (p + q)^{r-q}$, where $1 < q < r < p < 10$, then the value of $p + q + r$ is :
- (a) 31 (b) 21
(c) 15 (d) 12
- 4** If $a^2 + b^2 + c^2 + d^2 = 1$, then the maximum value of $a.b.c.d$ is :
- (a) 1 (b) 2
(c) $\frac{1}{16}$ (d) data insufficient
- 5** The value of n in the expression $n^2 - 2(n!) + n = 0$ for every $n \in N$ is :
- (a) 6 (b) 1
(c) 3 (d) both (b) and (c) are true
- 6** If m, n, p are in A.P. and $m^n = p^m = n! + p$; $m, n, p \in N$, then the value of $m.n.p$ is, where $2 < m, n < p < 10$:
- (a) 136 (b) 72
(c) 162 (d) none of these
- 7** The value of $(n!)^n$ if $n + (n-1) + (n-2) = n(n-1)(n-2)$, where $n^3 > 9$, a positive number :
- (a) 27 (b) 216 (c) 256 (d) 331776
- 8** If $(n)^3 - (n)^2 - n = n$, then the number of values of n that satisfy the given relation is :
- (a) 1 (b) 2
(c) 3 (d) can't be determined
- 9** If $k = (k_1, k_2, k_3, \dots, k_n) \in I$ and $k_1 k_2 k_3 + k_2 k_3 k_4 + k_3 k_4 k_5 + \dots + k_{(n-2)} k_{(n-1)} k_n = 0$ Then minimum how many entities i.e. $k_i (i = 1, 2, 3, \dots)$ must be zero? If there are total 12 terms in the above expression :
- (a) 3 (b) 4
(c) 6 (d) ${}^n C_3$
- 10** The given relation $n^{(n-1)} + n^{(n+1)} = (n^2 + 1)^2 - (n^2 + 1)$ is valid for every $n \in N$ if n equals to :
- (a) 3
(b) 5
(c) 1
(d) both (a) and (c)

- 11** The smallest possible number that can be expressed as the sum of cube of two natural numbers in two different combinations.
 (a) 1000 (b) 1728
 (c) 1729 (d) none of these
- 12** $8^6 - 5^6$ is individually divisible by :
 (a) 91 (b) 49
 (c) 129 (d) all of these
- 13** Total number of factors of a number is 24 and the sum of its 3 prime factors out of four is 25. The product of all 4 prime factors of this number is 1365. Then such a greatest possible number can be :
 (a) 17745 (b) 28561
 (c) 4095 (d) can't be determined
- 14** If p be any odd natural number greater than 3, then which digit will never appear as the last digit in the product of $(p^2 - 1)(p^2 + 1)$?
 (a) 9 and 7 (b) 5 and 3
 (c) 1 and 5 (d) all of these

Directions (for Q. Nos. 15 to 18) The relation $R(m, n)$ can be defined for every positive integer m, n as

$R(m, n) = m \times (m+1) \times (m+2) \times (m+3) \times \dots \times (m+n)$
 and the relation $R(1, n)$ is equal to $n!$ or can be written as $R(n)$.

- 15** The value of $\frac{R(135)}{R(100, 35)}$ is :
 (a) 99! (b) 100!
 (c) 270 (d) none of these
- 16** The value of $R(17) \cdot R(19, 62)$ is :
 (a) $\frac{81!}{18}$ (b) $(81!) \times 18$
 (c) $36!$ (d) $17 \times (19 + 62)$
- 17** The L.C.M. of $R(2, 995)$ and $R(996, 1)$ is :
 (a) 1994
 (b) 996!
 (c) 997!
 (d) can't be determined
- 18** The H.C.F. of $R(139, 2)$ and $R(141)$:
 (a) 141 (b) 2743860
 (c) 32,16,839 (d) 19599
- 19** A six digit number of the form $abcabc$ is written where $a, b, c \in I^+$, then which statement is true about this number ?
 (a) it is always divisible by 7 and 11
 (b) it is divisible by 143
 (c) it is divisible by 1001
 (d) all of (a), (b) and (c) are correct
- 20** How many natural numbers upto 1155 are divisible by either 5 or 7 but not by 11?
 (a) 105 (b) 330
 (c) 333 (d) none of these

- 21** The number of solution set (x, y) for the given equation $4x + 7y = 3$, such that $-99 \leq x \leq 99$ and $-100 \leq y \leq 100$, where $x, y \in I$:
 (a) 14 (b) 29
 (c) 15 (d) 30
- 22** When any two natural numbers N_1 and N_2 , such that $N_2 = N_1 + 2$, are multiplied with each other, then which digit appears least time as a unit digit if $N_2 \leq 1000$?
 (a) 0 (b) 9
 (c) 4 (d) both (a) and (c)
- 23** In the above problem (no. 22), if all such unit digits will be added the maximum sum can be :
 (a) 4491 (b) 4500
 (c) 3609 (d) 5400
- 24** A diamond expert cuts a huge cubical diamond into 960 identical diamond pieces in minimum number of 'n' cuts. If he wants to maximize the number of identical diamond pieces making same number of n cuts to it, so the maximum number of such diamond pieces are :
 (a) 1000 (b) 1331
 (c) 1200 (d) none of (a), (b), (c)
- 25** The sum of all the factors of 45000 which are exactly the multiples of 10 is :
 (a) 152295 (b) 141960
 (c) 600 (d) none of these
- 26** The unit digit of the expression $(1!)^{1!} + (2!)^{2!} + (3!)^{3!} + \dots + (100!)^{100!}$:
 (a) 0 (b) 1 (c) 2 (d) 7
- 27** When $(1!)^{1!} + (2!)^{2!} + (3!)^{3!} + \dots + (100!)^{100!}$ is divided by 5, the remainder obtained is :
 (a) 2 (b) 0
 (c) 4 (d) none of these
- 28** The digit at the tens place in the sum of the expression : $(1!) + (2!)^2 + (3!)^3 + (4!)^4 + (5!)^5 + \dots + (111!)^{111}$ is :
 (a) 0 (b) 1 (c) 8 (d) 9
- 29** A number is divided strictly into two unequal parts such that the difference of the squares of the two parts equals 50 times the difference between the two parts. The number is :
 (a) 100
 (b) 250
 (c) 50
 (d) can't be determined
- 30** A positive number p is such that $(p + 4)$ is divisible by 7. N being a smallest possible number larger than first prime number, which can make $(p + N^2)$ divisible by 7. The value of N is :
 (a) 3 (b) 9
 (c) 5 (d) 7

- 31** Anjali and Bhagwat fired 45 shots each. Total 66 bullets hit the target and the remaining bullets missed it. How many times does the Anjali hit the target if it is known that the number of hits per one miss shown by the Anjali is twice that of Bhagwat?
(a) 30 (b) 36 (c) 40 (d) 35
- 32** The remainder when n is divided by 3 is 1 and the remainder when $(n + 1)$ is divided by 2 is 1. The remainder when $(n - 1)$ is divided by 6 is :
(a) 2 (b) 3
(c) 5 (d) none of (a), (b), (c)
- 33** The expression $2222^{7777} + 7777^{2222}$ is divisible by :
(a) 99 (b) 101
(c) 13 (d) any two of these
- 34** The only library of a business school has total 91 books that are shared among all the students in such a way that every two students share a book on Organizational Behaviour, every three students share a book on Financial Management and every four students share a book on Operations Research. The number of students in the school is:
(a) 81 (b) 84
(c) 48 (d) can't be determined
- 35** After two successive raises the salary of an employee becomes equal to $15/8$ times of his initial salary. In terms of percentage raise, his first raise was half of the second raise, then by what percentage his salary was raised for the second time?
(a) 82.5 %
(b) 60 %
(c) 50 %
(d) can't be determined
- 36** The number of digits in the product of $5^{72} \times 8^{27}$ is :
(a) 77 (b) 75
(c) 99 (d) none of (a), (b), (c)
- 37** A student of 5th standard started writing down the counting numbers as 1, 2, 3, 4, ... and then he added all those numbers and got the result 500. But when I checked the result I have found that he had missed a number. What is the missing number?
(a) 25 (b) 32 (c) 30 (d) 28
- 38** A stairway of 20 ft height is such that each step accounts for half a foot upward and one foot forward. What distance will an ant travel if it starts from ground level to reach the top of the stairway?
(a) 59 (b) 60
(c) 58 (d) none of these
- 39** Paltry and Sundry, the two bird hunters went to woods. Paltry fires 5 shots when Sundry fires 7 shots. But Paltry kills 2 out of 5 while Sundry kills 3 out of 7. When Sundry has missed 32 shots, then how many birds has Paltry killed?
(a) 25 (b) 24 (c) 16 (d) 12
- 40** A cigarette pack is $\frac{5}{6}$ th full of its capacity, then 5 cigarettes were taken out and 2 another cigarettes were put inside the pack. Now it is $\frac{4}{5}$ full. How many cigarettes can this pack contain when it is full?
(a) 90 (b) 80
(c) 72 (d) can't be determined
- 41** Around a square table chairs are arranged in a sequence starting from one corner, numbered as 1, 2, 3, ... etc. The chair number 2 is opposite the chair number 14. How many chairs are there in all?
(a) 10 (b) 20
(c) 14 (d) can't be determined
- 42** The value of y for which the expression $p = \frac{1}{(|y - 1| - 3)}$ becomes undefined :
(a) {2, 8}
(b) {-2, 4}
(c) {-1, 3}
(d) {1, 2}
- 43** What is the sum of the following series?
$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{100 \times 101}$$

(a) $\frac{100}{101}$ (b) $\frac{1}{101}$
(c) $\frac{101}{100}$ (d) 101!
- 44** If Rupert has 4 more coins than Laxmi, Laxmi has 1 more coin than Bill and Bill has 1 more coin than Hawkins. Finally Hawkins has 4 more coins than Ajim. Then minimum number of coins that must be transferred if all of them wish to have an equal number of coins :
(a) 5 (b) 6
(c) 7 (d) none of these
- 45** If $f(x) = \frac{7}{x^7} + \frac{5}{x^5} + \frac{3}{x^3} + 1 + 3x^3 + 5x^5 + 7x^7$. Now, if the value of $f(2)$ is 1081.58 then the value of $f\left(\frac{1}{2}\right)$ is :
(a) 540.79 (b) $\frac{1}{1081.58}$
(c) 1081.58 (d) 367.42

- 46** The last digit of the expression $4 \times 9^2 \times 4^3 \times 9^4 \times 4^5 \times 9^6 \times \dots \times 4^{99} \times 9^{100}$ is :
 (a) 4 (b) 6
 (c) 9 (d) 1
- 47** The last digit of the expression $4 + 9^2 + 4^3 + 9^4 + 4^5 + 9^6 + \dots + 4^{99} + 9^{100}$ is :
 (a) 0 (b) 3
 (c) 5 (d) none of these
- 48** p, q, r are the decimal numbers (e.g., 5.8) and $\lfloor x \rfloor$ means the greatest integer less than or equal to x and $A = \lfloor p + q + r \rfloor$ and $B = \lfloor p \rfloor + \lfloor q \rfloor + \lfloor r \rfloor$, then the maximum value of $A - B$ is :
 (a) 0 (b) 2
 (c) 2.99 (d) none of these
- 49** If p, q, r be integers such that $p^2 = q^2 \cdot r$ then :
 (a) p is an even number (b) q is an even number
 (c) r is an even number (d) r is a perfect square
- 50** The sum of n positive integers $k_1, k_2, k_3, \dots, k_n$ is an even number, then number of odd integers involve in the expression is :
 (a) odd (b) even
 (c) $(n - 1)$ (d) none of these
- 51** If $1 + 2 + 3 + \dots + k = N^2$ and N is less than 100 then the value of k can be, where $N \in \text{Natural Numbers}$:
 (a) 8 (b) 1 and 49
 (c) 8 and 36 (d) both (a) and (b)
- 52** If a and b be two co-prime numbers, then $(a + b)$ and $(a - b)$:
 (a) are always co-primes
 (b) have atleast one common factor other than 1
 (c) if $(a + b)$ or $(a - b)$ is not a prime number, then their HCF is 2
 (d) none of the above
- 53** The G.M. of two positive numbers is 35 and the A.M. of the same number is $43\frac{3}{4}$, then the greater of these numbers is :
 (a) 28 (b) 30
 (c) 70 (d) 35
- 54** When a number is divided by 1, 2, 3, 4, 5, ..., $(n - 1)$, n individually it leaves 0, 1, 2, 3, 4, ..., $(n - 2)$, $(n - 1)$ respective remainders, then this number can be :
 (i) $n!$
 (ii) $(n! - 1)$
 (iii) $[(\text{L.C.M. of } 1, 2, 3, \dots, n) - 1]$
 (a) both (i) and (ii) (b) both (ii) and (iii)
 (c) only (iii) (d) only (i)
- 55** Four consecutive even numbers are such that 3 times of the first number is equal to twice the third number. The sum of all the four numbers is :
 (a) 20 (b) 40
 (c) 44 (d) can't be determined
- 56** In the morning batch at Lamamia we have observed that when five-five students took seat on a bench, 4 students remained unseated. But when eleven students took seat per bench, 4 benches remained vaccant. The number of students in our morning batch were?
 (a) 55 (b) 48
 (c) 26 (d) none of these
- 57** When $\left(\frac{m}{n}\right) = 25\left(\frac{n}{m}\right)$, then the value of $m : n$ is :
 (a) $1/25$ (b) 5
 (c) $1/5$ (d) 2.5
- 58** Which one of the following is the greatest one?
 (a) 3^{3322} (b) 33^{322}
 (c) 333^{22} (d) 22^{333}
- 59** The product of any two integers is 25, then the minimum possible sum is :
 (a) 5 (b) 10
 (c) 26 (d) none of these
- 60** If $4 \leq p \leq 5$ and $-10 \leq q \leq -9$, then the least value is given by the expression :
 (a) $p \cdot q$ (b) pq^6
 (c) p^6q (d) $(pq)^5$
- 61** The remainder obtained when $1! + 2! + 3! + \dots + 77!$ is divided by 7 is :
 (a) 0 (b) 5
 (c) 4 (d) can't be determined
- 62** If $(a, n) \in I^+$ and $(a, n) > 1$, then the remainder when $[(a + 1)^{2n-1} - 1]$ is divided by $(a - 1)$ is :
 (a) 1 (b) $a - 1$
 (c) n (d) none of these
- 63** A typist starts to type the serial numbers of candidates in a list, upto 500. Minimum how many times does he needs to press the keys of numerals only?
 (a) 1389 (b) less than 1000
 (c) 1392 (d) can't say
- 64** If a, b, c, d, e, f are sequentially the terms of an A.P. belong to set $\{1, 2, 3, \dots, 9\}$ where all the terms $a, b, c, \dots$ are in increasing order, then the last digit of $a^b \times c^d \times e^f$ is :
 (a) 5
 (b) 2
 (c) 7
 (d) either of (a) and (b)

- 65** Total number of factors of a greatest possible number which when divides 1313 and 621, the respective remainders obtained are 17 and 9 :
 (a) 9 (b) 10
 (c) 11 (d) can't be determined
- 66** The set of values of x for which $|(x - 5)x| > 0$ is :
 (a) all real numbers (b) $R - \{0\}$
 (c) $R - \{0, 5\}$ (d) $R - (0, 5)$
- 67** The sum of four prime numbers is 204. Each such number is a two digit number. The sum of first number p_1 and last number p_4 is same as the sum of second number p_2 and third number p_3 . The average of all the four numbers is not a prime number, but the product of two prime numbers. Further, $p_3 - p_2 = 2(p_2 - p_1) = 2(p_4 - p_3)$. Out of the four prime numbers (p_1, p_2, p_3, p_4) one of them is:
 (a) 23 (b) 89
 (c) 71 (d) can't be determined
- 68** Jai Bhan wanted to sell his mobilephone consists of the handset and a simcard, but Praveen who intended to buy it, asked the price of simcard only? Jai Bhan told him that the price of the simcard is ₹ 4000 less than the price of the handset but if he wished to buy the complete set he had to pay ₹ 5000 only. The price of the handset was :
 (a) 5500 (b) 2500
 (c) 4500 (d) can't be determined
- 69** If $|x + y| = |x - y|$ then the number of ordered pairs of (x, y) which satisfy the given condition is :
 (a) 1 (b) 4
 (c) infinite (d) none of these
- 70** If $292k + 7 = 23l$, where $(k, l) \in I$, then the value of l is :
 (a) 23 (b) 31
 (c) does not exist (d) none of these
- 71** If $ab + 4 = cd$ and $ba + 40 = dc$, where ab, cd, ba and dc are the two digit prime numbers. Further b and d are the prime number digits and a, c are neither prime nor composite. The value of $\frac{(ab + ba)}{(cd + dc)}$ is :
 (a) 1 (b) $\frac{1}{2}$
 (c) 2 (d) can't be determined
- 72** A man sells chocolates which are in the boxes. Only either full box or half a box of chocolates can be purchased from him. A customer comes and buys half the number of boxes which the seller had plus half a box more. A second customer comes and purchases half the remaining number of boxes plus half a box. After this the seller is left with no chocolate boxes. How many chocolate boxes did the seller has, initially?
 (a) 2 (b) 3 (c) 4 (d) 3.5
- 73** If $n^2 = 123454321$, then the value of n is :
 (a) 1001 (b) not a natural number
 (c) 111, 111 (d) none of these
- 74** $1^2 - 2^2 + 3^2 - 4^2 + \dots - 198^2 + 199^2$:
 (a) 19900 (b) 12321
 (c) 19998 (d) none of these
- 75** The quotient when L.C.M. is divided by the H.C.F. of a G.P. with first term ' a ' and common ratio ' r ' is :
 (a) r^{n-1} (b) r^n
 (c) $a^{-1}r^{n-2}$ (d) $(r^n - 1)$
- 76** Once I met two persons of the same parents namely Ashmit and Amisha. Meanwhile Ashmit told me that he has twice the number of sisters as the number of brothers. Further Amisha told me that she has twice the number of brothers as the number of sisters. Actually it was very confusing for me, so can you find the number of brothers and sisters in their family?
 (a) 4 (b) 5
 (c) 6 (d) can't be determined
- 77** If $p < q$, then $p @ q = p \# q$, else $p @ q = q \# p$, where $a \# b = \frac{a}{b}$. Then the value of $(4 @ 5) @ (6 @ 5)$ is :
 (a) $\frac{24}{25}$ (b) $\frac{2}{3}$
 (c) $3/4$ (d) none of these
- 78** A six digit number is such that every alternate digit is a prime digit and the three leftmost digits forms a G.P., while last three digits (i.e. hundreds, tens and unit) form an A.P. If it is expressed as $pqrst u$, where $p + q + r = u$, $q + r = t$, $p + r = s$, $\frac{r}{t} = \frac{2}{3}$ and $p \neq q \neq r \neq s \neq t \neq u$, then the sum of all the digits must be :
 (a) 25 (b) 16
 (c) 21 (d) can't be determined
- 79** Total number of digits in the product of $(4)^{111} \times (5)^{222}$ is :
 (a) 3333 (b) 2223
 (c) 2222 (d) can't be determined
- 80** If $p = N + 5$ when N is the product of any three consecutive positive integers. Then :
 (a) p is prime (b) p is odd
 (c) p is divisible by 6 (d) either of (b), (c)
- 81** If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers then the relation $\left| \frac{p}{q} \right| < \left| \frac{r}{s} \right|$ is :
 (a) always true (b) always false
 (c) never true (d) none of these

- 82** If $u^v + v^w + w^x = 0$ for every negative integer u, v, w, x , the value of $u \times v \times w \times x$ is necessarily be :
 (a) 0 (b) less than zero
 (c) even (d) odd
- 83** The unit digit of $2^{3^4} \times 3^{4^5} \times 4^{5^6} \times 5^{6^7} \times 6^{7^8} \times 7^{8^9}$ is :
 (a) 0 (b) 5
 (c) can't be determined (d) none of these
- 84** If $(a-7)(b-10)(c-12) = 1000$, the least possible value of $(a+b+c)$ equals :
 (a) 59
 (b) 29
 (c) 14
 (d) any integer less than 1000
- 85** If the number $23^{32} - 9$ is divided by 16, the remainder is :
 (a) 8 (b) 0
 (c) 6 (d) none of these
- 86** If $(x-5)(y+6)(z-8) = 1331$, the minimum value of $x+y+z$ is :
 (a) 40 (b) 33
 (c) 19 (d) not unique
- 87** If $x+y+z = 21$, the maximum value of $(x-6)(y+7)(z-4)$ is :
 (a) 343 (b) 216
 (c) 125 (d) not unique
- 88** The remainder R when $3^{37} + 4^{37}$ is divided by 7 is :
 (a) 0 (b) 1
 (c) $2 < R < 6$ (d) none of these
- 89** $7^{74} - 4^{74}$ is divisible by :
 (a) 3 (b) 11
 (c) 7 (d) both (a) and (b)
- 90** The factorial of a number n is exactly divisible by $(2^{11} \times 11^2)$ then the least possible value of n is :
 (a) 22 (b) 25
 (c) does not exist (d) none of these
- 91** The number of zeros at the end of the product of :
 $2^3 \times 3^4 \times 4^5 \times 5^6 + 3^5 \times 5^7 \times 7^9 \times 8^{10} + 4^5 \times 5^6 \times 6^7 \times 7^8$
 $- 10^2 \times 15^3 \times 20^4$ is :
 (a) 5 (b) 6
 (c) 28 (d) none of these
- 92** A nine digit number $abcdefghi$ is such that a is divisible by 1, ab is divisible by 2, abc is divisible by 3 and $abcd$ is divisible by 4 and so on where none of $a, b, c, d, \dots$ is same and every digit is a non-zero digit such a number is:
 (a) 123456789 (b) 381654729
 (c) 126453789 (d) 826435791

Directions (for Q. Nos. 93 to 95)

If Minimum $(x, y, z) = \text{Minimum of } (xy, yz, zx)$

Maximum $(x, y, z) = \text{Maximum of } (x^y, y^z, z^x)$

Labh $(x, y, z) = \text{Average of } (x, y, z)$

Hani $(x, y, z) = \text{Modulus of } (x - y - z) \text{ i.e., } |x - y - z|$

- 93** The value of Minimum $(1, 2, 3) + \text{Maximum } (1, 2, 3)$ is :
 (a) 10 (b) 8
 (c) 12 (d) 4
- 94** Labh [Hani $(1, 2, 3)$, Hani $(2, 3, 4)$, Hani $(3, 4, 5)$] is equal to :
 (a) 4
 (b) 5
 (c) 6
 (d) can't be determined
- 95** Minimum [Hani $(1, 2, 3)$, Hani $(2, 3, 4)$, Hani $(3, 4, 5)$] + Maximum [Labh $(1, 2, 3)$, Labh $(2, 3, 4)$, Labh $(3, 4, 5)$] is :
 (a) 101 (b) 111
 (c) 60 (d) none of these
- 96** When $(55)_{10}$ is represented in base 25 then the expression is :
 (a) $(25)_{25}$ (b) $(35)_{25}$
 (c) $(55)_{25}$ (d) none of these
- 97** For $(P, Q) > 0$ the number 340 PQQ 0 is divisible by both 3 and 8 then the total possible values of (P, Q) is :
 (a) 2 (b) 3
 (c) 6 (d) none of these
- 98** While typing the numbers from 600 to 799, a typist typed 8 whenever he was supposed to type 6. So the total number of times he has typed 8 is:
 (a) 300 (b) 230
 (c) 180 (d) none of these
- 99** If p and q are two distinct integers such that $p^2 - pq = 0$, then we can deduce that :
 (a) $p = q$
 (b) q is only negative integer
 (c) $p = 0$
 (d) either (a) or (c)
- 100** In a survey it was found that YTC sells the cigarettes of ₹ 15990 per day. If the cost of a pack is not less than ₹ 100, then what can be the price of each pack which it sells per day ?
 (a) 150 (b) 420 (c) 78 (d) 205
- 101** If $5n + 12 < 50$; where $n \in N$, then the value of n which satisfies the given inequality is :
 (a) $5 < n < 12$ (b) $3n^2 \leq 150$
 (c) $4n + 5 \leq 33$ (d) $4n^2 + 3 > 200$

102 If A and B are divided by D , then the remainder obtained are 13 and 31, respectively, where A, B, D are natural numbers. Further, $A + B$ is divided by the same divisor D , the remainder is 4, then the divisor D is :

- (a) 14 (b) 23
(c) 40 (d) 18

103 The unit digit of N^{2p} and $(N)^{(2p+1)}$ are 6 and 4 respectively. The greatest two digit number that follows the aforesaid condition is :

- (a) 49 (b) 84
(c) 94 (d) 98

104 If $f(x) = x^3 + 3x^2 - 4x$ then $f(x+1)$ is :

- (a) $x^3 + 7x^2 - 5$ (b) $x^3 + 6x^2 + 5x$
(c) $(x+1)^3$ (d) none of these

105 If $a + b = n$, $n \in I$ and $ab = 60$, then n cannot be equal to :

- (a) 33 (b) -19
(c) 17 (d) 32

Directions (for Q. Nos. 106 to 108) *If a sequence is as given below :*

1, 1, 2, 3, 5, 8, 13, 21, 34, ...

106 The unit digit of the 75th term of this sequence will be :

- (a) 0 (b) 5
(c) 7 (d) none of these

107 In the above sequence the 55th term will be :

- (a) an even number (b) an odd number
(c) either even or odd (d) can't be determined

108 The unit digit of the sum of the 88th and 89th term of the same sequence will be :

- (a) 5 (b) 2
(c) 0 (d) none of these

109 The unit digit of the product of n consecutive numbers of the form $2k + 1$, where $k \in I^+$ and n must be at least 5.

- (a) zero (b) any odd number
(c) 5 (d) both (b) and (c)

110 In the above question the sum of the n terms of the sequence is :

- (a) $n^2 - 1$ (b) $n(2n - 1)$
(c) $[(n - 1)!]n$ (d) none of these

111 If $n_1, n_2, n_3, \dots, n_k \in I$ such that $-5 \leq n_k \leq 25$, then the least possible product of any 3 numbers can be :

- (a) -60 (b) 0
(c) -3000 (d) none of these

112 If p, q, r are such that $q - p = r - q = 2$ for every $p, q, r \in I^+$, then the least possible value of $(p + q)(q + r)(r + p)$ is :

- (a) 192 (b) 720
(c) 129 (d) data insufficient

113 The H.C.F. of 38024 and 16296 is :

- (a) 1234 (b) 6431
(c) 5432 (d) none of these

114 For any positive real number R , $[R]$ is the greatest integer less than or equal to R , then the value of $\left[\frac{R}{2}\right] + \left[\frac{R+1}{2}\right]$ is

- (a) $[R]$ (b) $[2R]$
(c) $\left[R + \frac{1}{3}\right]$ (d) can't be determined

115 For the natural numbers n_1 and n_2 if $2n_1 + 3n_2 = n_1 \times n_2$, then the least possible value of the $2n_1 + 3n_2$ is :

- (a) 6 (b) 12
(c) 24 (d) can't be determined

116 A company offers total 150 pens to its customers. As per the scheme one pen will be offered on the purchase of a "Quantitative Aptitude" book. Out of 150 pens the cost of some pens is ₹ 3 and the cost of the remaining pens is ₹ 5. Maximum how many customers can avail a pen worth ₹ 5 as an offer from the company if the total cost of the pens cannot exceed ₹ 745.

- (a) 45 (b) 120
(c) can't be determined (d) none of these

Directions (for Q. Nos. 117 to 118) *A number of 109 digits is written as follows :*

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 ...

117 What is the least possible positive number that is required to added up to the given number to make it divisible by 2 and 5 both?

- (a) 0 (b) 1
(c) 3 (d) can't be determined

118 What is the least possible whole number which must be subtracted from the given number so that it will become divisible by 3 and 11 both?

- (a) 8 (b) 3
(c) 0 (d) none of these

119 Each family in Gyanpur village has atmost two adults and the total number of boys in this village is less than the number of girls. Similarly the number of girls is less than the number of adults in the village. Raghubir Singh, the chief of this village is the only adult in his family. The minimum number of families in his village is :

- (a) 2 (b) 3
(c) 4 (d) none of these

120 Which of the following is correct if $A = 3^{3^{3^3}}$, $B = 3^{3^{3^3}}$, $C = 3^{3^{3^3}}$ and $D = 3^{3^{3^3}}$?

- (a) $A > B = C > D$ (b) $C > A > B > D$
(c) $A > C > D > B$ (d) $C > B > D > A$

- 121** When two positive integers A and B are divided by another number D , then the remainders obtained were 23 and 3, respectively. Further when $(A + B)$ is divided by D , the remainder is 26, then the value of D is :
 (a) 62
 (b) 69
 (c) 26
 (d) Any value greater than 26
- 122** In how many ways can 729 be expressed as a difference of the square of whole numbers?
 (a) 4
 (b) 6
 (c) 8
 (d) none of these
- 123** In how many ways can 2310 be expressed as a product of 3 factors?
 (a) 41
 (b) 23
 (c) 56
 (d) 46
- 124** In the following triangular arrangement of numbers 0, 1, 2, 3, ... are placed along the lateral sides of the triangle and then interior numbers are obtained by adding the two adjacent numbers of the previous row. The following array depicts the rows 1 through 6.
- | | | | | | | |
|---|---|----|---|----|---|----|
| | | | 0 | | | |
| | | 1 | | 1 | | |
| | 2 | | 2 | | 2 | |
| 3 | | 4 | | 4 | | 3 |
| 4 | | 7 | | 8 | | 7 |
| | 5 | 11 | | 15 | | 11 |
| | | 15 | | 15 | | 11 |
| | | | 5 | | | |
- Let $f(n)$ denote the sum of the numbers in row n . What is the remainder when $f(100)$ is divided by 100?
 (a) 64
 (b) 74
 (c) 46
 (d) 36
- 125** For n being a positive integer, how many values of n satisfy $7^{21} \leq n^{28} \leq 3^{63}$?
 (a) 10
 (b) 7
 (c) 1
 (d) 2
- 126** From IIT Roorkee to IIM Kashipur in Uttarakhand, there are total 325 railway stations falling on a straight railway line. A local train stops at every 3rd station, an express train stops at every 5th station and a bullet train stops at every 8th railway station. How many pairs of stations are connected by exactly two different types of trains?
 (a) 25
 (b) 35
 (c) 36
 (d) 42
- 127** A set A contains 88 elements such that $A \equiv \{8, 88, 888, \dots\}$. Another set B contains all the elements of A such that the sum of no any two elements is divisible by 3. Find the maximum possible number of elements in B .
 (a) 31
 (b) 58
 (c) 59
 (d) 30
- 128** If $a < b$ and $(2)_a \times (17)_b = (32)_b$, find the number of possible values of a .
 (a) 9
 (b) 7
 (c) 6
 (d) none of these

- 129** Suppose the seed of any positive integer n is defined as follows :
 Seed $(n) = n$, if $n < 10$
 Seed $(n) = \text{Seed}(s(n))$, otherwise,
 Where $s(n)$ indicates the sum of digits of n .
 For example, seed $(6) = 6$, seed $(179) = 1 + 7 + 9 = \text{seed}(17) = \text{seed}(1 + 7) = \text{seed}(8) = 8$, etc.
 How many positive integers n such that $n < 999$, will have seed $(n) = 7$?
 (a) 99
 (b) 110
 (c) 111
 (d) 112

- 130** Prof. Viru Sahasrabuddhe wrote down a subtraction problem on the blackboard, while teaching at Lamamia, where he wrote the difference of two unknown numbers but replaced the leading digit by x and the last digit by y in the answer, as shown below.

$$\begin{array}{r} abcdef \\ - fedcba \\ \hline x4814y \end{array}$$

Then he asked his favorite student Chatur to find the sum of x and y , which is a two-digit number. Much to the professor's dismay Chatur was baffled and clueless. Then he asked the same question to Rancho whom he considers a careless and undisciplined student. Much to his utter surprise, Rancho told the correct answer immediately. What is the Rancho's answer?

- (a) 18
 (b) 19
 (c) 91
 (d) none of these
- 131** For every natural number a and b , such that $1 < a < b$ and $10!$ is divisible by $a! \times b!$. Find the number of sets of (a, b) .
 (a) 19
 (b) 9
 (c) 17
 (d) none of these
- 132** A regular polygon is a plane shape that is enclosed by 3 or more lines of equal lengths and it is always symmetric having all the interior angles equal to each other; examples include equilateral triangle, square, regular pentagon, regular hexagon and so on.
 Verma has 120 identical matchsticks and he wants to form a regular polygon, how many distinct polygons can he create using all the matchsticks together?
 (a) 10
 (b) 12
 (c) 14
 (d) 16
- 133** The total number of factors of N is 90, which has f distinct prime factors. What is the total number of factors of the product of all the possible values of f ?
 (a) 10
 (b) 8
 (c) 9
 (d) none of these
- 134** For every natural number n , the number of people in a village is $(n + 150)$ and each family of this village has $(n + 6)$ members. If in each village of this region the number of families is distinct, find the maximum number of villages in the region.
 (a) 8
 (b) 9
 (c) 14
 (d) none of these

- 135** Every artist of Rangmanch theatre adorns oneself with a two-piece attire - *dhori* and *kurta*. If N is the number of combinations of *dhori* and *kurta* that any actor of Rangmanch can wear and $N < 999$, which of the following best describes the value of N , for which the number of artists is maximum considering no two artists wear the same colour of *dhori* and *kurta* ?
- (a) $1 < N < 99$
 (b) $799 < N < 899$
 (c) $899 < N < 999$
 (d) $399 < N < 499$
- 136** Which of the following is necessarily a factor of $(1279 + 279 - 79 - 9)^{127} - (1279^{127} + 279^{127} - 79^{127} - 9^{127})$
- (a) 9 (b) 27 (c) 279 (d) 127
- 137** Find the number of non-negative integral values of x that satisfy the following equation, where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x .
- $$\left\lfloor \frac{x}{9} \right\rfloor = \left\lfloor \frac{x^2}{99} \right\rfloor$$
- (a) 13 (b) 14
 (c) 15 (d) none of these
- 138** If $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x , which one of the following could be a suitable value of x for which each term of the following expression is expressed in its simplest form?
- $$\frac{\lfloor x \rfloor + 3}{4}, \frac{\lfloor x \rfloor + 8}{5}, \frac{\lfloor x \rfloor + 15}{6}, \frac{\lfloor x \rfloor + 24}{7}, \dots, \frac{\lfloor x \rfloor + 168}{15}, \frac{\lfloor x \rfloor + 195}{16}$$
- (a) 9.99 (b) 11.11
 (c) 19.91 (d) 16.61
- 139** When 58123 and 59059 are divided by a three digit number N , the same remainder is obtained. How many values can N take?
- (a) 2 (b) 3 (c) 6 (d) 7
- 140** Find the smallest number which when divided by 3, 5, 7 and 11, it leaves remainders 1, 3, 5 and 2, respectively.
- (a) 337 (b) 418 (c) 838 (d) 912
- 141** Find the smallest number which when divided by 3, 4, 6, 7, 10 and 13, it leaves remainders 2, 3, 5, 4, 7 and 10, respectively.
- (a) 4337 (b) 3467
 (c) 5443 (d) 4547
- 142** Which of the following are the factors of $2^{387} + 3^{344}$?
- (i) 216 (ii) 643
 (iii) 735 (iv) 7073
- (a) Only (iii) (b) only (i), (ii) and (iv)
 (c) (ii), (iii) and (iv) (d) only (ii) and (iv)
- 143** After getting nominated as a class representative Narendra Bhai ordered some pizzas of the same shape, size, ingredients and taste to be shared with his 385 classmates, equally, in such a way that each of his classmates receives same amount of pizza. Also, whenever Narendra Bhai cuts a pizza, he makes sure that all the pieces of that pizza are indistinguishable and he never cuts a pizza into more than 12 pieces. Due to lack of sufficient funds he is not able to order 385 or more pizzas. Which of the following cannot be the number of pizzas that he shares with his 385 classmates?
- (a) 292
 (b) 312
 (c) 335
 (d) 381
- 144** Consider the equation, $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$; where x, y, z are natural numbers. If $x = 12$, find the total number of solutions to this equation.
- (a) 6 (b) 7
 (c) 18 (d) 9
- 145** Vijay and Siddhartha go to a pub and order a pitcher full of beer. In an hour together they finish half the pitcher of beer. Vijay alone can finish a pitcher of beer in x hours and Siddhartha alone can finish a pitcher of beer in y hours. If x and y are the integral numbers, then the maximum value of $x + y$ is
- (a) 4 (b) 8
 (c) 9 (d) 12
- 146** Three friends Babu, Sona and Janu after watching a movie want to order some meal, so they go straight to the food court of the Phoenix mall. Babu says if we order p number of pizzas it would be sufficient for three of us, Sona says if we order b number of burgers it would be sufficient for three of us and Janu says if we order s number of sandwiches it would be sufficient for three of us. The restaurant manager, who is listening to their conversation, says, "Guys you all are right, as whatever you go with, in each case you would have the same amount of meal. But what I suggest is that instead of having only one thing you can have a combo of pizza, burger and sandwich, and this combo would still give you the same amount of meal that you get when you order only one sort of dish and moreover you can save some bucks for the next movie ticket. All of them unanimously agree to his suggestion and order a combo meal. If the number of burgers is more than that of pizzas, but less than that of sandwiches, what is the total number of pizzas, burgers and sandwiches in the combo?
- (a) 9
 (b) 11
 (c) 14
 (d) data insufficient

Directions (for Q. Nos. 147 and 148) *Answer the following questions based on the given information.*

There are two fertility clinics in Chandigarh - 'Chaddha clinic and Chopra clinic; and Vicky Arora is the only suitable semen donor available in this city. In the morning he donates only to Chaddha clinic and in the evening he donates only to Chopra clinic. In a couple of weeks, after he started donating, he married his fiancée, so he had to discontinue his donation, but in all such weeks he had donated exactly 6 times a week and not more than once in a day. In each donation the amount of semen was exactly the same unit. In all these weeks, out of the total semen donated by Vicky to these two clinics, Chaddha clinic had received one-sixth of the semen in the last week while Chopra clinic had received one-ninth of the semen in the last week.

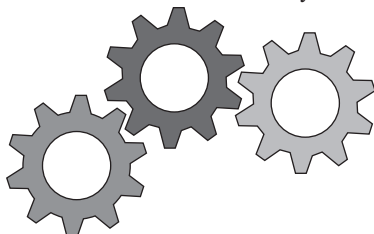
- 147** What could be the possible ratio of the amount of semen received by these two clinics in the last week?
 (a) 5:1 (b) 3:2
 (c) 2:1 (d) 1:1
- 148** For how many weeks he had donated his semen to these clinics?
 (a) 6
 (b) 7
 (c) 8
 (d) cannot be determined uniquely
- 149** On the last day of their college all the 300 girls had a girls' night out at Khadakwasla, where they saw some couples having fun around their resort. Reminiscent of their moments spent in the past three years at Pune campus most of them were quick to flaunt that they dated a guy in their college. However, none claimed that they dated more than one guy in the past 3 years. If a girl dated a guy for $0 < m \leq 12$ months she claims that she dated him for 1 year, similarly if a girl dated a guy for $12 < m \leq 24$ months she claims that she dated him for 2 years and so if a girl dated a guy for $24 < m \leq 36$ months she claims that she dated him for 3 years; whereas m denotes the number of months. There were all sorts of girls who boasted of dating a guy for 1 year, 2 years and even 3 years too. The girls who didn't date a guy at all can be counted on the fingers of one hand. The number of girls who dated a guy for at least 2 years was 36% more than those who dated a guy exactly for 1 year. What's the maximum number of girls who dated a guy for all the 3 years?
 (a) 135 (b) 136
 (c) 169 (d) 170
- 150** A 7 member team of doctors was summoned to treat the equal number of soldiers who got inflicted with wounds and airborne diseases leading to an epidemic in the military base. While the doctors were on the way to the military base, some of them lost their lives due to a landmine explosion.

The ones who could reach safely started treating the soldiers. Meanwhile 190 new soldiers were brought to the base for their treatment, so that each doctor had to treat 320 soldiers at the base. What was the total number of soldiers supposed to be treated, initially?

- (a) 1410 (b) 1400 (c) 770 (d) 1920

- 151** In a set of first 180 natural numbers find the number of prime numbers.
 (a) 48 (b) 24 (c) 41 (d) 38
- 152** In a set of first 1000 natural numbers find the number of prime numbers.
 (a) 176 (b) 168
 (c) 172 (d) none of these
- 153** There are three different teams engaged in laying the optical fiber cable under the ground. Team A is comprised of 11 men can lay 6 km cable in a certain time and team C is comprised of 19 men can lay 11 km cable in a certain time. Team B is less efficient than team A but more efficient than team C. Each member of a particular team is equally efficient. If team B lays n km cable in a certain time, where n is a natural number, then what could be the minimum number of men in team B?
 (a) 5 (b) 6
 (c) 9 (d) 7
- 154** The co-founder of a company called Lamamia thought that in the beginning it's practically impossible to take away some salary, so he devised an ingenious way to get some salary from his own company. Taking into confidence the other co-founders he decided to take ₹ n^2 in the n th month, starting from the very first month up to the first five years. After working for sometime he realized total ₹ 21740 from his company as his salary. However, he didn't take a single penny in a month when he was on bed-rest after he got diagnosed of severe back problem. In which month he didn't work for his company?
 (a) 18th (b) 24th
 (c) 20th (d) data insufficient
- 155** Two friends Azad and Bose are vacationing in India and they have some Indian currency in their wallets. Each one has some notes of denominations of ₹ 1000, ₹ 100, ₹ 10 and ₹ 1. The total number of notes with each one is same but the total amount with each one is distinct. None of them has 10 or more notes of any denomination. What information is necessary to find the total amount that they could have together in their wallets, if used together with the possible amount given in the respective options?
 (a) Azad has the same number of notes of denominations of ₹ 1000 and ₹ 100 as Bose has the notes of denominations of ₹ 100 and ₹ 1000. And Azad has the same number of notes of denominations of ₹ 10 and ₹ 1 as Bose has the notes of denominations of ₹ 1 and ₹ 10. The possible amount could be ₹ 14377

- (b) Azad has the same number of notes of denominations of ₹ 1000 and ₹ 100 as Bose has the notes of denominations of ₹ 10 and ₹ 1. And Azad has the same number of notes of denominations of ₹ 10 and ₹ 1 as Bose has the notes of denominations of ₹ 1000 and ₹ 100. The possible amount could be ₹ 24367
- (c) Azad has the same number of notes of denominations of ₹ 1000 and ₹ 10 as Bose has the notes of denominations of ₹ 10 and ₹ 1000. And Azad has the same number of notes of denominations of ₹ 100 and ₹ 1 as Bose has the notes of denominations of ₹ 1 and ₹ 100. The possible amount could be ₹ 44377
- (d) Azad has the same number of notes of denominations of ₹ 1000 and ₹ 1 as Bose has the notes of denominations of ₹ 1 and ₹ 1000. And Azad has the same number of notes of denominations of ₹ 100 and ₹ 10 as Bose has the notes of denominations of ₹ 10 and ₹ 100. The possible amount could be ₹ 44351
- 156** Ravikishan is a sales executive who has five different simcard for his mobile handset. Each simcard supports different talk-time. For each simcard the talk-time is always in integers and measured in minutes only. When he is working in the remote fields, he carries exactly five fully charged simcard with him. On ten different days he used different pairs of simcard and observed the total talk-time each day as 122, 124, 125, 126, 127, 128, 129, 130, 132, and 133 min. What is the highest talk time that a simcard supports?
- (a) 67 min
(b) 69 min
(c) 71 min
(d) 68 min
- 157** The following figure shows the three gears, which work in tandem. That means when anyone of the gears rotates, all other gears will also rotate. One gear has 36 teeth, another one has 25 teeth and the third one has 30 teeth. Though the number of teeth is not shown correctly in the given figure.



The gears are now in starting position. If you start turning them and continue to turn them, eventually all three will end up at their starting positions. Each gear makes x , y , z complete turns in order to have all three in their starting positions. What is the value of $x + y + z$?

- (a) 91
(b) 11
(c) 30
(d) 900

- 158** An integer n is called square-free if does not have a divisor of the form k^2 where $k \in \{2, 3, \dots, n\}$. Find the number of square-free integers between 1 and 120.
- (a) 45
(b) 60
(c) 75
(d) none of these
- 159** How many positive integral solutions exist for: $ab + cd = a + b + c + d$, where $1 \leq a \leq b \leq c \leq d$?
- (a) 0
(b) 2
(c) 3
(d) none of these
- 160** Ishaan Awasthi, a dyslexic boy, has some difficulties in reading but does not suffer from dyscalculia. Once his professor Mr. Nikumbh gave him a simple multiplication problem involving two distinct numbers. The numbers were exactly two-digit numbers and none of the four digits was same. He asked him to multiply both the numbers and find the answer. Due to inability in reading Ishaan interchanged the positions of tens and unit digits mutually in each of the two numbers and then he multiplied the resultant numbers. But the answer was still the same and correct. After realizing this amazing fact, now Nikumbh sir wants to know the number of pairs of two digit-numbers, which exists in our decimal number system.
- (a) 12
(b) 21
(c) 20
(d) none of these

Directions (for Q. Nos. 161 to 163) Answer the following questions based on the information given below.

Windows 8 - an operating system for Tablets and Mobile phones that can be accessed by the touch of a finger - is not just a revolutionary one in terms of technical advancements over its predecessors but also in terms of aesthetic value: user interface, look and feel.

Windows 8 comes with a magnificent start screen. The start screen of any Windows 8 device appears as a grid of tiles. These square shaped tiles are identical in size and each tile represents a different icon; for mail, calendar, music, photo, video, map, and other such features and applications.

The grid is always rectangular and by default the border tiles, in the outermost rows and columns, are occupied by the static icons of Windows applications only. And the remaining tiles are dynamic so that they can be replaced by the user with the icons of any third party application as per his/her choice. A grid has as many static tiles as dynamic tiles.

- 161** The total number of tiles on the start screen would be:
- (a) 36
(b) 48
(c) 60
(d) can't be determined
- 162** At most how many third party applications a Windows mobile user can have on his/her start screen?
- (a) 15
(b) 16
(c) 32
(d) 30

- 163** A tech savvy user of Windows mobiles prefers the shape in which the ratio of the length and the breadth of the screen is not too high, then the ratio of length and breadth of the grid of tiles on the start screen would be:

(a) 2:3 (b) 3:2 (c) 4:3 (d) 5:3

- 164** What is the sum of all integers directly above 160?

1					
4	7				
10	13	16			
19	22	25	28		
31	34	37	40	43	
...	...	...	...	...	...

(a) 136 (b) 133 (c) 148 (d) 127

- 165** What is the maximum integral value of m , if $n < 0$ and $m = n + \frac{285}{n}$?

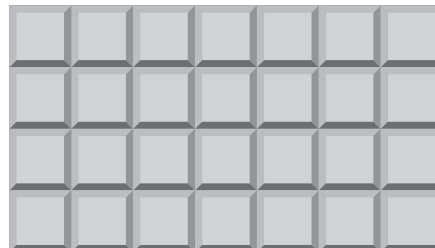
(a) -34 (b) -286 (c) 34 (d) -33

- 166** Find the value of abc , if a , b and c are integers and

$$a + \frac{1}{b + \frac{1}{c}} = \frac{37}{16}$$

(a) 30 (b) 24 (c) 48 (d) none

- 167** Minimum how many steps are required to break apart all the pieces of this chocolate bar, if it is performed gently without damaging any piece?



(a) 28 (b) 18
(c) 11 (d) none of these

- 168** For any set of real numbers $R = \{a, b, c\}$ let sum of pairwise product $S = ab + bc + ca$. If $a + b + c = 1$, then

(a) $S \leq \frac{1}{3}$ (b) $S < \frac{1}{3}$
(c) $S \leq \sqrt{3}$ (d) $S < \frac{1}{9}$

- 169** A number 4 can be expressed as an ordered sum of one or more positive integers in 8 ways namely 4, 3 + 1, 1 + 3, 2 + 2, 1 + 1 + 2, 1 + 2 + 1, 2 + 1 + 1, 1 + 1 + 1 + 1. Find the number of ways in which 8 can be expressed as above.

(a) 16 (b) 61
(c) 64 (d) none of these

LEVEL 03 > FINAL ROUND

- 1** Sania wanted to cut a cubical cake into 120 identical pieces applying minimum number of 'n' cuts. Later on she realised that she had to cut this cake into maximum number of identical pieces i.e., 125 pieces. Now she applies the number of cuts is :

(a) n (b) $n + 5$
(c) $n + 1$ (d) can't be determined

Directions (for Q. Nos. 2 to 4)

If $s_1 = (1);$
 $s_2 = (2), (3);$
 $s_3 = (4), (5), (6);$
 $s_4 = (7), (8), (9), (10);$
 $s_5 = (11), (12), (13), (14), (15) \dots etc.$

where $s_1, s_2, s_3, \dots etc.$ are the first, second and third terms ... of the given sequence.

- 2** The last digit and tens digit of the sum of all the elements of the first 21 terms of this sequence respectively are :

(a) 7, 0
(b) 6, 9
(c) 9, 6
(d) none of (a), (b), (c)

- 3** The number of zeros at the end of the product of the elements of s_{19} .

(a) 4 (b) 5
(c) 1 (d) none of these

- 4** The largest power of 10 that can exactly divide the product of all the elements of s_{19} and s_{20} is :

(a) 10 (b) 9
(c) 19 (d) can't be determined

- 5** A set 'S' contains first 50 elements of the form $2n; n \in N$. Further a subset 'P' of set 'S' is formed such that the product of any 3 elements of 'P' is not divisible by 16. Then maximum number of elements that 'P' can have is :

(a) 12 (b) 13
(c) 25 (d) none of (a), (b), (c)

- 6** A cuboid of dimensions 51, 85 and 102 cm is first painted by red colour then it is cut into minimum possible identical cubes. Now the total surface area of all those faces of cubes which are not red is :

(a) 119646 cm^2 (b) 52020 cm^2
(c) 18514 cm^2 (d) 36414 cm^2

- 7** A number 'p' is such that it is divisible by 7 but not by 2. Another number 'q' is divisible by 6 but not by 5, then the following expression which necessarily be an integer is :
 (a) $\frac{7p+6q}{42}$ (b) $\frac{5p+6q}{71}$
 (c) $\frac{6p+7q}{42}$ (d) none of these
- 8** If $p^q - q^r = (p+q)^{r-q}$; $p > r > q \in \text{Prime numbers less than 11}$ then $p+q$ is equal to :
 (a) $r(r-q)$ (b) $r(q-p)$
 (c) $r(p+q)$ (d) pq
- 9** To visit the Republic Day Parade on 26th January, 2005 the people from every nook and corner including intellectuals, artists, farmers and mathematicians thronged in New Delhi. There were 100 seats in front row numbered 1, 2, 3, ..., 100. But the smart mathematician chose not to sit on those seats which are the multiples of any number greater than unity. How many mathematicians could sit on these front row chairs?
 (a) 25 (b) 26
 (c) 33 (d) none of these
- Directions** (for Q. Nos. 10 to 12) *In South-Asia the New Desh follows a septarian calender in which every month starts with Monday and a week has 7 days. There are only four months, the first 3 months consists of 98 days each and the last month has 70 days only.*
- 10** Which day falls on the 88th day of the first month?
 (a) Sunday (b) Monday
 (c) Thursday (d) Saturday
- 11** Which date cannot fall on the Thursday of the first month?
 (a) 46 (b) 18
 (c) 81 (d) 64
- 12** Which day occurs maximum number of times in a year?
 (a) Monday (b) Saturday
 (c) Sunday (d) none of these
- 13** The last two digits in the expansion of $(1989)^{91}$ are :
 (a) 9, 1 (b) 8, 1
 (c) 6, 9 (d) 8, 9
- 14** Earlier when I have created my e-mail-ID, the password was consisting of first 4 prime numbers. Recently when I tried to check my emails I got dumbfounded since I could not remember my password exactly. So when I have written 2735, my computer indicated me that no digit is correctly placed. Again I tried 5273, I got the same response. So once again I have written only 3 as the left most digit for my password it again indicated me that it was wrong. Finally I have taken one more attempt and got the account open. The code of my password is :
 (a) 2537 (b) 7352
 (c) 7325 (d) none of these
- 15** The remainder when $(888!)^{999}$ is divided by 77 is :
 (a) 1 (b) 2
 (c) 3 (d) none of these
- 16** We publish a monthly magazine of 84 pages. Once I found that in a magazine 4 pages were missing. One out of them was page number 29. It is known that the page number of the last page of the magazine is 84, (including the coverpages). The numbers printed on the missing pages were :
 (a) 29, 52, 53 (b) 30, 55, 56
 (c) 28, 52, 53 (d) can't be determined
- 17** There are six locks exactly with one key for each lock. All the keys are mixed with each other. The maximum number of attempts needed to get the correct combination is :
 (a) 21 (b) 15
 (c) 6 (d) can't be determined
- 18** If n is an integer, how many values of n will give an integral value of $51n^2 + 17n + 6$?
 (a) 4 (b) 3
 (c) 2 (d) none of these
- 19** Sania always beats Plexur in tennis, but loses to Venus. Lindse usually beats Plexur and sometimes Sania, but cannot win against Venus. The worst player can be :
 (a) Venus (b) Plexur
 (c) Sania (d) can't say
- 20** Winner and Loser, the two brothers are playing a game in a recreational room at Amausi Airport, Lucknow. In this game each one in turn has to pick up a number m such that $1 \leq m \leq n$; $m, n \in I^+$. The game stops as soon as the sum of all the numbers picked so far attains the value of $2n+1$ or exceeds and thus the player who picked last number was loser. With which number winner starts if he were to pick up first to defeat necessarily the loser is?
 (a) $(n+1)$ (b) n (c) $(n-1)$ (d) $2n$
- 21** The sum of the last 10 digits of the sum of the expression :
 $(1^1 \times 2^2 \times 3^3 \times 4^4 \times 5^5) + (1^6 \times 2^7 \times 3^8 \times 4^9 \times 5^{10})$
 $+ (1^{11} \times 2^{12} \times 3^{13} \times 4^{14} \times 5^{15}) + \dots$
 $+ (1^{96} \times 2^{97} \times 3^{98} \times 4^{99} \times 5^{100})$ is :
 (a) 16 (b) 18
 (c) 20 (d) none of these
- 22** One day very early morning Ravishankar went to temple to offer some flowers as a part of *Puja*. He purchased some flowers but the seller offered him that if he would give him all his ₹ 2, he could get all the remaining 6 flowers and thus could gain 60 paise per dozen. If each time the transaction is possible only in rupees then how many flowers did Ravishankar purchase initially?
 (a) 6 (b) 3
 (c) 4 (d) 12

- 23** Maximum number of squares possible that can be constructed using 31 pencils of equal length on the table :
 (a) 30 (b) 20
 (c) 15 (d) 29

Directions (for Q. Nos. 24 and 25) *Kavita, a student of IIMA, told me that she did everyday 3 more passages of English than that of previous day and thus she completed all the passages in 10 days. Later on she told me that the number of passages she did on the last but one day were four times that she did on the second day.*

- 24** Number of passages she has done on the last day :
 (a) 30 (b) 41
 (c) 32 (d) none of these
- 25** Total number of passages that she has completed in those 10 days :
 (a) 84
 (b) 180
 (c) 175
 (d) can't be determined
- 26** Recently, a small village in Tamilnadu where only male shepherds reside with four sheep each, was devastated by Tsunami waves. Therefore 8 persons and 47 sheep were found to be dead and the people, who luckily survived, left the village with one sheep each. Since 21 sheep were too injured to move so have been left on their own luck in the village. The number of sheep which were earlier in the village is :
 (a) 84 (b) 120
 (c) can't be determined (d) none of these
- 27** The number of 3-digit numbers which consist of the digits in A.P., strictly in increasing order using the non-zero digits of the decimal system is :
 (a) 14 (b) 16
 (c) 15 (d) none of these
- 28** The sum of :
 $(2^2 + 4^2 + 6^2 + \dots + 100^2) - (1^2 + 3^2 + 5^2 + \dots + 99^2)$ is :
 (a) 5555 (b) 5050 (c) 888 (d) 222
- 29** If an integer p is such that $(8p + 1)$ is prime, where $p > 2$, then $(8p - 1)$ is :
 (a) divisible by 7 (b) divisible by 3
 (c) a prime number (d) none of these
- 30** The remainder when $3^0 + 3^1 + 3^2 + \dots + 3^{200}$ is divided by 13 is :
 (a) 0 (b) 12
 (c) 3 (d) none of these
- 31** The remainder when $4^0 + 4^1 + 4^2 + 4^3 + \dots + 4^{40}$ is divided by 17 is :
 (a) 0 (b) 16
 (c) 4 (d) none of these

- 32** In the problem number 31, if the divisor is 7 then the remainder is :
 (a) 4 (b) 5 (c) 6 (d) 1
- 33** A monkey wanted to climb on the smooth vertical pole of height of 35 metre. In the first one minute he climbed up 5 metre in the next one minute he slipped down by 2 metre. Further he repeated the same process till he had reached on the top of the pole. Minimum how many times did he have to go upward to reach the apex of the pole?
 (a) 35 (b) 12
 (c) 11 (d) can't say
- 34** In the above question the minimum time required for this job is :
 (a) 21 minute (b) 22 minute
 (c) 24 minute (d) none of these
- 35** In the same problem, if the height of the pole is 36 metre then the time taken by monkey to reach at the top of the pole is :
 (a) 22 min. 36 sec. (b) 22 min. 24 sec.
 (c) 23 min. 12 sec. (d) none of these
- 36** The remainder when $1^3 + 2^3 + 3^3 + \dots + 999^3 + 1000^3$ is divided by 13 is :
 (a) 7 (b) 11
 (c) 12 (d) none of these
- 37** If $22^3 + 23^3 + 24^3 + \dots + 87^3 + 88^3$ is divided by 110 then the remainder will be :
 (a) 55 (b) 1 (c) 0 (d) 44
- 38** The sum of the n terms of a series is $n! + n^2$ then the 6th term is, if $n \in N$:
 (a) 756 (b) 611
 (c) data insufficient (d) none of these
- 39** A smallest possible number which is divisible by either 3, 5 or 7 when represented by only two digits either 0 or 1, then the minimum number of digits required to represent it :
 (a) 6 (b) 5
 (c) 7 (d) can't be determined
- 40** The sum of first n odd numbers (i.e., $1 + 3 + 5 + 7 + \dots + 2n - 1$) is divisible by 11111 then the value of n is :
 (a) 12345 (b) 11111
 (c) can't be determined (d) none of these
- 41** Anjuli bought some chocolates from Nestle's exclusive shop and she gave Amit one less than half of what she had bought initially. Then she gave 3 chocolates to Bablu and then half of the chocolates which she had gave to Charles. Thus finally she gave one chocolate to Deepak and the remaining one she ate herself. The number of chocolates she had purchased.
 (a) 9 (b) 12 (c) 10 (d) 15

- 42 If $(a_1b_1c_1)^{100} + (a_2b_2c_2)^{100} + (a_3b_3c_3)^{100} + \dots + (a_{100}b_{100}c_{100})^{100}$, where $a_ib_jc_i$ is a three digit positive number and in the expression all the 100 numbers are any consecutive 3 digit numbers. The last digit is :
 (a) 0 (b) 1
 (c) 3 (d) none of these

Directions (for Q. Nos. 43 to 45) If $[x]$ is read as the greatest integer less than or equal to x , $\{x\}$ is the least integer greater than or equal to x . Further $f(x, y) = [x] + \{y\}$

and $g(x, y) = [x] - \{y\}$

and $P(x, y) = f(x, y) + g(x, y)$

and $Q(x, y) = f(x, y) - g(x, y)$

- 43 If $x = 16$ and $y = 25$, the value of $P(x, y) + Q(x, y)$ is :
 (a) 90 (b) 200
 (c) can't be determined (d) none of these

- 44 If $x^2 = 16$ and $y^2 = 25$, $P(x, y)Q(x, y)$ is :
 (a) 80 (b) -80
 (c) 72 (d) none of these

- 45 If $x, y \in I^+$ then $P(x, y) + Q(x, y)$ is always :
 (a) an even number (b) an odd number
 (c) can't say (d) none of these

- 46 Which of the following is/are true?

- (i) $43^3 - 1$ is divisible by 11
 (ii) $56^2 + 1$ is divisible by 19
 (iii) $50^2 - 1$ is divisible by 17
 (iv) $(729)^5 - 729$ is divisible by 5
 (a) (i) and (ii)
 (b) (iii) and (iv)
 (c) (ii), (iii) and (iv)
 (d) (ii) and (iii)

- 47 Capt. Manoj Pandey once decided to distribute 180 bullets among his 36 soldiers. But he gave n bullets to a soldier of n th row and there were same number of soldiers in each row. Thus he distributed all his 180 bullets among his soldiers. The number of soldiers in $(n-1)$ th row was :

- (a) 3 (b) 8
 (c) 9 (d) none of these

- 48 If $(n-5)$ is divisible by 17 for every $n \in I^+$ then the greatest integer which will necessarily divide $(n+12)(n+29)$ is :

- (a) 578
 (b) 289
 (c) such a number does not exist
 (d) none of the above

- 49 A certain number ' n ' can exactly divide $(3^{24} - 1)$, then this number can also divide the number :

- (a) $(3^{16} + 1)$ (b) $(3^8 - 1)$
 (c) $(3^{70} - 1)$ (d) $(3^{96} - 1)$

- 50 If a number ' n ' can exactly divide $(5^{14} - 1)$ then ' n ' can necessarily divide :

- (a) $(5^{28} - 1)$ (b) $(5^{42} - 1)$
 (c) $(5^{21} + 1)$ (d) both (a) and (b)

- 51 The n th term of a series of which all the terms are positive is defined as $T_n = n^2 + n$, then the sum of n terms of the series is :

- (a) $\frac{n(n+1)(n+2)}{6}$ (b) $\frac{n^3 + 4n}{n}$
 (c) $\frac{5n}{9}[n^2 + n]$ (d) $\frac{n(n+1)(n+2)}{3}$

- 52 The number of zeros at the end of the product of

$$222^{111} \times 35^{53} + (7!)^{61} \times (10!)^{51} + 42^{42} \times 25^{25} \text{ is :}$$

- (a) 42 (b) 53
 (c) 1055 (d) none of these

- 53 $\frac{12345}{12346} + \frac{12346}{12347} + \frac{12347}{12345}$ is equal to :

- (a) 2.67 (b) 6.27
 (c) 3 (d) 5

Directions (for Q. Nos. 54 to 56) The set $S_1 = \{1\}$, $S_2 = \{3, 5\}$, $S_3 = \{7, 9, 11\}$, etc. forms a sequence.

- 54 Sum of all the elements of S_{10} is :

- (a) 55 (b) 300
 (c) 3375 (d) none of these

- 55 The 11th element of the set S_{21} is :

- (a) 21 (b) 121
 (c) 221 (d) 441

- 56 The sum of the first and last element of the set S_{51} is :

- (a) 5202 (b) 5151
 (c) 5152 (d) 5102

- 57 During my studies once I brought a book from library which was written in early days, when there were only 9 digits i.e., the digit 0 did not exist. There was a sum in that book as follows :

$$k = 13 + 17 + 31 + 2$$

Then the value of k if $9 + 1 = 11$, $19 + 4 = 24$ etc.

- (a) 70 (b) 71
 (c) 63 (d) 64

- 58 According to that book, the sum of $4 + 16 - 5 + 12$ is

- (a) 27 (b) 26
 (c) 29 (d) none of these

- 59 The value of x for which the unit digits of $(2357)^{\log_{10} x}$ and $(5723)^x$ are same for $x > 1$.

- (a) 10 (b) 100
 (c) 1000 (d) none of these

- 60** The value of x for which the unit digits of the following two expressions $(1 + 2 + 3 + 4 + 7)^{x^2+x}$ and $(11 \times 11 \times 13)^x$ are same for $x > 0$:

(a) 1 (b) 2
(c) 3 (d) none of these

- 61** When any odd number greater than unity multiplied by even times by itself then dividing this product by 8, we get the remainder as :

(a) 1 (b) 7
(c) not unique (d) none of these

- 62** Stephen's birthday, this year falls on 2nd April, Wednesday. But coincidentally his marriage anniversary is 2 days before the 23rd of the same month. On which day he will celebrate his marriage anniversary?

(a) Monday
(b) Wednesday
(c) Friday
(d) can't be determined

- 63** In the above problem if there are only 6 days in a week i.e., there is no Sunday and the week starts with Monday and ends with Saturday then his marriage anniversary will fall on :

(a) Wednesday
(b) Thursday
(c) Friday
(d) data insufficient

- 64** An N.G.O. (non-government organisation) STRANGE working for the relief of Tsunami Victims in Srilanka consisting of 7 members S, T, R, A, N, G, E of the same family.

The eldest one 'S' spoke to me "I have deployed equal number of brothers and sisters for medical relief and psychological counselling under the supervision of myself". Later on the youngest member E spoke to me as "we have been working for rehabilitation and food supply as twice the number of sisters as the number of brothers have been deployed there, but I did not actually do any thing due to a severe injury in my leg". Then we can conclude that :

(a) youngest person is a lady
(b) eldest person's wife is youngest
(c) the brother of S is youngest
(d) nothing can be said

- 65** When the sum of n digits of an n digit number is subtracted from the number itself, where the number must be atleast two digit number, then the correct statement is :

(a) the difference is a prime number
(b) the resultant value is a perfect square
(c) the resultant value is an odd number
(d) the resultant value is a multiple of 9

- 66** For every natural number x and y the value of $x - y$ when $y + \frac{7}{x} + \frac{y}{13} = 6\frac{3}{143}$, is :

(a) 5 (b) 6
(c) 2 (d) not unique

Directions (for Q. Nos. 67 and 68) The $S = \{(1, 3, 5, 7, 9, \dots, 99) (102, 104, 106, \dots, 200)\}$ i.e., in the first part there are odd integers less than 100 and in the second part there are even integers greater than 100, but upto 200.

- 67** The highest power of 3 in the product of the element of the set is :

(a) 52 (b) 51
(c) 97 (d) can't be determined

- 68** The highest power of 5 that can exactly divide the product is :

(a) 25 (b) 24 (c) 30 (d) 26

- 69** The number of zeros at the end of the following expression :

$P = \{(2 \times 4 \times 6 \times 8 \times 10 \times \dots 50) \times (55 \times 60 \times 65 \times 70 \times 75 \times \dots 100)\}$
(a) less than 20 (b) 57
(c) 20 (d) 36

- 70** 1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, ... are the consecutive numbers written in base 5. The twenty fifth number in the above sequence would be :

(a) 52 (b) 100
(c) 25 (d) none of these

- 71** Watch India Corporation made a wrist watch in which the minute hand makes one complete round of dial in 12 minutes and accordingly the hour hand too. When I have set this watch at 12 : 00 noon on 1st February this year. What time will be shown by this watch at 3 O'clock on the same day.

(a) 2 : 30 P.M.
(b) 6 : 15 A.M.
(c) 3 O'clock
(d) can't be determined

- 72** The sum of first n numbers of the form $(5k + 1)$, where $k \in I^+$ is :

(a) $\frac{n}{2} [5n^2 - 3]$ (b) $n (20 - 3n)$
(c) $\frac{n}{2} (5n + 7)$ (d) none of these

- 73** A series is given as : 1, 4, 9, 16, 25, 36, ... Then the value of $T_{n+1} - T_n$ is, where T_n is the n th term of the series is :

(a) $n^2 - 1$ (b) $2n + 1$
(c) $n^2 + 1$ (d) none of these

- 74** The area of paper can be divided into 144 squares, but if the dimensions of each square were reduced by 2 cm each, then the number of squares so formed are 400. The area of the paper, initially, was :

(a) 544 cm² (b) 1444 cm²
(c) 3600 cm² (d) none of these

- 75** If $(ab)^2 = bcb$ and $(dd)^2 = cff$, where a, f, d are strictly in increasing order of G.P. and b, c, d are in increasing order of A.P. Then the value of f will be, where ab and bcb etc. are the two digit and 3 digit numbers etc.

(a) 9 (b) 8
(c) can't be determined (d) none of these

- 76** If $A = 555!$ and $B = (278)^{555}$ then which one of the following relations is appropriate?

(a) $A > B$ (b) $A = B$
(c) $A < B$ (d) can't say

Directions (for Q. Nos. 77 to 81) For any natural number n the sets $S_1, S_2, \dots$ are defined as below :

$$S_1 = \{1\}, S_2 = \{2, 3\}, S_3 = \{4, 5, 6\} \\ S_4 = \{7, 8, 9, 10\}, S_5 = \{11, 12, 13, 14, 15\} \dots \text{etc.}$$

- 77** The last element in the S_{24} is :

(a) 576 (b) 600
(c) 300 (d) 625

- 78** The middlemost element of the set S_{15} is :

(a) 196 (b) 169
(c) 131 (d) none of these

- 79** The sum of the elements of set S_{25} is :

(a) 7825 (b) 3125
(c) 3250 (d) none of these

- 80** In which set, there are maximum number of prime number elements among $S_1, S_2, S_3, \dots, S_{13}$ is :

(a) S_{12} (b) S_{13}
(c) S_{12} and S_{13} (d) S_9, S_{12}, S_{13}

- 81** Of which set the sum of all the elements of the set is even

(a) S_{39} (b) S_{50}
(c) S_{72} (d) S_{94}

Directions (for Q. Nos. 82 to 85) The sequence of sets $S_1, S_2, S_3, S_4, \dots$ is defined as $S_1 = \{1\}, S_2 = \{3, 5\}, S_3 = \{7, 9, 11\}, S_4 = \{13, 15, 17, 19\} \dots \text{etc.}$

- 82** The first element of the n th set S_n is :

(a) n (b) $n^2 - 1$
(c) $2n + 1$ (d) $n^2 - n + 1$

- 83** The last element of the set S_{100} is :

(a) 10099 (b) 9899
(c) 9900 (d) none of these

- 84** The middlemost element of an odd numbered set S_{125} is :

(a) 12500 (b) 15625
(c) 3125 (d) none of these

- 85** The sum of all the elements of S_{101} :

(a) 1531441 (b) 1189811
(c) 1030301 (d) none of these

- 86** The sum of the series :

$$S = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \dots + \frac{1}{99.100} \text{ is :}$$

(a) $\frac{98}{99}$ (b) $\frac{99}{100}$
(c) $S > 1$ (d) none of these

- 87** A number P when divided by D it leaves the remainder 18 and if another number Q is divided by the same divisor D it leaves the remainder 11. Further if we divide $P + Q$ by D then we obtain the remainder 4. Then the common divisor D is :

(a) 22 (b) 15
(c) 25 (d) can't be determined

- 88** If the product of $1 \times 2 \times 3 \times 4 \times \dots \times n$ contains 68 zeros in the end of the number. Then the maximum possible number of values of n is :

(a) 1 (b) 3 (c) 5 (d) 6

- 89**

The remainder when $6^{6^{6^{6^{\dots \infty \text{ times}}}}}$ is divided by 10 is :

(a) 3 (b) 6
(c) 0 (d) can't be determined

- 90** $53^3 - 46^3 - 7^3$ is divisible by :

(a) 6 and 9
(b) 2 and 21
(c) 21 and 23
(d) both (b) and (c)

- 91** A number of decimal system when written in base n , $2 < n < 10$, we get a two digit number. Further if we reverse the digits of the obtained number in base ' n ' we get a number which is twice of the original number in decimal system. The sum of original and resultant number both in decimal system for the largest possible value of n is :

(a) 45 (b) 63
(c) 77 (d) can't be determined

- 92** In the above question how many values of n are possible?

(a) 0 (b) 2 (c) 4 (d) 7

- 93** Pandavas won a hen in the war of the Mahabharat. They brought it on the 1st January, 2002. This hen gave birth to 7 new hens on the very first day. After it every new hen irrespective of its age everyday gave birth (only once in a lifetime) to 7 new hens. This process continued throughout the year, but no any hen had been died so far. On the 365th day all the Pandavas shared equally all the hens among all the five brothers. The remaining (if these can not be shared equally) hens were donated to Krishna. The number of hens which the Krishna had received is :

(a) 3 (b) 2
(c) can't be determined (d) none of these

- 94** Total number of natural numbers being the perfect square whose square root is equal to the sum of the digits of the perfect square is :
 (a) 0 (b) 1
 (c) 2 (d) 12
- 95** At our training Institute the number of boys is same as that of the girls. Last week, except $\frac{2}{3}$ rd of the girls all the students went to picnic, where they bought some samosas but later on they found exactly one dozen samosas were not fresh so those 12 samosas had been thrown away. After it the samosas were divided equally between boys and girls. Further when boys dealt out the samosas equally among themselves 39 samosas left undistributed, but when the girls dealt out the same number of samosas equally among themselves 12 samosas were still left undistributed. The number of students at our training institute is :
 (a) 60 (b) 156
 (c) 162 (d) can't be determined
- 96** Darwin Miya has 6 kinds of fruits in large amount and has sufficient number of identical boxes to store the fruits. He can put at least 10 and atmost 15 fruits in any box and he put only one kind of fruits in a box. Further not more than 5 boxes can contain same number of fruits. Maximum number of fruits that he can put in the boxes is :
 (a) 325
 (b) 375
 (c) 75
 (d) can't be determined
- 97** In the above question if he is allowed to put the equal number of fruits in atmost 7 boxes and he has only 33 boxes and now he can put any kind of fruit with any other kind of fruits. At least how many boxes are there in which the number of fruits are same if he fills every box to its maximum capacity.
 (a) 5
 (b) 6
 (c) 3
 (d) can't be determined
- 98** If $n \in 1, 3, 5, 7, \dots$ etc., then the value of $19^n - 23^n - 43^n + 47^n$ is necessarily divisible by :
 (a) 264 (b) 246
 (c) 76 (d) 129
- 99** The sum of the following series :
 $1.1^2 \left(1 - \frac{0}{1}\right) + 2.2^2 \left(1 - \frac{1}{2}\right) + 3.3^2 \left(1 - \frac{2}{3}\right)$
 $+ 4.4^2 \left(1 - \frac{3}{4}\right) + \dots$ upto n terms is :
 (a) $n \cdot n^2 \left(1 - \frac{1}{n}\right)$ (b) $\frac{n(n+1)(2n+1)}{6}$
 (c) $\frac{n(n+1)^2}{4}$ (d) none of these
- 100** The distance between the Sarvesh's house and Ravi's house is 900 km and the Sarvesh's house is at 100th milestone where as the Ravi's house is at 1000th milestone. There are total 901 milestones at a regular interval of 1 km each. When you go to Ravi's house from the Sarvesh's house which are on the same highway, you will find that if the last digit (*i.e.*, unit digit) of the 3 digit number on every milestone is same as the first (*i.e.*, hundreds digit) of the number on the next mile stone, then these milestones must be red and rest will be black. Total number of red milestones is :
 (a) 179 (b) 90
 (c) can't be determined (d) none of these
- 101** The highest power of 17 which can divide exactly the following expression :
 $(18^2 - 1)(18^4 - 1)(18^6 - 1)(18^8 - 1)(18^{10} - 1) \times \dots$
 $(18^{16} - 1)(18^{18} - 1)$ is :
 (a) 1 (b) 17
 (c) 9 (d) can't be determined
- 102** Help India Foundation and People for People Organisation decided to distribute the blankets among 22 men and 28 women who are Tsunami victims. When HIF and PPO distributed their respective blankets evenly among 28 women they were left with 24 and 16 blankets respectively. If they distributed their blankets evenly among 22 men they were left with 12 blankets each. So finally they decided to combine all their blankets and then distributed among 22 men and 28 women altogether then no any blanket remained undistributed. Minimum total blankets distributed by them were :
 (a) 960 (b) 700
 (c) 1300 (d) none of these
- 103** In the above problem the ratio of blankets between HIF and PPO is :
 (a) $\frac{43}{157}$ (b) $\frac{147}{179}$
 (c) $\frac{59}{101}$ (d) can't be determined
- 104** The total number of 3 digit numbers which have two or more consecutive digits identical is :
 (a) 171 (b) 170 (c) 90 (d) 180
- 105** For $p \neq 1$, the expression $(1 + p^{256}) \times (1 + p^{128}) \times (1 + p^{64}) \times (1 + p^{32}) \times (1 + p^{16}) \times (1 + p^8) \times (1 + p^4) \times (1 + p^2) \times (1 + p)$ is equivalent to :
 (a) $\frac{1 + p^{256}}{1 + p^{128}}$ (b) $\frac{1 - p^{512}}{1 - p}$
 (c) $\frac{1 - p^{256}}{1 - p}$ (d) p^{255}
- 106** For the given fixed perimeter of 50 cm, the total number of rectangles which must have its sides in integers (cm) is :
 (a) 50 (b) 25 (c) 12 (d) infinite

- 107** The total number of factors of a number is 24 and the product of the prime factors of the same number is 30. The square root of the ratio of such a smallest and the greatest possible numbers is :
 (a) $5/6$ (b) $6/9$ (c) $9/25$ (d) $4/25$
- 108** In the above problem, maximum how many numbers are possible?
 (a) 9 (b) 8
 (c) 7 (d) data insufficient
- 109** Mr. Oberai appeared in CAT for four consecutive years, but coincidentally each time his net score was 75. He told me that there was $\frac{1}{3}$ rd negative marking for every wrong answer and 1 mark was allotted for every correct answer. He has attempted all the questions every year, but certainly some answers have been wrong due to stress and conceptual problems. Which is not the total number of questions asked for CAT in any year, in that period?
 (a) 231 (b) 163
 (c) 150 (d) 123
- 110** A thief somehow managed to steal some golden coins from a bank's cash but while coming out of it at the first door he was caught by the watchman and he successfully dealt him by paying 1 coin plus half of the rest coins. Further he had to pay 2 coins, then half of the rest to the second watchman. Once again at the third gate (outermost) he gave 3 coins and then half of the rest. After it he was left with only one coin. How many coins had he stolen?
 (a) 32 (b) 36
 (c) 25 (d) none of these
- 111** The number $\log_2 7$ is :
 (a) an integer (b) a rational number
 (c) an irrational number (d) a prime number
- 112** The product of n positive numbers is unity. Then their sum is :
 (a) a positive integer (b) divisible by n
 (c) equal to $\left(n + \frac{1}{n}\right)$ (d) never less than n
- 113** Let $n > 1$, be a positive integer. Then the largest integer m , such that $(n^m + 1)$ divides $(1 + n + n^2 + n^3 + \dots + n^{127})$ is :
 (a) 127 (b) 63
 (c) 64 (d) 32
- 114** Number of divisors of the form $4n + 2$, $n \geq 0$ which can divide 240 is :
 (a) 4 (b) 8
 (c) 10 (d) 3
- 115** If the integers m and n are chosen at random from the set $\{1, 2, 3, \dots, 100\}$, the maximum how many distinct numbers of the form $7^m + 7^n$ would be divisible by 5?
 (a) 1250 (b) 10000
 (c) 2500 (d) none of these
- 116** If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then $M = (a + b)(c + d)$ satisfies the relation is :
 (a) $0 \leq M \leq 1$
 (b) $1 \leq M \leq 2$
 (c) $2 \leq M \leq 3$
 (d) $3 \leq M \leq 4$
- 117** A corrupt country receives some educational grant from the UNESCO in the form of digital notebooks. The total number of notebooks that this country receives is $N = \frac{P!}{Q!}$; where $0 \leq P \leq 99$ and $0 \leq Q \leq 99$ such that $P - Q = 10$. The country is divided into various levels – $L_1, L_2, L_3, \dots$ etc. – for administrative conveniences. Initially, all the notebooks are with the head of L_1 , who keeps half of the notebooks with himself and rest half of the notebooks he doles out equally to the each head of L_2 . Similarly, each head of L_2 keeps half of the notebooks with oneself and rest half of the notebooks he doles out to the each head of L_3 . This pattern occurs till the level L_x where it becomes impossible for the each head of L_x to divide the number of notebooks in two equal parts, so one does not dole out any notebook to any of the heads of L_{x+1} and thus to the expected beneficiary. That is, the general public, which is supposed to receive the notebooks directly from L_{x+1} never receives any notebook. Also, at any level the number of heads is never equal to 2. What's the maximum possible value of x ?
 (a) 10 (b) 11
 (c) 12 (d) none of these
- 118** In a far land of Lootokhao, an empowered group of ministers was supposed to allocate some coal blocks and 2G spectrums to the aspiring private companies. However, these ministers colluded for their personal gains by allocating the blocks and spectrums to a lobbying corporate coterie and that too after charging the bribe. The bribe fetched by each minister for allocating each block was ₹ 300 million and for each spectrum it was ₹ 500 million. Each minister allocated the same number of coal blocks and the same number of 2G spectrums. The total bribe they received in this process was ₹ 13.3 billion. How many coal blocks did each minister allocate to this corporate coterie by charging the bribe?
 (a) 1
 (b) 2
 (c) 3
 (d) can't be determined uniquely
- 119** Initially, a person had ₹ 20 when he sat down to play a game. If he wins he would get ₹ 3, but if he loses he would have to give away ₹ 1 to his opponent. If he played at least 1 game and at most 20 games, how many distinct amounts he would have at the end of the game?
 (a) 81 (b) 60
 (c) 80 (d) 78

- 120** During a counter insurgency operation, three paramilitary troops –A, B and C, fought bravely against the rebels and thwarted the attempt of a coup d'état. The fight was so intense and lethal that exactly half of the paramilitary forces lost their lives. Only 36% of troop A, 60% of troop B and 80% of the troop C could survive the fight. Which of the following is not the possible number of total soldiers in all the three troops together engaged in the Fight?
- (a) 40 (b) 50
(c) 60 (d) 90

Directions (for Q. Nos. 121 to 123) *At a bowling alley there are 25 pins, numbered 1 to 25, placed in a row. By using a bowling ball you have to knock over the pins, which are initially arranged in the order, 1, 2, 3, ..., 25. For no apparent reason, in any particular throw you hit exactly 5 contiguous pins out of the 25 pins. For any natural number m, if in the first throw your score is 16m, you are entitled to get the refund of what you pay for the game in advance. If out of the 5 pins, which you hit in a throw, the pin with the highest number is denoted by k, then $16m = \left[\frac{k!}{(k-5)!} \right]$; Where $5 \leq k \leq 25$.*

- 121** For how many distinct values of 16m you get back your money you pay for playing the game?
- (a) 15 (b) 16
(c) 12 (d) none of these
- 122** For how many distinct values of k you can get back your money you pay for playing the game?
- (a) 8 (b) 12
(c) 21 (d) none of these
- 123** How many distinct pins are there if you knock them over you won't get back your money you pay for playing the game?
- (a) 7 (b) 8
(c) 4 (d) none of these
- 124** For every natural number a, b, c and x, if $(x^a + 1)(x^b + 1)(x^c + 1)$ is the factor of $S = 1 + x + x^2 + x^3 + \dots + x^{111}$, what is the HCF of a, b and c
- (a) 7 (b) 3
(c) 14 (d) can't be determined
- 125** Find the value of k, if $15! = 1k07674368000$.
- (a) 0 (b) 3 (c) 2 (d) 1

Test of Your Learning 1

- 1** A monkey wants to climb up a pole of 50 metre height. He first climbs up 1 metre but he falls back by the same height. Again he climbs up 2 metre but he falls back by 1 metre. Once again he climbs up 3 metre but he falls back by 1 metre. Again he climbs up 4 metre but he falls back by 1 metre. In this way he reaches at the top of the pole. If it is known that the monkey needs 10 seconds for 1 metre in upward direction and 5 seconds for 1 metre in downward direction, then the total time required by monkey to reach at the top of the pole. For your kind information it must be clear to you that when he will reach on the top, he will not slip back. The total time required by this monkey to reach on the top of the pole is
- (a) 9 minutes 10 seconds (b) 10 minutes
(c) 8 minutes 20 seconds (d) none of these
- 2** Which one of the following is correct?
- (i) $(123)^{369} > (369)^{123}$ (ii) $(246)^{642} < (642)^{246}$
(iii) $(50)^{50!} = (50!)^{50}$ (iv) $3^{4^5} > 5^{4^3}$
- (a) (i) and (ii)
(b) (ii) and (iii)
(c) (i) only
(d) (i) and (iv)
- 3** The tens digit of the 2^{248} is :
- (a) 1 (b) 7
(c) 5 (d) none of these
- 4** For a community marriage 300 boys and 300 girls gathered for their marriage. But out of 600 people 200 were from America and 200 from France and rest 200 were from India itself. The rule was that no any person can marry with a person of different ethnicity or nationality. Minimum how many married couples will be formed, after marriage?
- (a) 100 (b) 150
(c) 200 (d) none of these
- 5** The 444th term of the following sequence 1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23, 30, 31, ... is :
- (a) 1331 (b) 17776 (c) 12330 (d) 13330
- 6** If $P = 3[x] + 5$ and $Q = 4[x + 3] + 6$ and $P : Q = 1 : 2$, where $[x]$ is the greatest integer less than or equal to x then the value of $\left[P + \frac{Q}{2} \right]$ is :
- (a) 34 (b) 51
(c) 40 (d) data insufficient
- 7** The maximum possible value of $a + b + c + d$ will be where $(a + b) = c$, $(b + c) = d$, $(c + d) = a$, for every $a, b, c, d \in I^+$:
- (a) -4 (b) 0
(c) 16 (d) can't be determined
- 8** The number of divisors of 720 which are multiples of perfect square of first prime number is :
- (a) 12 (b) 14
(c) 18 (d) can't be determined

- 9 The number of zeros which are not possible at the end of the $n!$ is :
 (a) 82 (b) 73 (c) 156 (d) none

Directions (for Q. Nos. 10 and 11) *The following sequence is given below :*

1, 3, 3, 3, 5, 5, 5, 5, 5, 7, 7, 7, 7, 7, 7, 9, 9, 9, 9, 9, 9, ...

- 10 The 111th term of this sequence will be :
 (a) 111 (b) 19 (c) 21 (d) none
- 11 The sum of the 1st, 3rd, 8th, 15th, 24th, 35th, ..., 99th term will be :
 (a) 100 (b) 101 (c) 121 (d) 81

Directions (for Q. Nos. 12 to 14) *The following sequence is given below :*

1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, ... etc.

- 12 The sum of the first 100 terms of this sequence is :
 (a) 945 (b) 358350
 (c) 100! (d) none of these
- 13 The 77th element is :
 (a) 10 (b) 12 (c) 15 (d) 13
- 14 The sum of the 1st, 2nd, 4th, 7th, 11th, 16th, ..., 211th terms is :
 (a) 221 (b) 400 (c) 231 (d) 211
- 15 Which one of the following is true?
 (i) The least positive value of n for which $n!$ can be divided by $(n + 1)$ is 5.
 (ii) The square root of 123454321 is 1111.
 (iii) The sum of the squares of the first 24 natural numbers is a perfect square.
 (a) (i) and (ii) (b) (ii) and (iii)
 (c) (i) and (iii) (d) all of these
- 16 The number of zeros at the end of $1^{99} \times 2^{98} \times 3^{97} \times \dots \times 99^1$ is
 (a) 1100 (b) 1111
 (c) 1099 (d) none of these

Directions (for Q. Nos. 17 to 20) *Mewalal is selling fruits in Charbagh, Lucknow, which were imported from Kesharbagh and Dayalbagh only. He has 13 apples, 25 bananas, 17 cherries, 29 goose berries, 35 mangoes and 42 pine apples. When I wanted to purchase some fruits from his shop, he told me that he can sell only in lots (i.e., either total apples or total bananas or total cherries etc., but anyhow he can't sell say 12 bananas or 10 apples) and he told me that he himself can import any type of fruits without any lot, say 20 mangoes from Kesharbagh and 15 mangoes from Dayalbagh and so on if he wished.*

- 17 Mewalal sold some fruits and then he told me that he is now left with twice the fruits of Kesharbagh than that of Dayalbagh. The number of fruits of Dayalbagh left after this selling is :
 (a) 35 (b) 43
 (c) 46 (d) can't be determined

- 18 In the above question (no. 17) how many lots or category of fruits from A, B, C, G, M, P he has sold?
 (a) 1 (b) 2
 (c) 3 (d) can't be determined

- 19 As per the given constraints if Mewalal has sold some fruits in such a way that he is now left with $\frac{3}{4}$ fruits of Dayalbagh than that of Kesharbagh. Maximum how many lots (i.e., categories) of fruits he can sell out in such a way?
 (a) 1 (b) 2
 (c) 3 (d) can't be determined

- 20 If all the fruits were bought by a single person and were distributed equally among maximum possible number of children and each child had definitely received more than one fruit, then minimum how many children had received different kind of fruits?
 (a) 2 (b) 3
 (c) 4 (d) can't be determined

- 21 Total number of factors of the expression $62^3 - 54^3 - 8^3$ is
 (a) 60 (b) 62
 (c) 46 (d) can't be determined

- 22 The least possible number with which $(180)!$ should be multiplied so that it can be divided by $(45)^{44}$ is :
 (a) 44 (b) 25 (c) 9 (d) none

- 23 If $P = abc$ and $Q = uv$ are three digits and 2 digits two natural numbers respectively, such that u and v must be distinct integers. How many pairs of P and Q are there in total which gives the same result when we multiply abc with uv as the product of cba with vu (i.e., the position of digits is interchanged) :
 (a) 2 (b) 8
 (c) 5 (d) can't be determined

- 24 There is a three digits number abc and a two digit number xy such that the product of abc with xy is same as the product of cba with yx , if x and y can be similar digits, then the number of such pairs of P and Q is :
 (a) 99 (b) 900
 (c) 810 (d) can't be determined

- 25 If $(N)_{10} = (ab)_n$ and $(3N)_{10} = (ba)_n$; where $3 < n < 10$, then the least possible value of n is :
 (a) 4 (b) 6
 (c) 7 (d) can't be determined

- 26 At his birthday party Pravesh invited his 100 friends and they took their seats numbered as 1, 2, 3, ..., 100. Starting from seat number 1 every third guest was served with Chapatis, every fourth guest was served with Poori and every fifth guest was served with Dahibara and the remaining guests were served with coca-cola only. How many guests enjoyed coca-cola in that party?
 (a) 33 (b) 13
 (c) 41 (d) can't be determined

- 27** In the above question (no. 26) if Pravesh served every guest who were sitting on the chair number 3, 6, 9, 12, 15, ... etc. the chapatis and to those who were, sitting on the chair number 5, 10, 15, 20, ... etc. the poori and to the rest of them the Idli. The number of guests who has not received the Idlis is :
 (a) 38 (b) 47
 (c) 41 (d) none of these
- 28** The number of ways in which 5^7 can be expressed as a product of three factors :
 (a) 7 (b) 8
 (c) 9 (d) none of these
- 29** If $[x]$ means the greatest integer less than or equal to x and $\{x\}$ means the least integer greater than or equal to x and $|x|$ means the absolute value of x , then $\{x\} - [x]$ equals to :
 (a) -1
 (b) 0
 (c) 1
 (d) any one of (a), (b), (c)
- 30** In the previous question if x is not an integer then the value of $|\{x\} - [x]|$ is :
 (a) 0 (b) -1
 (c) 1 (d) either (a) or (c)
- 31** If mnp is a three digit number such that :
 $m^3 + n^3 + p^3 = mnp = \Sigma(n!)$,
 then the value of $m \times n \times p$ is :
 (a) 30 (b) 21
 (c) 15 (d) none of these
- 32** A square field is fenced by fixing the polls all around it in such a way that in each side of the field the farmer fixed 19 polls. The total number of polls required to fence the field is :
 (a) 76 (b) 75
 (c) 72 (d) 80
- 33** Swarn Jayanti Park is a regular hexagonal park, around which there are 'n' trees on each side. Which of the following can't be the total number of trees around it?
 (a) 74070 (b) 81474
 (c) 59! (d) 325960
- 34** Pentabhai purchased the ticket of a movie for all his 200 friends. Luckily he was the first person to buy the ticket for the evening show. So he got the tickets number 1, 2, 3, 4, 6, 7, ... etc. Since he denied to have the tickets on which the digit '5' is printed as the part of ticket number. The last ticket number which he has purchased if no one else has bought the tickets until he has purchased for all his friends.
 (a) 244 (b) 242
 (c) 222 (d) can't be determined
- 35** The expression

$$N = \frac{1}{1+1 \times 1} + \frac{1}{2+2 \times 2} + \frac{1}{3+3 \times 3} + \frac{1}{4+4 \times 4} + \dots$$

$$\frac{1}{n+n \times n}, \text{ where } n = 100, \text{ then value of } N \text{ is :}$$

 (a) $\frac{100}{101}$ (b) $\frac{99}{100}$
 (c) 9900 (d) none of these
- 36** Anjuli has written all the letters (or characters) of English alphabets (i.e., A, B, C, D, ... Y, Z) horizontally in a single line. She started counting as A-1, B-2, C-3, D-4, ... Y-25, Z-26 and reversed back as Y-27, X-28, W-29, V-30, ... B-50, A-51 and further reversed back as B-52, C-53, D-54, ... etc. She continued counting till she reached the 777th character. At which letter or character she stopped counting?
 (a) Y (b) B
 (c) R (d) none of these
- 37** The reading style of Sunny is quite unusual. He reads one page on the first day, 2 pages on the second day, 3 pages on the third day etc. How many pages Sunny can read in 24 days?
 (a) 242 (b) 300
 (c) 276 (d) none of these
- 38** All the natural numbers less than 1000 but not containing the digit '5' are arranged horizontally in descending order. What is the 555th number on the list?
 (a) 111 (b) 212
 (c) 213 (d) none of these
- 39** A shopkeeper told me, when I have asked him the total number of cellphones in his shop, that he had all Kyocera except three, all Nokia except three, all samsung except three and all L.G. except three. The number of mobile phones he had is :
 (a) 24 (b) 256
 (c) can't be determined (d) none of these
- 40** The number of zeros at the end of the product of all the prime numbers between 1 and 1111 is :
 (a) 222 (b) 21
 (c) can't be determined (d) none of these

Test of Your Learning 2

- The remainder when $888222888222888222 \dots$ upto 9235 digits is divided by 5^3 is :
 (a) 103 (b) 38
 (c) can't be determined (d) none of these
- Shakuntala asked Aryabhatta to assume any two values of three digits say P and Q then she told him to multiply P by R and Q by S where the values of R and S were given by Shakuntala herself. Aryabhatta exactly told her the values of $PR + QS = 888222$. Then Shakuntala told him the value of P by Q (i.e., P/Q) is :
 (a) $1/4$ (b) 4
 (c) can't be determined (d) none of these
- In the above question the value of $P + Q$ is :
 (a) 1001 (b) 1110
 (c) 3108 (d) none of these
- The sum of the following series $(1^2 + 1) + (2^2 + 2) + (3^2 + 3) + (4^2 + 4) + \dots + (n^2 + n)$ is :
 (a) n^3 (b) $\frac{(n^2 + n)5}{3}$
 (c) $\frac{n(n+1)(n+2)}{3}$ (d) can't be determined

Directions (for Q. Nos. 5 to 8) *Earlier when I attended a close door meeting to discuss the outsourcing of CAT papers. Including the chairman there were n people viz. A, B, C, D, E, F, ... etc. As per the convention everyone got some chocolates in the following manner. As A, B, C, D, E, ... etc. received 1, 2, 3, 4, 5, ... etc. chocolates respectively. Before anyone had eaten a bit of chocolate, due to some urgent call, the chairman left the meeting with his chocolates. Later on the rest attendants recollected their chocolates in a box and then redistributed all the chocolates evenly among themselves and thus everyone received 13 chocolates.*

- Who is the Chairman of the meeting?
 (a) A (b) Q
 (c) M (d) can't be determined
- Total number of people, initially who have attended the meeting is :
 (a) 18 (b) 25
 (c) 30 (d) can't be determined
- Minimum number of people who attended the party can be :
 (a) 18 (b) 15
 (c) 24 (d) can't be determined
- As per the question number 7, maximum how many chocolates were there to be received by all of them, initially?
 (a) 300 (b) 676
 (c) 351 (d) none of these

Directions (for Q. Nos. 9 and 10) *Under the scheme of Kisan Vikas, the Govt. of U.P. purchased 't' number of tractors and these were allocated equally among 7 districts of U.P. and thus 4 tractors remained unallocated. Similarly every district allocated these tractors equally to 10 blocks in each district and still 3 tractors remained without allocation, in every district. Further every block assigned these tractors equally to every 16 village in each block and thus 2 tractors per block remained unallocated. According to the scheme every village must receive at least one tractor.*

- The number of tractors allocated to each block is :
 (a) 18 (b) 30
 (c) 34 (d) can't be determined

- If the state govt. of U.P. purchased 'k' tractors to provide exactly 1 tractor to each village as mentioned in the scheme but later on it has sent one tractor to each village of U.P. (as mentioned in the scheme) directly without any involvement of a block or a district. The number of tractors which the govt. of U.P. has saved more in comparison to the previous scheme.
 (a) 161 (b) 181
 (c) 24 (d) can't be determined

- If 2^n can exactly divide $p!$ Such that the quotient is an odd positive integer, then the value of n which is not possible is
 (a) 43 (b) 44
 (c) 45 (d) all of these

- Each of the numbers $x_1, x_2, x_3, x_4, x_5, \dots, x_n, x_n \geq 4$ is equal to -1 or 1 . If $x_1x_2x_3 + x_2x_3x_4 + x_3x_4x_5 + \dots + x_{n-3}x_{n-2}x_{n-1} + x_{n-2}x_{n-1}x_n = 0$, then the value of n is :
 (a) odd (b) even
 (c) prime (d) multiple of 3

- In the above question number 12, the minimum number of x_i (i.e., $x_1, x_2, x_3, \dots, x_n$) are equal to -1 , is :
 (a) $\frac{n}{3}$ (b) $\frac{n}{2}$
 (c) $\frac{(n-2)}{2}$ (d) can't be determined

- If in the above question (no. 13) the value of x_i is either 0 or 1 only (i.e., each of $x_1, x_2, x_3, \dots, x_n$ is either zero or 1) then the minimum number of x_i such that $x_i = 0$ is required, when $n = 6, 12, 18, 24, \dots$ etc.
 (a) $\frac{n}{3}$ (b) $\frac{(n+2)}{3}$ (c) $\frac{(n-2)}{2}$ (d) none

- If $m + n = mn - 5$, then the maximum number of ordered pairs of (m, n) for every m, n is a natural number.
 (a) 4 (b) 5
 (c) 6 (d) none of these

- 16** At the eve of marriage anniversary of Tristan and Iseult some special couples were invited. Out of them only 23 couples were there having one child per couple with them. When I observed it, I found that 13 men were without any child and 20 women were also without any child. It is known that there were only couples allowed to attend the ceremony. For your kind information we will not consider the couple (*i.e.*, Tristan and Iseult) for any calculation, since they are hosts. The total number of couples who attended the party (*i.e.*, anniversary).
- (a) 28 (b) 30
(c) 33 (d) can't be determined
- 17** The number of numbers less than or equal to 666 which are the products of exactly 4 distinct positive prime numbers is
- (a) 4 (b) 8
(c) 7 (d) can't be determined
- 18** Tata, Hutch and Idea started of with a game. The rule is that the loser of the game distributes $\frac{1}{3}$ rd of his money amongst the other two players in the ratio of the amount as they are holding with them. After playing three round game Tata, Hutch and Idea endup with Rs. 450, 150 and 300 respectively. The total amount of money what they were initially holding is
- (a) 575 (b) 750
(c) $\frac{5}{4}(6!)$ (d) can't be determined
- 19** Remainder when $4^{4^n} + 3$ is divided by 7 is :
- (a) 0 (b) 1
(c) 5 (d) none of these
- 20** All the soldiers are arranged in the form of an equilateral triangle *i.e.*, one soldier in the front and 2 soldiers in the second row and 3 soldiers in the third row, 4 soldiers in the fourth row and so on. If 669 more soldiers of another company are added in such a way that all the soldiers now are in the form of an square and each of the sides then contains 8 soldiers less than each side of equilateral triangle. Initially, how many soldiers were there?
- (a) 2056 (b) 1540 (c) 1400 (d) 1220
- 21** A natural number when divided by the quotient, which in turn obtained by dividing the leading digit by the unit digit of the same number, we get another number consisting of the same digits, but in reverse order. The original number is
- (a) 98658823 (b) 91703 (c) 87912 (d) 27918
- 22** One day my friend Dorsey told me from LA, that he gets the same salary in each month but when he adds up his six figure salary of two months, three months, four months, five months or six months each time the figures (or digits) of his salary remains the same, but not in a fix order. Later, he told me that when he adds up his salary of 7 months his total salary become $(10^6 - 1)$. What is the salary of Dorsey?
- (a) 123456 (b) 285614
(c) 142857 (d) can't be determined
- 23** A leading chocolate producing company produces 'abc' chocolates per hour (*abc* is a three digit positive number). In how many hours it will produce 'abcabc' chocolates?
- (a) *abc* (b) 101
(c) 1001 (d) can't be determined
- 24** A number D236D0 can be divided by 36 if D is :
- (a) 8 (b) 6
(c) 1 (d) more than one values are possible
- 25** In the Christmas eve of his 7th birthday anniversary, Martin, the eldest son of his parents, went to picnic with all his family members. There his family purchased the tickets for everyone. The entire family consists of 3 children, 1 grand father, 1 grand mother, two fathers and two mothers 1 brother and 1 sister. Also there is one daughter-in-law, one father in law, one mother in law. Minimum how many tickets has his family purchased for the entire family?
- (a) 5 (b) 6 (c) 8 (d) 14
- 26** In the above question (no. 25), if the rate of a ticket for a child (considered to be unmarried and below 18 years) is ₹ 6, while the price of a ticket for an adult (*i.e.*, married and above 18) is ₹ 10. So, what is the minimum price does he have to pay for the tickets?
- (a) ₹ 52 (b) ₹ 58
(c) ₹ 90 (d) data insufficient
- 27** Which one of the following is not the correct relation?
- (a) $(25)^3 + (38)^3 + (87)^3 = (90)^3$
(b) $(3)^3 + (4)^3 + (5)^3 = (6)^3$
(c) $(17)^3 + (29)^3 = (57)^3$ (d) $1^3 + 6^3 + 8^3 = 9^3$
- 28** In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers is :
- (a) at least 30 (b) at most 20
(c) 25 (d) none of (a), (b) and (c)
- 29** At East End Mall, burgers can be bought in quantities of either 6, 9 or 20 only. For example one can purchase 26 ($20 + 6$), 15 ($6 + 9$) and 24 ($9 \times 2 + 6$) etc. but can't purchase 10 burgers *i.e.*, without any lot. What is the largest number of burgers that can not be purchased?
- (a) 83 (b) 37
(c) can't be determined (d) none of (a), (b), (c)
- 30** Let there be a fraction whose denominator is one less than the square of its numerator. If we add 2 to both the numerator and denominator, the fraction will exceed to $\frac{1}{3}$ and if we subtract 3 from numerator and denominator the fraction will lie between 0 and $1/10$. The actual fraction is
- (a) $\frac{3}{8}$ (b) $\frac{7}{48}$
(c) $\frac{4}{17}$ (d) None of these

- 31** When a two digit number is subtracted from another two digit number consisting of the same digits in reverse order, then the resultant value we get is equal to the sum of the digits of that number. The square of sum of digits of that number can be :
 (a) 49 (b) 64 (c) 81 (d) 36
- 32** If $A = [k^3 + (k+2)^3 + (k+4)^3 + (k+6)^3 + (k+8)^3 + \dots + (k+38)^3]$ and
 $B = [k + (k+2) + (k+4) + (k+6) + (k+8) + \dots + (k+38)]$
 Now, if 'A' is divided by 'B', then the remainder will be (for every positive integer k) :
 (a) 0 (b) 1 (c) k (d) none of these
- 33** If $1^3 = 1, 2^3 = 3 + 5, 3^3 = 7 + 9 + 11, 4^3 = 13 + 15 + 17 + 19, 5^3 = 21 + 23 + 25 + \dots + 29$, etc.
 Then the value of $(100)^3$ is equal to :
 (a) $9901 + 9903 + \dots + 10099$
 (b) $9999 + 10001 + 10003 + \dots + 10199$
 (c) $9989 + 9991 + \dots + 10089$
 (d) Any two of (a), (b), (c)
- 34** If $\{x\}$ denotes the fractional part of x , then $\left\{\frac{5^{200}}{8}\right\}$ is :
 (a) $1/8$ (b) $1/4$ (c) $5/8$ (d) 0.625
- 35** Which one of the following is correct?
 (a) $25^{26} > 26^{25}$ (b) $100^{101} < 101^{100}$
 (c) $199^{198} > 198^{199}$ (d) $399^{400} < 400^{399}$
- 36** If x and y are positive prime numbers and if $x^2 - 2y^2 = 1$, then the value of $x + y$ is :
 (a) 7 (b) 24 (c) 13 (d) none of these
- 37** The greatest possible divisor of $3^{2n+3} - 24n - 27$ for every $n \in N$, which necessarily divides is :
 (a) 64 (b) 24 (c) 96 (d) none of these
- 38** If $(\sqrt{3} + 1)^5 = I + F$, where I is an integer and F is a proper fraction, then the value of $F \cdot (I + F)$ is :
 (a) $(\sqrt{3})^5$ (b) 32 (c) $(1.99)^5$ (d) 31.232
- 39** When the product of 'r' consecutive positive integers is divided by 'r!', then the quotient is :
 (a) any natural number (b) a perfect square (c) a proper fraction (d) either (b) or (a)
- 40** Two digits p, q , where $q > p > 1$ such that $p + q < pq$ and $(p + q), (p + q)^n, (p + q)^{n+1}$ have the same unit digits and $(p \cdot q), (p \cdot q)^n, (p \cdot q)^{n+1}$ have the same unit digits. The value of p is :
 (a) 2 (b) 3 (c) 4 (d) 5

Answers

✓ Introductory Exercise 1.1

1 (a)	2 (c)	3 (d)	4 (c)	5 (a)	6 (d)	7 (b)	8 (a)	9 (d)	10 (d)
11 (d)	12 (d)	13 (a)	14 (c)	15 (c)	16 (d)	17 (d)	18 (c)	19 (b)	20 (c)
21 (b)	22 (c)	23 (c)	24 (b)	25 (a)	26 (a)	27 (b)	28 (b)	29 (c)	30 (d)
31 (a)	32 (d)	33 (a)	34 (c)	35 (c)	36 (d)	37 (b)	38 (b)	39 (d)	40 (d)
41 (b)	42 (d)	43 (a)	44 (b)	45 (c)	46 (a)	47 (d)	48 (d)	49 (d)	50 (b)

✓ Introductory Exercise 1.2

1 (c)	2 (d)	3 (b)	4 (d)	5 (d)	6 (d)	7 (b)	8 (b)	9 (c)	10 (c)
11 (c)	12 (d)	13 (d)	14 (d)	15 (a)	16 (a)	17 (c)	18 (a)	19 (a)	20 (b)
21 (a)									

✓ Introductory Exercise 1.3

1 (a)	2 (d)	3 (b)	4 (d)	5 (a)	6 (d)	7 (d)	8 (b)	

✓ Introductory Exercise 1.4

1 (c)	2 (c)	3 (b)	4 (c)	5 (d)	6 (c)	7 (c)	8 (b)	9 (a)	10 (b)
11 (b)	12 (d)	13 (d)							

✓ Introductory Exercise 1.5

1 (c)	2 (c)	3 (c)	4 (a)	5 (d)	6 (d)	7 (a)	8 (c)	9 (a)	10 (a)
11 (c)	12 (d)	13 (a)	14 (c)	15 (c)	16 (b)				

✓ Introductory Exercise 1.6

1 (d)	2 (d)	3 (b)	4 (b)	5 (a)	6 (c)	7 (a)	8 (d)	9 (d)	10 (a)
11 (d)	12 (d)	13 (b)	14 (d)	15 (d)	16 (d)	17 (d)	18 (b)	19 (a)	20 (b)
21 (b)	22 (a)								

✓ Introductory Exercise 1.7

1 (a)	2 (b)	3 (d)	4 (d)	5 (b)	6 (a)	7 (c)	8 (c)	9 (d)	10 (a)
11 (d)	12 (d)	13 (c)	14 (a)	15 (d)	16 (a)	17 (c)			

✓ Introductory Exercise 1.8

1 (d)	2 (a)	3 (a)	4 (d)	5 (a)	6 (b)	7 (a)	8 (d)	9 (b)	10 (c)
11 (c)	12 (a)	13 (a)	14 (a)	15 (b)	16 (a)	17 (b)	18 (c)	19 (c)	20 (c)

✓ Introductory Exercise 1.9

1 (c)	2 (c)	3 (d)	4 (b)	5 (a)	6 (c)	7 (a)	8 (d)	9 (a)	10 (d)
11 (c)	12 (a)	13 (b)	14 (b)	15 (c)	16 (c)	17 (c)	18 (d)	19 (a)	20 (d)
21 (b)	22 (c)								

✓ Introductory Exercise 1.10

1 (a)	2 (b)	3 (d)	4 (d)	5 (a)	6 (a)	7 (c)	8 (c)	9 (a)	10 (a)
11 (d)									

✓ Introductory Exercise 1.11

1 (c)	2 (d)	3 (c)	4 (c)	5 (b)	6 (a)	7 (a)	8 (b)	9 (a)	10 (d)
11 (b)	12 (c)	13 (d)	14 (a)	15 (b)					

✓ Introductory Exercise 1.12

1 (b)	2 (d)	3 (b)	4 (a)	5 (b)	6 (c)	7 (d)	8 (d)	9 (b)

✓ Level 01 Basic Level Exercise

1 (b)	2 (b)	3 (b)	4 (b)	5 (c)	6 (c)	7 (a)	8 (d)	9 (d)	10 (d)
11 (c)	12 (b)	13 (d)	14 (b)	15 (d)	16 (b)	17 (c)	18 (b)	19 (c)	20 (c)
21 (d)	22 (b)	23 (d)	24 (c)	25 (b)	26 (a)	27 (a)	28 (d)	29 (d)	30 (a)
31 (c)	32 (b)	33 (b)	34 (d)	35 (d)	36 (c)	37 (d)	38 (c)	39 (c)	40 (c)
41 (b)	42 (c)	43 (c)	44 (d)	45 (a)	46 (c)	47 (b)	48 (b)	49 (c)	50 (a)
51 (a)	52 (b)	53 (b)	54 (c)	55 (b)	56 (d)	57 (b)	58 (b)	59 (c)	60 (d)
61 (c)	62 (c)	63 (b)	64 (c)	65 (d)	66 (c)	67 (d)	68 (c)	69 (c)	70 (d)
71 (b)	72 (b)	73 (b)	74 (d)	75 (b)	76 (b)	77 (b)	78 (a)	79 (c)	80 (d)
81 (c)	82 (b)	83 (c)	84 (c)	85 (b)	86 (b)	87 (c)	88 (c)	89 (d)	90 (b)
91 (b)	92 (b)	93 (d)	94 (b)	95 (d)	96 (a)	97 (a)	98 (d)	99 (b)	100 (c)
101 (d)	102 (b)	103 (c)	104 (b)	105 (a)	106 (b)	107 (b)	108 (c)	109 (b)	110 (b)
111 (b)	112 (c)	113 (c)	114 (a)	115 (b)	116 (a)	117 (c)	118 (a)	119 (a)	120 (b)
121 (d)	122 (b)	123 (c)	124 (d)	125 (d)	126 (a)	127 (c)	128 (d)	129 (c)	130 (b)
131 (d)	132 (c)	133 (b)	134 (c)	135 (b)	136 (c)	137 (c)	138 (c)	139 (c)	140 (b)
141 (a)	142 (c)	143 (d)	144 (d)	145 (d)	146 (c)	147 (d)	148 (a)	149 (c)	150 (a)
151 (a)	152 (a)	153 (d)	154 (d)	155 (d)	156 (d)	157 (d)	158 (d)	159 (b)	160 (b)

161 (a)	162 (b)	163 (c)	164 (c)	165 (d)	166 (b)	167 (c)	168 (c)	169 (a)	170 (c)
171 (a)	172 (d)	173 (a)	174 (c)	175 (b)	176 (b)	177 (c)	178 (d)	179 (b)	180 (a)
181 (a)	182 (b)	183 (b)	184 (a)	185 (c)	186 (a)	187 (a)	188 (a)	189 (a)	190 (d)
191 (d)	192 (d)	193 (d)	194 (c)	195 (c)	196 (d)	197 (a)	198 (b)	199 (c)	

✓ Level 02 Higher Level Exercise

1 (b)	2 (d)	3 (c)	4 (c)	5 (d)	6 (c)	7 (b)	8 (c)	9 (b)	10 (d)
11 (c)	12 (d)	13 (a)	14 (d)	15 (a)	16 (a)	17 (c)	18 (b)	19 (d)	20 (b)
21 (b)	22 (b)	23 (a)	24 (a)	25 (b)	26 (d)	27 (a)	28 (d)	29 (c)	30 (c)
31 (b)	32 (b)	33 (b)	34 (b)	35 (c)	36 (b)	37 (d)	38 (a)	39 (c)	40 (a)
41 (b)	42 (b)	43 (a)	44 (b)	45 (c)	46 (b)	47 (a)	48 (b)	49 (d)	50 (b)
51 (d)	52 (c)	53 (c)	54 (b)	55 (c)	56 (d)	57 (b)	58 (a)	59 (d)	60 (d)
61 (b)	62 (d)	63 (c)	64 (d)	65 (a)	66 (c)	67 (c)	68 (c)	69 (c)	70 (d)
71 (b)	72 (b)	73 (d)	74 (a)	75 (a)	76 (a)	77 (a)	78 (a)	79 (b)	80 (b)
81 (d)	82 (c)	83 (a)	84 (a)	85 (a)	86 (a)	87 (b)	88 (a)	89 (d)	90 (a)
91 (b)	92 (b)	93 (a)	94 (b)	95 (d)	96 (a)	97 (c)	98 (c)	99 (c)	100 (d)
101 (c)	102 (c)	103 (c)	104 (a)	105 (a)	106 (a)	107 (b)	108 (c)	109 (c)	110 (d)
111 (c)	112 (a)	113 (c)	114 (a)	115 (c)	116 (d)	117 (b)	118 (b)	119 (c)	120 (b)
121 (d)	122 (a)	123 (a)	124 (b)	125 (b)	126 (c)	127 (a)	128 (a)	129 (c)	130 (d)
131 (d)	132 (c)	133 (b)	134 (b)	135 (b)	136 (d)	137 (b)	138 (d)	139 (d)	140 (c)
141 (d)	142 (d)	143 (c)	144 (b)	145 (c)	146 (b)	147 (c)	148 (d)	149 (c)	150 (c)
151 (c)	152 (b)	153 (d)	154 (c)	155 (a)	156 (d)	157 (a)	158 (c)	159 (c)	160 (c)
161 (d)	162 (d)	163 (c)	164 (b)	165 (a)	166 (a)	167 (d)	168 (a)	169 (d)	

✓ Level 03 Final Round

1 (a)	2 (b)	3 (b)	4 (a)	5 (c)	6 (a)	7 (c)	8 (a)	9 (b)	10 (c)
11 (d)	12 (d)	13 (d)	14 (b)	15 (d)	16 (b)	17 (b)	18 (a)	19 (b)	20 (c)
21 (b)	22 (c)	23 (b)	24 (d)	25 (c)	26 (d)	27 (b)	28 (b)	29 (b)	30 (a)
31 (d)	32 (b)	33 (c)	34 (a)	35 (a)	36 (d)	37 (a)	38 (b)	39 (c)	40 (b)
41 (b)	42 (a)	43 (d)	44 (d)	45 (a)	46 (b)	47 (d)	48 (d)	49 (d)	50 (d)
51 (d)	52 (a)	53 (c)	54 (d)	55 (d)	56 (a)	57 (d)	58 (a)	59 (d)	60 (b)
61 (a)	62 (a)	63 (b)	64 (c)	65 (d)	66 (c)	67 (a)	68 (b)	69 (a)	70 (b)
71 (c)	72 (c)	73 (b)	74 (c)	75 (d)	76 (c)	77 (c)	78 (d)	79 (a)	80 (d)
81 (c)	82 (d)	83 (a)	84 (b)	85 (c)	86 (b)	87 (c)	88 (c)	89 (b)	90 (d)
91 (b)	92 (b)	93 (a)	94 (c)	95 (c)	96 (b)	97 (a)	98 (a)	99 (b)	100 (a)
101 (c)	102 (c)	103 (d)	104 (a)	105 (b)	106 (c)	107 (d)	108 (a)	109 (c)	110 (c)
111 (c)	112 (d)	113 (c)	114 (a)	115 (a)	116 (a)	117 (d)	118 (c)	119 (d)	120 (c)
121 (a)	122 (d)	123 (d)	124 (d)	125 (b)					

✓ Test for Your Learning 1

1 (a)	2 (d)	3 (c)	4 (a)	5 (c)	6 (a)	7 (d)	8 (c)	9 (b)	10 (c)
11 (a)	12 (a)	13 (b)	14 (c)	15 (c)	16 (a)	17 (d)	18 (d)	19 (b)	20 (a)
21 (a)	22 (d)	23 (b)	24 (c)	25 (c)	26 (a)	27 (b)	28 (b)	29 (d)	30 (c)
31 (c)	32 (c)	33 (d)	34 (b)	35 (a)	36 (a)	37 (b)	38 (c)	39 (d)	40 (d)

✓ Test for Your Learning 2

1 (a)	2 (c)	3 (b)	4 (c)	5 (d)	6 (d)	7 (c)	8 (c)	9 (d)	10 (a)
11 (d)	12 (b)	13 (c)	14 (a)	15 (a)	16 (a)	17 (c)	18 (c)	19 (a)	20 (b)
21 (c)	22 (c)	23 (c)	24 (a)	25 (b)	26 (a)	27 (c)	28 (c)	29 (d)	30 (d)
31 (c)	32 (a)	33 (a)	34 (a)	35 (a)	36 (d)	37 (d)	38 (b)	39 (a)	40 (a)

Hints & Solutions

Introductory Exercise 1.1

- 1 $1 + 2 + 3 + 7 + 5 = 18$, hence divisible by 3 and 9 both.

Again $(5 + 3 + 1) - (7 + 2) = 0$

Hence, divisible by 11 also. Thus (a).

- 2 The remainder will be the required answer.

Since $6 + 7 + 5 = 18$

$\therefore 6705$ will be divisible by 5 and 3 will be remainder.

Hence, (c).

- 3 Go through options

$\therefore 8 + 0 + 3 + 6 + 4 + 2 = 23$

So, the required number $= 27 - 23 = 4$

☞ **Alternatively** Divide 803642 and find the remainder.

Now the required number $= (9 - \text{Remainder})$

- 4 $171282 = 547 \times \text{divisor} + 71$

$\Rightarrow 171211 = 547 \times \text{divisor}$

$\Rightarrow \text{divisor} = 313$

Hence, (c) is the correct answer.

- 5 If the dividend is x , divisor be y and quotient be q , then

$$x = yq + r$$

Now, since $y = 8q$

and $y = 4r = 4 \times 12 = 48$ (r is remainder)

$\Rightarrow q = 6$

$\therefore x = 48 \times 6 + 12$

$$x = 300$$

- 6 Since we know that $111 = 3 \times 37$, so the whole number is divisible by 3, 37 and 111. Now since the digit 1 is being repeated even number of times so it is divisible by 11. Again since the given number is such that the digits are being repeated and can be broken into triplets, so it is also divisible by 1001.

- 7 Since $16 = 2^4$ so the last four digits, thus $x = 4$ or go back to the Basics.

- 8 $2 + 4 + 3 + x + 5 + 1 = 18$ (the only possible sum)

$\Rightarrow x = 3$

Hence, (a) is the correct choice.

- 9 Consider choice (d). $145 + 854 = 999$. It means 145854 is divisible by 999.

Hence choice (d) is the correct one.

- 10 99999999 is evidently divisible by 9. 99999999 can be grouped as $(99 + 99 + 99 + 99)$. Since $99 + 99 + 99 + 99 = 396$ is divisible by 99, so 99999999 is also divisible by 99.

99999999 can be grouped as $(9999 + 9999)$. Since $9999 + 9999 = 19998$ is divisible by 9999, so 99999999 is also divisible by 9999.

Hence, choice (d) is the correct one.

- 11 If $24AB4$ is divisible by 99 then, it must satisfy the following condition.

$$2 + 4A + B4 = 99m, \text{ for every } m = 1, 2, 3, \dots$$

For $m = 1$, $99m = 99$. Therefore

$$\begin{array}{r} 2 \\ 4A \\ + B4 \\ \hline 99 \end{array}$$

$$\Rightarrow 2 + A + 4 = 9 \text{ and } 4 + B = 9$$

$$\Rightarrow A = 3 \text{ and } B = 5$$

For $m = 2, 3, 4, \dots$ B would not be a single digit number, so it's not possible to consider $m = 2, 3, 4, \dots$

Thus, it can be concluded that $A = 3$ and $B = 5$ is the only possibility.

Hence, choice (d) is the answer.

- 12 As per the given choices first of all we check it by 99 since in the question largest possible number of two digits is being asked.

$$\text{Now, since } (2 + 7 + 7 + 2) - (8 + 9 + 1) = 18 - 18 = 0$$

Hence it is divisible by 11

$$\text{again } 2 + 1 + 7 + 9 + 7 + 8 + 2 = 36$$

Hence it is also divisible by 9.

Thus we can say that the given number 2179782 is divisible by 99 ($= 11 \times 9$)

So, the option (d) is correct answer.

- 13 A number which is multiple of 125 must be divisible by 125. So the number formed by last three digits must be divisible by 125 hence it can be 000, 125, 250, 375, 500, 675, 750, 875.

So the number which is divisible by 125 must be 9999875 since we have to subtract minimum possible number. Thus we see that we have to subtract 124.

$$[\because 9999999 - 9999875 = 124]$$

☞ **Alternatively** Divide 9999999 by 125 and then the obtained remainder is the required result.

- 14 As per the divisibility rule of 11, the difference of sum of the digits at odd places and sum of the digits at even places be either 0, 11, 22, 33, ... etc. So n must be an even number. Correct choice is (c).

☞ Alternatively Go through the options. Now when you write prime of any odd value of n you will find odd prime or odd number of nines then you will also observe that these numbers are not divisible by 11.

Further using option (c) you will find that it is always correct for any even number. Thus option (c) is correct.

- 15** If we look at the options we see that option (a) is not correct since in the first instance it is not divisible by 4. Again if we look at option (b) we see that it is not divisible by 3. Similarly option (d) is wrong because it is not divisible by 11 and at last when we check the option (c) then we find it is correct. Remember a number is divisible by 132 if the number is divisible by its factors as 3, 4, and 11 simultaneously as $132 = 3 \times 4 \times 11$. Thus option (c) is correct.

- 16** Since $80 = 8 \times 10$ or $80 = 16 \times 5$

Thus y (i.e., unit digit) must be zero.

$\therefore 653xy = 653x0$, where $653x0$ must be divisible by 16 or $653x$ is divisible by 8.

Thus the last 3-digit number $53x$ will be divisible by 8.

Hence, at $x = 6$, we get the required result.

$\therefore x + y = 6 + 0 = 6$

- 17** The best way to use the options

☞ Alternatively $(4 + 6 + 3) - (2 + 5 + k) = 0 / 11 / 22 \dots$

$$(13) - (7 + k) = 0 [\because 11/22 \dots \text{are not possible}]$$

$$\Rightarrow k = 6$$

Hence option (d) is correct.

- 18** The quickest method is to use appropriate option. As the divisor is 72 so the given number must be divisible 9 and 8 both. Therefore, it is clear that option (b) and (d) are useless, since unit digit can't be odd in order to be divisible by 8. Now option (a) and (c) can be tried out. Thus we can get the value of $k = 6$. Hence option (c) is correct.

- 19** The first number which is divisible by 7 is 7 and the last number is 994. It can be easily calculated just by dividing 1000 by 7 and then subtract the remainder from 1000. So total numbers between 1 and 1000 which are divisible by 7 are 142. The sequence is as follows 7, 14, 21, 28, 35, ... 987, 994.

- 20** The sequence is as follows 55, 60, 65, 70, ..., 550, 555. So the number of number divisible by 5 is

$$\left(\frac{555 - 55}{5} \right) + 1 = 101$$

NOTE we have added 1 from outside because here both the extremes (i.e., 55 and 555) are included.

SHORTCUT If D be the divisor of numbers lying between N_1 and N_2 , $N_2 > N_1$ then the number of numbers which are divisible by D (or multiple of D) is

(a) When no one extreme is divisible by such a divisor D :

$$\left(\frac{N_2 - N_1}{D} \right) - 1$$

(b) When any one of the extreme is divisible by D :

$$\left(\frac{N_2 - N_1}{D} \right)$$

(c) When both the extremes are divisible by D is :

$$\left(\frac{N_2 - N_1}{D} \right) + 1$$

- 21** $(200 - 100) + 1 = 101$

☞ Alternatively Since you know that you are counting the numbers from 100, so only 99 numbers (1, 2, 3, ..., 99) are not included

i.e., ~~1, 2, ... 99~~, 100, 101, 102, 103, ..., 200.

Hence total numbers = $200 - 99 = 101$.

- 22** The first number which is divisible by 3 is 300 and the last number which is divisible by 3 is 498.

$$\text{So, the total numbers} = \left(\frac{498 - 300}{3} \right) + 1 = 66 + 1 = 67$$

Hence (c) is the correct option.

☞ Alternatively When we form the sequence of numbers which are divisible by 3, the number 300 is lying at the 100th position and 498 at the 166th position (i.e., 3, 6, 9, 12, ... 297, 300, 303, 306, ..., 498) so there are total 166 numbers upto 498 but 99 numbers are not included in this sequence. Thus the required numbers are $166 - 99 = 67$ numbers, which are divisible by 3.

- 23** In the given range, the least number which is divisible by both 5 and 7. i.e., 35 is 210 and the highest number is 770. So the total number of numbers between 200 and 800 which are divisible by both 5 and 7 is

$$\left(\frac{770 - 210}{35} \right) + 1 = 17$$

Hence option (c) is correct.

- 24** Total numbers in the set are

$$(800 - 200) + 1 = 601$$

Total number of numbers which are divisible by 5

$$= \frac{(800 - 200)}{5} + 1 = 121$$

Total number of numbers which are divisible by 7

$$= \frac{(798 - 203)}{7} + 1 = 86$$

Total number of numbers which are divisible by both 5 and 7

$$= \frac{770 - 210}{35} + 1 = 17$$

So the total number of numbers which are either divisible by 5 or 7 or both = $(121 + 86) - 17 = 190$

Hence option (b) is correct.

25 Total numbers in the set $= (800 - 200) + 1 = 601$

Number of numbers which are divisible by 5

$$= \frac{(800 - 200)}{5} + 1 = 121$$

Number of numbers which are divisible by 7

$$= \frac{(798 - 203)}{7} + 1 = 86$$

Number of numbers which are divisible by both 5 & 7

$$= \left(\frac{770 - 210}{35} \right) + 1 = 17$$

$\therefore$ Number of numbers which are either divisible by 5 or 7 or both $= (121 + 86) - 17 = 190$

Thus the number of numbers in the given set which are neither divisible by 5 nor by 7 $= 601 - 190 = 411$.

Hence (a) is correct option.

26 To find the answer of this problem first of all we need to calculate total number of numbers in the given set which are divisible by any of 2, 3 or 5 then these numbers will be subtracted from the total number of the original set of numbers. Thus we get the total number of numbers which are neither divisible by 2, 3 or 5.

Total numbers in the set $= (3113 - 1331) + 1$

$$= 1782 + 1 = 1783$$

Total number of numbers in the set which are divisible by 2

$$= \left(\frac{3112 - 1332}{2} \right) + 1 = 891$$

Total numbers divisible by 3 $= \left(\frac{3111 - 1332}{3} \right) + 1 = 594$

Total numbers divisible by 5 $= \left(\frac{3110 - 1335}{5} \right) + 1 = 356$

Total numbers which are divisible by 2 and 3 both

$$= \left(\frac{3108 - 1332}{6} \right) + 1 = 297$$

Total numbers which are divisible by 3 and 5 both

$$= \left(\frac{3105 - 1335}{15} \right) + 1 = 119$$

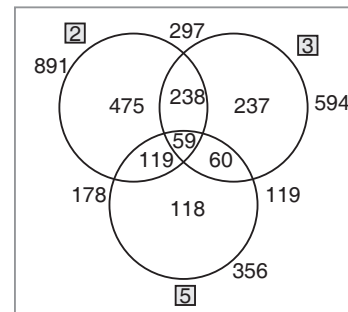
Total numbers which are divisible by 2 and 5 both

$$= \left(\frac{3110 - 1340}{10} \right) + 1 = 178$$

Total numbers which are divisible by 2, 3 and 5

$$= \left(\frac{3090 - 1350}{30} \right) + 1 = 59$$

So, the total number of numbers in the given set which are neither divisible by any of the 2, 3 or 5 $= 477$.



$$\therefore n(A \cup B \cup C) = [n(A) + n(B) + n(C)] - [n(A \cap B) + n(B \cap C) + n(A \cap C)] + n(A \cap B \cap C)$$

So the divisible numbers

$$= (891 + 594 + 356) - (297 + 119 + 178) + 59 = 1306$$

$\therefore$ Number of numbers which are not divisible by 2, 3 or 5.

$$= 1783 - 1306 = 477$$

Hence (a) is the correct option.

Alternatively Number of numbers in the given set which are only divisible by 2 $= 891$ the number of numbers which are only divisible by 3, but not by 2

$$= \frac{(3111 - 1335)}{6} + 1 = 297$$

The number of numbers which are only divisible by 5 but not by 2 or 3

$$= \left\{ \frac{(3105 - 1335)}{10} + 1 \right\} - \left\{ \frac{(3105 - 1335)}{30} + 1 \right\} = 178 - 60 = 118$$

So the total number of numbers which are divisible by any of the 2, 3 or 5

$$= 891 + 297 + 118 = 1306$$

Thus the total number of numbers which are neither divisible by 2, 3

$$\text{or } 5 = 1783 - 1306 = 477.$$

27 Just divide 434079 by 137 and then the obtained remainder is your correct answer which is 63, so (b) is the right choice.

28 To make 434079 divisible by 137 we have to add just the difference of remainder and the divisor. So the choice (b) is correct and the answer is 74.

29 Obviously (c) is the right choice because there can be maximum two numbers closest to any particular number. So out of 180 and 198 the number 198 is closer than the number 180 and 195 is not divisible by 18 also 108 is the farthest from 193.

- 30** Since when we multiply the least three digit numbers ($100 \times 100 = 10000$) we get a 5 digit number and when we multiply the greatest possible 3 digit numbers ($999 \times 999 = 998001$) we get a 6 digit number. So anyhow the product of a 3 digit number with another 3 digit number can never be a number of less than 5 digit or greater than 6 digit number. So the option (a) and (c) are ruled out because these are 4 digit and 7 digit numbers respectively.

Further whenever we multiply any two numbers whose unit digits are 7 and 5 individually then the unit digit of the resultant number is always 5 ($\because 5 \times 1 = 5, 5 \times 3 = 15, 5 \times 5 = 25, 5 \times 7 = 35, 5 \times 9 = 45$ and $5 \times 2 = 10, 5 \times 4 = 20, 5 \times 6 = 30, 5 \times 8 = 40$, and $5 \times 10 = 50$)

Thus the option (b) is also wrong since the required unit digit can never be 6 (i.e., an even number).

Hence the appropriate option is (d).

$$\begin{array}{r} 984 \\ + 4p3 \\ \hline 13q7 \end{array}$$

It is clear from the above expression that there is no any carry over obtained in the addition of unit digits. Further it is very simple to see that there is no carry over value taken while adding hundredths digit of both the numbers. So we get $8 + p = q$

Now if $13q7$ is divisible by 11 then

$$(7 + 3) - (1 + q) = 11k \quad \text{where, } k = 0, 1, 2, \dots$$

$$10 - (1 + q) = 0$$

$$\Rightarrow q = 9$$

here $k = 0$ is the only possible value because if

$k = 1$ then q must be either -2 or 20 .

but neither -2 nor 20 is possible because q is a single positive digit i.e., $q \in 1, 2, 3, 4, 5, 6, 7, 8, 9, 0$.

$$\begin{array}{l} \text{Thus the value of } p + q = 1 + 9 = 10 \\ \left[\begin{array}{l} \because p = q - 8 \\ \therefore p = 1 \end{array} \right] \end{array}$$

Hence, (a) is the correct option.

- 32** Since Dividend = Divisor $\times$ Quotient + Remainder
 $\therefore$ Dividend = $9235 \times 888 + 222$

Thus the number = 8200902

Hence (d) is the correct choice.

- 33** Option (a) is correct. There are only two relevant options (a) and (c). Option (b) is redundant since 999 is not divisible by 33. Now 990 and 1023 both are close to 1000 and both are divisible by 33 but it is obvious that 990 is much closer to 1000 than 1023.

- 34** Let this number be N then

$$N = 32 \times Q_1 + 29 \quad \dots(i)$$

$$\text{Again } N = 8 \times Q_2 + R \quad \dots(ii)$$

From Eq. (i) and (ii)

$$32Q_1 + 29 = 8Q_2 + R$$

(where R is the remainder)

$$8Q_2 - 32Q_1 = 29 - R$$

$$8(Q_2 - 4Q_1) = 29 - R$$

$$\text{or } (Q_2 - 4Q_1) = \frac{29 - R}{8}$$

Since Q_1, Q_2, R are integers also $Q_2 - 4Q_1$ is an integer. Therefore $29 - R$ must be divisible by 8.

So the probable values of R are 29, 21, 13 and 5.

But since the remainder can never be greater than the divisor so the only possible value of R is 5. Hence (c) is the correct option.

$$\textcircled{D} \text{ Alternatively } N = 32Q + 29 = 8 \times 4Q + 8 \times 3 + 5$$

(where Q is Quotient)

$$\Rightarrow N = 8(4Q + 3) + 5$$

$$\Rightarrow N = 8P + 5$$

(where p is any positive integer)

Thus the remainder is 5.

Shortcut In this kind of questions where the same number is divided by another divisor (proper factor of the original divisor), we just need to divide the original remainder by the new divisor to get the value of new remainder e.g. when we divide 29 by the new divisor '8' then we get new remainder 5.

- 35** Let the number be N and the quotient be Q then
 $N = 5Q + 4$

$$\text{Again } 2N = 2(5Q + 4) = 10Q + 8$$

$$\text{or } 2N = 5 \times 2Q + 5 + 3$$

$$\text{or } 2N = 5(2Q + 1) + 3$$

Hence, the remainder will be 3.

Alternatively Since the number becomes double then the remainder will also be double but when the remainder will become twice it will exceed the divisor which is not possible. So to get the correct remainder we further divide the twice of the remainder by the divisor and in this way we obtain the required remainder which is admissible. For example in the given problem the divisor is 5 and the original remainder is 4. So this remainder will become twice i.e., 8 (since the number i.e., dividend becomes twice) but since $8 > 5$ which is not possible so we divide 8 by 5 then we get the correct remainder 3.

- 36** In the first case $N = DQ + 14$

$$\text{In the second case } 3N = 3(DQ + 14) = 3DQ + 42$$

$$= 3DQ + 34 + 8$$

$$3N = (3DQ + 34) + 8$$

It shows that 34 must be divisible by 34 or its factor i.e., 1, 2, 17, 34. But 1 and 2 can't be values of D . Since the divisor D must be greater than the remainder (viz. 8, 14) Hence we are now left with two possible values of D viz. $D = 17$ and $D = 34$

Now, if remainder $D = 17$, then

$$N = 17Q + 14$$

$$4N = 4(17Q + 14) = 4 \times 17Q + 56$$

$$\Rightarrow 4N = 17 \times 4Q + 17 \times 3 + 5$$

$$\Rightarrow 4N = 17(4Q + 3) + 5$$

Thus the remainder will be 5.

But if we consider $D = 34$, then

$$N = 34Q + 14$$

$$\Rightarrow 4N = 4(34Q + 14)$$

$$\Rightarrow 4N = 34 \times 4Q + 56$$

$$\Rightarrow 4N = 34 \times 4Q + 34 + 22$$

$$\Rightarrow 4N = 34(4Q + 1) + 22$$

Thus the remainder will be 22.

Hence we can't say exactly whether the remainder will be 5 or 22. So (d) is the correct option.

37 Let the number be N then

$$N = 34Q + R, \quad \dots(i)$$

where Q is any quotient

Again $N = 5D$ and D is also a quotient

$$\text{but } D = R + 8$$

$$\text{so } N = 5(R + 8) \quad \dots(ii)$$

$$\therefore 5(R + 8) = 34Q + R$$

$$5R + 40 = 34Q + R$$

$$\Rightarrow 34Q - 40 = 4R$$

$$\Rightarrow 17Q - 2R = 20$$

So the minimum possible value of $Q = 2$ and the corresponding value of $R = 7$

$$\text{So } N = 34 \times 2 + 7$$

$$N = 75$$

Hence (b) is correct choice.

38 Let N be the number and d be the divisor then $N = dQ + 15$

$$\text{Again, } 10N = 10(dQ + 15)$$

$$\Rightarrow 10N = 10dQ + 150$$

$$\Rightarrow 10N = d \cdot 10Q + 144 + 6$$

$$\Rightarrow 10N = (d \times 10Q + 144) + 6$$

$$\Rightarrow 10N = d \left(10Q + \frac{144}{d} \right) + 6$$

$\therefore 144$ must be divisible by d . Thus the probable values of d are 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144 (as these numbers are the factors of 144).

But the value of divisor d must be greater than the remainder (here 6 and 15) so 1, 2, 3, 4, 6, 8, 9, 12 can not be the values of d . Hence only 7 values (greater than 15) are possible.

Thus the option (b) is correct.

39 $C = N_1Q_1 + 13$ and $C = N_2Q_2 + 1$

where Q_1 and Q_2 are the quotients and hence Q_1 and Q_2 must be integers

$$\text{So, } N_1Q_1 + 13 = N_2Q_2 + 1$$

$$N_2Q_2 - N_1Q_1 = 12$$

$$\Rightarrow \frac{4}{5}N_1Q_2 - N_1Q_1 = 12$$

$$N_1(4Q_2 - 5Q_1) = 60$$

where N_1 and $(4Q_2 - 5Q_1)$ both will be integers so that N_1 can be one of the values of factors of 60 i.e., 1, 2, 3, 4, 15, 20, 30 and 60. But N_1 can not be less than the remainder 13. So the possible values are 15, 20, 30 and 60. Thus we see that the corresponding values of N_2 are 12, 16, 24 and 48.

So there are more than one possible values of N_1 and N_2 . Thus there is no unique value of $N_1 + N_2$. Hence (d) is correct option.

40 Since we know that the values of N_1 and N_2 are not unique so the value of C is also not unique hence it is indeterminable.

$$\text{41 Number of parts} = \frac{\text{Total length}}{\text{length of each piece}} = \frac{1950}{65} = 30$$

($\because 19.5 \text{ m} = 1950 \text{ cm}$)

Thus option (b) is correct.

42 Since the 7, 11 and 13 all are the factors of such a number so (d) is the correct answer.

43 (a) is the correct option. Since 5321 is the greatest and 1235 is the least number then $5321 - 1235 = 4086$.

44 The possible values of k are 5 and 9. So the sum of $5 + 9 = 14$.

45 When we multiply 59829 with 13 we get the required number 777777. So (c) is the correct option.

46 First of all you must know that the given 90 digit number is divisible by 11. Then you must know that the new number will be divisible by 11 only when the three-digit number is also divisible by 11. If the three digit number is not divisible by 11, the new number cannot be divisible by 11. So, essentially, you have to find out that how many three digit numbers are not divisible by 11.

Total three digit numbers = 900

Total three digit number divisible by

$$11 = \left[\left(\frac{990 - 110}{11} \right) + 1 \right] = 81$$

Therefore the number of three digit numbers that cannot be divided by 11 = $900 - 81 = 819$

Hence choice (a) is the correct one.

47 $1125 = 3^2 \times 5^3$

A number which is divisible by 5^3 must have at least three 0s as the right most digits. So it has to contain three 0s. A number which is divisible by 3^2 must have the sum of all its digits divisible by 9. So it has to contain nine 1s. Thus the required number must have three 0s and nine 1s. Hence choice (d) is the correct one.

- 48 Just go through the given choices and test for the divisibility.

Hence choice (d) is the correct one.

☞ Alternatively Let us assume O stands for Odd and E stands for Even.

O	E	O	E	O	E	O	E	O	E

At first we have oeo5eo0. Now, we know that the first four digits have to be divisible by 4

So to do that with "oe" it has to be 12, 16, 32 or 36. That is, the fourth digit has to be 2 or 6. The same logic shows that the 8th digit has to be 2 or 6. So now we have two possibilities, oeo25eo6o0 or oeo65eo2o0.

In order to be divisible by 3, 6, 9, the sum of the first 3, first 6 and first 9 digits have to be a multiple of 3. It means each group of 3 digits is a multiple of 3. That means the middle 3 digits are either 258 or 654, and so we have either o4o258o6o0 or o8o654o2o0.

Now you have to fill in the remaining odd digits so that the number formed by initial three digits is divisible by 3. In this way, you can proceed and find the right number.

- 49 Any number divisible by 99 must be divisible by 9 and 11 both. A number will be divisible by 9 when the sum of all the digits of that number will be divisible by 9.

As, $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$, it implies that the number formed by using the digits 1, 2, 3, ..., 9 only once will be divisible by 9. Likewise, a number will be divisible by 11 when the difference of sum of the alternate digits of that number is multiple of 11 (i.e., 0, 11, 22, 33, etc.) Now, for a moment, let us denote 1, 2, 3, ..., 8, 9 by $a, b, c, \dots, h, i$. But it does not necessarily imply that $a = 1, b = 2, c = 3$ etc. It may be $a = 7, b = 3, c = 1$... etc.

Now the number $abcdefghi$ will be divisible by 11 when

$$(a + c + e + g + i) - (b + d + f + h) = 11m$$

where $m = 0, 1, 2, 3 \dots$

Further, if we denote $(a + c + e + g + i)$ by p and $(b + d + f + h)$ by q , then

$$p + q = 45 \quad \dots(i)$$

$$\text{and} \quad p - q = 1 \quad \dots(ii)$$

Now if $m = 0$, then the above two equations cannot be solved as p and q are integers. For $m = 1$, $p = 28$ and $q = 17$. Then we will have the following combinations of the digits of p and q .

p	q	Desired Number
$a + c + e + g + i$	$b + d + f + h$	$abcdefghi$
$3 + 4 + 6 + 7 + 8$	$1 + 2 + 5 + 9$	314265798
$3 + 4 + 5 + 7 + 9$	$1 + 2 + 6 + 8$	314256789
$2 + 5 + 6 + 7 + 8$	$1 + 3 + 4 + 9$	215364798
$2 + 4 + 6 + 7 + 9$	$1 + 3 + 5 + 8$	214365789
$2 + 4 + 5 + 8 + 9$	$1 + 3 + 6 + 7$	214356879
$2 + 3 + 6 + 8 + 9$	$1 + 4 + 5 + 7$	213465879
$1 + 5 + 6 + 7 + 9$	$2 + 3 + 4 + 8$	125364789
$1 + 4 + 6 + 8 + 9$	$2 + 3 + 5 + 7$	124365879
$1 + 3 + 7 + 8 + 9$	$2 + 4 + 5 + 6$	123475869

Out of the various nine-digit numbers the last number (123475869) is the smallest valid number divisible by 99. Remember that the top 6 numbers (or their combinations) are not required to consider (or write down) at all as they start with 2 or 3. So these numbers are obviously larger than those numbers that start with the digit 1. Thus, in order to save time just consider the last 3 combinations of p and q .

☞ NOTE For $m = 2, 4, 6, 8, \dots$ p and q cannot be determined from the given Eqs. (i) and (ii). And for $m = 3, 5, 7, 9, \dots$ there cannot be 5 digits for p and 4 digits for q in order to satisfy the respective values of p and q . For example, when $m = 3$, then $p = 39, q = 6$. But, we need any 5 distinct digits so that $p = a + c + e + g + i = 39$ and any 4 distinct digits so that $q = b + d + f + h = 6$, which is impossible as minimum value of sum of any four non-zero digits is $10 = 1 + 2 + 3 + 4$, implying that $q = (b + d + f + h) \neq 6$.

☞ Alternatively Any number divisible by 99 must be divisible by 9 and 11 both. Since sum of all the 9 digits is 45, which is divisible by 9. Therefore the required nine-digit number will also be divisible by 9. Thus it is implicit that the divisibility by 9 does not get affected with the position of digits in the given number.

On the other hand divisibility by 11 takes into consideration the positions of digits that is if the sum of odd positioned digits minus the sum of the even positioned digits is divisible by 11, then the number is divisible by 11. Now let us consider the smallest possible nine digit number containing all the 9 digits 1, 2, ..., 9 exactly once. That is 123456789.

Now, sum of odd positioned digits – sum of even positioned digits.

$$= (1 + 3 + 5 + 7 + 9) - (2 + 4 + 6 + 8) \\ = 25 - 20 = 5$$

But this difference should be either 11 or 22 or 33, ... Let us first try to create this difference equal to 11. To get this difference equal to 11, we will have to add 6 to our current difference 5.

This can be done through swapping the digits of 123456789 in such a way that sum of the odd positioned digits is increased by 3 and sum of the even positioned digits is decreased by 3. Therefore either we swap 1 with 4 or 3 with 6 or 5 with 8 to achieve the aforesaid desired results.

	Odd positioned digits	Even positioned digits	Odd positioned digits in increasing order	Even positioned digits in increasing order	Smallest possible number
Original Combination	1, 3, 5, 7, 9	2, 4, 6, 8	1, 3, 5, 7, 9	2, 4, 6, 8	123456789
Swap 1 $\leftrightarrow$ 4	4, 3, 5, 7, 9	2, 1, 6, 8	3, 4, 5, 7, 9	1, 2, 6, 8	314256789
Swap 3 $\leftrightarrow$ 6	1, 6, 5, 7, 9	2, 4, 3, 8	1, 5, 6, 7, 9	2, 3, 4, 8	125364789
Swap 5 $\leftrightarrow$ 8	1, 3, 8, 7, 9	2, 4, 6, 5	1, 3, 7, 8, 9	2, 4, 5, 6	123475869

Out of these 3 new numbers, which are divisible by 99, the last one that is 123475869 is the smallest required number.

NOTE The above solution is obtained when the difference between the sum of odd position digits and the sum of the even positioned digits is 11. This difference could have been 22, 33 or 44 etc. but unfortunately that is not possible as long as sum of all the digits is 45. A better explanation for this argument is already discussed in the previous solution of the same problem by assuming p and q .

Alternatively The smallest possible nine-digit number is 123456789. This number will be divisible only when it satisfies the following condition.

$$1 + 23 + 45 + 67 + 89 = 99m \text{ for every } m = 1, 2, 3$$

$$\begin{array}{r} 1 \\ 23 \\ 45 \\ 67 \\ +89 \\ \hline 225 \end{array}$$

For $m = 1$, $99m = 99$, which is very far away from 225. Now consider $m = 2$, $99m = 198$, which is pretty close to 225. Now as in 198 the unit digit is 8, but in 225 the unit digit is 5, so we have to increase unit digit by 3. This can be done by swapping either 1 by 4 or 3 by 6 or 5 by 8.

Remember that since the higher digits have low place value, so swapping higher digits, i.e. 5 with 8, would be the best option as it will increase the original number by least margin.

$$\begin{array}{r} 1 \\ 23 \\ 48 \\ 67 \\ +59 \\ \hline 198 \end{array}$$

Further we need to understand that we have to write the digits in the increasing order in unit's place and ten's place individually.

$$\begin{array}{r} 1 \\ 23 \\ 47 \\ 58 \\ +69 \\ \hline 198 \end{array}$$

Therefore the required number is 123475869

50 Please consider the following table to understand how the student marked the answer at www.lamamia.in

Q. No.	Preliminary Answer	First Change	Second Change	Third Change	Final Answers
1	A				A
2	A		C		C
3	A	B			B
4	A		C		C
5	A				A
6	A	B	C		C
7	A				A
8	A		C		C
9	A	B		D	D
10	A		C		C
11	A				A
12	A	B	C		C
13	A				A
14	A		C		C
15	A	B			B
16	A		C		C
17	A				A
18	A	B	C	D	D

Therefore, $A = 6, B = 2, C = 8, D = 2$
Hence, choice (b) is the correct answer.

Choices	For every Qn. No. divisible by X there will be	A	B	C	D
Preliminary Answers	For every Qn. No. divisible by 1 there will be 18As	18	0	0	0
After first change	For every Qn. No. divisible by 3 there will be 6Bs	12	6	0	0
After second change	For every Qn. No. divisible by 2 there will be 9Cs	6	3	9	0
After third change	For every Qn. No. divisible by 9 there will be 2Ds	6	2	8	2

Introductory Exercise 1.2

- 1 Option (a) and (b) are ruled out due to unit digit property. Now to check whether (c) is correct or not. Now it would be better to square 2506 instead of finding the square root of 6280036.

Let us find the square of 2506.

Step 1. $(2505)^2 + (2505 + 2506)$

Step 2. $(2500)^2 + 5(2500 + 2505) + 5011$

Step 3. $6250000 + 25025 + 5011$

Step 4. $6275025 + 5011 = 6280036$

Hence option (c) is correct.

- 2 From your basic knowledge you know that $(30)^2 = 900$ again $(40)^2 = 1600$. This shows that the square root lies between 30 and 40.

Further you know that $(35)^2 = 1225$. From this it is clear that only option (d) is correct.

- 3 Since you know that $(80)^2 = 6400$ and $(90)^2 = 8100$. It shows that the square root of 7744 must lie between 80 and 90. So now there is only one option to check i.e., 88. So we try to square 88 instead of finding the square root of 7744.

$$\text{So } (88)^2 = (11 \times 8)^2 = 11^2 \times 8^2 = 121 \times 64 = 7744$$

Hence option (b) is correct.

- 4 First of all option (a) is ruled out due to unit digit property. Again if we do some rough calculation we see that $(300)^2 = 90000$ then the value of $(323)^2$ will be very large even more than 90000 so this option is also ruled out. Now, if we find the square of 225 we will get $(225)^2 = 50625$ and the value of $(230)^2 = 52900$ since the value of 227 lies between 225 and 230, but 56169 does not lie between these values so this value is also not correct. Hence option (d) is correct.

Alternatively Initially, there will be 18 sums marked A. Since every third answer is changed to B. That is every question number which is divisible by 3 will be marked B. It implies that $18/3 = 6$ sums will be marked B and so only 12 sums will be marked A. Again, since every second answer is changed to C. That is every question number which is divisible by 2 will be marked C. It implies that $18/2 = 9$ sums will be marked C and only 6 sums will be marked A and only 3 sums will be marked B. Finally, since every ninth answer is changed to D. That is every question number which is divisible by 9 will be marked D. It implies that $18/9 = 2$ sums will be marked D and so only 6 sums will be marked A, only 2 sums will be marked B and only 8 sums will be marked C.

- 5 (d) is right option since 8 can't be the unit digit of the square of any natural number.
- 6 Since the factors of 11760 are $2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 7 \times 7$ so we need to multiply it with 3×5 because all the factors are paired but 3 and 5 are unpaired, hence (d) is the correct choice.
- 7 Since we know that there are two factors viz. 3 and 5 which are unpaired so if we remove them the number will become a perfect square. Hence we have to divide 11760 by 15 ($= 3 \times 5$) in order to make a perfect square number.
- 8 This problem can't be solved by factorisation because this is not a perfect square. So we have to solve it by division method as follows

	23.9
2	5 <u>75</u>
2	4
43	175
3	129
469	4600
9	4221

(If the number is not a perfect square then by putting decimal we can increase the zeros in pairs for further calculation). The result obtained is ≈ 23.9 .

So by adding some number we can make it the perfect square of 24. Now since we know that $(24)^2 = 576$. So we need to add 1 ($\because 576 - 575 = 1$)

Thus (b) is the correct option.

Alternatively Using options we can solve this problem as if we consider option (a) then 575 itself be a perfect square but its not a perfect square.

Again if we add 1 (i.e., using option (b)) we get the number 576 and then check it, we find that 576 is a perfect square. Hence (b) is correct.

☞ Alternatively Since we know that $(20)^2 = 400$ and $(25)^2 = 625$. It means the value of perfect square must lie in the range of 400 and 625. So we can try it manually and get that $(23)^2 = 529$ and $(24)^2 = 576$. So simply we need to add 1 to make a perfect square number.

- 9** From the discussion done in the previous question we know that we have to subtract 46 to make the perfect square number. Since the closest perfect square numbers are 529 and 576. Since we have to subtract the number therefore we choose 529 and then we get $(575 - 529 =) 46$. Hence (c) is the correct option.

☞ Alternatively If we subtract 5 then 570 is not a perfect square. Again if we subtract 38 then 538 is not a perfect square (simply due to unit digit concept of 2, 3, 7 and 8). Further if we subtract 46 we get 529 which is found to be a perfect square.

🔗 NOTE In the checking through option we have to follow the basic constraints imposed by the problem and here we have to find the least possible number so we start from the least option towards higher valued option i.e., 5 then 37 then 46 and then 50.

- 10** It is given that $A^2 = B^2 + C^2$
But $\frac{B}{C} = \frac{12}{5}$ and B, C are integers.
So the least possible values of B and C are 12 and 5 respectively.
Hence $A^2 = 12^2 + 5^2 \Rightarrow A^2 = 169 \Rightarrow A = 13$

☑ Introductory Exercise 1.3

- 1** Go back to the basics.

Since $2 - 2 = 0$

[Natural number - Natural number]

= Not a natural number

$$\frac{3}{2} = 1.5$$

$\frac{\text{Natural number}}{\text{Natural number}}$ = Not a natural number

Hence, it is not closed for subtraction and division.

- 2** Statement (1) and (2) are wrong since when p is a prime number so it does not have any factor so when all the factors (or numbers) before ' p ' do not involve in the product so it is

- 11** Since Lieutenant Kalia can arrange only 1444 soldiers in the square form. Now if he had arranged n soldiers in each of the n rows then

$$n \times n = 1444 \Rightarrow n = 38$$

Hence option (c) is correct.

- 12** $\frac{\sqrt{289}}{\sqrt{x}} = \frac{1}{5} \Rightarrow \frac{289}{x} = \frac{1}{25} \Rightarrow x = 7225$

- 13** Option (d) is the required answer.

- 14** By observation we get to know that

$$2 * 3 = \sqrt{2^2 + 3^2} = \sqrt{13}$$

$$\text{and } 3 * 4 = \sqrt{3^2 + 4^2} = 5$$

$$\text{then } 5 * 12 = \sqrt{5^2 + 12^2} = 13$$

- 15** $6 * 15 * 3 = \sqrt{\frac{(6+2)(15+3)}{(3+1)}} = \sqrt{\frac{8 \times 18}{4}} = 6$

- 16** $1780 + x = 1849$, which is the nearest perfect square.

$$\text{So, } x = 1849 - 1780; x = 69$$

Hence, (a) is the least possible number.

Remark If you have any problem in finding the value of nearest perfect square so try the following method.

$(40)^2 = 1600$, $(50)^2 = 2000$. It means your required value is lying somewhere between 40^2 & 50^2 . Now $45^2 = 2025$, so it is clear that the value is lying between $(40)^2$ & $(45)^2$. Now in this way you can attain the required value.

- 17** $\frac{100\sqrt{25}}{\sqrt{25+x}} = 50 \Rightarrow \frac{500}{5+x} = 50 \Rightarrow x = 5$

$$\therefore 5 = \sqrt{25}$$

Hence (c).

not divisible by p or any prime number greater than p . Statement (3) is wrong, since $1 \times 2 \times 3 \times 4 \times 5 \times 6$ is divisible by 5.

- 3** Statement (1) is true since all the prime numbers are either at a difference of 2 or 4. So for a suitable value of n it can be expressed. Statement (2) is correct since $(p-1)$ and $(p+1)$ both must be even numbers so $2k \times 2l = 4kl$. Thus it must be divisible by 4.

- 4** Go through options or Go back to the Basics.

The factors of 6 = 1, 2, 3, 6

Now, since $1 + 2 + 3 = 6$

Hence, (d) is correct choice.

- 5 Since $2^p + 1$ is a prime number so

$$2^0 + 1 = 2$$

$$2^1 + 1 = 3$$

$$2^2 + 1 = 5$$

$$2^{(2)^2} = 2^4 + 1 = 17$$

$$2^{(2)^3} = 2^8 + 1 = 257$$

Hence, the value of p is 2 or the power of 2.

✓ Introductory Exercise 1.4

1 $\frac{3}{4} \times \left(\frac{-2}{3} + \frac{3}{5} \right) = \frac{3}{4} \times \left(\frac{-1}{15} \right) = \frac{-1}{20}$

2 $4 \times 100 + 3 \times 10 + \frac{9}{1000} = 400 + 30 + 0.009$
 $= 430.009$ Hence, (c).

- 3 Let us assume some values as 4 and 2, then

$$4 > 2 \Rightarrow -4 < -2$$

Hence, (b) is correct.

 **NOTE** Other three option are wrong as

(a) $4 > 2 \Rightarrow -4 < -2$

(c) $4 > 2 \Rightarrow \frac{1}{4} < \frac{1}{2}$

(d) $4 > 2 \Rightarrow -\frac{1}{4} > -\frac{1}{2}$

- 4 $-\frac{1}{2} < 1$, hence (d) is wrong.

$-\frac{1}{2} < \frac{1}{2}$, hence, it is correct ($x \in I^-$ and $y \in I^+$)

$\therefore x < 0 \Rightarrow \frac{1}{x} < 0$ and $y > 0 \Rightarrow \frac{1}{y} > 0$

$\therefore \frac{1}{x} < \frac{1}{y}$ Hence, (c).

- 5 Go back to Basics.

6 $|x - 2| < 3$

$\Rightarrow (x - 2) < 3$ or $(2 - x) < 3$

$\Rightarrow x < 5$ or $x > -1$

$\Rightarrow -1 < x < 5$

- 7 Only (iii) and (iv) are always true.

Hence choice (c) is the correct one.

- 8 Choice (b) is not always true, as when one number is positive and the other one is negative, the given relationship becomes false. Hence choice (b) is the correct one.

- 6 Since, the prime number between 101 and 120 is 4 viz., 103, 107, 109, 113.

Hence, the number of composite numbers between 101 and 120 = $18 - 4 = 14$.

- 7 127, 131, 137, 139.

- 8 Except 2, all other prime numbers are odd. It means the unit digits of the prime numbers will be either 1, 3, 7 or 9, if we exclude 2 and 5. Hence choice (b) is the valid one.

- 9 Since a modulus value is always non-negative, therefore

$$||x - 2| - 4| - 6| < 10$$

$$\Rightarrow ||x - 2| - 4| - 6 < 10$$

$$\Rightarrow ||x - 2| - 4| < 16$$

$$\Rightarrow |x - 2| - 4 < 16$$

$$\Rightarrow |x - 2| < 20$$

$$\Rightarrow -20 < x - 2 < 20$$

$$\Rightarrow -18 < x < 20$$

Thus there are total 39 values of x , namely $-17, -16, \dots, -2, -1, 0, 1, 2, \dots, 20, 21$

Hence choice (a) is the correct one.

- 10 Since $|x| = x$ or $-x$

$\therefore |x - 1| + |x - 2| + |x - 3| \geq 6$

$\Rightarrow 3x - 6 \geq 6$ or $6 - 3x \geq 6$

$\Rightarrow x \geq 4$ or $x \leq 0$

Hence (b).

- 11 $(-1)^p$ is a negative value when p is odd and it is a positive value when p is even. To minimize the sum of $-(-1)^a + (-1)^b + (-1)^c + (-1)^d$ we need negative values, which is possible only when the powers are odd. However, since $a + b + c + d$ is 1947, which means either we can have only 1 of them odd integer or any 3 of them odd integers.

Case I. When out of 4 integers (a, b, c, d), one integer is odd,

$$(-1)^a + (-1)^b + (-1)^c + (-1)^d$$

$$= (-1) + (1) + (1) + (1) = 2$$

Case II. When out of 4 integers (a, b, c, d), three integers are odd,

$$(-1)^a + (-1)^b + (-1)^c + (-1)^d$$

$$= (-1) + (-1) + (-1) + (1) = -2$$

Hence choice (b) is the correct one.

Solutions (for Q. Nos. 12 and 13) Let the total number of questions attempted by the candidate is x and the number of wrong attempts is y , then $3x - 4y = 17 \Rightarrow y = \frac{3x - 17}{4}$

The possible values of x and y must be integer and do not exceed 30. Therefore the possible values of x are 7, 11, 15, 19, 23 and 27. Thus the corresponding values of y are 1, 4, 7, 10, 13 and 16.

Introductory Exercise 1.5

1 $6^4 \times 8^6 \times 10^8 \times 12^{10}$

$$\begin{aligned} &= (2 \times 3)^4 \times (2^3)^6 \times (2 \times 5)^8 \times (2^2 \times 3)^{10} \\ &= 2^4 \times 3^4 \times 2^{18} \times 2^8 \times 5^8 \times 2^{20} \times 3^{10} \\ &= 2^{50} \times 3^{14} \times 5^8 \end{aligned}$$

Thus, there are total 72 ($= 50 + 14 + 8$) prime factors.

2 Sum of factors of $2^m \times 3^n$ is

$$\begin{aligned} &\frac{[(2^{m+1} - 1)(3^{n+1} - 1)]}{[(2 - 1)(3 - 1)]} = 124 \\ \Rightarrow &(2^{m+1} - 1)(3^{n+1} - 1) = 248 \\ \Rightarrow &(2^{m+1} - 1)(3^{n+1} - 1) = 31 \times 8 \\ \Rightarrow &2^{m+1} - 1 = 31 \text{ and } (3^{n+1} - 1) = 8 \\ \Rightarrow &2^{m+1} = 32 \text{ and } 3^{n+1} = 9 \\ \Rightarrow &2^m = 16 \text{ and } 3^n = 3 \Rightarrow m = 4 \text{ and } n = 1 \end{aligned}$$

Hence, choice (c) is the correct one.

3 $500 = 2^2 \times 5^3$

$$\begin{aligned} \text{The sum of all the factors} &= \frac{(2^{2+1} - 1) \times (5^{3+1} - 1)}{(2 - 1)(5 - 1)} \\ &= \frac{7 \times 624}{1 \times 4} = 1092 \end{aligned}$$

Hence, (c) is the correct option.

4 Let l and b be the length and breadth of the rectangle, the perimeter of rectangle

$$\begin{aligned} &= 2(l + b) = 72 \\ \Rightarrow &l + b = 36 \\ \Rightarrow &(l + b) = (1 + 35), (2 + 34), (3 + 33), \dots (18 + 18) \end{aligned}$$

It implies that there are total 18 possible combinations of l and b , so there can be maximum 18 different areas whose perimeter is 72 cm.

Hence choice (a) is the correct one.

5 Let l and b be the length and breadth of the rectangle, the perimeter of rectangle $= l + b = 72$

$$\begin{aligned} \Rightarrow &l \times b = 2^3 \times 3^2 \\ \text{Total number of factors} &= (3 + 1)(2 + 1) = 12 \end{aligned}$$

Total number of ways of expressing 72 as a product of two factors $= \frac{1}{2} (\text{total number of factors}) = \frac{1}{2} (12) = 6$

12 In order to get 17 marks a student must attempt maximum 27 problems. Hence choice (d) is the correct one.

13 There can be at most 6 students who can score exactly 17 marks by attempting either 7 or 11 or 15 or 19 or 23 or 27 problems.

Hence choice (d) is the correct one.

Therefore 72 can be expressed as a product of two factors in 6 ways, as following.

$$(1 \times 72), (2 \times 36), (3 \times 24), (4 \times 18), (6 \times 12), (8 \times 9)$$

Now, since the value of $(l + b)$ is distinct for each of the 6 sets of $(l \times b)$, so there will be maximum 6 distinct perimeters of the rectangle with the area of 72 sq. cm.

Hence choice (d) is the correct one.

6 The 100 given numbers are 2, 4, 6, ..., 200. The number of perfect squares in the set $\{2, 4, 6, \dots, 200\} = 7$

Since all the perfect square numbers have odd number of factors, so the remaining numbers will have even number of factors. That means in this set there are 93 numbers which have even number of factors. Hence choice (d) is the correct one.

7 $350 = 2 \times 5^2 \times 7$

Number of natural numbers up to 350 which are not divisible by 5 $= 350 \times \left(1 - \frac{1}{5}\right) = 280$

Hence choice (a) is the correct one.

8 $350 = 2 \times 5^2 \times 7$

Number of natural numbers up to 350 which are not divisible by 7 $= 350 \times \left(1 - \frac{1}{7}\right) = 300$

Hence choice (c) is the correct one.

9 $350 = 2 \times 5^2 \times 7$

Number of natural numbers up to 350 which are neither divisible by 5 nor by 7 $= 350 \times \left(1 - \frac{1}{5}\right) \times \left(1 - \frac{1}{7}\right) = 240$

Hence choice (a) is the correct one.

10 $420 = 2^2 \times 3 \times 5 \times 7$

Number of natural numbers up to 420 which are divisible by none of 2, 3 or 5 $= 420 \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{5}\right) = 112$

Number of natural numbers up to 420 which are divisible by none of 2, 3, 5 or 7

$$= 420 \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{5}\right) \times \left(1 - \frac{1}{7}\right) = 96$$

Therefore, number of natural numbers up to 420, which are divisible by 7 but, not by 2, 3 or 5 $= 16$.

Hence choice (a) is the correct one.

11 $210 = 2 \times 3 \times 5 \times 7$

Number of co-prime numbers of 210, which are below 210

$$= 210 \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{5}\right) \times \left(1 - \frac{1}{7}\right) = 48$$

If you look deeper into the solution then you will realize that there are 48 numbers, which have no factors like 2, 3, 5 or 7. Since these 48 numbers have no factor like 2, 3, 5 or 7, then obviously 4, 6, 8, 9 and 10 will not be included in these 48 numbers. Now, since when 4, 6, 8, 9 and 10 are not included in these 48 numbers, so their multiples too won't be included in 48 numbers. It implies that these 48 numbers are free from the divisors of 2, 3, 4, ..., 10. Hence choice (c) is the correct one.

☞ **Alternatively** $210 = 2 \times 3 \times 5 \times 7$

And, $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$

$= 2^8 \times 3^4 \times 5^2 \times 7$. That means if a number is not divisible by 2, 3, 5 or 7, so it is not divisible by 2, 3, ..., 8, 9 or 10. Number of natural numbers up to 210 which are neither divisible by 2 nor 3 nor 5 nor 7

$$= 210 \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{5}\right) \times \left(1 - \frac{1}{7}\right) = 48$$

Therefore, number of natural numbers up to 210 which are not divisible by any of the nine numbers 2, 3, ..., 8, 9, 10 is 48.

- 12 Since 61 is a prime number, so it does not have any factor other than 1 and itself. But if we ignore 61, for a while, then 60 has factors like 2, 3 and 5. Let's separate the number 61 from the other 60 natural numbers. And then factorize 60.

$$60 = 2^2 \times 3 \times 5$$

Number of co-prime numbers of 60, which are below 60

$$60 \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{5}\right) = 16$$

Now you know that there are 16 such numbers in the set of first 60 numbers, which have no factors like 2, 3 or 5. Therefore including 61, there are total 17 numbers, which are divisible by neither 2 nor 3 nor 5.

Hence choice (d) is the correct one.

- 13 Since $123 = 3 \times 41$, it means we cannot use the method of co-prime numbers with 123 as only 3 is a required factor of 123, not the 2 and 5. As $2 \times 3 \times 5 = 30$, so we can use the method of co-prime numbers with 30 or any multiple of 30, i.e., 30, 60, 90, 120, 150, ...

The nearest multiple of 30 to 123 is 120. So for a while we will consider only first 120 natural numbers and ignore the last three numbers 121, 122 and 123. The number of co-prime numbers of 120, which are below 120

$$= 120 \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{5}\right) = 32$$

Therefore in a set of first 120 natural numbers there are 88 ($= 120 - 32$) natural numbers, which are divisible by either 2 or 3 or 5.

Further out of the remaining three numbers 121, 122 and 123 there are two more numbers 122 and 123, which are divisible by either 2 or 3. Therefore we have total 90 natural numbers, which are divisible by either 2 or 3 or 5. Hence choice (a) is the correct one.

14 $180 = 2^2 \times 3^2 \times 5$

Number of co-prime numbers of 180 which are below

$$180 = 180 \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{5}\right) = 48$$

Total number of numbers which are divisible by

$$7 = \left\lfloor \frac{180}{7} \right\rfloor = 75$$

These 25 numbers are 7, 14, 21, 28, 35, ..., 175.

Now out of 25 numbers there are 18 numbers, which are already divisible by 2, 3 or 5. It means there are 7 numbers which are divisible by 7 but not by 2, 3 or 5. These numbers are 7, 49, 77, 91, 119, 133 and 161. So if we exclude these 7 numbers from the 48 numbers we will get those numbers only which are divisible by neither 2 nor 3 nor 5 nor 7.

Thus the required numbers $= 48 - 7 = 41$

Hence choice (c) is the correct one.

Hint Total number of numbers, which are neither divisible by 2 nor 3 nor 5 nor 7 = (total number of numbers which are neither divisible by 2 nor by 3 nor by 5) – (total number of numbers which are divisible by 7 but not by 2 or 3 or 5).

Also $\left\lfloor \frac{180}{7} \right\rfloor$ indicates the number of times 180 can be fully

divided by 7 or it indicates the greatest possible integer that can be obtained when 180 is divided by 7.

☞ **Alternatively** $2 \times 3 \times 5 \times 7 = 210$

Number of co-primes of 210 which are below 210

$$= 210 \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{5}\right) \times \left(1 - \frac{1}{7}\right) = 48$$

It implies that there are 48 numbers below 210, which are divisible by neither 2 nor 3 nor 5 nor 7.

Now there are total 30 numbers in the set {181, 182, ..., 210}. But there are only 7 numbers in this set, which are divisible by neither 2 nor 3 nor 5 nor 7. Therefore, total numbers in the set {1, 2, 3, ..., 180} which are divisible by neither 2 nor 3 nor 5 nor 7 $= 48 - 7 = 41$.

Hint Since there are not too many numbers in the set 181, 182, ..., 210, so you can compute it manually. First of all remove all the even numbers since these are divisible by 2. Then there will be only 15 remaining numbers. Cancel out all those numbers, which are divisible by 3, then remaining by 5 and finally those which are divisible by 7. then there will be only 7 numbers, which are not divisible by either of 2, 3, 5 or 7. These are 181, 187, 191, 193, 197, 199 and 209.

- 15 There are 41 prime numbers in the set {1, 2, 3, ..., 180}

- 16 There are 168 prime numbers in the set {1, 2, 3, ..., 1000}

✓ Introductory Exercise 1.6

1 Go through option. It would be better to try to divide the given numbers. Then 19 divides 1007 and 1273 both. Hence, (d) is the correct option.

2 $5 \times 60 = 20 \times N \Rightarrow 15$, Hence (d)

3 $30k \times 30l = 5400$, where k and l are coprimes.

$$k \times l = 6$$

Now, since $6 = 1 \times 6$ and 2×3

Hence, there are two pairs viz., (30, 1800) and (60 and 90). Thus (b).

4 The pair will be of the form $(15k, 15l)$, where k and l must be coprimes. The values of k and l be such that $15k$ and $15l$ will remain between 40 and 100. So, the pairs are (45, 60), (45, 75), (60, 75), (75, 90).

5 The volume of tin is = HCF of 140, 260 and $320 = 20$

Hence, (a) is the correct answer.

6 Statement (2) is wrong since $3 \times (1) + 2 \times (-1) = 1$, here y is negative. Again since x and y depends on each other. Hence they are uniquely determined.

7 Go back to the Basics.

8 Let the numbers be $2a$ and $2b$ since 2 is the HCF of the numbers.

$$\text{Therefore } 2a \times 2b = 84$$

$$\Rightarrow ab = 21$$

$$\text{Now, since } 21 = 1 \times 21 = 3 \times 7$$

$$\therefore (a, b) = (1, 21) \text{ and } (3, 7)$$

Hence the values of pairs = (2, 42), (6, 14)

Thus there are only 2 pairs of numbers whose HCF is 2 and the product is 84.

9 Let the numbers be $6a$ and $6b$, since 6 is the HCF of the required numbers.

$$\text{Then } 6a \times 6b = 15120$$

$$\Rightarrow ab = 420$$

$$\text{Now } 420 = 1 \times 420$$

$$= 2 \times 210 \rightarrow \times$$

$$= 3 \times 140$$

$$= 4 \times 105$$

$$= 5 \times 84$$

$$= 6 \times 70 \rightarrow \times$$

$$= 7 \times 60$$

$$= 10 \times 42 \rightarrow \times$$

$$= 12 \times 35$$

$$= 14 \times 30 \rightarrow \times$$

$$= 15 \times 28$$

$$= 20 \times 21$$

Thus out of 12 pairs only 8 pairs are possible since the rest 4 pairs are not the coprimes.

Since 6 is the highest common factor of the two numbers

(i.e., pair of numbers)

So further there must not be any number (or factor) common in both the numbers otherwise the HCF will be changed so the only possible values of (a, b) are (1, 420), (3, 140), (4, 105), (5, 84), (7, 60), (12, 35), (15, 28) and (20, 21).

Thus the possible numbers can be

(6, 520), (18, 840), (24, 630), (30, 504), (42, 360), (72, 210), (90, 168), (120, 126).

Hence, there are 8 pairs of such numbers whose HCF is 6 and the product is 15120.

10 Only two pairs of numbers.

11 28 is the largest possible number by which we can divide the given numbers in order to obtain the same remainders and thus dividing by 28 we get the remainder 20 in each division.

12 Let the two numbers of a pair be $5a$ and $5b$ then

$$5a + 5b = 50 \Rightarrow 5(a + b) = 50$$

$$\Rightarrow (a + b) = 10$$

So the possible values of (a, b) can be

(1, 9), (2, 8), (3, 7), (4, 6), (5, 5) but (2, 8), (4, 6) and (5, 5) are inadmissible since these pairs are not the coprimes, hence they can change the value of HCF. Thus only two pairs [(1, 9) & (3, 7)] are possible. Thus the actual pairs of such numbers are (5, 45) and (15, 35).

13 The largest possible length of the tape = HCF of 525, 1050, 1155 = 105. Hence (b) is the correct answer.

14 Minimum number of attempts = $\frac{\text{Total length of cloths}}{\text{Max. possible length of tape}}$

$$= \frac{2730}{105} = 26$$

Hence in 26 attempts the whole length of cloth can be measured using the tape of 105 cm length.

15 In order to have minimum number of tiles we need the largest possible tiles of square shape (in area).

So the maximum possible dimension (length and breadth) of a tile is the HCF of the dimension of the room.

Thus the HCF of 462 and 360 is 6 cm.

Hence the area of a square tile be $6 \times 6 = 36 \text{ cm}^2$.

Now the number of tiles required

$$= \frac{\text{Area of floor of the room}}{\text{Area of one tile}} \\ = \frac{462 \times 360}{6 \times 6} = 77 \times 60 = 4620$$

Thus total 4620 tiles are required.

16 (d) is correct answer. The numbers will be 195 and 143.

17 (d) is the correct option. In the options (a), (b) and (c) the HCF is not 12.

- 18 The maximum capacity of the vessel = HCF of 1653, 2261 and 2527 = 19

Hence (c) is the correct option.

- 19 To be the minimum number of pencils the size of all the pencils must be maximum. So the length of each pencil

$$= \text{HCF of } 24 \text{ and } 42 = 6$$

Thus the number of pencils

$$= \frac{\text{Total length of pencils}}{\text{length of one pencil}} \\ = \frac{24 + 42}{6} = 11$$

Thus (b) is the correct choice since there can be min. 11 pencils of same size.

- 20 (a) The correct pair of numbers is (27, 189).

- 21 Minimum number of rows means max. number of trees per row, also equal number of trees per row is required so we need to find the HCF of 36, 144 and 234 to find the maximum number of trees in a row.

Thus HCF of 36, 144 and 234 = 18

$$\text{Thus the number of rows} = \frac{\text{Total no. of trees}}{\text{No. of trees in a row}} \\ = \frac{36 + 144 + 234}{18} = 23$$

Hence (b) is correct answer.

- 22 It is possible only when there is no common factor among p , q and r . Hence, the HCF of p , q , $r = 1$

and the LCM of p , q , $r = p \cdot q \cdot r$

$$\therefore (\text{HCF} \times \text{LCM}) \text{ of } p, q, r = 1 \times p \cdot q \cdot r = pqr$$

Introductory Exercise 1.7

1 $\therefore \frac{-2}{3} = -0.66$ and $\frac{5}{2} = 2.5$ and $\frac{-1}{6} = -0.16$

$\therefore$ The correct required order is

$$\frac{5}{2} > \frac{-1}{6} > \frac{-2}{3}$$

Thus (a) is the correct answer.

2 $\frac{-3}{7} = -0.42$ and $\frac{-1}{3} = -0.33$

So, the value in descending order will be as follows:

$$\frac{2}{3} > \frac{-1}{3} > \frac{-3}{7}$$

Hence, (b) is the correct option.

- 3 Express the fractions, as shown below

$$\frac{1029}{1025} = 1 + \frac{4}{1025}$$

$$\frac{1030}{1026} = 1 + \frac{4}{1026}$$

$$\frac{256}{255} = 1 + \frac{1}{255} = 1 + \frac{4}{1020}$$

$$\frac{1023}{1019} = 1 + \frac{4}{1019}$$

Since, we know that if the two or more fractions have the same numerator, the fraction with the least denominator would be the greatest one.

Hence choice (d) is the correct one.

- 4 Go back to the Basics.

- 5 Go through option : Consider the value of x such that $(5x + 7)$ must be divisible by 13. Since, the product is an integer and the value of y must be such that $(13y + 1)$ will be divisible by x .

Thus option (b) is correct.

- 6 Option (b) is clearly wrong. Again since $(28)^2 = 784$.

So, option (c) is also wrong. Now check for option (a)

$$\left(5\frac{15}{22}\right)^2 = \left(\frac{125}{22}\right)^2 = \frac{15625}{484} = 32\frac{137}{484}$$

Thus, (a) is the correct option.

7 $1 \div \frac{1}{1 \div \frac{1}{1 \div \frac{1}{3}}} = 1 \div \frac{1}{1 \div \frac{1}{3}} = 1 \div \frac{1}{\frac{1}{3}} = 3$

Hence, (c) is the correct answer.

8 $\therefore \frac{4\frac{1}{7} - 2\frac{1}{4}}{3\frac{1}{2} + 1\frac{1}{7}} = \frac{53}{130}$ and $\frac{1}{2 + \frac{1}{2 + \frac{1}{5 - \frac{1}{5}}}} = \frac{53}{130}$

$$\therefore \frac{53}{130} \div \frac{53}{130} = 1$$

Hence, (c).

- 9 Let the total journey be x kms then,

$$\frac{5x}{16} + \frac{7x}{20} + 10 = x \Rightarrow x = 29\frac{17}{27}$$

Hence, (d).

10 $x_1 + \frac{1}{x_2 + \frac{1}{x_3 + \frac{1}{x_4 + \dots}}} = \sqrt[3]{10}$

$$\Rightarrow x_1 + \frac{1}{x_2 + \frac{1}{x_3 + \frac{1}{x_4 + \dots}}} = 2.15$$

$$\Rightarrow x_1 + p = 2.15$$

$$\begin{aligned} \because 0 < p < 1, \therefore x_1 = 2 \Rightarrow P = 0.15 \\ \Rightarrow \frac{1}{x_2 + \frac{1}{x_3 + \frac{1}{x_4 + \dots}}} = 0.15 \\ \Rightarrow \frac{1}{x_2 + q} = 0.15 \end{aligned}$$

Since $0 < q < 1$, so $x_2 = 6$

Therefore, $\frac{x_1}{x_2} = \frac{1}{3}$

Hence choice (a) is the correct one.

Hint If the denominator (D) is any positive number, then $0 < \frac{1}{D} < 1$.

- 11 Statement (i)** Were we allowed to cut a guava into more than 6 pieces we would have cut every guava into 12 pieces and then we would have distributed 84 pieces equally among 12 people.

But, we are not allowed to make more than 6 pieces of a guava so we try to use the other method, as given below.

$$\begin{aligned} \frac{a}{b} + \frac{c}{d} &= \frac{7}{12} \\ \Rightarrow \frac{1}{3} + \frac{1}{4} &= \frac{7}{12} \end{aligned}$$

First of all take 3 guavas and cut each guava into 4 equal pieces and then divide these 12 pieces equally among 12 disciples.

Now, take the remaining 4 guavas and cut each guava into 3 equal pieces and then divide these 12 pieces equally among 12 disciples.

Statement (ii) Cut each guava into two equal parts and divide them among 12 disciples.

Statement (iii) Were we allowed to cut a guava into more than 6 pieces we would have cut every guava into 15 pieces and then we would have distributed 165 pieces equally among 15 people. But, we are not allowed to make more than 6 pieces of a guava so we try to use the other method, as given below.

$$\begin{aligned} \frac{a}{b} + \frac{c}{d} &= \frac{11}{15} \\ \Rightarrow \frac{a}{3} + \frac{b}{5} &= \frac{11}{15} \\ \Rightarrow 5a + 3b &= 11 \\ \Rightarrow a = 1, b &= 12 \\ \Rightarrow \frac{1}{3} + \frac{2}{5} &= \frac{11}{15} \end{aligned}$$

First of all take 5 guavas, and cut each guava into 3 equal pieces and then divide these 15 pieces equally among 15 disciples.

Now, take the remaining 6 guavas and cut each guava into 5 equal pieces and then divide these 30 pieces equally among 15 disciples.

Statement (iii) Were we allowed to cut a guava into more than 6 pieces we would have cut every guava into 15 pieces and then we would have distributed 105 pieces equally among 7 people. But, we are not allowed to make more than 6 pieces of a guava so we try to use the other method, as given below.

$$\begin{aligned} \frac{a}{b} + \frac{c}{d} &= \frac{7}{15} \\ \Rightarrow \frac{a}{3} + \frac{b}{5} &= \frac{7}{15} \\ \Rightarrow 5a + 3b &= 7 \end{aligned}$$

There are no natural numbers for a and b to satisfy the above equation. So we cannot divide the 7 guavas equally among 15 people by cutting 6 or less pieces of a guava. Therefore statement (iv) is false. Hence choice (d) is the correct one.

12 $33.33 \div 1.1 = \frac{33.33}{1.1} = \frac{33.33}{1.10}$
 $= \frac{3333}{110} = 30.3$

- 13** Go back to the Basics.

14 $\sqrt{900} + \sqrt{0.09} - \sqrt{0.000009}$
 $= 30 + 0.3 - 0.003 = 30.297$

Hence, (a) is the correct option.

15 $\frac{10\sqrt{6.25}}{\sqrt{6.25} - 0.5} = \frac{10 \times 2.5}{2.5 - 0.5} = \frac{25}{2} = 12.5$

16 $\because \sqrt{254016} = 504$
 and $\sqrt{10609} = 103$
 $\therefore \frac{\sqrt{25.4016} - \sqrt{1.0609}}{\sqrt{25.4016} + \sqrt{1.0609}} = \frac{5.04 - 1.03}{5.04 + 1.03}$
 $= \frac{4.01}{6.07} = \frac{401}{607}$

- 17** When 1 is divided by 2^x , such that $x = 0, 1, 2, \dots$, $\frac{1}{2^x}$ is a terminating decimal number.

Similarly, when 1 is divided by 5^y , such that $y = 0, 1, 2, \dots$, $\frac{1}{5^y}$ is a terminating decimal number.

Therefore, when 1 is divided by $2^x 5^y$, such that $x = 0, 1, 2, \dots$ and $y = 0, 1, 2, \dots$, $\frac{1}{2^x 5^y}$ is a terminating decimal number. Hence choice (c) is the correct one.

NOTE Choice (c) is better answer than choice (d), since choice (d) ignores $\frac{1}{10^0} = 1$, which is also a terminating decimal expansion.

Introductory Exercise 1.8

1 Go through options and put $x = 1$.

☞ Alternatively Solve with the help of quadratic equations.

2 Go back to the Basics.

$$3 \quad 3^n = 27 \Rightarrow 3^n = 3^3 \Rightarrow n = 3$$

$$\therefore 3^{n-2} = 3^1 = 3$$

Thus, (a) is the right choice.

$$4 \quad 4^{x+3} \times 2^{x-3} - 128 = 0$$

$$\Rightarrow 2^{2x+6} \times 2^{x-3} - 2^7 = 0$$

$$\Rightarrow 2^{3x+3} - 2^7 = 0$$

$$\Rightarrow 3x + 3 = 7$$

$$\Rightarrow 3x = 4 \Rightarrow x = \frac{4}{3}$$

Hence, (d) is the correct option.

$$5 \quad \therefore a^m \cdot a^n = a^{mn}$$

$$\Rightarrow a^{m+n} = a^{mn}$$

$$\Rightarrow m + n = mn$$

$$\begin{aligned} \therefore m(n-2) + n(m-2) &= mn - 2m + nm - 2n \\ &= (m+n) - 2m + (m+n) - 2n \\ &= 2m + 2n - 2m - 2n = 0 \end{aligned}$$

Hence, (a) is the correct option.

$$6 \quad \frac{6^6 + 6^6 + 6^6 + 6^6 + 6^6 + 6^6}{3^6 + 3^6 + 3^6} \div \frac{4^6 + 4^6 + 4^6 + 4^6}{2^6 + 2^6} = 2^n$$

$$\Rightarrow \frac{6(6^6)}{3(3^6)} \div \frac{4(4^6)}{2(2^6)} = 2^n$$

$$\Rightarrow 1 = 2^n \Rightarrow 2^0$$

$$\text{thus, } n = 0$$

Hence, (b) is the correct option.

$$7 \quad x^y = y^x \Rightarrow y = x^{y/x}$$

$$\Rightarrow 2x = x^{y/x} = x^{2x/x} = x^2$$

$$\Rightarrow 2x = x^2 \Rightarrow x = 2$$

Hence, (a) is the correct option.

☞ Alternatively Go through options and put $x = 2$ in both the values if they satisfy, then the presumed option is correct. As

$$y = 2x = 4$$

$$\text{then } 2^4 = 4^2$$

$$16 = 16$$

Hence, it is correct.

$$8 \quad 2^{x+3} \cdot 4^{2x-5} = 2^{3x+7}$$

$$\Rightarrow 2^{x+3} \cdot 2^{4x-10} = 2^{3x+7}$$

$$\Rightarrow 2^{5x-7} = 2^{3x+7}$$

$$\Rightarrow 5x - 7 = 3x + 7$$

$$\Rightarrow 2x = 14$$

$$x = 7$$

Hence, (d).

$$9 \quad 9^{3/2} \div (243)^{-2/3} = (3^2)^{3/2} \div \left(\frac{1}{3^5}\right)^{2/3}$$

$$= 3^3 \div 3^{-10/3} = 3^{3+\frac{10}{3}} = 3^{19/3}$$

10 At $x = 2$, both sides are equal.

$$\begin{aligned} 11 \quad & \left(\frac{x^a}{x^b}\right)^{(a^2+b^2+ab)} \cdot \left(\frac{x^b}{x^c}\right)^{(b^2+c^2+bc)} \cdot \left(\frac{x^c}{x^a}\right)^{(c^2+a^2+ac)} \\ &= x^{(a-b)(a^2+b^2+ab)} \cdot x^{(b-c)(b^2+c^2+bc)} \cdot x^{(c-a)(c^2+a^2+ac)} \\ &= x^{(a^3+ab^2+a^2b-a^2b-b^3-ab^2+b^3+bc^2+b^2c-b^2c-c^3-bc^2+c^3+a^2c+ac^2-ac^2-a^3-a^2c)} \\ &= x \\ &= x^0 = 1 \end{aligned}$$

Hence, (c).

$$12 \quad \frac{b}{a} = \frac{c}{b} = b^2 = ca$$

$$\text{Again, } a^x = b^y = c^z = k$$

$$\Rightarrow a = k^{1/x}, b = k^{1/y}, c = k^{1/z}$$

$$\therefore b^2 = ac$$

$$\Rightarrow k^{2/y} = k^{1/x} \cdot k^{1/z}$$

$$\Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z} = \frac{z+x}{zx}$$

$$\Rightarrow \frac{2z}{x+z} = \frac{y}{x}$$

$$13 \quad a^{1/m} = b^{1/n} = c^{1/p} = k$$

$$\Rightarrow a = k^m, b = k^n, c = k^p$$

$$\therefore a \cdot b \cdot c = 1$$

$$\Rightarrow k^m \cdot k^n \cdot k^p = 1$$

$$\Rightarrow k^{m+n+p} = 1 = k^0$$

$$\Rightarrow m + n + p = 0$$

Hence, (a) is the correct option.

$$14 \quad [(x^l)^{1-\frac{1}{l}}]^{\frac{1}{l-1}} = (x^{l-1})^{\frac{1}{l-1}} = x$$

Hence, (a) is the correct choice.

$$15 \quad \sqrt[4]{10} = (10)^{1/4} = (10)^{3/12} = (1000)^{1/12}$$

$$\sqrt[3]{6} = (6)^{1/3} = (6)^{4/12} = (1296)^{1/12}$$

$$\sqrt{3} = (3)^{1/2} = (3)^{6/12} = (729)^{1/12}$$

$\therefore \sqrt{3} < \sqrt[4]{10} < \sqrt[3]{6}$ is the correct order and hence (b) is correct.

$$\begin{aligned}
 16 \quad x &= \frac{1}{\sqrt{2}-1} = \frac{(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} \\
 \Rightarrow \quad x &= \sqrt{2}+1 \text{ and } \frac{1}{x} = \sqrt{2}-1 \\
 \therefore x^2 - 6 + \frac{1}{x^2} &= \left(x - \frac{1}{x}\right)^2 - 4 \\
 &= ((\sqrt{2}+1) - (\sqrt{2}-1))^2 - 4 = 0
 \end{aligned}$$

Hence choice (a) is the valid one.

$$\begin{aligned}
 17 \quad \frac{3}{5\sqrt[4]{27}} &= \frac{3}{5} \times \frac{1}{\sqrt[4]{27}} \\
 &= \frac{3}{5} \times \frac{1}{\frac{3}{3^{\frac{1}{4}}}} = \frac{3}{5} \times \frac{3^{\frac{1}{4}}}{3} = \frac{3^{\frac{1}{4}}}{5} = \frac{\sqrt[4]{3}}{5}
 \end{aligned}$$

Hence choice (b) is the valid one.

$$18 \quad \frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{18}+\sqrt{48}} = \frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{18}+\sqrt{48}} \times \frac{\sqrt{48}-\sqrt{18}}{\sqrt{48}-\sqrt{18}}$$

$$\begin{aligned}
 &= \frac{(7\sqrt{3}-5\sqrt{2})(\sqrt{48}-\sqrt{18})}{30} \\
 &= \frac{(7\sqrt{3}-5\sqrt{2})(4\sqrt{3}-3\sqrt{2})}{30}
 \end{aligned}$$

Hence choice (c) is the valid one.

$$\begin{aligned}
 19 \quad \sqrt{5+\sqrt[3]{x}} &= 3 \\
 \Rightarrow \quad 5+\sqrt[3]{x} &= 9 \\
 \Rightarrow \quad \sqrt[3]{x} &= 4 \\
 \Rightarrow \quad x &= 64
 \end{aligned}$$

Hence choice (c) is the valid one.

$$20 \quad A = \sqrt{2} = 2^{\frac{1}{2}} = 2^{\frac{6}{12}} = \sqrt[12]{64}$$

$$B = \sqrt[3]{3} = 3^{\frac{1}{3}} = 3^{\frac{4}{12}} = \sqrt[12]{81}$$

$$C = \sqrt[4]{4} = 4^{\frac{1}{4}} = 4^{\frac{3}{12}} = \sqrt[12]{64}$$

Hence choice (c) is the valid one.

Introductory Exercise 1.9

- 1 You know that 24^n gives last two digits as 24, if n is odd and it gives last two digits as 76, if n is even. Therefore,

$$\begin{aligned}
 64^{81} &= 2^{486} = (2^{10})^{48} \times 2^6 \\
 &= (1024)^{48} \times 64
 \end{aligned}$$

$$\Rightarrow 24^{48} \times 64 = 76 \times 64 = 4864$$

Thus the last two digit are 64.

Hence choice (c) is the correct one.

$$\begin{aligned}
 2 \quad \therefore \left(x + \frac{1}{x}\right)^2 &= x^2 + \frac{1}{x^2} + 2 \cdot \left(x \cdot \frac{1}{x}\right) \\
 \Rightarrow (2)^2 &= x^2 + \frac{1}{x^2} + 2(1)
 \end{aligned}$$

$$\left(\text{Substituting the value of } x + \frac{1}{x}\right)$$

$$\Rightarrow 4 = x^2 + \frac{1}{x^2} + 2 \Rightarrow x^2 + \frac{1}{x^2} = 2$$

Hence, (c) is the right choice.

$$\begin{aligned}
 3 \quad \therefore a^2 + b^2 + c^2 - ab - bc - ca \\
 &= \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2] \\
 &= \frac{1}{2}[(2)^2 + (-2)^2 + (4)^2] = 12
 \end{aligned}$$

Alternatively Substitute the values of a, b, c into given expression.

$$\begin{aligned}
 4 \quad \therefore (a^2 + b^2)^3 &= (a^3 + b^3)^2 \\
 \Rightarrow a^6 + b^6 + 3a^2b^2(a^2 + b^2) &= a^6 + b^6 + 2a^3b^3 \\
 \Rightarrow 3a^2b^2(a^2 + b^2) &= 2a^3b^3 \\
 \Rightarrow 3(a^2 + b^2) &= 2ab \\
 \Rightarrow \frac{a^2 + b^2}{ab} &= \frac{2}{3}
 \end{aligned}$$

$$\text{Now, } \left(\frac{a}{b} + \frac{b}{a}\right)^6 = \left(\frac{a^2 + b^2}{ab}\right)^6 = \left(\frac{2}{3}\right)^6 = \frac{64}{729}$$

Hence, (b) is the correct choice.

$$\begin{aligned}
 5 \quad \therefore (x-y)^3 &= x^3 - y^3 - 3xy(x-y) \\
 \therefore (1)^3 &= x^3 - y^3 - 3xy(1) \\
 [\text{Substituting the value of } (x-y)] \\
 \Rightarrow x^3 - y^3 - 3xy &= 1
 \end{aligned}$$

- 6 If $x = 1234, y = 4321, z = -5555$,

$$\begin{aligned}
 \text{The value of } \frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy} \\
 &= \frac{(x^3 + y^3 + z^3)}{xyz} = \frac{(3xyz)}{xyz} = 3
 \end{aligned}$$

Hence choice (c) is the correct one.

Hint Apply the formula

$$(a^3 + b^3 + c^3) - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\text{If } (a+b+c) = 0, (a^3 + b^3 + c^3) = 3abc$$

$$7 \quad 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 = 255$$

$$\text{And } 2^8 + 2^9 + 2^{10} + 2^{11} + 2^{12} + 2^{13} + 2^{14} + 2^{15} = 2^8(255)$$

Similarly

$$2^{16} + 2^{17} + 2^{18} + 2^{19} + 2^{20} + 2^{21} + 2^{22} + 2^{23} = 2^{16}(255)$$

.....

.....

$$\text{Therefore, } 2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^{255}$$

$$= 255 + 2^8(255) + 2^{16}(255) + 2^{24}(255) + \dots + 2^{248}(255)$$

$$= 255(1 + 2^8 + 2^{16} + 2^{24} + \dots + 2^{248})$$

$$= 255k; \text{ where } k \text{ is a positive integer.}$$

It shows that the given expression is divisible by 255, so there is no remainder.

Hence choice (a) is the correct one.

☞ Alternatively Using the formula for the sum of geometric progression, we have

$$2^0 + 2^1 + 2^2 + \dots + 2^{255} = \frac{2^0(2^{256} - 1)}{(2 - 1)} = (2^{256} - 1)$$

$$= [(2^8)^{32} - 1] = [(256)^{32} - 1]$$

The above expression is divisible by 255, so the required remainder will be 0.

$$\text{Hint } \frac{[(256)^{32} - 1]}{255} = \frac{(256)^{32}}{255} - \frac{1}{255}$$

$$= \frac{1^{32}}{255} - \frac{1}{255} = \frac{0}{255} = 0$$

$$8 \quad \because (1 - 81P^4) = (1 + 9P^2)(1 - 9P^2)$$

$$= (1 + 9P^2)(1 + 3P)(1 - 3P)$$

Thus, $(1 - 3P)$, $[1 - (3P)^2]$ and $[1 - (3P)^4]$ are the 3 factors possible.

$$9 \quad \text{Since } x^4 - y^4 = (x^2)^2 - (y^2)^2$$

$$= (x^2 + y^2)(x + y)(x - y)$$

Thus, $(x^4 - y^4)$ is exactly divisible by $(x - y)$ as one of the factors. Hence, no remainder will leave.

$$10 \quad \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2 \cdot x \cdot \frac{1}{x}$$

$$\Rightarrow 4 = \left(x^2 + \frac{1}{x^2}\right) - 2 \Rightarrow x^2 + \frac{1}{x^2} = 6$$

$$\text{Now } \left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2x^2 \cdot \frac{1}{x^2}$$

$$\Rightarrow 36 = x^4 + \frac{1}{x^4} + 2$$

$$\Rightarrow x^4 + \frac{1}{x^4} = 34$$

Hence, (d) is the correct answer.

$$11 \quad (x + y) = 25 \quad \text{and} \quad xy = 144$$

$$(x - y)^2 = (x + y)^2 - 4xy$$

$$(x - y)^2 = 625 - 576$$

$$(x - y)^2 = 49 \Rightarrow x - y = 7$$

Hence, (c) is correct.

$$12 \quad x + y = 9 \quad \text{and} \quad x^2 + y^2 = 41$$

$$(x + y)^2 = x^2 + y^2 + 2xy$$

$$81 = 41 + 2xy$$

$$\Rightarrow 40 = 2xy \Rightarrow xy = 20$$

$$\text{Now } (x - y)^2 = (x + y)^2 - 4xy$$

$$(x - y)^2 = 81 - 80$$

$$(x - y)^2 = 1 \Rightarrow x - y = 1$$

$$\text{Thus } x + y = 9 \quad \text{and} \quad x - y = 1$$

$$\text{Hence, } x = 5 \quad \text{and} \quad y = 4$$

Therefore (a) is the correct answer.

☞ Alternatively Go through options:

$$4 + 5 = 9$$

and

$$16 + 25 = 41$$

$$13 \quad \sqrt{\left(3\frac{1}{4}\right)^4 - \left(4\frac{1}{3}\right)^4} = \sqrt{\left(\frac{13}{4}\right)^4 - \left(\frac{13}{3}\right)^4}$$

$$= \sqrt{\left(\frac{13}{4}\right)^2 - \left(\frac{13}{3}\right)^2}$$

$$= \sqrt{\left(\frac{13}{4}\right)^2 + \left(\frac{13}{3}\right)^2}$$

$$= \sqrt{169\left(\frac{25}{144}\right)} = 13 \times \frac{5}{12}$$

$$\left[\because \frac{a^2 - b^2}{a - b} = \frac{(a + b)(a - b)}{(a - b)} = a + b; a = \left(\frac{13}{2}\right)^2 \text{ and } b = \left(\frac{13}{3}\right)^2 \right]$$

$$= \frac{65}{12} = 5\frac{5}{12}$$

$$14 \quad \frac{a^2 + ab + b^2}{a^3 - b^3} = \frac{a^2 + ab + b^2}{(a - b)(a^2 + ab + b^2)} = \frac{1}{(a - b)}$$

$$= \frac{1}{119 - 111} = \frac{1}{8}. \text{ Hence, (b).}$$

$$15 \quad \because \frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ac}$$

$$= \frac{(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)}{(a^2 + b^2 + c^2 - ab - bc - ac)}$$

$$= a + b + c$$

$\therefore$ The required answer

$$= 10 = (1.5 + 4.7 + 3.8)$$

$$16 \quad \because \frac{a^3 + b^3}{a^2 + b^2 - ab} = \frac{(a+b)(a^2 + b^2 - ab)}{(a^2 + b^2 - ab)} = (a+b)$$

Hence, the required number = $8.73 + 4.27 = 13$

$$17 \quad \because (a-b)^3 + (b-c)^3 + (c-a)^3 = 0 \Rightarrow (a-b) = 0 \Rightarrow a = b \\ \Rightarrow (b-c) = 0 \Rightarrow b = c, (c-a) = 0 \Rightarrow c = a \\ \text{Thus,} \quad a = b = c \quad \text{Hence, (c)}$$

$$18 \quad (a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ac) \\ 121 = 51 + 2(ab+bc+ac) \\ 70 = 2(ab+bc+ca) \Rightarrow (ab+bc+ca) = 35$$

$$19 \quad \frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy} = \frac{x^3}{xyz} + \frac{y^3}{xyz} + \frac{z^3}{xyz} \\ = \frac{x^3 + y^3 + z^3}{xyz} = \frac{3xyz}{xyz} = 3 \\ (\because a^3 + b^3 + c^3 = 3abc, \text{ when } a + b + c = 0)$$

$$20 \quad \left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3x \cdot \frac{1}{x} \left(x - \frac{1}{x}\right) \\ 125 = x^3 - \frac{1}{x^3} - 3(5)$$

$$\Rightarrow x^3 - \frac{1}{x^3} = 140 \quad \text{Hence, (d).}$$

$$21 \quad \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 = \frac{e^{2x} + e^{-2x} + 2}{4} - \frac{e^{2x} + e^{-2x} - 2}{4} = 1$$

$$22 \quad (1-x)(1+x)(1+x^2)(1+x^4)(1+x^8) \\ = (1-x^2)(1+x^2)(1+x^4)(1+x^8) \\ [\because (a-b)(a+b) = a^2 - b^2] \\ = (1-x^4)(1+x^4)(1+x^8) \\ = (1-x^8)(1+x^8) = (1-x^{16}) \quad \text{Hence, (c)}$$

Introductory Exercise 1.10

1 $(\sqrt{2})^2 = (2^{1/2})^2 = 2$ is a rational number.

2 $\frac{-18}{5} = -3.6$, which lies between -3 and -4 .

Hence, (b) is the correct option.

3 The same number can not be rational and irrational both, simultaneously. So (a) is wrong.

$$\text{Again} \quad (2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$$

So, (b) is wrong, as irrational + irrational = Rational.

$$\text{Once again,} \quad -3 < 2 \Rightarrow 9 > 4$$

For more information go back to the basics.

4 For example $\frac{3}{2} = 1.5$ which is not an integer, while 3, 2 are integers. Hence, (d).

5 Apply the properties of Rational and Irrational numbers. Even though 2 is a rational number and $\sqrt{3}$ is an irrational number, but $2 + \sqrt{3}$ and $2\sqrt{3}$ are irrational numbers. Therefore, choice (b) is wrong.

Similarly, we can prove that choice (c) and choice (d) are incorrect.

Hence choice (a) is the correct one.

$$6 \quad \frac{-24}{20} = \frac{x}{25} \Rightarrow x = -30$$

Hence, the equivalent rational number is $\frac{-30}{25}$.

7 Options (a) and (b) are wrong since when $c > a$ and $d > b$ still $\frac{a}{b} = \frac{c}{d}$ it means $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent rational numbers and $\frac{a}{b}$ is in standard form. Thus, $\frac{c}{d} = \frac{ka}{kb}$; $k > 1$

Hence, (c) is the correct answer.

8 A rational number $\frac{p}{q}$ can be either a terminating decimal or

a recurring decimal only, where p and q both are integers and $q \neq 0$. Further, a rational number $\frac{p}{q}$ can be a

terminating decimal number only when $q = 2^a 5^b$, where a and b are whole numbers. That is q is either 1 or 2 or 5 or any other composite number made up of 2 and 5 only.

Thus q cannot have any prime factor other than 2 and 5. So, the possible values of q are as following.

$\{1\}, \{2, 4, 8, 16, 32, 64\}, \{5, 25\}, \{10, 20, 40, 50, 80\}$

Therefore, we have total 14 values of q in the set of first 81 natural numbers such that every rational number $\frac{p}{q}$ is a

terminating decimal number.

Now, since 14 values of q form the terminating decimal numbers, so the remaining 67 ($= 81 - 14$) values of q will form the recurring decimal numbers.

Hence choice (c) is the correct one.

9 As you know that for a terminating decimal number numerator p can have any 81 natural numbers, but since p and q are co-prime so you have to figure it out carefully, as discussed below.

Case 1 When $q = 1$, p can have all the 81 values, as p and q are co-prime for all the 81 values of p .

Case 2 When q is an even number, p cannot have any even number. So for each even number q , there are only 41 acceptable values of p ; all the odd numbers 1, 3, 5, ..., 81.

As there are 6 even numbers (q) and for each such number there are 41 valid numbers, then the total required numbers = $6 \times 41 = 246$.

Case 3 When q is 5 or exponent of 5, p cannot have any number which is a multiple of 5. Since there are total 16 numbers, from 1 to 81, which are multiple of 5 so the acceptable values of $p = 81 - 16 = 65$

As, in this case, there are 2 values of q and for each such value there are 65 valid values of p , so the total required numbers $= 2 \times 65 = 130$

Case 4 When q is 10 or a multiple of 10, p cannot have any number which is a multiple of 2 or a multiple of 5. Since there are total $48 (= 40 + 16 - 8)$ numbers which are multiple of either 2 or 5, so the acceptable values of $p = 81 - 48 = 33$.

As, in this case, there are 5 values of q and for each such value there are 33 valid values of p , so the total required numbers $= 5 \times 33 = 165$. Thus the total number of rational numbers which are the terminating decimals $= 81 + 246 + 130 + 165 = 622$

Hence choice (a) is the correct one.

- 10** Since the terminating decimal numbers lie between 0 and 1, then $0 < p < q$, for every possible value of q . Further, as you know that, there are only 14 values of q in the set of first 81 numbers that can make $\frac{p}{q}$ a terminating decimal

number. Therefore, we can find the corresponding value of p for every acceptable value of q , as shown in the table.

q	The number of values of p which are co prime to q is
1	0
2	1
4	2
8	4
16	8
32	16
64	32

Introductory Exercise 1.11

- 1** Go through option

$$5 + 3 = 8 \text{ and } 53 - 35 = 18$$

Hence, option (c) is correct.

- 2** Go through options : $8 + 3 = 11$

and $83 - 38 = 45$ Hence, (d).

Alternatively $x + y = 11$

and $9(x - y) = 45$

$\Rightarrow x - y = 5$

Thus, $8 - 3 = 5$, hence 83.

3 $(10x + y) = k(x + y)$... (i)

Again $(10y - x) = l(x + y)$... (ii)

q	The number of values of p which are co prime to q is
5	4
25	20
10	4
20	8
40	16
50	20
80	32

Thus there are total 167 required rational numbers.
Hence choice (a) is the correct one.

- 11** $A = 3\overline{abbcabbcbcc} \dots$

$$A = 3\overline{abbc}$$

$$A = 3 + \frac{abbc}{9999}$$

$$A = 3 + \frac{abbc}{9999}$$

$$A = \frac{(29997 + abbc)}{9999}$$

Since, number of pigs $= A \times$ number of birds. The number of pigs must be in integer and it is possible only when the number of birds will be divisible by 9999.

Hence choice (d) is the correct one.

Alternatively $A = 3.\overline{abbcabbcbcc} \dots$

$$10000A = 3\overline{abbc.abbcabbcbcc}$$

$$(10000A) - (A)$$

$$= (3\overline{abbc.abbcabbcbcc} \dots) - (3.\overline{abbcabbcbcc} \dots)$$

$$9999A = 3\overline{abbc} - 3$$

$$A = \frac{3\overline{abbc} - 3}{9999}$$

Since, number of pigs $= A \times$ number of birds.

The number of pigs must be in integer and it is possible only when the number of birds will be divisible by 9999.

Now, adding equation (i) and (ii)

$$11(x + y) = (k + l)(x + y)$$

$$\Rightarrow k + l = 11 \Rightarrow l = (11 - k) \text{ Hence, (c).}$$

- 4** Going through options

Option (a) is invalid, since $5 > 2$

Try option (c) : $2 \times 40^1 + 5 \times 40^0 = 80 + 5 = 85$

Hence (c) is the correct option.

Alternatively $(25)_x = (85)_{10}$

$$\Rightarrow 2 \times x^1 + 5 \times x^0 = (85)_{10}$$

$$\Rightarrow 2x + 5 = 85 \Rightarrow 2x = 80 \Rightarrow x = 40$$

Hence (c) is the correct option.

5 Obviously option (b) is the required answer second line.
Since base must be greater than any integer of the number first line.

6 Going through option we find that option (b) is the invalid option. Option (d) is also wrong since $(53)_6 = 33$ which is not a prime number and option (c) is also wrong since $(53)_{12} = 60 + 3 = 63$ which is not a prime number.
Hence option (a) is the correct one. You can verify by assuming the value of n .

7 Since the last three digits in the given number are divisible by $2^3 (= 8)$. Hence the equivalent binary digits will be 000.

☞ Alternatively Divide the given number successively by 2 and get the last three digits as

2	365247728	
2	182623864	→ 0
2	91311932	→ 0 Remainders
	45655966	→ 0

Thus the last three digits in the binary representation will be 000.

8 Since last three digits (i.e., 956) are not divisible by $2^3 (= 8)$. Hence all the last three digits will not be zero. Now since 956 is divisible by 4. Hence the last two digits will be 00. Thus only option (b) is correct.

☞ NOTE Option (a) is invalid.

9

2	8009	
2	4000	→ 1
2	2000	→ 0 Remainders
2	1000	→ 0
	500	→ 0

Hence the last four digits in the binary representation of the given number are 0001.

Hence (a) is the correct option.

10 Go through options :

Let us assume option (c), then

2	8	
2	4	→ 0
2	2	→ 0
2	1	→ 0
	0	→ 1

Since $(1000)_2$ does not satisfy the given condition hence presumed option is wrong.

Now, consider option (d).

2	9	
2	4	→ 1
2	2	→ 0
2	1	→ 0
	0	→ 1

Thus $(9)_{10} = (1001)_2$, which satisfies all the given conditions.

Hence (d) is the correct option.

11 Let the tens digit and unit digit of the original number be a, b , then the original number $= 10a + b$

Again if the digits are reversed then the new number $= 10b + a$

So $(10a + b) - (10b + a) = 9(a - b)$

Now since it is given that $a + b = 9$ and $9(a - b) = 63$

So solving for $(a + b)$ and $(a - b)$, we get $a = 8$ and $b = 1$

It means the given two digit number is 81 and its reverse is 18.

Further $(81)_x = 5(18)_x$

$\Rightarrow 8x + 1 = 5[x + 8] \Rightarrow 8x + 1 = 5x + 40$

$\Rightarrow 3x = 39$

$x = 13$

Thus the value of $x = 13$ and hence option (b) is correct.

12

$$(25)_n \times (31)_n = (1015)_n$$

$$\Rightarrow (2n + 5)(3n + 1) = n^3 + n + 5$$

$$\Rightarrow 6n^2 + 17n + 5 = n^3 + n + 5$$

$$\Rightarrow n^3 - 6n^2 - 16n = 0$$

$$\Rightarrow n(n^2 - 6n - 16) = 0$$

$$\Rightarrow n = 0 \text{ or } n^2 - 6n - 16 = 0$$

$$\text{or } n^2 - 6n - 16 = 0$$

$$\Rightarrow (n - 8)(n + 2) = 0$$

$$\Rightarrow n = 8 \text{ or } n = -2$$

So the only admissible value of $n = 8$

$$\text{No. } (13)_8 \times (52)_8 = (11)_{10} \times (42)_{10} = (462)_{10} = (716)_8$$

Hence option (c) is correct.

$$\begin{aligned} 13 \quad (100000 \dots 001)_{16} &= 1 \times (16)^{24} + \underbrace{0 + 0 + \dots}_{23 \text{ zeros}} + 1 \times 16^0 \\ &= 16^{24} + 1 = (2^4)^{24} + 1 = 2^{96} + 1 \end{aligned}$$

14 Since 2^{111} is divisible by 2^3 (i.e., 8) hence the unit digit in octal system will be 0.

15 Option (b) is correct.

✓ Introductory Exercise 1.12

$$1 \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{5}\right) \dots \left(1 - \frac{1}{n}\right) \\ = \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \dots \times \frac{n-1}{n} = \frac{2}{n}$$

Hence, (b) is the correct option.

- 2 Let the three consecutive integers be $(n-1)$, n and $(n+1)$ then

$$(n-1) + n + (n+1) = 21 \\ \Rightarrow 3n = 21 \Rightarrow n = 7 \\ \therefore (n-1) + n = (7-1) + 7 = 13$$

3 $1^2 + 2^2 + \dots + 10^2 = \frac{10 \times 11 \times 21}{6} = 385$

$$\left[\because 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \right]$$

- 4 The arithmetic mean of first ' n ' odd natural number $= \frac{n^2}{n} = n$

Thus, the required answer is 50. Hence, (a) is correct.

5 Average area $= \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n}$

$$= \frac{\left(\frac{n(n+1)(2n+1)}{6} \right)}{n} \\ = \frac{(n+1)(2n+1)}{6} = \frac{(n+1)}{2} \cdot \frac{(2n+1)}{3}$$

Hence, (b) is the correct option.

- 6 Let a , l , d and n denote the first term, last term common difference and number of terms in an Arithmetic Progression, then $l = a + (n-1)d$

$$\Rightarrow 66 = 6 + (n-1)d$$

$$\Rightarrow (n-1)d = 60$$

$$\Rightarrow (n-1) = \frac{60}{d}$$

It is obvious that d must be the factor of 60, so we have the following values of d and n .

d	$n-1$	n
1	60	59
2	30	59
3	20	19
4	15	14
5	12	11
6	10	9
10	6	5
12	5	4
15	4	3
20	3	2
30	2	1
60	1	0

Since n cannot be less than 3, so out of 12 values of d only 9 values are possible.

Hence choice (c) is the correct one.

- 7 G. M. of first 27 terms = G. M. of 1st term and 27th term = G. M. of 2nd term and 26th term = ... = G. M. of 9th term and the 19th term = ...

Therefore, G. M. of first 27 terms = G. M. of 9th and 19th term $= \sqrt{12 \times 18} = 6\sqrt{6}$

- 8 AM of $a x_1 + b$, $a x_2 + b$, $a x_3 + b$, ..., $a x_n + b$

$$= \frac{(a x_1 + b) + (a x_2 + b) + (a x_3 + b) + \dots + (a x_n + b)}{n}$$

$$= \frac{(a x_1 + a x_2 + a x_3 + \dots + a x_n) + (b + b + b + \dots + b)}{n}$$

$$= \frac{a(x_1 + x_2 + \dots + x_n) + nb}{n}$$

$$= \frac{a(x_1 + x_2 + x_3 + \dots + x_n)}{n} + \frac{nb}{n} = a\bar{x} + b$$

- 9 Since all the numbers in the given series are of the form $4n + 2$, $n \in N$.

It means the number after division by 4 leaves remainder 2. So, the possible value is 338.

Hence, (b) is the correct option.

✓ Level 01 Basic Level Exercise

$$\begin{aligned}
 1 \quad & 5 \times (3, 13, 23, 33, 43, 53, 63, 73, 83, 93) \\
 & + 5 \times (30, 31, 32, 33, 34, 35, 36, 37, 38, 39) \\
 & + (301, 302, 303, 304, \dots, 399) \\
 & = 5 \times 10 + 5 \times 10 + 100 = 200
 \end{aligned}$$

Explanation In every set of 100 numbers there are 10 numbers whose unit digit is 3. Similarly in every set of 100 numbers there are 10 numbers whose tens digit is 3 and there are total 100 numbers whose hundreds digit is 3.

$$2 \quad 1146600 = 2^3 \times 3^2 \times 5^2 \times 7^2 \times 13$$

Thus total number of factors of

$$\begin{aligned}
 1146600 &= (3+1)(2+1)(2+1)(1+1) \\
 &= 4 \times 3 \times 3 \times 2 = 216
 \end{aligned}$$

So the number of ways in which 1146600 can be expressed as a product of two factors = $\frac{216}{2} = 108$

(for more discussion go back to the Basics)

- 3 The numbers which are divisible by 5 & 9 both means these numbers must be divisible by 45. So the total numbers which are divisible by 45 = 22 [45, 90, 135, ..., 990]

[i.e. all the numbers of the form $45k$, where $k = 1, 2, 3, \dots, 22$]

Now since these obtained numbers will not be divisible by 7, so the numbers of the form $45 \times 7l$ must be subtracted from these 22 numbers.

Thus the required number = $22 - 3 = 19$

($\because k = 7l$). Thus the possible values of $l = 1, 2, 3$ for $k = 1, 2, \dots, 22$

$$4 \quad \because |x| + |y| \leq 0 \text{ for } x, y \in R$$

$$\Rightarrow |x| = 0 \text{ and } |y| = 0$$

$$\Rightarrow x = 0 \text{ and } y = 0$$

So there is only one solution for the expression

i.e. $(x, y) = (0, 0)$

$$\begin{aligned}
 5 \quad & \frac{2^{39}}{39} \rightarrow \frac{(2^6)^6 \times 2^3}{39} \rightarrow \frac{64^6 \times 8}{39} \rightarrow \frac{25^6 \times 8}{39} \rightarrow \\
 & \frac{(625)^3 \times 8}{39} \rightarrow \frac{(1)^3 \times 8}{39} \rightarrow \frac{8}{39}
 \end{aligned}$$

Thus the remainder is 8.

(Since the final remainder can be obtained by dividing the factors of the given number). For more information GO BACK TO THE BASICS.

- 6 The unit digit of $(1!)^{99}$ is 1
 The unit digit of $(2!)^{98}$ is 4
 The unit digit of $(3!)^{97}$ is 6
 The unit digit of $(4!)^{96}$ is 6
 The unit digit of $(5!)^{95}$ is 0

Now since we know that all the factorial numbers starting with 5! has its unit digits 0. So we need not calculate it.

Thus the required unit digit

= unit digit of the sum of the unit digits

Thus the unit digit = 7 ($\because 1 + 4 + 6 + 6 + 0 = 17$)

- 7 The first four digit number which is divisible by 7 is 1001 and the last four digit number is 9996 and such total numbers are = $\frac{8995}{7} + 1 = 1286$

$$\begin{aligned}
 \text{So the sum of all these numbers} &= \left(\frac{9946 + 1001}{2} \right) \times 1286 \\
 &= 7071071
 \end{aligned}$$

[Since all these numbers are in A.P. with common difference 7]

- 8 Let the fraction be $\frac{x}{y}$ then,

$$\frac{x+2}{2y} = \frac{1}{2}$$

$$\Rightarrow \frac{x+2}{y} = 1$$

$$\Rightarrow x+2 = y$$

$$\Rightarrow y - x = 2$$

Thus any even number greater than 3 can be the sum of x and y . Where x and y are some integers where $y - x = 2$

Further we know that the even - even = even

or odd - odd = even

Again even + even = even

and odd + odd = even

Thus in all the cases $x + y$ must be an even number.

$$9 \quad pq = p - q + 9$$

$$\Rightarrow pq + q = p + 9$$

$$\Rightarrow q(p+1) = p+9$$

$$\Rightarrow q = \left(\frac{p+9}{p+1} \right)$$

Now since p is an integer such that q is a fraction.

So go through options and you will find that at $p = 7q$ is not a fraction. Hence (d) is the required answer

$$\because q = \frac{7+9}{7+1} = 2, \text{ which is inadmissible.}$$

- 10 Option (b) is clearly invalid. Now since the difference between these numbers = $792 = 99 \times 8$
 Hence the difference between unit digit and hundreds digit must be 8. Thus option (d) is the right choice.
 As $931 - 139 = 792$ and 1, 3, 9 are in G.P. also.

☞ Alternatively

$$(100a + 10ar + ar^2) - (100ar^2 + 10ar + a) = 792$$

$$\Rightarrow 100a(1 - r^2) + a(r^2 - 1) = 792$$

$$\Rightarrow 99a(1 - r^2) = 792 \Rightarrow a(1 - r^2) = 8$$

$$\text{But since } 8 = 1 \times 8 = 2 \times 4 = 4 \times 2 = 8 \times 1$$

$$\text{So at } a = 1, (1 - r^2) = 8$$

$$\Rightarrow r^2 = 9 \Rightarrow r = \pm 3$$

Therefore the digits are 1, 3 and 9.

(Note at $a = 2, 4$ and 8 , r^2 is not a perfect square, thus inadmissible)

- 11** The possible values can be obtained by substituting

$$k = 4, 6, 8, 10, \dots, 102$$

$$\text{Thus the number of required values} = \left(\frac{102 - 4}{2} \right) + 1 = 50$$

- 12** Here $k = 3, 5, 7, \dots, 103$

$$\text{Thus the number of required values} = \left(\frac{103 - 3}{2} \right) + 1 = 51$$

- 13** $\because a^2 - b^2 = (a + b)(a - b)$

Let us consider two odd prime numbers as 3 and 29.

$$\text{Then, } (29)^2 - 3^2 = (29 + 3)(29 - 3)$$

$$= 32 \times 26, \text{ which is divisible by 13.}$$

Again consider 7 and 29, then

$$(29)^2 - 7^2 = (29 + 7)(29 - 7)$$

$$= 36 \times 22, \text{ which is divisible by 11.}$$

Further consider 3 and 37, then

$$(37)^2 - (3)^2 = 40 \times 34, \text{ which is divisible by 17.}$$

Hence (d) is correct.

NOTE As we know odd + odd = even = odd - odd. Thus consider such two numbers whose difference must be even multiple of the required divisor. As

$$29 - 3 = 26 = 13 \times 2$$

$$29 - 7 = 22 = 11 \times 2$$

$$37 - 3 = 34 = 17 \times 2$$

- 14** Consider the following approach:

P	Q
101^{100}	100^{101}
101^{100}	$100^{100} \times 100$
$\frac{101^{100}}{100^{100}}$	$\frac{100^{100} \times 100}{100^{100}}$
1.01^{100}	100

Since, $1.01^{100} < 100$, therefore $P < Q$. Hence, choice (b) is the correct one

- 15** $k^2 - 25$ is an odd number

$$\Rightarrow k^2 \text{ must be even number} \Rightarrow k \text{ must be even}$$

Thus both (a) and (b) are even numbers so (d) is correct as (even)^{odd/even} $\rightarrow$ always even number and odd $\times$ even $\rightarrow$ even

- 16** $(a + 1)(b - 1) = 625$

$$\text{But } 625 = 1 \times 625 = (a + 1) \times (b - 1) \Rightarrow a = 0, b = 626$$

$$= 5 \times 125 = (a + 1) \times (b - 1) \Rightarrow a = 4, b = 126$$

$$= 25 \times 25 = (a + 1) \times (b - 1) \Rightarrow a = 24, b = 26$$

$$= 125 \times 5 = (a + 1) \times (b - 1) \Rightarrow a = 124, b = 6$$

$$= 625 \times 1 = (a + 1) \times (b - 1) \Rightarrow a = 624, b = 2$$

Thus $(a + b)$ is always equal to or greater than 50. Since the min $(a + b) = 50 = (24 + 26)$

$$\text{☞ Alternatively } (a + 1)(b - 1) = 625$$

$$ab + b - a - 1 = 625$$

$$\Rightarrow b(a + 1) - a = 626$$

$$\Rightarrow b(a + 1) = a + 626$$

$$\Rightarrow b = \frac{(a + 626)}{(a + 1)} = \frac{(a + 1) + 625}{(a + 1)} = \frac{625}{(a + 1)} + 1$$

Let us consider $a = 4$ then $b = 126$. Thus the only suitable option is (b) since option (c) and (d) are wrong.

And at $a = 24, b = 26 \Rightarrow a + b = 50$, which is least possible answer because in this case when the difference between a and b is the least then $(a + b)$ is also the least.

- 17** Consider the following approach

Value of P	Value of Q
$0 < p < 1$	$p + \frac{1}{p} > 2$
$p = 1$	$p + \frac{1}{p} = 2$
$p > 1$	$p + \frac{1}{p} > 2$

Therefore, if $p > 0, q \geq 2$.

Hence, choice (c) is the correct one.

- 18** $a^b = b^a; (a \neq b) > 1$

$$\text{Let us consider } a = 2 \text{ and } b = 4 \text{ then } 2^4 = 4^2$$

$$16 = 16$$

Hence (b).

NOTE $a + b = 5$ is not admissible since $2^3 \neq 3^2$

and $a + b = 7$ is also not possible since

$$2^5 \neq 5^2 \text{ and } 3^4 \neq 4^3; \text{ where } a \text{ and } b \text{ are greater than 1.}$$

- 19** $m^n - n^m = m + n$

$$\text{Consider } m = 2 \text{ and } n = 5, \text{ then } 2^5 - 5^2 = 5 + 2, 7 = 7$$

Thus options (a) and (b) are wrong and option (c) is correct.

- 20** Since 334 or 433 are not the perfect cubes hence (a) is wrong. Again 792, 279, 297, 972 or 927 are not the perfect cubes hence (b) is also wrong.

Further 512 is also a perfect cube, hence (c) is the correct answer.

☞ Alternatively See the perfect cubes of three digits as $5^3 = 125$, $6^3 = 216$, $7^3 = 343$, $8^3 = 512$, $9^3 = 729$

So the digits of $5^3 = 125$ and $8^3 = 512$ are same.

Hence (c) is the correct answer.

- 21** $n^4 - n^2 = n^2(n^2 - 1) = n^2(n + 1)(n - 1)$

Thus at $n = 3$ the given expression is divisible by 8, 4 and 12.

Hence it is divisible by all of a, b, c .

- 22** Since $(11)^4 = 14641$, hence b will always be less than 4 because 11 is the least two digit appropriate number but 14641 is a 5 digit number. Also $b \neq 1$, hence b lies between 1 and 4 i.e. b is equal to either 2 or 3.

Now try for suitable values of (aa) and b , we find that the only possible relation is

$$(11)^3 = 1331$$

Thus $1^3 \cdot 3^1 + 3^1 \cdot 1^3 = 6$

Hence (b) is the correct option.

- 23** $\frac{1}{6} : \frac{1}{5} : \frac{1}{3} : \frac{1}{2} = \frac{30}{6} : \frac{30}{5} : \frac{30}{3} : \frac{30}{2} = 5 : 6 : 10 : 15$

Therefore min. number of processors

$$= (5 + 6 + 10 + 15) = 36$$

Hence (d) is correct.

- 24** $(392)^n - (392)^{n-1} = (392)^{n-1} [392 - 1]$
 $= 392^{(n-1)}(391)$

Now since 13 is not the factor of either 391 or 392.

Hence (c) is the required answer.

- 25** Let the fair (i.e., charge) per km be ₹ x , then the total distance travelled by Mr. Chaalu is $(x + 1)$.

Hence the total expenses = $x \cdot (x + 1)$

but $x(x + 1) \leq 350$

Thus the greatest possible value of $x = 18$

Hence $18 \times 19 = 342$

Thus he is left with ₹ 8 only.

- 26** The numbers of times he pressed the keys for one digit number = 9

The number of times he pressed the keys for two digit numbers = $90 \times 2 = 180$

The numbers of times he pressed the keys for three digit numbers = $900 \times 3 = 2700$

The number of times he pressed the keys for the numbers 1000 to 1999 is $1000 \times 4 = 4000$

Then total number of times the keys are pressed = 6889

$$\begin{aligned} 27 \quad \frac{(20)^{23}}{17} &\rightarrow \frac{(3)^{23}}{17} \rightarrow \frac{(3^3)^7 \times 3^2}{17} \rightarrow \frac{(27)^7 \times 9}{17} \rightarrow \frac{(10)^7 \times 9}{17} \\ &\rightarrow \frac{(10^2)^3 \times 90}{17} \rightarrow \frac{(15)^3 \times 5}{17} \rightarrow \frac{225 \times 75}{17} \rightarrow \frac{4 \times 7}{17} \rightarrow 11 \end{aligned}$$

Hence the required remainder is 11.

- 28** Check for all the prime numbers between 3 and 50 you will find that option (d) is correct since all such numbers are always divisible by 8, 12 and 24.

Because $p^2 - 1 = (p + 1)(p - 1)$.

- 29** Since the unit digit of prime numbers = 1, 2, 3, 5, 7, 9

So the unit digit of the squares of prime numbers

$$= 1, 4, 9, 5, 9, 1$$

Thus $1 + 1 = 2$, $4 + 1 = 5$, $9 + 1 = 10$, $5 + 1 = 6$, $9 + 1 = 10$ and $1 + 1 = 2$

Hence the possible unit digits are 2, 5, 6, 0.

Thus 3, 7 or 9 are not possible hence (d) is correct.

- 30** The required number = Number of number, which are (divisible by 3 + divisible by 7 - divisible by 21)
 Number of numbers which are divisible by 3

$$= \left(\frac{198 - 3}{3} \right) + 1 = 66$$

Number of numbers which are divisible by 7

$$= \left(\frac{196 - 7}{7} \right) + 1 = 28$$

Number of numbers which are divisible by 21

$$= \left(\frac{189 - 21}{21} \right) + 1 = 9$$

Thus the number of divisible numbers = $(66 + 28 - 9) = 85$

Hence the number of numbers which are not divisible

$$= 200 - 85 = 115$$

Hence (a) is the correct answer.

- 31** Only one value of P is possible, which is 2. At $P = 2$, $P^2 + 3 = 7$, which is also prime.

Again at $P = 3, 5, 7, 11, \dots$

$P^2 + 3 =$ an even number which can not be a prime number.

- 32** Let there be x bangles at each side, then the total number of bangles he had = $x^2 + 38$

If he increases the size of the square by one unit at each side, then the total number of bangles

$$= (x + 1)^2 - 25$$

Thus $x^2 + 38 = (x + 1)^2 - 25$

$$x^2 + 38 = x^2 + 1 + 2x - 25$$

$$\Rightarrow 2x = 62 \Rightarrow x = 31$$

Thus total number of bangles = $x^2 + 38 = 961 + 38 = 999$

☞ Alternatively Go through options. Consider a suitable value from the options and check that in each case it must produce a perfect square number.

As $999 - 38 = 961$ is a perfect square

and $999 + 25 = 1024$ is also a perfect square.

33 Let $a = 20$ and $b = 15$

$a + b = 35$, which is not divisible by 10.

34 At $a = 7$ and $b = 2$, the following number becomes 755238792, which is divisible by both 7 and 2.

But the other values of (a, b) are also possible. Consider $a = 4$ and $b = 2$, $a = 8$, $b = 2$ etc.

So there is no any unique value of a and b . Hence (d).

35. Any number of the form “ $abcabc$ ” must be divisible by 1001 or its factors (i.e. $1001 = 7 \times 13 \times 11$).

Therefore, (a) and (b) are correct. Hence (d) is the most appropriate statement.

36 Let suppose $a \rightarrow$ odd, and $b \rightarrow$ even

then $\text{odd}^{\text{even}} \times \text{even}^{\text{odd}} = \text{odd} \times \text{even} = \text{even}$

Even if a is even and b is odd, the result is an even number.

37 Let there be n rows, then the number of trees in each row $= (n + 1)$

Thus total number of trees $= n(n + 1)$

$\therefore n(n + 1) = 5550$

Now, at this moment this problem can be solved in two ways. First by finding the roots of quadratic equation. Second by using the values from options.

Again since the value of $(n + 1)$ is given in the options so consider option (d)

$$74(75) = 5550.$$

Hence (d) is correct.

38 $3^a = 9^b$

$$\Rightarrow a = 2b$$

$$\text{and } 4^{(a+b+2)b} = 16^{ab}$$

$$\Rightarrow (a + b + 2)b = 2ab$$

$$\Rightarrow a + b + 2 = 2a$$

$$\Rightarrow a - b = 2$$

$$\Rightarrow b = 2 \quad (\because a = 2b)$$

$$\therefore a = 4$$

39. These numbers are 11, 13, 31, 17, 71, 37, 73, 79, 97.

40. Since he has covered twice the distance which he is yet to cover. It means he has covered $\frac{2}{3}$ of the whole journey and remaining journey is $\frac{1}{3}$ rd.

41. Let the height of my son increases x cm per year then the height at the end of 2002 $= 6x + 90$

and the height at the end of 2003 $= 7x + 90$

$$\text{but } 7x + 90 = (6x + 90) + (6x + 90) \frac{1}{9}$$

$$\Rightarrow (7x + 90) = (6x + 90) \times \frac{10}{9}$$

$$\Rightarrow 63x + 810 = 60x + 900$$

$$\Rightarrow 3x = 90$$

$$x = 30 \text{ cm}$$

Thus the height at the end of 2008 $= 90 + 12x$

$$= 90 + 12 \times 30 = 450 \text{ cm}$$

42. Let the cost of watch be ₹ x

Then the total amount for 12 months except ration $= 250 + x$

Therefore the amount for 9 months $= (250 + x) \times \frac{3}{4}$

$$\text{Thus } (250 + x) \frac{3}{4} = 270$$

$$\Rightarrow 750 + 3x = 1080$$

$$\Rightarrow 3x = 330$$

$$\Rightarrow x = 110$$

Therefore the cost of wrist watch $= 110$

(It can also be solved using options.)

$$\mathbf{43.} \quad 1^2 + 3^2 = 10$$

$$13 + 18 = 31$$

44. When two “two digit” numbers are added and the resultant value is a ‘three digit number’, it means there must be a carry over (i.e. the sum of the unit digits be greater than 9. Similarly the sum of the tens digit is also greater than 9.)

Hence (d) is correct as $72 + 27 = 99$ is inadmissible, but $64 + 46 = 110$ is an admissible value.

45. ‘A’ grade student $= 57.\overline{142857}\%$

$$= \left[57 + \frac{1}{7} \right] \%$$

$$= \frac{400}{7} \% = \frac{4}{7} \text{ of the total student}$$

‘B’ grade student $= 26.444\%$

$$= \left[26 + \frac{4}{9} \right] \% = \frac{238}{9} \%$$

$$= \frac{238}{900} \text{ of the total student}$$

Now the total students of ‘A’ and ‘B’ grade

$$= \frac{4}{7} + \frac{238}{900} = \frac{3600 + 1666}{6300} = \frac{5266}{6300}$$

$$= \frac{2633}{3150} \text{ of the total student}$$

∴ The number of students who achieved the grade 'C'

$$= 1 - \frac{2633}{3150} = 517 \text{ students}$$

Since the maximum number of students are below 3500.
So there cannot be the number of students in the multiples of 3150.

$$46 \quad 3 \overline{)16(5} \\ \underline{15} \\ 1$$

Remainder is odd.

$$47 \quad 551 + 552 + 553 + \dots + 559 + 560 \\ = (500 + 500 + 500 \dots 10 \text{ times}) \\ \quad \quad \quad + (51 + 52 + 53 + \dots + 60) \\ = 5000 + (50 + 50 + 50 \dots 10 \text{ times}) \\ \quad \quad \quad + (1 + 2 + 3 + \dots + 10) \\ = 5000 + 500 + 55 = 5555$$

$$48. \quad \quad \quad x^2 + x = x^3 - x \\ \Rightarrow \quad \quad \quad x^3 - x^2 - 2x = 0 \\ \Rightarrow \quad \quad \quad x(x^2 - x - 2) = 0 \\ \Rightarrow \quad \quad \quad x = 0 \quad \text{or} \quad x^2 - x - 2 = 0 \\ \text{If} \quad \quad \quad x^2 - x - 2 = 0 \\ \Rightarrow \quad \quad \quad (x - 2)(x + 1) = 0 \\ \Rightarrow \quad \quad \quad x = 2 \quad \text{or} \quad x = -1 \\ \text{Thus } x = 2 \text{ is the possible value of } x. \\ \text{Hence (b).}$$

☞ **Alternatively** Go through options
 $2^2 + 2 = 2^3 - 2$
 $6 = 6$

49 As we have seen in the above problem that the equation is a 3 degree equation. Hence there are 3 solutions for x , viz. $x = 0, 2$ and -1 .

50 The required number of digits written as the thousands digit = 101
 The required number of digits written as the hundreds digit = 99
 The required number of digits written as the tens digit = 10
 The required number of digits written as the unit digit = 10
 Thus total number of 3-digits = 220

$$51. \quad (2^{19} + 1) = (2^{19} + 1^{19}) \\ \text{Now, } 2^{19} + 1^{19} \text{ is divisible by } 3 (= 2 + 1) \\ \text{Since } (a^n + b^n) \text{ is always divisible by } (a + b), \text{ when 'n' is odd.} \\ \text{Again since } (2^{19} + 1) \text{ is an odd number, hence no any even number can divide it. Hence, (a) is the correct option.}$$

52. Except 467, all the remaining numbers are divisible by 3.

$$53. \quad \therefore \quad 1001 = 7 \times 11 \times 13 \\ \text{Hence (b).}$$

$$54. \quad \therefore \quad 5400 = 2^3 \times 3^3 \times 5^2$$

To get a perfect square, there must be even number of powers of each prime factor.

$$\text{So the least such perfect square} = 2^3 \times 3^3 \times 5^2 \times 2 \times 3 \\ = 32400$$

Thus we have to multiply it by 6 in order to get a perfect square.

55 At $x = 0$ and 1 the valid relation is

$$p^x q^{(1-x)} = px + q(1-x)$$

Hence (b).

$$56 \quad \quad \quad 3780 - 2835 = 945 \\ \text{Now} \quad \quad \quad \frac{3780 + 2835}{945} = \frac{6615}{945} = 7$$

$$57 \quad (x+y)^{-1} \cdot (x^{-1} + y^{-1}) = \frac{1}{(x+y)} \left(\frac{1}{x} + \frac{1}{y} \right) \\ = \frac{1}{(x+y)} \left(\frac{x+y}{xy} \right) = \frac{1}{xy} = (xy)^{-1}$$

58 In every set there is only one such relation exists, as if $p = 3$, $k = 2$

$$\text{Then the remainder is 1 when } \frac{k^2}{p} = \frac{4}{3}$$

$$\text{Similarly if } p = 7 \quad \text{and} \quad k = 6$$

$$\text{Then the remainder is 1 when } \frac{k^2}{p} = \frac{36}{7} \text{ etc.}$$

Hence k is always $(p-1)$.

59 Go through options Consider the correct option (c).

$$\text{Number of absent students} = 2700 \times \frac{1}{9} = 300$$

$$\therefore \text{ Number of students appeared in exam} = 2400$$

$$\text{Number of students who passed} = 2400 \times \frac{19}{24} = 1900$$

$$\text{Thus number of failed students} = 2400 - 1900 = 500$$

Hence presumed option is correct.

$$\text{☞ Alternatively} \quad \text{Failed students} = 500 = \frac{5}{24}$$

$$\therefore \quad \text{Passed students} = \frac{19}{24} = 1900$$

Thus the number of students appeared in exam

$$= 2400 = \frac{8}{9}$$

$$\therefore \text{ Total number of students registered} = \frac{2400 \times 9}{8} = 2700$$

☞ **Alternatively** Let the x students have registered for the exam then the number of students who have appeared in exam = $x - \frac{x}{9} = \frac{8x}{9}$

Again the number of passed students $= \frac{8x}{9} \times \frac{19}{24} = \frac{19x}{27}$

Thus the number of failed students $= \frac{5x}{27} = 500$

$$\Rightarrow x = 2700$$

$$60 \quad km < kn \Rightarrow \frac{km}{kn} < 1$$

Since for any value of k the above relation is true thus we can't specify that whether for $k < 0$, or $k > 0$ this is true. Hence (d).

61 Rational + Irrational = Irrational Number

62 In first 1122 years 'Decay' becomes 40 gm
In next 1122 years 'Decay' becomes 20 gm
In next 1122 years 'Decay' becomes 10 gm
and In last 1122 years 'Decay' becomes 5 gm

63 Vinod gets 210 marks and failed by 40 marks
It means the min. marks required $= 250 = \frac{2}{5}$ th of total.
Therefore maximum (or total) marks $= 250 \times \frac{5}{2} = 625$

$$64 \quad \text{Marks in QA : Marks in English} = \frac{3\frac{1}{2}}{1} = \frac{\frac{7}{2}}{1} = \frac{7}{2}$$

Thus his marks in QA $= 70 \left(= 90 \times \frac{7}{9} \right)$

$$65 \quad \frac{72}{+27} \quad \text{and} \quad \frac{72}{-27} \\ 99 \quad \quad \quad 45$$

But we are not sure whether it is larger or smaller number so (d) is the most appropriate answer.

66 Go back to the Basics. (Refer Indices)

$$67 \quad 12 \times 63 = 756 \text{ and } 21 \times 36 = 756$$

68 The best way to consider the values from given range in options. Thus option (a) is obviously ruled out since at $p = -1$ the relation is undefined. Similarly if we consider option (d) then at $p = -\frac{1}{2}$, $\frac{p}{p-1} = -1$, which is wrong.

Thus (b) is also wrong.

Hence (c) is correct.

69 The form of the required number is $14k + 8$

but when $14k + 8$ is divided by 7, then

$$\frac{14k + 8}{7} = \frac{14k + (7 + 1)}{7} = \frac{(14k + 7) + 1}{7}$$

Thus the remainder will be 1.

 **NOTE** $14k$ is already divisible by 7 and thus $14k + 7$ will also be divisible by 7.

Remember In this type of problem just divide the existing remainder and get the required answer. As $\frac{8}{7} \rightarrow$ Remainder is 1.

70 The unit digit of $(316)^{3^{4n}}$ is always 6.

So the unit digit of $(316)^{3^{4n}} + 1$ will be 7.

71 $a + b = 18$

So maximum of $(a \times b)$ will be only when $a = b$

$$\therefore a = b = 9$$

$$\therefore \text{Maximum of } a \times b = 9 \times 9 = 81$$

72 Let the smallest possible number be x , then

$$x = 15k + 12$$

and

$$x = 8l + 5$$

$$\Rightarrow 15k + 12 = 8l + 5$$

$$\Rightarrow 15k + 7 = 8l$$

$$\Rightarrow l = \frac{15k + 7}{8},$$

l must be an integer putting $k = 1, 2, 3, \dots$ etc.

But at $k = 7$, we get a number which on being divided by 8, gives 'l' as an integer.

$$\text{So } x = 15 \times 7 + 12$$

$$\Rightarrow x = 117$$

The next higher numbers = (L.C.M. of divisors) $m + 117$

$$= (\text{L.C.M. of 15 and 8}) m + 117$$

$$= 120m + 117$$

So consider the highest possible value of m such that $120m + 117 \leq 9999$ (largest possible number of four digits)
Thus at $m = 82$, the value of $120m + 117 = 9957$, which is the required number.

 **Alternatively** Go through options.

It is very obvious from the options that the options (a), (c) and (d) are wrong because when you multiply any number by 15, it will give the unit digit either 0 or 5 and when you add 12 to this product the unit digit will be either 2 or 7. Or in other way divide the values in the given options by 8 and it must leave the remainder 5.

73 Go through alternatives. Consider option (b)

$$\text{Total mobiles} = 600$$

$$\text{Indian mobiles} = 600 \times \frac{5}{12} = 250$$

$$\text{and imported mobiles} = 600 \times \frac{7}{12} = 350$$

$$\text{The coloured mobiles of Indian origin} = 250 \times \frac{1}{5} = 50$$

$$\text{The coloured mobiles which are imported} \\ = 350 \times \frac{2}{7} = 100$$

Thus total coloured mobiles $= 50 + 100 = 150$, which is same as given in the problem. Hence the presumed option is correct.

 **Alternatively** Solve through equation.

- 74** The minimum time interval when the call centre receives the calls from all the destinations at the same time

$$= \text{the L.C.M. of } 10, 12, 20 \text{ and } 25 \\ = 300 \text{ minutes} = 5 \text{ hours}$$

Thus $5 + 5 = 10$ A.M. and $10 + 5 = 15 = 3:00$ P.M.

Hence (d).

- 75** H.C.F. of $2^4, 8^2, 16^2, 20^3 = 2^4$

$$\text{L.C.M. of } 2^4, 8^2, 16^2, 20^3 = 2^8 \times 125 = 32,000$$

- 76** Go through options. Consider option (b)

$$15192327 - 84 = 15192243$$

$$\text{Now, } (3 + 2 + 9 + 5) - (4 + 2 + 1 + 1) = 11$$

$$\text{and } 1 + 5 + 1 + 9 + 2 + 2 + 4 + 3 = 27$$

Thus 15192243 is divisible by both 11 & 9, therefore by 99.

Thus the presumed remainder 84 is correct.

- 77.** The total numbers (integers) between 1 and 200 = 198

The number of numbers divisible by 2 = 99 (2, 4, 6, ..., 198)

The number of numbers which are purely divisible by 3 but not by 2.

(Since the division by 2 is already taken into account)

$$= \frac{(195 - 3)}{6} + 1 = 33 \quad (3, 9, 15, 21, \dots, 195)$$

The number of numbers which are purely divisible by 5 but not by 2 and 3

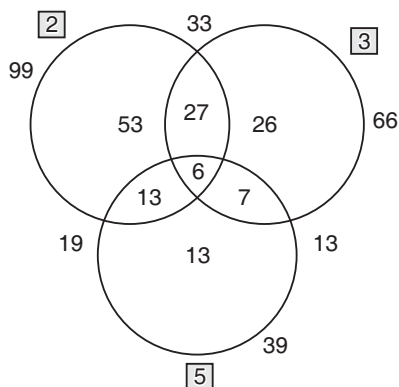
$$= \left\{ \frac{(195 - 5)}{10} + 1 \right\} - \left\{ \frac{195 - 15}{30} + 1 \right\} = 20 - 7 = 13$$

Thus the total number of numbers lying between 1 and 200, which are divisible by 2, 3 or 5 = $99 + 33 + 13 = 145$.

NOTE In III case we did not consider 10, 20, 30, ... etc. for the division of 5 because these numbers have already been divided by 2.

Alternatively So the required number

$$= (53 + 13 + 26) + (27 + 13 + 7) + 6 = 145$$



(for more clarification of the solution see the chapter of set theory or the example discussed in the theory of this same chapter.)

- 78**

$$x \Delta y \rightarrow x + y$$

$$x \nabla y \rightarrow x - y$$

$$x \nabla y \rightarrow x \times y$$

$$x \Delta y \rightarrow x \div y$$

$$4 \Delta 2 \nabla 3 \Delta 6 \Delta 12 = 4 \div 2 - 3 \times 6 + 12$$

$$= 2 - 3 \times 6 + 12 = 2 - 18 + 12 = -4$$

- 79** Go through options :

L.C.M. of 204 and 170 = 1020.

- 80** $1 - 3 + 5 - 7 + 9 - 11 + \dots + 197 - 199$

$$= (-2) + (-2) + (-2) + \dots + (-2) \text{ [50 times]}$$

$$= 50 \times (-2) = -100,$$

Hence (d).

$$\mathbf{81} \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{5}\right) \dots \left(1 - \frac{1}{n}\right)$$

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \dots \times \frac{(n-2)}{(n-1)} \times \frac{(n-1)}{n} = \frac{1}{n}$$

$$\mathbf{82} \text{ Minimum of } \frac{x}{y} = \frac{\text{Max. of } x}{\text{Min. of } y} = \frac{2}{32} = \frac{1}{16}$$

$$\text{and Maximum of } \frac{x}{y} = \frac{\text{Max. of } x}{\text{Min. of } y} = \frac{8}{16} = \frac{1}{2}$$

- 83** $56 = 1 \times 56 \Rightarrow$ H.C.F. is 1, therefore number of tiles = 56

$$2 \times 28 \Rightarrow \text{H.C.F. is 2, therefore number of tiles} = 1 \times 14 = 14$$

$$4 \times 14 \Rightarrow \text{H.C.F. is 2, therefore number of tiles} = 2 \times 7 = 14$$

$$7 \times 8 \Rightarrow \text{H.C.F. is 1, therefore number of tiles} = 7 \times 8 = 56$$

Therefore, the minimum number of tiles can be 14. Hence (c) is correct option.

$$\mathbf{84} \quad x + \frac{9x}{4} = 221 \Rightarrow x = 68 \text{ metre}$$

$$\text{and } \frac{9}{4}x = 153 \text{ metre}$$

Thus the difference between two parts

$$= 153 - 68 = 85 \text{ metre}$$

- 85** Total prime numbers between 1 and 100 = 25 }
And total prime numbers between 101 and 200 = 21 } 46

$$\mathbf{86} \quad \frac{6^{36}}{215} \rightarrow \frac{(6^3)^{12}}{215} \rightarrow \frac{(216)^{12}}{215} \rightarrow \frac{1^{12}}{215} \rightarrow 1 \text{ is the remainder.}$$

$$\mathbf{87} \quad \frac{12^{13}}{11} \rightarrow \text{Remainder is 1.}$$

$$\frac{23^{13}}{11} \rightarrow \text{Remainder is 1.}$$

$$\therefore \text{ The remainder of } \frac{(12^{13} + 23^{13})}{11} \text{ is } (1 + 1) = 2.$$

- 88** The least possible number = (L.C.M. of 4, 5, 6, and 7) + 3

$$= 420 + 3 = 423$$

$$\text{The next higher number} = (420m) + 3$$

Now put the least possible value of m such that

$$(420m + 3) \geq 1000$$

So at $m = 3$, the required value = $1260 + 3 = 1263$

☞ Alternatively Options (a) and (b) are ruled out since when we divide these numbers by 5 they do not leave the remainder 3.

Again check the option (c) instead of (d) because we are required to find out the smallest possible number.

$$\begin{aligned} 89 \quad 13^{31} &> 31^{13} \\ \frac{13^{31}}{13^{13}} &> \frac{31^{13}}{13^{13}} \\ 13^{18} &> (2.4)^{13} \end{aligned}$$

Hence it is correct.

$$\begin{aligned} \text{Again} \quad 10^{100} &< 100^{10} \\ 10^{100} &< (10^2)^{10} \\ 10^{100} &< 10^{20} \quad (\text{which is wrong}) \\ \text{Again,} \quad 2^{32} &< 32^2 \\ 2^{32} &< (2^5)^2 \\ 2^{32} &< 2^{10} \quad (\text{which is also wrong}) \end{aligned}$$

Hence (d) is the correct option.

90 Since it is not clear that which particular numbers are odd and which are even.

So we can't say about options (a), (c) and (d). Again in option (b) there are 1 more than half of the total numbers, so we can infer that there must be at least one even number.

Further we know that the product with even number is always even

$$\begin{aligned} \text{i.e.} \quad \text{even} \times \text{odd} &= \text{even} \\ \text{and} \quad \text{even} \times \text{even} &= \text{even} \end{aligned}$$

Thus option (b) is confirm.

$$\begin{aligned} 91 \quad (43 \text{ and } 387) \text{ and } (129, 301) \\ 43k + 43l &= 430 \\ \Rightarrow 43(k + l) &= 430 \\ \Rightarrow k + l &= 10 \end{aligned}$$

But k, l must be coprimes.

$$\text{So } (k, l) = (1, 9) \text{ and } (3, 7)$$

$$\begin{aligned} 92 \quad 2^{73} - 2^{72} - 2^{71} &= 2^{71} (2^2 - 2^1 - 1) \\ &= 2^{71} (4 - 2 - 1) = 2^{71} \end{aligned}$$

93 Since we know that if $a + b + c = 0$, then

$$a^3 + b^3 + c^3 = 3abc$$

$$\begin{aligned} \text{Hence } 55^3 + 17^3 - 72^3 &= 3 \times 55 \times 17 \times (-72) \\ &= -(3 \times 5 \times 11 \times 17 \times 2^3 \times 3^2) \end{aligned}$$

Thus the given number is divisible by 3, 17, 40, 11 and 15.
Hence (d) is the most appropriate answer.

$$\begin{array}{r} 94 \quad 13 \overline{)999999} \overline{)76923} \\ \underline{91} \\ 89 \\ \underline{78} \\ 119 \\ \underline{117} \\ 29 \\ \underline{26} \\ 39 \\ \underline{39} \\ \hline \end{array}$$

$$\text{Thus } abcde = 76923$$

$$\therefore a + b + c + d + e = 7 + 6 + 9 + 2 + 3 = 27$$

$$95 \quad n + (n + 2) + (n + 4) = 3n + 6 = 3(n + 2),$$

where $n = 2, 4, 6, \dots$

So it is always divisible by 12.

Hence (d).

96 Since we know that $(a^3 + b^3) = (a + b)(a^2 + ab + b^2)$, which is divisible by $(a + b)$. Therefore it leaves no any remainder.

$$97 \quad N = n(n + 1)(n + 2)(n + 3)(n + 4)$$

$$\text{at } n = 1, \quad N = 120 = 1 \times 2 \times 3 \times 4 \times 5$$

$$n = 2, \quad N = 720 = 2 \times 3 \times 4 \times 5 \times 6$$

$$n = 3, \quad N = 2520 = 3 \times 4 \times 5 \times 6 \times 7$$

$$n = 4, \quad N = 6720 = 4 \times 5 \times 6 \times 7 \times 8 \text{ etc.}$$

Therefore the possible number is 120.

$$98. \quad (x - y)^2 = x^2 + y^2 - 2xy = 25 - 24 \Rightarrow (x - y) = \pm 1$$

$$\text{Similarly } (x + y) = \pm 7$$

So, solving for x and y , we get

$$x = -3, -4, 3 \text{ and } 4$$

$$\text{and } y = -4, -3, 4 \text{ and } 3$$

$$\text{Thus } x^{-1} + y^{-1} = \frac{1}{x} + \frac{1}{y} = \frac{7}{12} \text{ and } \frac{-7}{12}$$

Hence (d).

$$99 \quad \frac{75^{75}}{37} = \frac{1^{75}}{37} \rightarrow \text{Remainder is 1.}$$

$$100 \quad \frac{p^2 + 17}{12} \rightarrow \frac{(p^2 + 5)}{12} \text{ Now since } p^2 = 12m + 1.$$

$$\text{Therefore } \frac{p^2 + 5}{12} \rightarrow \frac{(12m + 1) + 5}{12} \rightarrow \frac{12m + 6}{12}$$

Hence the remainder is 6.

101 The unit digit of $(12345k)^{72}$ is 6.

Means the unit digit of k^{72} is 6.

Now, go through options and verify

$$6^{72} \rightarrow \text{unit digit is 6}$$

$$8^{72} \rightarrow \text{unit digit is 6}$$

$$2^{72} \rightarrow \text{unit digit is 6}$$

Hence (d).

102 Go through options: If you subtract 2, 8 or 25 from 369, then you will not obtain a perfect cube number. So (b) is the correct answer.

Alternatively If you write down the nearest perfect cube such that $k \leq 369$, where k is a perfect cube, then $7^3 = 343$, which is just below 369.

$$\text{So } 369 - 343 = 26$$

103 $n(n+2) = 624$

From options, $24 \times 26 = 624$

Otherwise, find the roots of quadratic equation

$$n^2 + 2n - 624 = 0.$$

$$\begin{aligned} \text{104 } \frac{1719 \times 1715 \times 1713}{17} &\rightarrow \frac{2 \times 15 \times 13}{17} \rightarrow \frac{30 \times 13}{17} \rightarrow \frac{13 \times 13}{17} \\ &\rightarrow \frac{169}{17} \rightarrow 16, \text{ is the required remainder.} \end{aligned}$$

$$\text{105 } \frac{12.5}{1/6} = \frac{12.5 \times 6}{1} = 75$$

$$\text{106 } 32k - 23k = 315$$

$$\Rightarrow 9k = 315 \Rightarrow k = 35$$

$$\therefore \text{The original product} = 23 \times 35 = 805$$

107. Let the fraction be $\frac{x}{y}$, then

$$\left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2}$$

$$\text{then } \frac{\frac{2}{3}x^2}{\frac{1}{5}y^2} = \frac{10x^2}{3y^2}$$

$$\text{Thus } \frac{10x^2}{3y^2} = 2 \frac{x}{y}$$

$$\Rightarrow \frac{x}{y} = \frac{3}{5}$$

$$\text{Hence } x + y = 3 + 5 = 8$$

108 The required number

$$= \text{H.C.F. of } (221-104), (377-221), (377-104)$$

$$= \text{H.C.F. of } 117, 156, 273 = 39$$

$$\text{109 } (ab^2)^{1/3} = 125 = 5^3$$

$$\Rightarrow ab^2 = (5^3)^3$$

$$\Rightarrow ab^2 = 5^9$$

Now since $a \neq b \neq 1$ and $(a, b) > 1$.

Therefore a and b must be equal to some power of 5.

Again since $a > b$, so check the option (b).

$$\text{Consider } a = b^4$$

$$\therefore ab^2 = 5^9$$

$$\Rightarrow b^6 = 5^9$$

$$\Rightarrow b^2 = 5^3$$

$$\text{then } a = b^4 = 5^6$$

$$\therefore ab^2 = 5^6 \times 5^3 = 5^9, \text{ which is correct.}$$

Thus the presumed option (b) is correct.

Option (a) is clearly wrong and option (c) is also wrong.

Because $a^{3/2} = b^{2/3} \Rightarrow a^{9/6} = b^{4/6} \Rightarrow a < b$, which is inadmissible.

110 $24 = 8 \times 3$, so the given number must be divisible by both 8 & 3.

Now for 8, $n = 1, 5, 9$

Again for 3, when $n = 1$ then $m = 3, 6, 9$

and when $n = 5$ then $m = 2, 5, 8$

and when $n = 9$ then $m = 1, 4, 7$

Thus there are total 9 combinations of m and n .

111 Let the fraction be $\frac{x}{y}$, then its reciprocal be $\frac{y}{x}$,

$$\therefore \frac{\frac{x}{y}}{\frac{y}{x}} = \frac{x^2}{y^2}$$

$$\text{Now } \frac{x^2}{y^2} \times \frac{x}{y} = \frac{x^3}{y^3} = \frac{512}{125} = \frac{8^3}{5^3}$$

$$\Rightarrow \frac{x}{y} = \frac{8}{5}$$

$$\therefore \frac{y}{x} = \frac{5}{8}$$

$$\therefore \frac{x}{y} - \frac{y}{x} = \frac{8}{5} - \frac{5}{8} = \frac{64 - 25}{40} = \frac{39}{40}$$

112 Consider the values as $441 - 41 = 400$

$$\text{And, } 441 + 43 = 484$$

So, $41 + 43 = 84$, which is divisible by 4.

$$\text{113 } (11)^a = (19)^b = (209)^c = z$$

$$\Rightarrow 11 = z^{1/a}, 19 = z^{1/b}, 209 = z^{1/c}$$

$$\text{Therefore } 11 \times 19 = 209$$

$$z^{1/a} \times z^{1/b} = z^{1/c} = z^{(1/a + 1/b)} = z^{1/c}$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{1}{c}$$

$$\Rightarrow c(a+b) = ab$$

$$\text{114 } \frac{2^{54} - 1}{9} = \frac{(2^6)^9 - 1}{9} = \frac{(64)^9 - 1}{9}$$

$$\Rightarrow \frac{(1)^9 - 1}{9} \Rightarrow 0$$

Hence there will be no any remainder.

115 64 is the only number which is both a perfect square and a perfect cube.

$$\text{So } 6 + 4 = 10.$$

- 116** The only possible number is 729 which is the cube of 9 and square of 27.

So $7 \times 2 \times 9 = 126$

- 117** In order to make sure N be a least possible number $c < b < a$.

But, $c \neq 1$, since $a \neq b \neq c$.

So the least possible value of $c = 2$, then

$$c^8 = 2^8 = 256 = 4^4 = 16^2$$

Thus $a = 16$, $b = 4$ and $c = 2$ the required number is 256.

- 118** Go through options.

- 119** There are only 4 digits whose square also gives the same unit digit which are 0, 1, 5 and 6.

But the number whose unit digit is 0 (zero), this number also makes the tens digit zero after squaring. Again it means the tens digit of the original number be zero. Now if we square such a number whose tens and unit digits are zero, this number gives 4 zeros (at least) on the end of the square which is not possible since 000 can not be considered a relevant number. Thus zero cannot be the unit digit. Now, if we consider unit digit 1 and any digit except zero.

NOTE Zero shows the same behaviour as in the first case), then unit digit remains the same on being squared the given number but tens digit gets changed so, 1 as a unit digit is also impossible.

Now if we consider 6 as probable unit digit, similar behaviour is seen with 6 also.

Thus, if we consider 5 as a unit we always get 5 as a unit digit after squaring any number whose unit digit is 5. Also we always get the 2 as the tens digit when we square any number whose unit digit is 5. Now to maintain the tens digit '2' we consider a number whose tens and unit digits be respectively 2, 5. Now if we square to 25, we get a three digit number 625. It means if it is possible, then the three digit number must be 625.

Now, $(625)^2 = 390625$,

which satisfies the given conditions.

Now $3 \times 9 \times 0 \times 6 \times 2 \times 5 = 0$

Hence (a) is correct.

- 120.** $3^{6n} + 4^{6n} \rightarrow 9^{3n} + 16^{3n}$

Now $9^{3n} + 16^{3n}$ is divisible by 25 ($= 9 + 16$)

Since $(a^n + b^n)$ is divisible by $(a + b)$ only when 'n' is odd.

Hence n must be odd.

121.
$$\begin{array}{r} 5 \overline{) 100} \\ \underline{5 \ 20 \rightarrow} \\ 4 \rightarrow \end{array} 24$$

Number of zeros = 24

122.
$$\begin{array}{r} 3 \overline{) 41} \\ \underline{3 \ 13 \rightarrow} \\ 3 \ 4 \rightarrow \\ \underline{1 \rightarrow} \end{array} 18$$

$$41! = 3^{18} \times k^a \times l^b$$

- 123** Go through options

$$36 = 4 \times 9 = (2)^2 \times (3)^2$$

$$3, 6, 9 \text{ are in G.P.; } 9 = (3 + 6)$$

$$36 + 27 = 63$$

- 124** $7777 + 7777 \times 7777 \times (5 \div 77) \times (11 \div 35)$

$$= 7777 + 7777 \times 7777 \times \frac{5}{77} \times \frac{11}{35}$$

$$= 7777 + 1111 \times 1111$$

$$= 7777 + 1234321 = 1242098$$

Hence (d) is the answer.

- 125** Go through options

$$\frac{7}{8} + \frac{8}{7} = \frac{113}{56}$$

and $\frac{8}{7} - \frac{7}{8} = \frac{15}{56}$

Alternatively $a + \frac{1}{a} = \frac{113}{56}$

and $a - \frac{1}{a} = \frac{15}{56}$

Solve these equations.

- 126** Let $\frac{x}{y}$ be the fraction, then

$$\frac{x+4}{y} = \frac{1}{3}$$

and $\frac{x}{y+3} = \frac{1}{6} \Rightarrow 3x + 12 = y$

and $6x = y + 3$

$\Rightarrow x = 5$ and $y = 27$

Thus $x + y = 32$

- 127** Go through options and subtract the reasonable value from 7777 and see that whether the resultant number is a perfect square.

Alternatively $n^2 \leq 777 \Rightarrow n^2 = 7744$

Hence 33 ($= 7777 - 7744$) chairs are required to remove from that place.

128 $p^r \cdot p^{-1} \cdot p^s = (\sqrt{p^3})^2$

$\Rightarrow p^{r+s-1} = p^3$

$\Rightarrow r + s - 1 = 3$

$\Rightarrow r + s = 4$

Again $p^{3/2} \cdot p^r = p^s \cdot p^{-1/2}$

$\Rightarrow p^{r+3/2} = p^{s-1/2}$

$\Rightarrow r + \frac{3}{2} = s - \frac{1}{2}$

$\Rightarrow r - s = -2$

$\Rightarrow r = 1$ and $s = 3$

Thus $(r+s)^{r+s} = (4)^4 = 256$

Hence (d).

...(i)

...(ii)

$$129 \quad 5^{-k} = \frac{1}{l} \Rightarrow l = 5^k$$

$$5^{3k} = (5^k)^3 = (l)^3$$

Hence (c) is correct.

$$130 \quad 5^{x-1} + 5^x + 5^{x+1} = 775$$

$$\Rightarrow 5^{x-1} (1 + 5 + 5^2) = 775$$

$$\Rightarrow 5^{x-1} (31) = 775$$

$$\Rightarrow 5^{x-1} = 25 = 5^2$$

$$\Rightarrow x - 1 = 2 \Rightarrow x = 3$$

131 The perfect square digits are 1, 4 and 9.
So the number formed by using any of these two digits which is also a perfect square is 49.
where $(4 \times 9) + (4 + 9) = 49$
 $36 + 13 = 49 \Rightarrow 49 = 49$
So, $94 - (9 + 4) = 81$

132 Consider some consecutive values.

133 $25930800 = 2^4 \times 3^3 \times 5^2 \times 7^2$
So, if this number is divided by 3, then we get the quotient as a perfect square.

134 Let the Arun has x books and Prabhat has y books with them, then

$$2(x - 3) = (y + 3) \Rightarrow 2x - 6 = y + 3 \quad \dots(i)$$

$$\Rightarrow 2x - y = 9$$

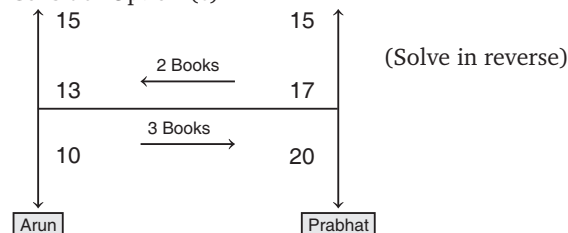
and $(x + 2) = (y - 2) \quad \dots(ii)$

$$\Rightarrow x - y = -4$$

Solving eqs. (i) and (ii), we get $x = 13$ and $y = 17$ thus $x + y = 30$.

Alternatively Go through options.

Consider Option (c)



136 Weight of Bottle + Total volume of water = 1600 gm

$$\text{Weight of Bottle} + \frac{1}{3} \text{ volume of water} = 900 \text{ gm}$$

$$\Rightarrow \frac{2}{3} \text{ volume of water} = 700 \text{ gm}$$

$$\Rightarrow \text{total volume of water} = 700 \times \frac{3}{2} = 1050 \text{ gm}$$

$$\therefore \text{The weight of empty bottle} = 550 \text{ gm}$$

$$= (1600 - 1050)$$

$$137 \quad (10a + b) \times (10c + d) - (10b + a) \times (10d + c)$$

$$= (100a.c + 10b.c + 10a.d + b.d)$$

$$- (100b.d + 10b.c + 10a.d + a.c)$$

$$= 99(a.c - b.d)$$

Now, in order to have the difference maximum, $a \times c$ will be maximum and $b \times d$ will be minimum. Therefore $a \times c = 9 \times 8 = 72$

$$\text{and } b \times d = 1 \times 2 = 2$$

$$\text{Hence } 99(ac - bd) = 99(72 - 2) = 99 \times 70 = 6930$$

$$138 \quad 12600 = 2^3 \times 3^2 \times 5^2 \times 7$$

$$\text{Required value} = 2^{(n-1)} = 2^{(4-1)} = 2^3 = 8$$

where $n \rightarrow$ number of prime factors.

Here 2, 3, 5, 7 are total 4 prime factors.

139 Go through options.

$$140 \quad (2^{123} - 2^{122} - 2^{121}) \times (3^{234} - 3^{233} - 3^{232})$$

$$= 2^{121} (2^2 - 2 - 1) \times 3^{232} (3^2 - 3 - 1)$$

$$= 2^{121} (1) \times 3^{232} \times 5$$

$$= 2^{121} \times 3^{232} \times 5^1$$

Therefore there will be only one zero at the end of the product.

$$141 \quad x + \frac{1}{x} = 3 \left(x - \frac{1}{x} \right) \Rightarrow \frac{x^2 + 1}{x} = 3 \left(\frac{x^2 - 1}{x} \right)$$

$$\Rightarrow 2x^2 = 4 \Rightarrow x^2 = 2 \Rightarrow x = \pm \sqrt{2}$$

Hence, (a) is correct.

142 Go through options Then consider option (c). As,

$$2 \times (14)^2 - 25 \times 14 = 3 \times 14$$

Alternatively Let the age of the lad be x then

$$2x^2 - 25x = 42$$

$$\Rightarrow 2x^2 - 25x - 42 = 0$$

Now, either solve this quadratic equation to get the answer or put the appropriate value from the option to get the required answer.

143 Check through options: As, $(5 + 3) + (5^2 + 3^2) = 42$

$$\text{and } 5 \times 3 = 15$$

144 Check through alternatives: As,

$$\frac{1}{0-1} = \frac{2}{0-2}$$

$$\Rightarrow \frac{1}{-1} = \frac{2}{-2} = \frac{1}{-1}$$

Option (a) and (b) are clearly inadmissible.

Alternatively $\frac{1}{x-1} = \frac{2}{x-2}$

$$\Rightarrow x - 2 = 2x - 2 \Rightarrow x = 0$$

$$145 \quad \therefore \frac{a}{b^2} \times \frac{b}{a^2} = \frac{1}{ab}$$

Now, consider option (d).

$$\frac{1}{2ab} + \frac{1}{2ab} = 2 \left(\frac{1}{2ab} \right) = \frac{1}{ab}$$

Hence, (d) is the appropriate option.

- 146** Substitute the value of x and y in the given expression

$$(6 + 3)^{6/3} = 9^2 = 81$$

- 147** Go back to the Basic. As $\sqrt[3]{2}$ is a real number but not a rational number.

- 148** Total number of lines (in one bound)

$$= [\text{Number of lines per page} \\ \times \text{Number of page per chapter} \\ \times \text{Number of chapter per book} \\ \times \text{Number of books per bound}]$$

$$= 60 \times 125 \times 5 \times 20 = 750000$$

$$\therefore \text{Number of bounds} = \frac{3000000}{750000} = 4$$

Hence, there are total 4 bounds.

149 $1 + \sqrt{1} = 2$

$$4 + \sqrt{4} = 6$$

$$9 + \sqrt{9} = 12$$

$$16 + \sqrt{16} = 20$$

$$25 + \sqrt{25} = 30$$

$$36 + \sqrt{36} = 42$$

Thus, we get the unit digits as 0, 2 and 6 only.

Hence (c) is the correct option.

150 Since $(10^N - 1) = (10^{2n} - 1)$

$$= (10^2)^n - 1 = (100)^n - 1$$

$$\text{So, } = 100^n - 1^n = (100 - 1)[X] = 99[X]$$

Thus, for any even power it is divisible by 11. i. e., $N = 2n$.

Hence, (a) is the correct option.

151 $\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{5}\right) \dots \left(1 + \frac{1}{n}\right)$

$$= \frac{4}{3} \times \frac{5}{4} \times \frac{6}{5} \times \dots \times \frac{n}{(n-1)} \times \left(\frac{n+1}{n}\right) = \frac{n+1}{3}$$

Hence, (a) is the correct option.

152 $(x)^3 = (2^{1/3} + 2^{-1/3})^3 = 2 + 2^{-1} + 3 \times 2^{1/3} \times 2^{-1/3} (x)$

$$= 2 + \frac{1}{2} + 3x$$

$$\Rightarrow x^3 - 3x = \frac{5}{2} \Rightarrow 2(x^3 - 3x) = 5$$

$$\Rightarrow 2x^3 - 6x = 5$$

Hence, (a) is the correct answer.

153 $\therefore (s-a)^2 + (s-b)^2 + (s-c)^2 + s^2 - a^2 - b^2 - c^2$

$$= s^2 + a^2 - 2as + s^2 + b^2 - 2bs + s^2 + c^2 - 2cs$$

$$+ s^2 - a^2 - b^2 - c^2$$

$$= 4s^2 + (a^2 + b^2 + c^2) - 2s(a+b+c) - (a^2 + b^2 + c^2)$$

$$= 4s^2 - 2s(a+b+c)$$

$$= 4s^2 - 4s^2 = 0 \quad [\because 2s = (a+b+c)]$$

Hence, (d) is the correct option.

- 154** Given that $ab + bc + ca = 0$

$$\text{Now } a^2 - bc = a^2 + ab + ca \quad [\because -bc = ab + ca]$$

$$\Rightarrow a^2 - bc = a(a+b+c)$$

$$\text{Similarly } b^2 - ca = b(a+b+c)$$

$$\text{and } c^2 - ab = c(a+b+c)$$

$$\therefore \frac{1}{a^2 - bc} + \frac{1}{b^2 - ca} + \frac{1}{c^2 - ab}$$

$$= \frac{1}{a(a+b+c)} + \frac{1}{b(a+b+c)} + \frac{1}{c(a+b+c)}$$

$$= \frac{1}{a+b+c} \left[\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right]$$

$$= \frac{1}{a+b+c} \left[\frac{bc+ac+ab}{abc} \right] = 0 \quad [\because ab+bc+ac=0]$$

- 155** Check the options for the given condition. 10080 fulfills all the requirements.

156 $s^2 + (s-1)^2 + (s-3)^2 + (s-5)^2$

$$= \left(\frac{9}{2}\right)^2 + \left(\frac{7}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{-1}{2}\right)^2 \quad \left[\because 2s=9 \Rightarrow s=\frac{9}{2}\right]$$

$$= \frac{81}{4} + \frac{49}{4} + \frac{9}{4} + \frac{1}{4} = 35$$

Hence, (d) is the correct option.

157 $\therefore x^a \times x^b \times x^c = 1 \Rightarrow x^{a+b+c} = 1 = x^0$

$$\Rightarrow a+b+c=0$$

$$\text{Now since } a^3 + b^3 + c^3 - 3abc$$

$$= (a+b+c)(a^2+b^2+c^2-ab-bc-ac)$$

$$\Rightarrow a^3 + b^3 + c^3 - 3abc = 0$$

$$\therefore a^3 + b^3 + c^3 = 3abc$$

Hence, (d) is the right choice.

158 $376 - 358 = 18, \quad 358 - 232 = 126, \quad 376 - 232 = 144$

Now, the HCF of 18, 126 and 144 = 18

Hence, 18 and all its factor can fulfill the required condition. But 9 is the greatest available such number.

Hence, (d) is the correct choice.

☞ Alternatively According to the given condition go through options starting with the greatest value from the choices available and see the remainders must be same.

159 The required number = (LCM of 2, 3, 4, 5, 6) $k+1 = 7l$

$$= 60k+1 = 7l \Rightarrow \frac{60k+1}{7} = l$$

Now put the least possible value of k such that l must be a positive integer. Hence at $k=5$, l is an integer. Thus, the required value is $60 \times 5 + 1 = 301$.

160 The required time = LCM of 36, 40 and 48

$$= 720 \text{ second} = 12 \text{ minutes}$$

Hence, (b) is the right choice.

$$161 \quad \left(1 - \frac{1}{2}\right) = \frac{1}{2}$$

$$\text{Now} \quad \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

$$\text{Thus} \quad 1 - \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$$

Thus, (a) is the correct answer.

162 See question number 63 or see the list of formulae and then solve it.

$$163 \quad a = b^x = (c^y)^x = (c^x)^y = (a^z)^y = a^{yz}$$

$$\text{Thus} \quad a = a^{yz} \Rightarrow xyz = 1$$

Hence, (c).

164 See the question number 37. Hence (c).

165

$$\begin{array}{r|l} \therefore & 2 \quad 7 \\ & 2 \quad 3 \rightarrow 1 \\ & 2 \quad 1 \rightarrow 1 \\ & 0 \rightarrow 1 \end{array}$$

$$\text{Thus} \quad (7)_{10} = (111)_2$$

166 The required number

$$= \text{HCF of } (398 - 7), (436 - 11) \text{ and } (542 - 15)$$

$$= \text{HCF of } (391, 425 \text{ and } 527) = 17$$

Hence, (b) is the right choice.

☞ **Alternatively** If your speed of calculation is very fast then pickup the suitable option and then divide the given numbers and see the required result.

167 Best way is to go through options. Consider option (c)

Correct answer = 20, marks for correct answer = 60

So wrong answer = 10, marks for wrong answer = -20

$$\therefore \text{Net marks} = 60 - 20 = 40$$

Hence, presumed option is correct.

$$\text{☞ Alternatively } 30 \times 3 - x \times 5 = 40$$

$$90 - 5x = 40$$

$$x = 10$$

Hence, the wrong answer = 10

Thus, the correct answer = 20

(for more detailed discussion see the example)

168 Total cost = Distance covered $\times$ Rate (or charge/km)

$$= x \times 10 + (200 - x) \times 2$$

$$= 10x + 400 - 2x = 8x + 400$$

Hence, (c).

$$169 \quad \therefore \quad 540 = 2^2 \times 3^3 \times 5$$

So, the least perfect square

$$= 2^2 \times 3^4 \times 5^2 = [(2^2 \times 3^3 \times 5) \times (3 \times 5)]$$

$$= 8100$$

Hence, (a).

$$170 \quad \frac{4}{5} + \frac{y-x}{y+x} = \frac{4}{5} + \frac{3-2}{3+2} \quad \left(\because \frac{x}{2} = \frac{y}{3} \Rightarrow \frac{x}{y} = \frac{2}{3} \right)$$

$$= \frac{4}{5} + \frac{1}{5} = \frac{5}{5} = 1$$

Hence, (c).

$$171 \quad 4A + \frac{5A}{3} = 85$$

$$\Rightarrow \quad A = 15 \Rightarrow \frac{A}{3} = 5$$

Hence, the cost of child ticket = ₹ 5. Thus (a) is the correct option.

$$172 \quad \text{Let the fraction be } \frac{x}{y}, \text{ then } \frac{x+1}{y+1} = 4 \quad \dots(i)$$

$$\text{and} \quad \frac{x-1}{y-1} = 7 \quad \dots(ii)$$

Solving eqs. (i) and (ii),

$$x = 15 \quad \text{and} \quad y = 3$$

Hence, (d) is correct.

173 The required number must be a factor of

$$= 34041 - 32506 = 1535$$

$$\text{Now since } 1535 = 5 \times 307$$

So, clearly 307 is the required number because it has 3 digits and is also a prime number.

$$174 \quad \text{Minimum number of corks} = \frac{403 + 434 + 465}{\text{HCF of } (403, 434, 465)}$$

$$= \frac{1302}{31} = 42$$

Hence, (c). For detailed discussion go back to examples.

175 The required value = HCF of 49.56 and 38.94 = 3.54

Therefore, the sum = ₹ 3.54.

Hence, (b) is correct.

176 The required value = LCM of 12, 30, 18, 48 = 720

Hence, (b).

177 Time taken for each of three persons A, B and C is

$$\text{respectively } \frac{6}{3}, \frac{6}{2\frac{1}{2}} \text{ and } \frac{6}{1\frac{1}{4}} \text{ hrs i. e., } \frac{2}{1}, \frac{12}{5} \text{ and } \frac{24}{5} \text{ hrs.}$$

$$\text{So, it is required to find the LCM of } \frac{2}{1}, \frac{12}{5}, \frac{24}{5} = \frac{24}{1} = 24 \text{ hr}$$

Hence, (c).

178 Since $\sqrt{3}$ is an irrational number.

$$179 \quad p > 0 \quad \text{and} \quad q < 0$$

$$\text{Now, let } p = 3 \quad \text{and} \quad q = -4$$

$$\text{therefore } p - q = 3 - (-4) = 7 > 0$$

Hence, (b) is correct.

180 The best way is to replace n with a suitable integral value. Let us say if we substitute $n = 2$, we find that the given expression become $(x^2 + y^2)(x^2 - y^2) = (x^4 - y^4)$.

Now, if we put $n = 2$ in the given options, we find that choice (a) becomes $(x^4 - y^4)$.

Hence, choice (a) is the valid one.

$$\begin{aligned} \mathbf{181} \quad x &= 2^{\frac{1}{3}} + 2^{-\frac{1}{3}} \Rightarrow x^3 = \left(2^{\frac{1}{3}} + 2^{-\frac{1}{3}}\right)^3 \\ \Rightarrow x^3 &= 2 + 2^{-1} + 3(2)(2^{-1})\left(2^{\frac{1}{3}} + 2^{-\frac{1}{3}}\right) \\ \Rightarrow x^3 &= 2 + \frac{1}{2} + 3(2)\left(\frac{1}{2}\right)(x) \\ \Rightarrow x^3 &= \frac{5}{2} + 3x \Rightarrow 2x^3 = 5 + 6x \end{aligned}$$

Hence, choice (a) is the valid one.

$$\begin{aligned} \mathbf{182} \quad x &= 3 + 3^{\frac{2}{3}} + 3^{\frac{1}{3}} \\ \Rightarrow x - 3 &= 3^{\frac{2}{3}} + 3^{\frac{1}{3}} \\ \Rightarrow (x - 3)^3 &= \left(3^{\frac{2}{3}} + 3^{\frac{1}{3}}\right)^3 \\ \Rightarrow x^3 - 3^3 - 3(x)(3)(x - 3) &= 3^2 + 3 + 3(3^{\frac{2}{3}})(3^{\frac{1}{3}})(3^{\frac{2}{3}} + 3^{\frac{1}{3}}) \\ \Rightarrow x^3 - 27 - 9x^2 + 27x &= 12 + 9(x - 3) \\ \Rightarrow x^3 - 9x^2 + 18x - 12 &= 0 \end{aligned}$$

Hence, choice (b) is the valid one.

183 We know that, $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
When $(a + b + c) = 0$, we have $a^3 + b^3 + c^3 = 3abc$

Therefore, $(x + y + z) = 3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}}$

$$\Rightarrow (x + y + z)^3 = \left(3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}}\right)^3$$

$$\Rightarrow (x + y + z)^3 = 27xyz$$

Hence, choice (b) is the valid one.

☞ **Alternatively** Let's assume $x^{\frac{1}{3}} = -1, y^{\frac{1}{3}} = -2, z^{\frac{1}{3}} = 3$

$$\Rightarrow x = -1, y = -8, z = 27$$

$$\therefore (x + y + z)^3 = (-1 - 8 + 27)^3 = 18^3$$

Now, by substituting the Values of x, y, z in the given options we find that

$$27xyz = 27(216) = 18^3$$

Hence, choice (b) is the valid one.

184

$$\begin{aligned} x^{x(\sqrt{x})} &= (x\sqrt{x})^x \\ \Rightarrow x^{\left(\frac{3}{2}x\right)} &= \left(\frac{3}{2}x\right)^x \\ \Rightarrow x^{\frac{3}{2}} &= \frac{3}{2}x \\ \Rightarrow x\sqrt{x} &= \frac{3}{2}x \\ \Rightarrow \sqrt{x} &= \frac{3}{2} \\ \Rightarrow x &= \frac{9}{4} \end{aligned}$$

Hence, choice (a) is the valid one.

185 The best way is to go through the options.

If we consider $(x, y) = (1, 1)$, we find the $5^x + 3^y = 8$ and $5^{x-1} + 3^{y-1} = 2$.

Hence choice (c) is the valid one.

186 Since $x^2 = y^3$, so choice (b) and choice (d) are invalid.

Now, let us consider choice (a), we have

$$\left(\frac{27}{8}\right)^{\frac{9}{4}} = \left(\frac{3}{2}\right)^{\frac{27}{4}}$$

$$\text{and} \quad \left(\frac{9}{4}\right)^{\frac{27}{8}} = \left(\frac{3}{2}\right)^{\frac{27}{4}}$$

Hence, choice (a) is the valid one.

$$\begin{aligned} \mathbf{187} \quad &\left(ax^{\left(\frac{1}{4}\right)} + 3a^{\left(\frac{1}{2}\right)}x^{\left(\frac{1}{2}\right)} + 4x^{\left(\frac{3}{4}\right)}\right)\left(a - 3a^{\frac{1}{2}}x^{\frac{1}{4}} + 4x^{\frac{1}{2}}\right) \\ &= x^{\left(\frac{1}{4}\right)}\left(a + 3a^{\frac{1}{2}}x^{\frac{1}{4}} + 4x^{\frac{1}{2}}\right)\left(a - 3a^{\frac{1}{2}}x^{\frac{1}{4}} + 4x^{\frac{1}{2}}\right) \\ &= x^{\left(\frac{1}{4}\right)}\left[\left(a + 4x^{\frac{1}{2}}\right)^2 - \left(3a^{\frac{1}{2}}x^{\frac{1}{4}}\right)^2\right] \\ &= x^{\left(\frac{1}{4}\right)}\left[\left(a^2 + 16x + 8ax^{\frac{1}{2}}\right) - \left(9ax^{\frac{1}{2}}\right)\right] \\ &= x^{\left(\frac{1}{4}\right)}\left[a^2 + 16x - ax^{\frac{1}{2}}\right] \\ &= a^2x^{\left(\frac{1}{4}\right)} + 16x^{\frac{5}{4}} - ax^{\frac{3}{4}} \end{aligned}$$

Hence, Choice (a) is the valid one.

188 Given that $a + b + c = 0$, we can consider

$$a = -1, b = 0, c = 1 \text{ and } x = 2$$

$$\therefore \frac{1}{x^b + x^{-c} + 1} + \frac{1}{x^c + x^{-a} + 1} + \frac{1}{x^a + x^{-b} + 1}$$

$$= \frac{1}{x^0 + x^{-1} + 1} + \frac{1}{x^1 + x^1 + 1} + \frac{1}{x^{-1} + x^0 + 1}$$

$$= \frac{2}{5} + \frac{1}{5} + \frac{2}{5} = \frac{5}{5} = 1$$

Hence, choice (a) is the valid one.

189 Given that $pqr = 1$, we can consider $p = \frac{1}{2}$, $q = 1$, $n = 2$

$$\therefore \frac{1}{1+p+q^{-1}} + \frac{1}{1+q+r^{-1}} + \frac{1}{1+r+p^{-1}}$$

$$= \frac{1}{\frac{3}{2}} + \frac{1}{2} + \frac{1}{2} = \frac{2}{3} + \frac{2}{3} = \frac{4}{3} \neq 1$$

Hence, choice (a) is the valid one.

190 $B + M + D + H + C + P = 100$

$$\Rightarrow M + D + H + C = 45$$

In order to minimize M , we have to maximize D, H and C , such that $\{D \neq H \neq C\} < M$.

Now, divide 45 by 4, as there are 4 cities (M, D, H, C). Then we get 11.25. Let us consider $M = 12$, then we cannot have D, H and C as three distinct integers less than 12.

However, if we consider $M = 13$, then D, H, C can be distinct integers less than 13.

Hence, choice (d) is the correct one.

191 Halwaai Babu has maximum number of gas connections. That means no one else has more than 7 connections.

It implies that even if 8 out of the 10 suspects have distinct number of gas connections $\{0, 1, 2, 3, 4, 5, 6, 7\}$, still there are two suspects who will have same number of connections as the other 8 suspects have.

Case (I) When the remaining two suspects have same number of connections, there must be three suspects with same number of gas connections.

Case (II) When the remaining two suspects have distinct number of connections, there must be at least two suspects with same number of connections.

Therefore, at least two suspects have the same number of gas connections.

Hence, choice (d) is the correct one.

192 $S(n) = \frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{n} + \sqrt{n+1}}$

$$\Rightarrow S(n) = (\sqrt{2} - \sqrt{1}) + (\sqrt{3} - \sqrt{2}) + (\sqrt{4} - \sqrt{3}) + \dots + (\sqrt{n+1} - \sqrt{n})$$

$$\Rightarrow S(n) = (\sqrt{n+1} - \sqrt{1})$$

$$\Rightarrow S(n) = \sqrt{n+1} - 1$$

Therefore $S(n)$ will be rational when $(n+1)$ will be a perfect square.

(i) $5! = 120$; $120 + 1 = 121 = 11^2$

(ii) $30 + 30 \times 30 + 30 = 960$; $960 + 1 = 961 = 31^2$

(iii) $\log 10^8 = 8 \log 10 = 8$; $8 + 1 = 9 = 3^2$

(iv) $2^{2^2} = 2^4 = 16$; $16 + 1 = 17$, it is, not a perfect square.

(v) $2^3 = 8$; $8 + 1 = 9 = 3^2$

Therefore (i), (ii), (iii) and (v) are valid.

Hence choice (d) is the correct one.

Hint $\frac{1}{\sqrt{a} + \sqrt{b}} = \frac{(\sqrt{a} - \sqrt{b})}{(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})} = \frac{(\sqrt{a} - \sqrt{b})}{(a - b)}$

193 The given summation can be expressed as following.

$$\sum_{n=1}^{99} D(n) = D(1) + D(2) + D(3) + \dots + D(97) + D(98) + D(99)$$

$$\sum_{n=1}^{99} (n) = 1 + 2 + 3 + \dots + (9 + 7) + (9 + 8) + (9 + 9)$$

It implies that you have to add all the digits of all the natural numbers up to 99.

Since, if you observe closely, then you find that each digit is appearing 20 times.

So the required sum = $20(1 + 2 + 3 + \dots + 7 + 8 + 9)$

$$= 20 \left(\frac{9 \times 10}{2} \right) = 900$$

Therefore, when 900 is divided by 99, it will leave the remainder 90

Hence, choice (d) is the correct one.

194 Since, $111111 = 1001 \times 111 = 7 \times 11 \times 13 \times 3 \times 37$

Therefore every such number having 6 or 12 or 18 etc. digits is divisible by $3 \times 7 \times 11 \times 13 \times 37$.

Now, we have

$$N = \underbrace{666666 \dots 666666}_{225 \text{ digits}}$$

$$N = \underbrace{\{666666 \dots 666666\}}_{222 \text{ digits}} \times 1000 + 666$$

$$N = \underbrace{\{666666 \dots 666666\}}_{37 \times 6 \text{ digits}} \times 1000 + 666$$

$$N = 6 \times \underbrace{\{111111 \dots 111111\}}_{37 \times 6 \text{ digits}} \times 1000 + 666$$

$$\text{The first term } 6 \times \underbrace{\{111111 \dots 111111\}}_{37 \times 6 \text{ digits}} \times 1000 \text{ is}$$

$$\text{divisible by } 2^4 \times 3^{38} \times 5^3 \times 7^{37} \times 11^{37} \times 13^{37} \times 37^{37}$$

So the first term is also divisible by 455 (since, $455 = 5 \times 7 \times 13$). Then, the first term will give remainder 0. And the second term 666 when divided by 455, it will give remainder 211.

Thus the final remainder when N is divided by 455, the remainder will be 211.

Hence, choice (c) is the correct one.

195 A term can be integer only when

$3n - 1$ is a factor of 420.

The factors of 420 are 1, 2, 3, 4, 5, 6, 7, 10, 12, 14, 15, 20, 21, 28, 30, 35, 42, 60, 70, 84, 105, 140, 210 and 420.

Now, we can figure out that only 2, 5, 14, 20, 35 and 140 can be written in the form of $3n - 1$.

Thus there are at most 6 values of $3n - 1$, which are the factors of 420, so there are at most 6 terms in the given sequence, which are integers.

Hence choice (c) is the correct one.

Solutions (for Q. Nos. 196 and 197) If the number of students found cheating be n and the mean of the ticket numbers be m , then $mn = 600$. Being consecutive natural numbers means all the ticket numbers form an AP.

Case I When n is odd.

In this case m will also be a natural number as m is the middlemost term of the arithmetic progression.

600 has 6 odd factors namely 1, 3, 5, 15, 25 and 75. But n must be greater than 1, so $n = 3, 5, 15, 25, 75$,

n	m	Consecutive natural numbers	Possibility
3	200	199, 200, 201	Yes
5	120	118, 119, 120, 121, 122	Yes
15	40	33, 34, 35, ..., 40, ..., 45, 46, 47	Yes
25	24	12, 13, 14, ..., 24, ..., 34, 35, 36	Yes
75	8	All the terms will not be the natural numbers	No

Case II When n is even

In this case m must be a decimal number of the form $x.5$, as m is the average of the even consecutive natural numbers.

Now, $600 = 2^3 \times 3 \times 5^2$

Since 8 is the factor of 600, so all the desired values of m will be obtained when n is 16 or its multiple, but excluding 32, 64, 128, 256, 512.

Therefore the desired values of

$$n = 16, 16(3), 16(5), 16(3 \times 5), 16(5 \times 5), 16(3 \times 5 \times 5)$$

That is $n = 16, 48, 80, 240, 400, 1200$

But since, $n = 1200 > 600$, so it's not a possible value of n .

Therefore, the possible values of $n = 16, 48, 80, 240, 400$

n	m	Consecutive natural numbers	Possibility
16	37.5	30, 31..., 37, 38, ...44, 45	Yes
48	12.5	All the terms will not be natural numbers	No
80	7.5	All the terms will not be natural numbers	No
240	2.5	All the terms will not be natural numbers	No
400	1.5	All the terms will not be natural number	No

196 Since there are various possibilities for n , so the unique answer cannot be determined.

Hence, choice (d) is the correct one.

197 This is possible when n is even. Here you have only one even value of $n = (16)$. In this case, the ticket number of Munnabhai will be 37 or 38 and that of Curkit will be 38 or 37.

Hence choice (a) is the correct one.

198 Required number of ordered pairs

$$(p, q) \text{ is } (2 \times 5 - 1)(2 \times 8 - 1)(2 \times 3 - 1) = 765$$

Hence choice (b) is the answer.

199 $16! = 2^{15} \times 3^6 \times 5^3 \times 7^2 \times 11^1 \times 13^1$

Total number of factors = $16 \times 7 \times 4 \times 3 \times 2 \times 2 = 5376$

Total number of odd factors which don't have unit digit 5 = $7 \times 3 \times 2 \times 2 = 84$

Number of factors which have unit digit 5 = $3 \times 84 = 252$

Number of factors which don't have unit digit 5 = $5376 - 252 = 5124$

Hence, choice (c) is the correct one.

Hint Any factor of $3^6 \times 7^2 \times 11^1 \times 13^1$ will be an odd factor, which has no unit digit 5.

If any such factor is multiplied by 5^1 or 5^2 or 5^3 , it would give a factor whose unit digit would be 5.

✓ Level 02 Higher Level Exercise

- 1 Since these bells tolls 18 times in 24 hrs

$$\text{So the min. time interval when they toll together} = \frac{24}{18}$$

$$= 80 \text{ minutes}$$

So the required number of bells

$$= \text{total number of different factors of 80}$$

$$\text{Now since } 80 = 2^4 \times 5$$

$$\therefore \text{Total number of factors} = (4 + 1)(1 + 1) = 10$$

Thus the maximum number of bells = 10

🔔 **NOTE** The time interval of all the bells individually will be the factor of 80.

- 2 Consider the following relation and compare with $p^q = q^r$

$$2^4 = 4^2 \Rightarrow p = 2, q = 4, r = 2$$

$$\Rightarrow q > r$$

$$\text{Again } 4^2 = 2^4 \Rightarrow p = 4, q = 2, r = 4$$

$$\Rightarrow q < r$$

So we can't say exactly whether $q < r$ or $q > r$.

🔔 **NOTE** If we know the relation between p and q then only we can say about q and r .

- 3 Hit and trial plus your intelligence and familiarity with the numbers will help you in finding the following relation.

Now follow the given directions :

$$7^3 - 3^5 = (7 + 3)^{5-3}$$

$$343 - 243 = 10^2$$

$$100 = 100$$

$$\text{Hence } p + q + r = 7 + 5 + 3 = 15.$$

Thus (c) is correct.

- 4 Given that $a^2 + b^2 + c^2 + d^2 = 1$

Now, the maximum value of $a.b.c.d$ will be only when

$$a = b = c = d = k$$

$$\text{Then } a^2 + b^2 + c^2 + d^2 = 4k^2 = 1$$

$$\Rightarrow k = \pm \frac{1}{2}$$

$$\text{Now } a.b.c.d = k^4 = \left(\pm \frac{1}{2}\right)^4 = \frac{1}{16}$$

Hence (c) is correct.

- 5 $n^2 - 2(n!) + n = 0$

$$\Rightarrow n^2 + n = 2(n!)$$

$$\Rightarrow n(n+1) = 2(n!)$$

Now, going through options, we see that 1 and 3 are the two possible values hence (d).

- 6 Try and find the following suitable values for the given relation

$$3^6 = 9^3 = (6!) + 9$$

$$729 = 729 = 720 + 9$$

$$729 = 729 = 729$$

$$\text{where } m = 3, n = 6 \text{ and } p = 9$$

$$\therefore m \cdot n \cdot p = 162$$

$$7 \quad n + (n-1) + (n-2) = n(n-1)(n-2)$$

$$\Rightarrow 3n - 3 = n(n-1)(n-2)$$

$$\Rightarrow 3(n-1) = n(n-1)(n-2)$$

$$\Rightarrow n(n-2) = 3$$

$$\Rightarrow n = 3, -1$$

But the only possible value is $n = 3$.

$$\text{Hence } (n!)^n = (3!)^3 = (6)^3 = 216$$

$$8 \quad n^3 - n^2 - n = n$$

$$\Rightarrow n^3 - n^2 - 2n = 0$$

$$\Rightarrow n(n^2 - n - 2) = 0$$

$$\Rightarrow n = 0 \text{ or } (n^2 - n - 2) = 0$$

$$\text{Now, if } (n^2 - n - 2) = 0$$

$$\Rightarrow n = -1, 2$$

$$\therefore n = -1, 0, 2$$

Hence (c).

$$9 \quad \frac{12}{3} = 4$$

First of all you should know that if $n_1 + n_2 + n_3 + \dots = 0$

$$\Rightarrow n_1 = n_2 = n_3 = \dots = 0$$

Again, each term is the product of three quantities (or numbers). So only one number which is equal to zero is enough to make the product zero. Thus a particular single number, which is equal to zero can make three terms zero, because it is present in three consecutive terms.

For example if $k_3 = 0$, then $k_1 k_2 k_3 = k_2 k_3 k_4 = k_3 k_4 k_5 = 0$

and if $k_6 = 0$, then $k_4 k_5 k_6 = k_5 k_6 k_7 = k_6 k_7 k_8 = 0$

and if $k_9 = 0$, then $k_7 k_8 k_9 = k_8 k_9 k_{10} = k_9 k_{10} k_{11} = 0$

and if $k_{12} = 0$, then $k_{10} k_{11} k_{12} = k_{11} k_{12} k_{13} = k_{12} k_{13} k_{14} = 0$ etc.

$$10 \quad 3^2 + 3^4 = (3^2 + 1)^2 - (3^2 + 1)$$

$$9 + 81 = 100 - 10$$

$$90 = 90$$

$$\text{and } 1^0 + 1^2 = (1^2 + 1)^2 - (1^2 + 1)$$

$$1 + 1 = 4 - 2$$

$$2 = 2$$

$$\text{🔗 Alternatively } n^{(n-1)} + n^{(n+1)} = (n^2 + 1)^2 - (n^2 + 1)$$

$$\Rightarrow n^{(n-1)}(1 + n^2) = (n^2 + 1)[(n^2 + 1) - 1]$$

$$n^{(n-1)} = [n^2]$$

$$\Rightarrow n - 1 = 2 \Rightarrow n = 3$$

$$\text{and if } n = 1, \text{ then } 1^0 = 1^2 = 1$$

$$\text{Thus } n = 1, 3.$$

$$11 \quad 1^3 + (12)^3 = (10)^3 + (9)^3$$

$$1 + 1728 = 1000 + 729$$

$$1729 = 1729$$

$$12 \quad 8^6 - 5^6 = (8^2)^3 - (5^2)^3 = (64)^3 - (25)^3 \\ = (64 - 25)(64^2 + 25^2 + 64 \times 25) = 39 \times 6321 \\ = 13 \times 3 \times 3 \times 7 \times 7 \times 43$$

$$\text{So it can be divided by } 13 \times 7 = 91, 7 \times 7 = 49, \\ 3 \times 43 = 129$$

Hence, all of these are possible.

$$13 \quad \text{Since the product of 4 prime factors} = 1365 \\ = 3 \times 5 \times 7 \times 13 \\ \text{and the sum of the 3 prime factors} = 25 = (5 + 7 + 13) \\ \text{Now, total number of factors of the required number} \\ N = 24$$

$$= 2^3 \times 3 = (1 + 1)(1 + 1)(1 + 1)(2 + 1)$$

Let N can be expressed as $N = 3^p \times 5^q \times 7^r \times 13^s$

Thus, in order to have N as the greatest possible number in the above expressed form, the power of the greatest prime factors will be greater.

$$\text{So} \quad N = 3 \times 5 \times 7 \times 13^2 \\ = 105 \times 169 = 17745$$

14 Obviously (as per the given options) no any odd digit will appear as the last (or unit digit).
Hence (d), since even $\times$ even = even.

$$15 \quad \frac{R(135)}{R(100, 35)} = \frac{(135)!}{100 \times 101 \times 102 \times 103 \times \dots \times 135} \\ = \frac{1 \times 2 \times 3 \times \dots \times 99 \times 100 \times 101 \times 102 \times \dots \times 135}{100 \times 101 \times 102 \times \dots \times 135} \\ = 1 \times 2 \times 3 \times \dots \times 99 = (99)!$$

$$16 \quad R(17) \cdot R(19, 62)$$

$$= (1 \times 2 \times 3 \times 4 \times 5 \times 6 \times \dots \times 17) \times (19 \times 20 \times 21 \times 22 \times \dots \times 81) \\ = \frac{(1 \times 2 \times 3 \times \dots \times 81)}{18} = \frac{81!}{18}$$

$$17 \quad R(2, 995) = 2 \times 3 \times 4 \times \dots \times 995 \times 996 \times 997 \\ R(996, 1) = 996 \times 997 \\ \therefore \text{L.C.M. of } R(2, 995) \text{ and } R(996, 1) \\ = 1 \times 2 \times 3 \times \dots \times 997 = 997!$$

$$18 \quad R(139, 2) = 139 \times 140 \times 141 \\ R(141) = 1 \times 2 \times 3 \times \dots \times 141 \\ \therefore \text{H.C.F. of } R(139, 2) \text{ and } R(141) = 139 \times 140 \times 141 \\ = 2743860$$

19 Since $abc \times 1001 = abcabc$
Thus the number $abcabc$ is always divisible by 1001 and its all factors. So (d) is the best option.
Since $1001 = 7 \times 11 \times 13$.

20 Numbers divisible by 5 upto 1155 = 231
Numbers divisible by 7 upto 1155 = 165
Numbers which are divisible by both 5 and 7 (i.e., 35) upto 1155 = 33
 $\therefore$ Numbers which are exactly divisible by 5 or 7 = $(231 + 165) - 33 = 363$
Numbers which are divisible by 5 and 11 both (i.e., 55) = 21
Numbers which are divisible by 7 and 11 both (i.e., 77) = 15
Numbers which are divisible by 5, 7 and 11 simultaneously = 3
 $\therefore$ Numbers which are only divisible by either 5 and 11 or 7 and 11

$$= 21 + 15 - 3 = 33$$

Hence, the total number of numbers which are divisible by 5 or 7 but not by 11 = $363 - 33 = 330$.

$$21 \quad 4x + 7y = 3$$

$$\Rightarrow y = \frac{3 - 4x}{7}; \quad x, y \in I$$

Now, in the given range of x and y , the least possible value of x is -99 and then $y = 57$.

Again the largest possible value of x is 97 and the corresponding value of y is -55 . So in this range total 29 solution sets are possible.
(Since the possible values of x are $-99, -92, -85, \dots, 99$ and the possible values of y are $57, 53, 49, \dots, -51, -55$)

22 See the following pattern :

$N_1 \times N_2 \rightarrow$ Unit digit	$N_1 \times N_2 \rightarrow$ Unit digit
$1 \times 3 \rightarrow 3$	$10 \times 12 \rightarrow 0$
$2 \times 4 \rightarrow 8$	$11 \times 13 \rightarrow 3$
$3 \times 5 \rightarrow 5$	$12 \times 14 \rightarrow 8$
$4 \times 6 \rightarrow 4$	$13 \times 15 \rightarrow 5$
$5 \times 7 \rightarrow 5$	$14 \times 16 \rightarrow 4$
$6 \times 8 \rightarrow 8$	$15 \times 17 \rightarrow 5$
$7 \times 9 \rightarrow 3$	$16 \times 18 \rightarrow 8$
$8 \times 10 \rightarrow 0$	$17 \times 19 \rightarrow 3$
$9 \times 11 \rightarrow 9$	$18 \times 20 \rightarrow 0$ etc.

Except to first column in all the rest columns the least occurring unit digits are 9 and 4 and they occur same number of times. But in the first column 9 is not occurring. Thus finally '9' as the unit digit occurs least number of times i.e., one times less as '4' occurs.

23 Except the first column the sum of all the unit digits
 $= 99(9 + 0 + 3 + 8 + 5 + 4 + 5 + 8 + 3 + 0)$
 $= 99(45) = 4455$

and the sum of all the unit digits of first column
 $= (3 + 8 + 5 + 4 + 5 + 8 + 3 + 0) = 36$

Thus the sum of all the unit digits = $4455 + 36 = 4491$
Hence (a).

- 24** Since a cube has to be cut from three sides (i.e., along 3 dimensions). Hence the number of cuts will be equal to the sum of the 3 factors of 960, when 960 is expressed as the product of 3 factors.

Again since the number of cuts to be applied are minimum. Hence the sum of these 3 factors of 960 must be minimum. So, in order that the sum of 3 factors of 960 to be minimum we have to have the minimum possible difference between the 3 factors of 960.

$$\text{Thus } 960 = 8 \times 10 \times 12$$

Hence the minimum possible number of cuts

$$= (8 + 10 + 12) = 30$$

Now, if we want to maximize the product of any 3 factors whose sum is constant i.e., 30, then it is possible only when all the 3 factors be equal. Thus a, b, c be the three factors such that :

$$a + b + c = 30$$

Then Maximum $(a \times b \times c) = 10 \times 10 \times 10 = 1000$

So, the maximum number of 1000 identical pieces can be formed by applying the same number of n cuts.

- 25** Since $45000 = 10 \times 4500$

So, the sum of all the factors of 45000 which are the multiples of 10 will be same as sum of all the factors of 4500 multiplied by 10.

$$\text{Now, } 4500 = 2^2 \times 3^2 \times 5^3$$

Therefore the sum of all the factors of

$$\begin{aligned} 4500 &= \frac{(2^3 - 1) \times (3^3 - 1)(5^4 - 1)}{(2 - 1)(3 - 1)(5 - 1)} \\ &= \frac{7 \times 26 \times 624}{1 \times 2 \times 4} = 14196 \end{aligned}$$

Thus the sum of all the products of 45000 which are the multiples of 10 $= 10 \times 14196 = 141960$.

- 26** The unit digit of $(1!)^{1!} = 1$

$$\text{The unit digit of } (2!)^{2!} = 4$$

$$\text{The unit digit of } (3!)^{3!} = 6$$

$$\text{The unit digit of } (4!)^{4!} = 6$$

$$\text{The unit digit of } (5!)^{5!} = 0$$

Now since we know that $5!, 6!, 7!, 8!, \dots$ all have their unit digits zero.

Thus the sum of all the unit digits = 7

$$(\text{Since } 1 + 4 + 6 + 6 + 0 = 17)$$

- 27** To know the remainder, when any number is divided by 5, we just need to know the unit digit of the dividend. Further from the previous question, we know that the unit digit of the sum of the whole expression is 7.

So divide 7 by 5 and get 2 as the remainder.

- 28** The last two digits of $(1!) = 01$

$$\text{The last two digits of } (2!)^2 = 04$$

$$\text{The last two digits of } (3!)^3 = 16$$

$$\text{The last two digits of } (4!)^4 = 76$$

$$\text{The last two digits of } (5!)^5 = 00$$

Now, we can conclude that the last two digits of the higher numbers e.g., $(6!)^6, (7!)^7, (8!)^8 \dots$ etc. are "00". So the last two digits of the whole expression = 9

$$(\text{Since } 01 + 04 + 16 + 76 + 00 = 97)$$

- 29** Let the two parts be x and y then $x^2 - y^2 = 50(x - y)$

$$\Rightarrow (x + y) = 50$$

Hence the number is 50.

- 30** If $p = 3$, then $n = 5$ (Since $n > 2$)

$$\text{If } p = 10, \text{ then } n = 5 \text{ also}$$

$$\text{If } p = 17, \text{ then } n = 5 \text{ also etc.}$$

Hence $n = 5$.

- 31** Go through options :

	Anjali	Bhagwat	
Total Shots $\rightarrow$	45	45	= 90
Total Hits $\rightarrow$	36	30	= 66
Total miss shots $\rightarrow$	9	15	
Since,	$\frac{36}{9} = 4$	and $\frac{30}{15} = 2, \frac{4}{2} = 2,$	

Hence, the presumed option is correct.

- 32** This is possible only when $n = 4, 10, 16, 22, 28, 34, \dots$

Therefore the remainder will be 3 when $(n - 1)$ will be divided by 6.

- 33** Option (b) is obvious, as 2222 and 7777 both numbers are divisible by 101.

Now we know that if the sum of the remainders of two or more numbers are divisible by the given divisor then the required expression is also divisible by the divisor. The remainder when $(2222)^{7777}$ is divided by 13 is 12 and the remainder when $(7777)^{2222}$ is divided by 13 is 9.

Hence the given expression is not divisible by 13.

$$(\text{Since } (12 + 9) \neq 13m \text{ for any positive integer } m)$$

Again the remainder when $(2222)^{7777}$ is divided by 99 is 44 and the remainder when $(7777)^{2222}$ is divided by 99 is 77.

Hence the whole expression cannot be divided by 99.

$$(\text{Since } 44 + 77 = 121 \text{ which is not divisible by } 99.)$$

- 34** Let there be x students, then $\frac{x}{2} + \frac{x}{3} + \frac{x}{4} = 91$

$$\Rightarrow \frac{6x + 4x + 3x}{12} = 91 \Rightarrow \frac{13x}{12} = 91$$

$$\Rightarrow x = 84 \text{ students}$$

- 35 Best way is to go through convenient options.

$$1 + \frac{1}{4} \times 1 = \frac{5}{4} \quad \left(\because \frac{1}{4} \rightarrow 25\% \right)$$

$$\frac{5}{4} + \frac{1}{2} \times \frac{5}{4} = \frac{15}{8} \quad \left(\frac{1}{2} \rightarrow 50\% \right)$$

Hence option (c) is correct.

☞ **Alternatively** Let the first time raise in salary be $x\%$, and the original salary be k then

$$k + \frac{kx}{100} = \frac{k(100+x)}{100}$$

$$\text{Again } \frac{k(100+x)}{100} + \frac{k(100+x)}{100} \times \frac{2x}{100} = \frac{k \times 15}{8}$$

$$\frac{k(100+x)}{100} \left[\frac{(100+2x)}{100} \right] = \frac{15k}{8}$$

$$\Rightarrow \frac{(100+x)(100+2x)}{100 \times 100} = \frac{15}{8}$$

So simplify and solve the quadratic equation and get the values of x , then twice the value of x will be the required value.

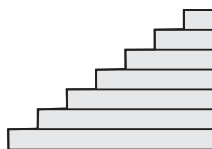
$$36 \quad 5^{72} \times 8^{27} = 5^{72} \times 2^{81} = (5^{72} \times 2^{72}) \times 2^9 \\ = 512 \times 10^{72}$$

Therefore there are total 75 ($= 3 + 72$) digits in the above expression.

- 37 Find the sum of n natural numbers, which is just greater than 500. So if we consider that he had to add 32 natural numbers then the ideal answer would have been 528. It means he had missed 28 during the process.

☞ **NOTE** For clarification of the concept just write the natural numbers in sequence and then add up them one by one but not as the child did, then you will must get the lucid explanation of this problem.

- 38 Total distance to be covered
= total horizontal movement
+ total upward movement
= $(20 \times 2 - 1) + (20)$
= $39 + 20 = 59$ feet



- 39
- | | Paltry | Sundry |
|--------------|--------|--------|
| Fired shots | 5 | 7 |
| Hit shots | 2 | 3 |
| Missed shots | 3 | 4 |

When Sundry missed 32 shots, it means Paltry missed 24 shots. When Paltry missed 24 shots, it means Paltry hit 16 shots.

- 40 It can be solved easily by options :

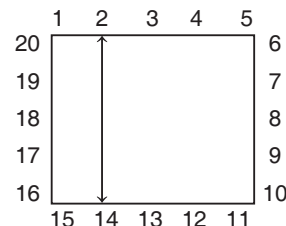
As consider option (a), then $90 \times \frac{5}{6} = 75$

$$75 - 5 + 2 = 72 \quad \leftarrow$$

Again $90 \times \frac{4}{5} = 72 \quad \leftarrow$

Hence, the assumed option is correct.

- 41 See the figure and visualise the actual situation.



- 42 Go through options and check the option (b).

Thus at $y = -2$ and $y = 4$, $p = \frac{1}{0}$, which is undefined.

$$43 \quad \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \dots + \frac{1}{100 \times 101} \\ = \left(\frac{1}{1} - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \left(\frac{1}{4} - \frac{1}{5} \right) + \dots \\ + \left(\frac{1}{100} - \frac{1}{101} \right) \\ = 1 - \frac{1}{101} = \frac{100}{101}$$

$$44 \quad \begin{array}{l} R \rightarrow x + 10 \\ L \rightarrow x + 6 \\ B \rightarrow x + 5 \\ H \rightarrow x + 4 \\ A \rightarrow x \end{array} \begin{array}{l} x \\ x \\ x \\ x \\ x \end{array} + \left(\begin{array}{l} 25 \\ 25 \\ 25 \\ 25 \\ 25 \end{array} \right) \begin{array}{l} x + 5 \\ x + 5 \\ x + 5 \\ x + 5 \\ x + 5 \end{array} \begin{array}{l} \textcircled{5}^+ \\ \textcircled{1}^+ \\ \textcircled{1}^- \\ \textcircled{1}^- \\ \textcircled{5}^- \end{array}$$

Thus total 6 coins have to be transferred.

- 45 As per the given expression the value of $f(x) = f\left(\frac{1}{x}\right)$

Try to put $\left(\frac{1}{x}\right)$ in place of x .

$$\text{Thus the value of } f\left(\frac{1}{2}\right) = f(2) = 1081.58$$

- 46 The unit digit of each pair is 4 and there are 50 such pairs which are mutually multiplied together. Thus finally we get 6 as unit digit. As

$$\text{Unit digit } \underbrace{4 \times 9^2}_{4} \times \underbrace{4^3 \times 9^4}_{4} \times \underbrace{4^5 \times 9^6}_{4} \times \dots \times \underbrace{4^{99} \times 9^{100}}_{4}$$

Again $4 \times 4 \times 4 \times 4 \dots 4$ (upto 50 times)

i.e., the unit digit of 4^{50} , which is 6

[Since unit digit of 4^{2n} is 6 for $n = 1, 2, 3, \dots$ etc.]

- 47 The unit digit of each pair is 5 and there are total 50 such pairs.

$$\begin{array}{ccccccc} \text{Unit} & \underbrace{4 + 9^2} & + & \underbrace{4^3 + 9^4} & + & \underbrace{4^5 + 9^6} & + \dots + \underbrace{4^{99} + 9^{100}} \\ \text{digit} & \downarrow & & \downarrow & & \downarrow & \downarrow \\ & 5 & & 5 & & 5 & 5 \end{array}$$

Thus $5 + 5 + 5 + \dots 5$ (50 times)

Hence the unit digit = 0

[Since $5 \times 50 = 250 \rightarrow$ unit digit is zero.]

- 48 Consider some appropriate values :

$$\text{As } p = 3.99, \quad q = 4.99, \quad r = 6.99$$

$$A = [p + q + r] = [3.99 + 4.99 + 6.99] = [15.97] = 15$$

$$\begin{aligned} B &= [p] + [q] + [r] = [3.99] + [4.99] + [6.99] \\ &= 3 + 4 + 6 = 13 \end{aligned}$$

$$\text{Hence } A - B = 2$$

- 49 Options (a), (b) and (c) are irrelevant.

$$\text{as } (25)^2 = (5)^2 \times 25$$

$$\text{or } (27)^2 = (9)^2 \times 9$$

$$\text{or } (12)^2 = (4)^2 \times 9$$

$$\text{or } (6)^2 = (3)^2 \times 4 \text{ etc.}$$

Hence (d) is correct.

NOTE If $a = b.c$ then $a^2 = b^2.c^2$, $a^2 = b^2.k$

Thus k must be a perfect square.

- 50 Since odd $\times$ even = even
even $\times$ even = even] even $\times$ even = even

It means when the odd numbers are added even number of times the resultant is always an even number for example :

$$3 + 5 = 8 \rightarrow \text{even}$$

$$3 + 5 + 9 = 17 \rightarrow \text{odd}$$

$$3 + 5 + 9 + 13 = 30 \rightarrow \text{even etc.}$$

Again, even + even = even

and odd + even = odd.

So the sum of odd numbers cannot be odd in order to be the resultant sum be an even number.

- 51 Go through options

$$1 + 2 + 3 + \dots + k = N^2 \quad (\text{a perfect square})$$

$$\frac{k(k+1)}{2} = N^2$$

$$\frac{1 \times 2}{2} = (1)^2 \rightarrow \text{a perfect square number,}$$

$$\frac{8 \times 9}{2} = 4 \times 9 = (2 \times 3)^2 \rightarrow \text{a perfect square number.}$$

$$\frac{49 \times 50}{2} = 49 \times 25 = (7 \times 5)^2 \rightarrow \text{a}$$

perfect square number.

- 52 Check through options consider some appropriate values.

$$(17 + 15) \text{ and } (17 - 15)$$

32 and 2, have a common factor 2.

$$(7 + 4) \text{ and } (7 - 4)$$

11 and 3, have no any common factor.

Hence options (a) and (b) are wrong and option (c) is correct.

- 53 If a and b be two such numbers, then

$$(ab)^{1/2} = 35 \Rightarrow ab = 1225$$

$$\text{and } \left(\frac{a+b}{2}\right) = 43 \frac{3}{4} \Rightarrow (a+b) = \frac{175}{2}$$

$$\therefore (a-b)^2 = (a+b)^2 - 4ab$$

$$(a-b)^2 = \frac{30625}{4} - 4900$$

$$(a-b) = \sqrt{\frac{30625 - 19600}{4}}$$

$$(a-b) = \frac{105}{2}$$

$$\text{Now solving } (a+b) = \frac{175}{2} \text{ and } (a-b) = \frac{105}{2}$$

$$\text{we get } a = 70 \text{ and } b = \frac{35}{2}$$

Hence (c) is correct.

Alternatively From option

$$\sqrt{70 \times x} = 35$$

$$\Rightarrow x = \frac{35}{2}$$

$$\text{and } \frac{70 + \frac{35}{2}}{2} = 43 \frac{3}{4}$$

Hence (c) is the right answer.

- 54 Such a least possible number = [L.C.M. of 1, 2, 3, ..., n] - 1

Now if the L.C.M. of (1, 2, 3, ..., n) = L

then the next higher number = $(m.L) - 1$

and $m.L = n!$ for a suitable value of m ; $m \in N$

Hence option (b) is the most appropriate one.

- 55 Let $n + (n + 2) + (n + 4) + (n + 6) = k$

$$\therefore 3n = 2(n + 4)$$

$$\Rightarrow n = 8$$

$$\therefore k = 8 + 10 + 12 + 14$$

$$k = 44$$

- 56 Go through options and match the conditions.

Alternatively Let there be k number of benches.

$$5k + 4 = 11(k - 4)$$

$$\Rightarrow 6k = 48$$

$$\Rightarrow k = 8$$

$$\text{Hence } 5k + 4 = 44$$

Thus there are 8 benches and 44 students.

$$57 \quad \frac{m}{n} = 25 \left(\frac{n}{m}\right)$$

$$\Rightarrow \left(\frac{m}{n}\right)^2 = 25 \Rightarrow \frac{m}{n} = \frac{5}{1} = 5$$

58 3^{3322} and 33^{332}

$$\Rightarrow (3^4)^{830} \times 3^2 \text{ and } 33^{332}$$

$$\text{Thus } (81)^{830} \times 3^2 > 33^{332}$$

$$\text{Again } 3^{3322} \text{ and } 33^{332}$$

$$\Rightarrow (3^6)^{553} \times 3^4 \text{ and } (333)^{22}$$

$$\text{Thus } (729)^{553} \times 3^4 > (333)^{22}$$

$$\text{Again } 3^{3322} \text{ and } 22^{333}$$

$$\Rightarrow (3^3)^{1107} \times 3 \text{ and } 22^{333}$$

$$\text{Thus } (27)^{1107} \times 3 > 22^{333}$$

Hence (a).

59 $a \times b = 25$

$$(-1) \times (-25) = 25$$

Thus the minimum required value of $a + b = (-1) + (-25)$
 $= -26$

Hence (d).

60 The least value of $p \cdot q = 5 \times (-10) = -50$

$$\text{The least value of } p \cdot q^6 = 4 \times (-9)^6 = 4 \times (729)^2$$

$$\text{The least value of } p^6 \cdot q = 5^6 \times (-10) = (-156250)$$

$$\text{The least value of } (pq)^5 = [5 \times (-10)]^5 = (-312500000)$$

Hence (d) is the correct option.

61 Please remember that all the factorial numbers greater than or equal to n are divisible by n . Hence $7!$, $8!$, $9!$, $10!$, ... etc. are divisible by 7. So we have to check only $1! + 2! + 3! + 4! + 5! + 6!$

$$\text{Hence } 1! + 2! + 3! + 4! + 5! + 6!$$

$$= 1 + 2 + 6 + 24 + 120 + 720 = 873$$

Thus 873 leaves a remainder of 5 when divided by 7.

Therefore $1! + 2! + 3! + 4! + 5! + \dots + 77!$ leaves the remainder 5.

62 It does not satisfy options a , b , c . Therefore option (d) is correct.

63 For one digit number $1 \times 9 = 9$

$$\text{for two digit numbers } 2 \times 90 = 180$$

$$\text{for three digit numbers upto } 500 = 3 \times 401 = 1203$$

$$\text{Thus the required value} = 9 + 180 + 1203 = 1392$$

64 If a , b , c , d , e , f all the six numbers form an A.P. then we can have only 9 values hence the common difference of the A.P. must be 1.

(Since common difference cannot be greater than 1.)

$\therefore$ If $a = 1$, then

$$1^2 \times 3^4 \times 5^6 \rightarrow \text{unit digit is 5}$$

If $a = 2$, then

$$2^3 \times 4^5 \times 6^7 \rightarrow \text{unit digit is 2}$$

If $a = 3$, then $3^4 \times 5^6 \times 7^8 \rightarrow \text{unit digit is 5}$

If $a = 4$, then

$$4^5 \times 6^7 \times 8^9 \rightarrow \text{unit digit is 2}$$

65 The greatest possible number

$$= \text{H.C.F. of } (1313 - 17) \text{ and } (621 - 9)$$

$$= \text{H.C.F. of } 1296 \text{ and } 612 = 36$$

Now the total number of factors of $36 = (2 + 1)(2 + 1) = 9$

$$(\text{Since } 36 = 2^2 \times 3^2)$$

66 Except $x = 0$ and $x = 5$ all the values are acceptable, hence the range of x is $R - \{0, 5\}$ where R represents all the real numbers.

$$67 \quad p_1 + p_2 + p_3 + p_4 = 204 \quad \dots(i)$$

$$p_1 + p_4 = p_2 + p_3 \quad \dots(ii)$$

$$\Rightarrow p_2 - p_1 = p_4 - p_3 \quad \dots(iii)$$

$$\text{and } p_3 - p_2 = 2(p_2 - p_1)$$

$$\Rightarrow p_3 - p_2 = 2(p_4 - p_3)$$

$$\Rightarrow p_1 + x = p_2$$

$$p_2 + 2x = p_3 \quad \dots(iv)$$

$$p_3 + x = p_4$$

$$\text{Also } \left(\frac{p_1 + p_2 + p_3 + p_4}{4} \right) = 51 = 17 \times 3$$

Thus from the relation (ii), we can deduce that

$$p_1 + p_4 = p_2 + p_3 = 102$$

Thus by hit and trial we will find the only possible values of p_1 , p_2 , p_3 and $p_4 = 31, 41, 61$ and 71 respectively.

 **NOTE** The unit digits of $(p_1 \text{ and } p_4)$ or $(p_2 \text{ and } p_3)$ can be only (1 and 1), (3 and 9), but not necessarily in the order. So consider only those two digit prime numbers which satisfy all the given conditions.

68 Let the price of simcard be x and handset be y

$$\text{then } x + y = 5000$$

$$\text{and } x = y - 4000$$

$$\Rightarrow (y - 4000) + y = 5000$$

$$\Rightarrow 2y = 9000$$

$$\Rightarrow y = 4500$$

 **Alternatively** Go through options

$$\begin{array}{cc} \text{handset} & \text{Simcard} \\ 4500 + 500 & = 5000 \end{array}$$

$$4500 - 500 = 4000$$

69 At $x = 0$ or $y = 0$, we get infinite number of ordered solutions set.

$$70 \quad l = \frac{292k + 7}{23}$$

Therefore at $k = 1$, $l = 13$.

Hence (d).

- 71** Since a, c are neither prime nor composite it means $a = c = 1$ and $b = 2, 3, 5, 7$ and $d = 2, 3, 5, 7$ but $b \neq d$.

Thus from $ab + 4 = cd$

$\Rightarrow ab = 13$ and $cd = 17$, the only possible combination.

$\therefore ba + 40 = dc$

$\Rightarrow 31 + 40 = 71$

Therefore $\frac{ab + ba}{cd + dc} = \frac{13 + 31}{17 + 71} = \frac{44}{88} = \frac{1}{2}$

- 72** The best way is to go through options.

Let there are initially 3 boxes then,

1st customer gets $= \frac{3}{2} + \frac{1}{2} = 2$

Remaining boxes $= 3 - 2 = 1$

2nd customer gets $= \frac{1}{2} + \frac{1}{2} = 1$

Remaining boxes $= 0$

Hence presumed option (b) is correct.

- 73** $123454321 = (11111)^2$

Hence (d).

- 74** $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \dots - 198^2 + 199^2$

$$= (1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) + \dots + (197^2 - 198^2) + 199^2$$

$$= (-3) + (-7) + (-11) + (-15) + \dots + (-395) + (199)^2$$

$$= \left[-\left(\frac{3+395}{2}\right) \times 99 \right] + 39601$$

$$= (-199 \times 99) + 39601$$

$$= (-19701) + 39601 = 19900$$

- 75** The G.P. is as following :

$a, ar, ar^2, ar^3, \dots$ etc.

then LCM of this G.P. $= ar^{n-1}$

and HCF of this G.P. $= a$

$$\therefore \frac{\text{LCM}}{\text{HCF}} = \frac{ar^{n-1}}{a} = r^{n-1}$$

- 76** From option (a)

Total number of brothers and sisters $= 4$

Also there are two brothers and 2 sisters.

- 77** $(4 @ 5) = (4 \# 5) = \frac{4}{5} \quad (\because 4 < 5)$

$$\text{and} \quad (6 @ 5) = (5 \# 6) = \frac{5}{6} \quad (\because 6 > 5)$$

$$\text{and} \quad \left(\frac{4}{5}\right) @ \frac{5}{6} = \frac{\frac{4}{5}}{\frac{5}{6}} = \frac{4}{5} \times \frac{6}{5} = \frac{24}{25} \quad (\because \frac{4}{5} < \frac{5}{6})$$

$$\therefore (4 @ 5) @ (6 @ 5) = \frac{\frac{4}{5}}{\frac{5}{6}} = \frac{24}{25}$$

- 78** The only possible values of p, q, r are 1, 2, 4

So $s = 5, t = 6, u = 7$.

Hence the number is 124567 therefore

$$p + q + r + s + t + u = 25$$

- 79** $4^{1111} \times 5^{2222} = 2^{2222} \times 5^{2222} = 1 \times 10^{2222}$

Hence there will be 2223 number of digits.

- 80** Since N is always even. Thus p is always odd

[As even + odd = odd]

- 81** Let $\frac{p}{q} = -7$ and $\frac{r}{s} = 3$

then $\left|\frac{p}{q}\right| = 7$ and $\left|\frac{r}{s}\right| = 3$

Thus $\left|\frac{p}{q}\right| > \left|\frac{r}{s}\right|$

Again if $\frac{p}{q} = 4$ and $\frac{r}{s} = 7$ then $\left|\frac{p}{q}\right| > \left|\frac{r}{s}\right|$, hence (d).

- 82** Since all the numbers u, v, w, x are negative, but when $u^v + v^w + w^x = 0$, then u^v . It means there must be some terms whose value will be positive and thus when these terms get added up with negative value they make the expression zero.

e.g., $(k) + (-k) = 0$

Hence we can say that there must be some even integers which converts a negative number into a positive number. Further we know that an even number when multiplied with any other (even or odd) number it finally makes an even number.

Explanation $u^v + v^w + w^x = 0$

$$\Rightarrow (-u)^{-v} + (-v)^{-w} + (-w)^{-x} = 0 \quad \{ \because u, v, w, x \in I^- \}$$

Again, if $k + l + m = 0$

$$\Rightarrow k + l = m \quad \text{or} \quad k + m = l \quad \text{or} \quad l + m = k$$

i.e., half of the numerical values will be positive and half of the numerical values will be negative.

Now, if $(-k)^{\text{even}} \rightarrow$ positive value

So, if there exists some even integer n , then

$k \times l \times m \times n \rightarrow$ even (though k, l, m can be odd).

- 83** The unit digit of 2^{3^4} is 2.

The unit digit of 3^{4^5} is 1.

The unit digit of 4^{5^6} is 4.

The unit digit of 5^{6^7} is 5.

The unit digit of 6^{7^8} is 6.

The unit digit of 7^{8^9} is 1.

Therefore the unit digit of :

$$2^{3^4} \times 3^{4^5} \times 4^{5^6} \times 5^{6^7} \times 6^{7^8} \times 7^{8^9} = 0$$

(Since $2 \times 1 \times 4 \times 5 \times 6 \times 1 = 240$)

☞ Alternatively 5 always gives unit digit 5 irrespective of its (positive) power and similarly 6 also gives 6 as a unit digit irrespective of its power.

So, $5 \times 6 = 30$. Hence you will get the unit digit zero without solving the complete expression.

- 84** In order that a, b, c be the least, then

$$(a - 7) = (b - 10) = (c - 12)$$

$$\text{Again } (a - 7)(b - 10)(c - 12) = 1000$$

$$\Rightarrow 10 \times 10 \times 10 = 1000$$

$$\Rightarrow a = 17, b = 20 \text{ and } c = 22$$

Therefore $(a + b + c) = 59$.

$$\begin{aligned} \textbf{85} \quad \frac{23^{32} - 9}{16} &\rightarrow \left(\frac{23^{32}}{16} - \frac{9}{16} \right) \rightarrow \left(\frac{7^{32}}{16} - \frac{9}{16} \right) \\ &\rightarrow \left[\frac{(7^2)^{16}}{16} - \frac{9}{16} \right] \rightarrow \left[\frac{1^{16}}{16} - \frac{9}{16} \right] \\ &\rightarrow \frac{1 - 9}{16} \rightarrow \frac{-8}{16} \rightarrow \frac{16 - 8}{16} \rightarrow \frac{8}{16} \rightarrow 8 \end{aligned}$$

- 86** Very similar to the problem number 84.

$$\text{Hence } (x - 5) = (y + 6) = (z - 8) = 11$$

$$\Rightarrow x = 16, y = 5, z = 19$$

$$\text{Thus } (x + y + z) = 40.$$

- 87** $(x - 6)(y + 7)(z - 4)$ will be maximum only when

$$(x - 6) = (y + 7) = (z - 4) = k \text{ (say), for a given value of}$$

$$(x + y + z) = 21$$

$$\Rightarrow x = k + 6$$

$$y = k - 7$$

$$z = k + 4$$

$$\therefore x + y + z = 21$$

$$\Rightarrow (k + 6) + (k - 7) + (k + 4) = 21$$

$$\Rightarrow 3k + 3 = 21$$

$$\Rightarrow k = 6$$

$$\text{Therefore, } (x - 6) \cdot (y + 7)(z - 4) = k^3 = (6)^3 = 216$$

$$\begin{aligned} \textbf{88} \quad \frac{3^{37} + 4^{37}}{7} &\rightarrow \left(\frac{3^{37}}{7} + \frac{4^{37}}{7} \right) \rightarrow \left[\frac{(3^2)^{18} \times 3}{7} + \frac{(4^2)^{18} \times 4}{7} \right] \\ &\rightarrow \left(\frac{2^{18} \times 3}{7} + \frac{2^{18} \times 4}{7} \right) \rightarrow \left(\frac{(2^3)^6 \times 3}{7} + \frac{(2^3)^6 \times 4}{7} \right) \\ &\rightarrow \left(\frac{3}{7} + \frac{4}{7} \right) = \left(\frac{7}{7} \right) \rightarrow \text{Remainder zero.} \end{aligned}$$

☞ Alternatively $a^n + b^n$ is divisible by $(a + b)$ if n is odd. Hence $(3^{37} + 4^{37})$ is divisible by 7 ($= 3 + 4$). Hence leaves no remainder.

- 89** $7^{74} - 4^{74}$ is divisible by 3

Since $a^n - b^n$ is divisible by $(a - b)$ and

$$\text{Again } 7^{74} - 4^{74} = (7^2)^{37} - (4^2)^{37} = 49^{37} - 16^{37}$$

Now, since $a^n - b^n$ is divisible by $(a - b)$ if n is odd, then $7^{74} - 4^{74}$ (i.e., $49^{37} - 16^{37}$) is divisible by 11.

Hence (d).

- 90** Try to find out 11^2 in less than 22!.

You can't find. So (a) is the correct choice.

- 91** The number of zeros at the end of $2^3 \times 3^4 \times 4^5 \times 5^6 = 6$

$$\text{The number of zeros at the end of } 3^5 \times 5^7 \times 7^9 \times 8^{10} = 7$$

$$\text{The number of zeros at the end of } 4^5 \times 5^6 \times 6^7 \times 7^8 = 6$$

$$\text{The number of zeros at the end of } 10^2 \times 15^3 \times 20^4 = 7$$

So the number of zeros at the end of the whole expression = 6.

- 93** Min. (1, 2, 3) + Max. (1, 2, 3)

$$= \text{Min. (2, 6, 3) + Max. (1, 8, 3) = 2 + 8 = 10}$$

- 94** Labh [Hani (1, 2, 3), Hani (2, 3, 4), Hani (3, 4, 5)]

$$= \text{Labh [4, 5, 6]} = \frac{4 + 5 + 6}{3} = 5$$

- 95** Min. [Hani (1, 2, 3), Hani (2, 3, 4), Hani (3, 4, 5)]

$$+ \text{Max. [Labh (1, 2, 3), Labh (2, 3, 4), Labh (3, 4, 5)]}$$

$$= \text{Min. [4, 5, 6] + Max. [2, 3, 4]}$$

$$= \text{Min. (20, 30, 24) + Max. (8, 81, 16)}$$

$$= 20 + 81 = 101$$

- 96** $\therefore (55)_{10} = (25)_{25}$

$$\begin{array}{r|l} 25 & 55 \\ \hline 25 & 2 \rightarrow 5 \\ \hline & 0 \rightarrow 2 \end{array}$$

- 97** $Q = 4, 8$

$$P = 3, 6, 9, 1, 4, 7; \quad \text{since } P, Q > 0$$

So there are total 6 values of (P, Q) .

- 98** Total number of 6s = Number of actual 6s

$$+ \text{Number of wrong 6s}$$

$$\text{Number of Actual 6s} = 20 \text{ (as unit digits)}$$

$$+ 20 \text{ (at tens digit)} + 100 \text{ (as hundreds digit)}$$

$$= 140 \text{ 6s}$$

$$\text{Number of wrong 6s (which are at the place of 8s)}$$

$$= 20 \text{ (at unit place)} + 20 \text{ (at tens place)} = 40$$

$$\text{Hence total number of 6s} = 180$$

- 99** If $p^2 - pq = 0 \Rightarrow p(p - q) = 0 \Rightarrow p = 0$ or $p = q$, but since $p \neq q$, then $p = 0$ is only possible solution.

- 100** $15990 = 2 \times 3 \times 5 \times 13 \times 41$

$$= 2 \times 3 \times 13 \times 5 \times 41 = 78 \times 205$$

Hence the only possible option is (d).

- 101** $n = 1, 2, 3, \dots, 7$.

- 102** If you go through options, then you will find option (c) is correct. Since divisor is always greater than remainder.

☞ Alternatively $\frac{A}{D} \rightarrow \text{Remainder } 13$
 $\frac{B}{D} \rightarrow \text{Remainder } 31$
 $\frac{A+B}{D} \rightarrow \text{Remainder should be } 44$

But it is given that the remainder is 4.
Hence, the divisor must be 40 ($= 44 - 4$).

103 Since $(N)^{2p} \rightarrow \text{unit digit } 6$
and $(N)^{(2p+1)} \rightarrow \text{unit digit } 4$

It means every even power of N has unit digit 6 and every odd power has unit digit 4. It means the unit digit of N is 4. Hence the largest two digit number is 94, whose unit digit is 4.

104 Just replace x by $(x + 1)$ and simplify or consider some numerical value of x then check through options.

105 $60 = \pm \begin{bmatrix} 1 \times 60 \\ 2 \times 30 \\ 3 \times 20 \\ 4 \times 15 \\ 5 \times 12 \\ 6 \times 10 \end{bmatrix}$
 $\Rightarrow a + b = \begin{cases} = (-4) + (-15) = -19 \\ = 5 + 12 = 17 \\ = 2 + 30 = 32 \end{cases}$

106 To find the required unit digit just add up the single unit digits of the two previous consecutive numbers successively and you will find that every 15th term of this sequence has unit digit zero. Hence the unit digit of the 75th term of this sequence is 0.

107 Every n th term has its unit digit 5 if n is divisible by 5 but it has unit digit zero (0) if n is divisible by 15. Since we have to find out the unit digit of 55th term. So here 55 is divisible by 5 but not by 15, hence the unit digit will be 5.

108 The unit digit of the sum of the 88th term plus 89th term will be equal to the unit digit of 90th term. Hence it will be zero. (See the question number 106).

109 Always 5. Since when we multiply a number whose unit digit is 5 with any odd number the unit digit of the resultant number is 5.

111 The least possible product of any 3 numbers can be found by multiplying numerically greatest possible number, but at least one number must be negative.

Thus the required product $= (-5) \times (24) \times (25) = (-3000)$
Hence (c).

112 Since $q - p = r - q = 2$

Therefore p, q, r are in A.P.

But the least possible value of p, q, r can be 1. So if $p = 1$, then $q = 3$ and $r = 5$.

Hence $(p + q)(q + r)(r + p) = 4 \times 8 \times 6 = 192$

114 Check for any value of R .

116 Options (a) and (b) are wrong since the maximum cost of total pens is nearly ₹ 745. Now, to maximize the number of pens of ₹ 5, we have to minimize the number of pens of ₹ 3 and the total cost cannot exceed ₹ 745.6
So by hit and trial get the required result. As :

Number of pens of ₹ 3 each	Number of pens of ₹ 5 each	Total Cost
$1 \times 3 = 3$	$149 \times 5 = 745$	748
$2 \times 3 = 6$	$148 \times 5 = 740$	746
$3 \times 3 = 9$	$147 \times 5 = 735$	744

Hence the maximum number of pens of ₹ 5 is 147.

117 The last digit of this number is 9.

Since $(1 \times 9) + (2 \times 50) = 109$

i.e., there are 9 digits of one digit number and 100 digits are of 50 numbers of 2 digit.

Again the 50th two digit number is 59, hence the last digit of the given number is 9.

Now, if we add 1 to this number it will become divisible by both 2 and 5.

118 If the sum of all the 109 digits of the given number is divisible by 3, then the given number itself is divisible by 3.

Now, the sum of all the 109 digits

$= [\text{sum of all the unit digits of all the 59 numbers}$
 $+ \text{sum of all the tens digit of all the 2 digit numbers upto 59}]$
 $= 6 \times (45) + 5 \times (15)$
 $= 270 + 75 = 345$

Now since the sum of all the digits is 345. Hence it is divisible by 3.

Again a number is divisible by 11 if the difference of sum of all the digits at even places and sum of all the digits at odd places is divisible by 11 or it is zero.

Now, 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 ... 575859.

The sum of all the digits at odd places

$= 5 \times (45) + (9 + 7 + 5 + 3 + 1) = 250$
[Counted from right side]

The sum of all the digits at even places

$= 10 \times (1 + 2 + 3 + 4 + 5) + (8 + 6 + 4 + 2)$
 $= 150 + 20 = 170$

Thus the required difference $= 250 - 170 = 80$

But if we subtract the least possible number i.e., 3 from the given number (1 2 3 4 5 ... 5859) the new number will become (1 2 3 4 5 ... 5856)

Now the required difference $= 247 - 170 = 77$

Since 77 is divisible by 11 hence the given number is also divisible by 11.

Again, the number (12345 ... 5856) is also divisible by 3. Since if a number is divisible by 3 and we subtract a multiple of 3 from this number then the resultant number is also divisible by 3.

119 Since $F < B < G < A$

Now, if $F = 1$, then $1 < 2 < 3 < 4$, adults cannot be in one family.

if $F = 2$, then $2 < 3 < 4 < 5$; 5 adults cannot be in 2 families.

if $F = 3$, then $3 < 4 < 5 < 6$, not possible since one family has only one adult.

Again, if $F = 4$ then $4 < 5 < 6 < 7$, possible since

$$(3 \times 2) + (1 \times 1) = 7$$

120

$$A = 3^{3^{3^3}} = 3^{3^{27}}$$

and

$$C = 3^{3^{3^3}} = 3^{3^{3^3}}$$

Hence $C > A$.

Hence either (b) or (d) option is correct.

Now $A = 3^{3^{3^3}} = 3^{3^{27}}$

and

$$D = 3^{3^{3^3}}$$

Hence $A > D$ (Since $3^{27} > 3^{3^3}$)

Thus the correct relation is $C > A > B > D$.

Hence, option (b) is correct.

121

$$\frac{A}{D} \rightarrow \text{Remainder } 23$$

and

$$\frac{B}{D} \rightarrow \text{Remainder } 3$$

$$\frac{A+B}{D} \rightarrow \text{Remainder } 26$$

Therefore it is clear that the divisor D cannot be less than or equal to 26. Thus the possible values of D are greater than 26.

122 $a^2 - b^2 = (a + b)(a - b) = m \times n$

$$\Rightarrow a = \frac{m+n}{2} \text{ and } b = \frac{m-n}{2}$$

Now, $729 = 1 \times 729 \Rightarrow a = 365, b = 364$

$$3 \times 243 \Rightarrow a = 123, b = 120$$

$$9 \times 81 \Rightarrow a = 45, b = 36$$

$$27 \times 27 \Rightarrow a = 27, b = 0$$

Thus we can express it in 4 ways, Hence (a).

☞ Alternatively The required answer is always equal to the number of ways in which a given product can be expressed as the product of two factors.

123 $2310 = 2 \times 3 \times 5 \times 7 \times 11$. Now since 2310 has to be expressed as a product of 3 factors, as $2310 = a \times b \times c$. Now, we exclude one prime factor say a then we find the rest part as a product of 2 factors.

Thus when $a = 1$, then $b \times c$

$$= \frac{(1+1)(1+1)(1+1)(1+1)(1+1)}{2} = 16 \text{ ways}$$

Now, if $a = 2$, then

$$b \times c = \frac{(1+1)(1+1)(1+1)(1+1)}{2} = 8 \text{ ways}$$

when $a = 3$, then

$$b \times c = \frac{(1+1)(1+1)(1+1)(1+1)}{2} = 8 \text{ ways}$$

Similarly when $a = 5, 7, 11$ the value of $b \times c = 8$ ways

But when $a = 2$, then the $(b \times c) = 8 - 1 = 7$ since one way of expression has been included in the case of $a = 1$.

Similarly when $a = 3, 5, 7$ and 11 then the number of ways will be $6 (= 8 - 2)$, $5 (= 8 - 3)$, $4 (= 8 - 4)$, $3 (= 8 - 5)$ respectively.

Thus the total number of ways in which 729 can be expressed as the product of 3 factors $= 16 + 7 + 6 + 5 + 4 + 3 = 41$ ways.

124 The sum of n th row is $f(n) = 2^n - 2$

Therefore, $f(100) = 2^{100} - 2$

Now, $\frac{2^{100} - 2}{100}$

$$\Rightarrow \frac{(2^{10})^{10} - 2}{100} \Rightarrow \frac{(1024)^{10} - 2}{100}$$

$$\Rightarrow \frac{24^{10} - 2}{100} \Rightarrow \frac{76 - 2}{100} \Rightarrow 74$$

Hence choice (b) is the correct one.

125 $7^{21} \leq n^{28} \leq 3^{63}$

$$\Rightarrow 7^{21} \leq n^{28} \leq 27^{21} \Rightarrow 7^{21} \leq n^{\left[\frac{4}{3}(21)\right]} \leq 27^{21}$$

$$\Rightarrow 7^{21} \leq \left(\frac{4}{n^3}\right)^{21} \leq 27^{21} \Rightarrow 7 \leq n^{\frac{4}{3}} \leq 27$$

$$\text{For, } 7 \leq n^{\frac{4}{3}} \Rightarrow 7^3 \leq n^4 \Rightarrow n \geq 5$$

$$\text{For, } n \leq (27)^{3/4} \Rightarrow n \leq 9\sqrt{3}$$

$$n \leq 11$$

It implies that $n = 5, 6, 7, 8, 9, 10, 11$

Hence choice (b) is the correct one.

126 Both a local train and an express train stop at every 15th station.

Therefore, number of pairs of stations connected by a local

and an express train $= \left\lfloor \frac{325}{15} \right\rfloor = 21$

Both a local train and a bullet train stop at every 24th station. Therefore, number of pairs of stations connected

by a local and a bullet train $= \left\lfloor \frac{325}{24} \right\rfloor = 13$

Both an express train and a bullet train stop at every 40th station.

Therefore, number of pairs of stations connected by an express and a bullet train = $\left\lfloor \frac{325}{40} \right\rfloor = 8$

All three types of trains stop at every 120th station.

Therefore, number of pairs of stations connected by all three trains = $\left\lfloor \frac{325}{120} \right\rfloor = 2$

Thus the required number of pairs of stations
 $= (21 - 2) + (13 - 2) + (8 - 2) = 36$

Hence choice (c) is the correct one.

127 Number of elements in A, which leaves remainder 0 when divided by 3 = 29

Number of elements in A, which leaves remainder 1 when divided by 3 = 29

Number of elements in A, which leaves remainder 2 when divided by 3 = 30

If the set B includes 30 elements of A which leaves remainder 2 and any one element which leaves remainder 0, the sum of no two element of set B will be divisible by 3.

That is, total number of required elements in set B = 31

Hence choice (a) is the correct one.

128 $(2)_a \times (17)_b = (32)_b$

$$\Rightarrow 2 \times (b + 7) = 3b + 2$$

$$\Rightarrow b = 12$$

Since $a < b$ and $a > 2$, so the possible values of a are 3, 4, 5, ..., 11. It shows that there are total 9 values of a .

Hence choice (a) is the correct one.

Hint Since the number $(2)_a$ is represented in base a , so base a must be greater than 2.

129 Seed (7) = 7

$$\text{Seed (16)} = \text{Seed (1 + 6)} = 7$$

$$\text{Seed (25)} = \text{Seed (2 + 5)} = 7$$

$$\text{Seed (34)} = \text{Seed (3 + 4)} = 7$$

.....

$$\text{Seed (997)} = \text{Seed (9 + 9 + 7)}$$

$$= \text{Seed (25)} = \text{Seed (2 + 5)} = 7$$

It implies that we get the desired number at the interval of 9 numbers. That is we have $n = 9k + 7$, where $k = 0, 1, 2, \dots, 110$

Therefore we have 111 positive integers less than 999 which have seed $(n) = 7$.

Hence choice (c) is the correct one.

130 The difference of the given numbers must be divisible by 9, if the two numbers have the same digits in any random order.

Further in this case, x and y cannot exceed beyond 9. Therefore,

Choice	$x + y$	$S = x + 4 + 8 + 1 + 4 + y$	Is S divisible by 9?	$x + y < 18$	Are both the conditions met?
(a)	18	35	No	No	No
(b)	19	36	Yes	No	No
(c)	91	108	Yes	No	No

Hence choice (d) is the correct one.

NOTE The correct value $x + y = 10$.

Alternatively The difference of the given numbers must be divisible by 9, if the two numbers have the same digits in any random order.

Therefore, $x + y = 1, 10, 19, 28, 37, \dots$

Since, $y \neq 0$, so $x + y \neq 1$

Again since, $x + y < 18$, so $x + y = 10$ is the only acceptable value.

Alternatively The digital root of the difference of two numbers should be 9.

That is the digital root of $x4814y$ should be 9.

Therefore, $x + 4 + 8 + 1 + 4 + y = x + y = 17$

Again, $(x + y) + (1 + 7) = x + y + 8$

But since the digital root of $x + y + 8$ is 9, so the digital root of $x + y$ will be 1.

So the choice (a) and (c) are ruled out.

Further, you know that $x + y < 18$, as $x < 9$ and $y \leq 9$.

So the choices (a) and (b) are ruled out.

Hence choice (d) is the correct one.

Hint you know that the digital root of $x + y$ is 1 so the possible values of (x, y) are

$(0, 1), (1, 0), (1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), (9, 1)$. But if (x, y) is $(0, 1)$ or $(1, 0)$, you would get $x + y$ as a 1-digit number, not a 2-digit number.

Also, $y \neq 0$, as $a \neq f$, so $(x, y) \neq (1, 0)$.

And if you consider all other possibilities of (x, y) , you would get $x + y = 10$ and that is a 2-digit number.

131 $10! = 2^8 \times 3^4 \times 5^2 \times 7^1$

$b!$	$a!$	Number of pairs (a, b)
10!	—	0
9!	2!	1
8!	2!, 3!	2
7!	2!, 3!, 4!, 5!, 6!	5
6!	2!, 3!, 4!, 5!	4
5!	2!, 3!, 4!	3
4!	2!, 3!	2
3!	2!	1
2!	—	0

Therefore total number of sets $(a, b) = 18$

Hence choice (d) is the answer.

- 132** Factors of 120 are 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60 and 120.

Number of matchsticks in each side of the polygon	Number of sides in the respective polygon
1	120
2	60
3	40
4	30
5	24
6	20
8	15
10	12
12	10
15	8
20	6
24	5
30	4
40	3
60	2
120	1

There are total 16 cases, however with the last 2 cases he cannot form any polygon as no polygon has less than 3 sides. Thus he can form only 14 distinct polygons using all the 120 matchsticks.

Hence choice (c) is the correct one.

☞ **Alternatively** $120 = 2^3 \times 3^1 \times 5^1$

Total number of factors of $120 = (3+1)(1+1)(1+1) = 16$
But these factors include 1 and 2, which have to be excluded, as there is no polygon with less than 3 sides.
Thus we have 14 possible cases.

- 133** The number N can be expressed as $N = a^p \times b^q \times c^r \times \dots$, where a, b, c are prime numbers and p, q, r are natural numbers.

Now since $90 = 2 \times 3 \times 3 \times 5$

Therefore, N can have either 1 or 2 or 3 or 4 distinct factors.

When $f = 1$, N can be written as $N = a^{89}$

When $f = 2$, N can be written as $N = a^1 \times a^{44}$ or $a^2 \times b^{29}$ or $a^4 \times b^{17}$, etc.

When $f = 3$, N can be written as $N = a^1 \times b^2 \times c^{14}$ or $a^1 \times b^8 \times c^4$, etc.

When $f = 4$, N can be written as $N = a^1 \times b^2 \times c^2 \times d^4$

Thus you see that the possible values of f are 1, 2, 3 and 4. So the product of 1, 2, 3, and 4 is 24. And therefore total number of factors of 24 can be computed as following.

$$24 = 2^3 \times 3^1$$

Total number of factor of $24 = (3+1) \times (1+1) = 4 \times 2 = 8$

Hence, choice (b) is the correct one.

- 134** The number of families in a village = $\frac{n+150}{n+6}$

$$= \frac{[(n+6)+144]}{n+6} = 1 + \frac{144}{n+6}$$

$n+6$ must be a factor of 144 as the number of families is always an integer. The factors of 144 are 1, 2, 3, 4, 6, 8, 9, 12, 16, 24, 36, 48, 72 and 144.

So the possible values of $(n+6)$ are 8, 9, 12, 16, 24, 36, 48, 72 and 144, such that $n \geq 1$. That is only 9 values are feasible.

Since each distinct value of n or $(n+6)$ determines the number of families in a village, so there will be total 9 villages.

Hence choice (b) is the correct one.

- 135** This problem is basically asking about the value of $N < 999$, for which N has the maximum number of factors. In order to maximize the number of factors, we need to consider.

(i) The number of prime factors as many as possible.

(ii) Face value of each prime factor as less as possible.

Therefore, let us consider the prime factors of N are 2, 3, 5, 7 and 11.

But, $2 \times 3 \times 5 \times 7 \times 11 > 999$, so the greatest factor 11 has to be removed, as N has to be less than 999.

Now, we are sure that $2 \times 3 \times 5 \times 7 = 210 < 999$

In order to maximize the value of N , we will try to increase power of the lower factors.

If we consider $2^2 \times 3 \times 5 \times 7 = 420 < 999$, the number of factors of N is 24.

If we consider $2^3 \times 3 \times 5 \times 7 = 840 < 999$, the number of factor of N is 32

Therefore for $N = 840$, we have the maximum number of factors of N , such that $N < 999$.

Thus any artist can wear 840 combinations of dhoti and kurta.

Hence choice (b) is the correct one.

$$\begin{aligned}
 & \mathbf{136} \quad (1297 + 279 - 79 - 9)^{127} \\
 & \quad - (1279^{127} + 279^{127} - 79^{127} - 9^{127}) \\
 & \Rightarrow (1279 + 279 - 79 - 9)^{127} \\
 & \quad - (1279 + 279 - 79 - 9) \\
 & \quad - (1279)^{127} + 279^{127} - 79^{127} - 9^{127} \\
 & \quad + (1279 + 279 - 79 - 9) \\
 & \Rightarrow (1279 + 279 - 79 - 9)^{127} \\
 & \quad - (1279 + 279 - 79 - 9) \\
 & \quad - (1279^{127} - 1279) - (279^{127} - 279) \\
 & \quad + (79^{127} - 79) + (9^{127} - 9)
 \end{aligned}$$

As per the Fermat's little theorem, if a is an integer and p is a prime number, $(a^p - a)$ is divisible by p .

Therefore each term in the above expression is definitely divisible by 127.

Hence choice (d) is the correct one.

137

$\lfloor \frac{x}{9} \rfloor$	x	$\lfloor \frac{x^2}{99} \rfloor$	x	Values of x that are common	Number of common values of x
0	(0, 1, ..., 8)	0	(0, 1, ..., 9)	{0, 1, 2, ..., 8}	9
1	{9, 10, ..., 17}	1	{10, 11, ..., 14}	{10, 11, ..., 14}	5
2	{18, 19, ..., 26}	2	{15, 16, 17}	None	0
3	{27, 28, ..., 35}	3	{18, 19}	None	0
4	{36, 37, ..., 44}	4	{20, 21, 22}	None	0
...	...	...	...	...	...
...	...	...	...	...	...

Thus there are total 4 (= 9 + 5) values of x that satisfy the given equation.

Hence, choice (b) is the correct one.

Hint Except 9 all other values from 0 to 14 satisfy the given relation.

138 $\frac{\lfloor x \rfloor + 3}{4}, \frac{\lfloor x \rfloor + 8}{5}, \frac{\lfloor x \rfloor + 15}{6}, \frac{\lfloor x \rfloor + 24}{7}, \dots, \frac{\lfloor x \rfloor + 168}{15}, \frac{\lfloor x \rfloor + 195}{16}$
 $\Rightarrow \frac{\lfloor x \rfloor + 3}{4}, 1 + \frac{\lfloor x \rfloor + 3}{5}, 2 + \frac{\lfloor x \rfloor + 3}{6}, 3 + \frac{\lfloor x \rfloor + 3}{7}, \dots, 11 + \frac{\lfloor x \rfloor + 3}{15}, 12 + \frac{\lfloor x \rfloor + 3}{16}$

A fraction is in its simplest form when numerator and denominator are co-prime. Thus, $\lfloor x + 3 \rfloor$ must be co-prime to 4, 5, 6, ..., 15, 16.

When $x = 9.99$, $\lfloor x + 3 \rfloor = 12$, which is not co-prime with 4, 6, 8, 9, 10, 12, 14, 15 and 16.

When $x = 11.11$, $\lfloor x + 3 \rfloor = 14$, which is not co-prime with 4, 6, 7, 8, 10, 12, 14 and 16.

When $x = 19.91$, $\lfloor x + 3 \rfloor = 22$, which is not co-prime with 4, 6, 8, 10, 11, 12, 14 and 16.

When $x = 16.61$, $\lfloor x + 3 \rfloor = 19$, which is co-prime with all the denominators, 4, 5, 6, ..., 15 and 16.

Hence choice (d) is the correct one.

139 $59059 - 58123 = 936$

Now, we know that $936 = 2^3 \times 3^2 \times 13$

$936 = (1 \times 936), (2 \times 468), (3 \times 312), (4 \times 234), (6 \times 156), (8 \times 117), (9 \times 104), (12 \times 78), (13 \times 72), \dots$

Thus we see that there are exactly 7 numbers (936, 468, 312, 234, 156, 117, 104) which can divide the difference of the given numbers, so there are 7 possible values of N . Hence choice (d) is the correct one.

140 The difference between the first three numbers and their respective remainders is same, so first of all we find the pertinent number of the first 3 numbers as follows.

$$(\text{LCM of } 3, 5, 7)k - 2 = 105k - 2$$

Now since the required number if divided by 11 leaves the remainder 2, so by hit and trial we can arrive at the following results.

When $k = 8, 19, 30, 41, \dots (105k - 2)$ will leave remainder, when divided by 11.

Thus the smallest possible such number is $105 \times 8 - 2 = 838$

Hence choice (c) is the correct one.

Alternatively Go through the given choices.

When 337 is divided by 5 it leaves remainder 2, which is not the same as required. So this choice is invalid.

When 418 is divided by 11, it leaves no remainder, which is not the same as required. So this choice is invalid.

When 912 is divided by 3 or 4, it leaves no remainder, which is not the same as required. So this choice is invalid.

Thus it is obvious that 838 should be the valid number.

Hence choice (c) is the correct one.

141 The difference between the first three numbers and their respective remainders is same, so first of all we find the pertinent number for the first three numbers as follows.

$$(\text{LCM of } 3, 4, 6)k - 1 = 12k - 1$$

Similarly, the difference between the last three numbers and their respective remainders is same, so first of all we find the pertinent number for the last 3 numbers as follows. (LCM of 7, 10, 13) $p - 3 = 910p - 3$

We get the required number when both the relations are satisfied simultaneously. Then we should have,

$$\begin{aligned} 12k - 1 &= 910p - 3 \\ \Rightarrow 12k &= 910p - 2 \\ \Rightarrow k &= \frac{910p - 2}{12} \end{aligned}$$

Therefore, when $p = 5$, k will be integer and then we will get the required value.

Thus the required minimum value = $910 \times 5 - 3 = 4547$

Hence choice (d) is the correct one.

Alternatively Go through the given choices.

When 4337 is divided by 4 it leaves remainder 1, which is not the same as required. So this choice is invalid.

When 3467 is divided by 7, it leaves remainder 2, which is not the same as required. So this choice is invalid.

When 5443 is divided by 10, it leaves remainder 3, which is not the same as required. So this choice is invalid.

Thus, it is obvious that 4547 should be the valid number.

Hence choice (d) is the correct one.

142 $2^{387} + 3^{344} = 2^{9 \times 43} + 3^{8 \times 43}$
 $= (2^9)^{43} + (3^8)^{43} = 512^{43} + 6561^{43}$

As we know that $a^n + b^n$ is always divisible by $(a + b)$ if n is odd.

So $512^{43} + 6561^{43}$ is divisible by 7073 ($= 512 + 6561$).

Now since, $7073 = 11 \times 643$, so the given expression is divisible by 643.

Since the unit digit of $(2^{387} + 3^{344})$ is 9, so it cannot be divided by 5.

Therefore, it cannot be divided by 735.

As you know that $(2^{387} + 3^{344})$ is an odd number, so it cannot be divided by any even number like 216.

Hence choice (d) is the correct one.

- 143** Since he cannot afford 385 pizzas, so he has to cut the pizzas in order to share it with his 385 classmates.

Since 385 has three unique prime factors so we have to consider the following cases :

Let us assume p is the number of pizzas that Narendra Bhai orders for his classmates.

Case I $\frac{a}{11} + \frac{b}{7} + \frac{c}{5} = \frac{p}{385}$

$$\Rightarrow 35a + 55b + 77c = p$$

For $a = 3$, $b = 2$ and $c = 1$, the above equation gives $105 + 110 + 77 = 292$, which is available in the given options.

That means he can take 105 pizzas, cut each pizza into 11 equal pieces and distribute equally among 385 classmates. Then he can take 110 pizzas, cut each pizza into 7 equal pieces and distribute equally among 385 classmates. Finally, he can take 77 pizzas, cut each pizza into 5 equal pieces and distribute equally among 385 classmates.

Therefore, choice (a) is valid.

For $a = 2$, $b = 3$ and $c = 1$, the above equation gives $70 + 165 + 77 = 312$.

That means we can cut 70 pizzas into 11 equal pieces, 165 pizzas into 7 equal pieces and 77 pizzas into 5 equal pieces in order to distribute equal amount of pizza to each of his 385 classmates.

Therefore, choice (b) is valid.

Case II $\frac{a}{11} + \frac{b}{35} = \frac{p}{385}$

$$\Rightarrow 35a + 11b = p$$

For, $a = 9$ and $b = 6$, the value of $p = 381$

Hence, it is possible for him to share 381 pizzas equally with his 385 friends by cutting each of the 35 pizzas into 11 equal pieces and 11 pizzas into 35 equal pieces.

Therefore, choice (d) is valid.

Case III $\frac{a}{7} + \frac{b}{55} = \frac{p}{385}$

$$\Rightarrow 55a + 7b = p$$

There is no set of value of (a, b) for which any of the value given for p holds true in the above equation.

Case IV $\frac{a}{5} + \frac{b}{77} = \frac{p}{385}$

$$\Rightarrow 77a + 5b = p$$

There is no set of values of (a, b) for which any of the values given for p holds true in the above equation.

Thus we see that except 335 all other values satisfy the value of p in the respective equations.

Hence choice (c) is the correct one.

144 Given that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$

$$\Rightarrow \frac{1}{x} < \frac{1}{z} \Rightarrow z < x$$

$$\Rightarrow 0 < z < 12$$

$$\therefore \frac{1}{x} + \frac{1}{y} = \frac{1}{z} \Rightarrow y = \frac{xz}{x-z}$$

$$\Rightarrow y = \frac{12z}{12-z}$$

Therefore the possible values of $(z, y) \equiv (3, 4), (4, 6), (6, 12), (8, 24), (9, 36), (10, 60), (11, 132)$

Thus we have total 7 solutions as following.

(x, y, z)

$\equiv (12, 4, 3) (12, 6, 4) (12, 12, 6) (12, 24, 8) (12, 36, 9), (12, 60, 10) \text{ and } (12, 132, 11)$

Hence choice (b) is the correct answer.

Alternatively

Given that

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z} \Rightarrow \frac{1}{x} < \frac{1}{z} \Rightarrow x > y$$

Similarly $\frac{1}{x} + \frac{1}{y} = \frac{1}{z} \Rightarrow \frac{1}{y} < \frac{1}{z} \Rightarrow y > z$

Again $\frac{1}{x} + \frac{1}{y} = \frac{1}{z} \Rightarrow \frac{x+y}{xy} = \frac{1}{z}$

$$\Rightarrow zx + zy = xy \Rightarrow y(x-z) = zx$$

$$\Rightarrow y = \frac{zx}{x-z}$$

Let us consider $x - z = k$; where k is any positive integer as $x > z$.

$$\Rightarrow y = \frac{(x-k)x}{k} = \frac{x^2 - kx}{k} = \frac{x^2}{k} - x$$

Now we can say that y will be an integer only when x^2 is properly divisible by k . That is k must be a factor of x^2 .

Since $x^2 = 144$, so $k = 1, 2, 3, 4, 6, 8, 9$ only; as $k < x$, so $k < 12$

It implies, $x - z = 1, 2, 3, 4, 6, 8, 9$

Or $z = 11, 10, 9, 8, 6, 4, 3$

And $y = 132, 60, 36, 24, 12, 6, 4$

- 145** Since, in an hour Vijay can finish x pitchers and Siddhartha can finish y pitchers, then

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$$

$$\Rightarrow \frac{x+y}{xy} = \frac{1}{2}$$

$$\Rightarrow y = \frac{2x}{x-2}$$

$$\Rightarrow y = 2 + \frac{4}{x-2}$$

Since x and y are positive integers, so we can have the following values of x and y as $x - y = 1$ or 2 or 4 , only.

$$x - 2 = 1 \Rightarrow x = 3, y = 6$$

$$x - 2 = 2 \Rightarrow x = 4, y = 4$$

$$x - 2 = 4 \Rightarrow x = 6, y = 3$$

Therefore, $x + y = 8$ or $x + y = 9$

Thus, the maximum value is 9.

Hence choice (c) is the correct answer.

Hint $y = \frac{2x}{x-2} = \frac{2x-4+4}{x-2}$

$$= \frac{2(x-2)+4}{x-2} = \frac{2(x-2)}{x-2} + \frac{4}{x-2}$$

146 As we know that p number of pizzas = 1 meal

And b number of burgers = 1 meal

Similarly, c number of sandwiches = 1 meal

Therefore, 1 pizza = $\frac{1}{p}$ meal

And 1 burger = $\frac{1}{b}$ meal

And 1 sandwich = $\frac{1}{s}$ meal

$\therefore \frac{1}{p} + \frac{1}{b} + \frac{1}{s} = 1$; where $p < b < s$ and $\{p, b, s\} \in N$

If $p = 1$, then $\left(\frac{1}{b} + \frac{1}{s}\right) = 0$. But, that's not possible, as b and s

both are positive integer.

If $p = 2$, then $\left(\frac{1}{b} + \frac{1}{s}\right) = \frac{1}{2}$

But, since $p < b < s$, so if we consider $b = 3$, then $s = 6$.

However, for $b > 3$, $\left(\frac{1}{b} + \frac{1}{s}\right) < \frac{1}{2}$; which is again not possible.

If $p \geq 3$, then $\left(\frac{1}{b} + \frac{1}{s}\right) \geq \frac{2}{3}$. So $b \geq 4$ and then

$\left(\frac{1}{b} + \frac{1}{s}\right) \leq \frac{9}{20} < \frac{2}{3}$, which is contradictory.

Thus we see that there is a unique set of value for p , b and s .

Therefore the required value of $p + b + s = 2 + 3 + 6 = 11$

Hence, choice (b) is the correct one.

Solutions (for Q. Nos. 147 and 148) Let us assume that Vicky donated his semen for ' w ' weeks and each day he donated 1 unit of semen.

Then the total amount of semen he had donated each week = 6 units. And the total amount of semen he had donated in w weeks = $6w$ units.

Now let us assume that the total amount of semen donated to Chaddha clinic is x units, then the total amount of semen donated to Chopra clinic is $(6w - x)$ units.

Therefore total semen donated to Chaddha clinic and Chopra clinic in the last week is

$$\frac{x}{6} + \frac{(6w - x)}{9} = 6 \text{ units}$$

$$\Rightarrow x = 108 - 12w$$

Since, $0 < x < 6w$, therefore

$$0 < 108 - 12w < 6w$$

$$\Rightarrow 0 < 108 < 18w \Rightarrow 6 < w$$

And since, $x = 108 - 12w$

$$\Rightarrow w = \frac{108 - x}{12}$$

$$\Rightarrow w = 9$$

(When x is minimum, w will be maximum)

Therefore, $6 < w < 9$

As w is an integer value, so $w = 7$ or 8 .

Therefore, $x = 108 - 12w$

$$\Rightarrow x = 24 \text{ or } 12$$

When $x = 24$, Vicky donates $\frac{24}{6} = 4$ units to Chaddha clinic

and $\frac{42 - 24}{9} = 2$ units to Chopra clinic, in the last week.

But, when $x = 12$, Vicky donates $\frac{12}{6} = 2$ units to Chaddha

clinic and $\frac{48 - 12}{9} = 4$ units to Chopra clinic, in the last week.

147 Thus we see that the possible ratio is either $\frac{2}{1}$ or $\frac{1}{2}$ depending upon $x = 24$ or 12 .

Hence choice (c) is the correct one.

148 Since $w = 7$ or 8 , so we don't have a definite answer.

Hence choice (d) is the correct one.

149 Let a , b , c and d denote the number of girls who dated a guy for 3 years, 2 years, 1 year and not at all in that order. Then,

$$a + b + c + d = 300 \quad \dots(i)$$

$$a + b = 1.36c \quad \dots(ii)$$

From the eqs. (i) and (ii)

$$2.36c + d = 300$$

$$\Rightarrow c = \frac{100}{236}(300 - d)$$

$$\Rightarrow c = \frac{25}{59}(300 - d)$$

Since d is the minimum possible value, so that $(300 - d)$ is the maximum possible multiple of 59.

Now, the maximum possible multiple of 59 below 300 is 295, which means $(300 - d) = 295$. So, we have $d = 5$ and the integral value of $c = 125$.

It implies that $a + b = 1.36(125) = 170$

But, since the minimum possible value of $b = 1$, so the maximum possible value of $a = 169$.

Hence choice (c) is the correct one.

- 150** Let the number of doctors reached to treat the soldiers be d and the initial number of soldiers at the military base be $7p$, then

$$\Rightarrow \quad \begin{aligned} 320d &= 7p + 190 \\ p &= \frac{320d - 190}{7} \end{aligned}$$

Since $0 < d < 7$, the only value of d that gives integral value of p is $d = 3$.

Therefore, $p = 110$ and $7p = 770$.

Hence choice (c) is the correct one.

Ⓢ Alternatively As each doctor was supposed to treat equal number of soldiers. So the initial number of patients must be divisible by 7. Again, you know that eventually each doctor treated 320 soldiers, so the total number of soldiers who got treated must be divisible by 320.

Now, if you look at the choice (b), 1400 is divisible by 7, but $1400 + 190 = 1590$ is not divisible by 320. So it's an invalid choice.

Again, if you look at choice (c), 770 is divisible by 7 and $770 + 190 = 960$ is divisible by 320. So it's a valid choice.

Hence choice (c) is the correct one.

- 151** $180 = 2^2 \times 3^2 \times 5$

Number of co-prime numbers of 180, which are below 180

$$= 180 \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{5}\right) = 48$$

So probable prime numbers = 48

Now, if P_i denotes the prime numbers other than 2, 3 and 5, then these 48 numbers include

- (i) P_i
- (ii) P_i^n ; such that $p^n \leq 180$; for each $n \geq 2$
- (iii) $P_1 P_2 P_3 \dots P_r \leq 180$
- (iv) 1

Now, find the relevant numbers in each of the four cases.

- (i) Other than 2, 3 and 5 we have following prime numbers; 7, 11, 13, 17, 19, 23, 29 ... etc.

- (ii) $7^2, 11^2, 13^2$

- (iii) $\{7 \times 11, 7 \times 13, 7 \times 17, 7 \times 19, 7 \times 23\}, \{11 \times 13\}$

- (iv) 1

So out of 48 probable prime numbers (ii), (iii), (iv) are not the prime numbers.

It implies that (i) = $48 - [(ii) + (iii) + (iv)]$

Thus net prime numbers = $48 - (3 + 6 + 1) = 38$.

But since 2, 3 and 5 are the prime numbers, so the total prime numbers = $38 + 3 = 41$

Ⓢ Alternatively If you know that the number of prime numbers in the set $\{1, 2, \dots, 100\} = 25$

And number of prime numbers in the set $\{101, 102, \dots, 200\} = 21$

And number of prime numbers in the set $\{181, 182, \dots, 200\} = 5$

Therefore number of prime numbers in the set of first 180 natural numbers = $25 + 21 - 5 = 41$

Hint $[\sqrt{200}] = [14.14] = 14$. It implies that you get the square root of 200 and consider only integral part, which is the left side of decimal point and ignore the decimal part, which is right side of the decimal point.

It implies that in order to know that whether a particular number in the set of $\{181, 182, \dots, 200\}$ is a prime or not you need to test these numbers by dividing all the prime numbers up to 14.

Since 181, 191, 193, 197 and 199 are not divisible by any of the prime numbers, 2, 3, 5, 7, 11, 13, so these 5 numbers are prime numbers in the set of numbers $\{181, 182, \dots, 200\}$.

- 152** $1000 = 2^3 \times 5^3$. It has only two prime factors namely 2 and 3.

Now if we consider $2 \times 3 \times 5 \times 7 = 210$, it involves four prime numbers. So we will try to bring 210 closer to 1000 by considering certain multiple of 210.

So if we multiply 210 by 5 we get 1050, the closest possible number to 1000, as a multiple of 210.

Now, $1050 = 2 \times 3 \times 5^2 \times 7$

Number of co-prime numbers of 1050 which are below 1050

$$= 1050 \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{5}\right) \times \left(1 - \frac{1}{7}\right) = 240.$$

So probable prime numbers = 240

Now, if P_i denotes the prime numbers other than 2, 3, 5 and 7 then these 240 numbers include

- (i) P_i
- (ii) P_i^n ; such that $p^n \leq 1050$; for each $n \geq 2$
- (iii) $P_1 P_2 P_3 \dots P_r \leq 1050$
- (iv) 1, as it's not a prime number.

Now, find the relevant numbers in each of the four cases.

- (a) Other than 2, 3, 5 and 7 we have following prime numbers : 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, ..., etc.

- (b) $\{11^2, 13^2, 17^2, 19^2, 23^2, 29^2, 31^2\}$

- (c) $\{11 \times 13, 11 \times 17, 11 \times 19, \dots, 11 \times 89\},$
 $\{13 \times 17, 13 \times 19, \dots, 13 \times 79\},$
 $\{17 \times 19, \dots, 17 \times 61\}, \{19 \times 23, \dots, 19 \times 53\},$
 $\{23 \times 29, \dots, 23 \times 43\}, \{29 \times 31\}$

- (d) 1

So out of 240 probable prime numbers (b), (c), (d) are not the prime numbers.

It implies that (a) = $240 - [(b) + (c) + (d)]$

Thus net prime numbers = $240 - (7 + 60 + 1) = 172$
 But since 2, 3, 5 and 7 are the prime numbers, so the total prime numbers in the set of first 1050 natural numbers = $172 + 4 = 176$

Now since the number of prime numbers in the set $\{1001, 1002, \dots, 1050\} = 8$

Therefore total number of prime numbers in the set of first 1000 natural numbers = $176 - 8 = 168$.

Hint Number of elements in (b) = 7 and number of elements in (c) = $19 + 16 + 11 + 8 + 5 = 60$ and elements in (d) = 1.

153 Let there be m men in team B, then $\frac{6}{11} < \frac{n}{m} < \frac{11}{19}$

$$\Rightarrow 0.5454 < \frac{n}{m} < 0.5784$$

n	m	$\frac{n}{m}$	Result
1	1, 2, 3, 4, 5, ...	$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$	None of the values lies between 0.5454 and 0.5784
2	1, 2, 3, 4, 5, ...	$\frac{2}{1}, \frac{2}{2}, \frac{2}{3}, \frac{2}{4}, \frac{2}{5}, \dots$	None of the values lies between 0.5454 and 0.5784
3	1, 2, 3, 4, 5, ...	$\frac{3}{1}, \frac{3}{2}, \frac{3}{3}, \frac{3}{4}, \frac{3}{5}, \frac{3}{6}, \dots$	None of the values lies between 0.5454 and 0.5784
4	1, 2, 3, 4, 5, 6, 7, ...	$\frac{4}{1}, \frac{4}{2}, \frac{4}{3}, \frac{4}{4}, \frac{4}{5}, \frac{4}{6}, \frac{4}{7}, \dots$	$\frac{4}{7} = 0.5714$, which lies between 0.5454 and 0.5784

Thus we can see that the minimum possible value of m is 7 for which $\frac{6}{11} < \frac{n}{m} < \frac{11}{19}$.

Hence choice (d) is the correct answer.

NOTE There may be other values of m for which the given relation will be valid, but those values of m will be higher than 7.

Alternatively $\frac{6}{11} < \frac{n}{m} < \frac{11}{19} \Rightarrow \frac{19n}{11} < m < \frac{11n}{6}$

n	$\frac{19n}{11}$	$\frac{11n}{6}$	m
1	$1\frac{8}{11}$	$1\frac{5}{6}$	No integral value of m is possible
2	$3\frac{5}{11}$	$3\frac{4}{6}$	No integral value of m is possible
3	$5\frac{2}{11}$	$5\frac{3}{6}$	No integral value of m is possible
4	$6\frac{10}{11}$	$7\frac{2}{6}$	$6\frac{10}{11} < 7 < 7\frac{2}{6}$

Thus at $n = 4$ the minimum value of $m = 7$ satisfies the given relation.

154 If he had worked in each and every month his salary in n months would have been

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} = 21740 + k^2$$

Where k denotes the month in which he didn't go to work. By hit and trial, you come to know that he has been working for more than 39 months as

$$1^2 + 2^2 + 3^2 + \dots + 39^2 = \frac{39(40)(79)}{6} = 20540 < 21740$$

Similarly, you also realize that he has been working for less than 41 months as

$$1^2 + 2^2 + 3^2 + \dots + 41^2 = \frac{41(42)(83)}{6} = 23821 > 21740$$

It's highly probable that he has been working for the last 40 months. In that case $1^2 + 2^2 + 3^2 + \dots + 40^2$

$$= \frac{40(41)(81)}{6} = 22140$$

Since $22140 - 21740 = 400$, it implies that he didn't work in the 20th month.

Hence choice (c) is the correct one.

155 One of the possible combinations with Azad could be $1000a + 100b + 10c + d$, where a, b, c and d denote the number of notes of various denominations mentioned. Similarly, there will be remaining 23 possible combinations of notes of various denominations with Bose. However, from the given limited information, it would be next to impossible to determine the total sum of amounts with Azad and Bose. The only way out is to use the given choices to get the better clue of what you need to answer.

Choice (a) $(1000a + 100b + 10c + d)$

$$+ (1000b + 100a + 10d + c) = 11(100a + 100b + c + d)$$

It shows that the possible total sum must be divisible by 11 and since 11 is the factor of 14377, so it can be a possible answer.

Choice (b) $(1000a + 100b + 10c + d)$

$$+ (1000c + 100d + 10a + b) = 101(10a + 10c + b + d)$$

It shows that the possible total sum must be divisible by 101 and since 101 is not the factor of 24377, so it CANNOT be a possible answer.

Choice (c) $(1000a + 100b + 10c + d)$

$$+ (1000c + 100d + 10a + b) = 11(10a + 10c + b + d)$$

It shows that the possible total sum must be divisible by 101, and since 101 is not the factor of 44377, so it CANNOT be a possible answer.

Choice (d) $(1000a + 100b + 10c + d)$

$$+ (1000d + 100c + 10b + a) = 11(91a + 91d + 10b + 10c)$$

It shows that the possible total sum must be divisible by 11 and since 11 is NOT the factor of 44351, so it CANNOT be a possible answer. Hence choice (a) is the correct one.

- 156** Let the talk-time of each simcard be a, b, c, d and e such that $a > b > c > d > e$. It is obvious that $a + b = 133$ and $d + e = 122$, since the combination of two highest talk-times will give the highest total and the combination of two lowest talk-times will give the lowest total.

It is known that $e + d = 122$ and $d \neq e$, so the probable values of d and e are as follows.

$(d, e) = (62, 60), (63, 59), (64, 58), (65, 57), \dots$ etc.

Now go through options. Consider $a = 71$, then $b = 62$, which is not feasible as b should be greater than d .

Again, consider $a = 69$, then $b = 64$. Therefore the possible value of $c = 63$, $d = 62$ and $e = 60$. But adding any two values of a, b, c, d and e we do not find many matching values which are given in the problem.

Once again consider another value of $a = 68$, then $b = 65$. Therefore the probable values of c are 64 and 63. Now if we consider $c = 64$, $d = 62$ and $e = 60$ for $a = 68$ and $b = 65$ we find that all the pairs of talk-times are matching.

Therefore the highest talk-time for a simcard is 68 minutes.

☞ Alternatively

a	b	c	d	e	Comment
71	62				d should be less than b which we can't get as the least possible of d is 62.
69	64	63	62	60	Even though individual values do not clash or violate the conditions, but few combinations (talk-times) do not match with the given ten values
68	65	64	62	60	Each and every condition is satisfied.
68	65	64	63	59	Even though individual values do not clash or violate the conditions, but few combinations (talk-times) do not match with the given ten values.
68	65	63	62	60	Even though individual values do not clash or violate the conditions, but few combinations (talk-times) do not match with the given ten values.

Therefore we can conclude that 68 is the highest talk-time of a given simcard.

- 157** To bring the three gears into starting position, each gear have to complete equal distance, which is equal to the LCM of 36, 25 and 30. That is 900.

$$\text{So } x = \frac{900}{36} = 25, y = \frac{900}{25} = 36, z = \frac{900}{30} = 30.$$

$$\text{Therefore } x + y + z = 25 + 36 + 30 = 91$$

Hence choice (a) is the answer.

- 158** An integer is square-free if and only if it does not have a divisor of the form p^2 for some prime p . Let A_p be the set of all integers between 1 and 120 divisible by p^2 . Then

$$A_2 = \left\lfloor \frac{120}{2^2} \right\rfloor = 30, A_3 = \left\lfloor \frac{120}{3^2} \right\rfloor = 13$$

$$A_5 = \left\lfloor \frac{120}{5^2} \right\rfloor = 4$$

$$A_7 = \left\lfloor \frac{120}{7^2} \right\rfloor = 2, A_{2 \times 3} = \left\lfloor \frac{120}{(2 \times 3)^2} \right\rfloor = 3,$$

$$A_{2 \times 5} = \left\lfloor \frac{120}{(2 \times 5)^2} \right\rfloor = 1$$

$$\text{But } A_{2 \times 7} = A_{3 \times 5} = A_{3 \times 7} = A_{5 \times 7} = A_{2 \times 3 \times 5} = A_{2 \times 5 \times 7} \\ = A_{2 \times 3 \times 7} = A_{3 \times 5 \times 7} = A_{2 \times 3 \times 5 \times 7} = 0$$

$$\text{Therefore } A_p = [(A_2 + A_3 + A_5 + A_7) - (A_{2 \times 3} + A_{2 \times 5})] \\ = 45$$

Thus total number of square-free integers between 1 and 120 = $120 - 45 = 75$

NOTE $[x]$ denotes the greatest integer less than or equal to x .

- 159** The equation can be rewritten as

$$(a-1)(b-1) + (c-1)(d-1) = 2$$

Now there are not many possibilities to consider! If the first product is 0, the second must be 2, and if the first product is 1, so is the second.

Case I If $a = 1$, then we need to have $(c-1)(d-1) = 2$

Since $1 \leq c \leq d$, this forces $c = 2, d = 3$. And b can be 1 or 2, giving the solutions (1, 1, 2, 3) and (1, 2, 2, 3).

Case II If $a > 1$, we need $a = 2$, else the left hand side is too big. That forces $b = c = d = 2$, giving the third solution (2, 2, 2, 2). Thus it has total 3 solutions.

Hence choice (c) is the correct one.

- 160** Let the given two numbers be $10a + b$ and $10x + y$. Then as per the given condition we have $(10a + b) \times (10x + y) = (10b + a) \times (10y + x) \Rightarrow ax = by \Rightarrow \frac{a}{b} = \frac{y}{x}$

Now we can get various possible combinations for $\frac{a}{b} = \frac{y}{x}$ as shown in the following table.

Possible values of $\frac{a}{b}$ and $\frac{y}{x}$	Basic possible pairs of $\begin{pmatrix} ab \\ xy \end{pmatrix}$	Basic acceptable pairs of $\begin{pmatrix} ab \\ xy \end{pmatrix}$	Total valid pairs
$\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$	$\begin{pmatrix} 12 \\ 42 \end{pmatrix}, \begin{pmatrix} 12 \\ 63 \end{pmatrix}, \begin{pmatrix} 12 \\ 84 \end{pmatrix}, \begin{pmatrix} 24 \\ 63 \end{pmatrix}, \begin{pmatrix} 24 \\ 84 \end{pmatrix}, \begin{pmatrix} 36 \\ 84 \end{pmatrix}$	$\begin{pmatrix} 12 \\ 63 \end{pmatrix}, \begin{pmatrix} 12 \\ 84 \end{pmatrix}, \begin{pmatrix} 24 \\ 63 \end{pmatrix}, \begin{pmatrix} 36 \\ 84 \end{pmatrix}$	$4 \times 2 = 8$
$\frac{1}{3}, \frac{2}{6}, \frac{3}{9}$	$\begin{pmatrix} 13 \\ 62 \end{pmatrix}, \begin{pmatrix} 13 \\ 93 \end{pmatrix}, \begin{pmatrix} 26 \\ 93 \end{pmatrix}$	$\begin{pmatrix} 13 \\ 62 \end{pmatrix}, \begin{pmatrix} 26 \\ 93 \end{pmatrix}$	$2 \times 2 = 4$
$\frac{1}{4}, \frac{2}{8}$	$\begin{pmatrix} 14 \\ 82 \end{pmatrix}$	$\begin{pmatrix} 14 \\ 82 \end{pmatrix}$	$1 \times 2 = 2$
$\frac{2}{3}, \frac{4}{6}, \frac{6}{9}$	$\begin{pmatrix} 23 \\ 64 \end{pmatrix}, \begin{pmatrix} 23 \\ 96 \end{pmatrix}, \begin{pmatrix} 46 \\ 96 \end{pmatrix}$	$\begin{pmatrix} 23 \\ 64 \end{pmatrix}, \begin{pmatrix} 23 \\ 96 \end{pmatrix}$	$2 \times 2 = 4$
$\frac{3}{4}, \frac{6}{8}$	$\begin{pmatrix} 34 \\ 86 \end{pmatrix}$	$\begin{pmatrix} 34 \\ 86 \end{pmatrix}$	$1 \times 2 = 2$

Since $\begin{pmatrix} ab \\ xy \end{pmatrix}$ and $\begin{pmatrix} ba \\ yx \end{pmatrix}$ give same result but they are distinct from each other.

As, $\begin{pmatrix} 12 \\ 63 \end{pmatrix}$ and $\begin{pmatrix} 21 \\ 36 \end{pmatrix}$ give same results but they are distinct from each other. That's why we have multiplied the basic number of pairs by 2.

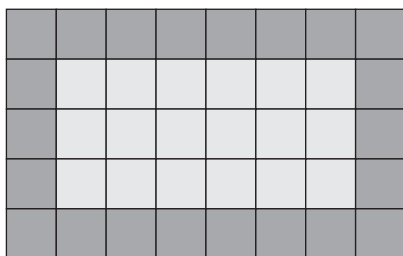
Therefore total required pairs = $8 + 4 + 2 + 4 + 2 = 20$
Hence choice (c) is the correct one.

NOTE We have excluded those combinations, which involve same digits, for example $\begin{pmatrix} 12 \\ 42 \end{pmatrix}$, since 12 and 42 have a common digit 2, so we can't consider this pair.

For your better understanding all the 20 valid pairs are mentioned below :

(12, 63), (21, 36), (12, 84), (21, 48), (24, 63), (42, 36), (36, 84), (63, 48), (13, 62), (31, 26), (29, 93), (62, 39), (14, 82), (41, 28), (23, 64), (32, 46), (23, 96), (32, 69), (34, 86), (43, 68)

Solutions (for Q. Nos. 160 to 162) The required grid of tiles will be similar in looks as the following one.



Let the number of dynamic tiles = $l \times b$
Then the number of static tiles = $(l + l) + (b + b) + 4$

As it is given that the no. of dynamic tiles = no. of static tiles

$$\text{So, } lb = 2l + 2b + 4$$

$$\Rightarrow lb - 2l - 2b = 4$$

$$\Rightarrow lb - 2l - 2b + 4 = 8$$

$$\Rightarrow (l-2)(b-2) = 8$$

Since l and b are positive integers, so $(l-2)$ and $(b-2)$ must be integers. It implies that $(l-2)$ and $(b-2)$ are the two factors of 8.

$$\text{Therefore } (l-2)(b-2) = 8 \times 1$$

$$\Rightarrow l = 10 \text{ and } b = 3$$

$$\text{And } (l-2)(b-2) = 4 \times 2$$

$$\Rightarrow l = 6 \text{ and } b = 4$$

Alternatively Let the number of dynamic tiles = $l \times b$

Then the number of static tiles = $(l + l) + (b + b) + 4$

As it is given that the no. of dynamic tiles = no. of static tiles

$$\text{So, } lb = 2l + 2b + 4 \quad \dots(i)$$

$$\text{Therefore, } l(b-2) = 2b + 4$$

$$\Rightarrow b > 2, \text{ as right hand side is strictly positive.}$$

$$\text{Similarly, } b(l-2) = 2l + 4$$

$$\Rightarrow l > 2, \text{ as right hand side is strictly positive.}$$

It implies that $l \geq 3$ and $b \geq 3$.

$$\text{Again rearranging the Eq. (i), we get } l = \frac{(2b+4)}{(b-2)} \quad \dots(ii)$$

$$\text{Therefore, } \frac{(2b+4)}{(b-2)} \geq 3$$

$$\Rightarrow (2b+4) \geq 3(b-2)$$

$$\Rightarrow b \leq 10$$

Now we know that $l \geq 3$ and $3 \leq b \leq 10$, so we can determine the integral values of l by substituting the value of b .

Thus, when $b = 3$, $l = 10$ and $b = 4$, $l = 6$

$$\text{161 Therefore total number of tiles} = 2(3 \times 10) = 60 \text{ or } 2(4 \times 6) = 48$$

As we don't have unique value for total number of tiles, so we are not able to answer the exact figure for total number of tiles in the grid.

Hence choice (d) is the correct answer.

$$\text{162 As we see the highest value of } l \times b \text{ is } 30, \text{ so the maximum number of dynamic tiles can be } 30.$$

Hence choice (d) is the correct answer.

$$\text{163 The dimensions of the required grid are } (l+2) \text{ and } (b+2), \text{ so the actual dimensions will be either } 12 \text{ and } 5 \text{ or } 8 \text{ and } 6.$$

Since $\frac{8}{6} < \frac{12}{5}$, so $\frac{8}{6}$ is the preferred ratio. As $\frac{8}{6}$ can be rationalized as $\frac{4}{3}$, so $\frac{4}{3}$ is the preferred ratio of length and

breadth of the grid.

Hence choice (c) is the correct answer.

164 Consider the following table.

	Number of terms	
1	1	
1 + 3 = 4	2	7 = 1 + 6
4 + 6 = 10	3	16 = 7 + 9
10 + 9 = 19	4	28 = 16 + 12
19 + 12 = 31	5	43 = 28 + 15
31 + 15 = 46	6	61 = 43 + 18
46 + 18 = 64	7	82 = 61 + 21
64 + 21 = 85	8	106 = 82 + 24
85 + 24 = 109	9	133 = 106 + 27
109 + 27 = 136	10	163 = 133 + 30

By observation you can confirm that 160 is the second last (or 9th) term of the 10th row. Further you know that the last term (9th term) of the 9th row is just above the second last term (9th term) of the 10th row.

Thus there is only one required term, which is 133. So the required sum is also 133.

Hence choice (b) is the correct answer.

165 The value of m can be integer only when n is a factor of 285.

Now, the factors of 285 are 1, 3, 5, 15, 19, 57, 95 and 285.

Case (i) When $n = 1$ or 285, $m = 286$

Case (ii) When $n = 3$ or 95, $m = 98$

Case (iii) When $n = 5$ or 57, $m = 62$

Case (iv) When $n = 15$ or 19, $m = 34$

Since $n < 0$, so the values of m will be $-286, -98, -62$ and -34 . Out of these four values of m , -34 is the highest value.

Hence choice (a) is the correct answer.

☞ **Alternatively** Since, for the positive real numbers, Arithmetic mean $\geq$ Geometric mean, therefore

$$\frac{\left(n + \frac{285}{n}\right)}{2} \geq \sqrt{n \times \left(\frac{285}{n}\right)}$$

$$\Rightarrow n + \frac{285}{n} \geq 2\sqrt{285} \Rightarrow n + \frac{285}{n} \geq 33.76$$

Since $n < 0$, that is n is negative, so we have

$$n + \frac{285}{n} \leq -33.76$$

But since m has to be integer, so we have

$$n + \frac{285}{n} \leq -34$$

☞ **Alternatively** Since, for the positive real numbers, Arithmetic mean $\geq$ Geometric mean, therefore

$$n + \frac{285}{n} \geq 2\sqrt{285}$$

Since n is negative, so we have,

$$-\left(n + \frac{285}{n}\right) \geq -2\sqrt{285}$$

$$\Rightarrow n + \frac{285}{n} \leq -2\sqrt{285}$$

$$\Rightarrow n^2 + 2\sqrt{285}n + 285 \leq 0$$

It implies that the only root of the above inequation is $n = -\sqrt{285}$

$$\therefore m = n + \frac{285}{n} = -\sqrt{285} + \frac{285}{-\sqrt{285}}$$

$$\Rightarrow m = -2\sqrt{285} = -33.76$$

Since m is an integer, so its highest value is -34 .

Hint To know more about this method, you may like to refer the chapter on theory of equations.

$$166 \quad a + \frac{1}{b + \frac{1}{c}} = \frac{37}{16} = 2 + \frac{5}{16} = 2 + \frac{1}{\frac{16}{5}} = 2 + \frac{1}{3 + \frac{1}{5}}$$

$$\Rightarrow a = 2, b = 3, c = 5$$

$$\therefore abc = 30$$

Hence choice (a) is the correct one.

167 From a simple bar you need 1 step to make 2 pieces.

Similarly you need 2 steps to make 3 pieces.

Similarly you need 3 steps to make 4 pieces.

.....
.....
.....

Thus to make 28 pieces you need 27 steps.

Hence choice (d) is the correct one.

Hint A sheet of $m \times n$ dimension requires $(mn) - 1$ steps to cut off into mn pieces of 1×1 dimensions.

168 Given that $a + b + c = 1$, so let us assume $a = \frac{1}{3} + x$,

$$b = \frac{1}{3} + y, c = \frac{1}{3} + z$$

$$\therefore a + b + c = \left(\frac{1}{3} + x\right) + \left(\frac{1}{3} + y\right) + \left(\frac{1}{3} + z\right)$$

$$\Rightarrow a + b + c = 1 + (x + y + z)$$

$$\Rightarrow 1 = 1 + (x + y + z)$$

$$\Rightarrow x + y + z = 0$$

$$\text{Now, } (x + y + z)^2 = 0$$

$$\Rightarrow x^2 + y^2 + z^2 + 2(xy + yz + zx) = 0$$

$$\Rightarrow (xy + yz + zx) = -\frac{(x^2 + y^2 + z^2)}{2}$$

$$\Rightarrow (xy + yz + zx) = -d; \text{ where } d \geq 0$$

$$\therefore ab + bc + ca$$

$$= \left(\frac{1}{3} + x\right)\left(\frac{1}{3} + y\right) + \left(\frac{1}{3} + y\right)\left(\frac{1}{3} + z\right) + \left(\frac{1}{3} + z\right)\left(\frac{1}{3} + x\right)$$

$$\Rightarrow ab + bc + ca = \frac{1}{3} + \frac{2}{3}(x + y + z) + (xy + yz + zx)$$

$$\Rightarrow ab + bc + ca = \frac{1}{3} - d \Rightarrow ab + bc + ca \leq \frac{1}{3}$$

Hence choice (a) is the correct one.

☞ **Alternatively** Let us consider $a = b = c = \frac{1}{3}$, as

$$a + b + c = 1. \text{ Therefore } ab + bc + ca = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$$

Now, by looking at the given choices, we can say that choice (a) can be the correct one.

✓ Level Final Round

- 1 $120 = 4 \times 5 \times 6$ (Since minimum number of cuts are possible only when all the three factors are very close)

Again $125 = 5 \times 5 \times 5$

But since $4 + 5 + 6 = 5 + 5 + 5 = 15$, hence the same number of cuts are required as previously. Hence 'n' number of cuts are required to get 125 identical pieces.

- 2 Note that the first element of n th term of the given sequence is $\left(\frac{n^2 - n + 2}{2}\right)$ and the last element of the n th element is $\frac{n(n+1)}{2}$.

So the last element of the 21st term of the sequence

$$= \frac{21 \times 22}{2} = 231$$

Now to find the sum of first 21 term we have to find the sum of first 231 natural numbers.

Hence $\frac{231 \times 232}{2} = 231 \times 116 = 26796$

Hence the last digit is 6 and tens digit is 9.

3. The elements of the set

$$S_{19} = \{172, 173, 174, 175, \dots, 190\}$$

Now $175 = 5^2 \times 7$

$$180 = 5 \times 36$$

$$185 = 5 \times 37$$

$$190 = 5 \times 38$$

Therefore the number of 5s contained in the product of the element of the set is 5. Note that the number of 2s are more than the number of 5s. So the total number of zeros at the end of the product of the element is 5.

4. Note the number of zeros in the product of S_{19} and S_{20} will depend upon the number of 5s contained in S_{19} and S_{20} .

Now, the $S_{20} = \{191, 192, 193, 194, 195, \dots, 210\}$

Thus $195 = 5 \times 39$

$$200 = 5 \times 5 \times 8$$

$$205 = 5 \times 41$$

$$210 = 5 \times 42$$

Therefore the total number of 5s contained in $S_{20} = 5$ hence the number of zeros at the end of the product is 5. Thus the number of zeros at the end of the product of S_{19} and $S_{20} = 10$.

Thus the largest power of 10 is 10, which can exactly divide the product of S_{19} and S_{20} .

- 169 A number N can be expressed as an ordered sum of one or more positive integers in $2^{(N-1)}$ ways. Therefore 8 can be expressed as an ordered sum of one or more positive integers in $2^{8-1} = 2^7 = 128$ ways.

Hence choice (d) is the correct one.

5. $S = \{2, 4, 6, 8, 10, 12, 14, 16, 18, \dots, 96, 98, 100\}$

If the product of any 3 elements contains 4 or more than 4 2s then it will be divisible by 16 (since $16 = 2^4$) otherwise it will not be divisible. Hence remove those elements which are multiples of either 4 or 8 or 16 or 32 or 64 in the given set. Since $(2k \times 2l \times 2m)$ is not divisible by 16 if k, l, m are odd numbers.

Thus there are only 25 elements in P .

$$\text{As } P = \{2, 6, 10, 14, 18, 22, 26, 30, \dots, 94, 98\}$$

6. Since $51 = 17 \times 3$

$$85 = 17 \times 5$$

$$102 = 17 \times 6$$

Thus the minimum possible number of cubes

$$= 3 \times 5 \times 6 = 90$$

The total surface area of non-red faces

$$= (\text{Total surface area of all the cubes}$$

$$- \text{total surface area of cuboid})$$

$$= (90 \times 6 \times 17 \times 17) - 2 \times (51 \times 85 + 85 \times 102 + 102 \times 51)$$

$$= 90 \times 6 \times 289 - 2 \times 289 (15 + 30 + 18)$$

$$= 90 \times 6 \times 289 - 2 \times 289 (63)$$

$$= 289 \times 18 (5 \times 6 - 7) = 289 \times 18 \times 23 = 119646 \text{ cm}^2$$

7. Go through options; $\frac{6p+7q}{42} \rightarrow \frac{6p}{42} + \frac{7q}{42}$

Since p is divisible by 7 and q is divisible by 6 thus

$$\frac{6p}{42} + \frac{7q}{42} \rightarrow \frac{6 \times 7m + 7 \times 6n}{42}$$

$$= \frac{42(m+n)}{42} = (m+n)$$

8. $p^q - q^r = (p+q)^{r-q}$; $11 > p > r > q \in \text{Prime numbers}$.

$$7^3 - 3^5 = (7+3)^{5-3} \Rightarrow 100 = 100$$

$$\therefore (p+q) = 7+3 = 10 = 5(5-3) = r(r-q)$$

9. Since there are 25 prime numbers upto 100 also 1 is not divisible by any number.

Hence there are only 26 seats available on the front row for the mathematicians.

10. Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
1	2	3	4	5	6	7
8	9	10	11	12	13	14

$$\frac{88}{7} = \text{Remainder is 4.}$$

So the 4th day in the week is Thursday.

11. Since Thursday will fall on those dates which can be expressed as $7k + 4$;

$$k = 0, 1, 2, 3, \dots$$

Now since 64 can't be expressed as $7k + 4$.

Hence (d) is the required answer.

12. According to the septarion calendar there are exactly 364 days and each day occurs equal number of times in whole year.

13. Find the pattern

Hence, $(89)^{91} \rightarrow (89^{10})^9 \times 89^1$

$$(89)^{91} \rightarrow 01 \times 89$$

$$(89)^{91} \rightarrow 89$$

Thus the last two digits of $(1989)^{91}$ are 8 and 9.

Since $(89)^1 \rightarrow 89$ $(89)^{11} \rightarrow 89$

$$(89)^2 \rightarrow 21$$
 $(89)^{12} \rightarrow 21$

$$(89)^3 \rightarrow 69$$
 $(89)^{13} \rightarrow 69$

$$\dots \dots \dots \dots$$

$$\dots \dots \dots \dots$$

$$89^{10} \rightarrow 01$$
 $(89)^{20} \rightarrow 01$

NOTE To find the last 'n' digits we find the pattern for last 'n' digits.

14. Step 1.

2	7	3	5

 $\rightarrow$ No any digit is placed at the right place.

- Step 2.

5	2	7	3

 $\rightarrow$ No any digit is correctly placed.

- Step 3.

3			

 $\rightarrow$ This digit is not acceptable at this place.

It means the first digit will be 7, since 2, 5 and 3 are inadmissible.

7			

Again since 5 and 3 are not acceptable at unit digit place and 7 is correctly filled up as the first digit, hence only one digit i.e., '2' is left for the unit digit place.

7			2

Now, since 3 and 7 are not acceptable and 2 has been used, hence only one digit i.e., '5' is left for the tens place.

7		5	2

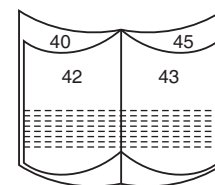
Thus, we can complete the password as

7	3	5	2

Alternatively Go through options.

15. $(888!)^{999} = (1 \times 2 \times 3 \times 4 \times \dots \times 76 \times 77 \times 78 \times \dots \times 888)^{999}$
Since it has a factor as 77. Thus it has no remainder.

16. The middle most pages of the magazines are shown in the figure numbered as 42 and 43.

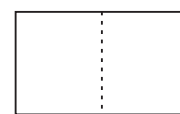


Did you notice that the front pages on the left side of the magazine (as shown in figure) are divisible by 2 and the odd numbers are on the back side of these numbers? Note that I am talking about the left side of the magazine when it is open as shown in figure.

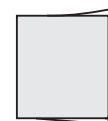
So the missing page numbers with 29 will be 30 and the corresponding number in the right side will be 51 and 52.

Alternatively Since the magazine has 84 pages it means there are 21 sheets of paper, which are folded in middle. Now see the pattern of page numbers :

Left side	Right side
1, 2	83, 84
3, 4	81, 82
5, 6	79, 80
7, 8	77, 78
9, 10	75, 76
$\vdots$	$\vdots$
29, 30	55, 56
$\vdots$	$\vdots$
41, 42	43, 44



(i)



(ii)

Thus we can say that the sum of the page numbers on a sheet in one side is always 85. Hence the page numbers printed on the sheet of paper is 29, 30 and 55, 56.

17. Let there be six locks A, B, C, D, E and F.

Then we need maximum 5 attempts to know the right key for 'A'. Again we need maximum 4 attempts to know the right key for 'B'. Similarly we need 3, 2 and 1 attempts to know the proper key for the locks C, D and E. Now 1 key is automatically left for the lock 'F'. Thus the total number attempts required

$$= 5 + 4 + 3 + 2 + 1 = 15$$

18. $\frac{51n^2 + 17n + 6}{n} = 51n + 17 + \frac{6}{n}$

Here the first two terms are divisible for any value of n , but the last term is divisible by only 1, 2, 3 and 6 i.e., only 4 values, which are actually the factors of 6.

19. $V \gg S \gg P$

$$V \gg L \gg P$$

$$V \gg L > S$$

Therefore, $V \gg L > S \gg P$

Thus the worst player can be Plexur.

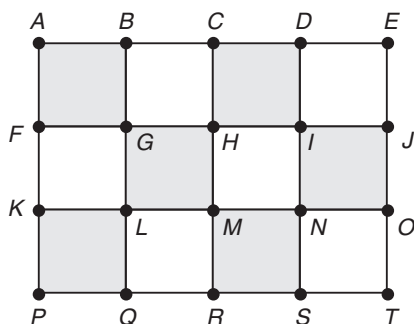
20. Obviously $(n - 1)$. Since if the Winner first picks up $(n - 1)$ sticks then there will be only one stick left, which must be picked up by the Loser and hence the last player will lose it.

- 21** The last ten digits of $1^1 \times 2^2 \times 3^3 \times 4^4 \times 5^5$ are
(00,86,40,00,00)
The last ten digits of $1^6 \times 2^7 \times 3^8 \times 4^9 \times 5^{10}$ are
(00,00,00,00,00)
The last ten digits of $1^{11} \times 2^{12} \times 3^{13} \times 4^{14} \times 5^{15}$ are
(00,00,00,00,00)
The last ten digits of $1^{96} \times 2^{97} \times 3^{98} \times 4^{99} \times 5^{100}$ are
(00,00,00,00,00)
Thus the sum of the last tens digits of the expression is
(00,86,40,00,00)
Hence the sum of the last ten digits of the sum of the
expression = 18. ($\because 8 + 6 + 4 = 18$)

- 22** Initially Ravi Shankar has only ₹ 2 with him. Since we know that the Ravi Shankar can pay the sum in rupees only it means he cannot pay in paise. Therefore he must have purchased initially the flowers of exactly ₹ 1.
Now from the options : Consider option (c).
The number of flowers he has purchased for ₹ 1 = 4.
Later on he could purchased the number of flowers for total ₹ 2 = 4 + 6 = 10.
Thus the initial cost of one dozen flowers = ₹ 3
and the changed cost of one dozen flowers = ₹ 2.40
Thus he could gain 60 paise per dozen. Hence the presumed option (c) is correct.

NOTE The initial number of flowers must be less than 6, for ₹ 1 only then he can get the benefit by purchasing 6 flowers for the next ₹ 1.

- 23** See the figure there are 31 pencils of equal length.



15 vertically and 16 horizontally.

Thus there are 12 small squares of the dimension of equivalent to 1 pencil and 6 squares of dimension of equivalent to 2 pencil and 2 squares of dimensions of equivalent to 3 pencils. Therefore total number of squares = 12 + 6 + 2 = 20.

- 24** Let she did x number of pages on the first day. It means second, third, fourth, ..., ninth and tenth day she did $(x+3)(x+6)(x+9)\dots(x+24)$ and $(x+27)$ passages.
Now since $(x+24) = 4(x+3)$
 $\Rightarrow 3x = 12 \Rightarrow x = 4$
Therefore the number of passages she did on the last day
 $= x + 27 = 31$.

- 25** Total number of passages

$$\begin{aligned} &= x + (x+3) + (x+5) + \dots + (x+27) \\ &= (4 \times 10) + (3+6+9+\dots+27) \\ &= 40 + 3(1+2+3+\dots+9) \\ &= 40 + 135 = 175 \end{aligned}$$

- 26** Let there be x people in the village then there must be $4x$ sheep.

Now the number of persons live = $(x-8)$

and the number of sheep to be carried with the survived people = $4x - (47+21) = 4x - 68$

Since we know that each person has carried only one sheep with him. Therefore

$$\begin{aligned} (x-8) &= 4x - 68 \\ \Rightarrow 3x &= 60 \\ \Rightarrow x &= 20 \\ \Rightarrow 4x &= 80 \end{aligned}$$

Hence there were total 80 sheep.

- 27** (123) (234) (345) (456) (567) (678) (789)
(135) (246) (357) (468) (579)
(147) (258) (369)
(159)

Thus there are total 16 such numbers.

- 28** $(2^2 + 4^2 + 6^2 + 8^2 + \dots + 100^2) - (1^2 + 3^2 + 5^2 + \dots + 99^2)$
 $= (2^2 + 4^2 + 6^2 + \dots + 100^2)$

$$\begin{aligned} &- [(1^2 + 2^2 + 3^2 + 4^2 + \dots + 99^2 + 100^2) \\ &\quad - (2^2 + 4^2 + 6^2 + \dots + 100^2)] \\ &= 2(2^2 + 4^2 + \dots + 100^2) - (1^2 + 2^2 + 3^2 + \dots + 100^2) \\ &= 8(1^2 + 2^2 + 3^2 + \dots + 50^2) \\ &\quad - (1^2 + 2^2 + 3^2 + 4^2 + \dots + 100^2) \\ &= 8 \left(\frac{50 \times 51 \times 101}{6} \right) - \left(\frac{100 \times 101 \times 201}{6} \right) \\ &= \frac{101 \times 100}{6} (51 \times 4 - 201) = \frac{10100}{6} \times (3) = 5050, \text{ Hence (b).} \end{aligned}$$

Alternatively $(2^2 + 4^2 + 6^2 + 8^2 + \dots + 100^2)$

$$\begin{aligned} &- (1^2 + 3^2 + 5^2 + \dots + 99^2) \\ &= (4 + 16 + 36 + 64 + \dots + 10000) \\ &\quad - (1 + 9 + 25 + \dots + 9801) \\ &= [(4-1) + (16-9) + (36-25) + \dots + (10000-9801)] \\ &= 3 + 7 + 11 + 15 + \dots + 199 = \left(\frac{3+199}{2} \right) \times 50 \\ &= \frac{202 \times 50}{2} = 101 \times 50 = 5050 \end{aligned}$$

- 29** Consider some proper values.

$$\begin{aligned} \mathbf{30} \quad S_n &= 3^0 + 3^1 + 3^2 + \dots + 3^{200} \\ S_n &= \frac{3^0 (3^{201} - 1)}{(3-1)} = \frac{(3^{201} - 1)}{2} \end{aligned}$$

$$\text{Now, } \frac{(3^{201} - 1)}{2} = \frac{(3^3)^{67} - 1^3}{2} = \frac{[(27)^{67} - 1]}{2}$$

Now since $a^n - b^n$ is divisible by $(a - b)$ therefore $[(27)^{67} - 1]$ is divisible by $(27 - 1) = 26$.

Hence it is also divisible by 13.

Thus the remainder is zero since the given expression is divisible by 13.

- 31** Let the sum of the expression be S_n then

$$S_n = 4^0 + 4^1 + 4^2 + 4^3 + \dots + 4^{40}$$

$$\Rightarrow S_n = (1 + 4 + 16 + 64) + 4^4(1 + 4 + 16 + 64) + \dots + 4^{36}(1 + 4 + 16 + 64) + 4^{40}$$

Since $(1 + 4 + 16 + 64) = 85$ is divisible by 17, hence except 4^{40} remaining expression is divisible by 17.

$$\therefore \frac{4^{40}}{17} \rightarrow \frac{(4^4)^{10}}{17} \rightarrow \frac{(1)^{10}}{17},$$

Hence the required remainder is 1.

- 32** Let $S_n = 4^0 + 4^1 + 4^2 + 4^3 + 4^4 + 4^5 + \dots + 4^{40}$

$$\Rightarrow S_n = (1 + 4 + 16) + 4^3(1 + 4 + 16) + 4^6(1 + 4 + 16) + \dots + 4^{40}$$

$$\Rightarrow S_n = 21 + 4^3(21) + 4^6(21) + 4^9(21) + \dots + 4^{36}(21) + (4^{39} + 4^{40})$$

Here we can see that except $(4^{39} + 4^{40})$ remaining expression is divisible by 7.

$$\text{Now } \frac{(4^{39} + 4^{40})}{7} \rightarrow \frac{4^{39}(5)}{7} \rightarrow \frac{(4^3)^{13} \times 5}{7} \rightarrow \frac{5}{7}$$

Hence the required remainder is 5.

- 33** The total distance which a monkey has to overcome is 35 metre. Since the monkey first climbs up 5 metre and then he slips down by 2 metre, it means he climbs up only 3 metre in one round of $2 (= 1 + 1)$ minutes.

Now, note that if the monkey has to just climb up 5 metre then he will not be slipped back. Thus he will cover the 30 metre height in 10 rounds and the rest 5 metre height of the top will cover in only one attempt. Since when he will touch (or reach) the top of the pole he will not be slipped back. Thus total number of attempts required = 11 ($= 10 + 1$).

NOTE In this type of question subtract the height which he/she climbs up in first attempt (i.e., 5 metre in this question) then assuming the resultant height and solve it as usual. Now the length of the top will be covered in only one attempt of upward direction and then add up all the results obtained.

- 34** Since first he covers 30 metres in 20 minutes and the rest height of 5 metre (at the top) covers in only one minute. Hence total time = $20 + 1 = 21$ minutes.

- 35** Since the actual height is 36 metre.

Therefore $36 - 5 = 31$, which is not divisible by 3.

Hence he will climb $33 (= 3 \times 11)$ metre in 11 attempts taking $22 (= 11 \times 2)$ minutes.

Now the rest height of 3 metre will be covered in $\frac{3}{5}$ minute

i.e., 36 seconds.

Hence total time taken to cover 36 metre height is 22 minutes 36 seconds.

- 36** $1^3 + 2^3 + 3^3 + 4^3 + \dots + 1000^3$

$$= \left[\frac{1000(1001)}{2} \right]^2 \left\{ \because 1^3 + 2^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2 \right\}$$

$$= (500 \times 1001)^2 = 250000 \times 1001 \times 1001,$$

which is divisible by 13. Hence the remainder is zero.

- 37** $(22)^3 + (23)^3 + (24)^3 + \dots + (87)^3 + (88)^3$

$$= (22^3 + 88^3) + (23^3 + 87^3) + (24^3 + 86^3) + (25^3 + 85^3) + \dots + (54^3 + 56^3) + 55^3$$

Now since we know that $a^n + b^n$ is divisible by $(a + b)$ when n is an odd number. Therefore all the terms, except $(55)^3$, is divisible by 110.

Now the remainder when $(55)^3$ is divided by 110 is 55.

Hence the required remainder of the whole expression is 55.

- 38** The value of n th term

$$= \text{Sum of } n \text{ terms} - \text{Sum of } (n - 1) \text{ term}$$

Therefore value of 6th term = Sum of 6 terms

$$- \text{Sum of 5 terms}$$

$$= (6! + 6^2) - (5! + 5^2) = (720 + 36) - (120 + 25) = 611$$

- 39** The smallest possible number which is divisible by either 3, 5 or 7 is 105. Now we express it in binary representation, then

2	105			
2	52	$\rightarrow 1$	$\rightarrow$ LSB	}
2	26	$\rightarrow 0$		
2	13	$\rightarrow 0$		
2	6	$\rightarrow 1$		
2	3	$\rightarrow 0$		}
2	1	$\rightarrow 1$		
0	$\rightarrow 1$	$\rightarrow$ MSB		

$$\text{Thus } (105)_{10} = (1101001)_2.$$

Hence the minimum 7 digits are required to represent it.

- 40** Go through option

$$S_n = 1 + 3 + 5 + 7 + \dots + 22221$$

$$S_{11111} = (11111)^2 \quad [\because S_n = n^2]$$

Hence it is divisible by 11111. Thus option (b) is correct.

- 41** Go through options :

Anjuli had chocolates = 12

$$\text{She gave to Amit} = \frac{12}{2} - 1 = 5$$

Now she is left with 7 chocolates

then she gave 3 chocolates to Bablu

Now she has 4 chocolates

then she gave 2 chocolates to Charles

Now she has only two chocolates.

Then she gave 1 chocolate to Deepak

and she is finally left with one chocolate.

☞ Alternatively Solve the problem in reverse.

Anjuli's chocolate = 1 (Finally)

Before she has given to Deepak = 2 chocolates

Before she has given to Charles = 4 chocolates

Before she has given to Bablu = 7 chocolates

Before she has given to Amit = 12 chocolates.

☞ Alternatively By forming the equation assuming any variable x as the original number of chocolates which Anjuli had.

- 42** Since $a_1b_1c_1, a_2b_2c_2, a_3b_3c_3, \dots$ etc. are consecutive 3 digit numbers. Hence $c_1, c_2, c_3, \dots$ etc. must be the consecutive unit digits. Hence there will be all the 10 digits (viz., 0, 1, 2, ..., 9) 10 times as unit digits.

It means the required unit digit is equal to the sum of all the unit digits.

$$\text{Thus } [(1)^{100} + (2)^{100} + (3)^{100} + \dots (9)^{100} + (10)^{100}] \times 10$$

$$= [1 + 6 + 1 + 6 + 5 + 6 + 1 + 6 + 1 + 0] \times 10 = [3] \times 10 = 0$$

Thus the unit digit of the whole expression is 0 (zero).

- 43** $P(x, y) + Q(x, y)$

$$= (f(x, y) + g(x, y)) + (f(x, y) - g(x, y))$$

$$= 2f(x, y) = 2 \times ([x] + \{y\})$$

$$= 2 \times ([16] + \{25\}) = 2 \times (16 + 25) = 2 \times 41 = 82$$

- 44** $P(x, y) \cdot Q(x, y) = (f(x, y) + g(x, y)) \cdot (f(x, y) - g(x, y))$

$$\text{Now since } f(x, y) = [x] + \{y\} = [\pm 4] + \{\pm 5\}$$

$$f(\pm 4, \pm 5) = \pm 9 \text{ or } \pm 1$$

$$\text{and } g(x, y) = [x] - \{y\} = [\pm 4] - \{\pm 5\}$$

$$g(\pm 4, \pm 5) = \pm 9 \text{ or } \pm 1$$

Now since there are too many combinations of $f(x, y)$ and $g(x, y)$.

So there is no unique value of $P(x, y) \cdot Q(x, y)$.

- 45** Since $P(x, y) + Q(x, y) = 2f(x, y)$

So the required value is always an even number.

- 46** $(50^2 - 1) = (50 + 1)(50 - 1) = (17 \times 3) \times (7 \times 7)$

hence divisible by 17.

$$\text{and } (729)^5 - 729 = 729(729^4 - 1)$$

$$= 729(729^2 - 1)(729^2 + 1)$$

$$= (729)(729 - 1)(729 + 1)(729^2 + 1)$$

$$= 729 \times 728 \times 730 \times (729^2 + 1)$$

Hence it is divisible by 5.

☞ Alternatively $(a^p - a)$ is divisible by p if p is a prime number.

- 47** Let there be n rows, then the number of soldier in each

$$\text{row} = \frac{36}{n}$$

(Since the number of soldier in each row is same)

Now the number of bullets he gave to the first row

$$= 1 \text{ bullet} \times \text{number of soldiers}$$

and the number of bullets he gave to the second row

$$= 2 \text{ bullets} \times \text{number of soldiers}$$

and the number of bullets he gave to the third row

$$= 3 \text{ bullets} \times \text{number of soldiers}$$

Hence total number of bullets

$$= 1 \times \frac{36}{n} + 2 \times \frac{36}{n} + 3 \times \frac{36}{n} + \dots + n \times \frac{36}{n}$$

$$180 = \frac{36}{n} (1 + 2 + 3 + \dots + n)$$

$$\text{or } 180 = \frac{36}{n} \times \frac{n(n+1)}{2}$$

$$\text{or } 180 = \frac{36(n+1)}{2}$$

$$\Rightarrow (n+1) = 10$$

$$\Rightarrow n = 9$$

Thus the number of soldiers in each row = $\frac{36}{9} = 4$

Hence, there are 4 soldiers in 8th row.

- 48** Given that $(n - 5)$ is divisible by 17 and let

$$(n - 5) = k = 17m$$

$$\text{So } (n + 12) \times (n + 29) = (n - 5 + 17) \times (n - 5 + 34)$$

$$= [(n - 5) + 17] \times [(n - 5) + 34]$$

$$= (k + 17) \times (k + 34)$$

$$= k^2 + 51k + 2(17)^2$$

Now since k is divisible by 17 so,

$$k^2 + 51k + 2(17)^2 = (17m)^2 + 51 \times 17m + 2(17)^2$$

$$= 17^2(m^2 + 3m + 2)$$

Now the least possible value of $m = 1$, therefore

$$(n + 12) \times (n + 29) \text{ is divisible by } (17)^2 \times 6 = 1734.$$

- 49** Go through options :

$$(3^{96} - 1) = (3^{48} + 1)(3^{48} - 1)$$

$$= (3^{48} + 1)(3^{24} + 1)(3^{24} - 1)$$

Thus if any number ' n ' can divide $(3^{24} - 1)$ it will certainly divide $(3^{96} - 1)$.

- 50** $(5^{28} - 1) = (5^{14} + 1)(5^{14} - 1)$

$$\text{and } (5^{42} - 1) = (5^{14})^3 - 1 = (5^{14} - 1)(5^{28} + 1 + 5^{14})$$

Hence both (a) and (b). Thus the appropriate answer is (d).

51 Check through some values.

☞ Alternatively $T_1 = 1^2 + 1$
 $T_2 = 2^2 + 2$
 $T_3 = 3^3 + 3$
 $\dots\dots\dots$
 $T_n = n^2 + n$

$$\begin{aligned} S_n &= T_1 + T_2 + T_3 + \dots + T_n \\ &= (1^2 + 1)(2^2 + 2)(3^2 + 3) + \dots + (n^2 + n) \\ &= (1^2 + 2^2 + 3^2 + \dots + n^2) + (1 + 2 + 3 + \dots + n) \\ &= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \\ &= \frac{n(n+1)}{2} \left[\frac{2n+1}{3} + 1 \right] = \frac{n(n+1)}{2} \left[\frac{2n+4}{3} \right] \\ S_n &= \frac{n(n+1)(n+2)}{3} \end{aligned}$$

52 The number of zeros at the end of $222^{11} \times 35^{53}$ is 53.

The number of zeros at the end of $(7!)^{61} \times (10!)^{51}$ is 960.

The number of zeros at the end of $42^{42} \times 25^{25}$ is 42.

Thus the number of zeros at the end of the whole expression is 42.

53 $\frac{12345}{12346} + \frac{12346}{12347} + \frac{12347}{12345}$
 $= 1 - \frac{1}{12346} + 1 - \frac{1}{12347} + 1 + \frac{2}{12345}$
 $= 3 + \frac{2}{12345} - \frac{1}{12346} - \frac{1}{12347} = 3$

54 The sum of all the elements of $S_n = n^3$

Therefore the sum of all the elements of

$$S_{10} = (10)^3 = 1000$$

55 The middle most element of any odd number of set

$$S_n = n^2$$

Therefore the 11th (i.e., the middle most) element of

$$S_{21} = (21)^2 = 441$$

56 The sum of the first and last elements of the odd number of set $S_n = 2n^2$.

Therefore the required answer $= 2 \times (51)^2$
 $= 2 \times 2601 = 5202$

57 $k = 13 + 17 + 31 + 2$

$$\begin{aligned} &= (9 + 3) + (9 + 7) + (9 + 9 + 9 + 1) + 2 \\ &= 9 + 9 + 9 + 9 + 9 + 7 + 2 + 3 + 1 \\ &= 9 + 9 + 9 + 9 + 9 + 9 + 4 = 64 \\ &[\because 9 + 9 = 19, 19 + 9 = 29, 29 + 9 = 39, 39 + 9 = 49, \\ &\quad 49 + 9 = 59, 59 + 4 = 64] \end{aligned}$$

58 $4 + 16 - 5 + 12 = 12 + 4 + 16 - 5$

$$\begin{aligned} &= 16 + 11 = (9 + 6) + (9 + 1) \\ &= 9 + 9 + 7 = 27 \end{aligned}$$

59 Check for options (a), (b) and (c) then you will find distinct unit digits. Now consider $x = 10^4$.

The unit digit of $(7)^4 = 1$ $[\because \log_{10}(10^4) = 4]$

and the unit digit of $(3)^{10000}$ is 1.

Hence (d).

60 $(1 + 2 + 3 + 4 + 7)^{x^2+x}$ and $(11 \times 11 \times 13)^x$

$$(17)^{x^2+x} \text{ and } (1573)^x$$

at $x = 2$ $x^2 + x = 6$

$\therefore$ The unit digit of $(17)^6$ is 9 and the unit digit of $(1573)^2$ is 9

Hence (b) is correct.

61 For example **Remainder**

$$\begin{aligned} \frac{3^2}{8} &\rightarrow 1 \\ \frac{3^4}{8} &\rightarrow 1 \\ \frac{5^6}{8} &\rightarrow 1 \\ \frac{7^2}{8} &\rightarrow 1 \\ \frac{11^2}{8} &\rightarrow 1 \text{ etc.} \end{aligned}$$

62

Wednesday

$$\begin{aligned} &2 \\ \text{Thus } (23 - 2) &= 21, &9 \\ \text{will fall on Monday} &16 \\ &23 \\ &30 \end{aligned}$$

63

Wednesday

$$\begin{aligned} &2 \\ &8 \\ \text{21 will fall on Thursday} &14 \\ &20 \\ &26 \end{aligned}$$

64 Eldest said $\rightarrow$ Equal number of brothers and sisters (excluding eldest one)

$\Rightarrow$ 3 brothers and 3 sisters (excluding eldest one)

Youngest said $\rightarrow$ Twice the number of sisters as the number of brothers

$\Rightarrow$ 4 sisters and 2 brothers (excluding youngest)

It means there are 3 male and 4 female persons.

Now, as the youngest said that there are 4 sisters and 2 brothers engaged in work. Therefore it is obvious that the youngest person itself is a male. Hence it can be inferred that the brother of S is youngest.

	F	F	F	F	M	M	M
As :	S	T	R	A	N	G	E
	↑						↑
	Eldest						Youngest

- 65** Since $(10x + y) - (x + y) = 9x$
 $(100x + 10y + z) - (x + y + z) = 9(11x + y)$
 $(1000w + 100x + 10y + z) - (w + x + y + z)$
 $= 9(111w + 11x + y)$ etc.

Hence all these numbers are divisible by 9.

66 $y + \frac{7}{x} + \frac{y}{13} = 6 \frac{3}{143}$
 $\Rightarrow \frac{14y}{13} + \frac{7}{x} = \frac{861}{143}$

Now, the value of x can be only 11 or 143 or its multiples, in order to satisfy the relation.

Let $x = 11$, then

$$\frac{14y}{13} = \frac{861}{143} - \frac{7}{11}$$

$$\Rightarrow \frac{14y}{13} = \frac{770}{143} \Rightarrow y = 5$$

Therefore $x - y = 7 - 5 = 2$

Again, $x = 143$, then

$$\frac{14y}{13} = \frac{861}{143} - \frac{7}{143}$$

$$\Rightarrow \frac{14y}{13} = \frac{854}{143}$$

$$\Rightarrow y = \frac{61}{11}, \text{ which is not a natural number.}$$

Therefore it is clear that when $x \neq 143$, then the multiples of 143 are also not acceptable.

Thus the only acceptable value is $x = 7$ and $y = 5$.

Hence $x - y = 2$, is the correct answer.

- 67** In the first subset, the number of elements
 Divisible by 3 $\rightarrow \{3, 9, 15, 21, \dots, 99\} \Rightarrow 17$
 Divisible by 9 $\rightarrow \{9, 27, 45, \dots, 99\} \Rightarrow 6$
 Divisible by 27 $\rightarrow \{27, 81\} \Rightarrow 2$
 Divisible by 81 is 162 only $\Rightarrow 1$

In the second subset, the number of elements
 divisible by 3 $\rightarrow \{102, 108, \dots, 198\} \Rightarrow 17$
 divisible by 9 $\rightarrow \{108, 126, \dots, 198\} \Rightarrow 6$
 divisible by 27 $\rightarrow \{108, 162\} \Rightarrow 2$
 divisible by 81 is 162 only $\Rightarrow 1$

Hence the greatest power of 3 is 52 which can divide the product of all the elements of the given set.

- 68** In the first subset, the number of elements
 divisible by 5 $\rightarrow \{5, 15, 25, \dots, 95\} \Rightarrow 10$
 divisible by 25 $\rightarrow \{25, 75\} \Rightarrow 2$

In the second subset, the number of elements
 divisible by 5 $\rightarrow \{110, 120, 130, \dots, 200\} \Rightarrow 10$
 divisible by 25 $\rightarrow \{150, 200\} \Rightarrow 2$

Thus the highest power of 5 is 24 that can divide the product of all the elements of the set.

- 69** In the first subset, the number of elements
 divisible by 5 $\rightarrow \{10, 20, 30, 40, 50\} \rightarrow 5$
 divisible by 25 $\rightarrow \{50\} \rightarrow 1$

In the second subset, the number of elements
 divisible by 5 $\rightarrow \{55, 60, 65, 70, \dots\} \rightarrow 10$

divisible by 25 $\rightarrow \{75, 100\} \rightarrow 2$

(Note, that in the whole set the number of 2's are greater than the number of 5's)

Hence the number of zeros at the end of the product of the element is 18. Hence (a).

- 70** Since the 5th number is 10,
 10th number is 20
 15th number is 30
 20th number is 40
 25th number will be 100

- 71** Since in normal one hour this watch completes 5 hours. So in normal 3 hours this watch will complete 15 hours hence it will show 3 o'clock As, $15 = 12 + 3$.

72 $T_1 = 5 \times 1 + 1$
 $T_2 = 5 \times 2 + 1$
 $T_3 = 5 \times 3 + 1$
 $\dots\dots\dots$
 $T_n = 5 \times n + 1$

$$\therefore S_n = 5(1 + 2 + 3 + \dots + n) + (1 + 1 + \dots + n \text{ times})$$

$$= 5 \left[\frac{n(n+1)}{2} \right] + n = \frac{5n^2 + 7n}{2} = \frac{n}{2}(5n + 7)$$

- 73** Go through options or solve similarly as Q. No. 72.

- 74** Let the original dimension of a square be x unit then the area will be x^2 , hence total area of paper = $144x^2$

Again the area of reduced square

$$= (x - 2)(x - 2) = (x - 2)^2$$

therefore the total area of paper = $400(x - 2)^2$

Thus $144x^2 = 400(x - 2)^2$
 $\Rightarrow 9x^2 = 25(x - 2)^2$
 $\Rightarrow 9x^2 = 25x^2 + 100 - 100x$
 $\Rightarrow 16x^2 - 100x + 100 = 0 \Rightarrow x = 5 \text{ cm}$

$$\therefore \text{Area of paper} = 5 \times 5 \times 144 = 3600$$

Alternatively $\frac{3600}{114} \rightarrow 25 \rightarrow \text{side of square} = 5 \text{ cm}$
 $\frac{3600}{400} \rightarrow 9 \rightarrow \text{side of new square} = 3 \text{ cm}$

Hence option (c) is correct.

- 75** $(ab)^2 = bcb$, where ab is a two digit number and bcb is a three digit number.

Now 'b' can be equal to 0, 1, 5 or 6, which are probable values of b as a unit digit.

at $b = 0$, bcb is not a three digit number.

at $b = 1$ either $a = b$ or $b = c$ or $b \neq b$, which is not possible.

at $b = 5$, c is always 2 and $b \neq b$, hence not possible.

at $b = 6$ and $a = 2$, $(ab)^2 = bcb$ i.e., $(26)^2 = 676$, the only possible value. Thus $a = 2$, $b = 6$, $c = 7$.

Since b, c, d are in AP hence $d = 8$

Therefore $(dd)^2 = (88)^2 = 7744 = ccff \Rightarrow f = 4$

$$\begin{aligned}
 76 \quad A &= 555! = 1 \times 2 \times 3 \times 4 \times \dots \times 277 \times 278 \times 279 \times \dots \times 554 \times 555 \\
 &= (1 \times 555) \times (2 \times 554) \times (3 \times 553) \times (4 \times 552) \times \dots \times (277 \times 279) \times (278) \\
 \text{and } B &= (278)^{555} = (278 \times 278) \times (278 \times 278) \times \dots \times (278 \times 278) \times (278)
 \end{aligned}$$

Hence $A < B$.

$$\begin{aligned}
 77 \quad \text{The last element of } n\text{th set} &= \frac{n(n+1)}{2} \\
 \text{Therefore last element of } S_{24} &= \frac{24 \times 25}{2} = 300
 \end{aligned}$$

$$\begin{aligned}
 78 \quad \text{The middlemost element of any odd number set is } &\frac{n^2 + 1}{2} \\
 \text{Therefore the middlemost element of the set } S_{15} \text{ is} & \\
 \frac{15^2 + 1}{2} &= 113
 \end{aligned}$$

$$\begin{aligned}
 79 \quad \text{The sum of all the elements of an odd numbered set} & \\
 S_n &= n \times (\text{middlemost term}) \\
 \text{Therefore, the sum of all the elements of set} & \\
 S_{25} &= 25 \times \left(\frac{25^2 + 1}{2} \right) = 25 \times 313 = 7825
 \end{aligned}$$

80 Write the elements and then find out.
 S_9, S_{12}, S_{13} have maximum number of prime number elements equally.

81 The sum of all the elements of the set S_n is even if n is divisible by 4. Hence (c) is the required answer.
 Thus sum of all the elements of S_{72} is even since 72 is divisible by 4.

82 Check the options.

83 The last element of the set $S_n = n^2 + n - 1$
 Therefore the last element of the set $S_{100} = 10099$

84 The middlemost element of an odd number set $S_n = n^2$
 Therefore the middlemost element of
 $S_{125} = (125)^2 = 15625$

85 The sum of all the elements of any set $S_n = n^3$
 Therefore the required sum of the set
 $S_{101} = (101)^3 = 1030301$

$$\begin{aligned}
 86 \quad S &= \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots + \frac{1}{99.100} \\
 &= \left(\frac{1}{1} - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \dots + \left(\frac{1}{99} - \frac{1}{100} \right) \\
 &= 1 - \frac{1}{100} = \frac{99}{100}
 \end{aligned}$$

$$87 \quad \frac{P}{D} \rightarrow \text{Remainder } 18$$

$$\frac{Q}{D} \rightarrow \text{Remainder } 11$$

$$\frac{P+Q}{D} \rightarrow \text{Remainder } 29, \text{ but the remainder is } 4.$$

Hence, the divisor $= (29 - 4) = 25$.

88 We know that $1 \times 2 \times 3 \times 4 \times \dots \times (n-1) \cdot n = n!$

Now if $n = 240, 241, 242, 243, 244$ (5 values)

then the number of zeros in the end of the product of $n!$ is 68.

Alternatively The number of zeros in the given expression will depend on number of 5s (but do not depend on number of 2s). Now, since you know that the number of 5s changes at the interval of 5 consecutive numbers. Then there will be no any change in the number of zeros (since number of 5s) for the five consecutive values of n .

89 Since $\frac{6}{10} \rightarrow$ Remainder is 6

$$\frac{6^6}{10} \rightarrow \text{Remainder is } 6$$

$$\frac{6^{6^6}}{10} \rightarrow \text{Remainder is } 6$$

90 Since we know that if $(a + b + c) = 0$, then

$$a^3 + b^3 + c^3 = 3abc$$

$$\text{Therefore } 53^3 - 46^3 - 7^3 = 3 \times 53 \times 46 \times 7$$

91 Let the original number in base ' n ' be $(xy)_n$ then its reverse number be $(yx)_n$

$$\begin{aligned}
 \text{Again } 2(nx + y) &= (ny + x) \\
 \Rightarrow \frac{x}{y} &= \frac{n-2}{2n-1}
 \end{aligned}$$

$$\text{So at } n = 3, \quad \frac{x}{y} = \frac{1}{5}$$

$$\text{at } n = 4, \quad \frac{x}{y} = \frac{2}{7}$$

$$\text{and at } n = 5, \quad \frac{x}{y} = \frac{3}{9} = \frac{1}{3}$$

$$\text{and at } n = 8, \quad \frac{x}{y} = \frac{6}{15} = \frac{2}{5}$$

But the values of x, y cannot be equal to or greater than their respective bases. Hence there are only two values of n are possible viz., $n = 5$ and $n = 8$.

Now since the greatest possible value of n is 8. Hence the numbers in base $n = 8$ are 25 and 52. Again when we convert them into base 10, then the numbers are 21 and 42. Thus the sum of these numbers in decimal system is 63.

92 $n = 5, 8$. Hence (b).

$$\begin{aligned}
 93 \quad 7^0 + 7^1 + 7^2 + 7^3 + \dots + 7^{365} \\
 = \frac{1(7^{366} - 1)}{(7 - 1)} = \frac{(7^{366} - 1)}{6},
 \end{aligned}$$

its divisible since $(a^n - b^n)$ is divisible by $(a - b)$.

$$\text{Now } \frac{(7^{366} - 1)}{5} \rightarrow \frac{(2^{366} - 1)}{5} \rightarrow \frac{(4)^{183} - 1}{5} \rightarrow \frac{4 - 1}{5} \rightarrow \frac{3}{5}$$

Since the remainder is 3.

Hence Krishna will receive 3 hens.

94 These numbers are 1 and 81.

95 $\frac{\text{Actual number of boys}}{\text{Actual number of girls}} = \frac{1}{1}$

Since $\frac{2}{3}$ rd girls did not go to picnic.

Hence the ratio of boys to the girls at picnic = 3 : 1.

Now, let there be k samosas to be distributed among boys or girls equally then

$\frac{k}{3x} \rightarrow \text{Remainder } 39 \text{ and } \frac{k}{x} \rightarrow \text{Remainder } 12$

Now, since $39 - 12 = 27$

Thus $x = 27$, or 3, or 9 [$\because 27 = 1 \times 27$ or $27 = 3 \times 9$]

but since x cannot be less than 12

hence $x = 27$

Thus the number of boys = $3x = 81$

and the number of girls = $3x = 81$

Hence total number of students = 162.

96 In any box the maximum number of fruits can be packed is 15 and since not more than 5 boxes contain same number of fruits. So the number of fruits in this case = $5 \times 15 = 75$
Again we can put 14 fruits in other 5 boxes = $5 \times 14 = 70$
Similarly 13 fruits in each of the 5 boxes can be contained
 $= 5 \times 13 = 65$

Since the minimum number of fruits we have to put in a box = 10

So the total number of fruits which can be put

$$= 5 \times 15 + 5 \times 14 + 5 \times 13 + 5 \times 12 + 5 \times 11 + 5 \times 10$$

$$= 5(10 + 11 + 12 + 13 + 14 + 15) = 375$$

97	To maximize the contents in the boxes first we fill up 15-15 fruits in maximum possible number (i.e., 7) of boxes.	7	15	15	15	15	15	15	15
		7	14	14	14	14	14	14	14
		7	13	13	13	13	13	13	13
		7	12	12	12	12	12	12	12
		5	11	11	11	11	11	11	

Then we fill up 14, 13, 12, in 7-7 boxes. Thus we filled up 28 boxes containing maximum number of fruits. Now in the rest 5 boxes we fill up 11 fruits in each box to its maximum possible capacity. Now we see that there are at least 5 boxes in which the number of fruits are same.

98 $(19)^n - (23)^n - (43)^n + (47)^n$

$$= [(47)^n + (19)^n] - [(43)^n + (23)^n]$$

$$= (47 + 19)k - (43 + 23)l$$

[since $a^n + b^n$ is divisible by $(a + b)$ if n is odd]

$$= 66[k - l]$$

Hence, it is divisible by 2, 3, 6, 11, 22, 33 and 66.

Again $(47)^n - (23)^n - (43)^n + (19)^n$

$$= (47)^n - (23)^n - [(43)^n - (19)^n]$$

$$= (47 - 23)(k) - (43 - 19)(l) = 24(k - l)$$

[$\because (a^n - b^n)$ is divisible by $(a - b)$]

Therefore it is divisible by 2, 12, 3, 8, 4, 6.

Thus the whole expression is divisible by 264.

Since $264 = 11 \times 24$.

99 $1 \cdot (1)^2 \left(1 - \frac{0}{1}\right) + 2 \cdot 2^2 \left(1 - \frac{1}{2}\right) + 3 \cdot 3^2 \left(1 - \frac{2}{3}\right)$

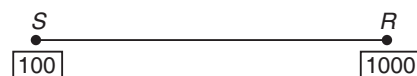
$$+ 4 \cdot 4^2 \left(1 - \frac{3}{4}\right)$$

$$= 1 \cdot (1)^2 \cdot 1 + 2 \cdot (2)^2 \cdot \frac{1}{2} + 3 \cdot (3)^2 \cdot \frac{1}{3} + 4 \cdot (4)^2 \cdot \frac{1}{4} + \dots$$

$$= 1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$

100 These numbers are (101, 102) (111, 112) (121, 122)



(131, 132) (141, 142) ... (202, 203) (212, 213) ... (303, 304) (313, 314) ... (808, 809) ... (898, 899, 900) ... (909, 910) (919, 920) ... (989, 990).

Thus there are total $20 \times 7 + 21 + 18 = 179$, milestones which are red, because the hundreds digit on next mile stone is same as the unit of previous milestone.

101 $(18^2 - 1) = (17)(18 + 1)$

$$(18^4 - 1) = (18^2 + 1)(18^2 - 1) = (18^2 + 1)(18 + 1)(18 - 1)$$

$$(18^6 - 1) = (18^3)^2 - 1 = (18^3 + 1)(18^3 - 1) = 17 \times k \text{ etc.}$$

Hence there will only 9 times 17 in the whole expression.

102 $\frac{M}{28} \rightarrow \text{Remainder } 24 \Rightarrow M = 28k + 24$

and $\frac{M}{22} \rightarrow \text{Remainder } 12 \Rightarrow M = 22l + 12$

$$\Rightarrow M = 28k + 24 = 22l + 12$$

$\Rightarrow$ at $k = 9$, $l (= 12)$, is an integer, for the least solution.

Hence, $M = 276$... (i)

and higher such numbers can be obtained as

$$M' = 308p_1 + 276 \quad \dots \text{(ii)}$$

Again $\frac{N}{28} \rightarrow \text{Remainder } 16$

$$\Rightarrow N = 28m + 16$$

and $\frac{N}{22} \rightarrow \text{Remainder } 12 \Rightarrow N = 22n + 12$

$$\Rightarrow N = 28m + 16 = 22n + 12$$

at $m = 3$, $n (= 4)$ is an integer, for the least solution.

Hence $N = 100$

and higher such numbers can be obtained as

$$N' = 308p_2 + 100$$

Now, $28 + 22 = 50$, hence $(M' + N')$ must be divisible by 5.

So $M' + N' = 308(p_1 + p_2) + 376$

In order that $M' + N'$ must be divisible by 50, either $p_1 = 1$ and $p_2 = 2$ or $p_1 = 2$ and $p_2 = 1$, for the least possible solution.

Hence, $M' + N' = 308 \times 3 + 376 = 1300$.

103 Since $M' + N' = 308(p_1 + p_2) + 376$

As we can observe that the values of p_1 and p_2 are not constant so we cannot determine uniquely the ratio of $M' : N'$.

104 In each set of 100 numbers, there are 10 numbers whose tens digit and unit digit are same. Again in the same set there are 10 numbers whose hundreds and tens digits are same. But there is one number in each set of 100 numbers whose Hundreds, Tens and Unit digit are same as 111, 222 etc. Hence there are exactly $(10 + 10 - 1) = 19$ numbers in each set of 100 numbers. Further there are 9 such sets of number. Hence total required number = 171.

☞ Alternatively

H	T	U		H	T	U
9	10	10	-	9	9	9

$$900 - 729 = 171$$

$$\begin{aligned} \text{105 } (1+p)^{256} \times (1+p^{128}) \times \dots \times (1+p^2)(1+p) &\times \frac{(1-p)}{(1-p)} \\ &= \frac{(1+p^{256})(1-p^{256})}{1-p} = \frac{(1-p^{512})}{1-p} \end{aligned}$$

106 Let the length and breadth of the given perimeter be l and b , respectively. Then $2(l+b) = 50$

$$\Rightarrow l + b = 25$$

Now

$$(b, l) = (1, 24), (2, 23), (3, 22), (4, 21), (5, 20), (6, 19), (7, 18), (8, 17), \dots, (12, 13)$$

Therefore the required number of rectangles = 12

107 The product of prime factors = $30 = 2 \times 3 \times 5$

Now the total number of prime factors = 24

$$= 2 \times 2 \times 6 \Rightarrow (1+1)(1+1)(5+1)$$

So the least possible number = $2^5 \times 3^1 + 5^1 = 480$

and the greatest possible number = $2^1 \times 3^1 \times 5^5 = 18750$

$$\text{So the ratio of} = \frac{480}{18750} = \frac{2^4}{5^4}$$

$$\therefore \text{The square root} = \frac{2^2}{5^2} = \frac{4}{25}$$

108 Let the powers of 2, 3, 5 be a , b , c respectively then the total number of factors = 24

$$= (a+1)(b+1)(c+1) = 24$$

So if $(a+1) = k$, $(b+1) = l$, $(c+1) = m$, then

$$k \times l \times m = 24, \quad \text{where } k, l, m \geq 2$$

$$2 \times 2 \times 6$$

$$2 \times 3 \times 4$$

$$2 \times 6 \times 2$$

$$2 \times 4 \times 3$$

$$3 \times 2 \times 4$$

$$3 \times 4 \times 2$$

$$4 \times 2 \times 3$$

$$4 \times 3 \times 2$$

$$6 \times 2 \times 2$$

Hence there are total 9 numbers possible.

109 The best way is to go through options. You will find that when you consider 150 question you can not get 75 marks any how.

☞ **Alternatively** Since 75 is an integer, hence he must have to do $(75 + 4x)$ problems, where x is a whole number.

Now put the values of x and satisfy the given options.

Explanation Since he gets the net marks in integers it means when he does $3n$ questions wrong, he loses actually $4n$ marks. For more clarification see the table below :

Total Questions	Correct	Marks	Wrong	Marks	Net marks
75	75	75	0	0	75
79	76	76	3	-1	75
83	77	77	6	-2	75
87	78	78	9	-3	75

110 Solve it through options

	Total given coins	Rest coins
$(25-1) \times \frac{1}{2} = 24 \times \frac{1}{2} = 12$	13	12
$(12-2) \times \frac{1}{2} = 10 \times \frac{1}{2} = 5$	7	5
$(5-3) \times \frac{1}{2} = 2 \times \frac{1}{2} = 1$	4	1

Hence he must have stolen 25 coins.

111 Let $y = \log_2 7 \Rightarrow 2^y = 7$ ($\because x^y = a \Rightarrow \log_x a = y, x \neq 1$)

[Clearly y neither can be an integer nor a fraction]

Now if y is a rational number, we can write as

$$y = \frac{p}{q}; \quad q \neq 0$$

$$\text{Thus } 2^{p/q} = 7 \Rightarrow 2^p = 7^q.$$

Since in L.H.S. the quantity is even while in the right hand side the given quantity is odd. Thus, y can't be a rational number.

112 Let us consider $2 \times \frac{1}{2} = 1$, then $2 + \frac{1}{2} = 2.5$

Thus the option (a) and (b) are ruled out.

Again, let us take some different values viz., $\frac{1}{3} \times \frac{3}{2} \times 2 = 1$

$$\text{Then, } \frac{1}{3} + \frac{3}{2} + 2 = \frac{23}{6} \neq 3 + \frac{1}{3} \left(= \frac{10}{3} = \frac{20}{6} \right)$$

Hence option (c) is ruled out.

Finally, option (d) is correct, as can be seen from the above illustrations.

☞ Alternatively Let $a_1, a_2, a_3, \dots, a_n$ be n positive real numbers such that $a_1 \cdot a_2 \dots a_n = 1$.

Now, we know that A.M. $\geq$ G.M. for some given quantities.

$$\text{So } \frac{(a_1 + a_2 + a_3 + \dots + a_n)}{n} \geq (a_1 \cdot a_2 \cdot a_3 \dots a_n)^{1/n} = 1$$

$$\Rightarrow (a_1 + a_2 + a_3 + \dots + a_n) \geq n$$

Hence option (d) is correct choice.

113 Let $S = 1 + n + n^2 + \dots + n^{127}$

$$\begin{aligned} &= \frac{1(n^{128} - 1)}{(n - 1)} \left[\because S_n = \frac{a(r^n - 1)}{r - 1}; r > 1 \right] \\ &= \frac{n^{128} - 1}{(n - 1)} = \frac{(n^{64} - 1)(n^{64} + 1)}{(n - 1)} \end{aligned}$$

Thus at $m = 64$ the given expression is divisible by $(n^m + 1)$. Hence (c).

114 We have $240 = 2^4 \times 3 \times 5$

$\therefore$ Number of divisors $= 5 \times 2 \times 2 = 20$, which are given below

1	2	4	8	16
3	6	12	24	48
5	10	20	40	80
15	30	60	120	240

Thus only 4 divisors are of the form $4n + 2$

Hence option (a) is correct.

115 We know that if a number is divisible by 5, then its unit digit must be either 0 or 5.

Again, the unit digits of $7^1 = 7, 7^2 = 9$

$$7^3 = 3 \text{ and } 7^4 = 1$$

i.e., we can get only 7, 9, 3, 1 as unit digits.

Now, the combinations of 7, 9, 3, 1 never gives unit digit 5, but $7 + 3 = 9 + 1 = 10$, these two combinations gives us unit digit zero.

So $7^m + 7^n$ is divisible where

case (i) $m = 1, 5, 9, \dots, 97$ correspondingly

$$n = 3, 7, 11, \dots, 99$$

case (ii) $m = 2, 6, 10, \dots, 98$ correspondingly

$$n = 4, 8, 12, \dots, 100$$

or the values of m and n can be reversed in each case mutually but then we get the same values.

Hence total distinct numbers so formed, divisible by 5,

$$\text{are } 1250 = 25 \times 25 + 25 \times 25$$

Hence option (a) is correct.

116 Let us assume the values of a, b, c, d such that

$$a + b + c + d = 2$$

$$\text{i.e., } \frac{1}{3} + \frac{2}{3} + \frac{1}{4} + \frac{3}{4} = 2$$

$$\text{then } M = (a + b)(c + d) = \left(\frac{1}{3} + \frac{2}{3}\right)\left(\frac{1}{4} + \frac{3}{4}\right) = 1$$

Hence option (c) and (d) are ruled out.

now, if a, b, c, d be $\frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{3}{4}$, then

$$M = (a + b)(c + d) = \left(\frac{1}{3} + \frac{1}{4}\right)\left(\frac{2}{3} + \frac{3}{4}\right) = \frac{119}{144} < 1$$

Hence option (b) is also ruled out. Thus only option (a) is correct.

☞ Alternatively Let $a + b = P$ and $c + d = Q$,

$$P + Q = 2 \text{ and } PQ > 0$$

(Since P and Q are positive real numbers)

Further we know that

$$AM \geq GM \Rightarrow \frac{P + Q}{2} \geq \sqrt{PQ}$$

$$\Rightarrow 1 \geq \sqrt{PQ} \Rightarrow 1 \geq PQ$$

$$\therefore PQ = M \leq 1$$

Hence option (a) is correct.

117 As per the given information, $N = \frac{P!}{Q!}$ implies that N is the

product of any 10 consecutive two-digit natural numbers, such that the largest number out of the 10 consecutive whole numbers is 99. The problem basically asks us to find out that maximum how many times can N be divided by 2.

Since the highest power of 2 among all the two-digit numbers is 64. So 64 must be one of the ten natural numbers.

Now you can consider any ten consecutive natural numbers say 64, 65, 66, 67, 68, 69, 70, 71, 72, and 73.

The number of times 64 can be divided by $2 = 6$

The number of times 66 can be divided by $2 = 1$

The number of times 68 can be divided by $2 = 2$

The number of times 70 can be divided by $2 = 1$

The number of times 72 can be divided by $2 = 3$

Therefore, N can be divided by 2 is 13 times. It means the maximum value of $x = 13$.

Hence, choice (d) is the answer.

☞ Alternatively You can find the total number of 2s in a set of 10 consecutive natural numbers in the following ways. Let S be the set of 10 consecutive natural numbers.

Then the max number of elements in S which are divisible by $2 = 5$

And the max number of elements in S which are divisible by $4 = 3$

And the max number of elements in S which are divisible by $8 = 2$

And the max number of elements in S which are divisible by $16 = 1$

And the max number of elements in S which are divisible by $32 = 1$

And the max number of elements in S which are divisible by $64 = 1$

Therefore N can be divided by 2^{13}

118 Let the total number of coal blocks allocated is x and total number of 2G spectrums allocated is y , then

$$300x + 500y = 13300$$

$$\Rightarrow y = \frac{133 - 3x}{5}$$

This gives us the following possibilities, provided x and y must be positive integers.

x	y	HCF of x and y
1	26	1
6	23	1
11	20	1
16	17	1
21	14	7
26	11	1
31	8	1
36	5	1
41	2	1

Whenever x and y are co-primes, there will be only one minister, but the problem talks about the involvement of several ministers. So we have to consider the case in which x and y are not the co-primes. Thus we can consider the case in which $x = 21$ and $y = 14$, as their HCF is 7. Now $21/7 = 3$ and $14/7 = 2$.

It implies that there are 7 ministers involved in the scam and each minister had allocated 3 coal blocks and 2 spectrums to the corporate coterie by accepting the stated amount of bribe.

Hence, (c) is the correct answer.

☞ Alternatively

Let the total number of coal blocks allocated is x and total number of 2G spectrums allocated is y , then

$$300x + 500y = 133000$$

$$\Rightarrow 3x + 5y = 7 \times 19$$

$$\Rightarrow (3 \times 7m) + (5 \times 7n) = 7 \times 19$$

$$\Rightarrow 7(3m + 5n) = 7 \times 19$$

$$\Rightarrow 3m + 5n = 19$$

From the given options only choice (d) satisfies the above equation. Otherwise, solve the following equation for positive integers m and n .

$$3m + 5n = 19 \Rightarrow n = \frac{19 - 3m}{5}$$

☞ **NOTE** $3x + 5y = 7 \times 19$ can be expressed in two ways as the RHS has two prime actors.

$$(i) (3 \times 7m) + (5 \times 7n) = 7 \times 19 \text{ and}$$

$$(ii) 3 \times 19m + 5 \times 19n = 7 \times 19$$

But, $3 \times 19m + 5 \times 19n = 7 \times 19 \Rightarrow 3m + 5n = 7$ does not yield any solution for any integral value of m and n .

119 His maximum loss = -20 (₹)

His maximum gain = 60 (₹)

Total possible distinct amounts including no profit no loss

$$= 20 + 60 + 1 = 81$$

However, he cannot gain by ₹ 59, ₹ 58, and ₹ 55.

Case I When he loses no game that he plays, his monetary gain could be 3, 6, 9, ... 54, 57 and 60.

In this case he can win max. 20 games and gain max. ₹ 60.

Case II When he loses one game out of all the games that he plays, his monetary gain could be 2, 5, 8, ... 50, 53 and 56.

In this case he can win max. 19 games and gain max. ₹ 56.

Case III When he loses two games out of all the games that he plays, his monetary gain could be 1, 4, 7, ..., 52.

In this case he can win at max 18 games and win max. ₹ 52.

By combining the data of case I, case II and case III, we can say that except 55, 58 and 59 all other values from 1 to 60 can be obtained.

Case IV Whenever the number of games lost is three times the number of games won his net gain is ₹ 0.

Case V When he loses all the matches that he plays his monetary loss is any amount from 0 to 20.

That means the player can have any amount from 0 to 80, except 75, 78 and 79.

So the net possible distinct values of amount = $81 - 3 = 78$.

Hence, choice (d) is the correct one.

☞ Alternatively

$$3x - 0 = 3, 6, 9, \dots, 45, 48, 51, 54, 57, 60.$$

$$3x - 1 = 2, 5, 8, \dots, 44, 47, 50, 53, 56.$$

$$3x - 2 = 1, 4, 7, \dots, 43, 46, 49, 52.$$

$$3x - 3 = 0, 3, 6, \dots, 42, 45, 48.$$

$$3x - 4 = -1, 2, 5, \dots, 41, 44.$$

$$3x - 5 = -2, 1, 4, \dots, 40.$$

$$\dots\dots\dots$$

$$\dots\dots\dots$$

$$3x - 18 = -18, -15, -12$$

$$3x - 19 = -19, -16$$

$$3x - 20 = -20$$

It clearly shows that ₹ 55, ₹ 58 and ₹ 59 cannot be gained.

☞ **Alternatively** Without losing any game player can have gain of 60, 57, 54, 51, 48, ..., 9, 6 and 3.

However, you can see that 59, 58, 56, 55, 53, 52, 50, 49, ..., 8, 7, 5, 4, 2, 1 numbers are missing.

In order to get the remaining numbers the player has to lose 1 game and 2 games. But, whenever the player loses 1 game, the maximum number of games he can win is 19, not the 20. Therefore he can't get a net gain of $60 - 1 = 59$.

Similarly, when the player loses 2 games, the maximum number of games he can win is 18, not the 19 and 20. Therefore he can't get a net gain of 55 and 58. Therefore out of 81 values $(-20, -19, -18, \dots, 80)$ 55, 58 and 59 are missing. Therefore, we can have maximum 78 distinct amounts.

120 Let a , b and c be the numbers of men engaged in fighting against the rebellions, then the total number of men alive after the fight got over

$$= \frac{36 \times a}{100} + \frac{60 \times b}{100} + \frac{80 \times c}{100} = \frac{50(a + b + c)}{100}$$

$$\Rightarrow 5(b + 3c) = 7a \Rightarrow a = \frac{5(b + 3c)}{7}$$

It implies that $b + 3c$ must be divisible by 7 and a is a multiple of 5.

We also know that 36% of a implies that a must be a multiple of 25. Similarly 60% of b and 80% of c imply that b and c must be the multiples of 5. Thus we can obtain the feasible values from the following table.

a	$b+3c$	c	b	$a+b+c$
25	35	5	20	50
25	35	10	5	40
50	70	5	55	110
50	70	10	40	100
50	70	15	25	90
50	70	20	10	80
75	105	5	90	170
75	105	10	75	160
75	105	15	60	150
75	105	20	45	140
75	105	25	30	130
75	105	30	15	120
...	...	..	...	..

From the above table, it is evident enough that $a + b + c \neq 60$. Hence, choice (c) is the correct one. Hence, choice (c) is the correct one.

Solutions (for Q. Nos. 121 to 123) *There are 25 pins each with distinct number as 1 to 25.*

All the 25 pins arranged, initially, in the order of 1, 2, 3, ... 25, from left to right. By throwing the ball, you have to knock over any 5 contiguous pins say pin number (1, 2, 3, 4, 5) or (2, 3, 4, 5, 6), or (3, 4, 5, 6, 7) etc.

The expression $\left[\frac{k!}{(k-5)!} \right]$ implies the product of any 5

consecutive natural numbers, since $k \geq 5$.

It implies that you will get refund, only when you hit 5 contiguous pins, whose product is the multiple of 16.

It means you have to find all the possible products of 5 consecutive natural numbers, such that the product is either 16 or 32 or 48, etc. You must know that any sequence of 5 consecutive numbers has two possibilities:

- The sequence which has 3 even numbers and 2 odd numbers
- The sequence which has 2 even numbers and 3 odd numbers

Since you are looking for 16 or its multiples, you have to focus on the number of 2s in the product of 5 consecutive numbers.

Case (i) When there are 3 consecutive even numbers in the sequence, the product must be divisible by 16. Since each of the 3 even numbers will be multiple of 2 and at least one number will be the multiple of 4, so there will be minimum four 2s. For example,

$$(2, 3, 4, 5, 6): 2 \times 3 \times 4 \times 5 \times 6 = 3 \times 5 \times (2 \times 4 \times 6) \\ = 3 \times 5 \times 3 \times (2 \times 2 \times 2 \times 2)$$

$$(4, 5, 6, 7, 8): 4 \times 5 \times 6 \times 7 \times 8 = 5 \times 7 \times (4 \times 6 \times 8) \\ = 5 \times 7 \times 3 \times (2 \times 2 \times 2 \times 2 \times 2 \times 2)$$

:::

$$(20, 21, 22, 23, 24): 20 \times 21 \times 22 \times 23 \times 24$$

$$= 21 \times 23 \times (20 \times 22 \times 24)$$

$$= 21 \times 23 \times 5 \times 11 \times 3 \times (2 \times 2 \times 2 \times 2 \times 2 \times 2)$$

In this case, there will be total 10 sequences.

Case (ii) When there are 2 consecutive even numbers in the sequence, it is not necessary that the product is divisible by 16. In this case, the product will be divisible by 16 only when one number must be 8 or its multiples. When out of two consecutive even numbers one is 8, total number of sequences is 2. (5, 6, 7, 8, 9) and (7, 8, 9, 10, 11). When out of two consecutive even numbers one is 16, total number of sequences is 2. (13, 14, 15, 16, 17) and (15, 16, 17, 18, 19). When out of two consecutive even numbers one is 24, total number of sequences is 1. (21, 22, 23, 24, 25). In this case, there are total 5 sequences.

121 Thus there are total 15 distinct sequences of 5 consecutive numbers such that the product of all the 5 numbers of the sequence is multiple of 16.

Hence, choice (a) is the correct one.

122 Since k is the highest number of each such sequence, which is unique for each sequence, so there will be total 15 values of k , as there are total 15 desired sequences. Hence choice, (d) is the correct one.

123 From the case (i) it can be inferred that all the numbers from 2 to 24 fall in one or the other sequence.

And from the case (ii) it can be inferred that the number 25 is also the part of a sequence. So the only number that is not the part of any desired sequence is 1. Thus, if you knock over pin number 1, you can't get back your money. Hence, choice (d) is the correct one.

$$124 \quad S = 1 + x + x^2 + x^3 + \dots + x^{111}$$

$$S = \frac{x^{112} - 1}{x - 1} \Rightarrow S = \frac{(x^{56} + 1)(x^{56} - 1)}{x - 1}$$

$$S = \frac{(x^{56} + 1)(x^{28} + 1)(x^{28} - 1)}{x - 1}$$

$$S = \frac{(x^{56} + 1)(x^{28} + 1)(x^{14} + 1)(x^{14} - 1)}{x - 1}$$

$$S = \frac{(x^{56} + 1)(x^{28} + 1)(x^{14} + 1)(x^7 + 1)(x^7 - 1)}{x - 1}$$

Therefore, a, b, c can assume any three of the four values (7, 14, 28, 56). So there are two distinct possibilities - When anyone of a, b, c is 7, HCF would be 7 and when none of a, b, c is 7, HCF would be 14.

Hence, choice (d) is the correct one.

125 The number 15! must be divisible by the prime numbers 2, 3, 5, 7, 11 and 13. The best way to check the divisibility of the 15! is by 9 or 11. So let's do it by 9.

$$\text{Now, } 1 + k + 0 + 7 + 6 + 7 + 4 + 3 + 6 + 8 + 0 + 0 + 0 = 42 + k$$

Since $42 + k$ should be divisible by 9, so $k = 3$

Hence, choice (b) is the correct one.

CHAPTER 02

Averages

Exams, such as CAT, XAT, IIFT, CMAT, GMAT, SSC CGL and Bank PO ask direct problems from this chapter. Looking at the historical data, we can say that every year they usually ask at least 1 problem in Quantitative Aptitude section and at least 4 problems in Data Interpretation section. Sometimes they ask a few questions which are application based and involve a great amount of logical thinking. It suggests that we need to develop a fresh logical approach to answer the problems of this chapter. Essentially, I won't recommend you to completely rely on formula-based approach.

2.1 Average

In general average is the central value of the given data. For example, if the heights of three persons A , B and C be 90 cm, 110 cm and 115 cm, respectively, then the average height of A , B and C together will be $\frac{90+110+115}{3} = 105$ cm. So, we can say that the height of each

person *viz.* A , B and C is near about 105 cm. Thus in layman's language it can be said that every one is almost 105 cm tall. Basically, the average is the arithmetic mean of the given data. For example, if the $x_1, x_2, x_3, x_4 \dots x_n$ be any ' n ' quantities (*i.e.*, data), then the average (or arithmetic mean) of these ' n ' quantities = $\frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$

Properties of Average

1. The average of any two or more quantities (or data) necessarily lies between the lowest and the highest values of the given data. *i.e.*, if x_l and x_h be the lowest and highest (or greatest) values of the given data ($x_1, x_2, \dots, x_l, \dots, x_h, \dots, x_n$) then $x_l < \text{Average} < x_h$; $x_l \neq x_h$ *i.e.* $x_l < \frac{(x_1 + x_2 + x_3 + x_l \dots + x_h \dots + x_n)}{n} < x_h$

Ex. 1) The average of 8, 9, 12, 13, 15, 9 is :

- (a) 11 (b) 6 (c) 16 (d) 18

Solution From the above mentioned property (1) we can say that options (b), (c) and (d) are invalid since 6, 16 and 18 are out of range *i.e.*, either below 8 (which is the least value) or above 15 (which is the highest value of the data). So, only option (a) can be acceptable.

☞ **Alternatively** Average = $\frac{8+9+12+13+15+9}{6} = \frac{66}{6} = 11$

Chapter Checklist

- Average
 - Properties of Average
 - Weighted Average
 - Problems based on Age
 - Problems based on Income and Salary
 - Problems based on Time, Speed and Distance
 - Value of an Overlapping Element
 - Average of Same Important Series of Numbers
- CAT Test

Ex. 2) *A* has 8 pencils, *B* has 10 pencils and *C* has 15 pencils, then the average number of pencils with them :

- (a) 8 (b) 10
(c) 15 (d) lies between 9 and 15

Solution Average number of pencils = $\frac{8+10+15}{3} = 11$

So, option (d) is correct.

2. If each quantity is increased by a certain value '*K*' then the new average is increased by *K*.

Ex. 3) *A*, *B*, *C*, *D* and *E* are the five electronic shops in the Naza market, which have 20, 30, 60, 80 and 50. T.V. sets with them respectively, then the average number of T.V. sets in each shop is :

- (a) 24 (b) 48 (c) 50 (d) 60

Solution Average number of T.V. sets = $\frac{20+30+60+80+50}{5} = 48$

So, option (b) is correct.

Ex. 4) In the previous example, if each of *A*, *B*, *C*, *D* and *E* have imported 12 new T.V. sets, then the average number of T.V. sets is:

- (a) 50 (b) 56 (c) 60 (d) 144

Solution Average number of T.V. sets
= $\frac{(20+12) + (30+12) + (60+12) + (80+12) + (50+12)}{5} = \frac{300}{5} = 60$

Alternatively

$$= \frac{(20+30+60+80+50) + (12+12+12+12+12)}{5}$$

$$= \frac{(240) + (5 \times 12)}{5} = 48 + 12 = 60$$

Thus, the new average will be 60.

Ex. 5) The average number of shirts with Salman, Ambani and Dalmiya is 60, if all of them reached a shopping mall (AMBI) in Gurgaon and purchased 6 shirts each of them then the average number of shirts each of them now has :

- (a) 66 (b) 63
(c) 62 (d) can't be determined

Solution Required average = Old average
+ New average = $60 + 6 = 66$

3. If each quantity is decreased by a certain value *K*, then the new average is also decreased by *K*.

Ex. 6) *A*, *B*, *C*, *D*, *E* and *F* are the only six families in Indira nagar. *A*, *B*, *C*, *D*, *E* and *F* has 7, 8, 10, 13, 6 and 10 members in their families respectively. If 1 member from all the six families left their respective families to accomodate themselves in the hostel of IIM Lucknow, then the average number of members now in each family of Indira nagar is :

- (a) 8 (b) 9 (c) 10 (d) 13

Solution Required average
= $\frac{(7-1) + (8-1) + (10-1) + (13-1) + (6-1) + (10-1)}{6}$
= $\frac{(7+8+10+13+6+10)}{6} - \frac{(6 \times 1)}{6} = 9 - 1 = 8$

Ex. 7) Last year Sahara, Tata, Singhanian and Birla each has 250 industries. This year everyone has sold out 10 factories due to the recession of the economy and poor turnout. The average number of industries which each of them now has :

- (a) 260 (b) 240
(c) 25 (d) none of these

Solution Required average = Old average – New average
= $(250) - (10) = 240$

4. If each quantity is multiplied by a certain value *K*, then the new average is the product of old average with *K*.

Ex. 8) In a flower shop there were 6 flowers in each bouquet. If the seller has doubled the number of flowers in each bouquet then the new average of flowers in each bouquet is :

- (a) 12 (b) 8
(c) 3 (d) none of these

Solution Now required average
= Old average $\times$ Multiplicand = $6 \times 2 = 12$

5. If each quantity is divided by a certain quantity '*K*', then the new average becomes $\frac{1}{K}$ times of the initial average, where $K \neq 0$.

Ex. 9) The average of 100, 200, 300, 400, ..., 1000 is 550. If each number is divided by 5, then the new average will be equal to :

- (a) 450
(b) 45
(c) 55
(d) none of the above

Solution Actual average = $\frac{(100+200+300+\dots+1000)}{10} = 550$

The new average = $\frac{550}{5} = 110$

Hence, (d) is correct.

6. If '*A*' be the average of $x_1, x_2, x_m, \dots, y_1, y_2, \dots, y_n$ where $x_1, x_2, \dots, x_m$ be the below *A* and $y_1, y_2, y_3, \dots, y_n$ be the above *A*, then

$$(A - x_1) + (A - x_2) + \dots (A - x_m)$$

$$= (y_1 - A) + (y_2 - A) + \dots (y_n - A)$$

i.e., the surplus above the average is always equal the net deficit below average.

Ex. 10) The salary of A, B, C, D and E is ₹ 8000, ₹ 5000, ₹ 11000, ₹ 7000, ₹ 9000 per month respectively, then the average salary of A, B, C, D and E per month is :

- (a) ₹ 7000 (b) ₹ 8000
(c) ₹ 8500 (d) ₹ 9000

Solution Average salary

$$= \frac{8000 + 5000 + 11000 + 7000 + 9000}{5} = ₹ 8000$$

NOTE The average salary = ₹ 8000 and the sum of salaries above ₹ 8000 = 11000 + 9000 = 20000

$$\text{Therefore, the average surplus} = \left(\frac{20000}{2} \right) - 8000 = 2000$$

$$\text{again, the sum of salaries below ₹ 8000} = 5000 + 7000 = 12000$$

$$\text{Therefore, the average deficit} = 8000 - \left(\frac{12000}{2} \right) = 2000$$

$$\text{Hence, the surplus above the average} \\ = \text{deficit below the average} = 2000$$

Alternatively Let the average be 10000

Then,

$$10000 - 8000 = 2000$$

$$10000 - 5000 = 5000$$

$$10000 - 7000 = 3000 \quad \text{and} \quad 11000 - 10000 = 1000$$

$$10000 - 9000 = 1000 \quad \text{Surplus} = \underline{1000}$$

$$\text{Deficit} \rightarrow \underline{11000}$$

$$\therefore \text{Net difference} = \text{Surplus} - \text{deficit} \\ = 1000 - 11000 = -10000$$

$$\therefore \text{Average variation} = \frac{-10000}{5} = -2000$$

$$\text{Hence, the actual average} = \text{assumed average} \\ + \text{average variation} \\ = 10000 - 2000 = 8000$$

Ex. 11) The average of 6 numbers 13, 17, 25, 11, 26, 10 is:

- (a) 20 (b) 17 (c) 15 (d) 25

Solution Let the average be 20.

$$\text{Then, } 20 - 13 = 7 \quad 20 - 25 = 5$$

$$20 - 17 = 3 \quad \text{and} \quad 20 - 26 = 6$$

$$20 - 11 = 9 \quad \underline{11}$$

$$20 - 10 = 10 \quad (\text{Surplus}) \\ \underline{29} \text{ (deficit)}$$

$$\text{The net difference (surplus} - \text{deficit)} = (11 - 29) = -18$$

$$\text{Therefore average variation} = \frac{-18}{6} = -3$$

$$\text{Hence, the actual average} = 20 + (-3) = 17$$

NOTE This method is very helpful in Data Interpretation section where there are very large values are to be calculated and most of the time we need just the lumpsum value.

7. Weighted Average : When the average of groups or sets, instead of individuals, having different number of elements is being calculated, then it is called the weighted average. Since in this case the number of elements is different for the different sets thus they carry different weightage. If the number of elements in n different groups be $K_1, K_2, K_3, K_4, \dots, K_n$ and the averages of the respective groups be $A_1, A_2, A_3, A_4 \dots A_n$ then the

$$\text{weighted average} = \frac{K_1 A_1 + K_2 A_2 + K_3 A_3 + \dots K_n A_n}{K_1 + K_2 + K_3 + \dots K_n}$$

Ex. 12) The average salary of 12 employees of STAR plus is ₹ 18000 per month and 15 employees of NDTV is ₹ 16000 per month. The average salary of all the 27 employees is :

- (a) ₹ 17000 (b) ₹ 16500
(c) ₹ 16888.88 (d) none of these

Solution Required average salary

$$= \frac{12 \times 18000 + 15 \times 16000}{12 + 15} = \frac{456000}{27} = ₹ 16888.88$$

Ex. 13) The number of students at $i4$ IIM in morning batch, evening batch and weekends batch is 30, 40 and 60 respectively and their respective average ages (in years) are 22, 21 and 25, then the average age of all the students (combined) is :

- (a) $22 \frac{1}{13}$ years (b) $23 \frac{1}{13}$ years
(c) 24.5 years (d) none of these

$$\text{Solution } \frac{30 \times 22 + 40 \times 21 + 60 \times 25}{130} = \frac{3000}{130} = 23 \frac{1}{13} \text{ years}$$

8. Problems Based on Age :

- If the average age of ' n ' members of a family is x years then K years back, the average age of the family is $(x - K)$ years, provided that no any person is expired or born in K years period.
- If the present age of ' n ' members of a family is x years then K years later the average age of the same family will be $(x + K)$ years.

Ex. 14) The average age of 7 members of Patel's family is 25 years. The average age of the same family 3 years ago was :

- (a) 21 years (b) 22 years
(c) 25 years (d) none of these

Solution Present average age of family = 25 years

$$3 \text{ years ago average age of family} = 25 - 3 = 22 \text{ years}$$

Ex. 15) The average age of all the 16 professors of Lucknow University is 52 years. Four years later the average age of all the 16 professors will be, if there is no any retirement or recruitment :

- (a) 68 (b) 64
(c) 56 (d) none of these

Solution Present average age = 52 years

4 years later the average age = $52 + 4 = 56$ years

Ex. 16) 6 months ago the present age of the student of class 10th was 14 years. 6 months hence, the age of the same students will be :

- (a) 15 years (b) $15\frac{1}{2}$ years
(c) 20 years (d) none of these

Solution Since the time difference between two dates is 1 year, hence the average age will be increased by 1 year. Thus the average age of the class 6 months hence will be 15 years.

Ex. 17) The average age of Priyambada's family consisting of 5 members 3 years ago was 35 years. One year ago a new baby was born in this family. Three years hence the average age of the family will be :

- (a) 36 years (b) $34\frac{5}{6}$ years
(c) $35\frac{4}{5}$ years (d) none of these

Solution 3 years ago total age of 5 members
= $5 \times 35 = 175$ years

At the time of birth of new baby the total age of family
= $175 + (2 \times 5) = 185$ years

The present age of family = $185 + (1 \times 6) = 191$ years

3 years hence, the average age of family

$$= \frac{191 + (3 \times 6)}{6} = 34\frac{5}{6} \text{ years}$$

Ex. 18) 10 years ago the average age of all the 25 teachers of the Girls college was 45 years. 4 years ago, the principal has retired from her post at the age of 60 year. So after one year a new principal whose age was 54 years recruited from outside. The present average age of all the teachers is, if principal is also considered as a teacher :

- (a) $54\frac{18}{25}$ years (b) $55\frac{17}{25}$ years
(c) $49\frac{1}{2}$ years (d) none of these

Solution 10 years ago average age of 25 teachers = 45 years
4 years ago (just before the retirement of principal) average age of 25 teachers = $45 + 6 = 51$ years

and the same time total age of 25 teachers = 51×25
= 1275 years

and the total age of remaining 24 teachers when just the principal has retired = $1275 - 60 = 1215$ years

1 year later (i.e., 3 years ago from present) total age of 24 teachers (just before the recruitment of new principal)

$$= 1215 + (1 \times 24) = 1239 \text{ years}$$

and the total age of 25 teachers including new principal just after the recruitment

$$= 1239 + 54 = 1293 \text{ years}$$

Thus the present age of all the 25 teachers

$$= 1293 + (3 \times 25) = 1368 \text{ years}$$

Hence, the present average age of the 25 teachers

$$= \frac{1368}{25} = 54\frac{18}{25} \text{ years}$$

Alternatively 10 years ago, the average age of 25 teachers = 45 years

Let us assume that the principal has not retired from her post then the present average age of all the 25 teachers,

$$= 45 + 10 = 55 \text{ years}$$

Thus the total age of all the 25 teachers

$$= 55 \times 25 = 1375 \text{ years}$$

Now, assume that the new principal has replaced the old principal 4 years ago instead of 3 years ago, when the age of new principal would have been 53 years. Thus the age of new principal was 7 years less than the age of old principal, which results in the reduction of total age of the group of 25 teachers by 7 years.

Thus the actual total age of the 25 teachers (presently)

$$= 1375 - 7 = 1368 \text{ years}$$

Hence, the present average age of the 25 teachers

$$= \frac{1368}{25} = 54\frac{18}{25} \text{ years}$$

Ex. 19) The ratio of the ages of the father and the daughter at present is 3 : 1. Four years ago the ratio was 4 : 1. The average age of the father and daughter 2 years hence will be :

- (a) 24 (b) 26 (c) 25 (d) 36

Solution Let the present ages of father be $3x$ and daughter be x .

So the 4 years ago father's age and daughter's age was

$$(3x - 4) \text{ and } (x - 4). \text{ Therefore } \left(\frac{3x - 4}{x - 4} \right) = \frac{4}{1}$$

$$\Rightarrow x = 12 \text{ years and } 3x = 36 \text{ years}$$

Hence, the present average age of father and daughter = 24 years and the average age 2 years hence will be 26 years.

9. Problems Based on Income/Salary

$$\text{Income} = \text{Expenditure} + \text{Savings}$$

Ex. 20) The average salary of Rajesh, Bahadur and Amir is ₹ 8000 per month. The average expenditure of the Rajesh, Bahadur and Amir per month is ₹ 5000. The average savings of all the 3 persons per month is :

- (a) ₹ 3000 (b) ₹ 5000 (c) ₹ 2500 (d) ₹ 9000

Solution Average saving

$$= \text{Average Income} - \text{Average Expenditure}$$

$$= 8000 - 5000 = 3000$$

Ex. 21) The average salary of A, B and C is ₹ 10000 and average expenditure of A is ₹ 6000 then the average savings of B and C is :

- (a) ₹ 5500 (b) ₹ 4500
(c) ₹ 4000 (d) can't be determined

Solution Total income = Total Expenditure + Total savings. We can not find the average savings of B and C , since data is insufficient.

Ex. 22) The average salary of A, B is ₹ 6000 and that of C, D and E is ₹ 8000. The average salary of all the 5 people is

- (a) ₹ 7200 (b) ₹ 7000
(c) ₹ 7500 (d) can't be determined

Solution Required average salary

$$= \frac{6000 \times 2 + 8000 \times 3}{(2 + 3)} = \frac{36000}{5} = ₹ 7200$$

Ex. 23) The average salary of all the 60 employees in an office is ₹ 12,000 per month. If the number of executives is twice the number of non-executive employees, then the average salary of all the non executive employees is :

- (a) ₹ 9000 (b) ₹ 8000
(c) ₹ 6000 (d) can't be determined

Solution $\frac{\text{No. of executives}}{\text{No. of non-executives}} = \frac{2}{1}$

Therefore number of executives = 40

and number of non-executive employees = 20

Now, go through the options

Total salary = 40 × salary of executive + 20

× salary of non-executive

$60 \times 12000 = 40 \times k + 20 \times l$, here k, l are unknowns

So we can't determine the required average salary.

Ex. 24) In the above problem, if the average salary of non-executives be $\frac{2}{5}$ th of the average salary of executives, then the average salary of non-executive employees is :

- (a) ₹ 9000 (b) ₹ 8000
(c) ₹ 6000 (d) data insufficient

Solution By options : $60 \times 12000 = 20 \times 6000 + 40 \times 15000$
Hence option (c) is correct.

☞ **Alternatively** $60 \times 12000 = 20 \times x + 40 \times \frac{5}{2}x \Rightarrow x = 6000$

10. Problems Based on Time, Speed and Distance :

Case 1. When the distance travelled in different time slots or parts is same *i.e.*, if a person or vehicle moves x km at a speed of u km/hr and further he goes or comes back the same distance x km at a speed of v km/hr.

Then the average speed = $\frac{2uv}{(u+v)}$. If there are 3 parts of

distance x km travelled with 3 different speeds *i.e.*, if a person goes first x km @ speed of u km/hr and next x km @ v km/hr and the last x km @ w km/hr.

Then the average speed = $\frac{3uvw}{(uv + vw + wu)}$.

☞ **NOTE** In this type of questions the average speed is independent of the distance travelled.

General Formula : Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

The above two formulae are derived with the help of the general formula of average speed.

Proportion Method : If the half of the distance is covered at u km/hr and the rest half of the journey is covered at v km/hr then the average speed can be found as follows :

Step 1. Divide the difference of u and v in the ratio of $u : v$ (where $u < v$)

Step 2. $u + \frac{(u-v) \times u}{(u+v)}$ or $v - \frac{(u-v) \times v}{u+v}$

Case 2. When the distances travelled at different speeds are different then we calculate the average speed with the help of general formula of average speed. *e.g.*, A person first goes x_1 km at the speed of u km/hr and x_2 km at the speed of v km/hr and x_3 km at the speed of w km/hr and so on, then the

$$\begin{aligned} \text{Average speed} &= \frac{\text{Total distance}}{\text{Total time}} = \frac{x_1 + x_2 + x_3 + \dots}{t_1 + t_2 + t_3 + \dots} \\ &= \frac{x_1 + x_2 + x_3 + \dots}{\frac{x_1}{u} + \frac{x_2}{v} + \frac{x_3}{w} + \dots} \end{aligned}$$

Ex. 25) Einstein goes from Ahmedabad to Lucknow at the speed of 40 km/hr and returned at the speed of 60 km/hr. The average speed of Einstein during the whole journey is :

- (a) 48 km/hr
(b) 24 km/hr
(c) 50 km/hr
(d) none of the above

Solution $\frac{2 \times 40 \times 60}{(40 + 60)} = \frac{4800}{100} = 48 \text{ km/hr}$

So the average speed = $40 + \frac{2}{5}(20) = 48 \text{ km/hr}$

or Average Speed = $60 - \frac{3}{5}(20) = 48 \text{ km/hr}$

Ex. 26) Sri Krishna took the chariot and started his journey from Mathura to Gokul by his chariot at the speed of 40 km/hr and then, the same distance he travelled on his foot at the speed of 10 km/hr from Gokul to Brindaban. Then he returned from Brindaban to Mathura via Gokul at the speed of 24 km/hr riding on the horse. The average speed of the whole trip is :

- (a) 20 km/hr
(b) 25 km/hr
(c) 19.2 km/hr
(d) 18.5 km/hr

Solution Since the distance from Mathura to Gokul is same as that of Gokul to Brindaban. So the average speed from Mathura to Brindaban = $\frac{2 \times 40 \times 10}{(40 + 10)} = 16 \text{ km/hr}$

Again since he returned on the same path, so the distance from Mathura to Brindaban is same in both the directions. Thus the required average speed = $\frac{2 \times 16 \times 24}{(16 + 24)} = 19.2 \text{ km/hr}$

Ex. 27) Anoop travels first $\frac{1}{3}$ rd of the total distance at the speed of 10 km/hr and the next $\frac{1}{3}$ rd distance at the speed of 20 km/hr and the last $\frac{1}{3}$ rd distance at the speed of 60 km/hr. The average speed of Anoop is :

- (a) 15 km/hr (b) 18 km/hr
(c) 25 km/hr (d) 30 km/hr

Solution Since all the three distances are same, hence the average speed

$$= \frac{3 \times 10 \times 20 \times 60}{(200 + 1200 + 600)} \quad \left(= \frac{3uvw}{uv + vw + wu} \right)$$

$$= \frac{36000}{2000} = 18 \text{ km/hr}$$

Ex. 28) Columbus started his journey from Lucknow to Kolkata, which is 200 km, at the speed of 40 km/h then he went to Bangalore which is 300 km, at the speed of 20 km/hr. Further he went to Ahmedabad which is 500 km, at the speed of 10 km/hr. The average speed of Columbus is :

- (a) $14 \frac{2}{7} \text{ km/hr}$ (b) $14 \frac{5}{7} \text{ km/hr}$
(c) 15.6 km/hr (d) none of these

Solution Average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{(200 + 300 + 500)}{\left(\frac{200}{40} + \frac{300}{20} + \frac{500}{10}\right)}$

$$= \frac{1000}{70} = \frac{100}{7} = 14 \frac{2}{7} \text{ km/hr} = 14.2857 \text{ km/hr}$$

11. Value of An Overlapping Element

- (i) $a + b = k$ and $b + c = l$ and $a + b + c = m$
then $[(a + b) + (b + c)] - (a + b + c) = (k + l) - m$
or $b = k + l - m$
- (ii) $a + b = k$, $d + e = l$ and $a + b + c + d + e = m$
then $c = (a + b + c + d + e) - [(a + b) + (d + e)] = m - [k + l]$

Ex. 29) The average weight of all the 11 players of Indian cricket team is 50 kg. If the average of first six lightest weight players is 49 kg and that of the six heaviest players is 52 kg. The average weight of the player which lies in the sixth position in the list of players when all the 11 players are arranged in the order of increasing or decreasing weights :

- (a) 56 kg (b) 52 kg
(c) 51 kg (d) none of these

Solution Let $A, B, C, D, E, F, G, H, I, J$ and K be the 11 players in the order of increasing weight then,

$$A + B + C + D + E + F = 49 \times 6 = 294$$

$$F + G + H + I + J + K = 52 \times 6 = 312$$

$$\text{and } A + B + C + D + E + F + G + H + I + J + K = 50 \times 11 = 550$$

$$\text{Therefore } F = (A + B + C + D + E + F) + (F + G + H + I + J + K)$$

$$- (A + B + C + D + \dots J + K) = 294 + 312 - 550 = 56$$

Hence the average weight of $F = 56 \text{ kg}$.

Ex. 30) The average presence of students in a class on Monday, Tuesday and Wednesday is 30 and on the Wednesday, Thursday, Friday and Saturday is 28 then number of students who attended the class on Wednesday is, if the average number of students on all the six days is 27

- (a) 24 (b) 25 (c) 20 (d) 40

Solution Since $W = (M + T + W) + (W + Th + F + S)$

$$= (30 \times 3) + (28 \times 4) - (M + T + W + Th + F + S)$$

$$= 202 - 162 = 40$$

Ex. 31) The average age of A, B, C, D and E is 40 years. The average age of A and B is 35 years and the average age of C and D is 42 years. The average age of E is :

- (a) 46 (b) 48
(c) 32 (d) none of these

Solution $A + B + C + D + E = 40 \times 5 = 200$

$$A + B = 35 \times 2 = 70; C + D = 42 \times 2 = 84$$

$$\text{Therefore, } E = (A + B + C + D + E) - [(A + B) + (C + D)]$$

$$= 200 - (70 + 84) = 46$$

Thus the average age of $E = 46 \text{ years}$.

Ex. 32) The average temperature on Monday, Tuesday and Wednesday is 38°C . The average temperature on Tuesday, Wednesday and Thursday is 43°C . If the average temperature on Monday and Thursday is 18.5°C . The average temperature on Monday is :

- (a) 11°C (b) 21°C (c) 35°C (d) 27°C

Solution $(M + T + W) = 38 \times 3 = 114$... (i)

$$(T + W + Th) = 43 \times 3 = 129$$
 ... (ii)

$$\text{Therefore, } Th - M = 15$$
 [(ii) - (i)]

$$\text{and } Th + M = 37$$

Thus the temperature on Monday is 11°C and on Thursday is 26°C .

12. Average of Some Important Series of Numbers :

- (i) Average of first ' n ' natural numbers = $\frac{n+1}{2}$
- (ii) Average of first ' n ' even numbers = $(n+1)$
- (iii) Average of first ' n ' odd numbers = n
- (iv) If there are $(p+q)$ elements in a set or group but the average of p elements is r and the average of q elements is s , then the average of all the elements of the set (or group) is $\frac{(pr + sq)}{(p + q)}$.

CAT-Test

Questions Helping you bell the CAT

LEVEL 01 > BASIC LEVEL EXERCISE

- 1 The average weight of a class of 20 students is 45 kgs. A new student whose weight is 40 kgs replaces an old student of this class. Hence, the average weight of the whole class decreases by 1 kg. The weight of the replaced student is :
(a) 55 kgs (b) 50 kgs
(c) 60 kgs (d) none of these
- 2 The average length of first 3 fingers is 3 inches and the average length of the other 2 fingers i. e., thumb and the index finger is 2.8 inches. If the length of the index fingers is 3 inches then the length of the thumb is :
(a) 2 inches (b) 2.6 inches
(c) 3 inches (d) none of these
- 3 Three types of rice whose rates are ₹ 38, ₹ 43 and ₹ 49 per kg are blended together to make a 15 kg of new blend of rice in which there are 8 kgs, 4 kgs and 3 kgs of the respective types of rice. The average price of the new blend of rice is :
(a) ₹ 41.53 (b) ₹ 43
(c) ₹ 40 (d) ₹ 43.3
- 4 Pankaj went to the post-office at the speed of 60 km/hr while returning for his home he covered the half of the distance at the speed of 10 km/hr, but suddenly he realized that he was getting late so he increased the speed and reached the home by covering rest half of the distance at the speed of 30 km/hr. The average speed of the Pankaj in the whole length of journey is :
(a) 5.67 km/hr
(b) 24 km/hr
(c) 22.88 km/hr
(d) 5.45 km/hr
- 5 123 typists typed 984 papers in $1/15$ hour. The number of papers typed per minute by an average typist is :
(a) 1 (b) 2
(c) 3 (d) 5
- 6 The cost of the Red, Green and Blue colours per kg is ₹ 15 and ₹ 18 respectively. Rang Mahal is a renowned building in which these three colours are being used in the ratio of 3 : 2 : 4. The average cost of all the three colours used per kg is :
(a) 18 (b) 20
(c) 17.66 (d) can't be determined
- 7 Rajeev earns $3/2$ times in January, April, July and October than his average earning of ₹ 600 per month in the rest of the months. Therefore, his savings in the month of January, April, July and October is $5/4$ times of the monthly savings of ₹ 400 for each of the remaining months of the year. The average expenditure of per month is :
(a) ₹ 266.66 (b) ₹ 250 (c) ₹ 233.33 (d) ₹ 433.33
- 8 The average of 3 prime numbers lying between 47 and 74 is $191/3$. The greatest possible difference between any two out of the 3 prime numbers is :
(a) 12 (b) 14
(c) 18 (d) can't be determined
- 9 The average of 9 numbers is 11. If each of these 9 numbers is multiplied by 5 and then 5 is added to each of these resultant numbers, then the new average is :
(a) 20 (b) 30 (c) 60 (d) 50
- 10 The average score of Dhoni after 48 innings is 48 and in the 49th innings Dhoni scores 97 runs. In the 50th innings the minimum number of runs required to increase his average score by 2 than it was before the 50th innings :
(a) 99 (b) 149
(c) 151 (d) can't be determined
- 11 The average age of Sachin and Ganguli is 35 years. If Kaif replaces Sachin, the average age becomes 32 years and if Kaif replaces Ganguli, then the average age becomes 38 years. If the average age of Dhoni and Irfan be half of the average age of Sachin, Ganguli and Kaif, then the average age of all the five people is :
(a) 28 years (b) 32 years (c) 25 years (d) none
- 12 Out of these five people (in question no. 11) whose age is the greatest?
(a) Sachin (b) Ganguli
(c) Kaif (d) can't be determined
- 13 In a village the average age of n people is 42 years. But after the verification it was found that the age of a person had been considered 20 years less than the actual age, so the new average, after the correction, increased by 1. The value of n is :
(a) 21 (b) 20
(c) 22 (d) none of these

- 14** The average rainfall in the months of January and February is 6 cm and in the months of March to June is 5 cm and July to October is 10 cm and in the November and December, it is 6 cm. The average rainfall for the whole year is :
 (a) 7 (b) 5.5
 (c) 7.5 (d) none of these
- 15** On an average 300 people watch the movie in Sahu Cinema hall on Monday, Tuesday and Wednesday and the average number of visitors on Thursday and Friday is 250. If the average number of visitors per day in the week be 400, then the average number of people who watch the movie in weekends (i.e., on Saturday and Sunday) is :
 (a) 500 (b) 600
 (c) 700 (d) none of these
- 16** The average weight of 11 players of Indian cricket team is increased by 1 kg, when one player of the team weighing 55 kg replaced by a new player. The weight of the new player is :
 (a) 55 kg (b) 64 kg
 (c) 66 kg (d) none of these
- 17** The average age of a family of 6 members 4 year ago was 25 years. Mean while a child was born in this family and still the average age of the whole family is same today. The present age of the child is :
 (a) 2 years (b) $1\frac{1}{2}$ years
 (c) 1 year (d) data insufficient
- 18** Amitabh's average expenditure for the January to June is ₹ 4200 and he spends ₹ 1200 in January and ₹ 1500 in July. The average expenditure for the months of February to July is :
 (a) ₹ 4250 (b) ₹ 4520
 (c) ₹ 4060 (d) none of these
- 19** The average of a , 11, 23, 17 is 15 and the average of a , b , 12, 25 is 16. The value of $\frac{a}{b}$ is :
 (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$
- 20** The average salary is being paid to all its employees by the Biotech corporation is ₹ 15500. The average salary of the senior employees is ₹ 18000 per month and the average salary of the junior employees is ₹ 12000 per month. If there are only two levels of employees viz junior and senior level, then what fraction of the total employees is the junior level employees are :
 (a) $\frac{7}{10}$ (b) $\frac{5}{12}$
 (c) $\frac{5}{10}$ (d) none of these
- 21** The average of any 5 consecutive odd numbers a , b , c , d and e is :
 (a) $\frac{(abcde)}{5}$ (b) $\frac{bd}{3}$
 (c) $\left(\frac{a+c+e}{5}\right)$ (d) none of these
- 22** The average age of 30 students of a class is 30 years. When the average age of class teacher is also included, the average age of the whole class increases by 1 year. The age of the class teacher is :
 (a) 31 years (b) 60 years
 (c) 61 years (d) none of these
- 23** There were five sections in MAT paper. The average score of Pooja in first 3 sections was 83 and the average in the last 3 sections was 97 and the average of all the sections (i.e., whole paper) was 92, then her score in the third section was :
 (a) 85 (b) 92
 (c) 88 (d) none of these
- 24** The average age of 18 pupils of Dronacharya was 25 years. If the age of Dronacharya was also included, the average age of 19 people becomes 26 years. The average age of the Dronacharya at that time was :
 (a) 33 (b) 44
 (c) 50 (d) 51
- 25** The average of 7 consecutive odd numbers if the smallest of those numbers is denoted by k :
 (a) $k + 4$ (b) $k + 7$
 (c) $k + 6$ (d) $7k$
- 26** If the average marks of $\frac{1}{4}$ th class is 85% and that of $\frac{1}{3}$ rd class is 70% and the average marks of the rest class is 56%, then the average of the whole class is (for the given subjects) :
 (a) 67.916% (b) 72.33%
 (c) 69.165% (d) can't be determined
- 27** The average length of any four fingers of my left hand is 600 mm. Then the average length of all the five fingers of my left hand is :
 (a) 800 mm (b) 750 mm
 (c) 480 mm (d) can't be determined
- 28** The average of $4\frac{3}{5}$, $2\frac{2}{3}$, $6\frac{8}{9}$, $7\frac{7}{15}$, $3\frac{5}{9}$ is :
 (a) $5\frac{3}{225}$ (b) $5\frac{8}{225}$ (c) $6\frac{3}{45}$ (d) $25\frac{8}{45}$
- 29** The average of 1000.0001, 100.001, 10.01, 1.1 is :
 (a) 277.777 (b) 322.222
 (c) 11.11 (d) 233.333
- 30** The average of 7 consecutive numbers which are positive integers is 10. The average of lowest and highest such numbers is :
 (a) 7 (b) 10
 (c) 15 (d) data insufficient
- 31** The average of first 100 natural number is :
 (a) 100 (b) 50
 (c) 50.50 (d) 55
- 32** The average of first 50 odd natural numbers is :
 (a) 50 (b) 55
 (c) 51 (d) 101

- 33** The average of first 99 even numbers is :
 (a) 9999 (b) 100
 (c) 9801 (d) 9009
- 34** The average of a , b and c is 79 and the average of a and c is also 79. Then the value of b is :
 (a) 0 (b) 79
 (c) -79 (d) none of these
- 35** The average value of property of Mittal, Ambani and Singhanian is ₹ 11111 crore. The property of Singhanian is as less as the property of Mittal is greater than the average property of both the Singhanian and Mittal. The value of property of Ambani is :
 (a) ₹ 111 crore (b) ₹ 11111 crore
 (c) ₹ 3703.7 crore (d) can't be determined
- 36** I went to Delhi @ speed of 200 km/hr but suddenly I returned to the same place @ speed of 600 km/hr. What is my average speed :
 (a) 300 km/hr (b) 400 km/hr
 (c) 366.66 km/hr (d) none of these
- 37** The average of A and B is 400 and the average of C and D is 600 the average of A , B , C and D is :
 (a) 500 (b) 450
 (c) 525 (d) 625
- 38** The average weight of liquid in 100 bottles is 500 gm. The total weight of all the bottles is 20 kg. The average weight of a bottle with liquid is :
 (a) 0.65 kg (b) 0.7 kg
 (c) 70 ml (d) none of these
- 39** The average score of Sehwag in 10 innings was 77 runs. In the 11th innings he had scored zero runs. The overall average score of Sehwag in all the 11 innings was :
 (a) 77 (b) 7.7
 (c) 11 (d) none of these
- 40** The average age of 3 children of Arihant Singh is 12 years and their ratio of ages is 3 : 4 : 5. The average age of the youngest and eldest child is if he had only 3 children :
 (a) 12 (b) 21
 (c) 8 (d) 9
- 41** The average income of all the Infosys employees is ₹ 20000 per month. Recently the company announced the increment of ₹ 2000 per month for all the employees. The new average of all the employees is :
 (a) ₹ 22000
 (b) ₹ 40000
 (c) ₹ 2200
 (d) data insufficient
- 42** The average age of 10 students in a class is 20 years, if a new student is also included, then the new average age of all the students increases by 1 year. The age of the new student is :
 (a) 21 years (b) 30 years
 (c) 31 years (d) none of these
- 43** The average weight of 20, four wheelers is 180 kg. If an old car is removed from this group of four wheelers, the new average weight decreases by 2 kg. The weight of the removed car is :
 (a) 220 (b) 218
 (c) 182 (d) none of these
- 44** The average price of 3 diamonds of same weights is ₹ 5 crore, where the average price of the two costliest diamonds is double the price of the cheapest diamond. The price of the cheapest diamond is :
 (a) 3 crore (b) 5 crore
 (c) 1.66 crore (d) can't be determined
- 45** In the previous question, the price of the costliest diamond is :
 (a) 5 crore (b) 6 crore
 (c) 8 crore (d) can't be determined
- 46** Praveen gets 40 marks out of 50 in Computer Science, 70 out of 100 in Manufacturing Science and 142 out of 150 in Professional communication. The average marks of Praveen (in percent) in all the three subjects is :
 (a) 84% (b) 76% (c) 71% (d) 60%
- 47** The average of all the prime and composite numbers upto 100 is :
 (a) 51 (b) 49.50
 (c) 50.50 (d) 55
- 48** The average of all the perfect squares upto 100 is :
 (a) 38.5 (b) 1000 (c) 100 (d) 385
- 49** The average of all the non-negative integers upto 99 is :
 (a) 50.49 (b) 49.50
 (c) 50.50 (d) 99
- 50** The average of 7, 14, 21, 28, ..., 77 is :
 (a) 7 (b) 11
 (c) 42 (d) 66
- 51** The average weight of A , B , C and D is 40 kg. A new person E is also included in the group, then the average weight of the group is increased by 1 kg. Again a new person F replaces A , then the new average of 5 persons becomes 42. The average weight of B , C , D and F is :
 (a) 42 (b) 41.25
 (c) 42.5 (d) none of these
- 52** The average of 3 consecutive natural numbers (which are in increasing order) is k . If two more consecutive number, just next the first set of numbers, is added, then the new average becomes :
 (a) $k + 2$ (b) $k + 1$ (c) $\frac{2k + 1}{2}$ (d) $2k - 1$
- 53** The average of any 5 consecutive odd natural numbers is k . If two more such numbers, just next to the previous 5 numbers are added, the new average becomes :
 (a) $\frac{2}{7}(k + 1)$ (b) $2k - 3$
 (c) $2k + 1$ (d) $k + 2$

- 54** The average weight of the 5 officers of a regiment is 42 kg. If a senior officer was replaced by a new officer and thus the average increased by 500 gm, the weight of the new officer is :
 (a) 44.5 kg (b) 45 kg
 (c) 42.5 kg (d) can't be determined
- 55** The average age of 6 servants in my farm house is 28 years. A new and young servant replaces an old servant, then the new average reduces by 1 year, the age of the new servant is :
 (a) 26 years (b) 22 years
 (c) 35 years (d) can't be determined
- 56** In the above question (no. 55) if the age of the replaced servant was 31 years, then the age of the new servant is :
 (a) 25 years (b) 35 years
 (c) 24 years (d) none of these
- 57** The average income of A, B and C is ₹ 12000 per month and the average income of B, C and D is ₹ 15000 per month. If the average salary of D be twice that of A , then the average salary of B and C is (in ₹) :
 (a) 8000 (b) 18000
 (c) 13500 (d) 9000
- 58** The average price of 80 computers in an electronic shop is ₹ 30000. If the highest and lowest price computers are sold out then the average price of the remaining 78 computers is ₹ 29500. The cost of the highest price computer is ₹ 80000. The cost of lowest price computer is
 (a) ₹ 19000 (b) ₹ 20000
 (c) ₹ 29000 (d) can't be determined
- 59** A has 50 coins of 10 paise denominations. While B has 10 coins of 50 paise denominations. C has 20 coins of 25 paise denominations while D has 25 coins of 20 paise denominations. The average number of paise per person is
 (a) 450 paise (b) 500 paise
 (c) 600 paise (d) can't be determined
- 60** A travel agency has three types of vehicles viz. four seater, autorickshaw, 10 seater maxi cab and 20 seater minibus. The rate of each passenger (irrespective of its age or weight or seniority) for the auto rickshaw is ₹ 12 and for the maxicab is ₹ 15 and for the minibus is ₹ 8 for the one round. The average occupancy of the seats is 100%, 80% and 75% respectively. If he has only one vehicle of each kind, then the average earning for one round of each vehicle is :
 (a) ₹ 96 (b) ₹ 90 (c) ₹ 86 (d) ₹ 70

LEVEL 02 > HIGHER LEVEL EXERCISE

- 1** A train normally covers a certain distance at a speed of 60 km/hr. However, if it were to halt for a fixed time interval in each hour its average speed reduced to 50 km/hr. What is the time interval for which the train halt in each hour?
 (a) 10 minutes (b) 20 minutes
 (c) 6 minutes (d) 12 minutes
- 2** If p, q and r be three positive numbers such that $p > q > r$ when the smallest number is added to the difference of the rest two numbers, then the average of the resultant number and the original numbers except to the smallest number is 21 more than the average of all the three original numbers. The value of $(p - q)$ is :
 (a) 7 (b) 14
 (c) 63 (d) 42
- 3** Progressive express left for New Delhi, increasing its speed in each hour. It started its journey from Lucknow, but after four hours of its journey it met with accident. Its speed in the fourth hour was $\frac{7}{5}$ times that of the third hour and the speed in the third hour was $\frac{10}{7}$ times that of the second hour and in the second hour it was $\frac{7}{5}$ times that of the first hour. If it would have been travelled with the half of the speed that of the third hour, then it would have gone 160 km less in the same time (i.e., in four hours). The average speed of the train during the journey of 4 hours was :
 (a) 50 km/hr
 (b) 90 km/hr
 (c) 80 km/hr
 (d) can't be determined
- 4** In a Mock CAT 123 students appeared and the average score obtained was 120. But later it was found that the top three students were repeaters, so their score has been eliminated and then the new average score was found to be decreased by 1.5. Also, it is known that all the students obtained the marks in integers and the scores of the toppers were distinct. If the second highest topper has scored more than 185 marks, then the highest possible score of the third highest topper is :
 (a) 166 (b) 167
 (c) 168 (d) 170
- 5** The average age of all the 100 employees in an office is 29 years, where $\frac{2}{5}$ employees are ladies and the ratio of average age of men to women is 5 : 7. The average age of female employees is :
 (a) 18 years (b) 35 years
 (c) 25 years (d) none of these

- 6 There are two houses in Parliament. One is Lok Sabha and the other one is Rajya Sabha and the member of Parliaments (MPs) in both the houses is 300 and 200 respectively. The average age of the members of Lok Sabha and Rajya Sabha is 40 years and 50 years respectively. A member of the Rajya Sabha when elected for the Lok Sabha also, he left the Rajya Sabha and becomes the member of the Lok Sabha. Thus the average age of both the houses increases. Which one of the following statements is true?
- The age of this member is greater than 50 years
 - The age of this member is less than 40 years
 - The age of this member is greater than 40 but less than 50 years
 - none of the above

Directions (for Q. Nos. 7 to 9) *Eight years ago there were 5 members in the Arthur's family and then the average age of the family was 36 years. Mean while Arthur got married and gave birth to a child. Still the average age of his family is same now.*

- 7 The present age of his wife is :
- 25 years
 - 26 years
 - 32 years
 - data insufficient
- 8 The age of his wife at the time of his child's birth was. If the difference between the age of her child and herself was 26 years :
- 25 years
 - 26 years
 - 20 years
 - can't be determined
- 9 The age of Arthur at the time of his marriage was :
- 22 years
 - 23 years
 - 26 years
 - can't be determined
- 10 Eleven years earlier the average age of a family of 4 members was 28 years. Now the age of the same family with six members is yet the same, even when 2 children were born in this period. If they belong to the same parents and the age of the first child at the time of the birth of the younger child was same as there were total family members just after the birth of the youngest members of this family, then the present age of the youngest member of the family is :
- 3 years
 - 5 years
 - 6 years
 - none of these
- 11 Mr. Patel walked 6 km to reach the station from his house, then he boarded a train whose average speed was 60 km/hr and thus he reached his destination. In this way he took total of 3 hours. If the average speed of the entire journey was 32 km/hr then the average speed of walking is :
- 3 km/hr
 - 4.5 km/hr
 - 4 km/hr
 - none of these

Directions (for Q. Nos. 12 and 13) *During the winter session all the women which constitutes $\frac{1}{5}$ th strength of the Lok Sabha, left the house (i.e., parliament) due to the rejection of their demand. Actually they were asking for the 50% reservation of seats for the women in the Lok Sabha. Thus the average age of the remaining members of the house (i.e., the Lok Sabha) increases by $\frac{1}{4}$ th than it was earlier when all the members (i.e., men and women) were present.*

- 12 The total strength of the Lok Sabha is :
- 300 MPs
 - 475 MPs
 - 526 MPs
 - none of these
- 13 If the average age of women be 30 years then the average age of male members in the Lok Sabha will be :
- 26
 - 34
 - 40
 - none of these
- 14 An artist in a circus moves along the hexagonal path of each side 20 metres in such a way that for the first 20 metres he goes with a speed of 20 m/s and the next 20 metres with a speed of 10 m/s. Similarly he continues for the rest of the hexagonal path with the same alternating speeds i.e., 20 m/s and 10 m/s. The average speed of the artist per round of the circus is :
- 13.33 m/s
 - 23.66 m/s
 - $39\frac{1}{3}$ m/s
 - 15 m/s
- 15 The average of 26, 29, n , 35 and 43 lies between 25 and 35. If n is always an integer and greater than the average of the given integers then the value of n is :
- $33 < n < 47$
 - $34 < n < 43$
 - $33 < n < 42$
 - none of these
- 16 The average of 4 distinct prime numbers a , b , c and d is 35, where $a < b < c < d$. a and d are equidistant from 36 and b and c are equidistant from 34 and a , b are equidistant from 30 and c and d are equidistant from 40. The difference between a and d is :
- 30
 - 14
 - 21
 - can't be determined
- 17 There are four numbers whose product is 9261000 and each of these four numbers is formed by 3 distinct prime numbers. The average of all the four numbers is :
- 61.75
 - 67.25
 - 82.33
 - data insufficient
- 18 A number is such that when it is multiplied by 8, it gives another number which is as much above from 270 as the original number (itself) is below 270. The average of the original number and the resultant number is :
- 33.75
 - 190
 - can't be determined
 - none of these

- 19** In the above question (no. 18) the average of the original no. and 270 is :
 (a) 165 (b) 185
 (c) 135 (d) none of these
- 20** In a particular week the average number of people who visited the Tajmahal is 40. If we exclude the holidays then the average is increased by 16. Further if we exclude also the day on which the maximum number of 112 people visited the Tajmahal, then the average becomes 42. The number of holidays in the week is :
 (a) 1 (b) 2
 (c) 3 (d) data insufficient
- 21** The average age of all the 20 students of a class is 24. The minimum age of a student is 18 and the maximum age of another student in the same class is 30 years. When the two students whose average age was 26 years restituted from the class but later on one of the restituted student was readmitted. Now the average age of the class is :
 (a) 23.89 years (b) 28.39 years
 (c) 25 years (d) can't be determined
- 22** In a set of prime and composite numbers, the composite numbers are twice the number of prime numbers and the average of all the numbers of the set is 9. If the number of prime numbers and composite numbers are exchanged then the average of the set of numbers is increased by 2. If during the exchange of the numbers the average of the prime numbers and composite numbers individually remained constant, then the ratio of the average of composite numbers to the average of prime numbers (initially) was :
 (a) $\frac{7}{13}$ (b) $\frac{13}{7}$
 (c) $\frac{9}{11}$ (d) none of these
- 23** The total age of all the guests in the party was 540 years. If a South Indian couple (guests) left the party, then the average of the remaining guests still remained unchanged, where the age of both the husband and wife (the South Indian couple) was same, then the average age of this couple and the total number of guests in the party, respectively, can be :
 (a) 18, 27 (b) 20, 27
 (c) 15, 38 (d) can't be determined
- 24** In the command hospital Lucknow the sum of the ages of all the 29 people i. e., physicians, surgeons and nurses is 696. If the age of each physician, each surgeon and each nurse be 1 year, 6 years and 3 years more, than the average age of the whole staff would have been 3 years more. If the number of surgeon is a square root of a two digit number which is also a perfect cube, then the number of nurses in the hospital is :
 (a) 12 (b) 15
 (c) 16 (d) none of these
- 25** The average expenditure of Sarvesh for the January to June is ₹ 4200 and he spends ₹ 1200 in January and ₹ 1500 in July. The average expenditure for the months of February to July is :
 (a) 4250 (b) 4520
 (c) 4060 (d) none of these
- 26** The average marks of Sameer decreased by 1, when he replaced the subject in which he has scored 40 marks by the other two subjects in which he has just scored 23 and 25 marks respectively. later he has also included 57 marks of Computer Science, then the average marks increased by 2. How many subjects were there initially?
 (a) 6 (b) 12
 (c) 15 (d) can't be determined
- 27** In a combined family the average age of 4 males and 7 females is 42 and 20 years respectively. If two persons whose average age is 13 years have left the family and other three people joined the family whose respective ages are 11, 15 and 28 years, then the average age of the new family is increased by :
 (a) 4 years (b) 1 year
 (c) 3 years (d) none of these
- 28** A teacher gave sum to his class to find the average of n numbers viz. 1, 2, 3, 4, 5, 6 ... etc. But when the teacher checked the solution, he has found that during the calculation a student just missed a number for the addition thus his average of the n numbers was 15. The value of n is :
 (a) 30 (b) 26
 (c) 31 (d) not unique
- 29** The average earning of a group of persons is ₹ 50 per day. The difference between the highest earning and lowest earning of any two persons of the group is ₹ 45. If these two people are excluded the average earning of the group decreases by ₹ 1. If the minimum earning of the person in the group lies between 42 and 47 and the number of persons initially in the group was equal to a prime number, with both its digits prime. The number of persons in the group initially was :
 (a) 29 (b) 53
 (c) 31 (d) none of these
- 30** There are three categories of jobs A, B and C. The average salary of the students who got the job of A and B categories is 26 lakh per annum. The average salary of the students who got the job of B and C category is 44 lakh per annum and the average salary of those students who got the job of A and C categories is 34 lakh per annum. The most appropriate (or closest) range of average salary of all the three categories (if it is known that each student gets only one category of jobs i. e., A, B and C) :
 (a) lies between 30 and 44
 (b) lies between 28 and 34
 (c) lies between 34 and 43
 (d) lies between 29 and 48

- 31** Out of the five integral numbers C is the average of A and D . B is greater than C and less than D . Also, B is the average of A and E . The middle most number in the sequence is :
 (a) A (b) B
 (c) C (d) D
- 32** The average age of Donald, his wife and their two children is 23 years. His wife is just 4 year younger than Donald himself and his wife was 24 years old when his daughter was born. He was 32 years old when his son was born. The average age of Donald and his daughter is :
 (a) 25 years
 (b) 22.5 years
 (c) 26 years
 (d) can't be determined
- 33** There are only five people in the Aman Verma's family. Aman, his wife, a son and two daughters. The younger daughter's age is $\frac{4}{5}$ th of the elder daughter's age. The age of eldest daughter is $\frac{3}{8}$ times that of her father Aman and the age of the son is $\frac{1}{5}$ th that of his father Aman. 4 years ago the age of her wife was 8 times that of his son and now the sum of the ages of the younger daughter and wife is same as the sum of the ages of Aman and his son. The average age of the family is :
 (a) 22.22 years (b) 25.4 years
 (c) 21.2 years (d) none of these
- 34** The average weight of a political party is decreased by 1, when some new politicians joined the party, whose strength is $\frac{1}{4}$ th of the existing (or old) politicians and the total weight of the new politicians is 209 kgs. What is the new average weight of all the politicians if it is known that in any case the number of politicians always must be greater than 50 but less than 100 :
 (a) 15 kgs (b) 16 kgs
 (c) 18 kgs (d) 19 kgs
- 35** Ravi went to Kanpur from Lucknow by his four wheeler. During the journey he had to use the spare wheel (i. e., stepney). Thus he finished his 160 km journey. The average distance covered by the wheels of his car is :
 (a) 40 km (b) 120 km
 (c) 128 km (d) 48 km
- 36** There are 6 consecutive odd numbers in increasing order. The difference between the average of the squares of the first 4 numbers and the last four numbers is 64. If the sum of the squares of the first and the last element (i. e., odd numbers) is 178, then the average of all the six numbers is :
 (a) 7 (b) 8
 (c) 9 (d) 10
- 37** The average age of board of directors of a company, having 10 directors was 48 years. Coincidentally when a director aged 53 resigned from the board of directors, another director died on the same day. So a new director joined the board of directors aged 34. Next year in the same month the average age of all the 9 directors was found to be 46 years. The age of the late (i. e., dead) director at the time of his death was :
 (a) 56 years (b) 53 years
 (c) 57 years (d) 61 years
- 38** In an office the average weight of 24 employees is 60 kg. If n employees were included whose average weight was 54 kg, then the total number of employees in the office, (given that the new average weight of all the $(24 + n)$ employees is a whole number) :
 (a) 36 (b) 30
 (c) 34 (d) 25
- 39** The average age of 100 nurses in a nursing home in 1982 was 50 years. In 1984, 20 nurses retired from their job, whose average age was 60 years. After a huge gap in 1987, 40 nurses were employed whose average age was 38 years. The average age of all the nurses in 1990 was :
 (a) 53 years
 (b) 51 years
 (c) 48.5 years
 (d) data insufficient

Directions (for Q. Nos. 40 to 44) *Bhartiya Idol is a talent search programme launched and run by TV Tarana. In this programme each participating candidate has to appear for the audition in such a manner that a candidate plays the Antakchhary with its aspiring opponent until he fails, otherwise he can continue and become winner. If a candidate fails on his part, he has to leave the contest and in place of it another candidate starts off with the same existing opponent. The number of points a candidate scores is equal to the number of times he responds correctly to his opponents. The candidate who finishes the game (or who wins over the last opponent) is declared as a winner, even if he/she scores less points and starts as the last participant. The points scored individually by all the 10 candidates are as shown below. Besides, if a person loses the contest only if he/she responds incorrectly.*

	Participants	Points
1.	Rajesh	8
2.	Radhe	10
3.	Harish	7
4.	Prajakta	12
5.	Aditya	25
6.	Rahul	24
7.	Shekhar	18
8.	Amit Tandon	23
9.	Amit Sana	29
10.	Abhijeet	30

- 40** The average score of all the 10 participants is :
 (a) 21.5 (b) 18.6
 (c) 18.3 (d) none of these
- 41** A candidate can face maximum 'n' opponents. The value of 'n' is :
 (a) 3 (b) 4
 (c) 1 (d) can't be determined
- 42** The average score of five participants who started earlier :
 (a) 23 (b) 12
 (c) 19.5 (d) can't be determined
- 43** If Abhijeet has not started off earlier and he is the winner then the minimum number of participants who has lost the game before he has started off :
 (a) 4 (b) 6
 (c) 5 (d) none of these
- 44** For the above question the maximum possible average of the opponents of Abhijeet could be :
 (a) 13.5 (b) 9.25 (c) 13.0 (d) 17.5
- 45** In an NGO, the daily average wages of 20 illiterate employees is decreased from ₹ 25 to ₹ 10, thus the average salary of all the literate (educated) and illiterate employees is decreased by ₹ 10 per day. The number of educated employees working in the NGO is :
 (a) 15 (b) 20
 (c) 10 (d) data insufficient
- 46** Mr. Tyagi while going from Meerut to Saharanpur covered half the distance by train at the speed of 96 km/hr then he covered half of the rest distance by his scooter at the speed of 60 km/hr and finally he covered the rest distance at the speed of 40 km/hr by car. The average speed at which Mr. Tyagi completed his journey is :
 (a) 64 km/hr (b) 56 km/hr
 (c) 60 km/hr (d) 36 km/hr
- 47** There are four types of candidates in our coaching preparing for the CAT. The number of students of Engineering, Science, Commerce and Humanities is 40, 60, 50 and 30 respectively and the respective percentage of students who qualified the CAT is 80%, 75%, 60% and 50% respectively the overall percentage of successful candidates in our institute is :
 (a) 67.77% (b) 66.66%
 (c) 68.5% (d) none of these
- 48** Mr. Manmohan calculated the average of 10, 'three digit numbers'. But due to mistake he reversed the digits of a number and thus his average increased by 19.8. The difference between the unit digit and hundreds digit of that number is :
 (a) 8
 (b) 4
 (c) 2
 (d) can't be determined
- 49** Once my peon went to the office of SKYLINE COURIER with 4 different envelopes. The clerk in the office measured the weights in all possible pairs. The weights obtained are 59 gm, 61 gm, 62 gm, 63 gm, 64 gm and 66 gm. The weight of the heaviest envelope is :
 (a) 35 gm (b) 36 gm
 (c) 34 gm
 (d) can't be determined
- 50** The average expenditure of the hotel when there are 10 guests is ₹ 60 per guests and the average expenditure is ₹ 40 when there are 20 guests. If it is known that there are some fixed expenses irrespective of the number of guests then the average expenditure per guest when there are 40 guests in the hotel :
 (a) ₹ 30 (b) ₹ 25
 (c) ₹ 20 (d) can't be determined

LEVEL 03 > Final Round

Directions (for Q. Nos. 1 to 15) *There are 3 sets of natural numbers 1 to 100. Set A contains all the natural numbers which are prime, upto 100. Set B contains all the non-prime even natural numbers upto 100. Set C contains all the non-prime odd natural numbers upto 100 i.e.,*

$$A = \{2, 3, 5, 7, 11, \dots 89, 97\}$$

$$B = \{4, 6, 8, 10, 12, \dots 98, 100\}$$

$$C = \{1, 9, 15, 21, 25, 27, 33, \dots 95, 99\}$$

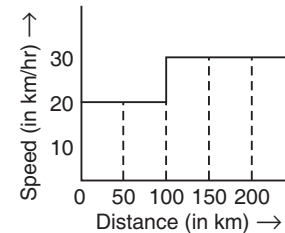
- 1** The average of all the elements of A, B and C is :
 (a) 49.50 (b) 50.50
 (c) 55 (d) none of these
- 2** The average of all the elements of B is :
 (a) 52 (b) 48
 (c) 49 (d) none of these
- 3** If the average (factually correct) of the set A is 42.46, then the average of the Set C is :
 (a) 52
 (b) 49.87
 (c) 55.46
 (d) can't be determined
- 4** The average of the elements of the Set A and C combined is:
 (a) 49.0588 (b) 49.0372
 (c) 50 (d) none of these
- 5** If an element less than 50 belongs to Set A is transferred to Set B, then the average of Set B :
 (a) increases (b) decreases
 (c) remains constant (d) can't be determined

- 6 If any two elements, greater than 50, belong to Set A are transferred to Set C, then the average of Set C :
- (a) remains constant (b) decreases
(c) increases (d) can't be determined
- 7 Any 10 elements of the Set A are transferred to the Set B, then the average of Set B :
- (a) increases (b) decreases
(c) remains constant (d) can't be determined
- 8 If a least and a greatest element of Set C are transferred from Set C to Set A then the average of Set A :
- (a) increases (b) decreases
(c) remains constant (d) can't be determined
- 9 If a smallest and a greatest element of the Set B is transferred to Set A, then the averages of A, B, C respectively :
- (a) decreases, decreases, increases
(b) decreases, constant and increased
(c) increases, constant, constant
(d) can't be determined
- 10 If an element 2 is also included in the Set B, then the average of B :
- (a) decreases by 3 (b) increases by 2
(c) decreases by 1 (d) can't be determined
- 11 The average of all the perfect squares of the Set C is :
- (a) 35 (b) 33
(c) 30.5 (d) none of these

Directions (for Q. Nos. 12 to 14) 5 elements below 25 from the Set A are transferred to Set B, and 10 elements lying between 25 and 50 from the Set B are transferred to Set C and 15 elements above 50 from the Set C are transferred to Set A.

- 12 The overall average of all the elements of A, B and C is :
- (a) 39.8 (b) 50.50
(c) 71.2 (d) can't be determined
- 13 The maximum increase in the average of set :
- (a) A (b) B
(c) C (d) can't be determined
- 14 When we intended to minimise the loss in average of Set B then the new average of Set B is :
- (a) 54.4772
(b) 45.74
(c) 51.84
(d) can't be determined
- 15 If 10-10 elements are transferred from Set A to Set B, then Set B to Set C and then from Set C to Set A but the received elements cannot be transferred to the next set. e.g. the elements obtained from Set A cannot be transferred to C through Set B. The average of which Set is maximum :
- (a) A (b) B
(c) C (d) can't be determined

Directions (for Q. Nos. 16 and 17) In the following graph the relation between speed and distance is given :



- 16 The average speed for the first 200 kms. (in km/hr)
- (a) 24 (b) 25 (c) 26 (d) 40
- 17 The average speed for the first 150 km :
- (a) 15 (b) 18
(c) 25 (d) none of these
- 18 Abhay working in Tele Bharti as a salesperson. His monthly salary is just ₹ 200. But he gets the bonus as per the given rule. If he sells the simcards of ₹ x then his bonus will be $\left[\left(\frac{x}{100} \right)^2 + 10 \right]$. In the first quarter of the year his average sale was ₹ 3000 per month. In the next 5 months his average sale was ₹ 5000 per month. In the last four months his average sale was ₹ 8000 per month. What is the average earning per month for the whole year?
- (a) ₹ 3350 (b) ₹ 3610
(c) ₹ 3560 (d) none of these
- 19 The price of Shirts at Sahara Ganj is defined as $\text{₹}(100 + 10x^2)$, where x is the number of shirts. Mallika purchased 5 shirts from the above shop. The average price of a shirt is :
- (a) ₹ 70 (b) ₹ 50
(c) ₹ 75 (d) none of these
- 20 In the above question if the sister of Mallika purchased k shirts but the average price was same as that of Mallika. The value of k could be :
- (a) 6 (b) 4
(c) 2 (d) none of these
- 21 There are 10 compartments in passenger train which carries on an average 20 passengers per compartment. If atleast 12 passengers were sitting in each compartment and no any compartment has equal number of passengers then maximum how many passengers can be accommodated in any compartment :
- (a) 64 (b) 45
(c) 56 (d) none of these
- 22 There are twice the number of two wheelers as there are three wheelers and the number of 4 wheelers are equal to the number of two wheelers. The average number of wheel per vehicle is :
- (a) 3 (b) 4
(c) 5 (d) none of these

- 23** Sone Lal has 'n' magical eggs whose average weight is 'k' gm. Each of the 'n' eggs produces 'n' eggs next day such that the average weight of 'n' eggs produced is same as that of the parental (previous generation) egg for each 'n' groups individually i.e., each egg produces 'n' eggs in its next generation and the average weight of all the 'n' eggs of next generation is same as the weight of the mother egg. This process is continued without any change in pattern. What is the total weight of all the eggs of r^{th} generation, where the initial number of eggs with Sone Lal are considered as the eggs of first generation :

(a) rnk (b) $n^r k$ (c) nk^r (d) $n^{r+1}k$

Directions (for Q. Nos. 24 to 27) A CAT training institute was established on January 1, 2004 with 3, 4, 5 and 6 faculty members in the Logical Reasoning (LR), Data Interpretation (DI), English Language and Quantitative Analysis (QA) areas respectively, to start with. No faculty member retired or joined the institute in the first three months, of the year 2004. In the next four years, the institute recruited one faculty member in each of the four areas. All these new faculty members, who joined the institute subsequently over the years were 25 years old at the time of their joining the institute. All of them joined the institute on April 1. During these four years, one of the faculty members retired at the age of 60. The following diagram gives the area-wise average age (in terms of number of completed years) of faculty members as on April 1 of 2004, 2005, 2006 and 2007.

Faculty	2004	2005	2006	2007
LR	49.33	44	45	46
DI	50.5	51.5	52.5	47.8
English	50.2	49	45	46
Quants	45	43	44	45

- 24** In which year did the new faculty member join as the faculty of English?
(a) 2004 (b) 2005 (c) 2006 (d) 2007
- 25** What was the age of the new faculty member, who joined the faculty of QA, as on April 1, 2007?
(a) 25 (b) 26 (c) 27 (d) 28
- 26** From which area did the faculty member retire?
(a) English (b) LR
(c) DI (d) Quants
- 27** Professors Sarvesh and Manish, two faculty members in the LR area, who have been with the Institute since its inception, share a birthday, which falls on 30th November. One was born in 1951 and the other one in 1954. On April 1, 2009, what was the age of the third faculty member, who has been in the same area since inception?
(a) 47 (b) 50
(c) 51 (d) 52

- 28** Asrani is a superintendent jailor in Ramgarh, who is assisted by 6 assistant jailors. Each of the seven jailors supervises a certain but distinct number of jail inmates. He has 30 inmates to look after. Jairaj, another jailor and a close aide of Asrani, supervises an average number of 32 inmates. Each of the seven jailors has to supervise not more than 45 inmates. Once Asrani heard a ruckus within the jail premises, he instructed his assistants that half of them go to his left and half of them go to his right and rest shall follow him. But, unfortunately, none was left to follow him. Gabbar Singh, a jail inmate, was curious to know that who went left and who went right. When he asked another inmate Kalia about the same, he got to know that the ones who supervise more inmates than that of Asrani went to the left and the ones who supervise fewer inmates than that of Asrani went to the right side. What is the maximum possible difference between the number of inmates supervised by any two jailors?

(a) 29 (b) 13
(c) 15 (d) 16

Directions (for Q. Nos. 29 to 31) Answer the following Questions based on the information given below :

Fair-Asia is well known for its queer marketing practices. To lure its customers, out of the blue, it comes up with a very lucrative offer. In that offer it promises to sell and the air tickets, for a one-way trip to Bangkok that originates from Jakarta, in such a manner that the n^{th} customer has to pay exactly Rs. n to buy an air ticket, but each prospective passenger has to buy a separate ticket for oneself. The airline claims that it had sold out all the seats at throw away prices in no time. However, at the last moment one of the customers had returned her ticket and got the full refund from the airline. Consequently, the average income of the airline from this particular flight is exactly ₹ 100 per passenger.

- 29** What could be the customer number, who returned her ticket to airline?
(a) 99 (b) 100
(c) 75 (d) 225
- 30** What could be the maximum number of passengers allowed to travel in this airline, if only one passenger occupies each seat and cabin crew or anyone else does not occupy any seat intended for the passengers?
(a) 320 (b) 240
(c) 200 (d) none of these
- 31** What's the loss incurred by the airline due to the refund of a ticket?
(a) ₹ 199 (b) ₹ 100
(c) ₹ 200 (d) Cannot be determined

Hints & Solutions

✓ Level 01 Basic Level Exercise

- 1 Initially the total weight = $20 \times 45 = 900$

Now, the total weight when a student has been replaced
 $= 20 \times 44 = 880$

It means the weight of the new student is 20 kgs less than the replaced student. Hence the weight of the replaced student = $40 + 20 = 60$ kgs

☞ **Alternatively** Since you know that there are total 20 students and when their average weight decreases by 1, it means on an average 1 kg weight gets reduced from each of the students. Thus there is 20 kg weight loss, in total. Again this happens due to the student whose weight is 40 kg who replaces an old student. Thus we can say that the weight of the old (or existing) student of the class was 60 kg, which is reduced by a 40 kg student.

- 2 The only useful data is that the average length of the thumb and index finger is 2.8 and the length of index finger is 3 inches. Now the total length of index finger and thumb

$$= 2.8 \times 2 = 5.6 \text{ inches}$$

Thus the length of thumb is $(5.6 - 3) = 2.6$ inches.

- 3 The average price of the new mixture
 $= \frac{(38 \times 8) + (43 \times 4) + (49 \times 3)}{15} = 41.53$

- 4 Average speed when Pankaj was returning
 $= \frac{2 \times 10 \times 30}{40} = 15 \text{ km/hr}$

Now the average speed of the whole journey

$$= \frac{2 \times 15 \times 60}{75} = 24 \text{ km/hr}$$

- 5 Use unitary method :

In $\frac{1}{15}$ hours, 123 typists can type 984 papers

In 1 minute, 123 typists can type $\frac{984}{4} = 246$ papers

In 1 minute, 1 typist can type $\frac{246}{123} = 2$ papers

Hint $\frac{1}{15}$ hours = 4 minutes

- 6 Average cost = $\frac{20 \times 3x + 15 \times 2x + 18 \times 4x}{9x} = 18$

- 7 Earning in the 8 months = $600 \times 8 = 4800$

Earning in the 4 months = $\left(600 \times \frac{3}{2}\right) \times 4 = 3600$

Total earning = ₹ 8400

Saving in 8 months = $400 \times 8 = 3200$

Saving in 4 months = $\left(400 \times \frac{5}{4}\right) \times 4 = 2000$

Total savings = 5200

Total expenditure for 12 months

$$= 8400 - 5200 = 3200$$

Therefore, average saving per month = $\frac{3200}{12} = 266.66$

- 8 There are 6 prime numbers between 47 and 74 as given below :

53, 59, 61, 67, 71, 73

There are only two combinations whose average is $\frac{191}{3}$ or whose total sum is 191.

These are, {53, 67, 71} and {59, 61, 71}

We observe that in the first set the difference between greatest and smallest prime number is 18.

Hence, choice (c) is correct.

- 9 Go back to the fundamentals and see the property number 4 and 2 of averages.

Hence, required average = $11 \times 5 + 5 = 60$

- 10 The average score after 48th innings = 48

The average score after 49th innings = 49

Since, $\frac{(48 \times 48 + 97)}{49} = 49$

Now the requirement of runs = $49 + (50 \times 2) = 149$

or $(50 \times 51) - (49 \times 49) = 149$

11

Average			Total
$S + G$	→	35	70
$K + G$	→	32	64
$S + K$	→	38	76
$S + K + G$	→	35	105
$D + I$	→	$\frac{35}{2}$	35

$$\therefore \frac{S + K + G + D + I}{5} = \frac{105 + 35}{5} = 28$$

12

Kaif → 35

Sachin → 41

Ganguli → 29

Dhoni < 35 and Irfan < 35

Hence, Sachin is the senior person.

- 13 It is the same as a person with 20 years of more age replaces an existing person of the group (or village)

Since the total age of the village having n persons, is being increased by 20 years and the average age of village is being increased by 1 year, hence there are total 20 people in the village.

☞ **Alternatively** $(n \times 42) + 20 = (n \times 43)$
 $n = 20$

14 Average rainfall = $\frac{2 \times 6 + 4 \times 5 + 4 \times 10 + 2 \times 6}{12} = 7$

15 $400 \times 7 = (300 \times 3) + (250 \times 2) + (n \times 2) \Rightarrow 700$

16 The new player must be 11 kg heavier than the replaced one. Hence the weight of the new person = $55 + 11 = 66$ kg.

	No. of family members	Average age	Total age
4 years ago	6	25	150
Presently	6	29	174

But the no. of family members (presently) = 7
and average age (presently) = 25
Therefore the total age = $25 \times 7 = 175$
Hence, the age of child = $175 - 174 = 1$ year

18 Amitabh's total expenditure for Jan-June
= $4200 \times 6 = 25200$
Expenditure for February-June = $25200 - 1200 = 24000$
Expenditure for the months of February-July
= $24000 + 1500 = 25500$

The average expenditure = $\frac{25500}{6} = 4250$

19. Total value of $a + 11 + 23 + 17 = 60 \Rightarrow a = 9$

Again $a + b + 12 + 25 = 64 \Rightarrow b = 18$

Therefore, $\frac{a}{b} = \frac{1}{2}$

20 By the method of Alligation

$$\begin{array}{ccc} 12000 & & 18000 \\ & \searrow & \nearrow \\ & 15500 & \\ & \nearrow & \searrow \\ 2500 & & 3500 \end{array}$$

Therefore the ratio of number of Junior level employee to the senior level employee = 5 : 7

Hence the required fraction = $\frac{5}{(5+7)} = \frac{5}{12}$

☞ Alternatively Go through options

$$\frac{J}{J+S} = \frac{5x}{12x} \Rightarrow \frac{J}{S} = \frac{5x}{7x}$$

Therefore $15500 \times 12x = 12000 \times 5x + 18000 \times 7x$
LHS = RHS, Hence correct

21 None of a, b, c is correct.

The correct answer can be $\frac{a+b+c+d+e}{5}$

or $\frac{a+e}{2} = \frac{b+d}{2} = c$

Where a, b, c, d and e are consecutive odd numbers.

22 $31 \times 31 - 30 \times 30 = 61$ years

23 $a + b + c + d + e = 5 \times 92 = 460$

$$a + b + c = 3 \times 83 = 249$$

$$c + d + e = 3 \times 97 = 291$$

$$\therefore c = (a + b + c) + (c + d + e) - (a + b + c + d + e)$$

$$\text{or } c = 540 - 460 \text{ or } c = 80$$

24 $19 \times 26 - 18 \times 25 = 44$ years

25 $\frac{k + k + 2 + k + 4 + k + 6 + k + 8 + k + 10 + k + 12}{7}$

$$= k + 6$$

26 $\frac{\frac{x}{4} \times \frac{85}{100} + \frac{x}{3} \times \frac{70}{100} + \frac{5x}{12} \times \frac{56}{100}}{x} \times 100 = 67.916$

Here, x is the total number of students in the whole class.

27 Since there are five possibilities or combinations (as ${}^5C_4 = 5$)

Therefore, average = $\frac{5 \times 600}{4} = 750$ mm

28 $\frac{\frac{23}{5} + \frac{8}{3} + \frac{62}{9} + \frac{112}{15} + \frac{32}{9}}{5} = 5 \frac{8}{225}$

29 $\frac{1000.0001 + 100.001 + 10.01 + 1.1}{4}$
= $\frac{1111.1111}{4} = 277.777$

30 $\frac{7 + 8 + 9 + 10 + 11 + 12 + 13}{7} = 10 \Rightarrow \frac{7+13}{2} = 10$

31 $\left(\frac{1+2+3+\dots+100}{100} \right) = \left(\frac{\frac{100 \times 101}{2}}{100} \right) = 50.50$

32 $\left(\frac{1+3+5+7+\dots+99}{50} \right) = \left(\frac{50 \times 50}{50} \right) = 50$

33 $\frac{2+4+6+\dots+198}{99} = \frac{99 \times 100}{99} = 100$

34 $a + b + c = 237$
 $a + c = 158$
 $\Rightarrow b = 79$

35 $\frac{M + A + S}{3} = 11111$

Also, $\frac{S + M}{2} = 11111$ ($\because M - 11111 = 11111 - S$)

$$\Rightarrow A = 11111$$

36 $\frac{2 \times 200 \times 600}{800} = 300$ km/hr

37 $\frac{A + B + C + D}{4} = \frac{400 \times 2 + 600 \times 2}{4} = 500$

38 Average weight = $\frac{20 + 50}{100} = 0.7$ kg ($\because 1 \text{ kg} = 1000 \text{ gm}$)

39 Average = $\frac{770}{11} = 70$

40 $\frac{3x + 4x + 5x}{3} = 12 \Rightarrow x = 3$

$\therefore$ The ages are 9, 12 and 15 years

and the required average = $\frac{9+15}{2} = 12$

41 Since the salary of each employee is being increased therefore the average salary will also increased by ₹ 2000.
Thus the required average = 22000.

42 $11 \times 21 - 10 \times 20 = 31$

43 $20 \times 180 - 19 \times 178 = 218$

44 Let the price of $A > B > C$

Then, $\frac{A+B}{2} = 2C$

$$A + B = 4C$$

Now, $A + B + C = 5 \times 3 = 15$ crore

$$\Rightarrow 5C = 15 \text{ crore}$$

$$\therefore C = 3 \text{ crore}$$

45 Since, we don't know about the price of B .

46 $\left(\frac{40 + 70 + 142}{50 + 100 + 150} \right) \times 100 = 84\%$

47 Since 1 is neither prime nor composite number. Thus there are only 99 number viz. 2, 3, 4, 5, 6, ..., 99, 100.

$$\begin{aligned} \text{Hence} &= \left(\frac{2 + 3 + 4 + 5 + 6 + \dots + 100}{99} \right) \\ &= \left(\frac{(1 + 2 + 3 + 4 + \dots + 100) - 1}{99} \right) \\ &= \frac{5050 - 1}{99} = \frac{5049}{99} = 51 \end{aligned}$$

48 $\frac{1 + 4 + 9 + 16 + \dots + 100}{10} = \left(\frac{10 \times 11 \times 21}{6 \times 10} \right)$

$$\left[\because 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \right] = 38.5$$

49 The non-negative integers upto 99 are 0, 1, 2, 3, ..., 99.

Therefore, average = $\left(\frac{0 + 1 + 2 + 3 + \dots + 99}{100} \right)$

$$= \left(\frac{99 \times \frac{100}{2}}{100} \right) \left[\because 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2} \right]$$

$$= 49.5$$

50 $\frac{7 + 14 + 21 + \dots + 77}{11} = \frac{7(1 + 2 + \dots + 11)}{11}$

$$= \frac{7 \times 11 \times 12}{11 \times 2} = 42 \quad \left[\because 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2} \right]$$

Alternatively Since, all the numbers are in A.P. Further there are odd number of numbers (*i.e.*, 11) in the sequence. Thus the middle most term is the average of the sequence, which is 42.

51	Average	Total
A, B, C, D	40	160
A, B, C, D, E	41	205
F, B, C, D, E	42	210

Thus, $E = 205 - 160 = 45$

Hence, $F, B, C, D = 210 - 45 = 165$

Therefore, average of $F, B, C, D = 41.25$

52 Three consecutive natural numbers whose average is k are $(k-1), (k), (k+1)$.

The next two numbers will be $(k+2)$ and $(k+3)$.

Therefore, average of $(k-1), (k), (k+1), (k+2)$ and $(k+3)$ is $(k+1)$.

Alternatively Consider any 3 consecutive natural number *eg.* 4, 5, 6 the average is 5.

Again 4, 5, 6, 7, 8 the average is 6.

To verify it consider some different numbers *eg.* 10, 11, 12 average = 11 and 10, 11, 12, 13, 14 average = 12

Hence, proved that the average is increased by 1.

53 The 5 consecutive odd numbers whose average is k are $(k-4), (k-2), k, (k+2), (k+4)$

Again the average of $(k-4), (k-2), (k), (k+2), (k+4), (k+6), (k+8)$ is $(k+2)$

Alternatively Consider some appropriate numbers.

54 The increase in weight = $(5 \times 42.5) - (5 \times 42) = 2.5$ kg

But we don't know the weight of the replaced officer. So, we can't determine.

55 The decrease in age = $6 \times (28 - 27) = 6$ years

But we don't know the age of the old servant which is being replaced. So, we cannot determine the average age of new servant.

56 The age of new servant = $31 - 6 = 25$ years

57 $A + B + C = 12000 \times 3$

$$B + C + D = 15000 \times 3$$

$$\Rightarrow D - A = 3000 \times 3$$

$$D - A = 9000$$

also $D = 2A$

$$\Rightarrow D = 18000 \text{ and } A = 9000$$

Therefore, average salary of B and $C = \frac{(45000 - 18000)}{2}$

$$= 13500$$

58 The price of the costliest and cheapest computer

$$= (80 \times 30000) - (78 \times 29500) = 99000$$

Therefore the price of the cheapest computer

$$= 99000 - 80000 = 19000$$

59 $\frac{10 \times 50 + 50 \times 10 + 20 \times 25 + 25 \times 20}{4} = 500$

60	Auto Rickshaw	Maxi. cab	Mini. Bus
No. of seats	4	10	20
No. of seats occupied	4	8	15
Rate per seat	12	15	8
Total amount (in ₹)	48	120	120

Therefore, average earning

$$= \frac{48 + 120 + 120}{3} = 96$$

✓ Level 02 Higher Level Exercise

- 1 For this type of questions take the LCM of speeds and assume the LCM as the distance.

Then the time taken @ speed of 60 km/hr = $\frac{300}{60} = 5$ hrs

Again the time taken @ speed of 50 km/hr = $\frac{300}{50} = 6$ hrs

Thus we see that in place of 5 hrs train takes 6 hours. It means the train takes 1 hour extra and this one hour is stopping period in the total time of 6 hours. Thus in 6 hour train halts for 1 hour. So in 1 hour train will stop for $\frac{1}{6}$ hours or 10 minutes.

④ **Alternatively** (short cut)

$$\begin{aligned}\text{Halting (or stopping) time} &= \left(1 - \frac{\text{slower speed}}{\text{faster speed}}\right) \text{ hours} \\ &= 1 - \frac{50}{60} = \frac{10}{60} = \frac{1}{6} \text{ hours} \\ &= 10 \text{ minutes} \\ &(\because 1 \text{ hour} = 60 \text{ minutes})\end{aligned}$$

$$\begin{aligned}2 \quad \frac{[r + (p - q)] + p + q}{3} &= 21 + \frac{p + q + r}{3} \\ \Rightarrow \frac{2p + r}{3} - 21 &= \frac{p + q + r}{3} \\ \Rightarrow \frac{p - q}{3} &= 21 \\ \Rightarrow p - q &= 63\end{aligned}$$

- 3 Let the speed for the first hour be x km/hr

Then the speed for the second hour be $\frac{7}{5}x$ km/hr

Then the speed for the third hour be $\frac{10}{7} \times \frac{7}{5}x = 2x$ km/hr

Then the speed for the fourth hour be

$$2x \times \frac{7}{5} = \frac{14x}{5} \text{ km/hr}$$

Therefore, total distance in four hours

$$= x + \frac{7}{5}x + 2x + \frac{14x}{5} = \frac{36x}{5} \text{ km}$$

$$\therefore \text{Average speed} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{\left(\frac{36x}{5}\right)}{4}$$

$$= \frac{9x}{5} \text{ km/hr}$$

Again the distance in 4 hours @ speed of x km/hr.

Which is half of the third hour's speed is $4x$ km.

$$\text{Hence, } \frac{36x}{5} - 4x = 160 \text{ km}$$

$$\Rightarrow x = 50$$

$$\text{Hence, the average speed} = \frac{9 \times 50}{5} = 90 \text{ km/hr}$$

- 4 The total score of 3 toppers

$$= 123 \times 120 - 120 \times 118.5$$

$$= 540$$

The highest possible score of the third highest topper is possible when the score of other two toppers was minimum

So,

$$\left. \begin{aligned}1^{\text{st}} \text{ rankers score} &= 187 \text{ (minimum)} \\ 2^{\text{nd}} \text{ rankers score} &= 186 \text{ (minimum)} \\ 3^{\text{rd}} \text{ rankers score} &= 167 \text{ (maximum)}\end{aligned} \right\} 540$$

- 5 Go through options : $40 \times 35 + 60 \times 25 = 29 \times 100$

Since there are 40 ladies and 60 gents

④ **Alternatively** $40 \times 7x + 60 \times 5x = 29 \times 100$

$$\Rightarrow x = 5 \therefore 7x = 35$$

- 6

	Lok Sabha	Rajya Sabha
No. of MPs	300	200
Ave. Age	40	50

Since, when a member of Rajya Sabha joins the Lok Sabha and the average age of both the houses increases, it means the average age of this member must lie between 40 and 50. When the age of this member is greater than 40, then the average age of the Lok Sabha increases. Again when the age of this member is less than 50, then after leaving it, the average age of the Rajya Sabha increases.

Solutions (for Q. Nos. 7 to 9)

	No. of family members	Average	Total
8 years ago $\rightarrow$	5	36	180
Presently $\rightarrow$	(if) 5	$(36 + 8) = 44$	220
	7	36	252

- 7 From the above explanation we have no any clue about his wife's age.

- 8 Since we know that the difference between the age of any two persons remains always constant, while the ratio of their ages gets changed as the time changes.

So, if the age of his child be x (presently)

Then the age of wife be $x + 26$ (presently)

Thus, the total age = $x + (x + 26) = 32$ [$\because 252 - 220 = 32$]

$$\Rightarrow x = 3$$

Therefore the age of her child is 3 years and her self is 29 years. Hence her age at the time of the birth of her child was 26 years.

④ **Alternatively** As we have mentioned above that the age difference remains always constant. Therefore her age at the time of her child's birth was 26 years.

- 9 Since there is no clue. So, we can't determine.

10	No. of family members	Average	Total
Eleven years earlier	4	28	112
Presently	if 4	39	156
	6	28	168

Since it is obvious that just after the birth of the youngest member (*i. e.*, child) was 6 family members in the family. Therefore at the time of the birth of the youngest child the elder child's age was 6 years.

$$\text{Now the sum of their ages} = x + (x + 6) = 12 \\ = (168 - 156)$$

$$\Rightarrow x = 3 \text{ and } (x + 3) = 9$$

- 11 Go through options

$$\frac{6}{4} = 1.5 \text{ hour, when he was walking}$$

Therefore, time of journey by train = $3 - 1.5 = 1.5$

$$\text{Now, the distance travelled by train} \\ = 1.5 \times 60 = 96 - 6 = 90$$

Hence correct.

☞ **Alternatively** Total distance = $32 \times 3 = 6 + 60 \times x$

$$\Rightarrow x = 1.5 \text{ hours}$$

$$\text{Thus, the speed of walking} = \frac{6}{1.5} = 4 \text{ km/hr}$$

- 12 Let the number of total MPs = n and their average age be x then

$$n \times x = \frac{4}{5} n \times \frac{5}{4} x + \frac{n}{5} \times y$$

$$\Rightarrow \text{either } n = 0 \text{ or } y = 0$$

[Since, there are only 80% MPs remained in the house which is equal to $\frac{4}{5}n$ and the increase in average age

$$= 20\% = \frac{5}{4}x]$$

Thus, there cannot be any possible value of n .

$$13 \quad nx = \frac{4}{5}n \times \frac{5}{4}x + \frac{n}{5} \times 30$$

$$\Rightarrow n = 0 \text{ which is impossible}$$

So, there is no any woman MP in the Lok Sabha.

$$14 \quad \text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} \\ = \frac{6 \times 20}{9} = 13.33 \text{ m/s}$$



Hexagonal path has six sides

- 15 Average of 26, 29, 35 and 43 is 33.25. Also the average of 26, 29, n , 35 and 43 lies between 25 and 35 *i. e.*,

$$25 < \frac{26 + 29 + n + 35 + 43}{5} < 35$$

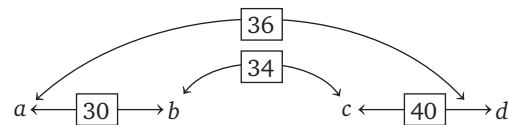
$$\Rightarrow 125 < 26 + 29 + n + 35 + 43 < 175$$

$$\Rightarrow 125 < 133 + n < 175$$

$$\Rightarrow n < 42$$

Since the value of n is an integer and greater than 33.25, then $33 < n < 42$; for every integer n .

- 16 Given that $a < b < c < d$



The only possible prime number between 30 and 34 is 31

Hence, $b = 31$, therefore $a = 29$

Similarly, $c = 37$ and $d = 43$

Therefore, $d - a = 43 - 29 = 14$

$$17 \quad 9261000 = 2^3 \times 3^3 \times 5^3 \times 7^3 \\ = (2 \times 3 \times 5) \times (2 \times 3 \times 7) \times (2 \times 5 \times 7) \times (3 \times 5 \times 7) \\ = 30 \times 42 \times 70 \times 105$$

Therefore the average of 30, 42, 70 and 105 is 61.75.

- 18 Since the number is as below 270 as its multiple is as above 270. It means these two numbers are equidistant from 270. Hence their average is 270.

☞ **Alternatively** Let the number be x then.

$$270 - x = 8x - 270$$

$$\Rightarrow x = 60 \text{ and } 8x = 480$$

Therefore the average of 60 and 480 is 270.

- 19 The average of 60 and 270 is 165

- 20 Number of days in a week = 7

Average number of visitors = 40

Total visitors = $280 (= 7 \times 40)$

Now, if n be the number of holidays in a week, then

$$(7 - n) \times 56 = 280 \quad (40 + 16 = 56)$$

$$\Rightarrow n = 2$$

The rest data is redundant or useless, since our problem is solved here without using it.

☞ **Alternatively** Go through options

$$40 \times 7 = 280 = 56 \times 5 \text{ hence proved.}$$

- 21 Since we don't know their ages individually, so we cannot calculate the average of the class when a student of unknown age readmitted in the class.

- 22 Let the average of prime numbers be P and average of composite numbers be C .

Again the number of prime numbers be x , then the number of composite numbers be $2x$.

$$\text{Then, } \frac{Px + 2Cx}{3x} = 9 \Rightarrow P + 2C = 27 \quad \dots(i)$$

$$\text{and } \frac{2Px + Cx}{3x} = 11$$

$$\Rightarrow 2P + C = 33 \quad \dots(ii)$$

On adding eqs. (i) and (ii) we get $P + C = 20$
 and on subtracting eqs. (i) from (ii), we get $P - C = 6$
 Therefore, $P = 13$ and $C = 7$
 Thus, $\frac{C}{P} = \frac{7}{13}$

23 Solve through option $\frac{540}{27} = 20$

$$\frac{540 - (2 \times 20)}{25} = \frac{500}{25} = 20$$

Hence option (b) is correct.

24 Number of members in the staff $\times$ average age = Total age

$$29 \times 24 = 696 \Rightarrow 29 \times 27 = 783$$

Hence, change in total age = 87

	Physician	Surgeon	Nurse
No. of members	$(21 - x)$	8	x
Increase in average age	1	6	3
Increase in total age	$(21 - x)$	48	$3x$

$$\text{Since } P + S + N = 29$$

Therefore, if there would be x nurses, then there must be $(21 - x)$ physicians. Again, total change (or increase) in age
 $= (21 - x) + 48 + 3x = 87 \Rightarrow x = 9$

25 Total exp. Jan-June = $4200 \times 6 = ₹ 25200$

$$\text{Total exp. Feb-June} = 25200 - 1200 = ₹ 24000$$

$$\text{Total exp. Feb-July} = 24000 + 1500 = ₹ 25500$$

$$\text{The average expenditure Feb-July} = \frac{25500}{6} = 4250$$

26 Let the number of subjects be n and average marks be x , then total marks = nx

$$\text{Again } (n + 1)(x - 1) = (nx - 40) + (23 + 25)$$

$$\Rightarrow x - n = 9 \quad \dots(i)$$

$$\text{Further } (n + 2)(x + 1) = (nx - 40) + (23 + 25) + 57$$

$$\Rightarrow nx + 2x + n + 2 = nx + 65$$

$$\Rightarrow 2x + n = 63 \quad \dots(ii)$$

On solving eqs. (i) and (ii), we get

$$n = 15$$

$$\text{and } x = 24$$

27 Initially, the total age of family = $4 \times 42 + 7 \times 20 = 308$

$$\text{and the average age} = \frac{308}{11} = 28$$

Now, the total age of family

$$= 308 - (2 \times 13) + (11 + 15 + 28)$$

$$= 308 + 28 = 336$$

$$\text{Now, the new average of the family} = \frac{336}{12} = 28$$

Since, the average age of the original family and that of new family is same (i.e., 28)

Hence, the average age of the new family is increased by 0 year.

28 Let there be n number and he missed a number k , then the

$$\text{average (which he has calculated)} = \frac{\left(\frac{n(n+1)}{2} - k\right)}{n} = 15$$

$$\Rightarrow n^2 - 29n = 2k$$

$$\Rightarrow n(n - 29) = 2k$$

Thus, at $n = 29$ or $n < 29$, the expression is invalid, since the value of k is neither zero nor negative, which is actually a natural number. So for the least possible value of $n = 30$

$$k = 15$$

$$\text{and for } n = 31, k = 31$$

Again for $n > 31$, k is beyond the range i.e., greater than n . Since, k cannot be greater than n . Hence, there are only two values of k . So there is no unique value of n .

29 Let there be n people (initially) in the group, then the total earning of the group = $n \times 50$

$$\text{Again, } n \times 50 = (n - 2) \times 49 + (2x + 45)$$

$$\Rightarrow n = 2x - 53; \text{ where } x \text{ is the lowest earning of any person.}$$

Now, since $42 < x < 47$ and $n \in$ prime numbers.

Then the only possible value of $n = 37$ for $x = 45$.

30 Let the number of students who got the jobs of A, B and C categories is a , b and c respectively,

$$\text{then the total salary} = \frac{26(a + b) + 44(b + c) + 34(c + a)}{2(a + b + c)}$$

$$= \frac{60a + 70b + 78c}{2(a + b + c)}$$

$$= \frac{30(a + b + c) + (5b + 9c)}{a + b + c}$$

$$= 30 + \text{some positive value}$$

So the minimum salary must be ₹ 30 lakh and the maximum salary cannot exceed 44, which is the highest of the three.

$$\mathbf{31} \quad D - C - A \quad \dots(i)$$

$$D > B > C \quad \dots(ii)$$

From eqs. (i) and (ii), we get

$$D > B > C > A \quad \dots(iii)$$

$$\text{Again, } E - B - A$$

But $B > A$, from eq. (iii)

$$\text{So, } E > D > B > C > A \quad \left[\begin{array}{l} \text{Since, } B \text{ is the average of} \\ E \text{ and } A \text{ so it is equidistant} \\ \text{from both } E \text{ and } A. \end{array} \right]$$

32 Let Donald be denoted by H (Husband).

His wife be denoted by W (Wife).

His daughter be denoted by D (Daughter).

His son be denoted by S (Son).

$$\text{The average age of 4 persons} = \frac{(H + W + D + S)}{4} = 23$$

$$\Rightarrow H + W + D + S = 92$$

$$\text{Again, } H = W + 4$$

So, At the time when daughter is born	H	W	D	S
	28	24	0	×
At the time when son is born	32	28	4	0

So, at the time of birth of his son, total age of his family = 64 years ($32 + 28 + 4 + 0 = 64$)
 and presently the total age of his family = 92 years
 It means total increase in age of the whole family = 28 years
 Thus the average increase in age = $\frac{28}{4} = 7$ years

It means the age of Donald = 39 years
 and age of his daughter = 11 years
 Therefore the average age of Donald and his daughter is 25 years.

- 33** Aman Wife Son EI. D Yg. D
 $5x$ x $5z$ $4z$
 $8y$ $3y$
 $\Rightarrow 40K$ W $8K$ $15K$ $12K$
 Again since, $Yg. D + W = A + S$ ($K = x \cdot y$)
 $\Rightarrow 12K + W = 40K + 8K$ $W \rightarrow$ Age of wife
 $\Rightarrow W = 36K$
 Thus, 4 years ago $(36K - 4) = 8(8K - 4)$
 $\Rightarrow 28K = 28 \Rightarrow K = 1$
 Therefore, the age of Aman = 40
 Wife = 36
 Son = 8
 Elder daughter = 15
 Younger daughter = 12

Hence, the average age of the family = $\frac{111}{5} = 22.22$ years

- 34** Let there be n politicians (initially) in the party and their average weight be x kg, then $nx + 209 = \frac{5}{4}n(x - 1)$

$$\begin{aligned} \frac{n}{4}(x - 5) &= 209 \\ \Rightarrow x &= \frac{209 \times 4}{n} + 5 \\ x &= \frac{4 \times 11 \times 19}{n} + 5 \end{aligned}$$

So the possible value of n is 76 ($= 19 \times 4$)
 Thus, $x = 16$
 Therefore the average weight of all the politicians is 15 kg.

- 35** Total distance covered by all the wheels = 4×160
 Number of wheels used = 5
 Therefore, average distance covered by each wheel
 $= \frac{4 \times 160}{5} = 128$ km

- 36** Let the numbers be
 $(a - 5), (a - 3), (a - 1), (a + 1), (a + 3), (a + 5)$,
 then their average
 $= \frac{(a - 5) + (a - 3) + (a - 1) + (a + 1) + (a + 3) + (a + 5)}{6} = a$

Again the value of 'a' can be found by using the last statement

$$\begin{aligned} i. e., \quad (a - 5)^2 + (a + 5)^2 &= 178 \\ \Rightarrow a^2 = 64 \Rightarrow a &= 8 \end{aligned}$$

	No. of Directors	Average Age	Total Age
Just before death and resignation	10	48	480
Just after death and resignation	9		$\{480 - (53 + x) + 34\}$
One year later	9	46	414

So, one year later, after the incident
 total age = $\{480 - (53 + x) + 34\} + 9 \times 1 = 414$
 $\Rightarrow x = 56$ years

Where x is the age of the dead person at the time of his death.

- 38** Go through options = $\frac{24 \times 60 + 12 \times 54}{(24 + 12)}$
 $= 58$, which is a whole number.

	Year/Time	No. of Nurses	Average Age	Total Age
	1982	100	50	5000
Just before retirement	1984	100	52	5200
Just after retirement	1984	80	50	$(5200 - 20 \times 60) = 4000$
Just before recruitment	1987	80	53	4240
Just after recruitment	1987	$(80 + 40) = 120$	48	$(4240 + 38 \times 40) = 5760$
	1990	120	51	6120

- 40** Average score
 $= \frac{8 + 10 + 7 + 12 + 25 + 24 + 18 + 23 + 29 + 30}{10} = \frac{186}{10} = 18.6$

- 41** The candidate who has scored maximum (means continuing for long time and number of responses were maximum) can be such a required person.
 Now, consider Abhijeet, whose score was 30.
 Again consider Rajesh, Radhe, Harish and one more candidate
 Since, $8 + 10 + 7 < 30 \Rightarrow 25 < 30$
 So there are exactly 3 persons which can be full time opponents.

Further there are 5 ($= 30 - 25$) more chances. So these chances can be utilised by any other candidate but he or she must be in the beginning or in the ending of the Abhijeet, since the score of all the candidates is greater than 5. Hence there can be maximum $3 + 1 = 4$ opponents of Abhijeet.

42 There is no such an information.

43 The minimum number of participants who have lost the game before Abhijeet started will be possible only when the number of participant with Abhijeet be maximum, which is 4.

Hence, excluding these 4 players (opponents of Abhijeet) and Abhijeet himself there are 5 people left. So, minimum 5 players have lost the game (or contest).

44 The opponents of Abhijeet (as per the requirement) are Rajesh, Radhe, Harish and Amit Sana

So, the average score $= \frac{8 + 10 + 7 + 29}{4} = 13.5$

45 Go through options or total employees $= \frac{(25 - 10) \times 20}{10} = 30$

Hence, number of educated employees $= 30 - 20 = 10$

46 Average speed of the later half journey
 $= \frac{2 \times 40 \times 60}{100} = 48 \text{ km/hr}$

Now the average speed of the whole journey
 $= \frac{2 \times 48 \times 96}{144} = 64 \text{ km/hr}$

47 $\frac{40 \times 0.8 + 60 \times 0.75 + 50 \times 0.6 + 30 \times 0.5}{180} \times 100$
 $= \frac{122}{180} \times 100 = 67.777 \dots \%$

48 Remember $\frac{abc}{99(a-c)} - \frac{cba}{99(a-c)}$

where abc and cba are the three digit numbers and $(a, c) \neq 0$

Again, since the difference in average $= 19.8$

Therefore the difference in total $= 19.8 \times 10 = 198$

Thus, $99 \times (a - c) = 198$

1 Since all the total 100 elements of Sets A, B, C are the natural numbers upto. Thus the average of these first 100 natural numbers is the required average.

$$\therefore \text{Average} = \frac{1 + 2 + 3 + 4 + \dots + 100}{100} = \frac{100 \times 101}{2 \times 100} = 50.50$$

2 Except to 2 there are all the even numbers upto 100

$$\text{So, the required average} = \frac{(2 + 4 + 6 + \dots + 100) - 2}{49} = \frac{50 \times 51 - 2}{49} = \frac{2548}{49} = 52$$

 **NOTE** There are only 49 elements in the Set B. Apply the formula of sum of first even numbers. Also use the property of AP.

$$\therefore (a - c) = 2$$

49 If the highest weight be 35 gm, then the second highest weight will be 31 gm. Again if the second highest will be 31, then the third highest will be 33 which is in admissible, since then $35 + 33 = 68$ which is not the greatest possible combination. Hence, wrong.

Similarly, 36 (i. e., option b) is also invalid

Highest	Sec. Highest	Third Highest
36	30	34

Thus, $36 + 34 = 70 > 66$, hence wrong.

The greatest possible combination cannot be greater than 66.

Now, consider option (c)

Highest	Sec. Highest	Third Highest
34	32	32 ✗ (since, weights are different)
	32	31 ✗ (since, 65 is not a combination)
	32	30 ✓

So, the highest weight 34

Sec. highest weight 32

Third highest weight 30

Lowest weight 29

Since, all the weights obtained give all the 6 different combinations, hence 34 is the highest possible weight of an envelope.

50 Let the fixed expenditure of the hotel be ₹ x and the variable expenditure (which is dependent on the guest) is ₹ y , then

$$x + 10y = 600 \quad \dots(i)$$

$$x + 20y = 800 \quad \dots(ii)$$

$$\Rightarrow 10y = 200$$

$$y = ₹ 20 \text{ and } x = 400$$

Hence the total expenditure when there are 40 guests

$$= 400 + 40 \times 20 = 1200$$

$$\text{Therefore, average expenditure} = \frac{1200}{40} = ₹ 30$$

3 The total value of all the 25 elements of the Set A
 $= 25 \times 42.4 = 1060$

Since, there are 25 prime numbers upto 100 in the Set A

Again in the Set A and C there are 50 odd numbers and one even number. So the sum of all the elements of A and C

$$= (1 + 3 + 5 + 7 + \dots + 99) + 2 = (50)^2 + 2 = 2502$$

Therefore the sum of all the elements of Set C

$$= 2502 - 1060 = 1442$$

Hence, the average of the Set C

$$= \frac{1442}{26} = 55.4615$$

- 4 The average of all the elements of the Set A and C

$$= \frac{2502}{51} = 49.0588$$

Solutions (for Q. Nos. 5 to 15)

Set	No. of elements	Average	Least element	Greatest element
A	25	42.4	2	97
B	49	52	4	100
C	26	55.46	1	99

- 5 Since the value of element which is transferred to Set B is less than 50, which in turn less than the average of Set B, hence the average of Set B decreases.
NOTE If a quantity which is less than the average of the group introduces from outside then the new average of the group decreases.
- 6 The least possible numbers of Set A which are greater than 50 are 53 and 59 whose average is always greater than the average of C. Hence the average of C will necessarily increase.
- 7 Can't say, since we don't know which 10 numbers are being transferred. Whether their average is greater, less or equal to the average of B.
- 8 Definitely increases, since the average of those numbers (viz. 1 and 99) is 50 which is greater than the average of Set A.
- 9 The average of those numbers (viz. 4 and 100) is 52. Hence average of A will increase and average of B will remain constant and the average of C remains unaffected because Set C is not involved.
NOTE If an element or average of some elements is equal to the average of the group then this element (or subset of elements) does not change the average of the group, when it joins the group or leaves the group.
- 10 After the insertion of new element viz. 2 in the Set B the new average = $\frac{2 + 4 + 6 + \dots + 100}{50} = 51$
Hence, the new average of Set B decreases by 1.
- 11 The perfect square number of the Set C are 1, 9, 25, 49, 81.
Hence, the average of these number = $\frac{165}{5} = 33$
- 12 Since there is no net change (i.e., all the elements even after being transferred are same). Hence their average is also same as in question no. 1.
- 13 Obviously A. Since the average of all those 15 elements which are joining the Set A is greater than the average of all those 5 elements which are leaving the Set A and this difference in average is largest in comparison to Set B or Set C. Even in Set C there is decrease in average.
- 14 To minimize the loss in average of Set B, we have to transfer the least possible values of the given range and have to bring the highest possible values from the Set A to the Set B.
Thus the absolute decrease in Set B

$$= (26 + 28 + 30 + 32 + \dots + 44) - (23 + 19 + 17 + 13 + 11) \\ = 350 - 83 = 267$$

Hence, the decrease in total value of Set B

$$= 2548 - 267 = 2281$$

Therefore, new average = $\frac{2281}{44} = 51.84$

NOTE Now there are only 44 elements in Set B.

- 15 There is no relevant information regarding the numbers which are being transferred from one set to another set.

16 Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

$$= \frac{200}{\left(5 + \frac{10}{3}\right)} = \frac{200 \times 3}{25} = 24 \text{ km/hr}$$

Since for the first 100 km time required is $\frac{100}{20} = 5$ hrs and

for the last 100 km time required = $\frac{100}{30} = \frac{10}{3}$ hrs

17 The average speed = $\frac{150}{5 + \frac{5}{3}} = \frac{150 \times 3}{20} = 22.5 \text{ km/hr}$

- 18 Average bonus for the first 3 months

$$= \left(\frac{3000}{100}\right)^2 + 10 = 910$$

Average bonus for the next 5 months

$$= \left(\frac{5000}{100}\right)^2 + 10 = 2510$$

Average bonus for the last 4 months

$$= \left(\frac{8000}{100}\right)^2 + 10 = 6410$$

His average bonus for the whole year

$$= \frac{910 \times 3 + 2510 \times 5 + 6410 \times 4}{12} = ₹ 3410$$

Hence his average earning per month

$$= 3410 + 200 = ₹ 3610$$

- 19 Total price of 5 shirts = ₹ [100 + 10 × (5)²] = ₹ 350

Hence, the average price = $\frac{350}{5} = ₹ 70$

- 20 Check the option (c).

$$\text{Total price} = 100 + 10 \times (2)^2 = ₹ 140$$

$$\text{Average price} = \frac{140}{2} = ₹ 70$$

Hence, the average price is same as that of Mallika.

- 21 Total number of passengers = 10 × 20 = 200

In the 9 compartments the total number of passengers

$$= 144 (= 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20)$$

So the no. of passengers in the 10th coach = 200 - 144 = 56

22	No. of 2 wheelers	No. of 3 wheelers	No. of 4 wheelers
No. of wheels	$2x$ $2 \times 2x = 4x$	x $3 \times x = 3x$	$2x$ $2x \times 4 = 8x$

Therefore average number of wheels = $\frac{4x + 3x + 8x}{5x} = 3$

- 23 The average weight of eggs of first generation is k gm and the no. of eggs is ' n '. Let $a_1, a_2, a_3, \dots, a_n$ be the weights of n eggs of the first generation

$$\therefore k = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n}$$

$$\therefore nk = a_1 + a_2 + a_3 + \dots + a_n \quad \dots(i)$$

where a_1 is the average weight of its ' n ' child eggs, a_2 is the average weight of its own ' n ' child eggs and so on. child egg is referred to the egg of next generation produced by its mother egg.

$$\therefore a_1 = \frac{a_1 + b_1 + c_1 + \dots + n_1}{n}$$

$$a_2 = \frac{a_2 + b_2 + c_2 + \dots + n_2}{n}$$

$$a_3 = \frac{a_3 + b_3 + c_3 + \dots + n_3}{n} \text{ etc.}$$

Substituting the values of $a_1, a_2, a_3 \dots$ in eq. (i)

$$nk = \left(\frac{a_1 + b_1 + c_1 + \dots}{n} \right) + \left(\frac{a_2 + b_2 + c_2 + \dots}{n} \right) + \left(\frac{a_3 + b_3 + c_3 + \dots}{n} \right) + \dots + \left(\frac{a_n + b_n + c_n + \dots}{n} \right)$$

$$\therefore nk = \frac{(a_1 + b_1 + c_1 + \dots) + (a_2 + b_2 + c_2 + \dots) + (a_3 + b_3 + c_3 + \dots)}{n}$$

$\Rightarrow$ Therefore n^2k is the total weight of all the eggs of second generation. Similarly, each of $a_1, b_1, c_1, a_2, b_2, c_2 \dots$ are the average weights of n eggs of their child eggs. Hence in the third generation total weight will be n^3k . Thus the total weight of all the eggs of r^{th} generation is n^rk .

Solutions (for Q. Nos. 24 to 27) Before going for the final solution we need to look for the fundamental concept of averages i.e., if a person of higher age than the average age of the group leaves the group, then the average age of the group decreases. Also if a person of less age than the average age of the existing group joins the group, then the average age of the group decreases. Besides it we also know that the average age of the same group after k years increases by K years.

Faculty of LR :

Year	No. of faculty	Average age	Total age
2004	3	49.33	148
2005	4	44	$176 = 148 + 3 + 25$
2006	4	45	$180 = 176 + 4$
2007	4	46	$184 = 180 + 4$

$176 = 148 + 3 + 25$, implies that due to 3 existing professors their total age will be increased by 3 years after one year time period and 25 years age will be added due to a new entrant in the faculty of LR.

Faculty of DI :

Year	No. of faculty	Average age	Total age
2004	4	50.5	202
2005	4	51.5	$206 = 202 + 4$
2006	4	52.5	$210 = 206 + 4$
2007	5	47.8	$239 = 210 + 4 + 25$

Faculty of English :

Year	No. of faculty	Average age	Total age
2004	5	50.2	251
2005	4	49	$196 = 251 + 5 - 60$
2006	5	45	$225 = 196 + 4 + 25$
2007	5	46	$230 = 225 + 5$

Faculty of Quants :

Year	No. of faculty	Average age	Total age
2004	6	45	270
2005	7	43	$301 = 270 + 6 + 25$
2006	7	44	$308 = 301 + 7$
2007	7	45	$315 = 308 + 7$

- 24 In the year 2006, a new faculty member joined the English faculty.

- 25 The new faculty member who joined on April 1, 2005 became 27 years old on April 1, 2007.

- 26 From the faculty of English a professor retired on April 1, 2005.

- 27 Age of Sarvesh on April 1, 2004 = 52 years

+ 4 months $\approx$ 52 years

Similarly age of Manish on April 1, 2004

= 49 years + 4 months $\approx$ 49 years

$\therefore$ Age of the third professor on April 1, 2004

= $148 - (52 + 49) = 47$ years

Hence the age of the third professor on April 1, 2009

= $47 + 5 = 52$ years

- 28 Since 32 is the average number of inmates, so total number of inmates = $32 \times 7 = 224$.

As 3 jailors have less than 30 inmates and 3 jailors have more than 30 inmates, so 30 is the median number of inmates.

To maximize the difference, you have to maximize the value of six figures so that you can minimize the value of the remaining figure.

Jailor	Jailor	Jailor	Asrani	Jairaj	Jailor	Jailor
X	28	29	30	32	44	45

So, the highest possible total number of inmates supervised by 6 jailors = $45 + 44 + 32 + 30 + 29 + 28 = 208$

Now, the remaining seventh jailor will be supervising 16 (= $224 - 208$) inmates.

Thus, the maximum possible difference = $45 - 16 = 29$

Solution (for Q. Nos. 29-31)

Let n be the total number of seats, then the total possible income of the airline = $1 + 2 + 1 + 3 + \dots + n = \frac{n(n+1)}{2}$

Now, if k th customer gets her refund, then the actual total income of the airline = $\left[\frac{n(n+1)}{2} \right] - k$

Therefore, the average income per passenger = 100

$$\Rightarrow \frac{\left[\frac{n(n+1)}{2} \right] - k}{(n-1)} = 100 \Rightarrow \frac{n(n+1)}{2} - k = 100(n-1)$$

$$\Rightarrow n^2 - 199n + 200 = 2k \quad \dots(i)$$

Now, go through the given choice and try to figure out which suitable value of k is a valid one.

If you choose $k = 99$, the eq. (i) becomes $n^2 - 199n + 2 = 0$ and then n is not an integer.

If you choose $k = 49$, the eq. (i) becomes $n^2 - 199n + 102 = 0$ and then n is not an integer.

If you choose $k = 225$, the eq. (i) becomes $n^2 - 199n - 250 = 0$ and then n is not an integer.

If you choose $k = 100$, the eq. (i) becomes $n^2 - 199n = 0$ and then $n = 199$ or 0 .

Thus, the possible customer number who had returned the ticket is 100. Hence, choice (b) is the correct one.

By hit and trial, you realize that if $n > 200$, then $k > n$, which is not possible. So, choice (c) is the correct one.

Since, you see that the average revenue per passenger is still ₹ 100 even if the customer number is different. From the little knowledge you have about the previous problems, you can see that whichever the customer number, either 100 or 200, takes the refund, the average revenue per passenger remains the same. So, you cannot conclude the exact loss, as it could be ₹ 100 or ₹ 200 or may be some other amount.

Hence, choice (d) is the correct one.

Alternatively Let n be the total number of seats, then the total possible income of the airline

$$= 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

Now, if k th customer gets her refund, then the actual total income of the airline = $\left[\frac{n(n+1)}{2} \right] - k$

Therefore, the average income per passenger = 100

$$\Rightarrow \frac{\left[\frac{n(n+1)}{2} \right] - k}{(n-1)} = 100$$

$$\Rightarrow \frac{n(n+1)}{2} - k = 100(n-1)$$

$$\Rightarrow n^2 - 199n + 200 = 2k$$

$$\Rightarrow n^2 - 199n + 2(100 - k) = 0$$

The value of n can be integer only when the discriminant (D) of the above quadratic equation is a perfect square. That is $D = 199^2 - 8(100 - k)$ is a perfect square.

$\sqrt{D} = \sqrt{199^2 - 8(100 - k)}$	k	n	Valid, if $1 \leq k \leq n$
197	1	198	Valid
199	100	199	Valid
201	200	200	Valid
203	301	201	Invalid
205	403	202	Invalid
...	...	...	...
...	...	...	...

We know that, $\sqrt{D}$ cannot be an even integer, as odd - even = odd. Also, $1 \leq k \leq n$ indicates that k must be greater than or equal to 1; and k must be less than or equal to m . Thus, there are three possible values of k (1, 100 and 200) and so the three possible values of n (198, 199 and 200).

Alternatively Let n be the total number of seats, then the total possible income of the airline = $\frac{n(n+1)}{2}$

Now, if k th customer gets her refund, then the actual total income of the airline = $\frac{n(n+1)}{2} - k$

Therefore, the average income per passenger = 100

$$\Rightarrow \frac{\frac{n(n+1)}{2} - k}{(n-1)} = 100$$

The minimum value of $k = 1$ and the maximum value of $k = n$. Therefore,

$$\frac{\frac{n(n+1)}{2} - 1}{(n-1)} \geq 100 \geq \frac{\frac{n(n+1)}{2} - n}{(n-1)}$$

$$\Rightarrow \frac{n^2 + n - 2}{2(n-1)} \geq 100 \geq \frac{n^2 + n - 2n}{2(n-1)}$$

$$\Rightarrow \frac{(n+2)(n-1)}{2(n-1)} \geq 100 \geq \frac{n(n-1)}{2(n-1)}$$

$$\Rightarrow \frac{(n+2)}{2} \geq \frac{100}{1} \geq \frac{n}{2}$$

$$\Rightarrow (n+2) \geq 200 \geq n \Rightarrow 198 \leq n \leq 200$$

It implies that there are three possible values of n (198, 199 and 200). And so, there are three possible values of k (1, 100 and 200).

29 Since $k = 100$ is one of the possible values, so choice (b) is the correct one.

30 Since the highest possible value of $n = 200$, so choice (c) is the correct one.

31 Since more than one value of k is possible, so choice (d) is the correct one.

CHAPTER 03

Alligations

The concept of Alligations is simply an extension of Averages. In fact, Allegation method is used to find the weighted average of mixture or different groups. It is also used to find out the percentage of impurity or reduction in the original quantity where the repeated dilution or depreciation takes place.

Exams, such as CAT, XAT, IIFT, CMAT and GMAT, ask the application based questions from this chapter. Though the number of problems asked in these exams is not very high, but it helps you fetch good marks in the exam if you understand the Alligation method really well. Nonetheless, exams such SSC CGL and Bank PO can ask good number of problems from this chapter. As far as its scope is concerned, we can use it in the chapters such as Simple and Compound interest, Profit and Loss, Ratio and Proportion to name just a few. In fact, it plays a very significant role in answering the problems based on Data Interpretation.

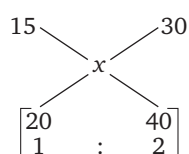
Exp. 1) The average weight of a class of 40 students is 30 and the average weight of a class of 20 students is 15. Find the average weight of both the combined classes :

- (a) 20 (b) 25 (c) 17.5 (d) 15

Solution To find the solution of this problem, we can use two widely used techniques :

1. Weighted average method The required average = $\frac{40 \times 30 + 20 \times 15}{(40 + 20)} = 25$

2. Alligation method



In this graphical representation of the solution, x is the weighted average where 15 and 30 are the averages of different class. So remember that the weighted average is always written in middle and the individual averages are written at the top, the smaller one in the left and the greater one in the right just for convenience and the number of elements (or the fixed quantities) are written below correspondingly. Now, the difference between the two averages is divided in the inverse ratio of the quantities written below. As in the above problem, the difference 15 ($= 30 - 15$) will be divided in the ratio of 2 : 1 (as 40 : 20) but not in the ratio of 1 : 2, which must be clear from the arrows indicating cross proportion. So, the value of $x = 15 + \frac{2}{3} \times (30 - 15) = 25$ or $x = 30 - \frac{1}{3} \times (30 - 15) = 25$.

Therefore, the average weight of both the classes is 25.

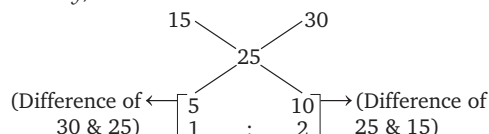
Chapter Checklist

- Problem Based Mixtures and Groups
- Problems Based on Time, Speed and Distance
- Problems Based on Dilution and Depreciation
- CAT Test

Exp. 2) If the average weight of a class of students is 15 and the average weight of another class of students is 30, then find the ratio of the students of the first class to the another class of 30 students when the average weight of both the classes is 25 :

- (a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 4

Solution Let the ratio of the students with 15 students to 30 students be $x : y$, then



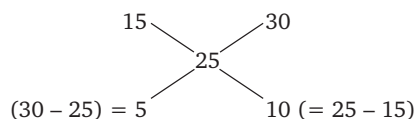
Here we know that the averages of individual classes is 15 and 30. Again we know that the averages of both the combined classes is 25. So the difference between 25 and 15 i.e., 10 and 30 & 25, i.e., 5 is written diagonally opposite.

Thus the required ratio = $\frac{5}{10} = \frac{1}{2}$

Exp. 3) The average weight of girls is 15 and the average weight of boys is 30 and the average weight of boys and girls both is 25. If the number of boys are 12, then the number of girls is :

- (a) 4 (b) 6 (c) 10 (d) 18

Solution



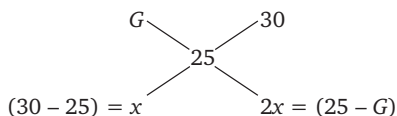
Since the ratio of girls to boys is $\frac{5}{10} = \frac{1}{2}$

Hence if there are 12 boys, so there will be 6 girls.

Exp. 4) The ratio of number of girls to number of boys is 1 : 2. If the average weight of the boys is 30 kg and the average weight of both the boys and girls be 25 kg, then the average weight of the girls is :

- (a) 15 kg (b) 20 kg (c) 35 kg (d) 40 kg

Solution



$$\Rightarrow \left(\frac{30 - 25}{25 - G} = \frac{x}{2x} \right)$$

$$\text{So, } \frac{(30 - 25)}{(25 - G)} = \frac{1}{2} \Rightarrow G = 15$$

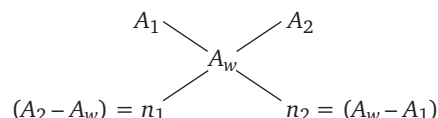
Therefore, the average weight of the girls is 15 kg. In general, if the average of group 1 be A_1 and the number of the elements be n_1 and the average of the group 2 be A_2 and the number of element be n_2 then the weighted average $A_w = \frac{n_1 A_1 + n_2 A_2}{n_1 + n_2}$

$$\Rightarrow n_1 A_w + n_2 A_w = n_1 A_1 + n_2 A_2$$

$$\Rightarrow n_1 (A_w - A_1) = n_2 (A_2 - A_w)$$

$$\Rightarrow \frac{n_1}{n_2} = \frac{(A_2 - A_w)}{(A_w - A_1)} \text{ (known as Alligation Equation)}$$

So, this can be represented in the graphical form as given below

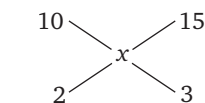


So, all these problems concerned to the topic can be solved either by the formula of weighted average or Alligation equation or graphical representation method, as discussed above.

Exp. 5) Two varieties of soda water with different prices is mixed in the ratio of 2 : 3. The price of first soda water is ₹ 10 per litre while the price of second soda water is ₹ 15 per litre, respectively. The average price of the mixture (per litre) is :

- (a) ₹ 12 (b) ₹ 13 (c) ₹ 14 (d) ₹ 15

Solution



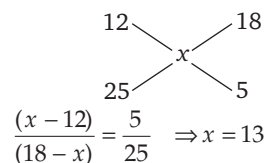
$$\Rightarrow \frac{(x - 10)}{(15 - x)} = \frac{3}{2} \Rightarrow 2(x - 10) = 3(15 - x)$$

$$\Rightarrow x = 13$$

Exp. 6) 5 kg of superior quality of sugar is mixed with 25 kg of inferior quality sugar. The price of superior quality and inferior quality sugar is ₹ 18 and ₹ 12 respectively. The average price per kg of the mixture is :

- (a) ₹ 13 (b) ₹ 15 (c) ₹ 18 (d) ₹ 21

Solution



$$\frac{(x - 12)}{(18 - x)} = \frac{5}{25} \Rightarrow x = 13$$

NOTE SHORTCUT The corresponding ratio is 25 : 5 = 5 : 1

Now reverse the ratio, which is 1 : 5

Now divide the differences of 12 and 18 in the ratio of 1 : 5

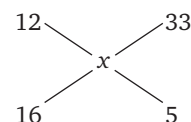
i.e., divide 6 into two parts in the ratio of 1 : 5.

Then the average price of mixture is $12 + 1 = 13$ or $18 - 5 = 13$

Exp. 7) 16 litres of kerosene is mixed with 5 litres of petrol. The price of kerosene is ₹ 12 per litre and the price of petrol is ₹ 33 per litre. The average price of the mixture per litre is :

- (a) ₹ 15 (b) ₹ 17 (c) ₹ 23 (d) ₹ 27

Solution



Now divide the difference of 12 and 33 in the ratio of 5 : 16 (not in the ratio of 16 : 5)

Here, the difference of 33 and 12 = 21

Again on dividing 21 in the ratio of 5 : 16, we get the first part = 5 and second part = 16

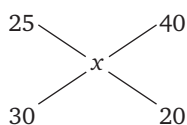
So, the average price = $12 + 5 = 17$ or $= 33 - 16 = 17$

NOTE This cross among the numbers written above shows that the difference between x and 12 is 5 and the difference between 33 and x is 16.

Exp. 8) Amit travels 30 minutes at the speed of 25 km/hr. Further he travels 20 minutes at the speed of 40 km/hr. Find his average speed.

- (a) 25 km/hr (b) 30 km/hr
(c) 31 km/hr (d) none of these

Solution



Since the actual ratio is 3 : 2, so reverse it, i.e., the required ratio is 2 : 3. Now divide the difference of both the speeds i.e., 25 and 40 in the ratio of 2 : 3.

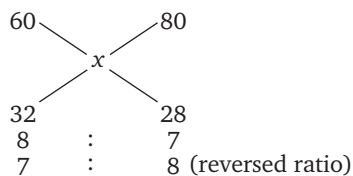
$$\text{i.e., } (40 - 25) \times \frac{2}{5} = 6 \quad \text{and} \quad (40 - 25) \times \frac{3}{5} = 9$$

So, the average speed is $25 + 6 = 31$ or $40 - 9 = 31$

Exp. 9) A milkman has two types of milk. In the first container the percentage of milk is 80% and in the second container the percentage of milk is 60%. If he mixes 28 litres of milk of the first container to the 32 litres of milk of the second container, then the percentage of milk in the mixture is :

- (a) 63.99 (b) 69.33 (c) 72.5 (d) 75.2

Solution



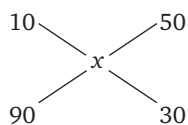
$$\text{So, } (80 - 60) \times \frac{7}{7 + 8} = 20 \times \frac{7}{15} = 9.33$$

Thus, the required percentage = $60 + 9.33 = 69.33$

Exp. 10) Modern electronic shop sold the 30% hardware at the profit of 50% and 90% software at the profit of 10%. The average profit per cent of the Modern electronic shop is, if it sells only these two kinds of things :

- (a) 15 (b) 20 (c) 25 (d) 45

Solution



The actual ratio is 3 : 1. So, the reversed ratio is 1 : 3.

Now divide the difference of 10 and 50 in the ratio of 1 : 3 then add the first part to the 10 or subtract the second part from the 50.

$$\text{i.e., } \text{Required average} = 10 + (50 - 10) \times \frac{1}{4} = 10 + 10 = 20$$

$$\text{or } 50 - (50 - 10) \times \frac{3}{4} = 20$$

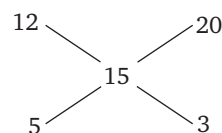
NOTE All the problems discussed above can be solved in any way, i.e., either by using the formula of weighted average or Alligation Equation. But we have emphasised on the graphical (cross proportion) method. Since this technique is quite handy, i.e., some times you need not to calculate on paper.

Exp. 11) Avinash covered 150 km distance in 10 hours. The first part of his journey he covered by car, then he hired a rickshaw. The speed of car and rickshaw is 20 km/hr and 12 km/hr respectively. The ratio of distances covered by car and the rickshaw respectively are :

- (a) 2 : 3 (b) 4 : 5
(c) 1 : 1 (d) none of these

Solution The average speed of Avinash

$$= \frac{150}{10} = 15 \text{ km/hr}$$



It means the rickshaw took 5/8 and car took 3/8 of the total time i.e., the ratio of time taken by rickshaw to car is 5 : 3.

So the ratio of distances covered by rickshaw to car is

$$5 \times 12 : 3 \times 20 \Rightarrow 1 : 1$$

NOTE

1. In this calculation (i.e., in alligation method) distance never involves directly. Only time and speeds are involved.
2. Since we have to find generally the average speed (not the average time) so speeds are written on the top and corresponding time taken is written below.

Exp. 12) A mixture of rice is sold at ₹ 3.00 per kg. This mixture is formed by mixing the rice of ₹ 2.10 and ₹ 2.52 per kg. What is the ratio of price of cheaper to the costlier quality in the mixture if the profit of 25% is being earned.

- (a) 5 : 2 (b) 2 : 7
(c) 2 : 5 (d) 15 : 8

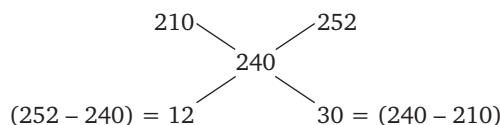
Solution Let the cost price of the mixture be ₹ x per kg, then

$$\text{selling price} = x + \frac{25 \times x}{100} = 3$$

$$\Rightarrow 1.25x = 3$$

$$\Rightarrow x = 2.4$$

Now the average cost price of mixture = ₹ 2.4



Therefore, the ratio of cheaper to costlier rice is 12 : 30, i.e., 2 : 5.

NOTE ₹ 1 = 100 paise. There is no change in the ratio when we change the rupees into paise, just for our convenience in calculation.

Exp. 13) A milkman has 20 litres of milk. If he mixes 5 litres of water, which is freely available, in 20 litres of pure milk. If the cost of pure milk is ₹ 18 per litre, then the profit of the milkman, when he sells all the mixture at cost price, is :

- (a) 20% (b) 25% (c) 33.33% (d) 18%

Solution When the water is freely available and all the water is sold at the price of the milk, then the water gives the profit on the cost of 20 litres of milk.

$$\text{Therefore, profit percentage} = \frac{5}{20} \times 100 = 25\%$$

$$\text{Since, the profit \%} = \frac{\text{profit}}{\text{cost price}} \times 100$$

NOTE Here the milkman cheats his customer by false practise of creating illusion that instead of 20 litres milk, there is 25 litres of milk. So this extra 5 litres of milk (actually water) is the part of profit.

Exp. 14) In what ratio should freely available water and a premium priced wine be mixed so that after selling the mixture at the cost price a profit of 33.33% is made?

- (a) 1 : 4 (b) 1 : 3 (c) 2 : 3 (d) 3 : 4

Solution 33.33% profit means there is one part water and 3 part is pure wine. So the required ratio of water and wine in the mixture is 1 : 3.

NOTE The above problem (No. 14) and the previous problem are basically related to the topics of ratio and profit-loss.

Exp. 15) In what ratio should freely available water be mixed with the wine worth ₹ 60 per litre so that after selling the mixture at ₹ 50 per litre, the profit will be 25%?

- (a) 1 : 2 (b) 2 : 3 (c) 3 : 4 (d) 4 : 5

Solution Selling price = ₹ 50

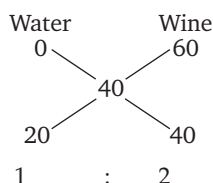
Therefore, the cost price

$$= ₹ 40, \text{ which is the average price.}$$

It means the wine, worth ₹ 60 becomes worth ₹ 40 when the water was mixed in it. So we can conclude that in the mixture of ₹ 60, there is wine worth ₹ 40 and the rest is water.

Therefore, the ratio of water and wine is 20 : 40 i.e., 1 : 2

Alternatively

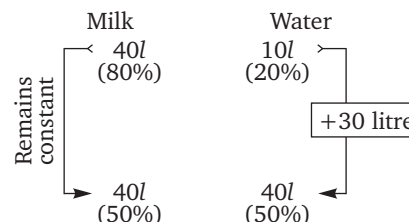


⇒

Exp. 16) A mixture of water and milk contains 80% milk. In 50 litres of such a mixture, how many litres of water is required to increase the percentage of water to, 50%?

- (a) 20
(b) 15
(c) 30
(d) none of the above

Solution Total amount of mixture is 50 litre



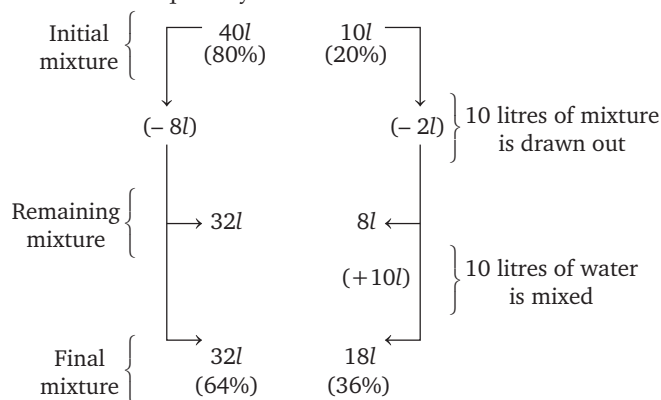
NOTE That, in this process the quantity of milk remains constant but its percentage decreases as the quantity (and percentage) of water increases.

Initially we have 40 litre milk and 10 litre water. Now we are required to have 50% water and remaining 50% milk in the new mixture. So we have to have 40 litre water, equal to the amount of pure milk (which is constant) available in the mixture. Thus we have to add up 30 litre (= 40 - 10) water in the original mixture.

Exp. 17) In a 50 litre mixture of water and milk, water is only 20%. The milkman gives 10 litres 'of this' mixture to a customer and then he adds up 10 litres of pure water in the remaining mixture. The percentage of water in the final mixture is :

- (a) 84% (b) 74%
(c) 26% (d) 36%

Solution Total quantity of mixture = 50 litre



See the chart for the Solution.

Initially there is 40 litres (80%) milk in the 50 litres mixture. When 10 litres of mixture is drawn out, it means 8 litres of milk is drawn out.

Now when the 10 litres of water is added, it means in the 50 litres of mixture, there is only 32 litres of milk, i.e., 64%. Hence the percentage of water is 36%.

Exp. 18) There are three types of milk, Parag, Amul and Nestle. The ratio of fat to the non-fat contents in milk is 4 : 5, 5 : 6, 6 : 7 respectively. If all these three types of milk is mixed in equal quantity, the ratio of fat to the non-fat contents in the mixture will be :

- (a) 1751 : 2110 (b) 175 : 543
(c) 3 : 5 (d) 10 : 18

Solution In this type of questions, we consider only one item (*viz.*, either fat or non-fat) as the fraction of the total quantity. Again we equate the denominators.

Let us consider fraction of 'fat'.

	Parag	Amul	Nestle
Fraction of fat →	$\frac{4}{9}$	$\frac{5}{11}$	$\frac{6}{13}$
	$\frac{4 \times 11 \times 13}{9 \times 11 \times 13}$	$\frac{5 \times 9 \times 13}{11 \times 9 \times 13}$	$\frac{6 \times 9 \times 11}{13 \times 9 \times 11}$
	$\frac{572}{1287}$	$\frac{585}{1287}$	$\frac{594}{1287}$

So the total fat in the total mixture

$$= \frac{572 + 585 + 594}{1287 + 1287 + 1287} = \frac{1751}{3861}$$

Hence, the ratio of fat to the non-fat contents in the mixture

$$= 1751 : 2110 \quad [\because 3861 - 1751 = 2110]$$

Exp. 19) Sharabi Chand purchased two different kinds of alcohol. In the first mixture the ratio of alcohol to water is 3 : 4 and in the second mixture it is 5 : 6. If he mixes the two given mixtures and makes a third mixture of 18 litres in which the ratio of alcohol to water is 4 : 5, the quantity of first mixture (whose ratio is 3 : 4) is required to make the 18 litres of the third kind of mixture is :

- (a) 6 (b) 7 (c) 8 (d) 9

Solution The fraction of alcohol in the different mixtures is as follows :

$$\begin{aligned} \text{First mixture } \frac{3}{7} &= \frac{297}{693} \\ \text{Second mixture } \frac{5}{11} &= \frac{315}{693} \\ \text{Final mixture } \frac{4}{9} &= \frac{308}{693} \end{aligned}$$

Therefore,

$$\begin{array}{ccc} \frac{297}{693} & & \frac{315}{693} \\ & \searrow \quad \swarrow & \\ & \frac{308}{693} & \\ & \swarrow \quad \searrow & \\ \frac{7}{693} & & \frac{11}{693} \end{array}$$

Hence the ratio of first mixture is to second mixture is 7 : 11. Thus he has to mix 7 litres of first type of alcohol to make 18 litres of required mixture.

Exp. 20) Some amount out of ₹ 6000 was lent out at 10% per annum and the rest amount @ at 20% per annum and thus in 4 years the total interest from both the amounts collected was ₹ 3400. What is the amount which was lent out @ 10% per annum?

- (a) ₹ 2500 (b) ₹ 2800
(c) ₹ 3200 (d) ₹ 3500

Solution The total interest of one year

$$= ₹ 850 = \left(\frac{3400}{4} \right)$$

Therefore, the average rate of interest = $14 \frac{1}{6} \% = \frac{85}{6} \%$

Thus,

$$\begin{array}{ccc} \left(\frac{10 \times 6}{6} \right) & & \left(\frac{20 \times 6}{6} \right) \\ & \searrow \quad \swarrow & \\ & \left(\frac{85}{6} \right) & \\ & \swarrow \quad \searrow & \\ \left(\frac{35}{6} \right) & & \left(\frac{25}{6} \right) \end{array}$$

Hence the ratio of amount which is lent @ 10% per annum to the amount lent @ 20% per annum is 35 : 25, *i.e.*, 7 : 5. Therefore, the amount which is lent out @ 10% per annum is ₹ 3500.

Exp. 21) From the 50 litres of milk, 5 litres of milk is taken out and after it 5 litres of water is added to the rest amount of milk. Again 5 litres of mixture of milk and water is drawn out and it was replaced by 5 litres of water. If this process is continued similarly for the third time, the amount of milk left after the third replacement :

- (a) 45 L (b) 36.45 L (c) 40.5 L (d) 42.5 L

Solution

Milk	Water	
50 L	0 L	50 litre (initially)
- 5 L	- 0 L	withdrawn amount
45 L	0 L	50 litre (after first replacement)
+ 0 L	+ 5 L	
- 4.5 L	- 0.5 L	withdrawn amount
40.5 L	4.5 L	50 litre (after second replacement)
+ 0 L	+ 5 L	
- 4.05 L	- 0.95 L	withdrawn amount
36.45 L	8.55 L	50 litre (after third replacement)
	+ 5 L	

Shortcut
$$50 \times \left(1 - \frac{5}{50} \right) \times \left(1 - \frac{5}{50} \right) \times \left(1 - \frac{5}{50} \right)$$

$$= 50 \times \frac{45}{50} \times \frac{45}{50} \times \frac{45}{50}$$

$$= 50 \times \left(\frac{45}{50} \right)^3$$

$$= 50 \times \left(\frac{9}{10} \right)^3 = 36.45 \text{ L}$$

General Formula

Final or reduced concentration

= Initial concentration

$$\left(1 - \frac{\text{amount being replaced in each operation}}{\text{total amount}}\right)^n$$

where n is the number of times the same operation is being repeated. The "amount being replaced" could be pure or mixture as per the case. Similarly, "total amount" could also be either pure or mixture. Here amount being replaced denotes the quantity which is to be withdrawn in each time.

Exp. 22) From a tank of petrol, which contains 200 litres of petrol, the seller replaces each time with kerosene when he sells 40 litres of petrol (or its mixture). Everytime he sells out only 40 litres of petrol (pure or impure). After replacing the petrol with kerosene 4th time, the total amount of kerosene in the mixture is :

- (a) 81.92 l (b) 96 l
(c) 118.08 l (d) none of these

Solution The amount of petrol left after 4 operations

$$= 200 \times \left(1 - \frac{40}{200}\right)^4 = 200 \times \left(\frac{4}{5}\right)^4$$

$$= 200 \times \frac{256}{625} = 81.92 \text{ litres}$$

Hence the amount of kerosene = $200 - 81.92 = 118.08$ litres

Exp. 23) From a container of wine, a thief has stolen 15 litres of wine and replaced it with same quantity of water. He again repeated the same process. Thus in three attempts the ratio of wine and water became 343 : 169. The initial amount of wine in the container was :

- (a) 75 litres (b) 100 litres
(c) 150 litres (d) 120 litres

Solution $\frac{\text{wine (left)}}{\text{water (added)}} = \frac{343}{169}$

It means $\frac{\text{wine (left)}}{\text{wine (initial amount)}} = \frac{343}{512}$ ($\because 343 + 169 = 512$)

Thus, $343x = 512x \left(1 - \frac{15}{K}\right)^3$

$$\Rightarrow \frac{343}{512} = \left(\frac{7}{8}\right)^3 = \left(1 - \frac{15}{K}\right)^3$$

$$\Rightarrow \left(1 - \frac{15}{K}\right) = \frac{7}{8} = \left(1 - \frac{1}{8}\right)$$

$$\Rightarrow K = 120$$

Thus the initial amount of wine was 120 litres.

Exp. 24) A jar was full with honey. A person used to draw out 20% of the honey from the jar and replaced it with sugar solution. He has repeated the same process 4 times and thus there was only 512 gm of honey left in the jar, the rest part of the jar was filled with the sugar solution. The initial amount of honey in the jar was :

- (a) 1.25 kg (b) 1 kg
(c) 1.5 kg (d) none of these

Solution Let the initial amount of honey in the jar was K , then

$$512 = K \left(1 - \frac{1}{5}\right)^4 \quad \left(\because 20\% = \frac{20}{100} = \frac{1}{5}\right)$$

or $512 = K \left(\frac{4}{5}\right)^4$

$$\Rightarrow K = \frac{512 \times 625}{256}$$

$$\therefore K = 1250$$

Hence, initially the honey in the jar = 1.25 kg.

CAT-Test

Questions Helping you bell the CAT

- 1 How much Pepsi at ₹ 6 a litre is added to 15 litre of 'dew' at ₹ 10 a litre so that the price of the mixture be ₹ 9 a litre?
(a) 5 (b) 8
(c) 10 (d) none of these
- 2 In a municipal parking there are some two wheelers and rest are 4 wheelers. If wheels are counted, there are total 520 wheels but the incharge of the parking told me that there are only 175 vehicles. If no vehicle has a stepney, then the no. of two wheelers is :
(a) 75 (b) 100 (c) 90 (d) 85
- 3 In my pocket there are ₹ 25 consisting of only the denominations of 20 paise and 50 paise. Thus there are total 80 coins in my pocket. The no. of coins of the denomination of 50 paise is :
(a) 30 (b) 70 (c) 50 (d) 25
- 4 There are some shepherds and their sheep in a grazing field. The no. of total heads are 60 and total legs are 168 including both men and sheep. The no. of sheep is :
(a) 18 (b) 26 (c) 24 (d) 36
- 5 In the 75 litres of mixture of milk and water, the ratio of milk and water is 4 : 1. The quantity of water required to make the ratio of milk and water 3 : 1 is :
(a) 1 litre (b) 3 litres (c) 4 litres (d) 5 litres
- 6 In my office the average age of all the female employees is 21 years and that of male employees is 32 years, where the average age of all the (male and female) employees is 28 years. The total no. of employees in my office could be :
(a) 35 (b) 78 (c) 231 (d) 90
- 7 A car agency has 108 cars. He sold some cars at 9% profit and rest at 36% profit. Thus he gains 17% on the sale of all his cars. The no. of cars sold at 36% profit is :
(a) 25 (b) 32 (c) 35 (d) 75
- 8 ₹ 69 were divided among 115 students so that each girl gets 50 paise less than a boy. Thus each boy received twice the paise as each girl received. The no. of girls in the class is :
(a) 92 (b) 42 (c) 33 (d) 23
- 9 In what proportion water be mixed with spirit to gain 12.5% by selling it at cost price?
(a) 3 : 5 (b) 1 : 8 (c) 2 : 7 (d) 1 : 9
- 10 A butler stole wine from a butt of sherry containing 50% of spirit, then he replenished it by different wine containing 20% spirit. Thus there was only 30% strength (spirit) in the new mixture. How much of the original wine did he steal?
(a) 1/3 (b) 2/3 (c) 1/2 (d) 1/4
- 11 Mr. Mittal purchased two steel factories, one in India and other one in Malaysia for total ₹ 72 crores. Later on he sold the Indian factory at 16% profit and Malasian factory at 24% profit. Thus he gained a total profit of 19%. The selling price of Indian factory is :
(a) 45 crore (b) 52.2 crore
(c) 8.55 crore (d) can not be determined
- 12 In a 25 litre mixture of milk and water, the water is only 20%. How many litres of water is required to increase the percentage of water to 90%?
(a) 45 litre (b) 70 litre
(c) 115 litre (d) 175 litre
- 13 A milkman sells the milk at the cost price but he mixes the water (freely available) in it and thus he gains 9.09%. The quantity of water in the mixture of 1 litre is :
(a) 83.33 mL (b) 90.90 mL
(c) 99.09 mL (d) can't be determined
- 14 The price of petrol is ₹ 60 per litre and the price of spirit is ₹ 40 per litre. In what ratio the petrol and spirit be mixed such that the profit after selling the mixture at ₹ 75 per litre be 25%?
(a) 1 : 1
(b) 3 : 2
(c) 5 : 1
(d) such a mixture is not possible
- 15 A trader sells total 315 TV sets. He sells black and white TV sets at a loss of 6% and colour TV sets at a profit of 15%. Thus he gains 9% on the whole. The no. of B/W TV sets, which he has sold, is :
(a) 126 (b) 216 (c) 135 (d) 90
- 16 In a class of 30 students, the average weight of boys is 20 kg and the average weight of the girls is 25 kg. The fraction of boys out of the total students of the class is :
(a) $\frac{4}{5}$ (b) $\frac{5}{6}$
(c) $\frac{3}{4}$ (d) data insufficient
- 17 Baniya sells two types of tea viz. Desi Chai and Videshi Chai. He sells Desi Chai at ₹ 18 per kg and incurs a loss of 10% whereas on selling the Videshi Chai at ₹ 30 per kg, he gains 20%. In what proportion should the Desi Chai and Videshi Chai be mixed such that he can gain a profit of 25% by selling the mixture at ₹ 27.5 per kg?
(a) 3 : 2 (b) 2 : 3 (c) 2 : 5 (d) 3 : 5

- 18** The average age of boys in a class is 16.66, while the average age of girls is 18.75. Thus the average age of all the 40 students of the class is 17.5. If the difference between the no. of boys and girls is 8, then the no. of girls in the class is :
 (a) 12 (b) 16
 (c) 18 (d) Data insufficient
- 19** The ratio of water and alcohol in two different containers is 2 : 3 and 4 : 5. In what ratio we are required to mix the mixtures of two containers in order to get the new mixture in which the ratio of alcohol and water be 7 : 5 ?
 (a) 7 : 3 (b) 5 : 3 (c) 8 : 5 (d) 2 : 7
- 20** The average marks of the students in four sections A, B, C and D together is 60%. The average marks of the students of A, B, C and D individually are 45%, 50%, 72% and 80% respectively. If the average marks of the students of sections A and B together is 48% and that of the students of B and C together is 60%. What is the ratio of number of students in sections A and D?
 (a) 2 : 3 (b) 4 : 3 (c) 5 : 3 (d) 3 : 5
- 21** The diluted wine contains only 8 litres of wine and the rest is water. A new mixture whose concentration is 30%, is to be formed by replacing wine. How many litres of mixture shall be replaced with pure wine if there was initially 32 litres of water in the mixture ?
 (a) 4 (b) 5
 (c) 8 (d) none of these
- 22** The average weight of boys in a class is 30 kg and the average weight of girls in the same class is 20 kg. If the average weight of the whole class is 23.25 kg, what could be the possible strength of boys and girls respectively in the same class ?
 (a) 14 and 26 (b) 13 and 27
 (c) 17 and 27 (d) none of these
- 23** The shopkeeper mixed 40 kg refined oil with vegetable oil worth ₹ 60 per kg. Thus he gains ₹ 10 after selling the mixture of the two oils. The price of the first oil is :
 (a) 20 (b) 25
 (c) 45 (d) can't be determined
- 24.** In a mixture of milk and water, there is only 26% water. After replacing the mixture with 7 litres of pure milk, the percentage of milk in the mixture become 76%. The quantity of mixture is :
 (a) 65 litre (b) 91 litre
 (c) 38 litre (d) none of these
- 25** The ratio of expenditure and savings is 3 : 2. If the income increases by 15% and the savings increases by 6%, then by how much per cent should his expenditure increases?
 (a) 25 (b) 21
 (c) 12 (d) 24
- 26** 4 kg of a metal contains $\frac{1}{5}$ copper and rest is zinc. Another 5 kg of metal contains $\frac{1}{6}$ copper and rest is zinc. The ratio of copper and zinc into the mixture of these two metals :
 (a) 49 : 221 (b) 39 : 231
 (c) 94 : 181 (d) none of these
- 27** 450 litres of a mixture of milk and water contain the milk and water in the ratio 9 : 1. How much water should be added to get a new mixture containing milk and water in the ratio 3 : 1?
 (a) 54 (b) 90
 (c) 45 (d) 63
- 28** The ratio of petrol and kerosene in the container is 3 : 2 when 10 litres of the mixture is taken out and is replaced by the kerosene, the ratio becomes 2 : 3. The total quantity of the mixture in the container is :
 (a) 25 (b) 30
 (c) 45 (d) cannot be determined
- 29** From a container, 6 litres milk was drawn out and was replaced by water. Again 6 litres of mixture was drawn out and was replaced by the water. Thus the quantity of milk and water in the container after these two operations is 9 : 16. The quantity of mixture is :
 (a) 15 (b) 16
 (c) 25 (d) 31
- 30** A milkman brings 100 litres pure milk from a dairy farmer and he sells 10 litres of it to the first customer, then he refills his vessel by adding 10 litres water. After this, he proceeds to the next house and sells 10 litres of it to the second customer and then he refills his vessel again by adding 10 litres of water. Thus, every time he sells 10 litres of milk - pure or impure - he keeps on replacing it with 10 litres of pure water.
 Maximum how many customers can get at least 50% milk in the mixture that they purchase from this milkman?
 (a) 5 (b) 6
 (c) 7 (d) None of these

Answers

1. (a)	2. (c)	3. (a)	4. (c)	5. (d)	6. (c)	7. (b)	8. (a)	9. (b)	10. (b)
11. (b)	12. (d)	13. (a)	14. (d)	15. (d)	16. (d)	17. (a)	18. (b)	19. (b)	20. (b)
21. (b)	22. (b)	23. (d)	24. (b)	25. (b)	26. (a)	27. (b)	28. (b)	29. (a)	30. (c)

Hints & Solutions

- 1 Let x litre pepsi is required.

$$\begin{array}{ccc} 6 & & 10 \\ & \searrow \quad \swarrow & \\ & 9 & \\ & \swarrow \quad \searrow & \\ x & & 15 \\ (10-9) = 1 & : & 3 = (9-6) \end{array}$$

Therefore $\frac{x}{15} = \frac{1}{3} \Rightarrow x = 5$ litre

Alternatively Go through options.

- 2 Go through options : $90 \times 2 + 85 \times 4 = 520$

If 2 wheelers be 90 then the four wheelers will be
 $85 = (175 - 90)$

Alternatively The average number of wheels per

vehicle = $\frac{520}{175}$

$$\begin{array}{ccc} 2 \times \frac{175}{175} & & 4 \times \frac{175}{175} \\ & \searrow \quad \swarrow & \\ & \frac{520}{175} & \end{array}$$

$$\begin{array}{ccc} \frac{180}{175} & & \frac{170}{175} \end{array}$$

$$\begin{array}{ccc} \Rightarrow & 180 & : & 170 \\ \Rightarrow & 90 & : & 85 \\ \Rightarrow & 18 & : & 17 \end{array}$$

Therefore the ratio of two wheelers to four wheelers is 18 : 17. Hence there are 90 "two wheelers".

- 3 Go through options : $30 \times 50 + 50 \times 20 = 2500$ paise

Alternatively Since the average price of a coin

$$= \frac{2500}{80} = 31.25 \text{ paise}$$

$$\begin{array}{ccc} 20 & & 50 \\ & \searrow \quad \swarrow & \\ & 31.25 & \\ & \swarrow \quad \searrow & \\ 18.75 & & 11.25 \end{array}$$

So the ratio of no. of 20 paise coins to the no. of 50 paise coins = $18.75 : 11.25 = 75 : 45 = 5 : 3$

Therefore, the no. of coins of the denominations of 50 paise is 30.

- 4 Go through option : $24 \times 4 + 36 \times 2 = 168$

Alternatively

$$\begin{array}{ccc} 2 & & 4 \\ & \searrow \quad \swarrow & \\ & 2.8 & \\ & \swarrow \quad \searrow & \\ 1.2 & & 0.8 \end{array}$$

$$\Rightarrow \quad \quad \quad 3 \quad : \quad 2$$

Therefore, the ratio of men and sheep is 3 : 2.

Alternatively Suppose there are only men, then the no. of legs = $60 \times 2 = 120$.

Now since there are $48 = (168 - 120)$ legs extra, it means there are $24 = \left(\frac{48}{2}\right)$ sheep, since a sheep has 2 extra legs than a man has.

- 5 Total quantity of mixture = 75 litre

Therefore

$$\begin{array}{ccc} \text{Milk} & & \text{Water} \\ 4 & : & 1 \\ \swarrow & & \swarrow \\ 60 \text{ L} & & 15 \text{ L} \\ \downarrow & & \downarrow \\ 60 \text{ L} & : & 20 \text{ L} \\ 3 & : & 1 \end{array}$$

(+5L)

- 6 Since the ratio of no. of female and male employees is 4 : 7, so the total no. of employees must be the multiples of 11. Hence the possible answer is 231.

$$\begin{array}{ccc} 21 & & 32 \\ & \searrow \quad \swarrow & \\ & 28 & \\ & \swarrow \quad \searrow & \\ 4 & & 7 \end{array}$$

- 7 Since the ratio of cars sold at profit of 9% to the 36% is 19 : 8, hence the no. of cars sold at 36% profit is 32.

$$\begin{array}{ccc} 9 & & 36 \\ & \searrow \quad \swarrow & \\ & 17 & \\ & \swarrow \quad \searrow & \\ 19 & & 8 \end{array}$$

- 8 Here each girl receives 50 paise and each boy receives 100 paise and the average receiving of each student

$$= \frac{6900}{115} = 60 \text{ paise}$$

$$\begin{array}{ccc} 50 & & 100 \\ & \searrow \quad \swarrow & \\ & 60 & \\ & \swarrow \quad \searrow & \\ 40 & & 10 \end{array}$$

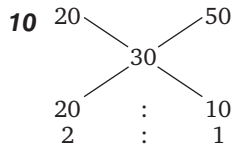
$$\Rightarrow \quad \quad \quad (\text{G}) 4 \quad : \quad (\text{B}) 1$$

Thus the no. of girls = 92 [Number of boys = 23]

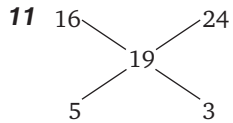
- 9 Profit = $12.5\% = \frac{1}{8}$.

Hence the ratio of water to spirit is 1 : 8

Since profit % = $\frac{\text{profit}}{\text{cost}} \times 100$.

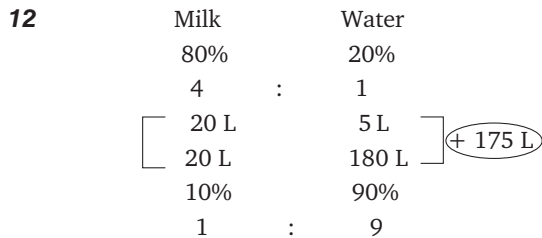


Since the ratio of 20% wine to 50% wine is 2 : 1, it means there is $\frac{2}{3}$ wine which is replaced with wine in which the concentration of spirit is 20%.



Thus the cost price of Indian factory is ₹ 45 crore.
Therefore, the selling price of Indian factory is

$$= 45 + \frac{45 \times 16}{100} = 52.2 \text{ crore}$$

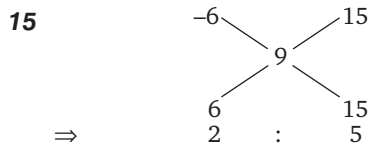


13 Profit (%) = $9.09\% = \frac{1}{11}$

Since the ratio of water and milk is 1 : 11,
Therefore the ratio of water is to mixture = 1 : 12
Thus the quantity of water in mixture of 1 litre
 $= 1000 \times \frac{1}{12} = 83.33 \text{ mL}$

14 The selling price of mixture = ₹ 75
∴ The cost price of mixture = ₹ 60

Now we know that if he mixes the spirit (worth ₹ 40) with petrol (worth ₹ 60), the cost price of mixture must be less than ₹ 60, which is impossible. Hence there is no spirit with the petrol.



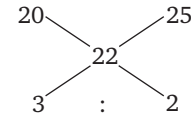
⇒ Thus the ratio of B/W TV sets to the no. of colour TV sets = 2 : 5

Therefore, no. of B/W TV sets = 90

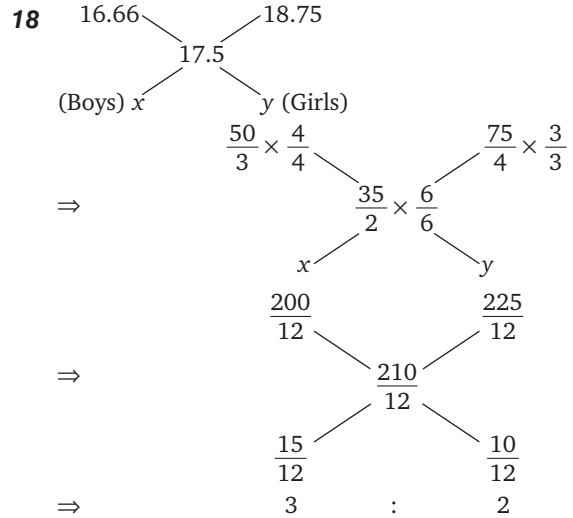
16 Since we do not know either the average weight of the whole class or the ratio of no. of boys to girls.

17 The S.P. of Desi Chai = ₹ 18
The S.P. of Videshi Chai = ₹ 30
The C.P. of Desi Chai = ₹ 20
The C.P. of Videshi Chai = ₹ 25
The S.P. of mixture = ₹ 27.5

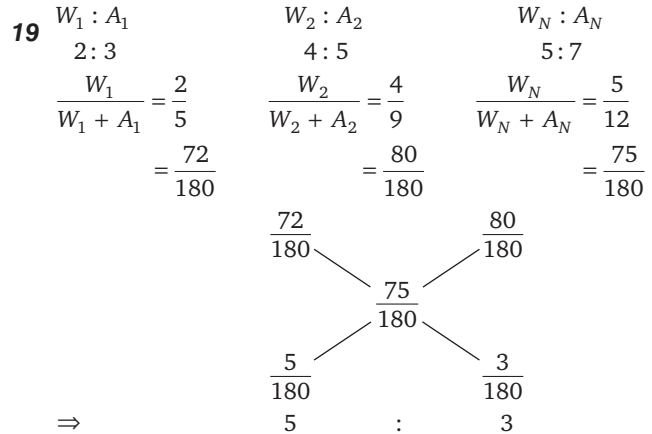
The C.P. of mixture = ₹ 22



Therefore, the ratio of Desi Chai is to Videshi Chai is 3 : 2

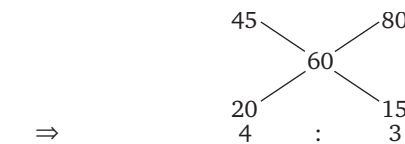


Thus the no. of girls = 16 and no. of boys = 24

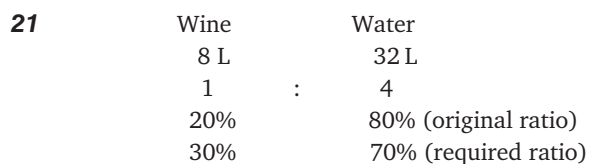


Therefore, the ratio is 5 : 3

20 Since the average marks of sections B and C together are equal the average marks of all the four sections (i.e., A, B, C and D), therefore the average marks of the remaining two sections A and D together will also be equal i.e., 60%.



⇒ Hence, the required ratio is 4 : 3.



In this case, the percentage of water being reduced when the mixture is being replaced with wine.

So the ratio of **left quantity** to the initial quantity is 7 : 8.

$$\text{Therefore } \frac{7}{8} = \left[1 - \frac{K}{40}\right] \Rightarrow \frac{7}{8} = \left[\frac{40 - K}{40}\right]$$

$$\Rightarrow K = 5 \text{ litre}$$

☞ **Alternatively** Go through options.

- 22 Therefore no. of boys : Number of girls = 13 : 27

$$\begin{array}{ccc} 20 & & 30 \\ & \searrow \quad \swarrow & \\ & 23.25 & \\ & \swarrow \quad \searrow & \\ 6.75 & & 3.25 \\ 27 & : & 13 \end{array}$$

- 23 Since there is insufficient data.

Milk	Water	
74%	26%	(initially)
76%	24%	(after replacement)

$$\text{Left amount} = \text{Initial amount} \left(1 - \frac{\text{replaced amount}}{\text{total amount}}\right)$$

$$24 = 26 \left(1 - \frac{7}{K}\right)$$

$$\Rightarrow \frac{12}{13} = \left(1 - \frac{7}{K}\right)$$

$$\Rightarrow \frac{1}{13} = \frac{7}{K}$$

$$\Rightarrow K = 91 \text{ litre}$$

☞ **NOTE** In case of replacement, the initial (pure) amount is equal to the amount of final mixture.

25

$$\begin{array}{ccc} 6 & & x \\ & \searrow \quad \swarrow & \\ & 15 & \\ & \swarrow \quad \searrow & \\ x-15 = 2k & & 3k = 9 \end{array}$$

Therefore $x = 21\%$.

- 26 Copper in 4 kg = $\frac{4}{5}$ kg

$$\text{and Zinc in 4 kg} = 4 \times \frac{4}{5} = \frac{16}{5} \text{ kg}$$

$$\text{Copper in 5 kg} = 5 \times \frac{1}{6} = \frac{5}{6} \text{ kg}$$

$$\text{Zinc in 5 kg} = 5 \times \frac{5}{6} = \frac{25}{6} \text{ kg}$$

$$\text{Therefore, Copper in mixture} = \frac{4}{5} + \frac{5}{6} = \frac{49}{30} \text{ kg}$$

$$\text{and Zinc in the mixture} = \frac{16}{5} + \frac{25}{6} = \frac{221}{30} \text{ kg}$$

Therefore, the required ratio = 49 : 221.

27

Milk	:	Water
9	:	1
405 L	:	45 L
		(+90L)
405 L	:	135 L
3	:	1

28. Petrol : Kerosene

$$3 : 2 \text{ (initially)}$$

$$2 : 3 \text{ (after replacement)}$$

$$\frac{\text{Remaining (or left) quantity}}{\text{Initial quantity}} = \left(1 - \frac{\text{replaced quantity}}{\text{total quantity}}\right)$$

$$\text{(for petrol)} \quad \frac{2}{3} = \left(1 - \frac{10}{K}\right) \Rightarrow \frac{1}{3} = \frac{10}{K}$$

$$\Rightarrow K = 30 \text{ litre}$$

Therefore, the total quantity of the mixture in the container is 30 litre.

29. $\frac{9}{25} = \left(1 - \frac{6}{K}\right)^2 \Rightarrow \frac{3}{5} = \left(1 - \frac{6}{K}\right)$

$$\Rightarrow K = 15 \text{ litre}$$

30. After the first customer, who gets absolutely pure milk, every next customer gets less pure milk than the previous customer. The purity of milk for the n th customer who receives only impure milk can be calculated as following.

$$50 \geq 100 \left(1 - \frac{10}{100}\right)^n$$

The highest value of n that satisfies the above relation is 6.

Therefore, we have 6 customers who get milk, which is less than 100% pure but more than 50% pure. However, there is one customer who gets 100% pure milk. So, there are actually 7 customers who get more than 50% pure milk.

Hence, choice (c) is the correct one.

☞ **Alternatively** We can do it manually, if we are really good at calculation.

The First customer gets 100% pure milk.

The Second customer gets 90% pure milk.

The Third customer gets 81% pure milk.

The Fourth customer gets 72.9% pure milk.

The Fifth customer gets 65.61% pure milk.

The Sixth customer gets 59.05% pure milk.

The Seventh customer gets 53.14% pure milk.

The eighth customer gets 47.83% pure milk.

Thus, we see that maximum 7 customers get at least 50% pure milk.

CHAPTER 04

Ratio, Proportion & Variation

The basic applications of the concepts involved in this chapter are comparisons of two or more quantities and changes in their magnitudes, *e.g.*, comparison of the ages, weights income, savings, heights, volume, density, temperature etc. So this chapter is very useful in solving the problems of Data interpretation. Also each and every year one or two problems from this chapter is/are asked in CAT, either directly or application based for QA section. Last but not the least, the concepts of ratio, proportion and variations are very useful in solving most of the arithmetic problems. That's why these problems are usually asked in most of the competitive exams like CAT, XAT, IIFT, CMAT, GMAT, SSC, CGL and Bank PO.

4.1 Ratio

The comparison between two quantities in terms of magnitude is called the ratio, *i.e.*, it tells us that the one quantity is how many times the other quantity.

For example, Amit has 5 pens and Sarita has 3 pens. It means the ratio of number of pens between Amit and Sarita is 5 is to 3. It can be expressed as '5 : 3'.

 **NOTE** It should be noted that in a ratio, the order of the terms is very important. For example, in the above illustration the required ratio is 5 : 3 while 3 : 5 is wrong.

So the ratio of any two quantities is expressed as $\frac{a}{b}$ or $a : b$.

The numerator ' a ' is called the **antecedent** and denominator ' b ' is called as **consequent**.

Rule of Ratio

The comparison of two quantities is meaningless if they are not of the same kind or in the same units (of length, volume or currency etc). We do not compare 8 boys and 6 cows or 15 litres and 5 toys or 5 metres and 25 centimetres. Therefore, to find the ratio of two quantities (of the same kind), it is necessary to express them in same units.

 **NOTE**

1. We do not compare 8 boys and 6 cows, but we can compare the number (8) of boys and number (6) of cows. Similarly, we cannot compare the number (15) of litres and the number (5) of toys etc.
2. Ratio has no units.

Chapter Checklist

- Ratio
- Proportion
- Variation
- Problems Based on Ages
- Partnership
- CAT Test

Properties of Ratios

1. The value of a ratio does not change when the numerator and denominator both are multiplied by same quantities

$$\text{i.e., } \frac{a}{b} = \frac{ka}{kb} = \frac{la}{lb} = \frac{ma}{mb} \text{ etc.}$$

$$\text{e.g., } \frac{3}{4} = \frac{6}{8} = \frac{9}{12} \dots \text{ etc. have the same ratio.}$$

2. The value of a ratio does not alter (or change) when the numerator and denominator both are divided by same quantities i.e.,

$$\frac{a}{b} = \frac{a/k}{b/k} = \frac{a/l}{b/l} = \frac{a/m}{b/m} \text{ etc.}$$

$$\text{e.g., } \frac{3}{4} = \frac{3/2}{4/2} = \frac{3/3}{4/3} = \frac{3/4}{4/4} \dots \text{ etc are same ratio.}$$

3. The ratio of two fractions can be expressed in ratio of integers. e.g.,

$$\frac{3/4}{5/4} = \frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$$

4. When two or more than two ratios are multiplied with each other, then it is called as compounded ratio e.g.,

$$\frac{2}{3} \times \frac{4}{5} \times \frac{6}{7} = \frac{16}{35} \text{ is the compounded ratio of } \frac{2}{3}, \frac{4}{5}, \frac{6}{7}$$

$$\text{So, } \frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} \dots = \frac{k}{l} \quad (\text{compounded ratio})$$

5. When the ratio is compounded with itself, it is called as duplicate, triplicate ratios etc. e.g.,

$$\frac{a}{b} \times \frac{a}{b} = \frac{a^2}{b^2} = \left(\frac{a}{b}\right)^2 \text{ is called as duplicate ratio of } \frac{a}{b} \text{ and}$$

$$\frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} = \left(\frac{a}{b}\right)^3 \text{ is called as triplicate ratio of } \frac{a}{b}.$$

$$\text{Similarly, } \sqrt{\left(\frac{a}{b}\right)} = \left(\frac{a}{b}\right)^{1/2} \text{ is called as sub-duplicate}$$

$$\text{ratio and } \sqrt[3]{\left(\frac{a}{b}\right)} = \left(\frac{a}{b}\right)^{1/3} \text{ is called as sub-triplicate}$$

$$\text{ratio of } \frac{a}{b}.$$

6. $\frac{a}{b} = \frac{c+am}{d+bm}$ if and only if $\frac{c}{d} = \frac{a}{b}$

$$\text{i.e., } \frac{30}{40} = \frac{30+3m}{40+4m} = \frac{33}{44} = \frac{36}{48} \dots \text{ etc at } m=1, 2, \dots$$

This property is very useful when we compare two fractions e.g., to compare between $\frac{10}{20}$ and $\frac{12}{22}$

We see that $10/20 = 1/2$

$$\text{Now if } \frac{10}{20} \text{ and } \frac{12}{22} \text{ are equal then } \frac{12}{22} = \frac{10+m}{22+2m}$$

Now putting $m=2$, we don't get the required fraction.

$$\text{So } \frac{12}{22} \neq \frac{10}{20} \text{ for any value of } m, \frac{12}{22} \neq \frac{10}{20} \text{ (in terms of ratio)}$$

$$7. \frac{a+k}{b+k} < \frac{a}{b} \text{ if for every positive } k, \frac{a}{b} > 1 \text{ and } \frac{a-k}{b-k} > \frac{a}{b}$$

$$8. \frac{a+k}{b+k} > \frac{a}{b} \text{ if for every positive } k, \frac{a}{b} < 1 \text{ and } \frac{a-k}{b-k} < \frac{a}{b}$$

$$9. \frac{a+c}{b+d} > \frac{a}{b} \text{ if } \frac{c}{d} > \frac{a}{b} \quad 10. \frac{a+c}{b+d} < \frac{a}{b} \text{ if } \frac{c}{d} < \frac{a}{b}$$

$$11. \text{ If } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h} = \dots = k \text{ then } \frac{a+c+e+g+\dots}{b+d+f+h+\dots} = k$$

$$12. \text{ Let } \frac{a}{b}, \frac{c}{d}, \frac{e}{f}, \frac{g}{h} \dots \text{ be some different ratios, then the}$$

$$\text{value of } \left(\frac{a+c+e+g+\dots}{b+d+f+h+\dots} \right) \text{ must lie between the}$$

lowest and highest ratios.

13. If $a:b$ and $b:c$ are given, then

$$a:b:c = \frac{a}{b} \times \frac{b}{c} = (a \cdot b):(b \cdot b):(b \cdot c)$$

14. If the ratios between $a:b, b:c, c:d, d:e$ are given individually, then the combined ratio of $a:b:c:d:e$.

$$\begin{array}{cccccc} a:b & a & b & b & b & b \\ b:c & b & b & c & c & c \\ c:d & c & c & c & d & d \\ d:e & d & d & d & d & e \end{array}$$

$$\text{i.e., } a:b:c:d:e = (a \cdot b \cdot c \cdot d):(b \cdot b \cdot c \cdot d):(b \cdot c \cdot c \cdot d):(b \cdot c \cdot d \cdot d):(b \cdot c \cdot d \cdot e)$$

Exp. 1) Find the ratio of 25 to 40.

$$\text{Solution } \frac{25}{40} = \frac{5 \times 5}{5 \times 8} = \frac{5}{8}$$

NOTE To get the ratio, we rationalize the fractions by cancelling out the common factors of numerator and denominator.

Exp. 2) Find the ratio of 90 cm to 1.5 m.

Solution 1.5 m = 150 cm (units must be same)

$$\text{So, the required ratio} = \frac{90}{150} = \frac{3}{5}$$

Exp. 3) The number of boys and girls in a school are 576 and 480 respectively. Express the ratio of the number of boys to that of girls in the simplest form.

$$\text{Solution Required ratio} = \frac{576}{480} = \frac{6}{5} \quad \left(\because \frac{576}{480} = \frac{96 \times 6}{96 \times 5} \right)$$

Exp. 4) Shukla earns ₹ 14000 per month and Mishra earns ₹ 18000 per month. Find the ratio of Shukla's salary to Mishra's salary.

$$\text{Solution Required ratio} = \frac{14,000}{18,000} = \frac{7}{9}$$

Exp. 5) Out of 144 persons working in an office, 56 are men and the remaining are women. Find the ratio of number of women to number of men.

Solution Ratio = $\frac{88}{56} = \frac{11}{7}$ ($\because$ number of women = $144 - 56$)

Exp. 6) In a club having 100 members, 20 play carrom, 24 play table-tennis and 16 play cricket and the remaining members do not play any game. No member plays more than one game. Find the ratio of the number of members who play.

- Carrom to the number of those who play table-tennis.
- Cricket to the number of those who play carrom.
- Cricket to the number of those who do not play any game.
- Table-tennis to the number of those who do not play any game.
- Some game to the number of those who do not play any game.

Solution Total members = 100

Carrom = 20

Table-tennis = 24

Cricket = 16

No any game = 40

$$(a) \frac{20}{24} = \frac{5}{6} \quad (b) \frac{16}{20} = \frac{4}{5} \quad (c) \frac{16}{40} = \frac{2}{5} \quad (d) \frac{24}{40} = \frac{3}{5}$$

$$(e) \frac{60}{40} = \frac{3}{2}$$

Exp. 7) A person earns ₹ 1200 per day and spends ₹ 800. Find the ratio of his savings to expenditure.

Solution $\frac{400}{800} = \frac{1}{2}$ (Savings = Income – Expenditure)
= 1 : 2

Exp. 8) Simplify the following ratios :

$$(a) \frac{1}{6} : \frac{1}{8} \quad (b) 2\frac{1}{3} : 3\frac{1}{2} \quad (c) \frac{5}{6} : \frac{3}{8} : 3\frac{3}{4}$$

Solution (a) $\frac{1}{6} : \frac{1}{8} = \frac{1/6}{1/8} = \frac{1}{6} \times \frac{8}{1} = \frac{4}{3}$ or 4 : 3

☞ **Alternatively** $\frac{1}{6} : \frac{1}{8} = \frac{48}{6} : \frac{48}{8} \quad \left(\text{or } \frac{24}{6} : \frac{24}{8} \right)$
= 8 : 6 = 4 : 3

☞ **NOTE** In case of fractions, convert them to whole numbers by multiplying each term by the LCM of their denominators.

$$(b) 2\frac{1}{3} : 3\frac{1}{2} = \frac{7}{3} : \frac{7}{2} = \frac{7/3}{7/2} = \frac{2}{3} \text{ or } 2 : 3$$

☞ **NOTE** In case of two fractions if numerators are same, then the required ratio is the inverse ratio of the fractions.

$$(c) \frac{5}{6} : \frac{3}{8} : 3\frac{3}{4} = \frac{5}{6} : \frac{3}{8} : \frac{15}{4} = \frac{5}{6} \times 24 : \frac{3}{8} \times 24 : \frac{15}{4} \times 24$$

$$= 20 : 9 : 90 \quad (\because \text{LCM of 6, 8 and 4 is 24})$$

Exp. 9) Divide 14 toffees among Ankita and Anshul in the ratio 5 : 2.

Solution According to the question, if there are 7 toffees then Ankita will have 5 and Anshul will have 2 toffees but since there are 14 toffees, which is twice of 7. So Ankita will have 10 toffees and Anshul will have 4 toffees.

Shortcut

Ankita	:	Anshul
5	:	2
$14 \times \frac{5}{(5+2)}$	:	$14 \times \frac{2}{(5+2)}$
10	:	4

Exp. 10) Three boys are aged 2 years, 4 years and 8 years. They want to divide seventy rupees in the ratio of their ages. How much money would each get?

Solution The ratio of their ages = A : B : C = 2 : 4 : 8 = 1 : 2 : 4

A	:	B	:	C
1	:	2	:	4
$70 \times \frac{1}{1+2+4}$	:	$70 \times \frac{2}{1+2+4}$	:	$70 \times \frac{4}{1+2+4}$
10	:	20	:	40

So A gets ₹ 10, B gets ₹ 20 and C gets ₹ 40 if the ratio of their ages is 2 : 4 : 8.

Exp. 11) An amount of ₹ 100 is being divided among two persons in the ratio $\frac{1}{10} : \frac{1}{15}$. How much money does each get?

Solution $\frac{1}{10} : \frac{1}{15} = \frac{1}{10} \times 30 : \frac{1}{15} \times 30 = 3 : 2$ (here, 30 is the LCM of 10 and 15). So, the ratio of amount of money = $\frac{3}{5} \times 100 : \frac{2}{5} \times 100 = 60$ and 40

Exp. 12) The lengths of sides of a triangle are in the ratio 2 : 3 : 4. If the perimeter of the triangle is 63 cm, find the lengths of the sides of the triangle.

Solution Let the sides of triangle be 2x, 3x and 4x, then $2x + 3x + 4x = 63 \Rightarrow 9x = 63 \Rightarrow x = 7$

$\therefore$ The sides of triangle = 2x, 3x, 4x = 14, 21, 28

☞ **Alternatively** $63 \times \frac{2}{(2+3+4)}$ and $63 \times \frac{3}{(2+3+4)}$
and $63 \times \frac{4}{(2+3+4)} = 14, 21 \text{ and } 28.$

Exp. 13) Divide 1224 into three parts such that first part be double that of second part and second part be $\frac{1}{3}$ of the third part.

Solution Let A, B and C be three parts respectively, then

$$A : B = 2 : 1 \text{ and } B : C = 1 : 3$$

$$\therefore A : B : C = 2 : 1 : 3 \therefore A = 1224 \times \frac{2}{6} = 408$$

$$B = 1224 \times \frac{1}{6} = 204 \Rightarrow C = 1224 \times \frac{3}{6} = 612$$

Exp. 14) If $A : B = 3 : 4$, $B : C = 5 : 2$ then find the value of $A : B : C$.

Solution $A : B = 3 : 4$

and $B : C = 5 : 2$

$$A : B : C = (3 \times 5) : (4 \times 5) : (4 \times 2)$$

or $A : B : C = 15 : 20 : 8$



Exp. 15) The ratio of $A : B = 1 : 3$, $B : C = 2 : 5$, $C : D = 2 : 3$. Find the value of $A : B : C : D$.

Solution $A : B = 1 : 3$

$$B : C = 2 : 5$$

$$C : D = 2 : 3$$

$$A : B : C : D = (1 \times 2 \times 2) : (3 \times 2 \times 2) : (3 \times 5 \times 2) : (3 \times 5 \times 3)$$

$$A : B : C : D = 4 : 12 : 30 : 45$$

Exp. 16) There are two types of mixtures of milk and water. In the first mixture, out of 12 litres of mixture, 5 litre is milk only and in the second mixture, 6 litre is milk and 12 litre is water. Which one mixture is better in terms of milk's strength?

Solution First Mixture Second Mixture

$\frac{5}{12}$	$\frac{6}{18}$	$(6 + 12 = 18)$
		(milk + water
$\frac{5}{12} \times 36$	$\frac{6}{18} \times 36$	= mixture)
15	12	

So the first mixture has more milk in comparison to water.

Exp. 17) The ratio of salary of $A : B = 1 : 2$, $B : C = 3 : 4$, $C : D = 5 : 6$ and $D : E = 7 : 8$. What is the ratio of salary of A and E ?

Solution $A : B = 1 : 2$

$$B : C = 3 : 4$$

$$C : D = 5 : 6$$

$$D : E = 7 : 8$$

$$A : B : C : D : E = (1 \times 3 \times 5 \times 7) : (2 \times 3 \times 5 \times 7) : (2 \times 4 \times 5 \times 7) : (2 \times 4 \times 6 \times 7) : (2 \times 4 \times 6 \times 8)$$

$$\text{or } A : B : C : D : E = 105 : 210 : 280 : 336 : 384$$

$$\text{So the ratio of salary of } A : E = 105 : 384 = 35 : 128$$

NOTE In every next step, we leave the left term and adopt right term.

Exp. 18) If $\frac{a}{b} = \frac{3}{4}$, then find the value of $7a - 4b : 3a + b$.

$$(a) 7 : 1$$

$$(b) 5 : 13$$

$$(c) 12 : 1$$

$$(d) \text{ none of these}$$

Solution Simply substitute the value of a and b as 3 and 4 in the given algebraic ratio

$$\text{as } (7 \times 3 - 4 \times 4) : (3 \times 3 + 4) = 5 : 13$$

Exp. 19) If $a : b = 3 : 2$ and $b : c = 6 : 5$ then $a : b : c$ is equal to
(a) 9 : 6 : 5 (b) 9 : 6 : 10 (c) 3 : 3 : 5 (d) 3 : 6 : 5

Solution $a : b = 3 : 2$

$$b : c = 6 : 5$$

$$a : b : c = (3 \times 6) : (2 \times 6) : (2 \times 5) = 18 : 12 : 10 = 9 : 6 : 5$$

Exp. 20) The sum of two natural numbers is 64. Which of the following cannot be the ratio of these two numbers?

$$(a) 3 : 5 \quad (b) 1 : 3 \quad (c) 7 : 9 \quad (d) 3 : 4$$

Solution Let the numbers be $3x$ and $5x$, then

$$3x + 5x = 64 \Rightarrow 8x = 64$$

$$\Rightarrow x = 8, \text{ which is possible}$$

The numbers are 24, 40.

$$\text{Take another option: } x + 3x = 64 \Rightarrow x = 16$$

The numbers are 16 and 48.

Check for option (c) :

$$7x + 9x = 64 \Rightarrow 16x = 64 \Rightarrow x = 4$$

The numbers are 28 and 36.

Check for option (d)

$$3x + 4x = 64 \Rightarrow 7x = 64$$

$$x = \frac{64}{7}$$

$$\text{Numbers are } \frac{3 \times 64}{7} \text{ and } \frac{4 \times 64}{7} \text{ or } \frac{192}{7} \text{ and } \frac{256}{7}$$

Which are not the natural numbers.

Hence option (d) is the required answer.

Exp. 21) Monthly incomes of A and B are in the ratio of 4 : 3 and their savings are in the ratio of 3 : 2. If the expenditure of each will be ₹ 600, then the monthly incomes of each are :

$$(a) 1800, 2400 \quad (b) 2400, 1600 \quad (c) 2400, 1800 \quad (d) 1600, 1200$$

Solution Income = Exp. + Savings

$$A \rightarrow 4x = 3y + 600$$

$$B \rightarrow 3x = 2y + 600$$

$$\text{Therefore, } 4x - 3y = 600 \text{ and } 3x - 2y = 600$$

$$\Rightarrow 4x - 3y = 3x - 2y \Rightarrow x = y$$

$$\therefore 4x - 3x = 600 \Rightarrow x = 600$$

$$\text{Then, the income of } A = 4 \times 600 = 2400$$

$$\text{and income of } B = 3 \times 600 = 1800$$

Alternatively Check the options. Consider (c)

Income	A	B
	2400	1800
	(4)	(3) $\Rightarrow$ correct

Again,	2400	1800
	$- 600$	$- 600$

Savings	1800	1200
	(3)	(2) $\Rightarrow$ correct

Hence, option (c) is correct. If you check other options, the ratio will not match.

Exp. 22) A, B and C have 40, x and y balls with them respectively. If B gives 20 balls to A , he is left with half as many balls as C . If together they had 60 more balls, each of them would have had 100 balls on an average. What is value of $x : y$?

- (a) 3 : 2 (b) 4 : 6 (c) 2 : 1 (d) 3 : 4

Solution From the last statement

$$\frac{100 + x + y}{3} = 100$$

$$\Rightarrow x + y = 200 \quad \dots(i)$$

$$\text{Again from first statement, } \frac{x - 20}{y} = \frac{1}{2}$$

$$\Rightarrow 2x - y = 40 \quad \dots(ii)$$

an solving eqs. (i) and (ii), we get

$$x = 80 \quad \text{and} \quad y = 120$$

Therefore, required ratio of $x : y = 2 : 3$

Hence, option (b) is correct.

Alternatively After forming the eqs. (i), we can go through the options. Let us assume option (b).

$$x + y = 200$$

$$\therefore x : y = 4 : 6$$

$$\Rightarrow x = 80 \quad \text{and} \quad y = 120$$

$$\text{Now from the first statement, } \frac{x - 20}{y} = \frac{1}{2}$$

$$\text{So } \frac{80 - 20}{120} = \frac{1}{2} \quad (\text{verified})$$

Hence option (b) is correct.

Exp. 23) The incomes of A, B and C are in the ratio of 12 : 9 : 7 and their spendings are in the ratio 15 : 9 : 8. If A saves 25% of his income. What is the ratio of the savings of A, B and C ?

Solution Income = Expenditure + Saving

$$A \rightarrow 12x = 15y + 3x \quad (3x = 25\% \text{ of } 12x)$$

$$B \rightarrow 9x = 9y + (9x - 9y)$$

$$C \rightarrow 7x = 8y + (7x - 8y)$$

$$\text{Therefore, } 12x - 3x = 15y$$

$$\Rightarrow \frac{x}{y} = \frac{5}{3} \Rightarrow y = \frac{3x}{5}$$

Therefore, savings = (income - expenditure)

$$A = 12x - 9x = 3x$$

$$B = 9x - \frac{27}{5}x = \frac{18}{5}x$$

$$C = 7x - \frac{24}{5}x = \frac{11}{5}x$$

$$\text{i.e., the ratio of savings of } A : B : C = 3x : \frac{18}{5}x : \frac{11}{5}x$$

$$= 15x : 18x : 11x = 15 : 18 : 11$$

Exp. 24) There are total 100 coins consisting of 20 paise, 50 paise and ₹ 1 in the ratio of 7 : 8 : 5. What is the no. of coins of 50 paise if the difference between the amount yielded by 20 paise and ₹ 1 coin is 18?

- (a) 32 (b) 40
(c) 26 (d) 56

Solution $(20 \times 7x) \sim (100 \times 5x) = 1800$ (₹ 18 = 1800 paise)

$$(140x) \sim (500x) = 1800$$

$$\Rightarrow 360x = 1800$$

$$\Rightarrow x = 5$$

$$\text{Therefore, number of coins of 50 paise} = 8 \times 5 = 40$$

Alternatively Let there be 40 coins of 50 paise denomination, then

$$\text{The no. of coins of } A, B \text{ and } C = 7x : 8x : 5x = 35 : 40 : 25$$

$$\text{Therefore, the amount from 20 paise coins} = 35 \times 20 = ₹ 7$$

$$\text{and amount from ₹ 1 coins} = 1 \times 25 = ₹ 25$$

$$\text{Hence difference} = ₹ 18 (25 - 7)$$

Thus the presumed option is correct.

Exp. 25) There are 43800 students in 4 schools of a city. If half of the first, two-third of the second, three-fourth of the third and four-fifth of the fourth are the same number of students, then find the ratio of number of students of A and D if A, B, C and D be the first, second, third and fourth schools respectively.

$$\text{Solution } \frac{A}{2} = \frac{2B}{3} = \frac{3C}{4} = \frac{4D}{5}$$

$$\text{Therefore, } \frac{A}{B} = \frac{4}{3} \text{ and } \frac{B}{C} = \frac{9}{8} \text{ and } \frac{C}{D} = \frac{16}{15}$$

$$\therefore A : B = 4 : 3$$

$$B : C = 9 : 8$$

$$C : D = 16 : 15$$

$$\therefore A : B : C : D = (4 \times 9 \times 16) : (3 \times 9 \times 16) : (3 \times 8 \times 16) : (3 \times 8 \times 15)$$

$$A : B : C : D = 576 : 432 : 384 : 360$$

Therefore, the ratio of number of students of

$$A \text{ and } D = 576 : 360 = 8 : 5$$

$$\text{Alternatively } \frac{A}{2} = \frac{2B}{3} = \frac{3C}{4} = \frac{4D}{5} = k$$

$$\text{then } A : B : C : D = 2k : \frac{3}{2}k : \frac{4}{3}k : \frac{5}{4}k = 24k : 18k : 16k : 15k$$

$$\therefore A : B : C : D = 24 : 18 : 16 : 15$$

$$\therefore A : D = 24 : 15 = 8 : 5$$

4.2 Proportion

An equality of two ratios is called a proportion and we say that four numbers are in proportion.

That is, if $\frac{a}{b} = \frac{c}{d}$, or $a : b = c : d$, then we say that a, b, c and d are in proportions and we write them as $a : b :: c : d$, where the symbol '::' indicates proportion and it is read as 'a is to b as c is to d'.

Here a and d are called **extremes** (or extreme terms) and b and c are called as **means** (or middle terms). Thus four numbers are said to be in proportion, if the ratio of the first to the second number is equal to the ratio of the third to the fourth number. For Example, $2 : 3 :: 4 : 6$.

Proportionality Test

If four numbers (quantities) are in proportion, then product of the extremes is equal to the product of the means and if these are not in proportion, then product of extremes is not equal to the product of the means.

That is if $a : b :: c : d$, then $a \times d = b \times c$

Thus it is clear that if three out of four terms of a proportion are given, we can find the fourth term by using this proportionality test.

Proportionality Theorems

- (a) **Invertendo:** If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$
- (b) **Alternando:** If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$
- (c) **Componendo:** If $\frac{a}{b} = \frac{c}{d} \Rightarrow \left(\frac{a+b}{b}\right) = \left(\frac{c+d}{d}\right)$
- (d) **Dividendo:** If $\frac{a}{b} = \frac{c}{d} \Rightarrow \left(\frac{a-b}{b}\right) = \left(\frac{c-d}{d}\right)$
- (e) **Componendo and Dividendo:** If $\frac{a}{b} = \frac{c}{d}$
- $$\Rightarrow \left(\frac{a+b}{a-b}\right) = \left(\frac{c+d}{c-d}\right)$$

NOTE

- Each of the two results (a) and (b) can be obtained by cross product.
- The result (e) can be obtained by dividing result (c) by result (d).

Continued Proportion

If a, b and c are three numbers such that $a : b = b : c$, then these numbers a, b and c are said to be in **continued proportion** or simply in proportion.

That is, if $a : b = b : c$ then $b^2 = ac$

Here b is said to be the **mean proportional** to a and c , and c is said to be the **third proportional** to a and b .

For Example, $3 : 9 :: 9 : 27$

Here, $9 \times 9 = 3 \times 27$. That means 3, 9 and 27 are in continued proportion.

NOTE Sometimes the above idea is also expressed by saying that the three numbers are in the ratio 3:9:27. Thus, if three quantities are proportionals, the first is to the third as the duplicate ratio of the first to the second. i.e., if $a : b = b : c$, then $a : c = a^2 : b^2$

$$\left[\because \frac{a}{b} = \frac{b}{c} \Rightarrow \frac{a^2}{b^2} = \frac{ab}{bc} \Rightarrow \frac{a^2}{b^2} = \frac{a}{c} \Rightarrow a : c = a^2 : b^2 \right]$$

Continued Proportionality Theorems

If a, b and c are three numbers such that $a : b = b : c$, that is $\frac{a}{b} = \frac{b}{c}$ then,

(a) $b^2 = ac$

Proof $\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$

(b) $\frac{b}{a} = \frac{c}{b}$

Proof $\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac \Rightarrow \frac{b}{a} = \frac{c}{b}$

(c) $\frac{a}{c} = \frac{a^2}{b^2}$ (Duplicate Ratio)

Proof $\frac{a}{b} = \frac{b}{c} \Rightarrow \frac{a}{b} \times \frac{a}{b} = \frac{b}{c} \times \frac{a}{b} \Rightarrow \frac{a^2}{b^2} = \frac{a}{c}$

(d) $\frac{a}{c} = \frac{b^2}{c^2}$ (Duplicate Ratio)

Proof $\frac{a}{b} = \frac{b}{c} \Rightarrow \frac{a}{b} \times \frac{b}{c} = \frac{b}{c} \times \frac{b}{c} \Rightarrow \frac{a}{c} = \frac{b^2}{c^2}$

(e) $\frac{a}{c} = \frac{a^2 + b^2}{b^2 + c^2}$

Proof Since $\frac{a}{c} = \frac{a^2}{b^2}$ and $\frac{a}{c} = \frac{b^2}{c^2}$

$$\Rightarrow \frac{a+a}{c+c} = \frac{a^2+b^2}{b^2+c^2} \Rightarrow \frac{a}{c} = \frac{a^2+b^2}{b^2+c^2}$$

Exp. 1) The first, second and fourth terms of a proportion are 5, 15 and 90 respectively. Find the third term.

Solution Let the third term be x , then 5, 15, x , 90 are in proportion

i.e., $5 : 15 :: x : 90$

$\Rightarrow 5 \times 90 = 15 \times x$ (by the 1st property)

$\therefore x = 30$

Exp. 2) The ratio of length to width of a rectangular sheet of paper is 5 : 3. If the width of the sheet is 18 cm, find its length.

Solution Let the length of sheet of paper be x cm. Then the ratio of length to width = $x : 18$

$$\text{Thus } x : 18 = 5 : 3 \Rightarrow x \times 3 = 18 \times 5 \Rightarrow x = 30$$

Hence the length of paper = 30 cm.

Exp. 3) If 81, x , x , 256 are in proportion, find x .

Solution $81 : x :: x : 256 \Rightarrow 81 \times 256 = x \times x \Rightarrow x = 144$

Exp. 4) The ratio between the number of men and women in an office is 5 : 7.

If the number of women working in the office is 56, find the number of men working in the office.

Solution $5 : 7 = x : 56$ (suppose number of men = x)

$$\Rightarrow x = 40 \quad (\text{by the first property})$$

Therefore, number of men in the office = 40

Exp. 5) The age of Chandi and Radhika are in the ratio 5 : 3. If Chandi's age is 20 years, find the age of Radhika.

Solution $5 : 3 = 20 : x$

$$\Rightarrow x = \frac{3 \times 20}{5} \quad (\text{by the first property})$$

$$\Rightarrow x = 12$$

Hence the age of Radhika is 12 years.

Exp. 6) The ratio of the number of boys to that of girls in a school is 9 : 11. If the number of girls in the school is 2035, find :

(a) number of boys in school

(b) number of students in school.

Solution (a) $9 : 11 = x : 2035 \Rightarrow x = \frac{9 \times 2035}{11}$

$$x = 1665 \quad (\text{number of boys})$$

(b) number of students

$$= \text{number of boys} + \text{number of girls} = 3700$$

Exp. 7) What is the least possible number which must be subtracted from 16, 19 and 23 so that the resulting numbers are in continued proportion?

(a) 2 (b) 4 (c) 6 (d) 7

Solution Going through options, we find option (d) is correct.

☞ **Alternatively** $(16 - x) : (19 - x) :: (19 - x) : (23 - x)$

$$\Rightarrow (19 - x)^2 = (16 - x)(23 - x)$$

By solving the above equation, we get $x = 7$

Exp. 8) If $(a + b) : (a - b) = 15 : 1$, then the value of $a^2 - b^2$ is :

(a) 56 (b) 15 (c) 112 (d) 8

Solution $\frac{(a + b)}{(a - b)} = \frac{15}{1} \Rightarrow \frac{a}{b} = \frac{8}{7}$ (by componendo and

dividendo)

$$\text{Therefore } a^2 - b^2 = 64 - 49 = 15$$

Exp. 9) The mean proportional between 8 and 98 is :

(a) 16 (b) 53 (c) 112 (d) 28

Solution $8 : x :: x : 98$

$$x^2 = 8 \times 98 \Rightarrow x = 28$$

Hence option (d) is correct.

Exp. 10) The students in three classes are in the ratio of 2 : 3 : 4. If 40 students are added in each class, the ratio becomes 4 : 5 : 6. Find the total number of students in all the three classes is :

(a) 270 (b) 180 (c) 126 (d) 135

Solution $2x + 40 = 4y$... (i)

$$3x + 40 = 5y \quad \dots (ii)$$

$$4x + 40 = 6y \quad \dots (iii)$$

$$\text{Therefore } 2x = 40 \Rightarrow x = 20$$

$$\text{Hence, total number of students} = 2x + 3x + 4x = 9x \\ = 9 \times 20 = 180$$

☞ **Alternatively** It can be solved through options also.

Exp. 11) The dimensions of a photograph are 4 and 1.8 cms. If the breadth of the enlarged photo is 4.5 cm and it was enlarged proportionally then what is the new length of new photograph?

(a) 6 (b) 5.4 (c) 10 (d) 9

Solution $4 : 1.8 = x : 4.5$

$$\therefore x = 10$$

Thus, the length of new photograph is 10 cm.

Exp. 12) Two equal containers are filled with the mixture of milk and water. The concentration of milk in each of the containers is 20% and 25% respectively. What is the ratio of water in both the containers respectively?

(a) 15 : 16 (b) 16 : 15 (c) 4 : 5 (d) 5 : 4

Solution

	Milk	20%	25%
Water	80%	75%	

$$\text{Therefore, required ratio} = \frac{80}{75} = \frac{16}{15} \text{ or } 16 : 15$$

Exp. 13) A cat takes 7 steps for every 5 steps of a dog, but 5 steps of a dog are equal to 6 steps of cat. What is the ratio of speed of cat to that of dog?

(a) 24 : 25 (b) 42 : 25 (c) 24 : 19 (d) 25 : 42

Solution

	CAT	DOG
Given speed	7 steps	5 steps

But the length of 5 steps of dog = length of 6 steps of cat

It means the ratio of length covered by dog is to cat = $\frac{6}{5}$

Therefore, in each step a dog will cover $\frac{6}{5}$ times distance than that of a cat.

Thus the ratio of actual speed of cat is to dog

$$= 7 : 5 \times \frac{6}{5} = 7 : 6$$

☞ **Alternatively** Actual speed of A : B

$$= \frac{\text{Given speed of A}}{\text{No. of steps of A in terms of length}} : \frac{\text{Given speed of B}}{\text{No. of steps of B in terms of length}}$$

$$= \text{CAT} : \text{DOG} = \frac{7}{6} : \frac{5}{5} = 7 : 6$$

Introductory Exercise 4.1

- If $A : B = 4 : 5$, $B : C = 3 : 4$, $C : D = 7 : 11$, then $A : D$ is :
(a) 3 : 4 (b) 21 : 55 (c) 21 : 44 (d) 7 : 5
- Mean proportional between 17 and 68 is :
(a) 51 (b) 24 (c) 4 (d) 34
- Third proportional between 16 and 36 is :
(a) 64 (b) 144 (c) 81 (d) 49
- If $a : b = 2 : 3$, then $(5a + b) : (3a + 2b)$ is :
(a) 13 : 12 (b) 15 : 17 (c) 12 : 13 (d) 13 : 11
- $a = 2b = 3c = 4d$, then $a : b : c : d$ is :
(a) 12 : 3 : 6 : 4 (b) 3 : 4 : 6 : 12
(c) 6 : 12 : 4 : 3 (d) 12 : 6 : 4 : 3
- The fourth proportional to 4, 7 and 20 is :
(a) 28 (b) 21 (c) 18 (d) 35
- If $\sqrt{2} : (1 + \sqrt{3}) :: \sqrt{6} : x$, then x is equal to :
(a) $1 + \sqrt{3}$ (b) $\sqrt{3} - 1$
(c) $\sqrt{3} + 3$ (d) $2\sqrt{3}$
- If $\frac{a}{3} = \frac{b}{4} = \frac{c}{5}$ then $\frac{a + b + c}{b} = ?$
(a) 2 (b) 3 (c) 4 (d) 5
- If $\frac{a}{b} = \frac{c}{d}$, then :
(a) $\frac{a + b}{a - b} = \frac{c + d}{c - d}$ (b) $\frac{a + b}{a^2} = \frac{c + d}{d^2}$
(c) $\frac{a + b}{a^2} = \frac{c + d}{c^2}$ (d) $ac = bd$
- If $a : b = b : c = c : d$ then $\frac{a}{b}, \frac{b}{c}, \frac{c}{d}$ are :
(a) in AP
(b) in continued proportion
(c) in GP
(d) both (b) and (c)
- If $(a + b) : (a - b) = 3 : 2$, the $(a^2 - b^2) : (a^2 + b^2)$ equals :
(a) 5 : 13 (b) 12 : 13
(c) 9 : 4 (d) none of these
- Two whole numbers, whose sum is 64, cannot be in the ratio :
(a) 1 : 7 (b) 3 : 5 (c) 5 : 11 (d) 1 : 2
- Two numbers are in the ratio 3 : 4. The difference between their squares is 28. Find the greater number
(a) 12 (b) 8 (c) 24 (d) 16
- If a, b, c, d, e, f and g are in continued proportion, then the value of $a.b.c.e.f.g$ is :
(a) d^3 (b) d^7
(c) d^6 (d) none of these
- If A and B shared ₹ 1300 in the ratio 1 : 12, how much did A get?
(a) 120 (b) 1200 (c) 100 (d) 1000
- ₹ 3960 are divided among A, B and C so that half of A's part, one-third of B's part and one-sixth of C's part are equal. Then C's part is :
(a) 720 (b) 2160 (c) 1080 (d) 810
- A sum of ₹ 21000 is divided among A, B and C such that shares of A and B are in the ratio of 2 : 3 and those of B and C are in the ratio 4 : 5. The amount received by A is :
(a) ₹ 6000 (b) ₹ 4500 (c) ₹ 4800 (d) ₹ 8400
- A certain amount was divided between A and B in the ratio 7 : 9. If B's share was ₹ 7200, the total amount was :
(a) ₹ 1280 (b) ₹ 6300 (c) ₹ 5600 (d) ₹ 12800
- ₹ 11250 are divided among A, B and C so that A may receive one-half as much as B and C together receive and B receives one-fourth of what A and C together receive. The share of A is more than that of B by :
(a) ₹ 2500 (b) ₹ 1500 (c) ₹ 1800 (d) ₹ 650
- A girl 1.2 metre tall casts a shadow 1.1 m at the time when a building casts a shadow 6.6 m long. The height of the building is :
(a) 2.7 m (b) 7.2 m (c) 6.0 m (d) 5.5 m
- The prices of Bajaj Scooter and Bajaj Pulser are in the ratio of 4 : 9. If the Bajaj Pulser costs ₹ 30000 more than a Bajaj Scooter, the price of Bajaj Pulser is :
(a) ₹ 63000 (b) ₹ 45000
(c) ₹ 54000 (d) ₹ 60000
- What is the ratio whose terms differ by 40 and the measure of which is $2/7$?
(a) 16 : 56 (b) 14 : 49
(c) 15 : 36 (d) 16 : 72

Exp. 14) A camel pursue an elephant and takes 5 leaps for every 7 leaps of the elephant, but 5 leaps of elephant are equal to 3 leaps of camel. What is the ratio of speeds of camel and elephant?

Solution Ratio of speed of camel and elephant
 $= \frac{5}{3} : \frac{7}{5} = \frac{5}{3} \times 15 : \frac{7}{5} \times 15 = 25 : 21$

- 23.** Two numbers are in the ratio 3 : 5. If 9 be subtracted from each, then they are in the ratio of 12 : 23. The second number is :
 (a) 53 (b) 54
 (c) 55 (d) 52
- 24.** Consider the following statements :
 (1) If both the terms of a ratio are multiplied or divided by the same natural number, then the ratio remains unaltered.
 (2) A statement which states that two ratios are equivalent is called proportion.
 (3) If 4 quantities are in proportion, the product of extremes is not equal to the means.
 (4) The mean proportion between any two numbers is equal to the square root of their product. The wrong one statements is/are :
 (a) 1 (b) 3
 (c) 3 and 4 (d) 1 and 4
- 25.** In a mixture of 120 litres, the ratio of milk and water is 2 : 1. If the ratio of milk and water is 1 : 2, then the amount of water (in litres) is required to be added is :
 (a) 20 (b) 40 (c) 80 (d) 120
- 26.** A quantity x varies inversely as the square of y . Given that $x = 4$, when $y = 3$, the value of x when $y = 6$ is :
 (a) 1 (b) 2 (c) 3 (d) 4
- 27.** Suppose y varies as the sum of two quantities of which one varies directly as x and the other inversely as x . If $y = 6$ when $x = 4$ and $y = 3\frac{1}{3}$ when $x = 3$, then the relation between x and y is :
 (a) $x = y + 4$ (b) $y = 2x + \frac{8}{x}$
 (c) $y = 2x - \frac{8}{x}$ (d) $y = 2x - \frac{4}{x}$
- 28.** The time period of a pendulum is proportional to the square root of the length of the pendulum. Consider the following statements :
 (1) If the length of the pendulum is doubled, then the time period is also doubled.
 (2) If the length is halved, then time period becomes one-fourth of the original time period.
 The correct assertions are :
 (a) 1 (b) 2
 (c) neither 1 nor 2 (d) both 1 and 2

4.3 Unitary Method

In this method, at first we find the value of one unit and, then we find the value of required number of units by multiplying the value of one unit with the required number of units.

For example, if the price of 10 bananas is ₹ 50, find the price of 25 bananas. Then, as per unitary method, first we find the price of one unit of bananas, which is ₹ 5. Thus, we can find the price of 25 bananas by multiplying the price of one unit with the number of desired units, which is ₹ 125.

Direct Proportion

Two quantities are said to be directly proportional if the increase (or decrease) in one quantity causes the increase (or decrease) in the other quantity by same proportion. *e.g.*,

- The cost of articles varies directly with the number of articles. More articles more cost, less articles less cost.
- The work done varies directly with the number of men (work force) at work. More men at work, more work done in the same time. Less men, less work done in the same time.

Inverse Proportion

Two quantities are said to vary inversely if the increase (or decrease) in one quantity causes the decrease (or increase) in the other quantity by same proportion. *e.g.*, The time taken to finish a work varies inversely to the number of men at work.

More men at work, less time taken to finish the same work. Less men at work, more time taken to finish the same work.

Exp. 1) If 6 note books cost ₹ 45, how much would 8 notebooks cost?

Solution More note books more cost; less note books, less cost

Note books	Cost
6	45
1	$\frac{45}{6}$
8	$\frac{8 \times 45}{6} = 60$

Hence 8 note books cost ₹ 60.

Exp. 2) If 45 students can consume a stock of food in 2 months, find for how many days the same stock of food will last for 27 students?

Solution More students, less days; less students, more days

Students	Days
45	60
1	60×45
27	$\frac{60 \times 45}{27} = 100$ days

Exp. 3) A man working 8 hours a day takes 5 days to complete a project. How many hours a day must he work to complete it in 4 days?

Solution 'More days, less hours; less days, more hours'

Days	Hours
5	8
1	5×8
4	$\frac{5 \times 8}{4} = 10$ hours a day

Introductory Exercise 4.2

- If 40 articles cost ₹ 180, the cost of 18 articles is :
(a) ₹ 18 (b) ₹ 36
(c) ₹ 81 (d) ₹ 40.5
- If 20 persons can do a piece of work in 7 days, then the number of persons required to complete the work in 28 days :
(a) 4 (b) 5
(c) 14 (d) 10
- If 20 men can reap a field in 38 days, in how many days will 19 men reap the field?
(a) 21 days (b) 19 days
(c) 76 days (d) 40 days
- A rope makes 260 rounds of a cylinder with base radius 20 cm. How many times can it go round a cylinder with base radius 26 cm?
(a) 130 (b) 300
(c) 200 (d) 150
- A garrison of 750 men has provision for 20 weeks. If at the end of 4 weeks they are reinforced by 450 men, how long will the remaining provision last?
(a) 10 weeks (b) 11 weeks
(c) 15 weeks (d) 16 weeks
- 40 men can build a wall 20 m high in 15 days. The number of men required to build a similar wall 25 m high in 6 days will be :
(a) 100 (b) 125
(c) 150 (d) 200
- Cost of erecting a fence round a square field of 625 hectares at 15 paise per metre is :
(a) ₹ 500 (b) ₹ 750 (c) ₹ 100 (d) ₹ 1500
- 56 workers can reap a field in 8 days. If the work is to be completed in 7 days, the extra workers needed are :
(a) 7 (b) 8 (c) 14 (d) 16
- If 5 persons weave 180 shawls in 12 days, how many shawls will 6 persons weave in 15 days?
(a) 260 (b) 370 (c) 270 (d) 360
- 15 men take 42 days of 4 hours each to do a piece of work. How many days of 6 hours each would 21 women take if 3 women do as much work as 2 men?
(a) 15 (b) 22 (c) 25 (d) 30
- 10 engines consume 80 litres diesel when each is running 9 hours a day. How much diesel will be required for 10 engines, each running 15 hours a day, whereas 6 engines of the former type consume as much as 5 engines of latter type?
(a) 160 litre (b) 75 litre
(c) 80 litre (d) $111\frac{1}{9}$ litre
- If $\frac{3}{5}$ th of a cistern is filled in 30 minutes, how much more time will be required to fill the rest of it?
(a) 50 minutes (b) 20 minutes
(c) 15 minutes (d) 45 minutes

4.4 Variation

When two or more quantities are dependent upon each other and then if any one of them is changed, the other (dependent) quantity is also changed.

For example :

- When the salary of a person increases, then its savings/expenditure increases.
- When the number of guests in a hotel/number of students in a hostel/number of employees changes, their respective expenses increases.

Basically, as it happens in direct proportion and inverse proportion, there are two types of variation :

- Direct variation
- Inverse variation

Direct Variation

A quantity A is said to vary directly if the increase (or decrease) in B yields increase (or decrease) in A but not in proportion. It is expressed as

$$A \propto B \Rightarrow A = KB,$$

where K is called proportionality constant

$$\Rightarrow K = \frac{A}{B}$$

Inverse Variation

A quantity A is said to vary inversely if the increase (or decrease) in B yields decrease (or increase) in A but not in same proportion. It is expressed as

$$A \propto \frac{1}{B} \Rightarrow A = \frac{K}{B}$$

or $K = AB$, K is called as proportionality constant.

NOTE

- If it is not mentioned that a particular quantity is inversely variable, then it means the given quantity is directly variable.
- A quantity sometimes vary jointly i.e., directly on any quantity and inversely on another quantity.

e.g., $A \propto B$ and $A \propto \frac{1}{C}$

It means $A \propto \frac{B}{C} \Rightarrow A = \frac{KB}{C}$

Here A varies directly as B but inversely as C .

Also it can vary as only directly or inversely as more than one quantities.

e.g., $A \propto BC \Rightarrow A = KBC$

and $A \propto \frac{1}{BC} \Rightarrow A = \frac{K}{BC}$

Exp. 1) A varies directly as B and inversely as C . A is 12 when B is 6 and C is 2. What is the value of A when B is 12 and C is 3?

Solution $A \propto B$ and $A \propto \frac{1}{C}$

$\Rightarrow A \propto \frac{B}{C} \Rightarrow A = K \frac{B}{C}$

When $A = 12$, $B = 6$, $C = 2$, then

$$12 = K \frac{6}{2} \Rightarrow K = 4$$

Again $A = K \frac{B}{C} = 4 \times \frac{12}{3} = 16 \Rightarrow A = 16$

Exp. 2) The value of a coin varies directly to the square of its radius, when its thickness is constant. The radius of a coin is 1.5 cm and its value is ₹ 2. What will be the radius of a coin if its value is ₹ 5?

Solution $V \propto r^2 \Rightarrow V = Kr^2$

$$2 = K \times (1.5)^2 \Rightarrow K = \frac{2}{2.25} \Rightarrow K = \frac{8}{9}$$

Again $5 = \frac{8}{9} \times r^2 \Rightarrow r^2 = \frac{5 \times 9}{8}$

$\therefore r = \frac{3}{2} \times \sqrt{\frac{5}{2}} = 1.5 \times \sqrt{2.5} = 1.5 \times 1.6$
 $r \approx 2.4$ cm

Hence, required radius = 2.4 cm

4.5 Problems Based on Ages

This article is very suitable as an appendix of ratio-proportion, since most of the questions based on ages involve the concept of ratio-proportion. e.g., the age of Ravi is 16 years and the age of her mother is twice etc. or the ratio of the ages of the father and son at present is 3:1. 4 years, earlier the ratio was 4:1. What are the present ages of the father and son etc.

Exp. 1) The ratio of ages of Krishna and Balram is 3:4. Four years earlier the ratio was 5:7. Find the present ages of Krishna and Balram :

- (a) 15 years, 20 years (b) 24 years, 32 years
 (c) 16 years, 20 years (d) 32 years, 24 years

Solution Let the present age of Krishna and Balram be $3x$ and $4x$, then four years ago their ages be $(3x - 4)$ and $(4x - 4)$

So $\frac{(3x - 4)}{(4x - 4)} = \frac{5}{7} \Rightarrow 7(3x - 4) = 5(4x - 4)$

$\Rightarrow 21x - 28 = 20x - 20$

$\Rightarrow x = 8$

$\therefore$ Present age of Krishna = $3x = 24$ years

and age of Balram = $4x = 32$ years

Exp. 2) The ratio of age of Aman and her mother is 3:11. The difference of their ages is 24 years. What will be the ratio of their ages after 3 years?

Solution Let the present age of Aman and her mother be $3x$ and $11x$ then 3 years later their ages will be $(3x + 3)$ and $(11x + 3)$ respectively.

Again $11x - 3x = 24$

$8x = 24$

$\Rightarrow x = 3$

Therefore the ratio of their ages 3 years after

$$= \frac{3x + 3}{11x + 3} = \frac{12}{36} = \frac{1}{3}$$

Exp. 3) The age of Sachin is 4 times that of his son. Five years ago Sachin was nine times as old as his son was at that time. The present age of the Sachin is :

- (a) 25 years (b) 36 years
 (c) 32 years (d) 48 years

Solution Let the age of son is x years, then the age of Sachin will be $4x$ years.

$\therefore (4x - 5) = 9(x - 5) \Leftrightarrow x = 8$

$\therefore$ Age of Sachin is 32 years.

Exp. 4) The ratio of Varun's age and his mother's age is 5:11. The difference of their ages is 18 years. The ratio of their ages after 5 years will be :

- (a) 19:59 (b) 2:3
 (c) 37:75 (d) 10:19

Solution Let their ages be $5x$ and $11x$.

$11x - 5x = 18$

$\Leftrightarrow x = 3$

So their present ages are 15 and 33 years. Therefore, ratio of their ages after 5 years = $20:38 = 10:19$.

Introductory Exercise 4.3

- Amit is as much younger to Barkha as he is older to Chaman. If the sum of the ages of Barkha and Chaman is 48 years, what is the present age of Amit?
(a) 18 years
(b) 36 years
(c) 24 years
(d) 28 years
- Bipin is 6 times old as Alok. Bipin's age will be twice of Chandan's age after 10 years. If Chandan's 7th birthday was celebrated 3 years ago, what is Alok's present age?
(a) 15 years
(b) 12 years
(c) 5 years
(d) none of these
- Renuka got married 8 years ago. Today her age is $1\frac{1}{3}$ times her age at the time of marriage. Her daughter's age is $\frac{1}{8}$ times her age. Her daughter's age is :
(a) 3 years (b) 4 years (c) 6 years (d) 8 years
- Ten years ago B was twice of A in age. If the ratio of their present ages is 4 : 3, what is the sum of their present ages?
(a) 25 years (b) 30 years
(c) 40 years (d) 35 years
- The sum of the ages of Aryabhatta and Shridhar is 45 years. Five years ago the product of their ages was 4 times the Aryabhatta's age at that time. The present ages of Aryabhatta and Shridhar respectively are :
(a) 25 and 20 (b) 35 and 10
(c) 36 and 9 (d) 40 and 5

4.6 Partnership

When two or more than two people run a business jointly by investing their money/resources, then it is called a joint venture or the business in partnership. All these people, who have invested their resources, are called as Partners.

Types of Partners

- Working partner** A partner who is directly involved with day-to-day activities of business is called as working partner.
- Sleeping partner** A partner who just invests his or her money is called as sleeping partner.

General Rules of Partnership

- If the partners invest different amounts for the same period of time, then the profits of all the partners are shared in the ratio of their investments.
- If the partners invest same amount for the different time periods, then the profits of all the partners are shared in the ratio of time periods for which their amounts were invested.
- If the partners invest different amounts for different time periods, then their profits are shared in the ratio of products of respective investments with the time period for each partner, individually. Thus gain or loss is divided in the ratio of 'money-time' capitals.

 **NOTE** Sometimes different problems are solved on the basis of partnership to find the expenses.

Exp. 1) Bhanu and Shafeeq started a business by investing ₹ 36000 and ₹ 63000. Find the share of each, out of an annual profit of ₹ 5500.

Solution Ratio of shares of Bhanu and Shafeeq
 $= 36000 : 63,000 = 4 : 7$

$$\therefore \text{Share of Bhanu} = 5500 \times \frac{4}{11} = ₹ 2000$$

$$\text{and Share of Shafeeq} = 5500 \times \frac{7}{11} = 3500$$

Exp. 2) A starts some business with ₹ 50000. After 3 months B joins him with ₹ 70000. At the end of the year, in what ratio should they share the profits?

Solution Ratio of amount of A and $B = 50000 : 70000$

Ratio of time periods for A and $B = 12 : 9$

$\therefore$ Ratio of their money-time capital

$$\text{Investments} = 50000 \times 12 : 70000 \times 9 \\ = 20 : 21$$

Exp. 3) Harsh Vardhan started a business by investing ₹ 36000. After 4 months Gyan Vardhan joined him with some investment. At the end of the year, the total profit was divided between them in the ratio of 9 : 7. How much capital was invested by Gyan Vardhan in the business?

Solution $\frac{36,000 \times 12}{x \times 8} = \frac{9}{7} \Rightarrow x = 42,000$

$\therefore$ Gyan Vardhan invested ₹ 42000 for 8 months only.

Exp. 4) A started some business with ₹ 26000. After 3 months B joined him with ₹ 16000. After some more time C joined them with ₹ 25000. At the end of the year, out of a total profit of ₹ 15453, C gets ₹ 3825 as his share. How many months after B joined the business did C join?

Solution Ratio (of share) of profits
 $= 26,000 \times 12 : 16000 \times 9 : 25000 \times C$
 $= 312 : 144 : 25C$
 Now C's share = $\frac{25C}{456 + 25C} = \frac{3825}{15453} \Rightarrow C = 6$

Therefore C joined 3 months later than B joined.

Exp. 5) A, B and C started a business with their investments in the ratio 1 : 2 : 4. After 6 months A invested the half amount more as before and B invested twice the amount as before while C withdrew $\frac{1}{4}$ th of the their

investments. Find the ratio of their profits at the end of the year.

Solution Let us assume their initial investments were x , $2x$ and $4x$ respectively.

Therefore, ratio of their investments during the whole year

$$= (x \times 6 + \frac{3x}{2} \times 6) : (2x \times 6 + 4x \times 6) : (4x \times 6 + 3x \times 6)$$

$$= 15x : 36x : 42x = 5x : 12x : 14x = 5 : 12 : 14$$

$\therefore$ Ratio of their profits = 5 : 12 : 14

Introductory Exercise 4.4

- A company make a profit of ₹ 9,00,000, 20% of which is paid as taxes. If the rest is divided among the partners P, Q and R in the ratio of $1 : 1\frac{1}{2} : 2$, then the shares of P, Q and R are respectively :
 (a) 240000; 320000; 160000
 (b) 320000; 240000; 160000
 (c) 160000; 320000; 240000
 (d) 160000; 240000; 320000
- We have to divide a sum of ₹ 13950 among three persons A, B and C. B must get the double of A's share and C must get ₹ 50 less than the double of B's share. The share of A will be :
 (a) ₹ 1950 (b) ₹ 1981.25
 (c) ₹ 2000 (d) ₹ 2007.75
- A started business with ₹ 45000 and B joined after-ward with ₹ 30000. If the profits at the end of one year were divided in the ratio 2 : 1 respectively, then B would have joined A for business after :
 (a) 1 month (b) 2 months
 (c) 3 months (d) 4 months
- A and B are partners in a business. They invest in the ratio 5 : 6, at the end of 8 months A withdraws. If they receive profits in the ratio of 5 : 9, find how long B's investment was used?
 (a) 12 months
 (b) 10 months
 (c) 15 months
 (d) 14 months
- Four milkmen rented a pasture. A put to graze 16 cows for 3 months, B 20 cows for 4 months, C 18 cows for 6 months and D 42 cows for 2 months. If A's share of rent be ₹ 2400, the rent paid by C is :
 (a) ₹ 3200 (b) ₹ 4200 (c) ₹ 4000 (d) ₹ 5400
- A, B and C subscribe ₹ 47000 for a business. If a subscribes ₹ 7000 more than B and B ₹ 5000 more than C, then out of total profit of ₹ 4700, C receives :
 (a) ₹ 1200
 (b) ₹ 4500
 (c) ₹ 1000
 (d) none of the above

Exp. 6) A started a business with ₹ 52000 and after 4 months B joined him with ₹ 39000. At the end of the year, out of the total profits B received total ₹ 20000 including 25% of the profits as commission for managing the business. What amount did A receive?

Solution Profit's share of A and B
 $= 52,000 \times 12 : 39,000 \times 8 = 2 : 1$

Let the profit be ₹ x , then B receives 25% as commission for managing business, the remaining 75% of the total profit x is shared between A and B in the ratio 2 : 1. Hence B will get $\frac{1}{3}$ rd part of this in addition to his commission. Hence his

$$\text{total earning} = 0.25x + \frac{1}{3} \times 0.75x = 0.5x = 20000 \Rightarrow x = 40000$$

So, the remaining profit goes to A, hence the profit of A is ₹ 20000.

Exp. 7) A working partner gets 20% as his commission of the profit after his commission is paid. If the working partner's commission is ₹ 8000, then what is the total profit in the business?

Solution Let the total profit be ₹ x . The remaining profit after paying 20% working partner's commission = $(x - 8000)$. Again since 20% of this is working partner's commission,

$$\text{therefore } \frac{20}{100} \times (x - 8000) = 8000 \Rightarrow x = 48000$$

$\therefore$ The total profit in the business is ₹ 48000.

CAT-Test

Questions Helping you bell the CAT

LEVEL 01 > BASIC LEVEL EXERCISE

- 1 Four numbers are in proportion. The sum of the squares of the four numbers is 50 and the sum of the means is 5. The ratio of first two terms is 1 : 3. What is the average of the four numbers?
(a) 2 (b) 3
(c) 5 (d) 6
- 2 A naughty student breaks the pencil in such a way that the ratio of two broken parts is same as that of the original length of the pencil to one of the larger part of the pencil. The ratio of the other part to the original length of pencil is
(a) $1 : 2\sqrt{5}$ (b) $2 : (3 + \sqrt{5})$
(c) $2 : \sqrt{5}$ (d) can't be determined
- 3 If $\frac{x+y}{x-y} = \frac{5}{3}$ and $\frac{x}{(y+2)} = 2$, then the value of (x, y) is
(a) (4, 1) (b) (2, 8)
(c) (1, 4) (d) (8, 2)
- 4 If $a^3 + b^3 : a^3 - b^3 = 133 : 117$; then find the value of $a : b$
(a) 2 : 3 (b) 5 : 4
(c) 5 : 2 (d) none of these
- 5 A student obtained equal marks in History and Sociology. The ratio of marks in Sociology and Geography is 2 : 3 and the ratio of marks in History and Philosophy is 1 : 2. If he has scored an aggregate of 55% marks. The maximum marks in each subject is same. In how many subjects did he score equal to or greater than 60% marks?
(a) 1 (b) 2
(c) 3 (d) none of these
- 6 The ratio of income of Anil and Mukesh is 2 : 3. The sum of their expenditure is ₹ 8000 and the amount of savings of Anil is equal to the amount of expenditure of Mukesh. What is the sum of their savings?
(a) 22000 (b) 4000
(c) 16000 (d) 12000
- 7 Hutch and Essar entered into a partnership just 5 months ago. The ratio of profit claimed by Hutch and Essar is 6 : 17. If Essar had just started his business 12 months ago with ₹ 1275, what is the amount contributed by Hutch?
(a) ₹ 980 (b) ₹ 1080
(c) ₹ 1200 (d) ₹ 998
- 8 A child has three different kinds of chocolates costing ₹ 2, ₹ 5 and ₹ 10. He spends total ₹ 120 on the chocolates. What is the minimum possible number of chocolates, he can buy, if there must be atleast one chocolate of each kind?
(a) 22 (b) 19 (c) 17 (d) 15
- 9 In the previous problem (no. 8) what is the maximum possible no. of chocolates?
(a) 52 (b) 53 (c) 55 (d) 60
- 10 Mr. Teremere and Mr. Meretere have 5 chocolates and 3 chocolates with them respectively. Meanwhile Mr. Khabbu Singh joined them and all 8 chocolates were distributed equally among all these three people. In turn Khabbu Singh gave ₹ 16 to Mr. Teremere and Mr. Meretere, since Khabbu Singh did not has any chocolate. What is the difference of amounts received by Teremere and Meretere? Given that the amount was shared in proportion of chocolates received by Khabbu Singh.
(a) ₹ 8 (b) ₹ 12 (c) ₹ 14 (d) ₹ 15
- 11 ₹ 4536 is divided among 4 men, 5 women and 2 boys. The ratio of share of a man, a woman and a boy is 7 : 4 : 3. What is the share of a woman?
(a) ₹ 336 (b) ₹ 498 (c) ₹ 166 (d) ₹ 256
- 12 The concentration of petrol in three different mixtures (petrol and kerosene) is $\frac{1}{2}$, $\frac{3}{5}$ and $\frac{4}{5}$ respectively. If 2 litres, 3 litres and 1 litre are taken from these three different vessels and mixed. What is the ratio of petrol and Kerosene in the new mixture?
(a) 4 : 5 (b) 3 : 2 (c) 3 : 5 (d) 2 : 3
- 13 Time period (T) of pendulum is directly proportional to the square root of length of string by which bob is attached to a fixed point and inversely proportional to the square root of gravitational constant 'g'. Time period of a bob is 3 seconds when the gravitational constant g is 4 m/sec² and length of string is 9 metre, what is the time period of a bob having a string of length 64 metre and gravitational constant 16 m/sec²?
(a) 4 seconds (b) 12 seconds
(c) 16 seconds (d) 10 seconds

- 14** In a milk shoppe there are three varieties of milk, 'Pure', 'Cure' and 'Lure'. The 'Pure' milk has 100% concentration of milk. The ratio of milk is to water in the 'Cure' is 2 : 5 and in the Lure it is 3 : 8 respectively. Sonali purchased 14 litres of Cure and 22 litres of Lure milk and mixed them. If she wanted to make the concentration of milk in the mixture of purchased milk to 50%. How many litres of 'Pure' milk she is needed?
 (a) 6 litres (b) 8 litres
 (c) 16 litres (d) 18 litres
- 15** In the squadron of Indian Air Force the ratio of Sukhoi to Mig and Jaguar together is 5 : 7 and the ratio of Jaguar to Sukhoi and Mig together is 1 : 2. Find the ratio of Sukhoi and Mig :
 (a) 2 : 7 (b) 3 : 5 (c) 3 : 1 (d) 5 : 3
- 16** During our campaign against child labour we have found that in three glass making factories A, B and C there were total 33 children aged below 18 were involved. The ratio of male to female in A, B and C was 4 : 3, 3 : 2 and 5 : 4 respectively. If the no. of female children working in the factories B and C be equal then find the no. of female children working in factory A :
 (a) 5 (b) 2 (c) 8 (d) 6
- 17** The value of a diamond is directly proportional to the square of its weight. A diamond unfortunately breaks into three pieces with weights in the ratio of 3 : 4 : 5 thus a loss of ₹ 9.4 lakh is incurred. What is the actual value of diamond :
 (a) 28.8 lakh (b) 13.5 lakh
 (c) 14.4 lakh (d) 18.8 lakh
- 18** In the Ruchika's wallet there are only ₹ 16, consisting of 10 paise, 20 paise and ₹ 1 coins. The ratio of no. of coins of 10 paise and 20 paise is 6 : 1. The minimum no. of ₹ 1 coin is :
 (a) 5 (b) 12 (c) 4 (d) 8
- 19** There are two vessels containing the mixture of milk and water. In the first vessel the water is $\frac{2}{3}$ of the milk and in the second vessel water is just 40% of the milk. In what ratio these are required to mix to make 24 litres mixture in which the ratio of water is to milk is 1 : 2 ?
 (a) 4 : 3 (b) 5 : 7
 (c) 5 : 2 (d) 7 : 5
- 20** Nehru Ji had 'n' chocolates. He distributed them among 4 children in the ratio of $\frac{1}{2} : \frac{1}{3} : \frac{1}{5} : \frac{1}{8}$. If he gave them each one a complete chocolate, the minimum no. of chocolates that he had :
 (a) 139 (b) 240
 (c) 278 (d) None of these
- 21** The ratio of working efficiency of A and B is 5 : 3 and the ratio of efficiency of B and C is 5 : 8. Who is the most efficient ?
 (a) A (b) B
 (c) C (d) Can't be determined
- 22** If $4A = 5B$ and $3A = 2C$, the ratio of $B : C$ is :
 (a) 4 : 3 (b) 5 : 8 (c) 8 : 15 (d) 10 : 15
- 23** Equal quantities of three mixtures of milk and water are mixed in the ratio of 1 : 2, 2 : 3 and 3 : 4. The ratio of water and milk in the mixture is :
 (a) 193 : 122 (b) 122 : 193
 (c) 61 : 97 (d) 137 : 178
- 24** The ratio of age of A and B is 8 : 9 and the age of B is $\frac{2}{3}$ of C's age and age of C is $\frac{9}{13}$ times the age of D. If the age of B is 18 years then the age of C is :
 (a) 36 years (b) 39 years
 (c) 27 years (d) 54 years
- 25** A milk man has a mixture of milk in which ratio of milk and water is 5 : 3. He sells 40 litres of milk i. e., mixture then he adds up 15 litres of pure water. Now the ratio of milk and water is 5 : 4. What is the new quantity of mixture?
 (a) 72 litres (b) 270 litres
 (c) 135 litres (d) Data insufficient
- 26** A and B are two alloys of copper and tin prepared by mixing the respective metals in the ratio of 5 : 3 and 5 : 11 respectively. If the alloys A and B are mixed to form a third alloy C with an equal proportion of copper and tin, what is the ratio of alloys A and B in the new alloy C?
 (a) 3 : 5 (b) 4 : 5 (c) 3 : 2 (d) 2 : 3
- 27** A hotel incurs two types of expenses, one which is fixed and others depend upon no. of guests. When there are 10 guests, total expenses of hotel are ₹ 6000. Also when there are 25 guests average expenses per guests are ₹ 360? What is the total expenses of hotel when there are 40 guests?
 (a) ₹ 8000 (b) ₹ 12000
 (c) ₹ 15500 (d) none of these
- 28** The ratio of third proportional to 21 and 42 and mean proportional to 16 and 49 is :
 (a) 3 : 1 (b) 2 : 3 (c) 4 : 3 (d) 1 : 3
- 29** The period of the pendulum is directly proportional to the square root of the length of the string. The period of such a pendulum with string of length 16 cm is 52 seconds. Find the length of the string if the period is 65 seconds :
 (a) 4.5 cm (b) 5 cm
 (c) 6 cm (d) none of these
- 30** For any two numbers m, n ; $(m + n) : (m - n) : mn = 7 : 1 : 60$, then find the value of $\frac{1}{m} : \frac{1}{n}$
 (a) 4 : 3 (b) 8 : 6 (c) 3 : 4 (d) 7 : 8
- 31** ₹ 960 were distributed among A, B, C and D in such a way that C and D together gets half of what A and B together gets and C gets one-third amount of B. Also D gets $\frac{5}{3}$ times as much as C. What is the amount of A?
 (a) ₹ 240 (b) ₹ 280
 (c) ₹ 320 (d) data insufficient

- 32** Find the value of $\frac{p^2 + q^2}{r^2 + s^2}$, if $p : q :: r : s$.
- (a) $\frac{1}{4}$ (b) $\frac{1}{9}$ (c) $\frac{ps}{rq}$ (d) $\left(\frac{p-q}{r-s}\right)^2$
- 33** Find the value of $\frac{p^2 + q^2}{r^2 + s^2}$, if $p : q :: r : s$.
- (a) $\frac{1}{4}$ (b) $\frac{pq}{rs}$
 (c) $\left(\frac{p+q}{r+s}\right)^2$ (d) Both (b) and (c)
- 34** The speeds of rickshaw, car and scooter are in the ratio of 3 : 5 : 6. What is the ratio of time taken by each one of them for the same distance?
- (a) 6 : 5 : 3
 (b) 10 : 6 : 5
 (c) 12 : 7 : 6
 (d) Data insufficient
- 35** Divide ₹ 6940 in such a way that A gets $\frac{2}{3}$ rd of what B gets and B gets $\frac{3}{5}$ th of what C gets? What is the share of A and B together?
- (a) ₹ 1982 (b) ₹ 1388
 (c) ₹ 3470 (d) none of these
- 36** The ratio of age of A and B is $x : y$. If A's age is increased by 3 years and B's age is increased by 2 years then new ratio of their ages becomes 24 : 25. Given that the sum of their actual ages is 93 years. Find the actual ratio of their ages.
- (a) 21 : 22 (b) 42 : 45
 (c) 45 : 48 (d) Can't be determined
- 37** $a : b = 4 : 9$ if 4 is added to both of the numbers then the new ratio becomes 21 : 46. What is the difference between a and b?
- (a) 80 (b) 100
 (c) 125 (d) 130
- 38** The ratio of ages of Rahul and Deepesh is 3 : 5. 10 years later this ratio becomes 5 : 7. What is the present age of Deepesh?
- (a) 20 years (b) 50 years
 (c) 25 years (d) 40 years
- 39** When 5 is added to the numerator and denominator both of a (positive) fraction, then the new ratio of numerator to denominator becomes 11 : 15. What is the original ratio?
- (a) 17 : 25 (b) 3 : 5
 (c) 28 : 40 (d) None of these
- 40** Five numbers a, b, c, d and e are in the ratio of 2 : 3 : 5 : 8 : 9 and their sum is 162. Find the average of all these numbers
- (a) 27 (b) 30
 (c) 32.4 (d) Can't be determined
- 41** 6 pumps of Kirlosker can fill a tank in 7 days and 2 similar pumps of USHA can fill the same tank in 18 days. What is the ratio of the efficiency of a Kirlosker pump and a USHA pump?
- (a) 6 : 7 (b) 7 : 6
 (c) 7 : 54 (d) can't be determined
- 42** 16 persons can reap $\frac{1}{5}$ th field in 6 days. How many persons (with same efficiency) are required to reap rest of the field in 8 days?
- (a) 27 (b) 54
 (c) 48 (d) 64
- 43** The LCM of two numbers is 210 and their ratio is 2 : 3. The sum of these numbers is :
- (a) 210 (b) 175
 (c) 315 (d) can't be determined
- 44** What number must be subtracted from each of the numbers 53, 21, 41, 17 so that the remainders are in proportion?
- (a) 1 (b) 3
 (c) 5 (d) none of these
- 45** The angles of a triangle are in the ratio of 2 : 3 : 4. Find the measurement of greatest angle :
- (a) 30° (b) 60°
 (c) 100° (d) 80°
- 46** In a wallet the ratio of 25 paise, 50 paise and ₹ 1 coins are in the ratio of 12 : 4 : 3, which amounts to ₹ 600. Find the no. of coins of 25 paise :
- (a) 200 (b) 225
 (c) 275 (d) none of these
- 47** $(x - a) : (x - b) : (x - c) = 11 : 9 : 5$, where $x = \frac{a + b + c}{2}$. What is the ratio of a, b, c ?
- (a) 10 : 8 : 7 (b) 3 : 5 : 9 (c) 7 : 8 : 10 (d) 6 : 5 : 3
- 48** Petrol is 7 times heavy than Kerosene and Castrol mobil is 18 times as heavy as Kerosene. What should be the ratio of petrol and mobil in the new mixture to get the mixture which must be 11 times as heavy as kerosene?
- (a) 3 : 4 (b) 7 : 4
 (c) 9 : 19 (d) 9 : 10
- 49** A girl buys 2 pigeons for ₹ 182. She sells one at a loss of 5% and another at a profit of 8%. But she neither gains nor loses on the whole. Find the price of pigeon which has sold at a profit :
- (a) ₹ 112 (b) ₹ 85
 (c) ₹ 70 (d) can't be determined
- 50** The ratio of prices of Cello and Rotomac pens in 2000 were in the ratio of 3 : 5. In 2005 the price of Cello pen trebles itself and the price of Rotomac pen is increased by ₹ 100, then the new ratio of prices of the same pens becomes 4 : 5. What was the original price of the Rotomac pen in 2000?
- (a) ₹ 60 (b) ₹ 80
 (c) ₹ 100 (d) ₹ 120

- 51** A goldsmith has 361 rings of gold. He sells some of them at a loss of 4% and rest at a profit of 15% making overall profit of 8%. Find the no. of rings sold at a profit of 15% :
 (a) 171 (b) 133
 (c) 218 (d) 228
- 52** Michel travelled from New York to New Jersey covering total distance of 250 mile in 8 h partly by car at 30 mile/h and rest by train at 35 mile/h. The distance travelled by car is :
 (a) 150 mile (b) 80 mile
 (c) 220 mile (d) 180 mile
- 53** A rabbit takes 22 leaps for every 17 leaps of cat and 22 leaps of a rabbit are equal to 17 leaps of the cat. What is the ratio of the speeds of rabbit and cat?
 (a) 1 : 1 (b) 484 : 289
 (c) 17 : 22 (d) none of these
- 54** ₹ 171 are divided among four friends in the ratio of $\frac{1}{3} : \frac{1}{4} : \frac{1}{5} : \frac{1}{6}$. What is the amount of the person who got the greatest share?
 (a) 14 (b) 40
 (c) 36 (d) 60
- 55** 10 years ago the age of Karishma was $\frac{1}{3}$ rd of the age of Babita. 14 years hence the ratio of ages of Karishma and Babita will be 5 : 9. Find the ratio of their present ages :
 (a) 13 : 29 (b) 11 : 27
 (c) 29 : 17 (d) 13 : 25
- 56** The ratio of numerator to a denominator of a fraction is $\frac{1}{5}$ when x and $5x$ are added to the numerator and denominator respectively to the given fraction then the ratio of the new fraction will be :
 (a) 1 : 1 (b) 1 : 25 (c) 1 : 5 (d) 2 : 7
- 57** x varies directly as y and x varies inversely as the square of z . When $y = 75$ and $x = 6$, then $z = 5$. Find the value of x when $y = 24$ and $z = 4$:
 (a) 1 (b) 2 (c) 3 (d) 4
- 58** x varies directly as $(y^2 + z^2)$. At $y = 1$ and $z = 2$, the value of x is 15. Find the value of z , when $x = 39$ and $y = 2$:
 (a) 2 (b) 3 (c) 4 (d) 6
- 59** Weight of a sumo is jointly varies as his height and his age. When height is 1.2 m and age is 20 years his weight is 48 kg. Find the weight of the sumo when his height is 1.5 metre and age is 30 years :
 (a) 60 kg (b) 72 kg (c) 90 kg (d) 58 kg
- 60** If $(a + b) : (b + c) : (c + a) = 5 : 6 : 9$ and $a + b + c = 10$. What is the value of c :
 (a) 2 (b) 3 (c) 5 (d) 7
- 61** A, B and C have amounts in the ratio of 3 : 4 : 5. First B gives $\frac{1}{4}$ th to A and $\frac{1}{4}$ th to C then C gives $\frac{1}{6}$ th to A. Find the final ratio of amount of A, B and C, respectively :
 (a) 4 : 3 : 5 (b) 5 : 4 : 3
 (c) 6 : 4 : 2 (d) 5 : 2 : 5

LEVEL 02 > HIGHER LEVEL EXERCISE

- 1** A tin contains a mixture of Dew and Sprite in the ratio of 7 : 3 and another tin contains the Dew and Sprite in the ratio of 5 : 4. In what proportion should the solution of two tins be mixed to achieve a perfect proportion of 2 : 1 (in which Dew is 2 times that of sprite).
 (a) 10 : 3 (b) 4 : 1
 (c) 3 : 10 (d) 3 : 1
- 2** There are two containers A and B filled with mentha oil with different prices and their volumes are 140 litres and 60 litres respectively. Equal quantities are drawn from both A and B in such a manner that the oil drawn from A is poured into B and the oil drawn from B is poured into A. The price per litre becomes equal in both A and B. How much oil is drawn from each of A and B :
 (a) 40 litre (b) 21 litre
 (c) 42 litre (d) can't be determined
- 3** A three digit number is such that this number itself is divisible by the sum of its digits. The sum of hundreds and unit digit is 6 while the sum of the tens and unit digit is 5. What is the ratio of unit and tens digit :
 (a) 1 : 2 (b) 2 : 3 (c) 3 : 4 (d) 2 : 7
- 4** Sachin bought 1.5 kg fresh grapes. The ratio of water is to pulp was 4 : 1. When his naughty child crushed these grapes, then some water get wasted. Now the ratio of water is to pulp is 3 : 2. What is the total amount of the crushed grapes?
 (a) 0.5 kg (b) 1 kg
 (c) 0.75 kg (d) none of these
- 5** A man bought 9 mangoes for a rupee and sold them at 6 mangoes for a rupee. What is the ratio of profit to the cost price?
 (a) $\frac{3}{10}$ (b) $\frac{3}{2}$
 (c) $\frac{1}{2}$ (d) none of these
- 6** Half of the volume of Petrol and kerosene mixture of ratio 7 : 5 is converted into a mixture of ratio 3 : 1 by the substitution (or replacement) method. While the mixture of ratio 7 : 5 was formed from the mixture of 7 : 3 by adding the Kerosene in it. If 240 litres petrol is required in the replacement method, what is the total amount of Kerosene was added to prepare the mixture of 7 : 5 ?
 (a) 100 litres (b) 400 litres
 (c) 50 litres (d) 200 litres

- 7 Sometimes ago in a Cinema hall a blockbuster movie was being shown. Due to excessive demand of the show, manager of the cinema hall increased the prices of tickets in all the three categories. 25% in first class, 12.5% in special class and 40% in balcony. If the collection of one show was ₹ 218925, then find the collection from the balcony only given that the ratio of price of ticket (increased price) for first class, special class and balcony is 5 : 9 : 14 :
 (a) 109462.5 (b) 58800
 (c) 14400 (d) data insufficient
- 8 In a mixture of petrol and Kerosene petrol is only 99 litres. if this same quantity of petrol would be presented in another mixture of petrol and Kerosene where total volume would be 198 litres less than the actual mixture then the concentration of petrol in the actual mixture would have been 13.33% point less than that of the new mixture. What is the concentration of petrol in actual mixture?
 (a) 20% (b) 16.66%
 (c) 26.66% (d) 8.33%
- 9 A drum of 20 litres is filled with milk. A milkman has only two measuring vessels of 3 litres and 5 litres without any calibration. He has to measure four litres of milk for a customer without using any other vessel. Minimum how many operations are required for this work, where an operation is counted if the milk is transferred from one vessel to another vessel?
 (a) 5 (b) 6
 (c) 8 (d) 11
- 10 The ages of Vinay, Varsha, Veera and Vikram are in arithmetic progression, but not in order. The ratio of ages of Vinay and Varsha is 6 : 5 and Veera is to Vikram is 7 : 8. Two years later the age of Varsha and Vikram will be 2 : 3. Find the ratio of ages of Vinay and Veera :
 (a) 7 : 6 (b) 5 : 8
 (c) 6 : 7 (d) 8 : 9
- 11 A container is filled with the mixture of milk and water. The ratio of milk and water is same. Bobby and Sunny increases the concentration to 60%. Bobby makes it by adding the milk and Sunny makes it by replacing the mixture with milk. What is the percentage of milk added by Bobby to that of milk replaced by Sunny :
 (a) 100% (b) 120%
 (c) 133.33% (d) None of these
- 12 There are two vessels A and B containing 25 litres each of pure milk and pure water respectively. 5 litres of milk from A is taken and poured into B, then 6 litres of mixture from B is taken and poured in A. What is the ratio of water in A and B respectively :
 (a) 4 : 5 (b) 1 : 4
 (c) 5 : 4 (d) 2 : 3
- 13 The ratio of age between A and B is 6 : 5 and the age of each C and D is $\frac{9}{10}$ times that of B. Age of F is less than A but greater than B. The ratio of ages between B and E is 2 : 3 also age of A is 3 years less than E. What is the ratio of ages of A and F if all the ages are in integers?
 (a) 12 : 11 (b) 9 : 7
 (c) 24 : 19 (d) 12 : 13
- 14 The ratio of students in a coaching preparing for B.Tech and MBA is 4 : 5. The ratio of fees collected from each of B. Tech and MBA student is 25 : 16. If the total amount collected from all the students is 1.62 lakh, what is the total amount collected from only MBA aspirants?
 (a) ₹ 62000 (b) ₹ 72000
 (c) ₹ 80000 (d) none of these
- 15 The cost of the marble varies directly with square of its weight. Marble is broken into 3 parts whose weights are in the ratio 3 : 4 : 5. If marble had been broken into three equal parts by weight then there would have been a further loss of ₹ 1800. What is the actual cost of the original (or unbroken) marble?
 (a) ₹ 3600 (b) ₹ 10800
 (c) ₹ 2160 (d) none of these
- Directions** (for Q. Nos. 16 and 17) Four friends A, B, C and D have some money among them one day they decided to equate the money, so first A gave B what B had initially, then B gave C what C had initially. Again C gave D what D had initially and finally D had doubled the money of A. Thus each of them had equal sum of ₹ 48.
- 16 What was the initial amount of B?
 (a) ₹ 36 (b) ₹ 54
 (c) ₹ 45 (d) ₹ 42
- 17 What was the amount with C after second transaction?
 (a) ₹ 45 (b) ₹ 69 (c) ₹ 72 (d) ₹ 84
- 18 Hari and Murli have 24 cows and 30 cows respectively. Both of them together hired a grazing field for the whole month of November. Initially Hari's cows used for grazing then for the remaining days of the month Murli's cows grazed it. If Hari has paid ₹ 3500 and Murli has paid ₹ 5000 for grazing then for how many days Hari used the grazing field :
 (a) 14 (b) 16 (c) 21 (d) 20
- 19 The speeds of scooter, car and train are in the ratio of 1 : 4 : 16. If all of them covers equal distance then the ratio of time taken/velocity for the each of the vehicle is :
 (a) 256 : 16 : 1 (b) 1 : 4 : 16
 (c) 16 : 4 : 1 (d) 16 : 1 : 4
- 20 Radhika purchased one dozen bangles. One day she slipped on the floor fell down. What cannot be the ratio of broken to unbroken bangles :
 (a) 1 : 2 (b) 1 : 3 (c) 2 : 3 (d) 1 : 5

- 21** If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then the value of $\left(\frac{a^n p + c^n q + e^n r}{b^n p + d^n q + f^n r}\right)^{1/n}$:
- (a) $\frac{ad}{bc}$ (b) $\frac{af}{be}$
 (c) $\frac{ck}{dk}$ (d) none of these
- 22** At a casino in Mumbai, there are 3 tables A, B and C. The pay-offs at A is 10 : 1, at B is 20 : 1 and at C is 30 : 1. If a man bets ₹ 200 at each table and win at two of the tables, what is the maximum and minimum difference between his earnings can be?
- (a) ₹ 2500 (b) ₹ 2000
 (c) ₹ 4000 (d) none of these
- 23** If $m = \frac{4pq}{p+q}$, then the value of $\frac{m+2p}{m-2p} + \frac{m+2q}{m-2q}$:
- (a) 2 (b) 4
 (c) $\frac{2mpq}{(p+q)}$ (d) none of these
- 24** The ratio of volumes of two cubes is 8 : 27. What is the ratio of surface area of these cubes respectively?
- (a) 2 : 3 (b) 4 : 9
 (c) 8 : 19 (d) 9 : 4
- 25** Dudheri Lal has two jars. Jar A is completely filled with milk and another jar B is totally empty. Before selling the milk in a town he transferred some milk in to the empty jar B then he fill the jar A with water. Once again he transferred the mixture of milk from A to B so that B is completely filled. Which one of the following is correct?
- (a) Concentration of milk in B cannot be less than 75%
 (b) Concentration of milk in B cannot be greater than 75%
 (c) Concentration of milk is always 75%
 (d) none of the above
- 26** In ABC corporation there are some management trainees. These trainees are divided into 3 groups A, B and C for 3 different projects in the ratio of 3 : 4 : 5 respectively, where P, Q, R are the projects-in-charge of A, B, C respectively. The difference between the no. of trainees in A and C is not greater than 3. Also P, Q, R belongs to the group of trainees. The no. of assistant of Q is less than the no. of assistance of R by :
- (a) 33.33% (b) 20%
 (c) 25% (d) 16.66%
- 27** If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \frac{e}{f} = \frac{1}{3}$, then find the value of $\left(\frac{a+b+c+d+e}{b+c+d+e+f}\right)$:
- (a) $\frac{1}{81}$ (b) $\frac{1}{27}$
 (c) $\frac{1}{3}$ (d) 1
- 28** An engine can move at the speed of $\frac{20}{3}$ m/s without any wagon attached. Reduction in the speed of the train is directly proportional to the square root of the no. of wagons attached to the engine. When there are only four wagons attached its speed is $\left(\frac{50}{9}\right)$ m/s. The greatest no. of wagons with which the engine can move is :
- (a) 144 (b) 143
 (c) 12 (d) none of these
- 29** Mrs. Annapurna per day sells exactly four quintal sugar at ₹ 2000 per quintal getting the profit of ₹ 25%. Since she mixes two varieties of sugar, one costs ₹ 14 per kg and another costs ₹ 22 per kg. One day due to huge demand in market she had only 3 quintal of the required mixture so she purchased the sugar costs ₹ 17 per kg at ₹ 18 per kg from the wholeseller on that day and then she mixed 300 kg mixture with 100 kg sugar costing ₹ 18 per kg to fulfill the demand of the market selling at the same price. How much percent less does she gain that she would have gained, if she had sufficient quantity of usual mixture of sugar?
- (a) 12.5% (b) 18.18%
 (c) 62.5% (d) can't be determined
- 30** Two vessels P and Q contain 'a' litres of petrol and 'b' litres of Kerosene respectively. 'c' litres of petrol and same quantity of Kerosene is taken out and then transferred to Q and P respectively. This process is repeated several times. If after the first operation the quantity of petrol or Kerosene in either of P and Q does not change. What is the value of 'c'?
- (a) $\frac{ab}{(a-b)}$ (b) $\frac{2ab}{(a+b)}$
 (c) $\frac{ab}{(a+b)}$ (d) $\left(\frac{a}{b}\right)^2$
- 31** Arvind Singh purchased a 40 seater bus. He started his services on route no. 2 (from Terhipuliya to Charbagh with route length of 50 km). His profit (P) from the bus depends upon the no. of passengers over a certain minimum number of passengers 'n' and upon the distance travelled by bus. His profit is ₹ 3600 with 29 passengers in the bus for a journey of 36 km and ₹ 6300 when there are 36 passengers. travelled for 42 km. What is the minimum no. of passengers are required so that he will not suffer any loss?
- (a) 12 (b) 20
 (c) 18 (d) 15
- 32** Three cats are roaming in a zoo in such a way that when cat A takes 5 steps, B takes 6 steps and cat C takes 7 steps. But the 6 steps of A are equal to the 7 steps of B and 8 steps of C. What is the ratio of their speeds :
- (a) 140 : 144 : 147 (b) 40 : 44 : 47
 (c) 15 : 21 : 28 (d) 252 : 245 : 240

- 33** In a family there were n people. The expenditure of rice per month in this family is directly proportional to the 5 times the square of no. of people in the family. If the elder son left the family to study in USA there was decrease in consumption of 95 kg rice per month. What is the value of n ?
- (a) 5 (b) 12
(c) 9 (d) 10
- 34** One day in summer I wanted to chill me out, I went to a cool corner. I gave him a note of ₹ 10 and asked for a coke costing ₹ 5 per jar and he did so, but he returned me ₹ 5, in the denomination of ₹ 1, 50 paise and 25 paise. What could be the ratio of no. of coins of ₹ 1, 50 paise and 25 paise respectively :
- (a) 2 : 3 : 1 (b) 1 : 7 : 2
(c) 6 : 1 : 3 (d) 2 : 1 : 2
- 35** If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}$ and $a+b+c \neq 0$ then the value of $\frac{b}{a+b+c}$ is :
- (a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) 1
- 36** The ratio of the density of 3 kinds of petrol P_1 , P_2 and P_3 is 9 : 7 : 5. The density of P_1 is 18 gm/cc and P_1 , P_2 and P_3 are mixed in the ratio of 6 : 5 : 4 by weight. If a litre of P_3 cost ₹ 40, then find the cost of P_3 in 450 kg of mixture of P_1 , P_2 and P_3 :
- (a) ₹ 380 (b) ₹ 480
(c) ₹ 355 (d) ₹ 448
- 37** Three persons Amar, Akbar and Anthony agree to pay their hotel bills in the ratio of 3 : 4 : 5. Amar pays the first day's bill which amounts to ₹ 26.65, Akbar pays the second day's bill which amounts to ₹ 42.75 and C pays the third day's bill which amounts to ₹ 53.00. When they settle their accounts, which of the following happens?
- (a) Amar gives ₹ 3 to C
(b) Akbar gives ₹ 2 to Amar
(c) Amar gives Akbar ₹ 1.95 and ₹ 2 to Anthony
(d) none of the above
- 38** Find the value of x if $(14x - 4) : (8x - 1) = (3x + 8) : (9x + 5)$:
- (a) 1 (b) $\frac{1}{2}$
(c) $\frac{3}{4}$ (d) none of these
- 39** Pooja, Shipra and Monika are three sisters. Pooja and Shipra are twins. The ratio of sum of the ages of Pooja and Shipra is same as that of Monika alone. Three years earlier the ratio of age of Pooja and Monika was 2 : 7. What will be the age of Shipra 3 years hence?
- (a) 21 years (b) 16 years
(c) 8 years (d) 12 years
- 40** A couple got married 9 years ago when the age of wife was 20% less than her husband. 6 years from now the age of wife will be only 12.5% less than her husband. Now they have six children including single, twins and triplets and the ratio of their ages is 2 : 3 : 4 respectively. What can be the maximum possible value for the present age of this family?
- (a) 110 years (b) 103 years
(c) 105 years (d) 83 years
- 41** The price of a necklace varies directly as the no. of pearls in it. Also, it varies directly as the square root of radius of a pearl. The price of a necklace was ₹ 150. When it had 75 pearls each of radius 1 cm. Find the radius of the pearl of a necklace having 100 pearls whose cost is ₹ 600.
- (a) 2 (b) 9 (c) 3 (d) 4
- 42** The price of a book varies directly as the no. of pages in it and inversely as the time periods in years that have elapsed since the date of purchasing. Two books cost the same, however, the no. of pages in the first book is triple of the second book. If the first book is sold on 18 years ago, how long ago was the second book sold?
- (a) 54 years (b) 9 years
(c) 6 years (d) 3 years
- 43** Akbar and Birbal who purchased the shares for the cost of their basic salaries which are in the ratio of 5 : 6. later on company gave them 40 additional shares to each, due to which the ratio changed to 7 : 8. If the worth of each share is ₹ 75, what is the basic salary of the person who got less shares?
- (a) ₹ 10500 (b) ₹ 7500
(c) ₹ 8800 (d) ₹ 9000
- 44** Distance covered by a train is directly proportional to the time taken and also it varies directly as the square root of fuel used and varies inversely as the no. of wagons attached to it. A train covers 192 km journey in 20 hours when there are 10 wagons attached to it and total fuel consumption was 256 litre of diesel. Find the consumption of fuel per km when a train goes 200 km in 25 hours with 15 wagons attached to it :
- (a) 1.5 l/km (b) 2 l/km
(c) 2.8 l/km (d) 20 l/km
- 45** At Sahara shopping centre, a person can purchase as much articles at a time as his or her age that is a person of n years age can purchase only n similar articles at a time. Amisha is younger to her elder brother who has just entered into his twenties. One day Amisha went to the Sahara shopping centre, she purchased same toffees at a particular rate on the ground floor. But when she reached on third floor she found that she could purchase double the no. of toffees with the same amount as she had spent on the ground floor. Also to purchase the same no. of toffees on the third floor she had to spend ₹ 2 less than that of on the first floor. How many toffees did she buy?
- (a) 6 (b) 12 (c) 18 (d) 15

- 46** A contractor deployed some men to plant 1800 trees in a certain no. of days. But in $\frac{1}{3}$ rd of the planned time 120 plants could be less planted so to fulfill the target for the rest of the days everyday 20 more plants were planted. Thus it saved one day out of the initially planned no. of days. How many plants he planned to plant each day initially?
(a) 180 (b) 100 (c) 120 (d) 160
- 47** A and B have to write 810 and 900 pages respectively in the same time period. But A completes his work 3 days ahead of time and B completes 6 days ahead of time. How many pages did A write per hour if B wrote 21 pages more in each hour?
(a) 45 (b) 72 (c) 54 (d) 100
- 48** Three friends A, B and C decided to share the soda water with D, who had no soda water. A contributed 2 tumbler more than that of B and B contributed 1 tumbler more than that of C and then all of them had equal amount of soda

water. In turn D paid money, which was divided among A, B and C in the ratio of their contribution to D. Thus A had gotten thrice as much money as B had gotten. The price of each tumbler of soda water was ₹ 15 and each transaction was integral in numbers either the sharing of money or contribution of soda water. What was the sum of money that B had gotten?

- (a) ₹ 15 (b) ₹ 18
(c) ₹ 22.5 (d) none of these
- 49** In Maa Yatri Temple every devotee offers fruits to the orphans. Thus every orphan receives bananas, oranges and grapes in the ratio of 3 : 2 : 7 in terms of dozen. But the weight of a grape is 24 gm and weight of a banana and an orange are in the ratio of 4 : 5, while the weight of an orange is 150 gm. Find the ratio of all the three fruits in terms of weight, that an orphan gets :
(a) 90 : 75 : 42 (b) 180 : 150 : 82
(c) 75 : 42 : 90 (d) none of these

LEVEL 03 > Final Round

- 1** In two alloys the ratio of Iron and copper is 4 : 3 and 6 : 1 respectively. If 14 kg of the first alloy and 42 kg of the second alloy are mixed together to form a new alloy, then what will be the ratio of copper to iron in the new alloy :
(a) 11 : 3 (b) 11 : 8
(c) 8 : 11 (d) none of these
- 2** In a zoo, there are rabbits and pigeons. If heads are counted, there are 340 heads and if legs are counted there are 1060 legs. How many pigeons are there?
(a) 120 (b) 150
(c) 180 (d) 170
- 3** 6 litre is taken out from a vessel full of Kerosene and substituted by pure petrol. This process is repeated two more times. Finally the ratio of petrol and Kerosene in the mixture becomes 1701 : 27. Find the volume of the original solution :
(a) 14 litre (b) 16 litre
(c) 8 litre (d) 42 litre
- 4** In three vessels, each of 25 litres capacity, mixture of milk and water is filled. The ratio of milk and water are 3 : 1, 2 : 3, 4 : 3 in the respective vessels. If all the three vessels are emptied into a single large vessel, then what will be the ratio of water to milk in the resultant mixture?
(a) 179 : 241 (b) 197 : 214
(c) 219 : 117 (d) 179 : 234
- 5** Two liquids are mixed in the ratio 4 : 3 and the mixture is sold at ₹ 20 with a profit of $33\frac{1}{3}\%$. If the first liquid is costlier than the second by ₹ 7. Find the sum of costs of both the liquids :
(a) ₹ 11 (b) ₹ 29 (c) ₹ 35 (d) ₹ 70
- 6** Two alloys made up of copper and tin. The ratio of copper and tin in the first alloy is 1 : 3 and in the second alloy it is 2 : 5. In what ratio the two alloys should be mixed to obtain a new alloy in which the ratio of tin and copper be 8 : 3 ?
(a) 3 : 5 (b) 4 : 7 (c) 3 : 8 (d) 5 : 11
- 7** Three vessels having volumes in the ratio of 2 : 3 : 5 are full of a mixture of water and milk. In the first vessel ratio of water and milk is 1 : 3, in second 2 : 3 and in third vessel, 2 : 5. If all the three vessels were poured out in a large container, what is the resulting ratio of milk and water?
(a) 43 : 96 (b) 438 : 962
(c) 348 : 962 (d) 962 : 438
- 8** The number of oranges in three baskets are in the ratio of 3 : 4 : 5. In which ratio the no. of oranges in first two baskets must be increased so that the new ratio becomes 5 : 4 : 3 ?
(a) 1 : 3 (b) 2 : 1
(c) 3 : 4 (d) 2 : 3
- 9** A vessel of capacity 2 litre has 25% alcohol and another vessel of capacity 6 litre had 40% alcohol. The total liquid of 8 litre was poured out in a vessel of capacity 10 litre and thus the rest part of the vessel was filled with the water. What is the new concentration of mixture?
(a) 31% (b) 71%
(c) 49% (d) 29%
- 10** Alloy A contains 40% gold and 60% silver. Alloy B contains 35% gold and 40% silver and 25% copper. Alloys A and B are mixed in the ratio of 1 : 4. What is the ratio of gold and silver in the newly formed alloy is?
(a) 20% and 30% (b) 36% and 44%
(c) 25% and 35% (d) 49% and 36%

✓ Level 02 Higher Level Exercise

1 (a)	2 (c)	3 (b)	4 (c)	5 (c)	6 (d)	7 (d)	8 (a)	9 (c)	10 (c)
11 (d)	12 (b)	13 (a)	14 (b)	15 (d)	16 (c)	17 (d)	18 (a)	19 (a)	20 (c)
21 (c)	22 (c)	23 (a)	24 (b)	25 (a)	26 (c)	27 (c)	28 (b)	29 (a)	30 (c)
31 (d)	32 (a)	33 (d)	34 (b)	35 (b)	36 (b)	37 (c)	38 (b)	39 (c)	40 (b)
41 (b)	42 (c)	43 (b)	44 (b)	45 (b)	46 (b)	47 (c)	48 (a)	49 (a)	

✓ Level 03 Final Round

1 (d)	2 (b)	3 (c)	4 (a)	5 (b)	6 (b)	7 (d)	8 (b)	9 (d)	10 (b)
11 (c)	12 (c)	13 (c)	14 (b)	15 (b)	16 (d)				

Hints & Solutions

✓ Level 01 Basic Level Exercise

1

$$a : b :: c : d$$

$$a^2 + b^2 + c^2 + d^2 = 50$$

$$b + c = 5$$

and

$$a : b = 1 : 3$$

If consider $a : b = 1 : 3$ as it is, then

$$c = 2 \quad (5 - 3 = 2)$$

and

$$d = 6 \quad (\because a : b :: c : d)$$

$$a^2 + b^2 + c^2 + d^2 = 1^2 + 3^2 + 2^2 + 6^2 = 50$$

Hence, the presumed values are correct.

$$\text{Thus, the average of } a, b, c \text{ and } d = \frac{a + b + c + d}{4}$$

$$= \frac{1 + 3 + 2 + 6}{4} = 3$$

Hence (b) is correct.

☞ Alternatively Assume option (b)

$$\therefore \frac{a + b + c + d}{4} = 3$$

$$\Rightarrow a + b + c + d = 12$$

$$\text{Now } \because b + c = 5$$

$$\Rightarrow a + d = 7$$

$$\text{Again } a : b = 1 : 3$$

$$\Rightarrow c : d = 2 : 6$$

Now verify that $a^2 + b^2 + c^2 + d^2 = 50$. Since it is correct.

Hence option (b) is correct.

☞ NOTE It can also be solved by forming quadratic equations.

2

$$\frac{a}{b} = \frac{a+b}{a}$$

$$\Rightarrow a^2 = ab + b^2$$

$$\Rightarrow a^2 - b^2 - ab = 0$$

$$\frac{a}{b}$$

Let $b = 1$, then $a : b = a : 1$ (by putting $b = 1$)

$$\therefore a^2 - a - 1 = 0$$

$$\Rightarrow a = \frac{1 \pm \sqrt{5}}{2} \quad \left(\begin{array}{l} \text{Solving quadratic equation} \\ \text{by Sridharacharya's formula} \end{array} \right)$$

$$\therefore a = \frac{1 + \sqrt{5}}{2} \quad (\text{negative value can't be considered})$$

$$a : b = \frac{1 + \sqrt{5}}{2} : 1$$

or

$$a : b = (1 + \sqrt{5}) : 2$$

$$\text{Therefore, } \frac{b}{a+b} = \frac{2}{1 + \sqrt{5} + 2} = \frac{2}{3 + \sqrt{5}}$$

Hence option (b) is correct.

3 The best way is to go through options

$$\frac{x+y}{x-y} = \frac{8+2}{8-2} = \frac{10}{6} = \frac{5}{3} \quad (\text{verified})$$

$$\text{Again } \frac{x}{y+2} = \frac{8}{2+2} = 2 \quad (\text{verified})$$

Hence option (d) is correct.

4 Best way is to go through options

$$\text{and } \frac{a^3 + b^3}{a^3 - b^3} = \frac{5^3 + 2^3}{5^3 - 2^3} = \frac{125 + 8}{125 - 8} = \frac{133}{117}$$

Hence option (c) is correct.

5

$$H : S = 1 : 1$$

and

$$S : G = 2 : 3$$

$$H : P = 1 : 2$$

$$\therefore H : S : G : P = 2 : 2 : 3 : 4$$

$$\text{Therefore } \frac{2x + 2x + 3x + 4x}{4} = \frac{11x}{4} = 55$$

$$\Rightarrow x = 20$$

Marks in History = 40

Sociology = 40

Geography = 60

Philosophy = 80

Hence, only in two subjects he scored 60% or above.

Hence option (b) is correct.

- 6 Let the incomes of A and M is $2x$ and $3x$.

Let the savings of A be K , then the expenditure of M be K

Also expenditure of A = $2x - K$

Given $(2x - K) + K = 8000 \Rightarrow x = 4000$

$\therefore$ Total income of

$$A \text{ and } B = 2x + 3x = 5x = 5 \times 4000 = 20000$$

$\therefore$ Total savings of A and B = $20000 - 8000 = ₹ 12,000$

- 7 Profit of Hutch

Profit of Essar

$$= \frac{\text{time period} \times \text{amount of Hutch invested}}{\text{time period} \times \text{amount of Essar invested}}$$

$$\frac{6}{17} = \frac{5 \times K}{12 \times 1275}$$

$$\Rightarrow K = \frac{6 \times 12 \times 1275}{17 \times 5} = 1080$$

- 8 Minimum number of chocolates are possible when he purchases maximum number of costliest chocolates.

Thus $2 \times 5 + 5 \times 2 = ₹ 20$

Now ₹ 100 must be spend on 10 chocolates as $100 = 10 \times 10$.

Thus minimum number of chocolates = $5 + 2 + 10 = 17$

- 9 Maximum number of chocolates are possible only when he purchases minimum number of costlier chocolates and maximum number of cheaper chocolates

$\therefore 2 \times 5 + 1 \times 10 = ₹ 20$

Now ₹ 100 must be spend on 50 chocolates as $100 = 2 \times 50$.

Thus maximum number of possible chocolates = $2 + 1 + 50 = 53$

- 10 Meretere $\rightarrow$ M, Teremere $\rightarrow$ T and Khabbu Singh $\rightarrow$ K

$$\frac{5+3}{3} = 2.66 \text{ (since all the 8 chocolates were shared by 3)}$$

It means M has given $5 - 2.66 = 2.33$ chocolates to K

and T has given $3 - 2.66 = 0.33$ chocolates to K

Thus M and T will receive the amount in the ratio of donations (i.e., share of chocolates)

$$\begin{array}{ccc} \text{M} & : & \text{T} \\ 2.33 & : & 0.33 \\ 2\frac{1}{3} & : & \frac{1}{3} \\ \frac{7}{3} & : & \frac{1}{3} \\ 7 & : & 1 \end{array}$$

So the M receives ₹ 14 and T receives ₹ 2

Thus the difference = ₹ 12

- 11 Share of a man, a woman and a boy = $7x$, $4x$ and $3x$

then the share of 4 men = $4 \times 7x = 28x$

then the share of 5 women = $5 \times 4x = 20x$

then the share of 2 boys = $2 \times 3x = 6x$

$$\begin{aligned} \text{Now, the share of all women} &= \frac{20x}{(28x + 20x + 6x)} \times 4536 \\ &= \frac{20}{54} \times 4536 = ₹ 1680 \end{aligned}$$

$$\text{Hence, the share of one woman} = \frac{1680}{5} = 336$$

- 12 Concentration of petrol in

$$\begin{array}{ccc} A & B & C \\ \frac{1}{2} & \frac{3}{5} & \frac{4}{5} \end{array}$$

Quantity of petrol taken from A = 1 litre out of 2 litre

Quantity of petrol taken from B = 1.8 litre out of 3 litre

Quantity of petrol taken from C = 0.8 litre out of 1 litre

Therefore total petrol taken out from

$$A, B \text{ and } C = 1 + 1.8 + 0.8 = 3.6 \text{ litre}$$

So, the quantity of Kerosene = $(2 + 3 + 1) - (3.6) = 2.4$ litre

$$\text{Thus, the ratio of petrol to Kerosene} = \frac{3.6}{2.4} = \frac{3}{2}$$

$$13 \quad T \propto \frac{\sqrt{l}}{\sqrt{g}} \Rightarrow T = k \sqrt{\frac{l}{g}}$$

$$\text{Therefore, } 3 = K \sqrt{\frac{9}{4}} \text{ or } 3 = K \frac{3}{2} \Rightarrow K = 2$$

$$\text{Again } T = K \sqrt{\frac{l}{g}} = 2 \times \sqrt{\frac{64}{16}} = 4$$

$$T = 4 \text{ seconds}$$

14 Pure	Cure	Lure
100%	40%	37.5%
$\frac{1}{1}$	$\frac{2}{5}$	$\frac{3}{8}$

	Milk	Water	Mixture
Cure	4l	10l	14l
Lure	6l	16l	22l
New mixture	Milk	Water	
	10l	26l	
	16l	26l	
	26l	26l	

$$\text{Required mixture } 1 : 1$$

Since in the required mixture the ratio of milk and water is 1 : 1 so she has to add up 16 litre of more milk (pure) to get it, for the fixed quantity of water.

$$15 \quad S : (M + J) = 5 : 7 \Rightarrow 7S = 5M + 5J \quad \dots(i)$$

$$J : (S + M) = 1 : 2 \Rightarrow 2J = S + M \quad \dots(ii)$$

By solving eqs. (i) and (ii), we get

$$S : M : J = 5 : 3 : 4$$

So

$$S : M = 5 : 3$$

16

	Male	Female
A	4x	3x
B	3y	2y
C	5z	4z

but

$$2y = 4z$$

 $\therefore$

	Male	Female
A	4x	3x
B	6z	4z
C	5z	4z

$$\text{Therefore, } 7x + 19z = 33$$

 but z can assume only one value i.e., $z = 1$

$$\text{Hence, } 7x + 19 = 33 \Rightarrow x = 2$$

 Thus the no. of female children in factory $A = 3x = 6$.

 17 The ratio of broken parts (by weight) = $3x : 4x : 5x$

Therefore value of broken parts of diamond

$$= (3x)^2 + (4x)^2 + (5x)^2 = 50x^2$$

$$\text{The value of original diamond} = (3x + 4x + 5x)^2 = 144x^2$$

$$\text{Therefore, loss in value} = 144x^2 - 50x^2 = 9.4 \text{ lakh}$$

$$\Rightarrow 94x^2 = 9.4 \text{ lakh}$$

$$\Rightarrow 94x^2 = 940000$$

$$\Rightarrow x^2 = 10000$$

$$\text{Hence, the actual value of the diamond} = 144x^2$$

$$= 144 \times 10000$$

$$= 14.4 \text{ lakh}$$

18.

$$10x + 20y + 100z = 1600$$

$$\text{Again since } x : y = 6 : 1$$

$$\therefore 60y + 20y + 100z = 1600$$

$$\Rightarrow 80y + 100z = 1600$$

$$\Rightarrow 4y + 5z = 80$$

$$\text{Putting } z = 1, 2, 3, 4, 5 \dots, \text{ we get at } z = 4, y = 15 \text{ (an integer)}$$

Hence, min. 4 coins of ₹ 1 will be there.

 19 Concentration of water in first vessel = $\frac{2}{5} = 40\%$

$$\text{Concentration of water in second vessel} = \frac{2}{7} = 28.57\%$$

By alligation

$$\begin{array}{ccc} \frac{2}{7} & & \frac{2}{5} \\ & \searrow \quad \swarrow & \\ & \frac{1}{3} & \\ & \swarrow \quad \searrow & \\ \left(\frac{2}{5} - \frac{1}{3}\right) & & \left(\frac{1}{3} - \frac{2}{7}\right) \\ \frac{1}{15} & : & \frac{1}{21} \\ 21 & : & 15 \\ 7 & : & 5 \end{array}$$

Therefore, ratio of first mixture to the second mixture = 5 : 7

20 LCM of 2, 3, 5, 8 = 120

$$\text{Therefore, } \frac{1}{2} \times 120 = 60$$

$$\frac{1}{3} \times 120 = 40$$

$$\frac{1}{5} \times 120 = 24$$

$$\frac{1}{8} \times 120 = 15$$

Therefore, minimum number of chocolates

$$= 60 + 40 + 24 + 15 = 139$$

21

$$A : B = 5 : 3$$

$$B : C = 5 : 8$$

$$\therefore A : B : C = 25 : 15 : 24$$

So, A is the most efficient.

22

$$A : B = 5 : 4 \Rightarrow 10 : 8$$

$$A : C = 2 : 3 \Rightarrow 10 : 15$$

$$\therefore A : B : C = 10 : 8 : 15$$

$$\therefore B : C = 8 : 15$$

23

	A	B	C
(Proportion of milk in mixture)	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{3}{7}$

$$\text{or } \frac{35}{105} : \frac{42}{105} : \frac{45}{105}$$

$$\text{Quantity of milk in new mixture} = 35 + 42 + 45 = 122l$$

$$\text{Quantity of water in new mixture} = (105 \times 3) - 122 = 193l$$

$$\text{Therefore, ratio of water is to milk} = 193 : 122$$

24

$$A : B = 8 : 9$$

$$B : C = 2 : 3$$

$$C : D = 9 : 13$$

$$A : B : C : D = 144x : 162x : 243x : 351x$$

But we need not to solve this, since we already know that

$$B : C = 2x : 3x$$

$$\therefore 2x : 3x :: 18 : k \Rightarrow k = 27 \text{ years}$$

25

Milk	Water	
$5x$	$3x$	(initially)
$(-25l)$	$(-15l)$	
$5x - 25$	$3x - 15$	(after selling 40l mixture)
	$(+15l)$	
$5x - 25$	$3x$	(after adding 15l water)
5	4	
:		
$\frac{5x - 25}{3x}$	$\frac{5}{4}$	$\Rightarrow x = 20$

$$\therefore \text{New quantity of mixture} = (5x - 25) + 3x = 135 \text{ litre}$$

26

A	B
C : T	C : T
5 : 3	5 : 11

$$\text{Concentration of copper in A} = \frac{5}{8}$$

Concentration of copper in B = $\frac{5}{16}$

By alligation

$$\begin{array}{ccc}
 \text{(B)} \frac{5}{16} & & \frac{5}{8} \text{ (A)} \\
 & \searrow \quad \swarrow & \\
 & \frac{1}{2} & \\
 & \swarrow \quad \searrow & \\
 \left(\frac{5}{8} - \frac{1}{2}\right) & & \left(\frac{1}{2} - \frac{5}{16}\right) \\
 \frac{2}{16} & : & \frac{3}{16} \\
 2 & : & 3
 \end{array}$$

So, the required ratio of A : B = 3 : 2 (Since B : A = 2 : 3)

$$\begin{aligned}
 27 \quad k + 10x &= 6000 \\
 k + 25x &= 9000 \quad (\because 25 \times 360 = 9000) \\
 \Rightarrow 15x &= 3000 \\
 \Rightarrow x &= 200 \quad \text{and} \quad k = 4000 \\
 \therefore k + 40x &= 4000 + 40 \times 200 = 12,000
 \end{aligned}$$

Where k is the fixed expenditure.

28 Third proportional of 21 and 42 is 84 and mean proportional to 16 and 49 is 28.

Therefore required ratio = $\frac{84}{28} = \frac{3}{1}$ or 3 : 1

$$\begin{aligned}
 29 \quad P &\propto \sqrt{l} \Rightarrow P = k\sqrt{l} \\
 \therefore 52 &= k\sqrt{16} \Rightarrow k = 13 \\
 \text{Again, } P &= k\sqrt{l} \\
 65 &= 13\sqrt{l} \\
 \Rightarrow l &= 25 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 30 \quad \frac{m+n}{m-n} &= \frac{7x}{x} \Rightarrow \frac{m}{n} = \frac{4x}{3x} \\
 \text{Again } mn &= 4x \times 3x = 12x^2 \\
 \text{and } mn &= 60x \\
 \text{So } 60x &= 12x^2 \Rightarrow x = 5 \\
 \therefore m &= 20 \quad \text{and} \quad n = 15 \\
 \text{Hence, } \frac{1}{m} : \frac{1}{n} &= \frac{1}{20} : \frac{1}{15} = 3 : 4
 \end{aligned}$$

Alternatively $\frac{m}{n} = \frac{4x}{3x}; \frac{1}{m} : \frac{1}{n} = 3 : 4$

$$\begin{aligned}
 31 \quad \underbrace{A+B}_2 : \underbrace{C+D}_1 \\
 \text{and } \frac{B}{3} : \frac{C}{1} \quad \text{and} \quad \frac{C}{3} : \frac{D}{5} \\
 \Rightarrow B : C : D \\
 9 : 3 : 5
 \end{aligned}$$

$$\begin{aligned}
 \text{Again } A+B : C+D \\
 \underbrace{7+9}_{16} : \underbrace{3+5}_8
 \end{aligned}$$

Thus $A+B=16 \Rightarrow A=7$ when $B=9$

$\therefore$ Therefore share of A = $\frac{7}{24} \times 9600 = ₹ 280$

32 The best way is to consider some values in the proportion and verify the option, but care should be taken that in this type of question you are required to verify a possible correct option atleast with 2-3 times with different values.

Since, if you consider $p : q :: r : s \Rightarrow 1 : 2 :: 3 : 6$

you will find answer (b) is also correct, but for only particular values. So it is not the general solution. Finally you will realize that option (d) is the most appropriate answer.

33 Solve by componendo and dividendo or as mentioned in the previous solution.

34 Since for the constant distance time is inversely proportional to the speed. So, the required ratio of time taken by each of the rickshaw, car and scooter is

$$\frac{1}{3} : \frac{1}{5} : \frac{1}{6} \Rightarrow 10 : 6 : 5$$

$$\left[\frac{1}{3} \times \frac{10}{10} : \frac{1}{5} \times \frac{6}{6} : \frac{1}{6} \times \frac{5}{5} = \frac{10}{30} : \frac{6}{30} : \frac{5}{30} = 10 : 6 : 5 \right]$$

$$35 \quad A : B = 2 : 3 \quad \text{and} \quad B : C = 3 : 5$$

$$\Rightarrow A : B : C = 2 : 3 : 5$$

$$\therefore (A+B) : C = 5 : 5 = 1 : 1$$

$$\text{Hence, Share of } (A+B) = \frac{1}{2} \times 6940 = 3470$$

$$36 \quad x + y = 93 \quad \dots(i)$$

$$\text{and } \frac{x+3}{y+2} = \frac{24}{25}$$

$$\Rightarrow 25x - 24y = (-27) \quad \dots(ii)$$

From eqs. number (i) and (ii)

$$x = 45 \quad \text{and} \quad y = 48$$

Alternatively Go through options.

$$37 \quad \frac{a}{b} = \frac{4x}{9x} \quad \text{and} \quad \frac{a+4}{b+4} = \frac{21}{46}$$

$$\Rightarrow a = 80 \quad \text{and} \quad b = 180$$

$$\therefore b - a = 100$$

$$38 \quad \frac{R}{D} = \frac{3x}{5x} \quad \text{and} \quad \frac{R}{D} = \frac{3x+10}{5x+10} = \frac{5}{7} \Rightarrow x = 5$$

$$\text{Hence, } \frac{R}{D} = \frac{15}{25}$$

Alternatively Go through options.

$$39 \quad \text{Let the fraction be } \frac{x}{y} \text{ then } \frac{x+5}{y+5} = \frac{11}{15} \Rightarrow 11y - 15x = 20$$

Since, we have only one equation in two variables, so we cannot find the solution.

$$40 \quad \frac{2x+3x+5x+8x+9x}{5} = \frac{162}{5} = 32.4$$

41 The number of days required by a single Kirlosker pump to fill the tank = $6 \times 7 = 42$ days and the number of days required by a single USHA pump to fill the same tanks = $2 \times 18 = 36$ days. Now, since efficiency is inversely proportional to the number of days. Hence,

$$\frac{\text{Efficiency of one K-pump}}{\text{Efficiency of U-pump}} = \frac{36}{42} = \frac{6}{7}$$

- 42 $\frac{1}{5}$ th field can be reaped by 96 man days
 $\therefore \frac{4}{5}$ th field can be reaped by $96 \times 4 = 384$ man days
 Now, since there are only 8 days so required
 number of men = $\frac{384}{8} = 48$ men

43 $210 = 2 \times 3 \times 5 \times 7$
 Therefore, $\frac{N_1}{N_2} = \frac{2k \times 5 \times 7}{3k \times 5 \times 7}$

Since, $N_1 : N_2 = 2 : 3$
 Therefore, $N_1 = 70$ and $N_2 = 105$
 $\therefore N_1 + N_2 = 70 + 105 = 175$

44 $\frac{(53-x)}{(21-x)} = \frac{(41-x)}{(17-x)} \Rightarrow x = 5$

☞ Alternatively Go through options.

45 $2x + 3x + 4x = 180 \Rightarrow x = 20 \therefore 4x = 80$

46 $(25 \times 12x) + (50 \times 4x) + (100 \times 3x) = 800x = 60000$

$\Rightarrow x = 75$

$\therefore$ Number of coins of 25 paise = $12x = 12 \times 75 = 900$

☞ Alternatively Go through options, choices (a), (b) and (c) are eliminated since neither of 200, 225, 275 is divisible by 12. Hence choice (d) is correct.

47 Go through options.

☞ Alternatively $\frac{(x-a)}{11} = \frac{(x-b)}{9} = \frac{(x-c)}{5} = k$

$\Rightarrow a = x - 11k$

$b = x - 9k$

$c = x - 5k$

$\therefore x = \frac{(x-11k) + (x-9k) + (x-5k)}{2}$

$2x = 3x - 25k \Rightarrow x = 25k$

$a = 14k \Rightarrow b = 16k$ and $c = 20k$

$\therefore a : b : c = 14 : 16 : 20 = 7 : 8 : 10.$

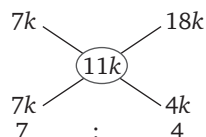
48 If the weight of kerosene be k kg/unit volume, then

Weight of petrol = $7k$ /unit volume

Weight of castrol = $18k$ /unit volume

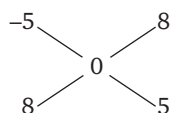
Required weight of the mixture = $11k$ /unit volume

By Alligation



The required ratio is $7 : 4$.

49 By Alligation



The ratio of price of pigeons sold at loss and profit respectively are in the ratio of $8 : 5$

$\therefore$ Cost price of profitable pigeon = $\frac{5}{8+5} \times 182 = ₹ 70.$

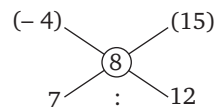
50

$\frac{C}{R} = \frac{3x}{5x}$

Again $\frac{C}{R} = \frac{3x \times 3}{(5x + 100)} = \frac{4}{5} \Rightarrow x = 16$

$\therefore$ Price of Rotomac pen in 2000 was ₹ 80.

51 By Alligation



Now, we get the ratio of no. of rings sold at a loss and profit is $7 : 12$

$\therefore$ Number of rings sold at profit = $\frac{12}{19} \times 361 = 228$

☞ Alternatively Go through options.

52 Best way is to go through options.

Consider option (d) $\frac{180}{30} + \frac{70}{35} = 6 + 2 = 8$ hrs

Hence, (d) is correct.

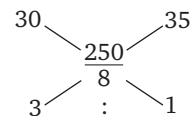
☞ Alternatively $\frac{x}{30} + \frac{250-x}{35} = 8$

$\Rightarrow x = 180$ miles

☞ Alternatively Since the Average Speed

$= \frac{\text{Total Distance}}{\text{Total Time}} = \frac{250}{8}$ miles/hr

$\therefore$ By Alligation



Therefore, the ratio of time taken @ 30 m/hr and @ 35 m/hr is in the ratio of $3 : 1$. It means he has travelled @ 30 m/hr for 6 hr.

Therefore, the distance travelled by car is 180.

($\therefore$ Distance = Speed $\times$ Time).

53 $\frac{22}{22} : \frac{17}{17} = 1 : 1$

54 $\frac{1}{3} : \frac{1}{4} : \frac{1}{5} : \frac{1}{6}$

$\Rightarrow \frac{20}{60} : \frac{15}{60} : \frac{12}{60} : \frac{10}{60}$ (by taking LCM)

$\Rightarrow 20 : 15 : 12 : 10$

$\therefore$ Largest share = $\frac{20}{(20 + 15 + 12 + 10)} \times 171$

$= \frac{20}{57} \times 171 = 60.$

55 Let the present age of Karishma and Babita be x and y then

$\frac{x-10}{y-10} = \frac{1}{3}$... (i)

Again $\frac{x+14}{y+14} = \frac{5}{9}$... (ii)

By solving eqs. (i) and (ii) we get $x = 26$ and $y = 58$

56 Let the fraction be $\frac{k}{5k}$, then $\frac{k+x}{5k+5x} = \frac{(k+x)}{5(k+x)} = \frac{1}{5}$

57 $x \propto y$ and $x \propto \frac{1}{z^2}$

$$x \propto \frac{y}{z^2} \Rightarrow x = k \frac{y}{z^2}$$

$$6 = k \times \frac{75}{(5)^2} \Rightarrow k = 2$$

Again $x = 2 \times \frac{24}{(4)^2} \Rightarrow x = 3$

58 $x \propto (y^2 + z^2) \Rightarrow x = k(y^2 + z^2)$
 $15 = k(1^2 + 2^2) \Rightarrow k = 3$

Again $39 = 3 \times (2^2 + z^2)$

$$\Rightarrow 13 = (4 + z^2) \Rightarrow z = 3$$

59 $W \propto HA \Rightarrow W = K \times H \times A$

Now, $48 = K \times 1.2 \times 20 \Rightarrow K = 2$

Again $W = 2 \times 1.5 \times 30$

$$\therefore W = 90$$

60 $(a+b):(b+c):(c+a) = 5x:6x:9x$

And $a+b+c = 10$

$$\Rightarrow (a+b) + (b+c) + (c+a) = 20x$$

$$\Rightarrow a+b+c = 10x$$

$$\Rightarrow a+b+c = 10 \quad (\because x=1)$$

$$\therefore c = (a+b+c) - (a+b)$$

or $c = 10 - 5 = 5$

61

A	B	C
3x	4x	5x
(3x+x)	2x	(5x+x)
= 4x	2x	6x
(4x+x)	2x	(5x)
5x	2x	5x
5	2	5

(B gives $\frac{1}{4}$ to A and $\frac{1}{4}$ to C)
 (C gives $\frac{1}{6}$ to A)

✓ Level 02 Higher Level Exercise

1

(M_1) (M_2)

D : S D : S

7 : 3 5 : 4

New Mixture

D S

2 : 1

Proportion of Dew in $M_1 = \frac{7}{10}$

Proportion of Dew in $M_2 = \frac{5}{9}$

Proportion of new mixture = $\frac{2}{3}$ (required)

By alligation rule

$$\begin{array}{ccc} \frac{7}{10} & & \frac{5}{9} \\ & \searrow \quad \nearrow & \\ & \frac{2}{3} & \\ & \nearrow \quad \searrow & \\ \left(\frac{2}{3} - \frac{5}{9}\right) & & \left(\frac{7}{10} - \frac{2}{3}\right) \end{array}$$

$$\frac{3}{27} \quad \frac{1}{30}$$

or $\frac{1}{9} : \frac{1}{30}$

or 10 : 3

2 The ratio of quantities = 140 : 60 = 7 : 3

$$\therefore \text{The quantity to be exchanged} = \frac{(140 \times 3) + (60 \times 7)}{2(7+3)} = 42 \text{ litre}$$

NOTE In this question individual prices are not required. You can solve it by forming equation.

Alternatively Go through options by assuming some prices.

3 Let the number be $100x + 10y + z$, then

$$x + z = 6 \text{ and } y + z = 5$$

Also, from the given options only option (b) is suitable

i.e., $y + z = 5$ or $3 + 2 = 5$

4

Water	Pulp
4x	x
3y	2y

But $x = 2y$ since pulp remains constant

$$5y = \text{loss in water} \rightarrow \begin{array}{cc} \text{Water} & \text{Pulp} \\ 8y & 2y \\ 3y & 2y \end{array}$$

But $10y = (8y + 2y) = 1.5 \text{ kg}$

$$\therefore 5y = (3y + 2y) = 0.75 \text{ kg}$$

5

Cost price	:	Selling price
$\frac{1}{9}$	:	$\frac{1}{6}$
or 2	:	3

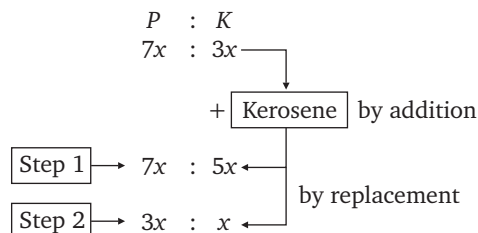
$$\text{CP} : \text{SP} = 2 : 3$$

$$\text{Since profit} = \text{SP} - \text{CP}$$

$$\therefore \text{Profit} = 3 - 2 = 1$$

$$\therefore \text{Profit} : \text{CP} = 1 : 2$$

6 Flow Chart :



By the replacement formula from alligation chapter

$$\frac{1}{4} = \frac{5}{12} \left(1 - \frac{240}{12x} \right) \Rightarrow \frac{3}{5} = \left(1 - \frac{240}{12x} \right)$$

$$\Rightarrow \frac{2}{5} = \frac{20}{x} \Rightarrow x = 50$$

Therefore, half of the initial amount = (350 + 150) litres
then, the required amount of kerosene
= $5x - 3x = 2x = 100$ litres

But for the whole amount required kerosene to be added
= 200 litres

- 7 Data insufficient. Since we don't know that how many persons bought ticket of all the categories individually
i.e., $5x + 9y + 14z = 213925$

But we don't know anything about x , y and z .

- 8
- | Petrol | Kerosene | Total Mixture |
|--------|-------------|---------------|
| 99 | x | $99 + x$ |
| 99 | $(x - 198)$ | $(x - 99)$ |

$$\text{Again } \frac{99}{(x - 99)} \times 100 - \frac{99}{(x + 99)} \times 100 = 13.33$$

$$\text{or } 9900 \left(\frac{x + 99 - x + 99}{x^2 - 99^2} \right) = 13.33$$

$$\text{or } \frac{9900(198)}{x^2 - 99^2} = \frac{40}{3}$$

$$\Rightarrow x^2 - 99^2 = 99^2 \times 15$$

$$\Rightarrow x^2 = (99)^2 \times (16)$$

$$\Rightarrow x = 99 \times 4 = 396 \text{ litres}$$

$$\text{Therefore the actual concentration of petrol} \\ = \frac{99}{(99 + 396)} = 20\%$$

- 9 Do it yourself.

- 10 Varsha : Vinay = 5 : 6 = $5x : 6x$

$$\text{Veera : Vikram} = 7 : 8 = 7y : 8y$$

But their ages are in A.P.

$$\text{Therefore, } 6x - 5x = 8y - 7y$$

$$\Rightarrow x = y$$

$$\text{Again, } \frac{5x + 2}{8y + 2} = \frac{2}{3}$$

$$\Rightarrow \frac{5x + 2}{8x + 2} = \frac{2}{3} \Rightarrow x = 2$$

Therefore, the ages of Varsha, Vinay, Veera and Vikram are 10, 12, 14 and 16 years respectively.

Therefore, the ratio of ages of Vinay and Veera = 6 : 7,

- 11
- | Bobby | | Sunny | |
|-------|-------|-----------------------|-------|
| Milk | Water | Milk | Water |
| 1 | 1 | 1 | 1 |
| 50% | 50% | 50% | 50% |
| ② | ② | By replacement method | |
| 60% | 40% | | |
| ③ | ② | | |

$$\frac{2}{5} = \frac{2}{4} \left(1 - \frac{k}{4} \right) \Rightarrow \frac{4}{5} = \left(1 - \frac{k}{4} \right)$$

$$\Rightarrow \frac{1}{5} = \frac{k}{4} \Rightarrow k = \frac{4}{5}$$

It means Bobby will add up 1 litre of milk, in 4 litre of initial mixture, to prepare 5 litres mixture in the ratio of 3 : 2

It means Sunny will replace $\frac{4}{5}$ litre of initial mixture by the same quantity of pure milk.

Hence, the percentage of milk added by Bobby to that of replace by Sunny = $\frac{1}{4/5} \times 100 = 125\%$

- 12

	A		B	
	Milk	Water	Milk	Water
After first operation	25l 20l	0	0 5l	25l 25l
			1 : 5	
After second operation	21l	5l	4l	20l

Therefore, the ratio of water in A and B is 1 : 4

- 13

$$B = \frac{5}{6} A \quad \dots(i)$$

$$\text{And } C = D = \frac{9}{10} B \quad \dots(ii)$$

$$\text{Also } B = \frac{2}{3} E \quad \dots(iii)$$

$$\text{And } E - A = 3 \quad \dots(iv)$$

$$\text{From eqs. (i) and (iii), we get } \frac{A}{E} = \frac{4}{5} \text{ or } E = \frac{5}{4} A \quad \dots(v)$$

$$\therefore E - A = \frac{5A}{4} - A = 3 \text{ From eqs. (iv) and (v), we get}$$

$$\Rightarrow A = 12 \text{ and } E = 15 \text{ and } B = 10$$

Also $C = D = 9$ and $F = 11$, since $B < F < A$ and F is integer

$$\therefore A : F = 12 : 11$$

- 14 The ratio of fees collected from B.Tech : MBA

$$= 4x \times 25y : 5x \times 16y$$

$$= 100xy : 80xy$$

$$= 5xy : 4xy = 5k : 4k$$

The amount collected only from MBA students

$$= \frac{4}{9} \times 1.62 \text{ lakh}$$

$$= ₹ 72,000$$

$$C \propto (W)^2$$

- 15

$$W_1 : W_2 : W_3 = 3 : 4 : 5$$

$$\text{Cost} = (3x)^2 + (4x)^2 + (5x)^2 = 50(x)^2$$

Again $W_1 : W_2 : W_3 = 4 : 4 : 4$ (when weights are equal)

$$\text{Cost} = (4x)^2 + (4x)^2 + (4x)^2 = 48x^2$$

$$\text{Loss} = 50x^2 - 48x^2 = 2x^2$$

$$1800 = 2x^2$$

$$x = 30$$

$$\begin{aligned}\therefore \text{Actual cost of unbroken marble} &= (4x + 4x + 4x)^2 \\ &= (12x)^2 = 144x^2 \\ &= 144 \times 30^2 \\ &= 144 \times 900 = 129600\end{aligned}$$

Solutions (for Q. Nos. 16 and 17)

	A	B	C	D
Initially	69	45	42	36
$A \rightarrow B$	24	90	42	36
$B \rightarrow C$	24	48	84	36
$C \rightarrow D$	24	48	48	72
$D \rightarrow A$	48	48	48	48

Solve it in reverse order (i.e., from the result side)

- 18** Ratio of price paid by Hari and Murli = 7 : 10

$$\text{or } (24 \times x) : 30 \times (30 - x) = 7 : 10$$

$$\Rightarrow x = 14$$

or go through options.

- 19** Since distance is constant. Therefore ratio of speeds of scooter, car and train = 1 : 4 : 16

and therefore, ratio of time taken = 16 : 4 : 1

$$\text{Therefore, required ratio} = \frac{16}{1} : \frac{4}{4} : \frac{1}{16}$$

$$= 16 : 1 : \frac{1}{16} = 256 : 16 : 1$$

- 20** Since there are 12 bangles, then the no. of broken to unbroken bangles cannot be 2 : 3, since $5x = (2x + 3x)$ can not divide 12 for any integral value of x i.e., all the sum of ratios which are the factors of 12 can possibly be the ratio of broken to unbroken.

$$\text{21 } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$$

$$\Rightarrow \frac{a^n}{b^n} = \frac{c^n}{d^n} = \frac{e^n}{f^n} = k^n$$

$$\Rightarrow \frac{a^n p}{b^n p} = \frac{c^n q}{d^n q} = \frac{e^n r}{f^n r} = k^n$$

$$\therefore \frac{a^n p + c^n q + e^n r}{b^n p + d^n q + f^n r} = (k^n)$$

$$\therefore \left(\frac{a^n p + c^n q + e^n r}{b^n p + d^n q + f^n r} \right)^{\frac{1}{n}} = (k^n)^{1/n}$$

$$\therefore \left(\frac{a^n p + c^n q + e^n r}{b^n p + d^n q + f^n r} \right)^{1/n} = k = \frac{a}{b} = \frac{c}{d} = \frac{e}{f}$$

Hence (c) is the possible answer.

- 22** Maximum earning will be only when he will won on the maximum yielding table.

$$A \rightarrow 10 : 1$$

$$B \rightarrow 20 : 1$$

$$C \rightarrow 30 : 1$$

i.e., he won on B and C but lost on A

$$20 \times 200 + 30 \times 200 - 1 \times 200 = 9800$$

Minimum earning will be when he won on table A and B and lose on that table 3.

$$\therefore 10 \times 200 + 20 \times 200 - 1 \times 200$$

$$6000 - 200 = 5800$$

Therefore, difference = 9800 - 5800 = 4000

Alternatively the difference

$$= [(30 + 20 - 1) - (10 + 20 - 1)] \times 200$$

$$= 20 \times 200 = ₹ 4000.$$

$$\begin{aligned}\text{23 } \frac{m+2p}{m-2p} + \frac{m+2q}{m-2q} \\ &= \frac{m}{2p} + \frac{m}{2q} \text{ (by componendo and dividendo)} \\ &= m \left(\frac{p+q}{2pq} \right) \\ &= \left(\frac{4pq}{p+q} \right) \left(\frac{p+q}{2pq} \right) \text{ substituting the value of} \\ &\quad m = \frac{4pq}{p+q} = 2\end{aligned}$$

$$\text{24 Volume of a cube} = (\text{side})^3 = (a)^3$$

$$\text{Surface area of a cube} = 6(a)^2 \text{ (a} \rightarrow \text{side)}$$

$$a_1^3 : a_2^3 = 8 : 27$$

$$\Rightarrow a_1 : a_2 = 2 : 3$$

$$\Rightarrow a_1^2 : a_2^2 = 4 : 9$$

$$\Rightarrow 6a_1^2 : 6a_2^2 = 4 : 9$$

- 25** Clearly it can not be less than 75%. For clarification of concept consider some values and then verify it.

$$\text{26 } A : B : C = 3x : 4x : 5x$$

$$\text{But } C - A = 5x - 3x \leq 3$$

$$\text{or } C - A = 2x \leq 3 \Rightarrow x = 1 \Rightarrow C - A = 2$$

$$\therefore A : B : C = 3 : 4 : 5$$

Therefore, number of assistant trainee (except project in charge) of Q = 3 and assistant trainee of R = 4

$$\begin{aligned}\text{Therefore, required percentage} &= \frac{4-3}{4} \times 100 \\ &= \frac{1}{4} \times 100 = 25\%\end{aligned}$$

$$\text{27 } b = 3a$$

$$c = 3b = 9a$$

$$d = 3c = 9b = 27a$$

$$e = 3d = 9c = 27b = 81a \text{ and } f = 3e = 243a$$

put the values and simplify.

$$\text{or we know that } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} \dots = \frac{a+c+e}{b+d+f}$$

- 28** Let w be the number of wagons and s be the speed of engine without wagon = $\frac{20}{3}$ m/s = 24 km/hr.

Then speed of the train = $s - k\sqrt{w}$

$$\therefore 20 = 24 - k\sqrt{4} \quad [\because \frac{50}{9} \text{ m/s} = 20 \text{ km/hr}]$$

$$\therefore k = 2$$

When train will stop its speed becomes zero

$$0 = 24 - 2\sqrt{w} \quad (\because k = 2)$$

$$\Rightarrow w = 144$$

Since at 144 wagons train will stop, so at 143 wagons train just can move with its least possible speed having maximum possible wagons.

- 29** Cost of 1 kg (mixture) of sugar = ₹ 16/kg

Since selling price is ₹ 20/kg

Therefore, ratio of quantity of sugar costing ₹ 14 and ₹ 22 per kg = 3 : 1

$$\begin{array}{ccc} 14 & & 22 \\ & \searrow & \nearrow \\ & 16 & \\ & \nearrow & \searrow \\ 6 & & 2 \\ 3 & : & 1 \end{array}$$

So, the available stock of mixture costing ₹ 16 = 3 quintal and the required stock of sugar costing ₹ 18 = 1 quintal

Therefore, the new price of mixture costing 3 quintal of sugar @ ₹ 16/kg and 1 quintal of sugar @ ₹ 18 per kg

$$\begin{array}{ccc} 16 & & 18 \\ & \searrow & \nearrow \\ & x & \\ & \nearrow & \searrow \\ 3 & : & 1 \end{array}$$

$$\therefore x = ₹ 16.5 \text{ per kg}$$

New price = ₹ 16.5 per kg

Now, original profit = $20 - 16 = ₹ 4$ per kg

New profit = $20 - 16.5 = ₹ 3.5$ per kg

$$\therefore \text{Percentage reduction in profit} = \frac{4 - 3.5}{4} \times 100 = 12.5\%$$

30

	P	Q
Initial amounts	a (petrol)	b (kerosene)
After one operation	$a - c$ (petrol) c (kerosene)	$b - c$ (kerosene) c (petrol)

Since there is no change in concentration in the second, third... etc operations.

In the vessel P; The fraction of kerosene in P is $\frac{c}{a}$

In the vessel Q ;

The fraction of kerosene in Q is $\left(\frac{b-c}{b}\right)$

$$\therefore \left(\frac{b-c}{b}\right) = \frac{c}{a} \Rightarrow c(a+b) = ab$$

$$\Rightarrow c = \frac{ab}{a+b},$$

Hence option (c) is correct.

Since at this moment the concentration in both vessels is same.

- 31** The minimum number of passengers n , at which there is no loss and number of passengers travelling = m and let the distance travelled is d , then

$$P \propto (m-n)d$$

or $p = k(m-n)d$; k is a constant.

when $P = 3600$, $m = 29$ and $d = 36$, then

$$3600 = k(29-n) \times 36 \quad \dots(i)$$

Again, when $p = 6300$, $m = 36$, $d = 42$ km

$$6300 = k(36-n) \times 42 \quad \dots(ii)$$

Dividing eqs. (ii) by (i), we get

$$\frac{6300}{3600} = \frac{k(36-n) \times 42}{k(29-n) \times 36}$$

$$\Rightarrow \frac{(36-n)}{(29-n)} = \frac{9}{6} \Rightarrow 3n = 45$$

$$\Rightarrow n = 15$$

Hence to avoid loss, minimum number of 15 passengers are required.

- 32** Frequency of step of $A : B : C = 5 : 6 : 7$

but in terms of size of step, $6A = 7B = 8C$

$$\begin{aligned} \therefore \text{Ratio of speeds of } A, B \text{ and } C &= \frac{5}{6} : \frac{6}{7} : \frac{7}{8} \\ &= 280 : 288 : 294 \\ &= 140 : 144 : 147 \end{aligned}$$

- 33** Expenditure = 5 (no. of family members)²

$$\Rightarrow E_1 = 5(n)^2 \quad \dots(i)$$

$$\text{Again } E_2 = 5(n-1)^2 \quad \dots(ii)$$

$$\text{But } E_1 - E_2 = 95$$

$$\therefore 5[n^2 - (n-1)^2] = 95$$

$$5[n^2 - (n^2 - 2n + 1)] = 95$$

$$n^2 - n^2 - 1 + 2n = 19$$

$$\Rightarrow 2n = 20$$

$$\Rightarrow n = 10$$

- 34** Finally I have ₹ 5

Consider option (a)

$$2 \times 100 + 3 \times 50 + 1 \times 25 = 375 \text{ paise}$$

Its integer multiple cannot give the value equals to 500. So cannot be true.

Consider option (b)

$$1 \times 100 + 7 \times 50 + 2 \times 25 = 500 \text{ paise, can be true}$$

But we also check the option (c) and (d) Now when you check the choices (c) and (d) you will find wrong as choice (a).

So, only choice (b) could be the best answer.

- 35 By componendo and dividendo :

$$\begin{aligned}\frac{a}{b+c} &= \frac{b}{c+a} \\ \frac{a+b+c}{a-b-c} &= \frac{b+c+a}{b-c-a} \\ \Rightarrow a-b-c &= b-c-a \\ \Rightarrow a &= b \\ \text{Similarly } a &= b = c \\ \therefore \frac{b}{a+b+c} &= \frac{1}{3}\end{aligned}$$

- 36 Density of P_1 , P_2 and P_3 are 18, 14 and 10 gm/cc

$$\text{Again since volume} = \frac{\text{weight}}{\text{density}}$$

$$\text{Now the weight of } P_3 \text{ in 450 kg mixture} = \frac{450 \times 4}{15} = 120 \text{ kg}$$

$$\text{Now the volume of } P_3 = \frac{120}{10} = 12 \text{ litre}$$

$$\therefore \text{The cost of 12 litre } P_3 \text{ petrol} = 12 \times 40 = ₹ 480$$

- 37 $26.65 + 42.75 + 53 = 122.40$

Proportionate amount of Amar, Akbar and Anthony in the ratio of 3 : 4 : 5 is ₹ 30.60, ₹ 40.80 and ₹ 51 respectively.

Now	Amar	Akbar	Anthony
	30.60	40.80	51.00
	26.65	42.75	53.00

So, Amar pays ₹ 1.95 to Akbar and ₹ 2 to Anthony.

- 38 The best way is to go through options

☞ **Alternatively** Solve by cross product rule and componendo.

- 39 Since Pooja and Shipra are twins so their ages be same. Let their ages be x and age of Monika be y , then,

$$\begin{aligned}x + x &= y & \dots(i) \\ \text{and } \frac{(x-3)}{(y-3)} &= \frac{2}{7}\end{aligned}$$

$$\Rightarrow 7x - 2y = 15$$

Now, from eqs. (i), we get

$$7x - 4x = 15 \Rightarrow x = 5$$

So the age of Shipra 3 years hence will be $5 + 3 = 8$ years.

- 40 $\frac{H-9}{W-9} = \frac{5}{4}$ and $\frac{H+6}{W+6} = \frac{8}{7}$

$\therefore$ Thus the present age of Husband is 34 and present age of his wife is 29 years.

Now, the maximum age of any child must be less than 9 years. Hence their ages can be 2, 3 and 4 years or 4, 6 and 8 years.

So the max. possible sum of age of this family

$$\begin{aligned}&= 34 + 29 + (1 \times 4 + 2 \times 6 + 3 \times 8) \\ &= 103 \text{ years}\end{aligned}$$

- 41 $P \propto n\sqrt{r} \Rightarrow P = kn\sqrt{r}$

Where P is the price of necklace, n is number of pearls and r is the radius of a pearl.

$$\text{Now } 150 = k \times 75 \times \sqrt{1} \Rightarrow k = 2$$

$$\text{Again } 600 = 2 \times 100 \times \sqrt{r}$$

$$\Rightarrow \sqrt{r} = 3 \Rightarrow r = 9 \text{ cm}$$

- 42 $P \propto \frac{N}{T} \Rightarrow P = K \frac{N}{T}$

$P \rightarrow$ Price of a book, $N \rightarrow$ Number of Pages, $T \rightarrow$ Time period

$$\begin{aligned}\frac{P_1}{T_1} &= \frac{P_2}{T_2} \\ \therefore K \frac{N_1}{T_1} &= K \frac{N_2}{T_2} \\ \Rightarrow \frac{3N}{18} &= \frac{N}{T} \\ T &= 6 \text{ years}\end{aligned}$$

- 43 $\frac{5x+40}{6x+40} = \frac{7}{8} \Rightarrow \frac{\text{Akbar}}{\text{Birbal}}$

$$\Rightarrow x = 20$$

$\therefore$ The actual number of shares of less salaried person

$$= 100 \quad \therefore (5 \times 20 = 100)$$

$\therefore$ The salary of Akbar $= 100 \times 75 = 7500$

- 44 $D \propto \frac{\sqrt{F} \times T}{W}$, $D \rightarrow$ Distance, $F \rightarrow$ Fuel, $T \rightarrow$ Time, $W \rightarrow$ No.

$$\text{of wagons } D = k \frac{\sqrt{F} \times T}{W}; 192 = k \frac{\sqrt{256} \times 20}{10} \Rightarrow k = 6$$

$$\text{Again } 200 = \frac{6 \times \sqrt{F} \times 25}{15}$$

$$\Rightarrow \sqrt{F} = 20 \Rightarrow F = 400 \text{ litre}$$

$$\therefore \text{Fuel used per km} = \frac{400}{200} = 2 \text{ l/km}$$

- 45 Just go through option and factorize the product into two factors such that the given conditions must satisfy.

$$\text{e.g., } 6 = 1 \times 6$$

$$2 \times 3$$

$$3 \times 2$$

$$6 \times 1$$

It is not true.

Again consider option (b)

$$12 = 1 \times 12$$

$$2 \times 6$$

$$3 \times 4 \rightarrow 3 \times 4 = 12$$

$$4 \times 3$$

$$6 \times 2 \rightarrow 6 \times 2 = 12$$

$$12 \times 1$$

Now you can see that the rate is being half from 4 to 2 so she can purchase double number of toffees as she was already purchasing on the ground floor. Again to purchase the same number of toffees she had to spend ₹ 2 less than the spending on the ground floor :

Rate	Number of toffee/₹	Total Number of toffee
4 ×	3	= 12
2 ×	6	= 12

and if you check other options (c) and (d) they will not satisfy the given conditions.

46 Go through options.

	Total no. of plants	No. of plants	Days
Previous scheme ⇒	1800	= 100 ×	18

Again after $\frac{1}{3}$ rd days, remaining plants

$$= \frac{1800 \times 2}{3} + 120 = 1320$$

$$1320 = (100 + 20) \times 11$$

This shows that in the second case 1 day was saved than the planned no. of day.

47 Go through options $\frac{810}{54} - \frac{900}{75} = 3 = (6 - 3 = 3)$

$$15 - 12 = 3; 3 = 3$$

Hence choice (c) is correct.

i.e., if A writes 54 page, then B writes 75 = (54 + 21) pages per hour.

48

	A	B	C	D
Initial amount of soda water	$(x + 3)$	$(x + 1)$	(x)	0
final amount of soda water	$\left(\frac{3x}{4} + 1\right)$	$\left(\frac{3x}{4} + 1\right)$	$\left(\frac{3x}{4} + 1\right)$	$\left(\frac{3x}{4} + 1\right)$

Therefore to be the integral value x must be $4m$; $m = 1, 2, 3, \dots$

So at $m = 1$

Hence option (a) is correct since B had contributed 1 tumbler.

	A	B	C	D
Initial amount	7	5	4	0
Final amount	$\frac{4}{3}$	$\frac{4}{1}$	$\frac{4}{0}$	4
	2	1	1	

49 Ratio of fruits (by dozen) = 3 : 2 : 7

Ratio of fruits by weight = 120 : 150 : 24

∴ Ratio of fruits (combined) by weight
= $3 \times 120 : 2 \times 150 : 7 \times 24 = 30 : 25 : 14$

Level 03 Final Round

1

	1st Alloy		2nd Alloy	
	Iron	Copper	Iron	Copper
	4	3	6	1
Proportion of iron in the alloys	$\frac{4}{7}$ ↓ $\frac{8}{14}$	$\frac{3}{7}$ ↓ $\frac{6}{14}$	$\frac{6}{7}$ ↓ $\frac{36}{42}$	$\frac{1}{7}$ ↓ $\frac{6}{42}$

So in the new alloy total iron will be 44 kg and copper will be 12 kg.

$$\therefore \text{Iron} = 8 + 36 = 44$$

$$\text{and Copper} = (14 + 42) - (8 + 36) = 12$$

$$\therefore \text{Ratio of copper to iron} = 12 : 44 = 3 : 11,$$

Hence option (d) is correct

2 Suppose there are all the pigeons then total no. of heads are 340 and total no. of legs are 680. Now since $380 = (1060 - 680)$ legs are extra, it means there will be $190 = \left(\frac{380}{2}\right)$ rabbits. As we know a rabbit has 2 extra legs

than that of a pigeon who has only two legs.

Therefore, number of rabbits = 190

and number of pigeons = $340 - 190 = 150$

Alternatively Go through options and consider choice (b)

	Pigeons	Rabbits
Heads (340)	150	190] 340
Legs (1060)	300	760] 1060

Alternatively $P + R = 340$... (i)

and $2P + 4R = 1060$... (ii)

Solve these two equations and you will get the answer.

Alternatively It can be solved through alligation rule.

3 By the replacement formula

Decreased amount

$$= \text{Original amount} \left(1 - \frac{\text{replacing amount}}{\text{original amount}} \right)^{\text{no. of times (n)}}$$

Now, since the ratio of petrol and kerosene is 1701 and 27 it means initially there was $(1701 + 27) = 1728$ unit of kerosene and the decreased amount of kerosene is 27 unit.

$$\therefore 27 = 1728 \left(1 - \frac{6}{k} \right)^3 \Rightarrow k = 8 \text{ litre}$$

NOTE Here 27 : 1728 just show the ratio not the exact amount.

4 The ratio of milk in 3 vessels

$$= \frac{3}{4} \times \frac{5 \times 7}{5 \times 7} : \frac{2}{5} \times \frac{4 \times 7}{4 \times 7} : \frac{4}{7} \times \frac{4 \times 5}{4 \times 5} = \frac{105}{140} : \frac{56}{140} : \frac{80}{140}$$

Remember, The value of 25 litre does not matter, the basic thing is that the amount of mixture in all the three quantities is same. So the total quantity of milk in mixture = $105 + 56 + 80 = 241$

So the total amount of water in mixture

$$= [(3 \times 140) - 241] = 179 \text{ litre}$$

Therefore ratio of water to milk in the new mixture

$$= 179 : 241$$

- 5 $\therefore$ Profit = $33\frac{1}{3}\%$, it means cost price = ₹ 15

Now, by alligation

$$\frac{(x+7)-15}{(15-x)} = \frac{3}{4} \Rightarrow x = 11$$

$$\therefore x = 11 \text{ and } (x+7) = 18$$

Thus the total value of both the prices = $11 + 18 = 29$

6

(A_1)	(A_2)
$C \quad T$	$C \quad T$
$1 \quad 3$	$2 \quad 5$
Copper $\rightarrow \frac{1}{4}$	Copper $\rightarrow \frac{2}{7}$

$$\text{Required copper} \rightarrow = \frac{3}{11}$$

So, the required ratio is 4 : 7

Since it is clear from the above values

($1 + 2 \rightarrow 3$ and $4 + 7 \rightarrow 11$)

④ Alternatively By Alligation

$$\begin{array}{c} \frac{1}{4} \quad \quad \frac{2}{7} \\ \quad \quad \swarrow \quad \searrow \\ \quad \quad 3 \\ \quad \quad \swarrow \quad \searrow \\ \left(\frac{2}{7} - \frac{3}{11}\right) \quad \quad \left(\frac{3}{11} - \frac{1}{4}\right) \end{array}$$

$$\Rightarrow \begin{array}{l} \frac{1}{77} : \frac{1}{44} \\ \frac{1}{7} : \frac{1}{4} \\ 4 : 7 \end{array}$$

7 Ratio of $W_1 \quad M_1 \quad W_2 \quad M_2 \quad W_3 \quad M_3$

$$1 : 3 \quad 2 : 3 \quad 2 : 5$$

$$\text{Proportion of water} \frac{1}{4} : \frac{2}{5} : \frac{2}{7}$$

$$\Rightarrow \frac{1}{4} \times \frac{5 \times 7}{5 \times 7} : \frac{2}{5} \times \frac{4 \times 7}{4 \times 7} : \frac{2}{7} \times \frac{4 \times 5}{4 \times 5} \Rightarrow \frac{35}{140} : \frac{56}{140} : \frac{40}{140}$$

Now since all these three mixtures are mixed in the ratio of 2 : 3 : 5.

$$\begin{aligned} \text{Therefore new ratio} &= \frac{35}{140} \times \frac{2}{2} : \frac{56 \times 3}{140 \times 3} : \frac{40 \times 5}{140 \times 5} \\ &= \frac{70}{280} : \frac{168}{420} : \frac{200}{700} \end{aligned}$$

Now, the amount of water = $70 + 168 + 200 = 438$

$\therefore$ The amount of milk = $(280 + 420 + 700) - 438 = 962$

$\therefore$ Ratio of milk to water = 962 : 438

8 $B_1 : B_2 : B_3 = 3x : 4x : 5x$

Again $B_1 : B_2 : B_3 = 5y : 4y : 3y$

Since there is increase in no. of oranges in first two basket only, it means the no. of oranges remains constant in the third basket

$$\therefore 5x = 3y$$

Hence $3x : 4x : 5x$

$$\Rightarrow \frac{9y}{5} : \frac{12y}{5} : \frac{15y}{5} = 9y : 12y : 15y$$

$$\text{and } 5y : 4y : 3y \Rightarrow 25y : 20y : 15y$$

Therefore, increase in first basket = 16

and increase in second basket = 8

$\therefore$ the required ratio = 2 : 1

9 Amount of alcohol in first vessel = $0.25 \times 2 = 0.5$ litre

Amount of alcohol in second vessel = $0.4 \times 6 = 2.4$ litre

Total amount of alcohol out of 10 litres of mixture is $0.5 + 2.4 = 2.9$ litre

Hence, the concentration of the mixture is 29%

$$= \left(\frac{2.9}{10} \times 100 \right)$$

10 Assume the weight of Alloy A is 100 kg

$\therefore$ The weight of Alloy B is 400 kg

	Gold	Silver	Copper
$\therefore$ A	40 kg	60 kg	0 kg
B	140 kg	160 kg	100 kg
$\therefore$ Total $\rightarrow$	180 kg	220 kg	100 kg

$$\therefore \text{Ratio of Gold and Silver in new alloy} = \frac{180}{500} : \frac{200}{500} = 36\% : 44\%$$

11

Urea				Dia			
N	P	K		N	P	K	
x	y	0		20%	70%	10%	
				Mixture			
				N	P	K	
				26%	68%	6%	

This 6% of K is obtained only from Dia.

Urea				Dia			
N	P	K		N	P	K	
x	y	0		120	420	60	
				Mixture			
				N	P	K	
				260	680	60	

$$N_U + N_D = N_M \Rightarrow N_U + 120 = 260$$

$N \rightarrow$ Nitrogen, $P \rightarrow$ Phosphorus

$$\text{and } P_U + P_D = P_M \Rightarrow P_U + 420 = 680$$

$U, D, M \rightarrow$ Urea, Dia and mixture

$\therefore$ Amount of Nitrogen in Urea = 140

and amount of Phosphorus in Dia = 260

$\therefore$ Ratio of $N : P = 7 : 13 \Rightarrow 35 : 65$

12

X				Y			
C	N			C	N		
2	7			5	4		
Copper $\rightarrow \frac{2}{9} = \frac{4}{18}$				Copper $\rightarrow \frac{5}{9} = \frac{10}{18}$			

$\therefore$ By alligation

$$\begin{array}{c} \frac{4}{18} \quad \quad \frac{10}{18} \\ \quad \quad \swarrow \quad \searrow \\ \quad \quad 9 \\ \quad \quad \swarrow \quad \searrow \\ 1 \quad \quad \quad 5 \end{array}$$

$$\therefore \text{Amount of X} = \frac{1}{6} \times 42 = 7 \text{ kg}$$

$$\text{and amount of Y} = \frac{5}{6} \times 42 = 35 \text{ kg}$$

13	First	Alloy	Second	Alloy
	C	Al	C	Al
	1	2	3	1

∴ Required Alloy
C Al
2 1

∴ Copper in first alloy = $\frac{1}{3}$

Copper in second alloy = $\frac{3}{4}$

Copper in required alloy (mixture) = $\frac{2}{3}$

Now, by alligation

$$\begin{array}{ccc} \frac{1}{3} & & \frac{3}{4} \\ & \searrow \quad \nearrow & \\ & \frac{2}{3} & \\ & \nearrow \quad \searrow & \\ \left(\frac{3}{4} - \frac{2}{3}\right) & & \left(\frac{2}{3} - \frac{1}{3}\right) \\ \frac{1}{12} & : & \frac{1}{3} \\ \Rightarrow & & \\ 1 & : & 4 \end{array}$$

Therefore, second alloy be mixed 4 times the first alloy.

14 Note in this type of question individual prices does not matter. To prove this solve it algebraically.

$$\text{Exchanged amount} = \frac{3 \times 150 + 5 \times 90}{2(3 + 5)} = \frac{450}{8} = 56.25 \text{ litre}$$

Here 3 and 5 are obtained from the ratio of amounts *i.e.*, from 90 and 150.

15 Here the ratio of mixtures (*i.e.*, milk, water) does not matter. But the important point is that whether the total amount (either pure or mixture) being transferred is equal or not. Since the total amount (*i.e.*, 5 cups) being transferred from each one to another, hence $A = B$.

Hint You can verify it very easily by considering a simple example.

16 CP. of rasgulla = ₹ 9 (since profit is 66.66%)

Now by alligation

$$\begin{array}{ccc} \text{(Flour) } 3x & & 7x \text{ (Sugar)} \\ & \searrow \quad \nearrow & \\ & 9 & \\ & \nearrow \quad \searrow & \\ 5 & & 3 \\ \frac{(9 - 3x)}{(7x - 9)} = \frac{3}{5} & \Rightarrow & x = 2 \end{array}$$

∴ Price of sugar = $7x = ₹ 14$ per kg.

CHAPTER 05

Percentages

It is one of the most important chapters which is the backbone of calculations either involved in commercial arithmetic or in real life. Personally I do maximum arithmetical calculation using percentage and others too. So in the context of calculation it is necessary to know the clear concepts of percentage which plays a very vital role in Data Interpretation besides Quantitative Aptitude section. On an average two problems i.e., nearly 4-5 % problems in QA only, are being asked in CAT every year. In other entrance/competitive exams like MAT, XAT and UPMCAT, etc there are too many questions asked from this chapter.

5.1 Percentage and Its Application

A fraction with denominator 100 is called a per cent. Per cent is an abbreviation for the latin word “percentum” meaning “per hundred ” or “hundredths” and is denoted by symbol %.

 **NOTE** A fraction with denominator 10 is called as decimal.

Since per cent is a form of fraction, we can express per cent as fractions (or decimals) and *vice-versa*.

5.2 Percentage Conversion

Conversion of a Fraction into Percentage

To convert a fraction into a percentage, multiply the fraction by 100 and put “%” sign.

Exp. 1) Convert the following fractions into percentages:

$$(i) \frac{1}{2} \quad (ii) \frac{3}{4} \quad (iii) \frac{4}{5} \quad (iv) \frac{7}{8}$$

$$\text{Solution } (i) \frac{1}{2} \rightarrow \frac{1}{2} \times 100 = 50\% \quad (ii) \frac{3}{4} \rightarrow \frac{3}{4} \times 100 = 75\%$$

$$(iii) \frac{4}{5} \rightarrow \frac{4}{5} \times 100 = 80\% \quad (iv) \frac{7}{8} \rightarrow \frac{7}{8} \times 100 = 87.5\%$$

Conversion of a Percentage into a Fraction

To convert a percentage into a fraction, replace the % sign with $\frac{1}{100}$ and reduce the fraction to simplest form.

Exp. 2) Express the following percentage as fraction:

$$(i) 20\% \quad (ii) 30\% \quad (iii) 45\% \quad (iv) 5\frac{1}{8}\% \quad (v) 155\%$$

$$\text{Solution } (i) 20\% = \frac{20}{100} = \frac{1}{5} \quad (ii) 30\% = \frac{30}{100} = \frac{3}{10} \quad (iii) 45\% = \frac{45}{100} = \frac{9}{20}$$

$$(iv) 5\frac{1}{8}\% = \frac{41}{8 \times 100} = \frac{41}{800} \quad (v) 155\% = \frac{155}{100} = \frac{31}{20} = 1\frac{11}{20}$$

Chapter Checklist

- Percentage and Its Application
- Percentage Conversion
- Percentage Change
- Percentage Point Change
- Special Cases of Percentage Change
- Problems Based on Population
- CAT Test

Conversion of a Percentage into a Ratio

To convert a percentage into a ratio, first convert the given percentage into a fraction in simplest form and then to a ratio.

Exp. 3) Solve the following :

- (i) 38% (ii) 25% (iii) 66.66%

Solution (i) $38\% = \frac{38}{100} = \frac{19}{50} = 19 : 50$ (ii) $25\% = \frac{25}{100} = \frac{1}{4} = 1 : 4$

(iii) $66.66\% = 66\frac{2}{3}\% = \frac{200}{3 \times 100} = \frac{2}{3} = 2 : 3$

Conversion of a Ratio into a Percentage

To convert a ratio into a percentage, first convert the given ratio into a fraction then to a percentage.

Exp. 4) Express the following ratios as percentage :

- (i) 1 : 5 (ii) 2 : 3 (iii) 4 : 9

Solution (i) $1 : 5 = \frac{1}{5} = \frac{1}{5} \times 100 = 20\%$ (ii) $2 : 3 = \frac{2}{3} = \frac{2}{3} \times 100 = 66.66\%$

(iii) $4 : 9 = \frac{4}{9} = \frac{4}{9} \times 100 = 44.44\%$

Conversion of a Percentage into a Decimal

To convert a percentage into a decimal remove the % sign and move the decimal point two places to the left.

Exp. 5) Convert the following percentages into decimals

- (i) 36% (ii) 250% (iii) 57.5% (iv) $17\frac{1}{5}\%$ (v) 7%

Solution (i) $36\% = 0.36$ (ii) $250\% = 250 = 25$

(iii) $57.5\% = 0.575$ (iv) $17\frac{1}{5}\% = 17.2\% = 0.172$

(v) $7\% = 0.07$

Conversion of a Decimal into a Percentage

To convert a decimal into a percentage, move the decimal point two place to the right (adding zeros if necessary) and put % sign.

Exp. 6) Convert the following decimals into percentages

- (i) 0.35 (ii) 8.12 (iii) 0.018

Solution (i) $0.35 = 35\%$ (ii) $8.12 = 812\%$

(iii) $0.018 = 1.8\%$

NOTE

1. Work out some more examples so that all these thing rest on your finger tips.

2. $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \dots = 50\%$ etc.

Percentage-Fraction Conversion Table

		Numerators											
Denominators		1	2	3	4	5	6	7	8	9	10	11	12
	1	100	200	300	400	500	600	700	800	900	1000	1100	1200
	2	50	100	150	200	250	300	350	400	450	500	550	600
	3	33.33	66.66	100	133.33	166.66	200	233.33	266.66	300	333.33	366.60	400
	4	25	50	75	100	125	150	175	200	225	250	275	300
	5	20	40	60	80	100	120	140	160	180	200	220	240
	6	16.66	33.33	50	66.66	83.33	100	116.66	133.33	150	166.66	183.33	200
	7	14.28	28.56	42.85	57.13	71.42	85.71	100	114.28	128.56	142.85	157.13	171.42
	8	12.5	25	37.5	50	62.5	75	87.5	100	112.5	125	137.5	150
	9	11.11	22.22	33.33	44.44	55.55	66.66	77.77	88.88	100	111.11	122.22	133.33
	10	10	20	30	40	50	60	70	80	90	100	110	120
	11	9.09	18.18	27.27	36.36	45.45	54.54	63.63	72.72	81.81	90.9	100	109.09
	12	8.33	16.66	25	33.33	41.66	50	58.33	66.66	75	83.33	91.66	100
	15	6.66	13.33	20	26.66	33.33	40	46.66	53.33	60	66.66	73.33	80

The Most Important Conversions

$$\begin{array}{ll} \frac{1}{2} \rightarrow 50\%, & \frac{1}{3} \rightarrow 33.33\%, \\ \frac{2}{3} \rightarrow 66.66\%, & \frac{1}{4} \rightarrow 25\%, \\ \frac{3}{4} \rightarrow 75\%, & \frac{1}{5} \rightarrow 20\%, \\ \frac{1}{6} \rightarrow 16.66\%, & \frac{5}{6} \rightarrow 83.33\% \end{array}$$

The Commonly Confusing Conversions

$$\begin{array}{ll} \frac{1}{7} = 14.28\% & \text{and} \quad \frac{1}{14} = 7.14\% \\ \frac{1}{6} = 16.66\% & \text{and} \quad \frac{1}{15} = 6.66\%, \\ \frac{1}{9} = 11.11\% & \text{and} \quad \frac{1}{11} = 9.09\% \\ \frac{1}{15} = 6.66\% & \text{and} \quad \frac{1}{16} = 6.25\%, \end{array}$$

$$\frac{1}{3} = 33.33\% \quad \text{and} \quad \frac{3}{10} = 30\%$$

NOTE With regard to percentage conversion, whenever you see 99.99%, you can safely assume that it is actually 100%. As, we know that $\frac{1}{9} = 11.11\%$, so we have $\frac{9}{9} = 99.99\%$ but $\frac{9}{9} = 100\%$

The Unforgettable Conversions

$$\frac{1}{13} = 7.69\%, \quad \frac{1}{17} = 5.88\%, \quad \frac{1}{19} = 5.26\%,$$

$$\frac{1}{21} = 4.76\%, \quad \frac{1}{23} = 4.35\%, \quad \frac{1}{24} = 4.166\%$$

Learnings from the Convergence Table

- This table is a first hand support as a percentage values of some frequently used fractions.
- All the percentage values whose decimal part is 0.33, 0.66, 0.00 contain the denominator 3 in the fraction.
- Similarly if there is 0.16, 0.33, 0.50, 0.66, 0.83, 0.00 it means there is a 6 in the denominator of the fraction.
- Similarly if there is 0.28, 0.56, 0.85, 0.13, 0.42, 0.71 it means there is a 7 as denominator.
- If there is 0.11, 0.22, 0.33, 0.44... 0.99 etc.
It means there is 9 as the denominator of the fraction.
- If there is 0.09, 0.18, 0.27... it means there is 11 as the denominator of the fraction.
- If two percentage values have different decimal values (there must be different denominators) then their addition or subtraction results always in decimal i.e., never as an integer.

Percentage of a Quantity

Exp. 1) Find the no. of male students (i.e., boys), if there are 42% male students in the school and the total no. of students in the school is 1000.

Solution Required number of male students

$$= 42\% \text{ of } 1000 = \frac{42}{100} \times 1000 = 420$$

Exp. 2) A student scored 85% marks in his exam. The maximum marks that any student can score in that exam are 400. How many marks did he score?

Solution Marks scored = $85\% \text{ of } 400 = \frac{85}{100} \times 400 = 340$

Exp. 3) In an orchard $16\frac{2}{3}\%$ of the trees are mango trees.

If the total number of trees in the orchard is 360, find the number of other types of trees in the orchard.

Solution Total number of trees = 360

$$\text{Number of mango trees} = 16\frac{2}{3}\% \text{ of } 360 = \frac{50}{100} \times 360 = 60$$

Therefore, the number of other trees = $360 - 60 = 300$

Alternatively Number of mango trees = $16\frac{2}{3}\%$

It means no. of other types of trees

$$= \left(100 - 16\frac{2}{3}\right)\% = 83\frac{1}{3}\%$$

Thus, number of other types of trees

$$= 83\frac{1}{3}\% \text{ of } 360 = 300$$

Practice Exercise

- Find the value of :
 - 25% of 200
 - 30% of 180
 - 37.5% of 300
 - 83.33% of 480
 - 100% of 2 quintal
 - 165% of 330 litre
 - $5\frac{1}{2}\%$ of ₹ 1600
 - 10% of 1 hour
 - 66.66% of 300
 - 20% of 1 million rupees
- Mr. Arvind Vidyarthi spends 30% of his money on education and he has total ₹ 15000. How many rupees he spends on education?
- Sonia purchased 80 metres of cloth, out of which 35% was used for making trousers. How much cloth was used by her for making trousers?
- The total no. of students in a school is 1250. 40% of the students are girls. Find the number of boys.
- William's monthly salary used to be ₹ 1140. Recently, his salary was increased by 16.66%.
 - How much his salary (in ₹) was increased?
 - Find his new salary, if it was increased by 33.33%.
- The population of Vatican city is 700. If it increases by 7.14% per annum (i.e., every year). Find the population of the Vatican city after one year.
- The usual speed of a car is 85 km/hr. If it is increased by 20%, what would be the increased speed of the car?
- A shopkeeper announces a reduction of 8.33% on all its prices after new year. If a wrist watch was earlier for ₹ 2400. How much would it cost now?
- 44% of the students in a class are enrolled as females and the number of students who are enrolled as males is 42. If the admitted students in that class are either males or females, find the total number of students in that class.
- 30% of a number is 225. Find the number.
- A horse costing ₹ 80,000 one year ago now costs 25% less. Find the changed price.
- 1700 students took an exam 85% students passed it. Find the number of students who failed in the exam.

CAT TIPS Now I would like to suggest you that all the problems mentioned in the exercise must be done using fractions instead of percentage, in order to make the calculation simple and handy.

Answers & Solutions

1. (i) 50, (ii) 54, (iii) 112.5
 (iv) 400, (v) 2 quintal (vi) 544.5
 (vii) 88 (viii) 6 minutes (ix) 200
 (x) 2 lakh rupees

2. $15000 \times 0.3 = 4500$ $\left(\because 30\% = \frac{30}{100} = 0.3 \right)$

Therefore, the required answer is ₹ 4500.

3. $80 \times 0.35 = 28$ $\left(\because 35\% = \frac{35}{100} = 0.35 \right)$

Therefore, the required answer is 28 meters

4. Total number of students = 1250

Total number of girls = $1250 \times 0.4 = 500$

Therefore, total number of boys in that school
 $= 1250 - 500 = 750$

Therefore, the required answer is 750.

5. Since, $16.66\% = \frac{1}{6}$

So, his salary was increased by $\frac{1}{6}$ times.

So, the increase in salary = $1140 \times \frac{1}{6} = ₹ 190$

Similarly, $33.33\% = \frac{1}{3}$

(i) Therefore the required answer is ₹ 190.

It means his salary was increased by $\frac{1}{3}$ times.

Therefore, new salary = $1 + \frac{1}{3} = \frac{4}{3}$ times

So, his new salary = $\frac{4}{3} \times 1140 = ₹ 1520$.

(ii) Therefore the required answer is ₹ 1520.

6. Since $7.14\% = \frac{1}{14}$

So, the new population of the city

$$= 700 \left(1 + \frac{1}{14} \right)$$

$$= 700 \left(\frac{15}{14} \right) = 750$$

Therefore, the required answer is 750.

7. Since $20\% = \frac{1}{5}$

Therefore, new speed = $1 + \frac{1}{5} = \frac{6}{5}$ times of the usual speed. That is new speed = $85 \times \frac{6}{5} = 102$ km/hr

Therefore, the required answer is 102 km/hr

8. Since, $8.33\% = \frac{1}{12}$

So, the reduced price of the watch

$$= 2400 \left(1 - \frac{1}{12} \right) = 2400 \left(\frac{11}{12} \right) = ₹ 2200$$

9. Refer the following solution

Female	Male	Total
44%	56%	100%
	42	

Since, $56\% = 42$, therefore, $1\% = \frac{42}{56}$

So, the $100\% = 100 \times \frac{42}{56} = 75$

Thus, the total number of students = 75

10. $30\% = 225 \Rightarrow 10\% = 75 \Rightarrow 100\% = 750$

Therefore, the required number is 750.

11. Since, $25\% = \frac{1}{4}$, therefore, $75\% = \frac{3}{4}$

So, the new cost of the horse = $\frac{3}{4}$ times of the original cost.

Therefore, the new cost of the horse
 $= 80000 \times \frac{3}{4} = ₹ 60000$

Therefore, the required number is 60000.

12. Refer the following solution

Total	Passed	Failed
100%	85%	15%
1700		

Number of students who failed the exam

$$= 15\% \text{ of } 1700 = 0.15 \times 1700 = 255$$

Therefore, the required answer is 255.

☞ **Alternatively** Number of students who failed the exam = 15% of 1700

$$= (10\% + 5\%) \times 1700 = (170 + 85) = 255$$

Expressing One Quantity as a Percentage of Another Quantity

Exp. 1) What per cent is number 3 of number 20 ?

Solution As per cent means out of 100. Then by unitary method
out of 20 $\rightarrow$ 3, out of 1 $\rightarrow \frac{3}{20}$, out of 100 $\rightarrow \frac{3}{20} \times 100 = 15\%$

Hence to find what per cent the first number is of second number, we divide the first number by the second number and multiply the result by 100.

Exp. 2) Ravi obtained 325 marks out of a maximum of 400 marks. Find the percentage of marks obtained by him.

Solution Required percentage of marks $= \frac{325}{400} \times 100 = 81.25\%$

Introductory Exercise 5.1

- What per cent is :
 - 30 out of 600 ?
 - 25 out of 160 ?
 - 75 out of 225 ?
 - 36 kg of 150 kg ?
 - 90 cm of 4.5 metre?
 - 60 litres of 40 litres?
 - 800 shirts out of 1200 shirts?
 - 875 m of 2 km?
- Express:
 - 20 as a percentage of 500.
 - 60 kg as a percentage of 80 kg .
 - 350 ml as a percentage of 5.6 litre.
 - ₹ 13 as a percentage of ₹ 39.
 - 15 seconds as a percentage of 1 hour.
 - 27° as a percentage of 360° .
- Manu scored 384 marks out of 450. What per cent marks did she get?
- In an election, out of 60,000 eligible voters 42000 cast their vote. Calculate the percentage of eligible voters casting their votes.
- A tin contains 24 litres of milk. Due to leakage 720 ml is lost. What per cent of milk is still present in the tin?
- Price of an item increased from ₹ 16.50 to ₹ 41.25 Find the percentage increase in price.
- The excise duty on a certain item has been reduced to ₹ 3480 from ₹ 5220 . Find the percentage reduction in the excise duty on that item.
- Out of total production of 6450 tonnes of a coalmine a quantity of 645 tonnes was lost during extraction. What per cent of the total production was the net coal extracted?
- A cricket team played 24 matches. The team won 9 matches and lost 3 matches. 12 matches ended in draw. What per cent of the total matches did the team lose?
- In a hostel, in a particular month, out of ₹ 50,000 budget, ₹ 10,000 were allocated for the food items. Further, out of the food budget, ₹ 2000 were allocated for the fruits. What percentage of the total monthly budget of the hostel was allocated for the fruits?

Exp. 3) In a factory of 150 workers, 18 were absent in a day what percentage were present?

Solution Present = $150 - 18 = 132$

$$\text{Percentage presence} = \frac{132}{150} \times 100 = 88\%$$

Exp. 4) Kurla obtained 480 marks out of 600 and Birla obtained 560 marks out of 800.

Whose performance is better?

Solution % marks of Kurla $= \frac{480}{600} \times 100 = 80\%$

$$\% \text{ marks of Birla} = \frac{560}{800} \times 100 = 70\%$$

So, obviously Kurla's performance is better than that of Birla even though getting less absolute marks.

5.3 Percentage Change

The application of percentage change is very diverse in nature e.g., profit and loss, simple interest and compound interest etc. All these are exactly based on the percentage increase/decrease of the original (or actual) value.

Percentage increase in a value

$$= \left(\frac{\text{increased value} - \text{original value}}{\text{original value}} \times 100 \right) \%$$

Percentage decrease in a value

$$= \left(\frac{\text{original value} - \text{decreased value}}{\text{original value}} \times 100 \right) \%$$

Exp. 1) Sometimes ago Abhimanyu's height was 110 cm. Now, his height is 120 cm. Find the percentage change in his height.

Solution $\frac{120 - 110}{110} \times 100 = 9.09\%$

☞ **Alternatively** $\frac{10}{110} \rightarrow \frac{1}{11} \rightarrow 9.09\%$

☞ **Alternatively** $\frac{12}{11} \rightarrow 109.09\%$, so increase = 9.09%

So, the increase in height = 109.09 – 100 = 9.09%

Exp. 2) The total expenses of a hostel were ₹ 8000 per month. Some students left the hostel due to which the new expenses come down by ₹ 1000. Find the percentage decrease in total expenses of the hostel.

Solution $\frac{1000}{8000} \rightarrow \frac{1}{8} \rightarrow 12.5\%$

Exp. 3) Raja's salary is ₹ 9,000 per month and Rani's salary is ₹ 10,000 per month.

(i) With respect to Raja's salary what percent is Rani's salary?

(ii) With respect to Rani's salary what percent is Raja's salary?

Solution (i) $\frac{10000}{9000} \times 100 = 111.11\%$

(ii) $\frac{9000}{10000} \times 100 = 90\%$

☞ **Alternatively** (i) $\frac{10}{9} \rightarrow \frac{9}{9} + \frac{1}{9} \rightarrow 100\% + 11.11\% \rightarrow 111.11\%$

(ii) $\frac{9}{10} \rightarrow 90\% \quad \left(\because \frac{1}{10} \rightarrow 10\% \right)$

5.4 Percentage Point Change

Last year Abhijeet's salary was ₹ 10,000 and Sonu's salary was ₹ 8,000. This year Abhijeet's salary is ₹ 12,000 while Sonu's salary is ₹ 10,000.

1. (i) What is the percentage increase in Abhijeet's salary?
- (ii) What is the percentage increase in Sonu's salary?
- (iii) Percentage increase in Sonu's salary is how much per cent greater than the percentage increase in Abhijeet's salary?
- (iv) What is the percentage point change in the salary of Sonu and Abhijeet?

Solution

(i) $\frac{12,000 - 10,000}{10,000} \times 100 = 20\%$

or $\frac{2}{10} \rightarrow \frac{1}{5} \rightarrow 20\%$

(ii) $\frac{2}{8} \rightarrow \frac{1}{4} \rightarrow 25\%$

(iii) Percentage increase in Sonu's salary = 25

Percentage increase in Abhijeet's salary = 20

So the required percentage

$$= \frac{25 - 20}{20} \times 100 = 25\%$$

It means percentage increase in Sonu's salary is 25% greater than the percentage increase in Abhijeet's salary.

(iv) Percentage point change

= (Percentage increase in Sonu's salary

– Percentage increase in Abhijeet's salary)

$$= 25 - 20 = 5 \text{ percentage point}$$

In fact, percentage point change is the difference between two percentage values.

5.5 Special Cases of Percentage Change

Getting Back the Original Value

- (A) If a value p is increased by $x\%$, then we have to decrease the resultant value by $\left(\frac{x}{x+100} \times 100 \right) \%$ to get back to the original value p .

$$\begin{array}{ccccc} \text{Original} & & \text{increasing value} & & \text{Increased value} \\ \text{value} & \longrightarrow & & \longrightarrow & \\ p & \longrightarrow & \frac{p \times x}{100} & \longrightarrow & \left(p + \frac{px}{100} \right) \\ & & & & = p \left(\frac{100+x}{100} \right) \end{array}$$

Now the percentage decrease

$$= \frac{\left[p \left(\frac{100+x}{100} \right) - p \right]}{p \left(\frac{100+x}{100} \right)} \times 100 = \left(\frac{x}{100+x} \times 100 \right) \%$$

In other words (*i.e.*, in terms of fraction) if a value is increased by $\frac{n}{d}$ then to get back the same number p from the resultant value, we have to decrease the increased value by $\left(\frac{n}{d+n} \right)$.

Exp. 1) Salary of Rajeev Ratan in 2001 was \$ 100 per day and his salary in 2002 was \$ 125 per day. Again in 2003 his salary was \$ 100 per day.

- What is the percentage increase in his salary in 2002?
- What is the percentage decrease in his salary in 2003 over 2002?

Solution (i) $\frac{125-100}{100} \times 100 = 25\%$ (ii) $\frac{125-100}{125} \times 100 = 20\%$

In fact there is same absolute change but percentage change is different due to different denominator (*i.e.*, base change). So the base change is as much important as the numerator or absolute change in quantity.

Exp. 2) Age of Ravi is 20 years. If the Shyam's age is 25% greater than that of Ravi then how much per cent Ravi's age is less than Shyam's age?

Solution Age of Ravi Age of Shyam

$$20 \quad \xrightarrow{(+25\%)} \quad 25$$

$$\quad \quad \quad \xleftarrow{(-20\%)} \quad$$

Solution using percentage change formula Here $x = 25$, then percentage decrease (or less) $= \frac{25}{125} \times 100 = 20\%$

Solution using fractional change formula Since increase $= \frac{1}{4}$

$$\therefore \left(25\% = \frac{1}{4} \right)$$

$$\text{Therefore decrease} = \frac{1}{4+1} = \frac{1}{5} \rightarrow 20\%$$

Exp. 3) Gautam has 33.33% more pencils than Rahul has. By how much per cent less pencils Rahul has than that of Gautam?

Solution

$$\begin{array}{ccc} & 33.33\% \rightarrow \frac{1}{3} & \\ \text{Rahul} & \xrightarrow{\hspace{1cm}} & \text{Gautam} \\ & -25\% = \frac{1}{4} = \frac{1}{3+1} & \end{array}$$

So Rahul has 25% less pencils than Gautam has.

Exp. 4) Height of Amitabh is 50% greater than the height of Amir Khan. Height of Amir Khan is how much per cent less than that of Amitabh?

Solution

$$\begin{array}{ccc} & +50\% \rightarrow \frac{1}{2} & \\ \text{Amir} & \xrightarrow{\hspace{1cm}} & \text{Amitabh} \\ & -33.33\% = \frac{1}{3} = \frac{1}{2+1} & \end{array}$$

Thus the height of Amir is 33.33% less than Amitabh.

Explanation of Concept

Assume A has 10 apples and B has 20 apples, then we say that B has 100% more apples than A has. Again we can say that A has 50% less apples than B has.

To calculate the percentage change :

$$\begin{array}{ccc} & 100\% \rightarrow \frac{1}{1} & \\ A & \xrightarrow{\hspace{1cm}} & B \\ & -50\% = \frac{1}{2} = \frac{1}{1+1} & \end{array}$$

Consider Another Example

Let us assume A has 6 chocolates and B has 10 chocolates then we can say that B has 66.66% more chocolates than A . So again we can also say that A has 40% less chocolates than B .

$$\begin{array}{ccc} & 66.66\% \rightarrow \frac{2}{3} & \\ A & \xrightarrow{\hspace{1cm}} & B \\ & -40\% = \frac{2}{5} = \frac{2}{3+1} & \end{array}$$

Remember the percentage change graphic is almost 70–80% better in giving quick and simple results. So instead of using the percentage formula, use the percentage change graphic method. For this you are just required to know the percentage fraction conversion table.

- (B) If a value P is first decreased by $x\%$ then to get back the original value P , we have to increase the decreased (resultant) value by $\left(\frac{x}{100-x} \times 100\right)\%$

$$\begin{aligned} &\text{original value } P \xrightarrow[\frac{Px}{100}]{\text{(decreasing value)}} \text{decreased value } \left(P - \frac{Px}{100}\right) = P \left(1 - \frac{x}{100}\right) \\ &\text{Now, the per cent increase} = \frac{\left[P - P \left(1 - \frac{x}{100}\right)\right]}{P \left(1 - \frac{x}{100}\right)} \times 100 \\ &= \frac{x}{100-x} \times 100 \end{aligned}$$

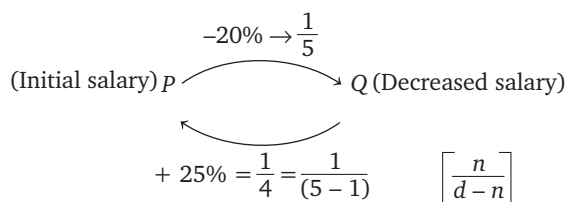
In terms of fraction, if a value P is first decreased by n/d then to get back the original number P . We have to increase the decreased (or resultant) value by $\frac{n}{d-n}$.

Exp. 5) Due to irregular working habits of Ramachandran, his salary was reduced by 20% but after some months his salary was increased to the original salary. What is the percentage increase in salary of Ramachandran?

Solution Let initial salary be ₹ 100. $100 \xrightarrow[-20\%]{+25\%} 80$

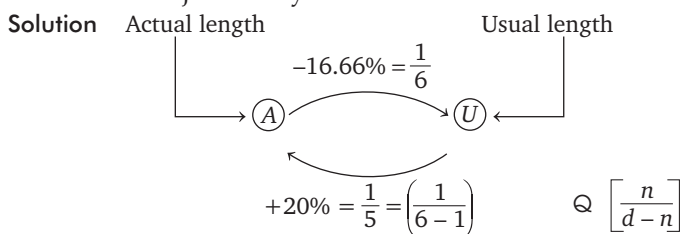
Alternatively Here $x = 20$
then % increase = $\frac{20}{(100-20)} \times 100 = 25\%$

Alternatively



Hence his salary will be increased by 25%.

Exp. 6) Kajol usually wears saree, which is 16.66% less than the actual length of the saree. By how much per cent the actual length of the saree is greater than the length of saree which kajol usually wears?



So the actual length of the saree is 20% greater than the usually used saree.

NOTE

- (1) If a value X is first increased by $p\%$ to Y then Y is again decreased to X by $q\%$ then p is always greater than q (for positive values).
- (2) If a value X is first decreased by $p\%$ to Y and then Y is increased by $q\%$ to X , then p is always less than q .
- (3) If a value A is increased by $p\%$ then again by $q\%$ once again it is increased by $r\%$, then the final value will be same as if you change the order of p, q, r i.e., A can be first increased by $r\%$ then by $q\%$ and then by $p\%$ still the result will be same.
- (4) The rule 3 is also applicable for the decreasing of the values. A value ' A ' is first decreased by $p\%$, then by $q\%$ and then by $r\%$ and so on, the resultant value will be same as when A is first decreased by $q\%$ then by $p\%$ and then by $r\%$ etc.
Note that in case 3 and 4 we are discussing the successive increase or decrease in the value.
- (5) A value ' A ' is first increased by $p\%$ then by $q\%$ and then it is reduced by $r\%$ will give the same results as when A is first decreased by $r\%$, then increased by $q\%$ and then by $p\%$ etc.

Exp. 7) Initially Ms. Rakhi Sawant has ₹ 200 in her wallet then she increased it by 20%. Once again she increased her amount by 25%. The final value of money in her wallet will be how much per cent greater than the initial amount?

Solution $200 \xrightarrow{+20\%} 240 \xrightarrow{+25\%} 300$
So the required % increase = $\frac{300-200}{200} \times 100 = 50\%$

Exp. 8) The age of B is 50% greater than the age of A . The age of C is 20% less than the age of B . By how much percentage the age of C is greater than the age of A ?

Solution $A \xrightarrow{+50\%} B \xrightarrow{-20\%} C$
 $100 \quad 150 \quad 120$
So the required percentage change = $\frac{20}{100} \times 100 = 20\%$

Reversing the change by same Percentage

If the value of a number is first increased by $x\%$ and then decreased by $x\%$, the net change is always a decrease (or loss) in original value. That is, % loss = $(x/10)^2\%$

If the value of a number is first decreased by $x\%$ and then the resultant value is increased by $x\%$, the net change is always a decrease (or loss) in the original value. That is % loss = $(x/10)^2$

Exp. 1) Shweta is very expert in bargaining. Once she went to a nearby shop. When Shweta asked the price of Shampoo Sachet the shopkeeper told her the price by increasing 15% of the original cost. But Shweta insisted to decrease the price by 15% so the shopkeeper sold it by decreasing the price by 15%. What is the loss or profit of shopkeeper and by how much per cent?

- (a) no loss (b) profit of 1.5%
(c) loss of 2.25% (d) none of these

Solution Let the actual price be 100.

$$100 \xrightarrow{+15\%} 115 \xrightarrow{-15\%} 97.75 \quad (\text{loss of } 2.25\%)$$

☞ **Alternatively** $\text{loss}\% = \left(\frac{15}{10}\right)^2 = 2.25\%$

There is always a loss.

Exp. 2) If the length and breadth of a rectangle are changed by +20% and -10%. What is the percentage change in area of rectangle?

- (a) 8% (b) 10.8%
(c) 20% (d) data insufficient

Solution $l \times b = \text{area}$

$$1 \times 1 = 1 \Rightarrow 1.2 \times 0.9 = 1.08$$

So there is 8% increase in the area of rectangle.

Exp. 3) A's monthly salary is 20% lower than B's monthly salary and C's monthly salary is 56.25% greater than A's salary. By how much percent B's salary is less than C's salary?.

- (a) 15% (b) 20%
(c) 25% (d) 33.33%

Solution $\begin{array}{ccc} A & B & C \\ 80 & \xleftarrow{(-20\%)} 100 & 125 \end{array}$ (Consider the salary of B = 100)

The required value = $\frac{25}{125} \times 100 = 20\%$

Product Constancy

It is the same as we know the inverse proportion in the chapter of Ratio, Proportion and Variation. For example, when the rate of a pencil is ₹ 1.25 then we can purchase 16 pencils by paying ₹ 20. If the rate of a pencil is decreased by ₹ 0.25 then we can purchase 20 pencils by paying ₹ 20.

Explanation : Rate $\times$ No. of pencils = Price

$$1.25 \times 16 = 20, \quad 1.00 \times 20 = 20$$

So you can see that here the product (20) is constant in both the cases. Thus it is clear that if we reduce the price of a pencil to ₹ 0.50, then we can purchase 40 pencils in ₹ 20.

Some more examples of product constancy :

- speed $\times$ time = distance
- rate $\times$ time = cost
- efficiency $\times$ time = work
- length $\times$ breadth = area
- average $\times$ no of elements = total value
- rate $\times$ quantity = price (or expenditure)

For example The price of sugar is increased by 25% then by how much per cent should a customer reduce the consumption (i.e., quantity used) of sugar so that he has not to increase his expenses on sugar.

$$\text{price} \times \text{quantity} = \text{expenditure}$$

$$100 \times 100 = 10,000$$

$$125 \times x = 10,000$$

$$\Rightarrow x = \frac{10,000}{125} = 80. \text{ Therefore \% reduction} = 20\%$$

or $1 \times 1 = 1$
 $1.25 \times k = 1 \Rightarrow k = 0.8$

Thus there will be 20% decrease in the consumption of sugar in order to maintain the same expenditure on sugar.

Product Constancy Conditions

- When one factor of a product is increased by $p\%$, then the other factor will be decreased by $\left(\frac{p}{100+p} \times 100\right)\%$.

It means when one factor of a product is increased by $\frac{n}{d}$

then the other factor is decreased by $\frac{n}{(d+n)}$.

- When one factor of a product is decreased by $p\%$ then the other factor will be increased by $\left(\frac{p}{100-p} \times 100\right)\%$.

It means when one factor of a product is decreased by $\frac{n}{d}$

then the other factor will must be increased by $\frac{n}{(d-n)}$.

Exp. 1) If the price of a commodity be raised by 20%, then by how much per cent a house holder reduce his consumption of the same commodity so that his expenditure does not increase.

Solution Since here product (i.e., expenditure) is constant
rate $\times$ consumption = expenditure

$$\text{initially} \rightarrow 1 \times 1 = 1$$

$$\text{After change} \quad 1.2 \times x = 1$$

$$\Rightarrow x = 0.833$$

$$\therefore \text{decrease in value} = 16.66\%$$

☞ **Alternatively**

Increase in rate

Decrease in consumption

$$+20\% = +\frac{1}{5}$$

→

$$-16.66\% = -\frac{1}{6}$$

Exp. 2) If the price of petrol falls down by 20% by how much per cent must a person increase its consumption, so as not to decrease the expenditure on this item?

Solution Since product is constant

decrease by

increase by

$$20\% = \frac{1}{5}$$

→

$$\frac{1}{4} = 25\%$$

$$\left[\therefore \frac{n}{d} \rightarrow \frac{n}{(d-n)} \right]$$

Exp. 3) Due to 50% increase in the price of rice . We purchased 5 kg less rice with the same amount of ₹ 60. What is the new price of rice?

- (a) ₹ 4.66 (b) ₹ 5 (c) ₹ 4 (d) ₹ 6

Solution Increase in price Decrease in amount
 $50\% = \frac{1}{2} \longrightarrow \frac{1}{3} = 33.33\%$

Since the new quantity of rice decreases by 33.33% which is equal to 5 kg it means initially there was 15 kg rice to be used.

So, the initial price = ₹ 4 $\therefore \left(\frac{60}{15} = 4\right)$

and final price = ₹ 6 $\therefore \left(\frac{60}{10} = 6\right)$

Alternatively From the options.

Let us consider choice (d).

Therefore $[6 \times 10 = 60]$ (finally)

Hence $[4 \times 15 = 60]$ (initially)

Exp. 4) The length of a plot is decreased by 33.33%. By how much % the breadth of the plot will be increased so that the area remains constant?

Solution (Decrease) (Increase)
 $33.33 = \frac{1}{3} \longrightarrow \frac{1}{2} = 50\% \quad \left[\frac{1}{2} = \frac{1}{3-1}\right]$

Difference Between 'by' and 'to'

Please note that there is a clear difference between “by” and “to”. For example., the income is reduced by 40% it means the new income is 60% of the original and the income is reduced to 40% means the new income is 40% of the original value. Thus “by” represents difference and “to” represents final value. For example., The income of Sarika is increased by 20% means its new income is $100 + 20 = 120\%$ of the original income. The income of Sarika is increased to 120% means the new income of Sarika is 120% of the original income.

Exp. 1) In an election between two candidates, the candidate who got 57% valid votes won by a majority of 420 votes. Find the total no. of valid votes .

Solution Winner Loser
 $0.57x \quad 0.43x \quad [100 - 57 = 43]$
 $\quad \quad \quad 0.14x \quad [57 - 43 = 14]$
 $0.14x = 420 \Rightarrow x = 3000$
 Hence total valid votes = 3000

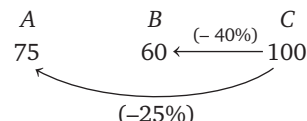
Exp. 2) Due to fall in manpower, the production in the factory decreases by 60%. By what per cent should the working hour be increased to restore the original production in the factory?

Solution Manpower $\times$ Working hours = Production
 $\downarrow \quad \quad \quad \uparrow$
 $(-60\%) = \frac{3}{5} \longrightarrow \frac{3}{2} = (+150\%) \quad \left[\frac{3}{5} \rightarrow \frac{3}{5-3} = \frac{3}{2}\right]$

Hence by 150% working hours will be increased. It means the new working hours will be 2.5 times (not 1.5 times) of the original time.

Exp. 3) Two numbers are respectively 25% and 40% less than a third number. What per cent is the second of the first ?

Solution Consider A, B, C three numbers and assume C = 100 (as a base)



Now $\frac{60}{75} \times 100 = 80\%$

Exp. 4) A person gives 10% to his wife 10% of the remaining to a hospital (as a donation) again 10% of the remaining to Prime Minister's Relief Fund. Then he has only ₹ 7290 with him. What was the initial sum of money with that person?

Solution Since he gives 10% so he is left with 90% of the original sum and since he does the same with the remaining (or left) amount. So it forms a chain.

$\therefore$ Remaining amount = $(x) \times 0.9 \times 0.9 \times 0.9 = 0.729x = 7290$
 $\Rightarrow x = 10,000$, where x is supposed to be initial amount.

Exp. 5) Initially a shopkeeper had n chocolates. A customer bought 10% chocolate from n then another customer bought 20% of the remaining chocolates, after that one more customer purchased 25% of the remaining chocolates. Finally shopkeeper is left with 270 chocolates in his shop. How many chocolates were there initially in his shop?

- (a) 300 (b) 450 (c) 500 (d) 600

Solution $n \times 0.9 \times 0.8 \times 0.75 = 270 \Rightarrow n = \frac{270 \times 10,000}{9 \times 8 \times 75} \Rightarrow n = 500$

NOTE These type of problems (see example no. 4 and 5) can be solved with a great convenience if we solve in reverse order, with the aid of given choices. Let us consider option (c)

$500 \times 0.9 = 450 \Rightarrow 450 \times 0.8 = 360 \Rightarrow 360 \times 0.75 = 270$

Hence presumed option is correct.

Difference Between Decreased value and Decreased in value

There is a huge difference between “decreased value” and “decrease in value” similarly, there is a huge between “increased value” and “increase in value”.

For example Initial value = 70

Final value = 90. It means increased value = 90 but increase in value = 20 and % increase in value = $\frac{90-70}{70} \times 100 = 28.57\%$

(1) If there is an increase of x/y in any value P then the increased value will be $P \left(1 + \frac{x}{y}\right)$.

(2) If there is a decrease of x/y in any value P , then the decreased value will be $P \left(1 - \frac{x}{y}\right)$.

Exp. 1) Nishith is now 20 years old. Some years later his age will increase by 50% of himself. What will be the new age at that time?

Solution $20\left(1 + \frac{1}{2}\right) = 30$ years $\left[\because 20 + 20 \times \frac{1}{2} = 20\left(1 + \frac{1}{2}\right)\right]$

Exp. 2) The average salary of Purushottam in Infosys is 20% less than that was in Microsoft. If the salary of Purushottam in Microsoft be \$ 80,000 per month then what is the salary of Purushottam in Infosys?

Solution $20\% = \frac{1}{5}$

So new salary = $80,000\left(1 - \frac{1}{5}\right) = 80,000 \times \frac{4}{5} = \$ 64,000$

NOTE Remember this type of problems can also be solved exactly through percentages.

As $80,000 \times 0.8$ ($100 - 20 = 80\%$) = 64,000

But the technique given above is not less important. Some where it becomes very necessary to solve through fractions.

So, keep your eyes open mind focussed and use your wits to solve the problems intelligently as per the situation.

Exp. 3) Which one of the following is greatest $\frac{3}{4}, \frac{7}{8}, \frac{16}{19}, \frac{13}{15}$?

Solution $\frac{3}{4} = 75\% \Rightarrow \frac{7}{8} = 87.5\%$
 $\frac{16}{19} = 84.21\% \Rightarrow \frac{13}{15} = 86.66\%$

So, $\frac{7}{8}$ is the greatest fraction (or rational number)

5.6 Problems Based on Population

If the original population of a locality (*i.e.*, region) be P and the annual growth rate be $r\%$.

The population after n years = $P\left(1 + \frac{r}{100}\right)^n$

change (or increase) in the population = $P\left[\left(1 + \frac{r}{100}\right)^n - 1\right]$

If there is decrease in population be $r\%$ then,

total population after n years = $P\left(1 - \frac{r}{100}\right)^n$

and decrease in population = $P\left[1 - \left(1 - \frac{r}{100}\right)^n\right]$

Exp. 1) If the annual increase in the population be 20% and the present population be 10,000. What will be the population after 3 years hence?

(a) 16,000 (b) 17,280 (c) 14,400 (d) 1,728

Solution $10,000\left(1 + \frac{20}{100}\right)^3 = 10,000\left(\frac{6}{5}\right)^3 = 10,000 \times (1.2)^3 = 17,280$

Hence (b) is correct.

Exp. 2) The population of a town in the first year increases by 10% in the next year it decreases by 10%. Once again in the third year it increase by 10% and in the fourth year it decrease by 10%. If the present population be 20,000 then the population after four years will be :

(a) 16,902 (b) 19,602
(c) 20,000 (d) none of these

Solution $20,000\left(1 + \frac{10}{100}\right)\left(1 - \frac{10}{100}\right)\left(1 + \frac{10}{100}\right)\left(1 - \frac{10}{100}\right)$

$= 20,000 (1.1) (0.9) (1.1) (0.9) = 20,000 \times 1.21 \times 0.81 = 19,602$

Thus option (b) is correct.

Practice Exercise

- What per cent of $\frac{3}{7}$ is $\frac{1}{105}$?
(a) 10% (b) 2.22% (c) 45% (d) 450%
- What per cent is 3% of 15%?
(a) 15% (b) 20% (c) 40% (d) 66.66%
- If the cost of a calculator worth ₹ 250 is increased by ₹ 100, the rate of increase is
(a) 100% (b) 40% (c) 25% (d) 90%
- A number increased by 37.5% gives 99, the number is
(a) 140 (b) 61.5 (c) 72 (d) 48
- When 40% of a number is added to 42, the result is the number itself. The number is:
(a) 105 (b) 72 (c) 70 (d) 82
- In an examination 52% of the candidates failed in Science, 42% in Mathematics and 17% in both. The no. of those who passed in both the subjects, is :
(a) 83% (b) 64% (c) 23% (d) 55.55%
- The price of an item is increased by 20% and then decreased by 20%. The final price as compared to original price
(a) 4% more (b) 20% more (c) 20% less (d) 4% less
- Two candidates fought an election. One got 65% of the votes and won by 600 votes. The total no. of votes polled is
(a) 12,000 (b) 10,000 (c) 2,000 (d) 8,000
- Out of a no. of electronic items, a person purchases 60% coloured TVs 5% of these were found to be defective. The percentage of defective TVs in all is
(a) 3% (b) 6%
(c) 12% (d) Can't be determined
- A's salary is half that of B. If A got a 50% raise in his salary and B got a 25% raise in his salary, then the percentage increase in combined salaries of both is
(a) 30% (b) 33.33% (c) 55% (d) 28%

Answers

1. (b) 2. (b) 3. (b) 4. (c) 5. (c)
6. (c) 7. (d) 8. (c) 9. (a) 10. (b)

CAT-Test

Questions Helping you bell the CAT

LEVEL 01 > BASIC LEVEL EXERCISE

- 1 In our Singapore office there are 60% female employees. 50% of all the male employees are computer literate. If there are total 62% employees computer literate out of the total 1600 employees, then the no. of female employees who are computer literate, provided that our employees are either male or female.
(a) 690 (b) 672
(c) 960 (d) Can't be determined
- 2 The price of a car depreciates in the first year by 25% in the second year by 20% in the third year by 15% and so on. The final price of the car after 3 years, if the present cost of the car is ₹. 10,00,000 :
(a) 7,80,000 (b) 5,10,000 (c) 6,90,000 (d) 1,70,000
- 3 A shopkeeper charges sales tax of $x\%$ up to ₹ 2,000 and above it he charges $y\%$. A customer pays total tax of ₹ 320, when he purchases the goods worth ₹ 6,000 and he pays the total tax of ₹ 680 for the goods worth ₹ 12,000. The value of $(x - y)$ is :
(a) 0 (b) - 2 (c) - 4 (d) 5
- 4 40% of a number when added to the square of the same number, then it is increased to 4040% of itself the actual number is :
(a) 175 (b) 400 (c) 40 (d) 120
- Directions** (for Q. Nos. 5-7) *In my office there are 30% female employees of whom 30% earn greater than ₹ 8000 per month and 80% of male employees earn less than ₹ 8000 per month. It is well known that my employees are either male or female. It is well known that my employees are either male or female.*
- 5 What is the percentage of employees who earn more than ₹ 8000 per month?
(a) 30% (b) 23%
(c) 60% (d) Can't be determined
- 6 What could be the maximum percentage of employees who earn more than ₹ 8000 per month?
(a) 44% (b) 23%
(c) 60% (d) Can't be determined
- 7 What could be the minimum percentage of employees who earn less than ₹ 8000 per month?
(a) 9% (b) 14%
(c) 56% (d) Can't be determined
- 8 600 students took the test on Physics and Chemistry. 35% students failed in Physics and 45% students failed in Chemistry and 40% of those who passed in Chemistry also passed in Physics, then how many students failed in both?
(a) 162 (b) 138
(c) 60 (d) none of these
- 9 In an examination 70% of the candidates passed in History and 50% in Geography and 20% students failed in both the subjects. If 500 students passed in both the subjects, then how many candidates appeared for the exam ?
(a) 1,000
(b) 1,500
(c) 2,500
(d) none of the above
- 10 My salary is ₹ 12,345 per month. The salary of my brother is 10% greater than that of mine. The salary of my only sister is 9.09% greater than my only brother. The salary of my wife is $56\frac{12}{23}\%$ less than the total salary of my brother and sister together, then the salary of my wife is :
(a) greater than my sister's salary
(b) $33\frac{11}{23}\%$ less than my sister's salary
(c) equal to my salary
(d) $44\frac{11}{23}\%$ greater than my own salary
- 11 MDTV is a very popular TV channel. It telecasts the programmes from 8 : 00 a.m. to 12 : 00 p.m. It telecasts 60 advertisements each of 8 seconds and 16 advertisements each of 30 seconds. What is the percentage of time devoted in a day for the advertisements?
(a) 1.5% (b) 1.66%
(c) 2% (d) 2.5%
- 12 Lagaan is levied on the 60% of the cultivated land. The revenue department collected total ₹ 3,84,000 through the lagaan from the village of Sukhiya. Sukhiya, a very rich farmer, paid only ₹ 480 as lagaan. The percentage of total land of Sukhiya over the total taxable land of the village is :
(a) 0.15% (b) 15%
(c) 0.125% (d) none of these

- 13** The actual area of a rectangle is 60 cm^2 , but while measuring its length a student decreases it by 20% and the breadth increases by 25%. The percentage error in area, calculated by the student is :
 (a) 5% (b) 25%
 (c) can't be determined (d) none of these
- 14** The cost of packaging of the mangoes is 40% the cost of fresh mangoes themselves. The cost of mangoes increased by 30% but the cost of packaging decreases by 50%, then the percentage change of the cost of packed mangoes, if the cost of packed mangoes is equal to the sum of the cost of fresh mangoes and cost of packaging :
 (a) 14.17% (b) 7.14%
 (c) 6.66% (d) none of these
- 15** Abhinav scores 80% in Physics and 66% in Chemistry and the maximum marks of both the papers are 100. What per cent does he score in maths which is of 200 marks, if he scores 80% marks in all the three subjects?
 (a) 74% (b) 84%
 (c) 87% (d) 83%
- 16** In the New York Stock Exchange there are 45% female employees and thus the number of male employees is exceeded by 72. Hence the total no. of employees in the New York Stock Exchange is :
 (a) 540 (b) 720
 (c) 7200 (d) 550
- 17** Three candidates A, B and C contested an election. Out of the total votes on a voter list 25% did not vote and 6.66% votes polled were invalid. C got 2450 valid votes, which were 40% more than that of B. If A got only 40% of the total votes, then who is the winner?
 (a) A (b) B
 (c) C (d) can't be determined
- 18** The cost of a car is 400% greater than the cost of a bike. If there is an increase in the cost of the car is 15% and that of bike is 20%. Then the total increase in the cost of the 5 cars and 10 bikes is :
 (a) 17.5% (b) $16\frac{3}{7}\%$
 (c) 18.5% (d) 18.25%
- 19** The square of a positive number is 2,000% greater than the number itself, then the square of that number is :
 (a) 1762 (b) 1635
 (c) 441 (d) 139
- 20** The monthly salary of Shahid and Kareena together is \$ 28,000. The salary of Shahid and Kareena is increased by 25% and 12.5% respectively then the new salary of Kareena becomes 120% of the new salary of Shahid. The new (or increased) salary of Shahid is :
 (a) \$ 15,000 (b) \$ 18,000
 (c) \$ 14,000 (d) \$ 16,000
- 21** 80% of a smaller number is 4 less than 40% of a larger number. The larger number is 85 greater than the smaller one. The sum of these two numbers is :
 (a) 325 (b) 425 (c) 235 (d) 500
- 22** 220% of a number 'X' is 44. What is 44% of 'X' ?
 (a) 88 (b) 8.8
 (c) 66 (d) Data insufficient
- 23** The shopkeeper increased the price of a product by 25% so that customer finds it difficult to purchase the required amount. But somehow the customer managed to purchase only 70% of the required amount. What is the net difference in the expenditure on that product ?
 (a) 10% more (b) 5% more
 (c) 12.5% less (d) 17.5% less
- 24** In the previous government, party Q was in the opposition. Now increasing the seats by 33.33% Q is the ruling party and thus party Q enjoys twice the majority than that of party P in the previous government. If there were only two parties P and Q and the fix no. of seats be 500 in the parliament of Hum-Tum, then the no. of seats of the Q in the new government is :
 (a) 225 (b) 200 (c) 275 (d) 300
- 25** In an examination a candidate got 30% marks and failed by 30 marks. If the passing marks are 60% of the total marks, then the maximum marks will be :
 (a) 450 (b) 600 (c) 300 (d) 100
- 26** In a school there are 1800 students. Last day except 4% of the boys all the students were present in the school. Today except 5% of the girls all the students are present in the school, but in both the days no. of students present in the school, were same. The no. of girls in the school is :
 (a) 1200 (b) 800 (c) 1000 (d) 600
- 27** In a library 60% of the books are in Hindi, 60% of the remaining are in English and rest of the books are in Urdu. If there are 3600 books in English, then the total no. of books in Urdu are :
 (a) 2400 (b) 2500
 (c) 3000 (d) none of these
- 28** In a test there are total n questions. Bhanu answers 20 out of 25 questions correctly in the first section. In the second section he answers 60% question correct and thus his total score is 66.66% in the test. Given that all the questions carry equal marks, without any negative marking. The total no. of question in the test is :
 (a) 50 (b) 60 (c) 75 (d) 100
- 29** In a class of MBA students 16.66% students are from Science background and 12.5% students are from commerce background and 6.66% students from arts background and rest are from Engineering background. The minimum possible students of engineering background are :
 (a) 45 (b) 77 (c) 100 (d) 120

- 30** An alloy contains the copper and aluminium in the ratio of 7 : 4. While making the weapons from this alloy, 12% of the alloy got destroyed. If there is 12 kg of aluminium in the weapon, then the weight of the alloy required is :
 (a) 48 kg (b) 40 kg
 (c) 37.5 kg (d) 14.4 kg
- 31** Hariharan goes to a shop to buy an FM radio costing ₹ 1404 including sales tax at 8%. He asks the shopkeeper to reduce the price of radio so that he can save the amount equal to the sales tax. The reduction of the price of the radio is :
 (a) ₹ 108 (b) ₹ 104
 (c) ₹ 112.32 (d) none of these
- 32** The average weight of a class of students is 67.5 kg. The weight of the class teacher is 25% more than the average weight of the class. The average weight of the class is less than the class teacher by $x\%$. The value of x is :
 (a) 33.33% (b) 25%
 (c) 20% (d) can't be determined
- 33** Last year in CAT, each section of the question paper had different weightage. The weightage of QA, DI and VA/RC sections were 8, 9 and 10 respectively. The maximum marks in all the three sections together was 810. Wrong answer did not carry negative marks as a penalty. If Padma had gotten 20% more marks in QA and 8% more marks in DI and 7.14...% more marks in VA/RC, then she must have gotten 100% marks in all the three sections. The total marks that Padma had scored :
 (a) 730 (b) 700
 (c) 750 (d) 775
- 34** A salesman gets commission on total sales at 9%. If the sale is exceeded ₹ 10,000 he gets an additional commission as bonus of 3% on the excess of sales over ₹ 10,000. If he gets total commission of ₹ 1380, then the bonus he received is :
 (a) ₹ 180 (b) ₹ 120
 (c) ₹ 480 (d) Data insufficient
- 35** In Veeru Bhai Pvt. limited company 60% of the employees are men and 48% of the employees are engineer and 66.6% of these are men. The percentage of women who are not engineers :
 (a) 33.33% (b) 60%
 (c) 52% (d) 46.66%
- 36** Initially Veer had 60% more love letters than that of Zara. In the last month the no. of love letters of Veer increased by 25% and that of Zara decreased by 25%. Again in the present month the no. of love letters of Veer decreased to 60% and that of Zara increased by 60%. Then which of the following statements is correct regarding the present no. of love letters?
 (a) Veer has 40% more letters than that of Zara
 (b) Zara has 20% less letters than that of Veer
 (c) Veer and Zara have equal no. of letters
 (d) Zara has 37.5% less letters than that of Veer.
- 37** The charges per hour of internet surfing is increased by 25% then find the percentage decrease in the time period of surfing of a user (a net savy) who can afford only a 10% increase in expenditure :
 (a) 22% (b) 12% (c) 15% (d) 9.09%
- 38** The average earning of each member of the Ambani family is 20% less than the average earning of each member of the Sahara family and the total earning of Ambani's family is 20% more than the total earning of Saharas's family. The no. of family members in the Sahara is what per cent of the no. of family members of Ambani?
 (a) 25% (b) 20%
 (c) 66.66% (d) None of these
- 39** From 2000 onwards, till 2003 the price of computers increased every year by 10%. After that due to government subsidy the price of computers decreases every year by 10%. The price of a computer in 2006 will be approx. how much per cent less than the price in 2000 if the same pattern of price is continued :
 (a) 2 (b) 3
 (c) 4 (d) none of these
- 40** A book consists of 30 pages, 25 lines on each page and 35 characters on each line. If this content is written in another note book consisting of 30 lines and 28 characters per line, then the required no. of pages will how much per cent greater than the previous pages?
 (a) 4.16% (b) 5%
 (c) 6.66% (d) none of these
- 41** The rate of increase of the price of sugar is observed to be two per cent more than the inflation rate expressed in percentage. The price of sugar on January 1, 2004 is ₹ 20 per kg. The inflation rates of the years 2004 and 2005 are expected to be 8% each. The expected price of sugar on January 1, 2006 would be :
 (a) ₹ 23.60 (b) ₹ 24.00 (c) ₹ 24.20 (d) ₹ 24.60
- 42** A club has raised 75% of the amount it needs for a new building by receiving an average donation of ₹ 600 from the people already solicited. The people already solicited represents 60% of the people the club will ask for donations. If the club is to raise exactly the amount needed for the new building, what should be the average donation from the remaining people to be solicited?
 (a) 250 (b) 300 (c) 400 (d) 600
- 43** A number x is mistakenly divided by 10 instead of being multiplied by 10. What is the percentage error in the result?
 (a) - 99% (b) + 99% (c) - 100% (d) + 100%
- 44** What is the percentage change in the result when we add 50 to a certain number x , instead of subtracting 50 from the same number x ?
 (a) 50% (b) 100%
 (c) 300% (d) can't be determined

- 45** In the Regional Science Centre, Lucknow the rate of ticket is increased by 50% to increase the revenue, but simultaneously 20% of the visitors decreased. What is percentage change in the revenue of Regional Science Centre. If it is known that the centre collects the revenue only from the visitors and it has no other financial supports :
 (a) + 20% (b) - 25%
 (c) + 30% (d) can't be determined
- 46** Recently when I visited a show room of shoes shopkeeper told me that he could reduce the price of Bata shoes by 49% and if I were to purchase Woodland shoes he could reduce the price to 51% of the original price. If the marked price (i.e., printed price) of both the shoes was same, then which shoes was cheap to buy :
 (a) Woodland (b) Bata (c) Both (d) Can't say
- 47** Selling price of a shirt and a coat is ₹ 4000. The cost price of a shirt is 58.33% of the cost price of a coat and so amount of profit on both the shirt and coat is same, then the price of the shirt could be :
 (a) ₹ 2100 (b) ₹ 2525 (c) ₹ 2499 (d) ₹ 1120
- 48** On the April 1, 2005 my salary increased from ₹ 10,000 to ₹ 16,000. Simultaneously the rate of income tax decreased by 37.5% , So the amount of income tax paid by me remains constant what is the value of income tax paid by me :
 (a) ₹ 3000 (b) ₹ 6000
 (c) ₹ 1600 (d) Can't be determined
- 49** In the previous question, if the difference in the rate of income tax be 9 (in per cent) then the income tax paid by me :
 (a) ₹ 2000 (b) ₹ 2400
 (c) ₹ 1600 (d) none of these
- 50** The average of a set of whole numbers is 27.2. When the 20% of the elements (ie numbers) are eliminated from the set of numbers then the average becomes 34. The number of elements in the new set of numbers can be :
 (a) 27 (b) 35 (c) 52 (d) 63
- 51** In a class, the no. of boys is more than the no. of girls by 12% of the total strength. The ratio of boys to girls is :
 (a) 15 : 11 (b) 11 : 14 (c) 14 : 11 (d) 8 : 11
- 52** The population of a village is 5000 and it increases at the rate of 2% every year. After 2 years, the population will be :
 (a) 5116 (b) 5202 (c) 5200 (d) 5204
- 53** A customer asks for the production of x number of goods. The company produces y number of goods daily. Out of which $z\%$ are unfit for sale. The order will be completed in :
 (a) $\frac{x}{100y(1-z)}$ days (b) $\frac{100yz}{x}$ days
 (c) $\frac{100x}{y(100-z)}$ days (d) $\frac{100}{y(z-1)}$ days

LEVEL 02 > HIGHER LEVEL EXERCISE

- 1** In the Awadh school Gomti Nagar, there are 500 students. 60% of the students are boys, 40% of whom play hockey and the girls don't play hockey, 75% of girls play badminton. There are only two games to be played. The number of students who don't play any game is :
 (a) 10% (b) 36%
 (c) 46% (d) can't be determined
- 2** A fraction in reduced form is such that when it is squared and then its numerator is increased by 25% and the denominator is reduced to 80% it results in $\frac{5}{8}$ of the original fraction. The product of the numerator and denominator is :
 (a) 6 (b) 12
 (c) 10 (d) 7
- 3** In the Chidambaram's family the ratio of expenses to the savings is 5 : 3. But his expenses is increased by 60% and income increases by only 25% thus there is a deficit of ₹ 3500 in the savings. The increased income of Mr. Chidambaram's family is :
 (a) ₹ 35,000 (b) ₹ 28,000
 (c) ₹ 25,000 (d) ₹ 18,500
- 4** In the Presidency College two candidates contested a presidential election. 15% of the voters did not vote and 41 votes were invalid. The elected contestant got 314 votes more than the other candidate. If the elected candidate got 45% of the total eligible votes, which is equal to the no. of all the students of the college. The individual votes of each candidate are :
 (a) 2250 and 1936 (b) 3568 and 3254
 (c) 2442 and 2128 (d) 2457 and 2143
- 5** The annual earning of Mr. Sikkawala is ₹ 4 lakhs per annum for the first year of his job and his expenditure was 50%. Later on for the next 3 years his average income increases by ₹ 40,000 per annum and the saving was 40%, 30% and 20% of the income. What is the percentage of his total savings over the total expenditure if there is no interest is applied on the savings for these four years?
 (a) $49\frac{37}{87}\%$
 (b) $41\frac{73}{83}\%$
 (c) 53%
 (d) none of the above

- 6 In an election only two candidates contested 20% of the voters did not vote and 120 votes were declared as invalid. The winner got 200 votes more than his opponents thus he secured 41% votes of the total voters on the voter list. Percentage votes of the defeated candidate out of the total votes casted is :
- (a) 47.5% (b) 41%
(c) 38% (d) 45%

Directions (for Q. Nos. 7 to 9) *Pujari ji, the chief of a temple's trust, has a beautiful daughter Nirjala and a son in law, Radhey. Pujarin, the wife of Pujari ji, lives her own life by receiving the alms from the devotees and receives 9.09% earning of her husband and the daughter together. The earning of Nirjala in each month is ₹ 8000 less than her husband Radhey. The earning of Pujari ji and Radhey together is ₹ 30,000 per month. The earning of Radhey and Nirjala together is ₹ 133.33% greater than that of Pujari ji.*

- 7 The average earning of each Pujari ji., Nirjala and Radhey is
(a) ₹ 13333.33 (b) ₹ 888.88
(c) ₹ 15,000 (d) none of these
- 8 What is the earning of Pujarin from the alms?
(a) ₹ 1800 (b) ₹ 2000
(c) ₹ 3600 (d) Can't be determined
- 9 The earning of Radhey is how much per cent greater than that of his wife?
(a) 50% (b) 80%
(c) $47\frac{11}{13}\%$ (d) none of these
- 10 A sales executive gets 20% bonus of the total sales value and 10% commission besides the bonus on the net profit after charging such commission. If the total sales value be ₹ 10 lakh per annum and the total profit of the company be ₹ 1.32 lakh, then his total earning per annum will be, given that he is not entitled to receive any fixed salary from the company
(a) 2.3 lakh
(b) 3.2 lakh
(c) 2.32 lakh
(d) 2.12 lakh
- 11 Mr Scindia after selling 5.5% stock at ₹ 92 releases ₹ 32200. Then he invested $\frac{1}{3}$ of the amount in 4.5% stock at ₹ 92, $\frac{2}{5}$ of the amount at ₹ 115 in 5% stock and the remaining in 6% stock at ₹ 56. The change in his income is :
(a) ₹ 56 loss (b) ₹ 78 profit
(c) ₹ 80 profit (d) ₹ 70 loss
- 12 Each edge of a cube is increased by 20% then the percentage increase in surface area of the cube is :
(a) 144% (b) 40%
(c) 44% (d) 72.8%

Directions (for Q. Nos. 13 to 14) *Pati, Patni and Woh (the three persons) were playing a game. At the beginning of the game Pati and Patni together had 100% more money than Woh. Patni and Woh together had 300% more than Pati. By the end of the game Pati and Patni together had 100% more money than Woh had and Pati had 12.5% less money than Patni and Woh together had. Finally Pati gained ₹ 800 by the end of the game.*

- 13 Who has suffered the loss?
(a) Patni (b) Woh
(c) Patni and Woh both (d) Can't be determined
- 14 The percentage change of money of Patni is :
(a) 40% (b) 30% (c) 57.1428% (d) 42.857%
- 15 The raw material and manufacturing cost formed individually 70% and 30% of the total cost and the profit percentage is 14.28% of the raw material. If the cost of raw material increase by 20% and the cost of manufacturing is increased by 40% and the selling price is increased by 80%, then the new profit percentage is :
(a) 57% (b) 65.8%
(c) 60% (d) can't determined
- 16 A, B, C and D purchased a cine-multiplex for ₹ 56 lakhs. The contribution of B, C and D together is 460% that of A, alone. The contribution of A, C and D together is 366.66% that of B's contribution and the contribution of C is 40% that of A, B and D together. The amount contributed by D is
(a) 10 lakh (b) 12 lakh (c) 16 lakh (d) 18 lakh
- 17 In a village three people contested for the post of village Pradhan. Due to their own interest, all the voters voted and no one vote was invalid. The losing candidate got 30% votes. What could be the minimum absolute margin of votes by which the winning candidate led by the nearest rival, if each candidate got an integral per cent of votes?
(a) 4 (b) 2
(c) 1 (d) none of these
- 18 Every day a mango seller sells half his stock, 10% of the stock overnight gets spoiled. If 1983 mangoes rotted over 3 nights then how many did he start with on the first day?
(a) 25,000 (b) 24,000 (c) 30,000 (d) 32,000
- 19 A man lost half of his initial amount in the gambling after playing 3 rounds. The rule of gambling is that if he wins he will receive ₹ 100, but he has to give 50% of the total amount after each round. Luckily he won all the three rounds. The initial amount with which he had started the gambling was :
(a) $\frac{500}{3}$ (b) $\frac{700}{3}$ (c) 300 (d) 600
- 20 In a factory there are three types of Machines M_1 , M_2 and M_3 which produces 25%, 35%, and 40% of the total products respectively. M_1 , M_2 and M_3 produces 2%, 4% and 5% defective products, respectively. What is the percentage of non-defective products?
(a) 89% (b) 97.1% (c) 96.1% (d) 86.1%

- 21** A company has 12 machines of equal efficiency in its factory. The annual manufacturing expenses are ₹ 24,000 and the establishment charges are ₹ 10,000. The annual output of the company is ₹ 48,000. The annual output and manufacturing costs are directly proportional to the no. of machines while the share holders get the 10% profit, which is directly proportional to the annual output of the company. If 8.33% machines remained close throughout the year. Then the percentage decrease in the amount of Share holders is

(a) 16.66% (b) 14.28%
(c) 8.33% (d) none of these

- 22** In every month Ravindra consumes 25 kg rice and 9 kg wheat. The price of rice is 20% of the price of wheat and thus he spends total ₹ 350 on the rice and wheat per month. If the price of wheat is increased by 20% then what is the percentage reduction of rice consumption for the same expenditure of ₹ 350? Given that the price of rice and consumption of wheat is constant.

(a) 36% (b) 40% (c) 25% (d) 24%

- 23** My friend Siddhartha Ghosh is working in the Life Insurance Corporation of India (LIC). He was hired on the basis of commission and he got the bonus only on the first years commission. He got the policies of ₹ 2 lakh having maturity period of 10 year. His commission in the first second, third, fourth and for the rest of the years is 20%, 16% 12% 10% and 4% respectively. The bonus is 25% of the commission. If the annual premium is ₹ 20,000 then what is his total commission if the completion of the maturity of all the policies is mandatory?

(a) ₹ 17,400 (b) ₹ 23, 600 (c) ₹ 15,000 (d) ₹ 15,500

Directions (for Q. Nos. 24 and 25) *DELL Computer has two branches : One in Ohio and second in Texas : The total no. of employees in Ohio office grew this year by 25% to 750 but the ratio of male to female employees is same as in the previous year. The no. of employees in the Texas office grew this year by 9.09% to 1200. The ratio of male to female employees last year in the Texas office was 5 : 6 and the no. of male employees in the Ohio office was 20% less than that of Texas office.*

- 24** The total no. of female employees this year in both the offices is :
- (a) 654 (b) 546
(c) 950 (d) can't be determined

- 25** The total no. of employees in both the offices last year was :
- (a) 1500 (b) 1700
(c) 1650 (d) can't be determined

- 26** A shepherd had n goats in the year 2000. In 2001 the no. of goats increased by 40%. In 2002 the no. of goats declined to 70%. In 2003 the no. of goats grew by 30%. In 2004, he sold 10% goats, then he had only 34,398 goats. The percentage increase of the no. of goats in this duration was :

(a) 14.66% (b) 16.66% (c) 20% (d) 33.33%

- 27** In the above question in which year the no. of goats was minimum?

(a) 2000 (b) 2001
(c) 2002 (d) 2004

Directions (for Q. Nos. 28 to 30) *In the IGNOU (Indira Gandhi National Open University) there are total 16,000 students pursuing MBA, which offers the specialization only in Finance, HR and Marketing. IGNOU accepts only Science, Commerce and Engineering students for the two years course of MBA. The number of Science students is 166.66% of the Commerce students. Number of Engineering students is equal to the number of Science and Commerce students together.*

Each student can specialize in only one of the marketing, HR and Finance. 20% of Science students opted the Finance, which is 16.66% less than the no. of Commerce students who opted Finance. The total Finance students is equal to 18% the total strength of the MBA students. 32% of Science students opted HR. Commerce students who opted HR is equal to 25% of total students specializing in Finance and Engineering students equal to 6.5% of the total strength of the MBA students opted HR.

- 28** The number of Engineering students who opted marketing is
- (a) 7850 (b) 7500
(c) 8850 (d) none of these

- 29** The percentage of Commerce students who opted HR over the total strength of the MBA students is :

(a) 6.6% (b) 42.5%
(c) 4.5% (d) 62.5%

- 30** The most preferable course among the MBA students is :

(a) Finance (b) HR
(c) Marketing (d) I. T and systems

- 31** $P\%$ of the students of a class passed the exam. $g\%$ of the passed students are girls and $b\%$ of the fail students are boys. The percentage of passed boys over the failed girls is:

(a) $\left(\frac{bg}{p} \times 100\right)$
(b) $\frac{100(100-g) \cdot p}{(100-p)(100-b)}$
(c) $\frac{(100-g)(100-b)}{(100-p)}$

(d) none of the above

- 32** In the Polo hospital some patients who were suffering from the Hepatitis-B were admitted for the treatment, but 9% of the patients were died within half an hour. After treatment, the percentage of patients cured out of the remaining was only 80%. Out of these patients only 70% are completely cured out and the remaining were partially cured out which were equal to 153 patients. The no. of patients (approx.) who were admitted for the treatment for the same was :

(a) 400 (b) 678 (c) 560 (d) 700

- 33** The total cost of setting up a sugar cane factory is ₹1 crore, which produces 5000 ton per annum. Sugar is being sold at ₹ 18 per kg. Manufacturing cost including raw material is ₹ 3.2 per kg, labour and packing charges are ₹ 1.8 per kg. Maintenance and utilities expenses are ₹ 2 per kg and the general expenses (ie all the rest charges) are ₹ 3 per kg, 20% taxes are being paid of the gross annual earnings, then the net profit of the production of the factory per annum is :
- (a) ₹ 4.2 crore (b) ₹ 3.2 crore
(c) ₹ 5.4 crore (d) none of these

- 34** A student appeared in the Mock CAT. The test paper contained 3 sections namely QA, DI and VA. The percentage marks in VA was equal to the average of the percentage marks in all the 3 sections. Coincidentally, if we reverse the digits of the percentage marks of QA we get the percentage marks of DI. The percentage marks in VA scored by the student could be :
- (a) 48 (b) 66
(c) 69 (d) 81

- 35** The pressure of a definite mass of a gas is directly proportional to the temperature and inversely proportional to the volume under the given conditions. If temperature is increased by 40% and the volume is decreased by 20% then the new pressure will:
- (a) be increased by 75% (b) reduce to 25%
(c) be increase by 20% (d) increase by 28%

- 36** A computer typist types a page with 20 lines in 10 minutes but he leaves 8% margin on the left side of the page. Now he has to type 23 pages with 40 lines on each page which he leaves 25% more margin than before. How much time is now required to type these 23 pages?
- (a) $7\frac{1}{2}$ hrs (b) $7\frac{2}{3}$ hrs
(c) $23\frac{1}{2}$ hrs (d) 3.916 hrs

- 37** A company made a cuboidal box of size $16 \times 12 \times 5$ to sell the ice cream, but later on it was found that the capacity of the box was 14.28% less than the required capacity while the height of the box was correct, which is 5 inches. As per the requirement he had to increase the length and breadth of the box in equal amount then the percentage increase in the area of the base of the box is
- (a) 12.5% (b) 6.66%
(c) 16.66% (d) none of these

- 38** In Sabarmati Express, there are as many wagons as there are the no. of seats in each wagon and not more than one passanger can have the same berth (seat). If the middlemost compartment carrying 25 passangers is filled with 71.428% of its capacity, then find the maximum no. of passangers in the train that can be accommodated if it has minimum 20% seats always vacant.
- (a) 500 (b) 786
(c) 980 (d) can't be determined

- 39** The prepaid card of Reliance Infocom gives 19% less talktime than the prepaid card of Tata Indicom, having same price. Again the post-paid card of same price of Tata Indicom gives 10% less talktime than its prepaid card. Similarly the post paid card of same price of Reliance gives 11.11% less talk time than its prepaid card. How much per cent less talk time we get from the Reliance post paid card than the post paid card of Tata Indicom?
- (a) 21.11% (b) 20%
(c) 30.11% (d) none of these

- 40** In the half yearly exam only 70% of the students were passed. Out of these (passed in half yearly) only 60% student are passed in annual exam. Out of those who did not pass the half yearly exam, 80% passed in annual exam. What per cent of the students passed the annual exam ?
- (a) 42% (b) 56%
(c) 66% (d) none of these

- 41** The marks obtained by the students of a school is given below : maximum marks are 50.

Marks	Percentage of Students
< 10	15%
< 20	32%
< 30	40%
< 40	70%
< 50	100%

The ratio of no. of boys to no. of girls who passed the exam is 7 : 6. It is known that a student can pass the exam only when he obtained at least 20 marks in the exam. The total no. of students in the school if the no. of girls who are passed is 408 :

(a) 1100 (b) 1200
(c) 1300 (d) 1430

Direction (for Q. Nos. 42 to 45) *After defeating Ravana, Ram and his family won a lot of valuable assets in the war. It consists of horses, chariots and some land of Ravans's kingdom. The cost of each horse and chariot was ₹ 20,000 and ₹ 8,000 respectively while the cost of 1 acre land was ₹ 5000. All the property was shared among the four persons in such a way that Ram and Sita got together the same wealth as Laxman and Urmila got together. Ram got more than Sita and Laxman got more than Urmila. Ram got $\frac{1}{3}$ rd horses and 20% chariots while Laxman received 50%chariots as the 50% of his total wealth. The no. of horses that Ram and Sita got together was 50% more than that of Laxman and Urmila together had. Sita got 8 horses and Urmila got 7 horses but the Ram and Sita got equal no. of Chariots and Urmila got 20 chariots less than that of Laxman. Urmila got twice the land than that of Sita but 20% less than Laxman.*

- 42** What is the difference between the wealth of Ram and wealth of Urmila?
- (a) 1.2 lakh (b) 1 lakh
(c) 1.4 lakh (d) can't be determined

- 43** If Laxman wanted to exchange all his chariots with the horses, then who can exchange with his/her horses in terms of wealth?
 (a) Ram (b) Sita
 (c) Urmila (d) can't be determined
- 44** The wealth of Urmila is how many per cent less than that of Laxman?
 (a) 42% (b) 45%
 (c) 35% (d) none of these
- 45** The wealth due to land and chariot together is how much greater, in per cent, than the wealth due to horses?
 (a) 25% (b) 20%
 (c) 33.33% (d) none of these
- 46** A big cube is formed by rearranging the 160 coloured and 56 non-coloured similar cubes in such a way that the exposure of the coloured cubes to the outside is minimum. The percentage of exposed area that is coloured is:
 (a) 25.9% (b) 44.44%
 (c) 35% (d) none of these
- 47** Selection into IIMs (Indian Institutes of Management) is quite simple. In our coaching institute some students qualified CAT (The first stage of entrance into IIMs) but coincidentally the no. of boys who qualified the CAT was equal to the no. of girls. Besides these boys and girls got the calls from only IIM Ahmedabad and IIM Bangalore, but each of these from both the IIMs. 60% of the boys failed in the group discussion (the second phase of the selection process) and thus equal no of boys (but distinct) appeared for the personal interview of IIM-A and IIM-B (interview is the third and final stage of selection of a candidate) but 20% of the boys who appeared for the interview of IIM-A

and 60% of the boys who appeared for the interview of IIM-B failed. If it is possible that a candidate can receive the calls from more than one IIMs but he/ she can face the interview of only one IIM.

Given that only 24 boys from our coaching institute were selected by the IIM-A and IIM-B also a candidate can appear for the next stage only if he/she qualifies the previous stage of the exam, then find the no. of girls who qualified the CAT (Common Admission Test).

- (a) 100 (b) 250 (c) 300 (d) 600
- 48** In an office there were initially n employees. The HR manager first hired $P\%$ employees then after a month $q\%$ employees left the office, then there were finally n employees remained in the office, the value of $p - q$ is :
 (a) pq (b) $\frac{pq}{100}$
 (c) $\frac{p}{q}$ (d) none of these
- 49** In the Garbar Jhala, Aminabad a shopkeeper first raises the price of a Jewellery by $x\%$ then he decreases the new price by $x\%$. After one such up down cycle, the price of a Jewellery decreased by ₹ 21025. After a second updown cycle the jewellery was sold for ₹ 484416. What was the original price of the jewellery?
 (a) ₹ 5,00,000 (b) ₹ 6,00,625
 (c) ₹ 5,25,625 (d) ₹ 5,26,000
- 50** The amount of work in a leather factory is increased by 50%. By what per cent is it necessary to increase the number of workers to complete the new amount of work in previously planned time, if the productivity of the new labour is 25% more.
 (a) 60% (b) 66.66% (c) 40% (d) 33.33%

LEVEL 03 > Final Round

Directions (for Q. Nos. 1 to 3) *The bar graph shown below reveals the data about the no. of students in different disciplines of MBA at a prominent Business School in India.*



- 1** The growth rate of Finance compared to that of marketing is:
 (a) less
 (b) equal
 (c) greater
 (d) none of the above
- 2** Total no. of MBA students in the session of 2004-05 if the no. of students in 2004-2005 is increasing by 9.09% over the previous year :
 (a) 555 (b) 600
 (c) 777 (d) none of these
- 3** In the session of 2002-03 what percentage of MBA students are studying International Business?
 (a) 13.33% (b) 25%
 (c) 6.66% (d) 24%

Directions (for Q. Nos. 4 to 7) *The following table gives the sales details of the books for CAT written by Sarvesh.*

Year	Quant. Aptitude	Data Interpretation	Verbal Ability	Log. Reasoning
2000	4000	3750	4140	4350
2001	4200	3870	4260	4400
2002	4370	3990	4255	4500
2003	4268	3868	5371	4690
2004	4750	4900	5476	4710
2005	4800	5000	5500	4800

- 4** What is the growth rate of sales of Quantitative Aptitude from 2000 to 2005?
 (a) 8% (b) 25%
 (c) 20% (d) 40%
- 5** Which of the categories shows the lowest growth rate from 2000 to 2005?
 (a) Q.A (b) DI
 (c) VA (d) LR
- 6** Which category had the highest growth rate in the period?
 (a) QA (b) DI
 (c) VA (d) LR
- 7** Which of the categories had either a consistent growth or consistent decline in the period shown?
 (a) QA (b) DI (c) VA (d) LR

Directions (for Q. Nos. 8 to 10) *In India there were only three bicycle making companies in the given period. The following table shows the production of units (in 000)*

Year	HERO	ATLAS	AVON
1990	2.97	1.75	3.77
91	4.22	2.48	4.55
92	5.95	3.14	4.5
93	6.28	3.01	4.76
94	6.33	4.12	4.74
95	8.50	4.21	4.26

UPTREND IN BICYCLES PRODUCTION IN INDIA

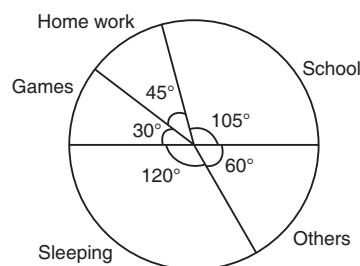
- 8** The simple average rate of growth of production of Hero cycles from 1990-95 :
 (a) 37% (b) 42% (c) 27% (d) 31.5%
- 9** Which of the following statements is/are correct ?
 (i) Atlas cycles have recorded the fastest growth rate.
 (ii) Total bicycles production was the highest in 1993.
 (iii) Hero cycles on an average account for 48% of total bicycles production
 (a) I only (b) II only
 (c) III only (d) none of these
- 10** Atlas cycles on an average account for what percentage of the total bicycles production :
 (a) 15% (b) 23.4% (c) 34.5% (d) 29%

Directions (for Q. Nos. 11 to 14) *Answer the questions based on the following information, which gives data about certain butter producer companies in India.*

Name of the Company	Production ('000) tonnes	Capacity Utilisation (%)	Sales ('000) tonnes	Total sales value (in crore)
Amul	1.54	59.35	1.47	17.45
Nestle	1.64	64.80	1.26	15.25
Parag	2.48	71.20	2.03	26.75
Amrit	2.97	76.50	2.55	31.15
Total (including others)	11.60	61.30	10.67	132.80

- 11** What is the maximum production capacity (in '000 tonnes) of Nestle for Butter?
 (a) 2.53 (b) 2.07 (c) 2.84 (d) 2.97
- 12** Which company out of the four companies mentioned above has the maximum unutilised capacity (in '000 tonnes)?
 (a) Amul (b) Nestle (c) Parag (d) Amrit
- 13** What is the approximate total production capacity (in '000 tonnes) for butter in India?
 (a) 7.8 (b) 18.9
 (c) 11.60 (d) Data insufficient
- 14** What per cent of the total market share (by sales value) is controlled by others?
 (a) 32% (b) 83%
 (c) 67% (d) Data insufficient

Directions (for Q. no. 15 to 19) *The following pie chart shows the hourly distribution of all the major activities of a student.*



- 15** The percentage of time which he spends in school is :
 (a) 38% (b) 30% (c) 40% (d) 25%
- 16** How much time (in per cent) he spends in games in comparison to sleeping?
 (a) 30% (b) 40%
 (c) 25% (d) none of these
- 17** If he spends the time in games equal to the home work and remains constant in other activities, then the percentage decrease in time of sleeping :
 (a) 15% (b) 12.5%
 (c) 20% (d) none of these

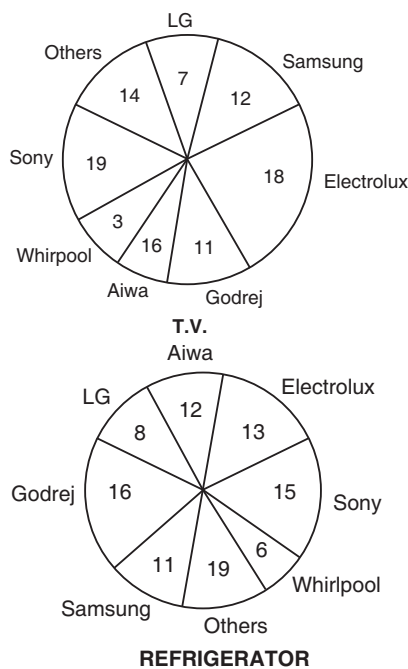
18 What is the difference in time (in hours) spends in school and in home work?

- (a) 2 (b) 3
(c) 4 (d) 8

19 If he spends $\frac{1}{3}$ rd time of home work in Mathematics then

- the no. of hours spends in rest of the subjects in homework
(a) 1 (b) 2
(c) 3 (d) 4

Directions (for Q. Nos. 20 to 23) *The two pie charts show the market share of different companies which produces TV and Refrigerator (both) in the first quarter of 2005-06.*



NOTE The graph is not shown to scale.

20 The difference in the angle subtended by Sony TV and refrigerator is :

- (a) 7.2° (b) 14.4°
(c) 21.6° (d) none of these

21 If the turn over of Samsung TV is ₹ 31 crores and of Samsung refrigerator is ₹ 9 crores, then what is the overall market share of Electrolux TV and refrigerator combined?

- (a) 33.33% (b) 28%
(c) 16.7% (d) 65%

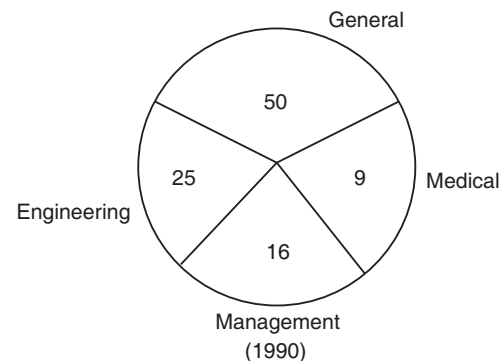
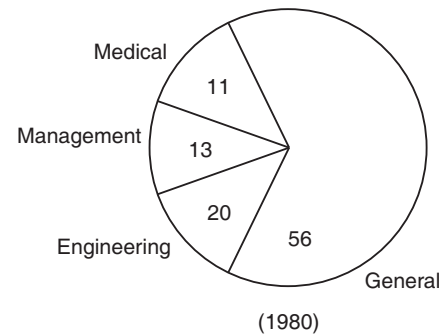
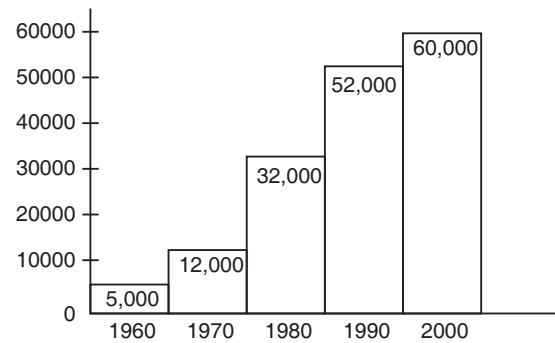
22 If the turnover in the first quarter of 2005-06 of Electrolux TV and Refrigerator is ₹ 42 crores and ₹ 6 crores respectively, then what is the average annual turnover of Godrej and Aiwa in both the product categories together?

- (a) ₹ 304 crores
(b) ₹ 284 crores
(c) ₹ 178 crores
(d) none of the above

23 Which of the following can't be deduced from the given data :

- (a) The angular difference between Samsung TV and Electrolux refrigerator is 3.6° .
(b) Sony is the market leader in the TV and Refrigerator segment combined.
(c) For every ₹ 100 turnover of Whirlpool Refrigerator, the difference in the turnover of Electrolux and Samsung Refrigerator is ₹ 33.34 crore.
(d) none of the above

Directions (for Q. Nos. 24 to 28) *Number of different colleges in India in different years is given in the graph below. Distribution of different colleges in the years 1980 and 1990 shown in the pie charts below :*



NOTE General courses include all the courses except Medical, Engineering and management.

- 24** What is percentage increase in the number of colleges from 1960 to 1990?
(a) 1000% (b) 940% (c) 1040% (d) 470%
- 25** The growth rate of no. of medical colleges in 1980 to 1990 is
(a) 33% (b) 52% (c) 36% (d) 39%
- 26** By what percentage did the no. of Engineering colleges go up from 1960 to 1970?
(a) 38% (b) 28%
(c) 42% (d) can't of be determind
- 27** Till 1990 what was the highest increase in the number of colleges in any decade?
(a) 150% (b) 166.66%
(c) 200% (d) 140%
- 28** If the projected increase in the number of colleges in 2000 over 1990 is to be the same in all the categories of colleges, the percentage of Medical colleges in 2000 will be :
(a) 9% (b) 11%
(c) 7% (d) 13%

Directions (for Q. Nos. 29 to 33) *Answer the questions based on the following information. The following table gives the tariff (in paise per kilo-litre) levied by the Lucknow Jal Nigam in 2003-04 in four sectors and the region within them. (Each sector is divided in 5 regions). The table gives the percentage change in the tariff as compared to 2000-01.*

	Region 1		Region 2		Region 3		Region 4		Region 5	
	P/kl	%	P/kl	%	P/kl	%	P/kl	%	P/kl	%
Sector 1	1000	+ 25	400	+ 14.28	250	– 16.66	625	+ 4.166	720	+ 20%
Sector 2	800	+ 33.33	375	+ 7.14	350	– 12.5	750	+ 7.14	360	– 10
Sector 3	625	– 16.66	525	– 12.5	400	+ 14.28	240	– 4%	320	– 20
Sector 4	575	– 4.166	800	– 20%	500	– 16.66	360	+ 20%	400	– 11.11

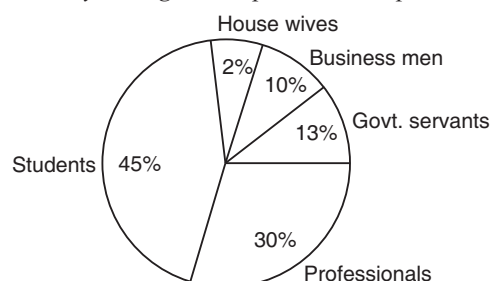
- 29** If the amount of water consumed by the various regions in sector 1 is the same, then as compared to 2000-01 the net tariff in 2003-04 :
(a) increased by 20% (b) increased by 13%
(c) decreased by 12% (d) decreased by 20%
- 30** What was the approximate average tariff in region 2 in 2000-01?
(a) 450 (b) 675 (c) 575 (d) 525

Directions (for Q. Nos. 31 to 33) *The Lucknow Jal Nigam supplies the water under four categories : Urban (25%) rural (15%) domestic (40%) industrial (20%). In 2003-04, the total water supplied by the Lucknow Jal Nigam was 20,000 kilo-litres.*

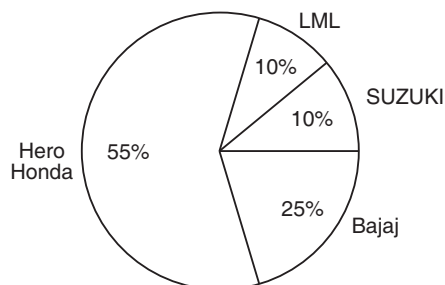
- 31** If in 2003-04, there was a 20% decrease in the domestic consumption of water as compared to that in 2000-01 what was the consumption of water in the industrial sector in 2000-01?
(a) 5,000 kl (b) 7500 kl (c) 10,000 kl (d) 6000 kl
- 32** In the given two years, what is the total tariff paid by the Urban sector?
(a) ₹ 16,000 (b) ₹ 48,000
(c) ₹ 23,000 (d) can't be determined
- 33** Which of the following statements is true?
(a) The average tariff in region 2 is 625
(b) The average tariff in region 4 is greater than the average tariff in region 5
(c) In 2000-01 the industrial sector contributed about 30% of the total revenue from water supply
(d) none of the above

Directions (for Q. Nos. 34 to 37) *A marketing company*

conducted a survey among 10,000 person in Kanpur :



It was observed that some people have more than one bike but from only one company i.e., a particular person can have more than one bike of Hero Honda, but not from bajaj etc. and vice versa. Thus there were total 12,000 bikes with 10,000 persons used for survey. There are only four companies operating in this market.



- 34** If each of Students, Govt. Servants and housewives use a

Hero Honda bike (motorcycle), what per cent of the remaining people drive Hero Honda bike?

- (a) 15% (b) 25%
(c) 20% (d) none of these

- 35** If the number of people who drive one, two and three bikes are in the ratio 15 : 3 : 1 what is the number of people in the survey who do not drive even a single bike?
(a) 750 (b) 400 (c) 600 (d) 500
- 36** If all the persons driving more than one bike drive only Hero Honda what is the number of people who drive single Hero Honda bike (the data can be used from previous question if necessary)?
(a) 2400 (b) 2100 (c) 4200 (d) 2600
- 37** If 20% of the persons who drive Bajaj's bike also drive another bike. What is the number of people who drive only Bajaj' bike ?
(a) 2400 (b) 2500
(c) 2660 (d) none of these

Directions (for Q. Nos. 38 to 42) A table below shows the production and imports of crude oil (in '000 tonnes). Domestic production of crude oil is total of on-shore and off-shore production, which is supplemented by imports to meet the total demand of crude oil in the country.

Year	2001	2002	2003
On shore	12,000	11,500	11,000
Off shore	11,000	19,000	16,000
Imports	21,000	24,000	30,000

(in '000 tonnes)

- 38** What was the percentage of domestic production of crude oil over imports in 2001?
(a) 80% (b) 140%
(c) 109.52% (d) none of these
- 39** What was the percentage change in domestic production of crude oil from 2001-2003 ?
(a) 14% (b) 27% (c) 17.4% (d) - 10%
- 40** What is the average of total demand of crude oil over the period?
(a) 185 million tonnes (b) 52 million tonnes
(c) 18.5 million tonnes (d) 35 million tonnes
- 41** What was the approx-percentage increase in imports of crude oil from 2001 to 2003?
(a) 49% (b) 65% (c) 43% (d) none of these
- 42** If in the year 2004, off-shore production declines by 12.5% production on-shore remains the same and total demand increases by 2% what will be the imports of crude oil in 2004?
(a) 33.14 million tonnes (b) 63 million tonnes
(c) 39 million tonnes (d) 25 million tonnes

Directions (Q. Nos. 43 to 50) Solve the following questions on the basis of given data in the following table :

Production of Rice in India

Year	Quantity (in tonnes)	Percentage change over the previous year
1920-21	1,34,300	+ 06.25%
1930-31	10,97,172	+ 12.50%
1940-41	2,64,280	+ 11.11%
1950-51	1,27,890	- 09.09%
1960-61	2,01,924	+ 20.00%
1970-71	1,12,325	- 16.66%
1980-81	2,13,465	- 25.00%
1990-91	1,69,368	+ 33.33%
2000-01	100,956	+ 50.00%
2010-11	23,800	- 83.33%

- 43** What is the production of rice in 1959-60?
(a) 1,84,250 (b) 1,68,270
(c) 242308.8 (d) none of these
- 44** What is the production of rice in 1949-50?
(a) 116263.63 (b) 1,23,460
(c) 1,40,679 (d) none of these
- 45** What is the total production of rice in 1919-20 and 1929-30 and 1939-40 ?
(a) 13,26,400 (b) 13,39,516
(c) 1142693.75 (d) can't be determined
- 46** The production of rice in 2000-01 forms what percentage of total production out of the given years?
(a) 3.12% (b) 3.23%
(c) 4.128% (d) 6.45%
- 47** What is the difference in production of rice in 1969-70 and 1979-80 ?
(a) 149830 (b) 175752
(c) 53,890 (d) none of these
- 48** The percentage decrease in production of rice from 1929-30 to 1949-50?
(a) 88.3% (b) 85.57%
(c) 66.66% (d) none of these
- 49** The production of rice in 1959-60 is what percentage of rice in 1960-61?
(a) 86.66% (b) 75.6% (c) 83.33% (d) 16.66%
- 50** Out of the given years which year has shown least increase (in amount) over the previous year?
(a) 1940-41 (b) 1960-61
(c) 1920-21 (d) 2000-01

Answers

✓ Introductory Exercise 5.1

1. (i) 5%, (ii) 15.625, (iii) 33.33%, (iv) 24%, (v) 20%, (vi) 150%, (vii) 66.66%, (viii) 43.75%
2. (i) 4%, (ii) 75%, (iii) 6.25%, (iv) 33.33%, (v) 0.4166%, (vi) 7.5%
3. 85.33% 4. 70% 5. 97% 6. 150% 7. $33\frac{1}{3}\%$ 8. 90% 9. 12.5% 10. 4%

✓ Level 01 Basic Level Exercise

1 (b)	2 (b)	3 (b)	4 (c)	5 (d)	6 (b)	7 (c)	8 (d)	9 (d)	10 (c)
11 (b)	12 (d)	13 (d)	14 (b)	15 (c)	16 (b)	17 (a)	18 (b)	19 (c)	20 (a)
21 (c)	22 (b)	23 (c)	24 (d)	25 (d)	26 (b)	27 (a)	28 (c)	29 (b)	30 (c)
31 (b)	32 (c)	33 (a)	34 (b)	35 (b)	36 (c)	37 (b)	38 (c)	39 (b)	40 (c)
41 (c)	42 (b)	43 (a)	44 (d)	45 (a)	46 (c)	47 (d)	48 (d)	49 (b)	50 (c)
51 (c)	52 (b)	53 (c)							

✓ Level 02 Higher Level Exercise

1 (d)	2 (c)	3 (a)	4 (d)	5 (d)	6 (d)	7 (a)	8 (d)	9 (b)	10 (d)
11 (c)	12 (c)	13 (a)	14 (c)	15 (a)	16 (d)	17 (c)	18 (b)	19 (b)	20 (c)
21 (b)	22 (a)	23 (a)	24 (d)	25 (b)	26 (a)	27 (c)	28 (d)	29 (c)	30 (c)
31 (b)	32 (d)	33 (b)	34 (b)	35 (a)	36 (a)	37 (c)	38 (c)	39 (b)	40 (c)
41 (c)	42 (a)	43 (a)	44 (c)	45 (b)	46 (b)	47 (a)	48 (b)	49 (c)	50 (c)

✓ Level 03 Final Round

1 (a)	2 (b)	3 (a)	4 (c)	5 (d)	6 (b)	7 (d)	8 (a)	9 (d)	10 (b)
11 (a)	12 (a)	13 (b)	14 (a)	15 (b)	16 (c)	17 (b)	18 (c)	19 (b)	20 (b)
21 (c)	22 (a)	23 (b)	24 (b)	25 (a)	26 (d)	27 (b)	28 (b)	29 (b)	30 (c)
31 (a)	32 (d)	33 (b)	34 (a)	35 (d)	36 (b)	37 (a)	38 (c)	39 (c)	40 (b)
41 (c)	42 (a)	43 (b)	44 (c)	45 (b)	46 (c)	47 (a)	48 (b)	49 (c)	50 (c)

Hints & Solutions

✓ Level 01 Basic Level Exercise

1

F	M	
60 x	40 x	
42x	20x	(comp. lit.)
$\swarrow \quad \searrow$ $\quad \quad 62x$		

$$\therefore \text{Female comp. literate} = 1600 \times \frac{42}{100} = 672$$

2 $(10,00,000) \times 0.75 \times 0.80 \times 0.85 = ₹ 5,10,000$

3 $2000 \times \frac{x}{100} + 4000 \times \frac{y}{100} = 320$

and $2000 \times \frac{x}{100} + 10,000 \times \frac{y}{100} = 680$

$$\therefore x = 4 \quad \text{and} \quad y = 6$$

$$\therefore x - y = -2$$

4 Go through option $(40 \times 0.4) + (40)^2 = 1616$

$$\left(100 \times \frac{1616}{40} = 4040\right)$$

☞ Alternatively $(x \times 0.4) + x^2 = \frac{x \times 4040}{100}$

$$\Rightarrow x^2 = 40x \Rightarrow x = 40$$

Directions for solutions (Q. 5-7) Let us assume that there are total number of employees at our office is 100.

	Female	Male	Total
Salary > 8000	30	70	100%
Salary ≤ 8000	9	No data	No data
Salary = 8000	21	No data	No data
Salary < 8000	No data	56	No data
Salary ≥ 8000	No data	14	No data

5 Since we don't have the desired data, so we cannot determine the required value.

Hence, choice (d) is the valid one.

6 To maximize the number of employees who earn more than ₹ 8000, we can assume there is no employee who earns exactly ₹ 8000. So there would be 14 male employees and 9 female employees who earn more than ₹ 8000. That is $9 + 14 = 23$

	Female	Male	Total
Salary > 8000	30	70	100%
Salary > 8000	9	14	23
Salary ≤ 8000	21	No data	No data
Salary = 8000	No data	0	No data
Salary < 8000	No data	56	No data

Hence, choice (b) is the valid one.

7 To minimize the number of employees who earn less than ₹ 8000, we have to maximize the number of employees who earn equal to or greater than ₹ 8000. We can assume that all the 21 female employees earn exactly ₹ 8000 and so, none of them earn less than ₹ 8000. However, there are 56 male employees who definitely earn less than ₹ 8000. Therefore, minimum 56 employees must earn less than ₹ 8000.

	Female	Male	Total
Salary > 8000	30	70	100%
Salary > 8000	9	No data	No data
Salary = 8000	21	No data	No data
Salary < 8000	0	56	56
Salary ≥ 8000	No data	14	No data

Hence, choice (c) is the valid one.

8 **Physics Chemistry**

Failed 35% 45%

Passed 65% 55%

Passed in both = 22% of total student

Percentage of students who are passed in any of the Physics or Chemistry or both = $(65 + 55) - 22 = 98\%$

So, the percentage of students who are failed in both = 2%

Therefore total failed (in both the subject) students = 12

9 **History Geography**

Pass 70 x 50 x

Fail 30 x 50 x

both 20 x

$$\therefore \text{Total failed candidates} = (30x + 50x) - 20x = 60x$$

$$\text{Passed in both} = (100x - 60x) = 40x = 500$$

$$\text{Therefore total students} = 100x = 1250 \Rightarrow x = 25/2$$

10 My salary = 100

Salary of my brother = 110

Salary of my sister = 120

$$\text{Salary of my wife} = 230 - \left(230 \times \frac{1300}{23 \times 100}\right) = 100$$

$$\therefore 56 \frac{12}{23} = \frac{1300}{23}$$

11 $\frac{60 \times 8 + 16 \times 30}{16 \times 60 \times 60} \times 100 = 1.66\%$

12 Total land of Sukhiya = $\frac{480x}{0.6} = 800x$

$$\therefore \text{Cultivated land of village} = 384000x$$

$$\therefore \text{Required percentage} = \frac{800x}{384000} \times 100 = 0.20833\%$$

- 13** Area = $l \times b$
 $1 = 1 \times 1 \Rightarrow 1 = 0.8 \times 1.25$
 So the area remained constant.
- 14** Cost of fresh Mangoes + Packaging cost = Total cost

$$\begin{array}{rcl} 1 & + & 0.4 = 1.4 \\ 1.3 & + & 0.2 = 1.5 \end{array}$$

 Percentage increase in cost = $\frac{0.1}{1.4} \times 100 = 7.14\%$

15 $\frac{80}{100} + \frac{66}{100} + \frac{x}{200} = \frac{320}{400}$
 $\Rightarrow x = 174 \Rightarrow 87\%$

16

Male	Female
55x	45x

 $\Rightarrow 10x = 72 \Rightarrow x = 7.2$
 $100x = 7.2 \times 100$

- 17** Let the total votes be x ,
 Polled votes = $0.75x$
 valid votes = $0.70x$
 $A + B + C = 0.70x$

$$\begin{array}{c} \downarrow \quad \downarrow \quad \downarrow \\ 1750 \quad 2450 \end{array}$$

 $0.4x$
 It means $B + C = 0.3x$ since given that $A = 0.4x$
 Hence A is the winner.

18

Bike	Car
x	$5x$
$1.2x$	$5.75x$

 Initially total cost = $25x + 10x = 35x$
 Changed cost = $28.75x + 12x = 40.75x$
 Percentage change = $\frac{5.75}{35} \times 100 = 16\frac{3}{7}\%$

- 20** Go through option

S	K
15,000	18,000
12,000	16,000

 Going in the reverse direction.
 Hence, the presumed option (a) is correct.

☞ Alternatively

Sahid	Kareena
$4x$	$\frac{48x}{9}$
$5x$	$6x$

 $\xrightarrow{+120\%}$
 $\Rightarrow$ Initially $S : K = 4x : \frac{48x}{9} = 3 : 4$
 $\Rightarrow$ Sahid's initial salary = \$12000
 Sahid's changed salary = \$15000

- 21** Let the smaller number be x and larger number be y .
 $0.8x + 4 = 0.4y$

$\Rightarrow 4y - 8x = 40$
 and $y - x = 85 \Rightarrow x = 75$ and $y = 160$
 $\therefore x + y = 235$

22 $\frac{x \times 220}{100} = 44$
 $\Rightarrow x = 20$ So $\frac{20 \times 44}{100} = 8.8$

23 Quantity $\times$ Rate = Price
 $1 \times 1 = 1$
 $0.7 \times 1.25 = 0.875$
 $\therefore$ Decrease in price = $\frac{0.125}{1} \times 100 = 12.5\%$

- 24** Go through options

P	Q
275	225

 Difference = 50

200	300

 Difference = 100
 Hence, the presumed option (d) is correct.

☞ **NOTE** It can also be solved through the equation and variables.

25 Passing marks are $0.6x$
 So $0.3x + 30 = 0.6x \Rightarrow x = 100$

26 Go through options

Girls	Boys
800	1000

 Now, $0.95 \times 800 + 1000 = 0.96 \times 1000 + 800$
 Hence, the presumed option (b) is correct.

27

H	E	U
$0.6x$	$0.24x$	$0.16x$

 $\downarrow$
 $3600 \longrightarrow 2400 (\because E : U = 24 : 16 = 3 : 2)$

28 Go through option

25	75	50
$\downarrow$		$\downarrow$
20		30
(60%)		
$\downarrow$		
$\frac{50}{75} \times 100 = 66.66\%$		

Hence, the presumed option (c) is correct.

29

Science	Commerce	Arts	Engineering
$\frac{x}{6}$	$\frac{x}{8}$	$\frac{x}{15}$	K

 $\left[\frac{x}{6} + \frac{x}{8} + \frac{x}{15} \right] = K$
 No. of Engineering student = $1 - \left(\frac{x}{6} + \frac{x}{8} + \frac{x}{15} \right) = k$
 $= \frac{77x}{120}$

When $x = 120$ (the least possible number)
 then the no of Engineering students = 77

- 30** Copper Aluminium
 7 4
 21 kg 12 kg → 33 kg
 Required total alloy = $\frac{33}{0.88} = 37.5$ kg

- 31** $1.08x = 1404$
 $x = 1300$
 Therefore reduction in price = $1404 - 1300 = ₹ 104$
 $\left[\text{Since } 1300 + 1300 \times \frac{8}{100} = 1404 \right]$

So you can solve it by using options.

- 32** Very fundamental question.

- 33** $8x + 9x + 10x = 810 \Rightarrow x = 30$

Total marks in QA → 240
 DI → 270
 VA → 300
 RC

Now her score in QA → $\frac{240}{1.2} = 200$

Her score in DI = $\frac{270}{1.08} = 250$

Her score in VA = $\frac{300}{1.0714} = 280$

Her total score = $200 + 250 + 280 = 730$

- 34** Commission up to 10000 = $10000 \times \frac{9}{100} = 900$

Again after 10000,

Commission : Bonus
 9 : 3
 $\Rightarrow$ 3x : x

$\therefore$ Bonus = $(1380 - 900) \times \frac{1}{4} = ₹ 120$

- 35** Men Women
 600x 400x
 Total Engineer = 480 x
 Male Engineer = $480 x \times 0.66 = 320 x$
 $\therefore$ Women who are Engineers = 160 x
 $\therefore$ Women who are not Engineers = $400 x - 160 x = 240 x$
 Required percentage = $\frac{240}{400} \times 100 = 60\%$

- 36** Veer Zara
 In the last month 160 100
 After the first change 200 75
 After final change 120 120

- 37** Time $\times$ Rate = Total charges
 $1 \times 1 = 1$
 $x \times 1.25 = 1.1$
 $x = \frac{1.1}{1.25} \times 100 = 88\%$

Thus decrease in time = 12%

- 38** Ambani Sahara
 Average earning $4x$ $5x$
 No. of family members $\frac{4x}{k}$ $\frac{5x}{l}$
 Total earning $6y$ $5y$
 Average earning $\times$ Number of family members
 = Total earning

$$l = \frac{5y}{5x} = \frac{y}{x}$$

$$k = \frac{6y}{4x}$$

$$\text{Required percentage} = \frac{l}{k} \times 100 = \frac{\frac{y}{x}}{\frac{6y}{4x}} \times 100$$

$$= \frac{4}{6} \times 100 = 66.66\%$$

☞ Alternatively After some steps you can use the options.

39	Year	Value
	2000	100
	2001	110
	2002	121
	2003	133.1
	2004	119.79
	2005	107.811
	2006	97.0299

Hint $(100) \times 1.1 \times 1.1 \times 1.1 \times 0.9 \times 0.9 \times 0.9 = 97.0299$

Now, $\frac{100 - 97.0299}{100} \times 100 \approx 3\%$

- 40** $30 \times 25 \times 35 = x \times 30 \times 28$

$$\Rightarrow x = 31.25$$

It means 32 pages.

So, the percentages increase in the no. of pages

$$= \frac{2}{30} \times 100 = 6.66\%$$

- 41** Rate of increase of the price
 = (rate of inflation + 2)% = $8 + 2 = 10\%$

Jan Jan Jan
 2004 2005 2006
 $20 \left\{ \begin{array}{l} \rightarrow 20 \\ +2 \end{array} \right\} \rightarrow 22 \left\{ \begin{array}{l} \rightarrow 22 \\ +2.2 \end{array} \right\} 24.2$

☞ Alternatively Expected price after 2 years
 = $20 \times 1.1 \times 1.1 = 24.2$

- 42** Let the total number of people = x
 then the amount donated by 0.6x people
 = $600 \times 0.6x = 360 x$

Now since ₹ 360x is equal to 75% of the required amount.
 Hence we need only 25% more amount from the rest of the

people i.e., from $0.4x$ people

$$\text{Hence, average requirement} = \frac{120x}{0.4x} = 300$$

☞ **Alternatively** $(600 \times 0.6 + k \times 0.4) \frac{3}{4} = 600 \times 0.6$

$$\Rightarrow k = 300$$

43 By mistake $= \frac{x}{10}$

$$\text{Actual value} = x \times 10$$

$$\% \text{ change} = \frac{10x - \frac{x}{10}}{10x} \times 100 = \frac{99}{100} \times 100 = 99\% \text{ (negative)}$$

Since actual value is greater than the wrong value

☞ **Alternatively**

$$\text{Actual result} = 10 \times 10 = 100 \quad (\text{suppose } x = 10)$$

$$\text{wrong result} = \frac{10}{10} = 1$$

$$\% \text{ change} = \frac{100 - 1}{100} \times 100 = 99\%$$

☞ **NOTE** Percentage error is always calculated on the basis of actual (i.e., correct) value.

44 Use some different values for x then verify.

$$\text{Let } x = 150 \text{ then } \% \text{ error} = \frac{200 - 100}{100} \times 100 = 100\%$$

$$\text{Again if } x = 100 \text{ then } \% \text{ error} = \frac{150 - 50}{50} \times 100 = 200\%$$

Hence, we cannot determine.

45 Number of visitors $\times$ Rate = Revenue collected

$$\begin{array}{ccc} \text{1} & \times & \text{1} \\ \text{(-20\%)} \swarrow & & \searrow \text{(+50\%)} \\ \text{0.8} & \times & \text{1.5} \\ & = & \text{1.2} \end{array}$$

Therefore percentage change in the revenue

$$= \frac{1.2 - 1}{1} \times 100 = 20\%$$

46

	Bata	Woodland	
M.P.	100	100	(Since the marked prices are same)
S.P.	51	51	

☞ **NOTE** M.P. $\rightarrow$ Marked price, S.P. $\rightarrow$ Selling price
decreased by 49% = reduced to 51%

47 Selling price = cost price + profit

$$\text{Shirt} = 7y + x \Rightarrow \text{Coat} = 12y + x$$

Since, the profit is same, so the selling price of shirt will certainly be less than ₹ 2000 (which is half of the total value) as it is clear that cost price of shirt is less than the cost price of coat.

☞ **NOTE** If selling price of shirt is equal to or greater than the selling price of coat then the C.P. of coat will be equal to or less than the C.P. of shirt which is wrong. Hence the only possible choice is (d).

48 Since we don't have sufficient data. Further any value is possible as the required income tax.

49 $2004 \rightarrow 10000 \times x\% = k$

$$2005 \rightarrow 16000 \times \frac{5}{8} x\% = k$$

$$\text{but } x - \frac{5}{8} x = 9$$

$$\Rightarrow x = 24\%$$

So, the income tax = 2400.

50 Only (c) is correct since it is divisible by 4.

Let the original number of element be x then the new no. of elements will be

$$\frac{4x}{5} = K$$

So K must be divisible by 4

$$\text{Since, } x = \frac{K \times 5}{4}$$

51 Go through option

Boys	Girls	Total
14	11	25
56	44	100

(12)

Since out of 100 students no. of boys are greater than the no. of girls by 12

i.e., 12% Hence correct.

☞ **Alternatively**

Let the no. of boys and girls be x and y respectively

$$\text{then } (x - y) = \frac{12 \times (x + y)}{100}$$

$$\Rightarrow \frac{x}{y} = \frac{14}{11}$$

52 Population after 2 years = $5000 \left(1 + \frac{2}{100}\right)^2$

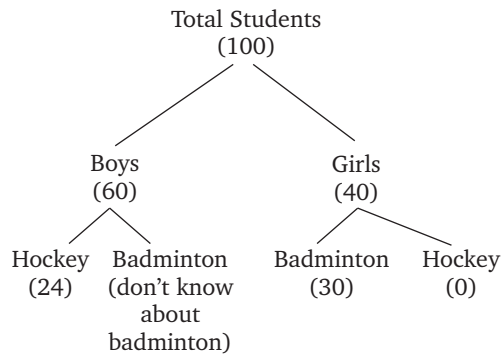
$$5000 \times \frac{51}{50} \times \frac{51}{50} = 5202$$

53 Daily supply = $(100 - z)\%$ of $y = \frac{(100 - z)y}{100}$

$$\therefore \text{Required no. of days} = \frac{x \times 100}{(100 - z)y}$$

✓ Level 02 Higher Level Exercise

1.



Since we do not have information that whether the rest of the boys playing badminton or not. So we cannot determine the total no. of students who are not playing any of the two games.

2 Go through option. Let us assume option (c)

$$10 = 2 \times 5 = 5 \times 2 = 1 \times 10 \times 10 \times 1$$

Consider the proper fraction = $\frac{2}{5}$

[Since the given percentage values are 25% and 20% that's why we have picked up option (c)].

$$\frac{2}{5} \rightarrow \frac{4}{25} \rightarrow \frac{5}{20}$$

To verify : $\frac{2}{5} \times \frac{5}{8} = \frac{1}{4} = \frac{5}{20}$

Hence presumed option (c) is correct.

☞ **Alternatively** $\frac{x}{y} \rightarrow \frac{x^2}{y^2} \rightarrow \frac{1.25 x^2}{0.8 y^2} = \frac{25 x^2}{16 y^2}$

Now since $\frac{25 x^2}{16 y^2} = \frac{5 \left(\frac{x}{y} \right)}{8} \Rightarrow \frac{x}{y} = \frac{2}{5}$

3

$$\text{Income} = \text{Expenditure} + \text{Savings}$$

$$\left(\begin{array}{l} 8x = \\ 10x = \end{array} \left[\begin{array}{l} 5x + 3x \\ 8x + 2x \end{array} \right] - x \right.$$

Now the deficit = $(3x - 2x) = x = 3500$

and $\therefore$ the new salary = $10x = 35,000$

☞ **Alternatively** Go through options.

4 Go through options

$$2457 - 2143 = 314 \quad (\text{True})$$

$$\text{Again } (2457 + 2143) + 41 = 4641$$

$$\text{Now } \frac{4641}{0.85} \rightarrow 5460$$

$$\text{Again } \frac{5460 \times 45}{100} = 2457 \quad (\text{True})$$

Hence the presumed option (d) is correct.

☞ **Alternatively** Let there be total x eligible voters, and the no. of votes goes to loser is k then

$$0.85x - 41 = 2k + 314 \quad \dots(1)$$

$$k + 314 = 0.45x \quad \dots(2)$$

Therefore $x = 5460$

then, $5460 \times 0.85 = 4641$

Again $4641 - 41 = 4600$

Again $k + (k + 314) = 4600$

$$\Rightarrow k = 2143$$

(loser)

and $k + 314 = 2457$

(winner)

5 Income $\rightarrow$ 4 4.4 4.8 5.2] 18.4 lakh
 Saving $\rightarrow$ 2 1.76 1.44 1.04] 6.24 lakh
 Exp. $\rightarrow$ 2 2.64 3.36 4.16] 12.16 lakh
 So, $\frac{6.24}{12.16} \times 100 = 51 \frac{6}{19} \%$

6 Let there be x voters and k votes goes to loser then

$$0.8x - 120 = k + (k + 200)$$

$$k + 200 = 0.41x$$

$$\Rightarrow k = 1440$$

and $(k + 200) = 1640$

Therefore $\frac{1440}{3200} \times 100 = 45\%$

Solutions (for 7 to 9)

$$P + R = 30,000 \quad \dots(i)$$

$$N = R - 8000 \quad \dots(ii)$$

$$(R + N) = 233.3 (P)$$

$$\Rightarrow 3(R + N) = 7P \quad \dots(iii)$$

$$\Rightarrow 6R - 7P = 24,000 \quad \dots(iv)$$

$$\therefore R = 18,000$$

and $P = 12,000$

and $N = 10,000$

$$7 \frac{P + R + N}{3} = \frac{40,000}{3} = 1333.33$$

8 Can't be determined

$$9 \frac{8}{10} \times 100 = 80\%$$

$$10 \text{ (Bonus) Commission} = \frac{20 \times 10,00,000}{100} = 2 \text{ lakh}$$

$$\text{but total profit} = \text{net profit} + \frac{10}{100} \times \text{net profit}$$

$$\Rightarrow 1.32 \text{ lakh} = 1.1 \times \text{net profit}$$

$$\Rightarrow \text{net profit} = 1.2 \text{ lakh} = 1,20,000$$

$$\therefore \text{commission} = \text{total profit} - \text{net profit} \\ = 1,32,000 - 1,20,000 = 12,000$$

$$\text{Hence total earning} = 2,00,000 + 12,000 = 2,12,000$$

11 Let Mr. Scindia has x shares of 5.5%

$$x \times 92 = 32,200 \Rightarrow x = 350 \text{ shares}$$

$$\text{Income} = 350 \times 5.5 = 1925$$

Now, after investment his income is

$$\left(\frac{1}{3} \times \frac{32200}{92} \times 4.5\right) + \left(\frac{2}{5} \times \frac{32200}{115} \times 5\right) + \left(\frac{4}{15} \times \frac{32200}{56} \times 6\right)$$

$$= 525 + 560 + 920 = 2005$$

$$\text{Profit} = 2005 - 1925 = ₹ 80$$

- 12** The surface area of a cube = $6a^2 = 6 \times (\text{side})^2$

$$\text{New surface area} = 6 \times 1.44 a^2$$

$$\therefore \frac{0.44 a^2}{a^2} \times 100 = 44\%$$

Solutions (for Q. Nos. 13 and 14)

Pati $\rightarrow Pt$, Patni $\rightarrow Pn$, Woh $\rightarrow W$

$$(Pt + Pn) = 2W \quad \dots(i)$$

$$(Pn + W) = 4Pt \quad \dots(ii)$$

Solving eqs. (i) and (ii) we get

$$\frac{Pn}{W} = \frac{7}{5} \quad \text{and} \quad \frac{Pt}{W} = \frac{3}{5}$$

$$\Rightarrow Pt : Pn : W = 3 : 7 : 5$$

$$\text{Again} \quad (Pt + Pn) = 2W \quad \dots(iii)$$

$$\text{and} \quad (Pn + W) \times 7 = 8 \times Pt \quad \dots(iv)$$

$$\text{Therefore} \quad \frac{Pn}{W} = \frac{3}{5}, \quad \frac{Pt}{W} = \frac{7}{5}$$

$$\Rightarrow Pt : Pn : W = 7 : 3 : 5$$

$$\text{Gain of Pati} = 7x - 3x = 4x = 800$$

$$\Rightarrow x = 200$$

	Pati	Patni	Woh
Amount at the begining of Game	= 600	1400	1000
Amount at the end of the game	= 1400	600	1000

- 13** Only Patni has suffered the loss

$$\text{14} \quad \frac{1400 - 600}{1400} \times 100 = 57.1428\%$$

- 15** RM + MC = Total cost

$$\text{Total cost} + \text{Profit} = \text{Sale price}$$

$$\begin{array}{l} 70 + 30 = 100 \quad 100 + 10 = 110 \\ 84 + 42 = 126 \quad 126 + 72 = 198 \end{array} \quad \left. \begin{array}{l} \nearrow \\ \nwarrow \end{array} \right\} +80\%$$

$$\text{Therefore profit \%} = \frac{72}{126} \times 100 = 57.14\%$$

- 16** $A + B + C + D = 56$

$$B + C + D = 4.6A$$

$$\Rightarrow A + B + C + D = 5.6A \quad (\text{adding } A \text{ in both sides})$$

$$56 \text{ lakh} = 5.6A$$

$$\Rightarrow A = 10 \text{ lakh}$$

$$\text{Similarly} \quad A + C + D = \frac{11}{3} B$$

$$\Rightarrow A + B + C + D = \frac{14}{3} B$$

$$\Rightarrow B = 12 \text{ lakh}$$

$$\text{Similarly} \quad 4(A + B + D) = C$$

$$\Rightarrow A + B + D = 2.5C$$

$$\Rightarrow A + B + C + D = 3.5C$$

$$\Rightarrow C = 16 \text{ lakh}$$

$$\text{Therefore } D = (A + B + C + D) - (A + B + C) = 18 \text{ lakh}$$

- 17** Losing candidate = $0.3x$

$$\therefore \text{Other two candidates} = 0.7x$$

$$\text{The share of winning candidate} = 0.36x$$

$$\text{and the second ranker} = 0.34x$$

$$\therefore \text{Margin (min. possible)} = 0.02x \Rightarrow 2\% \text{ of } x$$

Let the minimum possible voters be 50 then

$$\frac{2 \times 50}{100} = 1$$

Hence the minimum possible margin of votes = 1

18

Day	Initial amount	Sales	Remaining over night	Rotten	Stock for next day
I	x	$0.5x$	$0.5x$	$0.05x$	$0.45x$
II	$0.45x$	$0.225x$	$0.225x$	$0.0225x$	$0.2025x$
III	$0.2025x$	$0.10125x$	$0.10125x$	$0.010125x$	

$$\therefore \text{Total rotten amount} = 0.082625x = 1983$$

$$\Rightarrow x = 24,000$$

- 19** Check through option

Ⓢ Alternatively Let the initial amount be x (with gambler), then

$$\left(\left((x + 100) \frac{1}{2} + 100 \right) \frac{1}{2} + 100 \right) \frac{1}{2} = \frac{x}{2} \Rightarrow x = \frac{700}{3}$$

- 20** Non-defective products

$$\frac{25 \times 0.98 + 35 \times 0.96 + 40 \times 0.95}{100} \times 100 = 96.1$$

21

No. of Machines	Output	Manf. cost	Est. cost	Total cost	Profit
12	48,000	24,000	10,000	34,000	14,000
11	44,000	22,000	10,000	32,000	12,000

$$\text{Profit} = \text{Output} - \text{Total cost}$$

$$= 44,000 - 32,000 = 12,000$$

$$\text{Initial value of share holders} = 14,000 \times \frac{10}{100} = 1400$$

$$\text{Changed value of share holders} = 12,000 \times \frac{10}{100} = 1200$$

$$\% \text{ decrease} = \frac{200}{1400} \times 100 = 14.28\%$$

22

Rice	Wheat
25	9
$\times x$	$\times 5x$
25x	45x

$$70x = 350 \Rightarrow x = 5$$

Hence the price of Rice = ₹ 5 per kg

Price of wheat = ₹ 25 per kg

Now, the price of wheat = ₹ 30 per kg

Let the new amount of Rice be M kg, then

$$M \times 5 + 9 \times 30 = 350 \Rightarrow M = 16$$

Hence decrease (in%) of amount of rice

$$= \frac{25 - 16}{25} \times 100 = 36\%$$

23

Year	Rate of Commission	Commission in values
1	20%	$0.2 \times 20,000 = 4000$
	25% (bonus)	$0.25 \times 4000 = 1000$
2	16%	$0.16 \times 20,000 = 3200$
3	12%	$0.12 \times 20,000 = 2400$
4	10%	$0.1 \times 20,000 = 2000$
5-10	4%	$6 \times 0.04 \times 20,000 = 4800$

Total commission

$$= (4000 + 3200 + 2400 + 2000 + 4800) + 1000 = 17,400$$

Solutions (for Q. Nos. 24 and 25)

OHIO (M : F) TEXAS (M : F)
Last year : $600 \leftarrow (2 : 1)$ $(5 : 6) \rightarrow 1100$

This year : $750 \leftarrow (2 : 1)$ (unknown) $\rightarrow 1200$

24 Since we don't know the number of female employees in the Texas office this year so we can't determine.

25 $1100 + 600 = 1700$

26 There is no need to use the no. of goats i.e., (34, 398) let initially there be 1000 goats then

$$1000 \rightarrow 1400 \rightarrow 980 \rightarrow 1274 \rightarrow 1146.6$$

$$\text{Thus the \% increase} = \frac{1146.6 - 1000}{1000} \times 100 = 14.66\%$$

27 In 2002 (980 goats) as per the flow chart

Solutions (for Q. Nos. 28 to 30)

Optional	Science	Commerce	Engineering	Total
	5000	3000	8000	16,000
Finance	1000	1200	680	2880
HR	1600	720	1040	3360
Marketing	2400	1080	6280	9760

28 6280 students of Engineering opted marketing.

$$\text{29 } \frac{720}{16,000} \times 100 = 4.5\%$$

30 Marketing, since maximum students have opted marketing.

31 Consider some values and then verify the option.

32 Go through option :

$$\begin{array}{c} 700 \\ \downarrow \\ 91\% = (100 - 9)\% \end{array}$$

$$\begin{array}{c} \downarrow \\ 637 \\ \downarrow \\ 80\% \\ \downarrow \\ 509.6 \approx 510 \text{ persons} \\ \downarrow \\ 70\% \text{ (completely cured out)} \\ \downarrow \\ 357 \\ \downarrow \end{array}$$

$$\text{(Partially cured) } 153 = (510 - 357)$$

Hence, the presumed option (d) is correct.

33 Total expenditure per kg = $3.2 + 1.8 + 2 + 3 = 10$ = cost price

Selling price = ₹ 18 per kg

Gross profit = ₹ 8 per kg = $(18 - 10)$

$$\text{Net profit} = 8 \times \frac{80}{100} \text{ (since 20\% is tax)} = ₹ 6.4$$

Hence the net profit of the factory = $6.4 \times 50,00,000$

$$= ₹ 3,20,00,000 = ₹ 3.2 \text{ crore}$$

34 Let the percentage marks in QA = $(10a + b)\%$

Let the percentage marks in DI = $(10b + a)\%$

Let the percentage marks in VA = $x\%$

$$\text{then } \frac{(10a + b) + x + (10b + a)}{3} = x$$

$$\Rightarrow 11a + 11b + x = 3x \Rightarrow x = \frac{11}{2}(a + b)$$

Thus the percentage of the VA section is a multiple of 11.

35

$$P_1 = k \frac{T}{V}$$

$$P_2 = k \frac{1.4T}{0.8} = K \frac{7T}{4V}; \frac{P_2 - P_1}{P_1} = \frac{\frac{7T}{4V} - \frac{T}{V}}{\frac{T}{V}} = \frac{\frac{3T}{4V}}{\frac{T}{V}} = \frac{3}{4}$$

Hence, the new pressure will be increased by 75%.

$$\text{36 } 20 \times 0.92 \Rightarrow 10 \text{ minutes } \frac{23 \times 40 \times 0.90}{20 \times 0.92} = 45$$

Thus the required time is 45 times than the previous time

$$\text{Hence, } 450 \text{ minutes} = 7 \frac{1}{2} \text{ hrs.}$$

37 Original volume = $16 \times 12 \times 5 = 960 \text{ (inch)}^3$

Required capacity = 1120 (inch)^3

$$\text{Increase in area} = \frac{1120}{5} - 16 \times 12 = 224 - 192 = 32 \text{ (inch)}^2$$

$$\% \text{ increase} = \frac{32}{192} \times 100 = 16.66\%$$

38 The total passengers in each compartment = $25 \times \frac{7}{5} = 35$

Total no. of seats = $(35)^2 = 1225$

$$\text{Maximum available capacity} = 1225 \times \frac{80}{100} = 980 \text{ seats}$$

- 39**
- | | Tata | Reliance |
|----------|------|----------|
| Prepaid | 100 | 81 |
| Postpaid | 90 | 72 |
- Thus the % decrease in talk time = $\frac{90 - 72}{90} \times 100 = 20\%$

- 40**
- Half yearly exam**
- 100

Pass
(70)

Fail
(30)
- Annual exam**
- 70×0.6
42

30×0.8
24

∴ Total pass in annual exam = 42 + 24 = 66%

- 41** Percentage of passed students = 68% [100 - 32]%
 Number of girls passed the exam = 408
 Number of boys passed the exam = 476
 Total passed students = 884
 Therefore total no. of students = $\frac{884}{68} \times 100 = 1300$

Solution (for Q. Nos. 42 to 45)

Name	Horse	Chariot	Land	Total (in ₹)
Ram	2 lakh (10)	80,000 (10)	20 acre = 1 lakh	3.8 lakh
Sita	1.6 lakh (8)	80,000 (10)	8 acre = 40,000	2.8 lakh
Laxman	1 lakh (5)	2 lakh (25)	20 acre = 1 lakh	4 lakh
Urmila	1.4 lakh (7)	40,000 (5)	16 acre = 80,000	2.6 lakh

1. $R + S = L + U$ and $R > S$ and $L > U$
 2. Horses $\rightarrow (R + S) : (L + U) = 3x : 2x = 18x : 12x$
 Again Ram have $\frac{1}{3}$ rd horses. Therefore $30x \times \frac{1}{3} = 10x$
 Therefore the horse of Sita = $18x - 10x = 8x \Rightarrow x = 1$
 Therefore the horse of Ram = 10 and Laxman = 5
 No. of chariots of Sita = No. of chariots of Ram = $\frac{K}{5}$

and no. of chariots of Laxman = $\frac{K}{2}$

Hence the no. of chariots of Urmila
 $= K - \left(\frac{K}{5} + \frac{K}{5} + \frac{K}{2} \right) = \frac{K}{10}$

Again $\frac{k}{2} - \frac{k}{10} = 20 \Rightarrow k = 50$ chariots

Now the 50% property of Laxman = 25 chariots = 2,00,000
 Hence the total property of Laxman = 4,00,000
 Thus the area of Land of Laxman = $\frac{2,00,000 - 5 \times 20,000}{5000}$
 $= 20$ acre = (1 lakh)

Total property of Urmila
 $= 1,40,000 + 40,000 + 80,000 = 2,60,000$
 Thus the total property of Laxman and Urmila = 6.6 lakh

- 42** $3.8 - 2.6 = 1.2$ lakh

- 43** Value of chariots of Laxman = 2 lakh

Now since only Ram has the horses of worth ₹ 2 lakh. So only Ram can exchange with Laxman.

- 45** $\frac{7.2 - 6.0}{6.0} \times 100 = 20\%$

- 46** Total cubes $160 + 56 = 216$

Therefore the side of cube = 6 unit

No. of cubes without any exposure = $(6 - 2)^3 = 64$

Thus 64 cubes will be inside of the big cube

Now rest of the cubes = $160 - 64 = 96$

Again the no. of cubes with one face outside

$$= 6 \times (4 \times 4) = 96$$

Hence the required percentage = $\frac{96}{216} \times 100 = 44.44\%$

- 47**
- (Qualified CAT)
 ↓
 x boys and x girls
 ↓
 (Qualified G.D.)
 ↓
 0.4x boys
 ↓
 (Interview)

IIM-A
 [0.2x]
 ↓
 Qualified
 ↓
 $0.8 \times 0.2x$
 $= 0.16x$

IIM-B
 [0.2x] appeared
 ↓
 Qualified
 ↓
 $0.4 \times 0.2x$
 $= 0.08x$

Total boys qualified the final stage = 0.24%

Thus $0.24x = 24 \Rightarrow x = 100$

- 48** Go through option and consider some appropriate values

Alternatively $\frac{p}{100 + p} = \frac{q}{100}$

$$\Rightarrow 100(p - q) = pq \Rightarrow (p - q) = \frac{pq}{100}$$

- 49** Let the original price be P , then the decrease in value of P after one cycle = $p \left(\frac{x}{100} \right)^2 = 21025$... (i)

Again the final value after second cycle

$$\Rightarrow P \left(1 + \frac{x}{100} \right) \left(1 - \frac{x}{100} \right) \left(1 + \frac{x}{100} \right) \left(1 - \frac{x}{100} \right) = 484416$$

$$\Rightarrow P \left[1 - \left(\frac{x}{100} \right)^2 \right]^2 = 484416 \quad \dots (ii)$$

Dividing eq. (ii) by eq. (i)

$$\frac{\left[1 - \left(\frac{x}{100} \right)^2 \right]^2}{\left(\frac{x}{100} \right)^2} = \frac{484416}{21025} = \frac{2304}{100}$$

$$\Rightarrow \frac{1 - \left(\frac{x}{100}\right)^2}{\left(\frac{x}{100}\right)} = \sqrt{\frac{2304}{100}} = \frac{48}{10}$$

$$\text{Let } \frac{x}{100} = k,$$

$$\text{then } \frac{1 - k^2}{k} = \frac{48}{10}$$

$$\Rightarrow 10k^2 + 48k - 10 = 0$$

$$\Rightarrow 5k^2 - 24k - 5 = 0$$

$$\Rightarrow k = 5 \text{ or } k = -\frac{1}{5} \text{ (inadmissible value)}$$

$$\text{So } x = 20\%$$

$$\text{Hence, } P \left(\frac{x}{100} \right)^2 = 21025$$

$$\Rightarrow P = 525625$$

$$50 \text{ Men} \times \text{Time} = \text{Work}$$

$$100 \times 1 = 100 \text{ unit}$$

$$150 \times 1 = 150 \text{ unit}$$

Extra man power required = 50 but since new workers are $\frac{5}{4}$ times as efficient as existing workers.

$$\therefore \text{Actual no. of workers} = \frac{50}{5/4} = 40 \text{ men}$$

Hence, required percentage

$$= \frac{40}{100} \times 100 = 40\%$$

✓ Level 03 Final Round

- 1 Growth rate of Finance

$$= \frac{125 - 75}{75} \times 100 = \frac{2}{3} \times 100 = 66.66\%$$

Growth rate of Marketing

$$= \frac{250 - 100}{100} \times 100 = 150\%$$

- 2 Total no. of students in 2003-04 is

$$= (250 + 125 + 100 + 75) = 550$$

No. of student in 2004-05

$$= 550 \left(1 + \frac{1}{11} \right) = 600 \text{ students}$$

- 3 $\frac{50}{(150 + 100 + 75 + 50)} \times 100$

$$= \frac{50}{375} \times 100 = 13.33\%$$

- 4 $\frac{4800 - 4000}{4000} \times 100 = 20\%$

- 5 $\frac{450}{4350} \times 100 = 10.34\%$, which is lowest in comparison to others

Thus the growth rate of logical reasoning book is lowest.

- 6 DI $\rightarrow \frac{5000 - 3750}{3750} \times 100 = 33.33$

$$\text{and VA} \rightarrow \frac{5500 - 4140}{4140} \times 100 = 32.85\%$$

- 7 Only LR has consistent growth, others have been fluctuated (QA - 2003, DI - 2003, VA - 2002)

$$8 \left[\frac{8.5 - 2.97}{2.97} \times 100 \right] = 37.23\%$$

- 9 Growth rate of Hero = 37.23%

Growth rate of Atlas = 28.1%

Avon is least which is clear from the data.

Total bicycle production is highest in 95 hence false

$$\text{Hero cycle's share} = \frac{2.97 + 4.22 + \dots 8.5}{79.84} = 43\%$$

Hence, (d) since none of (a), (b) and (c) is correct.

- 10 $\frac{1.75 + 2.48 + \dots 4.21}{79.84} = 23.4\%$

- 11 $\frac{1.64}{64.8} \times 100 = 2.53$

- 12 Amul has max. unutilised capacity

$$\text{Hint } \left(\frac{1.54}{59.35} \times 100 \right) - 1.54 = 1.05$$

Similarly for others can also be find out

- 13 $\frac{11.6}{61.3} \times 100 \approx 18.9$

- 14 $\frac{42.20}{132.8} \times 100 = 31.81\%$

- 15 $\frac{105}{360} \times 100 = \frac{7}{24} \times 100 = 29.166\%$

- 16 $\frac{30}{120} \times 100 = 25\%$

- 17 $\frac{15}{120} \times 100 = 12.5\%$

- 18 $105 - 45 = 60^\circ = 4 \text{ hours}$

- 19 $15^\circ = 1 \text{ hour (Maths)}, 30^\circ = 2 \text{ hour (other subjects)}$

- 20 Sony TV $\rightarrow 19\%$ Sony Refrigerator $\rightarrow 15\%$

$$\text{difference} = 4\% = 14.4^\circ$$

$$(\text{Since } 1\% = 3.6^\circ)$$

- 21 $\left\{ \begin{array}{l} 12\% \rightarrow \text{Samsung TV} = 31 \text{ crore} \\ 18\% \rightarrow \text{Electrolux TV} = 46.5 \text{ crore} \\ 100\% \rightarrow \text{Total market} = 258 \text{ crore} \\ 11\% \rightarrow \text{Samsung Refrig.} = 9 \text{ crore} \\ 13\% \rightarrow \text{Electrolux Refrig.} = 10.6 \text{ crore} \\ 100\% \rightarrow \text{Total market} = 81.8 \text{ crore} \end{array} \right.$
 $\text{Market share} = \frac{46.5 + 10.6}{258 + 81.8} \approx 16.7\%$

- 22 18% = ₹ 42 crores, so 27% (= 16 + 11) = ₹ 63 crore
 13% = ₹ 6 crore, so 28% (= 12 + 16) = ₹ 13 crore
 Total ₹ 76 crores
 $\therefore$ Annual approx. turnover = $4 \times 76 = ₹ 304$ crores

- 23 (a) $(13 - 12) = 1\% = 3.6$
 (b) We don't know the turnover of TV and refrigerator market for each brand.
 (c) 6% total refrigerator market = 100 crore
 $\Rightarrow$ Total refrigerator market ≈ 1667 crore
 $\therefore$ Difference = 2% of 1667 = 33.34 crore
 Thus, (b) can't be inferred.

- 24 $\frac{52,000 - 5,000}{5000} \times 100 = 9.4 \times 100 = 940\%$

- 25 Medical college in 1980 = $\frac{11}{100} \times 32,000 = 3520$
 $1990 = \frac{9}{100} \times 52,000 = 4680$
 $\therefore \frac{4680 - 3520}{3520} \times 100 = 32.95\%$

- 26 We don't have the information about the proportion (share) of Engineering colleges in the given years.

- 27 $\frac{32,000 - 12,000}{12,000} \times 100 = 166.66\%$

- 28 Number of medical colleges in 1990 = $\frac{52,000 \times 9}{100} = 4680$

Increase in the total no. of colleges
 = $60,000 - 52,000 = 8000$

Increase in the no. of medical colleges = $8000 / 4 = 2000$
 Therefore, percentage of medical colleges in 2000

$$= \frac{4680 + 2000}{60,000} \times 100 = 11\%$$

- 29 If the amount of water consumed of sector 1 is the same then we can directly compare the tariffs to the two years.

Region	Tariff 2003-04	% change	Tariff 2000-01
Region 1	1000	+ 25%	800
Region 2	400	+ 14.28%	350
Region 3	250	-16.66%	300
Region 4	625	+ 4.166%	600
Region 5	720	+ 20%	600
	2995		2650

$$\begin{aligned} &= \frac{2995 - 2650}{2650} \times 100 \\ &= \frac{345}{2650} \times 100 \\ &= 13.01\% \end{aligned}$$

30

Sector	Tariff 2003-04	% change over 2000-01	Tariff 2000-01
Sector 1	400	+ 14.28%	350
Sector 2	375	+ 7.14%	350
Sector 3	525	-12.5%	600
Sector 4	800	-20%	1000
	2100		2300

$$\text{Average tariff} = \frac{2300}{4} = 575$$

- 31 In 2003-04 the water consumed by various sectors out of 20,000 kilo-litres can be given as follow:

Category	Percentage	Consumption in 2003-04
Domestic	40	8,000
Urban	25	5,000
Rural	15	3,000
Industrial	20	4,000
		20,000

Since there was a 20% decrease in the domestic consumption in 2003-04 the domestic consumption in

$$2000-01 = \frac{8,000}{0.8} = 10,000.$$

But this constitutes 40% of total water consumed in 2000-01 and the industrial consumption constitutes 20% of total water in 2000-01. Hence in 2000-01 the industrial consumption = $10,000 \times \frac{20}{40} = 5,000$ kilo-liters

- 32 We do not know the category-wise break up of tariffs i.e the rates of Urban sector is unknown.

- 33 The average of Region

$$2 = \frac{400 + 375 + 525 + 800}{4} = 525$$

Average tariff in Region 4

$$= \frac{62.5 + 750 + 240 + 360}{4} = 487.5$$

$$\text{Average tariff in Region 5} = \frac{220 + 360 + 320 + 400}{4} = 450$$

Statements (c) cannot be determined.

- 34 Total number of people = 10,000

Business men	10%	1000	LML	1200
Govt. Servant	13%	1300	SUZUKI	1200
Professionals	30%	3000	BAJAJ	3000
Students	45%	4500	HERO HONDA	6600
Housewives	2%	200		

The total number of Hero Honda bikes = 6600.

Total numbers of Government servants housewives and students = 6000

Total no. of Businessmen and professional = 4000

∴ Percentage of remaining (i.e., Businessmen and prof.)

$$\text{driving Hero Honda} = \frac{600}{4000} \times 100 = 15\%$$

- 35** Let the no. of people who drive one two and three bikes be 15k, 3k and k respectively.

$$\begin{aligned} \text{Number of bikes which are being driven} \\ = 15k + 2(3k) + 3(k) = 24k \end{aligned}$$

Since LML and Suzuki cannot be driven by same person and a person can drive maximum 3 bikes.

Total bikes which are being used to drive = 12,000

$$\therefore 24k = 12,000 \Rightarrow k = 500$$

$$\begin{aligned} \text{Total number of people driving the bikes} \\ = 15k + 3k + k = 19k = 9500 \end{aligned}$$

$$\begin{aligned} \therefore \text{Number of people who do not drive any bike} \\ = 10,000 - 9500 = 500 \end{aligned}$$

- 36** From the previous solution, number of people who drive more than 1 bike i.e., 2 bikes and 3 bikes are 1500 and 500 respectively. These people have total 4500 bikes

$$= (1500 \times 2 + 500 \times 3)$$

Hence, the remaining Hero Honda bikes

$$= 6600 - 4500 = 2100$$

$$\begin{aligned} \text{Thus, the number of persons who drive single Hero Honda} \\ = 2100 \end{aligned}$$

- 37** Since 20% drive other bikes 80% drive only bajaj bike number of people who drive only Bajaj bike

$$= 0.8 \times 3000 = 2400$$

$$\mathbf{38} \quad \frac{11000 + 12000}{21,000} \times 100 = 109.52\%$$

$$\mathbf{39} \quad \frac{27,000 - 23,000}{23,000} \times 100 = 17.4\%$$

- 40** Demand = Domestic Production + Imports

$$\text{Average demand} = \frac{44 + 54.5 + 57}{3} \approx 52 \text{ million tonnes}$$

$$\mathbf{41} \quad \frac{30,000 - 21,000}{21,000} \times 100 \approx 43\%$$

$$\mathbf{42} \quad \text{Offshore production in 2004} = 16,000 \times 0.875$$

$$= 14,000 \text{ thousand tonnes}$$

$$\text{Onshore production} = 11,000 \text{ thousand tonnes}$$

$$\text{Demand in 2004} = 57,000 \times 1.02 = 5814 \text{ thousand tonnes}$$

$$\text{Imports} = 33.14 \text{ million tonnes} \quad (\because 1 \text{ million} = 10^6)$$

$$\mathbf{43} \quad 201924 \times \frac{5}{6} = 33654 \times 5 = 168270$$

$$\mathbf{44} \quad 127890 \times \frac{11}{10} = 140679$$

$$\mathbf{45} \quad 126400 + 975264 + 237852 = 1339516$$

$$\mathbf{46} \quad \frac{100956}{2445480} \times 100 = 4.128\%$$

$$\textcircled{D} \text{ Alternatively } \frac{1}{24} \times 100 \approx 4.16\%$$

$$\mathbf{47} \quad \begin{array}{l} 1969 - 70 \rightarrow 134790 \\ 1979 - 80 \rightarrow 284620 \end{array} \quad \left. \vphantom{\begin{array}{l} 1969 - 70 \\ 1979 - 80 \end{array}} \right\} 149830$$

$$\mathbf{48} \quad 1929 - 30 \rightarrow 9,75,264, \quad 1949 - 50 \rightarrow 1,40,679$$

$$\% \text{ decrease} = \frac{975264 - 140679}{975264} \times 100 = 85.57\%$$

$$\mathbf{49} \quad 1959 - 60 \longrightarrow 1960 - 61$$

$$168020 \xrightarrow{+20\%} 201924$$

$$\text{Therefore } \frac{168020}{201924} \times 100 = 83.33\%$$

$\textcircled{D}$ Alternatively From percentage change graphic

Increase	Decrease
20%	16.66%
1/5	1/6

$$\therefore \text{The required value} = 83.33\% (100 - 16.66\%)$$

$$\textcircled{D} \text{ Alternatively } x \times \frac{6}{5} = k \Rightarrow x = \frac{5k}{6}$$

$$(x)\% = \frac{5k}{6} \times 100 = 83.33\% \text{ of } k$$

- 50** Only 7,900 tonnes increase over the 1920 – 21.

CHAPTER 06

Profit, Loss and Discount

This chapter is one of the important chapters in the Quantitative Aptitude section. Since calculating profit, loss and discount is our daily need, specially, if we are into business. Even calculating profit and loss is important for everyone who makes any buying or selling decisions of any sort of goods and services, because of one's propensity to earn more, spend less, increase profit and reduce loss. If people have multiple options to buy and sell they compare the prices and try to take the best decision. That's why there is hardly any competitive exam which does not include the problems based on this chapter. In fact, we can find a huge number of problems in Data Interpretation section.

6.1 Theory and Concepts

In day-to-day life we sell and purchase the things as per our requirement. A customer can get things in the following manner:

Manufacturer (or producer) → Whole-saler (dealer) → (Shopkeeper) Retailer (or sales person) → Customer

Terminology

Cost price (CP) The money paid by the shopkeeper to the manufacturer or whole-saler to buy the goods is called the cost price (CP) of the goods purchased by the shopkeeper.

NOTE If an article is purchased for some amount and there are some additional expenses on transportation labour, commission etc., these are to be added in the cost price. Such expenses are called overhead expenses or overheads.

Selling Price (SP) : The price at which the shopkeeper sells the goods is called the selling price (SP) of the goods sold by the shopkeeper.

Profit : If the selling price of an article is more than its cost price, then the dealer (or shopkeeper) makes a profit (or gain) *i.e.*, $\text{Profit} = \text{SP} - \text{CP}$; $\text{SP} > \text{CP}$

Loss : If the selling price of an article is less than its cost price, then the dealer suffers a loss.

i.e., $\text{loss} = \text{CP} - \text{SP}$; $\text{CP} > \text{SP}$

Chapter Checklist

- Theory and Concepts
- Mark up and Discount
- Common Gain or Loss Concept
- Selling Price and Cost Price Equality
- Profit-Loss Factors
- CAT Test

Important Formulae

$$(i) \text{ Profit} = \text{SP} - \text{CP} \quad (ii) \text{ Loss} = \text{CP} - \text{SP}$$

$$(iii) \text{ Profit percentage} = \frac{\text{profit}}{\text{cost price}} \times 100$$

$$(iv) \text{ Loss percentage} = \frac{\text{loss}}{\text{cost price}} \times 100$$

$$(v) \text{ SP} = \left(\frac{100 + \text{gain}\%}{100} \times \text{CP} \right) = \left(\frac{100 - \text{loss}\%}{100} \times \text{CP} \right)$$

$$(vi) \text{ CP} = \left(\frac{100}{100 + \text{gain}\%} \times \text{SP} \right) = \left(\frac{100}{100 - \text{loss}\%} \times \text{SP} \right)$$

$$(vii) \text{ SP} = (100 + k)\% \text{ of CP; when profit} = k\% \text{ of CP}$$

$$(viii) \text{ SP} = (100 - k)\% \text{ of CP; when loss} = k\% \text{ of CP}$$

NOTE Profit or loss is always calculated on the basis of cost price unless otherwise mentioned in the problem.

Exp. 1) A fruit seller buys 300 oranges at 5 for ₹ 8 and sold at 2 for ₹ 5. Find :

- the cost price of each orange
- the selling price of each orange
- profit or loss on selling one orange
- his total profit or loss on selling all the oranges.

Solution

- Since, the cost price of 5 oranges = ₹ 8
 $\therefore$ Cost price of one (or each) orange = ₹ $\frac{8}{5}$ = ₹ 1.60
- Since, the selling price of 2 oranges = ₹ 5
 $\therefore$ The selling price of one orange = ₹ $\frac{5}{2}$ = ₹ 2.50
- Since, SP is more than CP, there is a profit.
 So profit on selling one orange = SP - CP
 $= 2.5 - 1.6 = ₹ 0.90$
- Profit on selling all the oranges
 $= ₹ (0.90 \times 300) = ₹ 270$
 Hence, the total profit on selling all oranges = ₹ 270

Exp. 2) A shopkeeper buys 100 eggs at ₹ 1.20 per piece. Unfortunately 4 eggs got spoiled during transportation. The shopkeeper sells the remaining eggs at ₹ 15 a dozen. Find his profit or loss.

Solution Cost price of all eggs = ₹ $100 \times 1.2 = ₹ 120$

$$\text{Selling price of one egg} = \frac{15}{12} = ₹ 1.25$$

$$\therefore \text{Selling price of 96 eggs} = 96 \times \frac{15}{12} = ₹ 120$$

Hint After spoiling 4 eggs, only 96 eggs are left.

Now, since the total selling price and total cost price is same, the shopkeeper neither makes a profit nor suffers a loss.

Exp. 3) Aviral purchased a computer for ₹ 47,000. He had to sell it for ₹ 45,800. Find his profit or loss per cent.

Solution Since, SP < CP, there will be loss

$$\text{loss} (\%) = \frac{\text{loss}}{\text{CP}} \times 100 = \frac{47000 - 45800}{47000} \times 100 = \frac{1200}{47000} \times 100 = 2 \frac{26}{47} \%$$

Exp. 4) A dealer sold 600 quintals of sugar at a profit of 7%. If a quintal of sugar cost him ₹ 1600, find his total profit and the selling price.

Solution CP = $1600 \times 600 = ₹ 9,60,000$

Rate of Profit = 7%

$$\text{Profit} = 9,60,000 \times \frac{7}{100} = ₹ 67,200$$

$$\therefore \text{SP} = \text{CP} + \text{Profit} = ₹ 9,60,000 + ₹ 67,200 = ₹ 10,27,200$$

Thus, profit = ₹ 67,200 and selling price = ₹ 10,27,200

Exp. 5) A dealer buys 200 quintals of wheat at ₹ 1200 a quintal. He spends ₹ 10,000 on transportation and storage. Then, he sells the wheat at ₹ 13 per kg. Find his profit or loss. Also calculate it as a percentage.

Solution CP = $1200 \times 200 = ₹ 2,40,000$

Transportation and storage cost = ₹ 10,000

$$\begin{aligned} \text{Total CP} &= 2,40,000 + 10,000 \\ &= ₹ 2,50,000 \end{aligned}$$

$$\text{Total SP} = 13 \times 200 \times 100 = ₹ 2,60,000$$

$\therefore$ Now, since SP > CP, hence there will be profit.

$$\text{Profit} = \text{SP} - \text{CP} = ₹ (2,60,000 - 2,50,000) = ₹ 10,000$$

$$\text{Profit} (\%) = \frac{\text{Profit}}{\text{CP}} \times 100 = \frac{10,000}{2,50,000} \times 100 = 4\%$$

Exp. 6) Find the cost price of an article which is sold for ₹ 220 at a loss of 12%.

Solution SP = ₹ 220, Loss = 12%

Let CP = ₹ x

Then, SP = 88% of CP

$$220 = \frac{88}{100} \times x \Rightarrow x = 250$$

Therefore, cost price = ₹ 250

Exp. 7) By selling a colour TV for ₹ 23520, a dealer suffers a loss of 4%. What is the cost price of the colour TV? At what price should he sell it to gain 8%?

Solution SP = 96% of CP $\therefore$ (loss is 4%)

$$\therefore 23520 = \frac{96}{100} \times \text{CP}$$

$$\text{CP} = 24500$$

Now, gain % = 8%

$\therefore$ New selling price = $(100 + 8)\%$ of CP

$$= \frac{108}{100} \times 24500 = ₹ 26460$$

Introductory Exercise 6.1

1. A towel is sold for ₹ 198 at a gain of 10%. What is the cost price of the towel? At what price must it be sold to gain 25%?
2. A man sold a watch at ₹ 6000, at a loss of $33\frac{1}{3}\%$. Find the cost price?
3. By selling a shirt for ₹ 285 a shopkeeper loses 5%. At what price should he sell the shirt, so as to gain 15%?
4. Sufyan bought 1200 eggs at ₹ 16 a dozen. At what price per hundred must he sell the eggs, so as to earn a profit of 15%?
5. Ram Singh purchased two camels for ₹ 18,000 and ₹ 15,000 respectively. He sold them at a loss of 15% and a gain of 19% respectively. Find the selling price of each of the camels. Also find the overall gain or loss per cent in the transaction.

6.2 Markup and Discount

Marked price: Basically to avoid loss due to bargaining by the customer and to get the profit over the cost price trader increases the cost price by a certain value, this increase in value over cost price is known as markup and the increased price (*i.e.*, CP + Markup) is called the marked price or printed price or list price of the goods.

$$\text{Marked price} = \text{CP} + \text{Markup}$$

$$\text{Marked price} = \text{CP} + (\% \text{ markup on CP})$$

Generally goods are sold at marked price, if there is no further discount, then in this case selling price equals to marked price.

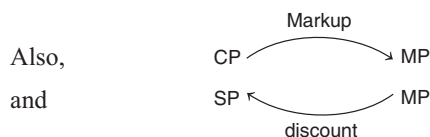
Discount : Discount means reduction of marked price to sell at a lower rate or literally discount means concession. Basically it is calculated on the basis of marked price.

$$\begin{aligned} \therefore \quad & \text{Selling price} = \text{Marked price} - \text{Discount} \\ \text{or} \quad & \text{Selling price} = \text{Marked price (MP)} \\ & \quad \quad \quad - (\% \text{ discount on MP}) \end{aligned}$$

Since, marked price = CP + % markup on CP

Remember **markup** is calculated on the basis of CP while **discount** is calculated on the basis of MP.

$$\begin{aligned} \text{In general, } & \text{CP} < \text{SP} < \text{MP} && \text{at profit} \\ & \text{CP} = \text{SP} < \text{MP} && \text{at no profit no loss} \\ & \text{SP} < \text{CP} < \text{MP} && \text{at loss} \end{aligned}$$



NOTE You will find wide application of percentage calculation, percentage change graphic etc. So, to move further you must know the concepts of percentages.

Exp. 8) If the cost price of an article is ₹ 300 and the per cent markup is 20%. What is the marked price?

$$\begin{aligned} \text{Solution} \quad \text{MP} &= \text{CP} + \% \text{ markup on CP} \\ &= 300 + 300 \times \frac{20}{100} \\ \text{MP} &= ₹ 360 \end{aligned}$$

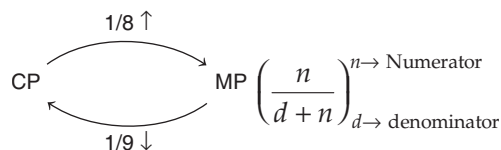
$$\text{Alternatively } \text{SP} = 300 \times 1.2 = 360$$

Exp. 9) If the marked price of an article is ₹ 450 and markup percentage is 12.5%, what is the cost price?

$$\begin{aligned} \text{Solution} \quad \text{MP} &= 112.5 \text{ of CP} \\ \therefore \quad 450 &= \frac{1125}{100} \times \text{CP} \\ 450 &= \frac{9}{8} \times \text{CP} \\ \Rightarrow \quad \text{CP} &= ₹ 400 \end{aligned}$$

Alternatively Change your outlook to visualize the problem differently for smarter calculation.

You can see that MP is 12.5% (*i.e.*, $1/8$ times) greater than CP. So, CP will be $1/9$ times less than MP. This percentage (or fraction) change rule has been thoroughly discussed through different illustrations in percentage chapter. In my opinion if you change your observation *i.e.*, you mould your thinking in a CAT oriented approach you will not change only your attitude and aptitude but the whole scenario will be changed. So, look this problem like me :



So, the CP will be $\frac{1}{9}$ times less than MP, which is rather too much easier to calculate than the traditional method given at the beginning of the solution.

Exp. 10) If the marked price of an article is ₹ 660 and the discount is 10%, then what is the selling price of the article?

Solution General Solution

$$SP = MP - \text{Discount}$$

$$SP = 660 - 660 \times \frac{10}{100} = 594$$

☞ **Alternatively** $SP = 90\%$ of MP

$$SP = 0.9 \times 660 = 594$$

☞ **Alternatively** We can see that when SP is 10% (*i.e.*, $\frac{1}{10}$) less

than MP , it means MP is $\frac{1}{9}$ times greater than SP .

$$\text{Therefore, } MP = \frac{10}{9} SP \quad \left(1 + \frac{1}{9} = \frac{10}{9}\right)$$

Ultimately I would like to say that you should see the core of the problem and solution both, instead of taking help from any formula. In other words formulae make you logically blind particularly in context of CAT where your intelligence is tested rather than your memory and theory of the maths.

Exp. 11) If the markup percentage of an article is 50% and discount percentage is also 20%, then the profit percentage will be :

- (a) 10% (b) 0% (c) 30% (d) 20%

Solution See the smartest calculation technique and follow me to be smarter.

$$\begin{array}{ccccc} & & \text{CP} & \text{SP} & \text{MP} \\ & & 100 & 120 & 150 \end{array}$$

Therefore, profit = 20%

$$\left(\frac{120 - 100}{100} \times 100 = 20\%\right)$$

Explanation: $MP = 150\%$ of CP ($\because$ markup is 50%)
 $SP = 80\%$ of MP ($\because$ discount is 20%)

Once again I am reiterating the gist of this chapter. What is that gist? The gist is that this chapter test nothing but your grip on the concepts of percentage change.

6.3 Common Gain or Loss Concept

When two articles are sold at the same price, but one article at a profit and the other at a loss and the percentage profit is the same as the percentage loss. In this case there is always a loss.

$$\text{loss } (\%) = \left(\frac{\text{Common gain or loss}}{10}\right)^2$$

Exp. 14) A man sells two wrist watches one at a profit of 10% and another at a loss of 10%, but the selling price of each watch is ₹ 200. Find the :

- (1) percentage profit or loss (2) net amount of profit or loss

Solution

- (1) Since, there is always loss. (**Logic:** To get the logic you should refer the percentage chapter)

I think CP , SP , MP etc., are just the words, the crux is that how CP , SP and MP etc., change with respect to each other. So those students who have grasped and internalised the most important concept of percentage change, they will not find any difficulty in understanding the concept of the profit-loss problems. Believe me when your visualisation will be improved, you will be master on this chapter. So try to have a different approach.

Exp. 12) A trader markup the goods by 10% and then gives a discount of 10%. What is the profit or loss percentage?

$$\begin{array}{ccccc} & & \text{CP} & \text{SP} & \text{MP} \\ & & 100 & 99 & 110 \end{array}$$

So, there is a loss of 1%.

Explanation: $MP = 110\%$ of CP (10% markup)
 $SP = 90\%$ of MP (10% discount)

Exp. 13) Successive discount of 10% and 5% is equivalent to

- (a) 16.5% (b) 15% (c) 15.5% (d) 14.5%

Solution

$$\begin{array}{c} 100 \\ \downarrow (-10\%) \\ 90 \\ \downarrow (-5\%) \\ 85.5 \end{array}$$

☞ **NOTE** There is no difference in two different cases *i.e.*, either you decrease 100 at first by 10% and then by 5% or decrease 100 at first by 5% and then by 10%.

$$\begin{array}{c} 100 \\ \downarrow (-5\%) \\ 95 \\ \downarrow (-10\%) \\ 85.5 \end{array}$$

Explanation : $100 \times 0.9 \times 0.95 = 85.5$

$\therefore$ Single discount equivalent to 10% and 5% is 14.5%

$$(100 - 85.5)\%$$

or $(100 \times 0.95 \times 0.9 = 85.5)$

$$\text{Now, } \text{loss } \% = \left(\frac{\text{Common gain or loss}}{10}\right)^2 \%$$

$$\text{loss } \% = \left(\frac{10}{10}\right)^2 \% = 1\%$$

(2) Amount of loss.

$$\text{Total } SP = ₹ 400 \quad (200 + 200)$$

$$\text{Again, total } SP = 99\% \text{ of } CP \quad (1\% \text{ loss})$$

$$400 = \frac{99}{100} \times CP$$

$$\Rightarrow CP = ₹ 404.04$$

$$\therefore \text{loss} = CP - SP$$

$$= ₹ 4.04 \quad (= 404.04 - 400)$$

Exp. 15) There were two articles and the sum of cost prices of these articles is ₹ 500. One of them was sold at a profit of 20% and another at a loss of 20%. Besides if the selling prices of both the articles were same. Find the amount of overall loss.

Solution $x \times \frac{120}{100} = (500 - x) \frac{80}{100} \Rightarrow x = 200$

So, the CP of profit yielding article = ₹ 200
 and the CP of loss giving article = ₹ 300 (500 - 200)
 and the common SP = ₹ 240 = (200 × 1.2 = 300 × 0.8)
 So the loss = CP - SP = 500 - 2 × 240 = ₹ 20

6.4 Selling Price and Cost Price Equality

When the total selling price of 'm' articles is same as the total cost price of 'n' articles, where each article is similar

Exp. 16) If the cost price of 15 apples is same as the selling price of 20 apples. What is the gain or loss per cent?

Solution CP of 15 apples = SP of 20 apples
 $\Rightarrow CP \times 15 = SP \times 20 \Rightarrow \frac{CP}{SP} = \frac{4}{3}$

So you can see that CP > SP, therefore, there will be loss.

Now consider CP = 4, then SP = 3

$\therefore \text{loss} = 1$
 $\therefore \text{loss} (\%) = \frac{\text{loss}}{CP} \times 100 = \frac{1}{4} \times 100 = 25\%$
 loss = 25%

Exp. 17) If the selling price of 10 CDs is the same as the cost price of 12 CDs. What is the profit or loss per cent?

Solution SP of 10 CDs = CP of 12 CDs
 $\Rightarrow SP \times 10 = CP \times 12 \Rightarrow \frac{SP}{CP} = \frac{12}{10} = \frac{6}{5}$

$\Rightarrow SP > CP$, therefore there will be a profit
 $\text{Profit} (\%) = \frac{(SP - CP)}{CP} \times 100 = \frac{(6 - 5)}{5} \times 100 = 20\% \text{ (profit)}$

When the articles are sold at the cost price, but the quantity bought and sold are distinct

When a person recovers the cost price of 'm' articles by selling 'n' articles ($n < m$), then

$$\text{Profit} (\%) = \frac{\text{goods left}}{\text{goods sold}} \times 100 = \frac{m - n}{n} \times 100$$

In this case money is equated in terms of number of (or amount) articles. For your convenience you can try assuming the CP of an article as ₹ 1. (or ₹ 100)

Exp. 18) A dealer by selling 12 oranges gets the cost price of 15 oranges. What is the percentage profit?

Solution $\text{Profit} (\%) = \frac{\text{goods left}}{\text{goods sold}} \times 100 = \frac{15 - 12}{12} \times 100 = 25\%$

Alternatively Suppose CP of one orange = ₹ 1
 then CP of 12 oranges = ₹ 12
 and SP of 12 oranges = CP of 15 oranges = ₹ 15
 $\therefore \text{profit} = \frac{15 - 12}{12} \times 100 = 25\%$

Exp. 19) By selling 8 bananas, a fruit seller gains the selling price of 1 banana. Calculate his gain per cent.

Solution Let the SP of one banana = ₹ 1,
 Then SP of 8 bananas = ₹ 8
 and profit = Re. 1
 $\therefore CP = 8 - 1 = ₹ 7$
 $\therefore \text{Profit} \% = \frac{1}{7} \times 100 = 14\frac{2}{7}\%$

Exp. 20) By selling 18 chocolates, a vendor loses the selling price of 2 chocolates. Find his loss per cent.

Solution Let the SP of 1 chocolate = ₹ 1
 $\therefore$ SP of 18 chocolate = ₹ 18 and loss = ₹ 2
 $\therefore CP = SP + \text{loss} = 18 + 2 = ₹ 20$
 $\therefore \text{Percentage loss} = \frac{\text{loss}}{CP} \times 100 = \frac{2}{20} \times 100 = 10\%$

Exp. 21) A trader sell all his articles at the cost price but gives 10% less amount as he should give. What is his percentage profit?

Solution $\text{Profit} (\%) = \frac{\text{goods left}}{\text{goods sold}} \times 100 = \frac{10}{90} \times 100 = 11\frac{1}{9}\%$

Since, if we assume that the CP of 1 article is ₹ 1.

Now, since he gives only 90% article instead of 100% and save 10% article. So, his profit will be the equal to the remaining articles (over the sold articles). It means when he sells the articles (actually) worth ₹ 90, then he gains by articles worth ₹ 10.

Hence, $\text{profit} \% = \frac{10}{90} \times 100 = 11\frac{1}{9}\%$
 (The selling price = $\frac{100}{90} = \frac{10}{9} = ₹ 1.11$)

Exp. 22) A trader by means of his false balance defrauds to the extent of 10% in buying goods and also defrauds to 10% in selling. Find his gain per cent.

Solution Let the actual CP of an article be ₹ 1, then the effective CP = $\frac{100}{110} = \frac{10}{11}$
 (Since, he purchases 110 articles by paying ₹ 100)
 Again, SP = $\frac{100}{90} = \frac{10}{9}$
 (Since, he sells only 90 articles charging the CP of 100 articles)
 $\therefore \text{Gain} \% = \frac{SP - CP}{CP} \times 100 = \frac{\frac{10}{9} - \frac{10}{11}}{\frac{10}{11}} \times 100$
 $= \frac{20}{99} \times \frac{11}{10} \times 100 = \frac{200}{9} = 22\frac{2}{9}\%$

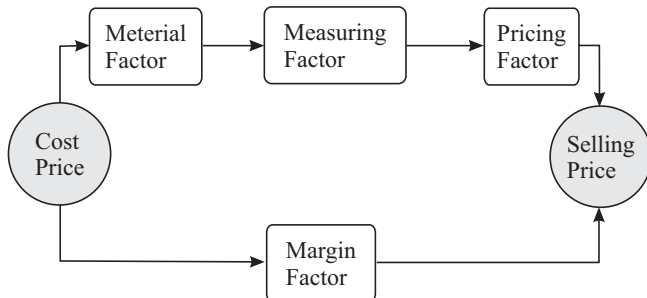
Introductory Exercise 6.2

- The cost price of a scooter is ₹ 20,000 and the profit per cent is 12%. What is the selling price?
(a) 2400 (b) 22040
(c) 2600 (d) 22400
- The SP of an article is ₹ 3200 and the profit per cent is $33\frac{1}{3}\%$. Find the cost price.
(a) ₹ 20000 (b) ₹ 2000
(c) ₹ 2400 (d) ₹ 3000
- The CP of an article is $\frac{5}{6}$ th of the SP. What is the percentage profit or loss?
(a) 20% loss (b) 16.66% profit
(c) 16.66% loss (d) 20% profit
- The MP of a camera is $\frac{3}{2}$ of the CP and SP is $\frac{9}{10}$ of MP. Find the percentage profit or loss.
(a) 25% profit (b) 35% profit
(c) 33.33% loss (d) none of these
- The MP of an article is 30% higher than its CP and 20% discount is allowed on this article, then the profit percentage :
(a) 10% (b) 14% (c) 4% (d) 26%
- An article is sold for ₹ 1980 at 10% profit. What is the cost price?
(a) ₹ 198 (b) ₹ 1800
(c) ₹ 1900 (d) ₹ 1600
- On selling an article for ₹ 576 a trader loses 4%. In order to gain $4\frac{1}{6}\%$, he must sell that article for :
(a) ₹ 636 (b) ₹ 676
(c) ₹ 625 (d) can't be determined
- The per cent profit made when an article is sold for ₹ 56 is thrice as when it is sold for ₹ 42. The cost price of the article is :
(a) ₹ 48 (b) ₹ 49
(c) ₹ 50 (d) ₹ 35
- A shopkeeper uses a weight of 460 g instead of 500 g and sells the articles at the cost price. What is the profit percentage?
(a) 40% (b) 23%
(c) $8\frac{16}{23}\%$ (d) 20%
- A trader uses a weight of 920 g instead of 1 kg and sells the articles at the marked price which is 15% above the cost price. Find the profit percentage.
(a) 20% (b) 23%
(c) 25% (d) can't be determined
- If a gift pack is sold at a gain of 6% instead of at a loss of 6%, then the seller gets ₹ 6 more. The cost price of the article is :
(a) ₹ 60 (b) ₹ 66
(c) ₹ 50 (d) ₹ 36
- A man sells a bicycle at a gain of 10%. If he had bought it at 10% less and sold it for ₹ 132 less, he would have still gained 10%. The cost price of the article is :
(a) ₹ 1000 (b) ₹ 1200
(c) ₹ 1500 (d) ₹ 1320
- An item costing ₹ 600 is being sold at 20% loss. If the price is further reduced by 12.5%, the selling price will be :
(a) ₹ 400 (b) ₹ 380
(c) ₹ 420 (d) ₹ 525
- While selling an electric fan, a dealer gives a discount of 5%. If he gives a discount of 8%, he earns ₹ 36 less as profit. The marked price of the fan is :
(a) ₹ 1000 (b) ₹ 1200
(c) ₹ 800 (d) none of these
- If a commission of 10% is given on the marked price of a book, the publisher gains 20%. If the commission is increased to 15%, the gain of publisher is :
(a) 13.33% (b) 15%
(c) 18% (d) data insufficient
- A retailer buys a cellphone at a discount of 15% and sells it for ₹ 5865. Thus, he makes a profit of 15%. The discount is :
(a) ₹ 200 (b) ₹ 850
(c) ₹ 750 (d) ₹ 900
- At what per cent above the cost price must an articles be marked, so as to gain 17% after allowing a discount of 10%?
(a) 34% (b) 70% (c) 30% (d) 27%
- A merchant marks his goods at ₹ 900 and allows a discount of 25%. If he still gain 12.5%, then the cost price of the article is :
(a) ₹ 500 (b) ₹ 600
(c) ₹ 720 (d) can't be determined
- A vendor buys oranges @ ₹ 2 for 3 oranges and sells them at a rupee each. To make a profit of ₹ 10, he must sell :
(a) 10 oranges (b) 20 oranges
(c) 30 oranges (d) 40 oranges

6.5 Profit-Loss Factors

There are various ways to earn profit in any business transaction. Some are fair means and some are unfair means. Using any such means a trader can create the difference between selling price and the cost price. In any business transaction there are three primary factors by which a businessperson makes a profit or loss.

One way is to change the price, the other way is to change the quantity and the third way is to change the quality. Essentially, a trader claims something else and he gives something else. That means the customer does not know if he is paying the right price or getting the right quantity or right quality that a trader claims to give to his customer. To enhance the margin or profit a trader can do multiple things in a single transaction. That means he can change the price, quantity and quality in a single transaction. That is how a lot of fraudulent traders enhance their margin between selling price and the cost price.



$$\begin{aligned}\text{Margin Factor} &= \frac{\text{Selling Price}}{\text{Cost Price}} \\ &= \frac{\text{Value received from your buyer}}{\text{Value given to your seller}}\end{aligned}$$

$$\begin{aligned}\text{Cost Price} \times \text{Material Factor} \times \text{Measuring Factor} \\ \times \text{Pricing Factor} &= \text{Selling Price} \\ \text{Margin Factor} &= \text{Material Factor} \times \text{Measuring Factor} \\ &\times \text{Pricing Factor}\end{aligned}$$

$$\text{Cost Price} \times \text{Margin Factor} = \text{Selling Price}$$

A trader earns a profit if the margin factor is greater than 1 and he makes a loss if the margin factor is less than 1 and he makes no profit or no loss when the margin factor is 1. Therefore,

$$\begin{aligned}\text{Profit Factor} &= \text{Margin Factor} - 1 \\ \text{Loss Factor} &= 1 - \text{Margin Factor} \\ \text{Profit (\%)} &= \text{Profit Factor} \times 100 \\ \text{Loss (\%)} &= \text{Loss Factor} \times 100\end{aligned}$$

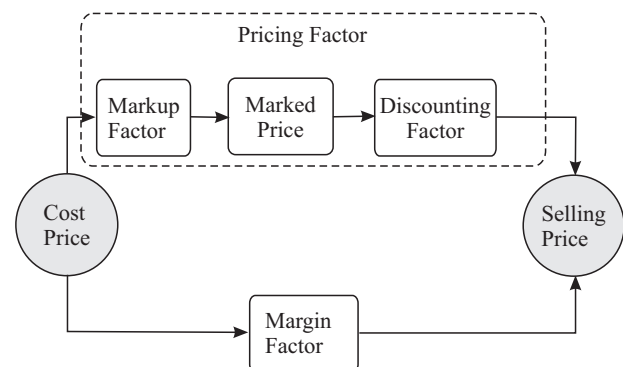
1. Profit by Manipulating The Price

- A trader simply marks up the cost price and sells the goods at the marked up price. Thus the marked-up price becomes the sales price.
- A trader marks up the price of the goods much higher, then he gives discounts on the marked up price. Thus the discounted price becomes the selling price.

$$\begin{aligned}\text{Markup Factor} &= \frac{\text{Marked Price}}{\text{Cost Price}} \\ &= \frac{\text{Value received from your buyer}}{\text{Value given to your seller}}\end{aligned}$$

$$\begin{aligned}\text{Discounting Factor} &= \frac{\text{Selling Price}}{\text{Marked Price}} \\ &= \frac{\text{Value received from your buyer}}{\text{Value given to your seller}}\end{aligned}$$

$$\begin{aligned}\text{Margin factor} &= \text{Pricing Factor} \\ &= \text{Markup Factor} \times \text{Discounting Factor}\end{aligned}$$



$$\begin{aligned}\text{Cost Price} \times \text{Markup Factor} &= \text{Marked Price} \\ \text{Marked Price} \times \text{Discounting Factor} &= \text{Selling Price} \\ \text{Cost Price} \times \text{Markup Factor} \times \text{Discounting Factor} \\ &= \text{Selling Price}\end{aligned}$$

$$\begin{aligned}\text{Cost Price} \times \text{Margin Factor} &= \text{Selling Price} \\ \text{Margin Factor} &= \text{Markup Factor} \times \text{Discounting Factor}\end{aligned}$$

$$\text{Markup factor} = 1 + \left(\frac{\text{markup \%}}{100} \right)$$

$$\text{Discounting Factor} = 1 - \left(\frac{\text{Discount \%}}{100} \right)$$

$$\text{Margin Factor} = \frac{\text{Selling Price}}{\text{Cost Price}}$$

When there are no price changes, these factors become 1.

2. Profit by Manipulating The Measurement

- When a trader cheats his customer - While selling his goods to his customer, the trader simply gives less quantity than what he charges for. In this case his customer is not aware of the fact that the trader is giving him less quantity. That means the customer perceives the displayed quantity to be true and so he pays for the quantity displayed.
- When a trader cheats his supplier - While buying goods from a wholesaler or a manufacturer, the trader simply takes more quantity than what he pays for. In this case supplier is not aware of the fact that the trader is taking away more quantity from him. That means the supplier perceives the displayed quantity to be true and so he charges for the displayed quantity.
- When a trader cheats both his customer and supplier. A trader takes away more goods from his supplier than what he pays for and the trader gives less goods to his customers than what he charges for.

Seller's Measuring Factor

$$= \frac{\text{Display Quantity}}{\text{Actual Quantity}}$$

$$= \frac{\text{Value received from your buyer}}{\text{Value given to your seller}}$$

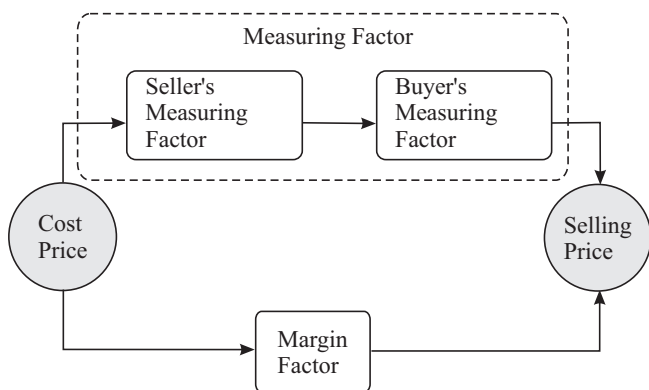
Buyer's Measuring Factor

$$= \frac{\text{Actual Quantity}}{\text{Display Quantity}}$$

$$= \frac{\text{Value received from your buyer}}{\text{Value given to your seller}}$$

Margin Factor = Seller's Measuring Factor

× Buyer's Measuring Factor



When there are no cheating, these factors become 1.

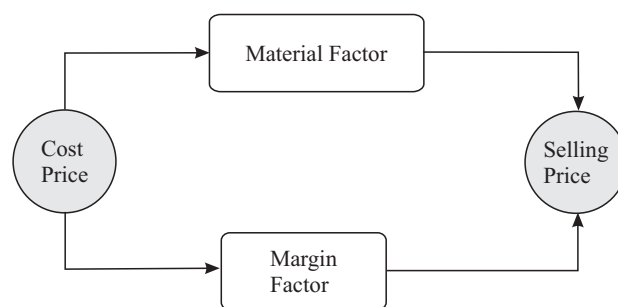
3. Profit by Manipulating The Quality of Material

- When a trader mixes up some freely available or a cheaper material to a costly material and he sells to his customers whereas customer pays the price of the material under the impression that it is a pure material. It enhances the profit of the trader.
- When a trader buys some material from a supplier who mixes up some freely available or a cheaper material to a costly material, however, the trader pays the price of the material under the impression that it is a pure material. It does not change the profit of the trader, though it enhances the profit of the supplier.

Since in this case where the end customer is not able to discern the impurity and therefore he is not in a position to claim against the damages made to him, the profit and loss of the trader depends on the difference of the amount that he receives from his customer and the amount that he pays to his supplier. Essentially, the effect of impurities added by supplier does not affect the profit or loss earned by the trader.

$$\text{Material Factor} = \frac{\text{Value received from your buyer}}{\text{Value given to your seller}}$$

Margin Factor = Material Factor



When there are no cheatings by trader, these factors become 1.

Exp. 23) A trader marks up his goods by 20% over the cost price, if he does not offer any discount, what would be his profit per cent?

- (a) 16.67 (b) 20 (c) 22.22 (d) 25

Solution It is given that the marked price is 20% more than the cost price. That means marked price is 120% of the cost price. Now, let us assume that the cost price of the goods = ₹ 100, the marked price of the goods = $100 \times \frac{120}{100} = ₹ 120$

Since the trader does not offer any discount, it means he sells his goods at the marked price only. That means the selling price = marked price = ₹ 120.

Therefore, the profit (%) = $\frac{120 - 100}{100} \times 100 = 20\%$

Hence choice (b) is the valid one.

☞ Alternatively Let the cost price of the goods be ₹ 1.

The marked price of the goods = $1 \times 1.2 = ₹ 1.2$

Since there is no any discount given by the trader, therefore selling price of the goods = ₹ 1.2

$$\text{Margin Factor} = \frac{\text{Selling Price}}{\text{Cost Price}} = \frac{1.2}{1} = 1.2$$

$$\text{Profit Factor} = \text{Margin Factor} - 1 = 0.2$$

Therefore Profit (%) = Profit Factor $\times$ 100 = 20.

Hence choice (b) is the valid one.

Exp. 24) A trader marks up his goods by 20% over the cost price, if he offers 10% discount over marked price, what would be his profit per cent?

- (a) 10 (b) 12 (c) 8 (d) 18

Solution Let the Cost Price = 1, Markup Factor = 1.2

$$\text{Marked Price} = 1 \times 1.2 = 1.2$$

$$\text{Discounting Factor} = 1 - \left(\frac{10}{100}\right) = 0.9$$

$$\text{Selling Price} = 1.2 \times 0.9 = 1.08$$

$$\text{Margin Factor} = \frac{1.08}{1} = 1.08$$

$$\text{Profit Factor} = \text{Margin Factor} - 1 = 0.08$$

$$\text{Profit} = \text{Profit Factor} \times 100 = 8\%$$

Hence choice (b) is the valid one.

☞ Alternatively Margin Factor

$$= \text{Markup Factor} \times \text{Discounting Factor}$$

$$\text{Margin Factor} = 1.2 \times 0.9 = 1.08$$

$$\text{Profit Factor} = 1.08 - 1 = 0.08$$

$$\text{Profit (\%)} = 0.08 \times 100 = 8\%$$

Exp. 25) A trader has a faulty balance that displays 25% more weight than what it should weigh. If he sells his goods at the cost price using his faulty balance, what is his profit per cent?

- (a) 25 (b) 20 (c) 33.33 (d) none

Solution

$$\text{Margin Factor} = \text{Measuring Factor}$$

$$\text{Margin Factor} = \frac{125}{100} = 1.25$$

$$\text{Profit Factor} = 1.25 - 1 = 0.25$$

$$\text{Profit (\%)} = 0.25 \times 100 = 25\%$$

Hence choice (a) is the valid one.

NOTE The profit in this transaction depends only on the false measurement. There is no role of pricing and quality. That's why all other factors are 1.

Exp. 26) A trader has a faulty balance that weight 25% less than what it should weigh. If he sells his goods at the cost price using his faulty balance, what is his profit per cent?

- (a) 25 (b) 20 (c) 16.67 (d) 33.33

Solution Margin Factor = Measuring Factor

$$\text{Margin Factor} = \frac{100}{75} = 1.3333$$

$$\text{Profit Factor} = 1.33 - 1 = 0.3333$$

$$\text{Profit (\%)} = 0.3333 \times 100 = 33.33\%$$

Hence choice (d) is the valid one

NOTE The profit in this transaction depends only on the false measurement. That's why pricing and quality factors are 1.

Exp. 27) A trader has a faulty balance that weighs 20% less than what it should weigh. Further he marks up the price of the goods by 60%, what is his profit per cent?

- (a) 80 (b) 120 (c) 200 (d) 100

Solution Margin Factor = Measuring Factor $\times$ Markup Factor $\times$ Discounting Factor

$$\text{Margin Factor} = \frac{100}{800} \times 1.6 \times 1 = 2$$

$$\text{Profit Factor} = 2 - 1 = 1$$

$$\text{Profit (\%)} = 1 \times 100 = 100\%$$

Hence choice (d) is the valid one

NOTE The profit in this transaction depends on the false measurement and the mark up. That's why discounting factor is 1.

Exp. 28) A trader has a faulty balance that weighs 20% less than what it should weigh. But, he offers a discount of 10% on the price, what is his profit per cent?

- (a) 8 (b) 10
(c) 12 (d) 12.5

Solution Margin Factor = Measuring Factor $\times$ Markup Factor $\times$ Discounting Factor

$$\text{Margin Factor} = \frac{100}{800} \times 1 \times 0.9 = 1.125$$

$$\text{Profit Factor} = 1.125 - 1 = 0.125$$

$$\text{Profit (\%)} = 0.125 \times 100 = 12.5\%$$

Hence choice (d) is the valid one

Exp. 29) A trader has a faulty balance that weighs 30% less than what it should weigh. Further he marks up the price of the goods by 16.67% and then he offers a discount of 10% on the marked price, what is his profit per cent?

- (a) 18.8 (b) 50
(c) 16.67 (d) 35

Solution Margin Factor = Measuring Factor $\times$ Markup Factor $\times$ Discounting Factor

$$\text{Margin Factor} = \frac{100}{70} \times \left(1 + \frac{16.67}{100}\right) \times 0.9$$

$$\text{Margin Factor} = \frac{10}{7} \times \frac{7}{6} \times \frac{9}{10} = 1.5$$

$$\text{Profit Factor} = 1.5 - 1 = 0.5$$

$$\text{Profit (\%)} = 0.5 \times 100 = 50\%$$

Hence choice (b) is the valid one.

Exp. 30) A trader earns a 25% profit by selling his goods at the cost price while using a faulty balance. How much per cent does his balance weigh less than what it should weigh?

- (a) 25 (b) 20
(c) 16.67 (d) 22.22

Solution

$$\text{Margin Factor} = \text{Measuring Factor} \times \text{Markup Factor} \times \text{Discounting Factor}$$

$$1.25 = \text{Measuring Factor} \times 1 \times 1$$

$$\frac{125}{100} = \frac{\text{Display Weight}}{\text{Actual Weight}}$$

$$\text{Required Percentage} = \frac{125 - 100}{125} \times 100 = \frac{25}{125} \times 100 = 20\%$$

Therefore, his balance weighs 20% less than what it should weigh.

Hence choice (b) is the valid one.

Exp. 31) A trader has a faulty balance that weighs 20% less than what it should weigh. What should be the minimum mark up per cent, if he wants to earn 50% profit?

- (a) 12.5 (b) 16.67
(c) 20 (d) 25

Solution Margin Factor = Measuring Factor $\times$ Markup Factor $\times$ Discounting Factor

$$1.5 = \frac{110}{80} \times \text{Markup Factor} \times 1$$

$$\text{Markup Factor} = \frac{120}{100}$$

$$1 + \frac{\text{Markup}\%}{100} = 1 + \frac{20}{100}$$

$$\text{Markup}\% = 20$$

Hence choice (c) is the valid one.

Exp. 32) A trader earns 40% profit by marking up his goods by 20%. Minimum how much per cent does his balance weigh less than that it should weigh?

- (a) 12 (b) 14.28
(c) 16.67 (d) none of these

Solution Margin Factor = Measuring Factor $\times$ Markup Factor

$$1.4 = \text{Measuring Factor} \times 1.2$$

$$\text{Measuring Factor} = \frac{1.4}{1.2} = \frac{7}{6}$$

$$\frac{\text{Display Weight}}{\text{Actual Weight}} = \frac{7}{6}$$

$$\text{Required Percentage} = \frac{7 - 6}{7} \times 100 = \frac{1}{7} \times 100 = 14.28\%$$

Therefore, his balance weighs 14.28% less than what it should weigh.

Hence choice (b) is the valid one.

Exp. 33) A trader has a faulty balance that weighs 20% less than what it should weigh. Maximum how much percentage discount can he offer, if he wants to earn 10% profit?

- (a) 12 (b) 12.5 (c) 8 (d) 16

Solution Margin Factor = Measuring Factor $\times$ Discounting Factor

$$\frac{110}{100} = \frac{100}{80} \times \text{Discounting Factor}$$

$$\text{Discounting Factor} = \frac{88}{100}$$

$$1 - \frac{\text{Discount \%}}{100} = 1 - \frac{12}{100}$$

$$\text{Discount} (\%) = 12$$

Hence choice (a) is the valid one.

Exp. 34) A trader earns 20% profit even after allowing 30% discount to his customers. Minimum how much per cent does his balance weigh less than what it should weigh?

- (a) 41.67 (b) 55.55 (c) 66.67 (d) 56

Solution Margin Factor = Measuring Factor $\times$ Discounting Factor

$$\frac{120}{100} = \text{Measuring Factor} \times \frac{70}{100}$$

$$\text{Measuring Factor} = \frac{12}{7}$$

$$\frac{\text{Display Quantity}}{\text{Actual Quantity}} = \frac{12}{7}$$

$$\text{Required Percentage} = \frac{12 - 7}{12} \times 100 = \frac{5}{12} \times 100 = 41.67\%$$

Therefore, his balance weighs 41.67% less than what it should weigh.

Hence choice (a) is the valid one.

Exp. 35) A trader has a faulty balance that weighs 20% less than what it should weigh. If he wants to earn 30% profit while marking up the price of the goods by 20%, maximum how much percentage discount can he offer?

- (a) 13.33 (b) 16.67
(c) 22.22 (d) 25

Solution Margin Factor = Measuring Factor $\times$ Markup Factor $\times$ Discounting Factor

$$\frac{130}{100} = \frac{100}{80} \times \frac{120}{100} \times \text{Discounting Factor}$$

$$\text{Discounting Factor} = \frac{13}{15}$$

$$1 - \frac{\text{Discount \%}}{100} = 1 - \frac{2}{15}$$

$$\text{Discount} (\%) = 13.33$$

Hence choice (a) is the valid one.

Exp. 36) A trader has a faulty balance that weighs 20% less than what it should weigh. If he wants to earn 25% profit, while offering a whopping 50% discount, by how much per cent should he mark up his goods?

- (a) 80 (b) 90 (c) 100 (d) 120

Solution Margin Factor = Measuring Factor

$$\times \text{Markup Factor} \times \text{Discounting Factor}$$

$$\frac{125}{100} = \frac{100}{80} \times \text{Markup Factor} \times \frac{50}{100}$$

$$\text{Markup Factor} = \frac{200}{100} \Rightarrow 1 + \frac{\text{Markup}\%}{100} = 1 + \frac{100}{100}$$

$$\text{Markup}\% = 100$$

Hence choice (c) is the valid one.

Exp. 37) A trader marks up the goods by 40% and allows a discount of 26.67% on the marked price. If he expects a decent 120% profit on the goods sold, how much per cent more does his balance display the weight than what it displays when it is not faulty?

- (a) 14.28 (b) 53.33 (c) 114.28 (d) 46.67

Solution Margin Factor = Measuring Factor

$$\times \text{Markup Factor} \times \text{Discounting Factor}$$

$$\frac{220}{100} = \text{Measuring Factor} \times \frac{140}{100} \times \frac{73.33}{100}$$

$$\frac{11}{5} = \text{Measuring Factor} \times \frac{7}{5} \times \frac{11}{15}$$

$$\text{Measuring Factor} = \frac{15}{7}$$

$$\text{Required Percentage} = \frac{15-7}{7} \times 100 = \frac{8}{7} \times 100 = 114.28\%$$

Therefore, his balance must display at least 114.28% more weight than what it displays when it is not faulty.

Hence choice (c) is the valid one.

Exp. 38) A retailer uses faulty balances to purchase and sell the goods. He uses his faulty balances in such a way that while buying the goods from wholesaler he gets 20% more of what he pays for, while selling his goods to his customer he gives 20% less of what he charges for. If he sells his goods at the cost price, how much profit percentage does he make?

- (a) 50 (b) 44 (c) 41 (d) 40

Solution Margin Factor = Measuring Factor

$$\text{Margin Factor} = \left(\frac{120}{100} \times \frac{100}{80} \right)$$

$$\text{Margin Factor} = \frac{12}{8} = 1.5$$

$$\text{Profit Factor} = 1.5 - 1 = 0.5$$

$$\text{Profit} (\%) = 0.5 \times 100 = 50\%$$

Hence choice (a) is the valid one.

NOTE The profit in this transaction depends on the false measurement while buying and selling the goods. That's why markup factor and discounting factor are 1.

Exp. 39) A retailer uses faulty balances to purchase and sell the goods. He uses his faulty balances in such a way that while buying the goods from wholesaler he gets $x\%$ more of what he pays for, while selling his goods to his customer he gives 40% less of what he charges for. If he sells his goods at the cost price and earns 100% profit, what is x ?

- (a) 60 (b) 140 (c) 20 (d) 75

Solution Margin Factor = Measuring Factor

$$\frac{200}{100} = \left(\frac{100+x}{100} \times \frac{100}{60} \right)$$

$$\frac{100+x}{100} = \frac{120}{100}$$

$$x = 20$$

Hence choice (c) is the valid one.

NOTE The profit in this transaction depends on the false measurement while buying and selling the goods. Also, since he sells the goods at the cost price, therefore the markup factor and discounting factor are 1. Actually, in this the Cost Price becomes the Selling Price and the Effective Cost Price becomes the Cost Price.

Exp. 40) A retailer uses faulty balances to purchase and sell the goods. He uses his faulty balances in such a way that while buying the goods from wholesaler he gets 20% more of what he pays for, while selling his goods to his customer he gives $x\%$ less of what he charges for. If he sells his goods at the cost price and earns 33.33% profit, what is x ?

- (a) 10 (b) 11.11 (c) 9.09 (d) 13.33

Solution Margin Factor = Measuring Factor

$$\frac{133.33}{100} = \left(\frac{120}{100} \times \frac{100}{100-x} \right)$$

$$\frac{100}{90} = \frac{100}{100-x}$$

$$x = 10$$

Hence choice (a) is the valid one.

NOTE The profit in this transaction depends on the false measurement while buying and selling the goods. Also, since he sells the goods at the cost price, therefore the markup factor and discounting factor are 1. Actually, in this the Cost Price becomes the Selling Price and the Effective Cost Price becomes the Cost Price.

Exp. 41) A retailer uses faulty balances to purchase and sell the goods. He uses his faulty balances in such a way that while buying the goods from wholesaler he gets 20% more of what he pays for, while selling his goods to his customer he gives 20% less of what he charges for. Further he marks up his goods by 33.33%. How much profit percentage does he make?

- (a) 25 (b) 50
(c) 66.67 (d) 100

Solution Margin Factor = Measuring Factor $\times$ Markup Factor

$$\text{Margin Factor} = \left(\frac{120}{100} \times \frac{100}{80} \right) \times \frac{133.33}{100}$$

$$\text{Margin Factor} = \frac{2}{1} = 2$$

$$\text{Profit Factor} = 2 - 1 = 1$$

$$\text{Profit (\%)} = 1 \times 100 = 100\%$$

Hence choice (d) is the valid one.

NOTE Since there is no discount allowed, so the discounting factor is 1. Remember margin factor = selling price/cost price, so in this problem profit equals to 100%.

Exp. 42) A retailer uses faulty balances to purchase and sell the goods. He uses his faulty balances in such a way that while buying the goods from wholesaler he gets 20% more of what he pays for, while selling his goods to his customer he gives 10% less of what he charges for. If he earns a profit of 60%, by how much per cent does he mark up his goods?

- (a) 40 (b) 44.44 (c) 20 (d) 56.67

Solution Margin Factor = Measuring Factor $\times$ Markup Factor

$$\frac{160}{100} = \left(\frac{120}{100} \times \frac{100}{90} \right) \text{Markup Factor}$$

$$\text{Markup Factor} = \frac{120}{100}$$

$$1 + \frac{\text{Markup}\%}{100} = 1 + \frac{20}{100}$$

$$\text{Markup \%} = 20$$

Hence choice (c) is the valid one.

Exp. 43) A retailer uses faulty balances to purchase and sell the goods. He uses his faulty balances in such a way that while buying the goods from wholesaler he gets $x\%$ more of what he pays for, while selling his goods to his customer he gives 20% less of what he charges for. Further he marks up his goods by 20%. If he earns a profit of 50%, what is x ?

- (a) 25 (b) 20 (c) 10 (d) 0

Solution Margin Factor = Measuring Factor $\times$ Markup Factor

$$\frac{150}{100} = \left(\frac{100 + x}{100} \times \frac{100}{80} \right) \times \frac{120}{100}$$

$$\frac{150}{120} = \frac{100 + x}{80} \Rightarrow x = 0$$

Hence choice (d) is the valid one

Exp. 44) A retailer uses faulty balances to purchase and sell the goods. He uses his faulty balances in such a way that while buying the goods from wholesaler he gets 5% more of what he pays for, while selling his goods to his customer he gives $x\%$ less of what he charges for. Further he marks up his goods by 14.28%. If he earns a profit of 100%, what is x ?

- (a) 60 (b) 50 (c) 40 (d) 25

Solution Margin Factor = Measuring Factor $\times$ Markup Factor

$$\frac{200}{100} = \left(\frac{105}{100} \times \frac{100}{100 - x} \right) \times \frac{114.28}{100}$$

$$\frac{200}{114.28} = \frac{105}{100 - x}$$

$$\frac{200}{800/7} = \frac{105}{100 - x}$$

$$x = 40$$

Hence choice (c) is the valid one

Exp. 45) A retailer uses faulty balances to purchase and sell the goods. He uses his faulty balances in such a way that while buying the goods from wholesaler he gets 4% more of what he pays for, while selling his goods to his customer he gives 13.33% less of what he charges for. Further he offers a discount of 16.67%. How much profit percentage does he make?

- (a) 100 (b) 33.33 (c) 25 (d) 0

Solution Margin Factor = Measuring Factor $\times$ Discounting Factor

$$\text{Margin factor} = \left(\frac{104}{100} \times \frac{100}{86.67} \right) \times \frac{83.33}{100}$$

$$\text{Margin Factor} = \frac{6}{5} \times \frac{5}{6} = 1$$

$$\text{Profit Factor} = 1 - 1 = 0$$

$$\text{Profit (\%)} = 0 \times 100 = 0\%$$

Hence choice (d) is the valid one.

Exp. 46) A retailer uses faulty balances to purchase and sell the goods. He uses his faulty balances in such a way that while buying the goods from wholesaler he gets 8% more of what he pays for, while selling his goods to his customer he gives 10% less of what he charges for. If he earns a profit of 28.56%, by minimum how much per cent does he markup or discount the price?

- (a) 6.67 (b) 10
(c) 12.5 (d) none of these

Solution Margin Factor = Measuring Factor $\times$ Markup Factor $\times$ Discounting Factor

$$\frac{128.56}{100} = \left(\frac{108}{100} \times \frac{100}{90} \right) \times \text{Markup Factor}$$

$$\times \text{Discounting Factor}$$

$$\text{Markup Factor} \times \text{Discounting Factor} = 15/14$$

Since 15/14 is greater than 1, therefore the retailer has certainly marked up the price of the goods. The minimum markup per cent is possible only when there is no discount allowed on the goods. That is when the discounting factor is 1.

$$\text{Thus the minimum markup factor} = \frac{15}{14}$$

$$1 + \frac{\text{Markup}\%}{100} = 1 + \frac{1}{14}$$

$$\text{Markup}\% = 7.14$$

Hence choice (d) is the valid one.

Exp. 47) A retailer uses faulty balances to purchase and sell the goods. He uses his faulty balances in such a way that while buying the goods from wholesaler he gets 25% more of what he pays for, while selling his goods to his customer he gives 25% less of what he charges for. If he earns a profit of 16.67%, by minimum how much per cent does he mark up or discount the price?

- (a) 20, markup
(b) 30, discount
(c) 50, discount
(d) 25, markup

Solution Margin Factor = Measuring Factor $\times$ Markup Factor $\times$ Discounting Factor $\frac{116.67}{100} = \left(\frac{125}{100} \times \frac{100}{75}\right) \times \text{Markup Factor} \times \text{Discounting Factor}$

$$\text{Markup Factor} \times \text{Discounting Factor} = \frac{7}{10}$$

Since $7/10$ is less than 1, therefore the retailer has certainly allowed the discount on the goods.

The minimum discount per cent is possible only when the price not marked up. That is when the markup factor is 1.

Thus the minimum discounting factor = $\frac{7}{10}$

$$1 - \frac{\text{Discount}\%}{100} = 1 - \frac{3}{10}$$

$$\text{Discount}\% = 30$$

Hence choice (b) is the valid one.

Exp. 48) A retailer uses faulty balances to purchase and sell the goods. He uses his faulty balances in such a way that while buying the goods from wholesaler he gets 15% more of what he pays for, while selling his goods to his customer he gives 8% less of what he charges for. Not only this, he marks up his goods by 20% and then to please his customer he offers her 42.84% discount on the marked price. How much profit or loss percentage does he make?

- (a) 14.28, profit
(b) 16.67, profit
(c) 14.28, loss
(d) 20, loss

Solution Margin Factor = Measuring Factor $\times$ Markup Factor $\times$ Discounting Factor

$$\text{Margin Factor} = \left(\frac{115}{100} \times \frac{110}{92}\right) \times \frac{6}{5} \times \frac{4}{7}$$

$$\text{Margin Factor} = \frac{5}{4} \times \frac{6}{5} \times \frac{4}{7} = \frac{6}{7}$$

$$\text{Loss Factor} = 1 - \frac{6}{7} = \frac{1}{7}$$

$$\text{Loss}(\%) = \frac{1}{7} \times 100 = 14.28\%$$

Hence choice (c) is the valid one.

NOTE Since the margin factor ($6/7$) is less than 1, therefore the retailer incur a loss. Profit is made only when the margin factor is greater than 1.

Exp. 49) An unscrupulous trader steals 4 bananas for every dozen whenever he purchases them from a farmer and his customers steal 2 bananas for every dozen whenever they purchase the bananas from this trader. What is the profit or loss percentage of the trader, if he sells the bananas at the price that is offered by the farmer to the trader?

- (a) 16.67 loss
(b) 22.22 profit
(c) 14.28 profit
(d) 12.5 loss

Solution Margin Factor = Measuring Factor

$$\text{Margin Factor} = \left(\frac{16}{12} \times \frac{12}{14}\right) = \frac{8}{7}$$

$$\text{Profit Factor} = \frac{8}{7} - 1 = \frac{1}{7}$$

$$\text{Profit}(\%) = \frac{1}{7} \times 100 = 14.28\%$$

Hence choice (c) is the valid one.

NOTE Since the margin factor ($8/7$) is greater than 1, therefore the retailer makes some profit. Loss is incurred only when the margin factor is less than 1.

Exp. 50) A seller buys a watch for ₹ 2000 and marks it up to ₹ 3120. Further, he gives three successive discounts of $x\%$, $y\%$ and $z\%$ in such a way that $x + y + z = 37.5$. If his overall profit/loss percentage is p , what could be the possible value of p ?

- (a) $62.5 < p < 87.5$
(b) $60 < p < 62.5$
(c) $-2.5 < p < 4.5$
(d) $60 < p < 71.5$

Solution When Cost Price = 2000, Marked Price = 5120

When Cost Price = 100, Marked Price = 256

Case I : when $x = 0$, $y = 0$ and $z = 37.5$.

$$\text{Selling Price} = 256 \left(100 - \frac{37.5}{100}\right) = 160$$

Therefore, Profit = 60%

Case II : When $x = y = z = 12.5$

$$\text{Selling Price} = 256 \left(100 - \frac{12.5}{100}\right) \left(100 - \frac{12.5}{100}\right)$$

$$\left(100 - \frac{12.5}{100}\right) = \frac{343}{2}$$

$$\text{Therefore, profit} = \left(\frac{343}{2} - 100\right) = 71.5\%$$

That is, $60 < p < 71.5$

Hence choice (d) is the valid one.

Exp. 51) A trader imported a bicycle for ₹ 40000 and marked it up by ₹ 60000. Further, he instructed his servant to given two successive discounts of $x\%$ and $y\%$ in such a way that $xy = 25$, $x, y \geq 1$. If his overall profit/loss percentage is p , what could be the possible value of p ?

- (a) $11.375 < p < 35.375$
(b) $85.625 \leq p \leq 125.625$
(c) $25 < p < 62.5$
(d) $60 < p < 71.5$

Solution When Cost Price = 40000, Marked Price = 100000

When Cost Price = 100, Marked Price = 250

Case I : when $x = 1$, $y = 25$.

$$\text{Selling Price} = 250 \left(100 - \frac{1}{100} \right) \left(100 - \frac{25}{100} \right) = 185.625$$

$$\text{Therefore, Profit} = (185.625 - 100) = 85.625\%$$

Case II : When $x = y = 5$

$$\text{Selling price} = 250 \left(100 - \frac{5}{100} \right) \left(100 - \frac{5}{100} \right) = 225.625$$

$$\text{Therefore, Profit} = (225.625 - 100) = 125.625\%$$

That is, $85.625 \leq p \leq 125.625$

Hence choice (b) is the valid one.

Exp. 52) A trader mixes some freely available drinking water with some quantity of pure milk that is procured from a reliable dairy and he sells out the mixture at 20% discount. If the quantity of water is 20% to that of pure milk, what is his overall profit/loss (%)?

- (a) 4%, loss (b) 4%, Profit
(c) No profit No loss (d) 4.4%, profit

Solution Margin Factor = Material Factor $\times$ Discounting factor

$$\text{Margin Factor} = \left(\frac{120}{100} \right) \times \frac{80}{100}, \text{ Margin Factor} = \frac{96}{100}$$

$$\text{Loss Factor} = 1 - \frac{96}{100} = \frac{4}{100}$$

$$\text{Loss (\%)} = \frac{4}{100} \times 100 = 4\%$$

Hence choice (a) is the valid one.

Exp. 53) Before selling the milk to his customers, a milkman mixes 2 litres of freely available potable water with every 8 litres of pure milk procured from a dairy. While selling it to his customers, he gives exactly 10% less than what he claims to give and mark the price up by 20%. What is the overall profit (%) earned by the milkman?

- (a) 60 (b) 66.67 (c) 50 (d) 55

Solution Margin Factor = Material Factor $\times$ Measuring Factor $\times$ Markup Factor

$$\text{Margin Factor} = \frac{10}{8} \times \frac{100}{90} \times \frac{120}{100}, \text{ Margin Factor} = \frac{5}{3}$$

$$\text{Profit Factor} = \frac{5}{3} - 1 = \frac{2}{3}$$

$$\text{Profit (\%)} = \frac{2}{3} \times 100 = 66.67\%$$

Hence choice (b) is the valid one.

Exp. 54) A petrol pump dealer adds 2 litres of kerosene with every 8 litres of petrol before selling the petrol to his customers. The kerosene is actually 25% cheaper than the petrol. While selling it to his customers, he gives 10% less than what he claims to give and mark the price up by 71% with comparison to the cost price of petrol. What is the overall profit (%) earned by the dealer?

- (a) 100 (b) 75 (c) 120 (d) 200

Solution Margin Factor = Material Factor $\times$ Measuring Factor $\times$ Markup Factor

$$\text{Margin Factor} = \frac{10 \times 100}{8 \times 100 + 2 \times 75} \times \frac{100}{90} \times \frac{171}{100}$$

$$\text{Margin Factor} = \frac{1000}{950} \times \frac{100}{90} \times \frac{171}{100}$$

$$\text{Margin Factor} = 2, \text{ Profit Factor} = 2 - 1 = 1$$

$$\text{Profit (\%)} = 1 \times 100 = 100\%$$

Hence choice (a) is the valid one.

Exp. 55) A milkman always purchases milk from a dairy wherein the farmer mixes up 1 litre of freely available water with every 9 litre of pure milk in such a way that the milkman is not able to identify the impurity. Now the milkman adds up 2 litres of freely available water to every 10 litres of milk purchased from the dairy and then he sells to his customers at the cost price. What is the profit or loss percentage earned/accrued by the milkman in this trading?

- (a) 8 (b) 12 (c) 20 (d) 33.33

Solution Margin Factor = Material Factor

$$\text{Margin Factor} = \frac{(10 + 2)}{10} = \frac{12}{10} = 1.2$$

$$\text{Profit Factor} = 1.2 - 1 = 0.2$$

$$\text{Profit (\%)} = 0.2 \times 100 = 20\%$$

Hence choice (c) is the valid one.

NOTE In this scenario it does not matter if the supplier has added any impurity, as the trader is paying him the full price believing that he is getting a pure material.

Alternatively Let the cost price of milk be ₹ 1 per litre.

Total amount paid to the dairy farmer = ₹ 10

Total amount received from the customer = ₹ 12

$$\text{Margin Factor} = \frac{(10 + 2)}{10} = \frac{12}{10} = 1.2$$

$$\text{Profit Factor} = 0.2$$

$$\text{Profit (\%)} = 20\%$$

Exp. 56) A milkman always purchases milk from a dairy wherein the farmer mixes up 3 litre of freely available water with every 7 litre of pure milk in such a way that the milkman is not able to identify the impurity. Now the milkman adds up 5 litres of freely available water to every 15 litres of milk purchased from the dairy and then he sells to his customers at the cost price. What is the profit or loss percentage earned/accrued by the milkman in this trading?

- (a) 8 (b) 12 (c) 20 (d) 33.33

Solution Margin Factor = Material Factor

$$\text{Margin Factor} = \frac{(15 + 5)}{15} = \frac{20}{15} = 1.33$$

$$\text{Profit Factor} = 1.33 - 1 = 0.33$$

$$\text{Profit (\%)} = 0.33 \times 100 = 33.33\%$$

Hence choice (d) is the valid one.

NOTE In this scenario it does not matter if the supplier has added any impurity, as the trader is paying him the full price believing that he is getting a pure material.

Alternatively Let the cost price of milk be ₹ 1 per litre.

Total amount paid to the dairy farmer = ₹ 15

Total amount received from the customer = ₹ 20

$$\text{Margin Factor} = \frac{20}{15} = 1.33$$

$$\text{Profit Factor} = 1.33 - 1 = 0.33$$

$$\text{Profit (\%)} = 0.33 \times 100 = 33.33\%$$

Exp. 57) A milkman purchased milk from a dairy wherein the farmer mixes up 3 litres of freely available water with every 7 litres of pure milk. Fortunately, the milkman caught him mixing up the water, but he could not figure out the exact amount of water in the mixture. So, the milkman paid him 40% less than the normal price he used to pay before catching him dilute the milk. Now the milkman adds up 3 litres of freely available water to every 15 litres of milk purchased from the dairy. However, the milkman sells the milk to his customers at the price that he usually pays to the dairy farmer. What is the profit or loss percentage earned/accrued by the milkman in this trading?

- (a) 60 (b) 75 (c) 90 (d) 100

Solution Margin Factor = Material Factor × Pricing Factor

$$\text{Margin Factor} = \frac{(15 + 3)}{15} \times \frac{100}{(100 - 40)} = 2$$

$$\text{Profit Factor} = 2 - 1 = 1$$

$$\text{Profit (\%)} = 1 \times 100 = 100\%$$

Hence choice (d) is the valid one.

NOTE In this scenario it does not matter if the supplier has added any impurity, as the trader is paying him the price based on mutual understanding. After all, all that matters is how much the trader pays the supplier for a particular quantity of mixture.

Alternatively Let the cost price of milk be ₹ 1 per litre.

Total amount paid to the dairy farmer for 10 litres = $10 - 4 = ₹ 6$

Total amount paid to the dairy farmer for 15 litres = ₹ 9

Total amount received from the customer = $15 + 3 = ₹ 18$

$$\text{Margin Factor} = \frac{18}{9} = 2$$

$$\text{Profit Factor} = 2 - 1 = 1$$

$$\text{Profit (\%)} = 1 \times 100 = 100\%$$

Exp. 58) A milkman purchased milk from a dairy wherein the farmer mixes up 3 litres of freely available water with every 7 litres of pure milk. Fortunately, the milkman caught him mixing up the water, but he could not figure out the exact amount of water in the mixture. So, the milkman paid him 40% less than the normal price he used to pay before catching him dilute the milk. Now the milkman adds up 3 litres of freely available water to every 15 litres of milk purchased from the dairy.

However, the milkman sells the milk to his customers at a price 50% higher than the one he usually pays to the dairy farmer. What is the profit or loss percentage earned/accrued by the milkman in this trading?

- (a) 60 (b) 200 (c) 90 (d) 300

Solution Margin Factor = Material Factor × Pricing Factor

$$\text{Margin Factor} = \frac{(15 + 3)}{15} \times \left(\frac{100 + 50}{100 - 40} \right) = 3$$

$$\text{Profit Factor} = 3 - 1 = 2$$

$$\text{Profit (\%)} = 2 \times 100 = 200\%$$

Hence choice (b) is the valid one.

NOTE In this scenario it does not matter if the supplier has added any impurity, as the trader is paying him the price based on mutual understanding. After all, all that matters is how much the trader pays the supplier for a particular quantity of mixture.

Alternatively Let the cost price of milk be ₹ 1 per litre.

Total amount paid to the dairy farmer for 10 litres = $10 - 4 = ₹ 6$

Total amount paid to the dairy farmer for 15 litres = ₹ 9

Total amount received from the customer = $(15 + 3) \times 1.5 = ₹ 27$

$$\text{Margin Factor} = \frac{27}{9} = 3$$

$$\text{Profit Factor} = 3 - 1 = 2$$

$$\text{Profit (\%)} = 2 \times 100 = 200\%$$

Exp. 59) A milkman purchased milk from a dairy wherein the farmer mixes up 3 litres of freely available water with every 7 litres of pure milk. Fortunately, the milkman caught him mixing up the water, but he could not figure out the exact amount of water in the mixture. So, the scared and embarrassed farmer gave him 25% extra quantity of mixture at the same price. Now the milkman adds up 3 litres of freely available water to every 15 litres of milk purchased from the dairy. However, the milkman sells the milk to his customers at a price 50% higher than the one he usually pays to the dairy farmer. What is the profit or loss percentage earned/accrued by the milkman in this trading?

- (a) 125 (b) 75 (c) 90 (d) 100

Solution Margin Factor = Measurement Factor × Material Factor × Pricing Factor

$$\begin{aligned} \text{Margin Factor} &= \frac{(100 + 25)}{100} \times \frac{(15 + 3)}{15} \times \frac{(100 + 50)}{100} \\ &= \frac{9}{4} = 2.25 \end{aligned}$$

$$\text{Profit Factor} = 2.25 - 1 = 1.25$$

$$\text{Profit (\%)} = 1.25 \times 100 = 125\%$$

Hence choice (a) is the valid one.

NOTE In this scenario it does not matter if the supplier has added any impurity, as the trader is paying the same price as he is supposed to pay for a pure material.

Alternatively Let the cost price of milk be ₹ 1 per litre.

Total amount paid to the dairy farmer for 5 litres = ₹ 4

Total amount paid to the dairy farmer for 15 litres = ₹ 12

Total amount received from the customer

$$= (15 + 3) \times \frac{150}{100} = ₹ 27$$

$$\text{Margin Factor} = \frac{27}{12} = 2.25$$

$$\text{Profit Factor} = 2.25 - 1 = 1.25$$

$$\text{Profit (\%)} = 1.25 \times 100 = 125\%$$

Exp. 60) A petrol pump dealer purchases the petrol from a petroleum company that adds 1 litre of kerosene with every 9 litres of petrol before selling the petrol to his customers. The petroleum company also gives 10% less quantity that it claims to sell to a dealer. The dealer adds 2 litres of kerosene with every 8 litres of petrol that he has purchased from the petroleum company. The dealer trying to impress the customers gives 10% extra quantity than what a customer pays for. However, he marks up the price by 10%. The kerosene is actually 50% cheaper than the petrol. What is the overall profit (%) earned by the dealer?

- (a) 10 (b) 11.11 (c) 12.1 (d) 0

Solution Margin Factor = Material Factor $\times$ Measuring Factor $\times$ Markup Factor

$$\text{Margin Factor} = \left(\frac{10 \times 100}{8 \times 100 + 2 \times 50} \right) \times \left(\frac{90}{100} \times \frac{100}{110} \right) \times \frac{110}{100}$$

$$\text{Margin Factor} = \frac{1}{1} = 1$$

$$\text{Profit Factor} = 1 - 1 = 0$$

$$\text{Profit (\%)} = 0 \times 100 = 0\%$$

That means there is neither a profit nor a loss.

Hence choice (d) is the valid one.

Exp. 61) A petrol pump dealer purchases the petrol from a petroleum company that adds 3 litres of kerosene with every 7 litres of petrol before selling the petrol to his customers. Further, the petroleum company gives 10% less quantity that it claims to sell to a dealer. However, it charges 20% less than the normal price. The dealer adds 2 litres of kerosene with every 10 litres of petrol that he has purchased from the petroleum company. The dealer trying to impress the customers gives 10% extra quantity than what a customer pays for. However, he marks up the price by 10%. The kerosene is actually 50% cheaper than the petrol. What is the overall profit (%) earned by the dealer?

- (a) 25 (b) 20 (c) 10 (d) 0

Solution Margin Factor = Material Factor $\times$ Measuring Factor $\times$ Pricing Factor

$$\text{Margin Factor} = \left(\frac{10 \times 100}{8 \times 100 + 2 \times 50} \right) \times \left(\frac{90}{100} \times \frac{100}{110} \right) \times \left(\frac{100}{80} \times \frac{110}{100} \right)$$

$$\text{Margin Factor} = \frac{5}{4}$$

$$\text{Margin Factor} = 1.25$$

$$\text{Profit Factor} = 1.25 - 1 = 0.25$$

$$\text{Profit (\%)} = 0.25 \times 100 = 25\%$$

Hence choice (a) is the valid one.

Exp. 62) A dishonest peddler claims to sell the sugar at 20% discount, but his scale weighs 58.33% less amount that it displays and he also adds some cheap quality sugar, which is 60% cheaper than the high quality sugar. Before selling the sugar he goes to buy standard sugar from a sugar mill.

The sugar mill had to give him 20% discount when the peddler caught the sugar mill red handed mixing freely available sand in the high quality sugar. Even the scale used to measure the weights at sugar mill shows 50% more than the actual weight of the sugar, which goes unnoticed by the trader. Nevertheless, peddler makes a profit of 100%. Which of the following statement is necessarily correct?

- (i) The mixture of sugar that sugar mill sells to the peddler has $1/3^{rd}$ amount of sand in the mixture.
 - (ii) The mixture of sugar that peddler sells has $1/3^{rd}$ amount of cheap quality sugar in the mixture.
 - (iii) Both the statements (i) and (ii) are true
 - (iv) Neither of the statements (i) and (ii) is true.
- (a) (i) (b) (ii)
(c) (iii) (d) (iv)

Solution Margin Factor = Material Factor $\times$ Measuring Factor $\times$ Pricing Factor

Margin Factor

$$= \left(\frac{(x + y) \times 100}{x \times 40 + y \times 100} \right) \times \left(\frac{100}{150} \times \frac{100}{41.67} \right) \times \left(\frac{100}{80} \times \frac{80}{100} \right)$$

$$\frac{(100 + 100)}{100} = \left(\frac{5x + 5y}{2x + 5y} \right) \times \left(\frac{2}{3} \times \frac{12}{5} \right) \times (1)$$

$$\frac{5}{4} = \left(\frac{5x + 5y}{2x + 5y} \right)$$

$$\frac{x}{y} = \frac{1}{2}$$

$$\frac{x}{x + y} = \frac{1}{3}$$

It shows that the peddler has $1/3^{rd}$ amount of the cheap quality sugar in the mixture that he sells to his customers. Therefore, statement (ii) is true.

Peddler's profit does not depend upon the impurity of the sugar sold by sugar mill, so the statement (i) is not necessarily true.

Hence choice (b) is the valid one

CAT-Test

Questions Helping you bell the CAT

LEVEL 01 > BASIC LEVEL EXERCISE

- 1 Abhishek and Bhanu both are dealers of KML scooters. The price of a KML Scooter is ₹ 28,000. Abhishek gives a discount of 10% on whole, while Bhanu gives a discount of 12% on the first ₹ 20,000 and 8% on the rest ₹ 8000. What is the difference between their selling prices?
(a) ₹ 240 (b) ₹ 420
(c) ₹ 640 (d) none of these
- 2 A trader sells two articles, one at a loss of 10% and another at a profit of 15% but finally there is no loss or gain. If the total sale price of these two articles is ₹ 30,000, find the difference between their cost prices :
(a) ₹ 5000 (b) ₹ 6000
(c) ₹ 7500 (d) none of these
- 3 A milkman purchases the milk at ₹ x per litre and sells it at ₹ $2x$ per litre still he mixes 2 litres water with every 6 litres of pure milk. What is the profit percentage?
(a) 116% (b) 166.66%
(c) 60% (d) 100%
- 4 60% goods are sold at 5% loss while rest are sold at 10% profit. If there is a total profit of ₹ 100, then the worth of goods sold is :
(a) ₹ 6000 (b) ₹ 5000
(c) ₹ 10000 (d) none of these
- 5 A retailer bought 20 kg tea at a discount of 10%. Besides 1 kg tea was freely offered to him by the wholesaler at the purchase of 20 kg tea. Now, he sells all the tea at the marked price to a customer. What is profit percentage of retailer?
(a) 30% (b) 12%
(c) 16.66% (d) none of these
- 6 Two articles are sold at the same price. One at a profit of 75% and another one at a loss of 30%. What is the overall profit or loss?
(a) 22.5% profit (b) 57.5% profit
(c) $13\frac{2}{7}$ % loss (d) none of these
- 7 What is percentage profit in selling an article at a discount of 20% which was earlier being sold at a 40% profit?
(a) 20% (b) 14%
(c) 28% (d) 12%
- 8 A man bought 18 oranges for a rupee and sold them at 12 oranges for a rupee. What is the profit percentage?
(a) 33.33% (b) 50%
(c) 66.66% (d) none of these
- 9 A dealer buys a product at ₹ 1920, he sells at a discount of 20% still he gets the profit of 20%. What is the selling price of that product?
(a) ₹ 2304 (b) ₹ 1536
(c) ₹ 2200 (d) it is not possible
- 10 Tinkawala purchased the articles for ₹ 123684. He sold 60% of those at a profit of 16.66% and rest at a loss. Find the loss percentage on the remaining if the overall loss is 14%?
(a) 20% (b) 30% (c) 60% (d) 66.66%
- 11 What should be the minimum markup percentage such that after giving a discount of $66\frac{2}{3}$ %, there will not be a loss?
(a) 200% (b) 133.33%
(c) 100% (d) 150%
- 12 The ratio of cost price and marked price of an article is 2 : 3 and ratio of percentage profit and percentage discount is 3 : 2. What is the discount percentage?
(a) 16.66% (b) 20% (c) 25% (d) 33.33%
- 13 A dealer gives as much discount (in per cent) as the markup (in per cent) above the cost price. What is the profit or loss per cent?
(a) 10% (b) 1%
(c) 4% (d) Can't be determined
- 14 In the above problem, what is the markup percentage when there is a loss in percentage be half of the percentage markup?
(a) 20% (b) 40%
(c) Can't be determined (d) none of these
- 15 A shopkeeper sold 12 cameras at a profit of 20% and 8 cameras at a profit of 10%. If he had sold all the 20 cameras at a profit of 15%, then his profit would have been reduced by ₹ 36. What is the cost price of each camera?
(a) 100 (b) 150 (c) 180 (d) 220

- 16** Mr. Mittal purchased a car for ₹ 3,00,000 and a bike for his son for ₹ 1,00,000. He sold the car at a profit of 10% and bike at a loss of 20%. What is the net gain or loss?
 (a) 2% gain (b) 1.5% loss
 (c) 2.5% loss (d) 2.5% gain
- 17** A trader sells 20 kg of sugar at ₹ 400. A customer asks 20% discount and he agrees to it but instead of 1 kg he gives 4% less sugar. What is the effective discount that the customer gets?
 (a) 16% (b) 16.66%
 (c) 15.5% (d) 19.6%
- 18** The profit percentage on the three articles A, B and C is 10%, 20% and 25% and the ratio of the cost price is 1 : 2 : 4. Also the ratio of number of articles sold of A, B and C is 2 : 5 : 2, then the overall profit percentage is :
 (a) 18.5% (b) 21%
 (c) 75% (d) none of these
- 19** The marked price of an article is increased by 25% and the selling price is increased by 16.66%, then the amount of profit doubles. If the original marked price be ₹ 400 which is greater than the corresponding cost price by 33.33%, what is the increased selling price?
 (a) 240 (b) 360
 (c) 420 (d) 600
- 20** A shopkeeper calculated his profit per cent on the selling price which comes out to be 30%. If it had been calculated as usual on the cost price, then what is the required percentage profit?
 (a) $42\frac{6}{7}\%$ (b) $4\frac{7}{27}\%$
 (c) $7\frac{6}{42}\%$ (d) none of these
- 21** An item was sold after giving two successive discount of 20% and 10% respectively. If the item was sold for ₹ 468. The marked price of that item is :
 (a) ₹ 600 (b) ₹ 500
 (c) ₹ 575 (d) ₹ 650
- 22** The cost price of an article 'A' is ₹ 160 and selling price of another article 'B' is ₹ 240. If the selling price of A will be equal to the cost price of B, then the profit after selling A is 20%. What is the profit on 'B'?
 (a) 16.66% (b) 50%
 (c) 25% (d) none of these
- 23** A single discount equivalent to three successive discounts of 5%, 10%, 20% is :
 (a) 68.4% (b) 35%
 (c) 31.6% (d) 32%
- 24** Ragini purchases oranges at ₹ 10 per dozen and sells them at ₹ 12 for every 10 oranges. What is the profit percentage?
 (a) 40% (b) 44%
 (c) 60% (d) 48%
- 25** A trader uses a weighing balance that shows 1250 g for a kilogram. He further marks-up his cost price by 20%. What is the profit percentage?
 (a) 5% (b) 45%
 (c) 50% (d) 30%
- 26** On selling an article for ₹ 240, a trader loses 4%. In order to gain 10% he must sell that article for :
 (a) ₹ 275 (b) ₹ 340 (c) ₹ 320 (d) ₹ 264
- 27** A merchant marks his goods at ₹ 300 and allows a discount of 25%. If he still gains 12.5%, then the cost price of article is :
 (a) ₹ 220 (b) ₹ 200
 (c) ₹ 240 (d) ₹ 260
- 28** An item costing ₹ 200 is being sold at 10% loss. If the price is further reduces by 5%, the selling price will be :
 (a) ₹ 170 (b) ₹ 171 (c) ₹ 180 (d) ₹ 181
- 29** A person sold two cows each for ₹ 9900. If he gained 10% on one and lost 20% on the other, then which of the following is true?
 (a) He gained ₹ 200
 (b) He lost ₹ 200
 (c) He neither gained nor lost
 (d) None of the above
- 30** Two third of a consignment was sold at a profit of 5% and the remainder at a loss of 2%. If the total profit was ₹ 400, the value of the consignment (in rupees) was :
 (a) 15000 (b) 20000 (c) 10000 (d) 12000
- 31** A fruit seller declares that he sells fruits at the cost price. However, he uses a weight of 450 g instead of 500 g. His percentage profit is :
 (a) 10% (b) $11\frac{1}{9}\%$ (c) 12% (d) $12\frac{2}{9}\%$
- 32** A person loses ₹ 20 by selling some bananas at the rate of ₹ 3 per banana and gains ₹ 30, if he sells them at ₹ 3.25 per banana. The number of bananas sold by him:
 (a) 100 (b) 200 (c) 120 (d) 2400
- 33** Due to an increase of 30% in the price of eggs, 3 eggs less are available for ₹ 9.10. The present rate per egg is :
 (a) 91 paise (b) 78 paise
 (c) 48 paise (d) 84 paise
- 34** By selling 12 apples for a rupee, a man loses 20%. How many for a rupee should he sell to gain 20%?
 (a) 8 (b) 10 (c) 15 (d) 16
- 35** A dealer buys a washing machine, listed at ₹ 10000 and gets 10% and 20% successive discounts. He spends 10% of his CP on transport. At what price (in rupees) should he sell the washing machine to earn a profit of 10%?
 (a) 8722 (b) 7892
 (c) 8712 (d) 8840

- 36** 6% more is gained by selling a coat for ₹ 1425 than by selling it for ₹ 1353. The cost price of the coat is :
 (a) ₹ 1000 (b) ₹ 1250
 (c) ₹ 1500 (d) ₹ 1200
- 37** By selling a wrist watch at ₹ 405 the shopkeeper incurs a loss of 10%. What is the gain or loss percentage if he sells the same watch at ₹ 465?
 (a) profit of 10% (b) loss of 6%
 (c) profit of 3.33% (d) no profit no loss
- 38** Titan sells a wrist watch to a wholesaler making a profit of 10%. The wholesaler, in turn, sells it to the retailer making a profit of 10%. A customer purchases it by paying ₹ 990. Thus, the profit of retailer is $2\frac{3}{11}\%$. What is the cost incurred by the Titan to produce it?
 (a) 768 (b) 750
 (c) 800 (d) 820
- 39** Pepsi and Coke, there are two companies, selling the packs of cold-drinks. For the same selling price Pepsi gives two successive discounts of 10% and 25%. While Coke sells it by giving two successive discounts of 15% and 20%. What is the ratio of their marked price?
 (a) 143 : 144 (b) 19 : 11
 (c) 136 : 135 (d) 73 : 77
- 40** When a shopkeeper reduces the selling price from 1080 to 1026 its loss increases by 4 percentage point. What is the selling price of this same article when it fetches a profit of 4%?
 (a) ₹ 1392 (b) ₹ 1404
 (c) ₹ 1450 (d) ₹ 1350
- 41** The difference between CP and SP of a table fan is ₹ 175 when it gives the profit of 14%. What is the selling price of that fan?
 (a) 1225 (b) 1450 (c) 1425 (d) 1275
- 42** A company instead of raising the mark-up by 20% discounted the cost price by 20% while stitching the price tag on its product. Further the company offers a discount of 6.25% to its customer. In this process company incurs a loss of ₹ 37.5 on a single article. What is the selling price of that article?
 (a) 417.5 (b) 112.5
 (c) 365.5 (d) none of these
- 43** When an article is sold for ₹ 703 loss incurred is 25% less than the profit earned on selling it at ₹ 836. What is the selling price of the article when it earns a profit of 20%?
 (a) 912 (b) 1576
 (c) 1532 (d) 1092
- 44** Arun bought toffees at 6 for a rupee. How many for a rupee he should sell to gain 20%?
 (a) 3 (b) 4
 (c) 5 (d) Can't be determined
- 45** A scientific calculator is available at Universal Shoppe in Hazratganj at 20% discount and the same is available at only 15% discount at Universal Shoppe Bhootnath Market. Ms. Aggarwal has just sufficient amount of ₹ 800 to purchase it at Universal Shoppe Hazratganj. What is the amount that Ms. Aggarwal has less than the required amount to purchase it at Universal Shoppe Bhootnath?
 (a) ₹ 70
 (b) ₹ 50
 (c) ₹ 100
 (d) data insufficient
- 46** A balance of a trader weighs 10% less than it should be. Still the trader marks-up his goods to get the overall profit of 20%. What is the markup on the cost price?
 (a) 40% (b) 8%
 (c) 25% (d) 16.66%
- 47** ITC sells one product at a profit of 20% another at a loss of 20% at the same selling price. What is the loss incurred by ITC?
 (a) 1% (b) 2%
 (c) 4% (d) 0%
- 48** In the previous question, if SP of each article be ₹ 200, what is the amount of loss?
 (a) ₹ 10 (b) ₹ 16
 (c) ₹ 16.66 (d) none of these
- 49** The cost price of 19 articles is same as the selling price of 29 articles. What is the loss %?
 (a) 35% (b) 34.48%
 (c) 52.63% (d) none of these
- 50** The selling price of 13 articles is same as the cost price of 23 articles. What is the profit percentage?
 (a) 43.47% (b) 74.83%
 (c) 78% (d) 76.92%
- 51** A trader can procure 34 pencils by selling 28 pencils. What is the ratio of cost price to the selling price of a pencil?
 (a) 2 : 3 (b) 14 : 17
 (c) 9 : 7 (d) 4 : 7
- 52** At style cloth emporium the shopkeeper measures 20% less for every metre of cloth also he marks-up goods by 20%. What is the profit percentage?
 (a) 50% (b) 80%
 (c) 75% (d) none of these
- 53** A bookseller procures 40 books for ₹ 3200 and sells them at a profit equal to the selling price of 8 books. What is the selling price of one dozen books, if the price of each book is same?
 (a) 720
 (b) 960
 (c) 1200
 (d) 1440

- 54** The profit percentage of A and B is same on selling the articles at ₹ 1800 each but A calculates his profit on the selling price while B calculates it correctly on the cost price which is equal to 20%. What is the difference in their profits?
 (a) ₹ 360 (b) ₹ 60
 (c) ₹ 540 (d) ₹ 450
- 55** Each of A and B sold their article at ₹ 1818 but A incurred a loss of 10% while B gained by 1%. What is the ratio of cost price of the articles of A to that of B ?
 (a) 101 : 90 (b) 85 : 89
 (c) 81 : 75 (d) none of these
- 56** A trader sold an article at a loss of 5% but when he increased the selling price by ₹ 65 he gained 3.33% on the cost price. If he sells the same article at ₹ 936, what is the profit percentage?
 (a) 15% (b) 16.66%
 (c) 20% (d) data insufficient
- 57** Even after a discount of $q\%$ on marked price a trader gains by $p\%$. What is the markup percentage over the cost price?
 (a) $\frac{p+q}{(q-p)} \times 100$ (b) $\frac{p+q}{(100-p)} \times 100$
 (c) $\frac{p+q}{(100-q)} \times 100$ (d) not possible
- 58** A milkman mixes 10% water in pure milk but he is not content with it so, he again mixes 10% more water in the previous mixture. What is the profit percentage of milkman if he sells it at cost price :
 (a) 11.11% (b) 20%
 (c) 21% (d) 12.1%
- 59** A person sold an electronic watch at ₹ 96 in such a way that his percentage profit is same as the cost price of the watch. If he sells it at twice the percentage profit of its previous percentage profit, then the new selling price will be :
 (a) ₹ 132 (b) ₹ 150
 (c) ₹ 192 (d) ₹ 180
- 60** A trader mixes 25% kerosene to his petrol and then he sells the whole mixture at the price of petrol. If the cost price of kerosene be 50% of the cost price of petrol, what is the net profit percentage?
 (a) $11\frac{1}{9}\%$ (b) $12\frac{1}{9}\%$ (c) $9\frac{1}{11}\%$ (d) 20%
- 61** A retailer cheats both to his whole-seller and his customer by 10% by his faulty balance i.e., he actually weighs 10% more while purchasing from wholesaler and weighs 10% less while selling to his customer. What is his net profit percentage, when he sells at CP?
 (a) $22\frac{2}{11}\%$ (b) $22\frac{2}{9}\%$
 (c) 20% (d) 21%
- 62** A trader procures his goods from a wholesaler, whose balance reads 1200 g for 1000 g. The trader sells all the procured goods to a customer after marking up the goods at 20% above the cost price. What is his overall percentage profit or loss in the whole transaction?
 (a) 38% profit
 (b) 50% profit
 (c) no profit no loss
 (d) none of the above
- 63** A person wants to reduce the trade tax, so he calculates his profit on the sale price instead of on the cost price. In this way by selling a article for ₹ 280 he calculates his profit as $14\frac{2}{7}\%$. What is his actual profit percentage?
 (a) 20% (b) 16.66%
 (c) 25% (d) data insufficient
- 64** A vendor sells his articles at a certain profit percentage. If he sells his articles at $\frac{1}{3}$ of his actual selling price, then he incurs a loss of 40%. What is his actual profit percentage?
 (a) 72% (b) 120%
 (c) 80% (d) none of these
- 65** A retailer increases the selling price by 25% due to which his profit percentage increases from 20% to 25%. What is the percentage increase in cost price?
 (a) 20% (b) 30%
 (c) 25% (d) 50%
- 66** Abhinav saves ₹ 25 by getting 6.66% discount on a textbook. What is the amount of money (in ₹) paid by him?
 (a) 450 (b) 350
 (c) 225 (d) 375
- 67** At kul-kul petrol pump the operator gives 5% less petrol but he sells it at the cost price. What is his profit in this way?
 (a) 5% (b) 5.6%
 (c) 5.26% (d) 4.78%
- 68** Due to reduction of 25% in price of oranges a customer can purchase 4 oranges more for ₹ 16. What is original price of an orange?
 (a) Re 1 (b) ₹ 1.33
 (c) ₹ 1.5 (d) ₹ 1.6
- 69** A reduction of 20% in the price of sugar enables a housewife to purchase 6 kg more for ₹ 240. What is the original price per kg of sugar?
 (a) ₹ 10 per kg (b) ₹ 8 per kg
 (c) ₹ 6 per kg (d) ₹ 5 per kg
- 70** A wholesaler sells toys at a profit of 20% to a retailer and retailer sells these toys to its customer at a profit of 25%. What is the profit percentage of the retailer?
 (a) 5% (b) 80%
 (c) 20% (d) 25%

LEVEL 02 > HIGHER LEVEL EXERCISE

- 1 An automobile agency launched a scheme that if a customer purchases two Jabaaj Discover bikes, one extra Jabaaj Discover will be free and if he purchases 3 Jabaaj Pulser he will get one extra Jabaaj Pulser free. If the cost price of 3 Jabaaj Discover and 4 Jabaaj Pulser be ₹ 67500 and ₹ 232500 respectively. If a customer purchases 2 bikes of Jabaaj Discover and 3 bikes of Jabaaj Pulser as per scheme he availed 1 bike free of each category, then at what price these bikes should be sold so, that the agency can get overall profit of 17.5% :
(a) 235250 (b) 352500 (c) 368000 (d) 268000
- 2 Rahul went to purchase a Nokia mobile handset, the shopkeeper told him to pay 20% tax if he asked the bill. Rahul manages to get the discount of 5% on the actual saleprice of the mobile and he paid the shopkeeper ₹ 3325 without tax. Besides he manages to avoid to pay 20% tax on the already discounted price, what is the amount of discount that he has got?
(a) 750 (b) 375 (c) 875 (d) 525
- 3 When a bicycle manufacturer reduced its selling price by 50%, the number of bicycles sold radically increased by 600%. Initially the manufacturer was getting only 140% profit. What is the percentage increase of his profit?
(a) 10% (b) 14%
(c) 0% (d) can't be determined
- 4 A trader marks his goods such that he can make 32% profit after giving 12% discount. However a customer availed 20% discount instead of 12%. What is the new profit percentage of trader?
(a) 20% (b) 44% (c) 30% (d) 28.8%
- 5 A retailer bought 3850 Linc pens and 1848 Cello pens at the same price. He sells Linc pens in such a way that he can buy 650 Linc pens with the sale price of 481 Linc pens. Again he can buy only 408 Cello pens with the sale price of 629 pens. What is the overall percentage of profit of the retailer?
(a) 4.8% (b) 9.6%
(c) 13% (d) none of these
- 6 The ratio of selling price of 3 articles A, B and C is 8 : 9 : 5 and the ratio of percentage profit is 8 : 7 : 14 respectively. If the profit percentage of A is 14.28% and the cost price of B is ₹ 400, what is the overall percentage gain?
(a) 14.28% (b) 14.87%
(c) 16.66% (d) none of these
- 7 Anna sold his car to Boney at a profit of 20% and Boney sold it to Chakori at a profit of 10%. Chakori sold it to mechanic at a loss of 9.09%. Mechanic spent 10% of his purchasing price and then sold it at a profit of 8.33% to Anna once again. What is the loss of Anna?
(a) 23% (b) 29% (c) 50% (d) 40%
- 8 In an office the number of employees reduces in the ratio of 3 : 2 and the wages increases in the ratio of 20 : 27. What is the profit percentage of employees over the previous wages?
(a) 10% (b) 9.09%
(c) 11.11% (d) none of these
- 9 I asked the shopkeeper the price of a wrist watch. I found that I had just the required sum of money. When the shopkeeper allowed me a discount of 25%, I could bought another watch worth ₹ 940 for my younger sister. What is the price which I have paid for my own watch?
(a) ₹ 2700 (b) ₹ 1800
(c) ₹ 2820 (d) ₹ 3760
- 10 A and B are two partners and they have invested ₹ 54,000 and ₹ 90,000 in a business. After one year A received ₹ 1800 as his share of profit out of total profit of ₹ 3600 including his certain commission on total profit, since he is a working partner and rest profit is received by B. What is the commission of A as a percentage of the total profit?
(a) 20% (b) 10%
(c) 5% (d) 25%
- 11 A trader sells goods to a customer at a profit of $k\%$ over the cost price, besides it he cheats his customer by giving 880 g only instead of 1 kg. Thus, his overall profit percentage is 25%. Find the value of k ?
(a) 8.33% (b) 8.25%
(c) 10% (d) 12.5%
- 12 A trader sells two brands of petrol; one is Extra Premium (EP) and other one is 'Speed' (SP). He mixes 12 litres of EP with 3 litres of speed and by selling this mixture at the price of EP he gets the profit of 9.09%. If the price of Extra Premium be ₹ 48 per litre, then the price of Speed (SP) is :
(a) ₹ 38 per litre
(b) ₹ 42 per litre
(c) ₹ 28 per litre
(d) none of these
- 13 A, B and C invest in the ratio of 3 : 4 : 5. The percentage of return on their investments are in the ratio of 6 : 5 : 4. Find the total earnings, if B earns ₹ 250 more than A :
(a) ₹ 6000 (b) ₹ 7250
(c) ₹ 5000 (d) none of these
- 14 Ajay bought a motor cycle for ₹ 50,000. 2 years later he sold it to Bijoy at 10% less of the cost price. Bijoy spend 5% of the purchasing price on its maintenance. Later Bijoy displayed the sale price of his motorcycle ₹ 50,000. Chetan wanted to purchase it at 15% discount but Bijoy gave him two successive discounts of 10% and 5% instead of 15% in one time. What is the actual discount availed by Chetan?
(a) 15% (b) 15.5%
(c) 14.5% (d) none of these

- 15** Kamal bought a house in Sushant city, whose sale price was ₹ 8 lakh. He availed 20% discount as an early bird offer and then 10% discount due to cash payment. After that he spent 10% of the cost price in interior decoration and lawn of the house. At what price should he sell the house to earn a profit of 25%?
 (a) ₹ 9 lakh (b) ₹ 7.99 lakh
 (c) ₹ 7.92 lakh (d) none of these
- 16** I wanted to purchase 10 chairs for the class room whose cost was ₹ 200 each. The trader offered me a discount if I were to purchase a set of 12 chairs. So, I calculated that if I assume the normal price of 10 chairs, then we can purchase 2 extra chairs which cost me only ₹ 80 each of two chairs at the cost price of 12 chairs after discount. What is the percentage discount?
 (a) 6% (b) 8% (c) 12% (d) 10%
- 17** The cost of servicing of a Maruti car at Maruti care Pvt. Ltd. is ₹ 400. Manager of service centre told me that for the second service within a year a customer can avail a 10% discount and further for third and fourth servicing he can avail 10% discount of the previous amount paid, within a year. Further if a customer gets more than 4 services within a year he has to pay just 60% of the servicing charges on these services. A customer availed 5 services from the same servicing station, what is the total percentage discount fetched by the customer?
 (a) 19.42% (b) 18.5% (c) 17.6% (d) 26%
- 18** The cost price of an article is C and the selling price of the same article is S , where Z is the profit or loss percentage. If the cost price and selling price both are increased by same amount, then which of the following is true :
 (a) Z increases (b) Z decreases
 (c) remains constant (d) none of these
- 19** Cost price of 12 oranges is equal to the selling price of 9 oranges and the discount on 10 oranges is equal to the profit on 5 oranges. What is the percentage point difference between the profit percentage and discount percentage?
 (a) 20 (b) 22.22 (c) 16.66 (d) 15
- 20** A car mechanic purchased four old cars for ₹ 1 lakh. He spent total 2 lakh in the maintenance and repairing of these four cars. What is the average sale price of the rest three cars to get 50% total profit if he has already sold one of the four cars at ₹ 1.2 lakh?
 (a) 1.5 lakh (b) 1.1 lakh (c) 1.2 lakh (d) 1.65 lakh
- 21** The cost of setting up a magazine is ₹ 2800. The cost of paper and ink etc is ₹ 80 per 100 copies and printing cost is ₹ 160 per 100 copies. In the last month 2000 copies were printed but only 1500 copies could be sold at ₹ 5 each. Total 25% profit on the sale price was realized. There is one more resource of income from the magazine which is advertising. What sum of money was obtained from the advertising in magazine?
 (a) ₹ 1750 (b) ₹ 2350 (c) ₹ 1150 (d) ₹ 1975
- 22** DSNL charges a fixed rental of ₹ 350 per month. It allows 200 calls free per month. Each call is charged at ₹ 1.4 when the number of calls exceeds 200 per month and it charges ₹ 1.6 when the number of calls exceeds 400 per month and so on. A customer made 150 calls in February and 250 calls in March. By how much per cent the each call is cheaper in March than each call in February?
 (a) 28%
 (b) 25%
 (c) 18.5%
 (d) None of the above
- 23** In the Bargain Bazar everyone purchases with a fair bargaining, so the traders markup the prices too much. A trader marked up an article at ₹ M expected huge profit if it is sold on the marked price. But a customer purchased it at $M/2$ with his fine bargaining skills, so the expected profit of the trader diminished by 66.66%. What is the percentage discount fetched by the customer through bargaining?
 (a) 33.33% (b) 50%
 (c) 66.66% (d) none of these
- 24** Tika Chand has a weighing balance in which there is a technical fault. The right pan of his balance measures always 200 g more than its left pan. Tika Chand as usual misutilise this balance in his business. While purchasing the articles he puts goods in the left pan and weight in the right pan while selling he reverse the order i.e., goods in the right pan and weight in the left pan. He uses only 2 kg weight for the measurement and to measure $2n$ kg weight he measures n times by 2-2 kg but he sells goods at cost price. What is his profit percentage?
 (a) 20% (b) $22\frac{2}{9}\%$
 (c) $18\frac{2}{11}\%$ (d) none of these
- 25** Akram Miya has two types of grapes. One is the fresh grapes containing 80% water and dry grapes containing 25% water. He sells 20 kg dry grapes, by adding water to the dry grapes, at cost price. What is the total profit percentage when after adding water the weight of 20 kg dry grapes increased in the proportion of water in fresh grapes?
 (a) 275% (b) 200%
 (c) 80% (d) 125%
- 26** Pankaj and Sushil invested some amount of money in the ratio of 3 : 5 for the same period in a business. They decided that at the end of year 20% profit was to be given to AIDS Control Society of India as a donation. Out of the remaining, 75% was to be reinvested and the rest of the profit was to be divided as interest on their capitals. If the difference in their shares is ₹ 1200. Find the total profit?
 (a) ₹ 18000 (b) ₹ 24000
 (c) ₹ 20000 (d) none of these

- 27** Jagran group launched a new magazine in January 2004. The group printed 10000 copies initially for ₹ 50000. It distributed 20% of its stock freely as specimen copy and 25% of the rest magazines are sold at 25% discount and rest at 16.66% discount whose printing price was ₹ 12 per copy. What is the overall gain or loss in the first month's issue of magazine, if the magazine could not realize the income from advertisements or other resources?
- (a) 56% profit
(b) 27% loss
(c) 16.66% profit
(d) 38% profit
- 28** Teenagers shoe company sells the shoes whose prices *i.e.*, cost prices and selling prices are the multiples of either 13, 14, 15, 16, 17, 18 or 19, starting from ₹ 399 to ₹ 699 (*i.e.*, $399 \leq \text{CP/SP} \leq 699$). What can be the maximum profit of the company?
- (a) ₹ 292
(b) ₹ 398
(c) ₹ 298
(d) ₹ 300
- 29** Jhun Jhunwala makes 1000 toys and incurs a cost of ₹ 1.2 for each toy. He marks-up the price in such a way that if he sells only 70% of the manufactured toys he will realize 16.66% overall profit. He sells only 750 articles at the marked price, since rest of the toys are found to be defective, so can't be sold. What is the net profit or loss of Jhun Jhunwala?
- (a) 14.44% loss
(b) 25% profit
(c) 33.33% profit
(d) none of these
- 30** Anupam sells a painting to Bhargava at $\frac{4}{5}$ th the rate of profit at which Bhargava sells it to Chaudhary. Further Chaudhary sells it to Dara Singh at half the rate of profit at which Anupam sold it to Bhargava. If Chaudhary earns a profit of 10% by selling it to Dara Singh for ₹ 2805. What is the cost price of painting for Bhargava?
- (a) 1896
(b) 2040
(c) 1680
(d) 2000
- 31** A dishonest retailer cheats his wholesaler and customer both. He purchases 19% more from the wholesaler and sells 15% less while selling to its customer. What is profit percentage by selling the goods at cost price?
- (a) 36.78%
(b) 34%
(c) 40%
(d) 36.85%
- 32** Rotomac produces very fine quality of writing pens. Company knows that on an average 10% of the produced pens are always defective, so are rejected before packing. Company promises to deliver 7200 pens to its wholesaler at ₹ 10 each. It estimates the overall profit on all the manufactured pens to be 25%. What is the manufacturing cost of each pen?
- (a) ₹ 6
(b) ₹ 7.2
(c) ₹ 5.6
(d) ₹ 8
- 33** Pratibha printers prepares diaries expecting to earn a profit of 40% by selling on the marked price. But during transportation 8% diaries were got spoiled due to at random rain and 32% could be sold only at 75% of the cost price. Thus, the remaining 60% diaries could be sold at the expected price. What is the net profit or loss in the whole consignment?
- (a) 6%
(b) 10%
(c) 8%
(d) can't be determined
- 34** Radhey Lal markup the prices of sweets by 40% and he sold only 40% of those at this price. He sells half of the rest at $14\frac{2}{7}\%$ discount and rest at 25% discount. What is the net profit of Radhey Lal?
- (a) 26.5%
(b) 23.5%
(c) 30%
(d) 28.6%
- 35** The price of an article reduces to 576 after two successive discounts. The markup is 80% above the cost price of ₹ 500. What is the new profit percentage if instead of two successive discounts the markup price was further increased successively two times by the same percentage?
- (a) 259.2%
(b) 59.2%
(c) 159.2%
(d) can't be determined
- 36** A trader marks-up his goods by 80% and gives discount of 25%. Besides it he weighs 10% less amount while selling his goods. What is the net profit of trader?
- (a) 50%
(b) 35%
(c) 45%
(d) 55%
- 37** A dishonest trader marks up his goods by 80% and gives discount of 25%. Besides he gets 20% more amount per kg from wholesaler and sells 10% less per kg to customer. What is the overall profit percentage?
- (a) 80%
(b) 60%
(c) 70%
(d) none of these
- 38** A dishonest dealer purchases goods at 20% discount of the cost price of Rs x and also cheats his wholesaler by getting 20% extra through false weighing, per kg. Then, he marks up his goods by 80% of x , but he gives a discount of 25% besides he cheats his customer by weighing 10% less than the required. What is his overall profit percentage?
- (a) 125%
(b) 100%
(c) 98.66%
(d) 120%
- 39** Anjali, Bhoomika and Chawla went to market to purchase the rings whose costs were same. But each ring was available with two successive discounts. Anjali availed two successive discounts of 5% and 20%. Bhoomika availed two successive discounts 10% and 15% while Chawla availed two successive discounts of 12% and 13%. Who gets the maximum possible discount?
- (a) Anjali
(b) Bhoomika
(c) Chawla
(d) all of these

- 40** An egg seller sells his eggs only in the packs of 3 eggs, 6 eggs, 9 eggs, 12 eggs etc., but the rate is not necessarily uniform. One day Raju (which is not the same egg seller) purchased at the rate of 3 eggs for a rupee and the next hour he purchased equal number of eggs at the rate of 6 eggs for a rupee. Next day he sold all the eggs at the rate of 9 eggs for ₹ 2. What is his percentage profit or loss?
 (a) 10% loss (b) 11.11% loss
 (c) 3% loss (d) 2.5% profit
- 41** Virendra and Gurindra purchased one camera each at the same prices. Later on Amrendra purchased both cameras at equal prices from Virendra and Gurindra. But the profit percentage of Virendra was P while the same of Gurindra was Q , since Gurindra calculated his profit on the selling price. Thus $Q = 41\frac{2}{3}\%$ of P . If Amrendra sells one of the camera to Dholakiya at $P\%$ profit, then what is the cost price for Dholakiya, while Amrendra purchased each of the camera at ₹ 240?
 (a) ₹ 676 (b) ₹ 500 (c) ₹ 576 (d) none
- 42** A merchant earns 25% profit in general. Once his 25% consignment was abducted forever by some goondas. Trying to compensate his loss he sold the rest amount by increasing his selling price by 20%. What is the new percentage profit or loss?
 (a) 10% loss (b) 12.5% loss
 (c) 12.5% profit (d) 11.11% loss
- 43** A milkman purchases 10 litres of milk at ₹ 7 per litre and forms a mixture by adding freely available water which constitutes 16.66% of the mixture. Later on he replaced the mixture by some freely available water and thus the ratio of milk is to water is 2 : 1. He then sold the new mixture at cost price of milk and replaced amount of mixture at twice the cost of milk, then what is the profit percentage?
 (a) 68%
 (b) 34%
 (c) 40%
 (d) none of the above
- 44** Profit on selling 10 candles equals selling price of 3 bulbs. While loss on selling 10 bulbs equals selling price of 4 candles. Also profit percentage equals to the loss percentage and cost of a candle is half of the cost of a bulb. What is the ratio of selling price of candle to the selling price of a bulb?
 (a) 5 : 4 (b) 3 : 2
 (c) 4 : 5 (d) 3 : 4
- 45** Cost price of two motorcycles is same. One is sold at a profit of 15% and the other for ₹ 4800 more than the first. If the net profit is 20%. Find the cost price of each motorcycle :
 (a) ₹ 48000 (b) ₹ 52000
 (c) ₹ 36000 (d) ₹ 42500

Answers

✓ Introductory Exercise 6.1

- 1 ₹ 180 and ₹ 225 2 ₹ 9000 3 ₹ 345 4 ₹ $153\frac{1}{3}$ per hundred 5 ₹ 15300, ₹ 17850, gain = $\frac{5}{11}\%$

✓ Introductory Exercise 6.2

1 (d)	2 (c)	3 (d)	4 (b)	5 (c)	6 (b)	7 (c)	8 (d)	9 (c)	10 (c)
11 (c)	12 (b)	13 (c)	14 (b)	15 (a)	16 (d)	17 (c)	18 (b)	19 (c)	

✓ Level 01 Basic Level Exercise

1 (a)	2 (b)	3 (b)	4 (c)	5 (c)	6 (d)	7 (d)	8 (b)	9 (a)	10 (c)
11 (a)	12 (a)	13 (d)	14 (d)	15 (c)	16 (d)	17 (b)	18 (b)	19 (c)	20 (a)
21 (d)	22 (c)	23 (c)	24 (b)	25 (c)	26 (a)	27 (b)	28 (b)	29 (d)	30 (a)
31 (b)	32 (b)	33 (a)	34 (a)	35 (c)	36 (d)	37 (c)	38 (c)	39 (c)	40 (b)
41 (c)	42 (b)	43 (a)	44 (c)	45 (b)	46 (b)	47 (c)	48 (c)	49 (b)	50 (d)
51 (b)	52 (a)	53 (c)	54 (b)	55 (a)	56 (c)	57 (c)	58 (c)	59 (a)	60 (a)
61 (b)	62 (c)	63 (b)	64 (c)	65 (a)	66 (b)	67 (c)	68 (b)	69 (a)	70 (d)

✓ Level 02 Higher Level Exercise

1 (b)	2 (c)	3 (c)	4 (a)	5 (d)	6 (d)	7 (a)	8 (a)	9 (c)	10 (a)
11 (c)	12 (c)	13 (b)	14 (c)	15 (c)	16 (d)	17 (a)	18 (b)	19 (b)	20 (b)
21 (d)	22 (a)	23 (b)	24 (b)	25 (a)	26 (b)	27 (a)	28 (c)	29 (b)	30 (b)
31 (c)	32 (b)	33 (c)	34 (b)	35 (c)	36 (a)	37 (a)	38 (a)	39 (a)	40 (b)
41 (c)	42 (c)	43 (a)	44 (b)	45 (a)					

Hints & Solutions

✓ Level 01 Basic Level Exercise

1		Abhishek	Bhanu
	Discount	2800	2400 + 640 = 3040

The difference in selling price is same as difference in discount which is ₹ 240 = (3040 - 2800)

- 2 ∴ 10 % of $x = 15\%$ of y , where $x + y = 30000$

$$\Rightarrow \frac{x}{y} = \frac{3k}{2k}$$

Hence, the difference = $k = 6000$

- 3 Let the cost price of 1 litre pure milk be ₹ 1, then
 $\left\{ \begin{array}{l} 6 \text{ litres (milk)} \rightarrow \text{CP} = ₹ 6 \\ 2 \text{ litres (water)} \rightarrow \text{CP} = ₹ 0 \end{array} \right\} \rightarrow \text{CP} = ₹ 6 \text{ only.}$

and 8 litre mixture $\rightarrow \text{SP} \rightarrow 8 \times 2 = ₹ 16$

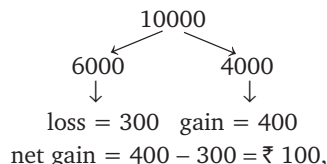
$$\text{Profit} = \frac{16 - 6}{6} \times 100 = \frac{1000}{6} = 166.66\%$$

- 4 SP of 60% goods = $0.6x \times 0.95 = 0.57x$
 SP of 40% goods = $0.4x \times 1.1 = 0.44x$ } Total SP = $1.01x$

$$\text{Profit} = 0.01x = 100$$

$$\Rightarrow x = 10000$$

☞ Alternatively From option (c)



Hence, option (c) is correct.

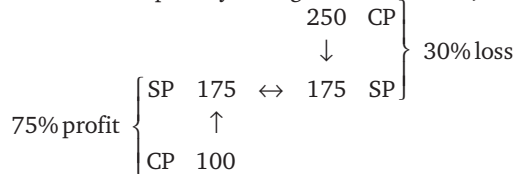
- 5 Let the MP of 1 kg tea be ₹ 1, then CP of 20 kg with discount = $20 \times 0.9 = ₹ 18$

Also 1 kg tea is free. So, the retailer gets tea worth ₹ 21 by paying ₹ 18 only.

$$\begin{aligned} \text{Profit \%} &= \frac{\text{goods left}}{\text{goods sold}} \times 100 \\ &= \frac{21 - 18}{18} \times 100 = 16.66\% \end{aligned}$$

(Since, the retailer earns ₹ 3 on each ₹ 18)

- 6 Let the CP of profit yielding article be ₹ 100, then



$$\text{Total CP} = 350, \quad \text{Total SP} = 350$$

So, there is no profit no gain.

☞ Alternatively $1.75x = 0.7y$

$$\Rightarrow \frac{x}{y} = \frac{2}{5}$$

$$\begin{array}{l} \text{Again} \quad \left[2 \times 1.75 = 3.5 \right] \\ \text{and} \quad \left[5 \times 0.7 = 3.5 \right] \end{array}$$

$$\text{Total cost price} = 2 + 5 = ₹ 7$$

$$\text{Total selling price} = 3.5 + 3.5 = ₹ 7$$

Hence, no loss no gain.

- 7
- | | CP | SP | MP |
|------------|-----|-----|-----|
| Initially | 100 | 140 | × |
| New prices | 100 | 112 | 140 |

$$\begin{array}{l} \text{profit} \\ = 12\% \end{array} \quad \begin{array}{l} \text{discount} \\ = 20\% \end{array}$$

- 8

$$\frac{\text{CP}}{\text{SP}} = \frac{2}{3}$$

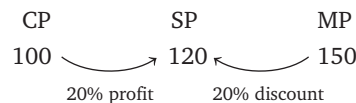
$$\therefore \text{profit (\%)} = \frac{1}{2} \times 100 = 50\%$$

☞ Alternatively $\text{CP} = \frac{100}{18} = 5.55 \text{ paise}$

$$\text{SP} = \frac{100}{12} = 8.33 \text{ paise}$$

$$\text{Profit} = \frac{8.33 - 5.55}{5.55} \times 100 = 50\%$$

- 9

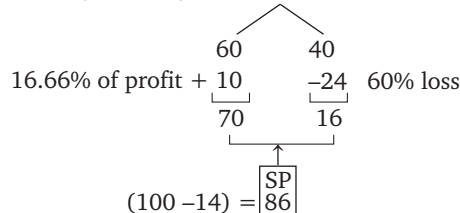


But it can be directly solved as $\text{SP} = 1.2 \text{ CP} = ₹ 2304$

🔔 NOTE There is no role of discount.

- 10 🔔 NOTE There is no role of cost price of article (₹ 123684)

(Consider) $\text{CP} \rightarrow 100$



Since, the overall % loss = 14%.

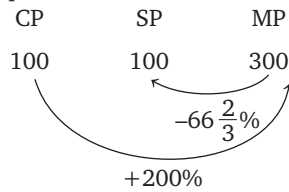
Thus, option (c) is correct.

☞ Alternatively $0.6x + k \times 0.4x = 0.86x$

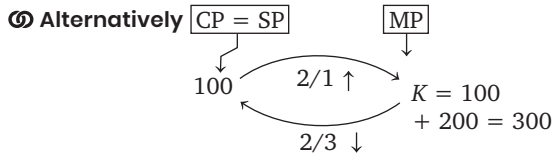
$$\Rightarrow 0.4kx = 0.16x \Rightarrow k = 0.4$$

Therefore, loss = $1 - 0.4 = 0.6$ i.e., 60%

11 Go through options



NOTE When there is no loss, $CP = SP$.



From percentage change graphic :

$$\begin{array}{cc} \text{Decrease} & \text{Increase} \\ \frac{2}{3} \downarrow & \frac{2}{3-2} = \frac{2}{1} \uparrow \\ = 66.66\% & = 200\% \end{array}$$

So, when $CP = 100$, markup % = 200, $MP = 300$

$$\text{Discount} = 66 \frac{2}{3}\% = 200, SP = 100 = CP$$

Alternatively $(CP = SP) = \frac{1}{3} MP$

$$\Rightarrow (CP = SP) : MP = 1 : 3$$

12 $CP : MP = 2x : 3x$

$$\Rightarrow \text{Profit} = x$$

$$(\%) \text{ profit} : (\%) \text{ discount} = 3 : 2$$

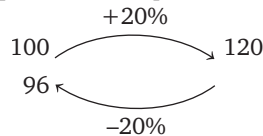
$$\text{Let } CP = 200, SP = 300$$

$$\text{But } \frac{3x}{100} \times 200 + \frac{2x}{100} \times 300 = 100$$

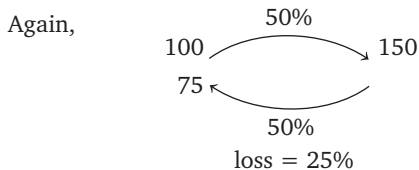
$$\Rightarrow x = 8.33\% \\ \text{Discount } 2x = 16.66\%$$

13 It is dependent upon the markup (%) or discount (%)

Since,



loss = 4%



loss = 25%

Hence, we can't determine.

14 $CP \rightarrow \text{Profit} (\%) \rightarrow MP$

$$\rightarrow 100 \rightarrow x\% \rightarrow 100 + \frac{100x}{100} = (100 + x)$$

Again, $MP \rightarrow \text{discount} \rightarrow SP$

$$\rightarrow (100 + x) \rightarrow x\% \rightarrow (100 + x) - (100 + x) \frac{x}{100}$$

$$= (100 + x) \left(1 - \frac{x}{100} \right)$$

$$\text{Now, } \frac{(100 + x)}{(100 + x) \left(1 - \frac{x}{100} \right)} = \frac{2}{1}$$

$$\Rightarrow \left(1 - \frac{x}{100} \right) = \frac{1}{2} \Rightarrow x = 50\%$$

This can be observed in the solution of the previous problem.

15 Go through option

$$180 \times 12 \times 1.2 + 180 \times 8 \times 1.1 = 180 [14.4 + 8.8] \\ = 180 (23.2) = 4176$$

$$\text{and } 180 \times 20 \times 1.15 = 4140$$

$$\text{Therefore, loss} = 4176 - 4140 = 36$$

Hence, option (c) is correct.

16 Gain = 30000

$$\text{loss} = 20000$$

$$\text{Net gain} = 10000 \text{ over } ₹ 4 \text{ lakh}$$

$$\text{Hence, profit} = \frac{10000}{400000} \times 100 = 2.5\%$$

17 Let the $MP = ₹ 1$ per kg, then

Weight	MP	Rate
100	100	1
96	$\frac{80}{SP}$	$\frac{80}{96}$

$$\text{Effective discount} = 1 - \frac{80}{96} = \frac{16}{96}$$

$$\% \text{ discount} = \frac{16}{96} \times 100 = 16.66\%$$

18 CP of $A + B + C = 2x \times y + 5x \times 2y + 2x \times 4y = 20xy$

$$\text{Profit of } A = 0.2xy$$

$$\text{Profit of } B = 2xy$$

$$\text{Profit of } C = 2xy$$

$$\text{Total profit} = 4.2xy$$

$$\text{Profit \%} = \frac{4.2xy}{20xy} \times 100 = 21\%$$

19 Initially

CP	Profit	SP	MP
100	x	$(100 + x)$	133.33

After change	100	2x	$(100 + x) \frac{7}{6}$

$$\text{Now, since } (100 + x) \frac{7}{6} - 100 = 2x$$

$$\Rightarrow x = 20\%$$

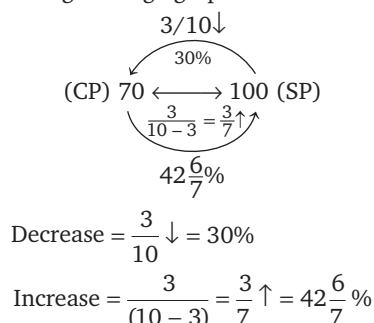
CP	Profit	SP	MP
100	20	120	133.33

So,	300	60	360	400

Again,	300	120	420

So, the increased selling price = ₹ 420

- 20 From percentage change graphic



- 21 Let the price be x , then

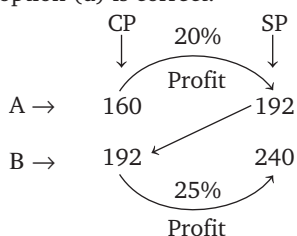
$$x \times 0.8 \times 0.9 = 468$$

$$\Rightarrow x = 650, \text{ therefore marked price} = ₹ 650$$

☞ **Alternatively** $650 \times 0.8 \times 0.2 = 468$

Hence, option (d) is correct.

- 22



- 23 Reduced price = $100 \times 0.95 \times 0.90 \times 0.80 = 68.40$

$$\therefore \text{Single discount} = (100 - 68.4)\% = 31.6\%$$

- 24 $CP = \frac{10}{12}$, $SP = \frac{12}{10}$

$$\text{Profit (\%)} = \frac{\frac{12}{10} - \frac{10}{12}}{\frac{10}{12}} \times 100 = 44\%$$

- 25 Let the cost price of one gram be ₹ 1, then the markup price be ₹ 1.2 per g. Now, he sells 1000 g which seems to be 1250 g so he charges to a customer $1250 \times 1.2 = ₹ 1500$ for 1000 g (or ₹ 1000)

$$\text{Thus, his profit \%} = \frac{1500 - 1000}{1000} \times 100 = 50\%$$

- 26 Let the CP be ₹ x , then SP be $0.96x$

$$\therefore 0.96x = 240 \Rightarrow x = 250$$

$$\text{Now, the new SP} = 250 \times 1.1 = 275$$

- 27
- | | | |
|-----|----------------|--------|
| CP | SP | MP |
| x | $112.5x = 225$ | 300 |
| | | (-25%) |

$$\therefore x = 200, \text{ hence the cost price be ₹ 200.}$$

☞ **NOTE** It can also be solved by using option.

First of all find the SP by decreasing MP by 25%, then this SP will be equal to 112.5% (12.5% is the profit) of the cost price, so the CP can be find as given above.

- 28
- | | | | | |
|-----|-------------------------|-------------------|------------------------|-------------------|
| CP | $\xrightarrow{(-10\%)}$ | (SP) ₁ | $\xrightarrow{(-5\%)}$ | (SP) ₂ |
| 200 | | 180 | | 171 |

- 29 The CP of profitable cow = $\frac{9900}{1.1} = 9000$

$$\text{and profit} = ₹ 900$$

$$\text{The CP of loss yielding cow} = \frac{9900}{0.8} = 12375$$

$$\text{and loss} = ₹ 2475$$

$$\text{So, the net loss} = 2475 - 900 = 1575$$

- 30 The best way is to go through option. Consider option (a)

15000	
$\left(\frac{2}{3}\right)$	$\left(\frac{1}{3}\right)$
10000	5000
(5% profit) ↓	↓ 2% loss
500	100
Net gain = ₹ 400 (= 500 - 100)	

Hence, option (a) is correct.

☞ **Alternatively** $\left[\left(1.05 \times \frac{2x}{3} + 0.98 \times \frac{x}{3} \right) - x \right] = 400$

$$\Rightarrow x = 15000$$

- 31 Profit (%) = $\frac{\text{goods left}}{\text{goods sold}} \times 100 = \frac{50}{450} \times 100 = 11\frac{1}{9}\%$

☞ **Alternatively** Suppose CP of 1 g is ₹ 1, then he sells goods worth ₹ 450 only and charges ₹ 500. So, the profit is ₹ 50 over the sale price of ₹ 450.

- 32 Go through options. Consider option (b)

$$200 \times 3.25 - 200 \times 3 = ₹ 50$$

$$= (30 + 20)$$

Hence, option (b) is correct.

☞ **Alternatively** ₹ 0.25 is the difference on 1 banana

$$\therefore ₹ 1 \text{ is the difference on } \frac{1}{0.25} = 4 \text{ bananas}$$

$$\therefore ₹ 50 \text{ is the difference on } 4 \times 50 = 200 \text{ bananas}$$

- 33 By percentage change (or fraction change) graphic

increase	decrease
$30\% = \frac{3}{10} \uparrow$	$\frac{3}{13} \downarrow$

[since, product i.e., price is constant]

It means now, $\frac{3}{13}$ times less eggs are available which is equal to 3 eggs.

$$\text{i.e., } \frac{3x}{13} = 3$$

$$\Rightarrow x = 13 \text{ eggs (initially)}$$

$$\text{Now, available eggs on the same price} = 13 - 3 = 10$$

$$\text{Thus, the new price} = \frac{910}{10} = 91 \text{ paise}$$

☞ **Alternatively** Go through options.

	New or changed	No. of eggs	Rate = total prices
		$\begin{pmatrix} 10 \\ 13 \end{pmatrix} \xrightarrow{(+3\%)}$	$\begin{pmatrix} 91 \\ 70 \end{pmatrix} \xrightarrow{(+30\%)} = 910 \text{ paise}$
Initial →			

Therefore, option (a) is correct.

- 34 $SP = \frac{100}{12}$ paise
 $CP = x$, $SP = 0.8x = \frac{100}{12}$
 $\Rightarrow x = \frac{100}{9.6}$ paise
 Therefore, $CP = \frac{100}{9.6}$ paise
 Thus, the new SP (with 20% profit)
 $= \frac{100}{9.6} \times 1.2 = \frac{100}{8}$ paise
 Hence, 8 apples can be purchased for ₹ 1 to gain 20%.
- 35 $10000 \xrightarrow{-10\%} 9000 \xrightarrow{-20\%} 7200 \xrightarrow{+10\%} 7920$
 $\xrightarrow{+10\%} 8712$
- 36 $1425 - 1353 = 72 = 6\%$ of CP
 $\Rightarrow CP = 1200$
- 37 $CP = \frac{405}{0.9} = 450$
 New SP = ₹ 465
 Profit (%) = $\frac{465 - 450}{450} \times 100 = 3.33\%$
- 38 $\left[((x \times 1.1) \times 1.1) \times \frac{1125}{1100} \right] = 990 \Rightarrow x = 800$
Alternatively Go through options or by the reverse process.
- 39 Best way is to go through option
Alternatively $P \times 0.9 \times 0.75 = C \times 0.85 \times 0.80$
 $\Rightarrow \frac{P}{C} = \frac{136}{135}$
- 40 Let the CP be x , then
 $0.04x = 54 = (1080 - 1026)$
 $\Rightarrow x = \frac{54}{0.04} = 1350$ (CP)
 $CP = 1350$
 $SP = 1350 \times 1.04 = 1404$
- 41 $SP - CP = 175 = 14\%$ of CP
 $\Rightarrow CP = 1250$
 $\therefore SP = 1250 + 175 = 1425$
- 42 $CP = 100$, then
 Tag price = 80
 SP, after discount = 75
 Total loss = 25
 $\therefore$ ₹ 25 are being less at CP of ₹ 100
 $\therefore$ ₹ 37.5 are being less at CP of ₹ $\frac{100}{25} \times 37.5 = 150$
 Hence, $SP = 150 - 37.5 = 112.5$
Alternatively $112.5 + 37.5 = 150 = CP$
 Again, $150 \times 0.8 \times 0.9375 = 112.5 = SP$
 Hence, option (b) is correct.

- 43 $CP + 4k = SP_1$ (gives profit)
 $CP - 3k = SP_2$ (gives loss)
 Since, loss (3k) is 25% less than profit (4k)
 $\therefore SP_1 - SP_2 = 7k = 836 - 703 = 133 \Rightarrow k = 19$
 $\therefore CP = SP_1 - 4k = SP_2 + 3k = 760$
 Therefore, required SP = $760 \times 1.2 = 912$
- 44 You can go through options.
 $CP = \frac{100}{6} = 16.66$ paise
 $SP = \frac{100}{5} = 20$ paise
 Profit (%) = $\frac{20 - 16.66}{16.66} \times 100 = \frac{3.33}{16.66} \times 100$
 $= \frac{1}{5} \times 100 = 20\%$
 Hence, option (c) is correct.
- Alternatively** $CP = \frac{100}{6}$, $SP = \frac{100}{6} \times 1.2 = \frac{100}{5}$
 Hence, he should sell 5 toffees for ₹ 1 (= 100 paise)
- 45 $CP \quad MP$
 $800 = 0.8x \quad x$
 20% discount
 $\Rightarrow x = ₹ 1000 = MP$
 $SP \text{ at Hazratganj} = ₹ 800$
 $SP \text{ at Bhootnath} = 1000 \times 0.85 = 850$
 So, she requires ₹ 50 more to purchase the same calculator at Bhootnath.
- 46 Let the CP be ₹ 1 per g
 But he weighs 900 g for every 1000 g
 $\therefore$ Value of goods sold = 900
 Now, let the markup be $x\%$
 $\therefore MP = 1000 + \frac{1000x}{100} = (1000 + 10x)$
 But, since $MP = SP$, $\therefore SP = (1000 + 10x)$
 Hence, profit (%) = $\frac{(1000 + 10x) - 900}{900} \times 100 = 20 \Rightarrow x = 8$
 Thus, the markup = 8%.
- 47 Loss % = $\left(\frac{\text{common gain or loss}}{10} \right)^2 \% = \left(\frac{20}{10} \right)^2 \% = 4\%$
- 48 Since, loss = 4%
 Now, assume total CP of both articles be x , then
 $SP = 0.96x = 400 \Rightarrow x = \frac{400}{0.96} = CP$
 $\therefore$ loss = 4% of CP
 $\frac{4}{100} \times \frac{400}{0.96} = ₹ 16.66$

49
$$\frac{CP}{SP} = \frac{29x}{19x}$$

$$\text{Loss \%} = \frac{29x - 19x}{29x} \times 100 = 34.48\%$$

 (To understand the concept assume CP of each article ₹ 29 and SP of each article = ₹ 19)

50
$$\frac{CP}{SP} = \frac{13}{23}, \text{ Profit (\%)} = \frac{23 - 13}{13} \times 100 = \frac{10}{13} \times 100 = 76.92\%$$

51
$$\frac{CP}{SP} = \frac{28}{34} = \frac{14}{17}$$

52 MP = ₹ 1.2, if CP = ₹ 1
 But he gives the cloth worth ₹ 0.8 instead of ₹ 1.
 Therefore, profit % = $\frac{1.2 - 0.8}{0.8} \times 100 = 50\%$

53
$$CP = ₹ 80 = \left(\frac{3200}{40} \right)$$

 Now, SP of 40 books = CP of 40 books + SP of 8 books
 $\Rightarrow$ SP of 32 books = 3200
 $\Rightarrow$ SP of 1 book = ₹ 100
 $\therefore$ Required SP of 1 dozen books = ₹ 1200

54 Profit of A = $1800 \times \frac{20}{100} = ₹ 360$

$$CP \text{ of } B = \frac{1800}{1.2} = 1500$$

 $\therefore$ Profit of B = ₹ 300 = (1800 - 1500)
 $\therefore$ Difference in profit of A and B = ₹ 60

55
$$CP \text{ of } A = \frac{1818}{0.9} = 2020$$

$$CP \text{ of } B = \frac{1818}{1.01} = 1800$$

$$\frac{CP \text{ of } A}{CP \text{ of } B} = \frac{2020}{1800} = \frac{101}{90}$$

56 $103.33 CP - 0.95 CP = 65 \Rightarrow CP = ₹ 780$

$$\text{Profit (\%)} = \frac{936 - 780}{780} \times 100 = 20\%$$

57 Best way is to assume some proper values, then verify it.

58 Let us assume initially he has 100 l of milk
 So, after 10% of water addition new amount of mixture = 110 l
 Again adding 10% more, then the final amount of mixture = 121 l

$$\text{Profit \%} = \frac{21}{100} \times 100 = 21\%$$

59
$$SP = x + \frac{x \times x}{100} = 96\% \Rightarrow x = 60$$

 New,
$$SP = 60 + \left(\frac{60 \times 120}{100} \right) = ₹ 132$$

60 $K \rightarrow 1 \text{ litre, price of } K \rightarrow 1$ $P \rightarrow 4 \text{ litre, price of } P \rightarrow 2$
 Price of total (new) mixture = $1 \times 1 + 4 \times 2 = ₹ 9$
 Price of pure petrol of same quantity = $5 \times 2 = ₹ 10$

$$\text{Percentage profit} = \frac{1}{9} \times 100 = 11 \frac{1}{9} \%$$

61 Amount purchased = 1100 g
 Amount sold = 900 g

$$\text{Profit \%} = \frac{200}{900} \times 100 = 22 \frac{2}{9} \%$$

62 If CP be ₹ 1 per g, then he pays ₹ 1200 for 1000 g
 Again, he obtains ₹ 1200 for 1000 g (by selling at 20% profit). Thus, there is no loss no gain.

63
$$\begin{array}{cc} CP & SP \\ 240 & 280 \end{array}$$

$$\left(-14 \frac{2}{7} \% \right)$$

$$\text{Actual profit (\%)} = \frac{40}{240} \times 100 = 16.66\%$$

NOTE Here you can apply base change concept.

$$\begin{array}{ccc} & \text{increase} & \text{decrease} \\ \uparrow 16.66\% & & 14 \frac{2}{7} \% \downarrow \\ \frac{1}{6} \uparrow & \longleftarrow & \frac{1}{7} \downarrow \end{array}$$

64 Go through options : $100 \xrightarrow{80\%} 180$

$$\text{New SP} = \frac{180}{3} = 60$$

Percentage loss = 40% (100 - 60)
 Hence, (c) is correct choice.

65
$$\begin{array}{ccc} CP & \text{Profit} & SP \\ 100 & 20 & 120 \\ x & 25 & 150 \end{array} \left. \begin{array}{l} \uparrow \\ \downarrow \end{array} \right\} +25\%$$

$$x \times 1.25 = 150 \Rightarrow x = 120$$

$$\% \text{ change} = \frac{120 - 100}{100} \times 100 = 20\%$$

66 $6.66\% \text{ of MP} = 25 \Rightarrow MP = 375$
 $\therefore$ $SP = MP - 25 = 350$

67
$$\text{Profit \%} = \frac{5}{95} \times 100 = 5.26\%$$

68 Recall it is based on inverse proportion or product constancy concept.

$$\begin{array}{cc} \text{Reduction in price} & \text{increase in amount} \\ 25\% \left(\frac{1}{4} \right) \downarrow & \uparrow \left(\frac{1}{3} \right) 33.33\% = 4 \text{ oranges} \end{array}$$

It means original number of oranges = $4 \times 3 = 12$
 $\therefore$ Original price of oranges = $\frac{16}{12} = ₹ 1.33$

69
$$\begin{array}{cc} \text{Reduction in price} & \text{increase in amount} \\ 20\% \left(\frac{1}{5} \right) \downarrow & \frac{1}{4} \uparrow (25\%) = 6 \text{ kg} \end{array}$$

It means original amount of sugar needed = $6 \times 4 = 24 \text{ kg}$
 $\therefore$ Original price of the sugar = $\frac{240}{24} = ₹ 10 \text{ per kg.}$

70 Just a sitter. Don't be bogged down. Its simple i.e., 25%, nothing else, which is very obvious.

Level 02 Higher Level Exercise

- 1 Just a sitter, but a logical problem.

$$\text{CP of 7 bikes} = 67500 + 232500 = 300000$$

Now, since we require 17.5% profit, so SP

$$= 300000 \times \frac{117.5}{100} = ₹ 352500$$

- 2 CP = 100, SP (with tax) = 120

$$\text{New SP} = 100 - 5 = 95$$

$$\therefore \text{Effective discount} = 120 - 95 = 25$$

So, at SP of 95 $\rightarrow$ discount = 25

$$\text{and at SP of 3325} \rightarrow \text{discount} = \frac{25}{95} \times 3325 = 875$$

- 3 Let the CP of a bicycle = ₹ 100

Now, since profit 140%

$$\therefore \text{SP} = 240$$

Now, 7 bicycles are being sold instead if 1 bicycle, but the sale price of new bicycle = ₹ 120

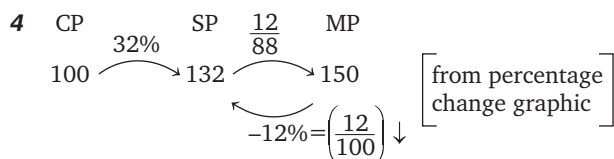
Therefore, total sale price of new sale of bicycles

$$= 7 \times 120 = 840$$

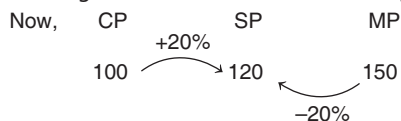
and the CP = $7 \times 100 = 700$

So, the new profit = $840 - 700 = ₹ 140$

Since, the initial profit is same as the new, so there is no increase in percentage.



NOTE In this case first of all find the SP, after adding profit percentage to CP, then find the MP through SP.



Here, first of all we subtract discount from the MP, then the resultant value will be SP.

- 5
- | | |
|-----------|------------|
| Linc pens | Cello pens |
| CP : SP | CP : SP |
| 37 : 50 | 37 : 24 |
- $$\text{Profit \%} = \frac{13}{37} \times 100 \text{ and Loss\%} = \frac{13}{37} \times 100$$

Since, Profit = loss

Hence, option (d) is correct.

- 6
- SP $\frac{1}{7} \left(\frac{8}{7} \right) \frac{1}{8} : \frac{1}{8} \left(\frac{9}{8} \right) \frac{1}{9} : \frac{1}{4} \left(\frac{5}{4} \right) \frac{1}{5}$
- Since, $14.28\% = \frac{1}{7}$

So, the ratio of profit percentage of

A	B	C	
8	: 7	: 14	(given)
$\downarrow$	$\downarrow$	$\downarrow$	
$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{4}$	

Thus, the ratio of CP of A : B : C

$$7 : 8 : 4$$

$$\text{Therefore, \% profit} = \frac{(8 + 9 + 5) - (7 + 8 + 4)}{(7 + 8 + 4)} \times 100$$

$$= \frac{3}{19} \times 100 = 15.78\%$$

- 7
- | | | | | |
|------------------|-----|-----|-----|------------------|
| | A | B | C | M |
| CP $\rightarrow$ | 100 | 120 | 132 | (120 + 12) = 132 |
| SP $\rightarrow$ | 120 | 132 | 120 | 143 |
| CP $\rightarrow$ | 143 | | | |

$$\text{Loss of A} = 143 - 120 = 23$$

$$\% \text{ loss of A} = \frac{23}{100} \times 100 = 23\%$$

- 8 Total wages = no. of employees $\times$ wage per employee

$$60xy = 3x \times 20y$$

$$54xy = 2x \times 27y$$

$$\text{Profit (\%)} = \frac{60 - 54}{60} \times 100 = 10\%$$

- 9 If I had ₹ 100. Discount = 25 = cost of my sister's watch

then cost of my own watch = 75. Thus, the ratio of cost of my own watch to that of my sister's watch = 3 : 1

- 10 Ratio of profit of A : B (excluding commission of A)

$$= 3 : 5 \quad (\because 54 : 90 = 3 : 5)$$

Now, the share of profit of B = $3600 - 1800 = ₹ 1800$

So, the share of profit of A (excluding commission) = ₹ 1080

So, the commission of A = $1800 - 1080 = 720$

Therefore, the required percentage = $\frac{720}{3600} \times 100 = 20\%$

- 11 Profit % = $\frac{25}{100} = \frac{120 + k}{880} \text{ (Profit)} \Rightarrow k = 100$

Therefore, net profit % = $\frac{100}{1000} \times 100 = 10\%$

- 12 SP = $\frac{12}{11}$ of CP

$$48 = \frac{12}{11} \text{ of CP} \Rightarrow \text{CP} = 44$$

Now, by alligation

48	$\swarrow$	$\searrow$	k
	44		
12	$\swarrow$	$\searrow$	3
4			1

$\therefore k = 28$

Thus, the price of speed brand is ₹ 28/litre.

13		A	B	C
Investment		3x	4x	5x
Rate of return		6y%	5y%	4y%
Return		$\frac{18xy}{100}$	$\frac{20xy}{100}$	$\frac{20xy}{100}$

$$\text{Total} = (18 + 20 + 20) = \frac{58xy}{100}$$

$$B's \text{ earnings} - A's \text{ earnings} = \frac{2xy}{100} = 250$$

$$\text{Total earning} = \frac{58xy}{100} = 7250$$

14	MP	after first discount	after second discount
	100	90	85.5

$$\text{So, the net discount} = 100 - 85.5 = 14.5\%$$

15	MP	CP	Total CP
	100	80	72
	- 20%	- 10%	+ 10%

$$SP = 125\% \text{ of CP}$$

$$SP = 1.25 \times 79.2$$

$$SP = 99$$

$$\text{So, initially marked price} = 100 \Rightarrow 8,00,000$$

$$\text{Final sale price} = 99 \Rightarrow 7,92,000$$

$$\text{16 Price of 10 chairs} = 10 \times 200 = 2000$$

$$\text{Price of 12 chairs (without discount)} = 12 \times 200 = 2400$$

$$\text{Price of 12 chairs with discount}$$

$$= 10 \times 200 + 2 \times 80 = 2160$$

$$\text{Therefore, discount} = 2400 - 2160 = 240$$

$$\text{Hence, discount \%} = \frac{240}{2400} \times 100 = 10\%$$

$$\text{17 Amount paid in Ist service} = 100 \text{ (suppose)}$$

$$\text{Amount paid in IInd service} = 90$$

$$\text{Amount paid in IIIrd service} = 81$$

$$\text{Amount paid in IVth service} = 72.9$$

$$\text{Amount paid in Vth service} = 60$$

$$\text{Total amount paid} = \underline{403.9}$$

$$\text{Discount} = 500 - 403.9 = 96.1$$

$$\text{Discount \%} = \frac{96.1}{500} \times 100 = 19.42\%$$

$$\text{18 Consider some proper values and then check out.}$$

19	CP	SP
	3	4

$$\text{Profit on 3 apples} = ₹ 1 \text{ (consider CP} = ₹ 1 \text{)}$$

$$\text{Profit} = 33.33\%$$

$$\text{and discount} = 11.11\%$$

Since,	CP	SP	MP
	3	4	4.5
	(1)	(0.5)	

Profit is double that of discount

$$\text{So, the percentage point difference} = 33.33\% - 11.11\% = 22.22\% \text{ point}$$

$$\text{20 Total cost of 4 cars} = 1 + 2 = 3 \text{ lakh}$$

$$\text{Total SP of 4 cars} = 3 \times 1.5 = 4.5 \text{ lakh}$$

$$\text{SP of 1 car} = 1.2 \text{ lakh}$$

$$\text{SP of rest 3 cars} = 4.5 - 1.2 = 3.3 \text{ lakh}$$

$$\text{Average SP of all the 3 cars} = 1.1 \text{ lakh}$$

$$\text{21 Setup cost} = ₹ 2800$$

$$\text{Paper etc.} = ₹ 1600$$

$$\text{Printing cost} = ₹ 3200$$

$$\text{Total cost} = ₹ 7600$$

$$\text{Total sale price} = 1500 \times 5 = 7500$$

$$\text{Let the amount obtained from advertising is } x, \text{ then}$$

$$(7500 + x) - (7600) = 25\% \text{ of } 7500$$

$$x = 1975$$

$$\text{22 Charge of 1 call in February} = \frac{350}{150} = \frac{7}{3}$$

$$\text{Charge of 1 call in March} = \frac{350 + 50 \times 1.4}{250} = \frac{420}{250} = \frac{42}{25}$$

$$\% \text{ cheapness of a call in March} = \frac{\frac{7}{3} - \frac{42}{25}}{\frac{7}{3}} \times 100 = 28\%$$

$$\text{23 Let the CP be 100 and \% markup be } k\%, \text{ then}$$

$$100 \xrightarrow{k\%} (100 + k) \text{ MP (also expected SP)}$$

$$\text{MP} = 100 + k$$

$$\text{but actual SP} = \frac{100 + k}{2}$$

$$\therefore \frac{\left[\frac{(100 + k)}{2} \right]}{k} = \frac{200}{3 \times 100} = (66.66\%)$$

$$\Rightarrow k = 300$$

$$\text{Therefore, CP MP (initially)}$$

$$100 \quad 400$$

$$\text{Finally SP} = \frac{400}{2} = 200$$

$$\therefore \text{Discount} = \frac{200}{400} \times 100 = 50\%$$

$$\text{Alternatively MP} \rightarrow M$$

$$\text{SP} \rightarrow \frac{M}{2}$$

$$\text{Discount (\%)} = \frac{M/2}{M} \times 100 = 50\%$$

$$\text{24 Let the CP and SP of 1 g} = ₹ 1, \text{ then}$$

$$\text{He spends ₹ 2000 and purchase 2200 g}$$

$$\text{and he charges ₹ 2000 and sells 1800 g}$$

$$\begin{aligned} \text{Profit (\%)} &= \frac{\text{goods left}}{\text{goods sold}} \times 100 \\ &= \frac{400}{1800} \times 100 = 22\frac{2}{9}\% \end{aligned}$$

25

Fresh grapes	
Water	Pulp
80%	20%
4	1
Dry grapes	
Water	Pulp
25%	75%
1	3
5 kg	15 kg] out of 20 kg dry grapes
80%	20%
4	1
+ (55 kg)	15 kg] Required proportion of water and pulp
60 kg	

Thus, to make dry grapes similar to the fresh grapes, Akram requires 55 kg water with 20 kg of dry grapes.

$$\text{So, the profit (\%)} = \frac{55}{20} \times 100 = 275\%$$

26 Let the total profit be 100

$$\text{Amount left after donation} = 80$$

$$\text{Amount left after reinvestment} = 20$$

$$\text{Now, } \frac{5x}{8} - \frac{3x}{8} = 1200$$

Where, x is the amount left after reinvestment

$$\frac{2x}{8} = 1200 \Rightarrow x = 4800$$

$$\text{Therefore, total profit} = 4800 \times 5 = 24000$$

☞ **Alternatively** Go through options.

27 Total cost = ₹ 50,000

$$\text{Total sale price (or revenue)}$$

$$= 2000 \times 9 + 6000 \times 10 = 78,000$$

$$\text{Profit (\%)} = \frac{28000}{50000} \times 100 = 56\%$$

28 The maximum possible profit

$$= \text{maximum possible difference in SP and CP.}$$

It means SP be maximum and CP be minimum

$$\text{CP}_{(\min)} = ₹ 399, 19m = 399$$

where m is an integer.

$$\text{Again, SP}_{(\max)} = ₹ 697, \text{ which is very close to } 699$$

Here, $697 = 17k$; k is a positive integer.

$$\text{So, the maximum profit} = 697 - 399 = ₹ 298$$

29 Total cost price = $1000 \times 1.2 = ₹ 1200$

$$\text{Expected selling price} = 700 \times x = 1200 \times 1.1666 = 1400$$

$$\Rightarrow x = ₹ 2 \text{ per toy}$$

$$\text{Now, the real selling price} = 750 \times 2 = ₹ 1500$$

$$\therefore \text{Profit} = ₹ 300 (= 1500 - 1200)$$

$$\therefore \text{Profit \%} = \frac{300}{1200} \times 100 = 25\%$$

30 Chaudhary's profit = 10%

$$\text{Anupam's profit} = 20\%$$

$$\text{Bhargava's profit} = 25\%$$

$$\text{i.e., } \begin{matrix} A & 20\% & B & 25\% & C & 10\% & D \\ 100 & \rightarrow & 120 & \rightarrow & 150 & \rightarrow & 165 \end{matrix}$$

$$\text{Now, } \begin{matrix} B & D \\ 120 & 165 \\ 17 \text{ times} & 17 \text{ times} \\ \downarrow & \downarrow \\ 2040 & 2805 \end{matrix}$$

31 From the statements it is clear that he purchases 119 g instead of 100 g and he sells 85 g instead of 100 g. Therefore, in this whole transaction he saves $19 + 15 = 34$ g

$$\text{Thus the profit} = \frac{34}{85} \times 100 = \left(\frac{\text{Goods left}}{\text{Goods sold}} \times 100 \right) = 40\%$$

☞ **NOTE** In this type of question always equate either money or weight for simple solution.

32 You must know that the company is able to deliver only 90% of the manufactured pens. So, let K be the manufacturing price of a pen, then total income (including 25% profit) $= (8000 \times K) \times 1.25$ also this same income is obtained by selling 90% manufactured pens at ₹ 10 which is equal to 7200×10 . Thus, $(8000 \times K) 1.25 = 7200 \times 10$
 $\Rightarrow K = ₹ 7.2$ (90% of 8000 = 7200)

33 Let the number of diaries (produced) be 100 and the cost price of a diary be ₹ 1, then

$$\text{Total cost incurred} = 100 \times 1 = 100$$

$$\text{Total sale price} = 32 \times 0.75 + 60 \times 1.4 = 108$$

$$\text{Therefore profit} = ₹ 8$$

Thus, there is 8% profit

☞ **NOTE** Marked price (i.e., expected) = 40% above the cost price.

34 Let the number of sweets be 100 and the cost price of one piece of sweet = ₹ 1

$$\text{then total cost price} = 100 \times 1 = ₹ 100$$

$$\text{Total sale price} = 40 \times 1.4 + 30 \times 1.2 + 30 \times 1.05 = 123.5$$

$$\therefore \text{profit (\%)} = 23.5\% = (123.5 - 100)$$

35

CP	SP	MP
500	576	900

$$\text{Again SP} = \text{MP} \left[\left(1 - \frac{r}{100} \right)^2 \right] \quad [r \rightarrow \text{rate of discount in \%}]$$

$$\Rightarrow 576 = 900 \left(1 - \frac{r}{100} \right)^2$$

$$\Rightarrow \frac{24}{30} = \left(1 - \frac{r}{100} \right) \Rightarrow r = 20\%$$

$$\text{Again, new SP} = \text{MP} \left(1 + \frac{r}{100} \right) = 900 \left(1 + \frac{20}{100} \right) = 1296$$

$$\begin{aligned}\text{New, profit percentage} &= \frac{SP - CP}{CP} \times 100 \\ &= \frac{1296 - 500}{500} \times 100 = 159.2\%\end{aligned}$$

- 36** Consider actual price of 1 g goods = ₹ 1, then he sells the product equals to ₹ 90 only
(10% less weighing)

Again, MP = ₹ 1.8 and SP = 1.35 for 1 g.

Thus, he gives the goods worth ₹ 90 and charges ₹ 135 after 25% discount. Thus, the profit %

$$= \frac{135 - 90}{90} \times 100 = 50\%$$

- 37** $CP = \frac{100}{120} = \frac{10}{12}$

(Since, he purchases 120 g and pays ₹ 100, by assumption actual CP of 1 g = ₹ 1)

$$SP = \frac{135}{90} = \frac{3}{2} = \frac{18}{12}$$

(Since, actual MP = 180, actual SP = 135, with 25% discount and he sells only 90 g instead of 100 g)

$$\text{profit (\%)} = \frac{18 - \frac{10}{12}}{\frac{10}{12}} \times 100 = 80\%$$

- 38** Let the actual cost price of an article be ₹ 1 (in place of x)

Now, he purchases goods worth ₹ 120 and pays ₹ 80, since 20% discount is allowed.

$$\text{So, the CP} = \frac{80}{120} = \frac{2}{3}$$

Again MP = 180, SP = 135 (since, 25% discount)

Thus, the trader sells goods worth ₹ 90 instead of 100 g and charges ₹ 135. Therefore, the effective SP = $\frac{135}{90} = \frac{3}{2}$

$$\therefore \text{Profit (\%)} = \frac{\frac{3}{2} - \frac{2}{3}}{\frac{2}{3}} \times 100 = 125\%$$

	Anjali	Bhoomika	Chawla
	100	100	100
	↓ - 20%	↓ - 15%	↓ - 12%
24	80	23.5	85
	↓ - 5%	↓ - 10%	↓ - 13%
	76	76.5	76.56

Thus, it is clear from the graphical solution that the maximum discount is availed by Anjali.

NOTE

It does not matter that we first decrease by 20% and then by 5% or *vice-versa*. This concept has been already illustrated in percentage chapter. Try to do it for your concept clarification.

- 40** CP of one egg (in first case) = $\frac{1}{3} = 33.33$ paise

$$\text{CP of one egg (in second case)} = \frac{1}{6} = 16.66 \text{ paise}$$

$$\text{Average CP of one egg} = \frac{(33.33 + 16.66)}{2} = 25 \text{ paise}$$

$$\text{SP of one egg} = \frac{200}{9} \quad (\text{₹ 1} = 100 \text{ paise})$$

$$\begin{aligned}\text{Profit-loss} &= \frac{25 - \frac{200}{9}}{25} \times 100 \quad \left(\text{loss\%} = \frac{CP - SP}{CP} \times 100 \right) \\ &= 11.11\% \text{ loss}\end{aligned}$$

- 41** This question is based on fundamental concept of percentage change.

$$\text{Virendra} = CP_V \xrightarrow{P\%} SP_V$$

$$\text{Gurindra} = CP_G \xrightarrow{Q\%} SP_G$$

Here, $CP_V = CP_G$ and $SP_V = SP_G$

But $P \neq Q$

also, $P\% \text{ of } CP_V = Q\% \text{ of } SP_G$

$$\text{also } Q = 41 \frac{2}{3} \% \text{ of } P = \frac{125}{3} \times \frac{P}{100}$$

$$\text{and } Q = \frac{P}{100 + P} \times 100$$

(From the concept of percentage change)

$$\therefore \frac{P}{100 + P} \times 100 = \frac{125}{3} \times \frac{P}{100} \quad \left(41 \frac{2}{3} = \frac{125}{3} \right)$$

$$\Rightarrow P = 140$$

$$\therefore CP = 100 \text{ when } SP = 240$$

Where $CP = CP_V = CP_G$ and $SP = SP_V = SP_G$

Again SP for Amrindra = $240 + 140\% \text{ of } 240 = 576$

- 42** Let the CP of one article be ₹ 1 then the SP be ₹ 1.25

Again, the new SP be $(1.25) \times 1.2 = 1.5$

Now, if he sell initially 100 articles, then

$$CP = 100 \times 1 = ₹ 100$$

$$SP = 100 \times 1.25 = ₹ 125$$

$$\text{New SP} = 75 \times 1.5 = 112.5$$

(since, 25% articles were abducted)

$$\therefore \text{New profit percentage} = 12.5\%$$

- 43** **NOTE** First of all the price of milk does not matter. You can assume any convenient price. Besides it instead of 10 l of milk you can consider 100 l of milk to avoid calculations in decimal.

Now, since water is 16.66% in the mixture of milk, therefore with 100 l pure milk 20 l water is added. Again note that in replacement method the quantity of mixture does not increase except to the variation in ratio of contents.

Again by replacement formula

$$\frac{80}{120} = \frac{100}{120} \left(1 - \frac{K}{120} \right) \Rightarrow K = 24 \text{ l}$$

Thus, he replaces 24 l of mixture with water.

(Note the required ratio of milk is to water is 2 : 1. It means in 3 l of new mixture, there will be 2 l of water)

Thus, if the price of new mixture be ₹ 1, then the price of replaced mixture be ₹ 2.

Therefore, total SP = $120 \times 1 + 24 \times 2 = 168$

and CP = $100 \times 1 = 100$

∴ Profit % = 68%

44	Candle	Bulb
CP	a	c
SP	b	d

and

$$c = 2a$$

$$\text{Profit} = 10 (b - a) = 3d$$

and

$$\text{Loss} = 10 (c - d) = 4b$$

$$\text{Profit (\%)} = \frac{3d}{10a} \times 100$$

and

$$\text{Loss (\%)} = \frac{4b}{10c} \times 100$$

Again,

$$\frac{3d}{10a} \times 100 = \frac{4b}{10c} \times 100$$

⇒

$$\frac{3d}{a} = \frac{4b}{c} \Rightarrow \frac{3d}{a} = \frac{4b}{2a} \quad (\because c = 2a)$$

⇒

$$\frac{b}{d} = \frac{3}{2}$$

45 Let the CP of each motorcycle be ₹ x , then

$$2 (1.15x) + 4800 = 2(1.2x)$$

⇒

$$0.1x = 4800 \Rightarrow x = 48000$$

CHAPTER 07

CI/ SI/ Instalments

Fundamentally, this chapter is the easiest one. It requires basic understanding of percentage and ratio-proportion along with the knack for simple calculations. If ever any problem is asked from this chapter, it is hardly a tricky one. That's why CAT is not a great fan of this chapter. However, this chapter holds relevance for most of the competitive exams and even for the Data Interpretation section. Nonetheless, Banking exams and SSC CGL exams and recruitment exams ask a significant number of problems from this chapter. In order to score more, we must rely on common sense and logic than that on formulae.

7.1 Important Definitions

Simple interest is nothing but the fix percentage of the principal (invested/borrowed amount of money). Some key words used in the concept of interest.

Principal (P): It is the sum of money deposited/loaned etc. also known as **capital**.

Interest : It is the money paid by borrower, calculated on the basis of **Principal**.

Time (T/n) : This is the duration for which money is lent/borrowed.

Rate of Interest (r/R) : It is the rate at which the interest is charged on principal.

Amount (A) = Principal + Interest

Simple Interest : When the interest is calculated uniformly only on the principal for the given time period.

Compound Interest : In this case for every next period of time the interest is charged on the total previous amount (which is the sum of principal and interest charged on it so far.) *i.e.*, every time we calculate successive increase in the previous amount.

7.2 Simple Interest (SI)

If the Principal (P), Rate of Interest (r) and Time Period (t) are given, we will have

$$\text{Simple Interest (SI)} = \frac{P \times r \times t}{100}$$

$$\text{Amount (A)} = P + \frac{Prt}{100} = P \left(1 + \frac{rt}{100} \right)$$

 **NOTE** Out of the five variables A, SI, P, r, t we can find any one of these, if we have the requisite information about the four variables directly or indirectly individually or jointly.

Chapter Checklist

- Important Definitions
- Simple Interest
- Compound Interest (CI)
- Depreciation
- Calculating the Population
- Instalments
- Important Amounts, Interest Rates and Time Periods
- CAT Test

Conversion of Time Period and rate of Interest

Given ($r\%$)	Given (t)	Required ($r\%$)	Required (t)
$r\%$ annual	t years	$\frac{r}{2}(\%)$ half-yearly	$2t$
$r\%$ annual	t years	$\frac{r}{4}\%$ quarterly	$4t$
$r\%$ annual	t years	$\frac{r}{12}\%$ monthly	$12t$

7.3 Compound Interest (CI)

1. $CI = A - P$
[A is the amount including interest and principal (P) both]

2. Amount (A) $= P \left(1 + \frac{r}{100} \right)^t$

3. When the rate of interest is half-yearly $A = P \left(1 + \frac{(r/2)}{100} \right)^{2t}$

4. When the rate of interest is quarterly $A = P \left(1 + \frac{r/4}{100} \right)^{4t}$

5. Difference between CI and SI for two years $= P \left(\frac{r}{100} \right)^2$

6. Difference between CI and SI for three years

$$= P \left(\frac{r}{100} \right)^2 \left(\frac{r}{100} + 3 \right)$$

7. Difference between CI and SI for n^{th} year

$$= \frac{Pr}{100} \left[\left(1 + \frac{r}{100} \right)^{n-1} - 1 \right]$$

8. For compound interest, if r denotes the rate of interest, the change in amount over the previous year can be calculated or following.

$$\frac{\text{increase in amount in } n^{\text{th}} \text{ year}}{\text{increase in amount in } (n+1)^{\text{th}} \text{ year}} = \frac{100}{(100+r)}$$

Similarly,

$$\frac{\text{decrease in amount in } n^{\text{th}} \text{ year}}{\text{decrease in amount in } (n+1)^{\text{th}} \text{ year}} = \frac{100}{(100-r)}$$

where r is rate of interest.

7.4 Depreciation

It is known that the prices of some articles depreciates in their values over a time period. When the value in terms of currency decreases, we say that the value of the article is

$$\text{depreciating } V_f = V_i \left(1 - \frac{r}{100} \right)^t.$$

V_i = Initial value of the article

V_f = Final (depreciated) value of article r , is the rate of interest by which the price of article decreases over the time period ' t '.

7.5 Calculating the Population

It has been observed that the population of a particular locality/nation etc. increases or decreases successively over its previous value *i.e.*, it increases or decreases as compound interest for the money. Thus we use similar formulae for the calculation of population.

Total population (P) $= P_0 \left(1 + \frac{r}{100} \right)^n$, when population

increases. Total population (P) $= P_0 \left(1 - \frac{r}{100} \right)^n$, when population decreases.

7.6 Instalments

When a borrower paid the sum in parts (*i.e.*, not in a single amount) then we say that he/she is paying in instalments. For example A borrowed ₹ 100 from B and he pays back it to B in several parts *i.e.*, ₹ 20 in 5 times or ₹ 50 in 2 times etc. But the important point is that borrower has to also pay the interest for using the borrowed sum/or purchased article. In general the value of each instalment is kept constant even when the interest charged on each instalment vary for each instalment.

For Simple Interest

Let A be the total amount to be paid, r be the rate of interest, x be the value of each instalment and n be the number of instalments, we have

$$A = \left[x + \left(x + \frac{x \times r \times 1}{100} \right) + \left(x + \frac{x \times r \times 2}{100} \right) + \left(x + \frac{x \times r \times 3}{100} \right) + \dots + \left(x + \frac{x \times r \times n}{100} \right) \right]$$

Let P be the principal amount, we have

$$A = P + \frac{P \times n \times r}{100}$$

Therefore, we have

$$P + \frac{P \times n \times r}{100} = \left[x + \left(x + \frac{x \times r \times 1}{100} \right) + \left(x + \frac{x \times r \times 2}{100} \right) + \left(x + \frac{x \times r \times 3}{100} \right) + \dots + \left(x + \frac{x \times r \times n}{100} \right) \right]$$

For Compound Interest

Let P be the principal amount, r be the rate of interest, x be the value of each instalment and n be the number of instalments, we have

$$P = \left[\frac{x}{\left(1 + \frac{r}{100}\right)} + \frac{x}{\left(1 + \frac{r}{100}\right)^2} + \frac{x}{\left(1 + \frac{r}{100}\right)^3} + \dots + \frac{x}{\left(1 + \frac{r}{100}\right)^n} \right]$$

Let A be the total amount paid back in n instalments, we have

$$A = P \left(1 + \frac{r}{100}\right)^n$$

Therefore, we have

$$P \left(1 + \frac{r}{100}\right)^n = \left[x + x \left(1 + \frac{r}{100}\right) + x \left(1 + \frac{r}{100}\right)^2 + \dots + x \left(1 + \frac{r}{100}\right)^{n-1} \right]$$

Exp. 1) Find the simple interest on ₹ 1000 at 12% per annum for 5 years

Solution $SI = \frac{Prt}{100} = \frac{1000 \times 12 \times 5}{100} = ₹ 600$
{Total amount = $P + SI = 1000 + 600 = ₹ 1600$ }

Exp. 2) Find the simple interest on ₹ 800 at 7% per annum and on ₹ 700 at 16% p.a. and on ₹ 500 at 4% p.a. for 2 years

Solution $SI = \frac{P_1 r_1 t_1}{100} + \frac{P_2 r_2 t_2}{100} + \frac{P_3 r_3 t_3}{100}$
 $= \frac{800 \times 7 \times 2}{100} + \frac{700 \times 16 \times 2}{100} + \frac{500 \times 4 \times 2}{100}$
 $= 112 + 224 + 40 = ₹ 376$

Exp. 3) A sum of money (P) doubles in 10 year In how many years it will be treble at the same rate of simple interest?

Solution $A = 2P$
 $\therefore SI = P$ ($SI = 2P - P$)
 $P = \frac{P \times r \times 10}{100} \Rightarrow r = 10\%$

So, the new amount = $3P$

But the new $SI = 2P = (3P - P)$; $2P = \frac{P \times 10 \times t}{100}$ ($r = 10\%$)
 $t = 20$ years

Exp. 4) A sum of money in 3 years becomes 1344 and in 7 years it becomes ₹ 1536. What is the principal sum where simple rate of interest is to be charged ?

- (a) 4000 (b) 1500
(c) 1200 (d) 2800

Solution It would be very time saving if we do it by unitary method.

$$1536 - 1344 = ₹ 192$$

₹ 192 is the interest for 4 years

So, ₹ 48 is the interest for 1 year

It means ₹ 144 is the interest for 3 years at actual principal.

$\therefore$ Principal = $1344 - 144 = ₹ 1200$

Exp. 5) A certain sum of money amounts to ₹ 15900 at simple rate of interest at 6% p.a. in 1 year. What is the value of principal sum ?

- (a) 12000 (b) 18000 (c) 15000 (d) 14000

Solution Best way is to go through option

$$\text{as } \frac{15000 \times 6 \times 1}{100} = 900$$

So, the total amount = 15900, hence (c) is correct.

Exp. 6) A sum of money becomes 3 times in 12 years. In how many years it will become 5 times at the same rate of simple interest?

- (a) 20 years (b) 16 years (c) 24 years (d) 30 years

Solution In this type of question consider only SI not the amount. So, (in first case) $SI = 2P$ for principal P

(in second case) $SI = 4P$ for principal P .

Now, since SI is directly proportional to the time at fixed rate of interest.

Therefore, to make SI two times it will require 12 years

So, to make SI four times it will require 24 years

Thus to make an amount 5 times of the principal requires 24 years at the given rate of interest as per question.

Exp. 7) What will be the amount when ₹ 10000 is deposited in a bank at 10% per annum compounded annually for 3 years?

Solution $A = P \left(1 + \frac{r}{100}\right)^n = 10000 \left(1 + \frac{10}{100}\right)^3$
 $= 10000 \times (1.1)^3 = ₹ 13310$

Exp. 8) A sum of amount at $r\%$ compound interest doubles in 3 years. In 9 years it will be k times of the original principal. What is the value of k ?

- (a) 10 (b) 9 (c) 6 (d) 8

Solution $A = P \left(1 + \frac{r}{100}\right)^n$

$$1^{\text{st}} \text{ case } 2P = P \left(1 + \frac{r}{100}\right)^3 \Rightarrow 2 = \left(1 + \frac{r}{100}\right)^3$$

$$2^{\text{nd}} \text{ case } (2)^3 = \left[\left(1 + \frac{r}{100}\right)^3 \right]^3 = \left(1 + \frac{r}{100}\right)^9 \Rightarrow k = (2)^3 = 8 \text{ times.}$$

Exp. 9) ₹ 20000 is being compounded at 20% p.a. If the rate of interest is charged half yearly. What will be the amount after 2 years ?

- (a) 28292 (b) 27292
(c) 29282 (d) 22358

Solution $20000 \left(1 + \frac{20/2}{100}\right)^{2 \times 2} = 20000 \left(1 + \frac{10}{100}\right)^4$
 $= 20000 (1.1)^4 = 20000 \times 1.4641$
 $= ₹ 29282$

Exp. 10) If the rate of interest is 10% p.a. and ₹ 12000 lent at the compound interest, half yearly. What is the equivalent simple rate of interest for first year?

Solution $A = P \left(1 + \frac{r}{100}\right)^n = 12000 (1.05)^2$
 $= 12000 \times 1.1025 = ₹ 13230$

Again $SI = 13230 - 12000 = 1230$

$$\therefore 1230 = \frac{12000 \times r \times 2}{2 \times 100}$$

$$\Rightarrow r = 10.25\%$$

Exp. 11) ₹ 1000 is being charged at 50% per annum. What is the interest for 3rd year at compound interest ?

- (a) 1122 (b) 1025
(c) 1125 (d) 625

Solution Total amount for two years $= 1000 (1.5)^2 = 2250$

$$\text{Now, interest for third year} = \frac{2250 \times 50 \times 1}{100} = 1125$$

Hence, (c) is correct.

Exp. 12) CI and SI for a certain sum at certain rate of interest are ₹ 220 and ₹ 200. What is the principal for these interests?

- (a) ₹ 2200 (b) ₹ 200
(c) ₹ 500 (d) ₹ 2000

Solution $220 - 200 = 20$

₹ 20 is the interest on the interest of the first year, which is

$$₹ 100 = \left(\frac{200}{2}\right)$$

$$\text{Thus, the rate of interest} = 20\% \quad \left(20 = \frac{100 \times r \times 1}{100}\right)$$

So, the actual principal $= P = 500$

$$\left(\text{since } 100 = \frac{P \times 20 \times 1}{100} \Rightarrow P = 500\right)$$

Hence, (c) is correct.

Exp. 13) The population of UP increases very rapidly @ 12% p.a. and 3 years ago its population was 1 million. What is present population?

Solution $A = 1000000 (1.12)^3 = 1404928$

Exp. 14) Jadeja purchased a maruti car 3 years ago for ₹ 2 lakh. Its value depreciated each year @ 25% p.a. What is the present value of the car ?

Solution $200000 \left(1 - \frac{25}{100}\right)^3 = 200000 \times \left(\frac{3}{4}\right)^3 = 84,375$

Exp. 15) The difference between CI and SI for 3 years @ 20% p.a. is ₹ 152. What is the principal lent in each case ?

- (a) 1200 (b) 1155
(c) 1187.5 (d) none of these

Solution Difference between CI and SI for 3 years = ₹ 152

$$P \left(\frac{r}{100}\right)^2 \left(\frac{r}{100} + 3\right) = 152$$

$$P \left(\frac{1}{25}\right) \left(\frac{16}{5}\right) = 152 \Rightarrow P = \frac{152 \times 25 \times 5}{16}$$

$$P = 9.5 \times 25 \times 5 = 1187.5$$

Exp. 16) A sum is being lent at 20% p.a. compound interest. What is the ratio of increase in amount of 4th year to 5th year?

- (a) 4 : 5 (b) 5 : 4
(c) 5 : 6 (d) can't be determined

Solution $\frac{P \left(1 + \frac{r}{100}\right)^4}{P \left(1 + \frac{r}{100}\right)^5} = \frac{1}{\left(1 + \frac{r}{100}\right)} = \frac{100}{100 + r} = \frac{100}{120} = \frac{5}{6}$

Exp. 17) ₹ 12000 amounts to ₹ 20736 in 3 years at $r\%$ p.a. of compound interest. What is the value of r ?

- (a) 10% (b) 25% (c) 12% (d) 20%

Solution $A = P \left(1 + \frac{r}{100}\right)^3 \Rightarrow 20736 = 12000 \left(1 + \frac{r}{100}\right)^3$
 $\Rightarrow \frac{20736}{12000} = \left(1 + \frac{r}{100}\right)^3 \Rightarrow \frac{1728}{1000} = \left(1 + \frac{r}{100}\right)^3$
 $\Rightarrow \left(\frac{12}{10}\right)^3 = \left(1 + \frac{r}{100}\right)^3 \Rightarrow \left(1 + \frac{2}{10}\right)^3 = \left(1 + \frac{r}{100}\right)^3 \Rightarrow r = 20\%$

☞ **Alternatively**

The best way for this problem is to go through options.

$$12000 \times 1.2 \Rightarrow 14400 \times 1.2 \Rightarrow 17280 \times 1.2 \Rightarrow 20736$$

Exp. 18) A certain sum amounts to ₹ 14641 in 4 years @ 10% p.a. compounded annually. What is the value of principal?

- (a) ₹ 6000 (b) ₹ 12000
(c) ₹ 10000 (d) data insufficient

Solution $14641 = P \left(1 + \frac{10}{100}\right)^4 \Rightarrow 14641 = P \left(\frac{11}{10}\right)^4$

$$P = 14641 \times \left(\frac{10}{11}\right)^4 = 10000$$

Exp. 19) A sum of ₹ 10000 is borrowed at 8% p.a. compounded annually which is paid back in 3 equal annual instalments.

What is the amount of each instalments?

$$\begin{aligned}\text{Solution } 10000 &= x \left[\frac{25}{27} + \left(\frac{25}{27} \right)^2 + \left(\frac{25}{27} \right)^3 \right] \\ &= x \times \frac{25}{27} \left[1 + \frac{25}{27} + \frac{625}{729} \right] \\ &= \frac{25x}{27} \left[\frac{2029}{729} \right] \\ \Rightarrow x &= ₹ 3880.335\end{aligned}$$

☞ Alternatively

$$\begin{aligned}10000 (1.08)^3 &= x [1 + (1.08) + (1.08)^2] \\ \Rightarrow x &= 3880.335\end{aligned}$$

Exp. 20) A scooty is sold by an automobile agency for ₹ 19200 cash or for ₹ 4800 cash down payment together with five equal monthly instalments. If the rate of interest charged by the company is 12% per annum find each instalment.

Solution Balance of the price to be paid through instalments
= ₹ 14400

Rate of interest (r) = 12% p.a.

$$\begin{aligned}\therefore \left(14400 + \frac{14400 \times 12 \times 5}{100 \times 12} \right) &= \left[x + \left(x + \frac{12x}{1200} \right) \right. \\ &\quad \left. + \left(x + \frac{12x \times 2}{12 \times 100} \right) + \dots + \left(x + \frac{12x \times 4}{12 \times 100} \right) \right]\end{aligned}$$

$\Rightarrow x = ₹ 2964.70$, where x is the value of each instalment.

📌 **NOTE** In the left hand side and right hand side given amounts are equal. Each amount is equal to the total amount payable after 5 months.

7.7 Important Amounts, Interest Rates and Time Periods

For quick and easy calculation to the problems related to this chapter you can learn and memorize the following values related to compound interest.

Rate of Interest	For ₹ 100, amount				
	after one year	after 2 years	after 3 years	after 4 years	after 5 years
5%	105	110.25	115.7625	121.550625	127.62815625
6%	106	112.36	119.1016	126.247696	133.82255776
8%	108	116.64	125.9712	136.048896	146.93280768
10%	110	121.00	133.1000	146.4100	161.051
12%	112	125.44	140.4928	157.351936	176.23416832
15%	115	132.25	152.0875	174.900625	201.13571875
20%	120	144.00	172.80	207.36	248.832
25%	125	156.25	195.3125	244.140625	305.17578125
30%	130	169.00	219.70	285.61	371.293
50%	150	225.00	337.50	506.25	759.375

CAT-Test

Questions Helping you bell the CAT

LEVEL 01 > BASIC LEVEL EXERCISE

- 1 A man borrows ₹ 4000 and pays back after 5 years at 15% simple interest. The amount paid by the man is :
(a) ₹ 1800 (b) ₹ 4800
(c) ₹ 7500 (d) ₹ 7000
- 2 What is the time period for which ₹ 8000 amounts to ₹ 12000 at 20% p.a. of simple interest?
(a) 4 years (b) 2.5 years
(c) 3.25 years (d) 6 years
- 3 What is the rate of simple interest at which ₹ 14,000 gives interest of ₹ 1960 in two years?
(a) 4% (b) 5%
(c) 7% (d) 10%
- 4 What is the sum of amount which gives ₹ 6300 as interest @ 7% per annum of simple interest in $7\frac{1}{2}$ years?
(a) 36000 (b) 24000
(c) 63000 (d) 12000
- 5 If the rate of simple interest is 12% per annum, the amount that would fetch interest of ₹ 6000 per annum is :
(a) ₹ 7200 (b) ₹ 48000
(c) ₹ 50000 (d) ₹ 72000
- 6 A sum was put at simple interest at a certain rate for 2 years. Had it been put at 4% higher rate, it would have fetched ₹ 112 more. The sum is :
(a) 1120 (b) 1400
(c) 1200 (d) 8000
- 7 In what time will a sum of money double itself @ 20% per annum (p.a.) simple interest?
(a) 10 years (b) 5 years
(c) 2 years (d) 4 years
- 8 At $r\%$ per annum a sum doubles after 20 years. The rate of interest per annum is :
(a) 4% (b) 5%
(c) 8% (d) 10%
- 9 A sum of money trebles (i.e., 3 times) in 15 years at $r\%$ of simple interest per annum. What is the value of r ?
(a) 12% (b) $\frac{40}{3}\%$
(c) $\frac{50}{3}\%$ (d) can't be determined
- 10 A sum of money doubles itself in 12 years. In how many years would it treble itself?
(a) 36 years (b) 18 years
(c) 24 years (d) 15 years
- 11 Out of a sum of ₹ 625, a part was lent at 5% SI and the other at 10% SI. If the interest on the first part after 2 years is equal to the interest on the second part after 4 years, then the second sum (in ₹) is :
(a) 250 (b) 300
(c) 125 (d) 275
- 12 A sum of ₹ 2500 is lent out in two parts; one at 12% p.a. and another at 12.5% p.a. for one year. If the total annual income is ₹ 306, the money lent at 12% is :
(a) 1000 (b) 1200 (c) 1500 (d) 1300
- 13 Akul lent ₹ 6000 to Bakul for 2 years and ₹ 1500 to camlin for 4 years and received altogether from both ₹ 900 as simple interest. The rate of interest is :
(a) 4% (b) 8% (c) 10% (d) 5%
- 14 A person takes a loan of ₹ 200 at 5% simple interest. He returns ₹ 100 at the end of one year. In order to clear his dues at the end of 2 years, he would pay :
(a) 125.50 (b) 110
(c) 115.50 (d) none of these
- 15 A lends a sum of money for 10 years at 5% SI. B ends double the amount for 5 years at the same rate of interest. Which of the following statements is true in this regard?
(a) A will get twice the amount of interest that B would get
(b) B will get twice the amount of interest that A would get
(c) A will get the same amount of interest that B would get
(d) B will get four times the amount of interest that A would get
- 16 Consider the following statements.
If a money is loaned at simple interest then the :
(i) money gets doubled in 5 years if the rate of interest is $16\frac{2}{3}\%$.
(ii) money gets doubled in 5 years if the rate of interest is 20%.
(iii) money becomes four times in 10 years if it gets doubled in 5 years of these statements :

- (a) (i) and (iii) are correct
 (b) (iii) alone is correct
 (c) (ii) alone is correct
 (d) (ii) and (iii) are correct
- 17** Pratibha invests an amount of ₹ 15,860 in the names of her three daughters A, B and C in such a way that they get the same interest after 2, 3 and 4 years respectively. If the rate of simple interest is 5% p.a., then the ratio of the amounts invested among A, B and C will be :
 (a) 5 : 10 : 12 (b) $\frac{1}{10} : \frac{1}{15} : \frac{1}{20}$
 (c) 6 : 7 : 8 (d) 6 : 5 : 4
- 18** What annual payment will discharge a debt of ₹ 580 in 5 years, the rate being 8% p.a.?
 (a) 120 (b) 100
 (c) 80 (d) 78
- 19** Find the amount of ₹ 2500 invested at 12% during the period from 4th February, 2005 to 18 April 2005 :
 (a) ₹ 3000 (b) ₹ 3800
 (c) ₹ 2560 (d) ₹ 2600
- 20** Find the amount of ₹ 1700 invested at 16% half yearly at simple interest for one year :
 (a) 2100 (b) 2244
 (c) 2200 (d) 2500
- 21** The compound interest on ₹ 1000 at 10% p.a. in 3 years is:
 (a) 331 (b) 1331 (c) 133 (d) 300
- 22** The compound interest on ₹ 10000 at 20% p.a. in 4 years:
 (a) 10736 (b) 736
 (c) 20736 (d) 7280
- 23** The compound interest on ₹ 4000 at 25% p.a. in 3 years :
 (a) 1235 (b) 5625
 (c) 3812.5 (d) 3750.5
- 24** The compound interest on ₹ 5000 at 30% per annum for 4 years :
 (a) 4280.5 (b) 6700
 (c) 9280.5 (d) 3857.5
- 25** A sum of ₹ 400 would become ₹ 441 after 2 years at $r\%$ compound interest, find the value of ' r ' :
 (a) 10% (b) 5%
 (c) 15% (d) 20%
- 26** At compound interest, if a certain sum of money doubles in n years, then the amount will be four fold in :
 (a) n^2 years (b) $2n^2$ years
 (c) $2n$ years (d) $4n$ years
- 27** ₹ 6000 amounts to ₹ 7986 in 3 years at CI. The rate of interest is :
 (a) 20% (b) 10%
 (c) 6% (d) 7.5%
- 28** The least number of complete years in which a sum of money put at 20% CI will be more than doubled is :
 (a) 4 (b) 5
 (c) 6 (d) 8
- 29** The CI on ₹ 5000 for 3 years at 8% for first year, 10% for second year and 12% for third year will be :
 (a) ₹ 1750 (b) ₹ 1652.80
 (c) ₹ 1575 (d) ₹ 1685.20
- 30** A sum of ₹ 2400 deposited at CI, doubled after 5 years After 20 years it will become :
 (a) ₹ 24000 (b) ₹ 38400
 (c) ₹ 19200 (d) can't be determine
- 31** A sum of ₹ 550 was taken as a loan. This is to be paid back in two equal annual instalments. If the rate of interest be 20% compounded annually, then the value of each instalments is :
 (a) 300 (b) 360
 (c) 250 (d) none of these
- 32** The difference between CI and SI on a sum of money lent for 2 years at 10% is ₹ 40. The sum is :
 (a) 1600 (b) 3000
 (c) 4000 (d) none of these
- 33** The difference between simple and compound interest on ₹ 6000 for 1 year at 20% per annum reckoned half yearly is :
 (a) 120 (b) 60
 (c) 180 (d) 72
- 34** A certain sum amounts to ₹ 8988.8 in two years and to ₹ 9528.128 in three years, at compound interest per annum. What is the principal and rate of interest?
 (a) ₹ 12,000, 5% (b) ₹ 6,000, 8%
 (c) ₹ 8,000, 6% (d) ₹ 10,000, 8.5%
- 35** The compound interest and the simple interest for two years on a certain sum of money at a certain rate of interest are ₹ 2257.58, ₹ 2100, respectively. Find the principal and rate per cent :
 (a) 6000, 7% (b) 7500, 8%
 (c) 14000, 10% (d) 7000, 15%
- 36** The compound interest on a certain sum at a certain rate of interest for the second year and third year is ₹ 21780 and ₹ 23958, respectively. What is the rate of interest?
 (a) 6% (b) 12%
 (c) 10% (d) 15%
- 37** Amit borrowed ₹ 800 at 10% rate of interest. He repaid ₹ 400 at the end of first year. What is the amount required to pay at the end of second year to discharge his loan which was calculated at compound interest?
 (a) 420 (b) 440 (c) 450 (d) 528
- 38** A sonata watch is sold for ₹ 440 cash or for ₹ 200 cash down payment together with ₹ 244 to be paid after one month. Find the rate of interest charged in the instalment scheme :
 (a) 10% (b) 15% (c) 20% (d) 25%
- 39** A cellphone is available for ₹ 600 or for 300 cash down payment together with ₹ 360 to be paid after two months. Find the rate of interest charged under this scheme :
 (a) 20% (b) 50%
 (c) 120% (d) none of these

- 40** Abhinav purchases a track suit for ₹ 2400 cash or for ₹ 1000 cash down payments and two monthly instalments of ₹ 800 each. Find the rate of interest :
- (a) 75% (b) 120%
(c) 50% (d) none of these
- 41** Indicom cell-phone is available for ₹ 2500 cash or ₹ 520 cash down payments followed by 4 equal monthly instalments. If the rate of interest charged is 25% per annum, calculate the monthly instalment :
- (a) 520 (b) 480
(c) 550 (d) none of these
- 42** An article is sold for ₹ 500 cash or for ₹ 150 cash down payments followed by 5 equal monthly instalments. If the rate of interest charged is 18% p.a., compute the monthly instalment :
- (a) 63.07% (b) 37.06%
(c) 75.0% (d) 73.06%
- 43** A sum of ₹ 390200 is to be paid back in three equal annual instalments. How much is each instalment, if the rate of interest charged is 4% per annum compounded annually?
- (a) ₹ 140608 (b) ₹ 120560
(c) ₹ 10000 (d) ₹ 18000
- 44** Purnima borrowed a sum of money and returned it in three equal quarterly instalments of ₹ 17576 each. Find the sum borrowed, if the rate of interest charged was 16% per annum compounded quarterly. Find also the total interest charged :
- (a) 46900 and 4700 (b) 48775 and 3953
(c) 68320 and 1200 (d) none of these
- 45** Sunidhi borrowed ₹ 10815, which is to be paid back in 3 equal half yearly instalments. If the interest is compounded half yearly at $\frac{40}{3}\%$ per annum, how much is each instalment?
- (a) 2048 (b) 3150 (c) 4096 (d) 5052
- 46** Sapna borrowed some money on compound interest and returned it in three years in equal annual instalments. If the rate of interest is 15% per annum and annual instalment is ₹ 486680, find the sum borrowed :
- (a) 1112220 (b) 1111200
(c) 1122000 (d) none of these
- 47** P and Q invest some amount under SI and CI, respectively but for the same period at 6% per annum. Each gets a total amount of ₹ 65,000 at the end of 6 years. Which of the following is definitely true?
- (i) Q's initial principal is less than that of P
(ii) Q's initial principal is equal to that of P
(iii) P's percentage earning is less than that of Q
- (a) (i) only (b) (ii) only
(c) (iii) only (d) (i) and (iii) only
- 48** In the above (i.e., previous) problem, what is the ratio of P's final amount to that of Q, if P and Q invest the same amounts?
- (a) $(1.06)^6$ (b) $\frac{136}{100} \times \left(\frac{100}{106}\right)^6$
(c) $\frac{100}{136} \times \left(\frac{106}{100}\right)^6$ (d) none of these
- 49** Mr. Lala Ram has lent some money to Aaju at 6% p.a. and the Baaju at 8% p.a. At the end of the year he has gain the overall interest at 7% per annum. In what ratio has he lent the money to Aaju and Baaju?
- (a) 2 : 3 (b) 1 : 1
(c) 5 : 6 (d) 4 : 3
- 50** The difference between simple and compound interest for the fourth year is ₹ 7280 at 20% p.a. What is the principal sum?
- (a) 10000 (b) 50000
(c) 1 lakh (d) 40000
- 51** The simple interest on certain sum at 5% for 9 month is ₹ 10 greater than the simple interest on the same sum @ 3% for 14 months. What is the sum of interest in both the cases (i.e., total sum of interest)?
- (a) ₹ 130 (b) ₹ 290
(c) ₹ 120 (d) ₹ 330
- 52** Mr. Bajaj invested $\frac{1}{7}$ of his total investment at 4% and $\frac{1}{2}$ at 5% and rest at 6% for the one year and received total interest of ₹ 730. What is the total sum invested?
- (a) ₹ 70000 (b) ₹ 14000
(c) ₹ 24000 (d) ₹ 38000
- 53** The rate of interest in two banks DNB and HBI are in the ratio of 7 : 8. If a person invested some amount in both the banks and received equal amounts from both the banks in two years. The ratio of amount invested in DNB and HBI, respectively is :
- (a) 15 : 1 (b) 8 : 7
(c) 7 : 8 (d) 108 : 107
- 54** The ratio of CI for 3 years and SI for 1 year for a fixed amount at a rate of $r\%$ is 3.64. What is the value of r ?
- (a) 10% (b) 15%
(c) 20% (d) none of these
- 55** A sum of money becomes $\frac{13}{5}$ times of itself in 32 years at $r\%$ of SI. What is the value of r ?
- (a) 6% (b) 7%
(c) 5% (d) 18%
- 56** The difference between interest received by A and B is ₹ 18 on ₹ 1500 for 3 years. What is the difference in rate of interest?
- (a) 1% (b) 2.5% (c) 0.5% (d) 0.4%

LEVEL 02 > HIGHER LEVEL EXERCISE

- 1 The compound interest on a certain sum for 2 years is ₹ 756 and SI (simple interest) is ₹ 720. If the sum is invested such that the SI is ₹ 900 and the number of years is equal to the rate per cent per annum, find the rate per cent :
 (a) 4 (b) 5
 (c) 6 (d) 1.0
- 2 Jalela and Dalela have to clear their respective loans by paying 3 equal annual instalments of ₹ 30000 each. Jalela pays @ 10% per annum of simple interest while Dalela pays 10% per annum compound interest. What is the difference in their payments?
 (a) ₹ 300 (b) ₹ 425
 (c) ₹ 245 (d) ₹ 333.33
- 3 Akbar lends twice the interest received from Birbal to Chanakya at the half of the interest at which he lent to Birbal. If Akbar lent ₹ P @ $r\%$ per annum for 1 year to Birbal then the interest received by Akbar from Chanakya is :
 (a) $\frac{Pr^2}{100}$ (b) $\left(\frac{Pr}{10}\right)^2$
 (c) $P\left(\frac{r}{10}\right)^2$ (d) $P\left(\frac{r}{100}\right)^2$
- 4 Equal amounts of each ₹ 43892 is lent to two persons for 3 years One @ 30% SI and second @ 30% CI annually. By how much per cent the CI is greater than the simple interest received in this 3 years duration?
 (a) 23% (b) 33%
 (c) 33.33% (d) none of these
- 5 ₹ 3500 was lent partly @ 4% and partly @ 6% SI. The total interest received after 3 years is 498. What is the amount lent @ 4% SI?
 (a) ₹ 1300 (b) ₹ 1800
 (c) ₹ 200 (d) ₹ 2200
- 6 The population of vultures in a particular locality is decreases by a certain rate of interest (compounded annually). If the current population of vultures be 29160 and the ratio of decrease in population for second year and 3rd year be 10 : 9. What was the population of vultures 3 years ago?
 (a) 30000 (b) 35000
 (c) 40000 (d) 50000
- 7 The ratio of the amount for two years under CI annually and for one year under SI is 6 : 5. When the rate of interest is same, then the value of rate of interest is :
 (a) 12.5% (b) 18%
 (c) 20% (d) 16.66%
- 8 A bicycle can be purchased on cash payment of ₹ 1500. The same bicycle can also be purchased at the down payment (initial payment, at the time of purchasing) of ₹ 350 and rest can be paid in 3 equal installments of ₹ 400 for next 3 months. The rate of SI per annum charged by the dealer is:
 (a) $23\frac{9}{17}\%$ (b) $17\frac{9}{23}\%$
 (c) $13\frac{9}{17}\%$ (d) none of these
- 9 Data Ram lends equal sum of money at the same rate of interest to A and B. The money lends to A becomes twice of the original amount in just four years at simple interest. While Data Ram lends to B for the first two years at compound interest and for the rest two years at simple interest. If the difference between the amount of A and B after 4 years is ₹ 2750. What is the amount of money that Data Ram lends to each one?
 (a) ₹ 40000 (b) ₹ 6000
 (c) ₹ 8000 (d) ₹ 80000
- 10 Akram Ali left an amount of ₹ 340000 to be divided between his two sons aged 10 years and 12 years such that both of them would get an equal amount when each attain 18 years age. What is the share of elder brother if the whole amount was invested at 10% simple interest :
 (a) 12000 (b) 16000
 (c) 160000 (d) 180000
- 11 Satyam took loan from IDIDI Bank for his 2 years course of MBA at IMD. He took the loan of ₹ 6 lakh such that he would be charged at 8% per annum at CI during his course and at 10% CI after the completion of course. He returned half of the amount which he had to be paid on the completion of his studies and remaining after 2 years What is the total amount returned by Satyam ?
 (a) ₹ 7.73323 lakh (b) ₹ 7.58 lakh
 (c) ₹ 7.336 lakh (d) none of these
- 12 We had 1000 goats at the beginning of year 2001 and the no. of goats each year increases by 10% by giving birth (compounded annually). At the end of each year we double the no. of goats by purchasing the same no. of goats as there is the no. of goats with us at the time. What is the no. of goats at the beginning of 2004?
 (a) 10600 (b) 10648
 (c) 8848 (d) 8226
- 13 ₹ 100000 was invested by Mohan in a fixed deposit @ 10% per annum at CI. However every year he has to pay 20% tax on the compound interest. How much money does Mohan has after 3 years?
 (a) 128414 (b) 108000
 (c) 126079.2 (d) none of these
- 14 A property dealer bought a rectangular plot (of land) in Noida 5 years ago at the rate of ₹ 1000 per m^2 . The cost of plot is increases by 5% in every 6 years and the worth of a rupee falls down at a rate of 2% in every 5 years. What is the approximate value of the land per meter² 25 years hence?
 (a) ₹ 995 (b) ₹ 1134
 (c) ₹ 1500 (d) ₹ 1495

- 15** A and B run a joint venture in which the profit earned by A and B are in the ratio 28 : 15. A invest his share at the start of the year and B joins in after 9 months of the same year. What is the ratio of their initial investment, respectively?
 (a) 7 : 15 (b) 8 : 13
 (c) 5 : 17 (d) 15 : 7
- 16** In the previous problem if A gets a profit of ₹ 4200 the amount invested by B is :
 (a) 2250 (b) 2600
 (c) 1350 (d) can't be determined
- 17** Arvind and Govind each invested ₹ 15000 for 3 years at the same rate of interest but Arvind's investment is compounded annually while Govind's investment is charged on simple interest. What amount did Arvind receive more than Govind?
 (a) ₹ 680 (b) ₹ 3450
 (c) data insufficient (d) none of these
- 18** Shyam Lal takes a loan of ₹ 10500 at 10% p.a. compounded annually which is to be repaid in two equal annual instalments. One at the end of one year and the other at the end of the second year. The value of each instalments is :
 (a) 5987 (b) 6050
 (c) 6352.5 (d) 5678.5
- 19** Hari Lal and Hari Prasad have equal amounts. Hari Lal invested all his amount at 10% compounded annually for 2 years and Hari Prasad invested $\frac{1}{4}$ at 10% compound interest (annually) and rest at $r\%$ per annum at simple interest for the same 2 years period. The amount received by both at the end of 2 year is same. What is the value of r ?
 (a) 14% (b) 12.5%
 (c) 10.5% (d) 11%
- 20** The annual sales of a company in the year 2000 was ₹ 1000 and in the year 2005 was ₹ 2490. Find the compounded annual growth rate (CAGR) of sales in the given period of the same company :
 (a) 14.289% (b) 10%
 (c) 15% (d) 20%
- 21** HDFC lends 1 million to HUDCO at 10% simple interest p.a. for 2 years and HUDCO lends the same amount to SAHARA STATES HOUSING corporation at 10% p.a. of compound interest for 2 years What is the earning of HUDCO in this way?
 (a) ₹ 133100 (b) ₹ 33100
 (c) ₹ 131000 (d) no profit no loss
- 22** ICICI lent ₹ 1 lakh to captain Ram Singh @ 6% per annum of simple interest for 10 years period. Meanwhile ICICI offered a discount in rate of interest for armed forces. Thus the rate of interest ICICI decreased to 4%. In this way Ram Singh had to pay total amount 1.48 lakh. After how many year Ram Singh got the discount in rate of interest?
 (a) 3 years (b) 4 years
 (c) 6 years (d) 5 years
- 23** Sanjay purchased a hotel worth ₹ 10 lakh and barkha purchased a car worth ₹ 16 lakh. The value of hotel every year increases by 20% of the previous value and the value of car every year depreciates by 25%. What is the difference between the price of hotel and car after 3 years?
 (a) ₹ 925000 (b) ₹ 1053000
 (c) remains constant (d) can't be determined

Answers

✓ Level 01 Basic Level Exercise

1. (d)	2. (b)	3. (c)	4. (d)	5. (c)	6. (b)	7. (b)	8. (b)	9. (b)	10. (c)
11. (c)	12. (d)	13. (d)	14. (c)	15. (c)	16. (c)	17. (b)	18. (b)	19. (c)	20. (b)
21. (a)	22. (a)	23. (c)	24. (c)	25. (b)	26. (c)	27. (b)	28. (a)	29. (b)	30. (b)
31. (b)	32. (c)	33. (b)	34. (c)	35. (d)	36. (c)	37. (d)	38. (c)	39. (c)	40. (b)
41. (a)	42. (d)	43. (a)	44. (b)	45. (c)	46. (b)	47. (d)	48. (b)	49. (b)	50. (b)
51. (b)	52. (b)	53. (b)	54. (c)	55. (c)	56. (d)				

✓ Level 02 Higher Level Exercise

1. (b)	2. (a)	3. (d)	4. (b)	5. (d)	6. (c)	7. (c)	8. (d)	9. (c)	10. (d)
11. (a)	12. (b)	13. (d)	14. (b)	15. (a)	16. (d)	17. (c)	18. (b)	19. (c)	20. (a)
21. (c)	22. (b)	23. (b)							

Hints & Solutions

✓ Level 01 Basic Level Exercise

1 $4000 + (4000 \times 5 \times 0.15) = ₹ 7000$

2 $(12000 - 8000) = \frac{8000 \times t \times 20}{100}$

$\Rightarrow t = \frac{5}{2}$ years

3 $1960 = \frac{14000 \times r \times 2}{100} \Rightarrow r = 7\%$

4 $6300 = \frac{P \times 7 \times 15}{100 \times 2}$

$\Rightarrow P = ₹ 12000$

5 $6000 = \frac{P \times 12 \times 1}{100}$

$\Rightarrow P = 50000$

6 In my opinion this question should be solved by unitary method instead of making complex solution

Years	Rate of Interest	Interest
2	4%	112
2	1%	28
1	1%	(14)

It means the principal sum is ₹ 1400

☞ Alternatively

$$\frac{P}{100} \times 2[(r + 4) - r] = 112$$

$\Rightarrow P = 1400$

7 $SI = 2P - P = P$ (Interest = Amount - Principal)

$$P = \frac{P \times 20 \times t}{100} \Rightarrow t = 5 \text{ years}$$

8 $P = \frac{P \times r \times 20}{100}$ (Interest = Amount - Principal)

$\Rightarrow r = 5\% \text{ p.a.}$

9 $2P = \frac{P \times 15 \times r}{100}$ ($2P = 3P - P$)

$$r = \frac{40}{3} \% \text{ p.a.}$$

10 $P = \frac{P \times r \times 12}{100} \Rightarrow r = \frac{100}{12} \% \text{ p.a.}$

Now, $2P = \frac{P \times 100 \times t}{12 \times 100} = 24 \text{ years}$

Shortcut (for SI) Since SI in second case is double the SI in first case, so the time period will also be double since SI is directly proportional to the time period, provided that rate of interest be same and principal as well.

11 $\frac{P_1 \times 5 \times 2}{100} = \frac{P_2 \times 10 \times 4}{100}$

$\Rightarrow P_1 : P_2 = 4 : 1$

Therefore, second principal is ₹ 125 $\left(= 625 \times \frac{1}{5} \right)$

☞ Alternatively Go through options.

12 Best way is to go through option

So, $1300 \times 0.12 + 1200 \times 0.125 = 306$

Hence, (d) is the correct option.

☞ Alternatively Solve through alligation

Therefore the ratio of first principal to the second principal is 13 : 12.

$$\begin{array}{ccc} 12 & & 12.5 \\ & \searrow & \nearrow \\ & 12.24 & \\ & \nearrow & \searrow \\ 0.26 & & 0.24 \\ 13 & : & 12 \end{array} \quad \left| \quad \begin{array}{l} 306 = \frac{2500 \times r \times 1}{100} \\ r = \frac{306}{25} = 12.24 \end{array} \right.$$

13 $\left(\frac{6000 \times 2 + 1500 \times 4}{100} \right) r = 900$

$\Rightarrow r = 5\%$

14 Amount to be paid in first year = $\frac{200 \times 5 \times 1}{100} + 200 = 210$

Amount left as a principal for the second year
 $= 210 - 100 = 110$

$\therefore$ Amount paid in second year

$$= 110 + \frac{110 \times 5 \times 1}{100} = 115.5$$

15

A	B
$P_A = P$	$P_B = 2P$
$r_A = 5\%$	$r_B = 5\%$
$t = 10 \text{ years}$	$t = 5 \text{ years}$
Interest of A = $\frac{P \times 5 \times 10}{100}$	Interest of B = $\frac{2P \times 5 \times 5}{100}$
$= \frac{P}{2}$	$= \frac{P}{2}$

Hence (c) is correct

16 Statement (I) $\frac{P \times 5 \times 50}{100 \times 3} \neq P$, hence wrong

Statement (II) $SI = \frac{P \times 5 \times 20}{100} = P$, hence correct

Statement (III) $P = \frac{P \times 5 \times r}{100} \Rightarrow r = 20\%$

Again $SI = \frac{P \times 10 \times 20}{100} = 2P$, hence wrong

$$17 \quad \frac{P_1 \times 2 \times 5}{100} = \frac{P_2 \times 3 \times 5}{100} = \frac{P_3 \times 4 \times 5}{100}$$

$$\Rightarrow 10P_1 = 15P_2 = 20P_3$$

$$\Rightarrow P_1 : P_2 : P_3 = 30 : 20 : 15 = \frac{1}{10} : \frac{1}{15} : \frac{1}{20}$$

$$18 \quad 580 = \left[(x) + \left(x + \frac{x \times 1 \times 8}{100} \right) + \left(x + \frac{x \times 2 \times 8}{100} \right) + \left(x + \frac{x \times 3 \times 8}{100} \right) + \left(x + \frac{x \times 4 \times 8}{100} \right) \right]$$

$$580 = 5x + \frac{8x}{10} = \frac{58x}{10}$$

$$\Rightarrow x = 100$$

$$19 \quad \text{Time period} = \frac{\text{Feb}}{24} + \frac{\text{March}}{31} + \frac{\text{April}}{18}$$

$$= 73 \text{ days} = \frac{1}{5} \text{ year}$$

$$\therefore 2500 + \frac{2500 \times 12 \times 1}{100 \times 5} = ₹ 2560$$

$$20 \quad 1700 + \frac{1700 \times 16 \times 2}{100} = ₹ 2244$$

$$21 \quad A = P \left(1 + \frac{r}{100} \right)^n$$

$$A = 1000 \left(1 + \frac{10}{100} \right)^3$$

$$A = 1000 \times (1.1)^3$$

$$A = 1331$$

$$\therefore \text{CI} = A - P = 1331 - 1000 = 331$$

$$22 \quad A = 10000 \times (1.2)^4$$

$$A = 10000 \times 2.0736$$

$$A = 20736$$

$$\text{CI} = A - P$$

$$\text{CI} = 20736 - 10000 = 10736$$

$$23 \quad \text{CI} = [4000 \times (1.25)^3] - (4000)$$

$$= 4000 \times 1.953125 - 4000$$

$$= 4000 \times 0.953125 = 3812.5$$

$$24 \quad \text{CI} = 5000 \times (1.3)^4 - 5000$$

$$= 5000 \times 2.8561 - 5000$$

$$= 5000 (1.8561) = 9280.5$$

$$25 \quad 441 = 400 \left(1 + \frac{r}{100} \right)^2$$

$$\Rightarrow \frac{21}{20} = 1 + \frac{r}{100}$$

$$\Rightarrow r = 5\%$$

$$26 \quad \text{Since, } A = 2P, \text{ then } 2P = P \left(1 + \frac{r}{100} \right)^n \Rightarrow 2 = \left(1 + \frac{r}{100} \right)^n$$

$$\Rightarrow 4 = (2)^2 = \left[\left(1 + \frac{r}{100} \right)^n \right]^2 = \left[\left(1 + \frac{r}{100} \right)^{2n} \right]$$

$$\Rightarrow \text{time period} = 2n \quad \{ \because (a^m)^n = a^{m \cdot n} \}$$

(this question can be easily done by considering some appropriate values)

$$27 \quad 7986 = 6000 \left(1 + \frac{r}{100} \right)^3$$

$$\frac{7986}{6000} = \left(1 + \frac{r}{100} \right)^3$$

$$\Rightarrow \frac{1331}{1000} = \left(1 + \frac{r}{100} \right)^3 \Rightarrow \frac{11}{10} = \left(1 + \frac{r}{100} \right)$$

$$\Rightarrow r = 10\%$$

☞ Alternatively Go through options.

$$6000 \xrightarrow{+10\%} 6600 \xrightarrow{+10\%} 7260 \xrightarrow{+10\%} 7986$$

Hence, assumed option is correct.

28 Go through option

$$1 \times (1.2) = 1.2$$

$$1 \times (1.2)^2 = 1.44$$

$$1 \times (1.2)^3 = 1.728$$

$$1 \times (1.2)^4 = 2.0736$$

Hence, minimum 4 years are required to double the sum.

$$29 \quad A = P \left(1 + \frac{R_1}{100} \right) \left(1 + \frac{R_2}{100} \right) \left(1 + \frac{R_3}{100} \right) \dots$$

$$A = 5000 (1.08) (1.1) (1.12)$$

$$A = 6652.8$$

$$\text{CI} = 1652.8 = (6652.8 - 5000)$$

$$30 \quad 2P = P \left(1 + \frac{r}{100} \right)^5 \Rightarrow 2 = \left(1 + \frac{r}{100} \right)^5$$

$$\text{Now, } (2)^4 = \left[\left(1 + \frac{r}{100} \right)^5 \right]^4 = \left(1 + \frac{r}{100} \right)^{20}$$

$\Rightarrow$ The amount becomes 16 ($= 2^4$) times

$\therefore$ Hence (b) is correct, $\because (2400 \times 16 = 38400)$

$$31 \quad 550 = x \left[\left(\frac{1}{1.2} \right) + \left(\frac{1}{1.2} \right)^2 \right] \Rightarrow 550 = x \left[\frac{2.2}{1.44} \right] \Rightarrow x = ₹ 360$$

$$32 \quad \text{Difference between CI and SI for 2 years} = P \left(\frac{r}{100} \right)^2$$

$$40 = P \left(\frac{10}{100} \right)^2 \Rightarrow P = 4000$$

☞ **Alternatively** Go through options

	First year	Second year
SI	400	400
CI	400	440

☞ **Alternatively** Go back in the reverse process.

₹ 40 at 10% implies that the principal for this interest was ₹ 400. Again by the same logic ₹ 400 as interest obtained at the principal of ₹ 4000 at 10%.

$$33 \quad 6000 \left(\frac{10}{100} \right)^2 = 60$$

$$34 \quad \frac{P \left(1 + \frac{r}{100} \right)^3}{P \left(1 + \frac{r}{100} \right)^2} = \left(1 + \frac{r}{100} \right)$$

$$\therefore \frac{9528.128}{8988.8} = \left(1 + \frac{r}{100} \right)$$

$$\Rightarrow \left(1 + \frac{84270}{1404500} \right) = \left(1 + \frac{r}{100} \right)$$

$$\Rightarrow \left(1 + \frac{6}{100} \right) = \left(1 + \frac{r}{100} \right) \Rightarrow r = 6\%$$

$$\text{So, } 8988.8 = P \left(1 + \frac{6}{100} \right)^3 \Rightarrow P = 8000$$

☞ **Alternatively** Best way is to go through options.

35 Difference between CI and SI for 2 years will be equal to the interest on SI for first year.

$$\text{Hence, } 6000 \times 0.07 \Rightarrow 420 \times 0.07 = 29.4 \neq 157.5$$

$$\text{and } 7500 \times 0.08 \Rightarrow 600 \times 0.08 = 48 \neq 157.5$$

$$\text{and } 14000 \times 0.1 \Rightarrow 1400 \times 0.1 = 140 \neq 157.5$$

$$\text{and } 7000 \times 0.15 \Rightarrow 1050 \times 0.15 = 157.5 = 157.5$$

Thus option (d) is correct.

$$36 \quad \frac{P \left(1 + \frac{r}{100} \right)^3}{P \left(1 + \frac{r}{100} \right)^2} = \left(1 + \frac{r}{100} \right) \Rightarrow \frac{23958}{21780} = \left(1 + \frac{r}{100} \right)$$

$$1 + \frac{2178}{21780} = 1 + \frac{r}{100} \Rightarrow r = 10\%$$

☞ **Alternatively** Remember the difference between compound interest of any two consecutive years will be same as the interest on the amount of total previous year

$$\text{So, } 23958 - 21780 = 2178$$

$$\text{Now, } r = \frac{2178}{21780} \times 100 \Rightarrow r = 10\%$$

37 Amount to be paid at the end of 2 year

$$= \frac{800 \times 10 \times 2}{100} + 800 = 880$$

Amount left as principal for the second year

$$= 480 = (880 - 400)$$

$$\text{Amount to be paid after 2nd year} = 480 + \frac{480 \times 10}{100} = ₹ 528$$

38 Principal for next month = 440 - 200 = 240

Amount paid after next month = 244

Therefore, interest charged at ₹ 240 = 4

$$\therefore 4 = \frac{240 \times r \times 1}{12 \times 100}$$

$$r = 20\% \text{ per annum}$$

39 Amount as a principal for first and

second month = 600 - 300 = ₹ 300

Now, interest = 360 - 300 = ₹ 60

$$\therefore 60 = \frac{300}{100} \times \frac{2}{12} \times r \Rightarrow r = 120\%$$

40 Amount as a principal for 2 month = 2400 - 1000 = 1400

At the rate of $r\%$ per annum after 2 months, ₹ 1400 will amount to

$$₹ \left(1400 + \frac{1400 \times r \times 2}{100 \times 12} \right) \quad \dots(i)$$

Again total amount for the 2 instalments at the end of second month will be

$$₹ \left[800 + \left(800 + \frac{800 \times r \times 1}{100 \times 12} \right) \right] \quad \dots(ii)$$

from Eqs. (i) and (ii), we get

$$1400 + \frac{2800r}{1200} = 1600 + \frac{800r}{1200}$$

$$\frac{2000r}{1200} = 200 \Rightarrow r = 120\%$$

41 Balance price to be paid in instalments = 1980

At the rate of $r\%$ per annum after 4 months, ₹ 1980 will

$$\text{amount to } ₹ \left(1980 + \frac{1980 \times 4 \times 25}{12 \times 100} \right) = ₹ 2145 \quad \dots(i)$$

Now, the total amount for the 4 instalments at the end of fourth month will be

$$\left[x + \left(x + \frac{25x \times 1}{12 \times 100} \right) + \left(x + \frac{25x \times 2}{12 \times 100} \right) + \left(x + \frac{25 \times 3x}{12 \times 100} \right) \right]$$

$$= 4x + \frac{25x}{1200} (1 + 2 + 3) = \frac{33x}{8} \quad \dots(ii)$$

$$\text{from, Eqs. (i) and (ii) we get } \frac{33x}{8} = 2145 \Rightarrow x = 520$$

42 Balance price to be paid in instalments = 350

At the rate of $r\%$ per annum after 5 months, ₹ 350 will amount to

$$₹ \left(350 + \frac{350 \times 18 \times 5}{12 \times 100} \right) = \left(350 + \frac{1750 \times 18}{1200} \right) \quad \dots(i)$$

Again, total amount for the 5 instalments at the end of 5th

$$\text{month will be } ₹ \left[x + \left(x + \frac{18x \times 1}{1200} \right) + \left(x + \frac{18x \times 2}{12 \times 100} \right) \right. \\ \left. + \left(x + \frac{18x \times 3}{12 \times 100} \right) + \left(x + \frac{18x \times 4}{12 \times 100} \right) \right]$$

$$= \left[5x + \frac{18x}{1200} (1 + 2 + 3 + 4) \right]$$

$$= 5x + \frac{180x}{1200} = \left(\frac{6180x}{1200} \right) \quad \dots (ii)$$

from Eqs. (i) and (ii), we get

$$\left(350 + \frac{1750 \times 18}{1200} \right) = \frac{6180x}{1200} \Rightarrow x = 73.058$$

43 Let each instalments be ₹ x

The amount to be paid instalments = 390200

The total value of all the three instalments is

$$₹ \left[x \left(\frac{25}{26} \right) + x \left(\frac{25}{26} \right)^2 + x \left(\frac{25}{26} \right)^3 \right]$$

and this must be equal to ₹ 390200

$$\text{Hence, } x \times \frac{25}{26} \left[1 + \frac{25}{26} + \frac{625}{676} \right] = 390200$$

$$x = ₹ 140608$$

44 Rate of interest = 16% annum

Actual rate of interest = 4% per quarter

Principal of all three instalments

$$= \left[17576 \left(\left(\frac{25}{26} \right) + \left(\frac{25}{26} \right)^2 + \left(\frac{25}{26} \right)^3 \right) \right]$$

$$= \frac{17576 \times 25 \times 1951}{26 \times 676} = 48775$$

Total amount paid = ₹ 17576 × 3 = 52728

Interest charged = 52728 – 48775 = 3953

45 Rate of interest = $\frac{40/3}{2} \% = \frac{20}{3} \%$ half yearly

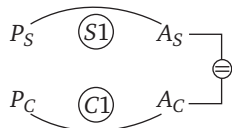
$$\therefore 10815 = x \left[\frac{15}{16} + \left(\frac{15}{16} \right)^2 + \left(\frac{15}{16} \right)^3 \right] \Rightarrow x = 4096$$

46 $P = 486680 \left[\frac{20}{23} + \left(\frac{20}{23} \right)^2 + \left(\frac{20}{23} \right)^3 \right]$

$$\Rightarrow P = \left[486680 \times \frac{20}{23} \left(1 + \frac{20}{23} + \frac{400}{529} \right) \right]$$

$$P = 1111200$$

47



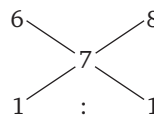
Obviously $P_S > P_C$, therefore percentage gain of P_C is greater than P_S .

48 $A_S = P + \frac{P \times 6 \times 6}{100} = P \left(\frac{136}{100} \right)$

$$A_C = P \left(1 + \frac{6}{100} \right)^6 = P \left(\frac{106}{100} \right)^6$$

$$\therefore \frac{A_S}{A_C} = \frac{136}{100} \times \left(\frac{100}{106} \right)^6$$

49



50 Difference between CI and SI for n^{th} year

$$= \frac{Pr}{100} \left[\left(1 + \frac{r}{100} \right)^{n-1} - 1 \right]$$

$$7280 = \frac{P \times 20}{100} [(1.2)^3 - 1] \Rightarrow P = 50000$$

Alternatively

	Initially	I st year	II nd year	III rd year	IV th year
SI	10000	12000	14000	16000	18000
CI	10000	12000	14400	17280	20736

CI for 4th year = 20736 – 17280 = 3456

SI for 4th year = 2000

Difference between CI and SI = 1456 for ₹ 10000

So, the difference of ₹ 7280 is for ₹ 50000

51 $\frac{P \times 5 \times 9}{100 \times 12} - \frac{P \times 14 \times 3}{100 \times 12} = 10 \Rightarrow P = ₹ 4000$

$$\text{Now, } \frac{4000}{100 \times 12} [5 \times 9 + 14 \times 3] = ₹ 290$$

52 Let the principal be x , then

$$\frac{\left(\frac{x}{7} \times 4 + \frac{x}{2} \times 5 + \frac{5x}{14} \times 6 \right)}{100} = 730 \Rightarrow x = 14000$$

Alternatively Go through suitable options.

Choose any middlemost option so that if the chosen option is not correct, then you can determine that whether you have to increase or decrease the value of the choices given.

53 $\frac{P_1 \times 2 \times 7x}{100} = \frac{P_2 \times 2 \times 8x}{100} \Rightarrow \frac{P_1}{P_2} = \frac{8}{7}$

$$P \left[\left(1 + \frac{r}{100} \right)^3 - 1 \right] = \left(1 + \frac{r}{100} \right)^3 - 1$$

54 $\frac{\frac{pr}{100}}{\frac{pr}{100}} = \frac{r}{r} = 3.64$

Now, go through options and verify

$$\frac{\left(1 + \frac{20}{100} \right)^3 - 1}{\frac{20}{100}} = \frac{(1.2)^3 - 1}{0.2} = \frac{0.728}{0.2} = 3.64$$

Hence (c) is correct.

55 $\frac{13p}{5} - p = \frac{8p}{5}$

$$\text{Now, } \frac{8}{5}P = \frac{P \times r \times 32}{100} \Rightarrow r = 5\%$$

56 $\frac{1500 \times 3}{100} (r_1 - r_2) = 18 \Rightarrow r_1 - r_2 = 0.4$

✓ Level 02 Higher Level Exercise

1 CI for 2 years = ₹ 756

SI for 2 years = ₹ 720

It means the interest on the interest of the first year = ₹ 36
(= 756 - 720)

This implies that the rate of interest is 10%

as $\frac{36}{360} \times 100 = 10\%$

It means the principal for first year was ₹ 3600

$\therefore \frac{P \times 10 \times 1}{100} = 360 \Rightarrow P = 3600$

Now, $\frac{P \times k \times k}{100} = \text{SI}$, where, $r = t = k$

$$\frac{3600 \times k^2}{100} = 900 \Rightarrow k = 5$$

2 $30000(1 + 1.1 + (1.1)^2) - 30000(1 + 1.1 + 1.2)$

$$\Rightarrow ₹ 300$$

$$30,000 \left(1 + \frac{10}{100}\right)^2 - \frac{30,000 \times 10 \times 2}{100} \Rightarrow ₹ 300$$

3 Interest received from Birbal = $\frac{Pr}{100}$

$$\text{Interest received from Chanakya} = \frac{\left(2 \times \frac{Pr}{100}\right) \times \frac{r}{2}}{100} = P \left(\frac{r}{100}\right)^2$$

4 $100(1.3)^3 = 219.7 \Rightarrow \text{CI} = 119.7$

and $\text{SI} = \frac{100 \times 3 \times 30}{100} = 90$

$\therefore \text{CI is greater than SI by ₹ 29.7} \quad (119.7 - 90)$

$\therefore \% \text{ increase} = \frac{29.7}{90} \times 100 = 33.0\%$

5 The best way is to go through options

$$\frac{2200 \times 4 \times 3}{100} + \frac{1300 \times 6 \times 3}{100} = ₹ 498$$

Hence, the presumed option is correct.

🔗 **NOTE** When we consider ₹ 2200 for 4%, then the rest amount i.e., ₹ 1300 = (3500 - 2200) will be considered automatically for 6%.

🔗 **Alternatively** Average % rate = $\frac{166}{35} \%$

$$\left[\because 498 = \frac{3500 \times r \times 3}{100} \right] \Rightarrow r = \frac{166}{35} \%$$

$$\therefore \begin{array}{ccc} \frac{35}{35} \times 4 & & 6 \times \frac{35}{35} \\ & \searrow \quad \nearrow & \\ & \frac{166}{35} & \\ & \nearrow \quad \searrow & \\ \frac{44}{35} & & \frac{26}{35} \\ 22 & : & 13 \end{array}$$

Thus the ratio of principal at 4% and 6% will be in the ratio of 22 : 13 respectively.

6 $\frac{\text{Decreases in second year}}{\text{Decreases in third year}} = \frac{100}{100 - r} = \frac{10}{9}$

$$\Rightarrow r = 10\%$$

Let the population of vultures 3 years ago be P , then

$$P \left(1 - \frac{10}{100}\right)^3 = 29160 \Rightarrow P = 40000$$

7 On the second year (in terms of CI) is

$$\frac{P \left(1 + \frac{r}{100}\right)^2}{\left(P + \frac{Pr}{100}\right)} = \frac{6}{5} \Rightarrow \left(1 + \frac{r}{100}\right) = \frac{6}{5}$$

$$\Rightarrow r = 20\%$$

8 Balance price to be paid in instalments

$$= ₹ 1150 \quad \dots(i)$$

$$\therefore (1500 - 350) = 1150$$

Now, the total amount for the next 3 instalments at the end of 3rd month will be

$$\left(1150 + \frac{1150 \times r \times 3}{12 \times 100}\right) = \left[400 + \left(400 + \frac{400 \times r \times 1}{100 \times 12}\right) + \left(400 + \frac{400 \times r \times 2}{100 \times 12}\right)\right]$$

$$\left(\frac{46000 + 115r}{40}\right) = \left(1200 + \frac{400 \times 3r}{1200}\right)$$

$$\Rightarrow r = \frac{80}{3} = 26.66\% \quad \dots(ii)$$

9 A : $P = \frac{P \times 4 \times r}{100}$

$$\Rightarrow r = 25\%$$

$$B : P \left(1 + \frac{25}{100}\right)^2 = \frac{25P}{16}$$

$$\text{Again } \frac{25P}{16} \times \frac{2 \times 25}{100} = \frac{25P}{32}$$

Therefore, total amount of A after 4 years = $2P$

$$\text{and total amount of B after 4 years} = \frac{25P}{16} + \frac{25P}{32} = \frac{75P}{32}$$

$$\text{Therefore, difference in amount} = \frac{75P}{32} - 2P = \frac{11P}{32} = 2750$$

$$\Rightarrow P = 8000$$

10 Go through options

$$1.8 + \frac{1.8 \times 6 \times 10}{100} = 1.6 + \frac{1.6 \times 8 \times 10}{100},$$

Hence (d) is correct.

$$\text{🔗 Alternatively } P_1 + \frac{P_1 \times 6 \times 10}{100} = P_2 + \frac{P_2 \times 8 \times 10}{100}$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{9}{8}$$

CHAPTER 08

Time and Work

This chapter is one of the easiest chapters for the students. Even an average student can perform better than that in other chapters. There is basically one concept involved in this chapter *i.e.*, concept of efficiency.

So most of the problems are very similar in their basic characteristic. Almost every aptitude exam asks the problems from this chapter. On an average 2-3 problems from this chapter have been asked in past years in CAT.

As it is very clear to all of us that the work is directly related with time. As one can say if a particular person or machine works for more time then more work will be done and if it devotes less time then it yields less work *i.e.*, output of a machine or person is directly proportional to time, provided he/she maintains his/her efficiency during the work.

8.1 Concept of Efficiency

Suppose a person can complete a particular work in 2 days then we can say that each day he does half of the work or 50% work each day.

Thus it is clear that his efficiency is 50% per day. Efficiency is generally considered with respect to the time. The time can be calculated either in days, hours, minutes or months etc. So if a person completes his work in 4 days, then his efficiency (per day) is 25%.

Since each day he works 1/4th of the total work (*i.e.*, 25% of the total work). I would like to mention that the calculation of percentage and conversion of ratios and fractions into percentage and vice versa is the prerequisite for this chapter.

Now, if a person can complete a work in n days then his one day's work = $1/n$ and this one day's work in terms of percentage is called his efficiency.

Also if a person can complete $\frac{1}{n}$ work in one day, then he can complete the whole work in n days.

Chapter Checklist

- Concept of Efficiency
- Relation Between Efficiency and Time
- Concept of Negative Work
- Applications of Inverse Proportion (or Product Constancy)
- Relation Between Efficiencies
- CAT Test

Relation between work of 1 unit of time and percentage efficiency A person can complete his work in n days, then his one day's work $= 1/n$, his percentage efficiency $= 1/n \times 100$

No. of days/hours etc. required to complete the whole work	Work of 1 day/hour	Percentage efficiency
n	$1/n$	$100/n$
1	$1/1$	100%
2	$1/2$	50%
3	$1/3$	$33.33\% = 33\frac{1}{3}\%$
4	$1/4$	25%
5	$1/5$	20%
6	$1/6$	$16.66\% = 16\frac{2}{3}\%$
7	$1/7$	$14.28\% = 14\frac{2}{7}\%$
8	$1/8$	12.5%
9	$1/9$	$11.11\% = 11\frac{1}{9}\%$
10	$1/10$	10%

This table is very similar to the percentage fraction table given in the chapter of percentage. This table just manifests as a model for efficiency conversion.

Basically for faster and smarter calculation you have to have your percentage calculation very smart. All the problems of this chapter can be solved through two methods :

1. Unitary method

2. Percentage efficiency

Unitary method is generally obsolete in respect to high level aptitude exam of CAT since it involves difficult calculation of LCM each and every time. But when the problems are solved through percentage efficiency it becomes inevitable to save the time which in turn helps to do a few more problems within the stipulated time. I admit that initially it might be difficult to solve for those students who are not so good and confident in percentage and fraction calculation, but a little bit of extra Practice will yield an unexpected result in quicker calculations. Now I have some good examples to show you both the methods of solving the same problems.

Exp. 1) A can do a job in 12 days and B can do the same job in 6 days, in how many days working together they can complete the job?

Solution A's 1 day's work $= \frac{1}{12}$, B's 1 day's work $= \frac{1}{6}$

$$\therefore (A + B)'s \text{ 1 day's work } = \frac{1}{12} + \frac{1}{6} = \frac{3}{12} = \frac{1}{4}$$

$$\therefore \text{Time taken by both to finish the whole work} = \frac{1}{1/4} = 4 \text{ days}$$

Alternatively efficiency of A $= \frac{100}{12} = 8.33\%$

and efficiency of B $= \frac{100}{6} = 16.66\%$

Combined efficiency of A and B both $= 8.33 + 16.66 = 25\%$

$\therefore$ Time taken by both to finish the work (working together)

$$= \frac{100}{25} = 4 \text{ days}$$

NOTE As per my experience, I have found that in this chapter only selected (numerals) numbers are always used and thus there are almost 20 – 25 numbers are frequently used. So one can very easily remember (and calculate) the percentage efficiency but with different combinations to calculate the LCM becomes a very tedious job. Still you can choose your own method, which is comfortable to you.

Exp. 2) A can do a job in 10 days, B can do the same job in 12 days and C can do the same job in 15 days. In how many days they will finish the work together?

Solution A's 1 day's work $= \frac{1}{10}$, B's 1 day's work $= \frac{1}{12}$

C's 1 day's work $= 1/15$

$$(A + B + C)'s \text{ one day's work } = \frac{1}{10} + \frac{1}{12} + \frac{1}{15} = \frac{15}{60} = \frac{1}{4}$$

Since they can complete $\frac{1}{4}$ work in 1 day. So they will finish the whole work in $\frac{1}{1/4} = 4$ days.

Alternatively A's efficiency $= 10\%$

B's efficiency $= 8.33\%$

C's efficiency $= 6.66\%$

Combined efficiency of A, B and C $= 10 + 8.33 + 6.66 = 25\%$

Hence, they will take $\frac{100}{25} = 4$ days to finish the job working together (Since in one day they complete 25% work)

8.2 Relation Between Efficiency and Time

Efficiency is inversely proportional to the time (*i.e.*, number of days, hours, minutes) etc.

For example if A is twice efficient as B, it means, A takes half the time to finish the same job as B requires working alone.

Exp. 1) A takes 16 days to finish a job alone, while B takes 8 days to finish the same job. What is the ratio of their efficiency and who is less efficient.

Solution Since A takes more time than B to finish the same job hence A is less efficient or efficiency of A $= \frac{100}{16} = 6.25\%$

and efficiency of B $= \frac{100}{8} = 12.5\%$

$$\text{ratio of efficiency of } A : B = \frac{1}{16} : \frac{1}{8} = 1 : 2$$

Hence, B is twice efficient as A.

Exp. 2) A is thrice efficient as B and A takes 20 days to do a job, then in how many days B can finish the same job?

Solution Ratio of efficiency of $A : B = 3 : 1$

$$\therefore \text{Ratio of required days of } A : B = \frac{1}{3} : \frac{1}{1} = 1 : 3$$

Now since A takes 20 days. So B will take 60 days to finish the same job.

NOTE Ratio of number of days is equal to the ratio of the reciprocals of efficiency and vice-versa.

Exp. 3) P is thrice as efficient as Q and is therefore able to finish a piece of work in 60 days less than Q . Find the time in which P and Q can complete the work individually.

Solution Efficiency of $P : Q = 3 : 1$

$$\text{Required number of days of } P : Q = 1 : 3$$

i.e., if P requires x days then Q requires $3x$ days

$$\text{but } 3x - x = 60 \Rightarrow 2x = 60 \Rightarrow x = 30 \text{ and } 3x = 90$$

Thus P can finish the work in 30 days and Q can finish the work in 90 days.

Exp. 4) A is twice as good a workman as B and is therefore able to finish a piece of work in 30 days less than B . In how many days they can complete the whole work; working together?

Solution Ratio of efficiency = $2 : 1$ ($A : B$)

$$\text{Ratio of required time} = 1 : 2 (A : B) \Rightarrow x : 2x$$

$$\text{but } 2x - x = 30 \Rightarrow x = 30 \text{ and } 2x = 60$$

now efficiency of $A = 3.33\%$ and efficiency of $B = 1.66\%$

Combined efficiency of A and B together = 5%

$$\therefore \text{time required by } A \text{ and } B \text{ working together to finish the work} \\ = \frac{100}{5} = 20 \text{ days.}$$

NOTE Efficiency $\propto \frac{1}{\text{number of time units}}$

$$\therefore \text{Efficiency} \times \text{time} = \text{constant work}$$

$$\text{Hence, Required time} = \frac{\text{work}}{\text{efficiency}}$$

Whole work is always considered as 1, in terms of fraction and 100%, in terms of percentage.

$$\text{In general, number of day's or hours} = \frac{100}{\text{efficiency}}$$

Exp. 5) A can do a works in x days while B can do the same work in y days then in how many days will they complete the work, working together?

Solution A 's one day's work = $\frac{1}{x}$, B 's one day's work = $\frac{1}{y}$

$$\text{So, both } A \text{ and } B \text{ completes } \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} \text{ work in one day.}$$

Now, by **Unitary method** $\frac{x+y}{xy}$ work can be completed in 1 day

$$\therefore 1 \text{ (means complete) work will be finished in } \frac{1}{\left(\frac{x+y}{xy}\right)} \text{ days}$$

$$= \frac{xy}{x+y} \text{ days}$$

$$(\because \text{Required time to complete the work} = \frac{1}{\text{one day's work}})$$

8.3 Concept of Negative Work

In this case one person works but another destroys it or cancels it. For example Sonu can write 20 pages per hour but his younger sister Rimjhim erases 10 pages per hour which Sonu writes. It means finally Sonu writes 10 pages per hour since each hour his sister erases 10 pages out of 20 written pages. Take an other example :

Exp. 1) A tub can be filled in 20 minutes but there is a leakage in it which can empty the full tub in 60 minutes. In how many minutes it can be filled?

Solution Filling efficiency = 5% $\left(\because 5 = \frac{100}{20}\right)$

$$\text{emptying efficiency} = 1.66\% \quad \left(\because 1.66 = \frac{100}{60}\right)$$

$$\text{Net efficiency} = 5 - 1.66 = 3.33\%$$

$$\therefore \text{Required time to fill the tub} = \frac{100}{3.33} = 30 \text{ minutes}$$

Alternatively In 1 minute tub is filling = $\frac{1}{20}$

$$\text{In 1 minute tub is emptying} = \frac{1}{60}$$

$$\therefore \text{In 1 minute, effective filling of tub} = \frac{1}{20} - \frac{1}{60} = \frac{2}{60} = \frac{1}{30}$$

Since $\frac{1}{30}$ part of tub is filled in 1 minute.

$$\text{Therefore complete tub will be filled in } \frac{1}{1/30} = 30 \text{ minutes}$$

8.4 Applications of Inverse Proportion (or Product Constancy)

As I have already discussed thoroughly the concept of inverse proportion and product constancy in ratio-proportion and it has been widely used in profit loss chapter also.

Since the efficiency or rate of work done in one unit of time (mentioned) is inversely proportional to the time i.e., if the rate of work done is greater then the time required will be less and if the rate of work done is less then the time required for the same amount of work will be more. This product constancy method is limited to the constant work, if the amount of work gets changed, then it does not work, then we have to take help from the unitary method. When more than one man/machine work on a particular work/project, the rate of work is calculated as the strength of workers working in a particular time. So the amount of work is defined in terms of man-days or man-hours or man-days-hours.

Exp. 1) If 20 persons can do a piece of work in 7 days then calculate the number of persons required to complete the work in 28 days

Solution Number of persons $\times$ days = work

$$20 \times 7 = 140 \text{ man-days}$$

$$\text{Now, } x \times 28 = 140 \text{ man-days} \Rightarrow x = 5$$

Therefore in second case the required number of person is 5.

Second Method : Since work is constant, therefore

$$M_1 \times D_1 = M_2 \times D_2 = \text{work done}$$

$$20 \times 7 = M_2 \times 28 \Rightarrow M_2 = 5$$

Third Method : Men $\times$ days = work, which is constant

$$\begin{array}{ccc} & 20 & 7 \\ \downarrow \frac{3}{4} & & \uparrow 3 \\ & 5 & 28 \end{array}$$

Since number of days is increased by 3 times (*i.e.*, 300%). So the number of men will be decreased by $\frac{3}{4}$ times (*i.e.*, 75%)

(remember percentage change graphic)

Exp. 2) If 25 men can do a piece of work in 36 days working 10 hours a day, then how many men are required to complete the work working 6 hours a day in 20 days?

Solution $M_1 \times D_1 \times H_1 = M_2 \times D_2 \times H_2$

$$25 \times 36 \times 10 = M_2 \times 20 \times 6 \Rightarrow M_2 = 75 \text{ persons}$$

Alternatively

$$\begin{array}{ccc} \text{Men} & \times & \text{Time} \\ 25 & & 360 \\ \uparrow 2 & & \downarrow \frac{2}{3} \\ 75 & & 120 \end{array} \quad \begin{array}{l} \text{[Since product (work) is} \\ \text{constant]} \end{array}$$

By percentage change graphic, when time is decreased by $\frac{2}{3}$ (*i.e.*, 66.66%), number of men is increased by 2 times (*i.e.*, 200%)

Exp. 3) A contractor employed 30 men to complete the project in 100 days. But later on he realised that just after 25 days only 20% of the work had been completed.

- How many extra days, than the scheduled time are required?
- To complete the work on the scheduled time how many men he has to increase?
- If the amount of work is also increased by 20% of the actual work, then how many extra days are required (in comparison with scheduled time) but the number of men remained constant.
- How many men should be increased so that the work will be completed in 25 days less than the scheduled time.

Solution

(a) Men $\times$ days = work done

$$30 \times 25 = 750 = 20\% \text{ of the actual work}$$

Now, the work to be done is 4 times than the work done but the number of days is only 3 times. So he is required 4 times the number of days, thus he has to work for extra 25 days.

$$(b) M_1 D_1 = M_2 D_2, 4(30 \times 25) = M_2 \times 75 \Rightarrow M_2 = 40$$

(Since, the work to be done is 4 times of the work done. Hence he requires 4 times man-days.)

Alternatively Since the new product (*i.e.*, work) is 4 times of the original product (*i.e.*, work). But the new product is being multiplied by 3. Thus to make it 4 times we have to multiply it by $\frac{4}{3}$. Thus without changing number of days we get the new value of number of men which is 40 (being multiplied by $\frac{4}{3}$) Therefore he has to increase 10 more men.

(c) New work = 3750 man-days and the available number of men = 30

$$\therefore \text{number of days required} = \frac{3750}{30} = 125$$

So, he has to work for extra 50 days where $50 = (125 - 75)$

(d) Work = 3000 unit (man-days)

$$\text{number of available days} = 50(75 - 25)$$

$$\therefore \text{number of men required} = \frac{3000}{50} = 60$$

Thus he has to increase $(60 - 30) = 30$ more men.

Exp. 4) 16 workers working 6 hours a day can build a wall of length 150 metres, breadth 20 m and height 12 m in 25 days. In how many days 12 workers, working 8 hours a day can build a wall of length 800 m, breadth 15 m and height 6 m.

Solution Here work is the volume of the wall. So the work force should be increased/decreased in the ratio of volume of the work. Therefore

$$\frac{L_1 B_1 H_1}{L_2 B_2 H_2} = \frac{M_1 D_1 T_1}{M_2 D_2 T_2}$$

where L, B, H are length, breadth and height of the wall respectively and M, D, T are men, days and time in hours per day, respectively. 1 indicates the first case, while 2 indicates second case. So the ratio of work force remains constant as the ratio of volume and work.

$$\therefore \frac{150 \times 20 \times 12}{800 \times 15 \times 6} = \frac{16 \times 6 \times 25}{12 \times 8 \times D_2} \Rightarrow D_2 = 50$$

hence the required number of days = 50

Please notice that the volume of work becomes twice (in the second case) so the work force will also be twice to the previous work force.

8.5 Relation Between Efficiencies

In this case the efficiencies of different persons are different but when they work in a group, so the efficiency of the group is required to know the time taken. For example 3 men can do a work in 4 days while 12 boys can do the same work in 3 days. It means we need $3 \times 4 = 12$ man-days *i.e.*, 12 men can finish the job in 1 day.

Similarly we need $12 \times 3 = 36$ boys days *i.e.*, 36 boys can finish the same job in 1 day.

Here we can see that to finish the work in only 1 day 12 men are needed while 36 boys are needed. Thus we can conclude that work of 12 men is equal to the work of 36 boys. Therefore efficiency of 12 men is equal to 36 boys *i.e.*, we can say the efficiency of 1 man is equal to 3 boys.

Thus a man is thrice efficient as a boy or we can say that a man is two times more efficient than a boy.

Exp. 1) 6 boys and 8 women finish a job in 6 days and 14 boys and 10 women finish the same job in 4 days. In how many days working together 1 boy and 1 woman can finish the work?

Solution In this kind of questions we find the work force required to complete the work in 1 day (or given unit of time) then we equate the work force to find the relationship between the efficiencies (or work rate) between the different workers.

Therefore $6B + 8W = 6 \text{ days}$
 $\Rightarrow 6(6B + 8W) = 1 \text{ day}$ (inversely proportional)
 $\Rightarrow 36B + 48W = 1$ (by unitary method)
 Again $14B + 10W = 4 \text{ days}$
 $\Rightarrow 56B + 40W = 1$

So, here it is clear that either we employ 36B and 48W to finish the work in 1 day or 56B and 40W to finish the same job in 1 day. Thus, we can say

$$\begin{aligned}\Rightarrow & 36B + 48W = 56B + 40W \\ \Rightarrow & 20B = 8W \\ \Rightarrow & W = 2.5B\end{aligned}$$

Thus a woman is 2.5 times as efficient as a boy.

$$\begin{aligned}\text{Now, since} \quad & 36B + 48W = 1 \\ \Rightarrow & 36B + 48 \times (2.5B) = 1 \\ \Rightarrow & 156B = 1\end{aligned}$$

i.e., to finish the job in 1 day 156 boys are required or the amount of work is 156 boys-days.

Again $1W + 1B = 2.5B + 1B = 3.5B$

Now, since 156 boys can finish the job in 1 day

So 1 boy can finish the job in 1×156 days

$\therefore$ 3.5 boys can finish the job in

$$\frac{1 \times 156}{3.5} = 44\frac{4}{7} \text{ days}$$

NOTE There is a great difference between 'and' & 'or' For example 4 men and 8 women can do a piece of work in 10 days : means it is unknown that who is faster or slower (*i.e.*, we don't know the relation between efficiencies of a man and a woman). Again, 4 men or 8 women can do a piece of work in 10 days : means 4 men are equal to 8 women. Hence a man is twice efficient as a woman.

Exp. 2) 6 men and 8 women can do a job in 10 days. In how many days can 5 men and 9 women do the same job?

Solution Since we don't know the relation between work rate (or efficiency) between a man and that of a woman so we can't find the required number of days.

Exp. 3) 6 men and 8 women can do a job in 10 days. In how many days can 3 men and 4 women finish the same job working together?

Solution Notice here we don't know the relation of efficiencies but we can solve the problem due to clear relation between the work force.

Since $6M + 8W = 10$ days
 $2(3M + 4W) = 10$ days
 $\Rightarrow (3M + 4W) = 20$ days

Since, the work force has become half of the original force so number of days must be double.

Thus required number of days = 20

Exp. 4) A can complete a work in 12 days, B in 15 days. Find the time taken by them :

- when A and B worked together.
- when A and B worked alternatively started by A .
- when A and B worked alternatively started by B .
- if A started two days later, in comparison to B .
- if B started two days later, in comparison to A .
- if A leaves two days before the actual completion of the work.
- if B leaves two days before the actual completion of the work.
- if A leaves two days before the scheduled completion of the work.
- if B leaves two days before the scheduled completion of the work.
- if B does negative work with his same work rate.

Solution (a) A 's efficiency = 8.33%

B 's efficiency = 6.66%

Combined efficiency = 15%

So, the required time = $\frac{100}{15} = 6\frac{2}{3}$ days.

- (b) In every two days A and B work 15%. So in 12 days they will complete 90% work. Now on the 13th day, A will finish 8.33% of the remaining (*i.e.*, 10% work) and the rest 1.66% will be finished by B on 14th day by taking time $= \frac{1.66}{6.66} = \frac{1}{4}$ day.

Thus, total required time = $12 + 1 + \frac{1}{4} = 13\frac{1}{4}$ days.

See the chart below :

Day 1 2 3 4 5 6 7 8 9 10 11 12 13 14
 Work 8.33 6.66 8.33 6.66 8.33 6.66 8.33 6.66 8.33 6.66 8.33 6.66 8.33 1.66

$\underbrace{\hspace{1.5cm}}_{15}$ $\underbrace{\hspace{1.5cm}}_{15}$ $\underbrace{\hspace{1.5cm}}_{15}$ $\underbrace{\hspace{1.5cm}}_{15}$ $\underbrace{\hspace{1.5cm}}_{15}$ $\underbrace{\hspace{1.5cm}}_{15}$

$\underbrace{\hspace{10.5cm}}_{15 \times 6 = 90\%}$ $\underbrace{\hspace{2.5cm}}_{10\%}$

- (c) Here the difference is in the last day/days only.

See the chart below :

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Work	6.66	8.33	6.66	8.33	6.66	8.33	6.66	8.33	6.66	8.33	6.66	8.33	6.66	3.33
	15		15		15		15		15		15		10	
	$15 \times 6 = 90\%$												10%	

So, after the completion of 90% work in 12 days, on the 13th day 6.66% work will be finished by B and then 3.33% (the last part of the work) will be finished by A by taking time = $\frac{3.33}{8.33} = \frac{2}{5}$ day.

Thus, the required time = $12 + 1 + \frac{2}{5} = 13\frac{2}{5}$ days.

- (d) It means B worked for 2 days more than A has worked for. Now, since B completes $2 \times 6.66 = 13.33\%$ work in two days. So the remaining work will be done by A and B together.

Therefore the time taken by A and B , working together

$$= \frac{86.66}{15} \frac{\text{(remaining work)}}{\text{(combined efficiency)}} = 5\frac{7}{9} \text{ days}$$

Thus the total required time to finish the whole work

$$= 2 + 5\frac{7}{9} = 7\frac{7}{9} \text{ days.}$$

- (e) It means A alone worked for 2 days which is equal to $8.33 \times 2 = 16.66\%$

So, the remaining (83.33%) work will be done by A and B together.

$\therefore$ Time taken together = $\frac{83.33}{15} = 5\frac{5}{9}$ days

Thus, the total required time = $2 + 5\frac{5}{9} = 7\frac{5}{9}$ days.

- (f) 'A leaves two days before the actual completion of the work' means in the last two days B has worked alone which is $6.66 \times 2 = 13.33\%$

Therefore the remaining work i.e., 86.66% work was only done by A and B together in $\frac{86.66}{15} = 5\frac{7}{9}$ days

Thus, the required time = $2 + 5\frac{7}{9} = 7\frac{7}{9}$ days.

NOTE This case is exactly same as part (d) of the same example. The difference is only in order of days. In part (d) B worked alone initially for 2 days and in part (f) B worked alone in the last 2 days.

- (g) Applying the same concept as above we can calculate the required number of days which is same as in case (e)

Work	$A + B$	A
	$15 \times 5\frac{2}{5}$	8.33×2
	83.33%	16.66%

- (h) 'A leaves two days before the scheduled completion of the work' means A works with B only for $4\frac{2}{3}$ days since the scheduled time is $6\frac{2}{3}$ [see the case (a)]

Scheduled time is the time in which work could be finished by working together without any change in efficiency or work force. Now since both A and B have worked together for $4\frac{2}{3}$ days it means the last 2 days

work (scheduled) which is equal to 30% (since if A and B working together can finish 30% work in 2 days) will be done by B only.

$\therefore$ required time to complete 30% work by B alone

$$= \frac{30}{6.66} = 45 \text{ days}$$

Thus the total required time to finish the whole work

$$= 4\frac{2}{3} + 4\frac{1}{2} = 9\frac{1}{6} \text{ days.}$$

- (i) Very similar to the case (h) A and B worked only for $4\frac{2}{3}$ days (2 days less than the scheduled time $6\frac{2}{3}$ days)

(It is obvious that in $4\frac{2}{3}$ days A and B , working together

completes 70% work, but we can directly calculate that they can complete 30% if they work together, in 2 days)

Thus 30% work is done by A alone

$\therefore$ time taken = $\frac{30}{8.33} = 3\frac{3}{5}$ days

Thus, the total time = $4\frac{2}{3} + 3\frac{3}{5} = 8\frac{4}{15}$ days.

- (j) Combined efficiency of A and $B = (8.33) + (-6.66)$
 $= 1.66\%$

$\therefore$ Time required by A and B working together

$$= \frac{100}{1.66} = 60 \text{ days}$$

(Negative work means, B works against A)

Exp. 5) A can complete a work in 10 days, B in 12 days and C in 15 days. All of them began the work together, but A had to leave the work after 2 days of the start and B 3 days before the completion of the work. How long did the work last?

Solution See the diagram and then interpret the language of the question.

Initial 2 days	Last 3 days	
$A + B + C$	$B + C$	C
25×2 $= 50\%$	$15\% \times 2$ $[100 - (50 + 20)]$	6.66×3 $= 20\%$

Since initially for 2 days all of them A , B and C work together so they complete the 50% work and for the last 3 days only C works which is equal to 20% work.

Thus, the remaining work = $30\% [100 - (50 + 20)]$

This 30% work was done by B and C in 2 days = $\left(\frac{30}{15}\right)$.

NOTE

Efficiency of $A = 10\%$
 Efficiency of $B = 8.33\%$
 Efficiency of $C = 6.66\%$

So, the total number of required days = $2 + 2 + 3 = 7$ days.

CAT-Test

Questions Helping you bell the CAT

LEVEL 01 > BASIC LEVEL EXERCISE

- 1 A can finish a piece of work in 12 days while B can do it in 15 days. If both work at it together, what time will they take to do the work?
(a) 6 days (b) 8 days
(c) $6\frac{2}{3}$ days (d) $10\frac{1}{5}$ days
- 2 A can do a piece of work in 10 days, B can do it in 12 days and C can do it in 15 days. In how many days can they complete the work, working together?
(a) 6 (b) 4 (c) 5 (d) 10
- 3 A can do a piece of work in 5 days, B can do it in 10 days. With the help of C, they finish the work in 2 days. In how many days can C alone do the whole work?
(a) 3 (b) 4 (c) 5 (d) 6
- 4 A can do a piece of work in 8 days, B can do it in 16 days, while C can do it in 80 days. In how many days can they complete the whole work, working together?
(a) 5 (b) 6 (c) $8\frac{2}{3}$ (d) $20\frac{2}{5}$
- 5 A can do a piece of work in 10 days, B can do it in 15 days working together they can finish the work in :
(a) 9 (b) 8 (c) 10 (d) 6
- 6 A can do a piece of work in 24 days, while B can do it in 30 days. With the help of C they can finish the whole work in 12 days. How much time is required for C to complete the work, alone?
(a) 100 days (b) 120 days
(c) 125 days (d) 72 days
- 7 A can do a piece of work in 10 days. B can do it in 24 days. If C also works with them then it takes only 6 days to complete the whole work. In how many days can C alone complete the whole work?
(a) 25 (b) 40 (c) 50 (d) 75
- 8 A can do a piece of work in 14 days while B can do it in 21 days. In how many days, working together will they complete the whole work?
(a) 10.5 (b) 8 (c) 8.4 (d) 9
- 9 A can do a piece of work in 24 days. If B is 60% more efficient than A, then the number of days required by B to do the same piece of work is :
(a) 10 (b) 15 (c) 12 (d) 9.6
- 10 A is twice as good a workman as B and together they finish a piece of work in 14 days. In how many days can A alone finish the work?
(a) 18 (b) 20 (c) 21 (d) 24
- 11 A is twice as good a workman as B and together they finish a piece of work in 7 days. In how many days can A alone finish the work?
(a) 10 (b) 10.5 (c) 12 (d) 11
- 12 A is thrice as good a workman as B and takes 10 days less to do a piece of work than B takes. The number of days taken by B to finish the work is :
(a) 9 (b) 18 (c) 15 (d) 12
- 13 A is twice as good a workman as B and therefore A takes 6 days less than B to finish the work individually. If A and B working together complete the work in 4 days, then how many days are required by B to complete the work alone?
(a) 12 (b) 18 (c) 8 (d) 6
- 14 A is thrice efficient as B and C is twice as efficient as B. What is the ratio of number of days taken by A, B and C, when they work individually?
(a) 2 : 6 : 3 (b) 2 : 3 : 6 (c) 1 : 2 : 3 (d) 3 : 1 : 2
- 15 Ajit is 3 times as efficient as Bablu, then the ratio of number of days required by each to work alone, completely?
(a) 3 : 1 (b) 1 : 3
(c) 6 : 3 (d) 3 : 6
- 16 X is 4 times more efficient as Y, then the number of days taken by X and Y are in the ratio :
(a) 8 : 1 (b) 1 : 5 (c) 6 : 8 (d) 5 : 1
- 17 If A takes $\frac{4}{5}$ days as B takes and working together they require $\frac{20}{3}$ days to complete the whole work. What is the efficiency of B ?
(a) $6\frac{2}{3}\%$ (b) 16% (c) 5.55% (d) $8\frac{1}{3}\%$
- 18 A is thrice as efficient as B. Working together they complete the work in 3 days. If B takes 8 days more than A, what is the number of days taken by A to finish the whole work, alone?
(a) 4 (b) 2
(c) 12 (d) 16

- 19** A and B can do a piece of work in 8 days, B and C can do the same work in 12 days and A and C complete it in 8 days. In how many days can A, B and C complete the whole work, working together?
 (a) 4 (b) 6
 (c) 12 (d) 9
- 20** A and B can do a piece of work in 12 days, B and C in 15 days C and A in 20 days. In how many days can C alone do it?
 (a) 60 (b) 50
 (c) 25 (d) 24
- 21** Ganga, Jamuna and Saraswati can do a piece of work, working together, in 1 day. Ganga is thrice efficient as Jamuna and Jamuna takes the twice the number of days as Saraswati takes to do it alone. What is the difference between the number of days taken by Ganga and Saraswati?
 (a) 1 (b) 2
 (c) 3 (d) 4
- 22** A can finish a work in 12 days and B can do it in 15 days. After A had worked for 3 days, B also joined A to finish the remaining work. In how many days, the remaining work will be finished?
 (a) 3 (b) 4
 (c) 5 (d) 6
- 23** Raja can do a piece of work in 14 days, while Rani can do the same work in 21 days. They started the work together but 3 days before the completion of the work, Raja left the work. The total number of days to complete the work is :
 (a) 7 (b) 8.5
 (c) 5 (d) $7\frac{1}{5}$
- 24** A and B can complete a task in 30 days when working together after A and B have been working together for 11 days, B is called away and A, all by himself completes the task in the next 28 days. Had A been working alone, the number of days taken by him to complete the task would have been :
 (a) $33\frac{3}{19}$ (b) $19\frac{6}{25}$
 (c) $44\frac{4}{19}$ (d) none of these
- 25** Sonu can do a piece of work in 20 days. He started the work and left after some days, when 25% work was done. After it Abhijeet joined and completed it working for 10 days. In how many days Sonu and Abhijeet can do the complete work, working together?
 (a) 6 (b) 8
 (c) 10 (d) 12
- 26** Efficiency of Asha is 25% more than Usha and Usha takes 25 days to complete a piece of work. Asha started a work alone and then usha joined her 5 days before actual completion of the work. For how many days did Asha work alone?
 (a) 9 (b) 11 (c) 10 (d) 15
- 27** Krishna can do a work in 10 days while Mohan can do the same work in 20 days. They started work together. After 3 days Krishna left the work and Mohan completed it. For how many days Mohan worked alone more than the number of days required when both worked together?
 (a) $4\frac{1}{3}$ (b) $3\frac{1}{4}$ (c) $2\frac{3}{5}$ (d) $3\frac{2}{3}$
- 28** Kareena can do a piece of work in 9 days and Karishma can do the same work in 18 days. They started the work. After 3 days Shahid joined them, who can complete alone the same whole work in 3 days. What is the total number of days in which they had completed the work?
 (a) 12 (b) 8
 (c) 4 (d) 6
- 29** Kavita, Babita and Samita started a work. 5 days later Samita left the work and Babita left the work after working 8 days. In how many more days Kavita would have completed the rest work if they take 20, 60 and 30 days individually to finish a work?
 (a) 4 (b) 5 (c) 6 (d) 8
- 30** The ratio of efficiency of A is to C is 5 : 3. The ratio of number of days taken by B is to C is 2 : 3. A takes 6 days less than C, when A and C completes the work individually. B and C started the work and left after 2 days. The number of days taken by A to finish the remaining work is :
 (a) 4.5 (b) 5 (c) 6 (d) $9\frac{1}{3}$
- 31** Anand can do a piece of work in 45 days, but Bahuguna can do the same work in 5 days less, than Anand, when working alone. Anand and Bahuguna both started the work together but Bahuguna left after some days and Anand finished the remaining work in 56 days with half of his efficiency but he did the work with Bahuguna with his complete efficiency. For how many days did they work together?
 (a) 6 (b) 8
 (c) 9 (d) 12
- 32** Chandni and Divakar can do a piece of work in 9 days and 12 days respectively. If they work for a day alternatively, beginning with Chandni, then in how many days will the work gets completed ?
 (a) $10\frac{1}{4}$ (b) $9\frac{1}{5}$ (c) 11.11 (d) 10
- 33** Fatima and Zahira can do a piece of work in 12 days and 15 days respectively. If they work for alternate day and Fatima starts the work first, then in how many days will the work gets completed ?
 (a) $12\frac{1}{5}$ (b) $13\frac{1}{4}$ (c) $13\frac{1}{5}$ (d) 15
- 34** In the previous question if Zahira starts first, then in how many days will the work gets completed ?
 (a) $14\frac{1}{5}$ (b) 14
 (c) $13\frac{1}{5}$ (d) $13\frac{2}{5}$

- 35** The number of days required by A , B and C to work individually is 6, 12 and 8 respectively. They started a work doing it alternatively. If A has started then followed by B and so on, how many days are needed to complete the whole work?
(a) 8 (b) 7.5 (c) 8.5 (d) $9\frac{1}{2}$
- 36** In the previous question if the order of working days be as $B, C, A, B, C, A \dots$ (starting with B and followed by C and A respectively), then in how many days the work will be completed?
(a) $7\frac{3}{4}$ (b) 8 (c) $8\frac{1}{4}$ (d) 9
- 37** A takes 6 days less than B to do a certain job and 2 days more than C . A and B together can do the work in the same time as C . In how many days B alone can do the complete work?
(a) 10 (b) 14 (c) 12 (d) 16
- 38** A and B undertook a work for ₹ 350. A got ₹ 150 more than that of B , when they worked together. B takes 9 days more than A , when they work individually. In how many days A and B working together can do the whole work :
(a) 5 (b) $4\frac{2}{7}$ (c) $4\frac{5}{7}$ (d) $5\frac{4}{7}$
- 39** Alen and Border can do a work individually in 21 and 42 days respectively. In how many days they can complete the work, working alternatively?
(a) 14 (b) 28 (c) 42 (d) 35
- 40** C takes twice the number of days to do a piece of work than A takes. A and B together can do it in 6 days while B and C can do it in 10 days. In how many days A alone can do the work?
(a) 60 (b) 30
(c) 6 (d) 7.5
- 41** When A , B and C are deployed for a task, A and B together do 70% of the work and B and C together do 50% of the work. Who is most efficient?
(a) A (b) B
(c) C (d) can't be determined
- 42** Colonel, Major and General started a work together for ₹ 816. Colonel and Major did $\frac{8}{17}$ of the total work, while Major and General together did $\frac{12}{17}$ of the whole work.
What is the amount of the least efficient person?
(a) ₹ 256 (b) ₹ 144
(c) ₹ 85 (d) can't be determined
- 43** Sharma is 20% less efficient than Kelkar. If Kelkar can do a piece of work in 24 days. The number of days required by Sharma to complete the same work alone?
(a) 20 (b) 30
(c) 28.8 (d) can't be determined
- 44** 30 persons can do a piece of work in 24 days. How many more people are required to complete the work in 20 days?
(a) 4 (b) 5
(c) 6 (d) none of these
- 45** 12 women can do a piece of work in 20 days. If the 4 women deny to work, then how many more days are required?
(a) 6 (b) 10
(c) 15 (d) none of these
- 46** 35 boys can do a piece of work in 15 days. The work was completed in 25 days. How many boys did not turn up for the job?
(a) 14 (b) 20
(c) 6 (d) 7
- 47** 24 men can complete a job in 40 days. The number of men required to complete the job in 32 days is :
(a) 30 (b) 40
(c) 25 (d) 50
- 48** 16 men finished one-third work in 6 days. The number of additional men are required to complete the job in next 6 days :
(a) 10 (b) 8
(c) 16 (d) 32
- 49** If 10 persons can do a job in 20 days, then 20 person with twice the efficiency can do the same job in :
(a) 5 days (b) 40 days
(c) 10 days (d) 20 days
- 50** A certain job was assigned to a group of men to do in 20 days. But 12 men did not turn up for the job and the remaining men did the job in 32 days. The original number of men in the group was :
(a) 32 (b) 36
(c) 42 (d) 40
- 51** 30 workers can finish a work in 20 days. After how many days should 9 workers leave the job so that the work is completed in total 26 days :
(a) 12 (b) 10
(c) 6 (d) none of these
- 52** 25 men can complete a job in 30 days. After how many days should the strength of work force be increased by 50 men so that the work will be completed in $\frac{2}{3}$ rd of the actual time:
(a) 15 (b) 10
(c) 18 (d) 5
- 53** A group of workers can complete a job in 120 days. If there were 4 more such workers then the work could be finished in 12 days less. What was the actual strength of workers?
(a) 30 workers (b) 40 workers
(c) 42 workers (d) 36 workers

- 54** Mr. Modi can copy 40 pages in 10 minutes, Mr Xerox and Mr. Modi both working together can copy 250 in 25 minutes. In how many minutes Mr. Xerox can copy 36 pages?
 (a) 5 minutes (b) 6 minutes
 (c) 3 minutes (d) 12 minutes
- 55** 20 persons completed $\frac{1}{3}$ rd of the work in 12 days. How many more person are required to finish the rest work in next 12 days?
 (a) 20 (b) 12 (c) 18 (d) 40
- 56** A contractor undertook a work to complete in 60 days. But just after 20 days he observed that only $\frac{1}{5}$ th of the project work had been completed. To complete the work in time (i.e., in rest days) minimum how many workers he had to increase, if there were initially 75 workers were deployed for the task?
 (a) 25 (b) 50
 (c) 75 (d) can't be determined
- 57** 6 men or 10 women can reap a field in 15 days, then the number of days that 12 men and 5 women will take to reap the same field is :
 (a) 5 (b) 6 (c) 8 (d) 12
- 58** If 2 men or 3 women or 4 boys can do a piece of work in 52 days, then the same piece of work will be done by 1 man, 1 woman and 1 boy in :
 (a) 48 days (b) 36 days
 (c) 45 days (d) none of these
- 59** 2 men or 5 women or 7 boys can finish a work in 469 days, then the number of days taken by 7 men, 5 women and 2 boys to finish the work is :
 (a) 134 (b) 106
 (c) 100 (d) 98
- 60** 6 children and 2 men complete a certain piece of work in 6 days. Each child takes twice the time taken by a man to finish the work. In how many days will 5 men finish the same work?
 (a) 6 (b) 8
 (c) 9 (d) 15
- 61** 2 men and 3 women finish 25% of the work in 4 days, while 6 men and 14 women can finish the whole work in 5 days. In how many days will 20 women finish it?
 (a) 20 (b) 25
 (c) 24 (d) 88

Directions (for Q. Nos. 62 and 63) *A can do a work in 15 days and B can do it in 18 days. With the help of C, all of them complete the work in 6 days.*

- 62** How long will it take C to finish the work alone?
 (a) 30 days (b) 22 days
 (c) $\frac{45}{2}$ days (d) 25 days

- 63** A, B and C received total ₹ 27,000 for the whole work. What is the share of C, if the money is distributed in the ratio of amount of work done, individually?
 (a) 2700 (b) 7200
 (c) 14400 (d) 6300

Directions : *Solve the following questions individually.*

- 64** 314 weavers weaves 6594 shawls in $1/6$ hours. What is the number of shawls weaved per hour by an average weaver?
 (a) 42 (b) 21 (c) 102 (d) 126
- 65** Three men and two women can do a piece of work in 4 days, while two men and three women can do the same work in 5 days. ₹ 44 is given to a woman for her contribution towards work, per day. What is the amount received by a man per day?
 (a) ₹ 88 (b) ₹ 144
 (c) ₹ 154 (d) can't be determined
- 66** 30 girls can finish a work in 40 days. After how many days should 10 girls leave the work so that it may be finished in 46 days, if all the 30 girls started the work?
 (a) 18 days (b) 28 days
 (c) 22 days (d) 30 days
- 67** If 8 women collect 200 kg of tea leaves in 10 hours. How many more (in kg) of tea leaves will 12 women collect in 8 hours?
 (a) 24 kg (b) 40 kg
 (c) 50 kg (d) 100 kg
- 68** 450 man-days of work can be completed by a certain number of men in some days. If the number of people (men) are increased by 27, then the number of day required to complete the same work is decreased by 15. The number of days are required to complete the three times work (than the previous/actual work) by 27 men?
 (a) 50 (b) 60 (c) 54 (d) 45
- 69** 4 boys and 5 girls can do a piece of work in 10 days. 6 boys and 6 girls can do the same work in 7 days. In how many days can 2 boys and 7 girls complete the same work, working together?
 (a) 15 days (b) 14 days
 (c) 21 days (d) 18 days
- 70** If 20 engineers and 20 workers can together construct a 20 km road in 20 days. 40 engineers and 40 workers together construct 40 km road in how many days?
 (a) 10 (b) 20
 (c) 40 (d) can't be determined
- 71** $(x - 2)$ men can do a piece of work in x days and $(x + 7)$ men can do 75% of the same work in $(x - 10)$ days. Then in how many days can $(x + 10)$ men finish the work?
 (a) 27 days (b) 12 days
 (c) 25 days (d) 18 days

- 72** A man, a woman and a girl worked for a contractor for the same period. A man is twice efficient as a woman and a woman is thrice efficient as a girl ₹ 10000 were given to all of them. What is the sum of money received by a woman and a girl together?
 (a) ₹ 5500 (b) ₹ 4500
 (c) ₹ 4000 (d) ₹ 6000
- 73** 33 men can do a job in 30 days. If 44 men started the job together and after every day of the work, one person leaves. What is the minimum number of days required to complete the whole work?
 (a) 21 (b) 42
 (c) 45 (d) none of these
- 74** Abhishek can do a piece of work in 40 days. He alone worked at it for 8 days and then Bacchhan completed alone the rest work in 24 days. In how many days they will complete the whole work, working together?
 (a) $17\frac{1}{7}$ days (b) $18\frac{1}{7}$ days
 (c) $9\frac{1}{6}$ days (d) 14 days
- 75** C is twice efficient as A. B takes thrice as many days as C. A takes 12 days to finish the work alone. If they work in pairs (i.e., AB, BC, CA) starting with AB on the first day then BC on the second day and AC on the third day and so on, then how many days are required to finish the work?
 (a) $6\frac{1}{5}$ days (b) 4.5 days
 (c) $5\frac{1}{9}$ days (d) 8 days
- 76** Ahluwalia and Bimal together take 6 days to finish the work. Bimal and Jalan together take 10 days to finish the work. What is the difference between number of days taken by Ahluwalia and Jalan when they worked alone to complete the whole work?
 (a) 12 days
 (b) 16 days
 (c) 15 days
 (d) can't be determined
- 77** B is twice efficient as A and A can do a piece of work in 15 days. A started the work and after a few days B joined him. They completed the work in 11 days, from the starting. For how many days they worked together?
 (a) 1 day (b) 2 days
 (c) 6 days (d) 5 days
- 78** A, B and C can complete a piece of work in 15, 30 and 40 days respectively. They started the work together and A left 2 days before the completion of the work and B left 4 days before the completion of the work. In how many days was the work completed?
 (a) $7\frac{3}{10}$ (b) $10\frac{2}{15}$
 (c) $10\frac{7}{30}$ (d) none of these
- 79** There was a leakage in the container of the refined oil. If 11 kg oil is leaked out per day then it would have lasted for 50 days, if the leakage was 15 kg per day, then it would have lasted for only 45 days. For how many days would the oil have lasted, if there was no leakage and it was completely used for eating purpose?
 (a) 80 days (b) 72 days
 (c) 100 days (d) 120 days
- 80** A contractor undertook to complete the work in 40 days and he deployed 20 men for his work. 8 days before the scheduled time he realised that $\frac{1}{3}$ rd of the work was still to be done. How many more men were required to complete the work in stipulated time?
 (a) 16 (b) 15 (c) 20 (d) 25
- 81** 7 Indian and 4 Chinese finish a job in 5 days. 7 Japanese and 3 Chinese finish the same job in 7 days. Given that the efficiency of each person of a particular nationality is same but different from others. One Indian, one Chinese and one Japanese will complete the work in :
 (a) $18\frac{3}{13}$ days (b) $20\frac{5}{12}$ days
 (c) $21\frac{6}{14}$ days (d) $20\frac{7}{12}$ days
- 82** A, B and C are three book binders. A takes 8 minutes, B takes 12 minutes and C takes 16 minutes to bind a book. If they work each day for 12 hours, then on an average, how many books each one bind per day?
 (a) 65 (b) 52 (c) 48 (d) 70
- 83** A piece of work can be completed by 10 men and 6 women in 18 days. Men works 9 hours per day while women works 7.5 hours per day. Per hour efficiency of a woman is $\frac{2}{3}$ rd of a man's efficiency. In how many days 10 men and 9 women complete the work?
 (a) 16 days (b) 20 days (c) 30 days (d) 25 days
- 84** B and C are equally efficient, but the efficiency of A is half of each B and C. A and B started a work and 3 days later C joined them. If A alone can do the work in 14 days, then in how many more days the work will be completed?
 (a) 1 (b) 2 (c) 3 (d) 4.5
- 85** A can do a piece of work in 10 days, B in 15 days. They work together for 5 days, the rest of the work is finished by C in two more days. If they get ₹ 3000 as wages for the whole work, what are the daily wages of A, B and C respectively (in ₹) :
 (a) 200,250,300 (b) 300,200,250
 (c) 200,300,400 (d) none of these
- 86** A can do a piece of work in 2 hours, B can do thrice the work in 8 hours and 'C' can do the same work as A in 8 hours. If all of them work together, how long it would take them to complete the work :
 (a) 1 hour (b) 2 hours
 (c) 3 hours (d) 4 hours

- 87** A is twice efficient as B and together they do the same work in as much time as C and D together. If C and D can complete the work in 20 and 30 days respectively, working alone, then in how many days A can complete the work individually :
 (a) 12 days (b) 18 days (c) 24 days (d) 30 days
- 88** 4 men and 2 boys can finish a piece of work in 5 days. 3 women and 4 boys can finish the same work in 5 days. Also 2 men and 3 women can finish the same work in 5 days. In how many days 1 man, 1 woman and one boy can finish the work, at their double efficiency?
 (a) $4\frac{8}{13}$ (b) $4\frac{7}{13}$
 (c) $3\frac{7}{13}$ (d) none of these
- 89** If m men can do a work in r days, then the number of days taken by $(m + n)$ men to do it is :
 (a) $\frac{m+n}{mn}$ (b) $\frac{m+n}{mr}$ (c) $\frac{mr}{(m+n)}$ (d) $\frac{(m+n)r}{mn}$
- 90** Pipe A can fill a tank in 36 minutes and pipe B can fill it in 45 minutes. If both the pipes are opened to fill an empty tank, in how many minutes will it be full?
 (a) 15 (b) 18 (c) 20 (d) 25
- 91** Tap A can fill the empty tank in 12 hours, but due to a leak in the bottom it is filled in 15 hours. If the tank is full and then tap A is closed then in how many hours the leak can empty it?
 (a) 45 hours (b) 48 hours
 (c) 52 hours (d) 60 hours
- 92** Pipe A and B can fill a cistern in 10 hours and 15 hours respectively. When a third pipe C which works as an outlet pipe is also open then the cistern can be filled in 18 hours. The outlet pipe can empty a full cistern in :
 (a) 12 hours (b) 8 hours
 (c) 9 hours (d) 14 hours
- 93** A cistern has a leak which would empty it in 6 hours. A tap is turned on which fills the cistern @ 10 liters per hour and then it is emptied in 15 hours. What is the capacity of the cistern?
 (a) 100 litres (b) 166.66 litres
 (c) 60.66 litres (d) none of these
- 94** Tap A fills a tank in 10 hours and B can fill it in 15 hours. Both are opened simultaneously. Sometimes later tap B was closed, then it takes total 8 hours to fill up the whole tank. After how many hours B was closed?
 (a) 2 (b) 3
 (c) 4 (d) 5
- 95** Tap A can fill a tank in 20 hours, B in 25 hours but tap C can empty a full tank in 30 hours. Starting with A, followed by B and C each tap opens alternatively for one hour period till the tank gets filled up completely. In how many hour the tank will be filled up completely?
 (a) $51\frac{11}{15}$ (b) $52\frac{2}{3}$
 (c) $24\frac{4}{11}$ (d) none of these
- 96** If one pipe A can fill a tank in 20 minutes, then 5 pipes, each of 20% efficiency of A, can fill the tank in :
 (a) 80 min (b) 100 min
 (c) 20 min (d) 25 min
- 97** Pipe A basically used as inlet pipe and pipe B is used as outlet pipe. Pipes A and B both are opened simultaneously, all the time. When pipe A fills the tank and B empty the tank, it will take double the time than when both the pipe fill the tank. When pipe B is used for filling the tank, its efficiency remains constant. What is the ratio of efficiency of pipe A and pipe B respectively?
 (a) 3 : 1 (b) 5 : 2
 (c) 1 : 3 (d) 3 : 2
- 98** Pipe A can fill the tank in 4 hours, while pipe B can fill it in 6 hours working separately. Pipe C can empty whole the tank in 4 hours. He opened the pipe A and B simultaneously to fill the empty tank. He wanted to adjust his alarm so that he could open the pipe C when it was half-filled, but he mistakenly adjusted his alarm at a time when his tank would be $\frac{3}{4}$ th filled. What is the time difference between both the cases, to fill the tank fully
 (a) 48 min (b) 54 min
 (c) 30 min (d) none of these
- 99** Two pipes A and B can fill a cistern in 15 hours and 10 hours respectively. A tap C can empty the full cistern in 30 hours. All the three taps were open for 2 hours, when it was remembered that the emptying tap had been left open. It was then closed. How many hours more would it take for the cistern to be filled?
 (a) 30 min (b) 1.2 hours
 (c) 24 min (d) 35 min
- 100** Pipe A can fill an empty tank in 30 hours while B can fill it in 45 hours. Pipe A and B are opened and closed alternatively i.e., first pipe A is opened, then B, again A and then B and so on for 1 hour each time without any time lapse. In how many hours the tank will be filled when it was empty, initially?
 (a) 36 (b) 54 (c) 48 (d) 60

LEVEL 02 > HIGHER LEVEL EXERCISE

- 1 A, B and C three weavers have to supply an order of 100 shawls. A can weave a shawl in 2 hours, B in 3 hours and C in 4 hours respectively. It is known that even being a joint contract each one weaves his own shawl completely i.e., no other weaver help to the rest weavers. In how many hours they will complete the order irrespective of day or night?
 - (a) 93 hours
 - (b) 100 hours
 - (c) $92\frac{4}{13}$ hours
 - (d) 94 hours
 - 2 Arun and Satyam can complete a work individually in 12 working days and 15 working days respectively with their full efficiencies. Arun does work only on Monday, Wednesday and Friday while Satyam does the work on Tuesday, Thursday and Saturday. Sunday is always off. But Arun and Satyam both works with half of their efficiencies on Friday and Saturday respectively. If Arun started the work on 1st January which falls on Monday followed by Satyam on the next day and so on (i.e., they work collectively in alternate days), then on which day work will be completed?
 - (a) Tuesday
 - (b) Wednesday
 - (c) Thursday
 - (d) Friday
 - 3 Kaushalya can do a work in 20 days, while Kaikeyi can do the same work in 25 days. They started the work jointly. Few days later Sumitra also joined them and thus all of them completed the whole work in 10 days. All of them were paid total ₹ 700. What is the share of Sumitra?
 - (a) ₹ 130
 - (b) ₹ 185
 - (c) ₹ 70
 - (d) can't be determined
 - 4 A and B can complete the work individually in 24 days and 30 days respectively, working 10 hours a day. Work is to be done in two shift. Morning shift lasts for 6 hours and evening shift lasts for 4 hours. On the first day A works in the morning shift while B works in the evening shift. Next day A works in the evening shift while B works in the morning shift and so on. It means they work alternatively with respect to their shifts. Thus they work on this pattern till the work is completed. On which day the work got completed?
 - (a) 26th day
 - (b) 27th day
 - (c) 28th day
 - (d) 30th day
 - 5 In the previous question what per cent of the time of the morning shift was utilised?
 - (a) 33.33%
 - (b) 60%
 - (c) 100%
 - (d) none of these
 - 6 Working together B and C take 50% more number of days than A, B and C together take and A and B working together, take $\frac{8}{3}$ more number of days than A, B and C take together. If A, B and C all have worked together till the completion of the work and B has received ₹ 120 out of the total earning of ₹ 450, then in how many days did A, B and C together complete the whole work?
 - (a) 10
 - (b) 6
 - (c) 4
 - (d) 2
 - 7 At Technosys Pvt Ltd. there are some engineering students employed as trainee engineers, belong to two eminent institutions of India. One group belong to MIT and another to NIT. Each student of MIT works for 10 hours a day till 60 days and each student of NIT works for 8 hours till 80 days on the two same projects. The ratio of number of students of MIT and that of NIT is 4 : 5 respectively. Students of which institution is slower in work and by how much?
 - (a) Each student of MIT is 20% less efficient than that of NIT
 - (b) Each student of NIT is 33.33% less efficient than that of MIT
 - (c) Each student of NIT is 25% less efficient than that of MIT
 - (d) Each student of MIT is 33.33% less efficient than that of NIT
- Directions** (for Q. Nos 8 to 10) *At Arihant Prakashan every book goes through 3 phases (or stages) typing, composing and binding. There are 16 typists, 10 composers and 15 binders. A typist can type 8 books in each hour, a composer can compose 12 books in each hour and a binder can bind 12 books in each hour. All of the people at Arihant Prakashan works for 10 hours a day and each person is trained to do only the job of 1 category.*
- 8 How many books can be prepared in each day?
 - (a) 1500
 - (b) 1200
 - (c) 1440
 - (d) 1380
 - 9 If company has hired 12 more people, who can do any of the three jobs, then maximum how many books can be prepared in each day?
 - (a) 1500
 - (b) 1680
 - (c) 1800
 - (d) more than 2000
 - 10 If company wanted to reduce the number of employees by 3, then from which category it reduces the number of employees without reducing the amount of production of books?
 - (a) It should reduce two binders and 1 typist
 - (b) It should reduce three binders only
 - (c) It should reduce 1 typist, 1 composer and 1 binder
 - (d) both (a) and (b) are possible

- 11** Bunty and Babli working together completed a job in 8 days. If Bunty worked twice efficiently as he actually did and Babli worked $\frac{1}{3}$ as efficiently as she actually did, the work would have been completed in 6 days. Find the time taken by Bunty to complete the job alone :

- (a) 8 days (b) $\frac{38}{35}$ days
(c) $\frac{15}{2}$ days (d) $13\frac{1}{3}$ days

- 12** A single reservoir supplies the petrol to the whole city, while the reservoir is fed by a single pipeline filling the reservoir with the stream of uniform volume. When the reservoir is full and if 40,000 litres of petrol is used daily, the supply fails in 90 days. If 32,000 litres of petrol is used daily, it fails in 60 days. How much petrol can be used daily without the supply ever failing?

- (a) 64000 litres (b) 56000 litres
(c) 78000 litres (d) 60000 litres

- 13** Railneer is packaged in a water bottling plant, with the help of two machines M_1 and M_2 . M_1 and M_2 produces 400 and 600 bottles per minute. One day's production can be processed by M_1 operating alone for 9 hours, by M_2 operating alone for 6 hours or by both M_1 and M_2 operating simultaneously for 3 hours and 36 minutes. If one days production is processed by M_1 operating alone for $\frac{1}{3}$ of the time and M_1 and M_2 simultaneously operating for $\frac{2}{3}$ of the time, then in how many hours total production of one day will be completed?

- (a) 2 (b) 3
(c) 4.5 (d) 4.8

Directions (for Q. Nos. 14 and 15) *A contractor employed a certain number of workers to finish constructing a road in a certain scheduled time. Sometime later, when a part of work had been completed, he realised that the work would get delayed by three-fourth of the scheduled time, so he at once doubled the number of workers and thus he managed to finish the road on the scheduled time.*

- 14** How much work he had been completed, before increasing the number of workers?

- (a) 10% (b) $14\frac{2}{7}\%$
(c) 20% (d) can't be determined

- 15** Some-time after the new workers were introduced, all of the newly introduced workers left the work due to heavy rain and the efficiency of the remaining workers reduced by 20% due to which the work finally got completed by delay of 60% of the scheduled time then how much work still remained incomplete by the end of the scheduled time?

- (a) $17\frac{3}{5}\%$ (b) 21% (c) $27\frac{5}{7}\%$ (d) 28%

Directions (for Q. Nos. 16 to 20) *Mr. Stanley employed a certain number of typists for his project. 8 days later 20% of the typist left the job and it was found that it took as much time to complete the rest work from then as the entire work needed with all the employed typists. The average speed of a typist is 20 pages/hour.*

- 16** How many typists left the work?

- (a) 10 (b) 5
(c) 16 (d) can't be determined

- 17** Minimum how many typist could be employed?

- (a) 10 (b) 5
(c) 15 (d) 4

- 18** What could be the number of typists remained at work when 20% of the employed typists left the job?

- (a) 15 (b) 18
(c) 68 (d) 78

- 19** What is the actual number of days required, when it is done with actual work force, through out the completion :

- (a) 32 days (b) 48 days
(c) 40 days (d) can't be determined

- 20** 16 days after the 20% typist left the job it was decided to complete the work on time by increasing the work-force again. By how much percentage increase in work-force is required?

- (a) 100% (b) 50%
(c) 200% (d) none of these

- 21** Five tailors A, B, C, D and E stitch 1800 shirts in 90 days working alternatively. Find the minimum possible number of shirts that can be stitched in a single day by working together :

- (a) 100 (b) 20 (c) 50 (d) 4

Directions (for Q. Nos. 22 and 23) *8 men and 5 women working 6 hours a day can complete a work in 4 days. Also 4 men and 5 women working for 8 hours a day can complete the same job in 5 days. Similarly 5 boys working 8 hours a day can complete the same job in 30 days.*

- 22** If 4 men, 3 women and 4 boys worked together everyday for 5 hours, then in how many days they have completed the work?

- (a) 3 (b) 4
(c) 8 (d) 6

- 23** If women and children (boys) can't be employed, then minimum how many men are required to complete the job in 6 days if the working hours per day cannot exceed 9?

- (a) 4 (b) 5
(c) 6 (d) 7

- 24** Eklavya can do the 6 times the actual work in 36 days while Faizal can do the one-fourth of the original work in 3 days. In how many days will both working together complete the 3 times of the original work?

- (a) 6 (b) 10
(c) 12 (d) 15

- 25** Progressive Company Pvt. Ltd. hired some employees in a fix pattern. On the first day it hired one person, on the second day one more joined him. On the third, fourth etc (i.e., every next day) one more person increased in this group. The capacity of each person was same. The whole work was completed on the 24th day then out of total ₹ 5000, maximum how much a person had earned?
 (a) ₹ 500
 (b) ₹ 400
 (c) ₹ 200
 (d) ₹ 250
- 26** Two persons having different productivity of labour, working together can reap a field in 2 days. If one-third of the field was reaped by the first man and rest by the other one working alternatively took 4 days. How long did it take for the faster person to reap the whole field working alone?
 (a) 12
 (b) 8
 (c) 6
 (d) 3
- 27** The total number of men, women and children working in a factory is 18. They earn ₹ 4000 in a day. If the sum of the wages of all men, all women and all children is in the ratio of 18 : 10 : 12 and if the wages of an individual man, woman and child is in the ratio 6 : 5 : 3, then how much a woman earn in a day?
 (a) ₹ 400
 (b) ₹ 250
 (c) ₹ 150
 (d) ₹ 120
- 28** A group of workers was put on a job. From the second day onwards, one worker was withdrawn each day. The job was finished when the last worker was withdrawn. Had no worker been withdrawn at any stage, the group would have finished the job in 55% of the time. How many workers were there in the group?
 (a) 50
 (b) 40
 (c) 45
 (d) 10
- 29** Two workers undertake to do a job. The second worker started working 2 hours after the first. Five hours after the second worker has begun working there is still $\frac{9}{20}$ of the work to be done. When the assignment is completed, it turns out that first worker has done 60% of the work, while second worker has done rest of the work. How many hours would it take each one to do the whole job individually?
 (a) 10 hours and 12 hours
 (b) 15 hours and 10 hours
 (c) 20 hours and 25 hours
 (d) 18 hours and 20 hours
- 30** A group of men decided to do a job in 4 days. But since 20 men dropped out every day, the job completed at the end of the 7th day. How many men were there at the beginning?
 (a) 240
 (b) 140
 (c) 280
 (d) 150
- 31** Brahma, Vishnu and Mahesh are three friends with different productivity. Brahma working alone needs as much time as Vishnu and Mahesh working together, while Vishnu himself needs 8 hours more working alone than when he works with Mahesh. Brahma working alone needs 8 hours less than Vishnu needs working alone. In how much time Brahma, Vishnu and Mahesh working together can complete the job?
 (a) 4 hours (b) 5 hours (c) 6 hours (d) 8 hours
- 32** Milinda takes $8\frac{1}{3}$ hours more when she works alone in comparison of when she works with Bill. While Bill takes $5\frac{1}{3}$ hours more when he work alone in comparison to the time, when he works with Milinda. How long it will take Bill to complete the work alone?
 (a) 10 hours (b) 15 hours (c) 18 hours (d) 12 hours
- 33** Pascal and Rascal are two workers. Working together they can complete the whole work in 10 hours. If the Pascal worked for 2.5 hours and Rascal worked for 8.5 hours, still there was half of the work to be done. In how many hours Pascal working alone, can complete the whole work?
 (a) 24 hours
 (b) $17\frac{1}{7}$ hours
 (c) 40 hours
 (d) can't be determined
- 34** Boston, Churchill and David are three workers, employed by a contractor. They completed the whole work in 10 days. Initially all of them worked together, but the last 60% of the work was completed by only Churchill and David together. Boston worked with Churchill and David only for initial two days then he left the work due to his poor health. Also Churchill takes 20% less time to finish the work alone than that of David working alone. If they were paid ₹ 3000 for the entire work, then what is the share of least efficient person?
 (a) ₹ 900
 (b) ₹ 1200
 (c) ₹ 1000
 (d) none of these
- 35** There are three boats B_1 , B_2 and B_3 working together they carry 60 people in each trip. One day an early morning B_1 carried 50 people in few trips alone. When it stopped carrying the passengers B_2 and B_3 started carrying the people together. It took a total of 10 trips to carry 300 people by B_1 , B_2 and B_3 . It is known that each day on an average 300 people cross the river using only one of the 3 boats B_1 , B_2 and B_3 . How many trips it would take to B_1 to carry 150 passengers alone?
 (a) 15
 (b) 30
 (c) 25
 (d) 10
- 36** Three men and 5 women together can finish a job in 3 days. Working on the same job 3 women take 5 days more than the time required by 2 men. What is the ratio of efficiency of a man to a woman?
 (a) 2 : 1
 (b) 3 : 2
 (c) 5 : 2
 (d) 4 : 1

- 37** Henry and Ford are two different persons, but when they work together, they complete it in 10 days. Had Henry worked at half of his efficiency and Ford at five times of his efficiency it would have taken them to finish the job in 50% of the scheduled time. In how many days can Ford complete the job working alone?

(a) 12 (b) 24
(c) 15 (d) 30

- 38** Anne, Benne and Cenne are three friends. Anne and Benne are twins. Benne takes 2 days more than Cenne to complete the work. If Anne starts a work and 3 days later Benne joins him, then the work gets completed in 3 more days. Working together Anne, Benne and Cenne can complete thrice the original work in 6 days. In how many days Benne can complete twice the original work with double the efficiency working alone?

(a) 2 (b) 3
(c) 4 (d) 6

- 39** Three typists A, B and C working together 8 hours per day can type 900 pages in 20 days. In a day B types as many pages more than A as C types as many pages more than B. The number of pages typed by A in 4 hours equal to the number of pages typed by C in 1 hour. How many pages C types in each hour?

(a) 1 (b) 2
(c) 3 (d) 4

Directions (for Q. Nos. 40 and 41) Four pipes A, B, C and D can fill a cistern in 20, 25, 40 and 50 hours respectively.

- 40** The first pipe A was opened at 6:00 am, B at 8:00 am, C at 9:00 am and D at 10:00 am. When will the cistern be full?

(a) 4:18 pm (b) 3:09 pm
(c) 12:15 pm (d) 11:09 am

- 41** If A and B are opened as inlet pipe into the cistern and C and D are opened as outlet pipes from the cistern and all the four pipes are opened simultaneously, how many hours will it take to fill the cistern completely?

(a) 20 hours
(b) $11\frac{1}{9}$ hours
(c) $22\frac{2}{9}$ hours
(d) 45 hours

- 42** A tank is connected with four pipes A, B, C and D of which two are filling the tank and other two are emptying it. The time taken by A, B, C and D to finish their jobs are 10 hours, 15 hours, 20 hours and 30 hours respectively. All four pipes are opened. When the tank was empty, it took 12 hours to fill it completely. Which two are the outlet pipes?

(a) A and B (b) C and D
(c) A and C (d) B and D

- 43** Two pipes A and B can fill a tank in 24 hours and $\frac{120}{7}$ hours

respectively. Harihar opened the pipes A and B to fill an empty tank and some times later he closed the taps A and B, when the tank was supposed to be full. After that it was found that the tank was emptied in 2.5 hours because an outlet pipe C connected to the tank was open from the beginning. If Harihar closed the pipe C instead of closing pipes A and B the remaining tank would have been filled in :

(a) 2 hours (b) 8 hours (c) 6 hours (d) 4 hours

- 44** Pipe A can fill a tank in 12 hours and pipe B can fill it in 15 hours, separately. A third pipe C can empty it in 20 hours. Initially pipe A was opened, after one hour pipe B was opened and then after 1 hour when pipe B was opened pipe C was also opened. In how many hours the tank will be full?

(a) $9\frac{2}{3}$ hours (b) $6\frac{2}{3}$ hours
(c) 10 hours (d) none of these

Directions (for Q. Nos. 45 and 46) A tank has an inlet and outlet pipe. The inlet pipe fills the tank completely in 2 hours when the outlet pipe is plugged. The outlet pipe empties the tank completely in 6 hours when the inlet pipe is plugged.

- 45** If both pipes are opened simultaneously at a time when the tank was one-third filled, when will the tank fill thereafter?

(a) $\frac{3}{2}$ hours (b) $\frac{2}{3}$ hour (c) 2 hours (d) $1\frac{2}{3}$ hours

- 46** If there is a leakage also which is capable of draining out the liquid from the tank at half of the rate of outlet pipe, then what is the time taken to fill the empty tank when both the pipes are opened?

(a) 3 hours (b) $3\frac{2}{3}$ hours
(c) 4 hours (d) none of these

- 47** An inlet pipe can fill a tank in 5 hours and an outlet pipe can empty the same tank in 36 hours, working individually. How many additional number of outlet pipes of the same capacity are required to be opened, so that the tank never over flows?

(a) 3 (b) 6 (c) 8 (d) 7

Directions (for Q. Nos. 48 and 49) In a public bathroom there are n taps 1, 2, 3... n . Tap 1 and tap 2 take equal time to fill the tank while tap 3 takes half the time taken by tap 2 and tap 4 takes half the time taken by tap 3. Similarly each next number of tap takes half the time taken by previous number of tap i.e., K th tap takes half the time taken by $(K - 1)$ th tap.

- 48** If the 10th tap takes 2 hours to fill the tank alone then what is the ratio of efficiency of 8th tap and 12th tap, respectively?

(a) 4 : 1 (b) 5 : 3
(c) 16 : 1 (d) 1 : 16

- 49** If the 8th tap takes 80 hours to fill the tank then the 10th and 12th taps working together take how many hours to fill the tank?
 (a) 2 hours
 (b) 4 hours
 (c) 6 hours
 (d) none of the above
- 50** Pipe A takes $\frac{3}{4}$ of the times required by pipe B to fill the empty tank individually. When an outlet pipe C is also opened simultaneously with pipe A and pipe B, it takes $\frac{3}{4}$ more time to fill the empty tank than it takes, when only pipe A and pipe B are opened together. If it takes to fill 33 hours when all the three pipes are opened simultaneously, then in what time pipe C can empty the full tank operating alone?
 (a) 66 hours
 (b) 50 hours
 (c) 44 hours
 (d) can't be determined

Directions (for Q. Nos. 51 to 53) *A contractor undertook a project to complete it, in 20 days which needed 5 workers to work continuously for all the days estimated. But before the start of the work the client wanted to complete it earlier than the scheduled time, so the contractor calculated that he needed to increase 5 additional men every 2 days to complete the work in the time the client wanted it :*

- 51** How many men were working on the day the project was completed as per the date of client wanted to complete it?
 (a) 5 (b) 10
 (c) 20 (d) none of these
- 52** Find the number of days in which client wanted to complete his work.
 (a) 15 (b) 10
 (c) 8 (d) can't be determined
- 53** If the work was further increased by 50% but the contractor continues to increase the 5 workers on every 2 days then how many more days are required over the initial time specified by the client?
 (a) 1 day (b) 2 days
 (c) 5 days (d) none of these
- 54** A tank is connected with 8 pipes. Some of them are inlet pipes and rest work as outlet pipes. Each of the inlet pipe can fill the tank in 8 hours, individually, while each of those that empty the tank i.e., outlet pipe, can empty it in 6 hours individually. If all the pipes are kept open when the tank is full, it will take exactly 6 hours for the tank to empty. How many of these are inlet pipes?
 (a) 2 (b) 4
 (c) 5 (d) 6

- 55** A tank has two inlet pipes which can fill the empty tank in 12 hours and 15 hours working alone and one outlet pipe which can empty the full tank in 8 hours working alone. The inlet pipes are kept open for all the time but the outlet pipe was opened after 2 hours for one hour and then again closed for 2 hours then once again opened for one hour. This pattern of outlet pipe continued till the tank got completely filled. In how many hours the tank has been filled, working on the given pattern?
 (a) 8 hours 24 minutes (b) 10 hours 15 minutes
 (c) 9 hours 10 minutes (d) 9 hours 6 minutes
- 56** Working together, two pipes A and B can fill 7 empty tanks of the same capacity in 48 hours. When a tank was initially empty, pipe B was kept open for as much time as was required by pipe A to fill $\frac{1}{3}$ rd of the empty tank by itself. Then, pipe A was kept open for as much time as was required by pipe B to fill up $\frac{1}{4}$ th of the empty tank by itself. It was then found that the tank was $\frac{7}{12}$ full. The least time in which any of the pipes can fill an empty tank fully is
 (a) 8.4 hours (b) 14.4 hours (c) 12 hours (d) 14 hours
- 57** Four men and three women can do a job in six days. When five men and six women work on the same job, the work gets completed in four days. How long will two women and three men take to do the job?
 (a) 18 days (b) 10 days (c) 9 days (d) 7 days
- 58** There are five water pipes P_1, P_2, P_3, P_4, P_5 connected to a tank, which can fill or empty the tank individually in 45, 40, 30, 20, 15 minutes, respectively. A few of them are inlet pipes and others are the outlet pipes. If the combination of an inlet pipe and an outlet pipe can either empty or fill the whole tank in 1 hour, which one of the following can be the feasible combination of pipes as indicated by {inlet pipe, outlet pipe} in that order only?
 (a) $\{P_3, P_4\}$ and $\{P_4, P_5\}$ (b) $\{P_1, P_2\}$ and $\{P_4, P_5\}$
 (c) $\{P_4, P_3\}$ and $\{P_5, P_4\}$ (d) $\{P_4, P_3\}$ and $\{P_4, P_5\}$
- 59** There are four water tanks each with equal capacity of 1200 liters. The water is being pumped from one tank to another as per the following scheme.
 From P to Q @ 52 liters/second
 From P to S @ 48 liters/second
 From Q to R @ 36 liters/second
 From R to P @ 76 liters/second
 From R to S @ 24 liters/second
 From S to Q @ 48 liters/second
 If all the pumps are started simultaneously, when each of the tanks has 384 liters of water which tank would get emptied first?
 (a) P (b) Q (c) R (d) S

LEVEL 03 › Final Round

Directions (for Q. Nos. 1 to 5) *In a nut-bolt factory 180 workers are working for 6 hours a day. Out of 180 workers there are some men, some women and rest are boys. All the workers can produce either nut or bolt or both of them. A man can produce 60 nuts and 80 bolts in each hour and a woman can produce 30 nuts and 60 bolts per hour. A man is thrice as efficient as a boy and $\frac{3}{2}$ times as efficient as a woman. Given that all men, all women and all boys produce equal number of articles of one kind (i.e., either nut or bolt) per hour.*

- Working 6 hours a day, how many nuts they can produce with 52500 bolt in each day?
(a) 17500 (b) 26250
(c) 50850 (d) can't be determined
- In how many hours can 15 men, 12 women and 8 boys can produce 12000 nuts and 8200 bolts?
(a) 3 (b) 5
(c) 6 (d) none of these
- If 30 women and 50 boys can produce 5400 bolts in one hour then to produce equal number of nuts in one hour how many men are required?
(a) 30 (b) 40
(c) 50 (d) none of these
- If the efficiency of each boy is doubled then what is the increase in production per hour?
(a) 100% (b) 50%
(c) 33.33% (d) none of the above
- If the manager of factory wanted equal number of men, women and boys in his factory but the efficiency of a man, woman and a boy remains constant, then the change in production is :
(a) increased by 10% (b) decreased by 10%
(c) increased by 8.33% (d) none of the above
- Ram Lal is a renowned packager of fruits in Varanasi. He packs 70 mangoes or 56 guavas every day working 7 hours per day. His wife also helps him. She packs 30 mangoes or 24 guavas working 6 hours per day. Ram Lal has to pack 3300 mangoes and 2400 guavas with the help of his wife. They work alternatively, each day 10 hours. His wife started packaging on the first day and works on every alternate days. Similarly Ram Lal started his work on second day and worked alternatively till the completion of the work. In how many days the work will be finished?
(a) 85 days (b) $85\frac{2}{5}$ days
(c) 84 days (d) none of these

Directions (for Q. Nos. 7 to 11) *A company produces three products. The products are processed on 3 different machines. The time required to manufacture one unit of each the three products and the daily capacity of three machines are given in the table below :*

Machine	Time per unit (in min.)			Machine capacity (min./day)
	Product 1	Product 2	Product 3	
M_1	2	2	3	450
M_2	2	5	–	410
M_3	3	–	4	480

- How many units of product 1 can be produced in one day?
(a) 160 (b) 205
(c) 225 (d) 64
- If minimum 20 units of P_1 and 30 units of P_2 have to be produced, then what is the maximum units of P_3 that can be produced in a day?
(a) 116 (b) 105
(c) 205 (d) 220

Directions (for Q. Nos. 9 to 11) *Read the following additional data for question number 9, 10 and 11. The profit per unit for product 1, 2 and 3 is ₹ 3, ₹ 4 and ₹ 5.*

- What combination of P_1, P_2 and P_3 will yield maximum profit, under the manufacturing constraints?
1. $P_1 - 25, P_2 - 50, P_3 - 100$
2. $P_1 - 20, P_2 - 60, P_3 - 80$
3. $P_1 - 100, P_2 - 0, P_3 - 50$
4. $P_1 - 0, P_2 - 80, P_3 - 100$
(a) 4 (b) 2
(c) 3 (d) 1
- Which of the machine if it breaks down will affect profitability the least?
(a) Machine 1
(b) Machine 2
(c) Machine 3
(d) Machine 1 or 3
- If no production of product 2 is scheduled today and it is decided to only produce one type of product today, then what is the maximum profits that can be had today?
(a) ₹ 480 (b) ₹ 750
(c) ₹ 600 (d) none of these

Directions (for Q. Nos. 12 to 15) *Ready Tailoring Services* is very well known in its quality and time bound services. The company (*Ready Tailoring Services*) received a large order for stitching military uniforms. It has two different orders to prepare the shirts one for Officers and second for Jawans (non-officers). It has three cutters who will cut the fabric. Six tailors who will do the stitching and 3 assistant to stitch the buttons and iron the shirts. Each of these 12 persons will work for exactly 8 hours a day. Each of the Officers uniform requires 20 minutes for cutting the fabric, 1 hour for stitching and 20 minutes for stitching buttons and ironing the shirts. Whereas the Jawan's uniform requires 15 minutes for cutting the fabric and 60 minutes for stitching and 10 minutes for buttons and ironing.

- 12** If the company has to supply 40 officers uniforms only and no other on a particular day, how many man-hours are utilised on that day?

(a) $33\frac{1}{3}$ hours (b) $66\frac{2}{3}$ hours
(c) 40 hours (d) 60 hours

- 13** If the number of tailors will be increased by 50% then maximum how many uniforms for officers can be completed in one day?

(a) 48 (b) 50
(c) 60 (d) 72

- 14** If the company can increase maximum 3 employees of any category then for which category should it hire to get maximum increase in production capacity, assuming that it needs to stitch only officer's uniform :

(a) cutter (b) tailor
(c) assistant (d) can't be determined

- 15** If the company has to produce the shirts for only one category then of which category it can produce maximum number of uniforms?

(a) Officers
(b) Jawans
(c) either (a) or (b)
(d) none of these

Answers

✓ Level 01 Basic Level Exercise

1 (c)	2 (b)	3 (c)	4 (a)	5 (d)	6 (b)	7 (b)	8 (c)	9 (b)	10 (c)
11 (b)	12 (c)	13 (a)	14 (a)	15 (b)	16 (b)	17 (a)	18 (a)	19 (b)	20 (a)
21 (a)	22 (c)	23 (d)	24 (c)	25 (b)	26 (b)	27 (a)	28 (c)	29 (c)	30 (c)
31 (b)	32 (a)	33 (b)	34 (d)	35 (a)	36 (c)	37 (c)	38 (b)	39 (b)	40 (d)
41 (a)	42 (b)	43 (b)	44 (c)	45 (b)	46 (a)	47 (a)	48 (c)	49 (a)	50 (a)
51 (c)	52 (a)	53 (d)	54 (b)	55 (a)	56 (c)	57 (b)	58 (a)	59 (d)	60 (a)
61 (a)	62 (c)	63 (b)	64 (d)	65 (c)	66 (b)	67 (b)	68 (a)	69 (b)	70 (b)
71 (b)	72 (c)	73 (d)	74 (a)	75 (c)	76 (d)	77 (b)	78 (b)	79 (b)	80 (c)
81 (b)	82 (a)	83 (a)	84 (a)	85 (b)	86 (a)	87 (b)	88 (d)	89 (c)	90 (c)
91 (d)	92 (c)	93 (a)	94 (b)	95 (a)	96 (c)	97 (a)	98 (b)	99 (c)	100 (a)

✓ Level 02 Higher Level Exercise

1 (a)	2 (c)	3 (c)	4 (b)	5 (c)	6 (c)	7 (c)	8 (b)	9 (c)	10 (d)
11 (d)	12 (b)	13 (c)	14 (b)	15 (c)	16 (d)	17 (b)	18 (c)	19 (c)	20 (d)
21 (a)	22 (c)	23 (b)	24 (c)	25 (b)	26 (d)	27 (b)	28 (d)	29 (c)	30 (b)
31 (c)	32 (d)	33 (b)	34 (c)	35 (a)	36 (c)	37 (d)	38 (d)	39 (c)	40 (b)
41 (c)	42 (b)	43 (b)	44 (a)	45 (c)	46 (c)	47 (d)	48 (d)	49 (b)	50 (c)
51 (c)	52 (c)	53 (b)	54 (b)	55 (c)	56 (c)	57 (c)	58 (d)	59 (c)	

✓ Level 03 Final Round

1 (c)	2 (b)	3 (a)	4 (c)	5 (c)	6 (c)	7 (a)	8 (b)	9 (d)	10 (b)
11 (c)	12 (b)	13 (d)	14 (b)	15 (c)					

Hints & Solutions

✓ Level 01 Basic Level Exercise

1 Efficiency of A = $\frac{100}{12} = 8.33\%$

Efficiency of B = $\frac{100}{15} = 6.66\%$

Combined efficiency of A and B = $8.33 + 6.66 = 15\%$

Number of days taken by A and B, when worked together

$$= \frac{100}{15} = 6\frac{10}{15} = 6\frac{2}{3} \text{ days}$$

🔗 **NOTE** Efficiency $\times$ Time period = Fixed amount of work

Also, in terms of percentage total work to be done is considered as 100% (in fraction it is 1), unless otherwise stated.

🔗 **Alternatively**

It can be done through unitary method, also.

2 Efficiency of A = $\frac{100}{10} = 10\%$

Efficiency of B = $\frac{100}{12} = 8.33\%$

Efficiency of C = $\frac{100}{15} = 6.66\%$

Combined efficiency of A, B and C = $10 + 8.33 + 6.66 = 25\%$

$\therefore$ Required number of days, when A, B and C worked

together = $\frac{100}{25} = 4$ days

3 Efficiency of A = $20\% \left(= \frac{100}{5} \right)$

Efficiency of B = $10\% \left(= \frac{100}{10} \right)$

Efficiency of A, B and C = $50\% \left(= \frac{100}{2} \right)$

$$\begin{aligned} \therefore \text{Efficiency of C} &= (\text{Efficiency of A, B and C}) \\ &\quad - (\text{Efficiency of A and B}) \\ &= (50) - (20 + 10) = 20\% \end{aligned}$$

$\therefore$ Number of days required by C to work alone

$$= \frac{100}{20} = 5 \text{ days}$$

🔗 **Alternatively**

Go through options and satisfy the values.

	A	B	C	(A + B + C)
Days	5	10	5	2
Efficiency	20%	10%	20%	50%

Consider option (c)

4 Efficiency of A = 12.5%

Efficiency of B = 6.25%

Efficiency of C = 1.25%

Efficiency of (A + B + C) = $20\% (= 12.5 + 6.25 + 1.25)$

Required number of days = $\frac{100}{20} = 5$ days

5 Efficiency of A = 10%, Efficiency of B = 6.66%

$\therefore$ Required number of days = $\frac{100}{16.66} = 6$ days

6 Efficiency of A = $\frac{100}{24} = 4.16\%$

Efficiency of B = $\frac{100}{30} = 3.33\%$

Efficiency of (A + B + C) = $\frac{100}{12} = 8.33\%$

$\therefore$ Efficiency of C = $(8.33) - (4.16 + 3.33) = 0.83\%$

$\therefore$ Number of days required by C to complete the work alone

$$= \frac{100}{0.83} = \frac{100}{5/6} = 120 \text{ days}$$

🔗 **NOTE** $\frac{5}{6} = 0.833$

7 Efficiency of A = 10%

Efficiency of B = 4.16%

Efficiency of (A + B + C) = 16.66%

Efficiency of C = $(16.66) - (10 + 4.16) = 2.5\%$

Number of days required by C alone to finish the work

$$= \frac{100}{2.5} = 40 \text{ days}$$

8 Efficiency of A = 7.14%

Efficiency of B = 4.76%, Efficiency of A + B = 11.9%

$\therefore$ Number of days required by A and B, working together

$$= \frac{100}{11.9} = 8.4 \text{ days}$$

Hint You can see that there is only one option between 8 and 9 which is 8.4 hence (c) is the correct choice.

Explanation For 8 days denominator should be 12.5 and for 9 days denominator should be almost 11.

🔗 **Alternatively** One day's work of A and B

$$= \frac{1}{14} + \frac{1}{21} = \frac{5}{42}$$

$\therefore$ Required number of days = $\frac{42}{5} = 8.4$ days

- 9 Efficiency of A = 4.16%

Efficiency of B = $1.6 \times 4.16 = 6.66\%$

$\therefore$ Number of days required by B = $\frac{100}{6.66} = 15$ days

☞ Alternatively

	A	B
Efficiency	$\rightarrow x$	$1.6x$
Days	$\rightarrow 1.6k$	k
(Since number of days are inversely proportional to efficiency)		

Now $1.6k = 24 \Rightarrow k = 15$ day

☞ Alternatively A's one day's work = $\frac{1}{24}$

$\therefore$ B's one day's work = 160% of $\frac{1}{24} = \frac{1}{15}$

$\therefore$ Required number of days = 15

- 10 Efficiency of A + B = $\frac{100}{14} = 7.14\%$

Again the ratio of efficiency of A and B = 2 : 1

$\therefore$ Efficiency of A = $\frac{2}{3} \times 7.14 = 4.76\%$

$\therefore$ required number of days by A = $\frac{100}{4.76} = 21$ days

🔔 **NOTE** Only choice (c) lies between 20 and 24.

Explanation For 20 days denominator should be 5 and for 24 days, denominator should be 4.16 also the only choice (c) gives a value which is multiple of 7 and very close to the answer (as appears).

☞ Alternatively One day's work of

$$A \text{ and } B = \frac{1}{x} + \frac{1}{2x} = \frac{1}{14} \Rightarrow x = 21$$

Since A is twice efficient as B so A will take half of the days taken by B.

- 11 Efficiency of A + B = $14.28\% \left(= \frac{100}{7} \right)$

Now, since the ratio of efficiency of A and B is 2 : 1

So, the efficiency of A = $\frac{2}{3} \times 14.28 = 9.52\%$

$\therefore$ Number of days required by A = 10.5 days

Hint Only choice (b) is correct since there is no other value (in option) lies between 10 and 11.

Explanation For 10 days, denominator should be 10 and for 11 days, denominator should be 9.09.

☞ Alternatively

Let A takes x days and then B takes $2x$ days

Then 1 day's work of A and B = $\frac{1}{x} + \frac{1}{2x} = \frac{1}{7}$

$\Rightarrow x = 10.5$

Thus A takes 10.5 days.

12

	A	B
Efficiency	$\rightarrow 3$	$\rightarrow 1$
Days	$\rightarrow 1$	$\rightarrow 3$

(Since number of days are inversely proportional to the efficiency)

Now if A requires x days, so B requires $3x$ days

$\therefore$ Difference of required days $(= 3x - x) = 2x = 10 \Rightarrow x = 5$

Hence the number of days required by

$$B = 3x = 3 \times 5 = 15 \text{ days}$$

13

	A	B
Efficiency	$\rightarrow 2$	$\rightarrow 1$
Days	$\rightarrow 1$	$\rightarrow 2$
$\left(\text{Days} \propto \frac{1}{\text{Efficiency}} \right)$		

Now, let A requires x days, then B requires $2x$ days

$\therefore$ difference in number of days $(= 2x - x) = x = 6 \Rightarrow x = 6$

$\therefore$ B requires $2x = 2 \times 6 = 12$ days

☞ Alternatively If A takes x days, then B takes $x + 6$ days

Now, A's 1 day's work = $\frac{1}{x}$

$$B's \text{ 1 day's work} = \frac{1}{x+6}$$

$$\therefore \frac{1/x}{1/(x+6)} = \frac{2}{1}$$

(Since A does twice the work as B does)

$$\Rightarrow x = 6$$

$\therefore$ B takes $2x = 12$ days.

☞ Alternatively $\frac{1}{x} + \frac{1}{(x+6)} = \frac{1}{4}$

$\Rightarrow x = 6$ and $2x = 12$ days (required by B)

☞ Alternatively Go through option and satisfy all the conditions.

14

	A	B	C
Ratio of efficiency	3	1	2
Ratio of number of days	$\left\{ \frac{1}{3} \right\}$	$\left\{ \frac{1}{1} \right\}$	$\left\{ \frac{1}{2} \right\}$
or	2	6	3

Hence, (a) is correct $\left[\because \text{Time} \propto \frac{1}{\text{Efficiency}} \right]$

15

	Ajit	Bablu
Efficiency	3	1
No. of days	1	3

16

	X	Y
Efficiency	5	1
No. of days	1	5

- 17 Efficiency of A + B = $\frac{100}{\frac{20}{3}} = 15\%$

	A	B
No. of days	4	5
Efficiency	5	4
$\therefore$ Efficiency of B	$= \frac{4}{9} \times 15 = \frac{20}{3} = 6\frac{2}{3}\%$	

18 Efficiency of $A + B = 33.33\% \left(= \frac{100}{3} \right)$

Ratio of efficiency of A and B = 3 : 1

$\therefore$ Efficiency of A = $\frac{3}{4} \times 33.33 = 25\%$

$\therefore$ Number of days taken by A = $4 \left(= \frac{100}{25} \right)$

☞ Alternatively

	A	:	B
Efficiency	3	:	1
No. of days	1 (x)	:	3 (x)
$\therefore$	Difference in days = $2x = 8$		
$\Rightarrow$	$x = 4$ and $3x = 12$		

Therefore number of days taken by A, working alone = 4 days

☞ Alternatively $\frac{1}{x} + \frac{1}{(x+8)} = \frac{1}{3}$

Now, you can use the options, or solve the equations to get the value of x which is equal to 4.

19 Efficiency of $(A + B) = 12.5\%$

Efficiency of $(B + C) = 8.33\%$

Efficiency of $(C + A) = 12.5\%$

$\therefore$ Efficiency of $[(A + B) + (B + C) + (C + A)]$
= 33.33%

$\therefore$ Efficiency of $(A + B + C) = 16.66\%$

$\therefore$ Number of days required by A, B and C together = 6 days
 $\left(= \frac{100}{16.66} \right)$

20 Efficiency of $(A + B) = 8.33\%$

Efficiency of $(B + C) = 6.66\%$, Efficiency of $(C + A) = 5.00\%$

$\therefore$ Efficiency of $A + B + C = \frac{1}{2} (8.33 + 6.66 + 5) = 10\%$

$\therefore$ Efficiency of C = Efficiency of $[(A + B + C) - (A + B)]$
= $(10 - 8.33) = 1.66\%$

$\therefore$ Number of days required by C alone = $\frac{100}{1.66} = 60$ days

21 Remember Time $\propto \frac{1}{\text{Efficiency}}$

Now,

	Ganga	:	Jamuna	:	Saraswati
Efficiency $\rightarrow$	3	:	1	:	2
No. of days $\rightarrow$	2	:	6	:	3

Again, efficiency of Ganga, Jamuna and Saraswati

$= \frac{100}{1} = 100\%$

$\therefore$ Efficiency of Ganga = $\frac{3}{6} \times 100 = 50\%$

Efficiency of Saraswati = $\frac{2}{6} \times 100 = 33.33\%$

Now, number of days taken by Ganga = $\frac{100}{50} = 2$

Number of days taken by Saraswati = $\frac{100}{33.33} = 3$

$\therefore$ Difference in number of days taken by Ganga and Saraswati
= $3 - 2 = 1$ day

22 Day 1 2 3 4 5 6 7 8
 $\frac{A}{12} \frac{A}{12} \frac{A}{12} \frac{(A+B)}{12} \frac{(A+B)}{12} \frac{(A+B)}{12} \frac{(A+B)}{12} \frac{(A+B)}{12}$

$3 \times \frac{1}{12} = \frac{3}{12} = 25\%$ } Remaining work
 work completed } $= 75\% = (100 - 25)$

Now, efficiency of $A + B = 15\% = (8.33 + 6.66)$

$\therefore$ Number of days required by $A + B = \frac{75}{15} = 5$

to complete rest work (= 75%)

23 '3 days before the completion of the work Raja left the work' means in last 3 days only Rani has worked alone.

So, in last 3 days worked done by Rani = $3 \times \frac{1}{21} = \frac{1}{7}$

So, the rest $\left(1 - \frac{1}{7} \right) = \frac{6}{7}$ work was done by Raja and Rani both.

Number of days in which Raja and Rani worked together

$= \frac{6/7}{5/42} = \frac{36}{5} = 7 \frac{1}{5}$ days

☞ NOTE Work done by Raja and Rani in one day = $\frac{1}{14} + \frac{1}{21} = \frac{5}{42}$
 $\left(\text{Also, number of days} = \frac{\text{Total work to be done}}{\text{Work to be done in one day}} \right)$

24 Work done in 11 days = $\frac{11}{30}$

Rest work = $\frac{19}{30}$

1 day's work of A = $\frac{19/30}{28} = \frac{19}{30 \times 28} = \frac{19}{840}$

Total number of days required to complete the whole work

alone = $\frac{1}{19/840} = \frac{840}{19} = 44 \frac{4}{19}$ days

25 Efficiency of Sonu = $5\% = \left(\frac{100}{20} \right)$

Rest work = 75%

$\therefore$ Efficiency of Abhijeet = $\frac{75}{10} = 7.5\%$

$\therefore$ Combined efficiency of Sonu and Abhijeet
= $12.5\% = (7.5 + 5)$

$\therefore$ Number of days required by Sonu and Abhijeet, to work together = $\frac{100}{12.5} = 8$ days

	Asha	Usha
Efficiency	5	4
No. of days	4x	5x = 25

∴ Number of days required by Asha to finish the work alone
 $= 20$ ∴ $(4x = 4 \times 5)$

(Alternatively, from percentage change graphic, number of days taken by Asha will be 20% less than Usha, if efficiency of Asha is 25% more than Usha)

Now, since Asha and Usha did work together for last 5 days
 $= 5 \times 9 = 45\%$

(Since efficiency of Asha = 5% and Usha's efficiency = 4%)

It means Asha completed 55% work alone

∴ Number of days taken by Asha to complete 55% work
 $= \frac{55}{5} = 11$ days

27 Krishna's efficiency = 10%, Mohan's efficiency = 5%

Work done by Krishna and Mohan together in 3 days
 $= 15 \times 3 = 45\%$

Now, number of days in which B completed rest (55%) work alone = $\frac{55}{5} = 11$

Total number of days in which B worked = $3 + 11 = 14$

Now number of days required by B, when A and B both worked together = $\frac{100}{15} = 6\frac{2}{3}$

∴ Required difference in number of days = $(11) - \left(6\frac{2}{3}\right)$
 $= \frac{13}{3} = 4\frac{1}{3}$ days

28 Efficiency of Kareena and Karishma

$$= 11.11 + 5.55 = 16.66\%$$

Work done in 3 days = $3 \times 16.66 = 50\%$

Rest work done by Kareena, Karishma and Shahid = $\frac{50}{50}$

$$= 1 \text{ day}$$

(Since efficiency of Shahid = 33.33%)

Thus in 4 (= 3 + 1) days they have completed the work.

29 Efficiency of Kavita = 5%

Efficiency of Babita = 1.66%

Efficiency of Samita = 3.33%

Work done in 5 days by K + B + S = $5 \times (10) = 50\%$

Work done in 3 days by K + B = $3 \times (6.66) = 20\%$

Remaining work (30%) done by Kavita alone = $\frac{30}{5} = 6$ days

30

		A	:	B	:	C
Efficiency	$\rightarrow$	10	:	9	:	6
No. of days	$\rightarrow$	$9x$	:	$10x$	:	$15x$
Now,		$15x - 9x = 6$				
$\Rightarrow$		$x = 1$				

∴ Number of days taken by A = 9

Number of days taken by B = 10

Number of days taken by C = 15

Work done by B and C in initial 2 days = $\frac{2 \times 1}{6} = \frac{1}{3}$

$$\text{rest work} = \frac{2}{3}$$

Number of days required by A to finish $\frac{2}{3}$ work = $\frac{2/3}{1/9} = 6$ days

31

	Anand	Bahuguna
No. of days	45	40
Efficiency	$2.22\% \left(= \frac{1}{45} \right)$	$2.5\% \left(= \frac{1}{40} \right)$

Anand did the work in 56 days

$$= 56 \times \frac{1}{45 \times 2} = \frac{28}{45}$$

∴ Rest work $\left(\frac{17}{45} \right)$ was done by Anand and Bahuguna

$$= \frac{17/45}{17/360} = 8 \text{ days}$$

(Since Anand and Bahuguna do the work in one day
 $= \frac{1}{45} + \frac{1}{40} = \frac{17}{360}$)

32 Efficiency of Chandni = 11.11%

Efficiency of Divakar = 8.33%

They do 19.44% work in 2 days

∴ They need 10 days to do 97.22% work,

Now the rest work (2.78) was done by Chandni in

$$\frac{2.78}{11.11} = \frac{1}{4} \text{ day}$$

Therefore total number of days required

$$= 10 + \frac{1}{4} = 10\frac{1}{4} \text{ days}$$

☞ Alternatively Chandni's one day's work = $\frac{1}{9}$

$$\text{Divakar's one day's work} = \frac{1}{12}$$

Chandni's and Divakar's (1 + 1) = 2 day's work

$$= \frac{1}{9} + \frac{1}{12} = \frac{7}{36}$$

So, in 10 days they do = $\frac{7 \times 5}{36} = \frac{35}{36}$ work

So, the remaining $\frac{1}{36} \left(= 1 - \frac{35}{36} \right)$ work will be done by

$$\text{Chandni} = \frac{1/36}{1/9} = \frac{1}{4} \text{ day}$$

Thus total number of required days = $10 + \frac{1}{4} = 10\frac{1}{4}$ days

Hint

Day	1	2	3	4	5	6	7	8	9	10	11
Turn	C	D	C	D	C	D	C	D	C	D	C
Work	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{36}$
	$\frac{7}{36}$		$\frac{7}{36}$		$\frac{7}{36}$		$\frac{7}{36}$		$\frac{7}{36}$		$\frac{1}{36}$
	$\frac{35}{36}$										$\frac{1}{36}$

33	Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		F	Z	F	Z	F	Z	F	Z	F	Z	F	Z	F	Z
		$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{12}$	$\frac{1}{15}$
		$\frac{9}{60}$	$\frac{9}{60}$	$\frac{9}{60}$	$\frac{9}{60}$	$\frac{9}{60}$	$\frac{9}{60}$	$\frac{9}{60}$	$\frac{9}{60}$	$\frac{9}{60}$	$\frac{9}{60}$	$\frac{9}{60}$	$\frac{9}{60}$	$\frac{9}{60}$	$\frac{9}{60}$
		$\frac{54}{60}$												$\frac{6}{60}$	

In two days Fatima and Zahira do = $\frac{1}{12} + \frac{1}{15} = \frac{9}{60}$ work. In

12 days they do = $\frac{6 \times 9}{60} = \frac{54}{60}$ work

So, the remaining work = $\frac{6}{60} = \frac{1}{10}$

Now, $\frac{1}{12}$ work will be done by Fatima in 13th day

So, the remaining work = $\frac{1}{10} - \frac{1}{12} = \frac{1}{60}$

This $\frac{1}{60}$ work will be done by Zahira = $\frac{1/60}{1/15} = \frac{1}{4}$ day

Thus the total number of days required

$$= 12 + 1 + \frac{1}{4} = 13\frac{1}{4} \text{ days}$$

☞ **Alternatively** A's efficiency = 8.33%
B's efficiency = 6.66% 15%

Thus in 12 days 90% work will be done and in 13th day 8.33% more work will be done so the rest work 1.66% will be done by Zahira

$$= \frac{1.66}{6.66} = \frac{1}{4} \text{ day}$$

Thus, total number of required days = $12 + 1 + \frac{1}{4} = 13\frac{1}{4}$

34 The difference will arise only on the last two days only. The work done on 13th day = $\frac{1}{15}$

$$\text{rest work} = \frac{1}{10} - \frac{1}{15} = \frac{1}{30}$$

This rest work $\left(\frac{1}{30}\right)$ will be done by Fatima = $\frac{1/30}{1/12} = \frac{2}{5}$ day

Thus total number of required days = $12 + 1 + \frac{2}{5} = 13\frac{2}{5}$ days.

35	1	2	3	4	5	6	7	8
	A	B	C	A	B	C	A	B
	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{12}$
	$\frac{3}{8}$			$\frac{3}{8}$			$\frac{1}{4}$	

$$\frac{6}{8} = \frac{3}{4}$$

In 3 days A, B, C do $\frac{3}{8}$ work

In 6 days A, B, C do $\frac{3}{4}$ work

Rest work = $\frac{1}{4}$, which is less than $\frac{3}{8}$

On the 7th day, $\frac{1}{6}$ more work will be done by A

$$\text{Now rest work} = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$$

Now, this rest work $\left(\frac{1}{12}\right)$ will be done by B in 1 complete day.

Thus, total number of days = $6 + 1 + 1 = 8$ days

☞ **Alternatively** Efficiency of A = 16.66%

Efficiency of B = 8.33%

Efficiency of C = 12.5%

Efficiency of A + B = 25%

Efficiency of A + B + C = 37.5%

In 3 days A, B, C completes 37.5% work

In 6 days A, B, C completes 75% work

Rest work = 25%

This 25% work will be completed by A and B in next 2 days

Thus total $6 + 2 = 8$ days are needed.

36 From the previous solution 75% work will be completed in 6 days. In the next two days (i.e., on 7th and 8th day) B and C will complete 20.83% ($12.5 + 8.33$) more work.

So the remaining work = 4.16%

This 4.16% work will be completed by A = $\frac{4.16}{16.66} = \frac{1}{4}$ day

So, the total number of required days = $6 + 2 + \frac{1}{4} = 8\frac{1}{4}$ days.

37 Let B takes x days to complete the work individually.

Then, the B's 1 day's work = $\frac{1}{x}$

A's 1 day's work = $\frac{1}{x-6}$

C's 1 day's work = $\frac{1}{x-8}$

$$\therefore \frac{1}{x-6} + \frac{1}{x} = \frac{1}{x-8}$$

Now either solve the equation or satisfy the equation from the choices given in the question.

Thus option (c) is correct.

☞ **Alternatively** Select an appropriate option and then solve through % efficiency.

A's share = ₹ 250

B's share = ₹ 100

It means the ratio of efficiency of A : B = 250 : 100 = 5 : 2

$\therefore$ Ratio of days taken by A and B = $2x : 5x$

Now, $5x - 2x = 9$

$\Rightarrow x = 3$

$\therefore$ Number of days taken by A = 6 (efficiency = 16.66%)

Number of days taken by B = 15 (efficiency = 6.66%)

Therefore number of days taken by A and B, working together

$$= \frac{100}{23.33} = \frac{300}{70} = 4\frac{2}{7} \text{ days}$$

- 39 Alen's one day's work = $\frac{1}{21}$
 Border's one day's work = $\frac{1}{42}$
 (working alternatively) Alen and Border's two days work

$$= \frac{1}{21} + \frac{1}{42} = \frac{1}{14}$$

 So, Alen and Border do $\frac{1}{14}$ work in 2 days

So, they complete the work in $14 \times 2 = 28$ days.

- 40 Efficiency of A and B = 16.66%

Efficiency of B and C = 10%

But efficiency of A is twice that of C

$\therefore$ Therefore, $A = 2C$

Now $A + B = 16.66$ and $B + C = 10$

$$\Rightarrow 2C + B = 16.66 \quad \dots(i)$$

$$\text{and } C + B = 10 \quad \dots(ii)$$

$\therefore$ from eq. (i) and (ii) $C = 6.66\%$

$\therefore A = 13.33\%$

$\therefore$ Number of days taken by A to complete the work alone

$$= \frac{100}{13.33} = 7\frac{1}{2} \text{ days}$$

- 41 $A + B = 70\%$

$$B + C = 50\% \quad \left[\begin{array}{l} \therefore A + B + B + C - (A + B + C) = B \\ 70 + 50 - 100 = 20\% \end{array} \right]$$

$$\Rightarrow B = 20\%$$

$$A = 50\%$$

$$\text{and } C = 30\%$$

Hence, A is most efficient.

- 42 $C + M = \frac{8}{17}$
 $M + G = \frac{12}{17}$
 $\Rightarrow M = \frac{3}{17}$

$$[\therefore (C + M + M + G) - (C + M + G) = M]$$

$$\left[\left(\frac{8}{17} + \frac{12}{17} \right) - 1 = \frac{3}{17} \right]$$

$$\therefore C = \frac{5}{17}, G = \frac{9}{17}$$

So the whole amount will be distributed in the ratio of 5 : 3 : 9 among C, M and G respectively.

Now since M is least efficient so he get his own share

$$= \frac{3}{17} \times 816 = ₹ 144$$

- 43
- | | Sharma | Kelkar |
|---------------|--------------------------------|-------------|
| Efficiency | $0.8x$ | x |
| Number of day | k | $0.8k = 24$ |
| $\therefore$ | $0.8k = 24 \Rightarrow k = 30$ | |

Thus Sharma requires 30 days, to complete the work, alone.

- 44
- | | Man | Days |
|------------------------|--|---|
| $\frac{1}{5} \uparrow$ | $\left(\begin{array}{c} 30 \\ 36 \end{array} \right)$ | $\left(\begin{array}{c} 24 \\ 20 \end{array} \right) \frac{1}{6} \downarrow$ |

Applying product constancy method.

For a constant work when days are reduced by $\frac{1}{6}$, then number of men is increased by $\frac{1}{5}$. Hence 6 men will increase.

☞ Alternatively $24 \times 30 = 20 \times x$

$$\Rightarrow x = 36$$

Therefore 6 more men are required.

- 45
- | | Women | Days |
|--------------------------|---|---|
| $\downarrow \frac{1}{3}$ | $\left(\begin{array}{c} 12 \\ 8 \end{array} \right)$ | $\left(\begin{array}{c} 20 \\ 30 \end{array} \right) \frac{1}{2} \uparrow$ |

Thus 10 more days are required.

☞ Alternatively $12 \times 20 = 8 \times x$

$$\Rightarrow x = 30$$

$\therefore$ 10 more days are required.

- 46
- | | Boys | Days |
|--------------------------|--|---|
| $\frac{2}{5} \downarrow$ | $\left(\begin{array}{c} 35 \\ 21 \end{array} \right)$ | $\left(\begin{array}{c} 15 \\ 25 \end{array} \right) \frac{2}{3} \uparrow$ |

Thus 14 boys did not turn up for the job.

☞ Alternatively $35 \times 15 = 25 \times x$

$$\Rightarrow x = 21$$

$\therefore 35 - 21 = 14$ boys did not turn up for the job.

- 47 $x \times 32 = 24 \times 40$

$$x = 30$$

$$(M_1 D_1 = M_2 D_2)$$

- 48
- | |
|-------------------------------|
| $M \times D = W$ |
| $16 \times 6 = \frac{1}{3} W$ |
| Rest work = $\frac{2}{3} W$ |

For double work in same time we need double men. So 16 more men are required.

☞ Alternatively $2 \times (16 \times 6) = 6 \times M$

$$\Rightarrow M = 32$$

$\therefore$ 16 more men are required.

- 49 $M \times D = 10 \times 20 = 200$ Man-days

$$\text{New Man-days} = (20 \times 2) \times x$$

$$200 = 20 \times 2 \times x$$

$$x = 5 \text{ days}$$

or

$$M_1 D_1 = M_2 D_2$$

$$10 \times 20 = (20 \times 2) \times x$$

$\Rightarrow$

$$x = 5$$

- 50

$$M_1 D_1 = M_2 D_2$$

$$M_1 \times 20 = (M_1 - 12) \times 32$$

$\Rightarrow$

$$M_1 = 32$$

Also, using the above concept you can go through options.

- 51 Go through options. Consider option (c)

$$30 \times 20 = 30 \times 6 + 21 \times 20$$

600 = 600, hence presumed option is correct.

☞ Alternatively $30 \times 20 = 30 \times x + 21 \times (26 - x)$

$$\Rightarrow x = 6$$

- 52 Go through options. Consider choice (a)

$$25 \times 30 = 25 \times 15 + 75 \times 5$$

750 = 750, hence choice (a) is correct.

☞ Alternatively $25 \times 30 = 25 \times x + 75 \times (20 - x)$

$$\left[\because 30 \times \frac{2}{3} = 20 \right]$$

- 53 Go through options. Consider choice (d)

$$36 \times 120 = 40 \times 108$$

$$4320 = 4320$$

Hence, choice (d) is correct.

☞ Alternatively

$\frac{1}{9} \uparrow$	Men x $\downarrow$ $\left(x \times \frac{10}{9}\right)$	Days 120 $\downarrow$ (12)	$\left(\frac{1}{10}\right) \downarrow$

From percentage change (product constancy) graphic when number of days are decreased by $\frac{1}{10}$ then the number

of men are increased by $\frac{1}{9}$ and $\frac{1}{9}$ is equivalent to 4 men so the actual number of men are $9 \times 4 = 36$.

☞ Alternatively $M_1 D_1 = M_2 D_2$
 $x \times 120 = (x + 4) \times 108$

$$\Rightarrow x = 36$$

- 54 Efficiency (per minute) of Modi = 4 copies/min

Efficiency of Modi and Xerox together = 10 pages/min

$\therefore$ Efficiency of Xerox alone = $10 - 4 = 6$ pages/min

$\therefore$ Mr. Xerox needs 6 minutes to copy 36 pages.

- 55 Work done = $\frac{1}{3}$

$$\text{Remaining work} = \frac{2}{3}$$

$$2 \times (20 \times 12) = 12 \times x$$

$$\Rightarrow x = 40$$

So, 20 men will be increased.

- 56 Work done = $\frac{1}{5}$

$$\text{remaining work} = \frac{4}{5}$$

$$\therefore 4(20 \times 75) = 40 \times x$$

$$x = 150$$

Therefore 75 men should be increased.

- 57 Work done by 6 men = Work done by 10 women.

$$\Rightarrow \text{Work done by 1 man} = \text{work done by } \frac{10}{6} = \frac{5}{3} \text{ women}$$

$$\therefore 12 \text{ men} + 5 \text{ women} = 12 \times \frac{5}{3} + 5 = 25 \text{ women}$$

$$\therefore W_1 \times D_1 = W_2 \times D_2, W = \text{women}, D = \text{days}$$

$$10 \times 15 = 25 \times D_2 \Rightarrow D_2 = 6$$

- 58 Work done by 2 men = 3 women = 4 boys

$$\therefore 1 \text{ man} = 2 \text{ boys}$$

$$1 \text{ woman} = \frac{4}{3} \text{ boys}$$

$$\therefore \text{boys} \times \text{days} = 4 \times 52 \text{ (boys-days)}$$

$$\text{Again } 1 \text{ man} + 1 \text{ woman} + 1 \text{ boy} = 2 + \frac{4}{3} + 1 = \frac{13}{3} \text{ boys}$$

$$B_1 \times D_1 = B_2 \times D_2, B = \text{boys}, D = \text{days}$$

$$4 \times 52 = \frac{13}{3} \times D_2$$

$$D_2 = 48 \text{ days}$$

- 59 2 men = 7 boys $\Rightarrow 1 \text{ man} = \frac{7}{2} \text{ days}$

$$5 \text{ women} = 7 \text{ boys} \Rightarrow 1 \text{ woman} = \frac{7}{5} \text{ boys}$$

$$7 \text{ men} + 5 \text{ women} + 2 \text{ boys} = 7 \times \frac{7}{2} + 5 \times \frac{7}{5} + 2 = \frac{67}{2} \text{ boys}$$

$$\text{Now, } B_1 \times D_1 = B_2 \times D_2$$

$$7 \times 469 = \frac{67}{2} \times D_2$$

$$\Rightarrow D_2 = 98 \text{ days}$$

- 60 $6C + 2M = 6 \text{ days}$

$$\Rightarrow 36C + 12M = 1 \text{ days}$$

$$\text{Again } 1M = 2C$$

$$\therefore 36 + 12 \times 2 = 1 \text{ day}$$

60 children can do the work in 1 day

Now, 5 men = 10 children

$\therefore$ 10 children can do the work in 6 days.

- 61 $8M + 12W = 4 \text{ days (whole work)}$

$$\Rightarrow 32M + 48W = 1 \text{ day} \quad \dots(i)$$

$$\text{Again } 6M + 14W = 5 \text{ days}$$

$$\Rightarrow 30M + 70W = 1 \text{ day} \quad \dots(ii)$$

From eq. (i) and (ii)

$$32M + 48W = 30M + 70W$$

$$\Rightarrow 2M = 22W$$

$$\Rightarrow 1M = 11W$$

$$\text{Now, } 30M + 70W = 1 \text{ day}$$

$$(30 \times 11 + 70)W = 1 \text{ day}$$

Therefore 400W requires 1 day to complete the whole work.

Thus 20W needs 20 days to complete the whole work.

- 62 Efficiency of A = 6.66%

Efficiency of B = 5.55%

Efficiency of A + B + C = 16.66%

$\therefore$ Efficiency of C = 4.44%

Now, number of days required by

$$C = \frac{100}{4.44} = \frac{100}{(10 \times 4)/9} = \frac{45}{2} \text{ days}$$

- 63** Ratio of efficiencies of A, B and C = 6 : 5 : 4

$$\therefore \text{Share of C} = \frac{4}{15} \times 27000 = 7200$$

- 64** In 1 hour 314 weavers weave = 6594 × 6 shawls

$$\begin{aligned} \text{In 1 hour 1 weaver weaves} &= \frac{6594 \times 6}{314} \text{ shawls} \\ &= 126 \text{ shawls} \end{aligned}$$

- 65** $3M + 2W = 4 \text{ days}$

$$\Rightarrow 12M + 8W = 1 \text{ day} \quad \dots(i)$$

$$\text{Again, } 2M + 3W = 5D$$

$$\Rightarrow 10M + 15W = 1D \quad \dots(ii)$$

From eq. (i) and (ii)

$$12M + 8W = 10M + 15W$$

$$\Rightarrow 2M = 7W$$

$$\Rightarrow \frac{M}{W} = \frac{7}{2}$$

Since the ratio of efficiency of women : men = 2 : 7

So, the amount of a man per day = ₹ 154

$$\left[\left(\frac{7}{2} \times 44 \right) = 154 \right]$$

- 66** Go through options.

☞ **Alternatively**

$$30 \times 40 = 30 \times x + 20 \times (46 - x) \Rightarrow x = 28 \text{ days}$$

- 67** Women × hours = 8 × 10 = 80 women hours

Now, women × hours = 12 × 8 = 96 women hours

Since new work force is 20% greater than previous work force.

So, the new quantity of tea leaves will be increased by 20% which is equal to $200 \times \frac{20}{100} = 40 \text{ kg}$, hence (b)

- 68** New work = 3 × 450 man-day

$$\therefore 3 \times 450 = 27 \times x$$

$$x = 50 \text{ days}$$

- 69** $4B + 5G = 10$

$$\Rightarrow 40B + 50G = 1 \quad \dots(i)$$

$$\text{Again, } 6B + 6G = 7$$

$$\Rightarrow 42B + 42G = 1 \quad \dots(ii)$$

Comparing eq. (i) and (ii)

$$40B + 50G = 42B + 42G$$

$$\Rightarrow 2B = 8G \Rightarrow 1B = 4G$$

Now, $(42 \times 4 + 42) \text{ girls} = 1 \text{ day}$

210 girls can do a work in 1 day

Again $2B + 7G = 15 \text{ girls}$

$$\text{So, } 15 \text{ girls require } \frac{210}{15} = 14 \text{ days}$$

- 70** Equate the man-days

For 20 km road, $20 \times 20 = 400 \text{ man-days}$ are required

$\therefore$ For 40 km road 800 man-days are required

$$\text{So, } 800 = 40 \times x \Rightarrow x = 20 \text{ days}$$

71 $\frac{3}{4} \times (x - 2) x = (x + 7)(x - 10)$

$$\Rightarrow x^2 - 6x - 280 = 0$$

$$\Rightarrow x = 20 \text{ and } x = -14$$

So, the acceptable values is $x = 20$

$$\therefore \text{Total work} = (x - 2) \times x = 18 \times 20 = 360 \text{ unit}$$

$$\text{Now } 360 = 30 \times k \quad \therefore (30 = 20 + 10)$$

$$\Rightarrow k = 12 \text{ days}$$

- 72** Efficiency of a man : woman : girl = 6 : 3 : 1

$$\begin{aligned} \therefore \text{Share of a woman and girl} &= \frac{(3+1)}{(6+3+1)} \times 10,000 \\ &= \frac{4}{10} \times 10000 = ₹ 4000 \end{aligned}$$

- 73** Total work = 33 × 30 = 990 man-days

$$\text{First day's work} = \frac{1}{990} \times 44$$

$$\text{Second day's work} = \frac{1}{990} \times 43$$

$$\text{Third day's work} = \frac{1}{990} \times 42 \text{ and so on}$$

$$\begin{aligned} \text{So, the total work in 44 days} &= \frac{1}{990} (44 + 43 + 42 + \dots) \\ &= \frac{1}{990} \times \frac{44 \times 45}{2} = 1 \end{aligned}$$

Hence in 44 days total work will be completed.

- 74** Efficiency of Abhishek = 2.5% = $\left(\frac{100}{40} \right)$

Work done in 8 days = 20%

Rest work = 80%

$$\text{Efficiency of Bacchhan} = \frac{80}{24} = 3.33\%$$

$$\begin{aligned} \therefore \text{Required number of days} &= \frac{100}{(2.5 + 3.33)} \\ &= \frac{100}{5.83} = \frac{100 \times 6}{35} = 17 \frac{1}{7} \end{aligned}$$

$$\text{Hint } 0.83 = \frac{5}{6}, \text{ so, } 5.83 = 5 + \frac{5}{6} = \frac{35}{6}$$

- 75**

	A	B	C
Efficiency	3	2	6
Number of days	2	3	1

$\therefore$ Number of days taken by A = 12

Number of days taken by B = 18

Number of days taken by C = 6

$$1 \text{ day's work of } (A + B) = \frac{5}{36}$$

$$1 \text{ day's work of } (B + C) = \frac{8}{36}$$

$$1 \text{ day's work of } (C + A) = \frac{9}{36}$$

Day	→	1	2	3	4	5	6
Work	→	5/36	8/36	9/36	5/36	8/36	1/36

In 5 days total work done = $\frac{35}{36}$

Now, the rest work (1/36), which is done by AC.

∴ Number of days taken by AC for the rest work
 $= \frac{1/36}{9/36} = \frac{1}{9}$

Therefore, total time = $5 + \frac{1}{9} = 5\frac{1}{9}$ days

76 Efficiency of A + B = 16.66, Efficiency of B + J = 10

We have no further relevant informations, so we cannot determine.

	A	B
Efficiency	x	2x
Number of days	15	15/2
A's one day's work	$\frac{1}{15}$	$\frac{2}{15}$
B's one day's work	$\frac{2}{15}$	$\frac{3}{15}$
(A + B)'s one day's work	$\frac{3}{15}$	

Now, let us assume B joined A after (11 - x) days, then

$$\frac{(11-x)}{15} + \frac{x \times 3}{15} = 1 \Rightarrow (11-x) + 3x = 15 \Rightarrow x = 2$$

It means they worked together for 2 days.

☞ **Alternatively** Go through options and check easily with the percentage efficiency.

$$9 \times 6.66 + 2 \times 20 = 100\%$$

78 Efficiency of A = 6.66%
 Efficiency of B = 3.33%
 Efficiency of C = 2.5%

Work done in last two days (only C do it) = $2 \times 2.5 = 5\%$

Work done in the 3rd and 4th day from the last day
 (only A and C do it) = $2 \times 9.16 = 18.33\%$

Remaining work = $100 - (5 + 18.33) = 76.66\%$

This 76.66% work was done by all of A, B and C.

∴ Number of days taken by them = $\frac{76.66}{12.5} = \frac{460}{75} = 6\frac{2}{15}$

∴ Total time required = $6\frac{2}{15} + 2 + 2 = 10\frac{2}{15}$ days

79 Let x kg of oil is used for eating purpose, daily, then

$$(x + 11) \times 50 = (x + 15) \times 45$$

$$x = 25$$

∴ Total quantity of oil = $(25 + 11) \times 50 = 1800$

∴ Required number of days = $\frac{1800}{25} = 72$ days

80 Work done = $\frac{2}{3}$

Remaining work = $\frac{1}{3}$, which is half of $\frac{2}{3}$

∴ $\frac{1}{2} \times (20 \times 32) = 8 \times x \Rightarrow x = 40$ men

Therefore, 20 more men were required.

81 One day's work of 7 Indian with 4 Chinese = $\frac{1}{5}$

and one day's work of 7 Japanese and 3 Chinese = $\frac{1}{7}$

Therefore, one day's work of 7 Indian, 7 Chinese and 7 Japanese
 $= \frac{1}{5} + \frac{1}{7} = \frac{12}{35}$

Therefore, one day's work of 1 Indian, 1 Chinese and 1 Japanese
 $= \frac{12}{35} \times \frac{1}{7} = \frac{12}{35 \times 7}$

Therefore, number of days required by 1 Indian, 1 Chinese and 1 Japanese

$$= \frac{1}{12/(35 \times 7)} = \frac{35 \times 7}{12} = 20\frac{5}{12} \text{ days}$$

82 In 1 minute A, B and C bind = $\frac{1}{8} + \frac{1}{12} + \frac{1}{16} = \frac{13}{48}$ book

In 12 hours A, B and C bind = $12 \times 60 \times \frac{13}{48} = 195$ books

∴ Average number of books bind by each = $\frac{195}{3} = 65$ books

83 Work of a man for 1 hour = $\frac{3}{2}$ women's work for 1 hour

Again, work of a man for 1 day

$$= \left(\frac{3}{2} \times \frac{9}{7.5}\right) \text{ women's work for 1 hour}$$

⇒ Work of a man for 1 day = $\frac{9}{5}$ women's work for 1 day

⇒ 1 man = $\frac{9}{5}$ women

∴ 10 men + 6 women = $10 \times \frac{9}{5} + 6 = 24$ women

∴ 10 men + 9 women = $10 \times \frac{9}{5} + 9 = 27$ women

Now,

$$D_1 \times W_1 = D_2 \times W_2$$

$$18 \times 24 = D_2 \times 27 \Rightarrow D_2 = 16 \text{ days}$$

84 Number of days taken by A to complete work alone

= 14 days

Number of days taken by B to complete work alone = 7 days

Number of days taken by C to complete work alone = 7 days

One day's work of A and B = $\frac{1}{14} + \frac{1}{7} = \frac{3}{14}$

and one day's work of A, B and C = $\frac{1}{14} + \frac{1}{7} + \frac{1}{7} = \frac{5}{14}$

3 day's work of A and B = $3 \times \frac{3}{14} = \frac{9}{14}$

remaining work = $\frac{5}{14} \left(1 - \frac{9}{14}\right)$

This remaining work will be done by A, B and C

$$= \frac{5/14}{5/14} = 1 \text{ day}$$

85

A's 5 days work = 50%

B's 5 days work = 33.33%

C's 2 days work = 16.66% [100 - (50 + 33.33)]

Ratio of contribution of work of A, B and

$$C = 50 : 33\frac{1}{3} : 16\frac{2}{3} = 3 : 2 : 1$$

A's total share = ₹ 1500

B's total share = ₹ 1000

C's total share = ₹ 500

A's one day's earning = ₹ 300

B's one day's earning = ₹ 200

and C's one day's earning = ₹ 250

86 Efficiency of A = 50% $\left(\frac{100}{2}\right)$

Efficiency of B = 37.5% $\left(\frac{100}{8/3}\right)$

Efficiency of C = 12.5% $\left(\frac{100}{8}\right)$

Combined efficiency of A, B and C = 100%

So, they complete the work in 1 hour.

87

	A	+	B	=	C	+	D
	↓		↓		↓		↓
Ratio of efficiency	10x	+	5x		9x	+	6x
	15x				15x		

Therefore, ratio of efficiency of A : C = 10 : 9

Therefore, ratio of days taken by A : C = 9 : 10

Therefore, number of days taken by A = 18 days

88 Efficiency of 4 men and 2 boys = 20%

Efficiency of 3 women and 4 boys = 20%

Efficiency of 2 men and 3 women = 20%

∴ Efficiency of 6 men, 6 women and 6 boys = 60%

∴ Efficiency of 1 man, 1 woman and 1 boy = 10%

Now, since they will work at double their efficiency

∴ Efficiency of 1 man, 1 woman and 1 boy = 20%

∴ Required number of days = 5

89 $M_1 \times D_1 = M_2 \times D_2$

$$m \times r = (m + n) \times D_2$$

$$\Rightarrow D_2 = \frac{mr}{(m + n)}$$

90 Efficiency of A = 2.77%, Efficiency of B = 2.22%

∴ Combined efficiency of A and B = 5% = (2.77 + 2.22)

Thus, it will take total of 20 minutes $\left(\because 20 = \frac{100}{5}\right)$

91 Efficiency of A = 8.33%

Effective efficiency = 6.66%, when there is leakage

∴ Efficiency of leakage = 1.66% = (8.33 - 6.66)

It means due to leakage a full tank will be empty in 60 hours.

92 Efficiency of A + B = 10 + 6.66 = 16.66%

Efficiency of A + B + C = 5.55%

∴ Efficiency of C (outlet pipe) = 16.66 - 5.55 = 11.11%

It means outlet pipe C can empty in 9 hours.

93 Efficiency of only leakage = 16.66%

Effective efficiency of leakage = 6.66%

It means the capacity of filling pipe = 10%

Therefore, the inlet pipe can fill the tank in 10 hours hence the capacity of tank = 100 l.

94 Efficiency of tap A and B = 16.66% = (10 + 6.66)

$$\therefore 16.66x + 10 \times (8 - x) = 100 \Rightarrow x = 3$$

95 Efficiency of A = 5%, Efficiency of B = 4%

Efficiency of C = -3.33%

It means in every 3 consecutive hours tops A, B and C can fill 5.66% (= 5 + 4 - 3.33). Therefore in 51 hours (= 3 × 17) tops A, B and C can fill 96.33% (= 5.66 × 17)

∴ the remaining part i.e., 3.66% (= 100 - 96.33) can be filled up by A in $\frac{11}{15}$ hours $\left(= \frac{3.66}{5}\right)$, since it is now A's turn.

$$\text{Hence, the total time required} = 51 + \frac{11}{15} = 51\frac{11}{15}$$

97 Efficiency when both pipes used to fill = A + B

and efficiency when pipe A is used to fill and pipe B is used to empty the tank = A - B

$$\therefore \frac{A+B}{A-B} = \frac{2}{1} \Rightarrow \frac{A}{B} = \frac{3}{1} \quad (\text{By componendo and dividendo})$$

Thus, the ratio of efficiency of pipe A and B = 3 : 1.

98 In ideal case:

$$\text{Time taken to fill the half tank by A and B} = \frac{50}{41.66} = \frac{6}{5} \text{ hours}$$

Time taken by A, B and C to fill rest half of the tank

$$= \frac{50}{16.66} = 3 \text{ hours}$$

$$\text{Total time} = \frac{6}{5} + 3 = 4 \text{ hours 12 minutes}$$

In second case:

$$\text{Time taken to fill } \frac{3}{4} \text{ tank by A and B} = \frac{75}{41.66} = \frac{9}{5} \text{ hours}$$

$$\text{Time taken by A, B and C to fill rest } \frac{1}{4} \text{ tank} = \frac{25}{16.66} = \frac{3}{2} \text{ hours}$$

$$\text{Total time} = \frac{9}{5} + \frac{3}{2} = 3 \text{ hours 18 minutes}$$

Therefore, difference in time = 54 minutes

99 Time taken by pipes A and B to fill the whole tank

$$= \frac{100}{16.66} = 6 \text{ hours}$$

$$\text{Capacity filled in 2 hours by pipes A, B and C} = 2 \times 13.33 = 26.66\%$$

Remaining capacity = 73.33%

This remaining capacity can be filled by A and B

$$= \frac{73.33}{16.66} = 4\frac{2}{5} \text{ hours}$$

$$\text{So, the total time required} = 2 + 4\frac{2}{5} = 6 \text{ hours 24 minutes}$$

Thus, in this case 24 minutes extra are required.

- 100 In one hour pipe A can fill = $\frac{1}{30}$ part of the tank
 In one hour pipe B can fill = $\frac{1}{45}$ part of the tank
 In two hour pipes A and B can fill = $\frac{1}{18}$ part of the tank

✓ Level 02 Higher Level Exercise

- 1 LCM of 2, 3 and 4 = 12

In 12 hours A will make 6 shawls
 " " " B will make 4 shawls
 " " " C will make 3 shawls

i.e., in 12 hours they will weave 13 shawls

so, in 84 hours they will weave 91 shawls

Now, in 9 hours A will make 4 shawls
 in 9 hours B will make 3 shawls
 in 9 hours C will make 2 shawls

So, they will complete 100 shawls in 93 hours.

NOTE Since, they cannot share each-others work so B will take completely 9 hours to make 3 shawls, even when A and C stay idle for the last 1 hour till B completes his own work.

2

	M	T	W	Th	F	S
	A	S	A	S	A	S
Work →	15%		15%		7.5%	
	37.5%					

{This pattern continued for total 2 weeks only till 75% work got completed.}

Thus in 2 weeks they will complete 75% work.

Now 15% of the remaining (25% of the work) will be done in the third week in Monday and Tuesday. Again 10% work remained undone. Out of this 8.33% work will be done by Arun on Wednesday and remaining 1.66% work will be completed on Thursday by Satyam.

Final week

M	T	W	Th
A	S	A	S
15%		8.33%	1.66%
25%			

- 3 Efficiency of Kaushalya = 5%

Efficiency of Kaikeyi = 4%

Thus, in 10 days working together they will complete only 90% of the work.

$$[(5 + 4) \times 10] = 90$$

Hence, the remaining work will surely done by Sumitra, which is 10%.

Thus, Sumitra will get 10% of ₹ 700, which is ₹ 70.

Therefore in 36 hours the tank will be completely filled.

☞ **Alternatively** Efficiency of pipe A = 3.33%

Efficiency of pipe B = 2.22%

and

Combined efficiency = 5.55%

Therefore in 2 hours pipe A and B fill 5.55%.

Thus to fill 100% tank, these pipe will take 36 hours.

Solutions (for Q. Nos. 4 and 5)

	1st day	2nd day
Morning shift}	A	B
Evening shift}	B	A

It is clear that in two days finally they work very similar to the alternate days i.e., finally A work for 10 hours and B also works for 10 hours. Thus in every two days they will complete 7.5% work. So, in 26 days they will complete 97.5% of the total work. Now, the remaining work = 2.5%

Now, this is the turn of A,

Since A does 4.16% work in 10 hours.

So, he will do 2.5% work in 6 hours.

- 4 Thus, the work will be finished on 27th day.

- 5 Since, A does 2.5% work in 6 hours, which is the actual duration of morning shift. So 100% time of the morning shift was utilised.

- 6 From the first statement :

	(B + C)	(A + B + C)
Efficiency	2x	3x
Days	3y	2y

Thus, we can say that efficiency of A is $\frac{1}{3}$ the efficiency of (A + B + C).

From the last statement:

$$\text{Share of B out of total amount} = \frac{120}{450} = \frac{4}{15}$$

From these two results we can conclude that:

	A	:	B	:	C
Ratio of efficiency	$\frac{5}{15} \left(= \frac{1}{3} \right)$	:	$\frac{4}{15}$	:	$\frac{6}{15}$
	$\Rightarrow 5$	:	4	:	6
Ratio of number of days	$\Rightarrow \frac{1}{5}$	:	$\frac{1}{4}$	:	$\frac{1}{6}$
	$\Rightarrow 12x$	:	$15x$	:	$10x$

$$\text{One day's work of A and B} = \frac{1}{12x} + \frac{1}{15x} = \frac{9}{60x}$$

∴ A and B will take $\frac{60x}{9}$ days to complete the whole work,

Again one day's work of A, B and C

$$= \frac{1}{12x} + \frac{1}{15x} + \frac{1}{10x} = \frac{15}{60x}$$

$\therefore$ A, B and C working together complete the work in $\frac{60x}{15}$ days

$$\therefore \frac{60x}{9} - \frac{60x}{15} = \frac{8}{3}$$

(Since, A and B take $\frac{8}{3}$ days more than A, B and C)

$$\Rightarrow x = 1$$

$\therefore$ Number of days required to complete the whole work by A, B and C = $4x = 4 \times 1 = 4$ days

☞ **Alternatively** You can solve through option.

☞ **Alternatively** Ratio of efficiencies of A, B and C = $5x : 4x : 6x$

$$\therefore \text{Number of days required by A and B} = \frac{100}{9x} \quad \dots(i)$$

$$\text{and number of days required by A, B and C} = \frac{100}{15x} \quad \dots(ii)$$

$$\therefore \frac{100}{9x} - \frac{100}{15x} = \frac{8}{3} \Rightarrow x = \frac{5}{3}$$

From eq. (ii) number of days required by A, B and C working together

$$= \frac{100}{15x} = \frac{100}{15 \times \frac{5}{3}} = \frac{100}{25} = 4 \text{ days}$$

$$\begin{array}{ccc} 7 & \text{MIT} & \text{NIT} \\ & 4 \times 10 \times 60 \times E_1 & = 5 \times 8 \times 80 \times E_2 \\ \Rightarrow & \frac{E_1}{E_2} = \frac{4}{3} \end{array}$$

where E_1 and E_2 are the respective working efficiencies per hour.

$\therefore$ Each engineer from NIT is 25% less efficient than each engineer from MIT.

8	T	C	B	
	16	10	15	
	8	12	12	
	128	120	180	← in one hour
	1280	1200	1800	← in 10 hour

Since, restriction is imposed by composers i.e., since only 1200 books can be composed in 10 hours so not more than 1200 books can be finally prepared.

9 To maximise the production we locate 5 persons for composing and 7 persons for typing. Only then we can maximise our production which is 1800 books per day.

	T	C	B
	(16 + 7)	(10 + 5)	15
	8	12	12
	184	180	180
	1840	1800	1800

10	1st case	T	C	B
		15	10	13
		8	12	12
		120	120	156
		1200	1200	1560
		No change in critical value		

2nd case	T	C	B
	16	10	12
	8	12	12
	128	120	144
	No change in critical value*		

☞ **NOTE** In third case it will reduce the production below 1200 books per day (or 120 books per hour)

So, option (d) is correct.

Critical value means the minimum amount of job which the restriction.

11 Efficiency of Bunty and Babli (jointly) = 12.5%

Now, go through options and satisfy the conditions.

Consider option (d).

	Bunty	Babli
Efficiency	7.5%	5%
Days	$\frac{40}{3}$	20

Now, the new efficiency of Bunty = 15%

and the new efficiency of Babli = $\frac{5}{3}\%$

$$\therefore \text{Combined efficiency} = \frac{50}{3}\%$$

$$\therefore \text{Number of days taken by them} = \frac{100}{50/3} = 6 \text{ days}$$

Hence, the presumed option (d) is correct.

☞ **NOTE** Without solving the complete problem we can say that only option (d) is true since other 3 options gives the efficiency of Bunty equal to or more than 12.5% which is inadmissible i.e., cannot be equal to or greater than the combined efficiency of both persons together.

12 Let x litre be the per day filling and v litre be the capacity of the reservoir, then

$$90x + v = 40000 \times 90 \quad \dots(i)$$

$$\text{and } 60x + v = 32000 \times 60 \quad \dots(ii)$$

Solving eq. (i) and (ii), we get

$$x = 56000$$

Hence, 56000 litres per day can be used without the failure of supply.

13 One day's production = $400 \times 9 \times 60 = 2160000$ bottles per day

Ratio of time utilised by M_1 and $(M_1 + M_2) = 1 : 2$

Now, the production of bottles by M_1 in 1 minute = 400

and the production of bottles by M_1 and M_2 together in 2 minutes = 2000

Thus total 2400 bottles can be processed in 3 minutes

$\therefore$ 216000 bottles can be processed in

$$216000 \times \frac{3}{2400} = 4.5 \text{ hours}$$

14 Let he initially employed x workers which works for D days and he estimated 100 days for the whole work and then he doubled the worker for $(100 - D)$ days.

$$D \times x + (100 - D) \times 2x = 175x \Rightarrow D = 25 \text{ days}$$

NOTE $175 = 100 + \frac{3}{4} \times 100$, since required number of days are $75\% \left(\text{i.e., } \frac{3}{4} \right)$ more than the estimated number of days.

Now, the work done in 25 days = $25x$
 Total work = $175x$
 $\therefore$ Work done before increasing the number of workers
 $= \frac{25x}{175x} \times 100 = 14 \frac{2}{7} \%$

- 15 NOTE** For easier calculation consider some convenient value of x (i.e., number of workers). Let initially there were 20 workers employed.

It means work done till 25 days = $25 \times 20 = 500$ man-days
 Now, since delay in works is 60%.
 It means the work was completed in 160 days. Let the increased workers worked for k days then
 $40 \times k + 16 \times (135 - k) = 75 \times 40$
 $\Rightarrow k = 35$ days
 $\therefore$ 40 workers work for only 35 days
 Here 40 means twice the work force of 20
 and 16 means 80% efficiency of the original work-force
 and 135 means $(160 - 25)$ days
 and 75 means $(100 - 25)$ days
 Since, number of days are increased by 60 in which only 16 workers work.
 $\therefore$ Remaining work after 100 days = $60 \times 16 = 960$
 $= \frac{960}{3500} \times 100 = 27 \frac{5}{7} \%$

Solution (for Q. Nos. 16 to 20) Let the actual number of typists be n required to work for D days, then

$$nD = (n \times 8) + \left(\frac{4n}{5} \times D \right) \Rightarrow D = 40$$

- 16** Since, the above equation is independent of n (i.e., number of typists) so cannot be calculated. It means there are many possible values.
- 17** Since $20\% \left(\text{i.e., } \frac{1}{5} \right)$ typists left the job. So, there can be any value which is multiple of 5 i.e., whose 20% is always an integer. Hence, 5 is the least possible value.
- 18** The remaining value must be divisible by 4.
 Since, $\frac{4x}{5} = k$ $x \rightarrow$ actual number of typists
 $\Rightarrow x = \frac{k \times 5}{4}$ $k \rightarrow$ remaining number of typists
 So, k must be divisible by 4, since a person cannot be a fraction. Hence, option (c) could be the possible answer.
- 19** Since, $D = 40$
- 20** Work done in 8 days = $8x$
 Work done in further 16 days = $16 \times \frac{4}{5} x = \frac{64x}{5}$

$$\text{Remaining work} = 40x - \left(8x + \frac{64x}{5} \right) = \frac{96x}{5}$$

$$\text{Remaining number of days} = 40 - (8 + 16) = 16$$

$$\therefore \text{New work-force} = \frac{96x/5}{16} = \frac{6x}{5}$$

$$\therefore \text{Change (or increase) in work-force} = \frac{\left(\frac{6x}{5} \right) - x}{x} \times 100 = 20\%$$

- 21** Let A, B, C, D and E stitches a, b, c, d and e shirts per day then

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{1}{e} = \frac{90}{1800} = \frac{1}{20}$$

when $a = b = c = d = e$, we get the minimum value

$$\therefore 5 \times \frac{1}{a} = \frac{1}{20}$$

$$\Rightarrow a = 100 = b = c = d = e$$

- 22** $(8M + 5W) \times 6 \times 4 = (4M + 5W) \times 8 \times 5$

$$\Rightarrow 4M = 10W$$

$$\Rightarrow 1M = 2.5W$$

Now, substituting the value M by W , we get total work-done

$$\text{Work} = (8 \times 2.5 + 5) \times 6 \times 4 = 600 \text{ women-days-hours} \\ = 240 \text{ man-days-hours}$$

$$\text{Again work} = 5 \times 8 \times 30 = 1200 \text{ boy-days-hours}$$

$$\therefore 1M = 2.5W = 5B$$

$$4M + 3W + 4B = 30 \text{ boys}$$

$$\therefore \text{Required number of days} = \frac{1200}{30 \times 5} = 8 \text{ days}$$

- 23** $9 \times 6 \times M = 240$

$$M = \frac{240}{6 \times 9} = 4.4 \text{ men}$$

Therefore, minimum 5 men are required.

- 24** Efficiency of Eklaya = 16.66%

$$\text{Efficiency of Faizal} = 8.33\%$$

$$\text{Total efficiency of Eklaya and Faizal} = 25\%$$

So, they can do actual work in 4 days

$\therefore$ 3 times work requires 12 days.

- 25** $1 + 2 + 3 + 4 + 5 + \dots + 24 = \frac{24 \times 25}{2} = 300$

$$\text{Total work} = 300 \text{ man-days}$$

But, the person who started the work on the first day works for 24 days. Hence, his share will be maximum which is equal to $\frac{24}{300} = \frac{2}{25}$. Thus, he will receive $5000 \times \frac{2}{25} = ₹ 400$.

- 26** Total efficiency of two persons = 50%

$$\text{Ratio of efficiencies of first person to the second person} = 1 : 2$$

Therefore, efficiency of second person = 33.33%

Hence, he will take 3 days to complete the work alone.

- 27 Ratio of number of men, women and children

$$= \frac{18}{6} : \frac{10}{5} : \frac{12}{3} = 3x : 2x : 4x$$

$$\therefore (3x + 2x + 4x) = 18$$

$$\therefore x = 2$$

Therefore, number of women = 4

$$\text{Share of all women} = \frac{10}{40} \times 4000 = ₹ 1000$$

$$(\because 18 + 10 + 12 = 40)$$

$$\therefore \text{Share of each woman} = \frac{1000}{4} = ₹ 250$$

- 28 It can be solved easily through option.

$$(10 + 9 + 8 + \dots + 1) = 10 \times \left(10 \times \frac{55}{100}\right)$$

$$\frac{55 = 55}{2} = n \times \frac{55n}{100} \quad \text{Hence correct.}$$

$$\text{Alternatively } \frac{n(n+1)}{2} = n \times \frac{55n}{100}$$

$$\Rightarrow n = 10$$

In both cases total work is 55 man-days.

- 29 Go through option. Consider choice (c).

Efficiency of first worker = 5%

Efficiency of second worker = 4%

$\therefore$ In 7 hours first worker completed 35% work

In 5 hours second worker completed 20% work

Thus, work completed = 55%

Remaining work = 45%

Hence, one condition is satisfied.

Again, they will take 5 more hours to complete 45% work

$$\left[\frac{45}{4 + 5} = 5 \right]$$

Thus, first person completes $7 \times 5 + 5 \times 5 = 60\%$ work

and second person completes $5 \times 4 + 5 \times 4 = 40\%$ work

Hence, second condition is also satisfied. Hence, correct option is (c).

- 30 Go through option

$$140 \times 4 = (140 + 120 + 100 + \dots + 20)$$

$$560 = 560$$

Alternatively Let n be the initial number of worker then

$$n \times 4 = n + (n - 20) + (n - 40) + \dots + (n - 120)$$

$$4n = 7n - 420 \Rightarrow 3n = 420 \Rightarrow n = 140 \text{ workers}$$

- 31 From the first statement

	B		$(V + M)$
Number of days	x	$=$	x
Efficiency	1	$:$	1

From the second statement

	V		$(V + M)$
Number of days $\rightarrow$	$(K + 8)$		K

From the third statement

	B		V
Number of days $\rightarrow$	$(n - 8)$		n

Now, go through option and consider option (c).

Efficiency of $(B + V + M) = 16.66\%$ (number of days = 6)

$\therefore$ Efficiency of $B = 8.33\%$ (number of days = 12)

and Efficiency of $(V + M) = 8.33\%$ (number of days = 12)

Therefore, B will take 12 days.

Now, from third statement V will take 20 days.

Hence, the efficiency of V is 5%.

Therefore, V will take 20 days. Hence, second statement is also true. Thus, the presumed option (c) is correct.

- 32 Consider option (d)

Time taken by Bill = 12 hours, Efficiency of Bill = 8.33%

Therefore, time taken by Bill and Milinda working together

$$= 6 \frac{2}{3} \text{ hours}$$

Hence, the efficiency of Milinda and Bill = 15%

Therefore efficiency of Milinda = $15 - 8.33 = 6.66\%$

Thus, the number of days taken by Milinda = 15, which is $8 \frac{1}{3}$ days more than when both work together.

Hence, all the conditions satisfied.

		M	$B + M$	B
Efficiency	$\rightarrow$	6.66%	15%	8.33%
Number of days	$\rightarrow$	15	$6 \frac{2}{3}$	12

- 33 Efficiency of Pascal and Rascal = 10%

Pascal worked for 2.5 hours and Rascal worked separately 8.5 hours. Which means it can be considered that Pascal and Rascal worked together for 2.5 hours and Rascal worked alone for 6 hours.

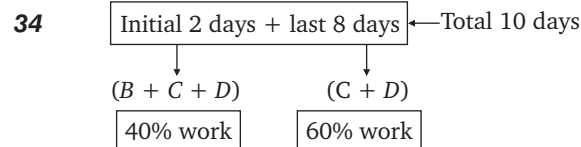
Thus, Pascal and Rascal in 2.5 hours can complete 25% work. It means the remaining $(50 - 25) = 25\%$ of the work was done by Rascal in 6 hours.

Therefore, Rascal can do 100% work in 24 hours. It means the efficiency of Rascal = 4.16%.

Therefore, efficiency of Pascal = $(10 - 4.16) = 5.83\%$

Thus, Pascal require $\frac{100}{5.83} = 17 \frac{1}{7}$ hours to complete the work alone.

Alternatively Go through option.



From the above diagram it is clear that efficiency of C and D is 7.5%, since C and D complete 60% work in 8 days and efficiency of B, C and D is 20%. It means efficiency of B alone is $12.5\% = (20 - 7.5)$.

Now	C	$:$	D
Number of days	$4x$	$:$	$5x$
Efficiency	$5y$	$:$	$4y$

$$\therefore \text{Efficiency of } C = \frac{5}{9} \times 7.5 = 4.16\%$$

and Efficiency of $D = \frac{4}{9} \times 7.5 = 3.33\%$

Thus, D is the least efficient person.

Now share of work done by David

$$(D) = 3.33\% \times 10 = 33.33\%$$

Hence, his share of amount = 33.33% of ₹ 3000 = ₹ 1000

- 35** Combined efficiency of all the three boats = 60 passenger/trip

Now, consider option (a).

15 trips and 150 passengers means efficiency of $B_1 = 10 \frac{p}{t}$

which means in carrying 50 passengers B_1 must have taken 5 trips. So the rest trips equal to 5 ($10 - 5 = 5$) in which B_2 and B_3 together carried remaining 250 ($300 - 50$) passengers.

Therefore the efficiency of B_2 and $B_3 = \frac{250}{5} = 50 \frac{p}{t}$

Since, the combined efficiency of B_1, B_2 and B_3 is 60. Which is same as given in the first statement hence option (a) is correct.

☞ **Alternatively** It can be solved by framing quadratic equation.

- 36** Efficiency of 3 men + 5 women = 33.33%

Required number of days by 2 men = x

∴ Required number of days by 3 women = $x + 5$

Now, consider option (c).

Therefore, $3M + 5W = 3M + 2M = 5$ men

Therefore, efficiency of a man = 6.66%

Hence, a man needs 15 days to finish the job, working alone.

Again $3M + 5W = 7.5W + 5W = 12.5W$

Therefore, efficiency of a woman = 2.66%

Therefore, a woman needs 37.5 days.

Thus, 2 men need 7.5 days to work alone ∴ $\left(7.5 = \frac{15}{2}\right)$

and 3 women need 12.5 days to work alone

∴ $\left(12.5 = \frac{37.5}{3}\right)$

Hence, the difference in number of days = 5 which is same as given in the problem. Hence correct option is (c).

- 37** Efficiency of Henry and Ford (combined) = 10%

Consider option (d).

Efficiency of Ford = 3.33% (30 days)

Therefore, Efficiency of Henry = 6.66%

Now, the new efficiency of Ford = 16.66%

and the new efficiency of Henry = 3.33%

Therefore, newly combined efficiency of H and F = 20%

Therefore, required number of days by Henry and Ford working together = 5

Since 5 is half of 10, hence the option (d) is correct.

- 38** From the last statement:

Efficiency of Anne (A), Benne (B) and Cenne (C) = 50%

From the first statement: Number of days taken by B is 2 more than C.

From the second statement: Anne had worked for 6 days and Benne had worked for 3 days only. Now, consider option (d).

Number of days taken by B = 6, Efficiency = 16.66%

It means Benne had completed $16.66 \times 3 = 50\%$ work in 3 days.

Therefore Anne had completed 50% work in 6 days.

Thus, the efficiency of Anne = $8.33\% \left(\frac{50}{6}\right)$

Hence, the efficiency of Cenne

$$= 50 - (16.66 + 8.33) = 25\%$$

Thus B takes 6 days (∵ efficiency = 16.66%)

and C takes 4 days (∵ efficiency = 25%)

which is true according to the first statement, hence option (d) is correct.

- 39** Number of pages typed by A, B and C together per day = 45

Now let the number of pages typed by B is x

then the number of pages typed by A = $x - d$

and the number of pages typed by C = $x + d$

$$\Rightarrow (x - d) + (x) + (x + d) = 45$$

$$\Rightarrow x = 15 \text{ pages per day.}$$

Again let C types k pages per day then A types $\frac{k}{4}$ pages per day.

Therefore, the ratio of typing of pages per day of A and C = 1 : 4

∴ Number of pages typed by C in one day

$$= \frac{4}{5} \times 30 = 24 \text{ pages} \quad (30 = 45 - 15)$$

∴ Number of pages typed by C per hour = $\frac{24}{8} = 3$ pages/hour

- 40** Efficiency of P = 5%

Efficiency of Q = 4%

Efficiency of R = 2.5%

Efficiency of S = 2%

Till 10 am pipe P filled 20%
Till 10 am pipe Q filled 8%
Till 10 am pipe R filled 2.5% } 30.5%

Thus, at 10 am pipe P, Q and R filled 30.5% of the cistern. Now, the time taken by P, Q, R and S together to fill the remaining capacity of the cistern

$$= \frac{69.5}{13.5} = \frac{139}{27} = 5 \text{ hours and 9 minutes (approx)}$$

Therefore, total time = 4 hours + 5 hours 9 minutes = 9 hours and 9 minutes

It means cistern will be filled up at 3 : 09 pm

- 41 Efficiency of $P + Q = 9\%$ (inlet pipes)

Efficiency of $R + S = 4.5\%$ (outlet pipes)

Net efficiency = 4.5%

So, the time taken = $\frac{100}{4.5} = 22\frac{2}{9}$ hours

- 42 Efficiency of $A = 10\%$, Efficiency of $B = 6.66\%$

Efficiency of $C = 5\%$, Efficiency of $D = 3.33\%$

Efficiency of $A + B + C + D = 8.33$ (time = 12 hours)

Now, go through options and consider A and B as inlet pipes and C and D as outlet pipes, then

$$(10 + 6.66) - (5 + 3.33) = 8.33$$

which is required hence it is certain that C and D are outlet pipes.

 **NOTE** There is no any other such combination.

- 43 Efficiency of inlet pipe $A = 4.16\%$ $\left(\frac{100}{24}\right)$

Efficiency of inlet pipe $B = 5.83\%$ $\left(\frac{100}{120/7}\right)$

$\therefore$ Efficiency of A and B together = 10% . (time = 10 hours)

Now, if the efficiency of outlet pipe be $x\%$ then in 10 hours the capacity of tank which will be filled = $10 \times (10 - x)$

Now, since this amount of water is being emptied by C at $x\%$ per hour, then

$$\frac{10 \times (10 - x)}{x} = 2.5 \text{ hours} \Rightarrow x = 8\%$$

Therefore in 10 hours 20% tank is filled only. Hence, the remaining 80% of the capacity will be filled by pipes A and

B in $\frac{80}{10} = 8$ hours.

- 44 Efficiency of pipe $A = 8.33\%$

Efficiency of pipe $B = 6.67\%$

Efficiency of pipe $C = 5\%$

When tap C was opened pipe A filled 16.66% capacity

When tap C was opened pipe B filled 6.67% capacity

Therefore rest capacity of the tank to be filled

$$= 100 - 23.34 = 76.66\%$$

Now, the net efficiency of A , B and $C = 10\%$

Hence, pipes A , B and C will take

$$= \frac{76.66}{10} = 7.66 = 7\frac{2}{3} \text{ hours}$$

$\therefore$ Total time = 2 hours + $7\frac{2}{3}$ hours = $9\frac{2}{3}$ hours

- 45 Efficiency of inlet pipe = 50%

Efficiency of outlet pipe = 16.66%

Net efficiency of pipes A and $B = 33.33\%$

Capacity of tank to be filled up = 66.66%

Hence, required time = $\frac{66.66}{33.33} = 2$ hours

- 46 Rate of leakage = 8.33% per hour

Net efficiency = $50 - (16.66 + 8.33) = 25\%$

Time required = $\frac{100}{25} = 4$ hours

- 47 Since, an inlet pipe is 7.2 times efficient than an outlet pipe. Therefore, in order to tank never overflow we will need total 8 outlet pipes.

Thus we need only 7 more ($8 - 1 = 7$) outlet pipes.

- 48 Time taken by 8th tap = $2 \times 2 \times 2 = 8$ hours

and time taken by 12th tap = $2 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$ hour

Ratio of time taken by 8th tap and 12th tap

$$= 8 : \frac{1}{2} = 16 : 1$$

$\therefore$ Ratio of efficiencies of 8th tap and 12th tap = $1 : 16$

- 49 Time taken by 10th tap = $80 \times \frac{1}{2} \times \frac{1}{2} = 20$ hours

Time taken by 12th tap = $80 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 5$ hours

Thus 10th tap and 12th tap together will take 4 hours.

- 50 Let pipe A fill the tank in $3x$ hours then pipe B fill it in $4x$ hours.

Therefore, in 1 hour they will fill = $\frac{1}{3x} + \frac{1}{4x} = \frac{7}{12x}$

i. e., they will take $\frac{12x}{7}$ hours

When an inlet pipe C is also opened then it takes

$$\begin{aligned} &= \frac{12x}{7} + \frac{12x}{7} \times \frac{3}{4} \\ &= \frac{12x}{7} \left(\frac{7}{4}\right) = 3x \text{ hours} \end{aligned}$$

Now, in one hour pipe A , B and C working together fill

$$= \frac{1}{3x} + \frac{1}{4x} - \frac{1}{C} = \frac{1}{3x}$$

$$\Rightarrow \frac{7}{12x} - \frac{1}{C} = \frac{1}{3x}$$

$$\Rightarrow \frac{1}{C} = \left(\frac{7}{12x} - \frac{1}{3x}\right)$$

$$\Rightarrow C = 4x$$

Hence in $4x$ hours pipe C can empty the whole tank.

Now, since $3x = 33 \Rightarrow x = 11$

$\therefore 4x = 4 \times 11 = 44$ hours

- 51 Total work = $5 \times 20 = 100$ man-days.

Let the client needed to complete it in n days then

$$(5 \times 2) + (10 \times 2) + (15 \times 2) + (20 \times 2) = 100$$

Hence, in 8 days all the work will be completed as per the requirement of client.

And on the 8th day 20 men were working.

- 52 It requires total 8 days by adding the work force successively.

- 53** Total work = 100 + 50 = 150 man-days

In 8 days 100 man-days work has been completed. Now on 9th and 10th day there will be 25 workers. So in 2 days they will complete additional 50 man-days work. Thus the work requires 2 more days.

- 54** Let there be 'n' inlet pipes then there must be (8 - n) outlet pipes.

$$\text{Therefore } (8 - n) \frac{1}{6} - n \times \frac{1}{8} = \frac{1}{6} \Rightarrow n = 4$$

☞ **Alternatively** $(8 - n) 16.66 - n \times 12.5 = 16.66$

$$\Rightarrow n = \frac{116.66}{29.16} = \frac{700}{175} = 4$$

☞ **Alternatively** It can be solve through options very easily.

- 55** Efficiency of two inlet pipes A and B = 8.33 + 6.66 = 15%

Efficiency of two inlet pipes A and B along with an outlet pipe

$$C = 8.33 + 6.66 - 12.5 = 2.5\%$$

Hours	1	2	3	4	5	6	7	8	9	10
Efficiency →	15	15	2.5	15	15	2.5	15	15	2.5	
	32.5			32.5			32.5			
	97.5%								2.5%	

In 9 hours 97.5% tank will be completely filled. On the 10th hour 2.5% (remaining capacity) will be fill by pipe A, B with its 15% efficiency.

$$\text{Thus, } A \text{ will take } = \frac{2.5}{15} = \frac{1}{6} \text{ hour} = 10 \text{ minutes}$$

Therefore, total time = 9 hours and 10 minutes.

- 56** Let pipe A and pipe B fill an empty tank in x hours and y hours, respectively. Therefore, the capacity filled by both the pipes in 1 hour is

$$\frac{1}{x} + \frac{1}{y} = \frac{7}{48} \quad \dots(i)$$

Further, we have

$$\frac{1}{y} \left(\frac{x}{3} \right) + \frac{1}{x} \left(\frac{y}{4} \right) = \frac{7}{12}$$

$$\Rightarrow 4 \left(\frac{x}{y} \right) + 3 \left(\frac{y}{x} \right) = 7$$

$$\Rightarrow \frac{x}{y} = \frac{3}{4} \text{ or } \frac{1}{1}$$

Now, if we consider $\frac{x}{y} = \frac{1}{1}$ then Eq. (i) will not be satisfied.

$$\text{So, we can consider } \frac{x}{y} = \frac{3}{4}$$

$$\text{That is } \frac{x}{y} = \frac{3k}{4k}$$

$$\text{Now, using Eq. (i), we have } \frac{1}{3k} + \frac{1}{4k} = \frac{7}{48}$$

$$\Rightarrow k = 4$$

$$\therefore x = 3k = 12 \text{ and } y = 4k = 16$$

Hence, choice (c) is the correct one.

- 57** Let us denote men by m, women by w and days by d.

Then,

$$4m + 3w = 6d \quad \dots(i)$$

$$\Rightarrow 8m + 6w = 3d \quad \dots(ii)$$

$$\text{and } 5m + 6w = 4d \quad \dots(iii)$$

Comparing Eq. (ii) and (iii) we can infer that 3 extra men working for 3 days save 1 day out of the 4 days.

That means 3 extra men working for 3 days, save 25% of the time.

It implies that 3 men working for 3 days can finish 25% of the total work.

So, to finish 100% work the number of man-days required = $3 \times 3 \times 4 = 36$

$$\text{Again, } 4m + 3w = 6d \quad \dots(iv)$$

$$\Rightarrow 24m + 18w = 1d$$

$$\text{But, } 24m + 18w = 36m$$

$$\Rightarrow 12m = 18w$$

$$\text{or } 2m = 3w \quad \dots(v)$$

$$\text{Therefore, } 3m + 2w = \frac{13}{3}m$$

Since, 1 man requires 36 days to finish the job

$$\text{So, } \frac{13}{3} \text{ men will require } \frac{36}{13/3} = \frac{108}{13}$$

$$= 8.3 \approx 9 \text{ days}$$

Hence, choice (c) is the required answer.

☞ **Alternatively**

$$4m + 3w = 6d \quad \dots(i)$$

$$\Rightarrow 8m + 6w = 3d \quad \dots(ii)$$

$$\Rightarrow 2.67m + 2w = 9d$$

$$\text{Again, } 4m + 3w = 6d$$

$$\Rightarrow 12m + 9w = 2d \quad \dots(iii)$$

$$\Rightarrow 3m + 2.25w = 8d \quad \dots(iv)$$

Now, we need to find the number of days required by $3m + 2w$.

If you look at Eq. (ii), then you can easily infer that the number of days required by $3m + 2w$ should be little less than 9.

But, if you look at the Eq. (iv), then you can infer that the number of days required should be little more than 8.

So, finally we can conclude that it will take more than 8 but less than 9 days.

It implies that the work will be finished on the 9th day only.

- 58** Let us consider the LCM of 45, 40, 30, 20 and 15, which is 360. Now, let us assume that the capacity of the tank be 360 liters. Therefore, the rate of flow of water of the pipes P_1, P_2, P_3, P_4, P_5 will be 8, 9, 12, 18 and 24 liter/min.

Outlet Pipes	Inlet Pipes				
	P_1 (8)	P_2 (9)	P_3 (12)	P_4 (18)	P_5 (24)
P_1 (-8)		1	4	10	16
P_2 (-9)	-1		3	9	15
P_3 (-12)	-4	-3		6	12
P_4 (-18)	-10	-9	-6		6
P_5 (-24)	-16	-15	-12	-6	

If we consider P_4 as inlet and P_3 and P_5 as outlet pipes, we see that $\{P_4, P_3\} = \{6\}$ and $\{P_4, P_5\} = \{-6\}$

Outlet Pipes	Inlet Pipes		
	P_3 (12)	P_4 (18)	P_5 (24)
P_3 (-12)		6	12
P_4 (-18)	-6		6
P_5 (-24)	-12	-6	

Therefore, $\{P_4, P_3\}$ will fill the tank in 1 hour and $\{P_4, P_5\}$ will empty the tank in 1 hour.

Hence, choice (d) is the correct one

- 59** Let's create a matrix as shown below.

		Incoming			
		P	Q	R	S
Outgoing	P		52		48 (100)
	Q			36	(36)
	R	76			24 (100)
	S		48		(48)
		76	100	36	72

Now, let's calculate the net flow of the water.

$$P : 76 - 100 = -24$$

$$Q : 100 - 36 = 64$$

$$R : 36 - 100 = -64$$

$$S : 72 - 78 = 24$$

It shows that R gives away 64 liters/second, so R will be empty first and the time taken = $\frac{384}{64} = 6$ seconds.

Hence, choice (c) is the correct one.

CHAPTER 09

Time, Speed and Distance

It is sure and certain that it is one of the most important chapters, which includes various fundamental and logical concepts and therefore most of the problems are complex in their nature. Every year 4-6 problems are generally asked in CAT. Besides CAT almost every aptitude exam contains the questions pertaining to the concepts of TSD.

This chapter includes the following :

- (a) Motion in a straight line (b) Circular motion and races
- (c) Problems based on trains, boats, river and clocks etc.

9.1 Concept of Motion

When a body moves from a point A to another point B at a distance of D , then it requires some time (T) to cover a distance (D) with a particular speed (S).

The relation between T , S and D is as follows : $T \times S = D$ i.e., Time $\times$ Speed = Distance

Therefore, when D is constant, $T \propto \frac{1}{S}$ and when T is constant, $D \propto S$ and when S is constant, $D \propto T$

 **NOTE** This relation of proportionality is very important.

Formulae : Distance = Speed $\times$ Time, Speed = $\frac{\text{Distance}}{\text{Time}}$ or Time = $\frac{\text{Distance}}{\text{Speed}}$

 **NOTE** To solve the problem, all the units involved in the calculation must be uniform i.e., either all of them be in metres and seconds or in kilometres and hours etc.

Conversion of unit: $1 \text{ km/h} = \frac{5}{18} \text{ m/s}$

$$1 \text{ m/s} = \frac{18}{5} \text{ km/h} \quad [1 \text{ km} = 1000\text{m}, \quad 1 \text{ h} = 60 \text{ min}, \quad 1 \text{ min} = 60 \text{ s}]$$

Hint Try to find these relations by unitary method.

$$1 \text{ mile} = 1609.30 \text{ m} = 1.6093 \text{ km}$$

and $1 \text{ km} = 0.621 \text{ mile}$, $1 \text{ yard} = 0.9144 \text{ m}$

Chapter Checklist

- Concept of Motion
- Relative Motion with Two or More Bodies
- To and Fro Motion in a Straight Line
- Concept Based on Trains
- Concept Based on Boats and Rivers (or streams)
- Races
- Circular Motion
- Clocks
- CAT Test

and 1 m = 1.0936 yards and 1 m = 39.4 inches

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time taken}}$$

When distances are equal :

$$\text{Average speed} = \frac{2xy}{x+y} \quad (\text{speeds } x, y)$$

$$\text{and Average speed} = \frac{3xyz}{xy + yz + zx} \quad (\text{speeds } x, y, z)$$

Exp. 1) Abhishek drives his bike at the speed of 150 km/h. What is the distance covered by him in 3 hours.

Solution $D = S \times T = 150 \times 3 = 450 \text{ km}$

Exp. 2) Udai travels half of his journey by train at the speed of 120 km/h and rest half by car at 80 km/h. What is the average speed?

Solution Let the total distance be $2D$ km, then

$$\begin{aligned} \text{Total time} &= \text{Time taken by train} + \text{Time taken by car} = \frac{D}{120} + \frac{D}{80} \\ \therefore \text{Average speed} &= \frac{2D}{\frac{D}{120} + \frac{D}{80}} = \frac{2D}{D\left(\frac{1}{120} + \frac{1}{80}\right)} = \frac{2}{\left(\frac{2+3}{240}\right)} = 96 \end{aligned}$$

km/h

Alternatively Consider the distance in numerals (since it is independent of the distance) as the LCM of the speeds.

Then, the total distance = LCM of 120 and 80 = 240

$$\text{Therefore, time taken by train} = \frac{120}{120} = 1 \text{ h}$$

$$\text{and time taken by car} = \frac{120}{80} = \frac{3}{2} \text{ h}$$

$$\begin{aligned} \therefore \text{Average speed} &= \frac{\text{Total distance}}{\text{Total time taken}} \\ &= \frac{240}{1 + \frac{3}{2}} = 96 \text{ km/h} \end{aligned}$$

$$\text{Alternatively Average speed} = \frac{2(xy)}{(x+y)}$$

where x and y are the speeds.

$$\therefore \text{Average speed} = \frac{2 \times 120 \times 80}{120 + 80} = 96 \text{ km/h}$$

NOTE This formula is applicable only when the distances travelled at the speed of x km/h and y km/h are same.

Alternatively Solve through alligations.

Exp. 3) Speed of Karan is 40 km/h and speed of Arjun is 60 km/h. What is the ratio of time taken by each to cover the same distance?

Solution	Karan		Arjun
Speed	40	:	60
$\Rightarrow$	2	:	3
$\therefore$ Time	3	:	2

$$\text{Alternatively Time taken} = \frac{1}{40} : \frac{1}{60} = 3 : 2$$

NOTE For the constant distance covered, the ratio of time taken by each person is the ratio of the reciprocal of their speeds and vice-versa.

If the ratio of speeds of A, B, C, D be $a : b : c : d$, then the ratio of time taken by A, B, C and D to cover the same distance

$$= \frac{1}{a} : \frac{1}{b} : \frac{1}{c} : \frac{1}{d}$$

and if the time taken by A, B, C and D be $p : q : r : s$, then the

$$\text{ratio of their speeds} = \frac{1}{p} : \frac{1}{q} : \frac{1}{r} : \frac{1}{s}$$

Exp. 4) A car takes half of the time taken by truck to go from Lucknow to Bombay. A truck takes 20 hours to go for the same journey. What is the speed of truck, if the speed of car be 120 km/h.

Solution	Car	Truck
Time $\rightarrow$	1	: 2
Speed $\rightarrow$	2	: 1

Therefore, speed of the truck = 60 km/h

Exp. 5) A cycle covers 75 km distance in 5 hours. What is the distance covered by the cycle in 6 hours?

$$\begin{aligned} \text{Solution Speed} &= \frac{\text{Distance}}{\text{Time}} \\ \therefore \text{Speed} &= \frac{75}{5} = 15 \text{ km/h} \end{aligned}$$

$$\begin{aligned} \text{Now, distance covered in 6 hours at the speed of 15 km/h} \\ = 15 \times 6 = 90 \text{ km} \end{aligned}$$

Exp. 6) The distance of the college and the home of Rajeev is 80 km. One day he was late by 1 hour than the normal time to leave for the college, so he increased his speed by 4 km/h and thus he reached to college at the normal time. What is the changed (or increased) speed of Rajeev?

(a) 28 km/h (b) 30 km/h (c) 40 km/h (d) 20 km/h

Solution Let the normal speed be x km/h, then

$$\begin{aligned} \frac{80}{x} - \frac{80}{(x+4)} &= 1 \\ \Rightarrow x^2 + 4x - 320 &= 0 \\ \Rightarrow x(x+20) - 16(x+20) &= 0 \\ (x+20)(x-16) &= 0 \\ x &= 16 \text{ km/h} \\ \therefore (x+4) &= 20 \text{ km/h} \end{aligned}$$

Therefore increased speed = 20 km/h

Alternatively If t_1 and t_2 be the original and changed time and S_1 and S_2 be the original and changed speeds, then

$$S_1 \times S_2 = \frac{\text{Distance} \times (S_1 \sim S_2)}{(t_1 \sim t_2)}$$

$$\text{Also, } t_1 \times t_2 = \frac{\text{Distance} \times (t_1 \sim t_2)}{(S_1 \sim S_2)}$$

$$\therefore S_1 \times S_2 = \frac{80}{1} \times 4 = 320 \quad \therefore S_1 \times S_2 = 320$$

$$\text{or } S_1 \times (S_1 + 4) = 320 \Rightarrow S_1 = 16 \quad \text{and} \quad S_2 = 20$$

NOTE Here you need not to solve necessarily quadratic equation. You can just try and find two factors of 320 in such a way that (in this particular problem) one factor must be greater than the other factor by 4.

Alternatively Go through options. Consider 20

$$t_2 = \frac{80}{20} = 4 \Rightarrow t_1 = 4 + 1 = 5$$

$$\therefore S_1 = \frac{80}{5} = 16$$

Since, S_1 is 4 km/h less than S_2 . Hence, option (d) is correct.

Alternatively If you find yourself comfortable then factorise 80 in two factors and check the correct combinations.

$$80 = \begin{bmatrix} 1 \times 80 \\ 2 \times 40 \\ 4 \times 20 \\ 5 \times 16 \\ 8 \times 10 \end{bmatrix} \begin{matrix} \\ \\ \textcircled{-1} \\ \\ \textcircled{+4} \end{matrix}$$

So, the increased speed = 20 km/h.

Exp.7) Shweta when increases her speed from 24 km/h to 30 km/h she takes one hour less than the usual time to cover a certain distance. What is the distance usually covered by Shweta?

- (a) 160 km (b) 240 km (c) 120 km (d) 90 km

Solution Let the original time be t hours, then

$$24t = 30 \times (t - 1) = D \quad (\text{distance})$$

$$\text{Then} \quad t = 5$$

$$\therefore \text{Distance} = 24 \times 5 = 120 \text{ km}$$

Alternatively Go through options.

$$\begin{matrix} \frac{120}{24} = 5 \text{ h} \\ \frac{120}{30} = 4 \text{ h} \end{matrix} \quad \left. \begin{matrix} \\ \end{matrix} \right\} \text{1 hour less}$$

Hence, the option (c) is correct.

Alternatively Since distance (D) is constant.

$$\text{Therefore, } D = S_1 \times t_1 = S_2 \times t_2$$

It means here we can apply product constancy

$$\begin{matrix} \text{Speed} & & \text{Time} \\ \frac{1}{4} \uparrow & & \frac{1}{5} \downarrow \end{matrix} = 1 \text{ hour}$$

$$\therefore \text{Original time taken} = 5 \times 1 = 5 \text{ hours}$$

$$\text{Therefore, Distance} = \text{Original speed} \times \text{Original time}$$

$$= 24 \times 5 = 120 \text{ km}$$

NOTE In the given exercise or in the whole chapter you have to solve maximum problems through product constancy concept described in the chapter of ratio proportion and variation. Solving through product constancy gives faster results.

Alternatively Let the distance be D , then

$$\frac{D}{24} - \frac{D}{30} = 1 \Rightarrow D = 120 \text{ km}$$

Exp. 8) Kriplani goes to school at 20 km/h and reaches the school 4 minutes late. Next time, she goes at 25 km/h and reaches the school 2 minutes earlier than the scheduled time. What is the distance of her school?

Solution Increase in speed = 5 km/h

$$\text{Decrease in time} = 6 \text{ min } (4 + 2)$$

By product constancy :

$$\begin{matrix} \text{Speed} & & \text{Time} \\ \frac{1}{4} \uparrow & & \frac{1}{5} \downarrow \end{matrix} = 6 \text{ min}$$

$$\text{It means original time} = 30 \text{ min} \quad \left(\because \frac{x}{5} = 6 \Rightarrow x = 30 \right)$$

$$\begin{aligned} \therefore \text{Total distance} &= \text{Original speed} \times \text{Original time} \\ &= 20 \times \frac{30}{60} = 10 \text{ km} \end{aligned}$$

Exp. 9) Amit covers a certain distance with his own speed, but when he reduces his speed by 10 km/h his time duration for the journey increases by 40 hours, while if he increases his speed by 5 km/h from his original speed he takes 10 hours less than the original time taken. Find the distance covered by him.

$$\begin{aligned} \text{Solution} \quad & \left. \begin{matrix} S & T \\ -10 & +40 \end{matrix} \right\} 40S - 10T = |-400| \quad \dots(i) \\ & \left. \begin{matrix} S & T \\ +5 & -10 \end{matrix} \right\} -10S + 5T = |-50| \quad \dots(ii) \end{aligned}$$

Solving eq. (i) and (ii), we get

$$S = 25 \quad \text{and} \quad T = 60$$

$$\therefore \text{Distance } (D) = S \times T = 25 \times 60 = 1500 \text{ km}$$

where $D \rightarrow$ Distance, $S \rightarrow$ Speed, $T \rightarrow$ Time

'+' means increase in value.

and '-' means decrease in value.

Alternatively Let distance be x km and usual speed be y km/h.

$$\frac{x}{(y-10)} - \frac{x}{y} = 40$$

$$\Rightarrow x \left[\frac{10}{y(y-10)} \right] = 40$$

$$\Rightarrow x = 4y(y-10) \quad \dots(i)$$

$$\text{and} \quad \frac{x}{y} - \frac{x}{(y+5)} = 10$$

$$\Rightarrow x = 2y(y+5) \quad \dots(ii)$$

from eq. (i) and (ii)

$$4y(y-10) = 2y(y+5)$$

$$2y - 20 = y + 5$$

$$y = 25 \text{ km/h}$$

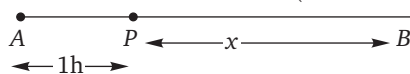
$$\therefore x = 1500 \text{ km}$$

Exp. 10) A train met with an accident 60 km away from Anantpur station. It completed the remaining journey at $\frac{5}{6}$ th of the previous speed and reached the Baramula station 1 hour 12 min late. Had the accident taken place 60 km further, it would have been only 1 hour late.

- (a) What is the normal speed of the train?
 (b) What is the distance between Anantpur and Baramula?

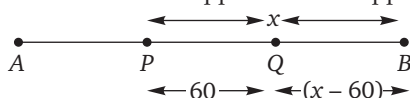
Solution

Case I. Since the speed is decreased by $\frac{1}{6}$. So, the time will be increased by $\frac{1}{5}$, which is equal to 1 hour 12 minutes. It means the normal time required for this remaining part (x) of the journey is $5 \times 72 \text{ min} = 360 \text{ min} = 6 \text{ h}$.
 ($\because 1 \text{ h } 12 \text{ min} = 72 \text{ min}$)



P is the place of accident.

Case II. When accident is supposed to be happened at Q .



Since, the speed is decreased by $\frac{1}{6}$, hence, the time will be increased by $\frac{1}{5}$, which is equal to 1 hour, hence the normal time required for this remaining part ($x - 60$) of journey $= 5 \times 1 = 5$ hours. Thus, it is clear that when the train runs 60 km of its normal speed it takes 1 hour less, which implies that in 1 hour the train can run 60 km with its normal speed. Thus, the normal speed of the train is 60 km/h.

- (b) Since the train requires 6 hours at its normal speed of 60 km/h for the x km. Hence, $x = 6 \times 60 = 360 \text{ km}$

Thus, the total distance = Distance travelled before accident
 + Distance travelled after accident
 $= 60 \times 1 + 60 \times 6 = 420 \text{ km}$

Exp. 11) A and B started simultaneously towards each other from P and Q respectively. The distance between P and Q is 600 km and the ratio of speeds of A to B is 5 : 7. If they meet at a point M:

- (i) Find the ratio of PM : QM. (ii) Find the distance PM.

Solution

- (i) Since the time is constant so, the distance covered by A and B is directly proportional to the speeds of A and B. Hence, $PM : QM = 5 : 7$
 (ii) Since, the ratio of their speeds (or their distances covered) is 5 : 7. Hence, A will cover $\frac{5}{12}$ of the total length.

$$\therefore PM = \frac{5}{12} \times 600 = 250 \text{ km}$$

$$\text{(Similarly } QM = \frac{7}{12} \times 600 = 350 \text{ km)}$$

9.2 Relative Motion with Two or More Bodies

(i) **When two bodies move in the same direction:** If the speeds of the two bodies A and B be S_A and S_B , then their relative speed $= S_A - S_B$ or $S_B - S_A$ i.e., in the same direction the relative speed or effective speed between two bodies is the difference of their speeds. (The difference is always considered as positive)

(ii) **When two bodies move in the opposite direction:** If the speeds of the two bodies A and B be S_A and S_B , then their relative speed $= S_A + S_B$. i.e., in the opposite direction the relative speed or effective speed between two bodies is the sum of their speeds.

Exp. 12) The distance between two places P and Q is 700 km. Two persons A and B started towards Q and P from P and Q simultaneously. The speed of A is 30 km/h and speed of B is 40 km/h. They meet at a point M which lies on the way from P to Q.

- (i) How long will they take to meet each other at M?
 (ii) What is the ratio of PM : MQ?
 (iii) What is the distance MQ?
 (iv) What is the extra time needed by A to reach at Q than to reach at P by B?
 (v) What is the ratio of time taken by A and B to reach their respective destinations after meeting at M?
 (vi) In how many hours will they be separated by only 560 km before meeting each other.
 (vii) How long will it take to separate them by 280 km from each other when they cross M (time to be considered after their meeting)?

Solution



- (i) Since, they are coming towards each other from opposite ends, therefore the relative speed will be the sum of their speeds $= 30 + 40 = 70 \text{ km/h}$.

Thus, the required time to meet at M

$$= \text{Time required to cover 700 km (combined)} \\ = \frac{700}{70} = 10 \text{ h}$$

Thus in 10 hours they will meet each other at M.

- (ii) The ratio of their distances covered to meet at M

$$= \text{Ratio of their speeds} = 3 : 4$$

(Since, time is constant i.e., same for each)

Thus $PM : MQ = 3 : 4$

$$\text{(iii) } MQ = \frac{4}{7} \times 700 = 400 \text{ km}$$

$$\text{(iv) Time required by A to reach at Q} = \frac{700}{30} = \frac{70}{3} \text{ h}$$

$$\text{Time required by B to reach at P} = \frac{700}{40} = \frac{70}{4} \text{ h}$$

$$\therefore \text{Extra time required by } A = \frac{70}{3} - \frac{70}{4} \\ = 70 \times \frac{1}{12} = 5 \text{ h } 50 \text{ min}$$

$$\begin{aligned} \text{(v) Time required by } A \text{ to cover } MQ &= \frac{400}{30} \\ \text{and time required by } B \text{ to cover } MP &= \frac{300}{40} \\ \therefore \text{Required ratio} &= \frac{400/30}{300/40} = \frac{16}{9} \end{aligned}$$

Remember If speed of A is S_A and speed of B is S_B and A takes t_A time to cover MQ and B takes t_B time to cover MP , then $\frac{S_A}{S_B} = \sqrt{\frac{t_B}{t_A}}$

(vi) It means they have to cover $(700 - 560) = 140$ km. Thus, the required time to cover 140 km distance $= \frac{140}{70} = 2$ h

(vii) Since in each hour they separate by 70 km from each other. Hence, to separate by 280 km, time required $= \frac{280}{70} = 4$ h

Exp. 13) Two places P and Q are 800 km apart from each other. Two persons start from P towards Q at an interval of 2 hours. Whereas A leaves P for Q before B . The speeds of A and B are 40 km/h and 60 km/h respectively. B overtakes (or catches or meets) A at M , which is on the way from P to Q .

- How long will B take to overtake?
- What is the distance from P , where B overtakes A (i.e., PM)?
- What is the ratio of time taken by A and B to meet at M ?
- What is the extra time required by A to reach at Q ?
- How many hours late A will reach at Q than that of B ?
- After how many hours A and B will be separated by 50 km before M , when both are moving?
- How many hours does B require to advance himself, by 100 km, in comparison to A ?



- (i) Since A moves 2 hours earlier than B at the speed of 40 km/h, so in 2 hours A will cover 80 km. Thus A will be 80 km away from B .

Now, since they are moving in the same direction therefore their relative speed will be $(60 - 40) = 20$ km/h i.e., with respect to A , B moves at 20 km/h. It means either they reduce (or create) the difference of 20 km in each hour between themselves.

Thus, the required time to overtake (or reduce the difference upto zero from 80 km)

$$= \frac{\text{Distance advanced}}{\text{Relative speed}} = \frac{80}{20} = 4 \text{ h}$$

- (ii) The distance between P and M
 $= (\text{Time required to overtake} \times \text{Speed of the faster body})$
 $= 4 \times 60 = 240 \text{ km}$

Since, B has to move for 4 hours at 60 km/h. Hence, distance covered $= 240$ km.

- (iii) Time taken by B to reach at $M = 4$ h
 Time taken by A to reach at $M = (4 + 2) = 6$ h
 Thus, the ratio of time taken by A and $B = 6 : 4 = 3 : 2$
 (Since, A has left 2 hours earlier)

- (iv) Time taken by A to reach at $Q = \frac{800}{40} = 20$ h

Time taken by B to reach at $Q = \frac{800}{60} = 13 \text{ h } 20 \text{ min}$

Hence, A takes 6 hours 40 min extra to reach at Q .

- (v) Since, A leaves 2 hours earlier, thus he will reach at Q only 4 hours 40 min late.

Since, A takes 6 hours 40 min extra to reach at Q .

- (vi) When B starts to move towards Q the difference between A and B is 80 km.

The required difference between A and $B = 50$ km

Hence, they have to reduce it by 30 km.

Thus, the required time $= \frac{(80 - 50)}{20} = 1.5 \text{ h} = 1 \text{ h } 30 \text{ min}$

Thus, after $3/2$ hours A will be only 50 metres ahead of B .

- (vii) When B starts to follow A (towards Q) A was 80 km ahead of B . Also B wants to overtake A and further go ahead of A by 100 km. Thus, the net difference (required) $= 180$ km.

$$\begin{aligned} \therefore \text{Required time} &= \frac{(\text{Distance advanced} + \text{Required difference})}{\text{Relative speed}} \\ &= \frac{80 + 100}{20} = \frac{180}{20} = 9 \text{ h} \end{aligned}$$

Thus, after 9 hrs (when B starts moving) B will be 100 km ahead of A i.e., they will be separated by 100 km from each other after crossing M .

9.3 To and Fro Motion in a Straight Line

This concept is just the extension of the previous concepts of relative motion between more than one dynamic (or moving) bodies.

(a) When two bodies start moving towards each other

Let the initial distance (or gap) between two bodies A and B be D , and their respective speeds be S_A and S_B , then A and B together have to cover D unit of distance for the first meeting, irrespective of their speeds.

- (i) To meet each other they cover the distances in the ratio of their individual speeds.

- (ii) If the initial distance (or gapping) between two bodies A and B is D , then A and B together have to cover D unit of distance for the first meeting.

- (iii) For the next number of meeting (e.g., second, third, fourth meeting and so on) both A and B together have to cover $2D$ distance more from the previous meeting *i.e.*, to meet for the fourth time they have to cover together $D + (3 \times 2D) = 7D$ unit of distances. Similarly for seventh meeting they have to cover together $D + (6 \times 2D) = 13D$ units of distance. Thus for each subsequent meeting they have to cover $2D$ distance extra from the previous one.

NOTE Individually they will cover the distances in the ratio of their speeds for any number of meeting. Thus, the total distance covered for the n^{th} meeting $= (2n - 1)D$.

(b) **When two bodies start moving towards the same direction**

Let the two bodies A and B be initially at the same end of the track, their respective speeds be S_A and S_B and D be the length of the track, then A and B together have to cover $2D$ unit of distance for the first meeting, irrespective of their speeds.

- (i) For the first meeting after they start to move they have to cover $2D$ distance, if the distance between two particular points (or places) be D unit. Since, the faster body reaches the next (or opposite) end first than the slower body and the faster body starts returning before the slower body reaches the same opposite end and thus the two bodies meet somewhere between the two ends covering individually the distances in their respective speeds.
- (ii) For every subsequent meeting they have to cover together $2D$ unit distance more from the previous meeting.

Thus, for n^{th} meeting they have to cover together $(n \times 2D)$ unit of distance.

- (iii) At any point of time the distances covered by the bodies will be equal to the ratio of their speeds.

Exp. 14) The distance between P and Q is 100 m and the speeds of A and B are 20 m/s and 30 m/s respectively. Initially A and B are at P . They move between P and Q . Calculate:

- (i) The distance covered by A in first meeting. (ii) The time required for the second meeting.
 (iii) The distance covered by B to meet for the third time. (iv) The ratio of distances covered by A and B till the fourth meeting.
 (v) The distance between P and the place of fifth meeting.
 (vi) The distance between Q and the point of third meeting.

Solution	Meeting	Total distance covered by A and B together	Distance covered by A	Distance covered by B	Point of meeting from P	Point of meeting from Q	Time (in second)
	1 st	200 m	80 m	120 m	80 m	20 m	4
	2 nd	400 m	160 m	240 m	40 m	60 m	8
	3 rd	600 m	240 m	360 m	40 m	60 m	12
	4 th	800 m	320 m	480 m	80 m	20 m	16
	5 th	1000 m	400 m	600 m	0 m (at P)	100 m	20

- (i) The distance covered by A in the first meeting $= 80$ m (ii) Time required for the second meeting $= 400/50 = 8$ s
 (iii) The distance covered by B for the third time meeting $= 12 \times 30 = 360$ m
 (iv) It is always (for any moment, after the starting of movement) will be in the ratio of their respective speeds.
 So, the required ratio of distances covered by A and $B = 20 : 30 = 2 : 3$ *i.e.*, $A : B = 20 : 30 = 2 : 3$
 (v) Since for the fifth meeting they have to cover $50 \times 20 = 1000$ m.
 (vi) 60 m.

Exp. 15) The distance between two points P and Q is 100 m. A is initially at P and B is at Q . The speeds of A and B is 20 m/s and 30 m/s. They move between P and Q to and fro :

- (i) Find the time required for the first meeting. (ii) Distance covered by A till the third meeting.
 (iii) Distance covered by B till the fifth meeting. (iv) The distance between P and the place of fourth meeting.
 (v) The distance between Q and the place of fifth meeting. (vi) The ratio of distances covered by each one till the third meeting.

Solution	No. of meeting	Distance covered by A and B together	Time (in second)	Distance covered by A	Distance covered by B	Distance between P and point of meeting	Distance between Q and point of meeting
	1 st	100 m	2	40 m	60 m	40 m	60 m
	2 nd	300 m	6	120 m	180 m	80 m	20 m
	3 rd	500 m	10	200 m	300 m	at P	at P
	4 th	700 m	14	280 m	420 m	80 m	20 m
	5 th	900 m	18	360 m	540 m	40 m	60 m

- (i) 2 second $\left(\text{Time} = \frac{100}{50} = 2 \right)$
 (ii) 200 m (Distance = Speed $\times$ Time = $20 \times 10 = 200$ m)
 (iii) 540 m ($D = 18 \times 30 = 540$ m)
 (iv) 80 m
 (v) 60 m
 (vi) 2 : 3 always equal to the ratio of respective speeds.

Exp. 16) A and B are two friends. A lives at a place P and B lives at another place Q. Everyday A goes to Q to meet B at 120 km/h. Thus, he takes 3 hours. On a particular day B started to meet A so he moved towards P. On that day A took only 2 hours to meet B on the way instead at Q.

- (i) What is the ratio of speeds of A is to B?
 (ii) What is the speed of B?

Solution Distance between P and Q = $120 \times 3 = 360$ km
 Let the speed of B be S_B , then

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$2 = \frac{360}{(120 + S_B)}$$

$$\Rightarrow S_B = 60 \text{ km/h}$$

Here, A and B are moving towards each other. So, the relative speed will be the sum of the speeds of A and B both.

Therefore, ratio of speeds of A : B = 2 : 1.

- (i) 2 : 1 (ii) 60 km/h

NOTE In this case ratio of speeds of

$$A : B = \frac{\text{Actual time required when B is also moving}}{\text{Time difference when B is also moving}} = \frac{(3-1)}{1} = \frac{2}{1}$$

Exp. 17) A lives at P and B lives at Q. A usually goes to meet B at Q. He covers the distance in 3 hours at 150 km/h. On a particular day B started moving away from A. While A was moving towards Q thus A took total 5 hours to meet B.

- (i) What is the speed of B?
 (ii) What is the ratio of speeds of A : B?

Solution Distance = $3 \times 150 = 450$ km

$$\text{Time} = \frac{\text{Distance}}{\text{Relative speed}}$$

$$5 = \frac{450}{(150 - S_B)} \quad (S_B \rightarrow \text{Speed of B})$$

$$\Rightarrow S_B = 60 \text{ km/h}$$

$$\text{Ratio of speeds of A : B} = \frac{150}{60} = \frac{5}{2} = 5 : 2$$

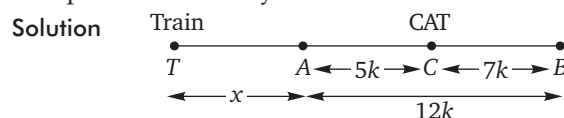
- (i) 60 km/h (ii) 5 : 2

In this case ratio of speeds of

$$A : B = \frac{\text{Actual time required when B is also moving}}{\text{Time difference}} = \frac{5}{2}$$

Exp. 18) A train approaches a tunnel AB. Inside the tunnel a cat is located at a point which is $\frac{5}{12}$ of the distance

AB measured from the entrance A. When the train whistles, the cat runs. If the cat moves to the entrance of the tunnel A, the train catches the cat exactly at the entrance. If the cat moves to the exit B, the train catches the cat at exactly the exit. The speed of the train is greater than the speed of the cat by what order?



Let the speed of train be u and the speed of cat be v and train whistles at a point T, x km away from A, then

$$\frac{u}{v} = \frac{x}{5k} = \frac{x + 12k}{7k}$$

$$\Rightarrow 7x = 5(x + 12k)$$

$$\Rightarrow \frac{x}{k} = \frac{30}{1}$$

$$\therefore \frac{u}{v} = \frac{30}{5 \times 1} = \frac{6}{1}$$

Alternatively $\frac{u}{v} = \frac{7k + 5k}{7k - 5k} = \frac{6}{1}$

NOTE Since time is constant, therefore distances covered by train and cat will be in the ratio of their respective speeds.

9.4 Concept Based on Trains

- When two trains (or bodies) are moving in opposite direction, their relative speed will be equal to the sum of their individual speeds.
- When two trains are moving in the same direction their relative speed will be equal to the difference of their speeds.
- Distance to be covered to cross each other is always equal to the sum of their individual lengths.
- Distance to be covered such as bridge, platform etc., is always equal to the sum of the length of train and the length of the particular object such as bridge, platform etc.
- Distance to be covered such as pole, man, tree etc, is always equal to the length of the train only.
- If a man is travelling in a train, then this man has to cover the distance to cross another train is equal to the length of the train which is passing or crossing him. In this case the relative speed of both the trains will be considered.

Exp. 19) A train crosses a tree in 10 seconds. If the length of the train be 150 m, then find the speed of the train.

Solution Distance = Length of train

$$= \text{Speed of train} \times \text{Time}$$

$$150 = \text{Speed} \times 10$$

$$\Rightarrow \text{Speed} = 15 \text{ m/s}$$

$$\text{Speed} = 15 \times \frac{18}{5} = 54 \text{ km/h}$$

NOTE A train starts to cross a stationary thin object (of inconsiderable thickness) when the engine of the train meets the object and it completes the crossing when the last wagon (or backend) of the train just crosses the object.

Exp. 20) A train crosses a man coming from the opposite direction in 7.5 seconds. If the speed of man be 10 m/s and speed of train is 20 m/s, find the length of the train.

Solution Length of train = Time $\times$ Relative speed

$$= 7.5 \times (10 + 20) = 7.5 \times 30 = 225 \text{ m}$$

Exp. 21) A train of length 250 m crosses a bridge of length 150 m in 20 seconds. What is the speed of train?

Solution (Length of train + Length of bridge)

$$= \text{Speed of train} \times \text{Time}$$

$$(250 + 150) = 20 \times \text{Speed}$$

$$\text{Speed} = \frac{400}{20} = 20 \text{ m/s} = 72 \text{ km/h}$$

Exp. 22) Two trains coming from the opposite sides crosses each other in 10 seconds if the lengths of first train and second train be 125 m and 175 m respectively, also the speed of first train be 36 km/h, find the speed of second train.

Solution Speed of first train = 36 km/h = 10 m/s
 Now,
$$\text{Time} = \frac{\text{Sum of length of the two trains}}{\text{Sum of their speeds}}$$

$$10 = \frac{125 + 175}{(10 + x)} \Rightarrow x = 20 \text{ m/s} = 72 \text{ km/h}$$

Exp. 23) A fast moving superfast express crosses another passenger train in 20 seconds. The speed of faster train is 72 km/hr and speeds of slower train is 27 km/h. Also the length of faster train is 100 m, then find the length of the slower train if they are moving in the same direction.

Solution
$$\text{Time} = \frac{\text{Sum of length of the two train}}{\text{Difference in speeds}}$$

$$20 = \frac{(100 + x)}{25/2} \Rightarrow x = 150 \text{ m}$$

NOTE Relative speed = $(72 - 27) = 45 \text{ km/h}$

$$= 45 \times \frac{5}{18} = \frac{25}{2} \text{ m/s}$$

9.5 Concept Based on Boats and Rivers (or streams)

(i) When the boat and stream (or current) of river move in the same direction, then the relative speed of the boat is the sum of the individual speeds of boat and river. It is known as **downstream speed**.

(ii) When the boat moves against the current of the river (*i.e.*, in opposite direction), then the relative speed of the boat is the difference of the speeds of the boat and stream (of the river). It is known as **upstream speed**.

Let the speed of boat in still water be B and speed of current of river be C then,

$$\begin{cases} \text{Downstream speed} = (B + C) \\ \text{Upstream speed} = (B - C) \end{cases} ; B > C$$

$$\begin{cases} \text{Speed of the boat in still water} = \frac{(D + U)}{2} \\ \text{Speed of current (or stream)} = \frac{(D - U)}{2} \end{cases}$$

where $D \rightarrow$ downstream speed of the boat
 and $U \rightarrow$ upstream speed of the boat

When the distance covered by boat in downstream (*i.e.*, with the flow of water) is same as the distance covered by boat in upstream (against the flow of the water) then,

$$\frac{\text{Time taken by boat in DS}}{\text{Time taken by boat in US}} = \frac{\text{Upstream speed}}{\text{Downstream speed}}$$

$DS \rightarrow$ Downstream, $US \rightarrow$ Upstream

Exp. 24) A boat can move at 5 km/h in still water (*i.e.*, when water is not flowing). The speed of stream of the river is 1 km/h. A boat takes 80 minutes to go from a point A to another point B and return to the same point.

- (i) What is the distance between the two points?
- (ii) What is the ratio of downstream speed and upstream speed?
- (iii) What is the ratio of time taken in downstream speed to the upstream speed?

Solution Downstream speed of boat = $(5 + 1) = 6 \text{ km/h}$
 Upstream speed of boat = $(5 - 1) = 4 \text{ km/h}$
 Therefore,
$$\frac{\text{Downstream speed}}{\text{Upstream speed}} = \frac{\text{Upstream time}}{\text{Downstream time}}$$

$$\frac{6}{4} = \frac{3}{2} = \frac{\text{Time taken in upstream direction}}{\text{Time taken in downstream direction}}$$

$$\therefore \text{Time taken in downstream} = \frac{2}{5} \times 80 = 32 \text{ min}$$
 and time taken in upstream direction = $\frac{3}{5} \times 80 = 48 \text{ min}$

$$\therefore \text{Distance between two points} = DS \text{ speed} \times DS \text{ time}$$

$= \text{US speed} \times \text{US time}$

where DS $\rightarrow$ Downstream and US $\rightarrow$ Upstream

$$D = \frac{6 \times 32}{60} = 3.2 \text{ km}$$

or $D = 4 \times \frac{48}{60} = 3.2 \text{ km}$

(i) 3.2 km (ii) 3 : 2 (iii) 2 : 3

Exp. 25) A man can row 9 km/h in still water. It takes him twice as long as to row up as to row down. Find the rate of stream of the river.

Solution $\frac{\text{Time taken in upstream}}{\text{Time taken in downstream}} = \frac{2}{1}$

$$\therefore \frac{\text{Downstream speed}}{\text{Upstream speed}} = \frac{2}{1} \text{ where } \frac{B + R}{B - R} = \frac{2}{1}$$

$B \rightarrow$ Speed of boat in still water

$R \rightarrow$ Speed of current

$$\Rightarrow \frac{B}{R} = \frac{3}{1} \quad (\text{By componendo and dividendo})$$

$$\Rightarrow \frac{9}{R} = \frac{3}{1} \Rightarrow R = 3 \text{ km/h}$$

9.6 Races

Terminology

(i) **Startup or headstart** When a runner allows to another runner to stay ahead in the same race, then it is said that there is a startup in the race.

For example if A allows B to go ahead before starting the race, then it is said that A gives startup to B and B has the startup. If before starting the race B goes ahead of x metre, then we can say A gives x metre startup to B or B has startup (or headstart) of x metre.

(ii) **Dead heat** When the runners reach the finishing line (or the final post) then it is said that these runners finish (or end) the race in dead heat.

Some More Useful Concepts

(i) When it is said that A can give B a start of x metre in y metre race, then it means in y metre race B runs x metre less than A in the same time.

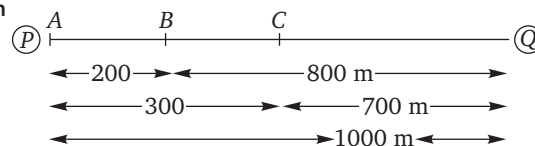
NOTE There is a great difference between 'can' and 'gives'.

(ii) When A beats B by t seconds in a race of y metre then it means B is the loser and A is the winner and when A reaches the finishing line B is still some distance back to A . Thus B takes t seconds to cover the remaining distance. Hence, we can calculate the speed of loser (B).

(iii) Throughout the race there is always a certain relationship among runners i.e., they always maintain the ratio of speeds. I think this is the nub of all the problems pertaining to the races. Go ahead...

Exp. 1) A can give B a 200 m startup and C a 300 m startup in a race of 1 km. How many metres startup can B give to C in a 1 km race.

Solution



Ratio of speeds of $A : B = 1000 : 800 = 5 : 4$

Ratio of speeds of $A : C = 1000 : 700 = 10 : 7$

Ratio of speeds of $B : C = 800 : 700 = 8 : 7$

Since, when B moves 8 m, C moves 7 metre. Therefore, when B moves 1000 m, C moves 875 metre. Thus, B can give C a start of $1000 - 875 = 125$ m.

Alternatively Since, C is 12.5% slower than B . So, C will cover 12.5% less distance than B in the same time.

(Since when time is constant, they cover the distances in the ratio of their speeds.)

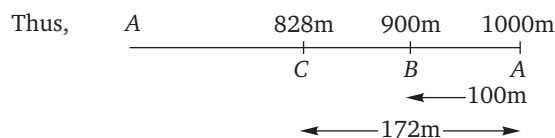
Thus, in 1000 m (or 1 km) when B runs 1000 m, C will run 125 m less than B . Hence, B can give a start of 125 m to C in a 1 km race.

Exp. 2) In a one km race A gives B a start of 100 m and in a one km race B gives a start of 80 m to C . In a 1 km race who will win and by how much distance from the worst performer between two losers?

Solution Ratio of speeds of $A : B = 1000 : 900 = 100 : 90$

Ratio of speeds of $B : C = 1000 : 920 = 100 : 92$

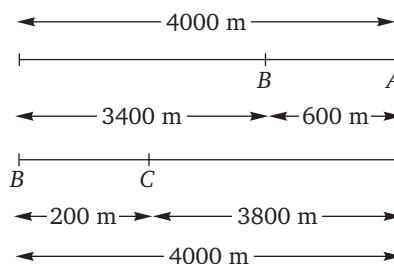
Therefore, when A moves 1000 m, B moves 900 m and when B moves 900 m, C moves 828 m.



Since, C moves 8% less than B in the same time. Thus, C is the worst performer and A will win by him by 172 m.

Exp. 3) In a 4 km race A wins by 600 m over B . B can give start of 200 m to C in a 4 km race. By how much distance C gets start up so that the race between A and C ends in dead heat in the same race of 4 km?

Solution



Ratio of speeds of $A : B = 20 : 17$

and Ratio of speeds of $B : C = 20 : 19$

$\therefore$ Ratio of speeds of $A : B : C = 400 : 340 : 323$

Therefore, in 4000 m race A run 4000 m, B run 3400 m and C run 3230 m. Thus C can get 770 m start up from A .

Exp. 4) In a 1500 m race A wins over B by 350 m and in 1500 m race C can give a startup of 250 m to B . By how much distance A give start up to C , so that A beats C by 50 metres?

Solution Ratio of speeds of $A : B = 30 : 23$

Ratio of speeds of $B : C = 5 : 6$

$\therefore$ Ratio of speeds of $A : B : C = 150 : 115 : 138$

So, when A moves 1500 m, B moves 1150 m and C moves 1380 m. Thus C moves 120 m less than A . To win A just by 50 m over C , A should give $120 - 50 = 70$ m startup to C .

Exp. 5) In a race of 2500 m, A beats B by 500 m and in a race of 2000 m, B beats C by 800 m. By what distance A gives startup to C so that they will end up with dead heat in 3 km race. Also find that by what distance A will win over C in a 1 km race?

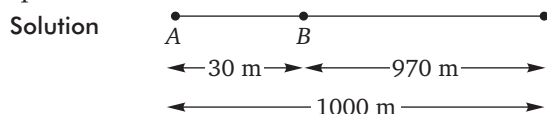
Solution Ratio of speeds of $A : B = 5 : 4$

Ratio of speeds of $B : C = 5 : 3$

$\therefore$ Ratio of speeds of $A : B : C = 25 : 20 : 12$

In 3 km race A run 3000 m, B run 2400 m, C run 1440 m, so to end up the race in dead heat A should give C the startup of 1560 m and therefore in 1 km the same will be 520 m.

Exp. 6) A gives B , a start of 30 m or 10 seconds and end up the race of 1 km in dead heat. What is the ratio of speeds of A and B ?



Since, either B has the startup of 30 m or 10 seconds. It means B runs 30 m in 10 seconds. Hence, the speed of B is 3 m/s.

NOTE Don't be confused that A 's speed = 3 m/s.

Also, it is very simple when A moves 1000 m, B moves 970 m.

Since, the ratio of speeds is equal to the ratio of distances covered by A and B in the same time. Thus, the speed of

$$A : B = 1000 : 970 = 100 : 97$$

Exp. 7) In a 1 km race A wins over B by 80 m or 20 seconds. B can give a start of 100 m to C in 1 km race. Find out that by how much time A will win over C ? Also, find the ratio of speeds of B and C .

Solution Ratio of speeds of $A : B = 100 : 92$

Ratio of speeds of $B : C = 10 : 9$

$\therefore$ Ratio of speeds of $A : B : C = 1000 : 920 : 828$

Also, Speed of $B = \frac{80}{20} = 4$ m/s

Therefore, Speed of $C = 3.6$ m/s

Now, C has to cover 172 m distance in extra time. So, the time taken by C to cover the remaining distance

$$= \frac{172}{3.6} = 47.77 \text{ s.}$$

Ratio of speeds of $B : C =$ Ratio of distances covered by $B : C$
 $= 1000 : 900 = 10 : 9$

$$\therefore \frac{\text{Speed of } B}{\text{Speed of } C} = \frac{10}{9}$$

Exp. 8) A can win B by 250 m in a 2 km race. What should be the change in distance of startup? So, that B must cover 20% less distance than that by A in the same time.

Solution $\frac{\text{Distance covered by } A}{\text{Distance covered by } B} = \frac{5}{4}$

$$\therefore \frac{\text{Speed of } A}{\text{Speed of } B} = \frac{5}{4}$$

So, when A moves 2000 m, B should move 1600 m. But since initially B moves 1750 m. Therefore the new startup will be increased by 150 m.

Exp. 9) The ratio of speeds of A and B is 4 : 7 and A loses the race by 270 m, then what is the length of the race course?

Solution When B moves 7 m, A moves only 4 m. Hence, A loses the race by 3 m.

Now, since B loses by 3 m in the race of 7 m.

$\therefore B$ will lose 270 m in the race of 630 m.

Exp. 10) The ratio of time taken to run a certain distance by Pythagoras and Hawkins is 4 : 3 and thus Hawkins wins the race by 360 m. What is the distance of race course?

Solution $\frac{\text{Time taken by Pythagoras}}{\text{Time taken by Hawkins}} = \frac{4}{3}$

$$\frac{\text{Speed of Pythagoras}}{\text{Speed of Hawkins}} = \frac{3}{4}$$

$$= \frac{\text{Distance covered by P}}{\text{Distance covered by H}}$$

Now, when Hawkins runs 4 m, Pythagoras runs 3 m and thus Hawkins wins by 1 m.

So, when Hawkins wins the race by 1 m, race course is 4 m. when Hawkins wins the race by 360 m, race course is

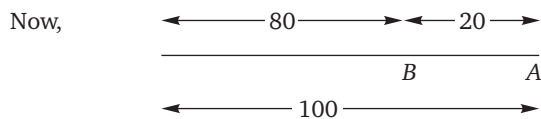
$$360 \times 4 = 1440 \text{ m}$$

Alternatively $360 = 25\%$ of the total length of race.

$\therefore$ length of race course = 1440 m.

Exp. 11) In a race of D km, A wins over B by $0.2D$ and in the same length of race B wins over C by $0.25D$. What should be the head-start to C , So that A and C finish the race at the same time.

Solution **NOTE** Instead of solving the problem using ' D ' as a distance convert it into 100 i.e., suppose $D = 100$. (It is just for your convenience.)



Now, Speed of $A : B = 100 : 80 = 5 : 4$

Speed of $B : C = 4 : 3$

$$\therefore \text{Speed of } A : B : C = 20 : 16 : 12$$

$$= 100 : 80 : 60$$

$$\quad \quad \quad \{40\}$$

So, A can give ' C ' a start of 40 m in 100 m race. Therefore, A can give 0.4 D . Start to ' C ' in the race of D unit.

Exp. 12) In a 3 km race the speeds of A and B are in the ratio of $6 : 5$ and A wins by 10 seconds. What is the time taken by B to finish the race. Also, to end the race in dead heat what per cent of total distance A should give the startup to B ?

Solution Ratio of distances covered by A and B when A just reaches the finishing line = $6 : 5$.

Thus, B has to cover 500 m distance in extra time and this 500 m distance is covered by B in 10 seconds.

So, the speed of $B = 50$ m/s.

Thus, the total time required by B to complete the 3 km race

$$= \frac{3000}{50} = 60 \text{ s} = 1 \text{ min}$$

Since, B covers 16.66% distance less than A covers in the same time, so A should give 16.66% of the total distance as a startup to B .

Exp. 13) In a 1 km race A gives B a startup of 5 seconds and still wins over B by 15 seconds. The ratio of speeds of A and B is $2 : 1$. Find the time taken by A to finish 2.5 km race.

Solution $\frac{\text{Speed of } A}{\text{Speed of } B} = \frac{2}{1}$

$$\frac{\text{Time taken by } A}{\text{Time taken by } B} = \frac{1}{2}$$

$$\frac{t}{(t + 20)} = \frac{1}{2}$$

$$\Rightarrow t = 20 \text{ s}$$

Thus, A needs 20 seconds to cover 1 km. Thus, to cover 2.5 km race he needs $20 \times 2.5 = 50$ seconds.

Exp. 14) X can beat Y by 200 m in a race of 2000 m. Y can beat Z by 100 m in a race of 2500 m. By how many metres can X beat Z in a race of 1000 m.

Solution Ratio of speeds of X and $Y = 10 : 9$ (2000 : 1800)

Ratio of speeds of Y and $Z = 25 : 24$ (2500 : 2400)

Ratio of speeds of X, Y and $Z = 250 : 225 : 216$

Since in a race of 250 m, X beats Z by 34 m.

So, in a race of 1000 m, X will beat Z by 136 m.

Exp. 15) In a 6 km race B has 250 m headstart and C has 500 m headstart by A , still A beats C and B by 235 m and 350 m respectively. How many metres startup can B give to C so as to end up the race at the same time with C in the race of 6 km. Also find the ratio of speeds of $A : B : C$.

Solution When A runs 6000 m, B runs 5400 m only and C runs 5265 m only.

So, when B runs 6000 m, C will run 5850 m. So, B can give 150 m startup to C .

$$\text{Ratio of speeds of } A : B : C = 6000 : 5400 : 5265$$

$$= 1200 : 1080 : 1053$$

$$= 400 : 360 : 351$$

Exp. 16) A can run 1 km in 2 min 20 second and B can run the same distance in 3 min. What is the distance travelled by B in the same time as A travels, when they start simultaneously in the race of 4.5 km.

Solution $\frac{\text{Time taken by } A}{\text{Time taken by } B} = \frac{140 \text{ s}}{180 \text{ s}} = \frac{7}{9}$

$$\therefore \frac{\text{Speed of } A}{\text{Speed of } B} = \frac{\text{Distance travelled by } A}{\text{Distance travelled by } B} = \frac{9}{7}$$

$$\text{Therefore, } B \text{ travels } \frac{7}{9} \times 4.5 = 3.5 \text{ km.}$$

Exp. 17) Shahrukh takes 4 min to cover the same distance for which Urmila takes 6 min 30 sec. What is the ratio of distances covered by Shahrukh and Urmila in the race of 2.6 km and by what distance Shahrukh wins over Urmila?

Solution $\frac{\text{Time taken by Shahrukh}}{\text{Time taken by Urmila}} = \frac{240}{390} = \frac{8}{13}$

$$\frac{\text{Distance covered by Shahrukh}}{\text{Distance covered by Urmila}} = \frac{13}{8}$$

Hence, Shahrukh will win the race by 1 km.

Exp. 18) A runs at the speed of $\frac{7}{4}$ times the speed of B . By calculation B finds that she has to run 300 m after A reaches to the winning post. What is the total distance of race?

Solution $\frac{\text{Speed of } A}{\text{Speed of } B} = \frac{7}{4}; \frac{\text{Distance of } A}{\text{Distance of } B} = \frac{7}{4}$

So, the total distance of race = 700 m.

$$(\because 7x - 4x = 300 \Rightarrow 7x = 700)$$

Exp. 19) Time taken by A is $\frac{5}{7}$ of B 's time for the same length of race. The speed of A is 84 m/s and A beats B by 240 m. What is the length of race course?

Solution $\frac{\text{Speed of } A}{\text{Speed of } B} = \frac{\text{Time taken by } B}{\text{Time taken by } A} = \frac{7}{5}$

$$\text{Now since, } 7x - 5x = 240$$

$$\Rightarrow 7x = 840 \text{ m}$$

Exp. 20) In a race of 3000 m, Michal beats Nicholas by 600 m and Nicholas in a race of 2000 m beats Oscar by 600 m. In a 1 km race by how much distance Michal beats Oscar?

Solution Ratio of speeds of $M : N : O = 100 : 80 : 56$
So, Michal (M) beats Oscar (O) by 440 m in 1 km race.

Exp. 21) In a race of 500 m, President runs at 5 m/s. If president gives chairman a start of 20 m and still beats him by 20 seconds, what is the chairman's speed?

Solution Chairman has to cover only 480 m. He takes total
 $100 + 20 = 120$ s
 Therefore, speed of chairman = $\frac{480}{120} = 4$ m/s

Exp. 22) In a race of 800 m Dholakiya gives Preetam a start of 200 m and then loses the race by 20 seconds. What is the speed of Preetam and Dholakiya respectively? If the ratio of respective speeds be 3 : 2.

Solution Let the speed of Dholakiya be S_D and speed of Preetam be S_P and let the time taken by Preetam be t second

$$\text{then, } \frac{S_D}{S_P} = \frac{\frac{800}{(t+20)}}{\frac{600}{t}} = \frac{2}{3}$$

$$\frac{800}{600} \times \frac{t}{(t+20)} = \frac{2}{3}; t = 20 \text{ s}$$

$$\therefore \text{Speed of Preetam} = \frac{600}{20} = 30 \text{ m/s}$$

$$\text{and speed of Dholakiya} = \frac{800}{40} = 20 \text{ m/s}$$

Exp. 23) In race of 1 km Sahara gives Birla a startup of 250 m still Sahara wins by 150 m. What is the ratio of speeds of Sahara and Birla?

Solution When Sahara covers 1000 m, Birla covers only 600 m. So, the ratio of speeds of Sahara and Birla = $1000 : 600 = 5 : 3$

Exp. 24) Priyambda wins the race over Kokilaben by 150 m in a race of 1 km but when she gives a startup of 5 seconds to Kokilaben she wins by 65 m. Find the speed of Kokilaben.

Solution In 5 seconds Kokilaben runs 85 m.
So, the speed of Kokilaben = 17 m/s.

9.7 Circular Motion

- When the bodies move in the opposite direction, their relative speed becomes equal to the sum of their individual speed.
- When two bodies move in the same direction their relative speed becomes equal to the difference of the individual speeds.

First Meeting

- Let A and B be two runners. Time taken by them to meet for the first time = $\frac{\text{Length of the circular track}}{\text{Relative speed}}$
- When there are more than two runners, then suppose A is the fastest runner and A meets B for the first time in t_{AB} hours and A meets C for the first time in t_{AC} hours and A meets D for the first time in t_{AD} seconds/hours and so on. Then time taken by all of them to meet for the first time is the LCM of t_{AB}, t_{AC}, t_{AD} etc.

First Meeting at the Starting Point

Let A takes t_A time to complete one round and B takes t_B time and C takes t_C time and so on, then the time taken to meet for the first time at the starting point = LCM of t_A, t_B, t_C etc.

Exp. 1) Arjun and Bhishma are running on a circular track of length 600 m (i.e., circumference of the track is 600 m). Speed of Arjun is 75 m/s and that of Bhishma is 45 m/s. They start running from the same point at the same time the same direction.

- When will they meet again for the first time?
- When will they meet again for the second time?
- When will they meet again for the tenth time?

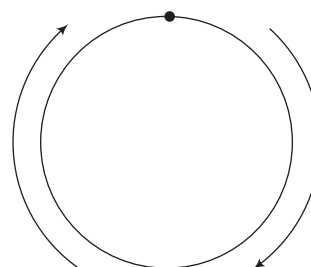
Solution Since, the two bodies are continuously and consistently running on the same circular path they will certainly meet again and again.

Under the given circumstances the two bodies will meet, after the start, when the difference in distance traversed by them is either 1 round or 2 rounds or 3 rounds and so on.

That is, their first meeting occurs when the faster body covers 1 extra round. And, second meeting occurs when the faster body covers 2 extra rounds. And, third meeting occurs when the faster body covers 3 extra rounds.

And so on so forth. In other words, the faster body has to have a lead of 1 round, 2 rounds, 3 rounds etc. depending on the number/order of meeting. And, thus the time required to have a lead of 1 round = $\frac{\text{Circumference}}{\text{Relative speed}}$. So, the time

required to have a lead of n rounds = $n \times \frac{\text{Circumference}}{\text{Relative speed}}$



- (i) In order to meet for the first time, after the start of the race, Arjun (faster man) has to have a lead of 600 m, because as soon as Arjun will cover 600 m extra than that of Bhishma, Arjun will overtake Bhishma.

And, you know that Arjun gets a lead of 30 m in every 1 second, so to have a lead of 600 m Arjun has to run for $600/30 = 20$ seconds.

$$\text{Otherwise, Time} = \frac{\text{Circumference}}{\text{Relative speed}} = \frac{600}{75 - 45} = \frac{600}{30} = 20$$

seconds

That is after 20 seconds they will meet for the first time, while running in the same direction.

- (ii) In order to meet for the second time, after the start of the race, Arjun (faster man) has to have a lead of 2 rounds.

$$\text{Time} = 2 \times \frac{\text{Circumference}}{\text{Relative speed}}$$

$$= 2 \times \frac{600}{75 - 45} = \frac{1200}{30}$$

$$= 60 \text{ seconds}$$

That is after 60 seconds they will meet for the second time, while running in the same direction.

- (iii) In order to meet for the tenth time, after the start of the race, Arjun (faster man) has to have a lead of 10 rounds.

$$\text{Time} = 10 \times \frac{\text{Circumference}}{\text{Relative speed}} = 10 \times \frac{600}{75 - 45} = \frac{6000}{30} = 200$$

seconds

That is after 200 seconds they will meet for the tenth time, while running in the same direction.

Exp. 2) Arjun and Bhishma are running on a circular track of length 600 m. Speed of Arjun is 75 m/s and that of Bhishma is 45 m/s. They start running from the same point at the same time in the opposite directions.

(i) When will they meet again for the first time?

(ii) When will they meet again for the second time?

(iii) When will they meet again for the tenth time?

Solution Since two bodies are continuously and consistently running on the same circular path they will certainly meet again and again.

Under the given circumstances the two bodies will meet, after the start, when the total distance traversed by them is either 1 round or 2 rounds or 3 rounds and so on.

That is their first meeting occurs when they together cover 1 round.

And, second meeting occurs when they together cover 2 rounds.

And, third meeting occurs when they together cover 3 rounds.

And so on so forth.

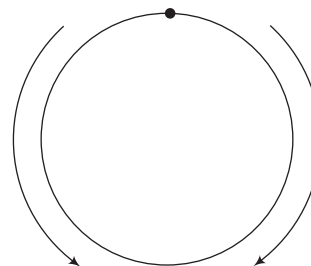
In other words, they have to complete together 1 round, 2 rounds, 3 rounds etc. depending on the number/order of meeting.

And, thus the time required to complete 1 round

$$= \frac{\text{Circumference}}{\text{Relative speed}}$$

So, the time required to complete n rounds

$$= n \times \frac{\text{Circumference}}{\text{Relative speed}}$$



- (i) In order to meet for the first time, after the start of the race, Arjun and Bhishma together will have to traverse together 600 m.

And, you know that they together traverse 120 m in every 1 second, so to traverse 600 m they have to run for $600/120 = 5$ seconds.

$$\text{Otherwise, Time} = \frac{\text{Circumference}}{\text{Relative speed}} = \frac{600}{75 + 45} = \frac{600}{120} = 5 \text{ seconds}$$

That is after 5 seconds they will meet for the first time, while running in the opposite directions.

- (ii) In order to meet for the second time, after the start of the race, Arjun and Bhishma together will have to traverse 2 times a round.

$$\text{Time} = 2 \times \frac{\text{Circumference}}{\text{Relative speed}}$$

$$= 2 \times \frac{600}{75 + 45} = \frac{1200}{120} = 10 \text{ seconds}$$

That is after 10 seconds they will meet for the second time, while running in the opposite directions.

- (iii) In order to meet for the tenth time, after the start of the race, Arjun and Bhishma together will have to traverse 10 times a round.

$$\text{Time} = 10 \times \frac{\text{Circumference}}{\text{Relative speed}}$$

$$= 10 \times \frac{600}{75 + 45} = \frac{6000}{120} = 50 \text{ seconds}$$

That is after 50 seconds they will meet for the tenth time, while running in the opposite directions.

Exp. 3) Arjun and Bhishma are running on a circular track of length 600 m. Speed of Arjun is 75 m/s and that of Bhishma is 45 m/s. They start running from the same point at the same time.

(i) When will they meet again for the first time at the starting point?

(ii) When will they meet again for the second time at the starting point?

(iii) When will they meet again for the tenth time at the starting point?

Solution Since, we are not interested in finding the meetings that may occur somewhere else on the track other than the starting point, so it does not matter whether they are moving in the same direction or in the opposite directions. It implies that we need to know when the two bodies are completing the rounds together, though it is not necessary and possible that the numbers of rounds to be same, as their speeds are distinct.

So, we have to find out when the two bodies complete their rounds and then we can find out the common time when both of them will complete their rounds.

So you can see that Arjun completes a round in every 8 (= $600/75$) seconds.

And, Bhishma completes a round in every $40/3$ (= $600/45$) seconds.

	Time taken to Complete the Different Rounds ($R_1, R_2, \dots$)									
	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}
Arjun	8	16	24	32	40	48	56	64	72	80
Bhishma	$\frac{40}{3}$	$\frac{80}{3}$	40	$\frac{160}{3}$	$\frac{200}{3}$	80	$\frac{280}{3}$	$\frac{320}{3}$	120	400

Thus, it is obvious that the two bodies meet at the starting point every 40 seconds, which is nothing but the LCM of 8 seconds and $40/3$ seconds.

Otherwise, Time taken by Arjun to complete 1 round

$$= \frac{600}{75} = 8 \text{ seconds}$$

And, time taken by Bhishma to complete 1 round

$$= \frac{600}{45} = \frac{40}{3} \text{ seconds}$$

LCM of 8 and $40/3 = 40$

- Therefore, the time required for the first meeting at the starting point = 40 seconds
- Therefore, the time required for the second meeting at the starting point = $2 \times 40 = 80$ seconds
- Therefore, the time required for the tenth meeting at the starting point = $10 \times 40 = 400$ seconds

Exp. 4) Arjun, Bhishma and Nakul run on the circular path at the speed of 20 m/s, 30 m/s and 50 m/s respectively in the same direction. The circumference of the track (or path) is 600 m.

- When will they be together again for the first time?
- When will they be together again for the first time at the starting point?

Solution

$$(i) \text{ Nakul meets Arjun after every } = \frac{600}{(50 - 20)} = 20 \text{ s}$$

$$\text{Nakul meets Bhishma after every } = \frac{600}{(50 - 30)} = 30 \text{ s}$$

Therefore, all of the three would meet after every 60 seconds. (60 = LCM of 20 and 30). Hence, they would all meet for the first time after 60 seconds.

- Arjun takes $600 / 20 = 30$ s to complete one round

$$\text{Bhishma takes } \frac{600}{30} = 20 \text{ s to complete one round and}$$

$$\text{Nakul takes } \frac{600}{50} = 12 \text{ s to complete one round}$$

Hence, they would meet for the first time at the starting point after 60 seconds.

Hint 60 = LCM of 30, 20 and 12.

Exp. 5) Arjun, Bhishma, Chaitanya and Duryodhana are running on a circular track of length 600 m. They start running from the same point at the same time in the same direction and the Speeds of Arjun, Bhishma, Chaitanya and Duryodhana are 45 m/s, 55 m/s, 65 m/s and 75 m/s, respectively.

- When will they meet again for the first time?
- When will they meet again for the seventh time?
- When will they meet again for the first time at the starting point?
- When will they meet again for the seventh time at the starting point?

Solution Let us, first of all, find the answers for (i) and (ii) parts.

Since, Duryodhana is the fastest runner, so we will find out the time taken by Duryodhana to meet Arjun, Bhishma and Chaitanya independently, then we will find out the common time when Duryodhana meets all the three together.

Time taken by Duryodhana to meet Arjun for the first time

$$= \frac{600}{75 - 45} = \frac{600}{30} = 20 \text{ seconds}$$

Time taken by Duryodhana to meet Bhishma for the first time

$$= \frac{600}{75 - 55} = \frac{600}{20} = 30 \text{ seconds}$$

Time taken by Duryodhana to meet Chaitanya for the first time

$$= \frac{600}{75 - 65} = \frac{600}{10} = 60 \text{ seconds}$$

Now, the LCM of 20, 30 and 60 is 60, so Duryodhana will meet all of them together in every 60 seconds.

- It can be conclude that all of them meet for the first time after 60 seconds.
- It can be conclude that all of them meet for the seventh time after 420 (= 7×60) seconds.

Now, let us solve it for the (iii) and (iv) part.

Since, we are interested in finding the meetings occurring at the starting point, so the direction of their motion does not matter. Therefore, we will find out the time taken by each of them to complete a round and then we will find out when all of them will complete the rounds together, though it is not necessary and possible that the number of rounds to be same, as their speeds are distinct.

Time taken by Arjun to complete 1 round $= \frac{600}{45} = \frac{40}{3}$

seconds

And, time taken by Bhishma to complete 1 round

$$= \frac{600}{55} = \frac{120}{11} \text{ seconds}$$

Time taken by Chaitanya to complete 1 round

$$= \frac{600}{65} = \frac{120}{13} \text{ seconds}$$

And, time taken by Duryodhana to complete 1 round

$$= \frac{600}{75} = 8 \text{ seconds}$$

Now, the LCM of $40/3$, $120/11$, $120/13$ and $8 = 120$

Thus, all of them will meet at the starting point in every 120 seconds.

- (iii) It can be concluded that all of them meet for the first time after 120 seconds.
- (iv) It can be concluded that all of them meet for the first time after 840 ($= 7 \times 120$) seconds

Exp. 6) Arjun, Bhishma, Chaitanya, Duryodhana and Eklavya are running on a circular track of length 600 m. They have started running from the same point at the same time and the Speeds of Arjun, Bhishma, Chaitanya, Duryodhana and Eklavya are 25 m/s, 35 m/s, 45 m/s, 60 m/s and 75 m/s, respectively. Arjun and Chaitanya are running clockwise, while Bhishma, Duryodhana and Eklavya are running anti-clockwise.

- (i) When will they meet again for the first time?
- (ii) When will they meet again for the fifth time?

Solution Since Eklavya is the fastest runner, so we will find out the time taken by Eklavya to meet Arjun, Bhishma, Chaitanya and Duryodhana independently, then we will find out the common time when Eklavya meets all of them together.

Time taken by Eklavya to meet Arjun for the first time

$$= \frac{600}{75 + 25} = \frac{600}{100} = 6 \text{ seconds}$$

Time taken by Eklavya to meet Bhishma for the first time

$$= \frac{600}{75 - 35} = \frac{600}{40} = 15 \text{ seconds}$$

Time taken by Eklavya to meet Chaitanya for the first time

$$= \frac{600}{75 + 45} = \frac{600}{120} = 5 \text{ seconds}$$

Time taken by Eklavya to meet Duryodhana for the first time

$$= \frac{600}{75 - 60} = \frac{600}{15} = 40 \text{ seconds}$$

Now, the LCM of 6, 15, 5 and 40 is 120, so Duryodhana will meet all of them together in every 120 seconds.

- (i) It can be conclude that all of them meet for the first time after 120 seconds.
- (ii) It can be conclude that all of them meet for the fifth time after 600 ($= 5 \times 120$) seconds.

Exp. 7) Two girls A and B run on a circular track, with constant speeds, such that the ratio of their speeds is 3:5.

- (i) Find the maximum number of distinct points where they can meet, if they run in the same direction.
- (ii) Find the maximum number of distinct points where they can meet, if they run in the opposite directions.

Solution When two bodies move in the same direction, the maximum number of distinct meeting points = difference in the ratio $= 5 - 3 = 2$. When two bodies move in the opposite directions, the maximum number of distinct meeting points = sum of the ratio $= 5 + 3 = 8$

Exp. 8) Two girls A and B run on a circular track, with the speeds of 80 m/s and 140 m/s.

- (i) Find the maximum number of distinct points where they can meet, if they run in the same direction.
- (ii) Find the maximum number of distinct points where they can meet, if they run in the opposite directions.

Solution When two bodies move in the same direction, the maximum number of distinct meeting points = difference in the ratio $= 7 - 4 = 3$. When two bodies move in the opposite directions, the maximum number of distinct meeting points = sum of the ratio $= 7 + 4 = 11$

Exp. 9) Two hands of a normal clock run at the speed of 30° per hour and 360° per hour. Find the number of distinct points where they meet on the dial.

Solution When two bodies move in the same direction, the maximum number of distinct meeting points = difference in the ratio $= 12 - 1 = 11$. It means, in a normal clock minute hand meets hour hand at 11 distinct points on the dial.

Exp. 10) There are two planets A and B revolving around a star in the two concentric orbits at the same plane. Planet A takes 366 days and B takes 30 days to complete a revolution around that star. But, whenever they pass each other the people living on the planet A observe the eclipse. Consider that a year has 366 days.

- (i) Find the number of times when planet A experiences the eclipse in a year, provided both the planets move in the same direction.
- (ii) Find the number of times when planet A experiences the eclipse in a year, provided the two planets move in the opposite directions.

Solution When two bodies move in the same direction, the maximum number of distinct meeting points = difference in the ratio $= 61 - 5 = 56$. Therefore, during a year planet A experiences 56 eclipses. When two bodies move in the opposite directions, the maximum number of distinct meeting points = sum of the ratio $= 61 + 5 = 66$

Therefore, during a year planet A experiences 66 eclipses.

Exp. 11) A tortoise can complete a full round on the circular track in 8 min, while the rabbit can do the same on the same circular track in 5 min. *A, B, C* and *D* are the four consecutive points on the circular track which are equidistant from each other. *A* is opposite to *C* and *B* is opposite to *D*.

- After how many minutes will they meet together for the first time, when both have started simultaneously from the same point in the same direction?
- After how many minutes will they meet together for the first time at the starting point, when both have started simultaneously from the same point in the same direction?
- If they start from a point *A*, simultaneously in the same direction then after how many minutes will they meet at *C* which is just opposite to *A*?
- If tortoise has a lead of 5 min, when would they meet for the first time, where they have started from the same point in the same direction?

Solution

- Ratio of time of rabbit and tortoise = 5 : 8

∴ Ratio of the speed of rabbit and tortoise = 8 : 5

Let the length of the circular track be 40 m, then the speed of rabbit = 8 m/min and speed of tortoise = 5 m/min. (40 is the LCM of 8 and 5, taken just for convenience in calculation)

Now, the time when they will be together for the first time

$$= \frac{\text{Circumference}}{\text{Relative speed}} = \frac{40}{3} \text{ min}$$

- Time taken by rabbit to complete one round = 5 min
Time taken by tortoise to complete one round = 8 min
∴ They will meet at the starting point, for the first time = LCM of 5 and 8 = 40 min

- Rabbit reaches point *C* in $\frac{20}{8} = 2.5$ min.

Rabbit reaches the same point *C* in 7.5, 12.5, 17.5, 22.5 min etc. Tortoise reaches after 4 min, 12 min, 20 min, ... etc.

They will never meet at *C* since time cannot coincide as time taken by rabbit is the decimal number and time taken by tortoise is a natural number.

- In 5 min tortoise travels 25 m. Now, rabbit starts chasing tortoise @ 8 m/min.

$$\therefore \text{Time taken by rabbit to overtake tortoise} = \frac{25}{(8-5)} = \frac{25}{3} \text{ min}$$

Exp. 12) Alfred and Bernard run on the circular track of 600 m. Speeds of Alfred and Bernard are 30 m/s and 20 m/s respectively. Initially they are diametrically opposite to each other.

- When will they meet for the first time if both move in the same direction?
- If both of them move in opposite directions, when will they meet for the second time?

Solution

- Relative speed of Alfred = 30 - 20 = 10 m/s
Bernard is 300 m ahead in the race.
So, time taken by Alfred to catch Bernard = 300/10 = 30 s

- To meet for the second time they have to cover
= 300 + 600 = 900 m

there relative speed = 30 + 20 = 50 m/s

(since direction is opposite)

$$\therefore \text{Time taken to meet for the second time} = \frac{900}{50} = 18 \text{ s}$$

Exp. 13) *A, B* and *C* run on a circular track of 800 m. Speeds of *A, B* and *C* are 20 m/s, 26 m/s and 33 m/s.

- When will they meet for the first time?
- What is the ratio of distances covered by each one to meet for the first time.

Solution

- Time taken by *C* to meet *A* for the first time = $\frac{800}{33-20} = \frac{800}{13} \text{ s}$

$$\text{Time taken by } C \text{ to meet } B \text{ for the first time} = \frac{800}{33-26} = \frac{800}{7} \text{ s}$$

$$\text{LCM of } \frac{800}{13} \text{ and } \frac{800}{7} = 800 \text{ s.}$$

Therefore, all of them (*i.e.*, *A, B* and *C*) meet after 800 seconds = 13 min 20 sec.

- The ratio of distances = ratio of speeds = 20 : 26 : 33

Exp. 14) Two friends – Choar and Poohlice start walking simultaneously from a point on the circular track. Their friend Pubhlik observes from a distance that when they walk in the same direction they take 7 times the time they take when they walk in the opposite direction for their first meeting to happen. If the speed of Choar, who walks faster, is 16 m/s, what is the speed of Poohlice?

- 8 m/s
- 10 m/s
- 12 m/s
- 14 m/s

Solution When they move in the same direction their effective speed is equal to the difference of their individual speeds and when they move in the same direction their effective speed is equal to the sum of their individual speeds. Also, speed is always inversely proportional to the time.

Now, let us assume that the speed of Poohlice be *p*, then

$$\frac{16+p}{16-p} = \frac{7}{1} \Rightarrow p = 12 \text{ m/s}$$

Hence, choice (c) is the correct one.

Hint Time taken by them to meet for the first time
= $\frac{\text{Length of the circular track}}{\text{Relative Speed}}$

And since length of the circular track is constant irrespective of the direction, so the ratio of time duration to meet for the first time would depend upon the two distinct relative speeds.

$$\left(\frac{\text{Length of the circular track}}{16-p} \right) = \frac{7}{\left(\frac{\text{Length of the circular track}}{16+p} \right)}$$

$$\Rightarrow \frac{16+p}{16-p} = \frac{7}{1}$$

Introductory Exercise 9.1

- P and Q start running on the circular track in opposite directions. They start from a point A and meet for the first, second and third time at B , C and A , respectively. What is the ratio of speeds of P and Q ?
 (a) 2 : 1 (b) 3 : 1
 (c) 4 : 1 (d) none of these
- P and Q run on a circular track of 100 m. Speed of P is 250 m/s and that of Q is 400 m/s. At how many points on a circular track will they meet if they start from the same point, simultaneously, in the same direction?
- (a) 1 (b) 2
 (c) 3 (d) 4
- A , B and C start walking around a circular track. A completes one round in $91/6$ seconds, B completes in $65/4$ seconds and C completes in $56/3$ seconds respectively. After how many seconds will they meet each other at the starting point again?
 (a) 3460 s
 (b) 1820 s
 (c) 3640 s
 (d) none of the above

9.8 Clocks

Actually the movement of hour-hand and minute-hand follows the relative motion. The dial of the clock behaves like a circular track and where minute-hand is a faster runner and hour-hand is a slower one.

For better understanding with the clocks, assume 60 minutes shown on the dial as 60 points. Here we give an arbitrary new unit of distance as 'point'.

So, minute-hand (MH) runs on the circular track of 60 points at 60 points per hour and hour-hand (HH) runs at 5 points per hour. Now we become familiar with the relative motion of two hands of a clock.

Here, 1 point = 6° and 60 point = 360°

also 1 point = $6^\circ = 1$ min

Exp. 1) How many times minute-hand coincides with hour-hand in 12 hours?

Solution Time taken by minute-hand to meet hour-hand for the first time = $\frac{60}{55} = \frac{12}{11}$ hours.

(Assume initially both hands are at 12 i.e., 12 O'clock is shown by them.)

Therefore, after every $12/11$ hours-minute hand and hour-hand meet each other (or coincide or overtake).

Now, since in $\frac{12}{11}$ hours they coincide 1 time

So, in 12 hours they will coincide = $\frac{12}{12/11} = \frac{12}{12} \times 11 = 11$ times

Remember Between 11 O'clock and 1 O'clock, two hands coincide only one time, that's why they coincide with each other only 11 times in 12 hours.

NOTE In every $\frac{12}{11}$ hours (or $65\frac{5}{11}$ min) two hands of a clock coincide.

Exp. 2) In 12 hours how many times the two hands of clock will be just opposite to each other i.e., they make a straight line having the difference of 180° between them?

Solution $180^\circ = 30$ points

For the first time minute-hand and hour-hand will be separated in $\frac{30}{55}$ h $\left(\text{Time} = \frac{\text{Required distance}}{\text{Relative speed}} \right)$ and for every

next time they will take $\frac{60}{55}$ minutes more to occur as opposite

to each other. (Since in $60/55$ hours they complete one round of clock's dial i.e., 60 points undergoing the relative motion, we have total 12 hours.)

Thus, $\frac{30}{55} + \frac{60}{55} + \frac{60}{55} + \frac{60}{55} + \dots = 12$

$\Rightarrow \frac{30}{55} + \frac{60}{55} + \frac{60}{55} + \dots = \frac{12 \times 55}{55}$

$\Rightarrow 6 + 12 + 12 + \dots = 12 \times 11 = 132$

$\Rightarrow 6 + 12n = 132$

$\Rightarrow 12n = 126 \Rightarrow n = 10$

only integral value is admissible.

Thus, total $10 + 1 = 11$ times both hands of a clock will be opposite to each other.

Remember: Between 5 O'clock and 7 O'clock the two hands make 180° angle only one time, that's why they make 180° angle only 11 times in 12 hours, i.e., at exactly 6 O'clock they are 180° apart.

Exp. 3) In 12 hours how many times a minute-hand and hour-hand of a clock makes 90° between them or becomes perpendicular to each other?

Solution $90^\circ = 15$ points

This problem can be solved in two parts.

- When minute-hand goes ahead of hour-hand.
- When hour-hand goes ahead of minute-hand.

- (i) For the first time minute-hand and hour-hand will make 90° (or 15 points) difference in $\frac{15}{55}$ hours.

$$\text{Since } \text{time} = \frac{\text{distance}}{\text{relative speed}}$$

Now, after every $\frac{60}{55}$ hours they will occur at 90° . Since,

in every $\frac{60}{55}$ hours they create a difference of 360° or

60 points (as the circumference of dial).

Now, we have 12 hours,

$$\therefore \frac{15}{55} + \frac{60}{55} + \frac{60}{55} + \dots = 12 \text{ h} = 12 \times \frac{55}{55}$$

$$\Rightarrow \frac{3}{11} + \frac{12}{11} + \frac{12}{11} + \dots = \frac{12 \times 11}{11}$$

$$\Rightarrow \frac{3}{11} + \left(\frac{12}{11} n\right) = \frac{12 \times 11}{11}$$

$$\Rightarrow 3 + 12n = 132$$

$$\Rightarrow n = \frac{129}{12} = 10$$

(only integral value of n is acceptable)

Therefore, $10 + 1 = 11$ times in 12 hours minute-hand makes 90° angle between the two hands, but when minute hand is ahead of hour-hand.

- (ii) For the first time minute-hand and hour-hand will make 90° (or 15 points) difference in $\frac{60 - 15}{55} = \frac{45}{55}$ hours.

Since, in this case minute-hand goes till it appears to be 15 points behind of hour-hand (consider initially they are showing 12 O'clock) then you will see that at 12 : 49 : 05 two hand are making 90° angle between them, while it appears to be hour-hand is ahead of minute-hand.

Now, for every next time after $\frac{60}{55}$ hours they will show

the same situation.

we have 12 hours.

$$\text{So, } \frac{45}{55} + \frac{60}{55} + \frac{60}{55} + \frac{60}{55} + \dots = 12 \text{ h} = 12 \times \frac{55}{55}$$

$$\Rightarrow \frac{3}{11} + \frac{4}{11} + \frac{4}{11} + \dots = \frac{4 \times 11}{11}$$

$$\Rightarrow 3 + 4 + 4 + \dots = 44$$

$$\Rightarrow 3 + 4n = 44$$

$$\Rightarrow n = \frac{41}{4} = 10, \text{ consider only integral value.}$$

Thus, total $10 + 1 = 11$ times they will make 90° angle.

Hence, in 12 hours both hands make 90° angle $(11 + 1) = 22$ times in different positions.

Remember At 3 O'clock and 9 O'clock they are at right angled. Since, except between 2–4 O'clock and 8–10 O'clock in each hour both hands make 90° angle 4 times while in the 2–4 O'clock and 8–10 O'clock two hands makes three-three times in every two hour.

Exp. 4) Between 2 O'clock and 3 O'clock when two hands of a clock overlap each other?

Solution To overlap or overtake minute-hand has to reduce the gap of 10 points. Since at 2 O'clock two hands are 10 point apart.

$$\therefore \text{Time} = \frac{\text{Distance advanced}}{\text{Relative speed}} \\ = \frac{10}{55} \text{ h} = \frac{10}{55} \times 60 \text{ min} = 10 \text{ min } 54 \text{ s}$$

Thus, at 2 : 10 : 54 both hands of a clock coincide.

Exp. 5) Between 6 am and 7 am when the two hands of a clock coincide.

$$\text{Solution } \text{Time} = \frac{30}{55} \text{ h} = \frac{30}{55} \times 60 \text{ min} = 32 \text{ min } 43 \text{ s}$$

$$(\text{Distance advanced} = 6 \times 5 = 30 \text{ points})$$

Thus at 6 : 32 : 43 two hands of a clock coincide.

Exp. 6) Between 11 O'clock and 12 O'clock when will they coincide.

$$\text{Solution } \text{Distance advanced at 11 O'clock} = 55 \text{ points} \\ \text{Relative speed} = 55 \text{ point/h}$$

$$\therefore \text{Time} = \frac{55}{55} = 1 \text{ h}$$

Hence, they will coincide at $(11 + 1) = 12$ O'clock.

Exp. 7) Between 3 O'clock and 4 O'clock when will the two hands make 36° angle between them:

(i) when hour-hand is ahead of minute-hand.

(ii) when minute-hand is ahead of hour-hand.

Solution

(i) $36^\circ = 6$ points

Now, at 3 O'clock two hands are separated by exactly 15 points to which we have to reduce upto 6 points. Thus, we have to reduce $15 - 6 = 9$ points distance, with the relative speed of 55 point/h.

$$\therefore \text{Time required} = \frac{9}{55} \text{ h} = \frac{9}{55} \times 60 = 9 \text{ min } 49 \text{ s}$$

Thus at 3 : 09 : 49 they are 36° apart from each other.

(ii) At 3 O'clock both hands are 15 points apart so to make them 6 points apart minute-hand has to move for $(15 + 6) = 21$ points, since minute-hand has to go 6 points ahead of hour hand, after crossing the hour-hand.

$$\therefore \text{Time} = \frac{21}{55} \text{ h} = \frac{21}{55} \times 60 \text{ min} = 22 \text{ min } 54 \text{ s}$$

Thus at 3 : 22 : 54, both hands will be 6 points (or 36°) apart from each other.

Did you notice something?

The same angle can be formed in two situations, one when hour-hand is ahead of minute-hand and when minute-hand is ahead of hour-hand. Thus, you can find the required time by dividing the required difference of points (which you have to either create or reduce) by the relative speed.

Exp. 8) What is the angle between the two hands at 3 : 10 am?

Solution Assume 60th point (*i.e.*, when it is 12 O'clock) as the origin.

Step 1. Find the distance of minute-hand from the origin.

Step 2. Find the distance of hour-hand from the origin.

Step 3. Take the difference between two values obtained in step 1 and step 2.

∴ **Step 1.** 10 point = 60°

Step 2. $90^\circ + 5^\circ = 95^\circ$ (In 10 min hour-hand moves 5°)

Step 3. $95^\circ - 60^\circ = 35^\circ$

Thus, at 3 : 10 am two hands are 35° apart.

Exp. 9) What is the angle between two hands of a clock at 7 : 35?

Solution **Step 1.** At 7 : 35, minute-hand is $35 \times 6 = 210^\circ$ away from origin.

Step 2. At 7 : 35 hour-hand is $7 \times 30 + 35 \times \frac{1}{2} = 210 + 17.5$
 $= 227.5^\circ$ away from the origin.

Step 3. $227.5 - 210 = 17.5^\circ$

Thus at 7 : 35, both hands make 17.5° angle between them.

NOTE A minute-hand moves 6° in one minute while a hour-hand moves $\left(\frac{1}{2}\right)^\circ$ in one minute.

CAT-Test

Questions Helping you bell the CAT

LEVEL 01 > BASIC LEVEL EXERCISE

- 'A' goes 10 km distance with average speed of 6 km/h while rest 20 km he travels with an average speed of 15 km/h. What is the average speed of 'A' during the whole journey?
 (a) 10 km/h (b) 12 km/h
 (c) 13 km/h (d) 14.5 km/h
- A covers half of his distance with 20 km/h and rest with 30 km/h. What is the average speed during the whole journey?
 (a) 20 km/h (b) 24 km/h
 (c) 25 km/h (d) 26 km/h
- A covers 1/3rd of his journey at the speed of 10 km/h and half of the rest at the speed of 20 km/h and rest at the speed of 30 km/h. What is the average speed of A?
 (a) $6\frac{2}{11}$ km/h (b) $16\frac{4}{11}$ km/h
 (c) $16\frac{4}{11}$ m/s (d) none of these
- A covers 1/4th of his journey at 20 km/h and 1/3rd of the rest at 25 km/h and half of the rest at 30 km/h and rest at the speed of 40 km/h. What is the average speed of A?
 (a) $13\frac{78}{89}$ km/h (b) 12 km/h
 (c) $26\frac{86}{89}$ km/h (d) 28 km/h
- A covered half of his journey at 20 km/h and rest at x km/h, then his average speed is 24 km/h. What is the value of x?
 (a) 30 (b) 32
 (c) 36 (d) 40
- A man covered half of the distance at 3x km/h and rest at 5x km/h. What is the average speed of the man?
 (a) 4x km/h (b) 3.5x km/h
 (c) 3.75x km/h (d) none of these
- A person goes to his office at 1/3rd of the speed at which he returns from his office. If the average speed during the whole trip (*i.e.*, one round) is 12 km/h. What is the speed of the person while he was going to his office?
 (a) 10 (b) 6
 (c) 8 (d) can't be determined
- A person X starts from Lucknow and another persons Y starts from Kanpur to meet each other. Speed of X is 25 km/h, while speed of Y is 35 km/h. If the distance between Lucknow and Kanpur be 120 km and both X and Y start their journey at the same time, when will they meet?
 (a) 1 h later (b) 2 h later
 (c) $\frac{1}{2}$ h later (d) 3 h later

9. In the above question (no. 8), what is the distance from Lucknow where they meet?

(a) 50 km (b) 60 km
(c) 100 km (d) 80 km

10. Two persons A and B started from two different places towards each other. If the ratio of their speeds be 3 : 5, then what is the ratio of distance covered by A and B respectively till the point of meeting?

(a) 1 : 2 (b) 3 : 4
(c) 3 : 5 (d) 5 : 3

Directions (for Q. Nos. 11 to 16) *A person P is at X and another person Q is at Y. The distance between X and Y is 100 km. The speed of P is 20 km/h. While the speed of Q is 60 km/h?*

11. If they first time meet at point Z somewhere between X and Y then the distance between X and Z is :

(a) 20 km (b) 40 km
(c) 25 km (d) 30 km

12. If they continue to move to and fro between X and Y then what is the distance covered by P when they meet second time?

(a) 105 km (b) 100 km
(c) 80 km (d) 75 km

13. If they continue to move to and fro between X and Y then what is the distance travelled by Q, when they meet each other for the third time?

(a) 375 km (b) 225 km
(c) 350 km (d) 445 km

14. If P and Q continue to move between X and Y in the given manner and if they meet for the fourth time at a place M somewhere between X and Y, then the distance between X and M is :

(a) 10 km (b) 90 km
(c) 75 km (d) 25 km

15. If P and Q continue to move between X and Y, then the ratio of distances covered by P and Q, when they meet for the 5th time?

(a) 1 : 4 (b) 1 : 3
(c) 2 : 3 (d) 3 : 4

16. If P and Q continue to move between X and Y, then the distance covered by P and Q together between any two consecutive meeting?

(a) 100 (b) 300
(c) 200 (d) can't be determined

Directions (for Q. Nos. 17, to 19) *A persons P starts his journey from A and another person Q starts his journey from B, towards each other. The speeds of P and Q are 16 km/h and 25 km/h respectively and they meet at point M somewhere between A and B when they start their journey simultaneously.*

17. What is the ratio of time taken by P and Q to reach at M?

(a) 1 : 4 (b) 1 : 1 (c) 4 : 5 (d) 16 : 25

18. What is the ratio of time taken by P and Q to reach B and A respectively?

(a) 16 : 25 (b) 1 : 1
(c) 25 : 16 (d) 4 : 5

19. What is the ratio of time taken by P and Q after meeting each other at M to reach B and A respectively?

(a) 25 : 16 (b) 625 : 256
(c) 16 : 25 (d) 4 : 5

20. The speed of Vimal and Kamal are 30 km/h and 40 km/h. Initially Kamal is at a place L and Vimal is at a place M. The distance between L and M is 650 km. Vimal started his journey 3 hours earlier than Kamal to meet each other. If they meet each other at a place P somewhere between L and M, then the distance between P and M is :

(a) 220 km (b) 250 km
(c) 330 km (d) 320 km

21. In the above question (no. 20) what is the distance between L and P ?

(a) 220 km (b) 320 km
(c) 330 km (d) none of the above

Directions (for Q. Nos. 22 to 27) *There are two places X and Y, 200 km apart from each other. Initially two persons P and Q both are at 'X'. The speed of P is 20 km/h and speed of Q is 30 km/h. Later on they starts to move to and fro between X and Y.*

22. If they start to move between X and Y, then for the first time when they will meet each other?

(a) after 12 hours (b) after 24 hours
(c) after 30 hours (d) after 8 hours

23. If they meet first time at a point M somewhere between X and Y, then what is the distance travelled by P?

(a) 160 km (b) 150 km
(c) 200 km (d) 210 km

24. If they meet second time each other at a point N somewhere between X and Y, then the distance travelled by Q is :

(a) 240 km (b) 480 km
(c) 360 km (d) none of these

25. If they meet third time each other at a point C, somewhere between X and Y, then the ratio of distances C X and C Y is

(a) 3 : 2 (b) 1 : 3
(c) 2 : 3 (d) 2 : 5

26. If they meet fourth time each other at a point D somewhere between X and Y, then what is the distance between D and X?

(a) 75 (b) 80
(c) 150 (d) 160

27. After starting their race, they meet each other for the n^{th} time at point X, then what is the minimum possible value of n ?

(a) 1 (b) 2
(c) 3 (d) 5

Directions (for Q. Nos. 28 to 33) *A person X started 3 hours earlier at 40 km/h from a place P, then another person Y followed him at 60 km/h, started his journey at 3 O'clock, afternoon.*

28. At what time will they meet to each other (or at what time Y will overtake X)?
 (a) 4:30 pm (b) 5 pm
 (c) 6 pm (d) 9 pm
29. At what time the difference between X and Y was 30 km, but before Y overtakes X?
 (a) 6:30 pm (b) 7:30 pm
 (c) 8:15 pm (d) none of these
30. At what time Y will be 30 km ahead of X, after overtaking it?
 (a) 6:45 pm (b) 7:30 pm
 (c) 10:30 pm (d) 8 pm
31. What is the distance travelled by Y to overtake X?
 (a) 180 km (b) 420 km
 (c) 320 km (d) 360 km
32. What distance should Y cover so that he may reach 360 km ahead of X?
 (a) 1440 km (b) 1200 km
 (c) 920 km (d) 750 km
33. What is difference in time when X was 30 km ahead of Y and when Y was 30 km ahead of X?
 (a) 2 (b) 3
 (c) 3.5 (d) 4.25
34. A postman goes with a speed of 36 km/h. What is the speed of postman in m/s?
 (a) 4.5 m/s
 (b) 6 m/s
 (c) 10 m/s
 (d) can't be determined
35. In the above question (no. 34) what is the speed in m/min?
 (a) 325 (b) 432
 (c) 360 (d) 600
36. In the above question (no. 34) what is the speed of postman in mile/h?
 (a) 22.37 (b) 30.08
 (c) 28.30 (d) 38.12
37. A train goes with a speed of 20 m/s. What is the speed of train in km/h?
 (a) 57 km/h (b) 72 km/h
 (c) 80 km/h (d) 120 km/h
38. As per the question (no. 37) what is the speed of train in km/min?
 (a) 1.2 (b) 12
 (c) 1200 (d) 120
39. A is twice fast as B and B is thrice as fast as C. The journey covered by C in 78 minutes will be covered by A in :
 (a) 12 min (b) 13 min
 (c) 15.5 min (d) none of these
40. The ratio of speeds of A is to B is 2 : 3 and therefore A takes 20 minutes less time than B takes. What is the ratio of time taken by A and B?
 (a) 2 : 3 (b) 2 : 5 (c) 3 : 2 (d) 3 : 5
41. What is the time taken by A (in the above question)?
 (a) 1 h (b) 1.2 h
 (c) 0.6 h (d) 30 min
42. A certain distance is covered at a certain speed. If half of this distance is covered in double the time, the ratio of the two speeds is :
 (a) 1 : 16 (b) 4 : 1
 (c) 2 : 1 (d) 2 : 8
43. Two runner start running together for a certain distance, one at 5 km/h and another at 3 km/h. The former arrives one and half an hour before the latter. The distance (in km) is :
 (a) 12 (b) 20
 (c) 25 (d) 36
44. The ratio between the rates of walking of A and B is 2 : 3. If the time taken by B to cover a certain distance is 48 minutes, the time taken (in minutes) by A to cover the distance is :
 (a) 52 min (b) 68 min
 (c) 72 min (d) 32 min
45. Two trains starting at the same time from two stations 300 km apart and going in opposite directions, cross each other at a distance of 160 km from one of them. The ratio of their speeds is :
 (a) 7 : 9 (b) 16 : 20 (c) 8 : 7 (d) 8 : 12
46. A and B travel the same distance at 9 km/h and 10 km/h respectively. If A takes 20 minutes longer than B, the distance travelled by each is :
 (a) 16 (b) 20
 (c) 30 (d) none of these
47. Abhinav leaves Mumbai at 6 am and reaches Bangalore at 10 am Praveen leaves Bangalore at 8 am and reaches Mumbai at 11:30 am. At what time do they cross each other?
 (a) 10 am (b) 8:32 am (c) 8:56 am (d) 9:20 am
48. Two persons, Alif and Laila start at the same time from Allahabad and Lucknow and proceed towards each other at 45 km/h and 54 km/h respectively. When they meet, it is found that one of them has travelled 72 km more than the other. The distance between the places (in km) is :
 (a) 729 (b) 792
 (c) 540 (d) none of these
49. Walking at $\frac{4}{5}$ of his normal speed, Dewang is 15 minutes late in reaching his club. What is the usual time taken by him to cover the distance?
 (a) 1 h (b) 1 h 20 min
 (c) 45 min (d) none of these

50. Walking at $\frac{3}{4}$ of her normal speed Malleshwari takes 2 hours more than the normal time. What is the normal time?
 (a) 4 h (b) 5 h
 (c) 6 h (d) 8 h
51. Walking at $\frac{3}{2}$ of his normal speed Shekhawat takes 40 minutes less than the usual time? What is the changed (new) time taken by Shekhawat?
 (a) 1 h (b) 1.2 h
 (c) 3 h (d) 2 h
52. A man reduces his speed from 20 km/h to 18 km/h. So, he takes 10 minutes more than the normal time. What is the distance travelled by him?
 (a) 30 km (b) 25 km
 (c) 50 km (d) 36 km
53. Osaka walks from his house at 5 km/h and reaches his office 10 minutes late. If this speed had been 6 km/h he would have reached 15 minutes early. The distance of his office from his house is :
 (a) 15 km (b) 12.5 km
 (c) 10.5 km (d) 18 km
54. Sachin and Mongiya starting from the same place walk at the rate of 7.5 km/h and 9 km/h respectively. What time will they take to be 7.5 km apart, if they walk in the same direction?
 (a) 3 h (b) 4 h
 (c) 5 h (d) none of these
55. Two aeroplanes start from the same place in opposite directions. One goes towards east at 320 km/h and the other goes towards west at 400 km/h what time will they take to be 720 km apart?
 (a) 4 h (b) 3 h (c) 1 h (d) 1.5 h
56. A man covers a certain distance by his own car. Had he moved 6 km/h faster he would have taken 4 hours less. If he had moved 4 km/h slower, he would have taken 4 hours more. The distance (in km) is :
 (a) 240 km (b) 640 km
 (c) 480 km (d) none of these
57. If Deepesh had walked 20 km/h faster he would have saved 1 hour in the distance of 600 km. What is the usual speed of Deepesh?
 (a) 100 (b) 120
 (c) 150 (d) none of these
58. Harsha takes 3 hours more than Ashok, who drives his car 5 km/h faster than Harsha drives, to cover 180 km distance. What is the speed of Harsha?
 (a) 12 km/h (b) 15 km/h
 (c) 30 km/h (d) 40 km/h
59. A mini bus takes 6 hours less to cover 1680 km distance, if its speed is increased by 14 km/h? What is the usual time taken by mini bus?
 (a) 15 h (b) 24 h
 (c) 25 h (d) 30 h
60. The driver of an ambulance sees a school bus 40 m ahead of him. After 20 second, the school bus is 60 metre behind. If the speed of the ambulance is 30 km/h, what is the speed of the school bus?
 (a) 10 km/h (b) 12 km/h
 (c) 15 km/h (d) 22 km/h
61. A postman riding a bicycle at 15 km/h can reach a village in 4 hours. If he is delayed by 1 hour at the start, then in order to reach his destination in time, he should ride with a speed of:
 (a) 20 km/h (b) 16 km/h
 (c) 14 km/h (d) 12 km/h
62. What is the time required by a train of length of 350 m to cross an electric pole with a speed of 70 m/s?
 (a) 3 s (b) 3.5 s
 (c) 5 s (d) 10 s
63. A train 270 m long is running over a bridge of length of 130 m with a speed of 40 m/s. What is the time taken by this train to cross the bridge?
 (a) 6 s (b) 16 s
 (c) 10 s (d) none of these
64. If a train 225 m long passes a telegraphic pole in 9 seconds, then the time taken (in seconds) by it to cross a tunnel 450 m long is :
 (a) 8 s (b) 10 s
 (c) 27 s (d) none of these
65. A train 350 m long is running at the speed of 36 km/h. If it crosses a tunnel in 1 minute, then the length of the tunnel (in metres) is :
 (a) 200 m (b) 250 m
 (c) 150 m (d) none of these
66. If a 250 m long train crosses a platform of the same length as that of the train in 25 seconds, then the speed of the train is :
 (a) 150 m/s (b) 200 m/s
 (c) 20 km/h (d) 72 km/h
67. Sabarmati express takes 18 seconds to pass completely through a station 162 m long and 15 seconds through another station 120 m long. The length of the Sabarmati express is :
 (a) 132 m (b) 100 m (c) 80 m (d) 90 m
68. A train 200 m long travels at the speed of 72 km/h. A man is running at 3.6 km/h in the same direction in which the train is going. The train will pass the man in :
 (a) 10 s (b) $12\frac{3}{19}$ s
 (c) $10\frac{10}{19}$ s (d) none of these
69. A train 350 m long is moving at the speed of 20 km/h. It will cross a man coming from the opposite direction at the speed of 1 km/h in :
 (a) 27 s (b) 35 s
 (c) 45 s (d) 60 s

70. The length of Lucknow mail is 120 m and that of Punjab mail is 80 m. These two trains are running in the same direction with velocities of 40 km/h and 50 km/h respectively. The time taken by them to cross each other is :
 (a) 8 s (b) 72 s
 (c) 11.5 s (d) 12.5 s
71. In the above question if the trains are running in opposite directions. The time taken by them to cross each other is :
 (a) 8 s (b) 72 s
 (c) 12.5 s (d) none of these
72. A train passes an electric pole in 10 seconds and a platform 120 m long in 18 seconds. Its length in metres is
 (a) 150 m (b) 130 m
 (c) 240 m (d) 180 m
73. A 175 m long train crosses a man walking at a speed of 9 km/h in the opposite direction in 10 sec. The speed of the train (in km/h) is :
 (a) 45 (b) 54
 (c) 72 (d) 68
74. A train of length 100 m takes $\frac{1}{6}$ minute to pass over another train 150 m long coming from the opposite direction. If the speed of first train is 60 km/h, the speed of the second train is :
 (a) 45 km/h (b) 28 km/h
 (c) 30 km/h (d) none of these
75. A train overtakes two girls who are walking in the opposite direction in which the train is going at the rate of 3 km/h and 6 km/h and passes them completely in 36 seconds and 30 seconds respectively. The length of the train (in metres) is :
 (a) 120 m (b) 150 m
 (c) 125 m (d) none of these
76. A coolie standing on a railway platform observes that a train going in one direction takes 4 seconds to pass him. Another train of same length going in opposite direction takes 5 seconds to pass him. The time taken (in seconds) by the two trains to cross each other will be :
 (a) 35 (b) 36.5
 (c) $\frac{40}{9}$ (d) none of these
77. Pushpak express leaves Lucknow at 6 am and two hours later another train Bhopal express leaves Lucknow. Both trains arrive Bhopal at 4 pm on the same day. If the difference between their speeds be 10 km/h, what is the average speeds of both the trains over entire route :
 (a) 40 km/h
 (b) $44\frac{4}{9}$ km/h
 (c) $42\frac{3}{5}$ km/h
 (d) none of the above

Directions (for Q. Nos. 78 and 79) *Two trains leave Meerut at the difference of 4 hours. The first train leaves at 8 am at 40 km/h and the faster train leaves later at 60 km/h in the same direction.*

78. When the faster train will overtake the slower train?
 (a) 4 pm (b) 2 pm (c) 8 pm (d) 6:30 pm
79. What is the distance from Meerut, where one train overtakes another train?
 (a) 480 km (b) 420 km (c) 360 km (d) 250 km
80. The distance between Lucknow and Delhi is 700 km. Rajdhani express starts from Delhi for the Lucknow at 60 km/h. 50 minutes later Lucknow express leaves Lucknow for Delhi on the parallel tracks at 70 km/h. How far from Lucknow will they cross each other?
 (a) 250 km (b) 360 km (c) 350 km (d) 475 km
81. Patna express travels first 560 km in 7 hours and rest 360 km in 9 hours. What is the average speed of the train?
 (a) 39 km/h (b) 43 km/h
 (c) 63 km/h (d) 57.5 km/h
82. Jammutavi express leaves Jammu for Kanya Kumari at 120 km/h and returns to Jammu at 80 km/h. What is the average speed of the train during the whole journey?
 (a) 47.5 km/h (b) 96 km/h
 (c) 38 km/h (d) 57.5 km/h
83. In the above question if the total time taken by the train is 25 hours, what is the distance between these two places?
 (a) 1365.5 km (b) 1369 km
 (c) 1200 km (d) can't be determined
84. Roorkee express normally reaches its destination at 50 km/h in 30 hours. Find the speed at which it travels to reduce the time by 10 hours?
 (a) 38 km/h (b) 76 km/h (c) 75 km/h (d) 60 km/h
85. Two trains A and B start simultaneously in the opposite direction from two points P and Q and arrive at their destinations 16 and 9 hours respectively after their meeting each other. At what speed does the second train B travel if the first train travels at 120 km/h per hour :
 (a) 90 km/h (b) 160 km/h
 (c) 67.5 km/h (d) none of these
86. There are two trains running on two parallel tracks. Length of each train is 120 m. When they are running in opposite directions, they cross each other in 4 seconds and when they are running in the same direction they cross in 12 seconds. What is the speed of the faster train?
 (a) 80 km/h (b) 72 km/h
 (c) 120 km/h (d) 144 km/h
87. Two trains are travelling in the same direction at 22.5 km/h and 7.5 km/h respectively. The faster train crosses a man in the slower train in 18 seconds. What is length of the faster train?
 (a) 87.5 m (b) 75 m
 (c) 122.5 m (d) none of these

88. A train covers a certain distance moving at a speed of 60 km/h. However if it were to halt for a fixed time every hour, its average speed comes out to be 50 km/h. For how much time does the train halt for every hour?
 (a) 6 min (b) 10 min
 (c) 12 min (d) none of these
89. Two horses start trotting towards each other, one from A to B and another from B to A. They cross each other after one hour and the first horse reaches B, $\frac{5}{6}$ hour before the second horse reaches A. If the distance between A and B is 50 km. What is the speed of the slower horse?
 (a) 30 km/h (b) 15 km/h
 (c) 25 km/h (d) 20 km/h
90. Pankaj walked at 5 km/h for certain part of the journey and then he took an auto for the remaining part of the journey travelling at 25 km/h. If he took 10 hours for the entire journey. What part of journey did he travelled by auto if the average speed of the entire journey be 17 km/h :
 (a) 750 km (b) 100 km
 (c) 150 km (d) 200 km
91. A car travelled first 36 km at 6 km/h faster than the usual speed, but it returned the same distance at 6 km/h slower than the usual speed. If the total time taken by car is 8 hours, for how many hours does it travelled at the faster speed?
 (a) 4 (b) 3
 (c) 2 (d) 1
92. A dog starts chasing to a cat 2 hours later. It takes 2 hours to dog to catch the cat. If the speed of the dog is 30 km/h, what is the speed of cat?
 (a) 10 km/h (b) 15 km/h
 (c) 20 km/h (d) can't be determined
93. Prachi starts from Barabanki at 6 am at constant speed of 60 km/h. She halts at Lucknow for half an hour and then drives at 40 km/h. If she reaches Kanpur at 9 : 30 am, which is 160 km from Barabanki, how far is Barabanki from Lucknow?
 (a) 75 km (b) 80 km
 (c) 100 km (d) 120 km
94. Two trains whose respective lengths are 200 m and 250 m cross each other in 18 s, when they are travelling in opposite direction and in 1 min, when they are travelling in the same direction. What is the speed of the faster train (in km/h)?
 (a) 38.5 (b) 48.5
 (c) 54 (d) 58.5
95. Abhinav started for the station half a km from his home walking at 1 km/h to catch the train in time. After 3 minutes he realised that he had forgotten a document at home and returned with increased, but constant speed to get it succeeded in catching the train. Find his latter speed in km/h :
 (a) 1.25 (b) 1.1
 (c) $\frac{11}{9}$ (d) 2

Directions (for Q. Nos. 96 and 97) *The ratio of speeds at which Anil and Mukesh walk is 3 : 4. Anil takes 30 minutes more than the time taken by Mukesh in reaching the destination.*

96. If Anil drives the car at twice the speed of his walking then the time required to reach his destination by car is :
 (a) 45 min
 (b) 60 min
 (c) 1.5 h
 (d) 1 h 20 min
97. What is the total distance travelled by each of them, if the average of speeds of Anil and Mukesh is 28 km/h?
 (a) 48 km (b) 60 km
 (c) 17 km (d) 70 km
98. Train X starts from point A for point B at the same time that train Y starts from B to A. Point A and B are 300 km apart. The trains are moving at a constant speed atleast at 25 km/h. The trains meet each other 3 hours after they start. If the faster train takes atleast 2 more hours to reach the destination. By which time will the slower train have definitely reached its destination? (Ignoring the length of trains in crossing)
 (a) 4 hours after the start
 (b) 7.5 hours after the start
 (c) 6 hours after the start
 (d) none of the above
99. In reaching the Purnagiri a man took half as long again to climb the second third as he did to climb the first third and a quarter as long again for the last third as for the second third. He took altogether 5 hr 50 minutes. Find the time he spent on the first third of the journey?
 (a) 72 min (b) 80 min
 (c) 81 min (d) 88 min
100. Walking at four fifth of his usual speed Vijay Malya reaches his office 15 minutes late on a particular day. The next day, he walked at $\frac{5}{4}$ of his usual speed. How early would he be to the office when compared to the previous day?
 (a) 27 min (b) 32 min
 (c) 30 min (d) none of these
101. Abdul starts in a car from Ahmedabad towards Bangalore. After sometime he realises that he will cover only 75% of the distance in the scheduled time and he therefore doubles his speed immediately and thus manages to reach Bangalore exactly on time. Find the time after which Abdul changed his speed, given that he could have been late by 3 hours if he had not changed his speed :
 (a) 3 h (b) 4 h
 (c) 5 h (d) 6 h
102. A man travels the first part of his journey at 20 km/h and the next at 70 km/h, covering the entire journey at an average speed of 50 km/h. What is the ratio of the distance that he covered at 20 km/h to that he covered at 70 km/h?
 (a) 4 : 21 (b) 3 : 22
 (c) 1 : 4 (d) 3 : 5

- 103.** Anjali fires two bullets from the same place at an interval of 6 minutes but Bhagwat sitting in a car approaching the place of firing hears the second fire 5 minute 32 seconds after the first firing. What is the speed of car, if the speed of sound is 332 m/s?
 (a) 56 m/s (b) 102 m/s
 (c) 28 m/s (d) 32 m/s
- 104.** A car crosses a man walking at 6 km/h. The man can see the things upto 450 m only in one direction due to fog. He sees the car which was going in the same direction for 4.5 minutes. What is the speed of the car?
 (a) 9 km/h (b) 12 km/h
 (c) 12.5 km/h (d) 15 km/h
- 105.** A man takes 4 h 20 minutes in walking to a certain place and riding back. If he walk on both sides he loses 1 h. The time he would take by riding both ways is :
 (a) 2 h 20 min (b) 3 h 20 min
 (c) 2 h (d) 4 h 40 min
- 106.** A train met with an accident 120 km from station A. It completed the remaining journey at $\frac{5}{6}$ of its previous speed and reached 2 hours late at station B. Had the accident taken place 300 km further, it would have been only 1 hour late? What is the speed of the train?
 (a) 100 km/h (b) 120 km/h
 (c) 60 km/h (d) 50 km/h
- 107.** For the above question what is the total distance between A and B ?
 (a) 480 km (b) 520 km
 (c) 600 km (d) 720 km
- 108.** The wheel of an engine of 300 cm in circumference makes 10 revolutions in 6 seconds. What is the speed of the wheel (in km/h)?
 (a) 18 (b) 20
 (c) 27 (d) 36
- 109.** A man can row downstream at 12 km/h and upstream at 8 km/h. What is the speed of man in still water?
 (a) 12 km/h (b) 10 km/h
 (c) 8 km/h (d) 9 km/h
- 110.** A man can row upstream at 15 km/h and downstream at 21 km/h. The speed of water current of the river is :
 (a) 8 km/h (b) 6 km/h
 (c) 3 km/h (d) 5 km/h
- 111.** A boat moves downstream at 1 km in 5 minutes and upstream at 1 km in 12 minutes. What is the speed of current?
 (a) 4.5 km/h (b) 3.5 km/h
 (c) 2 km/h (d) 2.5 km/h
- 112.** A man rows downstream 60 km and upstream 36 km, taking 4 hours each time. The speed of the man is :
 (a) 15 km/h (b) 16 km/h
 (c) 8 km/h (d) 12 km/h
- 113.** A man can row 5 km/h in still water. If the rate of current is 1 km/h, it takes $\frac{5}{4}$ hours to row to a place and back. How far is the place?
 (a) 2 km (b) 2.5 km
 (c) 3 km (d) 4 km
- 114.** A man can swim 5 km/h in still water. If the speed of current be 3 km/h, the time taken by him to swim to a place 16 km upstream and back is :
 (a) 8 h (b) 7.5 h
 (c) 6.66 h (d) 10 h
- 115.** A boat covers 48 km upstream and 72 km downstream in 12 hours, while it covers 72 km upstream and 48 km downstream in 13 hours. The speed of stream is :
 (a) 2 km/h (b) 2.2 km/h
 (c) 2.5 km/h (d) 4 km/h
- 116.** A motor boat takes 2 hours to travel a distance of 9 km downstream and it takes 6 hours to travel the same distance against the current. The speed of the boat in still water and that of the current (in km/h) respectively are :
 (a) 6, 5 (b) 3, 1.5 (c) 8, 5 (d) 9, 3
- 117.** A man can row 15 km/h in still water and he finds that it takes him twice as much time to row up than as to row down the same distance in the river. The speed of the current (in km/h) is :
 (a) 6 km/h (b) 6.5 km/h
 (c) 4.5 km/h (d) 5 km/h
- 118.** A motor boat takes 12 hours to go downstream and it takes 24 hours to return the same distance. What is the time taken by boat in still water?
 (a) 15 h (b) 16 h (c) 8 h (d) 20 h
- 119.** The speed of a boat in upstream is $\frac{2}{3}$ that of downstream. Find the ratio of speed of boat in still water and to the average speed of boat in downstream and upstream?
 (a) $\frac{24}{25}$ (b) $\frac{25}{24}$
 (c) $\frac{5}{12}$ (d) none of these
- 120.** The difference between downstream speed and upstream speed is 3 km/h and the total time taken during upstream and downstream is 3 hours. What is the downstream speed, if the downstream and upstream distance are 3 km each?
 (a) 2.5 km/h (b) 4.33 km/h
 (c) 4 km/h (d) 3.3 km/h
- 121.** A boat which sails at 10 km/h in still water starts chasing, from 10 km behind, another one which sails at 4 km/h in the upstream direction. After how long will it catchup if the stream is flowing at 2 km/h :
 (a) 4 h (b) 2.5 h
 (c) 2 h (d) 3.5 h

Directions (for Q. Nos. 122 and 123) *A motor boat went downstream for 120 km and immediately returned. It took the boat 15 hours to complete the round trip. If the speed of the river were twice as high, the trip downstream and back would take 24 hours.*

- 122.** What is the speed of the boat in still water?
 (a) 20 km/h (b) 18 km/h
 (c) 15 km/h (d) 16 km/h
- 123.** What is the speed of the stream?
 (a) 3.5 km/h (b) 4 km/h
 (c) 6 km/h (d) 8 km/h
- 124.** A boat sails 15 km of a river towards upstream in 5 hours. How long will it take to cover the same distance downstream, if the speed of current is one-fourth the speed of the boat in still water :
 (a) 1.8 h (b) 3 h
 (c) 4 h (d) 5 h
- 125.** A boat takes 5 hours more while going back in upstream than in downstream. If the distance between two places is 24 km and the speed of boat in still water be 5.5 km/h. What must be the speed of boat in still water so that it can row downstream, 24 km, in 4 hours?
 (a) 1.5 km/h (b) 3.5 km/h
 (c) 4.5 km/h (d) 3 km/h
- 126.** A boat takes 7 hours to go from *P* to *R*, through a midpoint *Q*, but it takes 8 hours to go from *P* to *Q* and then return from *Q* to *P*. How long it would take to go from *R* to *P*?
 (a) 7 h (b) 8 h
 (c) 9 h (d) none of these
- 127.** Mallah can row 40 km upstream and 55 km downstream in 13 h and 30 km upstream and 44 km downstream in 10 hours. What is the speed of Mallah in still water?
 (a) 6 km/h (b) 12 km/h
 (c) 3 km/h (d) 8 km/h

Directions (for Q. Nos. 128 and 129) *In a kilometre race, A can give B a start of 20 m and also in a half kilometre race C beats A by 50 m.*

- 128.** If *A*, *B* and *C* run a 2 km race, what is the difference between the distances covered by the two losers, when the winner finishes the race?
 (a) 64 m (b) 36 m (c) 32 m (d) 58 m
- 129.** *B* and *C* run a half km race, who should give a start to the slower runner and of how many metres so that they both finish the race at the same time?
 (a) *C*, 59 m (b) *B*, 34 m
 (c) *C*, 48 m (d) *B*, 56 m
- 130.** In a 1600 m race, *A* beats *B* by 80 m and *C* by 60 m. If they run at the same time then by what distance will *C* beat *B* in a 400 m race?
 (a) $5\frac{15}{77}$ m (b) $5\frac{20}{76}$ m
 (c) $15\frac{5}{77}$ m (d) none of these

- 131.** Aman can run a distance in 190 seconds and Shakti can run the same distance in 200 seconds. If they start together, by what distance Aman can beat Shakti in 1 km race?
 (a) 48 m (b) 25 m
 (c) 24 m (d) 50 m
- 132.** *A* runs $\frac{7}{4}$ times as fast as *B*. If *A* gives *B* a start of 300 m, how far must the winning post be if both *A* and *B* have to end the race at same time?
 (a) 1400 m (b) 700 m
 (c) 350 m (d) 210 m
- 133.** *A* beats *B* by 100 m in a race of 1200 m and *B* beats *C* by 200 m in a race of 1600 m. Approximately by how many metres can *A* beat *C* in a race of 9600 m?
 (a) 1600 m (b) 1800 m
 (c) 1900 m (d) 2400 m
- 134.** In a 1000 m race Ameesha gives a headstart of 100 m to Bipasha and beats her by 200 m. In the same race Ameesha gives a headstart of 100 m to Celina and beats her by 300 m. By how many metres would Bipasha beat Celina in a 50 m race?
 (a) 6.66 m (b) 7.143 m
 (c) 8 m (d) none of these
- 135.** In a 1000 metres race Ravi gives Vinod a start of 40 m and beats him by 19 seconds. If Ravi gives a start of 30 seconds then Vinod beats Ravi by 40 m. What is the ratio of speed of Ravi to that of Vinod?
 (a) 4 : 5 (b) 6 : 5
 (c) 3 : 8 (d) 5 : 4
- 136.** In a race, the man who came two places ahead of the last man finished one place ahead of the man who came three places behind the man just ahead of the one who stood second. How many men finished the race?
 (a) 6 (b) 5 (c) 4 (d) 8
- 137.** Vinay and Versha run a race with their speeds in the ratio of 5 : 3. They prefer to run on a circular track of circumference 1.5 km. What is the distance covered by Vinay when he passes Versha for the seventh time?
 (a) 25.25 km (b) 26.25 km
 (c) 132 m (d) none of these
- 138.** *A* gives both *B* and *C* a start of 60 m in a 1500 m race. However, while *B* finishes with him, *C* is 15 m behind them when *A* and *B* cross the finishing line. How much start can *B* give *C* for the 1500 m race course?
 (a) $7\frac{6}{23}$ m (b) $15\frac{5}{8}$ m
 (c) $7\frac{11}{16}$ m (d) $5\frac{5}{24}$ m
- 139.** In a 600 m race Prabhat has a start of 200 m and the ratio of speeds of Prabhat and Nishith is 4 : 5, then the distance by which Prabhat wins by :
 (a) 100 m (b) 80 m
 (c) 120 m (d) none of these

- 140.** In the game of billiards, A can give B , 20 points in 80 and B can give C , 16 points in 80. How many points can A give C in a game of 200?
 (a) 64 (b) 72
 (c) 80 (d) none of these
- 141.** In a day how many times the minute and hour-hands coincide (or overlap or come together)?
 (a) 12 (b) 20
 (c) 22 (d) 24
- 142.** In a day how many times the minute-hand and hour-hand make right angle between them?
 (a) 12 (b) 20
 (c) 22 (d) 44
- 143.** In a day how many times the minute-hand and hour-hand are opposite to each other?
 (a) 12 (b) 24 (c) 11 (d) 22
- 144.** The relative speed of minute-hand with respect to hour-hand is :
 (a) $\left(5\frac{1}{2}\right)^\circ$ per minute
 (b) $\left(\frac{11}{12}\right)$ minute per minute
 (c) 6° per minute
 (d) both (a) and (b)
- 145.** When do the hands of a clock coincide between 5 and 6?
 (a) 5 : 30 (b) 5 : 27 : 16
 (c) 5 : 32 : 16 (d) 5 : 28 : 56
- 146.** What is the angle between hour-hand and minute-hand at 2 : 25 pm?
 (a) 77.5° (b) 68°
 (c) 67.5° (d) none of these
- 147.** When do the two hands of a clock of just after 3 pm make 30° angle between them?
 (a) 3 : 15 : 00 (b) 3 : 10 : 54
 (c) 3 : 01 : 59 (d) none of these
- 148.** At what time are the hands of clock together between 7 pm and 8 pm?
 (a) 7 : 45 : 54 (b) 7 : 36 : 27
 (c) 7 : 37 : 49 (d) 7 : 38 : 11
- 149.** What is the angle between the hands of a clock when the time is 15 minutes past 6?
 (a) 97.5° (b) 87.5°
 (c) 77° (d) none of these
- 150.** What are the possible times when a clock shows 35° angle between two hands between 3 pm and 4 pm?
 (a) 20 min 25 sec (b) 25 min 20 sec
 (c) 22 min 43 sec (d) none of these

LEVEL 02 > HIGHER LEVEL EXERCISE

Directions (for Q. Nos. 1 to 4) *Aishwarya is going to cover a distance of 360 km from Ambala to Khandala. The first one-third of the distance she covers on a cycle. The second one-third she covers by an auto-rickshaw and the remaining distance she travels by car. The average speed of the journey by a car is 5 times the average speed by cycle and 20 km/h more than the average speed by auto-rickshaw, but she took 1 hour more by auto-rickshaw than by car.*

- What is the average speed of the whole journey?
 (a) 15 km/h (b) 24 km/h
 (c) 20 km/h (d) none of these
- What is the time taken in the whole journey?
 (a) 10 h (b) 12 h
 (c) 15 h (d) none of these
- What is the distance covered by her in last five hours of her journey?
 (a) 250 km
 (b) 240 km
 (c) 200 km
 (d) can't be determined
- Instead of travelling the first one-third by cycle if she travels by same auto-rickshaw with the same average speed, then what is the percentage decrease/increase in time taken during the entire journey?
 (a) 46.66% (b) 33.33% (c) 50% (d) 25%
- Bipasha and Mallika leave towns Kolkata and Ambala at 6 am and travel towards Ambala and Kolkata respectively. Speed of Bipasha is 60 km/h and speed of Mallika is 120 km/h. Rani leaves Kolkata for Ambala sometime later and travels at a speed of 90 km/h. If the distance between Kolkata and Ambala is 1080 km and all three meet at the same point on the way, at same time, then at what time did Rani leave Kolkata?
 (a) 7 am (b) 8 am (c) 7:30 am (d) 10 am
- A passenger sitting in a train of length l m, which is running with speed of 60 km/h passing through two bridges, notices that he crosses the first bridge and the second bridge in time intervals which are in the ratio of 7 : 4 respectively. If the length of first bridge be 280 m, then the length of second bridge is
 (a) 490 m (b) 220 m
 (c) 160 m (d) can't be determined
- Pathik and Rahi started from two places Andheri and Bhavnagar towards Bhavnagar and Andheri respectively at 8 : 20 am. The speeds of Pathik and Rahi are in the ratio of 4 : 5. They meet at Chandni Chowk, somewhere between Andheri and Bhavnagar, spent some-time together enjoyed coffee and burger and then both started towards their destination at 9 : 27 am. If Pathik reaches Bhavnagar at 10 : 32 am, how much time did they spend together?
 (a) 8 min (b) 12 min
 (c) 15 min (d) can't be determined

8. A train with 120 wagons crosses Arjun who is going in the same direction, in 36 seconds. It travels for half an hour from the time it starts overtaking the Arjun (he is riding on the horse) before it starts overtaking Srikrishna (who is also riding on his horse) coming from the opposite direction in 24 seconds. In how much time (in seconds) after the train has crossed the Srikrishna do the Arjun meets to Srikrishna?
 (a) 3560 sec (b) 3600 sec
 (c) 3576 sec (d) can't be determined
9. Kareena and Shahid start from Kurla and Worli towards Worli and Kurla respectively, at the same time. After they meet at Shantakruz on the way from Kurla to Worli, Kareena reduces her speed by 33.33% and returns back to Kurla and Shahid increases his speed by 33.33% and returns back to Worli. If Kareena takes 2 hours for the entire journey, what is the time taken by Shahid for the entire journey?
 (a) 96 min (b) 84 min
 (c) 168 min (d) can't be determined
10. Due to the technical snag in the signal system two trains start approaching each other on the same rail track from two different stations, 240 km away from each other. When the train starts a bird also starts moving to and fro between the two trains at 60 km/h touching each time each train. The bird is initially sitting on the top of the engine of one of the trains and it moves so till these trains collide. If these trains collide one and a half hour after the start, then how many kilometres bird travels till the time of collision of trains?
 (a) 90 km (b) 130 km
 (c) 120 km (d) none of these
11. Einstein walks on an escalator at a rate of 5 steps per second and reaches the other end in 10 seconds. While coming back, walking at the same speed he reaches the starting point in 40 seconds. What is the number of steps on the escalator?
 (a) 40 (b) 60 (c) 120 (d) 80
12. A girl while walking diametrically across a semicircular playground, takes 3 minutes less than if she had kept walking round the circular path from A to B. If she walks 60 metres a minute, what is the diameter of the play ground
 (a) 60 m (b) 48 m (c) 84 m (d) 315 m
13. Two trains start simultaneously from two stations Howrah and Bandra, respectively towards each other on the same track. The distance between the two stations is 560 km and the speed of trains are 30 and 40 km/h. Simultaneously with the trains, a sparrow sitting on the top of one of the train starts towards the other and reverses its direction on reaching the other train and so on. If the speed of sparrow is 80 km/h then the distance that the sparrow flies before being crushed between the train is :
 (a) 70 km (b) 560 km
 (c) 640 km (d) 650 km
14. A surveillance plane is moving between two fixed places Pukhwara and Kargil at 120 km/h. The distance between two places is 600 km. After 18 hour what will be the distance between the Kargil and its position if it starts moving from Pukhwara?
 (a) 360 km (b) 300 km
 (c) 240 km (d) none of these
15. The speed of a car during the second hour of its journey is thrice that in the first hour. Also its third hours speed is the average speed of the first two hours. Had the car travelled at the second hours speed during all the first three hours, then it would have travelled 150 km more. Find the percentage reduction in time in the second case for the first three hours :
 (a) $33\frac{1}{3}\%$ (b) 40% (c) 25% (d) 50%
16. There are three runners Tom, Dick and Harry with their respective speeds of 10 km/h, 20 km/h and 30 km/h. They are initially at P and they have to run between the two points P and Q which are 10 km apart from each other. They start their race at 6 am and end at 6 pm on the same day. If they run between P and Q without any break, then how many times they will be together either at P and Q during the given time period?
 (a) 5 (b) 7 (c) 4 (d) 12
- Directions** (for Q. Nos. 17 and 18) *Arjun and Sri-krishna go by chariot from Mathura to Kurukshetra which is on the way to Hastinapur. Abhimanyu goes from Hastinapur to Kurukshetra. The distance between Mathura to Hastinapur is 700 km and the distance between Hastinapur and Kurukshetra is 300 km. Speed of Arjun and Srikrishna's chariot is 25 km/h and speed of Abhimanyu is 10 km/h. All the three persons start their journey at 10 am. After travelling some miles Srikrishna sees Duryodhan going (by riding on his horse) at 20 km/h to Kurukshetra. Arjun and Srikrishna go ahead meet Abhimanyu and pick him up. Then they return immediately to Kurukshetra and thus all the four reach at the same time.*
17. What is the total distance travelled by Arjun?
 (a) 400 (b) 500
 (c) 600 (d) can't be determined
18. What is the total time taken to reach Kurukshetra?
 (a) 10 h (b) 15 h (c) 18 h (d) 24 h
19. Priyanka, Akshay and Salman started out on a journey to watch the newly released movie "Mujhse Shaadi Karogi", which was being shown at wave cine-multiplex. The multiplex was 120 km away from their starting point of journey. Priyanka and Salman went by car at the speed of 50 km/h, while Akshay travelled by Tonga (horse cart) at 10 km/h. After a certain distance Salman got off and travelled the rest distance by another Tonga at 10 km/h, while Priyanka went back for Akshay and reached the destination at the same time that Salman arrived. The number of hours required for the trip was :
 (a) 4 h (b) 5 h
 (c) 4.8 h (d) can't be determined

Directions (for Q. Nos. 20 and 21) *Ajay and Kajol start towards each other at the same time from Barabanki and Fatehpur for their destinations Fatehpur and Barabanki respectively which are 300 km apart. They meet each other 120 km away from Barabanki.*

20. Shahrukh starts from Barabanki to Fatehpur, 1 hour after Ajay starts. Shahrukh meets Kajol 1.5 hours after Shahrukh starts. If the speed of Shahrukh is at least 20 km/h faster than the speed of Kajol.

Which of the following statements is true?

- (a) The minimum possible speed of Ajay is 45 km/h
 (b) The maximum possible speed of Ajay is 45 km/h
 (c) The minimum possible speed of Kajol is 60 km/h
 (d) The maximum possible speed of Kajol is 60 km/h
21. What is the minimum speed of Shahrukh to overtake Ajay, before he meets Kajol? (Use the data from previous question, if necessary)
- (a) 30 (b) 40
 (c) 60 (d) none of these

Directions (for Q. Nos. 22 to 24) *Raghupati goes at a speed of 60 km/h. Raghav goes at a speed of 36 km/h. Raja Ram can go from Azamgarh to Bareilly in 2 hours. The distance between Azamgarh to Bareilly is equal to the distance between Azamgarh to Chandoli. Raghav takes the same time travelling from Bareilly to Azamgarh as from Bareilly to Chandoli at his regular speed which is twice the speed of Raja Ram.*

22. What is the distance between Azamgarh and Chandoli?
- (a) 60 km (b) 27 km
 (c) 36 km (d) 18 km
23. How much time will Raghupati take to complete a round trip of the three cities?
- (a) 1 h 12 min (b) 1 h 48 min
 (c) 1 h 30 min (d) 1 h 36 min
24. If Raghupati and Raja Ram travel towards each other from Bareilly and Chandoli respectively, how far from Bareilly will they meet each other?
- (a) $\frac{60}{13}$ km (b) $27\frac{9}{13}$ km
 (c) $37\frac{9}{13}$ km (d) $\frac{360}{9}$ km

Directions (for Q. Nos. 25 and 26) *Mohan, Namit and Pranav travel from Shantipur to Hulchulpur. They have a two seater bike which can be driven by only Mohan. It is known that due to very stringent traffic rules only two persons can ride at a time. Hulchulpur is 180 km away from Shantipur. All of them can walk at 6 km/h, but reach to Hulchulpur simultaneously also they started their journey simultaneously.*

25. If the speed of the bike is 36 km/h, then what is the total distance that the bike travels?
- (a) 400 km
 (b) 380 km
 (c) 200 km
 (d) 320 km

26. If the speed of the bike is 42 km/h, then what is the shortest possible time in which all three of them can complete the journey?

- (a) $7\frac{1}{3}$ h (b) $9\frac{4}{7}$ h
 (c) $9\frac{3}{7}$ h (d) can't be determined

27. While walking down on the pavements of New York city. I notice that every 20 minute there is a city bus coming in the opposite direction and every 30 minute there is a city bus overtaking me from behind. What is the time gap between one city bus passing a stationary point known as Local Bus Stop beside the route and the immediately next city bus in the same direction passing the same stationary point?

- (a) 27 min (b) 24 min
 (c) 25 min (d) can't be determined

28. Abhinav and Brijesh start from Allahabad and Barabanki respectively with uniform velocities. Abhinav is headed towards Barabanki and Brijesh towards Allahabad and both cities are 600 km apart. Abhinav rests whenever Brijesh is on the move and Brijesh rests whenever Abhinav is on the move. Abhinav's speed is 25 km/h and Brijesh's speed is 30 km/h. If Abhinav starts first and reaches Barabanki in 36 hours, then find the least time that Brijesh would take to reach his destination after Abhinav makes a start

- (a) 20 h (b) 36 h
 (c) 44 h (d) none of these

29. A man can cross a downstream river by steamer in 40 minutes and same by boat in 1 hour. If the time of crossing the river in upstream direction by steamer is 50% more than downstream time by the steamer and the time required by boat to cross the same river by boat in upstream is 50% more than the time required in downstream by boat. What is the time taken for the man to cross the river downstream by steamer and then return to same place by boat half the way and by steamer the rest of the way?

- (a) 85 min (b) 115 min (c) 120 min (d) 125 min

Directions (for Q. Nos. 30 and 31) *Awadh express and Bokaro express start simultaneously from Lucknow and Jamshedpur towards each other and continuously shuttle between these two places. Every time these trains meet each other, they turn back after exchanging their respective speeds, the initial ratio of their speeds is 2 : 1.*

30. What is the number of distinct places at which they will meet?
- (a) 1 (b) 2
 (c) 5 (d) none of these
31. Let these two trains first time meet at Patna, then what is the ratio of distances covered by Awadh express and Bokaro express till they meet for the third time at the same place Patna :
- (a) 1 : 1 (b) 14 : 13
 (c) 10 : 11 (d) none of these

32. Mahindra starts a journey for his office, which is in the north east of his home. An hour after starting meets with a minor accident. He takes one hour in resuming his journey. After that he proceeds at $\frac{5}{6}$ th of his former speed and arrives at the office 1 hour 36 minutes late than the scheduled time. Had the accident occurred 80 kms further from the actual place of accident, he would have arrived 1 hour 20 minutes beyond the scheduled time. What is the distance between his office and his home?
 (a) 180 km (b) 240 km
 (c) 250 km (d) 300 km
33. A soldier fired two bullets at an interval of 335 seconds moving at a uniform speed v_1 . A terrorist who was running ahead of the soldier in the same direction, hears the two shots at an interval of 330 seconds? If the speed of sound is 1188 km/h, then who is the faster and by how much?
 (a) Soldier, 22 km/h (b) Terrorist, 25 km/h
 (c) Soldier, 18 km/h (d) Terrorist, 20 km/h
34. A hunter fired two shots from the branch of a tree at an interval of 76 seconds. A tiger separating too fast hears the two shots at an interval of 83 seconds. If the velocity of the sound is 1195.2 km/h, then find the speed of tiger?
 (a) 112.8 km/h (b) 100.8 km/h
 (c) 80.16 km/h (d) none of these
35. A man goes to the fair in Funcity with his son and faithful dog. Unfortunately man misses his son which he realises 20 minutes later. The son comes back towards his home at the speed of 20 m/min and man follows him at 40 m/min. The dog runs to the son (child) and comes back to the man (father) to show him the direction of his son. It keeps moving to and fro at 60 m/min between son and father, till the man meets the son. What is the distance travelled by the dog in the direction of the son?
 (a) 800 m (b) 1675 m
 (c) 848 m (d) 1000 m
36. Amarnath express left Amritsar for Gorakhpur. Two hours later Gorakhnath express left from Amritsar to Gorakhpur. Both trains reached Gorakhpur simultaneously. If Amarnath express had started from Amritsar and Gorakhnath express had started from Gorakhpur at the same time and travelled towards each other they would meet in 1 h 20 min. Find the time taken by Amarnath express to travel from Amritsar to Gorakhpur (in hours) :
 (a) 2 (b) 4 (c) 5 (d) 6
37. Akbar and Birbal set out at the same time to walk towards each other respectively from Agra and Banaras 144 km apart. Akbar walks at the constant speed of 8 km/h, while Birbal walks 4 km in the first hour, 5 km in the second hour, 6 km in the third hour and so on. Then the Akbar and Birbal will meet :
 (a) in 6 h
 (b) in 8 h
 (c) midway between Agra and Banaras
 (d) 80 km away from Banaras
38. A tiger is 50 of its own leaps behind a deer. The tiger takes 5 leaps per minute to the deer's 4. If the tiger and the deer cover 8 m and 5 m per leap respectively, what distance will the tiger have to run before it catches the deer?
 (a) 600 m (b) 700 m
 (c) 800 m (d) 1000 m
39. Soniya and Priyanka started from Amethi and Bellari for Bellari and Amethi, which are 645 km apart. They meet after 15 hours. After their meeting, Sonia increased her speed by 3 km/h and Priyanka reduced her speed by 3 km/h, they arrived at Bellari and Amethi respectively at the same time. What is their initial speeds?
 (a) 24 km/h and 30 km/h
 (b) 25 km/h and 18 km/h
 (c) 18 km/h and 21 km/h
 (d) 20 km/h and 23 km/h
40. Den Bosch and Eastbourne are two famous cities 300 km apart. Maradona starts from Den Bosch at 8 : 24 am. An hour later Pele starts from Den Bosch. After travelling for 1 hour, Pele reaches Nottingham that Maradona had passed 40 minutes earlier. Nottingham falls on the way from Den Bosch to Eastbourne. If Pele and Maradona just reaches Eastbourne at the same time, what are the speeds of the Maradona and Pele respectively?
 (a) 100 km/h, 125 km/h (b) 60 km/h, 80 km/h
 (c) 60 km/h, 75 km/h (d) 75 km/h, 100 km/h
41. A thief sees a jeep at a distance of 250 m, coming towards him at 36 km/h. Thief takes 5 seconds to realise that there is nothing but the police is approaching him by the jeep and start running away from police at 54 km/h. But police realise after 10 seconds, when the thief starts running away, that he is actually a thief and gives chase at 72 km/h. How long after thief saw police did police catchup with him and what is the distance police had to travel to do so?
 (a) 50 s, 1000 m (b) 65 s, 1150 m
 (c) 65 s, 1300 m (d) 45 s, 1050 m
42. Inspired by the 'Golden quadrilateral project' UP Government recently accomplished a diamond triangular project. Under this project the State Government laid down 6 lane roads connecting three cities Ayodhya, Banaras and Chitrakoot, which are equally separated from each other i. e., in terms of geometry they form an equilateral triangle. Angad and Bajrang start simultaneously from Ayodhya and Banaras respectively, towards Chitrakoot. When Angad covers 100 kms, Bajrang covers such a distance that the distance between Angad and Bajrang makes 90° angle with the road joining Banaras and Chitrakoot. When Bajrang reaches Chitrakoot, Angad is still 150 km away from Chitrakoot. What is the distance between Ayodhya and Banaras?
 (a) 250 km
 (b) 450 km
 (c) 300 km
 (d) none of the above

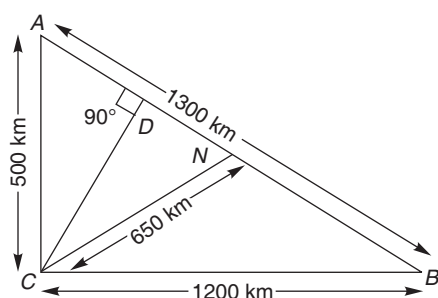
43. Two trains Ajanta express and Barouni express simultaneously started on two parallel tracks from Meerut to Nagpur, which are 390 km apart. The ratio of the speed of Ajanta express and Barouni express is 6 : 7. After how long (in kms) travelling, Barouni express exchanges the speed with Ajanta express so that both the trains reach at their destination simultaneously :

(a) 150 km (b) 190 km
(c) 210 km (d) can't be determined

44. In a circus there was a leopard and a tiger walking in the two different rings of same radii. There I observed that when leopard moved 3 steps, tiger moved 5 steps in the same time, but the distance traversed by leopard in 5 steps is equal to the distance traversed by tiger in 4 steps. What is the number of rounds that a leopard made when tiger completed 100 rounds?

(a) 120 (b) 48
(c) 75 (d) none of these

Directions (for Q. Nos. 45 to 48) In the following figure the route is shown which is followed by Professor Jai and Professor Jaya, who are visiting faculty at IIM-A and IIM-B respectively. A, B denote IIM-A and IIM-B respectively and C denotes the residence of Prof. Jai and Prof. Jaya. They leave home for classes at the same time and their driving speeds are $\frac{500}{13}$ km/h and $\frac{1200}{13}$ km/h respectively. Also they finish the classes at the same time to reach home.



The path adopted by Jai and Jaya is CADC and CBDC respectively. Prof. Jai and Prof. Jaya are husband and wife respectively.

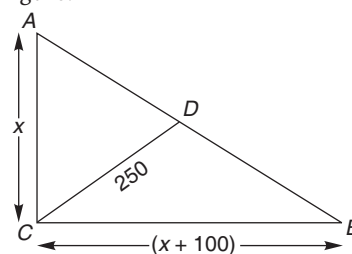
45. If both of them start and finish the classes at the same time, then who returned home earlier than other, if no one of them halts for anywhere in the route and they just leave the institution as soon as they finish the lectures?
(a) Prof. Jai (b) Prof. Jaya
(c) Return at the same time (d) can't be determined
46. In the shown figure N and D denotes Noida and Delhi respectively, who returned home late and by how much time, if Jaya turned from Noida instead of Delhi :
(a) Jai, 9 h 10 min
(b) Jaya, 9 h 50 min
(c) Jai, 2 h 55 min
(d) Jai, 16 h 10 min

47. In the above question how many per cent time Jaya saved in going via Noida of the total time taken previously :
(a) 10% (b) 25% (c) 50% (d) 17%

48. If Mrs. Jaya wants to watch the premier show of a movie at Wave Cinema in Noida while returning from institute through BNC. When will she return home given that she spends total time 3 hours at wave cinema?

(a) at the same time as normal
(b) 5 min late than her husband
(c) at the same time when her husband returns
(d) can't be determined

49. Preetam and Devi start running a race on the given track as shown in figure.



Where AC and BC are mutually perpendicular and CD is the median of triangular paths ABC. BC is 100 km longer than that of AC, again CD is 250 km. The speeds of Preetam and Devi are 30 km/h and 40 km/h, initially and their respective paths of running are CADC and CBDC. After how much time they reverse their speeds, so that they return C at the same time?

(a) $\frac{50}{7}$ h (b) $\frac{120}{7}$ h
(c) $\frac{80}{11}$ h (d) none of these

Directions (for Q. Nos. 50 to 52) The markets $M_1, M_2, M_3, \dots$ etc of Vyapaar city are lying besides the circular paths $P_1, P_2, P_3, \dots$ etc. All the circular paths are concentric at centre 'O' and their distances are 1 km, 2 km, 3 km, ... etc from the centre 'O' respectively. At the centre 'O' there is a 'Khoob Khao' tiffin agency which supplies the tiffins to all the markets $M_1, M_2, M_3, \dots$ etc. A tiffin carrier starts from 'O' goes directly east of the shop and then on reaching the circular path it moved 1 km in counter clock direction on it. After completing its 1 km distance on P_1 the carrier moves to P_2 in the radial direction. Then it goes 2 km on P_2 . Similarly 3 km on P_3 and 4 km on P_4 etc in counter clock direction moving radially from P_2 to P_3 and P_3 to P_4 etc and motion of the carrier continued in this manner till it reaches exactly in the east direction.

50. After reaching east of the shop it can't move on further than the given distance on the current path. For how many markets can it supply its tiffins directly?
(a) 4 (b) 5
(c) 7 (d) can't be determined
51. The total distance covered by the carriers in providing the tiffins from centre 'O' to the last point in one way only is :
(a) 30 km (b) 28 km
(c) 35 km (d) none of these

52. The ratio of distances covered on the circular path P_2 to that on the last path, where the carrier reaches directly eastward of its shop is :

(a) 1 : 1 (b) 2 : 7
(c) 2 : π (d) none of these

Directions (for Q. Nos. 53 to 57) A train enters into a tunnel AB at A and exits at B . A jackal is sitting at O in another by passing tunnel AOB , which is connected to AB at A and B , where OA is perpendicular to OB . A cat is sitting at P inside the tunnel AB making the shortest possible distance between O and P , such that $AO : PB = 30 : 32$. When a train before entering into the tunnel AB makes a whistle (or siren) somewhere before A , the jackal and cat run towards A , they meet with accident (with the train) at the entrance A . Further if the cat moves towards B instead of A it again meets with accident at the exit of the tunnel by the same train coming from the same direction.

53. What is the ratio of speeds of jackal and cat?

(a) 4 : 3 (b) 5 : 3
(c) 1 : 1 (d) can't be determined

54. The ratio of speeds of jackal is to train is :

(a) 5 : 1 (b) 3 : 5
(c) 1 : 5 (d) can't be determined

55. If jackal moves towards OPA , it will meet with train at M_1 then AM_1 is :

(a) 20 km (b) 16 km
(c) 10 km (d) can't be determined

56. If jackal moves towards OPB and cat moves towards POA who will not meet with accident with the train?

(a) Jackal (b) Cat
(c) Both (a) and (b) (d) can't be determined

57. The ratio of time taken by cat and jackal in moving $OAPO$ and $PBOP$ respectively given that they do not meet with accident :

(a) 1 : 1 (b) 3 : 4
(c) 5 : 4 (d) none of these

58. A candle of 6 cm long burns at the rate of 5 cm in 5 h and another candle of 8 cm long burns at the rate of 6 cm in 4 h. What is the time required by each candle to remain of equal lengths after burning for some hours, when they start to burn simultaneously with uniform rate of burning?

(a) 1 h (b) 1.5 h
(c) 2 h (d) none of these

59. Two boats start at the same instant to cross a river W metre wide. The faster boat reaches the other bank and returns back immediately. What are the distances travelled by them when they meet, where the speeds of these boats are b_1 & b_2 ?

(a) $\frac{2W}{(b_1 + b_2)}$, $\frac{2W}{(b_1 - b_2)}$
(b) $\frac{2W}{(b_1 + b_2)}$ b_1 and $\frac{2W}{(b_1 + b_2)}$ b_2
(c) $\frac{W}{(b_1 + b_2)}$ b_1 , $\frac{W}{(b_1 + b_2)}$ b_2
(d) data insufficient

60. Mariya was travelling in her boat when the wind blew her hat off and the hat started floating back downstream. The boat continued to travel upstream for twelve more minutes before Mariya realized that her hat had fallen off and turned back downstream. She caught up with that as soon as it reached the starting point. Find the speed of river if Mariya's hat flew off exactly 3 km from where she started :

(a) 5 km/h
(b) 6 km/h
(c) 7.5 km/h
(d) can't be determined

61. Akbar, Birbal and Chanakya run around a circular track of length 500 m. Akbar and Birbal run with the speeds of 15 m/s and 20 m/s in the same direction respectively and Chanakya being very intelligent run in the opposite direction with a speed of 25 m/s. If all three of them start at the same time, then :

(a) Akbar meets Chanakya more frequently than Birbal does
(b) Akbar and Chanakya meets as frequently as Birbal and Chanakya
(c) Akbar meets Birbal least frequently
(d) Nothing can be concluded

62. Arun and Barun run with the speeds of 30 m/s and 20 m/s around a circular track of 600 m. They participate in a 3000 m race. What is the distance covered by Arun when he passes Barun for the 5th time?

(a) 2200 m
(b) 2250 m
(c) 2850 m
(d) none of the above

63. Akkal and Bakkal are running on a circular track of radius 175 metres. Akkal can complete a round in 100 seconds and the speed of Bakkal is twice the speed of Akkal. They started simultaneously towards each other from two points 350 metres diametrically opposite on the circular path. If they first meet at a point they called it love point, which is between the two points P and Q from where they have started their race, after how much time from the start do they meet at love point for the third time?

(a) $218\frac{2}{5}$ s (b) $216\frac{2}{3}$ s
(c) 221 s (d) none of these

64. Arti and Barkha start swimming towards each other from the deep end and shallow end respectively of a swimming pool in Funcity. They start their swimming simultaneously in the length of 300 m pool. The ratio of their speeds is 1 : 2 respectively. Each swimmer rests for 6 seconds once she reaches the other end and starts swimming back. Where will they meet for the second time in the still water of swimming pool?

(a) 30 m from the shallow end
(b) at the shallow end
(c) at the deep end
(d) can't be determined

65. A and B runs around a circular track. A beats B by one round or 10 minutes. In this race, they had completed 4 rounds. If the race was only of one round, find the A's time over the course :
- (a) 8 min (b) 7.5 min
(c) 12.5 min (d) 12 min
66. A, B and C participated in a race. A covers the same distance in 49 steps, as B covers in 50 steps and C in 51 steps. A takes 10 steps in the same time as B takes 9 steps and C takes 8 steps. Who is the winner of the race?
- (a) A (b) B
(c) C (d) can't be determined
67. Shambhu drives his car very fast at 360 m/s. Moving ahead for some hours he finds some problem in headlights of the car. So he takes 20 seconds in changing the bulb of the head- light by stopping the car. Mean while he notices that another car which was 400 m back is now 200 m ahead of his car. What is the speed of this car?
- (a) 100 km/h (b) 92 km/h
(c) 108 km/h (d) 300 km/h
68. Two persons start from the opposite ends of a 90 km straight track and run to and fro between the two ends. The speed of first person is 30 m/s and the speed of other is $125/6$ m/s. They continue their motion for 10 hours. How many times they pass each other?
- (a) 10 (b) 9
(c) 12 (d) none of these
69. At what time after 3 : 10 am, the acute angle made by the minute and hour-hand is double to that of at 3 : 10 am, for the first time?
- (a) 4 h 43 min (b) 3 h 48 min
(c) $3\text{ h } \frac{320}{11}\text{ min}$ (d) none of these
70. If the two incorrect watches are set at 12 : 00 noon at correct time, when will both the watches show the correct time for the first time given that the first watch gains 1 min in 1 hour and second watch loses 4 min in 2 hours :
- (a) 6 pm, 25 days later
(b) 12 : 00 noon, 30 days later
(c) 12 noon, 15 days later
(d) 6 am 45 days later
71. Rajeev and Sanjeev are too close friends Rajeev's watch gains 1 minute in an hour and Sanjeev's watch loses 2 minutes in an hour. Once they set both the watches at 12 : 00 noon, with my correct watch. When will the two incorrect watches of Rajeev and Sanjeev show the same time together?
- (a) 8 days later (b) 10 days later
(c) 6 days later (d) can't be determined
72. At a railway station a 24 hour watch loses 3 minutes in 4 hours. If it is set correctly on Sunday noon when will the watch show the correct time?
- (a) 6 pm after 40 days (b) 12 noon after 75 days
(c) 12 pm after 100 days (d) 12 noon after 80 days
73. A swiss watch is being shown in a museum which has a very peculiar property. It gains as much in the day as it loses during night between 8 pm to 8 am. In a week how many times will the clock show the correct time?
- (a) 6 times (b) 14 times
(c) 7 times (d) 8 times
74. A wrist watch which is running 12 minutes late on a Sunday noon is 16 minutes ahead of the correct time at 12 noon on the next Sunday. When is the clock 8 minutes ahead of time?
- (a) Thursday 10 am
(b) Friday noon
(c) Friday 8 pm
(d) Tuesday noon
75. A clock loses 2 minutes in an hour and another clock gains 2 minutes in every 2 hours. Both these clocks are set correctly at a certain time on Sunday and both the clocks stop simultaneously on the next day with the time shown being 9 am and 10 : 06 am. What is the correct time at which they stopped?
- (a) 9 : 54 am (b) 9 : 44 pm
(c) 9 : 46 am (d) 9 : 44 am
76. David sets his watch at 6 : 10 am on Sunday, which gains 12 minutes in a day. On Wednesday if this watch is showing 2 : 50 pm. What is the correct time?
- (a) 1 : 50 pm (b) 2 : 10 pm
(c) 2 : 30 pm (d) 3 : 30 pm
77. Ramu purchased a second hand Swiss watch which is very costly. In this watch the minute-hand and hour hand coincide after every $65\frac{3}{11}$ minutes. How much time does the watch lose or gain per day?
- (a) 4 min (b) 5 min
(c) 4 min, 20 sec (d) none of these
78. My watch was 8 minutes behind at 8 pm on Sunday but within a week at 8 pm on Wednesday it was 7 minutes ahead of time. During this period at which time this watch has shown the correct time :
- (a) Tuesday 10 : 24 am
(b) Wednesday 9 : 16 pm
(c) It cannot show the correct time during this period
(d) None of the above
79. Out of the following four choices which does not show the coinciding of the hour hand and minute-hand :
- (a) 3 : 16 : 2 (b) 6 : 32 : 43
(c) 9 : 59 : 05 (d) 5 : 27 : 16
80. Kumbhakarna starts sleeping between 1 am and 2 am and he wakes up when his watch shows such a time that the two hands (i.e., hour-hand and minute-hand) interchange the respective places. He wakes up between 2 am and 3 am on the same night. How long does he sleep?
- (a) $55\frac{5}{13}$ min (b) $110\frac{10}{13}$ min
(c) $54\frac{6}{13}$ min (d) none of these

81. Kalmadi, Raja and Sharad must cross a river to visit their friend Kwatrochi who lives on the other side. They have a boat; but they also have a problem. Kalmadi and Raja each weigh 50 kg and Sharad weighs 75 kg. The boat will carry only 100 kg at a time. Raja moans on the inability that they all can't get across the river in the same boat due to excessive weight which this boat can't carry. The boat takes 90 minutes to get across from one end to another in each direction. What is the minimum time required to transport all of them, provided boat stops at any end for a negligible time?
- (a) 3 hrs (b) 6 hrs
(c) 7 hrs 30 min (d) 4 hrs 30 min
82. A traveler, named as Hamrahi, spends 6 hrs. in walking along a level road, up a hill, and back home again. His pace on the level being 4 mph, up-hill 3 mph, and down-hill 6 mph. Find the distance he walked along.
- (a) 8 miles (b) 12 miles
(c) 36 miles (d) 24 miles
83. Rohingya sneaks into an unfamiliar territory by walking over a rail-bridge. At a moment when he is just 16 meters before reaching the middle of the bridge, he notices that a train is coming from behind. At that moment, the train, which travels at a speed of 120 km/h, is exactly as far away from the bridge as the bridge measures in length. Without wasting a moment, Rohingya rushes straight towards the train to get off the bridge. Thereby, he misses the train by just 20 meters. If Rohingya had rushed in the other direction as fast as he had rushed towards the train, the train would have hit him 10 meters before the end of the bridge. What is the length of the rail-bridge?
- (a) 60 meters (b) 56 meters
(c) 96 meters (d) data insufficient
84. On a fateful night some drunken youngsters molested a girl in a moving bus and threw her out devastated on an aloof road. Soon she called the police control room (PCR) on 112. At the same time she asked an auto rickshaw that was coming behind the bus to rush her to the hospital, but he left her pleading. The incident occurred on the road that goes straight to the hospital, even the control room lies somewhere between these two places on the same road. A PCR van rushed to the spot from the control room that reached her in 45 minutes after getting her distress call. The PCR van took her to the hospital as quickly as possible. Upon reaching the hospital doctors pronounced her dead. They said that if she would have arrived at least 30 minutes earlier she would have been saved, for sure. In that response a cop from the PCR van who took her to the hospital said that if the auto-rickshaw would have helped her we could have picked her on the way from the auto-rickshaw and thus we could have reached timely to the hospital. If the speed of PCR van was 80 km/hr, what's the minimum speed of the auto-rickshaw that the cop assumes to be enough to save her life?
- (a) 60
(b) 20
(c) 40
(d) None of these
85. Faizal Khan and Danish Khan, the two sons of Sardar Khan, were supposed to kill their archrivals. Ramadhir Singh and his son JP Singh. Initially, Faizal Khan was in Wasseyypur while Danish Khan was waiting to kill Ramadhir Singh and his son in Dhanbad. But, when Sardar Khan got to know that JP Singh was seen in his factory at Ranchi, so he changed the plan and told his sons, on the phone, to move as he wanted both his rivals get killed at the same time. Faizal and Danish left simultaneously for Dhanbad and Ranchi, respectively. While Danish was somewhere on the way to Ranchi, Faizal overtook him and handed over some bullets and bombs. He immediately returned for Dhanbad and reached at the same time when Danish reached Ranchi. Dhanbad is equidistant from Wasseyypur and Ranchi and all the three cities are connected through a single straight road. If the distance between Wasseyypur and Ranchi is 180 km, what's the distance traveled by Faizal?
- (a) $90(\sqrt{2} + 2)$
(b) $90(2\sqrt{2} - 1)$
(c) $90(\sqrt{2} + 1)$
(d) data insufficient

SPEED TEST > TSD

Directions (for Q. Nos. 1 to 3) *Arun took part in a triathlon, an athletic event. He had to swim, run and bicycle to 10 km, 24 km and 30 km, respectively and return the same way. Arun's average speed for the triathlon is 4 km/h. He took a total of 4 min for swimming and 20 min for bicycling in the triathlon.*

- His speed of running is:
 (a) 5 km/min (b) 6 km/min
 (c) 7.5 km/min (d) $\frac{8}{3}$ km/min
- He finishes race in:
 (a) 16 min (b) 23 min
 (c) 32 min (d) 35 min
- How much time he took while returning for the bicycle if the time taken for the return of each phase (i.e., each event) is 50% greater than that of taken initially:
 (a) 12 min (b) 11 min
 (c) $11\frac{1}{9}$ min (d) none of these

Directions (for Q. Nos. 4 and 5) *A, B and C start running a race from the same point P in the same direction. A runs around a path which is an equilateral triangle PQX and B runs around a square path PQYZ and C runs on the regular hexagonal path PQRSTU. Where the one side PQ of each path is common.*

- If all of them complete one equal round at the same time then which of the following is true?
 (a) Speed of C is twice that of B
 (b) Speed of A is half that of C
 (c) Speed of B is 50% more than that of A
 (d) none of the above
- If each of them run at a same constant speed, what is the maximum number of rounds anyone of them would have completed when they meet for the first time?
 (a) 1 (b) 2 (c) 3 (d) 4

Directions (for Q. Nos. 6 to 9) *City X and City Y are connected to a straight road. A and B start moving simultaneously towards each other. After travelling some distance, B takes a 60° turn to his left. Two hours later after B turns, A takes a 60° turn to his right. A travels 60 km after turning, before he meets B. A and B meet 10 hours after they start their journey. A and B together travel 200 km before turning and they arrive at the meeting point simultaneously.*

- How many hours after turning does A meet B ?
 (a) 1 (b) 3 (c) 4 (d) 5
- What distance does A travel before turning 60 degree to his right?
 (a) 140 km (b) 150 km
 (c) 160 km (d) 170 km
- If A and B had not turned, after how many hours would they have meet?
 (a) $7\frac{1}{8}$ h (b) $9\frac{1}{8}$ h (c) $6\frac{1}{8}$ h (d) $8\frac{1}{8}$ h
- What is B's speed?
 (a) 10 km/h (b) 12.5 km/h
 (c) 12 km/h (d) 15 km/h
- Chetak and Ashwa, two horses, start galloping from Patna to Ranchi which are 80 km apart. The speed of Chetak is 160 km/h and that of Ashwa is 150 km/h. They start galloping simultaneously, from Patna to Ranchi. Chetak reached to Ranchi and returned to Patna but Ashwa returned from Jamshedpur (which is somewhere between Patna and Ranchi) to Patna at the same time. What is the ratio of distances between Patna and Jamshedpur and Jamshedpur and Ranchi?
 (a) 15 : 1 (b) 3 : 25
 (c) 15 : 2 (d) none of these

Answers

✓ *Introductory Exercise 9.1*

1 (a)	2. (c)	3. (c)

✓ *Level 01 Basic Level Exercise*

1 (a)	2. (b)	3. (b)	4. (c)	5. (a)	6. (c)	7. (c)	8. (b)	9. (a)	10. (c)
11. (c)	12. (d)	13. (a)	14. (d)	15. (b)	16. (c)	17. (b)	18. (c)	19. (b)	20. (c)
21. (b)	22. (d)	23. (a)	24. (b)	25. (c)	26. (d)	27. (d)	28. (d)	29. (b)	30. (c)
31. (d)	32. (a)	33. (b)	34. (c)	35. (d)	36. (a)	37. (b)	38. (a)	39. (b)	40. (c)
41. (a)	42. (b)	43. (b)	44. (c)	45. (c)	46. (c)	47. (c)	48. (b)	49. (a)	50. (b)
51. (b)	52. (a)	53. (b)	54. (c)	55. (c)	56. (c)	57. (a)	58. (b)	59. (d)	60. (b)
61. (a)	62. (c)	63. (c)	64. (c)	65. (b)	66. (d)	67. (d)	68. (c)	69. (d)	70. (a)
71. (a)	72. (a)	73. (b)	74. (c)	75. (b)	76. (c)	77. (b)	78. (c)	79. (a)	80. (c)
81. (d)	82. (b)	83. (c)	84. (c)	85. (b)	86. (d)	87. (b)	88. (b)	89. (d)	90. (c)
91. (c)	92. (b)	93. (d)	94. (d)	95. (c)	96. (b)	97. (a)	98. (b)	99. (b)	100. (a)
101. (d)	102. (a)	103. (c)	104. (b)	105. (b)	106. (c)	107. (d)	108. (a)	109. (b)	110. (c)
111. (b)	112. (d)	113. (c)	114. (d)	115. (a)	116. (b)	117. (d)	118. (b)	119. (b)	120. (b)
121. (b)	122. (b)	123. (c)	124. (b)	125. (b)	126. (c)	127. (d)	128. (b)	129. (a)	130. (a)
131. (d)	132. (b)	133. (c)	134. (b)	135. (b)	136. (b)	137. (b)	138. (b)	139. (a)	140. (c)
141. (c)	142. (d)	143. (d)	144. (d)	145. (b)	146. (a)	147. (b)	148. (d)	149. (a)	150. (c)

✓ *Level 02 Higher Level Exercise*

1 (b)	2. (c)	3. (b)	4. (a)	5. (b)	6. (c)	7. (c)	8. (c)	9. (b)	10. (a)
11. (d)	12. (d)	13. (c)	14. (c)	15. (a)	16. (b)	17. (c)	18. (d)	19. (c)	20. (b)
21. (d)	22. (c)	23. (b)	24. (b)	25. (b)	26. (c)	27. (b)	28. (c)	29. (b)	30. (b)
31. (c)	32. (b)	33. (c)	34. (b)	35. (d)	36. (b)	37. (c)	38. (c)	39. (d)	40. (d)
41. (b)	42. (c)	43. (c)	44. (b)	45. (c)	46. (c)	47. (d)	48. (b)	49. (b)	50. (c)
51. (a)	52. (a)	53. (b)	54. (c)	55. (c)	56. (b)	57. (c)	58. (d)	59. (b)	60. (c)
61. (c)	62. (d)	63. (b)	64. (b)	65. (b)	66. (a)	67. (c)	68. (c)	69. (c)	70. (b)
71. (b)	72. (d)	73. (d)	74. (b)	75. (d)	76. (b)	77. (a)	78. (a)	79. (c)	80. (a)
81. (c)	82. (d)	83. (b)	84. (c)	85. (c)					

✓ *Speed Test TSD*

1 (b)	2. (c)	3. (b)	4. (b)	5. (d)	6. (b)	7. (a)	8. (d)	9. (c)	10. (a)

Hints & Solutions

✓ Level 01 Basic Level Exercise

$$1. \text{ Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{10 + 20}{\frac{10}{6} + \frac{20}{15}}$$

$$= \frac{30}{\left(\frac{90}{30}\right)} = \frac{30}{3} = 10 \text{ km/h}$$

$$2. \text{ Average speed (when distances are same)} = \frac{2uv}{u+v}$$

$$= \frac{2 \times 20 \times 30}{(20 + 30)} = 24 \text{ km/h}$$

where u, v are the different speeds.
(or use the general formula—Total distance/Total time)

$$3. \text{ Average speed} = \frac{3uvw}{uv + vw + uw}$$

$$= \frac{3 \times 10 \times 20 \times 30}{200 + 600 + 300} = \frac{18000}{1100}$$

$$= 16 \frac{4}{11} \text{ km/h}$$

☞ **Alternatively** Suppose the total distance 3 times the LCM of the given speeds, then solve by general formula.

$$\text{Total distance} = 180 \text{ km (say)}$$

$$\text{Then Total time} = \frac{60}{10} + \frac{60}{20} + \frac{60}{30}$$

$$= 6 + 3 + 2 = 11 \text{ h}$$

$$\therefore \text{ Average speed} = \frac{180}{11} = 16 \frac{4}{11} \text{ km/h}$$

☞ **Alternatively** Suppose the total distance equals to $3x$ km then solve as above.

4. Suppose the total distance equals to 4 times the LCM of the speeds.

$$\therefore \text{ Total distance} = 2400 \text{ km}$$

$$\therefore \text{ Total time} = \frac{600}{20} + \frac{600}{25} + \frac{600}{30} + \frac{600}{40}$$

$$= 30 + 24 + 20 + 15 = 89 \text{ h}$$

$$\therefore \text{ Average speed} = \frac{2400}{89} = 26 \frac{86}{89} \text{ km/h}$$

$$5. \frac{2 \times 20 \times x}{(20 + x)} = 24$$

$$\Rightarrow x = 30 \text{ km/h}$$

☞ **Alternatively** Go through options.

$$6. \text{ Average speed} = \frac{2uv}{u+v}$$

$$= \frac{2 \times 3x \times 5x}{(3x + 5x)} = 3.75x \text{ km/h}$$

$$7. u = k, v = 3k$$

$$\therefore \frac{2uv}{u+v} \Rightarrow \frac{2 \times k \times 3k}{(k + 3k)} = 12$$

$$\Rightarrow 1.5k = 12$$

$$\therefore k = 8 \text{ km/h}$$

☞ **Alternatively** Go through options.

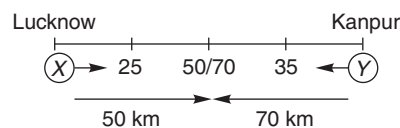
☞ **Alternatively** Solve through alligations.

$$8. \text{ Effective speed} = 25 + 35 = 60 \text{ km/h}$$

Total distance to be covered = 120 km

$$\therefore \text{ Time taken} = \frac{120}{60} = 2 \text{ h}$$

Hint Since in each hour X and Y together covers 60 km.

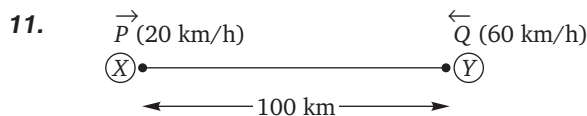


9. Since they take 2 hours to meet each other. Thus in 2 h X is 50 km away from Lucknow.

☞ **Alternatively** Both X and Y will cover the respective distances in the ratio of their speeds.

$$\text{So, distance from Lucknow} = \frac{5}{12} \times 120 = 50 \text{ km}$$

10. When time is constant the distances covered by A and B will be in the ratio of their speeds, respectively.



To meet each other they have to cover 100 km distance together and the ratio of distances covered by each one is directly proportional to the ratio of their speeds, respectively. Since the time taken by each one is same (i. e., constant).

$$\text{Hence, } \frac{XZ}{YZ} = \frac{20}{60} = \frac{1}{3}$$

$$\therefore XZ = \frac{1}{4} \times 100 = 25 \text{ km}$$

12. To meet the second time they have to cover 300 km distance together [for n^{th} time distance = $(2n - 1)d$]
Time taken by them to meet each other, for the second time

$$= \frac{300}{80} = 3 \frac{3}{4} \text{ h}$$

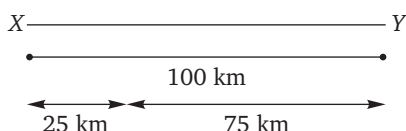
$$\text{Distance covered by } P = 20 \times 3 \frac{3}{4} = 75 \text{ km}$$

☞ Alternatively The ratio of distance covered
= Ratio of their speeds
 $\therefore$ Distance covered by $P = \frac{1}{4} \times 300 = 75$ km

13. The ratio of distances covered by each of them
= Ratio of their respective speeds
 $\therefore$ Distance covered by Q for the third meeting = $\frac{3}{4} \times 500$
= 375 km

14. Consider the distance travelled by any one of them, then find the distance between X and M , where they meet.
 $\therefore$ The distance travelled by P for the fourth meeting
= $\frac{1}{4} \times 700 = 175$ km

Therefore P will be 75 km from Y . It means P will be 25 km away from X .



15. This is constant for any number of meeting and is equal to the ratio of their speeds. Hence, 1 : 3.
16. It is always twice the length of race course. Hence, between any two consecutive meeting they have to cover total 200 km distance to meet each other for the next meeting.
Hint For the first meeting they will cover 100 km.
For the second meeting they will cover 300 km.
For the third meeting they will cover 500 km.
For the fourth meeting they will cover 700 km and so on.
17. To meet each other they will take equal time since they start their journey simultaneously.
18. To reach their destination the time taken by A and B is equal to the ratio of reciprocal to their speeds. Since when distance is constant time is inversely proportional to the respective speeds.

Hence, time taken by A and $B = \frac{1}{16} : \frac{1}{25} = 25 : 16$

19. $\frac{A's \text{ speed}}{B's \text{ speed}} = \frac{\sqrt{\text{Time taken by } B (t_B)}}{\sqrt{\text{Time taken by } A (t_A)}} \Rightarrow \frac{16}{25} = \sqrt{\frac{t_B}{t_A}}$
 $\Rightarrow \frac{t_B}{t_A} = \frac{256}{625} \Rightarrow \frac{t_A}{t_B} = \frac{625}{256}$

20.
(Kamal) 40 km/h (Vimal) 30 km/h

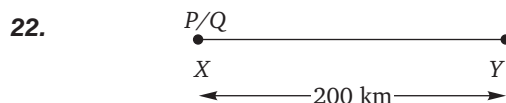
In the first 3 hours Vimal covers 90 km.
So, the rest distance = 560 km
Now, Kamal and Vimal both travels together, towards each other.

So, the time = $\frac{\text{Distance}}{\text{Speed}} = \frac{560}{70} = 8$ h

Thus, Vimal travels total = 3 + 8 = 11 h

Thus, the distance travelled by Vimal = 11 $\times$ 30 = 330 km

21. The distance covered by Kamal = 8 $\times$ 40 = 320 km



To meet the first time they have to cover
200 $\times$ 2 = 400 km

Now, the time taken to meet each other = $\frac{400}{50} = 8$ h

23. Distance travelled by $P = 20 \times 8 = 160$ km

24. Distance travelled by $Q = 30 \times \frac{800}{50} = 480$ km

25. Distance travelled by $P = 20 \times \frac{1200}{50} = 480$ km

So, P will be 80 km away from X . Therefore ratio of distances

$$= \frac{80}{120} = \frac{2}{3}$$

26. The distance travelled by $P = \frac{1600}{50} \times 20 = 640$ km

Thus, P will be 160 km away from X .

[Since 640 = (200 $\times$ 2) + 240]

$\therefore$ Distance between D and X is 160 km.

27. This is possible only when the distance covered by P in n round be the multiple of 400.

28. X started at 12 : 00 noon

Y started at 3 pm

$\therefore$ Required time = $\frac{\text{Distance advanced in 3 h}}{\text{Relative speed}}$

$$= \frac{40 \times 3}{20} = 6 \text{ h} \quad (20 = 60 - 40)$$

Hence, Y will overtake X at 9 pm. (3 + 6 = 9)

29. Required time = $\frac{\text{Distance advance} - \text{Required difference}}{\text{Relative speed}}$

$$= \frac{120 - 30}{20}$$

$$= \frac{90}{20} = 4.5 \text{ h}$$

Thus, at 7 : 30 pm X and Y will be 30 km apart.

30. Required time = $\frac{\text{Distance advanced} + \text{Required difference}}{\text{Relative speed}}$

$$= \frac{120 + 30}{20} = \frac{150}{20} = 7.5 \text{ h}$$

Thus at 10 : 30 pm X and Y will be 30 km apart.

31. Distance travelled by Y to overtake X
= Time taken $\times$ Speed of Y
= 6 $\times$ 60 = 360 km

Thus, Y will overtake X at a distance of 360 km from P .

32. Required distance

$$= \left(\frac{\text{Distance advanced} + \text{Required difference}}{\text{Relative speed}} \right) \times (\text{Speed of } Y)$$

$$= \frac{120 + 360}{20} \times 60 = 1440 \text{ km}$$

33. Time (when X was 30 km ahead of Y) = $\frac{120 - 30}{20} = 4.5$ h

Time (when Y was 30 km ahead of X) = $\frac{120 + 30}{20} = 7.5$ h

Thus, required difference in time = 3 h

34. Speed = 36 km/h

$$= 36 \times \frac{5}{18} = 10 \text{ m/s}$$

35. Since in 60 minutes postman goes 36000 metre.

So in 1 minute postman goes $\frac{36000}{60} = 600$ metre

Thus, his required speed = 600 m/min

36. 1 mile = 1609.3 m = 1.6093 km

$$\therefore 1 \text{ km} = \frac{1}{1.6093} \text{ mile} = 0.6213882 \text{ mile}$$

$$\therefore \text{Required speed} = 36 \times 0.6213882 \text{ mile/h} = 22.37 \text{ mile/h}$$

37. Speed = 20 m/s = $20 \times \frac{18}{5} = 72$ km/h

38. In one hour train goes 72 km. So in one minute train will go $\frac{72}{60}$.

$$\therefore \text{Speed} = \frac{6}{5} = 1.2 \text{ km/min}$$

39. The ratio of speeds of $A, B, C = 6 : 3 : 1$

$$\therefore \text{The ratio of time taken by } A, B, C = 1 : 2 : 6$$

$$\therefore \text{Time taken by } A = 13 \text{ min}$$

40. $A : B$

Speed 2 : 3

Time $3x : 2x$

41. $\therefore 3x - 2x = 20 \Rightarrow x = 20$

$$\therefore 3x = 60 \text{ min} = 1 \text{ h}$$

42. Let the original speed be S_1 and time t_1 and distance be D

Now, $\frac{D/2}{2t_1} = S_2$

$$\therefore S_2 = \frac{D}{4t_1} \text{ and } S_1 = \frac{D}{t_1}$$

$$\therefore \frac{S_1}{S_2} = \frac{D/t_1}{D/4t_1} = \frac{4}{1}$$

43. You can go through options to check the required difference.

☞ Alternatively Required distance

$$= \frac{S_1 S_2}{(S_1 - S_2)} \times \text{Time difference}$$

$$= \frac{5 \times 8}{3} \times \frac{3}{2} = 20 \text{ km}$$

☞ Alternatively Take the LCM of distances then solve by unitary method.

$$\therefore \text{LCM of } 5, 8 = 40$$

Now, consider 40 km as a distance, then there is a 3 hours difference in 40 km. So, $3/2$ hours difference will be in 20 km.

☞ Alternatively Let x be the distance, then

$$\frac{x}{5} - \frac{x}{8} = \frac{3}{2}$$

$$\Rightarrow x = 20 \text{ km}$$

44. $A : B$

Speed 2 : 3

Time 3 : 2

$$\therefore B \text{ takes } 48 \text{ minutes so } A \text{ will take } 72 \text{ minutes} = 1 \text{ h } 12 \text{ min}$$

45. The ratio of distances = 160 : 140 = 8 : 7

$$\therefore \text{The ratio of speeds} = 8 : 7$$

46. $\frac{x}{9} - \frac{x}{10} = \frac{20}{60}$

$$x = 30 \text{ km}$$

47. Time taken by Abhinav = 4 h

Time taken by Praveen = 3.5 h

For your convenience take the product of times taken by both as a distance.

Then the distance = 14 km

Since, Abhinav covers half of the distance in 2 hours (*i.e.*, at 8 am). Now, the rest half (*i.e.*, 7 km) will be covered by both Praveen and Abhinav.

$$\text{Time taken by them} = \frac{7}{7.5} = 56 \text{ min}$$

Thus, they will cross each other at 8 : 56 am.

48. 9 km difference arises in the 99 km distance.

$$\therefore 72 \text{ km difference will arise in the } 792 \text{ km distance.}$$

49. Apply the product constancy concept

$$\begin{array}{cc} \text{Speed} & \text{Time} \\ \frac{1}{5} \downarrow & \frac{1}{4} \uparrow = 15 \text{ min} \end{array}$$

$$\text{Since, } \frac{x}{4} = 15 \text{ min} \Rightarrow x = 60 \text{ min} = 1 \text{ h}$$

$$\text{So, the original (or usual) time} = 60 \text{ min} = 1 \text{ h}$$

50. $\begin{array}{cc} \text{Speed} & \text{Time} \\ \frac{1}{4} \downarrow & \frac{1}{3} \uparrow = 2 \text{ h} \end{array}$

$$\Rightarrow \text{Usual time} = 2 \times 3 = 6 \text{ h}$$

51. $\begin{array}{cc} \text{Speed} & \text{Time} \\ \frac{1}{2} \uparrow & \frac{1}{3} \downarrow = 40 \text{ min} \end{array}$

$$\Rightarrow \text{Usual time} = 3 \times 40 = 120 \text{ min} = 2 \text{ h}$$

52. $\begin{array}{cc} \text{Speed} & \text{Time} \\ \frac{2}{20} = \frac{1}{10} \downarrow & \frac{1}{9} \uparrow = 10 \text{ min} \end{array}$

$$\Rightarrow \text{Usual time} = 9 \times 10 = 90 \text{ min} = \frac{3}{2} \text{ h}$$

$$\therefore \text{Distance travelled} = \text{Speed} \times \text{Time} = 20 \times \frac{3}{2} = 30 \text{ km}$$

53. Increase in speed = 1 km

Change in time = 25 min

$$\begin{array}{ccc} \text{Speed} & & \text{Time} \\ \frac{1}{5} \uparrow & & \frac{1}{6} \downarrow = 25 \end{array}$$

$$\Rightarrow \text{Usual (normal) time} = 6 \times 25 = 150 \text{ min} = \frac{5}{2} \text{ h}$$

$$\therefore \text{Distance} = \text{Normal speed} \times \text{Normal time} \\ = 5 \times \frac{5}{2} = 12.5 \text{ km}$$

54. Since, they are 1.5 km apart in each hour.

$\therefore$ 7.5 km apart they will be in 5 hours.

(Since, in the same direction speeds are subtracted)

55. In each hour they will be 720 km apart.

(Since in opposite direction speeds are added)

56. Let the distance = x km and usual rate = y km/h

$$\therefore \frac{x}{y} - \frac{x}{(y+6)} = 4 \text{ h} \quad \dots(i)$$

$$\frac{x}{(y-4)} - \frac{x}{y} = 4 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{x}{y} - \frac{x}{(y+6)} = \frac{x}{(y-4)} - \frac{x}{y}$$

$$\Rightarrow y = 24$$

Now putting the value of $y = 24$ in Eq (i), we get

$$x = 480$$

☞ Alternatively

$$\begin{array}{c} S \swarrow \nearrow T \\ +6 \quad -4 \end{array} \Rightarrow -4S + 6T = |-4 \times 6|$$

$$\begin{array}{c} S \swarrow \nearrow T \\ -4 \quad +4 \end{array} \Rightarrow 4S - 4T = |-4 \times 4|$$

$$\therefore -4S + 6T = 24 \quad \dots(i)$$

$$4S - 4T = 16 \quad \dots(ii)$$

$\therefore$ Solving these two equations, we get

$$T = 20 \text{ and } S = 24$$

$$\therefore \text{Distance} = \text{Speed} \times \text{Time}$$

$$= 24 \times 20 = 480 \text{ km}$$

57. Let the original speed be x km/h, then

$$\frac{600}{x} - \frac{600}{(x+20)} = 1$$

$$\Rightarrow 600 \left(\frac{x+20-x}{x(x+20)} \right) = 1$$

$$\Rightarrow x^2 + 120x - 12000 = 0$$

$$\Rightarrow (x-120)(x-100) = 0$$

$$\Rightarrow x = 100 \text{ and } x = 120$$

$$\therefore \text{Original speed} = 100 \text{ km/h}$$

☞ Alternatively

$$S_1 \times S_2 = \text{Distance} \times \frac{\text{Difference in speed}}{\text{Difference in time}}$$

$$= 600 \times \frac{20}{1} = 600 \times 20 = 12000$$

$$\Rightarrow S_1 \times S_2 = 12000$$

$$\therefore S_1 = 100 \text{ and } S_2 = 120$$

(Factorise 12000 in such a way that the difference be 20)

S_1 is the original speed and S_2 is the changed speed.

58. Very similar to question number 57.

$$180 \times \frac{5}{3} = 60 \times 5$$

$$S_1 \times S_2 = 300 = 15 \times 20$$

$$S_1 = 15 \text{ km/h}$$

and

$$S_2 = 20 \text{ km/h}$$

$$\text{☞ Alternatively } S_1 \times (S_1 + 5) = 300$$

$$\Rightarrow S_1 = 15 \text{ km/h}$$

59. Very similar to previous problems

$$t_1 \times t_2 = \text{Distance} \times \frac{\text{Difference in time}}{\text{Difference in speed}}$$

$$\Rightarrow t_1 \times t_2 = 1680 \times \frac{6}{14}$$

$$t_1 \times t_2 = 120 \times 6 = 720 = 30 \times 24$$

$$\Rightarrow t_1 = 30 \text{ and } t_2 = 24$$

$$[\text{since } t_2 = (t_1 - 6) \Rightarrow t_1 \times (t_1 - 6) = 720 \Rightarrow t_1 = 30]$$

Therefore, usual time = 30 h

60. Relative speed = $\frac{\text{Total distance}}{\text{Total time}}$

$$= \frac{60 + 40}{20} = 5 \text{ m/s}$$

$$\therefore 5 \text{ m/s} = 5 \times \frac{18}{5} = 18 \text{ km/h}$$

Now, Relative speed = 18 km/h

$$= \text{Speed of ambulance} - \text{Speed of school bus}$$

$$18 = 30 - \text{speed of school bus}$$

$$\text{Speed of school bus} = 12 \text{ km/h}$$

61. $\begin{array}{cc} \text{Time} & \text{Speed} \\ \frac{1}{4} \downarrow & \frac{1}{3} \uparrow \end{array}$

Since he has to increase his speed by $\frac{1}{3}$ rd of the original

speed. So, the new speed = $15 + 5 = 20 \text{ km/h}$

$$\text{☞ Alternatively } 15 \times 4 = 3 \times x$$

$$\Rightarrow x = 20 \text{ km/h}$$

62. Time (required) = $\frac{\text{Length of train}}{\text{Speed of train}} \quad \left(T = \frac{D}{S} \right)$

$$= \frac{350}{70} = 5 \text{ s}$$

63. Time taken to cross the bridge = $\frac{\text{Total length}}{\text{Speed of train}}$

$$= \frac{270 + 130}{40} = \frac{400}{40}$$

$$= 10 \text{ s}$$

64. Speed of train = $\frac{225}{9} = 25$ s

Now, time taken to cross the tunnel = $\frac{225 + 450}{25} = 27$ s

65. Let the length of the tunnel be x m, then

$$\text{Time} = \frac{\text{Length of train} + \text{Length of tunnel}}{\text{Speed}}$$

$$60 = \frac{350 + x}{10} \quad \left(\text{speed} = 36 \times \frac{5}{18} = 10 \text{ m/s} \right)$$

$\Rightarrow x = 250$ m

66. Speed of train = $\frac{\text{Length of train} + \text{Length of platform}}{\text{Time}}$

$$= \frac{250 + 250}{25} = 20 \text{ m/s}$$

$$= 72 \text{ km/h}$$

67. Let the length of the Sabarmati express is x metre then

$$\frac{x + 162}{18} = \frac{x + 120}{15} \text{ Speed of train}$$

$$\Rightarrow x = 90 \text{ m}$$

68. Time = $\frac{\text{Length of train}}{\text{Relative speed}}$

$$= \frac{200}{19} = 10 \frac{10}{19} \text{ s}$$

(relative speed = $72 - 3.6 = 68.4 \text{ km/h} = 19 \text{ m/s}$)

69. Time = $\frac{\text{Length of train}}{\text{Relative speed}}$

$$= \frac{350}{35} \times 6 = 60 \text{ s}$$

(relative speed = $20 + 1 = 21 \text{ km/h} = 21 \times \frac{5}{18} = \frac{35}{6} \text{ m/s}$)

70. Relative speed = $50 - 40 = 10 \text{ km/h} = \frac{50}{18} \text{ m/s}$
 $\therefore \text{Time taken} = \frac{\text{Sum of length of the trains}}{\text{Relative speed}}$

$$= \frac{200}{50} \times 18 = 72 \text{ s}$$

71. Relative speed = $50 + 40 = 90 \text{ km/h}$

$$= 90 \times \frac{5}{18} = 25 \text{ m/s}$$

$$\therefore \text{Time taken} = \frac{\text{Sum of lengths of train}}{\text{Relative speed}}$$

$$= \frac{200}{25} = 8 \text{ s}$$

72. Let the length of train be x m, then

$$\frac{x}{10} = \frac{120 + x}{18}$$

$$\Rightarrow x = 150 \text{ m}$$

73. Let the speed of train be x km/h, then

$$\frac{175}{10} = (9 + x) \times \frac{5}{18}$$

$$\Rightarrow x = 54 \text{ km/h}$$

74. Relative speed = Sum of speeds of two trains

$$= (60 + x)$$

Time = $\frac{\text{Sum of length of two trains}}{\text{Relative speed}}$

$$10 = \frac{250}{(60 + x) \times 5} \times 18$$

$$(60 + x) = 90$$

$$x = 30 \text{ km/h}$$

75. Let the length of the train be x metre, and let the speed of train be y km/h, then

$$x = (y + 3) \times \frac{5}{18} \times 36 \quad \dots(i)$$

and
$$x = (y + 6) \times \frac{5}{18} \times 30 \quad \dots(ii)$$

From eqs. (i) and (ii), we get

$$(y + 3) \times 36 = (y + 6) \times 30$$

$$y = 12 \text{ km/h}$$

$$\therefore x = (y + 3) \times \frac{5}{18} \times 36$$

or
$$x = 15 \times \frac{5}{18} \times 36$$

or
$$x = 150 \text{ m}$$

76. Let the length of each train be x m, then

$$S_1 = \frac{x}{4} \text{ and } S_2 = \frac{x}{5}; S_1 \text{ and } S_2 \text{ are speeds}$$

Now, required time = $\frac{2x}{\frac{x}{4} + \frac{x}{5}} = \frac{40}{9} \text{ s}$

77.

	Pushpak		Bhopal
Time $\rightarrow$	10	:	8
Speed $\rightarrow$	8	:	10
$\Rightarrow$	$4x$	:	$5x$
Since,	$5x - 4x = 10$		
$\Rightarrow$	$5x = 50 \text{ and } 4x = 40$		
$\therefore$	Average speed = $\frac{2 \times 40 \times 50}{40 + 50} = 44 \frac{4}{9} \text{ km/h}$		

78. Required time = $\frac{\text{Distance advanced}}{\text{Relative speed}} = \frac{4 \times 40}{20} = 8 \text{ h}$

Thus, the faster train will overtake at 8 pm.

79. Required distance = Time taken in overtaking $\times$ Faster's speed

$$= 8 \times 60 = 480 \text{ km from Meerut}$$

80. In 50 minute Rajdhani express can cover 50 km. So, the rest distance = 650 km, which will be jointly covered by both trains.

$$\therefore \text{Time taken} = \frac{650}{(60 + 70)} = 5 \text{ h}$$

Distance from Lucknow = $5 \times 70 = 350 \text{ km}$

81. Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

$$= \frac{560 + 360}{16} = 57 \frac{1}{2} \text{ h}$$

82. Average speed = $\frac{2 \times 80 \times 120}{200} = 96 \text{ km/h}$

83. Distance = Average speed $\times$ Average time

or Distance = $96 \times \frac{25}{2}$

or Distance = 1200 km

84. $50 \times 30 = x \times 20$

$\Rightarrow x = 75 \text{ km/h}$

85. $\frac{S_1}{S_2} = \sqrt{\frac{t_2}{t_1}}$

$\frac{120}{S_2} = \sqrt{\frac{9}{16}} = \frac{3}{4}$

$\Rightarrow S_2 = 160 \text{ km/h}$

86. $S_1 + S_2 = \frac{240}{4} = 60$

$S_1 - S_2 = \frac{240}{12} = 20$

$\therefore S_1 = 40 \text{ m/s}$ and $S_2 = 20 \text{ m/s}$

$\therefore S_1 = 40 \times \frac{18}{5} = 144 \text{ km/h}$

87. Time taken to cross the man = $\frac{\text{Length of the faster train}}{\text{Relative speed}}$

$18 = \frac{x}{15 \times \frac{5}{18}} \Rightarrow x = 75 \text{ m}$

88. Suppose the total distance be 300 km (LCM of 50 and 60) then in the first case it takes only 5 hours and in the second case it takes 6 hours.

Thus, in 6 hours trains halts for 1 hour.

Therefore in 1 hour train halts for $\frac{1}{6}$ hour = 10 m

☞ **Alternatively** Difference in speeds = 10 km/h

Faster speed = 60 km/h

$\therefore$ Required time per hour = $\frac{10}{60} = \frac{1}{6} \text{ h} = 10 \text{ min}$

89. If the speed of faster horse be f_s and that of slower horse be

s_s , then $f_s + s_s = \frac{50}{1} = 50$

and $\frac{50}{s_s} - \frac{50}{f_s} = \frac{5}{6}$

Now, you can go through options.

The speed of slower horse is 20 km/h.

Since, $20 + 30 = 50$

and $\frac{50}{20} - \frac{50}{30} = \frac{5}{6}$

90. Let he walked for x hours, then

$5x + 25(10 - x) = 17 \times 10$

$\Rightarrow x = 4$

$\therefore 10 - x = 6 \text{ h}$

Hence, distance travelled by auto = $25 \times 6 = 150 \text{ km}$.

91. Let the original speed be $x \text{ km/h}$ then

$\frac{36}{(x-6)} + \frac{36}{(x+6)} = 8$

Now, you can go through options or solve it as follows

$\frac{(x+6) + (x-6)}{(x^2-36)} = \frac{8}{36}$

$\Rightarrow x = 12$ and $x = -3$

Thus, the possible value of $x = 12$

$\therefore$ Time taken by faster speed = 2 h

92. Time = $\frac{\text{Distance advanced}}{\text{Relative speed}}$

$2 = \frac{2 \times x}{(30-x)}$

$\Rightarrow x = 15 \text{ km/h}$

93. Prachi travels only for 3 hours, since half an hour she halts at Lucknow.

Now, the average speed (except the halt) = $\frac{160}{3} \text{ km/h}$

Therefore, by alligation

$$\begin{array}{ccc} 40 & \searrow & 60 \\ & 160 & \\ 20 & \nearrow & 40 \\ \frac{20}{3} & 3 & \frac{40}{3} \end{array}$$

$\Rightarrow 1 : 2$

Therefore, the ratio of time taken at 40 km/h and at 60 km/h is 1 : 2.

Thus, the distance between Lucknow and Kanpur

$= 2 \times 60 = 120 \text{ km}$

94. Let the speed of the faster train be f_s and slower train be s_s , then

$f_s + s_s = \frac{200 + 250}{18} = 25 \text{ m/s}$

and $f_s - s_s = \frac{200 + 250}{60} = \frac{450}{60} = 7.5 \text{ m/s}$

$\therefore f_s = 16.25 \text{ m/s}$
 $= 16.25 \times \frac{18}{5} = 58.5 \text{ km/h}$

95. Distance covered in 3 minutes = $3 \times \frac{1000}{60} = 50 \text{ m}$

Now he has to cover (500 + 50) metres in (30 - 3) minutes

$\therefore$ New speed = $\frac{550/1000}{27/60} = \frac{11}{9} \text{ km/h}$

96. **Anil** **Mukesh**

Speed $\rightarrow$ 3 : 4
 Time $\rightarrow$ 4 : 3

But $4x - 3x = \frac{1}{2} \text{ h}$

$\Rightarrow 4x = 2 \text{ h}$ and $3x = 1.5 \text{ h}$

Now, since Anil doubles the speed so time will be half of the actual time. Hence, new time will be 1 hour.

97. Average speed of Anil and Mukesh = $\frac{3x + 4x}{2} = 28$

$\Rightarrow x = 8$

$\therefore$ Speed of Sameer = $3 \times 8 = 24$ km/h

$\therefore$ Distance travelled = $2 \times 24 = 48$ km

98. Let the speed of X and Y be the x km/h and y km/h respectively. Since they meet after 3 hours, so $x + y = 100$. Since, the faster train takes at least $3 + 2 = 5$ hours to complete the 300 km journey. Hence, minimum possible speed for the slower train = 40 km/h at which speed it will take 7.5 h to complete the journey. $\left(7.5 = \frac{300}{40}\right)$

99. Let the time taken in first third part of the journey be x minutes, then the time required in second third part of the journey is $\frac{3x}{2}$ and in the last third part of the journey time required is $\frac{15x}{8}$.

Therefore, $x + \frac{3x}{2} + \frac{15x}{8} = 350$ min

$\Rightarrow x = 80$ min

100.

Speed	Time
$\frac{1}{5} \downarrow$	$\frac{1}{4} \uparrow = 15$ min

Therefore usual time = $4 \times 15 = 60$ min

Now,

Speed	Time
$\frac{1}{4} \uparrow$	$\frac{1}{5} \downarrow = 12$ min

(since original time = 60 min)

Therefore he will be $15 + 12 = 27$ minutes early in comparison to the previous day.

101. Let the original speed be s km/h and scheduled time = t hours

and total distance = D km

then $s \times t = \frac{3}{4} D$... (i)

and $s \times (t + 3) = D$... (ii)

From eqs. (i) and (ii), we get

$st = \frac{3}{4} [s(t + 3)] \Rightarrow t = 9$ h

and let $s = 1$ km/h, then $D = 12$ km

Again, since he doubles his speed after k hours then,

$s_1 t_1 + s_2 t_2 = D$

$1 \times k + 2 \times (9 - k) = 12$

$\Rightarrow k = 6$ h

102. By alligation rule

20		70
	50	
20		30
2	:	3

Ratio of time = $2 : 3$

$\therefore$ Ratio of distances = $2 \times 20 : 3 \times 70 = 4 : 21$

Alternatively $\frac{x}{20} + \frac{y}{70} = \frac{x+y}{50}$

$\Rightarrow \frac{76x + 20y}{1400} = \frac{x+y}{50}$

$\Rightarrow 42x = 8y \Rightarrow \frac{x}{y} = \frac{4}{21}$

103. $\frac{(\text{Speed of wind})}{(\text{Speed of car})} = \frac{(\text{Time utilised})}{(\text{Time saved})}$

$\frac{332}{x} = \frac{332}{28}$

$\Rightarrow x = 28$ m/s

104. Time = $\frac{\text{Total distance}}{\text{Relative speed}}$

$\frac{4.5}{60} = \frac{450/1000}{x} \Rightarrow x = 6$ km/h

Relative speed = Speed of car - Speed of man

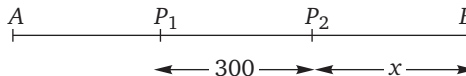
$6 = x - 6$

$\Rightarrow x = 12$ km/h

105. $W + R \rightarrow 4$ h 20 min

$W + W \rightarrow 5$ h 20 min

$\therefore R + R \rightarrow 3$ h 20 min

106. 

$P_1 \rightarrow$ Place of accident

$P_2 \rightarrow$ Imaginary place of accident

For the distance x

Speed	Time
$\frac{1}{6} \downarrow$	$\frac{1}{5} \uparrow = 1$ h

Thus, the usual time required for the distance x km is $5 \times 1 = 5$ hours.

For the distance $(x + 300)$

Speed	Time
$\frac{1}{6} \downarrow$	$\frac{1}{5} \uparrow = 2$ h

Thus, the usual time required for the distance $(x + 300)$ km is $5 \times 2 = 10$ h.

It means he covers 300 km distance in 5 h

$\therefore$ Speed = $\frac{300}{5} = 60$ km/h (normal speed)

107. Since, he can cover x km at 60 km/h in 5 h it means $x = 300$ km.

Therefore, total distance = $(120 + 300 + 300) = 720$ km.

108. Circumference means one revolution.

Therefore, distance covered in 10 revolutions = $300 \times 10 = 3000$ m

i. e., 30 metre in 6 seconds.

$\therefore$ Speed of wheel = $\frac{30}{6}$ m/s = 5 m/s

$\therefore$ 5 m/s = $5 \times \frac{18}{5} = 18$ km/h

$$109. \text{ Speed of man in still water } = \frac{S_D + S_U}{2} \\ = \frac{12 + 8}{2} = 10 \text{ km/h}$$

$S_D \rightarrow$ Speed in downstream (= Boat + River)

$S_U \rightarrow$ Speed in upstream (= Boat - River)

$$110. \text{ Speed of water current } = \frac{S_D - S_U}{2} = \frac{21 - 15}{2} = 3 \text{ km/h}$$

$$111. S_D = 12 \text{ km/h}$$

$$S_U = 5 \text{ km/h}$$

$$\text{Speed of current} = \frac{12 - 5}{2} = 3.5 \text{ km/h}$$

$$112. S_D \rightarrow 15 \text{ km/h}$$

$$S_U \rightarrow 9 \text{ km/h}$$

$$\text{Speed of man} = \frac{15 + 9}{2} = 12 \text{ km/h}$$

113. Let the required distance be D km, then

$$\frac{D}{6} + \frac{D}{4} = \frac{5}{4}$$

$$D \left(\frac{10}{24} \right) = \frac{5}{4} \Rightarrow D = 3 \text{ km}$$

$$114. \frac{16}{8} + \frac{16}{2} = 10 \text{ h}$$

115. Let the downstream speed be D and upstream speed be U , then

$$\frac{48}{U} + \frac{72}{D} = 12$$

$$\text{and } \frac{48}{D} + \frac{72}{U} = 13$$

$$\therefore 48m + 72n = 12 \quad \dots(i)$$

$$\text{and } 48n + 72m = 13 \quad \dots(ii)$$

on solving eqs. (i) and (ii), we get

$$m + n = \frac{5}{24} \quad \text{and} \quad m - n = \frac{1}{24}$$

$$\therefore D = 12 \text{ km/h} \quad \text{and} \quad U = 8 \text{ km/h}$$

$$\therefore \text{Speed of current} = 2 \text{ km/h}$$

$$116. D_S = \frac{9}{2} = 4.5 \text{ km/h}$$

$$U_S = \frac{9}{6} = 1.5 \text{ km/h}$$

$$\therefore \text{Speed of boat in still water} = \frac{4.5 + 1.5}{2} = 3 \text{ km/h}$$

$$\text{and Speed of river in still water} = \frac{4.5 - 1.5}{2} = 1.5 \text{ km/h}$$

$$117. \frac{D_T}{U_T} = \frac{1}{2} \Rightarrow \frac{D_S}{U_S} = \frac{2}{1} \Rightarrow \frac{B+S}{B-S} = \frac{2}{1} \Rightarrow \frac{B}{S} = \frac{3}{1}$$

Hint D_T and U_T are the downstream and upstream times and D_S and U_S are the downstream and upstream speeds. Here we can use componendo and dividendo.

$$\therefore \frac{\text{Speed of boat}}{\text{Speed of stream}} = \frac{3}{1} = \frac{15}{x}$$

$$\therefore \text{Speed of stream} = 5 \text{ km/h}$$

118. If t_1 and t_2 are the upstream and downstream times. Then time taken in still water is given by

$$\frac{2 \times t_1 \times t_2}{t_1 + t_2} = \frac{2 \times 12 \times 24}{36} = 16 \text{ h}$$

Alternatively $D = (B + S) \times 12$

and $D = (B - S) \times 24$

Where $(B + S)$ is downstream speed and $(B - S)$ is upstream speed

$$\Rightarrow \frac{B+S}{B-S} = \frac{2}{1}$$

$$\Rightarrow \frac{B}{S} = \frac{3}{1} \quad (\text{by componendo and dividendo})$$

$$\text{Now, } D = 4S \times 12 = 48S$$

$$D = 48S = 16B \quad (\text{Distance} = \text{Time} \times \text{Speed})$$

$\therefore$ Required time = 16 h

$$119. \frac{D_S}{U_S} = \frac{B+S}{B-S} = \frac{3}{2}$$

Where $B \rightarrow$ Speed of boat in still water

$S \rightarrow$ Speed of current/stream

$$\Rightarrow \frac{2B}{2S} = \frac{5}{1} \quad (\text{by componendo and dividendo})$$

$$\Rightarrow \frac{B}{S} = \frac{5/2}{1/2} \Rightarrow B = \frac{5}{2}$$

Average speed of downstream and upstream

$$= \frac{2 \times 3 \times 2}{3+2} = \frac{12}{5}$$

$$\therefore \text{Required ratio} = \frac{5/2}{12/5} = \frac{25}{24}$$

120. Let x be the upstream speed, then the downstream speed will be $(x + 3)$.

$$\therefore \frac{3}{x} + \frac{3}{x+3} = 3$$

$$\Rightarrow x^2 + x - 3 = 0$$

$$\Rightarrow x = \frac{-1 + \sqrt{13}}{2} \\ = \frac{-1 + 3.6}{2} = 1.3 \text{ km/h}$$

$$\therefore (x + 3) = 4.3 \text{ km/h}$$

121. Upstream speed of first boat = 8 km/h

Upstream speed of second boat = 4 km/h

$$\therefore \text{Relative speed} = 4 \text{ km/h} \quad (= 8 - 4)$$

$$\therefore \text{Required time} = \frac{10}{4} = 2.5 \text{ h}$$

122. Downstream speed = $B + S$ $B \rightarrow$ Speed of boat

Upstream speed = $B - S$ $S \rightarrow$ Speed of stream

$$\therefore \frac{120}{B+S} + \frac{120}{B-S} = 15$$

$$\Rightarrow \frac{B}{B^2 - S^2} = \frac{1}{16} \quad \dots(i)$$

$$\text{Again } \frac{120}{B+2S} + \frac{120}{B-2S} = 24$$

$$\Rightarrow \frac{B}{B^2 - 4S^2} = \frac{1}{10} \quad \dots(ii)$$

From eqs. (i) and (ii), we get

$$B = (B^2 - S^2) 16$$

$$\text{and } B = (B^2 - 4S^2) 10$$

$$\Rightarrow 10B^2 - 40S^2 = 16B^2 - 16S^2$$

$$\Rightarrow \frac{B^2}{S^2} = 9 \Rightarrow \frac{B}{S} = 3$$

$$\text{i.e., } B : S = 3 : 1$$

Now, you can go through options or solve by equations. Since now you know the ratio of speeds of boat and stream. The correct choice is (b).

124. Upstream speed = $B - S$

Downstream speed = $B + S$

$$B - S = \frac{15}{5} = 3 \text{ km/h}$$

$$\text{Again } B = 4S$$

$$\therefore B - S = 3 = 3S$$

$$\Rightarrow S = 1 \text{ and } B = 4 \text{ km/h}$$

$$\therefore B + S = 5 \text{ km/h}$$

$$\therefore \text{Time during downstream} = \frac{15}{5} = 3 \text{ h}$$

$$125. \frac{24}{(5.5 - R)} - \frac{24}{(5.5 + R)} = 5$$

$$\Rightarrow R = 2.5 \text{ km/h; } R \rightarrow \text{Speed of river/current}$$

$$\text{Again } (B_2 + R) = \frac{24}{4} = 6$$

$$\Rightarrow (B_2 + 2.5) = 6 \Rightarrow B_2 = 3.5 \text{ km/h}$$

$$126. \begin{array}{c} P \quad \quad \quad Q \quad \quad \quad R \\ \bullet \quad \quad \quad \parallel \quad \quad \parallel \quad \quad \bullet \\ PQ = QR \end{array}$$

$$P \rightarrow Q \rightarrow R \text{ (7 h)}$$

It means $P \rightarrow Q$ (3.5 h)

Again $\{P \rightarrow Q \text{ and } Q \rightarrow P\}$ (8 h)

It means $Q \rightarrow P$ (4.5 h)

Therefore $R \rightarrow Q$ (4.5 h)

Thus, from R to P boat will take 9 hours.

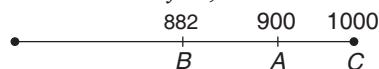
Hint $P \rightarrow R$ (Downstream) $R \rightarrow P$ (Upstream)

$$127. \frac{40}{(B - S)} + \frac{55}{(B + S)} = 13$$

$$\frac{30}{(B - S)} + \frac{44}{(B + S)} = 10$$

Hint Go through options for quicker answer and prefer the value which can help in dividing 44 and 55.

Solutions (for Q. Nos. 128 and 129) B is 2% slow than A and A is 10% slow than C . Therefore, in 1 km race.

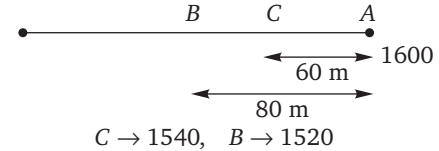


128. Since in 1000 m (1 km) race difference between A and B is 18 m.

So in 2 km race it will be 36 m.

129. In a 1 km race C can give B a start of 118 m. Therefore in a half km race C can give B a start of 59 m.

130. In a 1600 m race



The ratio of speeds of $C : B$ is 77 : 76. It means in 77 m race C beats B by 1 m. So, in 400 m race C will beat B by

$$400 \times \frac{1}{77} \text{ m} = 5 \frac{15}{77} \text{ m}$$

131.

	Aman	Shakti
Time	190	200
Speed	$20x$	$19x$

$$\therefore 20x = 1000 \text{ m}$$

$$\Rightarrow x = 50 \text{ m}$$

$$\text{Again } 20x - 19x = x = 50 \text{ m}$$

So, Aman can beat Shakti by 50 m.

132.

	A	:	B
Speed	7	:	4
Time	4	:	7
Distance	4	:	7 (since distance $\propto$ time)

$$\text{Now, } 7x - 4x = 300$$

$$3x = 300$$

$$\Rightarrow x = 100$$

$$\therefore 7x = 700$$

Thus, the winning post be 700 m away from the starting point.

133. Ratio of speeds of $A : B = 12 : 11$

and ratio of speeds of $B : C = 8 : 7$

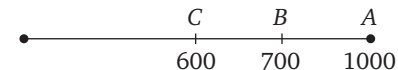
Therefore ratio of speeds of $A : B : C = 96 : 88 : 77$

So in 9600 m race A will beat C by 1900 m.

134. When Ameesha runs 1000 m, then Bipasha runs only 700 m $[1000 - (100 + 200)]$

when Ameesha runs 1000 m, then Celina runs only 600 m $[1000 - (100 + 300)]$

Therefore,



Now, since in 700 m race Bipasha beats Celina by 100 m. So, in 50 m race Bipasha will beat Celina by

$$50 \times \frac{100}{700} = 7.14 \text{ m}$$

Alternatively Bipasha beats Celina by 14.28% of the distance then in 50 m race Bipasha will beat Celina by 7.14 m.

135.		Distance	Time
Case I:	Ravi	1000	t_1
	Vinod	960	$t_1 + 19$
Case II:	Ravi	960	t_2
	Vinod	1000	$t_2 + 30$

Therefore, $\frac{1000}{t_1} = \frac{960}{t_2} = \text{Speed of Ravi}$

$\Rightarrow t_1 = \frac{25}{24} t_2$

Also $\frac{960}{t_1 + 19} = \frac{1000}{t_2 + 30} = \text{Speed of Vinod}$

$\therefore (t_2 + 30) 24 = 25 \left[\frac{25}{24} t_2 + 19 \right]$

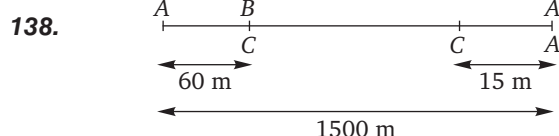
$\Rightarrow 49t_2 = 5880 \Rightarrow t_2 = 120 \text{ s}$

$\therefore \text{Required ratio} = \frac{960/t_2}{1000/t_2 + 30} = \frac{6}{5}$

- 137.** Since, the speeds of Vinay and Versha are in the ratio of 5 : 3 i. e., when Vinay covers 5 rounds, then Versha covers 3 rounds, but first time Vinay and Versha meet when Vinay completes $2\frac{1}{2}$ round and Versha completes $1\frac{1}{2}$ round.

For Vinay to pass Versha 7th time, Vinay would have completed $7 \times 2\frac{1}{2}$ rounds. Since, each round is $1\frac{1}{2}$ km, the distance covered by Vinay is

$$7 \times 2\frac{1}{2} \times 1\frac{1}{2} = 7 \times \frac{5}{2} \times \frac{3}{2} = 26\frac{1}{4} \text{ km}$$



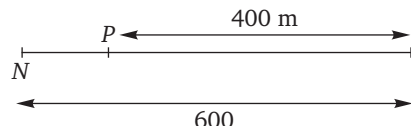
In the same time, when A covers 1500 m, B covers 1440 m and C covers 1425 m.

So, in 1440 m race B can give a start of 15 m.

$\therefore$ In 1500 m race B will give a start of

$$\frac{15}{1440} \times 1500 = 15\frac{5}{8} \text{ m}$$

- 139.** In 600 m race Prabhat can have only start of 120 m. Now since he has more than 200 m start up so he will win the race.



Now, when Prabhat will cover 400 m distance, then in the same time Nishith will cover only 500 m. So, Prabhat will win by 100 m.

- 140.** B is 25% slower than A and C is 20% slower than B. Therefore in a game of 200 points B can have 150 points and C can have 120 points. Thus, A can give C 80 points.

- 141.** Since between 11 am and 1 pm and 11 pm and 1 am two hands of a clock coincide only once, each time.

- 142.** Since between 2 am and 3 am (2 pm and 3 pm) and 8 am and 10 am (8 pm and 10 pm) two hands of a clock make 90° angle only 3 times in rest of the each hour two hands make 90° angle 2 times.

- 143.** Since between 5 am and 7 am (5 pm and 7 pm) this happens only once. In rest each of the hours it happens one time.

- 144.** Both (a) and (b) are correct.

$$\text{Relative speed} = \text{Speed of minute-hand} - \text{Speed of hour-hand}$$

$$= 6^\circ - \frac{1^\circ}{2} = 5\frac{1}{2}^\circ$$

and $1 \text{ min} - \frac{1}{12} \text{ min} = \frac{11}{12} \text{ min}$

145. $\frac{5 \times 30}{11/2} = \frac{300}{11} = 27\frac{3}{11} \text{ min} = 27 \text{ min } 16 \text{ s}$

Therefore, required time = 5 : 27 : 16

- 146.** The angle made by minute-hand = $5 \times 30 = 150^\circ$

The angle made by hour-hand = $2 \times 30 + 25 \times \frac{1}{2} = 72.5^\circ$

Hence, required angle = $150 - 72.5 = 77.5^\circ$

147. $\frac{90^\circ - 30^\circ}{5.5} = \frac{60}{11} \times 2 = \frac{120}{11} = 10\frac{10}{11} \text{ minute}$

$$= 10 \text{ min } 54 \text{ s}$$

$\therefore$ Required time = 3 : 10 : 54

148. $\frac{210}{5.5} = \frac{210}{11} \times 2 = \frac{420}{11} = 38\frac{2}{11} \text{ min} = 38 \text{ min } 11 \text{ s}$

Therefore, required time = 7 : 38 : 11

- 149.** The angle made by minute-hand = 90°

The angle made by hour-hand

$$= 6 \times 30 + 15 \times \frac{1}{2} = 187.5^\circ$$

$\therefore$ Required difference = 97.5°

150. $\frac{90 - 35}{5.5} = \frac{55}{11} \times 2 = 10 \text{ min}$

So, the required time = 3 : 10 : 00

Again $\frac{90 + 35}{5.5} = \frac{125}{11} \times 2 = \frac{250}{11} = 22\frac{8}{11} \text{ min}$

$$= 22 \text{ min } 43 \text{ s}$$

Level 02 Higher Level Exercise

1.	Cycle	Auto	Car	
Speed	x	$(5x - 20)$	$5x$	
Time		$(t + 1)$	t	
Distance	120	120	120	(in km)
$\therefore$	$\frac{120}{(5x - 20)} - \frac{120}{5x} = 1$			

$$\Rightarrow x^2 - 4x - 96 = 0$$

$$\Rightarrow x = 12$$

$$\therefore \text{Average speed} = \frac{360}{(10 + 3 + 2)} = 24 \text{ km/h}$$

2. Time taken by Cycle = $\frac{120}{12} = 10 \text{ h}$

Time taken by Auto = $\frac{120}{40} = 3 \text{ h}$

Time taken by Car = $\frac{120}{60} = 2 \text{ h}$

Total time = 15 h

3. In last 5 hours she covers 240 km (120 + 120)

4. New time = 3 + 3 + 2 = 8 h

Hence, decrease in time = 7 h (15 - 8)

$\therefore$ Percentage change = $\frac{7}{15} \times 100 = 46.66\%$

5. Time taken to meet Bipasha and Mallika = $\frac{1080}{(60 + 120)} = 6 \text{ h}$

So, in 6 hours Bipasha covers 360 km and this 360 km distance Rani covers in $\frac{360}{90} = 4 \text{ h}$.

Hence, Rani leaves Kolkata 2 hours later than Bipasha i. e., at 8 am. Rani leaves Kolkata.

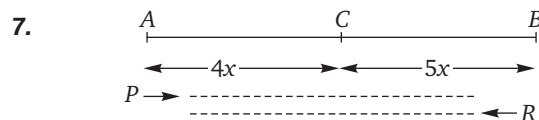
NOTE The distance 360 covered by Bipasha to meet Mallika can also be calculated by the ratio of their speeds.

6. Note here the length of the train in which passenger is travelling is not considered since we are concerned with the passenger instead of train. So, the length of the bridge will be directly proportional to the time taken by the passenger respectively.

Therefore, $\frac{t_1}{t_2} = \frac{l_1}{l_2}$ $t \rightarrow$ Time $l \rightarrow$ Length of bridge

$$\frac{7}{4} = \frac{280}{x}$$

$\Rightarrow x = 160 \text{ m}$



Note that the distances covered by them to meet at C are in the direct ratio of their speeds. Therefore

$$AC : BC = 4x : 5x$$

Now, for any particular person (say Pathik) the time required to cover different distances is directly proportional to the different distances. So, time taken by Pathik to cover AC and BC are the ratio of 4 : 5 (excluding staying or halt time at Chandni Chowk).

Thus time required to cover AC is 52 minutes only since he covers BC in 65 minutes.

But since he leaves Chandni Chowk for Bhavnagar at 9 : 27 am i. e., 67 minutes later, when he left Andheri. It means he must have stayed at C for (67 - 52) = 15 minutes.

8. Let the length of the train be L metres and speeds of the train Arjun and Srikrishna be R , A and K respectively, then

$$\frac{L}{R - A} = 36 \quad \dots(i)$$

and $\frac{L}{(R + K)} = 24 \quad \dots(ii)$

From eqs. (i) and (ii), We get

$$3(R - A) = 2(R + K)$$

$$\Rightarrow R = 3A + 2K$$

In 30 minutes (i. e., 1800 seconds), the train covers 1800R (distance) but the Arjun also covers 1800A (distance) in the same time.

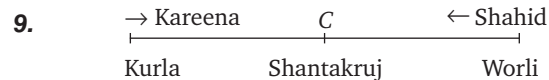
Therefore distance between Arjun and Srikrishna, when the train has just crossed Srikrishna

$$= 1800(R - A) - 24(A + K)$$

$$\therefore \text{Time required} = \frac{1800(R - A) - 24(A + K)}{(A + K)}$$

$$= (3600 - 24) = 3576 \text{ s}$$

$$(\text{since } R = 3A + 2K)$$



Let the time taken by Kareena in going from K to S is x minutes and the time taken by Shahid in going from Worli to Shantakruj be y min.

Since, the new speed of Kareena is $\frac{2}{3}$, therefore time taken

in returning = $\frac{3}{2}x$.

$$\therefore x + \frac{3}{2}x = 120$$

$$\Rightarrow x = 48 \text{ min}$$

But $x = y$

Again since the new speed of Shahid is $\frac{4}{3}$, therefore the

time taken in returning = $\frac{3}{4}y$.

$$\therefore \text{Total time} = y + \frac{3}{4}y$$

$$= 48 + 36 = 84 \text{ min}$$

10. Time taken to collide the two trains = $\frac{3}{2} \text{ h}$

So, in $\frac{3}{2} \text{ h}$ bird travels $\frac{3}{2} \times 60 = 90 \text{ km}$

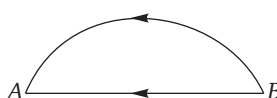
11. Let there be l steps in the escalator and x be the speed (in steps/second) of escalator, then

$$\frac{l}{(5+x)} = 10 \quad \text{and} \quad \frac{l}{(5-x)} = 40$$

then
$$\frac{5+x}{(5-x)} = \frac{40}{10} \Rightarrow x = 3$$

$\therefore$ Number of steps in the escalator $= l = 8 \times 10 = 80$

12. Let the radius be r , then difference in the distance $= (\pi r - 2r) = r(\pi - 2)$



$$= r \left(\frac{22}{7} - 2 \right) = 60 \times 3 \Rightarrow 2r = 315 \text{ m}$$

[$\pi r \rightarrow$ semiperimeter and $2r \rightarrow$ diameter]

13. Time taken by trains to collide $= \frac{560}{70} = 8 \text{ h}$

In 8 h sparrow will cover $8 \times 80 = 640 \text{ km}$

14. 

In 18 h plane will cover $18 \times 120 = 2160 \text{ km}$

Now, $2160 = (600 \times 2) + 600 + 360$

So, the plane will be 360 km away from Kargil it means it will be 240 km $(600 - 360)$ away from Pukhware.

	First hour	Second hour	Third hour	Total
Initial speed	x	$3x$	$2x$	$6x$
New speed	$3x$	$3x$	$3x$	$9x$

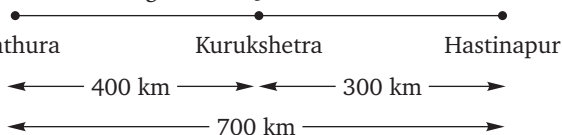
$\therefore$ Percentage increase in speed $= \frac{3x}{6x} \times 100 = 50\%$

Since speed is increased by $(50\%) \frac{1}{2}$

Therefore, time will reduce by $(33.33\%) \frac{1}{3}$

16. 

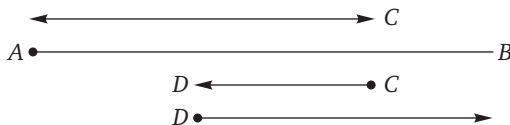
They will be together at every two hours. Therefore in 12 h they will be $(6 + 1) = 7$ times together at P and they will never meet altogether at Q.

17. 

Consider only one person either Arjun or Srikrishna since their speed is same and move together.

Now, the distance covered by Arjun and Abhimanyu is in the ratio of their speeds. So, Arjun will cover 500 km to meet Abhimanyu and thus Arjun has to return back 100 km for Kurukshetra. Therefore, Arjun will cover total 600 km distance.

18. Total time $= \frac{600}{25} = 24 \text{ h}$

19. 

A is the starting point of journey.

B is the destination.

C $\rightarrow$ where Salman has got off.

D $\rightarrow$ where Priyanka picks up Akshay

Let $AD = l$ and $BC = k$ and $CD = x$

then
$$\frac{CD + DB}{BC} = \frac{50}{10}$$

$$\frac{2x + k}{k} = \frac{5}{1}$$

$$\Rightarrow \frac{x}{k} = \frac{2}{1}$$

Again
$$\frac{AC + CD}{AD} = \frac{50}{10}$$

$$\frac{2x + l}{l} = \frac{5}{1}$$

$$\Rightarrow \frac{x}{l} = \frac{2}{1}$$

$$\Rightarrow x = 2k = 2l \quad \text{or} \quad k = l = \frac{x}{2}$$

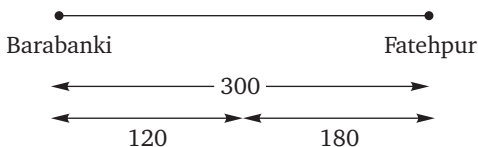
$$\therefore k + x + l = 120$$

$$\Rightarrow k = 30 \text{ km}, \quad x = 60 \text{ km} \quad \text{and} \quad l = 30 \text{ km}$$

Total distance travelled $= AC + CD + DB$

$$= l + x + x + k = 240 \text{ km}$$

$$\therefore \text{Time (required)} = \frac{240}{50} = 4.8 \text{ h}$$

20. 

Let the speeds of Ajay, Kajol and Shahrukh be x , y and z respectively, then

$$\frac{y}{x} = \frac{180}{120}$$

$$\Rightarrow x = \frac{2y}{3}$$

NOTE Kajol is faster since she covers 180 km while Ajay covers only 120 km in the same time.

Shahrukh meets Kajol 1.5 hours after Shahrukh himself starts and 2.5 hours after Kajol starts.

Hence, $2.5y + 1.5z = 300$

$$\Rightarrow z = \frac{600 - 5y}{3}$$

$$\text{Since } z \geq (y + 20) \Rightarrow \frac{600 - 5y}{3} \geq (y + 20)$$

$$\Rightarrow y \leq 67.5$$

$$\text{or } x \leq 45 \text{ km/h}$$

21. Let t be the time after Kajol starts, when she meets Ajay, then

$$t = \frac{300}{(x+y)}$$

This should be less than 2.5 or $(x+y) > 120$

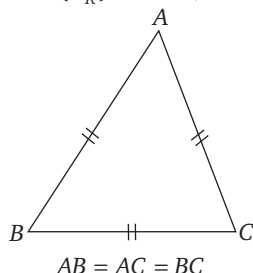
Since $y = \frac{3x}{2} \Rightarrow y > 72$

This ($y > 72$) is greater than 67.5 km/h and hence Shahrukh will always overtake Ajay before he meets Kajol.

22. Speed of Raghupati (R_P) = 60 km/h

Speed of Raghav (R_V) = 36 km/h

Speed of Raja Ram (R_R) = 18 km/h



Time taken to cover AB by (RR) is 2 hours

$\therefore$ Time taken to cover AB by Raghav is 1 hour

$\therefore$ Time taken to cover AB by Raghupati = 36 min

$$\left(t_{RP} : t_{RV} : t_{RR} = \frac{1}{S_{RP}} : \frac{1}{S_{RV}} : \frac{1}{S_{RR}} \right)$$

$t \rightarrow$ Time, $S \rightarrow$ Speed

$$AB = 2 \times 18 = 36 \text{ km}$$

23. Time = $\frac{3 \times 36}{60} = \frac{9}{5} \text{ h} = 1 \text{ h } 48 \text{ min}$

24. Distance from Bareilly = $\frac{60}{(60+18)} \times 36$
 $= \frac{360}{13} = 27 \frac{9}{13} \text{ km}$

- 25.

Since the speed of bike and walking are different. So, two people partially travelled by bike and rest by walking since all the three persons take equal time to reach the destination. It means initially Mohan will carry either Namit or Pranav to a point A , then this person reach to H by walking and Mohan return to B where he will pick up the third person and reach at H at the same time as the second person.

Let $SB = k$, $AB = x$ and $AH = l$

Now, $\frac{SA + AB}{SB} = \frac{36}{6}$

$$\frac{2x + k}{k} = \frac{6}{1}$$

$$\Rightarrow \frac{x}{k} = \frac{5}{2}$$

and

$$\frac{AB + BH}{AH} = \frac{36}{6}$$

$$\Rightarrow \frac{2x + l}{l} = \frac{6}{1}$$

$$\Rightarrow \frac{x}{l} = \frac{5}{2}$$

$$\therefore x : k : l = 5 : 2 : 2$$

$$\Rightarrow x + k + l = 180$$

$$\Rightarrow x = 100, k = 40 \text{ and } l = 40 \text{ km}$$

Total distance travelled by bike = $SA + AB + BH$
 $= k + 3x + l = 380 \text{ km}$

- 26.

$$\frac{2x + k}{k} = \frac{42}{6} = \frac{7}{1}$$

$$\Rightarrow \frac{x}{k} = \frac{3}{1}$$

Similarly $\frac{x}{l} = \frac{3}{1}$

$$\therefore x : k : l = 3 : 1 : 1$$

$$\therefore x = 108, k = 36, l = 36 \text{ km}$$

Total distance travelled = $k + 3x + l = 396 \text{ km}$

$$\therefore \text{Required time} = \frac{396}{42} = 9 \frac{3}{7} \text{ h}$$

27. Let the buses leave from both the stations at time intervals of T , then the distance between any two consecutive buses coming opposite to me = the distance between any two consecutive buses coming in the same direction as me = VT .

(where V is the velocity of the buses)

Let the speed of walking be W , then

$$\frac{VT}{V+W} = 20 \text{ and } \frac{VT}{V-W} = 30$$

$$\therefore \frac{V+W}{V-W} = \frac{30}{20} = \frac{3}{2}$$

$$\Rightarrow \frac{V}{W} = \frac{5}{1}$$

$$\therefore \frac{VT}{V+W} = 20 \Rightarrow \frac{5}{6} \times T = 20$$

$$\therefore T = 24 \text{ min}$$

28. Time taken by Abhinav = 36 h

Ideal time required by Abhinav = $\frac{600}{25} = 24 \text{ h}$

It means Abhinav rests for $(36 - 24) = 12 \text{ h}$

Now, the required time for Brijesh = $\frac{600}{30} = 20 \text{ h}$

But Brijesh utilised those 12 hours in which Abhinav rests, so he needs only $(20 - 12) = 8$ hours extra.

Thus, the total time taken by Brijesh = $36 + 8 = 44 \text{ h}$

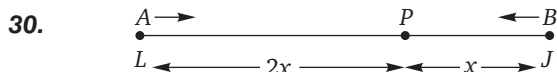
29. Downstream (Steamer) = 40 min

Downstream (Boat) = 60 min

Upstream (Steamer) = 60 min

Upstream (Boat) = 90 min

$$\therefore \text{Required time} = 40 + 30 + 45 = 115 \text{ min}$$



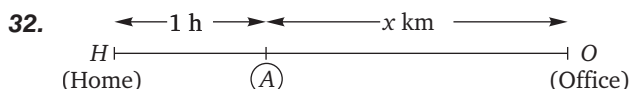
These two trains meet only at P and L i. e., there are only two points.

31. For the first meeting they have to cover only $2x + x = 3x$ distance and for the further meeting for each next meeting they have to cover $6x$ distance together.

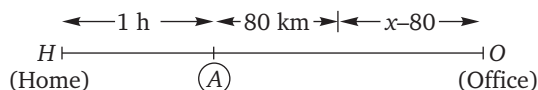
Distance covered by A	$2x$	$2x$	$4x$	$2x$
Distance covered by B	x	$4x$	$2x$	$4x$
Point of meeting	P	L	P	P
Total distance travelled	$3x$	$6x$	$6x$	$6x$

When A and B meet at P for the third time A goes $10x$ and B goes $11x$.

Thus, the required ratio = $10 : 11$



$\Rightarrow$ actual time required to cover x km is $5 \times 36 = 180$ min



$\Rightarrow$ actual time required for $(x - 80)$ km = $5 \times 20 = 100$ min

It means he can move = $x - (x - 80) = 80$ km in $(180 - 80) = 80$ min

It means his actual speed = 60 km/h

Thus, the total distance from his home to his office = $60 \times 1 + 60 \times 3 = 240$ km

NOTE Since 1 hour he lost at the place of accident, so the actual delay due to reduced speed is always 1 hour less than it is found to be in both the cases. It means due to reduced speed he becomes late only 36 minutes and 20 minutes in respective cases.

33. $\frac{\text{Speed of wind (Sound)}}{\text{Relative speed of soldier and terrorist}}$
 $= \frac{\text{Time utilised}}{\text{Difference in time}}$
 $\frac{1188}{x} = \frac{330}{5}$
 $\Rightarrow x = 18$ km/h

34. In case of increasing gap between two objects.

$$\frac{\text{Speed of sound}}{\text{Speed of tiger}} = \frac{\text{Time utilised}}{\text{Difference in time}}$$

$$\frac{1195.2}{x} = \frac{83}{7} \Rightarrow x = 100.8 \text{ km/h}$$

35. In 20 minutes the difference between man and his son = $20 \times 20 = 400$ m

Distance travelled by dog when he goes towards son

$$= \frac{400}{40} \times 60$$

= 600 m and time required is 10 minutes

In 10 minutes the remaining difference between man and son.

$$400 - (20 \times 10) = 200 \text{ m}$$

NOTE Relative speed of dog with child is 40 km/h and the same with man is 100 km/h.

$$\text{Time taken by dog to meet the man} = \frac{200}{100} = 2 \text{ min}$$

In 2 min the remaining distance between child and man

$$200 - (2 \times 20) = 160 \text{ m}$$

Now, the time taken by dog to meet the child again

$$= \frac{160}{40} = 4 \text{ min}$$

In 4 minutes he covers $4 \times 60 = 240$ m distance while going towards the son.

In 4 minute the remaining distance between man and child

$$= 160 - (4 \times 20) = 80 \text{ m}$$

Time required by dog to meet man once again

$$= \frac{80}{100} = 0.8 \text{ min}$$

In 0.8 min remaining distance between man and child

$$= 80 - (0.8 \times 20) = 64 \text{ m}$$

Now, time taken by dog to meet the child again

$$= \frac{64}{40} \times \frac{8}{5} \text{ min}$$

$\therefore$ Distance travelled by dog

$$= \frac{8}{5} \times 60 = 96 \text{ m}$$

Thus, we can observe that every next time dog just go $2/5$ th of the previous distance to meet the child in the direction of child.

So, we can calculate the total distance covered by dog in the direction of child with the help of GP formula.

Here, first term (a) = 600 and common ratio (r) = $\frac{2}{5}$

$$\therefore \text{Sum of the infinite GP} = \frac{a}{1 - r}$$

$$= \frac{600}{1 - \frac{2}{5}} = \frac{600}{3/5} = 1000 \text{ m}$$

36. Let Amarnath express takes x hours, then Gorakhnath express takes $(x - 2)$ hours.

$$\therefore \frac{1}{x} + \frac{1}{(x-2)} = \frac{60}{80} \Rightarrow x = 4 \text{ h}$$

37. Distance travelled by them in first hour = 12 km

Distance travelled by them in second hour = 13 km

Distance travelled by them in third hour = 14 km and so on

Thus, in 9 hours they will cover exactly 144 km and in 9 h each will cover half-half the total distance.

$$(8 \times 9 = 72 \text{ and } 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 = 72)$$

38. Speed of tiger = 40 m/min

Speed of deer = 20 m/min

Relative speed = $40 - 20 = 20$ m/min

Difference in distances = $50 \times 8 = 400$ m

$$\therefore \text{Time taken in overtaking (or catching)} = \frac{400}{20} = 20 \text{ min}$$

$$\therefore \text{Distance travelled in 20 min} = 20 \times 40 = 800 \text{ m}$$

39. The sum of their speeds = $\frac{615}{15} = 43$ km/h

Notice that they are actually exchanging their speeds. Only then they can arrive at the same time at their respective destinations. It means the difference in speeds is 3 km/h.

$$\text{Thus, } x + (x + 3) = 43$$

$$\Rightarrow x = 20 \text{ and } x + 3 = 23$$

The concept is very similar to the case when after meeting each other they returned to their own places of departure. It can be solved through option also.

40. Let Pele covers x km in 1 hour. So Maradona takes $(2 \text{ h} - 40 \text{ min}) = 1 \text{ h } 20 \text{ min}$ to cover x km. Let speed of Maradona and Pele be M and P respectively then

$$x = M \times \frac{4}{3} \text{ and } x = P \times 1$$

$$\Rightarrow \frac{M}{P} = \frac{3}{4}$$

$$\text{Again } \frac{300}{M} - \frac{300}{P} = 1 \Rightarrow \frac{300}{3k} - \frac{300}{4k} = 1$$

$$\Rightarrow k = 25$$

$$\Rightarrow M = 3k = 75 \text{ km/h}$$

$$\text{and } P = 4k = 100 \text{ km/h}$$

(Through option it is very easy to solve.)

41. Initial speed of police = 10 m/s

Increased speed of police = 20 m/s

Speed of thief = 15 m/s

Initial difference between thief and police = 250 m

After 5 seconds difference between thief and police

$$= 250 - (5 \times 10) = 200 \text{ m}$$

After 10 seconds more the difference between thief and police = $200 + (5 \times 10) = 250$ m.

Now, the time required by police to catch the thief

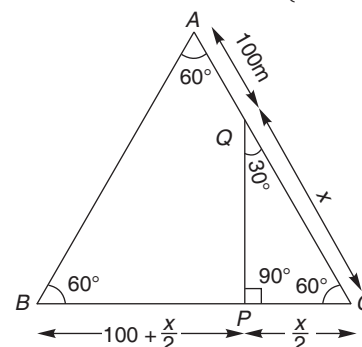
$$= \frac{250}{5} = 50 \text{ s}$$

Distance travelled = $50 \times 20 = 1000$ m

Total time = $50 + 15 = 65$ s

Total distance = $1000 + (15 \times 10) = 1150$ m

42.



$$\frac{100 + \frac{x}{2}}{100} = \frac{100 + x}{(100 + x - 150)} = \frac{\text{Speed of Bajrang}}{\text{Speed of Angad}}$$

$$\frac{200 + x}{200} = \frac{(100 + x)}{(x - 50)}$$

$$\Rightarrow (200 + x)(x - 50) = 200(100 + x)$$

$$\Rightarrow x^2 + 150x - 10000 = 20000 + 200x$$

$$\Rightarrow x^2 - 50x - 30000 = 0$$

$$\Rightarrow (x - 200)(x + 150) = 0 \Rightarrow x = 200 \text{ km}$$

Therefore distance between Ayodhya and Banaras is 300 km since $AB = BC = AC$.

(With the help of trigonometry we can find the value of PC in terms of x i. e., $\cos 60^\circ = \frac{PC}{QC} = \frac{1}{2}$.)

$$\text{Hence } PC = \frac{x}{2}$$

43. Basically they will exchange their speeds just after half of the time required for the whole journey. It means after covering 210 km distance they will exchange their speeds. Check it out graphically for more clarification.

44. The ratio of speeds

= The ratio of distances, when time is constant

$\therefore$ The ratio of distances covered by leopard to the tiger = 12 : 25

Again, ratio of rounds made by leopard to the tiger = 12 : 25

Hence, leopard makes 48 rounds, when tiger makes 100 rounds.

45. Length of $DC = \frac{6000}{13}$ (for this, refer geometry section)

Total distance covered in the returning by Jai

$$= AD + CD$$

$$= \frac{2500}{13} + \frac{6000}{13} = \frac{8500}{13} \text{ km}$$

$$\text{Required time} = \frac{8500/13}{500/13} = 17 \text{ h}$$

Total distance covered by Jaya while returning

$$= BD + DC$$

$$= \frac{14400}{13} + \frac{6000}{13}$$

$$\therefore \text{Required time} = \frac{20400/13}{1200/13} = 17$$

Hence, both will reach at the same time.

☞ Alternatively Since the ratio of speeds is same as that of distances. So, they will take same time to reach the home.

46. The distance of route $ADC = \frac{8500}{13}$
and the distance of route $BNC = 1300$
and the time taken by Jai is $\frac{8500/13}{500/13} = 17$ h
and the time taken by Jaya is $\frac{1300}{1200/13} = \frac{169}{12}$ h $= 14 \frac{1}{12}$ h
 $= 14$ h 05 min

Hence, option (c) is correct.

47. Time saved in percentage $= \frac{175}{1020} \times 100 = 17.15\%$
48. Husband takes 17 hours and she takes 14 h 05 min + 3h
 $= 17$ h 05 min
So, she becomes late by 05 min than her husband.
49. $x^2 + (x + 100)^2 = (500)^2$ (Using Pythagoras theorem)
 $\Rightarrow x = 300$ km
Now, let they change their speeds after t_1 hours and then the rest time is t_2 then
 $30t_1 + 40t_2 = 800$... (i)
 $40t_1 + 30t_2 = 900$... (ii)
On solving eqs. (i) and (ii), we get
 $t_1 = \frac{120}{7}$ and $t_2 = \frac{50}{7}$

50. Since it moves only one radian on every path and it has to move 2π radian to reach directly eastward. Hence, it has to run on more than 6 paths i.e., the last path is 7th one (or P_7) ($\because n \times 1 \text{ radian} \geq 2\pi \text{ radian}$)
 $\Rightarrow n \geq 2\pi$
or $n = 7$, for interger values
Hence, option (c) is correct.

51. Since it stops directly eastward of the shop so the total distance covered so for
 $= 7 + (1 + 2 + 3 + 4 + 5 + 6 + 2) = 30$ km

NOTE Total radial movement = 7 km
Again on the last path it will move only 2 km.

Actually it has to cover total 2π radian distance but on 6 paths it covers only 6 radian hence the remaining distance which will be covered on the 7th path i.e., $2\pi - 6$

$$= 2 \times \frac{22}{7} - 6 = \frac{2}{7} \text{ radian}$$

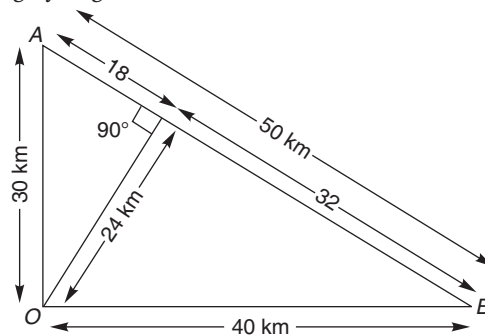
But, the radius of the last path (i.e., P_7) = 7 km

Hence, the distance covered in km $= \frac{2}{7} \times 7 = 2$ km

$$\left(\theta = \frac{\text{Arc}}{\text{Radius}} \right)$$

Thus, on the last path it moves only 2 km. Hence, (a) is the correct choice.

52. The ratio of distance covered on P_2 and $P_7 = \frac{2}{2} = \frac{1}{1}$
53. Since it is clear from the statement itself that $\triangle AOB$ is a right angle triangle and further OP must be perpendicular to AB then we can find that $AO = 30$ km and $BO = 40$ km by using Pythagoras theorem and its corollaries.



Hint $OP^2 = OA^2 - AP^2$
 $\therefore OP^2 = 900 - x^2$
and $OB^2 = OP^2 + BP^2$
 $\therefore OB^2 = 900 - x^2 + 1024$
and $AB^2 = OA^2 + OB^2$
 $\therefore (x + 32)^2 = 900 + 900 - x^2 + 1024$
 $\Rightarrow x = 18$ km

Hence, $AP = 18$, $OA = 30$ and $OB = 40$ and $OP = 24$ km

Now, since jackal and cat reaches A at the same time, so the ratio of speeds = ratio of distances covered by them.

$$\therefore \frac{\text{Speed of jackal}}{\text{Speed of cat}} = \frac{30}{18} = \frac{5}{3}$$

54. Again, since jackal and train both arrive at A at the same time and let the train was x km away from A, before entering into the tunnels, i.e., when it makes a whistle then the ratio of distances covered by train and jackal.

$$= \frac{x}{30} = \frac{x + 50}{40} \Rightarrow x = 150 \text{ km}$$

Thus, the ratio of speeds of Jackal is to train is 1 : 5.

55. Since, when the train arrive at A, the jackal can move 30 km. So, at the time when train is at A the jackal will cover 6 km from P on PA in addition to 24 km at OP. Now, the rest distance at AP is 12 km this remaining distance will be covered by train and jackal according to their respective speeds.

So, distance covered by train = $12 \times \frac{5}{6} = 10$ km

and distance covered by jackal = $12 \times \frac{1}{6} = 2$ km

Hence, jackal will meet with train at M_1 which is 10 km away from A (inside AB).

NOTE It can be solved using options in lesser time.

- 56.** It is obvious from the path of cat that if cat moves in the POA direction it will never meet with accident and now jackal follows the path OPB. Again when the train is at A then jackal will cover 30 km (i.e., 24 (OP) + 6 km on PB). So, the ratio of distances covered by jackal is to train = ratio of their respective speeds.

Now let the jackal and train meet each other at AB, $(6 + x)$ km away from P towards B, then

$$\frac{x}{24 + x} = \frac{1}{5}$$

$$\Rightarrow 4x = 24 \Rightarrow x = 6$$

Hence, train meets with jackal at $(18 + 6 + 6) = 30$ km away from A.

Alternatively $\frac{150 + 18 + 6 + x}{30 + x} = \frac{5}{1} \Rightarrow x = 6$

Hence, $18 + 6 + 6 = 30$ km.

Thus, option (b) is correct.

- 57.** The ratio of time taken by cat and jackal = $\frac{72/3}{96/5} = \frac{5}{4}$

Hence, option (c) is correct.

- 58.** $(6 - x) = (8 - 1.5x) \Rightarrow x = 4$ cm

So, it will take 4 hours to burn in such a way that they remain equal in length.

- 59.** Total distance covered by them when they meet = $2W$

and total time = $\frac{2W}{b_1 + b_2}$

$$\therefore d_1 = \frac{2W}{(b_1 + b_2)} b_1 \quad \text{and} \quad d_2 = \frac{2W}{(b_1 + b_2)} b_2$$

- 60.** Let the speed of boat be B and that of river be R . In 12 minutes the distance between boat and hat

$$= 12(B - R) + 12R = 12B$$

Now time taken by boat to reach to the hat

$$= \frac{12B}{(B + R) - R} = 12 \text{ min}$$

Total time = 24 min

In 24 minutes had flown off = 3 km

$$\therefore \frac{24}{60} \times R = 3 \Rightarrow R = 7.5 \text{ km/h}$$

- 61.** Akbar meets Birbal once = $\frac{500}{20 - 15} = 100$ s

$$\text{Birbal meets Chanakya once} = \frac{500}{20 + 25} = 11 \frac{1}{9} \text{ s}$$

$$\text{Akbar meets Chanakya once} = \frac{500}{15 + 25} = 12.5 \text{ s}$$

- 62.** Time taken by them to meet = $\frac{600}{30 - 20} = 60$ s

Time taken to meet 5th time = $5 \times 60 = 300$ s

$$\text{Total duration of race} = \frac{3000}{30} = 100 \text{ s}$$

So, they will not meet 5th time in the race of 3000 metre.

- 63.** Length of the track = $2 \times \frac{22}{7} \times 175 = 1100$ m

Distance to be covered for the first meeting = 550 m

$$\text{Speed of Akkal} = \frac{1100}{100} = 11 \text{ m/s}$$

$$\text{Speed of Bakkal} = \frac{1100}{50} = 22 \text{ m/s}$$

Time taken from the start of the first meeting

$$= \frac{550}{(11 + 22)} = \frac{50}{3} \text{ s}$$

Time taken for Akkal and Bakkal to meet again at Love point = LCM of times taken by them to go around the track once.

$$= \text{LCM of } \frac{1100}{11} \text{ and } \frac{1100}{22}$$

$$= \text{LCM of } 100 \text{ and } 50$$

$$= 100 \text{ s}$$

$$\begin{aligned} \text{So, the total required time} &= \frac{50}{3} + 100 + 100 \\ &= \frac{650}{3} \\ &= 216 \frac{2}{3} \text{ s} \end{aligned}$$

- 64.** Since both rest for 6 seconds so when B is just about to start the journey A reaches there at the shallow end so they meet at that shallow end.

- 65.** B runs around the track in 10 min.

i.e., Speed of B = 10 min per round

$\therefore$ A beats B by 1 round

Time taken by A to complete 4 rounds

$$= \text{Time taken by B to complete 3 rounds}$$

$$= 30 \text{ min}$$

$$\therefore \text{A's speed} = \frac{30}{4} \text{ min per round}$$

$$= 7.5 \text{ min per round}$$

Hence, if the race is only of one round A's time over the course = 7 min 30 sec.

- 66.** The ratio of speeds of A, B, C = $\frac{10}{49} : \frac{9}{50} : \frac{8}{51}$

Hence, A is the fastest.

- 67.** Speed of this car = $\frac{400 + 200}{20} \times \frac{18}{5} \text{ km/h}$
= 108 km/h

- 68.** The speeds of two persons is 108 km/h and 75 km/h. The first person covers 1080 km in 10 hours and thus he makes 12 rounds. Thus, he will pass over another person 12 times in any one of the direction.

69. Angle between two hands at 3 : 10 am

$$= (90 + 5) - 60 = 35^\circ$$

So, the required angle = 70° , after 3 : 10 am

Total time required to make 70° angle when minute-hand

$$\text{is ahead of hour-hand} = \frac{90 + 70}{11/2} = \frac{320}{11} \text{ min}$$

So, at 3 h $\frac{320}{11}$ min the required angle will be formed.

☞ **Alternatively** Check through options.

70. **For the first watch:** When a watch creates the difference of 12 hours, it shows correct time.

So to create the difference of 12 h required time

$$= \frac{60 \times 12}{24} = 30 \text{ days}$$

For the second watch: To create the difference of 12 h required time

$$= \frac{30 \times 12}{24} = 15 \text{ days}$$

So, after 30 days at the same time both watches show the correct time.

Hint Take the LCM of 30 and 15.

71. To show the same time together the difference between two watches must be 12 h.

Now, since they create 3 min difference in 1 h

$$\text{So they will create 12 h difference in } \frac{1}{3} \times \frac{12 \times 60}{24}$$

$$= 10 \text{ days later}$$

72. To show the correct time again, watch must create 24 h difference. (Since in one round hour-hand covers 24 h.)

$$\text{So, the required time} = \frac{4}{3} \times \frac{60 \times 24}{24} = 80 \text{ day}$$

73. $(n + 1)$ times in n days

74. Actually the watch gains $(12 + 16)$

$$= 28 \text{ min in } 7 \times 24 \times 60 \text{ min.}$$

Thus, it gains 1 min in 360 minutes.

$$\text{Therefore, it will gain } (12 + 8) \text{ min in } \frac{20 \times 360}{60 \times 24} = 5 \text{ day}$$

Hence, (b) is the correct choice.

75. Actually they create a difference of 3 min per hour and the two watches are showing a difference of 66 minutes. Thus, they must have been corrected 22 hours earlier.

Now, the correct time can be found by comparing any one of the watch. Since, second watch gains 1 min in 1 hour so it will must show 22 min extra than the correct time in 22 hours. Hence, the correct time can be found by subtracting 22 min from 10 : 06.

Hence, (d) is the correct answer.

☞ **NOTE** For quick answer go through options.

76. Incorrect watch covers 1452 min in 1440 min

$$\text{So, it will cover 1 min in } \frac{1440}{1452} \text{ min}$$

$$\text{Therefore it will cover 4840 min in } \frac{1440}{1452} \times 4840$$

$$= 4800 \text{ min} = 80 \text{ h}$$

Therefore 80 h = 3 days and 8 h.

77. You must know that a correct watch coincide just after $65\frac{5}{11}$ min.

Therefore in every $65\frac{5}{11}$ hours the watch gains $\frac{2}{11}$.

$$\text{Hence, in 24 hours it will gain } \frac{2}{11} \times \frac{11}{720} \times 24 \times 60 = 4 \text{ min}$$

78. In 72 hours my watch gains $(8 + 7) = 15$ min. To show the correct time watch must gain 8 minutes.

Since the watch gains 15 min in 72×60 min.

$$\text{Therefore, the watch will gain 8 min in } \frac{72 \times 60 \times 8}{15} \text{ min}$$

$$= \frac{72 \times 60 \times 8}{15} = 38 \text{ h } 24 \text{ min}$$

Hence, (a) is the correct choice.

80. To exchange the position both hands to cover 360° together.

In one minute, hour-hand moves $\frac{1^\circ}{2}$ and in one minute,

minute-hand moves 6° . Let the required time be t min,

$$\text{then } 6t + \frac{1}{2}t = 360$$

$$\Rightarrow t = \frac{360}{13} \times 2 = \frac{720}{13} = 55\frac{5}{13} \text{ min}$$

81. Consider the following explanation.

	Action	Time
1	Kalmadi and Raja cross the river with the boat.	90
2	Kalmadi stays behind the other side, while Raja takes the boat back.	90
3	Raja then waits on shore, while Sharad takes the boat across by himself.	90
4	Then Sharad stays on the other side, while Kalmadi brings the boat back.	90
5	Raja gets in the boat with Kalmadi, and then both of them cross the river to join Sharad.	90

$$\text{Thus the total time needed} = 90 \times 5$$

$$= 450 \text{ min} = 7\text{hrs } 30 \text{ min.}$$

Hence, choice (c) is the correct answer.

82. A level mile takes 15 min, up-hill 20 min, down-hill 10 min. So in going and coming back a mile on a level road takes half an hour, and walking up-hill and down-hill a mile also takes half an hour.

So, in going and returning over the same mile, whether on the level or on the hill-side, takes half an hour. Hence in six hours he walked 12 miles on each side. Thus the total distance he travelled = 24 miles.

Hence, choice (d) is the correct answer.

83. Let the length of the rail-bridge be x meters. Running towards the train, Rohingya covers $(0.5x - 16)$ meters in the time that the train travels $(x - 20)$ meters. Running away from the train, Rohingya covers $(0.5x + 6)$ meters in the time that the train travels $(2x - 10)$ meters.

Because their speeds are constant, the following holds :

$$\frac{(0.5x - 16)}{(x - 20)} = \frac{0.5x + 6}{(2x - 10)}$$

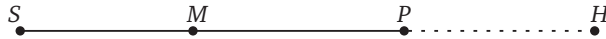
$$\Rightarrow x = \{10, 56\}$$

But, since bridge is longer than 10 meters, so the only valid length of the rail-bridge is 56 meters.

Hence, choice (b) is the correct one.

84. Let S denotes the spot where the victim was dumped, P denotes the location of police control room and H denotes the location of hospital. Also, M is the point where auto-rickshaw and PCR van meets, in case auto-rickshaw helps the victim.

If PCR van goes from P to S , it takes 45 minutes. But, if auto-rickshaw takes the victim to M , then PCR van doesn't need to go beyond M , and thus the PCR van saves total 30 minutes. It implies that the van takes 15 minutes from M to S and 15 minutes from S to M .



The distance between P and S

$$= 80 \times \frac{45}{60} = 60 \text{ km}$$

And the distance between S and M

$$= 80 \times \frac{15}{60} = 20 \text{ km}$$

And the distance between P and M

$$= 80 \times \frac{30}{60} = 40 \text{ km}$$

Since, the auto-rickshaw and PCR van start moving at the same time from their respective locations S and P , so they take the same time to reach M .

As you know now, that the time taken by both the auto-rickshaw and PCR van is same

So, their speeds will be in the ratio of the distances traversed to reach M .

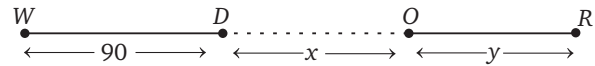
The distances traversed by van and auto-rickshaw are 40 km and 20 km, respectively.

So the speed of van and auto-rickshaw would be 80 and 40, respectively.

Hence, choice (c) is the correct one.

85. In the following diagram, W , D and R denote Wasseyapur, Dhanbad and Ranchi, respectively. O denotes the place where Faizal overtook Danish and handed over bullets and bombs. Assume $DO = x$ and $RO = y$.

The distance between W and D is 90 km and the distance between D and R is also 90 km.



Let the speeds of Faizal and Danish be f and g .

Now, you know that when Faizal covers $(90 + x)$ km,

Danish covers x km, then $\frac{f}{g} = \frac{90 + x}{x}$

Similarly, when Faizal covers $(90 + 2x)$ km, Danish covers

90 km, then $\frac{f}{g} = \frac{90 + 2x}{90}$

Therefore, you have $\frac{90 + x}{x} = \frac{90 + 2x}{90}$

$$\Rightarrow x = 45\sqrt{2} \text{ km}$$

Thus, the total distance travelled by Faizal Khan = $90 + 2x = 90(\sqrt{2} + 1)$ km.

Hence, choice (c) is the correct one.

CHAPTER 10

Mensuration

It is one of the easiest chapters, which contributes almost 6-8% problems in Quantitative Aptitude Section of CAT. Besides there are several other aptitude tests which include plethora of questions from this topic itself.

Therefore it is advised that those students who are not so good in other sections such as algebra or sort of logical questions they must emphasise on this chapter. Even the questions asked from this chapter are not as much complex as they are in Geometry.

10.1 Mensuration

Definition : Mensuration is a science of measurement of the lengths of lines, areas of surfaces and volumes of solids.

Planes : Planes are two dimensional i.e. these two dimensions are namely length and breadth. These occupy surface.

Solids : Solids are three dimensional, namely length, breadth and height. These occupy space.

Conversion of Important Units

$$\begin{aligned} 1 \text{ km} &= 10 \text{ hectometre} &= 100 \text{ decametre} \\ &= 1000 \text{ metre} &= 10,000 \text{ decimetre} \\ &= 1,00,000 \text{ centimetre} &= 10,00,000 \text{ millimetre} \end{aligned}$$

$$1 \text{ hectare} = 10,000 \text{ square metre}$$

$$1 \text{ are} = 100 \text{ square metre}$$

$$1 \text{ square hectometre} = 100 \text{ square decametre}$$

$$1 \text{ square decametre} = 100 \text{ square metre}$$

$$1 \text{ square metre} = 100 \text{ square decimetre}$$

$$1 \text{ square decimetre} = 100 \text{ square centimetre}$$

$$1 \text{ square centimetre} = 100 \text{ square millimetre}$$

$$\sqrt{2} = 1.414, \quad \sqrt{3} = 1.732, \quad \sqrt{5} = 2.236, \quad \sqrt{6} = 2.45$$

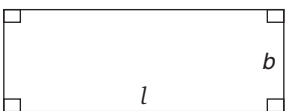
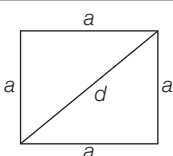
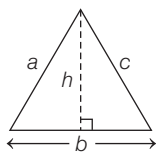
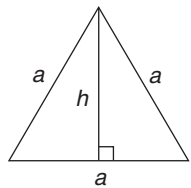
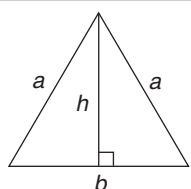
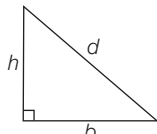
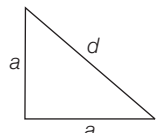
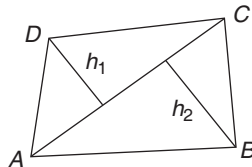
$$\text{Weight} = \text{Volume} \times \text{density}$$

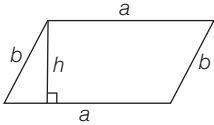
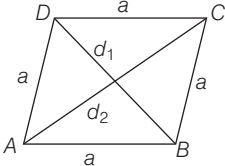
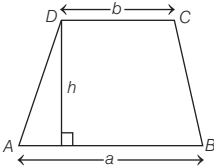
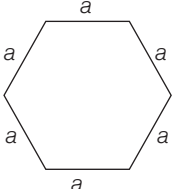
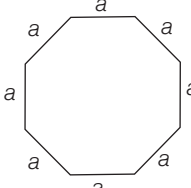
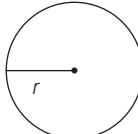
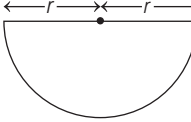
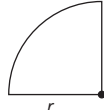
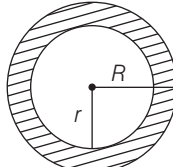
Chapter Checklist

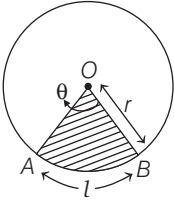
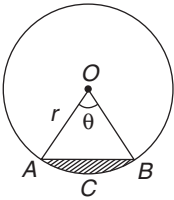
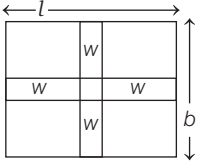
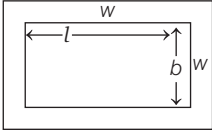
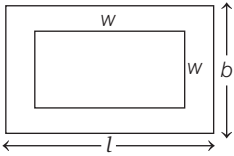
- Mensuration
- 2-D Figures (Planes)
- Rectangles and Squares
- Triangles
- Parallelogram, Rhombus and Trapezium
- Circles
- 3-D Figures (Solids)
- Cuboid and Cube
- Cylinder and Cone
- Sphere, Prism and Pyramid
- CAT Test

10.2 2-D Figures

Table 2-D Figures (Plane figures)

Name	Figure	Nomenclature	Area	Perimeter
Rectangle		$l \rightarrow$ length $b \rightarrow$ breadth	$l \times b = lb$	$2l + 2b = 2(l + b)$
Square		$a \rightarrow$ side $d \rightarrow$ diagonal $d = a\sqrt{2}$	(i) $a \times a = a^2$ (ii) $\frac{d^2}{2}$	$a + a + a + a = 4a$
Triangle (Scalene)		a, b and c are three sides of triangle and s the semiperimeter, where $s = \left(\frac{a + b + c}{2}\right)$ b is the base and h is the altitude of triangle	(i) $\frac{1}{2} \times b \times h$ (ii) $\sqrt{s(s-a)(s-b)(s-c)}$ (Heron's formula)	$a + b + c = 2s$
Equilateral triangle		$a \rightarrow$ side $h \rightarrow$ height or altitude $h = \frac{\sqrt{3}}{2} a$	(i) $\frac{1}{2} \times a \times h$ (ii) $\frac{\sqrt{3}}{4} a^2$	$3a$
Isosceles triangle		$a \rightarrow$ equal sides $b \rightarrow$ base $h \rightarrow$ height or altitude $h = \frac{\sqrt{4a^2 - b^2}}{2}$	(i) $\frac{1}{2} \times b \times h$ (ii) $\frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$	$2a + b$
Right angled triangle		$b \rightarrow$ base $h \rightarrow$ altitude/height $d \rightarrow$ diagonal $d = \sqrt{b^2 + h^2}$	$\frac{1}{2} \times b \times h$	$b + h + d$
Isosceles right angled triangle		$a \rightarrow$ equal sides $d \rightarrow$ diagonal $d = a\sqrt{2}$	$\frac{1}{2} a^2$	$2a + d$
Quadrilateral		AC is the diagonal and h_1, h_2 are the altitudes on AC , from the vertices D and B , respectively.	$\frac{1}{2} \times AC \times (h_1 + h_2)$	$AB + BC + CD + AD$

Name	Figure	Nomenclature	Area	Perimeter
Parallelogram		a and b are sides adjacent to each other. $h \rightarrow$ distance between the parallel sides	$a \times h$	$2(a + b)$
Rhombus		$a \rightarrow$ each equal side of rhombus d_1 and d_2 are the diagonals $d_1 \rightarrow BD$ $d_2 \rightarrow AC$	$\frac{1}{2} \times d_1 \times d_2$	$4a$
Trapezium		a and b are parallel sides to each other and h is the perpendicular distance between parallel sides	$\left(\frac{a + b}{2}\right) \times h$	$AB + BC + CD + AD$
Regular hexagon		$a \rightarrow$ each of the equal side	$\frac{3\sqrt{3}}{2} a^2$	$6a$
Regular octagon		$a \rightarrow$ each of equal side	$2a^2 (1 + \sqrt{2})$	$8a$
Circle		$r \rightarrow$ radius of the circle $\pi = \frac{22}{7} = 3.1416$ (approx)	πr^2	$2\pi r$ (called as circumference)
Semicircle		$r \rightarrow$ radius of the circle	$\frac{1}{2} \pi r^2$	$\pi r + 2r$
Quadrant		$r \rightarrow$ radius	$\frac{1}{4} \pi r^2$	$\frac{1}{2} \pi r + 2r$
Ring or circular path (shaded region)		$R \rightarrow$ outer radius $r \rightarrow$ inner radius	$\pi (R^2 - r^2)$	(outer) $\rightarrow 2\pi R$ (inner) $\rightarrow 2\pi r$

Name	Figure	Nomenclature	Area	Perimeter
Sector of a circle		$O \rightarrow$ centre of the circle $r \rightarrow$ radius $l \rightarrow$ length of the arc $\theta \rightarrow$ angle of the sector $l = 2\pi r \left(\frac{\theta}{360^\circ} \right)$	(i) $\pi r^2 \left(\frac{\theta}{360^\circ} \right)$ (ii) $\frac{1}{2} r \times l$	$l + 2r$
Segment of a circle		$\theta \rightarrow$ angle of the sector $r \rightarrow$ radius $AB \rightarrow$ chord $ACB \rightarrow$ arc of the circle	Area of segment ACB (minor segment) $= r^2 \left(\frac{\pi\theta}{360^\circ} - \frac{\sin \theta}{2} \right)$	$2r \left[\frac{\pi\theta}{360^\circ} + \sin \left(\frac{\theta}{2} \right) \right]$
Pathways running across the middle of a rectangle		$l \rightarrow$ length $b \rightarrow$ breadth $w \rightarrow$ width of the path (road)	$(l + b - w) w$	$2(l + b) - 4w$ $= 2[l + b - 2w]$
Outer pathways		$l \rightarrow$ length $b \rightarrow$ breadth $w \rightarrow$ widthness of the path	$(l + b + 2w) 2w$	(inner) $\rightarrow 2(l + b)$ (outer) $\rightarrow 2(l + b + 4w)$
Inner path		$l \rightarrow$ length $b \rightarrow$ breadth $w \rightarrow$ widthness of the path	$(l + b - 2w) 2w$	(outer) $\rightarrow 2(l + b)$ (inner) $\rightarrow 2(l + b - 4w)$

10.3 Rectangles and Squares

1. Area of a rectangle = length $\times$ breadth = $l \times b$

2. Area of a square = (side) $^2 = \frac{1}{2}$ (diagonal) ^{2}

$$= \frac{1}{2} d^2 = a^2$$

3. Diagonal of a rectangle

$$= \sqrt{(\text{length})^2 + (\text{breadth})^2} = \sqrt{l^2 + b^2}$$

4. Diagonal of a square = $\sqrt{\text{side}^2 + \text{side}^2} = \text{side} \sqrt{2} = a\sqrt{2}$

5. Perimeter of a rectangle = $2(\text{length} + \text{breadth}) = 2(l + b)$

6. Perimeter of a square = $4 \times \text{side} = 4a$

7. Area of four walls of a room = $2(l + b) \times h$

Exp. 1) The length and breadth of a rectangular room are 15 m and 8 m respectively :

(a) Find the perimeter of room.

(b) Find the area of the floor of room.

(c) Find the maximum possible length of the rod that can be put on the floor.

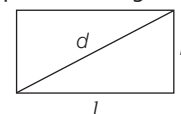
Solution

(a) Perimeter = $2(15 + 8) = 46$ m

(b) Area of floor = $15 \times 8 = 120$ m ^{2}

(c) Length of diagonal = $\sqrt{(15)^2 + (8)^2} = 17$ m

NOTE The maximum possible length of any rod that can be placed on a rectangular floor is equal to the diagonal of the floor of the room.



Here $d > l$

Exp. 2) The side of a square shaped garden is 20 m. Find the :

(a) area of the garden

(b) perimeter (or boundary) of the garden

(c) maximum possible distance between any two corners of the garden.

Solution (a) Area = (side) $^2 = (20)^2 = 400$ m ^{2} (square metre)

(b) Perimeter = $4 \times \text{side} = 4 \times 20 = 80$ m

(c) Diagonal = side $\sqrt{2} = 20 \times \sqrt{2} = 20 \times 1.414 = 28.28$ m

Exp. 3) One side of a rectangular lawn is 12 m and its diagonal is 13 m. Find the area of the field.

Solution $d = \sqrt{l^2 + b^2} \Rightarrow 13 = \sqrt{12^2 + b^2} \Rightarrow b = 5 \text{ m}$

$$\therefore \text{Area} = l \times b = 12 \times 5 = 60 \text{ m}^2$$

Exp. 4) The length of a rectangle is 1 cm more than its breadth. The diagonal is 29 cm. Find the area of the rectangle.

- (a) 481 cm² (b) 841 cm² (c) 420 cm² (d) 870 m²

Solution $l = (b + 1)$

$$\text{and } d = \sqrt{l^2 + b^2} = \sqrt{(b+1)^2 + b^2} = 29$$

$$\Rightarrow b^2 + b = 420 \Rightarrow b = 20$$

$$\therefore l = 21$$

$$\therefore \text{Area} = l \times b = 420 \text{ cm}^2$$

Exp. 5) The length of a wall is $\frac{5}{4}$ times of its height. If the area of the wall be 180 m². What is the sum of the length and height of the wall?

Solution Let the length be $5x$ and height be $4x$.

$$\text{Then, } l \times h = 180 = 5x \times 4x = 20x^2 \Rightarrow x = 3$$

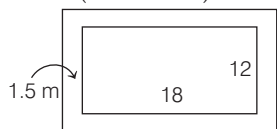
$$\therefore l + h = 15 + 12 = 27 \text{ m}$$

☞ **Alternatively** $1.25 h^2 = 180 \Rightarrow h = 12 \text{ m}$

$$\therefore l = 15 \text{ m; } l + h = 27 \text{ m}$$

Exp. 6) A rectangular grassy lawn is 18 m by 12 m. It has a gravel path 1.5 m wide all around it on the outside. What is the area of the path.

Solution Area of path (outside the lawn) = $(l + b + 2w) 2w$
 $= (18 + 12 + 3) 3 = 99 \text{ m}^2$



☞ **Alternatively** Area of lawn = $18 \times 12 = 216 \text{ m}^2$

$$\text{Total area (lawn + path)} = (18 + 3) \times (12 + 3) \\ = 21 \times 15 = 315 \text{ m}^2$$

$$\therefore \text{Area of path (only)} = 315 - 216 = 99 \text{ m}^2$$

Exp. 7) Find the cost of carpeting a room 17 m long and 9 m wide with a carpet 60 cm broad at 40 paise per metre.

Solution Area of carpet = Area of room

$$l \times 0.6 = 17 \times 9 \quad (60 \text{ cm} = 0.6 \text{ m})$$

$$\Rightarrow l = 255 \text{ m}$$

$$\therefore \text{Cost of carpeting the floor} = \text{rate} \times \text{length of carpet} \\ = 0.4 \times 255 = ₹ 102.00$$

Exp. 8) The dimensions of a lawn are in the ratio 4:1 and its area is $\frac{1}{4}$ hectares. What is the length of the lawn?

Solution $l \times b = 4x \times x = \frac{1}{4} \times 10000$

$$\Rightarrow x = 25$$

$$\Rightarrow \text{length} = 4x = 100 \text{ m}$$

Exp. 9) A room is 16 m long, 7 m broad and 8 m high. Find the cost of white washing the four walls of room at ₹ 7.5 per m², white washing is not to be done on the doors and windows, which occupy 65 m².

Solution Area of 4 walls of a room = $2(16 + 7) \times 8 = 368 \text{ m}^2$

$$\text{Net area of 4 walls} = 368 - 65 = 303 \text{ m}^2$$

$$\therefore \text{Cost of white washing} = 303 \times 7.5 = ₹ 2272.5$$

Exp. 10) The ratio between the sides of a room is 5:3. The cost of white washing the ceiling of the room at 50 Paise per square m is ₹ 270 and the cost of papering the walls at 10 P per square metre is ₹ 48. The height of the room is :

(a) 6 m (b) 8 m

(c) 5 m (d) 10 m

Solution Area of ceiling = $\frac{\text{Total cost}}{\text{Cost of 1 sq unit}} = \frac{270}{0.5} = 540 \text{ sq m}$

$$\text{Now since } l : b = 5x : 3x$$

$$\Rightarrow l \times b = 15x^2 = 540 \text{ m}^2$$

$$\Rightarrow l = 30 \text{ and } b = 18 \text{ m}$$

Now, Area of the 4 walls

$$= \frac{\text{Total cost}}{\text{Cost of 1 sq unit}} = \frac{48}{0.1} = 480 \text{ m}^2$$

$$\therefore \text{Height} = \frac{480}{2(30 + 18)} = 5 \text{ m}$$

Introductory Exercise 10.1

- A rectangular field has its length and breadth in the ratio of 16 : 9. If its perimeter is 750 cm. What is its area?
(a) 7500 cm^2 (b) 32400 cm^2
(c) 14400 cm^2 (d) 14000 cm^2
- A rectangular field costs ₹ 110 for levelling at 50 paise per square metre. If the ratio of length : breadth is 11 : 5. Find the length of the field.
(a) 16 m (b) 21 m
(c) 22 m (d) none of these
- A room is half as long again as it is wide. The cost of carpeting it at 62 paise per square metre is ₹ 2916.48. Find the cost of white washing the ceiling at 30 paise per metre.
(a) ₹ 2211.5 (b) ₹ 1114.2
(c) ₹ 1411.2 (d) can't be determined
- The length of a rectangular plot of ground is four times its breadth and its area is 4 hectares. How long will it take to a dog to walk round it at the rate of 3 km/h?
(a) 12 min (b) 20 min (c) 21 min (d) 18.5 min
- Find the length of the wire required to fence a square field 6 times having its area 5 hectares and 76 ares.
(a) 5760 m (b) 6760 m (c) 52500 m (d) 11760 m
- A room is 19 m long and 3.50 m broad. What will be the cost of covering its floor with a carpet of 70 cm wide at 95 paise per metre?
(a) ₹ 90.25 (b) ₹ 99.25
(c) ₹ 90.75 (d) none of these
- Find the cost of paving a courtyard $316.8 \text{ m} \times 65 \text{ m}$ with stones measuring $1.3 \text{ m} \times 1.1 \text{ m}$ at ₹ 0.5 per stone.
(a) ₹ 1440 (b) ₹ 7200
(c) ₹ 72,000 (d) none of these
- A rectangular garden 63 m long and 54 m broad has a path 3 m wide inside it. Find the cost of paving the path at ₹ $37/2$ per square metre.
(a) ₹ 12321 (b) ₹ 11100
(c) ₹ 74000 (d) none of these
- If the length of a rectangular field is doubled and its breadth is halved (i.e. reduced by 50%). What is percentage change in its area?
(a) 0% (b) 10% (c) 25% (d) 33.33%
- A path of uniform width runs all around the inside of rectangular field 116 m by 68 m and occupies 720 sq m . Find the width of the path.
(a) 1 m (b) 1.5 m (c) 2 m (d) 4 m
- A drawing room is 7.5 m long 6.5 m broad and 6 m high. Find the length of paper 2.5 dm wide to cover its walls allowing 8 sq m for doors.
(a) 368 m (b) 640 m (c) 625 m (d) 888 m
- A square field of 2 sq km is to be divided into two equal parts by a wall which coincides with a diagonal. Find the length of the wall.
(a) $\sqrt{2} \text{ km}$ (b) 1 km (c) 4.2 km (d) 2 km
- There are two square fields. Of the two square fields one contains 1 hectare area while the other is broader by 11 per cent. Find the difference in area expressed in sq m .
(a) 2321 sq m (b) 1210 sq m
(c) 2121 sq m (d) 7700 sq m
- The expenses of carpeting a half of the floor were ₹ 759, but if the length had been 6 m less than it was, the expenses would have been ₹ 561. What is the length?
(a) 21 m (b) 23 m (c) 45 m (d) 27 m
- If a roll of paper 1 km long has area $1/25$ hectare, how wide is the paper?
(a) 4 m (b) 40 cm (c) 40 dm (d) 25 cm
- The diagonal and one side of a rectangular plot are 289 m and 240 m respectively. Find the other side.
(a) 237 m (b) 181 m (c) 161 m (d) 159 m
- How many tiles 20 cm by 40 cm will be required to pave the floor of a prayer hall of a room 16 m long and 9 m wide?
(a) 18000 (b) 2700 (c) 1800 (d) 14400
- If the area of a square be 22050 sq cm . Find the length of diagonal.
(a) 201 cm (b) 220 cm
(c) 211 cm (d) 210 cm
- If requires 90 g paint for painting a door $12 \text{ cm} \times 9 \text{ cm}$, how much paint is required for painting a similar door $4 \text{ cm} \times 3 \text{ cm}$?
(a) 30 g (b) 27 g (c) 10 g (d) 45 g
- The area of a rectangular football field is 24200 sq m . It is half as broad as it is long. What is the approx minimum distance a man will cover if he wishes to go from one corner to the opposite one?
(a) 283 m (b) 246 m (c) 576 m (d) 289 m
- The area of the four walls of a room is 2640 sq m and the length is twice the breadth and the height is given as 11 m. What is the area of the ceiling?
(a) 2800 m^2 (b) 3200 m^2
(c) 320 m^2 (d) none of these
- If the perimeter of a square and a rectangle are the same, then the areas A and B enclosed by them would satisfy the inequality :
(a) $A > B$ (b) $A \geq B$ (c) $A < B$ (d) $A \leq B$

23. If the perimeter of a rectangle and a square each is equal to 80 cm and the difference of their areas is 100 sq cm, the sides of the rectangle are :
 (a) 25 cm, 15 cm (b) 28 cm, 12 cm
 (c) 30 cm, 10 cm (d) 35 cm, 15 cm
24. The number of square shaped tin sheets of side 25 cm that can be cut off from a square tin sheet of side 1 m, is :
 (a) 4 (b) 40 (c) 16 (d) 400
25. The length and breadth of a rectangular field are 120 m and 80 m respectively. Inside the field, a park of 12 m width is made around the field. The area of the park is :
 (a) 2358 m² (b) 7344 m² (c) 4224 m² (d) 3224 m²
26. A 5m wide lawn is cultivated all along the outside of a rectangular plot measuring 90m × 40 m. The total area of the lawn is :
 (a) 1441 m² (b) 1400 m² (c) 2600 m² (d) 420 m²
27. The length of a rectangle is 2 cm more than its breadth. The perimeter is 48 cm. The area of the rectangle (in cm²) is :
 (a) 96 (b) 128 (c) 143 (d) 144
28. The area of a rectangular field is 52000 m². This rectangular area has been drawn on a map to the scale 1 cm to 100 m. The length is shown as 3.25 cm on the map. The breadth of the rectangular field is :
 (a) 210 m (b) 150 m
 (c) 160 m (d) 123 m
29. If the length of diagonal BD of a square $ABCD$ is 4.8 cm, the area of the square $ABCD$ is :
 (a) 9.6 cm² (b) 11.52 cm²
 (c) 12.52 cm² (d) 5.76 cm²
30. If the side of square is increased by 20%, then how much per cent does its area get increased?
 (a) 40% (b) 20% (c) 44% (d) 24%
31. The ratio of the area of a square to that of the square drawn on its diagonal is :
 (a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 4
32. The length and breadth of a square are increased by 60% and 40% respectively. The area of the resulting rectangle exceeds the area of the square by :
 (a) 224% (b) 24%
 (c) 124% (d) 100%

10.4 Triangles

1. Area of a triangle = $\frac{1}{2} \times \text{base} \times \text{height}$ (General formula)
2. Area of a scalene triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
 where s is the semiperimeter of the triangle and a, b and c are the three sides of the triangle,
 $\left(s = \frac{a+b+c}{2}\right)$
- Scalene triangle :** A triangle whose all sides are different in length
3. Area of a right angle triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

Right angle triangle : A triangle in which two sides are perpendicular

Also, Hypotenuse = $\sqrt{(\text{base})^2 + (\text{height})^2}$

4. Area of an equilateral triangle

$$= \frac{1}{2} \times \text{side} \times \left(\frac{\sqrt{3}}{2} \text{side}\right) = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

Equilateral triangle : A triangle in which all the three sides are equal also all the three internal angles are equal

Height of an equilateral triangle (h) = $\frac{\sqrt{3}}{2} \times \text{side}$

An important property : In an equilateral Δ perpendiculars drawn from all the three vertices intersect each other in the

ratio of 2 : 1 from the vertex to the base.

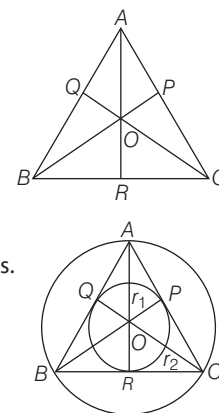
$$\therefore \frac{AO}{OR} = \frac{BO}{OP} = \frac{CO}{OQ} = \frac{2}{1} = \frac{r_2}{r_1}$$

$$OP = OQ = OR, \text{ all are the inradii}$$

$$OA = OB = OC,$$

all are the circumradii

NOTE Radii means more than one radius.
 O is the centre of two circles. Circle PQR is called as incircle (touching the sides) and circle ABC is called as circumcircle (touching the vertices)



$$\therefore \text{Inradius} = \frac{\text{side}}{2\sqrt{3}} \text{ and Circumradius} = \frac{\text{side}}{\sqrt{3}}$$

Exp. 1) The base of a right angled triangle is 8 cm and hypotenuse is 17 cm. Find its area.

Solution Hypotenuse = $\sqrt{(\text{base})^2 + (\text{altitude})^2}$

$$(17) = \sqrt{64 + (x)^2}$$

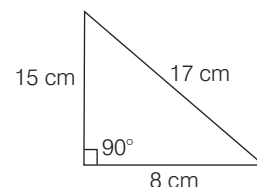
$$\Rightarrow 289 = 64 + x^2$$

$$\Rightarrow x^2 = 225 \Rightarrow x = 15 \text{ cm}$$

$$\therefore \text{Area} = \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$= \frac{1}{2} \times 8 \times 15 = 60 \text{ cm}^2$$

(height is called altitude also)



Exp. 2) Find the area of an equilateral triangle whose side is $4\sqrt{3}$ cm.

Solution Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{side})^2$
 $= \frac{\sqrt{3}}{4} \times (4\sqrt{3})^2 = \frac{\sqrt{3}}{4} \times 48 = 12\sqrt{3} \text{ cm}^2$

Exp. 3) The area of a right angled triangle is 24 cm^2 and the length of its hypotenuse is 10 cm. The length of the shorter leg is :

- (a) 5 cm (b) 4 cm
(c) 6 cm (d) 3 cm

Solution $H^2 = A^2 + B^2 = 100 \text{ cm}^2$

Area $= \frac{1}{2} \times A \times B$

$\Rightarrow \frac{1}{2} A \times B = 24 \text{ cm}^2$

$\Rightarrow A \times B = 48 \text{ cm}^2$

$\therefore (A + B)^2 = A^2 + B^2 + 2AB$

$(A + B)^2 = 100 \Rightarrow A + B = 10$

Again $(A - B)^2 = A^2 + B^2 - 2AB$

$\Rightarrow (A - B)^2 = 4 \Rightarrow A - B = 2$

Therefore by solving eqs. (i) and (ii), we get

$A = 8 \text{ and } B = 6$

Therefore, the shorter leg is 6 cm.

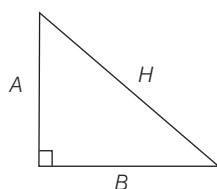
Alternatively Go through options.

$\therefore \frac{1}{2} \times A \times B = 24 \Rightarrow A \times B = 48$

Let us assume $B = 6$, then $A = \frac{48}{6} = 8$

Now, $A^2 + B^2 = 100, (8)^2 + (6)^2 = 100 \Rightarrow 100 = 100$

Hence, choice (c) is correct.



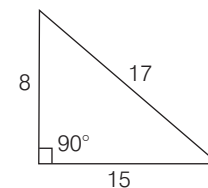
Exp. 4) Find the area and perimeter of a triangle whose sides are 17 cm, 8 cm and 15 cm long.

Solution Since, $8^2 + 15^2 = 17^2$
 $\Rightarrow 64 + 225 = 289$
 $289 = 289$

Hence, it is a right angled triangle.

$\therefore \text{Area} = \frac{1}{2} \times b \times h$
 $= \frac{1}{2} \times 15 \times 8 = 60 \text{ cm}^2$

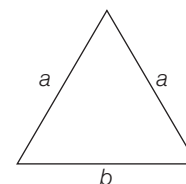
and perimeter $= 8 + 15 + 17 = 40 \text{ cm}$



Exp. 5) Find the area and perimeter of an isosceles triangle whose equal sides are 5 cm each and base is 6 cm.

Solution Area of an isosceles triangle

$= \frac{b}{4} \sqrt{4a^2 - b^2}$
 $= \frac{6}{4} \sqrt{4 \times 25 - 36}$
 $= \frac{3}{2} \times \sqrt{64} = 12 \text{ cm}^2$



Alternatively We know that the altitude CD bisects the base AB in the isosceles triangle ABC.

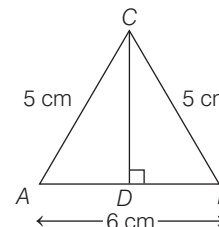
$\therefore AD = BD = 3 \text{ cm}$

Using Pythagoras theorem in $\triangle ADC$ we have,

$CD^2 = AC^2 - AD^2$

$\therefore CD = 4 \text{ cm}$

$\therefore \text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$
 $= \frac{1}{2} \times 6 \times 4 = 12 \text{ cm}^2$



Also, the perimeter of triangle $= 5 + 5 + 6 = 16 \text{ cm}$

Introductory Exercise 10.2

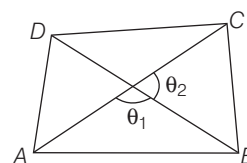
- What is the area of the triangle whose sides are 84 m, 80 m and 52 m?
 (a) 1620 sq. m (b) 2016 sq. m
 (c) 1818 sq. m (d) none of these
- Two poles 15 m and 30 m high stand upright in a play ground. If their feet be 36 m apart, find the distance between their tops.
 (a) 41 m (b) 36 m
 (c) 39 m (d) none of the above
- The sides of a triangle are 25 m, 39 m and 56 m respectively. Find the perpendicular distance from the vertex opposite to the side 56 m.
 (a) 15 m (b) 16.5 m
 (c) 18.6 m (d) 21 m
- ABC is a triangle and D, E, F are the mid-points of the sides BC, CA, AB respectively. The ratio of the areas of $\triangle ABC$ and $\triangle DEF$ is :
 (a) 4 : 1 (b) 5 : 1
 (c) 3 : 1 (d) can't be determined

5. The integral base of an isosceles triangle can be whose area is 60 cm^2 and the length of one of the equal sides is 13 cm :
 (a) 20 cm (b) 10 cm
 (c) 16 cm (d) data insufficient
6. A ladder is resting with one end in contact with the top of a wall of height 60 m and the other end on the ground is at a distance of 11 m from the wall. The length of the ladder is :
 (a) 61 m (b) 71 m
 (c) 87 m (d) none of these
7. The base of a triangular field is three times its height. If the cost of cultivating the field is ₹ 36.72 per hectare is ₹ 495.72, find the height and base of the triangular field.
 (a) 480 m, 1120 m (b) 400 m, 1200 m
 (c) 300 m, 900 m (d) 250 m, 650 m
8. If every side of a triangle is doubled, then increase in area of the triangle is :
 (a) 200% (b) 300%
 (c) 400% (d) none of these
9. If the altitude of an equilateral triangle is $2\sqrt{3}$, then its area is :
 (a) $4\sqrt{3} \text{ cm}^2$ (b) $12\sqrt{3} \text{ cm}^2$
 (c) $\frac{8}{\sqrt{3}} \text{ cm}^2$ (d) none of these
10. If the perimeter of an equilateral triangle and a square is same and the area of equilateral triangle is P and the area of square is Q , then :
 (a) $P < Q$ (b) $P \leq Q$
 (c) $P > Q$ (d) $P \geq Q$

10.5 Parallelogram, Rhombus and Trapezium

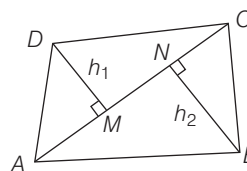
1. (a) Area of parallelogram = base (b) $\times$ height (h)
 (b) Area of parallelogram
 = product of any two adjacent sides
 $\times$ sine of the included angle
2. Perimeter = 2 (sum of any two adjacent sides)
3. (a) Area of rhombus = $\frac{1}{2} \times$ (product of diagonals)
 $= \frac{1}{2} \times d_1 d_2$
 (b) Area of rhombus = product of adjacent sides
 $\times$ sine of the angle included by them
4. Perimeter of rhombus = $4 \times$ side
5. Area of a trapezium = $\frac{1}{2} \times$ sum of parallel sides $\times$ height
 height $\rightarrow$ distance between the two parallel sides
6. Perimeter of trapezium = sum of all the four sides
7. (a) Area of quadrilateral = $\frac{1}{2} \times$ product of diagonal
 $\times$ sine of the angle between them

($AC = d_1$ and $BD = d_2$)



$$\therefore \text{Area} = \frac{1}{2} d_1 d_2 \sin \theta_1 = \frac{1}{2} d_1 d_2 \sin \theta_2$$

- (b) Area = $\frac{1}{2} \times$ diagonal $\times$ sum of the perpendiculars drawn from the opposite vertices on it.

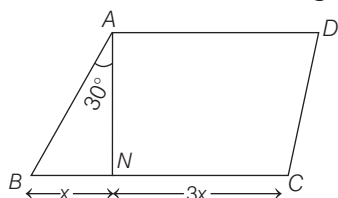


$$= \frac{1}{2} d \times (h_1 + h_2)$$

where, $MD = h_1$ and $BN = h_2$ and $AC = d$

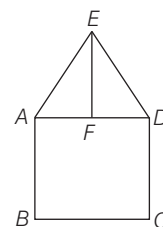
Introductory Exercise 10.3

- The adjacent sides of a parallelogram are 6 cm and 8 cm and the angle between them is 30° . What is the area of the parallelogram?
(a) 24 cm^2 (b) 12 cm^2
(c) 40 cm^2 (d) $24\sqrt{3} \text{ cm}^2$
- A parallelogram has sides 30 cm and 20 cm and one of its diagonal is 40 cm long. Then its area is :
(a) $75\sqrt{5} \text{ cm}^2$ (b) 245 cm^2
(c) $150\sqrt{15} \text{ cm}^2$ (d) 300 cm^2
- The distance of a 24 cm long side of a parallelogram from the opposite side is 22 cm. The area of the parallelogram is :
(a) 264 cm^2 (b) 246 cm^2
(c) 460 cm^2 (d) 528 cm^2
- The two adjacent sides of a parallelogram are 25 cm and 40 cm respectively. The altitude drawn on the longer side is 18 cm, then the area of the parallelogram is :
(a) 450 cm^2 (b) 720 cm^2
(c) 500 cm^2 (d) none of these
- In the adjoining figure, the ratio of the areas of the parallelogram ABCD and that of triangle ABN is :



- (a) 6 : 1 (b) 5 : 1
(c) 4 : 1 (d) 8 : 1
- If the perimeter of a rhombus is $4p$ and lengths of its diagonals are a and b , then its area is :
(a) $\frac{a}{b}$ (b) $\frac{ab}{2}$
(c) ab/p (d) $p(a^2 + b^2)$
 - The ratio of the lengths of the diagonal of a rhombus is 2 : 5. Then, the ratio of the area of the rhombus to the square of the shorter diagonal :
(a) 5 : 4 (b) 5 : 2
(c) 2 : 5 (d) none of these

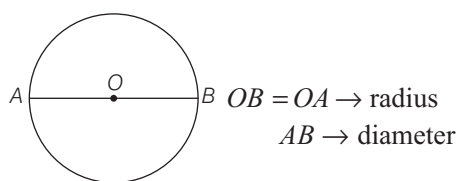
- Area of a rhombus is 256 cm^2 . One of the diagonal is half of the other diagonal. The sum of the diagonals is :
(a) 38 cm (b) 48 cm
(c) 28 cm (d) 56 cm
- The lengths of two parallel sides of a trapezium are 30 cm and 50 cm and its height is 16 cm. Its area is :
(a) 460 cm^2 (b) 750 cm^2
(c) 320 cm^2 (d) 640 cm^2
- ABCD is a trapezium in which $AB \parallel CD$ and $AB = 2CD$. If its diagonals intersect each other at O, then ratio of areas of triangles AOB and COD is :
(a) 1 : 4 (b) 1 : 2
(c) 4 : 1 (d) 2 : 1
- The area of a trapezium is 441 cm^2 and the ratio of parallel sides is 5 : 9. Also the perpendicular distance between them is 21 cm, the longer of parallel sides is :
(a) 36 cm (b) 27 cm
(c) 18 cm (d) 28 cm
- The cross-section of a canal is in the shape of a trapezium and the area of cross-section is 360 m^2 . If the canal is 12 m wide at the top and 8 m wide at the bottom the depth of the canal is :
(a) 36 m (b) 180 m
(c) 45 m (d) none of these
- The area of a hexagon whose one side is 4 m, is :
(a) $6\sqrt{3} \text{ m}^2$ (b) $24\sqrt{3} \text{ m}^2$
(c) $42\sqrt{3} \text{ m}^2$ (d) 24 m^2
- ABCD is a quadrilateral $AC = 19 \text{ cm}$. The lengths of perpendiculars from B and D on AC are 5 cm and 7 cm respectively. Then, the area of ABCD (in cm^2) is :
(a) 162 (b) 144
(c) 228 (d) 114
- ABCD is a square, $AC = BD = 4\sqrt{2} \text{ cm}$, $AE = DE = 2.5 \text{ cm}$. Find the area of the adjoining figure ABCDE.



10.6 Circles

1. Area of a circle = πR^2 ($R \rightarrow$ radius of the circle)
2. Circumference of the circle = $2\pi R$
3. Length of an arc = $2\pi R \left(\frac{\theta}{360^\circ} \right)$
4. Area of a sector = $\pi R^2 \left(\frac{\theta}{360^\circ} \right) = \frac{1}{2} (\text{arc} \times R)$
5. Area of segment = $\pi R^2 \left(\frac{\theta}{360^\circ} \right) - \frac{R^2}{2} \sin \theta$

Diameter = $2 \times$ radius



Exp. 1) The radius of a circular wheel is $1\frac{3}{4}$ m. How many revolutions will it make in travelling 11 km?

Solution Total distance (travelled) = 11 km = 11000 m

$$\begin{aligned} \text{Distance travelled in one revolution} \\ &= \text{circumference of the wheel} \\ &= 2 \times \pi \times r = 2 \times \frac{22}{7} \times \frac{7}{4} = 11 \text{ m} \end{aligned}$$

$$\begin{aligned} \therefore \text{Number of revolutions in 11 km} &= \frac{11000}{11} \\ &= 1000 \text{ revolution} \end{aligned}$$

Exp. 2) It takes 13.5 mL to paint the surface of the circular sheet of radius 17 cm. How much paint is required to paint a similar circular sheet with double the radius?

Solution Ratio of radii = 1 : 2

$\therefore$ Ratio of areas = 1 : 4

$$\left(\text{Since } \frac{\text{Area of } C_1}{\text{Area of } C_2} = \frac{\pi r_1^2}{\pi r_2^2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4} \right)$$

$\therefore$ Quantity of required paint is 4 times.

Thus we need $4 \times 13.5 = 54$ mL paint.

Exp. 3) A circular road runs round a circular garden. If the difference between the circumference of the outer circle and the inner circle is 44 m. Find the width of the road.

Solution Let R and r be radii of outer circle and inner circle respectively.

$$\therefore \text{Width of the road} = R - r$$

$$\therefore 2\pi R - 2\pi r = 44 \text{ m} \Rightarrow 2\pi (R - r) = 44 \text{ m}$$

$$\Rightarrow (R - r) = 7 \text{ m} \quad \left(\because \pi = \frac{22}{7} \right)$$

Exp. 4) The radius of a circle is 5 m. What is the radius of another circle whose area is 25 times that of the first?

Solution Ratio of areas = (ratio of radii)²

$$\frac{25}{1} = (\text{ratio of radii})^2 \Rightarrow \text{Ratio of radii} = \frac{5}{1}$$

Therefore radius of another circle is 5 times.

Hence, the required radius = 25 m

Exp. 5) What is the radius of a circle whose area is equal to the sum of the areas of two circles whose radii are 20 cm and 21 cm?

$$\begin{aligned} \text{Solution} \quad \pi R^2 &= \pi r_1^2 + \pi r_2^2 \\ \pi R^2 &= \pi (r_1^2 + r_2^2) \\ R^2 &= (400 + 441) \\ R^2 &= 841 \Rightarrow R = 29 \text{ cm} \end{aligned}$$

Exp. 6) In a circle of radius 28 cm, an arc subtends an angle of 108° at the centre.

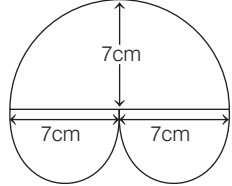
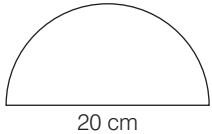
(a) Find the area of the sector.

(b) Find the length of the arc.

$$\begin{aligned} \text{Solution (a) Area of the sector} &= \pi r^2 \left(\frac{\theta}{360^\circ} \right) \\ &= \frac{22}{7} \times 28 \times 28 \times \frac{108^\circ}{360^\circ} \\ &= 22 \times 4 \times 28 \times \frac{3}{10} = 739.2 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{(b) Length of the arc} &= 2\pi r \left(\frac{\theta}{360^\circ} \right) \\ &= 2 \times \frac{22}{7} \times 28 \times \frac{108}{360} = 52.8 \text{ cm} \end{aligned}$$

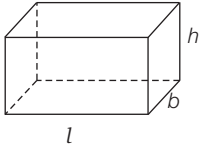
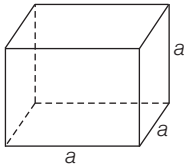
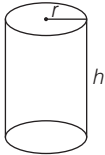
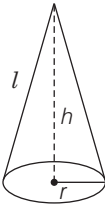
Introductory Exercise 10.4

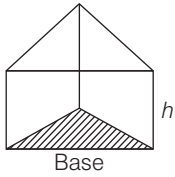
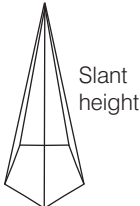
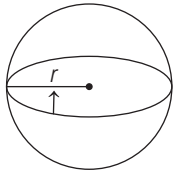
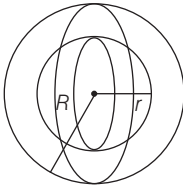
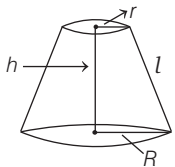
- If the circumference of a circle is 704 cm, then its area is :
 (a) 49324 m^2 (b) 39424 m^2
 (c) 3672 cm^2 (d) 39424 cm^2
- If the circumference of a circle is 4.4 m, then the area of the circle (in m^2) is
 (a) $49/\pi$ (b) 49π
 (c) 4.9π (d) none of these
- A circular wire of radius 4.2 m is cut and bent in the form of a rectangle whose longer side is 20% more than its shorter side. The longer side of the rectangle is :
 (a) 7.2 m (b) 72 cm
 (c) 8 m (d) none of these
- The inner circumference of a circular path around a circular lawn is 440 m. What is the radius of the outer circumference of the path, if the path is 14 m wide?
 (a) 96 m (b) 84 m
 (c) 70 m (d) 88 m
- The sum of the radius and the circumference of a circle is 51 cm. The area of the circle is :
 (a) 151 cm (b) 152 cm
 (c) 154 cm (d) data insufficient
- The difference between the circumference and the diameter of the circle is 15 m. What is the area of the circle?
 (a) 225 m^2 (b) 165 m^2
 (c) 156 m^2 (d) none of these
- The radius of a circle is increased by 2 cm from 5 cm to 7 cm. What is the percentage change in area of the circle?
 (a) 96% (b) 35%
 (c) 70% (d) 74%
- If the circumference of a circle is increased by 20%, then its area will be increased by :
 (a) 44% (b) 32%
 (c) 40% (d) none of these
- The area of a circular field is 124.74 hectares. The cost of fencing it at the rate of 80 paise per metre is
 (a) ₹ 3168 (b) ₹ 1584
 (c) ₹ 1729 (d) none of these
- Eldeco Housing Pvt. Ltd purchased a circular plot of land for ₹ 158400 at the rate of 1400 per sq. metre. The radius of the plot is :
 (a) 5 m (b) 6 m
 (c) 7 m (d) 14 m
- A figure consists of a square of side 'a' m with semicircles drawn on the outside of the square. The area (in m^2) of the figure so formed will be :
 (a) $a^2 (\pi + 1)$ (b) $a^2 \left(\pi + \frac{1}{4} \right)$
 (c) $a^2 + \frac{\pi a^2}{2}$ (d) none of these
- The length of a rope by which a buffalo must be tethered so that she may be able to graze a grassy area of 2464 sq m is :
 (a) 35 m (b) 27 m
 (c) 24 m (d) 28 m
- A circle of radius 'a' is divided into 6 equal sectors. An equilateral triangle is drawn on the chord of each sector to lie outside the circle. Area of the resulting figure is :
 (a) $3a^2 (\pi + \sqrt{3})$ (b) $3\sqrt{3} a^2$
 (c) $3(a^2\sqrt{3} + \pi)$ (d) $\frac{3\sqrt{3}\pi a^2}{2}$
- In the following figure, the area (in cm^2) is :

 (a) 115.5 (b) 228.5
 (c) 154 (d) none of these
- If a piece of wire 25 cm long is bent into an arc of a circle subtending an angle of 75° at the centre, then the radius of the circle (in cm) is :
 (a) $\frac{\pi}{120}$ (b) $\frac{60}{\pi}$
 (c) 60π (d) none of these
- Four horses are tethered at four corners of a square plot of 42 m so that they just cannot reach one another. The area left ungrazed is :
 (a) 378 m^2 (b) 438 m^2
 (c) 786 m^2 (d) none of these
- The circumference of the following figure is :

 (a) $(20 + 10\pi)$ (b) 20π
 (c) 10π (d) 30π

18. The area of a minor sector subtending the central angle at the centre 40° is 8.25 cm^2 . What is the area of the remaining part (i.e. major sector) of the circle?
 (a) 82.5 cm^2 (b) 74.25 cm^2
 (c) 66 cm^2 (d) none of these
19. The area of a sector of a circle of radius 8 cm, formed by an arc of length 5.6 cm, is :
 (a) 22.4 cm^2 (b) 2.24 cm^2
 (c) 56 cm^2 (d) none of these
20. How long will a man take to go, walking at 13.2 km/h, round a circular garden of 700 m radius?
 (a) 12 min (b) 30 min
 (c) 20 min (d) none of these
21. What is the radius of circular field whose area is equal to the sum of the areas of three smaller circular fields of radii 8 m, 9 m and 12 m respectively?
 (a) 17 m (b) 20 m
 (c) 21 m (d) 29 m
22. A rope by which a calf is tied is decreased from 23 m to 12 m. What is the decrease in area to be grazed by it?
 (a) 1110 m^2 (b) 1210 m^2
 (c) 1120 m^2 (d) 1221 m^2
23. A wire is bent in the form of a square of side 66 m. It is cut and again bent in the form of a circle. The diameter of this circle is :
 (a) 42 m (b) 84 m
 (c) 21 m (d) none of these
24. A wire is in the form of a circle of radius 42 m is cut and again bent in the form of a square. What is the diagonal of the square?
 (a) 66 m (b) $66\sqrt{3} \text{ m}$
 (c) $66\sqrt{2} \text{ m}$ (d) none of these
25. If the driving wheel of a bicycle makes 560 revolutions in travelling 1.1 km. Find the diameter of the wheel.
 (a) 31.5 cm (b) 30.5 cm
 (c) 62.5 cm (d) none of these

10.7 3-D Figures

3-D Figures are shown below

Name	Figure	Nomenclature	Volume	Curved/Lateral surface area	Total surface area
Cuboid		$l \rightarrow$ length $b \rightarrow$ breadth $h \rightarrow$ height	lbh	$2(l + b)h$	$2(lb + bh + hl)$
Cube		$a \rightarrow$ edge/side	a^3	$4a^2$	$6a^2$
Right circular cylinder		$r \rightarrow$ radius of base $h \rightarrow$ height of the cylinder	$\pi r^2 h$	$2\pi rh$	$2\pi r(h + r)$
Right circular cone		$r \rightarrow$ radius $h \rightarrow$ height $l \rightarrow$ slant height $l = \sqrt{r^2 + h^2}$	$\frac{1}{3} \pi r^2 h$	πrl	$\pi r(l + r)$

Name	Figure	Nomenclature	Volume	Curved/Lateral surface area	Total surface area
Right triangular prism		—	area of base $\times$ height	perimeter of base $\times$ height	lateral surface area + 2 (area of base)
Right pyramid		—	$\frac{1}{3} \times$ area of the base $\times$ height	$\frac{1}{2} \times$ perimeter of the base $\times$ slant height	lateral surface area + area of the base
Sphere		$r \rightarrow$ radius	$\frac{4}{3} \pi r^3$	—	$4\pi r^2$
Hemisphere		$r \rightarrow$ radius	$\frac{2}{3} \pi r^3$	$2\pi r^2$	$3\pi r^2$
Spherical shell		$r \rightarrow$ inner radius $R \rightarrow$ outer radius	$\frac{4}{3} \pi [R^3 - r^3]$	—	$4\pi [R^2 + r^2]$
Frustum of a cone		—	$\frac{\pi}{3} h (r^2 + Rr + R^2)$	$\pi (r + R) l$	lateral surface area + $\pi [R^2 + r^2]$

10.8 Cuboid and Cube

Cuboid : A cuboid has 6 faces, 12 edges, 8 vertices and 4 diagonals.

Faces :

$ABCD, EFGH, ABEH, CFGD, BCFE, ADGH$

Edges : $AB, BC, CD, AD, EF, FG, GH, EH, BE, CF, DG, AH$

Vertices : A, B, C, D, E, F, G, H

Diagonals : AF, BG, CH, DE

Formulae : Volume = $l \times b \times h$

($l \rightarrow$ length, $b \rightarrow$ breadth, $h \rightarrow$ height)

Total surface area = $2(lb + bh + hl)$

Diagonal (d) = $\sqrt{l^2 + b^2 + h^2}$

Cube : A cube has 6 equal faces, 12 equal edges, 8 vertices and 4 equal diagonals.

Formulae :

Volume = $(a)^3$

Total surface area = $6(a)^2$

($a \rightarrow$ edge of the cube)

Diagonal (d) = $a\sqrt{3}$

Euler's Theorem $\rightarrow (V + F) = (E + 2)$; where $V \rightarrow$ number of vertices, $F \rightarrow$ number of faces, $E \rightarrow$ number of edges

Exp. 1) The dimensions of a cuboid are 16 cm, 18 cm and 24 cm. Find :

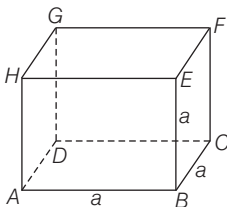
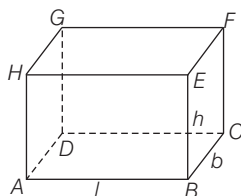
(a) volume (b) surface area (c) diagonal

Solution (a) Volume = $l \times b \times h = 16 \times 18 \times 24 = 6912 \text{ cm}^3$ (b)

Surface area = $2(lb + bh + hl)$

= $2(16 \times 18 + 18 \times 24 + 24 \times 16)$

= 2208 cm^2



$$\begin{aligned} \text{(c) Diagonal} &= \sqrt{l^2 + b^2 + h^2} \\ &= \sqrt{16^2 + 18^2 + 24^2} = \sqrt{1156} = 34 \text{ cm} \end{aligned}$$

Exp. 2) Edge of a cube is 5 cm. Find :

(a) volume (b) surface area (c) diagonal

Solution Volume = $a^3 = (5)^3 = 125 \text{ cm}^3$

Surface area = $6a^2 = 6 \times (5)^2 = 150 \text{ cm}^2$

Diagonal = $a\sqrt{3} = 5\sqrt{3} = 8.660 = 8.66 \text{ cm}$

Exp. 3) Three cubes of volumes, 1 cm^3 , 216 cm^3 and 512 cm^3 are melted to form a new cube. What is the diagonal of the new cube?

Solution Volume of new cube = $1 + 216 + 512 = 729 \text{ cm}^3$

$\therefore$ Edge of new cube = $\sqrt[3]{729} = 9 \text{ cm}$

$\therefore$ Surface area = $6a^2 = 6 \times (9)^2 = 486 \text{ cm}^2$

$\therefore$ Diagonal of the new cube = $a\sqrt{3} = 9\sqrt{3} = 15.6 \text{ cm (approx.)}$

Exp. 4) The surface area of a cube is 864 cm^2 . Find its volume.

Solution $6a^2 = 864 \Rightarrow a^2 = 144 \Rightarrow a = 12 \text{ cm}$

$\therefore a^3 = (12)^3 = 1728 \text{ cm}^3$

Exp. 5) Find the length of the longest pole that can be placed in a room 30 m long, 24 m broad and 18 m high.

Solution $d = \sqrt{l^2 + b^2 + h^2}$
 $d = \sqrt{900 + 576 + 324} \Rightarrow d = 30\sqrt{2} \text{ m}$

Exp. 6) A brick measures $20 \text{ cm} \times 10 \text{ cm} \times 7.5 \text{ cm}$. How many bricks will be required for a wall $20 \text{ m} \times 2 \text{ m} \times 0.75 \text{ m}$?

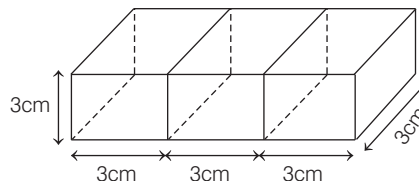
Solution Number of bricks = $\frac{\text{total volume of a wall}}{\text{volume of one brick}}$
 $= \frac{20 \times 2 \times 0.75 \times 100 \times 100 \times 100}{20 \times 10 \times 7.5} = 20000$

Introductory Exercise 10.5

- A cube of metal, each edge of which measures 4 cm, weighs 400 kg. What is the length of each edge of a cube of the same metal which weighs 3200 kg?
 - 64 cm
 - 8 cm
 - 2 cm
 - none of these
- The length of a tank is thrice that of breadth, which is 256 cm deep and holds 3000 L water. What is the base area of the tank? (1000 L = 1 cubic metre)
 - 111775 m^2
 - 1171.875 m^2
 - 1.171875 m^2
 - none of these
- The three co-terminus edges of a rectangular solid are 36 cm, 75 cm and 80 cm respectively. Find the edge of a cube which will be of the same capacity.
 - 60 cm
 - 52 cm
 - 46 cm
 - none of these
- A tank 10 m long and 4 m wide is filled with water. How many litres of water must be drawn off to make the surface sink by 1 m? (1000 L = 1 cubic metre)
 - 20 kilolitre
 - 40 kilolitre
 - 50 kilolitre
 - none of these

5. How many cubes each of surface area 24 sq. dm can be made out of a metre cube, without any wastage?
(a) 75 (b) 250
(c) 125 (d) 62
6. Three cubes of metal, whose edges are 3 cm, 4 cm and 5 cm respectively are melted to form a new cube. What is the surface area of the new cube?
(a) 216 cm^2 (b) 56 cm^2
(c) 36 cm^2 (d) none of these
7. A lid of rectangular box of sides 39.5 cm by 9.35 cm is sealed all around with tape such that there is an overlapping of 3.75 cm of the tape. What is the length of the tape used?
(a) 111.54 cm (b) 101.45 cm
(c) 110.45 cm (d) none of these
8. A cistern from inside is 12.5 m long, 8.5 m broad and 4 m high and is open at top. Find the cost of cementing the inside of a cistern at ₹ 24 per sq m.
(a) ₹ 6582 (b) ₹ 8256
(c) ₹ 7752 (d) ₹ 8752
9. 250 men took a dip in a water tank at a time, which is 80 m $\times$ 50 m. What is the rise in the water level if the average displacement of 1 man is 4 m^3 ?
(a) 22 cm (b) 25 cm (c) 18 cm (d) 30 cm
10. The edge of a cube is increased by 100%, the surface area of the cube is increased by :
(a) 100% (b) 200% (c) 300% (d) 400%
11. The edge of a cube is doubled. What will be the new volume?
(a) 2 times (b) 3 times
(c) 4 times (d) 8 times
12. The external dimensions of a wooden box closed at both ends are 24 cm, 16 cm and 10 cm respectively and thickness of the wood is 5 mm. If the empty box weighs 7.35 kg, find the weight of 1 cubic cm of wood.
(a) 10 g (b) 12.5 g
(c) 27 g (d) 15 g
13. The internal dimensions of a tank are 12 dm, 8 dm and 5 dm. How many cubes each of edge 7 cm can be placed in the tank with faces parallel to the sides of the tank. Find also, how much space is left unoccupied.
(a) 35; 113 dm^3 (b) 1313; 31.13 dm^3
(c) 1309; 31.013 dm^3 (d) 1309; 13.31 dm^3
14. The length, breadth and height of box are 2 m, 1.5 m and 80 cm respectively. What would be the cost of canvas to cover it up fully, if one square metre of canvas costs ₹ 25.00?
(a) ₹ 260 (b) ₹ 290
(c) ₹ 285 (d) none of these

15. Three cubes each of edge 3 cm long are placed together as shown in the adjoining figure. Find the surface area of the cuboid so formed :



- (a) 182 sq cm (b) 162 sq cm
(c) 126 sq cm (d) none of these
16. A room is 36 m long, 12 m wide and 10 m high. It has 6 windows, each 3 m $\times$ 2.5 m; one door 9.5 m $\times$ 6 m and one fire chimney 4 m $\times$ 4.5 m. Find the expenditure of papering its walls at the rate of 70 paise per metre, if the width of the paper is 1.2 m.
(a) ₹ 490 (b) ₹ 690
(c) ₹ 1000 (d) none of these
17. A school hall has the dimensions 30 m, 12 m by 6 m. Find the number of children who can be accommodated, if each child should get 8 m^3 of space.
(a) 240 (b) 270
(c) 250 (d) 150
18. When each side of a cube is increased by 2 cm, the volume is increased by 1016 cm^3 . Find the side of the cube. If each side of it is decreased by 2 cm, by how much will the volume decrease?
(a) 12 cm, 729 cm^3
(b) 8 cm, 512 cm^3
(c) 9 cm, 729 cm^3
(d) 12 cm, 728 cm^3
19. Three equal cubes are placed adjacently in a row. Find the ratio of the total surface area of the resulting cuboid to that of the sum of the total surface areas of the three cubes.
(a) 5 : 7 (b) 7 : 9
(c) 9 : 7 (d) none of these
20. A hollow square shaped tube open at both ends is made of iron. The internal square is of 5 cm side and the length of the tube is 8 cm. There are 192 cm^3 of iron in the tube. Find its thickness.
(a) 2 cm (b) 0.5 cm
(c) 1 cm (d) can't be determined
21. A cube of 11 cm edge is immersed completely in a rectangular vessel containing water. If the dimensions of base are 15 cm and 12 cm. Find the rise in water level in the vessel.
(a) 6.85 cm (b) 7 cm
(c) 7.31 cm (d) 7.39 cm

22. A rectangular tank 25 cm long and 20 cm wide contains water to a depth of 5 cm. A metal cube of side 10 cm is placed in the tank so that one face of the cube rests on the bottom of the tank. Find how many litres of water must be poured into the tank so as to just cover the cube.
 (a) 1 L (b) 1.5 L
 (c) 2 L (d) 2.5 L
23. A rectangular block has length 10 cm, breadth 8 cm and height 2 cm. From this block, a cubical hole of side 2 cm is drilled out. Find the volume and the surface area of the remaining solid.
 (a) 152 cm^3 , 512 cm^2 (b) 125 cm^3 , 215 cm^2
 (c) 152 cm^3 , 240 cm^2 (d) 125 cm^3 , 512 cm^2
24. A rectangular tank of dimensions $24 \text{ m} \times 12 \text{ m} \times 8 \text{ m}$ is dug inside a rectangular field 600 m long and 200 m broad. The earth taken out is evenly spread over the field. By how much will the level of the field rise?
 (a) 1.925 cm (b) 0.02 m
 (c) 0.2 cm (d) none of these
25. How many bricks (number near to next hundred) will be required to build a wall 30 m long, 30 cm thick and 5 m high with a provision of 2 doors, each $4 \text{ m} \times 2.5 \text{ m}$ and each brick being $20 \text{ cm} \times 16 \text{ cm} \times 8 \text{ cm}$ when one-ninth of the wall is filled with lime?
 (a) 13500 bricks (b) 13600 bricks
 (c) 20050 bricks (d) 18500 bricks
26. A rectangular water reservoir is 15 m by 12 m at the base. Water flows into it through a pipe whose cross-section is 5 cm by 3 cm at the rate of 16 m per second. Find the height to which the water will rise in the reservoir in 25 min.
 (a) 0.2 m (b) 2 cm
 (c) 0.5 m (d) none of these

27. Which of the following pairs is not correctly matched?

Geometrical objects	Number of vertices
(A) Tetrahedron	4
(B) Pyramid with rectangular base	5
(C) Cube	6
(D) Triangle	3
(a) (A)	(b) (B)
(c) (C)	(d) (D)

28. If the length of diagonal of a cube is $6\sqrt{3} \text{ cm}$, then the length of its edge is :
 (a) 2 cm (b) 3 cm
 (c) 6 cm (d) $\frac{36}{\sqrt{3}} \text{ cm}$
29. The length of longest pole that can be placed on the floor of a room is 12 m and the length of longest pole that can be placed in the room is 15 m. The height of the room is :
 (a) 3 m (b) 6 m
 (c) 9 m (d) none of these
30. The sum of length, breadth and depth of a cuboid is 12 cm and its diagonal is $5\sqrt{2} \text{ cm}$. Its surface area is
 (a) 152 cm^2 (b) 94 cm^2
 (c) 108 cm^2 (d) $60\sqrt{2} \text{ cm}^2$
31. The volume of a wall, 3 times as high as it is broad and 8 times as long as it is high, is 36.864 m^3 . The height of the wall is :
 (a) 1.8 m (b) 2.4 m
 (c) 4.2 m (d) none of these
32. If the areas of 3 adjacent sides of a cuboid are x , y , z respectively, then the volume of the cuboid is :
 (a) xyz (b) $\sqrt{xyz}$
 (c) $3xyz$ (d) none of these

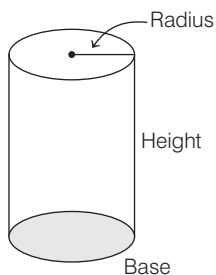
10.9 Cylinder and Cone

Cylinder :

$$\text{Volume} = \text{base area} \times \text{height} = \pi r^2 h$$

Curved surface area

$$= \text{perimeter} \times \text{height} = 2\pi rh$$

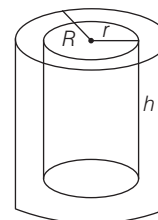


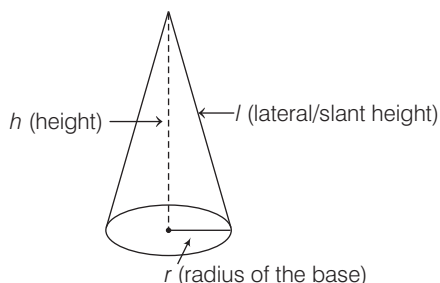
Total surface of the cylinder

$$= \text{curved surface area} + 2 (\text{base area})$$

$$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$$

$$\text{Volume of a hollow cylinder} = \pi h (R^2 - r^2)$$



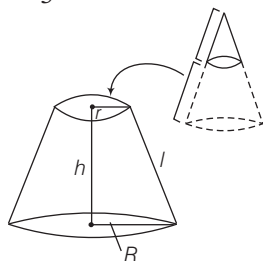
Cone :

$$\begin{aligned}\text{Volume (V)} &= \frac{1}{3} \times \text{base area} \times \text{height} \\ &= \frac{1}{3} \pi r^2 h\end{aligned}$$

Curved surface area = $\pi r l$ Total surface area = $\pi r l + \pi r^2 = \pi r (l + r)$ **Frustum of a cone :**

Volume of the frustum of a cone

$$= \frac{\pi}{3} h (r^2 + Rr + R^2)$$



Curved/Lateral surface area of the frustum of cone

$$= \pi l (r + R)$$

Exp. 1) The base radius of a cylinder is 14 cm and its height is 30 cm. Find :

- (a) volume (b) curved surface area
(c) total surface area

Solution (a) Volume of cylinder

$$\begin{aligned}&= \pi r^2 \times h = \frac{22}{7} \times 14 \times 14 \times 30 \\ &= 18480 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\text{(b) Curved surface area} &= 2\pi r h = 2 \times \frac{22}{7} \times 14 \times 30 \\ &= 2640 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{(c) Total surface area} &= 2\pi r (h + r) \\ &= 2 \times \frac{22}{7} \times 14 (30 + 14) \\ &= 2 \times \frac{22}{7} \times 14 \times 44 = 3872 \text{ cm}^2\end{aligned}$$

Exp. 2) How many cubic metres of earth must be dug to make a well 14 m deep and 4 m in diameter?**Solution** Earth to be dugout from the well
= volume of the cylindrical well

$$\begin{aligned}&= \pi r^2 h = \frac{22}{7} \times 2 \times 2 \times 14 \\ &= 176 \text{ m}^3\end{aligned}$$

Exp. 3) A hollow cylindrical tube open at both ends is made of iron 2 cm thick. If the external diameter be 50 cm and the length of the tube is 210 cm, find the number of cubic cm of iron in it.**Solution** External radius (R) = 25 cm

Internal radius (r) = (25 - 2) = 23 cm

$$\begin{aligned}\text{Volume of iron} &= \pi h (R^2 - r^2) \\ &= \frac{22}{7} \times 210 \times (25^2 - 23^2) \\ &= 63360 \text{ cm}^3\end{aligned}$$

Exp. 4) A well with 14 m inside diameter is dugout 15 m deep. The earth taken out of it has been evenly spread all around it to a width of 21 m to form an embankment. What is the height of the embankment?**Solution** Area of embankment $\times$ height of embankment

= volume of earth dugout

$$\pi (R^2 - r^2) \times h = \pi \times 7 \times 7 \times 15$$

$$\Rightarrow (28^2 - 7^2) h = 7 \times 7 \times 15$$

$$\Rightarrow (35 \times 21) \times h = 7 \times 7 \times 15$$

$$\Rightarrow h = 1 \text{ m}$$

Exp. 5) A cylindrical cistern whose diameter is 21 cm is partly filled with water. If a rectangular block of iron 14 cm in length, 10.5 cm in breadth and 11 cm in thickness is wholly immersed in water, by how many centimetres will the water level rise?**Solution** Volume of the block = $14 \times 10.5 \times 11 \text{ cm}^3$

$$\text{Radius of the tank} = \frac{21}{2} = 10.5 \text{ cm}$$

$$\text{Volume of the cylinder} = \pi r^2 h = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times h$$

$$\therefore \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times h = 14 \times 10.5 \times 11$$

$$h = \frac{14}{3} = 4 \frac{2}{3} \text{ cm}$$

Exp. 6) If the radius of cylinder is doubled, but height is reduced by 50%. What is the percentage change in volume?

$$\text{Solution} \quad \frac{r_1}{r_2} = \frac{r}{2r} \quad \text{and} \quad \frac{h_1}{h_2} = \frac{h}{h/2}$$

$$\therefore \text{Actual volume} = \pi r^2 h$$

$$\text{New volume} = \pi (2r)^2 \times \frac{h}{2} = 2\pi r^2 h$$

Therefore new volume is the twice of the original volume.

$$\text{Hence the change in volume} = \frac{2-1}{1} \times 100 = 100\%$$

Exp. 7) The radius of the base of a right cone is 35 cm and its height is 84 cm. Find :

- (a) slant height
- (b) curved surface area
- (c) total surface area
- (d) volume

Solution (a) Slant height (l) = $\sqrt{r^2 + h^2}$
 $(r \rightarrow \text{radius of the circular base})$
 $= \sqrt{35^2 + 84^2}$ $(h \rightarrow \text{height of the cone})$
 $= \sqrt{1225 + 7056}$
 $= \sqrt{8281} = 91 \text{ cm}$

(b) Curved surface area = $\pi r l = \frac{22}{7} \times 35 \times 91$
 $= 10010 \text{ cm}^2$

(c) Total surface area = lateral surface area + base area
 $= \pi r l + \pi r^2 = \pi r (l + r)$
 $= \frac{22}{7} \times 35 (91 + 35)$
 $= 110 \times 175 = 19250 \text{ cm}^2$

$$\begin{aligned} \text{(d) Volume} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \frac{22}{7} \times 35 \times 35 \times 84 = 107800 \text{ cm}^3 \end{aligned}$$

Exp. 8) Find the area of the iron sheet required to prepare a cone 20 cm high with base radius 21 cm.

Solution $r = 21 \text{ cm}, h = 20 \text{ cm}$
 $\therefore l = \sqrt{r^2 + h^2} = 29 \text{ cm}$
 $\therefore \text{Area of the sheet} = \text{total surface area of the cone}$
 $= \pi r l + \pi r^2 = \pi r (l + r)$
 $= \frac{22}{7} \times 21 [29 + 21] = 3300 \text{ cm}^2$

Exp. 9) A solid metallic cylinder of base radius 3 cm and height 5 cm is melted to make n solid cones of height 1 cm and base radius 1 mm. Find the value of n .

Solution $n = \frac{\text{volume of cylinder}}{\text{volume of one cone}}$
 $= \frac{\pi \times 3 \times 3 \times 5}{\frac{1}{3} \pi \times \frac{1}{10} \times \frac{1}{10} \times 1} = 13500$

Introductory Exercise 10.6

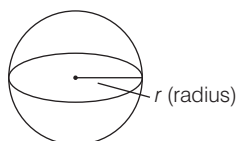
- How many cubic metres of water fill a pipe which is 3500 m long and 0.08 m in diameter?
 (a) 17.5 m^3 (b) 17.6 m^3 (c) 21 m^3 (d) 35 m^3
- A cube of metal, whose edge is 10 cm, is wholly immersed in water contained in cylindrical tube whose diameter is 20 cm. By how much will the water level rise in the tube?
 (a) 3.3 cm (b) $6\frac{3}{11} \text{ cm}$
 (c) $3\frac{2}{11} \text{ cm}$ (d) none of these
- Find the height of the cylinder whose volume is 511 m^3 and the area of the base is 36.5 m^2 :
 (a) 7 m (b) 10.5 m
 (c) 14 m (d) none of these
- The lateral surface area of a cylinder is 1056 cm^2 and its height is 16 cm. What is its volume?
 (a) 5566 cm^3 (b) 4455 cm^3
 (c) 5544 cm^3 (d) none of these
- There is a cubical block of wood of side 2 cm. If the cylinder of the largest possible volume is curved out from it. Find the volume of the remaining wood :
 (a) $\frac{7}{12} \text{ cm}^2$ (b) $\frac{12}{7} \text{ cm}^3$
 (c) $5\frac{5}{7} \text{ cm}^3$ (d) none of these
- The amount of concrete required to build a cylindrical pillar whose base has a perimeter of 8.8 m and whose curved surface area is 17.6 m^2 :
 (a) 12.32 m^3 (b) 12.23 m^3
 (c) 9.235 m^3 (d) 8.88 m^3
- If the diameter of the base of a closed right circular cylinder be equal to its height 'h', then its whole surface area is :
 (a) $\frac{2}{3} \pi h^2$ (b) $\frac{3}{2} \pi h^3$ (c) $\frac{3}{2} \pi h^2$ (d) πh^3
- A right circular cylindrical tunnel of diameter 4 m and length 10 m is to be constructed from a sheet of iron. The area of the iron sheet required :
 (a) $\frac{280}{\pi}$ (b) 40π
 (c) 80π (d) none of these
- The ratio between curved surface area and total surface area is 2 : 3. If the total surface area be 924 cm^2 , find the volume of the cylinder :
 (a) 2156 cm^3 (b) 1256 cm^3
 (c) 1265 cm^3 (d) none of these
- If the volume and curved surface area of a cylinder are 269.5 cm^3 and 154 cm^2 respectively, what is the height of the cylinder?
 (a) 6 (b) 3.5
 (c) 7 (d) can't be determined

11. If the curved surface area of a cylinder is 1320 cm^2 and its base radius is 21 cm, then its total surface area is :
 (a) 4092 cm^2 (b) 2409 cm^2
 (c) 4920 cm^2 (d) none of these
12. The ratio between the radius of the base and the height of a cylindrical pillar is 3 : 4. If its volume is 4851 m^3 , the curved surface area of the pillar is :
 (a) 924 m^2 (b) 1617 m^2
 (c) 425 m^2 (d) none of these
13. If the ratio of total surface area to the curved surface area of a cylinder be 4 : 1, what is the ratio of radius to the height?
 (a) 4 : 1 (b) 2 : 3
 (c) 3 : 2 (d) 3 : 1
14. The circumference of the base of a right cylinder is 33 cm and height is 330 cm. What is the volume of this cylinder?
 (a) 28586.25 cm^3 (b) 3344 cm^3
 (c) 4433 cm^3 (d) 3456 cm^3
15. The radius of an iron rod decreased to one-fourth. If its volume remains constant, the length will become :
 (a) 2 times (b) 8 times
 (c) 4 times (d) 16 times
16. The total surface area of the cylinder is 2640 m^2 and the sum of height and radius of base of cylinder is 30 m. What is the ratio of height and radius of the cylinder?
 (a) 7 : 9 (b) 9 : 7
 (c) 8 : 7 (d) 3 : 7
17. The radii of two cylinders are in the ratio of 3 : 5 and their heights are in the ratio 4 : 3. The ratio of their volumes is :
 (a) 12 : 25 (b) 13 : 25
 (c) 4 : 5 (d) 5 : 4
18. The heights of two cylinders are in the ratio of 3 : 1. If the volumes of two cylinders be same, the ratio of their respective radii are :
 (a) $\sqrt{3} : 1$ (b) $1 : \sqrt{3}$
 (c) 1 : 9 (d) none of these
19. The ratio of heights of two cylinders is 3 : 2 and the ratio of their radii is 6 : 7. What is the ratio of their curved surface areas?
 (a) 9 : 7 (b) 1 : 1
 (c) 7 : 9 (d) 7 : 4
20. A hollow garden roller 42 cm wide with a girth of 132 cm is made of iron 3 cm thick. The volume of the iron of the roller is :
 (a) 15544 cm^3
 (b) 15444 cm^3
 (c) 15545 cm^3
 (d) none of the above
21. A conical vessel has a capacity of 15 L of milk. Its height is 50 cm and base radius is 25 cm. How much milk can be contained in a vessel in cylindrical form having the same dimensions as that of the cone?
 (a) 15 L (b) 30 L
 (c) 45 L (d) none of these
22. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section made?
 (a) 20 cm (b) 18 cm (c) 27 cm (d) 15 cm
23. A tent is in the form of right circular cone 10.5 m high, the diameter of the base being 13 m. If 8 men are in the tent, find the average number of cubic metres of air space per man.
 (a) $32\frac{3}{58}$ (b) 59.75 (c) $36\frac{9}{13}$ (d) $58\frac{3}{32}$
24. The radius and height of a right circular cone are in the ratio of 5 : 12. If its volume is $314\frac{2}{7}\text{ m}^3$, its slant height is :
 (a) 26 m (b) 19.5 m
 (c) 13 m (d) none of these
25. The volume and height of a right circular cone are 1232 cm^3 and 24 cm respectively, the area of its curved surface (in cm^2) is :
 (a) 1100 (b) 225 (c) 616 (d) 550
26. The circumference of the base of a right circular cone is 220 cm^3 and its height 84 cm. The curved surface area of the cone is :
 (a) 20020 cm^2 (b) 2020 cm^2
 (c) 2200 cm^2 (d) 10010 cm^2
27. How many metres of cloth 10 m wide will be required to make a conical tent with base radius of 14 m and height is 48 m?
 (a) 110 m (b) 55 m (c) 77 m (d) 220 m
28. A cone of height 2.8 cm has a lateral surface area 23.10 m^2 . The radius of the base is :
 (a) 3.5 cm (b) 2 cm (c) 2.1 cm (d) 4 cm
29. The radii of two cones are equal and their slant heights are in the ratio 3 : 2. If the curved surface area of the smaller cone is 300 cm^2 , then the curved surface area of the bigger cone (in cm^2) is :
 (a) 250 (b) 450 (c) 150 (d) 200
30. The ratio of the volume of a right circular cylinder and a right circular cone of the same base and height will be :
 (a) 2 : 3 (b) 1 : 3 (c) 3 : 1 (d) 9 : 1
31. If the diameter of the base of right circular cone is equal to 8 cm and its slant height is 5 cm, then the area of its axial section is :
 (a) 9 cm^2 (b) 12 cm^2 (c) 20 cm^2 (d) 40 cm^2

32. If the base radius and the height of a right circular cone are increased by 40%, then the percentage increase in volume (approx) is :
 (a) 175% (b) 120%
 (c) 64% (d) 540%
33. From a circular sheet of paper of radius 25 cm, a sector area 4% is removed. If the remaining part is used to make a conical surface, then the ratio of the radius and height of the cone is :
 (a) 16 : 25 (b) 9 : 25
 (c) 7 : 12 (d) 24 : 7
34. If the radius of the base is doubled, keeping the height constant, what is the ratio of the volume of the larger cone to the smaller cone?
 (a) 2 : 1 (b) 3 : 1 (c) 4 : 1 (d) 4 : 3
35. A largest possible cone is cut out from a cube of volume 1000 cm^3 . The volume of the cone is :
 (a) 280 cm^3 (b) 261.9 cm^3
 (c) 269.1 cm^3 (d) 296.1 cm^3
36. If the height and the radius of a cone are doubled, the volume of the cone becomes :
 (a) 2 times (b) 8 times
 (c) 16 times (d) 4 times
37. A conical tent has 60° angle at the vertex. The ratio of its radius and slant height is :
 (a) 3 : 2 (b) 1 : 2
 (c) 1 : 3 (d) can't be determined
38. Water flows at the rate of 5 m per min from a cylindrical pipe 16 mm in diameter. How long will it take to fill up a conical vessel whose radius is 12 cm and depth is 35 cm?
 (a) 315 s (b) 365 s
 (c) 5 min (d) none of these
39. A reservoir is in the shape of a frustum of a right circular cone. It is 8 m across at the top and 4 m across at the bottom. It is 6 m deep its capacity is :
 (a) 224 m^3 (b) 176 m^3
 (c) 225 m^3 (d) none of these
40. A conical vessel whose internal radius is 10 cm and height 72 cm is full of water. If this water is poured into a cylindrical vessel with internal radius 30 cm, the height of the water level rises in it is :
 (a) $2\frac{2}{3} \text{ cm}$ (b) $3\frac{2}{3} \text{ cm}$
 (c) $5\frac{2}{3} \text{ cm}$ (d) none of these

10.10 Sphere, Prism and Pyramid

Sphere



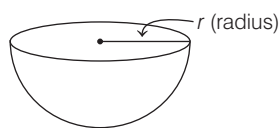
$$\text{Volume} = \frac{4}{3} \pi r^3, \text{ Surface area} = 4\pi r^2$$

Hemisphere :

$$\text{Volume} = \frac{2}{3} \pi r^3$$

$$\text{Curved surface area} = 2\pi r^2$$

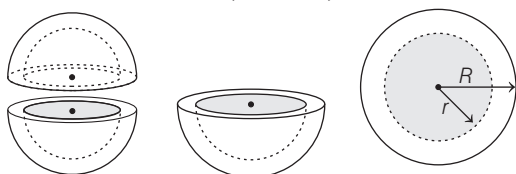
$$\text{Total surface area} = 3\pi r^2 = (2\pi r^2 + \pi r^2)$$



Spherical shell :

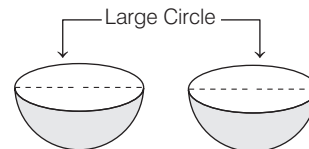
$$\text{Volume} = \frac{4}{3} \pi (R^3 - r^3)$$

$$\text{Total surface area} = 4\pi (R^2 + r^2)$$

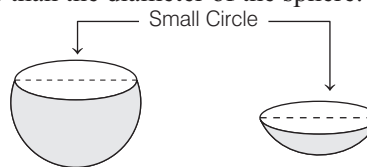


Circles in the Sphere

- (i) **Great Circle:** If a plane cuts the sphere at the centre, we get two hemispheres of equal volume and equal surface area. In this case, the circles that we get are called great circles, since these are the circles with the greatest possible chords. The longest chord (or diameter) of the great circle is same as the longest chord (or diameter) of the sphere.



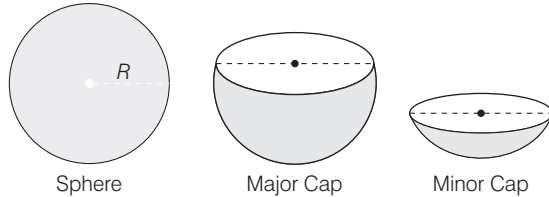
- (ii) **Small Circle:** If a plane cuts the spheres in such a way that the cut is not made through the centre of the sphere, then we get two frustums of unequal volume and unequal lateral surface area, but the area of the two circles would be same. However, these circles would be called as small circles, since the diameter of these circles is shorter than the diameter of the sphere.



Spherical cap of a Sphere

If a plane cuts the sphere into two portions then each portion is known as a cap.

The larger portion is known as major cap and the smaller portion is known as minor cap.



Let us consider the radius of sphere as R and the radius of the small circle as r and the height of the cap as h .

$$\text{Volume of a Spherical cap} = \frac{\pi h}{6} (h^2 + 3r^2) \quad (\text{If } r \text{ is known})$$

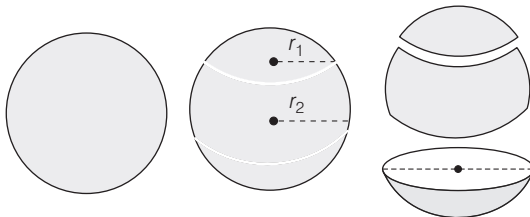
$$\text{Volume of a Spherical cap} = \frac{\pi h^2}{3} (3R - h) \quad (\text{If } R \text{ is known})$$

Lateral Surface Area of a Spherical Cap = perimeter of the sphere $\times$ height of the cap $= 2\pi R h$

$$\begin{aligned} \text{Total Surface Area of a Spherical Cap} &= 2\pi R h + \pi r^2 \\ &= \pi (2r^2 + h^2) \end{aligned}$$

Zone or Frustum of a Sphere

The portion of the surface of a sphere included between two parallel planes, which intersect the sphere, is called a zone. The distance between the two planes is called height or thickness of the zone.



Let us consider the radius of the sphere as R , height of the zone as h and the radii of the two small circles as r_1 and r_2 .

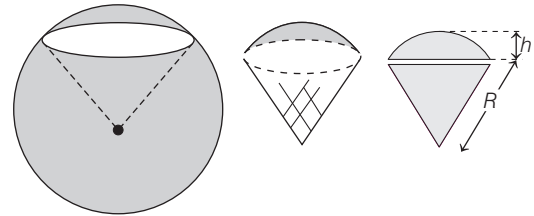
$$\text{Volume of the zone} = \frac{\pi h}{6} (h^2 + 3r_1^2 + 3r_2^2)$$

Lateral Surface Area of the zone
= perimeter of the sphere $\times$ height of the zone $= 2\pi R h$

$$\begin{aligned} \text{Total Surface Area of the zone} \\ &= 2\pi (rh + r_1^2 + r_2^2) \end{aligned}$$

Sector of a Sphere

A sector of a sphere is the solid subtended at the centre of the sphere by a segment. Essentially, a sector of a sphere is the combination of the spherical cap and the cone, where the base of spherical cap and sphere are the same and the vertex of the cone is the centre of the sphere. It is just like an ice-cream cone.



Let us consider the radius of the sphere as R and height of the spherical cap of the sphere as h .

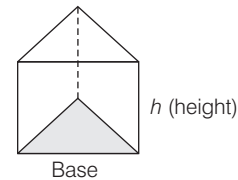
Volume of the Spherical Sector = Volume of the spherical

$$\text{cap} + \text{Volume of the cone} = \frac{2\pi R^2 h}{3}$$

Total Surface Area of the Spherical Sector = Lateral surface area of the spherical cap + Lateral surface area of the cone
 $= \pi R (2h + r)$

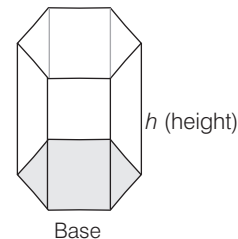
Prism

Volume = Base area $\times$ height



Lateral surface area

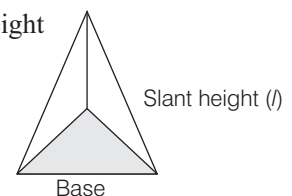
= perimeter of the base $\times$ height



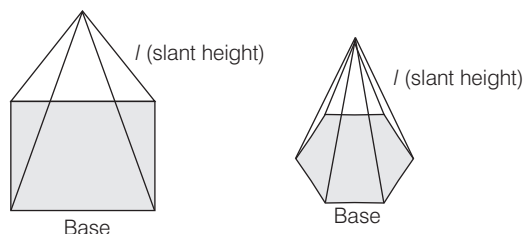
Pyramid

Volume = $\frac{1}{3} \times$ base area $\times$ height

Lateral surface area = $\frac{1}{2} \times$
perimeter of the base
 $\times$ slant height



Total surface area = lateral surface area + base area



Exp. 1) Find the volume and surface area of a sphere of radius 3.5 cm.

Solution Volume = $\frac{4}{3} \pi r^3$
 $= \frac{4}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5 = 179.66 \text{ cm}^3$
 Surface area = $4\pi r^2$
 $= 4 \times \frac{22}{7} \times 3.5 \times 3.5 = 154 \text{ cm}^2$

Exp. 2) Find the volume, curved surface area and total surface area of a hemisphere of diameter 7 cm.

Solution Volume = $\frac{2}{3} \pi r^3$
 $= \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} = 89.833 \text{ cm}^3$

Curved surface area = $2\pi r^2$
 $= 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 77 \text{ cm}^2$
 Total surface area = $3\pi r^2$
 $= 3 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$
 $= 115.5 \text{ cm}^2$

Exp. 3) How many bullets can be made from a sphere of 8 cm radius. The radius of each bullet must be 0.2 cm.

Solution Number of bullets = $\frac{\text{volume of sphere}}{\text{volume of 1 bullet}}$
 $= \frac{\frac{4}{3} \pi \times 8 \times 8 \times 8}{\frac{4}{3} \pi \times 0.2 \times 0.2 \times 0.2} = 64000$

Exp. 4) A sphere has the same curved surface as a cone of height 12 cm and base radius 5 cm. Find the radius to the nearest cm.

Solution $4\pi r^2 = \pi \times 5 \times 13$
 $\Rightarrow r^2 = \frac{65}{4}$
 $\Rightarrow r = 4 \text{ cm}$ (approx.)

Introductory Exercise 10.7

- A spherical ball of lead 6 cm in radius is melted and recast into three spherical balls. The radii of two of these balls are 3 cm and 4 cm. What is the radius of the third sphere?
 (a) 4.5 cm (b) 5 cm
 (c) 6 cm (d) 7 cm
- The radius of a copper sphere is 12 cm. The sphere is melted and drawn into a long wire of uniform circular cross-section. If the length of the wire is 144 cm, the radius of wire is :
 (a) 1 cm (b) 2 cm
 (c) 4 cm (d) none of these
- A hemispherical bowl of internal radius 6 cm contains alcohol. This alcohol is to be filled into cylindrical shaped small bottles of diameter 6 cm and height 1 cm. How many bottles will be needed to empty the bowl?
 (a) 36 (b) 27 (c) 16 (d) 4
- A hemisphere of lead of diameter 14 cm is cast into a right circular cone of height 14 cm. The radius of the base of the cone is :
 (a) 7 cm (b) 14 cm
 (c) 21 cm (d) none of these
- The volume of a spherical shell whose external and internal diameters are 14 cm and 10 cm respectively
 (a) $42\pi \text{ cm}^3$ (b) $\frac{872}{3} \pi \text{ cm}^3$
 (c) $118\pi \text{ cm}^3$ (d) $86\pi \text{ cm}^3$
- A solid metal ball of diameter 16 cm is melted and cast into smaller balls, each of radius 1 cm. The number of such balls is :
 (a) 256 (b) 2048 (c) 512 (d) 4096
- If a hemispherical dome has an inner radius 21 cm then its volume (in m^3) is :
 (a) 4910 m^3
 (b) 18354 m^3
 (c) 19404 m^3
 (d) none of the above
- A sphere of radius 9 cm is dropped into a cylindrical vessel partly filled with water. The radius of the vessel is 12 cm. If the sphere is submerged completely, then the surface of the water rises by :
 (a) 27.5 cm
 (b) 27 cm
 (c) 12 cm
 (d) 6.75 cm

9. If the height of a cone is half the radius of a sphere then the radius of the base of the cone, which has the same volume as a sphere of radius 7 cm is :
- (a) 14 m (b) $\frac{14}{\sqrt{2}}$ cm
(c) $14\sqrt{2}$ cm (d) none of these
10. From a solid sphere of radius 15 cm, a right circular cylindrical hole of radius 9 cm whose axis passing through the centre is removed. The total surface area of the remaining solid is :
- (a) 1188π cm² (b) 1080π cm²
(c) 1152π cm² (d) 1440π cm²
11. If a plane cuts the hemisphere parallel to the great circle of the hemisphere, the portion of hemisphere that has the great circle is called as
- (a) Cone (b) sector
(c) zone (d) spherical cap

Directions (for Q. Nos. 12 to 14) *Answer these questions based on the following information.*

A sphere of 5 cm radius is cut by two parallel planes which are 7 cm apart but on the opposite sides of the centre of the sphere. The radius of one of the ends of the zone is 3 cm.

12. What is the volume (in cu. cm.) of the zone?
- (a) $\frac{434}{3}\pi$ (b) $\frac{112}{3}\pi$
(c) $\frac{541}{3}\pi$ (d) 121π
13. What is the volume (in cu. cm.) of the smaller spherical cap of the two?
- (a) $\frac{21}{5}\pi$ (b) $\frac{17}{4}\pi$ (c) $\frac{14}{3}\pi$ (d) 6π
14. What is the lateral surface area (in sq. cm.) of the larger spherical cap of the two?
- (a) 10π (b) 16π
(c) 25π (d) 20π

Directions (for Q. Nos. 15 to 17) *Answer these questions based on the following information.*

A sphere of 17 cm radius is cut by two parallel planes which are 7 cm apart but on the same side of the centre of the sphere. The radius of one of the ends of the zone (or frustum) is 8 cm.

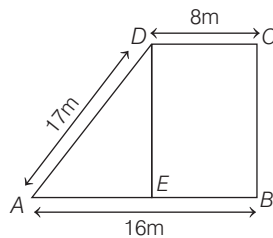
15. What is the volume of the zone?
- (a) 786π (b) $\frac{3206}{3}\pi$
(c) 1786π (d) $\frac{4096}{3}\pi$
16. What is the volume of the larger spherical cap of the two?
- (a) $\frac{15650}{7}\pi$ (b) $\frac{14400}{3}\pi$
(c) $\frac{16250}{3}\pi$ (d) none of these
17. What is the total surface area of the smaller spherical cap of the two?
- (a) 72π (b) 81π
(c) 68π (d) none of these
18. Find the volume of the sector of a sphere of radius 25 cm. The radius of the base of the conical base is 7 cm.
- (a) $344\sqrt{2}\pi$ (b) $\frac{1280}{3}\pi$
(c) $120\sqrt{3}\pi$ (d) $\frac{1250}{3}\pi$
19. The volume of a pyramid of base area 25 cm^2 and height 12 cm is :
- (a) 200 cm^3 (b) 100 cm^3
(c) 400 cm^3 (d) 800 cm^3
20. If the base of right rectangular prism remains constant and the measures of the lateral edges are halved, then its volume will be reduced by :
- (a) 50% (b) 33.33%
(c) 66.66% (d) none of these

CAT-Test

Questions Helping you bell the CAT

Multifaceted Exercise

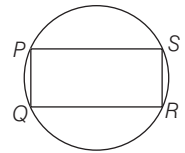
- If the side of an equilateral triangle is r , then the area of the triangle varies directly as :
(a) $\sqrt{r}$ (b) r (c) r^2 (d) r^3
- The length of the minute hand of the clock is 6 cm. The area swept by the minute hand in 30 minutes is :
(a) $\frac{1}{36\pi}$ cm² (b) $\frac{1}{18\pi}$ cm²
(c) 18π cm² (d) 36π cm²
- If the diagonals of a rhombus are 18 cm and 24 cm respectively, then its perimeter is :
(a) 15 cm (b) 42 cm (c) 60 cm (d) 70 cm
- If the ratio of diagonals of two squares is 3 : 2 then the ratio of the areas of two squares is :
(a) 4 : 5 (b) 6 : 5
(c) 9 : 4 (d) $\sqrt{3} : \sqrt{2}$
- In the given figure, $ABCD$ is a trapezium in which the parallel sides AB, CD are both perpendicular to BC . Find the area of the trapezium.
(a) 140 m²
(b) 168 m²
(c) 180 m²
(d) 156.4 m²



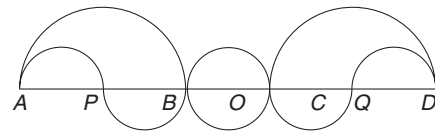
- One cubic metre piece of copper is melted and recast into a square cross-section bar, 36 m long. An exact cube is cut off from this bar. If cubic metre of copper cost ₹ 108, then the cost of this cube is :
(a) 50 paisa (b) 75 paisa
(c) 1 rupee (d) 1.50 rupee
- If ' h ' be the height of a pyramid standing on a base which is an equilateral triangle of side ' a ' units, then the slant height is :
(a) $\sqrt{h^2 + a^2/4}$ (b) $\sqrt{h^2 + a^2/8}$
(c) $\sqrt{h^2 + a^2/3}$ (d) $\sqrt{h^2 + a^2}$
- The area of the square base of a right pyramid is 64 cm². If the area of each triangle forming the slant surface is 20 cm², then the volume of the pyramid is :
(a) 64 cm³ (b) $\frac{128}{3}$ cm³
(c) $\frac{64}{3}\sqrt{3}$ cm³ (d) $64\sqrt{2}$ cm³

- If the surface areas of two spheres are in the ratio 4 : 9, then the ratio of their volumes is :
(a) 8 : 25 (b) 8 : 26
(c) 8 : 27 (d) 8 : 28
- The side of a rhombus are 10 cm and one of its diagonal is 16 cm. The area of the rhombus is :
(a) 96 cm² (b) 95 cm²
(c) 94 cm² (d) 93 cm²

- In the adjoining figure $PQRS$ is a rectangle $8\text{ cm} \times 6\text{ cm}$, inscribed in the circle. The area of the shaded portion will be :
(a) 48 cm²
(b) 42.50 cm²
(c) 32.50 cm²
(d) 30.5 cm²



- In the adjoining figure $AB = CD = 2BC = 2BP = 2CQ$. In the middle, a circle with radius 1 cm is drawn. In the rest figure all are the semicircular arcs. What is the perimeter of the whole figure?



- (a) 4π (b) 8π
(c) 10π (d) none of these
- In a shower 10 cm of rain fall the volume of water that falls on 1.5 hectares of ground is :
(a) 1500 m³ (b) 1400 m³
(c) 1200 m³ (d) 1000 m³
- The base of a prism is a right angle triangle and the two sides containing the right angle are 8 cm and 15 cm. If its height is 20 cm, then the volume of the prism is :
(a) 1600 cc (b) 300 cc
(c) 1200 cc (d) 600 cc
- A conical circus tent is to be made of canvas. The height of the tent is 35 m and the radius of the base is 84 m. If $\pi = \frac{22}{7}$, then the canvas required is :
(a) 24000 m² (b) 24004 m²
(c) 24014 m² (d) 24024 m²

- 16** The radius of base and the volume of a right circular cone are doubled. The ratio of the length of the larger cone to that of the smaller cone is :

(a) 1 : 4 (b) 1 : 2
(c) 2 : 1 (d) 4 : 1

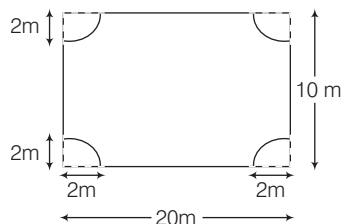
- 17** A cone and a hemisphere have equal base diameter and equal volumes. The ratio of their heights is :

(a) 3 : 1 (b) 2 : 1
(c) 1 : 2 (d) 1 : 3

- 18** A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted to form a solid cylinder of base diameter 8 cm. The height of the cylinder is approximately :

(a) 4.5 cm (b) 4.57 cm
(c) 4.67 cm (d) 4.7 cm

- 19** The perimeter of the figure given below correct to one decimal place is :



(a) 56 m (b) 56.6 m
(c) 57.2 m (d) 57.9 m

- 20** The sum of the radii of the two circle is 140 cm and the difference between their circumference is 88 cm. The radius of the larger circle is :

(a) 60 cm (b) 70 cm
(c) 63 cm (d) 77 cm

- 21** If the lateral surface of a right circular cone is 2 times its base, then the semi-vertical angle of the cone must be :

(a) 15° (b) 30° (c) 45° (d) 60°

- 22** There is a pyramid on a base which is a regular hexagon of side $2a$. If every slant edge of this pyramid is of length $\frac{(5a)}{2}$,

then the volume of this pyramid must be :

(a) $3a^3$ (b) $3a^3\sqrt{2}$
(c) $3a^3\sqrt{3}$ (d) $6a^3$

- 23** The slant height of a conical tent made of canvas is $\frac{14}{3}$ m.

The radius of tent is 2.5 m. The width of the canvas is 1.25 m. If the rate of canvas per metre is ₹ 33, then the total cost of the canvas required for the tent (in ₹) is :

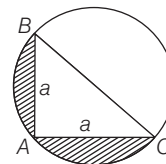
(a) 726 (b) 950
(c) 960 (d) 968

- 24** A hemispherical basin 150 cm in diameter holds water one hundred and twenty times as much a cylindrical tube. If the height of the tube is 15 cm, then the diameter of the tube (in cm) is :

(a) 23 (b) 24
(c) 25 (d) 26

- 25** If BC passes through centre of the circle, then the area of the shaded region in the given figure is :

(a) $\frac{a^2}{2} (3 - \pi)$
(b) $a^2 \left(\frac{\pi}{2} - 1 \right)$
(c) $2a^2 (\pi - 1)$
(d) $\frac{a^2}{2} \left(\frac{\pi}{2} - 1 \right)$



- 26** A river 3 m deep and 60 m wide is flowing at the rate of 2.4 km/h. The amount of water running into the sea per minute is :

(a) 6000 m³ (b) 6400 m³
(c) 6800 m³ (d) 7200 m³

- 27** A cone whose height is 15 cm and radius of base is 6 cm is trimmed sufficiently to reduce it to a pyramid whose base is an equilateral triangle. The volume of the portion removed is :

(a) 330 cm³ (b) 328 cm³
(c) 325 cm³ (d) 331 cm³

- 28** If a solid right circular cylinder is made of iron is heated to increase its radius and height by 1% each, then the volume of the solid is increased by :

(a) 1.01% (b) 3.03%
(c) 2.02% (d) 1.2%

- 29** The base of a prism is a regular hexagon. If every edge of the prism measures 1 m, then the volume of the prism is :

(a) $\frac{3\sqrt{2}}{2}$ m³ (b) $\frac{3\sqrt{3}}{2}$ m³
(c) $\frac{6\sqrt{2}}{5}$ m³ (d) $\frac{5\sqrt{3}}{2}$ m³

- 30** If the side of a square is 24 cm, then the circumference of its circumscribed circle (in cm) is :

(a) $24\sqrt{3}\pi$ (b) $24\sqrt{2}\pi$ (c) $12\sqrt{2}\pi$ (d) 24π

- 31** An isosceles right angled triangle has area 112.5 m². The length of its hypotenuse (in cm) is :

(a) 21.213 (b) 21.013
(c) 21.113 (d) 21.313

- 32** Two circles of unit radii, are so drawn that the centre of each lies on the circumference of the other. The area of the region, common to both the circles, is :

(a) $\frac{(4\pi - 3\sqrt{3})}{12}$ (b) $\frac{(4\pi - 6\sqrt{3})}{12}$
(c) $\frac{(4\pi - 3\sqrt{3})}{6}$ (d) $\frac{(4\pi - 6\sqrt{3})}{6}$

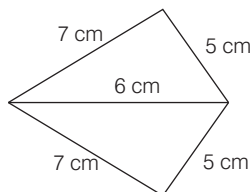
- 33** If the right circular cone is separated into three solids of volumes V_1 , V_2 and V_3 by two planes which are parallel to the base and trisects the altitude, then $V_1 : V_2 : V_3$ is :

(a) 1 : 2 : 3
(b) 1 : 4 : 6
(c) 1 : 6 : 9
(d) 1 : 7 : 19

- 34** Water flows at the rate of 10 m per minute from a cylindrical pipe 5 mm in diameter. A conical vessel whose diameter is 40 cm and depth 24 cm is filled. The time taken to fill the conical vessel is :

(a) 50 min (b) 50 min. 12 sec.
(c) 51 min. 12 sec (d) 51 min. 15 sec

- 35** The length of four sides and a diagonal of the given quadrilateral are indicated in the diagram. If A denotes the area and l the length of the other diagonal, then A and l are respectively :



(a) $12\sqrt{6}$, $4\sqrt{6}$ (b) $12\sqrt{6}$, $5\sqrt{6}$
(c) $6\sqrt{6}$, $4\sqrt{6}$ (d) $6\sqrt{6}$, $5\sqrt{6}$

- 36** If a regular square pyramid has a base of side 8 cm and height of 30 cm, then its volume is :

(a) 120 cc (b) 240 cc
(c) 640 cc (d) 900 cc

- 37** A cylinder circumscribes a sphere. The ratio of their volumes is

(a) 1 : 2 (b) 3 : 2 (c) 4 : 3 (d) 5 : 6

- 38** In triangle ABC , $BC = 8$ cm, $AC = 15$ cm and $AB = 17$ cm. The length of the altitude drawn from B on AC is :

(a) 6 cm (b) 7 cm
(c) 8 cm (d) 10 cm

- 39** The area of the largest possible square inscribed in a circle of unit radius (in square unit) is :

(a) 3 (b) 4 (c) $2\sqrt{3}\pi$ (d) 2

- 40** The area of the largest triangle that can be inscribed in a semicircle of radius r is :

(a) r^2 cm² (b) $\left(\frac{r}{3}\right)^2$ cm²
(c) $r\sqrt{2}$ cm² (d) $3\sqrt{3}r$ cm²

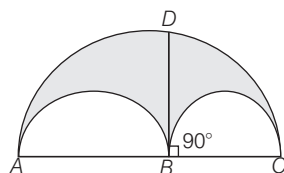
- 41** If a regular hexagon is inscribed in a circle of radius r , then its perimeter is

(a) $6\sqrt{3}r$ (b) $6r$
(c) $3r$ (d) $12r$

- 42** If a regular hexagon circumscribes a circle of radius r , then its perimeter is :

(a) $4\sqrt{3}r$ (b) $6\sqrt{3}r$
(c) $6r$ (d) $12\sqrt{3}r$

- 43** In the adjoining figure there are three semicircles in which $BC = 6$ cm and $BD = 6\sqrt{3}$ cm. What is the area of the shaded region



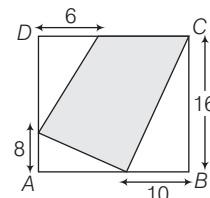
(in cm) :
(a) 12π (b) 9π
(c) 27π (d) 28π

- 44** Area of a rhombus is 144 cm² and the ratio of length of two diagonals is 1 : 2. The sum of lengths of its diagonals are :

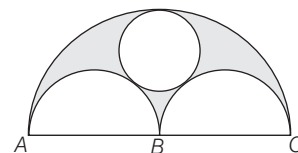
(a) 72 cm (b) 40 cm
(c) 36 cm (d) $18\sqrt{2}$ cm

- 45** Find the area of the shaded region in the given figure of square $ABCD$:

(a) 128 cm²
(b) 192 cm²
(c) 148 cm²
(d) 168 cm²



- 46** In the following figure $AB = BC$ and $AC = 84$ cm. The radius of the inscribed circle is 14 cm. B is the centre of the largest semi-circle. What is the area of the shaded region?



(a) 335 cm² (b) 770 cm²
(c) 840 cm² (d) 650 cm²

- 47** A tank 4 m long and 2.5 m wide and 6 m deep is dug in a field 10 m long and 9 m wide. If the earth dugout is evenly spread over the field, the rise in level of the field will be :

(a) 80 cm (b) 75 cm (c) 60 cm (d) 30 cm

- 48** An open box is made of wood 2 cm thick. Its internal length is 86 cm, breadth 46 cm and height is 38 cm. The cost of painting the outer surface of the box at ₹ 10 per m² is :

(a) ₹ 18.5 (b) ₹ 8.65
(c) ₹ 11.65 (d) ₹ 17.50

- 49** A rectangular tin sheet is 22 m long and 8 m broad. It is rolled along its length to form a cylinder by making the opposite edges just to touch each other. The volume of the cylinder (in m³) is :

(a) 385 (b) 204
(c) 280π (d) 308

- 50** The lateral surface of a cylinder is developed into a square whose diagonal is $2\sqrt{2}$ cm. The area of the base of the cylinder (in cm²) is :

(a) 3π (b) $1/\pi$
(c) π (d) 6π

- 51** If from a circular sheet of paper of radius 15 cm, a sector of 144° is removed and the remaining is used to make a conical surface, then the angle at the vertex will be :

(a) $\sin^{-1}\left(\frac{3}{10}\right)$ (b) $\sin^{-1}\left(\frac{6}{5}\right)$
(c) $2\sin^{-1}\left(\frac{3}{5}\right)$ (d) $2\sin^{-1}\left(\frac{4}{5}\right)$

- 52** A right circular cone of radius 4 cm and slant height 5 cm is carved out from a cylindrical piece of wood of same radius and height 5 cm. The surface area of the remaining wood is :

(a) 84π (b) 70π
(c) 76π (d) 50π

- 53** If h , s , V be the height, curved surface area and volume of a cone respectively, then $(3\pi Vh^3 + 9V^2 - s^2h^2)$ is equal to

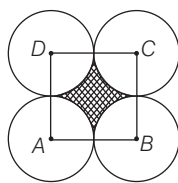
(a) 0 (b) π
(c) $\frac{V}{sh}$ (d) $\frac{36}{V}$

- 54** If a cone is cut into two parts by a horizontal plane passing through the mid point of its axis, the ratio of the volumes of the upper part and the frustum is :
 (a) 1 : 1 (b) 1 : 2
 (c) 1 : 3 (d) 1 : 7
- 55** A cone, a hemisphere and a cylinder stand on equal bases of radius R and have equal heights H . Their whole surfaces are in the ratio :
 (a) $(\sqrt{3} + 1) : 3 : 4$ (b) $(\sqrt{2} + 1) : 7 : 8$
 (c) $(\sqrt{2} + 1) : 3 : 4$ (d) none of these
- 56** If a sphere is placed inside a right circular cylinder so as to touch the top, base and the lateral surface of the cylinder. If the radius of the sphere is R , the volume of the cylinder is :
 (a) $2\pi R^3$ (b) $8\pi R^3$
 (c) $\frac{4}{3}\pi R^3$ (d) none of these
- 57** A cylinder is circumscribed about a hemisphere and a cone is inscribed in the cylinder so as to have its vertex at the centre of one end and the other end as its base. The volumes of the cylinder, hemisphere and the cone are respectively in the ratio of :
 (a) $3 : \sqrt{3} : 2$ (b) $3 : 2 : 1$ (c) $1 : 2 : 3$ (d) $2 : 3 : 1$
- 58** The base of a pyramid is a rectangle 40 m long and 20 m wide. The slant height of the pyramid from the mid-point of the shorter side of the base to the apex is 29 m. What is the volume of pyramid?
 (a) 5600 m^3 (b) 400 m^3
 (c) 6500 m^3 (d) $1753\sqrt{110} \text{ m}^3$
- 59** A copper wire when bent in the form of a square, encloses an area of 121 m^2 . If the same wire is bent to form a circle, the area enclosed by it would be :
 (a) 122 m^2 (b) 112 m^2 (c) 154 m^2 (d) 308 m^2

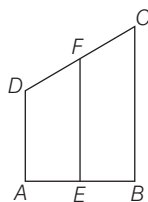
LEVEL 01 > BASIC LEVEL EXERCISE

- 1** The perimeter of a parallelogram with one internal angle 150° is 64 cm. Find the length of its sides when its area is maximum.
 (a) 16, 16 (b) 15, 17
 (c) 14, 18 (d) can't be determined
- 2** A sphere of 30 cm radius is dropped into a cylindrical vessel of 80 cm diameter, which is partly filled with water, then its level rises by x cm. Find x .
 (a) 27.5 cm (b) 22.5 cm
 (c) 18.5 cm (d) none of these
- 3** Amit walked 12 m toward east, then he turned to his right and walked 18 m. He then turned to his right and walked 12 m. He again turned to his right and walked 28 m then he again turned to his right and walked 24 m. At what distance is he from the starting point and in which direction?
 (a) 23 m north-east (b) 26 m north-east
 (c) 26 m west (d) 34 m north-east
- 4** Find the inradius of triangle if its area is 30 cm^2 and hypotenuse is 13 cm.
 (a) 1 cm (b) 2 cm
 (c) 2.5 cm (d) $2\sqrt{2} \text{ cm}$
- 5** Which of the following figure will have maximum area if the perimeter of all figures is same?
 (a) Square (b) Octagon
 (c) Circle (d) Hexagon
- 6** $ABCD$ is a trapezium with $\angle A = 90^\circ$ and AB parallel to CD . Then $\angle B$ is :
 (a) 90° (b) $90^\circ - \angle C$
 (c) $360^\circ - \angle C$ (d) $180^\circ - \angle C$
- 7** A square $ABCD$ has an equilateral triangle drawn on the side AB (interior of the square). The triangle has vertex at G . What is the measure of the angle CGB ?
 (a) 60° (b) 80°
 (c) 75° (d) 90°
- 8** There are two concentric circles whose areas are in the ratio of 9 : 16 and the difference between their diameters is 4 cm. What is the area of the outer circle?
 (a) 32 cm^2 (b) $64\pi \text{ cm}^2$
 (c) 36 cm^2 (d) 48 cm^2
- 9** A square and rhombus have the same base. If the rhombus is inclined at 60° , find the ratio of area of square to the area of the rhombus.
 (a) $2\sqrt{3} : 3$ (b) $1 : \sqrt{3}$
 (c) $\sqrt{3} : 2$ (d) none of these
- 10** Four isosceles triangles are cut off from the corners of a square of area 400 m^2 . Find the area of new smaller square (in m^2).
 (a) $200\sqrt{2}$ (b) $\frac{200}{\sqrt{2}}$
 (c) 200 (d) $100\sqrt{2}$
- 11** Altitude and base of a right angle triangle are $(x + 2)$ and $(2x + 3)$ (in cm). If the area of the triangle be 60 cm^2 , the length of the hypotenuse is :
 (a) 21 cm (b) 13 cm
 (c) 17 cm (d) 15 cm
- 12** Find the area of a regular octagon with each side ' a ' cm.
 (a) $2a^2(1 + \sqrt{2})$ (b) $\sqrt{2}a(1 + \pi)$
 (c) $a^2(\sqrt{2} + 2)$ (d) none of these

- 13** $ABCD$ is a square, 4 equal circles are just touching each other whose centres are the vertices A, B, C, D of the square. What is the ratio of the shaded to the unshaded area within square?



- (a) $\frac{8}{11}$ (b) $\frac{3}{11}$
(c) $\frac{5}{11}$ (d) $\frac{6}{11}$
- 14** $ABCD$ is a trapezium, in which $AD \parallel BC$, E and F are the mid-points of AB and CD respectively, then EF is :
- (a) $\frac{(AD + BC)}{2}$
(b) $\frac{(AB + CD)}{2}$
(c) $\frac{DF \times CF}{AE \times BE}$
(d) $\frac{AD + EF + BC}{2}$



- 15** A right circular cone resting on its base is cut at $\frac{4}{5}$ th its height along a plane parallel to the circular base. The height of original cone is 75 cm and base diameter is 42 cm. What is the base radius of cut out (top portion) cone?
(a) 4.2 cm (b) 2.8 cm (c) 3.5 cm (d) 8.4 cm
- 16** l, b are the length and breadth of a rectangle respectively. If the perimeter of this rectangle is numerically equal to the area of the rectangle. What is the value of $l - b$ (where $l > b$)?
(a) 1 (b) 2
(c) 3 (d) can't be determined

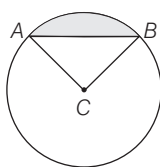
- 17** In the adjoining figure ABC is an equilateral triangle and C is the centre of the circle, A and B lie on the circle. What is the area of the shaded region, if the diameter of the circle is 28 cm?

(a) $\left(102\frac{2}{3} - 49\sqrt{3}\right) \text{ cm}^2$

(b) $\left(103\frac{2}{3} - 98\sqrt{3}\right) \text{ cm}^2$

(c) $(109 - 38\sqrt{3}) \text{ cm}$

(d) none of the above

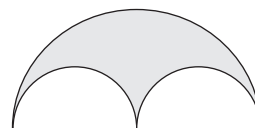


- 18** l_1, b_1 and l_2, b_2 are the lengths and breadths of the two rectangles respectively, but the areas of the rectangles are same. l_1 is increased by 25% and b_1 is decreased by 25%. Similarly l_2 is decreased by 25% and b_2 is increased by 25%. If A_1 and A_2 is the new areas of the two rectangles respectively, then :
(a) $A_1 > A_2$ (b) $A_1 < A_2$
(c) $A_1 = A_2$ (d) can't be determined
- 19** An acute angle made by a side of parallelogram with other pair of parallel sides is 60° . If the distance between these parallel sides is $6\sqrt{3}$, the other side is :
(a) 12 cm (b) $12\sqrt{3}$ cm
(c) $15\sqrt{3}$ cm (d) none of these

- 20** A solid sphere is melted and recast into a right circular cone with a base radius equal to the radius of the sphere. What is the ratio of the height and radius of the cone so formed?

- (a) 4 : 3 (b) 2 : 3
(c) 3 : 4 (d) none of these

- 21** In the given figure there are 3 semicircles, the radii of each smaller circle is equal. If the radius of the larger circle be 22 cm, then the area of the shaded region is :



- (a) $363\frac{\pi}{4}$ (b) $363\frac{\pi}{3}$ (c) 236.5π (d) 363π

- 22** A rectangular lawn $60 \text{ m} \times 40 \text{ m}$ has two roads each 5 m wide running in the middle of it, one parallel to length and the other parallel to breadth. The cost of gravelling the roads at 80 paise per sq. m is :

- (a) ₹ 380 (b) ₹ 385
(c) ₹ 400 (d) none of these

- 23** There are two rectangular fields of same area. The length of first rectangular field is $x\%$ less than the length of the second field and breadth of the first field is $(5x)\%$ greater than the breadth of the second field. What is the value of x ?

- (a) 15 (b) 25
(c) 50 (d) 80

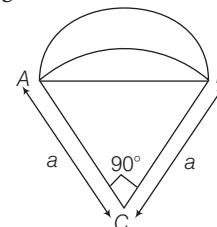
- 24** In the adjoining figure ACB is a quadrant with radius ' a '. A semicircle is drawn outside the quadrant taking AB as a diameter. Find the area of shaded region.

(a) $\frac{1}{4}(\pi - 2a^2)$

(b) $\left(\frac{1}{4}\right)(\pi a^2 - a^2)$

(c) $\frac{a^2}{2}$

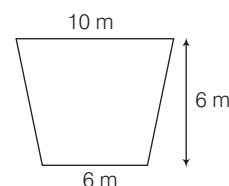
(d) can't be determined



- 25** Ravi made an error of 5% in excess while measuring the length of rectangle and an error of 8% deficit was made while measuring the breadth. What is the percentage error in the area?

- (a) - 3% (b) - 40%
(c) - 3.4% (d) can't be determined

- 26** In the adjoining figure the cross-section of a swimming pool is shown. If the length of the swimming pool is 120 m, then the amount of water it can hold is :



- (a) 5760 m^3
(b) 9600 m^3
(c) 7200 m^3
(d) none of the above

- 27 Around a circular garden a circular road is to be repaired which costs ₹ 22176 at the rate of ₹ 1 per sq m. If the inner radius is 112 m, find the width of the circular road.

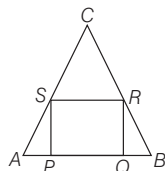
(a) 18 m (b) 28 m
(c) 14 m (d) none of these

- 28 An equilateral triangle is cut from its three vertices to form a regular hexagon. What is the percentage of area wasted?

(a) 20% (b) 50%
(c) 33.33% (d) 66.66%

- 29 ABC is an equilateral triangle and $PQRS$ is a square inscribed in the triangle in such a way that P and Q lie on AB and R, S lie on BC and AC respectively. What is the value of $RC : RB$?

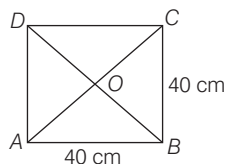
(a) $1 : \sqrt{2}$ (b) $1 : \sqrt{3}$
(c) $\sqrt{3} : 2$ (d) $1 : 2$



- 30 The area of a square and circle is same and the perimeter of square and equilateral triangle is same, then the ratio between the area of circle and the area of equilateral triangle is :

(a) $\pi : 3$ (b) $9 : 4\sqrt{3}$
(c) $4 : 9\sqrt{3}$ (d) none of these

- 31 Adjoining figure shows a square $ABCD$ in which O is the point of intersection of diagonals AC and BD . Four squares of maximum possible area are formed inside each of four triangles AOB , BOC , COD and AOD . What is the total area of these 4 squares?



(a) 400 cm^2 (b) 100 cm^2
(c) 80 cm^2 (d) none of these

- 32 What is the ratio of the area of circumcircle of equilateral triangle to the area of square with the same side length as the equilateral triangle?

(a) $\pi : 3$ (b) $\pi : \sqrt{3}$
(c) $\sqrt{3} : 2$ (d) none of these

- 33 It is required to construct a big rectangular hall that can accommodate 400 people with 25 m^3 space for each person. The height of the wall has been fixed at 10 m and the total inner surface area of the walls must be 1300 m^2 . What is the length and breadth of the hall (in metres)?

(a) 30, 20 (b) 45, 20
(c) 40, 25 (d) 35, 30

- 34 The perimeter of a rectangle and an equilateral triangle are same. Also, one of the sides of the rectangle is equal to the side of the triangle. The ratio of the areas of the rectangle and the triangle is :

(a) $\sqrt{3} : 1$ (b) $1 : \sqrt{3}$
(c) $2 : \sqrt{3}$ (d) $4 : \sqrt{3}$

- 35 If l, b, p be the length, breadth and perimeter of a rectangle and b, l, p are in GP (in order), then l/b is :

(a) $2 : 1$ (b) $(\sqrt{3} - 1) : 1$
(c) $(\sqrt{3} + 1) : 1$ (d) $2 : \sqrt{3}$

- 36 A spherical steel ball was silver polished then it was cut into 4 similar pieces. What is ratio of the polished area to the non polished area?

(a) $1 : 1$ (b) $1 : 2$
(c) $2 : 1$ (d) can't be determined

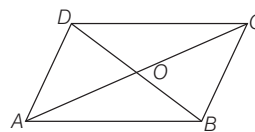
- 37 What is the total surface area of the identical cubes of largest possible size that are cut from a cuboid of size $85 \text{ cm} \times 17 \text{ cm} \times 5.1 \text{ cm}$?

(a) 26010 cm^2 (b) 21600 cm^2
(c) 26100 cm^2 (d) none of these

- 38 125 identical cubes are cut from a big cube and all the smaller cubes are arranged in a row to form a long cuboid. What is the percentage increase in the total surface area of the cuboid over the total surface area of the cube?

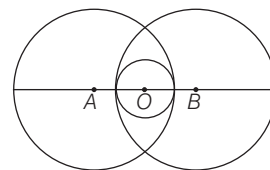
(a) $234\frac{2}{3}\%$ (b) $235\frac{1}{3}\%$
(c) $134\frac{2}{3}\%$ (d) none of these

- 39 In the adjoining figure a parallelogram $ABCD$ is shown. $AB = 24 \text{ cm}$ and $AO = BO = 13 \text{ cm}$. Find BC .



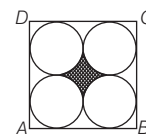
(a) 8 cm (b) 10 cm
(c) 11 cm (d) none of these

- 40 There are two circles intersecting each other. Another smaller circle with centre O , is lying between the common region of two larger circles. Centres of the circle (i.e., A, O and B) are lying on a straight line. $AB = 16 \text{ cm}$ and the radii of the larger circles are 10 cm each. What is the area of the smaller circle?



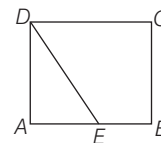
(a) $4\pi \text{ cm}^2$ (b) $2\pi \text{ cm}^2$
(c) $\frac{4}{\pi} \text{ cm}^2$ (d) $\frac{\pi}{4} \text{ cm}^2$

- 41 $ABCD$ is a square, inside which 4 circles with radius 1 cm, each are touching each other. What is the area of the shaded region?



(a) $(2\pi - 3) \text{ cm}^2$
(b) $(4 - \pi) \text{ cm}^2$
(c) $(16 - 4\pi) \text{ cm}^2$
(d) none of the above

- 42 $ABCD$ is a square, E is a point on AB such that $BE = 17 \text{ cm}$. The area of triangle ADE is 84 cm^2 . What is the area of square?



(a) 400 cm^2
(b) 625 cm^2
(c) 729 cm^2
(d) 576 cm^2

- 43 If the volume of a sphere, a cube, a tetrahedron and a octahedron be same then which of the following has maximum surface area?

(a) Sphere (b) Cube
(c) Octahedron (d) Tetrahedron

- 44 In a rectangle the ratio of the length is to breadth is same as that of the sum of the length and breadth to the length. If l and b be the length and breadth of the rectangle then which of the following is true?

(i) $\frac{l}{b} = \frac{l^2}{b^2} + 1$ (ii) $\frac{b}{l-b} = \frac{l+b}{l}$

(iii) $lb = (l+b)(l-b)$

- (a) only (i) is true
(b) only (ii) is true
(c) only (ii) and (iii) are true
(d) only (i) and (ii) are true

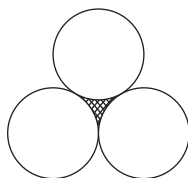
- 45 Three circles of equal radii touch each other as shown in figure. The radius of each circle is 1 cm. What is the area of shaded region?

(a) $\left(\frac{2\sqrt{3}-\pi}{2}\right) \text{ cm}^2$

(b) $\left(\frac{3\sqrt{2}-\pi}{3}\right) \text{ cm}^2$

(c) $\frac{2\sqrt{3}}{\pi} \text{ cm}^2$

- (d) none of the above



- 46 How many spheres of radius 1.5 cm can be cut out of a wooden cube of edge 9 cm?

- (a) 216 (b) 81
(c) 27 (d) can't be determined

- 47 Kaurav and Pandav have a rectangular field of area 20000 sq. m. They decided to divide it into two equal parts by dividing it with a single straight line. Kaurav wanted to fence their land immediately, so they incurred total expenses for the fencing all the four sides alone at ₹ 2 per metre. What is the minimum cost that Kaurav had to incur?

- (a) ₹ 800 (b) ₹ 1600 (c) ₹ 1200 (d) ₹ 600

- 48 There is a cone of height 12 cm, out of which a smaller cone (which is the top portion of the original cone) with the same vertex and vertical axis is cut out.

What is the ratio of the volume of the larger (actual) cone to the remaining part (frustum) of the cone, if the height of the smaller cone is 9 cm?

- (a) 3 : 1 (b) 9 : 1
(c) 64 : 37 (d) 16 : 7

- 49 Radhey can walk along the boundary of a rectangular field and also along the diagonals of the field. His speed is 53 km/h. The length of the field is 45 km. Radhey started from one corner and reached to the diagonally opposite corner in 1 hour. What is the area of the field?

- (a) 860 km² (b) 1260 km²
(c) 1060 km² (d) can't be determined

- 50 Charles has a right circular cylinder which he inserted completely into a right circular cone of height 30 cm. The vertical angle of the cone is 60° and the diameter of the cylinder is $8\sqrt{3}$ cm. What is the volume of the cone?

- (a) $\frac{3000}{7} \pi \text{ cm}^3$ (b) $3000\pi \text{ cm}^3$
(c) $4860\pi \text{ cm}^3$ (d) can't be determined

- 51 There are six faces in a cube. Rajeev fix one cube on each of the faces. The dimensions of all the cubes are same. What is the ratio of total surface area of the newly formed solid to the area of a single cube?

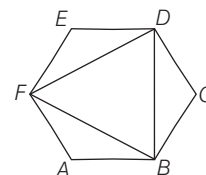
- (a) 7 : 1 (b) 6 : 1 (c) 5 : 1 (d) 41 : 9

- 52 If the ratio of diagonals of two cubes is 3 : 2, then the ratio of the surface areas of the two cubes respectively is :

- (a) 5 : 4 (b) 9 : 5
(c) 9 : 4 (d) can't be determined

- 53 $ABCDEF$ is a regular hexagon of side 6 cm. What is the area of triangle BDF ?

- (a) $32\sqrt{3} \text{ cm}^2$
(b) $27\sqrt{3} \text{ cm}^2$
(c) 24 cm^2
(d) none of the above



Directions (for Q. Nos. 54 and 55) *King Dashratha of Ayodhya had a rectangular plot of area 9792 m². He divided it into 4 square shaped plots by fencing parallel fences to the breadth of the rectangular plot. All the four sons got each square shaped plot. However, some area of plot was still left which could not be formed as a square shaped. So, four more square shaped plots were formed by fencing parallel to the longer side of the original plot. The king gave one smaller square shaped plot to each of his wives and one of the smaller square shaped plot retained with himself and then nothing left to divide.*

- 54 What is the ratio of the area of larger square shaped plot to the area of the smaller square shaped plot?

- (a) 17 : 1 (b) 25 : 9
(c) 16 : 1 (d) can't be determined

- 55 What are dimensions of the original plot?

- (a) 288 m, 34 m (b) 102 m, 96 m
(c) 306 m, 32 m (d) 204 m, 48 m

- 56 A scalene triangle PQR, such that PQ = 25, QR = 51 and PR = 74, is rotated completely about PQ, what is the volume of the solid formed in this way?

- (a) $3300\pi \text{ cm}^3$ (b) $4800\pi \text{ cm}^3$
(c) $3600\pi \text{ cm}^3$ (d) $4900\pi \text{ cm}^3$

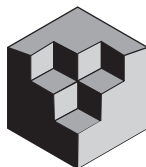
- 57 There are two squares $ABCD$ and $PQRS$ with the same area. Square $ABCD$ has the largest possible circle inscribed in it, while square $PQRS$ has four circles, tangent to each other, inscribed in it such that the maximum area is occupied by them. What is the ratio of the area occupied by the circle in $ABCD$ to the total area occupied by the circles in $PQRS$?

- (a) 1 : 2 (b) 1 : $\sqrt{2}$
(c) 2 : $\sqrt{3}$ (d) 1 : 1

- 58** A square paper is cut into smaller pieces such that each piece is exactly a square, not necessarily of the equal size. From each such square a circle of the largest possible area is cut out. Find the ratio of the area of all the possible circles to the area of the original square.

(a) 1 : 1
(b) π : 2
(c) π : 4
(d) data insufficient

- 59** The elder son of a carpenter picked up a solid wooden cube from his garage and then he removed a few cubical blocks from it so that he can sit on it with his friends comfortably and watch the football match in his house. If the volume of the original cube is 216 cubic ft. and the volume of each smaller cubic solid that has been removed is 8 cubic ft., what is the surface area (in cu. ft.) of the remaining solid?



(a) 216 (b) 192
(c) 180 (d) none of these

- 60** Find the area of the rectangle ABCD, if P is any point on AB such that DP = 18 mm, CP = 21 mm and $\angle DPC = 90^\circ$.

(a) 378 mm^2 (b) 189 mm^2
(c) 126 mm^2 (d) 136 mm^2

- 61** An ant has to go from one corner to the farthest corner of a canister of size $6 \times 6 \times 12$. What's the minimum distance it has to cover, (All the lengths are in cm)?

(a) $6(\sqrt{2} + 2)$ (b) $6\sqrt{3}$
(c) $12\sqrt{2}$ (d) $12(\sqrt{2} + 1)$

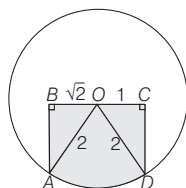
- 62** The ratio of semiperimeter of a parallelogram to its longer side is same as the ratio of its longer side to its shorter side. If the shorter side measures 2 cm, find the maximum possible area of the parallelogram.

(a) $2(\sqrt{5} + 2)$ (b) $2(1 + \sqrt{5})$
(c) $4(1 + \sqrt{5})$ (d) data insufficient

- 63** A semi-circle and a circle with same radius inscribe a square of greatest possible area. What is the ratio of area of square inscribed by semicircle to that by circle?

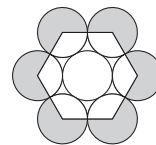
(a) 2:5 (b) 1:2
(c) 1:4 (d) 4:5

- 64** In the following figure the radii (OA and OD) are 2 cm and $\triangle ABO$ and $\triangle DCO$ are right angle triangles. Also, $OB = \sqrt{2}$ cm and $OC = 1$ cm. Find the area of the shaded region in the circle.



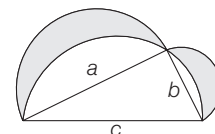
(a) $1 + \frac{5\pi}{6} \text{ sq cm}$
(b) $\frac{5\pi}{6} + \frac{\sqrt{3}}{2} \text{ sq cm}$
(c) $2 + \frac{5\pi}{6} - \frac{\sqrt{3}}{2} \text{ sq cm}$
(d) $1 + \frac{5\pi}{6} + \frac{\sqrt{3}}{2} \text{ sq cm}$

- 65** In the adjoining figure, 7 congruent circles are placed in such a way that one circle is at the centre and other 6 circles are tangent to it. A regular hexagon is drawn such that its vertices are the centres of the 6 circles, as shown here. If the area of circle is 1 sq. cm, what is the area of the hexagon?



(a) $\frac{7\sqrt{3}}{\pi} \text{ sq cm}$ (b) $\frac{2\pi\sqrt{3}}{7} \text{ sq cm}$
(c) $\frac{6\sqrt{3}}{\pi} \text{ sq cm}$ (d) none of these

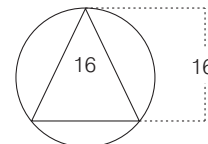
- 66** A right angle triangle whose hypotenuse is c and its perpendicular sides are a and b . Three semicircles are drawn along the three sides of this triangle in such a way that the diameter of each semicircle is equal to the length of the respective side of the triangle.



The semicircle drawn with the help of hypotenuse intersects the other two semicircles as shown in the concerned diagram. Find the area of the shaded region.

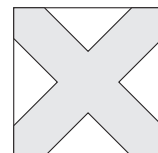
(a) $\frac{ab}{2}$ (b) $\frac{2ab\pi}{3}$
(c) $\frac{ab\pi}{c}$ (d) data insufficient

- 67** In the following diagram, the height and base of an isosceles triangle are 16 cm each. Find the area of the circumscribing circle.



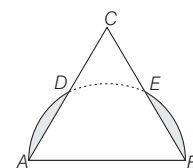
(a) 256 sq cm (b) 324 sq cm
(c) 314 sq cm (d) none of these

- 68** A large city is developed on a square plot that has two uniform roads connecting the opposite corners of the city, as shown in the diagram. If the total area of the roads is half the area of the square plot, what is the ratio of the length of the plot to the width of the each road?



(a) $5/2\sqrt{2}$ (b) $(4 + \sqrt{2})/3$
(c) $2\sqrt{2} + 2$ (d) none of these

- 69** The diameter of a semicircle and the base of an equilateral $\triangle ABC$ coincide in such a way that the other two sides of the triangle intersect the circumference of the semicircle at the points D and E, as shown in the diagram. If the perimeter of the triangle ABC is 6 cm, find the area of the shaded region.



(a) $\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) \text{ cm}^2$ (b) $\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right) \text{ cm}^2$
(c) $\pi \left(\frac{2 - \sqrt{3}}{3}\right)$ (d) none of these

LEVEL 02 > HIGHER LEVEL EXERCISE

Directions (for Q. Nos. 1 to 3) Each edge of an equilateral triangle is 'a' cm. A cone is formed by joining any two sides of the triangle.

- 1 What is the radius and slant height of the cone?

(a) $a, \frac{a}{2\pi}$ (b) $\frac{a}{\pi}, \frac{a}{2}$
 (c) $\frac{a}{2\pi}, a$ (d) $2a, \frac{a}{\pi}$

- 2 What is the volume of the cone?

(a) $\frac{a^2}{24\pi^3} \sqrt{4 - \pi^2}$ (b) $\frac{a^3}{24\pi^2} \sqrt{4\pi^2 - 1}$
 (c) $\frac{a^3}{8\pi^2} \sqrt{1 - 4\pi^2}$ (d) $\frac{a}{\sqrt{3}} \pi^2 \left(1 - \frac{2}{\pi}\right)$

- 3 If the cone is cut along its axis from the middle, the new shape we obtain after opening the paper is :

- (a) isosceles triangle (b) equilateral triangle
 (c) right angle triangle (d) none of these

- 4 If the sum of the radius and the height of a closed cylinder is 35 cm and the total surface area of the cylinder is 1540 cm^2 , then the circumference of the base of the cylinder is :

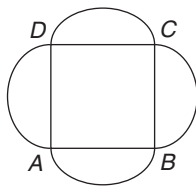
- (a) 66 cm (b) 44 cm
 (c) 56 cm (d) can't be determined

- 5 An iron pillar has some part in the form of a right circular cylinder and remaining in the form of a right circular cone. The radius of base of cone as well as cylinder is 21 cm. The cylindrical part is 80 cm high and conical part is 16 cm high. Find the weight of the pillar, if 1 cm^3 of iron weighs 8.45 g.

- (a) 999.39 kg (b) 111 kg
 (c) 1001 kg (d) 989 kg

- 6 ABCD is a square of side a cm. AB, BC, CD and AD all are the chords of circles with equal radii each. If the chords subtend an angle of 120° at their respective centres, find the total area of the given figure, where arcs are part of the circles :

(a) $\left[a^2 + 4 \left(\frac{\pi a^2}{9} - \frac{a^2}{3\sqrt{2}} \right) \right]$
 (b) $\left[a^2 + 4 \left(\frac{\pi a^2}{9} - \frac{a^2}{4\sqrt{3}} \right) \right]$
 (c) $[9a^2 - 4\pi + 3\sqrt{3}a^2]$
 (d) none of the above



- 7 A rectangle has a perimeter of 26. How many combinations of integral valued length are possible?

- (a) 4 (b) 8
 (c) 6 (d) 12

- 8 A hollow sphere with outer diameter 24 cm is cut into two equal hemispheres. The total surface area of one of the hemispheres is $1436 \frac{2}{7} \text{ cm}^2$. Each one of the hemispheres is

filled with water. What is the volume of water that can be filled in each of the hemispheres?

(a) $3358 \frac{2}{3} \text{ cm}^3$ (b) $3528 \frac{2}{3} \text{ cm}^3$
 (c) $2359 \frac{2}{3} \text{ cm}^3$ (d) $9335 \frac{2}{3} \text{ cm}^3$

- 9 A big cube of side 8 cm is formed by rearranging together 64 small but identical cubes each of side 2 cm. Further, if the corner cubes in the topmost layer of the big cube are removed, what is the change in total surface area of the big cube?

- (a) 16 cm^2 , decreases
 (b) 48 cm^2 , decreases
 (c) 32 cm^2 , decreases
 (d) remains the same as previously

- 10 A large solid sphere of diameter 15 m is melted and recast into several small spheres of diameter 3 m. What is the percentage increase in the total surface area of all the smaller spheres over that of the large sphere?

- (a) 200% (b) 400%
 (c) 500% (d) can't be determined

- 11 A cone is made of a sector with a radius of 14 cm and an angle of 60° . What is total surface area of the cone?

- (a) 119.78 cm^2 (b) 191.87 cm^2
 (c) 196.5 cm^2 (d) none of these

- 12 Kishan Chand is a very labourious farmer. He erected a fence around his paddy field in a square shape. He used 26 poles in each side, each at a distance of 4 m. What is the area of his field?

- (a) 1.6 hectare (b) 2.6 hectare
 (c) 5.76 hectare (d) 1 hectare

- 13 A rectangular lawn is surrounded by a path of width 2 m on all sides. Now, if the length of the lawn is reduced by 2 m the lawn becomes a square lawn and the area of path becomes $13/11$ times, what is the length of the original lawn?

- (a) 8 m (b) 9 m (c) 10 m (d) 12 m

Directions (for Q. Nos. 14 and 15) A cylinder with height and radius 2 : 1 is filled with soft drink and then it is tilted so as to allow some soft drink to flow off to an extent where the level of soft drink just touches the lowest point of the upper mouth.

- 14 If the 2.1 L soft drink is retained in the cylinder, what is the capacity of the cylinder?

- (a) 3.6 L (b) 4 L (c) 1.2 L (d) 4.2 L

- 15 If the quantity of soft drink is poured into a conical flask whose height and base radius are same as that of the cylinder so as to fill the conical flask completely, the quantity of soft drink left in the cylinder as a fraction of its total capacity is :

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{9}$ (d) $\frac{1}{10}$

- 16 An elephant of length 4 m is standing at one corner of a rectangular cage $16\text{ m} \times 30\text{ m}$ and facing towards the diagonally opposite corner. If the elephant starts moving towards the diagonally opposite corner it takes 15 s to reach the opposite corner. Find the speed of the elephant.

(a) 1 m/s (b) 2 m/s
(c) 1.87 m/s (d) can't be determined

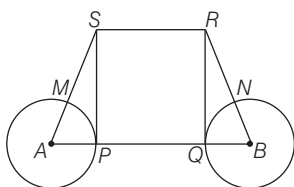
- 17 What is the height of the cone which is formed by joining the two ends of a sector of a circle with radius r and central angle 60° ?

(a) $\frac{\sqrt{35}}{6}r$ (b) $\frac{\sqrt{25}}{6}r$
(c) $\frac{r^2}{\sqrt{3}}$ (d) none of these

- 18 If a cube of maximum possible volume is cut off from a solid sphere of diameter d , the volume of the remaining (waste) material of the sphere would be equal to :

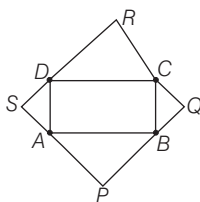
(a) $\frac{d^3}{3} \left(\pi - \frac{d}{2} \right)$ (b) $\frac{d^3}{3} \left(\frac{\pi}{2} - \frac{1}{\sqrt{3}} \right)$
(c) $\frac{d^2}{4} (\sqrt{2} - \pi)$ (d) none of these

- 19 In the adjoining figure $PQRS$ is a square and $MS = RN$ and the points A, P, Q and B lie on the same line. Find the ratio of the total area of two circles to the area of the square. Given that $AP = MS$.



(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{3\pi}{2}$ (d) $\frac{6}{\pi}$

- 20 $ABCD$ is a rectangle and there are four equilateral triangles. Area of $\triangle ASD$ is equal to area of $\triangle BQC$ and area of $\triangle DRC$ is equal to area of $\triangle APB$. The perimeter of the rectangle is 12 cm. Also the sum of the areas of the four triangles is $10\sqrt{3}\text{ cm}^2$, the total area of



the figure thus formed is :

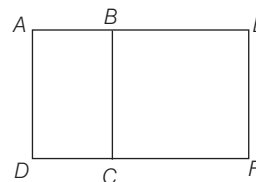
(a) $2(4 + 5\sqrt{3})\text{ cm}^2$
(b) $5(4 + 2\sqrt{3})\text{ cm}^2$
(c) $42\sqrt{3}\text{ cm}^2$
(d) none of the above

- 21 $ABCD$ and $EFGB$ are the squares of side 4 cm, each. In square $ABCD$, DMB and PMQ are the arcs of circles with centres at A and C , respectively. In square $AEFG$, the shaded region is enclosed by two arcs of circles with centres at A and F , respectively. What is the ratio of the shaded regions of the squares $ABCD$ and $AEFG$, respectively?

(a) $\frac{2 + \pi(\sqrt{2} - 2)}{(\pi - 2)}$ (b) $\frac{(\pi - 2)}{2(\sqrt{2} + 1 - \pi)}$
(c) $\frac{4}{3}$ (d) none of these

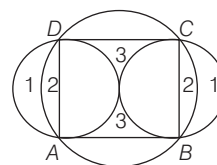
- 22 In the adjoining figure $\frac{AB}{BC} = \frac{AD}{DF}$, where $EBCF$ is a square.

Find the ratio of $\frac{AE}{EF}$.



(a) $\frac{(1 \pm \sqrt{7})}{3}$ (b) $\frac{(1 - \sqrt{7})}{2}$
(c) $\frac{(1 + \sqrt{5})}{2}$ (d) $\frac{(1 \pm \sqrt{5})}{2}$

Directions (for Q. Nos. 23 to 25) In the following figure $ABCD$ is a square. A circle $ABCD$ is passing through all the four vertices of the square. There are two more circles on the sides AD and BC touching each other inside the square. AD and BC are the respective diameters of the two smaller circles. Area of the square is 16 cm^2 .



- 23 What is the area of region 1?

(a) 2.4 cm^2 (b) $\left(2 - \frac{\pi}{4}\right)\text{ cm}^2$
(c) 8 cm^2 (d) $(4\pi - 2)\text{ cm}^2$

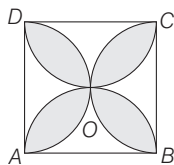
- 24 What is the area of region 2?

(a) $3(\pi - 2)\text{ cm}^2$ (b) $(\pi - 3)\text{ cm}^2$
(c) $(2\pi - 3)\text{ cm}^2$ (d) $4(\pi - 2)\text{ cm}^2$

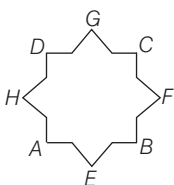
- 25 What is the area of region 3?

(a) $(4 - 4\pi)\text{ cm}^2$
(b) $4(4 - \pi)\text{ cm}^2$
(c) $(4\pi - 2)\text{ cm}^2$
(d) $(3\pi + 2)\text{ cm}^2$

- 26** In the adjoining figure $ABCD$ is a square. Four equal semicircles are drawn in such a way that they meet each other at 'O'. Sides AB, BC, CD and AD are the respective diameters of the four semicircles. Each of the sides of the square is 8 cm. Find the area of the shaded region.
- (a) $32(\pi - 2) \text{ cm}^2$ (b) $16(\pi - 2) \text{ cm}^2$
 (c) $(2\pi - 8) \text{ cm}^2$ (d) $\left(\frac{3}{4}\pi - 4\right) \text{ cm}^2$

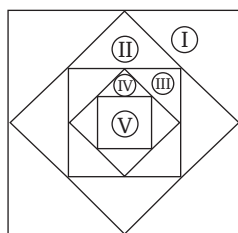


- 27** $ABCD$ is a square. Another square $EFGH$ with the same area is placed on the square $ABCD$ such that the point of intersection of diagonals of square $ABCD$ and square $EFGH$ coincide and the sides of square $EFGH$ are parallel to the diagonals of square $ABCD$. Thus a new figure is formed as shown in the figure. What is the area enclosed by the given figure if each side of the square is 4 cm?



- (a) $32(2 - \sqrt{2})$ (b) $16\left(\frac{3 + \sqrt{2}}{2 + \sqrt{2}}\right)$
 (c) $32\left(\frac{2 + \sqrt{2}}{3 + \sqrt{2}}\right)$ (d) none of these
- 28** A piece of paper is in the form of a right angle triangle in which the ratio of base and perpendicular is 3 : 4 and hypotenuse is 20 cm. What is the volume of the biggest cone that can be formed by taking right angle vertex of the paper as the vertex of the cone?
- (a) 45.8 cm^3 (b) 56.1 cm^3
 (c) 61.5 cm^3 (d) 48 cm^3
- 29** In a particular country the value of diamond is directly proportional to the surface area (exposed) of the diamond. Four thieves steal a cubical diamond piece and then divide equally in four parts. What is the maximum percentage increase in the value of diamond after cutting it?
- (a) 50% (b) 66.66%
 (c) 100% (d) none of these

Directions (for Q. Nos. 30 and 31) In the figure shown square II is formed by joining the mid-points of square I, square III is formed by joining the mid-points of square II and so on. In this way total five squares are drawn. The side of the square I is 'a' cm.



- 30** What is the perimeter of all the five squares?
- (a) $\frac{(4\sqrt{2} + 1)a}{(\sqrt{2} + 1)}$ (b) $\frac{(4\sqrt{2} - 1)a}{(\sqrt{2} + 1)}$
 (c) $\frac{5}{6}a$ (d) $(7 + 3\sqrt{2})a$
- 31** What is the total area of all the five squares?
- (a) $\frac{(4\sqrt{2} - 1)a^2}{(4\sqrt{2} - 1)}$ (b) $\frac{(4\sqrt{2} - 1)a^2}{4(\sqrt{2} - 1)}$
 (c) $\frac{31}{16}a^2$ (d) none of these

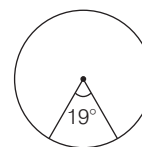
Directions (for Q. Nos. 32 to 35) Each edge of a cube is equally divided into n parts, thus there are total n^3 smaller cubes. Let,
 $N_0 \rightarrow$ Number of smaller cubes with no exposed surfaces
 $N_1 \rightarrow$ Number of smaller cubes with one exposed surfaces
 $N_2 \rightarrow$ Number of smaller cubes with two exposed surfaces
 $N_3 \rightarrow$ Number of smaller cubes with three exposed surfaces

- 32** What is the number of unexposed smaller cubes (N_0)?
- (a) $(n - 2)^3$ (b) n^3 (c) $n!$ (d) 8
- 33** What is the number of smaller cubes with one exposed surface (N_1)?
- (a) $4(n - 3)^3$ (b) $6(n - 2)^2$
 (c) $(n - 3)^2$ (d) $(n + 1)^2$
- 34** What is the value of (N_2)?
- (a) $8(n - 2)^2$ (b) $6(n - 2)$
 (c) $12(n - 2)$ (d) $3(n - 3)^2$
- 35** What is the value of N_3 ?
- (a) $(n - 1)!$ (b) $(n - 2)^2$
 (c) $\frac{n(n + 1)}{2}$ (d) 8

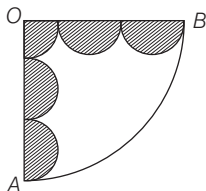
- 36** In a bullet the gun powder is to be filled up inside the metallic enclosure. The metallic enclosure is made up of a cylindrical base and conical top with the base of radius 5 cm. The ratio of height of cylinder and cone is 3 : 2. A cylindrical hole is drilled through the metal solid with its height two-third of the height of metal solid. What should be the radius of the hole, so that the volume of the hole (in which gun powder is to be filled up) is one-third of the volume of metal solid after drilling?

- (a) $\sqrt{\frac{88}{5}} \text{ cm}$ (b) $\sqrt{\frac{55}{8}} \text{ cm}$ (c) $\frac{55}{8} \text{ cm}$ (d) $33\pi \text{ cm}$

- 37** A sector of the circle measures 19° (see the figure). Using only a scale, a compass and a pencil, is it possible to split the circle into 360 equal sectors of 1° central angle?
- (a) Yes
 (b) No
 (c) Yes, only if radius is known
 (d) can't be determined

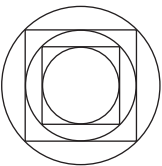


- 38 A circular paper is folded along its diameter, then again it is folded to form a quadrant. Then it is cut as shown in the figure. After it the paper was reopened in the original circular shape. Find the ratio of the original paper to that of the remaining paper? (The shaded portion is cut off from the quadrant. The radius of quadrant OAB is 5 cm and radius of each semicircle is 1 cm).



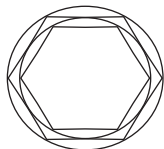
- (a) 25 : 16 (b) 25 : 9
(c) 20 : 9 (d) none of these
- 39 A cubical cake is cut into several smaller cubes by dividing each edge in 7 equal parts. The cake is cut from the top along the two diagonals forming four prisms. Some of them get cut and rest remained in the cubical shape. A complete cubical (smaller) cake was given to adults and the cut off part of a smaller cake is given to a child (which is not an adult). If all the cakes were given equally each piece to a person, total how many people could get the cake?
- (a) 343 (b) 448
(c) 367 (d) 456

Directions (for Q. Nos. 40 to 42) A square is inscribed in a circle then another circle is inscribed in the square then. Another square is inscribed in the circle. Finally a circle is inscribed in the innermost square. Thus there are 3 circles and 2 squares as shown in the figure. The radius of the outer-most circle is R .



- 40 What is the radius of the inner-most circle?
- (a) $\frac{R}{2}$ (b) $\frac{R}{\sqrt{2}}$
(c) $\sqrt{2}R$ (d) none of these
- 41 What is the sum of areas of all the squares shown in the figure?
- (a) $3R^2$ (b) $3\sqrt{2}R^2$
(c) $\frac{3}{\sqrt{2}}R^2$ (d) none of these
- 42 What is the ratio of sum of circumferences of all the circles to the sum of perimeters of all the squares?
- (a) $(2 + \sqrt{3})\pi R$ (b) $(3 + \sqrt{2})\pi R$
(c) $3\sqrt{3}\pi R$ (d) none of these

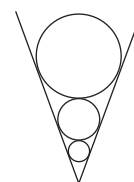
Directions (for Q. Nos. 43 to 45) A regular hexagon is inscribed in a circle of radius R . Another circle is inscribed in the hexagon. Now another hexagon is inscribed in the second (smaller) circle.



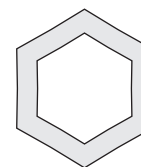
- 43 What is the sum of perimeters of both the hexagons?
- (a) $(2 + \sqrt{3})R$
(b) $3(2 + \sqrt{3})R$
(c) $3(3 + \sqrt{2})R$
(d) none of the above

- 44 What is the ratio of the area of inner circle to that of the outer circle?
- (a) 3 : 4 (b) 9 : 16
(c) 3 : 8 (d) none of these
- 45 If there are some more circles and hexagons inscribed in the similar way as given above, the ratio of each side of outermost hexagon (largest one) to that of the fourth (smaller one) hexagon is (fourth hexagon means the hexagon which is inside the third hexagon from the outside) :
- (a) $9 : 3\sqrt{2}$ (b) 16 : 9
(c) $8 : 3\sqrt{3}$ (d) none of these

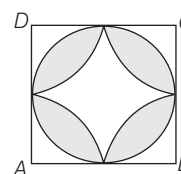
Directions (for Q. Nos. 46 and 47) Five spheres are kept in a cone in such a way that each sphere touches each other and also touches the lateral surface of the cone. It is due to increasing radius of the spheres starting from the vertex of the cone. The radius of the smallest sphere is 16 cm.



- 46 If the radius of the fifth (i.e., the largest) sphere be 81 cm, find the radius of the third (i.e., the middlemost) sphere.
- (a) 25 cm (b) $25\sqrt{3}$ cm
(c) 36 cm (d) data insufficient
- 47 What is the least distance between the smallest sphere and the vertex of the cone?
- (a) 64 cm (b) 80 cm
(c) 28 cm (d) none of these
- 48 Saumya has a pencil box of volume 60 cm^3 . What can be the maximum length of a pencil that can be accommodated in the box. Given that all the sides are integral (in cm) and different from each other?
- (a) $7\sqrt{2}$ cm (b) $\sqrt{905}$ cm
(c) $\sqrt{170}$ cm (d) $\sqrt{3602}$ cm
- 49 There are two concentric hexagons. Each of the side of both the hexagons are parallel. Each side of an internal regular hexagon is 8 cm. What is the area of the shaded region, if the distance between corresponding parallel sides is $2\sqrt{3}$ cm?
- (a) $120\sqrt{3} \text{ cm}^2$ (b) $148\sqrt{3} \text{ cm}^2$
(c) 126 cm^2 (d) none of these



- 50 $ABCD$ is a square. A circle is inscribed in the square. Also taking A, B, C, D (the vertices of square) as the centres of four quadrants, drawn inside the circle, which are touching each other on the mid-points of the sides of square. Area of square is 4 cm^2 . What is the area of the shaded region?
- (a) $\left(4 - \frac{3\pi}{2}\right) \text{ cm}^2$ (b) $(2\pi - 4) \text{ cm}^2$
(c) $(4 - 2\pi) \text{ cm}^2$ (d) none of these



- 51** In a factory there are two identical solid blocks of iron. When the first block is melted and recast into spheres of equal radii ' r ', the 14 cc of iron was left, but when the second block was melted and recast into sphere each of equal radii ' $2r$ ', the 36 cc of iron was left. The volumes of the solid blocks and all the spheres are in integers. What is the volume (in cm^3) of each of the larger spheres of radius ' $2r$ '?

(a) 176
(b) 12π
(c) 192
(d) data insufficient

- 52** There is a vast grassy farm in which there is a rectangular building of the farm-house whose length and breadth is 50 m and 40 m respectively. A horse is tethered at a corner of the house with a tether of 80 m long. What is the maximum area that the horse can graze?

(a) 5425π (b) 5245π
(c) 254π (d) none of these

- 53** A cube of side 6 cm is painted on all its 6 faces with red colour. It is then broken up into 216 smaller identical cubes. What is the ratio of $N_0 : N_1 : N_2$.

Where, $N_0 \rightarrow$ number of smaller cubes with no coloured surface.

$N_1 \rightarrow$ number of smaller cubes with 1 red face.

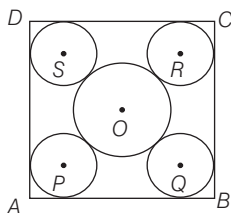
$N_2 \rightarrow$ number of smaller cubes with 2 red faces :

(a) 3 : 4 : 6 (b) 3 : 4 : 5
(c) 4 : 6 : 3 (d) can't be determined

- 54** Assume that a mango and its seed both are spherical. Now, if the radius of seed is $\frac{2}{5}$ of the thickness of the pulp. The seed lies exactly at the centre of the fruit. What per cent of the total volume of the mango is its pulp?

(a) $63\frac{3}{5}\%$ (b) 97.67%
(c) $68\frac{2}{3}\%$ (d) none of these

- 55** In the adjoining diagram $ABCD$ is a square with side ' a ' cm. In the diagram the area of the larger circle with centre ' O ' is equal to the sum of the areas of the remaining four circles with equal radii, whose centres are P, Q, R and S . What is the ratio between the side of square and radius of a smaller circle?



(a) $(2\sqrt{2} + 3)$ (b) $(2 + 3\sqrt{2})$
(c) $(4 + 3\sqrt{2})$ (d) can't be determined

- 56** Initially the diameter of a balloon is 28 cm. It can explode when the diameter becomes $\frac{5}{2}$ times of the initial diameter. Air is blown at 156 cc/s. It is known that the shape of balloon always remains spherical. In how many seconds the balloon will explode?

(a) 1078 s (b) 1368 s
(c) 1087 s (d) none of these

- 57** The radius of a cone is $\sqrt{2}$ times the height of the cone. A cube of maximum possible volume is cut from the same cone. What is the ratio of the volume of the cone to the volume of the cube?

(a) 3.18π
(b) 2.25π
(c) 2.35
(d) can't be determined

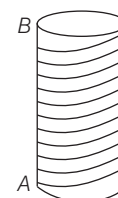
- 58** Raju has 64 small cubes of 1 cm^3 . He wants to arrange all of them in a cuboidal shape, such that the surface area will be minimum. What is the diagonal of this larger cuboid?

(a) $8\sqrt{2}\text{ cm}$ (b) $\sqrt{273}$
(c) $4\sqrt{3}\text{ cm}$ (d) $\sqrt{129}\text{ cm}$

- 59** The volume of a cylinder is 48.125 cm^3 , which is formed by rolling a rectangular paper sheet along the length of the paper. If a cuboidal box (without any lid i.e., open at the top) is made from the same sheet of paper by cutting out the square of side 0.5 cm from each of the four corners of the paper sheet, then what is the volume of this box?

(a) 20 cm^3
(b) 38 cm^3
(c) 19 cm^3
(d) none of the above

Directions (for Q. Nos. 60 to 62) Consider a cylinder of height $h\text{ cm}$ and radius $r = \frac{4}{\pi}\text{ cm}$ as shown in the figure. A string of certain length when wound on its cylindrical surface, starting at point A, gives a maximum of n turns.



- 60** What is the vertical spacing (in cm) between two consecutive turns?

(a) $\frac{h}{n}$ (b) $\frac{\sqrt{h}}{n}$
(c) $\frac{h^2}{n}$ (d) can't be determined

- 61** If there is no spacing between any two consecutive turns and the width of string be $x\text{ cm}$, then the required length of the string is :

(a) $\frac{8x}{h}\text{ cm}$ (b) $\frac{8h}{x}\text{ cm}$
(c) $8hx\text{ cm}$ (d) $2\sqrt{2}\frac{h}{x}\text{ cm}$

- 62 If the string is wound on the exterior four walls of a cube of side a cm starting at point C and ending at point D exactly above C , making equally spaced 4 turns. The side of the cube is :

(a) $a = \frac{2n}{\sqrt{255}}$ (b) $a = \frac{(n)^2}{16}$
 (c) $a = \frac{8n}{\sqrt{257}}$ (d) $a = 2\sqrt{15}n$

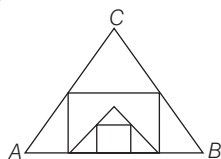
- 63 A blacksmith has a rectangular iron sheet 10 ft long. He has to cut out 7 circular discs from this sheet. What is the minimum possible width of the iron sheet if the radius of each disc is 1 ft?

(a) $2\sqrt{3}$ ft (b) $(2 + \sqrt{3})$ ft
 (c) $(3 + \sqrt{2})$ ft (d) $(2 + 2\sqrt{3})$ ft

- 64 The perimeter of a square, a rhombus and a hexagon are same. The area of square, rhombus and hexagon be s , r , h , respectively, which of the following is correct?

(a) $r > s > h$ (b) $s > h > r$
 (c) $h > s > r$ (d) data insufficient

- 65 In the following figure, ABC is an equilateral triangle inscribing a square of maximum possible area. Again in this square there is an equilateral triangle whose side is same as that of the square. Further the smaller equilateral triangle inscribes a square of maximum possible area. What is the area of the innermost square if the each side of the outermost triangle be 0.01 m?



(a) $(873 - 504\sqrt{3}) \text{ cm}^2$ (b) $(738 - 504\sqrt{3}) \text{ cm}^2$
 (c) $(873 - 405\sqrt{2}) \text{ cm}^2$ (d) none of these

- 66 A blacksmith has a rectangular sheet of iron. He has to make a cylindrical vessel of which both the circular ends are closed. When he minimises the wastage of the sheet of iron, then what is the ratio of the wastage to the utilised area of sheet?

(a) $\frac{1}{11}$ (b) $\frac{2}{17}$
 (c) $\frac{3}{22}$ (d) none of these

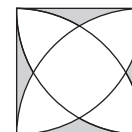
- 67 Barun needs an open box of capacity 864 m^3 . Actually, where he lives the rates of paints are soaring high so he wants to minimize the surface area of the box keeping the capacity of the box same as required. What is the base area and height of such a box?

(a) 36 m^2 , 24 m (b) 216 m^2 , 4 m
 (c) 144 m^2 , 6 m (d) none of these

- 68 There are two cylindrical containers of equal capacity and equal dimensions. If the radius of one of the containers is increased by 12 ft and the height of another container is increased by 12 ft, the capacity of both the container is equally increased by K cubic ft. If the actual heights of each of the containers be 4 ft, find the increased volume of each of the container.

(a) $1680 \pi \text{ cu ft}$ (b) $2304 \pi \text{ cu ft}$
 (c) $1480 \pi \text{ cu ft}$ (d) can't be determined

- 69 The four vertices of a unit square act as the centers of four quarter-circles. The arcs of these circles intersect each other within the square as depicted below. If the area of the region at the center of the square, which itself looks like a bloated square, is cordoned off by all the four arcs of circles is $\frac{\pi}{3} - \sqrt{3} + 1$, find the area of the shaded region.



(a) $4 - \sqrt{3} - \frac{2\pi}{3}$

(b) $4 - 2\sqrt{3} - \frac{2\pi}{3}$

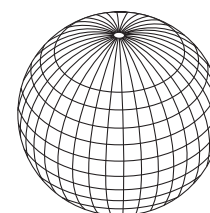
(c) $4 - \sqrt{3} - \frac{\pi}{3}$

(d) none of the above

- 70 Four small squares of equal size are cut off from each of the four corners of a square sheet of area 576 sq cm. The remaining sheet is then folded in such a way that it becomes an open cuboidal box having maximum holding capacity. Find the side of the smaller square that has been cut off.

(a) 6 (b) 3 (c) 2.4 (d) 4

- 71 The following diagram depicts the Earth. For the sake of convenience and simplicity let us consider that the earth is completely spherical. The topmost point (N), where all the great circles meet, is called the North Pole. South Pole is denoted by the point (S) is exactly opposite the North Pole. All the vertical circles are passing through each pole are called longitudes and all the horizontal circles that are parallel to each other are called as latitudes.



Let the centre of the earth be denoted by C and the radius of the earth is 6370 km. A man has to reach from the North Pole to a point D on the latitude with radius $3185\sqrt{3}$ km in the Southern hemisphere of the earth. Find the minimum distance along the surface of the earth that he will have to traverse.

(a) 40040 km (b) 32026.67 km
 (c) 13346.67 km (d) 14367.33 km

- 72 A rectangle of dimensions 32×49 has two circles inscribed in it. What's the maximum possible total area of the two circles?

(a) 337π (b) 306π
(c) 518π (d) 245π

- 73 A sheep and a goat are tethered at the diagonally opposite ends of a square field. The length of each rope they are attached by the post is 6m and the area of the field is 54 m^2 . Find the approx. area that cannot be grazed by either of them.

(a) 3m^2 (b) 4m^2 (c) 7m^2 (d) $\frac{8}{\pi}\text{m}^2$

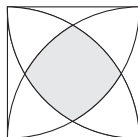
- 74 What is the area of the largest semicircle that can be inscribed in the square whose diagonal is $a\sqrt{2}$ units?

(a) $0.172\pi a^2$ (b) $0.125\pi a^2$
(c) $0.225\pi a^2$ (d) $1.25\pi a^2$

- 75 A semicircle of radius r is inscribed in a square such that the diameter of the semicircle makes 60° angle with one of the sides of the square. What is the least possible area of the square that can inscribe such a semicircle?

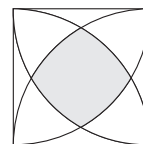
(a) $\left(\frac{7 + 4\sqrt{3}}{4}\right)r^2$ (b) $\left(\frac{11 + 6\sqrt{2}}{4}\right)r^2$
(c) $\left(\frac{4 + 2\sqrt{3}}{9}\right)r^2$ (d) $(5 + \sqrt{7})r^2$

- 76 There are four quadrants of 10 cm radius constructed within a square using the four vertices of the square as their centers. These four quadrants share a common region, which is shown by the shaded region in the adjoining figure. If the area of the given square is 100 sq cm, find the area of the largest possible square that can be inscribed in the common region occupied by four quadrants.



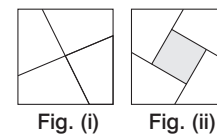
(a) $100(4 - \sqrt{3}) \text{ sq cm}$
(b) $100(2 - \sqrt{3}) \text{ sq cm}$
(c) $100(4 - 2\sqrt{3}) \text{ sq cm}$
(d) data insufficient

- 77 There are four quadrants of 10 cm radius constructed within a square using the four vertices of that square. If the area of the square is 100 sq. cm, find the common area (in sq cm) among all the four quadrants, shown by the shaded region.



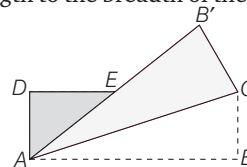
(a) $100\left(\frac{2\pi}{3} + 2 - \sqrt{3}\right)$ (b) $100\left(\frac{\pi}{3} + 1 - \sqrt{3}\right)$
(c) $100\left(\frac{\pi}{3} + 4 - 2\sqrt{3}\right)$ (d) none of these

- 78 A thin opaque square glass sheet ($30 \text{ cm} \times 30 \text{ cm}$) is cut into four pieces of equal area, as shown in figure (i). When these pieces are rearranged to form another square, as shown in figure (ii), it results in a square hole ($16 \text{ cm} \times 16 \text{ cm}$) at the centre of the new square. Find the length of each side of the new square, which is larger than the original one.



(a) 34 cm (b) 32 sq cm
(c) 42 sq cm (d) none of these

- 79 A rectangular paper is folded along its diagonal to form a polygon as shown in the following figure. If the ratio of the area of this polygon to that of the rectangle is $16/5$, the ratio of the length to the breadth of the original rectangle is



(a) $5/\sqrt{11}$ cm (b) $4/3$ cm
(c) $\sqrt{21/11}$ cm (d) data insufficient

Test of Your Learning

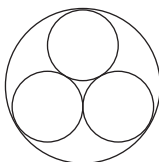
- 1 Three equal circles each of radius 1 cm are circumscribed by a larger circle. Find the perimeter of the circumscribing circle.

(a) $\frac{\sqrt{3}}{2}(2 - \sqrt{3})\pi \text{ cm}$

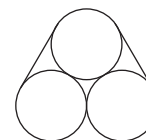
(b) $\left(\frac{2 + \sqrt{3}}{\sqrt{3}}\right)\text{cm}$

(c) $\frac{2}{\sqrt{3}}(2 + \sqrt{3})\pi \text{ cm}$

(d) none of the above



- 2 Three circular rings of equal radii of 1 cm each are touching each other. A string runs all around the set of rings very tightly. What is the minimum length of string required to bind all the three rings in the given manner?



(a) $\frac{6}{\pi} \text{ cm}$

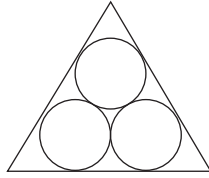
(b) $2(3 + \pi) \text{ cm}$

(c) $\frac{\pi}{6} \text{ cm}$

(d) can't be determined

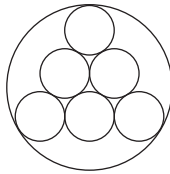
- 3 An equilateral triangle circumscribes all the three circles each of radius 1 cm. What is the perimeter of the equilateral triangle?

(a) $6(\sqrt{3} + 1)$ cm
 (b) $\sqrt{3}(8 + \sqrt{2})$ cm
 (c) $15(\sqrt{3} - 1)$ cm
 (d) none of the above



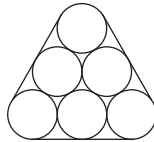
- 4 Six circles each of unit radius are being circumscribed by another larger circle. All the smaller circles touch each other. What is the circumference of the larger circle?

(a) $\left(\frac{\sqrt{3} + 4}{\sqrt{2}}\right) \pi$ cm
 (b) $4\sqrt{3}\pi$ cm
 (c) $2\left(\frac{4 + \sqrt{3}}{\sqrt{3}}\right) \pi$ cm
 (d) can't be determined



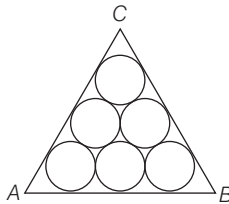
- 5 There are six circular rings of iron, kept close to each other. A string binds them as tightly as possible. If the radius of each circular iron ring is 1 cm. What is the minimum possible length of string required to bind them?

(a) $2(6 + 3\sqrt{3} + \pi)$ cm
 (b) $6(2 + \sqrt{3})\pi$ cm
 (c) $2(6 + \pi)$ cm
 (d) none of these



- 6 An equilateral triangle circumscribes all the six circles, each with radius 1 cm. What is the perimeter of the equilateral triangle?

(a) $6(2 + \sqrt{3})$ cm
 (b) $3(\sqrt{3} + 2)$ cm
 (c) $12(\sqrt{3} + 4)$ cm
 (d) none of the above



- 7 A cube of maximum possible volume is cut from the sphere of diameter $3\sqrt{3}$ cm. What is the ratio of volume of the sphere to that of cube?

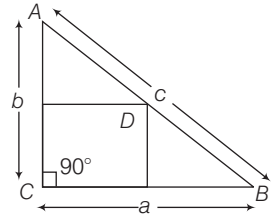
(a) $\frac{4\sqrt{3}}{\pi}$
 (b) $\frac{\sqrt{3}}{2}\pi$
 (c) $\frac{4}{\sqrt{3}}\pi$
 (d) none of these

- 8 A cube of maximum possible volume is cut from the solid right circular cylinder. What is the ratio of volume of cube to that of cylinder if the edge of a cube is equal to the height of the cylinder?

(a) $\frac{11}{7}$
 (b) $\sqrt{2}\frac{\pi}{7}$
 (c) $\frac{7}{11}$
 (d) none of these

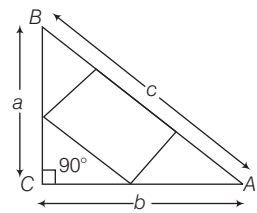
- 9 In a right angle triangle ABC , what is the maximum possible area of a square that can be inscribed when one of its vertices coincide with the vertex of right angle of the triangle?

(a) $\frac{a}{b}$
 (b) $\frac{ab}{a+b}$
 (c) $\frac{a+b}{ab}$
 (d) $\left(\frac{ab}{a+b}\right)^2$

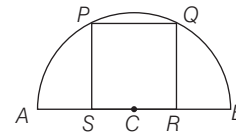


- 10 In the adjoining figure a square of maximum possible area is circumscribed by the right angle triangle ABC in such a way that one of its side just lies on the hypotenuse of the triangle. What is the area of the square?

(a) $\left(\frac{abc}{a^2 + b^2 + ab}\right)^2$
 (b) $\frac{a^2 + b^2 + c^2}{abc}$
 (c) $\frac{abc}{a^2 + b^2 + c^2}$
 (d) none of these



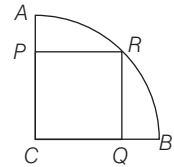
- 11 In the adjoining figure $PQRS$ is a square of maximum possible area which is circumscribed by the semicircle. Points R and S lie on the diameter AB . What is the area of the square if the radius of the circle is ' r '?



(a) $\frac{\sqrt{3}}{4}r^2$
 (b) $\frac{4}{5}r^2$
 (c) $\frac{3}{5}r^2$
 (d) $\frac{\sqrt{5}}{4}r^2$

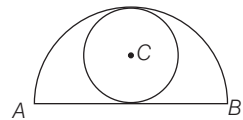
- 12 In the adjoining figure a quadrant (of circle) inscribes a square of maximum possible area. If the radius of the circle be ' r ' then what is the area of the square?

(a) $\frac{r^2}{2}$
 (b) $\frac{3r^2}{5}$
 (c) $\frac{r^2}{\sqrt{3}}$
 (d) $2\sqrt{6}r$

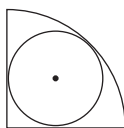


- 13 In the adjoining figure, AB is the diameter of a semicircle which inscribes a circle of maximum possible area. If the radius of the larger circle (i.e., semicircle) is r , the area of the inscribed circle is :

(a) $\frac{5r^2}{\pi}$
 (b) $\frac{2\pi}{\sqrt{3}}r^2$
 (c) $\frac{\pi r^2}{4}$
 (d) none of these



- 14** In a quadrant (of a circle) a circle of maximum possible area is given. If the radius of the circumscribing quadrant be r , then what is the area of the inscribed circle?



- (a) $(2 + 3\sqrt{2})r^2$ (b) $\frac{\pi r^2}{(3 + 2\sqrt{2})}$
(c) $\left(\frac{3 + 2\sqrt{2}}{r^2}\right)\pi$ (d) none of these

- 15** A cylindrical chocobar has its radius r unit and height ' h ' unit. If we wish to increase the volume by same unit either by increasing its radius alone or its height alone, then how many unit we have to increase the radius or height?

- (a) $\frac{r^2 + 2r}{h}$ (b) $\frac{r^2 - 2rh}{h}$
(c) $\frac{2r^2 - rh}{h^2}$ (d) $\frac{\pi r^2}{2h}$

- 16** A 12 cm long wire is bent to form a triangle with one of its angle as 60° . Find the sides of the triangle (in cm) when its area is largest?

- (a) 3, 4, 5
(b) 2, 4, 6
(c) 4, 4, 4
(d) 2.66, 3.66, 4.66

- 17** Let $S_1, S_2, \dots, S_n$ be the squares such that for each $n > 1$, the length of a side of S_n equals the length of the diagonal of $S_{(n+1)}$. If the length of a circle of S_1 is 10 cm, then for which of the following values of n the area of S_n is less than 1 square cm?

- (a) 4 (b) 8 (c) 6 (d) 7

- 18** Area of a regular hexagon and a regular octagon is same. Which one of the two has larger perimeter?

- (a) Hexagon (b) Octagon
(c) can't be determined (d) none of these

Answers

✓ Introductory Exercise 10.1

1 (b)	2 (c)	3 (c)	4 (b)	5 (a)	6 (a)	7 (b)	8 (a)	9 (a)	10 (c)
11 (b)	12 (d)	13 (a)	14 (b)	15 (b)	16 (c)	17 (c)	18 (d)	19 (c)	20 (b)
21 (b)	22 (a)	23 (c)	24 (c)	25 (c)	26 (b)	27 (c)	28 (c)	29 (b)	30 (c)
31 (b)	32 (c)								

✓ Introductory Exercise 10.2

1 (b)	2 (c)	3 (a)	4 (a)	5 (d)	6 (a)	7 (c)	8 (b)	9 (a)	10 (a)

✓ Introductory Exercise 10.3

1 (a)	2 (c)	3 (d)	4 (b)	5 (d)	6 (b)	7 (a)	8 (b)	9 (d)	10 (c)
11 (b)	12 (a)	13 (b)	14 (d)	15 (a)					

✓ Introductory Exercise 10.4

1 (d)	2 (d)	3 (a)	4 (b)	5 (c)	6 (d)	7 (a)	8 (a)	9 (a)	10 (b)
11 (c)	12 (d)	13 (b)	14 (a)	15 (b)	16 (a)	17 (a)	18 (c)	19 (a)	20 (c)
21 (a)	22 (b)	23 (b)	24 (c)	25 (c)					

✓ Introductory Exercise 10.5

1 (b)	2 (c)	3 (a)	4 (b)	5 (c)	6 (a)	7 (b)	8 (a)	9 (b)	10 (c)
11 (d)	12 (a)	13 (c)	14 (b)	15 (c)	16 (a)	17 (b)	18 (d)	19 (b)	20 (c)
21 (d)	22 (b)	23 (c)	24 (a)	25 (b)	26 (a)	27 (c)	28 (c)	29 (c)	30 (b)
31 (b)	32 (b)								

✓ **Introductory Exercise 10.6**

1 (b)	2 (c)	3 (c)	4 (c)	5 (b)	6 (a)	7 (c)	8 (b)	9 (a)	10 (c)
11 (a)	12 (a)	13 (d)	14 (a)	15 (d)	16 (c)	17 (a)	18 (b)	19 (a)	20 (b)
21 (c)	22 (a)	23 (d)	24 (c)	25 (d)	26 (d)	27 (d)	28 (c)	29 (b)	30 (c)
31 (b)	32 (a)	33 (d)	34 (c)	35 (b)	36 (b)	37 (b)	38 (a)	39 (b)	40 (a)

✓ **Introductory Exercise 10.7**

1 (b)	2 (c)	3 (c)	4 (a)	5 (b)	6 (c)	7 (c)	8 (d)	9 (c)	10 (c)
11 (d)	12 (a)	13 (c)	14 (d)	15 (b)	16 (c)	17 (a)	18 (d)	19 (b)	20 (a)

✓ **Multifaceted Exercise**

1 (c)	2 (c)	3 (c)	4 (c)	5 (c)	6 (a)	7 (c)	8 (a)	9 (c)	10 (a)
11 (d)	12 (c)	13 (a)	14 (c)	15 (d)	16 (b)	17 (b)	18 (c)	19 (b)	20 (d)
21 (b)	22 (c)	23 (d)	24 (c)	25 (d)	26 (d)	27 (d)	28 (b)	29 (b)	30 (b)
31 (a)	32 (c)	33 (d)	34 (c)	35 (a)	36 (c)	37 (b)	38 (c)	39 (d)	40 (a)
41 (b)	42 (a)	43 (c)	44 (c)	45 (a)	46 (b)	47 (b)	48 (c)	49 (d)	50 (b)
51 (c)	52 (c)	53 (a)	54 (d)	55 (c)	56 (a)	57 (b)	58 (a)	59 (c)	

✓ **Level 01 Basic Level Exercise**

1 (a)	2 (b)	3 (b)	4 (b)	5 (c)	6 (d)	7 (c)	8 (b)	9 (a)	10 (c)
11 (c)	12 (a)	13 (b)	14 (a)	15 (a)	16 (c)	17 (a)	18 (c)	19 (a)	20 (d)
21 (b)	22 (a)	23 (d)	24 (c)	25 (c)	26 (a)	27 (b)	28 (c)	29 (c)	30 (b)
31 (d)	32 (a)	33 (c)	34 (c)	35 (c)	36 (a)	37 (a)	38 (a)	39 (b)	40 (a)
41 (b)	42 (d)	43 (d)	44 (c)	45 (a)	46 (c)	47 (a)	48 (c)	49 (b)	50 (b)
51 (c)	52 (c)	53 (b)	54 (c)	55 (d)	56 (b)	57 (d)	58 (c)	59 (a)	60 (a)
61 (c)	62 (b)	63 (a)	64 (d)	65 (c)	66 (a)	67 (c)	68 (c)	69 (b)	

✓ **Level 02 Higher Level Exercise**

1 (c)	2 (b)	3 (c)	4 (b)	5 (a)	6 (b)	7 (c)	8 (a)	9 (d)	10 (b)
11 (a)	12 (d)	13 (c)	14 (d)	15 (b)	16 (b)	17 (a)	18 (b)	19 (b)	20 (a)
21 (a)	22 (c)	23 (c)	24 (d)	25 (b)	26 (a)	27 (a)	28 (b)	29 (c)	30 (d)
31 (c)	32 (a)	33 (b)	34 (c)	35 (d)	36 (b)	37 (a)	38 (a)	39 (b)	40 (a)
41 (a)	42 (d)	43 (b)	44 (a)	45 (c)	46 (c)	47 (a)	48 (b)	49 (a)	50 (b)
51 (a)	52 (a)	53 (c)	54 (b)	55 (b)	56 (a)	57 (b)	58 (c)	59 (a)	60 (a)
61 (b)	62 (c)	63 (b)	64 (c)	65 (a)	66 (a)	67 (c)	68 (b)	69 (a)	70 (d)
71 (c)	72 (a)	73 (b)	74 (a)	75 (a)	76 (b)	77 (b)	78 (a)	79 (a)	

✓ **Test for Your Learning**

1 (c)	2 (b)	3 (a)	4 (c)	5 (c)	6 (a)	7 (b)	8 (c)	9 (d)	10 (a)
11 (b)	12 (a)	13 (c)	14 (b)	15 (c)	16 (c)	17 (b)	18 (a)		

Hints & Solutions

Introductory Exercise 10.1

- 1 $2(16x + 9x) = 750$; $l = 16x$ and $b = 9x$
- 2 $\text{Area} = \frac{110}{0.5} = 220 \text{ sq m}$
and $11x \times 5x = 220$
- 3 Length : breadth = 3 : 2
Also, area of floor = area of roof
and 30 is almost 50% of 62. So you need not to solve it. Just look out for the appropriate option. Thus 1411.2 is almost 50% of 2916.48 and rest of the options are not satisfactory.
- 4 1 hectare = 10000 m², find length and breadth, then perimeter
Perimeter = 1000 m = 1 km
 $\therefore \text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{1}{3} \text{ hrs} = 20 \text{ min}$
- 5 5 hectares and 76 ares = 57600 m², find perimeter then multiply it by 6.
- 6 $l \times 0.7 = 19 \times 3.5 \Rightarrow l = 95 \text{ m}$ (70 cm = 0.7 m)
Also $(95)^2 = 9025$
So $95 \times 0.95 = 90.25$
- 7 Number of stones = $\frac{\text{Area of courtyard}}{\text{Area of one stone}} = 14400$
Cost = Rate $\times$ Number of stones
= (0.5×14400)
- 8 Area = $(63 + 54 - 6) \times 6$ [$\because \text{Area} = (l + b - 2w) 2w$]
Area = (111×6)
and $\text{Rate} = \frac{37}{2} = \frac{37 \times 3}{2 \times 3} = \frac{111}{6}$
 $\therefore \text{Cost} = (111 \times 6) \times \frac{111}{6} = 111 \times 111 = 12321$
- 9 Original area = $l \times b$
New area = $2l \times \frac{b}{2} = l \times b$
Hence, no change.
- 10 $(116 + 68 - 2w) 2w = 720$
Solve through quadratic equation.
☞ Alternatively Go through options and put $w = 2$ from option (c) you will find that both sides are equal. So the presumed choice is correct.
- 11 Net area = Total area of 4 walls – 8m²
and Area of 4 walls = $2(l + b) \times h$
- 12 $d^2 = 2 \times \text{area}$ ($\because \text{Area} = \frac{d^2}{2}$)
 $d^2 = 2 \times 2 \Rightarrow d = 2 \text{ km}$
- 13 $100 \times 100 = 10000 \text{ m}^2 = 1 \text{ hectare}$
 $111 \times 111 = 12321 \text{ m}^2$
 $\therefore \text{Difference in area} = 2321 \text{ m}^2$
- 14 Rate = $\frac{759 - 561}{6} = ₹ 33 \text{ per metre}$
- 15 $l \times b = \frac{1}{25} \times 10000 = 1000 \times b \Rightarrow b = 0.4 \text{ m}$
- 16 Side = $\sqrt{(289)^2 - (240)^2}$
- 17 $\frac{1600 \times 900}{40 \times 20} = 1800$
- 18 Area = $\frac{d^2}{2}$
- 19 $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
 $\therefore 90 \times \frac{1}{9} = 10 \text{ g}$ (It depends upon area)
- 20 $l : b = 2 : 1$
 $2x^2 = 24200$, find diagonal.
 $\sqrt{(110)^2 + (220)^2} = 110 \sqrt{1^2 + 2^2}$
= $110 \times \sqrt{5}$
= $110 \times 2.236 = 246 \text{ m}$
- 21 $2(2x + x) \times 11 = 2640$, find $2x \times x = ?$
- 22 Consider some appropriate values.
e.g. $4 \begin{array}{|c|} \hline 4 \\ \hline \end{array} 4$ $2 \begin{array}{|c|} \hline 6 \\ \hline \end{array} 2$
 $4a = 16 \text{ cm}$ $2(l + b) = 16 \text{ cm}$
but $a^2 = 16 \text{ cm}^2$ and $l \times b = 12 \text{ cm}^2$
 $\Rightarrow a^2 > l \times b$
Remember, when $a + b$ is constant, then the maximum value of $a \times b$ is at when $a = b$.
So, area of square (i. e., $l \times b = a \times a$) is always greater than area of rectangle with same perimeter.
☞ Alternatively Let each side of a square be a and length and breadth of the rectangle be l and b , then
 $4a = 2(l + b) \Rightarrow a = \left(\frac{l + b}{2}\right)$

$\therefore$ Area of the rectangle $= l \times b$

and Area of the square $= a^2 = \frac{1}{4} (l + b)^2$

But since we know that for any (different) certain values

$$AM > GM$$

$$\therefore \frac{l+b}{2} > \sqrt{l \times b}$$

$$\Rightarrow \left(\frac{l+b}{2} \right)^2 > l \times b$$

$\Rightarrow$ Area of square $>$ Area of rectangle

23 Best way is to go through options and verify the result.

☞ **Alternatively** $2(l+b) = 4a = 80$

$$\Rightarrow l+b = 40 \text{ and } a = 20 \Rightarrow a^2 = 400$$

$$\text{Also } a^2 - lb = 100$$

$$\Rightarrow 400 - lb = 100$$

$$\Rightarrow lb = 300$$

$$\text{Now } (l-b)^2 = (l+b)^2 - 4lb$$

$$(l-b)^2 = 1600 - 1200$$

$$\Rightarrow l-b = 20$$

(Solving $l+b = 40$ and $l-b = 20$)

$$\therefore l = 30 \text{ and } b = 10$$

$$\mathbf{24} \quad \frac{100 \times 100}{25 \times 25} = 16 \quad (1 \text{ m} = 100 \text{ cm})$$

$$\mathbf{25} \quad (120 + 80 - 24) \times 24 = 176 \times 24 = 4224$$

$$\mathbf{26} \quad (90 + 40 + 10) \times 10 = 1400$$

$$\mathbf{27} \quad 2[(x+2) + x] = 48 \Rightarrow x = 11 \text{ cm}$$

$$\therefore (x+2) = 13 \text{ cm}$$

$$\mathbf{28} \quad \frac{52000}{325} = 160$$

3.25 cm means 325 m, as per scale.

$$\mathbf{29} \quad \text{Area} = \frac{d^2}{2}; d \rightarrow \text{diagonal}$$

$$\mathbf{30} \quad 1.2 \times 1.2 = 1.44 \Rightarrow 44\% \text{ increase}$$

$$\mathbf{31} \quad a^2 : (a\sqrt{2})^2 = 1 : 2$$

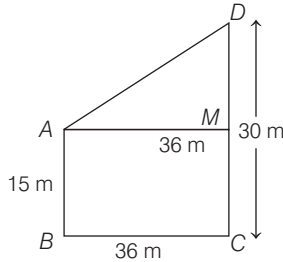
$$\mathbf{32} \quad 1.6 \times 1.4 = 2.24 \Rightarrow \text{increase} = 1.24 \Rightarrow 124\%$$

☑ Introductory Exercise 10.2

1 Use Hero's formula:

$$\text{Area of scalene triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

2



$$AM = BC = 36 \text{ m}$$

$$MD = CD - MC = 30 - 15 \quad (\because MC = AB = 15 \text{ m})$$

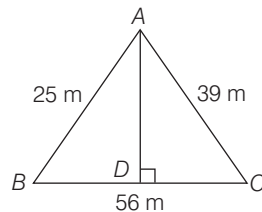
$$\therefore MD = 15 \text{ m}$$

Now using Pythagoras theorem

$$AD^2 = AM^2 + MD^2, \text{ find } AD$$

3 Find the area, using Hero's formula, then

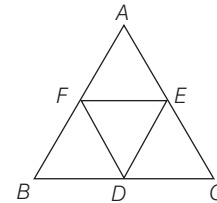
$$\begin{aligned} \text{Area} &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 56 \times AD \end{aligned}$$



4 All the triangles are similar to each other.

$$\therefore \frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \frac{4}{1}$$

(Since in $\triangle ABC$ there are 4 similar triangles having same area as $\triangle DEF$.)



5

$$60 = \frac{b}{4} \sqrt{4a^2 - b^2}$$

$$\Rightarrow 60 = \frac{b}{4} \sqrt{676 - b^2}$$

$$\Rightarrow 57600 = b^2 (676 - b^2)$$

By solving the above equation we can get the value of b .

☞ **Alternatively** You can go through option.

6 Use pythagorus theorem

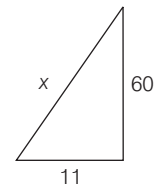
$$x^2 = (60)^2 + (11)^2$$

$$\left[\begin{array}{l} \text{Pythagorus theorem :} \\ (\text{Hypotenuse})^2 = (\text{base})^2 + (\text{height})^2 \end{array} \right]$$

$$\begin{aligned} \mathbf{7} \quad \text{Area of field} &= \frac{495.72}{36.72} \\ &= 13.5 \text{ hectare} = 135000 \text{ m}^2 \end{aligned}$$

Now, if $h = x$, $b = 3x$

$$\therefore \frac{1}{2} \times b \times h = 135000$$



8 Area of triangle = $\frac{1}{2} \times b \times h$

Let initially area of triangle = $1 \times 1 = 1$ unit

Now, the area of triangle = $2 \times 2 = 4$ unit

Increase in area = $\frac{4-1}{1} \times 100 = 300\%$

(For your convenience assume any value of b and h .)

9 Height of an equilateral triangle = $\frac{\sqrt{3}}{2} \times \text{side}$

$\therefore 2\sqrt{3} = \frac{\sqrt{3}}{2} \times \text{side} \Rightarrow \text{Side} = 4 \text{ cm}$

$\therefore \text{Area of an equilateral triangle} = \frac{\sqrt{3}}{4} \times (\text{side})^2$
 $= \frac{\sqrt{3}}{4} \times 4 \times 4 = 4\sqrt{3} \text{ cm}^2$

10 Let each side of a square be $3x$ and each side of an equilateral triangle be $4x$

then perimeter of square = $4 \times 3x = 12x$

and perimeter of equilateral triangle = $3 \times 4x = 12x$

Now area of square = $(3x)^2 = 9x^2$

Area of equilateral triangle = $\frac{\sqrt{3}}{4} \times (4x)^2 = 4\sqrt{3}x^2$

Since $9 > 4\sqrt{3} \Rightarrow 9x^2 > 4\sqrt{3}x^2$

Hence area of square is greater than area of equilateral triangle, when the perimeter of both is same.

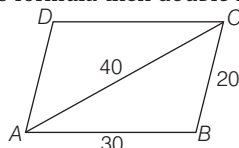
Alternatively Consider any suitable value and verify.

Introductory Exercise 10.3

1 Area = $6 \times 8 \sin 30^\circ$

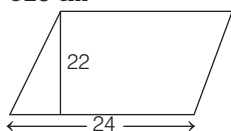
$= 6 \times 8 \times \frac{1}{2} = 24 \text{ cm}^2$ $\left(\sin 30^\circ = \frac{1}{2} \right)$

2 To find the area of parallelogram, first find the area of $\triangle ABC$ by Hero's formula then double it.

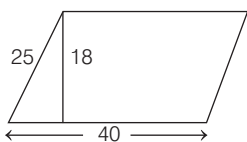


Since, area of $ABCD = 2 \times \text{Area of } \triangle ABC$
 $= 2 \times \text{Area of } \triangle ACD$

3 Area = $22 \times 24 = 528 \text{ cm}^2$



4 Area = $40 \times 18 = 720 \text{ cm}^2$



5 $\frac{\text{Area of parallelogram } ABCD}{\text{Area of triangle } ABN} = \frac{BC \times AN}{\frac{1}{2} \times BN \times AN} = \frac{2 \times 4x}{x} = \frac{8}{1}$

6 Area of rhombus = $\frac{1}{2} \times \text{product of diagonals}$
 $= \frac{1}{2} \times a \times b = \frac{ab}{2}$

7 Area of rhombus = $\frac{1}{2} \times 2x \times 5x = \frac{10x^2}{2} = 5x^2$
 and square of the shorter diagonal = $(2x)^2 = 4x^2$

$\therefore \frac{5x^2}{4x^2} = \frac{5}{4}$

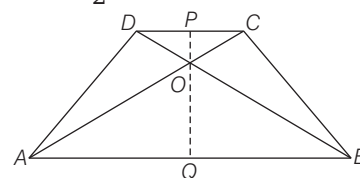
8 $\frac{1}{2} \times x \times 2x = 256 \Rightarrow x^2 = 256 \Rightarrow x = 16$

$\therefore 2x = 32$

Hence, $x + 2x = 16 + 32 = 48$

9 Area = $\frac{1}{2} \times (30 + 50) \times 16 = 640 \text{ cm}^2$

10 $\frac{\text{Area of } \triangle AOB}{\text{Area of } \triangle COD} = \frac{\frac{1}{2} \times AB \times OQ}{\frac{1}{2} \times CD \times PO}$



$= \frac{AB \times OQ}{CD \times PO} = \frac{2CD \times 2OP}{CD \times OP} = \frac{4}{1}$ $\left(\because AB = 2CD \right)$
 (and $OQ = 2PO$)

This is due to the similarity of triangles AOB and COD .

11 Area of trapezium = 441

$\Rightarrow \frac{1}{2} (5x + 9x) \times 21 = 441 \Rightarrow 14x = 42 \Rightarrow x = 3$

$\therefore 9x = 27 \text{ cm}$

12 $\frac{1}{2} \times (12 + 8) \times h = 360 \Rightarrow h = 36 \text{ m}$

13 $6 \times \frac{\sqrt{3}}{4} \times (\text{Side})^2 = \frac{3\sqrt{3}}{2} (\text{Side})^2 = \frac{3\sqrt{3}}{2} \times 4 \times 4 = 24\sqrt{3} \text{ m}^2$

14 Area of quadrilateral = $\frac{1}{2} \times 19 \times (5 + 7) = 114 \text{ cm}^2$

15 $AB = BC = CD = AD = 4 \text{ cm}$ $\left(\because \text{Side} = \frac{\text{Diagonal}}{\sqrt{2}} \right)$

and $EF = 1.5 \text{ cm}$ (By Pythagorus theorem)

$\therefore \text{Area of } ABCDE = \text{Area of } ABCD + \text{Area of } AED$

$= (4)^2 + \frac{1}{2} \times 4 \times 1.5 = 19 \text{ cm}^2$

Introductory Exercise 10.4

1 $2\pi r = 704 \Rightarrow r = 112$

$\therefore \pi r^2 = \frac{22}{7} \times 112 \times 112 = 39424 \text{ cm}^2$

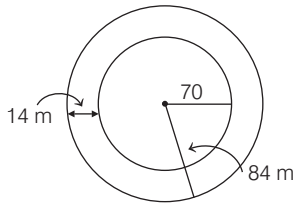
2 $2\pi r = 4.4 \Rightarrow r = \frac{7}{10} \text{ m}$

$\therefore \pi r^2 = 0.49\pi \text{ m}^2$

3 $2\pi r = 2 \times \frac{22}{7} \times 4.2 = 26.4 \text{ metre}$

$26.4 = 2(6x + 5x) \Rightarrow 6x = 7.2 \text{ m}$

4 $2\pi r = 440 \Rightarrow r = 70 \text{ m}$



$\therefore R = 70 + 14 = 84 \text{ m}$

5 $r + 2\pi r = 51 \Rightarrow r \left(1 + \frac{44}{7} \right) = 51 \Rightarrow r = 7$,

Find area.

6 $(2\pi r - 2r) = 15 \Rightarrow 2r \left(\frac{22}{7} - 1 \right) = 15$

$\Rightarrow r = \frac{7}{2}$

$\therefore \pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 38.5 \text{ m}^2$

7 $\frac{\text{New area}}{\text{Original area}} = \frac{\pi \times 7 \times 7}{\pi \times 5 \times 5} = \frac{49}{25}$

Change in area = $\frac{24}{25} \times 100 = 96\%$

8 $\frac{\text{New circumference}}{\text{Original circumference}} = \frac{6}{5} = \frac{\text{New radius}}{\text{Original radius}}$

$\therefore \frac{\text{New area}}{\text{Original area}} = \left(\frac{6}{5} \right)^2 = \frac{36}{25}$

$\therefore \text{Change in area} = \frac{11}{25} \times 100 = 44\%$

⑨ **Alternatively** Change = $(1.2 \times 1.2) - (1 \times 1) = 0.44$

$\therefore \text{Change} = 44\%$

9 $\pi r^2 = 124.74 \text{ hectare}$

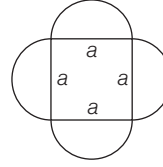
$\pi r^2 = 1247400 \text{ m}^2 \Rightarrow r = 630 \text{ m}$

$\therefore 2\pi r = 3960$

$\therefore \text{Cost} = 3960 \times 0.8 = ₹ 3168$

10 $\pi r^2 = \frac{158400}{1400} \Rightarrow r^2 = 36 \Rightarrow r = 6 \text{ m}$

11



Area of square = a^2

Area of circular parts = $4 \times \frac{1}{2} \pi \left(\frac{a}{2} \right)^2 = \frac{\pi a^2}{2}$

$\therefore \text{Total area} = a^2 + \frac{\pi a^2}{2} = a^2 \left(1 + \frac{\pi}{2} \right)$

12 $\pi r^2 = 2464 \Rightarrow r = 28 \text{ m}$

13 Basically there are 12 equilateral triangles each of side 'a'.

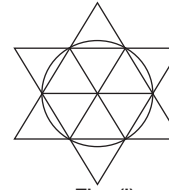


Fig. (i)

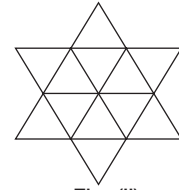


Fig. (ii)

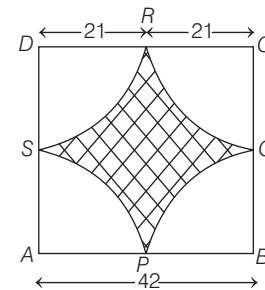
$\therefore 12 \times \frac{\sqrt{3}}{4} \times (a)^2 = 3\sqrt{3}a^2$

14 $\frac{1}{2} \times \pi \times (7)^2 + 2 \left[\frac{1}{2} \times \pi \times \left(\frac{7}{2} \right)^2 \right]$

Area of larger semicircle + 2 (area of smaller semicircle)

15 $2\pi \times R \times \frac{75}{360} = 25 \Rightarrow R = \frac{60}{\pi}$

16 Ungrazed area = Area of square – 4 (area of quadrants)



$= (42)^2 - 4 \times \frac{1}{4} \times \pi (21)^2 = (21)^2 [4 - \pi] = 378$

17 $20 + \frac{1}{2} \times [2\pi \times 10] = 20 + 10\pi$

18 $\frac{40}{360} = \frac{1}{9} \therefore \frac{(360 - 40)}{360} = \frac{8}{9}$

$\therefore \text{Area of major sector} = 8 \times 8.25 = 66 \text{ cm}^2$

19 Area of a sector = $\frac{1}{2} \times \text{arc} \times \text{radius} = \frac{1}{2} \times 8 \times 5.6 = 22.4 \text{ cm}^2$

- 20 $2\pi r = 2 \times \frac{22}{7} \times 700 = 4400 \text{ m} = 4.4 \text{ km}$
 $\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{4.4}{13.2} = \frac{1}{3} \text{ h} = 20 \text{ minute}$
- 21 $\pi R^2 = \pi [r_1^2 + r_2^2 + r_3^2]$, find R .
- 22 $\pi [23^2 - 12^2] = \frac{22}{7} \times [529 - 144] = 1210$

Introductory Exercise 10.5

- 1 Volume of original cube $= (4)^3 = 64 \text{ cm}^3$
 and its weight $= 400 \text{ kg}$
 Since weight of the larger cube is 8 times the weight of smaller cube. Hence, the volume of new cube will be 8 times the volume of smaller cube.
 Hence volume of required cube $= 8 \times 64 = (8)^3$
 $\therefore$ Edge of this cube $= 8 \text{ cm}$
- 2 Volume of the tank $= 3 \text{ m}^3$
 $\therefore$ Base area $\times$ height $= 3 \text{ m}^3$
 $\Rightarrow$ Base area $= \frac{3}{2.56} = 1.171875 \text{ m}^2$
 $[\therefore \text{Volume of cuboid} = (l \times b) \times h = (\text{base area}) \times \text{height}]$
- 3 Volume of cube $=$ Volume of cuboid
 $a^3 = lbh$
 $\Rightarrow a^3 = 36 \times 75 \times 80 = 216000$
 $\Rightarrow a = 60 \text{ cm}$
- 4 Base area $\times$ height $=$ Volume $10 \times 4 \times 1 = 40 \text{ m}^3$
 But $1 \text{ m}^3 = 1000 \text{ litre} = 1 \text{ kilolitre}$
 $\therefore 40 \text{ m}^3 = 40,000 \text{ litre} = 40 \text{ kilolitre}$
- 5 Volume of smaller (required) cube $= 8 \text{ (dm)}^3 = 0.008 \text{ m}^3$
 $\therefore$ Number of required cubes
 $= \frac{\text{Volume of larger cube}}{\text{Volume of each smaller cube}} = \frac{1}{0.008} = 125$
- 6 Volume of new cube $= (3)^3 + (4)^3 + (5)^3 = 216 \text{ cm}^3$
 $\therefore$ Each edge of the new cube $= 6 \text{ cm}$
 and hence, surface area $= 6 \times (a)^2 = 216 \text{ cm}^2$
- 7 Total length of tape $= 2(l + b) + 3.75$
 $= 2(39.5 + 9.35) + 3.75$
 $= 101.45 \text{ cm}$
- 8 Area of surface to be cemented $= 2 \times (l + b) \times h + (l \times b)$
i.e., area of four walls + area of floor
 $= 2 \times (21) \times 4 + (106.25) = 274.25 \text{ m}^2$
 $\therefore$ Cost of cementing $= 24 \times 274.25 = ₹ 6582$
- 9 Total volume of water displaced by 250 men
 $= 250 \times 4 = 1000 \text{ m}^3$
 $\therefore$ Rise in water level (h) $= \frac{\text{Volume}}{\text{Base area}} = \frac{1000}{80 \times 50} = 25 \text{ cm}$

- 23 $4a = 4 \times 66 = 264 = 2\pi r \Rightarrow r = 42$
 $\therefore d = 2r = 84 \text{ m}$
- 24 $r = 42 \therefore 2\pi r = 264 = 4a \Rightarrow a = 66$
 $\therefore d = a\sqrt{2} = 66\sqrt{2}$
- 25 Circumference $= \frac{1100}{560} = \frac{110}{56} = 2\pi r$

- 10 Let each edge of smaller cube $= 1 \text{ m}$
 $\therefore$ Each edge of larger cube $= 2 \text{ m}$
 and Surface area of smaller cube $= 6 \times (1)^2 = 6 \text{ m}^2$
 $\therefore$ Surface area of larger cube $= 6 \times (2)^2 = 24 \text{ m}^2$
 $\therefore$ % increase in surface area $= \frac{24 - 6}{6} \times 100 = 300\%$
NOTE It can be determined by using variables *e.g.*, x (edge of cube) instead of solving by assuming some numerals.
- Alternatively** $\frac{S_2}{S_1} = \left(\frac{e_2}{e_1}\right)^2 \Rightarrow \frac{S_2}{S_1} = \frac{4}{1}$
 $\therefore$ Percentage increase in surface area $= \frac{4 - 1}{1} \times 100 = 300\%$
 where, S = surface area, e = edge of cube.
- 11 $\frac{V_2}{V_1} = \left(\frac{e_2}{e_1}\right)^3 \quad \{\therefore V = (e)^3\}$
 $\therefore \frac{V_2}{V_1} = \left(\frac{2}{1}\right)^3 = \frac{8}{1}$
 $\therefore V_2 = 8V_1$
- 12 External volume of the box $= 24 \times 16 \times 10 = 3840 \text{ cm}^3$
 Thickness of the wood $= 5 \text{ mm} = 0.5 \text{ cm}$
 $\therefore$ Internal length of box $= 24 - 2 \times 0.5 = 23 \text{ cm}$
 Internal breadth of box $= 16 - 2 \times 0.5 = 15 \text{ cm}$
 Internal height of box $= 10 - 2 \times 0.5 = 9 \text{ cm}$
 $\therefore$ Internal volume of the box $= 23 \times 15 \times 9 = 3105 \text{ cm}^3$
 $\therefore$ Volume of the wood $= 3840 - 3105 = 735 \text{ cm}^3$
 Now, total weight of wood
 $= \text{Volume} \times \text{weight of } 1 \text{ cm}^3 \text{ wood}$
 $7350 = 735 \times \text{weight of } 1 \text{ cm}^3 \text{ wood}$
 $\therefore$ Weight of 1 cm^3 wood $= 10 \text{ gm}$
- 13 Length of tank $= 120 \text{ cm}$
 But since, $\frac{120}{7} = 17 \frac{1}{7}$, hence 17 cubes can be placed along length and breadth of tank $= 80 \text{ cm}$
 But since, $\frac{80}{7} = 11 \frac{3}{7}$, hence 11 cubes can be placed along breadth and height of tank $= 50 \text{ cm}$.
 But since, $\frac{50}{7} = 7 \frac{1}{7}$, hence only 7 cubes can be placed along height of the tank.

$$\therefore \text{Total volume occupied by these cubes} \\ = (7)^3 \times 17 \times 11 \times 7 = 448987 \text{ cm}^3$$

$$\text{Total volume of the tank} = 120 \times 80 \times 50 = 480000 \text{ cm}^3$$

$$\therefore \text{Area of unoccupied space} = 480000 - 448987 \\ = 31013 \text{ cm}^3 = 31.013 \text{ dm}^3$$

$$14 \text{ Surface area of the cuboid} = 2(lb + bh + hl) = 11.6 \text{ m}^2$$

$$\therefore \text{Cost of canvas} = 11.6 \times 25 = ₹ 290$$

$$15 \text{ Required surface area} = 2(9 \times 3 + 3 \times 3 + 3 \times 9) = 126 \text{ cm}^2$$

$$16 \text{ Area of 4 walls} = 2(36 + 12) \times 10 = 960 \text{ m}^2$$

$$\text{Total area of (windows + door + chimney)} = 120 \text{ m}^2$$

$$\therefore \text{Net area for papering} = 960 - 120 = 840 \text{ m}^2$$

$$\therefore \text{Length of required paper} = \frac{840}{1.2} = 700 \text{ m}$$

$$\text{Hence, cost of papering} = 700 \times 0.7 = ₹ 490$$

$$17 \text{ Number of children} \times \text{Space required for one child} \\ = \text{Volume of room}$$

$$\therefore \text{Number of children} = \frac{30 \times 12 \times 6}{8} = 270$$

$$18 \quad (x + 2)^3 - x^3 = 1016$$

$$\Rightarrow \quad x = 12 \text{ cm}$$

$$\text{and} \quad x^3 - (x - 2)^3 = (12)^2 - (10)^2 = 728$$

19 You can see the figure shown in the question number 15. Now let us consider that surface area of each face of the cube 1 cm^2 .

$$\therefore \text{Total surface area of the cuboid} = 14 \text{ cm}^2$$

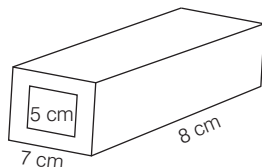
$$\text{and Total surface area of the 3 cubes} = 18 \text{ cm}^2$$

$$\text{Hence, required ratio} = 14 : 18 = 7 : 9$$

 **NOTE** In the arrangement of cuboid there are only 14 faces visible to us.

20 Iron used in the tube

$$= \text{Difference in external and internal volumes of the tube}$$



$$\therefore 192 = 8x^2 - 8(5)^2 \Rightarrow x = 7 \text{ cm}$$

$$\text{Hence, the thickness of the tube} = \frac{7 - 5}{2} = 1 \text{ cm}$$

21 Base area of vessel $\times$ rise in water level = Volume of cube

$$15 \times 12 \times h = 11 \times 11 \times 11 \Rightarrow h = 7.39 \text{ cm}$$

22 (Initial volume of water + required volume of water + volume of cube) = Base area of vessel $\times$ 10

$$\therefore 25 \times 20 \times 5 + \text{required volume of water} + 1000 \\ = 25 \times 20 \times 10$$

$$\Rightarrow \text{Required volume of water} = 1500 \text{ cm}^3 = 1.5 \text{ litre}$$

$$23 \text{ Net volume} = (10 \times 8 \times 2) - (2 \times 2 \times 2) = 152 \text{ cm}^3$$

$$\text{Net surface area} = 2(10 \times 8 + 8 \times 2 + 2 \times 10) \\ + 4(2 \times 2) - 2(2 \times 2) \\ = 240 \text{ cm}^2$$

24 Available area for spreading the earth

$$= (600 \times 200) - (24 \times 12) = 119712 \text{ m}^2$$

$$\text{Volume of the earth} = 119712 \times \text{rise in level}$$

$$24 \times 12 \times 8 = 119712 \times h$$

$$\Rightarrow h = \frac{2304}{119712} = 0.01924 \text{ m} = 1.92 \text{ cm}$$

25 Net volume of the wall

$$= \text{Total volume} - \text{Volume taken away due to doors}$$

$$= (30 \times 0.3 \times 5) - 2(4 \times 2.5 \times 0.3) = 39 \text{ m}^3$$

$$\text{Volume of the bricks} = 39 \times \frac{8}{9} \text{ (Since } \frac{1}{9} \text{ part is lime in the wall)}$$

$$\therefore \text{Number of bricks} = \frac{39 \times 8}{9 \times 0.2 \times 0.16 \times 0.08} \\ = 13541.66 \approx 13600$$

26 Volume of water which flow in 25 minutes

$$= 25 \times 60 \times 0.05 \times 0.03 \times 16 = 36 \text{ m}^3$$

$$\therefore \text{Rise in water level} = \frac{36}{15 \times 12} = \frac{1}{5} \text{ m} = 0.2 \text{ m}$$

27 A cube has 6 equal faces, 12 equal edges, 8 vertices and 4 equal diagonals.

(See the figure shown in article 10.8)

$$28 \quad a\sqrt{3} = 6\sqrt{3} \Rightarrow a = 6 \text{ cm}$$

$$29 \quad \sqrt{l^2 + b^2} = 12 \Rightarrow l^2 + b^2 = 144$$

$$\text{and} \quad \sqrt{l^2 + b^2 + h^2} = 15$$

$$\Rightarrow l^2 + b^2 + h^2 = 225$$

$$\Rightarrow h^2 = 81 \Rightarrow h = 9 \text{ m}$$

$$30 \quad l + b + h = 12 \text{ cm}, \quad \sqrt{l^2 + b^2 + h^2} = 5\sqrt{2}$$

$$\Rightarrow l^2 + b^2 + h^2 = 50$$

$$\text{Now,} \quad (l + b + h)^2 = l^2 + b^2 + h^2 + 2(lb + bh + hl)$$

$$\Rightarrow 144 = 50 + 2(lb + bh + hl)$$

$$\Rightarrow 2(lb + bh + hl) = 94 \text{ cm}^2$$

$$31 \quad h : b = 3 : 1 \text{ and } l : h = 8 : 1$$

$$\Rightarrow l : h : b = 24 : 3 : 1$$

$$\therefore 24x \times 3x \times x = 36.864$$

$$\Rightarrow x^3 = 0.512 \Rightarrow x = 0.8$$

$$\therefore h = 3x = 2.4 \text{ m}$$

$$32 \quad lb = x, \quad bh = y, \quad hl = z$$

$$\therefore lb \times bh \times hl = xyz$$

$$\Rightarrow (lbh)^2 = xyz$$

$$\Rightarrow lbh = \sqrt{xyz}$$

Introductory Exercise 10.6

$$1 \text{ Required volume of water} = \frac{22}{7} \times (0.04)^2 \times 3500 \\ = 17.6 \text{ m}^3$$

$$2 \quad h = \frac{10 \times 10 \times 10}{\pi \times 10 \times 10} = 3 \frac{2}{11} \text{ cm}$$

$$3 \quad h = \frac{511}{36.5} = 14 \text{ m}$$

$$4 \quad 2\pi rh = 1056 \text{ cm}^2$$

$$\Rightarrow r = \frac{33}{\pi} \text{ cm}$$

$$\therefore \pi r^2 h = \pi \times \left(\frac{33}{\pi}\right)^2 \times 16$$

$$\Rightarrow \text{Volume} = 5544 \text{ cm}^3$$

$$5 \text{ Diameter of the cylinder} = \text{edge of the cube} = 2 \text{ cm}$$

$$\text{Height of the cylinder} = \text{edge of the cube}$$

$$\therefore \text{Required volume} = (2)^3 - \pi \times (1)^2 \times 2 \\ = 8 - \left(\frac{44}{7}\right) = \frac{12}{7} \text{ cm}^3$$

$$6 \quad h = \frac{2\pi rh}{2\pi r} = \frac{17.6}{8.8} = 2 \text{ m}$$

$$\text{and } 2\pi r = 8.8$$

$$\Rightarrow r = 1.4 \text{ m}$$

$$\therefore \pi r^2 h = \frac{22}{7} \times (1.4)^2 \times 2 = 12.32 \text{ m}^3$$

$$7 \quad 2r = h \Rightarrow r = \frac{h}{2}$$

$$\therefore 2\pi r(h+r) = 2\pi \times \frac{h}{2} \left(h + \frac{h}{2}\right) = \frac{3}{2} \pi h^2$$

$$8 \quad 2\pi rh = 2 \times \pi \times 2 \times 10 = 40\pi \text{ m}^2$$

$$9 \quad \frac{2\pi rh}{2\pi r(h+r)} = \frac{2}{3} \Rightarrow \frac{h}{h+r} = \frac{2}{3} \Rightarrow \frac{h}{r} = \frac{2}{1}$$

$$\therefore 2\pi r(h+r) = 924$$

$$\therefore 2\pi rh = 924 \times \frac{2}{3} = 616 \text{ cm}^2$$

$$\Rightarrow 2 \times \pi \times x \times 2x = 616$$

$$\Rightarrow x = 7 \therefore r = 7 \text{ cm and } h = 14 \text{ cm}$$

$$\therefore \pi r^2 h = \frac{22}{7} \times (7)^2 \times 14 = 2156 \text{ cm}^3$$

$$10 \quad \frac{\pi \times r \times r \times h}{2 \times \pi \times r \times h} = \frac{269.5}{154} \Rightarrow r = 3.5 \text{ cm}$$

$$\text{Now, } 2\pi \times 3.5 \times h = 154 \Rightarrow h = 7 \text{ cm}$$

$$11 \quad 2\pi rh = 1320 \Rightarrow h = 10 \text{ cm}$$

$$\therefore 2\pi r(h+r) = 2 \times \frac{22}{7} \times 21 \times 31 = 4092 \text{ cm}^2$$

$$12 \quad \frac{r}{h} = \frac{3x}{4x}, \quad \pi r^2 h = 4851 \Rightarrow x = 3.5$$

$$\therefore r = 10.5 \text{ m and } h = 14 \text{ m}$$

$$\therefore 2\pi rh = 2 \times \frac{22}{7} \times 10.5 \times 14 = 924 \text{ m}^2$$

$$13 \quad \frac{2\pi r(h+r)}{2\pi rh} = \frac{4}{1}$$

$$\Rightarrow \frac{h+r}{h} = \frac{4}{1} \Rightarrow \frac{r}{h} = \frac{3}{1}$$

Alternatively Go through options.

$$14 \quad 2\pi r = 33 \Rightarrow r = \frac{21}{4}$$

$$\therefore \text{Volume} = \pi r^2 h = \frac{22}{7} \times \left(\frac{21}{4}\right)^2 \times 330 \\ = 28586.25 \text{ cm}^3$$

$$15 \quad \frac{r_1}{r_2} = \frac{4x}{x}, \text{ but } V_1 = V_2$$

$$\therefore \pi (4x)^2 \times h_1 = \pi (x)^2 h_2$$

$$\Rightarrow h_2 = 16h_1$$

$$16 \quad 2\pi r(h+r) = 2640$$

$$\Rightarrow 2\pi r(30) = 2640 \Rightarrow r = 14 \text{ m}$$

$$\Rightarrow h = 16 \text{ m } (\because r+h=30 \text{ m})$$

$$\therefore h:r=8:7$$

$$17 \quad \frac{V_1}{V_2} = \frac{\pi (r_1)^2 h_1}{\pi (r_2)^2 h_2} = \frac{3 \times 3 \times 4}{5 \times 5 \times 3} = \frac{12}{25}$$

$$18 \quad \frac{V_1}{V_2} = \frac{1}{1}$$

$$\left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2} = \frac{1}{1} \Rightarrow \left(\frac{r_1}{r_2}\right)^2 \times \frac{3}{1} = \frac{1}{1}$$

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{1}{3} \Rightarrow \frac{r_1}{r_2} = \frac{1}{\sqrt{3}}$$

$$19 \quad \frac{2\pi r_1 h_1}{2\pi r_2 h_2} = \frac{3}{2} \times \frac{6}{7} = \frac{9}{7}$$

$$20 \quad h = 42 \text{ cm, } 2\pi R = 132 \text{ cm}$$

$$\Rightarrow R = 21 \text{ cm}$$

$$\therefore r = 21 - 3 = 18 \text{ cm}$$

$$\therefore \text{Required volume} = \pi (R^2 - r^2) h \\ = \frac{22}{7} [(21)^2 - (18)^2] \times 42 = 15444 \text{ cm}^3$$

21 Since radius and height of the cylinder are same as that of cone. Therefore cylinder can contain $15 \times 3 = 45$ litre of milk.

Hint Volume of cone = $\frac{1}{3} \pi r^2 h$ and volume of cylinder = $\pi r^2 h$.

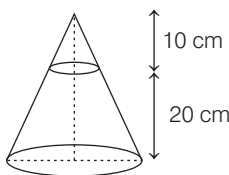
22 Best way is to go through option.

☞ Alternatively $\frac{V_1}{V_2} = \frac{1}{27} \Rightarrow \frac{(r_1)^2 h_1}{(r_2)^2 h_2} = \frac{1}{27}$

$$\Rightarrow \frac{x_1^3}{x_2^3} = \frac{(1)^3}{(3)^3} \Rightarrow \frac{x_1}{x_2} = \frac{1}{3}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{1}{3}$$

$$\therefore h_1 = \frac{h_2}{3} = \frac{30}{3} = 10 \text{ cm}$$



Hence, cone is cut off at $(30 - 10) = 20$ cm above the base.

Hint For more clarification of the concept seek help from geometry. (Similarity of triangles)

23 Volume of conical tent = $\frac{1}{3} \times \frac{22}{7} \times \frac{13}{2} \times \frac{13}{2} \times 10.5$

$$\therefore \text{Average space per man} = \frac{\text{Volume of tent}}{8} = 58 \frac{3}{32}$$

24 $\frac{1}{3} \times \pi \times (5x)^2 \times (12x) = 314 \frac{2}{7} = \frac{2200}{7}$

$$\Rightarrow x = 1 \therefore r = 5 \text{ and } h = 12$$

$$\therefore l = 13 \text{ m}$$

25 $\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 1232 \Rightarrow r = 7 \text{ cm}$

$$\therefore \pi r l = \frac{22}{7} \times 7 \times 25 = 550 \text{ cm}^2 \quad (\because l = \sqrt{r^2 + h^2})$$

26 $2\pi r = 220 \Rightarrow r = 35 \text{ cm}$

$$\therefore l = \sqrt{r^2 + h^2} = 91 \text{ cm}$$

$$\therefore \pi r l = \frac{22}{7} \times 35 \times 91 = 10010 \text{ cm}^2$$

27 Area of cloth = $\pi r l = \frac{22}{7} \times 14 \times 50 = 2200 \text{ m}^2$

$$\therefore \text{Length of cloth} = \frac{2200}{10} = 220 \text{ m}$$

28 $\pi r l = 23.10 \Rightarrow r l = 2.1 \times 3.5$

Now, we know that $h = 2.8$. So we can assume the value of r from the given option.

at $r = 2.1, l = 3.5 \quad (\because l = \sqrt{r^2 + h^2})$

☞ Alternatively $r^2 (r^2 + h^2) = (2.1 \times 3.5)^2$

$$\Rightarrow r^4 + (2.8)^2 r^2 = (7.35)^2 \Rightarrow r^4 + 7.84 r^2 = 54.0225$$

$$\Rightarrow k^2 + 7.84k - 54.0225, \quad k = r^2$$

Now, solve the above quadratic equation, if you can else substitute the value of r from the given choices.

29 $\frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{3}{2} \quad (\because r_1 = r_2)$

$$\therefore \frac{A_1}{A_2} = \frac{3}{2} \Rightarrow \frac{A_1}{300} = \frac{3}{2}$$

$$\therefore A_1 = 450 \text{ cm}^2$$

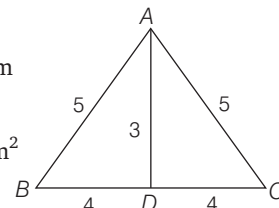
30 $\frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{\pi r^2 h}{\left(\frac{1}{3}\right) \pi r^2 h} = \frac{1}{\left(\frac{1}{3}\right)} = \frac{3}{1}$

31 By Pythagoras theorem

$$AD = 3 \text{ cm}$$

$\therefore$ Area of axial section

$$= \frac{1}{2} \times 8 \times 3 = 12 \text{ cm}^2$$



32 $\frac{V_2}{V_1} = \frac{(1.4)^3}{(1)^3} = \frac{2.744}{1}$

$$\therefore \% \text{ increase in volume} = \left[\frac{2.744 - 1}{1} \right] \times 100 = 174.4\%$$

33 Area of circular sheet = 625π

Since length of arc and area of sector are directly proportional to the central angle.

Therefore, length of remaining arc

$$= \frac{96}{100} \times 2 \times \pi \times 25 = 48\pi$$

But the remaining arc is equal to the circumference of the base of circular cone.

$$\therefore 2\pi R = 48\pi \Rightarrow R = 24 \text{ cm}$$

Now, since the slant height of cone is equal to the radius of the original circular sheet.

Hence, $l = 25 \text{ cm}$

$$h = 7 \text{ cm} \quad (\because l = \sqrt{r^2 + h^2})$$

$$\therefore \frac{\text{Radius}}{\text{Height}} = \frac{24}{7}$$

34 $\frac{V_2}{V_1} = \frac{(r_2)^2 h_2}{(r_1)^2 h_1} = \frac{(2r)^2}{(r)^2} \quad \{\because h_1 = h_2\}$

$$\Rightarrow \frac{V_2}{V_1} = \frac{4}{1}$$

35 Each edge of cube = 10 cm

Radius of base of cone = 5 cm

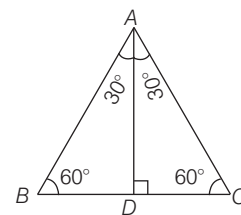
Height of cone = 10 cm

$$\therefore \text{Volume of cone} = \frac{1}{3} \times \frac{22}{7} \times 25 \times 10 = 261.9 \text{ cm}^2$$

36 $\frac{V_2}{V_1} = \frac{r_1^2 \times h_1}{r_2^2 \times h_2} = \frac{2 \times 2 \times 2}{1 \times 1 \times 1} = \frac{8}{1}$

37 $\frac{BD}{AB} = \cos 60^\circ$
 $\frac{BD}{AB} = \frac{1}{2}$

$BD = CD$, are the radii of the base and $AB = AC$ are the slant heights of the cone. A is the vertex and BC is the base.



38 Volume of cone = $\frac{1}{3} \pi \times 144 \times 35$

$$\begin{aligned} \text{Volume of water flowing per second} &= \pi \times (0.8)^2 \times \frac{500}{60} \\ \therefore \text{Required time} &= \frac{\left(\frac{\pi}{3}\right) \times 144 \times 35}{\pi \times 0.64 \times \left(\frac{500}{60}\right)} = 315 \text{ s} \end{aligned}$$

Introductory Exercise 10.7

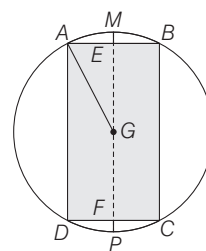
- 1 $\frac{4}{3} \pi (r_1^3 + r_2^3 + r_3^3) = \frac{4}{3} \pi (6)^3$
 $\Rightarrow 27 + 64 + r_3^3 = 216$
 $\Rightarrow r_3^3 = 125$
 $\Rightarrow r_3 = 5 \text{ cm}$
- 2 $\pi r^2 \times 144 = \frac{4}{3} \pi \times (12)^3$ ($\because$ Volume remains constant)
 $\Rightarrow r = 4 \text{ cm}$
- 3 Number of bottles $\times$ Volume of each bottle
 $=$ Volume of hemisphere
 $n \times \pi \times (3)^2 \times 1 = \frac{2}{3} \pi \times (6)^3 \Rightarrow n = 16$
- 4 $\frac{1}{3} \pi \times (r)^2 \times 14 = \frac{2}{3} \pi \times (7)^3 \Rightarrow r = 7 \text{ cm}$
- 5 Volume of the spherical shell $= \frac{4}{3} \pi (R^3 - r^3)$
 $= \frac{4}{3} \pi (7^3 - 5^3) = \frac{872}{3} \pi$
- 6 Since volume is constant
 $\therefore n \times \frac{4}{3} \pi (1)^3 = \frac{4}{3} \pi \times (8)^3 \Rightarrow n = 512$
- 7 Volume of hemisphere $= \frac{2}{3} \pi r^3$
 $= \frac{2}{3} \times \frac{22}{7} \times (21)^3 = 19404 \text{ m}^3$
- 8 Change in height (or level) of water
 $= \frac{\text{Volume of sphere}}{\text{Base area of cylinder}}$
 $= \frac{\frac{4}{3} \pi \times (9)^3}{\pi \times (12)^2} = \frac{27}{4} \text{ cm}$
- 9 Volume of cone $=$ Volume of sphere
 $\frac{1}{3} \pi r^2 \times \frac{7}{2} = \frac{4}{3} \pi (7)^3 \Rightarrow r = 14\sqrt{2} \text{ cm}$
- 10 Required surface area $=$ lateral surface area of cylinder + [Total surface Area of Sphere – 2 (lateral surface area of spherical cap)]

39 Use the given formula :

$$\text{Volume} = \frac{\pi}{3} h (r^2 + Rr + R^2)$$

where $\pi = \frac{22}{7}$, $h = 6$, $R = 4$ and $r = 2$

$$40 \frac{1}{3} \pi \times (10)^2 \times 72 = \pi \times (30)^2 \times h \Rightarrow h = \frac{8}{3} = 2\frac{2}{3} \text{ cm}$$



In the diagram, $AG = MG = PG =$ radius of sphere

Now, using Pythagoras Theorem in right angle $\triangle AEG$, we can find $EG = \sqrt{AG^2 - AE^2} = \sqrt{15^2 - 9^2} = 12 \text{ cm}$.

Therefore height of the cylinder $= EF = 2 EG = 24 \text{ cm}$

Now, Lateral surface area of cylinder

$$= 2\pi \times 9 \times 24 = 432 \pi \text{ sq cm}$$

Total surface area of original sphere

$$= 4\pi (\text{radius of sphere})^2 = 4\pi \times 15^2 = 900 \pi \text{ sq cm}$$

Lateral surface area of a spherical cap $=$ perimeter of the sphere $\times$ height of the spherical cap

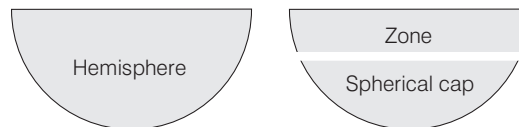
$$= 2\pi \times (\text{radius of the sphere}) \times (MG - EG)$$

$$= 2\pi \times 15 \times (15 - 12) = 90\pi$$

Therefore, required surface area

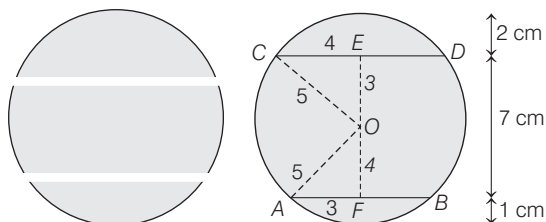
$$= 432 \pi + [900 \pi - 2(90 \pi)] = 1152 \pi \text{ sq cm}$$

11 Look at the following diagram and recall the definition of a zone.



Hence choice (d) is correct.

Solutions (for Q. Nos. 12 to 14) Given that AB and CD are parallel, if we draw a line passing from the centre and intersecting AB and CD , this line EF will be perpendicular to both the lines AB and CD . If you have any doubt please refer the Geometry chapter, especially, Lines and Angles section to understand this concept and even you can refer the Circles section also.



Now, we know that $\triangle AFO$ and $\triangle CEO$ are right angle triangles. We know that $EF = 7$ cm and $AF = 3$ cm.

Now in $\triangle OAF$, we can use Pythagoras theorem and find OF
 $= \sqrt{OA^2 - AF^2} = \sqrt{5^2 - 3^2} = 4$ cm.

Therefore, $OE = EF - OF = 3$ cm.

Now, in $\triangle CEO$, we can use Pythagoras theorem and find

$$CE = \sqrt{CO^2 - OE^2} = \sqrt{5^2 - 3^2} = 4 \text{ cm.}$$

Therefore, $AB = 2(AF) = 6$ cm and $CD = 2(CE) = 8$ cm. Now, we know that the segment with larger base radius is the larger segment, so the height of larger segment $= 5 - 3 = 2$ cm. Similarly, the height of the smaller segment $= 5 - 4 = 1$ cm

12 Volume of the zone

$$\begin{aligned} &= \frac{\pi h}{6} (h^2 + 3r_1^2 + 3r_2^2) = \frac{7\pi}{6} (7^2 + 3(3)^2 + 3(4)^2) \\ &= \frac{434}{3} \pi \end{aligned}$$

Hence, choice (a) is correct.

13 The smaller spherical cap has 3 cm radius and 1 cm height.

Therefore the volume of the smaller spherical cap
 $= \frac{\pi h}{6} (h^2 + 3r^2) = \frac{\pi (5-4)}{6} ((5-4)^2 + 3(3)^2) = \frac{14}{3} \pi$.

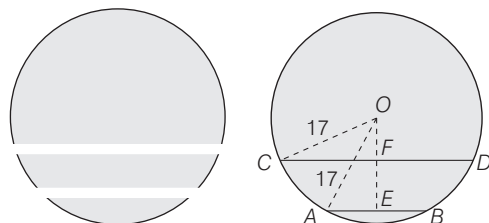
Hence, choice (c) is correct.

14 The larger spherical cap has 4 cm radius and 2 cm height.

Therefore, the lateral surface area of the larger spherical cap
 $= 2\pi (5) \times (2) = 20\pi$

Hence, choice (d) is correct.

Solutions (for Q. Nos. 15 to 17) Given that $OA = OC = 17$ cm. Now drop the perpendicular OF from the centre on the line CD and another perpendicular OE on the line AB . These perpendiculars will bisect the lines CD and AB respectively.



Now, in $\triangle AEO$, we have

$$OE = \sqrt{OA^2 - AE^2} = \sqrt{17^2 - 8^2} = 15 \text{ cm}$$

But, $OF = OE - EF = 15 - 7 = 8$ cm

Now, in $\triangle CFO$, we have

$$CF = \sqrt{CO^2 - OF^2} = \sqrt{17^2 - 8^2} = 15 \text{ cm}$$

The height of larger spherical cap

$$\begin{aligned} &= \text{radius of sphere} + \text{length of } OF \\ &= 17 + 8 = 25 \text{ cm} \end{aligned}$$

And the height of the smaller segment = radius of sphere
 $- \text{length of } OE = 17 - 15 = 2$ cm.

15 Volume of the zone

$$\begin{aligned} &= \frac{\pi h}{6} (h^2 + 3r_1^2 + 3r_2^2) \\ &= \frac{7\pi}{6} (7^2 + 3(8)^2 + 3(15)^2) = \frac{3206}{3} \pi \end{aligned}$$

Hence, choice (b) is correct.

16 Volume of the larger spherical cap

$$= \frac{\pi h}{6} (h^2 + 3r^2) = \frac{25\pi}{6} (25^2 + 3(15)^2) = \frac{16250}{3} \pi$$

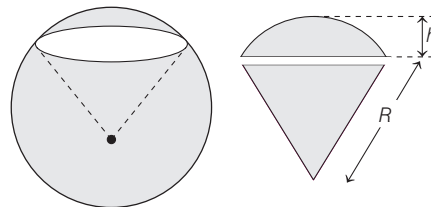
Hence, choice (c) is correct.

17 Total surface area of the smaller spherical cap

$$= 2\pi Rh + \pi r^2 = 2\pi (17 \times 2) + \pi (2)^2 = 72\pi$$

Hence, choice (a) is correct.

18 Volume of the spherical sector = Volume of the spherical cap + Volume of the conical section



But the height of the conical section $= \sqrt{25^2 - 7^2} = 24$ cm

Therefore, the height of the smaller spherical cap
 $= 25 - 24 = 1$ cm

Thus, the total surface area of the sector of the sphere

$$\begin{aligned} &= \frac{\pi h}{6} (h^2 + 3r^2) + \frac{1}{3} (\text{base area of cone} \times \text{height of cone}) \\ &= \frac{\pi}{6} (1 + 3(7)^2) + \frac{1}{3} (\pi \times 7^2) \times 24 = \frac{1250}{3} \pi \end{aligned}$$

Hence, choice (d) is correct.

$$\begin{aligned} \text{19 Volume of pyramid} &= \frac{1}{3} \times \text{base area} \times \text{height} \\ &= \frac{1}{3} \times 25 \times 12 = 100 \text{ cm}^3 \end{aligned}$$

Hence, choice (b) is correct.

20 Volume of prism = Base area $\times$ height

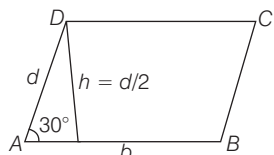
Since, base area is constant and height is being halved therefore volume will also be halved. Hence, its volume will be reduced by 50%.

Hence, choice (a) is correct.

✓ Level 01 Basic Level Exercise

- 1 With the given perimeter the area of parallelogram will be maximum when it will be a rhombus.
Hence, option (a) is correct.

☞ Alternatively



$$\text{Perimeter} = 2d + 2b = 4h + 2b = 64 \text{ cm}$$

$$\Rightarrow b = \frac{64 - 4h}{2}$$

$$\therefore \text{Area} = b \times h = \frac{64h - 4h^2}{2} = f(A)$$

$$\therefore f'(A) = \frac{1}{2} (64 - 8h) \text{ (Refer the differentiation)}$$

For the maximum area = $f'(A) = 0$

$$\therefore \frac{1}{2} (64 - 8h) = 0$$

$$\Rightarrow h = 8 \text{ cm and } d = 16 \text{ cm}$$

$$\therefore b = 16 \text{ cm}$$

- 2 Volume of water displaced = volume of sphere

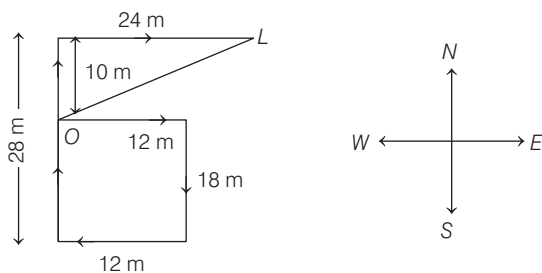
$$\pi \times (40)^2 \times h = \frac{4}{3} \pi \times (30)^3$$

$$\Rightarrow h = \frac{90}{4} = 22.5 \text{ cm}$$

Thus, the level of water rises by 22.5 cm.

☞ **NOTE** The volume of water will be calculated by considering it in the cylindrical shape since the water takes the shape of vessel in which it is filled.

- 3 O is the starting point and L is the end point.



$$(OL)^2 = (24)^2 + (10)^2$$

$$OL^2 = 676$$

$$\Rightarrow OL = 26 \text{ m North-East}$$

- 4 $a^2 + b^2 = 13^2 = 169$

$$\text{and } \frac{a \times b}{2} = 30$$

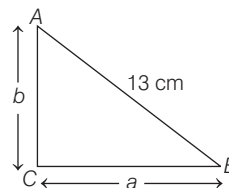
$$\text{Now } (a + b)^2 = a^2 + b^2 + 2ab$$

$$(a + b)^2 = 169 + 120$$

$\Rightarrow$

$$a + b = 17$$

...(i)



Again

$$(a - b)^2 = (a + b)^2 - 4ab$$

$$(a - b)^2 = 289 - 240$$

$\Rightarrow$

$$(a - b) = 7$$

...(ii)

From eq. (i) and (ii), we get

$$a = 12 \text{ cm and } b = 5 \text{ cm}$$

$$\text{Now, Inradius} = \frac{(\text{Altitude} + \text{Base} - \text{Hypotenuse})}{2}$$

$$= \frac{(12 + 5 - 13)}{2} = 2 \text{ cm}$$

$$\text{☞ Alternatively Inradius (r)} = \frac{\text{Area}}{\text{Semiperimeter}}$$

$$= \frac{30}{15} = 2 \text{ cm}$$

$$\left(\text{Semiperimeter} = \frac{(12 + 5 + 13)}{2} = 15 \right)$$

☞ **NOTE** If you are well aware with Pythagorean triplets then it is very easy to find the sides (i.e., altitude and base) of the right angle triangle.

- 5 **General phenomenon:** A convex polygon in which there is maximum number of sides, it has the greatest enclosed area when the perimeter of the polygon is constant.

Remember that in a circle there are infinite sides of minimum possible length. So, the area of circle will be maximum.

☞ **Alternatively** Perimeters of hexagon and circumference of circle are same i.e.

$$6a = 2\pi r \Rightarrow 21a = 22r$$

where a is the side of hexagon and r is the radius of circle.

then Area of circle = πr^2

$$\text{and Area of hexagon} = 6 \times \frac{\sqrt{3}}{4} \times a^2$$

$$= \frac{3\sqrt{3}}{4} \times \left(\frac{22r}{21} \right)^2$$

$$= \frac{3\sqrt{3}}{4} \times \frac{22}{7 \times 3} \times \frac{22}{7 \times 3} \times r^2$$

$$= \frac{\sqrt{3}}{4} \times \frac{22}{21} \pi r^2 = 0.45\pi r^2$$

Therefore, area of circle πr^2 is greater than area of hexagon. Similarly you can compare other figures.

☞ **Alternatively** Consider some suitable values. Let us assume perimeter of square and hexagon is 24 cm, then each side of square = 6 cm.

and $\text{Area} = 36 \text{ cm}^2$
 Similarly, each side of hexagon = 4 cm
 and area of hexagon = $\frac{3\sqrt{3}}{2} \times (4)^2 = 24\sqrt{3} \text{ cm}^2$

Thus, we can say that for a particular (or constant) value of perimeter, area of hexagon is greater than that of square.

Similarly each side of octagon = 3 cm $\left(= \frac{24}{8} \right)$

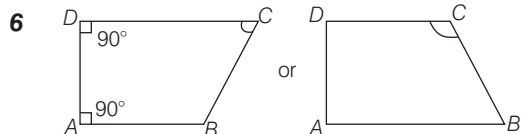
$$\therefore \text{Area of octagon} = 2a^2 (1 + \sqrt{2}) \\ = 2 \times 9 (1 + \sqrt{2}) \approx 43.5 \text{ cm}^2$$

Thus, the area of octagon is greater than the area of hexagon.

Similarly, $2\pi r = 24 \Rightarrow r = \frac{12}{\pi} \text{ cm}$

$$\therefore \text{Area of circle} = \pi \times r^2 = \pi \times \frac{12}{\pi} \times \frac{12}{\pi} = 45.81 \text{ cm}^2$$

Thus, we can say that ultimately the area of the circle is greatest.



Since sum of all the angles of a quadrilateral is 360° .

Therefore, $\angle A + \angle B + \angle C + \angle D = 360^\circ$

$$\Rightarrow \angle B + \angle C = 180^\circ$$

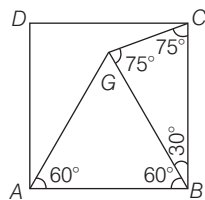
$$\Rightarrow \angle B = 180^\circ - \angle C$$

7 $\angle ABG = 60^\circ$

$$\therefore \angle GBC = 90^\circ - 60^\circ = 30^\circ$$

Again $\angle BGC + \angle BCG = 180^\circ - 30^\circ = 150^\circ$

Now, since $\angle BGC = \angle BCG$



$$\therefore \angle BGC = 75^\circ$$

NOTE In a triangle when two sides are equal, then the two angles opposite to these sides are also equal.

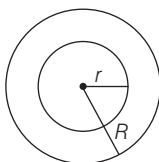
8 $\frac{r}{R} = \frac{3x}{4x}$

$$\therefore 4x - 3x = 2$$

$$\Rightarrow x = 2$$

$$\therefore \text{Outer radius} = 8 \text{ cm}$$

$$\therefore \text{Area of outer circle} = \pi \times (8)^2 = 64\pi \text{ cm}^2$$



9 $\frac{h}{a} = \sin 60^\circ$

$$\frac{h}{a} = \frac{\sqrt{3}}{2}$$

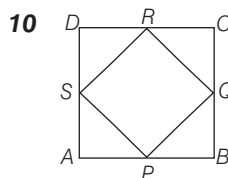
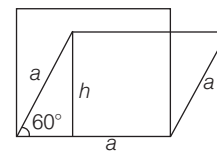
$$\Rightarrow h = \frac{a\sqrt{3}}{2}$$

$$\therefore \text{Area of rhombus}$$

$$= a \times h = \frac{a \times \sqrt{3}a}{2}$$

$$\text{and Area of square} = a^2$$

$$\therefore \text{Required ratio} = \frac{a^2}{\frac{a \times \sqrt{3}a}{2}} \times 2 \\ = \frac{2}{\sqrt{3}} \times \left(\frac{\sqrt{3}}{\sqrt{3}} \right) = \frac{2\sqrt{3}}{3}$$



ABCD is a square, $AB = BC = CD = AD = 20 \text{ cm}$

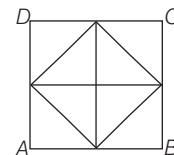
and P, Q, R and S are the mid-points.

$$\therefore PS = SR = RQ = PQ = 10\sqrt{2} \text{ cm } (\because AP = BP \text{ etc} = 10 \text{ cm})$$

$$\therefore \text{Area of square PQRS} = (10\sqrt{2})^2 = 200 \text{ cm}^2$$

Alternatively We can see that there are total 8 equal parts (or triangles) out of 8 we have taken only 4 parts i. e., 50% area has been cut out.

So, the remaining area = 50% of 400 = 200 cm^2 .

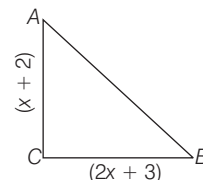


11 Area of right angle triangle

$$= \frac{(x+2) \times (2x+3)}{2} = 60$$

$$\Rightarrow 2x^2 + 7x + 6 = 120$$

$$\Rightarrow 2x^2 + 7x - 114 = 0$$



Solving the above quadratic equation, we get

$$x = 6$$

$$\therefore x + 2 = 8 \text{ cm}$$

$$\text{and } 2x + 3 = 15 \text{ cm}$$

$$\therefore \text{Hypotenuse } AB = \sqrt{(8)^2 + (15)^2} = 17 \text{ cm}$$

(Using Pythagoras theorem)

12 Let us assume PQRSTUWV is a regular octagon with each side a unit.

Again, if we produce all the sides of octagon in both sides we get a square. Since each of the side of octagon is 'a' then

$$UV = WP = QR = ST = a$$

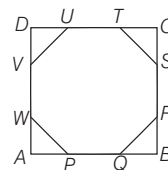
$$\therefore DU = DV (\text{etc.}) = \frac{a}{\sqrt{2}}$$

$$\therefore DU + UT + TC = \frac{a}{\sqrt{2}} + a + \frac{a}{\sqrt{2}}$$

$$= a(1 + \sqrt{2})$$

$$\therefore \text{Area of square} = a^2 (1 + \sqrt{2})^2$$

$$= a^2 (3 + 2\sqrt{2}) \text{ sq unit}$$



Again, area of each shaded portion (i.e., an isosceles right angle triangle)

$$= \frac{1}{2} \times \frac{a}{\sqrt{2}} \times \frac{a}{\sqrt{2}} = \frac{a^2}{4}$$

$$\therefore \text{Total area of all the shaded region} = 4 \times \frac{a^2}{4} = a^2$$

$$\begin{aligned} \therefore \text{Area of octagon} &= \text{Area of square } ABCD \\ &\quad - \text{Area of total shaded region} \\ &= a^2 (3 + 2\sqrt{2}) - a^2 \\ &= 2a^2 (1 + \sqrt{2}) \text{ sq. unit} \end{aligned}$$

- 13** Let the each side of the square be 2 cm, then
area of square = 4 cm² and
area of 4 quadrants of the four circles (i.e., unshaded part inside the square)

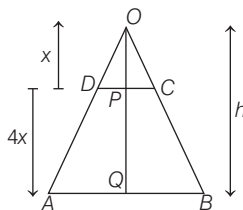
$$= 4 \times \frac{1}{4} \pi \times (1)^2 = \pi \text{ cm}^2$$

$$\therefore \text{Area of shaded region} = (4 - \pi) \text{ cm}^2$$

$$\text{Therefore, the required ratio} = \frac{4 - \pi}{\pi} = \frac{6/7}{22/7} = \frac{3}{11}$$

- 14** From the concept of mid point theorem, it is the average of the length of the parallel sides.

- 15** From the concept of similarity of triangles $\frac{OP}{OQ} = \frac{1}{5} = \frac{PC}{BQ}$



Since, the ratio in radii of the two cones is 1 : 5. Therefore, the radius of smaller cone ODC is $\frac{21}{5} = 4.2$ cm.

☞ Alternatively Solve it in the proper way, which is actually a very tedious process. Still you have to apply the concept of similarity of triangles, which you will study in Geometry.

$$\begin{aligned} 16 \quad l \times b &= 2(l + b) \\ \Rightarrow l &= \frac{2b}{(b-2)} = \frac{2(b-2) + 4}{b-2} = 2 + \frac{4}{(b-2)} \end{aligned}$$

Since, l is an integer, so 4 must be divisible by $(b-2)$. Thus, b can be 4 or 6 or 3.

Therefore, if $b = 4$, $l = 4$, it will be a square. If $b = 6$, $l = 3$ and if $b = 3$, $l = 6$.

$$\begin{aligned} \text{Hence,} \quad l &= 6 \quad \text{and} \quad b = 3 \\ \therefore \quad l - b &= 3 \end{aligned}$$

$$17 \quad \text{Area of } \triangle ABC = \frac{\sqrt{3}}{4} \times (14)^2 = 49\sqrt{3} \text{ cm}^2$$

Area of sector

$$ABC = \pi \times (14)^2 \times \frac{60}{360} = 102\frac{2}{3} \text{ cm}^2$$

$\therefore$ Area of the shaded region

$$= \left(102\frac{2}{3} - 49\sqrt{3} \right) \text{ cm}^2$$

$$18 \quad l_1 \times b_1 = l_2 \times b_2$$

New area of first rectangle

$$= \frac{5}{4} l_1 \times \frac{3}{4} b_1 = \frac{15}{16} l_1 b_1$$

$$\text{New area of the second rectangle} = \frac{3}{4} l_2 \times \frac{5}{4} b_2 = \frac{15}{16} l_2 b_2$$

Hence, areas of both the new rectangles are same.

$$19 \quad \frac{DM}{AD} = \sin 60^\circ$$

$$\frac{6\sqrt{3}}{AD} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow AD = 12 \text{ cm}$$

$$20 \quad \frac{4}{3} \pi r^3 = \frac{1}{3} \pi r^2 h$$

(Since radii of sphere and cone are same)

$$\Rightarrow 4r = h$$

$$\therefore \frac{h}{r} = \frac{4}{1} \Rightarrow h : r = 4 : 1$$

$$21 \quad \text{Area of the shaded region} = \left[\frac{1}{2} \pi (22)^2 - 2 \left(\frac{1}{2} \pi \times (11)^2 \right) \right]$$

$$= \frac{1}{2} \pi \times (11)^2 [4 - 2] = 121\pi \text{ cm}^2$$

$$22 \quad \text{Area of path} = (l + b - w) w$$

$$= (60 + 40 - 5) 5 = 475 \text{ m}^2$$

$$\text{Cost} = \text{Area} \times \text{Rate}$$

$$= 475 \times 0.8 = ₹ 380$$

- 23** Go through options. Let the length and breadth of the second rectangle is l and b respectively, then the area of second field = $l \times b$.

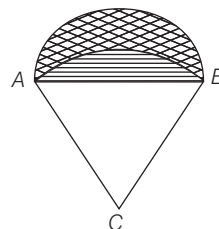
NOTE Now consider option (d) i.e., $x = 80$, then

$$\text{Area of first field} = \frac{1}{5} l \times 5b = lb$$

Hence, area of first field is same as that of second field, hence the presumed option is correct.

$$24 \quad \text{Area of quadrant} = \frac{1}{4} \pi a^2$$

$$\text{Area of triangle } ACB = \frac{a^2}{2}$$



 $\rightarrow$ Area of required region

 $\rightarrow$ Area of segment

$$\therefore \text{Area of segment} = \frac{\pi a^2}{4} - \frac{a^2}{2} = \frac{a^2}{4} [\pi - 2]$$

$$\text{Area of semi-circle} = \frac{1}{2} \pi \left(\frac{a\sqrt{2}}{2} \right)^2 = \frac{\pi a^2}{4}$$

$$\therefore \text{Area of required region} = \frac{\pi a^2}{4} - \frac{a^2}{4} [\pi - 2] = \frac{a^2}{2} \text{ sq unit}$$

- 25 Wrongly calculated area = $1.05 \times 0.92 = 0.966 = 96.6\%$

$$\therefore \% \text{ error} = 100 - 96.6 = 3.4\%$$

☞ **Alternatively** Actual area = $l \times b$

$$\text{Wrongly calculated area} = 1.05l \times 0.92b = 0.966lb$$

$$\text{Deficit in area} = lb - 0.966lb = 0.034lb$$

$$\% \text{ error in area} = \frac{0.034lb}{lb} \times 100 = 3.4\%$$

- 26 Volume of water = Area of cross-section $\times$ Length of pool

$$= \frac{(10 + 6)}{2} \times 6 \times 120 = 5760 \text{ m}^3$$

- 27 Area of path = $\frac{22176}{1} = 22176 \text{ m}^2$

$$\pi (R^2 - r^2) = 22176$$

$$(R^2 - r^2) = \frac{22176}{22} \times 7$$

$$R^2 - (112)^2 = 7056$$

$$\Rightarrow R^2 = 19600 \Rightarrow R = 140 \text{ m}$$

$$\therefore \text{Width of the path} = 140 - 112 = 28 \text{ m}$$

- 28 From the figure itself it is clear that there are total 9 equilateral (congruent) triangles. Out of 9 triangles, 3 triangles are cut out.

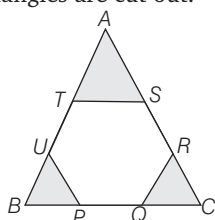


Fig. (i)

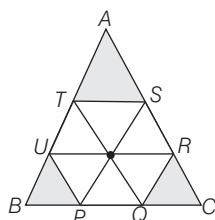


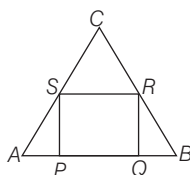
Fig. (ii)

It means $\frac{1}{3}$ (i. e., 33.33%) area has been removed.

- 29 $\frac{SP}{AS} = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$$\Rightarrow SP = \frac{\sqrt{3}}{2} AS$$

$$\text{Also, } SP = SR = CS = CR$$



$$(\text{since, } SR \parallel AB \therefore \angle CSR = \angle SRC = \angle RCS = 60^\circ)$$

$$\therefore CS = \frac{\sqrt{3}}{2} AS$$

$$\therefore \frac{CS}{AS} = \frac{\sqrt{3}}{2} = \frac{CR}{RB}$$

- 30 Let the each side of a square be a then its area will be a^2 . Therefore, area of circle will also be a^2 . Again since the perimeter of square and equilateral triangle is same then, the each side of equilateral triangle is $\frac{4a}{3}$.

$$\therefore \text{Area of equilateral triangle} = \frac{\sqrt{3}}{4} \left(\frac{4a}{3} \right)^2$$

$$= \frac{4\sqrt{3}}{9} a^2$$

$$\therefore \text{Required ratio of area} = \frac{a^2 \times 9}{4\sqrt{3} \times a^2} = 9 : 4\sqrt{3}$$

- 31 ABCD is a square O is the point of intersection of diagonals. P, Q, R and S are the mid-points on the sides AB, BC, CD, DA respectively.

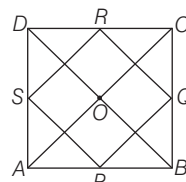


Fig. (i)

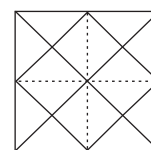


Fig. (ii)

In the above figure you can see that there are total 16 congruent isosceles right angled triangles.

In figure (iii) you can see that OMPN is a square of maximum possible area which is made up of 2 isosceles right angled triangles OMP and ONP. Thus, there are 4 smaller squares around O and thus the total area of these 4 squares is half of the larger square. Hence, the required area = 800 cm^2 .

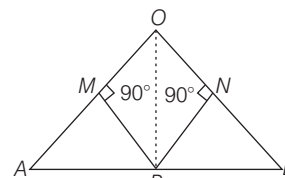


Fig. (iii)

☞ **Alternatively** ABCD is a square

$$AB = 40 \text{ cm}$$

$$OA = OB = 20\sqrt{2} \text{ cm}$$

and $OP = 20 \text{ cm}$

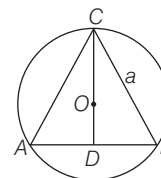
Square OMPN has maximum area inside the triangle and OP is the diagonal of square OMPN.

$$\therefore \text{Area of OMPN} = \frac{(20)^2}{2} = 200 \text{ cm}^2$$

Since, there are total 4 similar squares in each of the 4 triangles AOB, BOC, COD, DOA inside the larger square. So, the total required area of 4 smaller squares = $4 \times 200 = 800 \text{ cm}^2$

- 32 Let the each side of equilateral triangle be 'a', then circumradius of the circle = $\frac{a}{\sqrt{3}}$

$$\therefore \text{Area of circumcircle} = \pi r^2 = \pi \times \left(\frac{a}{\sqrt{3}} \right)^2 = \frac{\pi}{3} a^2$$



Area of square whose side is equal to that of equilateral triangle $= a^2$.

$$\therefore \text{Required ratio} = \frac{\frac{\pi}{3} a^2}{a^2} = \frac{\pi}{3}$$

NOTE A circumcircle always passes through the vertices of the inscribed figure (say triangle).

- 33** Best way is to go through option. Given that height of room = 10 m.

$$\text{Volume of room} = 25 \times 400 = 10000 \text{ m}^3$$

$$\text{and Surface area of walls} = 2h(l + b) = 1300 \text{ m}^2$$

Now, consider option (c) and verify it.

$$\text{where } l = 40 \text{ and } b = 25$$

- 34** Let the each side of equilateral triangle be a , then its perimeter $= 3a$

$$\text{Again, } 2(l + b) = 3a$$

$$\Rightarrow l + b = \frac{3}{2}a \Rightarrow a + b = \frac{3a}{2}$$

$$AB = l = a$$

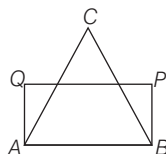
(for the rectangle)

$$\Rightarrow b = \frac{a}{2}$$

$$\therefore \text{Area of rectangle} = a \times \frac{a}{2} = \frac{a^2}{2}$$

$$\text{and Area of triangle} = \frac{\sqrt{3}}{4} \times a^2$$

$$\therefore \text{Required ratio} = \frac{a^2/2}{\sqrt{3}a^2/4} = \frac{2}{\sqrt{3}}$$



- 35** Since, b , l and $2(l + b)$ are in GP, therefore

$$\frac{l}{b} = \frac{2(l + b)}{l}$$

$$\text{Suppose } \frac{l}{b} = x,$$

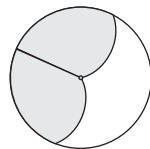
$$\text{then } x = 2\left(1 + \frac{1}{x}\right) \Rightarrow x^2 - 2x - 2 = 0$$

$$\Rightarrow x = \sqrt{3} + 1 = \frac{l}{b}$$

- 36** Non-polished area $= 4\pi(r)^2$

$$\text{Polished area} = 4 \times \left(2 \times \frac{\pi r^2}{2}\right) = 4\pi r^2$$

In the adjoining figure, one of the four parts of the sphere is shown (To understand it properly, take an apple and cut it in the four parts one across horizontal and another cut make vertical to it then you will notice that in a piece there are 2 semicircles.) Therefore, required ratio $= 1 : 1$.



- 37** Number of cubes $= \frac{85 \times 17 \times 5.1}{1.7 \times 1.7 \times 1.7} = 1500$

(1.7 is the HCF of 85, 17 and 5.1)

$$\text{Area of each cube} = 6 \times (1.7)^2$$

$$\therefore \text{Area of all the 1500 cubes} = 1500 \times 6 \times (1.7)^2 = 26010 \text{ cm}^2$$

- 38** Area of large cube $= 6 \times (5)^2 = 150 \text{ (unit)}^2$

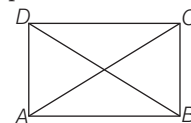
$$\text{Area of cuboid} = 2(1 \times 1 + 1 \times 125 + 125 \times 1) = 502 \text{ (unit)}^2$$

$$\text{Therefore, increase in surface area} = \frac{(502 - 150)}{150} \times 100 = 234 \frac{2}{3} \%$$

- 39** $AO = BO = 13 \text{ cm}$

$$\Rightarrow AC = BD = 26 \text{ cm} \quad (\because O \text{ is the point of bisector})$$

Now, since the diagonals are equal, it means the given figure is actually a rectangle.



$$\therefore BC^2 = AC^2 - AB^2$$

$$BC^2 = (26)^2 - (24)^2$$

$$BC^2 = 100$$

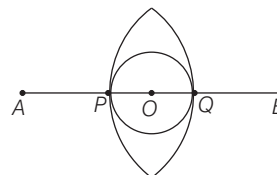
$$\Rightarrow BC = 10 \text{ cm}$$

- 40** $AB = 16 \text{ cm}$

$$AQ = 10 \text{ cm}$$

$$\text{and } AO = \frac{AB}{2} = 8 \text{ cm}$$

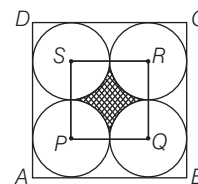
$$\therefore OQ = AQ - AO = 10 - 8 = 2 \text{ cm}$$



$OQ (= OP)$ is the radius of smaller enclosed circle between two arcs.

$$\therefore \text{Area of circle with centre } O \text{ is } \pi \times (2)^2 = 4\pi \text{ cm}^2.$$

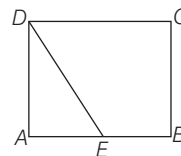
- 41** Area of the shaded region



$$= (\text{Area of square PQRS} - 4 \times \text{area of each quadrant of circles})$$

$$= \left[(2)^2 - 4 \times \frac{1}{4} \pi \times (1)^2 \right] = (4 - \pi) \text{ cm}^2$$

- 42** Let each side of the square be a , then



$$AE = AB - BE = (a - 17)$$

$$\begin{aligned}\therefore \text{Area of triangle} &= \frac{1}{2} \times AD \times AE \\ &= \frac{1}{2} a \times (a - 17) = 84\end{aligned}$$

$$\Rightarrow a^2 - 17a - 168 = 0$$

$$\Rightarrow a = 24 \text{ cm}$$

$$\therefore \text{Area of square} = (24)^2 = 576 \text{ cm}^2$$

- 43** The solid with the least number of sides will have maximum surface area. So, tetrahedron will have maximum surface area. Notice that in a sphere there are infinite number of sides with the least possible length. So, the surface area of sphere will be the least.

$$\begin{aligned}\frac{l}{b} &= \frac{(l+b)}{l} \\ \Rightarrow l^2 &= b(l+b) = lb + b^2 \quad \dots(A) \\ \Rightarrow l^2 - b^2 &= lb \quad \dots(i) \\ \Rightarrow (l+b)(l-b) &= lb \quad \dots(ii) \\ \text{and } \frac{(l+b)}{l} &= \frac{b}{(l-b)} \quad \dots(iii)\end{aligned}$$

Therefore, statements (ii) and (iii) are true from equation (A)

$$\begin{aligned}\frac{l^2}{b^2} &= \frac{bl + b^2}{b^2} \\ \Rightarrow \frac{l^2}{b^2} &= 1 + \frac{l}{b} \\ \Rightarrow \frac{l}{b} &= \frac{l^2}{b^2} - 1\end{aligned}$$

Hence, statement (i) is wrong.

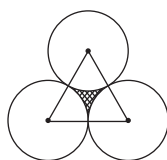
45 Area of triangle = $\frac{\sqrt{3}}{4} \times (2)^2 = \sqrt{3} \text{ cm}^2$

Area of 3 circles enclosed by the triangle

$$= 3 \times \pi \times (1)^2 \times \frac{60}{360} = \frac{\pi}{2} \text{ cm}^2$$

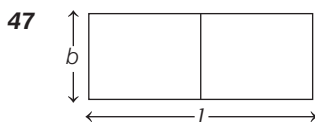
$\therefore$ Area of shaded region

$$= \sqrt{3} - \frac{\pi}{2} = \frac{(2\sqrt{3} - \pi)}{2} \text{ cm}^2$$



46 Diameter of cube = 3 cm

$$\therefore \text{Number of cubes} = \frac{9 \times 9 \times 9}{3 \times 3 \times 3} = 27 \text{ cubes}$$



$$l \times b = 20000$$

$$\Rightarrow \frac{l}{2} \times b = 10000, \text{ Area of Kaurav's land}$$

For the given area, a square gives the minimum perimeter.

$$\therefore \frac{l}{2} = b$$

$$\Rightarrow b \times b = 10000$$

$$\Rightarrow b = 100 \text{ m}$$

$$\therefore \text{Perimeter of Kaurav's land} = 4 \times 100 = 400 \text{ m}$$

$$\therefore \text{Expenses} = 2 \times 400 = ₹ 800$$

- 48** Ratio of height

$$\frac{h_1}{h_2} = \frac{9}{12} = \frac{3}{4}$$

$$\therefore \text{Ratio of volumes} = \frac{(3)^3}{(4)^3} = \frac{27}{64}$$

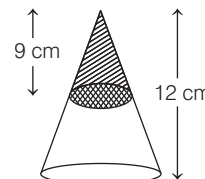
Hence, the volume of smaller cone

$$= 27x$$

and the volume of larger (actual) cone = $64x$

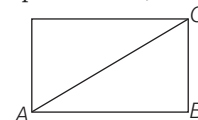
$$\therefore \text{the volume of frustum} = 64x - 27x = 37x$$

$$\therefore \text{Required ratio} = \frac{64x}{37x} = 64 : 37$$



- 49** $AB = 45 \text{ km}$, $AC = 53 \text{ km}$

(Since distance = speed $\times$ time)



$$\begin{aligned}\therefore BC &= \sqrt{AC^2 - AB^2} \\ BC &= \sqrt{(53)^2 - (45)^2}\end{aligned}$$

$$\text{or } BC = 28 \text{ km}$$

$$\therefore \text{Area of field} = AB \times BC = 45 \times 28 = 1260 \text{ km}^2$$

- 50**

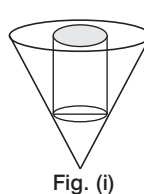


Fig. (i)

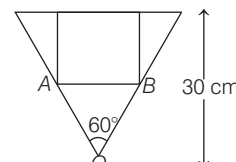


Fig. (ii)

$$\begin{aligned}\angle AOB &= 60^\circ \\ AB &= 8\sqrt{3} \text{ cm} \\ OC &= 30 \text{ cm} \\ \frac{OM}{MB} &= \tan 60^\circ = \sqrt{3} \\ OM &= \sqrt{3}MB \\ OM &= 12 \text{ cm}\end{aligned}$$

Now,

$$\frac{OM}{MB} = \frac{OC}{CD}$$

(Triangles OMB and OCD are similar)

$$\therefore CD = 10\sqrt{3} \text{ cm (which is the radius of cone)}$$

$$\therefore \text{Volume of cone} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi \times (10\sqrt{3})^2 \times 30$$

$$= 3000\pi \text{ cm}^3$$

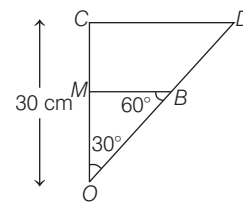


Fig. (iii)

- 51 Surface area of one cube = $6a^2$

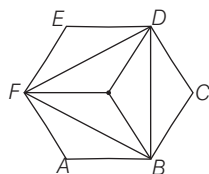
When 6 cubes are fixed on the 6 faces of a cube then only 5 faces of a cube are visible of each cube. Since, the central cube is completely covered. So, the only 6 cubes are visible each with 5 faces. Hence, the total surface area of this solid = $5a^2 \times 6$.

$$\therefore \text{Required ratio} = \frac{5a^2 \times 6}{6a^2} = \frac{5}{1} \Rightarrow 5:1$$

- 52 Ratio of areas = (Ratio of diagonals)²

$$= \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

- 53 You should know that



$$\frac{\text{Area of } \triangle BDF}{\text{Area of hexagon}} = \frac{1}{2}$$

Actually there is a perfect symmetry.

$$\therefore \text{Area of hexagon} = \frac{3\sqrt{3}}{2} \times (6)^2 = 54\sqrt{3} \text{ cm}^2$$

$$\therefore \text{Area of } \triangle BDF = 27\sqrt{3} \text{ cm}^2$$

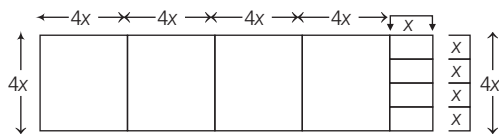
Alternatively Subtract the areas of three triangles DEF, BAF, BCD from the area of hexagon.

$$\text{Area of } \triangle DEF = \frac{1}{2} \times 6 \times 6 \sin 120^\circ = 9\sqrt{3} \text{ cm}^2$$

$$\therefore \text{Area of all the three triangles} = 27\sqrt{3} \text{ cm}^2$$

$$\text{Therefore, required area of } \triangle BDF = 54\sqrt{3} - 27\sqrt{3} = 27\sqrt{3} \text{ cm}^2$$

Solutions (for Q. Nos. 54 and 55)



Total length of plot = $17x$

and total breadth of the plot = $4x$

$$\therefore \text{Area of plot} = 17x \times 4x = 9792$$

$$\Rightarrow x = 12$$

$$\therefore l = 17 \times 12 = 204 \text{ m}$$

$$\text{and } b = 4 \times 12 = 48 \text{ m}$$

54 $\frac{4x \times 4x}{x \times x} = \frac{16}{1}$

$$\Rightarrow 16:1$$

55 $l = 204 \text{ m}$ and $b = 48 \text{ m}$

- 56 Consider the following Figure.

Let $RS = b$ and $QS = h$.

$$\text{Then, } RQ^2 = 51^2 = b^2 + h^2 \dots (i)$$

$$\text{and } RP^2 = RS^2 + (PQ + QS)^2$$

$$\Rightarrow 74^2 = b^2 + (25 + h)^2 \dots (ii)$$

Solving the above two equations we get $h = 45$ and $b = 24$.

Now, volume of cone (V_1) when $\triangle PRS$ is rotated about PS

$$= \frac{1}{3} \times \pi \times 24^2 \times 70 = 13440 \pi \text{ cubic cm}$$

And volume of cone (V_2) when $\triangle QRS$ is rotated about QS

$$= \frac{1}{3} \times \pi \times 24^2 \times 45$$

$$= 8640 \pi \text{ cubic cm}$$

Therefore the required volume of the solid (V_3)

$$= V_1 - V_2 = 4800 \pi \text{ cubic cm}$$

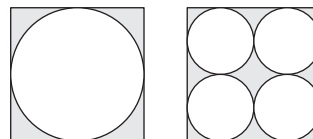
Hence, choice (b) is correct.

- 57 Let $4a$ be the side of each square.

Then the radius of the circle inscribed in the squares

$$ABCD = 2a$$

$$\text{So the area of this circle} = \pi (2a)^2 = 4\pi a^2$$



And the radius of each circle inscribed in the square PQRS

$$= a$$

$$\text{So the total area of four circles} = 4 \times \pi (a)^2 = 4\pi a^2$$

It shows that the area occupied by the circle(s) in each square is same. Therefore the required ratio = $1:1$

Hence, choice (d) is correct.

- 58 If $2a$ be the side of any square, the area of square would be $4a^2$.

And, the radius of the inscribed circle would be a , so the area of the circle would be πa^2 .

Therefore the ratio of area of each circle to the area of concerned square = $\pi:4$

Since the total area of all the smaller squares = total area of the Original Square.

$$\therefore \frac{\text{Area of all the circles}}{\text{Area of original square}} = \frac{\pi}{4}$$

Hence, choice (c) is correct.

- 59 After you cut out any cube or cuboid from the corner of a solid cube or cuboid, the surface area remains unchanged, which you can easily understand by looking at the diagram. So, the required area is 216 cubic ft.

Hence, choice (a) is correct.

- 60 Since $\angle DPC = 90^\circ$,

$$\text{Area of } \triangle DPC = \frac{1}{2} \times DP \times PC = 189$$

Now drop a perpendicular PQ from P on DC . Then you can see that the area of $\triangle APD = \triangle PQD$ and area of $\triangle PBC = \triangle PQC$.

Therefore, area of rectangle $ABCD$
 $= 2 \times \text{area of } \triangle DPC = 378 \text{ mm}^2$

Hence, choice (a) is the correct one.

☞ **Alternatively** Area of rectangle $ABCD = DC \times BC$
 $= DC \times PQ$
 $= DP \times PC$
 $= 18 \times 21 = 378$

Hint In right angle triangle DPC ,
 $DC \times PQ = DP \times PC = \text{area of } \triangle DPC$.
Hence, choice (a) is the correct one.

- 61 The shortest path that the ant will follow to reach the farthest corner from one particular corner is shown below in the diagram.

Thus the required length
 $= \sqrt{(12)^2 + (12)^2} = 12\sqrt{2}$

Hence, choice (c) is the correct one.

- 62 Let the longer side be x and the shorter side be y , the semiperimeter of the parallelogram would be $x + y$.

$$\begin{aligned} \therefore \frac{x}{y} &= \frac{x+y}{x} \Rightarrow \frac{x}{y} = \frac{\frac{x+y}{x}}{\frac{x}{x}} \\ \Rightarrow \frac{x}{y} &= \frac{1 + \left(\frac{y}{x}\right)}{1} \Rightarrow \frac{x}{y} = 1 + \frac{y}{x} \\ \Rightarrow \frac{x}{y} - \frac{y}{x} - 1 &= 0 \\ \Rightarrow k - \frac{1}{k} - 1 &= 0 \quad \left(\text{assuming } \frac{x}{y} = k\right) \\ \Rightarrow k^2 - k - 1 &= 0 \\ \Rightarrow k &= \frac{1 + \sqrt{5}}{2} \Rightarrow \frac{x}{y} = \frac{1 + \sqrt{5}}{2} \\ \therefore y &= 2 \text{ cm} \therefore x = 1 + \sqrt{5} \text{ cm.} \end{aligned}$$

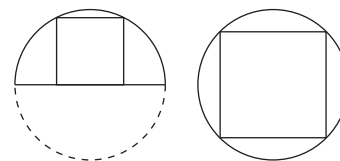
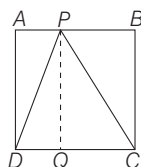
But you know that the maximum area can be obtained only when parallelogram is a rectangle. Therefore, area of the rectangle $= x \times y = (1 + \sqrt{5}) \times 2 = 2(1 + \sqrt{5})$

Hence, choice (b) is the correct one.

- 63 Let each side of the square inscribed by the semicircle is $2x$, then the radius of semicircle would be $\sqrt{5}x$.

It means the diameter of the circle would be $2\sqrt{5}x$.

It means the diagonal of the square inscribed by the circle is $2\sqrt{5}x$.

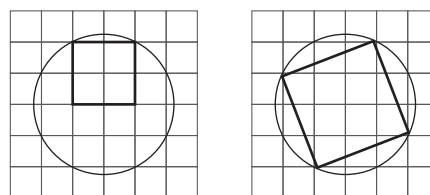


It means each side of the square inscribed by the circle is
 $= \frac{2\sqrt{5}x}{\sqrt{2}} = \sqrt{10}x$

Therefore, the required ratio $= \frac{(2x)^2}{(\sqrt{10}x)^2} = \frac{2}{5}$

Hence, choice (a) is the correct one.

☞ **Alternatively** Look carefully at the following diagrams. In the first diagram, the pertinent square covers 4 units of smaller squares. In the second diagram, the pertinent square covers 10 units of smaller squares.



Therefore, the required ratio $= 4/10 = \frac{2}{5}$.

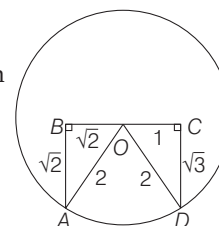
- 64 In $\triangle ABO$, $AB = \sqrt{OA^2 - OB^2} = \sqrt{2}$

Since $AB = OB$, so $\triangle ABO$ is an isosceles triangle.

Therefore, $\angle BAO = \angle BOA = 45^\circ$

In $\triangle OCD$,

$$CD = \sqrt{OD^2 - OC^2} = \sqrt{3}$$



Since, the three sides of the triangle are in the ratio $1 : \sqrt{3} : 2$, it implies that $\angle CDO = 30^\circ$ and $\angle COD = 60^\circ$

Therefore, $\angle AOD = 180 - (\angle BOA + \angle COD) = 75^\circ$

Now, area of the shaded region = area of $\triangle ABO$ + area of sector AOD + area of $\triangle OCD$

Area of $\triangle ABO = \frac{1}{2} \times \sqrt{2} \times \sqrt{2} = 1 \text{ sq cm}$

Area of sector $AOD = \pi (2)^2 \times \frac{75}{360} = \frac{5\pi}{6} \text{ sq cm}$

Area of $\triangle OCD = \frac{1}{2} \times 1 \times \sqrt{3} = \frac{\sqrt{3}}{2} \text{ sq cm}$

Therefore, the required shaded area $= 1 + \frac{5\pi}{6} + \frac{\sqrt{3}}{2} \text{ sq cm.}$

Hence, choice (d) is the correct one.

- 65 Since area of the circle = 1

$$\therefore \pi r^2 = 1 \Rightarrow r = 1/\sqrt{\pi}$$

$$\therefore \text{Each side of hexagon} = 2/\sqrt{\pi}$$

$$\therefore \text{Area of the hexagon} = \frac{3\sqrt{3}}{2} \left(\frac{2}{\sqrt{\pi}} \right)^2 = \frac{6\sqrt{3}}{\pi} \text{ sq cm.}$$

Hence, choice (c) is the correct one.

- 66 Area of the semicircle with a as its diameter $= \frac{\pi a^2}{4}$
 Area of the semicircle with b as its diameter $= \frac{\pi b^2}{4}$
 Area of the semicircle with c as its diameter $= \frac{\pi c^2}{4}$
 Area of right triangle $= \frac{ab}{2}$

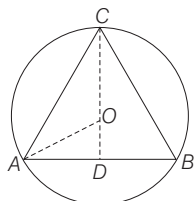
Therefore area of the shaded region

$$= \left(\frac{\pi a^2}{4} + \frac{\pi b^2}{4} + \frac{ab}{2} \right) - \frac{\pi c^2}{4} = \frac{\pi}{4} (a^2 + b^2 - c^2) + \frac{ab}{2}$$

$$= \frac{\pi}{4} (c^2 - c^2) + \frac{ab}{2} = \frac{ab}{2}$$

Hence, choice (a) is the correct one.

- 67 Let r be the circumradius of the circumcircle and O be the centre of the circle, where CD is the perpendicular bisector of AB , as $\triangle ABC$ is an isosceles triangle. Therefore, $AD = BD = 8$
 Now, since $AO = OC = r$
 Therefore, $OD = 16 - r$



Now, we have $AO^2 = AD^2 + OD^2$
 $\Rightarrow r^2 = 8^2 + (16 - r)^2 \Rightarrow r = 10$ cm
 $\therefore$ Area of circle $= 100\pi$ cm²

Hence, choice (c) is the correct one.

- 68 Let the length of each side of the square be a unit and the breadth of the road 1 unit.
 Then, we have $AE = AG = \frac{1}{\sqrt{2}} (EG) = \frac{1}{\sqrt{2}}$
 Therefore, $EF = AB - (AE + BF) = a - \sqrt{2}$

So, we have $EP = FP = \frac{1}{\sqrt{2}} (EF) = \frac{(a - \sqrt{2})}{\sqrt{2}}$

Therefore, total area of roads (shaded region)
 $= 4$ (area of $EP SG$) $+ 4$ (area of $\triangle AEG$) $+ 4$ (area of $PQRS$)

$$= 4 \left(\frac{(a - \sqrt{2})^2}{2} \right) + (1) + (1) = 2(\sqrt{2}a - 1)$$

And, the area of the square $= a^2$

But, it is given that $a^2 = 2[2(\sqrt{2}a - 1)]$

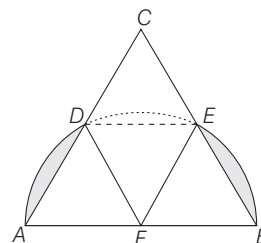
$$\Rightarrow a^2 - 4\sqrt{2}a + 4 = 0$$

$$\Rightarrow a = 2\sqrt{2} + 2$$

Please note that the other value of $a = (2\sqrt{2} - 2) < 1$. Since the length of the square cannot be less than the breadth of the road, so it is an inadmissible value.

Hence choice (c) is the correct one.

- 69 Point F is the centre of the semicircle. Now connect the points as shown in the diagram. Now, you see that there are four equilateral triangles of same height.



Therefore, $AF = DF = EF = BF = 1$ cm = radius of circle.

Area of shaded region $= \frac{2}{3}$ Area of semicircle - area of quadrilateral $ADEB$

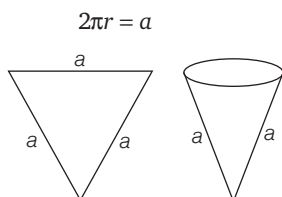
$$= \frac{2}{3} \left[\left(\frac{1}{2} \pi \times 1^2 \right) - \left(3 \times \frac{\sqrt{3}}{4} \times 1^2 \right) \right]$$

$$= \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) \text{ cm}^2$$

Hence, choice (b) is correct.

Level 02 Higher Level Exercise

1



Also, slant height (l) $= a$ $\therefore r = \frac{a}{2\pi}$

2 $\therefore$

$$l^2 = h^2 + r^2$$

$$\Rightarrow h^2 = l^2 - r^2 = a^2 - \left(\frac{a}{2\pi} \right)^2$$

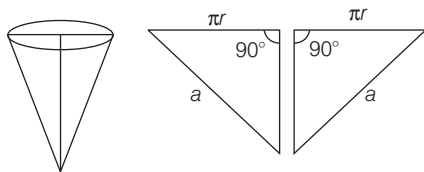
$$h^2 = a^2 \left[\frac{4\pi^2 - 1}{4\pi^2} \right]$$

$$\therefore h = \frac{a}{2\pi} \sqrt{4\pi^2 - 1}$$

$$\therefore \text{Volume} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \times \frac{a^2}{4\pi^2} \times \frac{a}{2\pi} \sqrt{4\pi^2 - 1}$$

$$= \frac{a^3}{24\pi^2} \sqrt{4\pi^2 - 1}$$

- 3 It will be in the form of a right angled triangle.



$$4 \quad 2\pi r(r+h) = 1540 \text{ cm}^2$$

$$\text{and} \quad (r+h) = 35 \text{ cm}$$

$$\therefore \quad 2\pi r = \frac{1540}{35} = 44 \text{ cm}$$

$$5 \quad \text{Total volume} = \pi r^2 h_1 + \frac{1}{3} \pi r^2 h_2$$

$$= \pi r^2 \left[h_1 + \frac{h_2}{3} \right]$$

$$= \frac{22}{7} \times (21)^2 \left[80 + \frac{16}{3} \right]$$

$$= \frac{22}{7} \times 441 \times \frac{256}{3}$$

$$\text{Weight} = \frac{22}{7} \times 441 \times \frac{256}{3} \times \frac{8.45}{1000} = 999.39 \text{ kg}$$

- 6 ABCD is a square, each side of square is 'a'.

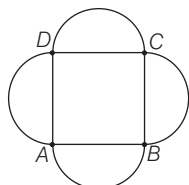


Fig. (i)

In figure (ii), $\angle DOC = 120^\circ$
and $\angle ODC = \angle OCD = 30^\circ$

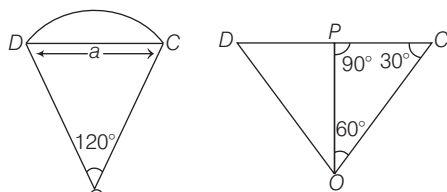


Fig. (ii)

Fig. (iii)

In figure (iii),

$$\frac{PC}{OC} = \sin 60^\circ$$

$$\frac{a/2}{OC} = \frac{\sqrt{3}}{2} \Rightarrow OC = \frac{a}{\sqrt{3}}$$

$\Rightarrow$ radius of the arc 'CD'.

$$\therefore \text{Area of triangle } OCD = \frac{1}{2} \times CD \times OP$$

$$= \frac{1}{2} \times a \times \frac{a}{2\sqrt{3}} = \frac{a^2}{4\sqrt{3}}$$

$$\left(\because \frac{OP}{PC} = \tan 30^\circ \text{ and } \tan 30^\circ = \frac{1}{\sqrt{3}} \right)$$

and area of sector COD (figure ii)

$$= \pi r^2 \frac{120}{360}$$

$$= \pi \times \left(\frac{a}{\sqrt{3}} \right)^2 \times \frac{1}{3} = \frac{\pi a^2}{9}$$

$\therefore$ Area of segment = (Area of sector - Area of triangle)

$$= \frac{\pi a^2}{9} - \frac{a^2}{4\sqrt{3}}$$

$$\text{Total area of all the four segments} = 4 \left(\frac{\pi a^2}{9} - \frac{a^2}{4\sqrt{3}} \right)$$

$$\text{and the total area of whole figure} = a^2 + 4 \left(\frac{\pi a^2}{9} - \frac{a^2}{4\sqrt{3}} \right)$$

$$7 \quad 2(l+b) = 26 \Rightarrow l+b = 13$$

$$12+1=13 \quad 11+2=13$$

$$10+3=13 \quad 9+4=13$$

$$8+5=13 \quad 7+6=13$$

Since, $l > b$, therefore, there are only 6 integral values of the length viz., 7, 8, 9, 10, 11 and 12.

$$8 \quad \text{Total surface area} = 2\pi R^2 + 2\pi r^2 + (\pi R^2 - \pi r^2)$$

$$= 3\pi R^2 + \pi r^2$$

$$= \pi (3R^2 + r^2)$$

$$\Rightarrow \quad 1436 \frac{2}{7} = \pi [3 \times (12)^2 + r^2]$$

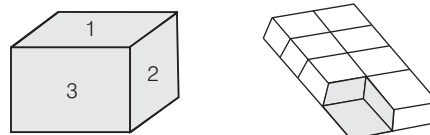
$$\Rightarrow \quad \frac{10054}{7} \times \frac{1}{\pi} = 432 + r^2 \Rightarrow r = 5 \text{ cm}$$

$$\therefore \text{Internal volume of hemisphere} = \frac{2}{3} \pi (R^3 - r^3)$$

$$= \frac{2}{3} \pi [(12)^3 - (5)^3]$$

$$= 3358 \frac{2}{3} \text{ cm}^3$$

- 9 Since, there are 3 faces which are visible in a corner cube. When the cube of a corner is removed then the 3 faces of other cubes will be visible from outside. So, there will not be any change in the surface area of this solid figure.



$$10 \quad \text{Number of spheres} = \frac{\frac{4}{3} \pi \left(\frac{15}{2} \right)^3}{\frac{4}{3} \pi \left(\frac{3}{2} \right)^3} = 125 \text{ spheres}$$

$$\text{Surface area of a large sphere} = 4\pi \times \left(\frac{15}{2} \right)^2$$

and surface area of a small sphere = $4\pi \left(\frac{3}{2}\right)^2$

and total surface area of all the smaller spheres

$$= 125 \times 4\pi \left(\frac{3}{2}\right)^2$$

% change in area

$$= \left[\frac{500\pi \left(\frac{3}{2}\right)^2 - 4\pi \left(\frac{15}{2}\right)^2}{4\pi \left(\frac{15}{2}\right)^2} \times 100 \right] = 400\%$$

④ **Alternatively** Surface area of larger sphere = $25x$

Surface area of smaller sphere = x

∴ The ratio of radii is 5 : 1.

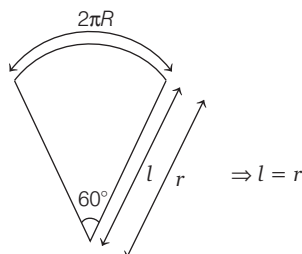
Therefore, ratio in surface areas = 25 : 1

Now, since there are 125 smaller spheres.

∴ Total surface area of smaller spheres = $125x$

∴ % change in area = $\frac{125x - 25x}{25x} \times 100 = 400\%$

11 Let the radius of cone be R and radius of sector = r ,



then the slant height of cone (l) = r

$$\text{and } 2\pi R = 2\pi r \times \frac{60}{360}$$

$$\Rightarrow R = \frac{r}{6} = \frac{14}{6} = \frac{7}{3} \text{ cm}$$

∴ Total surface area = $\pi r (l + r)$

$$= \frac{22}{7} \times \frac{7}{3} \left(14 + \frac{7}{3}\right) = 119.78 \text{ cm}^2$$

12 Between 26 poles, total length is $(26 - 1) \times 4 = 100$ m
It means the length of each side of a square field is 100 m.
∴ Area of field = $(100)^2 = 10,000 \text{ m}^2 = 1$ hectare

13 It is clear that length of the lawn is 2 m more than the breadth of lawn.
To solve this problem quickly, go through options. Let us take option (c).

$$l = 10 \text{ m} \Rightarrow b = 8 \text{ m}$$

$$\begin{aligned} \text{Area of path} &= (l + b + 2w) 2w \\ &= (10 + 8 + 4) 4 = 88 \text{ m}^2 \end{aligned}$$

and area of lawn = $10 \times 8 = 80 \text{ m}^2$

Reduced area of lawn = $8 \times 8 = 64 \text{ m}^2$

∴ New area of path = $88 + (80 - 64) = 104 \text{ m}^2$

∴ Ratio of areas of path = $\frac{104}{88} = \frac{13}{11}$

Hence, option (c) is correct.

④ **Alternatively** Let the breadth of the lawn be ' b ' then the length will be $(b + 2)$.

∴ Area of path = $(l + 4)(b + 4) - lb$

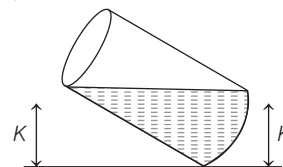
$$= (b + 6)(b + 4) - (b + 2)b = 8b + 24$$

and the new area of path = $(l + 4)(b + 4) - b \times b$

$$\begin{aligned} &= (b + 6)(b + 4) - b^2 \\ &= 10b + 24 \end{aligned}$$

$$\therefore \frac{(10b + 24)}{(8b + 24)} = \frac{13}{11} \Rightarrow b = 8 \text{ m} \quad \therefore l = 10 \text{ m}$$

14 From the figure you can see that just half of the liquid has been flown off and half the liquid is remained in the cylindrical jar.



Thus it is clear that the capacity (or volume) of the cylinder = $2 \times 2.1 = 4.2 \text{ L}$

15 When the height and base of the cone are same as that of cylinder, then the volume of cone is $\frac{1}{3}$ that of the cylinder.

Thus the capacity of cone = $\frac{1}{3} \times 4.2 = 1.4 \text{ l}$

Thus the remaining volume = $2.1 - 1.4 = 0.7 \text{ l}$

∴ The required ratio = $\frac{0.7}{4.2} = \frac{1}{6}$

$$16 \quad AC = \sqrt{(30)^2 + (16)^2}$$

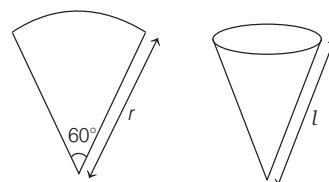
$$AC = 34 \text{ m}$$

But since elephant is itself 4 m long. So he has to travel only

$$(34 - 4) = 30 \text{ m.}$$

∴ the speed of elephant = $\frac{30}{15} = 2 \text{ m/s}$

$$17 \quad \text{Arc of sector} = \frac{2\pi r 60}{360} = \frac{2\pi r}{6}$$



This arc of sector will be equal to the perimeter of cone. Let the radius of cone be R , then $2\pi R = \frac{2\pi r}{6} \Rightarrow R = \frac{r}{6}$

Further the radius of sector will be equal to the slant height of cone.

$$\begin{aligned} \therefore l &= r \\ \text{Now, since } l^2 &= h^2 + R^2 \Rightarrow h = \sqrt{l^2 - R^2} \\ h &= \sqrt{r^2 - \left(\frac{r}{6}\right)^2} = \frac{\sqrt{35}}{6} r \end{aligned}$$

- 18** The diagonal of cube will be equal to the diameter of sphere.

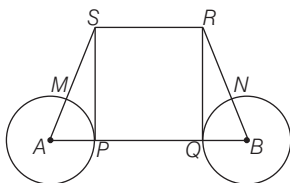
$$\therefore \text{Volume of sphere} = \frac{4}{3} \pi \left(\frac{d}{2}\right)^3 = \frac{\pi d^3}{6}$$

$$\text{and each side of cube} = a = \frac{d}{\sqrt{3}}$$

$$\therefore \text{Volume of cube} = a^3 = \frac{d^3}{3\sqrt{3}}$$

$$\therefore \text{Remaining volume} = \frac{\pi d^3}{6} - \frac{d^3}{3\sqrt{3}} = \frac{d^3}{3} \left(\frac{\pi}{2} - \frac{1}{\sqrt{3}} \right)$$

- 19** Let $AP = x$, then $AM = x$ and $MS = x$



$$\therefore AS = AM + MS$$

$$AS = 2x$$

$$\therefore PS = \sqrt{AS^2 - AP^2}$$

$$PS = \sqrt{3}x$$

$$\therefore \text{Area of square}$$

$$PQRS = (\sqrt{3}x)^2 = 3x^2$$

$$\text{Area of circle} = \pi r^2 = \pi \times x^2 = \pi x^2$$

$$\text{Area of both the circles} = 2\pi x^2$$

$$\therefore \text{Required ratio} = \frac{2\pi x^2}{3x^2} = \frac{2\pi}{3}$$

- 20** Let the length of rectangle be 'l' and breadth be 'b', then

$$2(l + b) = 12$$

$$\Rightarrow l + b = 6 \text{ cm}$$

and area of larger equilateral triangle

$$= \frac{\sqrt{3}}{4} l^2$$

$$\text{Similarly area of smaller equilateral triangle} = \frac{\sqrt{3}}{4} b^2$$

$$\therefore \text{Total area of all the 4 triangles}$$

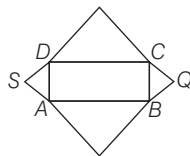
$$= 2 \times \frac{\sqrt{3}}{4} (l^2 + b^2) = 10\sqrt{3}$$

$$\Rightarrow l^2 + b^2 = 20$$

$$\therefore (l + b)^2 = l^2 + b^2 + 2lb$$

$$\Rightarrow 36 = 20 + 2lb$$

$$\Rightarrow lb = 8$$



$$\therefore (l - b)^2 = l^2 + b^2 - 2lb = 20 - 16 \text{ or } (l - b)^2 = 4$$

$$\Rightarrow l - b = 2$$

$$\therefore l + b = 6 \text{ and } l - b = 2$$

$$\therefore l = 4 \text{ and } b = 2$$

$$\therefore \text{Area of rectangle} = 4 \times 2 = 8 \text{ cm}^2$$

$$\therefore \text{Total area of the figure} = 8 + 10\sqrt{3} = 2(4 + 5\sqrt{3}) \text{ cm}^2$$

- 21** Area of each square = 16 cm^2

$$\text{Area of quadrant } ADMB = \frac{1}{4} \pi \times (4)^2 = 4\pi$$

and radius of smaller quadrant

$$\begin{aligned} CPMQ &= CM = AC - MA \\ &= 4\sqrt{2} - 4 = 4(\sqrt{2} - 1) \end{aligned}$$

$$\therefore \text{Area of smaller quadrant} = \frac{1}{4} \pi [4(\sqrt{2} - 1)]^2 = 4\pi(3 - 2\sqrt{2})$$

Area of shaded region inside the square ABCD

$$= 16 - [4\pi + 4\pi(3 - 2\sqrt{2})]$$

$$= 16 - [4\pi(1 + 3 - 2\sqrt{2})]$$

$$= 16 - [4\pi(4 - 2\sqrt{2})]$$

$$= 8[2 - 2\pi + \sqrt{2}\pi]$$

Now, area of quadrants = $AEG + EFG = 2AEG$

$$= 2 \times \frac{1}{4} \pi (4)^2 = 8\pi$$

$\therefore$ Area of shaded region inside the square EAGF

$$= 8\pi - 16 = 8(\pi - 2)$$

$$\begin{aligned} \therefore \text{Required ratio} &= \frac{8(2 - 2\pi + \sqrt{2}\pi)}{8(\pi - 2)} \\ &= \frac{[2 + \pi(\sqrt{2} - 2)]}{(\pi - 2)} \end{aligned}$$

- 22** Given that

$$\frac{AB}{BC} = \frac{AD}{DF}$$

Also

$$BE = BC$$

Let

$$AD = 1 \text{ and } AE = x$$

$$\therefore \frac{AE}{EF} = \frac{AE}{AD} = \frac{AE}{BC} = x$$

$$\therefore \frac{AE}{EF} = \frac{AD}{AB} \quad \left(\because AD = BC = BE \right. \\ \left. \text{and } AB = AE - BE \right)$$

$$\frac{x}{1} = \frac{1}{x - 1}$$

$$\Rightarrow x^2 - x - 1 = 0 \Rightarrow x = \frac{(1 \pm \sqrt{5})}{2}$$

$$\therefore x = \frac{(1 + \sqrt{5})}{2}$$

Since ratio of two sides can never be negative.

☞ **Alternatively** Since ratio of two side can never be negative therefore only option (c) is correct.

Solutions (for Q. Nos. 23 to 25)

$$AB = 4$$

$$\therefore AO = AC = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

$\therefore$ Area of circle

$$ABCD = \pi \times (2 \times \sqrt{2})^2 = 8\pi$$

Area of region 2 (only left part)

$$= \frac{\text{Area of circle} - \text{Area of square}}{4}$$

$$= \frac{8\pi - 16}{4} = (2\pi - 4)$$

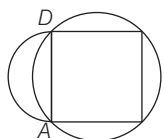
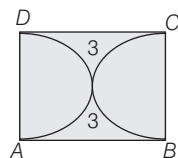
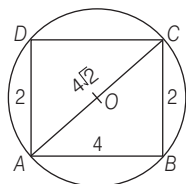
Area of region 3 = Area of square
- 2 (Area of semicircle)

$$= 16 - 2 \left(\frac{1}{2} \times \pi \times 4 \right)$$

$$= 16 - 4\pi = 4(4 - \pi) \text{ cm}^2$$

Area of region 1 = Area of semicircle AD
- Area of region 2

$$= \frac{1}{2} \pi \times (2)^2 - (2\pi - 4) = 4 \text{ cm}^2$$



23 Total area of region 1 = $2 \times 4 = 8 \text{ cm}^2$

24 Total area of region 2 = $2 \times (2\pi - 4) = 4(\pi - 2) \text{ cm}^2$

25 Total area of region 3 = $4(4 - \pi) \text{ cm}^2$

26 Total area of square = 64 cm^2

$$\therefore 4(x + y) = 64$$

$$\Rightarrow x + y = 16 \quad \dots(i)$$

Again in a semicircle

$$AOB = x + y + x = \frac{1}{2} \pi \times (4)^2$$

$$2x + y = 8\pi$$

From eqs. (i) and (ii), we get

$$x = 8\pi - 16$$

$$\text{Total area of shaded region} = 4(8\pi - 16)$$

$$= 32(\pi - 2) \text{ cm}^2$$

Alternatively Area of square - 2 (Area of semicircles)
= $2y$

$$(64 - 16\pi) = 2y$$

$$\therefore 4y = (128 - 32\pi)$$

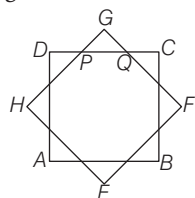
$\therefore$ Required area of shaded region

$$(4x) = (\text{Area of square} - 4y)$$

$$= 64 - (128 - 32\pi)$$

$$= 32\pi - 64 = 32(\pi - 2) \text{ cm}^2$$

27 You can see in the figure that the sides of one square is parallel to the diagonals of the other square.



Let $DP = a$, then

$$DC = DP + PQ + QC$$

$$= a + a\sqrt{2} + a$$

$$DC = a(2 + \sqrt{2})$$

$\therefore$ Area of

$$\Delta PGQ = \frac{1}{2} \times a \times a = \frac{a^2}{2}$$

$\therefore$ Area of all the triangles outside the square ABCD

$$= 4 \times \frac{a^2}{2} = 2a^2$$

But $DC = a(2 + \sqrt{2}) = 4 \text{ cm}$

$$\Rightarrow a = \frac{4}{(2 + \sqrt{2})}$$

$$\therefore 2(a)^2 = 2 \times \left(\frac{4}{2 + \sqrt{2}} \right)^2$$

$$= \frac{16}{(3 + 2\sqrt{2})} \times \frac{(3 - 2\sqrt{2})}{(3 - 2\sqrt{2})} = 16(3 - 2\sqrt{2})$$

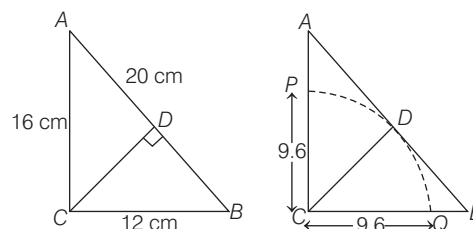
and Area of square = 16 cm^2

$$\therefore \text{Total area of the figure} = 16 + 16(3 - 2\sqrt{2})$$

$$= 16(4 - 2\sqrt{2})$$

$$= 32(2 - \sqrt{2}) \text{ cm}^2$$

28 When $l = CD$, then the volume of cone will be maximum, where l is the slant height of the cone and the largest possible angle at the vertex of cone is 90° .



$$CD = \frac{12 \times 16}{20} = 9.6 \text{ cm},$$

which is the radius of the sector.

Therefore, arc of the sector

$$= 2\pi \times 9.6 \times \frac{90}{360} = 4.8\pi$$

Let the radius of the cone be r , then

$$2\pi r = \text{arc of the sector}$$

$$2\pi r = 4.8\pi$$

$$r = 2.4$$

$\therefore$ Height of the cone (h) = $\sqrt{l^2 - r^2}$

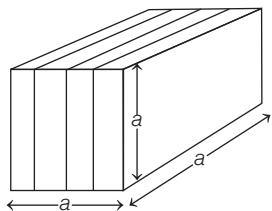
$$= \sqrt{(9.6)^2 - (2.4)^2} = 2.4 \sqrt{15} \text{ cm}$$

$$\therefore \text{Volume of the cone} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times (2.4)^2 \times 2.4 \sqrt{15}$$

$$= 56.1 \text{ cm}^3$$

- 29** To increase the value (or price of diamond) they should cut (divide) the diamond in such a way that the surface area will be maximum.



Thus, when four parts are parallel to each other.

In this way total surface area

$$= 6a^2 + 2a^2 + 2a^2 + 2a^2 = 12a^2$$

Actual surface area of cubical diamond = $6a^2$

Therefore, percentage increase in area

$$= \frac{12a^2 - 6a^2}{6a^2} \times 100 = 100\%$$

Remember that for the given volume, minimum surface area is possessed by a cube. So to maximize the area we have to increase the maximum possible difference between the edges of cuboid.

- 30** Side of square I = a

$$\text{Side of square II} = \frac{a}{\sqrt{2}}$$

$$\text{Side of square III} = \frac{a}{2}$$

$$\text{Side of square IV} = \frac{a}{2\sqrt{2}}$$

$$\text{Side of square V} = \frac{a}{4}$$

Therefore, sum of perimeters of all the squares

$$\begin{aligned} &= 4 \left(a + \frac{a}{\sqrt{2}} + \frac{a}{2} + \frac{a}{2\sqrt{2}} + \frac{a}{4} \right) \\ &= 4a \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2} + \frac{1}{2\sqrt{2}} + \frac{1}{4} \right) \\ &= 4a \left(\frac{4 + 2\sqrt{2} + 2 + \sqrt{2} + 1}{4} \right) = a(7 + 3\sqrt{2}) \end{aligned}$$

- 31** Total area of the five squares

$$\begin{aligned} &= a^2 + \left(\frac{a}{\sqrt{2}} \right)^2 + \left(\frac{a}{2} \right)^2 + \left(\frac{a}{2\sqrt{2}} \right)^2 + \left(\frac{a}{4} \right)^2 \\ &= a^2 \left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \right] \\ &= a^2 \left[\frac{16 + 8 + 4 + 2 + 1}{16} \right] = a^2 \times \frac{31}{16} = \frac{31a^2}{16} \end{aligned}$$

- 32** $(n-2)^3$

- 33** $6(n-2)^2$

- 34** $12(n-2)$

- 35** There are 8 cubes at the corners, which are always fix.

- 36** Volume of the whole body

$$V_1 = \frac{1}{3} \pi r_1^2 h_1 + \pi r_1^2 h_2$$

$$\text{but } \frac{h_1}{h_2} = \frac{2}{3}$$

$$\therefore V_1 = \pi r_1^2 \frac{11h_1}{6}$$

$$\text{and } h_3 = \frac{2}{3} (h_1 + h_2) = \frac{5h_1}{3}$$

Hence, volume of the hole (V_2) = $\pi r_2^2 h_3$

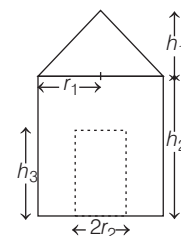
$$= \frac{5}{3} \pi r_2^2 h_1$$

$$\text{But it is given that } V_2 = \frac{V_1 - V_2}{3}$$

$$\therefore V_1 = 4V_2$$

$$\Rightarrow 4 \times \frac{5}{3} \pi r_2^2 h_1 = \pi r_1^2 \times \frac{11}{6} h_1$$

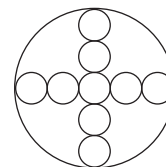
$$\Rightarrow r_2 = \sqrt{\frac{55}{8}} \text{ cm}$$



- 37** $19 \times 19 = 361$

Thus, we make equal 19 measurements each of 19° , then we get $(361 - 360) = 1^\circ$ angle at the centre. Thus, moving continuously in the similar fashion, we can get all the 360° angle i. e., 360 equal sectors of 1° .

- 38** When we open the paper after cutting it, we will find it as shown in the following figure.



Radius of the larger circle = 5 cm

$\therefore$ Area of larger circle = 25π

and the radius of each smaller circle is 1 cm.

Therefore, total area of all the 9 circles = $9 \times \pi \times (1)^2 = 9\pi$

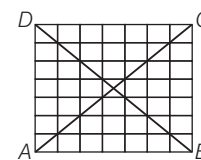
$\therefore$ Remaining area = $(25 - 9)\pi = 16\pi$

Hence, the required ratio = $25 : 16$

- 39** In the top layer we can see that total 13 cubes get a cut. So, in 7 layers total $13 \times 7 = 91$ cubes will get a cut and the remaining $(7^3 - 91) = 252$ cubes are without any cut.

Total number of pieces which are not a cube

$$= 12 \times 2 \times 7 + 4 \times 7 = 196$$



(Since 84 cubes are diagonally cut into two parts and 7 cubes which are in the centre are divided into 4 parts.)

Thus, total 196 children and teenagers will get one-one piece and 252 adults get one-one piece.

Thus total $252 + 196 = 448$ people can get a piece of cake.

NOTE It is clear that everyone get equal number of pieces but not according to the volume of pieces.

Solutions (for Q. Nos. 40 to 42) Diameter ($2R$) of the outermost circle is equal to the diagonal of larger square.

Hence, the side of square = $\frac{2R}{\sqrt{2}} = \sqrt{2}R$

Again the side of larger square is equal to the diameter of middle most circle.

Hence, the radius of mid-circle is $\frac{R}{\sqrt{2}}$.

Once again the diameter of the mid-circle is equal to the diagonal of smaller square. Hence, side of the smaller square = R . Similarly the diameter of innermost circle is equal to the side of the smaller square.

Hence, radius of the innermost circle = $\frac{R}{2}$.

40 $\frac{R}{2}$

41 Area of larger square = $(\sqrt{2}R)^2 = 2R^2$

and area of smaller square = R^2

$\therefore$ Total area of both squares = $3R^2$

42 Sum of all the circumferences = $2\pi \left(R + \frac{R}{\sqrt{2}} + \frac{R}{2} \right)$

$$= 2\pi R \left(\frac{2 + \sqrt{2} + 1}{2} \right)$$

$$= (3 + \sqrt{2}) \pi R$$

Sum of perimeters of all the squares = $4(\sqrt{2}R + R)$

$$= 4R(\sqrt{2} + 1)$$

$\therefore$ Required ratio = $\frac{(3 + \sqrt{2}) \pi R}{(\sqrt{2} + 1) 4R} = \frac{(3 + \sqrt{2}) \pi}{(\sqrt{2} + 1) 4}$

☞ Alternatively Since circumference and perimeter both has R . So R will be cancelled in the ratio of circumference to the perimeter. Thus, neither of the choices a, b and c are admissible. Hence, the only correct choice is (d).

Solutions (for Q. Nos. 43 to 45) Each side of outer (larger) hexagon is equal to the radius of circle which is R .

Now, $OC = ON = OD$
radii of the inner (smaller) circle

But $\frac{ON}{OA} = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\Rightarrow ON = \frac{\sqrt{3}}{2} OA = \frac{\sqrt{3}}{2} R$, radius of the

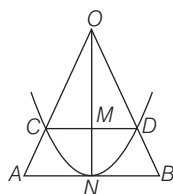
inner circle

and this is also equal to the side of the inner hexagon.

43 Sum of perimeters of both the hexagons = $6R + 6 \times \frac{\sqrt{3}}{2} R$

$$= 6R \left(1 + \frac{\sqrt{3}}{2} \right)$$

$$= 3(2 + \sqrt{3})R$$



44 $\frac{\text{Area of inner circle}}{\text{Area of outer circle}} = \frac{\pi \left[\left(\frac{\sqrt{3}}{2} R \right)^2 \right]}{\pi (R)^2} = \frac{3}{4}$

45 Each side of the first hexagon = R

Each side of the second hexagon = $\frac{\sqrt{3}}{2} R$

Each side of the third hexagon = $\frac{3}{4} R$

Each side of the fourth hexagon = $\frac{3\sqrt{3}}{8} R$

$\therefore$ Required ratio = $\frac{R}{\frac{3\sqrt{3}}{8} R} = \frac{8}{3\sqrt{3}}$

46 From the concept of similarity of triangles. All the five quadrilaterals viz., AOA' , BOB' , COC' , DOD' and EOE' are similar.

From the figure (ii),

$$\frac{r_2 - r_1}{r_2 + r_1} = \frac{r_3 - r_2}{r_3 + r_2}$$

$$= \frac{r_4 - r_3}{r_4 + r_3} = \frac{r_5 - r_4}{r_5 + r_4} = K$$

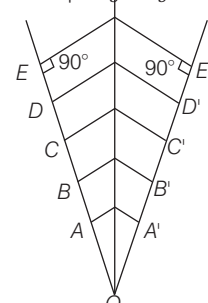


Fig. (i)

$\Rightarrow$

$$\frac{r_2}{r_1} = \frac{r_3}{r_2} = \frac{r_4}{r_3} = \frac{r_5}{r_4} = K$$

(By componendo and dividendo)

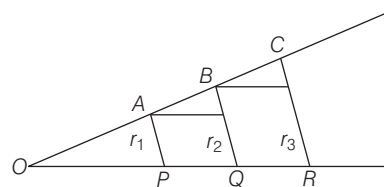


Fig. (ii)

It means all the radii are in GP.

Therefore, $\frac{r_5}{r_1} = (K)^4 = \frac{81}{16} = \left(\frac{3}{2} \right)^4$

$\Rightarrow K = \frac{3}{2} \therefore r_3 = r_1 (K)^2$

$r_3 = r_1 \times \frac{9}{4} = \frac{9r_1}{4} = \frac{9}{4} \times 16 = 36 \text{ cm}$

47 $\therefore r_1 = 16, r_2 = 24, r_3 = 36, \dots$ etc.

$$\therefore \frac{OP}{AP} = \frac{OQ}{BQ}$$

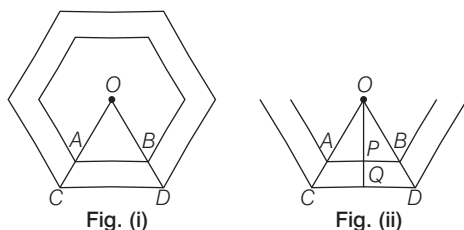
$$\frac{h + r_1}{r_1} = \frac{h + 2r_1 + r_2}{r_2} \Rightarrow \frac{h + 16}{16} = \frac{h + 56}{24}$$

$$\Rightarrow h = 64 \text{ cm}$$

48 $60 = 1 \times 1 \times 60$
 $= 1 \times 2 \times 30$
 $= 1 \times 3 \times 20$
 $= 1 \times 4 \times 15$
 $= 1 \times 5 \times 12$
 $= 1 \times 6 \times 10$
 $= 2 \times 2 \times 15$
 $= 2 \times 3 \times 10$
 $\dots \dots \dots$
 $\dots \dots \dots$
 $= 3 \times 4 \times 5$

Out of the given different combinations the first combination ($1 \times 1 \times 60$) gives maximum length of diagonal of cuboid, but in this case two of the edges are same. So, the second combination gives the proper value i. e., which gives the maximum length of diagonal whose all sides are different. Hence, the length of such a pencil is equal to the diagonal of cuboid $= \sqrt{1^2 + 2^2 + 30^2} = \sqrt{905}$

49



In figure (ii)

$$OP = \frac{\sqrt{3}}{2} OA = 4\sqrt{3} \text{ cm}$$

$$\text{Again } \frac{OP}{OQ} = \frac{OA}{OC}, \frac{4\sqrt{3}}{6\sqrt{3}} = \frac{8}{OC} \quad (OQ = OP + PQ = 4\sqrt{3} + 2\sqrt{3})$$

$$\Rightarrow OC = 12 \text{ cm}$$

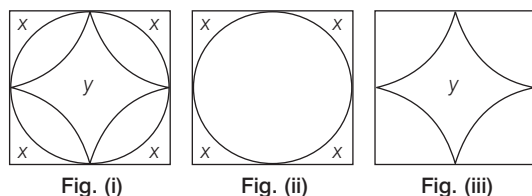
$\therefore$ Each side of the outer hexagon is 12 cm.

$$\therefore \text{Required area} = (\text{Area of outer hexagon} - \text{Area of inner hexagon})$$

$$= \frac{3\sqrt{3}}{2} [12^2 - 8^2] = 120\sqrt{3} \text{ cm}^2$$

50 Area of region x = Area of square – Area of inscribed circle

$$= (4 - \pi)$$



Area of region y = Area of square – 4 (area of quadrant)

$$= 4 - 4 \left(\frac{1}{4} \pi \times (1)^2 \right) = (4 - \pi)$$

$\therefore$ Required area (of shaded region)

$$= \text{Area of square} - [\text{Area of region } x + \text{Area of region } y]$$

$$= 4 - [4 - \pi + 4 - \pi] = 2\pi - 4$$

51 Let the volume of solid block be V and radius of the spheres formed from the first block be r_1 , then the volume of each sphere be V_1 .

Similarly, let the radius of each sphere obtained from second block be $r_2 (= 2r_1)$, then the volume of each sphere be

$$V_2 = (8V_1)$$

$$\therefore V = kV_1 + 14 \quad \dots(i)$$

$$\text{and } V = lV_2 + 36$$

$$\text{or } V = 8lV_1 + 36 \quad \dots(ii)$$

From eqs. (i) and (ii),

$$kV_1 + 14 = 8lV_1 + 36$$

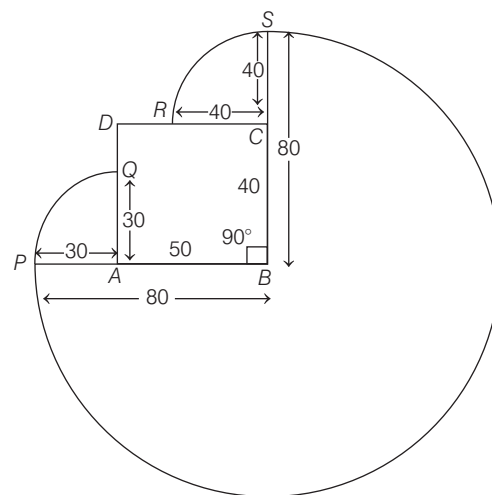
$$\Rightarrow V_1 (k - 8l) = 22$$

The possible value of $V_1 = 22, 11, 2$ or 1

But V_1 can never be equal to or less than 14 (since remainder is always less than divisor) So, the only possible value of $V_1 = 22$

$$\therefore V_2 = 8 \times V_1 = 176 \text{ cm}^3$$

52 The length of tether of the horse is 80 m.



Area grazed by horse

$$= \left[\pi \times (80)^2 \times \frac{270}{360} + \pi \times (30)^2 \times \frac{90}{360} + \pi \times (40)^2 \times \frac{90}{360} \right]$$

$$= \pi \left(6400 \times \frac{3}{4} + 900 \times \frac{1}{4} + 1600 \times \frac{1}{4} \right)$$

$$= \pi \left[\frac{21700}{4} \right] = 5425\pi \text{ m}^2$$

Hint When horse is tethered at B , then he can move freely 270° i. e., from P to S with the full length of his tether as the radius of the arc PS . Again when horse reaches P and tries to move further in the right direction i. e., towards D , his tether gets fixed at A and now only 30 m tether is free which works as a radius of the quadrant PAQ . Similarly when horse reaches S and tries to move left further in the direction of D , due to wall BC his tether gets fixed at C and now only 40 m tether is free to move further. At this moment point C behaves like a fixed point of tether (as centre of quadrant RSC).

NOTE You can tether horse at any of the four corners A, B, C and D area grazed by horse remains same.

- 53** Here each side is broken up into 6 parts i. e., $n = 6$

$$\text{Now, } N_0 = (n - 2)^3 = (4)^3 = 64$$

$$N_1 = 6(n - 2)^2 = 6 \times (4)^2 = 96$$

$$N_2 = 12(n - 2) = 12(4) = 48$$

$$\therefore N_0 : N_1 : N_2 = 64 : 96 : 48 = 4 : 6 : 3$$

- 54** Let the radius of seed be r and radius of the whole fruit (pulp + seed) be R , then thickness of the pulp $= (R - r)$

$$\text{Volume of mango fruit} = \frac{4}{3} \pi R^3$$

$$\text{and Volume of pulp} = \frac{4}{3} \pi (R^3 - r^3)$$

$$\text{but } = \frac{4}{3} \pi \left[R^3 - \left(\frac{2}{7} R \right)^3 \right]$$

$$\left[\because \frac{r}{R - r} = \frac{2}{5} \Rightarrow r = \frac{2}{7} R \right]$$

$\therefore$ Percentage of volume of pulp to the total volume of fruit

$$= \frac{\frac{4}{3} \pi R^3 \left[1 - \frac{8}{343} \right]}{\frac{4}{3} \pi R^3}$$

$$= \frac{335}{343} \times 100 = 97.66\%$$

- 55** Let the radius of each smaller circle is r and radius of the larger circle is R , then

$$\pi R^2 = 4\pi r^2 \Rightarrow R = 2r$$

$$OR = OP = R + r = 3r$$

$$\text{Also } PM = r$$

(PM is the perpendicular on AB)

$$\therefore AP = \sqrt{2}r$$

$$\therefore AO = AP + PO$$

$$= r\sqrt{2} + 3r = r(3 + \sqrt{2})$$

$$\therefore AC = 2AO = 2r(3 + \sqrt{2}), \text{ which is the diagonal of square}$$

$$\therefore \text{Required ratio} = \frac{2r(3 + \sqrt{2})}{\sqrt{2}r} = (2 + 3\sqrt{2})$$

- 56** Initial radius = 14 cm

Radius at a time when the balloon explodes = 35 cm

$$\text{Change in volume} = \frac{4}{3} \pi [(35)^3 - (14)^3]$$

$$= \frac{4}{3} \pi (7)^3 [125 - 8]$$

$$= \frac{4}{3} \pi \times 343 \times 117$$

$$= \frac{4}{3} \pi \times 343 \times 117$$

$$\text{Required time to explode} = \frac{\frac{4}{3} \pi \times 343 \times 117}{156} = 1078 \text{ s}$$

- 57** Let the each side of cube be a , then

$$CD = \sqrt{2}a$$

$$\therefore CQ = \frac{a}{\sqrt{2}}$$

Let the radius of cone be r and height be h , then

$$r = h\sqrt{2}$$

$\therefore$ In $\triangle APO$ and $\triangle CQO$ (Similar triangles)

$$\frac{AP}{PO} = \frac{CQ}{OQ} = \frac{r}{h} = \frac{\frac{a}{\sqrt{2}}}{(h - a)}$$

$$\Rightarrow \frac{\frac{a}{\sqrt{2}}}{(h - a)} = \sqrt{2}$$

$$\Rightarrow a = 2(h - a)$$

$$\Rightarrow h = \frac{3a}{2}$$

$$\therefore r = \frac{3a}{2} \times \sqrt{2} \text{ and } h = \frac{3a}{2}$$

$$\therefore \text{Volume of cone} = \frac{1}{3} \pi \times \left(\frac{3a\sqrt{2}}{2} \right)^2 \times \frac{3a}{2}$$

$$= \frac{9}{4} a^3 \pi$$

$$\text{and Volume of cube} = a^3$$

$$\therefore \text{Required ratio} = \frac{\frac{9}{4} \pi a^3}{a^3} = \frac{9}{4} \pi = 2.25\pi$$

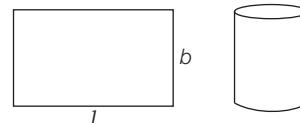
- 58** For the given volume, cube has minimum possible length of diagonal.

Therefore each side of cube = 4 cm

and its diagonal = $4\sqrt{3}$ cm.

- 59**

$$l = 2\pi r \Rightarrow r = \frac{l}{2\pi}$$

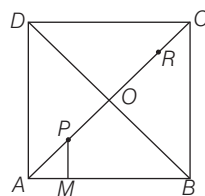
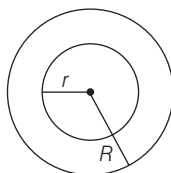


where r is the base radius of cylinder and l is the length of paper and $h = b$, where h is the height of cylinder and b is the breadth of the paper.

$$\therefore \text{Volume of cylinder} = \pi r^2 h = \pi \times \left(\frac{l}{2\pi} \right)^2 \times b$$

$$\Rightarrow \frac{\pi \times l^2 b}{4\pi^2} = 48.125 = \frac{385}{8}$$

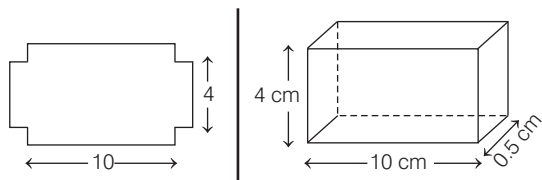
$$\Rightarrow l^2 b = 11 \times 11 \times 5$$



$$\Rightarrow l = 11 \text{ and } b = 5 \quad (\because l > b)$$

$$\therefore \text{Volume of the box} = l \times b \times 4$$

$$= 10 \times 4 \times 0.5 = 20 \text{ cm}^3$$



60 Vertical spacing between any two turns = $\frac{\text{Height of cone}}{\text{Number of turns}}$

$$= \frac{h}{n}$$

61 Number of turns = $\frac{h}{x}$

Length of string in each turn

$$= 2\pi r = 2\pi \times \frac{4}{\pi} = 8 \text{ cm}$$

$\therefore$ Total length of string in all the n turns

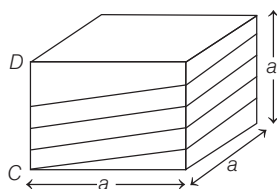
$$= \frac{h}{x} \times 8 = \frac{8h}{x} \text{ cm}$$

62 Total length of string = $8n$ cm

Since, total length of string

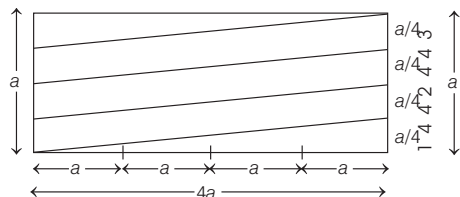
$$= \text{number of turns} \times \text{perimeter of cylinder}$$

$$= 8 \times n = 8n \text{ cm}$$



Length of string required for 1 turn (or round) = $\frac{8n}{4} = 2n$

but $2n = \sqrt{\left(\frac{a}{4}\right)^2 + (4a)^2}$



$$\Rightarrow a = \frac{8n}{\sqrt{257}}$$

where a is the side of cube.

63 From the sheet of 10 ft long, maximum $\frac{10}{2} = 5$ circular discs can be cut along the length of the iron sheet.

$$CM = \sqrt{AC^2 - AM^2} = \sqrt{4x^2 - x^2}$$

$$CM = x\sqrt{3} = \sqrt{3} \text{ ft} \quad (\text{Since } x = 1 \text{ ft})$$

$\therefore$ Width of the sheet = $AK + MC + CT$

$$= 1 + \sqrt{3} + 1 = (2 + \sqrt{3}) \text{ ft}$$

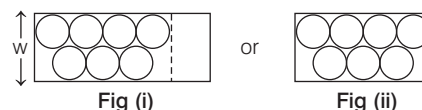


Fig (i)

Fig (ii)

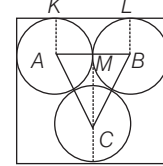


Fig. (iii)

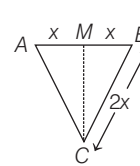
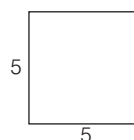


Fig. (iv)

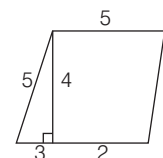
64 Recall that for a given perimeter the polygon of minimum number of sides has minimum area and the polygon of maximum number of sides has maximum area. So, the correct relation is $h > s > r$.

Thus, hexagon (6 sides) has maximum area.

Now, between square and rhombus, square has greater area than rhombus. For easier understanding consider some values.



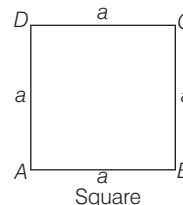
$$\text{Area} = 25 \text{ cm}^2$$



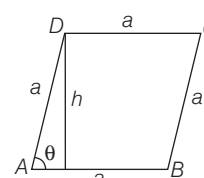
$$\text{Area} = \text{base} \times \text{height}$$

$$= 5 \times 4 = 20 \text{ cm}^2$$

Alternatively



Square



Rhombus

In rhombus ABCD: Area of rhombus = $a \times h$

$$= a \times a \sin \theta \quad \left(\frac{h}{a} = \sin \theta\right)$$

$$= a^2 \sin \theta$$

As you know the maximum value of $\sin \theta$ is 1 at $\theta = 90^\circ$ but at $\theta = 90^\circ$ rhombus will become a square. So except $\theta = 90^\circ$ for all the rest values the area of rhombus will be less than the area of square.

65

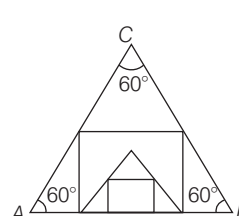


Fig. (i)

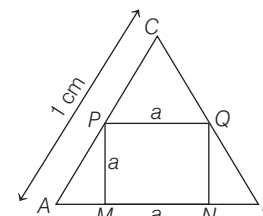


Fig. (ii)

PCQ is also an equilateral triangle

$$\therefore PC = PQ = PM = a$$

$$\begin{aligned}\therefore \frac{a}{PA} &= \frac{\sqrt{3}}{2} \Rightarrow PA = \frac{2a}{\sqrt{3}} \\ \therefore AC &= AP + PC = \frac{2a}{\sqrt{3}} + a = 1 \text{ cm} \\ \Rightarrow a &= \frac{\sqrt{3}}{(2 + \sqrt{3})} = \sqrt{3}(2 - \sqrt{3})\end{aligned}$$

Now, in figure (iii)

$$PM = MT = a$$

Let the each side of square RSYX be K , then $RT = K$ also (since RTS is an equilateral triangle)

$$\therefore \frac{K}{RM} = \frac{\sqrt{3}}{2}$$

$$\therefore RM = \frac{2K}{\sqrt{3}}$$

$$\therefore MT = RT + RM = K + \frac{2K}{\sqrt{3}}$$

$$MT = \frac{(\sqrt{3} + 2)}{\sqrt{3}} K$$

But

$$MT = a$$

$$\therefore a = \left(\frac{\sqrt{3} + 2}{\sqrt{3}} \right) K$$

$$\therefore K = \frac{\sqrt{3}a}{(\sqrt{3} + 2)}$$

But

$$a = \sqrt{3}(2 - \sqrt{3})$$

$$\therefore K = \frac{\sqrt{3}}{(\sqrt{3} + 2)} [\sqrt{3}(2 - \sqrt{3})]$$

$$K = \frac{3(2 - \sqrt{3})}{(2 + \sqrt{3})} \times \frac{(2 - \sqrt{3})}{(2 - \sqrt{3})}$$

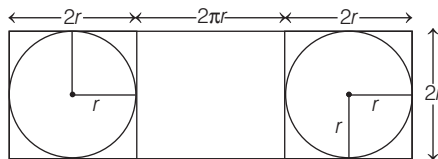
$$\Rightarrow K = \frac{3(2 - \sqrt{3})^2}{1} = 3(7 - 4\sqrt{3})$$

$$\therefore \text{Area of square RSYX} = K^2 = [3(7 - 4\sqrt{3})]^2$$

$$K^2 = 9(49 + 48 - 56\sqrt{3})$$

$$K^2 = (873 - 504\sqrt{3}) \text{ cm}^2$$

- 66** For the minimum wastage of sheet he has to cut the sheet in the given manner.



Total area of sheet required

$$(2\pi r + 4r) \times 2r = 4r^2(\pi + 2)$$

$$\text{Area of sheet utilised} = (2\pi r \times 2r) + 2(\pi r^2) = 6\pi r^2$$

$$\text{Area of wastage sheet} = 4r^2(\pi + 2) - 6\pi r^2$$

$$= 8r^2 - 2\pi r^2$$

$$\therefore \text{Required ratio} = \frac{8r^2 - 2\pi r^2}{6\pi r^2} = \frac{2r^2(4 - \pi)}{6r^2\pi} = \frac{1}{11}$$

- 67 Short cut:** Very quickly check the options. If all the options have values.

Alternatively The required capacity of box is 864 m^3 . Let the length of the base be l and height of the box is ' h ', then

$$864 = l^2 h \Rightarrow h = \frac{864}{l^2}$$

Now, surface area of the box $A = l^2 + 4lh$

$$= l^2 + \frac{4l \times 864}{l^2}$$

$$= l^2 + \frac{3456}{l}$$

Differentiating w.r. to l , we get

$$\frac{dA}{dl} = 2l - \frac{3456}{l^2}$$

For the minimum area $= \frac{dA}{dl} = 0$

$$\therefore 2l = \frac{3456}{l^2}$$

$$\Rightarrow l^3 = 1728$$

$$\Rightarrow l = 12$$

$$\therefore \text{Base area} = (12)^2 = 144 \text{ m}^2$$

$$\text{and Height} = \frac{864}{l^2} = 6 \text{ m}$$

- 68** Let the initial radius be r and volume be V , then

$$V = \pi r^2 \times 4$$

$$\text{Ist case: } V_1 = \pi(r + 12)^2 \times 4$$

$$\text{IInd case: } V_2 = \pi r^2 \times (4 + 12)$$

$$\text{But } V_1 = V + K$$

$$\text{and } V_2 = V + K$$

$$\therefore V_1 = V_2$$

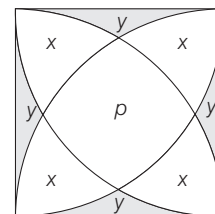
$$\Rightarrow \pi(r + 12)^2 \times 4 = \pi r^2 (16)$$

$$\Rightarrow r = 12 \text{ ft}$$

$$\therefore \text{Increased volume} = V_1 = V_2$$

$$= \pi \times (24)^2 \times 4 = 2304\pi \text{ cubic ft}$$

- 69** Let us denote the distinct regions of the square by x , y and p as shown in the following figure.



Now, from this figure we have

$$4x + 4y + p = 1 \quad (i)$$

And, $x + 2y = 1 - \frac{\pi}{4}$

$$\Rightarrow 4(x + 2y) = 4 - \pi \quad (ii)$$

But, it is given that $p = \frac{\pi}{3} - \sqrt{3} + 1$ (iii)

Now, substituting the values from eqs. (i) and (iii) in the equation (ii), we get

$$\begin{aligned} 4x + 8y &= 4 - \pi \\ \Rightarrow (4x + 4y) + 4y &= 4 - \pi \\ \Rightarrow 4y &= 4 - \sqrt{3} - \frac{2\pi}{3} \end{aligned}$$

Hence, choice (a) is correct.

- 70** Let each side of the square at the corner be x cm and each side of the base of the cuboid be b cm, then

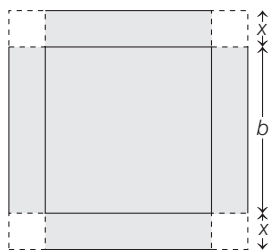
$$x + b + x = 24$$

$$\Rightarrow b = 24 - 2x$$

Therefore, each side of the base of the cuboid, which is $b = (24 - 2x)$ cm

Consequently, the height of the cuboid is $h = x$ cm

$$\begin{aligned} \text{Therefore, volume of the cuboid} &= (x)(24 - 2x)(24 - 2x) \\ &= 2(2x)(12 - x)(12 - x) \end{aligned}$$



We know that $b + 2x = 24$, is constant.

We also know that if $p_1 + p_2 + p_3 + \dots + p_n = \text{some constant}$, then $p_1 \cdot p_2 \cdot p_3 \dots p_n$ will be maximum when

$$p_1 = p_2 = p_3 = \dots = p_n$$

Accordingly, $2x + (12 - x) + (12 - x) = 24$ is a constant, therefore, the volume of cuboid will be maximum when $2x = (12 - x) = (12 - x)$

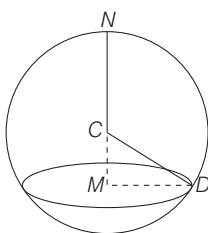
It implies that $x = 4$

So the volume of the cuboid will be maximum when $x = 4$ cm.

Hence, choice (d) is correct.

- 71** The man has to reach D from N . Let M is the centre of the latitude where the man is supposed to reach and MD is the radius of the pertinent latitude. Now, extend the radius NC to M so that MD becomes perpendicular to CM .

Given that $CD = 6370$ km and $MD = 3185\sqrt{3}$ km. That is $MD/CD = \sqrt{3}/2$.



It implies that $\angle MDC = 30^\circ$ and $\angle MCD = 60^\circ$, since $\angle CMD = 90^\circ$

Therefore $\angle NCD = 120^\circ$.

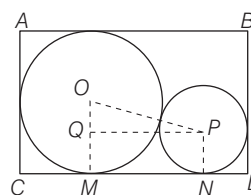
So the length of arc ND

$$= 2\pi \times 6370 \times \frac{120}{360} = 13346.67 \text{ km}$$

Hence, choice (c) is correct.

- 72 Case 1:** When the radius of each circle is same, then in this case radius of each circle would be $49/4$ and the total area of both the circles would be $2\pi (49/4)^2 \approx 300\pi$

Case 2: When there is one circle of the largest possible size and another circle is fitted in the remaining area.



Let the radius of larger circle be R and that of smaller circle be r .

Then, $OM = R = 16$

and $QM = PN = r$.

Therefore, $OQ = OM - QM = 16 - r$

And $OP = R + r = 16 + r$

Similarly,

$$PQ = MN = 49 - (CM + ND) = 49 - (16 + r) = 33 - r$$

Now, $OP^2 = OQ^2 + PQ^2$

$$\Rightarrow (16 + r)^2 = (16 - r)^2 + (33 - r)^2$$

$$\Rightarrow r^2 - 130r + 1089 = 0$$

$$\Rightarrow r = 9 \text{ or } 121$$

Since, 121 is greater than 16, so it's unacceptable. Therefore $r = 9$. Thus, the total area of the two circles $= \pi(16^2 + 9^2) = 337\pi$

As in case 2, the total arc is more than that in case 1, so 337π is max. possible area.

Hence, choice (a) is the correct one.

- 73** Since, area of square field $= 54 \text{ m}^2$,

so each side of the square $= 3\sqrt{6}$ m and

diagonal of the square $= 6\sqrt{3}$ m. Consider the following diagram, now.

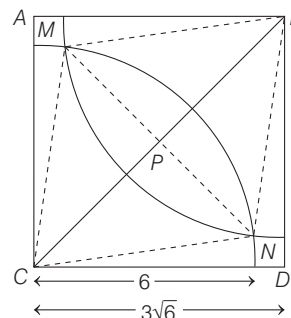
$$MB = BN = CN = CM = 6 \text{ m}$$

$$CP = 1/2(BC) = 3\sqrt{3} \text{ m}$$

Therefore, using Pythagoras theorem, you can find PM

$$PM = \sqrt{CM^2 - CP^2} = 3$$

$$\therefore MN = 2(PM) = 6.$$

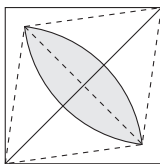


It implies that $\triangle MCN$ and $\triangle BMN$ are equilateral triangles.

Area of the common region = Area encapsulated between two arcs

$$= 2 \left[\pi 6^2 \left(\frac{60}{360} \right) - \frac{\sqrt{3}}{4} 6^2 \right]$$

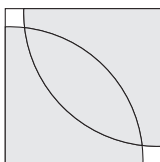
$$= 2 (6\pi - 9\sqrt{3})$$



Area grazed by sheep and goat = 2(Area of a quadrant) - area of the common region

$$= 2 \left(\frac{\pi}{4} (6)^2 \right) - 2 (6\pi - 9\sqrt{3})$$

$$= 6(\pi + 3\sqrt{3})$$



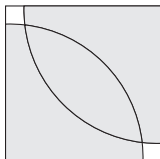
Area that cannot be grazed by sheep and goat = (total area of square - area grazed by sheep and square)

$$= 54 - 6(\pi + 3\sqrt{3}) = 3.97 \text{ m}^2$$

Hence, choice (b) is the correct one.

Alternatively From the given information, you can find that side of the square is $3\sqrt{6} = 7.4 \text{ m}$ but the length of rope = 6 m

If you look at the following figure you see that at the corner of the field there are two white (or un-grazed) regions. Roughly, each such region looks like a square. And the side of each such square = $7.35 - 6 = 1.35 \text{ m}$.



Therefore area of both the un-grazed regions

$$= 2 (1.35)^2 = 3.65 \approx 4$$

So, the total un-grazed area is approx. 4 m^2

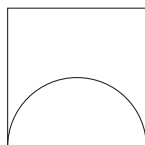
NOTE The actual area at the corners would be slightly more than the total area of the assumed squares at the two corners. That's why choices (a), (c) and (d) are invalid.

- 74** Given that each side of the square is a unit. Let r be the radius of such semicircle. Then, we can proceed as follows.

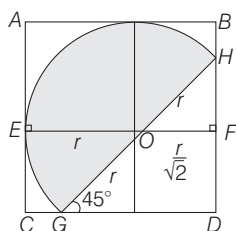
Case I: If the diameter of the semicircle coincides with one of the sides of the square, the maximum area of semicircle

$$= \frac{1}{2} \pi r^2 = \frac{1}{2} \pi \left(\frac{a}{2} \right)^2$$

$$= \frac{1}{8} (\pi a^2) = 0.125 \pi a^2$$



Case II: If the diameter of the circle makes some angle with one of the sides of the square.



Let O is the centre of the semicircle. Since square is a symmetric diagram, so both the sides of the square must be

tangent to the circle, as shown in the diagram. And, therefore, the diameter of the circle must make a 45° angle with the base in order to conform to the symmetry.

Now, the radii $GO = EO = HO = r$

Therefore, in $\triangle HOF$ either by using Pythagoras theorem or

Trigonometric ratio, we have $OF = HF = \frac{HO}{\sqrt{2}} = \frac{r}{\sqrt{2}}$

Therefore,

$$CD = EF$$

$$\Rightarrow CD = EO + OF$$

$$\Rightarrow a = r + \frac{r}{\sqrt{2}}$$

$$\Rightarrow r = a \left(\frac{\sqrt{2}}{1 + \sqrt{2}} \right)$$

$$r = a (2 - \sqrt{2})$$

Therefore, the required area of the semicircle

$$= \frac{1}{2} \pi r^2 = \frac{1}{2} \pi (a(2 - \sqrt{2}))^2$$

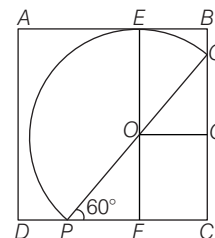
$$= \frac{1}{2} \pi a^2 (6 - 4\sqrt{2})$$

$$= \pi a^2 (3 - 2\sqrt{2}) \approx 0.172 \pi a^2$$

Case II gives the highest possible value.

Hence choice (a) is the correct one.

- 75** Consider the following diagram, in which O is the centre of the circle and OG is perpendicular to BC and EF is parallel to BC .



Since, PQ makes a 60° angle with DC , therefore $\angle QPC = \angle QOG = 60^\circ$ and $\angle PQC = \angle POF = 30^\circ$

Since $OP = OQ = OE = r$ therefore, using 30-60-90 degree theorem (or trigonometric ratios) in $\triangle OPF$, we can find that $PF = r/2$ and $OF = \frac{\sqrt{3}}{2} r$.

$$\text{Therefore, } AB = BC = EF = OE + OF = r + \frac{\sqrt{3}}{2} r = \frac{r(2 + \sqrt{3})}{2}$$

So, the area of the smallest possible square

$$= \left[\frac{r(2 + \sqrt{3})}{2} \right]^2 = \left(\frac{7 + 4\sqrt{3}}{4} \right) r^2$$

Hence, choice (a) is the correct one.

- 76** Look at the following figure (i), you will have $AB = RB$, as both of them are radii of the same circle centered at B . Similarly, $AB = AR$.

Therefore, $AR = BR = AB$. That is

$$\angle RAB = \angle RBA = \angle ARB = 60^\circ$$

Now, look at the figure (ii) and find all the important angles.

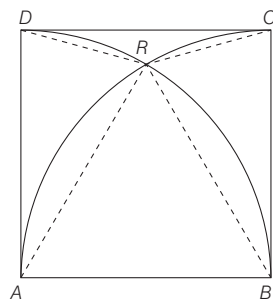


Fig (i)

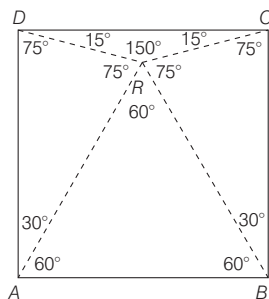


Fig (ii)

Now, with the help of fig (ii) find all the important angles in fig (iii).

$$\angle CDR = 15^\circ, \text{ similarly } \angle DSA = 150^\circ$$

$$\therefore \angle RDS = 90 - (15 + 15) = 60^\circ$$

But, since $DR = DS$

$$\therefore \angle DSR = \angle DRS = 60^\circ$$

That is triangle DRS is an equilateral triangle.

That's why all the four triangles $\triangle DSR$, $\triangle PBQ$, $\triangle RCQ$, $\triangle SAP$ in fig (iv) are equilateral ones.

Since, $RS = RQ = PQ = SP$ so the quadrilateral $SRQP$ is a square.

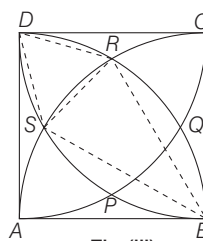


Fig (iii)

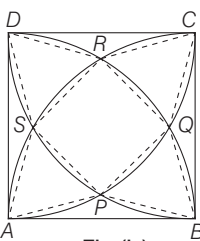


Fig (iv)

Now, look at the fig (v) and fig (vi).

Let us assume that each side of the square $SRQP$ is a , then

$$SR = SD = DR = a \text{ and } PQ = PB = QB = a$$

$$\therefore DM = BN = \frac{\sqrt{3}}{2} a$$

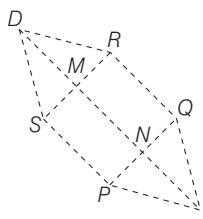


Fig (v)

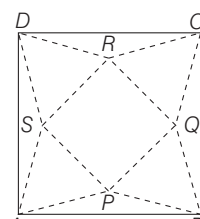


Fig (vi)

But,

$$DB = DM + MN + NB$$

$$\Rightarrow 10\sqrt{2} = \frac{\sqrt{3}}{2} a + a + \frac{\sqrt{3}}{2} a$$

$$\Rightarrow 10\sqrt{2} = (\sqrt{3} + 1)a$$

$$\Rightarrow a = \frac{10\sqrt{2}}{(\sqrt{3} + 1)}$$

$$\Rightarrow a^2 = \left(\frac{10\sqrt{2}}{\sqrt{3} + 1} \right)^2 = \frac{100}{2 + \sqrt{3}} = 100(2 - \sqrt{3}) \text{ sq cm}$$

Hence, choice (b) is the correct one.

77 Look at the following figure (i), you will have $AB = RB$, as both of them are radii of the same circle centered at B . Similarly, $AB = AR$.

Therefore, $AR = BR = AB$.

That is $\angle RAB = \angle RBA = \angle ARB = 60^\circ$

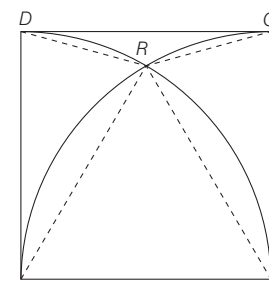


Fig (i)

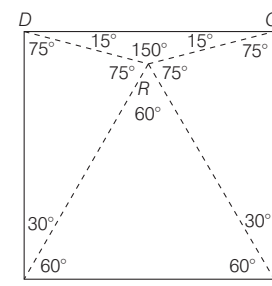


Fig (ii)

Now, with the help of fig (ii) find all important angles in fig (iii).

$$\angle CDR = 15^\circ, \text{ similarly } \angle DSA = 15^\circ$$

$$\therefore \angle RDS = 90^\circ - (15^\circ + 15^\circ) = 60^\circ$$

But, since $DR = DS$, $\therefore \angle DSR = \angle DRS = 60^\circ$

That is triangle DRS is an equilateral triangle.

That's why all the four triangles $\triangle DSR$, $\triangle PBQ$, $\triangle RCQ$, $\triangle SAP$ in fig (iv) are equilateral ones.

Since, $RS = RQ = PQ = SP$, so the quadrilateral $SRQP$ is a square.

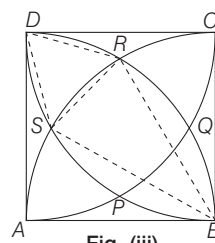


Fig. (iii)

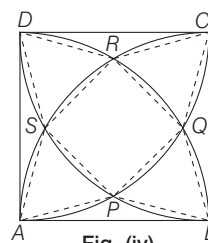


Fig. (iv)

Now, look at the fig (v) and fig (vi).

Let us assume that each side of the square $SRQP$ is a , then

$$SR = SD = DR = a \text{ and } PQ = PB = QB = a$$

$$\therefore DM = BN = \frac{\sqrt{3}}{2} a$$

But,

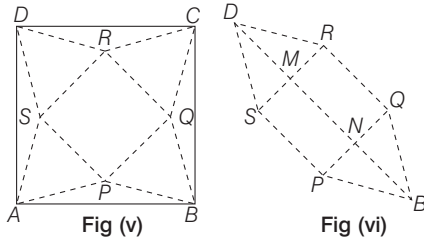
$$DB = DM + MN + NB$$

$$\Rightarrow 10\sqrt{2} = \frac{\sqrt{3}}{2} a + a + \frac{\sqrt{3}}{2} a$$

$$10\sqrt{2} = (\sqrt{3} + 1)a$$

$$\Rightarrow a = \frac{10\sqrt{2}}{(\sqrt{3} + 1)}$$

$$\Rightarrow a^2 = \left(\frac{10\sqrt{2}}{\sqrt{3} + 1} \right)^2 = \frac{100}{2 + \sqrt{3}} = 100(2 - \sqrt{3}) \text{ sq cm}$$



The area of the common region by two quadrants in fig (vii) is determined as follows. Area of common region = $2 \times$ area of each quadrant – area of the square

$$= 2 \times \left(\frac{1}{4} \pi \times 10^2 \right) - 10^2 = 50\pi - 100 \text{ sq cm}$$

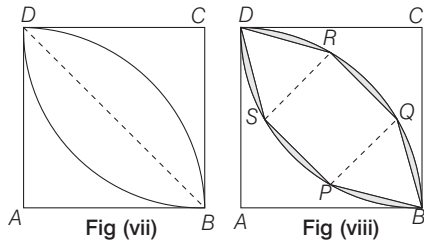
Now, look at the fig (viii). The total area of the 6 shaded segments = Total area of the common region between the two quadrants – Total area of the two equilateral triangles and one square

$$= (50\pi - 100) - \left[2 \times \frac{\sqrt{3}}{4} \times \left(\frac{10\sqrt{2}}{\sqrt{3}+1} \right)^2 + \left(\frac{10\sqrt{2}}{\sqrt{3}+1} \right)^2 \right] \text{ sq cm}$$

$$= (50\pi - 100) - \left[\frac{\sqrt{3}}{2} \times 100 (2 - \sqrt{3}) + 100 (2 - \sqrt{3}) \right] \text{ sq cm}$$

$$= (50\pi - 100) - 100 (2 - \sqrt{3}) \left[\frac{\sqrt{3}}{2} + 1 \right] \text{ sq cm}$$

$$= (50\pi - 100) - (50) \text{ sq cm} = 50 (\pi - 3) \text{ sq cm}$$



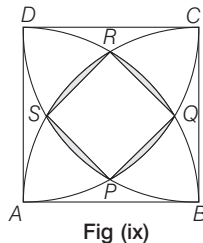
Therefore, the area of 4 shaded segments = $\frac{4}{6} \times 50 (\pi - 3)$ sq cm

$$= \frac{100 (\pi - 3)}{3} \text{ sq cm}$$

Thus, the required area = area of the 4 shaded segments around the square SRQP + area of the square SRQP inscribed in that region

$$= \frac{100 (\pi - 3)}{3} + 100 (2 - \sqrt{3}) \text{ sq cm}$$

$$= 100 \left(\frac{\pi}{3} + 1 - \sqrt{3} \right) \text{ sq cm}$$



Hence, choice (b) is the correct one.

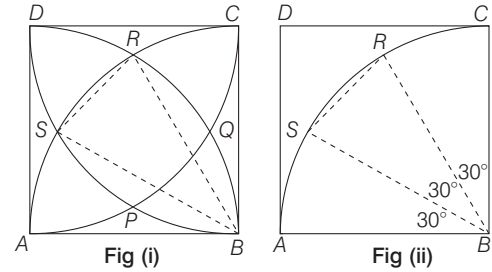
☞ **Alternatively** Look at the following figures: fig (i) and fig (ii).

$$\text{Area of the sector } BSR = \pi \times 10^2 \times \frac{30^\circ}{360^\circ} = \frac{25\pi}{3} \text{ sq cm}$$

$$\begin{aligned} \text{Area of isosceles triangle } BSR &= \frac{1}{2} \times 10^2 \sin 30^\circ \\ &= 25 \text{ sq cm} \end{aligned}$$

$$\begin{aligned} \text{Therefore, area of the segment} &= \frac{25\pi}{3} - 25 \text{ sq cm} \\ &= 25 \left(\frac{\pi}{3} - 1 \right) \text{ sq cm} \end{aligned}$$

Thus the total area of all the four segments around the inscribed square = $4 \times 25 \left(\frac{\pi}{3} - 1 \right) = 100 \left(\frac{\pi}{3} - 1 \right) \text{ sq cm}$



Now, using cosine rule you can obtain RS, the side of the inscribed square, as follows.

$$\cos 30^\circ = \frac{BS^2 + BR^2 - RS^2}{2 (BS) (BR)}$$

$$\frac{\sqrt{3}}{2} = \frac{10^2 + 10^2 - RS^2}{2 \times 10 \times 10}$$

$$= \frac{200 - RS^2}{200}$$

$$\sqrt{3} = \frac{200 - RS^2}{100}$$

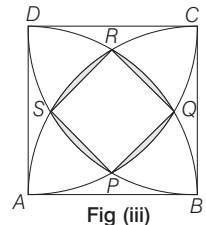
$$RS^2 = 100 (2 - \sqrt{3})$$

Therefore, area of the square SRQP = $SR^2 = 100 (2 - \sqrt{3}) \text{ sq cm}$

Thus, the required area

$$= 100 \left(\frac{\pi}{3} - 1 \right) + 100 (2 - \sqrt{3})$$

$$= 100 \left(\frac{\pi}{3} + 1 - \sqrt{3} \right) \text{ sq cm}$$

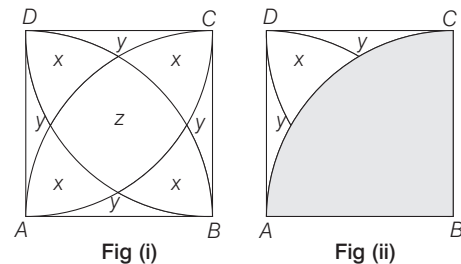


☞ **Alternatively** First of all, let us denote the different regions of the square as shown below.

Now, consider the fig (ii).

The area of the unshaded region = Area of square – area of the shaded quadrant $x + 2y = 100 - 25\pi$ (i)

Now, consider the fig (iv).



Area enclosed by the two overlapping quadrants = total area of the two quadrants – area of the square

$$2x + z = 2 \times 25\pi - 100$$

$$\Rightarrow 2x + z = 50\pi - 100 \quad \text{(ii)}$$

Now, consider the fig (iv).

Area of the shaded region = area of the equilateral triangle + 2(area of the segments outside the equilateral

triangle) $2x + y + z = \frac{\sqrt{3}}{4} \times 100 + 2(\text{area of the segments outside the equilateral triangle})$

outside the equilateral triangle)

$$\Rightarrow 2x + y + z = 25\sqrt{3} + 2(\text{area of the segments outside the equilateral triangle})$$

But the area of each segment = area of each sector with central angle 60° – area of equilateral triangle

$$= \frac{100\pi}{6} - 25\sqrt{3}$$

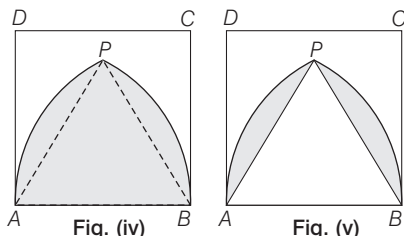


Fig. (iv)

Fig. (v)

Therefore, area of the shaded region in fig (iv) is

$$2x + y + z = 25\sqrt{3} + 2\left(\frac{100\pi}{6} - 25\sqrt{3}\right)$$

$$\Rightarrow 2x + y + z = \frac{100\pi}{3} - 25\sqrt{3} \quad \dots \text{(iii)}$$

Now, solving the eqs. (ii) and (iii), we get

$$x = 100 - 25\sqrt{3} - \frac{50\pi}{3}$$

Then, from the eq. (i), we get

$$y = \frac{25\pi}{3} + 50\sqrt{3} - 100$$

Again, from eq. (ii), we get

$$z = 100\left(\frac{\pi}{3} + 1 - \sqrt{3}\right)$$

- 78** Given that $AB = BC = CD = DA = 30$ cm. Due to symmetrical cutting, EF and GH are perpendicular. Also, $AE = BG = CF = DH$ and $EB = GC = FD = HA$.

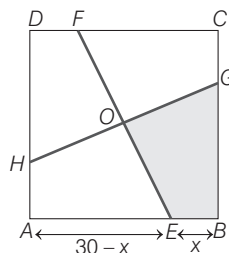


Fig. (i)

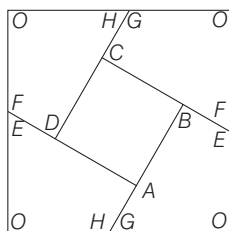


Fig. (ii)

In the fig (ii), we have $GB = 30 - x$ and $HA = GA = x$,

$$\therefore AB = GB - GA \Rightarrow 16 = 30 - 2x \Rightarrow x = 7 \text{ cm}$$

Now, from fig (i), we have $EB = GC = FD = HA = 7$ cm and $AE = BG = CF = DH = 23$ cm

Now, in fig (i), if we draw a perpendicular line FJ on AB , such that $FJ = AD = 30$

$$\begin{aligned} \text{Since, } AJ = DF = 7, \text{ therefore } JE &= AE - AJ = 23 - 7 \\ &= 16 \text{ cm} \end{aligned}$$

Therefore, in the right angle triangle FJE , we have

$$FE = \sqrt{30^2 + 16^2} = 34 \text{ cm}$$

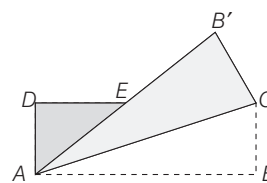
$$\text{But, since } OF = OE = \frac{FE}{2} = 17 \text{ cm}$$

Similarly, $OH = OG = 17$ cm

Therefore, the side of the new larger square = 34 cm.

Hence, choice (a) is correct.

- 79** Since, the ratio of area of polygon to that of rectangle is $16/25$, it means $9/25^{\text{th}}$ or 36% area is lost due to overlapping.



Therefore the lost area is equal to the area of triangle AEC .

It implies that area of triangle $AEC = 36\%$ of area of rectangle $ABCD$.

That is area of $\triangle AED + \triangle AEC + \triangle CEB' = 64\%$ of the area of rectangle $ABCD$

Since area of $\triangle AED = \triangle CEB'$ therefore,

$$\triangle AED = \triangle CEB' = 14\%(ABCD)$$

Now, see there are two triangles AEC and ECB' of same height.

$$\therefore \frac{\frac{1}{2}(AE \times CB')}{\frac{1}{2}(EB' \times CB')} = \frac{\triangle AEC}{\triangle ECB'}$$

$$\Rightarrow \frac{AE}{EB'} = \frac{36}{14} = \frac{18k}{7k}$$

$$\text{But, } EC = EA = 18k \text{ and } EB' = 7k$$

Therefore, by applying Pythagoras theorem in $\triangle ECB'$, we get $CB' = \sqrt{(18k)^2 - (7k)^2} = 5\sqrt{11}k$

$$\text{And, } AB' = AE + EB' = 18k + 7k = 25k$$

Therefore, the required percentage

$$= \frac{25k}{5\sqrt{11}k} = \frac{5}{\sqrt{11}}$$

Hence, choice (a) is correct.

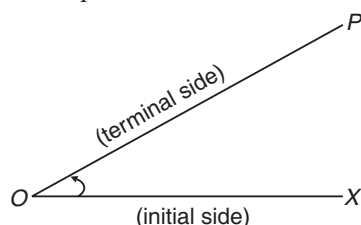
CHAPTER 11

Trigonometry

This chapter is basically written for those students who were either not so good in the secondary classes maths or who have not been in touch with the following concepts from 4-5 years. Hardly a question is asked in CAT, directly from this chapter, but almost every year there are one or two questions in which we can find the application of the concepts of trigonometry *e.g.*, sometimes in geometry or in height and distance problems. So just refresh the concepts of trigonometry. Therefore CAT aspirants are not required to waste so much of precious time on this chapter. But this chapter is useful for IIFT, NIFT, FMS, SSC, CGL etc.

11.1 Angle

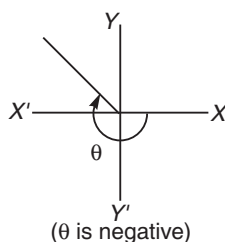
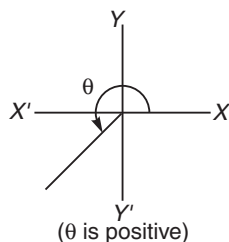
An angle is thought of as traced out by the rotation of a line (called the revolving line) from the initial position OX to a terminal position OP .



Then O is the vertex, OX is called the initial side and OP is called the terminal side of the angle.

11.2 Convention of Sign of Angles

It is customary to regard the angle traced out by a **counter-clockwise** rotation as positive and one traced out by a **clockwise** rotation as negative. The angle is indicated by using an arrow, starting from the initial line and ending at the terminal line.



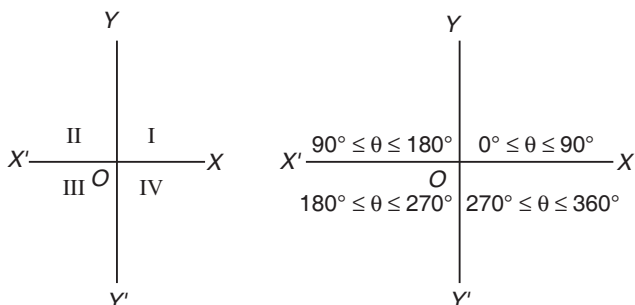
Chapter Checklist

- Angle
- Convention of Sign of Angles
- Quadrants
- Measures of Angles
- Important Facts
- Trigonometrical Ratios
- Trigonometrical Ratios of Negative and Associated Angles
- Sum, Difference and Product Formulae
- Trigonometrical Ratios of Multiple and Sub-multiple Angles
- Properties of Triangles
- Height and Distance
- CAT Test

11.3 Quadrants

Let XOX' and YOY' be two mutually perpendicular lines in any plane. These lines divide the plane into four regions called quadrants, numbered counter-clockwise.

Thus the quadrants XOY , YOX' , $X'OY'$ and $Y'OX$ are respectively called the first quadrant, the second quadrant, the third quadrant and the fourth quadrant.



NOTE θ will be more than 360° , if after making one complete revolution, the revolving line does not stop at the initial position OX but proceeds on further on.

11.4 Measures of Angles

There are three different systems of units for measurement in trigonometry, viz. (i) Sexagesimal system (ii) Centesimal system (iii) Circular system.

Sexagesimal System

In this system, 1 right angle is divided into 60 equal parts and each part (or division) is called a degree (i.e., 1° , read as 1 degree).

$$1^\circ = 60' \quad (60 \text{ sexagesimal minutes})$$

$$1' = 60'' \quad (60 \text{ sexagesimal seconds})$$

This system is called the Common or the English system.

Centesimal System

In this system, 1 right angle is divided into 100 equal parts and each part (or division) is called a grade (or 1^g).

$$90^\circ = 100^g$$

$$1^g = 100' \quad (K' \rightarrow K \text{ centesimal minute})$$

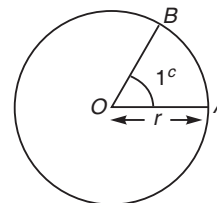
$$1' = 100'' \quad (K'' \rightarrow K \text{ centesimal second})$$

Circular Measure

The unit of measurement of angles in this system is a radian (or 1^c).

A **radian** is defined as the angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle and it is denoted by 1^c .

In the figure $OA = OB = r$, (radius of the circle) and $\angle AOB = 1 \text{ radian or } 1^c$.



11.5 Important Facts

- In all circles, the ratio of the circumference to its diameter is constant. (Circumference = $\pi \times \text{diameter}$) π is an incommensurable number, i.e., it cannot be expressed exactly either by a whole number or by a fraction. Its approximate value is $22/7$ or 3.1416 , correct to four places of decimal only.
- A radian is a constant angle.
1 radian (1^c) = a constant angle
- $\pi \text{ radian} = 180^\circ$ (or $1^\circ = \frac{\pi}{180} \text{ radian}$)
- If an arc of length 's' subtends an angle θ radian at the centre of a circle of radius r , then $s = r\theta$. $\left(\theta = \frac{\text{arc}}{\text{radius}}\right)$
- (i) Area of the sector $AOB = \frac{1}{2} r^2 \theta$
(ii) Area of the sector $AOB = \frac{1}{2} rs$ ($s \rightarrow$ length of arc AB)

Exp. 1) Express the following angles in radian measure and centesimal measure :

- (i) 45° (ii) $20^\circ 35'$ (iii) $50^\circ 38' 40''$

Solution

$$(i) \because 180^\circ = \pi \text{ rad} \quad \therefore 1^\circ = \frac{\pi}{180} \text{ rad}$$

$$\therefore 45^\circ = 45 \times \frac{\pi}{180} = \frac{\pi}{4} \text{ rad}$$

$$\text{Now, } 90^\circ = 100 \text{ grade}$$

$$\Rightarrow 45^\circ = 50 \text{ grade}$$

$$(ii) 20^\circ 35' = 20 \frac{35}{60} \text{ degree} = \frac{247}{12} \text{ degree}$$

$$\text{Now, } 180 \text{ degree} = \pi \text{ rad}$$

$$\therefore \frac{247}{12} \text{ degree} = \frac{247}{12} \times \frac{\pi}{180} = \frac{247}{2160} \pi \text{ rad}$$

$$\text{and } 90^\circ = 100 \text{ grade}$$

$$\therefore \left(\frac{247}{12}\right)^\circ = \frac{247}{12} \times \frac{100}{90} = \frac{2470}{108} \text{ grade}$$

$$\begin{aligned}
 &= \frac{1235}{54} \text{ grade} \\
 \text{(iii) } 50^\circ 38' 40'' &= 50^\circ 38 \frac{40}{60} \text{ min} = 50^\circ \frac{116}{3} \text{ min} = 50^\circ \frac{116}{3 \times 60} \text{ degree} \\
 &= \frac{2279}{45} \text{ degree}
 \end{aligned}$$

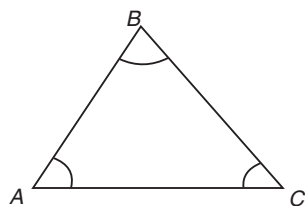
Now $180^\circ = \pi \text{ rad}$

$$\therefore \left(\frac{2279}{45} \right)^\circ = \frac{2279}{45} \times \frac{\pi}{180} = \frac{2279}{8100} \pi \text{ rad and } 90^\circ = 100^\circ$$

$$\Rightarrow \left(\frac{2279}{8100} \right)^\circ = \frac{2279}{8100} \times \frac{100}{90} = \frac{2279}{7290} \text{ grade.}$$

Exp. 2) One angle of a triangle is 54° and another angle is $\pi/4$ radian. Find the third angle in centesimal unit.

Solution Let $\angle A = 54^\circ$ and $\angle B = \frac{\pi}{4} \text{ rad}$



$$\therefore B = 45^\circ$$

$$\text{Thus, } \angle A + \angle B = 99^\circ$$

$$\therefore \angle C = 180 - 99 = 81^\circ \quad (\because \angle A + \angle B + \angle C = 180^\circ)$$

$$\text{Now, since } 90^\circ = 100 \text{ grade}$$

$$\Rightarrow 81^\circ = 81 \times \frac{100}{90} = 90 \text{ grade}$$

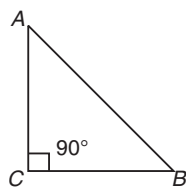
Exp. 3) The difference between two acute angles of a right-angled triangle is $\pi/6$ radians. Find the angle in degrees.

Solution $\angle A + \angle B = 90^\circ$

$$\text{and } \angle A - \angle B = \frac{\pi}{6} = 30^\circ$$

$$\therefore \angle A = 60^\circ \text{ and } \angle B = 30^\circ$$

Hence the two acute angles are 60° and 30° .



Exp. 4) In a circle of diameter 60 m, the length of a chord is 30 m. Find the length of the minor arc on one side of the chord. What is the length of the major arc?

Solution $2r = 60 \text{ m} \Rightarrow r = 30 \text{ m}$

$$\therefore OA = OB = AB = 30 \text{ m}$$

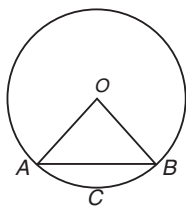
$\therefore \triangle OAB$ is an equilateral triangle.

$$\text{Thus, } \angle AOB = 60^\circ = \frac{\pi}{3} \text{ rad}$$

$$\text{Now, since } s = \theta r = \frac{\pi}{3} \times 30$$

$$= 10 \times 3.142$$

$$= 31.42 \text{ m (approx.)}$$



Thus the length of minor arc $ACB = 31.42 \text{ m}$ and the length of major arc $= (2\pi r - \text{minor arc})$
 $= 2\pi \times 30 - 31.42 = 157.1 \text{ m (approx.)}$

Exp. 5) An arc AB of a circle subtends an angle x radians at the centre O of the circle. Given that the area of the sector AOB is equal to the square of the length of the arc AB , find the value of x .

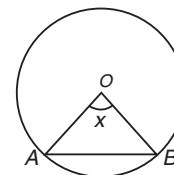
Solution Let arc $AB = s$

Given that area of the sector

$$AOB = (\text{arc } AB)^2$$

$$\therefore \frac{1}{2} r^2 x = s^2 \quad \text{and} \quad s = rx \quad (\because \theta = x)$$

$$\therefore \frac{1}{2} r^2 x = r^2 x^2 \quad \text{or} \quad x = \frac{1}{2} \text{ radian}$$



Exp. 6) Two arcs of two different circles are of equal lengths. If these arcs subtend angles of 45° and 60° at the centres of the circles. Find the ratio of the radii of the two circles.

Solution $45^\circ = \frac{\pi}{4} \text{ rad} \quad \text{and} \quad 60^\circ = \frac{\pi}{3} \text{ rad}$

Let r_1 and r_2 be the radii of the two circles and s be the length of each arc.

$$\therefore s = r_1 \frac{\pi}{4} = r_2 \frac{\pi}{3} \quad (\because s = r\theta)$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{4}{3}$$

Hence the required ratio of radii = 4 : 3

Exp. 7) The wheel of a railway carriage is 4 ft in diameter and makes 6 revolution per second, how fast is the train going? ($\pi = 3.14$)

Solution Radius (r) of the wheel = 2 ft

$$\therefore \text{Circumference of wheel} = 2 \times 3.14 \times 2 = 12.56 \text{ ft}$$

Since the wheel makes 6 revolutions per second,

$$\therefore \text{Distance traversed in 1 second} = 6 \times 12.56 = 75.36 \text{ ft.}$$

Hence the velocity of the train = 75.36 ft/s

$$= \frac{75.36 \times 60 \times 60}{3 \times 1760} = 51.38 \text{ mile/h}$$

Exp. 8) A horse trots uniformly along a circular track of radius 27 m. The angle subtended at the centre of the track by the arc passed over by the horse in 3 seconds is 70° . What distance will the horse pass over in $\frac{1}{2}$ minute.

Solution $r = 27 \text{ m}; \theta = 70^\circ = \frac{7\pi}{18} \text{ radian}$

$$\therefore s = r\theta = 27 \times \frac{7\pi}{18} = 33 \text{ m (approx.)}$$

$$\therefore \text{Distance passed over by the horse in } \frac{1}{2} \text{ minute}$$

$$= \frac{33}{3} \times 30 = 330 \text{ m (approx.)}$$

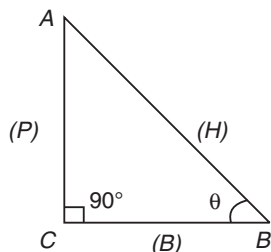
Introductory Exercise 11.1

- The perimeter of a certain sector of a circle is equal to the length of the arc of the semicircle having the same radius. The angle of the sector (approx.) is :
 (a) $65^\circ 27' 16''$ (b) $68^\circ 18' 19''$
 (c) $56^\circ 52' 18''$ (d) none of these
- The length of a pendulum is 8 m while the pendulum swings through 1.5 rad, find the length of the arc through which the tip of the pendulum passes :
 (a) 8 m (b) 9 m
 (c) 12 m (d) none of these
- The angles of a triangle are in AP and the greatest angle is 75° . Find all the three angles in degrees :
 (a) $55^\circ, 55^\circ, 70^\circ$ (b) $45^\circ, 60^\circ, 75^\circ$
 (c) $40^\circ, 65^\circ, 75^\circ$ (d) none of these
- A circular wire of radius 2.5 cm is cut and bent so as to lie along the circumference of a hoop whose radius is 1.29 m. Find (in degrees) the angle which is subtended at the centre of the hoop :
 (a) 9.67° (b) 7.69°
 (c) 6.97° (d) none of these
- A circle is drawn on AB as diameter. The centre of the circle is O and the length $AB = 13$ cm. P is a point on the circumference of the circle such that the chord $AP = 12$ cm. Calculate the value of the angles PAB and POB in radians :
 (a) 0.395, 0.789 (b) 0.786, 0.735
 (c) 0.398, 0.689 (d) 0.786, 0.753

11.6 Trigonometrical Ratios

$$\sin \theta = \frac{P}{H}, \cos \theta = \frac{B}{H}, \tan \theta = \frac{P}{B}$$

$$\operatorname{cosec} \theta = \frac{H}{P}, \sec \theta = \frac{H}{B}, \cot \theta = \frac{B}{P}$$



Remember

$\sin \theta$	$\cos \theta$	$\tan \theta$	
$\frac{P}{H}$	$\frac{B}{H}$	$\frac{P}{B}$	$\Rightarrow$ Pandit Badri Prasad
$\frac{H}{P}$	$\frac{H}{B}$	$\frac{B}{P}$	$\Rightarrow$ Hari Hari Bol
$\operatorname{cosec} \theta$	$\sec \theta$	$\cot \theta$	

$P \rightarrow$ Perpendicular (AC) $B \rightarrow$ Base (BC)

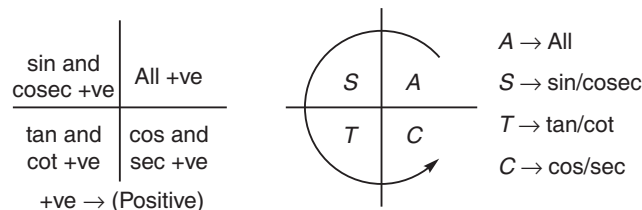
$H \rightarrow$ Hypotenuse (AB)

$$\begin{aligned} \text{Thus, } \sin \theta \operatorname{cosec} \theta &= 1 & \text{and } \tan \theta &= \frac{\sin \theta}{\cos \theta} \\ \cos \theta \sec \theta &= 1 & \tan \theta \cot \theta &= 1 \\ \cot \theta &= \frac{\cos \theta}{\sin \theta} \end{aligned}$$

NOTE $(\sin \theta)^{-1}$ is not written as $\sin^{-1} \theta$. Thus $\sin^{-1} \theta \neq (\sin \theta)^{-1}$ etc but $(\sin \theta)^2 = \sin^2 \theta$ and $(\sin \theta)^3 = \sin^3 \theta$

$$\begin{aligned} \sin^2 \theta + \cos^2 \theta &= 1 \Rightarrow \sec^2 \theta - \tan^2 \theta = 1 \\ \operatorname{cosec}^2 \theta - \cot^2 \theta &= 1 \end{aligned}$$

Sign of Trigonometrical Functions



Remember “Add Sugar To Coffee”

Values of T-Ratios

Angle	$\theta = 0$	$\theta = 30^\circ$	$\theta = 45^\circ$	$\theta = 60^\circ$	$\theta = 90^\circ$	$\theta = 180^\circ$
$\sin$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0
$\cos$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	-1
$\tan$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞	0
cosec	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1	∞
$\sec$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞	-1
$\cot$	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0	∞

Remember

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}, \sec \theta = \frac{1}{\cos \theta}, \cot \theta = \frac{1}{\tan \theta}$$

$$\text{and } \tan \theta = \frac{\sin \theta}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\text{and } \tan \theta = \frac{1}{\cot \theta}$$

$$\text{or } \cot \theta = \frac{1}{\tan \theta}$$

Range of Trigonometric Ratios

- (i) $-1 \leq \sin \theta \leq 1 \Rightarrow |\sin \theta| \leq 1$
 (ii) $-1 \leq \cos \theta \leq 1 \Rightarrow |\cos \theta| \leq 1$
 (iii) $\operatorname{cosec} \theta \leq -1$ and $\operatorname{cosec} \theta \geq 1 \Rightarrow |\operatorname{cosec} \theta| \geq 1$
 (iv) $\sec \theta \leq -1$ and $\sec \theta \geq 1 \Rightarrow |\sec \theta| \geq 1$
 (v) $-\infty < \tan \theta < \infty$
i.e., $\tan \theta$ may assume any value.

Increasing and Decreasing Functions of T-Ratios

	1 st quadrant	2 nd quadrant	3 rd quadrant	4 th quadrant
$\sin \theta$	increases from 0 to 1	decreases from 1 to 0	decreases from 0 to -1	increases from -1 to 0
$\cos \theta$	decreases from 1 to 0	decreases from 0 to -1	increases from -1 to 0	increases from 0 to 1
$\tan \theta$	increases from 0 to ∞	increases from $-\infty$ to 0	increases from 0 to ∞	increases from $-\infty$ to 0

Introductory Exercise 11.2

Directions (for Q. Nos. 1 to 7) Find the values of the following expressions.

- $\frac{\sec^2 \theta - 1}{\tan^2 \theta} = ?$
 (a) 1 (b) 2 (c) 3 (d) 4
- $\sin^4 \theta + \sin^2 \theta \cos^2 \theta = ?$
 (a) 0 (b) 1 (c) 2 (d) $\sin^2 \theta$
- $\frac{\sin \theta \operatorname{cosec} \theta \tan \theta \cot \theta}{\sin^2 \theta + \cos^2 \theta} = ?$
 (a) 0 (b) 1 (c) -1 (d) 2
- $\sin^2 A \cot^2 A + \cos^2 A \tan^2 A = ?$
 (a) -1 (b) 0 (c) 3 (d) 1
- $\tan \theta + \cot \theta$ is
 (a) 1 (b) $\tan \theta$
 (c) $\operatorname{cosec} \theta \cot \theta$ (d) $\sec \theta \operatorname{cosec} \theta$
- $\frac{3 - 4 \sin^2 \theta}{\cos^2 \theta} + \tan^2 \theta$ is
 (a) 1 (b) 2
 (c) 3 (d) none of these
- $\frac{\cot A + \tan B}{\cot B + \tan A}$ is
 (a) $\tan A \cot B$ (b) $\cot A \tan B$
 (c) 1 (d) none of these
- If $\sin \theta = \frac{21}{29}$, find the value of $\sec \theta + \tan \theta$, if θ lies between 0 and $\pi/2$
 (a) 1 (b) $\pi/2$
 (c) $5/2$ (d) none of these
- If A is in the fourth quadrant and $\cos A = \frac{5}{13}$, find the value of $\frac{13 \sin A + 5 \sec A}{5 \tan A + 6 \operatorname{cosec} A}$
 (a) $-2/37$ (b) $-3/27$
 (c) $2/37$ (d) can't be determined

- Find the value of,
 $\frac{4}{3} \cot^2 30^\circ + 3 \sin^2 60^\circ - 2 \operatorname{cosec}^2 60^\circ - \frac{3}{4} \tan^2 30^\circ :$
 (a) $10/3$ (b) $11/3$ (c) 4 (d) none
- Find the perimeter of a regular octagon inscribed in a circle of radius 100 cm :
 (a) 532.87 cm (b) 612.32 cm
 (c) 378.32 cm (d) 875 cm
- Find the altitude and base of an isosceles triangle whose vertical angle is 65° and whose equal sides are 415 cm :
 (a) 350 cm, 646 cm (b) 350 cm, 446 cm
 (c) 630 cm, 445 cm (d) none of these
- If $5 \sin^2 \theta + 3 \cos^2 \theta = 4$, find the value of $\sin \theta$ and $\cos \theta$
 (a) $\pm \frac{1}{\sqrt{2}}, \pm \frac{1}{\sqrt{2}}$ (b) $\pm \frac{\sqrt{3}}{2}, \pm \sqrt{2}$
 (c) $\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}$ (d) none of these
- Find the value of $\sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} + \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$:
 (a) $2 \sec \theta$ (b) $\sec \theta$
 (c) $2 \operatorname{cosec} \theta$ (d) none of these
- If $\tan \theta = \frac{a}{b}$, find the value of $\frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta}$
 (a) $\frac{a^2 - b^2}{a^2 + b^2}$ (b) $\frac{b^2 - a^2}{b^2 + a^2}$ (c) $\frac{a^2 + b^2}{a^2 - b^2}$ (d) none

Directions (for Q. Nos. 16 to 18) For the every θ , $0 < \theta \leq 90$, find the values of the following angles.

- $\tan \theta + \cot \theta = 2$
 (a) 30° (b) 45° (c) 60° (d) 55°
- $2 \sin^2 \theta + 4 \cos^2 \theta = 3 :$
 (a) 30° (b) 60° (c) 45° (d) 80°
- $2 \sin^2 \theta - 3 \sin \theta = -1 :$
 (a) 30° (b) 45°
 (c) 90° (d) both (a) and (c)

11.7 Trigonometrical Ratios of Negative and Associated Angles

Angle	$-\theta$	$(90 - \theta)$	$(90 + \theta)$	$(180 - \theta)$	$(180 + \theta)$	$(360 - \theta)$	$(360 + \theta)$
$\sin \theta$	$-\sin \theta$	$\cos \theta$	$\cos \theta$	$\sin \theta$	$-\sin \theta$	$-\sin \theta$	$\sin \theta$
$\cos \theta$	$\cos \theta$	$\sin \theta$	$-\sin \theta$	$-\cos \theta$	$-\cos \theta$	$\cos \theta$	$\cos \theta$
$\tan \theta$	$-\tan \theta$	$\cot \theta$	$-\cot \theta$	$-\tan \theta$	$\tan \theta$	$-\tan \theta$	$\tan \theta$

11.8 Sum, Difference and Product Formulae

- $\sin(A + B) = \sin A \cos B + \cos A \sin B$
- $\sin(A - B) = \sin A \cos B - \cos A \sin B$
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$
- $\cos(A - B) = \cos A \cos B + \sin A \sin B$
- $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
- $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$
- $\sin(A + B) \sin(A - B) = \sin^2 A - \sin^2 B$
 $= \cos^2 B - \cos^2 A$
- $\cos(A + B) \cos(A - B) = \cos^2 A - \sin^2 B$
 $= \cos^2 B - \sin^2 A$
- $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$
- $2 \cos A \sin B = \sin(A + B) - \sin(A - B)$
- $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$
- $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$
- $\cot(A + B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$
- $\cot(A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$
- $\sin C + \sin D = 2 \sin \frac{C + D}{2} \cos \frac{C - D}{2}$
- $\sin C - \sin D = 2 \cos \frac{C + D}{2} \sin \frac{C - D}{2}$
- $\cos C + \cos D = 2 \cos \frac{C + D}{2} \cos \frac{C - D}{2}$
- $\cos C - \cos D = 2 \sin \frac{C + D}{2} \sin \frac{D - C}{2}$

Introductory Exercise 11.3

- If $A + B = 45^\circ$, find the value of $\tan A + \tan B + \tan A \tan B$:
(a) -1 (b) $\frac{1}{2}$ (c) $\sqrt{3}$ (d) 1
- Find the value of $\tan 75^\circ$:
(a) $\frac{\sqrt{3} + 1}{2\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2 + \sqrt{2}}$
(c) $\frac{\sqrt{3} - 1}{2\sqrt{2}}$ (d) $\frac{2 + \sqrt{2}}{\sqrt{3}}$
- Find the value of $\cos 28^\circ \cdot \cos 32^\circ - \sin 28^\circ \cdot \sin 32^\circ$:
(a) 1 (b) $1/2$
(c) $1/3$ (d) can't be determined
- If $\cos A = \frac{3}{5}$ and $\sin B = \frac{5}{13}$, find the value of $\frac{\tan A + \tan B}{1 - \tan A \tan B}$:
(a) $\frac{63}{16}$ (b) $\frac{36}{16}$
(c) $\frac{61}{36}$ (d) none of these
- Find the value of $\sin^2(120^\circ - A) + \sin^2 A + \sin^2(120^\circ + A)$:
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{\sqrt{3}}{2}$
- If $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$ and α, β lie between 0° and 45° , find the value of $\tan 2\alpha$:
(a) $\frac{56}{63}$ (b) $\frac{56}{33}$
(c) $\frac{25}{33}$ (d) $\frac{65}{63}$
- Find the maximum and minimum values of $7 \cos \theta + 24 \sin \theta$:
(a) 25 and -25 (b) 16 and -9
(c) 25 and 0 (d) 36 and 25
- Find the value of $2 \cos 45^\circ \cdot \cos 15^\circ$:
(a) $\frac{(\sqrt{3} + 1)}{2\sqrt{2}}$ (b) $\frac{(\sqrt{3} - 1)}{2}$
(c) $\frac{\sqrt{3} + 1}{2}$ (d) none of these

9. Find the value of $\frac{\sin 75^\circ + \sin 15^\circ}{\cos 75^\circ + \cos 15^\circ}$:
- (a) 1 (b) 2
(c) $\frac{\sqrt{3}}{2}$ (d) can't be determined
10. Find the value of $\cos 20^\circ + \cos 100^\circ + \cos 140^\circ$:
- (a) -1 (b) 0 (c) 1 (d) none
11. $\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ$ is :
- (a) $\frac{1}{16}$ (b) $\frac{1}{50}$ (c) $\frac{1}{24}$ (d) $\frac{1}{25}$
12. What is the value of;

$$\frac{\sin(90^\circ - \theta) \sec(180^\circ - \theta) \sin(-\theta)}{\sin(180^\circ + \theta) \cot(360^\circ - \theta) \operatorname{cosec}(90^\circ + \theta)} :$$
 (a) $\sin \theta$ (b) $\cos \theta$ (c) 1 (d) $\frac{1}{\sqrt{2}}$
13. If $\sin(\alpha + \beta) = \frac{4}{5}$, $\sin(\alpha - \beta) = \frac{5}{13}$, find the value of $\tan 2\alpha$:
- (a) $\frac{16}{36}$ (b) $\frac{63}{16}$
(c) $\frac{36}{16}$ (d) none of these
14. Find the value of $\frac{\cos 2B - \cos 2A}{\sin 2A + \sin 2B}$:
- (a) $\tan(A - B)$
(b) $\cos(A - B)$
(c) $\cot(A - B)$
(d) $\tan(A + B)$
15. $\frac{\sin A + \sin 3A + \sin 5A + \sin 7A}{\cos A + \cos 3A + \cos 5A + \cos 7A}$ is :
- (a) $\tan 2A$ (b) $\tan 3A$
(c) $\tan 4A$ (d) $\tan 8A$

11.9 Trigonometrical Ratios of Multiple and Sub-multiple Angles

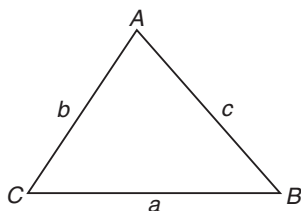
- $\sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$
- $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A$
 $= 2 \cos^2 A - 1 = \frac{1 - \tan^2 A}{1 + \tan^2 A}$
- $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$
- $\sin 3A = 3 \sin A - 4 \sin^3 A$
- $\cos 3A = 4 \cos^3 A - 3 \cos A$
- $\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$
- $\sin A = 2 \sin\left(\frac{A}{2}\right) \cos\left(\frac{A}{2}\right) = \frac{2 \tan(A/2)}{1 + \tan^2(A/2)}$
- $\cos A = \cos^2\left(\frac{A}{2}\right) - \sin^2\left(\frac{A}{2}\right)$ etc.
- $\tan A = \frac{2 \tan(A/2)}{1 - \tan^2(A/2)}$

Introductory Exercise 11.4

- If $\sin \theta = \frac{4}{5}$, find the value of $\sin 2\theta$:
 (a) 24/25 (b) 16/25
 (c) 9/20 (d) none of these
- $\frac{\sin 2A}{1 + \cos 2A}$ is :
 (a) $\tan 2A$ (b) $\cos 2A$
 (c) $\tan A$ (d) none of these
- $\cos^4 \theta - \sin^4 \theta$ is :
 (a) $\cos 2\theta$ (b) $\sin 2\theta$
 (c) $\tan 2\theta$ (d) none of these
- If $\cot x = \frac{a}{b}$, find the value of $a \cos 2x + b \sin 2x$
 (a) a (b) b
 (c) $\sin a$ (d) a/b
- Find the value of $\cot 3\theta$:
 (a) $\frac{\cot^3 \theta - 3 \cot \theta}{3 \cot^2 \theta}$ (b) $\frac{\cot^3 \theta - 3 \cot \theta}{3 \cot^2 \theta - 1}$
 (c) $\frac{3 \cot \theta - \cot^3 \theta}{3 \cot^2 \theta - 1}$ (d) none of these
- Find the value of $\sin\left(22\frac{1}{2}^\circ\right)$:
 (a) $\frac{\sqrt{(2-\sqrt{2})}}{2}$ (b) $\frac{\sqrt{(2-\sqrt{3})}}{3}$
 (c) $\frac{\sqrt{2-\sqrt{2}}}{4}$ (d) none of these
- If $2 \cos \theta = x + \frac{1}{x}$, find the value of $2 \cos 3\theta$
 (a) $x^3 + \frac{1}{x^3}$ (b) $x^2 + \frac{1}{x^2}$ (c) $x^3 - \frac{1}{x^3}$ (d) $\sqrt{x + \frac{1}{x}}$
- Find the value of $\tan A + \tan B + \tan C$, if $A + B + C = \pi$
 (a) $\tan A \tan B \tan C$ (b) 1
 (c) 0 (d) $\cot A \cot B \cot C$

11.10 Properties of Triangles

A triangle ABC has three angles and three sides. The three angles are denoted by A, B and C while the three sides are denoted by a, b and c respectively. The area of the triangle is denoted by Δ and its perimeter is denoted by $2s$, so that $a + b + c = 2s$. Thus 's' is the semiperimeter of the triangle and $s = \frac{a + b + c}{2}$. Also we denote the circumradius by R and inradius by r .



1. The law of sines

In any triangle the sides are proportional to the sines of the opposite angles *i.e.*, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

2. The law of cosines

The square on any side of a triangle is equal to the sum of the squares on the other two sides, minus twice the product of those two sides and the cosine of the included angle

i.e.,

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Rightarrow b^2 = c^2 + a^2 - 2ca \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

3. Area of a triangle

$$(i) \Delta = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B = \frac{1}{2} ab \sin C$$

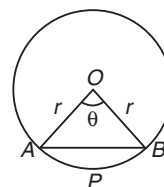
$$(ii) \Delta = \sqrt{s(s-a)(s-b)(s-c)} \quad (\text{Hero's formula})$$

4. The area of a segment of a circle

Area of the segment APB

$$= \text{Area of the sector } AOB - \text{Area of } \triangle AOB$$

$$= \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin \theta = \frac{1}{2} r^2 (\theta - \sin \theta)$$



5. Circumradius of a circle (R)

$$(i) R = \frac{a}{2 \sin A} = \frac{b}{2 \sin B} = \frac{c}{2 \sin C}$$

$$(ii) R = \frac{abc}{4\Delta}$$

6. Inradius of a circle (r)

$$r = \frac{\Delta}{s} \quad \left(s = \frac{a + b + c}{2} \right)$$

Introductory Exercise 11.5

- If $\cos B = \frac{\sin A}{2 \sin C}$, find the nature of triangle :
 (a) isosceles (b) scalene
 (c) can't be determined (d) none of these
- If the angles of a triangle are in the ratio $1 : 2 : 3$ and its circumradius is 10 cm, find the sides of the triangle (in cm) :
 (a) 8, 16, 24 (b) 18, 12, 6
 (c) 10, 20, $20\sqrt{3}$ (d) 10, 20, $10\sqrt{3}$
- If $a = 1, b = \sqrt{3}$ and $A = 30^\circ$, find the value of the angle B
 (a) 60° (b) 30°
 (c) 120° (d) either (a) or (c)
- Find B and C of a triangle ABC , if $b = 2$ cm, $c = 1$ cm and $A = 60^\circ$:
 (a) 90° and 45° (b) 90° and 30°
 (c) 60° and 30° (d) none of these
- In a triangle ABC , $a = 1$ cm, $b = \sqrt{3}$ cm and $C = \frac{\pi}{6}$, find the third side of the triangle :
 (a) 1 cm (b) 1.5 cm (c) 2 cm (d) $3\sqrt{3}$ cm
- If the angles of a triangle are in the ratio $1 : 2 : 3$, find the ratio between corresponding sides :
 (a) $3 : 2 : 1$ (b) $1 : \sqrt{3} : 2$ (c) $6 : 3 : 2$ (d) $1 : \sqrt{2} : \sqrt{3}$
- If $a = 2b$ and $A = 3B$, find the angles A, B, C of $\triangle ABC$:
 (a) $90^\circ, 30^\circ, 60^\circ$ (b) $30^\circ, 60^\circ, 90^\circ$
 (c) $45^\circ, 45^\circ, 90^\circ$ (d) data insufficient
- If in any triangle $a = 13$ cm, $b = 14$ cm and $c = 15$ cm, find the inradius of the circle :
 (a) 2 cm (b) 6.5 cm (c) 4 cm (d) 7 cm
- In a right angled triangle $a^2 + b^2 + c^2$ is :
 (a) R^2 (b) $4R^2$ (c) $8R^2$ (d) none
- Abhinav and Brijesh started walking simultaneously @ 3 km/h and 4 km/h from the same point 'C' on two different paths which diverge from each other at an angle of 120° . They walked for 6 hours and they stopped at A and B respectively. What is the least possible distance between Abhinav and Brijesh when they are at A and B respectively.
 (a) $36\sqrt{37}$ km (b) $6\sqrt{37}$ km
 (c) $12\sqrt{37}$ km (d) none of these

11.11 Height and Distance

Angle of Elevation

$\angle OPM$ is called as the angle of elevation.

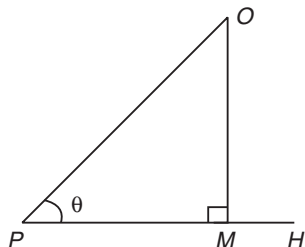


Fig. (i)

Suppose a person P is looking an object O which is at a higher level than P , then the angle θ ($\angle OPM$) is the angle of elevation for that person.

Angle of Depression

$\angle OPM$ in fig (ii) is called as angle of depression.

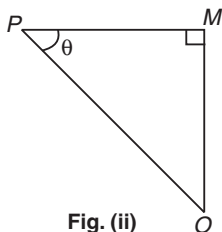


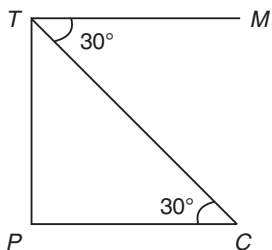
Fig. (ii)

Suppose a person P is looking downward an object O , which is at a lower level than P , then the angle θ ($\angle OPM$) is called as the angle of depression.

NOTE It should be noted that the angle of elevation of one position as seen from the other is equal to the angle of depression of the latter as seen from the former.

Exp. 1) From the top of a tower 180 m high, it was observed that the angle of depression of the bottom of a cat sitting on the ground was 30° . Find the distance of the cat from the foot of the tower.

Solution Let PT be the tower and cat is sitting at C



Then $\angle MTC = \angle TCP$

Now in $\triangle CPT$, $\angle TCP = 30^\circ$

$$\therefore \tan 30^\circ = \frac{TP}{PC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{180}{PC}$$

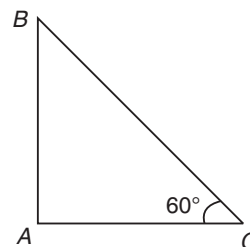
$$\Rightarrow PC = 180\sqrt{3} \text{ m} = 311.76 \text{ m}$$

Hence cat is 311.76 m away from the foot of the tower.

NOTE For a particular case, the angle of depression is equal to the angle of elevation.

Exp. 2) The angle of elevation of the top of a tower at a distance of 100 m from its foot on a horizontal plane is found to be 60° . Find the height of the tower.

Solution Let AB be the tower

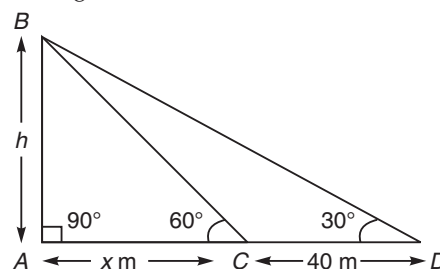


$$\text{then } \tan 60^\circ = \frac{AB}{AC}; \sqrt{3} = \frac{AB}{100} \Rightarrow AB = 100\sqrt{3} \text{ m} = 173.2 \text{ m}$$

Thus the height of the tower is 173.2 m.

Exp. 3) A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60° , when he retreats 40 m from the bank he finds the angle to be 30° . Find the height of the tree and the breadth of the river.

Solution Let height of the tree be AB and width of the river be AC .



$$\text{In } \triangle ABC, \tan 60^\circ = \frac{h}{x} \\ \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3} \quad \dots(i)$$

$$\text{and in } \triangle ABD, \frac{AB}{AD} = \tan 30^\circ \\ \frac{h}{(x+40)} = \frac{1}{\sqrt{3}} \quad \dots(ii)$$

From eqs. (i) and (ii), we get

$$\frac{x\sqrt{3}}{(x+40)} = \frac{1}{\sqrt{3}} \Rightarrow x = 20 \text{ m}$$

$$\therefore h = x\sqrt{3} = 20\sqrt{3} = 34.64 \text{ m (approx.)}$$

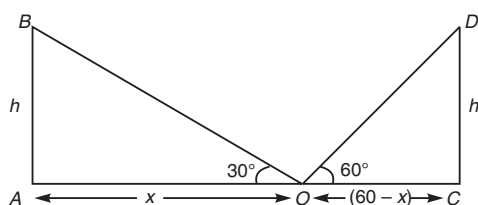
Thus the height of the tree is 34.46 m and breadth of the river is 20 m.

Exp. 4) Two towers of the same height stand on either side of a road 60 m wide. At a point on the road between the towers, the elevations of the towers are 60° and 30° . Find the height of the towers and the position of the point.

Solution In $\triangle AOB$

$$\frac{AB}{AO} = \frac{h}{x} = \frac{1}{\sqrt{3}} \quad \dots(i)$$

$$\Rightarrow h = \frac{x}{\sqrt{3}}$$



$$\text{and in } \triangle COD, \frac{CD}{OC} = \frac{h}{(60 - x)} = \sqrt{3} \quad \dots(ii)$$

from equation (i) and (ii)

$$\frac{x}{\sqrt{3}(60 - x)} = \sqrt{3} \Rightarrow x = 45 \text{ m}$$

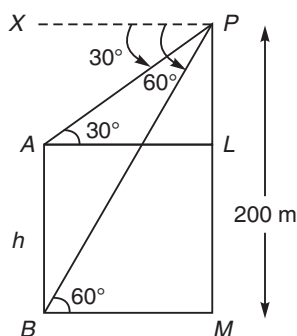
$$\therefore h = \frac{45}{\sqrt{3}} = 15\sqrt{3} = 15 \times 1.732 = 25.98 \text{ m}$$

Hence, the height of the towers is 25.98 m and the distance of the point 'O' from A is 45 m and C is 15 m.

Exp. 5) From the top of a cliff, 200 m high, the angle of depression of the top and bottom of a tower are observed to be 30° and 60° , find the height of the tower.

Solution Let the height of the tower be h m, then

$$\text{In } \triangle PBM, \tan 60^\circ = \frac{PM}{BM}$$



$$\sqrt{3} = \frac{200}{BM}$$

$$BM = \frac{200}{\sqrt{3}} \text{ m} = AL \text{ (also)}$$

$$\text{Now, in } \triangle ALP, \tan 30^\circ = \frac{PL}{AL} \Rightarrow \frac{1}{\sqrt{3}} = \frac{PL}{(200 / \sqrt{3})}$$

$$\Rightarrow PL = \frac{200}{3}$$

$$\therefore LM = PM - PL = 200 - \left(\frac{200}{3}\right)$$

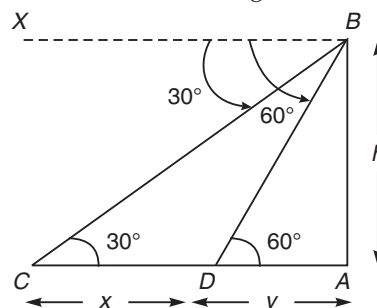
$$LM = \frac{400}{3}$$

$$\text{But } LM = AB = h$$

$$\therefore \text{Height of the tower } (h) = \frac{400}{3} \text{ m}$$

Exp. 6) A man on the top of a rock rising on a seashore observes a boat coming towards it. If it takes 10 minutes for the angle of depression to change from 30° to 60° , how soon the boat reach the shore?

Solution Let AB be the rock of height ' h ' metres.



Let C and D be the two positions of boat such that

$$\angle ACB = \angle XBC = 30^\circ$$

and

$$\angle ADB = \angle XBD = 60^\circ$$

Let $CD = x$ m and $AD = y$ m

$$\text{Now, in } \triangle ABD, \tan 60^\circ = \frac{h}{y}$$

$$\sqrt{3} = \frac{h}{y} \quad \dots(i)$$

$$\text{and in } \triangle ABC, \tan 30^\circ = \frac{h}{x + y} = \frac{1}{\sqrt{3}} \quad \dots(ii)$$

from equation (i) and (ii), we get

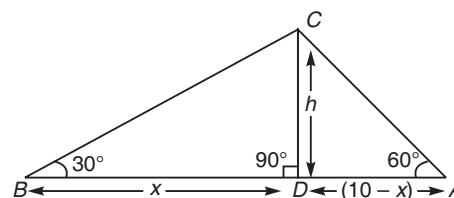
$$\frac{\sqrt{3}y}{x + y} = \frac{1}{\sqrt{3}} \Rightarrow x = 2y \Rightarrow y = \frac{x}{2}$$

Since the boat takes 10 minutes to cover x m, hence it will take $\frac{10}{2} = 5$ minutes to cover $y = \frac{x}{2}$ metres. Thus the required time = 5 minutes.

Exp. 7) The angles of elevation of an aeroplane from two places 10 km apart are found to be 60° and 30° respectively. Find the height of the aeroplane.

Solution Let A and B be the two places such that $AB = 10$ kms and let C be the position of the aeroplane at a height of h metres above AB . Let CD be perpendicular to AB .

In 1st case :



In $\triangle BCD$,

$$\frac{h}{x} = \frac{1}{\sqrt{3}} \quad \dots(i)$$

and $\frac{h}{10-x} = \sqrt{3} \quad \dots(ii)$

from equation (i) and (ii)

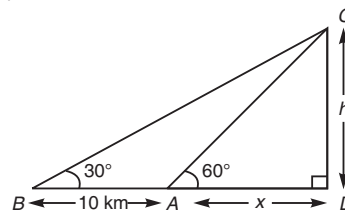
$$\frac{x}{\sqrt{3}(10-x)} = \sqrt{3}$$

$$\Rightarrow x = 7.5 \text{ km}$$

$$\therefore h = \frac{7.5}{\sqrt{3}} = \frac{7.5}{3} \times \sqrt{3} = 2.5 \times \sqrt{3}$$

$$= 4.33 \text{ km (approx.)}$$

In IInd case :



In $\triangle ACD$, $\frac{h}{x} = \sqrt{3} \quad \dots(i)$

In $\triangle BCD$, $\frac{h}{10+x} = \frac{1}{\sqrt{3}} \quad \dots(ii)$

Therefore from equation (i) and (ii)

$$h = 5\sqrt{3} \text{ km} \Rightarrow h = 8.66 \text{ km}$$

Introductory Exercise 11.6

- Two towers of the same height stand on opposite sides of a road 100 m wide. At a point on the road between the towers, the elevations of the towers are 30° and 45° . Find the height of the towers and the position of the point from the nearest tower :
 (a) 36.6 m and 63.4 m (b) 63.3 m and 63.4 m
 (c) 66.3 m and 63.4 m (d) 36.6 m and 86.4 m
- The elevation of a tower at a station A due north of it is 45° and at a station B due west of A is 30° . If $AB = 40$ m, find the height of the tower :
 (a) 26.26 m (b) 28.28 m
 (c) 38.5 m (d) none of these
- If the shadow of a tower is 30 m when the sun's altitude is 30° what is the length of the shadow when the sun's altitude is 60° ?
 (a) $10\sqrt{3}$ m (b) 20 m (c) 10 m (d) 12 m
- A helicopter is in a stationary position at a certain height over the lake. At a point 200 m above the surface of the lake, the angle of elevation of the helicopter is 45° . At the same time, the angle of depression of its reflection in the lake is 75° . Calculate the height of the helicopter from the surface of the lake :
 (a) $\frac{200}{\sqrt{3}}$ m (b) $200\sqrt{3}$ m (c) 200 m (d) 150 m
- A vertical tower stands on a horizontal plane and surmounted by a flagstaff of height 'h'. At a point on the plane, the angle of elevation of bottom of the flagstaff is α and that of the top of the flagstaff is β . Find the height of the tower :
 (a) $h \tan \alpha$ (b) $\frac{h \tan \alpha}{\tan \alpha - \tan \beta}$
 (c) $\frac{h \tan \alpha}{\tan \beta - \tan \alpha}$ (d) $\frac{\tan \alpha + \tan \beta}{h \tan \alpha}$
- A round balloon of radius 'r' subtends an angle α at the eye of the observer, while the angle of elevation of its centre is β . Find the height of the centre of balloon.
 (a) $r \operatorname{cosec} \left(\frac{\alpha}{2} \right) \sin \beta$ (b) $r \sin \alpha \operatorname{cosec} \beta$
 (c) $\frac{r}{2} \sin \beta$ (d) $r \sec \left(\frac{\alpha}{2} \right)$
- At the foot of a mountain the elevation of its summit is 45° , after ascending 1000 m towards the mountain up a stop of 30° inclination, the elevation is found to be 60° . Find the height of the mountain :
 (a) 1.3 km (b) 1.366 km
 (c) 2.72 km (d) none of these
- An aeroplane when 3000 m high passes vertically above another aeroplane at an instant when their angles of elevation at the same observing point are 60° and 45° respectively. How many metres higher is the one than the other?
 (a) 1248 m (b) 1188 m
 (c) 1752 m (d) 1268 m
- The angle of elevation of a cloud from a height h above the level of water in a lake is α and the angle of depression of its image in the lake is β . Find the height of the cloud above the surface of the lake :
 (a) $\frac{h \sin (\beta - \alpha)}{\sin (\alpha + \beta)}$ (b) $h \sin \alpha$
 (c) $\frac{h \sin (\alpha + \beta)}{\sin (\beta - \alpha)}$ (d) none of these
- A palm tree 90 ft high, is broken by the wind and its upper part meet the ground at an angle of 30° . Find the distance of the point where the top of the tree meets the ground, from its root :
 (a) 43.69 ft (b) 51.96 ft
 (c) 60 ft (d) 30 ft

CAT-Test

Questions Helping you bell the CAT

LEVEL 01 > BASIC LEVEL EXERCISE

1. If $0 < \theta < 90^\circ$, then $(\sin \theta + \cos \theta)$ is :
 (a) less than 1 (b) equal to 1
 (c) greater than 1 (d) greater than 2
2. The value of x satisfying the equation $\sin x + \frac{1}{\sin x} = \frac{7}{2\sqrt{3}}$ is :
 (a) 10° (b) 30°
 (c) 45° (d) 60°
3. If $\sin \theta - \cos \theta = 0$ and $0 < \theta \leq \pi/2$, then θ is equal to :
 (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$
 (c) $\frac{\pi}{6}$ (d) 0
4. Given that θ is acute and then $\sin \theta = \frac{3}{5}$. Let x, y be positive real numbers such that $3(x - y) = 1$, then one set of solutions for x and y expressed in terms of θ is given by :
 (a) $x = \sec \theta, y = \operatorname{cosec} \theta$ (b) $x = \cot \theta, y = \tan \theta$
 (c) $x = \operatorname{cosec} \theta, y = \cot \theta$ (d) $x = \sec \theta, y = \tan \theta$
5. Which one of the following pairs is correctly matched ?

If	then
(a) $x = \frac{1 + \sin 60^\circ - \cos 60^\circ}{1 + \sin 60^\circ + \cos 60^\circ}$	$x = \tan 60^\circ$
(b) $x = \frac{1 + \sin 90^\circ - \cos 90^\circ}{1 + \sin 90^\circ + \cos 90^\circ}$	$x = \tan 30^\circ$
(c) $x = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$	$x = \tan 60^\circ$
(d) $x = \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ}$	$x = \cos 60^\circ$
6. If $x \tan 45^\circ \cdot \cos 60^\circ = \sin 60^\circ \cot 60^\circ$, then x is equal to
 (a) 1 (b) $1/2$
 (c) $\sqrt{3}$ (d) $1/\sqrt{2}$
7. If θ lies in the second quadrant, then $\sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} + \sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}}$ is equal to :
 (a) $-2 \sec \theta$ (b) $2 \sec \theta$
 (c) $2 \operatorname{cosec} \theta$ (d) $2 \tan \theta$
8. $\sin^6 A + \cos^6 A$ is equal to :
 (a) $1 - 3 \sin^2 A \cos^2 A$ (b) $1 - 3 \sin A \cos A$
 (c) $1 + 3 \sin^2 A \cos^2 A$ (d) 1
9. If $\sec x = P, \operatorname{cosec} x = Q$, then :
 (a) $P^2 + Q^2 = PQ$ (b) $P^2 + Q^2 = P^2 Q^2$
 (c) $P^2 - Q^2 = P^2 Q^2$ (d) $P^2 + Q^2 = -P^2 Q^2$
10. $\sin^2 A \cos^2 B - \cos^2 A \sin^2 B$ simplifies to :
 (a) $\sin^2 A + \sin^2 B$ (b) $\cos^2 A + \cos^2 B$
 (c) $\sin^2 A - \sin^2 B$ (d) $\sin^2 A - \cos^2 B$
11. If $\sin 2x = n \sin 2y$, then the value of $\frac{\tan(x+y)}{\tan(x-y)}$ is :
 (a) $\frac{n+1}{n-1}$ (b) $\frac{n-1}{n+1}$ (c) $\frac{1-n}{n+1}$ (d) $\frac{1+n}{1-n}$
12. The least value of $2 \sin^2 \theta + 3 \cos^2 \theta$ is :
 (a) 1 (b) 2 (c) 3 (d) 5
13. The value of $\tan(180^\circ + \theta) \cdot \tan(90^\circ - \theta)$ is :
 (a) 1 (b) -1
 (c) 0 (d) none of these
14. $\log \tan 1^\circ + \log \tan 2^\circ + \dots + \log \tan 89^\circ$ is :
 (a) 1 (b) $1/\sqrt{2}$
 (c) 0 (d) -1
15. If we convert $\sin(-566^\circ)$ to same trigonometrical ratio of a positive angle lying between 0° and 45° , then we get :
 (a) $\cos 26^\circ$ (b) $-\cos 26^\circ$
 (c) $\sin 26^\circ$ (d) $-\sin 26^\circ$
16. From the mast head of ship, the angle of depression of a boat is 60° . If the mast head is 150 m, then the distance of the boat from the ship is :
 (a) 86.6 m (b) 68.6 m
 (c) 66.8 m (d) none of these
17. A portion of a 30 m long tree is broken by tornado and the top struck up the ground making an angle 30° with ground level. The height of the point where the tree is broken is equal to :
 (a) $30/\sqrt{3}$ m (b) 10 m
 (c) $30\sqrt{3}$ m (d) 60 m

18. Two posts are 25 m and 15 m high and the line joining their tips makes an angle of 45° with horizontal. The distance between these posts is :
 (a) 5 m
 (b) $10/\sqrt{2}$ m
 (c) 10 m
 (d) $10\sqrt{2}$ m
19. The angle of elevation of the top of a tower at a point G on the ground is 30° . On walking 20 m towards the tower the angle of elevation becomes 60° . The height of the tower is equal to :
 (a) $10/\sqrt{3}$ m
 (b) $20\sqrt{3}$ m
 (c) $20/\sqrt{3}$ m
 (d) $10\sqrt{3}$ m
20. If $x = \sec \theta + \tan \theta$, $y = \sec \theta - \tan \theta$, then the relation between x and y is :
 (a) $x^2 + y^2 = 0$
 (b) $x^2 = y^2$
 (c) $x^2 = y$
 (d) $xy = 1$
21. The value of θ for which $\sqrt{3} \cos \theta + \sin \theta = 1$ is
 (a) 0
 (b) $\pi/3$
 (c) $\pi/6$
 (d) $\pi/2$
22. If $\tan \theta = \frac{4}{3}$, then the value of $\sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}}$ is
 (a) 1
 (b) 2
 (c) 3
 (d) 4
23. If the arcs of the same length in two circles subtend angles of 60° and 90° at the centre, then the ratio of their radii is
 (a) $\frac{1}{3}$
 (b) $\frac{1}{2}$
 (c) $\frac{3}{2}$
 (d) 2
24. In the third quadrant, the values of $\sin \theta$ and $\cos \theta$ are :
 (a) positive and negative respectively
 (b) negative and positive respectively
 (c) both positive
 (d) both negative
25. The value of $\frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2} \frac{\cos 35^\circ}{\sin 55^\circ}$ is :
 (a) 1
 (b) -1
 (c) $\frac{1}{2}$
 (d) $-\frac{1}{2}$
26. The value of θ ($0 \leq \theta \leq \pi/2$) satisfying the equation $\sin^2 \theta - 2 \cos \theta + \frac{1}{4} = 0$ is :
 (a) $\frac{\pi}{2}$
 (b) $\frac{\pi}{3}$
 (c) $\frac{\pi}{4}$
 (d) $\frac{\pi}{6}$
27. If $\cos \theta = \frac{4}{5}$ and $0 < \theta < 90^\circ$, then the value of $\frac{3 \cos \theta + 2 \operatorname{cosec} \theta}{4 \sin \theta - \cot \theta}$ is
 (a) $-\frac{43}{2}$
 (b) $-\frac{41}{2}$
 (c) $\frac{43}{8}$
 (d) $-\frac{43}{6}$
28. Maximum value of $(\cos \theta - \sin \theta)$ is :
 (a) $\sqrt{2}$
 (b) 1
 (c) $\frac{1}{2}$
 (d) $\frac{1}{\sqrt{2}}$
29. The value of $\sin 105^\circ$ is
 (a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$
 (b) $\frac{\sqrt{3}-1}{\sqrt{2}}$
 (c) $\frac{\sqrt{3}+1}{2\sqrt{2}}$
 (d) $\frac{\sqrt{3}+1}{\sqrt{2}}$
30. If $\tan \theta = t$, then $\sin 2\theta$ is equal to
 (a) $\frac{1}{1+t^2}$
 (b) $\frac{2t}{1+t^2}$
 (c) $\frac{t^2}{1+t}$
 (d) $\frac{1+t^2}{1+t}$
31. If $\tan \theta = \sqrt{2}$, then the value of θ is
 (a) less than $\frac{\pi}{4}$
 (b) equal to $\frac{\pi}{4}$
 (c) between $\frac{\pi}{4}$ and $\frac{\pi}{3}$
 (d) greater than $\frac{\pi}{3}$
32. If $\tan \theta = 2 - \sqrt{3}$, then $\tan (90^\circ - \theta)$ is equal to :
 (a) $2 + \sqrt{3}$
 (b) $2 - \sqrt{3}$
 (c) $3 + \sqrt{2}$
 (d) $3 - \sqrt{2}$
33. If from point 100 m above the ground the angles of depression of two objects due south on the ground are 60° and 45° , then the distance between the objects is :
 (a) $\frac{50(3-\sqrt{3})}{3}$ m
 (b) $\frac{50(3+\sqrt{3})}{3}$ m
 (c) $\frac{100(3+\sqrt{3})}{3}$ m
 (d) $\frac{100(3-\sqrt{3})}{3}$ m
34. If the length of shadow of a vertical pole on the horizontal ground is $\sqrt{3}$ times of its height, then the angle of elevation of sun is :
 (a) 15°
 (b) 30°
 (c) 45°
 (d) 60°
35. A kite is flown with a thread of 250 m length. If the thread is assumed to be stretched and makes an angle of 60° with the horizontal, then the height of the kite above the ground is (approx.) :
 (a) 216.25 m
 (b) 215.25 m
 (c) 212.25 m
 (d) 210.25 m

Answers

✓ *Introductory Exercise 11.1*

1. (a)	2. (c)	3. (b)	4. (c)	5. (a)

✓ *Introductory Exercise 11.2*

1. (a)	2. (d)	3. (b)	4. (d)	5. (d)	6. (c)	7. (b)	8. (c)	9. (a)	10. (a)
11. (b)	12. (b)	13. (a)	14. (c)	15. (a)	16. (b)	17. (c)	18. (d)		

✓ *Introductory Exercise 11.3*

1. (d)	2. (a)	3. (b)	4. (a)	5. (b)	6. (b)	7. (a)	8. (c)	9. (a)	10. (b)
11. (a)	12. (a)	13. (b)	14. (a)	15. (c)					

✓ *Introductory Exercise 11.4*

1. (a)	2. (c)	3. (a)	4. (a)	5. (b)	6. (a)	7. (a)	8. (a)

✓ *Introductory Exercise 11.5*

1. (a)	2. (d)	3. (d)	4. (b)	5. (a)	6. (b)	7. (a)	8. (c)	9. (c)	10. (b)

✓ *Introductory Exercise 11.6*

1. (a)	2. (b)	3. (c)	4. (b)	5. (c)	6. (a)	7. (b)	8. (d)	9. (c)	10. (b)

✓ *Level 01 Basic Level Exercise*

1. (c)	2. (d)	3. (b)	4. (c)	5. (c)	6. (a)	7. (b)	8. (a)	9. (b)	10. (c)
11. (a)	12. (b)	13. (b)	14. (c)	15. (c)	16. (a)	17. (b)	18. (c)	19. (d)	20. (d)
21. (d)	22. (b)	23. (c)	24. (d)	25. (c)	26. (b)	27. (c)	28. (b)	29. (c)	30. (b)
31. (c)	32. (a)	33. (d)	34. (b)	35. (a)					

Hints & Solutions

✓ Level 01 Basic Level Exercise

1 Let $z = \sin \theta + \cos \theta$
 $\Rightarrow z^2 = 1 + \sin 2\theta$
 $\because 0 < \theta < 90^\circ$ so $\sin 2\theta < 1$, so that $z^2 < 2$
 Thus $z < \sqrt{2}$ i.e., z is greater than 1.

2 Go through option.

3 $\sin \theta - \cos \theta = 0$
 $\Rightarrow \sin \theta = \cos \theta$
 $\Rightarrow \tan \theta = 1 \Rightarrow \theta = \frac{\pi}{4}$

4 Go through option.

Hint $\sin \theta = \frac{3}{5}$ and $\operatorname{cosec} \theta = \frac{5}{3}$

7 $\sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} + \sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} = \frac{(1 - \sin \theta) + (1 + \sin \theta)}{\cos \theta}$
 $= 2 \sec \theta$

8 $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

Let $a = \sin^2 \theta$, $b = \cos^2 \theta$, so that

$$a + b = \sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \sin^6 \theta + \cos^6 \theta = 1 - 3 \sin^2 \theta \cos^2 \theta$$

9 $\cos x = \frac{1}{P}$ and $\sin x = \frac{1}{Q}$

$$1 = \cos^2 x + \sin^2 x = \frac{1}{P^2} + \frac{1}{Q^2}$$

$$\Rightarrow P^2 + Q^2 = P^2 Q^2$$

10 $\sin^2 A (1 - \sin^2 B) - (1 - \sin^2 A) \sin^2 B = \sin^2 A - \sin^2 B$

11 $\frac{n}{1} = \frac{\sin 2x}{\sin 2y}$

$$\Rightarrow \frac{n+1}{n-1} = \frac{\sin 2x + \sin 2y}{\sin 2x - \sin 2y} = \frac{2 \sin (x+y) \cos (x-y)}{2 \cos (x+y) \sin (x-y)}$$

$$= \frac{\tan (x+y)}{\tan (x-y)}$$

12 The value is least when $\theta = 90^\circ$.

14 $\log \tan 1^\circ + \log \tan 89^\circ = \log \tan 1^\circ + \log \tan (90 - 1)$
 $= \log \tan 1^\circ + \log \cot 1^\circ$
 $= \log \tan 1^\circ \cdot \cot 1^\circ = \log 1 = 0$

Similarly, $\log \tan 2^\circ + \log \tan 88^\circ = 0$

Also, $\log \tan 45^\circ = \log 1 = 0$

Thus, the value of the expression is 0.

15 $\sin (-566^\circ) = -\sin (566^\circ)$
 $= -\sin (90 \times 6 + 26) = \sin 26^\circ$

CHAPTER 12

Geometry

12.1 Introduction

I think geometry is all about visualization and creativity. By connecting some dots or extending some line segments or rotating some angles you can easily figure out the correct answers. Practically, you need not be an artist or a painter, but at least you must be capable of visualizing the geometrical figures. So before going for the tedious and complex geometrical problems you must make yourself familiar and comfortable with the fundamental concepts of geometry. Basically, geometry consists of a very large proportion of problems in CAT. Sometimes, 15-20% problems related to this chapter are asked in CAT. Apart from CAT other competitive exams such as XAT, IIFT and SSC also ask a plethora of questions from this chapter. Therefore, it is strongly advised that you must learn this chapter religiously for the sake of a decent score.

Mind me if you don't understand the theorems, axioms or concepts minutely you cannot perform well, as the intricacy of diagrams and concepts makes it a daunting task for you.

For the sake of convenience this chapter is divided into 5 parts

1. Lines and Angles
2. Triangles
3. Quadrilaterals
4. Polygons
5. Circles and Loci

Chapter Checklist

- Lines and Angles
- Triangles
- Quadrilaterals
- Polygons
- Circles
- Tangential Quadrilateral
- CAT Test

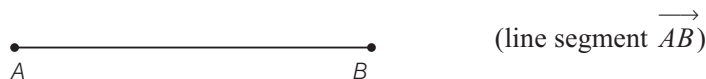
12.2 Lines and Angles

Point : The figure of which length, breadth and height cannot be measured is called a point. It is infinitesimal.

Line : A line is made up of a infinite number of points and it has only length *i.e.*, it does not has any thickness (or width). A line is endless *i.e.*, it can be extended in both directions.



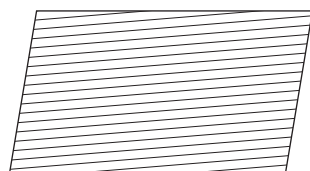
Line segment : A line segments has two end points, but generally speaking line segment is called a line.



Ray : A ray extends indefinitely in one direction from any given point. This is exhibited by an arrow. The starting point is called as the initial point.



Plane : It is a flat surface, having length and breadth both, but no thickness. It is a two dimensional figure.



(Plane)

Type of Lines	Definition	Diagram
Parallel lines	Two lines, lying in a plane and having no common intersecting point are called parallel lines. The distance between two parallel lines is constant.	 Symbol :
Perpendicular lines	Two lines, which lie in a single plane and intersect each other at right angle, are called perpendicular lines.	 Symbol : $\perp$
Concurrent lines	More than two straight lines intersecting at the same point.	

Points to Remember

1. A line contains infinitely many points.
2. The intersection of two different lines is a point.
3. Through a given point, there pass an infinite number of lines and these lines are called **concurrent lines**.
4. Only one line can pass through any two particular points.
5. When more than two points lie on a line, they are called as **collinear points** else they are called as **non-collinear points**.
6. Two lines can intersect maximum at one point. This point is called as **point of intersection** and these lines are called as **intersecting lines**.

Types of Angles	Property	Diagram
Acute	$0^\circ < \theta < 90^\circ$ ($\angle AOB$ is an acute angle)	
Right	$\theta = 90^\circ$ ($\angle AOB$ is a right angle)	

7. There are an infinite number of planes which pass through a single (particular) point.
8. When more than three points lie in the same plane, they are called as **coplanar**, else they are called as **non-coplanar**.
9. When more than one line lie in the same plane, then these lines are called as coplanar else they are called as non-coplanar.
10. When two planes intersect each other, they form a line *i.e.*, intersecting region is a line.
11. Two different lines which are perpendicular to the same (a third line) line, are necessarily parallel to each other, if all of them are lying on the same plane.
12. When two or more parallel lines are intercepted by some other intercepting lines, then the ratio of corresponding intercepts are equal. An intercepting line is generally called as a transversal.

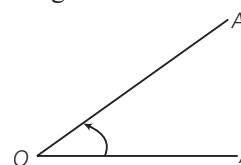


i.e.,

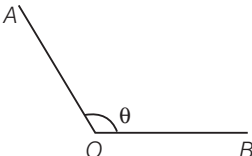
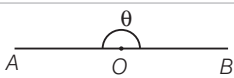
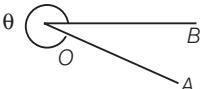
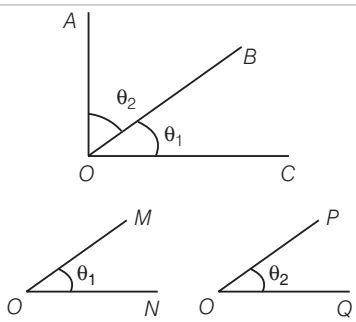
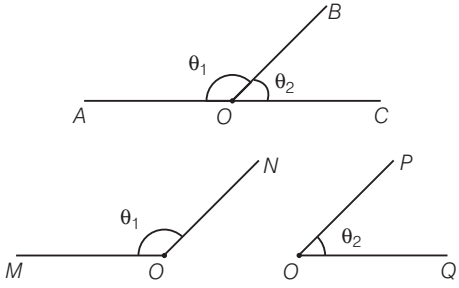
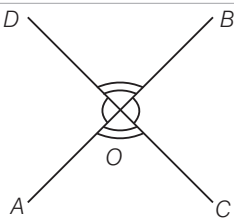
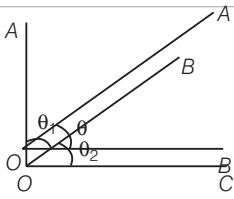
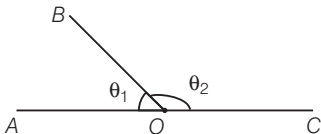
$$\frac{AB}{BC} = \frac{DE}{EF}$$

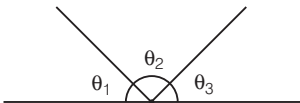
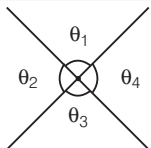
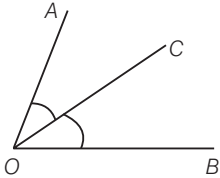
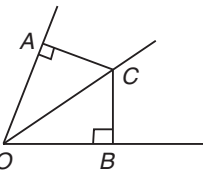
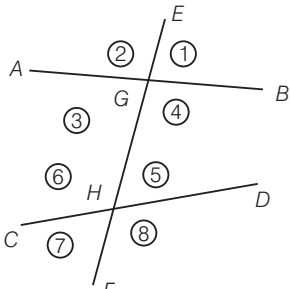
Angles

The amount of rotation about O , the vertex of the angle AOA' , is called the magnitude of the angle.

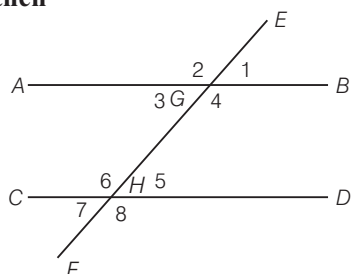


$m \angle AOA'$ denotes the measure of $\angle AOA'$. Angles are said to be congruent when their measure is same.
(symbol : $\cong$)

Types of Angles	Property	Diagram
Obtuse	$90^\circ < \theta < 180^\circ$ ($\angle AOB$ is an obtuse angle)	
Straight	$\theta = 180^\circ$ ($\angle AOB$ is a straight angle)	
Reflex	$180^\circ < \theta < 360^\circ$ ($\angle AOB$ is a reflex angle)	
Complementary	$\theta_1 + \theta_2 = 90^\circ$ Two angles whose sum is 90° , are complementary to each other	
Supplementary	$\theta_1 + \theta_2 = 180^\circ$ Two angles whose sum is 180° , are supplementary to each other.	
Vertically opposite	$\angle DOA = \angle BOC$ and $\angle DOB = \angle AOC$	
Adjacent angles	$\angle AOB$ and $\angle BOC$ are adjacent angles Adjacent angles must have a common side. (e.g., OB)	
Linear pair	$\angle AOB$ and $\angle BOC$ are linear pair angles. One side must be common (e.g., OB) and these two angles must be supplementary.	

Types of Angles	Property	Diagram
Angles on the one side of a line	$\theta_1 + \theta_2 + \theta_3 = 180^\circ$	
Angles round the point	$\theta_1 + \theta_2 + \theta_3 + \theta_4 = 360^\circ$	
Angle bisector	OC is the angle bisector of $\angle AOB$. <i>i.e.</i>, $\angle AOC = \angle BOC = \frac{1}{2} (\angle AOB)$ When a line segment divides an angle equally into two parts, then it is said to be the angle bisector (OC)	 <i>i.e.</i>, $AC = BC$ 
Corresponding angles	When two lines are intersected by a transversal, then they form four pairs of corresponding angles (a) $\angle AGE, \angle CHG \Rightarrow (\angle 2, \angle 6)$ (b) $\angle AGH, \angle CHF \Rightarrow (\angle 3, \angle 7)$ (c) $\angle EGB, \angle GHD \Rightarrow (\angle 1, \angle 5)$ (d) $\angle BGH, \angle DHF \Rightarrow (\angle 4, \angle 8)$	
Exterior angles	These are following four angles (i) $\angle AGE \Rightarrow \angle 2$ (ii) $\angle CHF \Rightarrow \angle 7$ (iii) $\angle EGB \Rightarrow \angle 1$ (iv) $\angle DHF \Rightarrow \angle 8$	
Interior angles	These are following four angles (i) $\angle AGH \Rightarrow \angle 3$ (ii) $\angle GHC \Rightarrow \angle 6$ (iii) $\angle BGH \Rightarrow \angle 4$ (iv) $\angle DHF \Rightarrow \angle 5$	
Alternate angles	These are two pairs angles as following: (i) $\angle AGH, \angle GHD (\angle 3, \angle 5)$ (ii) $\angle GHC, \angle BGH (\angle 6, \angle 4)$	

When two parallel lines are intersected by a transversal, then



1. The pairs of corresponding angles so formed are congruent.

i.e., $\angle 2 = \angle 6$

$$\angle 3 = \angle 7$$

$$\angle 1 = \angle 5$$

$$\angle 4 = \angle 8$$

NOTE 1. If one pair of corresponding angles is congruent, then the rest pairs of corresponding angles are also congruent.

2. The pairs of alternate angles so formed are congruent

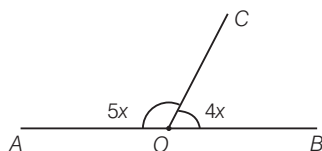
i.e., $\angle 3 = \angle 5$ and $\angle 4 = \angle 6$

3. The pair of interior angles (i.e., the interior angles on the same side of a transversal) are supplementary.

NOTE The converse of all the 3 rules is also true i.e., if the pair of corresponding angles, or alternate angles or interior angles is equal (i.e., congruent), then the two lines are parallel which are intersected by a transversal.

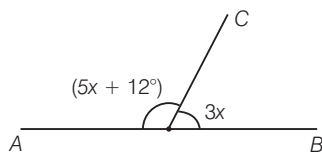
Introductory Exercise 12.1

1. What is the value of x in the following figure?



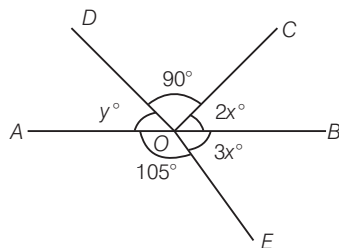
- (a) 80° (b) 40°
(c) 20° (d) 25°

2. What is the value of x in the following figure?



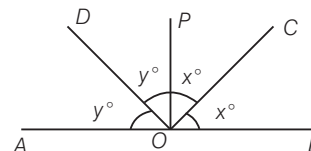
- (a) 18° (b) 20°
(c) 21° (d) 24°

3. In the following figure AB is a straight line. Find $(x + y)$.



- (a) 55° (b) 65°
(c) 75° (d) 80°

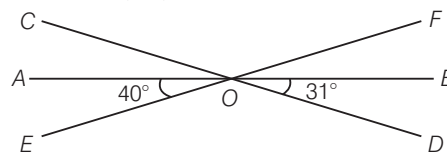
4. In the following figure $\angle BOP = 2x^\circ$, $\angle AOP = 2y^\circ$, OC and OD are angle bisectors of $\angle BOP$ and $\angle AOP$ respectively.



Find the value of $\angle COD$.

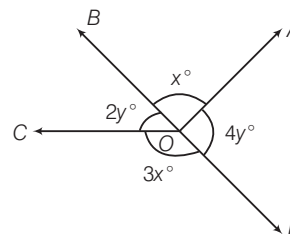
- (a) 75° (b) 90° (c) 100° (d) 120°

5. In the following figure find the value of $\angle BOC$.



- (a) 101° (b) 149° (c) 71° (d) 140°

6. Find y , if $x^\circ = 36^\circ$, as per the given diagram.



- (a) 36° (b) 16° (c) 12° (d) 42°

7. If $(2x + 17)^\circ$, $(x + 4)^\circ$ are complementary, find x .

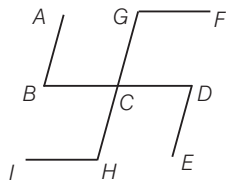
- (a) 63° (b) 53° (c) 35° (d) 23°

8. If $(5y + 62)^\circ$, $(22^\circ + y)$ are supplementary, find y .
 (a) 16° (b) 32° (c) 8° (d) 1°

9. If two supplementary angles are in the ratio 13 : 5, find the greater angle.
 (a) 130° (b) 65° (c) 230° (d) 28°

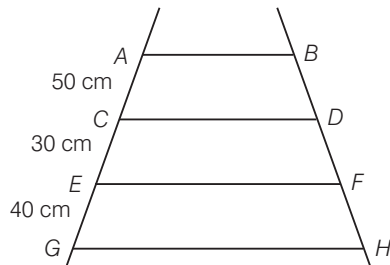
10. An angle is 30° more than one half of its complement. Find the angle in degrees.
 (a) 60° (b) 50° (c) 45° (d) 80°

11. In the given diagram $AB \parallel GH \parallel DE$ and $GF \parallel BD \parallel HI$, $\angle FGC = 80^\circ$. Find the value of $\angle CHI$.



- (a) 80° (b) 120°
 (c) 100° (d) 160°

12. In the given figure $AB \parallel CD \parallel EF \parallel GH$ and $BH = 100$ cm. Find the value of DF .



- (a) 26 cm (b) 40 cm
 (c) 25 cm (d) 24 cm

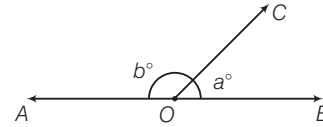
13. The complement of $65^\circ 50'$ is :
 (a) $24^\circ 50'$ (b) $24^\circ 10'$
 (c) $14^\circ 50'$ (d) $34^\circ 10'$

14. The supplement of $123^\circ 45'$ is :
 (a) $56^\circ 55'$
 (b) $56^\circ 15'$
 (c) $55^\circ 56'$
 (d) None of the above

15. If two angles are complementary to each other, then each angle is :
 (a) a right angle
 (b) a supplementary angle
 (c) an obtuse angle
 (d) an acute angle

16. How many degrees are there in an angle which equals one-fifth of its supplement?
 (a) 15° (b) 30°
 (c) 75° (d) 150°

17. In the given figure, $\angle a$ is greater than one-sixth of right angle, then.



- (a) $b > 165^\circ$ (b) $b < 165^\circ$
 (c) $b \leq 165^\circ$ (d) $b \geq 165^\circ$

18. Let D be the mid-point of a straight line AB and let C be a point different from D such that $AC = BC$, then.
 (a) $AC \perp AB$ (b) $\angle BDC = 90^\circ$
 (c) $\angle BDC$ is acute (d) $\angle BDC > 90^\circ$

19. Answer on the basis of the following statements.

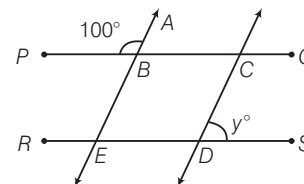
When two straight lines intersect, then :

1. adjacent angles are complementary
2. adjacent angles are supplementary
3. opposite angles are equal
4. opposite angles are supplementary

- (a) 2 and 3 are correct
 (b) 1 and 4 are correct
 (c) 1 and 3 are correct
 (d) 2 and 4 are correct

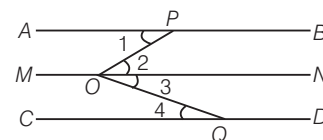
20. AB is a straight line and O is a point on AB , if a line OC is drawn not coinciding with OA or OB , then $\angle AOC$ and $\angle BOC$ are
 (a) equal
 (b) complementary
 (c) supplementary
 (d) together equal to 100

21. In the adjoining figure $AE \parallel CD$ and $BC \parallel ED$, then find Y .



- (a) 60° (b) 80°
 (c) 90° (d) 75°

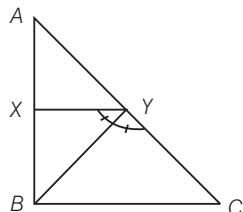
22. In the adjoining figure $\angle APO = 42^\circ$ and $\angle CQO = 38^\circ$. Find the value of $\angle POQ$.



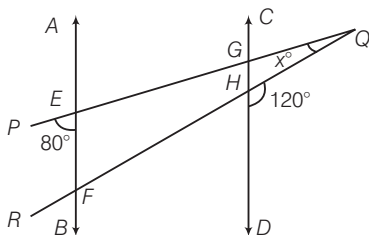
- (a) 68° (b) 72°
 (c) 80° (d) 126°

23. In a $\triangle ABC$, a line XY parallel to BC intersects AB at X and AC at Y :

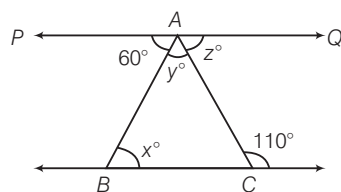
If BY bisects $\angle XYC$, then $m\angle CBY : m\angle CYB$ is :



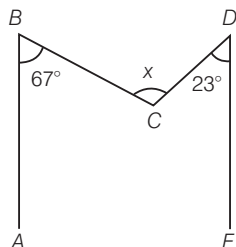
- (a) 5 : 4 (b) 4 : 5
(c) 1 : 1 (d) 6 : 5
24. In the adjoining figure $AB \parallel CD$ and PQ, QR intersects AB and CD both at E, F and G, H respectively. Given that $m\angle PEB = 80^\circ$, $m\angle QHD = 120^\circ$ and $m\angle PQR = x^\circ$, find the value of x .



- (a) 40° (b) 20°
(c) 100° (d) 30°
25. In the following figure, find the value of y .

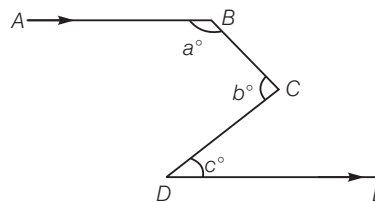


- (a) 70° (b) 60°
(c) 50° (d) 80°
26. In the adjoining figure $AB \parallel DE$, $\angle ABC = 67^\circ$ and $\angle EDC = 23^\circ$. Find $\angle BCD$.



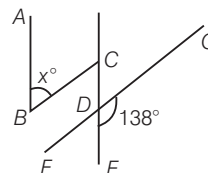
- (a) 90°
(b) 44°
(c) 46°
(d) none of the above

27. In the given figure $AB \parallel DE$. Find $a^\circ + b^\circ - c^\circ$:



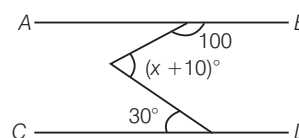
- (a) 160° (b) 120° (c) 180° (d) 210°

28. In the given figure $AB \parallel CE$ and $BC \parallel FG$. Find the value of x° .



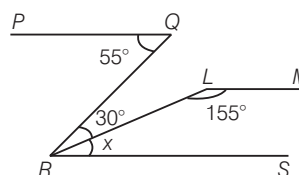
- (a) 52° (b) 32° (c) 42° (d) 36°

29. $AB \parallel CD$, shown in the figure. Find the value of x .



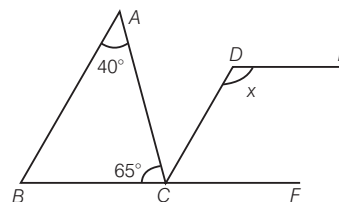
- (a) 100° (b) 90° (c) 110° (d) 140°

30. In the figure $PQ \parallel LM \parallel RS$. Find the value of $\angle LRS$.



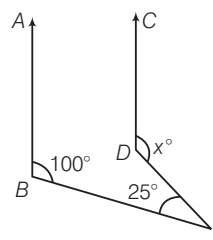
- (a) 30° (b) 25° (c) 35° (d) 40°

31. In the figure $AB \parallel DC$ and $DE \parallel BF$. Find the value of x .



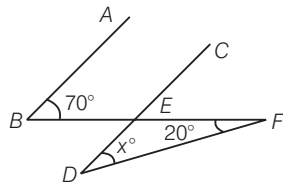
- (a) 140° (b) 155° (c) 105° (d) 115°

32. In the figure $AB \parallel CD$, $\angle ABE = 100$. Find $m\angle CDE$.



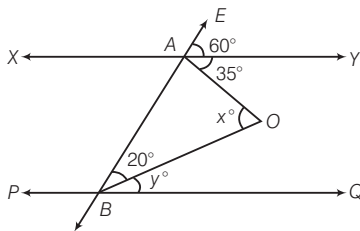
- (a) 125° (b) 55° (c) 65° (d) 75°

33. In the figure $AB \parallel CD$, find x° (i.e., $\angle CDF$).



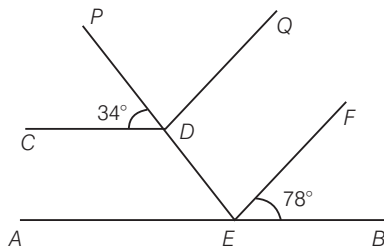
- (a) 50° (b) 90° (c) 30° (d) 70°

34. In the given figure $XY \parallel PQ$, find the value of x .



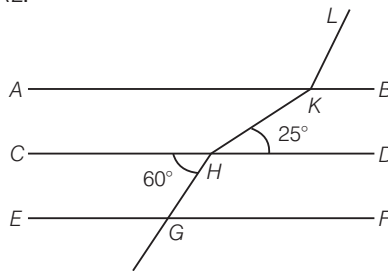
- (a) 70° (b) 40° (c) 75° (d) 15°

35. In the given figure $AB \parallel CD$ and $EF \parallel DQ$. Find the value of $\angle DEF$.



- (a) 68° (b) 78° (c) 34° (d) 39°

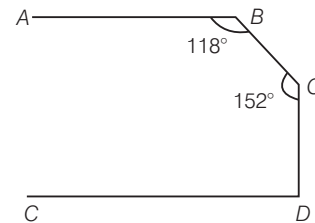
36. In the given figure $AB \parallel CD \parallel EF$ and $GH \parallel KL$. Find $m \angle HKL$.



- (a) 85° (b) 145° (c) 120° (d) 95°

37. Which one of the following statements is false?
 (a) Two straight lines can intersect at only one point.
 (b) Through a given point, only one straight line can be drawn.
 (c) A line segment can be produced to any desired length.
 (d) Through two given points, it is possible to draw one and only one straight line.

38. $AB \parallel CD$. $\angle ABO = 118^\circ$, $\angle BOD = 152^\circ$, find $\angle ODC$.



- (a) 70° (b) 80°
 (c) 90° (d) 34°

39. Two parallel lines AB and CD are intersected by a transversal EF at M and N respectively.

The lines MP and NP are the bisectors of interior angles $\angle BMN$ and $\angle DNM$ on the same side of the transversal. Then $\angle MPN$ is equal to :

- (a) 60° (b) 90°
 (c) 45° (d) 120°

40. If the arms of one angle are respectively parallel to the arms of another angle, then the two angles are :

- (a) either equal or supplementary
 (b) neither equal nor supplementary
 (c) equal but not supplementary
 (d) not equal but supplementary

41. AB and CD are two parallel lines. PQ cuts AB and CD at E and F respectively. EL is the bisector of $\angle BEF$. If $\angle LEB = 35^\circ$, then $\angle CFQ$ will be :

- (a) 70° (b) 55°
 (c) 110° (d) 125°

42. A plane figure is bounded by straight lines only. If n is the number of these lines, then the least value of n is :

- (a) 1 (b) 2
 (c) 3 (d) 4

43. How many planes can pass through any three arbitrary points?

- (a) 1 (b) 2
 (c) 3 (d) 0

44. Out of the four arbitrary non-collinear points three points are taken at a time, then the number of planes that can be drawn through the three points is :

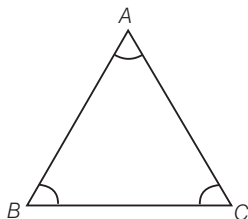
- (a) 3 (b) 4
 (c) 6 (d) 12

45. Maximum number of points of intersection of four lines on a plane is :

- (a) 4 (b) 6
 (c) 8 (d) 5

12.3 Triangles

Triangle : A three sided closed plane figure, which is formed by joining the three non-collinear points, is called as a triangle. It is denoted by the symbol Δ .



In the above Δ (triangle) ABC , A, B and C are three **vertices**, line segments AB, BC and AC are the three **sides** of the triangle. $\angle A, \angle B$ and $\angle C$ are the three **interior angles** of a triangle ABC .

In the adjoining figure $\angle FCB, \angle CBE, \angle ABD, \angle IAB, \angle HAC, \angle GCA$ are the **exterior angles** of the ΔABC .

Sum of the three interior angles of a triangle is always 180° .

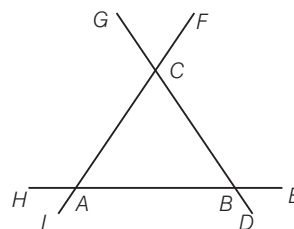
Exterior angle = Sum of two interior opposite angles

e.g., $\angle CBE = \angle CAB + \angle BCA$

Perimeter of triangle is equal to sum of all the three sides i.e., $a + b + c$

Semiperimeter of a triangle is half of the perimeter

i.e., $s = \frac{a + b + c}{2}$, a, b, c are the length of three sides of a triangle.



Types of Triangles

(A) According to interior angles

Types of Triangles	Property/Definition	Diagram
Acute angle triangle	Each of the angle of a triangle is less than 90° i.e., $a < 90^\circ, b < 90^\circ, c < 90^\circ$	 $\{\angle a, \angle b, \angle c\} < 90^\circ$
Right angled triangle	One of the angle is equal to 90° , then it is called as right angled triangle. Rest two angles are complementary to each other.	 $\angle C = 90^\circ$
Obtuse angle triangle	One of the angle is obtuse (i.e., greater than 90°), then it is called as obtuse angle triangle.	 $\angle C > 90^\circ$

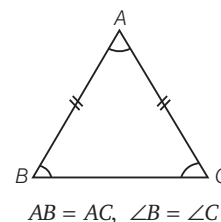
(B) According to the length of sides.

Types of Triangles	Property/Definition	Diagram
Scalene triangle	A triangle in which none of the three sides is equal is called a scalene triangle (all the three angles are also different).	 $a \neq b \neq c$

Isosceles triangle

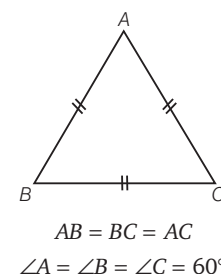
A triangles in which at least two sides are equal is called an isosceles triangle.

In this triangle, the angles opposite to the congruent sides are also equal.



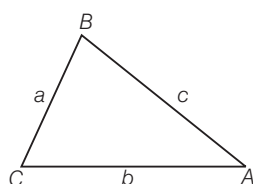
Equilateral triangle

A triangle in which all the three sides are equal called an equilateral triangle. In this triangle each angle is congruent and equal to 60° .



Fundamental Properties of Triangles

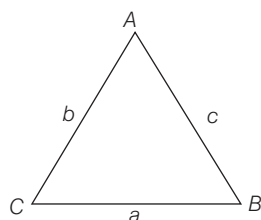
1. Sum of any two sides is always greater than the third side.
2. The difference of any two sides is always less than the third side.
3. Greater angle has a greater side opposite to it and smaller angle has a smaller side opposite to it *i.e.*, if two sides of a triangle are not congruent then the angle opposite to the greater side is greater.
4. Let a, b and c be the three sides of a $\triangle ABC$ and c is the largest side. Then



- (i) if $c^2 < a^2 + b^2$, the triangle is **acute angle triangle**
- (ii) if $c^2 = a^2 + b^2$, the triangle is **right angled triangle**
- (iii) if $c^2 > a^2 + b^2$, the triangle is **obtuse angle triangle**

5. **Sine rule :** In a $\triangle ABC$, if a, b, c be the three sides opposite to the angles A, B, C respectively, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



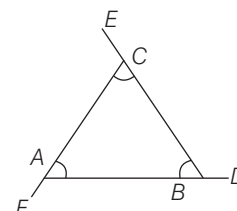
6. **Cosine rule :** In a $\triangle ABC$, if a, b, c be the sides opposite to angle A, B and C respectively, then

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \cos B = \frac{c^2 + a^2 - b^2}{2ca},$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

(These rules have been discussed already in trigonometry.)

7. The sum of all the three interior angles is always 180°



$$\text{i.e., } \angle CAB + \angle ABC + \angle BCA = 180^\circ$$

8. The sum of three (ordered) exterior angles of a triangle is 360°

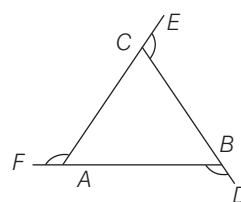


Fig. (i)

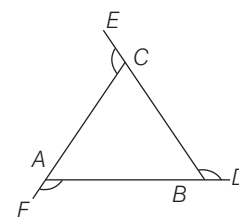


Fig. (ii)

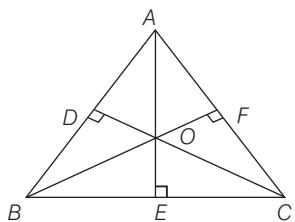
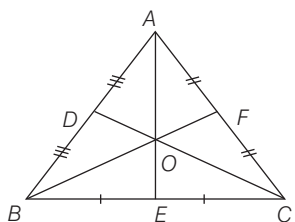
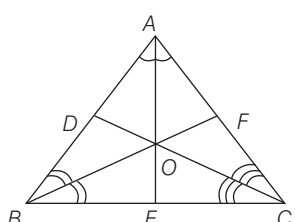
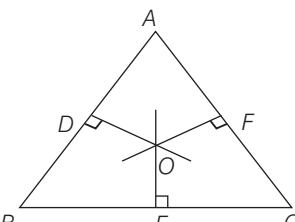
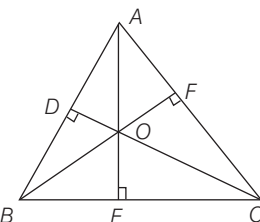
In fig. (i) : $(\angle FAC + \angle ECB + \angle DBA) = 360^\circ$

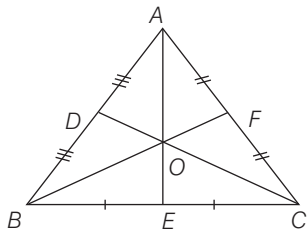
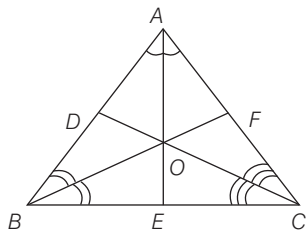
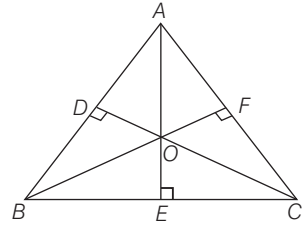
In fig. (ii) : $(\angle FAB + \angle DBC + \angle ECA) = 360^\circ$

9. The sum of an interior angle and its adjacent exterior angle is 180° .

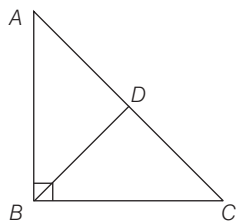
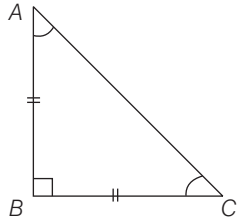
10. A triangle must have at least two acute angles.
 11. In a triangle, the measure of an exterior angle equals the sum of the measures of the interior opposite angles.
 12. The measure of an exterior angle of a triangle is greater than the measure of each of the opposite interior angles.

Important Definitions

Nomenclature	Property/Definition	Diagram
Altitude (or height)	The perpendicular drawn from the opposite vertex of a side in a triangle is called an altitude of the triangle. There are three altitudes in a triangle.	 AE, CD and BF are the altitudes
Median	The line segment joining the mid-point of a side to the vertex opposite to the side is called a median. - There are three medians in a triangle. - A median bisects the area of the triangle <i>i.e.</i>, $A(\triangle ABE) = A(\triangle AEC) = \frac{1}{2} A(\triangle ABC)$ etc.	 AE, CD and BF are the medians $(BE = CE, AD = BD, AF = CF)$
Angle bisector	A line segment which originates from a vertex and bisects the same angle is called an angle bisector. $(\angle BAE = \angle CAE = \frac{1}{2} \angle BAC)$ etc.	 AE, CD and BF are the angle bisectors.
Perpendicular bisector	A line segment which bisects a side perpendicularly (i.e., at right angle) is called a perpendicular bisector of a side of triangle. All points on the perpendicular bisector of a line are equidistant from the ends of the line.	 DO, EO and FO are the perpendicular bisectors.
Orthocentre	The point of intersection of the three altitudes of the triangle is called the orthocentre. $\angle BOC = 180 - \angle A$ $\angle COA = 180 - \angle B$ $\angle AOB = 180 - \angle C$	 'O' is the orthocentre

Types of Triangles	Property/Definition	Diagram
Centroid	The point of intersection of the three medians of a triangle is called the centroid. A centroid divides each median in the ratio 2 : 1 (vertex : base) $\frac{AO}{OE} = \frac{CO}{OD} = \frac{BO}{OF} = \frac{2}{1}$	 'O' is the centroid.
Incentre	The point of intersection of the angle bisectors of a triangle is called the incentre. Incentre O is always equidistant from all three sides i.e., the perpendicular distance between the sides and incentre is always same for all the three sides.	 'O' is the incentre.
Circumcentre	The point of intersection of the perpendicular bisectors of the sides of a triangle is called the circumcentre. $OA = OB = OC =$ (circum radius) Circumcentre O is always equidistant from all the three vertices A, B and C.	 'O' is the circumcentre.

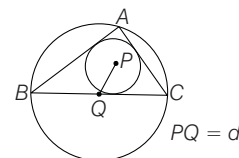
Important Theorems on Triangles

Theorem	Statement/Explanation	Diagram
Pythagoras theorem	The square of the hypotenuse of a right angled triangle is equal to the sum of the squares of the other two sides. i.e., $(AC)^2 = (AB)^2 + (BC)^2$ - The converse of this theorem is also true. - The numbers which satisfy this relation, are called Pythagorean triplets. e.g., (3, 4, 5), (5, 12, 13), (7, 24, 25), (8, 15, 17), (9, 40, 41), (11, 60, 61), (12, 35, 37), (16, 63, 65), (20, 21, 29), (28, 45, 53), (33, 56, 65) Note : All the multiples (or submultiples) of Pythagorean triplets also satisfy the relation. e.g., (6, 8, 10), (15, 36, 39), (1.5, 2, 2.5) etc	 $\angle B = 90^\circ$ $\overline{AC} \rightarrow$ Hypotenuse $AD = CD = BD$ (D is the mid-point of AC)
$45^\circ - 45^\circ - 90^\circ$ triangle theorem	If the angles of a triangle are 45°, 45° and 90°, then the hypotenuse (i.e., longest side) is $\sqrt{2}$ times of any smaller side. Excluding hypotenuse rest two sides are equal. i.e., $AB = BC$ and $AC = \sqrt{2}AB = \sqrt{2}BC$	 $\angle A = 45^\circ$, $\angle B = 90^\circ$, $\angle C = 45^\circ$

Theorem	Statement/Explanation	Diagram
30° – 60° – 90° triangle theorem	If the angles of a triangle are 30°, 60° and 90°, then the sides opposite to 30° angle is half of the hypotenuse and the side opposite to 60° is $\frac{\sqrt{3}}{2}$ times the hypotenuse. e.g., $AB = \frac{AC}{2}$ and $BC = \frac{\sqrt{3}}{2} AC$ ∴ $AB : BC : AC = 1 : \sqrt{3} : 2$	
Basic proportionality theorem (BPT) or Thales theorem	Any line parallel to one side of a triangle divides the other two sides proportionally. So if DE is drawn parallel to BC, it would divide sides AB and AC proportionally i.e., $\frac{AD}{DB} = \frac{AE}{EC} \quad \text{or} \quad \frac{AD}{AB} = \frac{AE}{AC}$ $\frac{AD}{DE} = \frac{AB}{BC} = \frac{AE}{DE} = \frac{AC}{BC}$	
Mid-point theorem	If the mid-points of two adjacent sides of a triangle are joined by a line segment, then this segment is parallel to the third side. i.e., if $AD = BD$ and $AE = CE$, then $DE \parallel BC$	
Apollonius theorem	In a triangle, the sum of the squares of any two sides of a triangle is equal to twice the sum of the square of the median to the third side and square of half the third side. i.e., $AB^2 + AC^2 = 2(AD^2 + BD^2)$	$BD = CD$ AD is the median
Interior angle bisector theorem	In a triangle the angle bisector of an angle divides the opposite side to the angle in the ratio of the remaining two sides. i.e., $\frac{BD}{CD} = \frac{AB}{AC} \quad \text{and} \quad BD \times AC = CD \times AB = AD^2$	
Exterior angle bisector theorem	In a triangle the angle bisector of any exterior angle of a triangle divides the side opposite to the external angle in the ratio of the remaining two sides i.e., $\frac{BE}{AE} = \frac{BC}{AC}$	

Euler's Theorem for a triangle

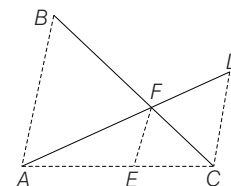
Let $\triangle ABC$ have circumradius R and inradius r . Let d be the distance between the circumcentre and the incentre. Then we have $d^2 = R(R - 2r)$



Crossed Ladder Theorem

Let the two line segments BC and AD intersect at a point F , such that the point E lies on AC and $AB \parallel CD \parallel EF$. Then, we have

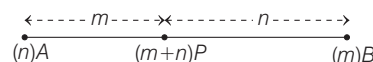
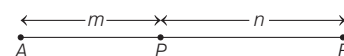
$$\frac{1}{AB} + \frac{1}{CD} = \frac{1}{EF}$$



Mass Point Geometry

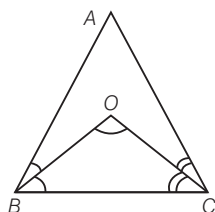
Let us consider a line AB which is intersected by a point P , such that $AP = m$ and $BP = n$, we have $\frac{wt(A)}{wt(B)} = \frac{n}{m}$

Therefore, the weight at A will be n , the weight at B will be m and the total weight at P will be $(m + n)$.

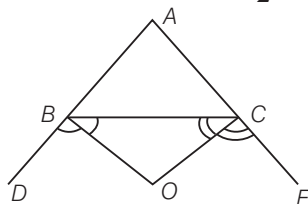


Useful Results

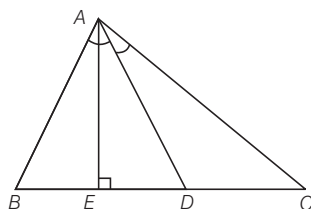
1. In a $\triangle ABC$, if the bisectors of $\angle B$ and $\angle C$ meet at O , then $\angle BOC = 90^\circ + \frac{1}{2} \angle A$



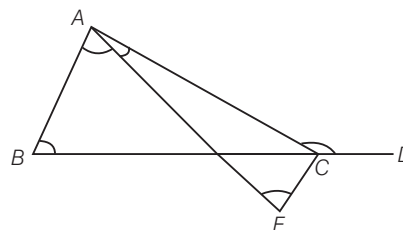
2. In a $\triangle ABC$, if sides AB and AC are produced to D and E respectively and the bisectors of $\angle DBC$ and $\angle ECB$ intersect at O , then $\angle BOC = 90^\circ - \frac{1}{2} \angle A$



3. In a $\triangle ABC$, if AD is the angle bisector of $\angle BAC$ and $AE \perp BC$, then $\angle DAE = \frac{1}{2} (\angle ABC - \angle ACB)$

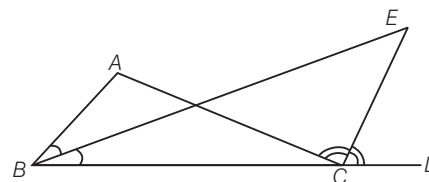


4. In a $\triangle ABC$, if BC is produced to D and AE is the angle bisector of $\angle A$, then



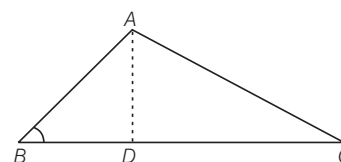
$$\angle ABC + \angle ACD = 2 \angle AEC.$$

5. In a $\triangle ABC$, if side BC is produced to D and bisectors of $\angle ABC$ and $\angle ACD$ meet at E , then



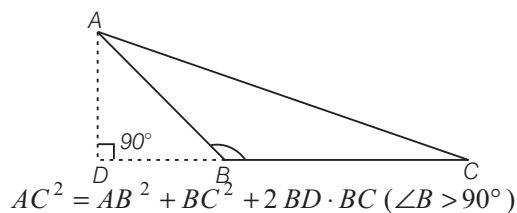
$$\angle BEC = \frac{1}{2} \angle BAC$$

6. In an acute angle $\triangle ABC$, AD is a perpendicular dropped on the opposite side of $\angle A$, then

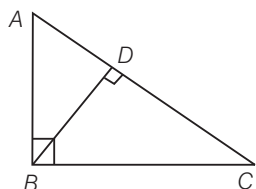


$$AC^2 = AB^2 + BC^2 - 2BD \cdot BC \quad (\angle B < 90^\circ)$$

7. In an obtuse angle $\triangle ABC$, AD is perpendicular dropped on BC . BC is produced to D to meet AD , then



8. In a right angle $\triangle ABC$, $\angle B = 90^\circ$ and AC is hypotenuse. The perpendicular BD is dropped on hypotenuse AC from right angle vertex B , then



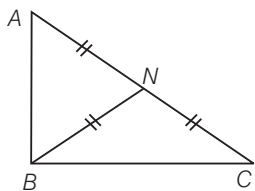
$$(i) BD = \frac{AB \times BC}{AC} \quad (ii) AD = \frac{AB^2}{AC}$$

$$(iii) CD = \frac{BC^2}{AC}$$

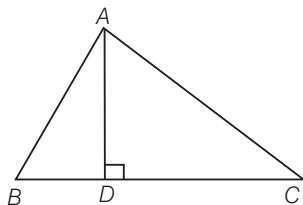
$$(iv) \frac{1}{BD^2} = \frac{1}{AB^2} + \frac{1}{BC^2}$$

In a right angled triangle, the median to the hypotenuse $= \frac{1}{2} \times \text{hypotenuse}$

$$\text{i.e., } BN = \frac{AC}{2} \quad (\text{as per the fig.})$$



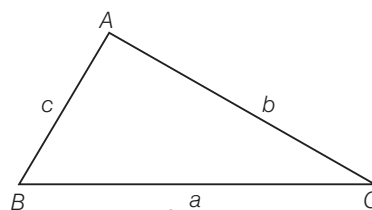
9. Area of a triangle (General formula)



$$A(\Delta) = \frac{1}{2} \times \text{base} \times \text{height}$$

$$A(\Delta) = \frac{1}{2} \times BC \times AD \quad (\text{as per the figure.})$$

10. Area of scalene triangle $= \sqrt{s(s-a)(s-b)(s-c)}$



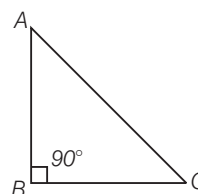
$$\text{Also, } A(\Delta) = r \times s = \frac{abc}{4R}$$

where a, b and c are the sides of the triangle.

$$s \rightarrow \text{semiperimeter} = \frac{a+b+c}{2}, \quad r \rightarrow \text{inradius}$$

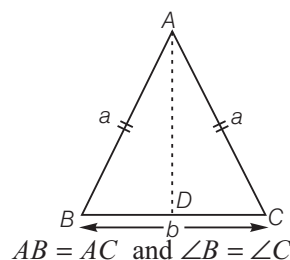
$R \rightarrow \text{circumradius}$

11. Area of right angled triangle



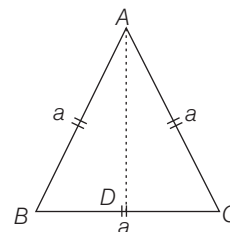
$$= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times BC \times AB \quad (\text{as per the figure})$$

12. Area of an isosceles triangle $= \frac{b}{4} \sqrt{4a^2 - b^2}$



$\triangle ABD \cong \triangle ACD$ ($AD \rightarrow$ Angle bisector, median, altitude and perpendicular bisector)

13. Area of an equilateral triangle $= \frac{\sqrt{3}}{4} a^2$



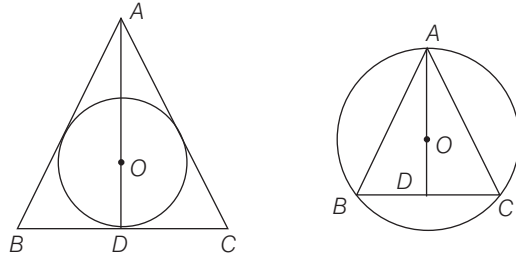
$$\left[A(\Delta) = \frac{1}{2} BC \times AD = \frac{1}{2} \times a \times \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{4} a^2 \right]$$

($a \rightarrow$ each side of the triangle)

$AD \rightarrow$ Altitude, median, angle bisector and perpendicular bisector also.

Inradius : $\frac{1}{3} \times \text{height} = \frac{\text{side}}{2\sqrt{3}}$

$OD \rightarrow$ Inradius



Circumradius $= \frac{2}{3} \times \text{height} = \frac{\text{side}}{\sqrt{3}}$

$OA \rightarrow$ Circumradius

NOTE In equilateral triangle orthocentre centroid, incentre and circumcentre coincide at the same point.

Circumradius $= 2 \times$ inradius

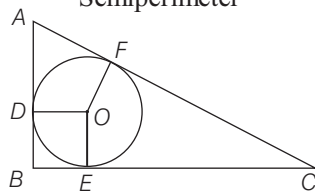
For the given perimeter of a triangle, the area of equilateral triangle is maximum.

For the given area of a triangle, the perimeter of equilateral triangle is minimum.

14. In a right angled triangle

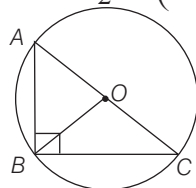
(i) Inradius $(r) = \frac{AB + BC - AC}{2}$

(ii) Inradius $(r) = \frac{\text{Area}}{\text{Semiperimeter}}$



$DO = EO = FO (r)$

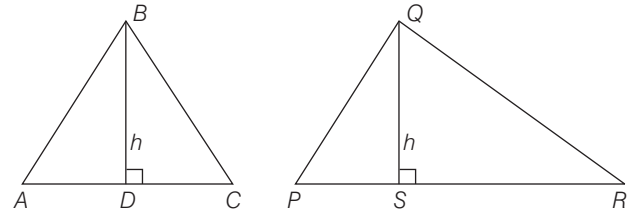
(iii) Circumradius $(R) = \frac{AC}{2} = \left(\frac{\text{hypotenuse}}{2} \right)$



$AO = CO = BO = (R)$

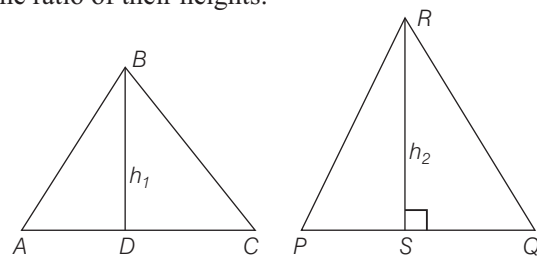
AC is the diameter.

15. The ratio of areas of two triangles of equal heights is equal to the ratio of their corresponding bases. i.e.,



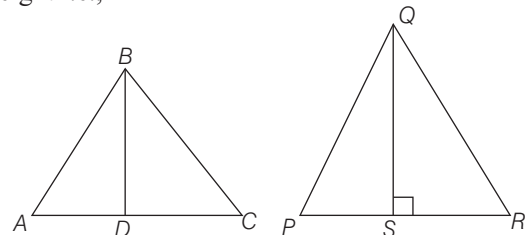
$$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{AC}{PR}$$

16. The ratio of areas of triangles of equal bases is equal to the ratio of their heights.



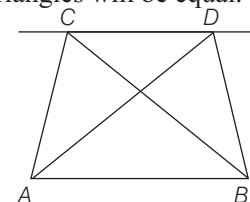
i.e., $\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{BD}{RS}$

17. The ratio of the areas of two triangles is equal to the ratio of the products of base and its corresponding height i.e.,



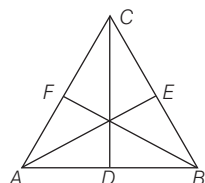
$$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{AC \times BD}{PR \times QS}$$

18. If the two triangles have the same base and lie between the same parallel lines (as shown in figure), then the area of two triangles will be equal.



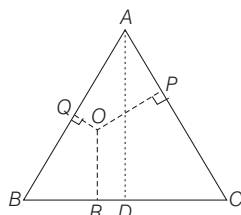
i.e., $A(\triangle ABC) = A(\triangle ADB)$

19. In a triangle AE , CD and BF are the medians, then

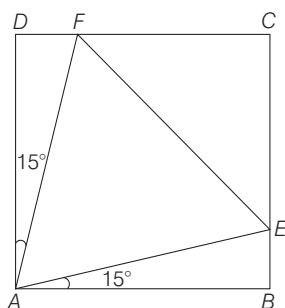


$$3(AB^2 + BC^2 + AC^2) = 4(CD^2 + BF^2 + AE^2)$$

20. In an equilateral triangle, if O is a point anywhere inside the equilateral triangle ABC , the sum of its distances from three sides is equal to the length of the altitude of the triangle. That is, as per the given diagram, $OP + OQ + OR = AD$.



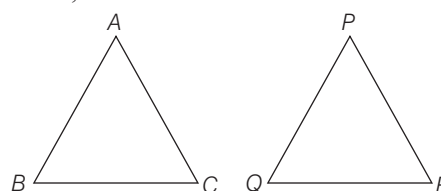
21. The largest possible area of an equilateral triangle inscribed in a unit square is $(2\sqrt{3} - 3)$.



- The maximum area can be enclosed only when one of the vertices of the triangle coincides with one of the vertices of the square and angle between the side of the triangle and the side of the rectangle at the point of coincidence is 15° .
- Also, the side of the equilateral triangle is $\sqrt{6} - \sqrt{2}$.
- Also, the area of $\triangle ADF = \triangle ABE = \frac{1}{2}(\triangle ECF)$.

Congruency of triangles

Two triangles are said to be congruent if they are equal in all respects. i.e.,



- Each of the three sides of one triangle must be equal to the three respective sides of the other.
- Each of the three angles of the one triangle must be equal to the three respective angles of the other.

$$\text{i.e., } \left. \begin{array}{l} AB = PQ \\ AC = PR \\ BC = QR \end{array} \right\} \text{ and } \left. \begin{array}{l} \angle A = \angle P \\ \angle B = \angle Q \\ \angle C = \angle R \end{array} \right\}$$

Tests for congruency

With the help of the following given tests, we can deduce without having detailed information about triangles that whether the given two triangles are congruent or not.

Test	Property	Diagram
S - S - S	(Side-Side-Side) If the three sides of one triangle are equal to the corresponding three sides of the other triangle, then the two triangles are congruent. $AB \cong PQ, AC \cong PR, BC \cong QR$ $\therefore \triangle ABC \cong \triangle PQR$	
S - A - S	(Side-Angle-Side) If two sides and the angle included between them are congruent to the corresponding sides and the angle included between them, of the other triangle, then the two triangles are congruent. $AB \cong PQ, \angle ABC \cong \angle PQR, BC \cong QR$ $\therefore \triangle ABC \cong \triangle PQR$	

Test	Property	Diagram
A – S – A	(Angle–Side–Angle) If two angles and the included side of a triangle are congruent to the corresponding angles and the included side of the other triangle, then the two triangles are congruent. $\angle ABC \cong \angle PQR, BC \cong QR, \angle ACB \cong \angle PRQ$ $\therefore \Delta ABC \cong \Delta PQR$	
A – A – S	(Angle–Angle–Side) If two angles and a side other than the included side of a one triangle are congruent to the corresponding angles and a corresponding side other than the included side of the other triangle, then the two triangles are congruent. $\angle ABC \cong \angle PQR, \angle ACB \cong \angle PRQ$ and $AC \cong PR$ (or $AB \cong PQ$)	
R – H – S	(Right angle–Hypotenuse–Side) If the hypotenuse and one side of the right angled triangle are congruent to the hypotenuse and a corresponding side of the other right angled triangle, then the two given triangles are congruent. $AC \cong PR, \angle B = \angle Q$ and $BC \cong QR$ $\therefore \Delta ABC \cong \Delta PQR$	

Similarity of triangles

Two triangles are said to be similar if the corresponding **angles** are **congruent** and their corresponding **sides** are in **proportion**. The symbol for similarity is ' $\sim$ '. If $\Delta ABC \sim \Delta PQR$, then $\angle ABC \cong \angle PQR, \angle BCA \cong \angle QRP, \angle BAC \cong \angle QPR$

Tests for Similarity

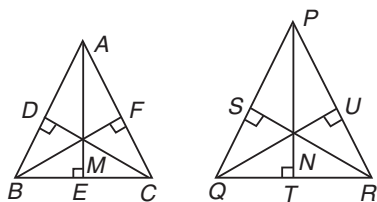
Through the tests for similarity we can deduce the similarity of triangles with minimum required information.

Test	Property/Definition	Diagram
A – A	Angle–Angle If the two angles of one triangle are congruent to the corresponding two angles of the other triangle, then the two triangles are said to be similar. $\angle ABC \cong \angle PQR$ $\angle ACB \cong \angle PRQ$ $\therefore \Delta ABC \sim \Delta PQR$	
S-A-S	Side–Angle–Side If the two sides of one triangle are proportional to the corresponding two sides of the other triangle and the angle included by them are congruent, then the two triangles are similar. i.e., $\frac{AB}{PQ} = \frac{BC}{QR}$ and $\angle ABC = \angle PQR$ $\therefore \Delta ABC \sim \Delta PQR$.	
S-S-S	Side–Side–Side If the three sides of one triangle are proportional to the corresponding three sides of the other triangle, then the two triangles are similar. i.e., $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ $\therefore \Delta ABC \sim \Delta PQR$	

NOTE When the corresponding sides are in proportion, then the corresponding angles are in proportion.

Properties of Similar Triangles

If the two triangles are similar, then for the proportional/corresponding sides we have the following results.



- Ratio of sides = Ratio of heights (altitudes)
= Ratio of medians
= Ratio of angle bisectors
= Ratio of inradii
= Ratio of circumradii

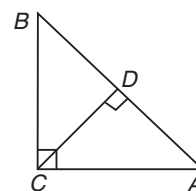
- Ratio of areas = Ratio of squares of corresponding sides.

i.e., if $\triangle ABC \sim \triangle PQR$, then

$$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}$$

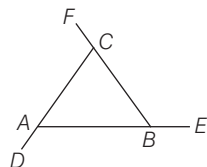
NOTE Rule 1 can also apply with rule 2.

- In a right angled triangle, the triangles on each side of the altitude drawn from the vertex of the right angle to the hypotenuse are similar to the original triangle and to each other too. i.e., $\triangle BCA \sim \triangle BDC \sim \triangle CDA$.

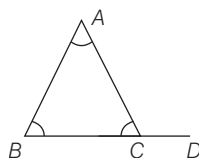


Introductory Exercise 12.2

- In a triangle ABC , if AB , BC and AC are the three sides of the triangle, then which of the statements is necessarily true?
(a) $AB + BC < AC$ (b) $AB + BC > AC$
(c) $AB + BC = AC$ (d) $AB^2 + BC^2 = AC^2$
- The sides of a triangle are 12 cm, 8 cm and 6 cm respectively, the triangle is :
(a) acute (b) obtuse
(c) right (d) can't be determined
- If the sides of a triangle are produced then the sum of the exterior angles i.e., $\angle DAB + \angle EBC + \angle FCA$ is equal to :

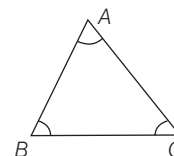


- (a) 180° (b) 270° (c) 360° (d) 240°
- In the given figure BC is produced to D and $\angle BAC = 40^\circ$ and $\angle ABC = 70^\circ$. Find the value of $\angle ACD$.

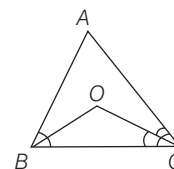


- (a) 30° (b) 40°
(c) 70° (d) 110°

- In a $\triangle ABC$, $\angle BAC > 90^\circ$, then $\angle ABC$ and $\angle ACB$ must be :

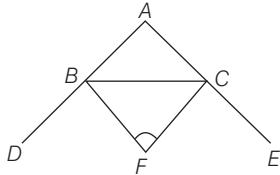


- (a) acute
(b) obtuse
(c) one acute and one obtuse
(d) can't be determined
- If the angles of a triangle are in the ratio $1 : 4 : 7$, then the value of the largest angle is :
(a) 135°
(b) 84°
(c) 105°
(d) none of the above
 - In the adjoining figure $\angle B = 70^\circ$ and $\angle C = 30^\circ$. BO and CO are the angle bisectors of $\angle ABC$ and $\angle ACB$. Find the value of $\angle BOC$.

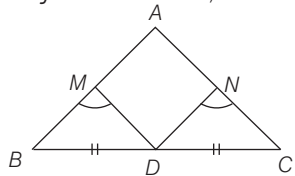


- (a) 30° (b) 40°
(c) 120° (d) 130°

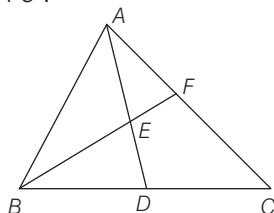
8. In the given diagram of $\triangle ABC$, $\angle B = 80^\circ$, $\angle C = 30^\circ$. BF and CF are the angle bisectors of $\angle CBD$ and $\angle BCE$ respectively. Find the value of $\angle BFC$.



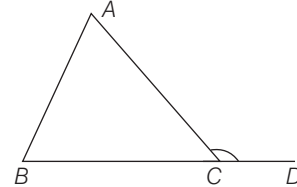
- (a) 110° (b) 50° (c) 125° (d) 55°
9. In an equilateral triangle, the incentre, circumcentre, orthocentre and centroid are :
 (a) conyclic (b) coincident
 (c) collinear (d) None of these
10. In the adjoining figure, D is the midpoint of BC of a $\triangle ABC$. DM and DN are the perpendiculars on AB and AC respectively and $DM = DN$, then the $\triangle ABC$ is :



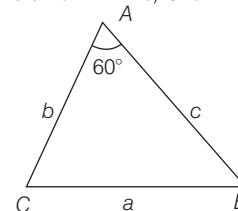
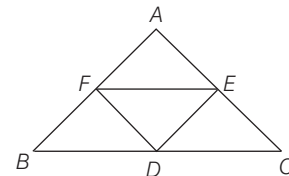
- (a) right angled (b) isosceles
 (c) equilateral (d) scalene
11. In the adjoining figure of $\triangle ABC$, AD is the perpendicular bisector of side BC . The triangle ABC is:
 (a) right angled
 (b) isosceles
 (c) scalene
 (d) equilateral
12. Triangle ABC is such that $AB = 9$ cm, $BC = 6$ cm, $AC = 7.5$ cm. Triangle DEF is similar to $\triangle ABC$. If $EF = 12$ cm then DE is :
 (a) 6 cm (b) 16 cm (c) 18 cm (d) 15 cm
13. In $\triangle ABC$, $AB = 5$ cm, $AC = 7$ cm. If AD is the angle bisector of $\angle A$. Then $BD : CD$ is :
 (a) $25 : 49$ (b) $49 : 25$ (c) $6 : 1$ (d) $5 : 7$
14. In a $\triangle ABC$, D is the mid-point of BC and E is mid-point of AD , BF passes through E . What is the ratio of $AF : FC$?



- (a) $1 : 1$ (b) $1 : 2$ (c) $1 : 3$ (d) $2 : 3$
15. In a $\triangle ABC$, $AB = AC$ and $AD \perp BC$, then :
 (a) $AB < AD$ (b) $AB > AD$
 (c) $AB = AD$ (d) $AB \leq AD$
16. The difference between altitude and base of a right angled triangle is 17 cm and its hypotenuse is 25 cm. What is the sum of the base and altitude of the triangle is :
 (a) 24 cm (b) 31 cm
 (c) 34 cm (d) can't be determined
17. If AB , BC and AC be the three sides of a triangle ABC , then which one of the following is true?
 (a) $AB - BC = AC$ (b) $(AB - BC) > AC$
 (c) $(AB - BA) < AC$ (d) $AB^2 - BC^2 = AC^2$
18. In the triangle ABC , side BC is produced to D . $\angle ACD = 100^\circ$ if $BC = AC$, then $\angle ABC$ is :

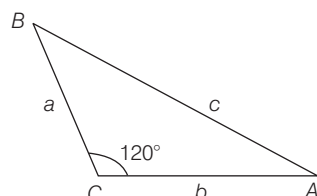


- (a) 40°
 (b) 50°
 (c) 80°
 (d) can't be determined
19. In the adjoining figure D , E and F are the mid-points of the sides BC , AC and AB respectively. $\triangle DEF$ is congruent to triangle :
 (a) ABC (b) AEF
 (c) CDE , BFD (d) AFE , BFD and CDE
20. In the adjoining figure, $\angle BAC = 60^\circ$ and $BC = a$, $AC = b$ and $AB = c$, then :

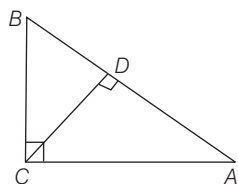


- (a) $a^2 = b^2 + c^2$
 (b) $a^2 = b^2 + c^2 - bc$
 (c) $a^2 = b^2 + c^2 + bc$
 (d) $a^2 = b^2 + 2bc$

21. In the adjoining figure of $\triangle ABC$, $\angle BCA = 120^\circ$ and $AB = c$, $BC = a$, $AC = b$, then:

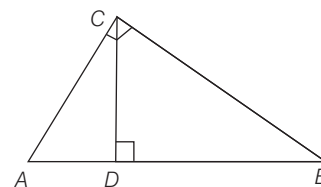


- (a) $c^2 = a^2 + b^2 + ba$
 (b) $c^2 = a^2 + b^2 - ba$
 (c) $c^2 = a^2 + b^2 - 2ba$
 (d) $c^2 = a^2 + b^2 + 2ab$
22. In a right angled $\triangle ABC$, $\angle C = 90^\circ$ and CD is the perpendicular on the hypotenuse AB , $AB = c$, $BC = a$, $AC = b$ and $CD = p$, then :



- (a) $\frac{p}{a} = \frac{p}{b}$ (b) $\frac{1}{p^2} + \frac{1}{b^2} = \frac{1}{a^2}$
 (c) $p^2 = b^2 + c^2$ (d) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$
23. If the medians of a triangle are equal, then the triangle is :
 (a) right angled (b) isosceles
 (c) equilateral (d) scalene
24. The incentre of a triangle is determined by the :
 (a) medians
 (b) angle bisector
 (c) perpendicular bisectors
 (d) altitudes
25. The circumcentre of a triangle is determined by the :
 (a) altitudes
 (b) median
 (c) perpendicular bisector
 (d) angle bisectors
26. The point of intersection of the angle bisectors of a triangle is :
 (a) orthocentre (b) centroid
 (c) incentre (d) circumcentre
27. A triangle PQR is formed by joining the mid-points of the sides of a triangle ABC . 'O' is the circumcentre of $\triangle ABC$, then for $\triangle PQR$, the point 'O' is :
 (a) incentre (b) circumcentre
 (c) orthocentre (d) centroid

28. If in a $\triangle ABC$, 'S' is the circumcentre, then :
 (a) S is equidistant from all the vertices of a triangle
 (b) S is equidistant from all the sides of a triangle
 (c) AS, BS and CS are the angular bisectors
 (d) AS, BS and CS produced are the altitudes on the opposite sides.
29. If AD, BE, CF are the altitudes of $\triangle ABC$ whose orthocentre is H, then C is the orthocentre of :
 (a) $\triangle ABH$ (b) $\triangle BDH$
 (c) $\triangle ABD$ (d) $\triangle BEA$
30. In a right angled $\triangle ABC$, $\angle C = 90^\circ$ and CD is the perpendicular on hypotenuse AB if $BC = 15$ cm and $AC = 20$ cm, then CD is equal to :



- (a) 18 cm (b) 12 cm
 (c) 17.5 cm (d) can't be determined
31. In an equilateral $\triangle ABC$, if a, b and c denote the lengths of perpendiculars from A, B and C respectively on the opposite sides, then :
 (a) $a > b > c$ (b) $a > b < c$
 (c) $a = b = c$ (d) $a = c \neq b$
32. What is the ratio of side and height of an equilateral triangle?
 (a) 2 : 1 (b) 1 : 1
 (c) 2 : $\sqrt{3}$ (d) $\sqrt{3}$: 2
33. The triangle is formed by joining the mid-points of the sides AB, BC and CA of $\triangle ABC$ and the area of $\triangle PQR$ is 6 cm^2 , then the area of $\triangle ABC$ is :
 (a) 36 cm^2 (b) 12 cm^2
 (c) 18 cm^2 (d) 24 cm^2
34. One side other than the hypotenuse of right angle isosceles triangle is 6 cm. The length of the perpendicular on the hypotenuse from the opposite vertex is :
 (a) 6 cm (b) $6\sqrt{2}$ cm
 (c) 4 cm (d) $3\sqrt{2}$ cm
35. Any two of the four triangles formed by joining the mid-points of the sides of a given triangle are :
 (a) congruent
 (b) equal in area but not congruent
 (c) unequal in area and not congruent
 (d) None of the above
36. The internal bisectors of $\angle B$ and $\angle C$ of $\triangle ABC$ meet at O. If $\angle A = 80^\circ$ then $\angle BOC$ is :
 (a) 50° (b) 160°
 (c) 100° (d) 130°

37. The point in the plane of a triangle which is at equal perpendicular distance from the sides of the triangle is :

(a) centroid (b) incentre
(c) circumcentre (d) orthocentre

38. Incentre of a triangle lies in the interior of :

(a) an isosceles triangle only
(b) a right angled triangle only
(c) any equilateral triangle only
(d) any triangle

39. In a triangle PQR , $PQ=20$ cm and $PR = 6$ cm, the side QR is :

(a) equal to 14 cm
(b) less than 14 cm
(c) greater than 14 cm
(d) none of the above

40. The four triangles formed by joining the pairs of mid-points of the sides of a given triangle are congruent if the given triangle is :

(a) an isosceles triangle
(b) an equilateral triangle
(c) a right angled triangle
(d) of any shape

41. O is orthocentre of a triangle PQR , which is formed by joining the mid-points of the sides of a $\triangle ABC$, O is :

(a) orthocentre (b) incentre
(c) circumcentre (d) centroid

42. In a $\triangle ABC$, a line PQ parallel to BC cuts AB at P and AC at Q . If BQ bisects $\angle PQC$, then which one of the following relation is always true :

(a) $BC = CQ$ (b) $BC = BQ$
(c) $BC \neq CQ$ (d) $BC \neq BQ$

43. Which of the following is true, in the given figure, where AD is the altitude to the hypotenuse of a right angled $\triangle ABC$?

(i) $\triangle CAD$ and $\triangle ABD$ are similar
(ii) $\triangle CDA$ and $\triangle ADB$ are congruent
(iii) $\triangle ADB$ and $\triangle CAB$ are similar

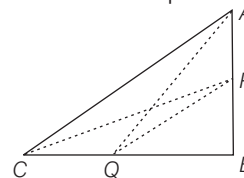
Select the correct answer using the codes given below :

(a) 1 and 2
(b) 2 and 3
(c) 1 and 3
(d) 1, 2 and 3

44. If D is such a point on the side BC of $\triangle ABC$ that $\frac{AB}{AC} = \frac{BD}{CD}$, then AD must be a/an :

(a) altitude of $\triangle ABC$
(b) median of $\triangle ABC$
(c) angle bisector of $\triangle ABC$
(d) perpendicular bisector of $\triangle ABC$

45. In right angled $\triangle ABC$, $\angle B = 90^\circ$, if P and Q are points on the sides AB and BC respectively, then :



(a) $AQ^2 + CP^2 = 2(AC^2 + PQ^2)$
(b) $AQ^2 + CP^2 = AC^2 + PQ^2$
(c) $(AQ^2 + CP^2) = \frac{1}{2}(AC^2 + PQ^2)$
(d) $(AQ + CP) = \frac{1}{2}(AC + PQ)$

46. If ABC is a right angled triangle at B and M, N are the mid-points of AB and BC , then $4(AN^2 + CM^2)$ is equal to :

(a) $4AC^2$ (b) $6AC^2$
(c) $5AC^2$ (d) $\frac{5}{4}AC^2$

47. If $\triangle ABC$ and $\triangle DEF$ are so related that $\frac{AB}{FD} = \frac{BC}{DE} = \frac{CA}{EF}$,

then which of the following is true ?

(a) $\angle A = \angle F$ and $\angle B = \angle D$
(b) $\angle C = \angle F$ and $\angle A = \angle D$
(c) $\angle B = \angle F$ and $\angle C = \angle D$
(d) $\angle A = \angle E$ and $\angle B = \angle D$

48. ABC is a right angle triangle at A and AD is perpendicular to the hypotenuse. Then $\frac{BD}{CD}$ is equal to :

(a) $\left(\frac{AB}{AC}\right)^2$ (b) $\left(\frac{AB}{AD}\right)^2$ (c) $\frac{AB}{AC}$ (d) $\frac{AB}{AD}$

49. Let ABC be an equilateral triangle. Let $BE \perp CA$ meeting CA at E , then $(AB^2 + BC^2 + CA^2)$ is equal to :

(a) $2BE^2$ (b) $3BE^2$ (c) $4BE^2$ (d) $6BE^2$

50. If D, E and F are respectively the mid-points of sides BC, CA and AB of a $\triangle ABC$. If $EF = 3$ cm, $FD = 4$ cm and $AB = 10$ cm, then DE, BC and CA respectively will be equal to :

(a) 6, 8 and 20 cm (b) 4, 6 and 8 cm
(c) 5, 6 and 8 cm (d) $\frac{10}{3}$, 9 and 12 cm

51. In the right angle triangle $\angle C = 90^\circ$. AE and BD are two medians of a triangle ABC meeting at F . The ratio of the area of $\triangle ABF$ and the quadrilateral $FDCE$ is :

(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 2 : 3

52. ABC is a triangle and DE is drawn parallel to BC cutting the other sides at D and E . If $AB = 3.6$ cm, $AC = 2.4$ cm and $AD = 2.1$ cm, then AE is equal to :

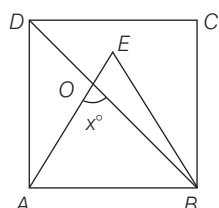
(a) 1.4 cm (b) 1.8 cm
(c) 1.2 cm (d) 1.05 cm

53. Consider the following statements :

- (1) If three sides of a triangle are equal to three sides of another triangle, then the triangles are congruent.
(2) If three angles of a triangle are equal to three angles of another triangle respectively, then the two triangles are congruent. Out of these statements :

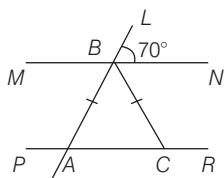
(a) 1 is correct and 2 is false
(b) both 1 and 2 are false
(c) both 1 and 2 are correct
(d) 1 is false and 2 is correct

54. In the figure $\triangle ABE$ is an equilateral triangle in a square $ABCD$. Find the value of angle x in degrees :



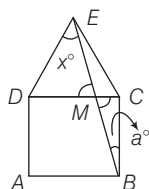
(a) 60° (b) 45° (c) 75° (d) 90°

55. In the given diagram $MN \parallel PR$ and $m\angle LBN = 70^\circ$, $AB = BC$. Find $m\angle ABC$:



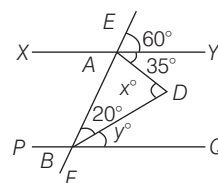
(a) 40° (b) 30°
(c) 35° (d) 55°

56. In the given diagram, equilateral triangle EDC surmounts square $ABCD$. Find the $m\angle BED$ represented by x where $m\angle EBC = \alpha^\circ$.



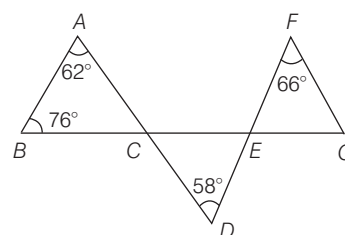
(a) 45° (b) 60°
(c) 30° (d) none of these

57. In the given diagram $XY \parallel PQ$. Find $m\angle x^\circ$ and $m\angle y^\circ$.



(a) 75° and 40° (b) 45° , 60°
(c) 75° , 45° (d) 60° and 45°

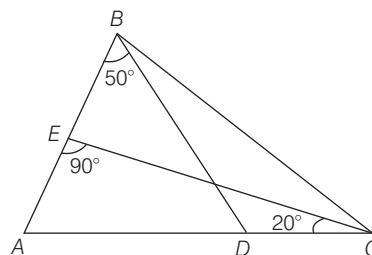
58. In the adjoining figure $m\angle CAB = 62^\circ$, $m\angle CBA = 76^\circ$
 $m\angle ADE = 58^\circ$ and $\angle DFG = 66^\circ$,



Find $m\angle FGE$.

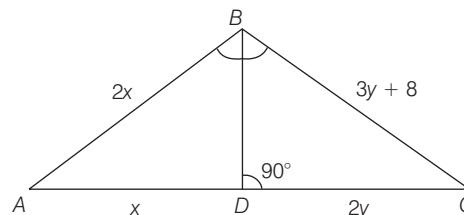
(a) 44° (b) 34°
(c) 36° (d) none of these

59. In the given figure $CE \perp AB$, $m\angle ACE = 20^\circ$ and $m\angle ABD = 50^\circ$. Find $m\angle BDA$.



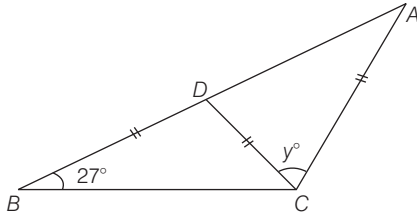
(a) 50° (b) 60°
(c) 70° (d) 80°

60. In the $\triangle ABC$, BD bisects $\angle B$, and is perpendicular to AC . If the lengths of the sides of the triangle are expressed in terms of x and y as shown, find the value of x and y .

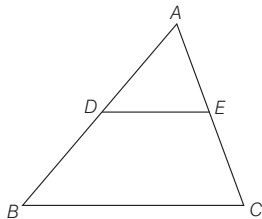


(a) 6, 12 (b) 10, 12
(c) 16, 8 (d) 8, 15

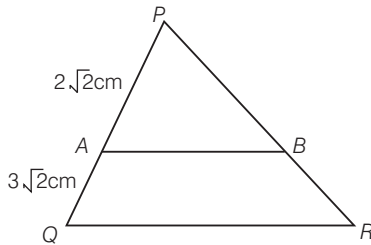
61. In the following figure $ADBC$. $BD = CD = AC$, $m\angle ABC = 27^\circ$, $m\angle ACD = y$. Find the value of y .



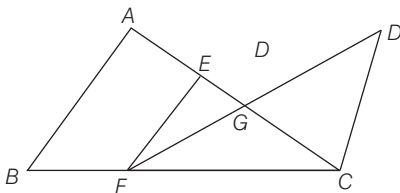
- (a) 27° (b) 54°
(c) 72° (d) 58°
62. ABC is an isosceles triangle with $AB = AC$. Side BA is produced to D such that $AB = AD$. Find $m\angle BCD$.
(a) 60° (b) 90°
(c) 120° (d) can't be determined
63. In $\triangle ABC$, $AC = 5$ cm. Calculate the length of AE where $DE \parallel BC$. Given that $AD = 3$ cm and $BD = 7$ cm.



- (a) 2 cm (b) 1 cm (c) 1.5 cm (d) 2.5 cm
64. In $\triangle PQR$, $AP = 2\sqrt{2}$ cm, $AQ = 3\sqrt{2}$ cm and $PR = 10$ cm, $AB \parallel QR$. Find the length of BR .

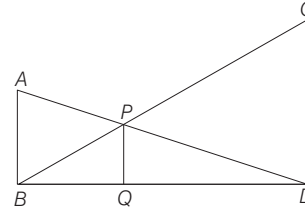


- (a) $6\sqrt{2}$ cm (b) 6 cm
(c) $5\sqrt{2}$ cm (d) none of these
65. In the adjoining figure (not drawn to scale) AB , EF and CD are parallel lines. Given that $EG = 5$ cm, $GC = 10$ cm and $DC = 18$ cm. Calculate AC , if $AB = 15$ cm.

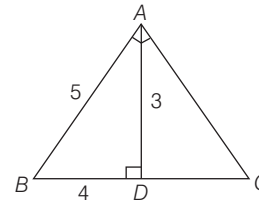


- (a) 21 cm (b) 25 cm
(c) 18 cm (d) 28 cm

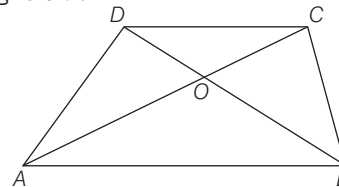
66. In the given figure, AB , PQ and CD are perpendiculars on the line segment BD . If $AB = x$, $CD = y$ and $BD = z$, find the length of PQ .



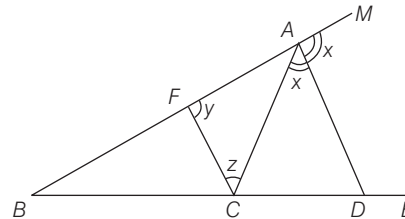
- (a) $\frac{3xyz}{x+y+z}$ (b) $\frac{x+y}{z}$ (c) $\frac{xy}{x+y}$ (d) $\frac{x+y+z}{xyz}$
67. In the adjoining figure the angle BAC and $\angle ADB$ are right angles. $BA = 5$ cm, $AD = 3$ cm and $BD = 4$ cm, what is the length of DC ?



- (a) 2.5 (b) 3 (c) 2.25 (d) 2
68. The areas of the similar triangles are in the ratio of 25 : 36. What is the ratio of their respective heights?
(a) 5 : 6 (b) 6 : 5 (c) 1 : 11 (d) 2 : 3
69. In the given diagram $AB \parallel CD$, then which one of the following is true?

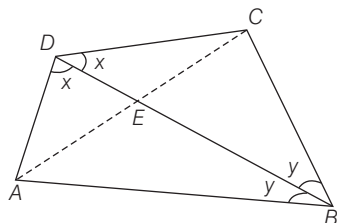


- (a) $\frac{AB}{CD} = \frac{AO}{OC}$ (b) $\frac{AB}{CD} = \frac{BO}{OD}$
(c) $\triangle AOB \sim \triangle COD$ (d) All of these
70. The bisector of the exterior $\angle A$ of $\triangle ABC$ intersects the side BC produced to D . Here CF is parallel to AD . Which one of the following is correct?

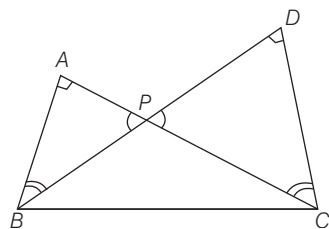


- (a) $\frac{AB}{AC} = \frac{BD}{CD}$ (b) $\frac{AB}{AC} = \frac{CD}{BD}$
(c) $\frac{AB}{AC} = \frac{BC}{CD}$ (d) none of these

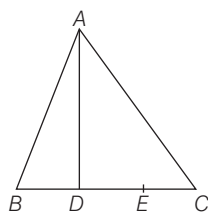
71. The diagonal BD of a quadrilateral $ABCD$ bisects $\angle B$ and $\angle D$, then :



- (a) $\frac{AB}{CD} = \frac{AD}{BC}$ (b) $\frac{AB}{BC} = \frac{AD}{CD}$
 (c) $AB = AD \times BC$ (d) none of these is true
72. Two right triangles ABC and DBC are drawn on the same hypotenuse BC on the same side of BC . If AC and DB intersect at P , then :

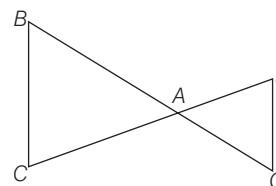


- (a) $\frac{AP}{PC} = \frac{BP}{DP}$ (b) $AP \times DP = PC \times BP$
 (c) $AP \times PC = BP \times DP$ (d) $AP \times BP = PC \times PD$
73. A man goes 150 m due east and then 200 m due north. How far is he from the starting point?
 (a) 200 m (b) 350 m
 (c) 250 m (d) 175 m
74. From a point O in the interior of a $\triangle ABC$ perpendiculars OD , OE and OF are drawn to the sides BC , CA and AB respectively, then which one of the following is true ?
 (a) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$
 (b) $AB^2 + BC^2 = AC^2$
 (c) $AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2$
 (d) $AF^2 + BD^2 + CE^2 = OD^2 + OE^2 + OF^2$
75. In an equilateral triangle ABC , the side BC is trisected at D . Find the value of AD^2 :

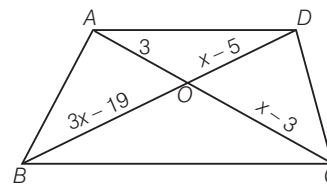


- (a) $\frac{9}{7} AB^2$ (b) $\frac{7}{9} AB^2$
 (c) $\frac{3}{4} AB^2$ (d) $\frac{4}{5} AB^2$

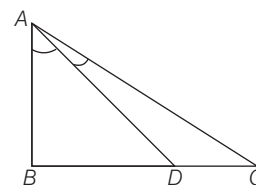
76. ABC is a triangle in which $\angle A = 90^\circ$.
 $AN \perp BC$, $AC = 12$ cm and $AB = 5$ cm. Find the ratio of the areas of $\triangle ANC$ and $\triangle ANB$.
 (a) 125 : 44 (b) 25 : 144
 (c) 144 : 25 (d) 12 : 5
77. A vertical stick 15 cm long casts its shadow 10 cm long on the ground. At the same time a flag pole casts a shadow 60 cm long. Find the height of the flag pole.
 (a) 40 cm (b) 45 cm
 (c) 90 cm (d) none of these
78. Vertical angles of two isosceles triangles are equal. Then corresponding altitudes are in the ratio 4 : 9. Find the ratio of their areas :
 (a) 16 : 49 (b) 16 : 81
 (c) 16 : 65 (d) none of these
79. In the figure $\triangle ACB \sim \triangle APQ$. If $BC = 8$ cm, $PQ = 4$ cm, $AP = 2.8$ cm, find CA :



- (a) 8 cm (b) 6.5 cm
 (c) 5.6 cm (d) none of these
80. In the figure $BC \parallel AD$. Find the value of x .

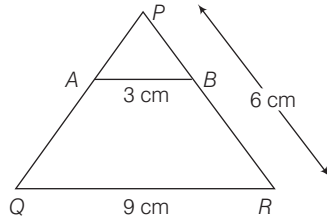


- (a) 9, 10 (b) 7, 8 (c) 10, 12 (d) 8, 9
81. In an equilateral triangle of side $2a$, calculate the length of its altitude.
 (a) $2a\sqrt{3}$ (b) $a\sqrt{3}$
 (c) $a\frac{\sqrt{3}}{2}$ (d) none of these
82. In figure AD is the bisector of $\angle BAC$. If $BD = 2$ cm, $CD = 3$ cm and $AB = 5$ cm. Find AC :



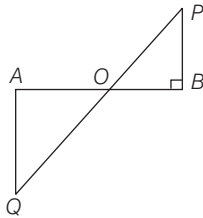
- (a) 6 cm (b) 7.5 cm
 (c) 10 cm (d) 15 cm

83. In the figure $AB \parallel QR$. Find the length of PB .



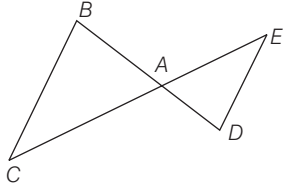
- (a) 2 cm (b) 3 cm (c) 2.5 cm (d) 4 cm

84. In the figure QA and PB are perpendicular to AB . If $AO = 10$ cm, $BO = 6$ cm and $PB = 9$ cm. Find AQ .



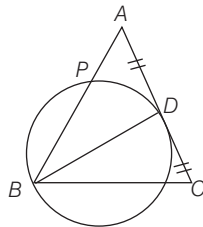
- (a) 8 cm (b) 9 cm (c) 15 cm (d) 12 cm

85. In the given figure $AB = 12$ cm, $AC = 15$ cm and $AD = 6$ cm. $BC \parallel DE$, find the length of AE .



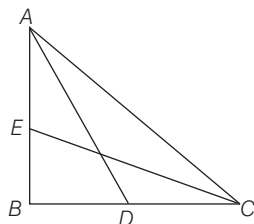
- (a) 6 cm (b) 7.5 cm (c) 9 cm (d) 10 cm

86. In the figure, ABC is a triangle in which $AB = AC$. A circle through B touches AC at D and intersects AB at P . If D is the mid-point of AC , Find the value of AB :



- (a) $2AP$
(b) $3AP$
(c) $4AP$
(d) none of the above

87. In figure, ABC is a right angled triangle, right angled at B . AD and CE are the two medians drawn from A and C respectively. If $AC = 5$ cm and $AD = 3\sqrt{5}/2$ cm, find the length of CE .



- (a) $2\sqrt{5}$ cm (b) 2.5 cm (c) 5 cm (d) $4\sqrt{2}$ cm

88. In a $\triangle ABC$, $AB = 10$ cm, $BC = 12$ cm and $AC = 14$ cm. Find the length of median AD . If G is the centroid, find length of GA .

- (a) $\frac{5}{3}\sqrt{7}$, $\frac{5}{9}\sqrt{7}$ (b) $5\sqrt{7}$, $4\sqrt{7}$
(c) $\frac{10}{\sqrt{3}}$, $\frac{8}{3}\sqrt{7}$ (d) $4\sqrt{7}$, $\frac{8}{3}\sqrt{7}$

89. $\triangle ABC$ is a right angled at A and AD is the altitude to BC . If $AB = 7$ cm and $AC = 24$ cm. Find the ratio of AD is to AM if M is the mid-point of BC .

- (a) 25 : 41 (b) 32 : 41
(c) 336/625 (d) 625/336

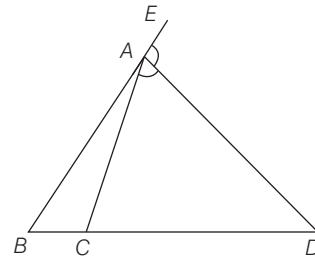
90. Area of $\triangle ABC = 30$ cm². D and E are the mid-points of BC and AB respectively. Find $A(\triangle BDE)$.

- (a) 10 cm² (b) 7.5 cm²
(c) 15 cm² (d) none of these

91. The three sides of a triangle are given which one of the following is not a right angle?

- (a) 20, 21, 29 (b) 16, 63, 65
(c) 56, 90, 106 (d) 36, 35, 74

92. In the following figure, AD is the external bisector of $\angle EAC$, and it intersects BC produced to D . If $AB = 12$ cm, $AC = 8$ cm and $BC = 4$ cm, find CD .

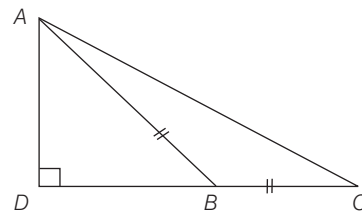


- (a) 10 cm (b) 6 cm
(c) 8 cm (d) 9 cm

93. In $\triangle ABC$, $AB^2 + AC^2 = 2500$ cm² and median $AD = 25$ cm, Find BC .

- (a) 25 cm
(b) 40 cm
(c) 50 cm
(d) 48 cm

94. In the given figure, $AB = BC$ and $\angle BAC = 15^\circ$, $AB = 10$ cm. Find the area of $\triangle ABC$.

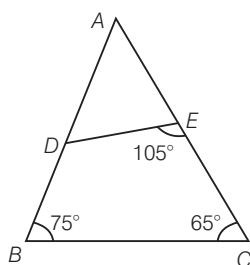


- (a) 50 cm² (b) 40 cm²
(c) 25 cm² (d) 32 cm²

95. In $\triangle ABC$, G is the centroid, $AB = 15$ cm, $BC = 18$ cm and $AC = 25$ cm. Find GD , where D is the mid-point of BC .

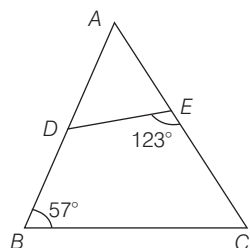
- (a) $\frac{1}{3}\sqrt{86}$ cm (b) $\frac{2}{3}\sqrt{86}$ cm
(c) $\frac{8}{3}\sqrt{15}$ cm (d) none of the above

96. In the given figure, if $\frac{DE}{BC} = \frac{2}{3}$ and if $AE = 10$ cm. Find AB .

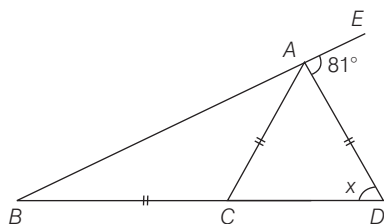


- (a) 16 cm (b) 12 cm (c) 15 cm (d) 18 cm
97. Find the maximum area that can be enclosed in a triangle of perimeter 24 cm.
- (a) 32 cm^2
(b) $16\sqrt{3} \text{ cm}^2$
(c) $16\sqrt{2} \text{ cm}^2$
(d) 27 cm^2

98. In the figure $AD = 12$ cm, $AB = 20$ cm and $AE = 10$ cm. Find EC .



- (a) 14 cm (b) 10 cm
(c) 8 cm (d) 15 cm
99. In the given figure, $BC = AC = AD$, $\angle EAD = 81^\circ$. Find the value of x .



- (a) 45° (b) 54° (c) 63° (d) 36°

100. What is the ratio of inradius to the circumradius of a right angled triangle?

- (a) 1 : 2 (b) $1 : \sqrt{2}$
(c) 2 : 5 (d) can't be determined

101. In a $\triangle ABC$, $AC = BC$. Point D lies on BC such that $CD = AD = AB$. Find $\angle ACB$.

- (a) 18° (b) 30°
(c) 36° (d) 48°

102. In $\triangle ABC$, AD is the angle bisector of $\angle A$ and a median to BC . If $AC = 15$ cm, find AB .

- (a) 12 cm (b) 18 cm
(c) 15 cm (d) can't be determined

103. In a $\triangle ABC$, $\frac{\angle BAC}{\angle ACB} = \frac{1}{2}$ and $AC = 4$ cm, $BC = 6$ cm. Find AB .

- (a) 6 (b) 10
(c) 9 (d) none of these

104. In a $\triangle ABC$, BD is the angle bisector of $\angle ABC$ and E is a point on AB such that $ED = AD$ and

$$\frac{(BD)(BC)}{CD} = \frac{(BE)(AB)}{AE}.$$

If $\angle A + \angle C = 66^\circ$, what is the value of $\angle A - \angle C$?

- (a) 10° (b) 22°
(c) 12° (d) 20°

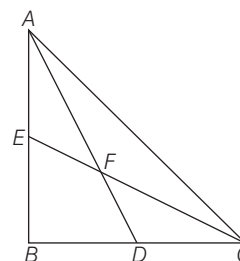
105. Find the area of an isosceles triangle ABC in which $\angle ABC = 120^\circ$ and each of the smaller side is 1 unit.

- (a) $\frac{\sqrt{3}}{4}$ (b) $\frac{\sqrt{3}}{2}$ (c) $2\sqrt{3}$ (d) $\frac{3\sqrt{3}}{10}$

106. If O is the incentre of a triangle ABC and $\angle AOC = 124^\circ$, find the $\angle ABC$.

- (a) 56 (b) 112
(c) 68 (d) can't be determined

107. The following diagram depicts the right angle triangle ABC , where line segments AD and CE are the medians intersecting at a point F . Find the ratio of the area of $\triangle AEF$ to that of quadrilateral $BEFD$.



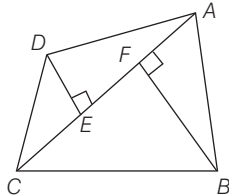
- (a) 2 : 3 (b) 4 : 9
(c) 1 : 2 (d) none of these

12.4 Quadrilaterals

A four sided closed figure is called a quadrilateral. It is denoted by symbol ' '.

Properties

1. Sum of four interior angles is 360° .
2. The figure formed by joining the mid-points of a quadrilateral is a parallelogram.



3. The sum of opposite sides of a quadrilateral circumscribed about a circle, is always equal.
4. Area of quadrilateral $= \frac{1}{2} \times \text{one of the diagonals} \times \text{sum of the perpendiculars drawn to the diagonals from the opposite vertices. i.e., } A(ABCD) = \frac{1}{2} \times AC \times (DE + BF)$

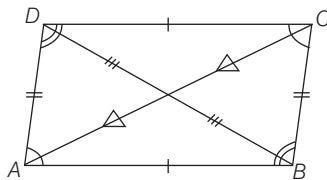
Types of Quadrilaterals

Parallelogram

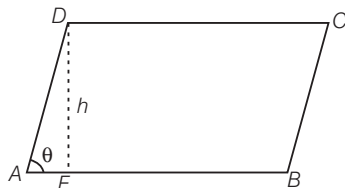
A quadrilateral whose opposite sides are parallel.

Properties

1. The opposite sides are parallel and equal.



2. Opposite angles are equal.
3. Sum of any two adjacent angles is 180° .
4. Diagonals bisect each other.



5. Diagonals need not be equal in length.
6. Diagonals need not bisect at right angle.
7. Diagonals need not bisect angles at the vertices.

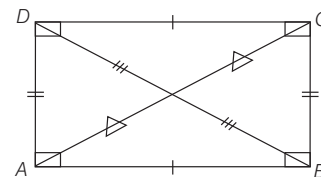
8. Each diagonal divides a parallelogram into two congruent triangles.
9. Lines joining the mid-points of the adjacent sides of a **quadrilateral** form a **parallelogram**.
10. Lines joining the mid-points of the adjacent sides of a **parallelogram** is a **parallelogram**.
11. The parallelogram that is inscribed in a circle is a rectangle.
12. The parallelogram that is circumscribed about a circle is a rhombus.
13. (a) Area of a parallelogram = base $\times$ height
(b) Area of parallelogram
= product of any two adjacent sides
 $\times$ sine of the included angles.
 $= AB \times AD \times \sin \theta$
14. Perimeter of a parallelogram = 2 (sum of any two adjacent sides)
15. $(AC)^2 + (BD)^2 = (AB)^2 + (BC)^2 + (CD)^2 + (AD)^2$
 $= 2(AB^2 + BC^2)$
16. Parallelogram that lie on the same base and between the same parallel lines are equal in area.
17. Area of a triangle is half of the area of a parallelogram which lie on the same base and between the same parallel lines.
18. A parallelogram is a rectangle if its diagonals are equal.

Rectangle

A parallelogram in which all the four angles at vertices are right (i.e., 90°), is called a rectangle.

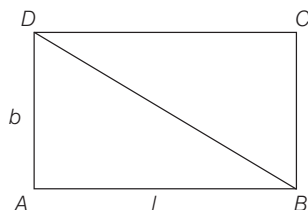
Properties

1. Opposite sides are parallel and equal.
2. Opposite angles are equal and of 90° .



3. Diagonals are equal and bisect each other, but not necessarily at right angles.
4. When a rectangle is inscribed in a circle, the diameter of the circle is equal to the diagonal of the rectangle.
5. For the given perimeter of rectangles, a square has maximum area.
6. The figure formed by joining the mid-points of the adjacent sides of a rectangle is a rhombus.

7. The quadrilateral formed by joining the mid-points of intersection of the angle bisectors of a parallelogram is a rectangle.
8. Every rectangle is a parallelogram.
9. Area of a rectangle = length $\times$ breadth ($= l \times b$)



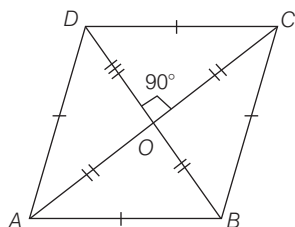
10. Diagonals of a rectangle $= \sqrt{l^2 + b^2}$
11. Perimeter of a rectangle $= 2(l + b)$
 $l \rightarrow$ length and $b \rightarrow$ breadth

Rhombus

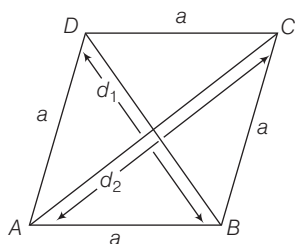
A parallelogram in which all sides are equal, is called a rhombus.

Properties

1. Opposite sides are parallel and equal.
2. Opposite angles are equal.
3. Diagonals bisect each other at right angle, but they are not necessarily equal.



4. Diagonals bisect the vertex angles.
5. Sum of any two adjacent angles is 180° .



6. Figure formed by joining the mid-points of the adjacent sides of a rhombus is a rectangle.
7. A parallelogram is a rhombus if its diagonals are perpendicular to each other.

8. (a) Area of a rhombus $= \frac{1}{2} \times$ product of diagonals
 $= \frac{1}{2} \times d_1 \times d_2$

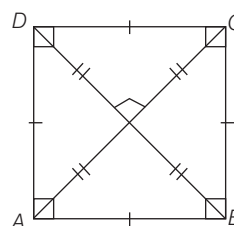
- (b) Area of a rhombus = Product of adjacent sides $\times$ sine of the included angle.

Square

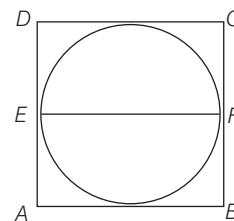
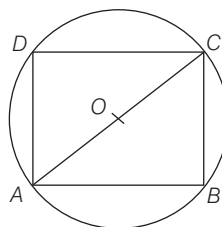
A rectangle whose all sides are equal or a rhombus whose all angles are equal is called a square. Thus each square is a parallelogram, a rectangle and a rhombus.

Properties

1. All side are equal and parallel.
2. All angles are right angles.
3. Diagonals are equal and bisect each other at right angle.

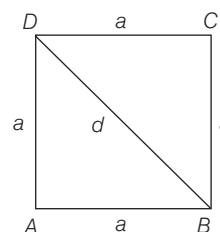


4. Diagonal of an inscribed square is equal to diameter of the inscribing circle.
5. Side of a circumscribed square is equal to the diameter of the inscribed circle.



6. The figure formed by joining the mid-points of the adjacent sides of a square is a square.

7. Area $= (\text{side})^2 = a^2 = \frac{(\text{diagonal})^2}{2} = \frac{d^2}{2}$



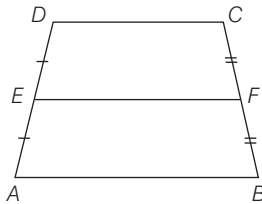
8. Diagonal $= \text{side} \sqrt{2} = a\sqrt{2}$
9. Perimeter $= 4 \times \text{side} = 4a$

Trapezium

A quadrilateral whose only one pair of sides is parallel and other two sides are not parallel.

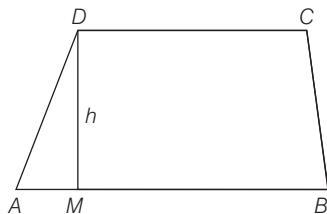
Properties

1. The line joining the mid-points of the oblique (non-parallel) sides is half the sum of the parallel sides and is called the median.



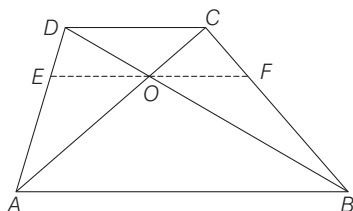
$$\begin{aligned} \text{(i.e., Median)} &= \frac{1}{2} \times \text{sum of parallel sides} \\ &= \frac{1}{2} \times (AB + DC) = EF \end{aligned}$$

2. If the non-parallel sides are equal then the diagonals will also be equal to each other.
3. Diagonals intersect each other proportionally in the ratio of lengths of parallel sides.
4. By joining the mid-points of adjacent sides of a trapezium four similar triangles are obtained.
5. If a trapezium is inscribed in a circle, then it is an isosceles trapezium with equal oblique sides.
6. Area of trapezium $= \frac{1}{2} \times (\text{sum of parallel sides} \times \text{height})$
 $= \frac{1}{2} \times (AB + CD) \times h$
7. $AC^2 + BD^2 = BC^2 + AD^2 + 2AB \cdot CD$



8. If a line segment EF , which is parallel to the two parallel sides of a trapezium $ABCD$, passes through a point O , which is the point of intersection of the two diagonals AC and BD , where point E lies on AD and point F lies on BC , we have

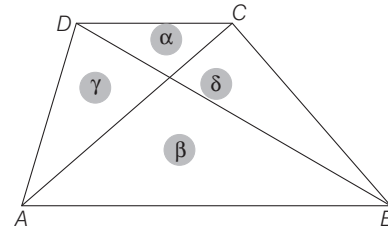
$$\text{(i) Point } O \text{ bisects the line } EF. \quad \text{(ii) } EF = \frac{2(AB)(CD)}{AB + CD}$$



9. In a trapezium $ABCD$, as shown in the diagram, where AB is parallel to CD and the diagonals AC and BD intersect each other at O , and if the total area of the trapezium is A , we have

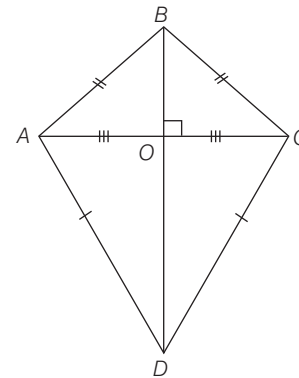
$$\text{(i) } \sqrt{A} = \sqrt{\alpha} + \sqrt{\beta}$$

$$\text{(ii) } \gamma = \delta = \sqrt{\alpha \cdot \beta}$$



Kite

In a kite two pairs of adjacent sides are equal.



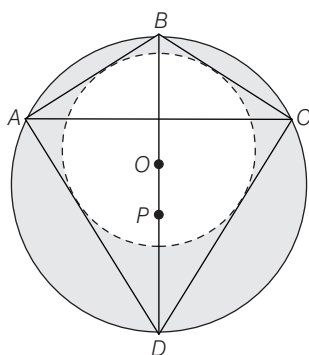
Properties

1. $AB = BC$ and $AD = CD$
2. Diagonals, AC and BD , are the perpendicular bisectors.
3. Longer diagonal (BD) bisects the shorter diagonal (AC).
4. Longer diagonal (BD) is the angle bisector of the pair of opposite angles.
5. Area of kite $= \frac{1}{2} \times \text{product of diagonals}$
 $= \text{inradius} \times \text{semi-perimeter}$
6. A kite is symmetrical about the longer diagonal BD .
7. The longer diagonal (BD) divides the kite into two congruent triangles ABD and CBD .
8. The shorter diagonal divides the kite into two isosceles triangles ABC and ADC .
9. The kites are exactly the quadrilaterals that are both tangential and orthodiagonal.
10. A kite can always inscribe an incircle and the incentre is the intersection of the angular bisectors of kite.

11. The inradius of the incircle

$$r = \frac{\text{Area of kite}}{\text{semiperimeter of kite}}$$

12. If a kite is equilateral, it must be a rhombus.
 13. If a kite is equiangular, it must be equilateral and thus it is a square.
 14. The kites that are also cyclic quadrilaterals are exactly the ones formed from two congruent right triangles. That is, for these kites the two equal angles on opposite sides of the axis of symmetry are each 90 degrees. These shapes are called right kites and they are in fact bi-centric quadrilaterals.



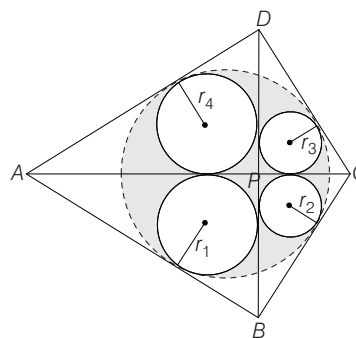
In the above diagram, points A, B, C, D are concyclic, so the quadrilateral $ABCD$ is a concyclic one and the centre of the larger circle is P . And the two congruent right triangles are $\triangle BAD$ and $\triangle BCD$. The centre of inscribed circle (smaller one) is O . Since the kite has two centers – incentre and circumcentre, that's why a right kite is bi-centric kite.

15. There are only eight polygons that can tile the plane in such a way that reflecting any tile across any one of its edges produces another tile; one of them is a right kite, with 60° , 90° , and 120° angles.

Condition for a Tangential Quadrilateral to Become a Kite

A tangential quadrilateral is a kite if and only if any one of the following conditions is/are true :

- The diagonals are perpendicular.
- The area is one half the product of the diagonals.
- The two line segments connecting opposite points of tangency have equal length.
- One pair of opposite tangent lengths has equal length.
- The bi-medians have equal length.
- The products of opposite sides are equal.
- The center of the incircle lies on a line of symmetry that is also a diagonal.
- If the diagonals in a tangential quadrilateral $ABCD$ intersect at P , and the incircles in triangles ABP, BCP, CDP, DAP have radii r_1, r_2, r_3 and r_4 , respectively, then the quadrilateral is a kite if and only if $r_1 + r_3 = r_2 + r_4$.

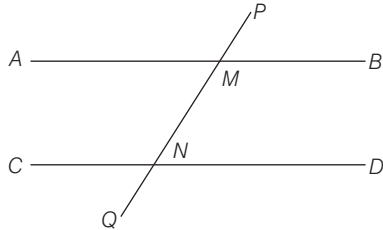


- If the excircles to the same four triangles opposite the vertex P have radii R_1, R_2, R_3 and R_4 , respectively, the quadrilateral is a kite if and only if $R_1 + R_3 = R_2 + R_4$.

Introductory Exercise 12.3

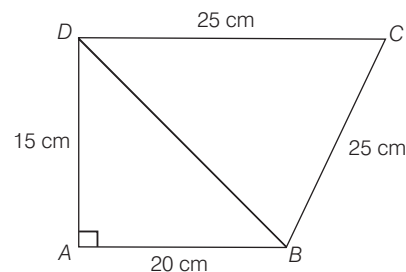
- The measures of the angles of a quadrilateral taken in order are proportional to $1 : 2 : 3 : 4$, then the quadrilateral is :
 (a) parallelogram (b) trapezium
 (c) rectangle (d) rhombus
- Find the measure of largest angle of a quadrilateral if the measures of its interior angles are in the ratio of $3 : 4 : 5 : 6$.
 (a) 60° (b) 120°
 (c) 90° (d) can't be determined
- $ABCD$ is a parallelogram, P and Q are the points on the diagonal AC such that $AP = QC$, then quadrilateral $BPDQ$ is a :
 (a) trapezium (b) parallelogram
 (c) square (d) none of these
- In a parallelogram $ABCD$, bisectors of consecutive angles A and B intersect at P . Find the measure of $\angle APB$:
 (a) 90° (b) 60°
 (c) 120° (d) data insufficient

5. AB and CD are two parallel lines and a transversal PQ intersects AB and CD at M and N respectively. The bisector of the interior angles form a quadrilateral :



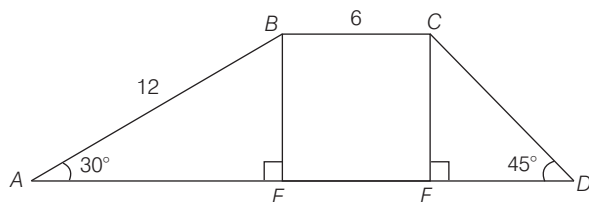
- (a) rectangle (b) square
(c) parallelogram (d) none of these
6. In the given figure $AE = BC$ and $AE \parallel BC$ and the three sides AB , CD and ED are equal in length. If $m\angle A = 102^\circ$, find measures of $\angle BCD$.
-
- (a) 138° (b) 162°
(c) 88° (d) none of these
7. $ABCD$ is a square, A is joined to a point P on BC and D is joined to a point Q on AB . If $AP = DQ$ and AP intersects DQ at R then, $\angle DRP$ is :
- (a) 60° (b) 120°
(c) 90° (d) can't be determined
8. A point X inside a rectangle $PQRS$ is joined to the vertices then, which of the following is true :
- (a) $A(\triangle PSX) = A(\triangle RXQ)$
(b) $A(\triangle PSX) + A(\triangle PXQ) = A(\triangle RSX) + A(\triangle RQX)$
(c) $A(\triangle PXS) + A(\triangle RXQ) = A(\triangle SRX) + A(\triangle PXQ)$
(d) None of the above
9. $ABCD$ is a parallelogram and $m\angle DAB = 30^\circ$, $BC = 20$ cm and $AB = 20$ cm. Find the area of parallelogram :
- (a) 150 cm^2 (b) 200 cm^2
(c) 400 cm^2 (d) 260 cm^2
10. The length of a side of a rhombus is 10 m and one of its diagonal is 12 m. The length of the other diagonal is :
- (a) 15 m (b) 18 m
(c) 16 m (d) can't be determined
11. $ABCD$ is a parallelogram and BD is a diagonal. $\angle BAD = 65^\circ$ and $\angle DBC = 45^\circ$, then $m\angle BDC$ is :
- (a) 65° (b) 70°
(c) 20° (d) none of these

12. If $ABCD$ is a parallelogram in which P and Q are the centroids of $\triangle ABD$ and $\triangle BCD$, then PQ equals :
- (a) AQ (b) AP (c) BP (d) DQ
13. Two parallelograms stand on equal bases and between the same parallels. The ratio of their areas is :
- (a) $1 : 1$ (b) $\sqrt{2} : 1$ (c) $1 : 3$ (d) $1 : 2$
14. If a rectangle and a parallelogram are equal in area and have the same base and are situated on the same side, then the ratio of perimeter of rectangle and that of parallelogram is k , then k is :
- (a) $k > 1$ (b) $k < 1$
(c) $k = 1$ (d) can't be determined
15. If area of a parallelogram with sides l and b is A and that of a rectangle with sides l and b is B , then :
- (a) $A < B$ (b) $A = B$
(c) $A > B$ (d) none of these
16. $ABCD$ is a parallelogram and M is the mid-point of BC . AB and DM are produced to meet at N , then :
- (a) $AN = \sqrt{3}AB$ (b) $AN = 2AB$
(c) $AN^2 = \frac{3}{2}AB^2$ (d) $AN^2 = 2AB^2$
17. In a rectangle $ABCD$, P, Q are the mid-points of BC and AD respectively and R is any point on PQ , then $\triangle ARB$ equals :
- (a) $\frac{1}{2}(\square ABCD)$ (b) $\frac{1}{3}(\square ABCD)$
(c) $\frac{1}{4}(\square ABCD)$ (d) none of these
18. Diagonals of a parallelogram are 8 m and 6 m respectively. If one of side is 5m, then the area of parallelogram is :
- (a) 18 m^2 (b) 30 m^2
(c) 24 m^2 (d) 48 m^2
19. In the given figure $AD = 15$ cm, $AB = 20$ cm and $BC = CD = 25$ cm. Find the area of $\square ABCD$.

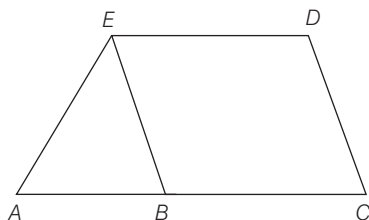


- (a) $\frac{25}{4}(24 + 25\sqrt{3}) \text{ cm}^2$
(b) $24(25 + 24\sqrt{3}) \text{ cm}^2$
(c) $\frac{25}{2}(24 + 25\sqrt{3}) \text{ cm}^2$
(d) none of the above

20. In the trapezium $ABCD$, $\angle BAE = 30^\circ$, $\angle CDF = 45^\circ$. $BC = 6$ cm and $AB = 12$ cm. Find the area of trapezium.



- (a) $18(3 + \sqrt{3}) \text{ cm}^2$
 (b) $36\sqrt{3} \text{ cm}^2$
 (c) $12(3 + 2\sqrt{3}) \text{ cm}^2$
 (d) none of the above
21. Area of quadrilateral $ACDE$ is 36 cm^2 , B is the mid-point of AC . Find the area of $\triangle ABE$ if $AC \parallel DE$ and $BE \parallel DC$.



- (a) 10 cm^2 (b) 9 cm^2
 (c) 12 cm^2 (d) can't be determined
22. A square and a rhombus have the same base and the rhombus is inclined at 30° . What is the ratio of area of the square to the area of the rhombus?
 (a) $\sqrt{2} : 1$ (b) $2 : 1$
 (c) $1 : 1$ (d) $2 : \sqrt{3}$
23. Find the area of a quadrilateral with sides 17, 25, 30 and 28 cm and one of its diagonal is 26 cm.
 (a) 450 cm^2 (b) 360 cm^2
 (c) 540 cm^2 (d) 720 cm^2
24. $PQRS$ is a parallelogram in which $\angle P = 70^\circ$, $\angle Q = 90^\circ$ and $\angle R = 100^\circ$. How many points in the plane of the quadrilateral are there such that a point is equidistant from its vertices?
 (a) 0 (b) 1
 (c) 2 (d) 3

25. $ABCD$ is a parallelogram. The diagonals AC and BD intersect at a point O . If E, F, G and H are the mid-points of AO, DO, CO and BO respectively, then the ratio of $(EF + FG + GH + HE)$ to $(AD + DC + CB + BA)$ is :

- (a) $1 : 1$ (b) $1 : 2$
 (c) $1 : 3$ (d) $1 : 4$

26. If $ABCD$ is a rhombus, then :

- (a) $AC^2 + BD^2 = 4AB^2$
 (b) $AC^2 + BD^2 = AB^2$
 (c) $AC^2 + BD^2 = 2AB^2$
 (d) $2(AC^2 + BD^2) = 3AB^2$

27. If P is a point within a rectangle $ABCD$, then :

- (a) $AP^2 + PC^2 = BP^2 + PD^2$
 (b) $AP^2 + BP^2 = PC^2 + PD^2$
 (c) $AP + PC = BP + PD$
 (d) $AP \times PC = BP \times PD$

28. $\square ABCD$ is a parallelogram, $AB = 14$ cm, $BC = 18$ cm and $AC = 16$ cm. Find the length of the other diagonal.

- (a) 24 cm (b) 28 cm
 (c) 36 cm (d) 32 cm

29. In a trapezium $ABCD$, where $AB \parallel CD$ and diagonal AC intersects the other diagonal BD at O . Which of the followings are always correct?

(i) $\frac{DO}{AO} = \frac{CO}{BO}$ (ii) $\frac{DO}{OC} = \frac{AO}{OB}$

(iii) $\frac{DO}{OB} = \frac{CO}{OA}$

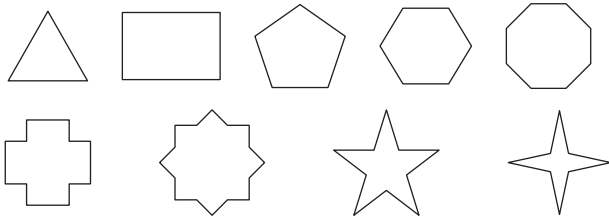
- (a) only (i) and (ii)
 (b) only (ii) and (iii)
 (c) only (i) and (iii)
 (d) only (iii)

30. In a trapezium $ABCD$, where $AB \parallel CD$ and diagonals AC intersects the other diagonal BD at O . If a line segment EF parallel to AB and CD passes through the point O , where E lies on AD and F lies on BC , find the length of EF , provided $AB = 6$ cm and $CD = 2$ cm.

- (a) 3 cm (b) 4 cm
 (c) 4.5 cm (d) none of these

12.5 Polygons

When three or more than three line segments are joined end to end on the same plane they form the closed area in various shapes. These shapes are called the polygons. Essentially, polygons are 2-dimensional plane figures. Look at the following figures to get the better picture of the shapes of polygons.

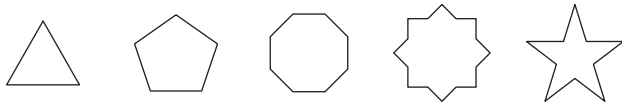


A polygon gets its name on the basis of the number of sides that it has. For example, a triangle has three sides, a quadrilateral has 4 sides, a pentagon has 5 sides, and so on.

Types of Polygon

Simple Polygon

A polygon in which no more than two line segments meet at any vertex. In other words, the sides of the polygon do not self-intersect each other.



Complex Polygon

A polygon in which more than two line segments meet at any vertex. In other words the sides of the polygon self-intersect each other. A complex polygon is neither convex nor concave.



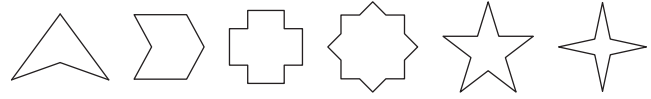
Convex Polygon

A polygon in which each of its interior angle is less than 180° is known as a convex polygon. A polygon is convex if and only if any line containing a side of the polygon doesn't contain a point in the interior of the polygon.



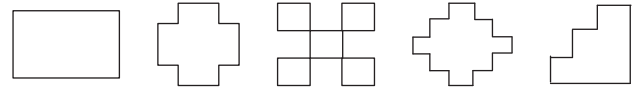
Concave Polygon

A polygon in which even if one interior angle is greater than 180° is known as a concave polygon.



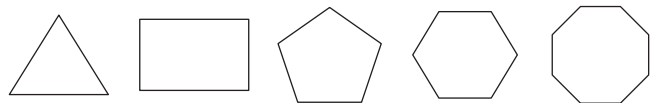
Rectilinear Polygon

A polygon whose sides meet at right angles, i.e., all its interior angles are 90° or 270° .



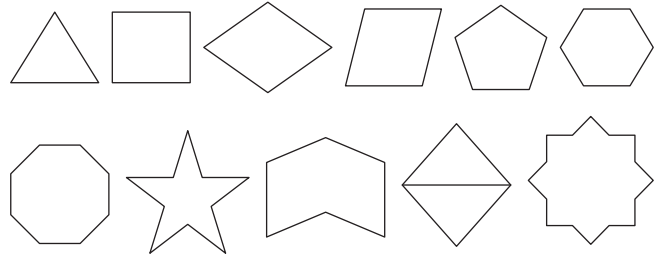
Equiangular Polygon

A polygon in which all its interior angles are equal is known as equiangular polygon.



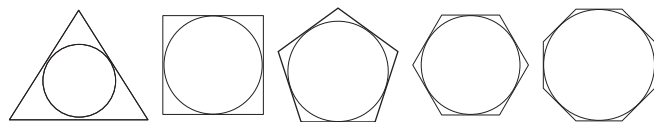
Equilateral Polygon

A polygon in which all sides (or edges) are of the same length.



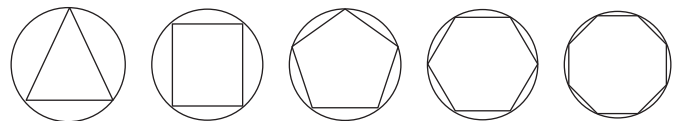
Tangential Polygon

A polygon whose all sides are tangent to an inscribed circle.



Cyclic Polygon

A polygon whose all the vertices lie on the circle.

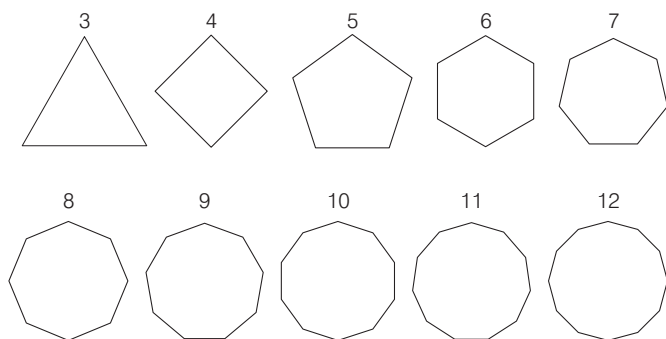


Regular Polygon

A polygon is regular if it is both cyclic and equilateral.

- A regular polygon has all its sides equal and all its interior angles equal.

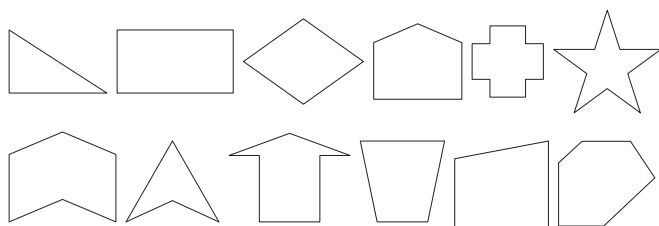
- A regular polygon has an incircle and a circumcircle. That is every regular polygon is tangential and cyclic polygon.



Irregular Polygon

A polygon which is not a regular one is known as an irregular polygon.

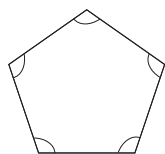
- If all the sides of a polygon are not equal, it's an irregular polygon.
- If all the interior angles of a polygon are not equal, it's an irregular polygon.



Basics of a Polygon

Interior Angle

Angle formed by two adjacent sides inside the polygon is known as interior angle.



Sum of all the interior angles of a regular polygon
 $= (n - 2) \times 180^\circ$

Each interior angle of a regular polygon

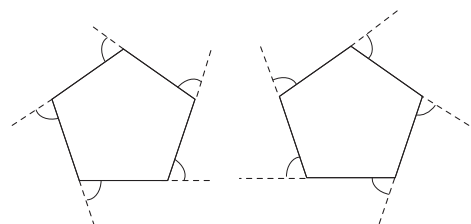
$$= \frac{(n - 2) \times 180^\circ}{n}$$

Each interior angle of a regular polygon

$$= (180^\circ - \text{exterior angle})$$

Exterior Angle

Angle formed by two adjacent sides outside the polygon.



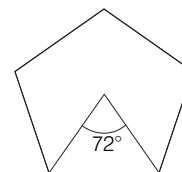
Sum of all exterior angles of a regular polygon $= 360^\circ$
 (always constant)

Each exterior angle of a regular polygon $= \frac{360^\circ}{n}$

Each exterior angle of a regular polygon
 $= (180^\circ - \text{interior angle})$

Central Angle

The angle at the center of the polygon formed by two radii, connected independently to the two adjacent vertices, is known as the central angle.

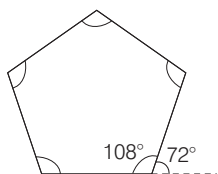


Each Central Angle $= \frac{2\pi}{n} = \frac{360^\circ}{n}$

The following table shows some angles of the various regular polygons.

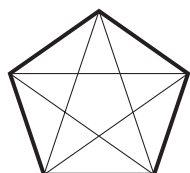
Number of Sides	Polygon	Sum of all the Angles	Each Interior Angle	Each Exterior Angle	Each Central Angle
3	Triangle	180°	60°	120°	120°
4	Quadrilateral	360°	90°	90°	90°
5	Pentagon	540°	108°	72°	72°
6	Hexagon	720°	120°	60°	60°
7	Heptagon	900°	$\left(128\frac{4}{7}\right)^\circ$	$\left(51\frac{3}{7}\right)^\circ$	$\left(51\frac{3}{7}\right)^\circ$
8	Octagon	1080°	135°	45°	45°
9	Nonagon	1260°	140°	40°	40°
10	Decagon	1440°	144°	36°	36°
...	...	...	...	...	...
...	...	...	...	...	...
n	n -gon	$(n - 2)180^\circ$	$\frac{(n - 2) \times 180^\circ}{n}$	$\frac{360^\circ}{n}$	$\frac{360^\circ}{n}$

In a polygon, Interior angle + Exterior angle = 180°



Diagonal

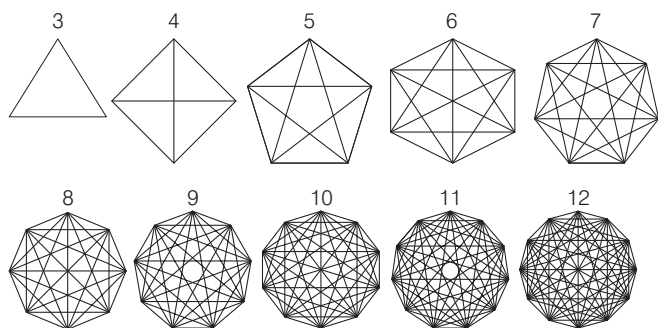
The line segments joining any two non-adjacent vertices are known as diagonals.



The number of diagonals in a polygon

$$= {}^nC_2 - n = \frac{n(n-3)}{2}; \text{ for } n \geq 3$$

Number of Sides	Polygon	Number of diagonals
3	Triangle	0
4	Quadrilateral	2
5	Pentagon	5
6	Hexagon	9
7	Heptagon	14
8	Octagon	20
9	Nonagon	27
10	Decagon	35
...	...	...
...	...	...
n	n -gon	$\frac{n(n-3)}{2}$



Longest Diagonal of a Regular Polygon

There are two different cases.

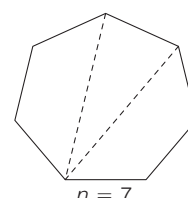
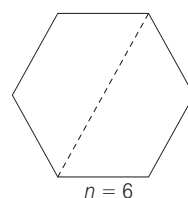
One when there is even number of sides in a polygon and the other one when there is odd number of sides in a polygon.

When n is even : The longest diagonal

$$= 2 \times \text{radius of the polygon}$$

When n is odd : The longest diagonal = $\frac{s}{2 \sin\left(\frac{90^\circ}{n}\right)}$

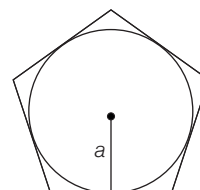
Where s denotes the length of each side and n denotes the number of sides in the regular polygon. And radius means circumradius.



Apothem (or Inradius)

It is the radius of the incircle of the regular polygon.

Apothem is perpendicular bisector of the side of the polygon.

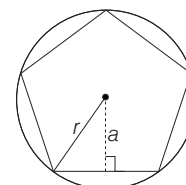


$$\begin{aligned} \text{Apothem of a regular polygon} &= \frac{1}{2} \times s \times \cot\left(\frac{\pi}{n}\right) \\ &= r \cos\left(\frac{\pi}{n}\right) \end{aligned}$$

(Where, s and r are the side and radius of the regular polygon.)

Radius (or Circumradius)

It is the radius of the circumcircle of the regular polygon.



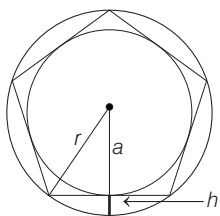
$$\text{Radius of a polygon} = \frac{1}{2} \times s \times \operatorname{cosec}\left(\frac{\pi}{n}\right) = a \sec\left(\frac{\pi}{n}\right)$$

Where, s and a are the side and apothem of a regular polygon.

Sagitta

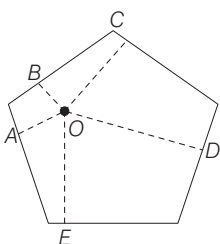
The perpendicular distance h from an arc's midpoint to the chord across it, equal to the radius r minus the apothem a .

That is $h = r - a$



Interior Point

For a regular n -gon (polygon of n -sides), the sum of the perpendicular distances from any interior point to the n sides is n times the apothem. Here, the apothem being the distance from the center to any side.



In the above diagram, if the apothem of the pentagon is a and since there are 5 sides in pentagon, so we have

$$AO + BO + CO + DO + EO = 5a$$

Perimeter

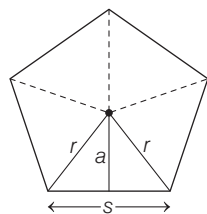
It is the total sum of the measurements of all the sides of a polygon.

The perimeter of a regular polygon
= number of sides $\times$ length of each side

Area

Area of a polygon is the region enclosed by all the n sides of the polygon.

An n -sided polygon can be divided into n congruent triangles having a common vertex at the centre.



Let s , a , p and r be the side, apothem, perimeter and radius, respectively, then

$$\text{Area of a regular polygon} = n \times \left(\frac{1}{2} \times s \times a \right) = \frac{1}{2} \times n \times s \times a$$

$$\text{Area of a regular polygon} = \frac{1}{2} \times a \times p$$

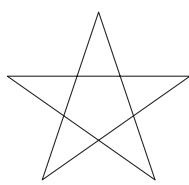
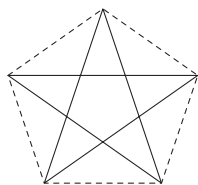
NOTE The other formulae for the area of polygon can be derived by using the trigonometric ratios, as following.

- Area of a regular polygon $= \frac{1}{4} \times n \times s^2 \cot \left(\frac{\pi}{n} \right)$
- Area of a regular polygon $= n \times a^2 \times \tan \left(\frac{\pi}{n} \right)$
- Area of a regular polygon $= \frac{1}{2} \times n \times r^2 \times \sin \left(\frac{2\pi}{n} \right)$

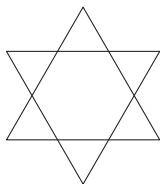
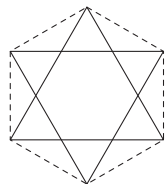
Regular Polygram (or Star Polygon)

A regular star polygon is a self-intersecting, equilateral equiangular polygon, created by connecting one vertex of a simple, regular, n -sided polygon to another, non-adjacent vertex and continuing the process until the original vertex is reached again. Thus, if out of total n vertices of an n -gon (i.e., an n -sided polygon) every k -th vertex is connected with each other, it is denoted by $\{n, k\}$, where $k < \frac{n}{2}$.

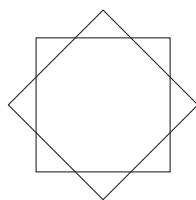
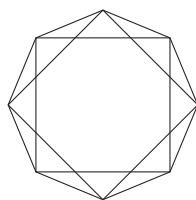
	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
$n = 3$					
$n = 4$					
$n = 5$					
$n = 6$					
$n = 7$					
$n = 8$					
$n = 9$					
$n = 10$					
$n = 11$					
$n = 12$					



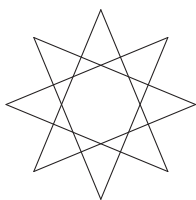
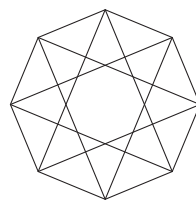
In the above figure out of 5 vertices of pentagon every 2nd vertex is connected with each other. It is denoted by $\left\{\frac{5}{2}\right\}$.



In the above figure out of 6 vertices of hexagon every 2nd vertex is connected with each other. It is denoted by $\left\{\frac{6}{2}\right\}$.



In the above figure out of 8 vertices of octagon every 2nd vertex is connected with each other. It is denoted by $\left\{\frac{8}{2}\right\}$.



In the above figure out of 8 vertices of octagon every 3rd vertex is connected with each other. It is denoted by $\left\{\frac{8}{3}\right\}$.

Under certain conditions a regular star polygon can be drawn in a single stroke such as polygon $\{5/2\}$, $\{8/3\}$ and it has to be drawn in more than one stroke as polygon $\{6/2\}$, $\{8/2\}$, shown above. A star polygon $\{n/k\}$ can be drawn in a single stroke (or it is connected or irreducible) if and only if n and k are relatively prime.

The following table depicts the various polygons and their respective polygrams $\{n/k\}$. Following is the list of some best-known polygrams (or star polygons).

Number of sides n	Symbol	Polygram
5	$\{5/2\}$	Pentagram
6	$\{6/2\}$	Hexagram
7	$\{7/2\}$	Heptagram
8	$\{8/2\}$	Star of Laxmi
8	$\{8/3\}$	Octagram
9	$\{9/3\}$	Nonagram
10	$\{10/3\}$	Decagram

Circumradius of a Polygram : If a star polygon $\{n, k\}$, with n, k positive integers such that $\text{HCF}(n, k) = 1$ and the edge length of the polygon is 1 unit, the circumradius of the star polygon is

$$R = \frac{\sin\left[\left(\frac{n-2k}{2n}\right)\pi\right]}{\sin\left[\left(\frac{2k}{n}\right)\pi\right]}$$

Sum of the Interior Angles of a Polygram : For any value of n , if $k = 2$, the sum of all the internal angles of such a polygram $= (n - 4)\pi$.

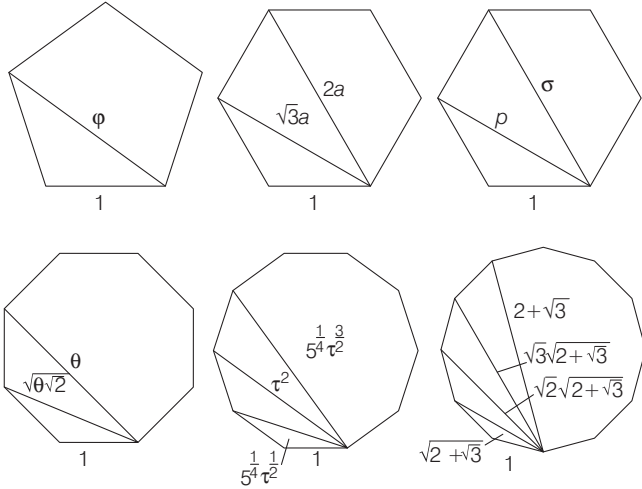
Edge of a Polygram (or Star Polygon) : The edge of a star polygon is equal to the length of the diagonal of a corresponding regular polygon depending upon the value of k . However, in case of even number of sides ($n = 4, 6, 8, \dots$) the longest possible diagonal of the regular polygon does not act in a manner to form a polygram so I have not mentioned this value in the following table.

Let us consider the edge of the regular polygon be 1 unit and $d_1, d_2, d_3, \dots, d_n$ denote the lengths of the diagonals of the regular polygon, such that $d_1 < d_2 < d_3 < \dots < d_n$, then we have the following values of various possible diagonals for the selected regular polygons.

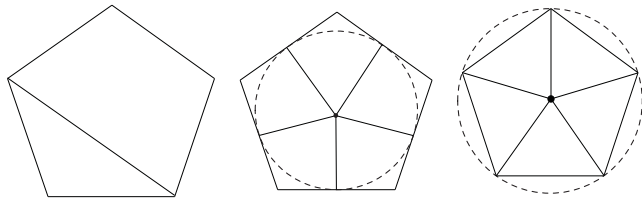
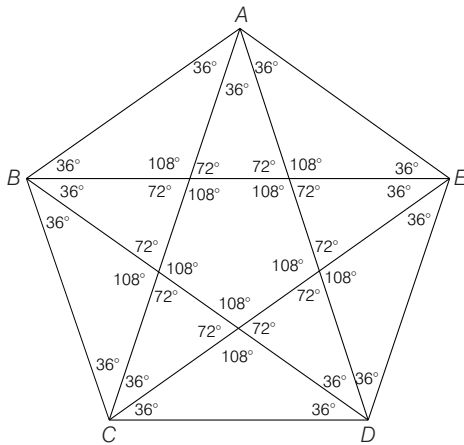
n	$k = 1$	$k = 2$	$k = 3$	$k = 4$
	d_1	d_2	d_3	d_4
5	ϕ			
6	$\sqrt{3}$			
7	ρ	σ		
8	$\sqrt{\theta\sqrt{2}}$	θ		
10	$5^{1/2} \times \tau^{1/2}$	τ^2	$5^{1/4} \times \tau^{3/2}$	
12	$\sqrt{2 + \sqrt{3}}$	$\sqrt{2}(\sqrt{2 + \sqrt{3}})$	$\sqrt{3}(\sqrt{2 + \sqrt{3}})$	$2 + \sqrt{3}$

$$\phi = \left(\frac{1 + \sqrt{5}}{2} \right), \quad \theta = (1 + \sqrt{2}) = 2.414213, \quad \rho = 1.8019377,$$

$$\sigma = 2.2469796$$



Pentagon



Let s, d, a, r, A denote side, diagonal, apothem, radius, area of a pentagon, then we have

$$d = \frac{1}{2} (1 + \sqrt{5}) s = \phi s$$

$$a = \frac{1}{10} (\sqrt{25 + 10\sqrt{5}}) s$$

$$r = \frac{1}{10} (\sqrt{50 + 10\sqrt{5}}) s$$

$$A = \frac{1}{4} (\sqrt{25 + 10\sqrt{5}}) s^2$$

$$a = \frac{1}{4} (1 + \sqrt{5}) r$$

$$s = 2\sqrt{r^2 - a^2}$$

Pentaflake

When five pentagons can be arranged around an identical pentagon, then it forms the first iteration of the pentaflake. This cluster of six pentagons has the shape of a pentagon with five triangular wedges removed, as you can see in the following figure.

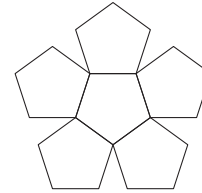


Fig. (i)

For a pentagon of side length 1, the first ring of pentagons has centers at radius

$$d_1 = 2a = \frac{1}{2} (1 + \sqrt{5}) (r)$$

(Where a is the apothem and r is the radius of each of the six pentagons)

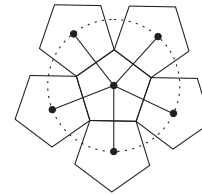


Fig. (ii)

For a pentagon of side length 1, the second ring of pentagons has centers at radius

$$d_2 = 2(r + h) = (2 + \sqrt{5}) r = \left[\frac{1}{2} (1 + \sqrt{5}) \right]^2 (r)$$

(Where a is the apothem and r is the radius of each of the six pentagons)

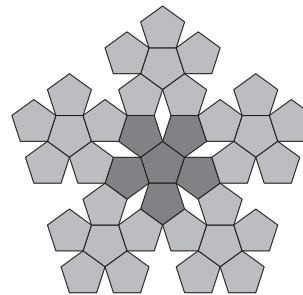


Fig. (iii)

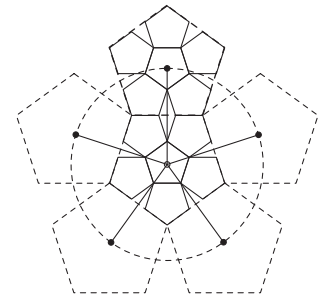


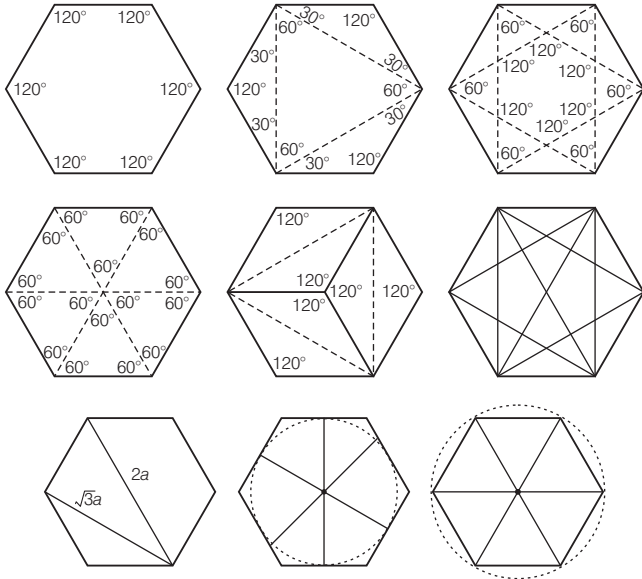
Fig. (iv)

For a pentagon of side length 1, the n -th ring of pentagons has centers at radius

$$d_n = (r) \left[\frac{1}{2} (1 + \sqrt{5}) \right]^{2n-1}$$

(Where a is the apothem and r is the radius of each of the six pentagons)

Hexagon



Let a, d, r, R, A denote side, diagonal, apothem (or inradius), radius (or Circumradius), area of a pentagon, then we have

$$d_1 = \sqrt{3}a; \quad d_2 = 2a$$

$$r = \frac{\sqrt{3}}{2}a; \quad R = a$$

$$A = \frac{3\sqrt{3}}{2}a^2; \quad \frac{r}{R} = \frac{\sqrt{3}}{2}$$

$$\frac{A_r}{A_R} = \frac{3}{4}$$

Hexaflakes

When six hexagons can be arranged around an identical hexagon, then it forms the first iteration of the hexaflake. This cluster of seven hexagons has the shape of a hexagon with six triangular wedges removed from the sides and six from the canters, as you can see in the first figure. Fig. (ii) shows the hexaflake of double iteration and Fig. (iii) shows the triple iteration.

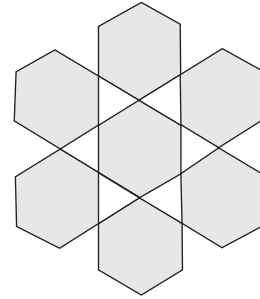


Fig. (i)

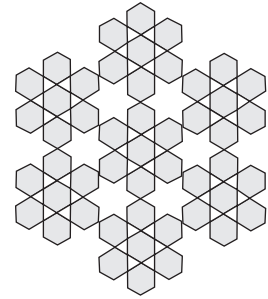


Fig. (ii)

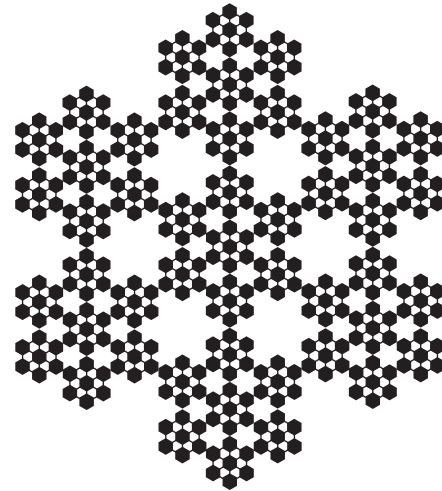
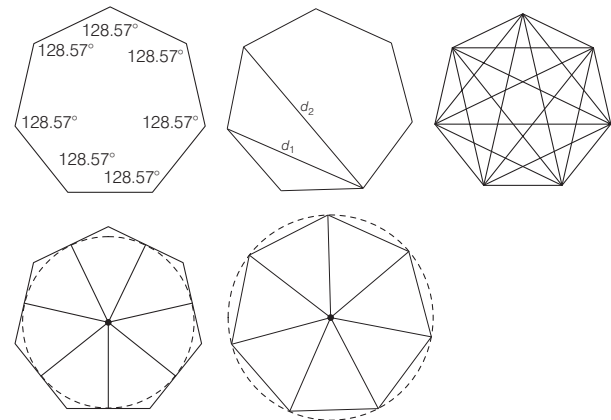


Fig. (iii)

Heptagon



$$\text{Area of a heptagon} = 3.633 s^2$$

(s is the side of the regular heptagon)

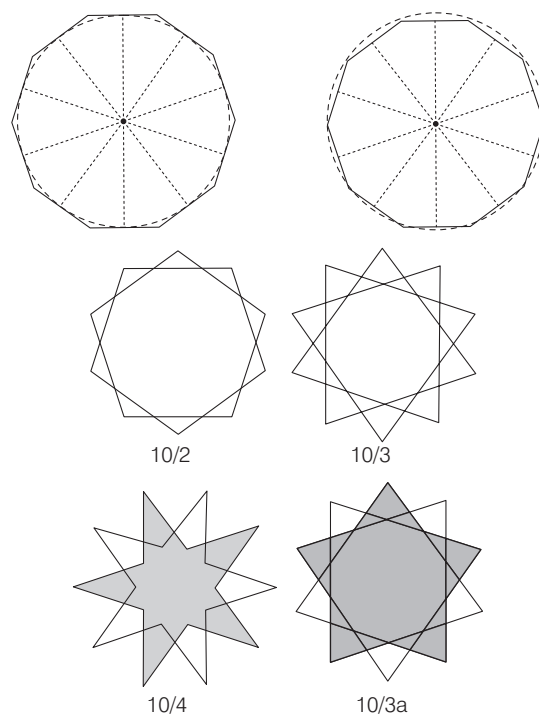
$$\text{Area of a heptagon} = 3.371 a^2$$

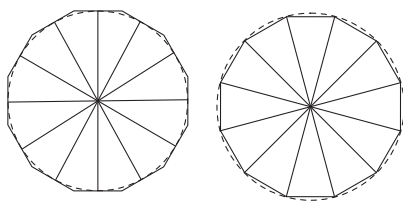
(a is the apothem of the regular heptagon)

$$\text{Area of a heptagon} = 2.7364 r^2$$

(r is the circumradius of the regular heptagon)

Figure 1 consists of eight sub-images labeled (a) through (h).
 (a) A regular octagon with all interior angles labeled 135° .
 (b) A regular octagon with a 3x3 grid of lines extending from its center.
 (c) A regular octagon with a diamond shape (square rotated 45 degrees) inscribed within it.
 (d) A regular octagon inscribed within a dashed square.
 (e) A regular octagon with three lines from one vertex to the opposite side, labeled d_1 , d_2 , and d_3 .
 (f) A regular octagon with a complex internal structure of many lines connecting vertices and midpoints.
 (g) A regular octagon with a dashed circle passing through its vertices and a central dot.
 (h) A regular octagon with a dashed circle passing through its vertices and a central dot, with dashed lines connecting the center to each vertex.

$$\begin{aligned} d_1 &= (\sqrt{2+\sqrt{2}})s & d_2 &= (1+\sqrt{2})s \\ d_3 &= (\sqrt{4+2\sqrt{2}})s & d_3 &= \sqrt{2}(d_1) \\ a &= \frac{(1+\sqrt{2})s}{2} & r &= \frac{(\sqrt{4+2\sqrt{2}})s}{2} \\ A &= 2(1+\sqrt{2})s^2 = d_2^2 - s^2 \\ A &= 2(d_3s) = 4as \end{aligned}$$

$$\begin{aligned} d_1 &= \frac{2as}{r} = \frac{2(\sqrt{5+2\sqrt{5}})s}{(1+\sqrt{5})} & d_2 &= \frac{(3+\sqrt{5})s}{2} = \varphi^2 s = \varphi r \\ d_3 &= (\sqrt{5+2\sqrt{5}})s = 2a & d_4 &= (1+\sqrt{5})s = 2\varphi s = 2r \\ a &= \frac{(\sqrt{5+2\sqrt{5}})s}{2} & r &= \frac{(1+\sqrt{5})s}{2} = \varphi \cdot s \\ A &= \frac{5(\sqrt{5+2\sqrt{5}})s^2}{2} = 5as \end{aligned}$$



Let s, d, a, r, A denote side, diagonal, apothem (inradius), radius (or circumradius), area of a decagon, then we have

$$d_1 = (\sqrt{2 + \sqrt{3}})s$$

$$d_2 = (\sqrt{3} + 1)s$$

$$d_3 = (\sqrt{2 + \sqrt{3}} + \sqrt{2})s = d_1 + \sqrt{2} = \sqrt{3}(d_1)$$

$$d_4 = (\sqrt{3} + 2)s$$

$$d_5 = 2(\sqrt{2 + \sqrt{3}})s = 2(d_1) = \sqrt{2}(d_2)$$

$$d_5 = (\sqrt{2} + \sqrt{6})s$$

$$a = \frac{(\sqrt{3} + 2)s}{2} = \frac{d_4}{2}$$

$$r = (\sqrt{2 + \sqrt{3}})s = d_1 = \frac{d_5}{2}$$

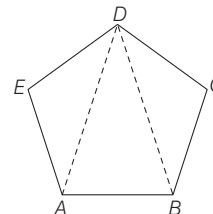
$$A = 3(\sqrt{3} + 2)s^2 = 6as = 3r^2$$

Introductory Exercise 12.4

- Each interior angle of a regular polygon is 140° . The number of sides is :
(a) 10 (b) 8 (c) 6 (d) 9
- Each angle of a regular hexagon is :
(a) 60° (b) 120°
(c) 90° (d) none of these
- If one of the interior angles of a regular polygon is equal to $5/6$ times of one of the interior angles of a regular pentagon, then the number of sides of the polygon is :
(a) 3 (b) 4 (c) 6 (d) 8
- The sum of the interior angles of a polygon is 1260° . The number of sides of the polygon is :
(a) 6 (b) 7 (c) 8 (d) 9
- If each interior angle of a regular polygon is 3 times its exterior angle, the number of sides of the polygon is :
(a) 4 (b) 5 (c) 6 (d) 8
- Difference between the interior and exterior angles of regular polygon is 60° . The number of sides in the polygon is :
(a) 5 (b) 6 (c) 8 (d) 9
- A polygon has 54 diagonals. The number of sides in the polygon is :
(a) 7 (b) 9
(c) 12 (d) none of these
- The ratio between the number of sides of two regular polygons is $1 : 2$ and the ratio between their interior angle is $3 : 4$. The number of sides of these polygons are respectively :
(a) 3, 6 (b) 4, 8 (c) 6, 9 (d) 5, 10
- The sum of all the interior angles of a regular polygon is four times the sum of its exterior angles. The polygon is :
(a) hexagon (b) triangle (c) decagon (d) nonagon
- The ratio of the measure of an angle of a regular nonagon to the measure of its exterior angle is :
(a) $3 : 5$ (b) $5 : 2$ (c) $7 : 2$ (d) $4 : 5$

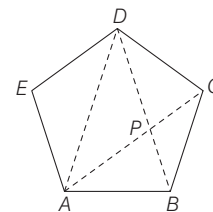
- In a regular polygon, if the number of diagonals is k times the number of sides and each interior angle is θ , then k is
(a) $\frac{3\theta - 2\pi}{2\pi - 2\theta}$ (b) $\frac{4\theta - \pi}{2\pi - 2\theta}$ (c) $\frac{3\theta - \pi}{2\pi - 2\theta}$ (d) $\frac{3(\theta - \pi)}{2(\pi - \theta)}$

- In the following figure, $ABCDE$ is a regular pentagon. Find $\angle ADB$,



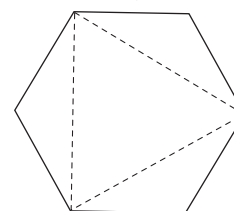
- (a) 36° (b) 30°
(c) 24° (d) cannot be determined

- In the following figure, $ABCDE$ is a regular pentagon. Find $\angle DPC$.



- (a) 54° (b) 72° (c) 24° (d) 60°

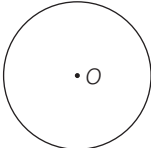
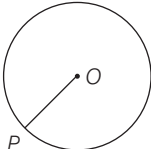
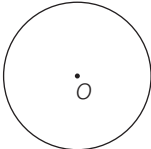
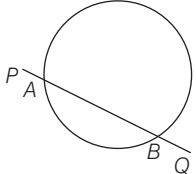
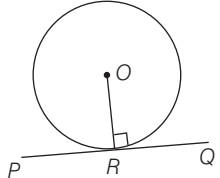
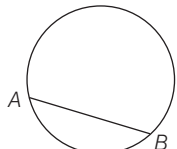
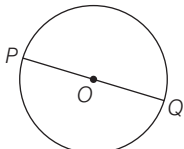
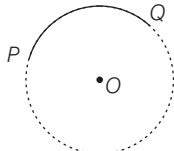
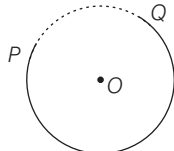
- In the following figure find the ratio of the area of the triangle to that of the regular hexagon.



- (a) $2 : 1$ (b) $\sqrt{3} : \sqrt{6}$
(c) $\sqrt{2} : \sqrt{3}$ (d) $1 : 2$

12.6 Circles

Definition : A circle is a set of points on a plane which lie at a fixed distance from a fixed point.

Nomenclature	Definition	Diagram
Centre	The fixed point is called the centre. In the given diagram 'O' is the centre of the circle.	
Radius	The fixed distance is called a radius. In the given diagram OP is the radius of the circle. (Point P lies on the circumference)	
Circumference	The circumference of a circle is the distance around a circle, which is equal to $2\pi r$. ($r \rightarrow$ radius of the circle)	
Secant	A line segment which intersects the circle at two distinct points, is called as secant. In the given diagram secant PQ intersects circle at two points A and B.	
Tangent	A line segment which has one common point with the circumference of a circle i.e., it touches only at one point is called as tangent of circle. The common point is called as point of contact. In the given diagram PQ is a tangent which touches the circle at a point R.	 (R is the point of contact) NOTE Radius is always perpendicular to tangent.
Chord	A line segment whose end points lie on the circle. In the given diagram AB is a chord.	
Diameter	A chord which passes through the centre of the circle is called the diameter of the circle. The length of the diameter is twice the length of the radius. In the given diagram PQ is the diameter of the circle. (O $\rightarrow$ is the centre of the circle)	
Arc	Any two points on the circle divides the circle into two parts the smaller part is called as minor arc and the larger part is called as major arc. It is denoted as $\widehat{PQ}$. In the given diagram $\widehat{PQ}$ is arc.	  PQ $\rightarrow$ minor arc PQ $\rightarrow$ major arc

Nomenclature	Definition	Diagram
Semicircle	A diameter of the circle divides the circle into two equal parts. Each part is called as semicircle.	
Central angle	An angle formed at the centre of the circle, is called the central angle. In the given diagram $\angle AOB$ is the central angle.	
Inscribed angle	When two chords have one common end point, then the angle included between these two chords at the common point is called the inscribed angle. $\angle ABC$ is the inscribed angle by the arc $\widehat{ADC}$.	
Measure of an arc	Basically it is the central angle formed by an arc. e.g., (a) measure of a circle = 360° (b) measure of a semicircle = 180° (c) measure of a minor arc = $\angle POQ$ (d) measure of a major arc = $360 - \angle POQ$	 $m(\text{arc } PRQ) = m\angle POQ$ $m(\text{arc } PSQ) = 360^\circ - m(\text{arc } PRQ)$
Intercepted arc	In the given diagram $\widehat{AB}$ and $\widehat{CD}$ are the two intercepted arcs, intercepted by $\angle CPD$. The end points of the arc must touch the arms of $\angle CPD$ i.e., CP and DP .	
Concentric circles	Circles having the same centre at a plane are called the concentric circles. In the given diagram there are two circles with radii r_1 and r_2 having the common (or same) centre. These are called as concentric circles.	
Congruent circles	Circles with equal radii are called as congruent circles.	

Nomenclature	Definition	Diagram
Segment of a circle	A chord divides a circle into two regions. These two regions are called the segments of a circle. (a) major segment (b) minor segment	
Cyclic Quadrilateral	A quadrilateral whose all the four vertices (A, B, C, D) lie on the circle. For a cyclic quadrilateral $ABCD$, we must have $(AB \times CD) + (BC \times AD) = (AC \times BD)$	
Tangential Quadrilateral	A tangential quadrilateral or circumscribed quadrilateral is a convex quadrilateral whose sides are all tangent to a single circle within the quadrilateral. For a tangential quadrilateral, we must have $AB + CD = BC + AD$	
Bicentric Quadrilateral	A quadrilateral is said to be bicentric if it is cyclic and tangential.	
Circumcircle	A circle which passes through all the three vertices of a triangle. Thus the circumcentre is always equidistant from the vertices of the triangle. $OA = OB = OC$ (circumradius)	
Incircle	A circle which touches all the three sides of a triangle <i>i.e.</i> , all the three sides of a triangle are tangents to the circle is called an incircle. Incircle is always equidistant from the sides of a triangle. $OP = OQ = OR$ (inradius of the circle)	
Curvature	Curvatures is a measure of how quickly a tangent line turns on a curve. We would expect the curvature to be 0 for a straight line, to be very small for curves which bend very little and to be large for curves which bend sharply. Larger the circle, smaller the curvature. Curvature of a circle = $\frac{1}{\text{radius}}$	

Theorems

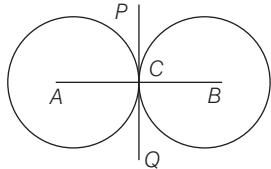
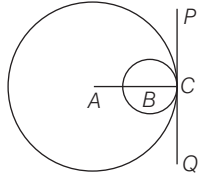
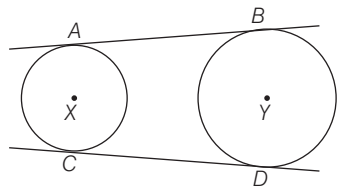
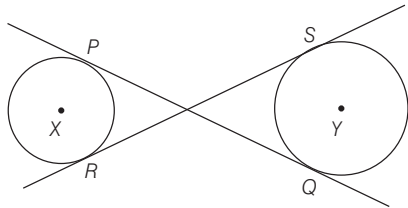
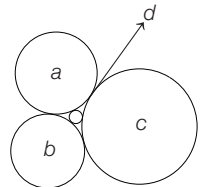
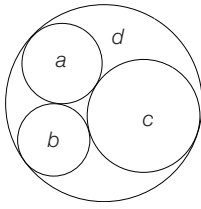
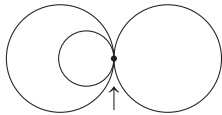
NOTE

- Two arcs of a circle (or of congruent circles) are congruent if their degree measures are equal.
- There is one and only one circle passes through three non-collinear points.

S. No.	Theorem	Diagram
1.	In a circle (or in congruent circles) equal chords are made by equal arcs. $\{OP = OQ\} = \{O'R = O'S\}$ and $PQ = RS$ $\therefore PQ = RS$	
2.	Equal arcs (or chords) subtend equal angles at the centre $\therefore PQ = AB$ $\therefore \angle POQ = \angle AOB$	
3.	The perpendicular from the centre of a circle to a chord bisects the chord <i>i.e.</i> , if $OD \perp AB$ $\therefore AB = 2AD = 2BD$	
4.	The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord. $\therefore AD = DB$ $\therefore OD \perp AB$	
5.	Perpendicular bisector of a chord passes through the centre. <i>i.e.</i> , if $OD \perp AB$ and $AD = DB$ $\therefore O$ is the centre of the circle.	
6.	Equal chords of a circle (or of congruent circles) are equidistant from the centre. $\therefore AB = PQ$ $\therefore OD = OR$	
7.	Chords which are equidistant from the centre in a circle (or in congruent, circles) are equal. $\therefore OD = OR$ $\therefore AB = PQ$	

S. No.	Theorem	Diagram
8.	The angle subtended by an arc (the degree measure of the arc) at the centre of a circle is twice the angle subtended by the arc at any point on the remaining part of the circle. $m \angle AOB = 2m \angle ACB.$	
9.	Angle in a semicircle is a right angle.	
10.	Angles in the same segment of a circle are equal. i.e., $\angle ACB = \angle ADB$	
11.	If a line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, then the four points lie on the same circle. $\angle ACB = \angle ADB$ $\therefore$ Points A, C, D, B are concyclic i.e., lie on the circle.	
12.	The sum of pair of opposite angles of a cyclic quadrilateral is 180°. $\angle DAB + \angle BCD = 180^\circ$ and $\angle ABC + \angle CDA = 180^\circ$ (Inverse of this theorem is also true)	
13.	Equal chords (or equal arcs) of a circle (or congruent circles) subtended equal angles at the centre. $AB = CD \text{ (or } AB = CD \text{)}$ $\therefore$ $\angle AOB = \angle COD$ (Inverse of this theorem is also true)	
14.	If a side of a cyclic quadrilateral is produced, then the exterior angle is equal to the interior opposite angle. $m \angle CDE = m \angle ABC$	

S. No.	Theorem	Diagram
15.	A tangent at any point of a circle is perpendicular to the radius through the point of contact. (Inverse of this theorem is also true)	
16.	The lengths of two tangents drawn from an external point to a circle are equal. i.e., $AP = BP$	
17.	If two chords AB and CD of a circle, intersect inside a circle (outside the circle when produced at a point E), then $AE \times BE = CE \times DE$	
18.	If PB be a secant which intersects the circle at A and B and PT be a tangent at T, then $PA \cdot PB = (PT)^2$	
19.	From an external point from which the tangents are drawn to the circle with centre O, then (a) they subtend equal angles at the centre. (b) they are equally inclined to the line segment joining the centre of that point. $\angle AOP = \angle BOP$ and $\angle APO = \angle BPO$	
20.	If P is an external point from which the tangents to the circle with centre O touch it at A and B, then OP is the perpendicular bisector of AB. $OP \perp AB$ and $AC = BC$	
21.	Alternate segment theorem : If from the point of contact of a tangent, a chord is drawn, then the angles which the chord makes with the tangent line are equal respectively to the angles formed in the corresponding alternate segments. In the adjoining diagram. $\angle BAT = \angle BCA$ and $\angle BAP = \angle BDA$	

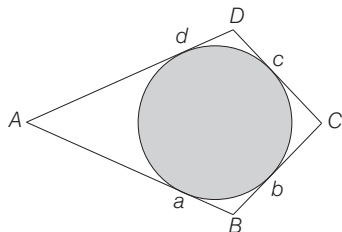
S. No.	Theorem	Diagram
22.	The point of contact of two tangents lies on the straight line joining the two centres. (a) When two circles touch externally, then the distance between their centres is equal to sum of their radii. <i>i.e.,</i> $AB = AC + BC$ (b) When two circles touch internally, the distance between their centres is equal to the difference between their radii. <i>i.e.,</i> $AB = AC - BC$	 
23.	For the two circles with centre X and Y and radii r_1 and r_2. AB and CD are two Direct Common Tangents (DCT), then the length of DCT $= \sqrt{(\text{distance between centres})^2 - (r_1 - r_2)^2}$	
24.	For the two circles with centre X and Y and radii r_1 and r_2. PQ and RS are two transverse common tangent, then length of TCT $= \sqrt{(\text{distance between centres})^2 - (r_1 + r_2)^2}$	
25.	Descartes' circle equation theorem : Given four mutually tangent circles with curvatures a, b, c and d, then $(a + b + c + d)^2 = 2(a^2 + b^2 + c^2 + d^2)$ If any one of the circles is circumscribing, say a circle of curvature d is circumscribing the circles of curvatures a, b and c, then $(a + b + c - d)^2 = 2(a^2 + b^2 + c^2 + (-d^2))$ In lay terms, if all the points of tangency are external, the curvatures are considered positive, but if one circle encompasses the others, that circle has negative curvature. Here, as all three circles are tangent to each other at the same point, Descartes' theorem does not apply.	  

12.7 Tangential Quadrilateral

A tangential quadrilateral or circumscribed quadrilateral is a convex quadrilateral whose sides are all tangent to a single circle within the quadrilateral. This circle is called the incircle of the quadrilateral; its center is the incenter and its radius is called the inradius.

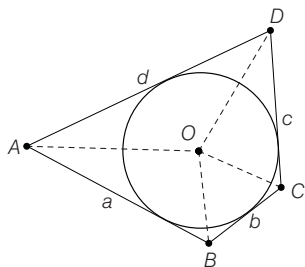
All triangles have an incircle, but not all quadrilaterals do. An example of a quadrilateral that cannot be tangential is a non-square rectangle.

If a quadrilateral is both tangential and cyclic, it is called a bicentric quadrilateral. Examples of tangential quadrilaterals are squares, rhombi, and kites.

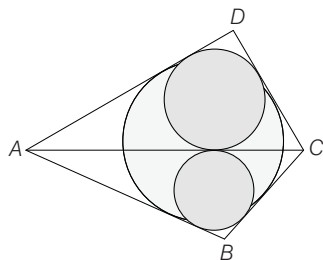


Properties

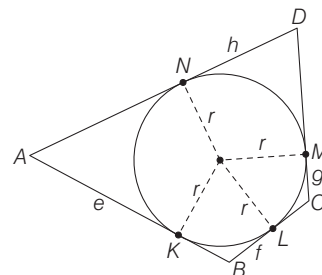
1. In a tangential quadrilateral, the four angle bisectors meet at the center of the incircle.
2. Conversely, a convex quadrilateral in which the four angle bisectors meet at a point must be tangential and the common point is the incenter.



3. The two pairs of opposite sides in a tangential quadrilateral add up to the same total length, which equals the semi perimeter s of the quadrilateral : $a + c = b + d = s$.
4. Conversely, a convex quadrilateral in which $a + c = b + d$ must be tangential.
5. A convex quadrilateral $ABCD$ is tangential, if and only if the incircles in the two triangles ABC and ADC are tangent to each other.



6. Area of the tangential quadrilateral —



- (i) $A = r \times s$; where r is the inradius and s is the semi-perimeter
- (ii) $A = \sqrt{(e + f + g + h)(efg + fgh + ghe + hef)}$; where e, f, g, h are the lengths of the tangents.
- (iii) $A = \frac{1}{2} \sqrt{p^2 q^2 - (ac - bd)^2}$; where p, q are the diagonals and a, b, c, d are the sides of tangential quadrilateral.
- (iv) $A = \sqrt{abcd - (eg - fh)^2}$; where a, b, c, d are the sides and e, f, g, h are the successive tangent lengths.
- (v) When the tangential quadrilateral is cyclic, that is bi-centric, then $eg = fh$ and so the tangential quadrilateral will have the maximum area $\sqrt{abcd}$.

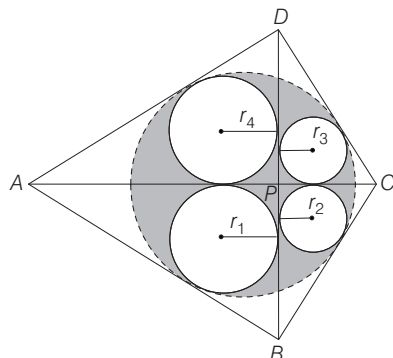
NOTE In the above diagram, it is considered as $AB = a, BC = b, CD = c, DA = d$ and $AK = e, BL = f, CM = g, DN = h$.

7. The maximum area of a tangential quadrilateral $A \leq \sqrt{abcd}$
8. The minimum area of a tangential quadrilateral $A \geq 4r^2$
9. The minimum value of the semi-perimeter $s \geq 4r$
10. The inradius of a tangential quadrilateral $r = A/s$; where A is the area and s is the semi-perimeter of the tangential quadrilateral.
11. If e, f, g and h are the tangent lengths from A, B, C and D , respectively, to the points where the incircle is tangent to the sides of a tangential quadrilateral $ABCD$, then the lengths of the diagonals $p = AC$ and $q = BD$ are

$$p = \sqrt{\left(\frac{e+g}{f+h}\right)[(e+g)(f+h) + 4fh]}$$

$$\text{and } q = \sqrt{\left(\frac{f+h}{e+g}\right)[(e+g)(f+h) + 4eg]}$$

12. In the non-overlapping triangles APB , BPC , CPD , DPA formed by the diagonals in a convex quadrilateral $ABCD$, where the diagonals intersect at P , there are the following characterizations of tangential quadrilaterals.



- (i) Let r_1, r_2, r_3 and r_4 denote the radii of the incircles in the four triangles APB , BPC , CPD and DPA respectively, then the quadrilateral is tangential if and only if $\frac{1}{r_1} + \frac{1}{r_3} = \frac{1}{r_2} + \frac{1}{r_4}$
- (ii) If h_1, h_2, h_3 and h_4 denote the altitudes in the four triangles APB , BPC , CPD and DPA respectively, (from the diagonal intersection to the sides of the quadrilateral), then quadrilateral is tangential if and only if

$$\frac{1}{h_1} + \frac{1}{h_3} = \frac{1}{h_2} + \frac{1}{h_4}$$

- (iii) If the exradii r_a, r_b, r_c and r_d in the four triangles APB , BPC , CPD and DPA respectively, (the four excircles are each tangent to one side of the quadrilateral and the extensions of its diagonals). A quadrilateral is tangential if and only if

$$\frac{1}{r_a} + \frac{1}{r_c} = \frac{1}{r_b} + \frac{1}{r_d}$$

- (iv) If R_1, R_2, R_3 and R_4 denote the radii in the circumcircles of triangles APB , BPC , CPD and DPA , respectively, then the quadrilateral $ABCD$ is tangential if and only if

$$R_1 + R_3 = R_2 + R_4$$

- (v) A convex quadrilateral $ABCD$, with diagonals intersecting at P , is tangential if and only if the four excenters in triangles APB , BPC , CPD and DPA opposite the vertices B and D are concyclic. If R_a, R_b, R_c and R_d are the exradii in the triangles APB , BPC , CPD and DPA , respectively, opposite the vertices B and D , then another condition is that the quadrilateral is tangential if and only if

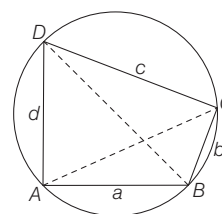
$$\frac{1}{R_a} + \frac{1}{R_c} = \frac{1}{R_b} + \frac{1}{R_d}$$

- (vi) A convex quadrilateral $ABCD$ with diagonals intersecting at P is tangential if and only if

$$\frac{a}{\Delta(APB)} + \frac{c}{\Delta(CPD)} = \frac{b}{\Delta(BPC)} + \frac{d}{\Delta(DPA)}$$

Cyclic Quadrilateral

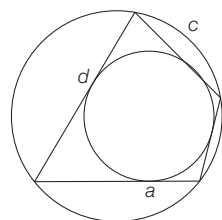
A cyclic quadrilateral is a quadrilateral for which a circle can be circumscribed so that it touches each vertex of the quadrilateral.



- (i) Semi perimeter $s = \frac{a + b + c + d}{2}$
- (ii) Area $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$
- (iii) Area $A = \frac{\sqrt{(ab+cd)(ac+bd)(ad+bc)}}{4R}$
- (iv) Circumradius $R = \frac{1}{4} \sqrt{\frac{(ab+cd)(ac+bd)(ad+bc)}{(s-a)(s-b)(s-c)(s-d)}}$

Bicentric Quadrilateral

A quadrilateral is said to be bicentric if it is cyclic and tangential.

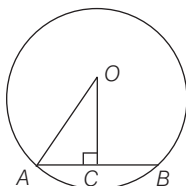


- (i) If the sides be a, b, c, d the area of a bicentric quadrilateral is $A = \sqrt{abcd}$
- (ii) Inradius, $r = \frac{2\sqrt{abcd}}{a+b+c+d} = \frac{\sqrt{abcd}}{s}$
- (iii) Circumradius, $R = \frac{1}{4} \sqrt{\frac{(ab+cd)(ac+bd)(ad+bc)}{abcd}}$
- (iv) In a bicyclic quadrilateral, the distance x between the incentre and circumcentre is given by

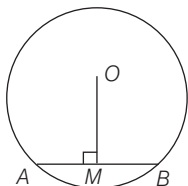
$$\frac{1}{(R-x)^2} + \frac{1}{(R+x)^2} = \frac{1}{(r)^2}$$

Introductory Exercise 12.5

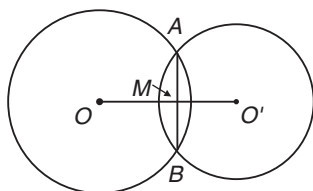
1. In the given figure, O is the centre of the circle. Radius of the circle is 17 cm. If $OC = 8$ cm, then the length of the chord AB is :



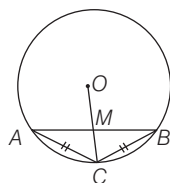
- (a) 35 cm (b) 30 cm (c) 15 cm (d) 18 cm
2. In the given figure $OM \perp AB$, radius of the circle is 5 cm and length of the chord $AB = 8$ cm. Find the measure of OM .



- (a) 3 cm (b) 2.5 cm (c) 2 cm (d) 6 cm
3. In the given figure, two circles with their respective centres intersect each other at A and B and AB intersects OO' at M , then $m \angle OMA$ is :

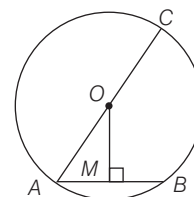


- (a) 60° (b) 80°
(c) 90° (d) can't be determined
4. In the above question (no. 3) what is the ratio of $AM : BM$?
- (a) 5 : 6 (b) 3 : 2
(c) 1 : 1 (d) can't be determined
5. In the given figure the two chords AC and BC are equal. The radius OC intersect AB at M , then $AM : BM$ is :

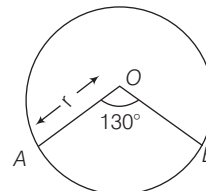


- (a) 1 : 1 (b) $\sqrt{2} : 3$
(c) $3 : \sqrt{2}$ (d) none of these

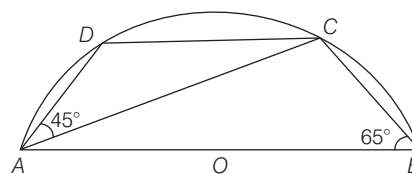
6. In the adjoining figure, O is the centre of circle and diameter $AC = 26$ cm. If chord $AB = 10$ cm, then the distance between chord AB and centre O of the circle is :



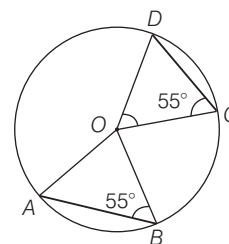
- (a) 24 cm (b) 16 cm
(c) 12 cm (d) none of these
7. In the adjoining circle $C(O, r)$ the degree measure of minor arc $AB = 130^\circ$. Find the degree measure of major arc.



- (a) 230° (b) 260°
(c) 310° (d) none of these
8. If the diagonals of a cyclic quadrilateral are equal, then the quadrilateral is :
- (a) rhombus (b) square
(c) rectangle (d) none of these
9. In the given figure, AB is diameter of the circle. C and D lie on the semicircle. $\angle ABC = 65^\circ$ and $\angle CAD = 45^\circ$. Find $m \angle DCA$.

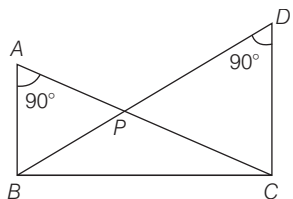


- (a) 45° (b) 25°
(c) 20° (d) none of these
10. In the given figure, chords AB and CD are equal. If $\angle OBA = 55^\circ$, then $m \angle COD$ is :

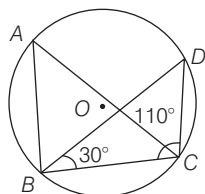


- (a) 65° (b) 55° (c) 70° (d) 50°

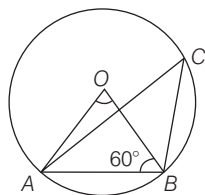
11. The quadrilateral formed by angle bisectors of a cyclic quadrilateral is a :
 (a) rectangle (b) square
 (c) parallelogram (d) cyclic quadrilateral
12. $\triangle ABC$ and $\triangle DBC$ have a common base and drawn towards one sides. $\angle BAC = \angle BDC = 90^\circ$. If AC and DB intersect at P , then :



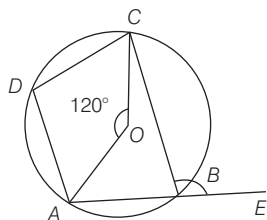
- (a) $AP \times PC = BP \times PD$
 (b) $AP \times BP = PC \times PD$
 (c) $AP \times PD = PC \times BP$
 (d) none of the above
13. In the given figure, $\angle BAC$ and $\angle BDC$ are the angles of same segments. $\angle DBC = 30^\circ$ and $\angle BCD = 110^\circ$. Find $m \angle BAC$.



- (a) 35° (b) 40°
 (c) 55° (d) 60°
14. In the given figure, O is the centre of the circle, $\angle ABO = 60^\circ$. Find the value of $\angle ACB$.

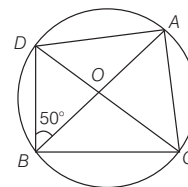


- (a) 40° (b) 60°
 (c) 50° (d) 30°
15. In the given figure, $\angle AOC = 120^\circ$. Find $m \angle CBE$, where O is the centre.

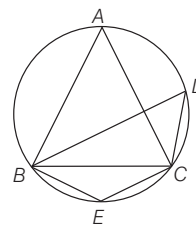


- (a) 60° (b) 100° (c) 120° (d) 150°

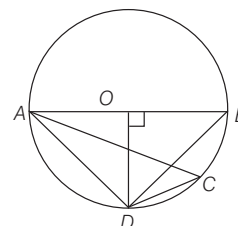
16. In the adjoining figure, O is the centre of the circle and $\angle OBD = 50^\circ$. Find the $m \angle BAD$.



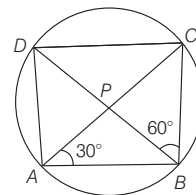
- (a) 60° (b) 40° (c) 80° (d) 45°
17. In the given figure, $\triangle ABC$ is an equilateral triangle. Find $m \angle BEC$.



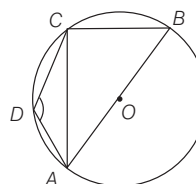
- (a) 120° (b) 60°
 (c) 80° (d) none of these
18. In the given figure, AB is the diameter of the circle. Find the value of $\angle ACD$.



- (a) 30° (b) 60° (c) 45° (d) 25°
19. In the given figure, $ABCD$ is a cyclic quadrilateral and diagonals bisect each other at P . If $\angle DBC = 60^\circ$ and $\angle BAC = 30^\circ$, then $\angle BCD$ is :

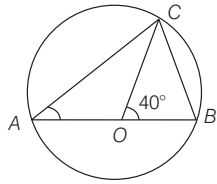


- (a) 90° (b) 60°
 (c) 80° (d) none of these
20. In the given figure, $ABCD$ is a cyclic quadrilateral and AB is the diameter. $\angle ADC = 140^\circ$, then find $m \angle BAC$.



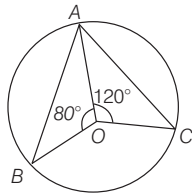
- (a) 45° (b) 40° (c) 50° (d) none of these

21. In the given figure, $\angle COB = 40^\circ$, AB is the diameter of the circle. Find $m \angle CAB$.



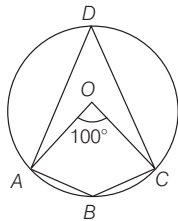
(a) 40° (b) 20° (c) 30° (d) none of these

22. In the given figure, O is the centre of circle. $\angle AOB = 80^\circ$ and $\angle AOC = 120^\circ$. Find $m \angle BAC$.



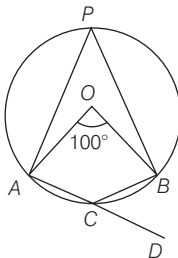
(a) 120° (b) 80° (c) 100° (d) none of these

23. In the given figure, O is the centre of the circle and $\angle AOC = 100^\circ$. Find the ratio of $m \angle ADC : m \angle ABC$.



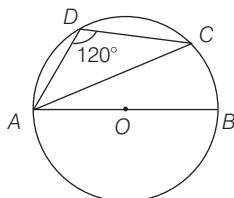
(a) 5 : 6 (b) 1 : 2
(c) 5 : 13 (d) none of these

24. In the given figure, O is the centre of circle, $\angle AOB = 100^\circ$. Find $m \angle BCD$.



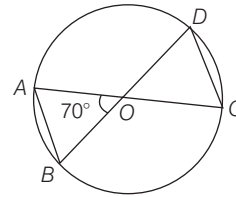
(a) 80° (b) 60° (c) 50° (d) 40°

25. In the given figure, AB is the diameter of the circle. $\angle ADC = 120^\circ$. Find $m \angle CAB$.



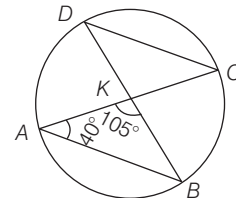
(a) 20° (b) 30°
(c) 40° (d) can't be determined

26. In the given figure, O is the centre of the circle. $\angle AOB = 70^\circ$, find $m \angle OCD$.



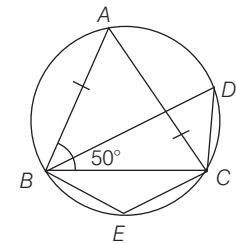
(a) 70° (b) 55°
(c) 65° (d) 110°

27. In the given figure, $\angle CAB = 40^\circ$ and $\angle AKB = 105^\circ$. Find $\angle KCD$.



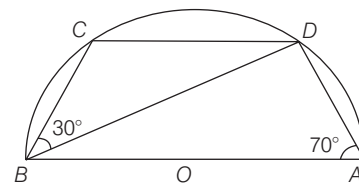
(a) 65° (b) 35° (c) 40° (d) 72°

28. In the given figure, ABC is an isosceles triangle in which $AB = AC$ and $m \angle ABC = 50^\circ$. Find $m \angle BDC$.



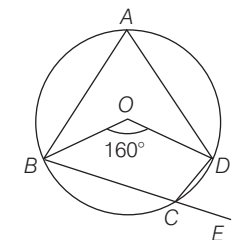
(a) 80° (b) 60° (c) 65° (d) 100°

29. In the given figure, AB is the diameter. $m \angle BAD = 70^\circ$ and $m \angle DBC = 30^\circ$. Find $m \angle BDC$.



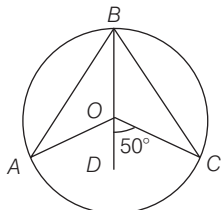
(a) 25° (b) 30° (c) 40° (d) 60°

30. Find the value of $\angle DCE$.

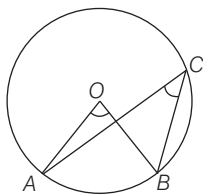


(a) 100° (b) 80°
(c) 90° (d) 75°

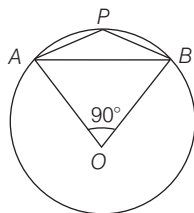
31. 'O' is the centre of the circle, line segment BOD is the angle bisector of $\angle AOC$, $m \angle COD = 50^\circ$. Find $m \angle ABC$.



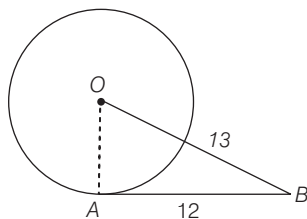
- (a) 25° (b) 50°
(c) 100° (d) 120°
32. In the given figure, O is the centre of the circle and $\angle ACB = 25^\circ$. Find $\angle AOB$.



- (a) 25° (b) 50°
(c) 75° (d) 60°
33. In the given figure, O is the centre of the circle. $\angle AOB = 90^\circ$. Find $m \angle APB$.

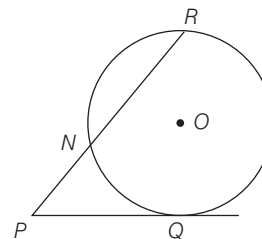


- (a) 130°
(b) 150°
(c) 135°
(d) can't be determined
34. In the given figure, O is the centre of the circle. AB is tangent. $AB = 12$ cm and $OB = 13$ cm. Find OA.

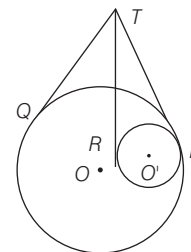


- (a) 6.5 cm
(b) 6 cm
(c) 5 cm
(d) none of the above

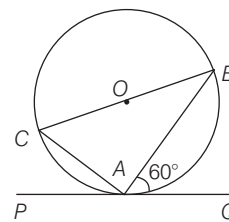
35. In the given figure, PQ is the tangent of the circle. Line segment PR intersects the circle at N and R. $PQ = 15$ cm, $PR = 25$ cm, find PN.



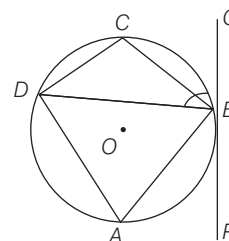
- (a) 15 cm (b) 10 cm (c) 9 cm (d) 6 cm
36. In the given figure, there are two circles with the centres O and O' touching each other internally at P. Tangents TQ and TP are drawn to the larger circle and tangents TP and TR are drawn to the smaller circle. Find $TQ : TR$.



- (a) 8 : 7 (b) 7 : 8
(c) 5 : 4 (d) 1 : 1
37. In the given figure, PAQ is the tangent. BC is the diameter of the circle. $m \angle BAQ = 60^\circ$, find $m \angle ABC$.

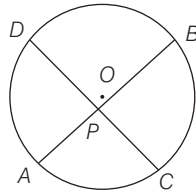


- (a) 25° (b) 30° (c) 45° (d) 60°
38. ABCD is a cyclic quadrilateral PQ is a tangent at B. If $\angle DBQ = 65^\circ$, then $\angle BCD$ is :

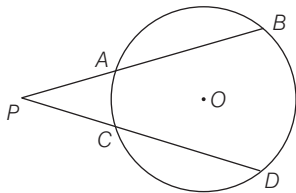


- (a) 35° (b) 85° (c) 115° (d) 90°

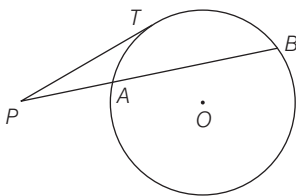
39. In the given figure, $AP = 2$ cm, $BP = 6$ cm and $CP = 3$ cm. Find DP .



- (a) 6 cm (b) 4 cm
(c) 2 cm (d) 3 cm
40. In the given figure, $AP = 3$ cm, $BA = 5$ cm and $CP = 2$ cm. Find CD .

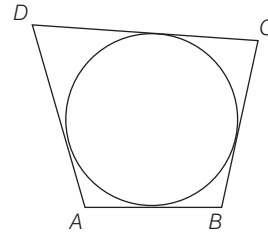


- (a) 12 cm (b) 10 cm
(c) 9 cm (d) 6 cm
41. In the given figure, tangent $PT = 5$ cm, $PA = 4$ cm, find AB .

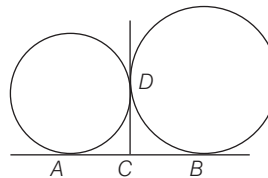


- (a) $\frac{7}{4}$ cm
(b) $\frac{11}{4}$ cm
(c) $\frac{9}{4}$ cm
(d) can't be determined
42. Two circles of radii 13 cm and 5 cm touch internally each other. Find the distance between their centres.
- (a) 18 cm (b) 12 cm
(c) 9 cm (d) 8 cm
43. Three circles touch each other externally. The distance between their centre is 5 cm, 6 cm and 7 cm. Find the radii of the circles.
- (a) 3 cm, 2 cm, 4 cm
(b) 3 cm, 4 cm, 1 cm
(c) 1 cm, 2.5 cm, 3.5 cm
(d) 1 cm, 2 cm, 4 cm

44. A circle touches a quadrilateral $ABCD$. Find the true statement

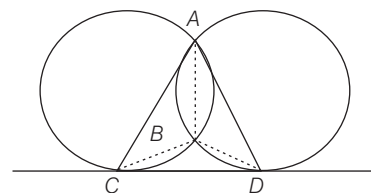


- (a) $AB + BC = CD + AD$ (b) $AB + CD = BC + AD$
(c) $BD = AC$ (d) none of these
45. O and O' are the centres of two circles which touch each other externally at P . AB is a common tangent. Find $\angle APO$.
- (a) 90° (b) 120°
(c) 60° (d) data insufficient
46. If AB is a chord of a circle, P and Q are two points on the circle different from A and B , then :
- (a) the angle subtended by AB at P and Q are either equal or supplementary.
(b) the sum of the angles subtended by AB at P and Q is always equal two right angles.
(c) the angles subtended at P and Q by AB are always equal.
(d) the sum of the angles subtended at P and Q is equal to four right angles.
47. In the given figure, AB and CD are two common tangents to the two touching circle. If $CD = 6$ cm, then AB is equal to :



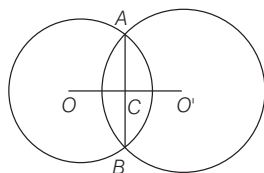
- (a) 9 cm (b) 15 cm
(c) 12 cm (d) none of these
48. In the given figure, CD is a direct common tangent to two circles intersecting each other at A and B , then :

$$\angle CAD + \angle CBD = ?$$



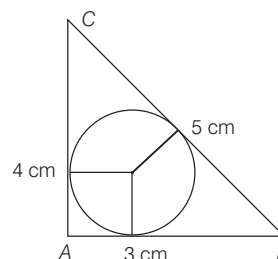
- (a) 120° (b) 90°
(c) 360° (d) 180°

49. O and O' are the centres of circle of radii 20 cm and 37 cm. $AB = 24$ cm. What is the distance OO' ?



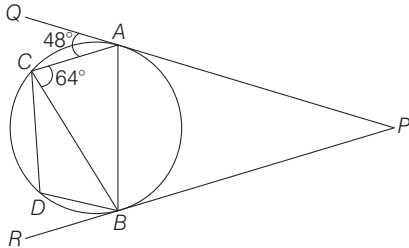
- (a) 51 cm (b) 45 cm
(c) 35 cm (d) 48 cm
50. In a circle of radius 5 cm, AB and AC are the two chords such that $AB = AC = 6$ cm. Find the length of the chord BC .
(a) 4.8 cm (b) 10.8 cm
(c) 9.6 cm (d) none of these
51. In a circle of radius 17 cm, two parallel chords are drawn on opposite sides of a diameter. The distance between the chords is 23 cm. If the length of one chord is 16 cm, then the length of the other is :
(a) 23 cm (b) 30 cm
(c) 15 cm (d) none of these
52. If two circles are such that the centre of one lies on the circumference of the other, then the ratio of the common chord of two circles to the radius of any of the circles is :
(a) $\sqrt{3} : 2$
(b) $\sqrt{3} : 1$
(c) $\sqrt{5} : 1$
(d) none of the above
53. Two circles touch each other internally. Their radii are 2 cm and 3 cm. The biggest chord of the other circle which is outside the inner circle, is of length :
(a) $2\sqrt{2}$ cm (b) $3\sqrt{2}$ cm
(c) $2\sqrt{3}$ cm (d) $4\sqrt{2}$ cm
54. Through any given set of four points P, Q, R, S it is possible to draw :
(a) atmost one circle
(b) exactly one circle
(c) exactly two circles
(d) exactly three circles
55. The distance between the centres of equal circles each of radius 3 cm is 10 cm. The length of a transverse tangent is :
(a) 4 cm (b) 6 cm (c) 8 cm (d) 10 cm
56. The number of common tangents that can be drawn to two given circles is at the most :
(a) 1 (b) 2 (c) 3 (d) 4

57. ABC is a right angled triangle $AB = 3$ cm, $BC = 5$ cm and $AC = 4$ cm, then the inradius of the circle is :

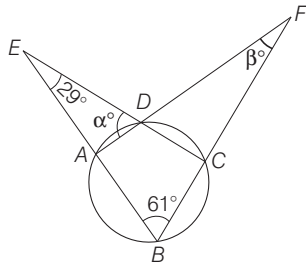


- (a) 1 cm (b) 1.25 cm
(c) 1.5 cm (d) none of these
58. A circle has two parallel chords of lengths 6 cm and 8 cm. If the chords are 1 cm apart and the centre is on the same side of the chords, then a diameter of the circle is of length :
(a) 5 cm (b) 6 cm
(c) 8 cm (d) 10 cm
59. Three equal circles of unit radius touch each other. Then, the area of the circle circumscribing the three circles is :
(a) $6\pi (2 + \sqrt{3})^2$ (b) $\frac{\pi}{6} (2 + \sqrt{3})^2$
(c) $\frac{\pi}{3} (2 + \sqrt{3})^2$ (d) $3\pi (2 + \sqrt{3})^2$
60. The radius of a circle is 20 cm. The radii (in cm) of three concentric circles drawn in such a manner that the whole area is divided into four equal parts, are :
(a) $20\sqrt{2}, 20\sqrt{3}, 20$
(b) $\frac{10\sqrt{3}}{3}, \frac{10\sqrt{2}}{3}, \frac{10}{3}$
(c) $10\sqrt{3}, 10\sqrt{2}, 10$
(d) 17, 14, 9
61. Points A, B, C and D are concyclic. BC is the diameter and O is the centre of the circle, $\angle BAO = 69^\circ$, find $\angle DAO + \angle DCO$.
(a) 138 (b) 111
(c) 121 (d) none of these
62. An isosceles right triangle ABC has an incircle with centre O , which touches the sides AB, BC and AC at D, E and F respectively. If the longest side AC is $\sqrt{2}$ unit, find DE .
(a) $\sqrt{2} - 1$ cm (b) $\sqrt{2} - \frac{1}{\sqrt{2}}$ cm
(c) $1 - \frac{1}{\sqrt{2}}$ cm (d) none of these

63. Consider the following diagram. In this diagram, A, B, D and C are cyclic. QP and RP are the tangents at A and B respectively and they meet at P . $\angle CAQ = 48^\circ$ and $\angle ACB = 64^\circ$. Find the value of $\angle BDC + \angle CBP + \angle APB$.



- (a) 248° (b) 276°
(c) 360° (d) none of these
64. In the following figure A, B, C and D are cyclic. BE, CE, AF and BF are straight lines

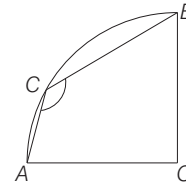


Find the value of $\alpha - \beta$.

- (a) 28° (b) 41°
(c) 32° (d) none of these
65. In a circle with centre O , there are two chords AB and BC , such that $\angle OAB = 24^\circ$ and $\angle OCB = 42^\circ$. Find the measure of $\angle AOC$.

- (a) 36° (b) 63°
(c) 228° (d) either (a) or (c)

66. In the following figure, $AOBC$ is a quadrilateral inscribed in a quadrant of a circle such that $\angle AOB$ is a right angle and point C falls on the arc of the quadrant. Find $\angle ACB$.



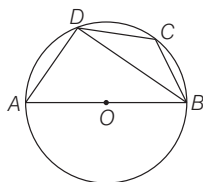
- (a) 120° (b) 135°
(c) 150° (d) data insufficient
67. The sides of a triangle $\triangle ABC$ are 15 cm, 41 cm and 52 cm. Find the diameter of the circumcircle of the $\triangle ABC$.
- (a) $\frac{205}{3}$ cm (b) $\frac{208}{3}$ cm
(c) 69 cm (d) 70 cm
68. The radius of a circle is 2 cm, what is its curvature?
- (a) 1 cm (b) 4 cm
(c) 0.5 cm (d) 1.414 cm
69. The curvatures of two externally tangent circles are 0.5 and 2, what is the distance between their centers?
- (a) 2.5 (b) 1
(c) 4 (d) 8
70. An equilateral triangle ABC is inscribed in a circle and a chord AD is connected to two other chords BD and CD , whereas point D is on the arc BC . If the side of the triangle is 30 cm and the chord AD is 32 cm, find $BD + CD$.
- (a) 31 cm (b) 32 cm
(c) $225/8$ cm (d) $256/15$ cm

CAT-Test

Questions Helping you bell the CAT

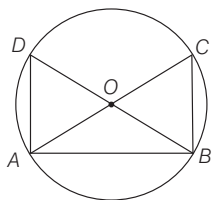
LEVEL 01 > BASIC LEVEL EXERCISE

- 1 In the adjoining figure AB is a diameter of the circle and $\angle BCD = 130^\circ$. What is the value of $\angle ABD$?



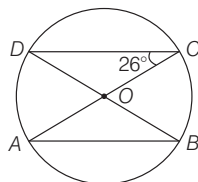
- (a) 30°
(b) 50°
(c) 40°
(d) none of the above

- 2 In the given figure O is the centre of the circle and $\angle BAC = 25^\circ$, then the value of $\angle ADB$ is :



- (a) 40°
(b) 55°
(c) 50°
(d) 65°

- 3 In the given figure O is the centre of the circle and $\angle OCD = 26^\circ$, find $\angle AOD$.

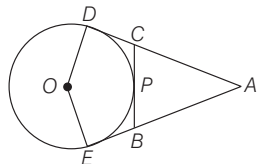


- (a) 52°
(b) 154°
(c) 128°
(d) data insufficient

- 4 Three sides of a triangle ABC are a, b, c . $a = 4700$ cm, $b = 4935$ cm and $c = 6815$ cm. The internal bisector of $\angle A$ meets BC at P , and the bisector passes through incentre O . What is the ratio of $PO : OA$?

- (a) $3 : 2$
(b) $2 : 3$
(c) $2 : 5$
(d) can't be determined

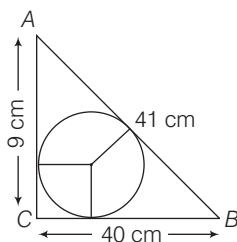
- 5 In the given circle O is the centre of the circle and AD, AE are the two tangents. BC is also a tangent, then :



- (a) $AC + AB = BC$
(b) $3AE = AB + BC + AC$
(c) $AB + BC + AC = 4AE$
(d) $2AE = AB + BC + AC$

- 6 What is the inradius of the incircle shown in the figure?

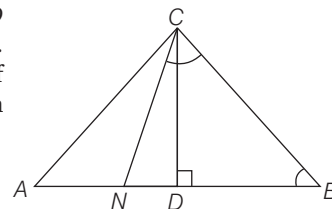
- (a) 9 cm (b) 4 cm
(c) can't be determined
(d) none of the above



- 7 One of the diagonal of a parallelogram is 18 cm, whose adjacent sides are 16 cm and 20 cm respectively. What is the length of other diagonal?

- (a) $2\sqrt{247}$ cm (b) 13 cm
(c) 15.2 cm (d) 28.5 cm

- 8 In the adjoining figure CD is perpendicular on AB . CN is the angle bisector of $\angle ACB$. $\angle ACN = 50^\circ$, then the angle NCD is :



- (a) $\frac{1}{3} \angle ACB$
(b) $\angle CAB - \angle CBA$
(c) $\frac{1}{2} (\angle CBA \sim \angle BAC)$
(d) can't be determined

- 9 In a ΔPQR , points M and N are on the sides PQ and PR respectively such that $PM = 0.6 PQ$ and $NR = 0.4 PR$. What percentage of the area of the triangle PQR does that of triangle PMN form?

- (a) 60% (b) 50% (c) 36% (d) 55%

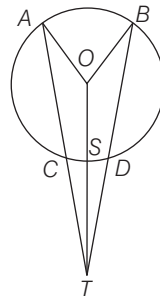
- 10 In a trapezium $ABCD$, the diagonals AC and BD intersect each other at O such that $OB : OD = 3 : 1$ then the ratio of areas of $\Delta AOB : \Delta COD$ is :

- (a) $3 : 1$ (b) $1 : 4$
(c) $9 : 1$ (d) can't be determined

- 11 Three cities Fatehpur, Barabanki and Lucknow are very famous. Fatehpur is 42 km, away from Barabanki and the distance between Fatehpur and Lucknow is 66 km. Which of the following cannot be the distance between Barabanki and Lucknow?

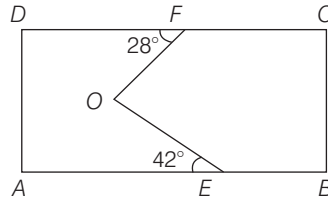
- (a) 28 km (b) 98 km (c) 32 km (d) 24 km

- 12 In the adjoining figure 'O' is the centre of circle AC and BD are the two chords of circle which meets at T outside the circle. OT bisects CD , $OA = OB = 8$ cm and $OT = 17$ cm. What is the ratio of distance of AC and BD from the centre of the circle?



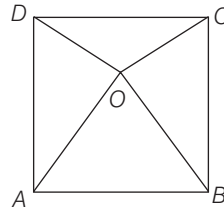
- (a) $15 : 17$
(b) $8 : 15$
(c) $8 : 9$
(d) none of the above

- 13** In the adjoining figure $ABCD$ is a rectangle and $DF = CF$ also, $AE = 3BE$. What is the value of $\angle EOF$, if $\angle DFO = 28^\circ$ and $\angle AEO = 42^\circ$?



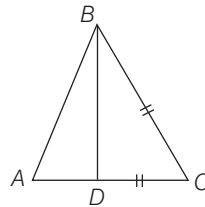
- (a) 14° (b) 42°
(c) 70° (d) 90°

- 14** $ABCD$ is a square and AOB is an equilateral triangle. What is the value of $\angle DOC$?



- (a) 120°
(b) 150°
(c) 125°
(d) can't be determined

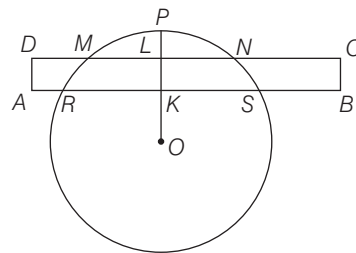
- 15** In the triangle ABC , $BC = CD$ and $(\angle ABC - \angle BAC) = 30^\circ$. The measure of $\angle ABD$ is :



- (a) 30°
(b) 45°
(c) 15°
(d) can't be determined
- 16** In a trapezium $ABCD$, AB is a parallel to CD . BD is perpendicular to AD . AC is perpendicular to BC . If $AD = BC = 15$ cm and $AB = 25$ cm, then the area of the trapezium is :

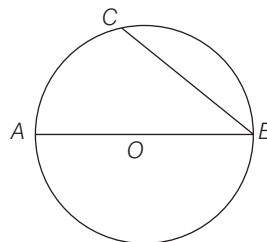
- (a) 192 cm^2
(b) 232 cm^2
(c) 172 cm^2
(d) none of the above

- 17** In the adjoining figure O is the centre of the circle. The radius OP bisects a rectangle $ABCD$, at right angle. $DM = NC = 2$ cm and $AR = SB = 1$ cm and $KS = 4$ cm and $OP = 5$ cm.



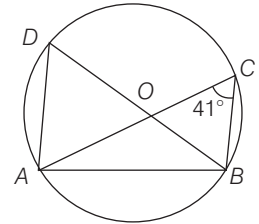
What is the area of the rectangle?

- (a) 8 cm^2 (b) 10 cm^2
(c) 12 cm^2 (d) none of these
- 18** In the given figure of circle AB is the diameter with length 20 cm and BC is 16 cm, then find the length of CO when CO is perpendicular on AB .



- (a) 9.6
(b) 8.4
(c) 10
(d) data insufficient

- 19** In the adjoining figure BD is the diameter of the circle and $\angle BCA = 41^\circ$. Find $\angle ABD$.

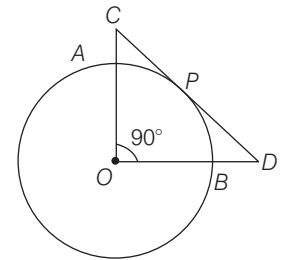


- (a) 41°
(b) 49°
(c) 22.5°
(d) 20.5°

- 20** Each interior angle of a regular polygon exceeds its exterior angle by 132° . How many sides does the polygon have ?

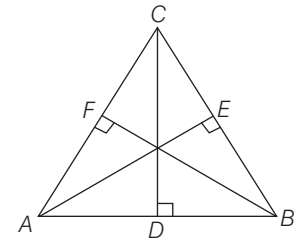
- (a) 9
(b) 15
(c) 12
(d) none of the above

- 21** In a circle O is the centre and $\angle COD$ is right angle. $AC = BD$ and CD is the tangent at P . What is the value of $AC + CP$, if the radius of the circle is 1 metre?



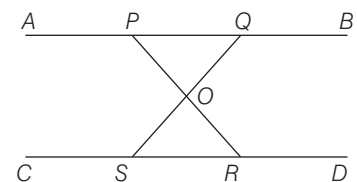
- (a) 105 cm
(b) 141.4 cm
(c) 138.6 cm
(d) can't be determined

- 22** In the given triangle ABC , CD , BF and AE are the altitudes. If the ratio of $CD : AE : BF = 2 : 3 : 4$, then the ratio of $AB : BC : CA$ is :



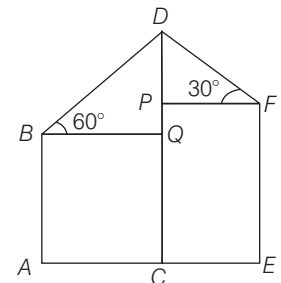
- (a) $4 : 3 : 2$
(b) $2 : 3 : 4$
(c) $4 : 9 : 16$
(d) $6 : 4 : 3$

- 23** In the adjoining figure $AB \parallel CD$ and $PQ = SR$, then :



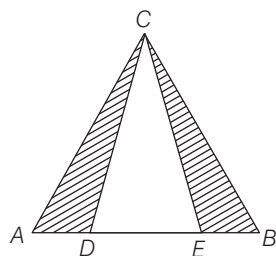
- (a) $PQ = PS$
(b) $SR = RP$
(c) $PS = QR$
(d) $AP = RD$

- 24** In the given figure AB , CD and EF are three towers. The angle of elevation of the top of the tower CD from the top of the tower AB is 60° and that from EF is 30° . $BD = 2\sqrt{3}$ m, $CD : EF = 5 : 4$ and $DF = 4$ m. What is the height of the tower AB ?



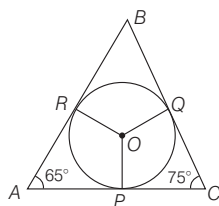
- (a) 6 m (b) 12 m
(c) 7 m (d) none of these

- 25** In the equilateral triangle ABC , $AD = DE = BE$, D and E lies on the AB . If each side of the triangle (i.e., AB , BC and AC) be 6 cm, then the area of the shaded region is :

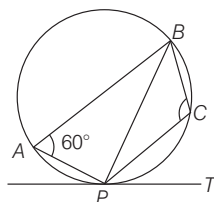


- (a) 9 cm^2
 (b) $6\sqrt{3} \text{ cm}^2$
 (c) $5\sqrt{3} \text{ cm}^2$
 (d) none of the above
- 26** In the above question (number 25) what is the perimeter of triangle CDE ?
- (a) $(3 + 2\sqrt{5}) \text{ cm}$
 (b) $2(1 + 2\sqrt{7}) \text{ cm}$
 (c) $2(1 + 3\sqrt{7}) \text{ cm}$
 (d) none of the above

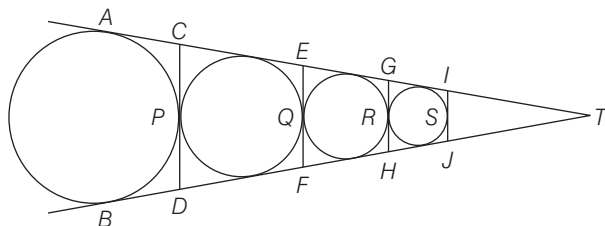
- 27** In a triangle ABC , O is the centre of incircle PQR , $\angle BAC = 65^\circ$, $\angle BCA = 75^\circ$, find $\angle ROQ$.
- (a) 80°
 (b) 120°
 (c) 140°
 (d) can't be determined



- 28** In the given figure, PT is a tangent at P and $ABCP$ is a quadrilateral. $\angle BAP$ is 60° , then the value of $\angle PCB$ is :

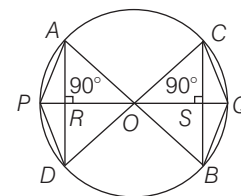


- (a) 60°
 (b) 90°
 (c) 120°
 (d) data insufficient
- 29** In the above question (number 28), what is the value of $\angle TPC$?
- (a) 30° (b) 60°
 (c) 90° (d) can't be determined
- 30** In the adjoining figure AT and BT are two tangents at A and B respectively CD is also a tangent at P . There are some circles touching each other and the tangents AT and BT also. Which one of the following is true?



- (a) $PC + CT = PD + DT$
 (b) $RG + GT = RH + HT$
 (c) $PC + QE = CE$
 (d) all of these

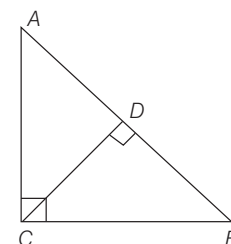
- 31** In the adjoining figure O is the centre of the circle.



$\angle AOD = 120^\circ$. If the radius of the circle be r , then find the sum of the areas of quadrilaterals $AODP$ and $OBQC$.

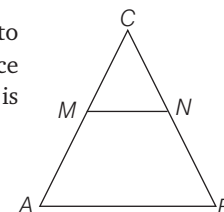
- (a) $\frac{\sqrt{3}}{2} r^2$
 (b) $3\sqrt{3} r^2$
 (c) $\sqrt{3} r^2$
 (d) none of the above

- 32** In a right angled triangle ABC , CD is the perpendicular on the hypotenuse AB . Which of the following is correct?



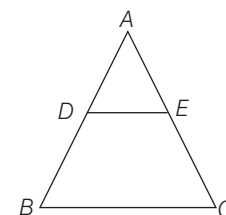
- (a) $CD = \frac{AC \times BC}{AB}$
 (b) $AD = \frac{AC \times AC}{AB}$
 (c) $BD = \frac{BC \times BC}{AB}$
 (d) all of the above

- 33** In the triangle ABC , MN is parallel to AB . Area of trapezium $ABNM$ is twice the area of triangle CMN . What is ratio of $CM : AM$?



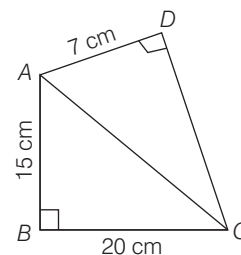
- (a) $\frac{1}{\sqrt{3} + 1}$
 (b) $\frac{\sqrt{3} - 1}{2}$
 (c) $\frac{\sqrt{3} + 1}{2}$
 (d) none of the above

- 34** In the given figure $AD = AE$, $\angle BDE = 100^\circ$, then what is the value of $\angle DBC + \angle BCE$?



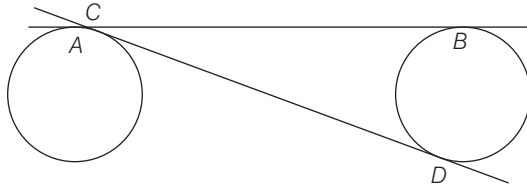
- (a) 200°
 (b) 160°
 (c) 80°
 (d) can't be determined

- 35** In the given quadrilateral $ABCD$, $AB = 15 \text{ cm}$, $BC = 20 \text{ cm}$ and $AD = 7 \text{ cm}$, $\angle ABC = \angle ADC = 90^\circ$. Find the length of side CD .

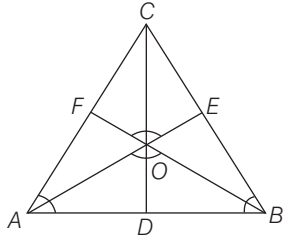


- (a) 12 cm
 (b) 18 cm
 (c) 24 cm
 (d) none of the above

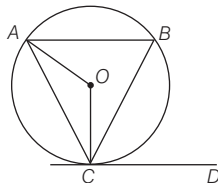
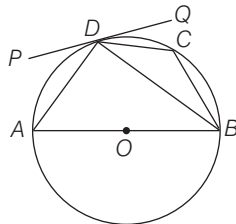
- 36 There are two circles each with radius 5 cm. Tangent AB is 26 cm. The length of tangent CD is :



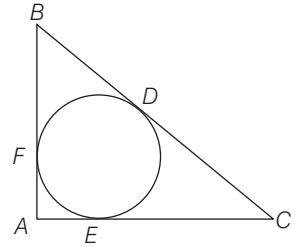
- (a) 15 cm (b) 21 cm
(c) 24 cm (d) can't be determined
- 37 One of diagonal of a parallelogram is 10 cm and an angle of the parallelogram is $\pi/4$. If its height be 8 cm, then find the area of the parallelogram.
(a) 112 cm^2 (b) 88 cm^2 (c) 92 cm^2 (d) 104 cm^2
- 38 ABC is a triangle in which $\angle CAB = 80^\circ$ and $\angle ABC = 50^\circ$, AE , BF and CD are the altitudes and O is the orthocentre. What is the value of $\angle AOB$?



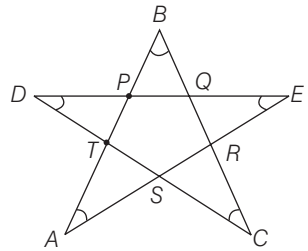
- (a) 65° (b) 70° (c) 50° (d) 130°
- 39 ABC is a triangle in which 35 times the smallest angle is equal to the 26 times largest angle. What is the measure of the second largest angle?
(a) 63° (b) 58° (c) 70° (d) 42°
- 40 In the adjoining figure O is the centre of the circle and AB is the diameter. Tangent PQ touches the circle at D . $\angle BDQ = 48^\circ$. Find the ratio of $\angle DBA : \angle DCB$.
(a) $\frac{22}{7}$
(b) $\frac{7}{22}$
(c) $\frac{7}{12}$
(d) can't be determined
- 41 In the given diagram O is the centre of the circle and CD is a tangent. $\angle CAB$ and $\angle ACD$ are supplementary to each other $\angle OAC = 30^\circ$. Find the value of $\angle OCB$.
(a) 30° (b) 20°
(c) 60° (d) none of these



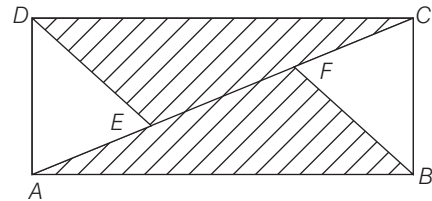
- 42 In the given diagram an incircle DEF is circumscribed by the right angled triangle in which $AF = 6 \text{ cm}$ and $EC = 15 \text{ cm}$. Find the difference between CD and BD .
(a) 1 cm
(b) 3 cm
(c) 4 cm
(d) can't be determined



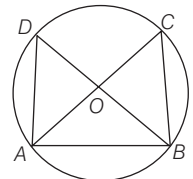
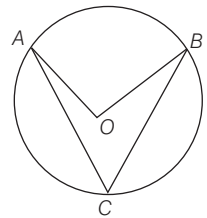
- 43 In the adjoining figure, a star is shown. What is the sum of the angles A, B, C, D and E ?
(a) 120°
(b) 180°
(c) 240°
(d) can't be determined



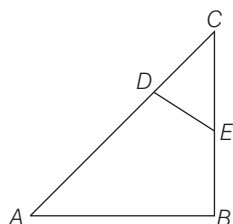
- 44 $ABCD$ is a rectangle of dimensions $6 \text{ cm} \times 8 \text{ cm}$. DE and BF are the perpendiculars drawn on the diagonal of the rectangle. What is the ratio of the shaded to that of unshaded region?



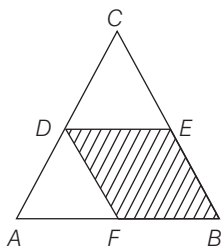
- (a) 7 : 3
(b) 16 : 9
(c) $4 : 3\sqrt{2}$
(d) data insufficient
- 45 In the adjoining figure 'O' is the centre of circle. $\angle CAO = 25^\circ$ and $\angle CBO = 35^\circ$. What is the value of $\angle AOB$?
(a) 55°
(b) 110°
(c) 120°
(d) data insufficient
- 46 In the given diagram $ABCD$ is a cyclic quadrilateral $\angle OCB = 50^\circ$ and $\angle BOC = 110^\circ$. Find the value of $\angle DAO$.
(a) 20°
(b) 30°
(c) 50°
(d) can't be determined



- 47 ABC and CDE are right angled triangle. $\angle ABC = \angle CDE = 90^\circ$, D lies on AC and E lies on BC . $AB = 24$ cm, $BC = 60$ cm. If $DE = 10$ cm, then CD is :



- (a) 28 cm (b) 35 cm
(c) 25 cm (d) can't be determined
- 48 In the above question (number 48) what is the ratio of $CE : BE$?
- (a) $\sqrt{29} : (12 - \sqrt{29})$ (b) $12 : \sqrt{29}$
(c) $7 : \sqrt{21}$ (d) none of these
- 49 In the given diagram $\triangle ABC$ is an isosceles right angled triangle, in which a rectangle is inscribed in such a way that the length of the rectangle is twice of breadth. Q and R lie on the hypotenuse and P, S lie on the two different smaller sides of the triangle. What is the ratio of the areas of the rectangle and that of triangle?
- (a) $\sqrt{2} : 1$ (b) $1 : \sqrt{2}$
(c) $1 : 2$ (d) $\sqrt{3} : 2$
- 50 An n sided polygon has ' n ' diagonals, then the value of n is :
- (a) 4 (b) 6
(c) 7 (d) 5
- 51 How many distinct equilateral triangles can be formed in a regular nonagon having at two of their vertices as the vertices of nonagon?
- (a) 72 (b) 36
(c) 66 (d) none of these
- 52 ABC is a triangle in which D, E and F are the mid-points of the sides AC, BC and AB respectively. What is the ratio of the area of the shaded to the unshaded region in the triangle?

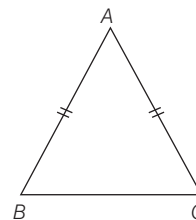


- (a) 1 : 1 (b) 3 : 4
(c) 4 : 5 (d) none of these

- 53 Three circles each of units radius intersects each other at P, Q and R . P, Q and R are the centres of the three circles. What is the sum of length of the arcs PQ, QR and PR ?

- (a) $\sqrt{3}\pi$ (b) $2\sqrt{2}\pi$
(c) π (d) none of these

- 54 ABC is an isosceles triangle in which $AB = AC$ and $(\angle A) = 2(\angle B)$. $AB = 4$ cm. What is the ratio of inradius to the circumradius?



- (a) 1 : 2 (b) $(\sqrt{2} - 1) : 1$
(c) none of these

- 55 In a triangle ABC , the two sides AB and AC are 36 cm and 30 cm, and the radius of its circumcircle is 45 cm. What's the length of perpendicular drawn from its vertex A on the sides BC ?

- (a) 12 cm (b) 18 cm
(c) 15 cm (d) 33 cm

- 56 The sides of a triangle are in the ratio $1 : \sqrt{3} : 2$, then the angles of the triangle are in the ratio

- (a) 1 : 3 : 5 (b) 2 : 3 : 4
(c) 3 : 2 : 1 (d) 1 : 2 : 3

- 57 If the angles of a triangles are in the ratio 4 : 1 : 1, then the ratio of the longest sides to the perimeter is

- (a) $\sqrt{3} : (2 + \sqrt{3})$ (b) 1 : 6
(c) $1 : (2 + \sqrt{3})$ (d) 2 : 3

- 58 Let ABC and ABC' be two non-congruent triangles with sides $AB = 4$, $AC = AC' = 2\sqrt{2}$ and $\angle B = 30^\circ$. The absolute value of the difference between the areas of these triangles is

- (a) 2 (b) $2\sqrt{2}$
(c) 4 (d) $3\sqrt{2}$

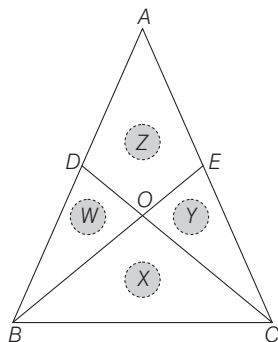
- 59 The sides of triangle ABC are 5, 5, 6 and the sides of triangle PQR are 5, 5, 8. Find the correct relation between the areas of two triangles.

- (a) $\triangle ABC > \triangle PQR$ (b) $\triangle ABC < \triangle PQR$
(c) $\triangle ABC = \triangle PQR$ (d) can't be determined

- 60 The sides AC, BC and AB of a triangle ABC are tangents to its incircle at D, E and F , respectively. Point D lies on the hypotenuse AC such that D is the circumcentre of the triangle ABC . Find $EF : AC$.

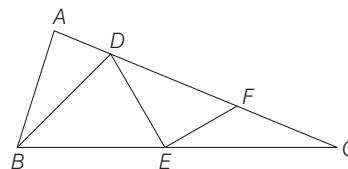
- (a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2 + \sqrt{2}}$
(c) $\frac{1}{2 - \sqrt{2}}$ (d) none of these

- 61** The inradius and circumradius of a right angle triangle are 6 cm and 16 cm. Find the area (in sq. cm) of the triangle.
 (a) 304 (b) 228
 (c) 288 (d) none of these
- 62** A circle is the circumcircle for the triangle DEF and the incircle for the triangle ABC . If the angles of $\triangle DEF$ are 76° , 56° and 48° , find the values of internal angles of $\triangle ABC$.
 (a) 32, 46, 92 (b) 28, 68, 84
 (c) 38, 48, 74 (d) data insufficient
- 63** In a $\triangle ABC$, a perpendicular OD is drawn on the side AC from the mid-point of the side AB . If $BC = 4\sqrt{3}$ cm, find the length of OD .
 (a) 2 cm (b) 3 cm
 (c) 4 cm (d) none of these
- 64** The hypotenuse AC of a right angle triangle ABC is 42 cm. This triangle has the maximum possible area with the given hypotenuse. A rectangle $EFGH$ is formed in such a way that the two vertices G and H lie on the hypotenuse AC while the vertices E and F lie on BC and AB , respectively. If $EH = 14$ cm, find the area of the largest possible circle that can be inscribed in the rectangle $EFGH$.
 (a) 155 sq cm (b) 140 sq cm
 (c) 154 sq cm (d) data insufficient
- 65** In an isosceles triangle ABC , side BC is distinct from rest of the two sides. CD and BE are the medians on AB and AC , respectively. If W , X , Y and Z represents distinct areas in the triangle, find the ratio of $W : X : Y : Z$.

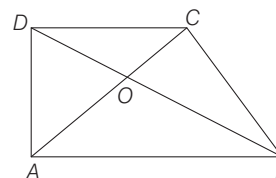
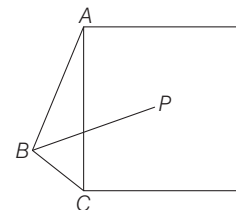


- (a) 1 : 2 : 1 : 3 (b) 1 : 3 : 1 : 4
 (c) 1 : 2 : 1 : 2 (d) 2 : 3 : 2 : 3
- 66** The perimeter of an isosceles triangle is 20 cm and if b is the length of the third side which is distinct from the other two equal sides, find b .
 (a) $\left(1, \frac{20}{3}\right) \cup \left(\frac{20}{3}, 10\right)$
 (b) $\left(0, \frac{20}{3}\right) \cup \left(\frac{20}{3}, 10\right)$
 (c) $0 < b < a$
 (d) $5 < b < 10$

- 67** In the following isosceles triangle, in which $AC = BC$ and $AB = BD = DE = EF = FC = 6$ cm, find BE .



- (a) 9 (b) 4.5
 (c) 7.5 (d) none of these
- 68** A fox, camouflaged in the bush, observes from the ground, that it takes 2 minutes for the angle of elevation of the squirrel to change from 30° to 60° . If the speed of fox is $\sqrt{3}$ times that of the squirrel, find the time taken by fox to reach the tree from the bush.
 (a) 1 minute (b) $\sqrt{3}$ minute
 (c) 3 minutes (d) $4/\sqrt{3}$ minutes
- 69** In an isosceles triangle ABC base $AB = 6$ cm and each lateral side is 5 cm. Find the circumcentre of the triangle ABC .
 (a) $\frac{22}{7}$ cm (b) $\frac{25}{8}$ cm (c) $\frac{21}{8}$ cm (d) $\frac{23}{7}$ cm
- 70** Let P be the centre of the square constructed on hypotenuse AC of the right angle triangle ABC . If $\angle BCA - \angle BAC = 20^\circ$. Find the value of $\angle ABP - \angle CBP$.
 (a) 10° (b) 20°
 (c) 5° (d) none of these
- 71** In a quadrilateral $ABCD$, $\angle B = \angle C = 60^\circ$ and $\angle D = 90^\circ$. Find the length of CD , if $AD = 3\sqrt{3}$ cm and $BC = 6\sqrt{3}$ cm.
 (a) $3(2\sqrt{3} - 1)$ cm (b) $6(\sqrt{3} - 1)$ cm
 (c) $2(3\sqrt{3} - 1)$ cm (d) $\sqrt{3}(2\sqrt{3} - 1)$ cm
- 72** In a trapezium $ABCD$, where $AB \parallel CD$ and diagonals AC intersects the other diagonal BD at O . Which of the following statements are always correct?
 (i) $\triangle AOB \sim \triangle COD$
 (ii) $\triangle DOA \sim \triangle COB$
 (iii) Area $\triangle DOA$ = Area $\triangle COB$
 (iv) Area $\triangle AOB$ = Area $\triangle COD$

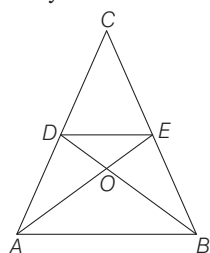


- (a) only (i), (ii) and (iii) (b) only (i), (ii), (iv)
 (c) only (i) and (iii) (d) only (i)

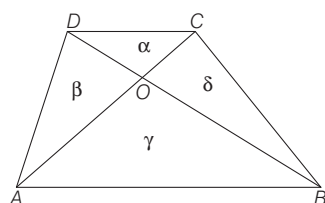
- 73** In an isosceles $\triangle ABC$, where, $AC = BC$. A line segment DE is drawn parallel to the side AB , where D and E are midpoints of AC and BC , respectively. The diagonals AE and DB of the trapezium $ABED$ intersect each other at O . Which of the following are necessarily true?

- (i) Area $\triangle DOA = \text{Area } \triangle EOB$
 (ii) Area $\triangle AOB = 4(\text{Area } \triangle DOE)$
 (iii) Area $\triangle CDE = 9(\text{Area } \triangle DOE)$
 (a) (i), (ii) and (iii) (b) (ii) and (iii)
 (c) (i) and (iii) (d) (i) and (ii)

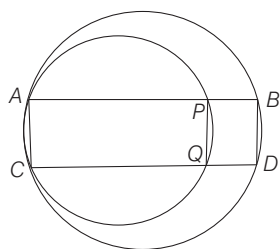
- 74** In an isosceles triangle ABC , such that $AC = BC$ and $DE \parallel AB$, where D and E lie on the side AC and BC , respectively. The area of $\triangle DOE = 40 \text{ cm}^2$ and area of $\triangle AOB = 250 \text{ cm}^2$, find the area (in cm^2) of $\triangle AOD$ and area of $\triangle ACB$, respectively.



- (a) 100 and 700 (b) 150 and 600
 (c) 175 and 600 (d) none of these
- 75** In a trapezium $ABCD$, $AB \parallel CD$. AD and BC are not necessarily equal. If $\alpha = 4 \text{ cm}^2$ and $\gamma = 9 \text{ cm}^2$. Find the value of $\frac{(\beta + \delta)}{(\alpha + \beta + \gamma + \delta)}$.



- (a) $\frac{9}{16}$ (b) $\frac{12}{25}$
 (c) $\frac{3}{5}$ (d) none of these
- 76** In the following figure quadrilaterals $ABDC$ and $APQC$ are concyclic ones. If AB and CD are two parallel lines, then $PBDQ$ is necessarily -



- (a) a square (b) a rectangle
 (c) a parallelogram (d) none of these

- 77** A circle is circumscribed by an isosceles trapezium in which one of the parallel sides is $49/16$ times the other parallel side. If the area of the trapezium is 3640 sq. cm , find the area of the circle.

- (a) 1890 sq. cm (b) 2464 sq. cm
 (c) 2884 sq. cm (d) 1694 sq. cm

- 78** If the opposite sides of a cyclic quadrilateral are equal and the sum of two of the adjacent sides is 10 cm , find the maximum possible area of the quadrilateral (in sq. cm).

- (a) 100 (b) 25
 (c) 24 (d) data insufficient

Directions (for Q. Nos. 79 and 80): Answer the following questions based on the information given below.

In a kite, the shorter diagonal intersects the longer diagonal in the ratio of $6 : 7$. And, the longer diagonal of the kite is 52 cm and perimeter of the kite is 208 cm .

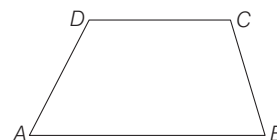
- 79** Find the area of the circle inscribed in this kite.

- (a) 2080 sq. cm (b) 1263 sq. cm
 (c) 1591 sq. cm (d) 2340 sq. cm

- 80** If there are four non-overlapping circles inscribed in each of the four triangles formed due to the intersection of two perpendicular diagonals of the kite, find the perimeter of the quadrilateral that is formed by joining the centers of these four circles.

- (a) 76 cm (b) $2(19 + \sqrt{362}) \text{ cm}$
 (c) $2(19 + \sqrt{360}) \text{ cm}$ (d) $4\sqrt{362} \text{ cm}$

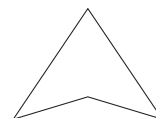
- 81** In the following diagram, $ABCD$ is a trapezium, whereas $AD = DC = BC = 1 \text{ unit}$ and $\angle ADC = \angle BCD = 108^\circ$. Find the value of line segment AB .



- (a) 2 (b) $\sqrt{3}$
 (c) $\sqrt{5} - 1$ (d) none of these
- 82** In a dart the reflex angle is 216° and the angle opposite the reflex angle is 72° . If each of the shorter sides of the dart is 1 cm , find the perimeter of the dart, in cm .

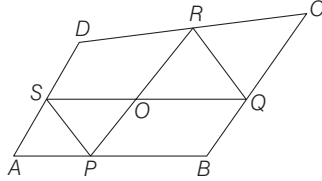
- (a) $3 + \sqrt{5}$ (b) $3 + \sqrt{6}$ (c) $4 + \sqrt{3}$ (d) $5 + \sqrt{3}$

- 83** In the following figure, the only reflex angle of the dart is 216° and the second largest angle is 72° . If each of the two smaller sides is 1 cm and each of the two larger sides is $\frac{(1 + \sqrt{5})}{2}$, find the area of the dart, in sq. cm .



- (a) $2\sqrt{5 - \sqrt{5}}$ (b) $\frac{\sqrt{5 - \sqrt{5}}}{\sqrt{2}}$
 (c) $\frac{\sqrt{5 + \sqrt{5}}}{2\sqrt{2}}$ (d) $\frac{3\sqrt{5 + \sqrt{5}}}{7\sqrt{2}}$

- 84** In the following quadrilateral $ABCD$, the four mid-points P, Q, R, S are joined in such a way that the quadrilateral is divided into six parts.



The ratio of the quadrilaterals $\square ASOP, \square POQB, \square ORCQ, \square SDRO$ is $4 : 5 : 7 : 6$ and the ratio of $\triangle ASP$ and $\triangle RCQ$ is $2 : 5$. If S is joined to R and P is joined to Q , find the ratio of $\square PQRS$ and $\square ABCD$.

- (a) $1 : 2$ (b) $5 : 11$
(c) $4 : 11$ (d) none of these
- 85** In a rhombus $ABCD$, $\angle ADC : \angle BCD = 1 : 2$ and $EF : BD = 1 : 3$, where E and F lie on the diagonal BD , inside the rhombus, such that $\square AECF$ is a quadrilateral. Find the ratio of areas of $\square AECF : \square ABCD$.
(a) $1 : 4$ (b) $2 : 5$
(c) $3 : 14$ (d) none of these
- 86** An isosceles trapezium whose parallel sides are in the ratio $1 : 4$. It inscribes a circle that touches all the sides of the trapezium. If the sum of the two parallel sides is 70 cm, find the circumference of the inscribed circle.
(a) 616 cm (b) 14π cm
(c) 140 cm (d) none of these
- 87** A 7×24 rectangular thin wall has a 2×2 square shaped window in such a way that the intersection of diagonals of rectangle and the intersection of diagonals of square coincide with each other. Which one of the following is correct?
(a) Diagonals of rectangle and that of square will be the same.
(b) The diagonals of wall will intersect the vertical sides of the window.
(c) The diagonals of wall will intersect the horizontal sides of the window.
(d) none of the above
- 88** A trapezium inscribes a circle that touches all its sides and the line connecting the mid-points of the non-parallel sides of the trapezium divides the trapezium in such a way that the area of one part is half the area of the other part. If the lengths of the non-parallel sides be 8 cm and 16 cm, find the length of the shortest side of the trapezium.
(a) 4 (b) 6
(c) 9 (d) none of these
- 89** A point O lies inside a square $ABCD$ such that $OA = 3, OB = 4$ and $OD = 5$, find the value of OC .
(a) $3\sqrt{3}$ (b) 6.67
(c) $3\sqrt{2}$ (d) $4\sqrt{2}$

- 90** In a $\triangle ABC$, CP and BQ intersect each other at R inside the triangle, where P and Q lie on the sides AB and AC respectively. $\angle BAC = 50^\circ$, $\angle ABC = 70^\circ$, $\angle ACB = \angle APQ$ and $PC = BC$. Find $\angle PBQ$.

- (a) 10° (b) 15°
(c) 20° (d) none of these

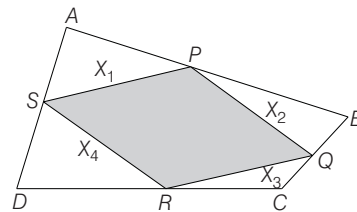
- 91** Which of the following statements is/are correct?

- (i) A kite always has two axes of symmetry
(ii) A square is both a kite and an isosceles trapezium
(iii) Every kite is an orthodiagonal
(iv) An isosceles trapezium can have a circumcircle
(v) A kite can always have an incircle
(vi) If the sum of two opposite sides is not the same as the sum of the other pair of opposite sides, it can never be a circumscribing quadrilateral.
(a) all, except (ii) and (iv)
(b) only (ii), (iii) and (v)
(c) only (i), (iii), (v)
(d) all, except (i)

- 92** Which of the following statements is/are correct?

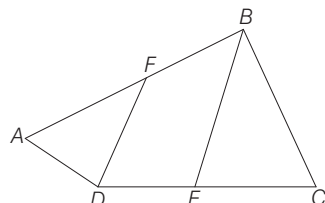
- (i) The intersections of the angle bisectors of the quadrilateral are the vertices of the cyclic quadrilateral.
(ii) If diagonals of a cyclic quadrilateral perpendicularly intersect at point P , then the line through P perpendicular to any side bisects the opposite side.
(iii) The product of the lengths of the two diagonals of a cyclic quadrilateral is equal to the sum of the products of opposite sides.
(a) (i) and (ii) (b) (ii) and (iii)
(c) (i) and (iii) (d) all of these

- 93** Which of the following is correct, considering the given figure, in which P, Q, R and S are the mid-points of AB, BC, CD and AD , respectively?

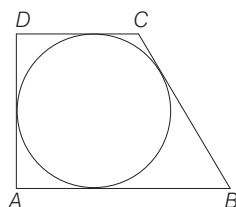


- (i) The perimeter of the parallelogram $PQRS$ is equal to the sum of the diagonals of the quadrilateral $ABCD$
(ii) $X_1 + X_3 = X_2 + X_4$
(iii) $A(PQRS) = \frac{1}{2} A(ABCD)$
(a) Only (i) and (iii) (b) only (ii) and (iii)
(c) All three are correct (d) none of the above

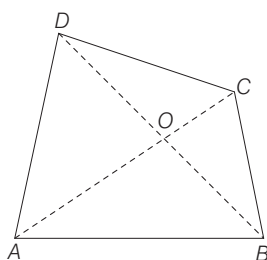
- 94 In the following quadrilateral $ABCD$, E and F are the mid-points of DC and AB , respectively. If the area of $\triangle ADF$ is 9 cm^2 and that of $\triangle EBC$ is 18 cm^2 , what is the area of quadrilateral $ABCD$?



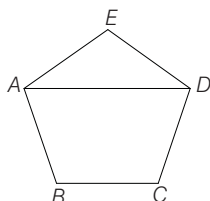
- (a) 45 cm^2 (b) 27 cm^2
(c) data insufficient (d) none of these
- 95 A trapezium $ABCD$ inscribes a circle. The two sides AB and CD are parallel and the diameter of the circle is equal to the length of side AD . If $DC = 5 \text{ cm}$, find the perimeter of the trapezium $ABCD$.



- (a) 45 cm (b) 50 cm^2
(c) 80 cm (d) none of these
- 96 In the following quadrilateral AC and BD are the diagonals intersecting at a point O inside the quadrilateral. The area of $\triangle AOD$ is 32 cm^2 and area of $\triangle BOC$ is 18 cm^2 . What is the minimum possible area of the quadrilateral $ABCD$?

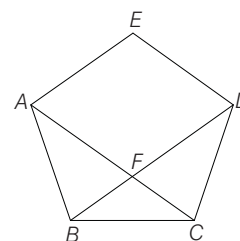


- (a) 144 cm^2 (b) 100 cm^2 (c) 98 cm^2 (d) 96 cm^2
- 97 In the following figure if $ABCDE$ is a regular pentagon, then $ABCD$ is necessarily

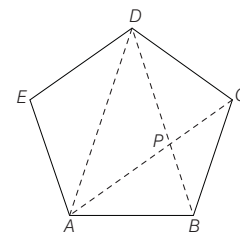


- (a) a cyclic quadrilateral and a tangential quadrilateral as well
(b) a tangential quadrilateral and a trapezium as well
(c) a trapezium and a cyclic quadrilateral as well
(d) all (a), (b) and (c)

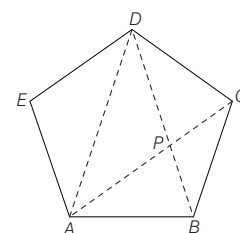
- 98 In the following figure if $ABCDE$ is a regular pentagon, then $AEDF$ is always



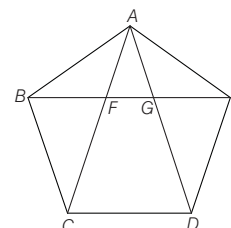
- (a) a cyclic quadrilateral and a parallelogram
(b) a tangential quadrilateral and a cyclic quadrilateral as well
(c) a rhombus and a kite but not a tangential quadrilateral
(d) a tangential quadrilateral as well as a rhombus
- 99 In the following figure, $ABCDE$ is a regular pentagon. If $AB = 2 \text{ cm}$, find the perimeter of the quadrilateral $APDE$.



- (a) $4(\sqrt{5} - 1) \text{ cm}$ (b) $8(\sqrt{3} - 1) \text{ cm}$
(c) $10(2 - \sqrt{3}) \text{ cm}$ (d) none of these
- 100 In the following figure, $ABCDE$ is a regular pentagon. If $AB = 2 \text{ cm}$, find PC .



- (a) $\sqrt{5} - 1 \text{ cm}$ (b) $\sqrt{3} - 1 \text{ cm}$
(c) $2 - \sqrt{3} \text{ cm}$ (d) 1.67 cm
- 101 In the following diagram, $ABCDE$ is a pentagon such that each side is of unit length, how many of the following relations is/are correct?
- (i) $BE \parallel CD$ and $AC \parallel ED$ and $AD \parallel BC$
(ii) $\triangle BCF \sim \triangle FAG$
(iii) $\triangle AFB \sim \triangle BAE \sim \triangle ABC$
(iv) $BF^2 = GF$
(v) $\triangle BCF : \triangle FAG = 1 : FG$
- (a) 4 (b) 3
(c) 2 (d) 5



- 102** If each side of a regular pentagon is 1 cm, find the length of its diagonal.

(a) $\frac{5\sqrt{2}-3}{2}$ (b) $\frac{-1+\sqrt{5}}{2}$
(c) $\frac{1+\sqrt{5}}{2}$ (d) data insufficient

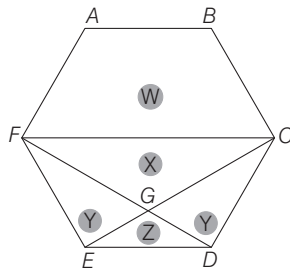
- 103** If the side of a regular pentagon be 2 cm, find the apothem of the pentagon, in cm.

(a) $\sqrt{\frac{5+2\sqrt{5}}{5}}$ (b) $\sqrt{\frac{5+2\sqrt{5}}{10}}$
(c) $\sqrt{\frac{5+4\sqrt{5}}{2}}$ (d) $\sqrt{\frac{10+\sqrt{5}}{4}}$

- 104** Let $A_0A_1A_2A_3A_4A_5$ be a regular hexagon inscribed in a circle of unit radius. Then the product of the lengths of the line segments A_0A_1 , A_0A_2 , A_0A_4 is

(a) $\frac{3}{4}$ (b) $3\sqrt{3}$ (c) 3 (d) $\frac{3\sqrt{3}}{2}$

- 105** In the following figure, a regular hexagon is divided into various parts. The area of respective part is indicated by W, X, Y, Z. Find $W : X : Y : Z$.



(a) 7 : 4 : 2 : 1 (b) 8 : 3 : 2 : 1
(c) 13 : 6 : 3 : 1 (d) 9 : 4 : 2 : 1

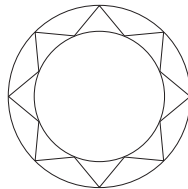
- 106** A circle intersects a hexagon such that the area of hexagon outside the circle is same as the area of circle outside the hexagon. If each side of the hexagon is a , what is the area of circle?

(a) $\frac{3\sqrt{2}}{\pi}a^2$ (b) $\frac{3\sqrt{3}}{2}a^2$
(c) $\frac{3\sqrt{3}\pi}{2}a^2$ (d) none of these

- 107** A regular dodecagon (a polygon with 12 sides) is inscribed in a circle. How many diagonals of this dodecagon would not pass through the centre of the circumscribing circle?

(a) 24 (b) 36
(c) 42 (d) none of these

- 108** There are two congruent squares placed in such a way that the sides of one square are parallel to the diagonals of the other square, whereas the point of intersection of diagonals of one square coincides with that of the diagonals of the other.



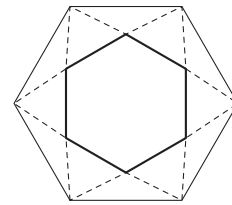
There is a circle passing through the points of intersection of sides of the two square. And there is another circle passing through the vertices of the two squares. If each side of the square is $2 + \sqrt{2}$ unit, find the ratio of the area of the smaller circle to that of the larger circle.

(a) $\frac{2+\sqrt{2}}{3+2\sqrt{2}}$ (b) $\frac{3+\sqrt{2}}{6+4\sqrt{2}}$
(c) $\frac{2+\sqrt{2}}{3+\sqrt{2}}$ (d) none of these

- 109** If the apothem of a regular hexagon is a , find each side of the hexagon.

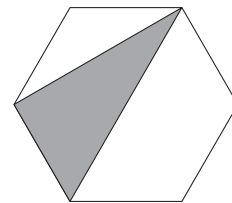
(a) $\frac{3a}{\sqrt{2}}$ (b) $\sqrt{3}a$
(c) $0.67a$ (d) $\frac{2}{\sqrt{3}}a$

- 110** In the following figure find the ratio of the area of the internal hexagon to that of the external regular hexagon.



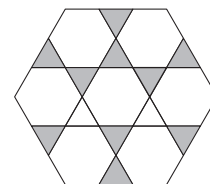
(a) 1 : 3 (b) $\sqrt{6} : \sqrt{2}$
(c) 7 : 2 (d) 4 : 1

- 111** In the following figure find the ratio of the shaded triangular area to that of the remaining area of the regular hexagon.



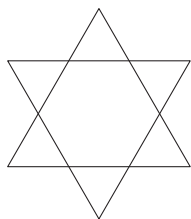
(a) 2 : 5 (b) $2 : 3\sqrt{7}$
(c) 3 : 8 (d) 1 : 2

- 112** In the following diagram, as shown below, a regular hexagon of side 3 cm inscribes 7 smaller regular hexagons. Find the area of the shaded region (of the triangles) within the larger hexagon.

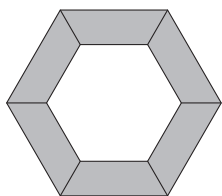


(a) 6 sq. cm (b) $4\sqrt{3}$ sq. cm
(c) $3\sqrt{3}$ sq. cm (d) none of these

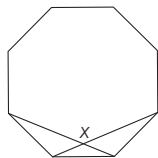
- 113** In the following figure a hexagram consists of a hexagon and six equilateral triangles outside the hexagon. If a circle is inscribed in the hexagon and the area of the inscribed circle is 3π sq. cm, find the area of the circumscribed circle of the regular hexagram.



- (a) 12π sq. cm (b) 9π sq. cm
(c) $6\sqrt{3}$ sq. cm (d) none of these
- 114** In the following hexagonal ring the area of shaded region is 3 times the area of the un-shaded region. Find the ratio of perimeter of the outer hexagon to that of the inner hexagon.



- (a) 3 : 1 (b) $\sqrt{3} : 1$
(c) 1 : 2 (d) none of these
- 115** The following figure shows a concave hexagon. Out of six internal angles it has each of the three acute angles equal to 36° and remaining three angles are reflex. If each side of the symmetrical concave hexagon is $1 + \sqrt{5}$ mm, find the total area of this hexagon, in sq. mm.
- (a) $\sqrt{3} + 3(\sqrt{5} + 2\sqrt{5})$ (b) $3\sqrt{3} + 3(\sqrt{5} + 2\sqrt{5})$
(c) $\sqrt{3} + 3(\sqrt{10} - 2\sqrt{5})$ (d) $2\sqrt{5} + 3(\sqrt{10} - 2\sqrt{5})$
- 116** If the side of a square is a , then find the side of the regular octagon that can be inscribed in this square.
- (a) $\frac{a(1 + \sqrt{2})}{\sqrt{2}}$ (b) $\frac{a}{3}$
(c) $\frac{a}{2 + \sqrt{2}}$ (d) none of these
- 117** Find the value of angle x in a regular octagon as shown below



- (a) 145° (b) 125°
(c) 136° (d) none of these

- 118** If each side of the octagon be a , then find the shortest diagonal of the octagon.

- (a) $(\sqrt{2} + \sqrt{2})a$ (b) $(1 + \sqrt{2})a$
(c) $2(1 + \sqrt{2})a$ (d) $(\sqrt{4 + 2\sqrt{2}})a$

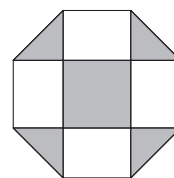
- 119** If each side of the octagon be a , then find the longest diagonal of the octagon.

- (a) $(\sqrt{2} + \sqrt{2})a$ (b) $(1 + \sqrt{2})a$
(c) $2(1 + \sqrt{2})a$ (d) $(\sqrt{4 + 2\sqrt{2}})a$

- 120** If d_1, d_2, d_3 be the three distinct diagonals of a regular octagon, such that $d_1 < d_2 < d_3$, find the ratio of the inradius to the circumradius of the octagon.

- (a) $\frac{d_1}{d_3}$ (b) $\sqrt{\frac{d_2}{d_3}}$ (c) $\frac{d_2}{d_3}$ (d) $\frac{d_1 d_3}{(d_2)^2}$

- 121** In the following figure the regular octagon has some shaded area and some unshaded area. Find the ratio of the shaded area to the unshaded area.



- (a) 1 : 1 (b) $\sqrt{2} : \sqrt{3}$ (c) $1 : \sqrt{2}$ (d) $\sqrt{2} : 1$

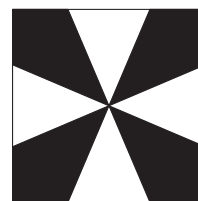
- 122** The midpoints of alternating sides of a regular octagon are joined to form a square. If each side of the octagon is a , find each side of the square.

- (a) $\frac{(2 + \sqrt{2})a}{2}$ (b) $\frac{(4 + \sqrt{2})a}{2}$
(c) $\frac{2(1 + \sqrt{2})a}{\sqrt{2}}$ (d) $(2 + \sqrt{2})a$

- 123** If each side of the octagon is a , find the area of the octagon.

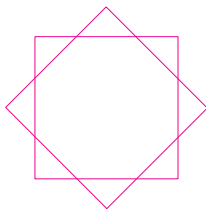
- (a) $2(\sqrt{2} - 1)a^2$ (b) $4(1 + \sqrt{2})a^2$
(c) $2(2 + \sqrt{2})a^2$ (d) $2(1 + \sqrt{2})a^2$

- 124** In the following figure a square inscribes 4 congruent kites and 4 congruent isosceles triangles. The area of each kite is same as the area of each triangle. Find the ratio of the longer diagonal to the shorter diagonal of each kite



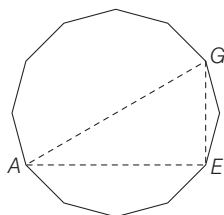
- (a) $\frac{2}{1}$ (b) $\frac{1 + \sqrt{2}}{\sqrt{2}}$
(c) $\frac{\sqrt{2}}{1}$ (d) data insufficient

- 125** In the following figure two congruent squares are placed in such a way that the diagonals of one square are parallel to the sides of the other square and all the four diagonals intersect at the same point. If the area of the shaded region is 4 cm^2 , find the area of the common region.



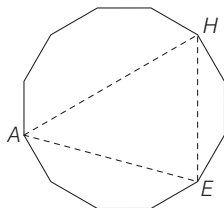
- (a) $\sqrt{4 + 2\sqrt{2}} \text{ cm}^2$ (b) $4(1 + \sqrt{2}) \text{ cm}^2$
(c) $\sqrt{2 + \sqrt{2}} \text{ cm}^2$ (d) none of these

- 126** Find the angle AGE in the regular dodecagon shown below.



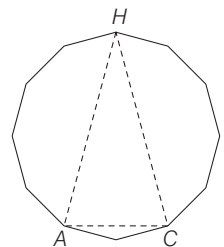
- (a) 30° (b) 45°
(c) 75° (d) none of these

- 127** Find the angle AHE in the regular dodecagon shown below.



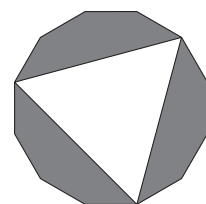
- (a) 45° (b) 50°
(c) 55° (d) none of these

- 128** Find the angle AHC in the regular dodecagon shown below.



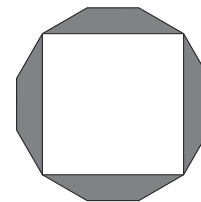
- (a) 15° (b) 22.5° (c) 30° (d) 45°

- 129** In the following figure the three equispaced vertices of a regular dodecagon are connected in such a way that they create a triangle, as shown below. If each side of the triangle is $\sqrt{3}\sqrt{2 + \sqrt{3}} \text{ cm}$, find the circumradius of the dodecagon.



- (a) $\sqrt{6 + \sqrt{3}} \text{ cm}$ (b) $\sqrt{3 + 2\sqrt{3}} \text{ cm}$
(c) $\sqrt{2 + \sqrt{3}} \text{ cm}$ (d) none of these

- 130** In the following figure the four equispaced vertices of a regular dodecagon are connected in such a way that they create a quadrilateral, as shown below. If each side of the quadrilateral is $\sqrt{3} + 1 \text{ cm}$, find the circumradius of the dodecagon.



- (a) $\frac{(\sqrt{3} + 1)}{\sqrt{2}} \text{ cm}$ (b) $\frac{2(\sqrt{6} + 1)}{\sqrt{3}} \text{ cm}$
(c) $\frac{(\sqrt{12} + 1)}{\sqrt{5}} \text{ cm}$ (d) none of these

- 131** A regular dodecagon is inscribed in a circle of radius 1 cm. Find the area of the dodecagon

- (a) 3 sq. cm (b) $3\sqrt{2}$ sq. cm
(c) $2\sqrt{3}$ sq. cm (d) none of these

- 132** Three circles with radii 3, 4 and 5 touch each other externally. If P is the only point of intersection of tangents to these circles, find the distance of P from the points of contact.

- (a) $\sqrt{5}$ (b) $\sqrt{6}$ (c) $\sqrt{7}$ (d) 3

- 133** Three points X , Y and Z are concyclic to the larger circle with radius 15 cm and there is another concentric circle with radius 8 cm such that XY and XZ are tangents to the smaller circle. At how many points does YZ intersect the larger circle?

- (a) 0 (b) 1
(c) 2 (d) can't say

Directions (for Q. Nos. 134 and 135): Answer the following questions based on the information given below.

A circle with centre O and radius 2 cm inscribes a triangle ABC . Two tangents AD and BD are drawn from a point D outside the circle and $\angle ABD = 60^\circ$.

- 134** Find the area of $\square OADB$, in cm^2 .

- (a) $4\sqrt{3}$ (b) $3\sqrt{3}$
(c) 4 (d) none of these

- 135** Find the maximum possible area of $\square ACBD$, in cm^2 .

- (a) $6\sqrt{3}$ (b) $8\sqrt{3}$
(c) $\frac{14}{\sqrt{3}}$ (d) none of these

- 136** Triangle ABC inscribes a circle with centre O and inradius $\sqrt{6} \text{ cm}$. The inradius OD divides side BC at D , such that $BD = 2 \text{ cm}$ and $DC = 8 \text{ cm}$. Find the ratio $AB : AC$.

- (a) 4 : 7 (b) 5 : 7
(c) 4 : 5 (d) cannot be determined

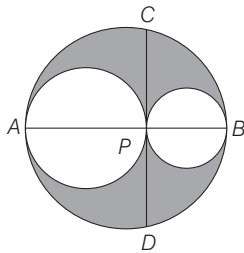
- 137** A circle with radius 12 cm has two perpendicular chords AB and CD intersecting at a point R , other than the centre O , where $RD = 6 \text{ cm}$ and $RB = 18 \text{ cm}$. Find the length of AR .

- (a) $6\sqrt{\frac{3}{5}}$ (b) $4\sqrt{\frac{3}{5}}$
(c) $4\sqrt{5}$ (d) Data inconsistent

- 138** There are two tangents AT and BT on a circle of radius 30 cm. A line OT ($= 50$ cm) connects the centre O with the external point T . Another tangent CD touches the circle at P such that C and D lie on the line segments AT and BT , respectively. Points P and T lie on the same side of the circle. Find the maximum possible area (in sq. cm) of the circle inscribed in the triangle CDT .

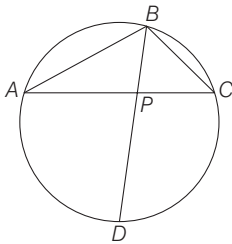
(a) 56.25π (b) 49π
(c) 42.25π (d) none of these

- 139** In the following figure there are two smaller circles inscribed in a larger circle touching each other. The centres of all the three circles fall on AB , and CD is tangent to both the interior circles. If $CD = 16$ cm, find the area of the shaded region.



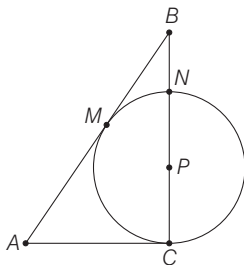
(a) 35π sq. cm (b) 28π sq. cm
(c) 32π sq. cm (d) data insufficient

- 140** In the following figure $AB = 21$ cm, $BC = 15$ cm, $AC = 24$ cm. Point P is the mid-point of arc AC , and chord BD bisects chord AC at P . Find the ratio BP/PD .



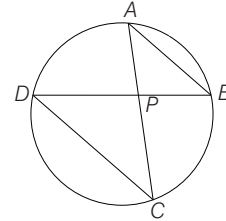
(a) $1 : 16$ (b) $1 : 21$
(c) $1 : \sqrt{21}$ (d) none of these

- 141** In the following figure one side of the $\triangle ABC$ passes through the centre P of the circle and the other two sides of this triangle are tangents to this circle. If $BN = 2$ cm and $AC = 6$ cm, find the radius of the circle.



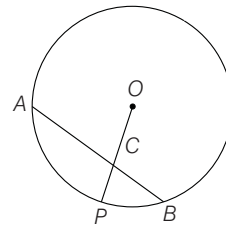
(a) 4 cm (b) 2 cm
(c) 2.5 cm (d) none of these

- 142** In the following figure there are four chords AB, CD, AC, BD in a circle. If $AB = 6$, $AP = 2$, $BP = 5$, $DP = 3$, find CD .



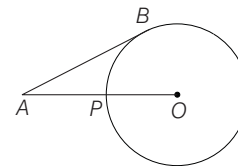
(a) 4 (b) 5
(c) 9 (d) none of these

- 143** In a circle, its radius OP is intersected by a chord AB at C , where O is the centre of the circle, such that $AC = 7$, $BC = 5$ and $PC = 1$, find the radius of the circle.



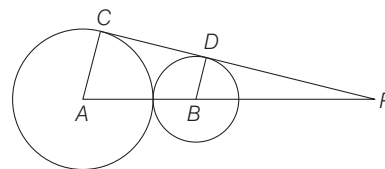
(a) 12 (b) 15
(c) data insufficient (d) none of the above

- 144** In a circle, its radius OP is extended to meet a point A outside the circle and a line segment AB is drawn from A to a point B on the circle. If O is the centre of the circle and $AP = 7$, $AB = 14$, find the circumference of the circle.



(a) 66 (b) 67
(c) 70 (d) none of these

- 145** In the following diagram, there are two circles externally tangent to each other and there is a common tangent CP touch the circles at C and D . AC and BD are the radii of the two circles. If $AC = 15$ and $BD = 6$, find the distance between C and D .

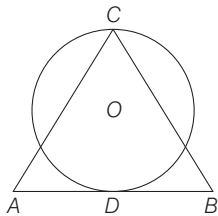


(a) 12 cm (b) $18 \sqrt{10}$
(c) 21 cm (d) $6\sqrt{10}$

- 146** A circle has its total area A . There are four equal circles tangent to this circle passing through the centre of this circle and tangent to this circle. The area of each such circle is B . The area of each region that is created by overlapping of any two of these circles is D and the area of each region that is not covered by these circles within the larger circle is C . If $D = \frac{\pi}{25}$, find the value of C .

- (a) $\frac{2\pi}{25}$ (b) $\frac{\pi}{5\sqrt{5}}$
(c) $\frac{\pi}{49}$ (d) none of these

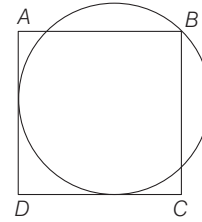
- 147** An equilateral triangle ABC is tangent to a circle, with centre O , at the midpoint D . If C lies on the circumference of the circle and its radius is 6 cm, find the area of the overlapping region.



- (a) $12\pi + 21\sqrt{2}$ sq. cm (b) $12\pi + 16\sqrt{3}$ sq. cm
(c) $12\pi + 18\sqrt{3}$ sq. cm (d) none of these

Directions (for Q. Nos. 148 and 149) Answer the following questions based on the information given below.

In the following figure the two non-parallel sides AD and CD of a square $ABCD$ are tangent to a circle of radius 1 cm in such a way that the vertex B lies on the circumference of the circle.



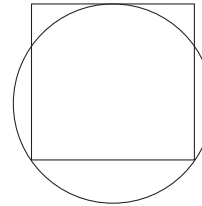
- 148** Find the side of the square.

- (a) $\frac{2 + \sqrt{2}}{2}$ cm (b) $\frac{2\sqrt{2} - 1}{\sqrt{2}}$ cm
(c) $\frac{1 + \sqrt{2}}{\sqrt{2}}$ cm (d) data insufficient

- 149** Find the common area between the circle and the square.

- (a) $\frac{3\pi - 2}{3}$ sq. cm (b) $\frac{\pi + 2}{2}$ sq. cm
(c) $\frac{3(\pi - 2)}{4}$ sq. cm (d) data insufficient

- 150** In the following figure one side of a square is tangent to the circle, as shown here. If each side of the square be 24 cm, what would be the radius of the circle?

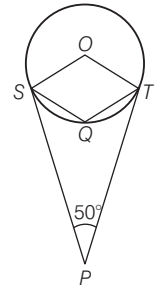


- (a) 12 cm (b) 16 cm
(c) can't be determined (d) none of these

LEVEL 02 > HIGHER LEVEL EXERCISE

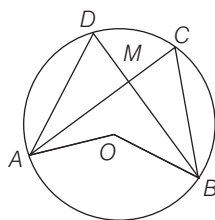
- 1** The semiperimeter of a right angled triangle is 126 cm and the shortest median is 53 cm. What is the area of a triangle which has the largest median as its longest side?
(a) 1560 cm^2 (b) 1260 cm^2
(c) 1060 cm^2 (d) none of these
- 2** In an isosceles right angled triangle ABC , $\angle B$ is right angle. Angle bisector of $\angle BAC$ is AN cut at M to the median BO . Point O lies on the hypotenuse. OM is 20 cm, then the value of AB is :
(a) 38.96 cm (b) 24.18 cm
(c) 34.134 cm (d) none of the above
- 3** In a right angled triangle $\angle B$ and $\angle A$ are acute angles. If $\angle A - \angle B = k$, where A and B are integers, then how many integer values can k take?
(a) 80 (b) 88 (c) 45 (d) 89

- 4** In the given figure O is the centre of the circle, SP and TP are the two tangents at S and T respectively. $\angle SPT$ is 50° , the value of $\angle SQT$ is :
(a) 125° (b) 65°
(c) 115° (d) none of the above



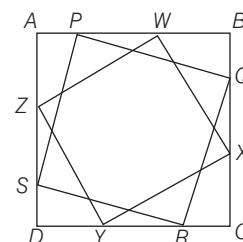
- 5** A 6.5 m long ladder is standing against a wall and the difference between the base of the ladder and wall is 5.2 m. If the top of the ladder now slips by 1.4 m, then by how much will the foot of the ladder slip?
(a) 1.2 m (b) 0.8 m
(c) 0.75 (d) can't be determined
- 6** In a triangle all the three angles A, B, C are in integers, then the number of values that A, B and C can take :
(a) 89 (b) 90 (c) 178 (d) 180

- 7 In the given figure of circle, 'O' is the centre of the circle and $\angle AOB = 130^\circ$. What is the value of $\angle DMC$?



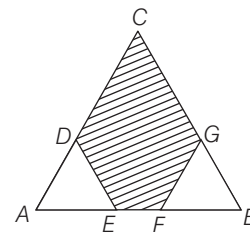
- (a) 65°
 (b) 125°
 (c) 85°
 (d) can't be determined
- 8 In a right angled triangle ABC , $\angle B$ is right angle, side AB is half of the hypotenuse. AE is parallel to median BD and CE is parallel to BA . What is the ratio of length of BC to that of EC ?
- (a) $\sqrt{2} : 1$ (b) $\sqrt{3} : 2$
 (c) $\sqrt{5} : \sqrt{3}$ (d) can't be determined
- 9 In an equilateral triangle ABC , AO , BO and CO are the angle bisectors meet at the incentre 'O'. D , E and F are the mid-points of AO , BO and CO respectively. A circle with centre O passes through D , E and F . Area of the circle is $3\pi \text{ cm}^2$. What is the perimeter of triangles (Δ) ABC ?
- (a) $12\sqrt{3} \text{ cm}$ (b) 18 cm
 (c) $6\sqrt{3} \text{ cm}$ (d) none of these
- 10 Two trains Punjab mail and Lucknow mail starts simultaneously from Patiala and Lakhimpur respectively towards each other with the speed of 40 km/h and 60 km/h respectively on the same track Lakhimpur is 500 km due east of Patiala. A plane starts flying at 200 km/h at the same time from Patiala to Jalandhar. Jalandhar is 100 km due north of Patiala. After travelling sometime two trains Punjab mail and Lucknow mail collide with each other. The plane moves continuously to and fro between Patiala to Jalandhar till the collision of the trains. How far would the plane have travelled?
- (a) 100 km (b) 1000 km
 (c) 2000 km (d) can't be determined
- 11 In the above question (number 10) what is the distance between the place of accident and the plane at the moment of accident of two trains?
- (a) 200 km (b) 250 km
 (c) 400 km (d) data insufficient
- 12 In the given figure $\angle B$ is right angle. $AD : BD = 3 : 2$ and $CE : BE = 5 : 2$ and $AF : FC = 1 : 1$. What is the area of ΔABC , if the area of ΔBDE is 20 cm^2 ?
- (a) 40 cm^2
 (b) 35 cm^2
 (c) 52.5 cm^2 (d) none of the above
- 13 In a triangle ABC with side $AB = AC$ and $\angle BAC = 20^\circ$, D is a point on side AC and $BC = AD$. Find $\angle DBC$.
- (a) 50° (b) 45° (c) 65° (d) 70°

- 14 In the adjoining figure $ABCD$, $PQRS$ and $WXYZ$ are three squares. Find number of triangles and quadrilaterals in the figure.



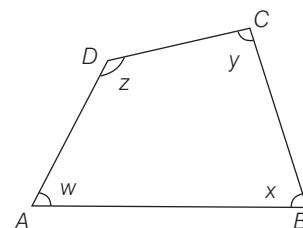
- (a) 24 and 16
 (b) 28 and 15
 (c) 27 and 16
 (d) none of the above

- 15 In the given figure ABC is a triangle in which $CDEFG$ is a pentagon. Triangles ADE and BFG are equilateral triangles each with side 2 cm and $EF = 2 \text{ cm}$. Find the area of the pentagon.



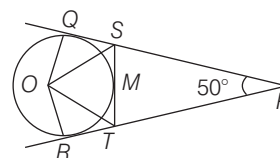
- (a) $8\sqrt{3} \text{ cm}^2$ (b) $7\sqrt{3} \text{ cm}^2$
 (c) $15\sqrt{3} \text{ cm}^2$ (d) 11.28 cm^2
- 16 $PQRS$ is a quadrilateral which is formed by joining the mid-points of a quadrilateral $ABCD$, such that $\angle A = 75^\circ$, $\angle B = 95^\circ$, $\angle C = 110^\circ$. If $\angle PQR = 90^\circ$, what is the value of $\angle PSR$?
- (a) 90° (b) 110°
 (c) 60° (d) 75°

- 17 $ABCD$ is a quadrilateral in which $\frac{z}{y} = \frac{y}{x} = \frac{x}{w} = r$ and k



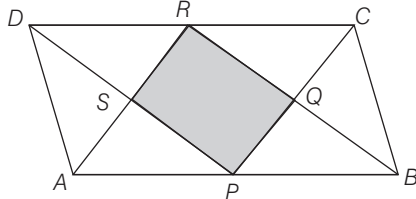
- is an integer. Also $\{w, x\} < 90$ and $\{y, z\} > 90$, then the difference between the greatest angle and smallest angle (i.e., $z - w$) is
- (a) 168° (b) 172°
 (c) 128° (d) 212°

- 18 In the adjoining figure, O is the centre of the circle and PQ , PR and ST are the three tangents. $\angle QPR = 50^\circ$, then the value of $\angle SOT$ is :

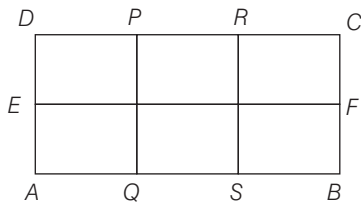


- (a) 30°
 (b) 75°
 (c) 65°
 (d) can't be determined
- 19 The number of points of intersection of the diagonals of a regular hexagon is :
- (a) 10 (b) 15
 (c) 18 (d) 19

- 20** In the adjoining figure $ABCD$, P and R are the mid-points of the sides AB and CD . $ABCD$ is a parallelogram. What is the ratio of the shaded to the unshaded region?

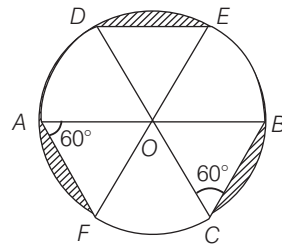


- (a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) none of these
- 21** In the adjoining figure $ABCD$ is a rectangle. Find the maximum number of rectangles including the largest possible rectangle.



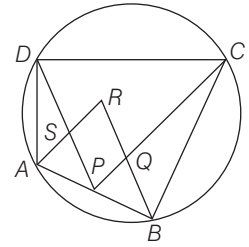
- (a) 16 (b) 7 (c) 18 (d) 24
- 22** ABC is an isosceles triangle and AC and BC are the tangents at M and N respectively. DE is the diameter of the circle. $\angle ADP = \angle BEQ = 100^\circ$. What is value of $\angle PRD$?
- (a) 60° (b) 50° (c) 20° (d) can't be determined
- 23** In the above question if OC is half of the AB , the value of $\angle ACB$ is :
- (a) 60° (b) 90° (c) 80° (d) can't be determined

- 24** In the adjoining figure, O is the centre of the circle with radius r . AB , CD and EF are the diameters of the circle. $\angle OAF = \angle OCB = 60^\circ$. What is the area of the shaded region?

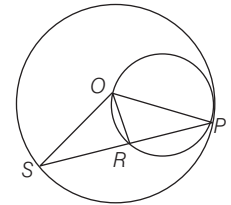


- (a) $\frac{r^2}{2} \left(\pi - \frac{3\sqrt{3}}{2} \right)$
(b) $\frac{r^2}{2} \left(\pi - \frac{3\sqrt{3}}{4} \right)$
(c) $\frac{r^2}{3} \left(\pi - \frac{2\sqrt{3}}{3} \right)$
(d) data insufficient

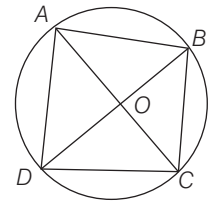
- 25** $ABCD$ is a cyclic quadrilateral. The angle bisector of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ intersect at P , Q , R and S as shown in the figure. These four points form a quadrilateral $PQRS$. Quadrilateral $PQRS$ is a :
- (a) square (b) rhombus
(c) rectangle (d) cyclic quadrilateral



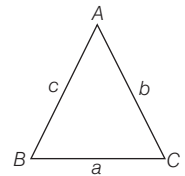
- 26** In the adjoining figure the diameter of the larger circle is 10 cm and the smaller circle touches internally the larger circle at P and passes through O , the centre of the larger circle. Chord SP cuts the smaller circle at R and OR is equal to 4 cm. What is the length of the chord SP ?



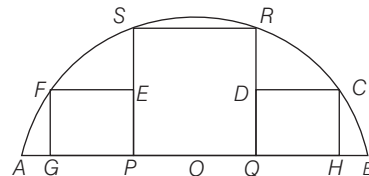
- (a) 9 cm (b) 12 cm (c) 6 cm (d) $8\sqrt{2}$ cm
- 27** In the given figure $ABCD$ is a cyclic quadrilateral, $DO = 8$ cm and $CO = 4$ cm. AC is the angle bisector of $\angle BAD$. The length of AD is equal to the length of AB . DB intersects diagonal AC at O , what is the length of the diagonal AC ?



- (a) 20 cm (b) 24 cm (c) 16 cm (d) none of these
- 28** In the given triangle ABC , the length of sides AB and AC is same (i.e., $b = c$) and $60^\circ < A < 90^\circ$, then the possible length of BC is :
- (a) $b < a < 2b$
(b) $\frac{c}{3} < a < 3a$
(c) $b < a < b\sqrt{3}$
(d) $c < a < c\sqrt{2}$

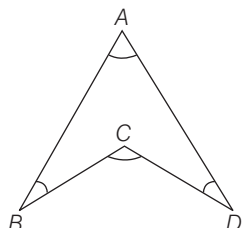


- 29** In the following figure there is a semicircle with centre 'O' and diameter $AB (= 2r)$. $PQRS$ is a square of maximum possible area. P and Q lie on the diameter AB and R, S lie on the arc of the semicircle. There are two more squares of maximum possible area $EFGP$ and $CDQH$. What is the sum of lengths of RC and FS ?

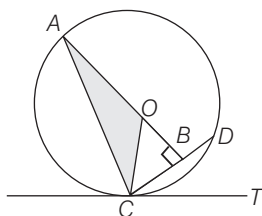


- (a) $\sqrt{\frac{2}{5}} (2r)$ (b) $\frac{2\sqrt{2}r}{5}$
(c) $\frac{\sqrt{2}}{5} r$ (d) none of these

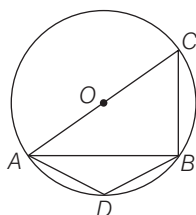
- 30 In the adjoining figure $\angle BAD = a$, $\angle ABC = b$ and $\angle BCD = c$ and $\angle ADC = d$, find the value of $\angle ABC$ in terms of a , c and d .



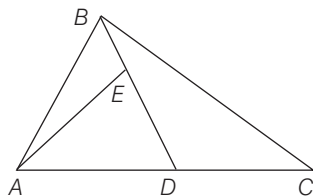
- (a) $c - (a + d)$ (b) $a - (c + d)$
 (c) $b - (c + d)$ (d) none of these
- 31 In the given diagram CT is tangent at C , making an angle of $\frac{\pi}{4}$ with CD . O is the centre of the circle. $CD = 10$ cm. What is the perimeter of the shaded region (ΔAOC) approximately?



- (a) 27 cm (b) 30 cm
 (c) 25 cm (d) 31 cm
- 32 In the given diagram, O is the centre of the circle and AC is the diameter. $\angle ADB$ is 120° . Radius of the circle is 6 cm, what is the area of the triangle ABC ?

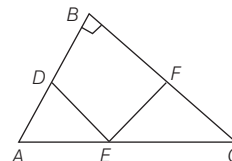


- (a) $18\sqrt{3} \text{ cm}^2$ (b) $24\sqrt{3} \text{ cm}^2$
 (c) 27 cm^2 (d) data insufficient
- 33 In the given figure ABC is a triangle in which $3AD = CD$ and E lies on BD , $DE = 2BE$. What is the ratio of area of ΔABE and area of ΔABC ?

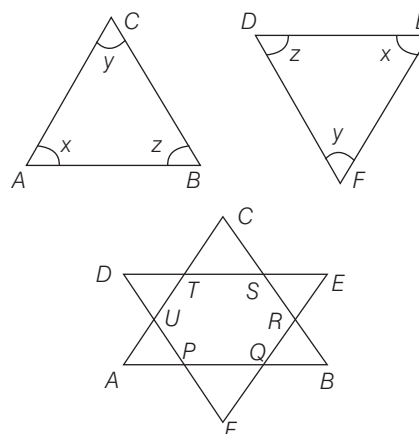


- (a) $1/12$ (b) $1/3$
 (c) $1/2$ (d) none of these

- 34 In the adjoining figure ABC is a right angle triangle, $BDEF$ is a square, $AE = 7.5$ cm and $AC = 18$ cm. What is the area of triangle ABC ?



- (a) 76.621 cm^2 (b) 70.054 cm^2
 (c) 83.25 cm^2 (d) 90.90 cm^2
- 35 There are two congruent triangles each with area 198 cm^2 . Triangle DEF is placed over triangle ABC in such a way that the centroids of both the triangles coincide with each other and $AB \parallel DE$, as shown in figure, thus forming a star. What is the area of the common region $PQRSTU$?



- (a) 99 cm^2 (b) 132 cm^2
 (c) 148 cm^2 (d) can't be determined
- 36 What is the sum of all the angles of a 9 pointed star (i.e., $\angle 1 + \angle 2 + \angle 3 + \dots + \angle 8 + \angle 9$)?
- (a) 909° (b) 900°
 (c) 720° (d) 540°
- 37 A circle is circumscribed by a rhombus which in turn is produced by joining the mid-points of a rectangle whose sides are 12 cm and 16 cm, respectively. What is the area of the circle?
- (a) $\frac{625}{26} \pi$ (b) $\frac{676}{25} \pi$
 (c) $\frac{576}{25} \pi$ (d) can't be determined
- 38 There are n rectangles each with area 200 cm^2 . If the dimensions of each n rectangles are in integers then the value of n is :
- (a) 4 (b) 6
 (c) 12 (d) none of these

- 39** There are 8 points on a plane, out of which 4 points lie on the circumference of the same circle and rest 4 points do not lie on a single circumference of a circle and also they are non-collinear. Maximum how many circles can be drawn such that each contains at least three of the given points?

(a) 53 (b) 32 (c) 35 (d) 56

- 40** ABC is an isosceles triangle a circle is such that it passes through vertex C and AB acts as a tangent at D for the same circle. AC and BC intersects the circle at E and F respectively $AC = BC = 4$ cm and $AB = 6$ cm. Also D is the mid-point of AB . What is the ratio of $EC : (AE + AD)$?

(a) 1 : 2 (b) 1 : 3
(c) 2 : 5 (d) none of these

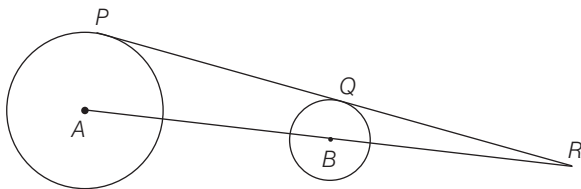
- 41** A smaller circle touches internally to a larger circle at A and passes through the centre of the larger circle. O is the centre of the larger circle and BA, OA are the diameters of the larger and smaller circles, respectively. Chord AC intersects the smaller circle at a point D . If $AC = 12$ cm, then AD is :

(a) 4 cm (b) 6 cm
(c) 5.6 cm (d) data insufficient

- 42** In the given figure $ADEC$ is a cyclic quadrilateral, CE and AD are extended to meet at B . $\angle CAD = 60^\circ$ and $\angle CBA = 30^\circ$. $BD = 6$ cm and $CE = 5\sqrt{3}$ cm, What is the ratio of $AC : AD$?

(a) $\frac{3}{4}$
(b) $\frac{4}{5}$
(c) $\frac{2\sqrt{3}}{5}$
(d) can't be determined

Directions (for Q. Nos. 43 and 44) In the following diagram A and B are the centres of the two different circles. PQR is a common tangent. Points A, B , and R lie on the straight line. Distance between A and B is 25 cm and the distance between P and Q is 24 cm. Diameter of the larger circle is 24 cm.



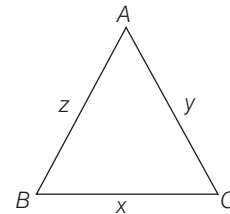
- 43** What is the ratio of $AB : BR$?

(a) 7 : 5
(b) 7 : 6
(c) 7 : 10
(d) data insufficient

- 44** What is the ratio of area of $\triangle APR$ and $\triangle BQR$?

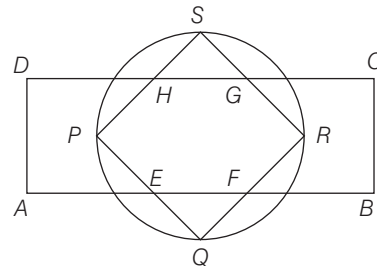
(a) 169 : 36
(b) 144 : 25
(c) 625 : 144
(d) can't be determined

- 45** If $x^2 + y^2 + z^2 = xy + yz + zx$, then the triangle is :



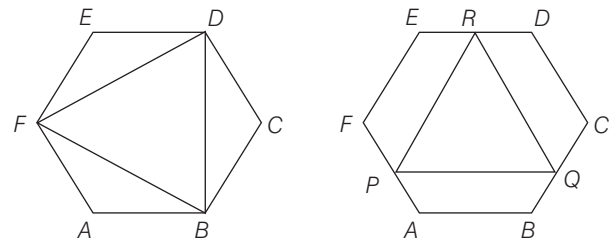
(a) isosceles (b) right angled
(c) equilateral (d) scalene

- 46** In the adjoining figure $ABCD$ is a rectangle in which length is twice of breadth. H and G divide the line CD into three equal parts. Similarly points E and F trisect the line AB . A circle $PQRS$ is circumscribed by a square $PQRS$ which passes through the points E, F, G and H . What is the ratio of areas of circle to that of rectangle?



(a) $3\pi : 7$ (b) 3 : 4
(c) $25\pi : 72$ (d) $32\pi : 115$

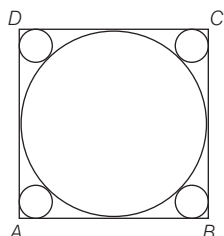
- 47** In the adjoining figure there are two congruent regular hexagons each with side 6 cm.



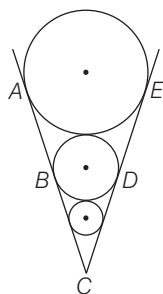
What is the ratio of area of $\triangle BDF$ and $\triangle PQR$, if P, Q and R are the mid-points of side AF, BC and DE ?

(a) 6 : 5 (b) 7 : 6
(c) 4 : 3 (d) 1 : 1

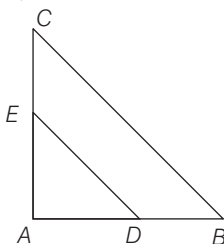
- 48 $ABCD$ is a square, in which a circle is inscribed touching all the sides of a square. In the four corners of square 4 smaller circles of equal radii are drawn, containing maximum possible area. What is the ratio of the area of larger circle to that of sum of the areas of four smaller circles ?



- (a) $1 : (68 - 48\sqrt{2})$ (b) $1 : 17\sqrt{2}$
 (c) $3 : (34 - 12\sqrt{2})$ (d) none of these
- 49 In the adjoining figure $\angle ACE$ is a right angle. There are three circles which just touch each other where AC and EC are the tangents to all the three circles. What is the ratio of radii of the largest circle to that of the smallest circle?

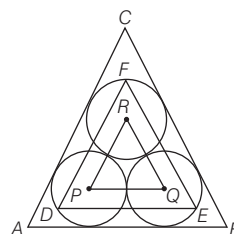


- (a) $17 : 12\sqrt{2}$ (b) $1 : (17 - 12\sqrt{2})$
 (c) $12 : 17\sqrt{2}$ (d) none of these
- 50 In a right angle triangle ABC , $\angle A$ is right angle, DE is parallel to the hypotenuse BC and the length of DE is 65% the length of BC , what is the area of $\triangle ADE$, if the area of $\triangle ABC$ is 68 cm^2 ?

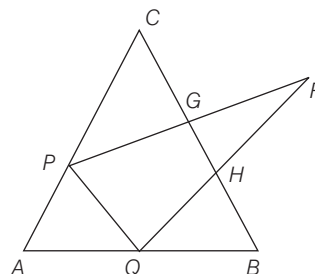


- (a) 27.83 cm^2 (b) 41.6 cm^2
 (c) 28.73 cm^2 (d) none of these
- 51 In the adjoining figure three congruent circles are touching each other. Triangle ABC circumscribes all the three circles. Triangle PQR is formed by joining the centres of the circle. There is a third triangle DEF . Points A, D, P and B, E, Q and C, F, R lie in the same straight lines respectively.

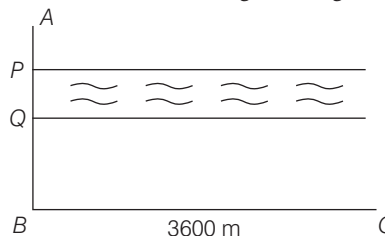
What is the ratio of perimeters of $\triangle ABC : \triangle DEF : \triangle PQR$?



- (a) $3\sqrt{2} : 2\sqrt{2} : 1$
 (b) $2(4 + \sqrt{3}) : (2 + \sqrt{3}) : \sqrt{3}$
 (c) $2(1 + \sqrt{3}) : (2 + \sqrt{3}) : 2$
 (d) $2(1 + \sqrt{3}) : 2\sqrt{3} : \sqrt{3}$
- 52 In the given figure, P and Q are the mid-points of AC and AB . Also, $PG = GR$ and $HQ = HR$. What is the ratio of area of $\triangle PQR$: area of $\triangle ABC$?

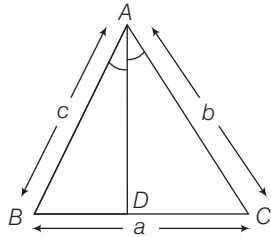


- (a) $1/2$ (b) $2/3$
 (c) $3/5$ (d) none of these
- 53 A trapezium $PQRS$ inscribes a circle which touches the circle at M, A, N and B . Radius of circle is 10 cm. The length of each non-parallel side is 21 cm. What is the perimeter of the trapezium?
 (a) 82 cm (b) 84 cm
 (c) 85.5 cm (d) can't be determined
- 54 In the given diagram, river PQ is just perpendicular to the national highway AB . At a point B highway just turns at right angle and reaches to C . $PA = 500 \text{ m}$ and $BQ = 700 \text{ m}$ and width of the uniformly wide river (i.e., PQ) is 300m. Also $BC = 3600 \text{ m}$. A bridge has to be constructed across the river perpendicular to its stream in such a way that a person can reach from A to C via bridge covering least possible distance. What is the minimum possible required distance from A to C including the length of the bridge?

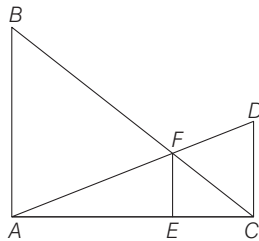


- (a) 4100 m (b) 3900 m
 (c) $3000\sqrt{2} \text{ m}$ (d) none of these

- 55 In a triangle ABC , AD is the angle bisector of $\angle BAC$ and $\angle BAD = 60^\circ$. What is the length of AD ?



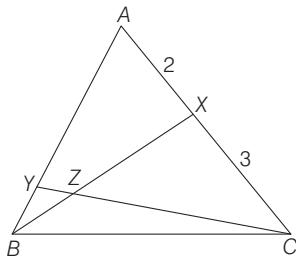
- (a) $\frac{b+c}{bc}$ (b) $\frac{bc}{b+c}$ (c) $\sqrt{b^2 + c^2}$ (d) $\frac{(b+c)^2}{bc}$
- 56 The top of the two parallel towers AB and CD can be accessed through two ladders $BC = 210$ m and $AD = 174$ m such that the foot of each ladder touches the foot of the other building. If there is a tree $EF = 70$ m somewhere between the points A and C , such that the peak of the tree F is the point of cross section of the two ladders, wherein points A, E and C are collinear, find the horizontal distance AC between the two towers.



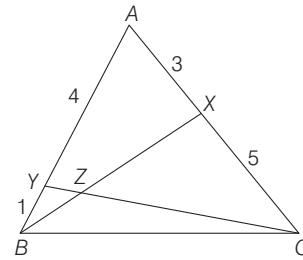
- (a) 125 (b) 126 (c) 127 (d) 135

Directions (for Q. Nos. 57 and 58) Answer the following questions based on the information given below.

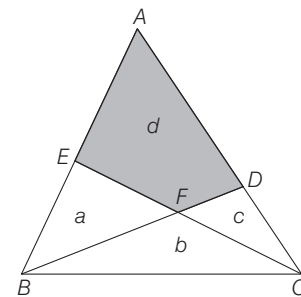
In the following triangle $\triangle ABC$, $AX : XC = 2 : 3$ and the weight at the point Z is 9.



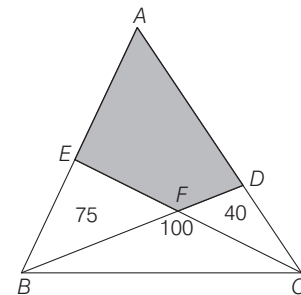
- 57 Find the weight at point B .
 (a) 1 (b) 3 (c) 4 (d) 5
- 58 Find the value of $AY : BY$.
 (a) 5 : 2 (b) 4 : 3 (c) 5 : 2 (d) 3 : 2
- 59 In the following triangle $\triangle ABC$, $AY : BY = 4 : 1$ and $AX : CX = 3 : 5$, where X lies on AC and Y lies on AB and Z is the point of intersection of BX and CY . Find the value of $BZ : CZ$.



- (a) 8 : 25 (b) 9 : 25
 (c) 9 : 64 (d) none of these
- 60 In a triangle ABC point D and E lie on the sides AB and AC , respectively. Line segments DC and BE intersect inside the triangle at O . The area of $\triangle BOC = 8$ sq. cm, $\triangle BDO = 7$ sq. cm and $\triangle CEO = 4$ sq. cm. Find the area of quadrilateral $ADOE$.
 (a) 21 sq. cm (b) 28 sq. cm
 (c) 25 sq. cm (d) 19 sq. cm
- 61 In a triangle ABC , the vertex B is connected to a point D on AC and the vertex C is connected to a point E on AB . The line segments BD and EC intersect at a point F inside the $\triangle ABC$, as shown in the diagram. If the area of $\triangle BEF$, $\triangle BCF$ and $\triangle CDF$ are a , b and c respectively. What is the area of quadrilateral $AEFD$ in terms of a , b , c ?

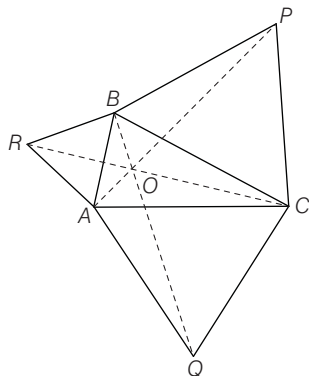


- (a) $\frac{ac(a+2b+c)}{b^2-ac}$ (b) $\frac{2b(a+2b+c)}{b^2-ac}$
 (c) $\frac{2b(a+b+c)}{b^2-ac}$ (d) $\frac{abc(a+b+c)}{b^2-ac}$
- 62 In the following triangle ABC , the area of smaller $\triangle BEF$ is 75 units, $\triangle BCF$ is 100 units and $\triangle FDC$ is 40 units. Find the area of quadrilateral $AEFD$.



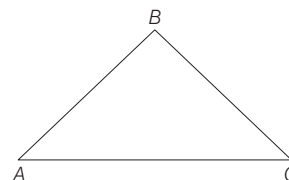
- (a) 125 (b) 135 (c) 145 (d) 90

- 63** Consider a triangle ABC and let a, b and c denote the lengths of the sides opposite the vertices A, B and C respectively, where $a = 6, b = 10$ and the area of the triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and r denotes the radius of the incircle of the triangle, then r^2 is equal to
(a) $\sqrt{3}$ (b) $2\sqrt{3}$ (c) 3 (d) 4.5
- 64** Given an isosceles triangle, whose one angle is 120° and radius of its incircle (r) = $\sqrt{3}$. Then the area of the triangle in sq. units is
(a) $7 + 12\sqrt{3}$ (b) $12 - 7\sqrt{3}$ (c) $12 + 7\sqrt{3}$ (d) 4π
- 65** Let ABC be an arbitrary triangle, and P be any point inside. Let d_1, d_2 and d_3 denote the perpendicular distances from P to the sides BC, CA and AB , respectively. Let h_1, h_2 and h_3 denote, respectively, the length of the altitude from A, B and C to the opposite sides of the triangle, then $\frac{d_1}{h_1} + \frac{d_2}{h_2} + \frac{d_3}{h_3}$ is
(a) $\sqrt{3}$ (b) 3 (c) 1 (d) π
- 66** On each side of an arbitrary triangle ABC , an equilateral triangle is constructed (protruding outside) as shown below. These equilateral triangles are ABR, BCP and ACQ . Then, which one of the following is CORRECT?

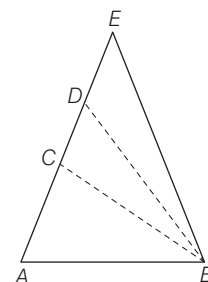


- (a) $\overline{AP} + \overline{BQ} + \overline{CR} = 2(\overline{AB} + \overline{BC} + \overline{CA})$
 (b) $\overline{AP} = \overline{BQ} = \overline{CR}$
 (c) Both (a) and (b)
 (d) none of the above
- 67** In a $\triangle ABC$, the points X, Y and Z are on the lines BC, CA and AB . P is a point inside the triangle such that AX, BY and CZ are concurrent at P if and only if
 (a) $\left(\frac{AZ}{BZ}\right)\left(\frac{BX}{CX}\right)\left(\frac{CY}{AY}\right) = 1$ (b) $\left(\frac{AZ}{AY}\right)\left(\frac{BZ}{BX}\right)\left(\frac{CY}{CX}\right) = 1$
 (c) $\left(\frac{AY}{AZ}\right)\left(\frac{BZ}{BX}\right)\left(\frac{CY}{CX}\right) = 1$ (d) none of these
- 68** In an isosceles right angle triangle the legs AB and BC have length $l = \sqrt{a + b\sqrt{2}}$; where a and b are natural numbers. Point O is inside the triangle such that $OA = 1, OB = 2$ and $OC = 3$. Find the value of $\frac{a}{b}$?
 (a) 2.5 (b) 2.33
 (c) 2.66 (d) none of these

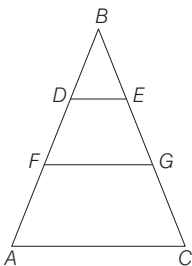
- 69** In a triangle PQR , S and T lie on PQ and PR , respectively, such that $\angle QSR = \angle PRQ$ and $\angle QTR = \angle PQR$. And, $PQ = 12, QR = 9, PR = 15$. Find the value of $PS \times PT$.
 (a) 50.4 (b) 52.64
 (c) 49.75 (d) none of these
- 70** In a $\triangle ABC$, AP is the median that intersects QR at O , where Q and R lie on the AB and AC , respectively. $2AQ = 3BQ$ and $3AR = 4CR$. Find the ratio of area of $\triangle QOP$ to that of $\triangle ROP$.
 (a) 21 : 20 (b) 9 : 8 (c) 2 : 1 (d) 16 : 15
- 71** If $\phi = \frac{1 + \sqrt{5}}{2}$ and the three sides of a triangle are 1, $\sqrt{\phi}$, ϕ , which one of the following statements is true about this triangle?
 (a) no such triangle exists (b) it's a right angle triangle
 (c) it's an isosceles triangle (d) nothing can be said
- 72** In an isosceles triangle ABC , the acute angle BAC is 36° . Point D lies on $\overline{AC}$ such that $\overline{BD}$ is the angle bisector of $\angle ABC$. Find $\frac{\overline{AB}}{\overline{BC}}$.
 (a) $\sqrt{5} : 2$ (b) $\frac{\sqrt{5} - 1}{2}$
 (c) $\frac{1 + \sqrt{5}}{2}$ (d) none of these
- 73** In the following diagram, $\triangle ABC$ is an isosceles triangle whereas $\angle B = 72^\circ$ and $AB = BC = 1$ unit. If $\phi = \frac{1 + \sqrt{5}}{2}$, which one of following choices is correct for the value of AC in terms of ϕ .
 (i) $\sqrt{3 - \phi}$ (ii) $\frac{\sqrt{\phi} - 1}{2}$
 (iii) $\sqrt{4 - \phi^2}$



- (a) all (i), (ii) and (iii) (b) none of (i), (ii) and (iii)
 (c) not all but more than one
 (d) data insufficient
- 74** In the following diagram $\triangle AEB$ is an isosceles triangle, whereas $\angle AEB = 36^\circ, \angle ADB = 54^\circ, \angle ACB = 72^\circ$. If $\phi = \frac{1 + \sqrt{5}}{2}$ and $AB = \phi$, find the ratio of $AC : CD : DE$.
 (a) $1 : \sqrt{\phi} : 1$
 (b) $1 : (\phi - 1) : 2$
 (c) $1 : \phi : \phi^2$
 (d) $\phi : 1 : \phi$

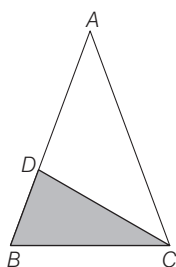


- 75** In an isosceles triangle ABC , line segments DE and FG are parallel to the base AC intersecting the other two sides AB and BC . Also given that $BD = BE = \phi$ and $\angle ABC = 36^\circ$. If $DE = EG$ and $FG = GC$, find AC in terms of ϕ , where $\phi = \frac{1 + \sqrt{5}}{2}$.



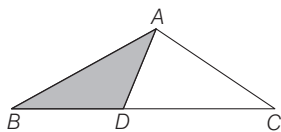
- (a) $(\phi + 1)^2$ (b) $\phi^2 + 1$
(c) $\phi^2 - 1$ (d) $\phi + 1$

- 76** In the following figure, $AB = AC$, $BC = CD$ and $\angle BAC = 36^\circ$. If $AD = 1 + \sqrt{5}$ cm, find the area of the shaded region (in sq. cm).



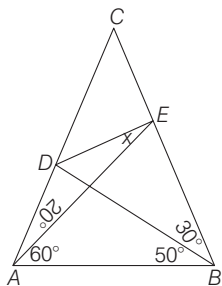
- (a) $\sqrt{5 + 2\sqrt{5}}$ (b) $2\sqrt{3 + \sqrt{5}}$
(c) $2\sqrt{7 - 2\sqrt{5}}$ (d) none of these

- 77** In the following figure $\triangle BAC$ and $\triangle BAD$ are isosceles triangles, such that $AB = AC$ and $AD = BD$. If $\angle ABC = 36^\circ$ and $BD = (\sqrt{5} - 1)$ cm, find the area of the $\triangle ABD$ (in sq. cm).



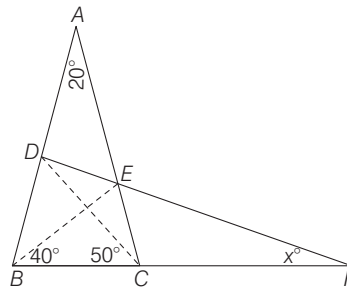
- (a) $\sqrt{5 - 2\sqrt{5}}$ (b) $2\sqrt{3 - \sqrt{5}}$
(c) $\frac{1}{2}\sqrt{7 - 2\sqrt{5}}$ (d) none of these

- 78** In the following diagram find the value of $\angle DEA$. Please note that the diagram is not drawn to scale. However, the measure of angles stated there are absolutely correct.



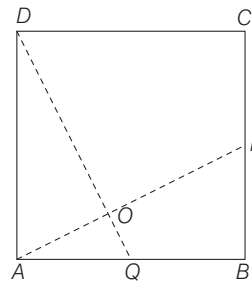
- (a) 10 (b) 20
(c) 30 (d) data insufficient

- 79** In an isosceles triangle $\triangle ABC$, $\angle A = 20^\circ$ and $AB = AC \neq BC$. Line segment BC is extended to a point F such that point D , point E and point F are collinear, whereas point D lies on AB and point E lies on AC . If $\angle EBC = 40^\circ$ and $\angle DCB = 50^\circ$, find the value of x as shown in the diagram.



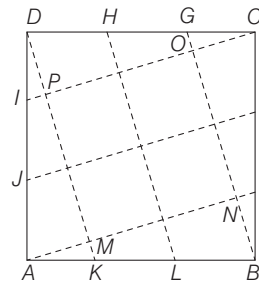
- (a) 30 (b) 20 (c) 15 (d) 10

- 80** In a square $ABCD$, a quadrilateral, P is the midpoint at BC and Q is the midpoint at AB . Line segments AP and DQ intersect each other at O . Find the ratio of the areas of two quadrilaterals $QBPO$ and $OPCD$.



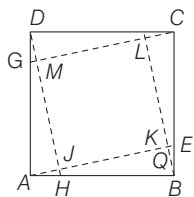
- (a) 9 : 25 (b) 4 : 9
(c) 4 : 11 (d) 16 : 25

- 81** In the following diagram, each side of the square $ABCD$ is trisected such that $DK \parallel HL \parallel GB$ and $IC \parallel JF \parallel AE$, where all the parallel lines are equispaced. If the area of the smallest triangle such as $\triangle DIP$ is 1 unit, what is the area of quadrilateral $MNOP$?

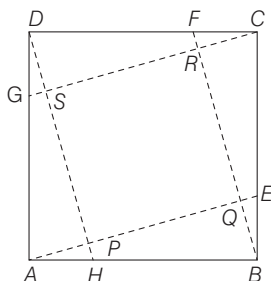


- (a) 21 units (b) 24 units
(c) 27 units (d) 32 units

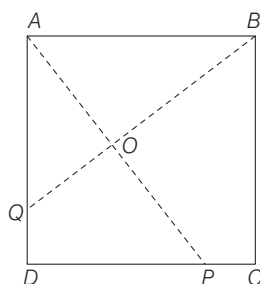
- 82 Each side of a square $ABCD$ is divided in the ratio $1 : 4$, as shown in the following diagram. What is the ratio of area of square $JKLM$ to that of square $ABCD$?



- (a) $8 : 13$ (b) $16 : 25$
(c) $8 : 15$ (d) $11 : 20$
- 83 In the following figure, each side of the square is divided in the ratio $2 : 3$, such that $\frac{DG}{GA} = \frac{AH}{HB} = \frac{BE}{EC} = \frac{CF}{FD} = \frac{2}{3}$. Find the ratio of the area of square $PQRS$ to that of square $ABCD$.

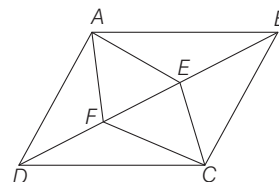


- (a) $4 : 9$ (b) $9 : 25$
(c) $9 : 29$ (d) $32 : 73$
- 84 In the given square $ABCD$, $\frac{AQ}{QD} = \frac{DP}{PC} = \frac{4}{1}$. Find the ratio of area of triangle AOQ to that of quadrilateral $OBCP$.



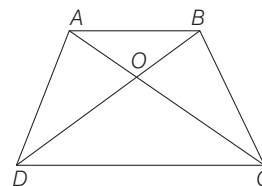
- (a) $16 : 25$ (b) $16 : 41$
(c) $24 : 75$ (d) $32 : 73$
- 85 A quadrilateral has one vertex on each side of a square of side-length 1. If the lengths of the quadrilateral are a, b, c, d , then
- (a) $2 \leq a^2 + b^2 + c^2 + d^2 \leq 4$
(b) $1 \leq a^2 + b^2 + c^2 + d^2 \leq 4$
(c) $\sqrt{2} \leq a^2 + b^2 + c^2 + d^2 \leq 4$
(d) $2\sqrt{2} \leq a^2 + b^2 + c^2 + d^2 \leq 4$

- 86 Consider the following diagram in which $\square ABCD$ is a rhombus and $AF = EF = EC$. A circle is inscribed in quadrilateral $AECF$ and a circle is inscribed in quadrilateral $ABCD$. Each of the circles is of the largest possible area.

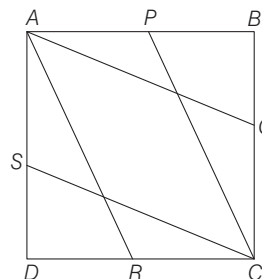


Find the ratio of the area of circle that lies in $\square AECF$ to the area of the circle inscribed in $\square ABCD$.

- (a) $1 : 2$ (b) $2 : 3$
(c) $1 : 4$ (d) data insufficient
- 87 Area of the trapezium $ABCD$ is 350 sq. cm and $AB = 21$ cm, $CD = 49$ cm. The two diagonals bisect each other at O , find the area of $\triangle AOD$.



- (a) 75 sq. cm (b) 73.5 sq. cm
(c) 72.5 sq. cm (d) data insufficient
- 88 Let $ABCD$ be a quadrilateral with area 18, with side AB parallel to the side CD and $AB = 2CD$. Let AD be perpendicular to AB and CD . If a circle is drawn inside the quadrilateral $ABCD$ touching all the four sides, then its radius is :
- (a) 3 (b) 2
(c) $\frac{3}{2}$ (d) 1
- 89 In the following diagram $ABCD$ is a square. The mid-points P, Q, R, S are connected in such a way that two congruent kites $SAPC$ and $CRAQ$ are formed. Area of each kite is 200 sq. cm. Find the area of common region contained by the two kites.



- (a) 100 sq. cm (b) 96 sq. cm
(c) $\frac{400}{3}$ sq. cm (d) none of these

- 90** Each of the sides of a quadrilateral is a natural number and the perimeter of the quadrilateral is always 12 units. Find the number of combinations of various lengths of the sides of the quadrilateral.

(a) 8 (b) 10
(c) 5 (d) none of these

- 91** If the side of a regular pentagon be 2 cm, find the circumradius of the pentagon, in cm.

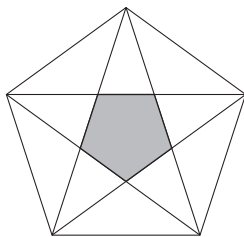
(a) $\frac{3 + \sqrt{5}}{\sqrt{5 + 2\sqrt{5}}}$ (b) $\frac{3 + \sqrt{5}}{\sqrt{10 - 2\sqrt{5}}}$
(c) $\frac{3 - \sqrt{5}}{\sqrt{10 - 2\sqrt{5}}}$ (d) $\frac{3 + 2\sqrt{5}}{\sqrt{5 + 2\sqrt{5}}}$

- 92** In a regular pentagon the side measures 6 cm and sum of the inradius and circumradius is 9 cm. Find the sum of the area of the circumcircle and the area of the incircle of the pentagon.

(a) $41\pi \text{ cm}^2$ (b) $45\pi \text{ cm}^2$
(c) $53\pi \text{ cm}^2$ (d) data insufficient

Directions (for Q. Nos. 93 and 94) *Answer the following questions based on the information given below.*

In the following diagram the larger pentagon constitutes a smaller pentagon at the centre. Each side of the larger pentagon is known to be S .



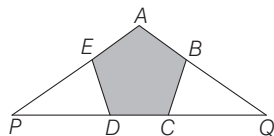
- 93** What would be the side of the smaller pentagon?

(a) $(6 - 2\sqrt{5})S$ (b) $\frac{(3 - \sqrt{5})}{2}S$
(c) $\frac{(5 - 2\sqrt{5})}{2}S$ (d) none of these

- 94** What would be the diagonal of the smaller pentagon?

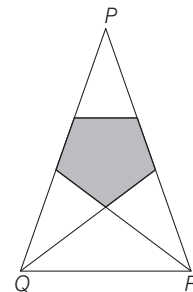
(a) $(7 - 2\sqrt{5})S$ (b) $\frac{3(\sqrt{5} - 2)}{2}S$
(c) $\frac{(\sqrt{5} - 1)}{2}S$ (d) none of these

- 95** A few of the sides of a regular pentagon are extended in such a way that it forms an isosceles triangle as shown in the given diagram. If the perimeter of the pentagon is 10 cm, what is the perimeter of the isosceles triangle APQ ?



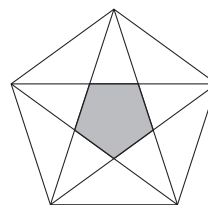
(a) $10 + 4\sqrt{5} \text{ cm}$ (b) $12 + 5\sqrt{5} \text{ cm}$
(c) 16 cm (d) $14 + 4\sqrt{5} \text{ cm}$

- 96** A few of the sides of a regular pentagon are extended in such a way that it forms an isosceles triangle as shown in the given diagram. If the perimeter of the pentagon is 10 cm, what is the perimeter of the isosceles triangle PQR ?



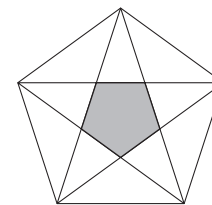
(a) $7 + 5\sqrt{5} \text{ cm}$
(b) $11 + 5\sqrt{5} \text{ cm}$
(c) 16 cm
(d) $16 + 3\sqrt{5} \text{ cm}$

- 97** In the following figure, if the area of the shaded region (the smaller pentagon) is $\left(\frac{7 - 3\sqrt{5}}{2}\right) \text{ sq. cm}$, find the area of the larger regular pentagon.



(a) 1 sq. cm (b) $\left(\frac{47 - 21\sqrt{5}}{2}\right) \text{ sq. cm}$
(c) $\left(\frac{7 + 3\sqrt{5}}{2}\right) \text{ sq. cm}$ (d) none of these

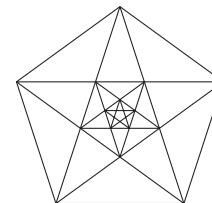
- 98** In the following figure when all the five diagonals of a regular pentagon are drawn they form another small pentagon at the centre which is shown as a shaded region. If each side of the larger (original) pentagon measures



1 inch, and $\phi = \frac{\sqrt{5} + 1}{2}$, find the ratio of the area of larger pentagon to the area of smaller pentagon in terms of ϕ .

(a) $\phi^2 : 1$ (b) $\phi^4 : 1$
(c) $1 : \sqrt{\phi}$ (d) $2 : \sqrt{\phi}$

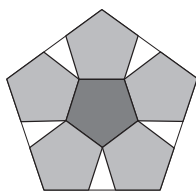
- 99** In the given figure the diagonals of a pentagon form another pentagon, and the diagonals of this pentagon form another pentagon, and then once again the diagonals of this pentagon form another pentagon at the centre. If the each side of the smallest pentagon is 1 unit and



$\phi = \frac{1 + \sqrt{5}}{2}$, find each side of the largest (original) pentagon in terms of ϕ .

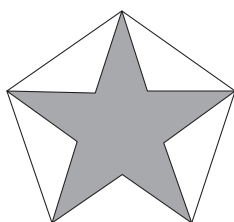
(a) $4\phi^4$ (b) ϕ^4
(c) ϕ^8 (d) ϕ^6

- 100** A larger regular pentagon contains 6 smaller pentagons of the same size, as shown in the following diagram. If each side of each of the congruent pentagons is 1 cm, find the total area A (in sq. cm) covered by the larger pentagon.



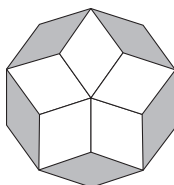
- (a) $10 < A < 11$ (b) $11 < A < 12$
(c) $12 < A < 13$ (d) $13 < A < 14$

- 101** In the following figure a regular pentagon inscribes a regular pentagram. If each side of the pentagon measures 1 mm, and $\phi = \frac{\sqrt{5} + 1}{2}$, find the area of pentagram in terms of ϕ .



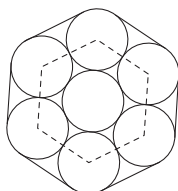
- (a) $\frac{1}{2}(2 - \phi)^{3/2}$ sq. mm (b) $\frac{1}{4}(2 - \phi)^{3/2}$ sq. mm
(c) $\frac{1}{2}(3 - \phi)^{3/2}$ sq. mm (d) $\frac{3}{2}(3 - \phi)^{3/2}$ sq. mm

- 102** In the following diagram a regular decagon consists of 10 parallelograms, out of which 5 parallelograms that are meeting at the centre are congruent to each other and other 5 parallelograms at the edges of the decagon are congruent to each other. If each side of the decagon is $1 + \sqrt{5}$ cm, find total area of the 5 parallelograms that are connected to the centre of the decagon.



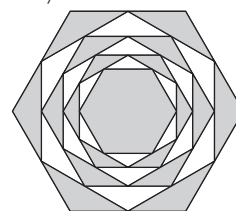
- (a) $5(1 + \sqrt{5})(\sqrt{25 - 2\sqrt{5}})$ sq. cm
(b) $15(\sqrt{25 + 4\sqrt{5}})$ sq. cm
(c) $5(1 + \sqrt{5})(\sqrt{5 + 2\sqrt{5}})$ sq. cm
(d) $5(6 + 2\sqrt{5})(\sqrt{1 + \sqrt{5}})$ sq. cm

- 103** Take seven circles and close-pack them together in a hexagonal arrangement, as shown below. Then a rubber band is wrapped around the seven circles. Find the perimeter of this arrangement, if the radius of each circle is r cm.



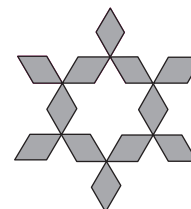
- (a) $2(6 + \pi)r$ (b) $3(4 + \pi)r$
(c) $6(2 + \pi)r$ (d) none of these

- 104** In the given diagram, there are total seven regular hexagons. Let's call the outermost (or the largest) hexagon as H_1 and so call the innermost (or the smallest) hexagon as H_7 . The hexagon H_2 is formed by joining the mid-points of the adjacent sides of H_1 . Similarly, H_3, H_4, H_5, H_6 and H_7 are formed by joining the mid-point of the adjacent sides of the H_2, H_3, H_4, H_5 and H_6 , respectively. If each side of the H_1 is 64 cm, find the each side of the H_7 .



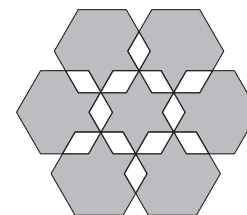
- (a) 27 cm (b) $9\sqrt{3}$ cm
(c) $18\sqrt{3}$ cm (d) none of these

- 105** In the given figure there are twelve rhombi placed in such a way that they form a regular hexagram at the centre and this hexagram is inscribed within a regular hexagon. If the area of the hexagon is 84 sq. cm, find the total area of all the twelve rhombi.



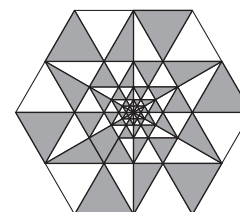
- (a) 56 sq. cm (b) 72 sq. cm
(c) 104 sq. cm (d) none of these

- 106** As shown in the given figure, there are total seven regular hexagons; six of them are placed around the seventh hexagon, partially overlapping, and then some rhombi are cut out of the whole arrangement. If the area of the unshaded region represented by the 12 rhombi is 84 sq. cm, find the area of the shaded region.



- (a) 420 sq. cm
(b) 484 sq. cm
(c) 488 sq. cm
(d) none of the above

- 107** If the area of all the shaded equilateral triangles of the largest possible size, as shown along the periphery of the regular hexagon, is $6\sqrt{3}$ sq cm, then find the total area of the shaded region of the whole figure.



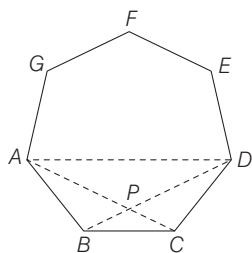
- (a) $18\sqrt{3}$ sq. cm
(b) 36 sq. cm
(c) $24\sqrt{3}$ sq. cm
(d) none of the above

- 108** There are seven concentric regular hexagons depicted in the adjacent figure such that they form a six alternating black and white hexagonal rings around a black hexagon. If the area of each hexagonal ring is same and equal to the black hexagon at the centre, find the ratio of the sides of the largest hexagon to that of the smallest hexagon of this arrangement.



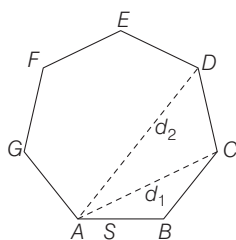
- (a) 7 : 1 (b) $1 : \sqrt{7}$
(c) 6 : 1 (d) none of these

- 109** In the following diagram, if the two shorter diagonals AC and BD intersect at P inside the regular heptagon and AD is one of the longer diagonals, which one of the following is correct?



- (a) $2(AB + AP) = 3AD$ (b) $AB + BP = AD$
(c) $AB + BD = AD + BP$ (d) $3(AB + AP) = 2AD$

- 110** In the following diagram, d_1, d_2 are the two distinct diagonals and s is the side of the regular polygon $ABCDEF$. Find the correct relation among the side and the two diagonals.

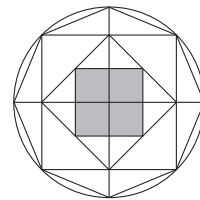


- (a) $\frac{d_2}{s} = \frac{s}{d_1}$ (b) $\frac{1}{s} = \frac{1}{d_2} - \frac{1}{d_1}$
(c) $\frac{d_2}{d_1} = \frac{d_1}{s}$ (d) $\frac{1}{s} = \frac{1}{d_1} + \frac{1}{d_2}$

- 111** Neither it's the longest one nor it's the shortest one of the lengths of the diagonal of an octagon you know is d and each side of the same octagon is s , find the area of the octagon in terms of the side and the known diagonal of the regular octagon.

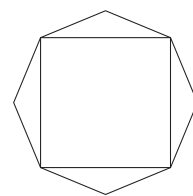
- (a) $3d^2 - \sqrt{3}s^2$
(b) $d^2 - s^2$
(c) $2ds$
(d) both (b) and (c)

- 112** In the given diagram, a regular octagon is inscribed in a circle in which four diagonals are connecting the opposite vertices of the octagon. After that a square is formed by connecting the alternate vertices of the octagon then another square is formed by joining the mid-points of the first square and then finally another square is formed by joining the mid-points of the previous square. This square is shown as the shaded region. If the area of the shaded region is 1 sq. cm, find the difference between the area of the circle and area of the octagon.



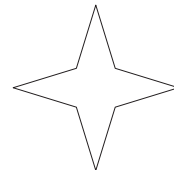
- (a) $2(\pi - 2\sqrt{2})$ sq. cm (b) $(2\pi - 3\sqrt{2})$ sq. cm
(c) $(3\pi - 4\sqrt{2})$ sq. cm (d) $4(\pi - 2\sqrt{2})$ sq. cm

- 113** In the given figure a quadrilateral is formed by joining the alternate vertices of a regular octagon. If the area of the octagon be $2(\sqrt{2} - 1)$ sq cm, find the area of the square.



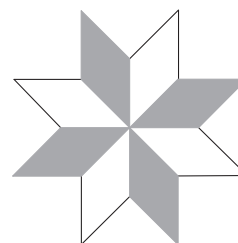
- (a) $\frac{2 + \sqrt{2}}{3 - 2\sqrt{2}}$ (b) $\frac{\sqrt{2 + \sqrt{2}}}{\sqrt{2} + 1}$
(c) $3 - 2\sqrt{2}$ (d) $\sqrt{2}(\sqrt{2} - 1)$

- 114** In the given figure, a concave equilateral octagon has each of its four acute internal angles equal to 30° and each of the two longer diagonals is 20 cm. Find the area of the octagon, if the diagonals connecting the opposite vertices are perpendicular and bisect each other.



- (a) $\frac{200}{\sqrt{3}}(\sqrt{3} - 1)$ (b) $\frac{100}{\sqrt{3}}(\sqrt{3} - 1)$
(c) $\frac{200}{\sqrt{3}}(\sqrt{3} - 2)$ (d) $\frac{200}{\sqrt{3}}(3 - \sqrt{3})$

- 115** In the given figure, there are 4 black and four white rhombi connected in such a way that one of their vertices is common and they share two of their sides with the rhombi of the either side without any overlapping. Each of the eight rhombi is congruent to each other. The total area of all the black rhombi (or shaded region) is 8 sq cm, find the area of the smallest possible octagon in which the following figure can be inscribed as it is.

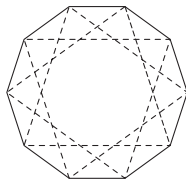


- (a) $8(2 + \sqrt{2})$ cm² (b) $8(\sqrt{4 - \sqrt{2}})$ cm²
(c) $8(4 - \sqrt{2})$ cm² (d) $8(\sqrt{2 + \sqrt{2}})$ cm²

Directions (for Q. Nos. 116 to 121) *Answer the following questions based on the information given below.*

The side of a regular decagon is 2 cm.

- 116** Find the circumradius of the decagon.
 (a) $1 + \sqrt{5}$ cm (b) $-1 + \sqrt{5}$ cm
 (c) $2 + \sqrt{3}$ cm (d) none of these
- 117** Find the apothem of the decagon.
 (a) $\sqrt{5 + 2\sqrt{5}}$ cm (b) $2\sqrt{5 - \sqrt{5}}$ cm
 (c) $2\sqrt{5 + \sqrt{5}}$ cm (d) $\sqrt{1 + \sqrt{5}}$ cm
- 118** Find the longest possible diagonal of the decagon.
 (a) $4(1 + \sqrt{5})$ cm (b) $2(1 - \sqrt{5})$ cm
 (c) $2(2 + \sqrt{5})$ cm (d) $2(1 + \sqrt{5})$ cm
- 119** Find the second longest diagonal of the decagon.
 (a) $\sqrt{5 + 2\sqrt{5}}$ cm (b) $2\sqrt{5 + 2\sqrt{5}}$ cm
 (c) $\sqrt{4 + 2\sqrt{5}}$ cm (d) $\sqrt{10 + 4\sqrt{5}}$ cm
- 120** Find the third longest diagonal of the decagon.
 (a) $2 + \sqrt{5}$ cm (b) $6 - \sqrt{5}$ cm
 (c) $3 + \sqrt{5}$ cm (d) $9 - 2\sqrt{5}$ cm
- 121** Find the third shortest diagonal of the decagon.
 (a) $\sqrt{10 + 2\sqrt{5}}$ cm (b) $\sqrt{12\sqrt{5}}$ cm
 (c) $\sqrt{5 + 4\sqrt{5}}$ cm (d) $\sqrt{15 - 2\sqrt{5}}$ cm
- 122** In the given diagram, some diagonals of a regular decagon are intersecting such that they form another small decagon inside the original decagon. If each side of the larger decagon be 2 cm, find the side of the smaller decagon.



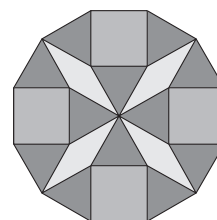
- (a) $\frac{2\sqrt{5}}{3}$ cm (b) $2\sqrt{5} - 3$ cm
 (c) $\sqrt{5} - 1$ cm (d) 1 cm

- 123** If each side of a regular dodecagon is 1 cm, find the area of the dodecagon.
 (a) 12 sq. cm (b) $6 + 3\sqrt{3}$ sq. cm
 (c) $12\sqrt{3}$ sq. cm (d) none of these
- 124** If each side of a regular dodecagon is 1 cm, find the second smallest diagonal of the dodecagon.
 (a) $3\sqrt{2} - 2$ sq. cm (b) $\sqrt{3} + 1$ sq. cm
 (c) $2\sqrt{3} + 1$ sq. cm (d) none of these
- 125** If each side of a regular dodecagon is 1 cm, find the circumradius of the dodecagon.
 (a) $\sqrt{2 + \sqrt{3}}$ cm (b) $3\sqrt{3}$ cm
 (c) $1 + 2\sqrt{3}$ cm (d) none of these
- 126** If each side of a regular dodecagon is 1 cm, find the smallest diagonal of the dodecagon.
 (a) $\sqrt{2 + \sqrt{3}}$ cm (b) $6 - 3\sqrt{3}$ cm
 (c) $2\sqrt{3} - \sqrt{2}$ cm (d) $\sqrt{3 + \sqrt{2}}$ cm

- 127** If each side of a regular dodecagon is 1 cm, find the third smallest diagonal of the dodecagon.

- (a) $\sqrt{2 + \sqrt{3}} + \sqrt{2}$ cm (b) $\sqrt{3}\sqrt{2 + \sqrt{3}}$ cm
 (c) $2\sqrt{3} - \sqrt{2}$ cm (d) both (a) and (b)

- 128** In the following figure of regular dodecagon find the area of all the four kites, where each of the four kites has two unequal diagonals and the side of each square, each triangle and each kite is 1 cm.



- (a) 2 sq. cm (b) $2\sqrt{2}$ sq. cm
 (c) $\sqrt{2 + \sqrt{3}}$ sq. cm (d) none of these

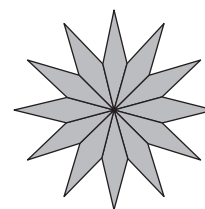
- 129** If each side of a regular dodecagon is 1 cm, find the second longest diagonal of the dodecagon.

- (a) $\sqrt{2 + \sqrt{3}} + \sqrt{3}$ cm (b) $\sqrt{2 + \sqrt{3}}$ cm
 (c) $2 + \sqrt{3}$ cm (d) both (a) and (c)

- 130** If each side of a regular dodecagon is 1 cm, find the longest diagonal of the dodecagon.

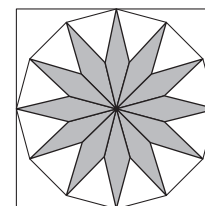
- (a) $2\sqrt{2 + \sqrt{3}}$ cm (b) $6 - 3\sqrt{3}$ cm
 (c) $2\sqrt{3} - \sqrt{2}$ cm (d) $\sqrt{3 + \sqrt{2}}$ cm

- 131** In the following figure 12 rhombi are joined in such a way that they have a common vertex, without overlapping each other, and every rhombus shares two of its sides with the adjacent rhombi, as shown below. Find the area of the following figure, if its perimeter is 24 cm.



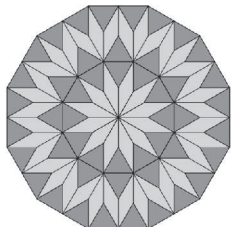
- (a) 6 sq. cm (b) 4 sq. cm
 (c) 8 sq. cm (d) data insufficient

- 132** In the given figure, a square inscribes a dodecagon and each side of the regular dodecagon is 1 cm. The dodecagon consists of 12 equilateral triangles and 12 rhombi. What is the ratio of area of dodecagon and the area of square?

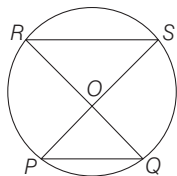


- (a) $\frac{3}{4}$ (b) $\frac{13}{16}$
 (c) $\frac{7}{8}$ (d) $\frac{7}{10}$

- 133** The following figure consists of two concentric dodecagons. Find the ratio of area of smaller (interior) dodecagon to that of larger (exterior) dodecagon.

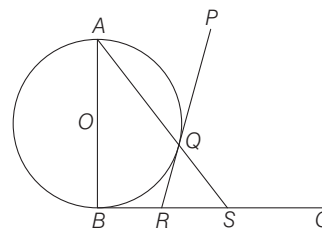


- (a) 1 : 3 (b) 1 : 4
(c) 1 : 6 (d) none of these
- 134** You have a regular hexagonal paper sheet and you are asked to make a regular dodecagon by cutting off the corners of the hexagonal sheet. If each side of the regular hexagonal sheet is 1 ft, minimum how much area of the sheet you would have to cut off?
- (a) $\left(\frac{63\sqrt{3}}{2} - 36\right)$ ft (b) $\left(\frac{21\sqrt{3}}{2} - 18\right)$ ft
(c) $\left(\frac{25\sqrt{3}}{4} - 6\right)$ ft (d) none of these
- 135** Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3} + 1$ apart. If the chords subtend, at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k > 0$. If $[k]$ denotes the largest integer less than or equal to k , the values of $[k]$ is
- (a) 1 (b) 2
(c) 3 (d) 6
- 136** The centers of two circles C_1 and C_2 , each of unit radii, are at a distance of 6 units from each other. Let P be the mid-point of the line segment joining the centers of C_1 and C_2 . A third circle C is drawn such that it is externally tangent to both the circles C_1 and C_2 at the same points where the lines passing through P are tangent to the circles C_1 and C_2 . The radius of the circle C is
- (a) 8 (b) 6
(c) $6\sqrt{2}$ (d) $4\sqrt{2}$
- 137** In the given diagram PQ is parallel to RS . For $PQ > OP$, when $\angle POQ$ and $\angle OPQ$ have integral values, find the greatest possible value of OPQ .



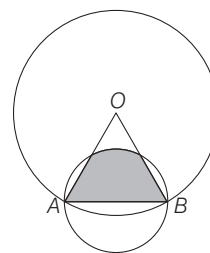
- (a) 119 (b) 61
(c) 59 (d) 58

- 138** In the following figure, BC and RP are tangents to the same circle with centre O and its diameter AB . Points R and S are the points on the tangent BC , such that AS passes through Q , where RP touches the circle.

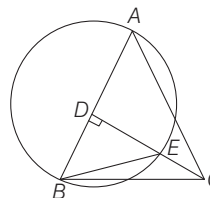


Which of the following is necessarily a correct relations?

- (a) $RS = QS$ (b) $BR = RS$
(c) $AB = 2AQ$ (d) none of these
- 139** In the concerned diagrams, chords AB and CD are parallel and radius OR bisects the chord AB at P and CD at Q . Radius OC when extended to a point T in order to meet the tangent TR it intersects the chord AB at S . If $OP : OS = 8 : 17$, find $OC : OT$
- (a) 9 : 17 (b) 8 : 15
(c) 17 : 23 (d) none of these
- 140** Consider the following figure in which $OA = OB = AB$. Chord AB is the longest possible chord of the smaller circle and O is the centre of the larger circle. Find the area of the shaded region, if the radius of the smaller circle is 6 cm.



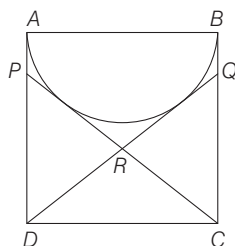
- (a) $6(\pi + 3\sqrt{3})$ sq. cm (b) $4(\pi + 3\sqrt{3})$ sq. cm
(c) $6(\pi + 2\sqrt{3})$ sq. cm (d) can't be determined
- 141** In the following figure $\triangle ABC$ is an isosceles triangle, where $AB = AC$ and $BC = 2\sqrt{2}$ cm. CD is perpendicular on the diameter AB which intersects the circle at E . Find BE .



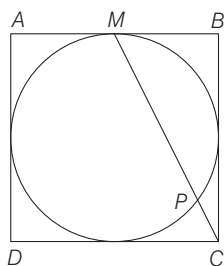
- (a) 2 cm (b) $\frac{5}{\sqrt{2}}$ cm (c) 2.16 cm (d) 1.67 cm

- 142** Two parallel chords in a circle have lengths 12 and 24, and the distance between them is 18. The chord parallel to these chords and midway between them is of length $\sqrt{a}$ where a is
- (a) 676 (b) 756
(c) 576 (d) 720

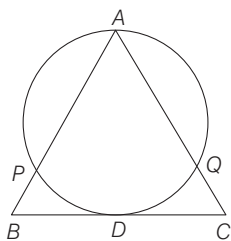
- 143** $ABCD$ is a square of area 16 sq. cm and AB is the diameter of the semicircle. The line segments CP and DQ intersect at R and touch the semicircle. Find the area of ΔDRC .



- (a) 3 sq. cm (b) 2 sq. cm
(c) 2.5 sq. cm (d) 5 sq. cm
- 144** In the following figure, a circle of radius 40 mm is inscribed in a square $ABCD$ in which M is the mid-point of AB . If the line segment MC intersects the circle at P , find PC .

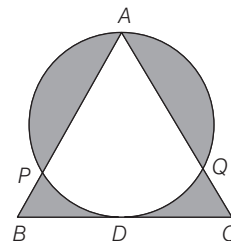


- (a) $8\sqrt{5}$ mm (b) $8\sqrt{10}$ mm
(c) $6\sqrt{5}$ mm (d) $\frac{16}{\sqrt{5}}$ mm
- 145** In the given figure, the perpendicular bisector AD of the equilateral triangle ABC is same as the diameter of the circle O , and the circle intersects the sides AB and AC at P and Q , respectively. If the height of the triangle be $10\sqrt{2}$ cm, find CQ .



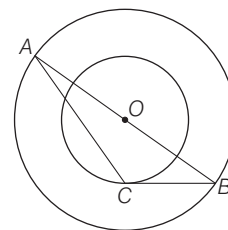
- (a) 3 cm (b) 6 cm
(c) 4 cm (d) 5 cm

- 146** In the given figure, the perpendicular bisector AD of the equilateral triangle ABC is same as the diameter of the circle O , and the circle intersects the sides AB and AC at P and Q , respectively. If the height of the triangle be $12\sqrt{2}$ cm, find CQ .



- (a) $24(\pi + \sqrt{3})$ sq. cm (b) $12(2\pi + 3\sqrt{3})$ sq. cm
(c) $12(4\pi + 5\sqrt{3})$ sq. cm (d) none of these

- 147** There are two concentric circles centered at O . Their radii are 21 cm and 29 cm. AB is the diameter of the larger circle and BC is the tangent to the smaller circle at C . Find AC .



- (a) 49.66 cm
(b) 46.52 cm
(c) 54.54 cm
(d) can't be determined

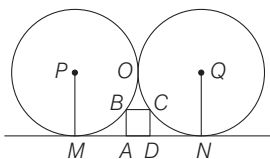
- 148** Let there be a circle with centre O and radius r , such that the four points A, B, C and D are cyclic while AC and BD are the two chords intersecting each other perpendicularly at a point P somewhere inside the circle. If A, B, C and D are variable points and $OP = d$, find the maximum and minimum area of the quadrilateral $ABCD$.

- (a) $4r^2$ and $2r\sqrt{r^2 - d^2}$
(b) $4d^2$ and $r^2\sqrt{r^2 - d^2}$
(c) $2r^2$ and $r^2\sqrt{r^2 - d^2}$
(d) $2r^2$ and $2r\sqrt{r^2 - d^2}$

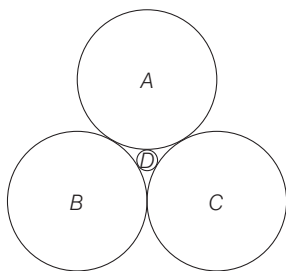
- 149** There are two circles C_1 and C_2 of radii r_1 and r_2 , respectively. They are mutually tangent to each other and to a line as well. There is one more circle of radius r_3 which is tangent to this line and the first two circles C_1 and C_2 . What is the relation between the three radii r_1, r_2 and r_3 ?

- (a) $\frac{1}{\sqrt{r_1}} - \frac{1}{\sqrt{r_2}} = \frac{1}{\sqrt{r_3}}$ (b) $\frac{\sqrt{r_1}}{\sqrt{r_2}} + \frac{\sqrt{r_2}}{\sqrt{r_1}} = \frac{1}{\sqrt{r_3}}$
(c) $\frac{1}{\sqrt{r_3}} = \frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}}$ (d) none of these

- 150** Two equal circles with centers P and Q are tangent at O . A common line is tangent to both the circles at M and N respectively. Point B lies on the arc OM and point C lies on the arc ON . Points A and D lie on the tangent MN . If the radius of each circle is r and each side of the square $ABCD$ is x , find x .

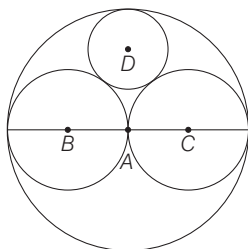


- (a) $0.25r$ (b) $0.3r$
(c) $0.33r$ (d) none of these
- 151** In the following diagram four circles are tangent to every other circle. The radius of each of the circles A , B and C is 1. Find the radius of circle D , which is tangent to all the three given circles and lies at the centre.



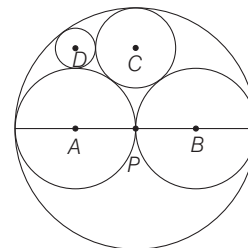
- (a) $\frac{1}{3 + 2\sqrt{3}}$ (b) $\frac{1}{3 - 2\sqrt{3}}$ (c) $2\sqrt{3} - 3$ (d) $\frac{1}{3 - 2\sqrt{2}}$

- 152** In the following diagram there are three circles packed in another circle, such that each circle is tangent to all other circles in this arrangement. The radius of the circumscribing circle A is 1 cm; and circle B and circle C are congruent. Find the radius of the circle D .



- (a) $\frac{1}{4}$ cm (b) $\frac{1}{3}$ cm (c) $\frac{2}{5}$ cm (d) $\frac{3}{10}$ cm

- 153** In the following diagram there are four circles A , B , C and D packed in a circle P . The radius of the circumscribing circle P is 1 cm. Circles A and B are tangent at P , the centre of the circumscribing circle. Circle D is tangent to circles A , C and P . Find the radius of the circle D .



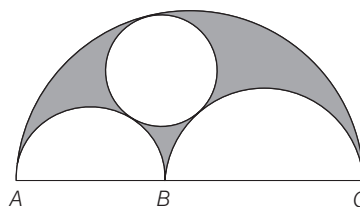
- (a) $\frac{1}{6}$ cm (b) $\frac{1}{2}$ cm
(c) $\frac{2}{15}$ cm (d) $\frac{1}{8}$ cm

- 154** There are three circles, with radii 1 cm, 2 cm and 3 cm, tangent to each other. Find the radius of the fourth circle, which is tangent to all the existing circles.

- (a) 6 cm (b) $\frac{23}{6}$ cm
(c) $\frac{6}{23}$ cm (d) both (a) and (c)

- 155** In the following diagram, a semicircle of radius r inscribes two semicircles, which are tangent to each other at a point B on the diameter AC of the larger semicircle. Then there is a full circle inscribed in the remaining area, which is tangent to all the three existing semicircles. Find the radius of this full circle, if the diameter $AB = \frac{1}{3}$ cm and

$$BC = \frac{2}{3} \text{ cm.}$$



- (a) $\frac{1}{7}$ cm
(b) $\frac{1}{9}$ cm
(c) $\frac{2}{15}$ cm
(d) none of these

TEST OF LEARNING

- 1 Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a , b and c denote the lengths of sides opposite A , B and C respectively. Find the values of x for which $a = (x^2 + x + 1)$, $b = (x^2 - 1)$ and $c = (2x + 1)$.

- (a) $-(2 + \sqrt{3}), (2 + \sqrt{3})$
 (b) $-(2 + \sqrt{3}), (2 - \sqrt{3})$
 (c) $(-2 + \sqrt{3}), (1 + \sqrt{3})$
 (d) $-(2 + \sqrt{3}), (1 + \sqrt{3})$

- 2 Let a , b and c be the sides of a triangle. No two of them are equal and $\lambda \in \mathbb{R}$. If the roots of the equation

$$x^2 + 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$$

are real, then

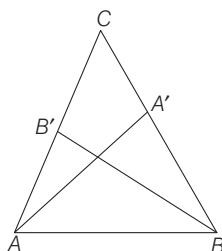
- (a) $\lambda < \frac{4}{3}$ (b) $\lambda > \frac{5}{3}$
 (c) $\frac{1}{3} < \lambda < \frac{5}{3}$ (d) $\frac{4}{3} < \lambda < \frac{5}{3}$

- 3 Which of the following best describes the value of k , if a , b and c are the lengths of the sides of the triangle and $k = \frac{a^2 + b^2 + c^2}{ab + bc + ca}$?

- (a) $1 \leq k < 2$ (b) $0 \leq k < 2$
 (c) $0 < k < \frac{9}{8}$ (d) $1 \leq k \leq 2$

- 4 In a triangle ABC , the median AA' and BB' are perpendicular to each other. Let $BC = a$, $AC = b$ and $AB = c$, which of the following is/are true?

- (a) $5c^2 = a^2 + b^2$
 (b) $\frac{a}{2} < b < 2a$
 (c) $a < b < 2a$
 (d) both (a) and (b)



- 5 In a right angle triangle, x and y are the perpendicular sides and z is the hypotenuse. Find the minimum value of $\frac{z}{x} + \frac{z}{y}$.

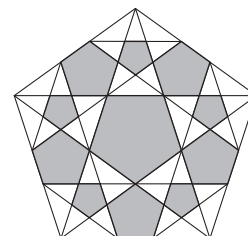
- (a) $2 + \sqrt{2}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $3\sqrt{2} - 2\sqrt{3}$ (d) $2\sqrt{2}$

- 6 In a triangle ABC , the median BD yields $\angle BDC = 45^\circ$ and $\angle ABD = \angle BCD = x^\circ$. Find the value of x .

- (a) 45° (b) 30°
 (c) 60° (d) none of these

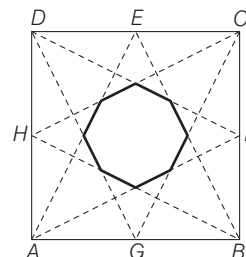
- 7 In the following diagram a regular pentagon is intersected by various diagonals forming another pentagon at the center, then by drawing parallel lines to the sides of the original pentagon while making them pass through the

vertices of the newly created central pentagon we form another set of 10 pentagons. Thus we can see there are four sizes of pentagons – one is the original size, second one is at the centre, the third ones are sharing the respective edges of the original pentagon and the fourth ones are the smallest in size. If each edge of the smallest shaded pentagon is 2 cm, what is the total area of all the shaded pentagonal regions?



- (a) $(\sqrt{5(5 + 2\sqrt{5})})(16 + 4\sqrt{5}) \text{ cm}^2$
 (b) $(\sqrt{5(5 - 2\sqrt{5})})(4 + 15\sqrt{5}) \text{ cm}^2$
 (c) $(\sqrt{5(5 + 2\sqrt{5})})(4 + 15\sqrt{5}) \text{ cm}^2$
 (d) none of the above

- 8 In the following figure A , B , C and D are the four vertices of the square and E , F , G and H are the four mid-points of the sides of the square $ABCD$. If each side of the square be a , then how many of the following statements are correct about the octagon formed at the centre by joining the mid-points with the opposite vertices?



- (i) It's an equiangular octagon
 (ii) It's an equilateral octagon
 (iii) It's a regular octagon
 (iv) It's each side is $\frac{\sqrt{5}a}{12}$

- (v) It's each side is $\frac{a}{2(\sqrt{4 + 2\sqrt{2}})}$

- (a) 1 (b) 2 (c) 3 (d) 4

- 9 A straight line through the vertex of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T . If S is not the centre of the circumcircle, then

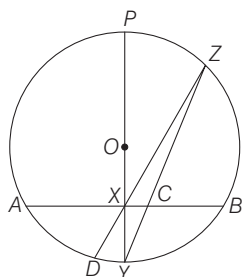
- (i) $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{QS \times SR}}$ (ii) $\frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}}$

- (iii) $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{QS}$ (iv) $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$

- (a) (i) and (iii) (b) (ii) and (iv)
 (c) (i) and (iv) (d) (ii) and (iii)

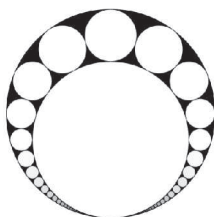
- 10** Let $ABCD$ be a square of side length 2 units. Let C_1 be the incircle and C_2 be the circumcircle of the square $ABCD$. If P is a point on C_1 and Q is a point on C_2 , the value of $\frac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2}$ is
- (a) 0.75 (b) 1.25 (c) 1 (d) 0.5

- 11** In a circle with centre O , the diameter PY is perpendicular to chord AB , as shown in the following figure. Find the correct relation between $\overline{DX}$ and $\overline{CY}$.



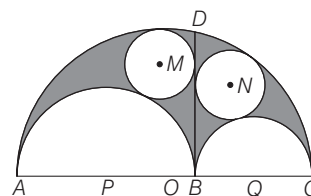
- (a) $\overline{XD} \geq \overline{CY}$ (b) $\overline{XD} \leq \overline{CY}$
 (c) $\overline{XD} = \overline{CY}$ (d) Can't be determined

- 12** In the following diagram, a circle of radius $1/3$ unit is internally tangent to another circle of radius $1/4$ unit. The region between these two circles is shaded in black. Within the shaded region there are numerous circles inscribed in such a way that each such triangle is tangent to at least 3 other triangles. If there are total 25 circles drawn in the shaded region, while maintaining the symmetry, find the sum of curvatures of all the 25 circles.
- (a) 2500 (b) 1600 (c) 1444 (d) 1786



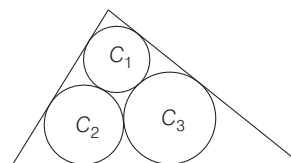
- 13** A circle C_1 with diameter $1/25$ cm is tangent to X -axis at $(2/5, 0)$ and the other circle with diameter $1/49$ cm is tangent to X -axis at $(3/7, 0)$. Moreover, these two circles are on the same side of X -axis and tangent to each other. There is another circle C_3 which is tangent to both of these circles and tangent to X -axis. Find, the distance between the Y -axis and the point of tangent of the circle C_3 on X -axis.
- (a) $\frac{26}{35}$ (b) $\frac{57}{144}$ (c) $\frac{17}{36}$ (d) $\frac{35}{84}$

- 14** In the following diagram a semicircle with centre O inscribes two semicircles with centers P and Q , which are tangent to each other at a point B on the diameter AC of the larger semicircle. $AB = p$ and $BC = q$ are the diameters of the inscribed semicircles, respectively. A perpendicular BD is drawn from D to B . D is a point on the largest semicircle. There are two circles with centre M and N inscribed on the different sides of BD such that these circles are tangent to BD as well as tangent to an inscribed semicircle and the largest semicircle. If the radii of these circles M and N be m and n , respectively, which of the followings is/are correct?



- (i) $m > n$ (ii) $m = n$
 (iii) $m < n$ (iv) $m + n = \frac{2pq}{p + q}$
 (a) only (i) is correct (b) only (iii) is correct
 (c) (ii) and (iv) are correct (d) none of these

- 15** There are three circles inscribed in a triangle such that each circle is tangent to the other two circles and the two sides of the triangle. Let r_1 , r_2 and r_3 be the radii of circles C_1 , C_2 and C_3 , find the inradius of the triangle inscribing these three circles.



- (a) $\frac{\sqrt{r_1 r_2 r_3}}{r_1 + r_2 + r_3 - 2\sqrt{r_1 + r_2 + r_3}}$
 (b) $\frac{2\sqrt{r_1 r_2 r_3}}{r_1 + r_2 + r_3 - \sqrt{3}\sqrt{r_1 + r_2 + r_3}}$
 (c) $\frac{2\sqrt{r_1 r_2 r_3}}{r_1 + r_2 + r_3 - \sqrt{r_1 + r_2 + r_3}}$
 (d) none of the above

Answers

✓ Introductory Exercise 12.1

1 (c)	2 (c)	3 (b)	4 (b)	5 (b)	6 (a)	7 (d)	8 (a)	9 (a)	10 (b)
11 (a)	12 (c)	13 (b)	14 (b)	15 (d)	16 (b)	17 (b)	18 (b)	19 (a)	20 (c)
21 (b)	22 (c)	23 (c)	24 (b)	25 (c)	26 (a)	27 (c)	28 (c)	29 (a)	30 (b)
31 (c)	32 (a)	33 (a)	34 (c)	35 (a)	36 (b)	37 (b)	38 (c)	39 (b)	40 (a)
41 (c)	42 (c)	43 (a)	44 (b)	45 (b)					

✓ Introductory Exercise 12.2

1 (b)	2 (b)	3 (c)	4 (d)	5 (a)	6 (c)	7 (d)	8 (d)	9 (b)	10 (b)
11 (b)	12 (c)	13 (d)	14 (b)	15 (b)	16 (b)	17 (c)	18 (b)	19 (d)	20 (b)
21 (a)	22 (d)	23 (c)	24 (b)	25 (c)	26 (c)	27 (c)	28 (a)	29 (a)	30 (b)
31 (c)	32 (c)	33 (d)	34 (d)	35 (a)	36 (d)	37 (b)	38 (d)	39 (c)	40 (d)
41 (c)	42 (a)	43 (c)	44 (c)	45 (b)	46 (c)	47 (a)	48 (a)	49 (c)	50 (c)
51 (a)	52 (a)	53 (a)	54 (c)	55 (a)	56 (a)	57 (a)	58 (b)	59 (b)	60 (c)
61 (c)	62 (b)	63 (c)	64 (b)	65 (b)	66 (c)	67 (c)	68 (a)	69 (d)	70 (a)
71 (b)	72 (c)	73 (c)	74 (a)	75 (b)	76 (c)	77 (c)	78 (b)	79 (c)	80 (d)
81 (b)	82 (b)	83 (a)	84 (c)	85 (b)	86 (c)	87 (a)	88 (d)	89 (c)	90 (b)
91 (d)	92 (c)	93 (c)	94 (c)	95 (b)	96 (c)	97 (b)	98 (a)	99 (b)	100 (d)
101 (c)	102 (c)	103 (d)	104 (a)	105 (a)	106 (c)	107 (c)			

✓ Introductory Exercise 12.3

1 (b)	2 (b)	3 (b)	4 (a)	5 (a)	6 (b)	7 (c)	8 (c)	9 (b)	10 (c)
11 (b)	12 (b)	13 (a)	14 (b)	15 (a)	16 (b)	17 (c)	18 (c)	19 (a)	20 (a)
21 (c)	22 (b)	23 (c)	24 (a)	25 (b)	26 (a)	27 (a)	28 (b)	29 (d)	30 (a)

✓ Introductory Exercise 12.4

1 (d)	2 (b)	3 (b)	4 (d)	5 (d)	6 (b)	7 (c)	8 (d)	9 (c)	10 (c)
11 (c)	12 (a)	13 (b)	14 (d)						

✓ Introductory Exercise 12.5

1 (b)	2 (a)	3 (c)	4 (c)	5 (a)	6 (c)	7 (a)	8 (c)	9 (c)	10 (c)
11 (d)	12 (a)	13 (b)	14 (d)	15 (c)	16 (b)	17 (a)	18 (c)	19 (a)	20 (c)
21 (b)	22 (b)	23 (c)	24 (c)	25 (b)	26 (b)	27 (b)	28 (a)	29 (c)	30 (b)
31 (b)	32 (b)	33 (c)	34 (c)	35 (c)	36 (d)	37 (b)	38 (c)	39 (b)	40 (b)
41 (c)	42 (d)	43 (a)	44 (b)	45 (a)	46 (a)	47 (c)	48 (d)	49 (a)	50 (c)
51 (b)	52 (b)	53 (d)	54 (a)	55 (c)	56 (b)	57 (a)	58 (d)	59 (c)	60 (c)
61 (b)	62 (a)	63 (b)	64 (c)	65 (d)	66 (b)	67 (a)	68 (c)	69 (a)	70 (b)

✓ **Level 01** *Basic Level Exercise*

1 (c)	2 (d)	3 (a)	4 (c)	5 (d)	6 (b)	7 (a)	8 (c)	9 (c)	10 (c)
11 (d)	12 (d)	13 (c)	14 (b)	15 (c)	16 (a)	17 (b)	18 (a)	19 (b)	20 (b)
21 (b)	22 (d)	23 (c)	24 (c)	25 (b)	26 (b)	27 (c)	28 (c)	29 (d)	30 (d)
31 (c)	32 (d)	33 (c)	34 (b)	35 (c)	36 (c)	37 (a)	38 (d)	39 (b)	40 (b)
41 (a)	42 (a)	43 (b)	44 (b)	45 (c)	46 (a)	47 (c)	48 (a)	49 (c)	50 (d)
51 (c)	52 (a)	53 (c)	54 (b)	55 (a)	56 (d)	57 (a)	58 (c)	59 (c)	60 (b)
61 (b)	62 (b)	63 (b)	64 (c)	65 (c)	66 (b)	67 (d)	68 (a)	69 (b)	70 (d)
71 (a)	72 (c)	73 (d)	74 (a)	75 (b)	76 (c)	77 (b)	78 (b)	79 (c)	80 (b)
81 (d)	82 (a)	83 (c)	84 (c)	85 (d)	86 (d)	87 (b)	88 (a)	89 (d)	90 (c)
91 (d)	92 (c)	93 (c)	94 (d)	95 (d)	96 (c)	97 (c)	98 (d)	99 (d)	100 (a)
101 (d)	102 (c)	103 (a)	104 (c)	105 (d)	106 (b)	107 (d)	108 (a)	109 (d)	110 (a)
111 (d)	112 (c)	113 (a)	114 (d)	115 (a)	116 (c)	117 (d)	118 (a)	119 (d)	120 (c)
121 (c)	122 (a)	123 (d)	124 (a)	125 (b)	126 (d)	127 (d)	128 (c)	129 (c)	130 (a)
131 (a)	132 (a)	133 (c)	134 (a)	135 (a)	136 (a)	137 (a)	138 (a)	139 (c)	140 (a)
141 (d)	142 (c)	143 (d)	144 (a)	145 (d)	146 (d)	147 (c)	148 (c)	149 (b)	150 (d)

✓ **Level 02** *Higher Level Exercise*

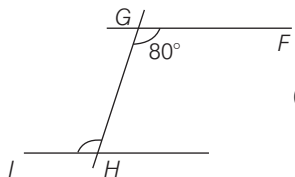
1 (b)	2 (d)	3 (d)	4 (c)	5 (b)	6 (c)	7 (d)	8 (b)	9 (b)	10 (b)
11 (a)	12 (d)	13 (d)	14 (b)	15 (b)	16 (a)	17 (a)	18 (c)	19 (d)	20 (b)
21 (c)	22 (c)	23 (b)	24 (a)	25 (d)	26 (c)	27 (a)	28 (d)	29 (a)	30 (a)
31 (a)	32 (a)	33 (a)	34 (a)	35 (b)	36 (b)	37 (c)	38 (b)	39 (a)	40 (b)
41 (b)	42 (a)	43 (a)	44 (b)	45 (c)	46 (c)	47 (c)	48 (a)	49 (b)	50 (c)
51 (c)	52 (a)	53 (b)	54 (a)	55 (b)	56 (b)	57 (c)	58 (b)	59 (a)	60 (a)
61 (a)	62 (b)	63 (c)	64 (c)	65 (c)	66 (b)	67 (a)	68 (a)	69 (a)	70 (a)
71 (b)	72 (c)	73 (c)	74 (d)	75 (d)	76 (a)	77 (a)	78 (c)	79 (b)	80 (c)
81 (b)	82 (a)	83 (c)	84 (d)	85 (a)	86 (c)	87 (b)	88 (b)	89 (c)	90 (a)
91 (a)	92 (a)	93 (b)	94 (c)	95 (a)	96 (b)	97 (a)	98 (b)	99 (d)	100 (b)
101 (c)	102 (c)	103 (a)	104 (a)	105 (d)	106 (a)	107 (d)	108 (d)	109 (c)	110 (d)
111 (d)	112 (a)	113 (d)	114 (a)	115 (a)	116 (a)	117 (a)	118 (d)	119 (b)	120 (c)
121 (a)	122 (c)	123 (b)	124 (b)	125 (a)	126 (a)	127 (d)	128 (a)	129 (c)	130 (a)
131 (a)	132 (a)	133 (b)	134 (b)	135 (c)	136 (a)	137 (c)	138 (b)	139 (d)	140 (a)
141 (a)	142 (b)	143 (a)	144 (a)	145 (d)	146 (a)	147 (b)	148 (d)	149 (c)	150 (d)
151 (a)	152 (b)	153 (a)	154 (d)	155 (a)					

✓ **Test for Your Learning**

1 (d)	2 (a)	3 (a)	4 (d)	5 (d)	6 (b)	7 (a)	8 (b)	9 (b)	10 (a)
11 (b)	12 (b)	13 (d)	14 (c)	15 (c)					

Hints & Solutions

Introductory Exercise 12.1

- 1 $4x + 5x = 180^\circ \Rightarrow x = 20^\circ$
($4x$ and $5x$ are linear pair angles)
- 2 $(5x + 12) + 3x = 180^\circ \Rightarrow x = 21^\circ$
- 3 $105 + 3x = 180 \Rightarrow x = 25$
and $y + 90 + 2x = 180^\circ \Rightarrow y = 40^\circ$
 $\therefore x + y = 65^\circ$
- 4 $2x + 2y = 180 \Rightarrow x + y = 90^\circ$
 $\therefore \angle COD = 90^\circ$
- 5 $\angle DOE = 180 - (40 + 31) = 109^\circ$
and $\angle BOF = 40^\circ$ ($\because \angle AOE = \angle BOF$)
 $\therefore \angle BOC = \angle BOF + \angle FOC$
 $= 40 + 109$ ($\because \angle FOC = \angle DOE$)
 $= 149^\circ$
- 6 $2y + 3x = 180 \Rightarrow y = 36^\circ$ ($\because x = 36^\circ$)
or $x + 4y = 180^\circ \Rightarrow y = 36^\circ$ ($\because x = 36^\circ$)
- 7 $(2x + 17) + (x + 4) = 90^\circ$
 $\Rightarrow 3x + 21 = 90^\circ \Rightarrow x = 23^\circ$
- 8 $(5y + 62)^\circ + (22^\circ + y) = 180$
 $\Rightarrow 6y + 84 = 180 \Rightarrow y = 16^\circ$
- 9 $13x + 5x = 180^\circ \Rightarrow x = 10^\circ$
 $\therefore 13x = 130^\circ$
- 10 Let the angle be x , then its complementary angle be $(90 - x)$
 $\therefore x - \frac{(90 - x)}{2} = 30 \Rightarrow x = 50$
Alternatively The best way is to solve through options.
- 11 
(Pair of alternate angles)
- 12 $\frac{AG}{BH} = \frac{CE}{DF}$
 $\frac{120}{100} = \frac{30}{DF} \Rightarrow DF = 25 \text{ cm}$
- 13 $65^\circ 50' + x = 90^\circ = 89^\circ 60'$ ($1^\circ = 60'$)
 $\Rightarrow x = 89^\circ 60' - 65^\circ 50' = 24^\circ 10'$
- 14 $123^\circ 45' + x = 180^\circ = 179^\circ 60'$ ($1^\circ = 60'$)
 $\Rightarrow x = 179^\circ 60' - 123^\circ 45'$
 $\Rightarrow x = 56^\circ 15'$

- 15 Since $x > 0$ and $y > 0$

Also, $x + y = 90^\circ$

Therefore, $0 < x < 90$ and $0 < y < 90^\circ$

- 16 Let the angle be x , then its supplement be $(180^\circ - x)$.

$\therefore x = \frac{1}{5}(180^\circ - x) \Rightarrow x = 30^\circ$

Alternatively Go through options.

$30 = \frac{1}{5}(180^\circ - 30^\circ)$

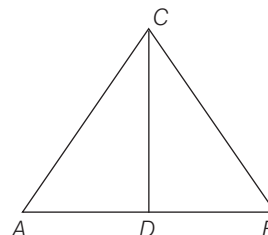
- 17 $a > \frac{90^\circ}{6} \Rightarrow a > 15^\circ$

$\therefore a + b = 180^\circ$

$\therefore b < 165^\circ$

($\because a > 15^\circ$)

- 18 $\therefore AC = BC$ and $AD = BD$

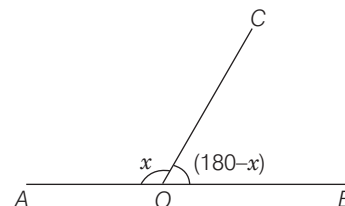


and CD is common

$\therefore \triangle CDA$ is similar to $\triangle CDB$ and AB is a straight line

$\therefore \angle CDA = \angle CDB = 90^\circ$

- 20



- 21 $\angle ABP = \angle CDR = 100^\circ$

$\therefore y = 80^\circ$ ($\because \angle CDR + \angle CDS = 180^\circ$)

- 22 $\angle APO = 42^\circ$ and $\angle CQO = 38^\circ$

$\angle POQ = \angle PON + \angle NOQ$
 $= \angle APO + \angle OQC = 42^\circ + 38^\circ = 80^\circ$

- 23 $\angle XYB = \angle YBC$ and $\angle XYB = \angle BYC$

$\Rightarrow \angle YBC = \angle BYC$

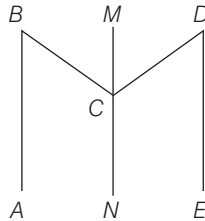
- 24 $\angle PGH = 80^\circ \Rightarrow \angle QGH = 100^\circ$

$\angle QHD = 120^\circ \Rightarrow \angle CHQ = 60^\circ$

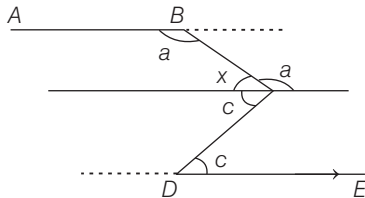
$\therefore \angle x + 100 + 60 = 180^\circ \Rightarrow x = 20^\circ$

25 $\angle z = 180^\circ - 110^\circ = 70^\circ$ ($\because \angle QAC = \angle BCA$)
 and $60^\circ + y^\circ + z^\circ = 180^\circ$
 $\therefore y = 180^\circ - (70^\circ + 60^\circ)$
 $y = 50^\circ$

26 $\angle BCD = \angle BCM + \angle DCM$
 $= \angle ABC + \angle EDC = 67^\circ + 23^\circ = 90^\circ$



27 $a + x = 180^\circ$; $c + x = b \Rightarrow x = b - c$

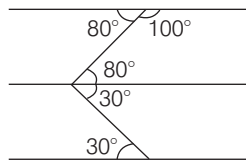


$\therefore a + (b - c) = 180^\circ \Rightarrow a + b - c = 180^\circ$

28 $\angle x = \angle ABC = \angle CDG = 180^\circ - 138^\circ = 42^\circ$

29 Use the same method as in question number 27.
 i.e., $(a + b - c) = 180^\circ$
 $\Rightarrow 100^\circ + (x + 10^\circ) - 30^\circ = 180^\circ$
 $\Rightarrow x = 100^\circ$

Alternatively

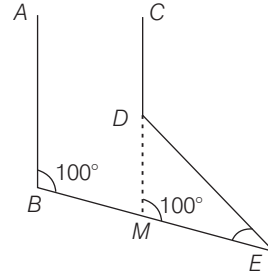


$\therefore x + 10^\circ = 80^\circ + 30^\circ \Rightarrow x = 100^\circ$

30 $\angle PQR = \angle QRS$
 $55^\circ = \angle QRS \Rightarrow 55^\circ = \angle QRL + \angle LRS$
 $55^\circ = 30^\circ + \angle LRS \Rightarrow \angle LRS = 25^\circ$

31 $\angle ABC = 180^\circ - (40^\circ + 65^\circ)$
 $\angle ABC = 75^\circ$ and $\angle DCF = \angle ABC = 75^\circ$
 $\therefore \angle EDC = 180^\circ - \angle DCF = 105^\circ$

32 Extend CD to M, then
 $\angle DME = \angle ABE = 100^\circ \Rightarrow \angle MED = 25^\circ$
 $\therefore \angle MDE = 180^\circ - (100^\circ + 25^\circ) = 55^\circ$
 $\therefore \angle CDE = 180^\circ - 55^\circ = 125^\circ$



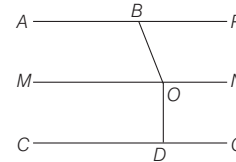
33 $\angle CEF = \angle ABF = 70^\circ$
 $\therefore \angle FED = 180^\circ - 70^\circ = 110^\circ$
 $\angle EDF + \angle DFE + \angle FED = 180^\circ$
 $\therefore \angle FDE = 180^\circ - (110^\circ + 20^\circ)$
 $= 50^\circ$

34 $\angle BAO = 180^\circ - (60^\circ + 35^\circ) = 85^\circ$
 $\angle AOB = 180^\circ - (85^\circ + 20^\circ) = 75^\circ$

35 $\angle AEP = \angle CDP = 34^\circ$
 $\angle AEP + \angle BEF + \angle DEF = 180^\circ$
 $34^\circ + 78^\circ + \angle DEF = 180^\circ$
 $\Rightarrow \angle DEF = 68^\circ$

36 $\angle AKH = \angle KHD = 25^\circ$
 $\angle EGH = \angle AKL = 180^\circ - 60^\circ = 120^\circ$
 $\therefore \angle HKL = 120^\circ + 25^\circ = 145^\circ$

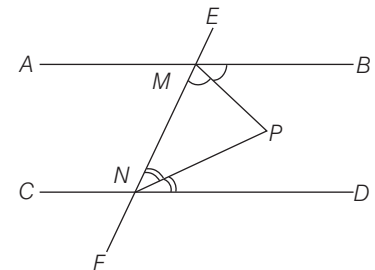
38



$\angle ABO = 118^\circ$
 $\therefore \angle MOB = 180^\circ - 118^\circ = 62^\circ$
 $\therefore \angle MOD = 152^\circ - 62^\circ = 90^\circ$
 $\therefore \angle ODC = 180^\circ - 90^\circ = 90^\circ$

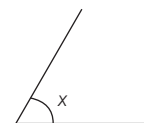
39 Let $\angle BMN = 2x$

and $\angle DNM = 2y$
 $\therefore 2x + 2y = 180^\circ$
 $\Rightarrow x + y = 90^\circ$
 $\therefore \angle MPN = 180^\circ - 90^\circ$
 $= 90^\circ$



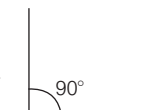
40

Equal



$x \neq 90^\circ$

Equal and Supplementary



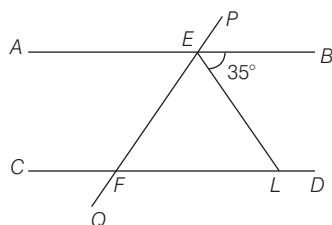
$x = 90^\circ$

41

$$\angle BEF = 2 \angle BEL = 70^\circ$$

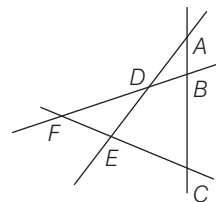
 $\therefore$

$$\begin{aligned}\angle CFQ &= \angle EFL = 180^\circ - 70^\circ \\ &= 110^\circ\end{aligned}$$



$$44 \quad {}^4C_3 = \frac{4!}{3!1!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 1} = 4$$

$$45 \quad {}^4C_2 = \frac{4!}{2!2!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1) \cdot (2 \cdot 1)} = 6$$



Introductory Exercise 12.2

1 The sum of the smaller sides cannot be equal to or less than the largest side.

2 $12^2 > 8^2 + 6^2 \Rightarrow$ obtuse triangle

3 Sum of exterior angles = 360°

4 $\angle ACD = \angle CBA + \angle BAC = 70^\circ + 40^\circ = 110^\circ$

5 $\angle ABC + \angle ACB < 90^\circ$

6 $x + 4x + 7x = 180^\circ$

$$\Rightarrow x = 15^\circ$$

$$\therefore 7x = 105^\circ$$

7 $\angle OBC + \angle OCB = \frac{70}{2} + \frac{30}{2} = 50^\circ$

$$\therefore \angle BOC = 180^\circ - 50^\circ = 130^\circ$$

8 $\angle BFC = 90^\circ - \frac{1}{2}(\angle A)$

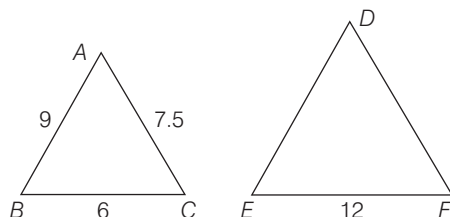
$$= 90^\circ - \frac{1}{2}[180^\circ - (80^\circ + 30^\circ)]$$

$$= 55^\circ$$

10 Prove with the help of similarity.

11 Prove with the help of the similarity.

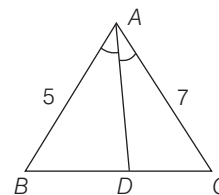
12 $\triangle ABC \sim \triangle DEF$


 $\therefore$

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\Rightarrow \frac{9}{6} = \frac{DE}{12} \Rightarrow DE = 18 \text{ cm}$$

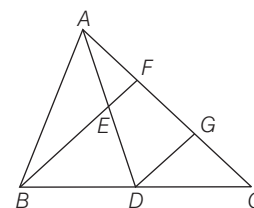
13 $\frac{AB}{AC} = \frac{BD}{CD} = \frac{5}{7}$ (By angle bisector theorem)



14 Let's draw a line segment DG , which is parallel to the line segment BF , such that G lies on AC .

Now look at the $\triangle ADG$, where $EF \parallel DG$, so we have

$$\frac{AF}{FG} = \frac{AE}{DE} = \frac{1}{1}$$



Now, let's have a look at the $\triangle BFC$, where $BF \parallel DG$, so we have

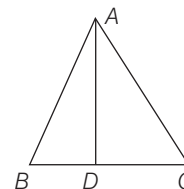
$$\frac{FG}{GC} = \frac{BD}{BC} = \frac{1}{1}$$

Thus we can say that $AF : FG : GC = 1 : 1 : 1$

Therefore, $AF : FC = 1 : 2$

15 AB is hypotenuse and AD is perpendicular in $\triangle ABD$,

$\therefore AB > AD$

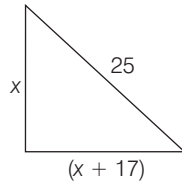


16 $x^2 + (x - 17)^2 = 25^2$

(Using Pythagoras theorem)

$\therefore x = 24 \text{ cm}$

$\therefore \text{Altitude} + \text{Base} = 24 + 7 = 31 \text{ cm}$



17 Go back to the basics.

18 $\angle ACB = 180^\circ - 100^\circ = 80^\circ$

and $\angle BAC + \angle ABC = 180^\circ - 80^\circ = 100^\circ$

$\therefore \angle ABC = 50^\circ$ ($\because \angle BAC = \angle ABC$)

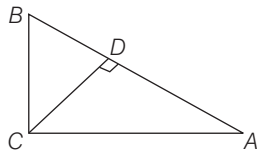
20 $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ (cosine rule)

$\frac{1}{2} = \frac{b^2 + c^2 - a^2}{2bc}$ ($\because \cos 60^\circ = 1/2$)

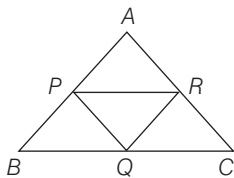
$\therefore a^2 = b^2 + c^2 - bc$

21 $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
 $-\frac{1}{2} = \frac{a^2 + b^2 - c^2}{2ab} \Rightarrow c^2 = a^2 + b^2 + ab$

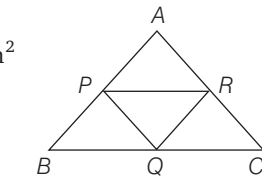
22 $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (Apply similarity of triangles)



27



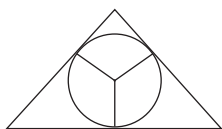
33 There are 4 congruent triangles.
Hence, $A(\triangle ABC) = 6 \times 4 = 24 \text{ cm}^2$



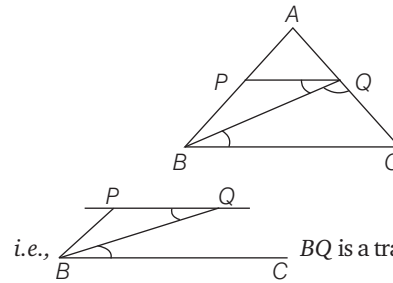
34 $BD = \frac{AB \times BC}{AC}$
 $= \frac{6 \times 6}{6\sqrt{2}} = 3\sqrt{2} \text{ cm}$

36 $\angle BOC = 90^\circ + \frac{1}{2} \angle A$
 $= 90^\circ + 40^\circ = 130^\circ$

37



42

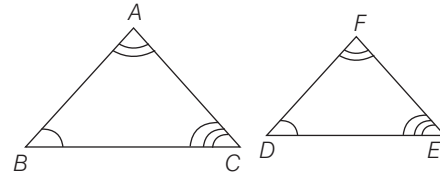


i.e., BQ is a transversal of PQ and BC .

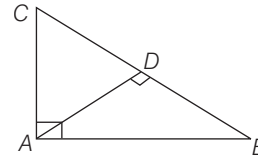
45 Apply Pythagoras theorem.

46 Apply Pythagoras theorem.

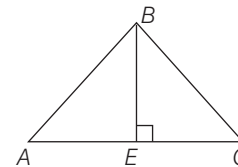
47



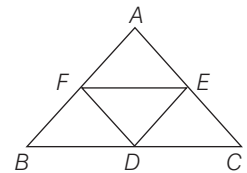
48 $\frac{BD}{CD} = \frac{(AB)^2}{(AC)^2}$



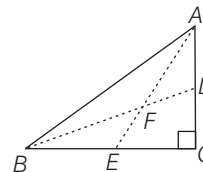
49 Apply Pythagoras theorem



50 $EF = 3 \text{ cm} \therefore BC = 6 \text{ cm}$
 $FD = 4 \text{ cm} \therefore AC = 8 \text{ cm}$
 $AB = 10 \text{ cm} \therefore DE = 5 \text{ cm}$

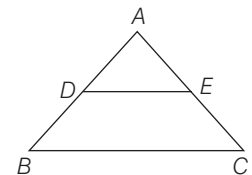


51



52

$\frac{AB}{AC} = \frac{AD}{AE}$
 $\frac{3.6}{2.4} = \frac{2.1}{AE}$
 $\Rightarrow AE = 1.4 \text{ cm}$



54

$\angle EAB = \angle OAB = 60^\circ$
 $\angle ABD = 45^\circ$
 $\therefore \angle AOB = 180^\circ - (60^\circ + 45^\circ) = 75^\circ$

- 55 $\angle LBN = \angle BAC = \angle BCA = 70^\circ$
 $\therefore \angle ABC = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$
- 56 $\angle ECB = 60^\circ + 90^\circ = 150^\circ$
 $\therefore \angle CEB + \angle EBC = 180^\circ - 150^\circ = 30^\circ$
 $\therefore \angle BEC = 15^\circ \quad (\because \angle BEC = \angle EBC)$
 $\therefore \angle DEC = 60^\circ$
 $\therefore \angle DEM = 60^\circ - 15^\circ = 45^\circ$

- 57 $\angle ABQ = \angle EAY$
 $\Rightarrow 20 + y = 60 \Rightarrow y = 40^\circ$
 Now, $\angle BAD = 180 - (60 - 35) = 85^\circ$
 $\therefore \angle ADB = 180 - (\angle BAD + \angle ABD)$
 $\Rightarrow \angle ADB = 180 - 105 = 75^\circ$

- 58 $\angle ACB = 42^\circ = \angle ECD$
 $\therefore \angle CED = 180^\circ - (58^\circ + 42^\circ) = 80^\circ = \angle FEG$
 $\therefore \angle FGE = 180^\circ - (66^\circ + 80^\circ) = 34^\circ$

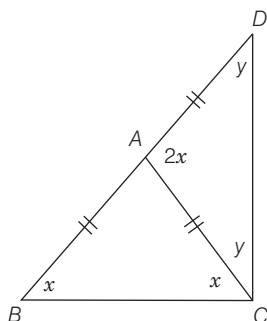
- 59 $\angle BAD = 70^\circ$
 $\therefore \angle BDA = 180^\circ - (70^\circ + 50^\circ) = 60^\circ$

- 60 $\frac{2x}{x} = \frac{3y + 8}{2y} \Rightarrow y = 8$

$\therefore x = 2y = 16$

- 61 $\angle BCD = 27^\circ$
 $\angle BDC = 180^\circ - (27^\circ + 27^\circ) = 126^\circ$
 $\therefore \angle ADC = 54^\circ = \angle DAC$
 $\therefore \angle ACD = 180^\circ - (54^\circ + 54^\circ) = 72^\circ$

- 62 Let us consider that $\angle ABC = x$, therefore $\angle ACB = x$
 Since $\angle CAD$ is the external angle of $\triangle ABC$, therefore,
 $\angle CAD = 2x$
 Now, we know that $AC = AD$, therefore
 $\angle ACD = \angle ADC = y$



But, since sum of all the angles of $\triangle ACD = 180^\circ$, therefore
 $2x + y + y = 180^\circ$

$\Rightarrow x + y = 90^\circ$

Thus, we have $\angle BCD = \angle ACB + \angle ACD$

$\Rightarrow \angle BCD = x + y = 90^\circ$

- 63 By mid point theorem, we can say that

$$\frac{AE}{AC} = \frac{AD}{AB} \Rightarrow \frac{AE}{AC} = \frac{AD}{(AD + BD)}$$

$$\Rightarrow \frac{AE}{5} = \frac{3}{10}$$

$$\Rightarrow AE = 1.5$$

64 $\frac{PB}{BR} = \frac{AP}{AQ}$

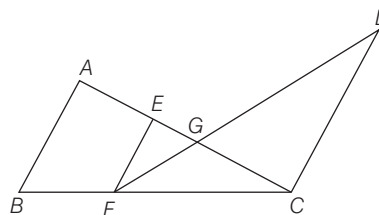
$$\Rightarrow \frac{PB}{BR} = \frac{2\sqrt{2}}{3\sqrt{2}} = \frac{2}{3}$$

$$\therefore \frac{PB}{BR} = \frac{2x}{3x} \therefore BR = 6 \text{ cm}$$

- 65 Since $EF \parallel DC$, so we have

$$\frac{EF}{DC} = \frac{EG}{CG} = \frac{FG}{DG} \Rightarrow \frac{EF}{18} = \frac{5}{10}$$

$$\Rightarrow EF = 9$$



Now, since $\triangle ABC$ and $\triangle EFC$ are similar triangles,
 therefore $\frac{AC}{AB} = \frac{EC}{EF}$

$$\Rightarrow \frac{AC}{15} = \frac{15}{9}$$

$$\Rightarrow AC = 25$$

- 66 Since $\triangle PQD$ and $\triangle ABD$ are similar, therefore

$$\frac{QD}{BD} = \frac{PQ}{AB} \quad \dots(i)$$

Similarly, $\triangle BPQ$ and $\triangle BCD$ are similar, therefore

$$\frac{BQ}{BD} = \frac{PQ}{CD} \quad \dots(ii)$$

By adding equations (i) and (ii), we will have

$$\frac{BD}{BD} = PQ \left(\frac{1}{AB} + \frac{1}{CD} \right)$$

$$\Rightarrow \frac{1}{PQ} = \frac{AB + CD}{AB \times CD}$$

$$\Rightarrow PQ = \frac{AB \times CD}{AB + CD} = \frac{xy}{x + y}$$

Hence, choice (c) is the correct one.

☞ **Alternatively** Consider $AB = CD = 2$, then by ladder theorem $PQ = 1$. Now substitute the values of x, y, z in the given options, you will see that only choice (d) is valid. In this question, z plays no role, so it's irrelevant.

☞ **Alternatively** Apply cross Ladder Theorem, we have

$$\frac{1}{PQ} = \frac{1}{AB} + \frac{1}{CD}$$

$$= \frac{1}{x} + \frac{1}{y} = \frac{xy}{x + y}$$

Hence choice (c) is the correct one.

- 67 Apply similarity of triangles, $\triangle ADB \sim \triangle CDA$

$$\begin{aligned} \frac{AD}{BD} &= \frac{CD}{AD} \\ \Rightarrow \frac{3}{4} &= \frac{CD}{3} \\ \Rightarrow CD &= 2.25 \end{aligned}$$

- 70 Since, $CF \parallel AD$, so we have,

$$\angle FCA = \angle DAC = x \text{ and } \angle AFC = \angle MAD = x.$$

$$\text{Therefore, } \angle AFC = \angle FCA = x$$

Thus, we can conclude that in the $\triangle AFC$ the two sides are equal.

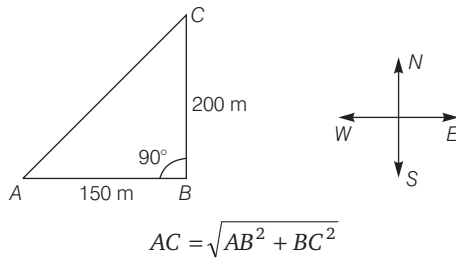
That is $AF = AC$.

Again, since $CF \parallel AD$, so we have,

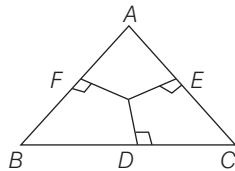
$$\begin{aligned} \frac{AB}{AF} &= \frac{BD}{CD} \\ \Rightarrow \frac{AB}{AC} &= \frac{BD}{CD} \quad (\because AF = FC) \end{aligned}$$

- 71 $\triangle ABD \cong \triangle CBD$

73



74



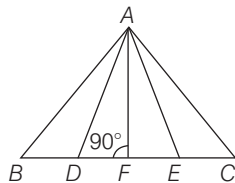
75

$$BD = 2DF$$

$$AB^2 = BF^2 + AF^2$$

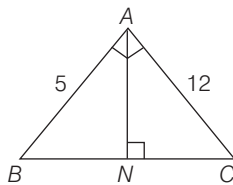
and

$$AD^2 = DF^2 + AF^2$$

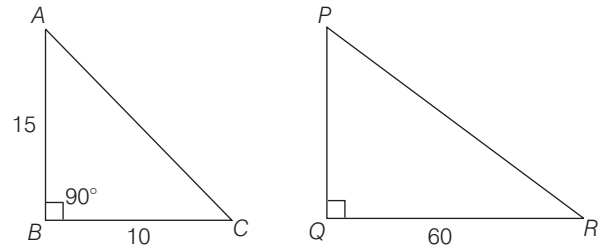


76

$$\begin{aligned} \frac{A(\triangle ANC)}{A(\triangle ANB)} &= \frac{\frac{AN \times NC}{2}}{\frac{AN \times BN}{2}} \\ &= \frac{NC}{NB} = \frac{(12 \times 12)}{5 \times 5} = \frac{144}{25} \end{aligned}$$



$$77 \quad \frac{AB}{BC} = \frac{PQ}{QR} \Rightarrow \frac{15}{10} = \frac{PQ}{60}$$



78 16 : 81

$$79 \quad \frac{BC}{PQ} = \frac{AC}{AP} = \frac{AB}{AQ}$$

$$80 \quad \frac{3}{x-3} = \frac{x-5}{3x-19} \Rightarrow x = 8 \text{ or } 9$$

$$81 \quad h = \frac{\sqrt{3}}{2} \text{ side} = \frac{\sqrt{3}}{2} \times 2a = a\sqrt{3}$$

$$82 \quad \frac{AB}{AC} = \frac{BD}{CD} \Rightarrow \frac{5}{AC} = \frac{2}{3} \Rightarrow AC = 7.5 \text{ cm}$$

$$83 \quad \frac{AB}{QR} = \frac{PB}{PR} \Rightarrow \frac{3}{9} = \frac{PB}{6} \Rightarrow PB = 2 \text{ cm}$$

84

$$\begin{aligned} \frac{AO}{BO} &= \frac{AQ}{BP} \\ \frac{10}{6} &= \frac{AQ}{9} \Rightarrow AQ = 15 \text{ cm} \end{aligned}$$

85

$$\begin{aligned} \frac{AB}{AC} &= \frac{AD}{AE} \Rightarrow \frac{12}{15} = \frac{6}{AE} \\ \therefore AE &= 7.5 \text{ cm} \end{aligned}$$

86

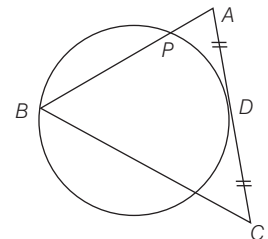
$$AB = AC \text{ and } AD = CD$$

$$\therefore AB = 2AD$$

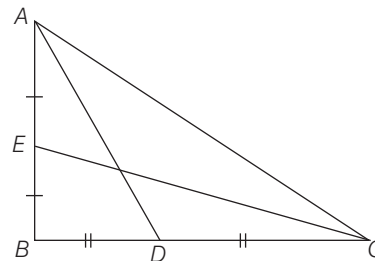
Now, since AD is a tangent

$$\therefore AD^2 = AP \times AB$$

$$\left(\frac{AB}{2}\right)^2 = AP \times AB \Rightarrow AB = 4AP$$



$$87 \quad AC = 5 \text{ cm} \Rightarrow AD = \frac{3\sqrt{5}}{2} \text{ cm}$$



$$AE = BE \text{ and } BD = CD$$

$$AB^2 = AC^2 - BC^2 = 25 - BC^2$$

...(i)

$$\begin{aligned} \text{and } AB^2 &= AD^2 - BD^2 = \left(\frac{3\sqrt{5}}{2}\right)^2 - BD^2 \\ &= \frac{45}{4} - \frac{BC^2}{4} \end{aligned} \quad \dots (ii)$$

From eq. (i) and (ii)

$$BC^2 = \frac{55}{3}$$

Now, from eq. (i)

$$AB^2 = 25 - \frac{55}{3} = \frac{20}{3}$$

$$\begin{aligned} \text{Also } CE^2 &= BE^2 + BC^2 = \left(\frac{1}{2}AB\right)^2 + BC^2 \\ &= \frac{AB^2}{4} + BC^2 = CE^2 = \frac{5}{3} + \frac{55}{3} = \frac{60}{3} = 20 \end{aligned}$$

$$\therefore CE = 2\sqrt{5} \text{ cm}$$

88 By Apollonius theorem

$$AB^2 + AC^2 = 2(AD^2 + BD^2)$$

$$100 + 196 = 2(AD^2 + 36)$$

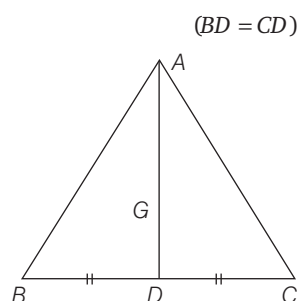
$$\Rightarrow AD^2 = 112$$

$$\therefore AD = 4\sqrt{7}$$

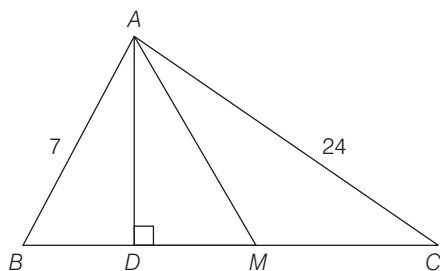
$$\text{Now, since } \frac{AG}{GD} = \frac{2}{1}$$

$$\Rightarrow AG = \frac{2}{3}AD$$

$$\therefore AG = \frac{2}{3} \times 4\sqrt{7} = \frac{8}{3}\sqrt{7} \text{ cm}$$



89 Since AM is the median of a right angled triangle.



$$\therefore AM = \frac{BC}{2} = \frac{25}{2} \text{ and } AD = \frac{AB \times AC}{BC} = \frac{7 \times 24}{25}$$

$$\therefore \frac{AD}{AM} = \frac{7 \times 24 \times 2}{25 \times 25} = \frac{336}{625}$$

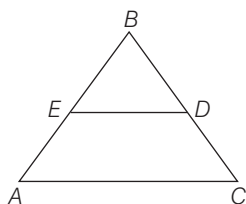
90

$$\triangle BED \sim \triangle BAC$$

$$\therefore \frac{BE}{BA} = \frac{ED}{AC} = \frac{BD}{BC} = \frac{1}{2}$$

$$\therefore \frac{A(\triangle BED)}{A(\triangle BAC)} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\therefore A(\triangle BDE) = 7.5 \text{ cm}^2$$



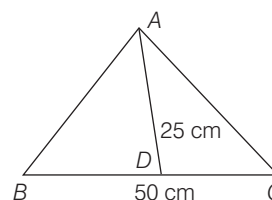
91 Apply Pythagoras theorem.

NOTE The sides given in option (d) cannot form any type of triangle.

$$\text{92 } \frac{AB}{BD} = \frac{AC}{CD} \Rightarrow \frac{12}{BC + CD} = \frac{8}{CD}$$

$$\frac{12}{4 + CD} = \frac{8}{CD} \Rightarrow CD = 8 \text{ cm}$$

$$\text{93 } AB^2 + AC^2 = 2 \left[AD^2 + \left(\frac{BC}{2}\right)^2 \right] \text{ (Appollonius theorem)}$$



$$2500 = 2 \left[625 + \left(\frac{BC}{2}\right)^2 \right] \Rightarrow BC = 50 \text{ cm}$$

94

$$\angle BAC = 15^\circ$$

$\therefore$

$$\angle BCA = 15^\circ \quad (\because AB = BC)$$

$\therefore$

$$\angle ABC = 180^\circ - (15^\circ + 15^\circ) = 150^\circ$$

$\therefore$

$$\angle ABD = 30^\circ \quad (180^\circ - 150^\circ)$$

$\therefore$

$$\sin 30^\circ = \frac{AD}{AB}$$

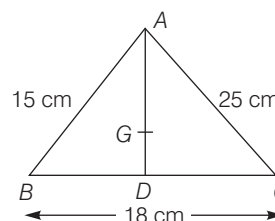
$$\frac{1}{2} = \frac{AD}{AB} \Rightarrow AD = 5 \text{ cm} \quad (\because AB = 10 \text{ cm})$$

$\therefore$

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} \times BC \times AD \\ &= \frac{1}{2} \times 10 \times 5 = 25 \text{ cm}^2 \end{aligned}$$

95

$$AB^2 + AC^2 = 2(AD^2 + BD^2)$$



$\therefore$

$$225 + 625 = 2(AD^2 + 81)$$

$\Rightarrow$

$$AD^2 = 344$$

$\Rightarrow$

$$AD = 2\sqrt{86} \text{ and } GD = \frac{1}{3}AD$$

$\therefore$

$$GD = \frac{2}{3}\sqrt{86} \text{ cm}$$

96

$$\angle BDE = 115^\circ$$

$\therefore$

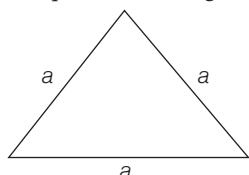
$$\angle ADE = 65^\circ \text{ and } \angle AED = 75^\circ$$

$\therefore$

$$\triangle AED \sim \triangle ABC$$

$$\begin{aligned}\therefore \frac{DE}{BC} &= \frac{AE}{AB} = \frac{AD}{AC} \\ \therefore \frac{2}{3} &= \frac{10}{AB} \\ \Rightarrow AB &= 15 \text{ cm}\end{aligned}$$

- 97** For the given perimeter of a triangle the maximum area is enclosed by an equilateral triangle.



$$\begin{aligned}\therefore 3a &= 24 \text{ cm} \\ \Rightarrow a &= 8 \text{ cm} \\ \therefore \text{Area} &= \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times (8)^2 = 16\sqrt{3} \text{ cm}^2\end{aligned}$$

- 98** $\triangle ADE \sim \triangle ACB$ (A-A-A property)

$$\begin{aligned}\therefore \frac{AE}{AB} &= \frac{AD}{AC} \\ \frac{10}{20} &= \frac{12}{AC} \\ \Rightarrow AC &= 24 \text{ cm} \\ EC &= AC - AE = 24 - 10 = 14 \text{ cm}\end{aligned}$$

99

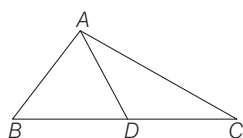
$$\begin{aligned}\angle ACD &= \angle ADC = x \\ \therefore \angle CAD &= (180^\circ - 2x) \\ \angle ABC &= \angle BAC = \frac{x}{2} \\ (\because \angle ABC + \angle BAC &= \angle ACD = x) \\ \therefore \angle BAC + \angle CAD + 81^\circ &= 180^\circ \\ \therefore \frac{x}{2} + (180^\circ - 2x) + 81^\circ &= 180^\circ \\ \therefore \frac{3}{2}x &= 81^\circ \\ \Rightarrow x &= 54^\circ\end{aligned}$$

☞ **Alternatively** Applying the same process go through options.

- 100** It is value (side) dependent i.e., it is different for different triangles unlike the case of equilateral triangle.

- 101** Let $\angle ACB = x$

$$\begin{aligned}\Rightarrow \angle CAD &= x \\ \Rightarrow \angle ADB &= 2x \\ \Rightarrow \angle ABD &= 2x\end{aligned}$$



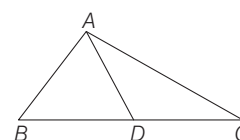
But since $\angle BAC = \angle ABC$

$$\begin{aligned}\Rightarrow \angle BAC &= 2x \\ \Rightarrow \angle BAD &= x \\ \therefore \angle BAC + \angle ABC + \angle ACB &= 180^\circ \\ \Rightarrow 2x + x + x &= 180^\circ \\ \Rightarrow x &= 36^\circ\end{aligned}$$

Hence choice (c) is the correct one.

- 102** As AD is the angle bisector, so

$$\frac{AB}{AC} = \frac{BD}{DC}$$



Further, since AD is the median, so

$$\begin{aligned}BD &= CD = k \text{ (say)} \\ \therefore \frac{AB}{AC} &= \frac{BD}{DC} = \frac{k}{k} = 1\end{aligned}$$

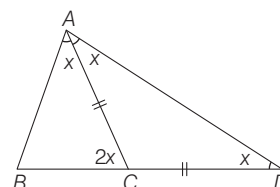
$$\Rightarrow AB = AC = 15 \text{ cm}$$

Hence choice (c) is the correct one.

- 103** Let $\angle BAC = x$, then $\angle ACB = 2x$. Now, extend BC till a point D such that when A and D are connected $\angle CAD$ becomes equal to $\angle BAC$.

But since $\angle ADC + \angle DAC = \angle ACB$

$$\Rightarrow \angle ADC = x$$



Therefore, $AC = DC = 4 \text{ cm}$. And, so $BD = BC + CD = 10 \text{ cm}$

Now you can see that $\triangle BAC$ and $\triangle BDA$ are similar triangles.

Therefore,

$$\begin{aligned}\frac{AB}{BC} &= \frac{BD}{AB} \\ \frac{AB}{6} &= \frac{10}{AB} \\ \Rightarrow AB &= \sqrt{60} = 2\sqrt{15} \text{ cm}\end{aligned}$$

Hence choice (d) is the correct one.

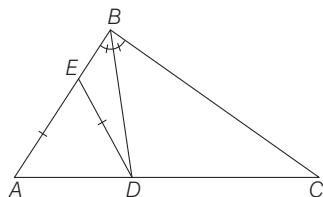
- 104** Since BD is the angle bisector of $\angle ABC$, therefore

$$\frac{AB}{BC} = \frac{AD}{CD} \Leftrightarrow AD = \frac{(AB)(CD)}{BC}$$

But, it is given that

$$\begin{aligned}\frac{(BD)(BC)}{CD} &= \frac{(BE)(AB)}{AE} \\ \Rightarrow \frac{(BD)(BC)}{(CD)(AB)} &= \frac{BE}{AE}\end{aligned}$$

$$\Rightarrow \frac{BD}{AD} = \frac{BE}{AE}$$



It implies that DE is the angle bisector of $\angle ADB$.

$$\therefore \angle BDE = \angle EDA = \angle EAD = x^\circ \text{ (say)}$$

$$\therefore \angle BDA + \angle DAB + \angle ABD = 2x + x + 66 = 180^\circ$$

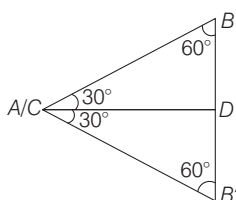
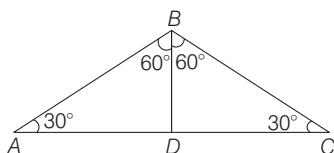
$$\Rightarrow x = 38^\circ$$

$$\Rightarrow \angle BAD = 38^\circ \text{ and } \angle ACB = 28^\circ$$

$$\therefore \angle BAD - \angle ACB = 10^\circ$$

Hence choice (a) is the correct one.

- 105** Consider the following figures. If you cut the first figure from the mid-point of its base along its altitude BD and then rotate it clockwise about the point D by 180° , then you will get an equilateral triangle, in which points A and C will overlap. Now it would become very easy to find the area of the given triangle as even though the shape of the triangle changes but its area remains the same.



$$\text{The area of the equilateral triangle} = \frac{\sqrt{3}}{4} \times a^2 = \frac{\sqrt{3}}{4} \quad (\because a = 1)$$

Hence choice (a) is the correct one.

Alternatively Consider the first figure. Since the triangle is an isosceles one, so the sides AB and AC would be of same length. And so the two angles BAC and BCA would be 30° each.

As the triangle ABC is an isosceles one, so the perpendicular dropped from the angle B on the opposite side AC will bisect the side AC as well as the angle ABC .

Then there will be two congruent triangles ABD and CBD . Now using 30-60-90 degree theorem (or

trigonometric ratios) we can find the base $AD = DC = \frac{\sqrt{3}}{2}$

and altitude $BD = \frac{1}{2}$.

$$\text{Therefore area of } \triangle ABC = \frac{1}{2} \times AC \times BD$$

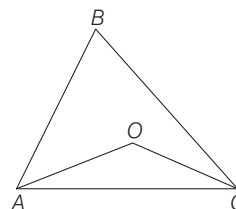
$$= \frac{1}{2} \times \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{4}$$

Alternatively Consider the first figure. Using

30-30-120 degree theorem or Cosine rule/Sine rule, we will have $AC = \sqrt{3}$ unit. After this we can use the formula for area of a triangle $\sqrt{s(s-a)(s-b)(s-c)}$, where s is the semiperimeter of the triangle and a, b, c are the sides of it.

$$\mathbf{106} \quad \angle AOC = 180 - \frac{1}{2}(\angle BAC + \angle BCA)$$

$$\Rightarrow (\angle BAC + \angle BCA) = 112^\circ$$

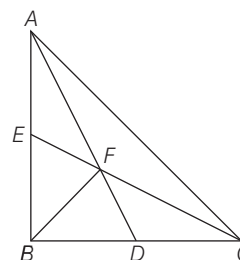


$$\text{and } \angle ABC = 180^\circ - (\angle BAC + \angle BCA)$$

$$\Rightarrow \angle ABC = 180^\circ - 112 = 68^\circ$$

Hence choice (c) is the correct one.

- 107** First of all, let's connect B to F . Now, we can easily figure out that the area of $\triangle BFD = \triangle CFD = x$, as the base and height of these two triangles are same. Similarly, we can find out that area of $\triangle AEF = \triangle BFE = y$.



But, we know that area of $\triangle ABD = \triangle EBC$.

$$\text{Therefore, } 2x + y = x + 2y$$

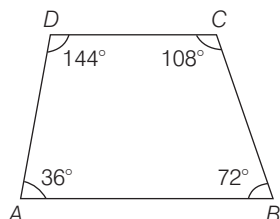
$$\Rightarrow x = y$$

Therefore, area of $\triangle AEF = x$ and area of quadrilateral $BEFD = 2x$.

Hence choice (c) is the correct one.

Introductory Exercise 12.3

1 $x + 2x + 3x + 4x = 360 \Rightarrow x = 36$

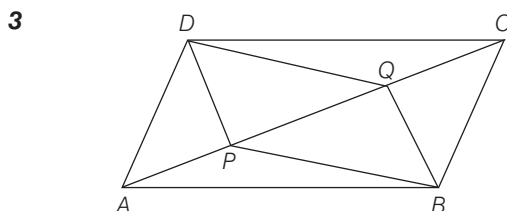


$\therefore$ The angles of quadrilateral (in order) are $36^\circ, 72^\circ, 108^\circ, 144^\circ$

Since opposite angles are supplementary, therefore $AB \parallel CD$. Hence it is a trapezium.

2 $3x + 4x + 5x + 6x = 360^\circ \Rightarrow x = 20 \therefore 6x = 120^\circ$

Hence, the largest angle of a quadrilateral is 120° .

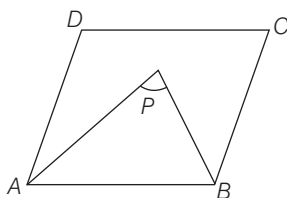


4 $\angle DAB = 2x \Rightarrow \angle ABC = 2y$

$2x + 2y = 180 \Rightarrow x + y = 90^\circ$

i.e., $\angle PAB + \angle PBA = 90^\circ$

$\therefore \angle APB = 90^\circ$



5 Prove that opposite sides are parallel, and diagonals are equal. Also all the interior angles are right angle.

6 $\angle BCE = 102^\circ$ and $\angle ECD = 60^\circ$

$\therefore \angle BCD = 102^\circ + 60^\circ = 162^\circ$

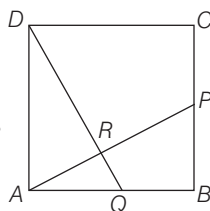
7 $\therefore \triangle ADQ \cong \triangle BAP$

and $\angle DCP = 90^\circ$

$\angle CDR$ and $\angle CPR$ are supplementary

$\therefore \angle DCP + \angle DRP = 180^\circ$

$\therefore \angle DRP = 90^\circ$

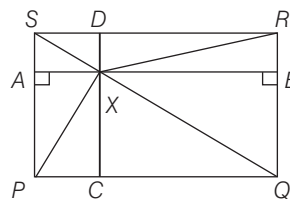


8 Area of $\triangle SXP = \frac{(SP) \times (AX)}{2}$

Area of $\triangle RXQ = \frac{(RQ) \times (BX)}{2}$

Area of $\triangle SXR = \frac{(SR) \times (DX)}{2}$

Area of $\triangle PXQ = \frac{(PQ) \times (CX)}{2}$



Area of $\triangle SXP + \triangle RXQ = \frac{SP \times AX}{2} + \frac{RQ \times BX}{2}$
 $= \frac{1}{2} (l \times b)$... (i)

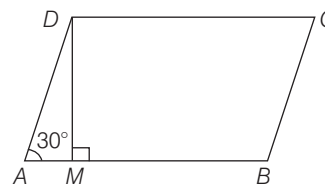
where $SP = RQ = b$ and $PQ = SR = l$

and $AX + BX = AB = PQ = SR = l$

and Area of $\triangle SXR + \triangle PXQ = \frac{(SR) \times (DX)}{2} + \frac{(PQ) \times (CX)}{2}$
 $= \frac{(SR) \times (DX)}{2} + \frac{(SR) \times (CX)}{2}$
 $= \frac{SR}{2} (DX + CX)$
 $= \frac{l}{2} \times b = \frac{lb}{2}$... (ii)

From eqs. (i) and (ii), we find that option (c) is correct.

9 $AD = BC = 20$ cm
 $AB = 20$



In $\triangle ADM$, $\frac{DM}{AD} = \sin 30^\circ = \frac{1}{2}$

$\therefore DM = \frac{1}{2} AD = 10$ cm

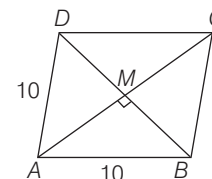
$\therefore$ Area of parallelogram = base $\times$ height
 $= 20 \times 10 = 200$ cm²

10 Let $DB = 12$ m

$\therefore BM = 6$ m

$\therefore AM = \sqrt{10^2 - 6^2}$

$AM = 8$ m



(Remember: Diagonals bisect each other at right angles)

(Bisect: Divide equally into two parts)

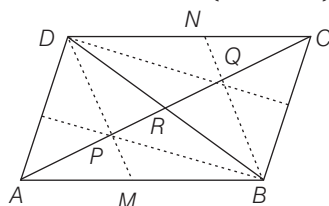
$\therefore AC = 8 \times 2 = 16$ m

11

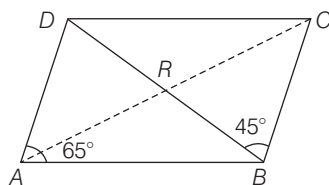
$$\angle BCD = \angle DAB = 65^\circ$$

 $\therefore$

$$\angle CDB = 180^\circ - (65^\circ + 45^\circ) = 70^\circ$$



- 12 Let AC and BD bisect each other at R, then $AP = 2PR$ and $QC = 2RQ$



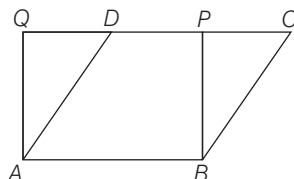
($\because$ AR and CR are the medians of $\triangle ADB$ and $\triangle BDC$)

and $AP = QC$ and $PR = RQ$ ($\because \triangle DAB \cong \triangle BCD$)

$\therefore AP = PQ$

- 13 Since their base and perpendicular heights are same.

- 14 ABCD is a parallelogram and ABPQ is a rectangle having same base and equal perpendicular height. Therefore the area of both is same.



Now, since $AD > AQ$

and $BC > BP$

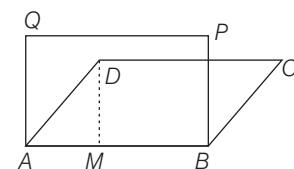
$\therefore 2(AB + BC) > 2(AB + BP)$

$\therefore$ Perimeter of parallelogram $>$ perimeter of rectangle.

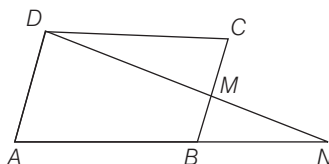
- 15 $AB = CD = PQ = l$

and $AQ = BP = AD = CB = b$

Bases of rectangle ABPQ and parallelogram ABCD is same, but perpendicular heights of parallelogram is less than that of rectangle. Therefore area of parallelogram is less than that of rectangle.



- 16 $AD = 2BM$ and $AD \parallel BM$

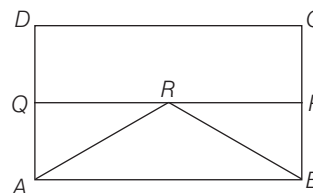


$\therefore$ From mid-point theorem

$$\therefore \frac{NM}{ND} = \frac{NB}{NA} = \frac{BM}{AD} = \frac{1}{2}$$

$$\therefore AN = 2AB$$

- 17 $A(\square ABPQ) = \frac{1}{2} A(\square ABCD)$

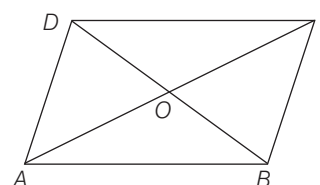


$$\text{and } A(\triangle ARB) = \frac{1}{2} A(\square ABPQ)$$

$$\therefore A(\triangle ARB) = \frac{1}{4} A(\square ABCD) \left[A(\triangle ARB) = \frac{1}{2} \times AB \times BP \right]$$

- 18 Let $BD = 6 \text{ m}$ and $AC = 8 \text{ m}$

$$\therefore AO = 4 \text{ m} \text{ and } BO = 3 \text{ m}$$



$$\text{Let } AB = 5 \text{ m} \therefore \angle AOB = 90^\circ$$

$$\Rightarrow \angle BOC = \angle AOD = \angle DOC = 90^\circ$$

$\therefore BC$ is also 5 m.

Hence, ABCD is a rhombus

$$\therefore \text{Area of rhombus } ABCD = \frac{AC \times BD}{2} = \frac{6 \times 8}{2} = 24 \text{ m}^2$$

- 19 $BD = \sqrt{15^2 + 20^2} = 25 \text{ cm}$

($\triangle ABD$ is right angled triangle)

$\therefore \triangle BCD$ is equilateral triangle.

$\therefore$ Area of ABCD = Area of $\triangle ABD$ + Area of $\triangle BCD$

$$= \frac{1}{2} \times 15 \times 20 + \frac{\sqrt{3}}{4} \times (25)^2 = \frac{25}{4} (24 + 25\sqrt{3}) \text{ cm}^2$$

20.

$$\frac{BE}{AB} = \sin 30^\circ = \frac{1}{2}$$

$$\therefore BE = \frac{1}{2} \times AB = 6 \text{ cm} = CF$$

$$\text{and } \frac{CF}{DF} = \tan 45^\circ = 1$$

$$\therefore DF = CF = 6 \text{ cm}$$

$$\text{and } \frac{AE}{AB} = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\therefore AE = 6\sqrt{3} \text{ cm}$$

$$\therefore AD = AE + EF + FD = 6\sqrt{3} + 6 + 6 = 6(2 + \sqrt{3}) \text{ cm}^2$$

$$\text{Area of trapezium } ADCB = \frac{1}{2} \times (AD + BC) \times BE$$

$$= \frac{1}{2} \times [6(2 + \sqrt{3}) + 6] \times 6$$

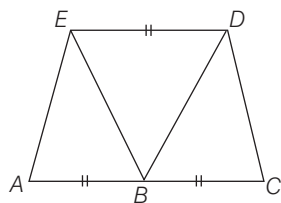
$$= \frac{6}{2} (2 + \sqrt{3} + 1) \times 6 = 18 \times (3 + \sqrt{3}) = 18(3 + \sqrt{3}) \text{ cm}^2$$

21 $A(\triangle ABE) = A(\triangle BDE)$

$$= A(\triangle BCE)$$

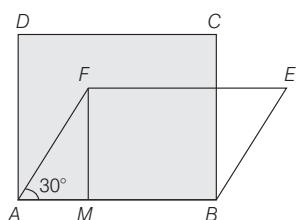
$$A(\triangle ABC) = \frac{1}{3} A(\square ACDE)$$

$$= \frac{1}{3} \times 36 = 12 \text{ cm}^2$$



22 $ABCD$ is a square and $ABEF$ is a rhombus.

$$\frac{FM}{AF} = \sin 30^\circ = \frac{1}{2} \Rightarrow FM = \frac{AF}{2}, \quad AF = AB$$



Area of square $= a^2$ $(AB = AD = a)$

Area of rhombus $= \frac{a \times a}{2}$ $\left(FM = \frac{a}{2}\right)$

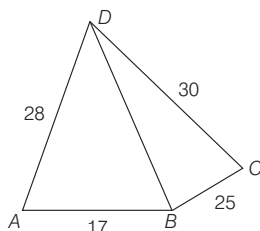
(Area of rhombus = base $\times$ height)

$$\therefore \frac{\text{Area of square}}{\text{Area of rhombus}} = \frac{2}{1}$$

23 Area of quadrilateral = Area of $\triangle ABD$ + Area of $\triangle BCD$

where

$$BD = 26 \text{ cm}$$



Hint Apply Hero's formula :

$$\text{Area of } \triangle = \sqrt{s(s-a)(s-b)(s-c)}$$

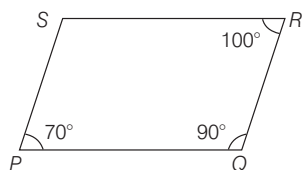
where

$$s = \frac{a+b+c}{2}$$

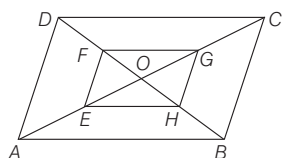
24 It is not possible.

$$\therefore \angle P + \angle R = 170^\circ \neq 180^\circ$$

$\therefore$ The given quadrilateral is not cyclic so there is no any point in the given plane which is equidistant from its vertices.



25 By mid-point theorem,

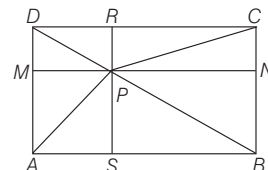


$$\frac{EF}{AD} = \frac{FG}{DC} = \frac{GH}{BC} = \frac{EH}{AB} = \frac{1}{2}$$

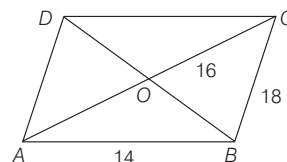
26 Since diagonals bisect each other at right angles and all sides are equal.

Apply Pythagoras theorem or Apollonius theorem.

27 Use Pythagoras theorem



28 Apply Apollonius theorem.



$$AB^2 + BC^2 = 2(BO^2 + AO^2)$$

$$\Rightarrow AB^2 + BC^2 = 2(BO^2 + 64)$$

$$\Rightarrow 196 + 324 = 2(BO^2 + 64)$$

$$\therefore BO^2 = 196 \Rightarrow BO = 14 \text{ cm} \Rightarrow BD = 2BO = 28 \text{ cm}$$

29 Consider the following diagram,

where AB is parallel to CD and AC and BD act as transversal lines.

$$\angle AOB = \angle COD \text{ (Opposite angles)}$$

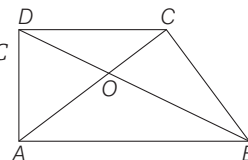
$$\angle OAB = \angle OCD \text{ (Alternate angles)}$$

$$\angle ABO = \angle CDO \text{ (Alternate angles)}$$

$$\therefore \triangle AOB \sim \triangle COD \text{ (AAA similarity)}$$

$$\text{Therefore } \frac{AB}{CD} = \frac{AO}{OC} = \frac{BO}{OD}$$

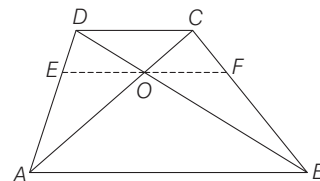
Hence choice (d) is correct.



30 Let us consider the following diagram. Where

$AB \parallel CD \parallel EF$. Further, let us consider $AB = m$ and

$$CD = n, \text{ we have, } \frac{OA}{OC} = \frac{m}{n} \text{ and } \frac{OB}{OD} = \frac{m}{n}.$$



$$\text{But, } EF = \frac{(AB)n + (CD)m}{m+n} = \frac{(m)n + (n)m}{(m+n)}$$

$$= \frac{2mn}{m+n} = \frac{2(AB)(CD)}{(AB+CD)}$$

$$\text{Therefore, } EF = \frac{2(AB)(CD)}{(AB+CD)}$$

$$= \frac{2(6 \times 2)}{6+2} = 3 \text{ cm}$$

Hence choice (a) is the correct one.

Introductory Exercise 12.4

- 1 Each interior angle = 140°
 $\therefore$ Exterior angle = $180^\circ - 140^\circ = 40^\circ$
 $\therefore$ Number of sides = $\frac{360^\circ}{\text{Exterior angle}} = \frac{360^\circ}{40^\circ} = 9$
- ☞ **Alternatively** It can be checked easily through options.
- 2 Exterior angle of a regular hexagon = $\frac{360^\circ}{6} = 60^\circ$
 $\therefore$ Interior angle = $180^\circ - 60^\circ = 120^\circ$
- 3 $\therefore$ Interior angle of pentagon = $180^\circ - \frac{360^\circ}{5} = 108^\circ$
 $\therefore$ Interior angle of required polygon = $5/6 \times 108^\circ = 90^\circ$
 $\therefore$ Each exterior angle of the required polygon = $180^\circ - 90^\circ = 90^\circ$
 $\therefore$ Number of sides = $\frac{360^\circ}{\text{Exterior angle}} = \frac{360^\circ}{90^\circ} = 4$
- ☞ **Alternatively** Best way is to go through options.
- 4 Sum of all the interior angles = $(n - 2)180^\circ$
 $1260^\circ = (n - 2)180^\circ$
 $\Rightarrow n = 9$ ($n \rightarrow$ number of sides)
- 5 Interior angle = $3 \times$ exterior angle
 $180^\circ - \frac{360^\circ}{n} = 3 \times \left(\frac{360^\circ}{n}\right) \Rightarrow n = 8$
- ☞ **Alternatively** Best way is to go through options.
- 6 Go through option. Exterior angle = $\frac{360^\circ}{6} = 60^\circ$
 Interior angle = $180^\circ - 60^\circ = 120^\circ$
 Difference between interior angle and exterior angle = $120^\circ - 60^\circ = 60^\circ$
- 7 $\frac{n(n-3)}{2} = 54^\circ \Rightarrow n^2 - 3n = 108^\circ \Rightarrow n = 12$
- ☞ **Alternatively** Put the value of n from the choices given below.
- 8 Go through options. Let us consider the correct option (d).

Number of sides	5	:	10
Exterior angle =	72°		36°
Interior angle =	$180^\circ - 72^\circ$		$180^\circ - 36^\circ$
	$(= 108^\circ)$		$(= 144^\circ)$
	3	:	4

 Hence, option (d) is correct.
- 9 Go through options
- ☞ **Alternatively** Sum of all exterior angles = 360°
 $\therefore$ Sum of interior angles = $4 \times 360^\circ = 1440^\circ$
 $\therefore 1440^\circ = (n - 2)180^\circ$
 $\Rightarrow n = 10$ (no. of sides)

10 Exterior angle = $\frac{360^\circ}{9} = 40^\circ$

Interior angle = $180^\circ - 40^\circ = 140^\circ$

$\therefore$ Interior angle : exterior angle = $140^\circ : 40^\circ = 7 : 2$

- 11 Let us consider a square which has 4 sides and 2 diagonals and each interior angle 90° . So it has $k = \frac{1}{2}$.

If you put $\theta = 90^\circ$ in choice (c) you will get $k = \frac{1}{2}$, but in other choices k would not be equal to $\frac{1}{2}$.

Else, you can assume another polygon say a regular hexagon which has 6 sides and 9 diagonals and each interior angle 120° . So it has $k = \frac{3}{2}$.

So, if you put $\theta = 120^\circ$ in choice (c) you will get $k = \frac{3}{2}$, but in other choices you won't get $k = \frac{3}{2}$.

These two tests suggest that choice (c) is the correct answer.

Hence choice (c) is the correct answer.

☞ **Alternatively** In an n -sided polygon the number of diagonals = $\frac{n(n-3)}{2}$

Therefore, $nk = \frac{n(n-3)}{2}$

$\Rightarrow k = \frac{(n-3)}{2}$

But, number of sides,

$n = \frac{(n-2)\pi}{\theta} \Rightarrow \frac{2\pi}{\pi - \theta}$

Therefore, $k = \frac{(n-3)}{2}$

$= \frac{\left(\frac{2\pi}{\pi - \theta}\right) - 3}{2} \Rightarrow k = \frac{3\theta - \pi}{2(\pi - \theta)}$

- 12 In $\triangle EAD$, $\angle AED = 108^\circ$ and $EA = ED$,

$\therefore \angle EAD = \angle EDA = 36^\circ$

It implies that $\angle DAB = 108 - 36 = 72^\circ$

Similarly, in $\triangle DCB$, $\angle DCB = 108^\circ$

and $CD = CB$,

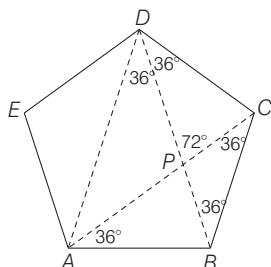
$\therefore \angle CDB = \angle CBD = 36^\circ$

It implies that $\angle DBA = 108 - 36 = 72^\circ$

$\therefore \angle ADB = 180 - (72 + 72) = 36^\circ$

Hence choice (a) is the correct one.

- 13** In $\triangle ACB$, $\angle ABC = 108^\circ$ and $AB = BC$,
 $\therefore \angle BAC = \angle BCA = 36^\circ$
 Similarly, in $\triangle DCB$, $\angle DCB = 108^\circ$ and $CD = CB$,
 $\therefore \angle CDB = \angle CBD = 36^\circ$



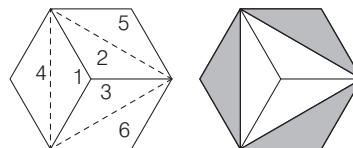
Now, in $\triangle BPC$, $\angle PBC = \angle PCB = 36^\circ$,
 $\therefore \angle BPC = 180 - (36 + 36) = 108^\circ$

Introductory Exercise 12.5

- 1** $AC = \sqrt{OA^2 - OC^2} = \sqrt{(17)^2 - (8)^2} = 15 \text{ cm}$
 $\therefore AB = 2AC = 30 \text{ cm} \quad (\because AC = BC)$
- 2** $AB = 8 \text{ cm}$
 $\therefore AM = 4 \text{ cm}$
 and $OA = 5 \text{ cm}$
 $\therefore OM = \sqrt{(OA)^2 - (AM)^2} = \sqrt{25 - 16} = 3 \text{ cm}$
- 3** $\therefore AB$ is the perpendicular bisector of OO' .
 $\therefore \angle OMA = 90^\circ$
- 4** $AM : BM : 1 : 1$
- 5** $\angle AOC = \angle BOC \quad (\because AC = BC)$
 $\therefore OC$ is the perpendicular bisector of AB .
 $\therefore AM = BM$
- 6** $OA = 13 \text{ cm} \Rightarrow AM = 5 \text{ cm}$
 $\therefore OM = \sqrt{(13)^2 - (5)^2} = 12 \text{ cm}$
- 7** Degree measure of major arc $= 360^\circ - 130^\circ = 230^\circ$
- 8** It is necessarily a rectangle.
- 9** $\angle ADC = 180^\circ - 65^\circ = 115^\circ$
 $\therefore \angle DCA = 180^\circ - (115^\circ + 45^\circ) = 20^\circ$
 ($ADCB$ is a cyclic quadrilateral)
- 10** $\angle OBA = \angle OAB = 55^\circ$
 $\therefore \angle AOB = 180^\circ - (55^\circ + 55^\circ) = 70^\circ$
 and $\angle AOB = \angle COD = 70^\circ$
- 12** $\triangle BAP \sim \triangle CDP \quad (\because \angle A = \angle D \text{ and } \angle APB = \angle DPC)$
 $\therefore \frac{AB}{CD} = \frac{AP}{DP} = \frac{BP}{CP}$
 $\therefore AP \times PC = BP \times DP$
- 13** $\angle BDC = 180^\circ - (110^\circ + 30^\circ) = 40^\circ$
 and $\angle BAC = \angle BDC = 40^\circ$ (angles of the same segment)

Thus, $\angle DPC = 180 - \angle BPC = 180 - 108 = 72^\circ$
 Hence choice (b) is the correct one.

- 14** Connect the centre of the hexagon with the three vertices of the given triangle, as shown below. Due to symmetry there are now total 6 congruent triangles, with 30-30-120 degree angles. Out of 6 such triangles exactly 3 triangles are comprised in the given triangle.



So the required ratio $= \frac{3}{6} = \frac{1}{2}$

Hence choice (d) is the correct one.

- 14** $\angle ABO = 60^\circ = \angle BAO \quad (\because AO = BO, \text{ radii of the circle})$
 $\therefore \angle AOB = 180^\circ - (60^\circ + 60^\circ) = 60^\circ$
 $\therefore \angle ACB = \frac{1}{2} \angle AOB = 30^\circ$
- 15** $\angle CBA = \frac{1}{2} \angle COA = 60^\circ$
 $\therefore \angle CBE = 180^\circ - \angle CBA$
 $180^\circ - 60^\circ = 120^\circ$
- 16** $\angle BDA = 90^\circ \Rightarrow \angle DBA = 50^\circ$
 $\therefore \angle DAB = 180^\circ - (90^\circ + 50^\circ) = 40^\circ$
- 17** $\angle BAC = 60^\circ$
 $\therefore \angle BEC = 180^\circ - 60^\circ = 120^\circ$
- 18** $\angle ABD = 45^\circ \therefore \angle ACD = 45^\circ$
- 19** $\angle BDC = \angle BAC = 30^\circ$
 $\therefore \angle BCD + \angle BDC + \angle DBC = 180^\circ$
 $\therefore \angle BCD = 180^\circ - (30^\circ + 60^\circ)$
 $\angle BCD = 90^\circ$
- 20** $\angle ADC = 140^\circ$
 $\therefore \angle ABC = 40^\circ \Rightarrow \angle ACB = 90^\circ$
 $\therefore \angle BAC = 180^\circ - (90^\circ + 40^\circ) = 50^\circ$
- 21** $\angle CAB = \frac{1}{2} \angle BOC = \frac{1}{2} \times 40^\circ = 20^\circ$
- 22** $\angle OBA = \angle OAB = \left(\frac{180^\circ - 80^\circ}{2} \right) = 50^\circ$
 and $\angle OCA = \angle OAC = \left(\frac{180^\circ - 120^\circ}{2} \right) = 30^\circ$
 $\therefore \angle BAC = \angle BAO + \angle OAC = 50^\circ + 30^\circ = 80^\circ$
- 23** $\angle ADC = \frac{100^\circ}{2} = 50^\circ$
 $\therefore \angle ABC = 180^\circ - 50^\circ = 130^\circ$
 $\therefore \frac{m \angle ADC}{m \angle ABC} = \frac{50^\circ}{130^\circ} = \frac{5}{13}$

$$24 \quad \angle APB = 50^\circ \quad \left(\because m \angle APB = \frac{1}{2} m \angle AOB \right)$$

$$\therefore \angle ACB = 180^\circ - 50^\circ = 130^\circ$$

$$\therefore \angle BCD = 180^\circ - 130^\circ = 50^\circ$$

$$25 \quad \angle ABC = 180^\circ - 120^\circ = 60^\circ$$

$$\angle ACB = 90^\circ$$

$$\therefore \angle CAB = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$$

$$26 \quad \angle COD = \angle AOB = 70^\circ$$

$$\therefore \angle OCD = \frac{180^\circ - 70^\circ}{2} = 55^\circ$$

$$27 \quad \angle ABK = 180^\circ - (105^\circ + 40^\circ) = 35^\circ$$

$$\therefore \angle ACD = \angle ABD = 35^\circ$$

$$28 \quad \angle BAC = 80^\circ \quad (\because \angle ABC = \angle ACB = 50^\circ)$$

$$\therefore \angle BDC = \angle BAC = 80^\circ$$

$$29 \quad \angle ADB = 90^\circ$$

$$\therefore \angle DBA = 20^\circ \quad [180^\circ - (90^\circ + 70^\circ)]$$

$$\therefore \angle CBA = 30^\circ + 20^\circ = 50^\circ$$

$$\therefore \angle CDA = 180^\circ - 50^\circ = 130^\circ$$

$$\therefore \angle CDB = 130^\circ - 90^\circ = 40^\circ$$

$$30 \quad \angle BAD = 80^\circ \therefore \angle DCE = 80^\circ$$

$$31 \quad \angle AOC = 2 \times 50 = 100^\circ$$

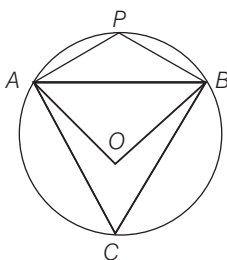
$$\therefore \angle ABC = \frac{100}{2} = 50^\circ$$

 **NOTE** There are some more ways to solve it. Find them.

$$32 \quad \angle AOB = 2 \angle ACB = 2 \times 25 = 50^\circ$$

$$33 \quad \angle APB = 180^\circ - \frac{1}{2} \times 90^\circ = 135^\circ$$

$$\left[\because \angle ACB = 45^\circ = \left(\frac{1}{2} \times 90^\circ \right) \right. \\ \left. \text{and } \angle APB + \angle ACB = 180^\circ \right]$$



$$34 \quad OA = \sqrt{(OB)^2 - (AB)^2}$$

$$\therefore OA = \sqrt{169 - 144} = 5 \text{ cm}$$

$$35 \quad PN \times PR = PQ^2 \Rightarrow PN \times 25 = (15)^2$$

$$\therefore PN = 9 \text{ cm}$$

$$36 \quad TQ = TP \text{ and } TP = TR$$

$$\therefore TQ = TP = TR$$

$$37 \quad \angle BAC = 90^\circ$$

$$\angle BCA = 60^\circ \quad (\because \angle BCA = \angle BAQ)$$

$$\therefore \angle ABC = 180^\circ - (90^\circ + 60^\circ)$$

$$\angle ABC = 30^\circ$$

$$38 \quad \angle DBQ = 65^\circ$$

$$\therefore \angle DAB = 65^\circ$$

$$\therefore \angle DCB = 180^\circ - 65^\circ = 115^\circ$$

$$39 \quad AP \times BP = DP \times CP$$

$$2 \times 6 = DP \times 3$$

$$\Rightarrow DP = 4 \text{ cm}$$

$$40 \quad PA \times PB = PC \times PD$$

$$3 \times (3 + 5) = 2 \times PD$$

$$\therefore PD = 12 \text{ cm}$$

$$\therefore CD = PD - PC = 12 - 2 = 10 \text{ cm}$$

$$41 \quad PT^2 = PA \times PB$$

$$5 \times 5 = 4 \times (4 + x)$$

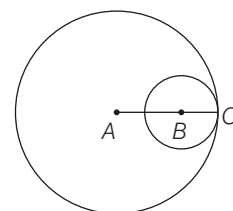
$$\Rightarrow x = \frac{9}{4} \text{ cm,}$$

where $AB = x$

42 Let them touch each other at O and A, B be the centres of the two circles, then $OA = 13 \text{ cm}$

and $OB = 5 \text{ cm}$

$$\therefore AB = OA - OB = 8 \text{ cm}$$



$$43 \quad AB = 5 \text{ cm} = x + y$$

$$BC = 6 \text{ cm} = y + z$$

$$AC = 7 \text{ cm} = z + x$$

$$\therefore 2(x + y + z) = 5 + 6 + 7 = 18 \text{ cm}$$

$$\Rightarrow x + y + z = 9 \text{ cm}$$

$$\Rightarrow x = (x + y + z) - (y + z) = 9 - 6 = 3 \text{ cm}$$

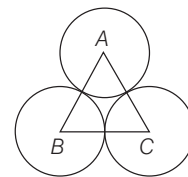
$$\therefore x + y = 5 \text{ cm}$$

$$\therefore y = 5 - x = 5 - 3 = 2 \text{ cm}$$

$$\therefore z = 6 - y = 6 - 2 = 4 \text{ cm}$$

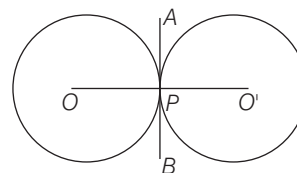
$$\therefore x = 3 \text{ cm}$$

$$y = 2 \text{ cm and } z = 4 \text{ cm}$$



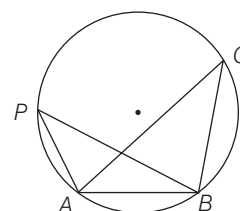
44 Prove with the help of property of tangents : "Tangents drawn from the same point on a circle are always equal."

45 Tangent is always perpendicular to the radius.



$$46 \quad \angle APB = \angle AQB$$

$$\text{when } \angle APB = \angle AQB = 90^\circ$$



then they are supplementary. Also they are supplementary when they are in different segments.

- 47** $CD = 6$ cm
 $\therefore AC = 6$ cm and $BC = 6$ cm
 ($\because AC = CD$ and $BC = CD$. Two tangents from the same point are always equal.)
 $\therefore AB = 12$ cm

- 48** $\angle CAB = \angle BCD$
 and $\angle DAB = \angle BDC$
 $\therefore \angle CAD = \angle CAB + \angle DAB = \angle BCD + \angle BDC$
 $\Rightarrow \angle CAD = \angle BCD + \angle BDC$
 $\therefore \angle CAD + \angle CBD = \angle BCD + \angle BDC + \angle CBD = 180^\circ$

- 49** $AB = 24$ cm
 $\therefore AC = BC = 12$ cm
 $OA = 20$ cm
 $\therefore OC = \sqrt{(OA)^2 - (AC)^2}$
 $= \sqrt{400 - 144} = 16$ cm
 and $O'A = 37$ cm
 $\therefore O'C = \sqrt{(O'A)^2 - (AC)^2}$
 $= \sqrt{(37)^2 - (12)^2} = 35$ cm
 $\therefore OO' = OC + CO' = 16 + 35 = 51$ cm

- 50** $AB = AC$, $OB = OC$

and BC is common and OA is common

also $\angle BOA = \angle COA$

$\therefore \triangle BOA \cong \triangle COA$

$\therefore BC$ and OA are perpendicular bisector to each other

$$[\text{Area of } \Delta = \sqrt{s(s-a)(s-b)(s-c)}]$$

$$s = \frac{a+b+c}{2} = \frac{5+5+6}{2} = 8 \text{ cm}$$

$$\text{Area of } \triangle BOA = \sqrt{8 \times 3 \times 3 \times 2} = 12 \text{ cm}^2$$

$$\text{Again area of } \triangle BOA = \frac{1}{2} OA \times BP = \frac{1}{2} \times 5 \times BP$$

$$12 = \frac{1}{2} \times 5 \times BP$$

$$\Rightarrow BP = 4.8 \text{ cm}$$

$$\therefore BC = 2BP = 2 \times 4.8 = 9.6 \text{ cm}$$

- 51** $OA = OC = 17$ cm

$$AP = \frac{1}{2} AB = \frac{1}{2} \times 16 = 8 \text{ cm}$$

$$\therefore OP = \sqrt{OA^2 - AP^2} = 15 \text{ cm}$$

$$\text{Now, } PQ = OP + OQ = 23 \text{ cm}$$

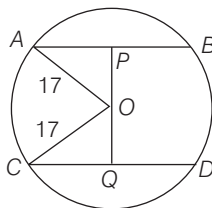
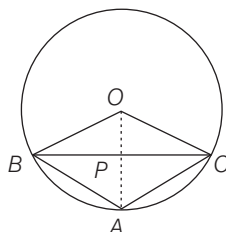
$$\therefore 15 + OQ = 23 \text{ cm}$$

$$\therefore OQ = 8 \text{ cm}$$

$$\text{Again, } CQ = \sqrt{(OC)^2 - (OQ)^2} \Rightarrow CQ = \sqrt{(17)^2 - (8)^2}$$

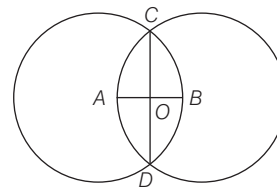
$$\therefore CQ = 15 \text{ cm}$$

$$\therefore CD = 2CQ = 30 \text{ cm}$$



- 52** $AB = r$ (say) then $AC = BC = r$, also

$$\therefore OA = OB = r/2 \quad (CD \text{ is a common chord})$$



$$\therefore OC = \sqrt{(AC)^2 - (OA)^2} = \sqrt{r^2 - \left(\frac{r}{2}\right)^2} = \sqrt{\frac{3r^2}{4}} = \frac{\sqrt{3}}{2} r$$

$$\therefore CD = 2CO = 2 \times \frac{\sqrt{3}}{2} r = \sqrt{3} r \therefore \frac{l(CD)}{l(AC)} = \frac{\sqrt{3} r}{r} = \frac{\sqrt{3}}{1}$$

- 53** Let A be the centre of larger circle and B be the centre of the smaller circle.

$$OA = 3 \text{ cm and } BO = 2 \text{ cm}$$

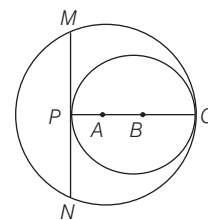
$$\text{and } AB = (OA - OB) = 1 \text{ cm}$$

$$\text{also } BP = 2 \text{ cm } \therefore AP = 1 \text{ cm}$$

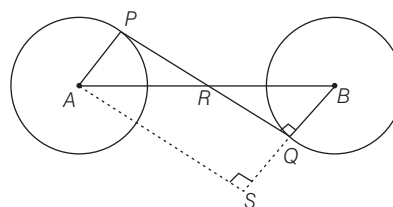
P is the point of contact of tangent MN .

$$\therefore MP = \sqrt{AM^2 - AP^2} = \sqrt{3^2 - 1^2} = 2\sqrt{2} \text{ cm}$$

$$\therefore MN = 2MP = 4\sqrt{2} \text{ cm}$$



- 55.** $PQ = AS$ (PQ is a transverse common tangent)



$$\text{and } BS = BQ + SQ = BQ + AP$$

$$BS = 2BQ \quad (BQ = AP = \text{radius})$$

$$BS = 6 \text{ cm}$$

$$\therefore AS = \sqrt{(AB)^2 - (BS)^2} = \sqrt{(10)^2 - (6)^2} = 8 \text{ cm}$$

$$\therefore AS = 8 \text{ cm } \therefore PQ = 8 \text{ cm}$$

☞ **Alternatively** The length of transverse common tangent

$$= \sqrt{(\text{distance between the centres})^2 - (r_1 + r_2)^2}$$

$$= \sqrt{(10)^2 - (3 + 3)^2} = 8 \text{ cm}$$

- 57** Inradius of an incircle of right angled triangle

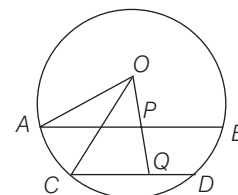
$$= \frac{\text{Base} + \text{Altitude} - \text{Hypotenuse}}{2}$$

$$= \frac{4 + 3 - 5}{2} = 1 \text{ cm}$$

☞ **Alternatively**

$$\text{Inradius} = \frac{\text{Area of triangle}}{\text{Semiperimeter of triangle}}$$

$$= \frac{(3 \times 4)/2}{(3 + 4 + 5)/2} = \frac{6}{6} = 1 \text{ cm}$$



- 58** Let AB and CD be the two chords, then $AB = 8$ cm and $CD = 6$ cm.

$\therefore AP = 4$ cm and $CQ = 3$ cm and $PQ = 1$ cm (given)

Let $OP = x$

$$\therefore AO^2 = AP^2 + OP^2$$

$$\therefore AO^2 = 4^2 + x^2 = 16 + x^2 \quad \dots(i)$$

$$\begin{aligned} \text{and } OC^2 &= CQ^2 + OQ^2 \\ &= CQ^2 + (OP + PQ)^2 = (3)^2 + (x + 1)^2 \\ OC^2 &= 9 + x^2 + 1 + 2x \quad \dots(ii) \end{aligned}$$

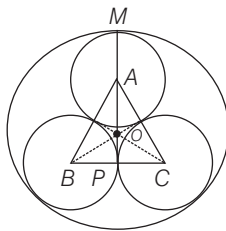
But $OA = OC$, are the radii of the same circle.

$$\therefore 16 + x^2 = 9 + x^2 + 1 + 2x \Rightarrow x = 3 \text{ cm}$$

$$\therefore OA^2 = 16 + x^2 = 16 + 9 = 25$$

$$\therefore OA = 5 \text{ cm}$$

$$\therefore \text{Diameter of the circle} = 2(OA) = 10 \text{ cm}$$



- 59** $AB = BC = AC = 2$ cm
($\because$ radius of each circle = 1 cm)

$$\therefore AP = \frac{\sqrt{3}}{2} \times 2 \Rightarrow AP = \sqrt{3} \text{ cm}$$

Let O be the centroid, then

$$OA = \frac{2}{3} \times \sqrt{3} = \frac{2}{\sqrt{3}} \text{ cm}$$

and $AM = 1$ cm

$$\therefore OM = (2/\sqrt{3}) + 1 = 2 + \sqrt{3}/\sqrt{3} \text{ cm}$$

OM is the radius of the larger circle.

$\therefore$ Area of the circumscribing circle

$$= \pi R^2 = \pi \times \left(\frac{2 + \sqrt{3}}{\sqrt{3}} \right)^2 = \frac{\pi}{3} (2 + \sqrt{3})^2$$

- 60** OA is the radius of the largest circle and O is the centroid of the all concentric circles. Let r_1, r_2, r_3 and r_4 be the radii of the concentric circles in increasing order then
Total area of given circle = $\pi \times (20)^2 = 400\pi$

$$\text{Area of each region} = \frac{1}{4} \times 400\pi = 100\pi$$

$$\therefore \text{Area of central region} = 100\pi = \pi r_1^2$$

$$\Rightarrow r_1 = 10 \text{ cm}$$

$$\text{Similarly, area of second region} = 100\pi = \pi (r_2^2 - r_1^2)$$

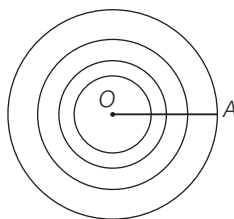
$$100\pi = \pi (r_2^2 - 100)$$

$$\Rightarrow r_2 = 10\sqrt{2} \text{ cm}$$

$$\text{Again, area of third region} = 100\pi = \pi (r_3^2 - r_2^2)$$

$$100\pi = \pi (r_3^2 - 200) \Rightarrow r_3^2 = 10\sqrt{3} \text{ cm}$$

$\therefore$ The required radii are 10, $10\sqrt{2}$ and $10\sqrt{3}$.



- 61** $\angle DAB + \angle DCB = 180^\circ$

$$\angle BAO + \angle OAD + \angle DCO = 180^\circ$$

$$\angle 69^\circ + (\angle OAD + \angle DCO) = 180^\circ$$

$$\angle OAD + \angle DCO = 111^\circ$$

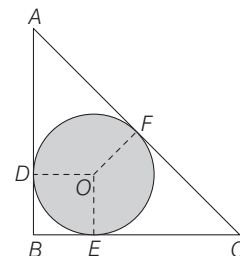
Hence choice (b) is the correct one.

- 62** $AC = \sqrt{2}$ cm, so $AB = BC = 1$ cm.

The inradius $OD = OE = OF$

$$= \frac{AB + BC - AC}{2} = \frac{2 - \sqrt{2}}{2} \text{ cm}$$

$$\begin{aligned} \text{Therefore } DE &= \sqrt{2} \left(\frac{2 - \sqrt{2}}{2} \right) \\ &= \sqrt{2} - 1 \text{ cm} \end{aligned}$$



Hence choice (a) is the correct one.

- 63** Using Alternate segment theorem, we get

$$\angle ABC = \angle QAC = 48^\circ$$

$$\angle BAP = \angle ACB = 64^\circ$$

$$\angle ABP = \angle ACB = 64^\circ$$

$$\therefore \angle APB = 180 - (\angle BAP + \angle ABP) = 52^\circ$$

$$\text{And } \angle BAC = 180 - (\angle ACB + \angle ABC) = 68^\circ$$

Opposite angles of a cyclic quadrilateral are always supplementary, therefore

$$\angle BDC = 180 - \angle BAC = 112^\circ$$

$$\text{Now, } \angle CBP = \angle ABC + \angle ABP = 48^\circ + 64^\circ = 112^\circ$$

$$\begin{aligned} \text{Thus, } \angle BDC + \angle CBP + \angle APB &= 112^\circ + 112^\circ + 52^\circ = 276^\circ \end{aligned}$$

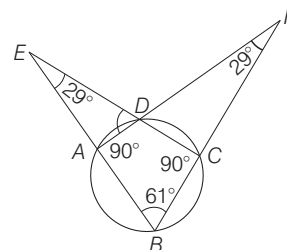
Hence choice (b) is the correct answer.

- 64** $\angle ECB = 180^\circ - (\angle CEB + \angle EBC) = 90^\circ$

$$\begin{aligned} \text{But since } A, B, C, D \text{ are cyclic, so } \angle DAB + \angle DCB &= 180^\circ \Rightarrow \angle DAB = 90^\circ \end{aligned}$$

$$\text{Again, } \angle AFB = 180^\circ - (\angle FAB + \angle FBA) = 29^\circ$$

$$\text{Since, } \angle EAD = 180^\circ - \angle BAD = 90^\circ$$



$$\text{and } \angle EDA = 180^\circ - (\angle DEA + \angle EAD) = 61^\circ$$

$$\therefore \alpha - \beta = 61 - 29 = 32^\circ$$

Hence choice (c) is the correct one.

☞ Alternatively

$$\alpha = \angle ADE = \angle ABC = 61^\circ$$

$$(\text{As } \angle ADE + \angle ADC = \angle ABC + \angle ADC = 180^\circ)$$

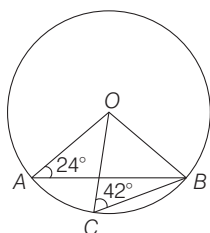
$$\therefore \angle CDF = 61^\circ$$

$$\text{and } \angle ACF = \angle BEC + \angle EBC = 90^\circ$$

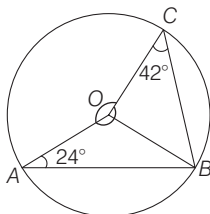
$$(\angle ACE \text{ is an external angle of } \triangle EBC)$$

$$\begin{aligned}\therefore \quad \beta &= \angle CFD = 180^\circ - (\angle CDF + \angle DCF) \\ &= 180^\circ - (61^\circ + 90^\circ) = 29^\circ \\ \therefore \quad \alpha - \beta &= 61^\circ - 29^\circ = 32^\circ\end{aligned}$$

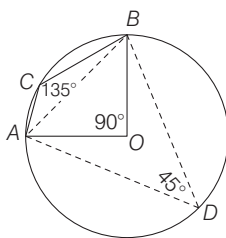
- 65 Case I :** $\angle OBC = \angle OCB = 42^\circ$
 $\therefore \quad \angle BOC = 180 - 2(42^\circ) = 96^\circ$
 and $\angle OBA = \angle OAB = 24^\circ$
 $\therefore \quad \angle AOB = 180 - 2(24^\circ) = 132^\circ$
 Thus, $\angle AOC = \angle AOB - \angle BOC$
 $\Rightarrow \quad \angle AOC = 132 - 96^\circ = 36^\circ$



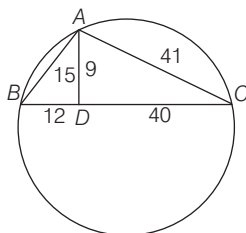
- Case II :** $\angle AOB = 180 - 2(24) = 132^\circ$
 and $\angle BOC = 180^\circ - 2(42) = 96^\circ$
 Thus, $\angle AOC = \angle AOB + \angle BOC$
 $\Rightarrow \quad \angle AOC = 132 + 96 = 228^\circ$
 Hence choice (d) is the correct one.



- 66** First of all complete the circle as shown below. Draw the chord AB and consider an arbitrary point D lying on the major arc of the circle.
 Now, central angle $AOB = 90^\circ$, so the angle in the major segment due to the same chord AB is angle $ADB = 45^\circ$.
 But since the quadrilateral $ADBC$ is a cyclic one, therefore
 $\angle ACB + \angle ADB = 180^\circ$
 $\Rightarrow \quad \angle ACB + 45^\circ = 180^\circ \Rightarrow \angle ACB = 135^\circ$
 Hence choice (b) is the correct one.



- 67** Let's drop a perpendicular AD on BC , we will have two right angled triangles $\triangle ADB$ and $\triangle ADC$.



By using Pythagoras theorem we can determine AD , BD and CD , as follows

$$\begin{aligned}AD^2 &= AB^2 - BD^2 \\ &= AB^2 - (BC - CD)^2 \\ &= 15^2 - (52 - CD)^2\end{aligned}\quad \dots(i)$$

$$\begin{aligned}\text{Also, } AD^2 &= AC^2 - CD^2 \\ &= 41^2 - CD^2\end{aligned}$$

Comparing eq. (i) and eq. (ii), we get
 $CD = 40$ cm, $BD = 12$ cm and $AD = 9$ cm

$$\begin{aligned}\text{Now, the area of triangle } ABC &= \frac{1}{2} \times AD \times BC \\ &= \frac{1}{2} \times 9 \times 52 = 234 \text{ sq cm}\end{aligned}$$

Let us consider that the circumradius of the triangle is R and area is A , then we have

$$\begin{aligned}A &= \frac{abc}{4R} \\ \Rightarrow \quad 234 &= \frac{15 \times 41 \times 52}{4R} \\ \Rightarrow \quad R &= \frac{205}{6} \text{ cm}\end{aligned}$$

Therefore, the diameter of the circumcircle = $\frac{205}{3}$ cm

Hence choice (a) is the correct one.

- 68** Curvature of a circle
 $= \frac{1}{\text{radius of the circle}} = \frac{1}{2} = 0.5$ cm

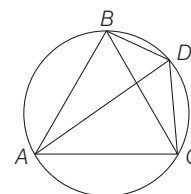
Hence choice (c) is the correct one.

- 69** Since the radius of a circle
 $= \frac{1}{\text{curvature of the circle}}$
 Therefore, radii of the circles are $\frac{1}{0.5} = 2$ and $\frac{1}{2} = 0.5$

Thus the required distance between the centers of the circles = $2 + 0.5 = 2.5$

Hence choice (a) is the correct one.

- 70** Draw the following diagram and you will find that $ABDC$ is a cyclic quadrilateral. However, the Ptolemy's theorem states that, "If a quadrilateral is inscribable in a circle then the product of the measures of its diagonals is equal to the sum of the products of the measures of the pairs of opposite sides."

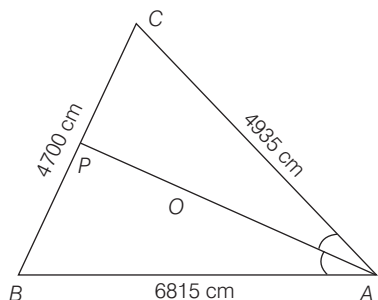


$$\begin{aligned}\text{Therefore, } AB \cdot CD + AC \cdot BD &= BC \cdot AD \\ \Rightarrow \quad 30(CD) + 30(BD) &= 30(AD) \\ \Rightarrow \quad CD + BD &= AD \\ \Rightarrow \quad CD + BD &= 32 \text{ cm}\end{aligned}$$

Hence choice (b) is the correct one.

✓ Level 01 Basic Level Exercise

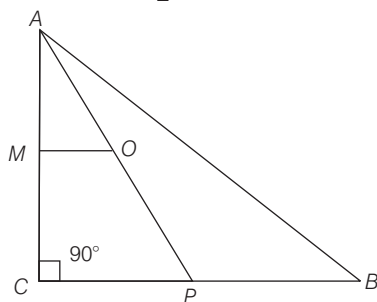
- 1 $\angle BAD = 180^\circ - 130^\circ = 50^\circ$ ($\because \square ABCD$ is cyclic)
 and $\angle ADB = 90^\circ$ (angle of semicircle)
 $\therefore \angle ABD = 180 - (90 + 50) = 40^\circ$
- 2 $\angle ACB = 180 - (90 + 25) = 65^\circ$ ($\because \angle ABC = 90^\circ$)
 But since $\angle ADB = \angle ACB$
 $\therefore \angle ADB = 65^\circ$
- 3 $\angle AOD = \angle ODC + \angle OCD$ ($\because \angle AOD$ is an external angle)
 $= 52^\circ$ ($\angle ODC = \angle OCD = 26^\circ$)
 ($\because OD = OC$)
- 4 Note that $4700 : 4935 : 6815 = 20 : 21 : 29$



$\therefore \triangle ACB$ is right angled triangle and $\angle ACB$ is a right angle.

$\therefore$ The inradius of the right angled triangle

$$= \frac{20 + 21 - 29}{2} = 6 \text{ cm}$$



$$\begin{aligned} \therefore MO &= MC = 6 \text{ cm} \\ \therefore AM &= 21 - 6 = 15 \text{ cm} \\ \therefore \frac{AM}{MC} &= \frac{AO}{OP} = \frac{15}{6} = \frac{5}{2} \Rightarrow \frac{PO}{AO} = \frac{2}{5} \end{aligned}$$

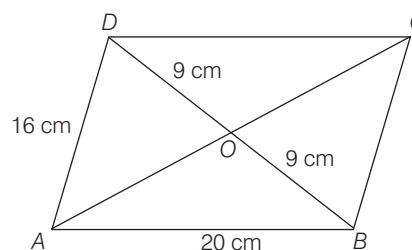
- 5 $CD = CP$ and $BE = BP$, since tangent from the same point are always same.

$$\begin{aligned} \therefore AE &= AD \\ \text{Now, } AD &= AC + CD = AC + CP \\ \text{and } AE &= AB + BE = AB + BP \\ \therefore AE + AD &= AC + CP + AB + BP \\ 2AE &= AC + AB + (CP + BP) \\ 2AE &= AB + AC + BC \end{aligned}$$

Hence option (d) is correct.

- 6 Inradius of a right angled triangle $= \frac{AC + BC - AB}{2}$
 $= \frac{9 + 40 - 41}{2} = 4 \text{ cm}$

- 7 Let $AD = 16 \text{ cm}$



then $AB = 20 \text{ cm}$ and $BD = 18 \text{ cm}$

$\therefore DO = BO = 9 \text{ cm}$

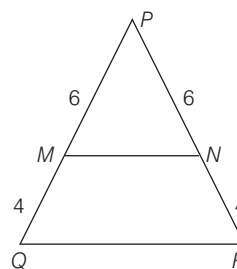
NOTE In a parallelogram the two diagonals bisect each other, therefore by Apollonius theorem

$$\begin{aligned} \therefore 16^2 + 20^2 &= 2(9^2 + AO^2) \\ \Rightarrow AO &= \sqrt{247} \\ \therefore AC &= 2\sqrt{247} \text{ cm} \end{aligned}$$

- 8 Go back to the basics

- 9 It is clear that

$$\frac{PM}{MQ} = \frac{PN}{NR} = \frac{6}{4} = \frac{3}{2}$$



Therefore $\triangle PMN$ and $\triangle PQR$ are similar

Now Ratio of areas of $\triangle PMN : \triangle PQR$

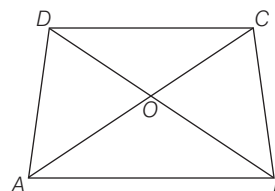
$$= 3 \times 3 : 5 \times 5 = 9 : 25$$

$\therefore$ Percentage of area of $\triangle PMN$ over

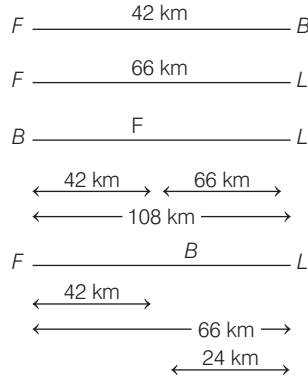
$$\triangle PQR = \frac{9}{25} \times 100 = 36\%$$

- 10 Since, $\triangle AOB$ is similar to $\triangle COD$.

$$\therefore \text{Ratio of areas of } \frac{\triangle AOB}{\triangle COD} = \left(\frac{3}{1}\right)^2 = \frac{9}{1}$$



- 11** The maximum possible distance between Barabanki and Lucknow is 108 km and minimum possible distance between Barabanki and Lucknow is 24 km. Hence (d) is the correct option.



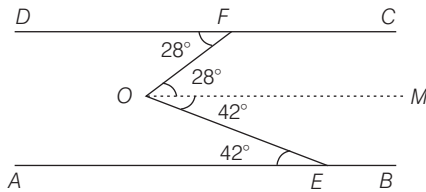
- 12** Since $CS = SD$

$\therefore$ The two chords must be equidistant from the centre 'O'.
Thus, the required ratio is 1 : 1

- 13**

$$\angle DFO = \angle FOM$$

and $\angle AEO = \angle EOM$ (since $CD \parallel AB$)



$$\therefore \angle FOE = (28^\circ + 42^\circ) = 70^\circ$$

- 14** $\angle AOB = 60^\circ$

$$\text{and } \angle AOD = \angle ADO = \frac{(180^\circ - 30^\circ)}{2} = 75^\circ$$

$$\text{Also } \angle BOC = 75^\circ$$

$$\therefore \angle COD = 360^\circ - (75^\circ + 60^\circ + 75^\circ) = 150^\circ$$

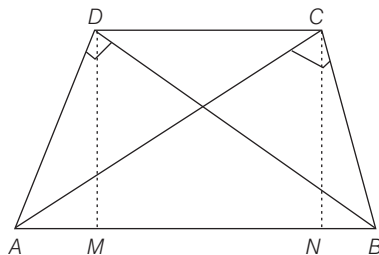
- 15** $\angle ABD = \angle ABC - \angle DBC$

$$= \angle ABC - \angle BDC = \angle ABC - (\angle ABD + \angle BAD)$$

$$\therefore 2(\angle ABD) = \angle ABC - \angle BAD = 30^\circ$$

$$\therefore \angle ABD = 15^\circ \quad (\because \angle BAD = \angle BAC)$$

- 16** $BD = AC = 20$ cm (by Pythagoras theorem)



$$\therefore DM = \frac{AD \times BD}{AB}$$

$$\therefore DM = \frac{15 \times 20}{25} = 12 \text{ cm, also } CN = DM$$

$$\therefore AM = \frac{AD^2}{AB}$$

$$\therefore AM = \frac{15 \times 15}{25} = 9 \text{ cm, also } AM = BN$$

$$\therefore MN = AB - (AM + BN) = 25 - (18) = 7 \text{ cm}$$

$$\text{but } MN = CD = 7 \text{ cm}$$

$$\therefore \text{Area of trapezium} = \frac{1}{2} \times (CD + AB) \times DM$$

$$= \frac{1}{2} \times 32 \times 12 = 192 \text{ cm}^2$$

- 17** $(OS)^2 = (OK)^2 + (KS)^2$

$$25 = OK^2 + 16 \Rightarrow OK = 3$$

$$\text{and } (OS)^2 = (OL)^2 + (LN)^2$$

$$25 = (OL)^2 + 9 \Rightarrow OL = 4 \text{ cm}$$

$$\therefore KL = OL - OK = 1 \text{ cm}$$

$$\therefore \text{Area of rectangle} = 1 \times 10 = 10 \text{ cm}^2$$

- 18**

$$\angle ACB = 90^\circ$$

$$\therefore AC = 12 \text{ cm}$$

Now, since CO is perpendicular on AB

$$\therefore CO = \frac{AC \times BC}{AB} = \frac{16 \times 12}{20} = 9.6 \text{ cm}$$

- 19** $\angle ADB = \angle ACB = 41^\circ$ (angles of the same segment)

$$\therefore \angle ABD = 180 - (\angle ADB + 90) = 49^\circ \quad (\angle DAB = 90^\circ)$$

- 20** Go through options for quicker answer

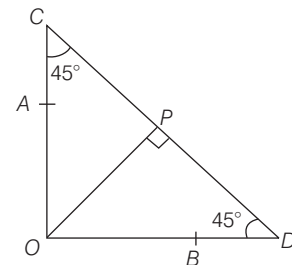
$$\text{Exterior angle} = \frac{360}{15} = 24^\circ \text{ (for } n = 15)$$

$$\therefore \text{Interior angle} = 180 - 24^\circ = 156^\circ$$

$$\therefore \text{Interior} - \text{Exterior} = 156 - 24 = 132^\circ$$

Hence, option (b) is correct.

- 21** $OC = OD$ and $OA = OP = OB$



$$OP = 1 \text{ m}$$

$$\therefore PC = 1 \text{ m} \Rightarrow OC = \sqrt{2} \text{ m}$$

$$\therefore AC = OC - OA = (\sqrt{2} - 1) \text{ m}$$

$$\text{and } AC + CP = (\sqrt{2} - 1) + 1$$

$$= \sqrt{2} \text{ m} = 1.414 \text{ m}$$

$$= 141.4 \text{ cm}$$

$$22 \quad AB : BC : CA = \frac{1}{CD} : \frac{1}{AE} : \frac{1}{BF} = \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6 : 4 : 3$$

23 $\triangle POQ$ and $\triangle ROS$ are similar

$$\therefore PS = QR$$

Option a, b and d are irrelevant.

$$24 \quad \frac{DQ}{BD} = \sin 60^\circ \Rightarrow DQ = BD \times \frac{\sqrt{3}}{2} = 3 \text{ m}$$

$$\text{and} \quad \frac{DP}{DF} = \sin 30^\circ = \frac{1}{2}$$

$$\therefore DP = 2 \text{ m} \quad (\because DF = 4 \text{ m})$$

$$\therefore PQ = 3 - 2 = 1 \text{ m}$$

$$\text{Let } AB = CQ = x \text{ m}$$

$$\therefore \frac{CD}{EF} = \frac{5}{4}$$

$$\frac{x+3}{x+1} = \frac{5}{4} \Rightarrow x = 7 \text{ m.}$$

25 Area of equilateral triangle

$$ABC = \frac{\sqrt{3}}{4} \times (6)^2 = 9\sqrt{3} \text{ cm}^2$$

$$\text{Area of } \triangle ADE = \frac{1}{2} \times DE \times CP$$

$$= \frac{1}{2} \times 2 \times \frac{\sqrt{3}}{2} \times 6 = 3\sqrt{3} \text{ cm}^2$$

$$\therefore \text{Area of shaded region} = 9\sqrt{3} - 3\sqrt{3} = 6\sqrt{3} \text{ cm}^2$$

$$26 \quad CD = CP^2 + DP^2 = (3\sqrt{3})^2 + (1)^2 = 2\sqrt{7} \text{ cm}$$

$$\text{and } CE = CD = 2\sqrt{7} \text{ and } DE = 2 \text{ cm}$$

$$\therefore \text{Perimeter of } \triangle CDE = 2(2\sqrt{7}) + 2 = 2(2\sqrt{7} + 1) \text{ cm}$$

$$27 \quad \angle ABC = 180 - (65 + 75) = 40^\circ$$

$$\angle ORB = \angle OQB = 90^\circ$$

$$\therefore \angle ROQ = 360 - (90 + 90 - 40)$$

$$\therefore \angle ROQ = 140^\circ$$

28 $\angle BAP + \angle BCP = 180^\circ$ ($\square ABCP$ is a cyclic quadrilateral)

$$\therefore \angle BCP = 180 - 60^\circ = 120^\circ$$

29 Since we don't have the requisite information.

$$30 \quad \therefore AT = BT,$$

($\because$ Tangents on the same circle from a fixed point is equal)

$$AC = PC \text{ and } BD = PD$$

$$\therefore AT = BT$$

$$\Rightarrow AC + CT = BD + DT$$

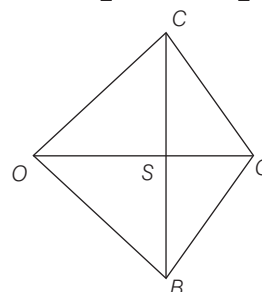
$$PC + CT = PD + DT$$

Similarly, all the relations can be verified.

$$31 \quad OQ = OB = OC = r \text{ (say); } \angle AOD = \angle BOC = 120^\circ$$

$$\therefore \angle BOQ = \angle COQ = 60^\circ$$

$$\therefore \frac{SB}{OB} = \sin 60^\circ = \frac{\sqrt{3}}{2} \Rightarrow SB = \frac{r\sqrt{3}}{2}$$



$$\therefore BC = 2SB = r\sqrt{3}$$

$\therefore$ Area of quadrilateral BQCO

$$= \frac{1}{2} \times BC \times OQ$$

$$= \frac{1}{2} \times r\sqrt{3} \times r = \frac{r^2\sqrt{3}}{2} \text{ cm}^2$$

$$\therefore \text{Area of both the quadrilateral} = 2 \left(\frac{r^2\sqrt{3}}{2} \right) = r^2\sqrt{3} \text{ cm}^2$$

32 Go back to the basics and apply similarity of triangle

$$33 \quad \frac{(A) \triangle CMN}{(A) \square ABNM} = \frac{1}{2}$$

$$\therefore \frac{(A) \triangle CMN}{(A) \triangle CAB} = \frac{1}{3}$$

$$\Rightarrow \frac{MN}{AB} = \frac{CM}{CA} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{CM}{MA} = \frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{2} \quad MA = (CA - CM)$$

$$34 \quad \angle ADE = \angle AED = 80^\circ$$

$$\therefore \angle BDE = \angle CED = 100^\circ$$

$$\therefore \angle DBC + \angle BCE = 360^\circ - (100^\circ + 100^\circ) = 160^\circ$$

$$35 \quad AC = \sqrt{(15)^2 + (20)^2} = 25 \text{ cm}$$

$$CD = \sqrt{(25)^2 - (7)^2} = 24 \text{ cm}$$

$$36 \quad AB = PQ = 26 \text{ cm}$$

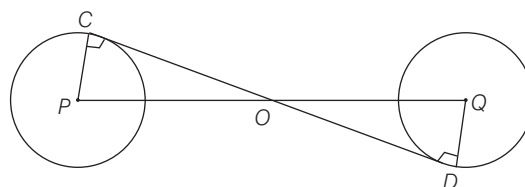
$$\text{and } PO = OQ = 13 \text{ cm}$$

$$CO = \sqrt{(PO)^2 - (PC)^2}$$

$$CO = \sqrt{(13)^2 - (5)^2}$$

$$CO = 12 \text{ cm}$$

$$\therefore CD = 2CO = 24 \text{ cm}$$



Alternatively Solve by using the formula of tangents.

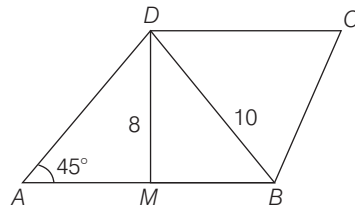
37

$$\angle DAM = 45^\circ$$

and
and

$$DM = 8 \text{ cm}$$

$$BD = 10 \text{ cm}$$



$$\therefore BM = 6 \text{ cm} \quad (BM = \sqrt{BD^2 - DM^2})$$

$$\text{and} \quad \frac{AM}{DM} = \tan 45^\circ = 1$$

$$\therefore AM = 8 \text{ cm}$$

$$\text{and} \quad AB = AM + MB = 8 + 6 = 14 \text{ cm}$$

$$\therefore \text{Area of parallelogram } ABCD = AB \times MD$$

$$= 14 \times 8 = 112 \text{ cm}^2$$

38

$$\angle ACB = 50^\circ \quad (\because \angle A + \angle B + \angle C = 180^\circ)$$

$$\angle CFO = \angle CEO = 90^\circ$$

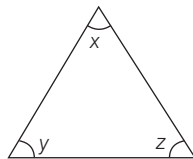
$$\therefore \angle FOE = 360^\circ - (90^\circ + 90^\circ + 50^\circ) = 130^\circ$$

$$\text{but} \quad \angle AOB = \angle FOE = 130^\circ$$

39

$$35x = 26y \Rightarrow \frac{x}{y} = \frac{26k}{35k}$$

$$\therefore x + y = 61k$$



$$\text{if } x = 1, \text{ then } z = 119^\circ$$

$$\text{But } x < z < y$$

$$\text{again consider } k = 2, \text{ then}$$

$$x = 52^\circ \text{ and } y = 70^\circ$$

$$\therefore z = 180^\circ - (70^\circ + 52^\circ) = 58^\circ$$

40

$$\angle DAB = \angle BDQ = 48^\circ$$

$$\therefore \angle DBA = 180^\circ - (90^\circ + 48^\circ) = 42^\circ \quad (\because \angle ADB = 90^\circ)$$

$$\angle DCB = 180^\circ - \angle DAB = 132^\circ$$

$$\therefore \frac{\angle DBA}{\angle DCB} = \frac{42}{132} = \frac{7}{22}$$

41

$$\angle OCD = 90^\circ \Rightarrow \angle OAC = \angle OCA = 30^\circ$$

$$\angle ACD = \angle ACO + \angle OCD = 30^\circ + 90^\circ = 120^\circ$$

$$\therefore \angle BAC = 180^\circ - 120^\circ = 60^\circ$$

$$\Rightarrow \angle BCD = 60^\circ \quad (\angle BCD = \angle BAC)$$

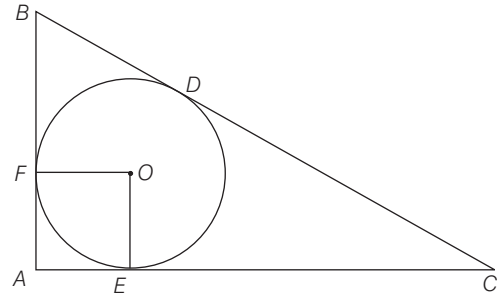
$$\Rightarrow \angle OCB = \angle OCD - \angle BCD$$

$$= 90^\circ - 60^\circ = 30^\circ$$

42

$$OE = OF \text{ (inradius)}$$

$$BF = BD$$



$$\text{and} \quad EC = DC = 15 \text{ cm}$$

$$\text{Let } BF = x \text{ cm}$$

$$\text{then } AB = (6 + x) \text{ cm}$$

$$\text{and } AC = 6 + 15 = 21 \text{ cm}$$

$$\therefore (BC)^2 = (AB)^2 + (AC)^2$$

$$(x + 15)^2 = (6 + x)^2 + (21)^2 \quad (\because BC = BD + CD)$$

$$\Rightarrow x = 14 \text{ cm}$$

$$\Rightarrow BD = 14 \text{ cm}$$

$$\therefore CD - BD = 15 - 14 = 1 \text{ cm}$$

43 $(\angle BPQ + \angle BQP) + (\angle EQR + \angle ERQ)$

$$+ (\angle CRS + \angle CSR) + (\angle TSA + \angle ATS)$$

$$+ (\angle DTP + \angle DPT) = 2 \times 360$$

$$\angle A + \angle D + \angle B + \angle E + \angle C = 5 \times 180^\circ - 2 \times 360^\circ = 180^\circ$$

$$44 \quad \frac{\text{Area of } \triangle DAE}{\text{Area of } \triangle DEC} = \frac{\frac{1}{2} \times DE \times AE}{\frac{1}{2} \times DE \times CE}$$

$$= \frac{AE}{CE} = \frac{(AD)^2}{(DC)^2} = \left(\frac{6}{8}\right)^2 = \frac{9}{16}$$

Similarly, in $\triangle ABC$,

$$\frac{\text{Area of } \triangle BCF}{\text{Area of } \triangle BFA} = \frac{9}{16}$$

$$\therefore \text{The area of shaded to unshaded region} = \frac{16}{9}$$

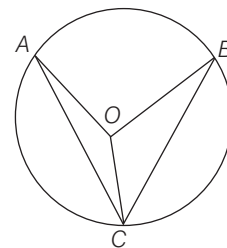
45

$$\angle OCA = \angle OAC = 25^\circ$$

$$\text{and } \angle OCB = \angle OBC = 35^\circ$$

$$\therefore \angle ACB = \angle ACO + \angle BCO = 25^\circ + 35^\circ = 60^\circ$$

$$\therefore \angle AOB = 2\angle ACB = 120^\circ$$



- 46 $\angle DOA = 110^\circ$
and $\angle ADB = 50^\circ$
 $\therefore \angle DAO = 180 - (110^\circ + 50^\circ) = 20^\circ$

47 $\triangle ABC$ is similar to $\triangle EDC$

$$\therefore \frac{AB}{ED} = \frac{BC}{DC} = \frac{AC}{EC}$$

$$\therefore \frac{AB}{DE} = \frac{BC}{DC} \Rightarrow \frac{24}{10} = \frac{60}{DC}$$

$$\Rightarrow DC = 25 \text{ cm}$$

- 48 $AC = \sqrt{(24)^2 + (60)^2} \quad (\because AC^2 = AB^2 + BC^2)$

$$AC = 12\sqrt{29}$$

$$\therefore \frac{BC}{DC} = \frac{AC}{EC} \Rightarrow \frac{60}{25} = \frac{12\sqrt{29}}{EC}$$

$$\Rightarrow EC = 5\sqrt{29}$$

$$\therefore BE = BC - EC = 60 - 5\sqrt{29}$$

$$\Rightarrow BE = 5(12 - \sqrt{29})$$

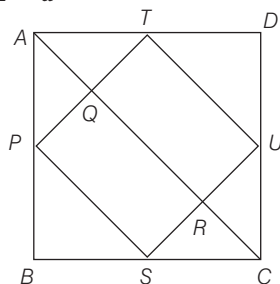
$$\therefore \frac{CE}{BE} = \frac{5\sqrt{29}}{5(12 - \sqrt{29})} = \frac{\sqrt{29}}{(12 - \sqrt{29})}$$

49 $PTUS$ is a square inscribed by a square $ABCD$.

Let each side of the square $ABCD$ be ' a ', then

Area of square $ABCD = a^2$

also $PU = ST = a$



$$\therefore \text{Area of square } PTUS = \frac{a^2}{2}$$

$$\therefore \frac{\text{Area of } PTUS}{\text{Area of } ABCD} = \frac{a^2/2}{a^2} = \frac{1}{2}$$

$$\therefore \frac{\text{Area of } \square PQRS}{\text{Area of } \triangle ABC} = \frac{1}{2}$$

50 Go through options. It is a pentagon.

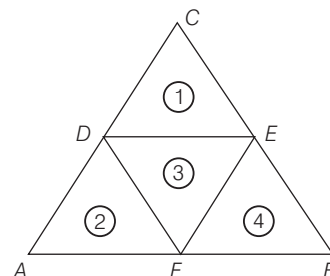
☞ Alternatively ${}^nC_2 - n = n \Rightarrow {}^nC_2 = 2n$

$$\Rightarrow \frac{n(n-1)}{2} = 2n \Rightarrow n = 5$$

- 51 Let $P_1, P_2, P_3, P_4, \dots, P_9$ be the vertices of the regular nonagon then, we can select two vertices out of 9 vertices in ${}^9C_2$ ways and ${}^9C_2 = 36$ ways and number of equilateral triangles formed $= 36 \times 2 = 72$. But the three triangles $\{P_1, P_4, P_7\}$, $\{P_2, P_5, P_8\}$ and $\{P_3, P_6, P_9\}$ are each counted 3 times i.e., counted as 9 triangles instead of 3 triangles. So, net number of equilateral triangles $= 72 - 6 = 66$.

52 There are 4 triangles (smaller) are congruent.

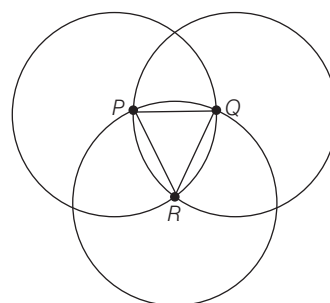
So, out of these 4 triangles 2 triangles are taken thus the ratio of the shaded to the unshaded region is 1 : 1 (since two triangles are shaded and 2 are unshaded).



53 $\triangle PQR$ is an equilateral triangle i.e., all the three angles are 60° each.

$\therefore$

$$PQ = QR = PR$$

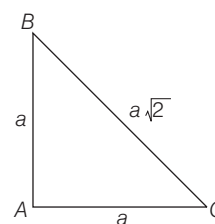


Now, $\text{arc } PQ = 2\pi \times (1) \times \frac{60}{360} = \frac{\pi}{3}$

$\therefore$ Sum of all three arcs $= 3 \times \frac{\pi}{3} = \pi$

54 $\angle A = 90^\circ$ and $\angle B = \angle C = 45^\circ$

$$\text{Inradius} = \frac{(a+a) - a\sqrt{2}}{2} = \frac{a(2-\sqrt{2})}{2}$$



and Circumradius $= \frac{a\sqrt{2}}{2}$

$$\begin{aligned} \therefore \frac{\text{Inradius}}{\text{Circumradius}} &= \frac{a(2-\sqrt{2})/2}{(a\sqrt{2})/2} = \frac{2-\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{2}(\sqrt{2}-1)}{\sqrt{2}} \\ &= \frac{(\sqrt{2}-1)}{1} \end{aligned}$$

- 55 Let the perpendicular be AD and circumradius be R , then

$$\Delta ABC = \frac{1}{2} \times BC \times AD = \frac{AB \times BC \times AC}{4R}$$

$$\frac{1}{2} \times BC \times AD = \frac{36 \times BC \times 30}{4 \times 45} \Rightarrow AD = 12 \text{ cm.}$$

Hence choice (a) is the correct one.

- 56 If you remember 30-60-90 degree theorem, it does not require any efforts to solve this problem.

Hence choice (d) is the correct one.

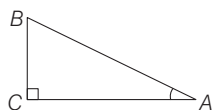
☞ **Alternatively**

$1^2 + (\sqrt{3})^2 = 2^2$. It implies that one angle is 90° .

Choice (b) : 1 : 3 : 5 means the angles are 20, 60, 100.

Choice (c) : 2 : 3 : 4 means the angles are 40, 60, 80.

It implies that choices (a) and (b) are not valid, as they do not have 90 degree angle.



Mind that if one angle is 90° , then the sum of the remaining two angles will also be 90° .

Now since we know that the angle opposite the longest side is the greatest and vice-versa, so if the order of sides is increasing one, the order of angles should also be the increasing one.

It implies that the order of the corresponding angles should be $1 < 2 < 3$.

Thus out of the remaining two choices (c) and (d), choice (d) is the valid one.

☞ **Alternatively** Let $a = x$, $b = \sqrt{3}x$, $c = 2x$. Then we see that $(x)^2 + (\sqrt{3}x)^2 = (2x)^2$.

It implies that $\angle C = 90^\circ$. Then, $\tan A = \frac{1}{\sqrt{3}}$

$$\Rightarrow \angle A = 30^\circ.$$

$$\text{Consequently, } \angle B = 60^\circ.$$

$$\text{Thus, } A : B : C = 1 : 2 : 3$$

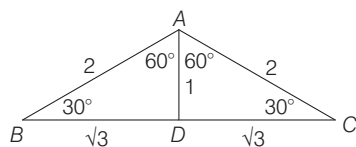
☞ **Alternatively** Using the Cosine rule you will get,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow \cos A = \frac{\sqrt{3}}{2} \Rightarrow A = 30^\circ$$

Similarly, $B = 60^\circ$ and $C = 90^\circ$.

- 57 $4x + x + x = 180^\circ \Rightarrow x = 30^\circ$. Therefore, the respective angles are 120° , 30° and 30° .



Drop a perpendicular AD on BC . Then AD will act as the median as well as angular bisector.

Let us consider $AD = p$, then using 30-60-90 degree theorem, we get $AB = AC = 2$ and $BD = CD = \sqrt{3}$.

Therefore, the required ratio = $BC : (AB + AC + BC)$

$$= 2\sqrt{3} : (4 + 2\sqrt{3})$$

$$= \sqrt{3} : (2 + \sqrt{3})$$

Hence choice (a) is the correct one.

☞ **Alternatively** Let's go through the given choices.

$$\text{Choice (a) : } \sqrt{3} : (2 + \sqrt{3})$$

$$\Rightarrow \sqrt{3} : (1 + 1 + \sqrt{3}), \text{ seems possible.}$$

$$\text{Choice (b) : } 1 : 6 \Rightarrow 1 : (2.5 + 2.5 + 1)$$

$\Rightarrow 2 : (5 + 5 + 2)$; NOT possible, as the largest side (2 unit) is smaller than the smaller side (5 unit).

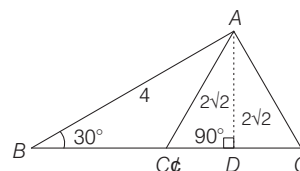
Choice (c) : $1 : (2 + \sqrt{3}) \Rightarrow 1 : (1 + 1 + \sqrt{3})$, which is not possible, as there are multiple inconsistencies and contradictions, as far as the existence of a triangle is concerned.

$$\text{Choice (d) : } 2 : 3 \Rightarrow 2 : (0.5 + 0.5 + 2)$$

$\Rightarrow 4 : (1 + 1 + 4)$, NOT impossible, as it does not follow the definition of a triangle which says that sum of any two sides of a triangle must be greater than the third side.

Hence choice (a) is the correct one.

- 58 Consider the following diagram consisting of triangles ABC and ABC' . Now draw a perpendicular AD on BC .



Thus using 30-60-90 degree theorem in the right angle ΔADB , we get $AD = 2$

And, by using Pythagoras theorem in right angle $\Delta ADC'$, we get $DC' = 2$

Or, if you could notice that there exists a 45-45-90 degree relation in the $\Delta ADC'$, as $\left(\frac{AD}{AC} = \frac{1}{\sqrt{2}}\right)$

AD bisects $C'C$, as AD is a perpendicular in the isosceles triangle $C'AC$.

$$\text{So, } C'C = 2(C'D) = 4.$$

Therefore the difference in the areas of ΔABC and $\Delta ABC'$

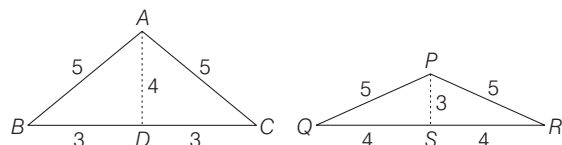
$$= ACC' = \frac{1}{2} (AD \times CC')$$

$$= \frac{1}{2} (2 \times 4) = 4 \text{ sq. unit}$$

Hence, choice (c) is the correct one.

Hint : The 30-60-90 degree theorem states that the sides opposite 30° , 60° , 90° angles will be in the ratio of $1 : \sqrt{3} : 2$. So, $AD : BD : AB = 2 : 2\sqrt{3} : 4$.

- 59 Simply drop the perpendiculars and use the Pythagoras theorem, you will find that these two triangles have the same area.



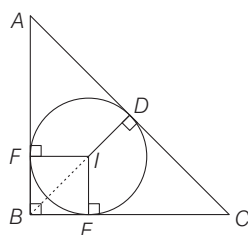
Hence choice (c) is the correct one.

- 60 Since AC is a hypotenuse, it means triangle ABC is a right angle triangle and $\angle B = 90^\circ$. Let I be the incentre of the triangle, then ID, IE, and IF will be perpendiculars on the AC, BC and AB, respectively.

Let the inradius be 1 unit, then

$$ID = IE = IF = BE = BF = IF = 1$$

Therefore, $BI = EF = \sqrt{2}$, as BI and EF are the diagonals of square BEIF.



Now, $BD = BI + ID = \sqrt{2} + 1$

But, since D is the circumcentre of the triangle, so $BD = CD = AD$

Therefore, $AC = 2(BD) = 2(\sqrt{2} + 1)$

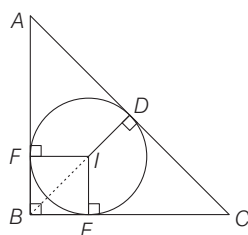
$$\begin{aligned} \text{Thus the required ratio} &= \frac{EF}{AC} \\ &= \frac{\sqrt{2}}{2(\sqrt{2} + 1)} = \frac{1}{2 + \sqrt{2}} \end{aligned}$$

Hence choice (b) is the correct one.

NOTE You can assume any value or you can simply consider r , as inradius. The method would remain the same.

- 61 If r and R denote the inradius and circumradius of a right angled triangle, the perimeter of the triangle

$$(2r + 4R) = (12 + 64) = 76 \text{ cm}$$



If s denotes the semi-perimeter of a triangle, the area of the triangle $= r \times s = 6 \times 38 = 228 \text{ sq cm}$.

Hence choice (b) is the correct one.

Hint : The perimeter of the triangle

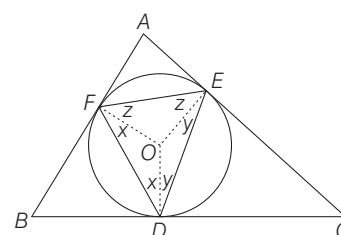
$$\begin{aligned} &= AF + AD + DC + CE + BE + BF \\ &= 2AD + 2CD + 2BE \\ &= 2(AD + CD) + 2BE \\ &= 2(AC) + 2BE = 2(2R) + 2r \\ &= 2r + 4R \end{aligned}$$

- 62 As we know that each radius of the same circle is equal, so we can find the following relations between angles.

In $\triangle FOD$, $OF = OD$, so $\angle OFD = \angle ODF = x$ (say)

In $\triangle EOD$, $OE = OD$, so $\angle OED = \angle ODE = y$ (say)

In $\triangle EOF$, $OF = OE$, so $\angle OFE = \angle OEF = z$ (say)



Then from the diagram we have,

$$\angle DFE = x + z = 76, \angle FDE = x + y$$

and

$$\angle DEF = y + z.$$

But

$$(x + y) + (y + z) + (z + x) = 180^\circ$$

$\Rightarrow$

$$x + y + z = 90^\circ$$

Now,

$$\angle DFE = 76$$

$\Rightarrow$

$$\angle DOE = 2(\angle DFE) = 152$$

$\therefore$

$$\angle DCE = 180 - \angle DOE = 28^\circ$$

Similarly,

$$\angle FED = 56$$

$\Rightarrow$

$$\angle FOD = 2(\angle FED) = 112$$

$\therefore$

$$\angle FBD = 180 - \angle DOE = 68^\circ$$

Similarly,

$$\angle FDE = 48$$

$\Rightarrow$

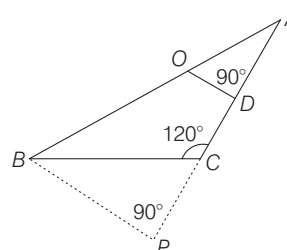
$$\angle FOE = 2(\angle FDE) = 96$$

$\therefore$

$$\angle FAE = 180 - \angle FOE = 84^\circ.$$

Hence choice (b) is the correct one.

- 63 Extend AC to a point P such that BP is a perpendicular on AP, as shown in the figure.



Since, OD and BP are parallel and O is the mid-point of AB, so D will be the mid-point of AP.

Now, in the $\triangle PBC$, $\angle PCB = 60^\circ$ and $\angle PBC = 30^\circ$. Using 30-60-90 degree theorem in this triangle, we get $BP = 6 \text{ cm}$, as $BC = 4\sqrt{3} \text{ cm}$.

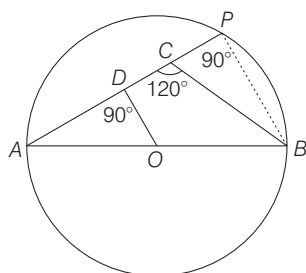
As $\triangle AOD$ and $\triangle ABP$ are similar triangles, so we can use the mid-point theorem and then we will get

$$\frac{AO}{AB} = \frac{AD}{AP} = \frac{OD}{BP} = \frac{1}{2}$$

Therefore, $OD = 3$ cm

Hence choice (b) is the correct one.

☞ Alternatively Since O is the mid-point of AB , so considering O as the centre you can draw a circle. Now, if you extend AC to the circumference and then connect AP and BP as shown in figure, then angle APB will be a right angle.



The rest of the solution will be similar to the above one.

- 64** A right angled triangle can have the largest possible area only when it is an isosceles one. That is $AB = BC$, so $\angle BAC = \angle ACB = 45^\circ$.

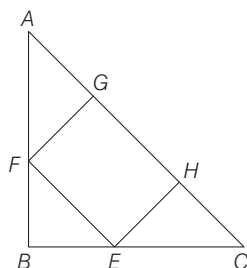
In $\triangle EHC$, $\angle EHC = 90^\circ$ and $\angle HCE = \angle HEC = 45^\circ$,

So, $HC = EH = 14$ cm

And, then, we have $AG = FG = EH = 14$ cm

Therefore, $EF = GH = AC - (AG + HC) = 14$ cm

It implies that $EFGH$ is a square with each side 14 cm.



Therefore, we can have a largest possible circle of radius 7 cm in the square $EFGH$.

And the area of this circle would be $\pi(7^2) = 154$ sq. cm.

Hence choice (c) is the correct one.

- 65** By mid-point theorem,

When CD is a median,

$$W + X = Y + Z \quad \dots(i)$$

And, when BE is a median

$$X + Y = W + Z \quad \dots(ii)$$

Subtracting the equation (ii) from (i), we get

$$W - Y = Y - W$$

$\Rightarrow$

$$W = Y$$

Therefore, from eqn. (i) or (ii)

$$X = Z$$

Again, since $BC = 2DE$, therefore

$$X = 2W = 2Y$$

Thus, $W : X : Y : Z = 1 : 2 : 1 : 2$

Hence choice (c) is the correct answer.

Hint To understand the ratios join DE and then apply mid point theorem in $\triangle ADE$ and $\triangle ABC$. Then, you see that $\frac{DE}{BC} = \frac{DO}{OC} = \frac{EO}{OB} = \frac{1}{2}$. In $\triangle BOD$ and $\triangle BOC$ the altitude is

same, but $\frac{OD}{OC} = \frac{1}{2}$, therefore $\frac{\triangle BOD}{\triangle BOC} = \frac{1}{2}$.

It implies $X = 2W$. Similarly, $X = 2Y$

Hence choice (c) is the correct one.

☞ Alternatively Apply Mass-Point Geometry, you get

wt. (A) = wt. (B) = 1 and wt. (D) = 2

Similarly, wt. (A) = wt. (C) = 1 and wt. (E) = 2

$$\text{Since, } \frac{\text{wt. (D)}}{\text{wt. (C)}} = \frac{CO}{DO} = \frac{2}{1}$$

$$\text{Therefore, } \frac{\text{area}(\triangle X)}{\text{area}(\triangle W)} = \frac{2}{1}$$

$$\text{Therefore, } \frac{\text{area}(\triangle X)}{\text{area}(\triangle Y)} = \frac{2}{1}$$

But, we also know that are $(\triangle W) + \text{area}(\triangle X)$
 $= \text{area}(\triangle Y) + \text{area}(\triangle Z)$.

Therefore, $(\triangle W) : (\triangle X) : (\triangle Y) : (\triangle Z) = 1 : 2 : 1 : 2$

- 66** Let the other two sides be a , then $2a + b = 20$.

We know that, for any triangle, the sum of any two sides must be greater than the third side. That is $2a > b$ and $a + b > a$.

(i) When $2a > b$

$$\Rightarrow 20 - b > b \quad (\because 2a = 20 - b)$$

$$\Rightarrow b < 10$$

(ii) When $a + b > a$

$$\Rightarrow b > 0$$

Also, when $a \neq b$

$$\Rightarrow 2a + b = 20$$

$$\Rightarrow b \neq \frac{20}{3}$$

$$\text{Thus, } b = (0, 10) - \left\{ \frac{20}{3} \right\}$$

$$\text{Or } b = \left(0, \frac{20}{3} \right) \cup \left(\frac{20}{3}, 10 \right)$$

Hence choice (b) is the correct one.

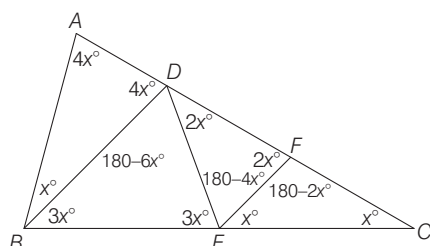
- 67** Let $\angle FCE = x$, then $\angle FEC = x$

$$\Rightarrow \angle EFC = 180 - 2x$$

$$\Rightarrow \angle DFE = 2x \Rightarrow \angle EDF = 2x$$

$$\Rightarrow \angle DEF = 180 - 4x$$

$$\begin{aligned}
 \Rightarrow \quad \angle DEB &= 3x \\
 &(\because \angle DEB + \angle DEF + \angle FEC = 180^\circ) \\
 \Rightarrow \quad \angle DBE &= 3x \\
 \Rightarrow \quad \angle BDE &= 180^\circ - 6x \\
 \Rightarrow \quad \angle ADB &= 4x \\
 &(\because \angle ADB + \angle BDE + \angle EDF = 180^\circ) \\
 \Rightarrow \quad \angle DAB &= 4x \\
 \Rightarrow \quad \angle ABD &= x \\
 &(\because \angle ABD + \angle DBE = 4x, \text{ as } \angle ABC = \angle BAC)
 \end{aligned}$$



$$\begin{aligned}
 \therefore \quad 4x + 4x + x &= 180^\circ \\
 \Rightarrow \quad x &= 20^\circ \\
 \Rightarrow \quad \angle DBE &= 60^\circ \text{ and } \angle DEB = 60^\circ
 \end{aligned}$$

It implies that $\triangle BDE$ is an equilateral triangle. Therefore $BE = 6$ cm. Hence choice (d) is the correct one.

- 68** Let TR be the tree. F is the initial position of fox, S be the initial position of squirrel and R be the final position of squirrel.

Let $TF = x$, then from 30-60-90 degree theorem,

$$ST = \frac{x}{\sqrt{3}} \text{ and } RT = x\sqrt{3}.$$

$$\begin{aligned}
 \text{Therefore, } RS &= RT - ST \\
 &= x\sqrt{3} - \frac{x}{\sqrt{3}} = \frac{2x}{\sqrt{3}}
 \end{aligned}$$

For the distance $\frac{2x}{\sqrt{3}}$ squirrel takes 2 minutes.

Therefore, for the distance $\frac{2x}{\sqrt{3}}$ fox will take $\frac{2}{\sqrt{3}}$ minutes.

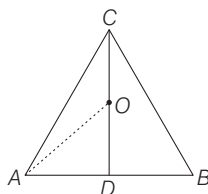
And thus for the distance x fox will take 1 minute.

Hence choice (a) is the correct one.

- 69** Drop a perpendicular bisector CD from the vertex C on the base AB . Now consider a point O on CD such that $CO = AO = R$, where R is the circumradius of the triangle ABC . Now $CD^2 = AC^2 - AD^2 = 25 - 9 = 16$

$$\Rightarrow CD = 4 \text{ cm.}$$

$$\begin{aligned}
 \text{Again, in } \triangle AOD, AO^2 &= OD^2 + AD^2 = (CD - CO)^2 + AD^2 \\
 &= (4 - CO)^2 + 9
 \end{aligned}$$



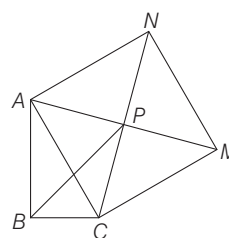
$$\begin{aligned}
 \Rightarrow \quad R^2 &= (4 - R)^2 + 9 \\
 \Rightarrow \quad R &= \frac{25}{8} \text{ cm}
 \end{aligned}$$

Hence choice (b) is the correct one.

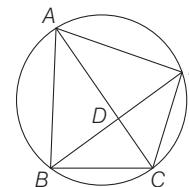
- 70** Follow the construction given below. Triangle ABC is a right angled triangle and $ACMN$ is a square. Point P is the intersection of diagonals of the square.

In the right angle $\triangle APC$, $AP = CP$, so the

$$\angle ACP = \angle CAP = 45^\circ.$$



Now, you can see that $\angle ABC = 90^\circ$ and $\angle APC = 90^\circ$. It implies that AC is the diameter of the circle such that $\angle ABC$ and $\angle APC$ are the angles of the semicircles, as shown below.



Now, using the concept of "angles in the same segment of a circle are equal" we can attain the following results.

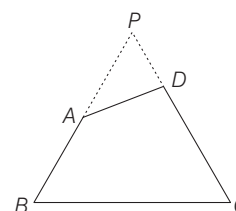
$$\angle ABP = \angle ACP = 45^\circ$$

$$\text{and } \angle CBP = \angle CAP = 45^\circ.$$

Hence choice (d) is the correct one.

- 71** Extend the sides BA and CD so as to meet at P . Now it will form an equilateral triangle BPC .

$$\text{Therefore, } BP = PC = BC = 6\sqrt{3} \text{ m.}$$



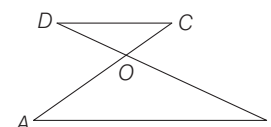
Further the triangle APD is a right angle triangle, where $\angle ADP = 90^\circ$, $\angle APD = 60^\circ$, $\angle PAD = 30^\circ$. So using 30-60-90 degree theorem in $\triangle APD$, we have $PD = 3$ and $AP = 6$.

$$\begin{aligned}
 \text{Therefore, } CD &= PC - PD = 6\sqrt{3} - 3 \\
 &= 3(2\sqrt{3} - 1) \text{ cm}
 \end{aligned}$$

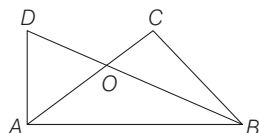
Hence choice (a) is the correct one.

- 72** Since, AB is parallel to CD , therefore $\angle CDO = \angle ABO$ and $\angle DCO = \angle BAO$.

Also, $\angle DOC = \angle AOB$. Therefore, $\triangle AOB \sim \triangle COD$.



However, we cannot say that $\triangle DOA$ is similar to $\triangle COB$ unless we have some necessary and exact information to prove the similarity. The area of $\triangle DOA$ will be same as the area of $\triangle COB$, when the trapezium $ABCD$ is a regular trapezium. That is when $AD = BC$.



Now, Area $\triangle DOA$ = Area $\triangle DAB$ – Area $\triangle AOB$.

Similarly, Area $\triangle COB$ = Area $\triangle CBA$ – Area $\triangle AOB$.

But, since the base and height of $\triangle DAB$ and $\triangle CBA$ are same, area of $\triangle DAB$ is same as the area of $\triangle CBA$. Thus Area $\triangle DOA$ = Area $\triangle COB$.

Now, we know that irrespective of the shape of the trapezium, $\triangle AOB$ is always similar to $\triangle COD$. But, it is not necessary that the area of $\triangle AOB$ is always same as that area of $\triangle COD$. The area of $\triangle AOB$ will be same as the area of $\triangle COD$, only when the trapezium $ABCD$ is a parallelogram. That is when $AB = CD$ and $AD = BC$.

Therefore choice (c) is correct.

- 73** If you drop the perpendiculars on AB from point D and point E , they will be of equal length, as DE is parallel to AB . Now, in both the triangles base AB is common. Therefore, we can say that the area of $\triangle ADB$ and area of $\triangle AEB$ is same. Further, since $\triangle AOB$ is common in both the triangles $\triangle ADB$ and $\triangle AEB$, so it is clear that area of $\triangle DOA$ is same as area of $\triangle EOB$. Since, DE is parallel to AB and

$$\frac{CD}{AC} = \frac{CE}{BC} = \frac{DE}{AB} = \frac{1}{2}$$

Further, due to AAA property of similarity, we know that $\triangle EOD \sim \triangle AOB$. But,

$$\text{Since } \frac{DE}{AB} = \frac{DO}{OB} = \frac{EO}{OA} = \frac{1}{2},$$

$$\text{Therefore } \frac{\text{area } \triangle DOE}{\text{area } \triangle AOB} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}.$$

That is area of $\triangle AOB$ = 4(area of $\triangle DOE$).

Now, (height of $\triangle DOE$)

$$= \frac{1}{2} (\text{height of } \triangle AOB) = \frac{1}{3} (\text{height of } \triangle ADB \text{ or } \triangle AEB)$$

And, height of $\triangle CDE$

$$= \text{height of } \triangle ADB = 3(\text{height of } \triangle DOE)$$

And, base of $\triangle CDE$ = base of $\triangle DOE$

$$\text{Then, we have } \frac{\text{Area } \triangle CDE}{\text{Area } \triangle DOE} = \frac{\frac{1}{2}(3h \times b)}{\frac{1}{2}(h \times b)} = \frac{3}{1}$$

$$\therefore \text{Area } \triangle CDE = 3(\text{Area } \triangle DOE)$$

Hence choice (d) is the correct one.

$$\begin{aligned} \text{74 } \frac{\text{Area } \triangle DOE}{\text{Area } \triangle AOB} &= \frac{40}{250} = \left(\frac{2}{5}\right)^2 \\ \Rightarrow \frac{DE}{AB} &= \frac{2}{5} = \frac{\text{height of } \triangle DOE}{\text{height of } \triangle AOB} \\ \Rightarrow \frac{\text{height of } \triangle ADB}{\text{height of } \triangle AOB} &= \frac{2+5}{5} = \frac{7}{5} \end{aligned}$$

But since the base of $\triangle ADB$ = base of $\triangle AOB$, therefore,

$$\frac{\text{area of } \triangle ADB}{\text{area of } \triangle AOB} = \frac{\frac{1}{2}(b \times 7h)}{\frac{1}{2}(b \times 5h)} = \frac{7}{5} = \frac{350}{250}$$

Thus the area of $\triangle AOD$ = 350 – 250 = 100 cm^2

Now, since the base of both the triangles $\triangle ACB$ and $\triangle ADB$ are same, but the height of $\triangle ACB$ is twice the height of $\triangle ADB$, therefore the area of $\triangle ACB$ = 2 (area of $\triangle ADB$) = 700 cm^2 .

Hence choice (a) is the correct one.

$$\begin{aligned} \text{Alternatively Area of region } \beta &= \sqrt{\alpha} \cdot \sqrt{\gamma} \\ &= \sqrt{40} \times \sqrt{250} = 100 \text{ cm}^2 \end{aligned}$$

And, area of $\triangle ADB$ = 100 + 250 = 350

Therefore area of $\triangle ACB$ = 2(350) = 700 cm^2

$$\begin{aligned} \text{75 } \beta &= \delta = \sqrt{\alpha} \times \sqrt{\gamma} = \sqrt{4} \times \sqrt{9} = 6 \text{ cm}^2 \\ \therefore \alpha + \beta + \gamma + \delta &= 4 + 6 + 9 + 6 = 25 \text{ cm}^2 \end{aligned}$$

$$\text{Thus, } \frac{(\beta + \delta)}{(\alpha + \beta + \gamma + \delta)} = \frac{12}{25}$$

Hence, choice (b) is correct one.

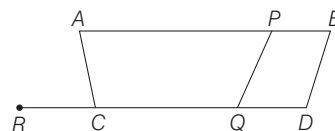
$$\text{Alternatively } \sqrt{(\alpha + \beta + \gamma + \delta)} = \sqrt{\alpha} + \sqrt{\gamma} = \sqrt{4} + \sqrt{9} = 5$$

$$\therefore \alpha + \beta + \gamma + \delta = (5)^2 = 25 \text{ cm}^2$$

$$\text{Thus } \frac{(\beta + \delta)}{(\alpha + \beta + \gamma + \delta)} = \frac{25 - (4 + 9)}{25} = \frac{12}{25}$$

- 76** Extend DC till a point R .

Now, $\angle ACR = \angle APQ = \angle ABD$



Therefore, $PQ \parallel BD$

But, given that $AB \parallel CD$

Thus it is proven that $PBDQ$ is definitely a parallelogram.

Hence choice (c) is the correct one.

- 77** Let's connect the mid-points P and M then the line PM will pass through the centre O of the circle, as the trapezium is symmetric. In fact, PM will act as a diameter of the circle and it will be perpendicular to the parallel sides AB and CD . Now, drop another perpendicular BN from B .

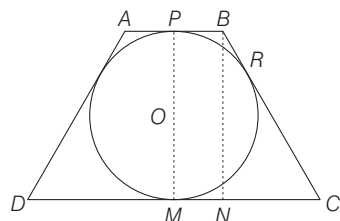
Let $PB = 16x$, then $BR = 16x$, as PB and BR are the two tangents from the same point.

And $MN = 16x$, as PM and BN are parallel.

As $PB = 16x$, then $MC = 49x$. It implies that $RC = 49x$ as MC and RC are two tangents from the same point.

Therefore, $NC = MC - MN = 49x - 16x = 33x$

And $BC = BR + RC = 16x + 49x = 65x$



Now, by using Pythagoras theorem in $\triangle BNC$, we have,

$$BN = \sqrt{BC^2 - NC^2} = \sqrt{(65x)^2 - (33x)^2} = 56x$$

$\therefore BN = PM = 56x$

Now, the area of trapezium $ABCD = \frac{1}{2} \times (AB + CD) \times PM$

$$\Rightarrow 3640 = \frac{1}{2} \times [2(PB) + 2(MC)] \times PM$$

$$\Rightarrow 3640 = \frac{1}{2} \times (32x + 98x) \times 56x$$

$$\Rightarrow 3640 = 3640x \Rightarrow x = 1$$

Thus the radius $= OP = OM = 28$ cm

And, therefore the area of circle $= \pi \times 28^2 = 2464$ sq. cm

Hence choice (b) is the correct one.

- 78** Since, opposite sides are equal, it means it is a parallelogram.

Since, it's parallelogram, opposite angles are equal. And since it is a cyclic, so opposite angles are supplementary. Therefore, a parallelogram inscribed in a circle is a rectangle.

Now, the area of a rectangle can be greatest only when all the four sides of the rectangle are same. Thus each side of the square = 5 cm.

Therefore required area = 25 sq. cm

Hence choice (b) is the correct one.

Solutions (for Q. Nos. 79 and 80)

- 79** Since, $\frac{DP}{BP} = \frac{6}{7}$, therefore, $DP = 24$ cm and $BP = 28$ cm.

Let $AD = CD = x$ and $AB = BC = y$

$$\therefore 2(x + y) = 208 \text{ cm}$$

$$\therefore x + y = 104 \text{ cm}$$

$$\Rightarrow y = (104 - x) \text{ cm}$$

$$\begin{aligned} \text{Now, } AP^2 &= AD^2 - DP^2 \\ &= x^2 - 24^2 \end{aligned} \quad \dots(i)$$

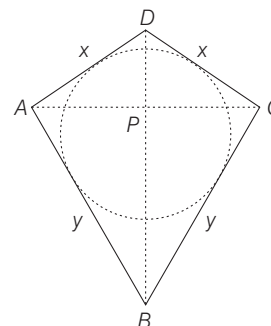
$$\text{Similarly, } AP^2 = AB^2 - BP^2$$

$$\begin{aligned} &= y^2 - 28^2 \\ &= (104 - x)^2 - 28^2 \end{aligned} \quad \dots(ii)$$

From the eqs. (i) and (ii), we have $x = 51$ and $y = 53$

$$\therefore AP = PC = 45 \text{ cm}$$

$$\Rightarrow AC = 90 \text{ cm}$$



$$\text{Area of kite} = \frac{1}{2} \times AC \times DB$$

$$= 90 \times 52 = 2340 \text{ sq. cm}$$

$$\text{And semi-perimeter of kite} = \frac{208}{2} = 104 \text{ cm}$$

As you know that a kite is always a tangential quadrilateral that is each the four sides touch the circle inscribed in it.

Now, you know that in a kite (or a tangential quadrilateral) the inradius

$$r = \frac{\text{area of kite}}{\text{semiperimeter of kite}} = \frac{2340}{104} = 22.5$$

$$\text{Therefore area of circle} = \pi(22.5)^2$$

$$= 1591.07 \approx 1591 \text{ sq cm}$$

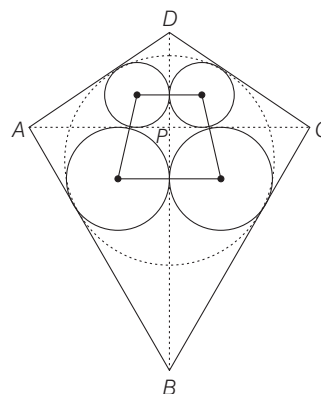
Hence choice (c) is the correct one.

- 80** In the right triangle APB , $AP = 45$, $BP = 28$ and $AB = 53$.

Then the inradius of the incircle inscribed in the $\triangle APB$ is $r_1 = 10$ cm

$$\text{But, } r_1 = r_2 = 10 \text{ cm}$$

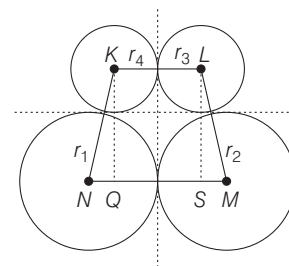
$$\text{Similarly, } r_3 = r_4 = 9 \text{ cm}$$



Now,
and

$$NM = r_1 + r_2 = 20 \text{ cm}$$

$$KL = r_3 + r_4 = 18 \text{ cm}$$



Let us drop the perpendiculars KQ and LS on NM from K and L , respectively.

Then we have, $QS = 18$ cm,

and $NQ = SM = 1$ cm

But, $KQ = LS = (r_1 + r_4) = (r_3 + r_4) = 19$ cm

Let us consider right triangle KQN , we have

$$KN^2 = NQ^2 + KQ^2 = 1 + 361 = 362$$

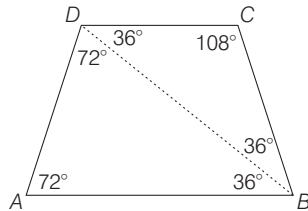
$$\Rightarrow KN = \sqrt{362} = LM$$

Therefore, perimeter of the quadrilateral

$$\begin{aligned} KLMN &= KL + LM + MN + KN \\ &= 18 + \sqrt{362} + 20 + \sqrt{362} \\ &= 38 + 2\sqrt{362} \\ &= 2(19 + \sqrt{362}) \text{ cm} \end{aligned}$$

Hence choice (b) is the correct one.

- 81** First of all draw a diagonal, say BD . Since $BC = CD$, it gives us $\angle CDB = \angle CBD = 36^\circ$ and $\angle ADB = 72^\circ$ and $\angle ABD = 36^\circ$.



Now, in $\triangle ABD$, since $AD = 1$, so $AB = BD = \frac{\sqrt{5} + 1}{2}$

Hence choice (d) is the correct one.

- 82** Let $ABCD$ be the dart with the given specifications (angles and sides). Extend BD to P such that when you connect PA and PC a kite $ADCP$ gets formed with each of its smaller angles equal to 36° and each of its larger angles equal to 144° .

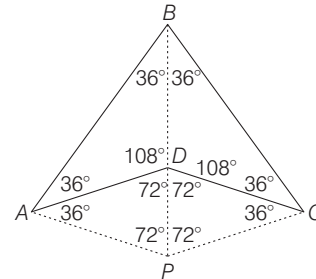
Now, you can see that both the triangles ABP and PBC are two isosceles and congruent triangles. Let us consider $\triangle ABP$ in which $BA = BP$ and $AP = AD = BD$.

As the two triangles $\triangle PAD$ and $\triangle ABP$ are similar ones,

$$\begin{aligned} \Rightarrow \frac{BP}{AP} &= \frac{AP}{DP} \\ \Rightarrow \frac{BP}{BD} &= \frac{BD}{DP} \\ \Rightarrow \frac{BP}{BD} &= \frac{BD}{BP - BD} \\ \Rightarrow \frac{BP}{BD} &= \frac{BD}{\frac{BP}{\frac{BP}{BD}} - \frac{BD}{\frac{BP}{\frac{BP}{BD}}}} \\ \Rightarrow \frac{BP}{BD} &= \frac{1}{\frac{BP}{\frac{BP}{BD}} - 1} \\ \Rightarrow x &= \frac{1}{x - 1} \end{aligned}$$

$$\Rightarrow x^2 - x - 1 = 0$$

$$\Rightarrow x = \frac{1 + \sqrt{5}}{2}$$



$$\Rightarrow \frac{BP}{AP} = \frac{1 + \sqrt{5}}{2}$$

$$\Rightarrow \frac{AB}{AD} = \frac{1 + \sqrt{5}}{2}$$

$$\therefore AD = CD = 1$$

$$\therefore AB = BC = \frac{1 + \sqrt{5}}{2}$$

$$\begin{aligned} \therefore AB + BC + CD + AD &= 2\left(\frac{1 + \sqrt{5}}{2}\right) + 2(1) \\ &= 3 + \sqrt{5} \end{aligned}$$

Thus the perimeter of the dart is $3 + \sqrt{5}$ cm.

Hence choice (a) is the correct one.

- 83** Since the dart is a sort of a kite, though it's a concave kite with a concave (or reflex) angle, so it's also symmetric about the diagonal DB . That's why DB acts as an angle bisector. Thus we can determine every angle of the dart $ABCD$ as shown in the diagram.

$$\angle ADB = \angle CDB = \frac{1}{2}(216^\circ) = 108^\circ$$

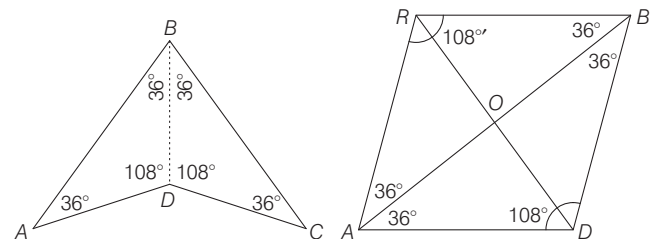
$$\text{And } \angle ABD = \angle CBD = \frac{1}{2}(72^\circ) = 36^\circ$$

$$\therefore \angle BAD = 180 - (108 + 36) = 36^\circ$$

$$\text{Similarly, } \angle BCD = 36^\circ$$

It shows that $\triangle ABD$ and $\triangle BCD$ are isosceles triangles.

Then, $AD = BD = CD = 1$ cm



Now cut the dart along the symmetric line BD , and then join the two congruent triangles as shown in the second diagram. What I have done is that keeping the $\triangle ADB$ as it is I have rotated the $\triangle BDC$ about the vertex B , and then I have attached it with the $\triangle ADB$. Now the vertices C and

A coincide, and the vertex D of $\triangle BDC$ has become the vertex R . Thus we get a rhombus of side 1 cm and its longer diagonal is $\frac{1 + \sqrt{5}}{2}$.

As we know that the diagonals of a rhombus perpendicularly bisect each other, so we can determine the shorter diagonal by using Pythagoras theorem, as discussed below.

$$\begin{aligned} RO^2 &= AR^2 - AO^2 \\ RO^2 &= 1^2 - \left(\frac{1 + \sqrt{5}}{4}\right)^2 = \frac{5 - \sqrt{5}}{8} \\ \Rightarrow RO &= \sqrt{\frac{5 - \sqrt{5}}{8}} \\ \Rightarrow RD &= 2\sqrt{\frac{5 - \sqrt{5}}{8}} = \sqrt{\frac{5 - \sqrt{5}}{2}} \end{aligned}$$

Therefore, area of rhombus

$$= \frac{1}{2} \times \frac{1 + \sqrt{5}}{2} \times \sqrt{\frac{5 - \sqrt{5}}{2}} = \frac{\sqrt{5 + \sqrt{5}}}{2\sqrt{2}}$$

Hence choice (c) is the correct one.

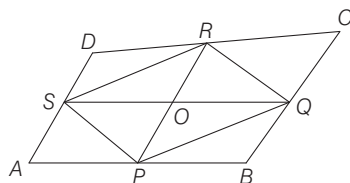
☞ Alternatively The area of dart will be less than 1 sq cm, as the second figure suggests that it is a rhombus with each side 1 cm. Had it been a square with each side 1 cm, the area of the second figure would have been maximum which is actually 1 sq. cm. Now, calculating the approximate value of all the choices, we see that only choice (c) is the valid one, as all other values are greater than 1.

- 84** You know that by joining the mid-points of any quadrilateral a parallelogram is formed. And you also know that the diagonals divide the parallelogram into four equal areas. That is area of $\triangle POQ = \triangle SOP = \triangle ORQ$

$$= \triangle SRO = k \text{ (say)}$$

Let area of quadrilaterals $\square ASOP$, $\square POQB$, $\square ORCQ$, $\square SDRO$ be $4x : 5x : 7x : 6x$, then we have,

$$\frac{\triangle ASP}{\triangle RCQ} = \frac{4x - k}{7x - k} = \frac{2}{5} \Rightarrow k = 2x$$



$$\therefore \frac{\text{Area}(\square PQRS)}{\text{Area}(\square ABCD)} = \frac{2x + x + 2x + 2x}{4x + 5x + 6x + 7x} = \frac{4}{11}$$

Hence choice (c) is the correct one.

NOTE As in any parallelogram the diagonals bisect each other, so they will act as medians and thus the parallelogram is divided into four triangles of equal areas.

- 85** Since $\angle ADC : \angle BCD = 1 : 2$ and $\angle ADC + \angle BCD = 180^\circ$. Therefore $\angle ADC = 60^\circ$ and $\angle BCD = 120^\circ$.

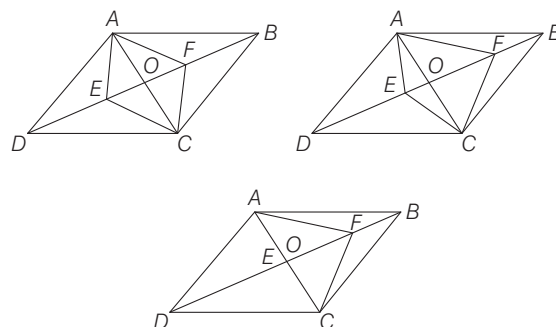
But as you know that AC and DB are the angle bisectors, so $\triangle DAC$ and $\triangle ABC$ are equilateral triangles.

Let us consider $AB = BC = CD = DA = AC = 6$, then $DO = OB = 3\sqrt{3}$, as DO and OB are the altitudes of equilateral triangles $\triangle DAC$ and $\triangle ABC$. Therefore, $BD = 6\sqrt{3}$ and $EF = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$.

Further, as you know that the diagonals of a rhombus always bisect each other perpendicularly, so the quadrilateral formed by the points A, F, C, E is either a parallelogram or a kite or an isosceles triangle. But in all such cases the area of the figure drawn using the four points

$$A, F, C, E = \frac{1}{2} \times AC \times EF = \frac{1}{2} \times 6 \times 2\sqrt{3} = 6\sqrt{3}$$

And the area of quadrilateral $ABCD = 2 \times \frac{\sqrt{3}}{4} \times 6^2 = 18\sqrt{3}$



Therefore the require ratio of area $= 6\sqrt{3} : 18\sqrt{3} = 1 : 3$
Hence choice (d) is the correct one.

- 86** Let $CD = 2a$, so $AB = 8a$

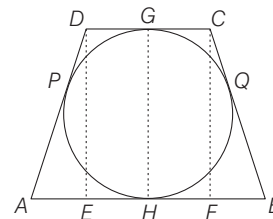
Thus, $AB + CD = 10a = 70$

$$\Rightarrow a = 7 \text{ cm}$$

$$\Rightarrow AB = 56 \text{ cm and } CD = 14 \text{ cm.}$$

Now, we have $DG = CG = 7$ cm. But, since DG and DP are two tangents from the same point on the same circle, so they must be equal.

$$\therefore DG = DP = 7 \text{ cm. Similarly, } CG = CQ = 7 \text{ cm.}$$



Further, $EH = DG = 7$ cm and $HF = CG = 7$ cm. Now, $AH = BH = 28$ cm.

But $AP = AH = 28$ cm, as both are the tangents drawn from the same point on the same circle.

Now, $BQ = BH = 28$ cm.

Thus we have, $AE = AH - EH = 21$ cm

and $DA = DP + AP = 7 + 28 = 35$ cm.

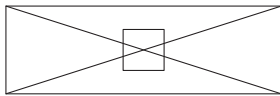
Now, using Pythagoras theorem in triangle ADE , we have
 $DE = \sqrt{(DA)^2 - (AE)^2} = 28$ cm

Therefore circumference of the inscribed circle

$$= 2\pi r = 2r(\pi) = 28\left(\frac{22}{7}\right) = 88 \text{ cm.}$$

Hence choice (d) is the correct one.

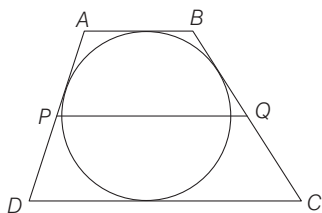
- 87** First of all we must understand that the square window will be at the centre of the rectangular wall, otherwise the diagonals of window and diagonals of wall won't intersect at the same point. Now, look at the figure or draw a proper diagram and you see that since the length of the wall is greater than its height, so the diagonal will have an acute angle with the base of the wall. Therefore the diagonals of the wall will intersect the vertical sides of the window.



Hence choice (b) is the correct one.

- 88** Let $AB = 2x$ and $CD = 2y$, then $PQ = \frac{1}{2}(AB + CD) = x + y$.

$$\begin{aligned} \frac{\text{Area of trapezium } ABQP}{\text{Area of trapezium } PQCD} &= \frac{\frac{1}{2} \times h \times (2x + x + y)}{\frac{1}{2} \times h \times (2y + x + y)} \\ &= \frac{3x + y}{x + 3y} = \frac{1}{2} \Rightarrow \frac{x}{y} = \frac{1}{5} \end{aligned}$$



Since $ABCD$ is a circumscribing trapezium,

So, $AB + CD = BC + DA$

$\Rightarrow AB + CD = 24$ cm

$\Rightarrow AB = 2x = 4$ cm,

$BC = 2y = 20$ cm

Hence choice (a) is the correct one.

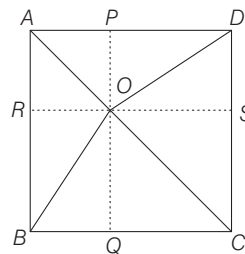
- 89** Given that $OA = 3$, $OB = 4$, $OD = 5$, then

$$OA^2 = AP^2 + OP^2 \quad \dots(i)$$

$$OD^2 = PD^2 + OP^2 \quad \dots(ii)$$

$$OB^2 = BR^2 + OR^2 = BR^2 + AP^2 \quad \dots(iii)$$

$$OC^2 = SC^2 + OS^2 = BR^2 + PD^2 \quad \dots(iv)$$



Adding eqs. (i) and (iv), we get

$$OA^2 + OC^2 = AP^2 + OP^2 + BR^2 + PD^2 \quad \dots(v)$$

Adding eqs. (ii) and (iii), we get

$$OD^2 + OB^2 = PD^2 + OP^2 + BR^2 + AP^2 \quad \dots(vi)$$

From eqs. (v) and (vi), we get

$$OA^2 + OC^2 = OD^2 + OB^2$$

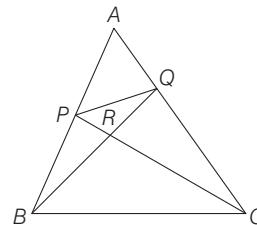
$$3^2 + OC^2 = 5^2 + 4^2$$

$$\Rightarrow OC = 4\sqrt{2}$$

Hence choice (d) is the correct one.

- 90** $\angle APQ = \angle ACB = 60^\circ = 180^\circ - (\angle BAC + \angle ABC)$

$$\therefore \angle AQP = 70^\circ = 180^\circ - (\angle PAQ + \angle APQ)$$



Now, you can see that $\angle BPQ = 120^\circ$ and $\angle PQC = 110^\circ$.

Thus you have, $\angle BPQ + \angle BCQ = 180^\circ$

and $\angle PBC + \angle PQC = 180^\circ$.

It implies that the quadrilateral $PQCB$ is a cyclic one.

$$\begin{aligned} \text{Therefore, } \angle PBQ &= \angle QCP = \angle ACB - \angle PCB \\ &= 60^\circ - 40^\circ = 20^\circ \end{aligned}$$

Hence Choice (c) is the correct one.

Hint $\angle PCB = 180^\circ - (\angle PBC + \angle BPC)$.

And $\angle PBC = \angle BPC$, as $BC = PC$

- 91** Statement (i) is incorrect. In the following diagram, you see that the kite diagram is symmetrical about only one axis KM .

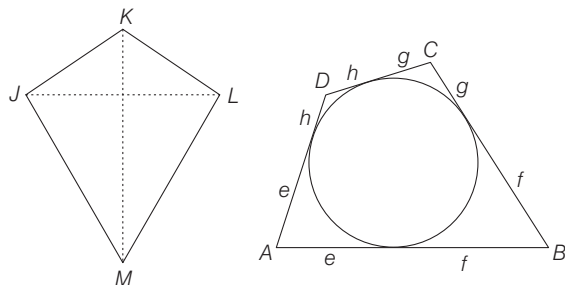
Statement (ii) is correct. Whenever the length of both the diagonals of a kite becomes equal, the kite becomes a square. Whenever all the angles of an isosceles trapezium become equal, the trapezium becomes a square.

Statement (iii) is correct, as the diagonals of a kite bisect each other at 90° .

Statement (iv) is correct. There are numerous possibilities.

Statement (v) is correct.

Statement (vi) is correct. A circumscribing quadrilateral has each of its sides tangent to a circle. Therefore,
 $AB + CD = AD + BC$.



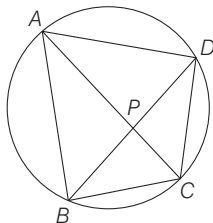
Hence choice (d) is the correct one.

92 Statement (i) is correct.

Statement (ii) is incorrect. When the cyclic quadrilateral is a kite whose main axis is the diameter of the circle, the two diagonals of the kite intersect each other perpendicularly, however, it is not necessary that the perpendicular line drawn through the point of intersection bisects the side of the cyclic quadrilateral. It is true only when the cyclic quadrilateral is a square.

Statement (iii) is correct.

$$AC \times BD = (AD \times BC) + (AB \times CD)$$



Hence choice (c) is the correct one.

93 Statement (i) is correct.

$$AC = 2PQ = 2SR \text{ and } BD = 2PS = 2QR$$

Statement (ii) is correct.

Area of $\triangle RCQ$ + Area of $\triangle SAP$ = Area of $\triangle SDR$ + Area of $\triangle PBQ$

$$\Rightarrow \frac{1}{4}(\text{Area of } \triangle DCB) + \frac{1}{4}(\text{Area of } \triangle DAB) = \frac{1}{4}(\text{Area of } \triangle ADC) + \frac{1}{4}(\text{Area of } \triangle ABC)$$

$$\Rightarrow \frac{1}{4}(\triangle DCB + \triangle DAB) = \frac{1}{4}(\triangle ADC + \triangle ABC)$$

$$\Rightarrow \frac{1}{4}(\square ABCD) = \frac{1}{4}(\square ABCD)$$

Statement (iii) is correct.

$$\text{Area of } X_1 + X_2 + X_3 + X_4 = \frac{1}{2}(\square ABCD)$$

$$\Rightarrow \square PQRS = \frac{1}{2}(\square ABCD)$$

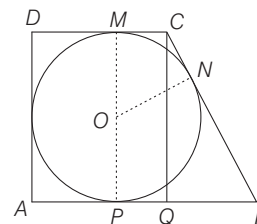
Hence choice (c) is the correct one.

94 Area of $\square ABCD = \triangle ABD + \triangle CBD$

$$= 2(\triangle AFD) + 2(\triangle CBE) \\ = 2(9) + 2(18) = 54 \text{ cm}^2$$

Hence choice (d) is the correct one.

95 First of all draw two lines MP and CQ parallel to DA and join the centre of circle O with N .



$$OP = OM = ON = 4 \text{ cm}$$

(Radii of same circle)

$$MP = CQ = DA = 8 \text{ cm.}$$

Since, CM and CN are tangents from the same point on the same circle, so $CM = CN$.

But, $CM = DC - DM = 5 - 4 = 1 \text{ cm}$,
 therefore, $PQ = CM = CN = 1 \text{ cm}$.

Now, since BP and BN are tangents from the same point on the same circle, so $BP = BN$.

Let us consider $BP = BN = x$, we have

$$BQ^2 + CQ^2 = BC^2$$

$$\Rightarrow (x - 1)^2 + 8^2 = (x + 1)^2 \Rightarrow x = 16$$

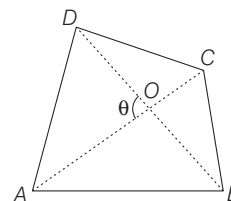
Thus the perimeter of the trapezium

$$ABCD = AB + BC + CD + DA \\ = 20 + 17 + 5 + 8 = 50 \text{ cm}$$

Since the unit of perimeter should be cm not the cm^2 . So choice (b) is incorrect.

Hence Choice (d) is the correct one.

96 We know that if the diagonals of a quadrilateral intersect each other, the product of areas of opposite triangles thus formed is equal.



$$\text{Area of } \triangle AOD = \frac{1}{2}(AO)(OD) \sin \theta$$

$$\text{Area of } \triangle COD = \frac{1}{2}(CO)(OD) \sin \theta$$

$$\text{Area of } \triangle BOC = \frac{1}{2}(BO)(OC) \sin \theta$$

$$\text{Area of } \triangle AOB = \frac{1}{2}(AO)(OB) \sin \theta$$

$$\begin{aligned} \therefore \frac{1}{2}(AO)(OD) \sin \theta &\times \frac{1}{2}(BO)(OC) \sin \theta \\ &= \frac{1}{2}(CO)(OD) \sin \theta \times \frac{1}{2}(AO)(OB) \sin \theta \end{aligned}$$

$$\Rightarrow \text{area of } \triangle AOD \times \text{Area of } \triangle BOC$$

$$= \text{Area of } \triangle COD \times \text{Area of } \triangle AOB$$

$$\Rightarrow 32 \times 18 = x \times y$$

We know that $x + y$ will be minimum when $x = y$

$$\text{Therefore, } 32 \times 18 = x \times x = x^2$$

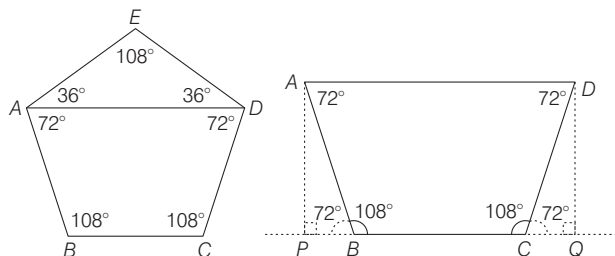
$$\Rightarrow x = 24$$

Thus the minimum area of quadrilateral

$$ABCD = 32 + 24 + 18 + 24 = 98 \text{ cm}^2$$

Hence Choice (c) is the correct one.

- 97** Extend the line segment BC on both the sides and drop the perpendiculars AP and DQ on the extended line PQ . The triangles ABP and DCQ are congruent, as $AB = DC$, and all the three angles in both the triangles, ABP and DCQ , are same. Therefore, $AP = DQ$. That is the distance between AD and PQ is constant. It shows that the line segments AD and PQ are parallel. Thus the quadrilateral $ABCD$ is a trapezium.



Now, you can see that in the quadrilateral $ABCD$ opposite angles are supplementary. Thus in any quadrilaterals if the opposite angles are supplementary, the quadrilateral is a cyclic one. Therefore $ABCD$ is a cyclic quadrilateral.

Now, it is obvious that $AB = BC = CD$, but $AD > BC$ so $AD + BC > AB + CD$. It proves that the sum of opposite sides is not equal, so it is not a tangential quadrilateral.

Hence choice (c) is the correct one.

- 98** In a regular pentagon each interior angle is 108° . Now in $\triangle ABC$, since $AB = BC$ and $\angle ABC = 108^\circ$, so $\angle CAB = \angle ACB = 36^\circ$.

Similarly, in $\triangle DCB$, we have $\angle BDC = \angle DBC = 36^\circ$.

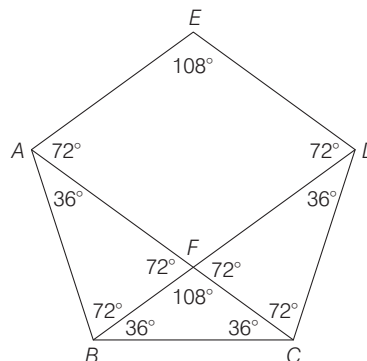
$$\begin{aligned} \text{Therefore, } \angle EAF &= \angle EAC = \angle EAB - \angle CAB \\ &= 108^\circ - 36^\circ = 72^\circ \end{aligned}$$

$$\text{Similarly, } \angle EDF = \angle EDB = 72^\circ.$$

So, in the quadrilateral $AEDF$, we have $\angle EAF = \angle EDF$.

As we know that the sum of all the four interior angles of a quadrilateral is always 360° , so in the quadrilateral $AEDF$ we have $\angle AFD = 360^\circ - (108^\circ + 72^\circ + 72^\circ) = 108^\circ$

It shows that $\angle AED = \angle AFD$.



Thus you can see that $AB = AF = AE = ED = DC = DF$. That is all the four sides of quadrilateral $AEDF$ are equal. So either it is a square or a rhombus; but since the internal angles are not equal to 90° , so it would certainly be a rhombus. Further you know that every rhombus is a kite, so $AEDF$ is a kite.

Now since the opposite angles of the rhombus $AEDF$ are not the supplementary angles, so $AEDF$ is not a cyclic quadrilateral.

However, since the sum of opposite sides is equal, that is $ED + AF = AE + DF$, so the rhombus $AEDF$ is a tangential quadrilateral.

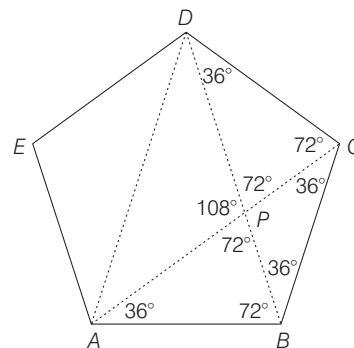
Hence choice (d) is the best answer.

- 99** In $\triangle ACB$, $\angle ABC = 108^\circ$ and $AB = BC$,

$$\therefore \angle BAC = \angle BCA = 36^\circ$$

Similarly, in $\triangle DCB$, $\angle DCB = 108^\circ$

$$\begin{aligned} \text{and } CD &= CB, \\ \therefore \angle CDB &= \angle CBD = 36^\circ \end{aligned}$$



Now in $\triangle PBC$, $\angle PBC = \angle PCB = 36^\circ$,

$$\text{So } \angle BPC = 108^\circ.$$

$$\text{Therefore, } \angle APB = \angle DPC = (180 - 108) = 72^\circ$$

Thus, in $\triangle ABP$, two angles are equal, so the two sides will also be equal. That is $AP = AB = 2 \text{ cm}$.

Similarly, in $\triangle DCP$, two angles are equal, so the two sides will also be equal.

That is $DP = DC = 2$ cm.

Therefore, the required perimeter of the quadrilateral $APDE = 2 + 2 + 2 + 2 = 8$ cm.

Hence choice (d) is the correct one.

100 In $\triangle ACB$, $\angle ABC = 108^\circ$ and $AB = BC$

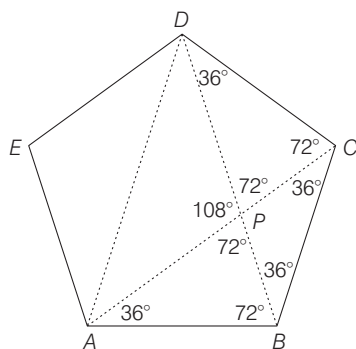
$$\therefore \angle BAC = \angle BCA = 36^\circ$$

Similarly, in $\triangle DCB$, $\angle DCB = 108^\circ$

and

$$CD = CB,$$

$$\therefore \angle CDB = \angle CBD = 36^\circ$$



Now, in $\triangle BPC$, $\angle PBC = \angle PCB = 36^\circ$,

$$\therefore \angle BPC = 180 - (36 + 36) = 108^\circ$$

As it is obvious that $\triangle ABC$ and $\triangle BPC$ are similar triangles, so

$$\frac{AB}{AC} = \frac{PC}{BC}$$

$$\Rightarrow \frac{2}{AP + PC} = \frac{PC}{2}$$

$$\Rightarrow \frac{2}{2 + PC} = \frac{PC}{2}$$

$$\Rightarrow PC = \frac{-2 \pm \sqrt{20}}{2} = -1 \pm \sqrt{5}$$

$$\Rightarrow PC = -1 + \sqrt{5}.$$

(Since, length cannot be a negative value, so $PC \neq -1 - \sqrt{5}$)

Hence choice (a) is the correct one.

101 It is known that the each interior angle of a regular pentagon is 108° ,

so, $\angle BAE = \angle AED = \angle EDC = \angle DCB = \angle CBA = 108^\circ$.

Since $\angle ABE = \angle AEB = 36^\circ$,

therefore $\angle EBC = \angle BED = 108^\circ - 36^\circ = 72^\circ$

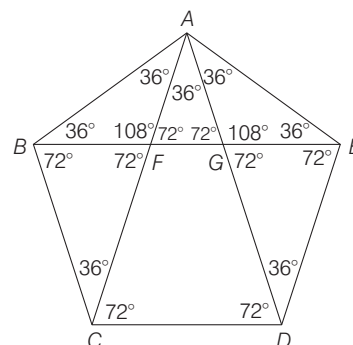
As the interior angles on the same side of a transversal are supplementary, that is $\angle EBC + \angle BCD = 180^\circ$, so the line segments BE and CD are parallel.

Similarly, $AC \parallel ED$ and $AD \parallel BC$.

$$\angle BCA = \angle BAC = 36^\circ$$

and $\angle FBC = 72^\circ$, so $\angle BFC = 72^\circ$.

$$\therefore \angle AFG = 72^\circ \text{ and } \angle AGF = 72^\circ,$$



$$\text{So, } \angle FAG = 36^\circ$$

Therefore, $\triangle BCF \sim \triangle FAG$.

Now, in $\triangle AFB$, $\angle BAF = 36^\circ$

and $\angle ABF = 36^\circ$,

$$\therefore \angle AFB = 108^\circ$$

and in $\triangle BAE$,

$$\angle BAE = 108^\circ \text{ and } \angle ABE = \angle AEB = 36^\circ.$$

Also in $\triangle ABC$, $\angle ABC = 108^\circ$

and $\angle BCA = \angle BAC = 36^\circ$

Thus, $\triangle AFB \sim \triangle BAE \sim \triangle ABC$

Now, since $\triangle BCF \sim \triangle FAG$

and $AF = BF$ and $BC = 1$

$$\therefore \frac{BC}{BF} = \frac{AF}{FG}$$

$$\Rightarrow \frac{BC}{BF} = \frac{BF}{FG}$$

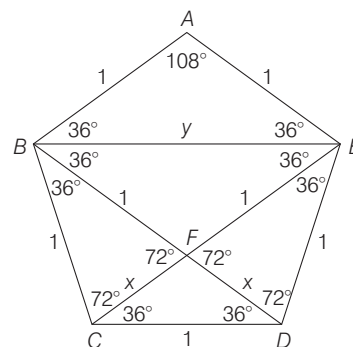
$$\Rightarrow \frac{1}{BF} = \frac{BF}{FG} = BF^2 = FG.$$

Once again, since $\triangle BCF \sim \triangle FAG$

$$\therefore \frac{\triangle BCF}{\triangle FAG} = \frac{BF^2}{FG^2} = \frac{FG}{FG^2} = \frac{1}{FG}$$

Hence choice (d) is the correct one.

102 Consider the following diagram



Now, triangles BFE and CDF are similar triangles,

$$\text{therefore } \frac{x}{1} = \frac{1}{y}$$

But $BE = BD = CE = y = x + 1$, therefore

$$\frac{x}{1} = \frac{1}{y} \Rightarrow \frac{x}{1} = \frac{1}{x+1} \Rightarrow x(x+1) = 1$$

$$\Rightarrow x^2 + x - 1 = 0$$

$$\Rightarrow x = \frac{-1 + \sqrt{5}}{2} \text{ and } x = \frac{-1 - \sqrt{5}}{2}$$

One of these lengths is negative, so it has to be discarded.

And the positive value of $x = \frac{-1 + \sqrt{5}}{2}$

$$\therefore y = x + 1 = \frac{1 + \sqrt{5}}{2}$$

Hence choice (c) is the correct one.

Hint For a standard quadratic equation $ax^2 + bx + c = 0$,

the two solutions of x are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

☞ **Alternatively** Choice (a) is wrong as the diagonal must be less than 2 cm.

Choice (b) is wrong, as the diagonal must be greater than 1 cm.

Choice (d) is absurd, as data is quite sufficient to determine the diagonal.

Hence choice (c) is the correct one.

☞ **Alternatively** Let us consider that each side of the pentagon is a and each diagonal of the pentagon is b .

Now, using Ptolemy's theorem in cyclic quadrilateral $BCDE$, we have

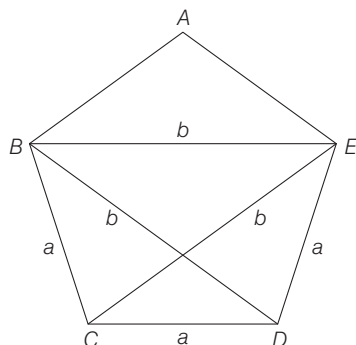
$$BC \times DE + BE \times CD = BD \times CE$$

$$\Rightarrow a^2 + ab = b^2$$

$$\Rightarrow b^2 - a^2 = ab$$

$$\Rightarrow \frac{b^2}{ab} - \frac{a^2}{ab} = \frac{ab}{ab}$$

$$\Rightarrow \frac{b}{a} - \frac{a}{b} = 1$$



$$\Rightarrow x - \frac{1}{x} = 1$$

$$\Rightarrow x^2 - 1 = x$$

$$\Rightarrow x^2 - x - 1 = 0$$

$$\Rightarrow x = \frac{1 + \sqrt{5}}{2} \text{ or } \frac{1 - \sqrt{5}}{2}$$

Since, $\frac{1 - \sqrt{5}}{2}$ is a negative value, which is inadmissible.

$$\text{Therefore, } x = \frac{b}{a} = \frac{1 + \sqrt{5}}{2}$$

$$\text{But, since } a = 1, \text{ therefore } b = \frac{1 + \sqrt{5}}{2}$$

103 First of all extend the sides DC and AB in order to meet at Q .

Now, the angle $BQC = 36^\circ$.

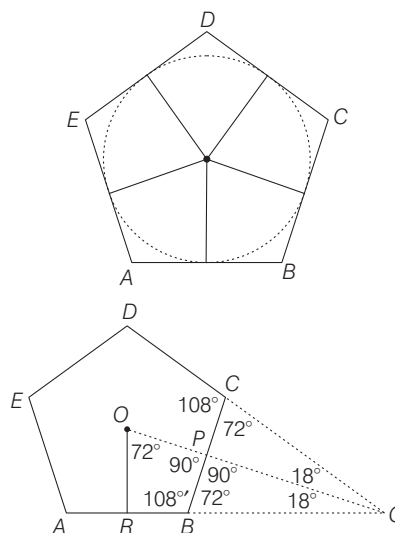
Since $\triangle BQC$ is an isosceles triangle so the perpendicular QP drawn from Q on the side BC will bisect the BC , and it will be the angular bisector of angle BQC , as well.

Similarly, the perpendiculars drawn from the incentre O of the regular pentagon $ABCDE$ will bisect the opposite sides as OR bisects AB at R and OP bisects BC at P .

Now as given that $BC = 2$ cm, therefore, using 36-72-72 degree theorem in $\triangle BQC$, we have $BQ = CQ = (1 + \sqrt{5})$ cm.

Now, in right angled triangle BPQ ,

$$PQ = \sqrt{BQ^2 - BP^2} = \sqrt{(1 + \sqrt{5})^2 - 1^2} = \sqrt{5 + 2\sqrt{5}}$$



But since $\triangle BPQ$ is similar to $\triangle ORQ$, therefore

$$\frac{BQ}{BP} = \frac{OQ}{OR} \Rightarrow \frac{BQ}{BP} = \frac{OP + PQ}{OR}$$

$$\Rightarrow \frac{1 + \sqrt{5}}{1} = \frac{a + \sqrt{5 + 2\sqrt{5}}}{a}$$

$$\Rightarrow a = \frac{\sqrt{5 + 2\sqrt{5}}}{\sqrt{5}} = \sqrt{\frac{5 + 2\sqrt{5}}{5}}$$

Hence choice (a) is the correct one.

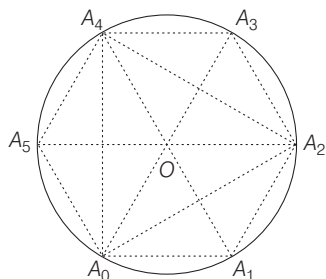
Hint Using the 72-72-36 degree theorem in $\triangle BQC$, we have $BQ = CQ = 1 + \sqrt{5}$ cm since $BC = 2$ cm.

☞ **Alternatively** Use trigonometric ratios.

- 104** Since radius of circle is 1 unit. So each side of the hexagon is 1 unit.

Therefore, $A_0A_3 = A_1A_4 = A_2A_5 = 2$ units.

$A_0A_2A_3$ is a right angle triangle as it is a triangle in the semi-circle. Since $A_2A_3 = 1$, $A_0A_3 = 2$, so $A_0A_2 = \sqrt{3}$.



Now, since $A_0A_4 = A_0A_2 = \sqrt{3}$.

Therefore, $A_0A_1 \times A_0A_2 \times A_0A_4 = 1 \times \sqrt{3} \times \sqrt{3} = 3$.

Hence choice (c) is the answer.

☞ **Alternatively** This problem can be solved using 30-60-90 theorem as well.

☞ **Alternatively** Problem can be solved by using the following concept too.

Area of parallelogram $A_0A_1A_2O$

$$= \text{Area of } \triangle A_0A_1O + \text{Area of } \triangle A_1A_2O$$

$$= \frac{1}{2} (A_0A_2 \times OA_1) = 2 \left(\frac{\sqrt{3}}{4} \times 1^2 \right)$$

$$\Rightarrow A_0A_2 = \sqrt{3}.$$

Therefore, $A_0A_1 \times A_0A_2 \times A_0A_4 = 1 \times \sqrt{3} \times \sqrt{3} = 3$.

☞ **Alternatively** Problem can be solved by using the following concept too.

$$\text{Area of } \triangle A_0A_2A_4 = \frac{A_0A_2 \times A_2A_4 \times A_4A_0}{4R}$$

$$\Rightarrow 3 \times \left(\frac{\sqrt{3}}{4} \times 1^2 \right) = \frac{k^3}{4} \Rightarrow k = \sqrt{3}$$

Therefore, $A_0A_1 \times A_2A_2 \times A_2A_4 = \sqrt{3} \times \sqrt{3} \times 1 = 3$

- 105** In $\triangle EDC$, $\angle EDC = 120^\circ$ and $ED = DC$

$$\Rightarrow \angle CED = \angle DCE = 30^\circ$$

Similarly, in $\triangle FED$, $\angle FED = 120^\circ$

and $FE = ED \Rightarrow \angle DFE = \angle FDE = 30^\circ$.

Therefore, in $\triangle GED$,

$$\angle GED = \angle GDE = 30^\circ$$

$$\Rightarrow \angle GED = 120^\circ \text{ and } GE = GD.$$

Now, using 30-30-120 degree theorem, we get the following results.

If $EG = GD = 1$, $ED = \sqrt{3}$. Therefore, $FD = \sqrt{3}ED = 3$. It shows that $FD = 3GD$.

It implies that $FG = 2GD$. Therefore, $Y : Z = 2 : 1$.

Now, in $\triangle CFD$, $FG = 2GD$. Therefore, $X : Y = 2 : 1$

Thus, $Z : Y : X = 1 : 2 : 4$.

$$\text{But } W = X + 2Y + Z = 4 + 2(2) + 1 = 9$$

$$\text{Therefore, } W : X : Y : Z = 9 : 4 : 2 : 1$$

Hence, Choice (d) is the answer.

☞ **Alternatively** Since $W = X + 2Y + Z$, therefore choice (a) is wrong.

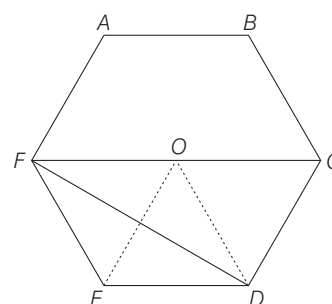
As $FC = 2ED$, $\Rightarrow X : Z = 4 : 1$, therefore choice (b) and (c) are wrong.

Hence choice (d) is correct.

☞ **Alternatively** As area of $\triangle FEO = \triangle EOD = \triangle OCD$ and area of $\triangle FDE = \triangle FDO$.

It implies that area of $\triangle FCD = 2(\triangle FED)$

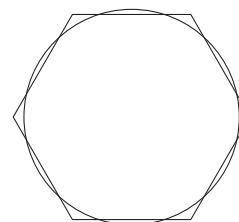
$$\Rightarrow X + Y = 2(Y + Z).$$



Therefore choice (b) and (c) are wrong.

Further, $W = X + 2Y + Z$, therefore choice (a) is also wrong.

- 106** It can be deduced from the problem that the area of circle and area of hexagon are same.



$$\text{Now, area of hexagon} = \frac{3\sqrt{3}}{2} a^2.$$

$$\text{Therefore, area of circle} = \frac{3\sqrt{3}}{2} a^2.$$

Hence choice (b) is the correct one.

$$\begin{aligned} \text{107 Total number of diagonals} &= \frac{n(n-3)}{2} \\ &= \frac{12 \times 9}{2} = 54 \end{aligned}$$

Number of diagonals will pass through the centre

$$= \frac{n}{2} = \frac{12}{2} = 6$$

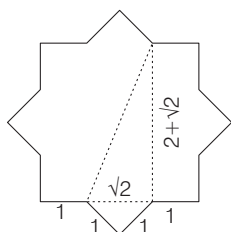
Therefore, the number of diagonals which do not pass through the center of the circumscribing circle $= 54 - 6 = 48$.

Hence choice (d) is the correct one.

- 108** The diameter of the smaller circle

$$\sqrt{(2 + \sqrt{2})^2 + (\sqrt{2})^2} = 2(\sqrt{2 + \sqrt{2}})$$

Therefore radius of the smaller circle = $\sqrt{2 + \sqrt{2}}$



The diameter of the larger circle = diagonal of the square
 $= \sqrt{2}(2 + \sqrt{2}) = 2(\sqrt{2} + 1)$

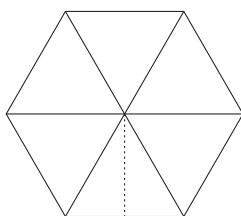
Therefore radius of the larger circle = $\sqrt{2} + 1$

The ratio of area of smaller circle to that of larger circle

$$= \frac{\pi(\sqrt{2 + \sqrt{2}})^2}{\pi(\sqrt{2} + 1)^2} = \frac{2 + \sqrt{2}}{3 + 2\sqrt{2}}$$

Hence choice (a) is the correct one.

- 109** In the following diagram, there are six equilateral triangles and you can see that apothem is the perpendicular angle bisector of the equilateral triangle which forms two congruent triangles of 30-60-90 degree. If the apothem of the regular hexagon is a , then by using 30-60-90 degree theorem (or trigonometric ratios or Pythagoras theorem) we get to know that its side will be $\frac{2}{\sqrt{3}}a$.



Hence choice (d) is the correct one.

Alternatively If s be the length of each side and n be the number of sides, the apothem of a regular polygon is

$$a = \frac{1}{2} \times s \times \cot\left(\frac{\pi}{n}\right)$$

$$\Rightarrow a = \frac{1}{2} \times s \times \cot\left(\frac{180}{6}\right)^\circ$$

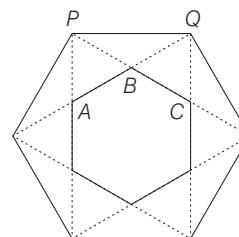
$$\Rightarrow a = \frac{1}{2} \times s \times \cot 30^\circ$$

$$\Rightarrow a = \frac{1}{2} \times s \times \sqrt{3}$$

$$\Rightarrow s = \frac{2}{\sqrt{3}}a$$

- 110** Let us assume that each side of the internal hexagon be 1 cm, that is $AB = BC = \dots = 1$ cm, then $AP = BP = BQ = CQ = 1$ cm, as $\triangle APB$ and $\triangle BQC$ are equilateral triangles.

But in $\triangle PBQ$, we have $\angle QPB = 30^\circ$, $\angle PQB = 30^\circ$ and $\angle PBQ = 120^\circ$, therefore either using cosine rule (or 30-30-120 degree theorem) we will have $PQ = \sqrt{3}$ cm. It means each side of the external hexagon is $\sqrt{3}$ cm.

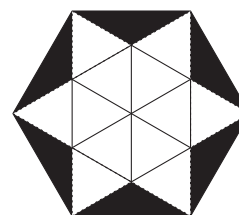


$$\therefore \frac{\text{Area of internal hexagon}}{\text{Area of external hexagon}} = \frac{\frac{3\sqrt{3}}{4}(1)^2}{\frac{3\sqrt{3}}{4}(\sqrt{3})^2} = \frac{1}{3}$$

Hence choice (a) is the correct one.

Alternatively : Draw three new lines passing through the centre of the hexagon, as shown below.

Now, you can easily observe that there are total 18 equilateral triangles as in 6 within the internal hexagon and $12 = (6 + 6)$ outside the internal hexagon. Please note that, in the following diagram, each of the 6 shaded obtuse triangles is equivalent to an equilateral triangle of side 1 cm.



Therefore, the required ratio of areas = $\frac{6}{18} = \frac{1}{3}$

Hint Consider the following triangles.

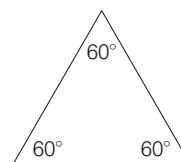


Fig. (i)

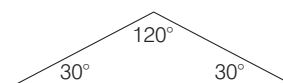
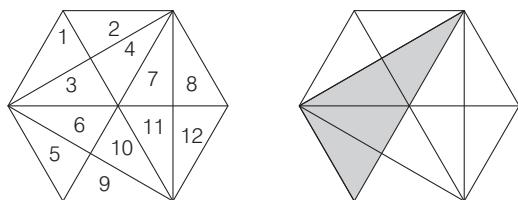


Fig. (ii)

The first triangle is an equilateral triangle with each of its sides 1 unit and fig. (ii) is an isosceles triangle with its angles 30-30-120 degree and each of the two smaller sides is 1 cm, then the area of both the triangles will be same.

- 111** Look closely at the following diagrams and you will find that there are total 12 isosceles right angled triangles each with equal area as each of these 12 triangles is congruent (30°-60°-90°).

Now, out of the 12 triangles 4 triangles fall under the shaded region and 8 triangles remain un-shaded.



Therefore, the required ratio $= \frac{4}{8} = \frac{1}{2}$.

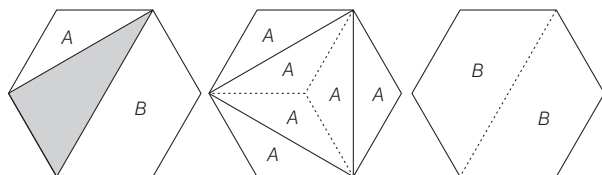
Hence choice (d) is the correct one.

☞ **Alternatively** From the following diagrams you can easily figure it out that the region A (isosceles triangle) is equal to $\frac{1}{6}$ of the total area of the hexagon.

Similarly, region B (trapezium) is equal to $\frac{1}{2}$ of the total area of the hexagon.

Therefore the total un-shaded region is $\frac{1}{6} + \frac{1}{2} = \frac{4}{6} = \frac{2}{3}$ of the total area of the hexagon.

So the shaded region $= 1 - \frac{2}{3} = \frac{1}{3}$ of the total area of the hexagon.



Thus the required ratio of areas of the shaded region to un-shaded region $= \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$.

☞ **Alternatively** Please note that the following two figures are exactly the same except the triangle in the fig. (ii) is rotated 60° anti-clockwise with relation to the fig. (i).

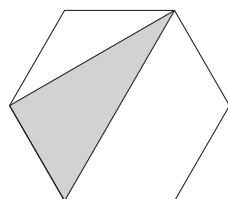


Fig. (i)

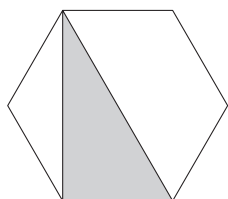


Fig. (ii)

Now you can see that the area of the shaded region is equal to a right angle triangle. Assume that the each side of the hexagon is 1 cm, so the smaller diagonal of the hexagon (which is also the altitude of the triangle) is $\sqrt{3}$ cm and the longer diagonal (which is also the hypotenuse of the triangle) is 2 cm.

Then the area of the shaded region

(or right angled triangle)

$$= \frac{1}{2} \times 1 \times \sqrt{3} = \frac{\sqrt{3}}{2} \text{ sq cm.}$$

And the area of the hexagon $= \frac{3\sqrt{3}}{2} \times (1)^2$

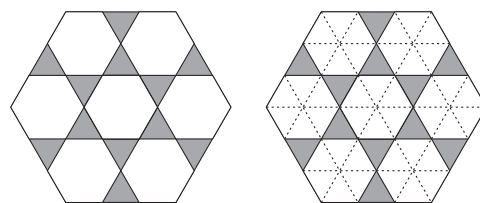
$$= \frac{3\sqrt{3}}{2} \text{ sq cm}$$

Therefore the area of un-shaded region

$$= \frac{3\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = \sqrt{3} \text{ sq cm}$$

Thus the required ratio $= \frac{\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}} = \frac{1}{2}$

- 112** In the following diagram you can easily notice that there are total 54 equilateral triangles of the same size. Out of 54 equilateral triangles 12 triangles are shaded and 42 ($= 7 \times 6$) triangles are un-shaded.



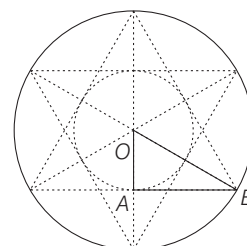
Even it is also obvious that each side of these 54 triangles is 1 cm.

Therefore area of the shaded region

$$= 12 \times \frac{\sqrt{3}}{4} \times (1)^2 = 3\sqrt{3} \text{ sq cm.}$$

Hence choice (c) is the correct one.

- 113** First of all join the opposite vertices of the hexagram, then you see that in the following diagram, $\triangle OAB$ is a right angled triangle, in which $\angle OBA = 30^\circ$, $\angle BOA = 60^\circ$, $\angle OAB = 90^\circ$.



Also, OA is the inradius and OB is the circumradius of the hexagram.

Since area of the incircle is 3π sq cm, so its inradius (OA) $= \sqrt{3}$ cm.

Using 30-60-90 degree theorem in $\triangle OAB$, we have $AB = 3$ cm and $OB = 2\sqrt{3}$ cm.

That is circumradius of the hexagram is $2\sqrt{3}$ cm.

Therefore, area of the circumcircle $= \pi(2\sqrt{3})^2 = 12\pi$ sq. cm.

Hence choice (a) is the correct one.

- 114** From the information given in the problem it can be concluded that the area of the outer hexagon is 4 times the area of the inner hexagon. That is, if a_0 and a_i be the sides of outer and inner hexagon, then

$$\frac{\text{Area of the outer hexagon}}{\text{Area of the inner hexagon}} = \frac{\frac{3\sqrt{3}}{2}(a_0)^2}{\frac{3\sqrt{3}}{2}(a_i)^2}$$

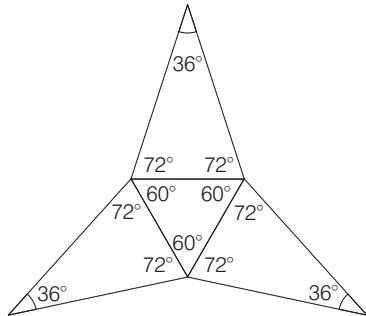
$$\Rightarrow \frac{4}{1} = \frac{(a_0)^2}{(a_i)^2} \Rightarrow \frac{a_0}{a_i} = \frac{2}{1}$$

$$\therefore \frac{\text{Perimeter of outer hexagon}}{\text{Perimeter of inner hexagon}} = \frac{6(a_0)}{6(a_i)} = \frac{2}{1}$$

Hence choice (d) is the correct one.

- 115** Connect the nearest three vertices of the hexagram which will form an equilateral triangle, as shown in the following diagram. Thus we can determine all the angles of this diagram.

Now, using 36-72-72 degree theorem we have the three sides of the isosceles triangles as $1 + \sqrt{5}$ mm, $1 + \sqrt{5}$ mm and 2 mm. It shows that the each side of the equilateral triangle is 2 mm.



Therefore, area of the equilateral triangle

$$= \frac{\sqrt{3}}{4} \times 2^2 = \sqrt{3} \text{ sq mm.}$$

Now the perpendicular of each isosceles triangle

$$= \sqrt{(1 + \sqrt{5})^2 - (1)^2} = \sqrt{5 + 2\sqrt{5}}$$

Therefore, area of each isosceles triangle

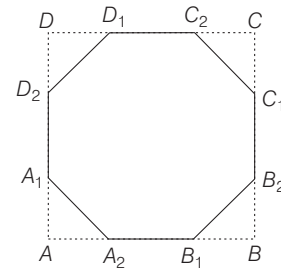
$$= \frac{1}{2} \times 2 \times \sqrt{5 + 2\sqrt{5}} = \sqrt{5 + 2\sqrt{5}}$$

Thus the total area of the hexagram = $\sqrt{3} + 3(\sqrt{5 + 2\sqrt{5}})$

Hence choice (a) is the correct one.

- 116** From the following diagram you can observe the symmetry of the structure in which each side of the octagon is equal and each of the four triangles at the corner is an isosceles right angle triangle. If the legs (or the equal sides) of the right angles are k , then

$$AA_1 = AA_2 = BB_1 = BB_2 = CC_1 = CC_2 = DD_1 = DD_2 = k$$



Therefore $A_1A_2 = B_1B_2 = C_1C_2 = D_1D_2 = \sqrt{2}k$

That is each side of the octagon = $\sqrt{2}k$

It is known that $AB = BC = CD = DA = a$

But Since $AB = AA_2 + A_2B_1 + B_1B$

$$\therefore a = k + \sqrt{2}k + k$$

$$\Rightarrow a = k(2 + \sqrt{2})$$

$$\Rightarrow k = \frac{a}{(2 + \sqrt{2})}$$

Hence choice (c) is the correct one.

- 117** In triangle ABC , $AB = BC$

$$\therefore \angle CAB = \angle ACB = \angle y = 22.5^\circ$$

$$\text{as } \angle ABC = 135^\circ$$

Similarly, in triangle BCD , $BC = CD$

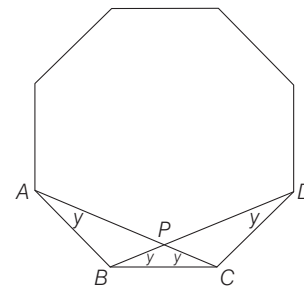
$$\therefore \angle DBC = \angle BDC = \angle y = 22.5^\circ$$

$$\text{as } \angle BCD = 135^\circ$$

Therefore, in triangle BPC ,

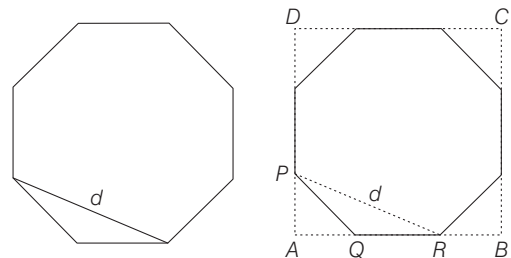
$$\angle BPC = 180^\circ - 2y = 135^\circ$$

$$\text{Thus, } \angle APD = \angle BPC = x = 135^\circ$$



Hence choice (d) is the correct one.

- 118** In the following diagram PR is the shortest possible diagonal of the regular octagon. Triangle PAQ is an isosceles right triangle.



Let us consider

$$AP = AQ = b$$

$$\therefore PQ = \sqrt{2}b$$

$$\text{But since } PQ = QR = \sqrt{2}b$$

$$\text{That is } a = \sqrt{2}b \Rightarrow b = \frac{a}{\sqrt{2}}$$

$$\therefore PR = \sqrt{PA^2 + AR^2}$$

$$\Rightarrow PR = \sqrt{b^2 + (b + \sqrt{2}b)^2}$$

$$\Rightarrow PR = (\sqrt{4 + 2\sqrt{2}})b = \left(\frac{\sqrt{4 + 2\sqrt{2}}}{\sqrt{2}} \right)a$$

$$= (\sqrt{2 + \sqrt{2}})a$$

Hence choice (a) is the correct one.

Alternatively As $\angle PQR = 135^\circ$, therefore by using cosine rule in the triangle ΔPQR , we have

$$\cos 135^\circ = \frac{PQ^2 + QR^2 - PR^2}{2(PQ)(QR)}$$

$$\Rightarrow \cos(90^\circ + 45^\circ) = \frac{a^2 + a^2 - d^2}{2(a)(a)}$$

$$\Rightarrow -\sin 45^\circ = \frac{a^2 + a^2 - d^2}{2(a)(a)}$$

$$\Rightarrow -\frac{1}{\sqrt{2}} = \frac{2a^2 - d^2}{2a^2}$$

$$\Rightarrow d = (\sqrt{2 + \sqrt{2}})a$$

- 119** In the following diagram PT is the longest possible diagonal of the regular octagon. Triangle RAS is an isosceles right triangle.

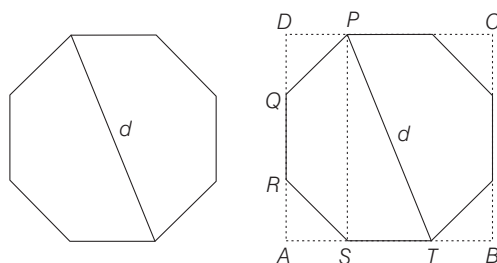
Let us consider $RA = AS = b$

$$\therefore RS = \sqrt{2}b$$

$$\text{But since } PQ = QR = RS = ST = \sqrt{2}b$$

$$\text{That is } a = \sqrt{2}b$$

$$\Rightarrow b = \frac{a}{\sqrt{2}}$$



$$\therefore PS = DA = DQ + QR + RA = b + \sqrt{2}b + b$$

$$= (2 + \sqrt{2})b = (1 + \sqrt{2})a$$

$$\therefore PT = \sqrt{PS^2 + ST^2}$$

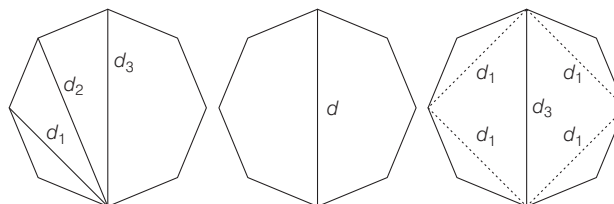
$$= \sqrt{[(1 + \sqrt{2})a]^2 + a^2} = (\sqrt{4 + 2\sqrt{2}})a$$

Hence choice (d) is the correct one.

NOTE Do you remember that in a regular octagon there are three diagonals of distinct lengths? So PS is one of the diagonals shorter than the longest diagonal and longer than the shortest diagonal.

Alternatively As you know that every regular octagon has three diagonals of distinct lengths, such that $d_1 < d_2 < d_3$. From the following diagrams, it is obvious that the required diagonal $d = d_3$

Even it is also obvious that $d_3 = \sqrt{2}(d_1)$

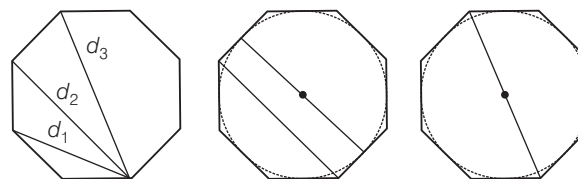


Therefore the required length of the diagonal is

$$d_3 = \sqrt{2}(d_1) = \sqrt{2} \times (\sqrt{2 + \sqrt{2}})a$$

$$= (\sqrt{4 + 2\sqrt{2}})a$$

- 120** If you observe closely, you will find that the inradius of the octagon is same as half of d_2 and circumradius of the octagon is same as half of d_3 .



$$\text{Therefore the required ratio} = \frac{\frac{1}{2}(d_2)}{\frac{1}{2}(d_3)} = \frac{d_2}{d_3}$$

Hence choice (c) is the correct one.

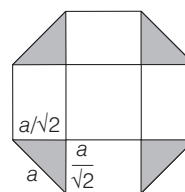
- 121** Let a be the side of the regular octagon, then each of the shorter sides of each of the isosceles right angled triangle at the four corners be $\frac{a}{\sqrt{2}}$.

Therefore area of all the four such triangles

$$= 4 \times \frac{1}{2} \times \left(\frac{a}{\sqrt{2}} \right) \left(\frac{a}{\sqrt{2}} \right) = a^2$$

And area of the square at the centre = a^2

Thus total shaded area = $2a^2$



Now area of each of the four rectangles $= a \times \left(\frac{a}{\sqrt{2}}\right) = \frac{a^2}{\sqrt{2}}$

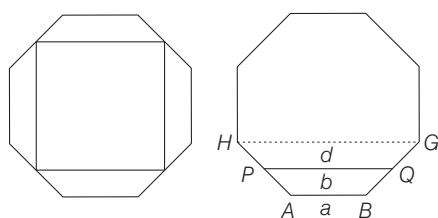
Therefore area of the unshaded region $= 4 \times \frac{a^2}{\sqrt{2}} = (2\sqrt{2})a^2$

Thus the required ratio $= \frac{2a^2}{(2\sqrt{2})a^2} = \frac{1}{\sqrt{2}}$

Hence choice (c) is the correct one.

- 122** In the following figures, you can easily observe that the side of the square, $b = \frac{d+a}{2}$

That is $(b) = \frac{[(1 + \sqrt{2})a] + a}{2} = \frac{(2 + \sqrt{2})a}{2}$



Hence choice (a) is the correct one.

Hint HG is parallel to AB . Since P and Q are the mid-points of HA and GB , so you can use mid-point theorem and thus PQ will be parallel to AB and HG ; and PQ will be half of the sum of the AB and HG .

Alternatively It is known that $HA = AB = BG$

And $PA = \frac{AH}{2} = \frac{a}{2}$

But since triangle PMA is an isosceles right angled triangle, so $PM = \frac{PA}{\sqrt{2}} = \frac{a/2}{\sqrt{2}} = \frac{a}{2\sqrt{2}}$

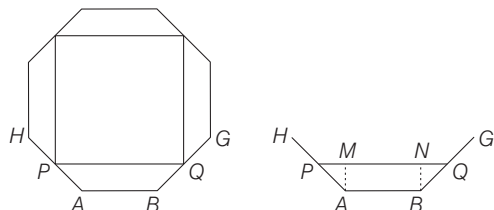
$\therefore PQ = PM + MN + NQ$

$\Rightarrow PQ = PM + AB + PM$

$(PM = MA = NB = NQ)$

$\Rightarrow PQ = \frac{a}{2\sqrt{2}} + a + \frac{a}{2\sqrt{2}} = \left(1 + \frac{1}{\sqrt{2}}\right)a$

$\Rightarrow PQ = \frac{(\sqrt{2} + 1)a}{\sqrt{2}} = \frac{(2 + \sqrt{2})a}{2}$

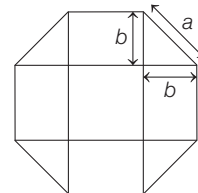


Hence choice (a) is the correct one.

- 123** In the following diagram of a regular polygon there are :

- (i) 4 isosceles right angled triangles each having its area $= \frac{1}{2} \times b \times b$

- (ii) 4 rectangles each having its area $= a \times b$
(iii) 1 square at the centre having its area $= a \times a$



Let each side of the isosceles triangle be b , then $a = \sqrt{2}b$
Therefore, total area of the octagon.

$$A = 4\left(\frac{1}{2} \times bb\right) + 4(ab) + (aa)$$

$$A = 2bb + 4ab + aa$$

$$A = 2\left(\frac{a}{\sqrt{2}} \times \frac{a}{\sqrt{2}}\right) + 4\left(a \times \frac{a}{\sqrt{2}}\right) + aa$$

$$A = a^2 + 2\sqrt{2}a^2 + a^2$$

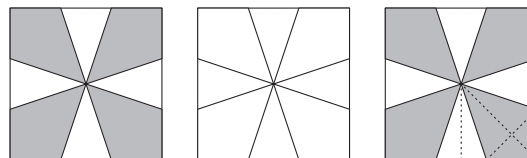
$$A = 2(1 + \sqrt{2})a^2$$

Hence choice (d) is the correct one.

- 124** Let the perpendicular height and base of each isosceles triangle be a and b , respectively; and the shorter diagonal and the longer diagonal of each kite be c and d , respectively.

Therefore area of each triangle $= \frac{1}{2} \times ab$

And, area of each kite $= \frac{1}{2} \times cd$



Now, if we assume that each side of the square is x , then we have $a = \frac{x}{2}$

But since area of each triangle is $\frac{1}{8}$ of the area of the square, therefore

$$\frac{1}{2} \times ab = \frac{1}{8} \times x \times x$$

$$\Rightarrow \frac{1}{2} \times \frac{x}{2} \times b = \frac{1}{8} \times x \times x$$

$$\Rightarrow b = \frac{x}{2}$$

Also, $d = \frac{x}{\sqrt{2}}$

Now, since $\frac{1}{2} \times ab = \frac{1}{2} \times cd$

$$\Rightarrow \frac{1}{2} \times \frac{x}{2} \times \frac{x}{2} = \frac{1}{2} \times c \times \frac{x}{\sqrt{2}}$$

$$\Rightarrow c = \frac{x}{2\sqrt{2}}$$

$$\text{Therefore, the required ratio} = \frac{d}{c} = \frac{\frac{x}{\sqrt{2}}}{\frac{x}{2\sqrt{2}}} = \frac{2}{1}$$

Hence choice (a) is the correct one.

- 125** You can easily observe that there are 8 isosceles right angled triangles, which are shaded. If each smaller side (legs) of the isosceles right angled triangle is a , then the hypotenuse of each triangle would be $\sqrt{2}a$ and thus each side of the octagon would also be $s = \sqrt{2}a$.

The area of 8 triangles = 4 sq cm

$$\Rightarrow 8 \times \frac{1}{2} \times a^2 = 4 \text{ sq cm}$$

$$\Rightarrow a = 1 \text{ cm}$$

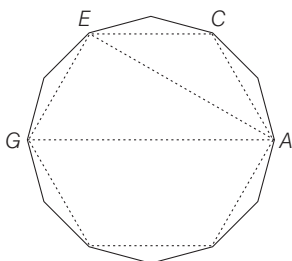
That is each side of the octagon is $s = \sqrt{2}a = \sqrt{2} \text{ cm}$

$$\begin{aligned} \text{Therefore area of the octagon} &= 2(1 + \sqrt{2})(s)^2 \\ &= 2(1 + \sqrt{2})(\sqrt{2})^2 = 4(1 + \sqrt{2}) \end{aligned}$$

Thus the area of the common region = $4(1 + \sqrt{2}) \text{ cm}^2$

Hence choice (b) is the correct one.

- 126** All the points are lying on the circle, so the quadrilateral $ACEG$ is a cyclic quadrilateral. Now, $\angle ACE = 120^\circ$, since $AC, CE, EG, \dots$ are the equal sides of a regular hexagon.

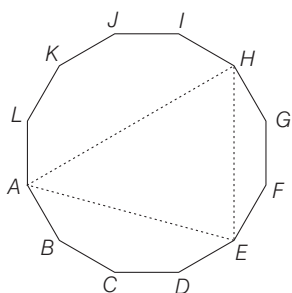


We know that the opposite angles of a cyclic quadrilateral are supplementary. That is $\angle ACE + \angle AGE = 180^\circ$.

Therefore, $\angle AGE = 180 - 120 = 60^\circ$.

Hence choice (d) is the correct one.

- 127** Consider the hexagon $ALKJIH$, in which $\angle ALK = \angle LKJ = \angle KJI = \angle JIH = 180 - 30 = 150^\circ$. Also, we know that the sum of all the interior angles of a hexagon = $(6 - 2) \times 180 = 720^\circ$.



Therefore, $\angle LAH + \angle IHA = 720 - 4(150) = 120^\circ$.

But, since $\angle LAH = \angle IHA$, therefore, $\angle IHA = 60^\circ$.

Consider the quadrilateral $EFGH$, $\angle HGF = \angle GFE = 150^\circ$.

Also, we know that the sum of all the interior angles of a quadrilateral = $(4 - 2) \times 180^\circ = 360^\circ$.

Therefore, $\angle EHG + \angle FEH = 360^\circ - 2(150) = 60^\circ$.

But, since $\angle EHG = \angle FEH$, therefore, $\angle EHG = 30^\circ$.

Now, we know that

$$\angle IHA + \angle AHE + \angle EHG = \angle IHG = 150^\circ.$$

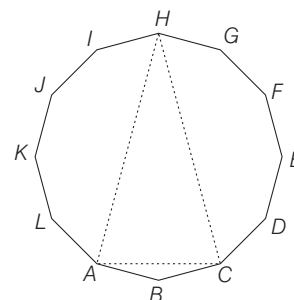
Therefore, $\angle AHE = 150^\circ - (60 + 30) = 60^\circ$

Hence choice (d) is the correct one.

- 128** Consider the hexagon $ALKJIH$, in which $\angle ALK = \angle LKJ = \angle KJI = \angle JIH = 180^\circ - 30^\circ = 150^\circ$. Also, we know that the sum of all the interior angles of a hexagon = $(6 - 2) \times 180 = 720^\circ$.

Therefore, $\angle LAH + \angle IHA = 720 - 4(150) = 120^\circ$.

But, since $\angle LAH = \angle IHA$, therefore, $\angle IHA = 60^\circ$.



Similarly, $\angle GHC = 60^\circ$.

But since

$$\angle IHA + \angle AHC + \angle GHC = \angle IHG = 150^\circ$$

Therefore, $\angle AHC = 150 - (60 + 60) = 30^\circ$.

Hence choice (c) is the correct one.

- 129** The circumradius of the dodecagon is same as the circumradius of the triangle.

The circumradius of the triangle

$$= \frac{1}{\sqrt{3}} (\sqrt{3}\sqrt{2+\sqrt{3}}) = \sqrt{2+\sqrt{3}} \text{ cm}$$

Hence choice (c) is the correct one.

- 130** The circumradius of the dodecagon is same as the circumradius of the square.

Since each side of the square is $\sqrt{3} + 1 \text{ cm}$, its diagonal would be $\sqrt{2}(\sqrt{3} + 1) \text{ cm}$ and so the circumradius would

$$\text{be } \frac{1}{2} (\sqrt{2}(\sqrt{3} + 1)) = \frac{(\sqrt{3} + 1)}{\sqrt{2}} \text{ cm.}$$

Hence choice (a) is the correct one.

- 131** The area of dodecagon = $3 \times \text{circumradius} = 3 \text{ sq cm}$.

Hence choice (a) is the correct one.

132 Consider the following figures.

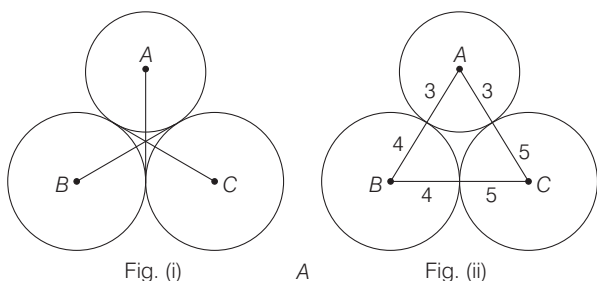


Fig. (i)

Fig. (ii)

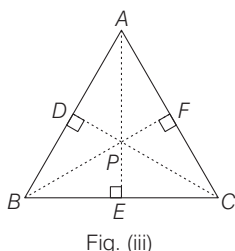


Fig. (iii)

In figure (iii), anyone can easily infer that $\triangle BDP$ and $\triangle BEP$ are congruent ones due to the Side-Angle-Side property. Consequently, it implies that BP (or BF) is the angle bisector as $\angle DBP = \angle EBP$.

Thus it can be generalized that all three tangents lines AE , BF , CD are angle bisectors. And, so the intersection of AE , BF and CD will be the incentre of $\triangle ABC$. It means PD , PE and PF each is the inradius of $\triangle ABC$. Let us assume that the inradius be r , such that $PD = PE = PF = r$.

Now you can use the relation,

area of $\triangle ABC = \text{inradius}(r) \times \text{semi-perimeter}(s)$

$$\therefore \sqrt{s(s-a)(s-b)(s-c)} = r \times s$$

$$\Rightarrow \sqrt{12 \times 5 \times 4 \times 3} = r \times 12$$

$$\Rightarrow r = \sqrt{5}$$

Hence choice (a) is the answer.

133 If the radius of smaller circle is exactly half of the radius of larger circle, YZ will be tangent to the smaller circle.

If the radius of smaller circle is less than half of the radius of the larger circle, YZ will neither touch nor intersect the smaller circle.

If the radius of smaller circle is greater half of the radius of larger circle, YZ will intersect the smaller circle at two distinct points.

Hence choice (c) is the correct one.

Solutions (for Q. Nos. 134 and 135)

134 Consider the following figure.

$$\angle ACB = \angle ABD = 60^\circ$$

(Using the Alternate Segment Theorem)

$$\angle AOB = 2(\angle ACB) = 120^\circ$$

(Using the concept of Central Angle)

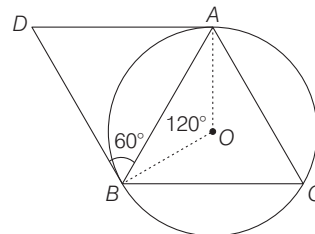
Now, since AD and BD are tangents, so $\angle OAD = 90^\circ$ and $\angle OBD = 90^\circ$.

Therefore, $\angle ADB = 360 - (\angle OAD + \angle AOB + \angle OBD) = 60^\circ$.

Now the $\angle DAB = 180^\circ - (\angle ABD + \angle ADB) = 60^\circ$

Thus $\triangle ADB$ is an equilateral triangle.

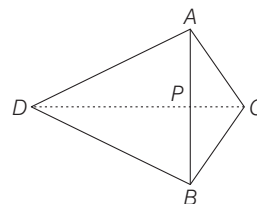
Now, we know that $\triangle AOB$ is an isosceles triangle as $OA = OB$, and $\angle AOB = 120^\circ$, so $\angle OAB = \angle OBA = 30^\circ$



Now consider the following figure, in which $OA = OB = 2$ cm and $AD = BD$ and AB is perpendicular to OD .

Now using 30-60-90° theorem, we get $OP = 1$ cm, and $AP = BP = \sqrt{3}$ cm.

Subsequently, $DP = 3$ cm and $AD = BD = 2\sqrt{3}$ cm.



Therefore, quadrilateral $OADB$ is a kite and so its area is $\frac{1}{2}(AB \times OD) = \frac{1}{2}(2\sqrt{3} \times 4) = 4\sqrt{3}$ sq. cm.

Hence choice (a) is the correct one.

Alternatively First of all determine the area of isosceles triangle AOB , then find the area of equilateral triangle ABD , and then add them up.

$$\text{Area of quadrilateral } OADB = \frac{1}{2}(AB \times OP) + \frac{\sqrt{3}}{4}(AB)^2$$

To determine AB and OP use 30-60-90 degree theorem or trigonometric ratios in $\triangle AOP$ or $\triangle BOP$, where $OP \perp AB$ and $OA + OB = 2$

135 The maximum possible area of quadrilateral $ACBD = \text{Area of equilateral triangle } \triangle ABD + \text{maximum area of } \triangle ABC$

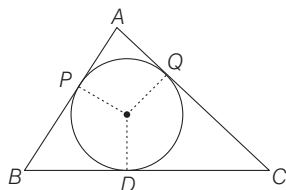
But, the maximum possible area of $\triangle ACB$ can be obtained only when the radius CO will be perpendicular to AB . In that case $\triangle ABC$ will be an equilateral triangle.

Therefore the maximum possible area of quadrilateral $ACBD$

$$\begin{aligned} &= \frac{\sqrt{3}}{4} \times (AB)^2 + \frac{\sqrt{3}}{4} \times (AB)^2 \\ &= 2 \left[\frac{\sqrt{3}}{4} \times (AB)^2 \right] \\ &= 2 \left[\frac{\sqrt{3}}{4} \times (2\sqrt{3})^2 \right] \\ &= 6\sqrt{3} \text{ sq. cm.} \end{aligned}$$

Hence choice (a) is the correct one.

- 136** Let OP and OQ be the other two in-radii on the sides AB and AC , respectively. Also, all the three in-radii will be perpendicular to the respective sides.



Since the two tangents drawn from the same point on the same circle are equal, so $BD = BP = 2$ and $CD = CQ = 8$. Similarly, $AP = AQ = x$ (say).

Semi-perimeter of the triangle (s)

$$= \frac{1}{2}(AB + BC + AC) = (10 + x)$$

And, area of the triangle

$$\begin{aligned} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{(10+x)(8)(2)(x)} \\ &= \sqrt{(10+x)(16x)} \end{aligned}$$

And, in-radius of the triangle $= \sqrt{6}$

$$\Rightarrow \frac{\text{Area of } \triangle ABC}{\text{Semi-perimeter of } \triangle ABC} = \sqrt{6}$$

$$\Rightarrow \frac{\sqrt{(10+x)(16x)}}{10+x} = \sqrt{6} \Rightarrow x = 6 \text{ cm}$$

Therefore, $AB = 2 + 6 = 8$ and $AC = 8 + 6 = 14$.

Thus the required ratio $4 : 7$.

Hence, choice (a) is the correct one.

- 137** Let $AR = 2x$, then

$$AB = AR + RB = 2x + 18.$$

$\therefore BP = x + 9$, as perpendicular OP bisects the chord AB at P .

But, you know that $AR \times RB = CR \times RD$

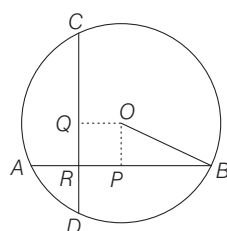
$$\therefore 2x \times 18 = CR \times 6$$

$$\Rightarrow CR = 6x$$

$$\therefore OP = QR = CR - CQ = CR - \left(\frac{CD}{2}\right)$$

$$= CR - \left(\frac{CR + RD}{2}\right)$$

$$\Rightarrow OP = 6x - \left(\frac{6x + 6}{2}\right) = 3(x - 1)$$



Now, since $\triangle OPB$ is a right angled triangle, therefore

$$OB^2 = OP^2 + BP^2$$

$$\begin{aligned} \Rightarrow 12^2 &= (3x - 3)^2 + (x + 9)^2 \\ \Rightarrow x &= \sqrt{\frac{27}{5}} \Rightarrow AR = 2x = 6\sqrt{\frac{3}{5}} \end{aligned}$$

Hence choice (a) is the correct one.

- 138** Since $\angle OAT = 90^\circ$, $\therefore AT = 40$ cm

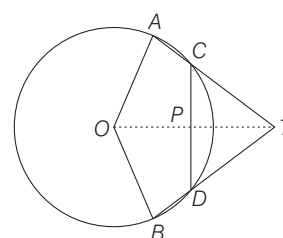
But you know that $AC = CP$,

$$\therefore CP + CT = 40 \text{ cm.}$$

Similarly, $PD + DT = 40$ cm

Thus the perimeter of $\triangle CDT = 80$ cm

The area of the required in circle would be greatest when the triangle is equilateral, which is not possible, so the triangle has to be isosceles. In that case, OT would bisect CD and pass through the point P , consequently $CP = DP$ and $\angle CPO = \angle DPT = 90^\circ$.



Since $\triangle OAT$ and $\triangle CPT$ are similar triangles,

So $PT = OT - OP = 20$ cm, $CP = 15$ cm, $CT = 25$ cm.

Now, the semi-perimeter of the

$$\triangle CDT = \frac{80}{2} = 40 \text{ cm}$$

$$\text{And area of the } \triangle CDT = \frac{1}{2} \times 30 \times 20 = 300 \text{ sq cm}$$

Therefore radius of the incircle that is inscribed in

$$\triangle CDT = \frac{300}{40} = 7.5 \text{ cm}$$

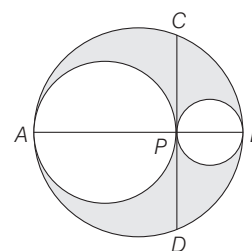
Thus the required area of the incircle $= 56.25 \pi$ sq cm.

Hence choice (a) is the correct one.

- 139** Let AB and CD intersect each other at P . Now, let us consider the radius of the smaller inscribed circle is r and that of larger inscribed circle is R , then $AP = 2R$, $BP = 2r$ and $AB = 2(R + r)$. Area of the shaded region

$$= \pi(R + r)^2 - (\pi R^2 + \pi r^2)$$

$$= \pi[R^2 + r^2 + 2Rr - R^2 - r^2] = 2\pi Rr$$



But,

$$AP \times BP = CP \times DP$$

$$\Rightarrow 2R \times 2r = 8 \times 8$$

$$\Rightarrow rR = 16 \text{ cm}$$

Therefore, the area of the shaded region = $2\pi Rr$
 $= 2\pi \times 16 = 32\pi \text{ sq cm.}$

Hence choice (c) is the correct one.

- 140** Since BP is the median of $\triangle ABC$, so you can use Apollonius theorem, as follows.

$$AB^2 + BC^2 = 2\left(BP^2 + \left(\frac{AC}{2}\right)^2\right)$$

$$\Rightarrow 441 + 225 = 2(BP^2 + 144)$$

$$\Rightarrow BP = 3\sqrt{21} \text{ cm}$$

Now, we have,

$$DP \times BP = AP \times PC$$

$$\Rightarrow DP \times 3\sqrt{21} = 12 \times 12$$

$$\Rightarrow DP = \frac{48}{\sqrt{21}} \text{ cm}$$

$$\text{Therefore, the required ratio} = \frac{BP}{PD} = \frac{3\sqrt{21}}{48\sqrt{21}} = \frac{1}{16}$$

Hence choice (a) is the correct one.

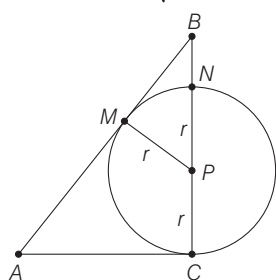
- 141** Let radius of the circle be r cm, then $PC = PM = PN = r$ cm.

Now, in $\triangle BMP$, we have $BM^2 + MP^2 = BP^2$

$$\Rightarrow BM^2 + r^2 = (BN + NP)^2$$

$$\Rightarrow BM^2 + r^2 = (2 + r)^2$$

$$\Rightarrow BM = 2\sqrt{r+1}$$



Now, in $\triangle ABC$, we have

$$AC^2 + BC^2 = AB^2$$

$$\Rightarrow AC^2 + (BN + NC)^2 = (AM + BM)^2$$

$$\Rightarrow 6^2 + (2 + 2r)^2 = (6 + 2\sqrt{r+1})^2$$

$$\Rightarrow r^2 + r = 6\sqrt{r+1}$$

Squaring both sides, we get

$$r^4 + 2r^3 + r^2 - 36r - 36 = 0 \quad \dots(i)$$

Now, the above equation has exactly two real roots; $r = -1$ and $r = 3$. But since negative value of r is inadmissible, as radius can never be negative, so only $r = 3$ is the valid answer. Otherwise, go through the choices and substitute the value of r in the above equation.

As none of the 4, 2 and 2.5 satisfies both side of the above equation (i), so choice (d) has to be the correct answer.

Hence choice (d) is the correct one.

Hint ($AM = AC$, as both are tangents on the same circle drawn from a common point)

NOTE The best way to solve this problem is to go through the given choices. For finding the roots (or roots) you may like to refer the chapter on Equations, Quadratic equations and Polynomial equations in Quantum CAT.

- 142** Since $\triangle APB$ and $\triangle DPC$ are similar triangles, so we have

$$\frac{AB}{CD} = \frac{AP}{DP} = \frac{BP}{CP}$$

$$\Rightarrow \frac{6}{CD} = \frac{2}{3} = \frac{5}{CP}$$

$$\Rightarrow CD = \frac{6 \times 3}{2} = 9$$

Hint $\angle APB = \angle DPC$, as they are vertically opposite angles.

$\angle BAC = \angle BDC$, as both the angles are on the same side of the chord BC .

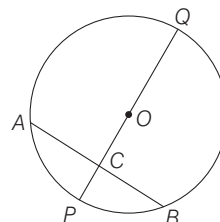
Similarly, $\angle ABD = \angle ACD$.

Hence choice (c) is the correct one.

- 143** First of all extend PO till Q a point on the circumference. Now using power of a point theorem you will have

$$AC \times BC = PC \times CQ$$

$$\Rightarrow 7 \times 5 = 1 \times CQ \Rightarrow CQ = 35$$



Now, if the radius of the circle be r , then $OQ = r$ and $OC = r - 1$

$$\therefore CQ = OQ + CO = 2r - 1$$

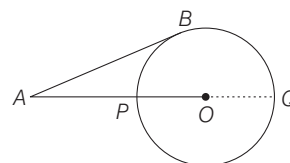
$$\text{That is } 2r - 1 = 35 \Rightarrow r = 18$$

Hence choice (d) is the correct answer.

- 144** First of all extend PO till Q , a point on the circumference. Now using power of a point theorem you will have

$$AB^2 = AP \times AQ \Rightarrow 196 = 7 \times AQ$$

$$\Rightarrow AQ = 28 \therefore PQ = AQ - AP = 21$$



Now, if the radius of the circle be r , then $OP = OQ = r = \frac{21}{2}$

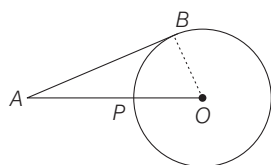
Therefore, circumference of the circle

$$= 2\pi r = 2\pi \times \frac{21}{2} = 66$$

Hence choice (a) is the correct answer.

☞ **Alternatively** First of all connect OB , then OB will be perpendicular on AB , as OB is the radius of the circle.

Let radius of the circle be r , then $OP = OB = r$.



Therefore, we have

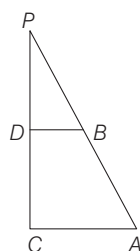
$$AB^2 + OB^2 = AO^2$$

$$\Rightarrow 14^2 + r^2 = (7 + r)^2 \Rightarrow r = \frac{21}{2}$$

Therefore circumference of the circle

$$= 2\pi r = 2\pi \left(\frac{21}{2}\right) = 66$$

- 145** Since CP is a common tangent, so AC and BD are perpendiculars to CP . Now, you can observe that the two triangles ($\triangle PDB$ and $\triangle PCA$) are similar triangles, as you can apply AAA rule.



$$\begin{aligned} \therefore \frac{DB}{PB} &= \frac{CA}{PA} \\ \Rightarrow \frac{6}{PB} &= \frac{15}{PB + AB} \\ \Rightarrow \frac{6}{PB} &= \frac{15}{PB + 21} \end{aligned}$$

$$PB = 14 \text{ cm and } PA = PB + AB = 35 \text{ cm}$$

Now, using Pythagoras theorem in $\triangle PDB$ and $\triangle PCA$, we have

$$PD = \sqrt{PB^2 - DB^2} = 4\sqrt{10}$$

and

$$\begin{aligned} PC &= \sqrt{PA^2 - CA^2} \\ &= 10\sqrt{10} \text{ cm} \end{aligned}$$

$\therefore$

$$\begin{aligned} CD &= PC - PD \\ &= (10 - 4)\sqrt{10} = 6\sqrt{10} \text{ cm} \end{aligned}$$

Hence choice (d) is the correct answer.

Hint $AB = 15 + 6 = 21 \text{ cm}$

And, AAA rules says that if the three angles of any two triangles are same, both the triangles are similar. In this problem $\angle P$ is common in both the triangles and one angles is right angles in both the triangles, so the third angle in both the triangles will naturally be same.

☞ **Alternatively** First of all draw a perpendicular BM from B on CA , so BM will be parallel to DC .

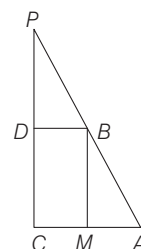
Now,

$$MA = CA - CM$$

$$= 15 - 6 = 9 \text{ cm}$$

And,

$$BA = 6 + 15 = 21 \text{ cm}$$



Now, applying Pythagoras theorem in $\triangle BMA$, you will have

$$BM^2 = BA^2 - MA^2$$

$$\Rightarrow BM^2 = 21^2 - 9^2 = 360$$

$$\Rightarrow BM = 6\sqrt{10} \text{ cm}$$

- 146** We can easily figure out that the radius of the larger circle A is double the radius of each of the smaller circle B .

That means area of circle $A = 4$ (area of each circle B)

$$\Rightarrow A = 4B$$

It implies that the area of four smaller circles is same as the area of larger circle.

It implies that area covered by region D is same as area covered by region C .

Hence choice (d) is the correct one.

- 147** First of all connect OC , OP and OQ .

Now, $OC = OP = OQ = 6 \text{ cm}$.

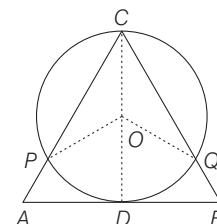
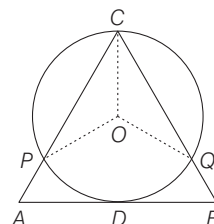
But, since D is the mid-point of AB , so CD is the median. And, since $AC = BC$, so CD is an angle bisector too. Also, CD is perpendicular to AB and must pass through the centre O .

Please note that $\triangle COP$ and $\triangle COQ$ are symmetric.

$$\angle PCO = \angle CPO = 30^\circ \text{ and } \angle COP = 120^\circ.$$

$$\text{Similarly, } \angle QCO = \angle CQO = 30^\circ \text{ and } \angle COQ = 120^\circ.$$

Therefore, $\angle POQ = 120^\circ$.

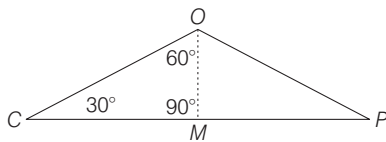


Now, area of the sector

$$POQ = \pi(6)^2 \times \frac{120^\circ}{360^\circ} = 12\pi$$

Now, drop a perpendicular on CP . Since $CO = PO = 6$ cm, therefore $OM = 3$ cm and $CM = 3\sqrt{3}$ cm, so $CP = 6\sqrt{3}$ cm.

$$\begin{aligned} \text{Therefore, area of } \triangle COP &= \frac{1}{2} \times CP \times OM \\ &= \frac{1}{2} \times 6\sqrt{3} \times 3 = 9\sqrt{3} \text{ sq cm.} \end{aligned}$$



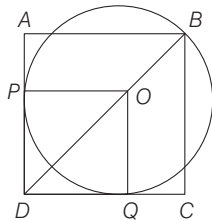
Similarly, the area of $\triangle COQ = 9\sqrt{3}$ sq cm.

Therefore, area of the common region

$$\begin{aligned} &= 12\pi + 9\sqrt{3} + 9\sqrt{3} \\ &= 12\pi + 18\sqrt{3} \text{ sq. cm.} \end{aligned}$$

Hence choice (c) is the correct one.

- 148** In the given figure, drop the two perpendiculars OP and OQ from the centre O of the circle and it is important to know that the centre O falls on the diagonal BD of the square $ABCD$.



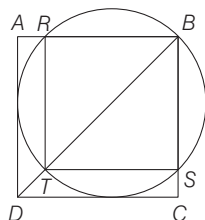
Now, let us assume that the side of the square is a cm. Therefore $AB = BC = CD = AD = a$ cm and $BD = \sqrt{2}a$ cm.

$$\begin{aligned} \text{But, } BD &= BO + OD \\ \Rightarrow \sqrt{2}a &= 1 + \sqrt{2} \\ \Rightarrow a &= \frac{1 + \sqrt{2}}{\sqrt{2}} \text{ cm} \end{aligned}$$

Hence, choice (c) is the correct one.

Hint : Since OB, OP, OQ are the radii of the same circle, so $OB = OP = OQ = 1$ cm, and since OD is the diagonal of the square $POQD$, so $OD = \sqrt{2}$ cm.

- 149** In the following figure, $RB = TS$ and $BS = RT$. But, since $RB = BS$, therefore, $RBST$ is a square. And, for that reason BT and BD coincide.



Now, the area of the square $RBST = \frac{BT^2}{2} = \frac{2^2}{2} = 2$ sq cm

And area of the circle $= \pi r^2 = \pi(1)^2 = \pi$

Total area between the circle and the square $RBST = \pi - 2$

Therefore, the area of the circle which is outside the square $ABCD = \frac{\pi - 2}{2}$

Thus, the required area of the common region between the square $ABCD$ and the given circle

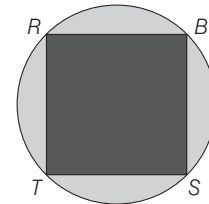
$$= \pi - \left(\frac{\pi - 2}{2} \right) = \frac{\pi + 2}{2}.$$

Hence choice (b) is the correct one.

Hint : In the following figure, area of circle $= \pi$ sq. cm

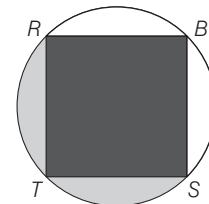
And, area of square (black region) $= 2$ sq. cm

And, the area of shaded grey region $= (\pi - 2)$ sq. cm



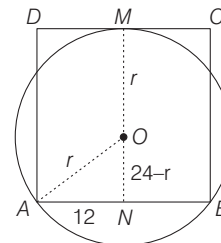
Therefore, the area of the common region in the following figure

$$\begin{aligned} &= \pi - \left(\frac{\pi - 2}{2} \right) \\ &= \frac{\pi + 2}{2} \text{ sq cm.} \end{aligned}$$



- 150** Let radius of the circle is r , then $AO = MO = r$.

Therefore, $ON = MN - MO = 24 - r$



Now, using Pythagoras theorem in $\triangle AON$, we have

$$OA^2 = AN^2 + ON^2$$

$$\Rightarrow r^2 = 12^2 + (24 - r)^2 \Rightarrow r = 15 \text{ cm}$$

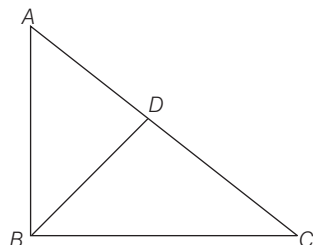
Hence choice (d) is the correct one.

Level 02 Higher Level Exercise

1 $BD = 53$ cm

$\therefore AD = CD = BD = 53$ cm

$\therefore AC = 2 \times 53 = 106$ cm



$AB + BC + AC = 2 \times 126$ cm = 252 cm

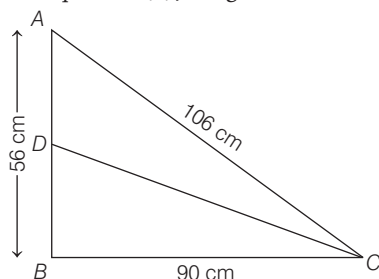
$\therefore AB + BC = 146$ cm

Let $AB = x$ cm

$\therefore BC = (146 - x)$ cm, $AB^2 + BC^2 = AC^2$

$x^2 + (146 - x)^2 = (106)^2 \quad \dots(1)$

Solving the equation (1), we get $x = 56$ and $x = 90$



Consider $AB = 56$ cm, then $BC = 90$ cm

Longest median will fall on the shortest side.

Now, the area of $\triangle BCD = \frac{1}{2} \times BD \times BC$

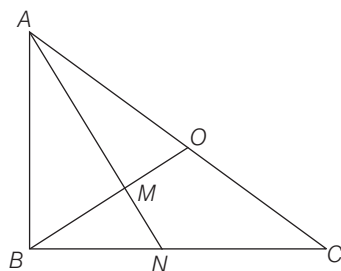
$= \frac{1}{2} \times 28 \times 90 = 1260$ cm²

2 Let $AB = BC = a$, then $AC = \sqrt{2}a$

$\therefore AO = OC = BO = \sqrt{2}a/2 = a/\sqrt{2}$

Now, by angle bisector theorem

$\frac{AB}{AO} = \frac{BM}{MO} \Rightarrow \frac{BM}{MO} = \frac{a}{a/\sqrt{2}} = \frac{\sqrt{2}}{1}$



$\therefore MO = 20$ cm

$\therefore BM = 20\sqrt{2}$ cm

$\therefore BO = 20 + 20\sqrt{2} = 20(1 + \sqrt{2})$ cm

Now, since $BO = a/\sqrt{2} = AB/\sqrt{2}$

$\therefore AB = \sqrt{2}(BO) = 1.414 [20(1 + 1.414)]$

$= 68.2843 = 68.28$ cm

3 $\angle A + \angle B = 90^\circ$

$\angle A - \angle B = 89 - 1 = 88$

$\angle A - \angle B = 88 - 2 = 86$

$\angle A - \angle B = 87 - 3 = 84$

$\dots \dots \dots$

$\angle A - \angle B = 45 - 45 = 0$

$\angle A - \angle B = 44 - 46 = -2$

$\dots \dots \dots$

$\angle A - \angle B = 1 - 89 = -88$

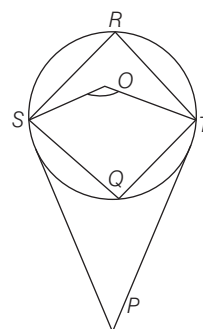
Thus k can assume total $44 + 1 + 44 = 89$ values

4 $\angle SPT$ and $\angle SOT$ are supplementary, then

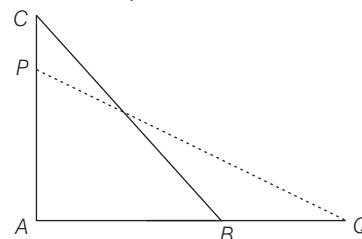
$\angle SOT = 180^\circ - 50^\circ = 130^\circ$

$\therefore \angle SRT = \frac{1}{2}(\angle SOT) = 65^\circ$

$\therefore \angle SQT = 180 - 65^\circ = 115^\circ$



5 Let BC be the ladder, then



$BC = 6.5$ m

and $AB = 5.2$ m

$\therefore AC = \sqrt{(BC)^2 - (AB)^2}$

$\Rightarrow AC = 3.9$ m

Now, $PQ^2 = PA^2 + AQ^2$

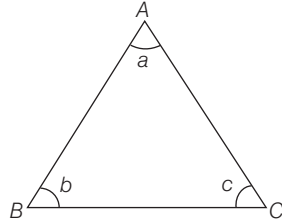
$(6.5)^2 = (2.5)^2 + AQ^2 \Rightarrow AQ = 6$ m

$\therefore BQ = AQ - AB = 6 - 5.2 = 0.8$ m

$\therefore$ The foot of the ladder will slip by 0.8 m

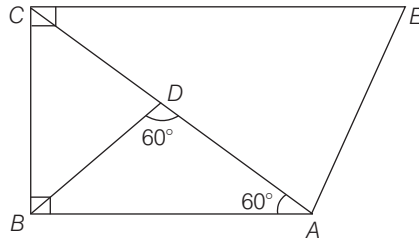
- 6 $\angle A + \angle B + \angle C = 180$

Any one of angle can possess the values from 1 to 178.



- 7 Cannot be determined.

- 8 $\triangle ABC$ is a right angled



and $\angle ABC = 90^\circ$

Let $AB = x$

then $AB = AD = CD = BD = x$

$\therefore \triangle ABD$ is equilateral triangle

$\therefore \angle CAE = 60^\circ$

$\therefore \angle BCA = 30^\circ \Rightarrow \angle ACE = 60^\circ$

$\therefore \angle CEA = 60^\circ$, also.

Hence, $\triangle ACE$ is an equilateral triangle

Thus, $AC = AE = CE = 2x$

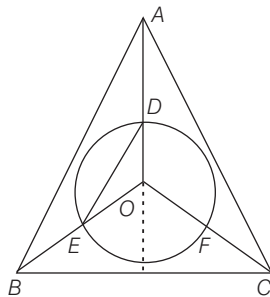
and $\frac{BC}{AB} = \tan 60^\circ = \sqrt{3} \Rightarrow BC = AB\sqrt{3} = x\sqrt{3}$

$\therefore \frac{BC}{EC} = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$

- 9 $\pi r^2 = 3\pi \Rightarrow r = \sqrt{3}$

$\therefore DE = 2r^2 - 2r^2 \cos 120^\circ$

$DE = r^2$ ($\because \sqrt{3} = r$)



But $AB = 2DE$ (By similarity of triangles)

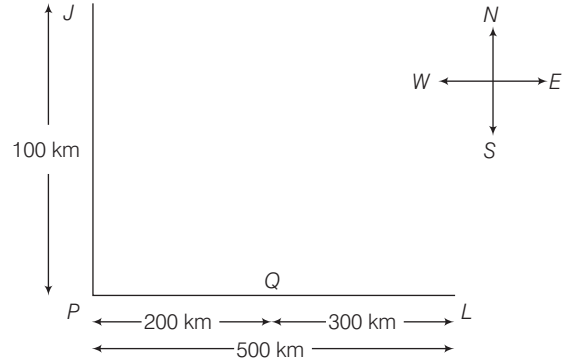
$\therefore AB = 2r^2 = 2 \times (\sqrt{3})^2 = 6$

($\because D$ and E are the mid-points of OA and OB)

$\therefore$ Perimeter of triangle $ABC = 3 \times 6 = 18$ units.

- 10 Time taken in the collision of the two trains

$$= \frac{500}{(40 + 60)} = 5 \text{ h}$$



In 5 hours, plane will cover $5 \times 200 = 1000$ km distance

- 11 Two trains meet with an accident at a place 200 ($= 40 \times 5$) km away from Patiala.

$\therefore$ The required distance = 200 km

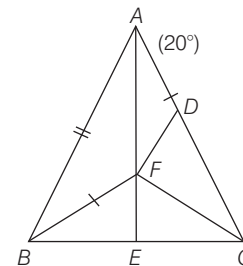
- 12 Area of $\triangle BDE = \frac{1}{2} \times \frac{2}{5} AB \times \frac{2}{7} BC$
 $= \frac{4}{35} \times \frac{1}{2} (AB \times BC) = \frac{4}{35}$ area of $\triangle ABC$

$\therefore$ Area of $\triangle ABC = \frac{35}{4} \times 20 = 175 \text{ cm}^2$

- 13 Let E be on BC and $BE = EC$

Let F be on AE so that triangle FBC is equilateral.

$\angle DAB = \angle ABF = 20^\circ$



and $DA = BF$

$\therefore$ Trapezoid $ADFB$ is isosceles.

$\angle FAD = \angle DBF = 10^\circ$

$\Rightarrow \angle DBC = 10 + 60 = 70^\circ$

- 14 Calculate them physically (or manually)

- 15 $AB = 6 \text{ cm}$, $\angle C = 60^\circ$ and $\angle A = \angle B = 60^\circ$

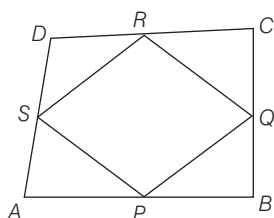
$\therefore \triangle ABC$ is an equilateral triangle

$\therefore$ Area of $\triangle ABC = \frac{\sqrt{3}}{4} \times (6)^2 = 9\sqrt{3}$

Area of $(\triangle ADE + \triangle BFG) = 2 \times \frac{\sqrt{3}}{4} \times (2)^2 = 2\sqrt{3}$

$\therefore$ Area of pentagon $= 9\sqrt{3} - 2\sqrt{3} = 7\sqrt{3} \text{ cm}^2$

- 16 Since, PQRS is a parallelogram



$$\therefore \angle PSR = 90^\circ \quad (\angle PSR + \angle PQR = 180^\circ)$$

- 17 From the given relation, we can say that $x = wr$, $y = wr^2$, $z = wr^3$.

$$\therefore z - w = w(r^3 - 1)$$

Now, since $y > 90$ and $x < 90$, therefore $r > 1$.

Further, it is known that $w + x + y + z = w(1 + r + r^2 + r^3) = 360$, which is integer. Ans, it is also given that r is an integer, so w , x , y and z should also be integers.

r	$1 + r + r^2 + r^3$	$w = \frac{360}{1 + r + r^2 + r^3}$	$z - w = w(r^3 - 1)$
2	15	24	168
3	40	9	236
4	75	Non-integer	—
5	156	Non-integer	—
6	259	Non-integer	—

Here, we see that for $r = \{2, 3\}$ we have w as an integer. For $r = 4, 5, 6$, we have w as a non-integer value. Also, for $r \geq 7$, $0 \leq w < 1$. That means w is a non-integer for $r \geq 7$.

Now, for $r = 2$, we have $w = 24$, $x = 48$, $y = 96$, $z = 112$, which satisfies the condition that $w, x < 90$ and $y, z > 90$.

But, for $r = 3$, we have $w = 9$, $x = 27$, $y = 81$, $z = 243$, which means $y < 90$. That is wrong.

Hence choice (a) is the correct one.

- 18 $\angle ROQ = 180 - 50 = 130^\circ$

$$\left(\begin{array}{l} \because \angle OQP + \angle ORP + \angle QOR + \angle QPR = 360^\circ \\ \text{and } \angle OQP = \angle ORP = 90^\circ \end{array} \right)$$

Now, since $RT = TM$ and $QS = SM$

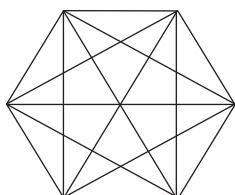
Also $OR = OM = OQ$

$$\therefore \angle ROT = \angle TOM \quad \text{and} \quad \angle MOS = \angle SOQ$$

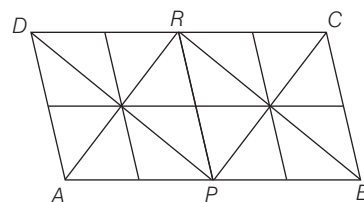
$$\therefore \angle SOT = \frac{1}{2} \angle ROQ$$

$$\therefore \angle SOT = \frac{130}{2} = 65^\circ$$

- 19 $12 + 6 + 1 = 19$



- 20 There are total 16 similar triangles each with equal area.



Here, 4 out of 16 triangles are taken. So the number of shaded triangles = 4 and number of unshaded triangles = 12

$$\therefore \text{Required ratio} = \frac{1}{3}$$

- 21 Number of total rectangles = ${}^4C_2 \times {}^3C_2 = 6 \times 3 = 18$

- 22 $\angle PDB = \angle QEA = 80^\circ$

$$\therefore \angle PED = \angle QDE = 10^\circ \quad (\because \angle DPE = \angle DQE = 90^\circ)$$

$$\therefore \angle DRE = 180 - (10 + 10) = 160^\circ$$

$$\therefore \angle PRD = 180 - \angle DRE = 20^\circ.$$

- 23 $OC = \frac{AB}{2}$

$$AO = OC = OB$$

$$\therefore \angle OAC = \angle OCA = \angle BCO = \angle OBC = 45^\circ$$

$$\therefore \angle ACB = 90^\circ$$

- 24 Please note that all the given triangles are equilateral.

$$\begin{aligned} \text{Area of shaded region} &= 3 \left[\pi r^2 \frac{60}{360} - \frac{\sqrt{3}}{4} \times r^2 \right] \\ &= \frac{r^2}{2} \left[\pi - \frac{3\sqrt{3}}{2} \right] \end{aligned}$$

- 26 $\angle ORP = 90^\circ$ ($\because OP$ is a diameter of smaller circle)

$$OS = 5 \text{ cm} \quad \text{and} \quad OR = 4 \text{ cm}$$

$$\therefore SR = \sqrt{(5)^2 - (4)^2} = 3 \text{ cm}$$

$$\therefore SP = 2(SR) = 6 \text{ cm}$$

(Since, OR passes through centre O and perpendicular to SP therefore OR bisects SP .)

- 27 $\frac{AD}{AB} = \frac{DO}{BO} = 1$

$$\Rightarrow OB = OD = 8 \text{ cm}$$

$\therefore ABCD$ is a cyclic quadrilateral

$$\therefore DO \times BO = CO \times AO$$

$$8 \times 8 = 4 \times AO \Rightarrow AO = 16 \text{ cm}$$

$$\therefore AC = 16 + 4 = 20 \text{ cm}$$

- 28 When $\angle A = 60^\circ$, $BC = b = c$.

$$\text{And, when } \angle A = 90^\circ, a = \sqrt{2}b = \sqrt{2}c$$

$$\therefore 60^\circ < A < 90^\circ, \quad c < a < c\sqrt{2}$$

- 29 $OR^2 = (OQ)^2 + (RQ)^2$

$$OR^2 = 5OQ^2 \quad [\because RQ = 2(OQ)]$$

$$\Rightarrow \text{radius } (r) = OQ\sqrt{5}$$

$$\begin{aligned} \therefore OQ &= \frac{r}{\sqrt{5}} \\ \text{Again, } OC^2 &= OH^2 + HC^2 \\ r^2 &= (OQ + QH)^2 + (QH)^2 \\ r^2 &= \left(\frac{r}{\sqrt{5}} + QH \right)^2 + (QH)^2 \quad (\because HQ = HC) \\ \Rightarrow QH &= \frac{r}{\sqrt{5}} \\ \therefore HC &= \frac{r}{\sqrt{5}} = \frac{RQ}{2} \Rightarrow RC = \sqrt{(RD)^2 + (DC)^2} \\ &= \sqrt{\left(\frac{r}{\sqrt{5}} \right)^2 + \left(\frac{r}{\sqrt{5}} \right)^2} = r\sqrt{\frac{2}{5}} \\ \therefore RC + FS &= 2r\sqrt{\frac{2}{5}} \end{aligned}$$

30 The best way is to consider some values and verify the results.

$$\begin{aligned} \mathbf{31} \quad \angle OCT &= 90^\circ, \quad \angle DCT = 45^\circ \\ \therefore \angle OCB &= 45^\circ \\ \therefore \angle COB &= 45^\circ \quad (\triangle OBC \text{ is a right angled triangle}) \\ \therefore \angle AOC &= 180^\circ - 45^\circ = 135^\circ \end{aligned}$$

$$\text{Now, } \because CD = 10 \Rightarrow BC = 5 \text{ cm} = OB$$

$$\Rightarrow OC = 5\sqrt{2} \text{ cm} = OA$$

$$\begin{aligned} \text{Again, } AC^2 &= OA^2 + OC^2 - 2OA \cdot OC \cos 135^\circ \\ &= 2(OA)^2 - 2(OA)^2 \cdot \cos 135^\circ \\ &= 2(5\sqrt{2})^2 - 2(5\sqrt{2})^2 \times \left(-\frac{1}{\sqrt{2}} \right) = 100 + \frac{100}{\sqrt{2}} \end{aligned}$$

$$AC^2 \approx 170.70 \Rightarrow AC \approx 13 \text{ cm}$$

$$\begin{aligned} \therefore \text{Perimeter of } \triangle OAC &= OA + OC + AC \\ &= 5\sqrt{2} + 5\sqrt{2} + 13 = 27 \text{ cm} \end{aligned}$$

$$\begin{aligned} \mathbf{32} \quad \angle ACB &= 60^\circ \quad (\because \angle ACB + \angle ADB = 180^\circ) \\ \text{and } \angle CAB &= 30^\circ \quad (\because \angle ACB + \angle CAB = 90^\circ) \end{aligned}$$

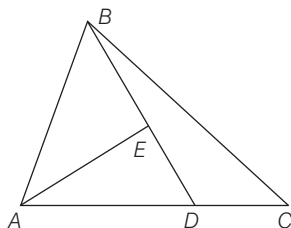
$$AC = 2 \times 6 = 12 \text{ cm}$$

$$\frac{BC}{AC} = \sin 30^\circ = \frac{1}{2} \Rightarrow BC = 6 \text{ cm}$$

$$\text{and } \frac{BC}{AB} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow AB = 6\sqrt{3} \text{ cm}$$

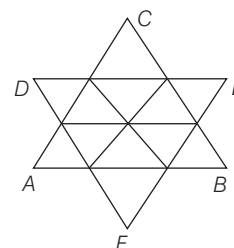
$$\therefore \text{Area of } \triangle ABC = \frac{1}{2} \times 6 \times 6\sqrt{3} = 18\sqrt{3} \text{ cm}^2$$

$$\mathbf{33} \quad \text{Area of } \triangle BAE = \frac{1}{4} AC \times \frac{1}{3} BD = \frac{1}{12} \text{Area of } \triangle ABC$$



$$\begin{aligned} \mathbf{34} \quad \frac{AE}{EC} &= \frac{AB}{BC} = \frac{7.5}{10.5} = \frac{5}{7} \\ \text{Now, } AB^2 + BC^2 &= AC^2 \\ (5k)^2 + (7k)^2 &= (18)^2 \\ 74k^2 &= 324 \Rightarrow k^2 = \frac{324}{74} \\ \text{Area of } \triangle ABC &= \frac{1}{2} \times AB \times BC = \frac{1}{2} \times 5k \times 7k \\ &= \frac{35}{2} k^2 = \frac{35}{2} \times \frac{324}{74} = 76.621 \text{ cm}^2 \end{aligned}$$

35 There are total 12 similar triangles each with equal area. But a larger triangle ABC (or DEF) has only 9 smaller triangles. Out of 9 triangles only 6 triangles are common.
 $\therefore$ Area of common region

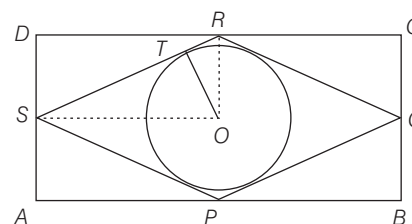


$$= \frac{6}{9} \times 198 = 132 \text{ cm}^2$$

$$\mathbf{36} \quad 9 \times 180 - 2 \times 360 = 180 \times 5 = 900^\circ$$

$$\mathbf{37} \quad OR = DS = \frac{AD}{2} = 6 \text{ cm}$$

$$\text{and } OS = DR = \frac{DC}{2} = 8 \text{ cm}$$



$$\begin{aligned} \therefore SR &= 10 \text{ cm} \\ \therefore \text{Area of } \triangle ORS &= \frac{OS \times OR}{2} = \frac{SR \times OT}{2} \\ \frac{8 \times 6}{2} &= \frac{10 \times OT}{2} \Rightarrow OT = \frac{48}{10} \text{ cm} \\ \therefore \text{Area of circle} &= \pi r^2 = \pi \times \left(\frac{48}{10} \right)^2 = \frac{576}{25} \pi \text{ cm}^2 \end{aligned}$$

$$\mathbf{38} \quad 200 = 2^3 \times 5^2$$

$$\text{Number of total factors} = (3 + 1) \times (2 + 1) = 12$$

$$\therefore \text{Total number of required rectangles} = \frac{12}{2} = 6$$

$$\text{Area} = b \times l$$

$$200 = 1 \times 200 = 2 \times 100 = 4 \times 50 = 5 \times 40$$

$$= 8 \times 25 = 10 \times 20$$

- 39 Since a circle can pass through any 3 non-collinear points, so from total 8 points we can have maximum possible number of circles = ${}^8C^3 = 56$

But, since it is given that 4 points lie on the same circle, therefore we will have fewer circles drawn.

Therefore, the actual number of circles drawn = ${}^8C^3 - {}^4C^3 + 1 = 53$

NOTE When we reduce ${}^4C^3$ circles from ${}^8C^3$, we actually reduce all the possible circles, but there must be 1 circle with 4 points on its circumference. That's why we are adding 1 after deducting ${}^4C^3$.

- 40 Here, AC and BC are the secants of the circle and AB is tangent at D

$$\therefore AE \times AC = AD^2$$

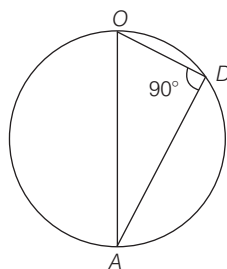
$$AE \times 4 = (3)^2 \Rightarrow AE = \frac{9}{4}$$

$$\therefore CE = 4 - \frac{9}{4} = \frac{7}{4}$$

$$\therefore CE : (AE + AD) = \frac{7}{4} : \left(\frac{9}{4} + 3\right) = \frac{7}{4} : \frac{21}{4} = 1 : 3$$

- 41 $\angle ADO$ is a right angle (angle of a semicircle)

New since OD is perpendicular on the chord AC and OD passes through the centre O of the larger circle ABC , then OD must bisect the chord AC at D .



$$\therefore AD = CD = 6 \text{ cm}$$

- 42 $\angle CED = 120^\circ$ ($\because ACED$ is cyclic)

$$\text{and } \angle BED = 60^\circ$$

$$\therefore \angle EDB = 90^\circ$$

$$\therefore \frac{BD}{BE} = \cos 30^\circ \Rightarrow \frac{6}{BE} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow BE = 4\sqrt{3} \text{ cm}$$

$$\therefore BC = BE + CE = 4\sqrt{3} + 5\sqrt{3} = 9\sqrt{3} \text{ cm}$$

Now, since AB and CB are the secants of the circle.

$$\therefore BD \times BA = BE \times BC$$

$$6 \times BA = 4\sqrt{3} \times 9\sqrt{3} \Rightarrow BA = 18 \text{ cm}$$

Again, $\triangle ACB$ is a right angled triangle ($\because \angle C = 90^\circ$)

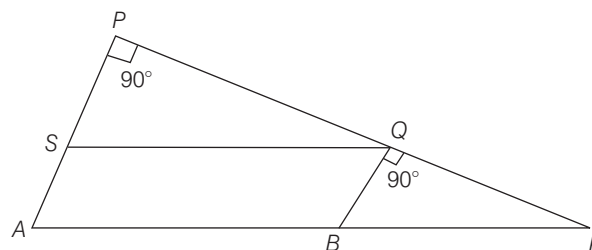
$$\therefore AC = AB \sin 30^\circ \quad (\sin 30^\circ = 1/2)$$

$$AC = 9 \text{ cm (Alternatively apply Pythagorus theorem).}$$

$$\text{and } AD = AB - BD = 12 \text{ cm}$$

$$\therefore \frac{AC}{AD} = \frac{9}{12} = \frac{3}{4}$$

- 43 $AB = SQ = 25 \text{ cm}$ and $PQ = 24 \text{ cm}$, therefore $SP = 7 \text{ cm}$



$$AP = 12 \text{ cm and } SP = 7 \text{ cm}$$

($\triangle PSQ$, $\triangle QBR$ and $\triangle PAR$ are similar)

$$\therefore AS (= BQ) = 12 - 7 = 5 \text{ cm}$$

$$\therefore \frac{PS}{SQ} = \frac{PA}{AR} \Rightarrow \frac{7}{25} = \frac{12}{AR} \Rightarrow AR = \frac{300}{7}$$

$$\therefore BR = AR - AB = \frac{300}{7} - 25$$

$$\therefore BR = \frac{125}{7} \Rightarrow \frac{AB}{BR} = \frac{25 \times 7}{125} = \frac{7}{5}$$

- 44

$$\frac{SP}{PQ} = \frac{BQ}{QR}$$

$$\Rightarrow \frac{7}{24} = \frac{5}{QR} \Rightarrow QR = \frac{120}{7}$$

$$\therefore \frac{\text{Area of } \triangle APR}{\text{Area of } \triangle BQR} = \frac{\frac{1}{2} \times AP \times PR}{\frac{1}{2} \times BQ \times QR} = \frac{12 \times \frac{288}{7}}{5 \times \frac{120}{7}} = \frac{144}{25}$$

$$(PR = PQ + QR)$$

Alternatively

$$\frac{\text{Area of } \triangle APR}{\text{Area of } \triangle BQR} = \left(\frac{AP}{BQ}\right)^2 = \left(\frac{12}{5}\right)^2 = \frac{144}{25}$$

- 45 $x^2 + y^2 + z^2 = xy + yz + zx$

$$\Rightarrow x^2 + y^2 + z^2 - xy - yz - zx = 0$$

$$\Rightarrow 2(x^2 + y^2 + z^2 - xy - yz - zx) = 0$$

$$\Rightarrow (x^2 + y^2) + (y^2 + z^2) + (z^2 + x^2) - 2xy - 2yz - 2zx = 0$$

$$\Rightarrow (x - y)^2 + (y - z)^2 + (z - x)^2 = 0$$

$$\Rightarrow x = y = z$$

$\therefore$ The given triangle is an equilateral triangle.

- 46 Let $AD = 3a$ and $DC = 6a$

$$\therefore DH = HG = GC = \frac{6a}{3} = 2a$$

$$HM = MG = \frac{2a}{2} = a = SM$$

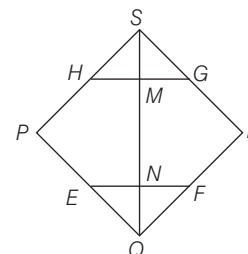
$$\therefore NQ = a \quad (\text{also})$$

$$\therefore SQ = SM + MN + NQ$$

$$= a + 3a + a = 5a$$

Since diagonal of square, $SQ = 5a$

But, $SQ = \text{diameter of circle } PQRS = \text{diagonal of square}$



PQRS

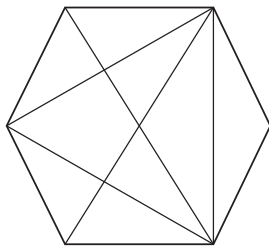
$$\therefore \text{Radius of the circle} = \frac{5a}{2}$$

$$\Rightarrow \text{Area of the circle} = \pi \times \left(\frac{5a}{2}\right)^2$$

$$\text{Here } \frac{\text{Area of circle}}{\text{Area of rectangle}} = \frac{\frac{25}{4}(a^2\pi)}{3a \times 6a} = \frac{25\pi}{72}$$

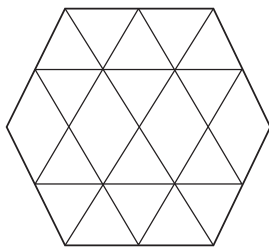
- 47** In the first diagram, join the opposite vertices of the hexagon. It gives us 12 congruent right angle triangles. Out of them 6 such triangles fall inside the triangle $\triangle BDF$.

Therefore, area of $\triangle BDF = \frac{1}{2}$ (area of hexagon)



Now, in the second diagram, we can observe that there are 24 congruent equilateral triangles. Out of them 9 such triangles fall inside the triangle $\triangle PQR$.

Therefore, area of $\triangle PQR = \frac{9}{24}$ (area of hexagon)



Therefore, we have

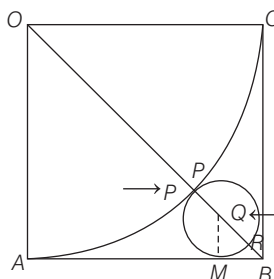
$$\frac{\text{Area of } \triangle BDF}{\text{Area of } \triangle PQR} = \frac{\frac{1}{2} (\text{Area of hexagon})}{\frac{9}{24} (\text{Area of hexagon})} = \frac{1/2}{9/24} = \frac{4}{3}$$

Hence choice (c) is the correct one.

- 48** $OA = AB = BC = OC = OP$

Let $OA = R$ (radius of the larger circle), then $OB = R\sqrt{2}$

Similarly $PQ = MQ = QR = r$ (radius of the smaller circle)



then $BQ = r\sqrt{2}$

Therefore, $BP = PQ + BQ = r + r\sqrt{2}$

Similarly, $BP = OB - OP = R\sqrt{2} = R\sqrt{2} - R$

$$\Rightarrow R(\sqrt{2} - 1) = r(\sqrt{2} + 1)$$

$$\Rightarrow r = R(\sqrt{2} - 1)^2$$

$$\Rightarrow r = R(3 - 2\sqrt{2})$$

$$\therefore \frac{\text{Area of larger circle}}{\text{Area of smaller circle}} = \frac{\pi R^2}{4\pi r^2} = \frac{R^2}{4(3 - 2\sqrt{2})^2 R^2}$$

$$= \frac{1}{4(17 - 12\sqrt{2})}$$

$$= \frac{1}{68 - 48\sqrt{2}}$$

- 49** AX , BY and CZ are perpendiculars drawn on the tangent

AD from the centers X , Y and Z of the respective circles.

Let $CZ = NZ = PZ = r_1$, $BY = MY = NY = r_2$, $X = MX = r_3$ and $DP = k$.

Since, $\angle ADG = 90^\circ$, therefore $\angle BYD = \angle AXD = 45^\circ$.

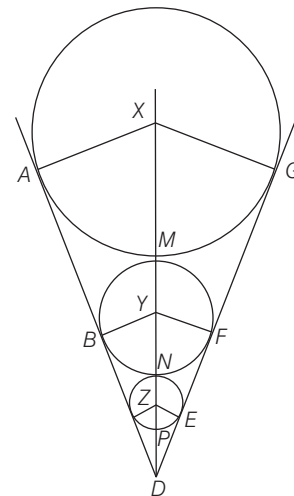
Now, since $CZ = r_1$, therefore $DZ = \sqrt{2}r_1$

Now, since $BY = r_2$, therefore

$$DY = \sqrt{2}r_2 = r_2 + r_1 + \sqrt{2}r_1$$

$$\Rightarrow r_2(\sqrt{2} - 1) = r_1(\sqrt{2} + 1)$$

$$\Rightarrow r_2 = \frac{r_1}{(\sqrt{2} - 1)^2}$$



Similarly, since $AX = r_3$, therefore $DX = \sqrt{2}r_3$

$$\Rightarrow \sqrt{2}r_3 = r_3 + r_2 + \sqrt{2}r_2$$

$$\Rightarrow \sqrt{2}r_3 = r_3 + r_2(\sqrt{2} + 1)$$

$$\Rightarrow \sqrt{2}r_3 = r_3 + \frac{r_1}{(\sqrt{2} - 1)^3}$$

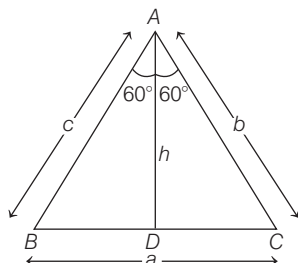
$$\Rightarrow r_2(\sqrt{2} - 1) = \frac{r_1}{(\sqrt{2} - 1)^3}$$

$$\Rightarrow \frac{r_3}{r_1} = \frac{1}{(\sqrt{2} - 1)^4}$$

- 55 Let $AD = h$ (say)

$$\text{then Area of } \triangle ABC = \frac{1}{2} bc \sin 120^\circ = \frac{\sqrt{3}}{4} bc$$

$$\text{Area of } \triangle BAD = \frac{1}{2} ch \sin 60^\circ = \frac{\sqrt{3}}{4} ch$$



$$\text{and area of } \triangle CAD = \frac{1}{2} bh \sin 60^\circ = \frac{\sqrt{3}}{4} bh$$

$$\text{Now, } A(\triangle ABC) = A(\triangle BAD) + A(\triangle CAD)$$

$$\frac{\sqrt{3}}{4} bc = \frac{\sqrt{3}}{4} ch + \frac{\sqrt{3}}{4} bh$$

$$\Rightarrow bc = h(b + c) \Rightarrow h = \frac{bc}{b + c}$$

- 56 Using Crossed ladders theorem, we have

$$\frac{1}{AB} + \frac{1}{CD} = \frac{1}{EF}$$

Further, using Pythagoras theorem in $\triangle BAC$, we have

$$AB = \sqrt{BC^2 - AC^2}$$

Similarly, using Pythagoras theorem in $\triangle DCA$, we have

$$CD = \sqrt{AD^2 - AC^2}$$

Therefore, we have

$$\frac{1}{\sqrt{BC^2 - AC^2}} + \frac{1}{\sqrt{AD^2 - AC^2}} = \frac{1}{EF}$$

$$\frac{1}{\sqrt{210^2 - AC^2}} + \frac{1}{\sqrt{174^2 - AC^2}} = \frac{1}{70}$$

Now, the best way is to go through the given options. Let us consider option (b) and substitute $AC = 126$.

$$\frac{1}{\sqrt{210^2 - AC^2}} + \frac{1}{\sqrt{174^2 - AC^2}}$$

$$= \frac{1}{168} + \frac{1}{120} = \frac{1}{70}$$

Hence choice (b) is the correct one.

- 57 We can solve this problem by applying Mass-Point Geometry. Since $AX : XC = 2 : 3$, therefore, weight at point $A = 3$ and weight at point $C = 2$.

That means weight at point X

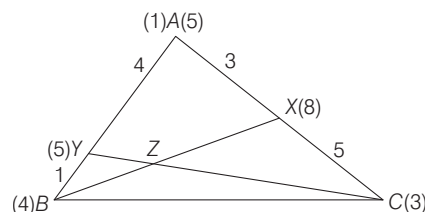
$$= \text{weight}(A) + \text{weight}(C) = 3 + 2 = 5.$$

Now, since the weight $(Z) = \text{weight}(B) + \text{weight}(X)$, therefore weight $(B) = 9 - 5 = 4$.

Hence choice (c) is the correct one.

- 58 Since the weight $(C) = 2$ and weight $(A) = 3$ and weight $(B) = 4$, therefore $AY : BY = 4 : 3$. Hence choice (b) is the correct one.

- 59 The best way is to apply Mass-Point Geometry concept. We can say that the mass (or weight) at point Y is $5 (= AY + BY)$ and the mass at X is $8 (= AX + BX)$.

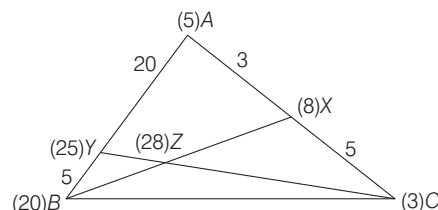


Further, the weight at A is 1 and weight at B is 4. Similarly, the weight at C is 3 and weight at A is 5.

Since, we are getting two different weights at A , so we need to take LCM of these two weights 1 and 5, which is 5.

Now, we need to revise the weights at all the points due to a new weight at A .

Considering the line segment AB , we need to multiply every value by 5, while all the values related to line segment AC will remain unchanged.



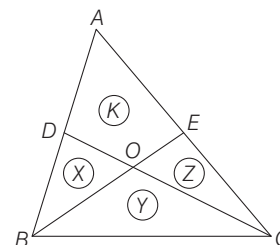
Now, the weight at point Z is 28, which can be obtained by $BZ + XZ$ or $CZ + YZ$.

Therefore, $BZ = 8$ and $XZ = 20$. Similarly, $CZ = 25$ and $YZ = 3$.

Thus the required ratio of $BZ : CZ = 8 : 25$.

Hence choice (a) is the correct one.

- 60 This problem can be solved by extending the crossed ladders theorem. However, if you do not want to go into details, you can always use this relation, as stated below.



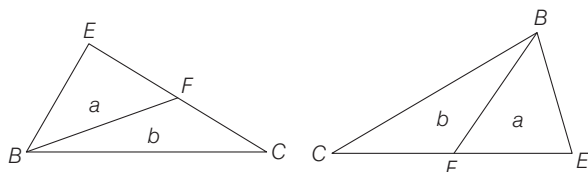
Let the area of the $\triangle ABC$ be T , then

$$\frac{1}{T} + \frac{1}{Y} = \frac{1}{X + Y} + \frac{1}{Y + Z} \Rightarrow \frac{1}{T} + \frac{1}{8} = \frac{1}{15} + \frac{1}{12}$$

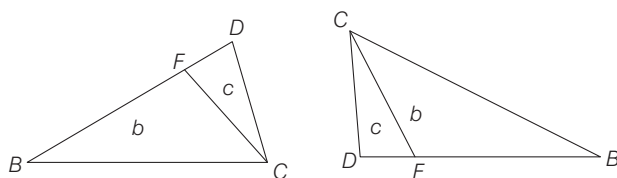
$$\Rightarrow T = 40 \Rightarrow K = 40 - (7 + 8 + 4) = 21 \text{ sq cm}$$

Hence choice (a) is the correct one.

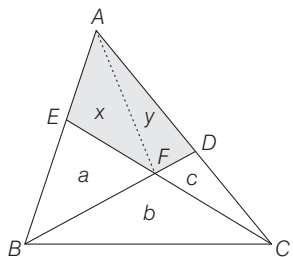
- 61 Since two triangles EFB and FCB have same height, therefore, $\frac{EF}{FC} = \frac{a}{b}$ (i)



Similarly, two triangles BFC and FDC have same height, therefore, $\frac{BF}{FD} = \frac{b}{c}$ (ii)



Now, the two triangles BFA and DFA have same height, therefore $\frac{BF}{FD} = \frac{a+x}{y}$



But, we know from equation (ii) that $\frac{BF}{FD} = \frac{b}{c}$

Therefore, $\frac{a+x}{y} = \frac{b}{c}$

$$\Rightarrow by = ca + cx \quad \dots (iii)$$

Similarly, we can find $\frac{x}{c+y} = \frac{a}{b}$

$$\Rightarrow bx = ca + ay \quad \dots (iv)$$

Solving the simultaneous equations (iii) and (iv) using substitution method, we get the following values of x and y .

$$x = \frac{ac(a+b)}{b^2-ac} \text{ and } y = \frac{ac(b+c)}{b^2-ac}$$

Therefore, total area of quadrilateral

$$AEFD = \frac{ac(a+2b+c)}{b^2-ac}$$

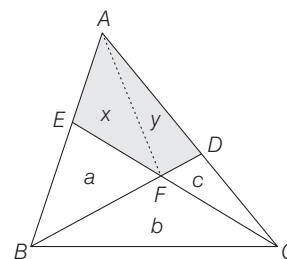
Hence choice (a) is the correct one.

☞ **Alternatively** Apply ladder theorem, and find the result as explained below.

$$\begin{aligned} \frac{1}{(a+b+c+d)} + \frac{1}{b} &= \frac{1}{(a+b)} + \frac{1}{(b+c)} \\ \Rightarrow d &= \frac{ac(a+2b+c)}{b^2-ac} \end{aligned}$$

- 62 From the previous problem, we know that $\frac{a+x}{y} = \frac{b}{c}$ and

$$\frac{x}{c+y} = \frac{a}{b}$$

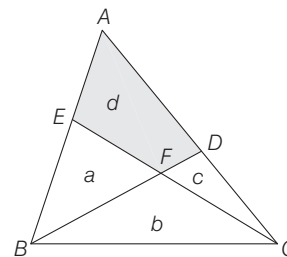


Therefore, we have $\frac{75+x}{y} = \frac{100}{40}$ and $\frac{x}{40+y} = \frac{75}{100}$

☞ **Alternatively** By solving these two equations, we get $x = 75$ and $y = 60$. Therefore the area of quadrilateral $AEFD = 135$ units.

Hence choice (b) is the correct one.

Apply ladder theorem, and find the result as explained below.



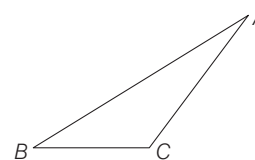
$$\begin{aligned} \frac{1}{(a+b+c+d)} + \frac{1}{b} &= \frac{1}{(a+b)} + \frac{1}{(b+c)} \\ \Rightarrow \frac{1}{(215+d)} + \frac{1}{100} &= \frac{1}{175} + \frac{1}{140} \\ \Rightarrow d &= 135 \end{aligned}$$

- 63 Area of triangle $ABC = \frac{1}{2} ab \sin C$

$$\Rightarrow 15\sqrt{3} = \frac{1}{2} \times 6 \times 10 \sin C$$

$$\Rightarrow \sin C = \frac{\sqrt{3}}{2}$$

$$\Rightarrow C = 120^\circ$$



Now use cosine rule : $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$$\Rightarrow \cos 120^\circ = \frac{36 + 100 - c^2}{2 \times 6 \times 10} \Rightarrow -\frac{1}{2} = \frac{136 - c^2}{120}$$

$$\Rightarrow c = 14$$

Now use the following relation :

$$\begin{aligned} \Delta(\text{area of triangle}) &= r (\text{inradius}) \times s(\text{semi-perimeter of triangle}) \\ \Rightarrow 15\sqrt{3} &= r \times \left(\frac{6+10+14}{2} \right) \\ \Rightarrow r &= \sqrt{3} \Rightarrow r^2 = 3 \end{aligned}$$

Hence choice (c) is the correct answer.

- 64** Since $\triangle ABC$ is an isosceles triangle, therefore, by dropping a perpendicular from B on AC , the perpendicular BD will bisect AC , and BD will also pass through the incentre E .

Now, drop another perpendicular EF on AB , then $EF = ED = r = \sqrt{3}$.

In right angle $\triangle EFB$, $\angle FBE = 60^\circ$ and $\angle BEF = 30^\circ$.

So, by using 30-60-90 degree theorem, we have $\frac{BE}{EF} = \frac{2}{\sqrt{3}}$

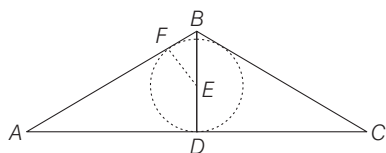
$$\Rightarrow BE = \frac{2}{\sqrt{3}} EF = \frac{2}{\sqrt{3}} r$$

$$\therefore BD = DE + BE = r + \frac{2}{\sqrt{3}} r = \sqrt{3} + 2 \quad \dots(i)$$

Again, using 30-60-90 degree theorem in $\triangle ABD$, we have

$$\frac{AD}{BD} = \frac{\sqrt{3}}{1}$$

$$\Rightarrow AD = \sqrt{3}(BD) = \sqrt{3}(\sqrt{3} + 2) \quad \dots(ii)$$



$$\begin{aligned} \therefore \text{Area of } \triangle ABC &= \frac{1}{2} (AC)(BD) \\ &= \frac{1}{2} (2AD)(BD) = (AD)(BD) \\ &= \sqrt{3}(\sqrt{3} + 2)^2 = 12 + 7\sqrt{3} \end{aligned}$$

Hence choice (c) is the answer.

Alternatively By applying 30-30-120 degree theorem, in the given triangle, if we assume $AB = BC = a$, then we have $AC = \sqrt{3}a$.

Either by dropping a perpendicular BD on AC or using any other technique we can have the area of triangle

$$\begin{aligned} \triangle ABC &= \frac{\sqrt{3} a^2}{4} \text{ and semi-perimeter of the triangle} \\ &= \frac{(2 + \sqrt{3})a}{2} \end{aligned}$$

Now using the relation, area = inradius $\times$ semi-perimeter, we have

$$\begin{aligned} \frac{\sqrt{3} a^2}{4} &= \sqrt{3} \times \frac{(2 + \sqrt{3})a}{2} \\ \Rightarrow a &= 2(2 + \sqrt{3}) \end{aligned}$$

Therefore area of the triangle

$$= \frac{\sqrt{3} a^2}{4} = \frac{\sqrt{3} [2(2 + \sqrt{3})]^2}{4} = 12 + 7\sqrt{3}$$

- 65** Let us consider an equilateral triangle. And assume point P is the incentre then $d_1 = d_2 = d_3 = k$ and $h_1 = h_2 = h_3 = 3k$. It implies that

$$\frac{d_1}{h_1} + \frac{d_2}{h_2} + \frac{d_3}{h_3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

To confirm the above result, let us take another example. Let us consider a right angle triangle with the sides 3, 4 and 5. Now assume that the point P is the incentre then, $d_1 = d_2 = d_3 = 1$ and $h_1 = 3$, $h_2 = 2.4$ and $h_3 = 4$. It implies that

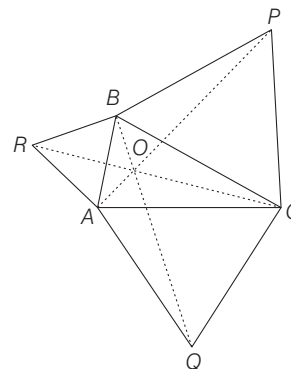
$$\frac{d_1}{h_1} + \frac{d_2}{h_2} + \frac{d_3}{h_3} = \frac{1}{3} + \frac{1}{2.4} + \frac{1}{4} = 1$$

As in both the considerations our result is unique.

It means 1 is the correct answer.

Hence choice (c) is the answer.

- 66** If you rotate $\triangle ARC$ clockwise about A by 60° , it will overlap the $\triangle ABQ$ completely, as $\angle RAC = \angle BAQ$. Also, $AR = AB$ and $AC = AQ$. Therefore, $\triangle RAC$ and $\triangle BAQ$ are congruent. It implies that $RC = BQ$.



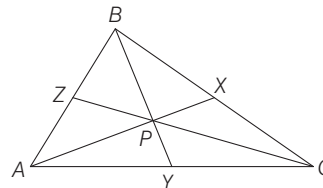
Similarly, you can prove that $RC = AP$.

Thus we can conclude that $AP = BQ = CR$

Hence choice (b) is the answer.

Hint : $\angle RAC = \angle RAB + \angle BAC = \angle BAC + \angle CAQ = \angle BAQ$

- 67** Let the altitude of the triangle ABC be h assuming that AC is the base.



$$\therefore \Delta(ABY) = \frac{1}{2} \times AY \times h \Rightarrow AY = 2 \times \frac{\Delta(ABY)}{h}$$

$$\text{Similarly, } CY = 2 \times \frac{\Delta(CBY)}{h}$$

$$\therefore \frac{AY}{CY} = \frac{\Delta(ABY)}{\Delta(CBY)}$$

In the same manner, we can have

$$\frac{AY}{CY} = \frac{\Delta(APY)}{\Delta(CPY)}$$

Now using the componendo-dividendo method, we have

$$\frac{AY}{CY} = \frac{\Delta(ABY) - \Delta(APY)}{\Delta(CBY) - \Delta(CPY)}$$

$$\Rightarrow \frac{AY}{CY} = \frac{\Delta(ABP)}{\Delta(CBP)}$$

Similarly, when AB is assumed to be the base, then we have

$$\frac{AZ}{BZ} = \frac{\Delta(ACP)}{\Delta(BCP)}$$

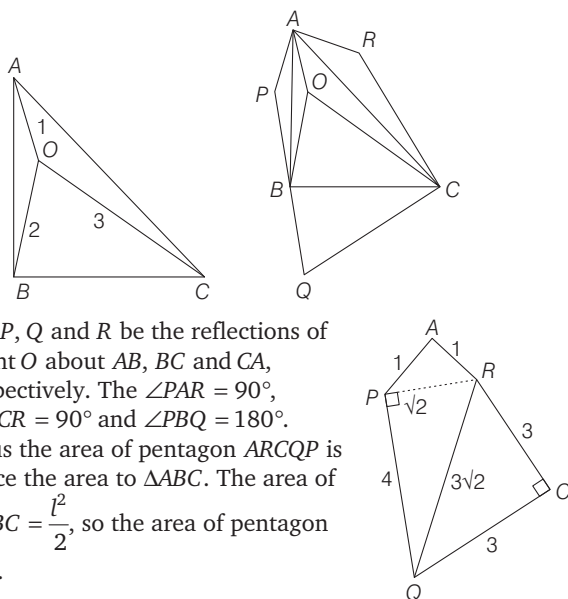
Similarly, when BC is assumed to be the base, then we have

$$\frac{BX}{CX} = \frac{\Delta(BAP)}{\Delta(CAP)}$$

$$\therefore \frac{AZ}{BZ} \cdot \frac{BX}{CX} \cdot \frac{CY}{AY} = \frac{\Delta(ACP)}{\Delta(BCP)} \cdot \frac{\Delta(BAP)}{\Delta(CAP)} \cdot \frac{\Delta(CBP)}{\Delta(ABP)} = 1$$

Hence eqs. choice (a) is the correct one.

- 68** Consider the following figures. Since AB and BC are the legs of the right angle triangle, so AC would be the hypotenuse and $\angle B = 90^\circ$ and $\angle BAC = \angle BCA = 45^\circ$



Let P , Q and R be the reflections of point O about AB , BC and CA , respectively. The $\angle PAR = 90^\circ$, $\angle QCR = 90^\circ$ and $\angle PBQ = 180^\circ$. Thus the area of pentagon $ARCQP$ is twice the area to $\triangle ABC$. The area of $\triangle ABC = \frac{l^2}{2}$, so the area of pentagon $= l^2$.

Since $\angle PAR = 90^\circ$, so

$$PR = \sqrt{2} \times AO; \quad (AP = AR = AO = 1)$$

Similarly, since $\angle QCR = 90^\circ$, so

$$QR = \sqrt{2} \times CO = 3\sqrt{2}; \quad (CQ = CR = CO = 3)$$

$$\text{And } PQ = (PB + BQ) = 2(BO) = 4$$

But, since $(\sqrt{2})^2 + (4)^2 = (3\sqrt{2})^2$. It implies that $\triangle QPR$ is a right angle triangle. Thus the total area of pentagon is comprised of three right angle triangles of which two are isosceles ones.

$$\frac{1}{2}(PA \times AR) + \frac{1}{2}(PR \times PQ) + \frac{1}{2}(RC \times CQ)$$

$$= 2\left(\frac{1}{2} \times AB \times BC\right)$$

$$\frac{1}{2}(1 \times 1) + \frac{1}{2}(\sqrt{2} \times 4) + \frac{1}{2}(3 \times 3) = 2\left(\frac{l^2}{2}\right)$$

$$5 + 2\sqrt{2} = l^2$$

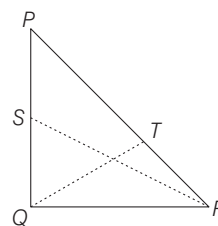
It implies that $a = 5$ and $b = 2$. Therefore $\frac{a}{b} = 2.5$

Hence choice (a) is the correct one.

- 69** Since, in $\triangle RQS$ and $\triangle PQR$, $\angle QSR = \angle PRQ$ and $\angle SQR = \angle PQR$, there $\triangle RQS \sim \triangle PQR$.

$$\therefore \frac{PQ}{RQ} = \frac{RQ}{SQ}$$

$$\Rightarrow SQ = \frac{RQ^2}{PQ} = \frac{81}{12} = 6.75$$



Similarly, $\triangle QTR \sim \triangle PQR$

$$\therefore \frac{QR}{PR} = \frac{RT}{QR} \Rightarrow RT = \frac{(QR)(QR)}{PR} = \frac{81}{15} = 5.4$$

Then we have,

$$PS \times PT = (PQ - SQ) \times (PR - RT)$$

$$= (12 - 6.75) \times (15 - 5.4) = 5.25 \times 9.6 = 50.4$$

Hence choice (a) is the correct one.

- 70** Given that $\frac{AQ}{BQ} = \frac{3}{2}$ and $\frac{AR}{CR} = \frac{4}{3}$

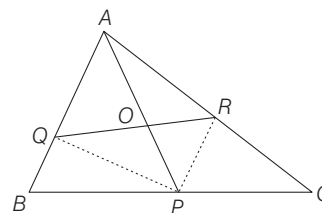
$$\therefore \frac{\text{Area}(\triangle AQP)}{\text{Area}(\triangle BQP)} = \frac{3}{2} = \frac{3m}{2m}$$

$$\text{and } \frac{\text{Area}(\triangle ARP)}{\text{Area}(\triangle CRP)} = \frac{4}{3} = \frac{4n}{3n}$$

$$\therefore \text{Area}(\triangle ABP) = 5m \text{ and } \text{Area}(\triangle ACP) = 7n$$

Since, AP is the median, therefore $\text{area}(\triangle ABP) = \text{area}(\triangle ACP)$

$$\Rightarrow 5m = 7n \Rightarrow \frac{m}{n} = \frac{7}{5} = \frac{7k}{5k}$$



Since height of the triangle AOQ and that of QOP are same and the height of triangle AOR and that of ROP are same, therefore $\frac{\Delta AOQ}{\Delta QOP} = \frac{\Delta AOR}{\Delta ROP}$

And let $\text{area}(\triangle QOP) = x$ and $\text{area}(\triangle ROP) = y$, then

$$\frac{3m - x}{x} = \frac{4n - y}{y}$$

$$\Rightarrow \frac{21k - x}{x} = \frac{20k - y}{y} \Rightarrow \frac{x}{y} = \frac{21}{20}$$

Hence choice (a) is the correct one.

$$71 \quad (1)^2 + (\sqrt{\phi})^2 = 1 + \phi = 1 + \frac{1 + \sqrt{5}}{2} = \frac{3 + \sqrt{5}}{2}$$

$$\phi^2 = \left(\frac{1 + \sqrt{5}}{2} \right)^2 = \frac{3 + \sqrt{5}}{2}$$

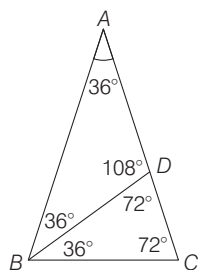
Since $1^2 + (\sqrt{\phi})^2 = \phi^2$, therefore it is a right angle triangle. Hence, choice (b) is the correct one.

72 From the given data it can be concluded that $BC = BD = AD$, as $\triangle CBD$ and $\triangle BDA$ are isosceles triangles.

$$\therefore \frac{AC}{BC} = \frac{BC}{DC}$$

$$\Rightarrow \frac{AC}{AD} = \frac{AD}{DC} = \frac{AD}{AC - AD} = \frac{\frac{AD}{AD}}{\frac{AC}{AD} - \frac{AD}{AD}} = \frac{1}{\frac{AC}{AD} - 1}$$

$$\Rightarrow x = \frac{1}{x-1} \Rightarrow x^2 - x - 1 = 0 \quad \left(\frac{AC}{AD} = x \right)$$



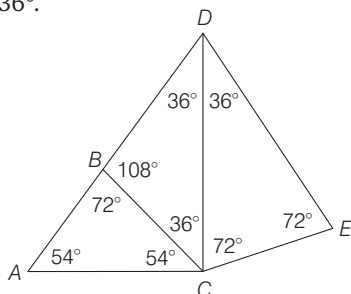
$$\Rightarrow x = \frac{1 + \sqrt{5}}{2} \Rightarrow \frac{AC}{AD} = \frac{1 + \sqrt{5}}{2}$$

$$\Rightarrow \frac{AB}{BC} = \frac{1 + \sqrt{5}}{2} \quad (\because AB = AC \text{ and } BC = AD)$$

Hence choice (c) is the correct one.

Hint : To find out how to solve a quadratic equation like $x^2 - x - 1 = 0$, you may like to refer the chapter on QUADRATIC EQUATIONS in this book only.

73 First of all extend AB to a point D such that $\angle ACD = 90^\circ$. Then draw a line segment DE equal to DC , such that $\angle CDE = 36^\circ$.



Now we know that in $\triangle CDE$, if $CE = 1$, $DC = DE = \phi$.

And, in $\triangle CBD$, if $CD = \phi$, $BC = BD = 1$.

Now, in $\triangle ABC$, $BC = AB = 1$.

Therefore, in right angle $\triangle ADC$, $AD = 2$ and $CD = \phi$.

$$\begin{aligned} \text{Thus we have } AC &= \sqrt{AD^2 - CD^2} \\ &= \sqrt{4 - \phi^2} = \sqrt{4 - (\phi + 1)} \end{aligned}$$

$$= \sqrt{3 - \phi}$$

Therefore (i) and (iii) are valid.

Hence choice (c) is the correct one.

74 Looking at the angles of $\triangle AEB$, we can say that $\angle EAB = \angle EBA = 72^\circ$ and $\angle AEB = 36^\circ$.

Therefore, $AB = \phi$ and $EA = EB = \phi^2$.

Now, by looking at the angles of $\triangle ADB$, we can say that $\angle ADB = \angle ABD = 54^\circ$ and $\angle DAB = 72^\circ$.

Therefore, $AD = AB = \phi$.

Again, by looking at the angles of $\triangle ACB$, we can say that $\angle ACB = \angle CAB = 72^\circ$ and $\angle ABC = 36^\circ$.

Therefore, $AB = BC = \phi$ and $AC = 1$.

Thus we see that $AC = 1$, $AD = \phi$ and $AE = \phi^2$.

$$\begin{aligned} \text{It implies that } CD &= \phi - 1 = \frac{1}{\phi} \text{ and } DE = \phi^2 - \phi \\ &= (\phi + 1) - \phi = 1. \end{aligned}$$

$$\text{Thus, we have } AC : CD : DE = 1 : \frac{1}{\phi} : 1 = \phi : 1 : \phi$$

Hence choice (d) is the correct one.

75 Given that $\angle ABC = 36^\circ$ and

$DE \parallel FG \parallel AC$, we have

$$\angle BDE = \angle BED = \angle BFG$$

$$= \angle BGF = \angle BAC = \angle BCA = 72^\circ.$$

In $\triangle BDE$, since $BD = BE = \phi$, we have $DE = 1$. Now connect DG and then we have $DE = EG = DF = 1$.

In $\triangle DFG$, since $DF = 1$, we have $DG = FG = \phi$.

Now connect FC and then we have $FG = GC = AF = \phi$.

In $\triangle FAC$, since $AF = \phi$, we have $FC = AC = \phi^2$.

But it can be established that $\phi^2 = \phi + 1$.

Hence choice (d) is the correct one.

76 We know that in this triangle, $\frac{AB}{AD} = \frac{1 + \sqrt{5}}{2}$

$$\Rightarrow BC = CD = AD = AB - BD = (1 + \sqrt{5})$$

Now, drop a perpendicular CP from C on BD , which will bisect the BD , as $BC = CD$.

$$\therefore CP = \sqrt{(1 + \sqrt{5})^2 - 1^2} = \sqrt{5 + 2\sqrt{5}}$$

$$\begin{aligned} \therefore \text{Area of isosceles triangle } \triangle BCD &= \frac{1}{2} \times 2 \times \sqrt{5 + 2\sqrt{5}} \\ &= \sqrt{5 + 2\sqrt{5}} \text{ cm}^2 \end{aligned}$$

Hence choice (a) is the correct one.

77 Let $AB = AC = CD = x$ and it is given that

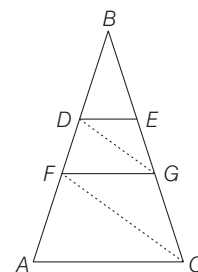
$$BD = CD = (\sqrt{5} - 1) \text{ cm}$$

Now, since the triangle $\triangle BDA \sim \triangle BAC$

$$\therefore \frac{BA}{BD} = \frac{BC}{BA}$$

$$\Rightarrow \frac{x}{\sqrt{5} - 1} = \frac{(\sqrt{5} - 1) + x}{x} \Rightarrow x = 2$$

Now, draw a perpendicular DP on AB , which will bisect AB .



The length of this perpendicular

$$= \sqrt{(\sqrt{5} - 1)^2 - (1)^2} = \sqrt{5 - 2\sqrt{5}} \text{ cm}$$

$$\begin{aligned} \text{Therefore the area of } \triangle ABD &= \frac{1}{2} \times 2 \times (\sqrt{5 - 2\sqrt{5}}) \\ &= \sqrt{5 - 2\sqrt{5}} \text{ cm}^2 \end{aligned}$$

Hence choice (a) is the correct one.

78 $\angle CAB = \angle CBA = 80^\circ$

$$\Rightarrow AC = BC \text{ and } \angle ACB = 20^\circ.$$

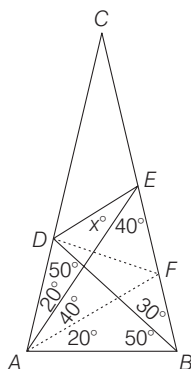
$$\text{And, } \angle ADB = \angle ABD = 50^\circ \Rightarrow AD = AB$$

Now draw a line AF from A on BC , such that $\angle AFB = 80^\circ$

$$\Rightarrow AF = AB$$

$$\therefore \angle BAF = 20^\circ \text{ and } \angle AFE = 100^\circ.$$

$$\Rightarrow \angle FAE = 40^\circ \text{ and } \angle AEF = 40^\circ \Rightarrow AF = EF$$



Now, we know that $AD = AB = AF = EF$ and $\angle DAF = 60^\circ$.

Therefore, $\triangle DAF$ is an equilateral triangle. That is $AD = AF = DF$.

Therefore, we have

$$\begin{aligned} \angle EDF = \angle DEF &= \frac{1}{2}(\angle DFB) \\ &= \frac{1}{2}(\angle DFA + \angle AFB) = \frac{1}{2}(60^\circ + 80^\circ) = 70^\circ \end{aligned}$$

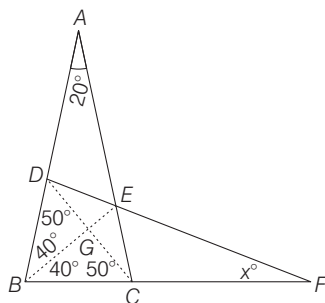
$$\text{But, } \angle x^\circ = \angle DEF - \angle AEF = 70^\circ - 40^\circ = 30^\circ$$

Hence choice (c) is the correct one.

79 Since $\angle ABC = \angle ACB = 80^\circ$, therefore, $\angle DBE = 40^\circ$ and $\angle DCE = 30^\circ$.

Now, in triangle DBC , $\angle DBC = 80^\circ$, $\angle BDC = 50^\circ$.

Therefore right angle triangle $\triangle DGB$ and $\triangle CGB$ are congruent.

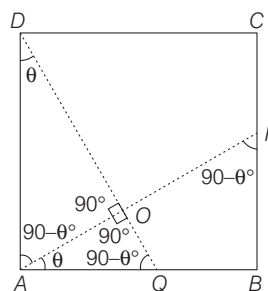


Now, consider the triangles $\triangle EDG$ and $\triangle ECG$, where EG is common, $DG = CG$ and $\angle DGE = \angle CGE = 90^\circ$. Therefore, the two triangles $\triangle EDG$ and $\triangle ECG$ are congruent. Now, since $\angle ECG = 30^\circ$, therefore, we have $\angle EDG = 30^\circ$.

Now, consider the $\triangle DBF$, where $\angle DBF = 80^\circ$ and $\angle BDF = 80^\circ$, so the $\angle DFB = 180^\circ - (80 + 80) = 20^\circ$.

Hence choice (b) is the correct one.

80 First of all try to observe the similarity. Let us consider $\angle ADQ = \theta$, we have $\angle AQD = 90 - \theta$. Since $AD = AB$, $AQ = BP$ and $\angle DAQ = \angle ABP = 90^\circ$, therefore $\triangle DAQ$ is congruent to $\triangle ABP$.



Now, in $\triangle AQO$, $\angle OAQ = \theta$ and $\angle OQA = 90 - \theta$, which implies that $\angle AOQ = 90^\circ$.

Thus we see that $\triangle DAQ$, $\triangle ABP$ and $\triangle AOQ$ are similar, where $\frac{DA}{AQ} = \frac{AB}{BP} = \frac{AO}{OQ} = \frac{2}{1}$.

Now, if we consider $OQ = 1$ cm, we have $AO = 2$ cm and $AQ = \sqrt{5}$ cm, $BP = \sqrt{5}$ cm and $OD = 4$ cm. Thus, we have $AB = BC = CD = DA = 2\sqrt{5}$ cm.

Now, area of square $ABCD = 20 \text{ cm}^2$, area of $\triangle DAQ = \triangle ABP = 4 \text{ cm}^2$ and area of $\triangle AOQ = 1 \text{ cm}^2$.

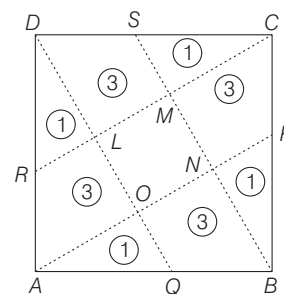
Therefore area of quadrilateral $QBPO = \triangle ABP - \triangle AOQ = 5 - 1 = 4 \text{ cm}^2$

And, area of quadrilateral $OPCD = \text{area of square } ABCD - (\triangle ABP + \triangle DOA) = 20 - (5 + 4) = 11 \text{ cm}^2$

Therefore the required ratio is 4 : 11.

Hence choice (c) is the correct one.

Alternatively Let's draw the lines RC and SB as shown in the diagram, where R and S are the mid-points on respective sides of the square $ABCD$. Also, you can see that RC is parallel to AP and DQ is parallel to SB .



As per mid-point theorem,

area of $\triangle DRL = \frac{1}{4}(\triangle DAO)$. Thus, if area of the $\triangle DRL$ is 1

unit, the area of $\triangle DAO$ will be 4 units. And, the same logic applies with other triangles too.

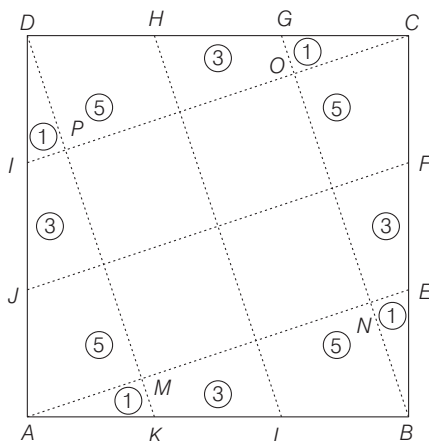
Now, if you join SQ , you will get four congruent triangles $\triangle DAQ$, $\triangle DSQ$, $\triangle SQB$ and $\triangle SCB$ with equal area.

Further, since, the area of triangle $\triangle DAQ$ is 5 units, the area of square $ABCD$ will be four times, that is, 20 units.

Therefore, the area of square $LMNO$
 $= 20 - (1 + 3 + 1 + 3 + 1 + 3 + 1 + 3) = 4$ units.

Therefore area of quadrilateral $QBPO = 1 + 3 = 4$ units
 and the area of quadrilateral $OPCD = 1 + 3 + 3 + 4 = 11$ units.
 Thus the required ratio is 4 : 11.

- 81** Since all the triangles are similar, so we can apply the rule of similarity and find the areas of all the triangles, as shown in the diagram.



Total area of square

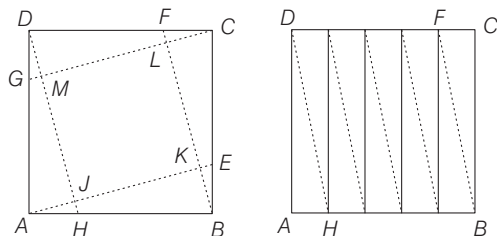
$$\begin{aligned} ABCD &= \triangle DAK + \square DHLK + \square HGBL + \triangle GCB \\ &= \triangle DAK = 2(\triangle DAK) + 2(\triangle DAK) + \triangle DAK \\ &= 6(\triangle DAK) = (1 + 3 + 5 + 1) \times 6 = 60 \text{ units.} \end{aligned}$$

Therefore, area of square $MNOP$ = area of square $ABCD - (\triangle DAM + \triangle ABN + \triangle BCO + \triangle CDP)$
 $= 60 - 4 \times (1 + 3 + 5) = 24$ units

Hence choice (b) is the correct one.

Hint If you join HK and GL , you see that the total area of the square $ABCD$ can be divided into 6 triangles of the same size as of $\triangle DAK$.

- 82** If the area of $\triangle DGM = 1$ unit, area of $\triangle DAJ = 25$ units and area of $\triangle DAH = 25 + 1 = 26$ units.



Area of square $ABCD = 26 \times 10 = 260$ units.

Area of square $JKLM$ = area of square $ABCD$
 $- (\text{area of } \triangle DAJ + \triangle ABK + \triangle BCL + \triangle CDM)$
 $= 260 - 4 \times (25) = 160$ units.

Therefore required ratio $= \frac{160}{260} = \frac{8}{13}$

Hence choice (a) is the correct one.

- 83** If the area of $\triangle DGS$ is 4 units, area of $\triangle DAP$ will be 25 units and area of $\triangle DAH$ will be 29 units.

$$\text{Area of square } ABCD = \left(\frac{5}{2}\right) \times 2 \times \triangle DAH = 145 \text{ units.}$$

Area of square $PQRS$ = area of square $ABCD - (\triangle DAP + \triangle ABQ + \triangle BCR + \triangle CDS)$

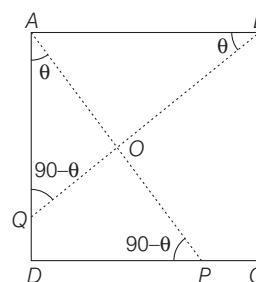
$$= 145 - 4(25) = 45 \text{ units}$$

$$\text{Thus the required ratio} = \frac{45}{145} = \frac{9}{29}$$

Hence choice (c) is the correct one.

- 84** $\triangle BAQ$ and $\triangle ADP$ are congruent, that's why $\angle AOQ = 90^\circ$.

Thus we see that AO is a perpendicular on BQ . It means,
 $\frac{\triangle AOQ}{\triangle AOB} = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$.



Since $\triangle AOQ$ and $\triangle BOA$ are similar, therefore

$$\frac{\text{area } \triangle AOQ}{\text{area } \triangle BOA} = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

$$\text{Now, } \frac{\triangle AOQ}{\triangle ADP} = \frac{\triangle AOQ}{\triangle BAQ}$$

$$= \frac{\triangle AOQ}{\triangle AOB + \triangle AOQ}$$

$$= \frac{16}{25 + 16} = \frac{16}{41}$$

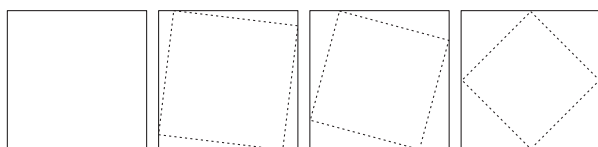
$$\text{Thus, we have } \frac{\triangle ADP}{\square ABCD} = \frac{\triangle ADP}{2(\triangle ADC)} = \frac{41}{2 \left[41 \left(\frac{5}{4} \right) \right]} = \frac{2}{5}$$

$$\begin{aligned} \text{Therefore, } \frac{\triangle AOQ}{\square OBCP} &= \frac{\triangle AOQ}{\square ABCD - (\triangle AOB + \triangle ADP)} \\ &= \frac{16}{\frac{205}{2} - (25 + 41)} \\ &= \frac{32}{73} \end{aligned}$$

Hence choice (d) is the correct one.

- 85** Maximum value of $a^2 + b^2 + c^2 + d^2$ will be obtained when the vertices of quadrilateral overlap the vertices of the square. In that case, actually a, b, c and d are the greatest possible values.

And, the minimum value of $a^2 + b^2 + c^2 + d^2$ will be obtained when the vertices of quadrilateral are the mid-points of the sides of the square. In this case actually a, b, c and d are the smallest possible values.



Hence choice (a) is the correct one.

- 86** As you know that $AF = CF$ and $AE = CE$, and since it is given that $AF = EF = EC$, therefore $\triangle AEF$ and $\triangle CEF$ are two equilateral triangle of same area.

Thus, the quadrilateral $AECF$ is a rhombus.

Let's assume that each side of this rhombus is a unit. Then the inradius of the circle inscribed in the rhombus

$$\square AECF = \frac{\text{Area of } \square AECF}{\text{semiperimeter of } \square AECF}$$

$$\text{Area of } \square AECF = 2 \times \frac{\sqrt{3}}{4} \times a^2$$

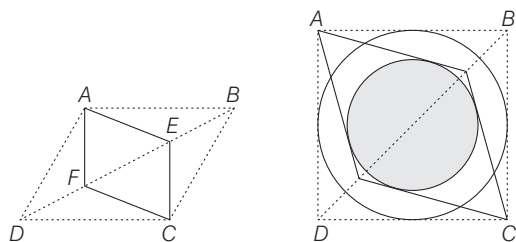
$$= \frac{\sqrt{3}}{2} \times a^2$$

And semi-perimeter of $\square AECF = 2a$

Therefore, the inradius of the incircle

$$= r = \frac{\frac{\sqrt{3}}{2} \times a^2}{2a} = \frac{\sqrt{3}}{4} a$$

Thus the area of the incircle of $\square AECF = \pi \left(\frac{\sqrt{3}}{4} a \right)^2 = \frac{3\pi}{16} a^2$



Now, the largest circle can be drawn in a rhombus only when it's a square. Then the diagonal of square $ABCD$ is $BD = AC = \sqrt{3}a$

But, this diagonal of square $ABCD$ will act as the diameter of the inscribed circle.

Therefore area of the circle that I inscribed in the square

$$ABCD = \pi \times \left(\frac{\sqrt{3}a}{2} \right)^2 = \frac{3\pi}{4} \times a^2$$

Thus the required ratio = 1 : 4

Hence choice (c) is the correct one.

- 87** If h be the height of the trapezium, the area of trapezium

$$\square ABCD = \frac{1}{2} \times (AB + CD) \times h$$

$$\Rightarrow 350 = \frac{1}{2} \times (21 + 49) \times h$$

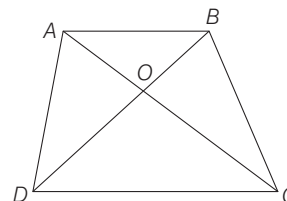
$$\Rightarrow h = 10 \text{ cm}$$

Since, $\triangle ABO$ and $\triangle CDO$ are similar triangles, therefore the heights of these triangles will be in the ratio of their

respective bases. That is if h_1 and h_2 be the heights of $\triangle ABO$ and $\triangle CDO$, then

$$\frac{h_1}{h_2} = \frac{AB}{CD} = \frac{21}{49} = \frac{3}{7}$$

But, since, $h_1 + h_2 = 10$, therefore $h_1 = 3$ cm and $h_2 = 7$ cm



$$\text{Thus the area of } \triangle ABO = \frac{1}{2} \times 21 \times 3 = \frac{63}{2}$$

$$\text{And, area of the } \triangle CDO = \frac{1}{2} \times 49 \times 7 = \frac{343}{2}$$

$$\text{Therefore Area of } \triangle AOD + \triangle BOC = \square ABCD - (\triangle AOB + \triangle COD)$$

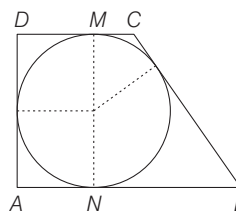
$$350 - (203) = 147 \text{ sq cm}$$

But, as you know that the area of $\triangle AOD = \triangle BOC$

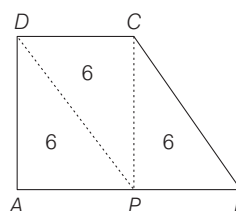
$$\text{Therefore, area of } \triangle AOD = \frac{147}{2} = 73.5 \text{ sq cm}$$

Hence choice (b) is the correct one.

- 88** Let's draw a diagram as per the given problem.



$DP \parallel CB$, as $AP = PB$. Thus $\triangle DAP = \triangle PDC = \triangle CPB = 6$



Let radius of the incircle is r . So $DA = CP = 2r$. And let us consider $CD = AP = PB = x$.

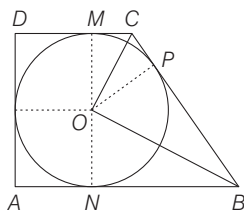
As $DM < CD \Rightarrow r < x$ and $CD < DA \Rightarrow x < 2r$.

$$\text{Since } \frac{1}{2}(2xr) = 6 \Rightarrow rx = 6$$

Choice	rx	r	x	$r < x$	$x < 2r$	Result
(a)	6	3	2	No	Yes	Impossible
(b)	6	2	3	Yes	Yes	Possible
(c)	6	3/2	4	Yes	No	Impossible
(d)	6	1	6	Yes	No	Impossible

Hence choice (b) is the answer.

☞ Alternatively Let's draw a diagram as per the given problem.



Then $\angle POC = \angle COM$, since PC and MC are tangents on the same circle from the same point C . Now if you assume $\angle COP = \theta$, then $\angle BOP = 90 - \theta$.

Therefore $\triangle CMO$ and $\triangle ONB$ are similar. That is $\frac{CM}{MO} = \frac{ON}{NB}$.

Let us assume $DC = x$ and radius is r , then $CM = x - r$ and $NB = 2x - r$.

$$\begin{aligned} \text{Therefore, } \frac{CM}{MO} &= \frac{ON}{NB} \\ \Rightarrow \frac{x-r}{r} &= \frac{r}{2x-r} \end{aligned}$$

$$\Rightarrow 2x = 3r \quad \dots(i)$$

Also the area of quadrilateral = 18

$$\Rightarrow \frac{1}{2}(2r \times 3x) = 18$$

$$\Rightarrow rx = 6 \quad \dots(ii)$$

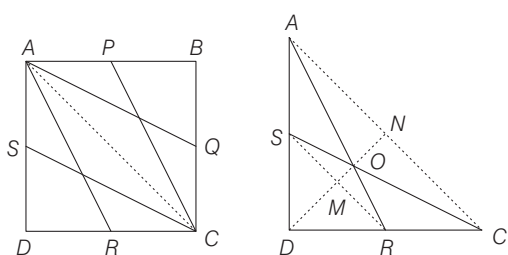
From eqs. (i) and (ii), $r = 2$

- 89** Since area of kite $CSAP$ is 200 sq cm, it means the area of $\triangle CSA = 100$ sq cm.

Which in turn implies that the area of $\triangle CDS = 100$ sq cm

Thus the area of $\triangle CDA = \triangle CDS + \triangle CSA = 200$ sq cm

But as $AD = CD$, so we have $AD = CD = 20$ cm



It means $AS = DS = DR = RC = 10$ cm.

Therefore, area of $\triangle SDR = 50$ sq cm

So, the area of parallelogram $ASRC = \triangle ADC - \triangle SDR = 150$ sq cm.

Now, if $OM = x$, then $ON = 2x$. That is $MN = 3x$, then $DM = 3x$

It implies that $DN = 6x = 10\sqrt{2}$

$$\Rightarrow x = \frac{5\sqrt{2}}{3}$$

Therefore, area of $\triangle AOC$

$$= \frac{1}{2} \times AC \times ON$$

$$\begin{aligned} &= \frac{1}{2} \times 20\sqrt{2} \times \frac{10\sqrt{2}}{3} \\ &= \frac{200}{3} \text{ sq cm.} \end{aligned}$$

Thus the area of common region

$$= 2 \times \frac{200}{3} = \frac{400}{3} \text{ sq cm.}$$

Hence choice (c) is the correct one.

- 90** We know that in any quadrilateral the sum of any three sides must be greater than the fourth side. Therefore we have, $a + b + c > d$, such that $a, b, c \leq d$.

Case	$a + b + c$	d
I	7	5
II	8	4
III	9	3
IV	10	2
V	11	1

Case	$a + b + c$	a	b	c	d	No. of combinations
I	7	1	1	5	5	4
		1	2	4		
		1	3	3		
		2	2	3		
II	8	1	3	4	4	3
		2	2	4		
		2	3	3		
III	9	3	3	3	3	1
IV	10	—	—	—	2	0
V	11	—	—	—	1	0

Case IV and Case V do not satisfy the required conditions.

Thus the total number of required combinations = $4 + 3 + 1 = 8$.

Hence choice (a) is the correct one.

- 91** Let us assume that $RD = h$, $OR = a$ and $OD = r$

First of all extend the sides DC and AB in order to make them meet at Q . Now, the angle $BQC = 36^\circ$.

Since $\triangle BQC$ is an isosceles triangle so the perpendicular QP drawn from Q on the side BC will bisect the BC , and it will be the angular bisector of angle BQC , as well.

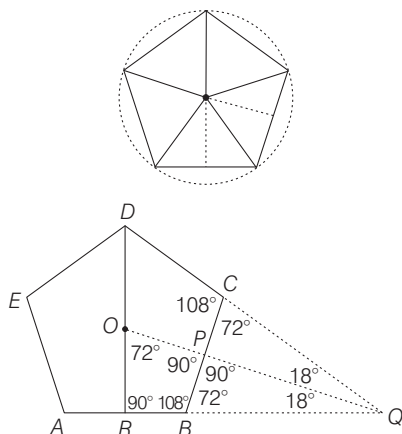
Similarly, the perpendiculars drawn from the incentre/circumcentre O of the regular pentagon $ABCDE$ will bisect the opposite sides as OR bisects AB at R and OP bisects BC at P .

Now as given that $BC = 2$ cm, therefore using 36-72-72 degree theorem in $\triangle BQC$, we have

$$BQ = CQ = (1 + \sqrt{5}) \text{ cm}$$

Now, in right angled triangle BPQ ,

$$PQ = \sqrt{BQ^2 - BP^2} = \sqrt{(1 + \sqrt{5})^2 - 1^2} = \sqrt{5 + 2\sqrt{5}}$$



But since $\triangle BPQ$ is similar to $\triangle ORQ$, therefore

$$\begin{aligned} \frac{BQ}{BP} &= \frac{OQ}{OR} \\ \Rightarrow \frac{BQ}{BP} &= \frac{OP + PQ}{OR} \\ \Rightarrow \frac{1 + \sqrt{5}}{1} &= \frac{a + \sqrt{5 + 2\sqrt{5}}}{a} \\ \Rightarrow a &= \frac{\sqrt{5 + 2\sqrt{5}}}{\sqrt{5}} \end{aligned}$$

Now, in right angle $\triangle DRQ$,

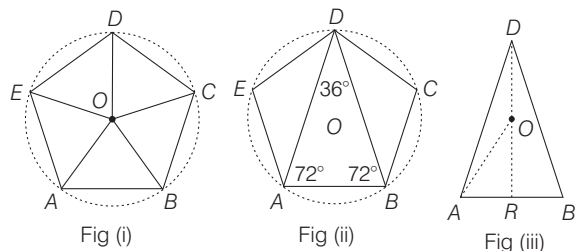
$$\begin{aligned} h &= DR = \sqrt{DQ^2 - RQ^2} \\ &= \sqrt{(DC + CQ)^2 - (RB + BQ)^2} \\ &= \sqrt{(3 + \sqrt{5})^2 - (2 + \sqrt{5})^2} = \sqrt{5 + 2\sqrt{5}} \\ \therefore r &= DO = DR - OR = h - a \\ &= \sqrt{5 + 2\sqrt{5}} - \frac{\sqrt{5 + 2\sqrt{5}}}{\sqrt{5}} \\ &= \sqrt{5 + 2\sqrt{5}} \left(1 - \frac{1}{\sqrt{5}} \right) = \frac{3 + \sqrt{5}}{\sqrt{5 + 2\sqrt{5}}} \end{aligned}$$

Hence choice (a) is the correct one.

☞ Alternatively

Looking at the following figures you can easily figure out that the circumradius of pentagon in fig (i) will be same as the circumradius of the isosceles triangle in fig (ii).

Using 36-72-72 degree theorem, in $\triangle DAB$, we have $AD = BD = 1 + \sqrt{5}$ cm, as $AB = 2$ cm.



Now, let us assume that $DR = h$ and $DO = AO = r$ and $OR = a$

We know that the circumradius of an isosceles triangle

$$\text{with the lateral sides } x \text{ and base } y \text{ is } \frac{x^2}{2\sqrt{x^2 - \frac{y^2}{4}}}$$

Therefore, the circumradius of the $\triangle ADB$

$$= \frac{(1 + \sqrt{5})^2}{2\sqrt{(1 + \sqrt{5})^2 - \frac{(2)^2}{4}}} = \frac{6 + 2\sqrt{5}}{2\sqrt{5 + 2\sqrt{5}}} = \frac{3 + \sqrt{5}}{\sqrt{5 + 2\sqrt{5}}}$$

☞ Alternatively Consider the figures intended for the previous alternate solution.

$$AR^2 + RD^2 = AD^2$$

$$\Rightarrow 1 + h^2 = (1 + \sqrt{5})^2 = 6 + 2\sqrt{5}$$

$$\Rightarrow h = \sqrt{5 + 2\sqrt{5}}$$

$$\text{But, } OR^2 = OA^2 - AR^2$$

$$\Rightarrow (h - r)^2 = r^2 - 1^2$$

$$\therefore h^2 + r^2 - 2hr = r^2 - 1 \Rightarrow 1 + h^2 = 2hr$$

$$\Rightarrow 6 + 2\sqrt{5} = 2(\sqrt{5 + 2\sqrt{5}})r$$

$$\Rightarrow r = \frac{3 + \sqrt{5}}{\sqrt{5 + 2\sqrt{5}}}$$

92 Let inradius be x and circumradius be y and side be z , then

$$x^2 + \left(\frac{z}{2}\right)^2 = y^2$$

$$\Rightarrow x^2 + 9 = y^2$$

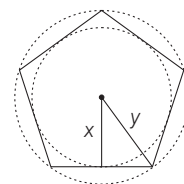
$$\Rightarrow y^2 - x^2 = 9$$

$$\Rightarrow (y + x)(y - x) = 9$$

$$\Rightarrow 9(y - x) = 9$$

$$\Rightarrow y - x = 1$$

$$\Rightarrow x = 4, y = 5$$



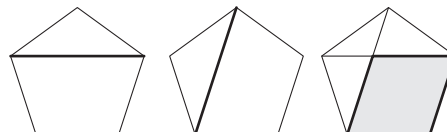
Therefore, area of circumcircle = 25π sq cm

And area of incircle = 16π sq cm

Thus the total required area = $25\pi + 16\pi = 41\pi$ sq cm

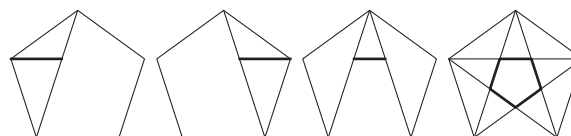
Hence choice (a) is the correct one.

93 In this pentagon the side S and the diagonal D are parallel, so we have a parallelogram as shown below in the fig (iii).



So the length of the highlighted segment in the fig (iv) and (v) is $d = D - S$

And the length of the highlighted section in the fig (vi) is $s = D - 2d = 2S - D$



But, we know that the diagonal (D) of a regular pentagon

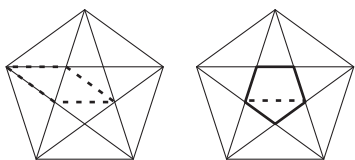
$$= \frac{1 + \sqrt{5}}{2} \times \text{side}$$

$$\therefore D = \frac{1 + \sqrt{5}}{2} \times S$$

$$\therefore s = 2S - D = 2S - \left(\frac{1 + \sqrt{5}}{2} \right) S = \frac{(3 - \sqrt{5})S}{2}$$

Hence choice (b) is the correct one.

- 94** In the following figure, there is a parallelogram shown by the dotted lines. And each side of this parallelogram is same and equal to the diagonal of the smaller pentagon.



Thus the diagonal of the smaller pentagon $= d = D - S$

$$= \left[\left(\frac{1 + \sqrt{5}}{2} \right) S \right] - S = \left(\frac{\sqrt{5} - 1}{2} \right) S$$

Hence choice (c) is the correct one.

- 95** $\angle PED = \angle PDE = \angle BCQ = \angle CBQ = 72^\circ$

and $\angle EPD = \angle BQC = 36^\circ$.

Now, we know that for a 72° - 36° - 72° triangle, the ratio of sides is $(1 + \sqrt{5}) : 2 : (1 + \sqrt{5})$.

That means, if $DE = 2$, $PE = PD = 1 + \sqrt{5}$.

- 96** We know that $\angle EAB = \angle ABC = \angle BCD = \angle CDE = \angle DEA = 108^\circ$

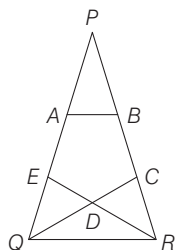
Therefore, $\angle PAB = \angle PBA = 72^\circ$ and $\angle APB = 36^\circ$.

Similarly, $\angle QED = \angle QDE = 72^\circ$ and $\angle EQD = 36^\circ$.

Also, $\angle RDC = \angle RCD = 72^\circ$ and $\angle DRC = 36^\circ$.

Thus, we have $\angle QDR = 108^\circ$ and $\angle DQR = \angle DRQ = 36^\circ$.

Therefore, $\triangle ERQ$ and $\triangle EDQ$ are similar triangles.



Now, we know that for a 72° - 36° - 72° triangle, the ratio of sides is $(1 + \sqrt{5}) : 2 : (1 + \sqrt{5})$

Therefore,

$$\begin{aligned} PQ = PR &= (1 + \sqrt{5}) + 2 + (1 + \sqrt{5}) \\ &= 4 + 2\sqrt{5}. \end{aligned}$$

And,

$$\begin{aligned} QR = ER = ED + DR \\ = 2 + (1 + \sqrt{5}) = 3 + \sqrt{5} \end{aligned}$$

Thus the perimeter of $\triangle PQR = PQ + PR + QR$
 $= 11 + 5\sqrt{5}$

Hence choice (b) is the correct one.

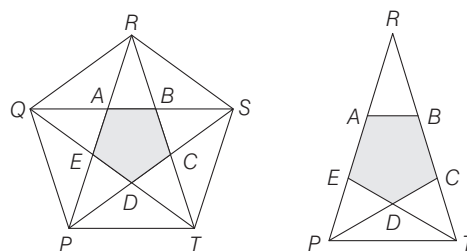
- 97** Since $\angle EPT = \angle PET = 72^\circ$, therefore,

$$PT = ET = ED + DT$$

But, we know that

$$DT = \left(\frac{1 + \sqrt{5}}{2} \right) DC = \left(\frac{1 + \sqrt{5}}{2} \right) ED$$

$$\begin{aligned} \text{Therefore, } PT &= ED + DT = ED + \left(\frac{1 + \sqrt{5}}{2} \right) ED \\ &= \left(\frac{3 + \sqrt{5}}{2} \right) ED \end{aligned}$$



$$\therefore PT = \left(\frac{3 + \sqrt{5}}{2} \right) ED$$

$\therefore$ Area of larger pentagon = area of smaller pentagon

$$\times \left(\frac{3 + \sqrt{5}}{2} \right)^2$$

$\Rightarrow$ Area of larger pentagon = area of smaller pentagon

$$\times \left(\frac{7 + 3\sqrt{5}}{2} \right)$$

$$\Rightarrow \text{Area of larger pentagon} = \left(\frac{7 - 3\sqrt{5}}{2} \right) \times \left(\frac{7 + 3\sqrt{5}}{2} \right)$$

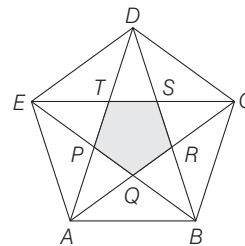
$\Rightarrow$ Area of larger pentagon = 1

Hence choice (a) is the correct one.

- 98** First of all let us understand a couple of relations before we proceed to solve this problem.

Given that $\phi = \frac{\sqrt{5} + 1}{2}$, so we have $\phi + 1 = \phi^2$ and

$$2\phi + 1 = \phi^3$$



Now, we know that $\frac{PQ}{QB} = \frac{1}{\phi}$

$$\text{and } AB = PB = PQ + QB = 1 + \phi = \phi^2$$

When $AB = \varphi^2$, $PQ = 1$

So, when $AB = 1$, $PQ = \frac{1}{\varphi^2}$

Now, we know that

$$\frac{\text{Area of } ABCDE}{\text{Area of } PQRST} = \frac{K(AB)^2}{K(PQ)^2} = \frac{(1)^2}{\left(\frac{1}{\varphi^2}\right)^2} = \frac{1}{\frac{1}{\varphi^4}} = \frac{\varphi^4}{1}$$

Hence choice (b) is the correct one.

- 99** There are clearly four distinct sizes of pentagon. Let's call the smallest pentagon P_1 and so the largest pentagon P_4 .

Each side of $P_1 = 1$

Each side of $P_2 = 1 + \varphi = \varphi^2$

Each side of $P_3 = \varphi^2 + \varphi^3 = \varphi^2(1 + \varphi) = \varphi^4$

Each side of $P_4 = \varphi^4 + \varphi^5 = \varphi^4(1 + \varphi) = \varphi^6$

Hence choice (d) is the correct one.

- 100** Since each side of the smaller pentagon is 1 cm, so each side of the larger pentagon will be

$$\left(1 + \frac{2}{1 + \sqrt{5}} + 1\right) = \frac{3 + \sqrt{5}}{2} \text{ cm.}$$

Thus the area of the larger pentagon

$$\begin{aligned} &= \frac{1}{4} (\sqrt{5(5 + 2\sqrt{5})}) \left(\frac{3 + \sqrt{5}}{2}\right)^2 \\ &= \frac{1}{4} (6.88)(6.85) = 11.72 \text{ sq. cm} \end{aligned}$$

Hence choice (b) is the correct one.

- 101** We know that if the side of a pentagon is s , the area of the inscribed pentagram is

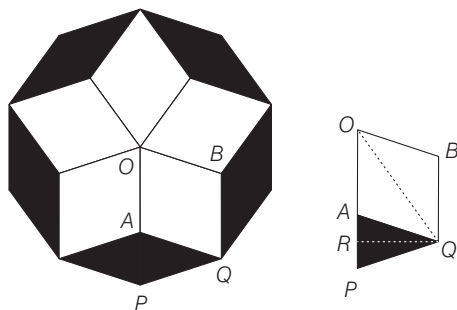
$$A = \frac{1}{2} \left[\sqrt{5(5 - 2\sqrt{5})} \right] s^2$$

$$\Rightarrow A = \frac{1}{2} \left[\sqrt{5(5 - 2\sqrt{5})} \right]$$

Now, the best way is to substitute the numerical value of $\varphi = \frac{\sqrt{5} + 1}{2}$ in the given options in order to find out the correct match.

Hence choice (c) is the correct one.

102



It is known that $PQ = AQ$ and $AQ = OA$.

So we have $PQ = AQ = OA = 1 + \sqrt{5}$ cm.

Now, applying 72° - 36° - 72° theorem, we get $AP = 2$ cm.

Now, let's drop a perpendicular RQ from Q on AP , which will bisect the AP . Now try to understand that the perpendicular RQ can act as the height of both the triangles $\triangle AQP$ and $\triangle OQP$.

$$\text{Therefore } RQ = \sqrt{AQ^2 - AR^2} = \sqrt{(1 + \sqrt{5})^2 - 1^2} = \sqrt{5 + 2\sqrt{5}}$$

$$\text{Thus, area of } \triangle OQA = \frac{1}{2} (OA) \times (RQ)$$

$$= \frac{1}{2} (1 + \sqrt{5}) (\sqrt{5 + 2\sqrt{5}})$$

Therefore, required area = $5 \times$ area of rhombus $OBQA$

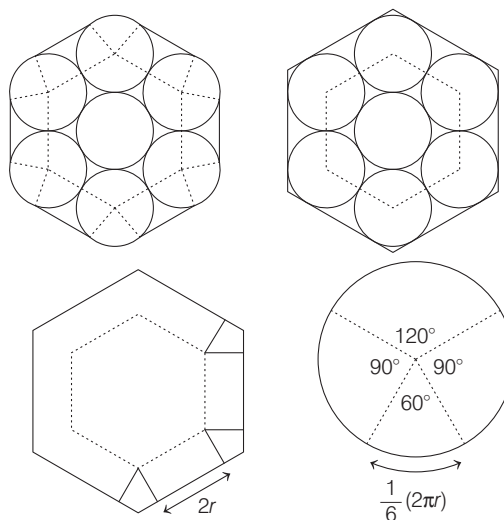
$$= 10 \times \text{area of } \triangle OQA$$

$$= 10 \left(\frac{1}{2} (1 + \sqrt{5}) (\sqrt{5 + 2\sqrt{5}}) \right)$$

$$= 5(1 + \sqrt{5}) (\sqrt{5 + 2\sqrt{5}})$$

Hence choice (c) is the correct one.

- 103** The perimeter obtained by wrapping a band around the circle will consist of six straight segments of length d (where d is the diameter) and 6 arcs, each with length $\frac{1}{6}$ th of a circle, as the arc angle is 60° .

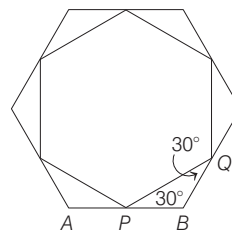


Therefore the required perimeter is

$$p = 6(d) + 6\left(\frac{2\pi r}{6}\right) = 6(2r) + 2\pi r = 2(6 + \pi)r$$

Hence choice (a) is the correct one.

- 104** Since the side of H_1 is 64 cm, that is $AB = 64$ cm, therefore $AP = BP = BQ = \dots = 32$ cm. Now since the angles of triangle PBQ are 30° , 30° , 120° , so $PQ = 32\sqrt{3}$ cm.



In the similar way, each side of $H_3 = 64 \times \left(\frac{\sqrt{3}}{2}\right)^2$

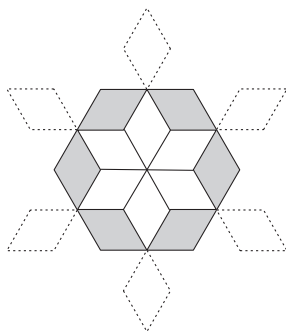
And, each side of $H_4 = 64 \times \left(\frac{\sqrt{3}}{2}\right)^3$

Therefore, each side of $H_7 = 64 \times \left(\frac{\sqrt{3}}{2}\right)^6 = 27 \text{ cm}$

Hence choice (a) is the correct one.

NOTE The answer should be given in cm, not in mm.

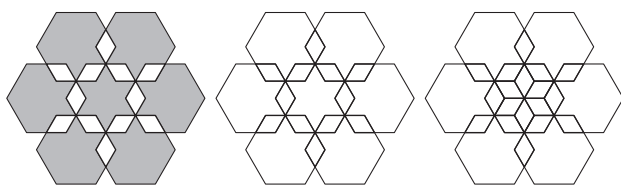
- 105** If you look closely at the figure, you will notice that the area comprised by the hexagram is equal to the 6 rhombi and thus the area comprised by the hexagon is equal to 12 rhombi.



It implies that the area of 12 rhombi
= area of hexagon = 84 sq cm.

Hence choice (d) is the correct one.

- 106** As per the figure, the area of the 12 rhombi
= area of each hexagon.
Therefore area of each rhombus = $\frac{84}{12} = 7 \text{ sq cm}$.



Total shaded area = shaded area of the 6 outer hexagons
– shaded area of the central hexagon

$$\begin{aligned} &= \text{area of 9 rhombi} \times 6 + \text{area of 6 rhombi} \\ &= (9 \times 6 \times 7) + (6 \times 7) = 9 \times (42) + (42) \\ &= 42 \times 10 = 420 \text{ sq cm.} \end{aligned}$$

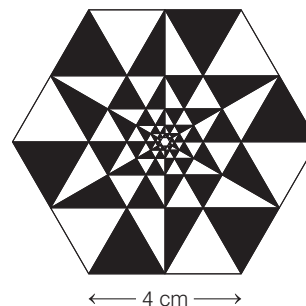
Hence choice (a) is the correct one.

- 107** There are total 6 shaded equilateral triangles along the perimeter of the hexagon.

So area of each such equilateral triangle is $\frac{6\sqrt{3}}{6} = \sqrt{3} \text{ sq cm}$.

Therefore, side of each such equilateral triangles is 2 cm.
Further, it implies that the side of the regular hexagon is 4 cm.

Therefore, area of the hexagon = $\frac{3\sqrt{3}}{2} \times 4^2 = 24\sqrt{3} \text{ sq cm}$.



As you can observe that the half of the area of the regular hexagon is shaded and rest half is un-shaded, so the total area of the un-shaded region = $\frac{1}{2} \times 24\sqrt{3} = 12\sqrt{3} \text{ sq cm}$.

Hence choice (d) is the correct one.

Hint Area of an equilateral triangle = $\frac{\sqrt{3}}{4} \times \text{side}^2$

And, area of a regular hexagon = $\frac{3\sqrt{3}}{2} \times \text{side}^2$

- 108** Let $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ be the side of the seven hexagons such that

$$a_1 > a_2 > a_3 > a_4 > a_5 > a_6 > a_7.$$

$$\begin{aligned} \text{Then we have, } & \frac{3\sqrt{3}}{2}(a_1)^2 - \frac{3\sqrt{3}}{2}(a_2)^2 \\ &= \frac{3\sqrt{3}}{2}(a_2)^2 - \frac{3\sqrt{3}}{2}(a_3)^2 \\ &= \frac{3\sqrt{3}}{2}(a_3)^2 - \frac{3\sqrt{3}}{2}(a_4)^2 \\ &= \frac{3\sqrt{3}}{2}(a_4)^2 - \frac{3\sqrt{3}}{2}(a_5)^2 \\ &= \frac{3\sqrt{3}}{2}(a_5)^2 - \frac{3\sqrt{3}}{2}(a_6)^2 \\ &= \frac{3\sqrt{3}}{2}(a_6)^2 - \frac{3\sqrt{3}}{2}(a_7)^2 \\ &= \frac{3\sqrt{3}}{2}(a_7)^2 \end{aligned}$$

$$\begin{aligned} \text{Or } \frac{3\sqrt{3}}{2}(a_1^2 - a_2^2) &= \frac{3\sqrt{3}}{2}(a_2^2 - a_3^2) \\ &= \frac{3\sqrt{3}}{2}(a_3^2 - a_4^2) \\ &= \frac{3\sqrt{3}}{2}(a_4^2 - a_5^2) \\ &= \frac{3\sqrt{3}}{2}(a_5^2 - a_6^2) = \frac{3\sqrt{3}}{2}(a_6^2 - a_7^2) = \frac{3\sqrt{3}}{2}a_7^2 \\ \text{Or } (a_1^2 - a_2^2) &= (a_2^2 - a_3^2) = (a_3^2 - a_4^2) = (a_4^2 - a_5^2) \\ &= (a_5^2 - a_6^2) = (a_6^2 - a_7^2) = a_7^2 \end{aligned}$$

$$\therefore (a_6^2 - a_7^2) = a_7^2$$

$$\Rightarrow a_6^2 = 2a_7^2$$

$$\text{And } (a_5^2 - a_6^2) = (a_6^2 - a_7^2)$$

$$\Rightarrow a_5^2 = 3a_7^2$$

$$\text{And } (a_4^2 - a_5^2) = (a_5^2 - a_6^2)$$

$$\Rightarrow a_4^2 = 4a_7^2$$

$$\text{And } (a_3^2 - a_4^2) = (a_4^2 - a_5^2)$$

$$\Rightarrow a_3^2 = 5a_7^2$$

$$\text{And } (a_2^2 - a_3^2) = (a_3^2 - a_4^2) \Rightarrow a_2^2 = 6a_7^2$$

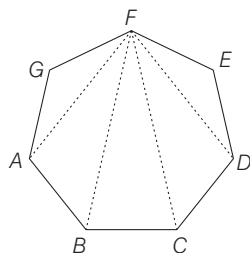
$$\text{And } (a_1^2 - a_2^2) = (a_2^2 - a_3^2) \Rightarrow a_1^2 = 7a_7^2$$

$$\therefore \frac{a_1}{a_7} = \frac{\sqrt{7}}{1}$$

Hence choice (d) is the correct one.

- 109** First of all you must know that as whenever a polygon is regular, it is necessarily a cyclic polygon. That is all its vertices lie on the circle. this ensures that all the inscribed angles must be equal, that is

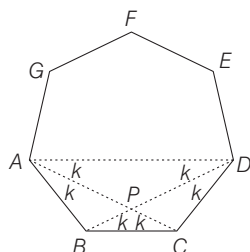
$$\begin{aligned} \angle GFA &= \angle AFB = \angle BFC = \angle CFD \\ &= \angle DFE = \frac{\pi}{7} = \frac{180^\circ}{7} \end{aligned}$$



So, based on the above logic, we have

$$\begin{aligned} \angle BAC &= \angle BCA = \angle BDC = \angle DBC = \angle CAD \\ &= \angle BDA = \frac{180}{7} = k \text{ (say)} \end{aligned}$$

That is $180^\circ = 7k$



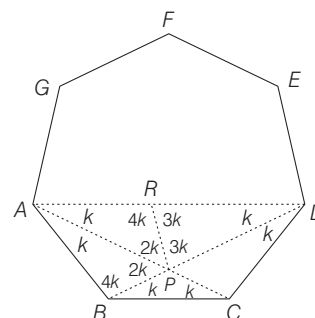
Now, due to symmetry triangle APD is isosceles, that is $AP = DP$ and therefore $\angle APD = 180 - (k + k) = 5k$

It implies that $\angle APB = 180 - \angle APD = 2k$

$$\begin{aligned} \text{Therefore, } \angle ABP &= 180 - (\angle BAP + \angle APB) \\ &= 180 - 3k = 4k \end{aligned}$$

Now choose a point R on AD such that $\angle APR = 2k$ and $\angle ARP = 4k$, that is $\triangle APB$ and $\triangle APR$ are congruent.

Therefore, $AR = AB$



$$\begin{aligned} \text{Now, } \angle RPD &= 180 - (\angle APB + \angle APR) \\ &= 7k - (2k + 2k) = 3k \end{aligned}$$

$$\begin{aligned} \text{And } \angle PRD &= 180 - (\angle RDP + \angle RPD) \\ &= 7k - (k + 3k) = 3k \end{aligned}$$

Therefore, $\angle PRD = \angle RPD = 3k$

It implies that $\triangle RDP$ is an isosceles triangle.

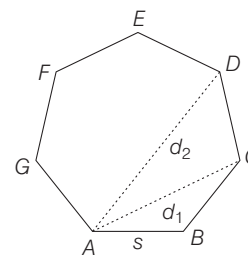
That is $RD = PD$

$$\begin{aligned} \therefore AB + BD &= AB + DP + PB = AB + RD + PB \\ &= AR + RD + BP = AD + BP \end{aligned}$$

Hence choice (c) is the correct one.

- 110** According to Ptolemy's theorem on the cyclic quadrilateral, "the sum of the products of the two pairs of opposite sides of a cyclic quadrilateral equals the product of its two diagonals." That is in the cyclic quadrilateral ACDE we have

$$\begin{aligned} \therefore AC \times DE + CD \times AE &= AD \times CE \\ \Rightarrow d_1 \times s + s \times d_2 &= d_1 d_2 \Rightarrow s(d_1 + d_2) = d_1 d_2 \\ \Rightarrow \frac{d_1 + d_2}{d_1 d_2} &= \frac{1}{s} \Rightarrow \frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{s} \end{aligned}$$

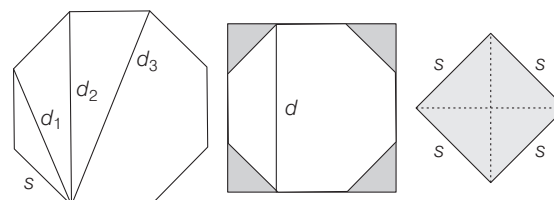


Hence choice (d) is the correct one.

Hint $AD = AE$ and $AC = CE$

- 111** Look at the following figures and find that the problem is talking about d_2 , the second largest diagonal of the octagon.

From the first two figures it can be seen that $d_2 = d$ And d is equal to the side of the circumscribing square. So, the area of the circumscribing square $= d^2$



Now, the area of the shaded region (in the circumscribing square)

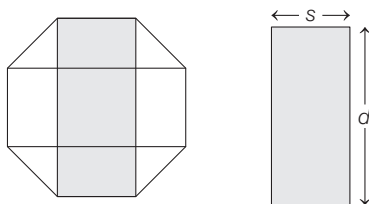
= area of the smaller square formed by joining the four corner triangles = s^2

Therefore, area of the octagon = area of the circumscribing square – area of the smaller square formed by joining the four corner triangles.

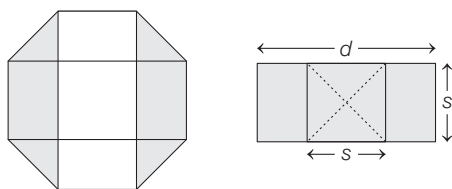
$$= d^2 - s^2$$

Now look at following diagrams.

The area of the vertical rectangle = ds



And the area of the horizontal rectangle = ds



Therefore, the total area of the octagon

$$= ds + ds = 2ds$$

Hence choice (d) is the correct one.

Ⓢ Alternatively

As you know $d = d_2 = (1 + \sqrt{2})s$

And area of octagon, $A = (1 + \sqrt{2})s^2$

Let us consider choice (b), then we will have

$$d^2 - s^2 = [(1 + \sqrt{2})s]^2 - s^2 = 2(1 + \sqrt{2})s^2 = A$$

Therefore, $A = d^2 - s^2$

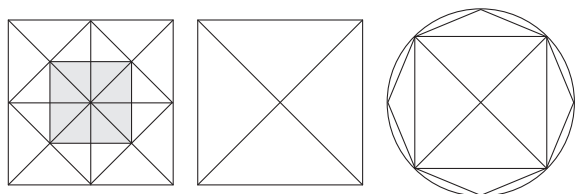
Now, let us consider choice (c), then we will have

$$2ds = 2 \times (1 + \sqrt{2})s \times s = 2(1 + \sqrt{2})s^2 = A$$

Therefore $A = 2ds$

Hence choice (d) is the most appropriate one.

- 112** Since area of the third square (shaded region) is 1 sq cm, so the area of the second square would be 2 sq cm. similarly, the area of the first square (the largest one) would be 4 sq cm.



That is the diameter of the circle is $2\sqrt{2}$ cm, so the radius of the circle is $\sqrt{2}$ cm and the area of the circle is 2π sq cm. Likewise, the longest diagonal of the octagon is $2\sqrt{2}$ cm.

So, the side of the octagon

$$= \frac{2\sqrt{2}}{\sqrt{4 + 2\sqrt{2}}} = \frac{2}{\sqrt{2 + \sqrt{2}}}$$

Therefore, area of the octagon

$$= 2(1 + \sqrt{2}) \left(\frac{2}{\sqrt{2 + \sqrt{2}}} \right) 4\sqrt{2} \text{ sq cm.}$$

Thus the difference between the areas of circle and octagon = $2\pi - 4\sqrt{2}$

$$= 2(\pi - 2\sqrt{2}) \text{ sq cm.}$$

Hence choice (a) is the correct one.

- 113** Area of the octagon = $2(\sqrt{2} - 1)$ sq cm

$$\Rightarrow 2(\sqrt{2} + 1)s^2 = 2(\sqrt{2} + 1)$$

$$\Rightarrow s^2 = \frac{\sqrt{2} - 1}{\sqrt{2} + 1}$$

$$\Rightarrow s^2 = \frac{\sqrt{2} - 1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} \Rightarrow s = \sqrt{2} - 1$$

Now, each side of the square = $(\sqrt{2 + \sqrt{2}})s$

$$= (\sqrt{2 + \sqrt{2}})(\sqrt{2} - 1)$$

Therefore, area of the square

$$= [(\sqrt{2 + \sqrt{2}})(\sqrt{2} - 1)]^2$$

$$= (2 + \sqrt{2})(3 - 2\sqrt{2}) = \sqrt{2}(\sqrt{2} - 1)$$

- 114** By joining A, C, E, G we get a square whose diagonals, AE and CG, are 20 cm each. Therefore, each side (AC, CE, EG, GA) of the square is $10\sqrt{2}$ cm and $AO = CO = EO = GO = 10$ cm.

Therefore, each of $\Delta AOC = \frac{1}{2} \times AO \times OC = 50 \text{ cm}^2$

Now, due to symmetry or uniformity of the figure the diagonals AE and CG will act as the angular bisector of $\angle A, \angle E, \angle C, \angle G$.

Therefore, $\angle OAB = \angle OCB = 15^\circ$

Thus the $\angle BAC = \angle OAC - \angle OAB = 45^\circ - 15^\circ = 30^\circ$.

Similarly, $\angle BCA = 30^\circ$.

Therefore in triangle ΔABC ,

$$\angle ABC = 180^\circ - (\angle BAC + \angle BCA) = 120^\circ$$

Now, using 30 – 30 – 120 degree theorem in ΔABC (or dropping a perpendicular BP on AC from B and using 30-60-90 degree theorem in ΔABP or ΔCBP) you can find

$$\text{that } AB = BC = \frac{10\sqrt{2}}{\sqrt{3}} \text{ and } BP = \frac{5\sqrt{2}}{\sqrt{3}}$$

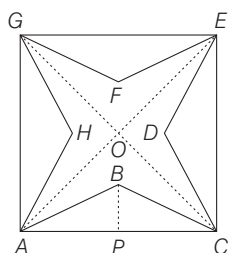
Therefore, area of triangle

$$ABC = \frac{1}{2} \times AC \times BP = \frac{50}{\sqrt{3}} \text{ cm}^2$$

Thus the area of the quadrilateral AOCB

= area of triangle AOC – area of triangle ABC

$$= 50 - \frac{50}{\sqrt{3}} \text{ cm}^2$$



Therefore, the area of the octagon
 = area of quadrilaterals
 $(OABC + OCDE + ODFG + OGHA)$
 = 4 (area of quadrilateral OABC)
 $= 4 \left(50 - \frac{50}{\sqrt{3}} \right) = \frac{200}{\sqrt{3}} (\sqrt{3} - 1)$

Hence choice (a) is the correct one.

- 115** Look at the second figure, which has two concentric octagons and the area of inner octagon is same as the area of the shaded region in the first octagon.

Now pay attention very closely in the second figure and try to figure out the fact that the circumradius of the outer octagon is double the inradius of the inner octagon. Therefore, if the circumradius of the outer octagon be R_0 and the inradius of the inner octagon be r_i and each side of the inner octagon be a , then

$$r_i = \frac{(1 + \sqrt{2})a}{2}$$

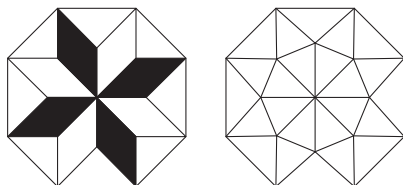
$$\therefore R_0 = 2r_i = (1 + \sqrt{2})a$$

Now, if the side of the outer octagon is s , then

$$\begin{aligned} s &= \frac{2R_0}{\sqrt{4 + 2\sqrt{2}}} = \frac{2(1 + \sqrt{2})a}{(\sqrt{4 + 2\sqrt{2}})} \\ &= \frac{\sqrt{2}(2 + \sqrt{2})a}{\sqrt{2}(\sqrt{4 + 2\sqrt{2}})} = (\sqrt{2 + \sqrt{2}})a \end{aligned}$$

Therefore, area of the outer octagon $= 2(1 + \sqrt{2})s^2$

$$\begin{aligned} &= 2(1 + \sqrt{2})[(\sqrt{2 + \sqrt{2}})a]^2 \text{ cm}^2 \\ &= 2(1 + \sqrt{2})a^2 [(\sqrt{2 + \sqrt{2}})]^2 \text{ cm}^2 \\ &= 2(1 + \sqrt{2})a^2 \times [(2 + \sqrt{2})] \text{ cm}^2 \end{aligned}$$



But, since area of the inner octagon $= 2(1 + \sqrt{2})a^2 = 8 \text{ cm}^2$

Therefore, area of the outer octagon
 $= 2(1 + \sqrt{2})a^2 \times (2 + \sqrt{2}) \text{ cm}^2$
 $= 8 \times (2 + \sqrt{2}) \text{ cm}^2$

Hence choice (a) is the correct one.

Alternatively Area of the 4 shaded rhombi $= 8 \text{ sq cm}$.

So the total area of all the 8 rhombi $= 16 \text{ sq cm}$.

Now we have to determine the area of 8 isosceles right angled triangles, each having one side common with the side of the octagon.

Let each side of the inner octagon be a , then the circumradius of the inner radius (say r)

$$= \frac{(\sqrt{4 + 2\sqrt{2}})a}{2} = \frac{(\sqrt{2 + \sqrt{2}})a}{\sqrt{2}}$$

And the area of the inner octagon $= 2(1 + \sqrt{2})a^2 = 8$

But the area of each isosceles triangle $= \frac{1}{2} r^2$

$$= \frac{1}{2} \left[\frac{(\sqrt{2 + \sqrt{2}})a}{\sqrt{2}} \right]^2 = \frac{(2 + \sqrt{2})a^2}{4}$$

Therefore area of all the 8 triangles

$$\begin{aligned} &= 8 \times \left[\frac{(2 + \sqrt{2})a^2}{4} \right] = 2(2 + \sqrt{2})a^2 \\ &= \sqrt{2}[2(1 + \sqrt{2})a^2] = 8\sqrt{2} \end{aligned}$$

Thus, the area of the outer octagon

$$= 16 + 8\sqrt{2} = 8(2 + \sqrt{2}) \text{ cm}^2$$

- 116** Look at the following figures. The central angle of the regular decagon is always 36° .

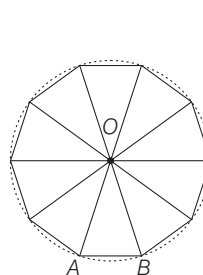


Fig (i)

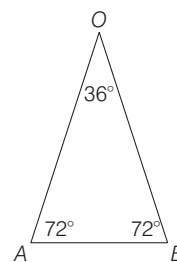


Fig (ii)

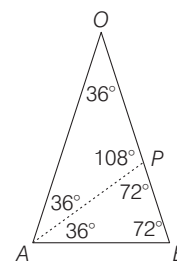


Fig (iii)

Then we will have 10 isosceles triangles which are congruent and their common vertex is the circumcentre of the decagon.

Now, look at the fig (iii) $AB = AP = OP = 2 \text{ cm}$

If $PB = x$, $OA = OB = (2 + x) \text{ cm}$

But since $\triangle AOB$ is similar to $\triangle BAP$, therefore we have

$$\frac{OA}{AB} = \frac{AB}{BP}$$

$$\Rightarrow \frac{2 + x}{2} = \frac{2}{x}$$

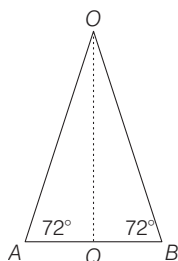
$$\Rightarrow x = \sqrt{5} - 1 \text{ cm}$$

$$\therefore OA = OB = 2 + x$$

$$\Rightarrow OA = 1 + \sqrt{5} \text{ cm}$$

Hence choice (a) is the correct one.

- 117** Since the $\triangle AOB$ is symmetrical, therefore the perpendicular OQ bisects AB such that $AQ = BQ$



Now, using Pythagoras theorem in $\triangle AOQ$ we get

$$OQ = \sqrt{OA^2 - AQ^2}$$

From the previous solution we know that $OA = 1 + \sqrt{5}$ cm, therefore

$$OQ = \sqrt{(1 + \sqrt{5})^2 - 1^2} = \sqrt{5 + 2\sqrt{5}} \text{ cm.}$$

Hence choice (a) is the correct one.

- 118** Look carefully at the fig (i) in the earlier solutions and you will find that the longest diagonal = $2 \times$ circumradius
 $= 2(1 + \sqrt{5})$ cm

Hence choice (d) is the correct one.

- 119** From the following figures, it is obvious that the second longest diagonal is exactly double the length of the apothem of the decagon.

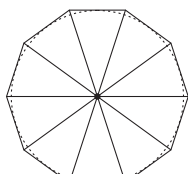


Fig (i)

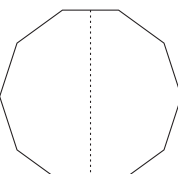


Fig (ii)

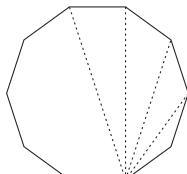
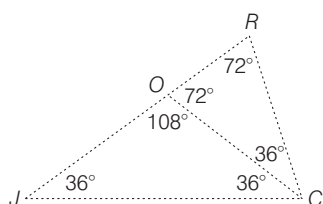
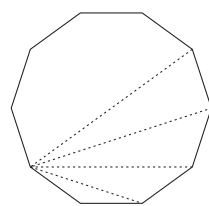
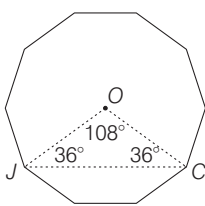
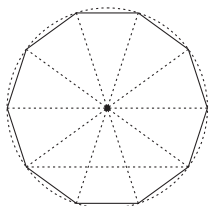


Fig (iii)

Therefore, the second longest diagonal = $2 \times$ apothem
 $= 2\sqrt{5 + 2\sqrt{5}}$ cm

Hence choice (b) is the correct one.

- 120** From the following figures, it is obvious that the third longest diagonal is equal to the longer side of the triangle OJC , which is formed by connecting the two vertices and the centre of the decagon as shown below.



Now, let us consider $JO = r$ and $RO = m$, therefore, $JR = r + m$ and $JO = OC = RC = r$

But, since $\triangle JRC$ and $\triangle COR$ are similar, so we have

$$\frac{JR}{RC} = \frac{RC}{RO}$$

$$\Rightarrow \frac{r + m}{r} = \frac{r}{m}$$

$$\Rightarrow m^2 + rm - r^2 = 0$$

As, we know that $r = 1 + \sqrt{5}$ cm, therefore

$$m^2 + rm - r^2 = 0$$

$$\Rightarrow m^2 + (1 + \sqrt{5})m - (1 + \sqrt{5})^2 = 0$$

$$\Rightarrow m = 2$$

$$\therefore JR = JC = r + m = 3 + \sqrt{5} \text{ cm}$$

Hence choice (c) is the correct one.

- 121** From the following figures let's figure out all the pertinent angles.

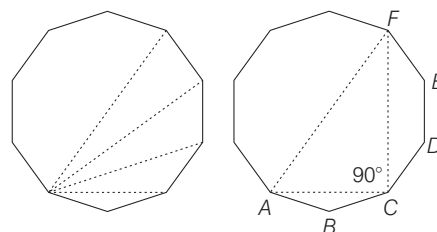
$$\angle ACB = \angle CAB = 18^\circ$$

And,

$$\angle FCD = \angle EFC = 36^\circ$$

$\therefore$

$$\angle ACF = 90^\circ$$



Otherwise, if you observe that AF acts as a diameter and vertex C falls on the circumference of the circle, so angle ACF will be a right angle.

Since each side of the decagon is 2 cm, so the longest diagonal $AF = 2 + 2\sqrt{5}$ cm and the third longest diagonal $CF = 3 + \sqrt{5}$ cm.

Now, as triangle ACF is right one, so the required shortest diagonal

$$AC = \sqrt{AF^2 - CF^2}$$

$$= \sqrt{(2 + 2\sqrt{5})^2 - (3 + \sqrt{5})^2} = \sqrt{10 + 2\sqrt{5}} \text{ cm}$$

Hence choice (a) is the correct one.

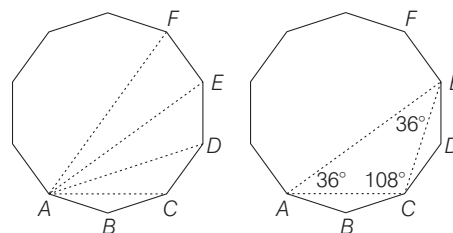
Alternatively From the following figures let's figure out all the pertinent angles.

$$\angle ACB = \angle CAB = \angle ECD = \angle CED = 18^\circ$$

$\therefore$

$$\angle ACE = 108^\circ$$

Since $AC = CE$, therefore triangle ACE is an isosceles triangle.



Since each side of the octagon is 2 cm, so the second longest diagonal

$$AE = 2\sqrt{5 + 2\sqrt{5}} \text{ cm}$$

Now, using the 36-36-108 degree theorem in triangle ACE, we have

$$\begin{aligned} \frac{AE}{AC} &= \frac{1 + \sqrt{5}}{2} \Rightarrow \frac{2\sqrt{5 + 2\sqrt{5}}}{AC} = \frac{1 + \sqrt{5}}{2} \\ \Rightarrow AC &= \frac{4(\sqrt{5 + 2\sqrt{5}})}{1 + \sqrt{5}} = (\sqrt{5 + 2\sqrt{5}})(\sqrt{5} - 1) \\ \Rightarrow AC &= \sqrt{(5 + 2\sqrt{5})(\sqrt{5} - 1)(\sqrt{5} - 1)} \\ &= \sqrt{10 + 2\sqrt{5}} \end{aligned}$$

Therefore, the required shortest diagonal = $\sqrt{10 + 2\sqrt{5}}$ cm

122 In the quadrilateral ABCD,

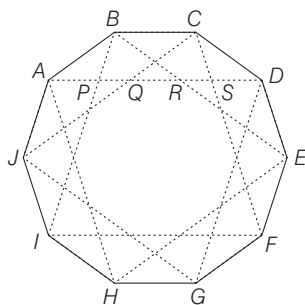
$$\angle ABC = \angle BCD = 144^\circ$$

so the

$$\angle BAD = \angle CDA = 36^\circ$$

By extending this logic, we can say that ABCQ is a rhombus, where $\angle BAQ = \angle BCQ = 36^\circ$ and $\angle ABC = \angle AQC = 144^\circ$.

That means $AQ = CQ = BC = AB = 2$ cm.



Since ABCD is a parallelogram, $BC \parallel AD$. Therefore,

$$\angle BRA = \angle CBR = 36^\circ$$

Therefore in $\triangle ABR$, $\angle BAR = \angle BRA = 36^\circ$

and

$$\angle ABR = 108^\circ.$$

Now in $\triangle ABR$, we can apply 36° - 72° - 36° theorem.

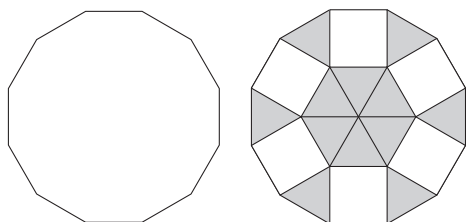
$$\therefore AR = \frac{(1 + \sqrt{5})}{2} AB$$

$$\Rightarrow AR = 1 + \sqrt{5}$$

$$\text{Thus, } QR = AR - AQ = (1 + \sqrt{5}) - 2 = \sqrt{5} - 1$$

Hence choice (c) is the correct one.

123 The given dodecagon can be partitioned into 12 congruent equilateral triangles and 6 congruent squares as shown below,



The area of each equilateral triangle

$$= \frac{\sqrt{3}}{4} \times 1^2 = \frac{\sqrt{3}}{4} \text{ sq cm.}$$

And, the area of each square = $1^2 = 1$ sq cm

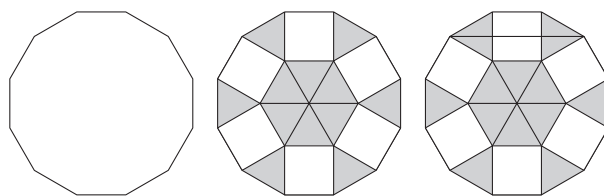
Therefore, the required total area of dodecagon

$$= 12 \times \frac{\sqrt{3}}{4} + 6 \times 1 = (3\sqrt{3} + 6) \text{ sq cm}$$

Hence choice (b) is the correct one.

124 The given dodecagon can be partitioned into 12 congruent equilateral triangles and 6 congruent squares as shown below.

Now, the length of the second smallest diagonal = (2 × height of the equilateral triangle) + (side of the square)



The height of each equilateral triangle

$$= \frac{\sqrt{3}}{2} \times 1 = \frac{\sqrt{3}}{2} \text{ cm}$$

And, the side of each square = 1 cm.

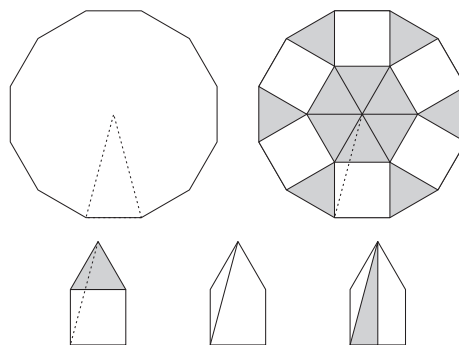
Therefore, the required total length of the diagonal

$$= 2 \times \frac{\sqrt{3}}{2} + 1 = (\sqrt{3} + 1) \text{ cm}$$

Hence choice (b) is the correct one.

125 The given dodecagon can be partitioned into 12 congruent equilateral triangles and 6 congruent squares as shown below.

Now, from the following figures the length of the circumradius = length of the diagonal of the pentagon.



The length of the diagonal of the pentagon

$$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(1 + \frac{\sqrt{3}}{2}\right)^2} = \sqrt{2 + \sqrt{3}}$$

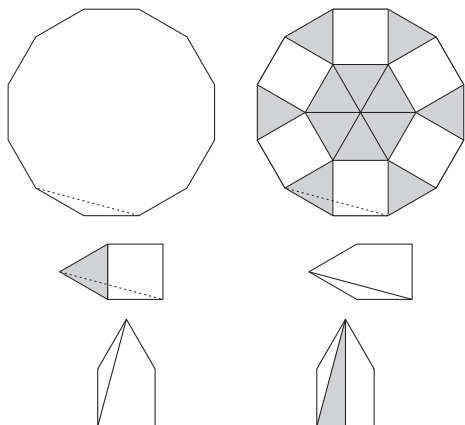
And, the side of each square = 1 cm

Therefore, the required total length of the circumradius = $\sqrt{2 + \sqrt{3}}$ cm

Hence choice (a) is the correct one.

- 126** The given dodecagon can be partitioned into 12 congruent equilateral triangles and 6 congruent squares as shown below.

Now, from the following figures the length of the smallest diagonal = length of the diagonal of the pentagon.



The length of the diagonal of the pentagon

$$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(1 + \frac{\sqrt{3}}{2}\right)^2}$$

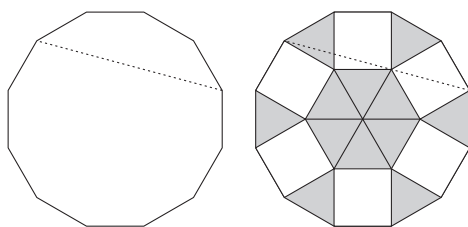
$$= \sqrt{2 + \sqrt{3}}$$

Therefore, the required total length of the diagonal

$$= (\sqrt{2 + \sqrt{3}}) \text{ cm}$$

Hence choice (a) is the correct one.

- 127** The given dodecagon can be partitioned into 12 congruent equilateral triangles and 6 congruent squares as shown below. Now, from the following figures the length of the third smallest diagonal = length of the diagonal of the pentagon + length of the diagonal of the square.



The length of the diagonal of the pentagon

$$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(1 + \frac{\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{2 + \sqrt{3}}$$

And, the length of the diagonal of square = $\sqrt{2}$

Therefore, the required total length of the diagonal

$$= (\sqrt{2 + \sqrt{3}}) + \sqrt{2} \text{ cm}$$

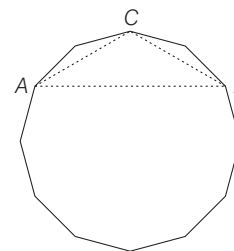
Hence choice (d) is the correct one.

- Alternatively** In the following diagram of a regular dodecagon $\angle ACE = 120^\circ$ and $\angle CAE = \angle CEA = 30^\circ$

Since each side of the dodecagon is 1 cm, so the smallest diagonal will be $\sqrt{2 + \sqrt{3}}$ cm. That is

$$AC = CE = \sqrt{2 + \sqrt{3}} \text{ cm.}$$

Therefore, using 30-30-120 degree theorem in $\triangle ACE$, we have $AE = \sqrt{3}(\sqrt{2 + \sqrt{3}})$



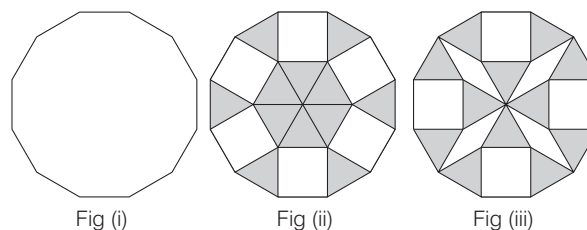
- 128** Look at the following figures carefully. Area of each dodecagon is same.

From the fig (ii) it is clear that the total area of the dodecagon = area of 12 equilateral triangles

+ area of 6 squares.

But, from fig (iii), the area of the dodecagon = area of 12 equilateral triangles + area of 4 squares + area of 4 kites

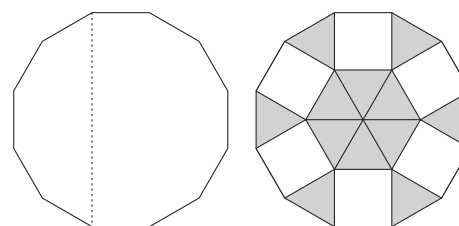
Therefore, area of 4 kites = area of 2 squares = 2 sq cm



Hence choice (a) is the correct one.

- 129** The given dodecagon can be partitioned into 12 congruent equilateral triangles and 6 congruent squares as shown below.

Now, from the following figures the length of the second longest diagonal = $2 \times$ length of the side of the square + $2 \times$ height of the equilateral triangle



The length of the diagonal of the dodecagon

$$= 2 \times 1 + 2 \times \frac{\sqrt{3}}{2}$$

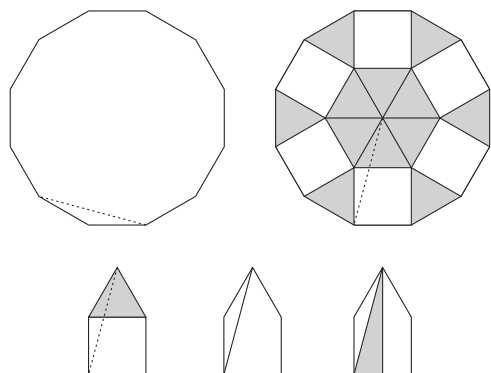
Therefore, the required total length of the diagonal

$$= 2 + \sqrt{3} \text{ cm}$$

Hence choice (c) is the correct one.

- 130** The given dodecagon can be partitioned into 12 congruent equilateral triangles and 6 congruent squares as shown below.

Now, from the following figures the length of the circumradius = length of the diagonal of the pentagon.



The length of the diagonal of the pentagon

$$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(1 + \frac{\sqrt{3}}{2}\right)^2} = \sqrt{2 + \sqrt{3}}$$

Therefore, the total length of the circumradius

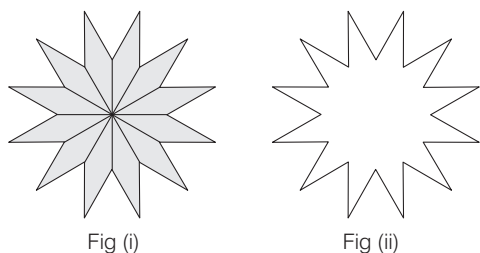
$$= \sqrt{2 + \sqrt{3}} \text{ cm}$$

Thus, the required length of the longest diagonal

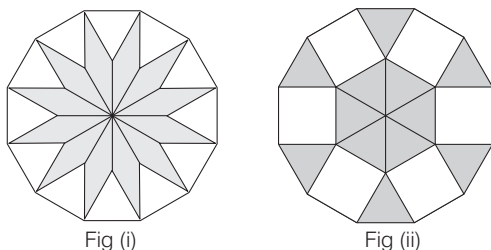
$$= 2 \times \text{length of the circumradius} = 2\sqrt{2 + \sqrt{3}} \text{ cm}$$

Hence (a) is the correct one.

- 131** Given that the perimeter of the figure is 24 cm, so each side of the each rhombus would be 1 cm. Fig (ii) shows the perimeter of the fig (i)



Now, when you connect all the outer (disjoint) vertices of the given figure you will get a regular dodecagon and each side of the dodecagon would be 1 cm, as there would be 12 equilateral triangles formed along the edge of the dodecagon.



Now, consider fig (iii) and fig (iv); each of them has equal area.

However, dodecagon in fig (iii) has 12 equilateral triangles and 12 rhombi.

And, dodecagon in fig (iv) has 12 equilateral triangles and 6 squares.

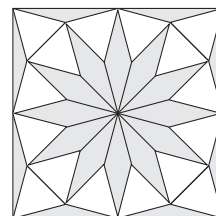
That means the area of 12 rhombi in fig (iii) is same as the total area of the 6 squares in fig (iv)

Therefore, the required area of 12 rhombi = total area of 6 squares = 6 sq cm

Hence choice (a) is the correct one.

- 132** From the following figure, it can be seen that the dodecagon consists of 12 equilateral triangles and 12 rhombi

And the square consists of 16 equilateral triangles and 16 rhombi.



Therefore, the required ratio

$$= \frac{12 (\text{equilateral triangles} + \text{rhombi})}{16 (\text{equilateral triangles} + \text{rhombi})} = \frac{3}{4}$$

Hence choice (a) is the correct one.

Hint : There are 8 isosceles triangles between the dodecagon and the square. These 8 triangles form the 4 rhombi. And each such rhombus is congruent with the each of the rhombi inside the dodecagon .

- 133** The inner dodecagon consists of 12 equilateral triangles and 12 rhombi, while the external consists of 48 equilateral triangles and 48 rhombi

Therefore, the required ratio

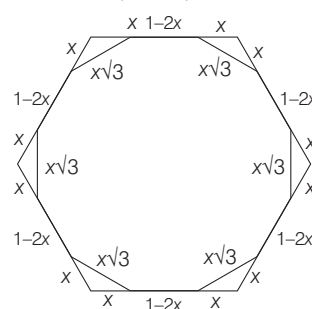
$$= \frac{12 (\text{equilateral triangles} + \text{rhombi})}{48 (\text{equilateral triangles} + \text{rhombi})} = \frac{1}{4}$$

Hence choice (b) is the correct one.

- 134** Let x be the length that you have to cut off from each corner, then the length of each side of hexagon = 1

The length of each side of the dodecagon

$$= 1 - (x + x) = 1 - 2x$$



But, from the corner triangle which is an isosceles triangle with the angles $30^\circ, 30^\circ, 120^\circ$, you can determine that the third side of the corner triangle is $x\sqrt{3}$.

However, you can observe that the third (longer) side of the corner triangle is same as the side of the dodecagon.

$$\text{Therefore, } x\sqrt{3} = 1 - 2x$$

$$\Rightarrow (2 + \sqrt{3})x = 1$$

$$\Rightarrow x = \frac{1}{2 + \sqrt{3}}$$

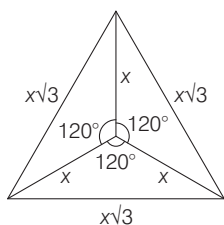
$$\Rightarrow x = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

$$\Rightarrow x = 2 - \sqrt{3}$$

$$\begin{aligned} \text{Now, the total area to be cut off} &= 6 \text{ isosceles triangles} \\ &= 2 \text{ equilateral triangles of side } x\sqrt{3} \\ &= 2 \times \frac{\sqrt{3}}{4} \times (x\sqrt{3})^2 = \frac{3\sqrt{3}}{2} x^2 \\ &= \frac{3\sqrt{3}}{2} (2 - \sqrt{3})^2 = \frac{21\sqrt{3}}{2} - 18 \end{aligned}$$

Hence choice (b) is the correct one.

NOTE When you join 3 isosceles triangles, which are congruent, whose 1 angle is 120° , you get an equilateral triangle, as depicted below.

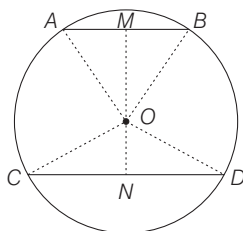


- 135** Since the distance between the two parallel chords is greater than the radius, so the chords will be lying on the opposite sides of the centre.

$$\text{Therefore, } \frac{\pi}{k} < \pi \Rightarrow k > 1$$

$$\text{And } \frac{2\pi}{k} < \pi \Rightarrow k > 2$$

Thus you can conclude that choices (a) and (b) are out of consideration.



In the given figure,

$$\angle AOB = \frac{\pi}{k}$$

$$\text{and } \angle COD = \frac{2\pi}{k}$$

Now, if you consider choice (c), that is $k = 3$, then

$$\angle AOB = \frac{\pi}{3} = 60^\circ$$

$$\text{and } \angle COD = \frac{2\pi}{3} = 120^\circ$$

Therefore $\angle AOB = \angle OAB = \angle OBA = 60^\circ$, so $OM = \sqrt{3}$

$$\text{And } \angle CON = 60^\circ$$

$\angle OCN = 30^\circ$, $\angle ONC = 90^\circ$, so using 30-60-90 degree theorem we get $ON = 1$.

$$\text{Therefore } MN = OM + ON = \sqrt{3} + 1$$

It implies that $k = 3$ is the valid answer. So

$$\lfloor k \rfloor = \lfloor 3 \rfloor = 3$$

Hence choice (c) is the correct one.

Alternatively

$$2 \cos \frac{\pi}{2k} + 2 \cos \frac{\pi}{k} = \sqrt{3} + 1$$

$$\text{Let } \frac{\pi}{2k} = \theta, \text{ Therefore}$$

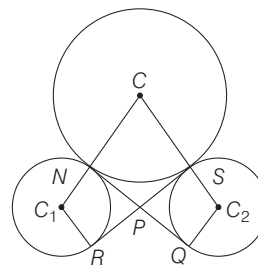
$$\begin{aligned} 2 \cos \theta + 2 \cos 2\theta &= \sqrt{3} + 1 \\ \Rightarrow 2 \cos \theta + 2(2 \cos^2 \theta - 1) &= \sqrt{3} + 1 \end{aligned}$$

$$\Rightarrow \cos \theta = \frac{-1 \pm (2\sqrt{3} + 1)}{4}$$

$$\Rightarrow \cos \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}$$

$$\Rightarrow \frac{\pi}{2k} = \frac{\pi}{6} \Rightarrow k = 3$$

- 136** Consider the following diagram with three distinct circles as shown below.



Since NQ and RS are tangents, so we have

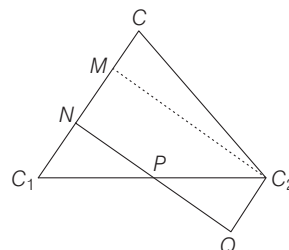
$$\begin{aligned} \angle C_1NQ &= \angle CNQ = \angle C_2SR = \angle CSR \\ &= \angle C_1RS = \angle C_2QN = 90^\circ \end{aligned}$$

$$\text{And, } C_1N = C_1R = C_2S = C_2Q = 1$$

$$\text{And, } C_1P = C_2P = 3$$

$$\therefore PN = PR = PS = PQ = \sqrt{3^2 - 1^2} = \sqrt{8}$$

Now, look at the following figure.



Now, let's draw a line C_2M parallel to NQ , then

$$\angle C_2MC = 90^\circ$$

$$\text{Therefore, } MN = C_2Q = 1.$$

Let r be the radius of circle C , then

$$MC_2 = \sqrt{(CC_2)^2 - (CM)^2}$$

$$\Rightarrow 2\sqrt{8} = \sqrt{(r+1)^2 - (r-1)^2} \Rightarrow r = 8$$

Hence choice (a) is the correct one.

NOTE $MC_2 = NQ = NP + NQ = \sqrt{8} + \sqrt{8} = 2\sqrt{8}$

And, $CC_2 = CS + SC_2 = r + 1$

Similarly, $CM = CN - MN = r - 1$

- 137** Since P, Q, S and R are concyclic, we have $\angle P = \angle R$ and $\angle Q = \angle S$.

Also, $\angle P = \angle S$ and $\angle Q = \angle R$, as $PQ \parallel RS$.

It implies that $\angle P = \angle Q = \angle R = \angle S$.

Now, $\angle P = \angle Q = \angle O = 180^\circ$

But, since $PQ > OP$, we have $\angle P = \angle Q < \angle O$.

$$\therefore \angle P + \angle Q + \angle O = 180^\circ$$

$$\Rightarrow 2(\angle P) + \angle O = 180^\circ$$

$$\Rightarrow 2(\angle P) + \angle P < 180^\circ$$

$$\Rightarrow 3(\angle P) < 180^\circ$$

$$\Rightarrow \angle P < 60^\circ \Rightarrow \angle P \leq 59^\circ$$

Hence choice (c) is the correct one.

- 138** Since BR and QR are the two tangents from a common point R to the same circle, therefore $BR = QR$

Now, since OB and OQ are the two radii of the same circle, therefore

$$OB = OQ$$

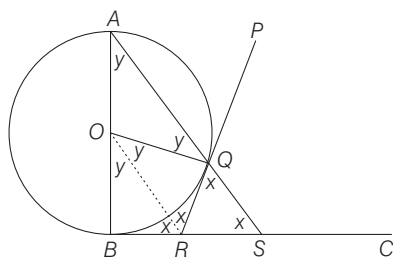
Thus, the quadrilateral $OBRQ$ is a kite.

$$\therefore \angle BOQ = 2(\angle BOR) = 2y \text{ (say)}$$

further, $\angle BOQ = 2(\angle BAQ) = 2y$

$$\Rightarrow \angle BOR = \angle BAQ$$

It implies that $OR \parallel AQ$



That means $\angle ORB = \angle ASB = \angle QSR = x$ (say)

Then, $\angle RQS = \angle ORQ = \angle ORB = x$

$$\therefore \angle RQS = \angle QSR = x$$

It implies that $RQ = RS$. Even $BR = RQ$, therefore $BR = RS$

Hence choice (b) is the correct one.

- 139** Since AB is parallel to CD , so we have

$$\frac{OP}{OS} = \frac{OQ}{OC} = \frac{8}{17}$$

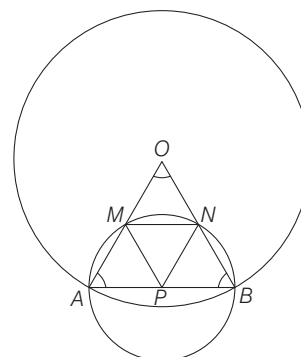
But, $OR = OC = 17$

It implies that $\frac{OQ}{OR} = \frac{8}{17}$

But, we have $\frac{OC}{OT} = \frac{OQ}{OR} = \frac{8}{17}$

Hence choice (d) is the correct one.

- 140** Triangle OAB is an equilateral triangle. Now consider the centre P of the smaller circle and join PM, PN and MN . Thus we see that the area of triangle OAB is divided into four equal parts.



The area of sector $PMN = \frac{1}{6} \pi (6)^2 = 6\pi$ sq. cm

Area of $\triangle AMP$ = Area of

$$\triangle PNB = \frac{\sqrt{3}}{4} (6)^2 = 9\sqrt{3} \text{ sq. cm}$$

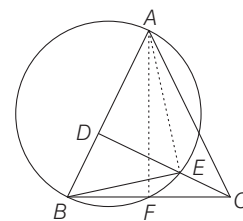
Total shaded area

$$= 6\pi + 2(9\sqrt{3}) = 6(\pi + 3\sqrt{3}) \text{ sq. cm}$$

Hence choice (a) is the correct one.

- 141** First of all join A to F . Now, you can see $\angle AFB = 90^\circ$, as it's an angle in a semicircle. That means AF is perpendicular on BC . But, since $AB = AC$, therefore AF will bisect BC . That is

$$BF = CF = \frac{2\sqrt{2}}{2} = \sqrt{2} \text{ cm.}$$



Now, $\triangle AEB$ is also a right angle triangle as $\angle AEB = 90^\circ$.

And, thus $\triangle DEB$ and $\triangle EAB$ are similar. Therefore,

$$\frac{BE}{BD} = \frac{AB}{BE}$$

$$\Rightarrow BE^2 = (BD)(AB)$$

But since $\triangle AFB$ and $\triangle CDB$ are similar. Therefore, we have

$$\frac{BD}{BC} = \frac{BF}{AB}$$

$$\Rightarrow \frac{BD}{BC} = \left(\frac{BC}{2} \right)$$

$$\Rightarrow BC^2 = 2(BD)(AB)$$

From eqs. (i) and (ii), we have

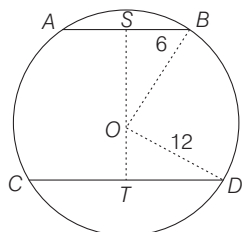
$$BC^2 = 2BE^2$$

$$\Rightarrow BE^2 = \frac{BC^2}{2} = \frac{(2\sqrt{2})^2}{2} = 4$$

$$\Rightarrow BE = 2 \text{ cm}$$

Hence choice (a) is the correct one.

- 142** Consider the following figure (i) in which let us assume that the radius is r . That is $OB = OD = r$, where O is the centre of the circle. As the chords AB and CD are parallel so the line passing through the centre of the circle will be perpendicular bisector of these chords. That is $AS = SB = 6$ and $CT = TD = 12$ and $\angle OSB = 90^\circ$ and $\angle OTD = 90^\circ$.



Now, using Pythagoras theorem in $\triangle OSB$, we have

$$OB^2 = OS^2 + SB^2$$

$$\Rightarrow r^2 = x^2 + 6^2 \quad \dots(i)$$

Similarly, using Pythagoras theorem in $\triangle OTD$, we have

$$OD^2 = OT^2 + TD^2$$

$$\Rightarrow r^2 = (18 - x)^2 + 12^2 \quad \dots(ii)$$

Now, comparing the eq. (i) and eq. (ii), we get

$$x^2 + 6^2 = (18 - x)^2 + 12^2$$

$$\Rightarrow x = 12$$

$$\Rightarrow OS = 12$$

$$\text{and } OT = 18 - 12 = 6$$

Now, substituting the value of x in eq. (i) or eq. (ii), we get the radius of the circle $r = 6\sqrt{5}$

Now, look at the figure (ii) in which $SM = MT = \frac{ST}{2} = 9$.

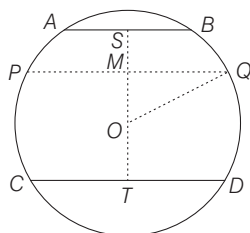
$$\therefore OM = OS - SM = 12 - 9 = 3$$

Then by using Pythagoras theorem in $\triangle OMQ$,

$$\text{we get } MQ = \sqrt{OQ^2 - OM^2}$$

$$\Rightarrow MQ = \sqrt{(6\sqrt{5})^2 - 3^2} \Rightarrow MQ = 3\sqrt{21}$$

$$\therefore PQ = 6\sqrt{21}$$

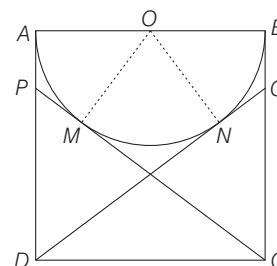


$$\text{That is } \sqrt{a} = 6\sqrt{21}$$

$$\text{Therefore, } a = 756$$

Hence choice (b) is the correct one.

- 143** Since, CP and DQ are tangents to the circle, so if O is the centre of the semi circle, then the two radii OM and ON will be perpendicular to CP and DQ , respectively, as shown in the diagram.



As DA and DN are the common tangents to the same circle, so $DA = DN = 4$ cm. Similarly, $CB = CM = 4$ cm.

Further, PA and PM are the common tangents to the same circle, so $PA = PM = x$ cm. Similarly, $BQ = NQ = x$ cm.

$$\text{Now, } DQ^2 = DC^2 + CQ^2$$

$$\text{or } (DN + NQ)^2 = DC^2 + (BC - BQ)^2$$

$$\text{or } (4 + x)^2 = 4^2 + (4 - x)^2$$

$$\text{or } x = 1 \text{ cm.}$$

It implies, $AP = BQ = 1$ cm and $DP = CQ = 3$ cm.

$$\begin{aligned} \text{Now, the area of } \triangle DRC &= \frac{1}{4} \text{ area of rectangle } PQCD \\ &= \frac{1}{4} (4 \times 3) = 3 \text{ sq. cm.} \end{aligned}$$

Hence choice (a) is the correct one.

Alternatively Go through options.

Choice (d): If area of triangle $DRC = 5$, then area of rectangle $PQCD = 4 \times 5 = 20$ sq cm. Since area of rectangle $PQCD$ is greater than the area of square $ABCD$, so it's an invalid choice.

Choice (a) : If area of triangle $DRC = 3$, then area of rectangle $PQCD = 4 \times 3 = 12$ sq cm. It implies that $CQ = 3$ cm and $BQ = 1$ cm.

Now, since $BQ = NQ = 1$ cm, therefore

$$DQ = DN + NQ = 5 \text{ cm as } DN = AD = 4 \text{ cm.}$$

As, $DQ^2 = DC^2 + CQ^2$, therefore the presumed choice (a) is valid one. However, when you consider the values of choices (b) and (c), the pythagorean relation between DC , QC and DQ can't be satisfied.

- 144** First of all drop a perpendicular MQ on CD . Now, join PQ , then PQ will be perpendicular to MC , as angle MPQ is an angle in a semicircle.

Since $QC = 40$ mm and $MQ = 80$ mm

$$\therefore MC = 40\sqrt{5} \text{ mm}$$

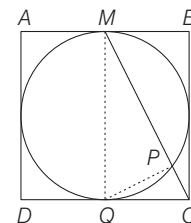
Now, using the similarity of triangles in $\triangle MPQ$ and $\triangle MQC$, we get

$$\frac{MP}{MQ} = \frac{MQ}{MC}$$

$$\Rightarrow \frac{MP}{80} = \frac{80}{40\sqrt{5}} \Rightarrow MP = 32\sqrt{5}$$

$$\begin{aligned} PC &= MC - MP = 40\sqrt{5} - 32\sqrt{5} \\ &= (40 - 32)\sqrt{5} = 8\sqrt{5} \end{aligned}$$

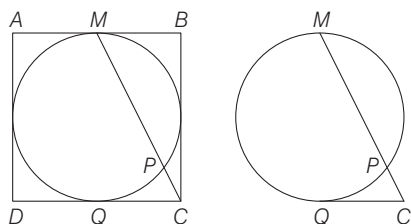
Hence choice (a) is the correct answer.



☞ Alternatively In the following figure, you can use the “power of a point theorem” as shown below

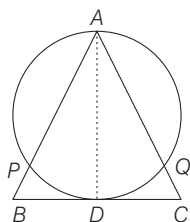
$$MC \times PC = QC^2$$

$$\Rightarrow 40\sqrt{5} \times PC = 40^2 \Rightarrow PC = 8\sqrt{5}$$



- 145** In the following figure, $\angle BAD = \angle DAC = 30^\circ$ and $\angle ABD = \angle ACD = 60^\circ$.

Now, using 30-60-90 degree theorem (or Pythagoras theorem) in $\triangle ADC$ we have $DC = 10$ cm and $AC = 20$ cm



Now, using Power of a point theorem in the above figure, we have $AC \times QC = DC^2$

$$\Rightarrow 20 \times QC = 10^2 \Rightarrow QC = 5 \text{ cm}$$

Hence choice (d) is the correct answer.

- 146** Consider the figure (i). Using the 30-60-90 degree theorem (or sine rule) in $\triangle ADC$ we get $AD = 12\sqrt{2}$, $CD = 4\sqrt{6}$ and $AC = 8\sqrt{6}$.

Therefore, area of triangle $ABC = 96\sqrt{3}$ sq cm.

$$(\triangle ABC = \frac{\sqrt{3}}{4} \times (8\sqrt{6})^2 = 96\sqrt{3})$$

$$\text{or } \triangle ABC = \frac{1}{2} \times 8\sqrt{6} \times 12\sqrt{2} = 96\sqrt{3}$$

Now, since the radius of the circle

$$= \frac{1}{2} \times AD = 6\sqrt{2} \text{ cm}$$

Therefore, area of the circle $= \pi(6\sqrt{2})^2 = 72\pi$

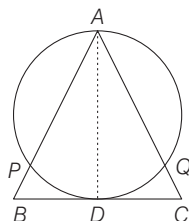


Fig (i)

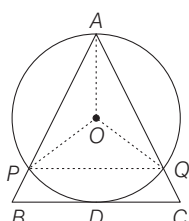


Fig (ii)

Now, consider the figure (ii). In the $\triangle PAQ$, $\angle PAQ = 60^\circ$, so $\angle POQ = 120^\circ$, as both the angles are formed by the same chord (PQ) on the same side of PQ, whereas one is the inscribed angle and other one is the central angle.

Further, since $AP = AQ$,

$$\therefore \angle APQ = \angle AQP = 60^\circ$$

Now, since $\angle APQ = \angle AQP = \angle PAQ = 60^\circ$, therefore triangle PAQ is an equilateral triangle.

(There are many ways to prove that $AP = AQ$. One approach is to observe that there is an obvious symmetry. And the other approach is to prove that $BP = CQ$ by applying Power of a Point theorem; since $BD = DC$ and $AB = AC$ therefore $BP = CQ$.)

Now, the side of the equilateral triangle PAQ is $6\sqrt{6}$ cm.

(To know that how you can find the side of triangle PAQ please refer the HINT at the end of the solution.)

Now, the area of triangle $PAQ = 54\sqrt{3}$

$$(\triangle PAQ = \frac{\sqrt{3}}{4} \times (6\sqrt{6})^2 = 54\sqrt{3})$$

$$\text{or } \triangle PAQ = \frac{1}{2} \times 6\sqrt{6} \times 9\sqrt{2} = 54\sqrt{3}$$

Now, consider the figure (iii), in which the area of common (or unshaded) region

= area of $(\triangle AOP + \triangle AOQ)$ + area of the sector $POQD$

= $\frac{2}{3}$ of area of triangle APQ + $\frac{1}{3}$ of area of the circle

$$= \frac{2}{3} \times 54\sqrt{3} + \frac{1}{3} \times 72\pi = 36\sqrt{3} + 24\pi$$

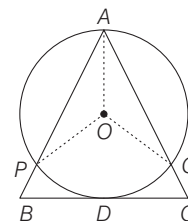


Fig (iii)

Thus, total area of the shaded region

= [total area of the circle – total area of the unshaded (or common) region] + [total area of the triangle ABC – total area of the unshaded (or common) region]

$$= [72\pi - (36\sqrt{3} + 24\pi)] + [96\sqrt{3} - (36\sqrt{3} + 24\pi)]$$

$$= 24(\pi + \sqrt{3})$$

Hence choice (a) is the correct answer.

Hint How to find the side of triangle PAQ ?

Method 1 : Consider the figure (iv), in which you can apply Power of Point theorem, as shown below.

$$CA \times CQ = CD^2$$

$$\Rightarrow 8\sqrt{6} \times CQ = (4\sqrt{6})^2$$

$$\Rightarrow CQ = 2\sqrt{6} \text{ cm}$$

$$\therefore AQ = AC - CQ$$

$$= 8\sqrt{6} - 2\sqrt{6} = 6\sqrt{6} \text{ cm}$$

Method 2 : Consider the figure (v), in which you can use 30-60-90 degree theorem (or cosine rule) and you will get $AP = AQ = PQ = \sqrt{3}(6\sqrt{2}) = 6\sqrt{6}$ cm.

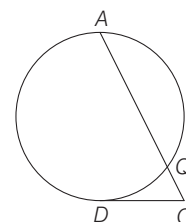


Fig (iv)



Now, using 30-60-90 degree theorem (or sine rule) you will have $AS = 9\sqrt{2}$, $SQ = 3\sqrt{6}$ and $AQ = 6\sqrt{6}$ cm.

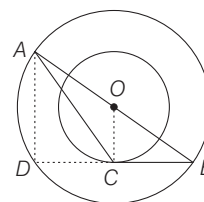


Thus we have, $\angle AOP = \angle AOQ = \angle POQ = 120^\circ$



= total area of the circle – total area of the unshaded region

147 First of all, extend BC to a point D on the larger circle and then join A with D and O with C . Now, since AB is the diameter of the larger circle, so $\triangle ADB$ is a right angle triangle and since BC is the tangent and OC is the radius of the smaller circle, so $\triangle OCB$ is a right angle triangle.

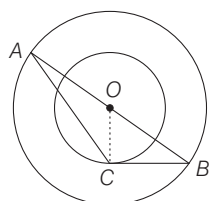


Hence choice (b) is the correct answer.

☞ Alternatively First of all join OC , which is the median of $\triangle ABC$, as $AO = BO$. Also, $\triangle OCB$ is a right angle triangle.

Therefore, using Pythagoras theorem, you can have

$$\begin{aligned} BC &= \sqrt{OB^2 - OC^2} \\ &= \sqrt{29^2 - 21^2} = 20 \text{ cm} \end{aligned}$$



Now, using Apollonius theorem in $\triangle ABC$, you can have

$$AC^2 + BC^2 = 2 \left(OC^2 + \left(\frac{AB}{2} \right)^2 \right)$$

$$\Rightarrow AC^2 + 20^2 = 2(21^2 + 29^2)$$

$$\Rightarrow AC^2 = 2164$$

$$\Rightarrow AC = 46.52 \text{ cm}$$

- 148** The best way is to go through the options considering the fact that the maximum area of the circle is πr^2 .

Choice (a) : Since $4r^2 > \pi r^2$, which is impossible, so this choice is invalid.

Choice (b) : Since d has no direct relationship with r , the radius of the circle, so d is interminable. Therefore we can not give the answer in terms of d alone. So, this choice is also an invalid one.

Choice (c) : If $r = d$, minimum area of the square = r^3 .

For $r > 2$, $2r^2 < r^3$, which means maximum area is less than the minimum area.

So, it is again an invalid choice. Essentially, the maximum area can be obtained only when the quadrilateral is a square and the minimum area can be obtained only when the quadrilateral is a square and the minimum area can be obtained only when the quadrilateral is a kite with its longer diagonal equal to the diameter of the circle and the smaller diagonal equal to $2r\sqrt{r^2 - d^2}$.

Hence choice (d) is the correct answer.

- 149** Consider the following figure. Let C_1, C_2 and C_3 be the centres of these circles, as shown in the figure. Let $AB = DC_3 = x$

$$\text{Therefore, } DC_3 = \sqrt{(C_1C_3)^2 - (DC_1)^2}$$

$$\Rightarrow x = \sqrt{(r_1 + r_3)^2 - (r_1 - r_3)^2}$$

$$\Rightarrow x = 2\sqrt{r_1 r_3}$$

$$\text{Now, let } BC = C_3E = y$$

$$\text{Therefore, } C_3E = \sqrt{(C_2C_3)^2 - (C_2E)^2}$$

$$\Rightarrow y = \sqrt{(r_2 + r_3)^2 - (r_2 - r_3)^2} \Rightarrow y = 2\sqrt{r_2 r_3}$$

$$\text{Thus, } AC = x + y = 2\sqrt{r_1 r_3} + 2\sqrt{r_2 r_3}$$

$$= 2(\sqrt{r_1 r_3} + \sqrt{r_2 r_3}) \quad \dots(i)$$

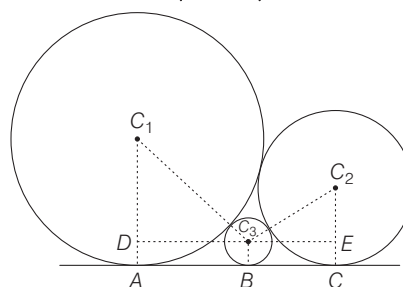


Fig (i)

Now, in the following figure (ii),

$$FC_2 = \sqrt{(C_1C_2)^2 - (FC_1)^2}$$

$$\Rightarrow x + y = \sqrt{(r_1 + r_2)^2 - (r_1 - r_2)^2}$$

$$\Rightarrow x + y = 2\sqrt{r_1 r_2} \quad \dots(ii)$$

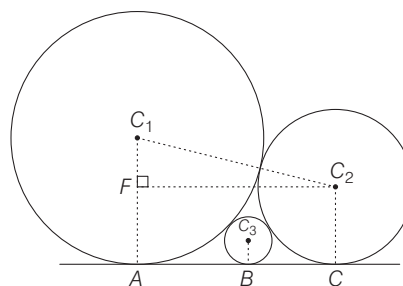


Fig (ii)

Now, comparing the equations (i) and (ii), we get

$$2(\sqrt{r_1 r_3} + \sqrt{r_2 r_3}) = 2\sqrt{r_1 r_2}$$

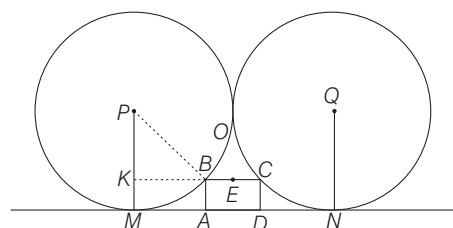
$$\Rightarrow \frac{\sqrt{r_1 r_3}}{\sqrt{r_1 r_2}} + \frac{\sqrt{r_2 r_3}}{\sqrt{r_1 r_2}} = 1$$

$$\Rightarrow \frac{\sqrt{r_3}}{\sqrt{r_2}} + \frac{\sqrt{r_3}}{\sqrt{r_1}} = 1$$

$$\Rightarrow \frac{1}{\sqrt{r_2}} + \frac{1}{\sqrt{r_1}} = \frac{1}{\sqrt{r_3}}$$

Hence choice (c) is the correct one.

- 150** Draw a line KB making it collinear with BC , such that K falls on PM . Looking at the information we can say that and $PK = PM - KM$ and $KB = KE - BE$, $PK^2 + KB^2 = PB^2$



$$\Rightarrow (r - x)^2 + \left(r - \frac{x}{2} \right)^2 = r^2$$

$$\Rightarrow 5x^2 - 12rx + 4r^2 = 0 \Rightarrow x = 0.4r$$

Hence choice (d) is the correct one.

151 The radius of circle $A = 1$ cm, so its curvature $= \frac{1}{1} = 1$

The radius of circle $B = \frac{1}{1}$ cm, so its curvature $= \frac{1}{1} = 1$

The radius of circle $C = \frac{1}{1}$ cm, so its curvature $= \frac{1}{1} = 1$

Now, using Descartes' circle equation theorem you will have

$$(a + b + c + d)^2 = 2(a^2 + b^2 + c^2 + d^2)$$

$$\Rightarrow (1 + 1 + 1 + d)^2 = 2(1^2 + 1^2 + 1^2 + d^2)$$

$$\Rightarrow (3 + d)^2 = 2(3 + d^2)$$

$$\Rightarrow d^2 - 6d - 3 = 0 \Rightarrow d = 3 + 2\sqrt{3}$$

$$\therefore \text{Radius of circle } D = \frac{1}{3 + 2\sqrt{3}} \text{ cm}$$

Hence choice (a) is the correct one.

Hint : To know how to obtain the value of d please refer Quadratic Equations in Quantum CAT.

152 The radius of circle $A = 1$ cm, so its curvature $= \frac{1}{1} = 1$

The radius of circle $B = \frac{1}{2}$ cm, so its curvature $= \frac{1}{\left(\frac{1}{2}\right)} = 2$

The radius of circle $C = \frac{1}{2}$ cm, so its curvature $= \frac{1}{\left(\frac{1}{2}\right)} = 2$

Now, using Descartes' circle equation theorem you will have

$$(a + b + c + d)^2 = 2((-a)^2 + b^2 + c^2 + d^2)$$

$$\therefore (-1 + 2 + 2 + d)^2 = 2((-1)^2 + 2^2 + 2^2 + d^2)$$

$$\Rightarrow (3 + d)^2 = 2(9 + d^2) \Rightarrow d^2 + 6d + 9 = 0$$

$$\Rightarrow (d + 3)^2 = 0 \Rightarrow d = -3$$

$$\therefore \text{Radius of circle } D = \frac{1}{3} \text{ cm}$$

Hence choice (b) is the correct one.

153 The radius of circle $P = 1$ cm, so its curvature $= \frac{1}{1} = 1$

The radius of circle $A = \frac{1}{2}$ cm, so its curvature $= \frac{1}{\frac{1}{2}} = 2$

The radius of circle $B = \frac{1}{2}$ cm, so its curvature $= \frac{1}{\frac{1}{2}} = 2$

Now, using Descartes' circle equation theorem you will have

$$(-p + a + b + c)^2 = 2((-p)^2 + a^2 + b^2 + c^2)$$

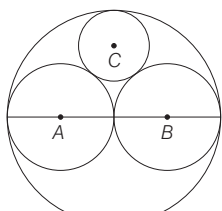


Fig (i)

$$\therefore (-1 + 2 + 2 + c)^2 = 2((-1)^2 + 2^2 + 2^2 + c^2)$$

$$\Rightarrow (3 + c)^2 = 2(9 + c^2)$$

$$\Rightarrow c^2 + 6c + 9 = 0$$

$$\Rightarrow (c + 3)^2 = 0 \Rightarrow c = -3$$

$$\therefore \text{Radius of circle } C = \frac{1}{3} \text{ cm}$$

So the curvature of the circle $C = 3$.

Now, consider the fig (ii), in which all the four circles are tangent to each other.

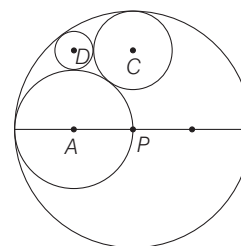


Fig (ii)

Now, using Descartes' circle equation theorem you will have

$$(-p + a + c + d)^2 = 2((-p)^2 + a^2 + c^2 + d^2)$$

$$\therefore (-1 + 2 + 3 + d)^2 = 2((-1)^2 + 2^2 + 3^2 + d^2)$$

$$\Rightarrow (4 + d)^2 = 2(14 + d^2)$$

$$\Rightarrow d^2 - 8d + 12 = 0 \Rightarrow d = 6 \text{ or } 2$$

$$\therefore \text{Radius of circle } D = \frac{1}{6} \text{ cm or } \frac{1}{2} \text{ cm. But, since } \frac{1}{2} \text{ cm is}$$

the radius of circle A , so it has to be less than $\frac{1}{2}$,

therefore, the required radius of circle $D = \frac{1}{6}$ cm.

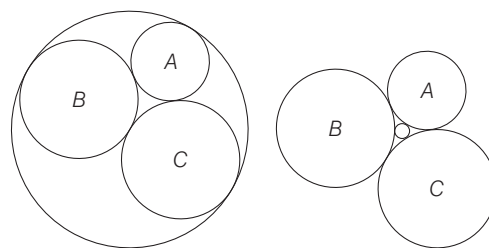
Hence choice (a) is the correct one.

154 From the statement given in the problem it is obvious that all the four circles are tangent to each other. Then, without any doubt, you can apply Descartes' circle theorem in order to get the radius of the circle.

Let a , b and c be the curvatures of the existing circles and d be the curvature of fourth circle, then

$$(a + b + c + d)^2 = 2(a^2 + b^2 + c^2 + d^2)$$

$$\therefore \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + d\right)^2 = 2\left(\left(\frac{1}{1}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2 + d^2\right)$$



$$\Rightarrow \left(\frac{11}{6} + d\right)^2 = 2\left(\frac{49}{36} + d^2\right)$$

$$\Rightarrow 36d^2 - 132d - 23 = 0$$

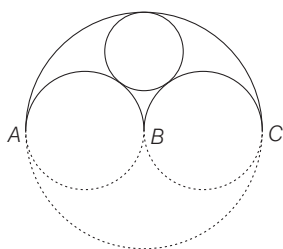
$$\Rightarrow d = -\frac{1}{6} \text{ or } \frac{23}{6}$$

The minus sign indicates that the circle is externally tangent.

$$\therefore \text{Radius of circle } D = 6 \text{ cm or } \frac{6}{23} \text{ cm.}$$

Hence choice (d) is the correct one.

- 155** First of all, complete all the semicircles as shown in the following figure. Now, since all the four circles are tangent to each other, so you can apply the Descartes' circle theorem in order to get the radius of the desired circle.



Let radius of the circle of diameter AC be r_1 ,

$$\text{then } r_1 = \frac{1}{2} \text{ cm}$$

So the curvature of the concerned circle $a = -2$

(Since the circle is circumscribing, that's why minus sign is applied)

And, let radius of the circles of diameter AB be r_2 , then

$$r_2 = \frac{1}{6} \text{ cm}$$

So the curvature of the concerned circle $AB = 6$

And, let radius of the circle of diameter BC be r_3 , then

$$r_3 = \frac{1}{3} \text{ cm}$$

So the curvature of the concerned circle $BC = 3$

Let a , b and c be the curvatures of the semicircles and d be the curvature of desired circle, then

$$(a + b + c + d)^2 = 2(a^2 + b^2 + c^2 + d^2)$$

That is

$$(-2 + 6 + 3 + d)^2 = 2((-2)^2 + 6^2 + 3^2 + d^2)$$

$$\Rightarrow (7 + d)^2 = 2(49 + d^2)$$

$$\Rightarrow d^2 + 14d + 49 = 0$$

$$\Rightarrow d = 7$$

$$\Rightarrow r_4 = \frac{1}{7} \text{ cm}$$

Hence choice (a) is the correct one.

TEST OF LEARNING

- 1** Use cosine rule in the given triangle,

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow \cos \frac{\pi}{6} = \frac{(x^2 + x + 1)^2 + (x^2 - 1)^2 - (2x + 1)^2}{2(x^2 + x + 1)(x^2 - 1)}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{2x^2 + 2x - 1}{2(x^2 + x + 1)}$$

$$\Rightarrow x = (1 + \sqrt{3}), -(2 + \sqrt{3})$$

Hence choice (d) is the answer.

- 2** The roots will be real when discriminant $D \geq 0$. That is

$$[2(a + b + c)]^2 - 12\lambda(ab + bc + ac) \geq 0$$

$$\Rightarrow (a + b + c)^2 \geq 3\lambda(ab + bc + ac)$$

$$\Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) \geq 3\lambda(ab + bc + ac)$$

$$\Rightarrow \frac{a^2 + b^2 + c^2 + 2(ab + bc + ca)}{3(ab + bc + ca)} \geq \lambda$$

$$\Rightarrow \lambda \leq \frac{a^2 + b^2 + c^2}{3(ab + bc + ca)} + \frac{2}{3}$$

Now, using the fundamental property of existence of a triangle; the sum of any two sides of a triangle is always greater than the third side. So

$$a + b > c \Rightarrow |c - b| < a$$

$$\Rightarrow b^2 + c^2 - 2bc < a^2 \quad \dots(i)$$

$$\Rightarrow b + c > a \Rightarrow |a - c| < b$$

$$\Rightarrow a^2 + c^2 - 2ac < b^2 \quad \dots(ii)$$

$$\Rightarrow c + a > b \Rightarrow |b - a| < c$$

$$\Rightarrow a^2 + b^2 - 2ab < c^2 \quad \dots(iii)$$

From eqs. (i), (ii) and (iii)

$$2(a^2 + b^2 + c^2) - 2(ab + bc + ca) < (a^2 + b^2 + c^2)$$

$$\Rightarrow \frac{a^2 + b^2 + c^2}{(ab + bc + ca)} < 2$$

$$\Rightarrow \frac{a^2 + b^2 + c^2}{3(ab + bc + ca)} < \frac{2}{3} \quad \dots(iv)$$

$$\therefore \lambda \leq \frac{a^2 + b^2 + c^2}{3(ab + bc + ca)} + \frac{2}{3}$$

$$\Rightarrow \lambda < \frac{2}{3} + \frac{2}{3}$$

$$\Rightarrow \lambda < \frac{4}{3}$$

Hence choice (a) is the answer.

NOTE If $a < 2$ and $b \leq 2$, $a = b < 4$ is correct and $a + b \leq 4$ is wrong.

- 3** As we know that, in a triangle the sum of the two sides is always greater than the third side, so we have the following 3 relations.

$$a + b > c \Rightarrow ac + bc > c^2$$

$$b + c > a \Rightarrow ab + ac > a^2$$

$$c + a > b \Rightarrow bc + ab > b^2$$

Adding the above 3 inequations, we get

$$2(ab + bc + ca) > a^2 + b^2 + c^2$$

$$\Rightarrow a^2 + b^2 + c^2 < 2(ab + bc + ca)$$

$$\Rightarrow \frac{a^2 + b^2 + c^2}{ab + bc + ca} < 2$$

Now, we know that the lengths of each side of triangle are always real numbers and if ab and c are real numbers then

$$(a - b)^2 + (b - c)^2 + (c - a)^2 \geq 0$$

$$\Rightarrow 2(a^2 + b^2 + c^2) - 2(ab + bc + ca) \geq 0$$

$$\Rightarrow a^2 + b^2 + c^2 \geq ab + bc + ca$$

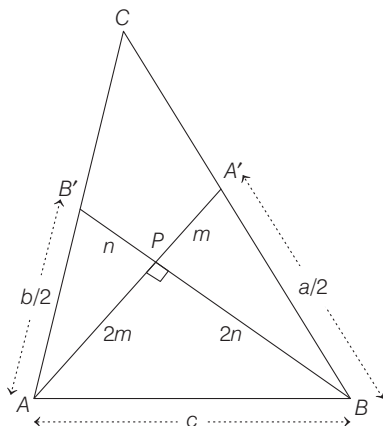
$$\Rightarrow \frac{a^2 + b^2 + c^2}{ab + bc + ca} \geq 1$$

Therefore, we can conclude that $1 \leq \frac{a^2 + b^2 + c^2}{(ab + bc + ca)} < 2$

$$\Rightarrow 1 \leq k < 2$$

Hence choice (a) is correct.

- 4 Since medians intersect each other in the ratio of 1 : 2, therefore $B'P = n$, $PB = 2n$ and $A'P = m$, $PA = 2m$.



$$\text{In } \triangle A'PB, m^2 + (2n)^2 = \left(\frac{a}{2}\right)^2 \Rightarrow m^2 + 4n^2 = \frac{a^2}{4}$$

$$\text{In } \triangle B'PA, n^2 = (2m)^2$$

$$\left(\frac{b}{2}\right)^2 \Rightarrow n^2 + 4m^2 = \frac{b^2}{4}$$

$$\therefore 5(m^2 + n^2) = \frac{(a^2 + b^2)}{4}$$

$$\text{Now, in } \triangle APB, (2m)^2 + (2n)^2 = c^2$$

$$\Rightarrow 4(m^2 + n^2) = c^2$$

$$\therefore c^2 = 4(m^2 + n^2) = \frac{a^2 + b^2}{5}$$

$$\Rightarrow 5c^2 = a^2 + b^2$$

Now, using the cosine rule in $\triangle ABC$, we have

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{a^2 + b^2 - \left(\frac{a^2 + b^2}{5}\right)}{2ab} = \frac{2(a^2 + b^2)}{5ab}$$

As a and b both are positive, so $0 < \cos C < 1$

$$\therefore \frac{2(a^2 + b^2)}{5ab} < 1 \Rightarrow 2a^2 - 5ab + 2b^2 < 0$$

Now, for a moment assuming the above inequality as the equality and then solving the quadratic equation in b , we get $b = \frac{a}{2}$, $2a$. That is $\frac{a}{2} < b < 2a$ is the requisite condition for the triangle's existence. Hence choice (d) is the correct one.

Alternatively Let us consider $AP = 6$, $BP = 8$, $AB = 10$, as $\triangle APB$ is a right angle triangle. Since, AA' and BB' are medians intersecting at P , so $A'P = 3$ and $B'P = 4$.

$$\text{Now, } AB' = \sqrt{AP^2 + B'P^2} = \sqrt{52}$$

$$\text{and } BA' = \sqrt{BP^2 + A'P^2} = \sqrt{73}$$

$$\therefore b = AC = 2\sqrt{52} \text{ and } a = BC = 2\sqrt{73}$$

$$\text{and } c = AB = 10$$

$$\text{Option (a) : } a^2 + b^2 = 4(52) + 4(73)$$

$$= 4(52 + 73) = 500 = 5(100) = 5c^2.$$

$$\text{Option (b) : } \frac{a}{2} = \sqrt{73}, 2a = 4\sqrt{73}, b = 2\sqrt{52}. \text{ It shows that}$$

$$\frac{a}{2} < b < 2a. \text{ Therefore option (b) is also correct.}$$

Hence option (d) is valid.

- 5 $\therefore AM \geq GM$

$$\frac{\frac{z}{x} + \frac{z}{y}}{2} \geq \sqrt{\left(\frac{z}{x}\right)\left(\frac{z}{y}\right)}$$

$$\frac{z}{x} + \frac{z}{y} \geq 2\sqrt{\left(\frac{z}{x}\right)\left(\frac{z}{y}\right)}$$

$$\frac{z}{x} + \frac{z}{y} \geq 2\sqrt{\frac{z^2}{xy}}$$

$$\frac{z}{x} + \frac{z}{y} \geq 2\sqrt{\frac{x^2 + y^2}{xy}}$$

$$\frac{z}{x} + \frac{z}{y} \geq 2\sqrt{\frac{x}{y} + \frac{y}{x}}$$

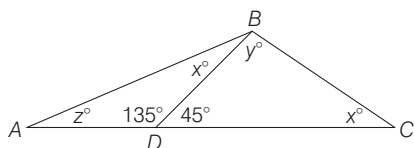
$$\frac{z}{x} + \frac{z}{y} \geq 2\sqrt{2} \quad \left[\because \left(\frac{y}{x} + \frac{x}{y}\right) \geq 2 \right]$$

Hence choice (d) is the correct one.

- 6 Since BD is a median, so $AD = DC$. Let us say

$AD = DC = 1$, and $\angle BAC = z^\circ$. Further, let us assume that $\angle DBC = y^\circ$, therefore $\angle DCB + \angle DBC = x + y = 135^\circ$. It implies that $\angle ABC = 135^\circ$. Then it is obvious that $\triangle ABC \sim \triangle ADB$.

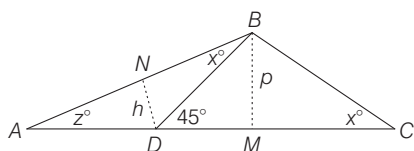
That is $\frac{AC}{AB} = \frac{AB}{AD}$
 $\Rightarrow \frac{2}{AB} = \frac{AB}{1} \Rightarrow AB = \sqrt{2}$



Now, we know that a median divide that total area of a triangle in two equal parts, therefore, $\Delta ABD = \Delta DBC$

Let us drop a perpendicular h on AB from D in ΔABD and a perpendicular p on DC from B in ΔDBC . Then we have, $\Delta ABD = \Delta DBC$

$$\Rightarrow \frac{1}{2} \times AB \times h = \frac{1}{2} \times CD \times p \Rightarrow \sqrt{2}h = p$$



As we see that in a right angle triangle ΔBMD , the hypotenuse $BD = 2h = 2ND$, therefore from 30-60-90 degree theorem $\angle NBD = 30^\circ$. That is $x = 30^\circ$.

Hence choice (b) is the correct one.

Ⓢ Alternatively Extend CB to E such that ΔAEC is a right angle triangle and AC is its hypotenuse, then $AD = ED = DC$, as AD, ED, DC will act as the radii of the circle, say it is r .

Now, as $\angle EBD$ is the external angle of ΔDBC , so $\angle EBD = (45 + x)^\circ$

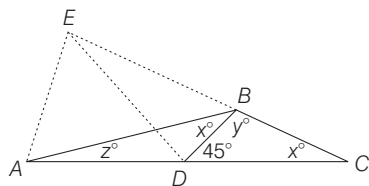
But, $\angle EBD = \angle EBA + \angle ABD$

$$\Rightarrow 45 + x = \angle EBA + x \Rightarrow \angle EBA = 45^\circ$$

Now, since in ΔAEB , $\angle AEB = 90^\circ$ and $\angle EBA = 45^\circ$, therefore $\angle EAB = 45^\circ$.

That is ΔAEB is an isosceles right angled triangle. It implies that $EA = EB$.

Let us assume $EA = EB = p \Rightarrow AB = \sqrt{2}p$



Now, since ΔABC is similar to ΔADB , therefore

$$\frac{AC}{AB} = \frac{AB}{AD} \Rightarrow \frac{2r}{\sqrt{2}p} = \frac{\sqrt{2}p}{r}$$

$$\Rightarrow r = p$$

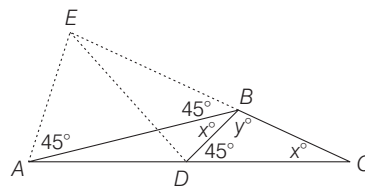
$$\therefore AE = EB = ED = AD = DC = r$$

That is in right angle triangle ΔAEC , we have $AC = 2AE$.

Therefore in ΔAEC by 30-60-90 degree theorem,

$$\angle ECA = 30^\circ \Rightarrow x = 30^\circ$$

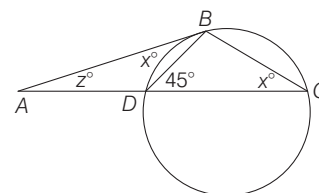
Ⓢ Alternatively Extend CB to E such that ΔAEB is a right angle triangle and AC is its hypotenuse, then $AD = ED = DC$, as AD, ED, DC will act as the radii of the circle, say it is r . In ΔDBC , we have $x + y = 135^\circ$, therefore in ΔABC we have $\angle ABC = 135^\circ$. It implies that $\angle EBA = 45^\circ$.



Thus we see that in right angle ΔEBA we have $\angle AEB = 90^\circ$, $\angle EBA = 45^\circ$ and $\angle EAB = 45^\circ$.

Therefore, $AE = EB = r$.

Ⓢ Alternatively Since $\angle BCD = \angle DBA$, therefore we can conclude that the angles in the alternate segment are same. It implies that AB is a tangent to a circle that passes through the points D, B and C .



Therefore, $AC \times AD = AB^2$. That is $2AD \times AD = AB^2$.

$$\Rightarrow AB = \sqrt{2}AD$$

Now the rest of the solution can be taken from any of the previous solutions to the same problem.

7 Applying 36° - 72° - 36° theorem, in triangle JFM , we have

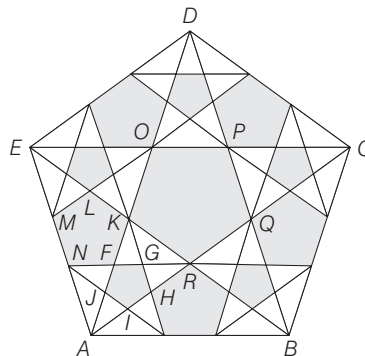
$$\frac{FM}{FJ} = \frac{1 + \sqrt{5}}{2}$$

Since each side of the pentagon $FGHIJ$ is 2 cm, therefore each side of the pentagon $FKLMN$ would be $1 + \sqrt{5}$ cm.

We know that $KR = FR = FG + GR = FG + \frac{1 + \sqrt{5}}{2} FG$. That

$$\text{is } \frac{KR}{FG} = \frac{3 + \sqrt{5}}{2}$$

Since each side of the pentagon $FGHIJ$ is 2 cm, therefore each side of the pentagon $KOPQR$ would be $3 + \sqrt{5}$ cm.



Therefore, total shaded area

$$\begin{aligned}
 &= 5 \left(\frac{1}{4} (\sqrt{5(5+2\sqrt{5})})^2 \right) + 5 \left(\frac{1}{4} (\sqrt{5(5+2\sqrt{5})}) (1 + \sqrt{5})^2 \right) \\
 &\quad + \left(\frac{1}{4} (\sqrt{5(5+2\sqrt{5})}) \right) (3 + \sqrt{5})^2 \\
 &= \frac{1}{4} (\sqrt{5(5+2\sqrt{5})}) [20 + (30 + 10\sqrt{5}) + (14 + 6\sqrt{5})] \\
 &= (\sqrt{5(5+2\sqrt{5})}) (16 + 4\sqrt{5}) \text{ cm}^2
 \end{aligned}$$

Hence choice (a) is the correct one.

- 8 Given that $AB = BC = CD = DA = a$

Let us consider $a = 12$ cm.

Now in triangle HAB and PGB ,

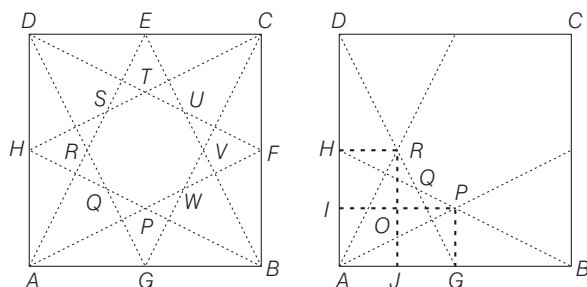
$$\frac{AH}{AB} = \frac{PG}{GB} = \frac{1}{2}$$

$\therefore AH = AG = GB = 6$ and $PG = 3$

Similarly, in triangle DAG and DHR ,

$$\frac{AG}{AD} = \frac{HR}{HD} = \frac{1}{2}$$

$\therefore AG = AH = DH = 6$ and $HR = 3$



Therefore, $RO = OP = 3$

Now, in triangle HIP and ROP ,

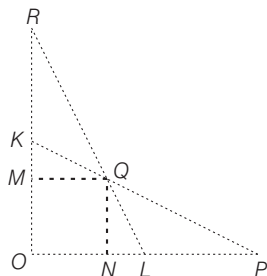
$$\frac{HI}{IP} = \frac{KO}{OP} = \frac{1}{2}$$

$\therefore OP = 3, OK = 1.5, KR = 1.5$

Similarly, in triangle GJR and POR ,

$$\frac{GJ}{JR} = \frac{OL}{OR} = \frac{1}{2}$$

$\therefore OR = 3, OL = 1.5, LP = 1.5$



Now, if we assume $KM = x$, then $MQ = 2x$

But $MQ = QN = 2x$

$\therefore NP = 4x$ and $LP = 3x$, as $NL = KM = x$

But $NL = KM = x$

and

$$PQ = \sqrt{QN^2 + NP^2}$$

$\Rightarrow$

$$PQ = \sqrt{(2x)^2 + (4x)^2} = 2x\sqrt{5}$$

$\therefore$

$$KM = x \Rightarrow OP = 6x$$

$\Rightarrow$

$$AB = 24x = 12$$

$\Rightarrow$

$$x = \frac{1}{2}$$

$\therefore$

$$PQ = 2 \times \frac{1}{2} \times \sqrt{5} = \sqrt{5}$$

Thus when $AB = 12, PQ = \sqrt{5}$, so when $AB = a, PQ = \frac{\sqrt{5}a}{12}$

There are 8 internal angles in the octagon, such that $\angle P = \angle R = \angle T = \angle V$ and $\angle Q = \angle S = \angle U = \angle W$. That is half of the internal angles are same of one value and rest half are of another value. So it's not an equiangular octagon, so it's not a regular octagon too.

However, each side of the octagon is same, so it's an equilateral octagon.

Thus statements (ii) and (iv) are correct.

Hence choice (b) is the correct one.

Alternatively Using the coordinate geometry to find the answer.

Let the coordinate of point A, B, C, D be

$(0, 0), (a, 0), (a, a), (0, a)$ respectively, then AF is $y = \frac{x}{2}$

and BH is $\frac{x}{a} + \frac{2y}{a} = 1$.

Therefore P is $\left(\frac{a}{2}, \frac{a}{4}\right)$

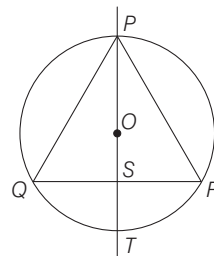
Similarly, DG is $\frac{2x}{a} + \frac{y}{a} = 1$

Equating BH and DG we get the coordinates of $Q = \left(\frac{a}{3}, \frac{a}{3}\right)$

$$\therefore PQ = \sqrt{\left(\frac{a}{2} - \frac{a}{3}\right)^2 + \left(\frac{a}{4} - \frac{a}{3}\right)^2} = \frac{\sqrt{5}a}{12}$$

Hence choice (b) is the correct one.

- 9 Considering the given conditions, the best you can do is assume an equilateral triangle, say of each side $2\sqrt{3}$. Then, from the following figure, $QS = SR = \sqrt{3}$ and $PS = 3, PO = 2, OS = 1$ and $ST = 1$.



$$\begin{aligned}
 \text{(i) } \frac{1}{PS} + \frac{1}{ST} &< \frac{2}{\sqrt{QS \times SR}} \\
 &\Rightarrow \frac{4}{3} < \frac{2}{3} \text{ That is invalid.}
 \end{aligned}$$

$$(ii) \frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}}$$

$$\Rightarrow \frac{4}{3} > \frac{2}{3}. \text{ That is valid.}$$

$$(iii) \frac{1}{PS} + \frac{1}{ST} < \frac{4}{QR}$$

$$\Rightarrow \frac{4}{3} < \frac{2\sqrt{3}}{3}. \text{ That is invalid.}$$

$$(iv) \frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$$

$$\Rightarrow \frac{4}{3} > \frac{2\sqrt{3}}{3}. \text{ That is valid.}$$

Hence choice (b) is the answer.

☛ Alternatively

Applying the theorem, you will get, $PS \times ST = QS \times SR$

Now, apply the principle of $AM \geq GM$ with $\frac{1}{PS} + \frac{1}{ST}$ as shown below.

$$\Rightarrow \frac{\frac{1}{PS} + \frac{1}{ST}}{2} \geq \sqrt{\frac{1}{PS} \times \frac{1}{ST}}$$

$$\Rightarrow \frac{1}{PS} + \frac{1}{ST} \geq \frac{2}{\sqrt{QS \times SR}}$$

$$\text{Similarly, } \frac{QS + SR}{2} \geq \sqrt{QS \times SR}$$

$$\Rightarrow \frac{QR}{2} \geq \sqrt{QS \times SR}$$

$$\Rightarrow \frac{1}{\sqrt{QS \times SR}} \geq \frac{2}{QR}$$

$$\Rightarrow \frac{1}{PS} + \frac{1}{ST} \geq \frac{4}{QR}$$

Therefore, you will have

$$\frac{1}{PS} + \frac{1}{ST} \geq \frac{2}{\sqrt{QS \times SR}}$$

$$\text{and } \frac{1}{PS} + \frac{1}{ST} \geq \frac{4}{QR}$$

But since S is not at the centre, so $PS \neq ST$ and $QS \neq SR$, so you will have the strict inequality, as following.

$$\frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}}$$

$$\text{and } \frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$$

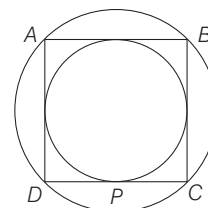
Hence choice (b) is the answer.

- 10** Consider P at the mid-point of CD which will also lie on C_1 , then

$$\begin{aligned} PA^2 + PB^2 + PC^2 + PD^2 \\ = 5 + 5 + 1 + 1 = 12 \end{aligned}$$

Now consider Q at D (Q coinciding with D), which will also lie on C_2 , then $QA^2 + QB^2 + QC^2 + QD^2$

$$\begin{aligned} &= DA^2 + DB^2 + DC^2 + DQ^2 \\ &= 4 + 8 + 4 + 0 = 16 \end{aligned}$$



$$\therefore \frac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2} = \frac{12}{16} = 0.75$$

Hence choice (a) is the answer.

Hint You must be wondering that why I have chosen P and Q at these places. If you are smart enough, you can figure it out, from the question, that the answer would not change if you change the locations of P and Q . That's why, for our convenience, I have chosen P on C_1 such that I can measure PA, PB, PC, PD easily and for the same reason I have chosen Q on C_2 .

- 11** When you shift Z to P, D will shift to Y , which makes $XD = XY$. Simultaneously, C will shift to X , which makes $CY = XY$. Therefore $DX = CY$. Now, when you shift Z to B, D will shift to A , which makes $XD = AX$. Simultaneously, C will shift to B , which makes $CY = BY$. But, we know that $AX = BX < BY$, so $XD < CY$.

It implies that $XD \leq CY$.

Hence choice (b) is the correct one.

- 12** Look at the following figures. First of all, consider the figure (ii), in which the radius of the circle inscribed in the shaded region is $\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{1}{12}$ cm.

Now, consider the fig (iii) and find the radius of the second circle in the shaded region. It can be done using the Descartes' circle theorem.

Let a, b and c be the curvatures of the existing circles and d be the curvature of desired circle, then

$$(a + b + c + d)^2 = 2(a^2 + b^2 + c^2 + d^2)$$

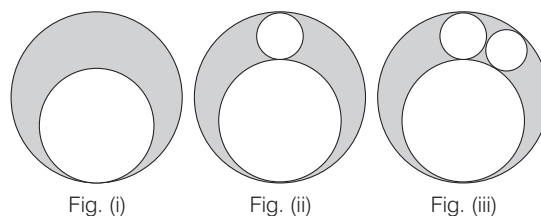
$$\text{That is } (-3 + 4 + 12 + d)^2 = 2((-3)^2 + 4^2 + 12^2 + d^2)$$

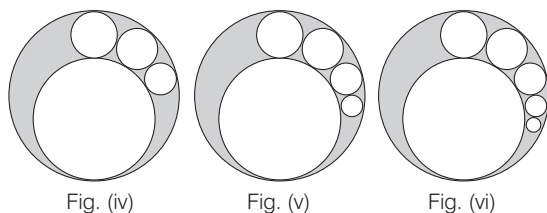
$$\Rightarrow (13 + d)^2 = 2(169 + d^2)$$

$$\Rightarrow d^2 - 26d + 169 = 0$$

$$\Rightarrow d = 13 \Rightarrow r_4 = \frac{1}{13} \text{ cm}$$

Using the same approach you can find the curvature of other new circles as shown in fig (iv).





Let a , b and d be the curvatures of the existing circles and e be the curvature of desired circle, then
 $(a + b + d + e)^2 = 2(a^2 + b^2 + d^2 + e^2)$

That is

$$(-3 + 4 + 13 + e)^2 = 2((-3)^2 + 4^2 + 13^2 + e^2)$$

$$\Rightarrow (14 + e)^2 = 2(194 + e^2)$$

$$\Rightarrow e^2 - 28e + 192 = 0 \Rightarrow e = 16$$

(Since $e = 12$ is not a possible value, as it is less than the value of the previous circle. Do you remember that the smaller circle has greater curvature?)

Now, consider the other figures (v) and (vi). Use the same approach as above, and you will find the other curvatures as 21 and 28.

Thus, you can see that the curvatures of these circles are as follows

12, 13, 16, 21 and 28.

Looking at the pattern you can find the values of all the curvatures, as given below

12, 13, 16, 21, 28, 37, 48, 61, 76, 93, 112, 133 and 156.

Out of the above 13 curvatures, except the first one (that is 12) all other curvatures repeat twice due to symmetry.

Therefore, total sum of all the 25 curvatures

$$= 12 + 2(13 + 16 + \dots + 156)$$

$$= 12 + 2(794) = 1600$$

Hence choice (b) is the correct one.

- 13** Given that the point of tangent of C_1 is $\frac{2}{5}$ cm (or 0.4 cm)

away from Y -axis and the point of tangent of C_2 is $\frac{3}{7}$ cm (or 0.4284 cm) away from the Y -axis, so the point of tangent of C_3 must be somewhere between $\frac{2}{5}$ and $\frac{3}{7}$ (or 0.4 cm and 0.4288 cm) away from the Y -axis.

Choice (a) : $\frac{26}{35} = 0.7428$ cm, which is more than

0.4284 cm

Choice (b) : $\frac{57}{144} = 0.3958$ cm, which is less than 0.4 cm

Choice (c) : $\frac{17}{36} = 0.4722$ cm, which is more than

0.4284 cm

Choice (d) : $\frac{35}{84} = 0.4167$ cm, which is between 0.4 cm and 0.4284 cm

Hence choice (d) is the correct one.

Alternatively If a circle C_1 is $\frac{a}{b}$ unit away from Y -axis and its corresponding diameter is $\frac{1}{b^2}$ and the other circle

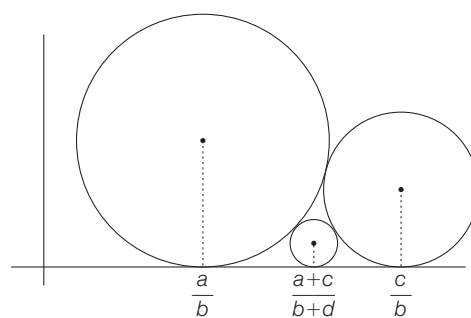
C_2 is $\frac{c}{d}$ unit away from Y -axis and its corresponding

diameter is $\frac{1}{d^2}$ and these two circles are tangent to each

other and have X -axis as their common tangent, the third circle C_3 will be tangent to the two circles C_1 and C_2 and

tangent to X -axis when it is $\frac{a+b}{c+d}$ unit away from Y -axis

and its diameter would be $\frac{1}{(c+d)^2}$.



Therefore, the required answer would be $\frac{2+3}{5+7} = \frac{5}{12} = \frac{35}{84}$

Hence choice (d) is the correct one.

Alternatively

Join the lines as shown in the diagram. Let us consider the radius of C_1 , C_2 and C_3 be r_1 , r_2 and r_3 , respectively.

Now, we have $BQ = \sqrt{AB^2 - AQ^2}$

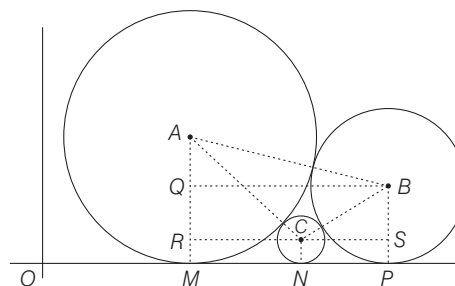
$$\Rightarrow BQ = \sqrt{(r_1 + r_2)^2 - (r_1 - r_2)^2}$$

$$\Rightarrow BQ = \sqrt{\left(\frac{1}{50} + \frac{1}{98}\right)^2 - \left(\frac{1}{50} - \frac{1}{98}\right)^2}$$

$$\Rightarrow BQ = \frac{1}{35}$$

Again,

$$\begin{aligned} BQ &= RS = RC + CS \\ &= \sqrt{(r_1 + r_3)^2 - (r_1 - r_3)^2} \\ &\quad + \sqrt{(r_2 + r_3)^2 - (r_2 - r_3)^2} \end{aligned}$$



$$\Rightarrow BQ = 2\sqrt{r_1 r_3} + 2\sqrt{r_2 r_3}$$

$$\Rightarrow BQ = 2(\sqrt{r_1 r_3} + \sqrt{r_2 r_3})$$

$$\Rightarrow \frac{1}{35} = 2\left(\frac{1}{5}\sqrt{\frac{r_3}{2}} + \frac{1}{7}\sqrt{\frac{r_3}{2}}\right)$$

$$\Rightarrow \frac{1}{35} = \frac{24}{35}\sqrt{\frac{r_3}{2}}$$

$$\Rightarrow r_3 = \frac{1}{288} \text{ cm}$$

But, we know that

$$MN = RC = \sqrt{(r_1 + r_3)^2 - (r_1 - r_3)^2}$$

$$\Rightarrow MN = RC = 2\sqrt{r_1 r_3} = 2\sqrt{\left(\frac{1}{50}\right)\left(\frac{1}{288}\right)} = \frac{1}{60}$$

Therefore, the required distance $= ON = OM + MN$

$$\Rightarrow ON = \frac{2}{5} + \frac{1}{60}$$

$$= \frac{25}{60} = \frac{5}{12} = \frac{35}{84}$$

- 14** Given that $AP = BP = p$ and $BQ = CQ = q$, therefore $PQ = p + q$.

Since $RB = MU = m$, so we have $PR = PB - RB = p - m$.

Also, $PM = p + m$

Let us consider $MR = UB = h$, we have

$$MR^2 = PM^2 - PR^2$$

$$\Rightarrow h^2 = (p + m)^2 - (p - m)^2 \quad \dots(i)$$

Now, we have,

$$OM = OW - MW = (p + q) - m$$

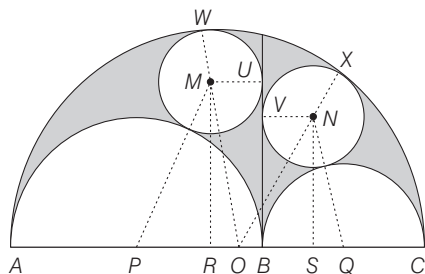
And, $RO = RB - OB = RB - (OC - BC)$

$$= m - (p + q - 2q) = m - p + q$$

Again, $MR^2 = OM^2 - RO^2$

$$\Rightarrow h^2 = (p + q - m)^2 - (m - p + q)^2 \quad \dots(ii)$$

Now, solving eqn. (i) and eqn. (ii) we get $m = \frac{pq}{p + q}$



Similarly, we can find out n , as shown below.

Let us consider $SN = j$, where $NQ = q + n$ and $SQ = BQ - BS = q - n$, we have $SN^2 = NQ^2 - SQ^2$

$$\Rightarrow j^2 = (q + n)^2 - (q - n)^2 \quad \dots(iii)$$

Now, we have, $ON = OX - NX = (p + q) - n$

And, $OS = OB + BS = (OC - BC) + BS$

$$= (p + q - 2q) + n = p - q + n$$

Again, $SN^2 = ON^2 - OS^2$

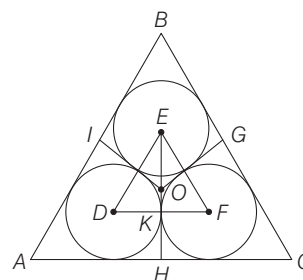
$$\Rightarrow j^2 = (p + q - n)^2 - (p - q + n)^2 \quad \dots(iv)$$

Now, solving eqs. (iii) and (iv) we get $n = \frac{pq}{p + q}$

Thus, we see that $m = n$ and $m + n = \frac{2pq}{p + q}$

Hence choice (c) is the correct one.

- 15** The best way is to convert this general scenario into a specific case by assuming an equilateral triangle where each of the three circles has same radius of 1 cm.



Then we have $DE = 2$ and $EK = \sqrt{3}$. And therefore, $EO = \frac{2}{\sqrt{3}}$ and $OK = \frac{1}{\sqrt{3}}$.

Also $KH = 1$.

Thus we have, inradius

$$OI = OG = OH = 1 + \frac{1}{\sqrt{3}}.$$

Now, you can substitute the value of $r_1 = r_2 = r_3 = 1$ in the given options and you would see that

Choice (a) : $\frac{\sqrt{r_1 r_2 r_3}}{r_1 + r_2 + r_3 - 2\sqrt{r_1 + r_2 + r_3}} = \frac{1}{3 - 2\sqrt{3}}$

It gives us a negative value, so it's invalid.

Choice (b) : $\frac{2\sqrt{r_1 r_2 r_3}}{r_1 + r_2 + r_3 - \sqrt{3}\sqrt{r_1 + r_2 + r_3}} = \frac{2}{3 - 3} = \frac{2}{0}$

It gives us an undefined value, so it's invalid.

Choice (c) : $\frac{2\sqrt{r_1 r_2 r_3}}{r_1 + r_2 + r_3 - \sqrt{r_1 + r_2 + r_3}} = \frac{2}{3 - \sqrt{3}}$

$$= \frac{2(3 + \sqrt{3})}{(3 - \sqrt{3})(3 + \sqrt{3})}$$

$$= \frac{2(3 + \sqrt{3})}{6} = 1 + \frac{1}{\sqrt{3}}$$

Hence choice (c) is the correct one.

CHAPTER 13

Elements of Algebra

Algebra plays a very crucial role in CAT since on an average 35–40% problems of ‘Quantitative Aptitude’ section are related to algebra and the questions asked from this section are not very calculative and can be solved quickly through options. According to the past years question papers, the questions of this section are very logical in spite of being algebraic.

Their level can be of High School Standard or IIT-JEE level. But you need not to go for an IIT coaching or for higher studies in maths. For this, first you have the command over the fundamentals, then solve the plenty of problems so that you must be aware with variety and diversity of problems. This section is written in this book as per the needs of those students who are really very poor in algebra and for the same, I have started from the zero level and created the standard of problems higher than CAT level. So in this journey of learning of algebra you have to learn maximum from the problems given in the chapter, because theory is never sufficient in itself.

13.1 Fundamental Concept of Algebra

- 1. Literal Numbers (or literals) :** Algebra is a branch of mathematics in which all the arithmetical operations are performed by using letters (like, $a, b, c, \dots, p, q, r, \dots x, y, z$) along with numerals. The letters used in algebra represent generalised numbers (or unknown) numbers. They are called literals.
- 2. Operations on Literals :** Since literals represent numbers, so they obey all the rules of operations of addition, subtraction, multiplication and division of numbers.

Addition of Literals and Numbers

- (i) $x + y = y + x$ (ii) $x + 3 = 3 + x$ (iii) $(x + y) + z = x + (y + z)$
(iv) $(x + 0) = x = (0 + x)$ (v) $x + (-x) = 0 = (-x) + x$

Subtraction of Literals and Numbers

- (i) $4 - x$ (ii) $x - 4$ (iii) $x - y = -(y - x)$
(iv) $y - x = -(x - y)$ (v) $x - 0 = x$

Multiplication of Literals and Numbers

- (i) $x \times y = xy$ ($xy = x + x + x + \dots$ y times)

Chapter Checklist

- Fundamental Concept of Algebra
- Algebraic Expression
- Polynomial
- Operations on Algebraic Expression
- Remainder Theorem
- Factor Theorem
- H.C.F. and L.C.M. of Polynomials
- Rational Expressions
- Basic Facts to Remember
- Linear Equations
- Simultaneous Linear Equations
- Algebraic Methods of Solving Simultaneous
- Equations in Two Variables
- CAT Test

- (ii) $4 \times x = 4x$ (not as $x4$) ($4x = x + x + x + x$)
 (iii) $1 \times x = x$ (not as $1x$) (iv) $xy = yx$
 (v) $(xy)z = x(yz)$ (vi) $1 \cdot x = x = x \cdot 1$
 (vii) $x(y + z) = xy + xz$ (viii) $0 \cdot x = 0 = x \cdot 0$

Division of Literals and Numbers

- (i) $4 \div x = \frac{4}{x}$ (ii) $x \div 4 = \frac{x}{4}$
 (iii) $x \div y = \frac{x}{y}$ (iv) $0 \div x = 0$

 **NOTE** $x \div 0$, i.e., $\frac{x}{0}$ is not defined.

Powers of a Literal

- (i) $x \times x = x^2$ or $x \times x \times x = x^3$ etc.
 i.e., $x \times x \times \dots n \times \text{times} = x^n$
 (ii) $x^1 = x$ (iii) $x^0 = 1$

 **NOTE** x^n is read as x raised to the power n or x to the power n or x power n .

$x \rightarrow$ base, $n \rightarrow$ exponent or index, $x^n \rightarrow$ exponential expression

Language of Algebra

Statement	Meaning
$a = b$	a is equal to b
$a \neq b$	a is not equal to b
$a < b$	a is less than b
$a > b$	a is greater than b
$a \leq b$	a is less than or equal to b
$a \geq b$	a is greater than or equal to b
$a \nless b$	a is not less than b
$a \ngtr b$	a is not greater than b .

Constraints and Variables

Constants : A symbol which has a fixed value is called a constant. e.g., 4, 5, 0, -3 , $\frac{2}{5}$, $3\frac{2}{7}$, etc.

Variable : A symbol which can assume various numerical values is called a variable.

e.g., Area of rectangle = length (l) $\times$ breadth (b) = $l \times b$.

Here, length (l) and breadth (b) can assume any possible numerical values.

Perimeter of a square = $4a$ ($a \rightarrow$ side of a square)

Here, a can assume any possible numerical value.

Price of 10 balls = $10x = 10 \times 2 = 20$ or $10 \times 5 = 50$, etc.

Here, x can assume any possible numerical value.

 **NOTE** In $5x$, 5 is constant and x is variable but together $5x$ is variable. Similarly, $5 + x$, $x - 5$, $5 - x$, $\frac{x}{5}$ and $\frac{5}{x}$ are variables. Hence, a combination of a constant and a variable is a variable.

13.2 Algebraic Expression

A collection of constants and literals connected by one or more of the operations of addition, subtraction, multiplication and division is called an “**algebraic expression**”.

The various parts of an algebraic expression separated by + or – sign are called “**terms**” of the algebraic expression.,

e.g.,

Algebraic expression	Number of terms	Terms
(i) $4x$	1	$4x$
(ii) $8x + 3y$	2	$8x, 3y$
(iii) $7 + 3x - 2y^2$	3	$7, 3x, -2y^2$
(iv) $\frac{7}{8}x^3 + 3x^2 + 4x - 9$	4	$\frac{7}{8}x^3, 3x^2, 4x, -9$

 **NOTE** Multiplication and division do not separate the terms of an algebraic expression, e.g., $7x$ is a single term, similarly $x/7$ is also a single term.

Types of Algebraic Expressions

- 1. Monomial :** An algebraic expression having only one term is called a monomial.

e.g., $3x, -5y, 12xy^2, \frac{3x}{y}, \frac{7x^2}{3y}$, etc.

- 2. Binomial :** An algebraic expression having two terms is called a binomial, e.g., $3x + 4y, 7x^2 + 2x, 9yz^2 - 4x^2, 3x - \frac{2}{3}y$, etc.

 **NOTE** A trinomial has three terms.

- 3. Polynomial (or multinomial) :** An algebraic expression having two or more than two terms is called a multinomial or polynomial, e.g.,

(i) $7x^2 - 3y$ is a polynomial having 2 terms

(ii) $7x^4 - 3x^2 + 5x$ is a polynomial having 3 terms

(iii) $3x^4 - 7x^3 + 3xy + 8$ is a polynomial having 4 terms, etc.

 **NOTE** Mono $\rightarrow$ single or one
 Tri $\rightarrow$ three
 and nomial $\rightarrow$ terms

Bi $\rightarrow$ two
 Poly/Multi $\rightarrow$ many

Product : When two or more constants or literals (or both) are multiplied, then the result so obtained is called the “**product**”, e.g., $7xy$ is the product of 7, x and y .

Factors : Each of the quantity (constant or literal) multiplied together to form a product is called a “**factor**” of the product.

A constant factor is called a **numerical factor** and any factor containing only literals is called a **literal factor**. e.g.,

- (i) In $7xy$, the numerical factor is 7 and literal factors are x , y and xy .
 (ii) In $-7x^2y$, the numerical factor is -7 and the literal factors are, x^2 , y , x , xy and x^2y .

Constant terms : The term of an algebraic expression having no literal factors is called its **constant term**. *e.g.*

In the algebraic expression $3x^2 + 7x + 8$, the constant term is 8. (Since, 8 is not associated with any literals)

Coefficients : In an algebraic expression any factor of a term is called the **coefficient** of the remaining factor of the term, *e.g.*, $4xy + 5x^2y - 17$. In the first term $4xy$,

- (i) numerical coefficient = 4 (ii) literal coefficient = xy
 (iii) coefficient of 4 = xy (iv) coefficient of $x = 4y$
 (v) coefficient of $y = 4x$ (vi) coefficient of $4x = y$
 (vii) coefficient of $4y = x$

Similarly, we can find the coefficient of the other two terms.

Like and unlike terms : The terms having same literal coefficients are called **like terms** else the terms are called **unlike terms**, *e.g.*,

- (i) $7xy, 4xy, 9xy, -\frac{3}{2}xy$ are like terms
 (ii) $3x^2y, 5x^2y, 8x^2y, \dots, -7x^2y$ are like terms
 (iii) $3x, 5x^2y, 9xy^2z$ are unlike terms, etc.

13.3 Polynomial

Polynomials in one variable : An algebraic expression of the form $a + bx + cx^2 + dx^3 + \dots$ where $a, b, c, d, \dots$ are constants and x is a variable, is called a polynomial in the variable x . The greatest power of x present in the polynomial is called the **degree of the polynomial**.

 **NOTE** No term in a polynomial can be of the form $\frac{1}{x}, \frac{1}{x^2}$, etc. *e.g.*,

- (i) $7 + 3x$ is a polynomial in x of degree 1.
 (ii) $7y^2 + 3y - 10$ is a polynomial in y of degree 2.
 (iii) $8k^3 + 3k^2 - 15$ is a polynomial in k of degree 3.
 (iv) $7x^5 + 3x^2 + 8$ is a polynomial in x of degree 5.
 (v) $3x^4 + 5x^3 + 7x - \frac{8}{x}$ is not a polynomial.
 (vi) 8 can be written as $8x^0$, so it is a polynomial in x of degree 0, *i.e.*, in general a single constant term is always of degree 0 (zero).

Polynomials in two or more variables

Any algebraic expression in two or more variables is called a polynomial if every variable present in it has only non-negative integral powers.

In case of a polynomial in two or more variables, the sum of the powers of the variables in each term is taken and the greatest sum is the **degree of the polynomial**.

 **NOTE** No term in a polynomial can be of the form $\frac{1}{x}, \frac{1}{x^2}, \frac{x}{y}$, etc.

Examples :

- (i) $8x^2 - 5xy + 3x - 7y + 2$ is a polynomial in two variables x and y . The degree of its terms are 2, (1 + 1), 1, 1, 0, so the degree of the polynomial is 2.
 (ii) $8a^3 + 9b^2a^3 + 12b^3 - \frac{5}{7}a + 15$ is a polynomial in two variables a and b . The degree of its terms are 3, (2 + 3), 3, 1, 0, so the degree of the polynomial is 5.

 **NOTE** The degree of a polynomial is equal to the maximum value of the sum of the powers of the variables in any term. Degree is always a whole number. It is not defined for polynomials having fractional and negative exponent.

Substitution

The process of finding the value of an algebraic expression by replacing the variables (literals) by their particular values is called substitution.

Exp. 1) If $x = 3$ and $y = 4$, find the value of

- (i) $3x + 2y$ (ii) $4x - 3y$ (iii) $18xy$ (iv) $\frac{15x}{12y}$

Solution

- (i) $3x + 2y = 3 \times 3 + 2 \times 4$ (substituting $x = 3$ and $y = 4$)
 $= 9 + 8 = 17$
 (ii) $4x - 3y = 4 \times 3 - 3 \times 4$ (substituting $x = 3$ and $y = 4$)
 $= 12 - 12 = 0$
 (iii) $18xy = 18 \times 3 \times 4$ (substituting $x = 3$ and $y = 4$)
 $= 216$
 (iv) $\frac{15x}{12y} = \frac{15 \times 3}{12 \times 4} = \frac{15}{16}$

Exp. 2) If $x = -3$, find the value of $4x^2 - 5x + 3$.

Solution $4x^2 - 5x + 3 = 4 \times (-3)^2 - 5 \times (-3) + 3$
 $= 36 + 15 + 3 = 54$

Exp. 3) If $a = 4, b = -3, c = 2$, find the value of $4a^2 + 5b - 12c$.

Solution $4a^2 + 5b - 12c = 4 \times (4)^2 + 5(-3) - 12(2)$
 $= 64 - 15 - 24 = 25$

13.4 Operations on Algebraic Expression

Addition of like terms : In any algebraic expression only like terms can be added or subtracted. Thus, the sum of two or more like terms is a like term whose numerical coefficient is the sum of coefficients of given like terms.

e.g., $2x = x + x$, $3x = x + x + x$, $4x = x + x + x + x$.

$\therefore 2x + 3x + 4x = (x + x) + (x + x + x) + (x + x + x + x) = 9x$

Similarly, $2x^2 + 3x^2 = (2 + 3)x^2 = 5x^2$

and $5xy + (-3xy) = [5 + (-3)]xy = 2xy$

Thus, we cannot further simplify $7x + 5x^2$ into single term, since both are unlike terms.

Addition of Algebraic Expressions

Exp. 1) Solve the following expression

$$(4a + 2b - 3c) + (3a - 2b + 4c)$$

Solution $(4a + 2b - 3c) + (3a - 2b + 4c)$

$$= (4a + 3a) + (2b - 2b) + (-3c + 4c)$$

$$= (4 + 3)a + (2 - 2)b + (-3 + 4)c = 7a + c$$

Exp. 2) Add $2x^2 + 5xy - 2y^2$, $3x^2 - 2xy + 5y^2$ and $-2x^2 + 8xy + 3y^2$.

Solution $(2x^2 + 5xy - 2y^2) + (3x^2 - 2xy + 5y^2) + (-2x^2 + 8xy + 3y^2)$
 $= (2x^2 + 3x^2 - 2x^2) + (5xy - 2xy + 8xy) + (-2y^2 + 5y^2 + 3y^2)$
 $= 3x^2 + 11xy + 6y^2$

Subtraction of Algebraic Expressions

Exp. 1) Subtract $3x + 2y - z$ from $4x - 5y + 3z$.

Solution $(4x - 5y + 3z) - (3x + 2y - z)$

$$= (4x - 3x) + (-5y - 2y) + [3z - (-z)] = x - 7y + 4z$$

Exp. 2) Subtract $7x^3 - 3x^2 + 8x + 16$ from

$$9x^3 + 6x^2 - 6x + 20.$$

Solution $(9x^3 + 6x^2 - 6x + 20) - (7x^3 - 3x^2 + 8x + 16)$

$$= (9x^3 - 7x^3) + [6x^2 - (-3x^2)] + [(-6x) - (8x)] + (20 - 16)$$

$$= 2x^3 + 9x^2 - 14x + 4$$

Multiplication of Algebraic Expressions

It follows the distributive property. In this operation all the numerals are multiplied with each other individually and all the literals (or variables) multiplied with each other as per the rule.

Exp. 1) Find the product of $6xy$ and $7x^2$.

Solution $6xy \times 7x^2 = (6 \times 7) \times (xy \times x^2)$

$$= (6 \times 7) \times (x \times y \times x \times x) = 42 \times x^3y$$

Exp. 2) Find the product of $3ab^2$ and $(4ab + 3b^2)$.

Solution $3ab^2 \times (4ab + 3b^2) = 3ab^2 \times 4ab + 3ab^2 \times 3b^2$

$$= 3 \times a \times b \times b \times 4 \times a \times b + 3 \times a \times b \times b \times 3 \times b \times b$$

$$= 12a^2b^3 + 9ab^4 = 3ab^3(4a + 3b)$$

Exp. 3) Multiply $(a + b)$ by $(c + d)$.

Solution $(a + b)(c + d) = a(c + d) + b(c + d) = ac + ad + bc + bd$

Exp. 4) Multiply $(5x - 4)$ by $(3x + 5)$.

Solution $(5x - 4) \times (3x + 5) = 5x \times (3x + 5) - 4(3x + 5)$
 $= 15x^2 + 25x - 12x - 20 = 15x^2 + 13x - 20$

Division of Algebraic Expressions

Exp. 1) Solve $x^6 \div x^4$

Solution $\frac{x^6}{x^4} = \frac{x \times x \times x \times x \times x \times x}{x \times x \times x \times x} = x \times x = x^2$

Exp. 2) Solve $35x^3y^2 \div 7x^2y^2$

Solution $\frac{35x^3y^2}{7x^2y^2} = \frac{7 \times 5 \times x \times x \times x \times y \times y}{7 \times x \times x \times y \times y} = 5x$

Thus, we can say $7x^2y^2$ and $5x$ are the two factors of $35x^3y^2$.

Exp. 3) Divide $14x^3y^2 + 8x^2y^3 - 32x^2y^5$ by $-2xy^2$.

Solution $\frac{14x^3y^2 + 8x^2y^3 - 32x^2y^5}{-2xy^2}$
 $= \frac{(14x^3y^2)}{(-2xy^2)} + \frac{(8x^2y^3)}{(-2xy^2)} + \frac{(-32x^2y^5)}{(-2xy^2)}$
 $= -7x^2 - 4xy + 16xy^3$

Exp. 4) Divide $(4x^2 + 3x + 2)$ by $(x + 2)$

Solution

$$\begin{array}{r} x+2 \overline{) 4x^2+3x+2} \\ \underline{4x^2+8x} \\ -5x+2 \\ \underline{-5x-10} \\ 12 \end{array}$$

Here $(x + 2)$ is divisor and $(4x - 5)$ is quotient and 12 is remainder and $(4x^2 + 3x + 2)$ is the dividend.

NOTE

- (i) If there is a '+' sign before a bracket, then the signs of the terms inside the bracket do not change.
- (ii) If there is a '-' sign before a bracket, then the signs of all the terms inside the bracket change.

General Form of a Polynomial

The expression of the form $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_n$ in which $a_0, a_1, a_2, \dots, a_n$ are constants ($a_0 \neq 0$) and n is a positive integer is called a polynomial (or rational integral function) in x of degree n . We may denote it by $f(x)$,

$$i.e., f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_n.$$

Algebraic Expression as a Function : A function is an operation which forms a relation between two variables.

e.g., $y = x^2$, $y = 3x$, $y = x^2 + 3$, etc.

This operation is represented by $f(x)$, i.e.,

$$y = f(x) \Rightarrow f(x) = x^2 \quad (f \text{ is called operand})$$

$$f(x) = 3x \Rightarrow f(x) = x^2 + 3$$

Exp. 1) If $f(x) = x^3 + 2x$, then find the value of $f(3)$.

Solution $\because f(x) = x^3 + 2x \therefore f(3) = (3)^3 + 2(3) = 27 + 6 = 33$

Exp. 2) If $f(x) = 2x^3 + 3x^2 + 5$, then find the value of $f(2)$.

Solution $\because f(x) = 2x^3 + 3x^2 + 5$
 $\therefore f(2) = 2(2)^3 + 3(2)^2 + 5 = 16 + 12 + 5 = 33$

Graphical Expression

Algebraic Expression

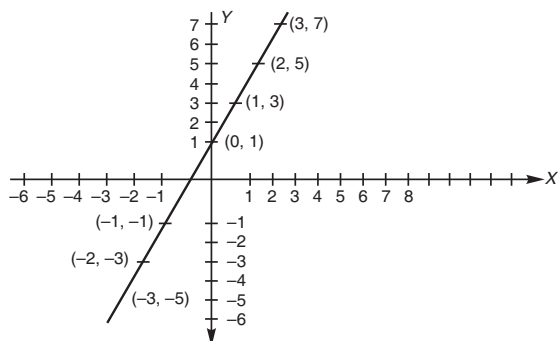
All the polynomial expressions that can be written as $y = f(x)$ can be represented in two dimensional co-ordinate system.

Exp. 1) $y = f(x) = 2x + 1$

Solution

x	-3	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5	7

Hint Find the corresponding values of y for each value of x . Generally we consider some +ve and some -ve values of x . Now we plot the graph as per the values obtained in the table.

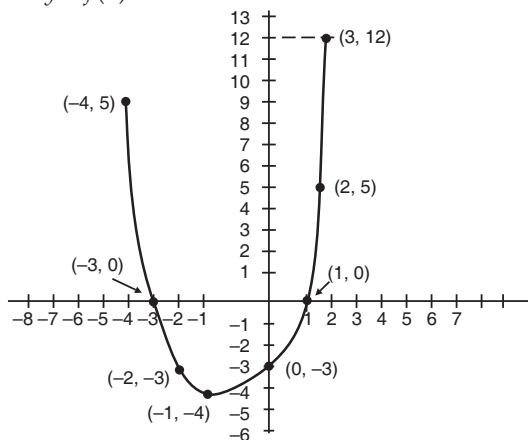


The points are represented as (x, y) and the values of (x, y) represent the points on the plane.

NOTE $y = 2x + 1$ is a linear equation, therefore the graph of the given equation shows a straight line.

Exp. 2) $f(x) = x^2 + 2x - 3$

Solution $y = f(x) = x^2 + 2x - 3$



x	-4	-3	-2	-1	0	1	2	3
y	5	0	-3	-4	-3	0	5	12

Since, $y = x^2 + 2x - 3$ is a quadratic equation, therefore the graph is a parabola.

13.5 Remainder Theorem

If a polynomial (*i.e.*, a rational integral function) $f(x)$ is divided by $(x - a)$, then the remainder is obtained by substituting a for x in $f(x)$, *i.e.*, the remainder is $f(a)$.

Exp. 1) Find the remainder when $x^2 - 6x + 5$ is divided by $(x + 2)$.

Solution Let $f(x) = x^2 - 6x + 5$. Thus by remainder theorem, the remainder when $f(x)$ is divided by $(x + 2)$, *i.e.*, $[x - (-2)]$ is $f(-2)$

$$\therefore f(-2) = (-2)^2 - 6(-2) + 5 = 4 + 12 + 5 = 21.$$

$\therefore$ Thus, the remainder is 21.

NOTE

1. If a polynomial $f(x)$ is divided by $(x + a)$, then remainder is $f(-a)$
2. If a polynomial $f(x)$ is divided by $(x - a)$, then remainder is $f(a)$
3. If a polynomial $f(x)$ is divided by $(ax + b)$, then the remainder is $f(-b/a)$

Hint To get the value of remainder first of all write the divisor equal to zero, *i.e.*, if divisor is $(x + a)$, then write $x + a = 0$. Then, find the value of x , *i.e.*, $x + a = 0 \Rightarrow x = -a$. Then, put the obtained value of x in the polynomial, *i.e.*, $f(x) = 3x^2 + 4x - 5$
 $\Rightarrow f(-a) = 3(-a)^2 + 4(-a) - 5$, then solve the function (or expression) and get the simplified value of remainder.

Exp. 2) If $2x^3 + 3x^2 + 7x + 6$ be divided by $2x + 1$, find the remainder.

Solution Let $f(x) = 2x^3 + 3x^2 + 7x + 6$.

Then the remainder when $f(x)$ is divided by $(2x + 1)$

$$\therefore 2x + 1 = 0$$

$$\Rightarrow x = -\frac{1}{2}$$

$$\begin{aligned} \therefore f\left(-\frac{1}{2}\right) &= 2\left(-\frac{1}{2}\right)^3 + 3\left(-\frac{1}{2}\right)^2 + 7\left(-\frac{1}{2}\right) + 6 \\ &= -\frac{1}{4} + \frac{3}{4} - \frac{7}{2} + 6 \\ &= \frac{-1 + 3 - 14 + 24}{4} = \frac{12}{4} = 3 \end{aligned}$$

Exp. 3) Find the remainder when the expression $3x^3 + 8x^2 - 6x + 1$ is divided by $(x + 3)$.

Solution Let $f(x) = 3x^3 + 8x^2 - 6x + 1$

$$\therefore x + 3 = 0$$

$$\Rightarrow x = -3$$

$$\begin{aligned} \therefore f(-3) &= 3(-3)^3 + 8(-3)^2 - 6(-3) + 1 \\ &= -81 + 72 + 18 + 1 = 10 \end{aligned}$$

Hence, the remainder is 10.

Exp. 4) When $x^3 + 3x^2 - kx + 4$ is divided by $x - 2$, the remainder is k . Find the value of the constant k .

Solution Let $f(x) = x^3 + 3x^2 - kx + 4$

Now, since $x - 2 = 0 \Rightarrow x = 2$

$$\therefore f(2) = (2)^3 + 3(2)^2 - k(2) + 4 = 24 - 2k$$

$$\therefore \text{Remainder} = 24 - 2k = k \Rightarrow k = 8$$

($\because$ the given remainder is k)

13.6 Factor Theorem

If $f(x)$ be a polynomial (i.e., rational integral function) in x and $f(a) = 0$, then $(x - a)$ is a factor of $f(x)$.

By remainder theorem if $f(x)$ be divided by $(x - a)$ the remainder is $f(a)$. But $f(a) = 0$, i.e., there is no remainder.

$\therefore f(x)$ is exactly divisible by $(x - a)$

Hence, $(x - a)$ is a factor of $f(x)$.

NOTE

1. If $f(x)$ be a polynomial and $f(-a) = 0$, then $(x + a)$ is a factor of $f(x)$.
2. If $f(x)$ be a polynomial and $f(-b/a) = 0$, then $(ax + b)$ is a factor of $f(x)$.

Exp. 1) Show that $(x - 2)$ is a factor of the polynomial $2x^2 - 5x + 2$.

Solution Let $f(x) = 2x^2 - 5x + 2$

$$\therefore (x - 2) = 0$$

$$\Rightarrow x = 2$$

$$\therefore f(2) = 2(2)^2 - 5(2) + 2$$

$$f(2) = 0$$

$\therefore$ By factor theorem $(x - 2)$ is a factor of $2x^2 - 5x + 2$

Exp. 2) Prove that $(2x + 5)$ is a factor of the polynomial $2x^2 + 11x + 15$.

Solution Let $f(x) = 2x^2 + 11x + 15$

$$\therefore 2x + 5 = 0 \Rightarrow x = -5/2$$

$$\therefore f(-5/2) = 2(-5/2)^2 + 11(-5/2) + 15$$

$$= \frac{25}{2} - \frac{55}{2} + 15$$

$$f(-5/2) = 0$$

Hence, $(2x + 5)$ is a factor of $f(x) = 2x^2 + 11x + 15$.

Exp. 3) Prove that $(a - b)$ is a factor of $(a^3 - b^3)$.

Solution Let $f(a) = a^3 - b^3$

$$\therefore (a - b) = 0 \Rightarrow a = b$$

$$\therefore f(b) = (b)^3 - b^3$$

$$f(b) = 0$$

$$\therefore f(a) = 0$$

Hence, $(a - b)$ is a factor of $a^3 - b^3$.

Exp. 4) If $(x - 3)$ and $(x + 4)$ are both the only factors of $f(x)$, then find $f(x)$.

Solution Since, there are only two factors of $f(x)$, therefore the product of $(x - 3)$ and $(x + 4)$ is equal to $f(x)$.

$$\therefore f(x) = (x - 3)(x + 4)$$

$$f(x) = x^2 + x - 12$$

13.7 H.C.F. and L.C.M. of Polynomials

1. A polynomial $D(x)$ is a divisor of the polynomial $P(x)$ if it is a factor of $P(x)$. Where $Q(x)$ is another polynomial such that $P(x) = D(x) \cdot Q(x)$
2. **HCF/GCD (Greatest Common Divisor)** : A polynomial $h(x)$ is called the HCF or GCD of two or more given polynomials, if $h(x)$ is a polynomial of highest degree dividing each one of the given polynomials.

NOTE The coefficient of highest degree term is always taken as positive.

3. **LCM (Least Common Multiple)** : A polynomial $P(x)$ is called the LCM of two or more given polynomials, if it is a polynomial of smallest degree which is divided by each one of the given polynomials.

For any two polynomials $P(x)$ and $Q(x)$. We have :

$$P(x) \times Q(x) = [\text{HCF of } P(x) \text{ and } Q(x)] \times [\text{LCM of } P(x) \text{ and } Q(x)]$$

Exp. 1) Find the HCF and LCM of the following polynomials $(x + 2)^2(x - 3)^2(x + 1)^2$, $(x + 1)^3(x + 2)^3(x - 3)$.

Solution Let $P(x) = (x + 2)^2(x - 3)^2(x + 1)^2$

$$\text{and } Q(x) = (x + 1)^3(x + 2)^3(x - 3)$$

$$\text{HCF} = (x + 1)^2(x + 2)^2(x - 3)$$

(For the HCF we take the highest power of factors common to both $P(x)$ and $Q(x)$) and $\text{LCM} = (x + 1)^3(x + 2)^3(x - 3)^2$

[For LCM we take the highest powers of the factors either of $P(x)$ or $Q(x)$]

Exp. 2) Find the HCF and LCM of the following polynomials $22x(x + 1)^2$, $36x^2(2x^2 + 3x + 1)$

Solution Let $P(x) = 22x(x + 1)^2$

$$\text{and } Q(x) = 36x^2(2x^2 + 3x + 1)$$

$$P(x) = 2 \times 11 \times x \times (x + 1)^2$$

$$\text{and } Q(x) = 2 \times 18 \times x^2 \times (x + 1)(2x + 1)$$

$$\therefore \text{HCF} = 2 \times x \times (x + 1) = 2x(x + 1)$$

$$\text{and } \text{LCM} = 2x(x + 1) \times 11 \times 18 \times x \times (x + 1)(2x + 1) = 396x^2(x + 1)^2(2x + 1)$$

(To find the LCM, first take the HCF and then take all the rest factors).

Exp. 3) Find the HCF and LCM of $(3 + 13x - 30x^2)$ and $(25x^2 - 30x + 9)$.

Solution Let $P(x) = (3 + 13x - 30x^2)$
and $Q(x) = (25x^2 - 30x + 9)$
or $P(x) = 3 + 13x - 30x^2$
 $= 3 + 18x - 5x - 30x^2$
 $= -(6x + 1)(5x - 3)$
 $Q(x) = 25x^2 - 30x + 9 = (5x - 3)^2$
 $\therefore$ HCF of $P(x)$ and $Q(x) = (5x - 3)$
and LCM of $P(x)$ and $Q(x) = -(5x - 3)^2(6x + 1)$.

Exp. 4) The HCF of two polynomials is $(x - 2)(x + 3)$ and their LCM is $(x - 2)^2(x + 3)(x + 1)$. If one of the polynomial is $x^2 - x - 2$, find the other.

Solution HCF $= (x - 2)(x + 3)$
and LCM $= (x - 2)^2(x + 3)(x + 1)$
 $\therefore$ HCF $\times$ LCM $= (x - 2)^3(x + 3)^2(x + 1)$
Let $P(x) = x^2 - x - 2 = x^2 - 2x + x - 2$
 $= (x - 2)(x + 1)$
 $\therefore P(x) \cdot Q(x) = \text{HCF} \times \text{LCM}$
 $\therefore Q(x) = \frac{\text{HCF} \times \text{LCM}}{P(x)}$
 $= \frac{(x - 2)^3(x + 3)^2(x + 1)}{(x - 2)(x + 1)}$
 $= (x - 2)^2(x + 3)^2$
 $\therefore$ The other polynomial [say $Q(x)$] is $(x - 2)^2(x + 3)^2$.

13.8 Rational Expressions

If $P(x)$ and $Q(x)$ are two polynomials such that $Q(x) \neq 0$, then the quotient $\frac{P(x)}{Q(x)}$ is called a rational expression.

- Every polynomial is a rational expression but a rational expression need not be a polynomial.
- The rational expression $\frac{P(x)}{Q(x)}$ is said to be in its simplest form (or in lowest terms) if the G.C.D. of $P(x)$ and $Q(x)$ is 1.
- To express a rational expression in its simplest form, express the polynomials in the numerator and in the denominators as product of simplest factors (*i.e.*, no further factorization is possible), and then cancel the common factors.

Exp. 1) Which of the following algebraic expressions are polynomials?

- (a) $x^2 - \sqrt{3}x + 4$ (b) $x^3 - 3x^2 + 4\sqrt{x}$
(c) $\sqrt{2}x^2 - \sqrt{3}x + \sqrt{5}$ (d) $x^3 - 2x + x^{-2}$

Solution

- (a) $x^2 - \sqrt{3}x + 4$ is a polynomial and (c) $\sqrt{2}x^2 - \sqrt{3}x + \sqrt{5}$ is also a polynomial
(b) $x^3 - 3x^2 + 4\sqrt{x}$ is not polynomial since $4\sqrt{x} = 4x^{1/2}$, *i.e.*, power of x is $\frac{1}{2}$ (*i.e.*, fractional)
(d) $x^3 - 2x + x^{-2}$ is not a polynomial since x^{-2} has negative power of x .

Exp. 2) Express the following in the lowest terms.
 $\frac{x^2 + 6x + 9}{x^2 - 9}$

Solution $\frac{x^2 + 6x + 9}{x^2 - 9} = \frac{(x + 3)(x + 3)}{(x + 3)(x - 3)} = \frac{x + 3}{x - 3}$

Important Formulae

1. $(a + b)^2 = a^2 + b^2 + 2ab$
2. $(a - b)^2 = a^2 + b^2 - 2ab$
3. $(a + b)^2 = (a - b)^2 + 4ab$
4. $(a - b)^2 = (a + b)^2 - 4ab$
5. $(a^2 + b^2) = \frac{1}{2}[(a + b)^2 + (a - b)^2]$
6. $a^2 - b^2 = (a + b)(a - b)$
7. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
8. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
9. $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$
10. $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$
11. $(a + b + c)^2 = (a^2 + b^2 + c^2) + 2(ab + bc + ac)$
12. $a^3 + b^3 + c^3 - 3abc$
 $= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$
13. $a^4 + b^4 + a^2b^2 = (a^2 + b^2 + ab)(a^2 + b^2 - ab)$
14. $a^4 - b^4 = (a - b)(a + b)(a^2 + b^2)$
15. $a^8 - b^8 = (a - b)(a + b)(a^2 + b^2)(a^4 + b^4)$

13.9 Basic Facts to Remember

1. To factorize the cyclic expressions :

- (a) Arrange the terms in decreasing or increasing powers of one of the letters (or literals)
- (b) Make one factor common by taking two or three terms together.
- (c) Rewrite the terms of the other factor in decreasing or increasing power of the next letter.
- (d) Repeat the process till all the factors are found out.

2. Sigma (Σ) notation :

$$\Sigma x = x + y + z, \text{ for letters } x, y, z$$

$$\Sigma xy = xy + yz + zx$$

$$\Sigma x^2(y - z) = x^2(y - z) + y^2(z - x) + z^2(x - y)$$

3. Some important results :

$$(a) \quad \Sigma(x - y) = 0 \quad (b) \quad \Sigma(x^2 - y^2) = 0$$

$$(c) \quad \Sigma(x^3 - y^3) = 0 \quad (d) \quad \Sigma x(y - z) = 0$$

$$(e) \quad \Sigma x^2(y^2 - z^2) = 0$$

$$(f) \quad \Sigma x^2(y - z) = -(x - y)(y - z)(z - x)$$

$$(g) \quad \Sigma x(y^2 - z^2) = (x - y)(y - z)(z - x)$$

$$(h) \quad \Sigma(x - y)^3 = 3(x - y)(y - z)(z - x)$$

Exp. 1) Factorize

$$x(y^2 + z^2) + y(z^2 + x^2) + z(x^2 + y^2) + 2xyz$$

$$\begin{aligned} \text{Solution} \quad & x(y^2 + z^2) + y(z^2 + x^2) + z(x^2 + y^2) + 2xyz \\ & = xy^2 + xz^2 + yz^2 + x^2y + x^2z + y^2z + 2xyz \end{aligned}$$

Arranging in decreasing powers of x

$$\begin{aligned} & = x^2y + x^2z + xy^2 + xz^2 + 2xyz + y^2z + yz^2 \\ & = x^2(y + z) + x(y^2 + z^2 + 2yz) + yz(y + z) \\ & = x^2(y + z) + x(y + z)^2 + yz(y + z) \\ & = (y + z)[x^2 + x(y + z) + yz] \\ & = (y + z)(x^2 + xy + xz + yz) \\ & = (y + z)[x(x + y) + z(x + y)] \\ & = (y + z)(x + y)(x + z) \\ & = (x + y)(y + z)(z + x) \end{aligned}$$

Exp. 2) Factorize $\Sigma xy(x + y) + 2xyz$

$$\begin{aligned} \text{Solution} \quad & \Sigma xy(x + y) + 2xyz \\ & = xy(x + y) + yz(y + z) + zx(z + x) + 2xyz \\ & = x^2y + x^2z + y^2z + y^2x + z^2x + z^2y + 2xyz \\ & = x^2(y + z) + x(y^2 + z^2 + 2yz) + yz(y + z) \\ & = x^2(y + z) + x(y + z)^2 + yz(y + z) \\ & = (y + z)[x^2 + x(y + z) + yz] \\ & = (y + z)[x(x + y) + z(x + y)] \\ & = (y + z)(x + y)(y + z) \end{aligned}$$

Introductory Exercise 13.1

1. Sunny Gupta scored x marks in Maths, y marks in DI/DS and 40 marks in English. What is his total score in these three subjects?

$$(a) 100 - (x + y) \quad (b) 100 - (x + y + 40)$$

$$(c) x + y - 40 \quad (d) x + y + 40$$

2. Quotient of z by 4 is multiplied by thrice the number y .

$$(a) \frac{3y}{4z} \quad (b) \frac{3z}{4y}$$

$$(c) \frac{3yz}{4} \quad (d) \frac{3}{4yz}$$

3. Express $6 \times p \times q \times r \times p \times p \times p$ in the exponential form

$$(a) 6(p^4 + q + r) \quad (b) 6 \times 4p \times q \times r$$

$$(c) 6p^4qr \quad (d) \text{none of these}$$

4. Find the number of terms in $2x^2 - \frac{3}{x} + \frac{5}{x^2} + 9$.

$$(a) 1 \quad (b) 2$$

$$(c) 3 \quad (d) 4$$

5. Find the number of terms in

$$4xyz - \frac{3}{x^2} + 6xy^2 - 5z^3 + 6x.$$

$$(a) 3 \quad (b) 4$$

$$(c) 5 \quad (d) 6$$

6. Find the number of terms in $7 \times a + b + 3 - c + b$.

$$(a) 3 \quad (b) 5$$

$$(c) 4 \quad (d) 6$$

7. Find the coefficient of z^3 in $-7xy^2z^3$.

$$(a) xy^2 \quad (b) -7$$

$$(c) 7xy^2 \quad (d) -7xy^2$$

8. In the following, which pair contains like terms?

$$(a) 2x^2y, 2xy^2 \quad (b) 2xy, -3yx$$

$$(c) 6, 6x \quad (d) 2xy, 3xyz$$

9. Find the degree of the polynomial $4x^3 - 2x^2y^3 + 6$.

$$(a) 3 \quad (b) 4$$

$$(c) 5 \quad (d) 6$$

10. Simplify the following :

$$2x^2 + 3y^2 - 5xy + 5x^2 - y^2 + 6xy - 3x^2$$

$$(a) 7x^2 + 4y^2 - 11xy \quad (b) 7x^2 + 2y^2 - 30xy$$

$$(c) 4x^2 + 2y^2 - 11xy \quad (d) 4x^2 + xy + 2y^2$$

11. If sum of two polynomials is $5x^2 + 3x - 1$. If one of them is $3x^3 - 2x + 7$. Find the other.

$$(a) 3x^3 + 5x^2 + 5x - 8 \quad (b) -3x^3 + 5x^2 + 5x - 8$$

$$(c) 2x^2 + x + 6 \quad (d) \text{none of these}$$

12. Multiply $\frac{1}{2}a^2b - \frac{2}{3}ab^2 + 5$ by $6abc$:

$$(a) 3a^3b^2c - 4a^2b^3c + 30abc$$

$$(b) 3a^3b^3c - 4a^3b^2c + 6abc$$

$$(c) 6a^3b^2 + 18ab^2c + 30abc$$

$$(d) \text{none of the above}$$

13. Find the product of $\left(\frac{1}{2}x^2 - \frac{1}{3}y^2\right)$ and $\left(\frac{1}{2}x^2 + \frac{1}{3}y^2\right)$.
 (a) $\frac{1}{9}y^4 - \frac{1}{4}x^4$ (b) $\frac{1}{4}x^4 - \frac{1}{9}y^4$
 (c) $4x^2 - 9y^4$ (d) $\frac{1}{4}x^4 - \frac{1}{9}y^9$
14. Divide $36x^2y^5 + 42xy^3 - 24x^3y^2 - 12y^5$ by $-6y^2$.
 (a) $6x^2y^3 - 7xy + 4x^3 + 2y^3$
 (b) $-6x^2y^3 - 7xy^2 + 4x^2 + 2y^3$
 (c) $-6x^2y^3 - 7xy + 4x^3 + 2y^3$
 (d) none of the above
15. Find the remainder when the expression $3x^2 + 8x^2 - 6x + 1$ is divided by $x + 3$.
 (a) 1 (b) 10
 (c) 6 (d) 0
16. Find the value of a if the division of $ax^3 + 9x^2 + 4x - 10$ by $(x + 3)$ leaves a remainder 5.
 (a) 1 (b) 2
 (c) 3 (d) 4
17. If $(x + 1)$ and $(x - 2)$ are factors of $x^3 + ax^2 - bx - 6$, then find the values of a and b respectively.
 (a) 2, 3 (b) 3, 5
 (c) 5, 3 (d) 2, 5
18. If $(5x^2 + 14x + 2)^2 - (4x^2 - 5x + 7)^2$ is divided by $(x^2 + x + 1)$, then, quotient ' q ' and remainder ' r ' are given by :
 (a) $q = (x^2 + 19x - 5), r = 0$
 (b) $q = 9(x^2 + 19x - 5), r = 0$
 (c) $q = (x^2 + 19x - 5), r = 1$
 (d) $q = 9(x^2 + 19x - 5), r = 1$
19. On dividing $(x^3 - 6x + 7)$ by $(x + 1)$, then the remainder is :
 (a) 2 (b) 0
 (c) 12 (d) 6
20. When $(x^4 - 3x^3 + 2x^2 - 5x + 7)$ is divided by $(x - 2)$, then the remainder is :
 (a) 0 (b) -3
 (c) 2 (d) 3
21. If $x^3 + 5x^2 + 10k$ leaves remainder $-2x$ when divided by $x^2 + 2$, then the value of k is :
 (a) -2 (b) -1 (c) 1 (d) 2
22. If $(x^{11} + 1)$ is divided by $(x + 1)$, then the remainder is :
 (a) 0 (b) 2 (c) 1 (d) 10
23. If $5x^3 + 5x^2 - 6x + 9$ is divided by $(x + 3)$, then the remainder is :
 (a) 0 (b) -32
 (c) 23 (d) -63
24. If $f(x)$ is divided by $(2x + 3)$, then the remainder is :
 (a) $f(2/3)$ (b) $f(-2/3)$
 (c) $f(3/2)$ (d) $f(-3/2)$
25. When $(x^3 - 2x^2 + px - q)$ is divided by $(x^2 - 2x - 3)$, the remainder is $(x - 6)$. The values of p and q are :
 (a) $p = -2, q = -6$ (b) $p = 2, q = -6$
 (c) $p = -2, q = 6$ (d) $p = 2, q = 6$
26. If $(x - 2)$ is a factor of $(x^2 + 3qx - 2q)$, then the value of q is :
 (a) -2 (b) -1
 (c) 1 (d) 2
27. Find the value of k , if $(x + 2)$ exactly divides $x^3 + 6x^2 + 4x + k$.
 (a) 4 (b) 6
 (c) -8 (d) -10
28. Which one of the following is a factor of $x^4 - 5x^3 + 5x^2 - 10x + 24$?
 (a) $(x + 2)$ (b) $(x + 4)$
 (c) $(x - 2)$ (d) none of these
29. If $(x + k)$ is a common factor of $(x^2 + px + q)$ and $(x^2 + lx + m)$, then the value of k is :
 (a) $l + p$ (b) $m - q$ (c) $\frac{l - p}{m - q}$ (d) $\frac{m - q}{l - p}$
30. If $(x - a)$ is a factor of $(x^3 - 3x^2a + 2a^2x + b)$, then the value of b is :
 (a) 0 (b) 1
 (c) 2 (d) 3
31. $(x^{29} - x^{25} + x^{13} - 1)$ is divisible by :
 (a) $(x + 1)$ but not by $(x - 1)$
 (b) $(x - 1)$ but not by $(x + 1)$
 (c) both $(x + 1)$ and $(x - 1)$
 (d) neither $(x - 1)$ nor $(x + 1)$
32. One of the factors of $3x^3 + x^2 - 12x - 4$ is :
 (a) $3x - 2$ (b) $3x - 1$
 (c) $3x + 1$ (d) $3x + 2$
33. $x^{100} + 2x^{99} + k$ is divisible by $(x + 1)$, then the value of k is :
 (a) -3 (b) -2
 (c) 1 (d) 2
34. If $(x - 1)$ is a factor of $(x^3 - m)$, then the value of m is :
 (a) -6 (b) -5
 (c) 1 (d) 3
35. If the polynomial $f(x)$ is such that $f(-3) = 0$, then a factor of $f(x)$ is :
 (a) -3 (b) 3
 (c) $\frac{1}{3}$ (d) $x + 3$

36. If $\left(x + \frac{1}{x}\right) = 2$, then $\left(x^2 + \frac{1}{x^2}\right)$ is equal, to :
- (a) 2 (b) 4
(c) 5 (d) 6
37. If $\left(x - \frac{1}{x}\right) = 4$, then the value of $\left(x^2 + \frac{1}{x^2}\right)$ is :
- (a) 16 (b) 18
(c) 14 (d) none of these
38. If $\left(x + \frac{1}{x}\right) = 2\sqrt{3}$, then the value of $\left(x^3 + \frac{1}{x^3}\right)$ is :
- (a) $12\sqrt{3}$ (b) 18
(c) $18\sqrt{3}$ (d) none of these
39. If $\left(x + \frac{1}{x}\right) = 3$, then the value of $\left(x^3 + \frac{1}{x^3}\right)$ is equal to :
- (a) $18\sqrt{3}$ (b) 18
(c) $9\sqrt{3}$ (d) 12
40. If $\left(x + \frac{1}{x}\right) = 2$, then the value of $\left(x^6 + \frac{1}{x^6}\right)$ is :
- (a) 2 (b) 4
(c) 8 (d) none of these
41. If $\left(x^2 + \frac{1}{x^2}\right) = 6$, then the value of $\left(x - \frac{1}{x}\right)$ is :
- (a) 2 (b) 3
(c) -2 (d) both (a) and (c)
42. If $\left(x^3 - \frac{1}{x^3}\right) = 36$, then the value of $\left(x - \frac{1}{x}\right)$ is :
- (a) 1 (b) 2
(c) 3 (d) 6
43. If $\left(x^3 + \frac{1}{x^3}\right) = 2$, then the value of $\left(x + \frac{1}{x}\right)$ is :
- (a) 1 (b) 2
(c) 3 (d) 4
44. If $\left(x^4 + \frac{1}{x^4}\right) = 34$, then the value of $\left(x - \frac{1}{x}\right)$ is :
- (a) 1 (b) 2 (c) 3 (d) 4
45. If $\left(x^4 + \frac{1}{x^4}\right) = 119$, then the value of $\left(x^3 - \frac{1}{x^3}\right)$ is :
- (a) 49 (b) 27
(c) 36 (d) 63
46. If $\left(3x - \frac{2}{x}\right) = 5$, then the value of $\left(9x^2 - \frac{4}{x^2}\right)$ is :
- (a) 25 (b) 35
(c) 30 (d) none of these
47. If $m^2 - 4m + 1 = 0$, then the value of $\left(m^3 + \frac{1}{m^3}\right)$ is :
- (a) 48 (b) 52
(c) 64 (d) none of these
48. If $x + y = 13$ and $xy = 40$, then the value of $(x^2 + y^2)$ is :
- (a) 69 (b) 80
(c) 89 (d) none of these
49. If $(x + y) = 13$ and $xy = 36$, then the value of $(x^3 + y^3)$ is :
- (a) 369 (b) 936
(c) 693 (d) none of these
50. If $(x + y) = 8$ and $xy = 15$, then the value of $(x^3 - y^3)$ is :
- (a) 120 (b) 60
(c) 98 (d) 82
51. If $(x + y + z) = 6$ and $(xy + yz + zx) = 11$, then the value of $(x^3 + y^3 + z^3 - 3xyz)$ is :
- (a) 81 (b) 54
(c) 18 (d) none of these
52. If $(x + y + z) = 0$, then the value of $(x^3 + y^3 + z^3)$ is :
- (a) $3x^3y^3z^3$ (b) $3xyz$
(c) xyz (d) $3(x + y + z)$
53. If $a^{1/3} + b^{1/3} + c^{1/3} = 0$, then :
- (a) $a + b + c = 0$ (b) $a + b + c = 3abc$
(c) $a^3 + b^3 + c^3 = 0$ (d) $(a + b + c)^3 = 27abc$
54. If $2^x - 2^{x-1} = 8$, then the value of x^3 is :
- (a) 46 (b) 27
(c) 64 (d) 8
55. If $3^x + 3^{x+1} = 36$, then the value of x^x is :
- (a) 64 (b) 3125
(c) 81 (d) 4
56. If a and b are non zero rational unequal numbers, then $\frac{(a+b)^2 - (a-b)^2}{a^2b - ab^2}$ is equal to :
- (a) $\frac{1}{a-b}$ (b) $\frac{2}{a-b}$
(c) $\frac{4}{a-b}$ (d) $\frac{1}{ab}$
57. If $\frac{x}{(b-c)(b+c-2a)}$

$$= \frac{y}{(c-a)(c+a-2b)} = \frac{z}{(a-b)(a+b-2c)},$$
then the value of $(x + y + z)$ is :
- (a) $a + b + c$ (b) 0
(c) $a^2 + b^2 + c^2$ (d) can't be determined
58. $(a^4 + 5a^3 + 6a^2)$ is equal to :
- (a) $a(a^2 + 3)(a + 2)$ (b) $a(a + 3)(a^2 + 2)$
(c) $a^2(a + 3)(a + 2)$ (d) $a^2(a - 2)(a - 3)$
59. The factors of $(x^4 + 16)$ are :
- (a) $(x^2 + 4)(x^2 - 4)$ (b) $(x^2 + 4)(x^2 + 4)$
(c) $x(x^2 + 2)(x^4 + 4)$ (d) do not exist

60. If $(a^{3/2} - ab^{1/2} + a^{1/2}b - b^{3/2})$ is divided by $(a^{1/2} - b^{1/2})$, then the quotient is :
 (a) $a + b$ (b) $a - b$ (c) $a^{1/2} + b^{1/2}$ (d) $a^2 - b^2$
61. The factors of $x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)$ are :
 (a) $(x - y)(y - z)(z - x)$ (b) $(x + y)(y + z)(z + x)$
 (c) $(y - x)(z - y)(x - z)$ (d) $(x + y)(z - y)(x - z)$
62. $(a + b)^3 - (a - b)^3$ can be factorized as :
 (a) $2a(a^2 + 3b^2)$ (b) $2a(3a^2 + b^2)$
 (c) $2b(3a^2 + b^2)$ (d) $2a(a^2 + 3b^2)$
63. The factors of $(x^4 + 4)$ are :
 (a) $(x^2 + 2)^2$ (b) $(x^2 + 2x + 2)(x^2 - 2x + 2)$
 (c) $(x^2 + 2)(x^2 - 2)$ (d) none of these
64. The factors of $x^3 - 7x + 6$ are :
 (a) $(x - 1)(x + 3)(x - 2)$ (b) $(x + 1)(x + 2)(x - 3)$
 (c) $x(x - 6)(x - 1)$ (d) $(x^2 - 6)(x - 1)$
65. The factors of $(64x^3 - 216y^3)$ are :
 (a) $(4x - 6y)(16x^2 - 36y^2)$
 (b) $(4x - 6y)[16x^2 + 36y^2 + 24xy]$
 (c) $(4x + 6y)(4x - 6y)$
 (d) none of the above
66. The factors of $(a^2 + 4b^2 + 4b - 4ab - 2a - 8)$ are :
 (a) $(a - 2b - 4)(a - 2b + 2)$
 (b) $(a - b + 2)(a - 4b - 4)$
 (c) $(a + 2b - 4)(a + 2b + 2)$
 (d) none of the above
67. The factors of $(a - b)^3 + (b - c)^3 + (c - a)^3$ are :
 (a) $(a + b + c)(a^2 + b^2 + c^2 + ab + bc + ac)$
 (b) $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
 (c) $3(a - b)(b - c)(c - a)$
 (d) $(a - b)(b - c)(c - a)$
68. If $a + b + c = 0$, then $a^2 + b^2 + c^2$ is :
 (a) 0 (b) $2(a^2 - bc)$
 (c) $-4(ab + bc + ca)$ (d) $-2(ab + bc + ca)$
69. Which of the following statements are true?
 1. LCM of $10a^2bc, 15abc, 20ab^2c^2$ is $120a^2b^2c^2$
 2. HCF of $(x^2 - 6x + 9)$ and $(x^3 - 27)$ is $(x - 3)$
 3. HCF of $(6x^2 - 7x - 3)$ and $(2x^2 + 11x - 21)$ is $2x - 3$.
 (a) 1, 2, 3 (b) 1, 2 (c) 1, 3 (d) 2, 3
70. The LCM of $(2x^2 - 3x - 2)$ and $(x^3 - 4x^2 + 4x)$ is :
 (a) $x(2x^2 + 1)(x^2 + 2)$ (b) $x(2x + 1)(x - 2)^2$
 (c) $x(2x^2 + 1)(x - 1)^2$ (d) $x(2x + 1)(x^2 - 1)$
71. The GCD of $2(x^2 - y^2)$ and $5(x^3 - y^3)$ is :
 (a) $2(x^2 - y^2)$ (b) $x + y$
 (c) $x - y$ (d) $10(x^2 - y^2)(x^2 + y^2 + xy)$
72. The GCD of $(x^4 - 4x^2 + 3)$ and $(x^4 - x^2 - 6)$ is :
 (a) $(x - 1)$ (b) $(x + 1)$ (c) $(x^2 - 1)$ (d) $(x^2 - 3)$
73. The HCF of $(x^2 - 4)$, $(x^2 - 5x - 6)$ and $(x^2 + x - 6)$ is :
 (a) 1 (b) $x + 2$ (c) $x - 2$ (d) $(x^2 + x - 6)$
74. The GCD of $[x^2 - ax - (a + 1)]$ and $[ax^2 - x - (a + 1)]$ is :
 (a) $(x - 1)$ (b) $(x + 1)$
 (c) $(x + a + 1)$ (d) $(x - a - 1)$
75. LCM of the polynomials P and Q , where
 $P = (x - 2)(x + 1)^2(x + 3)^2$
 $Q = (x + 1)^2(x + 3)(x + 4)$ is given by :
 (a) $(x - 2)(x + 1)(x + 3)^2(x + 4)$
 (b) $(x - 2)(x + 4)(x + 3)^2(x + 1)^2$
 (c) $(x - 2)(x + 1)^2(x + 3)(x + 4)$
 (d) $(x + 1)(x - 2)(x + 3)(x + 4)$
76. The LCM of $(a^3 + b^3)$ and $(a^4 - b^4)$ is :
 (a) $(a^3 + b^3)(a^2 + b^2)(a - b)$
 (b) $(a^3 + b^3)(a^2 + b^2)(a + b)$
 (c) $(a^3 + b^3)(a^2 + b^2 + ab)(a + b)$
 (d) $(a^3 + b^3)(a^2 - b^2)(a - b)$
77. The HCF of $(x^4 - 1)$ and $(x^3 + x^2 + x + 1)$ is :
 (a) $(x^2 - 1)(x^2 + 1)$ (b) $(x^2 + 1)(x + 1)$
 (c) $(x + 1)(x^2 - 1)$ (d) $(x + 1)(x^2 + 1)(x^3 + 1)$
78. The GCD of $(2x^2 - 4x)$, $(3x^4 - 12x^2)$ and $(2x^5 - 2x^4 - 4x^3)$ is :
 (a) $x(x - 2)$ (b) $2x(x - 2)$ (c) $2x(2 - x)$ (d) $2x(2 + x)$
79. The HCF of two expressions P and Q is 1. Their LCM is :
 (a) $(P + Q)$ (b) $(P - Q)$
 (c) PQ (d) $(P)^Q$
80. The LCM of $(x + 2)^2(x - 2)$ and $(x^2 - 4x - 12)$ is :
 (a) $(x + 2)(x - 2)^2$ (b) $(x - 2)(x + 2)(x - 6)$
 (c) $(x - 2)(x + 2)$ (d) $(x - 2)(x - 6)(x + 2)^2$
81. The HCF of $a^2 - ab - 2b^2$ and $2a^2 - ab - b^2$ is :
 (a) $(a + b)$ (b) $(a - b)$
 (c) $(2a - 3b)$ (d) none of these
82. HCF and LCM of $a^2b^3c^4$ and $a^5b^4c^3$ are :
 (a) $a^2b^3c^4, a^5b^4c^3$ (b) $a^2b^3c^3, a^2b^3c^4$
 (c) $a^2b^3c^3, a^5b^4c^4$ (d) $a^5b^3c^3, a^5b^4c^3$
- Directions** (for Q. Nos. 83 to 86) Express each of the following as a rational expressions.
83. $\frac{(x + 3)}{(x - 2)} - \frac{(x + 1)}{(x - 3)}$:
 (a) $\frac{x - 7}{(x^2 + 5x - 6)}$ (b) $\frac{7 - x}{(x^2 - 5x + 6)}$
 (c) $\frac{x - 7}{2x^2 - 5x + 6}$ (d) $\frac{x - 7}{x^2 - 5x + 6}$

84. $\frac{x+1}{x-1} + \frac{x-1}{x+1} :$

(a) $\frac{1}{2} \frac{(x^2+1)}{(x^2-1)}$ (b) $\frac{(x^2+1)}{(x^2-1)}$ (c) $\frac{2(x^2+1)}{(x^2-1)}$ (d) $\frac{x^2-1}{x^2+1}$

85. $\frac{x^2-5x+6}{x^2-9x+20} \div \frac{x^2-3x+2}{x^2-5x+4} :$

(a) $\frac{(x-4)}{(x-3)}$ (b) $\frac{(x-3)}{(x-5)}$

(c) $\frac{(x-2)}{(x-5)}$ (d) $\frac{(x-5)}{(x-3)}$

86. Sum of $\frac{2x^2+3}{x-3}$ and its reciprocal is :

(a) $\frac{4x^4+13x^2-6x+18}{2x^3-6x^2+3x-9}$ (b) $\frac{4x^4-13x^2+6x-18}{3x^2-6x^3+3x+9}$

(c) $\frac{4x^4-13x^2-6x+18}{4x^3-6x^2+3x-9}$ (d) none of these

Directions (for Q. Nos. 87 to 89) Express the following in the lowest terms.

87. $\frac{(x-3)(x^2-5x+4)}{(x-4)(x^2-2x-3)} :$

(a) $\frac{x-1}{x+1}$ (b) $\frac{x+1}{x-1}$ (c) $\frac{x-1}{x-2}$ (d) $\frac{x+1}{x+3}$

88. $\left(\frac{2x^2+1}{x-1} + \frac{x-1}{x+1} \right) \times \left(\frac{x^2-1}{2x} \right) :$

(a) $\frac{2x^2+3x}{2}$ (b) $\frac{2x^3+3x^2-x+2}{2x}$

(c) $\frac{2x^3-3x^2+x-2}{2x}$ (d) none of these

89. $\sqrt{\frac{(x^2+3x+2)(x^2+5x+6)}{x^2(x^2+4x+3)}} :$

(a) $\frac{x+2}{2}$ (b) $\frac{x+2}{x}$

(c) $\frac{x-2}{x}$ (d) $\frac{x+4}{x+3}$

90. Simplify $\frac{1}{(a-b)(a-c)} + \frac{1}{(b-c)(b-a)} + \frac{1}{(c-a)(c-b)}$

(a) 0 (b) 1
(c) 3 (d) none of these

91. Express $\frac{1}{(1-x)} + \frac{1}{(1+x)} + \frac{2}{(1+x^2)} + \frac{4}{(1+x^4)}$ as a rational expression :

(a) 1 (b) $\frac{1-x^4}{1+x^4}$
(c) $\frac{8}{1-x^8}$ (d) none of these

13.10 Linear Equations

Equations : A statement of equality which involves literal number(s) is called an equation.

e.g., $4x=12$, $4+x=10$, $7-2x=5$ etc.

NOTE The literal numbers involved in each equation are called its variables (unknowns). Usually the variables are denoted by letters towards the end of English alphabet, e.g., x, y, z, u, v, w , etc.

Linear Equation : An equation in which the highest power of the variables involved is one, is called a linear equation. e.g., $x+y=10$, $7x=21$, $\frac{x}{3}=8$.

Linear Equation in one Variable : An equation containing only one variable (literal) with highest power 1 is called a linear equation in one variable. e.g., $17x=51$, $23+x=30$, $5y=30$, etc.

Linear Equation in two Variables : An equation of the form $ax+by=c$, where a, b and c are real numbers is called a linear equation in two variables x and y .

The graph of a linear equation $ax+by=c$ is a straight line. $3x+2y=18$, is an example of a linear equation in two variables.

• The value of the variables that satisfy the equation is called the solution (or solution set) of the equation.

An equation can be solved by using the following rules :

- The same number may be added to both sides of the equation.
- The same number may be subtracted from both sides of the equation.
- Both sides of the equation may be multiplied by same non zero number.
- Both sides of the equation may be divided by same non-zero number.
- A term may be transposed from one side of the equation to the other side, but its sign will change. This is called rule of transposition.

Important Points About Linear Equation in Two Variables:

- The graph of an equation of the type $x=k$ (where k is a constant) is a straight line parallel to the y -axis at a distance of k units from the y -axis.

- (ii) The graph of an equation of the type $y = k$ (where k is a constant) is a straight line parallel to the x -axis at a distance of k units from the x -axis.
- (iii) The point of intersection of the two lines gives the solution of the two equations.
- (iv) A single linear equation in two variables has infinite no. of solutions.

13.11 Simultaneous Linear Equations

Two or more linear equations in two variables form a system of linear simultaneous equations.

e.g., $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$

Consistent System : A system consisting of two simultaneous linear equations is said to be consistent, if it has at least one solution.

Inconsistent System : A system consisting of two simultaneous linear equations is said to be inconsistent, if it has no solution. If there are two simultaneous equations :

$$a_1x + b_1y = c_1 \quad \text{and} \quad a_2x + b_2y = c_2, \text{ then}$$

Unique solution $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (Independent equation)	Infinite solution $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (Dependent equation)	No solution $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (Inconsistent equation)

- The homogeneous system has a non-zero solution only when $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ and in this case, the system has an infinite number of solution.
- A system of equations has unique solution, when only one variable satisfies the equation.
- For a system of equations a unique solution is possible only when the number of variables is equal to or less than number of independent and consistent equations.

e.g., $2x + 3y = 5$ and $7x + 5y = 20$

and $2x + 3y = 5$, $7x + 5y = 12$, $5x + 8y = 13$

- The equation of the type $ax + by = c$ and $kax + kby = kc$ are known as dependent equations.

e.g., $2x + y = 11$... (i)

$6x + 3y = 33$... (ii)

Here eq. (ii) is the multiple of eq. (i). Thus, there are basically both the equations same, i.e., eq. (ii) is dependent on eq. (i). In this case there are infinite number of solutions.

- The equation of the type $ax + by = c$ and $kax + kby = lc$ are known as inconsistent equations.

e.g., $3x + 2y = 8$... (i)

and $6x + 4y = 6$... (ii)

Properties of graphs of $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$

- (i) intersecting if the system has a unique solution.
- (ii) coincident if the system has infinite number of solution
- (iii) parallel if the system has no solution.

Graphical Representation of Simultaneous Linear Equations

- (i) intersecting lines (unique solution)

e.g., $x + y = 4$... (i)

and $3x + 2y = 11$... (ii)

$\therefore 3(x + y) = 3 \times 4 \Rightarrow 3x + 3y = 12$

$\therefore 3x + 3y = 12$

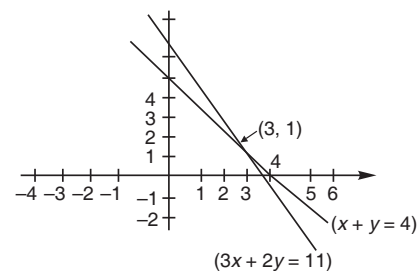
$3x + 2y = 11$

$\underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}}$
 $y = 1$

$\therefore x + y = 4 \Rightarrow x + 1 = 4 \Rightarrow x = 3$

$\therefore x = 3$ and $y = 1$

Thus, two given lines intersect at $x = 3, y = 1$.

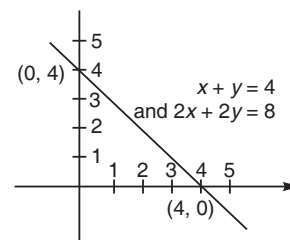


- (ii) Coincident lines (infinite number of solutions)

e.g., $x + y = 4$... (i)

$2x + 2y = 8$... (ii)

The two equations are dependent, therefore the graph of two lines will coincide.

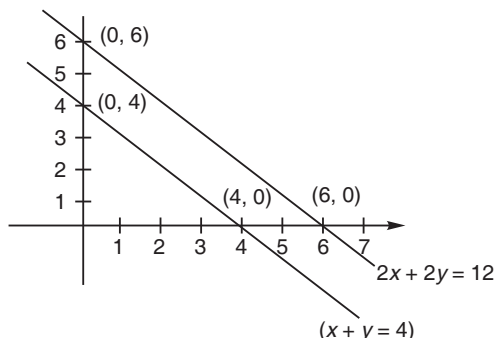


- (iii) Parallel lines (no solution)

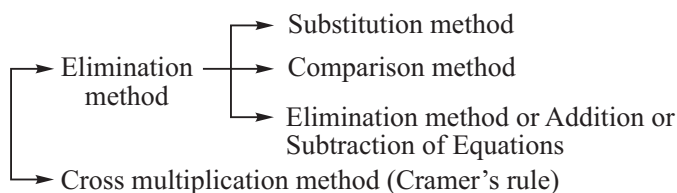
e.g., $x + y = 4$... (i)

$$2x + 2y = 12 \quad \dots(ii)$$

Since, the system of equations is inconsistent therefore there is no any solution and in this case we obtain two different parallel lines.



13.12 Algebraic Methods of Solving Simultaneous Equations in Two Variables



Exp. 1) Solve the following equations $3x + 5y = 9$ and $2x + 3y = 4$

- (a) by substitution method (b) by elimination method
(c) by comparison method

Solution (a) $3x + 5y = 9 \quad \dots(i)$

$2x + 3y = 4 \quad \dots(ii)$

Express y in terms of x from eq. (i)

$$y = \frac{9 - 3x}{5} \quad \dots(iii)$$

Substitute this value of y in eq. (ii)

$$2x + 3\left(\frac{9 - 3x}{5}\right) = 4$$

$$\Rightarrow 10x + 27 - 9x = 20 \Rightarrow x = -7$$

Substitute $x = -7$ in eq. (iii)

$$\therefore x = -7, y = 6.$$

NOTE We can express x in terms of y for one equation and substitute this value in other.

(b) $3x + 5y = 9 \quad \dots(i)$

$2x + 3y = 4 \quad \dots(ii)$

Multiply eq. (i) by 3 and (ii) by 5.

$9x + 15y = 27 \quad \dots(iii)$

$10x + 15y = 20 \quad \dots(iv)$

$$\begin{array}{r} 9x + 15y = 27 \\ 10x + 15y = 20 \\ \hline -x + 0 = 7 \\ \hline \Rightarrow x = -7 \end{array}$$

Substitute $x = -7$ in eq. (i)

$$3(-7) + 5y = 9$$

$$\Rightarrow -21 + 5y = 9$$

$$\Rightarrow y = 6$$

$$\therefore x = -7 \text{ and } y = 6$$

NOTE We can eliminate x by multiplying (i) by 2 and (ii) by 3 and then subtract the resulting equations.

(c) $3x + 5y = 9 \quad \dots(i)$

$2x + 3y = 4 \quad \dots(ii)$

From eq. (i) $x = \frac{9 - 5y}{3} \quad \dots(iii)$

and from eq. (ii) $x = \frac{4 - 3y}{2} \quad \dots(iv)$

Comparing eq. (iii) and (iv)

$$\frac{9 - 5y}{3} = \frac{4 - 3y}{2}$$

$$\Rightarrow 2(9 - 5y) = 3(4 - 3y) \Rightarrow 18 - 10y = 12 - 9y$$

$$\Rightarrow y = 6 \quad \dots(v)$$

Substitute the value of y in eq. (iii)

$$x = \frac{9 - 5(6)}{3} = \frac{-21}{3} = -7$$

$$\therefore x = -7 \text{ and } y = 6$$

NOTE We can compare the value of y in terms of x .

Exp. 2) Solve the equations $3x + 5y = 9$ and $2x + 3y = 4$ by cross multiplication method.

Solution If there are two simultaneous equations

$$a_1x + b_1y = c_1 \text{ and } a_2x + b_2y = c_2, \text{ where } c_2 \neq 0,$$

Then

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{-1}{a_1b_2 - a_2b_1}$$

$$\left[\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{-1}{a_1b_2 - a_2b_1} \right]$$

Here, $a_1 = 3, b_1 = 5, c_1 = 9$

$a_2 = 2, b_2 = 3, c_2 = 4$

$$\therefore \frac{x}{(20 - 27)} = \frac{y}{(18 - 12)} = \frac{-1}{(9 - 10)} \Rightarrow \frac{x}{-7} = \frac{y}{6} = 1$$

$$\Rightarrow x = -7 \text{ and } y = 6.$$

Exp. 3) Find the correct choice.

(A) $\begin{cases} 5x + 2y = 16 \\ 7x - 4y = 2 \end{cases} \quad (P) \text{ no solution}$

(B) $\begin{cases} 5x + 2y = 16 \\ 3x + \frac{6}{5}y = 2 \end{cases} \quad (Q) \text{ unique solution}$

(C) $\begin{cases} 5x + 2y = 16 \\ \frac{15}{2}x + 3y = 24 \end{cases} \quad (R) \text{ infinite number of solution}$

(a) $A - P, B - Q, C - R$ (b) $A - R, B - P, C - Q$

(c) $A - Q, B - P, C - R$ (d) $A - Q, B - R, C - P$

Solution

$$(A) \quad \frac{5}{7} \neq \frac{2}{-4}.$$

$\therefore$ The system of equations has unique solution.

$$(B) \quad \frac{5}{3} = \frac{2}{6/5} \neq \frac{16}{2}.$$

$$\Rightarrow \quad \frac{5}{3} = \frac{5}{3} \neq \frac{8}{1}$$

$\therefore$ The system of equations has no solution.

$$(C) \quad \frac{5}{15/2} = \frac{2}{3} = \frac{16}{24} \Rightarrow \frac{2}{3} = \frac{2}{3} = \frac{2}{3}$$

$\therefore$ The system of equations has infinite number of solution.

Exp. 4) Solve for x and y

$$\begin{aligned} 4(x-3) &= 3(y+3) \\ 17-2x &= 8-(2y+1) \end{aligned}$$

Solution

$$4(x-3) = 3(y+3)$$

$$\Rightarrow \quad 4x - 3y = 21 \quad \dots(i)$$

$$\text{and} \quad 17 - 2x = 8 - (2y + 1)$$

$$\Rightarrow \quad x - y = 5 \quad \dots(ii)$$

Multiply eq. (ii) by 3 and subtract from eq.

$$\begin{array}{r} 4x - 3y = 21 \\ 3x - 3y = 15 \\ \hline - \quad + \quad - \\ x = 6 \end{array}$$

Substitute $x = 6$ in eq. (ii), we get $y = 1$

$\therefore \quad x = 6 \quad \text{and} \quad y = 1.$

Exp. 5) Solve for x and y

$$\begin{aligned} 0.04x + 0.02y &= 5 \\ 0.5(x-2) - 0.4y &= 29 \end{aligned}$$

Solution

$$0.04x + 0.02y = 5$$

$$\Rightarrow \quad 4x + 2y = 500 \quad (\text{multiply both sides by } 100)$$

$$\Rightarrow \quad 2x + y = 250 \quad \dots(i)$$

$$\text{and} \quad 0.5(x-2) - 0.4y = 29$$

$$\Rightarrow \quad 5(x-2) - 4y = 290 \quad (\text{multiply both sides by } 10)$$

$$\Rightarrow \quad 5x - 4y = 300 \quad \dots(ii)$$

Multiply eq. (i) by 4 and add to eq. (ii)

$$\begin{array}{r} 8x + 4y = 1000 \\ 5x - 4y = 300 \\ \hline 13x = 1300 \end{array}$$

$$\Rightarrow \quad x = 100$$

Substitute $x = 100$, in eq. (i), we get

$$y = 50$$

$\therefore \quad x = 100 \quad \text{and} \quad y = 50.$

Exp. 6) Solve for x and y

$$\frac{3x-y}{5} = 2y-1, \quad \frac{3x}{8} - \frac{y}{4} = \frac{1}{2}$$

Solution

$$3x - y = 5(2y - 1)$$

$$\Rightarrow \quad 3x - 11y = -5 \quad \dots(i)$$

and

$$\frac{3x}{8} - \frac{y}{4} = \frac{1}{2} \Rightarrow 3x - 2y = 4 \quad \dots(ii)$$

Subtract eq. (ii) from (i)

$$\begin{array}{r} 3x - 11y = -5 \\ 3x - 2y = 4 \\ \hline - \quad + \quad - \\ -9y = -9 \Rightarrow y = 1 \end{array}$$

Substitute $y = 1$ in eq. (i), we get

$$x = 2 \quad \therefore \quad x = 2 \quad \text{and} \quad y = 1$$

Exp. 7) Solve for x and y $\frac{2}{x} + \frac{3}{y} = 2, \quad \frac{8}{x} - \frac{6}{y} = 2$

Solution

$$\frac{2}{x} + \frac{3}{y} = 2 \quad \dots(i)$$

$$\frac{8}{x} - \frac{6}{y} = 2 \quad \dots(ii)$$

$$\text{Let} \quad \frac{1}{x} = A \quad \text{and} \quad \frac{1}{y} = B$$

$$\therefore \quad 2A + 3B = 2 \quad \dots(iii)$$

$$\text{and} \quad 8A - 6B = 2 \quad \dots(iv)$$

Multiply eq. (iii) by 2

$$\begin{aligned} 4A + 6B &= 4 \Rightarrow 8A - 6B = 2 \\ 12A &= 6 \Rightarrow A = 1/2 \end{aligned}$$

Substitute $A = \frac{1}{2}$ in eq. (i), we get

$$B = \frac{1}{3}$$

$\therefore \quad x = 2 \quad \text{and} \quad y = 3.$

Exp. 8) Solve for x and y $\frac{2}{3x+2y} + \frac{3}{3x-2y} = \frac{17}{5}$

$$\frac{5}{3x+2y} + \frac{1}{3x-2y} = 2$$

$$\text{Solution} \quad \text{Let} \quad \frac{1}{3x+2y} = A \quad \text{and} \quad \frac{1}{3x-2y} = B$$

$$\therefore \quad 2A + 3B = \frac{17}{5} \Rightarrow 10A + 15B = 17 \quad \dots(i)$$

$$\text{and} \quad 5A + B = 2 \quad \dots(ii)$$

Multiply eq. (ii) by 2 and subtract from eq. (i)

$$\begin{array}{r} 10A + 15B = 17 \\ 10A + 2B = 4 \\ \hline 13B = 13 \Rightarrow B = 1 \end{array}$$

Substitute $B = 1$ in eq. (ii), we get $A = \frac{1}{5}$

$$\text{Now,} \quad A = \frac{1}{3x+2y} = \frac{1}{5} \Rightarrow 3x + 2y = 5 \quad \dots(iii)$$

$$\text{and} \quad B = \frac{1}{3x-2y} = 1 \Rightarrow 3x - 2y = 1 \quad \dots(iv)$$

$\therefore$ Add eq. (iii) to eq. (iv) $3x + 2y = 5$

$$3x - 2y = 1 \Rightarrow 6x = 6 \Rightarrow x = 1$$

Substitute $x = 1$ in eq. (iii), we get $y = 1$

$\therefore \quad x = 1 \quad \text{and} \quad y = 1.$

Introductory Exercise 13.2

Directions (for Q. Nos. 1 to 4) *Solve the following system of equations*

- $x - y = 0.9, \frac{11}{2(x+y)} = 1$:
(a) 3.2, 3.5 (b) 3.2, 2.3 (c) 2.3, 2.5 (d) 4, 3.1
- $\frac{x+y}{xy} = 2, \frac{x-y}{xy} = 6$:
(a) $-\frac{1}{2}, \frac{1}{4}$ (b) $\frac{1}{2}, -\frac{1}{4}$ (c) $\frac{1}{2}, -\frac{3}{2}$ (d) $-\frac{1}{4}, -\frac{3}{4}$
- $\frac{1}{2x} - \frac{1}{y} = -1, \frac{1}{x} + \frac{1}{2y} = 8$:
(a) $\frac{2}{3}, \frac{2}{5}$ (b) $\frac{1}{4}, \frac{1}{5}$ (c) $\frac{1}{6}, \frac{1}{4}$ (d) $-\frac{1}{6}, -\frac{1}{4}$
- $\frac{2x}{a} + \frac{y}{b} = 2, \frac{x}{a} - \frac{y}{b} = 4$:
(a) $\frac{2}{a}, \frac{2}{b}$ (b) $2a, -2b$ (c) $-2a, 2b$ (d) $\frac{a}{2}, -\frac{b}{2}$
- For what value of k , the following equations have no solutions?
 $9x + 4y = 9; 7x + ky = 5$
(a) 3 (b) 4.7 (c) 28/9 (d) 9/28
- For what value of k , the system of equations $kx + 2y = 2$ and $3x + y = 1$ will be coincident?
(a) 2 (b) 3 (c) 5 (d) 6
- For what value of k , will the equations $2x + 3y - 5 = 0$, $6x + ky - 15 = 0$ have an infinite number of solutions?
(a) 0 (b) 9 (c) 2 (d) 3
- The total cost of 8 buckets and 5 mugs is ₹92 and the total cost of 5 buckets and 8 mugs is ₹77. Find the cost of 2 mugs and 3 buckets.
(a) ₹ 35 (b) ₹ 70 (c) ₹ 30 (d) ₹ 38
- The sum of two numbers is 8. If their sum is four times their difference, find the numbers.
(a) 6, 2 (b) 7, 1 (c) 5, 3 (d) 6, 3
- The numerator of a fraction is 4 less than its denominator. If the numerator is decreased by 2 and the denominator is increased by 1, then the denominator is eight times the numerator. Find the fraction.
(a) 3/7 (b) 4/8 (c) 2/7 (d) 3/8
- Points A and B are 90 km apart from each other on a highway. A car starts from A and another from B at the same time. If they go in the same direction, they meet in 9 hours and if they go in opposite directions, they meet in 9/7 hours. Find their speed (in km/h).
(a), 25, 45 (b) 20, 50 (c) 30, 40 (d) 35, 65
- Priyanka has only 25 paise coins and 50 paise coins in her purse. If in all she has 40 coins totalling ₹12.75, how many coins of 25 paise does she have?
(a) 30 (b) 29 (c) 32 (d) 26
- Astha has pens and pencils which together are 40 in number. If she has 5 more pencils and 5 less pens, the number of pencils would have become 4 times the number of pens. Find the original number of pens with Astha.
(a) 11 (b) 12 (c) 13 (d) 14
- A father is three times as old as his son. After twelve years, his age will be twice as that of the age of his son, then find their present ages.
(a) 10 y, 30 y (b) 12 y, 36 y
(c) 6 y, 36 y (d) 124 y, 24 y
- Five years ago, A was three times as old as B and ten years later, A shall be twice as old as B. What are the present ages of A and B (in years)?
(a) 45, 15 (b) 30, 40 (c) 50, 30 (d) 50, 20
- Astha and Saumya each have certain number of oranges. Astha says to Saumya, "If you give me 10 of your oranges, I will have twice the number of oranges left with you." Saumya replies, "If you give me 10 of your oranges, I will have the same number of oranges as left with you." Find the number of oranges with Astha and Saumya respectively.
(a) 60, 40 (b) 70, 50 (c) 60, 80 (d) 70, 90
- Shubham travels 760 km to his home partly by train and partly by car. He takes 8 hours if he travels 160 km by train and the rest by car. He takes 12 minutes more if he travels 240 km by train and the rest by car. Find the speed of the train and the car respectively (in km/hr.).
(a) 40, 80 (b) 60, 120 (c) 80, 100 (d) 100, 120
- A person invested some amount at the rate of 12% simple interest and some other amount at the rate of 10% simple interest. He received yearly interest of ₹130. But if he had interchanged the amounts invested he would have received ₹ 4 more as interest. How much amount did he invest at 10% simple interest?
(a) ₹ 700 (b) ₹ 500 (c) ₹ 800 (d) ₹ 400
- The solution of the equations $\frac{3x - y + 1}{3} = \frac{2x + y + 2}{5} = \frac{3x + 2y + 1}{6}$ given by :
(a) $x = 2, y = 1$ (b) $x = 1, y = 1$
(c) $x = -1, y = -1$ (d) $x = 1, y = 2$

20. The course of an enemy submarine as plotted on a set of rectangular axes gives the equation $2x + 3y = 5$. On the same axes, the course of destroyer is indicated by $x - y = 10$. The point (x, y) at which the submarine can be destroyed is :

(a) $(-3, 7)$ (b) $(7, -3)$ (c) $(-7, 3)$ (d) $(3, -7)$

21. Let a, b, c be the positive numbers. The following system of equations in x, y and z .

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1; \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \text{ has}$$

- (a) no solution
(b) unique solution
(c) infinitely many solutions
(d) finitely many solutions

22. If the system of equations, $x - ky - z = 0$, $kx - y - z = 0$, $x + y + z = 0$, has a non zero solution, then the possible values of k are :

(a) $-1, 2$ (b) $1, 2$ (c) $0, 1$ (d) $-1, 1$

CAT-Test

Questions Helping you bell the CAT

LEVEL 01 > BASIC LEVEL EXERCISE

- If $a + b + c = 13$, what is the maximum value of $(a - 3)(b - 2)(c + 1)$?
(a) 26 (b) 27
(c) 30 (d) 19
- If $abcd = 81$, what is the minimum value of $a + b + c + d$?
(a) 18 (b) 9
(c) 12 (d) 36
- Which one of the following is correct?
(a) $x + \frac{1}{x} \leq -2$ (b) $x + \frac{1}{x} = 0$
(c) $x + \frac{1}{x} \geq 2$ (d) both (a) and (c)
- If a, b and c are all distinct positive numbers, then $(a + b)(b + c)(c + a)$:
(a) less than 4 (b) less than $4abc$
(c) greater than $8abc$ (d) greater than $(abc)^3$
- If a, b and c are the distinct positive numbers, then $(a + b + c)(ab + bc + ac)$ is :
(a) greater than $9abc$ (b) less than $8abc$
(c) equal to $10abc$ (d) greater than $25abc$
- If $x + y = 25$ and $x^2y^3 + y^2x^3 = 25$, what is the value of xy ?
(a) 0 (b) ± 1
(c) 5 (d) 4
- If x and y are both positive, then the minimum value of $(x + y)\left(\frac{1}{x} + \frac{1}{y}\right)$ is :
(a) 0 (b) 1 (c) 2 (d) 4
- If a, b, c and d are four positive numbers such that $a + b + c + d = 4$, then what is the maximum value of $(a + 1)(b + 1)(c + 1)(d + 1)$?
(a) 32 (b) 8 (c) 16 (d) 81
- If x, y and z are three distinct positive real numbers such that $x + y + z = 1$, then the value of $\left(\frac{1}{x} - 1\right)\left(\frac{1}{y} - 1\right)\left(\frac{1}{z} - 1\right)$ is :
(a) 16 (b) 8 (c) 4 (d) 1
- If x, y and z are real numbers such that $x + y + z = 4$ and $x^2 + y^2 + z^2 = 6$, then x, y, z lie in :
(a) $\left[\frac{3}{2}, 2\right]$ (b) $\left[\frac{2}{3}, 2\right]$
(c) $\left[0, \frac{2}{3}\right]$ (d) none of these
- If $x = 7 + 4\sqrt{3}$ and $xy = 1$, then the value of $\left(\frac{1}{x^2} + \frac{1}{y^2}\right)$ is
(a) 194 (b) 57 (c) $85\sqrt{3}$ (d) $\frac{7 + 4\sqrt{3}}{7 - 4\sqrt{3}}$

12. If a, b and c are positive real numbers, the least value of $(a + b + c) \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$ is :
- (a) 1 (b) 9
(c) 12 (d) none of these
13. If $a > 0, b > 0$ and $ab = 1$, then the least value of the expression $(1 + a)(1 + b)$ is :
- (a) 2 (b) 4
(c) 1 (d) $1/4$
14. If a, b, c are all positive and not equal, then the value of $\frac{(a + b + c)(ab + bc + ca)}{abc}$ is :
- (a) less than 9 (b) greater than 9
(c) less than or equal to 9 (d) can't be determined
15. If a, b, c are all positive, then the minimum value of the expression $\frac{(a^2 + a + 1)(b^2 + b + 1)(c^2 + c + 1)}{abc}$ is :
- (a) 3 (b) 9
(c) 27 (d) 1
16. If $1 \leq x \leq 3$ and $2 \leq y \leq 4$, what is the maximum value of $\left(\frac{x}{y} \right)^2$?
- (a) $\frac{2}{3}$ (b) 4
(c) $\frac{3}{2}$ (d) 2
17. If $a + b + c = 3, a^2 + b^2 + c^2 = 6$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$ where a, b, c are all non-zero, then abc is :
- (a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $\frac{3}{2}$ (d) 1
18. $2^x = 4^y = 8^z$ and $xyz = 288$, then value of $\frac{1}{2x} + \frac{1}{4y} + \frac{1}{8z}$ is :
- (a) $\frac{11}{12}$ (b) $\frac{11}{96}$
(c) $\frac{29}{96}$ (d) none of these
19. If $a^x = b, b^y = c, c^z = a$, then the value of xyz is :
- (a) 3 (b) 0
(c) 1 (d) can't be determined
20. Let $x, y \in N$ and $7x + 12y = 220$. The number of solutions is :
- (a) 1 (b) 2
(c) 3 (d) infinitely many
21. If $a^x = (x + y + z)^y, a^y = (x + y + z)^z, a^z = (x + y + z)^x$, then :
- (a) $3(x + y + z) = a$ (b) $2a = x + y + z$
(c) $x + y + z = 0$ (d) $x = y = z = \frac{a}{3}$
22. If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$, then $xy + yz + zx$ is equal to :
- (a) $\frac{(a + b + c)^2}{x^2 + y^2 + z^2}$
(b) $\frac{x^2(a + b + c)^2 - a^2(x^2 + y^2 + z^2)}{2a^2}$
(c) $\frac{ax + by + cz}{(a + b + c)^2}$
(d) none of the above
23. In the farm-house of Bhuwan Kisan, there were total 14 deer and ducks. One day a thief entered secretly in the farm-house and after killing all the deer and ducks he took away all the stuff except their legs. When Bhuwan noticed it he has found there 38 legs of deer and ducks. The number of deer in the farm house was :
- (a) 5 (b) 8 (c) 9 (d) 10
24. In an autofair at Pragati Maidan, New Delhi, there are 600 vehicles consisting of only two wheelers and four wheelers. A world renowned tyre agency visits the fair to survey the tyre market which finds that there are total 2000 tyres are fixed in all the 600 vehicles, but without any stepney (or spare tyre). The number of two wheelers in the autofair is :
- (a) 300 (b) 400 (c) 180 (d) 200
25. In the battle of Mahabharat Veer Arjun destroyed all the 125 raths (horse-carts). Out of which few were driven by 3 horses each and rest were driven by 5 horses each. Thus, he captured 125 damaged raths along with 575 horses. The raths which were driven by 5 horses belonged to the Kaurav and the raths which were driven by 3 horses belonged to the allied army of Kaurav. It is known that no any horse was injured in the battle. The number of raths originally belonged to the Kaurav (not to their allied army) is :
- (a) 60 (b) 25 (c) 75 (d) 100
26. In an electric appliances shop there were some fans and coolers containing 3 blades and 4 blades each respectively along with a motor each. The cost of a fan is ₹ 1200 and that of cooler is ₹ 3000. One day due to short circuit the shop caught the fire. After extinguishing the fire, shopkeeper has estimated that 90 sets of motor bodies and 320 blades were burned and then nothing was left useful in his shop. What is the loss incurred from the coolers alone?
- (a) ₹ 1,50,000 (b) ₹ 12,000
(c) ₹ 4800 (d) can't be determined
27. In the air talent show in the eve of independence day 90 aircraft (only Sukhoi and Mig-21) participated. It is known that for every one hour flight a Sukhoi and a Mig-21 consume 2 litres and 3 litres of fuel respectively. The price of fuel for Sukhoi is \$ 10 per litre and price for the fuel of Mig-21 is \$ 15 per litre. Thus, all the aircrafts accounted \$ 3050 for the fuel in one hour show. The number of Sukhoi planes is :
- (a) 20 (b) 30 (c) 40 (d) 50

Directions (for Q. Nos. 28 to 30) *In the fruit market adjacent to IIM Lucknow, a fruit vendor has total 20 kg of fruits containing only mangoes and apples. The vendor told to Ritika, an IIM student, that one kg apple contains only 7 apples while 1 kg of mangoes contains 10 mangoes and it is known that all the mangoes and apples are uniform in size individually. The vendor has initially 176, pieces of apples and mangoes, in all. Ritika purchased 55 fruits to distribute equally among the group of 11 students, irrespective of the kind of fruit i.e., mango or apple, and then the vendor is left with only 13 kg of fruits.*

- 28.** The number of apples remaining with the vendor when Ritika has purchased 7 kg of fruits, is :
 (a) 21 (b) 35
 (c) 80 (d) none of these
- 29.** If the selling price of mangoes and apples be ₹ 35 per kg and ₹ 40 per kg respectively, then how many rupees Ritika has spent for the 7 kg of fruits?
 (a) ₹ 262.5 (b) ₹ 255 (c) ₹ 270 (d) ₹ 155
- 30.** If the vendor has received 16.66% profit in selling of mangoes and 33.33% profit in the selling of apples to Ritika, but for the rest of the selling he exchanges the profit

per cent between mangoes and apples. What is his overall profit per cent for the rest of the fruits? (Data can be used from the previous question if necessary)

- (a) 29.48% (b) 5.92%
 (c) no any profit (d) can't be determined

- 31.** One day my assistant Rajesh went to a stationary shop to purchase marker pens. He purchased the pens for ₹ 52 and he gave a ₹ 100 note to the shopkeeper. The shopkeeper offered Rajesh to take back ₹ 48 in the denominations of only ₹ 1, ₹ 2 and ₹ 5.

So, Rajesh received total 26 coins from the shopkeeper. Minimum number of ₹ 1 coins which Rajesh has with him if he has at least one coin of each denomination offered by the shopkeeper :

- (a) 4 (b) 5
 (c) 7 (d) 10

- 32.** What is the solutions of the following simultaneous equations?

$$x + y + z = 6, \quad x + 2y + 3z = 14, \quad x + 3y + z = 10$$

- (a) $x = 1, y = 2, z = 4$ (b) $x = 1, y = 2, z = 3$
 (c) $x = 2, y = 1, z = 3$ (d) none of these

Answers

✓ Introductory Exercise 13.1

1. (d)	2. (c)	3. (c)	4. (d)	5. (c)	6. (c)	7. (d)	8. (b)	9. (c)	10. (d)
11. (b)	12. (a)	13. (b)	14. (c)	15. (b)	16. (b)	17. (d)	18. (b)	19. (c)	20. (b)
21. (c)	22. (a)	23. (d)	24. (d)	25. (c)	26. (b)	27. (c)	28. (c)	29. (d)	30. (a)
31. (b)	32. (c)	33. (c)	34. (c)	35. (d)	36. (a)	37. (b)	38. (c)	39. (b)	40. (a)
41. (d)	42. (c)	43. (b)	44. (b)	45. (c)	46. (b)	47. (b)	48. (c)	49. (c)	50. (c)
51. (c)	52. (b)	53. (d)	54. (c)	55. (d)	56. (c)	57. (b)	58. (c)	59. (d)	60. (a)
61. (a)	62. (c)	63. (b)	64. (a)	65. (b)	66. (a)	67. (c)	68. (d)	69. (d)	70. (b)
71. (c)	72. (d)	73. (a)	74. (b)	75. (b)	76. (a)	77. (b)	78. (a)	79. (c)	80. (d)
81. (d)	82. (c)	83. (d)	84. (c)	85. (b)	86. (a)	87. (a)	88. (b)	89. (b)	90. (a)
91. (c)									

✓ Introductory Exercise 13.2

1. (b)	2. (a)	3. (c)	4. (b)	5. (c)	6. (d)	7. (b)	8. (a)	9. (c)	10. (a)
11. (c)	12. (b)	13. (c)	14. (b)	15. (d)	16. (b)	17. (c)	18. (a)	19. (b)	20. (b)
21. (d)	22. (d)								

✓ Level 01 Basic Level Exercise

1. (b)	2. (c)	3. (d)	4. (c)	5. (a)	6. (b)	7. (d)	8. (c)	9. (b)	10. (b)
11. (a)	12. (b)	13. (b)	14. (b)	15. (c)	16. (c)	17. (c)	18. (b)	19. (c)	20. (c)
21. (d)	22. (b)	23. (a)	24. (d)	25. (d)	26. (a)	27. (c)	28. (a)	29. (c)	30. (a)
31. (c)	32. (b)								

Hints & Solutions

Introductory Exercise 13.1

- Total score = $x + y + 40$
- Quotient = $\frac{z}{4} \therefore 3y \times \frac{z}{4} = \frac{3yz}{4}$
- $6 \times p \times q \times r \times p \times p \times p = 6 \times p \times p \times p \times p \times q \times r$
 $= 6 \times p^4 \times q \times r = 6p^4qr$
- (i) $2x^2$ (ii) $-\frac{3}{x}$ (iii) $\frac{5}{x^2}$ (iv) 9
- (i) $4xyz$ (ii) $-\frac{3}{x^2}$ (iii) $6xy^2$ (iv) $-5z^3$ (v) $6x$
- $7 \times a + b \div 3 - c + b = 7a + \frac{b}{3} - c + b$
 $\therefore$ (i) $7a$ (ii) $\frac{b}{3}$ (iii) $-c$ (iv) b
- $\frac{-7xy^2z^3}{z^3} = -7xy^2$
- $4x^3 \rightarrow$ degree 3
 $2x^2y^3 \rightarrow$ degree $(2 + 3) = 5$
 $6x^0y^0 \rightarrow$ degree 0
 $\therefore$ The degree of whole expression is 5
- $2x^2 + 3y^2 - 5xy + 5x^2 - y^2 + 6xy - 3x^2$
 $= 2x^2 - 3x^2 + 5x^2 + 3y^2 - y^2 - 5xy + 6xy$
 $= 4x^2 + 2y^2 + xy$
- $(5x^2 + 3x - 1) - (3x^3 - 2x + 7)$
or $5x^2 + 3x - 1$
 $\begin{array}{r} -3x^3 - 2x + 7 \\ \hline -3x^3 + 5x^2 + 5x - 8 \end{array}$ (In subtraction we change the sign)
- $\left(\frac{1}{2}a^2b - \frac{2}{3}ab^2 + 5\right) \times (6abc)$
 $= \frac{1}{2}a^2b \times 6abc - \frac{2}{3}ab^2 \times 6abc + 5 \times 6abc$
 $= 3a^3b^2c - 4a^2b^3c + 30abc$
- $\left(\frac{1}{2}x^2 - \frac{1}{3}y^2\right) \times \left(\frac{1}{2}x^2 + \frac{1}{3}y^2\right)$
 $= \frac{1}{2}x^2 \left(\frac{1}{2}x^2 + \frac{1}{3}y^2\right) - \frac{1}{3}y^2 \left(\frac{1}{2}x^2 + \frac{1}{3}y^2\right)$
 $= \left(\frac{1}{2}x^2 \times \frac{1}{2}x^2 + \frac{1}{2}x^2 \times \frac{1}{3}y^2\right) - \frac{1}{3}y^2 \times \frac{1}{2}x^2 - \frac{1}{3}y^2 \times \frac{1}{3}y^2$
 $= \frac{1}{4}x^4 + \frac{1}{6}x^2y^2 - \frac{1}{6}x^2y^2 - \frac{1}{9}y^4 = \frac{1}{4}x^4 - \frac{1}{9}y^4$

$$14. \frac{36x^2y^5 + 42xy^3 - 24x^3y^2 - 12y^5}{-6y^2}$$

$$= \frac{36x^2y^5}{-6y^2} + \frac{42xy^3}{-6y^2} - \frac{24x^3y^2}{-6y^2} - \frac{12y^5}{-6y^2}$$

$$= -6x^2y^3 - 7xy + 4x^3 + 2y^3$$

- Since, divisor is $x + 3$
 $\therefore x + 3 = 0 \rightarrow x = -3$
 $\therefore$ put $x = -3$ in the expression
 $3x^3 + 8x^2 - 6x + 1 = 3(-3)^3 + 8(-3)^2 - 6(-3) + 1$
 $= -81 + 72 + 18 + 1 = 10$

- Put $x = -3$ in the given expression
 $ax^3 + 9x^2 + 4x - 10$
 $\therefore a(-3)^3 + 9(-3)^2 + 4(-3) - 10 = 5$
 $\therefore -27a + 81 - 12 - 10 = 5$
 $\Rightarrow -27a = -54 \Rightarrow a = 2$

- Put $x = -1$, then
 $(-1)^3 + a(-1)^2 - b(-1) - 6 = 0$
or $-1 + a + b - 6 = 0$
or $a + b - 7 = 0 \quad \dots(i)$
and put $x = 2$, then
 $(2)^3 + a(2)^2 - b(2) - 6 = 0$
 $\Rightarrow 8 + 4a - 2b - 6 = 0$
 $\Rightarrow 4a - 2b + 2 = 0 \quad \dots(ii)$

By solving eqs. (i) and (ii)
 $a + b - 7 = 0 \Rightarrow a + b = 7$
and $4a - 2b + 2 = 0 \Rightarrow 2a - b = -1$
 $\therefore 3a = 6 \Rightarrow a = 2$
Substituting $a = 2$ in eq. (i), we get $b = 5$
 $\therefore a = 2$ and $b = 5$

- $\therefore A^2 - B^2 = (A + B) \cdot (A - B)$
 $\therefore (5x^2 + 14x + 2)^2 - (4x^2 - 5x + 7)^2$
 $= (5x^2 + 14x + 2 + 4x^2 - 5x + 7)$
 $\quad \times (5x^2 + 14x + 2 - 4x^2 + 5x - 7)$
 $= 9(x^2 + 19x - 5)(x^2 + x + 1)$
 $\therefore \frac{(5x^2 + 14x + 2)^2 - (4x^2 - 5x + 7)^2}{(x^2 + x + 1)} = 9(x^2 + 19x - 5)$
 $\therefore q = 9(x^2 + 19x - 5)$ and $r = 0$.

19. Putting $x = -1$ in the given polynomial, we get

$$(-1)^3 - 6(-1) + 7 = -1 + 6 + 7 = 12 \quad (\because x + 1 = 0)$$

20. Putting $x = 2$, we get $(\because x - 2 = 0)$

$$(2)^4 - 3(2)^3 + 2(2)^2 - 5(2) + 7 \\ = 16 - 24 + 8 - 10 + 7 = -3$$

21. Since, divisor is $x^2 + 2$

$\therefore$ Substitute $x^2 = -2$, in the given expression

$x^3 + 5x^2 + 10k$ gives the remainder $-2x$

$$(-2)x + 5(-2) + 10k = -2x$$

$$\Rightarrow k = 1$$

22. Putting $x = -1$, we get

$$(-1)^{11} + 1 = -1 + 1 = 0$$

23. Putting $x = -3$, we get

$$5(-3)^3 + 5(-3)^2 - 6(-3) + 9 = -135 + 45 + 18 + 9 = -63$$

25. Best way is that to consider some appropriate value of x (say $x = 1$), then check through options :

Let $x = 1$, then

$$x^3 - 2x^2 + px - q = 1 - 2 + p - q \rightarrow \text{dividend}$$

$$\text{and } x - 6 = 1 - 6 = -5 \rightarrow \text{remainder}$$

$$\text{and } x^2 - 2x - 3 = 1 - 2 - 3 = -4 \rightarrow \text{divisor}$$

$\therefore$ Now assume choice (c) as $p = -2$ and $q = 6$, then

$$\frac{-1 + p - q}{-4} = \frac{-1 + (-2) - 6}{-4} = \frac{-9}{-4} \rightarrow \text{Remainder} = 5$$

 **NOTE** To avoid negative remainder you can assume $x = 7, 8, \dots$, etc.

 **Alternatively** Divide the given expression as conventionally and then compare the actual remainder with obtained remainder.

$$x^2 - 2x - 3 \overline{) x^3 - 2x^2 + px - q} \begin{matrix} \times & \times & \times & \times \\ \times & \times & \times & \times \end{matrix}$$

$$\text{Remainder} = (p + 3)x - q$$

$$\therefore (p + 3)x - q = x - 6$$

$$\therefore p + 3 = 1 \quad \text{and} \quad q = 6$$

$$\Rightarrow p = -2 \quad \text{and} \quad q = 6$$

26. Put $x = 2$ in the given expression as :

$$x^2 + 3qx - 2q = 0$$

$$(2)^2 + 3q(2) - 2q = 0$$

$$\Rightarrow 4 + 6q - 2q = 0 \Rightarrow 4 + 4q = 0$$

$$\Rightarrow q = -1$$

27. Put $x = -2$ in the given polynomial.

$$(-2)^3 + 6(-2)^2 + 4(-2) + k = 0$$

$$\Rightarrow k = -8$$

28. The best way is check by options. Let us consider choice (c). $(x - 2)$ is a factor of the given expression, then at $x = 2$ the given expression must be zero.

$$\therefore (2)^4 - 5(2)^3 + 5(2)^2 - 10(2) + 24 = 0$$

Hence, $(x - 2)$ is the factor of $x^4 - 5x^3 + 5x^2 - 10x + 24$.

29. Since, $(x + k)$ is a factor of each one of the given expression, $x = -k$ will make each zero.

$$\therefore k^2 - pk + q = k^2 - lk + m = 0$$

$$\Rightarrow k = \frac{m - q}{l - p}$$

30. Substitute $x = a$ in the given expression, since $(x - a)$ is a factor, which will make it equal to zero.

$$\therefore (a)^3 - 3a^3 + 2a^3 + b = 0 \Rightarrow b = 0$$

31. Go through options

32. Go through options

33. Put $x = -1$ in the given expression

34. Put $x = 1$ in the given expression

$$36. \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2x \cdot \frac{1}{x} \\ (2)^2 = x^2 + \frac{1}{x^2} + 2 \left(\text{putting } x + \frac{1}{x} = 2\right)$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 2$$

$$37. \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2x \cdot \frac{1}{x} \\ (4)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 18$$

$$38. \left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right)$$

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3 \left(x + \frac{1}{x}\right)$$

$$\Rightarrow (2\sqrt{3})^3 = x^3 + \frac{1}{x^3} + 3(2\sqrt{3})$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 18\sqrt{3}$$

$$39. \left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) \\ (3)^3 = x^3 + \frac{1}{x^3} + 3(3)$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 18$$

$$\begin{aligned}
 40. \quad \left(x + \frac{1}{x}\right)^2 &= x^2 + \frac{1}{x^2} + 2 \cdot x \cdot \frac{1}{x} \\
 (2)^2 &= x^2 + \frac{1}{x^2} + 2 \\
 x^2 + \frac{1}{x^2} &= 2 \\
 \text{Again, } \left(x^2 + \frac{1}{x^2}\right)^3 &= x^6 + \frac{1}{x^6} + 3x^2 \cdot \frac{1}{x^2} \left(x^2 + \frac{1}{x^2}\right) \\
 (2)^3 &= x^6 + \frac{1}{x^6} + 3(2) \\
 \Rightarrow x^6 + \frac{1}{x^6} &= 2
 \end{aligned}$$

$$\begin{aligned}
 41. \quad \left(x - \frac{1}{x}\right)^2 &= x^2 + \frac{1}{x^2} - 2x \cdot \frac{1}{x} \\
 \left(x - \frac{1}{x}\right)^2 &= 6 - 2 \\
 \left(x - \frac{1}{x}\right)^2 &= 4 \\
 \Rightarrow \left(x - \frac{1}{x}\right) &= \pm 2
 \end{aligned}$$

$$\begin{aligned}
 42. \quad \left(x - \frac{1}{x}\right)^3 &= x^3 - \frac{1}{x^3} - 3x \cdot \frac{1}{x} \left(x - \frac{1}{x}\right) \\
 \left(x - \frac{1}{x}\right)^3 &= 36 - 3 \left(x - \frac{1}{x}\right)
 \end{aligned}$$

$$\text{Let } \left(x - \frac{1}{x}\right) = P,$$

$$\text{then } P^3 = 36 - 3P$$

$$\text{or } P^3 + 3P = 36 \Rightarrow P^3 + 3P - 36 = 0$$

Clearly $P = 3$ satisfies the equation.

$$\therefore x - \frac{1}{x} = 3$$

☞ Alternatively Go through options

$$\begin{aligned}
 43. \quad \left(x + \frac{1}{x}\right)^3 &= x^3 + \frac{1}{x^3} + 3x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) \\
 \Rightarrow \left(x + \frac{1}{x}\right)^3 &= x^3 + \frac{1}{x^3} + 3 \left(x + \frac{1}{x}\right)
 \end{aligned}$$

Now, go through the given choices.

$$\begin{aligned}
 44. \quad x^4 + \frac{1}{x^4} &= \left(x^2 + \frac{1}{x^2}\right)^2 - 2 \\
 \therefore 34 &= \left(x^2 + \frac{1}{x^2}\right)^2 - 2 \\
 \Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 &= 36 \Rightarrow x^2 + \frac{1}{x^2} = 6 \\
 \text{Again, } \left(x - \frac{1}{x}\right)^2 &= x^2 + \frac{1}{x^2} - 2
 \end{aligned}$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 6 - 2 = 4$$

$$\Rightarrow x - \frac{1}{x} = 2$$

☞ Alternatively Go through options.

$$45. \quad \left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2$$

$$\Rightarrow q \quad \left(x^2 + \frac{1}{x^2}\right)^2 = 121$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 11$$

$$\text{Again, } \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\therefore \left(x - \frac{1}{x}\right)^2 = 9 \Rightarrow x - \frac{1}{x} = 3$$

$$\begin{aligned}
 \therefore \left(x - \frac{1}{x}\right)^3 &= x^3 - \frac{1}{x^3} - 3x \cdot \frac{1}{x} \left(x - \frac{1}{x}\right) \\
 (3)^3 &= x^3 - \frac{1}{x^3} - 3(3)
 \end{aligned}$$

$$\Rightarrow x^3 - \frac{1}{x^3} = 36$$

$$46. \quad \left(3x - \frac{2}{x}\right)^2 = 25$$

$$9x^2 + \frac{4}{x^2} - 12 = 25 \Rightarrow 9x^2 + \frac{4}{x^2} = 37$$

$$\text{Again, } 9x^2 + \frac{4}{x^2} + 12 = 37 + 12 = 49$$

$$\Rightarrow \left(3x + \frac{2}{x}\right)^2 = (7)^2 \Rightarrow 3x + \frac{2}{x} = 7$$

$$\therefore \left(9x^2 - \frac{4}{x^2}\right) = \left(3x - \frac{2}{x}\right) \left(3x + \frac{2}{x}\right) = 5 \times 7 = 35$$

$$47. \quad m^2 - 4m + 1 = 0 \Rightarrow m^2 + 1 = 4m \Rightarrow \frac{m^2 + 1}{m} = 4$$

$$\Rightarrow m + \frac{1}{m} = 4$$

$$\therefore \left(m + \frac{1}{m}\right)^3 = m^3 + \frac{1}{m^3} + 3 \left(m + \frac{1}{m}\right) = 64$$

$$\Rightarrow m^3 + \frac{1}{m^3} = 52$$

$$\begin{aligned}
 48. \quad (x + y)^2 &= x^2 + y^2 + 2xy \\
 169 &= x^2 + y^2 + 2 \times 40
 \end{aligned}$$

$$\Rightarrow x^2 + y^2 = 89$$

$$\begin{aligned}
 49. \quad (x + y)^3 &= x^3 + y^3 + 3xy(x + y) \\
 2097 &= x^3 + \frac{1}{x^3} + 3 \times 36(13)
 \end{aligned}$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 2097 - 1404$$

$$x^3 + \frac{1}{x^3} = 693$$

$$50. (x - y)^2 = (x + y)^2 - 4xy = 64 - 60 = 4$$

$$\Rightarrow (x - y) = 2$$

$$\therefore x^3 - y^3 = (x - y)^3 + 3xy(x - y)$$

$$= 8 + 45(2)$$

$$x^3 - y^3 = 98$$

$$51. (x^3 + y^3 + z^3 - 3xyz)$$

$$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= (x + y + z)[(x + y + z)^2 - 3(xy + yz + zx)]$$

$$= 6 \times [36 - 3 \times 11] = 6 \times 3 = 18$$

$$52. x + y + z = 0 \Rightarrow x^3 + y^3 + z^3 = 3xyz.$$

$$53. \therefore x + y + z = 0 \Rightarrow x^3 + y^3 + z^3 = 3xyz$$

$$\therefore a^{1/3} + b^{1/3} + c^{1/3} = 0$$

$$\Rightarrow a + b + c = 3a^{1/3}b^{1/3}c^{1/3}$$

$$\therefore (a + b + c)^3 = 27abc$$

$$54. 2^x - \frac{2^x}{2} = 8 \Rightarrow k - \frac{k}{2} = 8 \Rightarrow k = 16$$

$$\therefore 2^x = 16 \Rightarrow (\text{say } 2^x = k)$$

$$\Rightarrow 2^x = 2^4 \Rightarrow x = 4.$$

$$\therefore x^3 = (4)^3 = 64$$

$$55. 3^x + 3^{x+1} = 36 \Rightarrow 3^x + 3 \cdot 3^x = 36$$

$$\text{Let } 3^x = k$$

$$\therefore k + 3k = 36 \Rightarrow k = 9$$

$$\therefore 3^x = 9 \Rightarrow x = 2$$

$$\therefore x^x = 2^2 = 4.$$

$$56. \frac{(a+b) - (a-b)^2}{a^2b - ab^2} = \frac{(a^2 + b^2 + 2ab) - (a^2 + b^2 - 2ab)}{ab(a-b)}$$

$$= \frac{4ab}{ab(a-b)} = \frac{4}{(a-b)}$$

$$57. \text{ Let } \frac{x}{(b-c)(b+c-2a)} = \frac{y}{(c-a)(c+a-2b)}$$

$$= \frac{z}{(a-b)(a+b-2c)} = k, \text{ then}$$

$$x = k(b-c)(b+c-2a), y = k(c-a)(c+a-2b),$$

$$z = k(a-b)(a+b-2c)$$

$$\therefore (x + y + z) = k[(b^2 - c^2 + c^2 - a^2 + a^2 - b^2) - 2\{a(b-c) + b(c-a) + c(a-b)\}]$$

$$= 0$$

$$58. a^4 + 5a^3 + 6a^2 = a^2(a^2 + 5a + 6) = a^2(a+3)(a+2)$$

$$60. a^{3/2} - ab^{1/2} + a^{1/2}b - b^{3/2}$$

$$= a(a^{1/2} - b^{1/2}) + b(a^{1/2} - b^{1/2})$$

$$= (a+b)(a^{1/2} - b^{1/2})$$

$$\therefore \frac{(a+b)(a^{1/2} - b^{1/2})}{(a^{1/2} - b^{1/2})} = (a+b)$$

$$61. x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)$$

$$= x^2(z-y) - x(z^2 - y^2) + yz(z-y)$$

(Writing in descending power of x)

$$= (z-y)[x^2 - x(z+y) + yz]$$

$$= (z-y)(x-z)(x-y)$$

☞ Alternatively Consider $x = 1, y = 2$ and $z = 3$ and then verify the correct option.

$$62. \text{ Since, } x^3 - y^3 = (x-y)(x^2 + y^2 + xy)$$

$$\therefore (a+b)^3 - (a-b)^3$$

$$= [(a+b) - (a-b)][(a+b)^2 + (a-b)^2 + (a+b)(a-b)]$$

$$= 2b[2(a^2 + b^2) + (a^2 - b^2)] = 2b(3a^2 + b^2)$$

$$63. (x^2 + 4) = (x^2 + 2^2) = (x)^2 + (2)^2 + 4x^2 - 4x^2$$

$$= (x^2 + 2)^2 - (2x)^2$$

$$= (x^2 + 2 + 2x)(x^2 + 2 - 2x)$$

$$64. \text{ Best way is to go through options.}$$

$$(x-1)(x+3)(x-2) = (x^2 + 2x - 3)(x-2)$$

$$= x^3 + 2x^2 - 3x - 2x^2 - 4x + 6$$

$$= x^3 - 7x + 6$$

Hence, option (a) is correct.

$$65. (64x^3 - 216y^3) = (4x)^3 - (6y)^3$$

$$= (4x - 6y)[(4x)^2 + (6y)^2 + 24xy]$$

$$66. a^2 + 4b^2 + 4b - 4ab - 2a - 8$$

$$= a^2 + 4b^2 - 4ab + 4b - 2a - 8$$

$$= (a - 2b)^2 - 2(a - 2b) - 8$$

$$= k^2 - 2k - 8,$$

where $k = a - 2b$

$$= k^2 - 4k + 2k - 8$$

$$= k(k - 4) + 2(k - 4)$$

$$= (k - 4)(k + 2)$$

$$= (a - 2b - 4)(a - 2b + 2)$$

$$67. \text{ Let } (a-b) = x, (b-c) = y \text{ and } (c-a) = z$$

$$\text{Then, } x + y + z = 0$$

$$\therefore x^3 + y^3 + z^3 = 3xyz$$

$$\text{Hence, } (a-b)^3 + (b-c)^3 + (c-a)^3 = 3(a-b)(b-c)(c-a)$$

$$68. (a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ac)$$

$$0 = a^2 + b^2 + c^2 + 2(ab + bc + ac)$$

$$\therefore a^2 + b^2 + c^2 = -2(ab + bc + ac)$$

69. First of all factorize the expression, then find the HCF or LCM as per the requirement.

(1) LCM of $10a^2b^2$, $15abc$, $20ab^2c^2$ is

$$3 \times 2^2 \times 5a^2b^2c^2 = 60a^2b^2c^2, \text{ Hence (a) is wrong.}$$

$$(2) \quad (x^2 - 6x + 9) = (x - 3)^2$$

$$\text{and} \quad (x^3 - 27) = (x - 3)(x^2 + 3x + 9)$$

$$\therefore \quad \text{HCF} = (x - 3)$$

$$(3) \quad 6x^2 - 7x - 3 = (2x - 3)(3x + 1)$$

$$(2x^2 + 11x - 21) = (x + 7)(2x - 3)$$

$$\therefore \quad \text{HCF} = (2x - 3).$$

Hence, 2 and 3 are correct.

$$70. \quad 2x^2 - 3x - 2 = 2x^2 - 4x + x - 2$$

$$= (x - 2)(2x + 1)$$

$$\text{and} \quad (x^3 - 4x^2 + 4x) = x(x^2 - 4x + 4) = x(x - 2)^2$$

$$\therefore \quad \text{LCM} = x(x - 2)^2(2x + 1)$$

$$71. \quad 2(x^2 - y^2) = 2(x - y)(x + y)$$

$$\text{and} \quad 5(x^3 - y^3) = 5(x - y)(x^2 + y^2 + xy)$$

$$\therefore \quad \text{HCF} = (x - y)$$

$$72. \quad (x^4 - 4x^2 + 3) = x^4 - 3x^2 - x^2 + 3$$

$$= x^2(x^2 - 3) - 1(x^2 - 3)$$

$$= (x^2 - 3)(x^2 - 1)$$

$$= (x^2 - 3)(x - 1)(x + 1)$$

$$\text{and} \quad x^4 - x^2 - 6 = x^4 - 3x^2 + 2x^2 - 6$$

$$= x^2(x^2 - 3) + 2(x^2 - 3)$$

$$= (x^2 - 3)(x^2 + 2)$$

$$\therefore \quad \text{HCF} = (x^2 - 3)$$

$$73. \quad x^2 - 4 = (x + 2)(x - 2)$$

$$(x^2 - 5x - 6) = (x + 1)(x - 6)$$

$$(x^2 + x - 6) = (x - 2)(x + 3)$$

Since, there is no common factor. So, HCF is 1.

$$74. \quad x^2 - ax - (a + 1) = x^2 - (a + 1)x + x - (a + 1)$$

$$= (x + 1)(x - a + 1)$$

$$\text{and} \quad ax^2 - x - (a + 1) = ax^2 - a - x - 1$$

$$= a(x^2 - 1) - 1(x + 1)$$

$$= (x + 1)(ax - a - 1)$$

$$\therefore \quad \text{HCF} = (x + 1)$$

$$75. \quad P = (x - 2)(x + 1)^2(x + 3)^2$$

$$\text{and} \quad Q = (x + 1)^2(x + 3)(x + 4)$$

$$\therefore \text{LCM of } P \text{ and } Q = (x - 2)(x + 1)^2(x + 3)^2(x + 4)$$

$$76. \quad a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$$

$$(a^4 - b^4) = (a - b)(a + b)(a^2 + b^2)$$

$$\therefore \quad \text{LCM} = (a - b)(a + b)(a^2 + b^2)(a^2 + b^2 - ab)$$

$$= (a - b)(a^2 + b^2)(a + b)(a^2 + b^2 - ab)$$

$$= (a - b)(a^2 + b^2)(a^3 + b^3)$$

$$77. \quad (x^4 - 1) = (x - 1)(x + 1)(x^2 + 1)$$

$$(x^3 + x^2 + x + 1) = x^2(x + 1) + 1(x + 1) = (x^2 + 1)(x + 1)$$

$$\therefore \quad \text{HCF} = (x + 1)(x^2 + 1)$$

$$78. \quad 2x^2 - 4x = 2x(x - 2)$$

$$3x^4 - 12x^2 = 3x^2(x^2 - 4)$$

$$= 3x^2(x + 2)(x - 2)$$

$$2x^5 - 2x^4 - 4x^3 = 2x^3(x^2 - x - 2)$$

$$= 2x^3(x - 2)(x + 1)$$

$$\therefore \quad \text{HCF} = x(x - 2)$$

79. Since, all the factors are distinct

$$80. \quad (x + 2)^2(x - 2) \quad \text{and} \quad x^2 - 4x - 12 = (x - 6)(x + 2)$$

$$\therefore \quad \text{LCM} = (x - 6)(x + 2)^2(x - 2)$$

$$81. \quad a^2 - ab - 2b^2 = (a^2 - b^2) - (ab + b^2)$$

$$= (a + b)(a - b - b) = (a + b)(a - 2b)$$

$$\text{and} \quad 2a^2 - ab - b^2 = (a^2 - ab) + (a^2 - b^2)$$

$$= (a - b)a + (a^2 - b^2) = (a - b)(2a + b)$$

Since, there is no common factor, hence HCF = 1.

$$82. \quad \text{HCF} = a^2b^3c^3$$

$$\text{LCM} = a^5b^4c^4$$

$$83. \quad \frac{(x + 3)}{(x - 2)} - \frac{(x + 1)}{(x - 3)} = \frac{(x + 3)(x - 3) - (x - 2)(x + 1)}{(x - 2)(x - 3)}$$

$$= \frac{(x^2 - 9) - (x^2 - x - 2)}{(x^2 - 5x + 6)} = \frac{x - 7}{(x^2 - 5x + 6)}$$

$$84. \quad \frac{x + 1}{x - 1} + \frac{x - 1}{x + 1} = \frac{(x + 1)^2 + (x - 1)^2}{(x - 1)(x + 1)} = \frac{2(x^2 + 1)}{(x^2 - 1)}$$

$$85. \quad \frac{x^2 - 5x + 6}{x^2 - 9x + 20} \div \frac{x^2 - 3x + 2}{x^2 - 5x + 4}$$

$$= \frac{x^2 - 5x + 6}{x^2 - 9x + 20} \times \frac{x^2 - 5x + 4}{x^2 - 3x + 2}$$

$$= \frac{(x - 2)(x - 3)}{(x - 4)(x - 5)} \times \frac{(x - 1)(x - 4)}{(x - 1)(x - 2)}$$

$$= \left(\frac{x - 3}{x - 5} \right)$$

$$86. \quad \frac{2x^2 + 3}{x - 3} + \frac{x - 3}{2x^2 + 3}$$

$$= \frac{(2x^2 + 3)^2 + (x - 3)^2}{(x - 3)(2x^2 + 3)} = \frac{4x^4 + 9 + 12x^2 + x^2 + 9 - 6x}{2x^3 - 6x^2 + 3x - 9}$$

$$= \frac{4x^4 + 13x^2 - 6x + 18}{2x^3 - 6x^2 + 3x - 9}$$

$$87. \frac{(x-3)(x^2-5x+4)}{(x-4)(x^2-2x-3)} = \frac{(x-3)(x-1)(x-4)}{(x-4)(x-3)(x+1)} = \frac{(x-1)}{(x+1)}$$

$$88. \frac{(2x^2+1)(x+1)+(x-1)^2}{(x^2-1)} \times \frac{(x^2-1)}{2x} \\ = \frac{2x^3+2x^2+x+1+(x-1)^2}{(2x)} = \frac{2x^3+3x^2-x+2}{2x}$$

$$89. \sqrt{\frac{(x^2+3x+2)(x^2+5x+6)}{x^2(x^2+4x+3)}} \\ = \sqrt{\frac{(x+1)(x+2)(x+2)(x+3)}{x^2(x+1)(x+3)}} \\ = \sqrt{\frac{(x+2)^2}{x^2}} = \frac{x+2}{x}$$

$$90. \frac{1}{(a-b)(a-c)} + \frac{1}{(b-c)(b-a)} + \frac{1}{(c-a)(c-b)} \\ \frac{(b-c)(b-a)(c-a)(c-b) + (a-b)(a-c)(c-a)(c-b) + (a-b)(a-c)(b-c)(b-a)}{(a-b)(a-c)(b-c)(b-a)(c-a)(c-b)} = 0$$

$$91. \frac{1}{(1-x)} + \frac{1}{(1+x)} + \frac{2}{(1+x^2)} + \frac{4}{(1+x^4)} \\ = \frac{(1+x)+(1-x)}{(1-x^2)} + \frac{2}{(1+x^2)} + \frac{4}{(1+x^4)} \\ = \frac{2}{(1-x^2)} + \frac{2}{(1+x^2)} + \frac{4}{(1+x^4)} \\ = \frac{4}{(1-x^4)} + \frac{4}{(1+x^4)} = \frac{8}{1-x^8}$$

Introductory Exercise 13.2

$$1. \quad x-y = \frac{9}{10} \quad \text{and} \quad (x+y) = \frac{11}{2}$$

$$\left. \begin{array}{l} x-y = \frac{9}{10} \\ x+y = \frac{11}{2} \end{array} \right\} \text{By addition}$$

$$2x = \frac{64}{10}$$

$$\Rightarrow x = \frac{32}{10} = 3.2$$

$$\Rightarrow \therefore y = \frac{23}{10} = 2.3$$

$$2. \quad \frac{x+y}{xy} = 2 \Rightarrow \frac{1}{x} + \frac{1}{y} = 2 \quad \dots(i)$$

$$\text{and} \quad \frac{x-y}{xy} = 6 \Rightarrow \frac{1}{y} - \frac{1}{x} = 6 \quad \dots(ii)$$

By adding eqs. (i) and (ii), we get

$$\frac{2}{y} = 8 \Rightarrow y = \frac{1}{4} \therefore x = -\frac{1}{2}$$

$$\therefore x = -\frac{1}{2} \quad \text{and} \quad y = \frac{1}{4}$$

Alternatively

$$x+y = 2xy \quad \text{and} \quad x-y = 6xy \quad \dots(i)$$

$$\Rightarrow 3(x+y) = 6xy \quad \text{and} \quad x-y = 6xy$$

$$\Rightarrow 3x+3y = x-y$$

$$\Rightarrow 2x = -4y$$

$$\Rightarrow x = -2y$$

Substituting $x = -2y$ in the Eq. (i), we get

$$-3y = 6xy \Rightarrow x = -\frac{1}{2}$$

$$\text{and} \therefore y = \frac{1}{4}$$

$$3. \quad \frac{1}{2x} - \frac{1}{y} - 1 \quad \text{and} \quad \frac{1}{x} + \frac{1}{2y} = 8$$

$$\Rightarrow \frac{1}{2x} - \frac{1}{y} = -1 \quad \text{and} \quad \frac{2}{x} + \frac{1}{y} = 16$$

By adding these two equations, we get

$$\frac{5}{2x} = 15 \Rightarrow x = \frac{1}{6}$$

$$\therefore y = \frac{1}{4}$$

$$4. \quad \frac{2x}{a} + \frac{y}{b} = 2 \quad \dots(i)$$

$$\frac{x}{a} - \frac{y}{b} = 4 \quad \dots(ii)$$

Adding eqs. (i) and (ii), we get

$$\frac{3x}{a} = 6 \Rightarrow \frac{x}{a} = 2 \Rightarrow x = 2a$$

Now put $x = 2a$ in eq. (ii)

$$\frac{y}{b} = -2$$

$$\therefore x = 2a \quad \text{and} \quad y = -2b.$$

5. For no solution

$$\frac{9}{7} = \frac{4}{k} \neq \frac{9}{5}$$

$$\Rightarrow k = \frac{28}{9}$$

6. Two linear equations will coincide if there are infinite number of solutions,

$$\text{i.e.,} \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\text{for} \quad a_1x + b_1y = c_1 \quad \text{and} \quad a_2x + b_2y = c_2$$

$$\therefore \frac{k}{3} = \frac{2}{1} = \frac{2}{1} \Rightarrow k = 6$$

7. For an infinite number of solutions

$$\frac{2}{6} = \frac{3}{k} = \frac{-5}{-15} \Rightarrow k = 9$$

8. $8B + 5M = 92$... (i)

$5B + 8M = 77$... (ii)

Multiply eq. (i) by 8 Eq. (ii) by 5, we get

$$\begin{array}{r} 64B + 40M = 736 \\ 25B + 40M = 385 \quad \text{[subtracting]} \\ \hline 39B = 351 \end{array}$$

$$B = 9$$

$$\therefore M = 4$$

$$\therefore 2M + 3B = 35$$

$\therefore$ The required cost of 2 mugs and 3 buckets is ₹ 35.

(Here, $M \rightarrow$ Price of a mug and $B \rightarrow$ Price of a bucket)

9. Let the two numbers be x and y ,

then $x + y = 8$

and $(x + y) = 4(x - y)$

$$\Rightarrow x - y = 2$$

$$\therefore x + y = 8 \quad \text{and} \quad x - y = 2$$

$$2x = 10 \Rightarrow x = 5$$

$$\therefore y = 3$$

10. Let numerator be x and denominator be y ,

then $x = y - 4$... (i)

$8(x - 2) = (y + 1)$... (ii)

The above equations can be written as

$y - x = 4$... (iii)

and $y - 8x = -17$

$$\begin{array}{r} - \quad + \quad + \\ 7x = 21 \end{array}$$

$$\Rightarrow x = 3 \therefore y = 7$$

$$\therefore \text{The required fraction} = \frac{x}{y} = \frac{3}{7}$$

11. Distance = 90 km

Let the speeds of car be x km/h and y km/h

$$\therefore \text{Distance} = \text{Speed} \times \text{Time}$$

$$\therefore 90 = (x + y) \times \frac{9}{7} \Rightarrow x + y = 70$$

and $90 = (x - y) \times 9 \Rightarrow x - y = 10$

$$\therefore x = 40 \quad \text{and} \quad y = 30$$

$\therefore$ The required speeds of cars = 40 km/h and 30 km/h

12. Let there be x coins of 25 paise denomination and y coins of 50 paise denomination, then

$$x + y = 40 \Rightarrow 25x + 25y = 1000 \quad \dots (i)$$

and $25x + 50y = 1275 \quad \dots (ii)$

$\therefore$ Subtracting eqs. (i) from (ii), we get

$$25y = 275 \Rightarrow y = 11$$

$$\therefore x = 29.$$

13. Let there be x pencils and y pens, then

$$\therefore x + y = 40 \Rightarrow 4x + 4y = 160$$

$$(x + 5) = 4(y - 5)$$

$$\Rightarrow x - 4y = -25$$

$$\therefore 4x + 4y = 160$$

$$x - 4y = -25$$

$$5x = 135$$

$$\Rightarrow x = 27 \therefore y = 13$$

14. Let the present age of the son be ' x ' years and the present age of his father be ' y ' years.

Then, $y = 3x$... (i)

Again, $(y + 12) = 2(x + 12)$... (ii)

Substituting $y = 3x$ in eq. (ii), we get

$$x = 12$$

$$\therefore y = 36$$

15. Let the present age of A is x

and the present age of B is y (in years).

Then, $(x - 5) = 3(y - 5) \Rightarrow x - 3y = -10$... (i)

$$(x + 10) = 2(y + 10) \Rightarrow x - 2y = 10 \quad \dots (ii)$$

Subtracting eq. (ii) from eq. (i), we get

$$-y = -20 \Rightarrow y = 20$$

$$\therefore x = 50$$

16. Let Astha will have x number of oranges and

Saumya will have y number of oranges. Then,

$$(x + 10) = 2(y - 10) \Rightarrow x - 2y = -30 \quad \dots (i)$$

and $(x - 10) = (y + 10) \Rightarrow x - y = 20 \quad \dots (ii)$

Subtracting eq. (ii) from eq. (i), we get

$$-y = -50 \Rightarrow y = 50$$

$$\therefore x = 70$$

17. Total distance = 760 km.

Let the speed of car be x km/h and the speed of train be y km/h

$$\therefore \text{Distance} = \text{Speed} \times \text{Time}$$

$$\Rightarrow T = \frac{D}{S}$$

$$\therefore \frac{600}{x} + \frac{160}{y} = 8 \quad \dots (i)$$

and $\frac{520}{x} + \frac{240}{y} = 8 \frac{1}{5} \quad \dots (ii)$

$\therefore$ eq. (i) can be written as

$$600A + 160B = 8 \quad \dots (iii)$$

and eq. (ii) can be written as

$$520A + 240B = \frac{41}{5} \quad \dots (iv)$$

where, $\frac{1}{x} = A$ and $\frac{1}{y} = B$

∴ Solving eqs. (iii) and (iv), we get

$$\begin{array}{r} 1800A + 480B = 24 \\ 1040A + 480B = \frac{82}{5} \\ \hline 760A = \frac{38}{5} \end{array}$$

⇒ $A = \frac{1}{100} \Rightarrow x = 100 \text{ km/h}$

∴ $B = \frac{1}{80} \Rightarrow y = 80 \text{ km/h}$

18. Let he has invested ₹ x at 12% S.I. and ₹ y at 10% SI

∴ $\left(\because \text{SI} = \frac{P \times r \times t}{100} \right)$

$$\frac{x \times 12 \times 1}{100} + \frac{y \times 10 \times 1}{100} = 130$$

⇒ $12x + 10y = 13000 \quad \dots(i)$

⇒ $10x + 12y = 134000 \quad \dots(ii)$

∴ $x = 500$ and $y = 700$

19. $\frac{3x - y + 1}{3} = \frac{2x + y + 2}{5}$

⇔ $5(3x - y + 1) = 3(2x + y + 2)$

⇔ $9x - 8y = 1 \quad \dots(i)$

⇔ $\frac{2x + y + 2}{5} = \frac{3x + 2y + 1}{6}$

⇔ $6(2x + y + 2) = 5(3x + 2y + 1)$

⇔ $3x + 4y = 7 \quad \dots(ii)$

Multiplying eq. (ii) by 2 and adding to eq. (i), we get

$$15x = 15 \text{ or } x = 1$$

Putting $x = 1$ in (i), we get

$$9 \times 1 - 8y = 1 \Rightarrow y = 1.$$

20. Submarino can be destroyed when it meets the destroyer. Thus,

$$2x + 3y = 5 \quad \dots(i)$$

$$x - y = 10 \quad \dots(ii)$$

∴ $x = 7$ and $y = -3$

So, the required point is $(7, -3)$

21. $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \quad \dots(i)$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad \dots(ii)$$

$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad \dots(iii)$$

Adding eqs. (i), (ii) and (iii), we have

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 3 \quad \dots(iv)$$

∴ $\frac{x^2}{a^2} = 1 \Rightarrow x = \pm a$

$$\frac{y^2}{b^2} = 1 \Rightarrow y = \pm b$$

$$\frac{z^2}{c^2} = 1 \Rightarrow z = \pm c$$

Hence, (d) is correct.

Level 01 Basic Level Exercise

- 1 If $x + y + z$ is constant, the product xyz takes maximum value when each of x, y, z takes equal value.

∴ $a + b + c = 13$

∴ $(a - 3) + (b - 2) + (c + 1) = 13 - 3 - 2 + 1 = 9$

For the maximum value of $(a - 3)(b - 2)(c + 1)$

$$= (a - 3) = (b - 2) = (c + 1) = \frac{9}{3} = 3$$

So, $(a - 3)(b - 2)(c + 1) = 3 \times 3 \times 3 = 27$

- 2 If xyz is constant, then the sum of x, y, z (i.e., $x + y + z$) takes minimum value when each of x, y, z takes equal value.

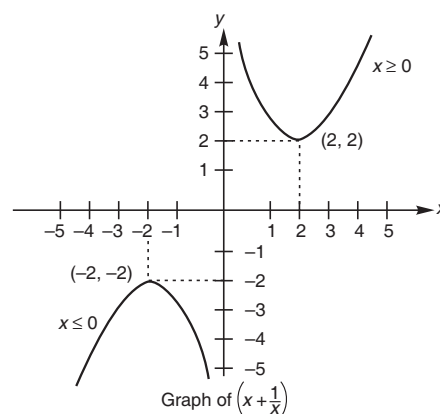
∴ Minimum value of $a + b + c + d$ for given constant product $abcd$ will be when $a = b = c = d$

∴ $a = b = c = d = 3$

∴ Minimum value of $a + b + c + d = 3 + 3 + 3 + 3 = 12$

- 3 For $x \geq 0$, $x + \frac{1}{x} \geq 2$

and for $x \leq 0$, $x + \frac{1}{x} \leq 2$



4 This is the standard inequality formula.

5 This is the standard inequality formula.

 **NOTE** The result can be proved by considering suitable values.

$$\begin{aligned}
 6 \quad & x^2y^3 + y^2x^3 = 25 \\
 \Rightarrow & x^2y^2(x+y) = 25 \\
 \Rightarrow & (xy)^2(x+y) = 25 \\
 \Rightarrow & (xy)^2 = 1 \quad (\because x+y=25) \\
 \Rightarrow & xy = \pm 1
 \end{aligned}$$

7 $x > 0$ and $y > 0$

$$\begin{aligned}
 \therefore (x+y) \left(\frac{1}{x} + \frac{1}{y} \right) &= 2 + \frac{x}{y} + \frac{y}{x} \\
 &= 2 + \left(k + \frac{1}{k} \right), \text{ where } k = \frac{x}{y}
 \end{aligned}$$

Since, the minimum value of the expression $\left(k + \frac{1}{k} \right)$ is 2.

$\therefore$ Minimum value of the given expression is 4.

8 If $a+b+c+d$ is constant, then the product $abcd$ is maximum when $a=b=c=d$.

$$\therefore (a+1) = (b+1) = (c+1) = (d+1)$$

$$\text{Given that } (a+1) + (b+1) + (c+1) + (d+1) = 8$$

$$\therefore 4(a+1) = 8$$

$$\Rightarrow (a+1) = 2$$

$$\therefore \text{Maximum value} = 2 \times 2 \times 2 \times 2 = 16$$

9 As $x+y+z=1$

$$\left(\frac{1}{x} - 1 \right) \left(\frac{1}{y} - 1 \right) \left(\frac{1}{z} - 1 \right) = \frac{y+z}{x} \cdot \frac{z+x}{y} \cdot \frac{x+y}{z}$$

$$\text{But } \frac{y+z}{2} \geq yz \text{ etc.}$$

$$\text{Hence, } \text{LHS} \geq \frac{8xyz}{xyz} = 8$$

10 $x+y+z=4$ and $x^2+y^2+z^2=6$

$$\therefore y+z=4-x$$

$$\text{and } y^2+z^2=6-x^2$$

$$yz = \frac{1}{2} \{ (y+z)^2 - (y^2+z^2) \}$$

$$= \frac{1}{2} \{ (4-x)^2 - (6-x^2) \}$$

$$\Rightarrow yz = x^2 - 4x + 5$$

Hence, y and z are the roots of

$$t^2 - (4-x)t + (x^2 - 4x + 5) = 0$$

Since, the roots y and z are real

$$(4-x)^2 - 4(x^2 - 4x + 5) \geq 0$$

$$\Rightarrow 3x^2 - 8x + 4 \leq 0$$

$$\Rightarrow (3x-2)(x-2) \leq 0$$

$$\Rightarrow x \in \left[\frac{2}{3}, 2 \right]$$

$$\text{By symmetry } y, z \text{ also } \in \left[\frac{2}{3}, 2 \right]$$

$$\begin{aligned}
 11 \quad & \frac{1}{x^2} + \frac{1}{y^2} = \frac{x^2+y^2}{(xy)^2} \\
 &= \frac{(7+4\sqrt{3})^2 + (7-4\sqrt{3})^2}{[(7+4\sqrt{3})(7-4\sqrt{3})]^2} = \frac{2(49+48)}{(1)^2} = 194
 \end{aligned}$$

$$\begin{aligned}
 12 \quad & \text{AM} \geq \text{GM} \Rightarrow \frac{a+b+c}{3} \geq (abc)^{1/3} \\
 & (\text{AM} \rightarrow \text{Arithmetic mean, GM} \rightarrow \text{Geometric mean})
 \end{aligned}$$

$$\text{and } \frac{1}{3} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \geq \left(\frac{1}{abc} \right)^{1/3}$$

$$\Rightarrow \frac{1}{3} (a+b+c) \frac{1}{3} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \geq (abc)^{1/3} \left(\frac{1}{abc} \right)^{1/3} = 1$$

$$\Rightarrow (a+b+c) \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \geq 9$$

Putting $a=b=c=1$, expression takes the value 9, which is therefore, its least value.

13 If ab is constant, then $(a+b)$ takes minimum value when

$$a=b$$

$$\therefore a=b=1$$

$$\therefore (1+a)(1+b) = (1+1)(1+1) = 4$$

$$\begin{aligned}
 14 \quad & \frac{(a+b+c)(ab+bc+ac)}{abc} = (a+b+c) \left(\frac{ab}{abc} + \frac{bc}{abc} + \frac{ac}{abc} \right) \\
 &= (a+b+c) \left(\frac{1}{c} + \frac{1}{a} + \frac{1}{b} \right) \\
 &= (a+b+c) \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) > 9
 \end{aligned}$$

(See the problem number 12 in this exercise.)

15 The expression will have minimum value of the expression when $a=b=c$.

Therefore, the required minimum value

$$= \frac{(1+1+1)}{1} \times \frac{(1+1+1)}{1} \times \frac{(1+1+1)}{1} = 27$$

$$16 \quad \text{Max} \left(\frac{x}{y} \right) = \frac{\text{Max}(x)}{\text{Min}(y)} = \frac{3}{2}$$

$$17 \quad \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$$

$$\Rightarrow \frac{bc+ac+ab}{abc} = 1$$

$$\Rightarrow bc+ac+ab=abc \quad \dots(i)$$

$$\text{Again, } (a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ac)$$

$$(a+b+c)^2 = a^2+b^2+c^2+2abc$$

$$(3)^2 = 6+2abc \Rightarrow abc = \frac{3}{2}$$

18 $2^x = 4^y = 8^z \Rightarrow 2^x = 2^{2y} = 2^{3z}$

$\Rightarrow x = 2y = 3z = k$ (say)

Then, $xyz = \frac{k^3}{6} = 288 \Rightarrow k = 12$

$\therefore x = 12, y = 6, z = 4$

$\Rightarrow \frac{1}{2x} + \frac{1}{4y} + \frac{1}{8z} = \frac{11}{96}$

19 $a = c^z$

$\Rightarrow a = (b^y)^z \quad (\because c = b^y)$

$\Rightarrow a = b^{yz}$

$\Rightarrow a = (a^x)^{yz} \quad (\because b = a^x)$

$\Rightarrow a = a^{xyz}$

$\Rightarrow a^1 = a^{xyz} \Rightarrow xyz = 1$

20 $7x + 12y = 220 \Rightarrow 7x = 220 - 12y$

$x = \frac{220 - 12y}{7} = \frac{4(55 - 3y)}{7}$

It means $55 - 3y$ must be divisible by 7. Since, 4 is not divisible by 7.

$\therefore$ at $y = 2, 9, 16$

we get $x = 28, 16, 4$

Thus, we have 3 solutions of x, y

$(x, y) = (28, 2), (16, 9)$ and $(4, 16)$

21 $a^x \cdot a^y \cdot a^z = (x + y + z)^{x+y+z}$

$\Rightarrow a^{x+y+z} = (x + y + z)^{(x+y+z)}$

$\Rightarrow a = x + y + z \quad \dots(i)$

$\therefore (x + y + z)^y = a^x = (x + y + z)^x$

$\Rightarrow x = y$

Similarly, $y = z$ and $z = x$

$\therefore x = y = z = \frac{a}{3} \quad [\text{using eq. (i)}]$

22 Let $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = k$

$\therefore x = ak, y = bk, z = ck$

$\therefore (x + y + z) = k(a + b + c)$

$\therefore (x + y + z)^2 = k^2(a + b + c)^2$

$\therefore x^2 + y^2 + z^2 + 2(xy + yz + zx) = k^2(a + b + c)^2$

$\therefore 2(xy + yz + zx) = k^2(a + b + c)^2 - (x^2 + y^2 + z^2)$

$\therefore xy + yz + zx = \frac{k^2}{2}(a + b + c)^2 - \frac{1}{2}(x^2 + y^2 + z^2)$
 $= \frac{x^2(a + b + c)^2 - a^2(x^2 + y^2 + z^2)}{2a^2} \quad \left(\because k = \frac{x}{a} \right)$

23 Let x and y be the number of deer and ducks respectively.

$\therefore x + y = 14 \quad \dots(i)$

and $4x + 2y = 38 \quad \dots(ii)$

($\because$ A deer has 4 legs and a duck has 2 legs)

By solving the above two eq. (i) and (ii), we get

$x = 5$ and $y = 9$

Thus, the number of deer is 5.

Alternatively Assume that there are only ducks (because a duck has less number of legs than that of deer). Hence, there must be $9 \times 2 = 18$ legs, but in this way there are 10 legs = $(28 - 18)$ more.

So, there must be some deer which fulfill this gap. Now this difference of 10 legs is compensated by 5 deer because a deer has 2 extra legs (*i.e.*, 2 more legs than a duck has).

Hence, there are 5 deer and $14 - 5 = 9$ ducks.

In short: Step 1. $2 \times 14 = 28$

Step 2. $38 - 28 = 10$

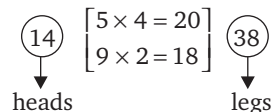
Step 3. $10 \div 2 = 5$

Step 4. $14 - 5 = 9$

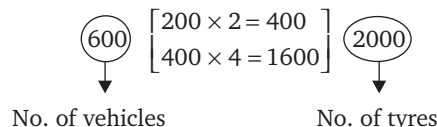
Hence, there were 5 deer and 9 ducks.

Alternatively Going through options. First of all option (d) is ruled out. Now we have to check just 3 options. Again we can leave option (b) and (c) also.

So, we can check the option (a).



24 Going through options, we find that option (d) is correct *i.e.*,



Alternatively

Step 1. $2 \times 600 = 1200$

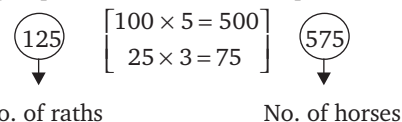
($2 \rightarrow$ number of tyres in two wheelers)

Step 2. $2000 - 1200 = 800$

Step 3. $800 \div 2 = 400 \rightarrow$ number of 4 wheelers

Step 4. $600 - 400 = 200 \rightarrow$ number of 2 wheelers

25 Go through options. Let us consider option (d).



Alternatively

Step 1. $3 \times 125 = 375$ Number of horses in a rath carried by 3 horses

Step 2. $575 - 375 = 200$

Step 3. $200 \div 2 = 100$ Number of extra horses in a rath carried by 5 horses.

Step 4. $125 - 100 = 25$

$\therefore$ Number of raths of Kaurav = 100

Number of raths of allied army of Kaurav = 25

- 26 Let us assume option (a) is correct

Then, the number of coolers

$$= \frac{150000}{3000} = 50$$

∴ the number of coolers = 50

and the number of fans = $40 = (90 - 50)$

Now, $\begin{matrix} \textcircled{90} & \left[\begin{matrix} 50 \times 4 = 200 \\ 40 \times 3 = 120 \end{matrix} \right] & \textcircled{320} \end{matrix}$

Thus, the assumed option (a) is correct.

Alternatively Step 1. $3 \times 90 = 270$
 Step 2. $320 - 270 = 50$
 Step 3. $50 \div 1 = 50$
 Step 4. $90 - 50 = 40$

Hence, number of fans = 40

and number of coolers = 50

∴ Cost of coolers = $50 \times 3000 = 150000$

- 27 Let us assume option (c), then

$$\begin{matrix} \textcircled{90} & \left[\begin{matrix} 40 \times 2 = 80 \rightarrow 80 \times 10 \\ 50 \times 3 = 150 \rightarrow 150 \times 15 \end{matrix} \right] & \begin{matrix} 800 \\ 2250 \end{matrix} & \textcircled{3050} \end{matrix}$$

Hence, assumed option (c) is correct.

Alternatively Per hour expenditure in fuel for Sukhoi
 $= 2 \times 10 = \$ 20$

and per hour expenditure in fuel for Mig-21 = $3 \times 15 = \$ 45$

Now, Step 1. $90 \times 20 = 1800$

Step 2. $3050 - 1800 = 1250$

Step 3. $1250 \div 25 = 50$ $(25 = 45 - 20)$

Step 4. $90 - 50 = 40$

Hence, the number of Sukhoi is 40.

- 28 Step 1. $20 \times 7 = 140$

Step 2. $176 - 140 = 36$

Step 3. $36 \div 3 = 12 \rightarrow$ weight of mangoes, initially

Step 4. $20 - 12 = 8 \rightarrow$ weight of apples, initially

∴ Number of mangoes = $12 \times 10 = 120$

and Number of apples = $8 \times 7 = 56$

Again, since she is left with 13 kg of mangoes and apples containing 121 fruits ($176 - 55 = 121$)

∴ Step 1. $13 \times 7 = 91$

Step 2. $121 - 91 = 30$

Step 3. $30 \div 3 = 10 \rightarrow$ weight of mangoes

Step 4. $13 - 10 = 3 \rightarrow$ weight of apples

Thus, the number of apples left with vendor
 $= 3 \times 7 = 21$.

Alternatively In the first instance option (c) is ruled out
 Since,

$$\frac{80}{7} + \frac{41}{10} \neq 13 \quad \begin{matrix} 176 - 55 = 121 \\ \text{and } 121 - 80 = 41 \end{matrix}$$

Again, $\frac{35}{7} + \frac{86}{10} \neq 13$ kg, option (b) is wrong.

Once again $\frac{21}{7} + \frac{100}{10} = 13$ kg

Hence, option (a) is correct.

- 29 Since, we know that Ritika has purchased 2 kg mangoes and 5 kg apples. Thus, she has spent $(2 \times 35 + 5 \times 40) = ₹ 270$.

	Mangoes	Apples
SP	35	40
CP	30	30
New SP	40	35

Therefore, $CP = 30 \times 10 + 30 \times 3 = 390$

and $New SP = 40 \times 10 + 35 \times 3 = 505$

∴ Profit = $505 - 390 = 115$

and Profit percentage = $\frac{115}{390} \times 100 = 29.48\%$

- 31 **Solving with the help of options:** In this type of questions we start with least valued options and tend towards higher valued option.

Option (a): If we take 4 coins of ₹ 1, then

$$(5 \times 1) + (2 \times 21) = 47 \neq 44 \quad (48 - 4 = 44)$$

$$5 \times 2 + 2 \times 20 = 50 \neq 44 \quad \text{and } (26 - 4 = 22)$$

Thus, we need to move further since the value of money is exceeding.

Option (b): If we take 5 coins of ₹ 1, then

$$5 \times 1 + 2 \times 20 = 45 \neq 43 \quad (43 = 48 - 5)$$

$$5 \times 2 + 2 \times 19 = 48 \neq 43 \quad \text{and } (21 = 26 - 5)$$

Option (c): If we take 7 coins of ₹ 1, then

$$5 \times 1 + 2 \times 18 = 41 = 41 \quad \left\{ \begin{matrix} (48 - 7 = 41) \\ \text{and } (26 - 7 = 19) \end{matrix} \right\}$$

Thus, the option (c) is correct.

Alternatively $5a + 2b + c = 48$ $a \rightarrow ₹ 5$ coin
 $a + b + c = 26$ $b \rightarrow ₹ 2$ coin
 $- \quad - \quad -$
 $4a + b = 22$ $c \rightarrow ₹ 1$ coin

$(a, b) \rightarrow (c)$

$(1, 18) \rightarrow 7$

$(2, 14) \rightarrow 10$

$(3, 10) \rightarrow 13$

$(4, 6) \rightarrow 16$

$(5, 2) \rightarrow 19$

- 32 Go through options.

CHAPTER 14

Theory of Equations

As per the past years CAT papers, on an average two-three questions are being asked. The problems of this chapter are not so much difficult to solve. Since this chapter is concerned with secondary level syllabus therefore it is important in JMET, XAT and SNAP etc. In this chapter we study quadratic equation, inequation and higher degree polynomial expressions.

14.1 Definition of the Standard Quadratic Expression

For the real numbers a, b and c , the expression $ax^2 + bx + c$ is called the quadratic expression, if $a \neq 0$. Here a is the coefficient of x^2 , b is the coefficient of x and c is a constant term.

- If $a = 0$, then quadratic expression $ax^2 + bx + c$ will reduce to a linear expression $bx + c$. That means if a (the coefficient of x^2) is zero, the given expression will no longer be a quadratic expression.
- When we say that a, b and c are real numbers, it means a, b and c can be any sort of numbers like rational numbers or irrational numbers or fractions or integers.

Nomenclature	Format	Restrictions
Quadratic Expression	$ax^2 + bx + c$	—
Quadratic Function	$y = ax^2 + bx + c$	Any value
Quadratic Function	$f(x) = ax^2 + bx + c$	Any value
Quadratic Equation	$ax^2 + bx + c = 0$	Strictly equal to zero
Quadratic Inequation	$ax^2 + bx + c \geq 0$	Non-negative values
Quadratic Inequation	$ax^2 + bx + c > 0$	Positive values
Quadratic Inequation	$ax^2 + bx + c \leq 0$	Non-positive values
Quadratic Inequation	$ax^2 + bx + c < 0$	Negative values

Thus by substituting the arbitrary numbers in place of x in the function you can obtain various values of y or $f(x)$.

NOTE You can use y and $f(x)$ interchangeably as per the requirement or convenience. It means, conceptually, y and $f(x)$ both represent the same thing. Technically, the relation $y = f(x)$ represents a function, which means by substituting the values of x you can get the corresponding values of y .

Chapter Checklist

- Definition of the Standard Quadratic Expression
- Characteristics of the Graph of the Quadratic Function
- Different Ways to Express the Quadratic Equation
- Solutions or Roots of the Quadratic Equations
- Methods of Solving the Quadratic Equation
- Formation of a Quadratic Equation From the Known Roots
- Nature of the Roots of the Quadratic Equation
- Maximum or Minimum value of a Quadratic Equation
- Condition for Common Roots Between two Quadratic Equations
- Basics of Inequality
- Position of Roots of a Quadratic Equation with Respect to One or Two Real Numbers
- Relation Between the Roots of Two Quadratic Equations
- Polynomial Equations or Functions of Higher Degree
- End (or Long Term) Behaviour of a Polynomial Function
- Solutions (or Roots) of a Polynomial Equation
- Polynomial Inequality
- Relation between Roots and Coefficients in a Polynomial
- Rational Polynomials
- Rational Polynomial Inequalities
- Maximum and Minimum value of a Rational Expression
- CAT Test

Variants of the Quadratic Function

All of the following functions are quadratic functions since the degree of each equation is 2. That means in each function there is always a term x^2 or ax^2 for every non-zero value of a . Here x is a variable and a, b, c are the constant numbers.

$$y = x^2$$

$$y = ax^2$$

$$y = ax^2 + x$$

$$y = ax^2 + bx$$

$$y = ax^2 + c$$

$$y = ax^2 + x + c$$

$$y = ax^2 + bx + c$$

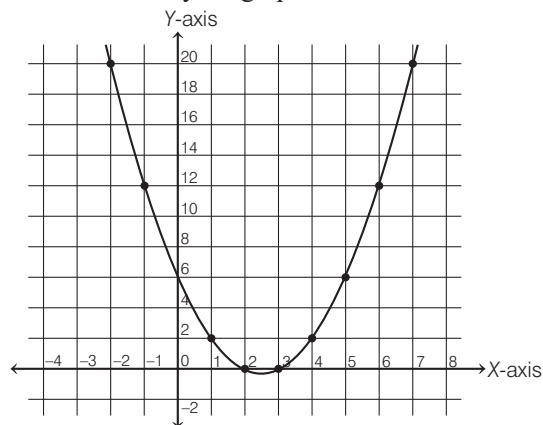
Graphical Illustration

Let us consider a quadratic function $y = x^2 - 5x + 6$. By substituting the arbitrary numerical values of x in the above function we get the corresponding values of y as shown in the following table

x	-2	-1	0	1	2	3	4	5	6	7
y	20	12	6	2	0	0	2	6	12	20

Now if we connect all the points (x, y) on the coordinate plane, we will get a quadratic graph as shown below.

Please take note of the fact that you can consider any real number for x to draw your graph.



Properties of the Quadratic Equations

1. The degree of any quadratic equation is always 2.
2. The value of x that satisfies the relation $ax^2 + bx + c = 0$ is called the root or zero or solution of this equation.
3. A quadratic equation has exactly two roots (or solutions or zeros).

4. A quadratic equation cannot have more than two different roots.
5. A quadratic equation can have either two or zero REAL roots.
6. If α is a root of the quadratic equation $ax^2 + bx + c = 0$, then $(x - \alpha)$ is a factor of $ax^2 + bx + c = 0$.
7. If α and β are the two roots of the quadratic equation $ax^2 + bx + c = 0$, then $ax^2 + bx + c = 0$ can be expressed as the product of two factors $(x - \alpha)$ and $(x - \beta)$ as following $ax^2 + bx + c = (x - \alpha)(x - \beta) = 0$

NOTE Even though there CANNOT be more than two roots of any given quadratic equation, however, from any two given roots you can have an infinite number of quadratic equations. To get all the possible quadratic equations with both the roots common, just multiply or divide the original (or basic) quadratic equation that you have. So it follows that $ax^2 + bx + c \Rightarrow k(x - \alpha)(x - \beta) = 0$; Where k is any real number.

Therefore, you have to use your own discretion with regard to the problem that k should be 1 or other values of k are possible/valid/required.

8. If $(x - \alpha)$ and $(x - \beta)$ are the two factors of the quadratic equation $ax^2 + bx + c = 0$, then $(x - \alpha)(x - \beta) = 0$

$$(x - \alpha) = 0 \Rightarrow x = \alpha \quad \text{or} \quad (x - \beta) = 0 \Rightarrow x = \beta$$

Where α and β are the roots of the equation $ax^2 + bx + c = 0$

9. If quadratic equation is satisfied by more than two distinct numbers (real or imaginary) then it becomes an identity i.e., $a = b = c = 0$.

Becoming an identity implies that whatever value you substitute for x , the equation will be satisfied for every value of x .

Practice Exercise

1. Which of the following is/are the solution(s) of $x^2 - 7x + 12 = 0$?
(a) 2 (b) 3
(c) 4 (d) both 3 and 4
2. A quadratic equation $ax^2 + bx + c = 0$ may have
(a) two real roots
(b) two non-real roots
(c) one real and one non-real root
(d) Either (a) or (b)
3. If 5 and -6 are the roots of the quadratic equation, then what will be the factors of the quadratic equation?
(a) $(x - 5)$ and $(x - 6)$ (b) $(x - 5)$ and $(x + 6)$
(c) $(x + 5)$ and $(x - 6)$ (d) $(x + 5)$ and $(x + 6)$
4. If $3/2$ and 4 are the two roots of a quadratic equation, then which one of the following is not the correct quadratic equation?
(a) $2x^2 - 11x + 6 = 0$ (b) $6x^2 - 33x + 18 = 0$
(c) $-10x^2 - 55x - 30 = 0$ (d) $4x^2 - 41x + 24 = 0$

5. If $(x - 7)$ and $(x - 17)$ are the factors of a quadratic equation, then which one of the following is the correct equation?
- (a) $x^2 - 10x + 24 = 0$
 (b) $x^2 - 33x + 119 = 0$
 (c) $x^2 + 24x - 119 = 0$
 (d) $x^2 - 24x + 119 = 0$
6. For what value of p , the equation $(p - 1)(2p + 1)x^2 + (p^2 - 1)x + (p - 1)(p - 3) = 0$ is an identity (or has more than two solutions)?
- (a) -1 (b) $-1/2$
 (c) 1 (d) $1/3$
7. In the given equation $ax^2 + bx + c$, if $a = 0$ and $b \neq 0$, then
- (a) The equation represents a linear equation
 (b) The equation represents a quadratic equation
 (c) The equation represents a bi-quadratic equation
 (d) none of the above

Answers

1. (d)	2. (d)	3. (d)	4. (d)	5. (d)
6. (c)	7. (a)			

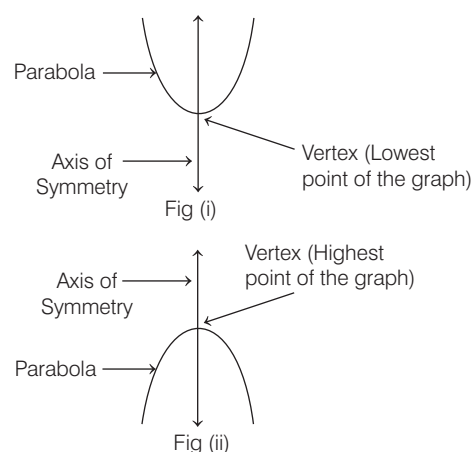
14.2 Characteristics of the Graph of the Quadratic Function

- A graph of a quadratic equation is exactly a parabola as shown below.
- The parabola is always symmetric.
- The line that splits the parabola through the middle is called the "axis of symmetry".
- Axis of symmetry is obtained by $x = -b/2a$. It implies that how far the axis of symmetry is from the Y -axis.
- The axis of symmetry may or may not overlap the Y -axis, which depends on the quadratic function. But axis of symmetry is always parallel to Y -axis.
- The point on the axis of symmetry that bisects the parabola is called the "vertex", and it is the point where the curvature is greatest.
- The parabola can open UP or DOWN only.
- If the parabola opens up, the vertex will be the lowest point and if it opens down, the vertex will be the highest point of the graph.
- The lowest point of the graph is called the minimum (plural is minima) and the highest point of the graph is called the maximum (plural is maxima).
- The vertex of the quadratic graph is obtained by $y = \frac{(4ac - b^2)}{4a}$.

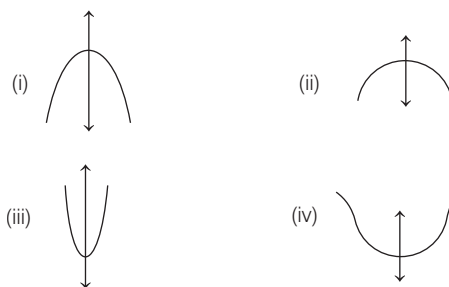
It implies that how far the vertex is from the X -axis. The positive (or negative) value of vertex y indicates that the vertex is above (or below) the X -axis.

11. The parabolic graph must intersect Y -axis but only once; except in rare cases where the values of $x = 0$ is not allowed, it may not intercept. It implies that the graph of quadratic function, in general, may exist anywhere across the whole X -axis, depending on the quadratic function.

NOTE For more info about the axis and Cartesian plane you can refer Coordinate Geometry.

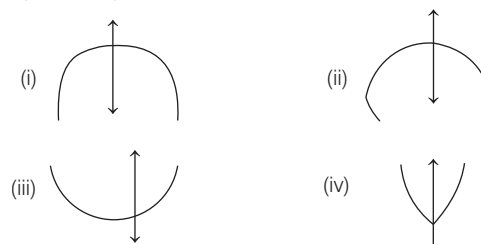
**Practice Exercise**

1. Which of the following is not a graph of a quadratic equation?



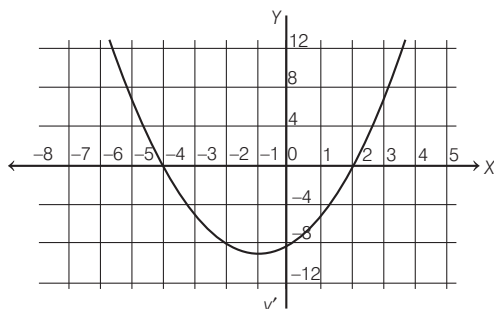
- (a) only (iii) (b) (i) and (iii) (c) (ii) and (iv) (d) only (iv)

2. Which of the following is a correct graph of a quadratic equation, where the vertical line represents the axis of symmetry?



- (a) Only (iii) (b) (i) and (iii)
 (c) (i) and (iv) (d) none of the graphs

3. Consider the following graph of a quadratic function. Then which of the following facts are true regarding this quadratic graph?



- (i) Y-intercept is -8 (ii) X-intercepts are -4 and 2 .
 (iii) $a > 0$ (iv) $D > 0$
 (v) $D > 32a$ (vi) $b = 2a$
 (vii) Vertex of the graph is less than -8 .
 (viii) The difference between the Y-axis and axis of symmetry is 1 unit.
 (ix) The equation of axis of symmetry is $x = -1$.
 (x) The value of y is same, if it is equidistant from $x = -1$.
 (a) (i), (ii), (iii) and (vi) (b) (ii), (iii), (iv) and (x)
 (c) Except (v), (vi) and (x) (d) all of these
4. For the quadratic equation $3x^2 + 12x + 10 = 0$, what is the distance between the Y-axis and the axis of symmetry of the quadratic graph?
 (a) 2 units (b) 4 units (c) 3 units (d) 6 units
5. The two roots of a quadratic equation are -6 and 12 , what is the distance between the axis of symmetry of the quadratic graph and the Y-axis?
 (a) 6 units (b) 1 unit (c) 2 units (d) 18 units
6. For the quadratic equation $x^2 + 6x - 16 = 0$, what will be the position of the axis of symmetry of the quadratic graph with reference to the Y-axis?
 (a) 2 units left (b) 3 units right
 (c) 3 units left (d) can't be determined
7. The axis of symmetry, of a quadratic graph, is always parallel to
 (a) X-axis (b) Y-axis
 (c) Z-axis (d) none of these
8. For a quadratic graph, the axis of symmetry and Y-axis coincide (or overlap) then,
 (a) $a - b = c$ (b) $2a - b = 0$
 (c) $b = 0$ (d) $c = 0$
9. For the quadratic equation $3x^2 + 12x + 10 = 0$, what is the vertex of the graph?
 (a) $y = -2$ (b) $y = -4$ (c) $y = 3$ (d) 6
10. For the quadratic equation $x^2 + 12x + 20 = 0$, how far is the vertex from the X-axis?
 (a) 8 unit (b) $16/3$ unit (c) 16 unit (d) $8/3$ unit

11. For the quadratic equation $x^2 - 11x + 24 = 0$, what will be the position of the vertex of the quadratic graph with reference to the X-axis?
 (a) $25/4$ units below (b) $25/4$ units above
 (c) $21/4$ units below (d) can't be determined
12. For the quadratic equation $x^2 + 9x + 18 = 0$, what will be the position of the vertex of the quadratic graph with reference to the origin (or intersection of both the axes)?
 (a) $9/2$ units left, $113/4$ units below
 (b) $9/2$ units left, $113/4$ units above
 (c) 9 units right, 81 units below
 (d) 9 units right, 9 units below
13. Find the value of y where the graph of $3x^2 + 19x - 18 = 0$ intersects the Y-axis.
 (a) -18 (b) 0
 (c) 4 (d) 16
14. Which of the following is false?
 (i) If only the positive values of x are taken, the quadratic graph does not intersect the Y-axis.
 (ii) If only negative values are taken, the quadratic graph does not intersect the Y-axis.
 (iii) The interception of Y-axis can be known when we substitute $x = 0$, in the quadratic function.
 (iv) There can be minimum zero and maximum two x -intercepts.
 (a) (i) and (iv) (b) (i), (iii), (iv)
 (c) (ii), (iii), (iv) (d) none of these
15. If D be the discriminant of the quadratic equation $ax^2 + bx + c = 0$, what will be the coordinates of its vertex?
 (a) $-b/2a, -D/4a$ (b) $-b/2a, D/4a$
 (c) $b/2a, -D/2a$ (d) $-b/2a, \sqrt{D}/4a$

Answers

1. (d)	2. (d)	3. (d)	4. (a)	5. (b)
6. (c)	7. (b)	8. (c)	9. (a)	10. (b)
11. (a)	12. (a)	13. (a)	14. (d)	15. (a)

14.3 Different Ways to Express the Quadratic Equation

A quadratic equation can be expressed in the following forms

(1) $ax^2 + bx + c = 0$

(2) $x^2 + \left(\frac{b}{a}\right)x + \left(\frac{c}{a}\right) = 0$;

where a , b and c are the standard notations.

(3) $x^2 - (\alpha + \beta)x + (\alpha\beta) = 0$; where α and β are the roots of the equation.

Exp.) We can express a quadratic equation in three different forms, whose roots are (-2) and $(1/2)$.

$$(1) 2x^2 + 3x - 1 = 0$$

$$(2) x^2 + \left(\frac{3}{2}\right)x + \left(-\frac{1}{2}\right) = 0; \text{ where } a = 2, b = 3, c = -1$$

$$\Rightarrow x^2 + \frac{3}{2}x - \frac{1}{2} = 0$$

$$(3) x^2 - \left(-\frac{3}{2}\right)x + \left(-\frac{1}{2}\right) = 0; \text{ where } \alpha = -2, \beta = \frac{1}{2}$$

$$\Rightarrow x^2 + \frac{3}{2}x - \frac{1}{2} = 0$$

Discriminant of the Quadratic Equations

For the given quadratic equation $ax^2 + bx + c = 0$, the discriminant is denoted by D and expressed as $D = b^2 - 4ac$.

Exp.) Determine the discriminant of the following quadratic equations.

$$(i) x^2 + 10x + 6$$

$$(ii) 8x + 17 + 5x^2$$

$$(iii) -x^2 - 8x + 15$$

$$(iv) x^2 + 6x$$

$$(v) x^2 - 16$$

$$(vi) 9x^2$$

Solution (i) Discriminant of the equation $x^2 + 10x + 6$ is

$$10^2 - 4 \times 1 \times 6 = 76$$

(ii) Discriminant of the equation $5x^2 + 8x + 17$ is

$$8^2 - 4 \times 5 \times 17 = -276$$

(iii) Discriminant of the equation $-x^2 - 8x + 15$ is

$$(-8)^2 - 4 \times (-1) \times 15 = 124$$

(iv) Discriminant of the equation $x^2 + 6x$ is $6^2 - 4 \times 1 \times 0 = 36$

(v) Discriminant of the equation $x^2 - 16$ is $0^2 - 4 \times 1 \times (-16) = 64$

(vi) Discriminant of the equation $9x^2$ is $0^2 - 4 \times 9 \times 0 = 0$

Condition that the Quadratic Equation is Factorisable into two Linear Factors

When the discriminant $D \geq 0$, then the equation $ax^2 + bx + c = 0$ is factorisable into two linear factors. That is $ax^2 + bx + c = 0$

$\Rightarrow (x - \alpha)(x - \beta) = 0$; where α and β are the roots of the quadratic equation.

Exp.) Which of the following quadratic expressions are factorisable into two linear factors?

$$(i) 2x^2 + 10x + 6$$

$$(ii) 5x^2 + 8x + 17$$

$$(iii) x^2 - 16$$

$$(iv) 9x^2$$

Solution

(i) Discriminant of the equation $x^2 + 10x + 6$ is $76 > 0$, so it is factorisable into two linear factors as $x^2 + 10x + 6 = (x + \sqrt{19})(x - \sqrt{19})$.

(ii) Discriminant of the equation $5x^2 + 8x + 17$ is $-276 < 0$, so it cannot be factorised into two linear factors.

(iii) Discriminant of the equation $x^2 - 16$ is $64 > 0$, so it is factorisable into two linear factors as $x^2 - 16 = (x + 4)(x - 4)$

(iv) Discriminant of the equation $9x^2$ is 0 , so it is factorisable into two linear factors as $9x^2 = (3x)(3x)$ or $9x^2 = x(9x)$.

How to Factorize a Given Quadratic Equation

Step 1 First of all write down the given quadratic equation in its standard form of $ax^2 + bx + c = 0$

Step 2 If any of the coefficients a, b and c is a rational number, make it integer by multiplying whole equation by the denominator of that rational number.

Step 3 Find the product ac .

Step 4 Find all the factor pairs of ac , such that the product of any pair is equal to ac .

Step 5 Choose one such pair, say (m, n) , such that if ac is positive then $m + n = b$ and if ac is negative then $m - n = -b$, provided $m \leq n$.

Step 6 Write down the expression as $ax^2 + (m + n)x + c = 0$ or $ax^2 + (m - n)x + c = 0$, as per the nature of b .

Step 7 Now make two pairs out of four terms considering there will be something common in the pairs as shown below. If $ax^2 + (m + n)x + c = 0$

$$\Rightarrow (ax^2 + m) + (n + c) = 0$$

$$\text{or } (ax^2 + n) + (m + c) = 0$$

$$\text{If } ax^2 + (m - n)x + c = 0$$

$$\Rightarrow (ax^2 + m) + (-n + c) = 0$$

$$\text{or } (ax^2 - n) + (m + c) = 0$$

Step 8 Take out common if any, and then simplify.

NOTE This technique has some limitations that you will come across when you try to find the factors of an equation whose roots are irrational. Therefore, you may use other techniques to find the factors, like using *sum and product of roots* method or *Sridharacharya's* method.

Exp. 1) Factorize the given quadratic equation $2x^2 + 3x - 9 = 0$.

Solution Product $a \cdot c = 2 \times -9 = -18$

Now, $18 = 1 \times 18$ or 2×9 or 3×6

Since $a \cdot c$ is negative, therefore we will take the difference of 3 and 6 and which is 3 = b . Therefore we choose 3 and 6.

Now we write the equation, as following $2x^2 + 3x - 9 = 0$

$$\Rightarrow 2x^2 + (6x - 3x) - 9 = 0 \Rightarrow (2x^2 + 6x) - (3x + 9) = 0$$

$$\Rightarrow 2x(x + 3) - 3(x + 3) = 0 \Rightarrow (x + 3)(2x - 3) = 0$$

Exp. 2) Factorize the given quadratic equation

$$3 - \frac{5}{2}x - 3x^2 = 0.$$

Solution The given quadratic equation can be expressed in the standard form as $-3x^2 - \frac{5}{2}x + 3 = 0$

Since the value of b is a rational number therefore we will multiply the whole equation by the denominator of b that is 2 and then we will get the whole equation with integer coefficients $-6x^2 - 5x + 6 = 0$. Since product $a \cdot c = -6 \times 6 = -36$

Therefore, $36 = 1 \times 36$ or 2×18 or 3×12 or 4×9 or 6×6

Now, since $a \cdot c$ is negative, therefore we will take the difference of 4 and 9 which is 5 = b . Therefore, we choose 4 and 9. Now, we write the equation as following

$$\begin{aligned} -6x^2 - 5x + 6 &= 0 \Rightarrow -6x^2 + (4x - 9x) + 6 = 0 \\ \Rightarrow (4x - 6x^2) + (6 - 9x) &= 0 \\ \Rightarrow 2x(2 - 3x) + 3(2 - 3x) &= 0 \Rightarrow (2 - 3x)(2x + 3) = 0 \end{aligned}$$

Exp. 3) Factorize the given biquadratic equation

$$x^4 + 2x^2 - 8 = 0.$$

Solution The given biquadratic equation can be expressed in the standard form as $(x^2)^2 + 2x^2 - 8 = 0$

Product $a \cdot c = 1 \times -8 = -8$

Now, $8 = 1 \times 8$ or 2×4

Since $a \cdot c$ is negative, therefore we will take the difference of 2 and 4 and which is 2 = b . Therefore, we choose 2 and 4.

Now we write the equation as following $(x^2)^2 + 2x^2 - 8 = 0$

$$\begin{aligned} \Rightarrow (x^2)^2 + (2x^2 - 4x^2) - 8 &= 0 \\ \Rightarrow [(x^2)^2 + 2x^2] + (-4x^2 - 8) &= 0 \\ \Rightarrow x^2(x^2 + 2) - 4(x^2 + 2) &= 0 \Rightarrow (x^2 + 2)(x^2 - 4) = 0 \\ \Rightarrow (x^2 + 2)(x + 2)(x - 2) &= 0 \end{aligned}$$

Exp. 4) Factorize the given quadratic equation

$$x^6 + 6x^3 + 5 = 0.$$

Solution The given quadratic equation can be expressed in the standard form as $(x^3)^2 + 6x^3 + 5 = 0$. Product $a \cdot c = 1 \times 5 = 5$

Now, $5 = 1 \times 5$ is the only pair, so it has got to be easy.

Since $a \cdot c$ is positive, therefore we will take the addition of 1 and 5 and which is 6 = b . Now we write the equation as following

$$\begin{aligned} (x^3)^2 + 6x^3 + 5 &= 0 \\ \Rightarrow (x^3)^2 + (x^3 + 5x^3) + 5 &= 0 \\ \Rightarrow [(x^3)^2 + x^3] + (5x^3 + 5) &= 0 \\ \Rightarrow [(x^3)^2 + x^3] + (5x^3 + 5) &= 0 \\ \Rightarrow x^3(x^3 + 1) + 5(x^3 + 1) &= 0 \\ \Rightarrow (x^3 + 5)(x^3 + 1) &= 0 \\ \Rightarrow (x^3 + 5)(x^3 + 1^3) &= 0 \\ \Rightarrow (x^3 + 5)(x + 1)(x^2 - x + 1) &= 0 \end{aligned}$$

Practice Exercise

Find the factors of following equations.

- $x^{2/3} - x^{1/3} - 56$
- $x + 9x^{1/2} + 8$
- $4x^4 - 81$
- $(x - 3)^4 + 2(x - 3)^2 - 8$
- $6x^2 - xy - 12y^2$
- $20x^2 + 17x - 91$
- $35x + 84 - 21x^2$

Answers

1. Assume $x^{1/3} = p$, then the given equation will become like $p^2 - p - 56$

$$\begin{aligned} \text{Then, } p^2 - p - 56 &= p^2 - 8p + 7p - 56 \\ &= (p + 7)(p - 8) \end{aligned}$$

$$\text{Therefore, } x^{2/3} - x^{1/3} - 56 = (x^{1/3} + 7)(x^{1/3} - 8)$$

2. Assume $x^{1/2} = p$, then the given equation will become like $p^2 + 9p + 8$

$$\text{Then, } p^2 + 9p + 8 = p^2 + p + 8p + 8 = (p + 8)(p + 1).$$

$$\text{Therefore, } x + 9x^{1/2} + 8 = (x^{1/2} + 8)(x^{1/2} + 1)$$

3. We can apply $(a^2 - b^2) = (a + b)(a - b)$

$$4x^4 - 81 \Rightarrow (2x)^2 - 9^2 = (2x^2 + 9)(2x^2 - 9)$$

4. Assume $(x - 3)^2 = p$, then the given equation will become like $p^2 + 2p - 8$.

$$\text{Then, } p^2 + 2p - 8 = p^2 + 4p - 2p - 8 = (p + 4)(p - 2).$$

5. $6x^2 - xy - 12y^2 = 6x^2 - 9xy + 8xy - 12y^2$

Then find a pair of factors of 6×12 , such that their difference is 1. That is $6 \times 12 = 9 \times 8$

$$\begin{aligned} &= 3x(2x - 3y) + 4y(2x - 3y) \\ &= (2x - 3y)(3x + 4y). \end{aligned}$$

6. Multiply 20 and 91. That is $2 \times 2 \times 5 \times 7 \times 13$. Then split these prime factors in two groups such that the difference of these two factors is 17. That is,

$$(2 \times 2 \times 13)(5 \times 7) = 52 \times 35$$

Now, we have, $20x^2 + 17x - 91$

$$= 20x^2 + (52x - 35x) - 91$$

$$= (4x - 7)(5x + 13)$$

7. $35x + 84 - 21x^2 = -21x^2 + 35x + 84$

$$= -7(3x^2 - 5x - 12)$$

Now multiply 3 and 12. That is $3 \times 2 \times 2 \times 3$

Then find the pair of factors of $3 \times 2 \times 2 \times 3$ such that the difference of these two factors is 5.

That is

$$3 \times 2 \times 2 \times 3 = 4 \times 9.$$

Now, $-21x^2 + 35x + 84$

$$= -7(3x^2 - 5x - 12)$$

$$= -7(3x^2 - 9x + 4x - 12)$$

$$= -7[(x - 3)(3x + 4)]$$

14.4 Solutions or Roots of the Quadratic Equations

The value of x that satisfies the relation $ax^2 + bx + c = 0$ is called the root or solution of this equation. Simply, the root of the quadratic function $y = ax^2 + bx + c = 0$ is the value of x for which y becomes 0.

NOTE It is preferable to use the term zero while referring a quadratic function and use the term root while referring to a quadratic equation.

Exp. 1) The roots of the quadratic equation $x^2 - 8x + 15 = 0$ are 3 and 5. As, by substituting 3 or 5 in place of x , both sides of the equation become 0.

Exp. 2) The roots of the quadratic function $y = x^2 - 8x + 15$ are 3 and 5. As by substituting 3 or 5 in place of x , the value of y becomes 0.

Exp. 3) The roots of the quadratic equation $x^2 - 8x + 22 = 7$ are 3 and 5. The given equation can be simplified as $x^2 - 8x + 15 = 0$. As, by substituting 3 or 5 in place of x , both sides of the equation become 0.

Exp. 4) The roots of the quadratic equation $x^2 + 9x - y = 17x - 15$ are 3 and 5. The given equation can be simplified as $y = x^2 - 8x + 15$.

As, by substituting 3 or 5 in place of x , the value of y becomes 0.

Sum and Product of the Roots of a Quadratic Equation

If the two roots of the quadratic equation $ax^2 + bx + c = 0$ are α and β ,

$$(i) \text{ Sum of the roots } = (\alpha + \beta) = \frac{-b}{a}$$

$$(ii) \text{ Product of the roots } = (\alpha\beta) = \frac{c}{a}$$

Exp. 1) Find the sum and product of the roots of the quadratic equation $x^2 - 11x + 28 = 0$.

Solution Comparing the given equation with the standard form of the quadratic equation $ax^2 + bx + c = 0$, we get $a = 1$, $b = -11$, $c = 28$. Therefore, sum of roots

$$= \frac{-b}{a} = \frac{-(-11)}{1} = 11 \text{ and product of the roots } = \frac{c}{a} = \frac{28}{1} = 28.$$

Exp. 2) Find the sum and product of the roots of the quadratic equation $3x^2 + 15x + 12 = 0$.

Solution Comparing the given equation with the standard form of the quadratic equation $ax^2 + bx + c = 0$, we get $a = 3$, $b = 15$, $c = 12$. Therefore, sum of roots $= \frac{-b}{a} = \frac{-15}{3} = -5$ and product of the roots $= \frac{c}{a} = \frac{12}{3} = 4$.

Exp. 3) If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, show that bc^2, ca^2, ab^2 are in Arithmetic Progression (AP). It is well known that if any three consecutive terms, form a sequence, are such that the difference between any two consecutive terms is same, then they are in AP.

For Example if x, y, z are in AP, then $z - y = y - x = d$ (say).

Solution If α and β be the roots of the quadratic equation

$$ax^2 + bx + c = 0, \text{ then } \alpha + \beta = \frac{-b}{a} \quad \dots(i)$$

$$\alpha\beta = \frac{c}{a} \quad \dots(ii)$$

$$\text{Given that, } \alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$\Rightarrow \alpha + \beta = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} \Rightarrow \alpha + \beta = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$\Rightarrow \frac{-b}{a} = \frac{\left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\left(\frac{c}{a}\right)^2} \Rightarrow \frac{-b}{a} = \frac{b^2 - 2ac}{c^2}$$

$$\Rightarrow bc^2 + ab^2 = 2ca^2 \Rightarrow bc^2 - ca^2 = ca^2 - ab^2$$

Therefore bc^2, ca^2, ab^2 are in Arithmetic Progression.

NOTE For better understanding of Arithmetic Progression you may refer the chapter Sequence, Series and Progression.

Practice Exercise

- If one root of a quadratic equation is zero, which one of the following is correct?
(a) $ax^2 + bx + c = 0$ (b) $ax^2 + c = 0$
(c) $ax^2 + bx = 0$ (d) $bx + c = 0$
- If both the roots of a quadratic equation are equal in magnitude but opposite in nature, which one of the following is correct?
(a) $x^2 + \left(\frac{b}{a}\right)x - \left(\frac{c}{a}\right) = 0$ (b) $ax^2 + bx = 0$
(c) $x^2 = 0$ (d) $ax^2 + c = 0$
- If both the roots of a quadratic equation are coincident, which one of the following is not correct?
(a) $x^2 - 2\sqrt{c}x + c = 0$ (b) $x^2 + 2kx + k^2 = 0$
(c) $x^2 - 2bx - b^2 = 0$ (d) $x^2 - 2kx + k^2 = 0$
- If one root of the equation is reciprocal to the other root, which of the following is correct?
(a) $ax^2 + bx = 0$ (b) $ax^2 + bx + 1 = 0$
(c) $ax^2 - bx = 0$ (d) $x^2 + x - c = 0$
- If one root of the quadratic equation $x^2 - x - 1 = 0$ is α , the other root is
(a) $\alpha^3 + 3\alpha$ (b) $\alpha^3 - 3\alpha$ (c) $\alpha^2 + 3$ (d) $\alpha^2 - 3$

Answers

1. (c) | 2. (d) | 3. (c) | 4. (b) | 5. (b)

14.5 Methods of Solving the Quadratic Equation

- (1) Factorization Method
- (2) Sridharacharya Method
- (3) Square Completion Method
- (4) Sum and Product of Roots Method

Factorization Method

Step 1 If the equation is not given in the standard form then express the given equation in the standard form of $ax^2 + bx + c = 0$.

Step 2 Factorize the equation $ax^2 + bx + c = 0$ into two linear factors.

Step 3 Equate each linear factor with zero.

Step 4 Simplify each linear factor.

Exp.) Find the roots of $\frac{55}{24}x + x^2 - 1 = 0$.

Solution The given expression can be written as $x^2 + \frac{55}{24}x - 1 = 0$

$$\Rightarrow 24x^2 + 55x - 24 = 0 \Rightarrow (3x + 8)(8x - 3) = 0$$

$$\Rightarrow \left(x + \frac{8}{3}\right)\left(x - \frac{3}{8}\right) = 0 \Rightarrow x + \frac{8}{3} = 0 \text{ or } x - \frac{3}{8} = 0$$

$$\Rightarrow x = -\frac{8}{3} \text{ or } x = \frac{3}{8}$$

Therefore, there are two roots $-\frac{8}{3}$ and $\frac{3}{8}$.

Sridharacharya Method

Step 1 If the equation is not given in the standard form then express the given equation in the standard form of $ax^2 + bx + c = 0$.

Step 2 Substitute the values of a, b and c in the formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Step 3 Then one root (say α) is equal to $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and

another root (say β) is equal to $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$

Exp.) Find the roots of $\frac{55}{24}x + x^2 - 1 = 0$.

Solution The given expression can be written as $x^2 + \frac{55}{24}x - 1 = 0$

$$\Rightarrow 24x^2 + 55x - 24 = 0$$

Here, $a = 24$, $b = 55$ and $c = -24$.

$$\text{Now, } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-55 \pm \sqrt{55^2 - 4 \times 24 \times (-24)}}{2 \times 24}$$

$$\begin{aligned} \Rightarrow \frac{-55 \pm \sqrt{5329}}{48} &= \frac{-55 \pm 73}{48} \\ \Rightarrow \frac{-55 + 73}{48} \text{ and } \frac{-55 - 73}{48} \\ \Rightarrow -\frac{8}{3} \text{ and } \frac{3}{8}. \end{aligned}$$

Square Completion Method

Step 1 If the equation is not given in the standard form then express the given equation in the standard form of $ax^2 + bx + c = 0$.

Step 2 Modify the equation as $ax^2 + bx = -c$

Step 3 If $a \neq 1$, divide the whole equation by a

Step 4 Add $\left(\frac{b}{2}\right)^2$ to both the sides.

Step 5 Find the square root of both the sides.

Step 6 Simplify the resulting equation.

Exp. 1) Find the roots of $\frac{55}{24}x + x^2 - 1 = 0$.

Solution The given expression can be written as $x^2 + \frac{55}{24}x - 1 = 0$.

$$\text{Then, } x^2 + \frac{55}{24}x + \left(\frac{55}{48}\right)^2 = 1 + \left(\frac{55}{48}\right)^2$$

$$\Rightarrow x^2 + 2\left(\frac{55}{48}\right)x + \left(\frac{55}{48}\right)^2 = \left(\frac{73}{48}\right)^2$$

$$\Rightarrow \left(x + \frac{55}{48}\right)^2 = \left(\frac{73}{48}\right)^2$$

$$\Rightarrow x + \frac{55}{48} = \pm \frac{73}{48} \Rightarrow x = -\frac{55}{48} \pm \frac{73}{48}$$

$$\Rightarrow x = -\frac{128}{48} \text{ and } \frac{18}{48} \Rightarrow x = -\frac{8}{3} \text{ and } \frac{3}{8}$$

Exp. 2) Find the roots of $2x^2 - 3x - 1 = 0$.

Solutions The given expression can be written as $x^2 - \frac{3}{2}x - \frac{1}{2} = 0$.

$$\Rightarrow x^2 - \left(\frac{3}{2}\right)x - \frac{1}{2} = 0 \Rightarrow x^2 - 2\left(\frac{3}{4}\right)x = \frac{1}{2}$$

$$\Rightarrow x^2 - 2\left(\frac{3}{4}\right)x + \left(\frac{3}{4}\right)^2 = \frac{1}{2} + \left(\frac{3}{4}\right)^2$$

$$\Rightarrow \left(x - \frac{3}{4}\right)^2 = \frac{17}{16} \Rightarrow \left(x - \frac{3}{4}\right) = \pm \frac{\sqrt{17}}{4}$$

$$\Rightarrow x = \frac{3}{4} \pm \frac{\sqrt{17}}{4}$$

$$\Rightarrow x = \frac{3}{4} + \frac{\sqrt{17}}{4} \text{ and } \frac{3}{4} - \frac{\sqrt{17}}{4}$$

$$\Rightarrow x = \frac{3 + \sqrt{17}}{4} \text{ and } \frac{3 - \sqrt{17}}{4}$$

Exp. 3) Find the roots of $x^2 + 2x + 5 = 0$.

Solution The given expression can be written as $x^2 + 2x = -5$.

$$\begin{aligned}\Rightarrow x^2 + 2x + 1 &= -5 + 1 \\ \Rightarrow (x + 1)^2 &= -4 \Rightarrow (x + 1) = \pm \sqrt{-4} \\ \Rightarrow (x + 1) &= \pm 2i \Rightarrow x = -1 \pm 2i \\ \Rightarrow x &= -1 + 2i \text{ and } -1 - 2i\end{aligned}$$

Sum and Product of Roots Method

Step 1 If the equation is not given in the standard form then express the given equation in the standard form of $ax^2 + bx + c = 0$.

Step 2 Find sum of the roots $(\alpha + \beta) = -\frac{b}{a}$ and product of the roots $(\alpha\beta) = \frac{c}{a}$.

Step 3 Substitute the values of $\alpha + \beta$ and $\alpha\beta$ in the formula $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$.

Step 4 Find the value of $\alpha - \beta$.

Step 5 Solve the equations $\alpha + \beta$ and $\alpha - \beta$ to get the roots α and β .

Exp.) Find the roots of $\frac{55}{24}x + x^2 - 1 = 0$.

Solution The given expression can be written as $x^2 + \frac{55}{24}x - 1 = 0$.

The sum of the roots $= \alpha + \beta = -\frac{55}{24}$ and the product of the roots $= \alpha\beta = -1$.

Therefore using $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$, we get

$$(\alpha - \beta)^2 = \left(-\frac{55}{24}\right)^2 - 4(-1) \Rightarrow \alpha - \beta = \pm \frac{73}{24}$$

Solving $\alpha + \beta$ and $\alpha - \beta$, we will get $(\alpha, \beta) = \left(-\frac{8}{3}, \frac{3}{8}\right)$.

Solutions of Equations Reducible to Quadratic Form

Equation which are not quadratic at a glance but can be reduced to quadratic equations by suitable transformations. Some of the common types are :

Type 1. $ax^4 + bx^2 + c = 0$. This can be reduced to a quadratic equation by substituting $x^2 = y$.
i.e., $ay^2 + by + c = 0$

Type 2. $px + \frac{q}{x} = r$, $p(x - a) + \frac{q}{(x - b)} = r$. Multiply both sides by the LCD of LHS to get a quadratic equation $px + \frac{q}{x} = r \Rightarrow px^2 - rx + q = 0$

$$\text{and } p(x - a) + \frac{q}{(x - b)} = r$$

$$\Rightarrow p(x - a)(x - b) - r(x - b) + q = 0$$

Type 3. $\sqrt{a - x^2} = bx + c$. Square both sides and simplify.

$$\text{i.e., } a - x^2 = b^2x^2 + 2bcx + c^2$$

$$\Rightarrow (1 + b^2)x^2 + 2bcx + (c^2 - a) = 0$$

Type 4. $\sqrt{ax + b} + \sqrt{cx + d} = e$ Transform one of the radicals to RHS and square $\sqrt{ax + b} = e - \sqrt{cx + d}$, such equations may require squaring and your solution must satisfy $ax + b \geq 0$ and $cx + d \geq 0$.

Type 5. $a\left(x^2 + \frac{1}{x^2}\right) + b\left(x + \frac{1}{x}\right) + c = 0$ For this type of equations we use the following identity

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$$

$$\text{Thus } a\left[\left(x + \frac{1}{x}\right)^2 - 2\right] + b\left(x + \frac{1}{x}\right) + c = 0$$

Now put $x + \frac{1}{x} = y$ to get a quadratic equation

$$\text{i.e., } ay^2 + by + (c - 2a) = 0$$

Type 6. $a\left(x^2 + \frac{1}{x^2}\right) + b\left(x - \frac{1}{x}\right) + c = 0$

Use the following identity $x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$

$$\text{Thus, } a\left[\left(x - \frac{1}{x}\right)^2 + 2\right] + b\left(x - \frac{1}{x}\right) + c = 0, \text{ put}$$

$x - \frac{1}{x} = y$ to get a quadratic equation in y .

$$\text{i.e., } ay^2 + by + (c - 2a) = 0$$

Type 7. (i) $x^{2a} + x^a + b = 0$ (ii) $x^a + x^{-a} = b$

Put $x^a = y$ to get a quadratic equation in y ,

$$\text{i.e., (i) } y^2 + y + b = 0 \quad \text{(ii) } y + \frac{1}{y} = b$$

NOTE In all the equations involving radical signs, the answer must be checked by substituting in the original equation.

Type 8. $(x + a)(x + b)(x + c)(x + d) + k = 0$. When sum of the quantities a, b, c, d is equal to the sum of the other two, can be solved as shown in example 16.

Exp. 1) Solve for y : $9y^4 - 29y^2 + 20 = 0$.

Solution $9y^4 - 29y^2 + 20 = 0$

Put $y^2 = x$.

$$9x^2 - 29x + 20 = 0 \Rightarrow 9x^2 - 20x - 9x + 20 = 0$$

$$\Rightarrow (x-1)(9x-20) = 0 \Rightarrow x = 1 \text{ or } x = \frac{20}{9}$$

$$\Rightarrow y^2 = 1 \text{ or } y^2 = \frac{20}{9} \Rightarrow y = \pm 1 \text{ and } y = \pm \frac{2\sqrt{5}}{3}$$

Exp. 2) Solve for x : $x^6 - 26x^3 - 27 = 0$.

Solution $x^6 - 26x^3 - 27 = 0$

Let $x^3 = y$

then, $y^2 - 26y - 27 = 0$

$$\Rightarrow y^2 - 27y + y - 27 = 0 \Rightarrow (y+1)(y-27) = 0$$

$$\Rightarrow y = -1 \text{ or } y = 27 \Rightarrow x^3 = -1 \text{ or } x^3 = 27$$

$$\Rightarrow x = -1 \text{ or } x = 3$$

Exp. 3) Solve: $2x - \frac{3}{x} = 5$.

Solution $2x - \frac{3}{x} = 5 \Rightarrow 2x^2 - 3 = 5x \Rightarrow (2x+1)(x-3) = 0$

$$\Rightarrow x = -\frac{1}{2} \text{ or } x = 3.$$

Exp. 4) Solve $\sqrt{2x+9} + x = 13$.

Solution $\sqrt{2x+9} + x = 13 \Rightarrow \sqrt{2x+9} = 13 - x$

$$\text{Squaring both sides } 2x + 9 = (13 - x)^2 \Rightarrow x^2 - 28x + 160 = 0$$

$$\Rightarrow (x-8)(x-20) = 0 \Rightarrow x = 8, x = 20$$

NOTE Here we must have $2x + 9 \geq 0$ thus for $x = 8, \sqrt{2x+9} > 0$

Also for $x = 20, \sqrt{2x+9} > 0$

$\therefore$ Both the values satisfy the condition.

Exp. 5) Solve $\sqrt{2x+9} - \sqrt{x-4} = 3$.

Solution $\sqrt{2x+9} - \sqrt{x-4} = 3$

$$\Rightarrow \sqrt{2x+9} = 3 + \sqrt{x-4}$$

Squaring both sides and simplifying, we get

$$x + 4 = 6\sqrt{x-4}$$

Again squaring both sides,

$$(x+4)^2 = 36(x-4)$$

$$\Rightarrow x^2 - 28x + 160 = 0 \Rightarrow (x-8)(x-20) = 0$$

$$\Rightarrow x = 8, x = 20$$

Verification: $2x + 9 \geq 0$

$$\text{and } x - 4 \geq 0 \Rightarrow x \geq \frac{-9}{2}$$

$$\text{and } x \geq 4$$

Since the values $x = 8$ and 20 satisfy both these conditions

$$\therefore x = 8, x = 20$$

Exp. 6) Solve for x : $2\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) + 14 = 0$.

Solution Put $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$$\therefore 2\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) + 14 = 0$$

$$\Rightarrow 2\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 9\left(x + \frac{1}{x}\right) + 14 = 0$$

Substitute $x + \frac{1}{x} = y$

$$\therefore 2(y^2 - 2) - 9y + 14 = 0 \Rightarrow 2y^2 - 4 - 9y + 14 = 0$$

$$\Rightarrow (y-2)(2y-5) = 0 \Rightarrow y = 2 \text{ or } y = \frac{5}{2}$$

$$\text{Since } x + \frac{1}{x} = 2 \Rightarrow x^2 - 2x + 1 = 0$$

$$\Rightarrow (x-1)^2 = 0 \Rightarrow x = 1$$

$$\text{Also, } y = \frac{5}{2} \Rightarrow x + \frac{1}{x} = \frac{5}{2}$$

$$\Rightarrow 2x^2 - 5x + 2 = 0 \Rightarrow x = 2 \text{ or } x = \frac{1}{2}$$

$$\therefore x = \frac{1}{2}, 1, 2$$

Exp. 7) Solve $6\left(x^2 + \frac{1}{x^2}\right) - 25\left(x - \frac{1}{x}\right) + 12 = 0$.

Solution Put $x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$

$$\therefore 6\left[\left(x - \frac{1}{x}\right)^2 + 2\right] - 25\left(x - \frac{1}{x}\right) + 12 = 0$$

$$\text{Let } x - \frac{1}{x} = y \Rightarrow 6(y^2 + 2) - 25y + 12 = 0$$

$$\Rightarrow y = \frac{3}{2}, y = \frac{8}{3}$$

$$\text{Since } y = x - \frac{1}{x} \text{ and } x - \frac{1}{x} = \frac{8}{3}$$

$$\therefore x - \frac{1}{x} = \frac{3}{2} \text{ and } x = -\frac{1}{3}, x = 3$$

$$\Rightarrow x = -\frac{1}{2} \text{ and } x = 2$$

$$\therefore x = -\frac{1}{3}, -\frac{1}{2}, 2, 3$$

Exp. 8) Solve for x : $\sqrt{x^2 + x - 6} - x + 2$.

$$= \sqrt{x^2 - 7x + 10}, x \in R$$

Solution $\sqrt{x^2 + x - 6} - x + 2 = \sqrt{x^2 - 7x + 10}$

$$\sqrt{(x+3)(x-2)} - (x-2) = \sqrt{(x-5)(x-2)}$$

$$\Rightarrow \sqrt{(x-2)} [\sqrt{(x+3)} - \sqrt{(x-2)} - \sqrt{(x-5)}] = 0$$

Either $\sqrt{x-2} = 0 \Rightarrow x = 2$

or $\sqrt{x+3} - \sqrt{x-2} - \sqrt{x-5} = 0$

$$\Rightarrow \sqrt{x+3} - \sqrt{x-2} = \sqrt{x-5}$$

Squaring both sides

$$x^2 + 12x + 36 = 4(x^2 + x - 6)$$

$$\Rightarrow x = 6, \quad x = -\frac{10}{3}$$

Since the equation involves radical therefore substituting $x = 2, 6$ and $-\frac{10}{3}$ in the original equation, we find that $x = -\frac{10}{3}$ does not satisfy the equation.

$$\therefore x = 2, 6$$

Exp. 9) Solve for x : $3^{x+2} + 3^{-x} = 10$.

Solution $3^{x+2} + 3^{-x} - 10 = 0 \Rightarrow 3^x \cdot 3^2 + \frac{1}{3^x} - 10 = 0$

Let $3^x = y \Rightarrow 9y + \frac{1}{y} - 10 = 0$

$$\Rightarrow 9y^2 - 10y + 1 = 0$$

$$\Rightarrow (9y - 1)(y - 1) = 0$$

$$\Rightarrow y = \frac{1}{9} \text{ or } y = 1$$

When $y = \frac{1}{9} \Rightarrow 3^x = \frac{1}{3^2} \Rightarrow x = -2$

when $y = 1 \Rightarrow 3^x = 1$

$$\Rightarrow x = 0$$

$$\therefore x = -2, 0$$

Exp. 10) Solve for x : $(x+1)(x+2)(x+3)(x+4) = 24$.

Solution $(x+1)(x+2)(x+3)(x+4) = 24$

$$\Rightarrow [(x+1)(x+4)][(x+2)(x+3)] = 24$$

$$\Rightarrow (x^2 + 5x + 4)(x^2 + 5x + 6) = 24$$

Let $x^2 + 5x = y$

$$\therefore (y+4)(y+6) = 24 \Rightarrow y^2 + 10y = 0$$

$$\Rightarrow y = 0 \text{ and } y = -10$$

Now, $y = x^2 + 5x$

When $y = 0, x^2 + 5x = 0 \Rightarrow x = 0, x = -5$

Again when $y = -10, x^2 + 5x = -10$

$$\Rightarrow x^2 + 5x + 10 = 0$$

Since LHS expression cannot be factorized, therefore we should use the formula for finding the value of x .

Here $D = b^2 - 4ac = 25 - 4 \times 1 \times 10 = -15$

Since $D < 0$, the equation $x^2 + 5x + 10 = 0$ has no real solution.

$$\therefore x = 0, -5$$

Practice Exercise

Find the solutions of the following equations.

1. $x^4 - 7x^2 + 12 = 0$ 2. $x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 15 = 0$

3. $p^{-6} - 9p^{-3} + 8 = 0$ 4. $t - 9\sqrt{t} + 14 = 0$

5. $y^{-4} - 4 = 0$ 6. $2x^{10} - x^5 - 4 = 0$

7. $x = \sqrt{x+6}$ 8. $x + \sqrt{x-4} = 4$

9. $m + \sqrt{2m-3} = 1$ 10. $y = \sqrt{5y+6} - 2$

11. $\sqrt{2h-1} - \sqrt{h-4} = 2$ 12. $\sqrt{3-r} = \sqrt{r+7} + 2$

Answers

1. ± 2 and $\pm \sqrt{3}$

2. -27 and 125

3. 1 and $\frac{1}{2}$

4. 4 and 49

5. $\pm \sqrt{2}i$

6. 1.11014 and -1.03473

7. 3

8. 4

9. There is no solution.

10. -1 and 2

11. 5 and 13

12. -6

14.6 Formation of a Quadratic Equation

If you know the two roots, say α and β , then you can form the possible quadratic equation using these two roots. Why I'm saying "possible" quadratic equation because there can be innumerable (or infinite) number of quadratic equations having the same two roots. The underlying logic is that if you multiply or divide a given quadratic equation by any non-zero real number, the equation gets changed (*i.e.*, the coefficients/constants get multiplied or divided), however, the roots remain the same for every such quadratic equation. So it implies that the roots of $ax^2 + bx + c = 0$ and $k[ax^2 + bx + c] = 0$ are same.

Now to form a quadratic equation from the known roots you can use the following approach. If α and β are the two roots then possible quadratic equation will be $(x - \alpha)(x - \beta) = 0$.

But since you are working backward, that is from roots to equation, therefore you never know about the exact quadratic equation these roots belong to. Thus to include all the possibilities you can multiply the above equation by any non-zero real number as it's given below

$$\Rightarrow k(x - \alpha)(x - \beta) = 0; \text{ Where } k \text{ is a non-zero real number.}$$

$$\Rightarrow k[x^2 - (\alpha + \beta)x + \alpha\beta] = 0$$

$$\Rightarrow k[x^2 - (\text{sum of the roots})x + (\text{product of the roots})] = 0$$

Exp. 1) Form the quadratic equation whose roots are -7 and $3/2$.

Solution Sum of the roots $= -7 + \left(\frac{3}{2}\right) = \frac{-11}{2}$ and product of the roots $= \frac{-21}{2}$. Therefore, the possible required equation is

$$k \left[x^2 - \left(-\frac{11}{2}\right)x + \left(\frac{-21}{2}\right) \right] = 0$$

$$\Rightarrow k[2x^2 + 11x - 21] = 0$$

Here k is a non-zero real number.

Exp. 2) Form the quadratic equation whose roots are $5 + 2\sqrt{7}$ and $5 - 2\sqrt{7}$.

Solution Sum of the roots $= 5 + 2\sqrt{7} + 5 - 2\sqrt{7} = 10$ and product of the roots $= 5^2 - (2\sqrt{7})^2 = -3$.

Therefore, the required equation is

$$K[x^2 - (10)x + (-3)] = 0$$

$$K[x^2 - 10x - 3] = 0$$

Here, k is a non-zero real number.

Exp. 3) Form the quadratic equation whose roots are $5 + 2\sqrt{7}$ and $5 - 2\sqrt{7}$ and the constant term is 45.

Solution Sum of the roots $= 5 + 2\sqrt{7} + 5 - 2\sqrt{7} = 10$ and product of the roots $= 5^2 - (2\sqrt{7})^2 = -3$.

Therefore, the required equation is $k[x^2 - (10)x + (-3)] = 0$

$$\Rightarrow k[x^2 - 10x - 3] = 0$$

$$\Rightarrow 15x^2 - 150x - 45 = 0; \text{ (since the constant term is 45)}$$

Exp. 4) If α and β are the roots of the equation $ax^2 + bx + c = 0$, find the values of the following expressions in terms of a , b and c .

$$(a) \alpha^2 + \beta^2 \quad (b) \frac{1}{\alpha} + \frac{1}{\beta} \quad (c) \alpha^4 - \beta^4$$

Solution

(a) Let $ax^2 + bx + c = 0$ and α, β be the roots of the equation,

$$\text{then } \alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

$$\therefore \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \left(-\frac{b}{a}\right)^2 - 2\frac{c}{a} = \frac{b^2}{a^2} - \frac{2c}{a}$$

$$= \frac{b^2 - 2ac}{a^2}$$

$$(b) \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-b/a}{c/a} = -\frac{b}{c}$$

$$(c) \alpha^4 - \beta^4 = (\alpha^2 + \beta^2)(\alpha + \beta)(\alpha - \beta)$$

$$= [(\alpha + \beta)^2 - 2\alpha\beta](\alpha + \beta) \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$$

$$= \left(\frac{b^2 - 2ac}{a^2}\right) \left(-\frac{b}{a}\right) \sqrt{\frac{b^2}{a^2} - 4\frac{c}{a}}$$

$$= -\frac{b}{a^4} (b^2 - 2ac) \sqrt{b^2 - 4ac}$$

Exp. 5) If α, β are the roots of the equation $2x^2 - 3x + 2 = 0$, form the equation whose roots are α^2, β^2 .

Solution $2x^2 - 3x + 2 = 0$, and α, β are the roots $\alpha + \beta = \frac{3}{2}, \alpha\beta = 1$

For the new equation, roots are α^2 and β^2

$$\therefore \text{Sum of the roots } \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \left(\frac{3}{2}\right)^2 - 2(1)$$

$$= \frac{9}{4} - 2 = \frac{1}{4}$$

$$\text{and product of the roots } = \alpha^2\beta^2 = (\alpha\beta)^2 = (1)^2 = 1$$

$\therefore$ the required equation is

$$x^2 - (\text{sum of roots})x + \text{product of roots} = 0$$

$$\Rightarrow x^2 - \frac{1}{4}x + 1 = 0$$

$$\Rightarrow 4x^2 - x + 4 = 0$$

Exp. 6) Two candidates attempt to solve a quadratic equation of the form $x^2 + px + q = 0$. One starts with a wrong value of p and finds the roots to be 2 and 6. The other starts with a wrong value of q and finds the roots to be 2 and -9 . Find the correct roots and the equation.

Solution When p is wrong i.e., $\frac{-b}{a} (= \alpha + \beta)$ is wrong but

$$\frac{c}{a} (= \alpha \cdot \beta) \text{ is correct.}$$

$$\text{Hence } \alpha\beta = \frac{c}{a} = 2 \times 6 = 12$$

Again when q is wrong i.e., $\frac{c}{a} (= \alpha\beta)$ is wrong

$$\text{but } \frac{-b}{a} = \alpha + \beta \text{ is correct.}$$

$$\therefore \frac{-b}{a} = \alpha + \beta = 2 + (-9) = -7$$

$\therefore$ the required correct quadratic equation is

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (-7)x + 12 = 0$$

$$\Rightarrow x^2 + 7x + 12 = 0$$

and the correct roots of this equation are $-3, -4$.

Practice Exercise

- If the two roots of a quadratic equation are $-\frac{3}{2}$ and $-\frac{5}{3}$, which one of the following is the concerned quadratic equation?
 (a) $6x^2 + 19x + 15$ (b) $19x + 6x^2 + 15$
 (c) $30x^2 + 95x + 75$ (d) both (a) and (c)
- If the two roots of any quadratic equation are $\sqrt{3}$ and $-\sqrt{3}$, how many such equations are possible?
 (a) 0 (b) 1
 (c) 2 (d) infinite
- If the two factors of any quadratic equation are $(x + \sqrt{3})$ and $(x - \sqrt{3})$, how many such equations are possible?
 (a) 0 (b) 1 (c) 2 (d) infinite
- If a quadratic equation is multiplied by 4, the roots of the new equation will be multiplied by
 (a) 16 (b) 4
 (c) 2 (d) 1
- If one root is $3 - \sqrt{5}$ and the other root is $3 + \sqrt{5}$, find the possible sum of coefficients of x^2 and x .
 (a) -20 (b) -6
 (c) 14 (d) 11

Answers

1. (d) | 2. (d) | 3. (d) | 4. (d) | 5. (a)

14.7 Nature of the Roots of the Quadratic Equation

For a quadratic equation $ax^2 + bx + c = 0$; where a, b, c are real numbers and $a \neq 0$, the nature of the roots can be easily determined by knowing the value of the discriminant ($D = b^2 - 4ac$) of the above quadratic equation.

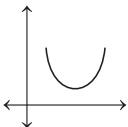
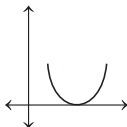
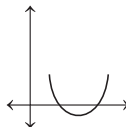
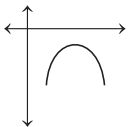
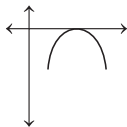
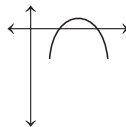
$D < 0$	$D = 0$	$D > 0$ (D is a perfect square)	$D > 0$ (D is not a perfect square)
Complex Non-zero imaginary parts Unequal (Conjugate pairs)	Real Rational Equal	Real Rational Unequal	Real Irrational Unequal (Conjugate Pairs)

- Roots are real only when D is non-negative.
- Roots are complex (or imaginary) when D is negative.
- Roots are rational only when D is a perfect square number like 0, 1, 4, 9, 16, ... etc.
- Roots are equal only when $D = 0$.
- The equal roots are called Double Root.

- When the roots are irrational they are in conjugate pairs as if one root is $p + \sqrt{q}$, the other root will be $p - \sqrt{q}$. Here, $\sqrt{q}$ is the irrational part of the root.
- When the roots are complex they are in conjugate pairs as if one root is $p + iq$, the other root will be $p - iq$. Here iq is the imaginary part of the root.
- If $D > 0$; $a = 1$; $b, c \in \mathbb{Z}$ (integer numbers) and roots are rational, the roots are integers.
- If a quadratic equation has one real root and $a, b, c \in \mathbb{R}$, other root is also real.
- If the roots are real and equal, the graph will touch the X -axis.
- If the roots are real and unequal, the graph will intercept the X -axis.
- If the roots are non-real, the graph does not touch the X -axis.
- The points on the X -axis, where the quadratic graph touches or intercepts are called the x -intercepts.

Graphical Illustration

Following graphs are depicted on the Cartesian plane, where horizontal arrow and vertical arrow represent X -axis and Y -axis, respectively. In these illustrative graphs my main consideration is the position of parabolic graph with relation to X -axis. In order to avoid complications I have not discussed about position of Y -axis and which is out of context as well.

	$D < 0$	$D = 0$	$D > 0$
When $a > 0$, graph will open upward and its vertex will be the lowest point of the graph.			
When $a < 0$, graph will open downward and its vertex will be the highest point of the graph.			

- The points, where parabola (quadratic graph) meets the X -axis, are called the real roots of the equation.
- When roots are real, the parabolic graph must touch or intersect the X -axis.
- When roots are non-real, the parabolic graph will not touch or pass through the X -axis.
- Either both the roots will be real or both the roots will be non-real.

- (v) When both the roots are equal, the vertex of parabolic graph tangentially passes through the X -axis.
- (vi) Double root has a multiplicity of 2. That means two roots are same and this is the reason why the quadratic graph just kisses the X -axis and bounces back to the same side of X -axis.
- (vii) In case of real roots when the roots are unequal the parabolic graph intersect the X -axis at two different points. These points are called the X -intercepts.
- (viii) The quadratic graph does not intersect the Y -axis more than once. Also the quadratic graph does not intersect the X -axis more than two times.

Exp. 1) Find the nature of the roots of the quadratic equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ without really knowing the exact values of the roots.

- (a) Rational (b) Irrational
- (c) Non-real (d) Both (a) and (b)

Solution Let us consider $x = 1$, therefore

$$(a-b) + (b-c) + (c-a) = 0$$

It implies that $x=1$ is a root of the given equation. Since one root of this equation is real and rational, so the other root definitely has to be real and rational. As the non-real roots and irrational roots occur in pairs only so there is no scope for the other root to be non-real or irrational.

Hence choice (a) is the answer.

Alternatively Let us consider $a = 3, b = 2, c = 1$ keeping in mind that a, b, c are rational numbers.

$$\text{Then } (a-b)x^2 + (b-c)x + (c-a) = 0 \Rightarrow x^2 + x - 2 = 0$$

The discriminant (D) of the above quadratic equation

$$= (1)^2 - 4(1 \times -2) = 9$$

We know that when D is a perfect square number the roots are real and rational.

Exp. 2) If $(a+b+c=0)$, find the nature of the roots of the equation $(c^2-ab)x^2 - 2(a^2-bc)x + (b^2-ac) = 0$.

- (a) Imaginary (b) Real and equal
- (c) Non-real (d) Irrational

Solution Since it is given that $a+b+c=0$, we can consider

$$a = -1, b = 0, c = 1$$

$$\text{Now, we have } (c^2-ab)x^2 - 2(a^2-bc)x + (b^2-ac) = 0$$

$$\Rightarrow x^2 - 2x + 1 = 0$$

The discriminant (D) of the above equation $= (-2)^2 - 4(1 \times 1) = 0$

We know that when $D=0$, the roots of the quadratic equation are real, rational and equal. Hence, choice (b) is the correct one.

Exp. 3) Determine k for which the roots of the equation $9x^2 + 2kx + 4 = 0$ are equal.

Solution Let the roots be α and α .

$$\therefore \text{ Sum of roots} = \alpha + \alpha = 2\alpha = -\frac{2k}{9}$$

$$\Rightarrow \alpha = -\frac{k}{9}$$

$$\text{and product of the roots} = \alpha^2 = \frac{4}{9}$$

$$\therefore \left(-\frac{k}{9}\right)^2 = \frac{4}{9} \Rightarrow \frac{k^2}{81} = \frac{4}{9}$$

$$\Rightarrow k^2 = 36 \Rightarrow k = \pm 6.$$

Alternatively In order that roots of a quadratic equation are equal, its discriminant must be zero.

$$\text{i.e., } b^2 - 4ac = 0$$

$$\therefore (2k)^2 - 4 \times 9 \times 4 = 0$$

$$\Rightarrow k = \pm 6.$$

Exp. 4) Which of the following quadratic expression can be expressed as a product of real linear factors?

- (a) $x^2 - 2x + 3$
- (b) $3x^2 - \sqrt{2}x - \sqrt{3}$
- (c) $2x^2 + 3x - 4$
- (d) $\sqrt{2}x^2 - \sqrt{5}x + 3$

Solution (a) $D = b^2 - 4ac < 0$ cannot be expressed

(b) $D = b^2 - 4ac > 0$ can be expressed

(c) $D = b^2 - 4ac > 0$ can be expressed

(d) $D = b^2 - 4ac < 0$, can't be expressed.

NOTE If $D < 0$, then the quadratic equation cannot be expressed into two linear factors.

Exp. 5) Find the set of values of p for which the quadratic equation has real linear factors.

$$9x^2 - px + 4$$

Solution For any quadratic polynomial to have real linear factors, we must have $D \geq 0$

$$\therefore b^2 - 4ac \geq 0$$

$$p^2 - 4 \times 9 \times 4 \geq 0$$

$$\Rightarrow p^2 - 144 \geq 0$$

$$\Rightarrow p^2 \geq 144$$

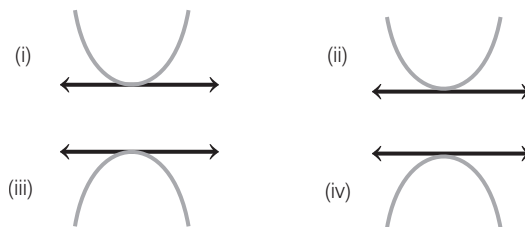
$$\Rightarrow p \geq \pm 12$$

Therefore, either $p \leq -12$ or $p \geq 12$.

Practice Exercise

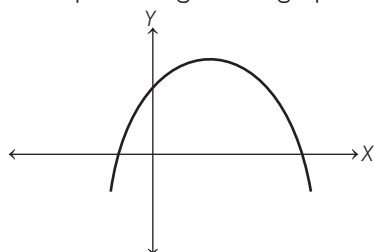
- If the roots of a quadratic equation are real and rational, which one of the following best describes the value of the discriminant D ?
 (a) $D > 0$ (b) $D = 0$
 (c) $D < 0$ (d) $D \geq 0$
- If the roots of a quadratic equation are real and unequal, which one of the following is necessarily true?
 (a) $D < 0$ (b) $D = 0$
 (c) $D > 0$ (d) $D \geq 0$
- If the roots of a quadratic equation are unequal, which one of the following cannot be true?
 (a) $D > 0$ (b) $D < 0$
 (c) $D = 0$
 (d) Roots are conjugate also
- Which one of the following is a correct statement?
 (a) When roots are real, these are definitely equal.
 (b) When roots are rational, these are definitely real.
 (c) When roots are unequal, these are definitely complex roots.
 (d) When roots make the conjugate pair, these are either real or complex ones.
- If one root of the quadratic equation is $2 + 5\sqrt{7}$, find the other root of this equation.
 (a) $5 + 2\sqrt{7}$ (b) $-2 + 5\sqrt{7}$
 (c) $-2 - 5\sqrt{7}$ (d) none of these
- If one root of the quadratic equation is $3 - 2i$, find the sum of the roots.
 (a) 0 (b) 9 (c) 6 (d) $-4i$
- Given the two roots $\sqrt{3}$ and $-3\sqrt{3}$, how many quadratic equations can be formed?
 (a) 2 (b) 1
 (c) 0 (d) none of these
- If one root of a quadratic equation is $\sqrt{8}$, then what would, definitely, be the other root?
 (a) $\sqrt{-8}$ (b) $2\sqrt{2}$
 (c) $i\sqrt{8}$ (d) none of these
- Roots can be conjugate except when
 (a) $D < 0$ (b) $a < 0$
 (c) $D > 0$ (d) $D = 0$
- If the roots are complex conjugate, then the sum of the roots is
 (a) An integer (b) An irrational
 (c) An imaginary (d) none of these
- If the roots are real and equal, then which of the following is always valid?
 (a) $D > 0$ (b) $D < 0$
 (c) $D = 0$ (d) D is a perfect square
- If the roots are real and rational, then which of the following is the most appropriate one?
 (a) $D > 0$ (b) $D < 0$
 (c) $D = 0$ (d) D is a perfect square
- If the two roots are real, equal and irrational, then which of the following is the most appropriate one?
 (a) $D > 0$ (b) $D < 0$
 (c) $D \leq 0$ (d) doesn't exist
- If the roots of a quadratic equation are rational, which of the following cannot be the value of discriminant D ?
 (a) 0.01 (b) 6.25 (c) 0.036 (d) 1.44
- A quadratic equation has double root, then
 (a) the two roots lie on the different sides of the Y-axis.
 (b) the quadratic equation cannot be expressed in the form of a perfect square.
 (c) the product of the roots is always negative.
 (d) the sum of the roots may be positive or negative.
- If roots are real and irrational, then which of the following is the most appropriate one?
 (a) $D > 0$, except rational numbers
 (b) $D > 0$, except perfect squares
 (c) $D \leq 0$, except rational numbers
 (d) none of the above
- If the roots are unequal, then the possible values of D is/are
 (a) $D = 0$ (b) $D - \{0\}$
 (c) $D > 0$ (d) $D < 0$
- If $D = 7744$, then roots are
 (a) equal and real (b) irrational
 (c) conjugate pairs (d) rational and unequal
- If $D = 4477$, then the roots are
 (a) equal and real (b) irrational
 (c) complex conjugate (d) rational and unequal
- If a quadratic equation has one real root then the other root will necessarily be
 (a) real (b) equal
 (c) complex conjugate (d) rational and unequal
- If the roots are same, then the roots must be
 (a) negative (b) rational
 (c) irrational (d) complex conjugate
- Which one of the following is false about the equation $4x^2 - 12x + 9 = 0$?
 (a) It can be expressed as a perfect square.
 (b) The solution of this equation is a double root.
 (c) The multiplicity of this equation is 2.
 (d) The roots are 1.5 units away from the Y-axis in the left side.

23. When the parabola opens upward, then which of the following condition must be satisfied?
(a) $D > 0$ (b) $a < 0$ (c) $a > 0$ (d) $a > b > c$
24. When the parabola kisses the X -axis then which of the following condition must be satisfied?
(a) $D > 0$ (b) $a > 0$ (c) $D = 0$ (d) $b^2 < 4ac$
25. When the parabola never intersects the X -axis, then which of the following is certainly true?
(a) $D > 0$ (b) $a < 0$ (c) $D = 0$ (d) $b^2 \leq 4ac$
26. The intersection of X -axis and parabola primarily depends on the value of
(a) D (b) a
(c) b (d) none of these
27. The parabola intersects the Y -axis at most
(a) 4 times (b) 3 times
(c) 2 times (d) 1 time
28. For a standard quadratic equation the parabola intersects the Y -axis at most
(a) 4 times (b) 3 times (c) 2 times (d) 1 time
29. If the quadratic graph opens downward, then the roots are always
(a) imaginary (b) real
(c) equal (d) cannot be determined
30. If the graph intersects X -axis at two distinct points, then the roots are necessarily
(a) complex (b) irrational
(c) rational (d) real
31. If a given quadratic equation is multiplied by -2 , now the parabola will open
(a) upward (b) downward
(c) in the original direction only
(d) the direction will be reversed
32. If the comprehensive graph of quadratic function lies in all the four quadrants, then roots are necessarily
(a) real (b) non-real
(c) real and equal (d) equal and irrational
33. If the graph intersects Y -axis below the X -axis then which of the following can be valid?
(a) Roots are real and parabola opens downward
(b) Roots are non-real and parabola opens upward
(c) Roots are real only
(d) If the roots are real then parabola opens upward and if the roots are non-real then parabola opens downward
34. If the quadratic graph intersects both the X and Y axes at the same point, which of the following is not correct?
(a) Roots are imaginary
(b) Roots are neither negative nor positive
(c) $b = c = 0$
(d) Graph may open in either side-up and down
35. If the two real roots are distinct, then which of the following may be correct?
(a) Both are negative
(b) One is positive and another one is negative
(c) Both are positive
(d) all of the above
36. The graph of a quadratic equation is symmetrical about Y -axis, which ones of the following can't be the roots of a quadratic equation?
(a) $(0, 0)$ (b) $\left(-\frac{2}{3}, \frac{2}{3}\right)$
(c) $(2\sqrt{3}, -2\sqrt{3})$ (d) $(4 + \sqrt{2}, 4 - \sqrt{2})$
37. If the roots of a quadratic equation are $(4, 10)$, then what is the distance between Y -axis and the axis of symmetry of the pertinent graph?
(a) 7 unit
(b) 6 unit
(c) $5/2$ units
(d) cannot be determined
38. The distance between Y -axis and the axis of symmetry is 4 and one of the two roots is -9 , then what can be the other root?
(a) 9 or 19
(b) 4 or 14
(c) 1 or 17
(d) -5 or 13
39. In a set of 200 quadratic equations, the discriminant D for different equations is such that $D = -99, -98, \dots, 99, 100$. Then which one of the following is not true?
(a) There are exactly 45% equations that have irrational roots.
(b) There are at most 21 distinct rational roots.
(c) The roots of 66% equations are in conjugate pairs.
(d) Number of non-real roots is less than that of real roots.
40. Which one(s) of the following graphs is/are correctly sketched for the given quadratic equation which has double root (i.e., both the roots are equal), considering that all of these graphs appear considerably thick when seen with a magnifying lens? The horizontal lines, in these graphs, represent the X -axis.

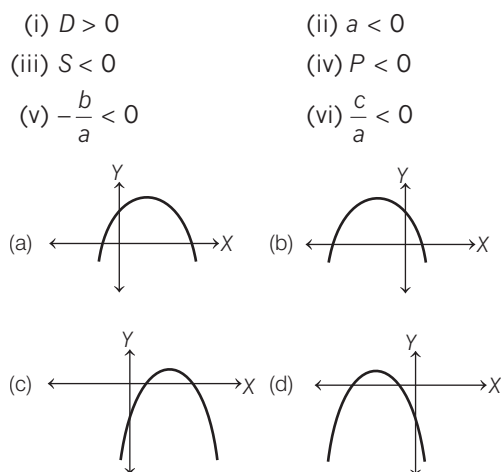


- (a) (i) and (ii) (b) (ii) and (iv)
(c) (i) and (iii) (d) (ii) and (iii)

41. Graph of the equation $ax^2 + bx + c = 0$ is shown below. If D denotes the discriminant of this equation; S and P denote the sum and product of the roots of this equation, respectively. Then which of the following facts is/are true pertaining to this graph?



- (i) $D < 0$ (ii) $a < 0$
 (iii) $S > 0$ (iv) $P < 0$
 (v) $-\frac{b}{a} > 0$ (vi) $\frac{c}{a} > 0$
 (a) (ii), (iii) and (iv)
 (b) (ii), (iii) and (vi)
 (c) Except (i) and (vi)
 (d) none of the above
42. Some facts are given below for a quadratic equation $ax^2 + bx + c = 0$, and then four graphs are also drawn. If D denotes the discriminant of this equation; S and P denote the sum and product of the roots of this equation, respectively, which one of the following graphs is the best representative of all these facts?



Answers

1. (d)	2. (c)	3. (c)	4. (d)	5. (d)
6. (c)	7. (c)	8. (d)	9. (d)	10. (a)
11. (c)	12. (d)	13. (d)	14. (c)	15. (d)
16. (b)	17. (b)	18. (d)	19. (b)	20. (a)
21. (b)	22. (d)	23. (c)	24. (c)	25. (d)
26. (a)	27. (d)	28. (c)	29. (d)	30. (d)
31. (d)	32. (a)	33. (d)	34. (a)	35. (d)
36. (d)	37. (a)	38. (c)	39. (c)	40. (c)
41. (c)	42. (b)			

Sign of the Roots of a Quadratic Equation

A quadratic equation $ax^2 + bx + c = 0$ can be expressed using the roots (α, β) as following

$$ax^2 + bx + c = 0 \Rightarrow x^2 + \left(\frac{b}{a}\right)x + \left(\frac{c}{a}\right) = 0$$

$$\Rightarrow x^2 - \left(-\frac{b}{a}\right)x + \left(\frac{c}{a}\right) = 0 \Rightarrow x^2 - (\alpha + \beta)x + (\alpha\beta) = 0$$

Assume $|\alpha| < |\beta|$, then we have the four conditions as shown below, provided $a > 0$.

Case	$\alpha\beta$	$\alpha + \beta$	α	β
1	+	+	+	+
2	+	-	-	-
3	-	+	-	+
4	-	-	+	-

Exp. 1) If α and β are the roots of the quadratic equation $x^2 + 5x + 6 = 0$, what are the signs of the roots?

Solution Since $\alpha\beta$ is positive and $\alpha + \beta$ is negative, therefore both the roots are negative.

Exp. 2) If α and β are the roots of the quadratic equation $24x^2 + 55x - 1 = 0$, what are the signs of the roots?

Solution Since $(\alpha\beta)$ is negative and $(\alpha + \beta)$ is also negative, therefore one root is positive and another one is negative. More precisely, as we know $|\alpha| < |\beta|$ therefore α is positive and β is negative.

Practice Exercise

- If sum of the roots is -2 and product of the roots is -15 ,
 - both the roots lie on the right-side of the Y -axis.
 - both the roots lie on the left-side of the Y -axis.
 - roots lie on both the sides of the Y -axis.
 - one root is zero and another root is imaginary number.
- If sum of the roots is -11 and product of the roots is 24 ,
 - both the roots lie right-side of the Y -axis
 - both the roots lie left-side of the Y -axis.
 - roots lie on both the sides of the Y -axis.
 - one root is zero and another root is imaginary number.
- If the quadratic equation is $x^2 - 34x + 93 = 0$.
 - both the roots are positive.
 - both the roots are negative.
 - one root is positive and another one is negative.
 - one root is rational and another root is irrational.
- If the quadratic equation is $-x^2 + 28x + 93 = 0$,
 - both the roots are positive.
 - both the roots are negative.
 - one root is positive and another one is negative.
 - one root is rational and another root is irrational.

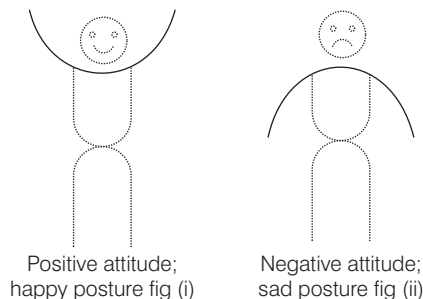
Answers

1. (c) | 2. (b) | 3. (a) | 4. (c)

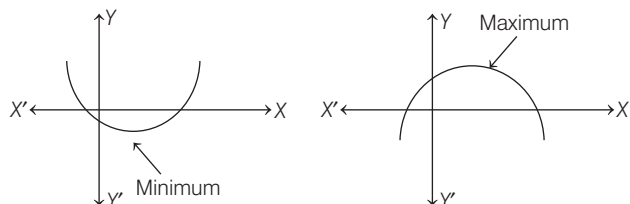
14.8 Maximum or Minimum Value of a Quadratic Equation

A quadratic equation, as you know, when drawn on a plane takes the shape of a parabola. This parabolic graph opens only in two directions –

- (a) Upward (like hands-up) when a person is happy.
- (b) Downward (like hands-down) when a person is sad.



These graphs can be related with the situations as in a person is happy when his attitude a is positive and he is sad when his attitude a is negative. So when in the quadratic equation $y = ax^2 + bx + c$, the coefficient of x^2 i.e., a is positive the parabolic graph opens upward like the fig (i) and it gives the minimum value of y . Similarly, when in the quadratic equation $y = ax^2 + bx + c$, the coefficient of x^2 i.e., a is negative the parabolic graph opens downward like the fig (ii) and it gives the maximum value of y .



Graphically, the maxima (or minima) denote the maximum (or minimum) distance between X -axis and the vertex of the graph.

In the equation $y = ax^2 + bx + c$, graph opens when $a > 0$	Parabolic upward	Graph may continue upwards the infinitely, so you cannot determine the maximum value of y , unless the value of x is restricted.	Graph attains the minimum value of y and it cannot go below this minimum value .

In the equation $y = ax^2 + bx + c$, graph opens when $a < 0$	Parabolic downward	Graph may continue downwards the infinitely, so you cannot determine the minimum value of y , unless the value of x is restricted	Graph attains the maximum value of y and it cannot go above this maximum value .

NOTE Essentially a quadratic equation $y = ax^2 + bx + c$ either gives minimum or maximum value, but not both.

Both the minimum and maximum values can be obtained in a particular case only when there is a restriction on the values of x and that tells nothing but it tells the range of the quadratic function.

Techniques for Obtaining the Minimum/Maximum Value of a Quadratic Equation

At $x = -\frac{b}{2a}$, we get the minimum/maximum value of y , where $y = ax^2 + bx + c$.

Approach 1 By substituting the value of $x = -\frac{b}{2a}$, in the quadratic equation $y = ax^2 + bx + c$ we can get the minimum/maximum value of y .

Approach 2 By substituting the values of a, b, c in the following formula we can get minimum/maximum value of y .

The minimum/maximum value of

$$y = \frac{4ac - b^2}{4a}$$

Approach 3 Differentiation method of Calculus.

NOTE You don't have to use the calculus at all in order to avoid the unnecessary complications. Your objective is to get the answer quickly and simply. That's it.

Exp.) Consider the following quadratic expressions and determine

- (i) Which of these quadratic expressions give minima and which of these give maxima?
- (ii) The value of x at which these expressions will yield minimum or maximum value.
- (iii) The minimum or maximum value of these quadratic expressions.

(1) $9x^2 + 28x - 4$	(2) $-7x^2 + 10x$
(3) $x^2 - 9x$	(4) $17x + 1 - 4x^2$
(5) $-2x^2 - 11x + 3$	(6) $10 + 9x - x^2$
(7) $x^2 - x$	(8) $x^2 + 16$
(9) $-\frac{3}{2}x^2 + 7x - 1$	
(10) $6x - 9x^2$	
(11) $-x^2 + 8x + 10$	
(12) x^2	

Solution Consider the following table for the answers of all the 12 problems.

	(i)	(ii)	(iii)
Answer Number	Value of coefficient	Minima / Maxima	Value of x for which equation gives minimum / maximum value of y
1.	9	Minima	$x = -\frac{b}{2a} = -\frac{14}{9}$
2.	-7	Maxima	$x = -\frac{b}{2a} = \frac{5}{7}$
3.	1	Minima	$x = -\frac{b}{2a} = \frac{9}{2}$
4.	-4	Maxima	$x = -\frac{b}{2a} = \frac{17}{8}$
5.	-2	Maxima	$x = -\frac{b}{2a} = \frac{11}{-4}$
6.	-1	Maxima	$x = -\frac{b}{2a} = \frac{9}{2}$
7.	1	Minima	$x = -\frac{b}{2a} = \frac{1}{2}$
8.	1	Minima	$x = -\frac{b}{2a} = 0$
9.	$-\frac{3}{2}$	Maxima	$x = -\frac{b}{2a} = \frac{7}{3}$
10.	9	Minima	$x = -\frac{b}{2a} = \frac{1}{3}$
11.	-1	Maxima	$x = -\frac{b}{2a} = 4$
12.	9	Minima	$x = -\frac{b}{2a} = 0$

NOTE Practice to get the min/max value of a quadratic function through both the techniques for superior confidence and better speed of calculation.

Practice Exercise

- What is the minimum distance between the vertex of the quadratic graph and X -axis when the roots are equal?
(a) 0 (b) 1
(c) ∞ (d) data insufficient
- What is the minimum distance between the vertex of the quadratic graph and X -axis when the roots are real and irrational?
(a) 1 (b) 0
(c) -1 (d) data is sufficient
- The axis of symmetry is 3 units away from Y -axis and the minimum is 4 units, then what is the minimum distance between minimum (extremum) of the graph and origin?
(a) 1 (b) -1 (c) 5 (d) none

- If the quadratic equation has negative real roots and its maximum (extremum) of the graph is 10 unit, then the distance between the X -axis and maximum of the graph is
(a) 10 (b) -10
(c) 10 and -10 both (d) data is insufficient
- If the minimum value of the quadratic function is 0, then which of the following is not definitely true about the roots?
(a) Real (b) Non-Real
(c) Equal (d) Rational
- If one of the roots is 0 and another one is real positive, then which of the following is certainly incorrect?
(a) minimum value of the function is -1
(b) maximum value of the function is 1
(c) minimum of the graph is at X -axis
(d) the graph intersects Y -axis below the X -axis
- For a quadratic equation $ax^2 + bx + c = 0$, if you have double roots and positive coefficient of x^2 , then the minimum value of y is
(a) any negative real number
(b) the lowest negative number
(c) the lowest whole number
(d) cannot be determined
- For a quadratic equation $ax^2 + bx + c = 0$, if you have equal roots and negative coefficient of x^2 , then the maximum value of y is
(a) the highest non-positive integer
(b) the lowest negative number
(c) depends on roots
(d) none of the above
- There are two quadratic equations; none of them has integer or complex roots. If each root of a quadratic equation is three times the root of another quadratic equation, then the maximum of this equation will be m times to that of the other equation. What's the value of m ?
(a) 1 (b) 3
(c) $\sqrt{3}$ (d) 9
- There are two quadratic equations E_1 and E_2 . Each root of E_1 is four times to that of E_2 , and the minimum of E_1 will be m times to that of E_2 . Then m is
(a) 0
(b) 2
(c) 16
(d) Cannot be determined uniquely
- The two roots of a quadratic equation are -7 and -3, what will be the maximum value of this equation?
(a) 1
(b) -1
(c) 4
(d) Cannot be determined uniquely

12. Given the two certain roots of any quadratic equation, what is the locus of the vertices of all such quadratic graphs?
 (a) Circle (b) Parabola
 (c) Axis of symmetry (d) A line parallel to X-axis
13. What is the locus of the vertices of all the quadratic equations whose roots are m times the certain given roots, say α and β ?
 (a) Polygon (b) Parabola
 (c) A line parallel to X-axis (d) Nothing can be said
14. The factors of a quadratic equation are $(x - 4)$ and $(x - 8)$, what will be the vertex of the quadratic graph?
 (a) -4 (b) 4
 (c) 6 (d) -6
15. For a quadratic equation, the roots are $(-1, 3)$ and the minimum is 4 units, then the triangle formed by connecting the roots and the minimum value of y is
 (a) right angle triangle
 (b) scalene triangle
 (c) equilateral triangle
 (d) isosceles triangle
16. For a quadratic equation, the roots are $(-3, 3)$ and the maximum of y is $3\sqrt{3}$ units, then the triangle formed by connecting the roots and the maximum value of y is
 (a) right angle triangle (b) scalene triangle
 (c) equilateral triangle (d) isosceles triangle
17. If the roots are $(-3, 9)$, then the highest/lowest value of y will be attained at x equal to
 (a) -27 (b) 6
 (c) -3 (d) 3
18. For all the non-negative real x , what is the minimum and maximum non-negative value of y , where $y = -x^2 + 8x + 20$?
 (a) $0, 36$ (b) $18, 36$
 (c) $0, 20$ (d) none of these
19. For the non-positive real x , such that $x \geq -11$, what is the minimum and maximum value of y , where $y = x^2 + 12x + 27$?
 (a) $-9, 16$ (b) $-9, 27$
 (c) $0, 27$ (d) none of these
20. If $y = 2x^2 + 12x + 35$, the distance between the Y-axis and the minimum of y is
 (a) 1 (b) -1 (c) 0.5 (d) 1.5
21. For $-\frac{8}{3} \leq x \leq \frac{8}{3}$, the minimum and maximum values of $f(x) = x^2 - 6x + 18$ are
 (a) $81, \frac{370}{9}$ (b) $\frac{11}{9}, \frac{26}{9}$ (c) $83, 173$ (d) $\frac{82}{9}, \frac{370}{9}$
22. For $-3 < x < 2$, find the range of $y = -3x^2 + 10x - 21$.
 (a) $\left(-78, -\frac{38}{3}\right)$ (b) $[-78, -13]$
 (c) $\{-78, -13\}$ (d) none of these
23. The minimum and maximum values of $(7 - x)^2 - 8$ are
 (a) $-8, 15$ (b) $-15, 15$
 (c) $-8, 41$ (d) none of these
24. The maximum value of $49 - (7 - x)^2$ is
 (a) 49 (b) 42
 (c) 0 (d) none of these
25. The equation $x^2 - 3(x - 3)^2 - 2 = 0$ has
 (a) one minimum point
 (b) one maximum point
 (c) Both maximum and minimum points
 (d) None of the above
26. A quadratic equation which attains its minimum -4 at $x = 5$ and its y -intercept is 21 , the largest root of this equation must be less than
 (a) 10 (b) 5
 (c) 6 (d) 3
27. A quadratic equation which attains its maximum 4 at $x = -4$, and $y = 3$ at $x = -3$ what will be the value of equation at $x = -5$?
 (a) 5 (b) 2
 (c) 3 (d) none of these
28. A quadratic equation attains minimum at $x = \frac{3}{2}$. Also, $y = \frac{3}{4}$ at $x = \frac{1}{2}$, what's the value of the equation when $x = \frac{5}{2}$?
 (a) $\frac{3}{4}$ (b) $\frac{5}{4}$
 (c) $\frac{1}{4}$ (d) cannot be determined
29. If one root of a quadratic equation is -2 . At $x = -5, y = 27$ and at $x = 7, y = 27$. Then what's the minimum value of the equation?
 (a) 5 (b) 0
 (c) -9 (d) none of these

Answers

1. (a)	2. (d)	3. (c)	4. (a)	5. (b)
6. (c)	7. (c)	8. (a)	9. (d)	10. (d)
11. (d)	12. (c)	13. (c)	14. (a)	15. (d)
16. (c)	17. (d)	18. (a)	19. (b)	20. (c)
21. (d)	22. (a)	23. (d)	24. (a)	25. (b)
26. (a)	27. (c)	28. (a)	29. (c)	

14.9 Condition for Common Roots Between Two Quadratic Equations

Consider two quadratic equations,

$$ax^2 + bx + c = 0; \text{ (where } a \neq 0) \quad \dots(i)$$

And $a'x^2 + b'x + c' = 0; \text{ (where } a' \neq 0) \quad \dots(ii)$

(A) If one root is common, then the following condition must be satisfied.

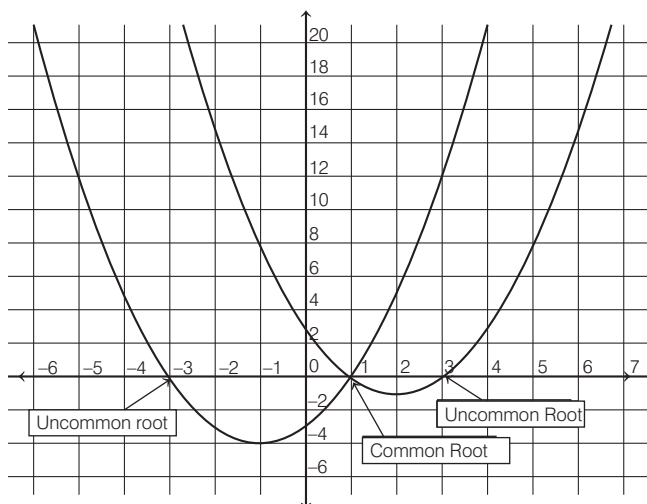
$$(ab' - a'b)(bc' - b'c) = (ca' - c'a)^2$$

(B) If both the roots are common, then the following condition must be satisfied.

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$$

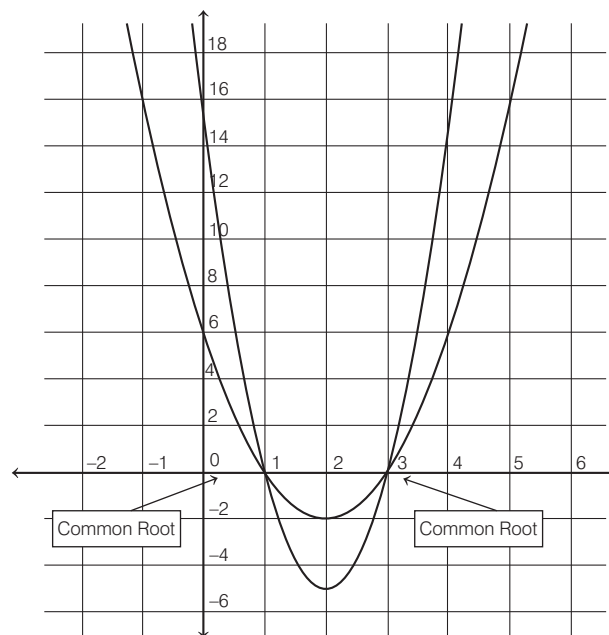
Please note that the above techniques help in determining the common roots without explicitly determining the individual roots of the two quadratic equations.

Graphical Illustration 1 The two quadratic equations $x^2 + 2x - 3$ and $x^2 - 4x + 3$ have two roots each where one root is common in both of them.



When exactly one root is common, then the graphs of two quadratic equations intersect at exactly one point on a co-ordinate plane.

Graphical Illustration 2 The two quadratic equations $3x^2 - 12x + 9$ and $5x^2 - 20x + 15$ have two roots each where both the roots are common in both of them.



When both the roots are common in the two given quadratic equations, then the graphs of two quadratic equations intersect at exactly two points on the same coordinate plane. It means the two graphs (parabolas), which have both the roots common they do not overlap necessarily except at the roots; and the two parabolas can open in opposite directions or even in the same direction with different degree of stretching.

Additional Points about Common Roots

Case 1 When $c = c' = 0$, then the equations reduce to

$$ax^2 + bx = 0 \text{ and } a'x^2 + b'x = 0$$

$$\Rightarrow x(ax + b) = 0 \text{ and } x(a'x + b') = 0$$

This implies that $x = 0$ is a common root.

The other root for the first equation is $x = -\frac{b}{a}$ and for

the second equation is $x = -\frac{b'}{a'}$

Case 2 When $\frac{a}{a'} = \frac{b}{b'}$, but $\frac{a}{a'} = \frac{b}{b'} \neq \frac{c}{c'}$, then no root is common.

Finding the Common Root

When α is the common root, then this root must satisfy both the equations

$$a\alpha^2 + b\alpha + c = 0 \quad \dots(i)$$

$$\text{And} \quad a'\alpha^2 + b'\alpha + c' = 0 \quad \dots(ii)$$

$$\therefore \frac{\alpha^2}{(bc' - b'c)} = \frac{\alpha}{(ca' - c'a)} \\ = \frac{1}{(ab' - a'b)}$$

From Eqs. (i) and (ii), we get

$$\alpha = \frac{(bc' - b'c)}{(ca' - c'a)}$$

And from the second and third we get

$$\alpha = \frac{(ca' - c'a)}{(ab' - a'b)}$$

Finally by equating both the values of α , we get the condition for a common root. That is

$$(ab' - a'b)(bc' - b'c) = (ca' - c'a)^2$$

Exp. 1) Determine that whether there is any common root exists between the following quadratic equations :

$$x^2 - 8x + 15 = 0 \quad \dots(i)$$

$$x^2 - 9x + 18 = 0 \quad \dots(ii)$$

Solution Assuming that one root is common, then

$$(-9 + 8)(-144 + 135) = (15 - 18)^2$$

$$\Rightarrow (-1)(-9) = (-3)^2$$

$$\Rightarrow 9 = 9$$

This proves that one root is common in between the given quadratic equation.

Again assuming that both the roots of the given quadratic equations are common, then

$$\frac{1}{1} \neq \frac{8}{9} \neq \frac{15}{18}$$

This proves that both the roots are not common.

Thus we find that only one root is common.

Exp. 2) Determine the value of the common root of the quadratic equations :

$$x^2 - 8x + 15 = 0 \quad \dots(i)$$

$$x^2 - 9x + 18 = 0 \quad \dots(ii)$$

Solution Since one root (or solution) is common in between the two equations, therefore

$$x^2 - 8x + 15 = x^2 - 9x + 18$$

$$\Rightarrow x = 3$$

Therefore, the value of the common root is 3.

Exp. 3) If the equations $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root, which of the following can be true?

$$(a) p - q = 0 \quad (b) p + q = -1 \quad (c) pq = -1 \quad (d) pq = 0$$

Solution Since one root is common, then

$$(q - p)(p^2 - q^2) = (q - p)^2$$

$$\Rightarrow (p^2 - q^2) = (q - p)$$

$$\Rightarrow [(p + q)(p - q)] = -(p - q)$$

$$\Rightarrow [(p + q)(p - q)] + (p - q) = 0$$

$$\Rightarrow (p - q)[p + q + 1] = 0$$

This implies that either $p - q = 0$

$$\text{or } p + q + 1 = 0$$

Therefore, (a) and (b) are valid relations.

Exp. 4) If the equations $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root, which of the following can be the common root?

$$(a) 1 \quad (b) -1$$

$$(c) -2 \quad (d) \frac{-1}{2}$$

Solution Let us say α is the common root, then

$$\alpha^2 + p\alpha + q = \alpha^2 + q\alpha + p$$

$$\Rightarrow p\alpha + q = q\alpha + p$$

$$\Rightarrow \alpha(p - q) = (p - q)$$

$$\Rightarrow \alpha = 1$$

Therefore choice (a) gives the appropriate answer.

Exp. 5) If the equations $x^2 + bx + c = 0$ and $x^2 + dx + e = 0$ have a common root, which of the following can be the common root?

$$(a) \frac{c-e}{d-b} \quad (b) \frac{be-cd}{c-e}$$

$$(c) \frac{c-e}{be-cd} \quad (d) \frac{d-b}{c-e}$$

Solution Since one root is common, then

$$(d - b)(be - cd) = (c - e)^2$$

$$\frac{be - cd}{c - e} = \frac{c - e}{d - b} \quad \dots(i)$$

Let us say α is the common root, then

$$\alpha^2 + b\alpha + c = \alpha^2 + d\alpha + e$$

$$\Rightarrow b\alpha + c = d\alpha + e$$

$$\Rightarrow \alpha(b - d) = (e - c)$$

$$\Rightarrow \alpha = \frac{e - c}{b - d} \Rightarrow \alpha = \frac{c - e}{d - b}$$

Now from the Eq. (i), we get

$$\alpha = \frac{c - e}{d - b} = \frac{be - cd}{c - e}$$

Therefore (a) and (b) are valid relations.

Exp. 6) Find the value of m if the equations $x^2 + 2x + 3m = 0$ and $2x^2 + 3x + 5m = 0$ have a common root.

- (a) 15 (b) 8 (c) 2 (d) -1

Solution Since one root is common, then

$$(3 - 4)(10m - 9m) = (6m - 5m)^2$$

$$\Rightarrow -m = m^2 \Rightarrow m = -1$$

Therefore choice (d) gives the appropriate answers.

Exp. 7) For any real numbers a, b and c the equations $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root, which of the following relations is NOT true?

- (a) $abc = 1$ (b) $a + b + c = 0$
(c) $a = b = c$ (d) $a^3 + b^3 + c^3 = 3abc$

Solution Since one root is common, therefore

$$(ac - b^2)(ab - c^2) = (bc - a^2)^2$$

$$\Rightarrow a(a^3 + b^3 + c^3 - 3abc) = 0$$

But $a \neq 0$;

$$\therefore a^3 + b^3 + c^3 - 3abc = 0 \quad \dots(i)$$

$$\Rightarrow (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac) = 0$$

$$\therefore a + b + c = 0 \quad \dots(ii)$$

$$\text{Or } a^2 + b^2 + c^2 - ab - bc - ac = 0$$

$$\Rightarrow 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac = 0$$

$$\Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$$

$$\Rightarrow a - b = 0, b - c = 0, c - a = 0$$

$$\Rightarrow a = b = c \quad \dots(iii)$$

Therefore choice (a) is appropriate one.

Hint $a^3 + b^3 + c^3 - 3abc = (a + b + c)$

$$(a^2 + b^2 + c^2 - ab - bc - ac)$$

Exp. 8) If the equation $x^2 - ax + b = 0$ and $x^2 - a'x + b' = 0$, have one root in common and the second equation has equal roots, then which one of the following is a valid relation?

- (a) $2(b' - b) = aa'$ (b) $2(a + a') = bb'$
(c) $2(a' - a) = bb'$ (d) $2(b + b') = aa'$

Solution Since the equation $x^2 - a'x + b' = 0$ has equal roots, we can consider them α and α , then

$$\text{Sum of the roots} = \alpha + \alpha = 2\alpha = a' \Rightarrow \alpha = \frac{a'}{2}$$

$$\text{And product of the roots} = \alpha \cdot \alpha = \alpha^2 = b'$$

Now since one root is common, then

$$\alpha^2 - a\alpha + b = \alpha^2 - a'\alpha + b'$$

$$\Rightarrow \alpha(-a + a') = (b' - b) \Rightarrow \alpha(a' - a) = (b' - b)$$

$$\Rightarrow a'(a' - a) = 2(b' - b) \quad (\text{since } \alpha = \frac{a'}{2})$$

$$\Rightarrow (a')^2 - aa' = 2(b' - b)$$

$$\Rightarrow 4\alpha^2 - aa' = 2(b' - b) \quad (\text{since } a' = 2\alpha)$$

$$\Rightarrow 4b' - aa' = 2(b' - b) \quad (\text{since } \alpha^2 = b')$$

$$\Rightarrow aa' = 2(b + b')$$

Therefore choice (d) is valid.

Exp. 9) If each pair of the three equations $x^2 + b_1x + c_1 = 0$, $x^2 + b_2x + c_2 = 0$ and $x^2 + b_3x + c_3 = 0$ have a common root, which one of the following is true?

$$(a) (b_1^2 + b_2^2 + b_3^2) = 2(b_1 \cdot b_2 + b_2 \cdot b_3 + b_3 \cdot b_1) - 4(c_1 + c_2 + c_3)$$

$$(b) (b_1^2 + b_2^2 + b_3^2)$$

$$= 2(b_1 \cdot b_2 + b_2 \cdot b_3 + b_3 \cdot b_1) + 4(c_1 + c_2 + c_3)$$

$$(c) (b_1^2 + b_2^2 + b_3^2)$$

$$= 2(c_1 \cdot c_2 + c_2 \cdot c_3 + c_3 \cdot c_1) - 4(b_1 + b_2 + b_3)$$

$$(d) (b_1^2 + b_2^2 + b_3^2)$$

$$= 4(b_1 \cdot b_2 + b_2 \cdot b_3 + b_3 \cdot b_1) - 2(c_1 + c_2 + c_3)$$

Solution The three equations are

$$x^2 + b_1x + c_1 = 0 \quad \dots(i)$$

$$x^2 + b_2x + c_2 = 0 \quad \dots(ii)$$

$$x^2 + b_3x + c_3 = 0 \quad \dots(iii)$$

Let $(\alpha, \beta), (\beta, \gamma)$ and (γ, α) be the roots of Eqs. (i), (ii) and (iii) respectively, then

$$\alpha + \beta = -b_1 \text{ and } \alpha \cdot \beta = c_1 \quad \dots(iv)$$

$$\beta + \gamma = -b_2 \text{ and } \beta \cdot \gamma = c_2 \quad \dots(v)$$

$$\gamma + \alpha = -b_3 \text{ and } \gamma \cdot \alpha = c_3 \quad \dots(vi)$$

From eqs. (iv), (v) and (vi), we get

$$\alpha + \beta + \gamma = -\frac{1}{2}(b_1 + b_2 + b_3)$$

$$\text{and } (\alpha \cdot \beta + \beta \cdot \gamma + \gamma \cdot \alpha) = c_1 + c_2 + c_3 \quad \dots(vii)$$

$$\text{Now, } (\alpha + \beta)^2 + (\beta + \gamma)^2 + (\gamma + \alpha)^2 = b_1^2 + b_2^2 + b_3^2$$

$$\Rightarrow 2(\alpha^2 + \beta^2 + \gamma^2) + 2(\alpha \cdot \beta + \beta \cdot \gamma + \gamma \cdot \alpha) = b_1^2 + b_2^2 + b_3^2$$

$$\Rightarrow 2[(\alpha + \beta + \gamma)^2 - 2(\alpha \cdot \beta + \beta \cdot \gamma + \gamma \cdot \alpha)]$$

$$+ 2(\alpha \cdot \beta + \beta \cdot \gamma + \gamma \cdot \alpha) = b_1^2 + b_2^2 + b_3^2$$

$$\Rightarrow 2(\alpha + \beta + \gamma)^2 - 2(\alpha \cdot \beta + \beta \cdot \gamma + \gamma \cdot \alpha) = b_1^2 + b_2^2 + b_3^2$$

$$\Rightarrow 2\left[-\frac{1}{2}(b_1^2 + b_2^2 + b_3^2)\right]^2 - 2(c_1 + c_2 + c_3) = b_1^2 + b_2^2 + b_3^2$$

$$\Rightarrow \frac{1}{2}[(b_1^2 + b_2^2 + b_3^2) + 2(b_1 \cdot b_2 + b_2 \cdot b_3 + b_3 \cdot b_1)]$$

$$- 2(c_1 + c_2 + c_3) = b_1^2 + b_2^2 + b_3^2$$

$$\Rightarrow \frac{1}{2}(b_1^2 + b_2^2 + b_3^2) + (b_1 \cdot b_2 + b_2 \cdot b_3 + b_3 \cdot b_1)$$

$$- 2(c_1 + c_2 + c_3) = (b_1^2 + b_2^2 + b_3^2)$$

$$\Rightarrow (b_1^2 + b_2^2 + b_3^2) + 2(b_1 \cdot b_2 + b_2 \cdot b_3 + b_3 \cdot b_1)$$

$$- 4(c_1 + c_2 + c_3) = 2(b_1^2 + b_2^2 + b_3^2)$$

$$\Rightarrow (b_1^2 + b_2^2 + b_3^2) = 2(b_1 \cdot b_2 + b_2 \cdot b_3 + b_3 \cdot b_1) - 4(c_1 + c_2 + c_3) = (b_1^2 + b_2^2 + b_3^2).$$

Therefore choice (a) is the appropriate one.

Hint $(a + b + c)^2 = (a^2 + b^2 + c^2) + 2(a \cdot b + b \cdot c + c \cdot a)$

Exp. 10) Find the value of $\frac{q}{p}$, if $px^2 + x - 15 = 0$ and

$18x^2 + 3x + q = 0$ have both the roots common.

- (a) $-\frac{15}{2}$ (b) -9 (c) $-\frac{27}{4}$ (d) $\frac{14}{3}$

Solution Since both the roots are common, therefore

$$\frac{p}{18} = \frac{1}{3} = \frac{-15}{q} \Rightarrow p = 6 \text{ and } q = -45$$

Therefore choice (a) is the appropriate one.

Exp. 11) If the equation $x^2 + px + qr = 0$ and $x^2 + qx + rp = 0$ have a common root, the other roots, which are not common in these equations, will be the roots of the equation :

- (a) $x^2 + pqx + r = 0$ (b) $x^2 - rx - pq = 0$
 (c) $px^2 + rx + q = 0$ (d) $x^2 + rx + pq = 0$

Solution The given equations are

$$x^2 + px + qr = 0 \quad \dots(i)$$

$$x^2 + qx + rp = 0 \quad \dots(ii)$$

Let (α, β) and (α, γ) be the roots of eqs. (i) and (ii) respectively, then

$$\alpha^2 + p\alpha + qr = \alpha^2 + q\alpha + rp \Rightarrow p\alpha + qr = q\alpha + rp$$

$$\Rightarrow \alpha = r \quad \dots(iii)$$

Since the given eqs. (i) and (ii) have a common root, then

$$(q-p)(p^2r - q^2r) = (qr - rp)^2$$

$$\Rightarrow (q-p)r(p^2 - q^2) = r^2(q-p)^2 \Rightarrow (p^2 - q^2) = r(q-p)$$

$$\Rightarrow r = \frac{(p^2 - q^2)}{-(p-q)} = -(p+q) \Rightarrow r = -(p+q) \quad \dots(iv)$$

Now from the eq. (i), the product of the roots $= \alpha\beta = qr$

But since one of the roots is $\alpha = r$, it means the other root $\beta = q$.

Similarly from the eq. (ii) the product of the roots $= \alpha\gamma = rp$

But since one of the roots is $\alpha = r$, it means the other root

$$\gamma = p$$

Therefore the new equation whose roots are β and γ is

$$x^2 - (\beta + \gamma)x + \beta\gamma = 0 \Rightarrow x^2 - (p + q)x + pq = 0$$

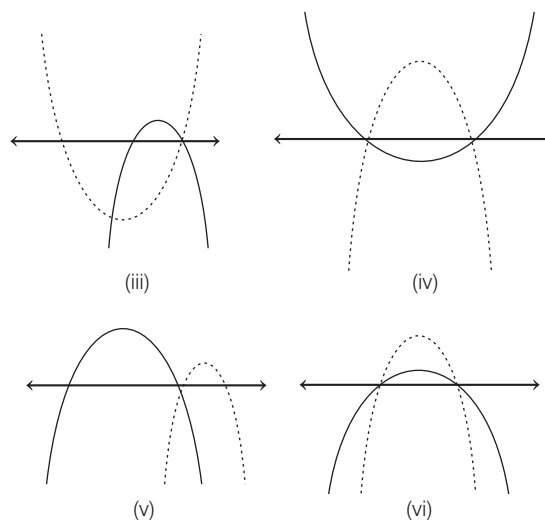
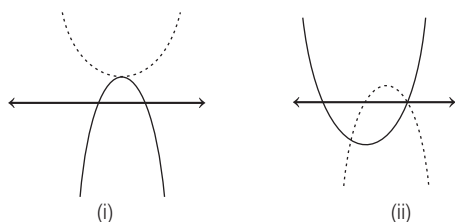
$$\Rightarrow x^2 + rx + pq = 0$$

Therefore choice (d) is the appropriate one.

Exp. 12) There are six pairs of quadratic equations as mentioned below.

1. $-x^2 - x + 6 = 0$ and $-x^2 + 7x - 10 = 0$
2. $x^2 + 3x - 4 = 0$ and $-x^2 - x + 2 = 0$
3. $-x^2 + 8x - 12 = 0$ and $x^2 - 3x - 18 = 0$
4. $-x^2 + 5x - 4 = 0$ and $-3x^2 + 15x - 12 = 0$
5. $x^2 - x - 6 = 0$ and $-4x^2 + 4x + 24 = 0$
6. $4 - 2x^2 = 0$ and $x^2 + 4 = 0$

And there are six sets of representative quadratic graphs as shown below, where the solid parabola depicts the first equation and dotted parabola depicts the second equation for each pair of quadratic equations.



Then which one of the following is correct?

- (a) 1-iv, 2-iii, 3-ii, 4-vi, 5-v, 6-i
 (b) 1-v, 2-ii, 3-iii, 4-vi, 5-iv, 6-i
 (c) 1-ii, 2-iii, 3-i, 4-v, 5-vi, 6-iv
 (d) 1-v, 2-ii, 3-iii, 4-iv, 5-vi, 6-i

Solution Use the following tricks to figure out the right match.

Both the coefficients of x^2 in pair (1) are negative; it means the parabolas will open downwards. But since there is only one root common in the equations of pair (1) so the appropriate graphs are given by graph (v).

Similarly, both the coefficients of x^2 in pair (4) are negative; it means the parabolas will open downwards. But since both the roots are common in the equations of (4) so the right graphs are given by graph (vi).

Equations in pair (5) have both the roots common and one has positive and another has negative coefficient of x^2 , so the graphs will open in opposite directions. Thus the appropriate graphs are represented by graph (iv).

In pair (6) one equation has non-real roots and another one has real roots. Also, one equation has minimum 4 and another has maximum 4. Also, the axes of symmetry for both the equations are Y-axis only; whereas the axis of symmetry is given by $x = \frac{-b}{2a}$. Thus the graph (i) is appropriate for pair (6).

Now, in pair (2) only one root is common and the first equation gives minimum and the second equation gives maximum, therefore graph (ii) is the appropriate match for pair (2).

Lastly, in pair (3) only one root is common and the first equation gives maximum and the second equation gives minimum, therefore graph (iii) is the appropriate match for pair (3).

Hence, choice (b) is correct.

14.10 Basics of Inequality

1. Square of any real quantity is always non-negative. In general, even power of any real quantity is always non-negative.

That means for every real number R , $R^{2n} \geq 0$; where n is any natural number.

2. If a, b, c are three real numbers such that $a \geq b$, then $a \pm c \geq b \pm c$.
3. (i) If a, b, c are three real numbers such that $a \geq b$ and $c > 0$, then $ac \geq bc$.
(ii) If a, b, c are three real numbers such that $a \leq b$ and $c > 0$, then $ac \leq bc$.
4. (i) If a, b, c are three real numbers such that $a \geq b$ and $c < 0$, then $ac \leq bc$.
(ii) If a, b, c are three real numbers such that $a \leq b$ and $c < 0$, then $ac \geq bc$.
5. (i) If a and b are two real numbers and $ab > 0$, then either each of a and b are positive or each of them are negative.
(ii) If a and b are two real numbers and $ab < 0$, then a and b are of opposite signs *i.e.*, either $a > 0$ and $b < 0$ or $a < 0$ and $b > 0$.

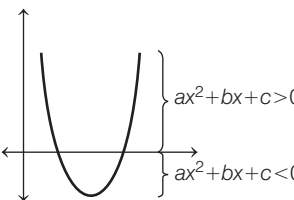
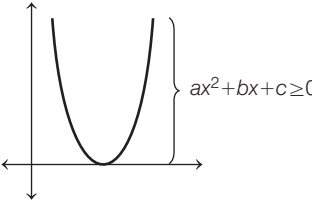
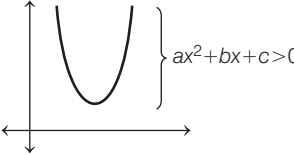
Quadratic Inequality

Essentially a quadratic expression is said to be a quadratic inequation when $ax^2 + bx + c \neq 0$. Thus $ax^2 + bx + c > 0$ and $ax^2 + bx + c < 0$ are the quadratic inequations. A quadratic equation talks about the zeros of y , while a quadratic inequation talks about either positive values of y or negative values of y .

For $y = ax^2 + bx + c$		
$y > 0$	y is strictly greater than 0.	Quadratic inequation
$y \geq 0$	y is greater than or equal to 0.	Quadratic inequation
$y = 0$	y is strictly equal to 0.	Quadratic equation
$y \leq 0$	y is less than or equal to 0.	Quadratic inequation
$y < 0$	y is strictly less than 0.	Quadratic inequation

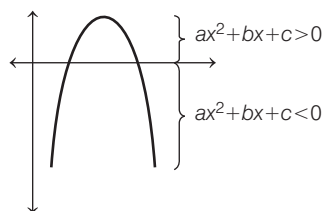
Graphically speaking a quadratic inequation states the position of the quadratic graph either above or below the X -axis. To know the nature (or position) of y we need to know the following things

- Whether the discriminant D is positive, negative or zero
- Whether the coefficient of x^2 is positive or negative
- Roots of the given quadratic equation, in case of real and distinct roots.

For $ax^2 + bx + c$, Such that $a > 0$		
$D > 0$		$y > 0$, for $x < \alpha$ and $x > \beta$ $y \geq 0$, for $x \leq \alpha$ and $x \geq \beta$ $y < 0$, for $\alpha < x < \beta$ $y \leq 0$, for $\alpha \leq x \leq \beta$ Where $y = ax^2 + bx + c$ Assuming α and β are the two roots such that $\alpha < \beta$.
$D = 0$		$y > 0$, for all the values of x , except $x = \alpha = \beta$ $y \geq 0$, for all the values of x Where $y = ax^2 + bx + c$ Assuming α and β are the two roots but they are always equal when $D = 0$.
$D < 0$		$y > 0$, for all the values of x Where $y = ax^2 + bx + c$

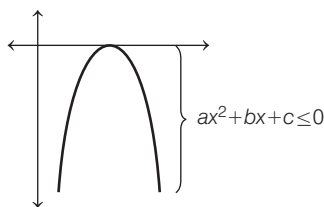
For $ax^2 + bx + c$, such that $a < 0$

$D > 0$



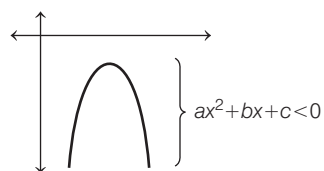
$y > 0$, for $\alpha < x < \beta$
 $y \geq 0$, for $\alpha \leq x \leq \beta$
 $y < 0$, for $x < \alpha$ and $x > \beta$
 $y \leq 0$, for $x \leq \alpha$ and $x \geq \beta$, Where $y = ax^2 + bx + c$
 Assuming α and β are the two roots such that $\alpha < \beta$.

$D = 0$



$y < 0$, for all the values of x , except $x = \alpha = \beta$
 $y \leq 0$, for all the values of x , Where $y = ax^2 + bx + c$
 Assuming α and β are the two roots but they are always equal when $D = 0$.

$D < 0$



$y < 0$, for all the value of x , Where $y = ax^2 + bx + c$

Sign of Quadratic Expression $ax^2 + bx + c$

In the preceding discussion you must have seen that sometimes graph intersects the X -axis and therefore some part of the graph is above the X -axis and rest of the graph is below the X -axis. So here our objective is to determine that in general when the graph $[y = f(x)]$ is positive and when it is negative.

1. If $D < 0$, i.e., α and β are imaginary, then $ax^2 + bx + c > 0$, if $a > 0$ and $ax^2 + bx + c < 0$, if $a < 0$
2. If $D = 0$, i.e., α and β are real and equal, then $ax^2 + bx + c \geq 0$, if $a > 0$ and $ax^2 + bx + c \leq 0$, if $a < 0$
3. If $D > 0$, i.e., α and β are real and unequal ($\alpha < \beta$), then the sign of the expression $ax^2 + bx + c$, $x \in R$ is determined as follows :



Exp. 1) If $a > 0$ and roots are $5 + 2i$ and $5 - 2i$, determine the nature of the quadratic graph.

Solution Since roots are imaginary and $a > 0$, therefore all the values of the quadratic function will be positive for every value of x .

Exp. 2) For a quadratic equation, $a < 0$ and $b^2 = 4ac$, determine the nature of the graph.

Solution Since $D = 0$ and $a < 0$, therefore all the values of the function will be negative except the maximum value and that will be equal to 0.

Exp. 3) For a quadratic equation $10x^2 + x - 21 = 0$, what is sign of the function for every $x < -\frac{3}{2}$?

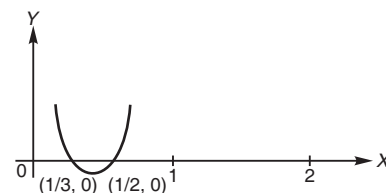
Solution The roots of the equation are $-\frac{3}{2}$ and $\frac{7}{5}$ and $D = 841 > 0$. Also $a > 0$, therefore all the values of function will be positive for every $x < -\frac{3}{2}$.

Exp. 4) Find the sign of $6x^2 - 5x + 1$ for all real values of x .

Solution $\therefore D = b^2 - 4ac \Rightarrow D = 25 - 4 \times 6 \times 1 = 1$

$\therefore$ The roots are real and different and the roots are $\frac{5 \pm \sqrt{25 - 24}}{12}$ i.e., $\frac{1}{2}$ and $\frac{1}{3}$

$\therefore$ The given expression has the same sign as the coefficient of x^2 ,
i.e., positive for all real values of x except for those which lie between $\frac{1}{3}$ and $\frac{1}{2}$



Solutions of Quadratic Inequations

Remember that in quadratic equation we are interested in knowing that for what values of x , the value of y will be zero but in quadratic inequation we are interested in knowing that for what values of x , the value of y will be positive or negative.

	Objective	Condition
Quadratic Equation	To know the values of x	So that $y = 0$
Quadratic Inequation	To know the values of x	So that $y > 0$ or $y < 0$

By having a look at the previous graphs, you can summarize the following facts regarding the quadratic expression $ax^2 + bx + c$ whose roots are α and β and R is the set of real numbers.

Discriminant	Coefficient of x	Values of y	Values of x	Alternative forms of expressing the values of x
$D < 0$	$a > 0$	$y > 0$	for every x	R
	$a < 0$	$y < 0$	for every x	R
$D = 0$	$a > 0$	$y > 0$	for every x , except $x = \alpha$	$R - \{\alpha\}$ or $(-\infty, \alpha) \cup (\alpha, \infty)$
		$y \geq 0$	for every x	R
	$a < 0$	$y < 0$	for every x , except $x = \alpha$	$R - \{\alpha\}$ or $(-\infty, \alpha) \cup (\alpha, \infty)$
		$y \leq 0$	for every x	R
$D > 0$	$a > 0$	$y > 0$	for $x < \alpha$ and $x > \beta$	$R - [\alpha, \beta]$ or $(-\infty, \alpha) \cup (\beta, \infty)$
		$y \geq 0$	for $x \leq \alpha$ and $x \geq \beta$	$R - (\alpha, \beta)$ or $(-\infty, \alpha] \cup [\beta, \infty)$
		$y < 0$	for $\alpha < x < \beta$	(α, β)
		$y \leq 0$	for $\alpha \leq x \leq \beta$	$[\alpha, \beta]$
	$a < 0$	$y > 0$	for $\alpha < x < \beta$	(α, β)
		$y \geq 0$	for $\alpha \leq x \leq \beta$	$[\alpha, \beta]$
		$y < 0$	for $x < \alpha$ and $x > \beta$	$R - [\alpha, \beta]$ or $(-\infty, \alpha) \cup (\beta, \infty)$
		$y \leq 0$	for $x \leq \alpha$ and $x \geq \beta$	$R - (\alpha, \beta)$ or $(-\infty, \alpha] \cup [\beta, \infty)$

So essentially when $D > 0$, then only you have to use the following methods to know that for what values of x , the value of y will be positive or negative. In other cases like $D < 0$ or $D = 0$, just by looking at the value (positive/negative) of a , you can tell the answer. Basically there are following three methods to determine the values of x for a quadratic inequation.

(A) Graphical Method (B) Interval Method (C) Algebraic Method

(A) Graphical Method

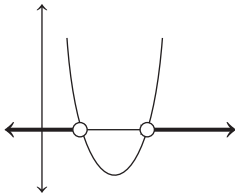
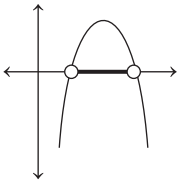
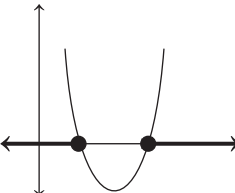
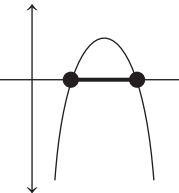
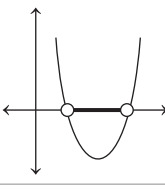
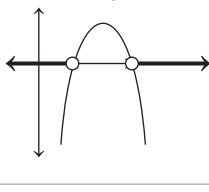
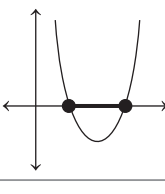
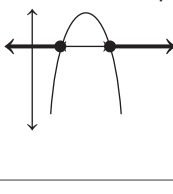
Step 1 Draw the graph (parabola)

Step 2 Determine the roots (intersections at X -axis)

Step 3 Consider the appropriate values of x corresponding to the given y (or $ax^2 + bx + c$) as shown in the diagram by shading the X -axis.

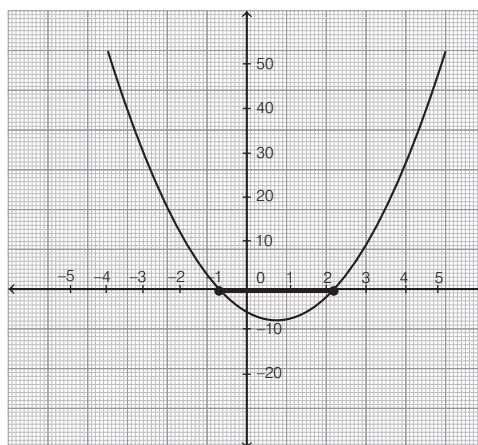
In the following diagrams :

The sign $\circ$ indicates that $x = \alpha$ and $x = \beta$ are excluded. The sign $\bullet$ indicates that $x = \alpha$ and $x = \beta$ are included.

Inequality	Coefficient of x^2	
	$a > 0$	$a < 0$
$ax^2 + bx + c > 0$	$x < \alpha$ and $x > \beta$ 	$\alpha < x < \beta$ 
$ax^2 + bx + c \geq 0$	$x \leq \alpha$ and $x \geq \beta$ 	$\alpha \leq x \leq \beta$ 
$ax^2 + bx + c < 0$	$\alpha < x < \beta$ 	$x < \alpha$ and $x > \beta$ 
$ax^2 + bx + c \leq 0$	$\alpha \leq x \leq \beta$ 	$x \leq \alpha$ and $x \geq \beta$ 

Exp.) Find the values of x for $3x^2 - 3x - 6 \leq 0$.

Solution Let's consider for a while that $3x^2 - 3x - 6 = 0$. Then the roots of this equation are -1 and 2 . Now sketch the graph as shown below. Since $a = 3$, that means $a > 0$, therefore the parabola will open up and it will pass through the points -1 and 2 .



Now you have to see where the graph of $f(x) \leq 0$. Looking at the above graph you can figure out that when $-1 \leq x \leq 2$, then $3x^2 - 3x - 6 \leq 0$.

(B) Interval Method

Step 1 Find the roots, α and β , of the given quadratic inequations considering for a while that the given quadratic expression is equal to zero.

Step 2 Draw a number line and mark the roots on it as you do it on the X -axis; smaller root on the left hand side and bigger root on the right hand side.

Step 3 These two roots divide the number line in 3 contiguous regions as shown below.

Step 4 Follow the table for the final steps.

	For, $a > 0$	For, $a < 0$
$y > 0$	Consider the positive region. (That means all the values less than α and all the values greater than β .)	Consider the positive region. (That means all the values between α and β .)
$y < 0$	Consider the negative region. (That means all the values between α and β .)	Consider the negative region. (That means all the values less than α and all the values greater than β .)

Exp. 1) Find the values of x for the following inequations.

(i) $3x^2 - 3x - 6 \leq 0$

(ii) $3x^2 - 3x - 6 < 0$

(iii) $3x^2 - 3x - 6 \geq 0$

(iv) $3x^2 - 3x - 6 > 0$

Solution The roots of given equations are -1 and 2 . Now, mark the roots on the number line then the line will be divided into 3 sections (or intervals) one section is before -1 , and second section is between -1 and 2 , and the third section is after 2 . Since $a > 0$, therefore the outer sections will be positive and the section lying between roots will be negative, so mark the sections appropriately using $+$ and $-$ signs.



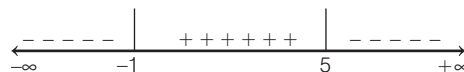
- Since you are required to find the values of x , on the number line, for which $3x^2 - 3x - 6 \leq 0$. That means you are looking for non-positive region. So, it is obvious from the above that $x \geq -1$ and $x \leq 2$. It implies that $-1 \leq x \leq 2$.
- In this case, it is a strict inequality, so you have to exclude the roots. For the inequation $3x^2 - 3x - 6 < 0$, the possible values are $-1 < x < 2$.
- Since, you are required to find the values of x on the number line, for which $3x^2 - 3x - 6 \geq 0$. That means you are looking for non-negative region. So, it is obvious from the above that $x \leq -1$ and $x \geq 2$. It implies that x is $(-\infty, -1] \cup [2, \infty)$ or you can say $x \in R - (-1, 2)$.
- In this case, it is a strict inequality so you have to exclude the roots. For the inequation $3x^2 - 3x - 6 > 0$, the possible values are $x < -1$ and $x > 2$. It implies that x is $(-\infty, -1) \cup (2, \infty)$ or you can say $x \in R - [-1, 2]$.

Exp. 2) Find the values of x for the following inequations.

(i) $-2x^2 + 8x + 10 \geq 0$

(ii) $-2x^2 + 8x + 10 \leq 0$

Solution The roots of this equation are -1 and 5 . Now, mark the roots on the number line then the line will be divided into 3 sections (or intervals) : one section is before -1 , and second section is between -1 and 5 , and the third section is after 5 . Since $a < 0$, therefore the outer sections will be negative and the section lying between roots will be positive, so mark the sections appropriately using $+$ and $-$ signs.



- Since you are required to find the values of x , on the number line, for which $-2x^2 + 8x + 10 \geq 0$. That means you are looking for non-negative region. So it is obvious from the above that $x \geq -1$ and $x \leq 5$. It implies that $-1 \leq x \leq 5$.
- Since, you are required to find the values of x on the number line, for which $-2x^2 + 8x + 10 \leq 0$. That means you are looking for non-positive region. So it is obvious from the above that $x \leq -1$ and $x \geq 5$. It implies that x is $(-\infty, -1] \cup [5, \infty)$ or you can say $x \in R - (-1, 5)$.

(C) Algebraic Method

As far as CAT is concerned, I won't recommend this method at all because it is slightly tedious and more time consuming in comparison to the previously discussed methods. So please avoid this method for competitive exams. However, this method may not be so oblivious to you because it has been inadvertently used on several occasions without specifying the name of it.

NOTE Once you understand the nature of a quadratic graph your most of the problem is solved. You are just expected to visualize and draw a rough sketch of a quadratic graph just by looking at certain things of a quadratic function (or equation or inequality) like the sign of a ($+$ or $-$) value of D ($D > 0$ or $D = 0$ or $D < 0$), value of roots etc. Then looking at the graph – its position, intersections at X -axis and opening direction – you can tell a lot about the quadratic equation/inequation/function.

Practice Exercise

Directions (for Q. Nos. 1 to 11) Find the values of x for which of the following equations are satisfied.

1. $x^2 - 4x + 3 \geq 0$
 (a) $[1, 3]$ (b) $(-\infty, -3] \cup [1, \infty)$
 (c) $(-\infty, 1) \cup (3, \infty)$ (d) $R - (1, 3)$
2. $-x^2 + 2x + 8 \geq 0$
 (a) $(-2, 6)$ (b) $(2, -4)$
 (c) $[-2, 4]$
 (d) $(-\infty, -2] \cup (-2, 4) \cup [4, \infty)$
3. $-x^2 + 2x + 8 < 0$
 (a) $[-\infty, -2) \cup (4, \infty]$ (b) $(-\infty, -2) \cup (4, \infty)$
 (c) $(-2, 4)$ (d) $(-\infty, -4) \cup (2, \infty)$
4. $2x^2 + 5x - 3 > 0$
 (a) $(-\infty, -3] \cup \left(\frac{1}{2}, \infty\right)$ (b) $(-\infty, -3] \cup [2, \infty]$
 (c) $R - \left[-3, \frac{1}{2}\right]$ (d) $R - \left(-3, \frac{1}{2}\right)$
5. $x^2 + 2x + 5 > 0$
 (a) $(-\infty, \infty)$ (b) $[-\infty, \infty]$
 (c) $(-\infty, 0) \cup (0, \infty)$ (d) $(-\infty, -2) \cup (-5, \infty)$
6. $x^2 + x - 12 < 0$
 (a) $-\infty < x - 4$ and $3 < x < \infty$ (b) $[-4, 3]$
 (c) $(-\infty, 4) \cup (3, \infty)$ (d) $(-4, 3)$
7. $x^2 - 4 < 0$
 (a) $-2 \leq x \leq 2$ (b) $-2 < x < 2$
 (c) $(-4, 4)$ (d) none of these
8. $x^2 + 1 < 0$
 (a) $-1 \leq x \leq 1$ (b) $-1 < x < 0$
 (c) Does not exist (d) none of these
9. $9 - x^2 \geq 0$
 (a) $-\sqrt{3} \leq x \leq \sqrt{3}$ (b) $-3 \leq x \leq 3$
 (c) $0 < x \leq 3$ (d) $R - (-3, 3)$
10. $9x^2 - 30x + 25 \leq 0$
 (a) $-3 \leq x \leq 5$ (b) $-5 \leq x \leq 3$
 (c) $\frac{5}{3}$ (d) $x \leq \frac{5}{3}$
11. $x^2 - 10x - 1 \geq 0$
 (a) $(-\infty, \infty)$ (b) $[5 - \sqrt{26}, 5 + \sqrt{26}]$
 (c) $(-\infty, -10) \cup (-1, \infty)$
 (d) $(-\infty, 5 - \sqrt{26}] \cup [5 + \sqrt{26}, \infty)$
12. For a quadratic inequation $ax^2 + bx + c < 0$, having negative discriminant, the values of x that satisfy the given inequation
 (a) R^- (b) R
 (c) R^+ (d) can't be determined
13. For a quadratic inequation $ax^2 + bx + c > 0$, having negative discriminant, the values of x that satisfy the given inequation
 (a) R
 (b) only positive real numbers
 (c) does not exist
 (d) Either R or none
14. For a quadratic inequation $ax^2 + bx + c \geq 0$; $D < 0$ and $a < 0$, the values of x that satisfy the given inequation
 (a) R^+ (b) R^-
 (c) $R - \{0\}$ (d) does not exist
15. For a quadratic inequation $ax^2 + bx + c > 0$, if you have double roots, the values of x that satisfy the given inequation
 (a) $-\infty \leq x \leq \infty$
 (b) $R - \{\alpha, \beta\}$ where α is twice of β
 (c) $-\infty < x < \infty$
 (d) all the real values except one value
16. For a quadratic inequation $ax^2 + bx + c \geq 0$, if you have double roots, the values of x that satisfy the given inequation
 (a) $-\infty \leq x \leq \infty$
 (b) $R - R^-$
 (c) $-\infty < x < \infty$
 (d) all the real numbers except one value
17. Find the valid interval of x for the inequality $-6 \leq x^2 - 5x \leq 6$.
 (a) $[-1, 2] \cup [3, 6]$ (b) $(-\infty, 2] \cup [3, \infty)$
 (c) $[-1, 6]$ (d) none of these
18. Find the values of x that satisfy the inequation $x^4 - 13x^2 + 36 \leq 0$.
 (a) $[-3, -2] \cup [2, 3]$ (b) $[-3, 3]$
 (c) $[-2, 2]$ (d) $(-\infty, -3] \cup [3, \infty)$
19. Find the values of x that satisfy the inequation $x^6 - 19x^3 - 216 > 0$.
 (a) $[-3, -2] \cup [2, 3]$ (b) $(-\infty, -2] \cup [3, \infty)$
 (c) $(-\infty, -2) \cup (3, \infty)$ (d) $(-\infty, -8] \cup [27, \infty)$
20. Find the values of m that satisfy the inequation $m + 3\sqrt{m} - 4 > 0$.
 (a) $m \leq -4$ and $m \geq 1$
 (b) $m > 1$
 (c) $(-\infty, -4) \cup (1, \infty)$
 (d) $(-\infty, -16] \cup [1, \infty)$

Answers

1. (d)	2. (c)	3. (b)	4. (c)	5. (a)
6. (d)	7. (b)	8. (c)	9. (b)	10. (c)
11. (b)	12. (d)	13. (b)	14. (d)	15. (d)
16. (c)	17. (a)	18. (a)	19. (c)	20. (b)

Hints

5. $(-\infty, \infty)$. Since a , the coefficient of x^2 , is positive and the roots are non-real. So, for all the values of x , the inequation is positive.
6. $(-4, 3)$ 7. $-2 < x < 2$
8. Does not exist; since x^2 is always positive and so $x^2 + 1$ is also positive.
9. $-3 \leq x \leq 3$
12. Since the whole graph will lie below the X-axis and we need just the opposite.
13. Double roots means both the roots are same and since you need to obey the 'strict inequality'. So, you have to EXCLUDE the zeros (roots) of this quadratic.
16. Maximum value of this expression can't be determined as it is keep on increasing when $x < 7$ or $x > 7$. However, minimum value is determinable which is attained at $x = 7$ making $(7 - x)^2 = 0$, the minimum possible value of a perfect square. Thus, the minimum value of the whole expression is -8 .
17. When $(7 - x)^2$ is minimum, i.e. zero, then $49 - (7 - x)^2$ will be maximum.

14.11 Position of Roots of a Quadratic Equation with Respect to One or Two Real Numbers

Before moving towards the core of the topic the first thing you need to be sure of is quadratic function. As in, how does a change in x , the change in function $f(x)$ occurs. The quadratic function $f(x)$ is defined as $f(x) = ax^2 + bx + c$, where $a \neq 0$. For example consider an arbitrary quadratic function $f(x) = x^2 - 8x + 12$. Now put some arbitrary numerical values, preferably integers to enjoy the convenience in the given function and then notice the values of $f(x)$.

$$x = 0, x^2 - 8x + 12 = 12 \Rightarrow f(0) = 12$$

$$x = 2, x^2 - 8x + 12 = 0 \Rightarrow f(2) = 0$$

$$x = 4, x^2 - 8x + 12 = -4 \Rightarrow f(4) = -4$$

$$x = 6, x^2 - 8x + 12 = 0 \Rightarrow f(6) = 0$$

$$x = 8, x^2 - 8x + 12 = 12 \Rightarrow f(8) = 12$$

$$x = 1, x^2 - 8x + 12 = 5 \Rightarrow f(1) = 5$$

$$x = 3, x^2 - 8x + 12 = -3 \Rightarrow f(3) = -3$$

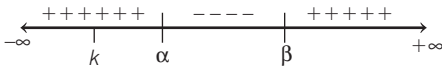
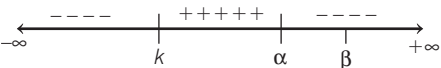
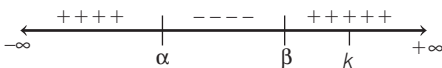
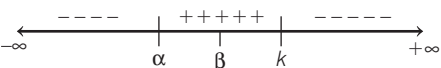
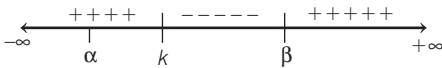
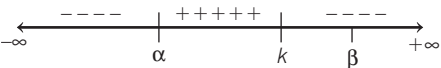
$$x = 5, x^2 - 8x + 12 = -3 \Rightarrow f(5) = -3$$

$$x = 7, x^2 - 8x + 12 = 5 \Rightarrow f(7) = 5$$

It can be easily seen that $f(2) = f(6) = 0$. These are zeros of the function. And, $f(0) > 0, f(1) > 0, f(7) > 0, f(8) > 0$. These are the positive values of the function. And, $f(3) < 0, f(4) < 0, f(5) < 0$. These are the negative values of the function.

Now, the actual point that I'm going to discuss is how to determine the position of a root (or both the roots) with respect to one (or two) given point(s) on the X -axis. Let $f(x) = ax^2 + bx + c$, where $a, b, c \in R, a \neq 0$. Let α, β be the two roots of this equation such that $\alpha < \beta$. Let $k, k_1, k_2 \in R$ such that $k_1 < k_2$.

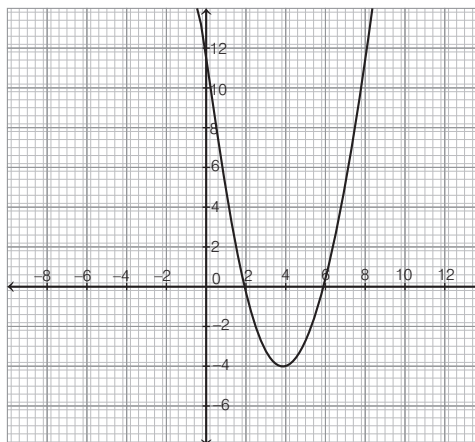
1. Position of real roots with respect to one real number k :

	For, $a > 0, (\alpha < \beta)$	For, $a < 0, (\alpha < \beta)$
If $k < \alpha < \beta$	 (i) $D \geq 0$ (ii) $af(k) > 0$ (iii) $k < -\frac{b}{2a}$	 (i) $D \geq 0$ (ii) $af(k) > 0$ (iii) $k < -\frac{b}{2a}$
If $\alpha < \beta < k$	 (i) $D \geq 0$ (ii) $af(k) > 0$ (iii) $k > -\frac{b}{2a}$	 (i) $D \geq 0$ (ii) $af(k) > 0$ (iii) $k > -\frac{b}{2a}$
If $\alpha < k < \beta$	 (i) $D > 0$ (ii) $af(k) < 0$	 (i) $D > 0$ (ii) $af(k) < 0$

2. Position of real roots with respect to two real numbers k_1, k_2 :

	For, $a > 0, (\alpha < \beta)$	For, $a < 0, (\alpha < \beta)$
If $k_1 < \alpha < k_2 < \beta$ Or $\alpha < k_1 < \beta < k_2$		
	(i) $D > 0$ (ii) $f(k_1) \cdot f(k_2) < 0$	(i) $D > 0$ (ii) $f(k_1) \cdot f(k_2) < 0$
If $k_1 < \alpha < \beta < k_2$		
	(i) $D \geq 0$ (ii) $af(k_1) > 0, af(k_2) > 0$	(i) $D \geq a$ (ii) $af(k_1) > 0, af(k_2) > 0$ (iii) $k_1 < -\frac{b}{2a} < k_2$
	(iii) $k_1 < -\frac{b}{2a} < k_2$	
If $\alpha < k_1 < k_2 < \beta$		
	(i) $D > 0$ (ii) $af(k_1) < 0, af(k_2) < 0$	(i) $D > 0$ (ii) $af(k_1) < 0, af(k_2) < 0$

Now, if we consider the above example and sketch a quadratic graph, we can better understand the significance of the above discussion.



In this function $f(x) = x^2 - 8x + 12$, $a = 1, D > 0$. Suppose you have to determine whether a particular point (say 8) is in between the roots or outside the roots, then by doing some simple calculation you will be able to figure out the relative position. Since $a > 0$,

$$(i) D > 0 \quad (ii) af(k) = 1 \cdot f(8) = 1 \cdot 12 = 12 \Rightarrow af(k) > 0 \quad (iii) k = 8 \text{ and } -\frac{b}{2a} = -\frac{-8}{2} = 4 \Rightarrow k > -\frac{b}{2a}$$

It implies that 8 is greater than both the roots (*i.e.*, 2 and 6), which is same as shown in the above graph.

Practice Exercise

- What are the essential conditions to get assured that a randomly chosen particular point k , lies between the roots of the quadratic equation $ax^2 + bx + c = 0$?
 (i) $D > 0$ (ii) $D \geq 0$
 (iii) $af(k) < 0$ (iv) $af(k) > 0$
 (v) $k > -\frac{b}{2a}$ (vi) $k < -\frac{b}{2a}$
 (a) (i), (iii) and (v) only (b) (i) and (iv) only
 (c) (i) and (iii) only (d) (ii), (iv) and (vi) only
- What can be the correct possible set of factors that determines the location of roots of $ax^2 + bx + c = 0$ of a quadratic equation which has a double root with respect to a randomly chosen point k on the X -axis?
 (a) $D \geq 0, af(k) < 0, k < -\frac{b}{2a}$
 (b) $D \geq 0, af(k) < 0, k > -\frac{b}{2a}$
 (c) $D > 0, af(k) < 0$
 (d) $D \geq 0, af(k) > 0, k < -\frac{b}{2a}$
- Which of the following may not be a possible set of determinants of relative position of roots of a quadratic equation $ax^2 + bx + c = 0$ with respect to a randomly chosen point k on the X -axis ?
 (a) $D \geq 0, af(k) > 0, k < -\frac{b}{2a}$
 (b) $D \geq 0, af(k) < 0, k > -\frac{b}{2a}$
 (c) $D > 0, af(k) < 0$
 (d) $D \geq 0, af(k) > 0, k < -\frac{b}{2a}$
- Which of the following facts are the most essential to know if both the roots of $ax^2 + bx + c = 0$ are either greater or less than the randomly chosen number k ?
 (i) D (ii) $a \cdot f(k)$
 (iii) Equation of axis of symmetry of the graph
 (iv) Equation of the vertex of the graph
 (v) Average of the real roots
 (a) (i), (ii) and (iv)
 (b) (i), (ii) and (iii) or (v)
 (c) (i), (ii), (iii) and (iv)
 (d) Only (i) and (ii)
- If the randomly chosen point k lies between the roots of the equation $ax^2 + bx + c = 0$, then which of the following is correct ?
 (a) $af(k) > 0$ (b) $af(k) \geq 0$
 (c) $af(k) < 0$ (d) all of these
- If both the roots of the equation $ax^2 + bx + c = 0$ lie inside the randomly chosen real points k_1, k_2 then which of the following facts are the most appropriate and valid ones?
 (i) $af(k_1) > 0, af(k_2) > 0$ (ii) $f(k_1) \cdot f(k_2) < 0$
 (iii) $af(k_1) < 0, af(k_2) < 0$ (iv) $D \geq 0$
 (v) $D > 0$ (vi) $k_1 < -\frac{b}{2a} < k_2$
 (vii) $f(k_1) \cdot f(k_2) > 0$
 (a) (i), (v) and (vi) (b) (i), (iv) and (vi)
 (c) (iii), (iv) and (vii) (d) (ii) and (v) only
- The following facts assure that the roots of the equation $ax^2 + bx + c = 0$ lie inside the randomly chosen real points k_1, k_2 , then which of the following facts are the most appropriate and valid ones ?
 (a) $af(k_1) > 0, af(k_2) > 0$ (b) $f(k_1) \cdot f(k_2) > 0$
 (c) $af(k_1) < 0, af(k_2) < 0$ (d) all of these
- Which of the following facts are correct if only one of the roots of the equation $ax^2 + bx + c = 0$ lies inside the randomly chosen real points k_1, k_2 ?
 (i) $D \geq 0$ (ii) $D > 0$
 (iii) $f(k_1) \cdot f(k_2) < 0$ (iv) $f(k_1) \cdot f(k_2) > 0$
 (v) $k_1 < -\frac{b}{2a} < k_2$
 (a) (i), (iii) and (iv) only (b) (i), (iv) and (v) only
 (c) (ii) and (iv) only (d) (ii) and (iii) only
- The value of axis of symmetry of the quadratic graph of $ax^2 + bx + c = 0$ is required to know the position of roots with respect to one or two real points when
 (a) the randomly chosen point is either side of the roots
 (b) if both the roots lie inside the randomly chosen points
 (c) both (a) or (b)
 (d) There are more possibilities other than those mentioned in (a) and (b)
- While determining the position of the roots with respect to one or two randomly chosen real point(s), the discriminant D is strictly greater than zero for the quadratic equation $ax^2 + bx + c = 0$ when
 (a) the randomly chosen point lies between both the roots
 (b) the randomly chosen both the points lie between the roots
 (c) only one root lies between the two randomly chosen points
 (d) all of the above

11. Find all the parameters p for which both the roots of the equation $x^2 - 6px + (3 - 4p + 9p^2) = 0$ exceed the number 4.
 (a) $p > \frac{3}{4}$ (b) $p > \frac{4}{3}$ (c) $p < 1$ (d) $p > \frac{19}{9}$
12. Find all the values of p for which both roots of the equation $x^2 + x + p = 0$ exceed the quantity p .
 (a) $p < -\frac{1}{2}$ (b) $p < \frac{1}{4}$ (c) $p > 0$ (d) $p < -2$
13. Determine all the values of p for which both roots of the equation $(p^2 + p - 2)x^2 - (p + 5)x - 2 = 0$ exceed the greatest negative integer.
 (a) $(-\infty, -2) \cup (-1, -\frac{1}{2}) \cup (1, \infty)$
 (b) $(-\infty, -2) \cup (1, \infty)$
 (c) $(-\infty, -1) \cup (-\frac{1}{2}, \infty)$
 (d) $(-\infty, -1) \cup (1, \infty)$
14. Find the value of p for which one root of the equation $x^2 - (p + 1)x + p^2 + p - 8 = 0$ is more than 2 and another root is less than 2.
 (a) $[2, 3]$ (b) $(-\frac{11}{3}, 3)$
 (c) $(-\frac{11}{3}, -2)$ (d) $-2 < p < 3$
15. Find all the values of p for which roots of the quadratic equation $(p^2 + p + 1)x^2 + (p - 1)x + p^2 = 0$ are located on either side of 4.
 (a) $p > -1$ (b) $-17 < p < 1$
 (c) $-\frac{17}{12} < p < 20$ (d) none of these
16. Find the set of values of u for which exactly one root of the equation $x^2 - ux + (u^2 + 6u) = 0$ lies in $(-2, 0)$.
 (a) $(-6 - 2) \cup (-2, 0)$ (b) $(-8, 0)$
 (c) $(-6, -2) \cup \{2\}$ (d) none of these
17. If α, β are the zeros of the quadratic function $f(x) = x^2 + 2(p - 3)x + 9$ such that $\alpha \neq \beta$ and $-6 < \{a, \beta\} < 1$, then find the value of p .
 (a) $(-4, \frac{27}{4})$ (b) $(-2, \frac{27}{4})$
 (c) $(6, \frac{27}{4})$ (d) $(6, \infty)$
18. Find all the values of m for the equation $(m - 4) - 2mx + (m - 5)x^2 = 0$, so that one root is less than 1 and another one is greater than 2.
 (a) $(\frac{20}{9}, 24)$ (b) $(6, 24)$
 (c) $(5, 24)$ (d) $(\frac{20}{9}, \infty)$
19. Find the values of w for which the equation $x^2 + 2(w - 1)x + w + 5 = 0$ has at least one positive root.
 (a) $(-\infty, -5) \cup (-5, -1]$ (b) $w \leq -1$
 (c) $w < 1$ (d) $(-\infty, -5) \cup (4, \infty)$
20. Find the number of integral values of p if the roots of the equation $2x^2 + p^2 - (5x + 6p) + 8 = 0$ are of opposite sign.
 (a) 1 (b) 2 (c) 3 (d) 6
21. For what values of p exactly one root of the equation $2^p x^2 - 4^p x + 2^p - 1 = 0$ lies between 1 and 2?
 (a) $\left[2 - \left(\frac{5 - \sqrt{17}}{4}\right)\right] < p < \left[2 - \left(\frac{5 + \sqrt{17}}{4}\right)\right]$
 (b) $\log_2\left(\frac{5 - \sqrt{17}}{4}\right) < p < \log_2\left(\frac{5 + \sqrt{17}}{4}\right)$
 (c) $\log_5\left(\frac{5 - \sqrt{17}}{4}\right) < p < \log_5\left(\frac{5 + \sqrt{17}}{4}\right)$
 (d) $\frac{-5 + \sqrt{17}}{4} < p < \frac{5 + \sqrt{17}}{4}$

Answers

1. (c)	2. (d)	3. (b)	4. (b)	5. (c)
6. (b)	7. (a)	8. (d)	9. (c)	10. (d)
11. (d)	12. (d)	13. (b)	14. (d)	15. (d)
16. (a)	17. (c)	18. (c)	19. (b)	20. (a)
21. (b)				

Hints

11. Since both the roots exceed the arbitrary point k (or greater than k) then you must have to satisfy the following three conditions.

$$(i) D \geq 0 \quad (ii) af(k) > 0 \quad (iii) k < -\frac{b}{2a}$$

$$\begin{aligned} \text{So here it goes } (-6p)^2 - 4 \times 1 \times (3 - 4p + 9p^2) &\geq 0 \\ \Rightarrow 36p^2 - 12 + 16p - 36p^2 &\geq 0 \Rightarrow 16p \geq 12 \\ \Rightarrow p &\geq \frac{3}{4} \quad \dots(i) \end{aligned}$$

$$\begin{aligned} \text{And } a \cdot f(k) > 0 &\Rightarrow 1 \cdot f(4) > 0 \\ \Rightarrow 1 \times (16 - 24p + 3 - 4p + 9p^2) &> 0 \\ \Rightarrow 9p^2 - 28p + 19 > 0 &\Rightarrow (p-1)(9p-19) > 0 \\ \Rightarrow p \in (-\infty, 1) \cup \left(\frac{19}{9}, \infty\right) &\quad \dots(ii) \end{aligned}$$

$$\text{And } k < -\frac{b}{2a} \Rightarrow 4 < -\frac{-6p}{2} \Rightarrow 4 < 3p \Rightarrow p > \frac{4}{3} \quad \dots(iii)$$

The intersection of (i), (ii) and (iii) is $p > \frac{19}{9}$.

Hence choice (d) is the answer.

12. Since both the roots exceed the arbitrary point k (or greater than k) then you must have to satisfy the following three conditions.

$$D \geq 0 \Rightarrow 1^2 - 4 \times 1 \times p \geq 0 \Rightarrow p \leq \frac{1}{4} \quad \dots(i)$$

$$\begin{aligned} af(k) &> 0 \\ \Rightarrow 1 \times f(p) > 0 &\Rightarrow p^2 + 2p > 0 \Rightarrow p(p+2) > 0 \\ \Rightarrow p \in (-\infty, -2) \cup (0, \infty) &\quad \dots(ii) \end{aligned}$$

$$k < -\frac{b}{2a} \Rightarrow p < -\frac{1}{2} \quad \dots(iii)$$

The intersection of (i), (ii) and (iii) is $p < -2$.

Hence choice (d) is the answer.

13. Since both the roots exceed the arbitrary point k (or greater than k) then you must have to satisfy the following three conditions $[-(p+5)]^2 - 4 \times (p^2 + p - 2) \times (-2) \geq 0$

$$\begin{aligned} \Rightarrow (p+1)^2 &\geq 0 \Rightarrow p \in (-\infty, -1) \cup (-1, \infty) \\ \text{And } af(k) > 0 &\Rightarrow (p^2 + p - 2) \cdot f(-1) > 0 \\ \Rightarrow (p^2 + p - 2) \cdot (p^2 + 2p + 1) &> 0 \\ \Rightarrow (p+2)(p-1)(p+1)(p+1) &> 0 \\ \Rightarrow (p+2)(p-1)(p+1)^2 &> 0 \\ \Rightarrow p \in (-\infty, -2) \cup (1, \infty) &\quad \dots(ii) \end{aligned}$$

$$\text{And } k < -\frac{b}{2a} \Rightarrow -1 < -\frac{-(p+5)}{2(p^2 + p - 2)}$$

$$\begin{aligned} \Rightarrow -2(p^2 + p - 2) &< (p+5) \\ \Rightarrow 2p^2 + 3p + 1 &> 0 \\ \Rightarrow (p+1)(2p+1) &> 0 \\ \Rightarrow p \in (-\infty, -1) \cup \left(-\frac{1}{2}, \infty\right) \end{aligned}$$

The intersection of (i), (ii) and (iii) is $p \in (-\infty, -2) \cup (1, \infty)$.

Hence choice (b) is the answer.

14. Since one root is less than 2 and another is greater than 2. It implies that 2 lies between the roots, then it must satisfy the following two conditions.

$$\begin{aligned} D &> 0 \\ \Rightarrow [-(p+1)]^2 - 4 \times 1 \times (p^2 + p - 8) &> 0 \\ \Rightarrow -3p^2 - 2p + 33 &> 0 \quad \dots(i) \\ \Rightarrow \left\{ \frac{-11}{3} < p < 3 \right\} \end{aligned}$$

$$\text{And } af(k) < 0 \Rightarrow 1 \cdot f(2) < 0$$

$$\Rightarrow p^2 - p - 6 < 0 \Rightarrow -2 < p < 3$$

The intersection of (i) and (ii) is $-2 < p < 3$

Hence choice (d) is the answer.

15. Since 4 lies between the roots, then it must satisfy the following two conditions.

$$(i) af(k) < 0 \quad (ii) D > 0$$

Now, $af(k) < 0$

Where $a = p^2 + p + 1$

The discriminant of $p^2 + p + 1$ is negative and the coefficient of p^2 is positive, it means $p^2 + p + 1 > 0 \Rightarrow a > 0$.

Similarly, $f(4) = 17p^2 + 20p + 12$

The discriminant of $17p^2 + 20p + 12$ is negative and the coefficient of p^2 is positive, it means $17p^2 + 20p + 12 > 0 \Rightarrow f(4) > 0$.

Since $a > 0$ and $f(4) > 0$. It implies that $af(4) > 0$.

As $af(k) \neq 0$, so 4 does not lie between the two roots.

Hence choice (d) is the answer.

16. Since exactly one root lies between two randomly chosen points, then it must satisfy the following conditions.

$$(i) D > 0 \quad (ii) f(k_1) \cdot f(k_2) < 0$$

Then we have,

$$\begin{aligned} D > 0 &\Rightarrow (-u)^2 - 4 \times 1 \times (u^2 + 6u) > 0 \\ \Rightarrow -3u^2 - 24u &> 0 \\ \Rightarrow -3(u^2 + 8u) &> 0 \\ \Rightarrow u^2 + 8u < 0 &\Rightarrow u(u+8) < 0 \\ \Rightarrow u \in (-8, 0) &\quad \dots(i) \end{aligned}$$

$$\text{And } f(k_1) \cdot f(k_2) < 0 \Rightarrow f(-2) \cdot f(0) < 0$$

$$\Rightarrow (u^2 + 4u + 4)(u^2 + 6u) < 0$$

$$\Rightarrow (u+2)^2(u^2 + 6u) < 0$$

$$\Rightarrow u \in (-6-2) \cup (-2, 0) \quad \dots(ii)$$

The intersection of (i) and (ii) is $(-6-2) \cup (-2, 0)$

Hence choice (a) is the answer.

17. Since both the roots lie between k_1 and k_2 therefore the following conditions must be satisfied.

(i) $D > 0$, since it is given that $\alpha \neq \beta$.

(ii) $af(k_1) > 0$, $af(k_2) > 0$

(iii) $k_1 < -\frac{b}{2a} < k_2$

Then we have $D > 0 \Rightarrow [2(p-3)]^2 - 4 \times 1 \times 9 > 0$

$$\Rightarrow p(p-6) > 0 \quad \dots(i)$$

$$\Rightarrow p \in (-\infty, 0) \cup (6, \infty)$$

$$\text{And } af(k_1) > 0 \Rightarrow 1 \cdot f(-6) > 0$$

$$\Rightarrow 36 + 2(p-3) \times (-6) + 9 > 0$$

$$\Rightarrow p < \frac{27}{4} \quad \dots(ii)$$

$$\text{Also } af(k_2) > 0$$

$$\Rightarrow 1 \cdot f(1) > 0$$

$$\Rightarrow 1 + 2(p-3) + 9 > 0$$

$$\Rightarrow p > -2 \quad \dots(iii)$$

$$\text{And } -\frac{b}{2a} = -\frac{2(p-3)}{2} = -(p-3)$$

$$\text{So, } k_1 < -\frac{b}{2a} < k_2$$

$$\Rightarrow -6 < -(p-3) < 1 \Rightarrow 2 < p < 9 \quad \dots(iv)$$

The intersection of (i), (ii), (iii) and (iv) is $\left(6, \frac{27}{4}\right)$.

Hence choice (c) is the answer.

18. Since one root is less than k_1 and another root is greater than k_2 , then the following conditions must be satisfied.

$$(i) D > 0 \quad (ii) af(k_1) < 0, af(k_2) < 0$$

Then we have

$$D > 0 \Rightarrow (-2m)^2 - 4(m-5)(m-4) > 0$$

$$\Rightarrow m > \frac{20}{9} \quad \dots(i)$$

$$\text{And } af(k_1) < 0 \Rightarrow (m-5)f(1) < 0$$

$$\Rightarrow (m-5)(-9) < 0 \Rightarrow m > 5 \quad \dots(ii)$$

$$\text{Also } af(k_2) > 0$$

$$\Rightarrow (m-5)f(2) < 0$$

$$\Rightarrow (m-5)(m-24) < 0$$

$$\Rightarrow 5 < m < 24 \quad \dots(iii)$$

The common values of m from (i), (ii) and (iii) is (5, 24).

Hence choice (c) is the answer.

19. Since roots are real, so discriminant cannot be negative. That is $D \geq 0$.

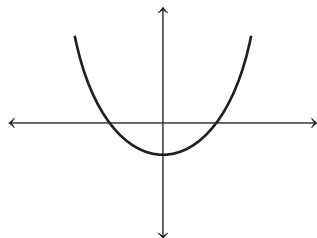
$$\therefore [2(w-1)]^2 - 4 \times 1(w+5) \geq 0$$

$$\Rightarrow w \in (-\infty, -1] \cup [4, \infty) \quad \dots(i)$$

Case 1 When exactly one root is positive. That means exactly one root is greater than 0. Then

$$af(0) < 1 \Rightarrow 1 \cdot f(0)$$

$$\Rightarrow w + 5 < 0 \Rightarrow w < -5$$



Case 2 When both the roots are positive. That means both the roots are greater than 0. Then following three conditions must be satisfied.

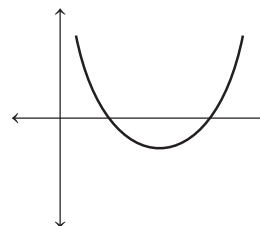
$$(i) af(0) > 0 \quad (ii) -\frac{b}{2a} > 0$$

Then we have,

$$1 \cdot f(0) > 0 \Rightarrow w + 5 > 0 \Rightarrow w > -5 \quad \dots(iii)$$

$$\text{And } \left(-\frac{b}{2a} = \frac{-2(w-1)}{2} = 1-w\right) - \frac{b}{2a} > 0 \Rightarrow 1-w > 0$$

$$\Rightarrow w < 1 \quad \dots(iv)$$



$$\text{Combining (iii) and (iv) we get } w \in (-5, 1) \quad \dots(v)$$

Now the intersection of (i), (ii) and (v) is $w \in (-\infty, -1]$

Hence choice (b) is the answer.

20. The given equation $2x^2 + p^2 - (5x + 6p) + 8 = 0$ can be expressed as $2x^2 - 5x + p^2 - 6p + 8 = 0$

Since, one root is greater than 0 and another one is less than 0, therefore the given equation must satisfy the following condition.

$$(i) D > 0 \Rightarrow (-5)^2 - 4 \times 2 \times (p^2 - 6p + 8) > 0$$

$$\Rightarrow -8p^2 + 48p - 39 > 0 \Rightarrow 3 - \frac{\sqrt{66}}{4} < p < 3 + \frac{\sqrt{66}}{4}$$

$$(ii) af(k) < 0 \Rightarrow 2f(0) < 0 \Rightarrow 2(p^2 - 6p + 8) < 0 \Rightarrow 2 < p < 4$$

Now the intersection (or common) values of (i) and (ii) will hold true. Thus the required value of p is (2, 4). That is 3 is the only one integral value that satisfy the required condition. Hence choice (a) is correct.

21. Let's assume $f(x) = 2^p x^2 - 4^p x + 2^p - 1$.

Since exactly one root lies between 1 and 2, so $f(1)f(2) < 0$.

$$\text{Therefore, } (2^p - 4^p + 2^p - 1) = (2^p 2^2 - 4^p \cdot 2 + 2^p - 1) < 0$$

$$\Rightarrow (-4^p + 2 \cdot 2^p - 1)(2^p \cdot 4 - 2^{2p} \cdot 2 + 2^p - 1) < 0$$

$$\Rightarrow -(4^p - 2 \cdot 2^p + 1)(2^p \cdot 5 - 2^{2p} \cdot 2 - 1) < 0$$

$$\Rightarrow (4^p - 2 \cdot 2^p + 1)(2 \cdot 2^{2p} - 5 \cdot 2^p + 1) < 0$$

$$\Rightarrow (2^p - 1)^2 (2 \cdot 2^{2p} - 5 \cdot 2^p + 1) < 0$$

$$\Rightarrow (2 \cdot 2^{2p} - 5 \cdot 2^p + 1) < 0; \text{ since } (2^p - 1)^2 \text{ is non negative.}$$

$$\Rightarrow [2(2^p)^2 - 5 \cdot (2^p) + 1] < 0$$

$$\Rightarrow \left[2^p - \left(\frac{5 + \sqrt{17}}{4}\right)\right] \left[2^p - \left(\frac{5 - \sqrt{17}}{4}\right)\right] < 0$$

$$\left(\frac{5 - \sqrt{17}}{4} < 2^p < \frac{5 + \sqrt{17}}{4}\right)$$

$$\Rightarrow \log_2 \left(\frac{5 - \sqrt{17}}{4}\right) < p < \log_2 \left(\frac{5 + \sqrt{17}}{4}\right)$$

Hence choice (b) is the answer.

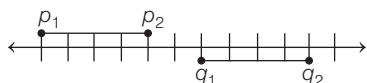
14.12 Relation Between The Roots of Two Quadratic Equations

Let us assume that there are two quadratic equations $ap^2 + bp + c = 0$ and $uq^2 + vq + w = 0$. Now, there can be any of the five possibilities between p and q

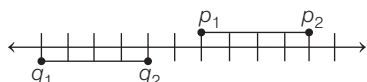
(a) $p < q$ (b) $p > q$ (c) $p \leq q$ (d) $p \geq q$ (e) $p = q$

If none of the above conditions are met, we cannot find a conclusive relation between p and q . In order to determine the relation between p and q , we can follow the following method. First of all find the real roots of the equation $ap^2 + bp + c = 0$ as $\{p_1, p_2\}$ such that $p_1 \leq p_2$ and the real roots of the equation $uq^2 + vq + w = 0$ as $\{q_1, q_2\}$ such that $q_1 \leq q_2$. Now, place the roots on the number line and connect the smallest root with the largest root of each equation. It gives us two sections - one is $p_1 p_2$ and another one is $q_1 q_2$. Now, compare the two sections created between the smallest and the largest roots of each quadratic equation, as shown below.

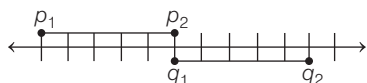
Case 1 In this case, $p < q$, as all the roots of first equation are smaller than all the roots of second equation. That is no any point is common between these two sections. So clearly each value of p is less than each value of q .



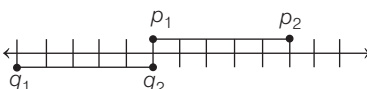
Case 2 In this case, $p > q$, as all the roots of first equation are greater than all the roots of second equation. That is no any point is common between these two sections. So clearly each value of p is greater than each value of q .



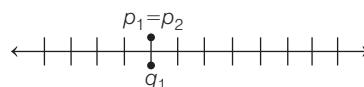
Case 3 In this case, $p \leq q$, as the largest root of the first equation is equal to the smallest root of the second equation and except this no other part of the two sections is common.



Case 4 In this case, $p \geq q$, as the smallest root of the first equation is equal to the largest root of the second equation and except this no other part of the two sections is common.



Case 5 In this case, $p = q$, as all the four roots of both the equations are same. It is a unique case when each equation has double root and not only this the value of these double roots is same.

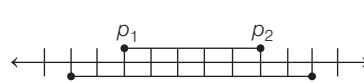
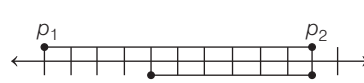
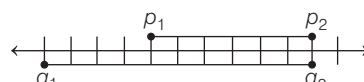
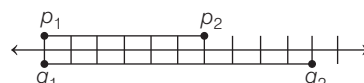
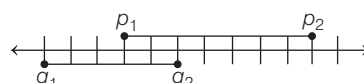
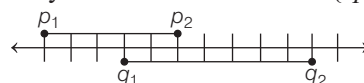


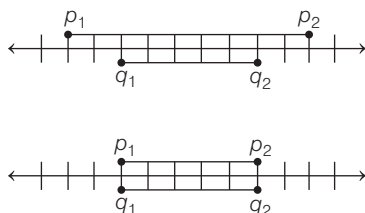
Thus, we have learnt that when no value is common between two sections, we can say either $p < q$ or $p > q$. And, when nothing is common on the number line except one value, we can say $p = q$. If more than one value is common the answer becomes indeterminable. This is where the challenge of comparing the two sections arises.

There are following possibilities of such situations where we cannot conclude the exact relation between the two sections.

In each such situation, either one or both the roots of a quadratic equation lie(s) between the two roots of the other quadratic equation. Essentially, whenever there is more than one point common between two quadratic equations, we cannot determine whether $p < q$ or $p > q$ or $p \leq q$ or $p \geq q$.

When relation is uncertain, we can express it by the symbol $\lessgtr$ to denote the relationship between p and q . If we don't want to use that symbol we can write it as $\{ p = q \text{ or } p \lessgtr q \}$.





So you see, whenever there is overlapping, the relation between p and q cannot be determined. In other words, this kind of relation can be expressed by $p \lesseqgtr q$. Also, when the roots of any or both of the equations are non-real, we cannot find the exact relation between p and q .

Tricks to Identify the Signs of the Roots Let us consider a quadratic equation $ax^2 + bx + c = 0$, whose roots are α and β such that $(x - \alpha)(x - \beta) = 0$. The signs of the two roots will depend upon the signs of b and c , where b denotes the coefficient of x and c denotes the constant term of the quadratic equation.

b	c	Signs of Roots	α	β
+	+	Both are always negative	-	-
+	-	One is positive and another one is negative	-	+
-	+	Both are always positive.	+	+
-	-	One is positive and another one is negative	+	-

Tricks to Determine the Relation Between the Roots of Two Quadratic Equations

It is very useful when the given equations are such that their roots are difficult to obtain. Let us consider two quadratic equations:

(i) $ap^2 + bp + c = 0$ (ii) $rq^2 + sq + t = 0$

- I. If any or both the roots of a quadratic equation lie between the two roots of another quadratic equation, the relation between the roots of the two quadratic equations cannot be established.
- II. If the signs of both the constant terms c and t are negative, the relation between the roots of the two quadratic equations cannot be established.
- III. If the roots of any or both the quadratic equation(s) are non-real, the relation between the roots of the two quadratic equations cannot be established.
- IV. When the constant terms of both the equations are positive and the sign of coefficients of p and q are opposite, we can definitely determine the relation between p and q . If the sign of the coefficient of p is positive and sign of the coefficient of q is negative, then we have $p < q$.

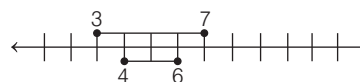
Exp. 1) There are two quadratic equations. Solve these equations and find the relation between p and q .

I. $p^2 - 10p + 21 = 0$ II. $q^2 - 10q + 24 = 0$

- (a) $p < q$
 (b) $p > q$
 (c) $p \leq q$
 (d) Cannot be determined

Solution Let us find roots of the equations.

$$\begin{aligned} p^2 - 10p + 21 &= 0 \\ \Rightarrow (p - 3)(p - 7) &= 0 \Rightarrow p = \{3, 7\} \\ q^2 - 10q + 24 &= 0 \Rightarrow (q - 4)(q - 6) = 0 \\ \Rightarrow q &= \{4, 6\} \end{aligned}$$



As we see that the two roots of the second equation lie between the two roots of the first equation. Therefore, we cannot determine the relation between p and q . Hence choice (d) is correct.

Alternatively Create a table as shown below and compare the values.

	$q_1 = 4$	$q_2 = 6$
$p_1 = 3$	$p_1 < q_1$	$p_1 < q_2$
$p_2 = 7$	$p_2 > q_1$	$p_2 > q_2$

Since there is no clarity whether $p < q$ or $p > q$, we cannot determine the exact relation between p and q .

This method of using table can be applied in every problem to test the relationship between the values of two quadratic equations.

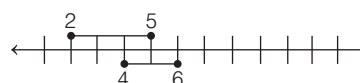
Exp. 2) There are two quadratic equations. Solve these equations and find the relation between p and q .

I. $p^2 - 7p + 10 = 0$ II. $q^2 - 10q + 24 = 0$

- (a) $p < q$
 (b) $p > q$
 (c) $p \leq q$
 (d) Cannot be determined

Solution Let us find the roots of the equations.

$$\begin{aligned} p^2 - 7p + 10 &= 0 \\ \Rightarrow (p - 2)(p - 5) &= 0 \Rightarrow p = \{2, 5\} \\ q^2 - 10q + 24 &= 0 \\ \Rightarrow (q - 4)(q - 6) &= 0 \\ \Rightarrow q &= \{4, 6\} \end{aligned}$$



As we see that the one root of the one equation lies between the two roots of the other equation. Therefore, we cannot determine the relation between p and q .

Hence choice (d) is correct.

Exp. 3) There are two quadratic equations. Solve these equations and find the relation between p and q

I. $p^2 - 12p + 32 = 0$ II. $q^2 - 10q + 24 = 0$

(a) $p < q$

(b) $p > q$

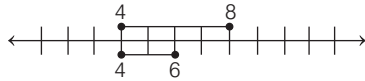
(c) $p \leq q$

(d) Cannot be determined

Solution Let us find the roots of the equations $p^2 - 12p + 32 = 0$

$$\Rightarrow (p-4)(p-8) = 0 \Rightarrow p = \{4, 8\}$$

$$q^2 - 10q + 24 = 0 \Rightarrow (q-4)(q-6) = 0 \Rightarrow q = \{4, 6\}$$



As we see that the one root of the second equation lies between the two roots of the first equation. Therefore, we cannot determine the relation between p and q . Hence choice (d) is correct.

Exp. 4) There are two quadratic equations. Solve these equations and find the relation between p and q .

I. $p^2 + 2p - 15 = 0$ II. $q^2 - 4q - 12 = 0$

(a) $p < q$

(b) $p > q$

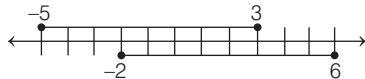
(c) $p \leq q$

(d) Cannot be determined

Solution Let us find the roots of the equations $p^2 + 2p - 15 = 0$

$$\Rightarrow (p+5)(p-3) = 0 \Rightarrow p = \{-5, 3\}$$

$$q^2 - 4q - 12 = 0 \Rightarrow (q+2)(q-6) = 0 \Rightarrow q = \{-2, 6\}$$



As we see that the one root of the one equation lies between the two roots of the other equation. Therefore, we cannot determine the relation between p and q .

Hence choice (d) is correct.

Alternatively Since the signs of the constant terms of both the equations are negative, therefore we can definitely say that each equation has one positive and one negative root. It implies that at least one root of the one equation will definitely lie between the two roots of the other equation, as these roots lie on the both the sides of the Y-axis. That's why their sections will overlap.

Exp. 5) There are two quadratic equations. Solve these equations and find the relation between p and q .

I. $-p^2 + 14p - 45 = 0$ II. $2q^2 - 28q + 90 = 0$

(a) $p < q$

(b) $p > q$

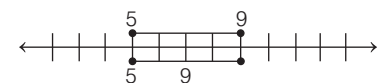
(c) $p \leq q$

(d) Cannot be determined

Solution Let us find the roots of the equations.

$$-p^2 + 14p - 45 = 0 \Rightarrow (p-5)(p-9) = 0 \Rightarrow p = \{5, 9\}$$

$$2q^2 - 28q + 90 = 0 \Rightarrow (q-5)(q-9) = 0 \Rightarrow q = \{5, 9\}$$



As we see that there is more than one point common between the two sections, it means the relation between p and q is indeterminable. Therefore, we cannot determine the relation between p and q . Hence choice (d) is correct.

Alternatively Create a table as shown below and compare the values.

	$q_1 = 5$	$q_2 = 9$
$p_1 = 5$	$p_1 = q_1$	$p_1 < q_2$
$p_2 = 9$	$p_2 > q_1$	$p_2 = q_2$

Since there is no clarity whether $p < q$ or $p = q$ or $p > q$, we cannot determine the exact relation between p and q .

Exp. 6) There are two quadratic equations. Solve these equations and find the relation between p and q .

I. $3p^2 + 5p + 5 = 0$ II. $q^2 - 4q - 12 = 0$

(a) $p < q$

(b) $p > q$

(c) $p \leq q$

(d) Cannot be determined

Solution Let us find the roots of the equations.

Since the discriminant of the $3p^2 + 5p + 5 = 0$ is -35 , which is negative. So it does not give us real roots. Therefore, we cannot determine the relation between p and q .

Hence choice (d) is correct.

Exp. 7) There are two quadratic equations. Solve these equations and find the relation between p and q .

I. $p^2 - 8p + 15 = 0$ II. $q^2 + 8q + 12 = 0$

(a) $p < q$

(b) $p > q$

(c) $p \leq q$

(d) Cannot be determined

Solution Let us find the roots of the equations $p^2 - 8p + 15 = 0$

$$\Rightarrow (p-3)(p-5) = 0 \Rightarrow p = \{3, 5\}$$

$$q^2 + 8q + 12 = 0 \Rightarrow (q+6)(q+2) = 0 \Rightarrow q = \{-6, -2\}$$



As we see that both the roots of the first equation are greater than both the roots of the second equation. Therefore, we can conclude that $p > q$. Hence choice (b) is correct.

Alternatively Since the constant terms of both the equations are positive and the sign of coefficients of p and q are opposite, we can definitely determine the relation between p and q . Further, since the sign of the coefficient of p is negative and sign of the coefficient of q is positive, then we have $p > q$.

Exp. 8) There are two quadratic equations. Solve these equations and find the relation between p and q .

I. $p^2 - 5p + 4 = 0$ II. $q^2 + 5q - 6 = 0$

(a) $p \leq q$

(b) $p > q$

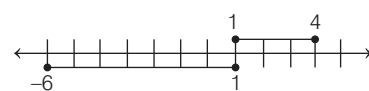
(c) $p \geq q$

(d) Cannot be determined

Solution Let us find the roots of the equations $p^2 - 5p + 4 = 0$

$$\Rightarrow (p-1)(p-4) = 0 \Rightarrow p = \{1, 4\}$$

$$q^2 + 5q - 6 = 0 \Rightarrow (q+6)(q-1) = 0 \Rightarrow q = \{-6, 1\}$$



As we see that all the values of p are greater than all the values of q , except one value. Therefore, we can conclude that $p \geq q$. Hence, choice (c) is correct.

Practice Exercise

Directions (for Q. Nos. 1 to 15) In each of these questions, two equations (I) and (II) are given. You have to solve both the equations and give answer regarding the relationship between p and q .

1. I. $p^2 - 361 = 0$ II. $q^2 - 40q + 399 = 0$
(a) $p < q$ (b) $p > q$ (c) $p \leq q$ (d) $p \geq q$
(e) $p = q$
2. I. $-p^2 + 529 = 0$ II. $2q^2 - 1058 = 0$
(a) $p < q$ (b) $p > q$ (c) $p \leq q$ (d) $p \geq q$
(e) cannot be determined
3. I. $5p^2 - 120p + 720 = 0$ II. $3q - 16 = \frac{q^2}{8}$
(a) $p < q$ (b) $p > q$ (c) $p \leq q$ (d) $p \geq q$
(e) $p = q$
4. I. $p^2 - 17161 = 0$ II. $q^2 + 17161 = 0$
(a) $p < q$ (b) $p > q$ (c) $p \leq q$ (d) $p \geq q$
(e) cannot be determined
5. I. $225p^2 - 1740p + 2835 = 0$
II. $225q^2 + 690q - 2835 = 0$
(a) $p < q$ (b) $p > q$ (c) $p \leq q$ (d) $p \geq q$
(e) $p \leq q$
6. I. $p^4 - 1296 = 0$ II. $q^2 = \sqrt{1296}$
(a) $p < q$ (b) $p > q$ (c) $p \leq q$ (d) $p \geq q$
(e) $p = q$ or $p \leq q$
7. I. $p^3 - 9p^2 + 18p = 0$ II. $q^3 - q^2 = 0$
(a) $p < q$ (b) $p > q$ (c) $p \leq q$ (d) $p \geq q$
(e) $p = q$ or $p \leq q$
8. I. $14p^2 + 27p + 9 = 0$ II. $56q^2 - 3q - 9 = 0$
(a) $p < q$ (b) $p > q$ (c) $p \leq q$ (d) $p \geq q$
(e) $p = q$
9. I. $p^2 - 42p - 343 = 0$ II. $\frac{16}{\sqrt{q}} + \frac{9}{\sqrt{q}} = 5\sqrt{q}$
(a) $p < q$ (b) $p > q$ (c) $p \leq q$ (d) $p \geq q$
(e) $p \leq q$
10. I. $7p^2 + 4\sqrt{7}p - 5 = 0$
II. $\sqrt{77}q^2 - (10\sqrt{7} + \sqrt{11})q + 10 = 0$
(a) $p < q$ (b) $p > q$ (c) $p \leq q$ (d) $p \geq q$
(e) can't be determined
11. I. $p^5 - 74p^3 + 1225p = 0$ II. $q^2 + 14q + 48.99 = 0$
(a) $p < q$ (b) $p > q$
(c) $p \leq q$ (d) $p \geq q$
(e) can't be determined
12. I. $25p^2 - 55\sqrt{11}p + 198 = 0$
II. $25p^2 + 55\sqrt{11}p + 198 = 0$
(a) $p < q$ (b) $p > q$ (c) $p \leq q$ (d) $p \geq q$
(e) can't be determined

13. I. $25p^2 - 35\sqrt{11}p - 198 = 0$
II. $25p^2 + 35\sqrt{11}p - 198 = 0$
(a) $p < q$ (b) $p > q$
(c) $p \leq q$ (d) $p \geq q$
(e) can't be determined
14. I. $p^2 - 8\sqrt{3} + 45 = 0$ II. $q^2 - 12\sqrt{3} + 105 = 0$
(a) $p < q$ (b) $p > q$
(c) $p \leq q$ (d) $p \geq q$
(e) can't be determined
15. I. $p^2 + 2|p| + 35 = 0$ II. $q^2 + 2q + 35 = 0$
(a) $p < q$ (b) $p > q$
(c) $p \leq q$ (d) $p \geq q$
(e) can't be determined

Answers

1. (c)	2. (e)	3. (e)	4. (e)	5. (d)
6. (e)	7. (e)	8. (a)	9. (e)	10. (c)
11. (e)	12. (b)	13. (e)	14. (c)	15. (e)

Hints

1. $p = \{-19, 19\}$ and $q = \{19, 21\}$
2. $p = \{-23, 23\}$ and $q = \{-23, 23\}$
3. $p = \{12, 12\}$ and $q = \{12, 12\}$
4. $p = \{131\}$ and $q = \text{non-real roots}$
5. $p = \left\{\frac{7}{3}, \frac{27}{5}\right\}$ and $q = \left\{-\frac{27}{5}, \frac{7}{3}\right\}$
6. $p = \{-6, 6\}$ and $q = \{-6, 6\}$
7. $p = \{0, 3, 6\}$ and $q = \{0, 1\}$
8. $p = \left\{-\frac{3}{2}, -\frac{3}{7}\right\}$ and $q = \left\{-\frac{3}{8}, \frac{3}{7}\right\}$
9. $p = \{-7, 49\}$ and $q = \{5\}$
10. $p = \left\{-\frac{5}{\sqrt{7}}, \frac{1}{\sqrt{7}}\right\}$ and $q = \left\{\frac{1}{\sqrt{7}}, \frac{10}{\sqrt{11}}\right\}$
11. $p = \{\pm 5, \pm 7\}$ and $q = \{-6.9, -7.1\}$
12. $p = \left\{\frac{2\sqrt{11}}{5}, \frac{9\sqrt{11}}{5}\right\}$ and $q = \left\{-\frac{9\sqrt{11}}{5}, -\frac{2\sqrt{11}}{5}\right\}$
13. $p = \left\{-\frac{2\sqrt{11}}{5}, +\frac{9\sqrt{11}}{5}\right\}$ and $q = \left\{\frac{2\sqrt{11}}{5}, -\frac{9\sqrt{11}}{5}\right\}$
14. $p = \{3\sqrt{3}, 5\sqrt{3}\}$ and $q = \{5\sqrt{3}, 7\sqrt{3}\}$
15. $p = \{-5, 5\}$ and $q = \{-7, 5\}$

14.13 Polynomial Equations or Functions of Higher Degree

From the discussion of the previous topics and chapters we got to know that what a polynomial is and what the degree of a term or polynomial is. However, I will start the topic with some basic points.

Function	Standard Form	Degree	Terms
Zero Function	$f(x) = 0$	Undefined	Undefined
Constant Function	$f(x) = a; (a \neq 0)$	0	1
Linear Function	$f(x) = ax + b; (a \neq 0)$	1	2
Quadratic Function	$f(x) = ax^2 + bx + c; (a \neq 0)$	2	3
Cubic Function	$f(x) = ax^3 + bx^2 + cx + d; (a \neq 0)$	3	4
Bi-quadratic Function	$f(x) = ax^4 + bx^3 + cx^2 + dx + e; (a \neq 0)$	4	5
...	...	...	...
...	...	...	...

Thus the polynomial function of degree n is expressed in the standard form as shown below $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_3 x^3 + a_2 x^2 + a_1 x + a_0$ ($a_n \neq 0$)

A polynomial function of degree n can be expressed as a product of linear factors (or product of quadratic factors may be along with linear factors) as following.

$$f(x) = (x - \alpha_1)(x - \alpha_2)(x - \alpha_3) \dots (x - \alpha_{n-2})(x - \alpha_{n-1})(x - \alpha_n); \text{ where } \alpha_i (i=1, 2 \dots n) \text{ is the root of the polynomial.}$$

- The constants $a_n, a_{n-1}, \dots, a_3, a_2, a_1, a_0$ are the coefficients of the polynomial.
- The term $a_n x^n$ is the leading term and a_0 is the constant term.
- The highest power of variable x is called the 'degree' and n is a non-negative integer.
- A polynomial function of degree n has at most $(n-1)$ local extrema (or lumps).
- A polynomial function of degree n has at most n zeros (or roots or solutions).

Properties of a Polynomial Graph

- The function is continuous. Thus the graph has no discrete jumps or breaks, or no sharp corners.
- It has at most n roots. Thus the graph cuts the X -axis in at most n points. In general, the graph cuts any X -axis in at most n points.
- The function has at most $n-1$ critical points. It follows that it has at most $n-1$ local extreme points, called extrema.
- The domain is the whole real line. In particular, the graph has no vertical asymptotes.
- The asymptotes are defined and explained in the upcoming topics.
- Since you have already learnt about the low degree polynomials such as linear and quadratic functions, so in the upcoming discussion I will primarily zero in on the polynomials of higher degree, such as degree 3, 4, 5, etc.

Deriving the Common Polynomials From the Standard Polynomial of Degree n

By substituting $n=1$ in the Standard Polynomial Expression you can obtain the linear function as shown below.

$$f(x) = a_1 x + a_0 \quad (a_1 \neq 0)$$

Similarly by substituting $n=2$ in the Standard Polynomial Expression you can obtain the quadratic function as shown below.

$$f(x) = a_2 x^2 + a_1 x + a_0 \quad (a_2 \neq 0)$$

Again, by substituting $n=3$ in the Standard Polynomial Expression you can obtain the cubic function as shown below.

$$f(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0 \quad (a_3 \neq 0)$$

Once again, by substituting $n=4$ in the Standard Polynomial Expression you can obtain the biquadratic function as shown below.

$$f(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 \quad (a_4 \neq 0)$$

In this way, using the standard polynomial function you can express the polynomial of any degree.

14.14 End (or Long Term) Behaviour of a Polynomial Function

If you look at any of the following polynomial graphs you will notice two vital things :

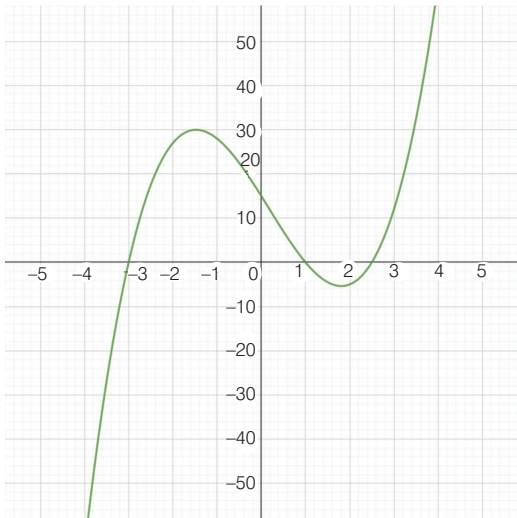
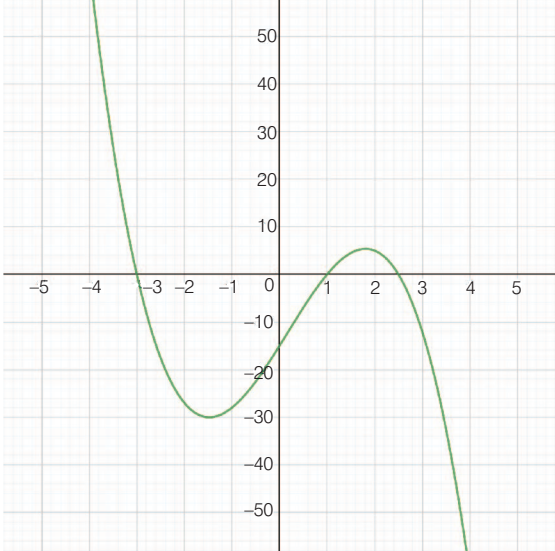
- (1) The typical graph of a polynomial function looks like a **wave** because of the roots of the polynomials. These roots are responsible for this wave-like graph. However, this wave shape ceases to exist as soon as the graph moves beyond the region of roots. Here region of the roots imply the stretch (or section) of the X -axis that falls between the smallest and the largest root. The numerically smallest root is always the left-most root and the largest root is always the right-most root along the X -axis.
- (2) As soon as the graph moves beyond the region of the roots (that means towards the extreme values of X -axis) the graph keeps on moving away from the X -axis and never returns back towards it. Thus it ends the phenomenon of wave-like graph after a certain point. And this is your area of interest at this time.

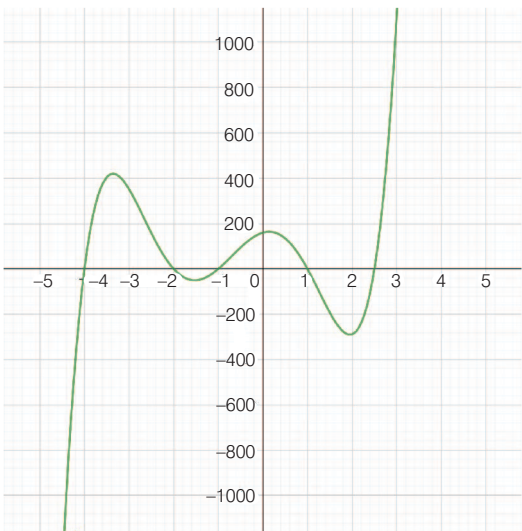
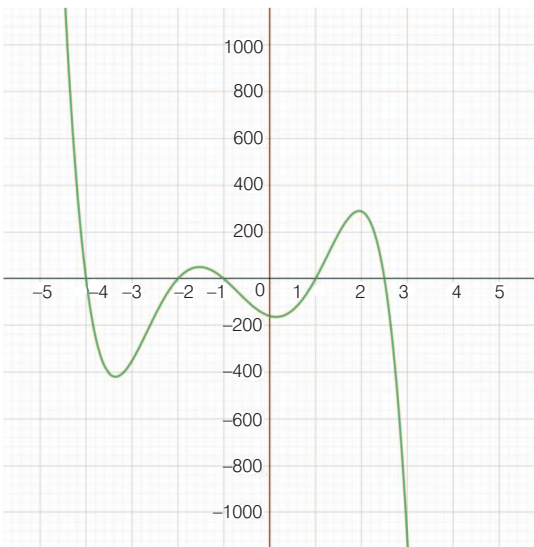
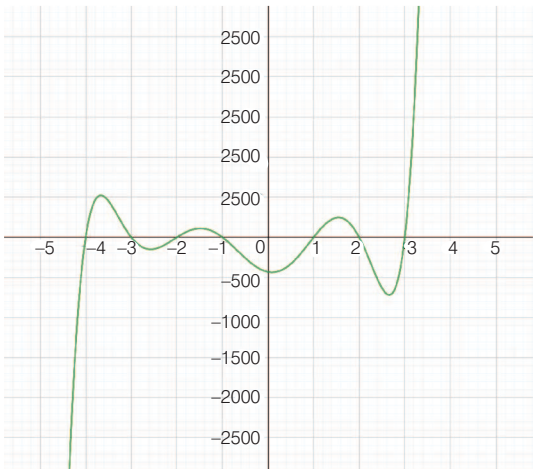
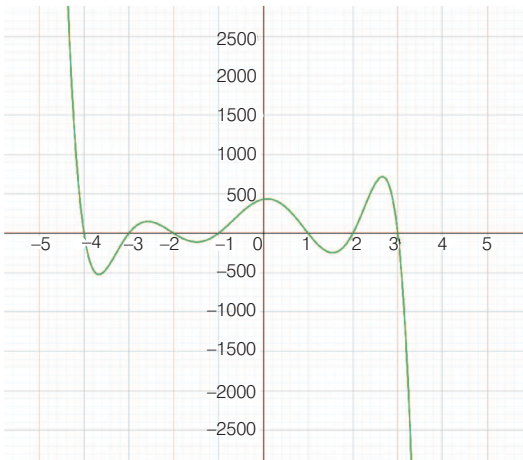
So you got it right that you are going to study that what happens or how does a graph behave when you keep on moving endlessly on the either side of the X -axis. The importance of this end behaviour of the graph is critical to sketch the complete graph (or to connect all the roots). If you know the end behaviour of a polynomial graph, it becomes pretty easy to sketch the complete graph; especially it becomes obvious to differentiate between peaks and valleys. The positive extrema are called the peaks and the negative extrema are called the valleys.

In this section you will get to know the tricks to find out the direction in which the two ends of the graph move - either both ends go up or both ends go down, or one end goes up and the other end goes down. Remember that we are here not interested in knowing that what happens between the two end roots (the smallest and the biggest root) of the polynomial.

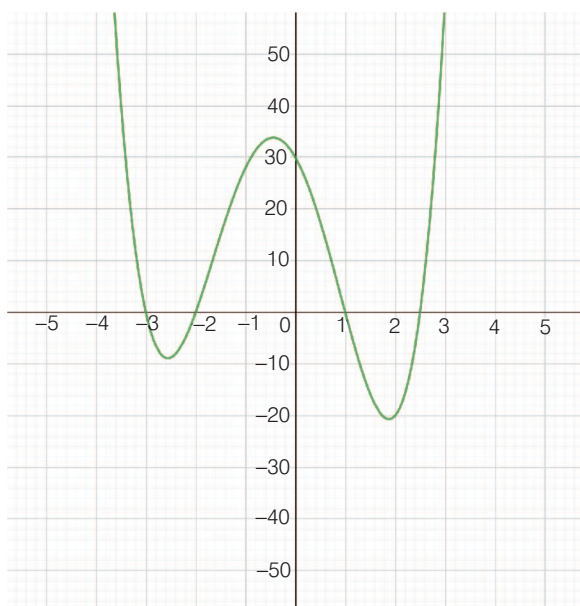
The following illustrative graphs suggest that the long term behaviour of a polynomial function basically depends on the leading term $a_n x^n$ of the polynomial.

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0.$$

n	$a_n > 0$	$a_n < 0$
Odd	 $f(x) = 2x^3 - x^2 - 16x + 5$ $\Rightarrow f(x) = 2(x+3)(x-1)\left(x - \frac{5}{2}\right)$	 $f(x) = -2x^3 + x^2 + 16x - 15$ $\Rightarrow f(x) = -2(x+3)(x-1)\left(x - \frac{5}{2}\right)$

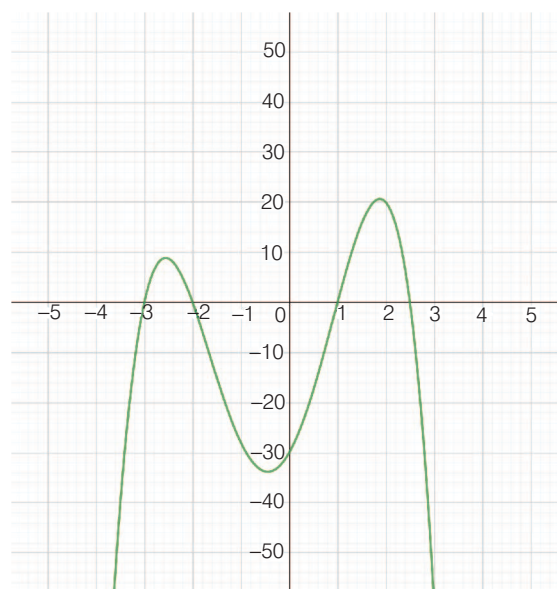
n	$a_n > 0$	$a_n < 0$
Odd	 $f(x) = 8x^5 + 28x^4 - 64x^3 - 188x^2 + 56x + 160$ $\Rightarrow f(x) = 8(x+4)(x+2)(x+1)(x-1)\left(x - \frac{5}{2}\right)$	 $f(x) = -8x^5 - 28x^4 + 64x^3 + 188x^2 - 56x - 160$ $\Rightarrow f(x) = -8(x+4)(x+2)(x+1)(x-1)\left(x - \frac{5}{2}\right)$
Odd	 $f(x) = 3x^7 + 12x^6 - 42x^5 - 168x^4 + 147x^3 + 588x^2 - 108x^2 - 422$ $\Rightarrow f(x) = 3(x+4)(x+3)(x+2)(x+1)(x-1)(x-2)(x-3)$	 $f(x) = -3x^7 - 12x^6 + 42x^5 + 168x^4 - 147x^3 - 588x^2 + 108x + 422$ $\Rightarrow f(x) = -3(x+4)(x+3)(x+2)(x+1)(x-1)(x-2)(x-3)$

Even



$$f(x) = 2x^4 + 3x^3 - 18x^2 - 17x + 30$$

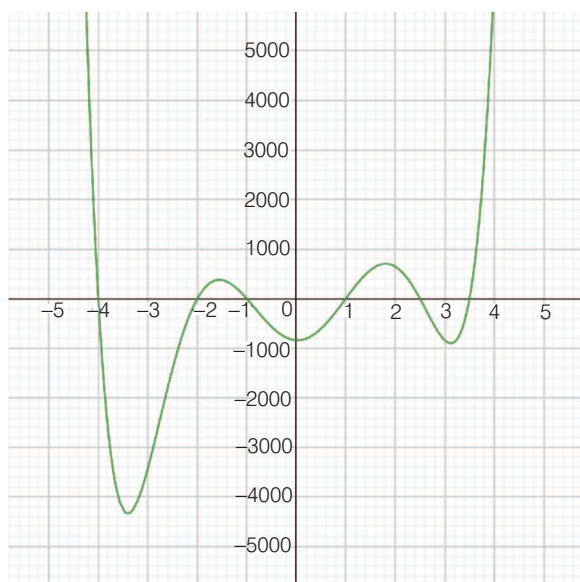
$$\Rightarrow f(x) = 2(x+3)(x+2)(x-1)\left(x - \frac{5}{2}\right)$$



$$\Rightarrow f(x) = -2x^4 - 3x^3 + 18x^2 + 17x - 30$$

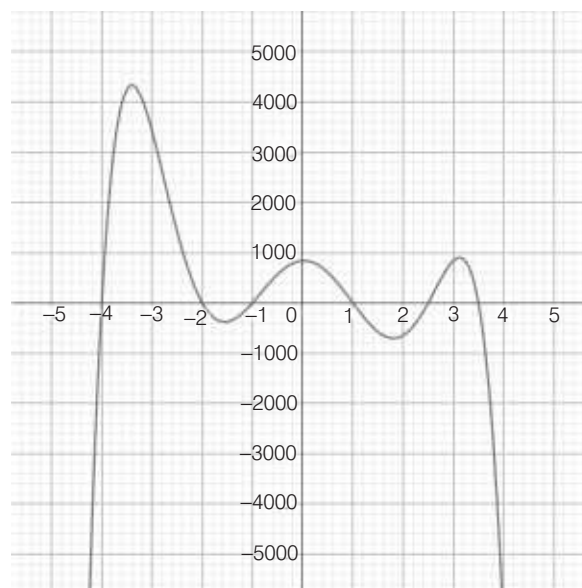
$$\Rightarrow f(x) = -2(x+3)(x+2)(x-1)\left(x - \frac{5}{2}\right)$$

Even



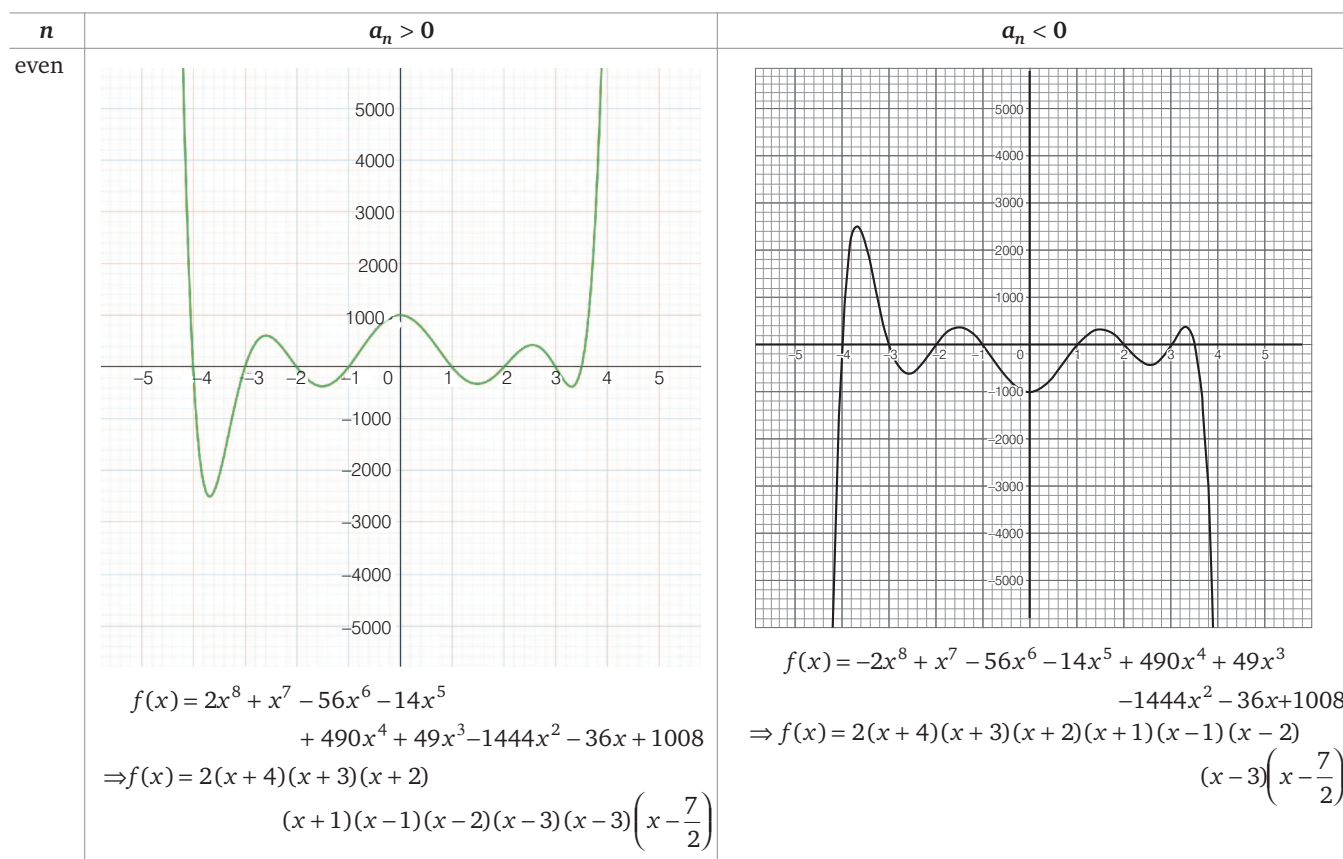
$$f(x) = 12x^6 - 243x^4 + 54x^3 + 1071x^2 - 54x - 840$$

$$\Rightarrow f(x) = 12(x+4)(x+2)(x+1)(x-1)\left(x - \frac{5}{2}\right)\left(x - \frac{7}{2}\right)$$



$$f(x) = -12x^6 + 243x^4 - 54x^3 - 1071x^2 + 54x + 840$$

$$\Rightarrow f(x) = -12(x+4)(x+2)(x+1)(x-1)\left(x - \frac{5}{2}\right)\left(x - \frac{7}{2}\right)$$

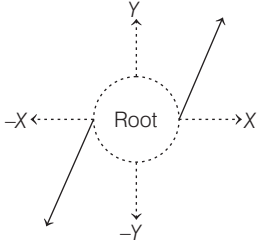
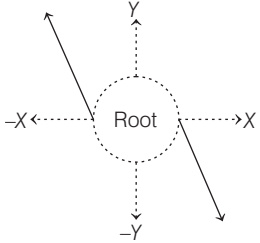


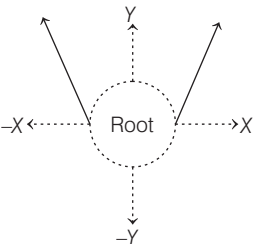
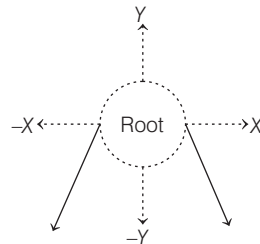
The graphical description of the end behaviour of the graph can be summarised as below, where $a_n x^n$ is the leading term of the polynomial $y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$.

x^n	$a_n > 0$	$a_n < 0$
n is ODD	As $x \rightarrow +\infty$, y increases As $x \rightarrow -\infty$, y decreases	As $x \rightarrow +\infty$, y decreases As $x \rightarrow -\infty$, y increases
n is EVEN	As $x \rightarrow +\infty$, y increases As $x \rightarrow -\infty$, y increases	As $x \rightarrow +\infty$, y decreases As $x \rightarrow -\infty$, y decreases

Sketchy Illustration

The circles in the following diagram hide all the real roots of the polynomial in order to give a clear picture of how the two ends of the graph move, when the values of x are extremely high on either side of the X -axis.

n	$a_n > 0$	$a_n < 0$
For extremely high values of x , when n is ODD , the two ends of the graph move in the OPPOSITE direction		
	Thus beyond the range of the roots when you increase x , then y will also increase and when you decrease x , then y will also decrease.	Thus beyond the range of the roots when you increase x , then y will decrease and when you decrease x , then y will increase.

n	$a_n > 0$	$a_n < 0$
For extremely high values of x , when n is EVEN , the two ends of the graph move in the SAME direction.		
	Thus beyond the range of the roots when you either increase x , or decrease x , the value of y is always increase.	Thus beyond the range of the roots when you either increase x , or decreases x , the value of y will always decrease.

Exp. 1) Determine the end behaviour of

$$f(x) = 10x + 2x^3 - 7x^2 + 24$$

Solution The end behaviour of any polynomial graph depends on the leading term which is supposed to be the term with the highest power, so for the purpose of convenience you may rearrange the terms in decreasing order as $f(x) = 2x^3 - 7x^2 + 10x + 24$.

Now since the power of leading term $2x^3$ is 3, i.e. odd, therefore the two ends of the graph will go in opposite directions. Further the coefficient of x^3 is 2 (i.e., positive) therefore when x increases, y also increases and likewise when x decreases, y also decreases, provided x is considerably far from the roots.

Exp. 2) For the extremely high values of x , what will be the tendency of the graph of $-3x^4 - 2x - 11$?

Solution The end behaviour of any polynomial graph depends on the leading term, which is supposed to be the term with the highest power. Now since the power of leading term $-3x^4$ is 4 (i.e., even) therefore the two ends of the graph will move in the same direction. Further the coefficient of x^4 is -3 (i.e., negative) therefore either when x increases or it decreases, y always decreases, provided x is considerably far from the roots.

Exp. 3) When x approaches $-\infty$, then what will be the tendency of the graph of $5x^8 + 3x^7 - 18x^4 - 7x - 30$?

Solution The end behaviour of any polynomial graph depends on the leading term, which is supposed to be the term with the highest power. So for the purpose of convenience you may rearrange the terms in decreasing order as

$$f(x) = 5x^8 + 3x^7 - 18x^4 - 7x - 30$$

Now since the power of leading term $5x^8$ is 8 (i.e., even) therefore the two ends of the graph will go in the same direction. Further the coefficient of x^8 is 5 (i.e., positive) therefore when x decreases, y also decreases, provided x tends to $-\infty$.

Exp. 4) Which one of the following is true about the polynomial function $y = -7x^5 + 3x^3 - 18x^2 - 7x + 23$?

- (a) $y \rightarrow +\infty$, when $x \rightarrow -\infty$ (b) $y \rightarrow -\infty$, when $x \rightarrow -\infty$
 (c) $y \rightarrow +\infty$, when $x \rightarrow +\infty$ (d) none of the above

Solution The end behaviour of any polynomial graph depends on the leading term, which is supposed to be the term with the highest power.

Now since the power of leading term $-7x^5$ is 5, i.e. odd, therefore the two ends of the graph will go in opposite directions. Further the coefficient of x^5 is -7 (i.e., negative) therefore when x increases, y decreases and likewise when x decreases, y increases, provided x is considerably far from the roots. Hence, choice (a) is true.

14.15 Solutions (or Roots) of a Polynomial Equation

The values of the variable satisfying the given polynomial equation are called its roots or solutions or zeros. That is α is a root of the polynomial equation $f(x) = 0$ if $f(\alpha) = 0$.

There are various methods to find out the roots of the polynomial equations, however in your competitive exams you are not expected to know these sophisticated methods at all.

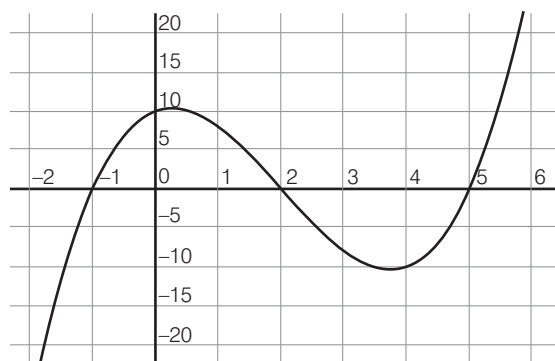
What all you are expected to do is use your reasoning, logic and common sense to figure out the correct answer regarding polynomial expressions.

Examiners are not interested in knowing the higher mathematics much into depth; rather they try to test your logical prowess in this kind of problems. That's why I'm not going to unnecessarily discuss the methods of finding the roots.

Exp. 1) In the following diagram you can see the graph of a cubic polynomial $f(x) = x^3 - 6x^2 + 3x + 10$. Since the degree of this polynomial is 3, therefore it has three roots namely $-1, 2, 5$.

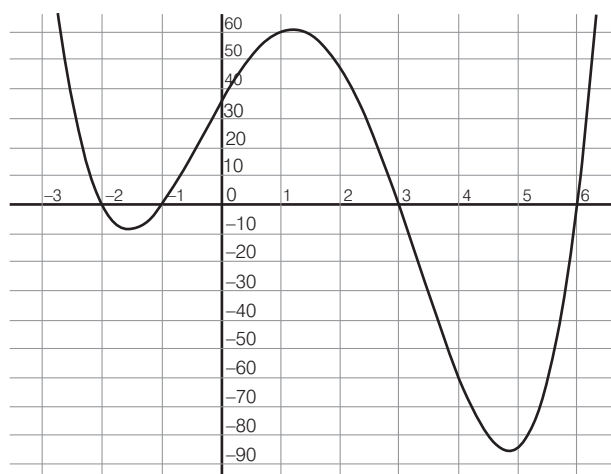
This shows that if you substitute $x = -1$ or 2 or 5 in the given function $f(x) = x^3 - 6x^2 + 3x + 10$, then you will get $f(x) = 0$.

And there are two local extrema too, one above the X-axis and one below the X-axis.



Exp. 2) In the following diagram you can see the graph of a biquadratic polynomial $f(x) = x^4 - 6x^3 - 7x^2 + 36x + 36$. Since the degree of this polynomial is 4, therefore it has four roots namely $-2, -1, 3, 6$.

This shows that if you substitute $x = -2$ or -1 or 3 or 6 in the given function $f(x) = x^4 - 6x^3 - 7x^2 + 36x + 36$, then you will get $f(x) = 0$. And there are three local extrema too, one above the X-axis and two below the X-axis. That means there are one local maximum and two local minima.



Basic Properties of Roots

1. An equation of degree n has exactly n roots; may be all real or all imaginary or the combination of both.
2. Imaginary roots (also known as complex roots) occur in conjugate pairs. That is if $a + ib$ is a root of the polynomial, then $a - ib$ is also a root of that polynomial.
3. Irrational roots always occur in pairs. That means if $a + \sqrt{b}$ is a root of a polynomial equation then $a - \sqrt{b}$ is also a root of that polynomial.
4. Total number of roots $n = \text{Real roots } (R) + \text{Complex roots } (2K)$; where $K = 0, 1, 2, 3, \dots \Rightarrow$ Number of real roots $= n - 2K$, since complex roots occur in pairs.
5. For an odd degree equation (i.e., if n is odd) there must be at least one real root. Since n is odd and $2K$ is even.

6. For an even degree equation (i.e., if n is even) whose constant term is negative and $a_n > 0$ (i.e., the coefficient of highest degree term is positive) there must be at least two real roots, one positive and one negative.
7. A real root must intersect or touch the X-axis.
8. The number of intersections or touching points will always be less than or equal to the number of real roots. If it is less than the real roots, it means there is a multiplicity of the roots.

Exp. 1) If two of the roots of the polynomial equation $ax^4 + bx^3 + cx^2 + dx + e = 0$, $a \neq 0$, are $7 + 5\sqrt{2}$ and $5 - 7\sqrt{2}$, then what will be the sum of all the roots of this polynomial?

Solution Since this is a polynomial of degree four, so it will have maximum four roots. Further since there are two distinct irrational roots, it implies that there must be the conjugates of these two roots. The conjugate of these two roots will be $7 - 5\sqrt{2}$ and $5 + 7\sqrt{2}$ respectively. Therefore the sum of all the roots is 24.

Exp. 2) A polynomial equation of degree 5 has complex roots and irrational roots. How many real roots does this equation have?

Solution Since complex roots and irrational roots occur in conjugate pairs, so if there is minimum one pair of each, then there will be two complex roots and two irrational roots. Since there is only one root unknown, so it must be real and rational root. However since irrational roots are also the real roots. So there are total 3 real roots and two non-real roots.

Exp. 3) What cannot be the exact number of real roots in a polynomial of degree 9?

- (a) 5 (b) 6 (c) 7 (d) 9

Solution Since the total number of roots are odd and the non-real roots (i.e., complex roots) occur in pairs (i.e., 0, 2, 4, 6...) therefore the number of real roots will be either 9 or 7 or 5 or 3 or 1, depending on the number of pairs of complex roots.

Refer the following table.

Total Number of Roots	Number of Complex Roots	Number of Real Roots
9	0	9
9	2	7
9	4	5
9	6	3
9	8	1

Thus, we can say that there cannot be exactly 6 real roots.

Multiplicity of a Root of a Polynomial Function

A normal polynomial equation in which all the roots are real and distinct can be expressed in the form of linear factors as

$$(x - \alpha_1)(x - \alpha_2)(x - \alpha_3) \dots (x - \alpha_n).$$

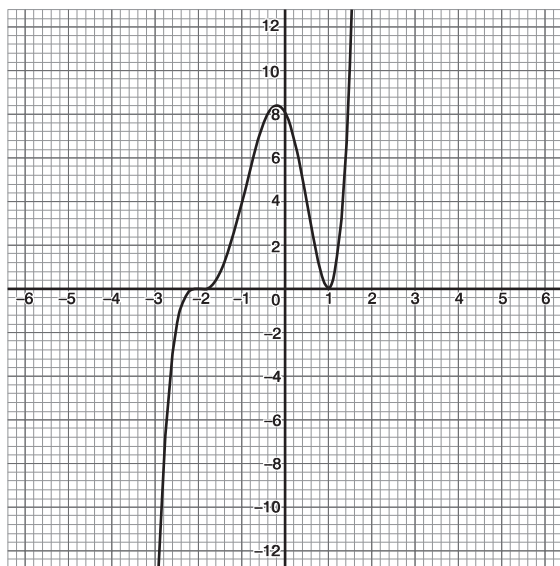
However when certain roots are same, *i.e.*, roots are repeated then we call it the multiplicity of the root. For example

$$(x - \alpha)(x - \alpha)(x - \alpha)(x - \alpha)(x - \beta)(x - \beta)(x - \beta) \dots (x - \gamma_1) \dots (x - \gamma_k) \\ = (x - \alpha)^4 (x - \beta)^3 (x - \gamma_1) \dots (x - \gamma_k)$$

In the above example α occurs 4 times, so α is a root of multiplicity 4 and β occurs 3 times so β is a root of multiplicity 3.

Definition If f is a polynomial function and $(x - \alpha)^m$ is a factor of f but $(x - \alpha)^{m+1}$ is not, then α is a root of multiplicity m of f .

Exp.) The following graph shows the x -intercepts of $(x + 2)^3 (x - 1)^2$. Even though there are five real roots, but due to multiplicity of roots you see that there are only two points on the X -axis where the graph intersects or touches the X -axis.



Odd Multiplicity If a polynomial function f has a real root α of odd multiplicity, then the graph of f crosses the X -axis at $(\alpha, 0)$ and the value of f changes sign at $x = \alpha$.

In the polynomial function $f(x) = (x + 2)^3 (x - 1)^2$, $x = -2$ is a root of multiplicity 3, which is a root of odd multiplicity. Therefore at $x = -2$ the function (or graph) crosses the X -axis and changes the sign.

Even Multiplicity If a polynomial function f has a real root α of even multiplicity, then the graph of f does not cross the X -axis at $(\alpha, 0)$ and the value of f does not change the sign at $x = \alpha$. In the polynomial function $f(x) = (x + 2)^3 (x - 1)^2$, $x = 1$ is a root of multiplicity 2, which is a root of even multiplicity. Therefore at $x = 1$ the function (or graph) does not cross the X -axis; however the graph kisses the X -axis and bounces back at $x = 1$.

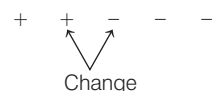
Sign Rules of a Polynomial Equation

For a polynomial of any degree written in the decreasing order – highest degree term first and lowest degree term in the last – you can determine the maximum possible positive and negative real roots by the following rules.

1. The maximum number of POSITIVE real roots of a polynomial equation $f(x) = 0$ is the same (or less than by an even number) as the number of changes of signs – from positive to negative and/or negative to positive – in the coefficients of $f(x)$. If the maximum number of changes in the signs is p , the maximum number of positive real roots of this polynomial equation would be $p - 2k \geq 0$, such that $k = 0, 1, 2, 3, \dots$
2. The maximum number of NEGATIVE real roots of a polynomial equation $f(x) = 0$ is the same (or less than by an even number) as the number of changes of signs – from positive to negative and/or negative to positive – in the coefficients of $f(-x)$. If the maximum number of changes in the signs is n , the maximum number of positive real roots of this polynomial equation would be $n - 2k \geq 0$, such that $k = 0, 1, 2, 3, \dots$

Exp. 1) Find the possible number of positive and negative real roots of $x^4 + 7x^3 - 4x^2 - x - 7 = 0$.

Solution For positive real roots, you have to determine the number of times the change happens in the sign of coefficients of the given polynomial when written in the decreasing order of degree of its terms. So write down the signs of coefficients as shown below and mark the change in sign.



Since there is only one change in sign, therefore there will be maximum one positive real root. Now if you try to replace the real roots with complex roots then you have to replace them in pairs. It means one positive real root cannot be replaced by 2 complex roots. So finally there will have to be exactly one real root. For negative real roots, you have to determine the number of times the change occurs in sign of coefficients of the polynomial, where x is replaced by $-x$ and the terms are written in the decreasing order of degree.

So first you have to replace every x with $-x$ as shown below.

$$\begin{aligned} & (-x)^4 + 7(-x)^3 - 4(-x)^2 - (-x) - 7 = 0 \\ \Rightarrow & x^4 - 7x^3 - 4x^2 + x - 7 = 0 \end{aligned}$$

Now write down the signs of coefficients as shown below and mark the change in sign.



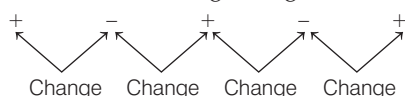
Since there are three changes in the sign, therefore there will be maximum three negative real roots. So finally there will be either 3 or 1 negative real root.

Total Number of Roots	Number of Complex Roots	Number of Positive Roots	Number of Negative Roots
4	0	1	3
4	2	1	1

NOTE Please take cognizance of the fact that if you consider 4 complex roots then you have to have 1 negative root, which is impossible.

Exp. 2) Find the possible number of positive and negative real roots of $2x^4 - x^3 + 4x^2 - 5x + 3 = 0$.

Solution For positive real roots, you have to determine the number of times the change happens in the sign of coefficients of the given polynomial when written in the decreasing order of degree of its terms. So write down the signs of coefficients as shown below and mark the change in sign.



Since there are only 4 changes in sign, therefore there will be maximum four positive roots. Thus there could be 4, or 2 or 0 positive real roots.

For negative real roots, you have to determine the number of times the change occurs in sign of coefficients of the polynomial, where x is replaced by $-x$ and the terms are written in the decreasing order of degree.

So first you have to replace every x with $-x$ as shown below.

$$2(-x)^4 - (-x)^3 + 4(-x)^2 - 5(-x) + 3 = 0$$

$$\Rightarrow 2x^4 + x^3 + 4x^2 + 5x + 3 = 0$$

Now write down the signs of coefficients as shown below and mark the change in sign.

+ + + + +

Since there is no change in sign, so there won't be any negative real root.

Total Number of Roots	Number of Complex Roots	Number of Positive Roots	Number of Negative Roots
4	0	4	0
4	2	2	0
4	4	0	0

Exp. 3) Find the possible number of positive and negative real roots of $x^5 + x^4 + 4x^3 + 32x^2 + x + 1 = 0$.

Solution For positive real roots, you have to determine the number of times the change happens in the sign of coefficients of the given polynomial when written in the decreasing order of degree of its terms. So write down the signs of coefficients as shown below and mark the change in sign.

+ + + + +

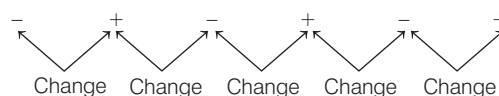
Since there is no change in sign, so there won't be any positive real root. For negative real roots, you have to determine the number of times the change occurs in sign of coefficients of the polynomial, where x is replaced by $-x$ and the terms are written in the decreasing order of degree.

So first you have to replace every x with $-x$ as shown below.

$$(-x)^5 + (-x)^4 + 4(-x)^3 + 3(-x)^2 + (-x) + 1 = 0$$

$$\Rightarrow -x^5 + x^4 - 4x^3 + 3x^2 - x + 1 = 0.$$

Now write down the signs of coefficients as shown below and mark the change in sign.

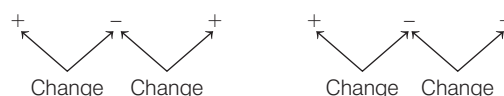


Since there are five changes in the sign, therefore there will be maximum five negative real roots. So finally there will be either 5 or 3 or 1 negative real root.

Total Number of Roots	Number of Complex Roots	Number of Positive Real Roots	Number of Negative Real Roots
5	0	0	5
5	2	0	3
5	4	0	1

Exp. 4) Find the possible number of positive and negative real roots of $x^5 - x^4 + 3x^3 + 9x^2 - x + 5 = 0$.

Solution For positive real roots, you have to determine the number of times the change happens in the sign of coefficients of the given polynomial when written in the decreasing order of degree of its terms. So write down the signs of coefficients as shown below and mark the change in sign.



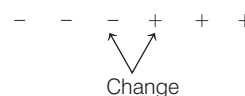
Since there are four changes in sign, so there will be at most four positive real roots. Thus finally there will be either 4 or 2 or 0 positive real root. For negative real roots, you have to determine the number of times the change occurs in sign of coefficients of the polynomial, where x is replaced by $-x$ and the terms are written in the decreasing order of degree.

So first you have to replace every x with $-x$ as shown below.

$$(-x)^5 - (-x)^4 + 3(-x)^3 + 9(-x)^2 - (-x) + 5 = 0$$

$$\Rightarrow -x^5 - x^4 - 3x^3 + 9x^2 + x + 5 = 0.$$

Now write down the signs of coefficients as shown below and mark the change in sign.



Since there is only one change in the sign, therefore there will be maximum one negative real root.

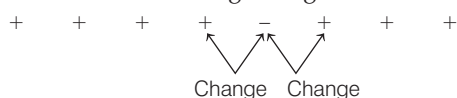
So finally there will be exactly 1 negative real root.

Total Number of Roots	Number of Complex Roots	Number of Positive Real Roots	Number of Negative Real Roots
5	0	4	1
5	2	2	1
5	4	0	1

Exp. 5) Find the possible number of positive and negative real roots of

$$4x^7 + 3x^6 + x^5 + 2x^4 - x^3 + 9x^2 + x + 1 = 0$$

Solution For positive real roots, you have to determine the number of times the change happens in the sign of coefficients of the given polynomial when written in the decreasing order of degree of its terms. So write down the signs of coefficients as shown below and mark the change in sign.



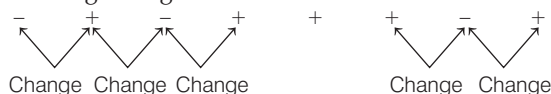
Since there are two changes in sign, so there will be at most two positive real roots. Thus finally there will be either 2 or 0 positive real root. For negative real roots, you have to determine the number of times the change occurs in sign of coefficients of the polynomial, where x is replaced by $-x$ and the terms are written in the decreasing order of degree.

So first you have to replace every x with $-x$ as shown below.

$$4(-x)^7 + 3(-x)^6 + (-x)^5 + 2(-x)^4 - (-x)^3 + 9(-x)^2 + (-x) + 1 = 0$$

$$\Rightarrow -4x^7 + 3x^6 - x^5 + 2x^4 + x^3 + 9x^2 - x + 1 = 0$$

Now write down the signs of coefficients as shown below and mark the change in sign.



Since there are five changes in the sign, therefore there will be maximum five negative real roots. So finally there will be either 5 or 3 or 1 negative real root.

Total Number of Roots	Number of Complex Roots	Number of Positive Real Roots	Number of Negative Real Roots
7	0	2	5
7	2	0	5
7	2	2	3
7	4	0	3
7	4	2	1
7	6	0	1

14.16 Polynomial Inequality

In a polynomial equation you have to find the values of x , which make $f(x)$ equal to zero. However, in a polynomial inequality you have to find the values of x which make $f(x)$ less than or greater than zero.

In a sense you have to find out the conditions when the graph (or function) will be positive, negative, non-negative, or non-positive. To get the valid values of x , first of all determine the real roots and mark them on the number line as per convention; means from left to right in increasing order.

Then you will get $n + 1$ intervals, if there are n real roots. In the next step what you have to do is to pick any convenient number in each interval and substitute it for x in the given function and see whether this test gives a positive number or negative number.

Then you can decide which intervals are to be considered for your answer and it depends on your given inequation.

$f(x) > 0$	All positive values of x except the roots
$f(x) \geq 0$	All positive values of x including the roots
$f(x) < 0$	All negative values of x except the roots
$f(x) \leq 0$	All negative values of x including the roots

How to Express Intervals

When roots are to be included in the desired interval

$$[\alpha_1, \alpha_2] \text{ or } \alpha_1 \leq x \leq \alpha_2$$

When roots are not to be included in the desired interval:

$$(\alpha_1, \alpha_2) \text{ or } \alpha_1 < x < \alpha_2$$

When only root α_1 is to be included in the desired interval

$$[\alpha_1, \alpha_2) \text{ or } \alpha_1 \leq x < \alpha_2$$

When only root α_2 is to be included in the desired interval:

$$(\alpha_1, \alpha_2] \text{ or } \alpha_1 < x \leq \alpha_2$$

Infinity is not an exact point or position, so it's always expressed as $(-\infty, \infty)$ or $-\infty < x < \infty$.

How to Express Various Intervals and Individual Values Jointly

Let us assume that $\alpha_1 < \alpha_2 < \alpha_3 < \alpha_4 < \dots < \alpha_n$.

$$[\alpha_1, \alpha_2] \cup [\alpha_2, \alpha_3] = [\alpha_1, \alpha_3] = \alpha_1 \leq x \leq \alpha_3$$

$$(\alpha_1, \alpha_2) \cup (\alpha_2, \alpha_3) = (\alpha_1, \alpha_3) - \{\alpha_2\}$$

$$= \alpha_1 < x < \alpha_3 \text{ except } \alpha_2$$

$$(\alpha_1, \alpha_2) \cup \{\alpha_2\} \cup (\alpha_2, \alpha_3) = (\alpha_1, \alpha_3) = \alpha_1 < x < \alpha_3$$

$$[\alpha_1, \alpha_2] \cup [\alpha_2, \alpha_3] = (\alpha_1, \alpha_3) = \alpha_1 < x < \alpha_3$$

$$(\alpha_1, \alpha_2) \cup (\alpha_3, \alpha_4) = (\alpha_1, \alpha_4) - [\alpha_2, \alpha_3]$$

$$= \alpha_1 < x \leq \alpha_4, \text{ except } \alpha_2 \leq x \leq \alpha_3$$

$$[\alpha_1, \alpha_2] \cup [\alpha_3, \alpha_4] = [\alpha_1, \alpha_4] - (\alpha_2, \alpha_3)$$

$$= \alpha_1 \leq x \leq \alpha_4, \text{ except } \alpha_2 < x < \alpha_3$$

$$(-\infty, \alpha_1) \cup (\alpha_2, \infty) = (-\infty, \infty) - [\alpha_1, \alpha_2]$$

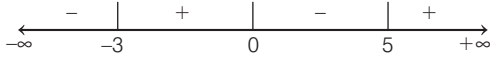
$$(-\infty, \alpha_1] \cup [\alpha_2, \infty) = (-\infty, \infty) - (\alpha_1, \alpha_2)$$

$$(-\infty, \alpha_1] \cup [\alpha_2, \infty) = [-\infty, \infty] - (\alpha_1, \alpha_2)$$

$$\{\alpha_1\} \cup \{\alpha_2\} \cup \{\alpha_3\} \cup \dots \cup \{\alpha_n\} = \{\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\}$$

Exp. 1) Find the values that satisfy the polynomial inequation $x(x+3)(x-5) > 0$.

Solution The roots of this inequation are $-3, 0$ and 5 . So, now you have to mark these roots on the number line. Since there are three real roots, so there will be four intervals on the number line as shown below.



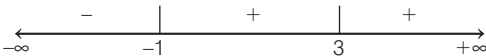
The four intervals are $(-\infty, -3)$, $(-3, 0)$, $(0, 5)$ and $(5, \infty)$. Now check the first interval. Consider any value, say -5 which lies in the first interval $(-\infty, -3)$, you will get $x(x+3)(x-5) = -5(-2)(-9) = -90 < 0$. So the first interval gives negative value.

Similarly you can consider $x = -1$ to test the second interval, since -1 lies in the second interval $(-3, 0)$. This will give you $x(x+3)(x-5) = (-1)(2)(-5) = 10 > 0$. So the second interval is positive. Again the third interval test yields negative values and the fourth interval test yields the positive values.

Therefore you see that the second interval and fourth interval satisfy the given inequality. Thus the required answer is $(-3, 0) \cup (5, \infty)$ or $-3 < x < 0$ and $5 < x < \infty$.

Exp. 2) Find the values that satisfy the polynomial inequation $(x+1)(x-3)^2 > 0$.

Solution The roots of this inequation are -1 and 3 . So now you have to mark these roots on the number line. Since there are only two distinct real roots, so there will be exactly three intervals on the number line as shown below.



The three intervals are $(-\infty, -1)$, $(-1, 3)$ and $(3, \infty)$. Now to check the first interval you may consider any value between that interval, say -2 , then you will get $(x+1)(x-3)^2 = (-1)(25) = -25 < 0$. So the first interval $(-\infty, -1)$ is negative.

Similarly you can consider any value from the second interval, say 0 , then you will get $(x+1)(x-3)^2 = (1)(9) = 9 > 0$. It means the second interval $(-1, 3)$ is positive.

Finally you have to test the third and last interval by considering any value from that interval, say 4 , then you will get $(x+1)(x-3)^2 = (5)(1) = 5 > 0$. Thus the third interval is also positive.

Since you are looking for positive values of the function, therefore only second and third intervals satisfy the inequation. Thus the required answer is $(-1, 3) \cup (3, \infty)$.

NOTE At $x = 3$, the inequation is invalid. That's why you can't include 3 in this solution set.

Exp. 3) Find the values that satisfy the polynomial inequation $(x^2+7)(2x^2+1) > 0$.

Solution The given inequation is $(x^2+7)(2x^2+1) > 0$. Since $(x^2+7) > 0$ for all real numbers. Similarly $(2x^2+1) > 0$ for all real numbers. Therefore $(x^2+7)(2x^2+1) > 0$ for all real numbers. Thus the desired values of x are $(-\infty, \infty)$.

Exp. 4) Find the values that satisfy the polynomial inequation $(x^2+7)(2x^2+1) < 0$.

Solution Since $(x^2+7) > 0$ for all real numbers, and, similarly $(2x^2+1) > 0$ for all real numbers, therefore the product of two positive real numbers can never be negative. Therefore there is no solution for this inequation or you can say the solution set is empty. Graphically speaking the graph of this function never exists below the X-axis; even it does not touch or cross the X-axis at all.

Exp. 5) Find the values that satisfy the polynomial inequation $(x^2-3x+3)(2x+5)^2 \geq 0$.

Solution The given inequation is $(x^2-3x+3)(2x+5)^2 \geq 0$. If you can assume for a while that it's an equation then there will be total 4 roots, out of them only one

root $\left(x = \frac{-5}{2}\right)$ is a real root of multiplicity 2. The even

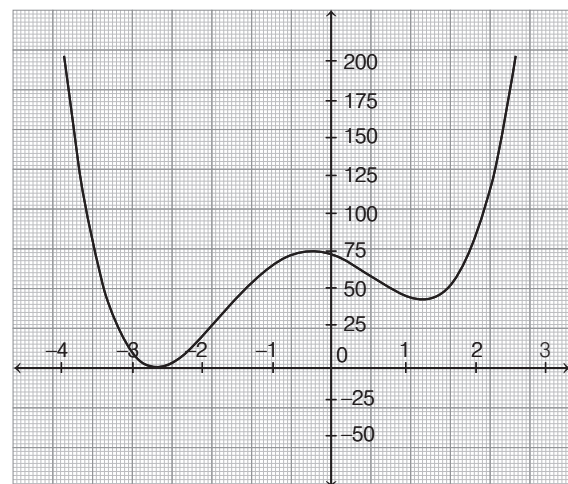
multiplicity is an indicator of the fact that the graph at $x = \frac{-5}{2}$

will not intersect the X-axis, instead it will bounce back.

Further, if you simplify the expression then the leading term will be $4x^4$. The even power of x suggests that both the ends of the graph will move in the same direction, provided x is very-very large.

Further the coefficient of x^4 is 4, which is positive. That means when x approaches ∞ , y also approaches ∞ . Also, when x approach $-\infty$, y again approaches $+\infty$.

That means whole graph never goes below the X-axis. Therefore, for every x , such that $-\infty \leq x \leq \infty$ you will have $(x^2-3x+3)(2x+5)^2 \geq 0$. Or other way round, the solution of $(x^2-3x+3)(2x+5)^2 \geq 0$ is $-\infty \leq x \leq \infty$.



Alternatively You see that $(2x+5)^2 = 0$ for $x = \frac{-5}{2}$ and $(2x+5)^2 \geq 0$ for all real numbers. And (x^2-3x+3) produces two complex roots. But the complex roots never change the sign, since they never intersect the X-axis.

Thus you can conclude that the graph of the product $(x^2 - 3x + 3)(2x + 5)^2$ never goes below the X-axis. Thus, $(x^2 - 3x + 3)(2x + 5)^2 \geq 0$, for all real numbers. That means $-\infty \leq x \leq \infty$.

Exp. 6) Find the value that satisfy the polynomial inequation $(x^2 - 3x + 3)(2x + 5)^2 > 0$.

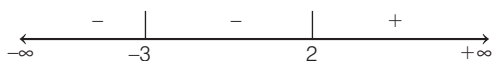
Solution Since you know that $(x^2 - 3x + 3)(2x + 5)^2 \geq 0$ for every $x \in (-\infty, \infty)$ and $(x^2 - 3x + 3)(2x + 5)^2 = 0$ at $x = \frac{-5}{2}$.

Therefore $(x^2 - 3x + 3)(2x + 5)^2 > 0$

when $x \in (-\infty, \infty) - \left\{ \frac{-5}{2} \right\}$

Exp. 7) Find the value that satisfy the polynomial inequation $2(x - 2)^3(x + 3)^2 < 0$.

Solution The roots of this inequation are -3 and 2 . So now you have to mark these roots on the number line. Since there are two roots, so there will be three intervals on the number line as shown below.



The three intervals are $(-\infty, -3)$, $(-3, 2)$, $(2, \infty)$. Now to check the first interval you may consider any value between that interval, say -4 , then you will get $2(x - 2)^3(x + 3)^2 = 2(-216)(1) = -432 < 0$. So the first interval $(-\infty, -3)$ is negative. Similarly you can consider any value from the second interval, say 0 , then you will get $2(x - 2)^3(x + 3)^2 = 2(-8)(9) = -144 < 0$. It means the second interval $(-3, 2)$ is also negative.

Finally you have to test the third and last interval by considering any value from that interval, say 4 , then you will get $2(x - 2)^3(x + 3)^2 = 2(8)(49) = 784$. Thus the third interval is positive.

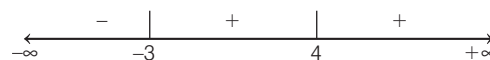
Since you are looking for strictly negative values of the function, therefore only first and second intervals satisfy the inequation. Thus the required answer is $(-\infty, -3) \cup (-3, 2)$, which you can express as $(-\infty, 2) - \{-3\}$

Alternatively Here 2 is the root of multiplicity 3 (i.e., odd multiplicity) so the graph will intercept at $x = 2$ that will lead to change in sign of the graph (or function). Again, -3 is the root of multiplicity 2 (i.e. even multiplicity) so the graph will bounce back at $x = -3$ that will not lead to any change in sign of the graph.

The leading term of this function is $2x^5$, which suggests that the one end of the graph will be below X-axis and one end will be above the X-axis. Further the coefficient of the leading term is positive, so the graph will be positive when x is positive and greater than 2 . Similarly the graph will be negative when x is less than 2 , except when x is -3 . Thus it can be concluded that the graph will be negative for $x = (-\infty, 2) - \{-3\}$

Exp. 8) Find the values that satisfy the polynomial inequation $(x + 3)(x^2 + 1)(x - 4)^2 < 0$.

Solution The irreducible quadratic factor $(x^2 + 1)$ gives complex conjugate roots. The only two distinct real roots are -3 and 4 . So now you have to mark these two real roots on the number line. Since there are two roots, so there will be three intervals on the number line as shown below.



The three intervals are $(-\infty, -3)$, $(-3, 4)$, $(4, \infty)$. Now to check the first interval you may consider any value between that interval, say -4 , then you will get $(x + 3)(x^2 + 1)(x - 4)^2 = (-1)(17)(64) = -1088 < 0$. So the first interval $(-\infty, -3)$ is negative. Similarly you can consider any value from the second interval, say 0 , then you will get $(x + 3)(x^2 + 1)(x - 4)^2 = (3)(1)(16) = 48 > 0$. It means the second interval $(-3, 4)$ is positive.

Finally you have to test the third and last interval by considering any value from that interval, say 5 , then you will get $(x + 3)(x^2 + 1)(x - 4)^2 = (8)(26)(1) = 208 > 0$. Thus the third interval is also positive.

Since you are looking for strictly negative values of the function, therefore only first interval satisfies the inequation. Thus the required answer is $(-\infty, -3)$.

Alternatively At $x = 4$, due to even multiplicity of the root the graph touches the x -axis and bounces back retaining the sign. At $x = -3$, due to odd multiplicity the graph intersects the X-axis and changes the sign. Since the leading term, after simplification into standard polynomial form, is x^5 , which implies that if x increases y also increases; and when x decreases y also decreases. Thus you can conclude that the graph will be negative when $x < -3$ and graph will be non-negative when $x > -3$. So the answer is $x < -3$.

Exp. 9) Find the values that satisfy the following polynomial inequation $(x + 5)^2(x + 1)(x - 1) \leq 0$.

Solution This function gives three real distinct roots -5 , -1 and 1 . So now you have to mark these roots on the number line. Since there are three roots, so there will be four intervals on the number line as shown below.



The four intervals are $(-\infty, -5)$, $(-5, -1)$, $(-1, 1)$, $(1, \infty)$.

Now check the first interval. Consider any value, say -6 which lies in the first interval $(-\infty, -5)$, you will get

$$(x + 5)^2(x + 1)(x - 1) = (1)(-5)(-7) = 35 > 0$$

So the first interval gives positive value. Similarly you can consider $x = -2$ to test the second interval, since -2 lies in the second interval $(-5, -1)$.

This will give you $(x + 5)^2(x + 1)(x - 1) = (9)(-1)(-3) = 27 > 0$

So the second interval is also positive.

Again you can consider $x=0$ to test the third interval, since 0 lies in the third interval $(-1, 1)$. This will give you $(x+5)^2(x+1)(x-1) = -25 < 0$.

So the third interval gives negative value. Finally you can consider $x=2$ to test the fourth interval. since 2 lies in the fourth interval $(1, \infty)$.

It will give you $(x+5)^2(x+1)(x-1) = (49)(3)(1) = 147$.

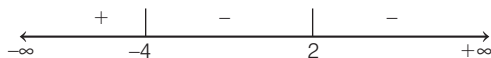
So the fourth interval is again positive. Therefore you see that the third interval and all the roots satisfy the given inequality. Thus the required answer is $\{-5\} \cup [-1, 1]$ or $x = -5$ and $-1 \leq x \leq 1$.

Alternatively At $x = -5$, due to even multiplicity of the root the graph touches the x-axis and bounces back retaining the sign. At $x = -1$, due to odd multiplicity the graph intersects the X-axis and changes the sign. Similarly at $x = 1$, due to odd multiplicity the graph intersects the X-axis and changes the sign once again.

Since the leading term, after simplification into standard polynomial form, is x^4 , which implies that both the ends will move in the same direction when you move on the X-axis too far in either direction. Further if x increases y also increases; but when x decreases y increases. Thus you can conclude that the graph will be non-positive when $x = -5$ and $-1 \leq x \leq 1$ and graph will be positive when $x < -5$ and $-5 < x < -1$ and $x > 1$. So the answer is $\{-5\} \cup [-1, 1]$.

Exp. 10) Find the values that satisfy the polynomial inequation $3(x-2)^2 2(x+4)^3 (-x^2-2) < 0$.

Solution You can modify the given inequation as, $-3(x-2)^2(x+4)^3(x^2+2)$. The given inequation if equated with zero, it gives only two real distinct roots -4 and 2 . The irreducible quadratic factor (x^2+2) yields only non real zeros, which don't affect the sign of the graph. So now you have to mark these two real roots on the number line. Since there are two roots, so there will be three intervals on the number line as shown below.



The three intervals are $(-\infty, -4)$, $(-4, 2)$, $(2, \infty)$. Now to check the first interval you may consider any value between that interval, say -5 , then you will get

$$-3(x-2)^2(x+4)^3(x^2+2) = -3(49)(-1)(27) = 3969 > 0.$$

So the first interval $(-\infty, -4)$ is positive. Similarly you can consider any value from the second interval, say 0 , then you will get $-3(x-2)^2(x+4)^3(x^2+2) = -3(4)(64)(2) = -1536 < 0$. It means the second interval $(-4, 2)$ is negative.

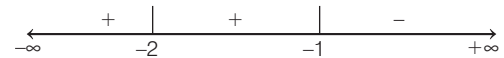
Finally you have to test the third and last interval by considering any value from that interval, say 3 , then you will get $-3(x-2)^2(x+4)^3(x^2+2) = -3(1)(343)(11) = -11319 < 0$. Thus the third interval is also negative.

Since you are looking for negative values of the function, therefore only second and third intervals satisfy the inequation. Thus the required answer is $(-4, 2) \cup (2, \infty)$.

Alternatively At $x = -4$, due to odd multiplicity the graph intersects the X-axis and changes the sign. At $x = 2$, due to even multiplicity of the root the graph touches the X-axis and bounces back retaining the sign. Since the leading term, after simplification into standard polynomial form, is $-3x^7$ which implies that the two ends of the graph will be in opposite directions and if x increases y decreases; and when x decreases y increases. Thus you can conclude that the graph will be positive when $x < -4$ and it will be negative when $-4 < x < 2$ and $2 < x < \infty$. So the answer is $(-4, 2) \cup (2, \infty)$ or $(-4, \infty) - \{2\}$.

Exp. 11) Find the values that satisfy the following polynomial inequation $-(x+2)^4(x+1)(2x^2+x+1) \geq 0$.

Solution The given inequation if equated with zero, it gives only two real distinct roots -2 and -1 . The irreducible quadratic factor $(2x^2+x+1)$ yields only non-real zeros, which don't affect the sign of the graph. So now you have to mark these two real roots on the number line. Since there are two roots, so there will be three intervals on the number line as shown below.



The three intervals are $(-\infty, -2)$, $(-2, -1)$, $(-1, \infty)$. Now after testing each of the three intervals for their respective signs you will get that the first and second intervals are positive and the last interval is negative; also the graph touches at $x = -2$. Since you are looking for non-negative values of the function, therefore only first and second intervals will satisfy the inequation. Thus the required answer is

$$(-\infty, -2) \cup \{-2\} \cup (-2, -1) \cup \{-1\} \text{ or } (-\infty, -1]$$

Alternatively At $x = -2$, due to even multiplicity of the root the graph touches the X-axis and bounces back retaining the sign. At $x = -1$, due to odd multiplicity the graph intersects the X-axis and changes the sign. Since the leading term, after simplification into standard polynomial form, is $-2x^7$ which implies that the two ends of the graph will be in opposite directions and if x increases y decreases; and when x decreases y increases. Thus you can conclude that the graph will be positive when $-\infty < x < -2$ and $-2 < x < -1$ and it will be negative when $x > -1$. So the answer is $(-\infty, -1]$.

Practice Exercise

- Find the values that satisfy the polynomial inequation $x^4 + 4x^3 - 12x^2 \leq 0$.
- Find the values that satisfy the polynomial inequation $x^4 + 4x^3 - 12x^2 < 0$.
- Find the values that satisfy the polynomial inequation $x^4 + 4x^3 - 12x^2 > 0$.
- Find the values that satisfy the polynomial inequation $x^4 + 4x^3 - 12x^2 \geq 0$.

Answers

- $[-6, 2]$
- $(-6, 0) \cup (0, 2)$
- $(-\infty, -6) \cup (2, \infty)$
- $(-\infty, 6] \cup \{0\} \cup [2, \infty)$

14.17 Relation Between Roots and Coefficients in a Polynomial

If $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_3 x^3 + a_2 x^2 + a_1 x + a_0$
(where $a_n \neq 0$)

And if $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ be the roots of the polynomial, then

$$S_1 = \alpha_1 + \alpha_2 + \alpha_3 + \dots + \alpha_n = \sum \alpha_i = -\frac{a_{n-1}}{a_n}$$

$$S_2 = \alpha_1 \alpha_2 + \alpha_1 \alpha_3 + \dots = \sum_{i \neq j} \alpha_i \alpha_j = (-1)^2 \frac{a_{n-2}}{a_n}$$

$$S_3 = \alpha_1 \alpha_2 \alpha_3 + \alpha_2 \alpha_3 \alpha_4 + \dots$$

$$= \sum_{i \neq j \neq k} \alpha_i \alpha_j \alpha_k = (-1)^3 \frac{a_{n-3}}{a_n}$$

... ..

... ..

$$S_{n-1} = \alpha_1 \alpha_2 \dots \alpha_{n-1} + \alpha_2 \alpha_3 \dots \alpha_n$$

$$= \sum_{i \neq j \neq \dots \neq (n-1)} \alpha_i \alpha_j \dots \alpha_{n-1} = (-1)^{n-1} \frac{a_1}{a_n}$$

$$S_n = \alpha_1 \alpha_2 \alpha_3 \dots \alpha_n = \prod_{i=1}^n \alpha_i = (-1)^n \frac{a_0}{a_n}$$

Thus a **quadratic equation** $ax^2 + bx + c = 0$, having the roots α and β , then

$$S_1 = \alpha + \beta = -\frac{b}{a} \text{ and } S_2 = \alpha\beta = \frac{c}{a}$$

Similarly, a **cubic equation** $ax^3 + bx^2 + cx + d = 0$, having the roots α, β and γ , then

$$S_1 = \alpha + \beta + \gamma = -\frac{b}{a} \text{ and } S_2 = \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$S_3 = \alpha\beta\gamma = -\frac{d}{a}$$

Similarly, a **biquadratic equation**

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$

having α, β, γ and δ , then

$$S_1 = \alpha + \beta + \gamma + \delta = -\frac{b}{a}$$

$$S_2 = \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a}$$

$$S_3 = \alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -\frac{d}{a}$$

$$S_4 = \alpha\beta\gamma\delta = \frac{e}{a}$$

Formation of a Polynomial Equation from the Given Roots If $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ be the roots of the n th degree

polynomial and S_k ($k=1, 2, 3, \dots, n$) denotes the sum of the products of roots taken k at a time, then the equation will be

$$x^n - S_1 x^{n-1} + S_2 x^{n-2} - S_3 x^{n-3} + \dots + (-1)^n S_n = 0.$$

Formation of a Quadratic Equation from the Given Roots

$$x^2 - S_1 x + S_2 = 0 \Rightarrow x^2 - (\alpha + \beta)x + (\alpha\beta) = 0$$

Formation of a Cubic Equation from the Given Roots

$$x^3 - S_1 x^2 + S_2 x - S_3 = 0$$

$$\Rightarrow x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \alpha\gamma + \beta\gamma)x - (\alpha\beta\gamma) = 0$$

Formation of a Biquadratic Equation from the Given Roots

$$x^4 - S_1 x^3 + S_2 x^2 - S_3 x + S_4 = 0$$

$$\Rightarrow x^4 - (\alpha + \beta + \gamma + \delta)x^3 + (\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta)x^2 - (\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta)x + (\alpha\beta\gamma\delta) = 0$$

Exp. 1) Let $P(x) = x^3 + ax^2 + bx + c$ be a polynomial with real coefficients, $c \neq 0$ and x_1, x_2, x_3 be the roots of $P(x)$. Determine the polynomial $Q(x)$ whose roots are $\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}$.

Solution $x_1 + x_2 + x_3 = -a, x_1 x_2 + x_2 x_3 + x_3 x_1 = b$

and

$$\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} = \frac{x_1 x_2 + x_2 x_3 + x_3 x_1}{x_1 x_2 x_3} = \frac{b}{c}$$

$$\frac{1}{x_1 x_2} + \frac{1}{x_2 x_3} + \frac{1}{x_3 x_1} = \frac{x_1 + x_2 + x_3}{x_1 x_2 x_3} = \frac{a}{c}$$

$$\text{and } \frac{1}{x_1 x_2 x_3} = -\frac{1}{c}$$

$$\text{Therefore } Q(x) = x^3 - \left(\frac{-b}{c}\right)x^2 + \left(\frac{a}{c}\right)x - \left(-\frac{1}{c}\right)$$

$$\Rightarrow Q(x) = cx^3 + bx^2 + ax + 1$$

Exp. 2) The real number x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + bx + c = 0$ are in AP. Find the intervals in which b and c lie.

Solution Let the roots be $a-d, d, a+d$, then

$$\text{Sum of roots} = (a-d) + (a) + (a+d) = 1$$

$$\Rightarrow a = \frac{1}{3}$$

$$\text{and } (a-d)(a) + a(a+d) + (a-d)(a+d) = b$$

$$\Rightarrow 3a^2 - d^2 = b \Rightarrow 3\left(\frac{1}{9}\right) - d^2 = b$$

$$\Rightarrow d^2 = \frac{1}{3} - b \quad [\text{since the minimum value of } d^2 = 0]$$

$$\Rightarrow \frac{1}{3} - b \geq 0 \Rightarrow b \leq 1/3$$

$$\text{Now product of roots} = (a-d)(a)(a+d) = -c$$

$$\begin{aligned} \Rightarrow a(a^2 - d^2) &= -c \\ \Rightarrow \frac{1}{3} \left(\frac{1}{9} - d^2 \right) &= -c \\ \Rightarrow c &= \frac{d^2}{3} - \frac{1}{27} \\ \Rightarrow c &\geq -\frac{1}{27} \end{aligned}$$

[since the minimum value of $\frac{d^2}{3} = 0$]

Therefore $-\infty < b \leq 1/3$ and $-\frac{1}{27} \leq c < \infty$

14.18 Rational Polynomials

Conceptually, a rational polynomial function is a quotient of polynomials. Therefore a rational polynomial function $f(x)$ is expressed as following

$$f(x) = \frac{p(x)}{q(x)}$$

Where $p(x)$ and $q(x)$ are the polynomial functions, such that $q(x) \neq 0$.

- The rational function $f(x)$ is not defined for any x which is the root of $q(x)$.
- The numerator and denominator can be polynomials of any order.

Critical Numbers

The critical numbers for a rational function are all the values of x where the function $f(x)$ is zero or undefined, That is, all the x where the numerator $p(x)$ and denominator $q(x)$ are zero.

- A rational expression will be zero only when its numerator is zero and the denominator is not zero for the same value of x . That is $f(x) = 0$, if $p(x) = 0$ but $q(x) \neq 0$ for the same x .
- For critical points you have to consider only real numbers. So, if upon solving the polynomial in the numerator you will get non-real numbers, these non-real values won't be considered as critical points.
- It's not necessary that all the functions will have critical points; it may occur when a polynomial function has no real root and the denominator is defined for all the values of the given interval.
- However, in this course most of the functions that you will be looking at do have critical points. That is only because those problems make for more interesting examples. Do not let this fact lead you to expect that a function will always have critical points.

Exp. 1) Find the critical numbers of the rational polynomial $\frac{x^2 - 6x + 8}{x + 2}$.

Solution $\frac{x^2 - 6x + 8}{x + 2} = \frac{(x - 2)(x - 4)}{(x + 2)}$

Since the roots of $x^2 - 6x + 8$ are $\{2, 4\}$ and that of $x + 2$ is $\{-2\}$. Therefore the critical points are $-2, 2$ and 4 .

Exp. 2) Find the critical numbers of the rational polynomial $\frac{x^2 + 4x + 3}{x^2 - x - 20}$.

Solution $\frac{x^2 + 4x + 3}{x^2 - x - 20} = \frac{(x + 3)(x + 1)}{(x + 4)(x - 5)}$

Since the roots of $x^2 + 4x + 3$ are $\{-3, -1\}$ and that of $x^2 - x - 20$ are $\{-4, 5\}$. Therefore the critical points are $-1, -4, -3$ and 5 .

Exp. 3) Find the critical numbers of the following rational polynomial $\frac{x^2 + 2x - 15}{x^2 - 9}$.

Solution $\frac{x^2 + 2x - 15}{x^2 - 9} = \frac{(x + 5)(x - 3)}{(x + 3)(x - 3)}$

Since the roots of $x^2 + 2x - 15$ are $\{-5, 3\}$ and that of $x^2 - 9$ are $\{-3, 3\}$. Therefore that critical points are $-5, -3$ and 3 .

Exp. 4) Find the critical numbers of the rational polynomial $\frac{x^2 + 1}{x^2 - x - 6}$.

Solution $\frac{x^2 + 1}{x^2 - x - 6} = \frac{(x^2 + 1)}{(x + 2)(x - 3)}$

Since the roots of $x^2 + 1$ are non-real, so these are not considerable. But the roots of $x^2 - x - 6$ are $\{-2, 3\}$, which are real. Therefore the critical points are -2 and 3 .

Root or Solution of a Rational Polynomial Equation

The root of a rational polynomial equation $f(x) = 0$ is the value of x , for which $p(x) = 0$, but $q(x) \neq 0$.

NOTE The terms root, zero and solution of a rational polynomial can be interchangeably used for one another. And the root/zero/solution can be called x-intercept too, if the root/zero/solution is a real number.

Exp. 1) Find the roots of the rational polynomial equation $\frac{(2x + 1)}{(x + 3)(x - 1)} = 0$.

Solution $\frac{(2x + 1)}{(x + 3)(x - 1)} = 0 \Rightarrow 2x + 1 = 0 \Rightarrow x = -1/2$

Therefore there is only one solution of the above rational equation, which is $x = -1/2$.

Exp. 2) Find the roots of the rational polynomial equation $\frac{x^2 - 6x + 8}{x + 2} = 0$

Solution $\frac{x^2 - 6x + 8}{x + 2} = 0 \Rightarrow \frac{(x - 4)(x - 2)}{(x + 2)} = 0$

$\Rightarrow (x - 2)(x - 4) = 0 \Rightarrow x = 2 \text{ or } 4.$

Therefore there are two solutions of the above rational equation, which are $x = 2$ and 4 .

Exp. 3) Find the solutions of the rational polynomial equation $\frac{x^2 - 4}{x^3 - x^2 - 6x} = 0$

Solution $\frac{x^2 - 4}{x^3 - x^2 - 6x} = 0 \Rightarrow \frac{(x + 2)(x - 2)}{x(x + 2)(x - 3)} = 0$

$\Rightarrow \frac{(x - 2)}{x(x - 3)} = 0 \quad x$

$\Rightarrow (x - 2) = 0$

$\Rightarrow x = 2$

Therefore there is only one solution of the above rational equation, which is $x = 2$

Exp. 4) Find the solutions of the rational equation

$$\frac{x^2(x^4 - 81)(x + 2)^3}{x^5 - x^4 - 6x^3} = 0$$

Solution $\frac{x^2(x^4 - 81)(x + 2)^3}{x^5 - x^4 - 6x^3} = 0$

$\Rightarrow \frac{x^2(x^2 + 9)(x + 3)(x - 3)(x + 2)^3}{x^3(x - 3)(x + 2)} = 0$

$\Rightarrow \frac{(x^2 + 9)(x + 3)(x + 2)^2}{x} = 0$

$\Rightarrow (x^2 + 9)(x + 3)(x + 2)^2 = 0$

$\Rightarrow (x^2 + 9) = 0 \text{ or } (x + 3) = 0 \text{ or } (x + 2)^2 = 0$

If $x^2 + 9 = 0 \Rightarrow x = \pm 3i$, which are complex (*i.e.*, non-real) roots. And if $x + 3 = 0 \Rightarrow x = -3$ which is a real root.

Again, if $(x + 2)^2 = 0 \Rightarrow x = -2$, which is a real root.

Therefore there are two non-real roots and two real roots of the above rational equation and thus there are total four solutions.

NOTE Whenever there is a non-real root, the graph or the function does not intersect or touch the X-axis.

Domain of a Rational Polynomial Equation

The domain of a rational function includes only points where both the numerator and the denominator are defined for real values of x .

Essentially, the domain of a rational function does not contain any value that would make the denominator equal to zero.

NOTE

- Any value of x that makes both the numerator and denominator 0, simultaneously, will either create a 'hole' or a 'vertical asymptote'. And it happens when some factors are common between numerator and denominator.
- A hole in the graph shows the absence of the graph or a miniscule break in the graph, without changing its direction.
- The vertical asymptote is nothing but a virtual line that creates a boundary for the graph and thus the graph cannot cross the boundary and it runs parallel to this boundary. So a vertical asymptote changes the normal path of the graph apart from breaking it.

Exp. 1) Find the domain of the rational polynomial

$$f(x) = \frac{x^2 + x - 20}{x^2 - 3x - 18}$$

Solution $\frac{x^2 + x - 20}{x^2 - 3x - 18} = \frac{(x + 5)(x - 4)}{(x + 3)(x - 6)}$

Since at $x = -3$ and 6 , the denominator will be zero, *i.e.* the rational polynomial will be undefined at these points. So the domain of this polynomial will be all the real numbers except $x = -3$ and 6 .

That is $R - \{-3, 6\}$ or $(-\infty, -3) \cup (-3, 6) \cup (6, \infty)$.

Exp. 2) Find the domain of the rational polynomial

$$f(x) = \frac{2x^2 - 5x - 3}{(x^2 + 4)}$$

Solution $\frac{2x^2 - 5x - 3}{(x^2 + 4)} = \frac{(x - 3)\left(x + \frac{1}{2}\right)}{(x^2 + 4)}$

Since $(x^2 + 4)$ cannot be equal to zero for any real value of x . So this factor $(x^2 + 4)$, in the denominator, gives non-real zeros and a non-real zero never affects the domain. So the domain of this rational function will include all the real numbers. That is R or $(-\infty, \infty)$.

Exp. 3) Find the domain of the rational polynomial

$$f(x) = \frac{x^2 - 1}{2x^2 - 5x}$$

Solution $\frac{x^2 - 1}{2x^2 - 5x} = \frac{(x + 1)(x - 1)}{x(2x - 5)}$

Since at $x = 0$ and $5/2$, the denominator will be zero, *i.e.* the rational polynomial will be undefined at these points.

So the domain of this polynomial will be all the real numbers except

$$x = 0 \text{ and } \frac{5}{2}$$

That is $R - \left\{0, \frac{5}{2}\right\}$

or $(-\infty, 0) \cup \left(0, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$.

Exp. 4) Find the domain of the rational polynomial

$$f(x) = \frac{x^2 - 4}{x^3 - x^2 - 6x}$$

Solution. $\frac{x^2 - 4}{x^3 - x^2 - 6x} = \frac{(x+2)(x-2)}{x(x+2)(x-3)}$

Since at $x = 0, -2$ and 3 , the denominator will be zero, *i.e.* the rational polynomial will be undefined at these points.

So the domain of this polynomial will be all the real numbers except $x = 0, -2$ and 3 .

That is $R - \{-2, 0, 3\}$

or $(-\infty, -2) \cup (-2, 0) \cup (0, 3) \cup (3, \infty)$.

Exp. 5) Find the domain of the rational polynomial

$$f(x) = \frac{x^3 + 2x^2 - 4x - 8}{x^4 - x^3 - 6x^2}$$

Solution $f(x) = \frac{x^3 + 2x^2 - 4x - 8}{x^4 - x^3 - 6x^2} = \frac{(x+2)^2(x-2)}{x^2(x+2)(x-3)}$

Since at $x = 0, -2$ or 3 , the denominator will be zero, *i.e.* the rational polynomial will be undefined at these points.

So the domain of this polynomial will be all the real number except $x = 0, -2$ and 3 .

That is $R - \{0, -2, 3\}$ or $(-\infty, 0) \cup (0, -2) \cup (-2, 3) \cup (3, \infty)$.

Exp. 6) Find the domain of the rational polynomial

$$f(x) = \frac{(x+3)(x+2)(x-2)^3}{(x+2)^2(x-2)(x-3)}$$

Solution $f(x) = \frac{(x+3)(x+2)(x-2)^3}{(x+2)^2(x-2)(x-3)}$

Since at $x = -2, 2$ and 3 , the denominator will be zero, *i.e.* the rational polynomial will be undefined at these points.

So the domain of this polynomial will be all the real numbers except $x = -2, 2$ and 3 .

That is $R - \{-2, 2, 3\}$ or $(-\infty, -2) \cup (-2, 2) \cup (2, 3) \cup (3, \infty)$.

Graphical Features of a Rational Polynomial

A rational function is a function that can be written as a fraction of two polynomials where the denominator is not zero.

Let the rational polynomial function be

$$f(x) = \frac{p(x)}{q(x)} = \frac{a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \dots + b_1 x + b_0}$$

Or in the factor form a rational function can be expressed as

$$f(x) = \frac{(x - \alpha_1)(x - \alpha_2) \dots (x - \alpha_{n-1})(x - \alpha_n)}{(x - \beta_1)(x - \beta_2) \dots (x - \beta_{m-1})(x - \beta_m)}$$

Where n is the degree of numerator $p(x)$ and m is the degree of denominator $q(x)$ remove comma and α_i and β_i are the roots of the polynomials $p(x)$ and $q(x)$, respectively.

Asymptote

An asymptote is an imaginary line that a curve approaches but does not touch. There are basically two types of asymptotes: vertical and non-vertical asymptotes. The non-vertical asymptotes are also called as end-behavior asymptotes.

Asymptote	Condition	Value of Asymptote
Vertical	Roots of the denominator of the rational polynomial, in the lowest term.	Vertical asymptote, $x = c$
Horizontal	If $n < m$ If $n = m$ If $n > m$	Horizontal asymptote, $y = 0$ Horizontal asymptote, $y = a_n / b_m$ No horizontal asymptote
Oblique	If $n > m$, but $n = m + 1$	Oblique asymptote, $y = k(x)$; where $p(x) = q(x)k(x) + r(x)$
Curvilinear	If $n > m$, but $n > m + 1$	Curvilinear asymptote, $y = k(x)$; Where $p(x) = q(x)k(x) + r(x)$

NOTE A rational polynomial in the 'lowest term' implies that all the common factors have been cancelled out and thus there is no common factor between the numerator and denominator in the rational polynomial when it is in the reduced or lowest term.

Mind the fact the even after cancelling out the common factors the relation between n and m remains intact, since equal number of factors get cancelled out from the numerator and the denominator.

Vertical Asymptotes

1. A rational polynomial function may have none, one, or several vertical asymptotes.
2. The graph of the rational function never intersects the vertical asymptote.
3. The vertical asymptotes occur at all the zeros of the denominator when the rational function is expressed in the lowest term.
4. To determine the vertical asymptote factorize both the numerator and denominator. If there are any common factors between the numerator and denominator, then cancel out the common factors and then set the denominator equal to zero and solve. Any real solution will represent a vertical asymptote. The equation of a vertical asymptote is $x = c$.

Holes

Sometimes, as mentioned above, a factor may appear in both the numerator and the denominator. Then there is a possibility that there may not be a vertical asymptote and so it may create a hole in the graph. A **hole** is simply a single point on the graph that doesn't actually exist due to a restriction.

Let us assume that the factor $(x - c)^u$ is in the numerator and $(x - c)^v$ is the denominator. If

- (i) $u < v$, there will be a vertical asymptote $x = c$.
- (ii) $u > v$, there will be a hole in the graph on the X -axis at $x = c$. there is no vertical asymptote there.
- (iii) $u = v$, there will be a hole in the graph at $x = c$, but not on the X -axis. The y -value of the hole can be found by cancelling the factors and substituting $x = c$ in the reduced function.

NOTE A rational function may have both - holes and vertical asymptotes-for different values of x . However, for a certain value of x the function cannot have both the hole and the vertical asymptote.

Exp. 1) Find all the vertical asymptotes and holes of $\frac{(x+3)(x-7)}{(2x-5)}$.

Solution Since $2x - 5 = 0 \Rightarrow x = 5/2$

Therefore there is only one vertical asymptotes, $x = 5/2$.

Since there is no common factor in numerator and denominator, so there won't be any hole in the graph.

Exp. 2) Find all the vertical asymptotes and holes of

$$\frac{(2x+9)(5x-4)}{(x-3)(x-5)(2x-11)}$$

Solution Since $(x-3)(x-5)(2x-11) = 0 \Rightarrow x = 3$ or 5 or $11/2$.

Therefore there are three vertical asymptotes, $x = 3$, $x = 5$ and $x = 11/2$

Exp. 3) Find all the vertical asymptotes and holes of

$$\frac{(x+2)(x+1)(x-3)}{x^2(x-3)(3x+8)}$$

Solution Since $(x-3)$ is common in both the numerator and denominator, so you have to cancel it out.

Then the simplified (or reduced) rational function is $\frac{(x+2)(x+1)}{x^2(3x+8)}$.

Now since $x^2(3x+8) = 0 \Rightarrow x = 0$ or $x = -8/3$.

Therefore there are two vertical asymptotes, $x = 0$ and $x = -8/3$. If you look at the original rational function in which the power of x in the numerator is lower than that of denominator, so there will be no hole in the graph.

The other way to look at this issue is that the reduced function has no common factor in numerator and denominator, so there won't be any hole in the graph.

The possibility of hole arises only when numerator and denominator of the reduced function has some common factor. In this example, the reduced function has no common factor between numerator and denominator.

Exp. 4) Find the vertical asymptotes and holes of $\frac{3(2x+5)^3(8x-3)^2(x-3)}{2(8x-3)^2(2x+5)}$.

Solution. First of all cancel out all the common factors between the numerator and denominator.

Then the simplified rational function is

$$\frac{3(2x+5)^2(x-3)}{2} = \frac{3}{2}(2x+5)^2(x-3)$$

Since, there is no factor or root in the denominator of simplified polynomial. Therefore there is no vertical asymptote.

Since the power of $(2x+5)$ in the numerator is higher than that of denominator, so there will be a hole in the graph on the X -axis at $x = -5/2$. Again since the power of $(8x-3)$ in the numerator is same as that of denominator, so there will be a hole in the graph at $x = 3/8$, but not on the X -axis. The y -value of the hole can be found by canceling the factors and substituting $x = 3/8$ in the reduced function as following.

$$\begin{aligned} y &= \frac{3}{2}(2x+5)^2(x-3) = \frac{3}{2}\left(2 \times \frac{3}{8} + 5\right)^2\left(\frac{3}{8} - 3\right) \\ \Rightarrow &= \frac{-33327}{256} = -130.18 \end{aligned}$$

Exp. 5) Find all the vertical asymptotes and holes of $\frac{(x-3)^2(x-4)}{(x-5)(x-3)}$.

Solution. Since $(x-3)$ is common in both the numerator and denominator, so you have to cancel it out.

Then the simplified rational function is $\frac{(x-3)(x-4)}{(x-5)}$.

Now, since $(x-5) = 0 \Rightarrow x = 5$. Therefore there is only one vertical asymptote, $x = 5$. Since the power of $(x-3)$ in the numerator is higher than that of denominator, so there will be a hole in the graph on the X -axis at $x = 3$.

Exp. 6) Find all the vertical asymptotes and holes of $\frac{(x+2)(x+4)^3}{(x+4)(x+2)^3(2x+3)(x-4)}$.

Solution. First of all cancel out all the common factors between the numerator and denominator.

Then the simplified rational function is $\frac{(x+4)^2}{(x+2)^2(2x+3)(x-4)}$

Now, Since $(x+2)^2(2x+3)(x-4) = 0$

$$\Rightarrow x = -2$$

$$\text{or } x = -3/2 \text{ or } x = 4.$$

Therefore there are three vertical asymptotes,

$$x = -2, x = -3/2 \text{ and } x = 4.$$

However, since the power of $(x+4)$ in the numerator is higher than that of denominator, so there will be a hole in the graph on the X -axis at $x = -4$.

Exp. 7) Find the vertical asymptotes and holes of $\frac{x(x+5)^4(x-2)^6}{(x+5)^6(x-2)^3(x-5)}$.

Solution. First of all cancel out all the common factors between the numerator and denominator.

Then the simplified rational function is $\frac{x(x-2)^3}{(x+5)^2(x-5)}$

Now, since $(x+5)^2(x-5) = 0 \Rightarrow x = -5$ or $x = 5$.

Therefore there are two vertical asymptotes, $x = -5$ and $x = 5$.

However, since the power of $(x-2)$ in the numerator is higher than that of denominator, so there will be a hole in the graph on the X-axis at $x = 2$.

Exp. 8) Find the vertical asymptotes and holes of $\frac{x^2(x+7)^2(x-2)^3}{x(x^2+7)}$.

Solution. First of all cancel out all the common factors between the numerator and denominator.

Then the simplified rational function is $\frac{x(x+7)^2(x-2)^3}{x(x^2+7)}$

Now, since $x^2+7=0 \Rightarrow x = \pm\sqrt{-7}$, which is not a real solution.

Therefore there is no vertical asymptote.

Looking at the original function we can say that the power of x in the numerator is higher than that of denominator, so there will be a hole in the graph on the X-axis at $x = 0$.

Exp. 9) Find the vertical asymptotes and holes of $\frac{(x-10)(x-20)^2(x-30)^3(x-40)}{(x-10)^3(x-20)^2(x-30)(x-50)}$.

Solution. First of all cancel out all the common factors between the numerator and denominator.

Then the simplified rational function is $\frac{(x-30)^2(x-40)}{(x-10)^2(x-50)}$

Now since $(x-10)^2(x-50) = 0$

$\Rightarrow x = 10$ or $x = 50$.

Therefore, there are two vertical asymptotes, $x = 10$ and $x = 50$.

Looking at the original function, we can say that the power of $(x-30)$ in the numerator is higher than that of denominator, so there will be a hole in the graph on the X-axis at $x = 30$.

Again since the power of $(x-20)$ in the numerator is same as that of denominator, so there will be a hole in the graph at $x = 20$, but not on the X-axis.

The y -value of the hole can be found by canceling out the common factors and substituting $x = 20$ in the reduced function as following.

$$y = \frac{(x-30)^2(x-40)}{(x-10)^2(x-50)} = \frac{(20-30)^2(20-40)^2}{(20-10)^2(20-50)} \\ = \frac{2}{3} = 0.67$$

Non-Vertical Asymptotes

(or End Behaviour Asymptotes)

Since the non-vertical asymptotes determines the end behaviour or long term behaviour of a rational function, that's why they are called as end behaviour asymptotes. These asymptotes give the glimpse of what a rational graph look like for higher values of x or in short these asymptotes give you a big picture of a rational function.

1. There are three types of End-behaviour asymptotes: Horizontal, Slant and Curvilinear.
2. A rational polynomial function may have at most one non-vertical asymptote.
3. The graph of the rational polynomial may intersect the non-vertical asymptote.

(a) Horizontal Asymptotes

The graph of a rational function may also have horizontal asymptote. To determine the horizontal asymptotes look for the highest power in the numerator (*i.e.*, n) and denominator (*i.e.*, m) of the rational function.

1. If the degree of the numerator is higher than that of denominator (*i.e.*, $n > m$), then there is no horizontal asymptote.
2. If the degree of the numerator is less than the degree of denominator (*i.e.*, $n < m$), then the X-axis is the horizontal asymptote. That means the horizontal asymptote is $y = 0$
3. If the degree of the numerator and the denominator is same (*i.e.*, $n = m$), then the horizontal asymptote is given by the following formula.

$$y = \left(\frac{\begin{array}{c} \text{the coefficient of the highest power} \\ \text{in the numerator} \end{array}}{\begin{array}{c} \text{the coefficient of the highest power in} \\ \text{the denominator} \end{array}} \right)$$

That means if the rational Function is

$$f(x) = \frac{P(x)}{q(x)} = \frac{a_n x^n + \dots + a_1 x + a_0}{b_m x^m + \dots + b_1 x + b_0}$$

Then the horizontal asymptote is $y = \frac{a_n}{b_m}$

Exp. 1) Find the horizontal asymptote of $\frac{(x+3)(x-7)}{(2x-5)}$

Solution. $\frac{(x+3)(x-7)}{(2x-5)} = \frac{x^2 - 4x - 21}{2x - 5}$

Since $n = 2$ and $m = 1$. It implies that $n > m$. So there won't be any horizontal asymptote.

Exp. 2) Find the horizontal asymptote of

$$\frac{(2x+9)(5x-4)}{(x-3)(x-5)(2x-11)}$$

Solution. $\frac{(2x+9)(5x-4)}{(x-3)(x-5)(2x-11)} = \frac{10x^2 + 37x - 36}{2x^3 - 27x^2 + 118x - 165}$

Since $n = 2$ and $m = 3$. It implies that $n < m$. Then X-axis (*i.e.*, $y = 0$) is the horizontal asymptote.

Exp. 3) Find all the vertical asymptotes of

$$\frac{(x-2)^2(x+2)(x+1)(x-3)}{x^3(x-3)(3x+8)}$$

Solution $\frac{(x-2)^2(x+2)(x+1)(x-3)}{x^3(x-3)(3x+8)} = \frac{7x^5 + \dots}{3x^5 + \dots}$

Since $n = 5$ and $m = 5$. It implies that $n = m$.

Then the horizontal asymptote is $y = \frac{7}{3}$.

Exp. 4) Find The horizontal asymptote of

$$\frac{(2x-3)^2(5x-4)}{x(3x-5)(7x-3)}$$

Solution $\frac{(2x-3)^2(5x-4)}{x(3x-5)(7x-3)} = \frac{20x^3 + \dots}{21x^3 + \dots}$

Since $n = 3$ and $m = 3$. it implies that $n = m$

Then the horizontal asymptote is $y = \frac{20}{21}$

(b) Slant (or Oblique) Asymptotes

The graph of a rational function may also have a slant asymptote. When the degree of the numerator is exactly one more than the degree of the denominator (*i.e.* $n = m + 1$), the graph of the rational function will have an oblique asymptote.

To find the equation of the oblique asymptote, perform long division (or synthetic division if it works).

As x gets very large (this is the far left or far right position), the remainder portion becomes very small, almost zero. So, to find the equation of the oblique asymptote, perform the long division and discard the remainder. The slant asymptote is $y = k(x)$; where $p(x) = q(x)k(x) + r(x)$. Here $k(x)$ is a linear function.

Exp. 1) Find the slant asymptote of $\frac{(x+3)(x+7)}{(x-5)}$

Solution $\frac{(x+3)(x+7)}{(x-5)} = \frac{x^2 + 10x + 21}{x-5}$

Since $n = 2$ and $m = 1$. It implies that $n = m + 1$. So a slant asymptote is possible. After dividing the numerator by denominator you will get quotient $k(x) = x + 15$. Therefore the slant asymptote is $y = x + 15$.

Hint $x^2 + 10x + 21 = (x-5)(x+15) + 96$

Exp. 2) Find the slant asymptote of

$$\frac{(x+3)(x-7)(2x+3)^2}{(x-5)^2(2x+3)}$$

Solution $\frac{(x+3)(x-7)(2x+3)^2}{(x-5)^2(2x+3)} = \frac{(x+3)(x-7)(2x+3)}{(x-5)^2}$
 $= \frac{2x^3 - 5x^2 - 54x - 63}{x^2 - 10x + 25}$

Since $n = 3$ and $m = 2$. It implies that $n = m + 1$. So a slant asymptote is possible. After dividing the numerator by denominator you will get quotient $k(x) = 2x + 15$, Therefore the slant asymptote is $y = 2x + 15$

Hint $2x^3 - 5x^2 - 54x - 63 = (x^2 - 10x + 25)(2x + 15) + (46x - 438)$

(c) Curvilinear Asymptotes

The graph of a rational function may also have a curvilinear asymptote. When the degree of the numerator is higher than the degree of the denominator and the difference in the degree is more than one (*i.e.* $n > m + 1$), the graph of the rational function will have a curvilinear asymptote.

To find the equation of the curvilinear asymptote, perform long division (or synthetic division if it works). As x gets very large (this is the far left or far right position), the remainder portion becomes very small, almost zero. So, to find the equation of the curvilinear asymptote, perform the long division and discard the remainder.

The slant asymptote is $y = k(x)$; where $p(x) = q(x)k(x) + r(x)$. Here $k(x)$ is a polynomial function of at least degree 2.

Exp. 1) Find the curvilinear asymptote of

$$\frac{(x-3)(x+7)(x-8)}{(x-5)}$$

Solution $\frac{(x+3)(x+7)(x-8)}{(x-5)} = \frac{x^3 + 2x^2 - 59x - 168}{(x-5)}$
 $= (x^2 + 7x - 24) + \frac{-288}{(x-5)}$

Since $n = 3$ and $m = 1$, It implies that $n > m + 1$. So a curvilinear asymptote is possible. After dividing the numerator by denominator you will get quotient $k(x) = x^2 + 7x - 24$. Therefore the curvilinear asymptote is $y = x^2 + 7x - 24$.

Hint $(x+3)(x+7)(x-8) = x^3 + 2x^2 - 59x - 168$
 $= (x-5)(x^2 + 7x - 24) - 288$

Exp. 2) Find the curvilinear asymptote of

$$\frac{3x^5 - x^4 + 2x^2 + x + 1}{(x^2 + 1)}$$

Solution $\frac{3x^5 - x^4 + 2x^2 + x + 1}{(x^2 + 1)} = (3x^3 - x^2 - 3x + 3) + \frac{4x - 2}{(x^4 + 1)}$

Since $n = 5$ and $m = 2$. It implies that $n - m > 1$. So a curvilinear asymptote is possible. After dividing the numerator by denominator you will get quotient $k(x) = 3x^3 - x^2 - 3x + 3$.

Therefore the curvilinear asymptote is $y = 3x^3 - x^2 - 3x + 3$

Hint

$$3x^5 - x^4 + 2x^2 + x + 1 = (x^2 + 1)(3x^3 - x^2 - 3x + 3) + (4x - 2)$$

X-intercepts and Y-intercepts

These are the points where the rational polynomial graph intercepts the X -axis and Y -axis.

	x-intercepts	y-intercepts
What to determine	Values of x	Value of y
Favorable condition	at $p(x) = 0$	at $x = 0$
Restriction	but $q(x) \neq 0$, for the same x	if $q(0) \neq 0$

X-intercepts

These occur at the real zeros of the numerator, which are not the zeros of the denominator too. That is x -intercepts are the values of x when $p(x) = 0$, but $q(x) \neq 0$ for the same value of x . Graphically speaking, these are the points on the X -axis where the rational function intercepts or touches the X -axis.

Look at the given rational function, that is, original function in which you have not yet cancelled the common factors, if any, between numerator and denominator. So whenever there is a common factor between numerator and denominator of the given rational function there will be a discontinuity (either a Hole or a Vertical Asymptote) in the graph of the rational function. And, wherever, there is a Hole or a Vertical Asymptote there cannot be any x -intercept. Therefore, while finding the x -intercepts we need to be careful about the discontinuity of the graph. If there are holes and asymptotes coinciding with x -intercepts, please discard these holes and asymptotes, as they cannot be considered as the x -intercepts.

Exp. 1) Find all the x -intercepts of $\frac{(x+3)(x-7)}{(2x-5)}$.

Solution For x -intercepts $p(x) = 0$

$$\text{That means } (x+3)(x-7) = 0$$

$$\Rightarrow (x+3) = 0 \text{ or } (x-7) = 0 \Rightarrow x = -3 \text{ or } x = 7$$

Therefore there are two x -intercepts namely -3 and 7 .

Exp. 2) Find all the x -intercepts of

$$f(x) = \frac{(x+2)(x-1)(x-4)^2}{(x+2)^2(x-1)(x-4)}$$

Solution Given that $f(x) = \frac{(x+2)(x-1)(x-4)^2}{(x+2)^2(x-1)(x-4)}$

For x -intercepts, $f(x) = \frac{p(x)}{q(x)} = 0$, such that $p(x) = 0$. but for the same values of x $q(x) \neq 0$.

Now, we see that for $x = -2, 1, 4$ the numerator of the function is zero, but for all these values of $x = -2, 1, 4$ the denominator is also zero.

Therefore, we don't have any x -intercepts for the given function.

NOTE Since the power of $(x+2)$ in the numerator is less than that of denominator, therefore at $x = -2$, there is a vertical asymptote.

Since the power of $(x-4)$ in the numerator is higher than that of denominator, therefore at $x = 4$, there is a hole.

Since the power of $(x-1)$ in the numerator is same as that of denominator, so there will be a hole in the graph at $x = 1$, but not on the X -axis.

The Y -value of the hole can be found by canceling the factors and substituting $x = 1$ in the reduced function as following.

$$\begin{aligned} y &= \frac{(x-4)}{(x+2)} \\ &= \frac{(1-4)}{(1+2)} \\ &= \frac{-3}{3} = -1 \end{aligned}$$

Exp. 3) Find all the x -intercepts of $\frac{(x^2+2)(x^4+4)}{(2x-25)}$

Solution For x -intercepts $p(x) = 0$

$$\text{That means } (x^2+2)(x^4+4) = 0$$

$$\Rightarrow (x^2+2) = 0 \text{ or } (x^4+4) = 0$$

Now you can see that there are no real roots in the numerator.

Therefore there are no x -intercepts given by this function.

Exp. 4) Find all the x -intercepts of

$$f(x) = \frac{(x-3)^4(x-6)^4(x-4)(5x-8)}{(2x-5)(x-3)(x-6)^2}$$

Solution Given that $f(x) = \frac{(x-3)^4(x-6)^4(x-4)(5x-8)}{(2x-5)(x-3)(x-6)^2}$

$$\Rightarrow f(x) = \frac{(x-3)^3(x-6)^2(x-4)(5x-8)}{(2x-5)}$$

For x -intercepts $p(x) = 0$

$$\text{That means } (x-3)^3(x-6)^2(x-4)(5x-8) = 0$$

$$\Rightarrow (x-3)^3 = 0 \text{ or } (x-6)^2 = 0$$

$$\text{or } (x-4) \text{ or } (5x-8) = 0$$

$$\Rightarrow x = 3 \text{ or } x = 6$$

$$\text{or } x = 4 \text{ or } x = 8/5$$

But $x = 3$ and $x = 6$ cannot be the x -intercepts, as when $x = 3$ and $x = 6$ the denominator of the given rational function becomes 0 (zero). Essentially, at $x = 3$ and $x = 6$, there are holes on the X -axis.

Thus there are only two x -intercepts namely 4 and $\frac{8}{5}$.

Y-intercept

This is the value of $f(0)$, if defined. Graphically speaking, this is the point on the Y-axis where the rational function intercepts the Y-axis.

Exp. 1) Find the y-intercept of $\frac{(x+3)(x-7)}{(2x-5)}$

Solution For y-intercept, $f(0) = \frac{(0+3)(0-7)}{(2 \times 0 - 5)} = \frac{21}{5}$

Therefore the y-intercept is $21/5$.

Exp. 2) Find the y-intercept of $\frac{(x+2)(x+1)(x-3)}{x^2(x-3)(3x+8)}$.

Solution For y-intercept $f(0) = \frac{(0+2)(0+1)(0-3)}{(0)^2(0-3)(3 \times 0 + 8)} = \frac{-6}{0}$

That means the given function is not defined at $x = 0$.

Therefore there is no y-intercept for this rational function.

Exp. 3) Find the y-intercept of $\frac{x^2(x+7)^2(x-2)^3}{x(x^2+7)}$.

Solution For Y-intercept $f(0) = \frac{0(0+7)^2 \times (0-2)^3}{0(0+7)}$

$$= \frac{0 \times (49)(-8)}{0 \times (7)} = \frac{0}{0}$$

That means the given function is not defined at $x = 0$

Therefore there is no Y-intercept for this, rational function

Graphing the Rational Functions

To sketch the graph of a rational function you may need to follow the ensuing pointers in order to keep it infallible and glitch free.

1. Find the domain
2. Find the Vertical Asymptotes and holes
3. Find the Horizontal/Slant/Curvilinear asymptote
4. Find the X-intercepts and Y-intercepts
5. Find the points where graph intercepts the non-vertical asymptote
6. Finally determine where the graph will be positive or negative in different intervals to draw the graph then you can easily sketch it.

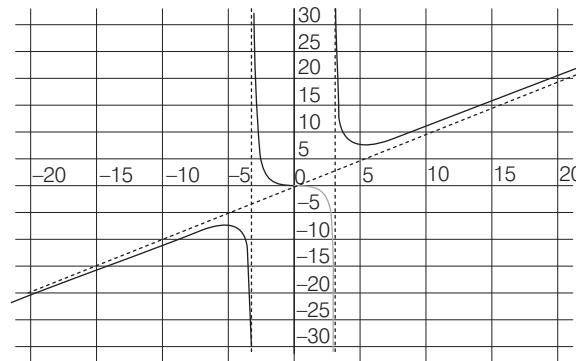
Exp. 1) Find the asymptotes, intercepts and sketch the graph of the function $f(x) = \frac{x^3}{x^2-9}$

Solution. Factoring the denominator, you will get the roots of the denominator -3 and 3 . Consequently, $x = -3$ and $x = 3$ are the vertical asymptotes of the given function $f(x)$.

The degree of the numerator is greater than the degree of the denominator by 1, so there will be a slant asymptote. Using polynomial long division, we obtain

$$f(x) = \frac{x^3}{x^2-9} = x + \frac{9x}{x^2-9}$$

So the quotient polynomial is $k(x) = x$. That is $y = x$ is a slant asymptote. The only zero (root) of the numerator is 0, as $f(0) = 0$ and thus we see that the point $(0,0)$ is the only x-intercept and the y-intercept of the graph of $f(x)$.



The graph of $f(x)$ in the above figure passes through $(0,0)$ where $x = -3$ and $x = 3$ are the vertical asymptotes and the slant asymptote is $y = x$.

Exp. 2) Find the asymptotes, intercepts and sketch the graph of the function $f(x) = \frac{2x+6}{x^2-2x-15}$.

Solution The given rational function can be expressed as following.

$$f(x) = \frac{2x+6}{x^2-2x-15} = \frac{2(x+3)}{(x+3)(x-5)}$$

- (i) The Domain: $(-\infty, -3) \cup (-3, 5) \cup (5, \infty)$. Since, at $x = -3$ and $x = 5$, the given function is not defined.
- (ii) Vertical Asymptotes/Holes: One asymptote is there at $x = 5$. Also, there is one hole in the graph. Since the power of $(x+3)$ in the numerator is same as that of denominator, so there will be a hole in the graph at $x = -3$, but not on the X-axis. The y-value of the hole can be found by canceling out the common factors and substituting $x = -3$ in the reduced function as following.

$$y = \frac{2}{(x-5)} = \frac{2}{(-3-5)} = \frac{1}{-4} = 0.25$$

- (iii) Non-vertical Asymptote: Since the degree of numerator is less than that of denominator, therefore there will be a horizontal asymptote.

Then the horizontal asymptote is the X-axis itself, That is $y = 0$.

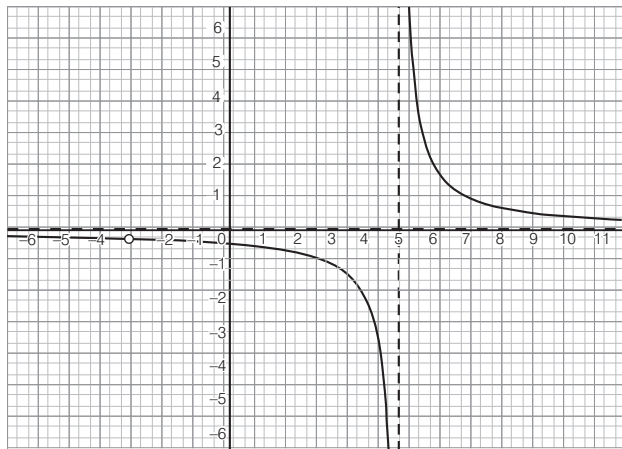
- (iv) X-intercepts: It is the value of x when $P(x) = 0$, but $q(x) \neq 0$, for the same value of x . Therefore, no x-intercept is there because at $x = -3$ the function $f(x)$ is not defined.

- (v) Y-intercept: It is the value of y when the value of $x = 0$.
 Therefore y-intercept is $y = \frac{2(0+2)}{(0+3)(0-5)} = -\frac{6}{15} = -\frac{2}{5}$.

Thus the coordinates of y-intercept are $\left\{0, -\frac{2}{5}\right\}$.

- (vi) Points where the graph intercepts the non-vertical asymptote: Since the horizontal asymptote is X-axis itself.

So the horizontal asymptote overlaps the X-axis. Now, since there are no x -intercepts, so there is no such point where graph intercepts the horizontal asymptote.



The above graph manifests that when $x = -3$ there is a hole in the graph at $y = -0.25$ and being a vertical asymptote at $x = 5$, the graph approaches the Y-axis but does not intercept it.

For, all the values of $x < 5$, all the values of $y < 0$ and for all the values of $x > 5$, all the values of $y > 0$.

Exp. 3) Find the asymptotes, intercepts and sketch the graph of the function $f(x) = \frac{x^2 - x - 6}{x - 3}$.

Solution The given rational function can be expressed as following.

$$f(x) = \frac{x^2 - x - 6}{x - 3} = \frac{(x + 2)(x - 3)}{(x - 3)}$$

(i) The Domain: $(-\infty, 3) \cup (3, \infty)$. Since, at $x = 3$, the function is not defined.

(ii) Vertical Asymptotes/Holes: There is no vertical asymptote since after canceling out the common factors, there is no factor at all in the denominator.

$$f(x) = \frac{x^2 - x - 6}{x - 3} = \frac{(x + 2)(x - 3)}{(x - 3)} = (x + 2)$$

Since the power of $(x - 3)$ in the numerator is same as that of denominator, so there will be a hole in the graph at $x = 3$, but not on the X-axis. The value of the hole can be found by canceling out the common factors and substituting $x = 3$ in the reduced function as following.

$$y = (x + 2) = (3 + 2) = 5$$

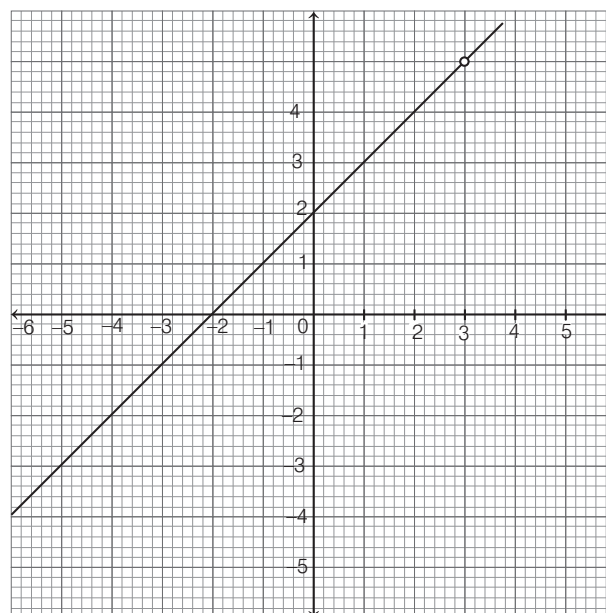
(iii) Non-vertical Asymptote: Since the degree of numerator is one more than the denominator, therefore the graph will experience a slant asymptote, not the horizontal one.

The value of the slant asymptote is equal to the quotient of the function when numerator is divided by the denominator. So, the quotient $= (x + 2)$. Therefore, the slant asymptote is also $y = (x + 2)$.

(iv) X-intercepts: It is the value of x when $p(x) = 0$, but $q(x) \neq 0$, for the same value of x . Therefore x -intercept is obtained at $x = -2$. Thus the coordinates of x -intercept are $\{-2, 0\}$.

(v) Y-intercept: It is the value of y when the value of $x = 0$. Therefore y -intercept is $y = (0 + 2) = 2$. Thus the coordinates of y -intercept are $\{0, 2\}$.

(vi) Points where the graph intercepts the non-vertical asymptote: Since the reduced function is same as the equation for slant asymptote, it implies that the graph of the given function will overlap the slant asymptote.



The only difference between the slant asymptote of the rational function and the rational function itself is that the rational function isn't defined at $x = 3$. To account for this, you have to leave an open circle in the graph corresponding to the point $x = 3$, which indicates that this point is not actually included on the graph, because of the zero in the denominator of the rational.

Exp. 4) Find the asymptotes, intercepts and sketch the graph of the function $f(x) = \frac{x^3 - 4x}{x^2 - 4x + 4}$.

Solution The given rational function can be expressed as following.

$$f(x) = \frac{x^3 - 4x}{x^2 - 4x + 4} = \frac{x(x + 2)(x - 2)}{(x - 2)^2}$$

(i) The domain: $(-\infty, 2) \cup (2, \infty)$. Since, at $x = 2$, the given function is not defined.

(ii) Vertical Asymptotes/Holes: There is one vertical asymptote $x = 2$

(iii) Non-vertical Asymptote: Since the degree of numerator is one more than the denominator, therefore the graph will experience a slant asymptote, not the horizontal one.

The value of the slant asymptote is equal to the quotient of the function when numerator is divided by the denominator. Since

$$x^3 - 4x = (x^2 - 4x + 4)(x + 4) + (8x - 16),$$

so, the quotient is $(x + 4)$. Therefore, the slant asymptote is also $y = (x + 4)$.

- (iv) X-intercepts: It is the value of x when $p(x) = 0$, but $q(x) \neq 0$, for the same value of x . Therefore x -intercept is obtained at $x = 0$ and $x = -2$. Thus the coordinates of x -intercept are $\{0, 0\}$ and $\{-2, 0\}$.

- (v) Y-intercept: It is the value of y when the value of $x = 0$. Therefore y -intercept is

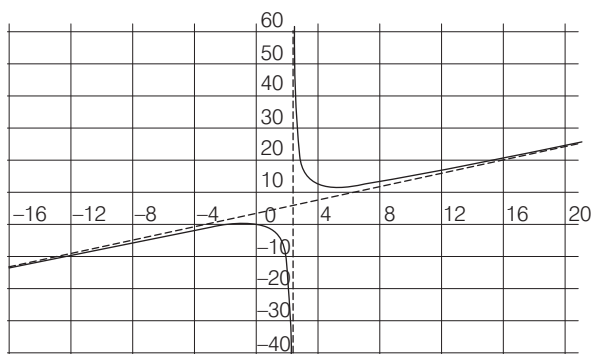
$$y = [0(0+2)(0-2)] / [(0-2)(0-2)] = 0.$$

Thus the coordinates of y -intercept are $\{0, 0\}$.

- (vi) Points where the graph intercepts the non-vertical asymptote: To determine such points you need to equate the given function with the equation of slant asymptote.

$$\text{Then } \frac{x^3 - 4x}{x^2 - 4x + 4} = x + 4$$

Since you don't get any solution here, it shows that the graph never intersects the slant asymptote.



Exp. 5) Find the asymptotes, intercepts and sketch the graph of the function $f(x) = \frac{x^2 + 9}{x^2 - 6x + 9}$

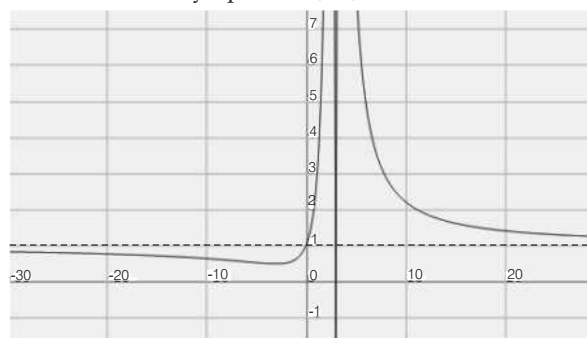
Solution The given rational function can be expressed as following $f(x) = \frac{x^2 + 9}{x^2 - 6x + 9} = \frac{x^2 + 9}{(x-3)^2}$

- The domain: $(-\infty, 3) \cup (3, \infty)$. Since, at $x = 3$, the given function is not defined.
- Vertical Asymptotes/Holes: There is one vertical asymptote $x = 3$.
- Non-vertical Asymptote: Since the degree of numerator and denominator is same, therefore there will be a horizontal asymptote $y = 1$.
- X-intercepts: It is the value of x when $p(x) = 0$, but $q(x) \neq 0$, for the same value of x . Now, when $x^2 + 9 = 0$, you won't get the real roots, so there won't be any x -intercept by this graph.

- (v) Y-intercept: It is the value of y when the value of $x = 0$. Therefore y -intercept is $y = 9 / 9 = 1$. Thus the coordinates of y -intercept are $\{0, 1\}$.

- (vi) Points where the graph intercepts the non-vertical asymptote: To determine such points you need to equate the given function with the equation of horizontal asymptote. Then $\frac{x^2 + 9}{x^2 - 6x + 9} = 1 \Rightarrow x = 0$

Therefore the graph of the given function will intercept the horizontal asymptote at $\{0, 1\}$.



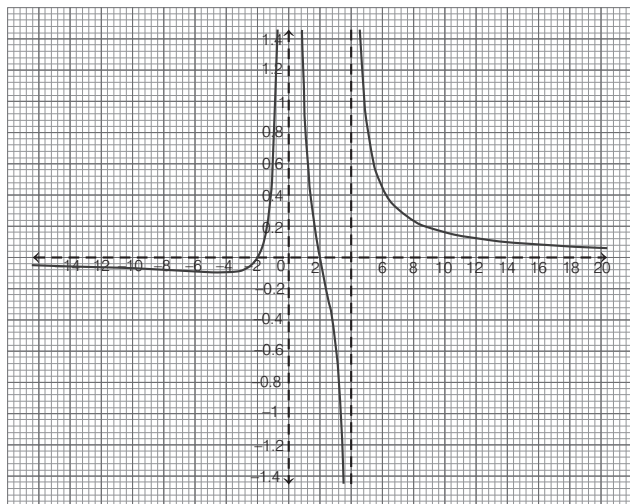
Please mind the fact that this rational function has two non-real roots and no real roots. That's why in the absence of any real root the graph does not have any x -intercepts.

Exp. 6) Find the asymptotes, intercepts and sketch the graph of the function $f(x) = \frac{x^2 - 4}{x^3 - 4x^2}$.

Solution The given rational function can be expressed as following $f(x) = \frac{x^2 - 4}{x^3 - 4x^2} = \frac{(x+2)(x-2)}{x^2(x-4)}$

- The domain: $(-\infty, 0) \cup (0, 4) \cup (4, \infty)$. Since, at $x = 0, 4$ the given function is not defined.
- Vertical Asymptotes/Holes: There are two vertical asymptotes $x = 0$ and $x = 4$.
- Non-vertical Asymptote: Since the degree of numerator is less than that of denominator, therefore there will be a horizontal asymptote $y = 0$.
- X-intercepts: It is the value of x when $p(x) = 0$, but $q(x) \neq 0$, for the same value of x . Therefore x -intercept is obtained at $x = -2$ and $x = 2$. Thus the coordinates of X-intercept are $\{-2, 0\}$ and $\{2, 0\}$.
- Y-intercept: It is the value of y when the value of $x = 0$. Since at $x = 0$, the function is not defined, therefore there won't be any y -intercept.
- Points where the graph intercepts the non-vertical asymptote: To determine such points you need to equate the given function with the equation of horizontal asymptote.

$$\text{Then } \frac{x^2 - 4}{x^3 - 4x^2} = 0 \Rightarrow x = \pm 2$$



Therefore the graph of the given function will intercept the horizontal asymptote at $\{0, -2\}$ and $\{0, 2\}$.

Exp. 7) Find the asymptotes, intercepts and sketch the graph of the function $f(x) = \frac{x^3 + 2x^2 - 59x - 168}{x - 5}$

Solution The given rational function can be expressed as following

$$f(x) = \frac{x^3 + 2x^2 - 59x - 168}{x - 5} = \frac{(x + 3)(x + 7)(x - 8)}{(x - 5)}$$

- (i) The domain: $(-\infty, 5) \cup (5, \infty)$. Since, at $x = 5$ the given function is not defined.
- (ii) Vertical Asymptotes/Holes: There is one vertical asymptote $x = 5$.
- (iii) Non-vertical Asymptote: Since the degree of numerator is 2 more than that of denominator, therefore there will be a curvilinear asymptote. Now since

$$f(x) = \frac{x^3 + 2x^2 - 59x - 168}{x - 5} = (x^2 + 7x - 24) + \frac{-288}{(x - 5)}$$

Therefore the curvilinear asymptote is $y = x^2 + 7x - 24$.

- (iv) X-intercepts: It is the value of x when $p(x) = 0$, but $q(x) \neq 0$, for the same value of x .

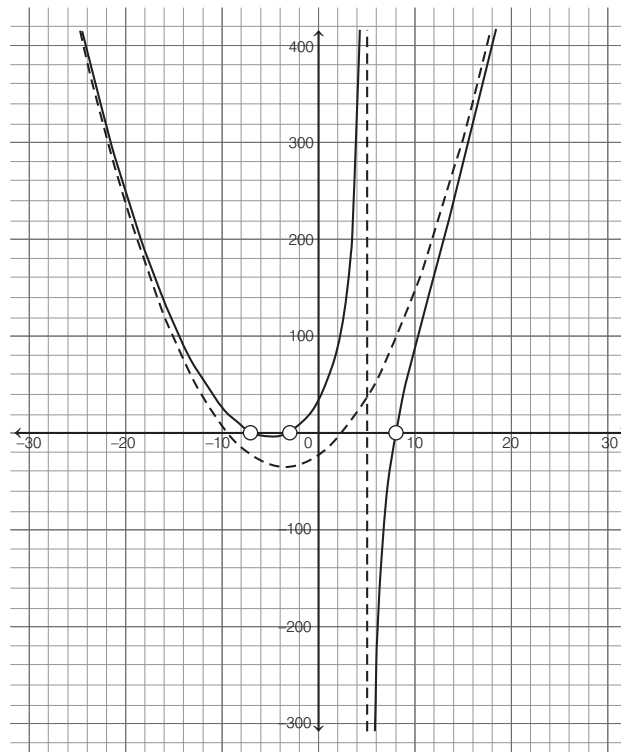
Therefore x -intercept is obtained at $x = -3, x = -7$ and $x = 8$. Thus the coordinates of x -intercept are $\{-3, 0\}, \{-7, 0\}$ and $\{8, 0\}$.

- (v) Y-intercept: It is the value of y when the value of $x = 0$. Therefore the y -intercept is $168/5$. And so the respective coordinates are $\{0, 168/5\}$.

- (vi) Points where the graph intercepts the horizontal/slant/curvilinear asymptote: To determine such points you need to equate the given function with the equation of horizontal asymptote.

$$\text{Then } \frac{x^3 + 2x^2 - 59x - 168}{x - 5} = x^2 + 7x - 24$$

There is no solution for the above equation. Therefore the graph of the given function does not intercept the curvilinear asymptote.



Practice Exercise

- Which of the following is not true about the features of a rational function?
 - Can be written as a polynomial over polynomial (like a fraction)
 - Variables do not have fractional or imaginary exponents
 - A variable must at least be in the denominator of a ratio
 - Variables cannot be exponents themselves
 - Variables cannot exist within absolute value delimiters
 - Variables cannot be applied to a trigonometric function
 - (i) and (iii)
 - (iv) and (vi)
 - (ii) and (v)
 - none of these
- Which of the following is not true regarding a rational function?
 - The equation of vertical asymptote is $x = k$, for some constant k
 - The equation of horizontal asymptote is $y = k$, for some constant k
 - The equation of slant asymptote is $y = mx + c$, for some constants m and c
 - There can't be both the vertical and horizontal asymptotes together
- Which of the following is not true regarding a rational function?
 - There can be only one of the asymptotes, either horizontal or slant or curvilinear at a time
 - It's not necessary that there is always a non-vertical asymptote
 - There cannot be more than one non-vertical asymptote
 - The rational graph cannot intersect a non-vertical asymptote
- Which of the following is true for the following rational function?

$$f(x) = \frac{8x^7 - 32x^6 + 3x^5 - 12x^4 - 10x + 40}{2x^2 - 32}$$
 - There is one vertical asymptote at $x = -4$
 - The horizontal asymptote is $y = 4$.
 - The slant asymptote is $y = 7/2$.
 - There are two vertical asymptotes.
- Which of the following is correct for the rational function

$$f(x) = \frac{3x^2 + 27x + 25}{3x + 4}$$
 - The horizontal asymptote is $y = 0$
 - The slant asymptote is $y - 1 = 0$
 - The vertical asymptote is $y = -4/3$
 - The slant asymptote is $x + 7$.
- The number of times the graph intercepts the horizontal asymptote of the following function.

$$f(x) = \frac{(x^2 - 4)(x^2 - 25)}{x^5(x^2 - 4x)}$$
 - 0
 - 1
 - 4
 - none of these
- Which of the following is/are always true regarding a polynomial rational function?
 - There is no need to know about the numerator since y -intercepts can be determined just by finding all the zeros of denominator.
 - All the roots of the numerator yield the x -intercepts of the rational function.
 - In case of the X -axis acting as the horizontal asymptote and you know the x -intercepts, then there is no need to further work out the points of interception between the graph and the horizontal asymptote as they are already known.
 - There is at most one point of interception between the graph of the rational function and horizontal/slant/curvilinear asymptote.
 - Whenever the X -axis acts as a horizontal asymptote there would not be any x -intercept.
 - Only (iv)
 - (ii) and (v)
 - (i), (ii) and (iv)
 - Only (iii)
- Which of the following is/are never true about the rational function?
 - Wherever there is no domain there is either a vertical asymptote or a hole in the graph.
 - All the critical points either have x -intercept or vertical asymptote or a hole in the graph.
 - Wherever there is no domain there are vertical asymptotes and holes together for the same value of x .
 - Union of all the roots of numerator and denominator forms the set of critical numbers.
 - A rational graph may intercept or touch the X -axis at $x = k$, even when there is a hole on the X -axis at $x = k$.
 - (iv) and (v)
 - (iii), (iv) and (v)
 - (i), (ii) and (iii)
 - (ii) and (iii)
- For the following rational function which one of the given statements is/are incorrect?

$$f(x) = \frac{(x-1)(x-3)^2(x-7)^3}{(x-1)^3(x-3)^2(x-7)}$$
 - Domain does not include $x = 1, 3$ and 7 .
 - Critical points are $x = 1, 3, 7$
 - The only vertical asymptote is $x = 1$

- (iv) There is one hole in the graph at $x = 7$ and another hole is at $\{x, y\} = \{3, 4\}$
- (v) There is only one root or one x -intercepts at $x = 7$.
- (vi) There is one horizontal asymptote $y = 1$.
- (vii) The y -intercept is $y = 49$.
- (a) (iii) and (iv)
- (b) (v) and (vii)
- (c) (iv) and (v)
- (d) (v)

10. There are six different rational functions as shown below.

$$f(x) = \frac{x^2}{x}, g(x) = \frac{x}{x}, h(x) = \frac{x}{x^2}$$

$$j(x) = \frac{x^4 - 16}{x^2 + 4}, k(x) = \frac{x^2 + 4}{x^2 + 4}$$

$$l(x) = \frac{x^2 + 4}{x^4 - 16}$$

The following table gives the facts from (i) to (x) regarding the above functions. How many facts are correct for all the functions?

Facts	$f(x)$	$g(x)$	$h(x)$	$j(x)$	$k(x)$	$l(x)$
(i) Domain	$R - \{0\}$	$R - \{0\}$	$R - \{0\}$	R	R	$R - \{-2, 2\}$
(ii) Critical points	0	0	0	-2, 2	No	-2, 2
(iii) Vertical asymptotes	No	No	$x = 0$	No	No	$x = -2, 2$
(iv) Holes	$x = 0$	$\{x, y\} = \{0, 1\}$	No	No	No	No
(v) Roots	No	No	No	-2, 2, $\pm 2i$	No	No
(vi) x -intercepts	No	No	No	-2, 2, $\pm 2i$	No	No
(vii) Horizontal asymptote	No	$y = 1$	$y = 0$	No	$y = 1$	$y = 0$
(viii) Slant asymptote	$y = x$	No	No	No	No	No
(ix) Curvilinear asymptote	No	No	No	$x^2 - 4$	No	No
(x) y -intercept $\{x, y\}$	No	No	No	$\{0, -4\}$	$\{0, 1\}$	$\{0, -1/4\}$

- (a) 6
- (b) 7
- (c) 8
- (d) 10

11. There are some facts presented in the table regarding three different rational functions.

	Function $f(x)$	Function $g(x)$	Function $h(x)$
Critical points	$-5/2, -5/3$	$2/3, 3/5$	$-9, 0, 1, 6$
Domain	$R - \{-5/2, -5/3\}$	$R - \{2/3\}$	$R - \{-9, 0, 1\}$
Vertical asymptotes	$x = -5/2$	No	$x = \{-9, 1\}$
Holes	$x = -5/3$	$\{x, y\} = \{2/3, 1/242\}$	$x = 0$
Roots	$\pm 4i$	$3/5$	6
x -intercepts	No	$3/5$	6
Horizontal asymptote	$y = 0$	$y = 125$	No
Slant asymptote	No	No	No
Curvilinear asymptote	No	No	$y = x^2 - 5x - 14$
y -intercept	4	$3/2$	No

Which of the following functions represent the given description correctly?

- (a) $f(x) = \frac{(2x+5)(3x+5)^2(x^2+16)}{(3x+5)(2x+5)^3(x^4+64)}$,
 $g(x) = \frac{(3x-2)(5x-3)^5}{(3x^4+34)(3x-2)}$, $h(x) = \frac{x^7-7x^6+6x^5}{(x^4+9x^3)(x-1)^2}$
- (b) $f(x) = \frac{(2x+5)^2(3x+5)^2(x^2+16)^2}{(3x+5)(2x+5)^3(x^4+4)}$,
 $g(x) = \frac{(3x-2)^3(5x-3)^4}{(4x^4+4)(3x-2)}$, $h(x) = \frac{x^7-7x^6+6x^5}{(x^3+9x^2)(x-1)}$
- (c) $f(x) = \frac{(2x+5)(3x+5)^2(x^2+16)}{(3x+5)(2x+5)^2(x^4+4)}$,
 $g(x) = \frac{(3x-2)(5x-3)^4}{(5x^4+54)(3x-2)}$,
 $h(x) = \frac{x^7-7x^6+6x^5}{(x^3+9x^2)(x-1)^2}$
- (d) $f(x) = \frac{(2x-5)(3x-5)^2(x^2-16)}{(3x+5)(2x+5)^2(x^4+4)}$,
 $g(x) = \frac{(3x+2)(5x-3)^4}{(5x^4+54)(3x+2)}$, $h(x) = \frac{x^7-7x^6-6x^5}{(x^3+9x^2)(x-1)^2}$

Answers

1. (d)	2. (d)	3. (d)	4. (a)	5. (d)
6. (c)	7. (d)	8. (b)	9. (d)	10. (d)
11. (c)				

14.19 Rational Polynomial Inequalities

There are four different ways of representing the rational polynomial inequations:

$$\begin{aligned} \text{(i)} \quad \frac{p(x)}{q(x)} &> 0 & \text{(ii)} \quad \frac{p(x)}{q(x)} &\geq 0 \\ \text{(iii)} \quad \frac{p(x)}{q(x)} &< 0 & \text{(iv)} \quad \frac{p(x)}{q(x)} &\leq 0 \end{aligned}$$

Here $p(x)$ and $q(x)$ are polynomials of same or varying degrees:

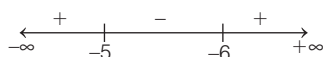
Solving Rational Inequalities

1. First of all bring all the values towards the left side of the inequality sign and leave just 0 in the right side.
2. Now, simplify all the terms in the left side of the inequality sign making a single rational function.
3. Now what you need to do is to factorize numerator and denominator, assuming that the numerator and denominator each are equal to 0; and determine the critical points. Critical points are all those points which are either the real roots of the rational function or the points on the X-axis where the rational function is not defined.
4. Then mark all the critical points on the number line in the conventional order, which in turn these critical points will create intervals.
5. Now test these intervals for signs; which intervals give positive and which ones give negative values.
6. If your inequality is positive, choose all the positive intervals and if it is negative inequality, choose the negative intervals. And then combine all such desired intervals.
7. Do not forget to consider the limit of the interval; if it's a strict inequality, do not include limits of the interval and if it's not a strict inequality, include the limits of the interval.

NOTE Coincidentally, you may notice that a critical point has alternating signs on the different sides of it. However, it is very uncommon in the rational functions. So make sure that you have tested the sign in each interval before concluding the results.

Exp. 1) Find the values of x which satisfy this rational inequation $\frac{x+5}{x-6} \leq 0$

Solution. The critical points of this rational function are -5 and 6 .



After arranging the critical points on the number line, we are now supposed to test the sign in each interval.

Let us assume $x = -10$, then $\frac{x+5}{x-6} = \frac{-10+5}{-10-6} = \frac{5}{4}$

Therefore the given function is positive for the first interval. Again let us assume $x = 0$, then

$$\frac{x+5}{x-6} = \frac{0+5}{0-6} = -\frac{5}{6}$$

Therefore the given function is negative for the second interval. Now let us assume $x = 10$, then $\frac{x+5}{x-6} = \frac{10+5}{10-6} = \frac{15}{4}$

Therefore the given function is positive for the third and last interval. Since, our inequality is non-positive so we will not consider the first and the third intervals. Further since it's not a strict inequality, so we have to include the boundary values in the interval.

Therefore the required values of x are $[-5, 6)$ or $-5 \leq x < 6$.

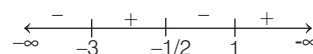
NOTE Since at $x = 6$, the rational function is undefined so we cannot say that whether the function will be positive or negative. Therefore it's better to ignore this boundary value from the interval consideration. However at $x = 5$ the given function is zero, which is acceptable by the inequality so we have to include it.

Exp. 2) Find the values of x which satisfy this rational inequation $\frac{5}{x+3} < \frac{-3}{x-1}$.

Solution. The given inequality can be simplified as shown below.

$$\frac{5}{x+3} < \frac{-3}{x-1} \Rightarrow \frac{5}{x+3} + \frac{3}{x-1} < 0 \Rightarrow \frac{4(2x+1)}{(x+3)(x-1)} < 0$$

The critical points of this rational function are $-\frac{1}{2}$, -3 and 1 .



After arranging the critical points on the number line, we are now supposed to test the sign in each interval.

Let us assume $x = -5$, then

$$\frac{4(-10+1)}{(-5+3)(-5-1)} = -3$$

Therefore the given function is negative for the first interval.

Again let us assume $x = -2$, then $\frac{4(-4+1)}{(-2+3)(-2-1)} = 4$

Therefore the given function is positive for the second interval.

Now let us assume $x = 0$, then $\frac{4(0+1)}{(0+3)(0-1)} = -\frac{4}{3}$

Therefore the given function is negative for the third interval.

Finally let us assume $x = 2$,

$$\text{then } \frac{4(4+1)}{(2+3)(2-1)} = 4$$

Therefore the given function is positive for the fourth and last interval.

Since, our inequality is negative so we will consider the first and the third intervals only. Further since it's a strict inequality, so we have to exclude the boundary values from the interval.

Therefore the required values of x are

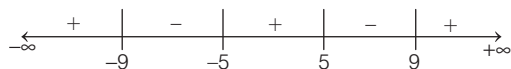
$$(-\infty, -3) \cup \left(-\frac{1}{2}, 1\right)$$

Exp. 3) Find the values of x which satisfy this rational inequation $\frac{x^2 - 81}{x^2 - 25} \leq 0$.

solution The given inequality can be simplified as shown below.

$$\begin{aligned} \frac{x^2 - 81}{x^2 - 25} &\leq 0 \\ \Rightarrow \frac{(x+9)(x-9)}{(x+5)(x-5)} &\leq 0 \end{aligned}$$

The critical points of this rational function are $-9, 9, -5$, and 5 .



Now testing each of the 5 intervals we get the results as shown in the above number line.

Here we need non-positive intervals so we exclude all the positive intervals and combine the others keeping in mind the restriction on limits of the intervals.

Therefore the valid values of x are $[-9, -5) \cup (5, 9]$.

NOTE The critical points $-5, 5$ are not included in the domain since the function is not defined there and $-9, 9$ are included since they satisfy the given relation.

Exp. 4) Find the values of x which satisfy this rational inequation $\frac{x^2 + 5x + 4}{x^2 - 16} \geq 0$.

Solution The given inequality can be simplified as shown below.

$$\begin{aligned} \frac{x^2 + 5x + 4}{x^2 - 16} &\geq 0 \\ \Rightarrow \frac{(x+4)(x+1)}{(x+4)(x-4)} &\geq 0 \end{aligned}$$

The critical points of this rational function are $-4, -1$, and 4 .



Now testing each of the 4 intervals we get the results as shown on the above number line.

Here we need non-negative intervals so we exclude all the negative intervals and combine the others keeping in mind the restriction on limits of the intervals. Therefore the valid values of x are

$$(-\infty, -4) \cup (-4, -1] \cup (4, \infty)$$

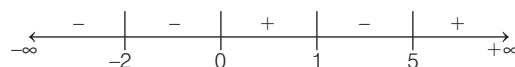
NOTE The critical points $-4, 4$ are not included in the domain since the function is not defined there and -1 is included since it satisfies the given relation.

Exp. 5) Find the values of x which satisfy this rational inequation $\frac{x^3 + 4x^2 + 4x}{x^2 - 6x + 5} > 0$.

Solution The given inequality can be simplified as shown below.

$$\frac{x^3 + 4x^2 + 4x}{x^2 - 6x + 5} > 0 \Rightarrow \frac{x(x+2)^2}{(x-1)(x-5)} > 0$$

The critical points of this rational function are $-2, 0, 1$, and 5 .



Now testing each of the 4 intervals we get the results as shown on the above number line.

Here we need positive intervals so we exclude all the negative intervals and combine the others keeping in mind the restriction on limits of the intervals.

Therefore the valid values of x are $(0, 1) \cup (5, \infty)$.

NOTE The critical points -1 and 5 are not included in the domain since the function is not defined there and the critical point 0 is also not included because there is a strict inequality.

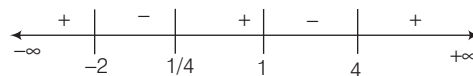
Exp. 6) Solve the rational inequality

$$\frac{x-2}{x+2} > \frac{2x-3}{4x-1}$$

Solution The given inequality can be simplified as shows below

$$\begin{aligned} \frac{x-2}{x+2} &> \frac{2x-3}{4x-1} \\ \Rightarrow \frac{x-2}{x+2} - \frac{2x-3}{4x-1} &> 0 \\ \text{or } \frac{(x-2)(4x-1) - (x+2)(2x-3)}{(x+2)(4x-1)} &> 0 \\ \text{or } \frac{2(x^2 - 5x + 4)}{(x+2)(4x-1)} &> 0 \\ \text{or } \frac{2(x-1)(x-4)}{(x+2)(4x-1)} &> 0 \end{aligned}$$

The critical points of this rational function are $-2, \frac{1}{4}, 1$ and 4 .



Now testing each of the 5 intervals we get the results as shown on the above number line.

Here we need positive intervals so we exclude all the negative intervals and combine the others keeping in mind the restriction on limits of the intervals.

Therefore the valid values of x are $(-\infty, -2) \cup \left(\frac{1}{4}, 1\right) \cup (4, \infty)$.

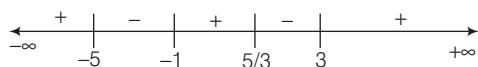
NOTE The critical points -2 and $1/4$ are not included in the domain since the function is not defined there. And, the critical point 1 and 4 are also not included because there is a strict inequality.

Exp. 7) Determine the range of values of x for which $\frac{x^2 - 2x + 5}{3x^2 - 2x - 5} > \frac{1}{2}$.

Solution The given inequality can be simplified as shown below

$$\begin{aligned} \frac{x^2 - 2x + 5}{3x^2 - 2x - 5} &> \frac{1}{2} \quad \text{or} \quad \frac{x^2 - 2x + 5}{3x^2 - 2x - 5} - \frac{1}{2} > 0 \\ \text{or} \quad \frac{-x^2 - 2x + 15}{3x^2 - 2x - 5} &> 0 \Rightarrow \frac{x^2 + 2x - 15}{3x^2 - 2x - 5} \\ \text{or} \quad \frac{(x+5)(x-3)}{(x+1)(3x-5)} &< 0 \end{aligned}$$

The critical points of this rational function are $-5, -1, \frac{5}{3}$ and 3 .



Now testing each of the 5 intervals we get the results as shown on the above number line.

Here we need the negative intervals so we exclude all the positive intervals and combine the others keeping in mind the restriction on limits of the intervals. Therefore the valid values of x are $(-5, -1) \cup \left(\frac{5}{3}, 3\right)$

NOTE The critical points -1 and $5/3$ are not included in the domain since the function is not defined there. And, the critical point -5 and 3 are also not included because there is a strict inequality.

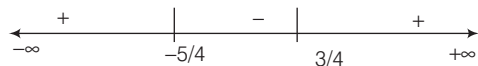
Exp. 8) Find all the real values of x for which

$$\frac{x-1}{(4x+5)} < \frac{x-3}{4x-3}$$

Solution The given inequality can be simplified as shown below $\frac{x-1}{4x+5} < \frac{x-3}{4x-3} \Rightarrow \frac{x-1}{4x+5} - \frac{x-3}{4x-3} < 0$

$$\begin{aligned} \Rightarrow \frac{(x-1)(4x-3) - (x-3)(4x+5)}{(4x-3)(4x+5)} &< 0 \\ \text{or} \quad \frac{(4x^2 - 7x + 3) - (4x^2 - 7x - 15)}{(4x+5)(4x-3)} &< 0 \\ \text{or} \quad \frac{18}{(4x+5)(4x-3)} &< 0 \end{aligned}$$

The critical points of this rational function are $-5/4$ and $3/4$.



Now testing each of the 3 intervals we get the results as shown on the above number line.

Here we need the negative intervals so we exclude all the positive intervals and combine the others keeping in mind the restriction on limits of the intervals.

Therefore the valid values of x are $\left(-\frac{5}{4}, \frac{3}{4}\right)$.

NOTE The critical points $-5/4$ and $3/4$ are not included in the domain since the function is not defined there.

Practice Exercise

Directions (for Q. Nos. 1 to 3) Find the values of x for which of the following equations are satisfied.

- $\frac{x^2 + 4x + 3}{x-1} > 0$
- $\frac{x^2 - 16}{(x-1)^2} < 0$
- $\frac{x-8}{x} + x \leq 3$

Answers

- $(-3, -1) \cup (1, \infty)$
- $(-4, 1) \cup (1, 4)$
- $(-\infty, 2] \cup (0, 4]$

14.20

Maximum and Minimum Value of a Rational Expression

Exp. 1) For any real value of x , find the minimum value of the rational expression $\frac{x^2 + x + 1}{x^2 - x + 1}$.

- $-1/6$
- $1/3$
- $2/9$
- 1

Solution Let $\frac{x^2 + x + 1}{x^2 - x + 1} = k$

$$\Rightarrow x^2 + x + 1 = k(x^2 - x + 1) \Rightarrow (1-k)x^2 + (1+k)x + (1-k) = 0$$

Since x is a real number, so the discriminant $D \geq 0$. That is

$$\begin{aligned} (1+k)^2 - 4(1-k)(1-k) &\geq 0 \\ \Rightarrow -3k^2 + 10k - 3 &\geq 0 \Rightarrow 3k^2 - 10k + 3 \leq 0 \\ \Rightarrow (k-3)(3k-1) &\leq 0 \\ \Rightarrow \frac{1}{3} &\leq k \leq 3 \end{aligned}$$

Hence the minimum value of the given expression is $1/3$.

Exp. 2) If x be real, find the maximum value of $\frac{(x+2)}{(2x^2 + 3x + 6)}$.

Solution Let $y = \frac{x+2}{2x^2 + 3x + 6}$

$$\text{then } 2x^2y + (3y-1)x + 6y - 2 = 0$$

$$\text{For } x \text{ to be real, } (3y-1)^2 - 8y(6y-2) \geq 0$$

$$\text{or } (1+13y)(1-3y) \geq 0 \text{ or } (13y+1)(3y-1) \leq 0$$

Putting each factor equal to zero, we get $y = -\frac{1}{13}, \frac{1}{3}$

$$\text{If } y < -\frac{1}{13}, (1+13y)(1-3y) < 0$$

$$\text{If } -\frac{1}{13} < y < \frac{1}{3}, (1+13y)(1-3y) > 0$$

$$\text{If } y > \frac{1}{3}, (1+13y)(1-3y) < 0$$

Thus, y will lie between $-\frac{1}{13}$ and $\frac{1}{3}$.

Hence the maximum value of y is $\frac{1}{3}$ and minimum value is $-\frac{1}{13}$.

Exp. 3) If x is real, prove that the value of the expression $\frac{(x-1)(x+3)}{(x-2)(x+4)}$ cannot be between $\frac{4}{9}$ and 1.

Solution Let $\frac{(x-1)(x+3)}{(x-2)(x+4)} = y$, then $x^2 + 2x - 3 = y(x^2 + 2x - 8)$

$$\text{or } x^2(1-y) + 2x(1-y) + (8y-3) = 0$$

For x to be real, its discriminant $D \geq 0$

$$\text{or } [2(1-y)]^2 - 4(1-y)(8y-3) \geq 0$$

$$\text{or } (1-y)(4-9y) \geq 0 \text{ or } (1-y)\left(\frac{4}{9}-y\right) \geq 0$$

It shows that $(1-y)$ and $(4/9-y)$ should be of the same sign.

$$\therefore \text{ we get } y = \frac{4}{9}, 1$$

$$\text{If } y < \frac{4}{9}, \quad (1-y)\left(\frac{4}{9}-y\right) > 0$$

$$\text{If } \frac{4}{9} < y < 1, \quad (1-y)\left(\frac{4}{9}-y\right) < 0$$

$$\text{If } y > 1 \quad (1-y)\left(\frac{4}{9}-y\right) > 0$$

Hence for x to be real, y (i.e., the value of the given expression) cannot lie between $\frac{4}{9}$ and 1.

Introductory Exercise 14.1

Directions (for Q. Nos. 1 to 12) Find the roots of the following equations.

1. $15x^2 - 7x - 36 = 0$:

- (a) $\frac{5}{9}, -\frac{4}{3}$ (b) $\frac{9}{5}, -\frac{4}{3}$
(c) $\frac{9}{5}, -\frac{3}{4}$ (d) none of these

2. $\sqrt{7}y^2 - 6y - 13\sqrt{7} = 0$:

- (a) $\sqrt{7}, 2\sqrt{7}$ (b) $3, \frac{2}{\sqrt{7}}$
(c) $\frac{13}{\sqrt{7}}, -\sqrt{7}$ (d) none of these

3. $6x^2 + 40 = 31x$:

- (a) $\frac{3}{8}, \frac{2}{5}$ (b) $\frac{3}{8}, \frac{3}{2}$ (c) $0, \frac{8}{3}$ (d) $\frac{8}{3}, \frac{5}{2}$

4. $6\left(\frac{y-3}{2y+1}\right) + 1 = 5\sqrt{\frac{y-3}{2y+1}}, y \neq \frac{1}{2}$:

- (a) $4, \frac{13}{2}$ (b) $4, \frac{11}{2}$
(c) $\frac{13}{2}, \frac{9}{2}$ (d) none of these

5. $(x+2)(x-5)(x-6)(x+1) = 144$:

- (a) $-1, -2, -3$ (b) $7, -3, 2$
(c) $2, -3, 5$ (d) none of these

6. $\sqrt{x+5} + \sqrt{x+21} = \sqrt{6x+40}$:

- (a) 1 (b) 2 (c) 4 (d) 6

7. $2\left(x^2 + \frac{1}{x^2}\right) - 3\left(x + \frac{1}{x}\right) - 1 = 0$:

- (a) $2, \frac{1}{2}$ (b) $-2, 4$ (c) $2, \frac{3}{2}$ (d) none

8. $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = \frac{13}{6}$:

- (a) $\frac{13}{4}, 5$ (b) $\frac{4}{13}, \frac{9}{13}$ (c) $\frac{4}{13}, 6$ (d) none

9. $\sqrt{2x^2 - 2x + 1} - 2x + 3 = 0$:

- (a) 2 (b) 3 (c) 4 (d) 5

10. $\sqrt{x^2 - 9x + 20} - \sqrt{x^2 - 12x + 32} = \sqrt{2x^2 - 25x + 68}$:

- (a) 4, 9 (b) 3, 9 (c) 2, 4 (d) 2, 6

11. $\left(x - \frac{1}{x}\right)^2 + 8\left(x + \frac{1}{x}\right) = 29, x \neq 0$:

- (a) $-5, 4$ (b) $3, 2$
(c) $1, 3$ (d) $\frac{3+\sqrt{5}}{2}, \frac{-11 \pm 3\sqrt{13}}{2}$

12. $\left(\frac{1}{x+1} + \frac{1}{x+5}\right) = \left(\frac{1}{x+2} + \frac{1}{x+4}\right)$:

- (a) 7 (b) -3 (c) -5 (d) 4

13. Determine k such that the quadratic equation $x^2 + 7(3+2k) - 2x(1+3k) = 0$ has equal roots :

- (a) 2, 7 (b) 7, 5
(c) $2, \frac{-10}{9}$ (d) none of these

14. Discriminant of the equation $-3x^2 + 2x - 8 = 0$ is :

- (a) -92 (b) -29 (c) 39 (d) 49

15. The nature of the roots of the equation $x^2 - 5x + 7 = 0$ is

- (a) no real roots (b) 1 real root
(c) can't be determined (d) none of these

16. The roots of $a^2x^2 + abx = b^2, a \neq 0$ are :

- (a) equal (b) non-real
(c) unequal (d) none of these

17. The equation $x^2 - px + q = 0, p, q \in R$ has no real roots if:

- (a) $p^2 > 4q$ (b) $p^2 < 4q$
(c) $p^2 = 4q$ (d) none of these

18. Determine the value of k for which the quadratic equation $4x^2 - 3kx + 1 = 0$ has equal roots :

- (a) $\pm\left(\frac{2}{3}\right)$ (b) $\pm\left(\frac{4}{3}\right)$ (c) ± 4 (d) ± 6

19. Find the value of k such that the equation $x^2 - (k + 6)x + 2(2k - 1) = 0$ has sum of the roots equal to half of their product
(a) 3 (b) 4 (c) 7 (d) 10
20. Find the value of k so that the sum of the roots of the quadratic equation is equal to the product of the roots :
 $(k + 1)x^2 + 2kx + 4 = 0$
(a) -2 (b) -4 (c) 6 (d) 8
21. If -4 is a root of the quadratic equation $x^2 - px - 4 = 0$ and the quadratic equation $x^2 - px + k = 0$ has equal roots, find the value of k :
(a) 9/4 (b) 1
(c) 2.5 (d) 3
22. Find the value of k such that the sum of the squares of the roots of the quadratic equation $x^2 - 8x + k = 0$ is 40 :
(a) 12 (b) 2 (c) 5 (d) 8
23. Find the value of p for which the quadratic equation $x^2 + p(4x + p - 1) + 2 = 0$ has equal roots :
(a) -1, $\frac{2}{3}$ (b) 3, 5 (c) 1, $-\frac{4}{3}$ (d) $\frac{3}{4}$, 2
24. If α and β are the roots of the equation $3x^2 + 7x + 3 = 0$ Find the value of $\alpha\beta$:
(a) 1 (b) 0
(c) 2 (d) 3
25. If α and β are the roots of the equation $x^2 - 3x + 2 = 0$. Find the quadratic equation whose roots are $-\alpha$ and $-\beta$
(a) $x^2 - 3x + 2 = 0$ (b) $x^2 + 3x + 2 = 0$
(c) $x^2 + 3x - 2 = 0$ (d) none of these
26. Find the quadratic equation whose roots are $\sqrt{3}$ and $2\sqrt{3}$:
(a) $x^2 + 3\sqrt{3}x - 6 = 0$ (b) $x^2 - 3\sqrt{3}x + 6 = 0$
(c) $x^2 + 3\sqrt{3}x + 5 = 0$ (d) none of these
27. If α and β are the roots of equation $6x^2 + x - 2 = 0$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$:
(a) $-\frac{25}{12}$ (b) $-\frac{12}{75}$ (c) $-\frac{4}{9}$ (d) none
28. If a and c are such that the quadratic equation $ax^2 - 5x + c = 0$ has 10 as the sum of the roots and also as the product of the roots, find a and c respectively :
(a) $\frac{1}{2}$, 6 (b) $\frac{1}{2}$, 5
(c) $\frac{1}{2}$, 8 (d) none of these
29. If α and β are the roots of the equation $x^2 - x - 4 = 0$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$:
(a) $\frac{16}{7}$ (b) $\frac{8}{5}$ (c) $\frac{15}{4}$ (d) none
30. If α and β are the roots of the equation $x^2 - 2x - 1 = 0$, find the value of $\alpha^2\beta + \alpha\beta^2$
(a) -8 (b) -5 (c) 6 (d) -2
31. If α and β are the roots of $x^2 - x - 2 = 0$, find the quadratic equation in x whose roots are $(2\alpha + 1)$ and $(2\beta + 1)$:
(a) $x^2 + 5x - 4 = 0$ (b) $x^2 - 4x - 5 = 0$
(c) $x^2 + 4x - 5 = 0$ (d) none of these
32. If α , β be the roots of the quadratic equation $3x^2 - 6x + 4 = 0$, find the value of $\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$:
(a) 6 (b) 8 (c) 7 (d) 5
33. If α , β be the roots of the quadratic equation $x^2 - 5x + k = 0$, find the value of k such that $\alpha - \beta = 1$:
(a) 2 (b) 4 (c) 6 (d) 8
34. If one root of the quadratic equation $ax^2 + bx + c = 0$ is double the other then which one of the following is correct?
(a) $b^2 = 3ac$ (b) $2b^2 = 5ac$
(c) $2b^2 = 9ac$ (d) $2b^2 > 9ac$
35. The length of a hypotenuse of a right triangle exceeds the length of its base by 2 cm and exceeds twice the length of the altitude by 1 cm. Find the length of each side of the triangle (in cm) :
(a) 6, 8, 10 (b) 7, 24, 25
(c) 8, 15, 17 (d) 7, 40, 41
36. A two digit number is such that the product of its digits is 12. When 9 is added to the number, the digits interchange their places, find the number :
(a) 62 (b) 34 (c) 26 (d) 43
37. A plane left 40 minutes late due to bad weather and in order to reach its destination, 1600 km away in time, it had to increase its speed by 400 km/h from its usual speed. Find the usual speed of the plane :
(a) 600 km/h (b) 750 km/h
(c) 800 km/h (d) none of these
38. The sum of the squares of two consecutive positive odd numbers is 290. Find the sum of the numbers :
(a) 14 (b) 24 (c) 28 (d) none
39. A shopkeeper buys a number of books for ₹ 80. If he had bought 4 more for the same amount, each book would have cost ₹ 1 less. How many books did he buy?
(a) 8 (b) 16 (c) 24 (d) 28
40. Two square have sides x cm and $(x + 4)$ cm. The sum of their areas is 656 cm^2 . Find the sides of the square :
(a) 8 cm, 12 cm (b) 12 cm, 15 cm
(c) 6 cm, 10 cm (d) 16 cm, 20 cm

CAT-Test

Questions Helping you bell the CAT

LEVEL 01 > BASIC LEVEL EXERCISE

- 1 If p and $q (\neq 0)$ are the roots of the equation $x^2 + px + q = 0$, then the least value of $x^2 + px + q$ ($x \in R$) is :
 (a) $-\frac{1}{4}$ (b) $\frac{1}{4}$ (c) $-\frac{9}{4}$ (d) $\frac{9}{4}$
- 2 The value of p for which the sum of the squares of the roots of the equation $x^2 - (p-2)x - p - 1 = 0$ assumes the least value is :
 (a) -1 (b) 1 (c) 0 (d) 2
- 3 If the equation $x^2 + 2(p+1)x + 9p - 5 = 0$ has only negative roots, then :
 (a) $p \leq 0$ (b) $p \leq -6$ (c) $p \geq 6$ (d) $p \geq 36$
- 4 The ratio of the roots of the equation $ax^2 + bx + c = 0$ is same as the ratio of the roots of the equation $px^2 + qx + r = 0$. If D_1 and D_2 are the discriminants of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$, respectively, then $D_1 : D_2$ is equal to
 (a) $\frac{a^2}{q^2}$ (b) $\frac{a^2}{p^2}$
 (c) $\frac{b^2}{q^2}$ (d) none of these
- 5 If every pair from among the equations $x^2 + px + qr = 0$, $x^2 + qx + rp = 0$ and $x^2 + rx + pq = 0$ has a common root, then the sum of the three common roots is :
 (a) $2(p+q+r)$ (b) $p+q+r$
 (c) $-(p+q+r)$ (d) pqr
- 6 If the roots of the equation, $ax^2 + bx + c = 0$, are of the form $\frac{\alpha}{\alpha-1}$ and $\frac{\alpha+1}{\alpha}$ then the value of $(a+b+c)^2$ is :
 (a) $b^2 - 2ac$ (b) $b^2 - 4ac$
 (c) $2b^2 - ac$ (d) $4b^2 - 2ac$
- 7 Equation $ax^2 + 2x + 1$ has one double root if :
 (a) $a = 0$ (b) $a = -1$
 (c) $a = 1$ (d) $a = 2$
- 8 The number of quadratic equations, which remain unchanged by squaring their roots, is :
 (a) 0 (b) 2
 (c) 4 (d) infinitely many
- 9 The value of $\sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots \infty}}}}$ is :
 (a) 2 (b) 3
 (c) 4 (d) 5
- 10 If α, β, γ are the roots of the equation $x^3 + a_0x^2 + a_1x + a_2 = 0$, then $(1-\alpha^2)(1-\beta^2)(1-\gamma^2)$ is equal to :
 (a) $(1-a_1)^2 + (a_0-a_2)^2$ (b) $(1+a_1)^2 - (a_0+a_2)^2$
 (c) $(1+a_1)^2 + (a_0+a_2)^2$ (d) none of these
- 11 If $x \in R$, and $K = \frac{(x^2 - x + 1)}{(x^2 + x + 1)}$, then :
 (a) $x \leq 0$ (b) $\frac{1}{3} \leq k \leq 3$
 (c) $k \geq 5$ (d) none of these
- 12 If $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots \infty}}}}$ then x is :
 (a) 1 (b) 2
 (c) 3 (d) none of these
- 13 If α, β, γ are the roots of the equation $x^3 - 3x + 11 = 0$, then the equation whose roots are $(\alpha + \beta)$, $(\beta + \gamma)$ and $(\gamma + \alpha)$ is :
 (a) $x^3 + 3x + 11 = 0$
 (b) $x^3 - 3x + 11 = 0$
 (c) $x^3 + 3x - 11 = 0$
 (d) $x^3 - 3x - 11 = 0$
- 14 If $x^2 - ax - 21 = 0$ and $x^2 - 3ax + 35 = 0$; $a > 0$ have a common root, then a is equal to :
 (a) 1 (b) 2
 (c) 4 (d) 5
- 15 If α, β, γ are such that $\alpha + \beta + \gamma = 2$, $\alpha^2 + \beta^2 + \gamma^2 = 6$, $\alpha^3 + \beta^3 + \gamma^3 = 8$, then $\alpha^4 + \beta^4 + \gamma^4$ is equal to :
 (a) 10 (b) 12
 (c) 18 (d) none of these

- 16** The real values of a for which the quadratic equation $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$ possesses roots of opposite signs are given by :
 (a) $a > 6$ (b) $a > 9$
 (c) $0 < a < 4$ (d) $a < 0$
- 17** If α, β are the roots of $ax^2 + bx + c = 0$ and $\alpha + k, \beta + k$ are the roots of $px^2 + qx + r = 0$, then k is equal to
 (a) $-\frac{1}{2}\left(\frac{a}{b} - \frac{p}{q}\right)$ (b) $\left(\frac{a}{b} - \frac{p}{q}\right)$
 (c) $\frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$ (d) $(ab - pq)$
- 18** If $\alpha, \beta \in R$ are the roots of the equation $ax^2 + bx + c = 0$, $k \in R$ lies between α and β , if :
 (a) $a^2k^2 + bk + c < 0$ (b) $a^2k^2 + abk + ac < 0$
 (c) $ak^2 + abk + ac > 0$ (d) None of these
- 19** If the equation $(3x)^2 + (27 \times 3^{1/k} - 15)x + 4 = 0$ has equal roots, then k is equal to
 (a) -2 (b) $-\frac{1}{2}$
 (c) $\frac{1}{2}$ (d) 0
- 20** The least possible integral value of a for which the equation $x^2 - 2(a-1)x + (2a+1) = 0$ has both the roots positive is :
 (a) 1 (b) 3
 (c) 4 (d) 6
- 21** The number of real solutions of the equation $2^{3x^2-7x+4} = 1$ is :
 (a) 0 (b) 4
 (c) 2 (d) infinitely many
- 22** The number of real solutions of the equation $2|x|^2 - 5|x| + 2 = 0$ is :
 (a) 0 (b) 4
 (c) 2 (d) none of these
- 23** The number of real roots of the equation $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$:
 (a) 0 (b) 2
 (c) 3 (d) 6
- 24** The equation $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ has :
 (a) no solution
 (b) one solution
 (c) two solutions
 (d) more than two solutions
- 25** If α, β are the roots of the equation $8x^2 - 3x + 27 = 0$, then the value of $[(\alpha^2/\beta)^{1/3} + (\beta^2/\alpha)^{1/3}]$ is :
 (a) $1/3$ (b) $1/4$
 (c) $1/5$ (d) $1/6$
- 26** The quadratic equation $3x^2 + 2(a^2 + 1)x + a^2 - 3a + 2 = 0$ possesses roots of opposite sign, then a lies in :
 (a) $(-\infty, 0)$ (b) $(-\infty, 1)$ (c) $(1, 2)$ (d) $(4, 9)$
- 27** The expression $\frac{(x-a)(x-b)}{(x-c)}$ will assume all real values for every $x \in R$, if :
 (a) $a < b < c$ (b) $a > b > c$
 (c) c lies between a and b (d) none of these
- 28** If α and β are the roots of the equation $ax^2 + bx + c$, where $(a, b, c) > 0$, then α and β are :
 (a) rational numbers (b) real and negative
 (c) negative real parts (d) none of these
- 29** The number of solutions of the equation $3^{x-1} + 5^{x-1} = 34$:
 (a) 0 (b) 1
 (c) 2 (d) none of these
- 30** If $(2 + \sqrt{3})^{x^2-2x+1} + (2 - \sqrt{3})^{x^2-2x-1} = \frac{2}{2-\sqrt{3}}$, then x is equal to :
 (a) 0 (b) 1
 (c) 2 (d) both (a) and (c)
- 31** The values of a for which the quadratic equations $(1-2a)x^2 - 6ax - 1 = 0$ and $ax^2 - x + 1 = 0$ have at least one root in common are :
 (a) $\frac{1}{2}, \frac{2}{9}$ (b) $0, \frac{1}{2}$
 (c) $\frac{2}{9}$ (d) $0, \frac{1}{2}, \frac{2}{9}$
- 32** The integer k for which the inequality $x^2 - 2(4k-1)x + 15k^2 - 2k - 7 > 0$ is valid for any x is :
 (a) 2 (b) 3 (c) 4 (d) 6
- 33** The values of a for which $2x^2 - 2(2a+1)x + a(a+1) = 0$ may have one root less than a and other root greater than a are given by :
 (a) $-1 < a < 0$ (b) $0 < a < 1$
 (c) $a \geq 0$ (d) $a < -1$ or $a > 0$
- 34** The condition that $x^3 - ax^2 + bx - c = 0$ may have two of its roots equal to each other but of opposite signs is :
 (a) $ab = c$ (b) $\frac{2}{3}a = bc$
 (c) $a^2b = c$ (d) none of these
- 35** If the roots of $ax^2 + bx + c = 0$, $a > 0$, be each greater than unity, then :
 (a) $a + b + c = 0$ (b) $a + b + c > 0$
 (c) $a + b + c < 0$ (d) none of these
- 36** The number of real solutions of $x - \frac{1}{x^2-4} = 2 - \frac{1}{x^2-4}$ is :
 (a) 0 (b) 1
 (c) 2 (d) infinite

- 37** The number of real solutions of the equation $2^{x/2} + (\sqrt{2} + 1)^x = (5 + 2\sqrt{2})^{x/2}$ is :
 (a) one (b) four
 (c) six (d) infinite
- 38** If $xy = 2(x + y)$, $x \leq y$ and $x, y \in N$, the number of solutions of the equation :
 (a) 2 (b) 3
 (c) 0 (d) infinitely many
- 39** If $a, b, c \in R$ and equality $ax^2 + bx + c = 0$ has complex roots which are reciprocal to each other, then one has :
 (a) $a = c$ (b) $|b| \leq |c|$
 (c) $|b| \leq |a|$ (d) all of these
- 40** The equation $|x + 1| |x - 1| = a^2 - 2a - 3$ can have real solutions for x if a belongs to :
 (a) $(-\infty, -1] \cup [3, \infty)$
 (b) $[1 - \sqrt{5}, 1 + \sqrt{5}]$
 (c) $[1 - \sqrt{5}, -1] \cup [3, 1 + \sqrt{5}]$
 (d) none of the above
- 41** $p, q, r \in R$ and k is a prime number such that $p + qk^{1/3} + rk^{2/3} = 0$, then :
 (a) $p = \frac{q}{r}$ (b) $p = q^{1/3} \neq r^{2/3}$
 (c) $p = q = r = 0$ (d) none of these
- 42** The expression $x^4 + 2px^3 + x^2 + 2px + 1 = 0$ has at least two distinct negative roots if :
 (a) $p < 1$ (b) $p = 0$
 (c) $p > \frac{3}{4}$ (d) $p \in \left(-\frac{1}{2}, \frac{1}{2}\right)$
- 43** The roots of the expression $x^2 - 2a|x - a| - 3a^2 = 0$; $a < 0$:
 (a) $(1 + \sqrt{2})a, (-1 - \sqrt{6})a$ (b) $(1 - \sqrt{2})a, (\sqrt{6} - 1)a$
 (c) $(1 \pm \sqrt{2})a$ (d) none of these
- 44** The possible values of x for the equation $2^{|x+2|} - |2^{x+1} - 1| - 2^{x+1} = 1$ is :
 (a) $x = -3$ (b) $x \geq -1$
 (c) $x = 0$ (d) $(x \geq -1) \cup (x = -3)$
- 45** The values of x which satisfy the expression :
 $(5 + 2\sqrt{6})^{x^2-3} + (5 - 2\sqrt{6})^{x^2-3} = 10$
 (a) $\pm 2 \pm \sqrt{3}$ (b) $\pm \sqrt{2}, \pm 4$
 (c) $\pm 2, \pm \sqrt{2}$ (d) $2, \sqrt{2}, \sqrt{3}$
- 46** If $a, b \in \{1, 2, 3, 4\}$, then the number of quadratic equations of the form $ax^2 + bx + 1 = 0$, having real roots is :
 (a) 6 (b) 7
 (c) 8 (d) none of these
- 47** The minimum value of the expression $a + \frac{1}{a}$; $a > 0$ is :
 (a) 0 (b) 1
 (c) 2 (d) 4
- 48** If $a^2 + b^2 + c^2 = 1$, then $ab + bc + ac$ lies in the interval :
 (a) $[1, 2/3]$ (b) $[-1/2, 1]$
 (c) $[-1, 1/2]$ (d) $[2, -4]$
- 49** The integral values of x satisfying the equation $2^{|x+1|} - 2^x = |2^x - 1| + 1$ is :
 (a) $(-\infty, 0)$ (b) $[1, \infty)$
 (c) $[0, \infty)$ (d) $2n : n \in I$
- 50** Number of values of x satisfying the equation $(15 + 4\sqrt{14})^t + (15 - 4\sqrt{14})^t = 30$, where $t = x^2 - 2|x|$:
 (a) 0 (b) 2 (c) 4 (d) 6
- 51** The inequality $|2x - 3| < 1$ is valid when x lies in the interval :
 (a) $(-4, 3)$ (b) $(-1, 2)$
 (c) $(1, 2)$ (d) $(3, 4)$
- 52** Let α and β be the roots of the equation $x^2 + x + 1 = 0$. The equation whose roots are α^{19}, β^7 is :
 (a) $x^2 - x - 1 = 0$ (b) $x^2 - x + 1 = 0$
 (c) $x^2 + x - 1 = 0$ (d) $x^2 + x + 1 = 0$
- 53** The value of p for which the difference between the roots of the equation $x^2 + px + 8 = 0$ is 2 are :
 (a) ± 2 (b) ± 4
 (c) ± 6 (d) ± 8
- 54** If $x > 0$, $xy = 1$, minimum value of $x + y$ is :
 (a) 1 (b) 0
 (c) 2 (d) $\sqrt{2}$
- 55** If $x = 7 + 4\sqrt{3}$ and $xy = 1$, then the value of $\frac{1}{x^2} + \frac{1}{y^2}$ is :
 (a) 64 (b) 128
 (c) 184 (d) 194
- 56** If $x \in R$, and $\alpha = \frac{x^2}{(1 + x^4)}$, then :
 (a) $0 \leq \alpha \leq 2$ (b) $0 \leq \alpha \leq 1$
 (c) $0 \leq \alpha \leq \frac{1}{4}$ (d) $0 \leq \alpha \leq \frac{1}{2}$
- 57** The set of values for which $x^3 + 1 \geq x^2 + x$ is :
 (a) $x \geq 0$ (b) $x \leq 0$
 (c) $x \geq -1$ (d) $-1 \leq x \leq 1$
- 58** If $a^2 = 5a - 3$ and $b^2 = 5b - 3$, ($a \neq b$) find the quadratic equation whose roots are $\frac{a}{b}$ and $\frac{b}{a}$.
 (a) $3x^2 - 19x + 1 = 0$ (b) $2x^2 - 39x + 27 = 0$
 (c) $3x^2 - 19x + 3 = 0$ (d) $3x^2 + 19x + 3 = 0$
- 59** If $f(x) = ax^2 + bx + c$ and $g(x) = -ax^2 + bx + c$, where $ac \neq 0$, the new equation $f(x) \cdot g(x) = 0$ has n real roots. Find the minimum value of n .
 (a) 0 (b) 1 (c) 2 (d) 4

- 60** Given that $x^2 + ux + v = 0$ and $x^2 + jx + k$; where $uj = 2(v + k)$ and u, v, j, k are real numbers. Also, $f(x) = x^2 + ux + v$ and $g(x) = x^2 + jx + k$. Then which one of the following is always correct?
 (a) $f(x)$ has real roots (b) $g(x)$ has real roots
 (c) $f(x)$ and $g(x)$ both have real roots
 (d) either $f(x)$ or $g(x)$ have real roots
- 61** If one root of a quadratic equation $ax^2 + bx + c = 0$ is equal to the n th power of the other, then
 (a) $\left[(ac^n)^{\frac{1}{n+1}} \right] + \left[(ca^n)^{\frac{1}{n+1}} \right] + b^n = 0$
 (b) $\left[(ac^n)^{\frac{1}{n+1}} \right] + \left[(ca^n)^{\frac{1}{n+1}} \right] - b = 0$
 (c) $\left[(ac^n)^{\frac{1}{n+1}} \right] + \left[(ca^n)^{\frac{1}{n+1}} \right] + b = 0$
 (d) $\left[(ac^n)^{\frac{1}{n+1}} \right] + \left[(ca^n)^{\frac{1}{n+1}} \right] + \frac{1}{b^{n+1}} = 0$
- 62** If r be the ratio of the roots of the equation $ax^2 + bx + c = 0$, then which one of the following is correct?
 (a) $ac(1 + r)^2 = (rb)^2$ (b) $ac(1 + r)^2 = rb^2$
 (c) $ab(1 + r)^2 = rc^2$ (d) $ac(1 + r)^2 = br^2$
- 63** If c, d are the roots of the quadratic equation $(x - a)(x - b) - k = 0$, then, a, b are the roots of the equation
 (a) $(x - c)(x - d) - k = 0$ (b) $(x + c)(x + d) - k = 0$
 (c) $(x - c)(x - d) + k = 0$ (d) $(x + a)(x + b) + k = 0$
- 64** In a quadratic polynomial $a_2x^2 + a_1x + a_0$, the roots are r_1 and r_2 . Defining $S_k = r_1^k + r_2^k$ and $P = r_1r_2$. Then which one of the following relation is true?
 (i) $S_1^3 = S_3 + 3PS_1$ (ii) $S_3 = S_1(S_1^2 - 3P)$
 (a) only (i) is true (b) only (ii) is true
 (c) both are true (d) none are true
- 65** The coefficient of x in the equation $x^2 + bx + c = 0$ was wrongly written as 17 in place of 13 and the roots thus found were -2 and -15 . Find the roots of the correct equation.
 (a) $-7, -3$ (b) $-13, 1$ (c) $-10, -3$ (d) $13, -1$
- 66** Find the possible value(s) of p if the equation $3x^2 - 2x + p = 0$ and $6x^2 - 17x + 12 = 0$ have a common root.
 (a) $7, -12$ (b) $-8/3, -15/4$
 (c) $-3/8, -4/15$ (d) data insufficient
- 67** If a, b, c belong to R ; and equations $x^2 + 2x + 9 = 0$ and $ax^2 + bx + c = 0$ have a common root, then $a : b : c$ is
 (a) $1:2:9$ (b) $2:1:7$ (c) $1:3:9$ (d) $1:2:4$
- 68** If $x^2 - ax - 45 = 0$ and $x^2 - 3ax + 27 = 0$; ($a > 0$) has a common root then the value of a is
 (a) 5 (b) ± 7 (c) ± 8 (d) 4
- 69** If the equations $x^2 - px + q = 0$ and $x^2 + qx - p = 0$ have only one common root, then which one of the following is necessarily true?
 (a) $p - q = 1$ (b) $p + q = -1$
 (c) $p - q = -1$ (d) $p + q = 1$
- 70** There are three quadratic equations $x^2 - 2m + 5 = 0$, $x^2 - 2n + 7 = 0$ and $x^2 - 2p + 35 = 0$. If each pair of equations has a common positive real root, find the value of $m + n + p$.
 (a) 13 (b) 15 (c) 47 (d) 23
- 71** If $(a^2 - 1)x^2 + (a - 1)x + a^2 - 4a + 3 = 0$ be an identity in x , then find the value of a .
 (a) -1 (b) 0
 (c) 1 (d) 3
- 72** Consider the following equation.

$$\frac{(x + b)(x + c)}{(b - a)(c - a)} + \frac{(x + c)(x + a)}{(c - b)(a - b)} + \frac{(x + a)(x + b)}{(a - c)(b - c)} = 1$$

 Then what is the most appropriate answer regarding the above equation?
 (a) It has only two real roots
 (b) It has no roots
 (c) It's an identity
 (d) none of the above
- 73** If a, b, c belong to $R, a \neq 0$, and the quadratic equation $ax^2 + bx + c = 0$ has no real roots. Then, which one of the following is correct?
 (a) $(a + b + c)c > 0$ (b) $(a + b + c)c < 0$
 (c) $(a + b + c)c = 0$ (d) $(a + b + c)c \leq 0$
- 74** Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ are in Harmonic Progression (HP).
 It is known that if any three terms a, b, c have a common difference and are in increasing or decreasing order, they are in Arithmetic Progression (AP) and their reciprocals $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in HP.
 (a) $27r^2 - 9pqr - 2q^3 = 0$
 (b) $27r^2 - 9pqr + 2q^3 = 0$
 (c) $27r^2 - 9pqr + 3q^3 = 0$
 (d) $27r^2 - 9pqr + 3p^3 = 0$
- 75** Find the equation whose roots are negative of the roots of the equation $x^3 - 3x^2 + x + 1 = 0$
 (a) $x^3 + 3x^2 + x - 1 = 0$
 (b) $x^3 - 3x^2 + x - 1 = 0$
 (c) $x^3 + 3x^2 - x - 1 = 0$
 (d) $-x^3 + 3x^2 + x + 1 = 0$

- 76** Find the equation whose roots are square of the roots of the equation $x^3 - 2x^2 + 3x + 1 = 0$
- (a) $x^3 + 2x^2 + 13x - 1 = 0$
 (b) $x^3 - 13x^2 + 2x - 1 = 0$
 (c) $x^3 + 3x^2 - 12x - 1 = 0$
 (d) $-x^3 + 2x^2 + 13x + 1 = 0$
- 77** Find the equation whose roots are cubes of the roots of the equation $ax^3 + bx^2 + cx + d = 0$
- (a) $a^3x^3 + x^2(abd^2 + ab - cd) - x(3ad^2 - 3bcd - c^2) + d^3 = 0$
 (b) $a^3x^3 + x^2(ab^2 + 4cd - ad) - x(3ad^2 - 3bcd - c^2) + d^3 = 0$
 (c) $a^3x^3 + x^2(b - 4ac + abcd) - x(3ad^2 - 3bcd + c^2) + d^3 = 0$
 (d) $a^3x^3 + x^2(3a^2d - 3abc + b^3) + x(3ad^2 - 3bcd + c^3) + d^3 = 0$
- 78** What are the values of x , if $(7 + 4\sqrt{3})^{x^2-8} + (7 - 4\sqrt{3})^{x^2-8} = 14$?
- (a) $3, \sqrt{7}$ (b) $\pm 3, \pm 4$
 (c) $\pm 3, \pm \sqrt{7}$ (d) $\pm 4, \pm \sqrt{6}$
- 79** If $a + b + c = 0$, then $3ax^2 + 2bx + c = 0$ has always
- (a) real and distinct roots (b) imaginary roots
 (c) rational roots (d) real and equal roots
- 80** Find the roots of the equation $2x^4 + x^3 - 11x^2 + x + 2 = 0$
- (a) $\frac{-8 - \sqrt{5}}{2}, \frac{-8 + \sqrt{5}}{2}, \frac{1}{2}, 2$
 (b) $\frac{-3 - \sqrt{5}}{2}, \frac{-3 + \sqrt{5}}{2}, -2, 2$
 (c) $\frac{-3 - \sqrt{7}}{2}, \frac{-3 + \sqrt{7}}{2}, \frac{1}{2}, 2$
 (d) $\frac{-3 - \sqrt{5}}{2}, \frac{-3 + \sqrt{5}}{2}, \frac{1}{2}, 2$
- 81** Find the solution of $(x)^2 = [x]^2 + 2x$; where (x) is the integer just less than or equal to x and $[x]$ is the integer just greater than or equal to x .
- (a) $0, n + \frac{1}{2}$ (b) $0, n - \frac{1}{2}$
 (c) $0, 1/2$ (d) none of these
- 82** For every a, b, c being real numbers the root of $ax^2 + 2bx + c = 0$ are real numbers. If m, n are real numbers such that $m^2 > n > 0$, then the roots of the equation $ax^2 + 2mbx + nc = 0$ are
- (a) real
 (b) complex
 (c) both real and complex
 (d) cannot be determined
- 83** If the graph of the function $16x^2 + 8(a + 5)x - 7a - 5 = 0$ is strictly above the X -axis, then
- (a) $2 < a < 15$ (b) $-15 < a < -2$
 (c) $17 < a < 30$ (d) $a > 0$
- 84** Find the value of x , if
- $$x = 1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{3 + \frac{1}{2 + \dots \infty}}}}$$
- (a) $\pm \sqrt{5/3}$ (b) $\pm \sqrt{5/2}$
 (c) $\sqrt{5/3}$ (d) $\sqrt{8/5}$
- 85** The number of integral solutions of $y^4 = 2x^4 + 1402$.
- (a) 4 (b) 2
 (c) 1 (d) none
- 86** Let a, b, c be real, if $ax^2 + bx + c = 0$ has two real roots α and β , where $\alpha < -1$ and $\beta > 1$, then the value of $1 + \frac{c}{a} + \left| \frac{b}{a} \right|$ is
- (a) less than zero (b) greater than zero
 (c) equal to zero (d) equal to $b^2 - 4ac$
- 87** Find all the values x that satisfy $3x^3 = [(x^2 + \sqrt{18}x + \sqrt{32})(x^2 - \sqrt{18}x - \sqrt{32})] - 4x^2$
- (a) $3 \pm \sqrt{17}, \frac{-3 \pm \sqrt{7}i}{2}$
 (b) $3 \pm \sqrt{17}, \frac{-3 \pm \sqrt{17}i}{2}$
 (c) $3 \pm \sqrt{7}, \frac{-3 \pm \sqrt{7}i}{2}$
 (d) $3 \pm \sqrt{17}, \frac{3 \pm \sqrt{17}i}{2}$
- 88** For how many values of x , the following equation is an identity?
- $$\frac{(x-a)(x-b)}{(c-a)(c-b)} + \frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-c)(x-a)}{(b-c)(b-a)} = 1$$
- (a) 0 (b) 6 (c) 2 (d) 3
- 89** The number of solutions of the following equation is
- $$\frac{a^2(x-b)(x-c)}{(a-b)(a-c)} + \frac{b^2(x-c)(x-a)}{(b-c)(b-a)} + \frac{c^2(x-a)(x-b)}{(c-a)(c-b)} = x^2$$
- (a) 0 (b) 2
 (c) 3 (d) 4
- 90** Number of positive integral solutions of the following inequation
- $$\frac{x^3(2x-3)^2(x-4)^6}{(x-3)^3(3x-8)^4} \leq 0$$
- (a) 0 (b) 3 (c) 5 (d) 4

- 91** Find the most appropriate solution of the following rational inequation.

$$\frac{x^2 + 6x - 7}{|x + 3|} < 0$$

- (a) $(-3, -1) \cup (-1, 7)$
 (b) $(-7, -3) \cup (-3, 1) \cup (1, 3)$
 (c) $(-7, 3) \cup (3, 1)$
 (d) $(-7, -3) \cup (-3, 1)$
- 92** If x is real, then find the minimum and maximum value of the following rational function $f(x)$
- $$f(x) = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$$
- (a) $1/7, 7$ (b) $-1/7, 7$
 (c) $-1/7, -7$ (d) $-1, 1/7$
- 93** If one root of the equation $24x^3 - 14x^2 - 63x + 45 = 0$ is double the other, then the third root of this equation is
 (a) $-3/2$ (b) $3/2$
 (c) $-5/3$ (d) $-7/2$
- 94** If sum of the two roots of the equation $4x^3 + 16x^2 - 9x - 36 = 0$ is the first whole number, then the third root of this equation is:
 (a) $-3/4$ (b) $3/2$ (c) $-3/2$ (d) -4
- 95** Find a cubic equation in which each root is greater by unity than a root of the equation $x^3 - 5x^2 + 6x - 3 = 0$. Which of the following could be the required equation?
 (a) $p^3 - 8p^2 + 19p - 15 = 0$
 (b) $p^3 - 19p^2 + 8p - 15 = 0$
 (c) $p^3 - 8p^2 - 19p - 15 = 0$
 (d) $p^3 - 19 - 8p + 15 = 0$
- 96** Find a cubic equation in which each root is the cube of the roots of $x^3 + 3x^2 + 2 = 0$. Which of the following could be the required equation?
 (a) $p^3 + 33p^2 + 12p + 8 = 0$
 (b) $p^3 - 19p^2 + 8p - 15 = 0$
 (c) $p^3 - 8p^2 - 19p - 15 = 0$
 (d) $p^3 - 19 - 8p + 15 = 0$
- 97** Let a_1, a_2, a_3 be the roots of the polynomial $x^3 + x^2 + 3x + 1$. Find the value of $a_1 + a_2 + a_3 + a_1a_2 + a_2a_3 + a_3a_1 + a_1a_2a_3$
 (a) 5 (b) 6
 (c) 3 (d) 1
- 98** For a cubic polynomial, $a_3x^3 + a_2x^2 + a_1x + a_0$, the roots r_1, r_2, r_3 come in three symmetric combinations : $C_1 = r_1 + r_2 + r_3, C_2 = r_1r_2 + r_2r_3 + r_3r_1, C_3 = r_1r_2r_3$. The sum of the k^{th} powers of the roots is defined as $S_k = r_1^k + r_2^k + r_3^k$. Then
 (i) $a_3S_1 + a_2 = 0$ (ii) $a_3S_2 + a_2S_1 + 2a_1 = 0$
 (a) only (i) is true (b) only (ii) is true
 (c) both are true (d) none are true

- 99** Find the real a if the equation $x^3 - 3x^2 + ax - 1 = 0$ has three positive solutions, then

(a) 4 (b) $10/3$ (c) $-10/3$ (d) 3

- 100** The equation $x^4 + ax^3 + bx^2 - 4x + 1 = 0$ has four positive roots, then

(a) $a > b$ (b) $a > 0$
 (c) $a < 0$ (d) $a - b > 0$

- 101** If $f(x) = 4x^2 + px + (p - 3)$ is negative for at least one x , find all possible values of p .

(a) $(-\infty, -12] \cup [-4, \infty)$ (b) $(-\infty, 4] \cup [12, \infty)$
 (c) $(4, 12)$ (d) $(-\infty, 4) \cup (12, \infty)$

- 102** If $f(x) = (p^2 + 3)x^2 + (p + 2)x - 5 < 0$, for at least one x , find all possible values of p .

(a) $(-\infty, -3) \cup (8, \infty)$ (b) $(-\infty, \infty)$
 (c) $(-3, 5)$ (d) none of these

- 103** Let $a, b, c \in \mathbb{R}$ and $a \neq 0$. Let α, β be the roots of equation $ax^2 + bx + c = 0$, where $\alpha < -p$ and $\beta > p$. Then, for every $n \in \mathbb{N}$, which one of the following is correct?

(a) $1 + \frac{c}{ap^2} + \frac{1}{p} \left| \frac{b}{a} \right| < 0$ (b) $1 + \frac{c}{ap^2} + \frac{1}{p} \left| \frac{b}{a} \right| > 0$
 (c) $1 + \frac{c}{ap^2} + \frac{1}{p} \left| \frac{b}{a} \right| \leq 0$ (d) $1 + \frac{c}{ap^2} + \frac{1}{p} \left| \frac{b}{a} \right| \geq 0$

- 104** Let α be the root of $ax^2 + bx + c = 0$ and β be the root of $-ax^2 + bx + c = 0$; where $(\alpha, \beta \neq 0)$. Let there be a quadratic function $f(x) = \frac{a}{2}x^2 + bx + c = 0$. Then, which of the following is certainly correct?

(a) One root of $f(x)$ must lie between α and β
 (b) Both the roots lie between α, β
 (c) Both the roots are greater than α, β
 (d) Both the roots are less than α, β

- 105** Find the number of solutions of the equation $x^2 - 3|x| + 2 = 0$

(a) 0 (b) 2
 (c) 4 (d) none of these

- 106** Find all the values of x that satisfy $3|x^2 + 4x + 2| = 5x + 16$

(a) $\{1, -10/3\}$ (b) $(-2, 1)$
 (c) $\{-2, -11/3\}$ (d) $\{-2, 1\}$

- 107** Find all the values of x that satisfy $2^{|x|} - |2^{x-1} - 1| = 2^{x-1} + 1$

(a) $[-2, 2]$
 (b) $[2, 8]$
 (c) $[0, 9]$
 (d) $[-1, \infty) - (-1, 1)$

- 108** Find all the values of x that satisfy

$(|x - 1| - 3)(|x + 2| - 5) < 0$
 (a) $(-7, -2)$ (b) $(3, 4)$
 (c) $(-7, 4) - (-2, 3)$ (d) $(-7, -2) \cup (3, 4)$

- 109** Find all the values of x that satisfy the following inequation

$$\left| \frac{x^2 - 5x + 4}{x^2 - 4} \right| \leq 1$$

- (a) $(-2, 8/5) \cup [2, \infty)$
 (b) $[0, 8/5] \cup [5/2, \infty)$
 (c) $(-\infty, -2) \cup (0, \infty)$
 (d) $(0, 8/5) \cup (2, 5/2) \cup (5/2, \infty)$

- 110** Find all the values of x that satisfy the following inequation

$$\left| \frac{x^2 - 3x - 1}{x^2 + x + 1} \right| < 3$$

- (a) $(-\infty, -1) \cup (-2, \infty)$
 (b) $(-\infty, -2) \cup (-1, \infty)$
 (c) $(-\infty, \infty)$
 (d) $(-9, 6)$

- 111** For a given quadratic/polynomical function $f(x)$ having all the real coefficients, if $f(p)$ and $f(q)$ have opposite signs, then $f(x)=0$ have

- (a) at least one real root between p and q
 (b) all the real roots between p and q
 (c) none of the roots between p and q
 (d) none of the above

- 112** Consider the polynomial function $f(x) = (x-2)(x+3)(2x-1)^2(3x-10)^5$. Then, which of the following relations is/are incorrect regarding this polynomial.

- (i) $f(-10^{10})f(10^{10}) < 0$ (ii) $f(-2)f(3) < 0$
 (iii) $f(-2)f(-4) > 0$ (iv) $f(-2)f(1) < 0$
 (v) $f(-2)f(-1) < 0$ (vi) $f(1)f(3)f(5) < 0$
 (a) only (ii), (iii) and (v)
 (b) only (ii), (iv), (v) and (vi)
 (c) Except (i), (iii) and (v)
 (d) Except (i) and (ii)

- 113** Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of the following rational function is:

$$f = \frac{a_{10} - 2a_8}{2a_9}$$

- (a) 1 (b) 2
 (c) 3 (d) 4

- 114** If $a > b$, where a and b are the roots of the equation $3x^3 - 8x - 14 = 0$ and $f_n = a^n - b^n$, find the value of $\frac{3f_9 - 14f_6}{f_7}$.

- (a) 8 (b) 3
 (c) 11 (d) $3/7$

Directions (for Q. Nos. 115 and 116) Answer the following questions on the basis of the information given below.

Consider the cubic polynomial $f(x) = 4x^3 + 3x^2 + 2x + 1$.

- 115** At least one real root of $f(x)$ certainly lies in the interval

- (a) $(-1/4, 0)$ (b) $(-11, -3/4)$
 (c) $(-3/4, -1/2)$ (d) $(0, 1/4)$

- 116** The number of positive real roots is

- (a) 2 (b) 1
 (c) 0 (d) cannot be determined

- 117** Let p and q be real numbers such that $p \neq 0$, $p^3 \neq \pm q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -\frac{p}{q}$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

as its roots is

- (a) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$
 (b) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$
 (c) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$
 (d) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

- 118** The maximum value of the function

$$f(x) = 2x^3 - 15x^2 + 36x - 48 \text{ on the set}$$

$$A = \{x \mid x^2 + 20 \leq 9x\} \text{ is}$$

- (a) 7 (b) 6 (c) 9 (d) none

- 119 Statement I (Assertion)** The curve

$$y = \frac{-x^2}{2} + x + 1 \text{ is symmetric with respect to the line } x = 1$$

Statement II (Reason) Because a parabola is symmetric about its axis.

- (a) Statement I is true, statement II is true; statement II is a correct explanation for statement I.
 (b) Statement I is true, statement II is true; statement II is not a correct explanation for statement I.
 (c) Statement I is true; statement II is false.
 (d) Statement I is False; statement II is true.

- 120** Consider the rational function $f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6}$. Match the conditions/expressions in **Column I** with statements in **Column II** where more than one item of column II can match with an item of column I.

Column I	Column II
(P) If $-1 < x < 1$, then $f(x)$ satisfies	(p) $0 < f(x) < 1$
(Q) If $1 < x < 2$, then $f(x)$ satisfies	(q) $f(x) < 0$
(R) If $3 < x < 5$, then $f(x)$ satisfies	(r) $f(x) > 0$
(S) If $x < 5$, then $f(x)$ satisfies	(s) $f(x) < 1$

- (a) P (p,r,s); Q(q, s); R(q,s); S(p, r, s)
 (b) P(p,r); Q(q); R(q, s); S(r,s)
 (c) P(p,s); Q(s); R(q); S(p, r, s)
 (d) P(p); Q(q, s); R(s); S(p,s)

- 121** For a, b, c and d are distinct numbers, if roots of the equation $x^2 - 10cx - 11d = 0$ are a, b and those of $x^2 - 10ax - 11b = 0$ are c, d , then the value of $a + b + c + d$ is
 (a) 1221
 (b) 1100
 (c) 1210
 (d) cannot be determined
- 122** Let $f(x) = ax^3 + bx^2 + cx + d$, then $f'(x) = 3ax^2 + 2bx + c$. Also, if $f'(k) = 0$ then $f(k)$ will give maxima/minima. Also, if there are various peak values of a graph then the highest peak value is called the absolute or global maximum and rest are called the local maxima. Similarly absolute or global minimum and local minima too can be defined.
 $f(x)$ is cubic polynomial which has local maximum at $x = -1$. If $f(2) = 18$, $f(1) = -1$, and $f'(x)$ has local minimum at $x = 0$. Then
 (i) $f(x)$ has local minima at $x = 1$
 (ii) Number of real roots are 3
 (iii) $f(x)$ is increasing for $x = [1, 2\sqrt{5}]$
 (iv) Sum of the roots is zero
 (a) Only (ii) and (iv) are true
 (b) Only (i) and (iii) are false
 (c) All of the four statements are true
 (d) None of the four statements are true
- 123** Let $f(x) = ax^3 + bx^2 + cx + d$, then $f'(x) = 3ax^2 + 2bx + c$. Also, if $f'(k) = 0$, then $f(k)$ will give maxima/minima. Also, if there are various peak values of a graph then the highest peak value is called the absolute or global maximum and rest are called the local maxima. Similarly absolute or global minimum and local minima too can be defined.
 $f(x)$ is cubic polynomial which has local maximum at $x = -1$. If $f(-1) = 10$, $f(1) = -6$, and $f'(x)$ has local minimum at $x = 1$. Find the distance between the local maximum and local minimum of the curve given by $f(x)$.
 (a) $4\sqrt{65}$ (b) $7\sqrt{65}$ (c) $12\sqrt{35}$ (d) $65\sqrt{6}$
- 124** If α and β are the roots of $ax^2 + bx + c = 0$, and $\alpha + \beta, \alpha^2 + \beta^2, \alpha^3 + \beta^3$ are in GP (Geometric Progression) and $\Delta = b^2 - 4ac$, then
 (a) $\Delta \neq 0$ (b) $b\Delta = 0$ (c) $c\Delta = 0$ (d) $\Delta = 0$
- 125** Let $f(x) = ax^p + bx^q + cx + d$, then $f'(x) = p(ax^{p-1}) + q(bx^{q-1}) + c$. Also, if $f'(k) = 0$, then $f(k)$ will give local maxima or local minima. Also, if there are various peak values of a graph then the highest peak value is called the absolute or global maximum and rest are called the local maxima. Similarly absolute or global minimum and local minima too can be defined.
 $f(x) = x^3 + bx^2 + cx + d$ is a cubic polynomial such that $0 < b^2 < c$, then in the interval of all the real numbers $(-\infty, \infty)$
 (a) $f(x)$ is a strictly increasing function
 (b) $f(x)$ is a strictly decreasing function
 (c) $f(x)$ has a local maxima
 (d) $f(x)$ has a local minima
- 126** If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ such that $\min f(x) > \max g(x)$, then the relation between b and c is
 (a) No real value of b and c
 (b) $0 < c < b\sqrt{2}$
 (c) $|c| < |b|\sqrt{2}$
 (d) $|c| > |b|\sqrt{2}$
- 127** If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then
 (a) $a < 2$ (b) $2 \leq a \leq 3$
 (c) $3 < a \leq 4$ (d) $a > 4$
- 128** If $9 \leq x \leq 16$, then
 (a) $(x-9)(x-16) \leq 0$
 (b) $(x-9)(x-16) \geq 0$
 (c) $(x-9)(x-16) < 0$
 (d) $(x-9)(x-16) > 0$
- 129** The quadratic equations $f(x) = 0$ and $g(x) = 0$ have a common root such that $f(2) = g(7) = 0$ and $f(4) \times g(9) = 24$. Which among the following give (s) all the correct values of the possible common root?
 (i) 10, 3, -4, -9
 (ii) -6.5, 10, 0, -4
 (iii) -9, -8, -4.5, -4
 (iv) $2\sqrt{2}$, $3\sqrt{3}$, 8, 27
 (v) 3, 10, -5, -8
 (a) Only I and IV (b) Only I
 (c) Only V (d) Only II and III

LEVEL 02 > HIGHER LEVEL EXERCISE

- 1 The value of x satisfying the equation

$$|x - 1|^{\log_3 x^2 - 2 \log_x 9} = (x - 1)^7 :$$

- (a) $\sqrt{3}$ (b) 3^4
(c) 3 (d) $\log_4 3$
- 2 If the roots of $10x^3 - cx^2 - 54x - 27 = 0$ are in HP, then find the value of c :
- (a) 2 (b) 6
(c) 9 (d) none of these
- 3 Find the number of pairs for (x, y) from the following equations :

$$\log_{100} |x + y| = \frac{1}{2}$$

$$\log_{10} y - \log_{10} |x| = \log_{100} 4$$

- (a) 0 (b) 1
(c) 2 (d) none of these
- 4 The number of solutions for real x , which satisfy the equation $2 \log_2 \log_2 x + \log_{1/2} \log_2 (2\sqrt{2}x) = 1$:
- (a) 1 (b) 2
(c) 4 (d) none of these
- 5 The solution set for which the equation satisfies $|x^2 + 4x + 3| + 2x + 5 = 0$:
- (a) $(-1 + \sqrt{3}, -1 - \sqrt{3})$ (b) $[-2, (-1 + \sqrt{3})]$
(c) $[-4, (-1 - \sqrt{3})]$ (d) none of these
- 6 The real numbers x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + \beta x + \gamma = 0$ are in AP. Find the intervals in which β and γ lie, respectively :
- (a) $(-\infty, 1/3] [-1/27, \infty)$
(b) $(-\infty, 3)$
(c) $(-1/3, 1/3) (-1/27, 1/27)$
(d) None of the above

- 7 For all $x \in (0, 1)$:

- (a) $e^x < 1 + x$
(b) $\log_e (1 + x) < x$
(c) $\sin x > x$
(d) $\log_e x > x$

- 8 Let $p \geq 3$ be an integer and α, β be the roots of $x^2 - (p + 1)x + 1 = 0$, then the value of $\alpha^n + \beta^n$, where $n \in N$

- (a) is divisible by 'p' (b) is an integer
(c) is a rational number (d) both (b) and (c)

- 9 Let a, b, c be real, if $ax^2 + bx + c = 0$ has two real roots α, β , where $\alpha < -1$ and $\beta > 1$, then the value of $1 + \frac{c}{a} + \left| \frac{b}{a} \right|$ is :

- (a) less than zero
(b) greater than zero
(c) equal to zero
(d) equal to $b^2 - 4ac$

- 10 If p and q are the roots of $x^2 + px + q = 0$, then :

- (a) $p = 1$
(b) $p = 1$ or 0
(c) $p = -2$
(d) $p = -2$ or 0

- 11 If p, q, r are positive and are in AP, the roots of quadratic equation $px^2 + qx + r = 0$ are real for :

- (a) $\left| \frac{r}{p} - 7 \right| \geq 4\sqrt{3}$ (b) $\left| \frac{p}{r} - 7 \right| \geq 4\sqrt{3}$
(c) all p and r (d) no p and r

- 12 The sum of all the real roots of the equation $|x - 2|^2 + |x - 2| - 2 = 0$ is :

- (a) 2 (b) 3
(c) 4 (d) none of these

- 13 Let p and q be the roots of the equation $x^2 - 2x + A = 0$ and let r and s be the roots of the equation $x^2 - 18x + B = 0$. If $p < q < r < s$ are in arithmetic progression, then A, B respectively equal to :

- (a) 8, 17 (b) 3, 7
(c) -3, 11 (d) none of these

- 14 If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then :

- (a) $a < 2$ (b) $2 \leq a \leq 3$
(c) $3 < a \leq 4$ (d) $a > 4$

- 15 If α and β ($\alpha < \beta$), are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then :

- (a) $0 < \alpha < \beta$ (b) $\alpha < 0 < \beta < |\alpha|$
(c) $\alpha < \beta < 0$ (d) $\alpha < 0 < |\alpha| < \beta$

- 16 If $b > a$, then the equation $(x - a)(x - b) - 1 = 0$ has :

- (a) both roots in $[a, b]$
(b) both roots in $(-\infty, 0)$
(c) both roots in $(b, +\infty)$
(d) one root in $(-\infty, a)$ and other root in $(b, +\infty)$

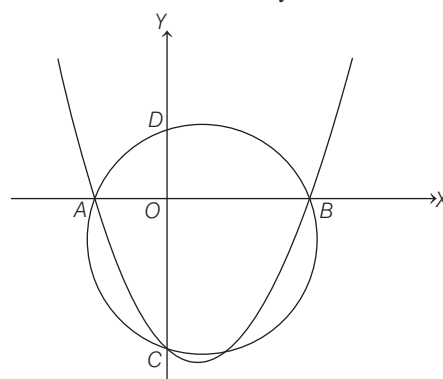
- 17 Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in GP, then the integral values of p and q respectively are :

- (a) -2, -32 (b) -2, 3
(c) -6, 3 (d) -6, -32

- 18 If α, β are the roots of the equation $x^2 - px + q = 0$, then find the quadratic equation, the roots of which are $(\alpha^2 - \beta^2)(\alpha^3 - \beta^3)$ and $(\alpha^3\beta^2 + \alpha^2\beta^3)$:

- (a) $px^2 - (5p + 7q)x - (p^6q^6 + 4p^2q^6) = 0$
(b) $x^2 - (p^5 - 5p^3q + 5pq^2)x + (p^6q^2 - 5p^4q^3 + 4p^2q^4) = 0$
(c) $x^2 - (p^3q - 5p^5 + p^4q) - (p^6q^2 - 5p^2q^6) = 0$
(d) all of the above

- 19** Given that α, γ are the roots of the equation $Ax^2 - 4x + 1 = 0$ and β, δ are the roots of the equation $Bx^2 - 6x + 1 = 0$, then the values of A and B respectively such that α, β, γ and δ are in HP :
- (a) -5, 9 (b) 3/2, 5
(c) 3, 8 (d) none of these
- 20** Let α, β be the roots of the equation $(x - a)(x - b) = c$; $c \neq 0$, then the roots of the equation $(x - \alpha)(x - \beta) + c = 0$ are :
- (a) a, c (b) b, c
(c) a, b (d) $a + c, b + c$
- 21** Let a, b, c be real numbers such that $a + b + c = 0$, then
- (a) $\left(\frac{a^5 + b^5 + c^5}{5}\right) > \left(\frac{a^3 + b^3 + c^3}{3}\right)\left(\frac{a^2 + b^2 + c^2}{2}\right)$
(b) $\left(\frac{a^5 + b^5 + c^5}{5}\right) < \left(\frac{a^3 + b^3 + c^3}{3}\right)\left(\frac{a^2 + b^2 + c^2}{2}\right)$
(c) $\left(\frac{a^5 + b^5 + c^5}{5}\right) = \left(\frac{a^3 + b^3 + c^3}{3}\right)\left(\frac{a^2 + b^2 + c^2}{2}\right)$
(d) $\left(\frac{a^5 + b^5 + c^5}{5}\right) = 6\left(\frac{a^3 + b^3 + c^3}{3}\right)\left(\frac{a^2 + b^2 + c^2}{2}\right)$
- 22** Given that p and q are prime, the equation $px^2 - qx + q = 0$ has
- (a) No rational roots (b) one rational root
(c) Two rational roots (d) Either (b) or (c)
- 23** The equation $x^4 - px^3 + q = 0$ has an integral root. Find this only integral root if p and q are prime numbers.
- (a) 6 (b) 5 (c) 7 (d) 1
- 24** If x, y, z are real and distinct, then $x^2 + 4y^2 + pz^2 - 2xy - 6yz - 3zx$ is always
- (a) Non-positive (b) non-negative
(c) negative (d) zero
- 25** The number of real solutions of $e^{\sin x} + e^{-\sin x} = 4$.
- (a) 0 (b) 1
(c) 2 (d) 4
- 26** If α and β are the roots of $x^2 + ux + v = 0$ and $x^{2n} + u^n x^n + v^n = 0$, where n is an even integer. What are the roots of $x^n + 1 + (x + 1)^n = 0$?
- (a) $(\alpha^n + 1)$ and $(\beta^n + 1)$
(b) $\frac{1}{\alpha}$ and $\frac{1}{\beta}$
(c) $(\alpha + \beta)$ and $(\alpha - \beta)$
(d) $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$
- 27** If sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then bc^2, ca^2, ab^2 are in
- (a) HP (b) GP
(c) AP (d) AGP
- 28** Let $f(x)$ be a quadratic expression, which is positive for all real x . Let $f(x) = ax^2 + bx + c$, $f'(x) = 2ax + b$, $f''(x) = 2a$; where $f'(x)$ and $f''(x)$ are the first and second derivatives of $f(x)$, respectively. If $g(x) = f(x) + f'(x) + f''(x)$, then for every real x
- (a) $g(x) > 0$ (b) $g(x) < 0$
(c) $g(x) \geq 0$ (d) $g(x) \leq 0$
- 29** Find the values of x , if
- $(x - 1)^3 + (x - 2)^3 + (x - 3)^3 + (x - 4)^3 + (x - 5)^3 = 0$.
- (a) $3, 2 \pm \sqrt{6}i$ (b) $3, 3 \pm \sqrt{6}i$
(c) $2, 3 \pm \sqrt{5}i$ (d) $0, 3 \pm \sqrt{5}i$
- 30** Find all the real solutions of $(x + 3)^5 - (x - 1)^5 \geq 244$.
- (a) $(-\infty, -1] \cup [1, \infty)$ (b) $(-\infty, -2] \cup [0, \infty)$
(c) $(-\infty, -2) \cup (0, \infty)$ (d) none of these
- 31** If α, β are the roots of $x^2 + px + q = 0$ and α^n, β^n are the roots of $x^{2n} + p^n x^n + q^n = 0$, and if $\left(\frac{\alpha}{\beta}\right), \left(\frac{\beta}{\alpha}\right)$ are the roots of $x^n + 1 + (x + 1)^n = 0$. Then n is
- (a) an integer (b) an odd integer
(c) an even integer (d) an irrational number
- 32** If P_i and p_i are the real numbers, then the number of non-real roots of the following equation is
- $\frac{P_1^2}{(x - p_1)} + \frac{P_2^2}{(x - p_2)} + \dots + \frac{P_k^2}{(x - p_k)} = x + 1$
- (a) 2 (b) 1
(c) 0 (d) data insufficient
- 33** If $a < b < c < d$ and $k \in \mathbb{R}$, then the roots of the equation $(x - a)(x - c) + k(x - b)(x - d) = 0$ are
- (a) real (b) non-real
(c) Either real or imaginary (d) none of these
- 34** For every $\{b, c\} \in \mathbb{R}^{\oplus}$, the quadratic curve $y = x^2 - bx - c$ is drawn. The roots are denoted by A and B .



The curve intersects Y -axis at C and another point D is taken on the Y -axis such that A, D, B and C are concyclic. Find the coordinates of D .

- (a) $(0, 1)$ (b) $(0, 3/2)$
(c) $(0, b/c)$ (d) cannot be determined

Answers

✓ Introductory Exercise 14.1

1 (b)	2 (c)	3 (d)	4 (a)	5 (b)	6 (c)	7 (a)	8 (b)	9 (c)	10 (a)
11 (d)	12 (b)	13 (c)	14 (a)	15 (a)	16 (c)	17 (b)	18 (b)	19 (c)	20 (a)
21 (a)	22 (a)	23 (a)	24 (a)	25 (b)	26 (b)	27 (a)	28 (b)	29 (c)	30 (d)
31 (b)	32 (b)	33 (c)	34 (c)	35 (c)	36 (b)	37 (c)	38 (b)	39 (b)	40 (d)

✓ Level 01 Basic Level Exercise

1 (c)	2 (b)	3 (c)	4 (c)	5 (b)	6 (b)	7 (c)	8 (c)	9 (b)	10 (b)
11 (b)	12 (b)	13 (d)	14 (c)	15 (c)	16 (c)	17 (c)	18 (b)	19 (b)	20 (c)
21 (c)	22 (b)	23 (a)	24 (a)	25 (b)	26 (c)	27 (c)	28 (c)	29 (b)	30 (d)
31 (c)	32 (b)	33 (d)	34 (a)	35 (b)	36 (a)	37 (a)	38 (a)	39 (d)	40 (a)
41 (c)	42 (c)	43 (b)	44 (d)	45 (c)	46 (b)	47 (c)	48 (b)	49 (d)	50 (c)
51 (c)	52 (d)	53 (c)	54 (c)	55 (d)	56 (d)	57 (c)	58 (c)	59 (c)	60 (d)
61 (c)	62 (b)	63 (c)	64 (c)	65 (c)	66 (b)	67 (a)	68 (d)	69 (d)	70 (a)
71 (c)	72 (c)	73 (a)	74 (b)	75 (a)	76 (a)	77 (d)	78 (c)	79 (a)	80 (d)
81 (d)	82 (a)	83 (b)	84 (c)	85 (d)	86 (a)	87 (a)	88 (d)	89 (c)	90 (b)
91 (d)	92 (a)	93 (c)	94 (d)	95 (a)	96 (a)	97 (d)	98 (c)	99 (d)	100 (c)
101 (d)	102 (b)	103 (a)	104 (a)	105 (c)	106 (d)	107 (d)	108 (d)	109 (b)	110 (b)
111 (a)	112 (d)	113 (c)	114 (a)	115 (c)	116 (c)	117 (b)	118 (a)	119 (a)	120 (a)
121 (c)	122 (c)	123 (a)	124 (c)	125 (a)	126 (d)	127 (a)	128 (a)	129 (c)	

✓ Level 02 Higher Level Exercise

1 (b)	2 (c)	3 (c)	4 (a)	5 (c)	6 (a)	7 (b)	8 (d)	9 (a)	10 (b)
11 (b)	12 (c)	13 (d)	14 (a)	15 (b)	16 (d)	17 (a)	18 (b)	19 (c)	20 (c)
21 (c)	22 (a)	23 (d)	24 (b)	25 (a)	26 (d)	27 (c)	28 (a)	29 (b)	30 (b)
31 (c)	32 (c)	33 (a)	34 (a)						

Hints & Solutions

Introductory Exercise

$$\begin{aligned}
 1 \quad \therefore x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 \therefore x &= \frac{7 \pm \sqrt{49 - 4 \times 15 \times (-36)}}{2 \times 15} \Rightarrow x = \frac{7 \pm 47}{30} \\
 \therefore x &= \frac{54}{30} \quad \text{or} \quad x = \frac{-40}{30} \\
 x &= \frac{9}{5} \quad \text{or} \quad x = -\frac{4}{3}
 \end{aligned}$$

NOTE (i) The above problem can also be solved by factorization.
(ii) The same problem can also be solved through options.

$$\begin{aligned}
 2 \quad \therefore x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 x &= \frac{6 \pm \sqrt{36 - 4 \times \sqrt{7} \times (-13\sqrt{7})}}{2 \times \sqrt{7}} \\
 \Rightarrow x &= \frac{6 \pm \sqrt{36 + 364}}{2\sqrt{7}} \Rightarrow x = \frac{6 \pm 20}{2\sqrt{7}} \\
 \therefore x &= \frac{13}{\sqrt{7}} \quad \text{or} \quad x = -\frac{7}{\sqrt{7}} \Rightarrow x = \frac{13}{\sqrt{7}} \quad \text{or} \quad x = -\sqrt{7}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad 6x^2 - 31x + 40 &= 0 \\
 \Rightarrow \left(x - \frac{8}{3}\right)\left(x - \frac{5}{2}\right) &= 0 \Rightarrow x = \frac{8}{3}, \frac{5}{2} \\
 4 \quad \text{Consider } \sqrt{\frac{y-3}{2y+1}} = t \text{ then, } 6t^2 + 1 &= 5t \\
 \Rightarrow 6t^2 - 5t + 1 &= 0 \quad \dots(i)
 \end{aligned}$$

get the values of t by solving the eq. (i) and then substitute the value of t in the relation

$$t = \sqrt{\frac{y-3}{2y+1}} \quad \dots(ii)$$

Solve the eq. (i) and then get the values of t and subsequently y .

$$\begin{aligned}
 5 \quad (x+2)(x-5)(x-6)(x+1) &= 144 \\
 \Rightarrow [(x+2)(x-6)][(x-5)(x+1)] &= 144 \\
 \Rightarrow (x^2 - 4x - 12)(x^2 - 4x - 5) &= 144 \\
 \Rightarrow \{(x^2 - 4x - 5) - 7\} \{x^2 - 4x - 5\} &= 144 \\
 \text{Let } t = x^2 - 4x - 5 &\quad \dots(i) \\
 \therefore (t-7)(t) &= 144 \Rightarrow t^2 - 7t - 144 = 0 \\
 \Rightarrow (t-16)(t+9) &= 0 \Rightarrow t = 16 \quad \text{or} \quad t = -9 \\
 \text{Substituting the values of } t \text{ in eq. (i), we get} \\
 x &= 7, -3, 2
 \end{aligned}$$

6 Check through options

Alternatively, square both the sides twice.

$$\begin{aligned}
 7 \quad 2\left(x^2 + \frac{1}{x^2}\right) - 3\left(x + \frac{1}{x}\right) - 1 &= 0 \\
 \Rightarrow 2\left[x^2 + \frac{1}{x^2} + 2 - 2\right] - 3\left(x + \frac{1}{x}\right) - 1 &= 0 \\
 \Rightarrow 2\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 3\left(x + \frac{1}{x}\right) - 1 &= 0 \\
 \Rightarrow 2\left(x + \frac{1}{x}\right)^2 - 3\left(x + \frac{1}{x}\right) - 5 &= 0 \\
 \Rightarrow 2t^2 - 3t - 5 &= 0 \quad \left(\text{Substituting } x + \frac{1}{x} = t\right)
 \end{aligned}$$

Now solve it and you will get, $t = -1$ and $t = \frac{5}{2}$

Now if $t = -1$, then $x + \frac{1}{x} = -1$.

$$\Rightarrow x^2 + 1 + x = 0 \Rightarrow x^2 + x + 1 = 0$$

$$\therefore x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$$

$$\text{and if } t = \frac{5}{2} \text{ then } x + \frac{1}{x} = \frac{5}{2} \Rightarrow 2x^2 - 5x + 2 = 0 \Rightarrow x = \frac{1}{2}, 2$$

$$8 \quad \text{Assume } \sqrt{\frac{x}{1-x}} = t$$

$$\therefore \sqrt{\frac{1-x}{x}} = \frac{1}{t}$$

$$\therefore t + \frac{1}{t} = \frac{13}{6}$$

Now, solve it as in the previous problem.

$$9 \quad \sqrt{2x^2 - 2x + 1} = 2x - 3$$

Square on both sides and simplify (**Hint** Type -3 problem)

10 **Hint** Type-3 problem. It requires double squaring.

Alternatively Go through options.

11 **Hint** Type 6 problem.

$$\begin{aligned}
 12 \quad \frac{1}{x+1} + \frac{1}{x+5} &= \frac{1}{x+2} + \frac{1}{x+4} \\
 \Rightarrow \left(\frac{1}{x+1} - \frac{1}{x+4}\right) + \left(\frac{1}{x+5} - \frac{1}{x+2}\right) &= 0 \\
 \Rightarrow \frac{3}{(x+1)(x+4)} - \frac{3}{(x+5)(x+2)} &= 0 \\
 \Rightarrow \frac{3[(x^2 + 7x + 10) - (x^2 + 5x + 4)]}{(x+1)(x+4)(x+5)(x+2)} &= 0 \\
 \Rightarrow (x^2 + 7x + 10) - (x^2 + 5x + 4) &= 0 \\
 \Rightarrow 2x + 6 = 0 \Rightarrow x &= -3
 \end{aligned}$$

13 For equal roots $D = 0$

$$\text{i.e., } b^2 - 4ac = 0$$

$$\Rightarrow [-2(1 + 3k)]^2 - 4 \times 1 \times 7 \times (3 + 2k) = 0$$

Solve it and get the value of k .

14 $D = b^2 - 4ac = 4 - 4 \times (-3) \times (-8) = -92$

15 $D = b^2 - 4ac = 25 - 4 \times 1 \times 7 = -3$

Since $D < 0$, therefore roots are not real, i.e., roots will be imaginary.

16 $a^2x^2 + abx - b^2 = 0$

$$D = b^2 - 4ac = (ab)^2 - 4 \times a^2 \times (-b^2) = (ab)^2 + 4a^2b^2 \\ = (ab)^2[1 + 4] = (ab)^2(5)$$

$\therefore$ The roots are real and unequal

17 For non-real roots $D < 0$

$$\therefore b^2 - 4ac < 0$$

$$\therefore (-p)^2 - 4 \times 1 \times q < 0$$

$$\Rightarrow p^2 - 4q < 0 \Rightarrow p^2 < 4q$$

18 $4x^2 - 3kx + 1 = 0 \Rightarrow D = b^2 - 4ac = 0$

$$\therefore 9k^2 - 4 \times 1 \times 4 = 0$$

$$\Rightarrow k^2 = \frac{16}{9} \Rightarrow k = \pm \frac{4}{3}$$

19 Let α, β be the roots of the equation, then

$$\alpha + \beta = \frac{1}{2} \alpha\beta \text{ and } -\frac{b}{a} = \frac{1}{2} \cdot \frac{c}{a}$$

$$\Rightarrow -b = \frac{c}{2} \Rightarrow (k + 6) = \frac{2(2k - 1)}{2} \Rightarrow k = 7$$

20 $\alpha + \beta = \alpha\beta$

$$\Rightarrow \frac{-b}{a} = \frac{c}{a} \Rightarrow -b = c$$

$$\therefore -2k = 4 \Rightarrow k = -2$$

21 Since -4 is a root of $x^2 - px - 4 = 0$

$$\therefore (-4)^2 - p(-4) - 4 = 0$$

$$\Rightarrow p = -3$$

$\therefore$ The equation becomes $x^2 + 3x + k = 0$

Since both the roots of this equation are equal.

$$\therefore \text{Sum of the roots } (\alpha + \alpha) = 2\alpha = -3 \Rightarrow \alpha = -\frac{3}{2}$$

$$\therefore \text{Product of the roots } (\alpha \cdot \alpha) = \alpha^2 = (-3/2)^2 = \frac{9}{4} = k$$

$$\therefore k = \frac{9}{4}$$

22 $\alpha^2 + \beta^2 = 40$

$$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 40 \Rightarrow (8)^2 - 2k = 40$$

$$\Rightarrow k = 12$$

23 For equal roots, $D = 0$

$$\text{i.e., } b^2 - 4ac = 0$$

$$\text{But now } x^2 + p(4x + p - 1) + 2 = 0$$

$$\Rightarrow x^2 + 4px + p^2 - p + 2 = 0$$

$$\Rightarrow x^2 + 4px + (p^2 - p + 2) = 0$$

$$\therefore (4p)^2 - 4 \times 1 \times (p^2 - p + 2) = 0$$

$$\Rightarrow 16p^2 - 4p^2 + 4p - 8 = 0 \Rightarrow 12p^2 + 4p - 8 = 0$$

$$\Rightarrow 12p^2 + 12p - 8p - 8 = 0 \Rightarrow 12p(p + 1) - 8(p + 1) = 0$$

$$\Rightarrow (p + 1)(12p - 8) = 0 \Rightarrow p = -1 \text{ or } p = \frac{8}{12} = \frac{2}{3}$$

24 $\alpha\beta = \frac{c}{a} = \frac{3}{3} = 1$

25 $x^2 - 3x + 2 = 0$

$$\Rightarrow x^2 - x - 2x + 2 = 0$$

$$\Rightarrow (x - 1)(x - 2) = 0 \Rightarrow \alpha = 1 \text{ and } \beta = 2$$

$$\therefore -\alpha = -1 \text{ and } -\beta = -2$$

$\therefore$ The required equation is

$$[x - (-\alpha)][x - (-\beta)] = 0$$

$$\Rightarrow (x + 1)(x + 2) = 0 \Rightarrow x^2 + 3x + 2 = 0$$

26 Method I. If α and β be the roots of a quadratic equation, then the equation will be

$$(x - \alpha)(x - \beta) = 0$$

$$\therefore (x - \sqrt{3})(x - 2\sqrt{3}) = 0 \Rightarrow x^2 - 3\sqrt{3}x + 6 = 0$$

Method II. The required equation

$$x^2 - (\alpha + \beta)x + (\alpha \cdot \beta) = 0$$

$$\Rightarrow x^2 - (\sqrt{3} + 2\sqrt{3})x + (\sqrt{3} \cdot 2\sqrt{3}) = 0$$

$$\Rightarrow x^2 - 3\sqrt{3}x + 6 = 0$$

27 $\alpha + \beta = \frac{-b}{a} = \frac{-1}{6} \Rightarrow \alpha\beta = \frac{c}{a} = \frac{-2}{6} = -\frac{1}{3}$

$$\therefore \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(-1/6)^2 - 2(-1/3)}{-1/3} = \frac{\frac{1}{36} + \frac{2}{3}}{-\frac{1}{3}} = \frac{\frac{25}{36}}{-\frac{1}{3}} = -\frac{25}{12}$$

Alternatively $6x^2 + x - 2 = 0$

$$\Rightarrow 6x^2 - 3x + 4x - 2 = 0$$

$$\Rightarrow 3x(2x - 1) + 2(2x - 1) = 0$$

$$\Rightarrow x = \frac{1}{2} \text{ or } x = -\frac{2}{3}$$

$$\therefore \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{1/2}{-2/3} + \frac{-2/3}{1/2}$$

$$= -\frac{3}{4} - \frac{4}{3} = -\frac{25}{12}$$

28 $\alpha + \beta = 10$ and $\alpha \cdot \beta = 10$

$$\Rightarrow -\frac{b}{a} = 10 \text{ and } \frac{c}{a} = 10$$

$$\Rightarrow \frac{5}{a} = 10 \Rightarrow a = \frac{1}{2}$$

$$\therefore \frac{c}{1/2} = 10 \Rightarrow c = 5$$

$$\therefore a = \frac{1}{2} \text{ and } c = 5$$

29 $\alpha + \beta = -\frac{b}{a} = 1$ and $\alpha\beta = \frac{c}{a} = -4$

$$\begin{aligned} \therefore \frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta &= \frac{\alpha + \beta}{\alpha\beta} - \alpha\beta \\ &= \frac{1}{-4} - (-4) = 4 - \frac{1}{4} = \frac{15}{4} \end{aligned}$$

30 $\alpha + \beta = 2$ and $\alpha\beta = -1$

$$\therefore \alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta) = 2(-1) = -2$$

31 Sum of roots $= (2\alpha + 1) + (2\beta + 1)$

$$= 2\alpha + 2\beta + 2 = 2(\alpha + \beta) + 2$$

and product of roots $= (2\alpha + 1)(2\beta + 1)$

$$= 4\alpha\beta + 2\alpha + 2\beta + 1 = 4\alpha\beta + 2(\alpha + \beta) + 1$$

Now since $\alpha + \beta = 1$

and $\alpha\beta = -2$

$$\therefore \text{Sum of roots} = 2(\alpha + \beta) + 2 = 2(1) + 2 = 4$$

and product of roots $= 4\alpha\beta + 2(\alpha + \beta) + 1$

$$= 4 \times (-2) + 2(1) + 1 = -8 + 2 + 1 = -5$$

$\therefore$ Required equation is $x^2 - 4x - 5 = 0$.

32 $\alpha + \beta = \frac{6}{3} = 2$ and $\alpha\beta = \frac{4}{3}$

$$\therefore \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2\left(\frac{\alpha + \beta}{\alpha\beta}\right) + 3\alpha\beta$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} + 2\left(\frac{\alpha + \beta}{\alpha\beta}\right) + 3\alpha\beta$$

$$= \frac{4 - 8/3}{4/3} + 2\left(\frac{2}{4/3}\right) + 3 \times \frac{4}{3} = \frac{4/3}{4/3} + 3 + 4 = 8$$

33 $\alpha + \beta = 5 \Rightarrow \alpha - \beta = 1$

$$\therefore \alpha = 3 \text{ and } \beta = 2$$

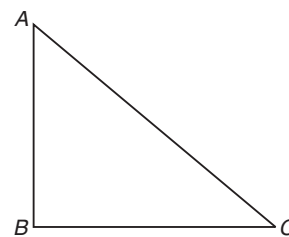
$$\therefore \alpha\beta = \frac{c}{a} = \frac{k}{1} = 3 \times 2 = 6 \Rightarrow k = 6$$

34 Let the one root be α and other root be β , then $\beta = 2\alpha$.

$$\therefore \alpha + \beta = 3\alpha = -\frac{b}{a} \Rightarrow \alpha = -\frac{b}{3a}$$

and $\alpha\beta = 2\alpha^2 = \frac{c}{a}$

$$\therefore 2\alpha^2 = \frac{c}{a}$$



$$\Rightarrow 2 \times \left(\frac{-b}{3a}\right)^2 = \frac{c}{a} \Rightarrow \frac{2b^2}{9a^2} = \frac{c}{a} \Rightarrow 2b^2 = 9ac$$

- 35** Let the base of the triangle be x cm, then $BC = x$ cm and hypotenuse $AC = (x + 2)$ cm and the perpendicular

$$AB = \frac{(x + 2) - 1}{2} = \frac{(x + 1)}{2}$$

[Let the length of AB be k , then

$$AC = 2k + 1 \Rightarrow k = \frac{AC - 1}{2}]$$

$\therefore$ By Pythagoras theorem

$$AB^2 + BC^2 = AC^2 \quad (\text{see the figure})$$

$$\left(\frac{x + 1}{2}\right)^2 + x^2 = (x + 2)^2$$

$$\Rightarrow \frac{x^2 + 1 + 2x}{4} + x^2 = x^2 + 4 + 4x$$

$$\Rightarrow x^2 + 1 + 2x + 4x^2 = 4x^2 + 16 + 16x$$

$$\Rightarrow x^2 - 14x - 15 = 0 \Rightarrow (x - 15)(x + 1) = 0$$

$$\Rightarrow x = 15 \text{ cm} \quad (\because x = -1 \text{ cm is inadmissible})$$

$$\therefore \text{base} = 15 \text{ cm}$$

$$\text{and perpendicular} = 8 \text{ cm and hypotenuse} = 17 \text{ cm}$$

- 36** Let the tens digit be x and unit digit be y . Then the two digit number $= 10x + y$

$$\text{but } x \times y = 12 \Rightarrow y = \frac{12}{x}$$

$$\therefore \text{the number is } \left(10x + \frac{12}{x}\right)$$

$$\text{Again } 10x + \frac{12}{x} + 9 = 10 \times \left(\frac{12}{x}\right) + x$$

$$\Rightarrow 10x^2 + 12 + 9x = 120 + x^2$$

$$\Rightarrow 9x^2 + 9x - 108 = 0 \Rightarrow x^2 + x - 12 = 0$$

$$\Rightarrow (x + 4)(x - 3) = 0$$

$$\Rightarrow x = -4, 3$$

but x cannot be negative

$$\therefore x = 3 \text{ only.}$$

$$\therefore y = \frac{12}{x} = \frac{12}{3} = 4$$

$$\therefore \text{the number} = 10x + y = 10 \times 3 + 4 = 34$$

37 Let the usual speed of plane be x km/hr

$$\therefore \text{Usual time taken} = \frac{1600}{x} \text{ hr}$$

$$\text{New speed} = (x + 400) \text{ km/hr}$$

$$\text{and the new time} = \frac{1600}{x} - \frac{40}{60} = \left(\frac{1600}{x} - \frac{2}{3} \right) \text{ hr}$$

$$\therefore \text{Distance} = \text{speed} \times \text{time}, 1600 = (x + 400) \left(\frac{1600}{x} - \frac{2}{3} \right)$$

$$1600 = 1600 - \frac{2x}{3} + \frac{640000}{x} - \frac{800}{3}$$

$$\Rightarrow 4800x = 4800x - 2x^2 + 1920000 - 800x$$

$$\Rightarrow 2x^2 - 1920000 + 800x = 0 \Rightarrow x^2 - 960000 + 400x = 0$$

$$\Rightarrow x^2 + 400x - 960000 = 0$$

$$\Rightarrow x^2 + 1200x - 800x - 960000 = 0$$

$$\Rightarrow x(x + 1200) - 800(x + 1200) = 0$$

$$(x + 1200)(x - 800) = 0$$

$$\Rightarrow x = 800 \text{ km/hr} \quad (\because x = -1200 \text{ is inadmissible})$$

38 Let the two consecutive odd numbers be x and $x + 2$

$$\therefore x^2 + (x + 2)^2 = 290$$

$$\Rightarrow x^2 + x^2 + 4 + 4x = 290 \Rightarrow 2x^2 + 4x + 4 = 290$$

$$\Rightarrow x^2 + 2x + 2 = 145$$

$$\Rightarrow x^2 + 2x - 143 = 0$$

$$\Rightarrow (x + 13)(x - 11) = 0$$

$$\Rightarrow x = 11 \text{ or } x = -13, \text{ but } x \text{ is positive}$$

$$\therefore x + 2 = 13$$

$$\therefore \text{Sum of the numbers} = 11 + 13 = 24$$

39 Let the shopkeeper buys x books

$$\therefore \text{Rate of the book} = \frac{80}{x}$$

$$\text{the new number of books} = (x + 4)$$

$$\therefore \text{the new price} = \frac{80}{(x + 4)}$$

$$\therefore \frac{80}{(x + 4)} + 1 = \frac{80}{x} \Rightarrow \left(\frac{1}{x} - \frac{1}{(x + 4)} \right) = \frac{1}{80}$$

$$\Rightarrow \frac{4}{x^2 + 4x} = \frac{1}{80} \Rightarrow x^2 + 4x - 320 = 0$$

$$\Rightarrow (x + 20)(x - 16) = 0$$

$$\Rightarrow x = 16, (x = -20, \text{ is inadmissible})$$

$$\therefore \text{He bought 16 books.}$$

40 $x^2 + (x + 4)^2 = 656$

$$\Rightarrow x^2 + x^2 + 16 + 8x = 656$$

$$\Rightarrow x^2 + 4x - 320 = 0$$

$$\Rightarrow (x + 20)(x - 16) = 0$$

$$\Rightarrow x = 16, (x = -20 \text{ is inadmissible})$$

$$\therefore \text{Sides of the square are } x = 16 \text{ cm}$$

$$\text{and } (x + 4) = 20 \text{ cm.}$$

✓ Level 01 Basic Level Exercise

$$1 \quad y_{\min} = \frac{-b^2 + 4ac}{4a} \quad \text{at } x = -\frac{b}{2a}$$

$$\therefore y_{\min} = \frac{-p^2 + 4 \times 1 \times q}{4 \times 1} = \frac{4q - p^2}{4}$$

Now, since p and q are the roots of the equation $x^2 + px + q = 0$.

$$\therefore p + q = -p$$

$$\Rightarrow q = -2p$$

$$\text{and } pq = q \Rightarrow p = 1 \quad (\because q \neq 0)$$

$$\therefore q = -2$$

$$\therefore y_{\min} = \frac{4q - p^2}{4} = \frac{4 \times (-2) - (1)^2}{4} = -\frac{9}{4}$$

2 Let α, β be the roots of the given equation, then, $\alpha + \beta = p - 2$ and $\alpha\beta = -(p + 1)$.

$$\text{Now, } \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (p - 2)^2 + 2(p + 1)$$

$$= p^2 - 2p + 6 = (p - 1)^2 + 5$$

Clearly $\alpha^2 + \beta^2 \geq 5$. So minimum value of $\alpha^2 + \beta^2$ is 5, which is attained at $p = 1$.

3 Let $f(x) = x^2 + 2(p + 1)x + 9p - 5$. Let α, β be the roots of $f(x) = 0$. The equation $f(x) = 0$ will have both negative roots, if

$$(i) D \geq 0 \quad (ii) a < 0, \beta < 0, \text{ i.e., } (\alpha + \beta) < 0$$

$$(iii) f(0) > 0$$

$$\text{Now, } D \geq 0 \Rightarrow 4(p + 1)^2 - 36p + 20 \geq 0$$

$$\Rightarrow p^2 - 7p + 6 \geq 0$$

$$\Rightarrow (p - 6)(p - 1) \geq 0$$

$$\Rightarrow p \leq 1 \text{ or } p \geq 6 \quad \dots(i)$$

$$\alpha + \beta < 0 \Rightarrow -2(p + 1) < 0$$

$$\Rightarrow p + 1 > 0 \Rightarrow p > -1 \quad \dots(ii)$$

$$\text{and } f(0) > 0 \Rightarrow 9p - 5 > 0$$

$$\Rightarrow p > \frac{5}{9} \quad \dots(iii)$$

from eqs. (i), (ii) and (iii), we get $p \geq 6$.

4 Let α_1, β_1 be the roots of $ax^2 + bx + c = 0$ and α_2, β_2 be the roots of $px^2 + qx + r = 0$, then,

$$\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$$

$$\begin{aligned}
 &\Rightarrow \frac{\alpha_1 + \beta_1}{\alpha_1 - \beta_1} = \frac{\alpha_2 + \beta_2}{\alpha_2 - \beta_2} \\
 &\Rightarrow \frac{(\alpha_1 + \beta_1)^2}{(\alpha_1 - \beta_1)^2} = \frac{(\alpha_2 + \beta_2)^2}{(\alpha_2 - \beta_2)^2} \\
 &\Rightarrow \frac{(\alpha_1 + \beta_1)^2}{(\alpha_1 + \beta_1)^2 - 4\alpha_1\beta_1} = \frac{(\alpha_2 + \beta_2)^2}{(\alpha_2 + \beta_2)^2 - 4\alpha_2\beta_2} = \frac{b^2/a^2}{b^2 - 4ac} \\
 &= \frac{q^2/p^2}{q^2 - 4rp} = \frac{b^2}{D_1} = \frac{q^2}{D_2} \Rightarrow \frac{D_1}{D_2} = \frac{b^2}{q^2} \\
 &\quad \frac{q^2}{p^2}
 \end{aligned}$$

☞ Alternatively Let 2, 3 be the roots of the equation $ax^2 + bx + c = 0$

$\therefore$ the equation is $x^2 - 5x + 6 = 0$

$\therefore a = 1, b = -5$ and $c = 6$

$\therefore D_1 = b^2 - 4ac = 25 - 4 \times 1 \times 6 = 1$

Again, if 4, 6 be the roots of the equation $px^2 + qx + r = 0$, then the equation will be $x^2 - 10x + 24 = 0$

$\Rightarrow p = 1, q = -10$ and $r = 24$

$\therefore D_2 = q^2 - 4pr = 100 - 4 \times 1 \times 24 = 4$

$\therefore \frac{D_1}{D_2} = \frac{1}{4}$

Now, check through option, only option (c) is correct, as

$$\frac{b^2}{q^2} = \frac{(-5)^2}{(-10)^2} = \frac{1}{4} = \frac{D_1}{D_2}$$

Caution : $\frac{\alpha_1}{\beta_1} \neq \frac{\alpha_2}{\alpha_2}$, when solving through numerical values.

5 The given equations are :

$$x^2 + px + qr = 0 \quad \dots(i)$$

$$x^2 + qx + rp = 0 \quad \dots(ii)$$

$$\text{and } x^2 + rx + pq = 0 \quad \dots(iii)$$

Let α, β be the roots of (i), β, γ be roots of (ii) and γ, α be the roots of (iii). Since β is a common root of (i) and (ii)

$$\therefore \beta^2 + p\beta + qr = 0$$

$$\text{and } \beta^2 + q\beta + rp = 0$$

$$\Rightarrow (p - q)\beta + r(q - p) = 0 \Rightarrow \beta = r$$

$$\text{Now, } \alpha\beta = qr \Rightarrow \alpha r = qr \Rightarrow \alpha = q$$

Since β and γ are roots of (ii). Therefore

$$\beta r = rp \Rightarrow \gamma r = rp \Rightarrow \gamma = p$$

$$\therefore \alpha + \beta + \gamma = q + r + p = p + q + r$$

NOTE $\alpha + \beta + \gamma$ can also be equal to 0 and $-\frac{1}{2}(p + q + r)$.

6 By hypothesis,

$$\frac{\alpha}{\alpha - 1} + \frac{\alpha + 1}{\alpha} = -\frac{b}{a} \quad \text{and} \quad \frac{\alpha}{\alpha - 1} \cdot \frac{\alpha + 1}{\alpha} = \frac{c}{a}$$

$$\Rightarrow \frac{2\alpha^2 - 1}{\alpha^2 - \alpha} = -\frac{b}{a} \quad \text{and} \quad \alpha = \frac{c + a}{c - a}$$

$$\Rightarrow (c + a)^2 + 4ac = -2b(c + a)$$

$$\Rightarrow (c + a)^2 + 2b(a + c) + b^2 = b^2 - 4ac$$

$$\Rightarrow (a + b + c)^2 = b^2 - 4ac$$

7 One double root means a single root appears two times i.e., two roots are equal.

$$\therefore D = 0 \Rightarrow (2)^2 - 4a \cdot 1 = 0$$

$$\Rightarrow a = 1.$$

8 Let α, β be the roots of a quadratic equation and α^2, β^2 be the roots of another quadratic, Since the quadratic remains the same, we have

$$\alpha + \beta = \alpha^2 + \beta^2 \quad \dots(i)$$

$$\text{and } \alpha\beta = \alpha^2\beta^2 \quad \dots(ii)$$

$$\text{Now, } \alpha\beta = \alpha^2\beta^2 \Rightarrow \alpha\beta(\alpha\beta - 1) = 0$$

$$\Rightarrow \alpha = 0 \quad \text{or} \quad \beta = 0 \quad \text{or} \quad \alpha\beta = 1.$$

If $\alpha = 0$, then $\beta = \beta^2$ [putting $\alpha = 0$ in Eq. (i)]

$$\Rightarrow \beta(1 - \beta) = 0 \Rightarrow \beta = 0, \beta = 1.$$

Thus, we get two sets of values of α and β viz. $\alpha = 0, \beta = 0$ and $\alpha = 0, \beta = 1$. Now, if $\alpha\beta = 1$,

$$\text{then } \alpha + \frac{1}{\alpha} = \alpha^2 + \frac{1}{\alpha^2} \quad [\text{putting } \beta = \frac{1}{\alpha} \text{ in Eq. (i)}]$$

$$\Rightarrow \alpha + \frac{1}{\alpha} = \left(\alpha + \frac{1}{\alpha}\right)^2 - 2$$

$$\Rightarrow \left(\alpha + \frac{1}{\alpha}\right)^2 - \left(\alpha + \frac{1}{\alpha}\right) - 2 = 0$$

$$\Rightarrow \alpha + \frac{1}{\alpha} = 2 \quad \text{or} \quad \alpha + \frac{1}{\alpha} = -1$$

$$\Rightarrow \alpha = 1 \quad \text{or} \quad \alpha = \omega, \omega^2$$

Putting $\alpha = 1$, in $\alpha\beta = 1$, we get $\beta = 1$, and putting $\alpha = \omega$ in $\alpha\beta = 1$, we get $\beta = \omega^2$

Putting $\alpha = \omega^2$ in $\alpha\beta = 1$, we get $\beta = \omega$. Thus, we get four sets of values of α, β viz., $\alpha = 0, \beta = 0$; $\alpha = 0, \beta = 1$; $\alpha = \omega, \beta = \omega^2$; $\alpha = 1, \beta = 1$.

Thus, there are four quadratics which remain unchanged by squaring their roots.

9 Let $x = \sqrt{6 + \sqrt{6 + \dots}} \Rightarrow x^2 = x + 6$

$$\Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow x = 3, -2$$

$$\therefore x > 0, \therefore x = 3.$$

- 10** The best way is to consider some values of the roots α , β and γ .

Let $\alpha = 2$, $\beta = 3$ and $\gamma = 4$, then

$$(x - \alpha)(x - \beta)(x - \gamma) = 0$$

$$\Rightarrow (x - 2)(x - 3)(x - 4) = 0$$

$$\Rightarrow x^3 - 9x^2 + 26x - 24 = 0$$

$$\Rightarrow a_0 = -9$$

$$a_1 = 26$$

$$a_2 = -24$$

$$\begin{aligned} \text{Now, } (1 - \alpha^2)(1 - \beta^2)(1 - \gamma^2) &= (1 - 4)(1 - 9)(1 - 16) \\ &= -3 \times -8 \times -15 = -360 \end{aligned}$$

Now, go through options and verify.

Consider option (b)

$$\begin{aligned} (1 + a_1)^2 - (a_0 + a_2)^2 &= (1 + 26)^2 - (-9 - 24)^2 \\ &= (27)^2 - (33)^2 \\ &= (27 + 33)(27 - 33) \\ &= 60 \times (-6) = -360 \end{aligned}$$

Hence option (b) is correct.

11 $k = \frac{x^2 - x + 1}{x^2 + x + 1}$

$$\Rightarrow kx^2 + kx + k = x^2 - x + 1$$

$$\Rightarrow (k - 1)x^2 + (k + 1)x + k - 1 = 0$$

Since x is real, the discriminant

$$D = (k + 1)^2 - 4(k - 1)^2 \geq 0$$

$$\Rightarrow (3k - 1)(-k + 3) \geq 0$$

$$\Rightarrow \left(k - \frac{1}{3}\right)(k - 3) \leq 0$$

$$\Rightarrow \frac{1}{3} \leq k \leq 3$$

12 Let $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots \infty}}}}$

$$\therefore x = \sqrt{2 + x}$$

$$\Rightarrow x^2 = 2 + x$$

$$\Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow x = \frac{1 \pm \sqrt{9}}{2} = \frac{1 \pm 3}{2} = 2, -1$$

$$\therefore x > 0, \therefore x = 2$$

13 We have, $\alpha + \beta + \gamma = 0$

$$\therefore \beta + \gamma = -\alpha, \gamma + \alpha = -\beta \text{ and } \alpha + \beta = -\gamma.$$

$\therefore$ The equation whose roots are $-\alpha, -\beta, -\gamma$ can be obtained by replacing x by $-x$ in the given equation.

Therefore the required equation is :

$$(-x)^3 - 3(-x) + 11 = 0$$

or $x^3 - 3x - 11 = 0$

- 14** Let α be a common root of the two given equations, then $\alpha^2 - 3\alpha + 35 = 0$ and $\alpha^2 - a\alpha - 21 = 0$. On subtracting we get $-2a\alpha + 56 = 0$ or $\alpha = \frac{28}{a}$.

As α is a root of $x^2 - ax - 21 = 0$,

$$\therefore \left(\frac{28}{a}\right)^2 - a\left(\frac{28}{a}\right) - 21 = 0$$

$$\text{or } a^2 = 4^2 \text{ or } a = \pm 4$$

$\therefore a > 0$, we get $a = 4$

15 $(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$$4 = 6 + 2(\alpha\beta + \beta\gamma + \gamma\alpha)$$

$$\Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = -1$$

$$\text{Also, } \alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma$$

$$= (\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha)$$

$$8 - (3\alpha\beta\gamma) = 2(6 + 1)$$

$$\Rightarrow \alpha\beta\gamma = -2$$

$$\text{Now, } (\alpha^2 + \beta^2 + \gamma^2)^2 = \Sigma\alpha^4 + 2\Sigma\beta^2\gamma^2$$

$$= \Sigma\alpha^4 + 2[(\Sigma\beta\gamma)^2 - 2\alpha\beta\gamma(\Sigma\gamma)]$$

$$\Rightarrow \Sigma\alpha^4 = 36 - 2[(-1)^2 - 2(-2)(2)] = 18$$

- 16** The roots of the given equation will be of opposite signs if they are real and their product is negative, i.e.,

$$D \geq 0 \text{ and product of roots } < 0$$

$$\Rightarrow (a^3 + 8a - 1)^2 - 8(a^2 - 4a) \geq 0$$

$$\text{and } \frac{a^2 - 4a}{2} < 0$$

$$\Rightarrow a^2 - 4a < 0$$

$$[\therefore a^2 - 4a < 0 \Rightarrow (a^3 + 8a - 1)^2 - 8(a^2 - 4a) \geq 0]$$

$$\Rightarrow 0 < a < 4.$$

17 We have $\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$

$$\text{and } \alpha + k + \beta + k = -\frac{q}{p}$$

$$\text{and } (\alpha + k)(\beta + k) = \frac{r}{p}$$

$$\Rightarrow \alpha + \beta + 2k = -\frac{q}{p}$$

$$\Rightarrow -\frac{b}{a} + 2k = -\frac{q}{p} \quad \left(\because \alpha + \beta = -\frac{b}{a}\right)$$

$$\therefore k = \frac{1}{2} \left(\frac{b}{a} - \frac{q}{p} \right)$$

 **NOTE** The problem can also be solved by assuming some appropriate values.

- 18** Let $f(x) = ax^2 + bx + c$. It is given that α, β are real roots of $f(x) = 0$. So, k lies between α and β , if

$$af(k) < 0 \Rightarrow a(ak^2 + bk + c) < 0$$

$$\Rightarrow a^2k^2 + abk + ac < 0$$

- 19** The given equation will have equal roots iff $D = 0$

$$\Rightarrow (27 \times 3^{1/k} - 15)^2 - 144 = 0$$

$$\Rightarrow 27 \times 3^{1/k} - 15 = \pm 12$$

$$\Rightarrow 27 \times 3^{1/k} = 27 \text{ or } 3$$

$$\Rightarrow 3^{1/k} = 1 \text{ or } 3^{1/k} = \frac{1}{9}$$

$$\Rightarrow \frac{1}{k} = 0 \text{ or } -2$$

But $\frac{1}{k}$ cannot be zero. So $k = -\frac{1}{2}$.

- 20** Let $f(x) = x^2 - 2(a-1)x + (2a+1) = 0$.

Then, $f(x) = 0$ will have both roots positive, if

(i) $D > 0$ (ii) Sum of the roots > 0 and (iii) $f(0) > 0$

$$\text{Now, } D \geq 0 \Rightarrow 4(a-1)^2 - 4(2a+1) \geq 0$$

$$\Rightarrow a^2 - 4a \geq 0 \Rightarrow a \leq 0 \text{ or } a \geq 4 \quad \dots(i)$$

Sum of the roots > 0

$$\Rightarrow 2(a-1) > 0 \Rightarrow a > 1 \quad \dots(ii)$$

and $f(0) > 0$

$$\Rightarrow (2a+1) \Rightarrow a > -\frac{1}{2} \quad \dots(iii)$$

From eqs. (i), (ii) and (iii) we get $a \geq 4$.

Hence, the least integral value of a is 4.

- 21** $2^{3x^2 - 7x + 4} = 1 = 2^0$

$$\Rightarrow 3x^2 - 7x + 4 = 0$$

$$\Rightarrow 3x^2 - 3x - 4x + 4 = 0$$

$$\Rightarrow 3x(x-1) - 4(x-1) = 0$$

$$\Rightarrow x = 1 \text{ or } x = \frac{4}{3}$$

$$\therefore x = 1, \frac{4}{3}$$

- 22** $2|x|^2 - 5|x| + 2 = 0$

$$\Rightarrow (2|x| - 1)(|x| - 2) = 0$$

$$\therefore |x| = \frac{1}{2}, 2$$

$$\therefore x = \pm \frac{1}{2}, \pm 2$$

- 23** $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0 \quad \dots(i)$

It is possible only when $x-1=0$, $x-2=0$, $x-3=0$

$$x = 1, x = 2, x = 3$$

but $x = 1, 2, 3$ do not satisfy eq. (i).

- 24** $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$

$$\therefore x+1 \geq 0, x-1 \geq 0, 4x-1 \geq 0$$

All these inequalities are satisfied when $x \geq 1$. Squaring both the sides of the equation we get

$$(x+1) + (x-1) - 2\sqrt{x^2-1} = 4x-1$$

$$\Rightarrow 2\sqrt{x^2-1} = 1-2x$$

Squaring again, we get

$$4(x^2-1) = 1-4x+4x^2 \Rightarrow x = \frac{5}{4}$$

$$\text{For } x = \frac{5}{4}, \quad \text{LHS} = \sqrt{\frac{5}{4}+1} - \sqrt{\frac{5}{4}-1} = 1$$

$$\text{and} \quad \text{RHS} = \sqrt{4\left(\frac{5}{4}\right)-1} = \sqrt{4} = 2$$

but LHS $\neq$ RHS

$\therefore x = \frac{5}{4}$ is not a root of the equation.

- 25** $\alpha + \beta = \frac{3}{8}, \alpha\beta = \frac{27}{8}$

$$\begin{aligned} \therefore \left(\frac{\alpha^2}{\beta}\right)^{1/3} + \left(\frac{\beta^2}{\alpha}\right)^{1/3} &= \left(\frac{\alpha^3}{\alpha\beta}\right)^{1/3} + \left(\frac{\beta^3}{\alpha\beta}\right)^{1/3} \\ &= \frac{\alpha + \beta}{(\alpha\beta)^{1/3}} = \frac{3/8}{(27/8)^{1/3}} = \frac{3/8}{3/2} = \frac{1}{4} \end{aligned}$$

- 26** The quadratic equation $3x^2 + 2(a^2+1)x + (a^2-3a+2) = 0$ will have two roots of opposite sign if it has real roots and the product of the roots is negative, i.e., if

$$D \geq 0 \text{ and } \alpha \cdot \beta < 0$$

$$\Rightarrow 4(a^2+1)^2 - 12(a^2-3a+2) \geq 0$$

$$\text{and} \quad \frac{a^2-3a+2}{3} < 0$$

$$\therefore a^2-3a+2 < 0,$$

$$\text{i.e., if } (a-1)(a-2) < 0 \text{ or } 1 < a < 2$$

- 27** Let $y = \frac{(x-a)(x-b)}{(x-c)} \Rightarrow x^2 - (a+b+y)x + ab + cy = 0$

Since x is real, then $D \geq 0$

$$\Rightarrow (a+b+y)^2 - 4(ab+cy) \geq 0$$

$$\Rightarrow y^2 + 2(a+b-2c)y + (a+b)^2 - 4ab \geq 0$$

$$\Rightarrow y^2 + 2(a+b-2c)y + (a-b)^2 \geq 0 \quad \dots(i)$$

Since y takes all real values, (i) is possible iff

$$4(a+b-2c)^2 - 4(a-b)^2 \leq 0$$

$$\Rightarrow (a+b-2c+a-b)(a+b-2c-a-b) \leq 0$$

$$\Rightarrow 4(a-c)(b-c) \leq 0 \Rightarrow (a-c)(b-c) \leq 0$$

This is possible if c lies between a and b , that is if $a < c < b$ or $a > c > b$.

28 $\because D = b^2 - 4ac$

and, if $D \geq 0$, then the roots of the equation are given by

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

As $D = b^2 - 4ac < b^2$ ($\because a > 0$ and $c > 0$)

It follows that the roots of the quadratic equation are negative.

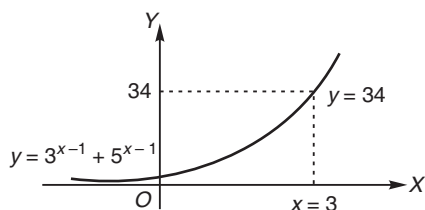
In case $D < 0$, then the roots of the equation are given by

$$x = \frac{-b \pm i\sqrt{-D}}{2a}$$

which have negative real parts.

29 It is very obvious that at $x = 3$ the given expression satisfies.

Now, $y = 3^{x-1}$ and $y = 5^{x-1}$ are both increasing functions of x (exponential functions with base greater than 1). Therefore their sum $y = 3^{x-1} + 5^{x-1}$ is also an increasing function of x . It means for $x < 3$, $y = 3^{x-1} + 5^{x-1} < 34$ and for $x > 3$, $y = 3^{x-1} + 5^{x-1} > 34$.



Thus, the equation has no other solution.

30 $\because 2 - \sqrt{3} = \frac{1}{2 + \sqrt{3}}$

$\therefore$ the given equation can be expressed as

$$(2 + \sqrt{3})^{x^2 - 2x + 1} + \frac{1}{(2 + \sqrt{3})^{x^2 - 2x - 1}} = 2(2 + \sqrt{3}) \quad \dots(i)$$

Let us put $y = (2 + \sqrt{3})^{x^2 - 2x + 1}$, then the eq. (i) can be expressed as

$$\begin{aligned} y + \frac{(2 + \sqrt{3})^2}{y} &= 2(2 + \sqrt{3}) \\ \Rightarrow y^2 - 2(2 + \sqrt{3})y + (2 + \sqrt{3})^2 &= 0 \\ \Rightarrow [y - (2 + \sqrt{3})]^2 &= 0 \Rightarrow y = 2 + \sqrt{3} \\ \therefore (2 + \sqrt{3})^{x^2 - 2x + 1} &= (2 + \sqrt{3}) \\ \Rightarrow x^2 - 2x + 1 &= 1 \Rightarrow x(x - 2) = 0 \\ \Rightarrow x &= 0 \text{ or } 2. \end{aligned}$$

31 For $a = 0$ or $a = \frac{1}{2}$ one of the quadratic equations becomes

linear, so $a \neq 0$, $a \neq \frac{1}{2}$.

Hence the only answer is $a = \frac{2}{9}$

32 Let $f(x) = x^2 - 2(4k - 1)x + 15k^2 - 2k - 7$. Then $f(x) > 0$

$$\Rightarrow D < 0 \quad [\because \text{coefficient of } x^2 > 0]$$

$$\Rightarrow 4(4k - 1)^2 - 4(15k^2 - 2k - 7) < 0$$

$$\Rightarrow k^2 - 6k + 8 < 0 \Rightarrow 2 < k < 4$$

33 According to the given condition a lies between the roots.

$$\text{Let } f(x) = 2x^2 - 2(2a + 1)x + a(a + 1).$$

For a to lie between the roots, we must have $D \geq 0$ and $f(a) < 0$

$$\text{Now, } D \geq 0 \Rightarrow 4(2a + 1)^2 - 8a(a + 1) \geq 0$$

$$\Rightarrow 8\left[a^2 + a + \frac{1}{2}\right] \geq 0, \quad \text{which is always true}$$

$$\therefore f(a) < 0 \Rightarrow 2a^2 - 2a(2a + 1) + a(a + 1) < 0$$

$$\Rightarrow -a^2 - a < 0 \Rightarrow a^2 + a > 0$$

$$\Rightarrow a > 0 \text{ or } a < -1$$

34 Let α, β and γ be the roots of the given equation such that $\alpha = -\beta$. Then,

$$\alpha + \beta + \gamma = a \Rightarrow \gamma = a$$

Since γ is a root of the given equation, so it satisfies the equation, i.e.,

$$\gamma^3 - a\gamma^2 + b\gamma - c = 0$$

$$\Rightarrow a^3 - a^3 + ab - c = 0 \Rightarrow ab = c$$

35 Let $f(x) = ax^2 + bx + c$. Since 1 lies outside the roots of $f(x) = 0$. So,

$$af(1) > 0 \Rightarrow f(1) > 0 \quad (\because a > 0)$$

$$\Rightarrow a + b + c > 0$$

36 $x - \frac{1}{x^2 - 4} = 2 - \frac{1}{x^2 - 4}$

$$\text{Here } x^2 - 4 \neq 0$$

$$\Rightarrow x \neq \pm 2 \Rightarrow (x - 2) = 0$$

$$\Rightarrow x = 2, \text{ which is inadmissible.}$$

37 $2^{x/2} + (\sqrt{2} + 1)^x = (5 + 2\sqrt{2})^{x/2}$

$$\Rightarrow \left(\frac{\sqrt{2}}{\sqrt{5 + 2\sqrt{2}}}\right)^x + \left(\frac{\sqrt{2} + 1}{\sqrt{5 + 2\sqrt{2}}}\right)^x = 1$$

which is of the form

$$\cos^x \alpha + \sin^x \alpha = 1$$

$$\therefore x = 2.$$

38 $xy = 2(x + y) \Rightarrow y(x - 2) = 2x$

$$\therefore y = \frac{2x}{x - 2} \text{ but } x, y \in N \text{ by trial, we get } x = 3, 4, 6$$

$$\therefore y = 6, 4, 3 \text{ But } x \leq y$$

$$\therefore x = 3, 4 \text{ and } y = 6, 4$$

Thus two solutions are possible.

39 Let the roots be α and $\frac{1}{\alpha}$

$$\therefore \alpha \cdot \frac{1}{\alpha} = \frac{c}{a} \Rightarrow c = a$$

$$\therefore (|a| - |b|)^2 \geq 0 \text{ or } |a| \geq |b|$$

$$\text{but } c = a \therefore |c| \geq |b|$$

40 $|x+1||x-1| = a^2 - 2a - 3$

$$\Rightarrow |x^2 - 1| = a^2 - 2a - 3$$

$$\Rightarrow x^2 - 1 = \pm (a^2 - 2a - 3)$$

$$\text{but } a^2 - 2a - 3 \geq 0$$

$$\therefore x^2 - 1 = a^2 - 2a - 3$$

$$\Rightarrow x^2 = a^2 - 2a - 2, -a^2 + 2a + 4 \geq 0$$

For real solutions,

$$a^2 - 2a - 2 \geq 0 \text{ and } -a^2 + 2a + 4 \geq 0$$

Thus we have $a \in (-\infty, -1] \cup [3, \infty)$

41 Since k is a prime number

$$\therefore k \neq 0$$

$$\text{Again } p = 0, qk^{1/3} = 0 \text{ and } rk^{2/3} = 0$$

$$\Rightarrow p = 0 \text{ and } q = 0 \text{ and } r = 0$$

$$\therefore p = q = r = 0$$

42 The equation $x^4 + 2px^3 + x^2 + 2px + 1 = 0$ cannot have 0 as a root. Dividing by x^2 , we can write as

$$x^2 + \frac{1}{x^2} + 2p\left(x + \frac{1}{x}\right) + 1 = 0$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 + 2p\left(x + \frac{1}{x}\right) - 1 = 0 \quad \dots(i)$$

Put $x + \frac{1}{x} = y$. Then we can write eq. (i) as

$$y^2 - 2py - 1 = 0 \quad \dots(ii)$$

Since $D > 0$, i.e., $4p^2 + 4 > 0$, hence eq. (ii) has two distinct real roots. These roots are

$$y = \frac{-2p \pm \sqrt{4p^2 + 4}}{2} = -p \pm \sqrt{p^2 + 1}$$

Since $-p + \sqrt{p^2 + 1} > 0$, the equation

$$x + \frac{1}{x} = -p + \sqrt{p^2 + 1} \quad \dots(iii)$$

has either positive roots or non-real complex roots.

Since $-p - \sqrt{p^2 + 1} < 0$, both the roots of the equation

$$x + \frac{1}{x} = -p - \sqrt{p^2 + 1} \text{ are negative.}$$

We can write Eq. (iii) as

$$\left(\sqrt{-x} - \frac{1}{\sqrt{-x}}\right)^2 = p - 2 + \sqrt{p^2 + 1}$$

As $\left(\sqrt{-x} - \frac{1}{\sqrt{-x}}\right)^2 > 0$, we must have,

$$p - 2 + \sqrt{p^2 + 1} > 0, \text{ i.e., } \sqrt{p^2 + 1} > |2 - p|.$$

$$\text{Since } p^2 + 1 = (p - 2)^2 + 4p - 3,$$

$$\sqrt{p^2 + 1} > |2 - p| \text{ iff } 4p - 3 > 0.$$

$$\text{i.e., iff } p > \frac{3}{4}.$$

43 Case 1. Suppose $x \geq a$. Thus the given equation becomes

$$x^2 - 2a(x - a) - 3a^2 = 0$$

$$\Rightarrow x^2 - 2ax - a^2 = 0$$

$$\Rightarrow x = \frac{2a \pm 2\sqrt{2}a}{2} = (1 \pm \sqrt{2})a$$

As $a < 0$ and $1 + \sqrt{2} > 1$, $(1 + \sqrt{2})a < a$.

Therefore, $x \neq (1 + \sqrt{2})a$.

Again as $1 - \sqrt{2} < 1$, $(1 - \sqrt{2})a > a$.

$$\therefore x = (1 - \sqrt{2})a.$$

Case 2. Suppose $x < a$, then the given equation becomes

$$x^2 - 2a(a - x) - 3a^2 = 0$$

$$\Rightarrow x^2 + 2ax - 5a^2 = 0$$

$$\Rightarrow x = \frac{-2a \pm \sqrt{24}a}{2} = (-1 \pm \sqrt{6})a$$

As $a < 0$, $-1 - \sqrt{6} < 0 < 1$, so $(-1 - \sqrt{6})a > a$,

$$\therefore x \neq (-1 - \sqrt{6})a \quad (\because x < a)$$

Again, as $a < 0$, $-1 + \sqrt{6} > 1$ and $(-1 + \sqrt{6})a < a$

$$\therefore x = (-1 + \sqrt{6})a = (\sqrt{6} - 1)a.$$

44 Case 1. Let $x + 2 \geq 0$.

$\therefore |x + 2| = x + 2$ and the equation becomes

$$2^{x+2} - 2^{x+1} - 1 - |2^{x+1} - 1| = 0$$

$$\Rightarrow 2^{x+1}(2 - 1) - 1 - |2^{x+1} - 1| = 0$$

$$\Rightarrow 2^{x+1} - 1 = |2^{x+1} - 1|$$

$$\Rightarrow 2^{x+1} - 1 \geq 0 \text{ or } x + 1 \geq 0 \text{ or } x \geq -1$$

Case 2. Let $x + 2 < 0$.

$\therefore |x + 2| = -(x + 2)$ and the equation becomes

$$2^{-(x+2)} - |2^{x+1} - 1| = 2^{x+1} + 1 \quad \dots(i)$$

Put $2^{x+1} = y$. Then eq. (i) becomes $\frac{1}{2y} - |y - 1| = y + 1$

$$\text{or } 2y^2 + 2y + |y - 1| 2y = 1 \quad \dots(ii)$$

If $y \geq 1$, then LHS of eq. (ii) becomes

$$2y^2 + 2y - 2y(y - 1) - 1 = 0$$

$$\Rightarrow 4y = 1 \text{ or } y = \frac{1}{4}$$

$$\text{Also } 2^{x+1} = 2^{-2} \text{ or } x = -3$$

45 Let $y = 5 + 2\sqrt{6}$, then $\frac{1}{y} = 5 - 2\sqrt{6}$. Thus the given

equation becomes $y^{x^2-3} + \left(\frac{1}{y}\right)^{x^2-3} = 10 \quad \dots(i)$

Again let $y^{x^2-3} = t$. Then eq. (i) becomes

$$t + \frac{1}{t} = 10 \Rightarrow t^2 - 10t + 1 = 0$$

Thus, $t = 5 \pm 2\sqrt{6}$

$$\therefore (5 + 2\sqrt{6})^{x^2-3} = 5 \pm 2\sqrt{6} = (5 + 2\sqrt{6})^{\pm 1}$$

$$\Rightarrow x^2 - 3 = 1 \quad \text{or} \quad x^2 - 3 = -1 \Rightarrow x^2 = 4 \text{ or } x^2 = 2$$

$$\Rightarrow x = \pm 2 \quad \text{or} \quad x = \pm \sqrt{2}$$

46 For the real roots we must have $D \geq 0$. i.e., $b^2 - 4ac \geq 0 \Rightarrow b^2 \geq 4a (\because c = 1)$

b	b^2	a	$4a$	
1	1	$\times$	$\times$	
2	4	1	4	$\rightarrow 1$
3	9	1, 2	4, 8	$\rightarrow 2$
4	16	1, 2, 3, 4	4, 8, 12, 16	$\rightarrow 4$

Thus, there are 7 possible equations of the given form.

47 Let $y = a + \frac{1}{a}$ $\dots(i)$

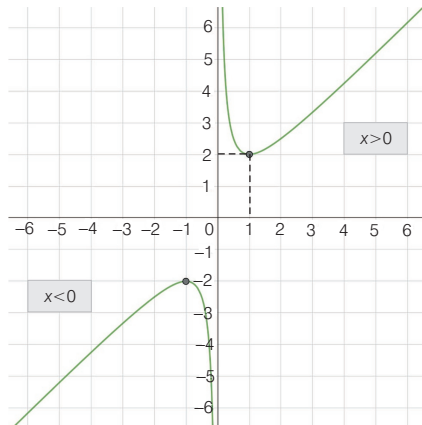
Differentiating above eq. (i), we get

$$1 - \frac{1}{a^2} = 0 \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$$

But, since $a > 0$, therefore, only $a = 1$ is acceptable. Now substituting $a = 1$ in the original Eq. (i), we get the required minimum value of the expression.

$$\text{Hence, minimum of } y = 1 + \frac{1}{1} = 2$$

Alternatively See the following graph of $y = x + \frac{1}{x}$.



Here, $x = a$

$$\therefore y = x + \frac{1}{x} = a + \frac{1}{a}$$

a	$\frac{1}{a}$	$a + \frac{1}{a}$
1	1	2
2	1/2	2.5
3	1/3	3.33
4	1/4	4.25
5	1/5	5.20
1/2	2	2.5
1/3	3	3.33
1/4	4	4.25

Alternatively Let us consider $y = a + \frac{1}{a}$; ($a > 0$)

$$\Rightarrow y = \frac{a^2 + 1}{a} \Rightarrow a^2 - ya + 1 = 0$$

For a to be a real number, $(-y)^2 - (4 \times 1 \times 1) \geq 0$

$$\Rightarrow y^2 \geq 4 \Rightarrow y \leq -2 \text{ or } y \geq 2$$

Given that $a > 0$, so $y > 0$.

$$\Rightarrow y \geq 2$$

It implies that the minimum value of $y = 2$

Thus the minimum value of $a = \frac{1}{a} = 2$

48 $(a + b + c)^2 \geq 0$

$$\therefore (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) \geq 0 \quad \dots(i)$$

$$\therefore 1 + 2(ab + bc + ca) \geq 0 \quad (\because a^2 + b^2 + c^2 = 1)$$

$$\Rightarrow (ab + bc + ca) \geq -\frac{1}{2} \quad \dots(ii)$$

$$\text{Also, } a^2 + b^2 \geq 2ab, \quad b^2 + c^2 \geq 2bc, \quad c^2 + a^2 \geq 2ac$$

$$(a^2 + b^2 + b^2 + c^2 + c^2 + a^2) \geq 2ab + 2bc + 2ac$$

$$\Rightarrow a^2 + b^2 + c^2 \geq ab + bc + ac \quad (\because a^2 + b^2 + c^2 = 1)$$

$$\Rightarrow ab + bc + ac \leq 1$$

$$\therefore -\frac{1}{2} \leq ab + bc + ac \leq 1.$$

49 Best way is to go through options. It satisfies for every even integer.

50 Let $(15 + 4\sqrt{14})^t = k$

$$\therefore (15 - 4\sqrt{14})^t = \frac{1}{k}$$

$\therefore$ The given equation can be written as

$$k + \frac{1}{k} = 30$$

$$\Rightarrow k^2 - 30k + 1 = 0 \Rightarrow k = 15 \pm 4\sqrt{14}$$

Now, the equation will hold only at $t = \pm 1$.

$$\therefore x^2 - 2|x| = \pm 1$$

$$\text{Again if } x^2 - 2|x| = 1 \Rightarrow x = \pm(1 + \sqrt{2})$$

$$\text{and if } x^2 - 2|x| = -1 \Rightarrow x = \pm 1$$

$$\text{Hence } x = \pm 1, \pm(1 + \sqrt{2})$$

51 We have $|2x - 3| < 1 \Leftrightarrow -1 < 2x - 3 < 1$

$$\Leftrightarrow 1 < x < 2 \Leftrightarrow x \in (1, 2)$$

52 The roots of the equation are ω and ω^2 .

Let $\alpha = \omega$ and $\beta = \omega^2$. Then

$$\alpha^{19} = \omega^{19} = \omega \text{ and } \beta^7 = \omega^{14} = \omega^2$$

Hence α^{19} and β^7 are roots of the same equation.

53 Let α, β be the roots of the equation $x^2 + px + 8 = 0$.

Then $\alpha + \beta = -p$ and $\alpha\beta = 8$

$$\text{But } \alpha - \beta = 2 \Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (2)^2$$

$$\Rightarrow p^2 - 32 = 4 \Rightarrow p = \pm 6.$$

54 $(x + y)^2 = x^2 + y^2 + 2xy$

$$\Rightarrow (x + y)^2 = x^2 + y^2 + 2 \quad (\because xy = 1)$$

For the minimum value of the expression, $x = y$

$$\therefore x = 1 \text{ and } y = 1$$

$$\text{Hence, } (x + y)_{\min} = \sqrt{x^2 + y^2 + 2} = \sqrt{4} = 2$$

$$(x + y)_{\min} = 2 \quad (\because x > 0 \text{ and } xy > 0 \therefore y > 0)$$

☞ Alternatively

xy	x	y	$\sqrt{x^2 + y^2 + 2} = (x + y)$
1	1/10	10	101/10
1	1/5	5	26/5
1	1/2	2	5/2
1	1	1	2
1	2	1/2	5/2
1	5	1/5	26/5
1	10	1/10	101/10

☞ Alternatively

We know that for any real number, $AM \geq GM$.

$$\therefore \frac{x+y}{2} \geq \sqrt{xy} \Rightarrow (x+y) \geq 2\sqrt{xy} \Rightarrow (x+y) \geq 2 \quad (\because xy = 1)$$

55 $x = 7 + 4\sqrt{3}$

$$\therefore y = \frac{1}{7 + 4\sqrt{3}} = 7 - 4\sqrt{3}$$

$$\begin{aligned} \therefore \frac{1}{x^2} + \frac{1}{y^2} &= \frac{x^2 + y^2}{(xy)^2} = \frac{(7 + 4\sqrt{3})^2 + (7 - 4\sqrt{3})^2}{[(7 + 4\sqrt{3})(7 - 4\sqrt{3})]^2} \\ &= \frac{2(49 + 48)}{1} = 194 \end{aligned}$$

56 For real x , the even power of x is always a non-negative number.

Therefore, we have $x^2/(1 + x^4) \geq 0$

$$\text{Also } 1 + x^4 - 2x^2 = (1 - x^2)^2 \geq 0 \Rightarrow 1 + x^4 \geq 2x^2$$

$$\therefore \left(\frac{x^2}{1 + x^4} \right) \leq \frac{1}{2}, \quad \text{So } 0 \leq \frac{x^2}{1 + x^4} \leq \frac{1}{2}$$

☞ Alternatively For real values of x , $x^2 + \frac{1}{x^2} \geq 2$

$$\Rightarrow \frac{x^4 + 1}{x^2} \geq 2 \Rightarrow \frac{x^2}{x^4 + 1} \leq \frac{1}{2}$$

And, for real values of x , $\frac{x^2}{x^4 + 1} \geq 0$

$$\text{Therefore, we have } 0 \leq \frac{x^2}{1 + x^4} \leq \frac{1}{2}$$

57 $x^3 + 1 \geq x^2 + x \Rightarrow x^3 - x^2 - x + 1 \geq 0$
 $\Rightarrow (x - 1)^2(x + 1) \geq 0 \Rightarrow x \geq -1$

58 Since $a^2 = 5a - 3$ and $b^2 = 5b - 3$ ($a \neq b$). It implies that a and b are the roots of $x^2 = 5x - 3$ or $x^2 - 5x + 3 = 0$.

Therefore $a + b = 5$ and $ab = 3$.

$$\text{Now } \frac{a}{b} + \frac{b}{a} = \frac{19}{3} \text{ and } \frac{a}{b} \times \frac{b}{a} = 1.$$

$$\text{So the required equation is } x^2 - \frac{19}{3}x + 1 = 0$$

$$\Rightarrow 3x^2 - 19x + 3 = 0$$

Hence, choice (c) is the answer.

59 Let D_1 and D_2 be the discriminants of $f(x)$ and $g(x)$ respectively.

$$\text{Then } D_1 = b^2 - 4ac \text{ and } D_2 = b^2 + 4ac$$

Since $ac \neq 0 \Rightarrow ac > 0$ or $ac < 0$.

If $ac > 0$, D_2 must be positive; D_1 may or may not be.

And if $ac < 0$, D_1 must be positive; D_2 may or may not be.

It implies that when $ac \neq 0$, at least one of D_1 and D_2 must be positive. That means at least one of the two equations $f(x)$ and $g(x)$ must have real roots.

Hence choice (c) is the answer.

☞ NOTE In any quadratic equation either both the roots are real or both the roots are non-real.

☞ Alternatively The four roots of $f(x) \cdot g(x) = 0$ are the two roots of $f(x)$ and two roots of $g(x)$. It implies that the number of real roots of $f(x) \cdot g(x) = 0$ will be either 4 or 2 or 0.

Now we have to find the minimum number of real roots of $f(x) \cdot g(x) = 0$

So let us consider that none of the equations have real roots. That means, both the discriminants D_1 and D_2 are negative. That is $D_1 < 0$ and $D_2 < 0$

$$\text{It implies that } b^2 - 4ac < 0 \text{ and } b^2 + 4ac < 0$$

But, we observe that, whether $ac > 0$ or $ac < 0$, at least one of the above discriminants is certainly invalid. That means one of the two discriminants has to be positive.

Thus, it is certain that one equation must have real roots. This, in turn, implies that at least 2 roots have to be real.

☞ Alternatively Let D_1 and D_2 be the discriminants of $f(x)$ and $g(x)$ respectively.

$$\text{Now, } D_1 + D_2 = (b^2 - 4ac) + (b^2 + 4ac)$$

$$\Rightarrow D_1 + D_2 = 2b^2 \Rightarrow D_1 + D_2 > 0$$

It implies that at least one of D_1 and D_2 must be positive. So, at least one of the two equations $f(x)$ and $g(x)$ must have real roots.

Therefore, the equation $f(x) \cdot g(x) = 0$ has at least two real roots.

- 60** Let D_1 and D_2 be the discriminants of $f(x)$ and $g(x)$ respectively.

$$\text{Therefore } D_1 + D_2 = (u^2 - 4v) + (j^2 - 4k)$$

$$\Rightarrow D_1 + D_2 = u^2 + j^2 - 4(v + k)$$

$$\Rightarrow D_1 + D_2 = u^2 + j^2 - 2ij \Rightarrow D_1 + D_2 = (u - j)^2$$

$$\Rightarrow D_1 + D_2 \geq 0$$

So at least one of D_1 and D_2 must be positive. Therefore it can be inferred that at least one of $f(x)$ and $g(x)$ must have real roots.

Hence choice (d) is the answer.

- 61** Let α and α^n be the roots of the given equation, then

$$\alpha + \alpha^n = \frac{-b}{a} \quad \dots(i)$$

$$\alpha \cdot \alpha^n = \frac{c}{a} \Rightarrow \alpha^{n+1} = \frac{c}{a}$$

$$\Rightarrow \alpha = \left(\frac{c}{a}\right)^{\frac{1}{n+1}} \quad \dots(ii)$$

From the eq. (i) we have, $\alpha\alpha + \alpha\alpha^n + b = 0$

$$\Rightarrow a\left(\frac{c}{a}\right)^{\frac{1}{n+1}} + a\left(\frac{c}{a}\right)^{\frac{n}{n+1}} + b = 0$$

$$\Rightarrow a\left(\frac{c}{a}\right)^{\frac{1}{n+1}} + a\left(\frac{c^n}{a^n}\right)^{\frac{1}{n+1}} + b = 0$$

$$\Rightarrow (ca^n)^{\frac{1}{n+1}} + (ac^n)^{\frac{1}{n+1}} + b = 0$$

Hence choice (c) is the answer.

- 62** Let α and β be the roots of this equation, then

$$\alpha + \beta = \frac{-b}{a} \quad \dots(i)$$

$$\text{and } \alpha\beta = \frac{c}{a} \quad \dots(ii)$$

$$\text{Also, } \frac{\alpha}{\beta} = r \quad \dots(iii)$$

Now go through the given choices and check their validity.

Let's consider choice (b), then

$$ac(1+r)^2 = rb^2$$

$$\Rightarrow ac\left(1 + \frac{\alpha}{\beta}\right)^2 = \frac{\alpha}{\beta} \times b^2 \Rightarrow ac\left(\frac{\alpha + \beta}{\beta}\right)^2 = \frac{\alpha}{\beta} \times b^2$$

$$\Rightarrow ac(\alpha + \beta)^2 \frac{1}{\beta^2} = \frac{\alpha}{\beta} \times b^2 \Rightarrow ac\left(-\frac{b}{a}\right)^2 = \alpha\beta \times b^2$$

$$\Rightarrow \frac{c}{a} \times b^2 = \frac{c}{a} \times b^2$$

Since both the sides are equal. Therefore the chosen relationship is true. Hence choice (b) is the answer.

- 63** Let's assume that $a = -2$, $b = 2$, and $k = 5$, then

$$(x+2)(x-2) - 5 = 0 \Rightarrow (x+3)(x-3) = 0$$

$$\Rightarrow c = -3, d = 3$$

Now substituting the values of c and d in the given choices, we try to find out the correct choice whose roots are a, b . That is $-2, 2$.

(a) $x^2 - 14$, which is not true.

(b) $x^2 - 14$, which is not true.

(c) $x^2 - 4$, which is true, since it gives the required roots $-2, 2$.

(d) $x^2 + 1$, which is not true.

Hence choice (c) is the answer.

☞ Alternatively $(x-a)(x-b) - k = 0$

$$\Rightarrow x^2 - (a+b)x + ab - k = 0 \quad \dots(i)$$

$$\Rightarrow c + d = a + b$$

$$\Rightarrow cd = ab - k \quad \dots(ii)$$

The new quadratic equation whose roots are a, b is

$$x^2 - (a+b)x + ab = 0$$

Now from eqs. (i) and (ii), we get

$$x^2 - (a+b)x + ab = 0$$

$$\Rightarrow x^2 - (c+d)x + cd + k = 0 \Rightarrow (x-c)(x-d) + k = 0$$

Hence choice (c) is the answer.

- 64** The best way to solve it is by considering arbitrary values of roots then framing a cubic equation as follows.

Let's assume that $r_1 = 1, r_2 = 2$, then we will have a corresponding equation to these roots as

$$(x-1)(x-2) = x^2 - 3x - 2$$

Then $S_1 = r_1 + r_2 + r_2 = 3, P = r_1 r_2 = 2$, and $S_3 = r_1^3 + r_2^3 = 9$

$$\therefore S_1^3 = S_3 + 3PS_1 \Rightarrow 27 = 9 + 3(2 \times 3), \text{ which is valid.}$$

Similarly $S_3 = S_1(S_1^2 - 3P) \Rightarrow 9 = 3(9 - 6)$, which is valid

Therefore (i) and (ii) both are true

Hence choice (c) is the answer

☞ Alternatively Use the following formulae

$$a^3 + b^3 = (a+b)(a^2 + b^2 - ab) = (a+b)[(a+b)^2 - 3ab]$$

and

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

- 65 If the two roots are α and β , then

$$x^2 + bx + c = 0$$

$$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

It implies that $\alpha + \beta = -17$ and $\alpha\beta = 30$

Thus the wrong equation is $x^2 + 17x + 30 = 0$

And the correct equation is $x^2 + 13x + 30 = 0$

Therefore the roots of correct equation can be computed by using any of the known methods as -10 and -3 .

Hence choice (c) is the answer.

NOTE Since, only the coefficient of x is wrong, it means the constant term c (or $\alpha\beta$) is correct.

- 66 The two roots of $6x^2 - 17x + 12 = 0$ are $\frac{3}{2}$ and $\frac{4}{3}$.

Now if $\frac{3}{2}$ is a common root in both the equations then it

will satisfy $3x^2 - 2x + p = 0$ also.

Therefore, $3x^2 - 2x + p = 0$

$$\Rightarrow 3\left(\frac{9}{4}\right) - 2\left(\frac{3}{2}\right) + p = 0$$

$$\Rightarrow p = -\frac{15}{4}$$

Again, if $\frac{4}{3}$ is a common factor in both the equations then

it will satisfy $3x^2 - 2x + p = 0$ also.

Therefore, $3x^2 - 2x + p = 0$

$$\Rightarrow 3\left(\frac{16}{9}\right) - 2\left(\frac{4}{3}\right) + p = 0$$

$$\Rightarrow p = -\frac{8}{3}$$

Hence choice (b) is the answer.

Alternatively Consider two quadratic equations,

$$ax^2 + bx + c = 0; \text{ (where } a \neq 0) \quad \dots(i)$$

$$\text{And } a'x^2 + b'x + c' = 0; \text{ (where } a' \neq 0) \quad \dots(ii)$$

If one root is common, then the following condition must be satisfied.

$$(ab' - a'b)(bc' - b'c) = (ca' - c'a)^2$$

$$\text{Therefore } [(3 \times -17) - (6 \times -2)][(-2 \times 12) - (-17 \times p)] \\ = [(p \times 6) - (12 \times 3)]^2$$

$$\Rightarrow 12p^2 + 77p + 120 = 0$$

$$\Rightarrow p = -\frac{8}{3}, -\frac{15}{4}$$

- 67 The roots of the equation $x^2 + 2x + 9 = 0$ are imaginary as the discriminant $D = -32$, which is negative. That means there is a pair of conjugate roots.

So, one root alone cannot be common, as the conjugate root of the common root be same for both the equations. It implies that both the roots of the equations will be common.

$$\therefore \frac{a}{1} = \frac{b}{2} = \frac{c}{9} \Rightarrow a : b : c = 1 : 2 : 9$$

Hence choice (a) is the answer.

- 68 Choices (b) and (c) can't be the required answer as they contain the minus sign while it is given that $a > 0$.

Now try the following approach and see if it works. Consider

$a = 4$, from the last choice.

$$x^2 - 4x - 45 = 0 \Rightarrow (x - 9)(x + 5)$$

$$x^2 - 12x + 17 = 0 \Rightarrow (x - 9)(x - 3)$$

It shows that when $a = 4$, there is one common root (9) between the two given equations.

Hence choice (d) is the answer.

69. Comparing the given equations you will get,

$$x^2 - px + q = x^2 + qx - p \Rightarrow (p + q) = x(p + q) \Rightarrow (p + q) \\ \Rightarrow x = 1$$

It implies that 1 is the common root, then substituting $x = 1$ in any of the given equations you will get, $p - q = 1$.

Hence choice (d) is the answer.

70. Since in each equation product of roots is clearly known, but the sum of roots is unknown. However, it is obvious that the roots will be odd as sum of roots is an even number.

With this little known information, we can assume, with some confidence, that (1, 5), (1, 7) and (5, 7) are the roots of respective equations. Then $x^2 - 6x + 5 = 0$, $x^2 - 8x + 7 = 0$ and $x^2 - 12x + 15 = 0$ be the possible equations.

Now comparing these equations with the given ones, we find that $m = 3$, $n = 4$ and $p = 6$. Therefore $m + n + p = 13$.

Hence choice (a) is the correct one.

Alternatively Let (a, b) , (b, c) and (c, a) be the roots of the given equations, respectively. Then $a \times b = 5$, $b \times c = 7$ and $c \times a = 35 \Rightarrow a \times b \times c = 35$

But since, a, b, c are positive real numbers, therefore $a = 5$, $b = 1$, $c = 7$.

Now, $2m = a + b = 6$, $2n = b + c = 8$, $2p = c + a = 12$

Therefore, $2m + 2n + 2p = 26 \Rightarrow m + n + p = 13$.

- 71 As it's an identity, so it must be satisfied by any value of x . So the coefficients and the constant term must be zero.

$$\text{Therefore, } a^2 - 1 = 0 \Rightarrow a = -1, 1$$

$$\text{And } a - 1 = 0 \Rightarrow a = 1 \text{ and } a^2 - 4a + 3 = 0 \Rightarrow a = 1, 3$$

Thus you can see that when $a = 1$, the constant term and the coefficients of x and x^2 are zero. So, for $a = 1$, the given relation will be an identity.

Hence choice (c) is the answer.

Hint Identity equations are the equations which are always true no matter what value is plugged in for the variable. If you simplify an identity equation, you'll always get a true statement.

When an equation becomes an identity in x , then the equation gets satisfied by every real number. And, this can happen only when all the 3 terms of the given equation become equal to zero. So, you can see when $a = 1$, each of the three terms given in the equation becomes zero. And, then whatever value you substitute for x , you'll always get LHS = RHS.

- 72** The given equation is satisfied for three distinct values of x ; $x = -a$, $x = -b$, $x = -c$.

The highest power of x in the given equation is 2 but this equation is satisfied by three distinct values of x . So it is an identity.

Hence choice (c) is the answer.

Hint Actually, when you simplify the given equation you will find that it's a quadratic equation, which should not have more than 2 roots. However, this equation has more than 2 roots (a , b and c). So, it is obvious that the given equation is an identity.

- 73** The best way is to consider any quadratic equation whose roots are not real. So, I'm assuming such an equation: $x^2 - x + 2 = 0$

Now, comparing this equation ($x^2 - x + 2 = 0$) with the given equation ($ax^2 + bx + c = 0$), we get $a = 1$, $b = -1$ and $c = 2$

Now, if you plug in the values of a , b and c in the given options, you will see that options (b), (c) and (d) do not satisfy.

Hence, choice (a) is the valid one.

Hint How can you find a quadratic equation whose roots are non-real?

(i) Have $b^2 < 4ac$

(ii) Directly consider any two non-real roots, say $\alpha = (1 + i)$ and $\beta = 1 - i$ and then

form the equation $x^2 - (\alpha + \beta)x + (\alpha\beta) = 0$

Alternatively Let $f(x) = ax^2 + bx + c$. Since, the equation has no real roots, the graph of $f(x)$ will lie either above the X -axis or below the X -axis. It clearly suggests that for all the arbitrary values of x , the value of $f(x)$ will have the same sign. So, you can claim that $f(1)$ and $f(0)$ will have the same sign.

Therefore $f(1) \cdot f(0) > 0 \Rightarrow (a + b + c)c > 0$

Hence choice (a) is the answer.

Hint Please recall that the product of two numbers is always positive if both of them have the same sign. So, if $f(0)$ and $f(1)$ have the same sign, then their product will certainly be positive.

Here, $f(0) = c$ and $f(1) = a + b + c$

- 74** Let us consider any three arbitrary roots for a cubic equation, which are in HP.

As 1, 1, 1 are in HP, so we can consider them the valid roots. Then we will have the cubic equation as following

$$(x - 1)(x - 1)(x - 1) = 0 \text{ or } (x - 1)^3 = 0$$

$$\text{Or } x^3 - 3x^2 + 3x - 1 = 0 \quad \dots(i)$$

Now, comparing the above eq. (i) with given equation $x^3 - px^2 + qx - r = 0$, we get $p = 3$, $q = 3$, $r = 1$

Now, if we substitute these values of p , q and r in the given choices, then only choice (b) will be satisfied.

Hence choice (b) is the answer.

NOTE Even if you consider the three roots as 2, 3, 6 (or any three roots which are in HP), the answer will always be choice (b).

Alternatively The equation whose roots are reciprocals of the roots of the given equation is given by

$$\frac{1}{x^3} - \frac{p}{x^2} + \frac{q}{x} - r = 0$$

$$\Rightarrow rx^3 - qx^2 + px - 1 = 0 \quad \dots(ii)$$

Since the roots of the given equation are in HP, the roots of the eq. (i) are in AP. Let the roots of eq. (i) be $(b - d)$, b , $(b + d)$.

Then, sum of the roots $= (b - d) + b + (b + d) = -\frac{(-q)}{r}$

$$\Rightarrow b = \frac{q}{3r}$$

Since b is the root of eq. (i), so

$$rb^3 - qb^2 + pb - 1 = 0$$

$$\Rightarrow r\left(\frac{q}{3r}\right)^3 - q\left(\frac{q}{3r}\right)^2 + p\left(\frac{q}{3r}\right) - 1 = 0$$

$$\Rightarrow 27r^2 - 9pqr + 2q^3 = 0$$

Hence choice (b) is the answer.

- 75** Replacing x by $-x$, we can get the desired equation.

$$(-x)^3 - 3(-x)^2 + (-x) + 1 = 0$$

$$\Rightarrow -x^3 - 3x^2 - x + 1 = 0$$

$$\Rightarrow x^3 + 3x^2 + x - 1 = 0$$

Hence choice (a) is the answer.

- 76** Replacing x by $\sqrt{x}$, we can get the desired equation.

$$(\sqrt{x})^3 - 2(\sqrt{x})^2 + 3(\sqrt{x}) + 1 = 0$$

$$\Rightarrow x\sqrt{x} - 2x + 3\sqrt{x} + 1 = 0$$

$$\Rightarrow \sqrt{x}(x+3) = (2x-1)$$

Squaring both sides and simplifying, we get

$$x^3 + 2x^2 + 13x - 1 = 0$$

Hence choice (a) is the answer.

77 Replacing x by $(x)^{1/3}$, we can get the desired equation.

$$a \left[(x)^{1/3} \right]^3 + b \left[(x)^{1/3} \right]^2 + c \left[(x)^{1/3} \right] + d = 0$$

$$\Rightarrow ax + d = - \left[b(x)^{2/3} + c(x)^{1/3} \right]$$

Cubing both the sides of the above equation and simplifying it we get,

$$a^3x^3 + x^2(3a^2d - 3abc + b^3) + x(3ad^2 - 3bcd + c^3) + d^3 = 0$$

Hence choice (d) is the answer.

☞ Alternatively Since the equation given in the problem and the equations given in the choices are purely in variables, so you have the flexibility to solve this problem by assuming any convenient numbers.

Now, let us assume that three roots of the given equation be 1, 2, 3. Then the original equation would be

$$(x-1)(x-2)(x-3) = 0$$

$$\Rightarrow x^3 - 6x^2 + 11x - 6 = 0 \quad \dots(i)$$

Now, form another equation whose roots are 1, 8, 27 (i.e., cube of 1, 2, 3).

Then the required equation would be

$$(x-1)(x-8)(x-27) = 0$$

$$\Rightarrow x^3 - 36x^2 + 251x - 216 = 0 \quad \dots(ii)$$

Now, if we put in the values of a, b, c, d in the given choices, only choice (d) will give the Eq. (ii).

Hence choice (d) is the answer.

78 The best way is to go through the given choices. Since 3 is there in each of the choices so it's a sure shot value for x . And you can easily notice that whether you have $x = 3$ or $x = -3$ in both the case $x^2 - 8 = 1$

So, for $x = 3$ or -3 , $(7 + 4\sqrt{3}) + (7 - 4\sqrt{3}) = 14$.

That is valid.

Now there are basically two values to choose from the given choices: ± 4 and $\pm\sqrt{7}$. If we consider $x = 4$, it does not lead to any potential possibility. So let's consider $x = \sqrt{7}$

So, for $x = \pm\sqrt{7}$, $x^2 - 8 = -1$. Therefore,

$$(7 + 4\sqrt{3})^{-1} + (7 - 4\sqrt{3})^{-1}$$

$$= \frac{1}{7 + 4\sqrt{3}} + \frac{1}{7 - 4\sqrt{3}} = 14 \text{ that is valid}$$

Hence choice (c) is the answer.

79 The best way is to assume some relevant numerical values for a, b, c such that $a + b + c = 0$. So, for the sake of simplicity, we can assume

$$a = 1, b = 0, c = -1$$

Now, substituting the values of a, b and c in the given equation, $3ax^2 + 2bx + c = 0$, it becomes as $3x^2 - 1 = 0$,

and its roots are $\pm \frac{1}{\sqrt{3}}$

Thus you can see that the roots are real and distinct.

Hence choice (a) is the answer.

☞ Alternatively If you assume $a = 1, b = 2, c = -3$, you will have $3x^2 + 4x - 3 = 0$, and then its discriminant will be a positive value, which ensures that the roots are real and distinct.

NOTE For any quadratic equation, the coefficient of x^2 cannot be zero, so you can assume any real value for x , but not zero.

Remember that roots can be rational only when the discriminant is a perfect square. It means whether roots are rational or not, but they must be real and distinct (unequal) when discriminant is a positive value. That's why choice (a) is always correct but choice (c) may or may not be true.

Please try to understand that choice (c) represents the subset (or a special case) of choice (a). So as per the given condition you can tell the general answer, but not the very specific one.

☞ Alternatively The discriminant of this equation is

$$D = 4b^2 - 12ac = 4(b^2 - 3ac)$$

$$\Rightarrow D = 4\{-(a+c)\}^2 - 3ac$$

$$\Rightarrow D = 4\{(a^2 + c^2 + 2ac) - 3ac\}$$

$$\Rightarrow D = 4(a^2 + c^2 - ac)$$

$$\Rightarrow D > 0$$

It implies that the roots are real and distinct.

But, as we can see that D is not a perfect square so the roots are irrational.

Hint We know that for any two real numbers, $AM \geq GM$.

that is $\frac{x+y}{2} \geq \sqrt{xy}$

$$\Rightarrow x + y \geq 2\sqrt{xy} \Rightarrow a^2 + c^2 \geq 2\sqrt{a^2c^2}$$

$$\Rightarrow a^2 + c^2 \geq 2ac \Rightarrow a^2 + c^2 > ac$$

$$\Rightarrow a^2 + c^2 - ac > 0$$

NOTE

Since, $a^2 + c^2 - ac > 0$, therefore $D > 0$.

Otherwise, the square of any real number is always non-negative. So, we have $(a-c)^2 \geq 0$

$$\Rightarrow a^2 + c^2 - 2ac \geq 0 \Rightarrow a^2 + c^2 \geq 2ac$$

$$\Rightarrow a^2 + c^2 \geq 2ac > ac \Rightarrow a^2 + c^2 > ac \Rightarrow a^2 + c^2 - ac > 0$$

- 80** Since $x = 0$ is not a solution of given equation. Dividing by x^2 , we get

$$2\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 11 = 0$$

$$\Rightarrow 2\left[\left(x + \frac{1}{x}\right)^2 - 2 \times x \times \frac{1}{x}\right] + \left(x + \frac{1}{x}\right) - 11 = 0 \quad \dots(i)$$

Putting $x + \frac{1}{x} = y$ in eq. (i), we get

$$2(y^2 - 2) + y - 11 = 0 \Rightarrow 2y^2 + y - 15 = 0$$

$$\Rightarrow y = -\frac{5}{3}, 2 \Rightarrow x + \frac{1}{x} = -\frac{5}{3} \text{ or } x + \frac{1}{x} = 2$$

$$\Rightarrow x = \frac{-3 - \sqrt{5}}{2}, \frac{-3 + \sqrt{5}}{2}, \frac{1}{2}, 2$$

Hence choice (d) is the answer.

- 81 Case 1:** If $x \in I$, then $(x) = [x]$

Therefore, $(x)^2 = [x]^2 + 2x \Rightarrow x = 0$

Case 2: $x \notin I$, then $(x) = [x] - 1$

Therefore, $(x)^2 = [x]^2 + 2x$

$$\Rightarrow [x]^2 + 1 + 2[x] = [x]^2 + 2x$$

$$\Rightarrow x = [x] + \frac{1}{2}$$

$$\Rightarrow x = n + \frac{1}{2}; n \in I$$

Hence choice (a) is the answer.

- 82** Since the roots of the equation $ax^2 + 2bx + c = 0$ are real. Therefore,

$$4b^2 - 4ac \geq 0$$

$$\Rightarrow b^2 \geq ac \quad \dots(i)$$

And given that $m^2 > n$... (ii)

Now the discriminant of the equation $ax^2 + 2mbx + nc = 0$ is $4m^2b^2 - 4anc$

From the eqs. (i) and (ii), we get $m^2b^2 > anc$

$$\therefore 4m^2b^2 > 4anc$$

Hence choice (a) is the answer.

- 83** $16x^2 + 8(a+5)x - 7a - 5 = 0$

Since the whole graph is strictly above the X -axis, so it implies that the roots of the graph are non-real. That means the discriminant of this equation D is negative.

Therefore,

$$[8(a+5)]^2 - 4[16 \times (-7a-5)] < 0$$

$$\Rightarrow a^2 + 17a + 30 < 0$$

$$\Rightarrow (a+15)(a+2) < 0$$

$$\Rightarrow -15 < a < -2$$

Hence choice (b) is the answer.

$$\mathbf{84} \quad x = 1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{3 + \frac{1}{2 + \dots \infty}}}}$$

$$\Rightarrow x + 1 = 2 + \frac{1}{3 + \frac{1}{2 + \frac{1}{3 + \frac{1}{2 + \dots \infty}}}}$$

$$\Rightarrow x + 1 = 2 + \frac{1}{3 + \frac{1}{(x+1)}}$$

$$\Rightarrow x + 1 = 2 + \frac{x+1}{3x+4} \Rightarrow x = 1 + \frac{x+1}{3x+4}$$

$$\Rightarrow x = \frac{4x+5}{3x+4} \Rightarrow x(3x+4) = 4x+5$$

$$\Rightarrow 3x^2 = 5 \Rightarrow x = \pm \sqrt{\frac{5}{3}}$$

But, x cannot be a negative value.

Hence choice (c) is the answer.

- 85** Let us assume that $x^2 = a$ and $y^2 = b$. Then,

$$y^4 = 2x^4 + 1402$$

$$\Rightarrow b^2 = 2a^2 + 1402 \quad \dots(i)$$

As the problem asks for integral solutions, then we can assume that x and y are integers, so a and b are also integers. Since perfect squares can never be negative, so $a^2 \geq 0$ and $b^2 \geq 0$. As we can see that a^2 and b^2 are perfect square integers, so the minimum possible value of b^2 must be a perfect square number just greater than 1402 in order to satisfy the eq. (i).

Therefore the minimum possible value of $b^2 = 1444$ and the minimum possible value of $b = 38$.

From eq. (i) we know that b is an even number, so b can be expressed as following. $b = 38 + 2k$; Where k is any whole number.

$$\therefore b^2 = 2a^2 + 1402$$

$$\Rightarrow (38 + 2k)^2 = 2a^2 + 1402$$

$$\Rightarrow a^2 = 2k^2 + 76k + 21$$

Since $2k^2 + 76k + 21$ is a perfect square, so its

discriminant D should be zero.

$$\text{That is } D = 76^2 - 4 \times 2 \times 21 \Rightarrow D > 0$$

It shows that $2k^2 + 76k + 21$ is not a perfect square.

Therefore a, b cannot be positive integers.

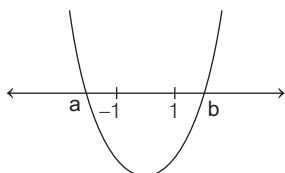
So, x, y cannot have integral solutions.

Hence choice (d) is the answer.

86 $ax^2 + bx + c = 0$

$$\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad \dots(i)$$

Let's draw a quadratic graph such that $\alpha < -1$ and $\beta > 1$



From the graph, we observe that $f(-1) < 0$

$$\Rightarrow 1 + \frac{b}{a} + \frac{c}{a} < 0 \quad \dots(ii)$$

Similarly from the graph, we observe that $f(1) < 0$

$$\Rightarrow 1 - \frac{b}{a} + \frac{c}{a} < 0 \quad \dots(iii)$$

From eqs. (ii) and (iii), we can conclude that

$$1 + \left| \frac{b}{a} \right| + \frac{c}{a} < 0$$

Hence choice (a) is the answer.

87 $3x^3 = [(x^2 + \sqrt{18}x + \sqrt{32})(x^2 - \sqrt{18}x - \sqrt{32})] - 4x^2$

$$\Rightarrow 3x^3 = x^4 - (\sqrt{18}x + \sqrt{32})^2 - 4x^2$$

$$\Rightarrow 3x^3 = x^4 - 2(3x + 4)^2 - 4x^2$$

$$\Rightarrow x^4 - 2(3x + 4)^2 - x^2(3x + 4) = 0$$

Now putting $3x + 4 = y$, the above equation becomes

$$x^4 - yx^2 - 2y^2 = 0$$

$$\Rightarrow (x^2 - 2y)(x^2 + y) = 0 \Rightarrow (x^2 - 2y) = 0$$

$$\text{or } (x^2 + y) = 0 \Rightarrow x^2 - (3x + 4) = 0$$

$$\Rightarrow x = 3 \pm \sqrt{17} \text{ or } x = \frac{-3 \pm \sqrt{7i}}{2}$$

Hence choice (a) is the answer.

88 For $x = a, b, c$ the given equation is an identity. Actually the given equation is a quadratic equation of degree two. But, since this equation has more than two solutions, therefore it is an identity.

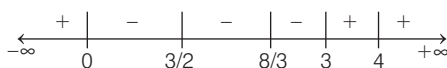
Hence choice (d) is the answer.

89 Since at $x = a, b, c$ both the sides are equal. It means there are 3 solutions.

Hence choice (c) is the answer.

NOTE Please bear in mind that the given equation is a quadratic equation which should have only two roots, but this equation has more than two roots, so it's an identity.

90 The critical points of this rational function are $0, 3/2, 8/3, 3$ and 4 .



Now testing each of the 6 intervals we get the results as shown on the above number line.

Here we need non-positive intervals so we exclude all the positive intervals and combine the others keeping in mind the restriction on limits of the intervals.

Therefore the valid values of x are

$$\left[0, \frac{8}{3} \right) \cup \left(\frac{8}{3}, 3 \right) \cup \{4\}.$$

The critical points $8/3, 3$ are not included in the domain since the function is not defined there and $3/2, 4$ are included since they satisfy the given relation. Since the problem asks only integral solutions so we have to look back in the domain of this function. Now looking in the domain we find that $1, 2, 4$ are the three integral solutions to this problem.

Hence choice (b) is the answer.

Alternatively The least critical number is 0 and the greatest critical number is 4. So first of all by putting $1, 2, 3$ and 4 in the above rational function you can check whether a particular integer gives the value of function $f(x) \leq 0$.

Now test the same by selecting any random and convenient integer number before 1 and after 4.

The result is that only $1, 2$ and 4 make the function $f(x) \leq 0$.

91 $\frac{x^2 + 6x - 7}{|x + 3|} < 0 \Rightarrow \frac{(x + 7)(x - 1)}{|x + 3|} < 0$

There are three critical points $-7, -3$ and 1 . So there will be four different regions on the number line. That means there will be four intervals as following.

$$(-\infty, -7) \cup (-7, -3) \cup (-3, 1) \cup (1, \infty)$$

Now you can test each interval and then you see that only two intervals $(-7, -3)$ and $(-3, 1)$ give negative regions, which is the requirement of the problem.

Hence choice (d) is the answer.

Hint While testing for the signs in each of the intervals you can test the intervals by testing the numerator only. You need not consider the denominator as for every real number denominator will give positive results only. Also keep in mind that at $x = -3$, domain is broken (or undefined).

92 In each of the numerator and denominator, the discriminant $D < 0$; and the coefficient of x^2 is positive. So both the numerator and denominator are positive. It implies that the ratio of numerator and denominator is also positive. Therefore choices (b), (c) and (d) are eliminated. Hence choice (a) is the answer.

93 The best way is to go through the given choices.

(a) Putting $\frac{-3}{2}$ in place of x does not satisfy the equation, so it's not a root of this equation at all.

(b) Putting $x = \frac{3}{2}$, the equation gets satisfied, so $\frac{3}{2}$ is one of the roots. Now put double of $\frac{3}{2}$ that is 3 in the equation, but this time it is not satisfied. So what you can do is put half of $\frac{3}{2}$ that is $\frac{3}{4}$ in the equation. This time it is satisfied, so $\frac{3}{4}$ is also the root of this equation.

So $\frac{3}{2}$ and $\frac{3}{4}$ are the roots of this equation and one root is double the other one. But we have to find the third root.

(c) Putting $\frac{-5}{3}$ in place of x , we see that the equation is satisfied. So clearly $\frac{-5}{3}$ is the third root.

Hence choice (c) is the answer.

- 94** Let α, β, γ be the roots of the given equation $4x^3 + 16x^2 - 9x - 36 = 0$, then sum of the roots

$$= \alpha + \beta + \gamma = -\frac{16}{4} = -4$$

$$\Rightarrow \gamma = -4 \text{ since } \alpha + \beta = 0$$

Hence choice (d) is the answer.

Hint Zero is the first whole number. And $\alpha + \beta + \gamma = -\frac{b}{a}$

- 95** Let p be one of the roots of the required equation, then $p = x + 1 \Rightarrow x = p - 1$

Now substituting $p - 1$ in the given equation, we get

$$(p - 1)^3 - 5(p - 1)^2 + 6(p - 1) - 3 = 0$$

$$\Rightarrow p^3 - 8p^2 + 19p - 15 = 0$$

Hence choice (a) is the answer.

Alternatively Let α, β, γ be the roots of the given equation

$$x^3 - 5x^2 + 6x - 3 = 0, \text{ then } \alpha + \beta + \gamma = -\frac{-5}{1} = 5$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{6}{1} = 6, \alpha\beta\gamma = -\frac{-3}{1} = 3$$

Now the roots of required equation are $(\alpha + 1), (\beta + 1), (\gamma + 1)$, then

$$S_1 = \Sigma\alpha = (\alpha + 1) + (\beta + 1) + (\gamma + 1)$$

$$= (\alpha + \beta + \gamma) + 3 = 8$$

$$S_2 = \Sigma\alpha\beta = (\alpha + 1)(\beta + 1) + (\beta + 1)(\gamma + 1) + (\gamma + 1)(\alpha + 1)$$

$$= (\alpha\beta + \beta\gamma + \gamma\alpha) + 2(\alpha + \beta + \gamma) + 3$$

$$= 6 + 2(5) + 3 = 19$$

$$S_3 = \Sigma\alpha\beta\gamma = (\alpha + 1)(\beta + 1)(\gamma + 1)$$

$$= \alpha\beta\gamma + (\alpha\beta + \beta\gamma + \gamma\alpha) + (\alpha + \beta + \gamma) + 3$$

$$= 3 + 6 + 3 + 3 = 15$$

Therefore the required equation is

$$p^3 - S_1p^2 + S_2p - S_3 = 0$$

$$\Rightarrow p^3 - 8p^2 + 19p - 15 = 0$$

- 96** Let p be one of the roots of the required equation, then $p = x^3$

$$\Rightarrow x = p^{\frac{1}{3}}$$

Now substituting $p^{\frac{1}{3}}$ in the given equation, we get

$$(p^{\frac{1}{3}})^3 + 3(p^{\frac{1}{3}})^2 + 2 = 0$$

$$\Rightarrow p + 3p^{\frac{2}{3}} + 2 = 0 \Rightarrow (p + 2)^3 = (-3p^{\frac{2}{3}})^3$$

$$\Rightarrow p^3 + 8 + 6p(p + 2) = -27p^2$$

$$\Rightarrow p^3 + 33p^2 + 12p + 8 = 0$$

Hence choice (a) is the answer.

Alternatively let α, β, γ be the roots of the given equation $x^3 + 3x^2 + 2 = 0$, then

$$\alpha + \beta + \gamma = -\frac{3}{1} = -3$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{0}{1} = 0$$

$$\alpha\beta\gamma = -\frac{2}{1} = -2$$

Now the roots of required equation are $\alpha^3, \beta^3, \gamma^3$ then

$$S_1 = \Sigma\alpha = \alpha^3 + \beta^3 + \gamma^3 = -33$$

$$S_2 = \Sigma\alpha\beta = \alpha^3\beta^3 + \beta^3\gamma^3 + \gamma^3\alpha^3 = 12$$

$$S_3 = \Sigma\alpha\beta\gamma = \alpha^3\beta^3\gamma^3 = (\alpha\beta\gamma)^3 = -8$$

Therefore the required equation is

$$p^3 - S_1p^2 + S_2p - S_3 = 0$$

$$\Rightarrow p^3 + 33p^2 + 12p + 8 = 0$$

Hint $(\alpha^3 + \beta^3 + \gamma^3) - 3(\alpha\beta\gamma)$

$$= (\alpha + \beta + \gamma)[(\alpha^2 + \beta^2 + \gamma^2) - (\alpha\beta + \beta\gamma + \gamma\alpha)]$$

$$\Rightarrow (\alpha^3 + \beta^3 + \gamma^3) - 3(-2)$$

$$= (-3)[(\alpha^2 + \beta^2 + \gamma^2) - (0)]$$

$$\text{But } (\alpha + \beta + \gamma)^2 = (\alpha^2 + \beta^2 + \gamma^2) + 2(\alpha\beta + \beta\gamma + \gamma\alpha)$$

$$\Rightarrow (-3)^2 = (\alpha^2 + \beta^2 + \gamma^2) + 2(0) \Rightarrow (\alpha^2 + \beta^2 + \gamma^2) = 9$$

$$\therefore (\alpha^3 + \beta^3 + \gamma^3) = -3(9) - 6 = -33$$

Similarly, $(\alpha^3\beta^3 + \beta^3\gamma^3 + \gamma^3\alpha^3)$

$$= (\alpha\beta)^3 + (\beta\gamma)^3 + (\gamma\alpha)^3$$

$$\therefore (\alpha\beta)^3 + (\beta\gamma)^3 + (\gamma\alpha)^3 - 3[(\alpha\beta)(\beta\gamma)(\gamma\alpha)]$$

$$= [(\alpha\beta) + (\beta\gamma) + (\gamma\alpha)]$$

$$[(\alpha\beta)^2 + (\beta\gamma)^2 + (\gamma\alpha)^2 - (\alpha\beta\beta\gamma) + (\beta\gamma\gamma\alpha) + (\gamma\alpha\alpha\beta)]$$

$$= [0][(\alpha\beta)^2 + (\beta\gamma)^2 + (\gamma\alpha)^2 - (\alpha\beta\beta\gamma) + (\beta\gamma\gamma\alpha) + (\gamma\alpha\alpha\beta)] = 0$$

$$\begin{aligned} &\Rightarrow (\alpha\beta)^3 + (\beta\gamma)^3 + (\gamma\alpha)^3 = 3[(\alpha\beta)(\beta\gamma)(\gamma\alpha)] \\ &\Rightarrow (\alpha\beta)^3 + (\beta\gamma)^3 + (\gamma\alpha)^3 = 3(\alpha\beta\gamma)^2 \\ &\Rightarrow (\alpha\beta)^3 + (\beta\gamma)^3 + (\gamma\alpha)^3 = 3(-2)^2 \Rightarrow \alpha^3\beta^3 + \beta^3\gamma^3 + \gamma^3\alpha^3 = 12 \end{aligned}$$

- 97 $\alpha_1 + \alpha_2 + \alpha_3 = -1$, $\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_1 = 3$, $\alpha_1\alpha_2\alpha_3 = -1$
 $\therefore \alpha_1 + \alpha_2 + \alpha_3 + \alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_1 + \alpha_1\alpha_2\alpha_3 = 1$
Hence choice (d) is the answer.

98. The best way to solve it is by considering arbitrary values of roots then framing a cubic equation as follows.

Let's assume that $r_1 = 1$, $r_2 = 2$, $r_3 = 3$, then we will have a corresponding equation to these roots as

$$(x-1)(x-2)(x-3) = x^3 - 6x^2 + 11x - 6$$

It implies that $a_3 = 1$, $a_2 = -6$,

$$a_1 = 11, a_0 = -6$$

Then $S_1 = r_1 + r_2 + r_3 = 6$

$$\text{And } S_2 = r_1^2 + r_2^2 + r_3^2 = 14$$

$$\therefore a_3S_1 + a_2 = 1 \times 6 + (-6) = 0$$

$$\text{Similarly } a_3S_2 + a_2S_1 + 2a_1 = 1 \times 14 + (-6) \times 6 + 2 \times 11 = 0$$

Therefore (i) and (ii) both are true.

Hence choice (c) is the answer.

99. Comparing the given equation with $x^3 - S_1x^2 + S_2x - S_3 = 0$, you will find that a is positive.

Let α , β and γ be the roots of the given equation, then $\alpha + \beta + \gamma = 3$ and $\alpha\beta + \beta\gamma + \gamma\alpha = a$ and $\alpha\beta\gamma = 1$

From A.M. - G.M. inequality,

$$\frac{(\alpha + \beta + \gamma)}{3} \geq (\alpha\beta\gamma)^{\frac{1}{3}}$$

Substituting the value of $\alpha + \beta + \gamma$ and $\alpha\beta\gamma$, you will find

$$\text{that } \frac{(\alpha + \beta + \gamma)}{3} = (\alpha\beta\gamma)^{\frac{1}{3}}$$

$\Rightarrow$ A.M. = G.M., when A.M = G.M then $\alpha = \beta = \gamma$

$$\text{Therefore } \alpha\beta + \beta\gamma + \gamma\alpha = 3$$

Hence, choice (d) is the answer.

☞ Alternatively

Since $\alpha + \beta + \gamma = 3$, $\alpha\beta\gamma = 1$ and $\alpha > 0$, $\beta > 0$, $\gamma > 0$

It is possible only when $\alpha = \beta = \gamma = 1$.

$$\text{Therefore } \alpha\beta + \beta\gamma + \gamma\alpha = 3$$

$$100. \text{ Let } f(x) = x^4 + ax^3 + bx^2 - 4x + 1.$$

Since there are four positive roots, it implies that there will be four changes in the sign among the various terms of the function, when the terms are written in increasing/decreasing order of degree. So a must be negative and b must be positive.

$$100 \text{ Let } f(x) = x^4 + ax^3 + bx^2 - 4x + 1.$$

Since there are four positive roots, it implies that there will be four changes in the sign among the various terms of the function, when the terms are written in increasing/decreasing order of degree. So a must be negative and b must be positive.
Hence choice (c) is the answer.

- 101 If you can find the real values of p so that $f(x) \geq 0$ for all x , then for the remaining real values of p the function $f(x) < 0$ for at least one x .

Now, since $f(x) \geq 0$ for all x and $a > 0$, then $D \leq 0$.

$$\text{Therefore } p^2 - 4 \times 4(p-3) \leq 0$$

$$\Rightarrow p^2 - 16p + 48 \leq 0 \Rightarrow (p-4)(p-12) \leq 0$$

$$\Rightarrow p \in [4, 12]$$

It implies that when $p \in [4, 12]$, the given function $f(x)$ is not negative for any value of x .

It means when $p \in (-\infty, 4) \cup (12, \infty)$, the given function $f(x)$ is negative for at least one x .

Hence choice (d) is the answer.

☞ Alternatively

Since $f(x) < 0$ for at least one value of x , and $a > 0$, then $D > 0$. Therefore $p^2 - 16p + 48 > 0$

$$\Rightarrow (p-4)(p-12) > 0$$

$$\Rightarrow p \in (-\infty, 4) \cup (12, \infty)$$

$$102 \text{ } f(x) = (p^2 + 3)x^2 + (p+2)x - 5 < 0 \quad \dots(i)$$

Since $f(x) < 0$ for at least one value of x , and $a > 0$, then $D > 0$.

$$\text{Therefore } (p+2)^2 - 4(p^2+3)(-5) > 0$$

$$\Rightarrow 21p^2 + 4p + 64 > 0 \quad \dots(ii)$$

Now you have to find out the values of p for which the above expression (ii) is positive. For this you would have to again find out the discriminant of expression (ii) which is negative. Since in quadratic expression (ii), the coefficient of x^2 is positive and discriminant is negative, the quadratic expression (ii) will be positive for all real numbers. That is for all real values of p , the discriminant, $D > 0$.

Thus the given function $f(x) < 0$ for at least one x , if

$$p \in (-\infty, \infty)$$

Hence choice (b) is the answer.

Hint $p^2 + 3 > 0$.

☞ Alternatively If you can find the real values of p so that $f(x) \geq 0$ for all x , then for the remaining real values of p the function $f(x) < 0$ for at least one x .

$$f(x) = (p^2 + 3)x^2 + (p+2)x - 5 \geq 0 \quad \dots(i)$$

Now, since $f(x) \geq 0$ for all x and $a > 0$, then $D \leq 0$.

Therefore $(p+2)^2 - 4(p^2+3) \times (-5) \leq 0$.

$$\Rightarrow 21p^2 + 4p + 64 \leq 0 \quad \dots(ii)$$

Since the coefficient of p^2 is positive and the discriminant of quadratic expression (ii) is negative, then the above quadratic inequality (ii) is not valid for any real value of p .

It implies that when $p \in (-\infty, \infty)$, the given function $f(x)$ is not positive for any value of x .

It means when $p \in (-\infty, \infty)$, the given function $f(x)$ is negative for at least one x . Hence choice (b) is the answer.

103 let $f(x) = ax^2 + bx + c$.

$$\Rightarrow f(x) = x^2 + \frac{b}{a}x + \frac{c}{a}$$

Since $\alpha < -p$ and $\beta > p$. Therefore $f(-p) < 0$ and $f(p) < 0$.
When $f(-p) < 0$

$$\Rightarrow p^2 - \frac{b}{a}p + \frac{c}{a} < 0 \quad \dots(i)$$

And when $f(p) < 0$

$$\Rightarrow p^2 + \frac{b}{a}p + \frac{c}{a} < 0 \quad \dots(ii)$$

From eqs. (i) and (ii), we can conclude that

$$f(x) = p^2 + \left| \frac{b}{a} \right| p + \frac{c}{a} < 0$$

$$f(x) = 1 + \left| \frac{b}{a} \right| \frac{1}{p} + \frac{c}{ap^2} < 0$$

Hence choice (a) is the answer.

104 Since α is a root of $ax^2 + bx + c = 0$, therefore α will satisfy it as $a\alpha^2 + b\alpha + c = 0$... (i)

Similarly since β is a root of $-ax^2 + bx + c = 0$, therefore β will satisfy it as $-a\beta^2 + b\beta + c = 0$... (ii)

$$\text{Let } f(x) = \frac{a}{2}x^2 + bx + c = 0 \Rightarrow f(\alpha) = \frac{a}{2}\alpha^2 + b\alpha + c = 0$$

$$\Rightarrow f(\alpha) = \frac{a}{2}\alpha^2 - a\alpha^2 \quad [\text{from eq. (ii)}]$$

$$\Rightarrow f(\alpha) = -\frac{a}{2}\alpha^2 \quad \dots(iii)$$

$$\Rightarrow f(\beta) = \frac{a}{2}\beta^2 + b\beta + c = 0 \quad [\text{from eq. (ii)}]$$

$$\Rightarrow f(\beta) = \frac{a}{2}\beta^2 + a\beta^2 \quad \dots(iv)$$

$$\Rightarrow f(\beta) = \frac{3a}{2}\beta^2$$

$$\text{Now } f(\alpha) \cdot f(\beta) = -\frac{3}{4}a^2\alpha^2\beta^2$$

$$\Rightarrow f(\alpha) \cdot f(\beta) < 0 \quad (\because \alpha, \beta \neq 0)$$

It implies that $f(\alpha)$ and $f(\beta)$, have opposite signs.

Therefore equation $f(x) = 0$ has exactly one root between α and β . Hence choice (a) is the answer.

105 Case (i) When $x > 0$, the given equation will be $x^2 - 3x + 2 = 0$ and then the roots will be 1, 2.

Case (ii) When $x < 0$, the given equation will be $x^2 + 3x + 2 = 0$ and then the roots will be -1, -2

Thus there are total four roots, viz. -2, -1, 1, 2

Hence choice (c) is the answer.

106 The given equation is $3|x^2 + 4x + 2| = 5x + 16$... (i)

Case (i) When $(x^2 + 4x + 2)$ is positive, the given equation will be $3x^2 + 12x + 6 = 5x + 16 \Rightarrow 3x^2 + 7x - 10 = 0$

The roots are 1 and $-\frac{10}{3}$.

Case (ii) When $(x^2 + 4x + 2)$ is negative, the given equation will be

$$-3x^2 - 12x - 6 = 5x + 16 \Rightarrow 3x^2 + 17x + 22 = 0$$

The roots are -2 and $-\frac{11}{3}$.

Since $-\frac{10}{3}$ and $-\frac{11}{3}$ do not satisfy the given Eq. (i), so

you have to discard them. Thus the valid values of x are 1 and -2. Hence choice (d) is the answer.

107 The best way is to go through the given choices.

All the values of choice (d) satisfy the equation, so it's the answer. Hence choice (d) is the answer.

108 Let $f(x) = (|x-1| - 3)$

$$\text{Now if } f(x) = 0$$

$$\Rightarrow (|x-1| - 3) = 0$$

Case (i) When $x-1 > 0$, then $(x-1) - 3 = 0$

$$\Rightarrow x = 4$$

Case (ii) When $x-1 < 0$, then

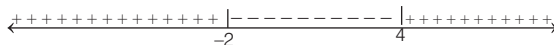
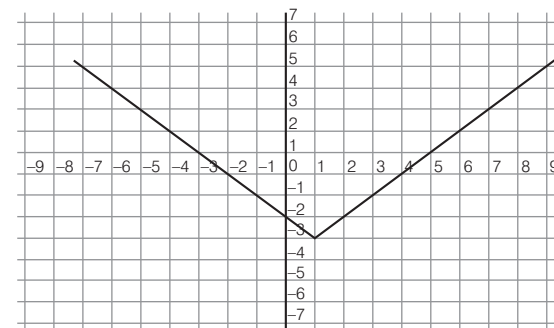
$$-(x-1) - 3 = 0$$

$$\Rightarrow x = -2$$

After testing for the signs you will get

$f(x) < 0$, if $x \in (-2, 4)$ and $f(x) > 0$, if

$$x \in (-\infty, -2) \cup (4, \infty) \quad \dots(i)$$



Let $g(x) = (|x + 2| - 5)$

Now if $g(x) = 0$

$\Rightarrow (|x + 2| - 5) = 0$

Case (i) When $x + 2 > 0$, then

$$f(x) = (x + 2) - 5 = 0$$

$\Rightarrow x = 3$

Case (ii) When $x + 2 < 0$, then

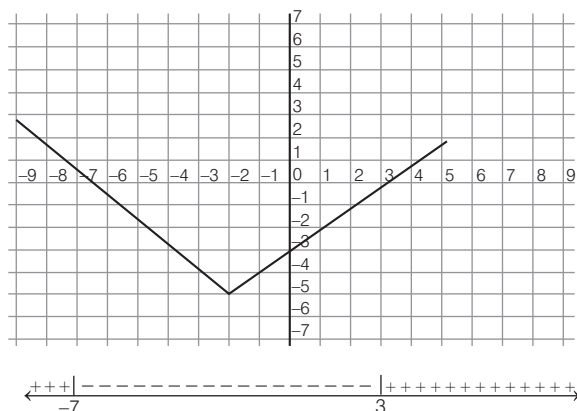
$$f(x) = -(x + 2) - 5 = 0$$

$\Rightarrow x = -7$

After testing for the signs you will get

$$g(x) < 0, \text{ if } x \in (-7, 3) \text{ and } g(x) > 0$$

if $x \in (-\infty, -7) \cup (3, \infty)$... (ii)

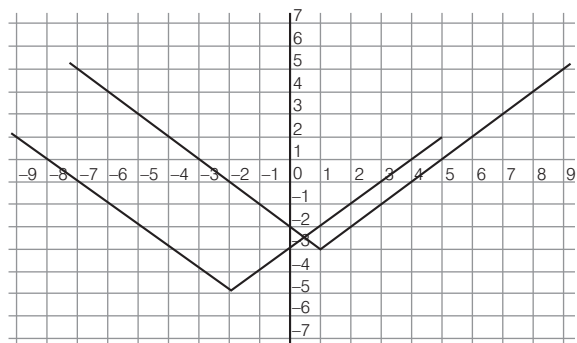


Now since you know that $f(x)g(x) < 0$, only when $f(x)$ and $g(x)$ are of opposite signs.

That means $f(x)g(x) < 0$, if $f(x) > 0$ but $g(x) < 0$ or $f(x) < 0$ but $g(x) > 0$.

So combining the results of (i) and (ii), we can conclude that $f(x)g(x) < 0$, if $x \in (-7, -2) \cup (3, 4)$.

The same result can be easily seen by the following graph.



Hence choice (d) is the answer.

☞ Alternatively The best way is to go through the given choices. All the values of x that lie between $(-7, -2)$ and $(3, 4)$ satisfy the given equation.

Hence choice (d) is the answer.

109 Case (i) When both the numerator and denominator have the same sign. That is

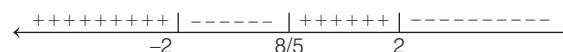
$$\frac{x^2 - 5x + 4}{x^2 - 4} \text{ or } \frac{-(x^2 - 5x + 4)}{-(x^2 - 4)}$$

Therefore the given inequation is $\frac{x^2 - 5x + 4}{x^2 - 4} \leq 1$

$$\Rightarrow \frac{x^2 - 5x + 4}{x^2 - 4} - 1 \leq 0 \Rightarrow \frac{-5x + 8}{(x + 2)(x - 2)} \leq 0$$

The critical points are -2 , 2 and $\frac{8}{5}$. So there are four

intervals $(-\infty, -2)$, $(-2, \frac{8}{5})$, $(\frac{8}{5}, 2)$, $(2, \infty)$



Since the rational function is non positive, therefore you

have to consider only two intervals: $(-2, \frac{8}{5})$, and $(2, \infty)$

Case (ii) when the signs of numerator and denominator are different. That is

$$\frac{-(x^2 - 5x + 4)}{(x^2 - 4)} \text{ or } \frac{(x^2 - 5x + 4)}{-(x^2 - 4)}$$

Therefore the given inequation is

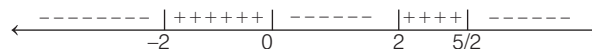
$$\frac{-(x^2 - 5x + 4)}{x^2 - 4} \leq 1$$

$$\Rightarrow \frac{-(x^2 - 5x + 4)}{x^2 - 4} - 1 \leq 0$$

$$\Rightarrow \frac{-x(2x - 5)}{(x + 2)(x - 2)} \leq 0$$

The critical points are -2 , 2 , 0 and $\frac{5}{2}$. So there are five

intervals $(-\infty, -2)$, $(-2, 0)$, $(0, 2)$, $(2, \frac{5}{2})$, $(\frac{5}{2}, \infty)$



Since the rational function is non-positive, therefore you have to consider only three

intervals: $(-\infty, -2)$, $(0, 2)$ and $(\frac{5}{2}, \infty)$

Therefore the intersection of both the cases will be your desired result.

That is $[0, 8/5] \cup [5/2, \infty)$.

Hence choice (b) is the answer.

☞ NOTE Since the given inequality is not a strict inequality, so the boundary values of the chosen intervals will have to be included. The best way is to cross check by the substitution of boundary values in the given (original) expression, in case of any uncertainty.

- 110 Case (i)** when both the numerator and denominator have the same sign. That is

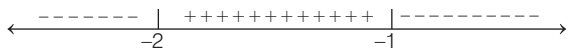
$$\frac{x^2 - 3x - 1}{x^2 + x + 1} \text{ or } \frac{-(x^2 - 3x - 1)}{-(x^2 + x + 1)}$$

Therefore the given inequation is

$$\frac{x^2 - 3x - 1}{x^2 + x + 1} < 3 \Rightarrow \frac{x^2 - 3x - 1}{x^2 + x + 1} - 3 < 0$$

$$\Rightarrow \frac{-2(x+2)(x+1)}{x^2 + x + 1} < 0$$

The critical points are -2 and -1 , since denominator does not yield any real root. So, there are three intervals $(-\infty, -2)$, $(-2, -1)$, $(-1, \infty)$



Since the rational function is negative, therefore you have to consider only negative values. That is $x \in (-\infty, -2) \cup (-1, \infty)$.

Case (ii) when the signs of numerator and denominator are different. That is

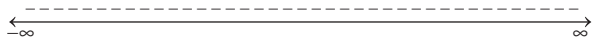
$$\frac{-(x^2 - 3x - 1)}{x^2 + x + 1} \text{ or } \frac{x^2 - 3x - 1}{-(x^2 + x + 1)}$$

Therefore the given inequation is

$$\frac{-(x^2 - 3x - 1)}{x^2 + x + 1} < 3$$

$$\Rightarrow \frac{-(x^2 - 3x - 1)}{x^2 + x + 1} - 3 < 0 \Rightarrow \frac{-2(2x^2 + 1)}{x^2 + x + 1} < 0$$

Since both the numerator and denominator are unable to give any real root, so there is no critical point for this rational function. Further since the coefficient of x^2 of the numerator is negative so the whole graph will be negative as roots are non-real.



Since the rational function is negative, therefore all the real values satisfy the inequality. That is $(-\infty, \infty)$

Therefore the intersection of both the cases will be your desired result. That is $(-\infty, -2) \cup (-1, \infty)$.

Hence choice (b) is the answer.

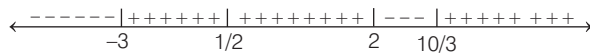
- 111** Since the sign of $f(p)$ and $f(q)$ are opposite, it means there must be a real root between p and q through which graph intercepts the X -axis and so it changes the sign of the graph from positive to negative or negative to positive.

As you know that a real root (with odd multiplicity) changes the sign of graph – that is the two points, one lying in the left side and the other one lying in the right side of the real root, have different signs.

Hence choice (a) is the answer.

- 112** The best way to solve this problem is to sketch a rough graph of the given polynomial or determine the sign of each interval on the number line (or X -axis).

The real roots of the given polynomial are 2 , -3 , $\frac{1}{2}$ and $\frac{10}{3}$.



Now there are two methods to determine the sign of the interval. First, either test each interval individually by substituting any suitable numerical value from that interval in the given polynomial or follow the ensuing technique.

After simplification, the highest power (or degree) of this equation is 9, that is odd. The coefficient of the highest power of x is a positive integral.

So the first interval (that is the left most) will be negative.

After that -3 is a root of odd multiplicity so after -3 , the sign of that interval will change to positive. But after that $-\frac{1}{2}$ is a root of even multiplicity so there won't be any

change in sign after $-\frac{1}{2}$. Further 2 is a root of odd

multiplicity so the sign will change after 2 from positive to negative. Finally, $\frac{10}{3}$ is again a root of odd multiplicity so

after $\frac{10}{3}$ sign will change from negative to positive. Now

since $x = -10^{10}$ lies in first interval which is negative and $x = 10^{10}$ lies in the last interval which is positive, so the product of negative and positive is negative, that is $f(-10^{10})f(10^{10}) < 0$. Therefore (i) is correct.

Similarly, $f(-2) > 0$ but $f(3) < 0$, so $f(-2)f(3) < 0$.

Therefore (ii) is also correct.

It means choices (a), (b) and (c) are wrong, as they say that (i) and (ii) are incorrect, which is not.

Hence choice (d) is the answer.

- 113** Since α and β are the roots of $x^2 - 6x - 2 = 0$. therefore

$$\alpha^2 - 6\alpha - 2 = 0 \Rightarrow \alpha^2 - 2 = 6\alpha \quad \dots(i)$$

$$\beta^2 - 6\beta - 2 = 0 \Rightarrow \beta^2 - 2 = 6\beta \quad \dots(ii)$$

$$\text{Now we have, } \frac{a_{10} - 2a_8}{2a_9} = \frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{(\alpha^{10} - 2\alpha^8) - (\beta^{10} - 2\beta^8)}{2(\alpha^9 - \beta^9)} = \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(6\alpha) - \beta^8(6\beta)}{2(\alpha^9 - \beta^9)} = \frac{6(\alpha^9 - \beta^9)}{2(\alpha^9 - \beta^9)} = 3$$

Hence choice (c) is the answer

$$114 \quad \frac{3f_9 - 14f_6}{f_7} = \frac{3(a^9 - b^9) - 14(a^6 - b^6)}{a^7 - b^7}$$

$$= \frac{a^6(3a^3 - 14) - b^6(3b^3 - 14)}{a^7 - b^7}$$

Since a and b are the roots of $3x^3 - 8x - 14 = 0$, therefore you will have $3a^3 - 8a - 14 = 0 \Rightarrow 3a^3 - 14 = 8a$

And, $3b^3 - 8b - 14 = 0 \Rightarrow 3b^3 - 14 = 8b$

$$\text{Therefore, } \frac{3f_9 - 14f_6}{f_7} = \frac{a^6(8a) - b^6(8b)}{a^7 - b^7}$$

$$= \frac{8a^7 - 8b^7}{a^7 - b^7} = \frac{8(a^7 - b^7)}{a^7 - b^7} = 8$$

Hence choice (a) is the correct one.

115 The best way is to solve through the given choices.

Choice (a)

$$f\left(-\frac{1}{4}\right) = 4\left(-\frac{1}{4}\right)^3 + 3\left(-\frac{1}{4}\right)^2 + 2\left(-\frac{1}{4}\right) + 1 = \frac{10}{16}$$

$$\Rightarrow f\left(-\frac{1}{4}\right) > 0$$

Similarly $f(0) > 0$

Since $f\left(-\frac{1}{4}\right) \cdot f(0) > 0$ therefore it implies that between $-\frac{1}{4}$ and 0 either there are two real roots or there is no real root at all.

Choice (b) $f(-11) < 0$ and $f\left(-\frac{3}{4}\right) < 0$

Since $f(-11) \cdot f\left(-\frac{3}{4}\right) > 0$, therefore it implies that between -11 and $-\frac{3}{4}$ either there are two real roots or there is not real root at all.

Choice (c) $f(-3/4) < 0$ and $f(-1/2) > 0$

Since $f\left(-\frac{3}{4}\right) \cdot f\left(-\frac{1}{2}\right) < 0$, therefore it implies that

between $-\frac{3}{4}$ and $-\frac{1}{2}$ either there are three real roots or

there is one real root. In essence, there is certainly at least one real root. Hence choice (c) is the answer.

Hint When $f(p) \cdot f(q) < 0$, then there is at least one real root between p and q .

Choice	$f(p)$	$f(q)$	$f(p) \cdot f(q)$	Possible number of roots	Conclusion
(a)	> 0	> 0	> 0	0 or 2	May or may not be
(b)	< 0	< 0	> 0	0 or 2	May or may not be
(c)	> 0	< 0	< 0	1 or 3	At least one root is there
(d)	> 0	> 0	> 0	0 or 2	May or may not be

116 The maximum number of positive real roots of a polynomial equation $f(x) = 0$ is the number of changes of signs – from positive to negative and / or negative to positive – in the coefficients of $f(x)$.

Equation	$4x^3 + 3x^2 + 2x + 1$			
Coefficient / constants	+ 4	+ 3	+ 2	+ 1
Sign	+	+	+	+

Since there is no change in sign of coefficients/constant values, so there won't be any positive real root.

Hence choice (c) is the answer.

$$117 \text{ Product of roots} = \frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = 1$$

Now since $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$$\Rightarrow q = (-p)[(\alpha + \beta)^2 - 3\alpha\beta] \Rightarrow q = (-p)[(-p)^2 - 3\alpha\beta]$$

$$\Rightarrow \alpha\beta = \frac{1}{3}\left(p^2 + \frac{q}{p}\right)$$

$$\text{Therefore sum of roots} = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{p^2 - \frac{2}{3}\left(p^2 + \frac{q}{p}\right)}{\frac{1}{3}\left(p^2 + \frac{q}{p}\right)} = \frac{p^3 - 2q}{p^3 + q}$$

$$\text{So the required equation is } x^2 - \left(\frac{p^3 - 2q}{p^3 + q}\right)x + 1 = 0$$

$$\Rightarrow (p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$$

Hence choice (b) is the answer.

$$118 \quad x^2 + 20 \leq 9x$$

$$\Rightarrow x^2 - 9x + 20 \leq 0 \Rightarrow (x - 4)(x - 5) \leq 0 \Rightarrow 4 \leq x \leq 5$$

$$f(4) = -4 \Rightarrow f(4) < 0 \text{ and } f(5) = 7 \Rightarrow f(5) > 0$$

It implies that there will be either only one real root or all the three roots between 4 and 5.

Using rule of signs you get to know that there is no negative root so all the roots are positive. If all the three roots lie between 4 and 5, then their sum will lie between 12 and 15. But since the sum of three roots is $15/2$ (or 7.5), it means only one root lies between 4 and 5.

Since $f(4) < f(5)$, it means the highest value of $f(x)$ will be given by $f(5)$, which is 7.

Hence choice (a) is the answer.

$$119 \quad y = -\frac{x^2}{2} + x + 1 \Rightarrow y - \frac{3}{2} = -\frac{1}{2}(x - 1)^2$$

It implies that the curve is symmetric about $x = 1$.

Hence choice (a) is the answer.

$$120 \quad f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6} \Rightarrow \frac{(x-1)(x-5)}{(x-2)(x-3)}$$

The critical points are 1, 2, 3 and 5.

The vertical asymptotes are $x = 2$, $x = 3$

Horizontal asymptote is $y = 1$.

Partial graph of the given function is shown below.

Otherwise, test the nature of the function in various intervals.

(P) When $-1 < x < 1 \Rightarrow 0 < f(x) < 1$

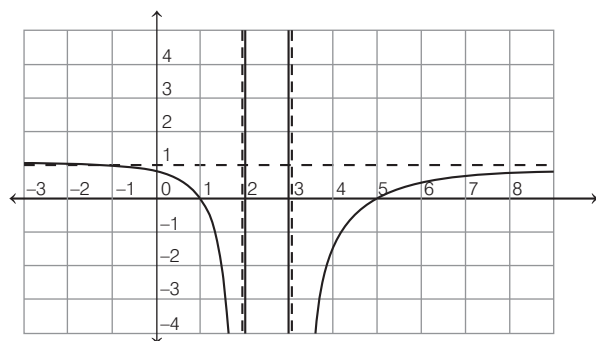
(Q) When $1 < x < 2 \Rightarrow f(x) < 0$

(R) When $3 < x < 5 \Rightarrow f(x) < 0$

(S) When $x > 5 \Rightarrow 0 < f(x) < 1$

Therefore (P) $\rightarrow p, r, s$; (Q) $\rightarrow q, s$; (R) $\rightarrow q, s$; (S)

$\rightarrow p, r, s$



Hence choice (a) is the answer.

$$121 \quad x^2 - 10cx - 11d = 0 \quad \dots(i)$$

$$a + b = 10c \text{ and } ab = -11d$$

$$x^2 - 10ax - 11b = 0 \quad \dots(ii)$$

$$c + d = 10a \text{ and } cd = -11b$$

$$\text{So } (a+b) + (c+d) = 10(a+c) \Rightarrow b+d = 9(a+c) \quad \dots(iii)$$

$$\text{And } (ab)(cd) = 121bd$$

$$\Rightarrow ac = 121 \quad \dots(iv)$$

$$\text{Now } a^2 - 10ac - 11d = 0 \text{ and } c^2 - 10ac - 11b = 0$$

$$\text{So } a^2 + c^2 - 20ac - 11(b+d) = 0$$

$$\Rightarrow (a+c)^2 - 22(121) - 99(a+c) = 0 \quad \dots(v)$$

$$\Rightarrow (a+c) = 121 \text{ or } (a+c) = -22$$

$$\Rightarrow a+b+c+d = 10(a+c) = 1210$$

Hence choice (c) is the answer.

NOTE Equation (v) is a quadratic Equation. The value of $(a+c) = -22$ has to be discarded otherwise $a = -11$ and $c = -11$, which is not possible as a and c have to be distinct.

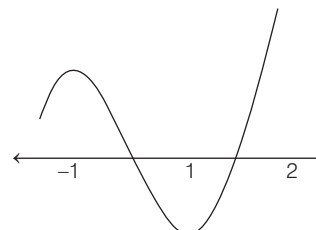
122 Since $f(1)f(2) < 0$, it implies that at least one real root of $f(x)$ lies between 1 and 2.

Also $f(-1) > 0$ and $f(1) < 0 \Rightarrow f(-1)f(1) < 0$, it implies at least one real root lies between -1 and 1 .

Since $f(x)$ is a cubic polynomial which either have 1 real root or 3 real roots. It means all 3 roots are real.

Therefore statement (ii) is true.

Using the given information you can sketch the graph of $f(x)$ similar to the one as shown below.



$f'(x) = 3ax^2 + 2bx + c$ is a quadratic function. It gives minimum / maximum when $x = \frac{-b}{2a}$. But it is given that

$f'(x)$ has local minimum at $x = 0$. It implies that $b = 0$

Therefore $f'(x) = 3ax^2 + c$

Since at $x = -1$, $f(x)$ has local maxima, it means at

$$x = -1, f'(x) = 0$$

$$\text{That is } 3a(-1)^2 + c = 0 \Rightarrow c = -3a$$

$$\text{Therefore } f'(x) = 3ax^2 - 3a = 3a(x^2 - 1)$$

$$\text{Since } a \neq 0, x^2 - 1 = 0 \Rightarrow x = \pm 1.$$

Since at $x = \pm 1$, $f'(x) = 0$, therefore at $x = \pm 1$, $f(x)$ will have its local maxima / minima.

As, $f(-1)$ already has local maxima, so $f(1)$ will have local minima.

It means $f(1) = -1$ is the local minima. So $f(x)$ is increasing for $[1, \infty)$. Therefore statement (i) and (iii) are true.

As you have just found that $b = 0$, it means $\frac{-b}{a} = 0 \Rightarrow$ Sum of roots is zero.

Therefore statement (iv) is true.

Hence choice (c) is the answer.

Hint As you have determined $b = 0$ and $c = -3a$, now you can easily find out the relevant polynomial as following

$$f(x) = ax^3 + bx^2 + cx + d \Rightarrow f(x) = ax^3 - 3ax + d$$

$$\therefore f(2) = 18 \Rightarrow 8a - 6a + d = 18$$

$$\Rightarrow 2a + d = 18 \text{ and } f(1) = -1 \Rightarrow a - 3a + d = -1$$

$$\Rightarrow -2a + d = -1 \Rightarrow a = \frac{19}{4}, d = \frac{17}{2}, c = -\frac{57}{4}$$

$$\text{Thus } f(x) = \frac{1}{4}(19x^3 - 57x + 34)$$

$$123 \quad f(x) = ax^3 + bx^2 + cx + d \quad \dots(i)$$

$$f(-1) = -a + b - c + d = 10 \quad \dots(ii)$$

$$f(1) = a + b + c + d = -6 \quad \dots(iii)$$

$$\text{From eqs. (ii) and (iii), } b + d = 2 \text{ and } a + c = -8 \quad \dots(iv)$$

Now, $f'(x) = 3ax^2 + 2bx + c$ is a quadratic function. The quadratic function has max / min at

$$x = \frac{-(\text{coefficient of } x)}{2(\text{coefficient of } x^2)}.$$

But it is given that $f'(x)$ has local minimum at $x = 1$.

That is $1 = \frac{-(2b)}{2(3a)} \Rightarrow b = -3a$... (v)

Therefore $f'(x) = 3ax^2 - 6ax + c$

Now, since $f(x)$ has local maxima at $x = -1$, it implies that at $x = -1$, $f'(x) = 0$. That is $f'(-1) = 0$

$\Rightarrow 3a + 6a + c = 0 \Rightarrow c = -9a$... (vi)

From eqs. (vi), (iv) and (v), we have

$$a = 1, b = -3, c = -9, d = 5$$

Therefore $f(x) = x^3 - 3x^2 - 9x + 5$ and

$$f'(x) = 3x^2 - 6x - 9.$$

Since $f'(x) = 0 \Rightarrow 3x^2 - 6x - 9 = 0 \Rightarrow 3(x+1)(x-3) = 0$

As $f'(x) = 0$ at $x = -1$. It means at $x = -1$, $f(x)$ is maximum, as already given.

Similarly, $f'(x) = 0$ at $x = 3$. It means at $x = 3$, $f(x)$ is minimum, as required.

$f(-1) = 10$ and $f(3) = -22$. Therefore distance between $(-1, 10)$ and $(3, -22)$ is $4\sqrt{65}$.

Hence choice (a) is the answer.

Hint The distance between two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

124 What if a root, say β , is zero! Then

$\alpha + \beta, \alpha^2 + \beta^2, \alpha^3 + \beta^3 \Rightarrow \alpha, \alpha^2, \alpha^3$, which are naturally in GP. So, $\alpha + \beta, \alpha^2 + \beta^2, \alpha^3 + \beta^3$ will also be in GP.

And when one root is zero, product of roots will be zero. That is $c = 0$.

So $\Delta = b^2 \Rightarrow c\Delta = 0(b^2) = 0$

Hence choice (c) is the answer.

Alternatively If x, y, z are in GP, then $y^2 = xz$.

Therefore,

$$\begin{aligned} (\alpha^2 + \beta^2)^2 &= (\alpha + \beta)(\alpha^3 + \beta^3) \\ \Rightarrow \alpha^4 + \beta^4 + 2\alpha^2\beta^2 &= \alpha^4 + \beta^4 + \alpha\beta(\alpha^2 + \beta^2) \\ \Rightarrow 2(\alpha\beta)^2 &= \alpha\beta(\alpha^2 + \beta^2) \Rightarrow \alpha\beta(\alpha^2 + \beta^2 - 2\alpha\beta) = 0 \\ \Rightarrow \alpha\beta[(\alpha + \beta)^2 - 4\alpha\beta] &= 0 \Rightarrow \frac{c}{a} \left[\left(\frac{-b}{a} \right)^2 - \frac{4c}{a} \right] = 0 \\ \Rightarrow \frac{c}{a} \left[\frac{b^2 - 4ac}{a^2} \right] &= 0 \Rightarrow c(b^2 - 4ac) = 0 \Rightarrow c\Delta = 0 \end{aligned}$$

125 $\therefore f(x) = x^3 + bx^2 + cx + d$... (i)

$\therefore f'(x) = 3x^2 + 2bx + c$... (ii)

Now the discriminant of $f'(x)$ is

$$D = 4b^2 - 12c$$

$\Rightarrow D = 4(b^2 - c) - 8c < 0$

$\Rightarrow D < 0$

Since the quadratic function $f'(x)$ has no real root, so the cubic function $f(x)$ will have no local maxima/minima. It implies that either $f(x)$ is strictly increasing or strictly decreasing.

But since the coefficient of x^3 in $f(x)$ is positive so the function $f(x)$ is strictly increasing in $(-\infty, \infty)$.

Hence choice (a) is the answer.

126 $\min f(x) = 2c^2 - b^2$ and $\max g(x) = b^2 + c^2$

Therefore, $2c^2 - b^2 > b^2 + c^2 \Rightarrow c^2 > 2b^2 \Rightarrow |c| > \sqrt{2}|b|$

Hence choice (d) is the answer.

Hint The quadratic function has max/min at

$$x = \frac{-(\text{coefficient of } x)}{2(\text{coefficient of } x^2)}.$$

Otherwise,

$$f(x) = x^2 + 2bx + 2c^2 = (x + b)^2 + 2c^2 - b^2$$

$f(x)$ is minimum when $(x + b)^2 = 0$ at $x = -b$

and $g(x) = -x^2 - 2cx + b^2$

$$b^2 + c^2 - (x + c)^2$$

$g(x)$ is maximum when $(x + c)^2 = 0$ at $x = -c$.

127 Since both the roots are less than the arbitrary point k then you must have to satisfy the following three conditions.

(i) $D \geq 0 \Rightarrow 4a^2 - 4(a^2 + a - 3) \geq a \geq 3$

(ii) $af(k) > 0 \Rightarrow 1. f(3) > 0$

$\Rightarrow 9 - 6a + a^2 + a - 3 \Rightarrow a \in (-\infty, 2) \cup (3, \infty)$

(iii) $k > -\frac{b}{2a} \Rightarrow 3 > -\frac{-2a}{2} \Rightarrow a < 3$

The intersection of (i), (ii) and (iii) is $a \in (-\infty, -2)$

Hence choice (a) is the answer.

128 $(x - 9)(x - 16) = x^2 - 25x + 144$

Since the coefficient of x^2 is positive, it means the graph will open upwards. So the values between roots will be included only when, $x^2 - 25x + 144 \leq 0$

Hence choice (a) is the answer.

129 Let the common root be r , then

$$f = (x - r)(x - 2) = 0 \Rightarrow f = m[(x - r)(x - 2)] = 0$$

and $g = (x - r)(x - 7) = 0 \Rightarrow g = n[(x - r)(x - 7)] = 0$

When, m and n are non-zero real numbers.

Since, $f(4) \times g(9) = 24$

$\Rightarrow m(4 - r)(4 - 2) \times n[(9 - r)(9 - 7)] = 24$

$\Rightarrow mn(4 - r) \times (9 - r) = 6 \Rightarrow mn(r^2 - 13r + 36) = 6$

$$\Rightarrow (r^2 - 13r + 36) = \frac{6}{mn} \Rightarrow r^2 - 13r + \left(36 - \frac{6}{mn}\right) = 0$$

As you can see that r depends on m and n , so there is no unique solution.

But since one root in each of the equations $f(x) = 0$ and $g(x) = 0$ is real and rational, so the other root, in each equation, must be real and rational.

A root is real only when discriminant,

$$D \geq 0. \text{ That is } b^2 - 4ac \geq 0$$

$$\therefore 169 - 4\left(36 - \frac{6}{mn}\right) \geq 0$$

$$\Rightarrow \left(25 + \frac{24}{mn}\right) \geq 0$$

✓ Level 02 Higher Level Exercise

- 1 The best way is to go through options.

Consider option (b)

$$|3^4 - 1|^{\log_3 (3^4)^2 - 2 \log_{81} 9} = (3^4 - 1)^7$$

$$|80|^{\log_3 3^8 - \log_{81} 81} = (80)^7 \Rightarrow \log_3 3^8 - \log_{81} 81 = 7$$

$$\Rightarrow 8 - 1 = 7 \text{ and } 7 = 7$$

Hence option (b) is correct.

- 2 Putting $x = \frac{1}{y}$, we get $27y^3 + 54y^2 + cy - 10 = 0 \dots(i)$

This above eq. (i) must be in AP.

Let the roots of equation in y be

$$\alpha - \beta, \alpha, \alpha + \beta \quad (\because \text{roots are in AP})$$

$$\therefore \Sigma \alpha = \alpha - \beta + \alpha + \alpha + \beta = 3\alpha$$

$$\Rightarrow 3\alpha = \frac{-54}{27} \Rightarrow \alpha = \frac{-2}{3}$$

Now $\alpha = \frac{-2}{3}$ will satisfy the Eq. (i) and we get

$$27 \times \frac{-8}{27} + 54 \times \frac{4}{9} - \frac{2c}{3} - 10 = 0 \Rightarrow c = 9$$

- 3 $\log_{100} |x + y| = \frac{1}{2} \Rightarrow (100)^{1/2} = |x + y|$

$$\Rightarrow |x + y| = 10 \dots(i)$$

$$\text{Again, } \log_{10} y - \log_{10} |x| = \log_{100} 4$$

$$\log_{10} y - \log_{10} |x| = \log_{10} 2 \Rightarrow \log_{10} \frac{y}{|x|} = \log_{10} 2$$

$$\Rightarrow y = 2|x| \dots(ii)$$

From eq. (ii) we can conclude that y is always positive.

Now, when $x > 0$ and $y > 0$ (always)

$$|x + y| = 10 \Rightarrow |x + 2|x|| = 10$$

$$\Rightarrow x + 2|x| = 10 \quad (\because x > 0)$$

$$\Rightarrow x + 2x = 10 \Rightarrow x = \frac{10}{3}$$

A root is rational only when discriminant D is a perfect square.

That is $D = 0, 1, 4, 9, \dots$

$$\therefore D = 25 + \left(\frac{24}{mn}\right) \Rightarrow mn = -\frac{24}{(25-D)}$$

It implies that $D \neq 25$.

$$\therefore r = \frac{-13 \pm \sqrt{D}}{2} \Rightarrow r \neq -4, -9$$

It means, (i), (ii) and (iii) are invalid.

Further, since D is a perfect square, so $\sqrt{D}$ will be a non-negative integer. Therefore the values of r would be rational numbers not the irrational ones. Therefore (iv) is also invalid. Hence choice (c) is the correct one.

$$\therefore y = \frac{20}{3}$$

Again, $x < 0$ and $y > 0$ (always positive)

$$|-x + 2| - x| = 10 \Rightarrow |-x + 2x| = 10$$

$$\Rightarrow |x| = 10 \Rightarrow x = -10 \quad (\because x < 0)$$

$$\therefore y = 20$$

$$\text{Hence, } x = -10, y = 20 \text{ and } x = \frac{10}{3} \text{ and } y = \frac{20}{3}.$$

- 4 $2 \log_2 \log_2 x + \log_{1/2} \log_2 (2\sqrt{2}x) = 1$

$$\Rightarrow 2 \log_2 \log_2 x - \log_2 \log_2 (2\sqrt{2}x) = \log_2 2$$

$$\Rightarrow \log_2 (\log_2 x)^2 - \log_2 [\log_2 2\sqrt{2}x] = \log_2 2$$

$$\Rightarrow \log_2 \frac{(\log_2 x)^2}{[\log_2 2\sqrt{2}x]} = \log_2 2 \Rightarrow \frac{(\log_2 x)^2}{\log_2 (2\sqrt{2}x)} = 2$$

$$\Rightarrow (\log_2 x)^2 = 2 \log_2 (2\sqrt{2}x) = 2 \log_2 (2^{3/2}x)$$

$$\Rightarrow (\log_2 x)^2 = 2[3/2 \log_2 x + \log_2 x] = 3 + 2 \log_2 x$$

$$\Rightarrow (\log_2 x)^2 - 2 \log_2 x - 3 = 0 \text{ or } \log_2 x = -1$$

$$\Rightarrow \log_2 x = 3 \Rightarrow x = \frac{1}{2} \text{ or } x = 8$$

But for $x = \frac{1}{2}$, $\log_2 \log_2 (1/2)$ is undefined

$\therefore$ Only possible value of $x = 8$.

- 5 Consider $x^2 + 4x + 3 \geq 0 \dots(i)$

$$\text{then } (x^2 + 4x + 3) + 2x + 5 = 0$$

$$\Rightarrow x = -2 \text{ and } x = -4$$

but $x = -2$ does not satisfy eq. (i)

$$\text{Now, if } x^2 + 4x + 3 < 0 \dots(ii)$$

$$\text{then } -(x^2 + 4x + 3) + 2x + 5 = 0$$

$$\Rightarrow x = -1 - \sqrt{3}$$

$$\text{or } x = -1 + \sqrt{3}$$

but only $x = -1 - \sqrt{3}$ satisfies the eq. (ii).
Hence the solution set of x is $(-4, -1 - \sqrt{3})$.

Alternatively, check the options by substituting the values from the options given in the question.

6 $\because x_1, x_2, x_3$ are in AP.

$$\Rightarrow x_1 = a - d, \quad x_2 = a, \quad x_3 = a + d$$

Where d is the common difference.

Now, since x_1, x_2, x_3 are the roots of the given equation

$$x^3 - x^2 + \beta x + \gamma = 0$$

$$\text{So, } \Sigma \alpha = x_1 + x_2 + x_3 = 1$$

$$\Rightarrow (a - d) + (a) + (a + d) = 1 \quad \dots(i)$$

$$\Sigma \alpha \beta = x_1 x_2 + x_2 x_3 + x_1 x_3$$

$$\Rightarrow \beta = (a - d)a + a(a + d) + (a - d)(a + d) \quad \dots(ii)$$

$$\text{and } \Sigma = \alpha \beta \gamma = x_1 x_2 x_3 = -\gamma = (a - d)(a)(a + d) \quad \dots(iii)$$

hence from eq. (i) we get $a = \frac{1}{3}$

and from eq. (ii) we get

$$\beta = 3a^2 - d^2 \Rightarrow \beta = \frac{1}{3} - d^2 \quad \left(\because a = \frac{1}{3} \right)$$

$$\text{Thus } \beta = \frac{1}{3} - d^2 \leq \frac{1}{3} \quad [\because d^2 \geq 0]$$

$$\Rightarrow \beta \leq \frac{1}{3} \text{ or } \beta \in \left(-\infty, \frac{1}{3} \right]$$

Again from eq. (iii), we get

$$a(a^2 - d^2) = -\gamma \Rightarrow \left(\frac{1}{27} \right) + \left(-\frac{d^2}{3} \right) = -\gamma$$

$$\Rightarrow \gamma = \frac{d^3}{3} - \frac{1}{27} \Rightarrow \gamma \geq \frac{-1}{27} \quad (\because d^2 \geq 0)$$

Hence option (a) is correct.

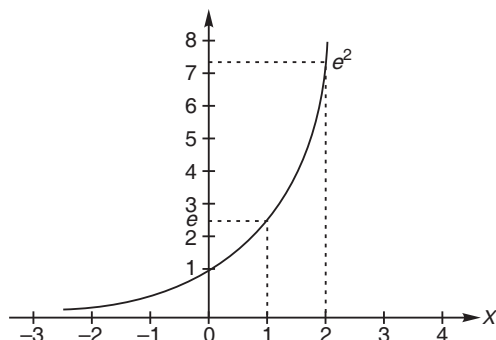
7 (a) $\rightarrow e^x < 1 + x$

$$(b) \rightarrow e^x > (1 + x) \Leftrightarrow \log_e (1 + x) < x$$

$$(c) \rightarrow \sin x > x$$

$$(d) \rightarrow e^x < x \Leftrightarrow \log_e x > x$$

NOTE $e \approx 2.714$ and $e^2 \approx 7.366$



From the graph it is clear that when $0 < x < 1$, $e^x > (1 + x)$

Option (b) is clearly wrong.

8 Let us consider some value of $p = 3$ (say), then

$$x^2 - 4x + 1$$

$$\text{and } (\alpha, \beta) = 2 \pm \sqrt{3} \quad (\alpha, \beta \text{ are the roots})$$

$$\text{Now, } \alpha^n + \beta^n = (2 + \sqrt{3})^n + (2 - \sqrt{3})^n \quad \dots(i) \quad (n \in \mathbb{N})$$

Then, $\alpha^n + \beta^n$ will always be an integer, for the validity of statement you can put $n = 1, 2, 3, \dots$ etc. in Eq. (i).

Similarly for $p = 4, 5, 6, \dots$ etc. we can conclude the same results.

NOTE In this question, the discriminant D is always positive, i.e., $b^2 - 4ac > 0$. So the roots will always be real, unequal and irrational.

But the fact is that $\alpha^n + \beta^n$, for $n \in \mathbb{N}$, always yields integral value. This can be easily proved by mathematical induction method. So if the answer is an integer, then it must be a rational number, hence option (d) is correct.

9 Just assume some values of α, β conforming the basic constraints of the problem.

e.g., $\alpha = -2, \beta = 8$, then the equation becomes

$$x^2 - 6x - 16 \Rightarrow b = -6 \text{ and } c = -16$$

$$\therefore 1 + \frac{c}{a} + \left| \frac{b}{a} \right| = 1 - 16 + 6 = -9$$

$\therefore$ The value of the expression is negative, hence choice (a) is correct.

NOTE Since $\alpha < 1$ and $\beta > 1$

$$\therefore \alpha \beta < 1 \Rightarrow \frac{c}{a} < 1$$

Further the product of any two numbers ($n_1, n_2 \neq 0$) is less than the sum of the number if any one of them is negative.

So, $\alpha \beta < \alpha + \beta$ ($\because$ Here $\alpha \beta$ is negative)

$$\therefore \frac{c}{a} < \left| \frac{b}{a} \right|; \text{ but } \frac{c}{a} \text{ is numerically greater than } \left| \frac{b}{a} \right|$$

$$\Rightarrow 1 + \frac{c}{a} + \left| \frac{b}{a} \right| < 0.$$

10 Since p and q are the roots of given equation $x^2 + px + q = 0$ then $p + q = -p \Rightarrow q = -2p$

$$\text{and } pq = q \Rightarrow p = 1$$

So, when $p = 1$, then $q = -2$

Again, when $q = 0$, then $p = 0$ hence,

$$p = 1, 0 \text{ and } q = -2, 0$$

Thus, option (b) is most appropriate.

11 $\because p, q, r$ are in AP.

$$\therefore q = \frac{p+r}{2} \quad [\because p+r=2q]$$

For the real roots $q^2 - 4pr \geq 0$

$$\Rightarrow \left(\frac{p+r}{2} \right)^2 - 4pr \geq 0 \Rightarrow p^2 + r^2 - 14pr \geq 0$$

$$\Rightarrow \left(\frac{p}{r}\right)^2 - 14\left(\frac{p}{r}\right) + 1 \geq 0$$

$$\Rightarrow \left(\frac{p}{r} - 7\right)^2 \geq 48$$

$$\Rightarrow \left|\frac{p}{r} - 7\right| \geq 4\sqrt{3}$$

- 12** The given equation is $|x - 2|^2 + |x - 2| - 2 = 0$.

Let us assume $|x - 2| = m$

Then, $m^2 + m - 2 = 0$ or $(m - 1)(m + 2) = 0$

Only admissible value is

$$m = 1 \quad (\because m \neq -2 \text{ as } m \geq 0)$$

$$\therefore |x - 2| = 1$$

$$\Rightarrow x - 2 = 1 \Rightarrow x = 3 \text{ or } -(x - 2) = 1 \Rightarrow x = 1$$

$$\text{Hence } x = 1, 3$$

$$\therefore \text{Sum of the roots of equation} = 1 + 3 = 4.$$

- 13** Just consider an option, and then substitute the values of A and B from assumed option, if the roots p, q, r, s are in AP, then the assumed option is correct, else not.

Thus we find that options a, b and c are incorrect.

Hence (d) is the right choice.

Alternatively Let us consider $p = a - 3d, q = a - d, r = a + d$ and $s = a + 3d$

$$\therefore 2a - 4d = 2 \quad \dots(i)$$

$$\text{and } 2a + 4d = 18 \quad \dots(ii)$$

Solving equations (i) and (ii), we get

$$a = 5, d = 2$$

$$\therefore p = -1, q = 3, r = 7, s = 11$$

$$\text{Hence } A = pq = -3 \text{ and } B = rs = 77$$

Thus (d) is the right choice

- 14** Let $f(x) = x^2 - 2ax + a^2 + a - 3$

Since $f(x)$ has real roots, both less than 3.

Therefore, $D > 0$ and $f(3) > 0$

$$\Rightarrow a^2 - (a^2 + a - 3) > 0 \text{ and } a^2 - 5a + 6 > 0$$

$$\Rightarrow a < 3 \text{ and } (a - 2)(a - 3) > 0$$

$$\Rightarrow a < 3 \text{ and } a < 2 \text{ or } a > 3 \Rightarrow a < 2$$

- 15** Considering the given constraints in the problem.

Let us consider $\alpha, \beta = (-3, 2)$

Then the given equation becomes

$$x^2 + x - 6 = 0 \Rightarrow b = 1$$

$$\text{and } c = -6$$

Now, we check for the given choices, which satisfy the aforesaid conditions

(a) It is clearly wrong (b) It is correct

(c) It is also wrong (d) It is also wrong

Hence option (b) is appropriate.

NOTE You should not worry about this technique. It must be clear to you that while assuming the values of α, β , you have to strictly follow the basic given constraints, i.e., α must be less than β and after forming the similar equation you should have $c < 0 < b$. So I think it is even very intelligent method to solve the problems rather than conventional methods.

- 16** Let us assume $a = 3, b = 4$, given that $a < b$ then the given equation becomes

$$(x - 3)(x - 4) - 1 = 0 \text{ or } x^2 - 7x + 11 = 0$$

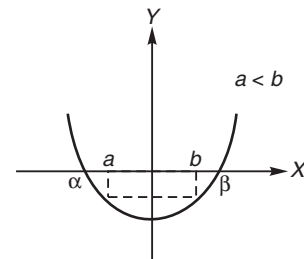
$$\therefore x = \frac{7 \pm \sqrt{49 - 44}}{2} \Rightarrow x = \frac{7 \pm \sqrt{5}}{2}$$

$$\Rightarrow x = \frac{7 + \sqrt{5}}{2} > 4 \text{ and } \frac{7 - \sqrt{5}}{2} < 3$$

Hence only option (d) is satisfied, hence correct.

Alternatively Since $f(x) = (x - a)(x - b) - 1 = 0$

$$\Rightarrow (x - a)(x - b) = 1$$



$$\alpha < a \text{ and } \beta > b$$

It means X -axis is shifted to -1 unit below the original position. So it is clear from the graph that $\alpha < a$ and $\beta > b$.

- 17** $\alpha\beta = p$ and $\gamma\delta = q$

Now since, $\alpha, \beta, \gamma, \delta$ are in GP and integral values. So the options (b) and (c) are ruled out, as they have no required proper integral factors. Now let us look for option (a). We see that

$$\alpha\beta = -2 = -1 \times 2$$

$$\gamma\delta = -32 = -4 \times 8$$

So, $-1, 2, -4, 8$ are in GP satisfying the above conditions.

Again in option (d) the two values don't have the factors with common ratio, hence it is wrong. Finally option (a) is correct.

Alternatively The sum of roots and product of roots is as follows :

$$\alpha + \beta = 1, \quad \alpha\beta = p$$

$$\gamma + \delta = 4, \quad \gamma\delta = q$$

Let r be the common ratio of the GP $\alpha, \beta, \gamma, \delta$.

$$\text{Then, } \alpha + \beta = 1 \text{ and } \gamma + \delta = 4$$

$$\alpha + \alpha r = 1 \text{ and } \alpha r^2 + \alpha r^3 = 4$$

$$\Rightarrow \alpha(1 + r) = 1 \text{ and } \alpha r^2(1 + r) = 4$$

$$\text{So } 1 + r = \frac{1}{\alpha} \text{ and } \alpha r^2 \left(\frac{1}{\alpha}\right) = 4 \Rightarrow r = \pm 2$$

When $r = 2$, then $\alpha = \frac{1}{3}$ which is inadmissible.

Hence, $r = -2$ is to be considered, then $\alpha = -1$

Thus $\alpha\beta = p = -2$ and $r\delta = q = -32$

- 18** When this problem will be solved by algebraic methods, it will take too much time to solve beyond the normal required time. So, the best way to get the correct and quick answer is to assume some simple roots (i.e., α and β) then go through options.

Let us take two arbitrary values $\alpha = -1, \beta = 2$, then the equation will be $x^2 - x - 2 = 0$

Comparing with the equation $x^2 - px + q = 0$

$$\Rightarrow p = 1, q = -2$$

Now, the sum of the roots of the required equation

$$= [(\alpha^2 - \beta^2)(\alpha^3 - \beta^3)] + [\alpha^3\beta^2 + \alpha^2\beta^3]$$

$$= 27 + 4 = 31$$

and product of roots $[(\alpha^2 - \beta^2)(\alpha^3 - \beta^3)] + [\alpha^3\beta^2 + \alpha^2\beta^3]$

$$= 27 \times 4 = 108$$

Hence equation is $x^2 - 31x + 108 = 0$

Now putting the values of p and q in the equation options a, b , and c we get option (b) is correct. as :

$$x^2 - [1 - \{5 \times 1(-2)\} + 5 \times 1 \times 4] + [1 \times 4 - \{5 \times 1 \times (-8)\}]$$

$$+ 4 \times 1 \times 16 = 0$$

$$= x^2 - 31x + 108 = 0$$

- 19** Let us consider choice (a). When we put the values of A and B respectively, we get the values of α, β, γ and δ as $-1, 1/3, 1/5, 1/3$, which are not in HP. So this option is not correct.

Now for our convenience we consider choice (c), then by substituting the values of A and B , we get the values of α, β, γ and δ as $1, 1/2, 1/3$ and $1/4$ which are in HP.

Hence this could be the correct choice.

Alternatively $Ax^2 - 4x + 1 = 0$... (i)

$$\alpha + \gamma = \frac{4}{A} \text{ or } \alpha\gamma = \frac{1}{A}$$

and $Bx^2 - 6x + 1 = 0$... (ii)

$$\beta + \delta = \frac{6}{B} \text{ or } \beta\delta = \frac{1}{B}$$

Since it is given that $\alpha, \beta, \gamma, \delta$ are in HP.

$$\therefore \beta = \frac{2\alpha\gamma}{\alpha + \gamma} = \frac{1}{2} \text{ and } \gamma = \frac{2\beta\delta}{\beta + \delta} = \frac{1}{3}$$

Again since β and γ are the roots of the given equation hence they must satisfy the equation. So

$$B\beta^2 - 6\beta + 1 = 0 \text{ and } A\gamma^2 - 4\gamma + 1 = 0 \Rightarrow B = 8 \text{ and } A = 3$$

Hence option (c) is correct.

- 20** Assume some convenient and appropriate values of a, b, c as

$$a = 3, b = 4, c = 6.$$

$$\text{Then, } (x - 3)(x - 4) - 6 = 0$$

$$[\because (x - a)(x - b) = c, c \neq 0]$$

$$\Rightarrow x^2 - 7x + 6 = 0 \Rightarrow \alpha = 6, \beta = 1$$

$$\text{Again } (x - 6)(x - 1) + 6 (\because (x - \alpha)(x - \beta) + c = 0)$$

$$x^2 - 7x + 6 + 6 = 0 \text{ or } x^2 - 7x + 12 = 0$$

$$\therefore \text{The roots } k_1 = 3 \text{ and } k_2 = 4$$

which are same as a and b .

Hence, option (c) is correct.

Alternatively $x^2 - (a + b)x + (ab - c) = 0$

$$\therefore \alpha + \beta = a + b \text{ and } \alpha\beta = ab - c$$

Again if k_1 and k_2 be the roots of the other equation, then

$$(x - \alpha)(x - \beta) + c = 0$$

$$\text{i.e., } x^2 - (\alpha + \beta)x + (\alpha\beta + c) = 0$$

$$\therefore k_1 + k_2 = \alpha + \beta = a + b \quad \dots (i)$$

$$\text{and } k_1 \cdot k_2 = \alpha\beta + c = (ab - c) + c = ab \quad \dots (ii)$$

Thus, from eqs. (i) and (ii) it is clear that the roots are a and b . Hence correct choice is c.

- 21** The best way is to assume any convenient arbitrary numerical values. And then verify the options.

Let us assume $a = 1, b = 2, c = -3$, as $a + b + c = 0$. Then we will see that only choice (c) has a valid relation as both the sides have same value.

Hence, choice (c) is the answer.

- 22** Given that $px^2 - qx + q = 0 \Rightarrow px^2 + q = qx$

As p and q are prime, so they are positive. It implies $x > 0$.

Let us consider now $x = \frac{a}{b}$ such that a and b are co-prime.

$$\therefore px^2 - qx + q = 0 \Rightarrow p \frac{a^2}{b^2} - q \frac{a}{b} + q = 0$$

$$\text{and } pa^2 - qab + qb^2 = 0 \quad \dots (i)$$

Now dividing eq. (i) by a we get $pa - qb + \frac{qb^2}{a} = 0$; as

$pa - qb$ is integer, so $\frac{qb^2}{a}$ too must be an integer. But,

since a and b are co-prime, so a cannot divide b . It implies that $a = 1$ or q .

Similarly by dividing eqn. (i) we get to know that $b = 1$ or p .

$$\text{Therefore } x = \frac{a}{b} = \frac{1}{1}, \frac{1}{p}, \frac{q}{1}, \frac{q}{p}$$

Case 1: If $x = 1, px^2 - qx + q = 0 \Rightarrow p = 0$.

That's impossible as p being a prime number cannot be zero.

Case 2: If $x = \frac{1}{p}$, $px^2 - qx + q = 0 \Rightarrow \frac{1}{p} - \frac{q}{p} + q = 0 \Rightarrow$

$$pq + 1 = q.$$

That's impossible as $pq > q$.

Case 3: If $x = q$, $px^2 - qx + q = 0 \Rightarrow pq^2 + q = q^2$.

That's impossible as $pq^2 > q^2$.

Case 4: If $x = \frac{q}{p}$, $px^2 - qx + q = 0 \Rightarrow q = 0$.

That's impossible as q being a prime number cannot be zero.

Therefore no rational solutions to the quadratic equation $px^2 - qx + q = 0$ exists.

Hence, choice (a) is the correct answer.

23. Let r be an integral root of the given equation, then

$$r^4 - pr^3 + q = 0 \Rightarrow r^3(r - p) + q = 0 \Rightarrow q = r^3(p - r)$$

Since q itself is a prime (which cannot be the product of two distinct primes), so either $p - r = \pm 1$ or $r^3 = \pm 1$,

If $p - r = \pm 1$, then $q = \pm r^3$ which is not possible as a prime number cannot be a perfect cube. So $r^3 = \pm 1$.

If $r = -1$, then $q = -1(p + 1)$

$\Rightarrow q + p = -1$, which is impossible as prime numbers are always positive.

Further, if $r = 1$, then $q = 1(p - 1) \Rightarrow p - q = 1$.

So we can infer that $p = 3$ and $q = 2$ when the root $r = 1$.

Therefore the required integral root is 1.

Hence choice (d) is the answer.

24. $x^2 + 4y^2 + 9z^2 - 2xy - 6yz - 3zx$

$$= \frac{1}{2} [2x^2 + 8y^2 + 18z^2 - 4xy - 12yz - 6zx]$$

$$= \frac{1}{2} [(x^2 + 4y^2 - 4xy) + (x^2 + 9z^2 - 6zx)$$

$$+ (4y^2 + 9z^2 - 12yz)]$$

$$= \frac{1}{2} [(x - 2y)^2 + (x - 3z)^2 + (2y - 3z)^2] \geq 0$$

That means the given expression is always non-negative.

Hence choice (b) is the correct one.

Hint Since a perfect square number is always a non-negative number.

$$\therefore (x - 2y)^2 \geq 0, (x - 3z)^2 \geq 0, (2y - 3z)^2 \geq 0.$$

25 Let $p = e^{\sin x}$, then the given equation becomes as shown below.

$$p + \frac{1}{p} = 4 \quad \Rightarrow p^2 + p - 4 = 0$$

$$\Rightarrow p = 2 - \sqrt{5} \quad \text{or} \quad p = 2 + \sqrt{5} \Rightarrow e^{\sin x} = 2 - \sqrt{5}$$

$$\text{or } e^{\sin x} = 2 + \sqrt{5}.$$

Now taking log on both the sides, we get

$$\sin x = \log(2 - \sqrt{5}) \text{ or } \sin x = \log(2 + \sqrt{5})$$

Here, $2 - \sqrt{5}$ is a negative number. Since log of negative number is not defined, so $\sin x = \log(2 - \sqrt{5})$ does not give the solution.

$$\text{Again, } \sin x = \log(2 + \sqrt{5}) > \log e$$

$$\Rightarrow \sin x > 1, \text{ which is not possible.}$$

Hence choice (a) is the answer.

Hint $\log e = 1$ and $e = 2.714$. Also, $-1 \leq \sin x \leq 1$.

26 Since α and β are the roots of $x^2 + ux + v = 0$.

Therefore

$$\alpha + \beta = -u \quad \dots(i)$$

$$\text{And } \alpha\beta = v \quad \dots(ii)$$

Again since α and β are the roots of $x^{2n} + u^n x^n + v^n = 0$.

Therefore α and β will satisfy this equation too, which is shown below.

$$(\alpha^n)^2 + u^n \alpha^n + v^n = 0 \quad \dots(iii)$$

$$(\beta^n)^2 + u^n \beta^n + v^n = 0 \quad \dots(iv)$$

From eqs. (iii) and (iv) we can infer that α^n and β^n are the roots of $p^2 + u^n p + v^n = 0$.

$$\text{So } \alpha^n + \beta^n = -p^n \quad \dots(v)$$

$$\text{And } \alpha^n \beta^n = q^n \quad \dots(vi)$$

From the eq. (i), we have

$$(\alpha + \beta)^n = (-p)^n = p^n, \text{ as } n \text{ is even.}$$

Now, from the eq. (v), we have

$$(\alpha + \beta)^n = -(-p^n) = -(\alpha^n + \beta^n)$$

$$\Rightarrow (\alpha^n + \beta^n) + (\alpha + \beta)^n = 0 \quad \dots(vii)$$

Now dividing eq. (vii) by β^n , we get

$$\left(\frac{\alpha}{\beta}\right)^n + 1 + \left(\frac{\alpha + \beta}{\beta}\right)^n = 0$$

$$\Rightarrow \left(\frac{\alpha}{\beta}\right)^n + 1 + \left(\frac{\alpha}{\beta} + 1\right)^n = 0 \quad \dots(viii)$$

Again dividing eq. (vii) by α^n , we get

$$\left(\frac{\beta}{\alpha}\right)^n + 1 + \left(\frac{\alpha + \beta}{\alpha}\right)^n = 0$$

$$\Rightarrow \left(\frac{\beta}{\alpha}\right)^n + 1 + \left(\frac{\beta}{\alpha} + 1\right)^n = 0 \quad \dots(ix)$$

From eqs. (viii) and (ix), we can infer that $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ are

the roots of $x^n + 1 + (x + 1)^n = 0$

Hence choice (d) is the answer.

- 27 Let α and β be the two roots, such that $\alpha + \beta = \left(\frac{1}{\alpha}\right)^2 + \left(\frac{1}{\beta}\right)^2$

$$\text{That is } \alpha + \beta = \frac{1}{(\alpha)^2} + \frac{1}{(\beta)^2}$$

$$\Rightarrow \alpha + \beta = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} \quad \dots(i)$$

$$\text{Now since, } \alpha + \beta = \frac{-b}{a} \quad \dots(ii)$$

$$\text{And } \alpha\beta = \frac{c}{a} \quad \dots(iii)$$

Therefore using eqs. (ii) and (iii), the Eq. (i) becomes as following.

$$\frac{-b}{a} = \frac{\left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\left(\frac{c}{a}\right)^2}$$

$$\Rightarrow -\frac{b}{a} = \frac{b^2 - 2ac}{c^2} \Rightarrow bc^2 + ab^2 = 2ca^2$$

Therefore bc^2, ca^2, ab^2 are in A.P.

Hence choice (c) is the answer.

☞ Alternatively Let us consider the two roots 1 and 1, as they satisfy the given condition $1 + 1 = \left(\frac{1}{1}\right)^2 + \left(\frac{1}{1}\right)^2$

Therefore, you will have the following quadratic equation,

$$(x-1)(x-1) = 0$$

$$\Rightarrow x^2 - 2x + 1 = 0$$

$$\Rightarrow a = 1, b = -2, c = 1$$

$$\text{Then, } bc^2 = -2, ca^2 = 1, ab^2 = 2$$

As, -2, 1, 2 are in AP, therefore choice (c) is the valid one.

- 28 Since $f(x)$ is positive for all real values of x , it means the graph of $f(x)$ is strictly above the X -axis. That means roots are not real.

So, the best way is to consider any quadratic equation whose roots are not real. So,

I'm assuming a desired equation: $x^2 - x + 2 = 0$

Now, comparing this equation ($x^2 - x + 2 = 0$) with the given equation ($ax^2 + bx + c = 0$), we get

$$a = 1, b = -1 \text{ and } c = 2$$

$$\text{But, } g(x) = ax^2 + (2a+b)x + (2a+b+c)$$

Now, if you plug in the values of a, b and c in $g(x)$, you will get $g(x) = x^2 + x - 3$. And, you see, as the discriminant of $g(x)$ is strictly negative and coefficient of x^2 is positive, so the roots of $g(x)$ will be non-real and the graph will be strictly above the X -axis. That is $g(x) > 0$.

Hence, choice (a) is the valid one.

☞ Alternatively Since $f(x)$ is positive for all real x .

$$\text{Then, } b^2 - 4ac < 0$$

$$\text{and } a > 0 \quad \dots(i)$$

$$\text{Now } g(x) = ax^2 + (2a+b)x + (2a+b+c)$$

The discriminant of $g(x)$ is

$$D = (2a+b)^2 - 4a(2a+b+c) \Rightarrow D = (b^2 - 4ac) - 4a^2$$

But using eq. (i) we can conclude that $(b^2 - 4ac) - 4a^2 < 0$

$$\therefore D < 0$$

And from eq. (i), we already know that $a > 0$. Therefore $g(x) > 0$, for all real x .

Hence choice (a) is the answer.

- 29 Since AM of $(x-1)$ and $(x-5)$ = AM of $(x-2)$ and $(x-4)$ = $(x-3)$. So, for our convenience, we can substitute y in place of $x-3$. Therefore

$$(x-1)^3 + (x-2)^3 + (x-3)^3 + (x-4)^3 + (x-5)^3 = 0$$

$$\Rightarrow (y+2)^3 + (y+1)^3 + y^3 + (y-1)^3 + (y-2)^3 = 0$$

$$\Rightarrow [(y+2)^3 + (y-2)^3] + y^3 + [(y+1)^3 + (y-1)^3] = 0$$

$$\Rightarrow 2(y^3 + 12y) + 2(y^3 + 3y) + y^3 = 0$$

$$\Rightarrow 5y^3 + 30y = 0 \Rightarrow 5y(y^2 + 6) = 0$$

$$\Rightarrow y = 0, \pm\sqrt{-6} \Rightarrow y = 0, \pm\sqrt{6}i$$

$$\therefore x = 3, 3 \pm \sqrt{6}i$$

Hence choice (b) is the answer.

- 30 The AM of $(x+3)$ and $(x-1)$ is $(x+1)$.

So, now putting $x+1 = y$, we get the given equation in the following form.

$$(y+2)^5 - (y-2)^5 \geq 244$$

$$\Rightarrow 2(10y^4 + 80y^2 + 32) \geq 244 \Rightarrow (y^2 + 9)(y^2 - 1) \geq 0$$

$$\Rightarrow (y^2 - 1) \geq 0, \text{ since } y^2 + 9 > 0 \text{ for every real value of } y.$$

The corresponding roots for $y^2 - 1 = 0$ are -1 and 1.

So the inequation $y^2 - 1 \geq 0$ is valid only when $y \leq -1$ or $y \geq 1$.

$$\begin{array}{c} + \quad - \quad + \\ \leftarrow \quad | \quad | \quad \rightarrow \\ -1 \quad 1 \end{array}$$

$$\text{Now if } y \leq -1 \Rightarrow x \leq -2 \quad \text{And if } y \geq 1 \Rightarrow x \geq 0$$

It implies that the real solutions of the given equation is

$$x \leq -2 \text{ or } x \geq 0$$

Therefore the solution set is $(-\infty, -2] \cup [0, \infty)$

Hence choice (b) is the answer.

$$\text{Hint } (y+2)^5 = y^5 + {}^5C_1 y^4 \times (2) + {}^5C_2 y^3 \times (2^2) + {}^5C_3 y^2 \times (2^3) + {}^5C_4 y \times (2^4) + {}^5C_5 y \times (2^5)$$

$$\text{and } (y-2)^5 = y^5 - {}^5C_1 y^4 \times (2) + {}^5C_2 y^3 \times (2^2) - {}^5C_3 y^2 \times (2^3) + {}^5C_4 y \times (2^4) - {}^5C_5 \times (2^5)$$

$$\begin{aligned}
 &\text{Therefore } (y+2)^5 - (y-2)^5 \\
 &= 2[{}^5C_1 y^4 \times (2) + {}^5C_3 y^2 \times (2^3) + {}^5C_5 \times (2)^5] \\
 &\Rightarrow (y+2)^5 - (y-2)^5 \\
 &= 2[5y^4 \times (2) + 10y^2 \times (8) + 1 \times (32)]
 \end{aligned}$$

- 31** From the given information we will have the following results.

$$\text{If } \alpha + \beta = -p \text{ and } \alpha\beta = q \quad \dots(i)$$

$$\text{And } \alpha^n + \beta^n = -p^n \text{ and } \alpha^n\beta^n = q^n \quad \dots(ii)$$

Now since $\frac{\alpha}{\beta}$ is root of $x^n + 1 + (x+1)^n = 0$, so it must satisfy this equation.

$$\begin{aligned}
 \therefore \quad &\left(\frac{\alpha}{\beta}\right)^n + 1 + \left(\frac{\alpha}{\beta} + 1\right)^n = 0 \\
 \Rightarrow \quad &\frac{\alpha^n}{\beta^n} + 1 + \left(\frac{\alpha + \beta}{\beta}\right)^n = 0 \\
 \Rightarrow \quad &\frac{\alpha^n + \beta^n}{\beta^n} + \frac{(\alpha + \beta)^n}{\beta^n} = 0 \\
 \Rightarrow \quad &\frac{-p^n}{\beta^n} + \frac{(-p)^n}{\beta^n} = 0 \Rightarrow \frac{-p^n + (-p)^n}{\beta^n} = 0 \\
 \Rightarrow \quad &-p^n + (-p)^n = 0
 \end{aligned}$$

It is valid only when n is an even integer.

Hence choice (c) is the answer.

- 32** Let's assume that $a + ib$ is an imaginary root of the given equation, then the conjugate of this root $a - ib$ is also a root of this equation. Putting $x = a + ib$ and $x = a - ib$ in the given equation we get

$$\begin{aligned}
 &\frac{P_1^2}{(a+ib-p_1)} + \frac{P_2^2}{(a+ib-p_2)} + \dots + \frac{P_k^2}{(a+ib-p_k)} \\
 &= a+ib+1 \quad \dots(i)
 \end{aligned}$$

$$\begin{aligned}
 &\frac{P_1^2}{(a-ib-p_1)} + \frac{P_2^2}{(a-ib-p_2)} + \dots + \frac{P_k^2}{(a-ib-p_k)} \\
 &= a-ib+1 \quad \dots(ii)
 \end{aligned}$$

Subtracting eq. (i) from eq. (ii), we get

$$\begin{aligned}
 &2ib \left[\frac{P_1^2}{(a-p_1)^2 + b^2} + \frac{P_2^2}{(a-p_2)^2 + b^2} + \dots + \frac{P_k^2}{(a-p_k)^2 + b^2} \right] = -2ib \\
 \Rightarrow 2ib \left[\frac{P_1^2}{(a-p_1)^2 + b^2} + \frac{P_2^2}{(a-p_2)^2 + b^2} + \dots + \frac{P_k^2}{(a-p_k)^2 + b^2} \right] + 2ib = 0
 \end{aligned}$$

$$2ib \left[1 + \left\{ \frac{P_1^2}{(a-p_1)^2 + b^2} + \frac{P_2^2}{(a-p_2)^2 + b^2} + \dots + \frac{P_k^2}{(a-p_k)^2 + b^2} \right\} \right] = 0$$

Since all the values inside the bracket are positive, so the expression in the bracket cannot be zero. Therefore

$$2ib = 0 \Rightarrow b = 0 \text{ since } i \neq 0$$

$$a + ib = a \text{ and } a - ib = a$$

Therefore all the roots are real.

Hence choice (c) is the answer.

- 33** $(x-a)(x-c) + k(x-b)(x-d) = 0$

$$\Rightarrow x^2 - (a+c)x + ac + k[x^2 - (b+d)x + bd] = 0$$

$$\Rightarrow (1+k)x^2 - [(a+c) + k(b+d)]x + (ac + kbd) = 0 \quad \dots(i)$$

Let's assume that the eq. (i) will have real root, then the discriminant $D \geq 0$.

$$\text{That is } [(a+c) + k(b+d)]^2 - 4[(1+k)(ac + kbd)] \geq 0$$

$$\Rightarrow [(b+d)^2 - 4bd]k^2 + [2(a+c)(b+d) - 4ac - 4bd]k + [(a+c)^2 - 4ac] \geq 0$$

$$\Rightarrow (b-d)^2 k^2 + 2(ab + ad + bc + cd - 2ac - 2bd)k + (a-c)^2 \geq 0 \quad \dots(ii)$$

Now, the expression (ii) looks like a quadratic inequality.

In the above inequation (ii) the coefficient of k^2

[i.e., $(b-d)^2$] is positive.

Let's assume that the discriminant of the expression (ii) is J . Now,

(a) If $J > 0$, for certain values of k , expression will be positive and for certain values of k , expression will be negative. So it won't be valid for all the real values of k .

(b) If $J = 0$, for all the values of k , expression will be non-negative, so it will be valid.

(c) If $J < 0$, for all the values of k , expression will be positive, so it will be valid.

Therefore the need arises to determine the discriminant J . Then,

$$J = 4(ab + ad + bc + cd - 2ac - 2bd)^2 - 4(b-d)^2(a-c)^2$$

$$\Rightarrow J = 4(ab + ad + bc + cd - 2ac - 2bd)^2 - 4[(b-d)(a-c)]^2$$

$$\Rightarrow J = 4[(ab + ad + bc + cd - 2ac - 2bd)^2 - \{(b-d)(a-c)\}^2]$$

$$\Rightarrow J = 4\{[(ab + ad + bc + cd - 2ac - 2bd + (b-d)(a-c)) \times \{ab + ad + bc + cd - 2ac - 2bd + (b-d)(a-c)\}]\}$$

$$\Rightarrow J = 16(b-c)(a-d)(a-b)(d-c)$$

$\Rightarrow J = 16(-ve)(-ve)(-ve)(+ve)$; since $a < b < c < d \Rightarrow J < 0$
 Since the discriminant is negative and the coefficient of k^2 is positive, it means the given inequation is valid for all the real values of k . This in turn implies that the roots of the given equation are real. Hence choice (a) is the answer.

Hint Since $b < d$, so $(b - d)^2$ is positive; as the square of any negative number is always positive.

☞ Alternatively

Let $f(x) = (x - a)(x - c) + k(x - b)(x - d) = 0$

Also it is given that $a < b < c < d$.

Now, $f(b) = (b - a)(b - c) \Rightarrow f(b) < 0$

($\because b - a > 0$ and $b - c < 0$)

And $f(d) = (d - a)(d - c)$

$\Rightarrow f(d) > 0$ ($\because d - a > 0$ and $d - c > 0$)

Since $f(b)$ and $f(d)$ have opposite signs therefore the given equation has one real root between b and d .

Since one root is real and a, b, c, d, k are also real, therefore the other root of the given equation will also be real.

Thus the given quadratic equation

$(x - a)(x - c) + k(x - b)(x - d) = 0$ has both the real roots.

34 Since the discriminant of the quadratic equation $x^2 - bx - c = 0$ is $b^2 + 4c$, which is positive, so there will be two real roots, say α and β .

The product of the roots $= \alpha\beta = -c$, so one root will be negative and one root will be positive.

Then the three coordinates will be

$A(\alpha, 0)$, $B(\beta, 0)$ and $C(0, c)$.

Therefore the line segment $AO = -\alpha$

(as the root is negative), $BO = \beta$ and $CO = c$.

As we know that in a circle, with two intersecting chords the product of the segments is

equal. That is $AO \times BO = CO \times DO$.

$\therefore -\alpha \times \beta = c \times DO$

$\Rightarrow c = c \times DO$

$\Rightarrow DO = 1$

Since D lies on the Y-axis, so the coordinates of D are $(0, 1)$, which is coincidentally independent of both b and c .

Hint When you put $x = 0$ in $y = x^2 - bx - c$, you will get $y = OC$

CHAPTER 15

Set Theory

It is one of the easiest and most interesting chapters. Almost every year 2-3 questions appear in CAT from this chapter, either in QA or in DI/DS section. The important feature of this chapter is Venn diagrams which play a vital role in the logical reasoning. That's why this chapter is interesting, logical and important in CAT, XLRI, FMS, CET etc. Since we have to solve most of the problems through Venn diagrams, therefore the whole emphasis is given on the theory and concepts of Venn diagrams. Formulae and algebraic theory is not important particularly in CAT, still we have to know some fundamentals of sets and their applications.

15.1 Set

A well defined collection of objects is called a set. The objects in a set are called its members or elements. If a is an element of a set A , then we write $a \in A$ (and say a belongs to A) i.e., ' a ' is a member of ' A '. If a does not belong to A , then we write $a \notin A$. By convention the sets are denoted by capital alphabets e.g., $A, B, C, \dots X, Y, Z$ etc and elements of a set are denoted by $a, b, c, \dots, x, y, z$ etc.

Important Notations of Standard Sets

N :	the set of natural numbers	Z :	the set of integers
Z^+ :	the set of positive integers	Q :	the set of all rational numbers
Q^+ :	the set of all positive rational numbers	R :	the set of real numbers
R^+ :	the set of all positive real numbers	C :	the set of all complex numbers.

15.2 Methods of Representation of Sets

1. Roster Method

In this method a set is described by listing all the elements, separated by commas, within braces $\{ \}$, e.g.

- The set of perfect squares upto 100 may be described as $\{0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$
- The set of all the days of a week may be described as $\{\text{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}\}$.

 **NOTE** The order of elements (i.e., position of elements) is not important. Hence any element can be written anywhere within braces $\{ \}$.

2. Set Builder Method

In this method, a set is described by listing its property in the given manner.

The set $P(x)$ is described as $P = \{x : P(x) \text{ holds}\}$ or $\{x \mid P(x) \text{ holds}\}$

Chapter Checklist

- Set
- Methods of Representation of Sets
- Types of Sets
- Venn Diagrams
- Operations on Sets
- Algebraic Laws of Sets
- Important Results on Number of Elements in Sets
- Problems Based on Venn Diagrams
- Concept of Maximum or Minimum
- CAT Test

or $P = \{x : x \text{ has the property } abc \dots\}$

which is read as 'the set of all x such that $P(x)$ holds'. The symbol ' $\{$ ' or ' $:$ ' is read as 'such that', e.g.,

1. The set of perfect squares upto 100 can be written as
 $P = \{x : x \text{ is a perfect square and } 0 \leq x \leq 100, x \in N\}$
or $P = \{x : x \in N, x = n^2, 0 \leq x < 100, n \in Z\}$
or $P = \{x^2 : x \in Z\}$
2. The set of all the odd natural numbers can be expressed as $A = \{x : x \in N, x = 2n - 1, n \in N\}$

15.3 Types of Sets

Singleton Set : A set consisting of a single element is called a singleton set.

- e.g. 1. $\{\text{cat}\}$, $\{7\}$ etc. 2. $\{x : x + 5 = 7\} = \{2\}$
3. $\{0\}$ is a singleton set.

Empty Set : A set containing no element at all is called an empty set or a null set or a void set and is denoted by ϕ . In set builder form it is expressed as $\phi = \{\}$

- e.g. 1. $\{x : x \in N, 9 < x < 10\} = \phi$ 2. $\{x : x \in R, x^2 = -8\} = \phi$

NOTE A set containing atleast one element is called a non-empty or non-void set.

Finite Set : If a set has finite number of elements that can be counted or listed i.e., the counting of elements surely comes to an end, is called a finite set, e.g.,

1. Set of all the management institutes in the world.
2. Set of all the recognised countries in the world.

Cardinal Number : The number of distinct elements in a finite set is called the 'cardinal number' of the set e.g., $n(A) \rightarrow n$ is the number of distinct elements in the set A .

Infinite Set : If the number of elements of a set is infinite i.e., which can't be counted by natural numbers, is called infinite set in other words if a set is not finite then it must be an infinite set, e.g.,

1. Set of all points on the arc of a circle
2. Set of all concentric circles with a given centre
3. Number of all lines in a particular plane.
4. $\{x : x \in R, 1 < x < 2\}$ and $\{x : x \in I, x < 10\}$

Equivalent Sets : Two finite sets A and B are equivalent if their cardinal numbers are same i.e. $n(A) = n(B)$

- e.g. $A = \{a, e, i, o, u\}$ and $B = \{1, 3, 5, 7, 9\} \Rightarrow n(A) = n(B)$

Equal Sets : Two sets A and B are said to be equal, if every element of A is in B and every element of B is in A and it is denoted as $A = B$. When the elements of A and B are distinct, then $A \neq B$ i.e., A and B are not equal e.g.,

1. $A = \{a, e, i, o, u\}$ and $B = \{e, o, i, a, u\} \Rightarrow A = B$
2. $A = \{p, q, r\}$, $B = \{q, p, r\}$ and $C = \{r, q, p\}$
 $\Rightarrow A = B = C$

$$3. A = \{C, A, T\}, B = \{C, A, T, A\}$$

$$C = \{A, C, A, A, T\} \Rightarrow A = B = C$$

Remark

1. Order of elements in the list of elements is not important i.e., meaningless.
2. The repetition of elements in a set is meaningless.
3. $\phi \neq \{0\} \neq 0$, since ϕ is a set containing no element at all, $\{0\}$ is a singleton set and 0 is not a set.

Remember that equivalent sets are not always equal but equal sets are always equivalent. Thus $\{1, 2, 3, 4\}$ and $\{W, X, Y, Z\}$ are equivalent but not equal.

Subsets : Let A and B be two sets such that every element of A is the element of B (i.e., every element of A is in B), then A is called a subset of B and is denoted as $A \subseteq B$ (Read it as- A is a subset of B)

Superset : If $A \subseteq B$, we say that B is a superset of A and we write $B \supseteq A$. Read it as- B is the superset of A .

- e.g. $B = \{2, 4, 6, 8, 10\}$, $A = \{4, 8\} \Rightarrow B \supseteq A$

Proper Subset : If $A \subseteq B$ and $A \neq B$, then A is called a proper subset of B and denoted as $A \subset B$ e.g.,

1. $\{5, 15, 25\} \subset \{5, 10, 15, 20, 25\}$
2. $N \subset W \subset I \subset Q \subset R$

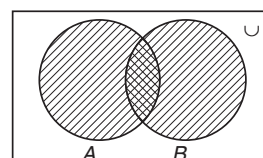
Comparable Sets : Two sets A and B are said to be comparable if either $A \subseteq B$ or $B \subseteq A$.

Results on Subsets

1. The empty set ϕ is a subset of every set.
2. Every set is a subset of itself.
3. The number of all possible subsets of a set containing n elements is 2^n .
4. The number of all proper subsets of a set containing n elements is $(2^n - 1)$.
5. The set of all subsets of a given set A is called the power set of A , denoted by $P(A)$. Thus, if A has n elements $P(A)$ has 2^n elements.

Universal Set : A set that contains all sets in a given context is called the universal set e.g.,

1. In a two dimensional coordinate geometry, the set of all points in x - y plane is the universal set.
2. Sets containing natural numbers have a universal set N .
3. If $A = \{1, 2, 4\}$, $B = \{2, 2, 4, 6\}$,
 $C = \{1, 2, 5, 6, 7\}$, then $U = \{1, 2, 3, 4, 5, 6, 7\}$



Power Set : The collection or family of all the subsets of a set A is called the power set of A and is denoted by $P(A)$.

NOTE The power set is always non-empty, because the empty set and the given set itself are the members or elements of the power set.

$A = \{1, 2, 3\}$, then the number of subsets of the $P(A)$ will be $2^3 = 8$
 $P(A) = \{\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}$

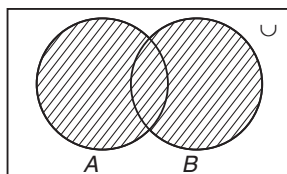
15.4 Venn Diagrams

First of all Swiss mathematician Euler elucidated the idea of representing the sets by points in a closed curve. Later on British mathematician Venn brought this into practice. That's why this whole concept is known as Euler-Venn diagram or simply Venn diagrams. In Venn diagrams the universal set U is represented by points within a rectangle and its subsets are represented by points in closed curves (usually circles) within a rectangle.

15.5 Operations On Sets

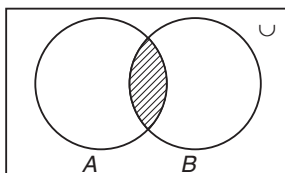
- 1. Union of Sets :** If A and B be two sets, then the union of A and B is the set of all those elements which belong either to A or to B or to both A and B . It is denoted as $A \cup B$ (read as ' A union B ') e.g.

$$A = \{2, 3, 5\}, \quad B = \{1, 2, 4, 5\}, \quad A \cup B = \{1, 2, 3, 4, 5\}$$



[The shaded region is representing the union of sets]

- 2. Intersection of Sets :** If A and B be two sets then the intersection of A and B is the set of all those elements that belong to both A and B . It is denoted by $A \cap B$ (read as " A intersection B ")



[The shaded region is representing the intersection of sets]

$$\text{e.g. } A = \{3, 6, 9, 12, 15, 18\}, B = \{4, 6, 8, 10, 12, 14, 16, 18\}$$

$$\therefore A \cap B = \{6, 12, 18\}$$

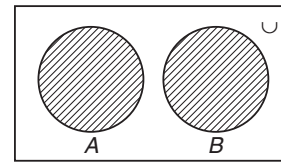
Disjoint Sets : Two sets A and B are said to be disjoint, if $A \cap B = \emptyset$. If $A \cap B \neq \emptyset$ then A and B are said to be intersecting or overlapping sets.

$$\text{e.g. } A = \{1, 3, 5, 7, 9\} \quad B = \{2, 4, 6, 8, 10\}, C = \{4, 8, 12\}$$

$$\text{Here } A \cap B = \emptyset \text{ and } B \cap C \neq \emptyset$$

$$\therefore A \text{ and } B \text{ are disjoint sets}$$

$\therefore B$ and C are intersecting sets.



Sets A and B have no elements common.

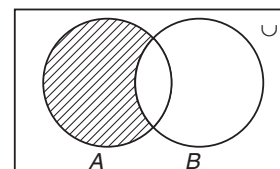
Difference of Sets : The difference of set A and set B (written as $A - B$) is the set of all those elements of A which do not belong to B . Thus, $A - B = \{x : x \in A \text{ and } x \notin B\}$

$$\text{or } A - B = \{x \in A : x \notin B\}$$

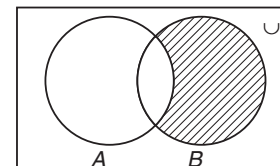
$$\text{Clearly } x \in A - B \Leftrightarrow x \in A \text{ and } x \notin B$$

$$\text{e.g. If } A = \{1, 2, 3, 4, 5, 6\}, \quad B = \{2, 4, 6, 8, 10\}$$

$$A - B = \{1, 3, 5\} \text{ and } B - A = \{8, 10\}$$

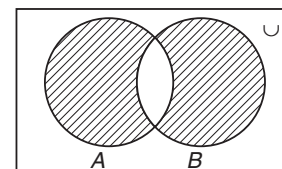


$(A - B)$ is represented by the shaded region



$(B - A)$ is represented by the shaded region

Symmetric difference of two Sets : The symmetric difference of sets A and B is the set $(A - B) \cup (B - A)$ and is denoted by $A \Delta B$

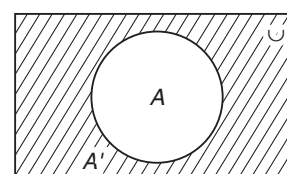


$$\text{Thus } A \Delta B = (A - B) \cup (B - A) = \{x : x \notin A \cap B\}$$

Complement of a Set : If U be the universal set and a set A is such that $A \subset U$ then, the complement of A with respect to U is denoted by A' or A^c or $U - A$ and is defined by the set of all those elements of U which are not in A .

$$\text{Thus } A' = \{x \in U : x \notin A\}$$

$$\text{Clearly } x \in A' \Leftrightarrow x \notin A$$



$[A'$ is shown by the shaded region]

e.g. $U = \{2, 3, 5, 7, 11, 13, \dots\}$, $A = \{2\}$

$$A' = U - A = \{3, 5, 7, 11, 13, \dots\}$$

15.6 Algebraic Laws of Sets

- Idempotent Laws** : For any set A , we have
 - $A \cup A = A$
 - $A \cap A = A$
- Identity Laws** : For any set A , we have
 - $A \cup \phi = A$
 - $A \cap U = A$
- Commutative Laws** : For any two sets A and B , we have
 - $A \cup B = B \cup A$
 - $A \cap B = B \cap A$
- Associative Laws** : If A , B and C are any three sets, then
 - $(A \cup B) \cup C = A \cup (B \cup C)$
 - $A \cap (B \cap C) = (A \cap B) \cap C$
- Distributive Laws** : If A , B and C are any three sets, then
 - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
 - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- De-Morgan's Laws** : If A and B are any two sets, then
 - $(A \cup B)' = A' \cap B'$
 - $(A \cap B)' = A' \cup B'$

15.7 Important Results on Operations on Numbers

- $A - B = A \cap B'$
- $B - A = B \cap A'$
- $A - B = A \Leftrightarrow A \cap B = \phi$
- $(A - B) \cup B = A \cup B$

$$5. (A - B) \cap B = \phi$$

$$6. A \subseteq B \Leftrightarrow B' \subseteq A'$$

$$7. (A - B) \cup (B - A) = (A \cup B) - (A \cap B)$$

15.8 Important Results on Number of Elements in Sets

If A , B and C are finite sets, and U be the finite universal set then

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B) = n(A) + n(B) \Leftrightarrow A, B$ are disjoint non-void sets.
- $n(A - B) = n(A) - n(A \cap B)$ i.e.,
 $n(A - B) + n(A \cap B) = n(A)$
- $n(A \Delta B) =$ Number of elements which belong to exactly one of A or $B = n(A) + n(B) - 2n(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- number of elements in exactly two of the sets A, B, C
 $= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$
- number of elements in exactly one of the sets A, B, C
 $= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$
- $n(A' \cup B') = n[(A \cap B)'] = n(U) - n(A \cap B)$
- $n(A' \cap B') = n[(A \cup B)'] = n(U) - n(A \cup B)$

Introductory Exercise 15.1

- If $A = \{S\}$, which of the following is a correct statement ?
 (a) $S \in A$ (b) $S \subset A$ (c) $\{S\} \in A$ (d) $A = S$
- Consider the following statements :
 1. Any set A is comparable with itself
 2. $\{0\}$ is a singleton set
 3. $\{\phi\}$ is an empty set.
 Of these statements, the correct ones are :
 (a) 1 and 2 (b) 2 and 3
 (c) 1 and 3 (d) 1, 2 and 3
- The total number of elements in the power set of a set A containing n elements is :
 (a) n^2 (b) 2^n
 (c) $2^n - 1$ (d) none of these
- Let $A = \{3, \pi, \sqrt{2}, -5, 3 + \sqrt{7}, 2/7\}$. The subset of A containing all the elements from it which are irrational numbers is :
 (a) $\{\pi, \sqrt{2}, 3 + \sqrt{7}\}$ (b) $\{3, \pi, 2/7, -5, 3 + \sqrt{7}\}$
 (c) $\{3, 2/7, -5\}$ (d) $\{3, -5\}$
- The set $\{x : (x - 3)(x - 5) > 0\}$ is equal to :
 (a) $\{x : 3 < x < 5\}$ (b) $\{x : x < 3\} \cup \{x : x < 5\}$
 (c) $\{x : x < 3\} \cup \{x : x > 5\}$ (d) none of these
- If A is the set of all integral multiples of 3 and B is the set of all integral multiples of 5, then $A \cap B$ is the set of all integral multiples of :
 (a) $3 + 5$ (b) $5 - 3$
 (c) $\text{GCD}(3, 5)$ (d) $\text{LCM}(3, 5)$
- If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n - 1) : n \in N\}$, then precisely :
 (a) $X \subset Y$ (b) $X \subseteq Y$
 (c) $X = Y$ (d) $X \supseteq Y$
- The number of elements of the set $\{x : x \in N, x^2 = 1\}$, where N is the set of all natural numbers is :
 (a) 2 (b) 1 (c) 0 (d) 3
- If $A = \{1, 2, 3, 4\}$ and $B = \{5, 6, 7\}$, then $A \cap B$ is :
 (a) $\{1, 2, 3\}$ (b) $\{5, 6, 7\}$
 (c) $\{4\}$ (d) ϕ

10. If $A = \{1, 2, 3, \dots, 9\}$, $B = \{2, 4, 6, 7, 8\}$ and $C = \{3, 4, 5, 8, 9, 10\}$, then $(A - B) \cup C$ is :
 (a) $\{1, 3, 4, 5, 8, 9, 10\}$ (b) $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
 (c) $\{2, 4, 6, 7, 8\}$ (d) $\{1, 3, 4, 5, 8, 9\}$
11. $A \cup B = A$, then :
 (a) $A \subset B$ (b) $B \subset A$
 (c) $A \not\subset B$ and $B \not\subset A$ (d) none of these
12. If A has 3 elements and B has 6 elements, then the minimum number of elements in $A \cup B$ is :
 (a) 3 (b) 6
 (c) 9 (d) 18
13. Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ be the universal set and $A = \{2, 4, 6\}$, $B = \{1, 3, 7\}$, then $A^c \cap B^c$ is equal to :
 (a) $\{2, 4, 5, 6, 8, 9, 10\}$ (b) $\{1, 3, 5, 7, 8, 9, 10\}$
 (c) X (d) $\{5, 8, 9, 10\}$
14. If A, B, C are any three sets, then :
 (a) $A - (B \cup C) = (A - B) \cup (A - C)$
 (b) $A - (B \cup C) = (A - B) \cap (A - C)$
 (c) $A - (B \cap C) = (A - B) \cup (A - C)$
 (d) $A - (B \cup C) = (A \cup B) - (A \cup C)$
15. Consider the following statements :
 For any two sets A and B
 (1) $(A - B) \cup B = A$ (2) $(A - B) \cup A = A$
 (3) $(A - B) \cap B = \phi$ (4) $A \subseteq B \Rightarrow A \cup B = B$
 Of these statements :
 (a) 1, 2, 3 are correct (b) 2, 3, 4 are correct
 (c) 1, 3, 4 are correct (d) 1, 2, 4 are correct
16. If X and Y are two sets and X' denotes the complement of X , then $X \cap (X \cup Y)$ equals :
 (a) X (b) Y (c) ϕ (d) none
17. If $B \subseteq A$, which of the following statements is correct?
 (a) $A \cup B = B \Rightarrow A = B$ (b) $A \cap B = B \Rightarrow A \supseteq B$
 (c) $A - B = A \Rightarrow A = B$ (d) $B - A = B \Rightarrow A = B$
18. If A, B and C be any three sets, then $A \cup (B \cap C)$ is the same as :
 (a) $(A \cap C) \cup (B \cap C)$ (b) $(A \cup B) \cap (A \cup C)$
 (c) $(A \cup B) \cap (B \cup C)$ (d) $(A \cup C) \cap (B \cup C)$
19. $A - (B \cup C)$ equals :
 (a) $(A - B) \cup (A - C)$ (b) $(A - B) \cap (A \cap C)$
 (c) $(A - B) \cup C$ (d) $(A - C) \cup B$

15.9 Problems Based on Venn Diagrams

Venn Diagram Containing Two Circles

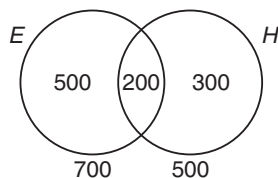
Exp. 1) In a group of 1000 people 700 can speak English and 500 can speak Hindi. If all the people speak atleast one of the two languages, find :

- (a) how many can speak both the languages?
 (b) how many can speak exactly one language?

Solution

$$(a) \quad n(E \cup H) = n(E) + n(H) - n(E \cap H)$$

$$1000 = 700 + 500 - n(E \cap H)$$



$$\Rightarrow n(E \cap H) = 200$$

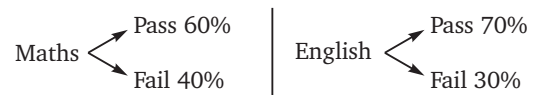
(b) People speaking exactly one language

$$n(E \cup H) - n(E \cap H) = 1000 - 200 = 800$$

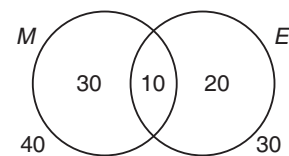
☞ **Alternatively** People speaking exactly one language
 $= 500 + 300 = 800$

Exp. 2) In an exam 60% of the candidates passed in Maths and 70% candidates passed in English and 10% candidates failed in both the subjects. If 300 candidates passed in both the subjects. The total number of candidates appeared in the exam, if they took test in only two subjects viz-Maths and English.

Solution



Fail in both subjects = 10%



Failed number of candidates

$$\therefore \text{Total number of candidates failed in atleast one subject} \\ = 30 + 10 + 20 = 60\%$$

$$\therefore \text{Total number of candidates passed in both the subjects} \\ = 100 - 60 = 40\%$$

Let total number of candidates appeared in exam = x

$$\therefore \frac{40x}{100} = 300$$

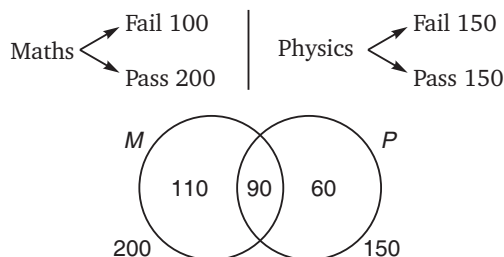
$$\Rightarrow x = 750$$

Hence, total 750 candidates appeared in exam.

Exp. 3) A class took two tests, one in Physics and other in Maths. $\left(\frac{1}{3}\right)^{rd}$ class failed in Maths and $\frac{1}{2}$ class failed in

Physics. 60% of those passed in Physics also passed in Maths. If 300 students took both the tests then how many students failed in both?

Solution

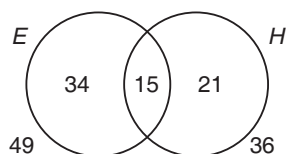


Total number of students passed in atleast one subject
 $= 110 + 90 + 60 = 260$

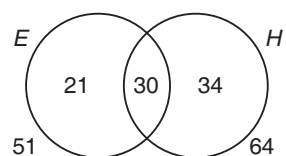
Therefore number of students failed in both the subjects
 $= 300 - 260 = 40$

Exp. 4) In an exam 49% candidates failed in English and 36% failed in Hindi and 15% failed in both subjects. If the total number of candidates who passed in English alone is 630. What is the total number of candidates appeared in exam?

Solution



Number of candidates failed in atleast one subject
 $= 34 + 15 + 21 = 70\%$



$\therefore$ Number of candidates passed in both the subjects
 $= 100 - 70 = 30\%$

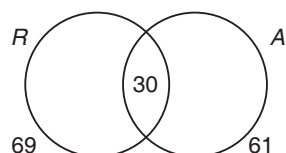
Number of candidates passed in English only
 $= 51 - 30 = 21\%$

Let the total number of candidates be x , then $\frac{21x}{100} = 630$

$$\Rightarrow x = 3000$$

Alternatively Since 21% candidates failed in Hindi only it means they are pass in only English. Hence, number of candidates who are passed only in English = 21%

Exp. 5) In a survey it was found that 69% of Indians like romantic movies and 61% like action movies. What percentage of Indians like both type of movies?

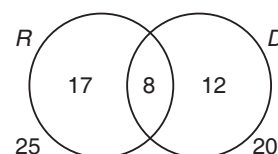
Solution $n(R \cap A) = (69 + 61) - 100$ 

NOTE In general we cannot determine the solution but this can be done if we assume that all the Indians like atleast one of these two types of movies.

Exp. 6) In a certain city only two cellular (mobile) phone services are available. One is Radiance and other is DSML. It is known that 25% of the city population uses the services of Radiance and 20% uses the services of DSML, while 8% uses both Radiance and DSML. It is also known that 30% of those who uses Radiance services but not services of DSML have prepaid cards and 40% of those who uses DSML but not Radiance have prepaid cards while 50% of those who uses the services of both Radiance and DSML have prepaid cards. What percentage of the population have prepaid cards?

(a) 12.5% (b) 10% (c) 13.9% (d) 20%

Solution

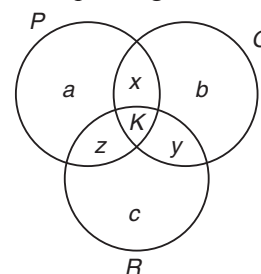


$\therefore$ Percentage of people have prepaid cards

$$= (30\% \text{ of } 17) + (40\% \text{ of } 12) + (50\% \text{ of } 8) = 13.9\%$$

Venn Diagrams Containing Three Circles

Explanation of the concept using an example

Let P, Q, R be three books, then

$a \rightarrow$ number of people reading only P

$b \rightarrow$ number of people reading only Q

$c \rightarrow$ number of people reading only R

$x \rightarrow$ number of people reading P and Q both but not R

$y \rightarrow$ number of people reading Q and R both but not P

$z \rightarrow$ number of people reading P and R both but not Q

$k \rightarrow$ number of people reading all the three books

$x + k \rightarrow$ number of people reading P and Q both

$y + k \rightarrow$ number of people reading Q and R both

$z + k \rightarrow$ number of people reading P and R both

$a + x + k + z \rightarrow$ number of people reading P

$b + x + k + y \rightarrow$ number of people reading Q

$c + y + k + z \rightarrow$ number of people reading R

$a + x + b \rightarrow$ number of people reading P or Q but not R

$b + y + c \rightarrow$ number of people reading Q or R but not P

$c + z + a \rightarrow$ number of people reading P or R but not Q .

$a + b + c \rightarrow$ number of people reading only one book

$x + y + z \rightarrow$ number of people reading only two books

$k \rightarrow$ number of people reading all of three books

$(a + b + c) + (x + y + z) + (k) \rightarrow$ number of people reading atleast one book
 $(x + y + z) + (k) \rightarrow$ number of people reading atleast two books
 $(a + b + c) + (x + y + z) + (k) \rightarrow$ total number of readers.

Finding the Values of Components

$$P + Q + R = (a + x + k + z) + (b + x + k + y) + (c + y + k + z)$$

$$P + Q + R = (a + b + c) + 2(x + y + z) + 3k$$

$$P + Q + R = \alpha + 2\beta + 3\gamma$$

where $\alpha = a + b + c$, $\beta = x + y + z$, $\gamma = k$

Again $(x + k) + (y + k) + (z + k) = (x + y + z) + 3k = \beta + 3\gamma$

$$\therefore (\alpha + 2\beta + 3\gamma) - (\beta + 3\gamma) = \alpha + \beta$$

and $(\alpha + \beta + \gamma) - (\alpha + \beta) = \gamma$

NOTE 'And' means intersection, 'or' means union

Exp. 1) In a group of 100 children, 32% play hockey, 64% play football and 40% play cricket. 14% play hockey and football, 15% play football and cricket, 13% play hockey and cricket. Only 6% play all the three games.

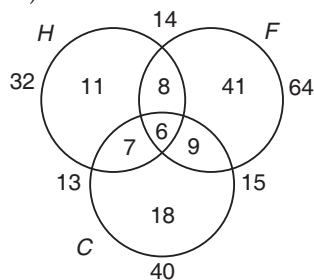
(a) How many children play only one game?

(b) How many children play atleast two game?

(c) How many children play hockey or football but not cricket?

Solution (a) $11 + 41 + 18 = 70$ (b) $(8 + 7 + 9) + 6 = 30$

(c) $(11 + 41) + 8 = 60$



Exp. 2) In a hostel of 100 students capacity, 42 are science graduate, 34 are law graduate and 58 are commerce graduate. 8 students are science and law graduate, 13 students are law and commerce graduate, 18 students are commerce and science graduate. How many students are only science graduate?

Solution $\alpha + \beta + \gamma = 100$; $\alpha + 2\beta + 3\gamma = 42 + 34 + 58 = 134$

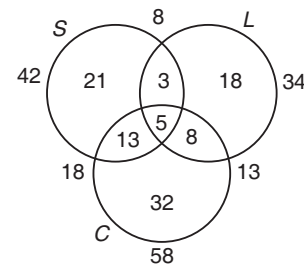
$$\beta + 3\gamma = 8 + 13 + 18 = 39$$

$$\alpha + \beta = (\alpha + 2\beta + 3\gamma) - (\beta + 3\gamma) = 134 - 39 = 95$$

$$\therefore \gamma = (\alpha + \beta + \gamma) - (\alpha + \beta) \Rightarrow \gamma = 100 - 95 = 5$$

$\therefore$ Number of students who are only science graduate

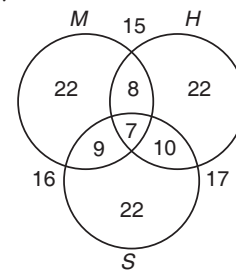
$$= 42 - [(18 + 8) - 5] = 42 - (13 + 3 + 5) = 21$$



Exp. 3) In a premier B-school having 100 students in the MBA final year, 15 students specialize in Marketing and HR. 17 students specialize in HR and Systems, 16 students specialize in Marketing and Systems. The number of students specialize in only one field is equal and is 22 in each field of specialization. How many students specialize in all three fields viz- Marketing, HR and Systems?

Solution $\alpha = a + b + c = 22 + 22 + 22 = 66$

$$\therefore \beta + \gamma = 100 - \alpha = 100 - 66 = 34$$



Again $\beta + 3\gamma = x + y + z + 3k = 15 + 16 + 17 = 48$

$$\therefore (\beta + 3\gamma) - (\beta + \gamma) = 2\gamma = 48 - 34 = 14 \Rightarrow \gamma = 7$$

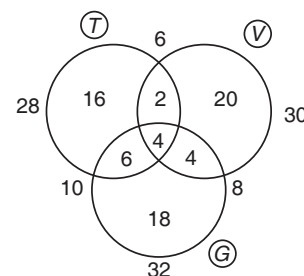
Hence, 7 students specialize in all the three fields.

Exp. 4) In a music school 28 students learn trumpet, 30 students learn violin and 32 students learn guitar. 6 students learn trumpet and violin, 8 students learn violin and guitar, 10 students learn guitar and trumpet. The number of students who learn only one instrument is 54. Also 20 students learn only violin. Every student learn atleast one instrument out of the three instruments.

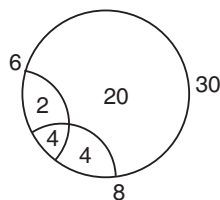
(a) Find the number of students in the music school.

(b) Find the number of students who learn trumpet and guitar but not violin?

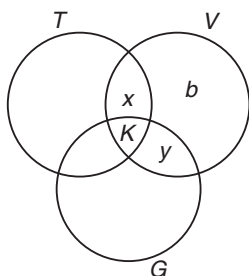
Solution



$$\begin{aligned}
 x + b + k + y &= 30 \\
 b &= 20 \\
 \Rightarrow x + k + y &= 10 \\
 \text{But, } (x + k) + (y + k) &= 14 \\
 \Rightarrow (x + y + k) + k &= 14 \\
 \Rightarrow 10 + k &= 14 \\
 \Rightarrow k &= 4
 \end{aligned}$$



- (a) Number of students in music school
 $= (16 + 20 + 18) + (2 + 4 + 6) + 4 = 70$
- (b) Number of students who learn trumpet and guitar but not violin = 6



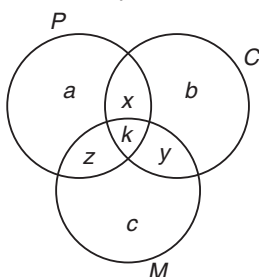
Exp. 5) Out of 180 students in a class 7 students failed in all the three subjects viz., Physics, Chemistry, Maths. Only 144 students pass in only one subject and 21 students pass in only two subjects.

- (a) How many students passed in all the three subjects?
- (b) How many students passed in atleast two subjects?
- (c) How many students passed in at most two subjects but atleast in one subject?

Solution $\alpha = a + b + c = 144 \Rightarrow \beta = x + y + z = 21$

$$\begin{aligned}
 \alpha + \beta + \gamma &= 180 - 7 = 173 \quad \therefore \gamma = (\alpha + \beta + \gamma) - (\alpha + \beta) \\
 \Rightarrow \gamma &= (173) - (144 + 21) \Rightarrow \gamma = 8 (\gamma = k)
 \end{aligned}$$

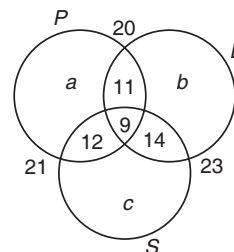
- (a) 8
- (b) Atleast two subjects $= \beta + \gamma = 21 + 8 = 29$
- (c) Atleast one subject but atmost two subjects.
 $= \alpha + \beta = 144 + 21 = 165.$



Exp. 6) In a marriage party total 278 guests were present. 20 guests took Pepsi and Dew, 23 guests took Dew and Sprite and 21 guests took Pepsi and Sprite and 9 guests took all the three cold drinks viz. Pepsi, Sprite and Dew. It is also known that there were equal number of bottles of each of three kinds viz. Pepsi, Dew and Sprite

- (a) How many guests took only one bottle cold drink?
- (b) How many guests enjoyed Dew or Sprite but not Pepsi?
- (c) How many guests enjoyed Pepsi and Dew but not Sprite?

Solution Number of guests $= (a + b + c) + (11 + 12 + 14) + (9)$
 $= 278$



Number of bottles $= (a + b + c) + 2(11 + 12 + 14) + 3(9)$
 $= [(a + b + c) + (11 + 12 + 14) + (9)] + (11 + 12 + 14) + 2(9)$
 $= 278 + 55 = 333$

$$\therefore \text{Each kind of bottles} = \frac{333}{3} = 111$$

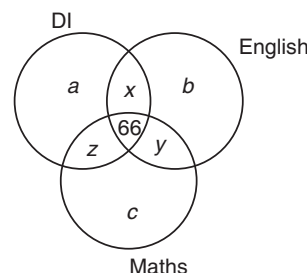
$$\begin{aligned}
 \therefore a &= 111 - (11 + 9 + 12) = 79 \\
 b &= 111 - (11 + 9 + 14) = 77 \\
 c &= 111 - (12 + 9 + 14) = 76
 \end{aligned}$$

- (a) Number of guests who took only one bottle $= a + b + c$
 $= (79 + 77 + 76) = 232$
- (b) Dew or Sprite but not Pepsi $= b + 14 + c$
 $= 77 + 14 + 76 = 167$
- (c) Dew and Pepsi but not Sprite $= 20 - 9 = 11$

Exp. 7) 250 students appeared in MOCK CAT consisting of three sections viz. Maths, Data Interpretation (DI) and English. 20 students did not qualify any of the three sections and 66 students qualified all the three sections, 24 students did not qualify DI only. 12 did not qualify DI and Maths only.

Those who qualified Maths only exceeded those who did not qualify any of the section by 16. Those who did not qualify Maths only exceeded those who did not qualify only DI and Maths by 8. Those who did not qualify English only were twice the number of students who did not qualify Maths only. Find the number of students who has qualified atleast two sections.

Solution Total students = 250



Number of students who did not qualify any of the three sections = 20

$\therefore$ Number of students who qualified atleast one section = $250 - 20 = 230$

Hint $y = 24$ (It means 24 students must have qualified English and Maths) $b = 12$ (It means they have qualified neither DI nor Maths but English only)

$$c = 36 = (20 + 16) \Rightarrow x = 20 = (12 + 8) \Rightarrow z = 40 = (20 \times 2)$$

$$\therefore (a + b + c) + (x + y + z) + 66 = 230$$

$$(a + 12 + 36) + (20 + 24 + 40) + 66 = 230 \Rightarrow a = 32$$

$$\text{but the required answer} = (x + y + z) + 66 = 84 + 66 = 150$$

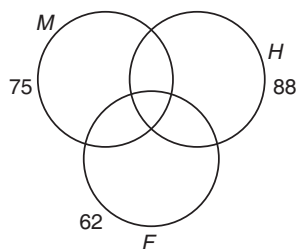
15.10 Concept of Maxima and Minima

Type 1. In a Business School there are 3 electives and atleast one elective is compulsory to opt. 75% students opted for Marketing, 62% students opted for Finance and 88% students opted for HR. A student can have dual or triple specialization.

- What is the minimum number of students that are specialize in all three streams?
- What is the maximum number of students that can specialize in all three streams?

Solution

- 75% students opted for Marketing, it means atleast 25% students opted for HR or Finance or HR and Finance both.

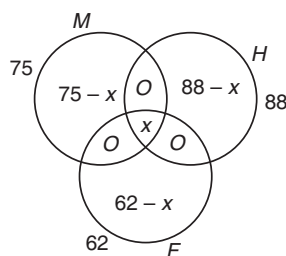


Similarly atleast 12% students opted for Marketing or Finance or Marketing and Finance both. Again atleast 38% students opted for Marketing or HR or Marketing and HR both.

$$\therefore 25 + 12 + 38 = 75\%$$

It means that if there is no intersection in these three courses, 75% would be maximum number of students in M , H , F alone or $(M \cap H)$, $(M \cap F)$, $(H \cap F)$. Thus it gives atleast 25% = $(100 - 75)$ students in all 3 departments.

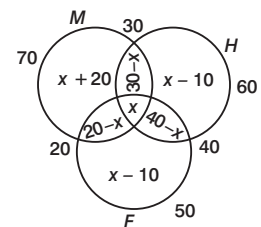
- $(75 - x) + (88 - x) + (62 - x) + x = 100 \Rightarrow x = 62.5\%$



Type 2. In a Business school there are 3 electives offered to the students, where students have a choice of not choosing any electives. 70% students opted for Marketing, 60% students opted for HR and 50% students opted for Finance. 30% students opted for Marketing and HR both, 40% opted for HR and Finance both, 20% opted for Marketing and Finance both.

- What is the minimum number of students opted for all 3 electives?
- What is the maximum number of students opted for all 3 electives?

Solution Total number of students = $70 + (x - 10) + (40 - x) + (x - 10) = 90 + x$



- The minimum value of x so that none of the categories becomes negative = 10
- The maximum value of x so that none of the categories becomes negative = 20

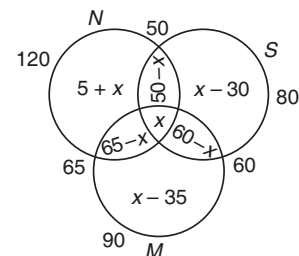
Type 3. In a hostel there are 250 students. 120 watch news channels, 80 watch sports channels and 90 watch music channels. Each channel telecasts the content as per their nature. 50 students watch both news and sports channel 60 students watch both sports and music channel and 65 students watch both news and music channels.

- What is the minimum number of students who watch atleast one of the given channel?
- What is the maximum number of students who watch atleast one of the given channels?
- What is the minimum number of students who watch all the three types of channels?
- What is the maximum number of students who watch all the three types of channels?

Solution Since the number of students cannot be negative

$$\therefore \begin{cases} x - 30 \geq 0 \\ x - 35 \geq 0 \end{cases}$$

$$\text{Also } \begin{cases} 50 - x \geq 0 \\ 60 - x \geq 0 \\ 65 - x \geq 0 \end{cases}$$



Total number of students playing atleast one game

$$= 120 + (x - 30) + (60 - x) + (x - 35) = 115 + x$$

- Minimum number of students watching atleast one channel = $115 + 35 = 150$
- Maximum number of students watching atleast one channel = $115 + 50 = 165$
- Minimum number of students watching all the 3 channels = 35
- Maximum number of students watching all the 3 channels = 50

CAT-Test

Questions Helping you bell the CAT

LEVEL 01 > BASIC LEVEL EXERCISE

- 1 In a city 45% of the people read English and Hindi newspapers. 15% read only Urdu newspaper and 75% read Hindi newspaper. If nobody reads all the three newspapers and everybody read atleast one newspaper out of English, Hindi and Urdu newspapers, how many read Urdu newspaper?
- (a) Max 45% (b) Max. 55%
(c) Min 25% (d) Min 30%

Directions (for Q. Nos. 2 to 5) *In our coaching institute there are total 170 students and they use different vehicles for transportation viz. bike, car and taxi.*

The ratio of students using all 3 vehicles to students using atleast 2 vehicles is 2 : 9. The ratio of students using only one vehicle to students using atleast 2 vehicles is 8 : 9.

Number of students using car only exceeds number of students using bike only by 14.

Number of students using taxi only exceeds number of students using bike only by 12. Number of students using taxi, bike, car is 90, 93, 97 respectively.

- 2 Number of students using all three vehicles is :
(a) 18 (b) 12
(c) 20 (d) none of these
- 3 Number of students using no more than one vehicles is :
(a) 76 (b) 80
(c) 60 (d) can't be determined
- 4 Number of students using exactly two vehicles is :
(a) 38 (b) 55
(c) 70 (d) none of these
- 5 The number of students who are using both bike and car but not taxi is :
(a) 23 (b) 40
(c) 36 (d) data insufficient
- 6 In a survey among B-school students, 68% of those surveyed were in favour of atleast one of the three magazines- A, B and C. 38% of those surveyed favoured magazine A, 26% favoured magazine B and 36% favoured magazine C. If 11% of those surveyed favoured all three magazines. What per cent of those surveyed favoured more than one of the three magazines?
(a) 25% (b) 33% (c) 21% (d) 26%

Directions (for Q. Nos. 7 and 8) *In a class of 80 students 25 passed in QA and DI, 25 passed in DI and English, 20 passed in QA and English. 10 student passed in all the three subjects.*

- 7 How many students passed only in Q.A.?
(a) 20 (b) 15
(c) 12 (d) can't be determined
- 8 If no student failed in all three subjects, what is the total number of students who passed in QA only, DI only and English only?
(a) 50 (b) 30
(c) 25 (d) can't be determined

Directions (for Q. Nos. 9 and 10) *A survey shows that 41%, 35% and 60% of the people watch 'Maine Pyaar Kiya' 'Maine Pyaar Kyun Kiya' and 'Pyaar to Hona Hi Tha' respectively. 27% people watch exactly two of the three movies and 3% watch none.*

- 9 What percentage of people watch all the three movies ?
(a) 40% (b) 6%
(c) 9% (d) 12%
- 10 If another survey indicates that 16% of the people watch Maine Pyaar Kiya and Pyaar to Hona Hi Tha, and 14% watch Maine Pyaar Kyun Kiya and Pyaar to Hona Hi Tha, then what percentage of the people watch only Maine Pyaar Kyun Kiya ? (Use the data from the previous question, if necessary)
(a) 10% (b) 8%
(c) 12% (d) 15%
- 11 Find the number of positive integers up to 100 which are not divisible by any 2, 3 and 5 ?
(a) 24 (b) 25
(c) 26 (d) 27
- 12 A survey was conducted at a coaching institution and it was found that there were 34 students who appeared in MAT. There were 37 students who appeared in CAT of which 17 students appeared in MAT. 30 students appeared in XAT of which 13 students appeared in MAT. Of the XAT applicants (i.e., appeared students) 14 appeared in CAT and these 6 appeared in MAT. How many students appeared in CAT but not in MAT or XAT ?
(a) 9 (b) 10
(c) 12 (d) none of these

- 13** A survey among 151 persons is conducted regarding their favourite channel of radio - Radio city, Radio mirchi and Radio life. It was found that every listener of Radio mirchi also listen either Radio city or Radio life. The number of persons listening all the radio channels is the same as the number of persons who listen none of the channels. 55 persons listen exactly two channels and 70 persons listen only one channel. The number of people who listen all the three channels ?

(a) 16 (b) 13
(c) 9 (d) data insufficient

- 14** In a group of 132 people 50, 60, 70 people like three different sweets-Barfi, Jalebi, Rasgulla, respectively. The number of people who like all the three sweets is half the number of people who like exactly two sweets. The number of people who like exactly any two out of the three sweets is the same as those who like exactly any other two of the three sweets. The number of people who like the three sweets

(a) 12 (b) 6
(c) 8 (d) none of these

- 15** In the previous question (number 14) find the number of people who like Rasgulla or Jalebi but not Barfi.

(a) 82 (b) 42 (c) 48 (d) 38

- 16** In a group of 80 employees, the number of employees who are engineers is twice that of the employees who are MBAs. The number of employees who are not engineers is 32 and that of those who are not MBAs is 56. The number of employees who are both engineers and MBAs is twice that of the employees who are only MBAs. How many employees are neither engineer (B. Tech) nor MBAs?

(a) 24 (b) 38
(c) 36 (d) can't be determined

Directions (for Q. Nos. 17 to 19) *There are 60 workers who work for M/s. Nottan Dibbawala Pvt. Ltd. Mumbai, out of which 25 are women. Also :*

- (i) 28 workers are married (ii) 26 workers are graduate
(iii) 20 married workers are graduate of which 9 are men
(iv) 15 men are graduate (v) 15 men are married.

- 17** How many unmarried women are graduate ?

(a) 20 (b) 8
(c) 0 (d) can't be determined

- 18** How many unmarried women work in the company ?

(a) 11 (b) 12
(c) 9 (d) none of these

- 19** How many graduate men are married ?

(a) 9 (b) 15
(c) 13 (d) none of these

Directions (for Q. Nos. 20 to 23) *In the year 222 B.C. 100 CAT aspirants appeared in CAT- (Common Admission Test for MBA in some prestigious institutions). They had to show their competency in all the following four areas viz- Maths, Data Interpretation (DI), Logical Reasoning (LR) and English.*

Number of aspirants who had qualified Maths is 55.

Number of aspirants who had qualified LR is 38

Number of aspirants who had qualified Maths and English is 30.

Number of aspirants who had qualified LR and English is 15.

Number of aspirants who had qualified Maths and LR is 20.

Number of aspirants who had qualified Maths, LR and English is 5.

Number of aspirants who had qualified DI is 22.

Number of aspirants who had qualified DI and LR is 5.

Number of aspirants who had qualified DI and Maths is 5.

Number of aspirants who had qualified DI, Maths and LR is 5.

Number of aspirants who had qualified English is 50.

Number of aspirants who had qualified English could not qualify DI section.

- 20** Number of aspirants who had qualified only in DI section

(a) 17 (b) 22
(c) 27 (d) none of these

- 21** Number of aspirants who had qualified in only Maths and LR sections :

(a) 20 (b) 10 (c) 28 (d) 38

- 22** Find the number of aspirants who had qualified in atleast two sections out of 4 sections.

(a) 45 (b) 35
(c) 55 (d) can't be determined

- 23** Find the number of aspirants who had qualified none of the sections.

(a) 12 (b) 18 (c) 7 (d) 0

- 24** In a survey among 80 people, 50 people like arrange marriage and 70 people like love marriage. What is the minimum and maximum number of people like both the marriages respectively?

(a) 40, 45 (b) 40, 50
(c) 30, 40 (d) can't be determined

- 25** In a car agency one day 120 cars were decorated with three different accessories viz., power window, AC and music system. 80 cars were decorated with power windows, 65 cars were decorated with AC and 80 cars were decorated with music systems. What is the minimum and maximum number of cars which were decorated with all of three accessories ?

(a) 10, 61 (b) 10, 45
(c) 25, 35 (d) none of these

- 26** In our coaching there were 200 students enrolled for DI, 150 for English and 150 for Maths. Of these 80 students enrolled for both DI and English. 60 students enrolled for Maths and English, while 70 students enrolled for DI and Maths. Some of these students enrolled for all the three subjects. Diwakar teaches those students who are enrolled for DI classes only. Priyanka teaches those students who are enrolled for English only and Varun teaches those students who are enrolled for Maths only. Sarvesh is a senior most faculty therefore, he can teach all the three subjects. Students always prefer a specialist for their respective subjects. If Diwakar teaches 80 students then the other three faculty can be arranged in terms of the number of students taught as

(a) Sarvesh > Varun > Priyanka
(b) Sarvesh > Priyanka > Varun
(c) Varun > Sarvesh > Priyanka
(d) none of the above

Directions (for Q. Nos. 27 to 29) on the basis of the information given below

Sanskaram Karoti (SK) is a spiritual organisation involved in performing spiritual rites. Currently it has 37 volunteers. They are involved in three jobs : Body Massage (BM), Yoga and Pooja. Each volunteer working with Sanskaram Karoti (SK) has to be involved in atleast one of the three jobs mentioned above.

- A maximum number of volunteers are involved in Yoga. Among them, the number of volunteers involved in Yoga alone is equal to the volunteers having additional involvement in the Pooja.
- The number of volunteers involved in Pooja alone is double the number of volunteers involved in all the three jobs.
- 17 volunteers are involved in Body Massage (BM).
- The number of volunteers involved in Body Massage alone is one less than the number of volunteers involved in Pooja alone.
- Ten volunteers involved in the Body Massage are also involved in atleast one more job.

- 27** Based on the information given above, the minimum number of volunteers involved in both Yoga and Body Massage, but not in the Pooja is :
 (a) 1 (b) 3 (c) 4 (d) 5
- 28** Which of the following additional information would enable to find the exact number of volunteers involved in various jobs?
 (a) Twenty volunteers are involved in Yoga
 (b) Four volunteers are involved in all the three jobs
 (c) Twenty three volunteers are involved in exactly one task
 (d) No need for any additional information
- 29** After some time, the volunteers who were involved in all the three tasks were asked to withdraw from one task. As a result, one of the volunteers opted out of the Body Massage and one opted out of the Pooja, while the remaining ones involved in all the three tasks opted out of the Yoga. Which of the following statements, then, necessarily follows?
 (a) The least number of volunteers is now performing Body Massage

- (b) More volunteers are now associated with Yoga as compared to Pooja
 (c) More volunteers are now performing Body Massage as compared to Pooja
 (d) None of the above

Direction for (Q 30–32) A modeling agency conducted a survey to figure out the most preferred brands of lingerie. In the survey total 969 models participated and the top three brands emerged are – Vagin Pouvoir, Victoria's Secret and Ravage. 169 models like all the three brands, 269 models like Vagin Pouvoir and Victoria's Secret, 369 models like Vagin Pouvoir. The number of models who like Ravage is 69 less than that of models who like Victoria's Secret.

- 30** The number of models who like Ravage is 69 less than that of models who like Victoria's Secret. What could be the maximum number of models who don't like any of these top three brands?
 (a) 369 (b) 331 (c) 300 (d) 269
- 31** The number of models who like Ravage is 69 less than that of models who like Victoria's Secret. If the number of models who like only Ravage is less than the number of models who like only Vagin Pouvoir then what could be the maximum number of models who like only Victoria's Secret?
 (a) 68 (b) 300 (c) 136 (d) 299
- 32** The number of models who like Ravage is 69 more than the number of models who like Victoria's Secret.
 (a) 269 (b) 300 (c) 231 (d) 369
- 33** Let $S = \{1, 2, 3, 4\}$. The total number of unordered pairs of disjoint subsets of S is equal to
 (a) 34 (b) 43 (c) 42 (d) 41
- 34** There are two sets A and B such that $A \in (1, 5, 9, 13, 17, \dots, 100\text{th term})$ and $B \in \{3, 10, 17, 24, 31, \dots, 100\text{th term}\}$. If there is another set $S = A \cup B$, find the total number of elements in S .
 (a) 186 (b) 165 (c) 135 (d) 172

Answers

✓ Introductory Exercise 15.1

1. (a)	2. (a)	3. (b)	4. (a)	5. (c)	6. (d)	7. (a)	8. (b)	9. (d)	10. (a)
11. (b)	12. (b)	13. (d)	14. (b)	15. (b)	16. (c)	17. (b)	18. (b)	19. (b)	

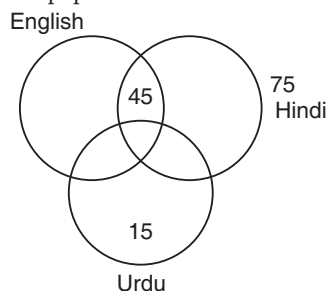
✓ Level 01 Basic Level Exercise

1. (b)	2. (c)	3. (b)	4. (c)	5. (a)	6. (c)	7. (d)	8. (b)	9. (b)	10. (c)
11. (c)	12. (c)	13. (b)	14. (a)	15. (a)	16. (a)	17. (c)	18. (b)	19. (a)	20. (a)
21. (b)	22. (c)	23. (d)	24. (b)	25. (d)	26. (a)	27. (c)	28. (a)	29. (b)	30. (c)
31. (a)	32. (c)	33. (d)	34. (a)						

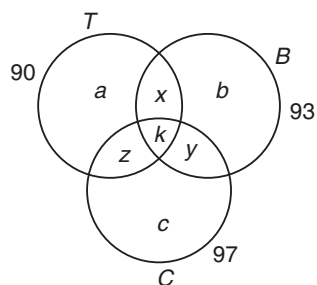
Hints & Solutions

✓ Level 01 Basic Level Exercise

- 1 It is clear that 45% people cannot read another third (i.e., Urdu) newspaper. Besides them all of the rest people can read Urdu newspaper.



Hence, maximum 55% (100 – 45) people can read Urdu newspaper.



$$a + b + c = \alpha, \quad x + y + z = \beta, \quad k = \gamma$$

$$\alpha + \beta + \gamma = 170$$

$$\alpha + 2\beta + 3\gamma = 90 + 93 + 97 = 280$$

$$\gamma : (\beta + \gamma) = 2 : 9$$

$$\Rightarrow \gamma : \beta = 2 : 7$$

$$\text{and } \alpha : (\beta + \gamma) = 8 : 9$$

$$\Rightarrow \alpha : \beta : \gamma = 8 : 7 : 2$$

$$\therefore \alpha = 80, \beta = 70 \text{ and } \gamma = 20$$

$$\Rightarrow a + b + c = 80, \quad x + y + z = 70 \quad \dots(i)$$

$$\text{and } k = 20$$

$$\text{Again } c - b = 14 \text{ and } a - b = 12 \quad \dots(ii)$$

On solving eq. (i) and (ii) we get $a = 30, b = 18, c = 32$

$$\begin{aligned} \text{Again } (a + x + k + z) - (a + k) &= (x + z) \\ &= 90 - (30 + 20) = 40 \end{aligned}$$

$$\text{and } (x + y + z) - (x + z) = y = 70 - 40 = 30$$

$$\text{similarly } x = 25 \text{ and } z = 15$$

Solutions (for Q. Nos. 2 to 5)

2 20

3 80

4 70

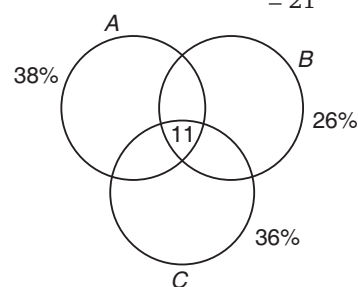
5 23

$$6 \quad \alpha + \beta + \gamma = 68$$

$$\alpha + 2\beta + 3\gamma = (38 + 26 + 36) = 100$$

$$\text{and } \gamma = 11$$

$$\begin{aligned} \therefore (\alpha + 2\beta + 3\gamma) - [(\alpha + \beta + \gamma) + \gamma] &= \beta + \gamma \\ &= 100 - [68 + 11] \\ &= 21 \end{aligned}$$

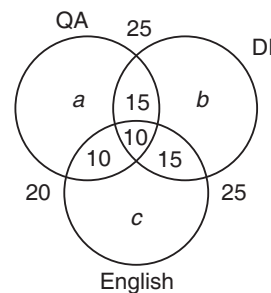


Hence, 21% favoured more than one magazine.

($\alpha \rightarrow$ favoured only one magazine, $\beta \rightarrow$ favoured only two magazines, $\gamma \rightarrow$ favoured all three magazines)

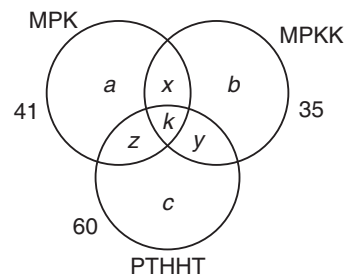
Solutions (for Q. Nos. 7 to 8)

- 7 Since we don't know how many students failed in all the three subjects, the questions cannot be answered. Hence (d).



$$8 \quad (a + b + c) = 80 - [(15 + 15 + 10) + (10)] = 30$$

9



$$\alpha + \beta + \gamma = 97\%$$

$$\alpha + 2\beta + 3\gamma = 41 + 35 + 60 = 136\%$$

$$\text{But } \beta = (x + y + z) = 27\%$$

$$\therefore (\alpha + 2\beta + 3\gamma) - (\alpha + \beta + \gamma) = \beta + 2\gamma = 39\%$$

$$\therefore (\beta + 2\gamma) - \beta = 2\gamma = 39 - 27 = 12\%$$

$$\therefore \gamma = 6\% = (k)$$

$\therefore$ 6% people watch all the three movies

$$10 \quad z + k = 16 \Rightarrow z = 10 \quad \{\because \gamma = k = 6\}$$

$$y + k = 14 \Rightarrow y = 8$$

$$\therefore x = 9 \quad (\because x + y + z = 27)$$

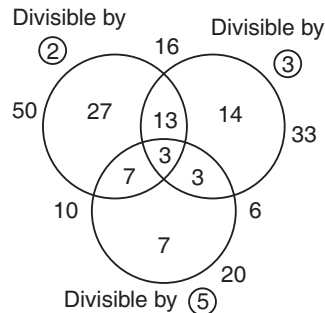
$$\therefore b = 35 - (x + k + y) = 35 - (9 + 6 + 8) = 12\%$$

11 Total numbers divisible by 2 upto 100 = 50

Total numbers divisible by 3 upto 100 = 33

Total numbers divisible by 5 upto 100 = 20

Total numbers divisible by 2 and 3 i.e., 6 upto 100 = 16



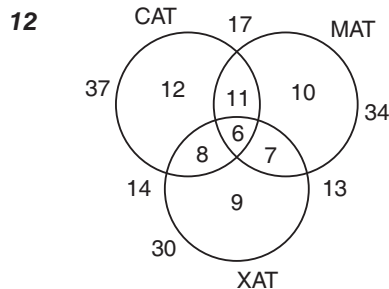
Total numbers divisible by 3 and 5 i.e., 15 upto 100 = 6

Total numbers divisible by 2 and 5 i.e., 10 upto 100 = 10

Total numbers divisible by 2, 3 and 5 i.e., 30 upto 100 = 3

$\therefore$ Total number of numbers upto 100 which are divisible by at least one of 2, 3 and 5 = 74

$\therefore$ Total number of numbers upto 100 which are not divisible by any 2, 3 or 5 = $100 - 74 = 26$

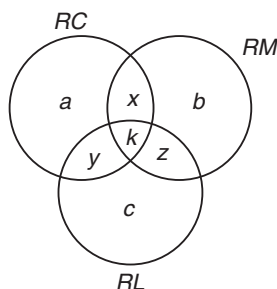


Hence, there are 12 students who appeared in CAT but not in MAT or XAT.

$$13 \quad \beta = (x + y + z) = 55$$

$$\alpha = (a + b + c) = 70$$

$$\gamma = k$$



Let m people listen none of the three channels, then $m = \gamma = k$

$$(\alpha + \beta + \gamma) + m = 151$$

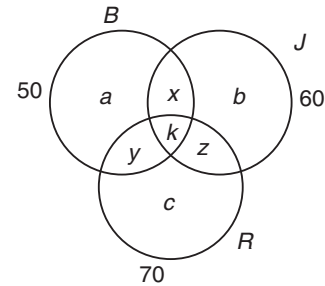
$$\Rightarrow \alpha + \beta + \gamma + \gamma = 151$$

$$\Rightarrow (55 + 70) + 2\gamma = 151$$

$$\Rightarrow \gamma = 13$$

Hence, there are 13 people listen all three channels.

$$14 \quad \alpha = a + b + c, \quad \beta = x + y + z, \quad \gamma = k$$



$$\text{Here} \quad \gamma = \frac{1}{2}\beta \Rightarrow 2\gamma = \beta$$

$$\text{Again} \quad x = y = z = p$$

$$\therefore \beta = 3p \Rightarrow \gamma = \frac{3}{2}p$$

$$\text{Now,} \quad \alpha + 2\beta + 3\gamma = 50 + 60 + 70 = 180$$

$$\Rightarrow \alpha + 4\gamma + 3\gamma = 180$$

$$\Rightarrow \alpha + 7\gamma = 180$$

...(i)

$$\text{Again} \quad \alpha + \beta + \gamma = 132$$

$$\alpha + 2\gamma + \gamma = 132$$

$$\Rightarrow \alpha + 3\gamma = 132$$

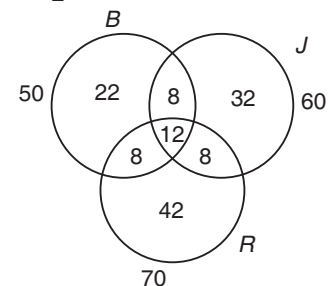
...(ii)

From eq. (i) and (ii), we get

$$\gamma = 12$$

Hence, 12 people like all 3 sweets.

$$15 \quad \gamma = \frac{3}{2}p \Rightarrow 12 = \frac{3}{2}p \Rightarrow p = 8$$

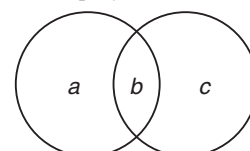


Hence, the number of persons who like Rasgulla or Jalebi but not Barfi = $32 + 8 + 42 = 82$

16 Let a be the number of engineers only

c be the number of MBAs only

b be the number of employees who are both engineers and



MBA's and d be the number of employees who are neither engineer nor MBA

$$\therefore a + b + c + d = 80 \quad \dots(i)$$

$$(a + b) = 2(b + c) \quad \dots(ii)$$

$$\Rightarrow (a - b) = 2c \quad \dots(iii)$$

$$\text{and } c + d = 32 \quad \dots(iv)$$

$$\text{and } a + d = 56 \quad \dots(v)$$

$$\text{and } b = 2c \quad \dots(vi)$$

From eq. (ii) and (v), we get

$$a = 2b \quad \dots(vi)$$

From eq. (i) and (iii), we get

$$a + b = 48$$

From eq. (vi), we get

$$b = 16$$

$$\therefore a = 32 \quad (\text{from eq. vi})$$

$$\text{and } c = 8 \quad (\text{from eq. v})$$

$$\text{and } d = 24$$

Hence, 24 employees are neither engineer nor MBAs.

Solutions (for Q. Nos. 17 to 19)

Total number of employees = 60

Women = 25

Men = 35

Married workers = 28

Graduate workers = 26

$a \rightarrow$ unmarried men who are not graduate

$b \rightarrow$ married women who are not graduate

$c \rightarrow$ unmarried women who are graduate

$x \rightarrow$ married men who are not graduate

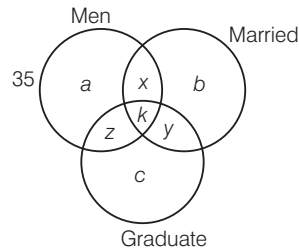
$y \rightarrow$ married women who are graduate

$z \rightarrow$ unmarried men who are graduate

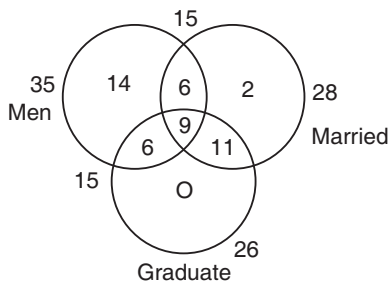
$k \rightarrow$ married men who are graduate

$p \rightarrow$ unmarried women who are not graduate

According to the given information the Venn diagram can be completed as given below.



- 17** No one unmarried woman is graduate. Hence (c)

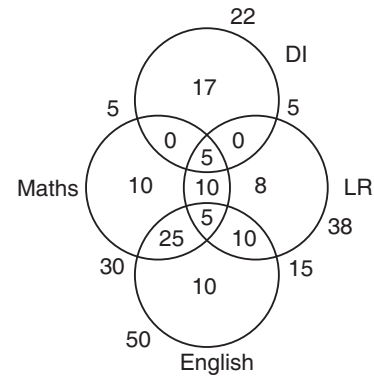


- 18** Number of unmarried women

$$= 60 - [14 + 2 + 6 + 6 + 11 + 9] = 12$$

- 19** There are 9 graduate men who are married.

Solutions (for Q. Nos. 20 to 23)



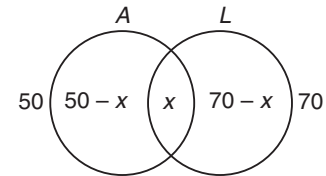
$$\mathbf{20} \quad 17$$

$$\mathbf{21} \quad 10$$

$$\mathbf{22} \quad 55$$

$$\mathbf{23} \quad 0$$

- 24** For the minimum value of x people who like only arrange marriage and only love marriage must be greater



$$\therefore x = (70 + 50) - 80 = 40$$

For the maximum value of x : $(50 - x)$ and $(70 - x)$ must not be negative, therefore max. possible value of x is 50.

- 25** 80 cars were decorated with power windows it means at least 40 cars were decorated with AC or music system or both.

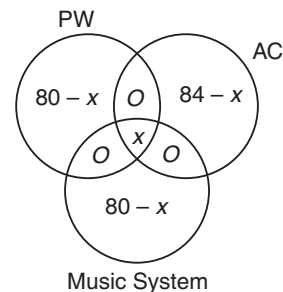
84 cars were decorated with ACs, it means atleast 36 cars were decorated with power windows or music systems.

80 cars were decorated with music system means at least 40 cars were decorated with power windows or ACs.

It means if there is no intersection in these three, then at most $40 + 36 + 40 = 116$ cars had been decorated with one or two accessories.

Hence at least 4 cars would have been decorated with all three accessories.

For maximum value of x :



$$\begin{aligned}\text{Total number of cars} &= (80 - x) + (84 - x) + (80 - x) + x \\ 120 &= 244 - 2x\end{aligned}$$

$$\Rightarrow 2x = 124$$

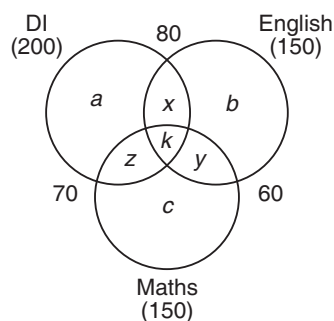
$$\Rightarrow x = 62$$

minimum $\rightarrow$ 4 cars and maximum $\rightarrow$ 62 cars

$$\mathbf{26} \quad a + x + k + z = 200 \quad \dots(i)$$

$$b + x + k + y = 150 \quad \dots(ii)$$

$$c + y + k + z = 150 \quad \dots(iii)$$



But since Diwakar teaches only 80 students of DI.

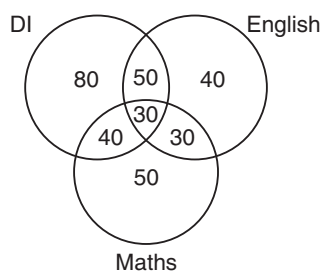
Therefore, $a = 80$

Hence, $x + k + z = 120$

but $(x + k) + (k + z) = 150$

$\therefore k = 30$

Hence, $x = 50, z = 40, y = 30, b = 40$



$\therefore$ No. of students taught by Diwakar $= a = 80$

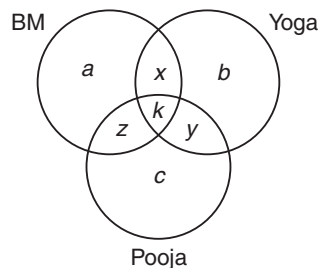
No. of students taught by Priyanka $= b = 40$

No. of students taught by Varun $= c = 50$

No. of students taught by Sarvesh $= x + y + z + k = 150$

Hence, choice (a) is correct.

Solutions (for Q. Nos. 27 to 29)



Remember Maximum number of volunteers are involved in Yoga.

$$\text{Now,} \quad b = k + y \quad \dots(i)$$

$$c = 2k \quad \dots(ii)$$

$$a + x + k + z = 17 \quad \dots(iii)$$

$$a = c - 1 \quad \dots(iv)$$

$$x + k + z = 10 \quad \dots(v)$$

From eq. (iii) and (v), we obtain

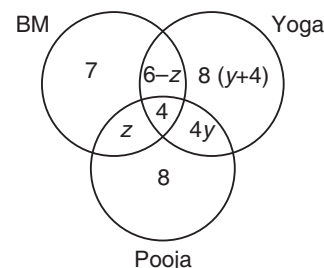
$$a = 7 \quad \dots(vi)$$

and from eq. (iv) and (vi), we obtain

$$c = 8 \Rightarrow k = 4$$

$$\therefore (z + x) = 6 \Rightarrow x = (6 - z)$$

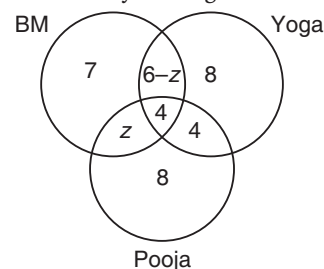
$$\therefore b = (y + 4), \quad \text{from eq. (i).}$$



$$\text{Again} \quad 7 + (6 - z) + 4 + z + (y + 4) + y + 8 = 37$$

$$\Rightarrow 29 + 2y = 37 \Rightarrow y = 4$$

27 Since no. of volunteers involved in Yoga are maximum so we can compare it from the no. of volunteers involved in Pooja and that of Body Massage.



Since,

$$6 - z > z \quad \forall \quad z \in (0, 1, 2, 3, \dots)$$

$$\therefore z = 0, 1, 2 \Rightarrow (6 - z) = 4, 5, 6$$

Hence, the minimum possible value of $(6 - z) = 4$.

28 See the venn diagram shown in solution no. 27, then you will notice that you are required to know the value of y .

Thus from the data provided by choice (a) enable us to calculate all the required details.

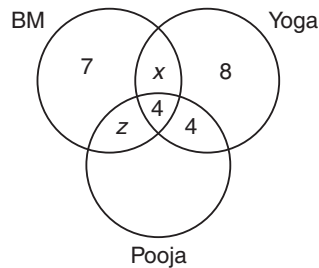
$$\therefore \{(6 - z) + 4 + 4 + 8\}_{\text{Yoga}} = 20$$

$$\Rightarrow z = 4$$

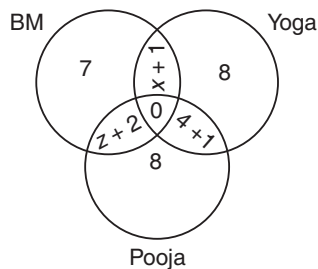
Hence, we can find the exact number of volunteers involved in various projects.

NOTE Coincidentally we obtain $z = 4$ in both the questions 27 and 28 but actually these two answers have no any relation between them.

29 Initially



After the withdraw of volunteers :



The volunteer who is opted out of the BM will be involved in the Yoga and Pooja.

Similarly the volunteer who is opted out of Pooja will be involved in the BM and Yoga.

and the remaining two volunteers who are opted out of Yoga will be involved in BM and Pooja.

NOTE Since initially 4 volunteers are involved in all the three jobs, but when a volunteer leave one job, he or she still continue to perform other two jobs.

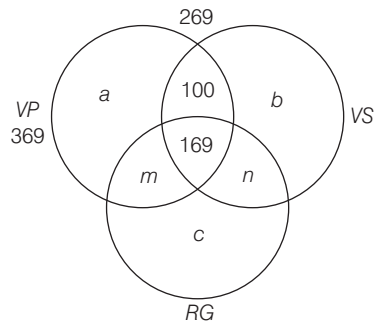
$$\begin{aligned} \text{Total no. of volunteers in BM} &= 7 + (x + 1) + 0 + (z + 2) \\ &= 16 \quad (\because x + z = 6) \end{aligned}$$

Since we know that $x = 4, 5, 6$

Therefore corresponding values of $z = 2, 1, 0$
 $(\because x + z = 6)$

$\therefore$ No. of volunteers involved in Yoga = 18, 19 or 20
 and No. of volunteers involved in Pooja = 17, 16 or 15
 Hence, it is clear that choice (b) is correct.

Solutions for (Q. 30-32) Consider the following diagram.



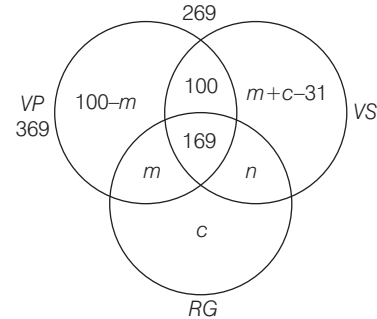
Given that, $a + m + 169 + 100 = 369$

$$\Rightarrow a = 100 - m$$

$$\text{And } (100 + b + 169 + n) - (169 + n + m + c) = 69$$

$$\Rightarrow (100 + b) - (m + c) = 69$$

$$\Rightarrow b = m + c - 31$$



30. In order to maximize the number of models who don't like any of the top three brands, you have to minimize the total value represented by the above Venn diagram.

Therefore, minimum number of models who like at least one of these three brands

$$= \min[(369) + (c) + (n) + (m + c - 31)] \quad \dots(i)$$

But you know that $c \geq 0, n \geq 0, m + c - 31 \geq 0, 100 - m \geq 0$

Since, you have to minimize the value of Eq. (i), so you can take $n = 0$, as there is no other restriction on n .

$$\text{Now, } 100 - m \geq 0 \Rightarrow m \leq 100.$$

$$\text{And } m + c - 31 \geq 0 \Rightarrow c \geq 31 - m$$

Therefore, maximum value of $m = 31$ and the minimum value of $c = 0$

Thus, the minimum value of Eq. (i) is 369.

So, the maximum number of models who don't like any of these top three brands = $669 - 369 = 300$

Hence, choice (c) is the correct answer.

31. Given that $c < a$

$$\Rightarrow c < 100 - m$$

$$\Rightarrow m + c < 100$$

$$\Rightarrow (m + c) - 31 < 100 - 31$$

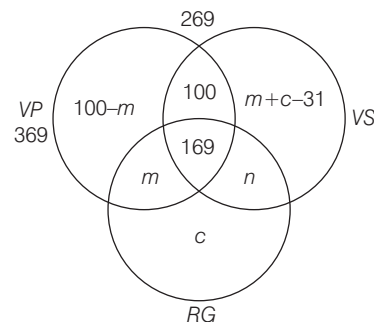
$$\Rightarrow (m + c) - 31 < 69$$

$$\therefore \text{Maximum of } (m + c - 31) = 68$$

Hence, choice (a) is the correct answer.

32. Given that $a + m + 169 + 100 = 369$

$$\Rightarrow a = 100 - m$$



$$\text{And } (169 + n + m + c) - (100 + b + 169 + n) = 69$$

$$\Rightarrow (100 + b) - (m + c) = 69$$

$$\Rightarrow b = m + c - 169$$

In order to maximize the number of models who don't like any of the top three brands, you have to minimize the total value represented by the above Venn diagram.

Therefore, minimum number of models who like at least one of these three brands

$$= \text{Min}[(369) + (c) + (n) + (m + c - 169)] \quad \dots(i)$$

But, you know that $c \geq 0$, $n \geq 0$, $m + c - 169 \geq 0$ and $100 - m \geq 0$

Since, you have to minimize the value of eqn. (i), so you can take $n = 0$, as there is no other restriction on n .

$$\text{Now, } 100 - m \geq 0 \Rightarrow m \leq 100.$$

$$\text{And } m + c - 169 \geq 0$$

$$\Rightarrow c > 169 - m$$

Therefore, minimum value of $c = 69$

Thus, the minimum value of Eq. (i) is 438 ($= 369 + 69$).

So, the maximum number of models who don't like any of these top three brands $= 669 - 438 = 231$

Hence, choice (c) is the correct answer.

33. Let us assume a random pair P_i having two sets X_i and Y_i , such that X_i and Y_i are the subsets of S , but no element is common in X_i and Y_i .

Now, each element of S can be the element of either X_i or Y_i or none of them. It implies that any element of S can be placed in 3 ways. So, all the 4 elements can be placed in $3 \times 3 \times 3 \times 3$ ways. It means the total pairs $= 3^4$.

But, this includes a pair P_i in which both the sets X_i and Y_i are empty.

So, the total number of ordered pairs having at least one element in at least one of X_i and $Y_i = 3^4 - 1$

But the problem is asking for unordered pairs, so it would be $\frac{3^4 - 1}{2}$ pairs.

Now, we will add back one more unordered pair P_i in which both X_i and Y_i are empty.

So, the total number of unordered disjoint pairs

$$= \left(\frac{3^4 - 1}{2} \right) + 1 = \frac{3^4 + 1}{2} = 41$$

Hence, choice (d) is the answer.

34. The common difference between any two consecutive elements of $A = 4$

And the common difference between any two consecutive elements of $B = 7$

Therefore, the difference between any two consecutive common terms $= \text{LCM of } (4, 7) = 28$.

The first common term is 17.

The 100th term of A is 397 and the 100th term of B is 696.

Since, the largest term of A is smaller than the largest term of B , it means there can't be any common term beyond 397.

Now, since all the common terms are in Arithmetic Progression, so you can use the formula $a + (n - 1)d = l$

$$\text{Therefore, } 17 + (n - 1)28 \leq 397$$

$$\Rightarrow (n - 1)28 \leq 380$$

$$\Rightarrow (n - 1) \leq 13$$

$$\Rightarrow n \leq 14$$

It implies that there are 14 common terms between A and B .

Therefore total number of elements in

$$S = (100 + 100) - 14 = 186$$

Hence, choice (a) is the correct one.

 **NOTE** To know more about the arithmetic series and number of terms you may like to refer Sequence, Series and Progressions chapter in Quantum CAT.

CHAPTER 16

Logarithm

Atleast one question from this chapter is necessarily being asked in CAT. Whenever a question appears related with logarithm, it is of applied nature and the logic behind the question is the application of logarithm. One more thing I would like to mention that many a student are scared of this chapter. But In my opinion they should not bother about the word ‘logarithm’ instead they should look the problems of logarithm as problems of exponents and most of the problems can be easily solved by plotting the graph. Notice that this chapter is important for SNAP, MAT, IIFT, JMET and XAT etc.

16.1 Exponential Function

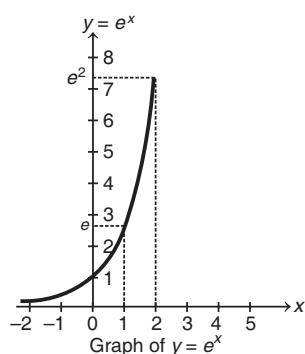
For every $x \in R$, $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$ or $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

Here e^x is called as exponential function and it is a finite number for every $x \in R$.

Properties

For every $x \in R$, e^x is defined, then

- (i) $e^x > 0$ for all $x \in R$ and $e^0 = 1$
- (ii) $e^a > e^b$ if $a > b$ and $a, b \in R$
- (iii) $e^a \times e^b = e^{(a+b)}$ for all $a, b \in R$
- (iv) $e^a \div e^b = e^{(a-b)}$ for all $a, b \in R$
- (v) $(e^a)^b = e^{ab}$ for all $a, b \in R$
- (vi) For each positive real number x there exists one and only one real number y such that $e^y = x$
- (vii) e^x is one-one function
- (viii) $e \approx 2.714$



Chapter Checklist

- Exponential Function
- Logarithm
- Properties of Logarithm
- Characteristics and Mantissa
- Points to Remember About Characteristics
- Important Conversions
- CAT Test

16.2 Logarithm

Let a, b be positive real numbers then $a^x = b$ can be written as

$$\begin{aligned} \log_a b &= x; & a &\neq 1, a > 0, b > 0 \\ \text{e.g., } 2^5 &= 32 & \Leftrightarrow & \log_2 32 = 5 \\ 10^3 &= 1000 & \Leftrightarrow & \log_{10} 1000 = 3 \\ 3^{-4} &= \frac{1}{81} & \Leftrightarrow & \log_3 \left(\frac{1}{81} \right) = -4 \text{ etc.} \end{aligned}$$

Types of Logarithm

- (i) **Natural Logarithm**: $\log_e N$ is called Natural logarithm or Napierian Logarithm, denoted by $\ln N$ i.e., when the base is 'e' then it is called as Natural logarithm.

e.g., $\log_e 5, \log_e \left(\frac{1}{81}\right), \log_e 72 \dots$ etc.

- (ii) **Common Logarithm**: $\log_{10} N$ is called common logarithm or Brigg's Logarithm i.e., when base of log is 10, then it is called as common logarithm.

e.g., $\log_{10}(100), \log_{10} 25, \log_{10} 248$ etc.

16.3 Properties of Logarithm

$$1. \log_a 1 = 0, \quad a > 0, a \neq 1$$

$$2. \log_a a = 1, \quad a > 0, a \neq 1$$

$$3. \log_a a^x = x \quad \forall x \in R, x > 0$$

$$4. a^{\log_a x} = x \quad \forall x \in R, x > 0$$

NOTE $\log_a a^x$ is the inverse function of a^x .

$$5. \log_a (m \cdot n) = \log_a m + \log_a n \quad \forall m, n > 0, a > 0, a \neq 1$$

$$6. \log_a (m/n) = \log_a m - \log_a n \quad \forall m, n > 0, a > 0, a \neq 1$$

$$7. \log_a (m^n) = n \log_a m \quad \forall m > 0, a > 0, a \neq 1$$

$$8. \log_a \left(\frac{1}{m}\right) = -\log_a m \quad \forall m > 0, a > 0, a \neq 1$$

$$9. \log_a b = \frac{1}{\log_b a} = \frac{\log_c b}{\log_c a} \quad \forall a, b, c > 0 \text{ and } a \neq 1, b \neq 1, c \neq 1$$

$$10. \text{If } \log_a b = x \quad \forall a > 0, a \neq 1, b > 0 \text{ and } x \in R$$

$$(i) \log_{(1/a)} b = -x \quad (ii) \log_a (1/b) = -x$$

$$(iii) \log_{1/a} (1/b) = x$$

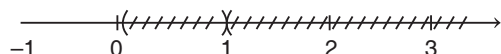
$$11. \log_{a^m} (b) = \frac{1}{m} \log_a b$$

$$12. \log_a x \text{ is a decreasing function, if } 0 < a < 1$$

$$13. \log_a x \text{ is an increasing function, if } a > 1$$

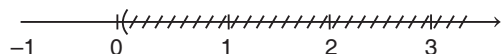
NOTE The base of logarithm can never be equal to 1. i.e., $\log_1 x$ is undefined $\forall x$.

14. On the real number line the base of logarithm can assume any positive real value except 1 (i.e., unity) Hence at $a = 1$, $\log_a x$ is not defined; $x > 0$



Thus at $a > 0$ except $a = 1$, $\log_a x$ is defined; $x > 0$

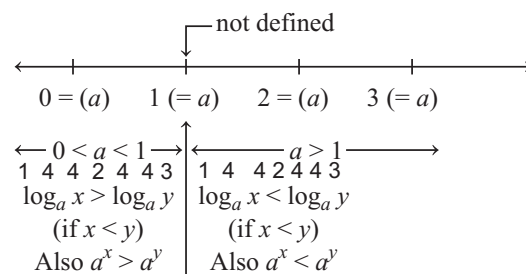
15. On the real number line x can assume any positive real value (number) for $\log_a x$; $a > 0, a \neq 1$



Thus the logarithm of only positive values is defined i.e., $\log_a x$ is defined for every $x > 0, a > 0, a \neq 1$. Hence for $f(x) = \log_a x$

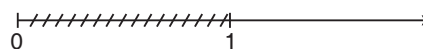
Domain $\rightarrow R^+$ and Range $\rightarrow R$

16.



at $a = 1$, $\log_a x$ is not defined.

17.



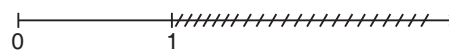
For $0 < a < 1$

$$\log_a x > 1 \text{ if } x < a \quad \forall x > 0$$

$$\log_a x = 1 \text{ if } x = a \quad \forall x > 0$$

$$\log_a x < 1 \text{ if } x > a \quad \forall x > 0$$

18.



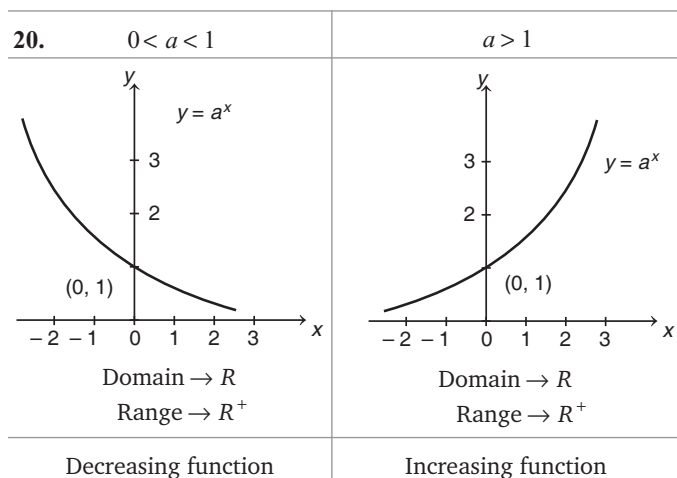
For $a > 1$

$$\log_a x < 1 \text{ if } x < a \quad \forall x > 0$$

$$\log_a x = 1 \text{ if } x = a \quad \forall x > 0$$

$$\log_a x > 1 \text{ if } x > a \quad \forall x > 0$$

19. $0 < a < 1$	$a > 1$
$\log_a x$ is a decreasing function	$\log_a x$ is an increasing function
$\log_a x > \log_a y \quad (x < y)$ i.e., inequality gets reversed	$\log_a x < \log_a y \quad (x < y)$ i.e., the inequality remains unchanged
Sign of $\log_a x$	Sign of $\log_a x$



21. For $|x| < 1$

- (i) $\log(1+x) = \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots\right)$
- (ii) $\log(1-x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots\right)$
- (iii) $\log\left(\frac{1+x}{1-x}\right) = 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \dots\right]$
- (iv) $\log 2 = \left(1 - \frac{1}{2} + \frac{1}{3} - \dots\right)$
- (v) $\log_{10} 2 = 0.3010$
- (vi) $\log_{10} 3 = 0.4771$

22. Logarithmic Inequalities

$0 < a < 1$	$a > 1$
$\log_a b \geq \log_a c \Leftrightarrow b \leq c$	$\log_a b \geq \log_a c \Leftrightarrow b \geq c > 0$
$\log_a b \geq c \Leftrightarrow b \leq a^c$	$\log_a b \geq c \Leftrightarrow b \geq a^c$

16.4 Characteristics and Mantissa

Characteristic : The integral part of logarithm is known as characteristic.

Mantissa : The decimal part is known as mantissa and is always positive. *e.g.*, In $\log_a x$, the integral part of x is called the characteristic and the decimal part of x is called the mantissa. For example : In $\log 3274 = 3.5150$, the integral part is 3 *i.e.*, characteristic is 3 and the decimal part is .5150 *i.e.*, mantissa is .5150.

To find the characteristic of common logarithm $\log_{10} x$:

- (a) When the number is greater than 1 *i.e.*, $x > 1$. In this case, the characteristic is one less than the number of digits in the left of the decimal point in the given number.

- (b) When the number is less than 1 *i.e.*, $0 < x < 1$. In this case the characteristic is one more than the number of zeros between the decimal point and the first significant digit of the number and is negative.

NOTE The characteristic may be positive or negative. If the characteristic is negative, it is always represented by putting a bar on the number *i.e.*, $-1 \rightarrow \bar{1} \Rightarrow -2 \rightarrow \bar{2} \Rightarrow -3 \rightarrow \bar{3}$

Exp.

Number	Characteristic	Number	Characteristic
8.3145	0	0.457	$\bar{1}$
74.8120	1	0.0546	$\bar{2}$
568.31	2	0.001324	$\bar{3}$

Remark

- (1) Students must note that $\bar{2}.784$ is not the same as -2.784
i.e., $\bar{2}.784 = (-2) + (.784) \Rightarrow -2.784 = (-2) - (.784)$
- (2) If the logarithm of a number is negative, then we have to make the **mantissa** positive.
 For this we subtract 1 from the integral part and add up 1 to the decimal part.
e.g., $-2.5436 = (-2) + (-.5436)$ Step 1.
 $\Rightarrow = (-2-1) + (-.5436+1)$ Step 2.
 $\Rightarrow = (-3) + (.4564)$ Step 3.
 $\Rightarrow = \bar{3}.4564$ Step 4.

16.5 Points to Remember About Characteristics

1. The characteristic of common logarithm of positive number less than unity (*i.e.*, 1) is negative.
2. The characteristic of common logarithm of a positive number greater than 1 is positive.
3. If the logarithm to any base a gives the characteristic 'n', then the number of possible integral values is given by $a^{n+1} - a^n$. For example $\log_{10} x = n.abcd$, then the number of integral values that x can have given by $10^{n+1} - 10^n$
e.g., $\log_{10} x = 2.abcd$ gives $10^{2+1} - 10^2 = 900$ integral values for x
i.e., $x = 100, 101, 102, 103, \dots, 998, 999$. For all these 900 integral numbers (100 – 999) all the characteristic is 2.
 Similarly $\log_{10} x = 3.abcd$ give $10^{3+1} - 10^3 = 9000$ integral values for x *i.e.*, $x = 1000, 1001, 1002, \dots, 9999$. For all these 9000 integral numbers (1000 – 9999) the characteristic is 3.

4. If the characteristic of $\log_{10} x$ is negative (i.e., $-n$), then the number of zeros between the decimal and the first significant number after the decimal is $(n-1)$

e.g., $\log_{10} x = -3$. $abcd$ has $a = 0$ and $b = 0$.

i.e., first two places after the decimal point are filled with zeros since characteristic is -3 .

Hence $\log_{10} x = -3$. $abcd = -3.00cd$

16.6 Important Conversions

(i) For $a > 1, a \neq 1, c > 0, a^b > c \Leftrightarrow \log_a c < b$

(ii) For $0 < a < 1, a \neq 1, c > 0, a^b > c \Leftrightarrow \log_a c > b$

Exp. 1) The value of $\log 6$ is equal to :

- (a) $\log 1 + \log 2 + \log 3$ (b) $\log (1 + 2 + 3)$
(c) $\log (1 \times 2 \times 3)$ (d) all of the above

Solution $\log 6 = \log (1 \times 2 \times 3) = \log 1 + \log 2 + \log 3$

Also $\log 6 = \log (1 + 2 + 3)$, Hence (d)

Exp. 2) Find the value of $\log_8 128$.

Solution Let $\log_8 128 = x \Rightarrow 8^x = 128 \Rightarrow 2^{3x} = 2^7 \Rightarrow 3x = 7$

$$\Rightarrow x = \frac{7}{3} \therefore \log_8 128 = \frac{7}{3}$$

Exp. 3) Find the value of $\log_{5\sqrt{5}}(125)$.

Solution Let $\log_{5\sqrt{5}}(125) = x \Rightarrow (5\sqrt{5})^x = 125 \Rightarrow 5^{(3/2)x} = 5^3$

$$\Rightarrow \frac{3}{2}x = 3 \Rightarrow x = 2$$

$$\therefore \log_{5\sqrt{5}}(125) = 2$$

Exp. 4) $c \log_5 \log_5(3125)$.

Solution $\log_5 \log_5(3125) = \log_5 \log_5 5^5 = \log_5 5 \log_5 5$
 $= \log_5 5.1 \quad (\because \log_5 5 = 1)$
 $= \log_5 5 = 1$

$$\therefore \log_5 \log_5(3125) = 1$$

Exp. 5) If $\log_{10} x - \log_{10} \sqrt{x} = \frac{2}{\log_{10} x}$, find the value of x .

Solution $\log_{10} x - \log_{10} \sqrt{x} = \frac{2}{\log_{10} x}$

$$\Rightarrow \log_{10} \left(\frac{x}{\sqrt{x}} \right) = \frac{2}{\log_{10} x} \Rightarrow \log_{10}(x)^{1/2} = \frac{2}{\log_{10} x}$$

$$\Rightarrow \frac{1}{2} \log_{10} x = \frac{2}{\log_{10} x}$$

$$\Rightarrow \frac{1}{2} (\log_{10} x)^2 = 2 \Rightarrow (\log_{10} x)^2 = 4$$

$$\Rightarrow \log_{10} x = \pm 2$$

$$\text{If } \log_{10} x = 2 \Rightarrow x = 10^2 \Rightarrow x = 100$$

$$\text{If } \log_{10} x = -2$$

$$\Rightarrow x = 10^{-2} \Rightarrow x = \frac{1}{100} \therefore x = 100, \frac{1}{100}$$

Exp. 6) Find the value of $\log_y x \times \log_z y \times \log_x z$.

- (a) 0 (b) 1 (c) $\log x$ (d) xyz

Solution $\log_y x \times \log_z y \times \log_x z = \frac{\log_a x}{\log_a y} \times \frac{\log_a y}{\log_a z} \times \frac{\log_a z}{\log_a x} = 1$

Exp. 7) If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, then find the value of $a^a \cdot b^b \cdot c^c$.

Solution We have, $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b} = k$ (say)

Then, $\log a = k(b-c)$, $\log b = k(c-a)$, $\log c = k(a-b)$

Let $A = a^a \cdot b^b \cdot c^c$

Then $\log A = a \log a + b \log b + c \log c$

$$= ak(b-c) + bk(c-a) + ck(a-b)$$

$$= abk - ack + bck - abk + cak - bck = 0$$

Thus $\log A = 0 = \log 1$

$$\therefore A = 1 \text{ i.e., } a^a \cdot b^b \cdot c^c = 1$$

Exp. 8) Find the value of $\log_8 25$, given that $\log_{10} 2 = 0.3010$

Solution $\log_8 25 = \frac{\log_{10} 25}{\log_{10} 8} = \frac{2 \log_{10} 5}{3 \log_{10} 2} = \frac{2 \log_{10}(10/2)}{3 \log_{10} 2}$
 $= \frac{2(\log_{10} 10 - \log_{10} 2)}{3 \times 0.3010} = \frac{2(1 - 0.3010)}{0.9030} = 1.5482$

Exp. 9) Find the value of $\frac{\log \sqrt{27} + \log 8 + \log \sqrt{1000}}{\log 120}$.

Solution $\frac{\log (27)^{1/2} + \log 2^3 + \log (10^3)^{1/2}}{\log (3 \times 2^2 \times 10)}$
 $= \frac{\frac{1}{2} \log 3^3 + 3 \log 2 + \frac{1}{2} \log (10)^3}{\log 3 + \log 2^2 + \log 10}$
 $= \frac{\frac{3}{2} \log 3 + \frac{3}{2} \cdot 2 \log 2 + \frac{3}{2} \log 10}{\log 3 + 2 \log 2 + \log 10}$
 $= \frac{\frac{3}{2} (\log 3 + 2 \log 2 + \log 10)}{\log 3 + 2 \log 2 + \log 10} = \frac{3}{2}$

Exp. 10) Solve for x , if $\log_x (8x - 3) - \log_x 4 = 2$

Solution $\log_x (8x - 3) - \log_x 4 = 2 \Rightarrow \log_x \left(\frac{8x - 3}{4} \right) = 2$
 $\Rightarrow x^2 = \frac{8x - 3}{4} \Rightarrow 4x^2 - 8x + 3 = 0 \Rightarrow x = \frac{3}{2} \text{ or } \frac{1}{2}$

Exp. 11) Find the value of

$$x^{\log y - \log z} \times y^{\log z - \log x} \times z^{\log x - \log y}$$

Solution Let $P = x^{\log y - \log z} \times y^{\log z - \log x} \times z^{\log x - \log y}$

Then $\log P = \log x^{\log y - \log z} + \log y^{\log z - \log x} + \log z^{\log x - \log y}$

$$= (\log y - \log z) \log x + (\log z - \log x) \log y + (\log x - \log y) \log z$$

$$= \log x \log y - \log z \log x + \log y \log z - \log x \log y \\ + \log z \log x - \log y \log z = 0$$

$$\text{i.e., } \log P = 0 = \log 1 \therefore P = 1$$

Exp. 12) If $\frac{xy \log(xy)}{x+y} = \frac{yz \log(yz)}{y+z} = \frac{zx \log(zx)}{z+x}$, then

show that $x^x = y^y = z^z$.

Solution $\frac{xy \log(xy)}{x+y} = \frac{yz \log(yz)}{y+z} = \frac{zx \log(zx)}{z+x} = k$ (say)

Then $xy \log xy = k(x+y)$

or $\log xy = k \left(\frac{1}{x} + \frac{1}{y} \right)$

or $\log x + \log y = k \left(\frac{1}{x} + \frac{1}{y} \right)$... (i)

Similarly, $\log y + \log z = k \left(\frac{1}{y} + \frac{1}{z} \right)$... (ii)

and $\log z + \log x = k \left(\frac{1}{z} + \frac{1}{x} \right)$... (iii)

Adding (i), (ii) and (iii), we get

$$2(\log x + \log y + \log z) = 2k \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right)$$

or $\log x + \log y + \log z = k \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right)$... (iv)

Subtracting eqs. (i), (ii) and (iii) successively from (iv), we get

$$\log z = k \cdot \frac{1}{z}, \quad \log x = k \cdot \frac{1}{x}, \quad \log y = k \cdot \frac{1}{y}$$

or $z \log z = k, \quad x \log x = k$

and $y \log y = k$ or $x \log x = y \log y = z \log z$

or $\log(x^x) = \log(y^y) = \log(z^z)$

Hence $x^x = y^y = z^z$

Exp. 13) If $\log_a bc = x$, $\log_b ca = y$, $\log_c ab = z$, prove that

$$\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = 1.$$

Solution $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$

$$= \frac{1}{(\log_a bc) + 1} + \frac{1}{(\log_b ca) + 1} + \frac{1}{(\log_c ab) + 1}$$

$$= \frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ca + \log_b b} + \frac{1}{\log_c ab + \log_c c}$$

$$= \frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$$

$$= \log_{abc} a + \log_{abc} b + \log_{abc} c \quad \left(\because \log_a b = \frac{1}{\log_b a} \right)$$

$$= \log_{abc}(abc) = 1$$

Exp. 14) Given $\log 2 = 0.3010$ and $\log 3 = 0.4771$, find the value of $\log 45$.

Solution $\log 45 = \log 3^2 \cdot 5 = \log 3^2 + \log 5 = 2 \log 3 + \log 5$

$$= 2 \times 0.4771 + \log \frac{10}{2} = 0.9542 + \log 10 - \log 2$$

$$= 0.9542 + 1 - 0.3010 = 1.6532$$

Exp. 15) Given $\log 2 = 0.3010$ and $\log 3 = 0.4771$. Find the value of $\log 0.0075$.

Solution $\log 0.0075 = \log \frac{75}{10000} = \log 75 - \log 10000$

$$= \log 3 \times 5^2 - \log 10^4 = \log 3 + 2 \log 5 - 4 \log 10$$

$$= \log 3 + 2[\log 10 - \log 2] - 4 \log 10$$

$$= 0.4771 + 2[1 - 0.3010] - 4 = -2.1249$$

Exp. 16) What is the number of digits in 3^{3^3} ? Given that $\log 3 = 0.47712$?

Solution Let $x = 3^{3^3} = (3)^{3^3}$

Then $\log x = 3^3 \log 3 = 27 \times 0.47712 = 12.88224$

Since the characteristic in the resultant value of $\log x$ is 12

$\therefore$ The number of digits in x is $(12 + 1) = 13$

Hence the required number of digits in 3^{3^3} is 13.

Exp. 17) If $\log 3 = 0.4771$, find the number of digits in 3^{43} .

Solution Let $x = 3^{43}$

$\therefore \log x = \log 3^{43} = 43 \log 3 = 43 \times 0.4771 = 20.5153$

Since the characteristic of $\log 3^{43}$ is 20, then number of digits in 3^{43} is $(20 + 1) = 21$.

Exp. 18) How many zeros are there between the decimal point and the first significant figure in $(0.5)^{100}$? Given that $\log \frac{1}{2} = \bar{1}.6990$

Solution Let $x = (0.5)^{100}$

$\therefore \log x = 100 \log (0.5) = 100 \times \bar{1}.6990$

$$= 100(-1 + 0.6990) = -100 + 69.90$$

$$= -31 + 0.90 = \bar{31}.90$$

Since the characteristics of $\log (0.5)^{100}$ is $\bar{31}$, the number of zeros between the decimal point and the first significant figure is $31 - 1 = 30$.

Exp. 19) If $\frac{x(y+z-x)}{\log x} = \frac{y(z+x-y)}{\log y} = \frac{z(x+y-z)}{\log z}$, then

- (a) $xy = yz = zx$ (b) $x^y y^x = z^y y^z = x^z z^x$
- (c) $xyz = 1$ (d) none of these

Solution Let $\frac{x(y+z-x)}{\log x} = \frac{y(z+x-y)}{\log y}$

$$= \frac{x(x+y-z)}{\log z} = \frac{1}{k} \quad (\text{say})$$

$\Rightarrow \log x = kx(y+z-x); \log y = ky(z+x-y)$

$$\log z = kz(x+y-z)$$

Hence $y \log x + x \log y = 2kxyz, y \log z + z \log y = 2kxyz$

$$z \log x + x \log z = 2kxyz$$

Thus $y \log x + x \log y = y \log z + z \log y = z \log x + x \log z$

$\Rightarrow \log(x^y y^x) = \log(z^y y^z) = \log(x^z z^x)$

$\Rightarrow x^y y^x = z^y y^z = x^z z^x$

Introductory Exercise 16.1

- If $x^y = k$, then :
 (a) $\log_y k = x$ (b) $\log_x y = k$
 (c) $\log_k x = y$ (d) $\log_x k = y$
- If $\log_m n = p$, then :
 (a) $m^n = p$ (b) $p^n = m$
 (c) $m^p = n$ (d) $n^p = m$
- $\log_a(mn)$ is equal to :
 (a) $n \log_a(m)$ (b) $\log_a m \times \log_a n$
 (c) $\log_a m + \log_a n$ (d) $\log_a m - \log_a n$
- $\log_a \left(\frac{m}{n}\right)$ is equal to :
 (a) $\log_a(m - n)$ (b) $\log_a m - \log_a n$
 (c) $\frac{(\log_a m)}{n}$ (d) $\log_a m \div \log_a n$
- The value of $\log_{81} 27$ is :
 (a) 3^7 (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$
- The value of $\log_{36} \frac{1}{216}$ is :
 (a) $\frac{2}{3}$ (b) $-\frac{3}{2}$
 (c) $\frac{1}{6}$ (d) none of these
- The value of $\log_{\sqrt{6}} 216$ is :
 (a) $\frac{1}{6}$ (b) $\sqrt{6}$ (c) 3 (d) 6
- The value of $\log_{3\sqrt{2}} 5832$ is :
 (a) 12 (b) 9
 (c) 6 (d) none of these
- The value of $\log_{(-1/5)} 625$ is :
 (a) $\frac{1}{4}$ (b) -5 (c) -4 (d) -3
- The value of $\log_{10}(0.0001)$ is :
 (a) $\frac{1}{1000}$ (b) -3
 (c) -4 (d) none of these
- The value of $\log_{(0.01)}(10000)$ is :
 (a) $1/2$ (b) -2 (c) $1/4$ (d) -4
- The value of $(\log_4 32 + \log_9 243)$:
 (a) 5 (b) 4
 (c) 3 (d) none of these
- The value of $(\log_2 8 + \log_3 9 + \log_5 25)$ is :
 (a) 5 (b) 6
 (c) 7 (d) none of these
- The value of $\log_2(\log_3 81)$ is :
 (a) 0 (b) 1 (c) 2 (d) 6
- The value of $\log_2 \log_2 \log_3 \log_5(125)^3$ is :
 (a) 0 (b) 1 (c) 2 (d) 3
- The value of $\log_2 \log_2 \log_2 \log_2(65536)$ is :
 (a) 4 (b) 2 (c) 1 (d) 0
- The value of $\log_{10} 1 + \log_{10} 10 + \log_{10} 100 + \dots + \log_{10} 10000000000$
 (a) 10 (b) 11
 (c) 11111111111 (d) 55
- The value of $\log_5 5 + \log_5 5^2 + \log_5 5^3 + \dots + \log_5 5^n$:
 (a) $n!$ (b) $n^2 - 1$
 (c) $\frac{(n+1)n}{2}$ (d) none of these
- The value of $27^{\log_3 81}$ is :
 (a) 27381 (b) 531441
 (c) 65631 (d) none of these
- The value of $_{216}\log_6 49$ is :
 (a) 117694 (b) 117649
 (c) 216649 (d) none of these
- The value of $25^{(-1/4 \log_5 25)}$ is :
 (a) $\frac{1}{5}$ (b) $-\frac{1}{25}$
 (c) -25 (d) none of these
- $(\log_{10} 500000 - \log_{10} 5)$ is equal to :
 (a) 5 (b) 50
 (c) 49995 (d) 100000
- If $\log_3[\log_3(\log_2 x)] = 1$, then x is :
 (a) 2^{234} (b) 243
 (c) 2^{243} (d) none of these
- If $\log_2 \log_{10} 4x = 1$, then the value of x is :
 (a) $1/25$ (b) 25
 (c) 5 (d) none of these
- If $\log_{27} x = \frac{4}{3}$, then the value of x is :
 (a) 9
 (b) 27
 (c) 81
 (d) none of the above
- If $\log_x \left(\frac{4}{9}\right) = -\frac{1}{2}$, then the value of x is :
 (a) $\frac{16}{81}$ (b) $-\frac{2}{3}$
 (c) $\frac{81}{16}$ (d) none of these

27. If $\log_{100} x = -4$, then the value of x is :
 (a) $\frac{1}{(10)^{-4}}$ (b) $(10)^{-4}$
 (c) $\frac{1}{(10)^8}$ (d) none of these
28. If $\log_{100} x = -\frac{1}{2}$, then the value of x is :
 (a) 0.001 (b) $\frac{1}{10}$
 (c) 0.00001 (d) 10^{-2}
29. If $\log_x(0.2) = -\frac{1}{4}$, then the value of x is :
 (a) 4^{-5} (b) 0.016
 (c) 5^4 (d) none of these
30. The value of x satisfying $\log_{243} x = 0.8$ is :
 (a) 81 (b) 1.8
 (c) 2.43 (d) none of these
31. If $\log_e(x-1) + \log_e x + \log_e(x+1) = 0$, then
 (a) $x^2 + e^{-1}$ (b) $x^3 - x - 1 = 0$
 (c) $x^2 + e - 1 = 0$ (d) $x^3 - x - e = 0$
32. The value of $\log_3 5 \times \log_{25} 9$ is :
 (a) 0 (b) 1
 (c) 2 (d) 4
33. The value of $\left(\log \frac{14}{3} + \log \frac{11}{5} - \log \frac{22}{15}\right)$ is :
 (a) $\log 77$ (b) $\log 11$
 (c) $\log 7$ (d) none of these
34. The value of $\left[3 \log \left(\frac{81}{80}\right) + 5 \log \left(\frac{25}{24}\right) + 7 \log \left(\frac{16}{15}\right)\right]$ is
 (a) $\log 3$ (b) $\log 5$
 (c) $\log 7$ (d) $\log 2$
35. $\left(\log \frac{a^3}{bc} + \log \frac{b^3}{ac} + \log \frac{c^3}{ab}\right)$ is equal to :
 (a) 1 (b) $\log abc$
 (c) abc (d) none of these
36. $\frac{1}{(\log_a bc) + 1} + \frac{1}{(\log_b ac) + 1} + \frac{1}{(\log_c ab) + 1}$ is equal to:
 (a) 1 (b) 2
 (c) 0 (d) abc
37. $\frac{1}{\log_{ab} abc} + \frac{1}{\log_{bc} abc} + \frac{1}{\log_{ca} abc}$ is equal to :
 (a) 0 (b) 1
 (c) 2 (d) 3
38. The value of $\left(\frac{1}{\log_5 210} + \frac{1}{\log_6 210} + \frac{1}{\log_7 210}\right)$ is :
 (a) 0 (b) 1
 (c) 18 (d) 21
39. The value of $\left[\frac{1}{\log_{a/b} x} + \frac{1}{\log_{b/c} x} + \frac{1}{\log_{c/a} x}\right]$ is :
 (a) 0 (b) 1 (c) abc (d) x^3
40. If $\log_{10} 2 = 0.3010$, then $\log_2 10$ is :
 (a) 3.010 (b) 1.5050
 (c) 3.3222 (d) none of these
41. If $2^x \cdot 3^{2x} = 100$, then the value of x is
 ($\log 2 = 0.3010, \log 3 = 0.4771$)
 (a) 2.3 (b) 1.59
 (c) 1.8 (d) 1.41
42. If $\log_{10} 2 = 0.3010$ and $5^x = 400$, then the value of x is:
 (a) 3.72 (b) 2.52
 (c) 0 (d) none of these
43. If $\log 2 = 0.3010$, the number of digits in 5^{20} is :
 (a) 16 (b) 14
 (c) 8 (d) none of these
44. If $\log(m+n) = \log m + \log n$, then :
 (a) $mn = 1$ (b) $m = -n$
 (c) $\frac{m}{(m-1)} = n$ (d) none of these
45. If $\log_{10} a + \log_{10} b = c$, then the value of a is :
 (a) bc (b) $\frac{c}{b}$
 (c) $\frac{(10)^c}{b}$ (d) $\frac{10b}{c}$
46. The mantissa of $\log 3274$ is 0.5150. The value of $\log(0.3274)$ is :
 (a) $\bar{1}.5150$ (b) 1.5150
 (c) $\bar{2}.5150$ (d) none of these
47. If $\log_{10} x = 1.9675$, then the value of $\log_{10}(100x)$ is :
 (a) 196.75 (b) 3.9675
 (c) 2.9675 (d) none of these
48. If $5^x = (0.5)^y = 1000$, then the value of $\left(\frac{1}{x} - \frac{1}{y}\right)$ is :
 (a) $\frac{1}{4}$ (b) $\frac{1}{3}$
 (c) $\frac{1}{2}$ (d) 1
49. If $\frac{1}{2} \log x + \frac{1}{2} \log y + \log 2 = \log(x+y)$, then :
 (a) $x+y = 0$ (b) $x = y$
 (c) $x=2, y=0$ (d) $x = \log y$
50. If $p = \log_3 5$ and $q = \log_{17} 25$, which one of the following is correct ?
 (a) $p < q$ (b) $p = q$
 (c) $p > q$ (d) can't say

51. If $x^2 + 4y^2 = 12xy$, then $\log(x + 2y)$ is equal to :

- (a) $\frac{1}{2}(\log x + \log y + 2 \log 2)$
 (b) $\frac{1}{2}(\log x + \log y + 4 \log 2)$
 (c) $\frac{1}{2}(\log x + \log y - \log 2)$
 (d) none of the above

52. The value of $(2^{\log_3 7} - 7^{\log_3 2})$ is :

- (a) 0 (b) 1
 (c) $\log_7 5$ (d) $\log_7 6$

53. $\frac{1}{\log_2 a} + \frac{1}{\log_4 a} + \frac{1}{\log_8 a} + \dots$ to n terms $= \frac{n(n+1)}{k}$, then k is equal to :

- (a) $\log_2 a^2$ (b) $\log_2 \left(\frac{a}{2}\right)$
 (c) $\log_a 2$ (d) none of these

54. The value of $(\log \tan 1^\circ + \log \tan 2^\circ + \dots + \log \tan 89^\circ)$

- is :
 (a) -1 (b) 0
 (c) $\frac{1}{\sqrt{2}}$ (d) 1

55. The value of x for which $\log_9 x - \log_9 \left(\frac{x}{10} + \frac{1}{9}\right) = 1$ is :

- (a) 2
 (b) 4
 (c) 9
 (d) 10

56. If $(x^4 - 2x^2y^2 + y^4)^{a-1} = (x - y)^{2a} (x + y)^{-2}$, then the value of a is :

- (a) $(x^2 - y^2)$ (b) $\frac{x - y}{x + y}$
 (c) $\frac{\log(x + y)}{\log(x - y)}$ (d) $\frac{\log(x - y)}{\log(x + y)}$

57. $\log_2 7$ is :

- (a) an integer
 (b) a prime number
 (c) a rational number
 (d) an irrational number

58. $\log_y x = ?$

- (a) $x \log_e y$ (b) $\frac{x}{\log_e y}$
 (c) $\frac{\log_e x}{\log_e y}$ (d) $\frac{\log_e y}{\log_e x}$

59. The value of

$$\log_{10} 2 + 16 \log_{10} \left(\frac{16}{15}\right) + 12 \log_{10} \left(\frac{25}{24}\right) + 7 \log_{10} \left(\frac{81}{80}\right)$$

is :

- (a) 0 (b) 1
 (c) 2 (d) 3

60. If $\log_{10} x = a$, $\log_{10} y = b$ and $\log_{10} z = c$, then $\text{antilog}(pa + qb - rc) = ?$

- (a) $\frac{pxqy}{rz}$ (b) $px + qy - rz$
 (c) $\frac{x^p y^q}{z^r}$ (d) $x^p y^q z^r$

61. $\log_{10} a^p \cdot b^q \cdot c^r = ?$

- (a) $pqr (\log_{10} abc)$
 (b) $p \log_{10} a + q \log_{10} b + r \log_{10} c$
 (c) $pqr (\log_{10} a + \log_{10} b + \log_{10} c)$
 (d) $(\log_{10} a)^p + (\log_{10} b)^q + (\log_{10} c)^r$

62. If a, b, c are in GP then $\log_{10} a, \log_{10} b, \log_{10} c$ are in :

- (a) GP (b) HP
 (c) AP (d) none of these

63. If $\log_{10} x, \log_{10} y, \log_{10} z$ are in AP then x, y, z are in :

- (a) AP (b) GP
 (c) HP (d) none of these

64. If a, b, c are in GP then $\frac{1}{\log_a x}, \frac{1}{\log_b x}, \frac{1}{\log_c x}$ are in :

- (a) AP (b) GP
 (c) HP (d) none of these

65. If $\log(x - 1) + \log(x + 1) = 3 \log 2$, then x is equal to :

- (a) ± 1 (b) 4
 (c) ± 3 (d) none of these

66. If a, b, c are three consecutive integers, then

$\log(ac + 1)$ has the value :

- (a) $\log b$ (b) $(\log b)^2$
 (c) $2 \log b$ (d) $\log 2b$

CAT-Test

Questions Helping you bell the CAT

LEVEL 01 > BASIC LEVEL EXERCISE

- 1 Find the value of $\log_{10} \sqrt[3]{13}$ if $\log_{10} 13 = 1.1139$:
 (a) 0.15913 (b) 0.5119
 (c) 1.15913 (d) none of these
- 2 Find the logarithm of 144 to the base $2\sqrt{3}$:
 (a) 8 (b) 4
 (c) $2\sqrt{3}$ (d) none of these
- 3 If $\log_a N = (\log_b N) \times P$, then find P in terms of a and b :
 (a) b^a (b) a^b
 (c) $\log_a b$ (d) none of these
- 4 The value of $\log(ab)^2 - \log(ac) + \log(bc^4) - 3 \log(bc)$ is :
 (a) 0 (b) $\log b$
 (c) $\log c$ (d) $\log a$
- 5 If $\log_2 x + \log_4 x + \log_{64} x = 5$, find x :
 (a) 8 (b) 16 (c) 7 (d) 2
- 6 If $\log_q(xy) = 3$ and $\log_q(x^2y^3) = 4$, find the value of $\log_q x$
 (a) 4 (b) 5 (c) 3 (d) 2
- 7 If $\frac{\log x}{l+m-2n} = \frac{\log y}{m+n-2l} = \frac{\log z}{n+l-2m}$, then xyz is equal to :
 (a) 0 (b) 1 (c) lmn (d) 2
- 8 If a, b, c be the p^{th}, q^{th}, r^{th} terms of a GP then the value of $(q-r) \log a + (r-p) \log b + (p-q) \log c$ is :
 (a) 0 (b) 1 (c) -1 (d) pqr
- 9 If $a^{3-x} b^{5x} = a^{x+5} b^{3x}$, then the value of $x \log \left(\frac{b}{a}\right)$ is :
 (a) $\log a$ (b) $\log b$
 (c) $\log x$ (d) 1
- 10 If $u = v^2 = w^3 = z^4$, then $\log_u(uvwz)$ is equal to :
 (a) $1 + \frac{1}{2} - \frac{1}{3} - \frac{1}{4}$ (b) 24
 (c) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ (d) $\frac{1}{24}$
- 11 Find the value of $\frac{\log \sqrt{27} + \log \sqrt{8} - \log \sqrt{125}}{\log 6 - \log 5}$:
 (a) $\frac{2}{3}$ (b) $\frac{1}{3}$
 (c) $\frac{3}{2}$ (d) none of these
- 12 The first term and the last term of a GP are a and k respectively. If the number of terms be n , then n is equal to ($r \rightarrow$ common ratio) :
 (a) $1 - \frac{\log k - \log a}{\log r}$ (b) $1 + \frac{\log a + \log k}{\log r}$
 (c) $1 + \frac{\log k - \log a}{\log r}$ (d) $\log r = \log k - \log a$
- 13 If a, b, c are in GP, then $\log_a d, \log_b d, \log_c d$ are in :
 (a) AP (b) HP
 (c) GP (d) none of these
- 14 Find the value of x for $\log_x 2 \cdot \log_{x/16} 2 = \log_{x/64} 2$:
 (a) 4 (b) 4, 16 (c) 4, 8 (d) 4, 8, 32
- 15 Find the value of x and y respectively for
 $\log_{10}(x^2y^3) = 7$ and $\log_{10}(x/y) = 1$:
 (a) $x = 10, y = 100$ (b) $x = 100, y = 10$
 (c) $x = 10, y = 20$ (d) none of these
- 16 If $y = a^{\left(\frac{1}{1-\log_a x}\right)}$ and $z = a^{\left(\frac{1}{1-\log_a y}\right)}$, then x is equal to :
 (a) $1 + \log yz$ (b) $\frac{1}{1 - \log_a z}$
 (c) $a^{\left(\frac{1}{1 - \log_a z}\right)}$ (d) $\frac{y}{z}$
- 17 If $x = \log_c b + \log_b c$, $y = \log_a c + \log_c a$ and $z = \log_b a + \log_a b$, then $x^2 + y^2 + z^2$ is equal to :
 (a) 1 (b) 2 (c) xyz (d) $xyz + 4$
- 18 Find the value of $\frac{1}{\log_3 e} + \frac{1}{\log_3 e^2} + \frac{1}{\log_3 e^4} + \dots$ is :
 (a) $\log_e 9$ (b) 0
 (c) 1 (d) $\log_e 270$
- 19 If $\log_{10} x^2 - \log_{10} \sqrt{y} = 1$, find the value of y , when $x = 2$
 (a) $2/9$ (b) $4/25$ (c) $25/4$ (d) $4/9$
- 20 Find the value of $(b^2)^{5 \log_b x}$:
 (a) 10^x (b) x^4 (c) $10x$ (d) x^{10}
- 21 Find the value of $(7^3)^{-2 \log_7 8}$:
 (a) 8^{-7} (b) 6^{-8}
 (c) 8^{-6} (d) none of these

- 22** $\log 12900$ is equal to :
 (a) $2 \log 129$ (b) $\log 129 + \log 2$
 (c) $4 + \log 1.29$ (d) none of these
- 23** $\log 0.786$ is equal to :
 (a) $\log 786 - \log 3$ (b) $\log 78.6 - \log 100$
 (c) $\log 786 - 3$ (d) both (b) and (c)
- 24** If $\log_5 x = y$, then 5^{5y} is equal to :
 (a) $\frac{x}{5}$ (b) $5x$ (c) $\log_x 5$ (d) x^5
- 25** If $A = \log_{13} 189$ and $B = \log_{23} 521$, then which one of the following is correct?
 (a) $A < B$ (b) $A = B$
 (c) $A > B$ (d) none of these
- 26** If $A = \frac{(\log_3 2187)}{5}$ and $B = \log_{243} 2187$, then which one of the following is correct?
 (a) $A < B$ (b) $A = B$
 (c) $A > B$ (d) can't be determined
- 27** If $(150)^x = 7$, then x is equal to :
 (a) $\frac{\log 7}{(\log 3) + (\log 5) + 1}$ (b) $\frac{\log 7}{(\log 3) + (\log 6)}$
 (c) $\frac{\log 7}{(\log 3) + (\log 5) + 10}$ (d) none of these
- 28** The value of x satisfying the following relation
 $\log_{1/2} x = \log_2 (3x - 2)$
 (a) $\frac{1}{3}$ (b) $-\frac{1}{3}$
 (c) 3 (d) none of these
- 29** If $\log_2 (x + y) = 3$ and $\log_2 x + \log_2 y = 2 + \log_2 3$ then the values of x and y are
 (a) $x = 1, y = 8$ (b) $x = 4, y = 4$
 (c) $x = 4, y = 8$ (d) $x = 2, y = 6$
- 30** The set of all the solutions of the equation
 $\log_5 x \log_6 x \log_7 x = \log_5 x \cdot \log_6 x + \log_6 x \cdot \log_7 x + \log_7 x \cdot \log_5 x$ is :
 (a) $\{0, 1\}$ (b) $\{1, 210\}$
 (c) $\{1, 5, 6, 7, 210\}$ (d) none of these
- 31** The value of $\frac{\log_3 54}{\log_{486} 3} - \frac{\log_3 1458}{\log_{18} 3}$ is :
 (a) 0 (b) 1 (c) 2 (d) 3
- 32** The number of solutions of $\log_9 (2x - 5) = \log_3 (x - 4)$ is :
 (a) 0 (b) 1 (c) 2 (d) 3
- 33** If $\log_3 2, \log_3 (2^x - 5)$ and $\log_3 (2^x - 7/2)$ are in AP then x is equal to :
 (a) 2 (b) 3 (c) 4 (d) 5
- 34** If $1, \log_y x, \log_z y, -15 \log_x z$ are in AP, then
 (a) $z^3 = x$ (b) $x = y^{-1}$
 (c) $z^{-3} = y$ (d) $x = y^{-1} = z^3$
- 35** If $\log_x a, a^{x/2}$ and $\log_b x$ are in GP, then x is equal to :
 (a) $\log_a (\log_b a)$ (b) $\log_a (\log_e a) - \log_a (\log_e b)$
 (c) $-\log_a (\log_a b)$ (d) both (a) and (b)
- 36** If $1, \log_9 (3^{1-x} + 2)$ and $\log_3 (4 \cdot 3^x - 1)$ are in AP, then x is equal to :
 (a) $\log_4 3$ (b) $\log_3 4$
 (c) $1 + \log_3 4$ (d) $\log_3 (3/4)$
- 37** The value of $\frac{1}{\log_{100} n} + \frac{1}{\log_{99} n} + \frac{1}{\log_{98} n} + \dots + \frac{1}{\log_2 n}$ is:
 (a) 1 (b) $\frac{1}{\log_{100!} n}$ (c) $\frac{1}{\log_n 99!}$ (d) $\frac{1}{\log_n 100!}$
- 38** If $\log_2 (1 + 3^{x-1}) + 2 = \log_2 (3^{2x-2} + 7)$, then x is :
 (a) 0 (b) 1
 (c) 2 (d) both (b) and (c)
- 39** $x^{\log_5 x} > 5$ implies :
 (a) $x \in (0, \infty)$ (b) $x \in (1, \infty)$
 (c) $x \in (1, 2)$ (d) $x \in (0, 1/5) \cup (5, \infty)$
- 40** Find x , if $\log x^3 - \log 3x = 2 \log 2 + \log 3$:
 (a) 3 (b) 6
 (c) 9 (d) none of these
- 41** If x satisfies $\log_5 (2x + 3) < \log_5 7$, then x lies in :
 (a) $\left(-\frac{3}{2}, 2\right)$ (b) $\left(\frac{3}{2}, 2\right)$ (c) $(0, \infty)$ (d) $(0, 2)$
- 42** Find x , if $\log_{2x} \sqrt{x} + \log_{2\sqrt{x}} x = 0$:
 (a) $1, 2^{-5/6}$ (b) $1, 2^{-6/5}$
 (c) 4, -2 (d) none of these
- 43** For a positive real x ($x > 1$), which one of the following is correct ?
 (a) $\frac{x}{1+x} < x < \log(x+1)$ (b) $x < \frac{x}{1+x} < \log(x+1)$
 (c) $\log(x+1) < \frac{x}{1+x} < x$ (d) $\frac{x}{1+x} < \log(x+1) < x$
- 44** For $x \in N, x > 1$, if $P = \log_x (x+1)$ and $Q = \log_{(x+1)} (x+2)$, then which one of the following is correct ?
 (a) $P < Q$ (b) $P = Q$
 (c) $P > Q$ (d) can't be determined
- 45** If $a = 1 + \log_x yz, b = 1 + \log_y zx$ and $c = 1 + \log_z xy$, then $ab + bc + ca$ is :
 (a) 1 (b) 0
 (c) abc (d) none of these
- 46** Find the values of x and y for the given equation
 $xy^2 = 4$ and $\log_3 (\log_2 x) + \log_{1/3} (\log_{1/2} y) = 1$:
 (a) $x = \frac{1}{8}, y = 64$ (b) $x = \frac{1}{4}, y = 48$
 (c) $x = 64, y = \frac{1}{4}$ (d) none of these
- 47** Find x if $\log_{1/\sqrt{2}} (1/\sqrt{8}) = \log_2 (4^x + 1) \cdot \log_2 (4^{x+1} + 4)$:
 (a) 0 (b) 1
 (c) 2 (d) none of these

- 48 The value of x satisfying

$$\log_3 4 - 2 \log_3 \sqrt{3x+1} = 1 - \log_3 (5x-2)$$

- (a) 1 (b) 2
(c) 3 (d) none of these

- 49 The solution set of $|3-4x| > 2$ is :

- (a) $\left(\frac{7}{4}, \infty\right)$ (b) $\left(-\frac{1}{4}, \frac{7}{4}\right)$
(c) $\left(-\infty, -\frac{1}{4}\right) \cup \left(\frac{7}{4}, \infty\right)$ (d) $\left(-\infty, \frac{1}{4}\right) \cup \left(\frac{5}{4}, \infty\right)$

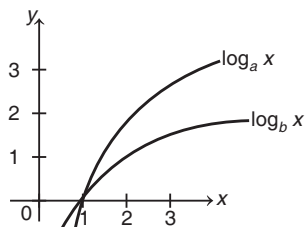
- 50 Solve the following equations for x and y

$$\log_{100} |x+y| = \frac{1}{2}, \log_{10} y - \log_{10} |x| = \log_{100} 4 :$$

- (a) $\left(\frac{8}{3}, \frac{16}{3}\right), (-8, -16)$
(b) $\left(\frac{10}{3}, \frac{20}{3}\right), (-10, 20)$
(c) $\left(-\frac{10}{3}, -\frac{20}{3}\right), (70, 20)$
(d) none of the above

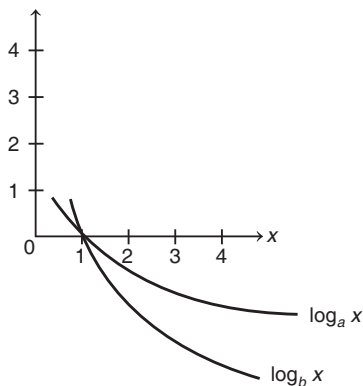
LEVEL 02 > HIGHER LEVEL EXERCISE

- 1 In the adjoining diagram there are two curvature graphs of $\log_a x$ and $\log_b x$ shown for $x > 0$. If $(a, b) > 0$ and $a \neq b \neq 1$, then :



- (a) $a < b$ (b) $a > b$
(c) can't be determined (d) none of the above

- 2 In the adjoining diagram graphs of $\log_a x$ and $\log_b x$ are shown for $x > 0$ and $0 < a < 1$, then :



- (a) $a < b$ (b) $a > b$
(c) can't be determined (d) none of these

- 3 Find the sum of ' n ' terms of the series.

$$\log_2 \left(\frac{x}{y}\right) + \log_4 \left(\frac{x}{y}\right)^2 + \log_8 \left(\frac{x}{y}\right)^3 + \log_{16} \left(\frac{x}{y}\right)^4 + \dots :$$

- (a) $\log_2 \left(\frac{x}{y}\right)^{4n}$ (b) $n \left(\log_2 \frac{x}{y}\right)$
(c) $\log_2 \left(\frac{x^{n-1}}{y^{n-1}}\right)$ (d) $\frac{1}{2} \log_2 \left(\frac{x}{y}\right)^{n(n+1)}$

- 4 Find the value of $\log m + \log m^2 + \log m^3 + \dots + \log m^n$:

- (a) $\frac{n(n+1)}{2}$ (b) $\frac{mn}{2}$
(c) $\frac{n(n+1)}{2} \log m$ (d) $n(n+1) \log m^2$

- 5 The greatest possible value of n could be if $9^n < 10^8$, given that $\log 3 = 0.4771$ and $n \in \mathbb{N}$:

- (a) 7 (b) 8 (c) 9 (d) 10

- 6 The set of solutions for all x satisfying the equation

$$x^{\log_3 x^2 + (\log_3 x)^2 - 10} = \frac{1}{x^2} \text{ is :}$$

- (a) $\{1, 9\}$ (b) $\{1, 4, 16\}$ (c) $\{1, 9, 81\}$ (d) $\left\{9, \frac{1}{81}, 1\right\}$

- 7 The set of all the solutions of the inequality

$$\log_{(2-x)}(x-3) \geq 1 \text{ is :}$$

- (a) $x < 2$ (b) $x > 3$
(c) $(x < 2) \cup (x > 3)$ (d) none of these

- 8 If $\log_3 30 = \frac{1}{a}$ and $\log_5 30 = \frac{1}{b}$ then the value of $3 \log_{30} 2$

is:

- (a) $3(1+a+b)$ (b) $2(1-a-b)$
(c) $3(1-a-b)$ (d) $3(1+a-b)$

- 9 The set of all values of x satisfying $x^{\log_x |3-x|^2} = 4$:

- (a) $(\sqrt{3}, 2)$ (b) $(1, 5)$ (c) $[1, \infty)$ (d) $\{1, 5\}$

- 10 The number of solutions of the expression satisfying $4^{x^2+2} - 9 \cdot 2^{x^2+2} + 8 = 0$ is :

- (a) 0 (b) 1
(c) 2 (d) none of these

- 11 The number of solutions satisfying, for $x \in \mathbb{R}$

$$x^{\left[(\log_3 x)^2 - \frac{9}{2} \log_3 x + 5\right]} = 3\sqrt{3}$$

- (a) 0 (b) 1 (c) 2 (d) 3

- 12 Find the values of x satisfying

$$\log_{x^2+6x+8} \log_{2x^2+2x+3} (x^2-2x) = 0 \text{ is :}$$

- (a) 0 (b) -1 (c) 2 (d) -3

- 13** For every p being real number the solution set of the inequality

$$\log_{10} p + (\log_{10} 10p)^2 + (\log_{10} 100p)^2 \leq \log_{10}(10)^9$$

- (a) $p \geq 10^{-4}$ (b) $p > 10^{1/2}$
 (c) $10^{-4} \leq p \leq 10^{1/2}$ (d) none of the above
- 14** The number of real solutions of the equation $2\log_2 \log_2 x + \log_{1/2} \log_2(2\sqrt{2}x) = 1$ is :
- (a) 1 (b) 2
 (c) 3 (d) more than 3
- 15** The number of solutions of the equation $\log_{x/2} x^2 + 40 \log_{4x} \sqrt{x} - 14 \log_{16x} x^3 = 0$ is :
- (a) 0 (b) 1 (c) 2 (d) 3
- 16** Find the solution set of x , for the given inequality $\log_m n > 1$, where $m = \frac{(5)^2 - (x)^2}{(4)^2}$ and $n = \frac{2(12-x) - x^2}{14}$:
- (a) $(-5, 4)$ (b) $\{-3, 3\}$
 (c) $(-3, 1) \cup (3, 4)$ (d) none of these
- 17** Find the values of x satisfying the following system of inequalities
- $$\frac{\sqrt{(2-x)(x-8)}}{\log_{3/10} \frac{10}{7} \left[\log_2 \left(\frac{5}{2} \right) \right]} \geq 0 \quad \text{and} \quad \frac{2^x}{8} - (2^5 - 1) > 0$$
- (a) 6 (b) 8 (c) 4 (d) (2, 8)
- 18** The least value of expression $2 \log_{10} x - \log_x(1/100)$ for $x > 1$ is :
- (a) 2 (b) 3 (c) 4 (d) 5

- 19** The equation $x^{[(3/4)(\log_2 x)^2 + \log_2 x - (5/4)]} = \sqrt{2}$ has :

- (a) at least one real solution
 (b) exactly three real solutions
 (c) exactly one irrational solution
 (d) all of these

- 20** The values of x for which the given equation satisfy $\log_{(2x+3)}(6x^2 + 23x + 21) = 4 - \log_{(3x+7)}(4x^2 + 12x + 9)$ are :

- (a) $(-8, 5)$ (b) $-\frac{1}{4}$ only (c) $-4, -\frac{1}{4}$ (d) $-\frac{1}{4}, -2$

- 21** Find all real values of x satisfying equation

$$|x-1|^{\log x^2 - 2 \log x} = (x-1)^7$$

- (a) (2, 9) (b) (9, 81)
 (c) (2, 81) (d) none of these

- 22** Find the value of x_0 , if (x_0, y_0) be the solution of the following equations

$$(2x)^{\ln 2} = (3y)^{\ln 3}; 3^{\ln x} = 2^{\ln y}$$

- (a) 1/6 (b) 1/3 (c) 1/2 (d) 6

- 23** The number $\log_{20} 3$ lies in

- (a) (3/4, 4/5) (b) (1/3, 1/2)
 (c) (1/2, 3/4) (d) (1/4, 1/3)

- 24** Find the number of consecutive zeros in the non-integral part of the number $\frac{5}{500^{50}}$ if $\log 5 = 0.4771$.

- (a) 125 (b) 124
 (c) 123 (d) 100

Answers

✓ Introductory Exercise 16.1

1 (d)	2 (c)	3 (c)	4 (b)	5 (c)	6 (b)	7 (d)	8 (c)	9 (c)	10 (c)
11 (b)	12 (a)	13 (c)	14 (c)	15 (a)	16 (c)	17 (d)	18 (c)	19 (b)	20 (b)
21 (a)	22 (a)	23 (c)	24 (b)	25 (c)	26 (c)	27 (c)	28 (b)	29 (c)	30 (a)
31 (b)	32 (b)	33 (c)	34 (d)	35 (b)	36 (a)	37 (c)	38 (b)	39 (a)	40 (c)
41 (b)	42 (a)	43 (b)	44 (c)	45 (c)	46 (a)	47 (b)	48 (b)	49 (b)	50 (c)
51 (b)	52 (a)	53 (a)	54 (b)	55 (d)	56 (d)	57 (d)	58 (c)	59 (b)	60 (c)
61 (b)	62 (c)	63 (b)	64 (a)	65 (d)	66 (c)				

✓ Level 01 Basic Level Exercise

1 (a)	2 (b)	3 (c)	4 (d)	5 (a)	6 (b)	7 (b)	8 (a)	9 (a)	10 (c)
11 (c)	12 (c)	13 (b)	14 (c)	15 (b)	16 (c)	17 (d)	18 (a)	19 (b)	20 (d)
21 (c)	22 (c)	23 (d)	24 (d)	25 (c)	26 (b)	27 (a)	28 (d)	29 (d)	30 (b)
31 (d)	32 (b)	33 (b)	34 (d)	35 (d)	36 (d)	37 (b)	38 (d)	39 (d)	40 (b)
41 (a)	42 (b)	43 (d)	44 (c)	45 (c)	46 (c)	47 (a)	48 (a)	49 (d)	50 (b)

✓ Level 02 Higher Level Exercise

1 (a)	2 (a)	3 (b)	4 (c)	5 (b)	6 (d)	7 (d)	8 (c)	9 (d)	10 (c)
11 (d)	12 (b)	13 (c)	14 (a)	15 (d)	16 (c)	17 (b)	18 (c)	19 (d)	20 (b)
21 (c)	22 (c)	23 (b)	24 (c)						

Hints & Solutions

Introductory Exercise 16.1

$$1 \quad x^y = k \Leftrightarrow \log_x k = y$$

$$2 \quad \log_m n = p \Leftrightarrow m^p = n$$

$$3 \quad \log_a(mn) \Leftrightarrow \log_a m + \log_a n$$

$$4 \quad \log_a \left(\frac{m}{n} \right) = \log_a m - \log_a n$$

$$5 \quad \log_{81} 27 = \frac{\log_{10} 27}{\log_{10} 81} = \frac{\log_{10} 3^3}{\log_{10} 3^4} = \frac{3 \log_{10} 3}{4 \log_{10} 3} = \frac{3}{4}$$

$$6 \quad \log_{36} \left(\frac{1}{216} \right) = \log_{36} 6^{-3} = \frac{\log 6^{-3}}{\log 36} \\ = \frac{-3 \log 6}{2 \log 6} = -\frac{3}{2}$$

$$7 \quad \log_{\sqrt{6}} 216 = \frac{\log 216}{\log \sqrt{6}} = \frac{\log (\sqrt{6})^6}{\log \sqrt{6}} = \frac{6 \log \sqrt{6}}{\log \sqrt{6}} = 6$$

$$8 \quad \log_{3\sqrt{2}} 5832 = \log_{3\sqrt{2}} (18)^3 = \log_{3\sqrt{2}} [(3\sqrt{2})^2]^3 \\ = \log_{3\sqrt{2}} (3\sqrt{2})^6 = 6 \quad (\because \log_a a^m = m)$$

$$9 \quad \text{Let } \log_{(-1/5)} 625 = x$$

$$\therefore \left(-\frac{1}{5} \right)^x = 625 \Rightarrow \left(-\frac{1}{5} \right)^x = (5)^4$$

$$\Rightarrow (-5)^{-x} = (5)^4 = (-5)^4 \Rightarrow x = -4$$

$$10 \quad \text{Let } \log_{10} (0.0001) = x$$

$$\therefore 10^x = 0.0001 = 10^{-4}$$

$$\Rightarrow x = -4$$

$$11 \quad \text{Let } \log_{0.01} (10000) = x$$

$$\therefore (0.01)^x = 10000 \Rightarrow (10^{-2})^x = 10000$$

$$\Rightarrow 10^{-2x} = 10^4 \Rightarrow x = -2$$

$$12 \quad \log_4 32 + \log_9 243 = \frac{\log 32}{\log 4} + \frac{\log 243}{\log 9} \\ = \frac{5 \log_2 2}{2 \log_2 2} + \frac{5 \log_3 3}{2 \log_3 3} = \frac{5}{2} + \frac{5}{2} = 5$$

$$13 \quad \log_2 2^3 + \log_3 3^2 + \log_5 5^2 = 3 + 2 + 2 = 7$$

$$14 \quad \log_2 (\log_3 81) = \log_2 (\log_3 3^4) = \log_2 (4 \log_3 3) \\ = \log_2 4 = \log_2 2^2 = 2$$

$$15 \quad \log_2 \log_2 \log_3 \log_5 (125)^3 = \log_2 \log_2 \log_3 \log_5 5^9 \\ = \log_2 \log_2 \log_3 9 \\ = \log_2 \log_2 (2) = \log_2 1 = 0$$

$$16 \quad \log_2 \log_2 \log_2 \log_2 (65536) = \log_2 \log_2 \log_2 \log_2 (2^{16}) \\ = \log_2 \log_2 \log_2 16 = \log_2 \log_2 4 \\ = \log_2 2 = 1$$

$$17 \quad \log_{10} 1 + \log_{10} 10 + \log_{10} 100 + \dots + \log_{10} 10000000000 \\ = \log_{10} (1 \times 10 \times 100 \times \dots \times 10000000000) \\ = \log_{10} 10^{(0+1+2+3+\dots+10)} = \log_{10} 10^{55} = 55$$

Alternatively

$$\log_{10} 1 + \log_{10} 10 + \log_{10} 10^2 + \dots + \log_{10} 10^{10} \\ = 0 + 1 + 2 + 3 + 4 + \dots + 10 = 55$$

$$18 \quad \log_5 5 + \log_5 5^2 + \log_5 5^3 + \dots + \log_5 5^n \\ = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$19 \quad 27^{\log_3 81} = 3^{3 \log_3 81} \\ = 3^{\log_3 (81)^3} = (81)^3 = 531441 \quad (\because a^{\log_a m} = m)$$

$$20 \quad 216^{\log_6 49} = 6^{3 \log_6 49} \\ = 6^{\log_6 (49)^3} = (49)^3 = 117649$$

$$21 \quad 25^{[(-1/4) \log_5 25]} = 5^{[2(-1/4) \log_5 25]} \\ = 5^{(-1/2) \log_5 25} = 5^{\log_5 (25)^{-1/2}} = 25^{-1/2} = \frac{1}{5}$$

$$22 \quad \log_{10} 500000 - \log_{10} 5 = \log_{10} \left(\frac{500000}{5} \right) = \log_{10} 100000 \\ = \log_{10} 10^5 = 5$$

$$23 \quad \log_5 [\log_3 (\log_2 x)] = 1 = \log_5 5 \\ \Rightarrow \log_3 (\log_2 x) = 5 = \log_3 3^5 \\ \Rightarrow \log_2 x = 3^5 = 243 \Rightarrow 2^{243} = x$$

$$24 \quad \log_2 \log_{10} 4x = 1 = \log_2 2 \\ \Rightarrow \log_{10} 4x = 2 = \log_{10} 10^2 = \log_{10} 100 \\ \Rightarrow 4x = 100 \Rightarrow x = 25$$

$$25 \quad \text{If } \log_{27} x = \frac{4}{3} \Rightarrow (27)^{4/3} = x \\ \Rightarrow (3^3)^{4/3} = x \Rightarrow 3^4 = x \Rightarrow x = 81$$

$$26 \quad \log_x \left(\frac{4}{9} \right) = -\frac{1}{2} \\ \Rightarrow x^{-1/2} = \frac{4}{9} \Rightarrow \frac{1}{x^{1/2}} = \frac{4}{9} \\ \Rightarrow \left(\frac{1}{x} \right)^{1/2} = \frac{4}{9} \Rightarrow \frac{1}{x} = \left(\frac{4}{9} \right)^2 \Rightarrow x = \frac{81}{16}$$

$$\begin{aligned} 27 \quad \log_{100} x &= -4 \Rightarrow (100)^{-4} = x \\ \Rightarrow x &= (10^2)^{-4} = 10^{-8} \Rightarrow x = \frac{1}{10^8} \end{aligned}$$

$$\begin{aligned} 28 \quad \log_{100} x &= -\frac{1}{2} \Rightarrow (100)^{-1/2} = x \\ \Rightarrow (10^2)^{-1/2} &= x \Rightarrow 10^{-1} = x \\ \Rightarrow x &= \frac{1}{10} \end{aligned}$$

$$\begin{aligned} 29 \quad \log_x 0.2 &= -\frac{1}{4} \\ \Rightarrow x^{-1/4} &= 0.2 = \frac{2}{10} \Rightarrow x^{-1} = \left(\frac{2}{10}\right)^4 \\ \Rightarrow x &= \left(\frac{10}{2}\right)^4 \Rightarrow x = 5^4 \end{aligned}$$

$$\begin{aligned} 30 \quad \log_{243} x &= 0.8 \\ \Rightarrow 243^{0.8} &= x \Rightarrow (3^5)^{4/5} = x \\ \Rightarrow 3^4 &= x \Rightarrow x = 81 \end{aligned}$$

$$\begin{aligned} 31 \quad \log_e(x-1) + \log_e x + \log_e(x+1) &= 0 \\ \Rightarrow \log_e(x-1) \times x \times (x+1) &= \log_e 1 \\ \Rightarrow \log_e(x^2-1)x &= \log_e 1 \\ \Rightarrow (x^2-1)x &= 1 \\ \Rightarrow x^3 - x - 1 &= 0 \end{aligned}$$

$$32 \quad \log_3 5 \times \log_{25} 9 = \frac{\log 5}{\log 3} \times \frac{\log 3^2}{\log 5^2} = \frac{\log 5}{\log 3} \times \frac{2 \log 3}{2 \log 5} = 1$$

$$\begin{aligned} 33 \quad \log \frac{14}{3} + \log \frac{11}{5} - \log \frac{22}{15} \\ = \log \left(\frac{14}{3} \times \frac{11}{5} \times \frac{15}{22} \right) &= \log 7 \end{aligned}$$

$$\begin{aligned} 34 \quad 3 \log \frac{81}{80} + 5 \log \frac{25}{24} + 7 \log \frac{16}{15} \\ = \log \left[\left(\frac{81}{80} \right)^3 \times \left(\frac{25}{24} \right)^5 \times \left(\frac{16}{15} \right)^7 \right] \\ = \log \left(\frac{3^{12} \times 5^{10} \times 2^{28}}{2^{12} \times 5^3 \times 2^{15} \times 3^5 \times 3^7 \times 5^7} \right) \\ = \log 2 \end{aligned}$$

$$35 \quad \log \frac{a^3}{bc} + \log \frac{b^3}{ac} + \log \frac{c^3}{ab} = \log \left(\frac{a^3 b^3 c^3}{ab \cdot bc \cdot ac} \right) = \log abc$$

$$\begin{aligned} 36 \quad \frac{1}{(\log_a bc) + 1} + \frac{1}{(\log_b ac) + 1} + \frac{1}{(\log_c ab) + 1} \\ = \frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ac + \log_b b} + \frac{1}{\log_c ab + \log_c c} \\ = \frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc} \\ = \log_{abc} a + \log_{abc} b + \log_{abc} c = \log_{abc} abc = 1 \end{aligned}$$

$$\begin{aligned} 37 \quad \frac{1}{\log_{ab} abc} + \frac{1}{\log_{bc} abc} + \frac{1}{\log_{ca} abc} \\ = \log_{abc}(ab) + \log_{abc}(bc) + \log_{abc}(ca) \\ = \log_{abc}(abc)^2 = 2 \end{aligned}$$

$$\begin{aligned} 38 \quad \frac{1}{\log_5 210} + \frac{1}{\log_6 210} + \frac{1}{\log_7 210} \\ = \log_{210} 5 + \log_{210} 6 + \log_{210} 7 \\ = \log_{210} (5 \times 6 \times 7) = \log_{210} 210 = 1 \end{aligned}$$

$$\begin{aligned} 39 \quad \frac{1}{\log_{a/b} x} + \frac{1}{\log_{b/c} x} + \frac{1}{\log_{c/a} x} \\ = \log_x \frac{a}{b} + \log_x \frac{b}{c} + \log_x \frac{c}{a} \\ = \log_x \left(\frac{a}{b} \times \frac{b}{c} \times \frac{c}{a} \right) = \log_x 1 = 0 \end{aligned}$$

$$40 \quad \log_2 10 = \frac{1}{\log_{10} 2} = \frac{1}{0.3010} = 3.3222$$

$$\begin{aligned} 41 \quad 2^x \cdot 3^{2x} &= 100 \\ \Rightarrow x \log 2 + 2x \log 3 &= \log 100 \\ \Rightarrow x (0.3010 + 2 \times 0.4771) &= 2 \\ \Rightarrow x &= \frac{2}{1.2552} = 1.59 \end{aligned}$$

$$\begin{aligned} 42 \quad \log_{10} 2 &= 0.3010 \quad \text{and} \quad 5^x = 400 \\ \therefore 5^x &= 400 \\ \Rightarrow x \log 5 &= \log 4 + \log 100 \\ \Rightarrow x (\log 10 - \log 2) &= 2 \log 2 + 2 \\ \Rightarrow x (1 - 0.3010) &= 2(0.3010 + 1) \\ \Rightarrow x &= 3.72 \end{aligned}$$

$$\begin{aligned} 43 \quad 5^{20} &= 20 \log 5 \\ &= 20 \log \left(\frac{10}{2} \right) = 20 (\log 10 - \log 2) \\ &= 20 \times [1 - 0.3010] \\ &= 20 \times 0.6990 = 13.98 \end{aligned}$$

Characteristic = 13

$$\therefore \text{Number of digits} = (13 + 1) = 14$$

$$\begin{aligned} 44 \quad \log(m+n) &= \log m + \log n \\ \Rightarrow \log(m+n) &= \log(mn) \\ \Rightarrow m+n &= mn \\ m &= n(m-1) \\ \Rightarrow n &= \frac{m}{(m-1)} \end{aligned}$$

$$\begin{aligned} 45 \quad \log_{10} a + \log_{10} b &= c \\ \Rightarrow \log_{10}(ab) &= c \Rightarrow 10^c = ab \\ \Rightarrow a &= \frac{(10)^c}{b} \end{aligned}$$

46 Since 0.3274 gives characteristic $\bar{1}$. Therefore value of $\log (0.3274) = \bar{1}.5150$

47 $\log_{10} (100x) = \log_{10} 100 + \log_{10} x$
 $= 2 + 1.9675 = 3.9675$

48 $(5)^x = 1000$
 $\Rightarrow 5 = 10^{3/x} \quad \dots(i)$

And $(0.5)^y = 1000 \Rightarrow \left(\frac{5}{10}\right)^y = 10^3$
 $\Rightarrow 5^y = 10^3 \times 10^y \Rightarrow 5 = 10^{\frac{(3+y)}{y}} \quad \dots(ii)$

From eq. (i) and (ii)

$$5 = 10^{3/x} = 10^{\frac{(3+y)}{y}} \Rightarrow \frac{3}{x} = \frac{3+y}{y}$$

$$\Rightarrow \frac{1}{x} - \frac{1}{y} = \frac{1}{3}$$

49 $\frac{1}{2} \log x + \frac{1}{2} \log y + \log 2 = \log (x + y)$
 $\Rightarrow \log (x^{1/2} y^{1/2} \times 2) = \log (x + y)$
 $\Rightarrow 2\sqrt{xy} = (x + y) \Rightarrow 4xy = (x + y)^2$
 $\Rightarrow (x + y)^2 - 4xy = 0 \Rightarrow (x - y)^2 = 0$
 $\Rightarrow x - y = 0 \Rightarrow x = y$

50 $q = \log_{17} (5)^2 = 2 \log_{17} 5$
 $\Rightarrow \frac{1}{q} = \frac{1}{2} \log_5 17$
 And $\frac{1}{p} = \log_5 3 = \frac{1}{2} \times (2 \log_5 3)$

$$= \frac{1}{2} (\log_5 9)$$

$$\therefore \frac{1}{p} < \frac{1}{q} \Rightarrow p > q$$

51 $x^2 + 4y^2 = 12xy$
 $\Rightarrow x^2 + (2y)^2 + 4xy = 12xy + 4xy$
 $\Rightarrow (x + 2y)^2 = 16xy$
 $\Rightarrow (x + 2y)^2 = (2^4) xy$
 $\Rightarrow 2 \log (x + 2y) = 4 \log 2 + \log x + \log y$
 $\Rightarrow \log (x + 2y) = \frac{1}{2} (\log x + \log y + 4 \log 2)$

52 $\log_3 7 = \frac{\log_2 7}{\log_2 3} = (\log_2 7) (\log_3 2)$

$$\log_3 7 = (\log_3 2) (\log_2 7) = \log_2 7^{\log_3 2}$$

$$\therefore 2^{\log_3 7} = 2^{\log_2 7^{\log_3 2}}$$

$$\Rightarrow 2^{\log_3 7} = 7^{\log_3 2}$$

$$\Rightarrow 2^{\log_3 7} - 7^{\log_3 2} = 0$$

53 $\frac{1}{\log_2 a} + \frac{1}{\log_4 a} + \frac{1}{\log_8 a} + \dots + \frac{1}{\log_{2^n} a} = \frac{n(n+1)}{k}$
 $= \log_a 2 + \log_a 2^2 + \log_a 2^3 + \dots + \log_a 2^n = \frac{n(n+1)}{k}$
 $\Rightarrow (\log_a 2) (1 + 2 + 3 + \dots + n) = \frac{n(n+1)}{k}$

$$\Rightarrow (\log_a 2) \left[\frac{n(n+1)}{2} \right] = \frac{n(n+1)}{k} \Rightarrow \frac{1}{2} \log_a 2 = \frac{1}{k}$$

$$\Rightarrow \log_a 2 = \frac{2}{k} \Rightarrow \log_2 a = \frac{k}{2}$$

$$\Rightarrow 2 \log_2 a = k \Rightarrow \log_2 a^2 = k$$

54 $\log \tan 1^\circ + \log \tan 2^\circ + \dots + \log \tan 89^\circ$
 $= (\log \tan 1^\circ + \log \tan 89^\circ) + (\log \tan 2^\circ + \log \tan 88^\circ)$
 $\quad + \dots + \log \tan 45^\circ$
 $= \log (\tan 1^\circ \cdot \tan 89^\circ) + \log (\tan 2^\circ \cdot \tan 88^\circ) + \dots \log 1$
 $= \log (\tan 1^\circ \cdot \cot 1^\circ) + \log (\tan 2^\circ \cdot \cot 2^\circ) + \dots \log 1$
 $= \log 1 + \log 1 + \dots + \log 1$
 $(\because \tan (90 - \theta) = \cot \theta \text{ and } \tan 45^\circ = 1) = 0$

55 $\log_9 x - \log_9 \left(\frac{x}{10} + \frac{1}{9} \right) = 1 = \log_9 9$

$$\Rightarrow \log_9 \left(\frac{x}{\left(\frac{x}{10} + \frac{1}{9} \right)} \right) = \log_9 9$$

$$\Rightarrow \frac{x}{\left(\frac{x}{10} + \frac{1}{9} \right)} = 9 \Rightarrow \frac{x \times 90}{9x + 10} = 9$$

$$\Rightarrow 10x = 9x + 10 \Rightarrow x = 10$$

Alternatively Go through options.

56 $[(x^2 - y^2)^2]^{a-1} = (x - y)^{2a} (x + y)^{-2}$
 $\Rightarrow (x^2 - y^2)^{2(a-1)} = (x - y)^{2a} (x + y)^{-2}$
 $\Rightarrow (x - y)^{2(a-1)} (x + y)^{2(a-1)} = (x - y)^{2a} (x + y)^{-2}$
 $\Rightarrow \frac{(x - y)^{2a-2} (x + y)^{2a-2}}{(x - y)^{2a} (x + y)^{-2}} = 1$
 $\Rightarrow (x - y)^{-2} (x + y)^{2a} = 1$
 $\Rightarrow -2 \log (x - y) + 2a \log (x + y) = \log 1$
 $\Rightarrow 2a \log (x + y) = 2 \log (x - y) \Rightarrow a = \frac{\log (x - y)}{\log (x + y)}$

57 Let $\log_2 7$ be a rational

$$\log_2 7 = \left(\frac{p}{q} \right) \quad q \neq 0$$

$$\Rightarrow 2^{(p/q)} = 7 \Rightarrow 2^p = 7^q$$

Here p and q being the integers cannot satisfy the above relation.

Hence, $\log_2 7$ is an irrational number.

(Clearly (a) and (b) are wrong)

$$58 \log \frac{x}{y} = \frac{\log_e x}{\log_e y} \text{ (see the theory)}$$

$$\begin{aligned} 59 \log_{10} 2 + \log_{10} \left(\frac{16}{15} \right)^{16} + \log_{10} \left(\frac{25}{24} \right)^{12} + \log_{10} \left(\frac{81}{80} \right)^7 \\ = \log_{10} 2 + \log_{10} \left(\frac{2^{64}}{3^{16} \times 5^{16}} \right) + \log_{10} \left(\frac{5^{24}}{2^{36} \times 3^{12}} \right) + \log_{10} \left(\frac{3^{28}}{2^{28} \times 5^7} \right) \\ = \log_{10} \frac{2^{65} \times 3^{28} \times 5^{24}}{2^{64} \times 3^{28} \times 5^{23}} = \log_{10} (2 \times 5) = \log_{10} 10 = 1 \end{aligned}$$

$$\begin{aligned} 60 (pa + qb - rc) &= p \log_{10} x + q \log_{10} y - r \log_{10} z \\ &= \log_{10} (x^p) + \log_{10} (y^q) - \log_{10} (z^r) = \log_{10} \left(\frac{x^p y^q}{z^r} \right) \\ \Rightarrow \text{antilog } (pa + qb - rc) &= \frac{x^p y^q}{z^r}. \end{aligned}$$

$$61 \log_{10} a^p \cdot b^q \cdot c^r = \log_{10} a^p + \log_{10} b^q + \log_{10} c^r \\ = p \log_{10} a + q \log_{10} b + r \log_{10} c$$

$$\begin{aligned} 62 \because a, b, c \text{ are in GP} \\ \Rightarrow b^2 = ac \Rightarrow 2 \log b = \log a + \log c \\ \Rightarrow \log_x a, \log_x b, \log_x c \text{ are in AP} \\ \Rightarrow \log_{10} a, \log_{10} b, \log_{10} c \text{ are in AP} \end{aligned}$$

$$\begin{aligned} 63 \log_{10} x, \log_{10} y, \log_{10} z \text{ are in AP} \\ \therefore 2 \log_{10} y = \log_{10} x + \log_{10} z \\ \Rightarrow \log_{10} y^2 = \log_{10} (xz) \\ \Rightarrow y^2 = xz \end{aligned}$$

$$\begin{aligned} 64 \because a, b, c \text{ are in GP} \\ \therefore \log_x a, \log_x b, \log_x c \text{ are in AP} \\ \Rightarrow \frac{1}{\log_a x}, \frac{1}{\log_b x}, \frac{1}{\log_c x} \text{ are in AP} \end{aligned}$$

$$\begin{aligned} 65 \log (x-1) + \log (x+1) &= 3 \log 2 = \log 2^3 \\ \Rightarrow \log [(x-1)(x+1)] &= \log 8 \\ \Rightarrow \log (x^2 - 1) &= \log 8 \\ \Rightarrow x^2 - 1 = 8 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3 \end{aligned}$$

But at $x = -3$ logarithm equation is not defined.

$$\text{Hence, } x = 3$$

$$\begin{aligned} 66 a, b, c \text{ are consecutive integers} \\ \therefore b = a + 1 \text{ and } c = a + 2 \\ \therefore \log (ac + 1) = \log [a(a+2) + 1] \\ = \log [(b-1)(b-1+2) + 1] \quad (\because a = b-1) \\ = \log b^2 = 2 \log b \end{aligned}$$

✓ Level 01 Basic Level Exercise

$$\begin{aligned} 1 \log_{10} \sqrt[7]{13} &\Rightarrow \log_{10} (13)^{1/7} \\ \Rightarrow \frac{1}{7} \log_{10} 13 &= \frac{1}{7} \times 1.1139 = 0.15913 \end{aligned}$$

$$2 \log_{2\sqrt{3}} 144 = \log_{2\sqrt{3}} (2\sqrt{3})^4 = 4$$

$$\begin{aligned} 3 \log_a N = \log_b N \times P \\ \Rightarrow \frac{\log_a N}{\log_b N} = P \Rightarrow \frac{\log_N a}{\log_N b} = P \end{aligned}$$

$$\Rightarrow \frac{\log_N b}{\log_N a} = P \Rightarrow \log_a b = P$$

$$\begin{aligned} 4 \log (ab)^2 - \log (ac) + \log (bc^4) - 3 \log (bc) \\ = \log \frac{(ab)^2}{ac} + \log \frac{(bc^4)}{(bc)^3} = \log \left[\frac{(ab)^2}{ac} \times \frac{(bc^4)}{(bc)^3} \right] = \log a \end{aligned}$$

$$\begin{aligned} 5 \log_2 x + \log_4 x + \log_{64} x &= 5 \\ \Rightarrow \frac{1}{\log_x (2)} + \frac{1}{2 \log_x (2)} + \frac{1}{6 \log_x (2)} &= 5 \\ \Rightarrow \frac{10}{6 \log_x (2)} = 5 \Rightarrow \log_x (2) &= \frac{1}{3} \\ \Rightarrow \log_2 x = 3 \Rightarrow 2^3 = x \therefore x &= 8 \end{aligned}$$

☞ Alternatively Go through options.

$$6 \log_q (xy) = 3 \text{ and } \log_q (x^2 y^3) = 4$$

$$\therefore \log_q (x^2 y^3) = 4 \Rightarrow \log_q [(xy)^2 \cdot y]$$

$$\Rightarrow \log_q (xy)^2 + \log_q y = 4$$

$$\Rightarrow 2 \log_q (xy) + \log_q y = 4$$

$$\Rightarrow 2 \times 3 + \log_q y = 4$$

$$\Rightarrow \log_q y = -2$$

$$\text{Again } \log_q (xy) = \log_q x + \log_q y = 3$$

$$\Rightarrow \log_q x + (-2) = 3$$

$$\Rightarrow \log_q x = 5$$

$$7 \frac{\log x}{l+m-2n} = \frac{\log y}{m+n-2l} = \frac{\log z}{n+l-2m} = k \text{ (say)}$$

$$\Rightarrow \log x = k(l+m-2n) \quad \dots(i)$$

$$\log y = k(m+n-2l) \quad \dots(ii)$$

$$\log z = k(n+l-2m) \quad \dots(iii)$$

$$\therefore \log x + \log y + \log z$$

$$= k(l+m-2n) + k(m+n-2l) + k(n+l-2m)$$

$$\log (xyz) = 0$$

$$\Rightarrow \log xyz = \log 1$$

$$\Rightarrow xyz = 1$$

- 8 Let m be the first term and k be the common ratio of GP, then

$$\begin{aligned} a &= mk^{p-1}, b = mk^{q-1}, c = mk^{r-1} \\ \therefore (q-r) \log a + (r-p) \log b + (p-q) \log c \\ &= \log [mk^{(p-1)q-r} + \log b [mk^{(q-1)r-p} + \log [mk^{(r-1)p-q} \\ &= \log (m^{q-r+p-p+q}) (k^{p-1})^{q-r} (k^{q-1})^{r-p} (k^{r-1})^{p-q} \\ &= \log m^0 k^0 = \log 1 = 0 \end{aligned}$$

- 9 $a^{3-x} \cdot b^{5x} = a^{x+5} \cdot b^{3x}$

$$\Rightarrow \frac{a^{3-x}}{a^{x+5}} = \frac{b^{3x}}{b^{5x}} \Rightarrow a^{-2(x+1)} = b^{-2x}$$

$$\Rightarrow a^{-2x} \cdot a^{-2} = b^{-2x} \Rightarrow \left(\frac{b}{a}\right)^{-2x} = a^{-2}$$

$$\Rightarrow \left(\frac{b}{a}\right)^x = a \Rightarrow \log \left(\frac{b}{a}\right)^x = \log a$$

$$\Rightarrow x \log \left(\frac{b}{a}\right) = \log a$$

- 10 $v = u^{1/2}, w = u^{1/3}, z = u^{1/4}$

$$\therefore uvwz = u^{(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4})}$$

$$\begin{aligned} \therefore \log_u uvwz &= \log_u u^{(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4})} = \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) \log_u u \\ &= \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) \end{aligned}$$

- 11 $\frac{\log \sqrt{27} + \log \sqrt{8} - \log \sqrt{125}}{\log 6 - \log 5}$

$$= \frac{\log (3^{3/2} \times 2^{3/2}) - \log (5)^{3/2}}{\log 6 - \log 5}$$

$$= \frac{\log 6^{3/2} - \log 5^{3/2}}{\log 6 - \log 5} = \frac{\frac{3}{2} \log \left(\frac{6}{5}\right)}{\log \left(\frac{6}{5}\right)} = \frac{3}{2}$$

- 12 Last term $k = a \cdot r^{n-1}$, $r \rightarrow$ common ratio

$$\therefore \log k = \log (a \cdot r^{n-1})$$

$$\Rightarrow \log k = \log a + (n-1) \log r$$

$$\Rightarrow \frac{\log k - \log a}{\log r} = n-1$$

$$\Rightarrow n = 1 + \left(\frac{\log k - \log a}{\log r}\right)$$

- 13 $\therefore a, b, c$ are in GP

$$\therefore b^2 = ac$$

$$\Rightarrow 2 \log b = \log a + \log c$$

$$\therefore \log a, \log b, \log c \text{ are in AP}$$

$$\text{i.e., } \log_d a, \log_d b, \log_d c \text{ are in AP}$$

$$\therefore \log_a d, \log_b d, \log_c d \text{ are in HP}$$

- 14 Best way is to go through options

- 15 Best way is to go through options

☞ Alternatively $\log_{10} x^2 y^3 = 7$

$$\Rightarrow x^2 y^3 = 10^7 \quad \dots(i)$$

and $\log_{10} \left(\frac{x}{y}\right) = 1$

$$\Rightarrow \frac{x}{y} = 10 \quad \dots(ii)$$

$$\therefore \frac{x^2 y^3}{(x/y)^2} = \frac{10^7}{(10)^2} \Rightarrow y^5 = 10^5$$

$$\Rightarrow y = 10 \quad \therefore x = 100$$

16 $y = a^{\left(\frac{1}{1 - \log_a x}\right)}$

$$\Rightarrow \log_a y = \frac{1}{1 - \log_a x} \quad (\text{taking log of both sides})$$

$$\Rightarrow \log_a x = 1 - \frac{1}{\log_a y} \quad \dots(i)$$

Again $z = a^{\left(\frac{1}{1 - \log_a y}\right)} \Rightarrow \log_a z = \frac{1}{1 - \log_a y}$

$$\Rightarrow \log_a y = 1 - \frac{1}{\log_a z} \quad \dots(ii)$$

$\therefore$ From eq. (i) and (ii), we get

$$\log_a x = 1 - \frac{1}{\left(1 - \frac{1}{\log_a z}\right)}$$

$$\Rightarrow \log_a x = \frac{1}{(1 - \log_a z)} \Rightarrow x = a^{\frac{1}{1 - \log_a z}}$$

☞ Alternatively Consider some numerical values suppose $a = 10$ and $x = \sqrt{10}$

$$\therefore y = a^{\left(\frac{1}{1 - \log_a x}\right)}$$

$$y = 10^{\left(\frac{1}{1 - \log_{10} \sqrt{10}}\right)} \Rightarrow y = 100$$

and $z = a^{\left(\frac{1}{1 - \log_a y}\right)}$

$$\therefore z = 10^{\left(\frac{1}{1 - \log_{10} 100}\right)} \Rightarrow z = \frac{1}{10}$$

Now, go through options. Consider option (c)

$$\begin{aligned} x &= a^{\frac{1}{1 - \log_a z}} = 10^{\left(\frac{1}{1 - \log_{10} (1/10)}\right)} \\ x &= 10^{1/2} = \sqrt{10} \end{aligned}$$

Hence, option (c) is correct.

☞ NOTE You can check other options but they do not satisfy.

17 Let $a = 10$, $b = 100$ and $c = 1000$, then

$$x = \log_{1000}(100) + \log_{100}(1000) = \frac{13}{6}$$

$$y = \log_{10} 1000 + \log_{1000} 10 = \frac{10}{3}$$

$$z = \log_{100} 10 + \log_{10} 100 = \frac{5}{2}$$

$$\therefore x^2 + y^2 + z^2 = \left(\frac{13}{6}\right)^2 + \left(\frac{10}{3}\right)^2 + \left(\frac{5}{2}\right)^2 = \frac{794}{36}$$

$$\text{and } xyz = \frac{13}{6} \times \frac{10}{3} \times \frac{5}{2} = \frac{650}{36}$$

$$\therefore xyz + 4 = \left(\frac{650}{36}\right) + 4 = \frac{794}{36}$$

Hence, option (d) is correct.

$$18 \frac{1}{\log_3 e} + \frac{1}{\log_3 e^2} + \frac{1}{\log_3 e^4} + \dots$$

$$= \frac{1}{\log_3 e} + \frac{1}{2\log_3 e} + \frac{1}{4\log_3 e} + \dots$$

$$= \frac{1}{\log_3 e} \left(1 + \frac{1}{2} + \frac{1}{4} + \dots\right) = \log_e 3 \left(\frac{1}{1-1/2}\right)$$

$$\text{(using sum of infinite GP)} \left(s_\infty = \frac{a}{1-r}; |r| < 1\right)$$

$$= 2 \log_e 3 = \log_e 3^2 = \log_e 9$$

$$19 \log_{10} x^2 - \log_{10} \sqrt{y} = 1$$

$$\Rightarrow \log_{10} \left(\frac{x^2}{\sqrt{y}}\right) = \log_{10} 10$$

$$\Rightarrow \frac{x^2}{\sqrt{y}} = 10 \Rightarrow \frac{4}{\sqrt{y}} = 10$$

$$\Rightarrow \sqrt{y} = \frac{4}{10} = \frac{2}{5} \Rightarrow y = \frac{4}{25}$$

$$20 (b^2)^5 \log_b x = b^{10} \log_b x = b^{\log_b(x)^{10}} = x^{10}$$

$$21 (7^3)^{-2} \log_7 8 = 7^{-6} \log_7 8 = 7^{(\log_7 8^{-6})} = 8^{-6} = \frac{1}{8^6}$$

$$22 \log 12900 = \log (1.29 \times 10000)$$

$$= \log 1.29 + \log 10000 = \log 1.29 + 4$$

$$23 \log 0.786 = \log \left(\frac{786}{1000}\right) = \log 786 - \log 1000$$

$$= \log 786 - 3$$

$$\text{Also } \log 0.786 = \log \frac{78.6}{100} = \log 78.6 - \log 100$$

$$24 \log_5 x = y \Rightarrow 5^y = x$$

$$\therefore (5^y)^5 = x^5 \Rightarrow 5^{5y} = x^5$$

$$25 A = \log_{13} 189 = \log_{13} (13^2 + 20)$$

$$\text{and } B = \log_{23} 521 = \log_{23} (23^2 - 8)$$

$$\therefore A > B \quad (\because A > 2 \text{ and } B < 2)$$

$$26 A = \frac{\log_3 2187}{5} = \frac{1}{5} \log_3 2187$$

$$= \frac{1}{5} \log_3 (3^7) = \frac{7}{5} \log_3 3 = \frac{7}{5}$$

$$\text{and } B = \log_{243} 2187 = \frac{\log_3 2187}{\log_3 243} = \frac{\log_3 3^7}{\log_3 3^5} = \frac{7}{5}$$

$$\therefore A = B$$

$$27 (150)^x = 7$$

$$\Rightarrow \log_{150} 7 = x \Rightarrow \frac{\log 7}{\log 150} = x$$

$$\Rightarrow x = \frac{\log 7}{\log (3 \times 5 \times 10)} = \frac{\log 7}{\log 3 + \log 5 + \log 10}$$

$$= \frac{\log 7}{\log 3 + \log 5 + 1}$$

$$28 \log_{1/2} x = \log_2 (3x - 2)$$

$$\Rightarrow -\log_2 x = \log_2 (3x - 2)$$

$$\Rightarrow \log_2 (x^{-1}) = \log_2 (3x - 2)$$

$$\Rightarrow x^{-1} = (3x - 2) \Rightarrow 3x^2 - 2x - 1 = 0$$

$$\Rightarrow 3x^2 - 3x + x - 1 = 0$$

$$\Rightarrow 3x(x - 1) + 1(x - 1) = 0$$

$$\Rightarrow x = 1, -1/3$$

but at $x = -1/3$, $\log x$ is not defined

The only admissible value of x is 1.

☞ Alternatively Go through options.

$$29 \log_2 (x + y) = 3$$

$$\text{and } \log_2 x + \log_2 y = 2 + \log_2 3$$

$$\therefore \log_2 (x + y) = 3 = \log_2 2^3 = \log_2 8$$

$$\Rightarrow x + y = 8,$$

Hence option (c) is wrong.

$$\text{Again, } \log_2 x + \log_2 y = 2 + \log_2 3 = \log_2 4 + \log_2 3$$

$$\Rightarrow \log_2 (xy) = \log_2 12 \Rightarrow xy = 12$$

Hence, option (d) is correct.

30 Go through option.

at $x = 0$, $\log x$ is not defined.

at $x = 1$, both sides are equal.

Again at $x \neq 1$

$$\log_5 x \log_6 x \log_7 x = \log_5 x \log_6 x + \log_6 x \cdot \log_7 x + \log_7 x \cdot \log_5 x$$

$$\Rightarrow 1 = \frac{1}{\log_7 x} + \frac{1}{\log_5 x} + \frac{1}{\log_6 x}$$

(By dividing both sides by $\log_5 x \log_6 x \log_7 x$)

$$\therefore 1 = \log_x 7 + \log_x 5 + \log_x 6$$

$$1 = \log_x (7 \cdot 5 \cdot 6) = \log_x 210$$

$$\Rightarrow \log_x 210 = 1$$

$$\Rightarrow x = 210$$

$$\text{Hence, } x = \{1, 210\}$$

31 Let $\log_3 18 = a$

$$\begin{aligned} \therefore \frac{\log_3 54}{\log_{486} 3} - \frac{\log_3 1458}{\log_{18} 3} \\ = \log_3(18 \times 3) \log_3(18 \times 27) - \log_3(18 \times 81) \log_3 18 \\ = (\log_3 18 + \log_3 3)(\log_3 18 + 3 \log_3 3) \\ \quad - (\log_3 18 + 4 \log_3 3) \log_3 18 \\ = (a+1)(a+3) - (a+4)a \\ = a^2 + 3 + 4a - a^2 - 4a = 3 \end{aligned}$$

32 $\log_9(2x-5) = \log_3(x-4)$

$$\begin{aligned} \Rightarrow \frac{1}{2} \log_3(2x-5) &= \log_3(x-4) \\ \Rightarrow \log_3(2x-5) &= \log_3(x-4)^2 \\ \Rightarrow (2x-5) &= (x-4)^2 \Rightarrow (2x-5) = x^2 + 16 - 8x \\ \Rightarrow x^2 - 10x + 21 &= 0 \Rightarrow (x-3)(x-7) = 0 \\ \Rightarrow x &= 3 \quad \text{or} \quad x = 7 \\ \text{but at } x &= 3, \log_3(x-4) \text{ is not defined since } (x-4) \\ &\text{becomes negative} \\ \text{Hence, } x &= 7 \text{ is the only possible solution.} \end{aligned}$$

33 Since we know that when a, b, c are in GP, then $\log a, \log b$ and $\log c$ are in AP.

Therefore $2, (2^x - 5)$ and $(2^x - 7/2)$ must be in GP

Now, going through options, we get

at $x = 3$ the three terms $2, (2^x - 5)$ and $(2^x - 7/2)$ are in GP

Hence $\log 2, \log(2^x - 5)$ and $\log(2^x - 7/2)$ are in AP.

Alternatively

$$\begin{aligned} \text{We have } 2 \log_3(2^x - 5) &= \log_3 2 + \log_3(2^x - 7/2) \\ \Rightarrow \log_3(2^x - 5)^2 &= \log_3 2(2^x - 7/2) \\ \Rightarrow (2^x - 5)^2 &= 2(2^x - 7/2) \\ \Rightarrow (2^x)^2 - 10 \cdot 2^x + 25 &= 2 \cdot 2^x - 7 \\ [10 \cdot 2^x + 2 \cdot 2^x &= 12 \cdot 2^x] \quad [\because (a-b)^2 = a^2 - 2ab + b^2] \\ \Rightarrow (2^x)^2 - 12 \cdot 2^x + 32 &= 0 \\ \Rightarrow (2^x - 8)(2^x - 4) &= 0 \\ \Rightarrow x &= 3, \quad x = 2 \\ \text{but at } x &= 2, \log(2^x - 5) \text{ is undefined} \\ \text{Hence } x &= 3 \end{aligned}$$

34 Go through options.

35 As $\log_x a, a^{x/2}, \log_b x$ are in GP.

$$\begin{aligned} \therefore (a^{x/2})^2 &= \log_x a \cdot \log_b x \\ \Rightarrow a^x &= \frac{\log a}{\log x} \cdot \frac{\log x}{\log b} = \frac{\log a}{\log b} = \log_b a \\ \Rightarrow a^x &= \log_b a \\ \Rightarrow x &= \log_a(\log_b a) = \log_a \left(\frac{\log_e a}{\log_e b} \right) \\ &= \log_a(\log_e a) - \log_a(\log_e b) \end{aligned}$$

36 $1, \log_9(3^{1-x} + 2), \log_3(4 \cdot 3^x - 1)$ are in AP

$1, \log_3(3^{1-x} + 2)^{1/2}, \log_3(4 \cdot 3^x - 1)$ are in AP

$\therefore \log_3 3, \log_3(3^{1-x} + 2)^{1/2}, \log_3(4 \cdot 3^x - 1)$, are in AP

$\therefore 3, (3^{1-x} + 2)^{1/2}, (4 \cdot 3^x - 1)$ are in GP

$$\begin{aligned} \therefore [(3^{1-x} + 2)^{1/2}]^2 &= 3 \cdot (4 \cdot 3^x - 1) \\ \Rightarrow 3^{1-x} + 2 &= 4 \cdot 3^{x+1} - 3 \\ \Rightarrow 4 \cdot 3^{x+1} - 3^{1-x} &= 5 \end{aligned}$$

Solving we get $x = \log_3 \left(\frac{3}{4} \right)$

Alternatively Go through options. Consider option (d)

$$\begin{aligned} x = \log_3 \frac{3}{4} \Rightarrow 3^x &= \frac{3}{4} \\ \Rightarrow 1, \log_9 \left(\frac{3}{4} + 2 \right), \log_3 \left(4 \cdot \frac{3}{4} - 1 \right) \end{aligned}$$

$\Rightarrow 1, \log_9 6, \log_3 2 \Rightarrow \log_9 9, \log_9 6, \log_9 4$ are in AP

Since $9, 6, 4$ are in GP.

$$\begin{aligned} \mathbf{37} \quad \frac{1}{\log_{100} n} + \frac{1}{\log_{99} n} + \dots + \frac{1}{\log_2 n} \\ = \log_n 100 + \log_n 99 + \dots + \log_n 2 \\ = \log_n (100 \times 99 \times 98 \times \dots \times 2) = \log_n 100! = \frac{1}{\log_{100!} n} \end{aligned}$$

38 Best way is to go through options.

At $x = 1, \log_2(1 + 3^0) + 2 = \log_2(3^0 + 7)$

$$\Rightarrow \log_2 2 + 2 = \log_2 8 \quad (\because x^0 = 1)$$

$$\Rightarrow 1 + 2 = \log_2 2^3 \Rightarrow 3 = 3$$

Similarly, at $x = 2$

$$\log_2(1 + 3) + 2 = \log_2(9 + 7)$$

$$\Rightarrow (\log_2 4) + 2 = \log_2(16) \Rightarrow 4 = 4$$

Hence, (d) is the correct answer.

39 $x^{\log_5 x} > 5$

Taking logarithm with base 5 we have

$$(\log_5 x)(\log_5 x) > 1$$

$$\Rightarrow (\log_5 x)^2 > 1 \Rightarrow (\log_5 x)^2 - 1 > 0$$

$$\Rightarrow (\log_5 x - 1)(\log_5 x + 1) > 0$$

$$\Rightarrow \log_5 x > 1 \quad \text{or} \quad \log_5 x < -1$$

$$\Rightarrow x > 5 \quad \text{or} \quad x < \frac{1}{5}$$

Also, x must be greater than zero

$$\therefore x \in (0, 1/5) \cup (5, \infty)$$

Alternatively Go through options

Let us assume $x = 25$

then $25^{\log_5 25} = 625 > 5$, which is true

Hence option (d) is correct.

Again consider $x = 5$, then

$$5^{\log_5 5} = 5, \text{ which is not greater than } 5.$$

Hence choices (a) and (b) are wrong.

Thus option (d) is correct.

- 40** $\log x^3 - \log 3x = 2 \log 2 + \log 3$
 $\log \frac{x^2}{3} = \log 12 \Rightarrow \frac{x^2}{3} = 12 \Rightarrow x = \pm 6$
 But at $x = -6$, $\log 3x$ and $\log x^3$ are not defined.
 Hence $x = 6$ is the only correct answer.

- 41** $\log_5(2x + 3) < \log_5 7$
 $\Rightarrow (2x + 3) < 7 \Rightarrow 2x < 4 \Rightarrow x < 2$
 But $(2x + 3) > 0 \Rightarrow x > -\frac{3}{2}$
 $\therefore x \in \left(-\frac{3}{2}, 2\right)$

- 42** Best way is to go through options.

☞ **Alternatively** Suppose $\log_2 x = t$, then

$$\begin{aligned}\log_{2\sqrt{x}} x &= \frac{\log_2 x}{\log_2 2\sqrt{x}} = \frac{t}{1 + \frac{t}{2}} \text{ and } \log_{2x} \sqrt{x} = \frac{\log_2 \sqrt{x}}{\log_2 2x} \\ &= \frac{\frac{1}{2} \log_2 x}{1 + \log_2 x} = \frac{t/2}{1 + t} \log_{2\sqrt{x}} x + \log_{2x} \sqrt{x} = \frac{2t}{2+t} + \frac{t}{2+2t} = 0 \\ &\Rightarrow 4t + 4t^2 + 2t + t^2 = 0 \Rightarrow 5t^2 + 6t = 0 \\ &\Rightarrow t(5t + 6) = 0 \Rightarrow t = 0 \text{ or } t = -\frac{6}{5} \\ &\Rightarrow \log_2 x = 0 \Rightarrow x = 2^0 = 1 \\ \text{and } \log_2 x = -\frac{6}{5} &\Rightarrow x = 2^{-6/5} \\ \therefore x = 1 \text{ and } x = 2^{-6/5}\end{aligned}$$

- 43** Clearly option (b) is wrong

$$\therefore \frac{x}{1+x} < x \text{ for every } x > 1$$

$$\text{Again } \frac{x}{1+x} < \log(x+1); x > 1$$

Hence option (c) is also wrong and $\log(x+1) < x$

$\therefore$ Option (d) is correct.

- 44** $\frac{k}{l} > \frac{k+1}{l+1}$ for $(k, l) > 0$ and $k > l$

$$\text{Let } k = x + 1 \text{ and } l = x$$

$$\therefore \frac{x+1}{x} > \frac{(x+1)+1}{(x)+1} \therefore (x+1) > x$$

$$\therefore \frac{\log(x+1)}{\log x} > \frac{\log(x+2)}{\log(x+1)} \Rightarrow \log_x(x+1) > \log_{x+1}(x+2)$$

- 45** $a = 1 + \log_x yz = \log_x x + \log_x yz = \log_x xyz$

Similarly $b = \log_y xyz$ and $c = \log_z xyz$

$$\text{Now, } ab + bc + ca = abc \left[\frac{1}{c} + \frac{1}{a} + \frac{1}{b} \right]$$

$$= abc \left[\frac{1}{\log_x xyz} + \frac{1}{\log_y xyz} + \frac{1}{\log_z xyz} \right]$$

$$= abc [\log_{xyz} x + \log_{xyz} y + \log_{xyz} z] = abc [\log_{xyz} xyz] = abc$$

- 46** $\log_3(\log_2 x) + \log_{1/3}(\log_{1/2} y) = 1$
 $\Rightarrow \log_3(\log_2 x) - \log_3(\log_{1/2} y) = 1$
 $\Rightarrow \log_3 \left(\frac{\log_2 x}{\log_{1/2} y} \right) = 1 = \log_3 3$
 $\Rightarrow \frac{\log_2 x}{\log_{1/2} y} = 3 \Rightarrow \log_2 x = 3 \log_{1/2} y = -3 \log_2 y$
 $\Rightarrow x = y^{-3} \Rightarrow xy^3 = 1 \quad \dots(i)$

Again dividing eq. (i) by $xy^2 = 4$, we get $y = \frac{1}{4}$

$$\therefore x = 64 \quad [\text{From eq. (i)}]$$

☞ **Alternatively** Go through options.

- 47** Best way is to check the options.

☞ **Alternatively**

$$\begin{aligned}\log_{(1/\sqrt{2})}(1/\sqrt{8}) &= \log_2(4^x + 1) \cdot \log_2(4^{x+1} + 4) \\ \Rightarrow \frac{\log_2(1/\sqrt{8})}{\log_2(1/\sqrt{2})} &= \log_2(4^x + 1) \cdot \log_2(4^{x+1} + 4) \\ \Rightarrow 3 &= [\log_2 4 + \log_2(4^x + 1)] [\log_2(4^x + 1)] \\ \Rightarrow (2+t)t &= 3, \text{ where } t = \log_2(4^x + 1) \\ \Rightarrow t &= -3, 1\end{aligned}$$

If $\log_2(4^x + 1) = -3$, then $4^x = -7/8$ which is not possible

If $\log_2(4^x + 1) = 1$, then $4^x + 1 = 2 \Rightarrow 4^x = 1 \Rightarrow x = 0$

- 48** $\log_3 4 - \log_3(3x + 1) = \log_3 \frac{3}{(5x - 2)}$

$$\begin{aligned}\log_3 \frac{4}{3x+1} &= \log_3 \frac{3}{(5x-2)} \\ \Rightarrow \frac{4}{(3x+1)} &= \frac{3}{(5x-2)} \Rightarrow 20x - 8 = 9x + 3 \\ \Rightarrow 11x &= 11 \Rightarrow x = 1\end{aligned}$$

☞ **Alternatively** Go through options.

- 49** $|3 - 4x| > 2 \Rightarrow (3 - 4x) > 2 \text{ or } -(3 - 4x) > 2$

$$\Rightarrow x < \frac{1}{4} \text{ or } x > \frac{5}{4}$$

$$\therefore x \in \left(-\infty, \frac{1}{4}\right) \cup \left(\frac{5}{4}, \infty\right)$$

- 50** $\log_{100} |x + y| = \frac{1}{2}$
 $\Rightarrow |x + y| = 10 \quad (-10 \text{ is inadmissible})$

$$\text{and } \log_{10} y - \log_{10} |x| = \log_{100} 4$$

$$\Rightarrow \log_{10} \left(\frac{y}{|x|} \right) = \log_{10} 2 \Rightarrow \frac{y}{|x|} = 2 \Rightarrow y = 2|x|$$

Hence we have $|x + 2|x|| = 10$

Now, if $x > 0$, then we have, $3x = 10 \Rightarrow x = \frac{10}{3}$

and if $x < 0$, then $|-x| = 10 \Rightarrow x = -10$

$$\text{Thus, } x = \frac{10}{3}, \quad y = \frac{20}{3}$$

$$\text{and } x = -10, \quad y = 20$$

✓ **Level 02 Higher Level Exercise**

1 See the theory of this chapter

$$\begin{aligned} 3 \log_2 \left(\frac{x}{y} \right) + \log_4 \left(\frac{x}{y} \right)^2 + \log_8 \left(\frac{x}{y} \right)^3 + \dots \\ = \log_2 \left(\frac{x}{y} \right) + \log_2 \left(\frac{x}{y} \right) + \log_2 \left(\frac{x}{y} \right) + \dots \\ = \log_2 \left(\frac{x}{y} \times \frac{x}{y} \times \frac{x}{y} \times \dots n \text{ times} \right) \\ = \log_2 \left(\frac{x}{y} \right)^n = n \log_2 \left(\frac{x}{y} \right) \end{aligned}$$

$$\begin{aligned} 4 \log m + \log m^2 + \log m^3 + \dots + \log m^n \\ = \log (m \cdot m^2 \cdot m^3 \dots m^n) = \log m^{(1+2+3+\dots+n)} \\ = \log (m)^{\frac{n(n+1)}{2}} = \frac{n(n+1)}{2} \log m \end{aligned}$$

5 Given that $9^n < 10^8$

Taking log to both sides

$$\begin{aligned} \Rightarrow \log 9^n < \log 10^8 \\ \Rightarrow 2n \log 3 < 8 \log 10 \\ \Rightarrow 2n \times 0.4771 < 8 \\ \Rightarrow n \times 0.9542 < 8 \\ \Rightarrow n < \frac{8}{0.9542} \\ n < 8.3839 \\ \therefore n = 8 \quad (\because n \in \mathbb{N}) \end{aligned}$$

6 Taking log of both sides with base 3 we have,

$$\begin{aligned} [\log_3 x^2 + (\log_3 x)^2 - 10] \log_3 x = -2 \log_3 x \\ \Rightarrow \log_3 x = 0 \\ \text{or } (\log_3 x)^2 + 2 \log_3 x - 8 = 0 \\ \Rightarrow x = 1, (\log_3 x + 4)(\log_3 x - 2) = 0 \\ \Rightarrow x = 1, \log_3 x = -4 \text{ and } \log_3 x = 2 \\ \Rightarrow x = 1, x = \frac{1}{81} \text{ and } x = 9 \\ \therefore x = \left\{ 1, \frac{1}{81}, 9 \right\} \end{aligned}$$

$$7 \quad (x-3) > 0 \Rightarrow x > 3$$

$$\text{and } (2-x) > 0 \text{ and } 2-x \neq 1$$

$$\therefore x < 2 \text{ and } x \neq 1$$

Clearly there is no single value for which these inequalities are satisfied. Thus the set of its solutions is empty.

$$\begin{aligned} 8 \log_3 30 = \frac{1}{a} \Rightarrow a = \log_{30} 3 \text{ and } \log_5 30 = \frac{1}{b} \\ \Rightarrow b = \log_{30} 5 \\ 3 \log_{30} 2 = 3[\log_{30}(30/15)] \\ = 3[\log_{30} 30 - \log_{30} 15] \\ = 3[\log_{30} 30 - (\log_{30} 3 + \log_{30} 5)] \\ = 3[1 - a - b] \end{aligned}$$

$$\begin{aligned} 9 \quad x^{\log_x |3-x|^2} = 4 \\ \Rightarrow |3-x|^2 = 4 \\ \Rightarrow |3-x| = 2 \quad (-2 \text{ is inadmissible}) \\ \Rightarrow (3-x) = 2 \text{ or } -(3-x) = 2 \\ \Rightarrow x = 1 \text{ or } x = 5 \end{aligned}$$

10 Let $2^{x^2+2} = t$, then

$$\begin{aligned} 4^{x^2+2} - 9 \cdot 2^{x^2+2} + 8 = 0 \text{ becomes} \\ t^2 - 9t + 8 = 0 \\ \Rightarrow t = 1, 8 \\ \Rightarrow 2^{x^2+2} = 1 \\ \Rightarrow x^2 + 2 = 0 \text{ but this has no solution} \\ \text{If } 2^{x^2+2} = 8 \Rightarrow x^2 + 2 = 3 \\ \Rightarrow x^2 = 1 \Rightarrow x = \pm 1 \end{aligned}$$

11 Taking log of both sides, we get

$$\begin{aligned} \left[(\log_3 x)^2 - \frac{9}{2} \log_3 x + 5 \right] \log_3 x = \frac{3}{2} \\ \Rightarrow 2(\log_3 x)^3 - 9(\log_3 x)^2 + 10 \log_3 x - 3 = 0 \\ \Rightarrow (\log_3 x - 1)(\log_3 x - 3)(2 \log_3 x - 1) = 0 \\ \Rightarrow \log_3 x = 1, \log_3 x = 3, 2 \log_3 x = 1 \\ \Rightarrow x = 3, x = 27, x = \sqrt{3} \\ \text{i.e., } x = (\sqrt{3}, 3, 27) \end{aligned}$$

$$12 \quad x^2 + 6x + 8 > 0 \text{ and } 2x^2 + 2x + 3 > 0$$

$$\begin{aligned} \Rightarrow (x+4)(x+2) > 0 \text{ and } \left(x + \frac{1}{2}\right)^2 + \frac{5}{4} > 0 \\ \Rightarrow x \in (-\infty, -4) \cup (-2, \infty) \quad \dots(i) \end{aligned}$$

The given equation can be written as

$$\begin{aligned} \log_{(2x^2+2x+3)}(x^2-2x) = 1 \\ \Rightarrow x^2 - 2x = 2x^2 + 2x + 3 \\ \Rightarrow x^2 + 4x + 3 = 0 \\ \Rightarrow x = -1, -3 \\ \text{but at } x = -3, \log_{(x^2+6x+8)} \text{ is not defined} \\ \text{Hence, } x = -1 \end{aligned}$$

13 Let $u = \log_{10} p$, then the given inequality reduces to

$$\begin{aligned} (2+u)^2 + (1+u)^2 + u \leq 9 \\ 2u^2 + 7u + 5 \leq 9 \\ 2u^2 + 7u - 4 \leq 0 \\ \Rightarrow 2u^2 + 8u - u - 4 \leq 0 \\ \Rightarrow 2u(u+4) - 1(u+4) \leq 0 \\ \Rightarrow (u+4)(2u-1) \leq 0 \end{aligned}$$

$$\Rightarrow -4 \leq u \leq \frac{1}{2}$$

$$\Rightarrow -4 \leq \log_{10} p \leq \frac{1}{2}$$

$$\Rightarrow 10^{-4} \leq p \leq 10^{1/2}$$

14 Let $u = \log_2 x$, then

$$2 \log_2 \log_2 x + \log_{1/2} \log_2 (2\sqrt{2}x) = 1$$

$$\Rightarrow 2 \log_2 u + \log_{1/2} (\log_2 2^{3/2} + u) = 1$$

$$\Rightarrow \log_2 u^2 - \log_2 (3/2 + u) = 1$$

$$\Rightarrow \log_2 \left(\frac{u^2}{3/2 + u} \right) = 1$$

$$\Rightarrow u^2 = 2 \left(\frac{3}{2} + u \right)$$

$$\Rightarrow u^2 - 2u - 3 = 0$$

$$\Rightarrow u = -1, 3$$

$$\Rightarrow x = \frac{1}{2}, 8$$

but at $x = \frac{1}{2}$, $2 \log_2 \log_2 x$ is undefined

Hence, $x = 8$

15 By changing the base to 2 the given equation becomes

$$\frac{\log_2 x^2}{\log_2 x/2} + \frac{40 \log_2 \sqrt{x}}{\log_2 4x} - 14 \frac{\log_2 x^3}{\log_2 16x} = 0$$

$$\Rightarrow \frac{2 \log_2 x}{\log_2 x - 1} + 20 \frac{\log_2 x}{2 + \log_2 x} - 42 \frac{\log_2 x}{4 + \log_2 x} = 0$$

Let $t = \log_2 x$, then we have

$$2t(4+t)(2+t) - 42t(t-1)(t+2) + 20t(t-1)(t+4) = 0$$

$$\Rightarrow 2t[t^2 + 6t + 8 - 21t^2 - 21t + 42 + 10t^2 + 30t - 40] = 0$$

$$\Rightarrow t[2t^2 - 3t - 2] = 0$$

$$\Rightarrow t = 0, \quad t = 2, \quad t = -\frac{1}{2}$$

$$\Rightarrow x = 1, \quad x = 4, \quad x = \frac{1}{\sqrt{2}}$$

16 $m > 0$ and $n > 0$ and $m \neq 1$

i.e., $25 - x^2 > 0$ and $x \neq \pm 3$

and $24 - 2x - x^2 > 0$

$\Rightarrow -5 < x < 5, x \neq \pm 3$

and $x^2 + 2x - 24 < 0$

$-5 < x < 5, x \neq \pm 3$

and $-6 < x < 4$

$x \in (-5, 4) - \{-3, 3\}$... (i)

Case 1. $0 < m < 1 \Rightarrow 9 < x^2 < 25$

$\Rightarrow x \in (-5, -3) \cup (3, 5)$... (ii)

$\therefore$ The given inequality can be written as

$$\frac{24 - 2x - x^2}{14} < \frac{25 - x^2}{16}$$

$$\Rightarrow x^2 + 16x - 17 > 0$$

$$\Rightarrow (x + 17)(x - 1) > 0$$

$$\Rightarrow x < -17 \text{ or } x > 1$$

From eqs. (i) and (ii), we have

$$x \in (3, 4)$$

Case 2. If $m > 1$, i.e., $\frac{25 - x^2}{16} > 1$

$$\Rightarrow x \in (-3, 3) \quad \dots (iii)$$

The given inequality reduces to

$$\frac{24 - 2x - x^2}{14} > \frac{25 - x^2}{16}$$

$$\Rightarrow x^2 + 16x - 17 < 0$$

$$\Rightarrow -17 < x < 1$$

Thus combining with (iii), we get

$$x \in (-3, 1)$$

but $x \in \{-5, 4\} \sim \{-3, 3\}$ by (1)

thus $x \in (-3, 1)$

Hence, the required value of x should lie in $(-3, 1) \cup (3, 4)$.

17 $\log_2 \left(\frac{5}{2} \right) = (\log_2 5) - 1$

but $\therefore (\log_2 5) - 1 > (\log_2 4) - 1$

$$\therefore \log_{3/10} \left[\frac{10}{7} (\log_2 5 - 1) \right] < \log_{3/10} \frac{10}{7} < \log_{3/10} 1 (= 0)$$

($\because \log_a x < \log_a y$ if $x > y$ for $0 < a < 1$)

Since $\log_{3/10} \frac{10}{7} (\log_2 5 - 1) < 0$

Hence, the first inequality is true only if

$$\sqrt{(x-8)(2-x)} = 0$$

$$\Rightarrow x = 8 \text{ or } x = 2$$

If $x = 8$, then $\frac{2^x}{8} - (2^5 - 1) = 1 > 0$

If $x = 2$ then $\frac{2^x}{8} - (2^5 - 1) = \frac{1}{2} - (2^5 - 1) < 0$

Hence, $x = 8$ is the required value.

18 $2 \log_{10} x - \log_x \frac{1}{100} = 2 \log_{10} x - \frac{\log_{10} 10^{-2}}{\log_{10} x}$

$$= 2 \log_{10} x + \frac{2}{\log_{10} x}$$

$$= 2 \left(\log_{10} x + \frac{1}{\log_{10} x} \right)$$

Since $x > 1 \Rightarrow \log_{10} x > 0$

But since $AM \geq GM$

$$\therefore \frac{\left(\log_{10} x + \frac{1}{\log_{10} x} \right)}{2} \geq \sqrt{\log_{10} x \times \frac{1}{\log_{10} x}}$$

$$\Rightarrow \log_{10} x + \frac{1}{\log_{10} x} \geq 2$$

$$\Rightarrow 2 \left(\log_{10} x + \frac{1}{\log_{10} x} \right) \geq 4$$

$$\text{For } x = 10, \quad 2 \left(\log_{10} x + \frac{1}{\log_{10} x} \right) \geq 4$$

Hence the least value of $\left(\log_{10} x - \log_x \frac{1}{100} \right)$ is 4.

19 We have, $x^{[(3/4)(\log_2 x)^2 + \log_2 x - (5/4)]} = \sqrt{2}$

$$\Rightarrow \log_x \sqrt{2} = \frac{3}{4} (\log_2 x)^2 + \log_2 x - \left(\frac{5}{4} \right)$$

$$\Rightarrow \frac{\log \sqrt{2}}{\log x} = \frac{3}{4} (\log_2 x)^2 + \log_2 x - \left(\frac{5}{4} \right)$$

$$\Rightarrow \log \sqrt{2} = \log x \left[\frac{3}{4} (\log_2 x)^2 + \log_2 x - \left(\frac{5}{4} \right) \right]$$

$$\Rightarrow \log_2 \sqrt{2} = \log_2 x \left[\frac{3}{4} (\log_2 x)^2 + \log_2 x - \left(\frac{5}{4} \right) \right]$$

$$\Rightarrow \frac{1}{2} = \alpha \left[\frac{3}{4} \alpha^2 + \alpha - \frac{5}{4} \right] \quad (\text{say } \alpha = \log_2 x)$$

$$\Rightarrow 2 = 3\alpha^3 + 4\alpha^2 - 5\alpha$$

$$\Rightarrow 3\alpha^3 + 4\alpha^2 - 5\alpha - 2 = 0$$

$$\Rightarrow (\alpha - 1)(3\alpha^2 + 7\alpha + 2) = 0$$

$$\Rightarrow \alpha = 1 \Rightarrow \log_2 x = 1 \Rightarrow x = 2$$

$$\text{Again } 3\alpha^2 + 7\alpha + 2 = 0$$

$$\Rightarrow \alpha = -2, -\frac{1}{3}$$

$$\Rightarrow \log_2 x = -2 \text{ and } \log_2 x = -\frac{1}{3}$$

$$\Rightarrow x = \frac{1}{4} \Rightarrow x = 2^{-1/3}$$

$$\text{Hence } x = 2, \frac{1}{4}, 2^{-1/3}$$

Thus option (d) is most appropriate.

20 $2x + 3 > 0$ and $2x + 3 \neq 1$

$$\Rightarrow x > -\frac{3}{2} \text{ and } x \neq -1$$

$$\text{And } 3x + 7 > 0 \text{ and } 3x + 7 \neq 1$$

$$\Rightarrow x > -\frac{7}{3} \Rightarrow x \neq -2$$

$$\text{Now, } \log_{(2x+3)}(6x^2 + 23x + 21)$$

$$= 4 - \log_{(3x+7)}(4x^2 + 12x + 9)$$

$$\Rightarrow \log_{(2x+3)}(2x+3)(3x+7) = 4 - \log_{(3x+7)}(2x+3)^2$$

$$\Rightarrow \log_{(2x+3)}(2x+3) + \log_{(2x+3)}(3x+7)$$

$$= 4 - 2 \log_{(3x+7)}(2x+3)$$

$$\Rightarrow 1 + \log_{(2x+3)}(3x+7) + 2 \log_{(3x+7)}(2x+3) - 4 = 0$$

$$\Rightarrow \frac{2 \log (2x+3)}{\log (3x+7)} + \frac{\log (3x+7)}{\log (2x+3)} - 3 = 0 \quad \dots(i)$$

Putting $\frac{\log (2x+3)}{\log (3x+7)} = y$ in eq. (i), we get

$$2y + \frac{1}{y} - 3 = 0 \Rightarrow 2y^2 - 3y + 1 = 0$$

$$(2y - 1)(y - 1) = 0$$

$$\Rightarrow y = \frac{1}{2} \text{ and } y = 1$$

Now, when $y = \frac{1}{2}$

$$\therefore \frac{\log (2x+3)}{\log (3x+7)} = \frac{1}{2}$$

$$\Rightarrow (2x+3)^2 = (3x+7)$$

$$4x^2 + 9x + 2 = 0$$

$$(4x+1)(x+2) = 0$$

$$x = -\frac{1}{4}, -2$$

Again if $y = 1$, then

$$\frac{\log (2x+3)}{\log (3x+1)} = 1$$

$$\Rightarrow 2x+3 = 3x+7$$

$$x = -4$$

Since we know that $x > -\frac{3}{2}$ and $x > -\frac{7}{3}$

therefore $x = -2$ and $x = -4$ are not admissible values

Again since $x \neq -1$ and $x \neq -2$

Hence $x = -2$ is also inadmissible value

Thus, $x = -\frac{1}{4}$ is only possible value.

$\therefore$ Option (b) is correct.

☞ Alternatively Go through options

21 $x > 0, x \neq 1$

Since exponential function assumes positive value, so we must have $(x-1)^7 > 0$ i.e., $x > 1$.

Taking logarithm on both sides, we get

$$(\log_3 x^2 - 2 \log_x 9) \log (x-1) = 7 \log (x-1)$$

Either $\log (x-1) = 0$ i.e., $x = 2$

$$\text{or } \log_3 x^2 - 2 \log_x 9 = 7$$

$$\Rightarrow 2(\log_3 x) - 4 \log_x 3 = 7$$

$$\Rightarrow 2t - \frac{4}{t} = 7, \quad (\because t = \log_3 x)$$

$$\Rightarrow 2t^2 - 7t - 4 = 0$$

$$\Rightarrow t = 4, -\frac{1}{2}$$

$$\log_3 x = 4 \Rightarrow x = 81$$

If $\log_3 x = -\frac{1}{2}$, then $x = 3^{-1/2} < 1$, which is not the case

Hence, $x = 2, 81$

$$\begin{aligned} 22 \quad (2x)^{\ln 2} &= (3y)^{\ln 3} & \dots(i) \\ 3^{\ln x} &= 2^{\ln y} & \dots(ii) \end{aligned}$$

Now, taking log of both the sides in Eq. (ii), you get

$$\begin{aligned} \log x \log 3 &= \log y \log 2 \\ \Rightarrow \log y &= \frac{\log x \log 3}{\log 2} & \dots(iii) \end{aligned}$$

Similarly, taking log of both the sides in (i), you get

$$\begin{aligned} \log 2(\log 2 + \log x) &= \log 3(\log 3 + \log y) \\ \Rightarrow (\log 2)^2 + \log 2 \log x &= (\log 3)^2 + \frac{(\log 3)^2 \log x}{\log 2} \quad [\text{from eq. (iii)}] \\ \Rightarrow (\log 2)^2 - (\log 3)^2 &= \frac{(\log 3)^2 - (\log 2)^2}{\log 2} (\log x) \\ \Rightarrow -\log 2 &= \log x \\ \Rightarrow x = \frac{1}{2} \Rightarrow x_0 &= \frac{1}{2} \end{aligned}$$

Hence, choice (c) is the answer.

$$\begin{aligned} 23 \quad 3 &< 20^{1/2} \\ \Rightarrow \log_{20} 3 &< \log_{20} (20)^{1/2} \\ \Rightarrow \log_{20} 3 &< \frac{1}{2} \\ \text{and} \quad 3 &> 20^{1/3} \\ \Rightarrow \log_{20} 3 &> \log_{20} (20)^{1/3} \end{aligned}$$

$$\begin{aligned} \Rightarrow \log_{20} 3 &< \frac{1}{3} \\ \therefore \frac{1}{3} &< \log_{20} 3 < \frac{1}{2} \end{aligned}$$

Hence, choice (b) is the answer.

$$24 \quad \text{Let } m = \frac{5}{500^{50}} \Rightarrow m = 5 \times 500^{-50}$$

Taking log on both the sides, we get

$$\begin{aligned} \log m &= \log(5 \times 500^{-50}) \\ \log m &= \log 5 + \log 500^{-50} \\ \log m &= \log 5 - 50 \log 500 \\ \log m &= \log 5 - 50 \log(5 \times 10^2) \\ \log m &= \log 5 - 50(\log 5 + 2 \log 10) \\ \log m &= -49 \log 5 - 100 \log 10 \\ \log m &= -49 \log 5 - 100 \\ \log m &= -49(0.4771) - 100 \\ \log m &= -123.3779 \\ \log m &= -123 - 0.3779 \\ \log m &= -124 + 0.6221 \\ \log m &= \overline{124.6221} \end{aligned}$$

Since, the characteristic of $\log m$ is $\overline{124}$, therefore the number of consecutive zeros immediately after the decimal point will be $124 - 1 = 123$.

Hence choice (c) is the correct one.

CHAPTER 17

Functions and Graphs

In the introduction of this chapter, I would like to mention that in the recent past this chapter contributed 3-12 questions in some years. Basically on an average 4-5 questions are being asked from this chapter. Besides most of the questions are based on the user defined (or self defined) functions which are rather easier in comparison to the problems based on the algebraic functions of pure mathematics. Still you are required to know some basic characteristic and applications of standard algebraic functions, since they are not uncommon in CAT. Remember that a little bit of practice on this chapter will give you an edge over your competitor, since the problems related to this chapter are very easy provided that these problems are being solved patiently. Moreover, you need not to remember any formula at all.

The structure of the chapter is as follows :

1. Function (An introduction)
2. Domain, range and graphs of different types of functions and operations on real valued functions.
3. Transformation of graphs.
4. Composite functions
5. Even and odd functions
6. Surjective and injective functions
7. Inverse functions
8. Binary operations
9. User defined (or self defined) operations (Also important for XAT)
10. Maxima and minima.

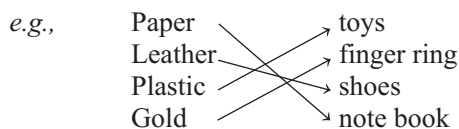
17.1 Functions

Suppose we have to produce some required products from the raw materials, then the raw material is known as input and the desired products are known as output. For this we have to look into two points.

1. Every input must give some output (to avoid wastage of inputs)
2. No input should give more than one output. (To avoid ambiguity or variation from the desired output). Thus, if the above conditions are satisfied by any input output relation, then this relation is called function,

Chapter Checklist

- Functions
- Domain & Graphs of Different Types of Functions
- Range
- Equal Function
- Type of Functions
- Algebraic Operations on Real Functions
- Transformation of Graphs
- Composite Functions
- Even and Odd Functions
- Surjective and Injective Functions
- Inverse Function
- Binary Operations
- User Defined Functions
- Maxima and Minima
- **CAT Test**



Here you can see that each input gives an output necessarily and no input give more than one output. The collection of raw materials (*i.e.*, input) is called the “Domain” and the collection of products (or output) is called the “Range”.

Definitions

- 1. Domain :** Set of inputs (also called pre-images) is called domain.
- 2. Codomain :** Set of possible outputs is called codomain.
- 3. Range :** Set of actual outputs (also called images) is known as range. Here, $\text{Range} \subseteq \text{Codomain}$.

Mathematical Definition of Function

Let A and B be two non-empty sets, then a function from ‘ A ’ to ‘ B ’ is a rule that assigns to each element in a set A , one and only one element of a set B . In general, the sets A and B need not be sets of real numbers. However, we consider only those functions for which A and B are both subsets of the real numbers. If the number of elements in A be n and the number of elements in B be m , then the total number of functions that can be defined from A to B is n^m .

$$A \xrightarrow{f} B \quad \text{or} \quad f : A \longrightarrow B$$

If a pre-image is denoted by x and an image is denoted by y then we can write $y = f(x)$.

Here, $f(x)$ is called the value of the function f at the point x .

Remember

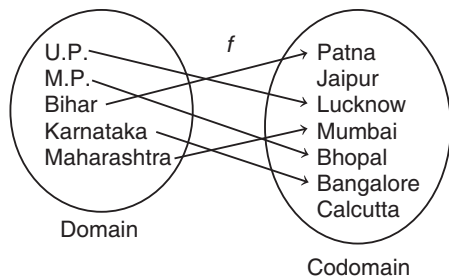
A function cannot be multiple-valued. For example, the expression $\pm\sqrt{x}$ does not define a function of x , since it assigns two values to each positive x . *i.e.*, $y = f(x)$

or $y = \pm\sqrt{x}$ is not a function since y assumes two values

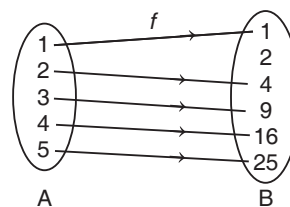
i.e., $y = +\sqrt{x}$ and $y = -\sqrt{x}$, which is impossible.

Methods of Representation of a Function

- 1. Verbal description :** For example, if f is a function which assigns to each state in the country its capital city.



2. Arrow diagram :



$$\text{Domain} = \{1, 2, 3, 4, 5\}$$

$$\text{Codomain} = \{1, 2, 4, 9, 16, 25\}$$

$$\text{Range} = \{1, 4, 9, 16, 25\}$$

3. Tabular form :

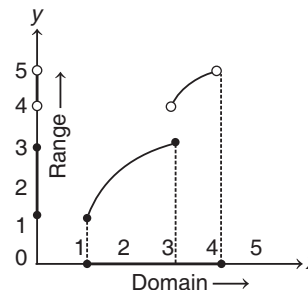
e.g.,

x	-1	-2	-3	0	1	2
y	-1	-8	-27	0	1	8

$$D = \{-1, -2, -3, 0, 1, 2\}, R = \{-1, -8, -27, 0, 1, 8\}$$

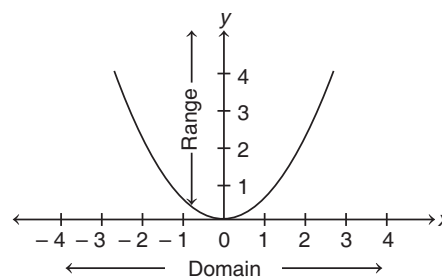
$$f(2) = 8, \quad f(-2) = -8 \text{ etc.}$$

4. Graph :



$$D = [1, 4], R = [1, 3] \cup (4, 5)$$

5. Equation (or formula) : $f : R \rightarrow R \quad f(x) = x^2 \quad \forall x \in R$



$$D = R \quad (\text{all real numbers}),$$

$$\text{Codomain} = R \quad (\text{All real numbers})$$

$$(\text{Range}) R = R^+ \cup \{0\}$$

$$(R^+ \rightarrow \text{set of positive real numbers})$$

17.2 Domain & Graphs of Different Types of Functions

A function $f : A \rightarrow B$ is called a real valued if the image of every element of A , under f is a real number *i.e.*, if $f(x) \in R \quad \forall x \in A; y = f(x)$

$x \rightarrow$ independent variable

$y \rightarrow$ dependent variable

Domain : The values of x must be such that for every x , y must be real is called the domain. In other words the value of x for which the values of $f(x)$ is not a real number cannot be included in the domain of the function $y = f(x)$. There are five different common situations where the domain of $f(x)$ for some particular value(s) is not defined.

(a) **When base of $(1/2n)^{\text{th}}$ power is negative for any**

$$n \in \mathbb{I}: \text{If } x^{1/2} = k \Rightarrow x = k^2$$

since k^2 is always positive even when x is negative, therefore this equality is not satisfied. Hence the domain of definition of $y = \sqrt{x}$ is defined for only $x \geq 0$ i.e., not defined for $x < 0$

Example

Function	Domain defined for	Domain not defined for
$(x-3)^{1/2}$	$(x-3) \geq 0 \Rightarrow x \geq 3$	$(x-3) < 0 \Rightarrow x < 3$
$(x+4)^{1/6}$	$(x+4) \geq 0 \Rightarrow x \geq -4$	$(x+4) < 0 \Rightarrow x < -4$
$(x -3)^{1/10}$	$ x -3 \geq 0 \Rightarrow x \geq 3$ $\Rightarrow x \leq -3 \text{ and } x \geq 3$	$ x -3 < 0 \Rightarrow x < 3$ $\Rightarrow -3 < x < 3$

(b) **When denominator of a function is zero**

If $\frac{x^2+1}{2x}$ is such that $x=0$, then $\frac{x^2+1}{2x}$ is not defined i.e., the

function $y = \frac{1}{x}$ is defined only for $x \neq 0$ (i.e., non zero real numbers) therefore the domain of function

$$f(x) = \frac{1}{x} \text{ is } (-\infty, 0) \cup (0, \infty) \text{ or } (-\infty, \infty) - \{0\}$$

or $-\infty < x < 0 \cup 0 < x < \infty$.

Examples

Function	Domain defined for	Domain not defined for
$\frac{1}{(x-3)}$	$x \in (-\infty, \infty) - \{3\}$	$x = 3$
$\frac{19}{(x^2-16)}$	$x \in (-\infty, \infty) - \{\pm 4\}$	$x = \pm 4$
$\frac{(x^2+4)}{ x -5}$	$x \in (-\infty, \infty) - \{\pm 5\}$	$x = \pm 5$
$\frac{1}{(l_n x) - 1}$	$x \in (0, \infty) - \{e\}$	$x = e$

(c) **When function becomes 0°**

Example $y = (|x|-3)^{(x-3)}$ is undefined for $x = 3$

(d) **When the value of x in $\log_a x$ becomes non-positive (i.e., negative or zero)**

$\therefore \log_{10}(-3)$ is not defined.

(e) **When the value of a in $\log_a x$ becomes 1 or 0 or negative.**

$\therefore \log_1 5$ is not defined. Similarly, $\log_{(-3)} 10$ is not defined.

Example

Function	Domain defined for	Domain not defined for
$\log_{10}(x-5)$	$(x-5) > 0 \Rightarrow x > 5$	$x-5 \leq 0 \Rightarrow x \leq 5$
$\log_{15}(7-x)$	$(7-x) > 0 \Rightarrow x < 7$	$(7-x) \leq 0 \Rightarrow x \geq 7$
$\log_{(x^2-4)} 12$	$(x^2-4) > 0$ and $x^2-4 \neq 1$ $\Rightarrow x < -2$ and $x^2 \neq 5$ and $x > 2$ and $x \neq \pm \sqrt{5}$ $\therefore x \in (-\infty, -2) \cup (2, \infty)$ excluding $\pm \sqrt{5}$	$x^2-4 \leq 0$ and $x^2-4 = 1$ $\Rightarrow -2 \leq x \leq 2$ and $x^2 = 5$ and $x = \pm \sqrt{5}$ $\therefore x \in [-2, 2] \cup \{\pm \sqrt{5}\}$

17.3 Range

The set of all images of all the elements of domain. It can be obtained by one of the following ways.

(i) Obtaining x in terms of y and then finding the values of y for which we get real x . e.g., $f(x) = \frac{x+2}{x+3} = y$

$$\Rightarrow yx + 3y = x + 2 \Rightarrow x(y-1) = (2-3y) \Rightarrow x = \frac{2-3y}{y-1}$$

for $y=1$, there is no real value of x for the above equation is satisfied hence.

Range = $R - \{1\}$ $R \rightarrow$ Set of real numbers

(ii) If the given function is continuous, then Range = $[l, m]$, where l is the least value of the function in its domain and m is the maximum value in its domain.

(iii) By sketching the graphs of the function.

17.4 Equal Function

Two functions f and g are said to be equal iff (i.e., if and only if)

(i) the domain of f = domain of g

(ii) the co-domain of f = co-domain of g , and

(iii) $f(x) = g(x)$ for every x belong to their common domain. If two functions f and g are equal then we write $f = g$, e.g.,

1. Let $A = \{1, 2\}$, $B = \{3, 6\}$ and $f: A \rightarrow B$ given by $f(x) = x^2 + 2$ and $g: A \rightarrow B$ given by $g(x) = 3x$. Then we observe that f and g have the same domain and co-domain also $f(1) = 3 = g(1)$; $f(2) = 6 = g(2)$

Hence, $f = g$

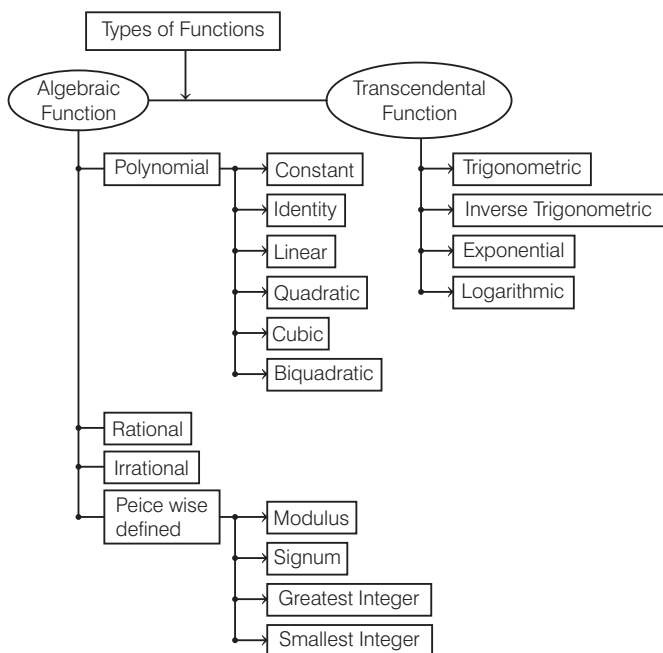
2. Let $f(x) = \sqrt{\frac{x-3}{x-5}}$ and $g(x) = \frac{\sqrt{x-3}}{\sqrt{x-5}}$ $f(x)$ is defined

when $x-3$ and $x-5$ both are negative, since $\frac{x-3}{x-5} > 0$

But when $x - 3$ and $x - 5$ are negative then both $\sqrt{x - 3}$ and $\sqrt{x - 5}$ are undefined. Hence $g(x)$ is undefined

$\therefore$ Domain of $f \neq$ Domain of g . Hence, $f(x)$ and $g(x)$ are different.

17.5 Types of Functions



Algebraic Functions

A function which contains finite number of terms having different powers of independent variable (x) and the operations $+$, $-$, $\times$, $\div$ is called algebraic function

e.g., $4x^2 - 3x^{1/2} + 7$, $5x^2 + 13x + 8$, $\frac{x^2 + 1}{x^2 - 2}$ etc.

A-1) Polynomial Functions A function $f(x)$ of the following form is known as polynomial function.

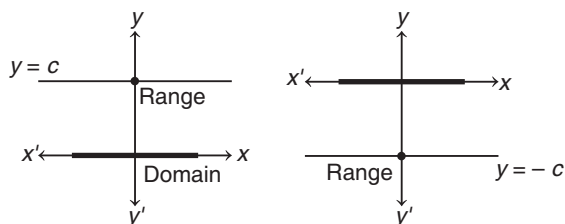
$$f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

where $n \in \mathbb{N}$ and $a_1, a_2, a_3 \dots a_n \in \mathbb{R}$

Domain : It is the set of real numbers

Range : It is the set of real numbers

(A-1.1) **Constant function :** $f(x) = c$, $c \in \mathbb{R}$, c is constant.

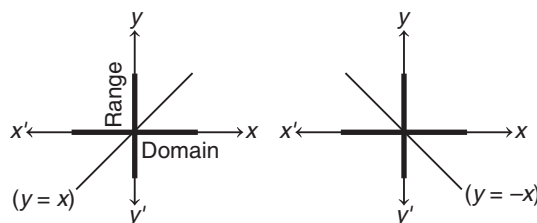


Domain : It is the set of real numbers

Range : It is a particular real number $\{c\}$

(A-1.2) **Identity function**

$$y = x \quad \text{i.e., } f(x) = x$$

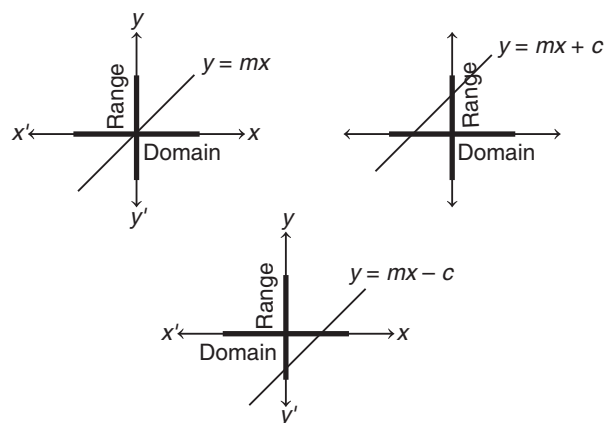


(i.e., input equals to output or pre image equals to image)

Domain $\rightarrow$ It is the set of real numbers.

Range $\rightarrow$ It is also the set of real numbers.

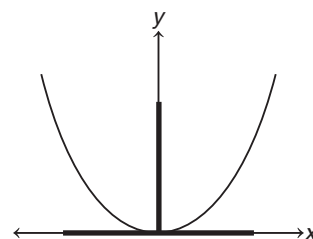
(A-1.3) **Linear function** $y = mx + c$ ($m, c \in \mathbb{R}$, m is positive)



Domain $\rightarrow$ It is the set of real numbers.

Range $\rightarrow$ It is also the set of real numbers.

(A-1.4.) **Quadratic function** $y = f(x) = x^2$



$$\therefore f(x) = f(-x)$$

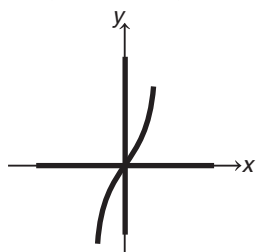
$\therefore$ Graph of $f(x)$ is symmetric about y-axis.

Domain $\rightarrow$ It is the set of real values.

Range $\rightarrow \mathbb{R}^+ \cup \{0\}$ i.e., non-negative real numbers

(A-1.5) **Cubic function** $y = f(x) = x^3$

$$\therefore f(-x) = -f(x)$$



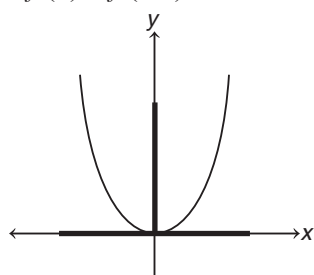
$\therefore$ Graph of $f(x)$ is symmetric about origin

Domain $\rightarrow R$ (set of real numbers)

Range $\rightarrow R$ (set of real numbers)

(A-1.6) Biquadratic function $y = f(x) = x^4$

$$\therefore f(x) = f(-x)$$



$\therefore$ Graph of $f(x)$ is symmetric about y-axis.

Domain $\rightarrow R$ (set of real numbers)

Range $\rightarrow R^+ \cup \{0\}$ i.e., set of non-negative real numbers.

NOTE Graph of $f(x) = x^n$ is symmetric about
(a) y-axis if n is even (b) origin if n is odd

Relation between the values of x , x^2 , x^3 ...etc.

For	Order of the values
$x \in (0, 1)$	$x > x^2 > x^3 > x^4 \dots$
$x \in (1, \infty)$	$x < x^2 < x^3 < x^4 \dots$
$x \in (-1, 0)$	$x < x^3 < x^5 \dots$ (-ve values)
	$x^2 > x^4 > x^6 \dots$ (+ve values)
$x \in (-\infty, -1)$	$x > x^3 > x^5 \dots$ (-ve values)
	$x^2 < x^4 < x^6 \dots$ (+ve values)

(A-2) Rational Function $f: A \rightarrow R; f(x) = \frac{P(x)}{Q(x)}$

Here, $P(x)$ and $Q(x)$ are polynomial functions and

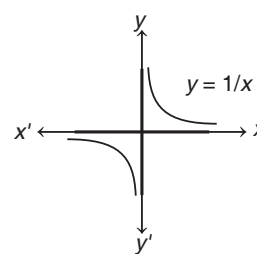
$$A = \{x : x \in R \text{ such that } Q(x) \neq 0\}$$

Examples

Function	Domain
1. $\frac{a_0 + a_1x + a_2x^2 + \dots + a_nx^n}{P_0 + P_1x + P_2x^2 + \dots + P_nx^n}$	Set of all the real numbers except the real roots of the equation $P_0 + P_1x + P_2x^2 + \dots + P_nx^n$
2. $\frac{x^2 + 5x + 8}{x^2 - 5x + 6}$	$R - \{2, 3\}$
3. $\frac{1}{x^n}; n \in N$	$R - \{0\}$

Graphs of Important Rational Functions

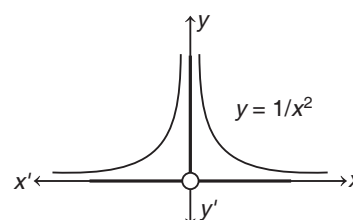
$$1. y = f(x) = \frac{1}{x} \Rightarrow f(x) = -f(-x)$$



Domain $\rightarrow R - \{0\}$; set of non-zero rational numbers

Range $\rightarrow R - \{0\}$; set of non-zero rational numbers

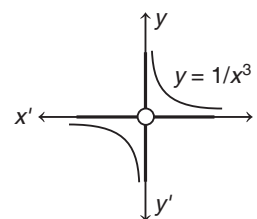
$$2. y = f(x) = \frac{1}{x^2} \Rightarrow f(x) = f(-x)$$



Domain $\rightarrow R - \{0\}$ i.e., set of non-zero rational numbers

Range $\rightarrow R^+$ i.e., set of positive rational numbers

$$3. y = f(x) = \frac{1}{x^3} \Rightarrow f(-x) = -f(x)$$



Domain $\rightarrow R - \{0\}$ i.e., set of non-zero rational numbers

Range $\rightarrow R - \{0\}$ i.e., set of non-zero rational numbers.

NOTE Graph of $f(x) = \frac{1}{x^n}$ ($n \in N$) is symmetric about
 (a) y axis if n is even (b) origin if n is odd.

Relation between the values of $\frac{1}{x}, \frac{1}{x^2}, \frac{1}{x^3}, \frac{1}{x^4} \dots$

For	The order of the values
$x \in (1, \infty)$	$\frac{1}{x} > \frac{1}{x^2} > \frac{1}{x^3} > \frac{1}{x^4} \dots$
$x \in (0, 1)$	$\frac{1}{x} < \frac{1}{x^2} < \frac{1}{x^3} < \frac{1}{x^4} \dots$
$x \in (-1, 0)$	$\frac{1}{x^2} < \frac{1}{x^4} < \frac{1}{x^6} \dots$ (positive values) $\frac{1}{x} > \frac{1}{x^3} > \frac{1}{x^5} \dots$ (negative values)
$x \in (-\infty, -1)$	$\frac{1}{x^2} > \frac{1}{x^4} > \frac{1}{x^6} \dots$ (positive values) $\frac{1}{x} < \frac{1}{x^3} < \frac{1}{x^5} \dots$ (negative values)

(A-3) **Irrational Functions**: The algebraic functions containing one or more terms having non-integral rational powers of x are called irrational functions.

e.g., $y = 4x^2 + \sqrt{2x}$, then y is undefined if $x < 0$

$y = \sqrt{x}$ then y is undefined if $x < 0$

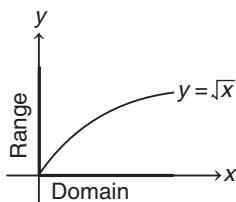
$y = \frac{5x^3 + 7x^2 + 3\sqrt{x}}{\sqrt{x+4}}$, then y is undefined if $x < 0$

and $\sqrt{x+4} \leq 0$

These functions are not defined for $f(x) < 0$. They are also not-defined if the denominator is zero.

Graphs of Important Irrational Functions

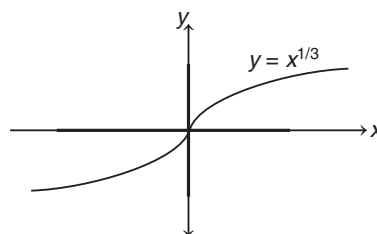
1. $y = f(x) = x^{1/2}$ (i.e., $\sqrt{x}$)



Domain $\rightarrow R^+ \cup \{0\}$ i.e., set of non-negative real numbers.

Range $\rightarrow R^+ \cup \{0\}$ i.e., set of non-negative real numbers

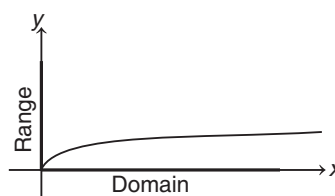
2. $y = f(x) = x^{1/3}$



Domain $\rightarrow$ (i.e., set of real numbers)

Range $\rightarrow R$ (i.e., set of real numbers)

3. $y = f(x) = x^{1/4}$



Domain $\rightarrow R^+ \cup \{0\}$ i.e., set of non-negative real numbers

Range $\rightarrow R^+ \cup \{0\}$ i.e., set of non-negative real numbers

Relation between the values of $x^{1/2}, x^{1/3}, x^{1/4} \dots$

For	Order of the values
$x \in (1, \infty)$	$x^{1/2} > x^{1/3} > x^{1/4} > x^{1/5} > \dots$
$x \in (0, 1)$	$x^{1/2} < x^{1/3} < x^{1/4} < x^{1/5} < \dots$
$x \in (-1, 0)$	$x^{1/3} > x^{1/5} > x^{1/7} \dots$
$x \in (-\infty, -1)$	$x^{1/3} < x^{1/5} < x^{1/7} \dots$

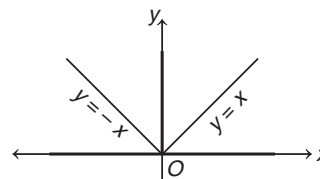
(A-4) Piecewise Defined Functions

(A-4.1) **Modulus function** (or absolute-value function)

$$y = f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$\therefore$

$$f(-x) = f(x)$$



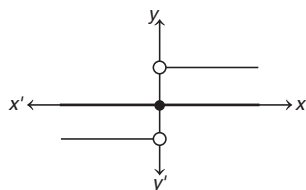
$\therefore f(x)$ is symmetric about y-axis.

Domain $\rightarrow R$ (i.e., set of real numbers)

Range $\rightarrow R \cup \{0\}$ (i.e., set of non-negative real numbers)

(A-4.2) **Signum function** [Sgn (x)]

$$y = f(x) = \text{Sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$



Domain $\rightarrow R$ Range $\rightarrow \{-1, 0, 1\}$

- Indicates that the points (0, 1) and (0, -1) are not included in the graphs.
- Indicates that the point (0, 0) is included in the graphs

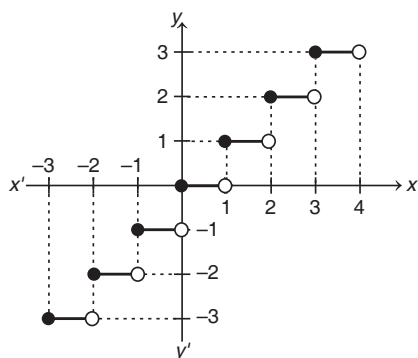
$$(a) \text{Sgn}(x) = \begin{cases} \frac{x}{|x|} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \quad (b) \text{Sgn}(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

(A-4.3) Greatest integer function

$y = [x]$ denotes the greatest integer less than or equal to x .

e.g., $[2.3] = 2$
 $[4.7] = 4$; $[5.2] = 5$
 $[-8.3] = -9$; $[-13.2] = -14$

i.e., $[n] = n$, if n is an integer and $[x] = n$, if $n \leq x < n+1$ Any real number x can be expressed as. Integral part + Fractional part i.e., $I + f$ where $0 \leq f < 1$ then $[x]$ gives the integral part of x .



Domain $\rightarrow R$ (Set of real numbers)

Range $\rightarrow$ (Integral values) $\{\dots -3, -2, -1, 0, 1, 2, 3, \dots \text{etc.}\}$
 (It is also called the step functions)

(A-4.4) Smallest (or least) integer function

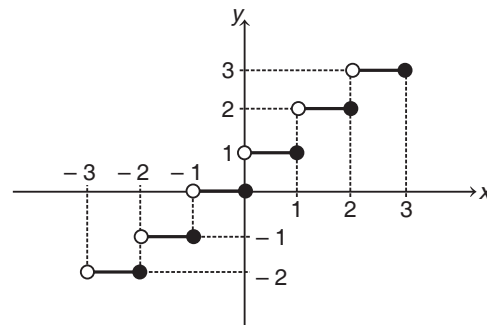
The function whose value at any number x is the smallest integer greater than or equal to x is called the least integer function

e.g., if $y = f(x) = \{x\}$ = Smallest integer, greater than or equal to x .

Then, $\{2.8\} = 3$

$$\{7.5\} = 8 \Rightarrow \{-3.6\} = -3 \Rightarrow \{-7.4\} = -7$$

i.e., $\{n\} = n$, if n is an integer and $\{x\} = n$, if $n < x \leq n+1$



Domain $\rightarrow R$ (Set of real numbers)

Range $\rightarrow I$ (Integral values)

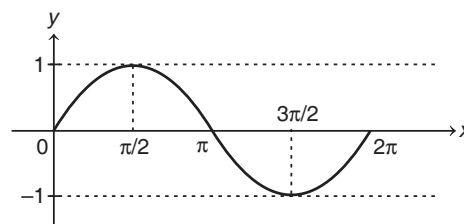
e.g., $\{-3, -2, -1, 0, 1, 2, 3 \text{ etc.}\}$

Transcendental Function

A function which is not an algebraic function is called a transcendental function. e.g., trigonometry, logarithmic and exponential function.

(T-1) Trigonometric Function :

(T-1.1) $y = f(x) = \sin x$

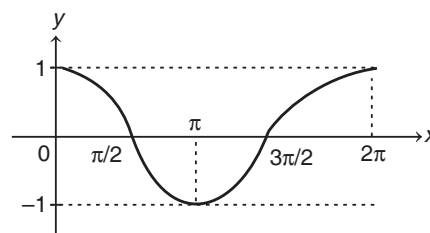


It is a periodic function with period 2π

Domain $\rightarrow R$ (Set of real numbers)

Range $\rightarrow [-1, 1]$ i.e., $\{x \in R : -1 \leq x \leq 1\}$

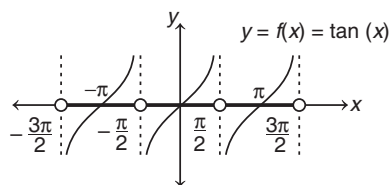
(T-1.2) $y = f(x) = \cos x$



Period = 2π

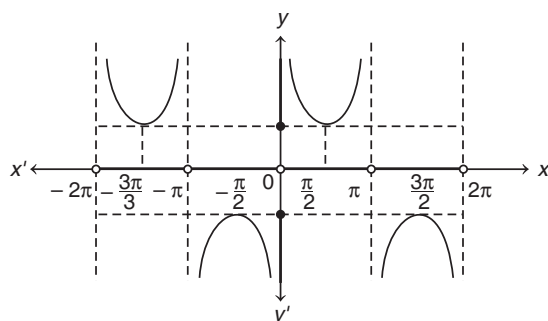
Domain $\rightarrow R$ Range $\rightarrow [-1, 1]$

(T-1.3)



Period = π Domain $\rightarrow R - \left\{ (2n+1)\frac{\pi}{2}, n \in I \right\}$

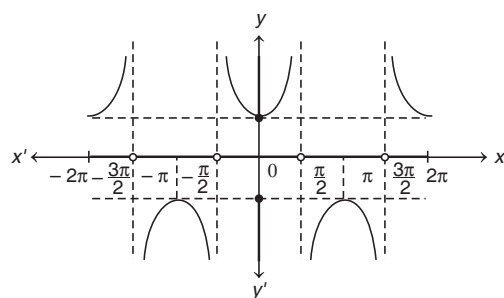
Range $\rightarrow R$ (Set of real numbers)

(T-1.4) $y = f(x) = \operatorname{cosec} x$ 

Period = 2π

Domain $\rightarrow R - \{n\pi : n \in I\}$

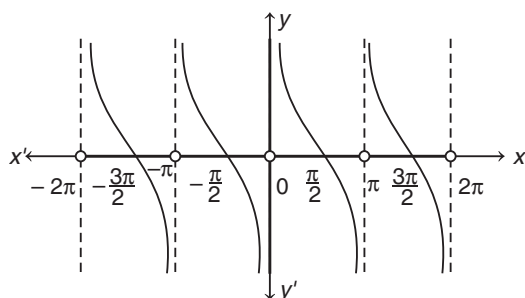
Range $(-\infty, -1] \cup [1, \infty)$

(T-1.5) $y = f(x) = \sec x$ 

Period = 2π

Domain $\rightarrow R - \left\{ (2n+1)\frac{\pi}{2} : n \in I \right\}$

Range $\rightarrow (-\infty, -1] \cup [1, \infty)$

(T-1.6) $y = f(x) = \cot x$ 

Period = π

Domain $\rightarrow \{n\pi : n \in I\}$

Range $\rightarrow R$ (set of real numbers)

(T-2.1) $y = \sin^{-1} x \Rightarrow x = \sin y \in [-1, 1]$

$\therefore$ Domain = $[-1, 1]$

(T-2.2) $y = \cos^{-1} x \Rightarrow x = \cos y \in [-1, 1]$

$\therefore$ Domain = $[-1, 1]$

(T-2.3) $y = \tan^{-1} x \Rightarrow x = \tan y \in R$

$\therefore$ Domain = R

(T-2.4) $y = \cot^{-1} x \Rightarrow x = \cot y \in R$

$\therefore$ Domain = R

(T-2.5) $y = \sec^{-1} x$

$\Rightarrow x = \sec y \in (-\infty, -1] \cup [1, \infty)$

$\therefore$ Domain = $(-\infty, -1] \cup [1, \infty)$

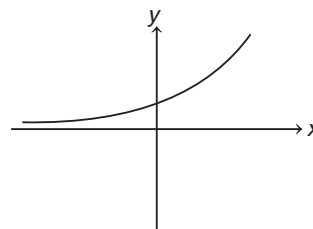
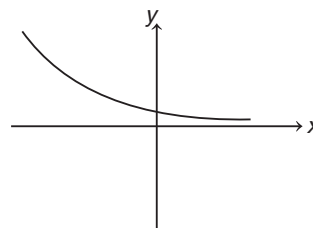
(T-2.6) $y = \operatorname{cosec}^{-1} x \Rightarrow x = \operatorname{cosec} y \in (-\infty, -1] \cup [1, \infty)$

$\therefore$ Domain = $(-\infty, -1] \cup [1, \infty)$

(T-3) Exponential Function $y = a^x$ (i) $y = a^x; a > 1$

Domain $\rightarrow R$

Range $\rightarrow R^+$

(ii) $y = a^x; 0 < a < 1$ 

Domain $\rightarrow R$

Range $\rightarrow R^+$

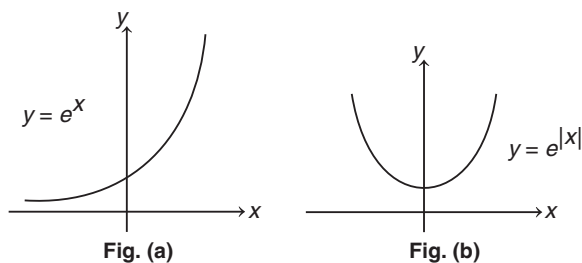


NOTE

$0 < a < 1$ function is decreasing i.e., $a^x > a^y$ if $x < y$

$a > 1$ function is increasing i.e., $a^x < a^y$ if $x < y$

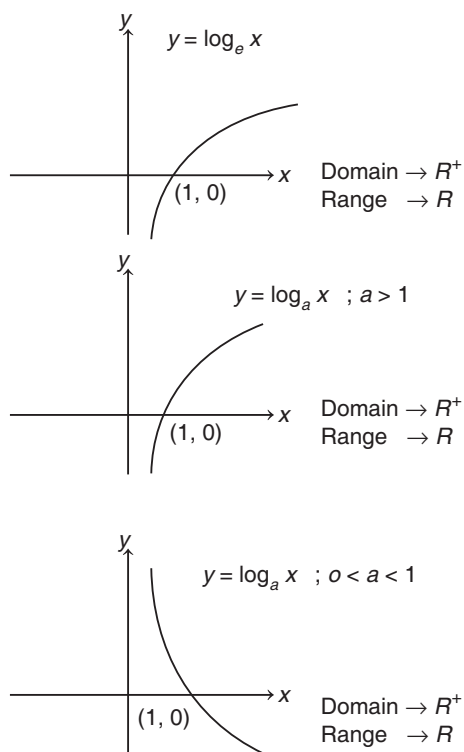
(iii) (a) $y = e^x$ and (b) $y = e^{|x|}$



(a) Domain $\rightarrow R$
Range $\rightarrow R^+$

(b) Domain $\rightarrow R$
Range $\rightarrow R^+$

(T-4) Logarithmic Function



Introductory Exercise 17.1

- The domain of the function $f(x) = x$ is :
(a) R (b) R^+
(c) $R - \{0\}$ (d) $R^+ \cup \{0\}$
- The domain of the function $f(x) = \sqrt{x}$ is :
(a) R (b) R^+
(c) $R^+ \cup \{0\}$ (d) $R - \{0\}$
- The domain of the function $f(x) = \frac{1}{x}$ is :
(a) R (b) R^+
(c) $R - \{0\}$ (d) $R^+ \cup \{0\}$

17.6 Algebraic Operations on Real Functions

Addition : If f and g are two functions, then the sum of the functions $(f + g)(x) = f(x) + g(x)$ is defined only for those values of x for which both f and g are defined i.e.,

$$x \in \text{Dom } f \cap \text{Dom } g$$

$$\therefore \text{Dom } (f + g) = \text{Dom } f \cap \text{Dom } g$$

Subtraction : If f and g are two functions, then the difference of the functions $(f - g)x = f(x) - g(x)$

Where $x \in \text{Dom } f \cap \text{Dom } g$

$$\therefore \text{Dom of } (f - g) = \text{Dom } f \cap \text{Dom } g$$

Multiplication : If k is any real number and f is a function, then kf is defined for all $x \in \text{Dom } f$ by $(kf)x = k f(x)$

Hence, if f and g are two functions, then the pointwise product fg is defined for all $x \in \text{Dom } f \cap \text{Dom } g$ by $(fg)x = f(x)g(x)$

$$\therefore \text{Dom of } (fg)x = \text{Dom } f \cap \text{Dom } g$$

Division : If f and g are two functions, then f/g is defined for all $x \in \text{Dom } f \cap \text{Dom } g \cap \{x : g(x) \neq 0\}$ by $(f/g)x = \frac{f(x)}{g(x)}$

$$\therefore \text{Dom } (f/g)x = \text{Dom } f \cap \text{Dom } g - \{x : x \in \text{Dom } g \text{ and } g(x) = 0\}$$

Composition : Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions, then $gof : A \rightarrow C$ defined by $(gof)x = g(f(x))$

Alternatively

$(gof(x)) = g f(x)$ i.e., gof is defined when

- input of f (i.e., $x \in \text{Domain of } f$ and
- input of g (i.e., $f(x) \in \text{Domain of } g$

Note that gof is defined, then fog may not be defined

- The domain of the function $f(x) = |x|$ is :
(a) R (b) R^+
(c) $R - \{0\}$ (d) $R^+ \cup \{0\}$
- The domain of the function $f(x) = \sqrt{|x|}$ is :
(a) $x \in (-\infty, \infty)$ (b) $x \in (0, \infty)$
(c) $x \in (-\infty, 0) \cup (0, \infty)$ (d) $x \in [0, \infty)$
- The domain of the function $f(x) = |\sqrt{x}|$ is :
(a) $x \in (-\infty, +\infty)$ (b) $x \in (0, \infty)$
(c) $x \in (9, 16)$ (d) $x \in [0, \infty)$

7. The domain of the function $f(x) = \frac{1}{\sqrt{x}}$ is :
 (a) $-\infty < x < \infty$ (b) $0 < x < \infty$
 (c) $R - \{0\}$ (d) $0 \leq x < \infty$
8. The domain of the function $f(x) = \sqrt{x^2}$ is :
 (a) $-\infty < x < \infty$ (b) $0 < x < \infty$
 (c) $R - \{0\}$ (d) $R^+ \cup \{0\}$
9. The domain of the function $f(x) = (\sqrt{x})^2$ is :
 (a) $-\infty < x < \infty$ (b) $0 < x < \infty$
 (c) $0 \leq x < \infty$ (d) none of these
10. The domain of the function $f(x) = \log_{10} x$ is :
 (a) $-\infty < x < \infty$ (b) $0 < x < \infty$
 (c) $0 \leq x < \infty$ (d) none of these
11. The domain of the function $f(x) = \log x^2$ is :
 (a) R (b) R^+
 (c) $R - \{0\}$ (d) $R^+ \cup \{0\}$
12. The domain of the function $f(x) = \frac{1}{\log_{10} x}$ is :
 (a) R (b) $R^+ - \{1\}$
 (c) $R^+ \cup \{0\}$ (d) $R - \{0\}$
13. The domain of the function $f(x) = \frac{1}{\sqrt{\log_{10} x}}$ is :
 (a) R (b) $R^+ - (0, 1]$
 (c) $R - \{0\}$ (d) none of these
14. The domain of the function $f(x) = \sqrt{3x - 4}$ is :
 (a) $[0, \infty)$ (b) $\left[\frac{3}{4}, \infty\right)$ (c) $\left[\frac{4}{3}, \infty\right)$ (d) $(4, \infty)$
15. The domain of the function $f(x) = \sqrt{4x - 3} + \sqrt{2x - 6}$ is :
 (a) $[0, \infty)$ (b) $\left[\frac{3}{4}, \infty\right)$ (c) $\left[\frac{4}{3}, \infty\right)$ (d) $[3, \infty)$
16. The domain of $y = \frac{1}{\sqrt{|x| - x}}$ is :
 (a) $[0, \infty)$ (b) $(0, \infty)$ (c) $(-\infty, 0)$ (d) $(-\infty, 0]$
17. The domain of $y = \frac{x}{\sqrt{x^2 - 3x + 2}}$
 (a) $(-\infty, \infty)$ (b) $[0, \infty)$
 (c) $(-\infty, 1] \cup [2, \infty)$ (d) none of these
18. The domain of $y = \log_{10}(\sqrt{6 - x} + \sqrt{x - 4})$ is :
 (a) R (b) $R - \{0\}$
 (c) $[6, \infty)$ (d) $(4, 6)$
19. The domain of definition of the function $y = \frac{1}{\log_{10}(1 - x)} + \sqrt{x + 2}$ is :
 (a) $[-2, 1)$ (b) $[-2, 0)$
 (c) $[-2, 0) \cup (0, 1)$ (d) $(-2, 1]$
20. The domain of definition of $f(x) = \frac{\log_2(x + 3)}{x^2 + 3x + 2}$ is :
 (a) $(-3, -2)$ (b) $(-2, -1]$
 (c) $(-3, \infty) \sim \{-1, -2\}$ (d) $[0, \infty)$
21. The domain of $f(x) = 2 \sin x + 3 \cos x + 4$ is :
 (a) R (b) R^+
 (c) $R - \{0\}$ (d) $R - \{n\pi : n \in I^+\}$
22. The domain of the function $y = \sqrt{\log \frac{1}{|\sin x|}}$ is :
 (a) R (b) $R \sim \left(\frac{n}{2}\pi; n \in I\right)$
 (c) $[0, \infty)$ (d) $R \sim \{n\pi, n \in I\}$
23. The domain of $f(x) = (x^2 + x + 1)^{-3/2}$ is :
 (a) $(-\infty, \infty)$ (b) $(0, \infty)$
 (c) $\left(\frac{3}{2}, \infty\right)$ (d) none of these
24. The domain of $f(x) = \log(5x - 6 - x^2)$ is :
 (a) $(5, 6)$ (b) $(2, 3)$
 (c) $[2, \infty)$ (d) none of these
25. The domain of $y = \log_x 5$ is :
 (a) $x > 5$ (b) $(0, \infty)$ (c) $(0, \infty) \sim \{1\}$ (d) $[5, \infty)$
26. The domain of $y = 9 - \sqrt{9 - x^2}$ is :
 (a) $x < -3$ (b) $-3 < x < 3$
 (c) $x \in [-3, 3]$ (d) none of these
27. The domain of $y = \frac{x}{\sqrt{x^2 - 5x + 6}}$ is :
 (a) $R \sim \{2, 3\}$ (b) $R \sim (2, 3)$
 (c) $R - [2, 3]$ (d) none of these
28. The range of the function $y = \frac{x^2}{1 + x^4}$ is :
 (a) $[0, \infty)$ (b) $[0, 2]$ (c) $\left(0, \frac{1}{2}\right)$ (d) $\left[0, \frac{1}{2}\right]$
29. Find the range of $f(x) = \frac{x^2 - 2}{x^2 - 3}$:
 (a) $(1, \infty)$ (b) $(-\infty, 2/3]$
 (c) $(-\infty, 2/3)$ (d) $(-\infty, 2/3] \cup (1, \infty)$
30. If $f(x) = \frac{x^2 + 2x + 3}{x}$, then the range of the function is
 (a) $[2\sqrt{3}, 3\sqrt{2}]$ (b) $R - (2 - 2\sqrt{3}, 2 + 2\sqrt{3})$
 (c) $R - (2, 2\sqrt{3})$ (d) none of these
31. Which of the following pairs are identical :
 (a) $f(x) = \sqrt{x^2}$, $g(x) = (\sqrt{x})^2$
 (b) $f(x) = \frac{1}{\sqrt{x^2}}$, $g(x) = \frac{x}{x^2}$
 (c) $f(x) = \log(x - 1) + \log(x - 2)$,
 $g(x) = \log(x - 1)(x - 2)$
 (d) none of the above

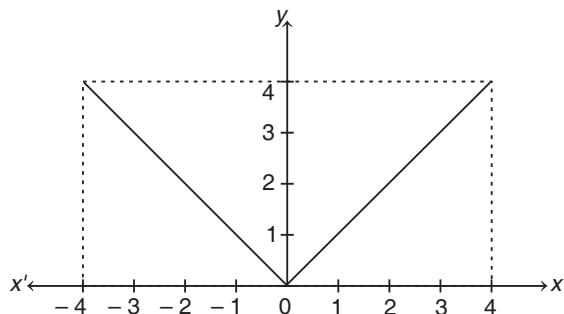
32. If $f(x) = \frac{1}{\sqrt{|x| - x}}$ and $g(x) = \frac{1}{\sqrt{x - |x|}}$, then
 (a) $\text{dom } f \neq \phi$ and $\text{dom } g = \phi$
 (b) $\text{dom } f = \phi$ and $\text{dom } g \neq \phi$
 (c) $\text{dom } f = \phi$ and $\text{dom } g = \phi$
 (d) f and g have the same domain
33. If $f(x) = x$, $g(x) = \sqrt{x^2}$ and $h(x) = \frac{x^2}{x}$, then the set of values for which the given functions $f(x)$, $g(x)$ and $h(x)$ are identical is :
 (a) $R - \{0\}$ (b) R
 (c) R^+ (d) $R^+ \cup \{0\}$
34. If $f(x) = \log x^4$ and $g(x) = 4 \log x$, then the domain for which $f(x)$ and $g(x)$ are identical ?
 (a) $(-\infty, \infty)$ (b) $[0, \infty)$
 (c) $(0, \infty)$ (d) none of these
35. If $f(x) = \text{Sgn } x$ and $g(x) = 1 + x - [x]$. Then for all x , $f(g(x))$ is equal to :
 (a) 0 (b) 1
 (c) x (d) $f(x)$
36. If $f(x) = 64x^3 + \frac{1}{x^3}$ and α, β are the roots of $4x + \frac{1}{x} = 2$, then :
 (a) $f(\alpha) = -24$ (b) $f(\alpha) = -16$
 (c) $f(\beta) = -16$ (d) both (b) and (c)
37. If $f(x) = \frac{x}{\sqrt{1+x^2}}$, then $f(f(f(x)))$ is :
 (a) $\frac{3x}{\sqrt{1+x^2}}$ (b) $\frac{x}{\sqrt{1+3x^2}}$
 (c) $\frac{x^3}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{3+3x^2}}$
38. If $f(x) = \frac{4^x}{4^x + 2}$, then the value of $f(x) + f(1-x)$ is :
 (a) 0 (b) -1
 (c) 1 (d) can't be determined
39. If $f(x) = 1 - f(1-x)$, then the value of $f\left(\frac{1}{999}\right) + f\left(\frac{2}{999}\right) + \dots + f\left(\frac{998}{999}\right)$ is :
 (a) 998 (b) 1
 (c) 499 (d) none of these
40. Let $f(x) = x^2$ and $g(x) = \sqrt{x}$, then :
 (a) $\text{gof}(3) = 9$ (b) $\text{gof}(-3) = 9$
 (c) $\text{gof}(3) = 3$ (d) $\text{gof}(-9) = 3$
41. If $g(x) = x^2 - x + 7$ and $\left(\frac{1}{9}\right) \text{gof}(x) = x^2 + x + 1$, then $f(x)$ is :
 (a) $2x - 3$ (b) $3x + 2$
 (c) $3x - 2$ (d) none of these
42. Let $f(x) = \frac{x}{x+3}$, then $f(x+1) = ?$
 (a) $\frac{3x+2}{x+2}$ (b) $\frac{x+1}{x+4}$ (c) $\frac{x+1}{x+3}$ (d) $\frac{2x+3}{x+3}$
43. Let $f(x) = \frac{x}{x+3}$, then $\frac{1}{f(x+1)} - f\left(\frac{1}{x+1}\right) = ?$
 (a) $\frac{3x^2+7x+4}{3(x^2+5x+5)}$ (b) $\frac{3(x^2+5x+5)}{3x^2+7x+4}$
 (c) $\frac{-x(2x+3)}{(x+3)^2}$ (d) $\frac{-(x+3)^2}{x(2x+3)}$
44. If $f(x) = \frac{x+1}{x-1}$, $x \neq 1$, find $f(f(f(f(2))))$
 (a) 2 (b) 3 (c) 4 (d) 6

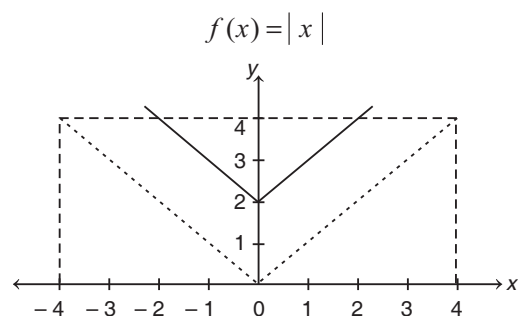
17.7 Transformation of Graphs

Here we study some techniques for the following transformation.

1. $f(x) \rightarrow f(x) + k$
2. $f(x) \rightarrow f(x) - k$
3. $f(x) \rightarrow k f(x)$
4. $f(x) \rightarrow \frac{1}{k} f(x)$
5. $f(x) \rightarrow -f(x)$
6. $f(x) \rightarrow |f(x)|$
7. $f(x) \rightarrow f(x+k)$
8. $f(x) \rightarrow f(x-k)$
9. $f(x) \rightarrow f(kx)$
10. $f(x) \rightarrow f\left(\frac{x}{k}\right)$
11. $f(x) \rightarrow f(-x)$
12. $f(x) \rightarrow f|x|$

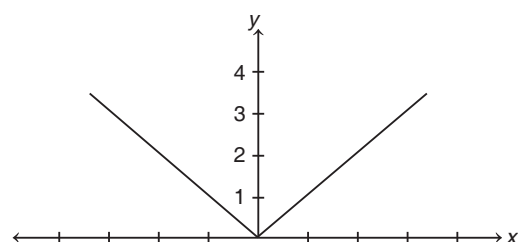
1. $f(x) \rightarrow f(x) + k$, k is a positive constant. Shift the graph of $f(x)$ upwards by ' k '.



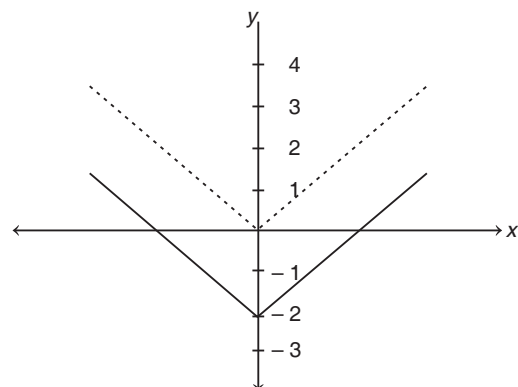


$$f(x) = |x| + 2$$

2. $f(x) \rightarrow f(x) - k$; k is a positive constant

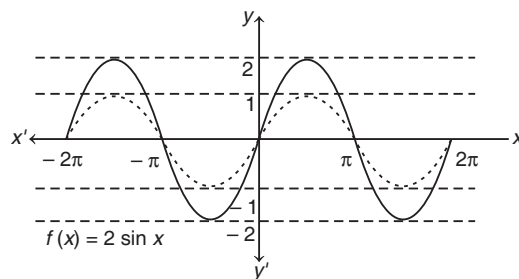
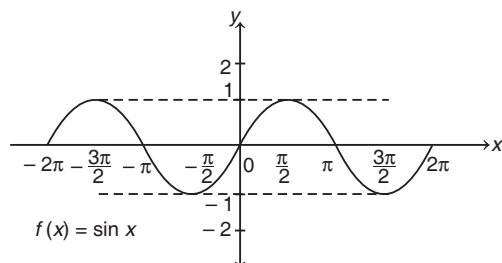


$$f(x) = |x|$$

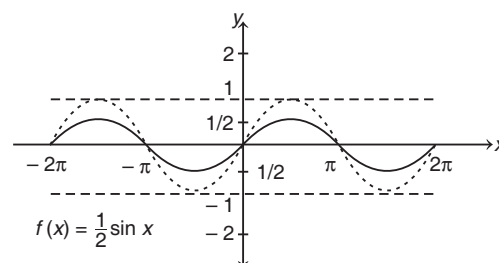
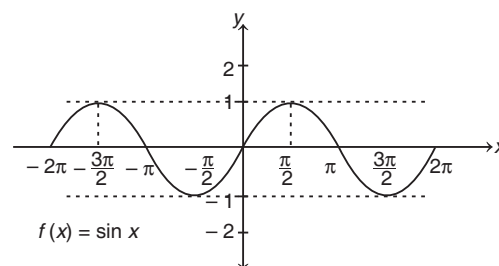


$$f(x) = |x| - 2$$

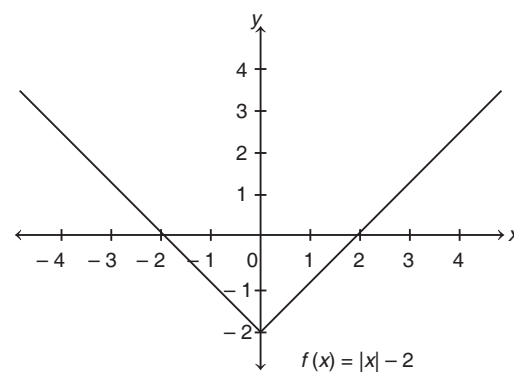
3. $f(x) \rightarrow kf(x)$; ($k > 1$) Stretch the graph of $f(x)$, k times along y -axis.

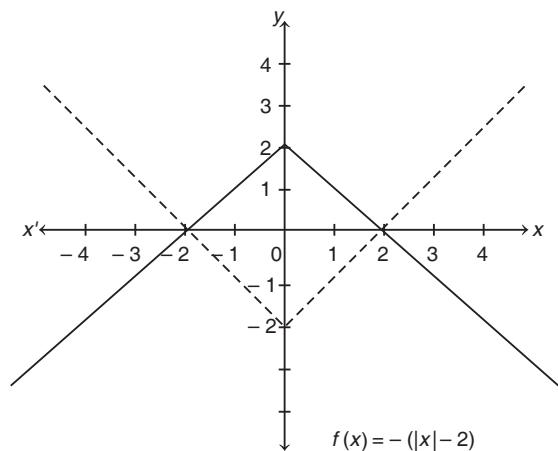


4. $f(x) \rightarrow \frac{1}{k} f(x)$; ($k > 1$) Shrink the graph of $f(x)$ ' k ' times along y -axis.

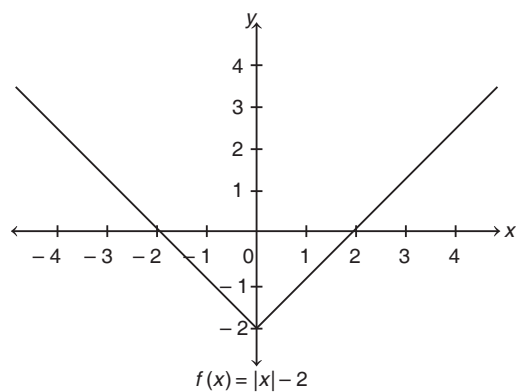


5. $f(x) \rightarrow -f(x)$ Turn the graph of $f(x)$ by 180° about x -axis. or Take the mirror image of $f(x)$ in x -axis.

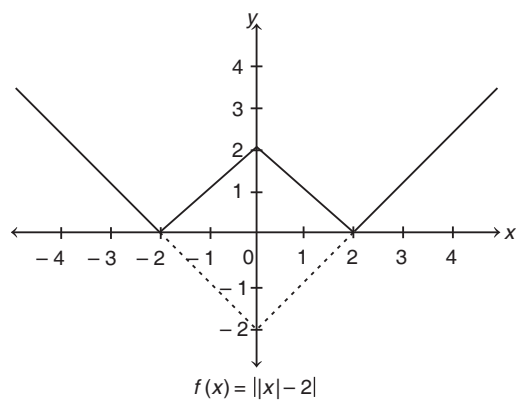




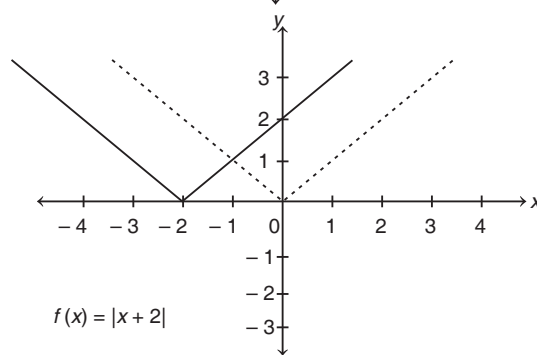
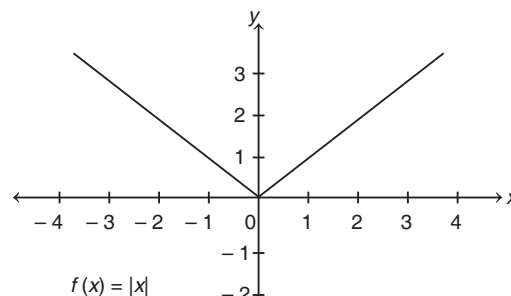
6. $f(x) = |f(x)|$, where $|f(x)| = \begin{cases} f(x) & \text{if } f(x) \geq 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}$



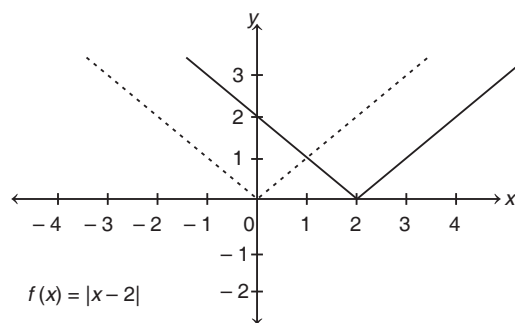
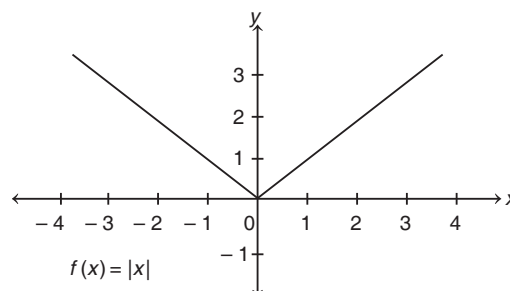
Turn the portion of the graph of $f(x)$ lying below x -axis by 180° about x -axis. Or, take the mirror image (in x -axis) of the portion of the graph of $f(x)$ which lies below x -axis.



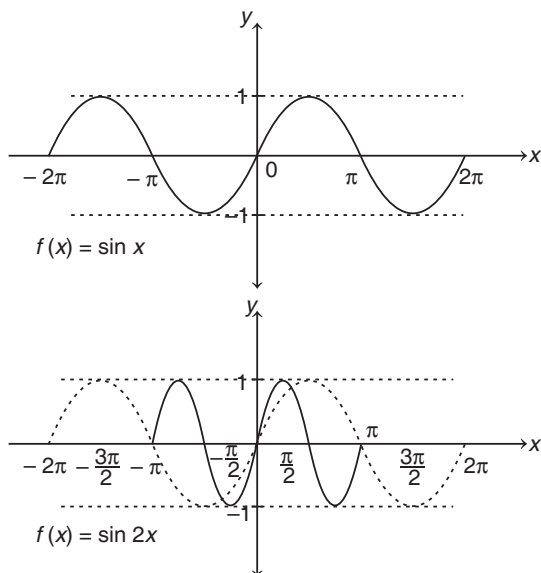
7. $f(x) \rightarrow f(x + k)$ Shift the graph of $f(x)$ towards left by ' k '.



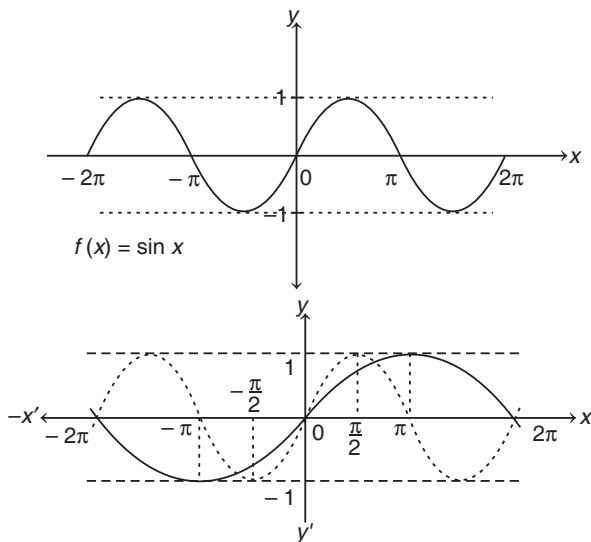
8. $f(x) \rightarrow f(x - k)$ Shift the graph of $f(x)$ towards right by ' k '.



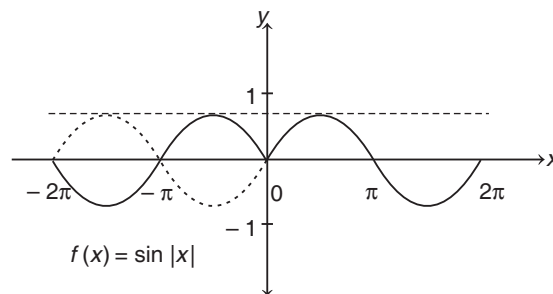
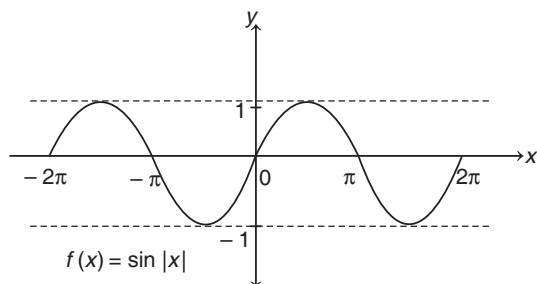
9. $f(x) \rightarrow f(kx)$; ($k > 1$) Shrink the graph of $f(x)$ 'k' times along x-axis.



10. $f(x) \rightarrow f\left(\frac{x}{k}\right)$ Stretch the graph of $f(x)$ 'k' times along x-axis.



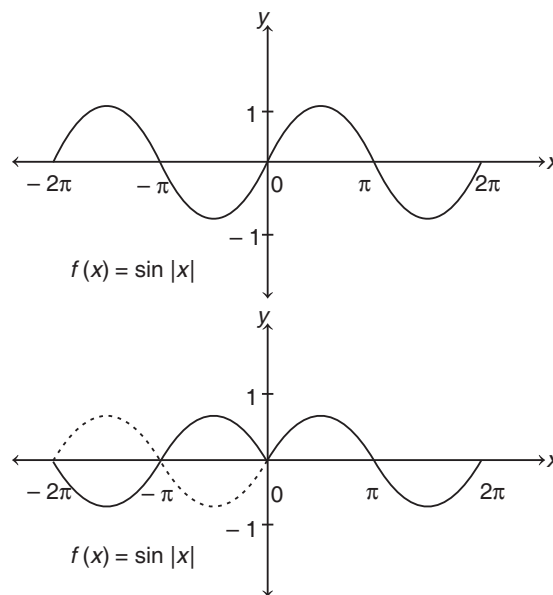
11. $f(x) \rightarrow f(-x)$ Turn the graph of $f(x)$ by 180° about y-axis or Take the mirror image of the graph of $f(x)$ in y-axis.



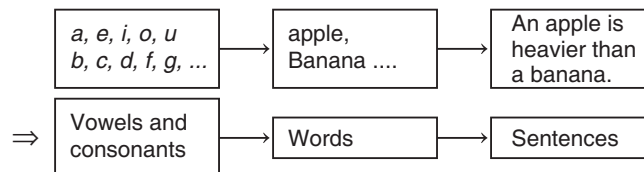
12. $f(x) \rightarrow f|x|$

$$\therefore f|x| = \begin{cases} f(x) & \text{if } x \geq 0 \\ f(-x) & \text{if } x < 0 \end{cases}$$

$\therefore$ On the right of y-axis, plot the graph of $f(x)$ as such and on the left of y-axis, plot the mirror image of the graph of $f(x)$ of the portion lying on the right of y-axis.



17.8 Composite Functions

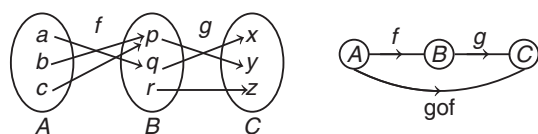


When the output of the first operation is the input for the second operation, then there is a composition of functions (operations).

If $f : A \rightarrow B$ and $g : B \rightarrow C$, then $(g \circ f) : A \rightarrow C$

$$\Rightarrow (g \circ f)(a) = g\{f(a)\}$$

Exp. 1)



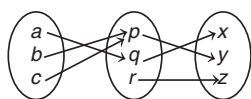
$$(g \circ f)(a) = g\{f(a)\} = g(p) = x$$

$$(g \circ f)(b) = g\{f(b)\} = g(q) = y$$

$$(g \circ f)(c) = g\{f(c)\} = g(r) = y$$

Properties

- Associative** : If $f_1 : A \rightarrow B, f_2 : B \rightarrow C, f_3 : C \rightarrow D$ then $f_1 \circ (f_2 \circ f_3) = (f_1 \circ f_2) \circ f_3$
- Non commutative** : Let $f(x) = \cos x$ and $g(x) = x^3$ then, $g \circ f(x) = g(f(x)) = g(\cos x) = \cos^3 x$ and $f \circ g(x) = f(x^3) = \cos x^3$ Hence $g \circ f \neq f \circ g$
- Range of $(g \circ f)$** may not be the same as the range of g



$$(g \circ f)(a) = x$$

$$(g \circ f)(b) = y$$

$$(g \circ f)(c) = y$$

∴

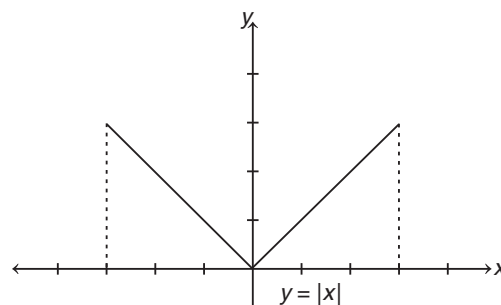
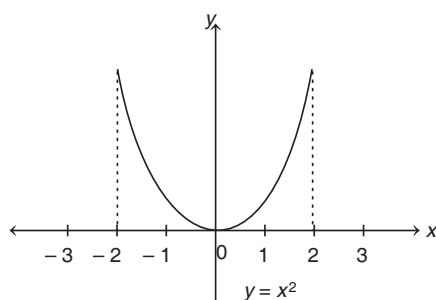
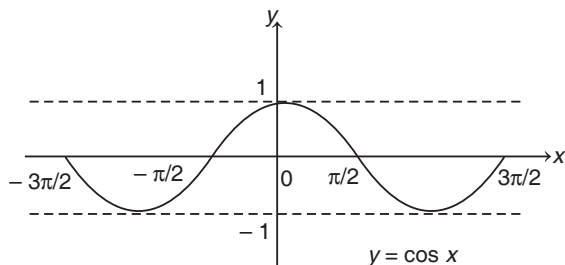
$$\text{Range of } (g \circ f) = \{x, y\}$$

and

$$\text{Range of } g = \{x, y, z\}$$

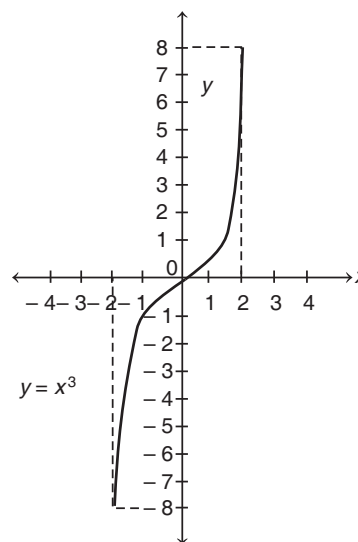
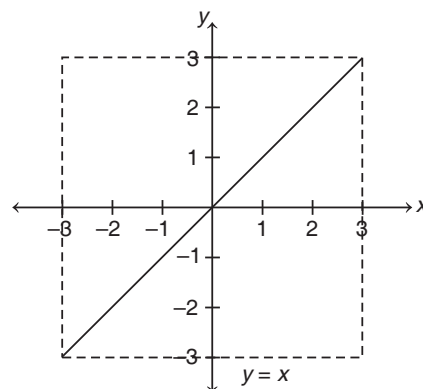
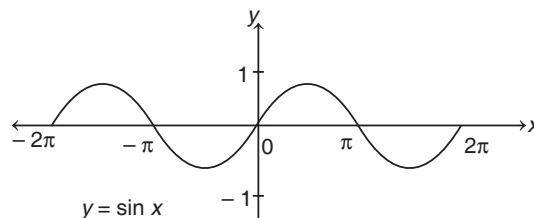
17.9 Even and Odd Functions

- Even function** : A function $y = f(x)$ is said to be an even function if $f(-x) = f(x) \forall x \in \text{Dom}(f)$ e.g., $y = \cos x, y = x^{2n} \forall n \in N, y = |x|$



NOTE Graph of an even function is symmetric about y-axis.

- Odd function** : A function $y = f(x)$ is said to be an odd function if $f(-x) = -f(x), \forall x \in \text{Dom}(f)$ e.g., $y = \sin x, y = x^{2n-1} \forall n \in N$



NOTE Graph of an odd function is symmetric about origin.

Properties of Even and Odd Functions

1. **Sum :** Even + Even = Even

$$\text{Odd} + \text{Odd} = \text{Odd}$$

$$\text{Even} + \text{Odd} = (\text{neither even nor odd})$$

$$\text{Odd} + \text{Even} = (\text{neither even nor odd})$$

2. **Difference :** Even - Even = Even

$$\text{Odd} - \text{Odd} = \text{Odd}$$

$$\text{Even} - \text{Odd} = (\text{neither even nor odd})$$

$$\text{Odd} - \text{Even} = (\text{neither even nor odd})$$

Remember Any function $y = f(x)$ can be expressed uniquely as the sum of an even and an odd function as follows :

$$f(x) = \frac{1}{2}[f(x) + f(-x)] + \frac{1}{2}[f(x) - f(-x)]$$

3. **Product:** Even $\times$ Even = Even

$$\text{Odd} \times \text{Odd} = \text{Even}$$

$$\text{Even} \times \text{Odd} = \text{Odd}$$

$$\text{Odd} \times \text{Even} = \text{Odd}$$

NOTE If the number of odd functions in the product is odd $\rightarrow$ odd function. If the number of odd functions in the product is even $\rightarrow$ even function

4. **Division :** Even $\div$ Even = Even

$$\div \text{Odd} = \text{Even}$$

$$\text{Even} \div \text{Odd} = \text{Odd}$$

$$\text{Odd} \div \text{Even} = \text{Odd}$$

5. **Composition :** Even (Even) = Even

$$\text{Even (Odd)} = \text{Even}$$

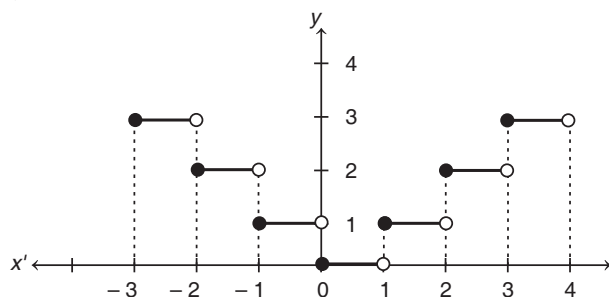
$$\text{Odd (Even)} = \text{Even}$$

$$\text{Odd (Odd)} = \text{Odd}$$

NOTE If all the functions in the composition are odd $\rightarrow$ Odd function. If at least one of the functions in the composition is even $\rightarrow$ even function.

Introductory Exercise 17.2

1. Which one of the following is correct for the following graph?



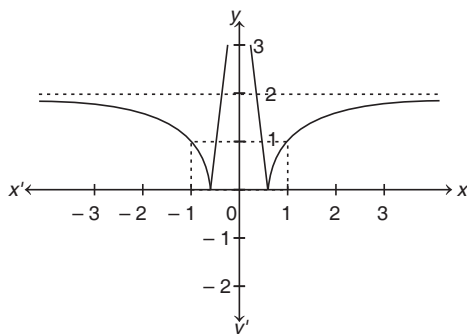
(a) $f(x) = \{x\}$

(b) $f(x) = [x]$

(c) $f(x) = |[x]|$

(d) none of these

2. Which one of the following is correct for the following graph?



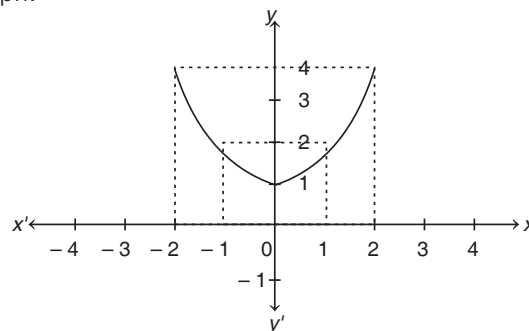
(a) $f(x) = \left| \frac{1}{x} - 2 \right|$

(b) $f(x) = \left(\frac{1}{x} \right)^2$

(c) $f(x) = \left| \frac{1}{x^3} \right|$

(d) $f(x) = \log |x|$

3. Which one of the following is correct for the following graph?



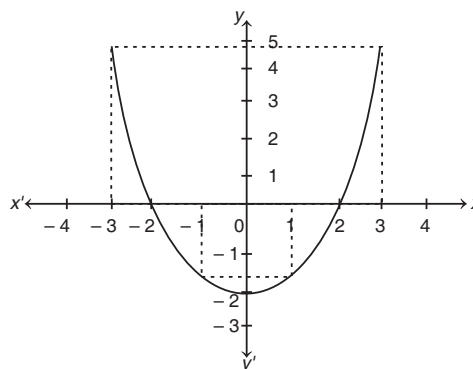
(a) $f(x) = e^x$

(b) $f(x) = \log |x|$

(c) $f(x) = 2^{|x|}$

(d) $f(x) = x^2$

4. Which one of the following is correct for the given graph?



(a) $f(x) = |x^3 - 2|$

(b) $f(x) = x^2 - 2$

(c) $f(x) = \left| \frac{x^3}{4} \right| - 2$

(d) $f(x) = \log |x - 1|$

5. The equation $\frac{x^2}{1 - |x - 2|} = 1$ has n real solutions, then
 (a) $n = 0$ (b) $n = 1$ (c) $n = 2$ (d) $n = 3$
6. The number of real solutions of the equation $\log_{0.5} |x| = 2 |x|$ is :
 (a) 0 (b) 1
 (c) 2 (d) none of these
7. The graph of the function $f: R \rightarrow R$ defined by $f(x) = \frac{|x|^3 + |x|}{1 + x^2}$ may lie in :
 (a) Ist and IInd quadrant (b) IInd and IIIrd quadrant
 (c) IIIrd and IVth quadrant (d) none of these
8. The two roots of the equation $f(x) = f\left(\frac{x+8}{x-1}\right)$ are :
 (a) 2, -2 (b) 4, -2
 (c) -4, -2 (d) 2, 4
9. If the functions f, g, h are defined from ' R ' to ' R ' by $f(x) = x^2 - 1, g(x) = \sqrt{x^2 + 1}, h(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ x, & \text{if } x \geq 0 \end{cases}$ then $h \circ f \circ g$ is equal to :
 (a) x^2 (b) $\sqrt{x^4 - 1}$
 (c) 0 (d) none of these
10. If $f(x) = (a - x^n)^{1/n}$, where $a > 0$, then $f(f(x))$ is :
 (a) x^n (b) x^2
 (c) $2x$ (d) none of these
11. If $f(x) = \left(\frac{x-1}{x+1}\right)$, then $f(f(ax))$ in terms of $f(x)$ is equal to :
 (a) $\frac{f(x)+1}{a(f(x)+1)}$ (b) $\frac{f(x)+1}{a(f(x)-1)}$
 (c) $\frac{f(x)-1}{a(f(x)+1)}$ (d) $\frac{f(x)+1}{a(f(x)-1)}$
12. If $f(x) = \frac{x}{\sqrt{1+x^2}}$, then $f \circ f \circ f(x)$ is equal to :
 (a) $\frac{x}{\sqrt{1+3x^2}}$ (b) $\frac{3x}{\sqrt{1-x^2}}$
 (c) $\frac{3x}{\sqrt{1+x^2}}$ (d) none of these
13. If $f(x) = \begin{cases} 1 + |x| & ; x < -1 \\ [x] & ; x \geq -1 \end{cases}$ Also, $[.]$ is greatest integer function then $f(f(-2.4))$ is equal to :
 (a) -3 (b) 2 (c) 3 (d) 4
14. If $f(x) = x^n, n \in N$ and $(g \circ f)(x) = ng(x)$, then $g(x)$ can be
 (a) $n |x|$ (b) $\sqrt[n]{x}$
 (c) e^x (d) $\log x$
15. Let $f: N \rightarrow R; f(x) = 2x - 1$
 $g: Z \rightarrow R; g(x) = \frac{x^2}{2}$, then $(g \circ f)(0)$ is :
 (a) $-\frac{1}{2}$ (b) $\frac{1}{2}$
 (c) 1 (d) none of these
16. Let $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}; (x \neq 0)$, then $f(x)$ is equal to :
 (a) $x^2 - 1$ (b) $x^2 - 2$
 (c) x^2 (d) none of these
17. Let $f(x) = \sqrt{x^5}$, then $f(5x)$ is equal to :
 (a) $5x\sqrt{5x}$ (b) $25x^2\sqrt{5x}$
 (c) $5x$ (d) $\sqrt{5x^5}$
18. If $f(x) = 4x - 5, g(x) = x^2$ and $h(x) = \frac{1}{x}$, then $f(g(h(x)))$ is :
 (a) $\frac{4}{(x-5)}$ (b) $\frac{1}{(4x-5)^2}$
 (c) $\left(\frac{x}{4} - 5\right)$ (d) $\left(\frac{4}{x^2} - 5\right)$
19. If $[x]$ is greatest integer function (GIF) and $\{x\}$ is smallest integer function (SIF), and $x \in R \sim I$, then $[x] - \{x\}$ is equal to :
 (a) -1 (b) 0
 (c) 1 (d) 2
20. If the graph of the function $f(x) = \frac{a^x - 1}{x^n (a^x + 1)}$ is symmetric about y -axis, then n is equal to :
 (a) $-\frac{1}{3}$ (b) $\frac{1}{4}$
 (c) $2/3$ (d) 2
21. $f(x) = \ln(x + \sqrt{x^2 + 1})$ is :
 (a) an even function (b) an odd function
 (c) neither even nor odd (d) none of these
22. If a function f satisfies the conditions $f(x + y) = f(x) + f(y) \forall x, y \in R$, then f is :
 (a) an even function (b) an odd function
 (c) neither even nor odd (d) none of these
23. Which of the following function is an even function ?
 (a) $f(x) = \frac{a^x + 1}{a^x - 1}$ (b) $f(x) = \frac{a^x - a^{-x}}{a^x + a^{-x}}$
 (c) $f(x) = x \frac{a^x + 1}{a^x - 1}$ (d) $f(x) = \sin x$

24. Which of the following function is odd ?

- (a) $\ln \left(\frac{1+x^3}{1-x^3} \right)$ (b) $x \cdot \frac{e^{ax} - 1}{e^{ax} + 1}$
 (c) $\frac{(1+2^x)^2}{2^x}$ (d) none of these

25. Which of the following function is even function ?

- (a) $f(x) = \log \frac{1-x}{1+x}$
 (b) $f(x) = x \cdot \frac{a^x + 1}{a^x - 1}$
 (c) $f(x) = \log \left[x + \sqrt{(1+x^2)} \right]$
 (d) $f(x) = \sqrt{(1+x+x^2)} - \sqrt{(1-x+x^2)}$

26. If $f(x) = \sqrt[3]{(1-x^2)} + \sqrt[3]{(1+x^2)}$, then $f(x)$ is:

- (a) an even function (b) an odd function
 (c) constant function (d) none of these

27. Which of the following function is an odd function ?

- (a) $f(x) = 2^{-x^2}$ (b) $f(x) = 2^{x-x^4}$
 (c) $f(x) = \cos x$ (d) none of these

28. The function $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$ is :

- (a) even (b) odd
 (c) constant (d) neither even nor odd

29. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then :

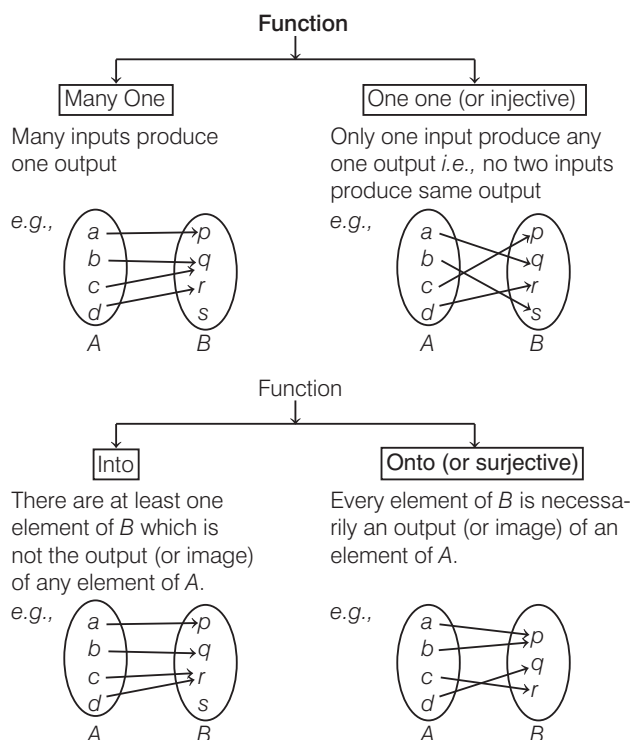
- (a) $f(x+2) = f(x-2)$ (b) $f(2+x) = f(2-x)$
 (c) $f(x) = f(-x)$ (d) none of these

30. Given $f(x) = \frac{1}{(1-x)}$, $g(x) = f(f(x))$ and $h(x) = f(f(f(x)))$,

then the value of $f(x) \cdot g(x) \cdot h(x)$ is :

- (a) 0 (b) -1
 (c) 1 (d) 2

17.10 Surjective and Injective Functions



One to One (or Injective) Functions

A function f is said to be one-to-one if it does not take the same values at two distinct points in its domain. For example, $f(x) = x^3$ is one to one whereas $f(x) = x^2$ is not, as $f(1) = 1$ and $f(-1) = 1$

In order to show that a function $f : A \rightarrow B$ is one-one we may take any $x, y \in A$ such that $f(x) = f(y)$ and try to show that $x = y$. Graphically, if every horizontal line intersects the graph of a given function at not more than one point, i.e., the function is strictly increasing (or decreasing) then the function is one-one.

NOTE 1. Functions which are not one to one are called many-one function.
 2. One many function does not exist.

Onto (or Surjective Function)

If a function $f : A \rightarrow B$ is such that each element in B is the f -image of at least one element in A , then we say that f is function of A 'onto' B . Equivalently a function f is an onto function if codomain of $f = \text{Range of } f$.

In order to show that a function $f : A \rightarrow B$ is onto we start with any $y \in B$ and try to find $x \in A$ such that $f(x) = y$.

NOTE If there exists at least one element in B which is not the f -image of any element of A , then f is called an into function.

Bijjective Function : If a function f is both one-to-one and onto then f is said to be a bijective function e.g., an identity function is a bijective function.

17.11 Inverse Function

If $f : A \rightarrow B$ is a bijection, we can define a new function from B to A which associates each element $y \in B$ to its pre-image $f^{-1}(y) \in A$.

Such a function is known as the inverse of function f and is denoted by f^{-1} .

- Inverse of a function is defined iff the function is one-to-one and onto.
- In the inverse function inputs and outputs interchange their roles.
- Inverse of a function is not defined if the given function is a many-one function or into function or both.
- If the graph of a function $f(x)$ is symmetric about the line $y=x$, then $f(x)$ and $f^{-1}(x)$ are identical function.
- The graph of $f(x)$ and $f^{-1}(x)$ is symmetric about

$$y = x$$

17.12 Binary Operations

Let S be a non-empty set. A function f from $S \times S$ to S is called a binary operation on S

i.e., $f : S \times S \rightarrow S$ is a binary operation on S .

Composition table : Let $S = \{a_1, a_2, a_3, \dots, a_n\}$ be finite set and $*$ be a binary operation on S . Then the composition table is constructed in the manner given below :

$*$	a_1	a_2	$\dots$	a_i	$\dots$	a_j	$\dots$	a_n
a_1	$a_1 * a_1$	$a_1 * a_2$	$\dots$	$a_1 * a_i$	$\dots$	$a_1 * a_j$	$\dots$	$a_1 * a_n$
a_2	$a_2 * a_1$	$a_2 * a_2$	$\dots$	$a_2 * a_i$	$\dots$	$a_2 * a_j$	$\dots$	$a_2 * a_n$
$\dots$								
a_i	$a_i * a_1$	$a_i * a_2$	$\dots$	$a_i * a_i$	$\dots$	$a_i * a_j$	$\dots$	$a_i * a_n$
$\dots$								
a_j	$a_j * a_1$	$a_j * a_2$	$\dots$	$a_j * a_i$	$\dots$	$a_j * a_j$	$\dots$	$a_j * a_n$
a_n	$a_n * a_1$	$a_n * a_2$	$\dots$	$a_n * a_i$	$\dots$	$a_n * a_j$	$\dots$	$a_n * a_n$

Example Consider the set $S = \{1, 2, 3, 4\}$. Define a binary operation $*$ on S as follows :

For any $a, b \in S$ define $a * b = r$, where ' r ' is the least non-negative remainder when ab is divided by 5.

Solution We have

$$\begin{aligned} 1 * 1 &= 1, 1 * 2 = 2, 1 * 3 = 3, 1 * 4 = 4 \\ 2 * 1 &= 2, 2 * 2 = 4, 2 * 3 = 1, 2 * 4 = 3 \\ 3 * 1 &= 3, 3 * 2 = 1, 3 * 3 = 4, 3 * 4 = 2 \\ 4 * 1 &= 4, 4 * 2 = 3, 4 * 3 = 2, 4 * 4 = 1 \end{aligned}$$

Thus, the following is the composition table for the binary operation $*$ on S .

$*$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

Generally binary operations are represented by the symbols $*$, $\oplus$ etc. instead of letters f , g etc.

Types of Binary Operations

- 1. Commutative binary operation :** A binary operation $*$ on a set S is said to be commutative if $a * b = b * a$ for all $a, b \in S$.
- 2. Associative binary operation :** A binary operation $*$ on a set S is said to be associative if $(a * b) * c = a * (b * c)$ for all $a, b, c \in S$.
- 3. Distributive binary operation :** Let $*$ and $\oplus$ be two binary operations on a set S . Then $*$ is said to be :
 - (i) Left distributive over $\oplus$ if $a * (b \oplus c) = (a * b) \oplus (a * c)$ for all $a, b, c \in S$,
 - (ii) Right distributive over $\oplus$ if $(b \oplus c) * a = (b * a) \oplus (c * a)$ for all $a, b, c \in S$.

If $*$ is both left and right distributive over $\oplus$, then $*$ is said to be distributive over $\oplus$.

17.13 User Defined functions

These functions are defined as per the requirement of user. Basically these functions are not standard function. There are some examples illustrated below :

Exp. 1) If $f(A, B) = A \# B = A^4 - B^3 + A^2 - B + AB$, then find the value of $f(3, 4)$.

$$\begin{aligned} \text{Solution } f(3, 4) &= 3 \# 4 = 3^4 - 4^3 + 3^2 - 4 + (3 \times 4) \\ &= 81 - 64 + 9 - 4 + 12 = 34 \end{aligned}$$

Exp. 2) If $A \# B = A + B + AB$. If for any A there is a number C such that $A \# C = A$, then $C = ?$

Solution $A \# C = A$, we get

$$\begin{aligned} A + C + AC &= A \\ \Rightarrow C(1 + A) &= 0 \\ \Rightarrow C = 0 \text{ or } A &= -1 \end{aligned}$$

Exp. 3) If $f(a, b) = \frac{a+b}{2}$; $g(a, b) = a^2 + b^2$ $h(a, b) = \max(a, b)$

Find the value of $f(g(3, 9), h(-1, 1))$.

Solution $f(g(3, 9), h(-1, 1)) = f((3^2 + 9^2), 1) = f(90, 1) = 45.5$

Exp. 4) $f_n = \begin{cases} f_{n-1} & \text{if } n \text{ is even} \\ 2f_{n-1} & \text{if } n \text{ is odd} \end{cases}$ and $f_0 = 1$, find the value of $f_4 + f_5$

Solution $f_4 + f_5 = f_3 + 2f_4 = 2f_2 + 2f_3 = 2(f_2 + f_3) = 2(f_1 + 2f_2)$
 $= 2(2f_0 + 2f_1) = 4(f_0 + f_1)$
 $= 4(1 + 2f_0) = 4(1 + 2) = 12$

Exp. 5) $f_n = \begin{cases} f_{n-1} & \text{if } n \text{ is even} \\ 2f_{n-1} & \text{if } n \text{ is odd} \end{cases}$ and $f_0 = 1$, find the value of f_{16} .

Solution $f_{16} = f_{15} = 2f_{14} = 2f_{13} = 2^2 f_{12} = 2^2 f_{11} = 2^3 f_{10} = \dots$
 $f_{16} = 2^8 f_0 = 2^8 = 256$

Exp. 6) If $f(n)$ defines the number of pages read by a person in n th hour, where $f(n) = \{\sqrt{n}\}$; $\{x\} \rightarrow$ least integer greater than or equal to x and n is the n^{th} hour. If a person reads the book for 10 hours, then find the total number of pages read by him.

Solution Total number of pages read by him

$$= f_1 + f_2 + f_3 + \dots + f_{10}$$

$$f_1 = \{\sqrt{1}\} = \{1\} = 1$$

$$f_2 = \{\sqrt{2}\} = \{1.414\} = 2$$

$$f_3 = \{\sqrt{3}\} = \{1.732\} = 2$$

$$f_4 = \{\sqrt{4}\} = 2$$

$$f_5 = \{\sqrt{5}\} = 3$$

$$f_6 = \{\sqrt{6}\} = 3$$

$$f_7 = \{\sqrt{7}\} = 3$$

$$f_8 = \{\sqrt{8}\} = 3$$

$$f_9 = \{\sqrt{9}\} = 3$$

$$f_{10} = \{\sqrt{10}\} = 4$$

$\therefore$ Total number of pages read by a person

$$= 1 + 2 + 2 + 2 + 3 + 3 + 3 + 3 + 3 + 4 = 26 \text{ pages}$$

17.14 Maxima and Minima

In this topic we study the maximum and minimum value of various functions. In advance mathematics, we find the maxima and minima through differentiation of the function (As in calculus). But for those students who are not well-versed with calculus, they can solve these problems with some unconventional methods.

Recall that we have been already studied the maxima and minima in the chapter “Theory of Equations.”

Exp. 1) What is the minimum value of the expression $x^2 + 8x + 10$?

Solution

Method 1. $x^2 + 8x + 10 = x^2 + 8x + 16 - 6$

$$\Rightarrow = (x + 4)^2 - 6$$

Now the smallest possible value of the above expression can be -6

Since, $(x + 4)^2$ can be minimum 0, when $x = -4$

Method 2. Compare the given expression with the standard quadratic equation $ax^2 + bx + c = 0$

$$\therefore a = 1, b = 8, c = 10$$

$$\text{Now, minima} = \frac{-(b^2 - 4ac)}{4a} \quad (D = b^2 - 4ac)$$

$$D \rightarrow \text{Discriminant}$$

$$= \frac{-(64 - 40)}{4} = -6$$

Method 3. By differentiating the given expression twice w.r. to the given variable, we get minima or maxima.

$$\text{Let } y = x^2 + 8x + 10$$

$$\therefore \frac{dy}{dx} = y' = 2x + 8$$

$$\text{Again } \frac{d^2y}{dx^2} = y'' = 2$$

Since, $\frac{d^2y}{dx^2} = 2$ (+ve value), hence the given

expression gives minima.

Now since $y' = 2x + 8$

$$\text{Let } y' = 0$$

$$\Rightarrow 2x + 8 = 0 \Rightarrow x = -4$$

$$\therefore y = x^2 + 8x + 10 = (-4)^2 + 8(-4) + 10 = -6$$

NOTE If $\frac{d^2y}{dx^2}$ is a -ve value then we get maxima.

Hence, the minimum value of the expression is -6 .

Exp. 2) What is the maximum value of the expression $5 - 6x - x^2$

Solution

Method 1. $5 - 6x - x^2 = 14 - (3 + x)^2$

Thus the expression can have maximum value 14, when $x = -3$

Since, the minimum value of $(3 + x)^2$ is zero.

Method 2. $-x^2 - 6x + 5 = 0$... (i)

Comparing the above expression with standard quadratic equation $ax^2 + bx + c = 0$, we get

$$a = -1, b = -6, c = 5$$

Since, a is negative, therefore the expression gives maxima.

$$\text{Now, } \frac{-D}{4a} = \frac{-(b^2 - 4ac)}{4a}$$

$$= \frac{-(36 + 20)}{-4} = 14$$

Method 3. Let $y = -x^2 - 6x + 5$... (i)

$$\therefore \frac{dy}{dx} = -2x - 6 \therefore \frac{d^2y}{dx^2} = -2$$

Since, $\frac{d^2y}{dx^2} = -2$ (-ve value), therefore we get maxima

$$\text{Now, let } \frac{dy}{dx} = 0 \Rightarrow -2x - 6 = 0 \Rightarrow x = -3$$

Substituting $x = -3$ in equation (1), we get the

$$\begin{aligned} \text{maxima} &= -(-3)^2 - 6(-3) + 5 \\ &= -9 + 18 + 5 = 14 \end{aligned}$$

Exp. 3) What is the maximum value of the function

$$y = \frac{2x^2 + 3x + 4}{x^2 + x + 3}$$

$$\text{Solution } y = \frac{2x^2 + 3x + 4}{x^2 + x + 3}$$

$$\Rightarrow (y - 2)x^2 + (y - 3)x + (3y - 4) = 0$$

$$\therefore (y - 3)^2 - 4(y - 2)(3y - 4) \geq 0$$

$$\Rightarrow 11y^2 - 34y + 23 \leq 0 \therefore 1 \leq y \leq \frac{23}{11}$$

$$\therefore \text{The maximum value of the expression is } \frac{23}{11}$$

(and minimum value is 1)

Points to remember

(i) When $a_1 + a_2 + a_3 + \dots + a_n = k$, then the maximum value of $a_1 \cdot a_2 \cdot a_3 \dots a_n$ is obtained when

$$a_1 = a_2 = a_3 = \dots = a_n.$$

(ii) When $a_1 \cdot a_2 \cdot a_3 \dots a_n = k$, then the minimum value of $a_1 + a_2 + a_3 + \dots + a_n$ is obtained when

$$a_1 = a_2 = a_3 = \dots = a_n.$$

Exp. 4) If $a + b = 20$, then the maximum value of ab is :

- (a) 19 (b) 75
(c) 100 (d) 120

$$\begin{aligned} \text{Solution } \text{Maximum of } (ab) &= 10 \times 10 & (\because a + b = 20) \\ &= 100 & (a = b = 10) \end{aligned}$$

Exp. 5) If $a + b + c = 24$, then the maximum value of abc is

- (a) 215 (b) 512 (c) 125 (d) 576

Solution abc can be maximum only when $a = b = c$ for a given sum $a + b + c = 24$

$$\therefore (abc)_{\max} = (8 \times 8 \times 8) \quad (\because a = b = c) = 512$$

Exp. 6) If $ab = 25$, then the minimum value of $a + b$ is :

- (a) 10 (b) 8 (c) 24 (d) 16

Solution Minimum value of $(a + b)$ can be obtained when $a = b$ then $a + b = 5 + 5$ ($\because ab = 25$ and $a = b$) = 10

Exp. 7) If $f(x) = \min(4x + 3, x + 4)$ for $x \in [0, 2]$, what is the maximum value of $f(x)$?

Solution $f(x) = \min(4x + 3, x + 4)$ is an increasing function for every $x \in [0, 2]$

It means the value of $f(x)$ increases as x increases.

$\therefore$ at $x = 2$, $f(x)$ will be maximum

$$f(x)_{\max} = \min(11, 6) \text{ at } x = 2$$

$$f(x)_{\max} = 6$$

Hence, the maximum value of the function is 6.

Exp. 8) If $f(x) = \max(4x + 3, 3x + 6)$ for $x \in [-6, 10]$, Find the maximum value of $f(x)$.

Solution Since the function $f(x) = \max(4x + 3, 3x + 6)$ is an increasing function. Therefore at $x = 10$ we get the maximum value.

$$f(x)_{\max} = \max(43, 36)$$

$$f(x)_{\max} = 43$$

Exp. 9) What is the maximum value of $f(x) = \min(4 - 5x, x - 3)$ for every $x \in (0, 4)$?

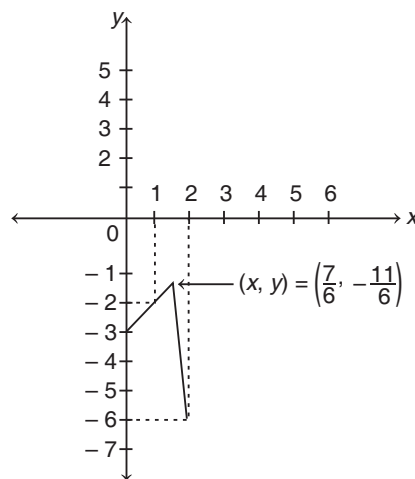
Solution At $x = 0$ $f(x) = \min(4, -3) = -3$

$$x = 1, f(x) = \min(-1, -2) = -2$$

$$x = 2, f(x) = \min(-6, -1) = -6$$

$$x = 3, f(x) = \min(-11, 0) = -11$$

$$x = 4, f(x) = \min(-16, 1) = -16 \text{ etc.}$$



It is clear from the graph that $f(x)_{\max}$ will lie between $x = 1$ and $x = 2$ i.e., when $x \in (1, 2)$ and the $f(x)_{\max}$ will be obtained when

$$4 - 5x = x - 3 \Rightarrow 6x = 7 \Rightarrow x = \frac{7}{6}$$

$\therefore$

$$\begin{aligned} f(x)_{\max} &= \min(4 - 5x, x - 3) \\ &= \min\left(-\frac{11}{6}, -\frac{11}{6}\right) = -\frac{11}{6} \end{aligned}$$

NOTE In case of $f(x)_{\max} = \min(x, y)$ $f(x)_{\max}$ will be obtained when $x = y$. Similarly in case of $f(x)_{\min} = \max(x, y)$ $f(x)_{\min}$ will be obtained when $x = y$.

Introductory Exercise 17.3

Directions (for Q. Nos. 1 to 4)

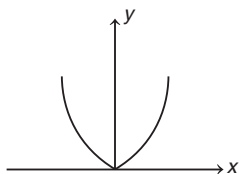
Mark (a) if function is one-one and into

Mark (b) if function is one-one and onto

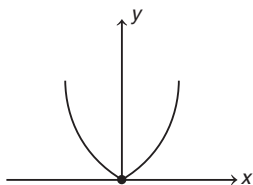
Mark (c) if function is many one and into

Mark (d) if function is many one and onto

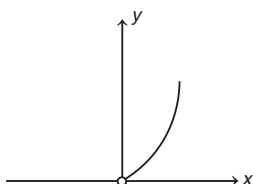
1. $f(x) = x^2$ $f : R \rightarrow R$



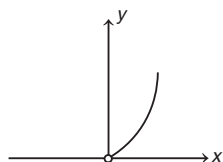
2. $f(x) = x^2$ $f : R \rightarrow R^+ \cup \{0\}$



3. $f(x) = x^2$ $f : R^+ \rightarrow R$



4. $f(x) = x^2$ $f : R^+ \rightarrow R^+$



5. Let $f : R \rightarrow R$ be a function defined by $f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$,

then :

- (a) f is both one-one and onto
- (b) f is one-one but not onto
- (c) f is onto but not one-one
- (d) f is neither one-one nor onto

6. Let $n(A) = 3$ and $n(B) = 5$, then the number of one-one functions from A to B is :

- (a) 15
- (b) 60
- (c) 125
- (d) 10

7. Let $A = R - \{3\}$, $B = R - \{1\}$. Let $f : A \rightarrow B$ be defined by

$$f(x) = \left(\frac{x-2}{x-3} \right), \text{ then}$$

- (a) f is bijective
- (b) f is one-one only
- (c) f is onto only
- (d) f is one-one but not onto

8. The function $f : N \rightarrow N$ (N is the set of natural numbers defined by $f(n) = 2n + 3$ is :

- (a) surjective
- (b) injective
- (c) bijective
- (d) none of these

9. The function $f : R \rightarrow R$ defined by

$$f(x) = (x-1)(x-2)(x-3)$$
 is :

- (a) one-one but not onto
- (b) onto but not one-one
- (c) both one-one and onto
- (d) neither one-one nor onto

10. If the function $f : R \rightarrow R$ be such that $f(x) = x - [x]$, where $[k]$ denotes the greatest integer less than or equal to k , then $f^{-1}(x)$ is :

- (a) $[x] - x$
- (b) $1/x - [x]$
- (c) not defined
- (d) none of these

11. If $f(x) = x^2 + 2$ then $f^{-1}(x)$ is :

- (a) $\sqrt{x-2}$
- (b) $\sqrt{x+2}$
- (c) $\sqrt{x} + 2$
- (d) $x + 4$

12. If $f(x) = 5^x$, then $f^{-1}(x)$ is :

- (a) x^5
- (b) 5^{-x}
- (c) $\log_5 x$
- (d) none of these

13. If $f(x) = \frac{x}{x+1}$, then $f^{-1}(x)$ is :

- (a) $\frac{x+1}{x}$
- (b) $\frac{x}{1-x}$
- (c) $\frac{x-1}{x}$
- (d) none of these

14. If $f(x) = \frac{x-1}{x+1}$, then $f^{-1}(x)$ is :

- (a) $\frac{1+x}{1-x}$
- (b) $\frac{x+1}{x-1}$
- (c) $\frac{1}{x+1}$
- (d) none of these

15. Let $f : (-\infty, 1] \rightarrow (-\infty, 1]$ such that $f(x) = x(2-x)$, then $f^{-1}(x)$ is :

- (a) $1 + \sqrt{1-x}$
- (b) $1 - \sqrt{1-x}$
- (c) $\sqrt{1-x}$
- (d) none of these

16. If $f(x)$ is a one to one function, where $f(x) = x^2 - x + 1$, then find the inverse of the $f(x)$:

- (a) $\left(x - \frac{1}{2}\right)$
- (b) $\sqrt{x - \frac{3}{4}} + \frac{1}{2}$
- (c) $\sqrt{x - \frac{3}{4}} - \frac{1}{2}$
- (d) none of these

17. In the above question find the value of $f\left(\frac{3}{4}\right) + f^{-1}\left(\frac{3}{4}\right)$

- (a) $\frac{21}{16}$
- (b) $\frac{4}{5}$
- (c) $\frac{16}{5}$
- (d) none of these

CAT-Test

Questions Helping you bell the CAT

LEVEL 01 > BASIC LEVEL EXERCISE

Directions (for Q. Nos. 1 and 2)

$P(A, B) = A \times B$, $S(A, B) = A - B$, $D(A, B) = A + B$

1 $D(P(3, 5), S(5, 3))$ is equal to

- (a) 15 (b) 17
(c) 0 (d) 30

2 $P(S(D(7, 3), P(2, 3)), P(6, 5))$ is equal to :

- (a) 120 (b) 60
(c) 30 (d) none of these

Directions (for Q. Nos. 3 to 5) For every $a, b \in N$

$a @ b = a^2$ when $(a + b)$ is even

$= a^2 - b^2$ when $(a + b)$ is odd

$a \# b = b^2$ when $a \times b$ is odd

$= a^2 - b^2$ when $a \times b$ is even

3 Find the value of $20 \# (21 @ 22)$:

- (a) 0 (b) 2249
(c) -1849 (d) can't be determined

4 What is the value of $(a \# a) @ (a @ a)$?

- (a) 0 (b) a^4
(c) either (a) or (b) (d) neither (a) nor (b)

5 Find the value of $(p \# q) - (q @ p)$; p is even and q is odd :

- (a) 0 (b) $2(p^2 + q^2)$
(c) $2(p^2 - q^2)$ (d) none of these

6 If $M(a, b) = a^2 + b^2 + ab$; $N(a, b) = a^2 + b^2 - ab$. Find the value of $M(7, N(9, 4))$.

- (a) 5197 (b) 3197
(c) 4197 (d) none of these

7 If $f(x) = x^3 - x^2$, then $f(x + 1)$ is :

- (a) $x^3 - x^2 + 1$ (b) $x(x + 1)^2$
(c) $x^2(x - 1)$ (d) none of these

Directions (for Q. Nos. 8 to 10)

$$a \cup b = \begin{cases} a + b, & \text{if } a < b \\ a \vee b, & \text{otherwise} \end{cases}$$

$$a \cup \cup b = \begin{cases} a \times b, & \text{if } a < b \\ a \vee \vee b, & \text{otherwise} \end{cases}$$

$$a \cup \cup \cup b = \begin{cases} a^b, & \text{if } a < b \\ a \vee \vee \vee b, & \text{otherwise} \end{cases}$$

$$a \vee b = \begin{cases} a - b, & \text{if } a \geq b \\ a \cup b, & \text{otherwise} \end{cases}$$

$$a \vee \vee b = \begin{cases} \frac{a}{b}, & \text{if } a \geq b \\ a \cup \cup b, & \text{otherwise} \end{cases}$$

$$a \vee \vee \vee b = \begin{cases} \frac{1}{a^b}, & \text{if } a \geq b \\ a \cup \cup \cup b, & \text{otherwise} \end{cases}$$

8 Find the value of $[(5 \cup 4) \vee \vee (2 \cup \cup \cup 3)] \cup (8 \vee 3)$:

- (a) 1 (b) 2
(c) 3 (d) 4

9 Which of the following is true?

- (a) $2 \cup \cup 8 = 2 \cup \cup \cup 4$ (b) $8 \cup \cup 2 = 4 \cup \cup \cup 2$
(c) $8 \cup \cup \cup 2 = 2 \vee \vee \vee 5$ (d) none of these

10 Which one of the following is/are true?

- (a) $a \cup b = a \vee b$ for all real values of a and b
(b) $a \vee \vee b = a \cup \cup b$ is true for integer values of a and b
(c) $a \vee \vee \vee b = a \cup \cup \cup b$ is true only for natural number a, b
(d) none of the above

Directions (for Q. Nos. 11 to 14)

$$x @ y = (x + y)^2$$

$$x \# y = (x - y)^2$$

11 Find the value of $(20 @ 4) \# 35$:

- (a) 280081 (b) 292681
(c) 262781 (d) 301756

12 Find the value of $12 \# (14 @ (12 \# 13))$:

- (a) 4539
(b) 42336
(c) 45369
(d) 89719

13 Find the value of $(9 \# 7) @ (21 \# 5)$:

- (a) 67600 (b) 62500
(c) 57600 (d) none of these

14 $(2 @ 2) + (3 \# 3) - (4 @ 4) + (5 \# 5)$ is equal to :

- (a) 64 (b) -48
(c) 28 (d) none of these

Directions (for Q. Nos. 15 and 16)

$$CAT(a, b) = \begin{cases} a^b, & \text{if } a > 0 \text{ and } b \in I \\ 0, & \text{otherwise} \end{cases}$$

$$MAT(a, b) = \begin{cases} b - a - 1, & \text{if } b > 0 \\ 1, & \text{otherwise} \end{cases}$$

$$XAT(a, b) = \begin{cases} \text{remainder}(b / a), & \text{if } a > 0 \\ 2, & \text{otherwise} \end{cases}$$

15 $\frac{XAT[MAT\{2, CAT(3, XAT[3, 2])\}, 5]}{CAT[4, XAT\{MAT(2, -5), 4\}]}$:

- (a) 0 (b) 2
(c) 5 (d) none of these

16 $\frac{XAT(r, s)}{MAT(r, s)} = 2$:

- (i) $r = 3, s = 5$ (ii) $r = -3, s = -5$
(a) (i) is true (b) (ii) is true
(c) both are true (d) none is true

Directions (for Q. Nos. 17 to 19) a and b are any numbers and x and y are any non negative integers.

$l(a)$ = the greatest integer less than or equal to a

$g(a)$ = the smallest integer greater than or equal to a

$r(x, y)$ = the remainder when x is divided by y .

17 $r(16, 7) - 2$ is equal to :

- (a) 0 (b) 1
(c) 2 (d) 3

18 $g(4.7) - l(2)$ is equal to :

- (a) 1 (b) 2
(c) 3 (d) 4

19 $r(x, y)$ with both x and y not equal to zero, is equal to :

- (a) $l\left(\frac{x}{y}\right)$ (b) $x - \left[yl\left(\frac{x}{y}\right)\right]$
(c) $y - \left[xl\left(\frac{x}{y}\right)\right]$ (d) $\left[l\left(\frac{x}{y}\right) - y\right]$

Directions (for Q. Nos. 20 to 22)

$$x \# y = \begin{cases} x + 2y; & \text{if } |x + y| \text{ is even} \\ 2x @ 2y; & \text{if } |x + y| \text{ is odd} \end{cases}$$

$$x @ y = \begin{cases} 3x - y; & \text{if } |x + y| \text{ is even} \\ 2x \# 2y; & \text{if } |x + y| \text{ is odd} \end{cases}$$

20 Find the value of $2 \# ((7 @ (4 \# 5)) @ 3)$:

- (a) 204 (b) 306
(c) 408 (d) 102

21 Which of the following is true?

- (i) If $q = 2p$ and $p + q$ is even, then $p \# q = q \# p$
(ii) If $q = 3p$, $p @ q$ will always be zero
(iii) If $q = 2p$ and p is odd, then $p @ q$ will be even
(a) (ii) only
(b) (iii) only
(c) (i) and (iii) only
(d) (ii) and (iii) only

22 Which of the following is true?

- (i) $6 @ 2 = 12 \# 2$
(ii) $6 \# 3 = (14 @ 12) + (2 @ 6)$
(iii) $(6 @ 2) + (6 \# 3) = (12 \# 2) + (14 @ 12) - (2 @ 6)$
(a) (i) and (ii) only (b) (i) and (iii) only
(c) (ii) and (iii) only (d) (i), (ii) and (iii)

Directions (for Q. Nos. 23 to 25)

$$a \# b = a^2 + b - 1$$

$$a \$ b = a^2 + b^2 + 1$$

$$\text{Max}(a, b) = |a + b|$$

$$\text{Min}(a, b) = |a - b|$$

23 $\text{Max}[(2 \# 3) \$ (3 \# 4), (3 \# 4) \$ (4 \$ 4)]$ is equal to :

- (a) 1234 (b) 1236
(c) 1335 (d) none of these

24 $\text{Max}[\text{Min}((1 \$ 2), (2 \$ 3)), \text{Max}((3 \# 4), (4 \# 5))]$ is equal to :

- (a) 24 (b) 40
(c) 42 (d) none of these

25 Find the value of

$$\text{Min}[(1 \# 2) \$ 3, 4 \# (5 \$ 6)] \$ \text{Max}(6 \# 7), (7 \# 8)] :$$

- (a) 13574 (b) 14573
(c) 13575 (d) 14674

26 If $a \Delta b = a^b + ab + \frac{a}{b}$, then $(9 \Delta 3)$ is equal to :

- (a) 729 (b) 927
(c) 759 (d) none of these

27 $a * b \# c = (ab)^c$:

$$a \# b * c = a^b \cdot c, \text{ then find the value of } 3 \# 4 * 5 + 3 * 4 \# 5$$

- (a) 35287 (b) 249237
(c) 12345 (d) none of these

Directions (for Q. Nos. 28 and 29)

$$a \# b = (-1)^{ab} (a^b + b^a)$$

$$f(x) = x^2 - 2x \text{ if } x \geq 0$$

$$= 0 \text{ if } x < 0$$

$$g(x) = 2x; \text{ if } x \geq 0$$

$$= 1; \text{ if } x < 0$$

28 Find the value of $f(2 \# 3) \# g(3 \# 4)$:

- (a) 2045 (b) $(17)^{18}$
(c) $(180)^{81}$ (d) none of these

29 Find the value of $f(g(2 \# 3)) + g(f(1 \# 2))$:

- (a) 9401 (b) 1049
(c) 1094 (d) none of these

Directions (for Q. Nos. 30 and 31)

$$a \oplus b = (a + b)^2 - ab$$

$$a * b = (ab)^2 - ab$$

$$a @ b = (a - b)^2 - ab$$

30 Find the value of $(5 \oplus 6) - (6 @ 3) + (2 * 4)$

- (a) 156 (b) 256
(c) 1056 (d) none of these

31 Find the value of $((1 \oplus 2) * 3) @ 20$:

- (a) 151600 (b) 141500
(c) 161500 (d) none of these

Directions (for Q. Nos. 32 and 33)

If $m < n$ then $m D n = m C n$ else $m D n = n C m$

If $m = n$ then $m E n = mn$ else $m E n = m C n$ where $m C n = \frac{m}{n}$

32 Find the value of $(3 D 4) D (12 D 5)$:

- (a) $\frac{9}{5}$ (b) $\frac{5}{9}$
(c) $\frac{2}{3}$ (d) $\frac{5}{3}$

33 Find the value of $(12 D 60) E (2 C 10)$:

- (a) 1 (b) $\frac{1}{5}$
(c) $1/25$ (d) 0

Directions (for Q. Nos. 34 to 38) The operands in the Devic Mathematics are as follows :

$+$ $\rightarrow$ multiplication

$-$ $\rightarrow$ division

$\times$ $\rightarrow$ addition

$/$ $\rightarrow$ subtraction

$\wedge$ $\rightarrow$ power (or index) for exponential expression.

But Devic Mathematics must follow the same conventional BODMAS rule of mathematics. Solve the following questions in Devic Mathematics.

34 $30 + 20 - 4 \times 6$ is equal to :

- (a) 156 (b) 276
(c) 330 (d) none of these

35 $50 / 40 - 5 + 4 \times 54$ is equal to :

- (a) 720 (b) 72
(c) 144 (d) none of these

36 $(5 \wedge 5) \times (4 \wedge 4) / (3 \wedge 3) \times (2 \wedge 2) / (1 \wedge 1)$ is equal to :

- (a) 2987 (b) 4013
(c) 3357 (d) none of these

37 The value of the following expression in Devic mathematics is

$$[(x^2 \times y^2) - (x + y)^2] + \left[(x + y)^3 - \left\{ \left(\frac{x}{y} \right)^2 \times 2 + x + y \right\} \right]$$

- (a) $\frac{x}{y}$ (b) $x + y$
(c) $x \times y$ (d) $x \wedge y$

38 The value of the following expression in Devic mathematics is $a + a + a - a + a + a / a + a - a$:

- (a) 1 (b) 0
(c) $1 - a$ (d) $\frac{1}{a}$

Directions (for Q. Nos. 39 and 40)

$P \Delta Q = P + Q$, if $PQ < 0$

$P \Delta Q = P - Q$, if $PQ > 0$

39 $[(2 \Delta 3) \Delta (-2 \Delta 3)]$ is equal to :

- (a) 0 (b) 1
(c) 2 (d) 3

40 If $(a \Delta b) \Delta (a \Delta -b) = 0$ and $a > 0$ then which of the following is true?

- (a) $a < b$ and $b > 0$ (b) $|a| > |b|$ and $b < 0$
(c) $a > b$ and $b > 0$ (d) all of these

41 If $f(x) = x^3 + 3x^2 + 3x$, then $f(x + 1)$ is equal to

- (a) $x^3 + 6x^2 + 12x + 7$ (b) $x^3 + 4x^2 + 5$
(c) $3x^3 + 12x^2 + 4x + 1$ (d) $2x^3 + 6x^2 + 3$

42 $x @ y = xy - 1$, if $x > y = 0$; otherwise

and $x \# y = \frac{x+y}{x-y}$ if $x \geq 0, y \geq 0 = 1$; otherwise.

The value of $\frac{(4 \# 2) @ (9 \# (-1))}{(2 @ 3) \# (5 @ 2)}$:

- (a) 1 (b) -2
(c) 0 (d) none of these

Directions (for Q. Nos. 43 and 44) The following functions are defined for any two distinct, non zero integers a and b

$$f_1(a, b) = |a| \times b^2; f_2(a, b) = a^2 \times |b|$$

$$f_3(a, b) = \frac{(a^2 + b^2)}{2}; f_4(a, b) = \left(\frac{a}{2} + \frac{b}{2} \right)$$

43 Which of the following is equal to $(a + b)^2$?

- (a) $f_3(a, b) + f_4(a, b)$ (b) $2 \times f_3(a, b) \times f_4(a \times b)$
(c) $4 \times f_3(a, b) \times f_4(a, b)$ (d) none of these

44 Which one of the following is necessarily greater than zero?

- (a) $f_1(a, b) \times f_2(a, b) \times f_3(a, b)$
(b) $f_1(a, b) \times f_2(a, b) \times f_4(a, b)$
(c) $f_1(a, b) \times f_3(a, b) \times f_4(a, b)$
(d) $f_2(a, b) \times f_3(a, b) \times f_4(a, b)$

Directions (for Q. Nos. 45 to 47)

$lt(p) \rightarrow$ the least possible integer greater than or equal to p .

$gt(p) \rightarrow$ the greatest possible integer less than or equal to p .

$$\Delta(p, q) = \text{Min}(lt(p), gt(q))$$

45 What is the value of $lt(8.3) - gt(5.3) + lt(6.8) + gt(9.4)$

- (a) 15 (b) 20
(c) 19 (d) none of these

46 Find the value of

$$\Delta(3, 2) - \Delta(2, 3) + gt(4.8) - lt(2.7) + \Delta(4, 5)$$

- (a) 4 (b) 5
(c) 6 (d) none of these

- 47 Which of the following statements is always true about $(a - b)$ where $a = \frac{p+q}{2}$ and $b = \frac{lt(p) + gt(q)}{2}$

- (a) $-\frac{1}{4} \leq (a - b) \leq \frac{1}{4}$, always
 (b) $-\frac{1}{2} \leq (a - b) \leq \frac{1}{2}$
 (c) $-1 \leq (a - b) \leq 1$
 (d) $-1 \leq (a - b) \leq 1$, except zero

Directions (for Q. Nos. 48 and 49)

$$a @ b = \frac{a}{a+b} \text{ and } b \# a = \frac{a+b}{b}$$

- 48 Which of the following is true ?

- (a) $\frac{a @ b}{b \# a} = \frac{a}{b}$ (b) $(a @ b)(b \# a) = a^2 b^2$
 (c) $(a @ b)(b \# a) = \frac{a}{b}$ (d) $\frac{a @ b}{b \# a} = \frac{a+b}{b+a}$

- 49 Which of the following is true?

- (a) $a @ b + b \# a = \frac{a^2 + b^2 + 3ab}{a(b+a)}$
 (b) $a @ b - b \# a = \frac{a^2 + b^2 + ab}{b(b+a)}$
 (c) $b \# a - a @ b = \frac{a^2 + b^2 + ab}{b(a+b)}$
 (d) $a @ b + b \# a = 1$

Directions (for Q. Nos. 50 and 51) The following operations are defined for any two real number a and b .

$$mn(p, q) = \min(p, q)$$

$$mx(p, q) = \max(p, q)$$

$$md(p) = |p|$$

- 50 Find the value of $mn(md(p), mx(q, mn(p, md(q))))$; if

- $p = -2, q = 5$:
 (a) -2 (b) 2
 (c) 3 (d) -7

- 51 Find the value of $md(mn(P, mx(md(q), mn(P, r))))$; if

- $p = -2, q = 5, r = 7$.
 (a) -1 (b) 0
 (c) 2 (d) none of these

Directions (for Q. Nos. 52 and 53)

$$a @ b = (a + b)(ab + b)$$

$$a \# b = (a - b)(ab - b)$$

$$a \Delta b = (a @ b) - (a \# b)$$

- 52 Find the value of $((1 @ (2 \Delta 3)) \# 4)$:

- (a) 35500 (b) 482420
 (c) 123400 (d) none of these

- 53 Find the value of $(1 @ 1) \Delta (2 \# 2)$

- (a) 5 (b) 22
 (c) 17 (d) none of these

Directions (for Q. Nos. 54 and 55)

$$a @ b = |a - b|$$

$$a * b = ab$$

$$a \# b = |a^2 - b^2|$$

$$a \Delta b = a^2 + b^2$$

where $a, b \in R$ and $a \neq b$ and the real algebraic operations are unchanged.

- 54 Find the value of $(a \Delta b) + (a \# b)$:

- (a) $2a^2$ (b) $2b^2$
 (c) $2(a^2 + b^2)$ (d) can't be determined

- 55 The value of the expression

$$[(a \# b) \div (a @ b)]^2 - 2(a * b)$$

- (a) $a \# b$
 (b) $a \Delta b$
 (c) $(a \# b)(a @ b)$
 (d) can't be determined

Directions (for Q. Nos. 56 to 58) A function

$f(x) = \log(g(x))$, where $g(x)$ is any function of x .

- 56 For what value of $g(x)$, $g(x) = g(f(x))$?

- (a) e (b) e^x
 (c) $\log x$ (d) none of these

- 57 For what value of $g(x)$ is the function of $f(x) = g(x)$?

- (a) $g(x) = e$ (b) $g(x) = e^x$
 (c) $g(x) = \log x$ (d) none of these

- 58 If $g(x) = e^x$, then $f(f(x))$ is equal to :

- (a) e (b) e^x
 (c) $\log x$ (d) none of these

- 59 If $f(x) = 2x + 3$ and $g(x) = 9x + 6$, then find

- $g[f(x)] - f[g(x)]$
 (a) 18 (b) 22
 (c) 20 (d) none of these

- 60 If $f(x) = x^2 + 2x + 2$; $x \geq -1$

$$= -x^3 + 3x + 1; x < -1$$

then find the value of $f(f(-2))$:

- (a) 15 (b) 17
 (c) 22 (d) none of these

Directions (for Q. Nos. 61 to 63)

$[m]$ is defined as the greatest integer less than m .

$\{m\}$ is defined as the least integer greater than m .

$$f(x, y) = \{x\} + [y]$$

$$g(x, y) = [x] - \{y\}$$

$$F(f(x, y)) = \{f(x, y)\} - [g(x, y)]$$

$$G(g(x, y)) = [f(x, y)] - \{g(x, y)\}$$

- 61 Given that $x^2 = 4$ and $y^2 = 9$ and

$|F(f(x, y))| = |G(g(x, y))|$ then find the value of $x + y$:

- (a) -1 (b) 0
 (c) 1 (d) 5

62 If x and y are consecutive integers, then find $g(x, y)$

- (a) -1 (b) -3
(c) -1 or -3 (d) none of these

63 Which of the following is false?

- (a) $f(x, y) + g(x, y) = F(f(x, y)) + G(g(x, y))$
(b) $F(f(x, y)) - G(g(x, y)) = -2[g(x, y)]$
(c) $F(f(x, y)) + G(g(x, y)) = 2\{f(x, y)\} + 2$
(d) none of the above

Directions (for Q. Nos. 64 to 66) A decimal number ' m ' (say) can be expressed as

$$m = I + D$$

where $I \rightarrow$ integral part

and $D \rightarrow$ decimal part, $0 \leq D < 1$

and $[m] =$ greatest integer less than or equal to m if $0 < D < 0.5$

also $\{m\} =$ least integer greater than or equal to m if $0.5 \leq D < 1$

$$\text{Hence, } f(x, y) = \begin{cases} [x + y] & \text{if } 0 < D < 0.5 \\ \{x + y\} & \text{if } 0.5 \leq D < 1 \\ (x + y) & \text{otherwise} \end{cases}$$

Also, $F(f(x, y)) = -f(x, y)$

and $G(f(x, y)) = -F(f(x, y))$

64 What is the value of the following

$$f(G(f(1.2, 1.3))), f(F(f(0.1))), G(f(0.8, 0.4)))?$$

- (a) 1 (b) 2 (c) 3 (d) 4

65 Which of the following statements is true if x and y are positive?

- (a) $F(f(x, y)) \cdot G(f(x, y)) = -F(f(x, y)) \cdot G(f(x, y))$
(b) $F(f(x, y)) \cdot G(f(x, y)) > -F(f(x, y)) \cdot G(f(x, y))$
(c) $F(f(x, y)) \cdot G(f(x, y)) \neq G(f(x, y)) \cdot F(f(x, y))$
(d) $F(f(x, y)) + G(f(x, y)) + F(x, y) = F(x, y)$

66 Which of the following expressions will yield a number which has a positive integral square root if x, y are positive

- (a) $G(f(1.2, 1.3)) \times f(F(f(2, 1), G(f(4, 1)))$
(b) $F(f(2, 2)) + G(f(2, 2))$
(c) $f(1.1, 1.2) \times G(f(1.1, 1.2))$
(d) $G(f(0, 1)) \times G(f(0, 1))$

67 $f(x) = \frac{1}{x}$; $g(x) = \left(x + \frac{1}{x}\right)$, then which of the following is true?

- (a) $f(g(x)) \cdot g(f(x)) = 1$ (b) $\frac{f(g(x))}{g(f(x))} = 1$
(c) $f(g(x)) + g(f(x)) = 1$ (d) none of these

68 If $f(x) = 2x^2 + 7x - 9$ and $g(x) = 2x + 3$, then find the value of $g(f(x))$ at $x = 2$

- (a) 92
(b) 39
(c) 29
(d) none of the above

69 Find $f(f(f(f(f(2))))))$ if $f(x) = \frac{x+1}{x-1}$; $x \neq 1$

- (a) 1 (b) 2
(c) 3 (d) 5

Directions (for Q. Nos. 70 and 71)

$$f(x) = 1 - h(x), g(x) = 1 - k(x), h(x) = f(x) + 1$$

$$j(x) = g(x) + 1, k(x) = j(x) + 1$$

70 Find the value of $\frac{j(f(x)) + k(h(x))}{h(k(x)) + f(j(x))}$:

- (a) 0 (b) 1
(c) 2 (d) none of these

71 $\frac{h(f(k(x))) + j(g(h(x))) + k(j(f(x)))}{[f(x) + j(x) + k(x)][f(x) \cdot j(x) \cdot k(x)]}$ is equal to:

- (a) 0 (b) 3
(c) $3/2$ (d) none of these

72 If $f(x)$ and $g(x)$ are odd functions of x , then which of the following is true?

- (a) $f + g$ is odd, fg is odd (b) $f + g$ is odd, fg is even
(c) $f + g$ is even, fg is odd (d) $f + g$ is even, fg is even

73 If $f(x)$ is an even function of x and $g(x)$ is an odd function then which of the following is true?

- (a) $f[g(x)]$ is odd, $g[f(x)]$ is even
(b) $f[g(x)]$ is odd, $g[f(x)]$ is odd
(c) $f[g(x)]$ is even, $g[f(x)]$ is even
(d) $f[g(x)]$ is even, $g[f(x)]$ is odd

74 If $y = \min(x^2 + 2, 6 - 3x)$, then the greatest value of y for $x > 0$

- (a) 1 (b) 2
(c) 3 (d) none of these

75 If $p^2 + q^2 + r^2 = 1$, then the maximum value of pqr is:

- (a) 1 (b) $\frac{1}{3\sqrt{3}}$
(c) 3 (d) none of these

Directions (for Q. Nos. 76 to 78) Following functions are defined for three distinct positive numbers, such that $x < y < z$.

$$f(x, y, z) = \min(\max(x + y, y + z), \max(z + x, x + y))$$

$$g(x, y, z) = \max(\min(x + y, y + z), \min(z + x, x + y))$$

$$h(x, y, z) = \max(\max(x + y, y + z), \max(z + x, x + y))$$

$$j(x, y, z) = \min(\min(x + y, y + z), \min(z + x, x + y))$$

$$k(x, y, z) = \text{int.}((x + y + z) / 3)$$

$$\text{such that } k(x, y, z) \leq (x + y + z) / 3$$

$$l(x, y, z) = \min(x, y, z); m(x, y, z) = \max(x, y, z)$$

76 Which of the following expression is necessarily equal to 1?

- (a) $\frac{(f(x, y, z) - m(x, y, z))}{(m(x, y, z) - l(x, y, z))}$ (b) $\frac{g(x, y, z)}{j(x, y, z)}$
(c) $\frac{(m(x, y, z) - f(x, y, z))}{k(x, y, z) + j(x, y, z)}$ (d) $\frac{(h(x, y, z) - f(x, y, z))}{f(x, y, z)}$

- 77 Which of the following expression gives negative value?

- (a) $\frac{(m(x, y, z) - f(x, y, z))}{h(x, y, z) \times k(x, y, z) \times l(x, y, z)}$
 (b) $\frac{(m(x, y, z) - l(x, y, z) + j(x, y, z))}{k(x, y, z)}$
 (c) $\frac{l(x, y, z) - m(x, y, z)}{g(x, y, z) - j(x, y, z)}$
 (d) $\frac{h(x, y, z) - g(x, y, z)}{k(x, y, z)}$

- 78 Which of the following expressions is necessarily greater than 1?

- (a) $\frac{[h(x, y, z) - j(x, y, z)]}{k(x, y, z)}$
 (b) $[g(x, y, z) - j(x, y, z)] \times m(x, y, z)$
 (c) $\frac{g(x, y, z)}{j(x, y, z)}$
 (d) $\frac{f(x, y, z) + h(x, y, z) - g(x, y, z)}{j(x, y, z)}$

Directions (for Q. Nos. 79 to 83)

$$A(x, y, z) = \min(x + y, y + z, z + x)$$

$$B(x, y, z) = \max(x - y, y - z, z - x)$$

$$C(x, y, z) = \max(A(x, y, z), B(x, y, z))$$

$$D(x, y, z) = \min(A(x, y, z), B(x, y, z))$$

- 79 The value of $C(1, 2, 3)$ is :

- (a) 1 (b) 2 (c) 3 (d) 4

- 80 The value of $D(0, 1, 2)$ is :

- (a) -1 (b) 1
(c) 0 (d) none of these

- 81 What is the value of $A(2, 3, 4) + B(2, 3, 4)$?

- (a) 13 (b) 4
(c) 7 (d) none of these

- 82 What is the value of $C(0, 1, D(1, 2, 3))$?

- (a) 1 (b) 2
(c) -2 (d) none of these

- 83 What is the value of $D(1, 2, C(0, 1, 2))$?

- (a) 1 (b) 2
(c) 3 (d) 4

- 84 What is the maximum possible value of xy , where

$$|x + 5| = 8 \text{ and } y = 9 - |x - 4|$$

- (a) 401
(b) 124
(c) 104
(d) none of these

- 85 For real x, y , such that $f(x) \neq 0$, if $f(x + y) = f(x) \cdot f(y)$, what's the value of $f(100x)$?

- (a) $(f(x))^{100}$ (b) $(f(x))^{99}$
(c) $99(f(x))^{100}$ (d) $\frac{(f(x))^{99}}{100}$

- 86 Find the number of solutions of $|x|e^{|x|} = 9$.

- (a) 0 (b) 1 (c) 2 (d) 3

LEVEL 02 > HIGHER LEVEL EXERCISE

- 1 Let a function is defined as $f(x) = \frac{a^x + a^{-x}}{2}$; $a > 0$, what is the value of $f(x + y) + f(x - y)$?

- (a) $f(x) + f(y)$ (b) $f(x)f(y)$
(c) $2f(x)f(y)$ (d) $4f(x)f(y)$

- 2 If $\left| \frac{30 - x}{5} \right| < 4$, then find x :

- (a) $-6 < x < 6$ (b) $10 < x < 50$
(c) $x > 10$ (d) none of these

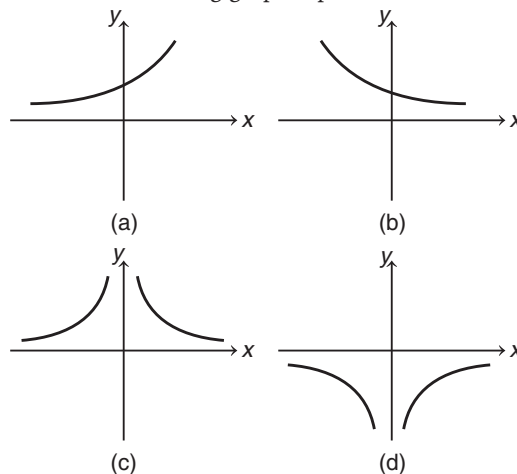
- 3 Let $f(x, y) = |x + y|$ and $g(x, y) = |x - y|$, then how many ordered pairs of the form (x, y) would satisfy $f(x, y) = g(x, y)$?

- (a) 1 (b) 2
(c) 4 (d) infinitely many

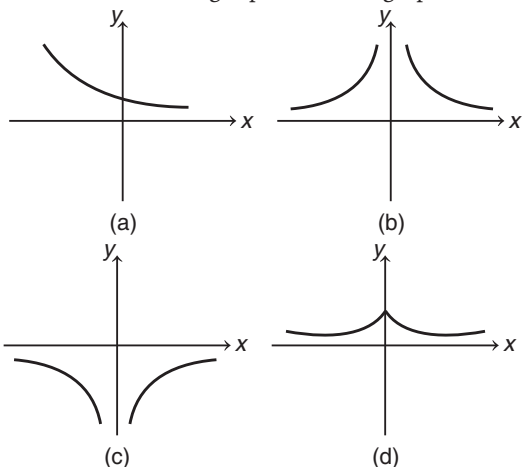
- 4 What is the value of x if $|x - 1| + |x| = 2$:

- (a) $-\frac{1}{2}$ (b) $\frac{3}{2}$
(c) both (a) and (b) (d) none of these

- 5 Which of the following graph represents e^{-x} ?

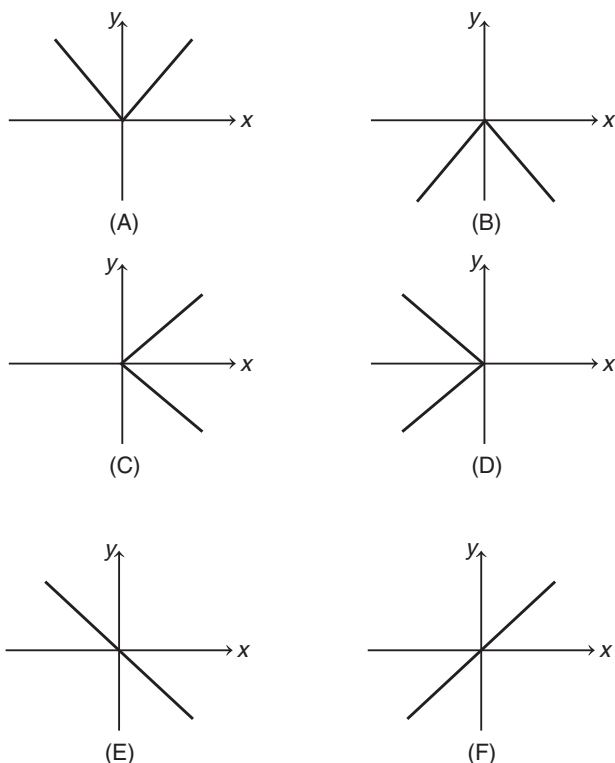


- 6 Which of the following represents the graph of e^{-x^2} :



- 7 A function is defined as $f(x) = a^x \forall x \in \mathbb{R}$ and $0 < a < 1$, then which of the following is true?
- (i) It is an increasing function
 - (ii) It is a decreasing function
 - (iii) All such functions must meet at $(0, 1)$
- (a) only (i) is true (b) only (ii) is true
(c) (i) and (iii) are true (d) (ii) and (iii) are true

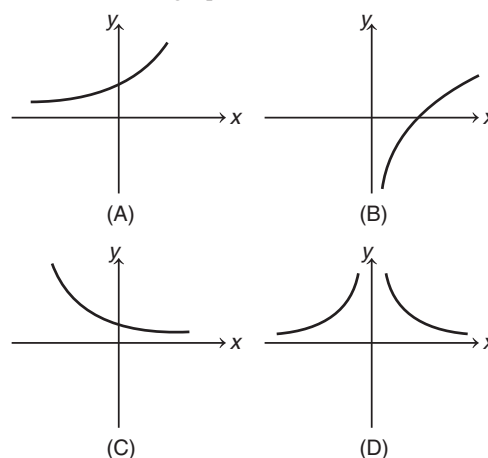
Directions (for Q. Nos. 8 to 13) The following figure represents various functions. Match the figure with the appropriate function.



- 8 $f(x) = -x$:
(a) B (b) C (c) E (d) D

- 9 $f(x) = |x|$:
(a) A (b) B (c) C (d) D
- 10 $f(x) = -|x|$:
(a) E (b) B (c) A (d) F
- 11 $f(x) = x$:
(a) E (b) F (c) A (d) B
- 12 $f(x) = |x| - x$:
(a) A (b) C
(c) B (d) none of these
- 13 $f(x) = ||x| - x|$:
(a) B (b) D
(c) A (d) none of these

- 14 If $x = 10^y$ then the graph of inverse of this function is :



- (a) A (b) B (c) C (d) D

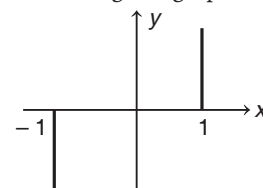
- 15 If $f(x) = \begin{cases} 1-x, & 0 \leq x \leq 2 \\ x-1, & 2 \leq x \leq 4, \\ 1, & 4 \leq x \leq 6 \end{cases}$

then find, $f(0) + f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{45}{18}\right)$:

- (a) 1 (b) 2
(c) 3 (d) none of these

- 16 Which one of the following is true about the given graph ?

- (a) $x = \frac{|y|}{y}, y \neq 0$
(b) $y = \frac{|x|}{x}, x \neq 0$
(c) $x = -\frac{|y|}{y}, y \neq 0$
(d) $y = \frac{-|x|}{x}, x \neq 0$



Directions (for Q. Nos. 17 and 18) In each question there are two graphs $f(x)$ and $g(x)$ for the real values of x .

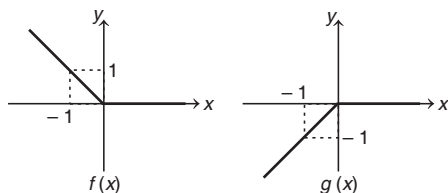
Mark (a) if $g(x) = f(-x)$

Mark (b) if $g(x) = -f(x)$

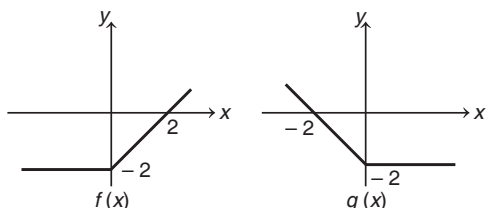
Mark (c) if $g(x) = |f(x)|$

Mark (d) if none of the above is true.

17



18



Directions (for Q. Nos. 19 to 22) In the following questions a function is represented by the graph.

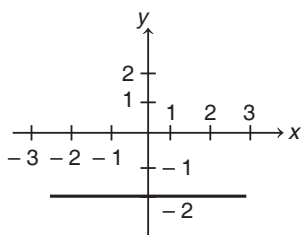
Mark (a) if $f(x) = f(-x)$

Mark (b) if $f(x) = -f(-x)$

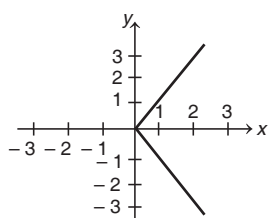
Mark (c) if $f(x)$ does not exist

Mark (d) if none of the above is true.

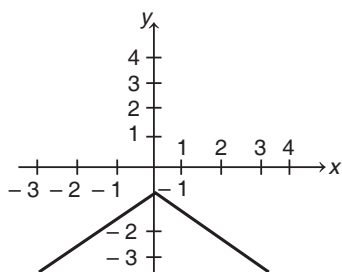
19



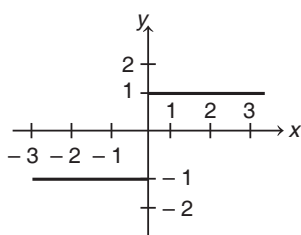
20



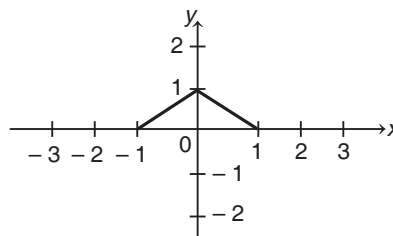
21



22



23 In the following graph for $x \in [-1, 1]$, $f(x)$ is defined by :



- (a) $|x| + 1$ (b) $-|x| + 1$
(c) $-|x + 1|$ (d) $-|x| - 1$

24 How many times does the graph of $y = x^3 - 3x^2 - x + 3$ intersects the x-axis :

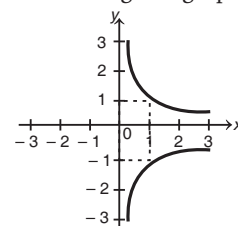
- (a) 1 (b) 2
(c) 3 (d) none of these

25 Which one of the following is true for $x \in \mathbb{R}$:

- (a) $f(x) = \log |x|$ (b) $f(x) = \frac{|x|}{\log |x|}$
(c) $f(x) = \log(x^2 + 2)$ (d) $f(x) = \log_x 100$

26 Which one of the following is correct about the given graph of $f(x)$:

- (a) $|xy| = 1$
(b) $|x|y = 1$
(c) $x|y| = 1$
(d) $|x||y| = 1$



27 A function is defined as $f(x) = \frac{25}{(x+5)^2}$, then the value of $f(x)$:

- (a) decreases as the value of x increases, only if x is negative
(b) increases as the value of x increases, only if x is negative
(c) decreases as the value of x increases, only if x is positive.
(d) both (a) and (c) are true

Directions (for Q. Nos. 28 and 29) The following functions are defined for the set of variables $x_1, x_2, x_3, \dots, x_n$

$$f(x_i, x_j) = \begin{cases} x_{i+j} & \text{if } i + j \leq n^2 \\ x_{i+j-n} & \text{if } i + j > n^2 \end{cases}$$

$$g(x_i, x_j) = x_m,$$

where m is the remainder when $i \times j$ is divided by n .

28 Find the value of $f(f(x_2, x_3), f(x_5, x_6))$ if $n = 3$

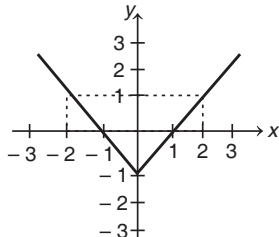
- (a) x_5
(b) x_{10}
(c) x_{13}
(d) none of the above

29 Find the value of $g(g(x_2, x_3), g(x_7, x_8))$ if $n = 5$

- (a) x_1 (b) x_2
(c) x_5 (d) none of these

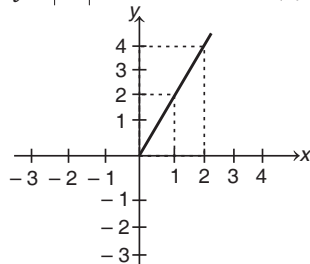
Directions (for Q. Nos. 30 to 34) In each of the following questions a graph of a function is being shown. Select the correct equation of the function of the graph.

30



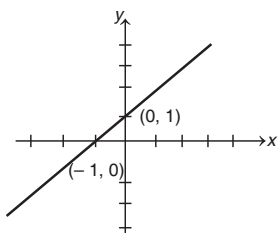
- (a) $y = |x - 1|$ (b) $x = |y| + 1$
(c) $y = |x| - 1$ (d) $y = |x| - 2$

31



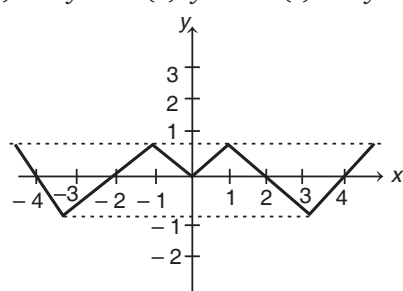
- (a) $x = |y| + y$ (b) $y = |x| + x$
(c) $y = |x|$ (d) $x = |y|$

32



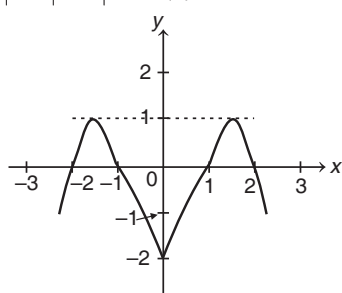
- (a) $x = y + 1$ (b) $y = x - 1$ (c) $x = y - 1$ (d) $y = 1 - x$

33



- (a) $y = |x - 1| |x - 2|$ (b) $y = ||x| - 1| - 2 - 1$
(c) $y = ||x| - 2| - 1$ (d) none of these

34

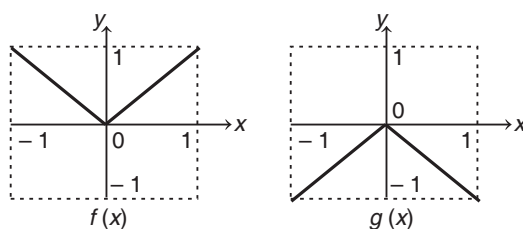


- (a) $y = |x - 1| |x - 2|$ (b) $y = -(|x| - 1)(|x| - 2)$
(c) $y = ||x| - 1| - 2$ (d) $y = (|x| - 1)(|x| - 2)$

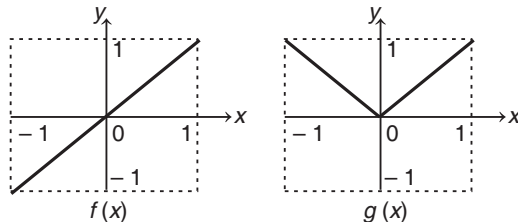
Directions (for Q. Nos. 35 to 37) In each of the following questions a pair of graphs $f(x)$ and $g(x)$ is given, defined for every $x \in (-1, 1)$. Choose the correct answer as

- (a) if $f(x) = -g(x)$ for every x
(b) if $f(x) = g(x)$ for $x < 0$ and $f(x) = -g(x)$ $x > 0$
(c) if $f(x) = -g(x)$ for $x < 0$ and $f(x) = g(x)$, $x > 0$
(d) none of the above

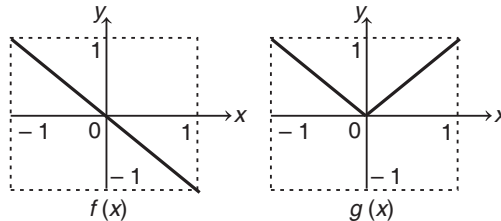
35



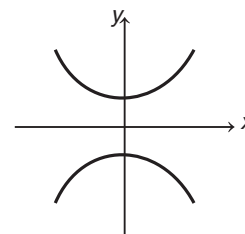
36



37



38 In the adjacent diagram two graphs of equal magnitude are being shown. Their maxima and minima are also same, then the correct choice for the two equation of functions is :



- (a) $y = ax^2 + b$ and $y = ax^2 - b$
(b) $f(x) = ax^2$ and $f(x) = bx^2$; $a \neq b$
(c) $f(x) = ax^2 + bx$ and $f(x) = -ax^2 + bx$
(d) $y + ax^2 + c = 0$ and $y - ax^2 - c = 0$

39 If $f(x) = \frac{1}{x}$; $x \neq 0$ and $f^{n+1}(x) = f^n(f(x))$, find the product

- of $f^{11}(1)$, $f^{33}(1)$, $f^{55}(1)$, $f^{77}(1)$, $f^{99}(1)$:
(a) 110 (b) 88
(c) 275 (d) 1

- 40 If $f(x) = \frac{1}{x}$, $x \neq 0$, $f^n(x) = f(f^{n-1}(x))$ and $f^{12} = 11$, what is the value of $f(x) = ?$

(a) 1 (b) 2
(c) $1/11$ (d) none of these

Directions (for Q. Nos. 41 to 43) A program module is given below with various steps involved, which starts with the values of a, b, c as 1, 2, 3 respectively. These values remain unchanged till step 1.

The program is executed with the help of following operators.

$$\text{Milan } 2(P) = P \times 2$$

$$\text{Milaap } 2(P) = P + 2$$

and $b = a$ means the value of a is assigned to b .

START

Step 1. Read (a, b, c)

Step 2. Milan $2(a)$

Step 3. Milaap $2(b)$

Step 4. $c = a + b$

Step 5. If, $c > 111$, then go to step 6 else go to step 2.

Step 6. STOP

- 41 How many times in the above program does c take a perfect square value ?
(a) 1 (b) 2
(c) 0 (d) none of these
- 42 In a particular cycle a, b and c get the values such that $9a = 8c$, then the value of b is :
(a) 6 (b) 8 (c) 16 (d) none of these
- 43 How many times is the program module repeated?
(a) 6 (b) 7
(c) 5 (d) none of these

Directions (for Q. Nos. 44 to 46) In the following algorithm x, y and z are variables which change their values in each step such that the value of the expression on the right hand side is assigned to the variable on the left hand side of the equation. Each cycle consists of 5 steps. $S_1, S_2, S_3, \dots$ are the sum of each corresponding cycle 1, 2, 3,...

START

Step 0. $x = 1, y = -2, z = 3$

Step 1. $x = y - z$

Step 2. $y = z - x$

Step 3. $z = x - y$

Step 4. $S = x + y + z$

Step 5. Go to step 1.

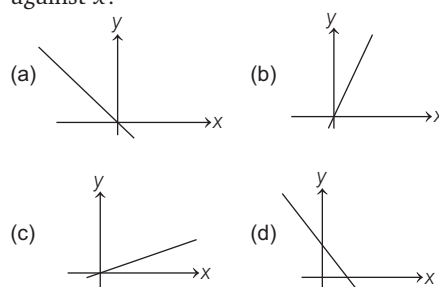
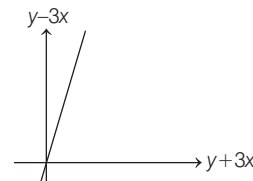
- 44 Which one of the following is correct for each cycle?
(a) $S = 2x$ (b) $S = 3x$
(c) S is an odd number (d) none of these
- 45 What is the value of S_4 ?
(a) 987 (b) 754
(c) -178 (d) none of these
- 46 Which of the following is correct?
(a) S_{11} is an odd number (b) S_{13} is a negative number
(c) S_{15} is an even number (d) both (b) and (c)

Directions (for Q. Nos. 47 and 48) There are three bottles of water K, L, M whose capacities are 9 litres, 5 litres and 4 litres respectively. For transferring water from one bottle to another and to drain out the bottles, there exists a piping system. The flow through these pipes is computer controlled. The computer that controls the flow through these pipes can be fed with three types of instructions, as explained below. Initially K is completely filled up with water and L, M are empty.

Instruction type	Explanation of the instruction
Fill (X, Y)	Fill bottle labelled X from the water in bottle labelled Y , where the remaining capacity of X is less than or equal to the amount of water in Y .
Empty (X, Y)	Empty out the water in bottle labelled X into bottle labelled Y , where the amount of water in X is less than or equal to remaining capacity of Y .
Drain (X)	Drain out all the water contained in bottle X .

- 47 How much water would be there in bottle labelled L after the execution of the sequence of the following four instructions :
First instruction : Fill (L, K)
Second instruction : Fill (M, L)
Third instruction : Empty (L, K)
Fourth instruction : Empty (M, L)
(a) 0 litre (b) 4 litres
(c) 5 litres (d) none of these
- 48 After executing a sequence of 6 instructions, bottle K has half of the water that the bottles L and M together have and the bottle L has half of the water that the bottle M has. Find the fourth instruction if the rest instructions are given below.
Instruction 1. Fill (L, K)
Instruction 2. Fill (M, L)
Instruction 3. Empty (M, K)
Instruction 5. Fill (L, K)
Instruction 6. Fill (M, L)
(a) Fill (L, M) (b) Empty (L, M)
(c) Fill (M, K) (d) Empty (M, L)

- 49 The graph of $y - 3x$ against $y + 3x$ is shown in the adjacent. All graphs in questions are drawn to scale and the same scale has been used on each axis. Which of the following shows the graph of y against x ?



LEVEL 03 > Final Round

(The questions in this exercise given are very similar to the questions which are asked in XAT for XLRI, XIM etc.)

Directions (for Q. Nos. 1 to 10) Solve the following questions on the basis of following functions

- (a) $P_{(n+1)} = P_{(n)} - P_{(n-1)}$, P is the term of the sequence and $P_{(0)} = 0, P_{(1)} = 1$
- (b) $Q_{(n+1)} = Q_{(n)} + Q_{(n+1)}$; Q is the term of the sequence and $Q_{(0)} = 0, Q_{(1)} = 1$
- What is the 10th term of series $Q_{(n+1)}$ starting from $n = 0$?
(a) 34 (b) 89
(c) -1 (d) 55
 - What is the 12th term of the series $P_{(n+1)}$ starting from $n = 0$?
(a) -1 (b) 0
(c) 1 (d) none of these
 - The value of Q cannot be :
(a) 144 (b) 55 (c) 98 (d) 34
 - The value of P cannot be :
(a) -1 (b) 0 (c) 1 (d) 4
 - Which of the following can be a positive value of $Q_n + P_{(n+1)}$?
(a) 55 (b) 234
(c) 146 (d) none of these
 - The value of $Q_n + P_n$ cannot be :
(a) 2 (b) 22 (c) 55 (d) 88
 - The value of $Q_{[Q(n+1)]}$ can be :
(a) 55 (b) 13 (c) 21 (d) 89
 - The value of $Q_n \times P_n$ can be :
(a) -34 (b) 89 (c) -233 (d) 377
 - The value of $[Q_{(n)}]^{P_n}$ can be :
(a) -1 (b) 0.2 (c) -13 (d) 5.5
 - The value of $Q_{(n-1)} + Q_{(n+1)} + L_{(n-1)} + L_{(n+1)}$ cannot be :
(a) 30 (b) 75
(c) 18 (d) 122

Directions (for Q. Nos. 11 to 14) Let S be a sequence of the form $[K(1), K(2), \dots, K(m)]$. Each term can be defined by the following four functions :

$$P[K(x)] = 3Q[K(x)] - 4$$

$$Q[K(x)] = 2R[K(x)] + R[K(2x)]$$

$$R[K(x)] = S[K(2x)] - S[K(x)]$$

$$S[K(x)] = \begin{cases} 0 & \text{if } x < 0 \\ 3x + 4 & \text{if } 0 \leq x \leq 6 \\ 4x + 3 & \text{if } x > 6 \end{cases}$$

Also if $Q[K(x)] = y$ then $PQ[K(x)] = P[K(y)]$

- What is the value of $P[K(3)]$?
(a) 3 (b) 5 (c) 32 (d) 137
- What is the value of $QRS[K(1)]$?
(a) 112 (b) 224
(c) 448 (d) none of these
- What is the value of $R[K(5)] - S[K(10)]$?
(a) 43 (b) -19
(c) 14 (d) none of these
- What is the value of $\sum_{x=-\infty}^{+\infty} R[K(x)]$ if $\sum_{x=1}^4 R[K(x)] = R[K(1)] \times R[K(2)] \times R[K(3)] \times R[K(4)]$?
(a) -1 (b) 0
(c) 1 (d) none of these

Directions (for Q. Nos. 15 to 20) These questions are based on the given data.

A polynomial can be represented in an equivalent sequence form.

The polynomial $P_1x^{k_1} + P_2x^{k_2} + P_3x^{k_3} + \dots + P_nx^{k_n}$ where $k_1 > k_2 > k_3 \dots > k_n$ and $a_i \neq 0$ for $1 \leq i \leq n$ will be represented as the sequence $(P_1, k_1, P_2, k_2, P_3, k_3, \dots, P_n, k_n)$

Also, we add, subtract, multiply as we do for corresponding polynomials. The resulting polynomial is again represented as a sequence.

- $(3, 4, 2, 2, 5, 1) + (2, 4, 3, 3, 7, 2)$ equals :
(a) $(5, 4, 3, 4, 9, 3, 5, 1)$ (b) $(5, 4, 5, 5, 12, 3)$
(c) $(5, 4, 3, 3, 9, 2, 5, 1)$ (d) none of these
- $(6, 5, 7, 4, 8, 3) - (3, 5, 5, 3, 7, 1)$ equals
(a) $(3, 5, 2, 2, 1, 2)$ (b) $(3, 5, 7, 4, 3, 3, -7, 1)$
(c) $(3, 5, 7, 3, 8, 2)$ (d) none of these
- $(1, 1, 2, 0) \times (1, 1, 2, 0) \times (1, 1, 2, 0)$ equals :
(a) $(1, 3, 6, 2, 12, 1, 8, 0)$ (b) $(1, 3, 2, 2, 3, 1, 4, 0)$
(c) $(1, 8, 2, 4, 3, 2)$ (d) none of these
- The expression $\frac{3, 3, -10, 2, 7, 1}{3, 2, -7, 1}$ equals :
(a) $(1, 1, -1, 1)$ (b) $(1, 1, -1, 0)$
(c) $(2, 2, -2, 0)$ (d) none of these
- Which of the following is incorrect?
(a) $(1, 1, 1, 0) \times (1, 2, -1, 1, 1, 0) = (1, 3, 1, 0)$
(b) $(2, 2, 1, 1) \times (4, 4, 4, 4) = (16, 6, 10, 5)$
(c) $\frac{(1, 2, 4, 1, 4, 0)}{(1, 1, 2, 0)} = (1, 1, 2, 0)$
(d) none of the above
- $(4, 4, 3, 3) \times (2, 2, 1, 1) + (2, 2, 1, 1) - (3, 5, 2, 4)$ equals :
(a) $(8, 6, 5, 5, 7, 4, 2, 3, 2, 1)$
(b) $(8, 6, 7, 5, 1, 4, 2, 2, 1, 1)$
(c) $(8, 6, 7, 5, 2, 4, 3, 2, 1, 1)$
(d) none of the above

Directions (for Q. Nos. 21 to 25)

If p, q, r, s be the distinct integers such that :

$f(p, q, r, s) = \text{maximum of } (p, q, r, s)$

$g(p, q, r, s) = \text{minimum of } (p, q, r, s)$

$h(p, q, r, s) = \text{remainder when } p \times q \text{ is divided by } r \times s \text{ if}$

$p \times q > r \times s$
 $= \text{remainder when } r \times s \text{ is divided by}$
 $p \times q \text{ if } r \times s > p \times q$

Also, the same operations are valid with two variable functions of the form $f(p, q)$

Also a function $fgh(p, q, r, s)$

$= f(p, q, r, s) \times g(p, q, r, s) \times h(p, q, r, s)$

21 What is the value of $h(3, 2, 8, 7) \div g(4, 7, 10, 8)$?

- (a) 1 (b) $1/2$
 (c) 24 (d) none of these

22 What is the value of $h(fg(2, 5, 7, 3), 9)$?

- (a) 2 (b) 3
 (c) 5 (d) none of these

23 What is the value of $h(h(7, 13, 5, 9), h(4, 6, 12, 14))$?

- (a) 1 (b) $1/7$
 (c) 7 (d) not defined

24 If $A = h(3, 7, 6, 5)$, $B = h(16, 11, 13, 3)$

$C = h(9, 4, 8, 7)$, $D = h(19, 14, 18, 7)$, then which of the following is true?

- (i) $A > B > C > D$ (ii) $D > A > C$
 (iii) $A < D < B$ (iv) $B = C < D$
 (a) only (i) is true (b) only (iii) is true
 (c) (ii) and (iii) are true (d) (iii) and (iv) are true.

25 If $a_k \times b_k$ is an integral multiple of

$c_k \times d_k$ or $c_k \times d_k$ is an integral multiple of $a_k \times b_k$, which of the following is indeterminable?

- (a) $f(g(a_1, b_1, c_1, d_1), h(a_2, b_2, c_2, d_2))$
 (b) $h(h(a_1, b_1, c_1, d_1), h(a_2, b_2, c_2, d_2))$
 (c) $h(h(a_1, b_1, c_1, d_1), g(a_2, b_2, c_2, d_2))$
 (d) none of the above

Directions (for Q. Nos. 26 to 32) Following questions are based on the given information for the following functions $f(x)$

$f(x) = 2bx + f(-x)$; if $x < 0$

$f(x) = a$ if $x = 0$; $f(x) = b + c - 2cx + f(x - 1)$; if $x > 0$

26 $f(8)$ equals :

- (a) $a + 8b - 32c$ (b) $a + 8(b - 8c)$
 (c) $8(a + b - c)$ (d) none of these

27 $f(-19)$ equals :

- (a) $a - 19b + 361c$ (b) $a + 19(b - 19c)$
 (c) $a - 19(b + 19c)$ (d) none of these

28 If $a = 15$, $b = 11$, $c = -3$, then $f(7)$ equals :

- (a) 239 (b) 115
 (c) -147 (d) none of these

29 If $a = 10$, $b = -7$, $c = 6$, then $f(-10)$ equals :

- (a) -660 (b) -520
 (c) -250 (d) none of these

30 If $a = 4$, $b = -17$ and $c = -18$, then for what value of x , $f(x) = 0$?

- (a) 4 or 9 (b) $\frac{1}{2}$ or $\frac{4}{9}$ (c) -1 or 18 (d) none

31 If $a = 12$, $b = 10$ and $c = 8$, then for what value of x , $f(x) < 0$?

- (a) $\frac{3}{4}$ (b) 1 (c) -2 (d) none

32 If $b = -a$, $c = a$, $f(f(1))$ equals :

- (a) $a + a^2 + a^3$ (b) $a + a^2 - a^3$
 (c) $a - a^2 + a^3$ (d) none of these

Directions (for Q. Nos. 33 to 37)

A function $f(z_1, z_2, z_3, \dots, z_n) = f(z_1, z_n) + f(z_2, z_3, \dots, z_{n-1})$
 $+ (z_1 + z_2 + \dots + z_n)$; for $n > 0$

$f(y, z) = f(z, 0) + f(0, y)$; $f(y, 0) = y + f(y - 1, 0)$

$f(0, y) = y - f(0, y - 1)$; $f(0, 0) = 1$

33 Find the value of $f(1, 2, 3, 4, \dots, n)$, where n is a perfect cube less than 50 and n is greater than 25 :

- (a) 1665 (b) 1089
 (c) 729 (d) not defined

34 Find the value of $f(0, 1, 0, 1)$:

- (a) 2 (b) 4
 (c) 8 (d) none of these

35 Find the value of $f(8, 8, 8, 2, 2, 2)$:

- (a) 22 (b) 88 (c) 77 (d) 66

36 Find the value of $f(1, 1, 3, 1, 1, 3)$:

- (a) 17 (b) 28 (c) 21 (d) none

37 $f(9, 2, k, 0, 9, 4) = 124$, then the value of k is :

- (a) 0 (b) 5 (c) 9 (d) none

Directions (for Q. Nos. 38 to 40) A function is defined as follows :

$f(a_1, a_2, a_3, \dots, a_n) = a_1 2^{n-1} + a_2 2^{n-2} + a_3 2^{n-3} + \dots + a_n 2^0$

The above function is repeated until the value of function reduces to a single digit number.

38 $f(128)$ equals :

- (a) 1 (b) 2 (c) 4 (d) 8

39 What is the value of $f[f(888222) + f(113113)]$:

- (a) 6 (b) 7 (c) 8 (d) 9

40 $f(9235) + f(9450)$ equals :

- (a) 5 (b) 3
 (c) 2 (d) none of these

41 Let f, g and h be real-valued functions defined on the interval $[0, 1]$ by $f(x) = e^{x^2} + e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$, $h(x) = x^2 e^{x^2} + e^{-x^2}$

If a, b and c denote, respectively, the absolute maximum of f, g and h on $[0, 1]$, then

- (a) $a = b$ but $b \neq c$ (b) $a = c$ but $a \neq b$
 (c) $a \neq b$ but $b \neq c$ (d) $a = b = c$

- 42 If the functions $f(x)$ and $g(x)$ are defined on $R \rightarrow R$ such that

$$f(x) = \begin{cases} 0, & x \in \text{rational} \\ x, & x \in \text{irrational} \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 0, & x \in \text{irrational} \\ x, & x \in \text{rational} \end{cases}$$

- (a) One-one and onto (b) neither one-one nor onto
(c) One-one but onto (d) onto but not one-one
- 43 If $\frac{x^2 - 1}{x^2 + 1}$, for every real number x , then the minimum value of f is

- (a) Does not exist (b) 1
(c) -1 (d) 0

- 44 Range of the function $f(x) = \frac{x^2 + x + 2}{x^2 + x + 1}$; $x \in R$ is

- (a) $(1, \infty)$ (b) $(1, 11/7)$ (c) $(1, 7/3)$ (d) $(1, 7/5)$

- 45 Let $f\left[x + \frac{1}{x}\right] = \left[x^2 + \left(\frac{1}{x^2}\right)\right]$; ($x \neq 0$), then $f(x)$ is equal to

- (a) $x^2 - 1$ (b) $x^2 - 2$
(c) x^2 (d) None of these

Answers

✓ Introductory Exercise 17.1

1. (a)	2. (c)	3. (c)	4. (a)	5. (a)	6. (d)	7. (b)	8. (a)	9. (c)	10. (b)
11. (c)	12. (b)	13. (b)	14. (c)	15. (d)	16. (c)	17. (d)	18. (d)	19. (c)	20. (c)
21. (a)	22. (d)	23. (a)	24. (b)	25. (c)	26. (c)	27. (c)	28. (d)	29. (d)	30. (b)
31. (d)	32. (a)	33. (c)	34. (c)	35. (b)	36. (d)	37. (b)	38. (c)	39. (c)	40. (c)
41. (b)	42. (b)	43. (b)	44. (b)						

✓ Introductory Exercise 17.2

1. (c)	2. (a)	3. (c)	4. (c)	5. (a)	6. (c)	7. (a)	8. (b)	9. (a)	10. (d)
11. (c)	12. (a)	13. (c)	14. (d)	15. (d)	16. (b)	17. (b)	18. (d)	19. (a)	20. (a)
21. (b)	22. (b)	23. (c)	24. (a)	25. (b)	26. (a)	27. (d)	28. (a)	29. (b)	30. (b)

✓ Introductory Exercise 17.3

1. (c)	2. (d)	3. (a)	4. (b)	5. (d)	6. (b)	7. (a)	8. (b)	9. (b)	10. (c)
11. (a)	12. (c)	13. (b)	14. (a)	15. (b)	16. (b)	17. (a)			

✓ Level 01 Basic Level Exercise

1. (b)	2. (a)	3. (d)	4. (c)	5. (c)	6. (c)	7. (b)	8. (c)	9. (a)	10. (a)
11. (b)	12. (c)	13. (a)	14. (b)	15. (c)	16. (c)	17. (a)	18. (c)	19. (b)	20. (b)
21. (d)	22. (d)	23. (d)	24. (b)	25. (a)	26. (c)	27. (b)	28. (d)	29. (c)	30. (a)
31. (a)	32. (b)	33. (c)	34. (a)	35. (b)	36. (c)	37. (b)	38. (d)	39. (a)	40. (d)
41. (a)	42. (b)	43. (d)	44. (a)	45. (b)	46. (b)	47. (b)	48. (c)	49. (d)	50. (b)
51. (c)	52. (d)	53. (d)	54. (d)	55. (b)	56. (b)	57. (d)	58. (d)	59. (a)	60. (b)
61. (a)	62. (c)	63. (a)	64. (c)	65. (d)	66. (c)	67. (a)	68. (c)	69. (c)	70. (c)
71. (d)	72. (b)	73. (c)	74. (c)	75. (b)	76. (b)	77. (a)	78. (d)	79. (c)	80. (b)
81. (c)	82. (b)	83. (a)	84. (c)	85. (a)	86. (c)				

✓ Level 02 Higher Level Exercise

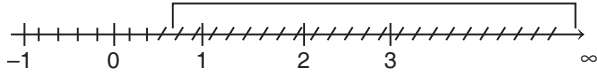
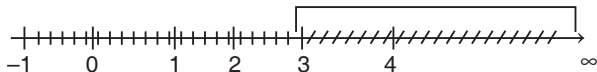
1. (c)	2. (b)	3. (d)	4. (c)	5. (b)	6. (d)	7. (d)	8. (c)	9. (a)	10. (b)
11. (b)	12. (d)	13. (d)	14. (b)	15. (c)	16. (a)	17. (b)	18. (a)	19. (a)	20. (c)
21. (a)	22. (b)	23. (b)	24. (c)	25. (c)	26. (c)	27. (c)	28. (b)	29. (a)	30. (c)
31. (b)	32. (c)	33. (b)	34. (b)	35. (a)	36. (c)	37. (b)	38. (d)	39. (d)	40. (c)
41. (d)	42. (c)	43. (a)	44. (a)	45. (b)	46. (d)	47. (b)	48. (b)	49. (a)	

✓ Level 03 Final Round

1. (d)	2. (b)	3. (c)	4. (d)	5. (b)	6. (a)	7. (c)	8. (d)	9. (b)	10. (b)
11. (d)	12. (c)	13. (b)	14. (b)	15. (c)	16. (b)	17. (a)	18. (b)	19. (b)	20. (b)
21. (b)	22. (c)	23. (d)	24. (b)	25. (b)	26. (b)	27. (c)	28. (a)	29. (b)	30. (b)
31. (c)	32. (b)	33. (d)	34. (b)	35. (c)	36. (b)	37. (b)	38. (d)	39. (b)	40. (a)
41. (d)	42. (a)	43. (c)	44. (c)	45. (b)					

Hints & Solutions

Introductory Exercise 17.1

- 1** x is defined for all real numbers (R)
 $\therefore$ Domain of x is R
- 2** $\because \sqrt{x}$ is not defined for negative values of x
hence $x \geq 0$
 $\therefore x \in R^+ \cup \{0\}$
- 3** Since a rational number is not defined if denominator is zero.
Hence $\frac{1}{x}$ is defined for only non zero numbers.
Thus $x < 0$ and $x > 0$
 $\Rightarrow (-\infty < x < 0) \cup (0 < x < \infty)$
 $\Rightarrow x \in (-\infty, 0) \cup (0, \infty)$
 $\Rightarrow x \in R - \{0\}$
- 4** $\because x$ is defined for all real values
 $\therefore |x|$ is also defined for all real values
Hence $|x| \in R$
- 5** $\because |x| \geq 0 \forall x \in R$
 $\therefore \sqrt{|x|}$ is defined for all real values of x .
Hence, domain of $\sqrt{|x|}$ is R .
- 6** Since domain of $\sqrt{x}$ is $[0, \infty)$
therefore domain of $|\sqrt{x}|$ is also $[0, \infty)$
- 7** $\sqrt{x}$ is defined only when $x \geq 0$
but $\frac{1}{\sqrt{x}}$ is defined only when $x > 0$
Since rational expressions are not defined for zero denominator so, $\sqrt{x} \neq 0$
 $\therefore$ Domain of $\frac{1}{\sqrt{x}}$ is $(0, \infty)$ i.e., R^+
- 8** Since x^2 is positive for all values of x
i.e., $x^2 \geq 0 \forall x \in R$
 $\therefore \sqrt{x^2}$ is defined for all values of R
Hence domain of $\sqrt{x^2}$ is R .
- 9** Since $\sqrt{x}$ is defined for only non-negative real numbers.
Hence domain of $(\sqrt{x})^2$ is $[0, \infty)$
- 10** $\log_a x$ is defined only for $x > 0$
Hence the domain of $\log_{10} x$ is $(0, \infty)$
- 11** x^2 is positive for all values of x except at $x = 0$
Since $\log_a b$ is defined only if $b > 0$
 $\therefore$ Domain of $\log_{10} x^2$ is $R - \{0\}$
- 12** $\log_{10} x$ is defined for only $x > 0$
also denominator must be non-zero
 $\therefore$ Domain of $f(x) = \frac{1}{\log_{10} x}$ is $R^+ - \{1\}$
- 13** $\log x$ is defined for only $x > 0$
Also denominator be non-zero real number
Again square root of negative numbers is not defined.
Hence domain of $f(x) = \frac{1}{\sqrt{\log_{10} x}}$ is $1 < x < \infty$
i.e., $R^+ - (0, 1]$
- 14** $f(x) = \sqrt{3x - 4}$ ($\because \sqrt{k}$ is not defined for $k < 0$)
 $\Rightarrow 3x - 4 \geq 0$
 $\Rightarrow 3x \geq 4 \Rightarrow x \geq \frac{4}{3}$
 $\therefore \frac{4}{3} \leq x < \infty$
 $\Rightarrow x \in \left[\frac{4}{3}, \infty\right)$
- 15** $f(x) = \sqrt{4x - 3} + \sqrt{2x - 6}$
 $\therefore 4x - 3 \geq 0$ and $2x - 6 \geq 0$
 $\Rightarrow 4x \geq 3$ and $2x \geq 6$
 $\Rightarrow x \geq \frac{3}{4}$ and $x \geq 3$
 $\therefore x \in [3, \infty)$
(since if $x < 3$, $\sqrt{2x - 6}$ is not defined)
Therefore we consider only common values of domain.
- 
- Domain of $\sqrt{4x - 3} = \left[\frac{3}{4}, \infty\right)$
- 
- Domain of $\sqrt{2x - 6} = [3, \infty)$
- 16** $|x| - x > 0$ ($\because |x| - x \neq 0$)
 $\Rightarrow |x| > x$ also $|x| - x > 0$ for the condition of square root.
Since $|x|$ is positive value and $|x|, x$ are numerically equal.
Therefore $x < 0$
Hence, the domain of $f(x) = \frac{1}{\sqrt{|x| - x}}$ is $(-\infty, 0)$

17 $x \in R$

$$\begin{aligned} \text{but } x^2 - 3x + 2 &> 0 \\ \Rightarrow (x-1)(x-2) &> 0 \quad (\because x^2 - 3x + 2 \neq 0) \\ \Rightarrow x < 1 \text{ and } x > 2 \end{aligned}$$

$\therefore$ Domain of $f(x) = \frac{x}{\sqrt{x^2 - 3x + 2}}$ is $(-\infty, 1) \cup (2, \infty)$

18

$$\begin{aligned} (\sqrt{6-x} + \sqrt{x-4}) &> 0 \\ \text{Also } (6-x) &> 0 \text{ and } (x-4) > 0 \\ \Rightarrow 6 > x \text{ and } x > 4 \\ \therefore 4 < x < 6 \end{aligned}$$

$\therefore$ domain of given function is $(4, 6)$.

19

$$\begin{aligned} (1-x) > 0 &\Rightarrow 1 > x \\ \Rightarrow x < 1 &\dots(i) \\ \text{Also } \log_{10}(1-x) &\neq 0 \\ \Rightarrow 10^0 &\neq (1-x) \Rightarrow 1 \neq 1-x \\ \Rightarrow 0 &\neq -x \Rightarrow x \neq 0 \dots(ii) \\ \text{Again } (x+2) &\geq 0 \Rightarrow x \geq -2 \dots(iii) \\ \therefore x &\in [-2, 0) \cup (0, 1) \\ \text{or } x &\in [-2, 1) - \{0\} \\ \text{or } -2 &\leq x < 1, \text{ excluding zero.} \end{aligned}$$

20

$$\begin{aligned} f(x) &= \frac{\log_2(x+3)}{x^2 + 3x + 2} \\ (x+3) &> 0 \Rightarrow x > -3 \dots(i) \\ \text{Also } x^2 + 3x + 2 &\neq 0 \\ \Rightarrow (x+1)(x+2) &\neq 0 \\ \Rightarrow x &\neq -1 \text{ or } x \neq -2 \dots(ii) \\ \therefore \text{Domain of } f(x) &= (-3, \infty) - \{-1, -2\} \\ \text{or } (-3, -2) &\cup (-2, -1) \cup (-1, \infty) \end{aligned}$$

NOTE

'o' indicates absence of domain
'////' indicates presence of domain on the real number line.

21 Domain of $\sin x$ is R and Domain of $\cos x$ is also R

$\therefore$ Domain of given function is R

22

$$\begin{aligned} \log_{10} \frac{1}{|\sin x|} &\geq 0 \\ \Rightarrow 10^0 &\leq \frac{1}{|\sin x|} \Rightarrow 1 \leq \frac{1}{|\sin x|} \\ \Rightarrow |\sin x| &\leq 1 \\ \Rightarrow -1 &\leq \sin x \leq 1 \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} x &\in R \\ \text{but } \frac{1}{|\sin x|} &> 0 \\ \Rightarrow |\sin x| &> 0 \\ \Rightarrow x &\in R (\because |\sin x| \text{ is always positive}) \\ \text{Again } |\sin x| &\neq 0 \\ \Rightarrow x &\neq n\pi, n \in I \end{aligned}$$

$\therefore$ Domain of $f(x)$ is $R - \{n\pi, n \in I\}$

23

$$\begin{aligned} f(x) &= (x^2 + x + 1)^{-3/2} \\ \text{Now } x^2 + x + 1 &\neq 0 \\ \Rightarrow \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} &\neq 0 \end{aligned}$$

$\left(x + \frac{1}{2}\right)^2$ is always positive for all the values of real numbers and the whole expression is never zero and never negative for any value of R .

Hence domain of $f(x) = (x^2 + x + 1)^{-3/2}$ is R .

24

$$\begin{aligned} 5x - 6 - x^2 &> 0 \\ \Rightarrow x^2 - 5x + 6 &< 0 \\ \Rightarrow (x-2)(x-3) &< 0 \\ \Rightarrow 2 < x < 3 \\ \Rightarrow x &\in (2, 3) \end{aligned}$$

25

$$\begin{aligned} y &= \log_x 5 = \frac{\log 5}{\log x} \\ \log x &\neq 0 \Rightarrow x \neq 1 \end{aligned}$$

and

$$\begin{aligned} x &> 0 \\ \therefore x &\in (0, \infty) - \{1\} \\ \text{or } x &\in (0, 1) \cup (1, \infty) \end{aligned}$$

26

$$\begin{aligned} f(x) &= 9 - \sqrt{9 - x^2} \\ \therefore 9 - x^2 &\geq 0 \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} x^2 - 9 &\leq 0 \\ \Rightarrow (x-3)(x+3) &\leq 0 \\ \Rightarrow -3 &\leq x \leq 3 \\ \Rightarrow x &\in [-3, 3] \\ x^2 - 5x + 6 &\geq 0 \end{aligned}$$

27

$$\begin{aligned} x^2 - 5x + 6 &\geq 0 \\ \Rightarrow x^2 - 5x + 6 &\neq 0 \\ \therefore x^2 - 5x + 6 &> 0 \end{aligned}$$

$$\begin{aligned} \Rightarrow & (x-2)(x-3) > 0 \\ \Rightarrow & x < 2 \text{ or } x > 3 \\ \Rightarrow & x \in (-\infty, 2) \cup (3, \infty) \\ \text{or} & x \in (-\infty, \infty) \sim [2, 3] \end{aligned}$$

28 Clearly $\frac{x^2}{1+x^4} \geq 0$ for any $x \in R$

$$\begin{aligned} \text{Again} & (x^2-1)^2 \geq 0 \\ \Rightarrow & x^4 + 1 - 2x^2 \geq 0 \\ \Rightarrow & x^4 + 1 \geq 2x^2 \\ \therefore & \frac{x^2}{1+x^4} \leq \frac{x^2}{2x^2} \\ \Rightarrow & \frac{x^2}{1+x^4} \leq \frac{1}{2} \end{aligned}$$

Hence, the range of f is $\left[0, \frac{1}{2}\right]$.

29 Let $y = \frac{x^2-2}{x^2-3}$

$$\begin{aligned} \Rightarrow & yx^2 - 3y = x^2 - 2 \\ \Rightarrow & x^2(y-1) = 3y-2 \\ \Rightarrow & x^2 = \frac{3y-2}{y-1} \\ \therefore & \frac{3y-2}{y-1} \geq 0 \\ \therefore & y \in \left(-\infty, \frac{2}{3}\right] \cup (1, \infty) \end{aligned}$$

30 Let $y = \frac{x^2+2x+3}{x}$

$$\begin{aligned} \Rightarrow & x^2 + (2-y)x + 3 = 0 \\ \therefore & x \in R \\ \therefore & (2-y)^2 - 4 \times 1 \times 3 \geq 0 \\ \Rightarrow & y^2 - 4y + 4 - 12 \geq 0 \\ \text{or} & y^2 - 4y - 8 \geq 0 \\ \text{or} & [y - (2-2\sqrt{3})][y - (2+2\sqrt{3})] \geq 0 \\ \Rightarrow & y \in (-\infty, 2-2\sqrt{3}] \cup [2+2\sqrt{3}, \infty) \\ \text{i.e.,} & y \in R - (2-2\sqrt{3}, 2+2\sqrt{3}) \end{aligned}$$

31 Domain of $f(x) = \sqrt{x^2}$ is R i.e., $x \in (-\infty, \infty)$

Domain of $g(x) = (\sqrt{x})^2$ is $R^+ \cup \{0\}$ i.e., $x \in [0, \infty)$

$$\therefore \sqrt{x^2} \neq (\sqrt{x})^2$$

Again co-domain of $f(x) = \frac{1}{\sqrt{x^2}}$ is $R^+ \sim \{0\}$

and co-domain of $g(x) = \frac{x}{x^2}$ is R^+ i.e., $g(x) \in (0, \infty)$

$$\text{i.e., } g(x) \in (-\infty, 0) \cup (0, \infty)$$

$$\therefore \frac{1}{\sqrt{x^2}} \neq \frac{x}{x^2}$$

Again, Domain of $f(x) = \log(x-1) + \log(x-2)$ is $(2, \infty)$

and Domain of $g(x) = \log(x-1)(x-2)$ is

$$(-\infty, 1) \cup (2, \infty)$$

$$\therefore \log(x-1) + \log(x-2) \neq \log(x-1)(x-2)$$

Since in each point domain of definition of $f(x)$ is unequal, therefore the given pairs of functions are not identical.

Hint See the concept of "Equal functions".

32 Domain of $f(x) = \frac{1}{\sqrt{|x|-x}}$ is $(-\infty, 0)$

$$\text{and domain of } g(x) = \frac{1}{\sqrt{x-|x|}} \text{ is } \phi$$

$$\therefore \text{dom } f \neq \phi$$

$$\text{and } \text{dom } g = \phi$$

Hint $\phi \rightarrow$ There is no any single value of x , which satisfy the given function.

33 Dom. of $f(x) = x$ is $(-\infty, \infty)$

Dom. of $g(x) = \sqrt{x^2}$ is $(-\infty, \infty)$, but Range $\equiv [0, \infty)$

Dom. of $h(x) = \frac{x^2}{x}$ is $(-\infty, \infty) \sim \{0\}$

$\therefore$ The common domain is $[-0, \infty) \sim \{0\} = R^+$

(Since for every $f(x)$, $g(x)$, $h(x) \in R^+$ the range of each $f(x)$, $g(x)$ and $h(x)$ is same)

34 Dom. $f(x)$ is $R - \{0\}$ i.e., $(-\infty, 0) \cup (0, \infty)$

and Dom. $g(x)$ is R^+ i.e., $(0, \infty)$

$\therefore$ Common domain of $f(x)$ and $g(x)$ is $(0, \infty)$

Hence if $x \in (0, \infty)$, then $f(x) = g(x)$

35 Since $[x] \leq x$ or $x \geq [x]$

$$\Rightarrow x - [x] \geq 0$$

$$\therefore 1 + x - [x] > 0$$

$$\therefore f(g(x)) = f(1 + x - [x]) = 1$$

Hint See the definition of Signum function.

36 $f(x) = 64x^3 + \frac{1}{x^3} = \left(4x + \frac{1}{x}\right)^3 - 3.4\left(4x + \frac{1}{x}\right)$

$$\text{So } f(\alpha) = 2^3 - 3.4.2 = -16$$

$$\text{Also } f(\beta) = 2^3 - 3.4.2 = -16$$

Hence option(d).

$$\begin{aligned}
 37 \quad \text{Let } y &= \frac{x}{\sqrt{1+x^2}} \\
 \therefore f(f(x)) &= f(y) = \frac{y}{\sqrt{1+y^2}} = \frac{x/\sqrt{1+x^2}}{\sqrt{1+\left(\frac{x}{\sqrt{1+x^2}}\right)^2}} \\
 &= \frac{x}{\sqrt{1+2x^2}} = z \text{ (say)} \\
 \therefore f(f(f(x))) &= f(z) = \frac{z}{\sqrt{1+z^2}} = \frac{x/\sqrt{1+2x^2}}{\sqrt{1+\left(\frac{x}{\sqrt{1+2x^2}}\right)^2}} \\
 &= \frac{x}{\sqrt{1+3x^2}}
 \end{aligned}$$

$$\begin{aligned}
 38 \quad f(x) &= \frac{4^x}{4^x+2} \therefore f(1-x) = \frac{4^{1-x}}{4^{1-x}+2} = \frac{2}{2+4^x} \\
 \therefore f(x) + f(1-x) &= \frac{4^x}{4^x+2} + \frac{2}{4^x+2} = 1
 \end{aligned}$$

Hence (c) is correct

$$\begin{aligned}
 39 \quad f(x) &= 1 - f(1-x) \Rightarrow f(x) + f(1-x) = 1 \\
 \text{Now, } f\left(\frac{1}{999}\right) + f\left(\frac{2}{999}\right) + \dots + f\left(\frac{997}{999}\right) + f\left(\frac{998}{999}\right) \\
 &= \left[f\left(\frac{1}{999}\right) + f\left(\frac{998}{999}\right) \right] + \left[f\left(\frac{2}{999}\right) + f\left(\frac{997}{999}\right) \right] + \dots \\
 &= \left[f\left(\frac{1}{999}\right) + f\left(1 - \frac{1}{999}\right) \right] + \left[f\left(\frac{2}{999}\right) + f\left(1 - \frac{2}{999}\right) \right] + \dots \\
 &= 1 + 1 + 1 + \dots 499 \text{ times} = 499
 \end{aligned}$$

$$\begin{aligned}
 40 \quad f(3) &= 9 \\
 \therefore g(f(3)) &= g(9) = 3
 \end{aligned}$$

$$\text{Alternatively } g \circ f = g(f(x)) = g(x^2) = \sqrt{x^2} = x$$

$$\begin{aligned}
 41 \quad \text{Solve through options} \\
 \text{Let us consider } f(x) &= 3x + 2 \\
 \therefore g(f(x)) &= (3x + 2)^2 - (3x + 2) + 7 \\
 \Rightarrow g \circ f(x) &= 9(x^2 + x + 1) \\
 \Rightarrow \left(\frac{1}{9}\right) g \circ f(x) &= x^2 + x + 1
 \end{aligned}$$

Hence choice (b) is correct.

$$42 \quad f(x+1) = \frac{(x+1)}{(x+1)+3} = \frac{x+1}{x+4}$$

$$\begin{aligned}
 43 \quad \frac{1}{f(x+1)} - f\left(\frac{1}{x+1}\right) &= \frac{x+4}{x+1} - \frac{1/(x+1)}{1/(x+1)+3} \\
 &= \frac{x+4}{x+1} - \frac{1/(x+1)}{(3x+4)/(x+1)} \\
 &= \frac{x+4}{x+1} - \frac{1}{(3x+4)} = \frac{3(x^2+5x+5)}{3x^2+7x+4}
 \end{aligned}$$

$$\begin{aligned}
 44 \quad f(2) &= \frac{2+1}{2-1} = 3 \Rightarrow f(f(2)) = f(3) = \frac{3+1}{3-1} = 2 \\
 f(f(f(2))) &= f(f(3)) = f(2) = \frac{2+1}{2-1} = 3 \\
 f(f(f(f(2)))) &= f(3) = \frac{3+1}{3-1} = 2 \\
 f(f(f(f(f(2))))) &= f(2) = \frac{2+1}{2-1} = 3
 \end{aligned}$$

Introductory Exercise 17.2

- 1 Choice (a) is ruled out since the given graph is the graph of GIF (greatest integer function). Again the graph of $[x]$ is parallel to the graph of $y = x$. But since in the left part (i.e., left of y -axis) all the values of y are positive which are transformed by the modulus.

Go through options and put the values (e.g., $-3, -2, -1, 0, 1, 2, 3$, etc) and then verify the correct option.

- 2 Best way is to check the option by substituting the values of x e.g., $x = -3, -2, -1, 0, 1, 2, 3, \dots$

$$\text{choice (b) is wrong since at } x = 2, f(x) = \frac{1}{4}$$

$$\text{choice (d) is wrong since at } x = 1, f(x) = 0$$

$$\text{choice (c) is wrong since at } x = 2, f(x) = \frac{1}{8}$$

$$\therefore \text{choice (a) is correct, since at } x = 1, f(x) = 1$$

$$\text{at } x = 2, f(x) = \frac{3}{2}$$

$$\text{at } x = \frac{1}{2}, f(x) = 0, \text{ etc.}$$

- 3 Choice (a) is wrong, since graph is even also at $x = 1, e^x = f(x) = 2.714$

$$\text{choice (d) is wrong since at } x = 0, f(x) = 0$$

$$\text{choice (b) is wrong since at } x = 1, f(x) = 0$$

$$\text{also at } x = 0, f(x) \text{ is not defined}$$

$$\text{choice (c) is correct, since at } x = 0, f(x) = 1$$

$$\text{at } x = 1, f(x) = 2$$

$$\text{at } x = 2, f(x) = 4$$

$$\text{at } x = -1, f(x) = 2$$

$$\text{at } x = -2, f(x) = 4$$

- 4 Choice (a) is wrong since at $x = 1$, $f(x) = 1$
 at $x = 2$, $f(x) = 6$
 choice (b) is wrong since at $x = 2$, $f(x) = 2$
 choice (d) is wrong since at $x = 0$, $f(x) = 0$
 choice (c) is correct since at $x = 0$, $f(x) = -2$
 at $x = 2$, $f(x) = 0$
 at $x = 3$, $f(x) = 4.25$
 at $x = -2$, $f(x) = 0$
 at $x = -3$, $f(x) = 4.25$

5 Algebraic solution

x^2 is always positive $\forall x \in R$

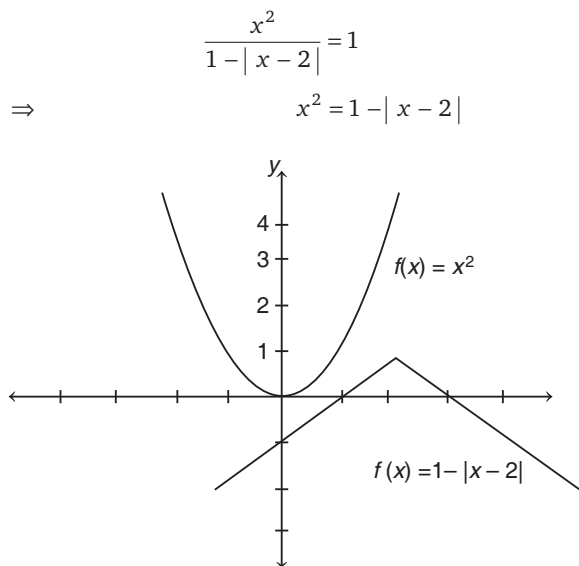
$1 - |x - 2|$ is positive, if $1 < x < 3$ ($x - 2 \neq 1$)

but when $1 < x < 3$, $x^2 \in (1, 9)$

and $(1 - |x - 2|) \in (0, 1)$

therefore there is no real solution of the given expression.

Graphical solution :

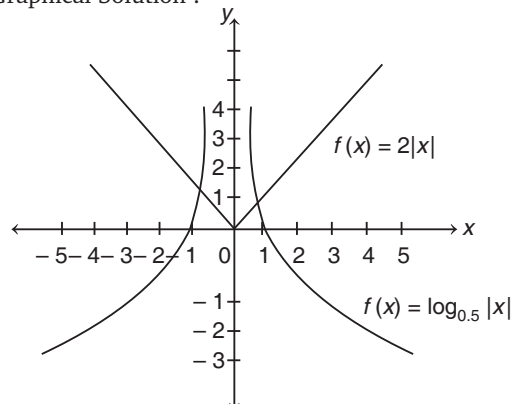


Hence it is clear from the graph that there is no real solution, since the two graphs do not cut each other at any point.

6 Algebraic solution :

$$\begin{aligned} \Rightarrow \log_{0.5} |x| &= 2|x| \\ \Rightarrow (0.5)^{2|x|} &= |x| \\ \Rightarrow \left(\frac{1}{2}\right)^{2|x|} &= |x| \\ \Rightarrow \left(\frac{1}{2}\right)^{|x|} &= |x|^{1/2} \\ \Rightarrow x &= -\frac{1}{2}, \frac{1}{2} \end{aligned}$$

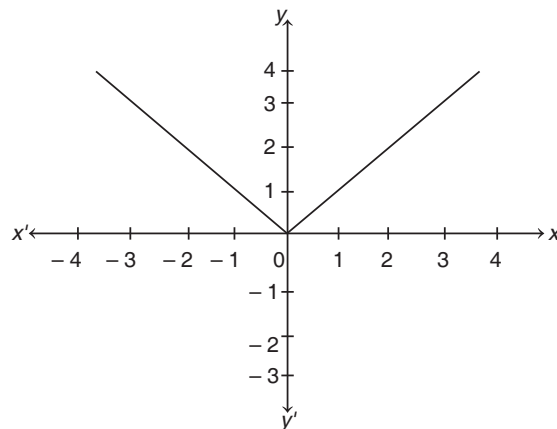
Graphical Solution :



Clearly at $x = -\frac{1}{2}$ and $x = \frac{1}{2}$ the graphs of $\log_{(0.5)} |x|$ and $2|x|$ intersect each other.

Thus there are two solutions.

7



$$f(x) = \frac{|x|^3 + |x|}{1 + x^2}$$

$$f(x) = f\left(\frac{x+8}{x-1}\right)$$

$$\Rightarrow x = \frac{x+8}{x-1}$$

$$\Rightarrow x^2 - 2x - 8 = 0 \Rightarrow x = 4, -2$$

Alternatively Go through options

9

$$g(x) = \sqrt{x^2 + 1}$$

$$f(g(x)) = (\sqrt{x^2 + 1})^2 - 1$$

$$\Rightarrow f(g(x)) = x^2$$

$$\therefore h(f(g(x))) = h(x^2) = x^2 \quad (\because h(x) = x, x \geq 0)$$

10

$$f(x) = (a - x^n)^{1/n}$$

$$\therefore f(f(x)) = (a - (f(x))^n)^{1/n}$$

$$= (a - ((a - x^n)^{1/n})^n)^{1/n}$$

$$= (a - (a - x^n))^{1/n} = (x^n)^{1/n} = x$$

$$\begin{aligned}
 11 \quad \text{At} \quad & y = f(x) = \frac{x-1}{x+1} \\
 \Rightarrow & y(x+1) = x-1 \\
 \Rightarrow & x = \frac{y+1}{1-y} \quad \dots(i)
 \end{aligned}$$

$$\text{Now, } f(f(x)) = f\left(\frac{x-1}{x+1}\right) = \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1} = -\frac{1}{x}$$

$$\begin{aligned}
 \therefore f(f(ax)) &= -\frac{1}{ax} = -\frac{1}{a\left(\frac{y+1}{1-y}\right)} \\
 &= \frac{y-1}{a(y+1)} = \frac{f(x)-1}{a(f(x)+1)}
 \end{aligned}$$

$$\begin{aligned}
 12 \quad f(x) &= \frac{x}{\sqrt{1+x^2}} \\
 \therefore f(f(x)) &= \frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\left(\frac{x}{\sqrt{1+x^2}}\right)^2}} = \frac{x}{\sqrt{1+2x^2}} \\
 \therefore f(f(f(x))) &= \frac{\frac{x}{\sqrt{1+2x^2}}}{\sqrt{1+\left(\frac{x}{\sqrt{1+2x^2}}\right)^2}} = \frac{x}{\sqrt{1+3x^2}}
 \end{aligned}$$

☞ **Alternatively** Consider some value for x , then match the correct option

$$\begin{aligned}
 f(2) &= \frac{2}{\sqrt{1+4}} = \frac{2}{\sqrt{5}} \\
 \therefore f(f(2)) &= f\left(\frac{2}{\sqrt{5}}\right) = \frac{\frac{2}{\sqrt{5}}}{\sqrt{1+\left(\frac{2}{\sqrt{5}}\right)^2}} = \frac{2}{3} \\
 \therefore f(f(f(2))) &= f\left(\frac{2}{3}\right) = \frac{\frac{2}{3}}{\sqrt{1+\left(\frac{2}{3}\right)^2}} = \frac{2}{\sqrt{13}}
 \end{aligned}$$

Now, verify the options.

Consider option (a)

$$f(f(f(2))) = \frac{2}{\sqrt{1+3(2)^2}} = \frac{2}{\sqrt{13}}$$

Hence option (a) is correct.

$$\begin{aligned}
 13 \quad f(-2.4) &= 1 + |-2.4| \\
 (\because -2.4 < -1); f(-2.4) &= 3.4 \\
 \therefore f(f(-2.4)) &= f(3.4) = [3.4] \quad (\because 3.4 > -1) \\
 &= 3
 \end{aligned}$$

14 Go through options

Let us consider $g(x) = \log x$,

$$\therefore f(x) = x^n$$

$$\begin{aligned}
 \text{and } g(f(x)) &= g(x^n) = \log x^n \\
 &= n \log x = ng(x)
 \end{aligned}$$

Hence, choice (d) is correct.

15 Since $f: N \rightarrow R$

but $0 \notin N$

$\therefore f(0)$ is not defined

Hence (d) is correct

16 Go through options.

Let us consider $f(x) = x^2 - 2$

$$\begin{aligned}
 \therefore f\left(x + \frac{1}{x}\right) &= \left(x + \frac{1}{x}\right)^2 - 2 \\
 &= x^2 + \frac{1}{x^2} + 2 - 2 = x^2 + \frac{1}{x^2}
 \end{aligned}$$

Hence choice (b) is correct.

$$17 \quad f(x) = \sqrt{x^5}$$

$$\therefore f(5x) = \sqrt{(5x)^5} = 25x^2\sqrt{5x}$$

$$18 \quad h(x) = \frac{1}{x}$$

$$g(h(x)) = g\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^2$$

$$f(g(h(x))) = f\left(\left(\frac{1}{x}\right)^2\right) = \left(4 \cdot \frac{1}{x^2}\right) - 5 = \frac{4}{x^2} - 5$$

19 Let $x = 2.6$, then $[x] - \{x\} = 2 - 3 = -1$

and $x = 3.8$, then $[x] - \{x\} = 3 - 4 = -1$

and $x = -3.6$, then $[x] - \{x\} = (-4) - (-3) = -1$

and $x = -7.3$, then $[x] - \{x\} = (-8) - (-7) = -1$

Hence (a) is the correct choice

$$20 \quad f(x) = \frac{a^x - 1}{x^n(a^x + 1)} = \frac{1}{\underbrace{x^n}_{g(x)}} \cdot \frac{a^x - 1}{\underbrace{a^x + 1}_{h(x)}}$$

(By breaking the function)

$f(x)$ is even since graph of $f(x)$ is symmetric about y-axis.

$$\text{Now, } h(-x) = \frac{a^{-x} - 1}{a^{-x} + 1} = \frac{1 - a^x}{1 + a^x} = -h(x)$$

i.e., $h(x)$ is an odd function.

Hence, $g(x) \times \text{odd function} = \text{even function}$

$\therefore g(x)$ is an odd function

($\because \text{odd} \times \text{odd} = \text{Even}$ but $\text{even} \times \text{odd} = \text{odd}$)

$\therefore g(x) = \frac{1}{x^n}$ is an odd function, which is positive only when

$$n = -\frac{1}{3}$$

$$\begin{aligned}
 21 \quad & f(x) = \ln(x + \sqrt{x^2 + 1}) \\
 \therefore & f(-x) = \ln(-x + \sqrt{(-x)^2 + 1}) \\
 & = \ln(-x + \sqrt{x^2 + 1})
 \end{aligned}$$

Here it is not obvious whether f is an odd or even function. So in such case we check for

$$\begin{aligned}
 f(x) + f(-x) &= 0 \Rightarrow f \text{ is an odd function} \\
 f(x) - f(-x) &= 0 \Rightarrow f \text{ is an even function}
 \end{aligned}$$

$$\begin{aligned}
 \text{Now, } f(x) + f(-x) &= \ln(x + \sqrt{x^2 + 1}) \\
 & \quad + \ln(-x + \sqrt{x^2 + 1}) \\
 &= \ln((\sqrt{x^2 + 1} + x)(\sqrt{x^2 + 1} - x)) \\
 &= \ln(x^2 + 1 - x^2) \\
 & \quad (\because \ln x + \ln y = \ln x \cdot y) \\
 &= \ln 1 = 0 \quad (\because \ln 1 = 0)
 \end{aligned}$$

Hence, f is an odd function.

$$\begin{aligned}
 23 \quad & f(x) = \frac{a^x + 1}{a^x - 1} \\
 \therefore & f(-x) = \frac{a^{-x} + 1}{a^{-x} - 1} = \frac{1 + a^x}{1 - a^x} \\
 \Rightarrow & f(x) \neq f(-x), \text{ hence } f(x) \text{ is not an even function} \\
 \text{Again} \quad & f(x) = \frac{a^x - a^{-x}}{a^x + a^{-x}} \\
 \therefore & f(-x) = \frac{a^{-x} - a^x}{a^{-x} + a^x} \\
 \therefore & f(x) \neq f(-x), \text{ hence } f(x) \text{ is not an even function}
 \end{aligned}$$

$$\begin{aligned}
 \text{Again} \quad & f(x) = x \cdot \frac{a^x + 1}{a^x - 1} \\
 \therefore & f(-x) = (-x) \cdot \frac{a^{-x} + 1}{a^{-x} - 1} = (-x) \cdot \frac{\frac{1 + a^x}{a^x}}{\frac{1 - a^x}{a^x}} \\
 &= (-x) \cdot \frac{1 + a^x}{1 - a^x} = (-x) \cdot \frac{1 + a^x}{-(a^x - 1)} \\
 &= x \cdot \frac{1 + a^x}{a^x - 1} = x \cdot \frac{a^x + 1}{a^x - 1}
 \end{aligned}$$

$$\therefore f(x) = f(-x)$$

Hence, $f(x)$ is an even function.

$$\begin{aligned}
 24 \quad & f(x) = \ln\left(\frac{1 + x^3}{1 - x^3}\right) \quad (\ln \rightarrow \text{natural logarithm}) \\
 \therefore & f(-x) = \ln\left(\frac{1 + (-x)^3}{1 - (-x)^3}\right) = \ln\left(\frac{1 - x^3}{1 + x^3}\right)
 \end{aligned}$$

$$\begin{aligned}
 \therefore -f(-x) &= -\ln\left(\frac{1 - x^3}{1 + x^3}\right) \\
 &= \ln\left(\frac{1 - x^3}{1 + x^3}\right)^{-1} = \ln\left(\frac{1 + x^3}{1 - x^3}\right)
 \end{aligned}$$

$$\Rightarrow f(x) = -f(-x)$$

Hence $f(x) = \ln\left(\frac{1 + x^3}{1 - x^3}\right)$ is an odd function.

$$25 \quad f(x) = \log\left(\frac{1-x}{1+x}\right) \text{ is an odd function.}$$

$$f(x) = x \cdot \frac{a^x + 1}{a^x - 1} \text{ is an even function.}$$

$$f(x) = \log[x + \sqrt{1 + x^2}] \text{ is an odd function.}$$

$$f(x) = \sqrt{1 + x + x^2} - \sqrt{1 - x + x^2} \text{ is an odd function}$$

$$26 \quad f(x) = \sqrt[3]{1 - x^2} + \sqrt[3]{1 + x^2}$$

$$\therefore f(-x) = \sqrt[3]{1 - (-x)^2} + \sqrt[3]{1 + (-x)^2}$$

$$\text{or } f(-x) = \sqrt[3]{1 - x^2} + \sqrt[3]{1 + x^2} \Rightarrow f(x) = f(-x)$$

Hence $f(x)$ is even function

$$27 \quad f(x) = 2^{-x^2}$$

$$\therefore f(-x) = 2^{-(-x)^2} = 2^{-x^2}$$

$$\therefore f(x) = f(-x), \text{ hence } f(x) \text{ is an even function}$$

$$\text{Again } f(x) = 2^{x-x^4} = \frac{2^x}{2^{x^4}}$$

$$\therefore f(-x) = 2^{-x-(-x)^4} = 2^{-x-x^4} = \frac{1}{2^{x+x^4}}$$

$$\therefore f(x) \neq f(-x), \text{ Hence } f(x) \text{ is not an odd function.}$$

Again $f(x) = \cos x$ is an even function

Hence (d).

$$28 \quad f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$$

$$\begin{aligned}
 \therefore f(-x) &= \frac{-x}{e^{-x} - 1} + \frac{(-x)}{2} + 1 \\
 &= \frac{-xe^x}{1 - e^x} - \frac{x}{2} + 1 = \frac{xe^x}{e^x - 1} - \frac{x}{2} + 1
 \end{aligned}$$

If $f(x) - f(-x) = 0$, then $f(x)$ is an even function.

$$\begin{aligned}
 \therefore f(x) - f(-x) &= \left(\frac{x}{e^x - 1} + \frac{x}{2} + 1\right) - \left(\frac{xe^x}{e^x - 1} - \frac{x}{2} + 1\right) \\
 &= \frac{x(1 - e^x)}{(e^x - 1)} + \left(\frac{x}{2} + \frac{x}{2}\right) + (1 - 1) \\
 &= \frac{-x(e^x - 1)}{(e^x - 1)} + x + 0 = -x + x
 \end{aligned}$$

$f(x) - f(-x) = 0$, hence the given function is an even function.

$$30 \quad f(x) = \frac{1}{1-x} \quad \dots(i)$$

$$\therefore g(x) = f(f(x)) = f\left(\frac{1}{1-x}\right)$$

$$\Rightarrow g(x) = \frac{1}{1 - \left(\frac{1}{1-x}\right)} = \frac{1-x}{-x} = \frac{x-1}{x} \quad \dots(ii)$$

$$\therefore h(x) = f(f(f(x))) = f(g(x)) = f\left(\frac{x-1}{x}\right)$$

$$\Rightarrow h(x) = \frac{1}{1 - \left(\frac{x-1}{x}\right)}$$

$$h(x) = x \quad \dots(iii)$$

$$\therefore f(x) \cdot g(x) \cdot h(x) = \frac{1}{1-x} \times \frac{(1-x)}{-x} \times x = -1$$

Introductory Exercise 17.3

$$1 \quad f(x) = x^2$$

$$f(1) = 1$$

$$\text{and } f(-1) = 1$$

$\Rightarrow f(x)$ is many-one

$$\therefore f: R \rightarrow R$$

$$(x) \quad (y)$$

but the negative values of y are not the actual output (or images) of any element of x . Hence function is into.

$$2 \quad f: R \rightarrow R^+ \cup \{0\}$$

$$f(x) = x^2; f(1) = 1$$

$$\text{and } f(-1) = 1$$

Hence $f(x)$ is many-one

$$\text{again } f: \underbrace{R}_{(x)} \rightarrow \underbrace{R^+ \cup \{0\}}_{(y)}$$

Since each element of y is the actual output of some element of x therefore function is onto.

$$3 \quad f: R^+ \rightarrow R$$

$$f(x) = x^2 \Rightarrow f(0) = 0, f(1) = 1, f(2) = 4 \text{ etc.}$$

Since non-positive values of x (i.e., input) are not in the domain therefore no two inputs produce same output hence function is one-one.

Also function is into since non-positive values of y are not the images (output) of any value of x (i.e., input)

$$4 \quad f: R^+ \rightarrow R^+$$

$$f(x) = x^2$$

$$f(1) = 1$$

$$f(2) = 4; f(3) = 9 \text{ etc.}$$

Since there are no two inputs which produce same output hence the function is one-one.

Again the function is onto since every element of y is an actual output/image of some element of x .

$$5 \quad f \text{ is not one-one as } f(0) = 0 \text{ and } f(-1) = 0$$

f is not onto as for $y = 1$, there is no $x \in R$

such that $f(x) = 1$

If there is such a $x \in R$ then $e^{|x|} - e^{-x} = e^x + e^{-x}$ clearly $x \neq 0$ ($\because$ at $x = 0$, $f(x) \neq 1$)

Again for $x > 0$, this equation gives $-e^{-x} = e^{-x}$ which is not possible

Again for $x < 0$, the above equation gives $-e^{-x} = e^x$, which is also not possible.

$$6 \quad \text{If } n(A) = m \text{ and } n(B) = n \geq m, \text{ then the number of one-one functions from } A \text{ to } B \text{ is } {}^n P_m : n > m$$

$${}^n P_m = \frac{n!}{(n-m)!}$$

$$\text{Hence } n = 5 \text{ and } m = 3$$

$\therefore$ Number of one-one function from A to B

$$= {}^5 P_3 = \frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2} = 60$$

$$7 \quad \text{Let } x, y \in A \text{ be such that } f(x) = f(y)$$

$$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$$

$$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$$

$$\Rightarrow -2y + 3y = 3x - 2x \Rightarrow x = y$$

Hence f is one-one.

Let $y \in B$, we want to get $x \in A$ such that $f(x) = y$,

$$\text{i.e., } \frac{x-2}{x-3} = y$$

$$\Rightarrow x = \frac{3y-2}{y-1} \in A, \text{ Hence } f \text{ is onto}$$

Thus f is bijective.

$$8 \quad \text{For any } x, y \in N, \text{ we have}$$

$$f(x) = f(y) \Rightarrow 2x + 3 = 2y + 3 \Rightarrow x = y$$

Also, f is not surjective

$\because 1 \in N$ does not have its pre image.

$$9 \quad \text{We have, } f(x) = (x-1)(x-2)(x-3)$$

$$\Rightarrow f(1) = f(2) = f(3) = 0$$

$\Rightarrow f(x)$ is not one-one

For every $y \in R$, there exists $x \in R$ such that

$f(x) = y$. Therefore f is onto.

Hence $f: R \rightarrow R$ is onto but not one-one.

$$10 \quad \text{Since } x - [x] = 0 \text{ for all integral values of } x.$$

Therefore, the function is many-one and therefore inverse function is not defined.

- 11** $y = x^2 + 2 \Rightarrow x^2 = y - 2$
 $\Rightarrow x = \sqrt{y - 2}$
 $\Rightarrow f^{-1}(x) = \sqrt{x - 2}$
- 12** $y = 5^x \Rightarrow \log_5 y = x \Rightarrow x = \log_5 y$
 $\Rightarrow f^{-1}(x) = \log_5 x$
- 13** $y = \frac{x}{x+1} \Rightarrow yx + y - x = 0$
 $\Rightarrow x(y - 1) = -y$
 $\Rightarrow x = -\frac{y}{y-1} = \frac{y}{1-y} \Rightarrow f^{-1}(x) = \frac{x}{1-x}$
- 14** $y = \frac{x-1}{x+1} \Rightarrow yx + y = x - 1$
 $\Rightarrow yx - x = -(y+1)$
 $\Rightarrow x(y-1) = -(y+1)$
 $\Rightarrow x = \frac{-(y+1)}{(y-1)} = \frac{y+1}{1-y} = \frac{1+y}{1-y} \Rightarrow f^{-1}(x) = \frac{1+x}{1-x}$
- 15** Let $f(x) = y = x(2-x)$
 $\Rightarrow x^2 - 2x - y = 0$

$$\Rightarrow x = \frac{2 \pm 2\sqrt{1-y}}{2} = 1 \pm \sqrt{1-y}$$

But $x < 1$
 So $x = 1 - \sqrt{1-y}$
 $\therefore f^{-1}(x) = 1 - \sqrt{1-x}$

16 $y = x^2 - x + 1$
 $\Rightarrow y = x^2 - x + \frac{1}{4} + \frac{3}{4} \Rightarrow y = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$
 $\Rightarrow y - \frac{3}{4} = \left(x - \frac{1}{2}\right)^2 \Rightarrow \left(x - \frac{1}{2}\right) = \sqrt{y - \frac{3}{4}}$
 $\Rightarrow x = \sqrt{y - \frac{3}{4}} + \frac{1}{2} \Rightarrow y = \sqrt{x - \frac{3}{4}} + \frac{1}{2}$

17 $f(x) + f^1(x) = (x^2 - x + 1) + \sqrt{x - \frac{3}{4}} + \frac{1}{2}$
 $= \left(\frac{3}{4}\right)^2 - \frac{3}{4} + 1 + \sqrt{\frac{3}{4} - \frac{3}{4}} + \frac{1}{2}$
 $= \frac{9}{16} + \frac{1}{4} + \frac{1}{2} = \frac{9+4+8}{16} = \frac{21}{16}$

✓ Level 01 Basic Level Exercise

- 1** $P(3, 5) = 3 \times 5 = 15$
 $S(5, 3) = 5 - 3 = 2$
 $D(P(3, 5), S(5, 3)) = D(15, 2) = 15 + 2 = 17$
- 2** $P(S(10, 6), 30) = P(4, 30) = 120$
- 3** $20 \# (-43) (\because 21 @ 22 = -43)$
 We cannot determine further the value of $20 \# (-43)$ since we don't know $a \# b$ takes what value in case of negative product as a, b is defined only for natural numbers.
 Hence (d)
- 4** $(a \# a) @ (a @ a)$
 Since, it is given that a and b are natural numbers. Two cases arise.
 Case I. a is even
 $a \# a = a^2 - a^2 = 0$
 $a @ a = a^2$
 $\therefore 0 @ a^2 = 0 \quad (\because 0 + a^2 \text{ is even})$
 Case II. a is odd
 $a \# a = a^2$, which is odd
 $\therefore a^2 @ a^2 = a^4 \quad (\because a^2 + a^2 \text{ will be even})$
 Hence (c) is correct.
- 5** $(p \# q) - (q @ p)$
 Since p is even and q is odd
 $p \# q = p^2 - q^2$

$q @ p = q^2 - p^2$
 $\therefore p^2 - q^2 - (q^2 - p^2)$
 $= 2p^2 - 2q^2 = 2(p^2 - q^2)$

6 $M(7, N(9, 4)) = M(7, (9^2 + 4^2 - 9.4))$
 $= M(7, 61) = 7^2 + 61^2 + 7.61 = 4197$

7 $f(x) = x^3 - x^2$
 $\therefore f(x+1) = (x+1)^3 - (x+1)^2$
 $= (x+1)^2(x+1-1) = (x+1)^2(x)$

8 $[(5 \cup 4) \vee \vee (2 \cup \cup \cup 3)] \cup (8 \vee 3) = [1 \vee \vee 8] \cup (5)$
 $= 8 \cup 5 = 3$

9 $2 \cup \cup 8 = 2 \times 8 = 16$
 and $2 \cup \cup \cup 4 = 2^4 = 16$

Hence (a) is true.

10. $a \cup b = a \vee b$ if $a \geq b$

$a \vee b = a \cup b$ if $a < b$

$\therefore a \cup b = a \vee b$ for all real values.

11. $(20 @ 4) \# 35 = 576 \# 35$
 $= (541)^2 = 292681$

12. $12 \# (14 @ (12 \# 13)) = 12 \# (14 @ 1)$
 $= 12 \# 225 = (-213)^2 = 45369$

13. $(9 \# 7) @ (21 \# 5) = 4 @ 256 = (260)^2 = 67600$

14. $(2 @ 2) + (3 \# 3) - (4 @ 4) + (5 \# 5)$

$$= 16 + 0 - 64 + 0 = -48$$

15. $\frac{\text{XAT}[\text{MAT}\{2, \text{CAT}(3, \text{XAT}[3, 2])\}, 5]}{\text{CAT}[4, \text{XAT}\{\text{MAT}(2, -5), 4\}]}$

$$= \frac{\text{XAT}[\text{MAT}\{2, \text{CAT}(3, 2)\}, 5]}{\text{CAT}[4, \text{XAT}\{1, 4\}]}$$

$$= \frac{\text{XAT}[\text{MAT}\{2, 9\}, 5]}{\text{CAT}[4, 0]}$$

$$= \frac{\text{XAT}[6, 5]}{1} = 5$$

16. $\frac{\text{XAT}(3, 5)}{\text{MAT}(3, 5)} = \frac{2}{1} = 2$ hence true

and $\frac{\text{XAT}(-3, -5)}{\text{MAT}(-3, -5)} = \frac{2}{1} = 2$ hence true

Thus, (c) is correct.

17. $r(16, 7) - 2 = 2 - 2 = 0$

18. $g(4.7) - l(2) = 5 - 2 = 3$

19. $\therefore \text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$

$$\therefore x = y \times k + r \quad (k \text{ is a quotient and } k \in I)$$

But $k = l\left(\frac{x}{y}\right)$

$$\therefore r = x - yk$$

$$\Rightarrow r = x - \left[y \cdot l\left(\frac{x}{y}\right) \right]$$

20. $2 \# ((7 @ (4 \# 5)) @ 3) = 2 \# ((7 @ 14) @ 3)$

$$= 2 \# (70 @ 3)$$

$$= 2 \# 152 = 306$$

21. (i) $\begin{cases} p \# q = p \# 2p = 5p \\ q \# p = 2p \# p = 4p \end{cases} \quad (\because p + q \text{ is even})$

(ii) $p @ q = p @ 3p = 0 \quad (\because p + 3p = 4p \text{ is even})$

(iii) $p @ q = p @ 2p \quad (p + 2p \text{ is odd})$

$$= 2p \# 4p \quad (2p + 4p = 6p \text{ is even})$$

$$= 2p + 8p$$

$$= 10p, \text{ which is even}$$

Hence (ii) and (iii) are true

Thus (d) is correct.

22. $6 @ 2 = 18 - 2 = 16$

$$12 \# 2 = 12 + 4 = 16$$

Hence (i) is true

Again $6 \# 3 = 12 @ 6 = 36 - 6 = 30$

$$(14 @ 12) + (2 @ 6) = 30 + 0 = 30$$

Hence (ii) is true

Again $(6 @ 2) + (6 \# 3) = 16 + 30 = 46$

$$(12 \# 2) + (14 @ 12) - (2 @ 6) = 16 + 30 - 0 = 46$$

Hence (iii) is also true

Thus (d) is correct option.

23. $\text{Max}[(2 \# 3) \$ (3 \# 4), (3 \# 4) \$ (4 \# 4)]$

$$= \text{Max}[(6 \$ 12), (12 \$ 33)]$$

$$= \text{Max}[181, 1234]$$

$$= |181 + 1234| = 1415$$

24. $\text{Max}[\text{Min}((1 \$ 2), (2 \$ 3)), \text{Max}((3 \# 4), (4 \# 5))]$

$$= \text{Max}[\text{Min}(6, 14), \text{Max}(12, 20)]$$

$$= \text{Max}[8, 32] = 40$$

25. $\text{Min}[(1 \# 2) \$ 3, 4 \# (5 \$ 6)] \$ \text{Max}[(6 \# 7), (7 \# 8)]$

$$= \text{Min}[(2 \$ 3), (4 \# 62)] \$ \text{Max}[42, 56]$$

$$= \text{Min}[14, 77] \$ [98]$$

$$= 63 \$ 98 = 13574$$

26. $9 \Delta 3 = 9^3 + 9 \times 3 + \frac{9}{3} = 729 + 27 + 3 = 759$

27. $3 \# 4 * 5 + 3 * 4 \# 5$

$$= 3^4 \times 5 + (3 \times 4)^5 = 81 \times 5 + (12)^5$$

$$= 405 + 248832 = 249237$$

28. $f(2 \# 3) \# g(3 \# 4) = f(17) \# g(145)$

$$= 255 \# 290$$

$$= 255^{290} + 290^{255}$$

29. $f(g(2 \# 3)) + g(f(1 \# 2)) = f(g(17)) + g(f(3))$

$$= f(34) + g(3)$$

$$= 1088 + 6 = 1094$$

30. $(5 \oplus 6) - (6 @ 3) + (2 * 4) = 91 - (-9) + 56$

$$= 91 + 9 + 56 = 156$$

31. $((1 \oplus 2) * 3) @ 20 = (7 * 3) @ 20$

$$= (420 @ 20)$$

$$= 160000 - 8400$$

$$= 151600$$

32. $(3 D 4) D (12 D 5) = \left(\frac{3}{4}\right) D \left(\frac{5}{12}\right) = \frac{\frac{5}{12}}{\frac{3}{4}} = \frac{5}{9}$

33. $(12 D 60) E (2 C 10) = \left(\frac{1}{5}\right) E \left(\frac{1}{5}\right) = \frac{1}{25}$

34. $30 + 20 - 4 \times 6 = 30 \times 20 \div 4 + 6$

$$= 30 \times 5 + 6 = 156$$

35. $50/40 - 5 + 4 \times 54$

$$= 50 - 40 \div 5 \times 4 + 54 = 50 - 8 \times 4 + 54$$

$$= 50 - 32 + 54 = 72$$

36. $(5 \wedge 5) \times (4 \wedge 4) / (3 \wedge 3) \times (2 \wedge 2) / (1 \wedge 1)$

$$= (5^5) + (4^4) - (3^3) + (2^2) - (1^1)$$

$$= 3125 + 256 - 27 + 4 - 1 = 3357$$

$$\begin{aligned}
 37 \quad & [(x^2 \times y^2) - (x + y)^2 + \{(x + y)^3 - \{(x/y)^2 \times 2 + x + y\}\}] \\
 & = [(x^2 + y^2) + (xy)^2] \times [(xy)^3 + \{(x - y)^2 + 2xy\}] \\
 & = \left[\frac{x^2 + y^2}{(xy)^2} \right] \times \left[\frac{(xy)^3}{(x - y)^2 + 2xy} \right] \\
 & = \left[\frac{(x^2 + y^2)}{(xy)^2} \times \frac{(xy)^3}{(x^2 + y^2)} \right] = xy \\
 & = x + y \text{ (In Devic Mathematics)}
 \end{aligned}$$

$$\begin{aligned}
 38 \quad & a + a + a - a + a + a / a + a - a \\
 & = a \times a \times a + a \times a \times a - a \times a + a \\
 & = a^3 + a^3 - a^2 + a = 1 - a \\
 & = \frac{1}{a} \text{ (In Devic maths)}
 \end{aligned}$$

$$\begin{aligned}
 39 \quad & [(2 \Delta 3) \Delta (-2 \Delta 3)] = [(2 - 3) \Delta (-2 + 3)] \\
 & [(-1) \Delta (1)] = -1 + 1 = 0
 \end{aligned}$$

$$\begin{aligned}
 40 \quad & \therefore a \Delta a = 0 \quad \forall a \in R \\
 \text{Given} \quad & a > 0 \\
 \text{If} \quad & b > 0 \\
 (a \Delta b) \Delta (a \Delta (-b)) &= (a - b) \Delta (a - b) \quad (\because a \times -b < 0) \\
 &= 0
 \end{aligned}$$

Again if $b < 0$, then

$$\begin{aligned}
 (a \Delta b) \Delta (a \Delta (-b)) \quad & (\because b \text{ is negative, } \therefore -b \text{ is positive}) \\
 &= (a + b) \Delta (a + b) = 0
 \end{aligned}$$

Hence, (d).

$$\begin{aligned}
 41 \quad & f(x) = x^3 + 3x^2 + 3x \\
 \therefore f(x + 1) &= (x + 1)^3 + 3(x + 1)^2 + 3(x + 1) \\
 &= x^3 + 1 + 3x(x + 1) + 3(x^2 + 1 + 2x) + 3x + 3 \\
 &= x^3 + 1 + 3x^2 + 3x + 3x^2 + 3 + 6x + 3x + 3 \\
 &= x^3 + 6x^2 + 12x + 7
 \end{aligned}$$

$$42 \quad \frac{(4 \# 2) @ (9 \# (-1))}{(2 @ 3) \# (5 @ 2)} = \frac{3 @ 1}{0 \# 9} = \frac{-2}{-1} = 2$$

43 Solve the options as per the directions.

44 Since, f_4 can assume negative values also, but f_1 , f_2 and f_3 always give positive values only.
Hence choice (b), (c) and (d) are not possible.

$$\begin{aligned}
 45 \quad & lt(8.3) - gt(5.3) + lt(6.8) + gt(9.4) \\
 &= 9 - 5 + 7 + 9 = 20
 \end{aligned}$$

$$\begin{aligned}
 46 \quad & \Delta(3, 2) - \Delta(2, 3) + gt(4.8) - lt(2.7) + \Delta(4, 5) \\
 &= 2 - 2 + 4 - 3 + 4 = 5
 \end{aligned}$$

$$\begin{aligned}
 47 \quad & a = \frac{p + q}{2}, b = \frac{lt(p) + qt(q)}{2} \\
 \text{Now,} \quad & lt p = p + k; \quad 0 \leq k \leq 1 \\
 & gt q = q - m; \quad 0 \leq m \leq 1 \\
 & \text{where } p, q \notin I
 \end{aligned}$$

Also $k + m = 1$ (Actually it is just less than 1)

$$\begin{aligned}
 \therefore (a - b) &= \frac{p + q}{2} - \frac{(p + k) + (q - m)}{2} \\
 \Rightarrow (a - b) &= \frac{p + q}{2} - \frac{p + q + k - m}{2} \\
 \Rightarrow (a - b) &= \frac{k - m}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{Now,} \quad (a - b) &= \frac{k - (1 - k)}{2} \quad (\because k + m = 1) \\
 &= k - \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{and} \quad (a - b) &= \frac{(1 - m) - m}{2} \quad (\because k + m = 1) \\
 &= \frac{1}{2} - m \\
 (a - b) &= k - \frac{1}{2}
 \end{aligned}$$

$$\Rightarrow \frac{1}{2} \leq (a - b) \leq \frac{1}{2} \quad (\because 0 \leq k \leq 1)$$

$$\text{(Similarly } -\frac{1}{2} \leq (a - b) \leq -\frac{1}{2}, \text{ since } 0 \leq m \leq 1)$$

NOTE The correct result (in practical) is $-\frac{1}{2} < (a - b) < \frac{1}{2}$ since $0 \leq k < 1$ and $0 \leq m < 1$.

$$48 \quad (a @ b)(b \# a) = \frac{a}{a + b} \times \frac{a + b}{b} = \frac{a}{b}$$

Hence (c) is correct.

$$\begin{aligned}
 49 \quad b \# a - a @ b &= \frac{(a + b)}{b} - \frac{a}{(a + b)} = \frac{(a + b)^2 - ab}{b(a + b)} \\
 &= \frac{a^2 + b^2 + 2ab - ab}{b(a + b)} = \frac{a^2 + b^2 + ab}{b(a + b)}
 \end{aligned}$$

$$\begin{aligned}
 50 \quad mn(md(-2), mx(5, mn(-2, md(5)))) \\
 &= mn(2, mx(5, mn(-2, 5))) \\
 &= mn(2, mx(5, -2)) = mn(2, 5) = 2
 \end{aligned}$$

$$\begin{aligned}
 51 \quad md(mn(-2, mx(md(5), mn(-2, -7)))) \\
 &= md(mn(-2, mx(5, -7))) \\
 &= md(mn(-2, 5)) = md(-2) = 2
 \end{aligned}$$

$$\begin{aligned}
 52 \quad ((1 @ (2 \Delta 3)) \# 4) &= ((1 @ ((2 @ 3) - (2 \# 3)) \# 4) \\
 &= ((1 @ (45 - (-3))) \# 4) \\
 &= ((1 @ 48) \# 4) \\
 &= (4704 \# 4) = 88416400
 \end{aligned}$$

$$53 \quad (1 @ 1) \Delta (2 \# 2) = 4 \Delta 0 = (4 @ 0) - (4 \# 0) = 0 - 0 = 0$$

$$54 \quad (a \Delta b) + (a \# b) = (a^2 + b^2) + (|a^2 - b^2|)$$

Case I. When $(a^2 - b^2)$ is positive

$$(a^2 + b^2) + (a^2 - b^2) = 2a^2$$

Case II. When $(a^2 - b^2)$ is negative

$$(a^2 + b^2) - (a^2 - b^2) = 2b^2$$

Hence we cannot determine the required value.

Hint $|k| \Rightarrow \pm k$

$$\begin{aligned}
 55 \quad & [(a \# b) \div (a @ b)]^2 - 2(a * b) \\
 & = [|a^2 - b^2| \div |a - b|]^2 - 2ab \quad (\because |k| \geq 0) \\
 & = (|a + b|)^2 - 2ab \\
 & = a^2 + b^2 = a \Delta b
 \end{aligned}$$

56 Consider $g(x) = e^x$ (from the given options)

$$\begin{aligned}
 \therefore \quad & e^x = g(f(x)) \\
 \Rightarrow \quad & e^x = g \log(g(x)) \\
 \Rightarrow \quad & e^x = g \log(e^x) \\
 \Rightarrow \quad & e^x = g(x) \\
 \Rightarrow \quad & e^x = e^x
 \end{aligned}$$

Hence, choice (b) is correct.

57 Going through options, we find that none of the choices (a), (b) and (c) satisfy the relation, hence (d)

58 None of the options (a), (b) and (c) satisfy the given conditions.

$$\begin{aligned}
 59 \quad & g[f(x) - f[g(x)]] = g(2x + 3) - f(9x + 6) \\
 & = [9(2x + 3) + 6] - [2(9x + 6) + 3] \\
 & = [18x + 27 + 6] - [18x + 12 + 3] = 18
 \end{aligned}$$

$$\begin{aligned}
 60 \quad & f(f(-2)) = f(8 - 6 + 1) \\
 & (\because -2 < -1) \\
 & = f(3) \\
 & = (9 + 6 + 2) \quad (\because 3 > -1) \\
 & = 17
 \end{aligned}$$

$$\begin{aligned}
 61 \quad & x^2 = 4 \Rightarrow x = \pm 2; y^2 = 9 \Rightarrow y = \pm 3 \\
 & \begin{array}{cccc} & (2, 3) & (-2, 3) & (-2, -3) & (2, -3) \\ f(x, y) & = & 5 & 1 & -5 & -1 \\ g(x, y) & = & -3 & -7 & -1 & 3 \\ F(f(x, y)) & = & 10 & 10 & -2 & -2 \\ G(g(x, y)) & = & 2 & -6 & -6 & 2 \end{array} \\
 & \text{Also, } |F(f(x, y))| = |G(g(x, y))|, \text{ which is true for } x = 2 \\
 & \text{and } y = -3
 \end{aligned}$$

$$\therefore x + y = -1$$

$$62 \text{ Let } x = 8, y = 9$$

$$\therefore [x] = 7 \text{ and } \{y\} = 10$$

$$\therefore g(x, y) = -3$$

$$\text{Again if } x = 9 \text{ and } y = 8, \text{ then}$$

$$[x] = 8 \text{ and } \{y\} = 9 \therefore g(x, y) = -1$$

Hence (c) is correct choice.

Hint There are two cases (i) when $x < y$ (ii) when $x > y$

Alternatively Let $x = n \therefore y = n + 1$

$$\therefore g(x, y) = (n - 1) - (n + 1 + 1) = -3$$

$$\text{Also, if } x = n, y = n - 1$$

$$g(x, y) = (n - 1) - (n - 1 + 1) = -1$$

Hence (c).

$$\begin{aligned}
 63 \quad & f(x, y) = x + 1 + y - 1 = x + y \\
 & g(x, y) = x - 1 - y - 1 = x - y - 2 \\
 & F(f(x, y)) = x + y + 1 - x + y + 2 + 1 = 2y + 4 \\
 & G(g(x, y)) = x + y - 1 + x - y - 2 + 1 = 2x - 2 \\
 & \text{Hence, only (a) is false.}
 \end{aligned}$$

$$\begin{aligned}
 64 \quad & F(f(x, y)) = -f(x, y) \\
 & G(f(x, y)) = -F(f(x, y)) = f(x, y) \\
 \therefore & f(G(f(1.2, 1.3)), f(F(f(0, 1))), G(f(0.8, 0.4))) \\
 & = f(f(1.2, 1.3), f(-f(0, 1)), f(0.8, 0.4)) \\
 & = f(\{2.5\}, f(-1), [1.2])) \\
 & = f(3, f(-1, 1)) = f(3, 0) \\
 & = 3, \text{ hence (c) is correct.}
 \end{aligned}$$

$$65 \quad F(f(x, y)) + G(f(x, y)) = 0$$

Hence (d) is correct.

$$\begin{aligned}
 66 \quad & f(1.1, 1.2) \times G(f(1.1, 1.2)) = f(1.1, 1.2) \times f(1.1, 1.2) \\
 & = [2.3] \times [2.3] = 2 \times 2 = 4
 \end{aligned}$$

$$\begin{aligned}
 67 \quad & f(g(x)) = f\left(x + \frac{1}{x}\right) = \frac{1}{x + \frac{1}{x}} = \frac{x}{x^2 + 1} \\
 & g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{x} + x = \frac{x^2 + 1}{x} \\
 \therefore & f(g(x)) \cdot g(f(x)) = \frac{x}{x^2 + 1} \times \frac{x^2 + 1}{x} = 1
 \end{aligned}$$

$$68 \quad g(f(x)) = g(f(2)) = g(13) = 29$$

$$\therefore f(2) = 2 \cdot (2)^2 + 7(2) - 9 = 13$$

$$\text{and } g(13) = 2 \times 13 + 3 = 29$$

$$\begin{aligned}
 69 \quad & f(f(f(f(f(2)))))) = f(f(f(f(3)))) \quad \left(\because f(2) = \frac{2+1}{2-1} = 3\right) \\
 & = f(f(f(2))) \quad \left(\because f(3) = \frac{3+1}{3-1} = 2\right) \\
 & = f(f(3)) = f(2) = 3
 \end{aligned}$$

Solutions (for Q. Nos. 70 and 71)

$$f(x) = 1 - h(x) \quad \dots(i)$$

$$g(x) = 1 - k(x) \quad \dots(ii)$$

$$h(x) = f(x) + 1 \quad \dots(iii)$$

$$j(x) = g(x) + 1 \quad \dots(iv)$$

$$k(x) = j(x) + 1 \quad \dots(v)$$

From eqs. (i) and (iii)

$$f(x) + h(x) = 1 - h(x) + f(x) + 1$$

$$\Rightarrow h(x) = 1 \quad \dots(vi)$$

$$\Rightarrow f(x) = 0 \quad \dots(vii)$$

From eqs. (ii) and (iv)

$$g(x) = 1 - k(x) = 1 - (j(x) + 1)$$

$$g(x) = -j(x) \quad \dots(viii)$$

From eqs. (iv) and (viii)

$$j(x) - g(x) = 1$$

$$\begin{aligned}
 &\text{and} & g(x) &= -j(x) \\
 \Rightarrow & & j(x) &= \frac{1}{2} & \dots(\text{ix}) \\
 \therefore & & g(x) &= -\frac{1}{2} & \dots(\text{x}) \\
 \therefore & k(x) &= j(x) + 1 &= \frac{3}{2} & \dots(\text{xi}) \\
 \text{Hence} & & f(x) &= 0 \\
 & & g(x) &= -\frac{1}{2} \\
 & & h(x) &= 1 \\
 & & j(x) &= \frac{1}{2} \\
 & & k(x) &= \frac{3}{2}
 \end{aligned}$$

Thus all the functions are constant.

$$70 \quad \frac{j(f(x)) + k(h(x))}{h(k(x)) + f(j(x))} = \frac{\frac{1}{2} + \frac{3}{2}}{1 + 0} = \frac{2}{1} = 2$$

$$71 \quad \frac{1 + \frac{1}{2} + \frac{3}{2}}{\left(0 + \frac{1}{2} + \frac{3}{2}\right)\left(0 + \frac{1}{2} + \frac{3}{2}\right)} = \frac{3}{0} = \infty$$

i.e., not defined

72 Since, odd function + odd function = odd function
and (odd function) (odd function) = even function.

73 Even (Odd) = Even and Odd (Even) = Even

74 y is maximum when $x^2 + 2 = 6 - 3x$

$$\text{i.e., } x^2 + 3x - 4 = 0 \Rightarrow x^2 + 4x - x - 4 = 0$$

$$\Rightarrow x(x + 4) - 1(x + 4) = 0$$

$$\Rightarrow x = 1 \quad \text{or} \quad x = -4$$

$\therefore$ at $x = 1, y = 3$ ($\because x > 0$)

75 $p^2 \times q^2 \times r^2$ is maximum when $p^2 = q^2 = r^2 = \frac{1}{3}$

$$\therefore (pqr)^2_{\max} = \frac{1}{27} \Rightarrow (pqr)_{\max} = \frac{1}{3\sqrt{3}}$$

$$76 \quad \frac{g(x, y, z)}{j(x, y, z)} = \frac{3}{3} = 1$$

$$77 \quad \frac{(m(x, y, z) - f(x, y, z))}{h(x, y, z) \times k(x, y, z) \times l(x, y, z)} = \frac{3 - 4}{5 \times 2 \times 1} = -\frac{1}{10}$$

$$78 \quad \frac{f(x, y, z) + h(x, y, z) - g(x, y, z)}{j(x, y, z)} = \frac{4 + 5 - 3}{3} = 2 > 1$$

$$79 \quad C(1, 2, 3) = \max(A(1, 2, 3), B(1, 2, 3)) = \max(3, 2) = 3$$

$$80 \quad D(0, 1, 2) = \min(A(0, 1, 2), B(0, 1, 2)) = \min(1, 2) = 1$$

$$81 \quad A(2, 3, 4) + B(2, 3, 4) = 5 + 2 = 7$$

$$\begin{aligned}
 82 \quad C(0, 1, D(1, 2, 3)) &= C(0, 1, \min(A(1, 2, 3), B(1, 2, 3))) \\
 &= C(0, 1, \min(3, 2)) = C(0, 1, 2) \\
 &= \max(A(0, 1, 2), B(0, 1, 2)) \\
 &= \max(1, 2) = 2
 \end{aligned}$$

$$\begin{aligned}
 83 \quad D(1, 2, C(0, 1, 2)) &= D(1, 2, 2) \quad (\because C(0, 1, 2) = 2) \\
 &= \min(A(1, 2, 2), B(1, 2, 2)) \\
 &= \min(3, 1) = 1
 \end{aligned}$$

$$84 \quad |x + 5| = 8 \Rightarrow x = 3 \quad \text{and} \quad x = -13$$

$$\text{and} \quad 9 - |x - 4| = y = f(x)$$

$$\text{for} \quad x = 3, \quad y = f(x) = 8$$

$$\text{and for } x = -13, \quad y = f(x) = -8$$

$$\text{Max. } (x \cdot y) = (-13) \times (-8) = 104$$

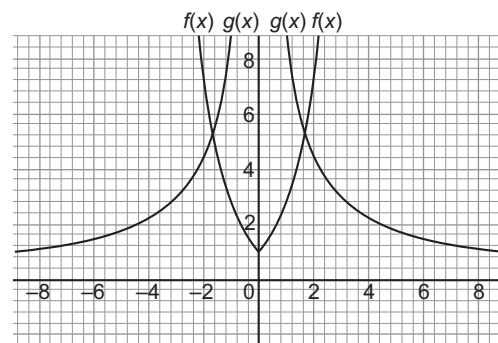
$$\begin{aligned}
 85 \quad f(100x) &= f(x + 99x) \\
 &= f(x) \cdot f(99x) = \dots = \underbrace{f(x) \cdot f(x) \dots f(x)}_{100 \text{ times}} \\
 \Rightarrow f(100x) &= f(x)^{100}
 \end{aligned}$$

Hence, choice (a) is the correct one.

$$86 \quad |x| \cdot e^{|x|} = 9 \Rightarrow e^{|x|} = \frac{9}{|x|}$$

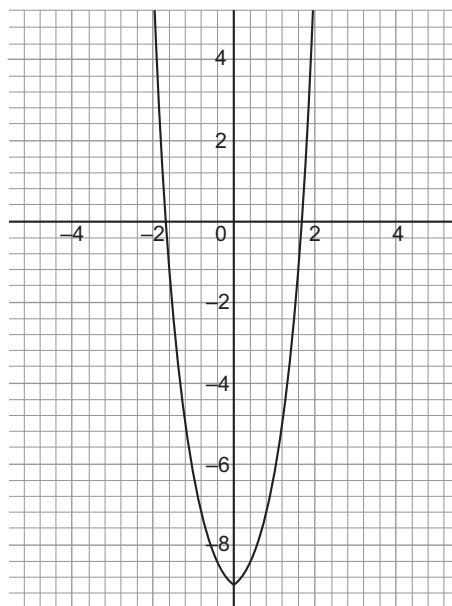
$$\text{Let } f(x) = e^{|x|} \text{ and } g(x) = \frac{9}{|x|}$$

Now, look at the graphs of $f(x)$ and $g(x)$, as given below.



Since the graphs intersect at 2 distinct points, so the number of solutions is 2. Hence, choice (c) is the correct one.

NOTE The actual graph of the given equation is shown below.



Level 02 Higher Level Exercise

$$1 \quad f(x) = \frac{a^x + a^{-x}}{2}, \quad f(y) = \frac{a^y + a^{-y}}{2}$$

$$\therefore f(x+y) = \frac{a^{x+y} + a^{-(x+y)}}{2} \quad \dots(i)$$

$$\text{and} \quad f(x-y) = \frac{a^{x-y} + a^{-(x-y)}}{2} \quad \dots(ii)$$

adding eqs. (i) and (ii), we get

$$f(x+y) + f(x-y) = \frac{a^{x+y} + a^{-(x+y)}}{2} + \frac{a^{x-y} + a^{-(x-y)}}{2}$$

$$= \frac{1}{2} [a^x (a^y + a^{-y}) + a^{-x} (a^y + a^{-y})]$$

$$= \frac{1}{2} (a^x + a^{-x}) (a^y + a^{-y}) = 2f(x)f(y)$$

$$2 \quad \left| \frac{30-x}{5} \right| < 4$$

$$\Rightarrow |30-x| < 20$$

$$\Rightarrow -20 < (30-x) < 20$$

$$\Rightarrow -20 < 30-x < 20$$

$$\Rightarrow -50 < -x < -10$$

$$\Rightarrow 50 > x > 10 \text{ or } 10 < x < 50$$

$$3 \quad |x+y| = |x-y| \Rightarrow (x+y)^2 = (x-y)^2 \Rightarrow 4xy = 0$$

$$\Rightarrow \text{either } x = 0 \text{ or } y = 0$$

If $x = 0$ and y is any real number, we have infinite possible values of y as $|0+y| = |0-y| \Rightarrow |y| = |-y|$
 Similarly $(x, 0)$ where x can be any real number also satisfies $|x+0| = |x-0|$
 So there are infinite number of solutions.

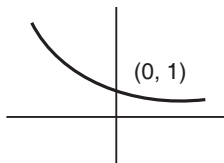
$$6 \quad e^{-x^2} = \frac{1}{e^{x^2}}$$

Since x^2 is an even function therefore e^{-x^2} is also an even function, hence it is symmetric about y -axis.
 Also it is always positive.

$$\text{and at } x = 0, \quad e^{-x^2} = 1$$

Hence (b) is not true. Thus only choice (d) is correct.

$$7 \quad y = a^x \Rightarrow \text{at } x = 0, y = 1$$



Hence (iii) statement is true.

$$\text{at } a = \frac{1}{2}, f(x) = \left(\frac{1}{2}\right)^x$$

$$\text{e.g., } a = \frac{1}{2}, \text{ then } f(x) = a^x = \left(\frac{1}{2}\right)^x$$

Hence the function is decreasing.

14 Inverse of an exponential function is a logarithmic function.

$$\therefore x = 10^y \Rightarrow \log_{10} x = y$$

graph (A) is an exponential function

graph (B) is a logarithmic function

graph (C) is also an exponential function

graph (D) represents inverse function like $\left| \frac{1}{x} \right|$

Logarithmic function is defined only for positive values hence choice (a), (c) and (d) are wrong.
 Also the inverse function (i.e., $f^{-1}(x)$ of a function $f(x)$) is symmetric about $x = y$

$$15 \quad f(0) = 1 - 0 = 1, \quad f\left(\frac{1}{2}\right) = 1 - \frac{1}{2} = \frac{1}{2} = 0.5$$

$$f(1) = 1 - 1 = 0, \quad f\left(\frac{45}{18}\right) = 2.5 - 1 = 1.5$$

$$\therefore f(0) + f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{45}{18}\right) = 1 + 0.5 + 0 + 1.5 = 3$$

$$16 \quad x = \frac{|y|}{y}, y \neq 0 \text{ if } y > 0, x = \frac{y}{y} = 1$$

$\therefore$ for all values of $y > 0, x = 1$

and for all values of $y < 0, x = -1$

17 Since the reflection of $g(x)$ is about x -axis therefore $g(x) = -f(x)$

18 Since the reflection of $g(x)$ is about y -axis

$$\therefore g(x) = f(-x)$$

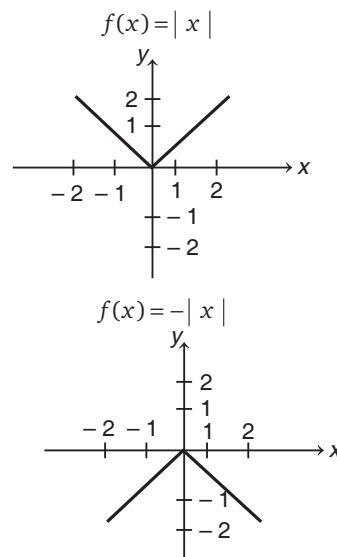
19 $f(x)$ is an even function

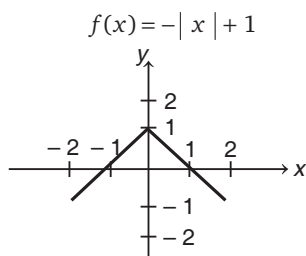
20 Since for each x there are more than 1 value of $f(x)$

21 $f(x)$ is an even function

22 $f(x)$ is an odd function

23

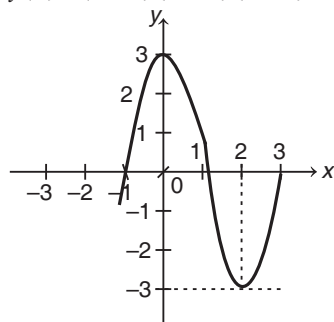




24

$$f(x) = x^3 - 3x^2 - x + 3$$

$$f(x) = (x-1)(x+1)(x-3)$$



$$\Rightarrow x = -1, 1, 3$$

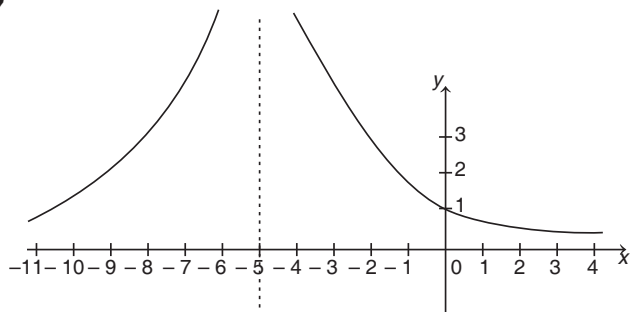
Thus the graph of $f(x)$ intersects the x -axis three times viz., at $x = -1, 1, 3$

- 25 Choice (a) $\rightarrow$ at $x = 0$ $f(x)$ is not defined
 Choice (b) $\rightarrow$ at $x = 0$ and $x = 1$, $f(x)$ is not defined
 Choice (d) $\rightarrow$ x is defined for only positive real numbers except 1.
 Hence choice (c) is correct as x^2 is always positive and $x^2 + 2 \neq 0$

- 26 Since x is always positive and y can assume both positive and negative values therefore (c) is the correct choice.

$$x|y| = 1 \Rightarrow x = \frac{1}{|y|} \quad \text{for } x > 0$$

27



28

$$f(x_2, x_3) = x_{2+3} = x_5 \quad (\because 2+3 < (3)^2)$$

$$f(x_5, x_6) = x_{5+6-3} = x_8 \quad (\because 5+6 > (3)^2)$$

$$\therefore f(f(x_2, x_3), f(x_5, x_6)) = f(x_5, x_8) = x_{5+8-3} = x_{10} \quad (\because 5+8 > (3)^2)$$

29

$$g(x_2, x_3) = x_1 \quad \left[\because \frac{2 \times 3}{5} \rightarrow m = 1 \right]$$

$$g(x_7, x_8) = x_1 \quad \left[\because \frac{7 \times 8}{5} \rightarrow m = 1 \right]$$

$$\therefore g(g(x_2, x_3), g(x_7, x_8))$$

$$= g(x_1, x_1) = x_1 \quad \left[\because \frac{1 \times 1}{5} \rightarrow m = 1 \right]$$

30 $\therefore$ at $x = 1$ $y = 0$

$$x = -1 \quad y = 0$$

$$x = 0 \quad y = -1$$

$$x = 2 \quad y = 1$$

$$x = -2 \quad y = 1$$

31

$$y = |x| + x$$

$$\text{at } x = 0 \quad y = 0$$

$$x = 1 \quad y = 2$$

$$x = 2 \quad y = 4$$

$$x = -1, \quad y = 0$$

$$x = -2, \quad y = 0$$

32

$$x = y - 1$$

$$\Rightarrow y = x + 1$$

$$\text{at } x = 0, \quad y = 1$$

$$x = -1, \quad y = 0$$

$$x = 1, \quad y = 2$$

$$x = -2, \quad y = 1$$

33 Option (a) is wrong since at $x = 1, 2$, $y = 0$

Option (c) is wrong since at $x = 0$, $y = 1$

Option (b) is correct since at $x = -2, 2$; $y = 0$

also $x = 0$, $y = 0$ etc. Also the graph is symmetric about y -axis hence $|x|$ must be appeared.

34 The graph intersects x -axis at four different points therefore choice (c) is wrong. Since graph is symmetric about y -axis therefore choice (a) is wrong since at $x = 1$, $y = 0$ but at $x = -1$, $y = 6$
 choice (d) is wrong since at $x = 0$, $y = +2$
 Hence choice (b) is correct.

$$35 \quad f(x) = \begin{cases} -x & x < 0 \\ x & x > 0 \end{cases}$$

$$g(x) = \begin{cases} x & x < 0 \\ -x & x > 0 \end{cases} \quad \therefore f(x) = -g(x)$$

$$36 \quad f(x) = \begin{cases} x & \forall x < 0 \\ x & \forall x > 0 \end{cases}$$

$$g(x) = \begin{cases} -x & x < 0 \\ x & x > 0 \end{cases}$$

$$\therefore f(x) = -g(x) \text{ for } x < 0$$

$$\text{and } f(x) = g(x) \text{ for } x > 0$$

$$37 \quad f(x) = \begin{cases} -x & x < 0 \\ -x & x > 0 \end{cases} \Rightarrow g(x) = \begin{cases} -x & x < 0 \\ x & x > 0 \end{cases}$$

$$\therefore f(x) = g(x) \text{ for } x < 0$$

$$\text{and } f(x) = -g(x) \text{ for } x > 0$$

- 38** Choice (b) is wrong since at $x = 0$, $f(x) = 0$. Choices (a) and (c) are also wrong. Now, choice (d) is correct, since

$$y = -(ax^2 + c) \quad \text{and} \quad y = (ax^2 + c)$$

These two equations give equal but opposite values.

- 39** $f(1) = 1$; $f^2(1) = f(f(1)) = 1$
 $f^3(1) = f^2(f(1)) = f^2(1) = 1$
 $f^4(1) = f^3(f(1)) = f^3(1) = 1$
 therefore $f^{11}(1) \cdot f^{33}(1) \cdot f^{55}(1) \cdot f^{77}(1) \cdot f^{99}(1)$
 $= 1 \times 1 \times 1 \times 1 \times 1 = 1$

- 40** $f^2(x) = f(f(x)) = f\left(\frac{1}{x}\right) = x$
 $f^3(x) = f(f^2(x)) = f(x) = \frac{1}{x}$
 $\therefore f^2(x) = f^4(x) = f^6(x) = f^8(x) \dots = x$
 and $f(x) = f^3(x) = f^5(x) = f^7(x) \dots = \frac{1}{x}$
 $\therefore f^{12}(x) = f^2(x) = 11 \quad \therefore f(f(x)) = 11$
 $\Rightarrow f\left(\frac{1}{x}\right) = 11 \quad \Rightarrow f(x) = \frac{1}{11}$

Solutions (for Q. Nos. 41 to 43)

Cycle	Values of		
	a	b	c
Start	1	2	3
1	2	4	6
2	4	6	10
3	8	8	16
4	16	10	26
5	32	12	44
6	64	14	78
7	128	16	144

- 41** 16 and 144
42 $b = 16$
43 Basically the module takes 7 cycles to reach step 6 (i.e., to stop), Hence the number of repetitions
 $= 7 - 1 = 6$

Solutions (for Q. Nos. 44 to 47)

Cycle	x	y	z	Sum
C_0	1	-2	3	$2 = S_0$
C_1	-5	8	-13	$-10 = S_1$
C_2	21	-34	55	$42 = S_2$
C_3	-89	144	-233	$-178 = S_3$
C_4	377	-610	987	$754 = S_4$

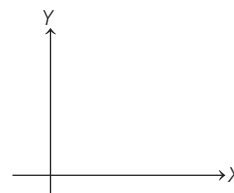
- 44** We can see from the above table that sum of each cycle is twice the each corresponding value of x in the same cycle.
45 $S_4 = 754$

- 46** $S_1, S_3, S_5, S_7 \dots$ etc are negative integers. Hence S_{13} is negative and each sum $S_1, S_2, S_3, S_4, S_5, S_6 \dots$ is an even number. Hence (d)

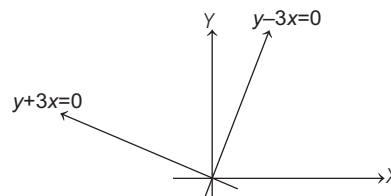
	K	L	M
initially	9	0	0
Fill (L, K)	4	5	0
Fill (M, L)	4	1	4
Empty (L, K)	5	0	4
Empty (M, L)	5	4	0

$\therefore$ L has 4 litres of water.

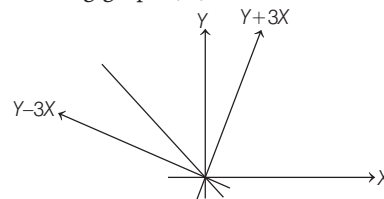
- 48** Note that we can feed the instruction of Empty (L, M) since the amount of water in L is less than the remaining capacity of M.
49 In the normal coordinate plane the equation of X-axis corresponds to $y = 0$ and the equation of Y-axis corresponds to $x = 0$.



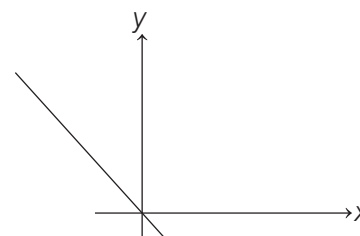
But $y - 3x = 0 \Rightarrow y = 3x$ and $y + 3x = 0 \Rightarrow y = -3x$. And these two lines are perpendicular to each other. So you can show them in the $x - y$ coordinate plane as shown below, fig. (ii).



Now, since $y - 3x = 0$ corresponds to $Y + 3X$ axis and $y + 3x = 0$ corresponds to $Y - 3X$ axis. Therefore comparing the given graph with the above graph you will get the following graph (iii).



Thus from the above graph (iii) you can filter out the requisite information as shown in the following graph (iv).



Hence, choice (a) is the correct one.

Level 03 Final Round

Solutions (for Q. Nos. 1 to 10)

P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}
0	1	1	0	-1	-1	0	1	1	0	-1	-1	0
Q_0	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	Q_{12}
0	1	1	2	3	5	8	13	21	34	55	89	144

5 $Q_{13} = 233, P_{14} = 1$

$\therefore Q_{13} + P_{14} = 234$

6 $Q_{10} + P_{10} = 55 + (-1) = 54$

7 $Q_6 = 8 \therefore Q_8 = 21$

i. e. $Q_{[Q_6]} = Q_{[8]} = 21$

8 $\therefore$ (a), (b) and (c) are wrong

9 $(Q_5)^{P_5} = (5)^{-1} = \frac{1}{5} = 0.2$

Solutions (for Q. Nos. 11 to 14)

11 $P[k(3)] = 3Q[k(3)] - 4 = 3(2R[k(3)] + R[k(6)]) - 4$
 $= 3(2(S[k(6)]) - S[k(3)] + (S[k(12)] - S[k(6)])) - 4$
 $= 3(2(22 - 13) + (51 - 22)) - 4$
 $= 3(18 + 29) - 4 = 141 - 4 = 137$

12 $S[k(1)] = 7$

$R[k(7)] = S[k(14)] - S[k(7)]$
 $= 59 - 31 = 28$

$Q[k(28)] = 2R[k(28)] + R[k(56)]$
 $= 2(S[k(56)] - S[k(28)] + S[k(112) - S[k(56)]])$
 $= 2(227 - 115) + (451 - 227)$
 $= 2(112) + (224) = 448$

$\therefore QRS[k(1)] = QR[k(7)] = Q[k(28)] = 448$

13 $R[k(5)] = S[k(10)] - S[k(5)]$

$= 43 - 19 = 24$

and $S[k(10)] = 43$

$\therefore R[k(5)] - S[k(10)] = 24 - 43 = -19$

14 $\forall x < 0, R[k(x)]$ and $S[k(x)]$ are equal to zero. Therefore the whole product will be zero.

Solutions (for Q. Nos. 15 to 20)

15 $(3x^4 + 2x^2 + 5x) + (2x^4 + 3x^3 + 7x^2)$
 $= 5x^4 + 3x^3 + 9x^2 + 5x$
 $= (5, 4, 3, 3, 9, 2, 5, 1)$

16 $(6, 5, 7, 4, 8, 3) - (3, 5, 5, 3, 7, 1)$
 $= (6x^5 + 7x^4 + 8x^3) - (3x^5 + 5x^3 + 7x)$
 $= (3x^5 + 7x^4 + 3x^3 - 7x)$
 $= (3, 5, 7, 4, 3, 3, -7, 1)$

17 $(1, 1, 2, 0) \rightarrow (x + 2)$

$\therefore (x + 2)^3 = x^3 + 6x^2 + 12x + 8$
 $= (1, 3, 6, 2, 12, 1, 8, 0)$

18 $(3, 3, -10, 2, 7, 1) \div (3, 2, -7, 1)$
 $= (3x^3 - 10x^2 + 7x) \div (3x^2 - 7x)$
 $= x(3x^2 - 10x + 7) \div x(3x - 7)$
 $= x(x - 1)(3x - 7) \div x(3x - 7)$
 $= (x - 1) = (1, 1, -1, 0)$

20 $(4x^4 + 3x^3) \times (2x^2 + x) + (2x^2 + x) - (3x^5 + 2x^4)$
 $= (8x^6 + 10x^5 + 3x^4) + (2x^2 + x - 3x^5 - 2x^4)$
 $= (8x^6 + 7x^5 + x^4 + 2x^2 + x)$
 $= (8, 6, 7, 5, 1, 4, 2, 2, 1, 1)$

Solutions (for Q. Nos. 21 to 25)

21 $h(3, 2, 8, 7) \div g(4, 7, 10, 8) = 2 \div 4 = \frac{1}{2}$

22 $h(fg(2, 5, 7, 3), 9) = h(f(2, 5, 7, 3) \times g(2, 5, 7, 3), 9)$
 $= h(7 \times 2, 9) = h(14, 9) = 5$

23 $h(h(7, 13, 5, 9), h(4, 6, 12, 14)) = h(1, 0)$
 $= \frac{1}{0}$, which is not defined

24 $A = 9, B = 20, C = 20, D = 14$

$\therefore B = C > D > A$

Hence (b) is the appropriate answer

25 $h(h(a_1, b_1, c_1, d_1), h(a_2, b_2, c_2, d_2)) = h(0, 0)$
 $= \frac{0}{0}$ is not defined, while $0 \times 0 = 0$ is defined

Solutions (for Q. Nos. 26 to 32) In case of $x > 0$, we get the following pattern.

$f(1) = b + c - 2c + a = a + b - c$

$f(2) = b + c - 4c + a + b - c = a + 2b - 4c$

$f(3) = b + c - 6c + a + 2b - 4c = a + 3b - 9c$

$f(4) = b + c - 8c + a + 3b - 9c = a + 4b - 16c$

(i.e., $f(x) = a + bx - cx^2$)

26 Hence $f(8) = a + 8b - 64c = a + 8(b - 8c)$

Hence (b) is correct.

27 $f(-19) = 2b \times (-19) + f(-(-19))$
 $= -38b + f(19)$
 $= -38b + a + 19b - 361c$
 $= a - 19b - 361c = a - 19(b + 19c)$

Hint For $x < 0$ i.e., $f(-x) = a + b(-x) - c(-x)^2$

28 $f(7) = a + 7b - 49c$

$\therefore$ when $a = 15, b = 11$ and $c = -3$
 $f(7) = 15 + 7 \times 11 - 49(-3)$
 $= 15 + 77 + 147 = 239$

29 $f(-10) = a - 10b - 100c$

$\therefore$ at $a = 10, b = -7$ and $c = 6$

$$f(-10) = 10 - 10(-7) - 100 \times 6 \\ = 10 + 70 - 600 = -520$$

30 $f(x) = a + b(x) - c(x)^2$ for every x

$\therefore 0 = 4 - 17x + 18x^2$

Now, for convenience go through options.

31 $f(x) < 0$

$\Rightarrow a + b(x) - c(x)^2 < 0$

$\Rightarrow 12 + 10(x) - 8(x)^2 < 0 \quad \dots(i)$

Now, for convenience go through options.

☞ **Alternatively** Solve the quadratic inequality (i) and then get the required set of values.

32 $f(1) = a + b - c = -a \quad (\because b = -a, c = a)$

$$\therefore f(f(1)) = f(-a) \\ = a + b(-a) - c(-a)^2 \\ = a - ab - ca^2 \\ = a + a^2 - a^3$$

Solutions (for Q. Nos. 33 to 37)

$$f(y, 0) = y + f(y-1, 0) \\ = y + (y-1) + f(y-2, 0) \\ = y - (y-1) + (y-2) + \dots + 1 + f(0, 0) \\ = \frac{y(y+1)}{2} + 1$$

and $f(0, y) = y - f(0, y-1)$

$$= y - [(y-1) - f(0, y-2)] \\ = y - (y-1) + f(0, y-2) \\ = 1 + (y-2) - f(0, y-3) \\ = 1 + (y-2) - (y-3) + f(0, y-4) \\ = 2 + f(0, y-4)$$

Thus, $f(0, y) = \begin{cases} \frac{y-1}{2}, & \text{if } y \text{ is odd} \\ \frac{y+2}{2}; & \text{if } y \text{ is even} \end{cases}$

33 $f(y_1, y_2, y_3, \dots, y_n)$ is not defined for every odd n .

Here $n = 27$

34 $f(0, 1, 0, 1) = f(0, 1) + f(1, 0) + (0 + 1 + 0 + 1)$

$$= 0 + 2 + 2 = 4$$

35 $f(8, 8, 8, 2, 2, 2) = f(8, 2) + f(8, 8, 2, 2)$

$$+ (8 + 8 + 8 + 2 + 2 + 2) \\ = f(2, 0) + f(0, 8) + [f(8, 2) + f(8, 2) \\ + (8 + 8 + 2 + 2)] + 30$$

$$= 4 + 5 + [2f(8, 2) + 20] + 30 \\ = 9 + [2(9) + 20] + 30 = 9 + [38] + 30 = 77$$

36 $f(1, 1, 3, 1, 1, 3)$

$$= f(1, 3) + f(1, 3, 1, 1) + (1 + 1 + 3 + 1 + 1 + 3) \\ = f(3, 0) + f(0, 1) + [f(1, 1) + f(3, 1) + (1 + 3 + 1 + 1)] + 10 \\ = 7 + 0 + [f(1, 0) + f(0, 1) + f(1, 0) + f(0, 3) + 6] + 10 \\ = 17 + [2 + 0 + 2 + 1 + 6] = 17 + 11 = 28$$

37 $f(9, 2, k, 0, 9, 4)$

$$= f(9, 4) + f(2, k, 0, 9) + (9 + 2 + k + 0 + 9 + 4) \\ = f(4, 0) + f(0, 9) + [f(2, 9) + f(k, 0) \\ + (2 + k + 0 + 9)] + 24 + k \\ = 11 + 4 + [f(9, 0) + f(0, 2)] + f(k, 0) \\ + (11 + k)] + 24 + k$$

$$= 15 + [48 + f(k, 0) + (11 + k)] + 24 + k \\ = 98 + 2k + f(k, 0) = 98 + 2k + \left[\frac{k(k+1)}{2} + 1 \right] = 124$$

$\Rightarrow k^2 + 5k - 50 = 0$

$\Rightarrow k = -10 \text{ or } k = 5$

Since k is a positive integer, hence $k = 5$

38 $f(128) = 1 \cdot 2^2 + 2 \cdot 2^1 + 8 \cdot 2^0 = 4 + 4 + 8 = 16$

$\therefore f(16) = 1 \cdot 2^1 + 6 \cdot 2^0$

$$= 2 + 6 = 8$$

39 $f(888222) = 8 \cdot 2^5 + 8 \cdot 2^4 + 8 \cdot 2^3 + 2 \cdot 2^2 + 2 \cdot 2^1 + 2 \cdot 2^0$

$$= 2^8 + 2^7 + 2^6 + 2^3 + 2^2 + 2^1 \\ = 2^6(7) + 14 \\ = 448 + 14 = 462$$

$f(462) = 4 \cdot 2^2 + 6 \cdot 2^1 + 2 \cdot 2^0 = 30$

$f(30) = 3 \cdot 2^1 + 0 \cdot 2^0 = 6$

Again $f(113113) = 1 \cdot 2^5 + 1 \cdot 2^4 + 3 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 3 \cdot 2^0$

$$= 32 + 16 + 24 + 4 + 2 + 3 = 81$$

$f(81) = 8 \cdot 2^1 + 1 \cdot 2^0$

$= 16 + 1 = 17$

$f(17) = 1 \cdot 2^1 + 7 \cdot 2^0$

$= 2 + 7 = 9$

$\therefore f[f(888222) + f(113113)] = f(6 + 9)$

$$= f(15) = 1 \cdot 2^1 + 5 \cdot 2^0 \\ = 2 + 5 = 7$$

40 $f(9235) = 9 \cdot 2^3 + 2 \cdot 2^2 + 3 \cdot 2^1 + 5 \cdot 2^0$

$$= 72 + 8 + 6 + 5 = 91 \\ f(91) = 9 \cdot 2^1 + 1 \cdot 2^0 = 19 \\ f(19) = 1 \cdot 2^1 + 9 \cdot 2^0 = 11s$$

$$f(11) = 1 \cdot 2^1 + 1 \cdot 2^0 = 3$$

$$f(9450) = 9 \cdot 2^3 + 4 \cdot 2^2 + 5 \cdot 2^1 + 0 \cdot 2^0$$

$$= 72 + 16 + 10 = 98$$

$$f(98) = 9 \cdot 2^1 + 8 \cdot 2^0 = 26$$

$$f(26) = 2 \cdot 2^1 + 6 \cdot 2^0 = 10$$

$$f(10) = 1 \cdot 2^1 + 0 \cdot 2^0 = 2$$

$$\therefore f(9235) + f(9450) = 3 + 2 = 5$$

$$41 \quad f(x) = e^{x^2} + e^{-x^2} \Rightarrow f(x) = e^{x^2} + \frac{1}{e^{x^2}}$$

$$g(x) = xe^{x^2} + e^{-x^2} \Rightarrow g(x) = xe^{x^2} + \frac{1}{e^{x^2}}$$

$$h(x) = x^2e^{x^2} + e^{-x^2} \Rightarrow h(x) = x^2e^{x^2} + \frac{1}{e^{x^2}}$$

$f(x)$, $g(x)$ and $h(x)$ all of them are the increasing functions in the interval $[0, 1]$, so the maximum value of f , g and h will be attained at $x = 1$.

$$\therefore f_{\max} = f(1) = e + \frac{1}{e} \text{ and } g_{\max} = g(1) = e + \frac{1}{e}$$

$$\text{and } h_{\max} = h(1) = e + \frac{1}{e}$$

$$\therefore f_{\max} = g_{\max} = h_{\max} \Rightarrow a = b = c$$

Hence, choice (d) is the answer.

Hint e , e^x , e^{x^2} all are increasing functions. So the other terms, given in the problem, are also increasing. The best way to test the increasing/decreasing nature of the function, in case you don't know differentiation method of calculus, is to substitute the values from the given interval in the stated function.

To get the maximum value replace x by 1, the highest value of the interval $[0, 1]$, as the functions are increasing.

$$42 \quad \text{Let } h(x) = (f - g)(x)$$

$$\Rightarrow h(x) = f(x) - g(x)$$

$$\Rightarrow h(x) = \begin{cases} x, & x \in \text{irrational} \\ -x, & x \in \text{rational} \end{cases}$$

It implies that the function $h(x)$ is one-one and onto.

Hence, choice (a) is the answer.

$$43 \quad f(x) = \frac{x^2 - 1}{x^2 + 1}$$

$$\Rightarrow f(x) = 1 - \frac{2}{x^2 + 1}$$

When $x = 0$, $x^2 + 1$ will be minimum, so $\frac{2}{x^2 + 1}$ will be

maximum and so $1 - \frac{2}{x^2 + 1}$ will be minimum.

$$\text{Therefore, minimum of } f(x) = 1 - \frac{2}{x^2 + 1} = 1 - 2 = -1$$

Hence, choice (c) is the answer.

$$44 \quad f(x) = \frac{x^2 + x + 2}{x^2 + x + 1} = 1 + \frac{1}{x^2 + x + 1} = 1 + \frac{1}{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

Now, when the denominator $\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$ will be highest,

$f(x)$ will be lowest which can be achieved when x is highest (close to infinity) and thus $f(x) > 1$

Similarly, when the denominator $\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$ will be

least, $f(x)$ will be greatest which can be achieved when $x = -\frac{1}{2}$, and thus $f(x) = \frac{7}{3}$

Therefore, the range of $f(x)$ is $(1, 7/3)$

Hence, choice (c) is the answer.

$$45 \quad \text{The best way is to go through the choice. Let us consider choice (b), then } f(x) = x^2 - 2$$

$$\Rightarrow f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^2 - 2 = x^2 + \frac{1}{x^2}$$

Hence, choice (b) is the answers

CHAPTER 18

Sequence, Series & Progressions

This is one of the chapters which contributes really very logical problems which can be solved easily by some innovative methods rather than conventional methods (like applying formulae etc.) Basically CAT is an exam where logic prevails over pure mathematics.

Therefore the students who use their ingenuity to solve these problems score better than those who rely on the formula based approach. In the starting of the chapter I have given the proper mathematical methods, then later on I have gradually moved towards novel techniques for faster and smarter calculation. Remember that almost 1–2 questions are necessarily asked in CAT and this chapter is also important for rest of the entrance tests eg. XAT, FMS, IIFT, CET, MAT etc. Once again I would like to advise you that maximum problems must be attempted with the help of given choices. Besides most of the problems can be solved by substituting the values of variables *e.g.*, n, x , etc. for n th term or sum of the series/progression. The above method is so smart that it reduces the time of problem solving upto 80% for some complex and typical problems.

18.1 Sequence

A set of numbers occurring in a definite order or by a rule is called a sequence. Thus any set of numbers $a_1, a_2, a_3, \dots, a_n$ such that to each positive integer n , there corresponds a number a_n , is sequence. The numbers $a_1, a_2, a_3, \dots, a_n$ are called elements (or members) of the sequence.

e.g., $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots, \frac{1}{n}$ is a sequence, since the number occur in a definite order. A sequence

can be finite or infinite according to the number of elements in it. If there are finite number of elements, then it is a finite sequence and if there are infinite number of elements in it, then it is an infinite sequence and is denoted by $\{a_n\}$.

18.2 Series

Let $u_1, u_2, u_3, \dots, u_n$ be a given sequence of n numbers, where n is a specified positive integer. Then their sum *i.e.* $u_1 + u_2 + u_3 + \dots + u_n$ is called a finite series and the numbers $u_1, u_2, u_3, \dots, u_n$ are called terms of the series.

If $S_n = u_1 + u_2 + u_3 + \dots + u_n$, then S_n is called the sum to n terms (or the sum of the first n terms) of

Chapter Checklist

- Sequence
- Series
- Progressions
- Arithmetic Progression (AP)
- Geometric Progression (GP)
- Harmonic Progression
- Arithmetico-Geometric Sequence
- CAT Test

the series and is denoted by $\sum_{r=1}^n u_r$ or Σu_r . e.g.,

$2 + 4 + 6 + \dots + 30$ is a finite series and can be denoted as $\sum_{n=1}^{15} (2n)$ ($\Sigma \rightarrow$ Sigma, summation notation)

18.3 Progressions

Sequences of special types are called progressions. The sequences $a_1, a_2, \dots, a_n \dots$ or $\{a_n\}$ is known when its n th term a_n is given. **For example :**

(i) If $a_n = 2n$, then the sequence is 2, 4, 6, 8, ...

(ii) If $a_n = 2n - 1$, then the sequence is 1, 3, 5, 7, ...

But to define a sequence we may not always have a formula for the n th term. For example, the sequence of the prime numbers is 2, 3, 5, 7, 11, 13, ... for which a formula for the n th term may not be given. In this chapter we study basically Arithmetic Progression (A.P.) and Geometric Progression (G.P.) and get a overview of Harmonic Progression (H.P.) also.

18.4 Arithmetic Progression (AP)

A sequence $a_1, a_2, a_3, \dots, a_n$ is said to be in arithmetic progression, when $a_2 - a_1 = a_3 - a_2 = \dots$ i.e., when the quantities increase or decrease continuously by a common quantity and this constant (or common) quantity is called the common difference of the AP e.g.,

Progressions	Common Difference
1, 3, 5, 7, 9, ...	$\rightarrow 2$
12, 9, 6, 3, 0, -3, -6, ...	$\rightarrow -3$
7, 12, 17, 22, ...	$\rightarrow 5$
-6, -4, -2, 0, 2, 4, 6, ...	$\rightarrow 2$

General Term (or n th term) of an AP

Let us look at the following : 1, 4, 7, 10, 13, ...

Each next term in the above sequence is being increased by 3 (which is the common difference). So you can see that from the second term onwards 3 is being added to every next term i.e.,

$$\begin{aligned} 1 &= 1 \\ 4 &= 1 + 3 \\ 7 &= 1 + 3 + 3 \\ 10 &= 1 + 3 + 3 + 3 \\ 13 &= 1 + 3 + 3 + 3 + 3 \text{ etc.} \end{aligned}$$

Let if a be the first term of the progression and d be the common difference, then the

First term	$\longrightarrow a$
Second term	$\longrightarrow a + d$
Third term	$\longrightarrow a + 2d$
Fourth term	$\longrightarrow a + 3d$
Fifth term	$\longrightarrow a + 4d$

And n th term $\longrightarrow a + (n - 1)d$

The general term or n th term is denoted by T_n .

Hence, $T_n = a + (n - 1)d$.

Also $l = a + (n - 1)d$ where l is the last term of AP

Exp. 1) Find the 11th term of -7, -2, 3, 8, 13, ...

Solution $T_{11} = -7 + (11 - 1)5 = 43$ ($\because a = -7, d = 5$ and $n = 11$)

Sum of First n terms of an AP Let us look at the following :

1, 4, 7, 10, 13, ...

Let the sum of ' n ' terms is denoted by S_n , then

$$\begin{aligned} S_1 &= 1 \\ S_2 &= 1 + 4 = 5, \quad S_3 = 1 + 4 + 7 = 12 \\ S_4 &= 1 + 4 + 7 + 10 = 22 \\ S_5 &= 1 + 4 + 7 + 10 + 13 = 35 \\ \therefore S_n &= T_1 + T_2 + T_3 + \dots + T_n \\ &= (a) + (a + d) + (a + 2d) + \dots + (a + (n - 1)d) \\ &= na + \frac{n(n - 1)d}{2} \Rightarrow S_n = \frac{n}{2} [2a + (n - 1)d] \end{aligned}$$

Alternatively $S_n = \left(\frac{a + l}{2} \right) n$,

($l \rightarrow$ last term of the progression)

NOTE In my own experience the alternative formula of S_n is better for problem solving since in this formula we have to just take the product of average of first and last term with number of terms. Also for our convenience to use this formula we can consider any term as last term.

Exp. 2) Find the sum of 11 terms of -7, -2, 3, 8, ...

Solution $a = -7$

$$l = -7 + (10) \cdot 5 = 43 \therefore S_{11} = \frac{-7 + 43}{2} \times 11$$

$$S_{11} = 198$$

NOTE Personally I hardly use the formula $S_n = \frac{n}{2} [2a + (n - 1)d]$

Since, I prefer $S_n = \left(\frac{a + l}{2} \right) n$.

Sum of First n Terms of an A.P.

Let us look at the following :

1, 4, 7, 10, 13, ...

Let the sum of ' n ' terms is denoted by S_n , then

$$\begin{aligned} S_1 &= 1 \\ S_2 &= 1 + 4 = 5 \\ S_3 &= 1 + 4 + 7 = 12 \\ S_4 &= 1 + 4 + 7 + 10 = 22 \\ S_5 &= 1 + 4 + 7 + 10 + 13 = 35 \\ \therefore S_n &= T_1 + T_2 + T_3 + \dots + T_n \\ &= (a) + (a + d) + (a + 2d) + \dots + (a + (n - 1)d) \\ &= na + \frac{n(n - 1)d}{2} \\ S_n &= \frac{n}{2} [2a + (n - 1)d] \end{aligned}$$

Arithmetic Mean (AM)

When three quantities are in AP, the middle one is called arithmetic mean (A.M.) of the other two.

Let a, m, b are in AP, then A.M. of a and b i.e., $m = \frac{a+b}{2}$

$$\therefore \text{A.M.} = \frac{a+b}{2}$$

In general, if $a, m_1, m_2, m_3, \dots, m_n, b$ are in AP then the intermediate numbers $m_1, m_2, m_3, \dots, m_n$ are called the n arithmetic means between a and b also the AM of $a_1, a_2, a_3, \dots, a_n$, which are in AP is

$$\text{AM} = \frac{(a_1 + a_2 + a_3 + \dots + a_n)}{n}$$

How to Insert n Arithmetic Means Between a and b If a and b are the first and last term respectively between which we have to insert n arithmetic means $m_1, m_2, m_3, \dots, m_n$,

then common difference $d = \frac{b-a}{n+1}$. Thus, $m_1 = a + \left(\frac{b-a}{n+1}\right)$

$$\Rightarrow m_2 = a + 2\left(\frac{b-a}{n+1}\right) \Rightarrow m_3 = a + 3\left(\frac{b-a}{n+1}\right)$$

.....

.....

$$m_n = a + n\left(\frac{b-a}{n+1}\right)$$

Exp. 3) Insert 3 arithmetic means between 5 and 21.

Solution $d = \frac{21-5}{3+1} = 4$

$\therefore$ First A.M. $= a + d = 5 + 4 = 9$

Second A.M. $= a + 2d = 5 + 2(4) = 13$

Third A.M. $= a + 3d = 5 + 3(4) = 17$

Hence the new sequence is 5, 9, 13, 17, 21, where all the terms are in AP

Properties of Arithmetic Progressions (AP)

1. If a certain number is added or subtracted to each term of a given AP, then the resulting sequence is also an AP and it has the same common difference as that of the given AP

e.g., 1, 4, 7, 10, 13, 16, ..., then

3, 6, 9, 12, 15, 18, ... is also an AP

and 2, 5, 8, 11, 14, 17, ... is also an AP

In the second sequence each term is increased by 2 than that of the first sequence and each term of the third sequence is decreased by 1 than that of second sequence.

2. If each term of an AP is multiplied by a fixed constant (or divided by a non-zero fixed constant), then the resulting sequence is also an AP e.g., 6, 12, 18, 24, 30, 36, ..., then 9, 18, 27, 36, 45, 54, ... is also an AP and 3, 6, 9, 12, 15, 18, ... is also an AP.

In the second sequence each term is $3/2$ times that of the first sequence and in the third sequence each term is half of the each term of the first sequence.

3. If $m_1, m_2, m_3, \dots, m_k$ and $n_1, n_2, n_3, \dots, n_k$ are in AP, then $(m_1 + n_1), (m_2 + n_2), (m_3 + n_3), \dots, (m_k + n_k)$ are also in AP

e.g., $S_1 = 3, 7, 11, 15, 19, \dots$ and $S_2 = 5, 11, 17, 23, 29, \dots$,

then $S_1 + S_2 = (3+5), (7+11), (11+17), (15+23), (19+29) \dots \Rightarrow S_1 + S_2 = 8, 18, 28, 38, 48, \dots$ is also an AP

4. If $m_1, m_2, m_3, \dots, m_k$ and $n_1, n_2, n_3, \dots, n_k$ are in AP, then $(m_1 - n_1), (m_2 - n_2), (m_3 - n_3), \dots, (m_k - n_k)$ are also in AP e.g., $S_1 = 7, 12, 17, 22, 27, \dots$, $S_2 = 4, 13, 22, 31, 40, \dots$. Then $S_1 - S_2 = (7-4), (12-13), (17-22), (22-31), (27-40) \dots$

$\Rightarrow S_1 - S_2 = 3, -1, -5, -9, -13, \dots$ is also an AP

5. If there are odd number of terms in an AP as $a_1, a_2, a_3, \dots, a_n$, then the middle most term is the A.M. of the given sequence. e.g., 4, 10, 16, 22, 28

Then, $\frac{4+10+16+22+28}{5} = 16$

Also $\frac{4+28}{2} = \frac{10+22}{2} = 16$ (A.M.)

(It is the most important point for logical problems.)

6. If there are even number of terms in an AP as $a_1, a_2, a_3, \dots, a_n$, then

$$a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = a_4 + a_{n-3} \text{ etc.}$$

e.g., 4, 10, 16, 22, 28, 34, 40, 46

Then, $4 + 46 = 10 + 40 = 16 + 34 = 22 + 28$

also $\frac{4+46}{2} = \frac{10+40}{2} = \frac{16+34}{2} = \frac{22+28}{2} = 25$ (A.M.)

7. The sum of an AP with n terms is given by

$$S_n = n(\text{A.M.})$$

e.g. (1) 3, 7, 11, 15, 19, then $S_5 = 5 \times 11 = 55$

(2) 5, 8, 11, 14, 17, 20

$$\therefore S_6 = 6 \times \frac{25}{2} = 75$$

8. If we have to take three terms in AP it is convenient to take them as $(m-k), m, (m+k)$ where $(m-k), m$ and $(m+k)$ are consecutive terms of an AP and k is the common difference. Similarly, we can take five terms in AP as $(m-2k), (m-k), m, (m+k), (m+2k)$.

Exp. 4) If the sum of five terms of an AP is 75, then find the third term of the AP

Solution $(m - 2k) + (m - k) + m + (m + k) + (m + 2k) = 75$

$$\Rightarrow 5m = 75$$

$$\Rightarrow m = 15$$

Hence, the third term (*i.e.* middle most term) is 15.

NOTE In general to take odd number of terms, we assume m as the middlemost term and take other terms with common difference k , as per their position.

9. If we have to take 4 terms in AP, it is convenient to take them as $(m - 3k)$, $(m - k)$, $(m + k)$, $(m + 3k)$

where $(m - 3k)$, $(m - k)$, $(m + k)$, $(m + 3k)$ are consecutive terms of an AP and ' $2k$ ' is the common difference. Similarly, we can take six terms in AP as $(m - 5k)$, $(m - 3k)$, $(m - k)$, $(m + k)$, $(m + 3k)$, $(m + 5k)$.

Exp. 5) If the sum of 4 terms of an AP is 36, then find the A.M. of the AP

Solution $(m - 3k) + (m - k) + (m + k) + (m + 3k) = 36$

$$\Rightarrow 4m = 36$$

$$\therefore S_4 = 4m = 36$$

$$\therefore \text{A.M.} = \frac{S_4}{4} = \frac{4m}{4} = \frac{36}{4} = 9$$

10. The sum of first n natural numbers

$$\Sigma n = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

Exp. 6) Find the sum of first 10 natural numbers.

Solution $\Sigma 10 = 1 + 2 + 3 + \dots + 10 = \frac{10 \times 11}{2} = 55$

11. The sum of squares of first ' n ' natural numbers

$$\Sigma n^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Exp. 7) Find the sum of squares of first 11 natural numbers.

Solution $\Sigma n^2 = 1^2 + 2^2 + 3^2 + \dots + 11^2 = \frac{11 \times 12 \times 23}{6} = 506$

12. The sum of cubes of first ' n ' natural numbers

$$\Sigma n^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2 = [\Sigma n]^2$$

Exp. 8) Find the sum of cubes of first 10 natural numbers.

Solution $\Sigma (10)^3 = 1^3 + 2^3 + 3^3 + \dots + 10^3 = \left(\frac{10 \times 11}{2} \right)^2 = 3025$

13. The sum of first ' n ' odd numbers

$$\text{i.e., } 1 + 3 + 5 + 7 + \dots + (2n - 1) = n^2$$

Exp. 9) Find the sum of $1 + 3 + 5 + 7 + \dots + 11$

Solution $1 + 3 + 5 + 7 + \dots + 11 = (11)^2 = 121$

14. The sum of first n even numbers

$$\text{i.e., } 2 + 4 + 6 + \dots + 2n = n(n+1)$$

Exp. 10) Find the sum of first 10 even numbers.

Solution $2 + 4 + 6 + 8 \dots 20 = 10 \times 11 = 110$

Introductory Exercise 18.1

- Find the 41st term of the progression 3, 8, 13, 18, ... :
(a) 102 (b) 203
(c) 304 (d) none of these
- Find the 25th term of the AP 10, 6, 2, -2, -6, -10, ... :
(a) -86 (b) 106
(c) 96 (d) none of these
- What term of the AP 2, 5, 8, ... is 56 ?
(a) 20 (b) 21
(c) 19 (d) 15
- What is the 10th term of the sequence 2, 4, ... ?
(a) 18 (b) 20
(c) 1024 (d) can't be determined
- If the 3rd and 7th terms of an AP are 17 and 27 respectively. Find the first term of the AP :
(a) 9 (b) 12
(c) 14 (d) none of these
- Find the n th term of an AP whose 6th and 8th terms are 12 and 22 respectively :
(a) $7n - 30$ (b) $n^2 - 24$
(c) $5n - 18$ (d) none of these
- If 7 times the seventh term of an AP is equal to 11 times its eleventh term, the value of eighteenth term of the AP is
(a) 0 (b) -8 (c) 18 (d) 77
- If the p th, q th and r th terms of an AP are a , b , c respectively, then the value of $a(q - r) + b(r - p) + c(p - q)$ is :
(a) 0 (b) 1 (c) abc (d) pqr
- If the p th term of an AP is q and its q th term is p , then its m th term is :
(a) $\frac{pq}{m}$ (b) $pq - m$ (c) $p + q - m$ (d) pqm
- Find the sum of the AP 11, 13, 15, ..., 99 :
(a) 2475 (b) 2500 (c) 1122 (d) 1580

11. Find the number of terms in the AP 22, 28, 34, ..., 616
(a) 80 (b) 78 (c) 99 (d) 100
12. Find the sum of 222, 224, 226, ..., 888 :
(a) 185370 (b) 195300 (c) 183000 (d) 899000
13. If the second and seventh terms of an AP are 2 and 22 respectively. Find the sum of first 35 terms :
(a) 2310 (b) 3210
(c) 2130 (d) none of these
14. The 12th term of an AP is -13 and the sum of the first four terms of it is 24. Find the sum of its first ten terms
(a) -48 (b) -26 (c) 0 (d) 52
15. The third term of an AP is $\frac{1}{5}$ and the 5th term is $\frac{1}{3}$. Find the sum to 15 terms of the AP :
(a) $\frac{1}{15}$ (b) $\frac{3}{5}$ (c) 8 (d) 15
16. How many terms of the AP 1, 4, 7, ... are needed to give the sum 925 ?
(a) 20 (b) 22 (c) 24 (d) 25
17. How many terms of the series $20 + 16 + 12 + \dots$ amounts to 48 ?
(a) 3 (b) 5
(c) 8 (d) both a and c
18. p, q, r, s, t are first five terms of an AP such that $p + r + t = -12$ and $p \cdot q \cdot r = 8$. Find the first term of the above AP :
(a) 3 (b) 2 (c) 4 (d) -4
19. The sum of all the terms of the AP 7, 10, 13, ... / is 1242, where / is the last term of the AP Find the value of /.
(a) 67 (b) 79 (c) 85 (d) 102
20. Find the sum of all the integers between 55 and 5555 which are divisible by 7 :
(a) 678 (b) 786
(c) 876 (d) none of these
21. How many terms are there in AP whose first and fifth terms are -14 and 2 respectively and the sum is 40 :
(a) 12 (b) 10 (c) 16 (d) 8
22. The first and last terms of an AP are -7 and 233 and the sum of the AP is 9153. Find the number of terms in the AP :
(a) 83 (b) 81 (c) 49 (d) 99
23. The sum of three numbers in AP is 12 and sum of their cubes is 408. Find the product of the numbers :
(a) 17 (b) 35 (c) 28 (d) 36
24. The series of natural numbers is written as follows :

$$\begin{array}{ccccccc} & & 1 & & & & \\ & 2 & 3 & 4 & & & \\ 5 & 6 & 7 & 8 & 9 & & \\ \dots & \dots & \dots & \dots & \dots & \dots & \\ \dots & \dots & \dots & \dots & \dots & \dots & \end{array}$$
- Find the sum of the numbers in the n th row :
(a) $n^3 + 1$ (b) $(2n)^2 - 1$
(c) $n^3 + (n-1)^3$ (d) none of these
25. If you save ₹ 1 today, ₹ 2 the next day, ₹ 3 the succeeding day and so on, what will be your total savings in 365 days ?
(a) 66579 (b) 66795
(c) 56795 (d) none of these
26. The ratio of the 7th to the 3rd terms of an AP is 12 : 5, find the ratio of the 13th to 4th term :
(a) 13 : 7 (b) 4 : 3
(c) 10 : 3 (d) none of these
27. Find the sum of the first hundred even natural numbers divisible by 5 :
(a) 50500 (b) 55000
(c) 50050 (d) 50005
28. If m times the m th term of an AP is equal to n times its n th term, find $(m+n)$ th term of the AP :
(a) -1 (b) 0 (c) mn (d) 4
29. The sum of the first fifteen terms of an AP is 105 and the sum of the next fifteen terms is 780. Find the common difference of AP :
(a) 4 (b) 3 (c) 6 (d) 5
30. If the first term of an AP is 2 and the sum of the first five terms is equal to one-fourth of the sum of the next five terms, then find the 20th term ?
(a) -114 (b) -82
(c) -112 (d) none of these
31. The sum of the first six terms of an AP is 42. The ratio of the 10th term to the 30th term of AP is $\frac{1}{3}$. Find the 40th term of the AP :
(a) -60 (b) 20 (c) 39 (d) 80
32. The sum of n terms of two arithmetic series are in the ratio of $\frac{7n+1}{4n+27}$. Find the ratio of their 11th terms :
(a) 4 : 3 (b) 5 : 4
(c) 7 : 4 (d) none of a, b, c
33. The sum of three numbers in AP is 15 and sum of their squares is 93. Find the greatest number :
(a) 6 (b) 10
(c) 8 (d) 7
34. If the n th term of an AP is $4n - 1$, find the 30th term and the sum of first 30 terms.
(a) 119 and 1830 (b) 69 and 1665
(c) 119 and 1380 (d) none of these
35. The sum of n terms of a series is $3n^2 + 5n$. Find the value of n if n th term is 152 :
(a) 15 (b) 18
(c) 20 (d) 25

36. Find the number of terms of the AP 98, 91, 84, ... must be taken to give a sum of zero :
 (a) 14 (b) 15 (c) 31 (d) 29
37. What is the greatest possible sum of the AP 17, 14, 11, ... :
 (a) 51 (b) 73 (c) 57 (d) 75
38. What is the least possible sum of the AP - 23, - 19, - 15, ... :
 (a) - 3 (b) - 78 (c) - 87 (d) none
39. Find the sum of all odd numbers of four digits which are divisible by 9 :
 (a) 2784491 (b) 2478429
 (c) 2754000 (d) 2448729
40. If a, b, c be the p th, q th and r th terms of an AP, then $p(b - c) + q(c - a) + r(a - b)$ equals to :
 (a) 0 (b) 1 (c) 3 (d) $\frac{abc}{pqr}$
41. If a, b, c be respectively the sum of first p, q, r terms of an AP, then $\frac{a}{p}(q - r) + \frac{b}{q}(r - p) + \frac{c}{r}(p - q)$ equals
 (a) pqr
 (b) $ap + bq + cr$
 (c) 0
 (d) $p + q + r$
42. Divide 20 into four parts which are in AP and such that the product of the first and fourth is to the product of the second and third in the ratio 2 : 3. Find the product of the first and fourth term of the AP :
 (a) 12 (b) 16
 (c) 20 (d) 25
43. The sum of the first p terms of an AP is q and the sum of the first q terms is p . Find the sum of the first $(p + q)$ terms :
 (a) pq (b) $p - q$ (c) $-(p + q)$ (d) 0

18.5 Geometric Progression (G.P.)

Geometric Progression

A sequence $a_1, a_2, a_3, \dots, a_n$ is said to be in Geometric Progression when

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \dots r \text{ (say)}$$

and $a_1, a_2, a_3, \dots, a_n$ are all non-zero numbers and r is said to be the common ratio of the G.P. e.g.,

- (i) 2, 4, 8, 16, ... $\rightarrow r = 2$
 (ii) 1, 3, 9, 27, 81 ... $\rightarrow r = 3$
 (iii) - 3, 6, - 12, 24, - 48, 96 ... $\rightarrow r = - 2$
 (iv) $\frac{1}{2}, \frac{3}{2}, \frac{9}{2}, \frac{27}{2}$... $\rightarrow r = 3$
 (v) $k, kr, kr^2, kr^3, kr^4, \dots \rightarrow r = r$
 (vi) 64, 16, 4, 1, $\frac{1}{4}, \frac{1}{16}, \dots \rightarrow r = \frac{1}{4}$

General term (or n th term) of a G.P.

Let us look at the following 1, 3, 9, 27, 81, 243, 729, 2187, ...

Each next term in the above sequence is becoming thrice (i.e. 3 times) which is called as common ratio.

$$\begin{aligned} 1 &= 1 \\ 3 &= 1 \times 3 \\ 9 &= 1 \times 3 \times 3 \\ 27 &= 1 \times 3 \times 3 \times 3 \\ 81 &= 1 \times 3 \times 3 \times 3 \times 3 \\ 243 &= 1 \times 3 \times 3 \times 3 \times 3 \times 3 \\ 729 &= 1 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \end{aligned}$$

and n th term $= a \times r \times r \times r \times \dots \times (n - 1)$ times $T_n = a \cdot r^{(n-1)}$

$\therefore$ The general term or n th term is represented by T_n

Hence, $T_n = a \cdot r^{(n-1)}$

Also $l = a \cdot r^{(n-1)}$

where l is the last term of the G.P.

Exp. 1) Find the 7th term of the sequence 2, 10, 50, 250, ...

Solution The given sequence is a G.P. with common ratio

$$= 5 \quad \left(\because \frac{10}{2} = \frac{50}{10} = 5 \right)$$

and first term $(a) = 2$

$$\therefore T_7 = 2 \cdot (5)^6 \quad (\because T_n = a \cdot r^{n-1})$$

$$T_7 = 31250$$

Sum of First n terms of a G.P.

Let us look at the following : 3, 6, 12, 24, 48, 96, ...

Let the sum of n terms is denoted by S_n , then

$$S_1 = 3$$

$$S_2 = 3 + 6 = 9, S_3 = 3 + 6 + 12 = 21$$

$$S_4 = 3 + 6 + 12 + 24 = 45$$

$$S_5 = 3 + 6 + 12 + 24 + 48 = 93$$

$$\therefore S_n = T_1 + T_2 + T_3 + \dots + T_n$$

$$\therefore S_n = \frac{a \cdot (r^n - 1)}{(r - 1)} \text{ if } r > 1$$

$$\text{and } S_n = \frac{a \cdot (1 - r^n)}{(1 - r)} \text{ if } r < 1 \text{ also, } S_n = \frac{lr - a}{r - 1}$$

NOTE The above formulae for S_n fails when $r = 1$, as the form $\frac{0}{0}$ is undefined. However,

$$S_n = a + a + a + \dots \text{ to } n \text{ terms} = n \cdot a$$

Exp. 2) Find the sum of the first 6 terms of the G.P.

$$3, \frac{9}{2}, \frac{27}{4}, \frac{81}{8}, \dots$$

Solution Here $a = 3, r = \frac{3}{2}$ and $n = 6$, then

$$\begin{aligned} S_6 &= \frac{3 \cdot \left[\left(\frac{3}{2} \right)^6 - 1 \right]}{\left(\frac{3}{2} - 1 \right)} = \frac{3 \cdot \left(\frac{729}{64} - 1 \right)}{\frac{1}{2}} = \frac{3 \cdot \left(\frac{665}{64} \right)}{\frac{1}{2}} \\ &= \frac{3 \times 665 \times 2}{64 \times 1} = 62.34375 \end{aligned}$$

Exp. 3) Find the sum of first five terms of the G.P.

1, 2, 4, ...

Solution $S_5 = \frac{1 \cdot (2^5 - 1)}{(2 - 1)} = 31$

Geometric Mean (G.M.)

If three quantities are in G.P., the middle one is called the geometric mean (G.M.) of the other two quantities.

If a, m, b are in G.P., then m is the G.M. of a and b

and since $\frac{m}{a} = \frac{b}{m}$

$$\therefore m^2 = ab \Rightarrow m = \pm \sqrt{ab} = \pm (ab)^{1/2}$$

In general, if $a_1, a_2, a_3, \dots, a_n$ are in G.P., then

$$\text{G.M.} = (a_1 \cdot a_2 \cdot a_3 \cdot \dots \cdot a_n)^{1/n}$$

Exp. 4) Find the G.M. of 4, 12, 36, which are in G.P.

Solution G.M. of 4, 12, 36 $= (4 \times 12 \times 36)^{1/3}$
 $= (1728)^{1/3} = (12^3)^{1/3} = 12$

Exp. 5) Find the G.M. of 3, 6, 12, 24, 48.

Solution G.M. $= (3 \times 6 \times 12 \times 24 \times 48)^{1/5} = (12^5)^{1/5} = 12$

How to Insert n Geometric Means between a and b If a and b are the first and last term respectively between which we have to insert n geometric means $m_1, m_2, m_3, \dots, m_n$, then

$$\text{Common ratio, } r = \left(\frac{b}{a} \right)^{\frac{1}{n+1}}$$

$$\text{Thus, } m_1 = a \cdot \left(\frac{b}{a} \right)^{\frac{1}{n+1}}$$

$$m_2 = a \cdot \left(\frac{b}{a} \right)^{\frac{2}{n+1}}$$

$$m_3 = a \cdot \left(\frac{b}{a} \right)^{\frac{3}{n+1}}$$

... ..

$$m_n = a \cdot \left(\frac{b}{a} \right)^{\frac{n}{n+1}}$$

Exp. 6) Insert 3 geometric means between 3 and 48.

$$\text{Solution } r = \left(\frac{48}{3} \right)^{\frac{1}{3+1}} = (2^4)^{\frac{1}{4}} = 2$$

$$\begin{aligned} \therefore m_1 &= 3 \cdot 2 = 6 \\ \Rightarrow m_2 &= 3 \cdot (2)^2 = 12 \\ m_3 &= 3 \cdot (2)^3 = 24 \end{aligned}$$

Hence the new sequence is 3, 6, 12, 24, 48 where all the terms are in G.P.

Properties of Geometric Progression

1. If each term of a G.P. is multiplied or divided by a certain non-zero constant, then the resulting sequence is also a G.P. e.g., 3, 6, 12, 24, 48, 96, ... then 9, 18, 36, 72, 144, 288, ... is also a G.P. and $\frac{9}{2}, 9, 18, 36, 72, 144, \dots$ is also a G.P.

In the second sequence each term is multiplied by 3 and each term of the third sequence is divided by 2 (compare with the second sequence)

2. If $a_1, a_2, a_3, \dots$ and $b_1, b_2, b_3, \dots$ are two geometric progressions, then the sequence $a_1 b_1, a_2 b_2, a_3 b_3, \dots$ is also a G.P.
e.g., 2, 6, 18, 54, 162, ... and 1, 2, 4, 8, 16, ... then $(2 \times 1), (6 \times 2), (18 \times 4), (54 \times 8), (162 \times 16) \dots$
i.e. 2, 12, 72, 432, 2592 is also a G.P.

$$\text{(Similarly } \frac{2}{1}, \frac{6}{2}, \frac{18}{4}, \frac{54}{8}, \frac{162}{16}, \dots$$

$$\text{i.e., } 2, 3, \frac{9}{2}, \frac{27}{4}, \frac{81}{8}, \dots \text{ is also a G.P.)}$$

3. If there are odd number of terms in a G.P. as $a_1, a_2, a_3, \dots, a_n$ then the middle most term is the G.M. of the given sequence. e.g., 2, 4, 8, 16, 32 then $(2 \times 4 \times 8 \times 16 \times 32)^{1/5} = (2^{15})^{1/5} = 2^3 = 8$

$$\text{Also, } (2 \times 32)^{1/2} = (4 \times 16)^{1/2} = 8$$

NOTE In this case without knowing all the terms we can find the G.M. of the G.P.

4. If there are even number of terms in a G.P. as $a_1, a_2, a_3, \dots, a_n$, then
 $\sqrt{a_1 \cdot a_n} = \sqrt{a_2 \cdot a_{n-2}} = \sqrt{a_3 \cdot a_{n-3}} = \dots$ G.M.
5. If we have to take three terms in G.P., it is convenient to take them as $\frac{m}{k}, m, mk$; where $\frac{m}{k}, m$ and mk are consecutive terms of G.P. and k is the common ratio. Similarly we can take five terms of G.P. as $\frac{m}{k^2}, \frac{m}{k}, m, mk, mk^2$.

Exp. 7) If the product of five consecutive terms of G.P. is 1024, then find the third term of this set.

Solution $\frac{m}{k^2} \times \frac{m}{k} \times m \times mk \times mk^2 = 1024$

$$\Rightarrow m^5 = 1024 = 2^{10} = 4^5 \Rightarrow m = 4$$

Hence the third term (i.e. middle most term) is 4.

In general to take odd number of terms, we assume m as the middle most term and take other terms with common ratio k , as per their position.

6. If we have to take 4 terms in G.P. it is convenient to

take them as $\frac{m}{k^3}, \frac{m}{k}, mk, mk^3$ where $\frac{m}{k^3}, \frac{m}{k}, mk$ and

mk^3 are four consecutive terms of a G.P. and k^2 is the common ratio. Similarly we can take six terms in G.P.

as $\frac{m}{k^5}, \frac{m}{k^3}, \frac{m}{k}, mk, mk^3, mk^5$

Exp. 8) If the product of 4 terms of a G.P. is 729, then find the G.M. of the G.P.

Solution $\frac{m}{k^3} \times \frac{m}{k} \times mk \times mk^3 = 729$

$$\Rightarrow m^4 = 729 = 3^6 \Rightarrow m = 3^{6/4} = 3^{3/2} \Rightarrow m = \sqrt{27}$$

Introductory Exercise 18.2

- Find the 22nd term of the G.P. $-2, 2, -2, \dots$:
(a) -22 (b) -2
(c) 2 (d) none of these
- Find the 7th term of the series $-\frac{1}{8} + \frac{1}{4} - \frac{1}{2} + 1 \dots$:
(a) 8 (b) -16
(c) -8 (d) none of these
- The 5th, 8th and 11th terms of a G.P. are a, b, c respectively, then which one of the following is true?
(a) $2b = ac$ (b) $b^2 = ac$
(c) $a + b + c = 0$ (d) none of these
- The 5th and 12th terms of a G.P. are 32 and 4096 respectively. Find the n th term of the G.P.:
(a) 2^n (b) n^2
(c) $2n^2$ (d) none of these
- What is the least number of terms of the G.P. $5 + 10 + 20 + \dots$ whose sum would surely exceed 10^6 ?
(a) 17 (b) 18
(c) 19 (d) 21
- The A.M. of two positive numbers is 15 and their G.M. is 12. What is the smaller number?
(a) 8 (b) 12
(c) 6 (d) 24

7. If $a_1, a_2, a_3, \dots, a_n$ are in G.P. then $a_1^m, a_2^m, a_3^m, \dots, a_n^m$ are also in G.P.

e.g. 1, 2, 4, 8, 16, ... then 1, 4, 16, 64, 256, ... are also in G.P.

8. If $a_1, a_2, a_3, \dots, a_n$ are in G.P. (for every $a_i > 0$; $i \in I^+$) then $\log a_1, \log a_2, \log a_3, \dots$ are in AP

(The converse of the above is also true)

Sum of an Infinite G.P.

$$S_\infty = \frac{a}{1-r}; |r| < 1$$

where a is the first term and r is the common ratio of the G.P. of infinite terms.

Exp. 9) Find the sum of an infinite G.P. whose first term is 28 and the second term is 4.

Solution $a = 28, T_2 = ar = 4 \Rightarrow r = \frac{1}{7}$

$$\therefore S_\infty = \frac{a}{1-r} = \frac{28}{1-\frac{1}{7}} = \frac{28}{\frac{6}{7}} = \frac{28}{6} \times 7 = \frac{98}{3}$$

- The sum of 3 numbers in G.P. is 38 and their product is 1728. Find the greatest number:
(a) 24 (b) 18 (c) 16 (d) 8
- The sum of first three terms of a G.P. is to the sum of the first six terms is 125 : 152. Find the common ratio of the G.P.:
(a) $3/5$ (b) 3
(c) $2/5$ (d) $5/8$
- The sum of three numbers in G.P. is 14. If the first two terms are each increased by 1 and the third term is decreased by 1, the resulting numbers are in AP. Find the product of these three numbers:
(a) 125 (b) 64
(c) 216 (d) 124
- The third term of a G.P. is 4. The product of first five terms is:
(a) 4^3 (b) 4^4
(c) 4^5 (d) none of these
- The sum of first three terms of a G.P. is 21 and sum of their squares is 189. Find the common ratio:
(a) $\frac{1}{2}$ or 2 (b) 3 or $\frac{1}{3}$
(c) 4 or $\frac{1}{4}$ (d) none of these

12. The sum of the first and the third terms of a G.P. is 15 and that of the 5th and the 7th terms is 240. Find the 9th term
 (a) 678 (b) 786
 (c) 867 (d) 768
13. Sum of three consecutive terms in a G.P. is 42 and their product is 512. Find the largest of these numbers
 (a) 28 (b) 16
 (c) 32 (d) none of these
14. The sum of three numbers in G.P. is 70, if the two extremes be multiplied each by 4 and the mean by 5, the new numbers so formed are in AP Find the product of original numbers :
 (a) 8000
 (b) 6000
 (c) 7000
 (d) none of the above
15. The sum of four terms in G.P. is 312. The sum of first and fourth terms is 252. Find the product of second and third terms :
 (a) 500 (b) 150
 (c) 60 (d) none of these
16. A bouncing tennis ball rebounds each time to a height equal to one half the height of the previous bounce. If it is dropped from a height of 16 m, find the total distance it has travelled when it hits the ground for the 10th time :
 (a) $47\frac{15}{16}$ (b) $37\frac{5}{16}$
 (c) $67\frac{11}{16}$ (d) none of these
17. Find the sum to n terms of the series
 $7 + 77 + 777 + 7777 + \dots$
 (a) $\frac{7}{9} \{10(10^n - 1) - n\}$
 (b) $\frac{7}{9} \left\{ \frac{10}{9} (10^n - 1) - n \right\}$
 (c) $7 \left\{ \frac{10}{9} (10^n - 1) - n \right\}$
 (d) none of the above
18. Find the sum to n terms of $0.8 + 0.88 + 0.888 + \dots$
 (a) $\frac{8}{9} \left[\frac{1}{9} \{1 - (0.1)^n\} \right]$ (b) $\frac{8}{9} \left[\frac{1}{10} \{1 - (0.1)^n\} \right]$
 (c) $\frac{8}{9} \left[n - \frac{1}{9} \{1 - (0.1)^n\} \right]$ (d) none of these
19. If the p th, q th and r th terms of a G.P. be respectively a , b , and c then the value of $a^{q-r} \cdot b^{r-p} \cdot c^{p-q}$ is :
 (a) 0 (b) 1
 (c) 3 (d) $\frac{abc}{pqr}$
20. If a , b , c are respectively the x th, y th and z th terms of a G.P, then the value of
 $(y - z) \log a + (z - x) \log b + (x - y) \log c$:
 (a) 0 (b) 1
 (c) 3 (d) none of these
21. There are four numbers such that the first three of them form an Arithmetic Progression and the last three form a Geometric Progression. The sum of the first and the third is 2 and that of second and fourth is 26. What is the sum of first and fourth numbers?
 (a) $85\frac{13}{25}$ (b) $25\frac{73}{85}$
 (c) 83 (d) none of these
22. Find the sum to n terms of the series
 $1 + 3 + 7 + 15 + \dots$
 (a) $[(2^n - 1) - n]$ (b) $[(2^n - 1) + n]$
 (c) $[2(2^n - 1) - n]$ (d) none of these
23. Find the sum of $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \dots + \frac{2n-1}{2^n}$:
 (a) $\left(\frac{2n-3}{2^n} \right)$ (b) $\left(3 - \frac{2n+3}{2^n} \right)$
 (c) $\left(\frac{2n+3}{2^n} \right)$ (d) none of these
24. Find the sum to n terms of the series
 $11 + 102 + 1003 + 10004 + \dots$:
 (a) $(10^n - 1) + \frac{n(n+1)}{2}$
 (b) $\frac{10}{9} (10^n - 1) + \frac{n(n+1)}{2}$
 (c) $10^n + n^2 - 1$
 (d) none of the above
25. Find the sum to first n groups of
 $(1) + (1 + 3) + (1 + 3 + 9) + (1 + 3 + 9 + 27) + \dots$
 (a) $\frac{1}{2} (3^n - 1)$ (b) $\frac{3}{4} (3^n - 1) - \frac{n}{2}$
 (c) $n^2 + 1$ (d) none of these
26. Find the sum to n terms of the following series :
 $2 + 5 + 14 + 41 + \dots$
 (a) $\frac{3}{4} (3^{n-1}) + \frac{n}{2}$ (b) $\frac{3}{4} (3^n - 1) + \frac{n}{2}$
 (c) $(3n + 1)$ (d) none of these
27. Find the sum to n terms :
 $1 + 2x + 3x^2 + 4x^3 + \dots; x \neq 1$:
 (a) $1 + nx^n$ (b) $\frac{x(x+1)}{2} + x^n$
 (c) $\frac{1-x^n}{(1-x)^2}$ (d) $\left\{ \frac{1-x^n}{(1-x)^2} - \frac{nx^n}{(1-x)} \right\}$

28. Find the sum to n terms of $1 + 3x + 5x^2 + 7x^3 + \dots$, $x \neq 1$.
- (a) $2(1 - x^{n-1})$ (b) $\frac{2x(1 - x^{n-1})}{(1 - x^2)}$
- (c) $\frac{2x(1 - x^{n-1})}{(1 - x)^2} + \frac{1 - (2n - 1)x^n}{(1 - x)}$
- (d) none of these
29. Find the sum to first n terms :
- $$1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \dots$$
- (a) $\frac{3}{4} - \frac{3 + 2n}{4(3^{n-1})}$ (b) $\frac{9}{4} - \frac{3}{4} \left[\frac{3 + 2n}{3^n} \right]$
- (c) $2n - \left(\frac{1}{n} \right)^2$ (d) none of these
30. Find the sum to n terms of
- $$3 \cdot 2 + 5 \cdot 2^2 + 7 \cdot 2^3 + \dots$$
- (a) $2^{n+2} - 2^{n+1} - 2$ (b) $n \cdot 2^{n+2} - 2^{n+1} + 2$
- (c) $n \cdot 2^{n+2} + 2^{n-2}$ (d) none of these
31. Find the sum of the series :
- $$1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \dots \text{ to } n \text{ terms :}$$
- (a) $\frac{35}{16} + \frac{7n}{5}$ (b) $\frac{12n - 7}{16(5^{n-1})}$
- (c) $\frac{35}{16} - \left[\frac{12n + 7}{16(5^{n-1})} \right]$ (d) none of these
32. Find the sum of the series
- $$1 \cdot 3^2 + 2 \cdot 5^2 + 3 \cdot 7^2 + \dots \text{ to } 20 \text{ terms :}$$
- (a) 12896 (b) 187898
- (c) 98970 (d) 188090
33. Find the sum of the series
- $$\frac{1^3}{1} + \frac{1^3 + 2^3}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + \dots \text{ to } 16 \text{ terms}$$
- (a) 224 (b) 446
- (c) 2356 (d) none of these
34. In a set of four numbers, the first three are in G.P. and the last three are in AP with common difference 6. If the first number is the same as the fourth, find the third number :
- (a) -8 (b) 4
- (c) 2 (d) can't be determined
35. The sum of an infinite G.P. is 16 and the sum of the squares of its terms is $153\frac{3}{5}$. Find the fourth term of the progression :
- (a) $\frac{7}{16}$ (b) $\frac{5}{32}$
- (c) $\frac{3}{16}$ (d) none of these
36. If $x = 1 + a + a^2 + \dots$ to ∞ and $y = 1 + b + b^2 + \dots$ to ∞ , and $|a| < 1, |b| < 1$, then the value of :
- $$1 + ab + a^2b^2 + \dots \text{ to } \infty \text{ is :}$$
- (a) $\frac{xy}{x + y}$ (b) $\frac{x + y + 1}{xy}$
- (c) $\frac{xy}{x - y}$ (d) $\frac{xy}{x + y - 1}$
37. A person is entitled to receive an annual payment which for each year is less by one tenth of what it was for the year before. If the first payment is 100, then find the maximum possible payment which he can receive, however long he may live :
- (a) 900
- (b) 9999
- (c) 1000
- (d) none of the above
38. Find the sum of the series $\frac{1}{4} - \frac{3}{16} + \frac{9}{64} - \frac{27}{256} + \dots$
- (a) $\frac{1}{3}$ (b) $\frac{1}{4}$
- (c) $\frac{1}{7}$ (d) none of these
39. The sum of first two terms of a G.P. is $\frac{5}{3}$ and the sum to infinity of the series is 3. Find the first term :
- (a) 1 (b) $\frac{2}{3}$
- (c) 5 (d) both (a) and (c)
40. A ball is dropped from a height of 96 ft and it rebounds $\frac{2}{3}$ of the height it falls. If it continues to fall and rebound, find the total distance that the ball can travel before coming to rest.
- (a) 240 ft (b) 360 ft
- (c) 480 ft (d) none of these

18.6 Harmonic Progression

A series of quantities $a_1, a_2, a_3, a_4, \dots, a_n$ is said to be in H.P. when their reciprocals $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \dots, \frac{1}{a_n}$ are in AP

NOTE The converse is also true.

$\therefore$ If a, b, c are in H.P., then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP

Simply there is no general formula for the sum of any number of quantities in H.P. All questions in H.P. are generally solved by inverting the terms and making use of the properties of the corresponding AP

Harmonic Mean (H.M.)

When three quantities are in H.P., the middle one is called the Harmonic Mean (H.M.) between the other two.

If a, H, b are in H.P., then H is the H.M. of a, b and $\frac{1}{a}, \frac{1}{H}, \frac{1}{b}$ are in AP

$$\therefore \frac{1}{H} - \frac{1}{a} = \frac{1}{b} - \frac{1}{H} \text{ or } \frac{2}{H} = \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} \therefore H = \frac{2ab}{a+b}$$

Exp. 1) Find the 10th term of the H.P. $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \dots$

Solution Since $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots$ are in H.P.

$\therefore 1, 3, 5, 7, \dots$ are in AP

$\therefore T_{10} = 1 + 9 \times 2, T_{10} = 19$

Since 10th term of the AP is 19

therefore 10th term of the H.P. is $1/19$.

Exp. 2) If the 4th and the 7th terms of an H.P. are $1/2$ and $2/7$ respectively. Find the first term.

Solution 4th term in H.P. = $1/2$

$\therefore$ 4th term in AP = 2, 7th term in H.P. = $2/7$

$\therefore$ 7th term in AP = $7/2$

$\therefore T_4 = a + 3d = 2$ and $T_7 = a + 6d = \frac{7}{2} \Rightarrow d = \frac{1}{2}$

$\therefore a = \frac{1}{2}$ (which is the first term of the AP)

$\therefore$ the first term of H.P. = 2

18.7 Arithmetico-Geometric Sequence

If $a_1, a_2, a_3, \dots, a_n$ are in AP and $b_1, b_2, b_3, \dots, b_n$ are in G.P., then the sequence $a_1 b_1, a_2 b_2, a_3 b_3, \dots, a_n b_n$ is said to be in arithmetico-geometric sequence.

An Arithmetico-Geometric sequence is of the form $ab, (a+d)br, (a+2d)br^2, (a+3d)br^3, \dots$

$$\therefore S_n = \frac{ab}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]br^n}{1-r}$$

$$\text{and } S_\infty = \frac{ab}{1-r} + \frac{dbr}{(1-r)^2}$$

Exp. 3) Find the sum to n terms of the series

$$1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \dots$$

Solution This is an A.G. series

$$\therefore S_n = 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \dots + \frac{3n-2}{5^{n-1}}$$

$$\therefore \frac{S_n}{5} = \frac{1}{5} + \frac{4}{5^2} + \frac{7}{5^3} + \dots + \frac{3n-5}{5^{n-1}} + \frac{3n-2}{5^n}$$

subtracting we get

$$\frac{4}{5} S_n = 1 + \frac{3}{5} + \frac{3}{5^2} + \frac{3}{5^3} + \dots + \frac{3}{5^{n-1}} - \frac{3n-2}{5^n}$$

$$\text{or } \frac{4}{5} S_n = 1 + \frac{3\left(1 - \frac{1}{5^{n-1}}\right)}{1 - \frac{1}{5}} - \frac{3n-2}{5^n}$$

$$= \frac{35}{16} - \frac{12n+7}{16(5^{n-1})}$$

Exp. 4) In the above example find the sum to infinite number of terms.

$$\text{Solution } S_\infty = \frac{ab}{1-r} + \frac{dbr}{(1-r)^2} = \frac{1 \times 1}{1 - \frac{1}{5}} + \frac{3 \times 1 \times \frac{1}{5}}{\left(1 - \frac{1}{5}\right)^2}$$

$$= \frac{1}{\frac{4}{5}} + \frac{\frac{3}{5}}{\frac{16}{25}} = \frac{5}{4} + \frac{15}{16} = \frac{35}{16}$$

Introductory Exercise 18.3

1. Which one of the following is true about A.M., G.M. and H.M.

- (a) H.M. $\geq$ G.M. $\geq$ A.M.
- (b) A.M. $\geq$ H.M. $\geq$ G.M.
- (c) A.M. $\geq$ G.M. $\geq$ H.M.
- (d) G.M. $\geq$ H.M. $\geq$ A.M.

2. If a, b, c are in AP b, c, d are in G.P. and c, d, e are in H.P., then which one of the following is true :

- (a) $2c = a + e$ (b) $ae = c^2$ (c) $c = \frac{2ae}{a+e}$ (d) $ab = de$

3. If A.M. and G.M. of two numbers is 10 and 8 respectively then find their H.M. :

- (a) 9 (b) 12 (c) 6 (d) 6.4

CAT-Test

Questions Helping you bell the CAT

LEVEL 01 > BASIC LEVEL EXERCISE

- Find the number of terms in the G.P. whose first term is 3, sum is $\frac{4095}{1024}$ and the common ratio is $\frac{1}{4}$:
 (a) 4 (b) 5
 (c) 6 (d) none of these
- Find the common ratio of the G.P. whose first and last terms are 5 and $\frac{32}{625}$ respectively and the sum of the G.P. is $\frac{5187}{625}$:
 (a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{5}{3}$ (d) $\frac{4}{5}$
- Find the sum of the series $2\sqrt{3}, 2\sqrt{2}, \frac{4}{\sqrt{3}}, \dots$
 (a) $6(\sqrt{3} + \sqrt{2})$ (b) $6(\sqrt{3} - \sqrt{2})$
 (c) $\frac{6}{(\sqrt{3} + \sqrt{2})}$ (d) none of these
- The number of terms in an A.P. is even, the sum of odd terms is 63 and that of even terms is 72 and the last term exceeds the first term by 16.5. Find the number of terms :
 (a) 8 (b) 12 (c) 9 (d) 10
- There are two pockets, each containing 3 coins of different denominations, which are in A.P. and the total value of coins in each pocket is ₹ 21. The common difference of the first set of coins is greater than that of the second set by 1, and the product of the first set is the product of the second set as 8 to 9. Find the value of the coin of largest denomination, among the six coins :
 (a) 9 (b) 8
 (c) 11 (d) 10
- Find the sum of the three numbers in G.P. whose product is 216 and the sum of the products of them taken in pairs is 126 :
 (a) 28 (b) 21
 (c) $35/4$ (d) none of these
- The sum of four consecutive terms in A.P. is 36 and the ratio of product of the first and fourth is to the product of the second and third is 9:10. Find the largest of the numbers :
 (a) 9 (b) 10
 (c) 8 (d) 12
- The sum of four integers in A.P. is 24 and their product is 945. Find the product of the smallest and greatest integers :
 (a) 30 (b) 27 (c) 35 (d) 39
- In an A.P. consisting of 23 terms, the sum of the three terms in the middle is 114 and that of the last three is 204. Find the sum of first three terms :
 (a) 14 (b) 42 (c) 24 (d) 69
- The sum of an infinite G.P. is 4 and the sum of their cubes is 192. Find the first term :
 (a) 4 (b) 8 (c) 6 (d) 2
- Vibhor joined as an area manager of Quick Corporation in the pay scale of ₹ 12,000 – 500 – 18,500. Minimum how many years he has to work in the corporation to avail the salary of ₹ 18,500 per month :
 (a) 12 years (b) 10 years
 (c) 13 years (d) 11 years
- How many terms are common in two arithmetic progression 1, 4, 7, 10. ... upto 63 terms and 3, 7, 11, 15, ... upto 47 terms :
 (a) 12 (b) 16
 (c) 15 (d) none of these
- The value of $3^{1/3} \cdot 9^{1/18} \cdot 27^{1/81} \dots$:
 (a) 3 (b) 9
 (c) 27 (d) none of these
- The sum to n terms of the series
 $1 + (1 + 3) + (1 + 3 + 5) + \dots$ is
 (a) $\left[\frac{n(n+1)}{2}\right]^2$ (b) n^2
 (c) $\frac{n(n+1)(2n+1)}{6}$ (d) none of these
- The sum to n terms of the series
 $1^2 + (1^2 + 3^2) + (1^2 + 3^2 + 5^2) + \dots$ is :
 (a) $\frac{1}{3}(n^4 + 2n^2)$ (b) $\frac{1}{3}(n^3 + 3n^2 - n)$
 (c) $\frac{1}{6}n(n+1)(2n^2 + 2n - 1)$ (d) none of these
- If x, y, z are in G.P. and $a^x = b^y = c^z$, then :
 (a) $\log_b a = \log_c b$ (b) $\log_b a = \log_a c$
 (c) $\log_c b = \log_a c$ (d) none of these

- 17** The sum of integers from 113 to 113113 which are divisible by 7 is :
 (a) 92358576 (b) 913952088
 (c) 94501895 (d) 912952066
- 18** The sum of n terms of a progression is $3n^2 + 5$. The number of terms which equals 123 is :
 (a) 12 (b) 18 (c) 21 (d) 23
- 19** If a, b, c are in A.P., $b - a, c - b$ and a are in G.P., then $a : b : c$ is :
 (a) $4 : 3 : 2$ (b) $2 : 3 : 4$ (c) $1 : 2 : 3$ (d) $3 : 4 : 5$
- 20** The sum of first n terms of the series $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \dots$
 (a) $2^n - 1$ (b) $1 - 2^{-n}$
 (c) $2^n - n + 1$ (d) $n + 2^{-n} - 1$
- 21** The sum of all 4 digit numbers which when divided by 4 leave the remainder 1 :
 (a) 12372750 (b) 12327027
 (c) 12370705 (d) 109980
- 22** If n arithmetic means are inserted between two quantities a and b , then their sum is equal to :
 (a) $n(a + b)$ (b) $\frac{n}{2}(a + b)$ (c) $2n(a + b)$ (d) $\frac{n}{2}(a - b)$
- 23** The product of n geometric means between two given numbers a and b is :
 (a) $(ab)^n$ (b) $(ab)^{2n}$
 (c) $(ab)^{n/2}$ (d) none of these
- 24** If $a_1, a_2, a_3, \dots$ is an A.P. such that
 $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$,
 then $a_1 + a_2 + a_3 + \dots + a_{23} + a_{24}$ is equal to :
 (a) 999 (b) 900 (c) 1225 (d) none
- 25** The sum to n terms of the series, where n is an even number
 $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \dots$
 (a) $n(n + 1)$ (b) $\frac{n(n + 1)}{2}$
 (c) $-\frac{n(n + 1)}{2}$ (d) none of these
- 26** $\underbrace{(666 \dots 6)}_{n\text{-digits}} + \underbrace{(888 \dots 8)}_{n\text{-digits}}$ is equal to :
 (a) $14(n^2 - 1)$ (b) $\frac{48}{9}(10^{2n} - 1)$
 (c) $\frac{14}{9}(10^n - 1)$ (d) none of these
- 27** The cubes of the natural numbers are grouped as $1^3, (2^3, 3^3), (4^3, 5^3, 6^3), \dots$ then the sum of the numbers in the n th group is :
 (a) $\frac{n^3}{12}(n^2 + 2)(n^2 + 4)$ (b) $\frac{1}{8}n^3(n^2 + 1)(n^2 + 3)$
 (c) $\frac{n^3}{6}(n^2 + 1)(n^2 + 2)$ (d) none of these
- 28** Three non-zero real numbers form an A.P. and the squares of these numbers taken in the same order form a G.P. Then the number of all possible common ratio of the G.P. is :
 (a) 1 (b) 2
 (c) 3 (d) none of these
- 29** If three positive real numbers a, b, c are in A.P. such that $a \cdot b \cdot c = 4$, then the minimum value of b is :
 (a) $2^{1/2}$ (b) $2^{1/3}$
 (c) $2^{2/3}$ (d) $2^{3/2}$
- 30** Three numbers form an increasing G.P. If the middle number is doubled, then the new numbers are in A.P. The common ratio of the G.P. is :
 (a) $2 - \sqrt{3}$ (b) $2 + \sqrt{3}$
 (c) $\sqrt{3} - 2$ (d) $3 + \sqrt{2}$
- 31** If a, b, c ($a > 0$) are three successive terms of a G.P. with common ratio r , the value of r for which $c > 4b - 3a$ holds given by :
 (a) $1 < r < 3$ (b) $-3 < r < -1$
 (c) $r < 1$ and $r > 3$ (d) none of these
- 32** Let a_n be the n th term of an A.P. and $a_7 = 22$, then the value of the common difference (d) that would make $a_3 \cdot a_7 \cdot a_{11}$ greatest is :
 (a) 4 (b) 2
 (c) 0 (d) 7
- 33** If the A.M. of two positive numbers a and b , ($a > b$), is twice their G.M., then $a : b$ is :
 (a) $2 : \sqrt{3}$ (b) $2 : 7 + 4\sqrt{3}$
 (c) $2 + \sqrt{3} : 2 - \sqrt{3}$ (d) $7 + 4\sqrt{3} : 7 - 4\sqrt{3}$
- 34** The number of common terms to the two sequences 17, 21, 25, ..., 417 and 16, 21, 26, ..., 466 is :
 (a) 19 (b) 20
 (c) 21 (d) 84
- 35** The sum to n terms of the series
 $\frac{1}{\sqrt{1} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{5}} + \frac{1}{\sqrt{5} + \sqrt{7}} + \dots$ is :
 (a) $\sqrt{2n + 1}$ (b) $\frac{1}{2}\sqrt{2n + 1}$
 (c) $\sqrt{2n - 1}$ (d) $\frac{1}{2}\{\sqrt{2n + 1} - 1\}$
- 36** Let T_r be the r th term of an A.P. for $r = 1, 2, 3, \dots$. If for some positive integers m, n we have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then T_{mn} equals :
 (a) $\frac{1}{mn}$ (b) $\frac{1}{m} + \frac{1}{n}$
 (c) 1 (d) 0
- 37** Find the sum to n terms of the series $3 + 6 + 10 + 16 + \dots$
 (a) $\frac{n(n - 1)}{2} - 1$ (b) $n(n + 1) + 2^n - 1$
 (c) $n(n + 2) + 1$ (d) $3(2n + 1) - 2^n$

- 38** In a certain colony of cancerous cells, each cell divides into two every minute. How many cells will be produced from a single cell if the rate of division continues for 12 minutes :
(a) 9180 (b) 8190 (c) 8910 (d) none
- 39** Divakar and Subhank set out to meet each other from Delhi and Saharanpur, 330 km apart. Divakar travels 30 km on the first day 28 km on the second day, 26 km on the third day and so on. Subhank travels 20 km on the first day, 24 km on the second day, 28 km on the third day and so on. In how many days they meet, if they started moving towards each other at the same time ?
(a) 4 (b) 5
(c) 6 (d) none of these
- 40** Find the sum to n terms of the series
 $1 + (1 + 2) + (1 + 2 + 3) + (1 + 2 + 3 + 4) + \dots :$
 (a) $\frac{n(n+1)(2n+1)}{6}$ (b) $\frac{n(n+1)(n+2)}{6}$
 (c) $\frac{n(n+1)(n+2)}{12}$ (d) $\frac{n(n+1)}{2}$
- 41** If a, b, c are in H.P., then the value of
 $\left(\frac{1}{b} + \frac{1}{c} - \frac{1}{a}\right)\left(\frac{1}{c} + \frac{1}{a} - \frac{1}{b}\right)$ is
 (a) $\frac{3}{b^2} - \frac{2}{ab}$ (b) $a^2 + b^2 + c^2$
 (c) $\frac{2}{bc} - \frac{1}{b^2}$ (d) both (a) and (c)
- 42** Find the sum to n terms of the series
 $1 \cdot 3 \cdot 5 + 3 \cdot 5 \cdot 7 + 5 \cdot 7 \cdot 9 + \dots$ is :
 (a) $n(8n^3 + 11n^2 - n + 3)$ (b) $n(2n^3 + 8n^2 + 7n - 2)$
 (c) $8n^3 + 12n^2 - 2n - 3$ (d) none of the above
- 43** What is the sum of 100 terms of the series
 $1 - 2 + 3 - 4 + 5 - 6 + \dots ?$
 (a) 100 (b) 50
 (c) 550 (d) none of these
- 44** An infinite G.P. has first term x and its sum is 5, then the value of x is
 (a) $x < -10$ (b) $-10 < x < 0$
 (c) $0 < x < 10$ (d) $x > 10$
- 45** The next term of the sequence 1, 5, 14, 30, 55, ... is
 (a) 91 (b) 85
 (c) 90 (d) 95
- 46** In an AP the total number of terms is 51. The 9th term of the AP is $7\frac{3}{17}$ and 26th term of the same AP is $21\frac{9}{17}$. What is the ratio of the sum of the first 17 terms and that the last 34 terms of the AP?
 (a) 1 : 3 (b) 1 : 8
 (c) 1 : 9 (d) 1 : 4
- 47** What's the sum of
 $3 \cdot 4 \cdot 5 + 4 \cdot 5 \cdot 6 + 5 \cdot 6 \cdot 7 + \dots + 28 \cdot 29 \cdot 30?$
 (a) 300003 (b) 188760
 (c) 287695 (d) 178764
- 48** Find the value of $p^{21} + p^{22} + p^{23} + \dots + p^{399}$, if p is one of the roots of $x^7 - 1 = 0$, where $p \neq 1$.
 (a) p (b) 400
 (c) 21 (d) 1
- 49** If $0 < a < 1$ and $9 \leq b \leq 10$, then
 (a) $9 \leq a + b \leq 11$ (b) $9 \leq a + b < 11$
 (c) $9 < a + b \leq 11$ (d) $9 < a + b < 11$

LEVEL 02 > HIGHER LEVEL EXERCISE

- 1** An A.P. whose first term is unity and in which the sum of the first half of any even number of terms to that of the second half of the same number of terms is in constant ratio, the common difference ' d ' is given by :
 (a) 1 (b) 2
 (c) 3 (d) 4
- 2** The number of divisors of 187, 637 and 1001 are in :
 (a) A.P. (b) G.P.
 (c) H.P. (d) none of these
- 3** The sum to n terms of the series
 $1^2 + (1^2 + 3^2) + (1^2 + 3^2 + 5^2) + \dots$ is :
 (a) $\frac{1}{3}(n^3 + n^2 + 1)$ (b) $\frac{1}{6}n(n+1)(2n^2 + 2n - 1)$
 (c) $\frac{1}{3}(2n^2 + 2n - 1)$ (d) none of these
- 4** If $|a| < 1$ and $|b| < 1$, then the sum of the series
 $a(a+b) + a^2(a^2+b^2) + a^3(a^3+b^3) + \dots$ upto ∞ is :
 (a) $\frac{a^2}{b} + 1$ (b) $\frac{a^2}{1-ab}$
 (c) $\frac{a^2}{1-a^2} + \frac{ab}{1-ab}$ (d) $\frac{a^2}{b^2} + \frac{1}{ab}$
- 5** The sum to n terms of the series
 $\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \dots$ is :
 (a) $\frac{n^2 - 2n}{(n-1)^2}$ (b) $\frac{n^2 + 2n}{(n+1)^2}$
 (c) $\frac{2n^2 + 1}{n}$ (d) none of these

- 6 The natural numbers are written as follows :

$$\begin{array}{ccccccc} & & & & 1 & & \\ & & & & 2 & 3 & \\ & & & 4 & 5 & 6 & \\ & & 7 & 8 & 9 & 10 & \\ & \dots & \dots & \dots & \dots & \dots & \\ & \dots & \dots & \dots & \dots & \dots & \end{array}$$

The sum of numbers in the n th row is :

- (a) $\frac{n(n+1)}{2}$ (b) $\frac{n^2+1}{2}$
 (c) $\frac{n(n^2+1)}{2}$ (d) none of these
- 7 The number $\underbrace{1111\dots1}_{119 \text{ times}}$ is a :
 (a) prime number (b) composite number
 (c) multiple of 3 (d) none of these
- 8 The interior angles of a polygon are in A.P. If the smallest angle is 120° and the common difference is 5° , then the number of sides in the polygon is :
 (a) 8 (b) 12 (c) 9 (d) 6
- 9 If $n \in N$ and n is an odd number, then $10^0 + 10^4 + 10^8 + 10^{12} + \dots + 10^{4n}$ is a :
 (a) prime number (b) composite number
 (c) a multiple of 10 (d) none of these
- 10 The value of $0.\overline{123}$ is :
 (a) $\frac{61}{495}$ (b) $\frac{71}{485}$ (c) $\frac{62}{719}$ (d) $\frac{123}{999}$
- 11 If n is a positive integer, then $\underbrace{111\dots1}_{2n \text{ times}} - \underbrace{222\dots2}_{n \text{ times}}$ is :
 (a) a perfect square (b) a perfect cube
 (c) prime number (d) none of these
- 12 One side of an equilateral triangle is 24 cm. The mid-points of its sides are joined to form another triangle whose mid points are in turn joined to form still another triangle. This process continues indefinitely. Find the sum of the perimeters of all the triangles :
 (a) 144 cm (b) 72 cm
 (c) 536 cm (d) 676 cm
- 13 The odd positive integers are arranged in a triangle as follows :

$$\begin{array}{ccccccc} & & & & 1 & & \\ & & & & 3 & 5 & \\ & & & 7 & 9 & 11 & \\ & & 13 & 15 & 17 & 19 & \\ & 21 & 23 & 25 & 27 & 29 & \\ & \dots & \dots & \dots & \dots & \dots & \\ & \dots & \dots & \dots & \dots & \dots & \end{array}$$

Find the sum of the numbers in the n th row of this arrangement :

- (a) $n^2 + n$ (b) $2n^2 - 2 + n$
 (c) n^3 (d) none of these

- 14 Find the sum to n terms of the series

$$\log a + \log \frac{a^3}{b} + \log \frac{a^5}{b^2} + \log \frac{a^7}{b^3} + \dots$$

- (a) $\log \left(\frac{a^{2n}}{b^{n-1}} \right)^{n/2}$ (b) $\log \frac{a^{2n-1}}{b^{n-1}}$
 (c) $\log \frac{a^{2n}}{b^n}$ (d) none of these

- 15 The value of $(0.2)^{\log_{\sqrt{5}} \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots \right)}$:

- (a) 2 (b) $\frac{1}{2}$
 (c) 4 (d) none of these

- 16 If a, b, c are in H.P., then $\frac{b+a}{b-a} + \frac{b+c}{b-c}$ equals:

- (a) 1 (b) 2 (c) $\frac{b-c}{a-b}$ (d) $\frac{ab}{c}$

- 17 Let a, b, c are in G.P. with common ratio r ($0 < r < 1$). Then $a, 2b, 3c$ are in A.P., if r equals :

- (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$
 (c) $\frac{1}{3}$ (d) none of these

- 18 The sum of the series

$$\frac{5}{13} + \frac{55}{(13)^2} + \frac{555}{(13)^3} + \frac{5555}{(13)^4} + \dots \infty :$$

- (a) $\frac{55}{39}$ (b) $\frac{65}{39}$ (c) $\frac{110}{169}$ (d) $\frac{65}{36}$

- 19 The sum of the infinite series

$$\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \dots \infty \text{ is equal to :}$$

- (a) $\frac{1}{3}$ (b) $\frac{1}{4}$
 (c) $\frac{38}{27}$ (d) none of these

- 20 A sequence $a_1, a_2, a_3, \dots, a_n$ of real numbers is such that $a_1 = 0$, $|a_2| = |a_1 + 1|$, $|a_3| = |a_2 + 1|$, ..., $|a_n| = |a_{n-1} + 1|$, then the A.M. $\left(i. e. \frac{a_1 + a_2 + a_3 + \dots + a_n}{n} \right)$ can not be less than :

- (a) $\frac{1}{2}$ (b) $-\frac{1}{2}$
 (c) 1 (d) can't be determined

- 21 Find the A.M. of second and third term of a G.P. of real numbers such that the sum of its first four terms is equal to 30 and the sum of the squares of the first four terms is 340.
 (a) 5 (b) 6 (c) 8 (d) 10

- 22 A square is drawn by joining the mid points of the sides of a given square. A third square is drawn inside the second square in the same way and this process continues indefinitely. If a side of the first square is 4 cm, determine the sum of the areas of all the squares :

- (a) 32 cm^2 (b) 16 cm^2
 (c) 20 (d) none of these

- 23** An A.P. and a G.P. with positive terms have the same number of terms and their first terms as well as last terms are equal. Then what can be said about the sum of the A.P. and G.P. if A_s and G_s denote the sum of the A.P. and G. P. respectively :

(a) $A_s \geq G_s$ (b) $A_s \leq G_s$
(c) $A_s = G_s$ (d) can't be determined

- 24** An A.P. and a G.P. have all the terms positive. Also the first and second terms of the progressions are equal, If a_i and g_i denote the terms of A.P. and G.P. respectively for every $i \in N$ and $i > 2$, then which one of the following is true ?

(a) $a_i \geq g_i$ (b) $a_i \leq g_i$
(c) $a_i = g_i$ (d) none of these

- 25** $1^k + 2^k + 3^k + \dots + n^k$ is divisible by $1 + 2 + 3 + \dots + n$ for every $n \in N$, then k is :

(a) even (b) odd
(c) only prime (d) none of these

- 26** If p th, q th, r th and s th terms of an A.P. are in G.P. then $p - q$, $q - r$, $r - s$ are in :

(a) A.P. (b) G.P.
(c) H.P. (d) none of these

- 27** In our country 'Dreamland' the currency consists of a set of notes of different denominations. If the sum of all the denominations is 63 units of currency then what is the minimum number of denominations that can be there so that all the transactions can be made (in integers).

(a) 3 (b) 5 (c) 6 (d) 9

- 28** A retailer has n stones by which he can measure (or weigh) all the quantities from 1 kg to 121 kg (in integers only. e.g., 1 kg, 2 kg, 3 kg, etc) keeping these stones on either side of the balance. What is the minimum value of n ?

(a) 3 (b) 4 (c) 5 (d) 11

- 29** What will be the 33rd term of the sequence 1, 7, 25, 79, ...?

(a) $33^3 - 1$ (b) $3^{33} - 2$
(c) $3(33^3 - 2)$ (d) none of these

- 30** All the four angles of a quadrilateral are in G.P. with common ratio r ($r \in N$). Any two angles are acute and other two angles are obtuse, then the measure of the smallest angle of the quadrilateral is :

(a) 18° (b) 32° (c) 24° (d) 36°

Directions (for Q. Nos. 31 and 32) *In the kingdom of YAMRAJ, there are certain number of souls in a particular day. Where a single soul can merge with another soul in every second and thus two souls reduced to a single soul and if any soul can not merge with anyother soul in a particular second then it has to become a ghost and suffer in the hell. Thus every soul tries to avoid to become a ghost, but after a certain time it has to go to hell after maximum possible survival.*

- 31** In a particular day there were $(2^{43200} - 1)$ souls after how many seconds will all the souls become ghost :

(a) 43200 (b) 43199
(c) 43201 (d) 21600

- 32** If after n seconds there are k souls left, then what is the maximum number of souls there can be initially in a particular day ?

(a) $2^{k \cdot n}$ (b) $2^{kn} + 2k$
(c) $2^n k + 2^n - 1$ (d) $2^k n + 2^n + 1$

- 33** The income of HBI on the n th day is ₹ $(n^2 + 2)$ and the expenditure of HBI on the n th day is ₹ $(2n + 1)$.

Also income = expenditure + savings

In how many days his total saving will be ₹ 1240. :

(a) 10 (b) 12 (c) 15 (d) 16

Directions (for Q. Nos. 34 and 35) *In the zoological park Lucknow there are four kinds of animals viz. Elephant, monkey, lion and tiger which are in increasing G.P. and in the local zoo in Kanpur there are the same kinds of animals but in A.P. The number of elephants is least and equal in each of the places. Also the number of monkeys in each of the places is same but just greater than that of elephants. Total number of animals in zoological park is 50% more than that of local zoo. Also the common ratio of the G.P. is same as the common difference of the A.P. Number of tigers in both the places is maximum.*

- 34** What is the number of elephants in each of the places ?

(a) 1 (b) 2 (c) 3 (d) 4

- 35** What is the number of lions in zoological park Lucknow :

(a) 5 (b) 6
(c) 8 (d) can't be determine

- 36** What is the sum of n terms of the series

$$-1 + 1^2 - 2 + 2^2 - 3 + 3^2 \dots ?$$

(a) $\frac{n(n^2 + 1)}{3}$ (b) $n^2 + n$
(c) $\frac{n(n^2 - 1)}{3}$ (d) none of these

- 37** Find the sum of $1 + \frac{2}{3} + \frac{4}{9} + \frac{6}{27} + \frac{8}{81} + \frac{10}{243} + \dots \infty$:

(a) $\frac{2}{3}$ (b) $\frac{5}{2}$ (c) $\frac{19}{45}$ (d) $\frac{81}{17}$

- 38** Sum of n terms of the series

$$\log a + \log \left(\frac{a^2}{b} \right) + \log \left(\frac{a^3}{b^2} \right) + \log \left(\frac{a^4}{b^3} \right) + \dots \text{ is :}$$

(a) $\log \left\{ \frac{a^n}{b^{n-1}} \right\}^{n/2}$ (b) $\log \left\{ \frac{a^{n+1}}{b^{n-1}} \right\}^{n/2}$
(c) $\log \left\{ \frac{a^n}{b^n} \right\}^{n/2}$ (d) $\log \left\{ \frac{a^{n+1}}{b^n} \right\}^{n/2}$

- 39** If $a_1, a_2, a_3, \dots, a_n$ are in A.P. where $a_i > 0$ for all i , then the

value of $\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}}$:

(a) $\frac{n-1}{\sqrt{a_1} + \sqrt{a_2}}$ (b) $\frac{n+1}{\sqrt{a_1} + \sqrt{a_2}}$
(c) $\frac{n}{\sqrt{a_1} - \sqrt{a_2}}$ (d) none of these

40 If $\log_5 2$, $\log_5(2^x - 3)$ and $\log_5\left(\frac{17}{2} + 2^{x-1}\right)$ are in A.P. then the value of x is :

- (a) 2 (b) 3 (c) $\log_2 3$ (d) $\log_2 5$

41 If 1 , $\log_y x$, $\log_x y$, $-15\log_x z$ are in A.P., then which is correct ?

- (a) $x = z^3$ (b) $x = \frac{1}{y}$
(c) $y = \frac{1}{z^3}$ (d) all of these

42 If x , y , z are in G.P. and $a^x = b^y = c^z$, then $\log_b a \cdot \log_b c$ is equal to :

- (a) 0 (b) 1
(c) ac (d) none of these

43 If $\log_2(5 \cdot 2^x + 1)$, $\log_4(2^{1-x} + 1)$ and 1 are in A.P. then x equals :

- (a) $\log_2 5$ (b) $1 - \log_5 2$
(c) $\log_5 2$ (d) none

44 If 1 , $\log_9(3^{1-x} + 2)$ and $\log(4 \cdot 3^x - 1)$ are in A.P. then x is equal to :

- (a) $\log_4 3$ (b) $\log_3 4$ (c) $1 - \log_3 4$ (d) $\log_3\left(\frac{1}{4}\right)$

45 If $x > 1$, $y > 1$, $z > 1$ are in G.P., then

$$\frac{1}{1 + \ln x}, \frac{1}{1 + \ln y}, \frac{1}{1 + \ln z} \text{ are in :}$$

- (a) A.P. (b) H.P.
(c) G.P. (d) none of the above

46 Let $a_1, a_2, \dots, a_{10}$ be in A.P. and $h_1, h_2, \dots, h_{10}$ be in H.P. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then $a_4 h_7$ is:

- (a) 2 (b) 3 (c) 5 (d) 6

47 Let $S_1, S_2, \dots$ be squares such that for each $n \geq 1$, the length of a side of S_n equals the length of a diagonal of S_{n+1} . If the length of a side of S_1 is 10 cm, then for which of the following values of n is the area of S_n less than 1 square cm ?

- (a) 7 (b) 8 (c) 6 (d) 4

Directions (for Q. 48-50) Answer the following three questions based on the information given below :

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$, let A_{n-1} and H_{n-1} has arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

48 Which of the following statements is correct?

- (a) $G_1 > G_2 > G_3 > \dots$
(b) $G_1 < G_2 < G_3 < \dots$
(c) $G_1 = G_2 = G_3 = \dots$
(d) $G_1 < G_3 < G_5 < \dots$ and $G_2 > G_4 > G_6 > \dots$

49 Which of the following statements is correct?

- (a) $A_1 > A_2 > A_3 > \dots$
(b) $A_1 > A_3 > A_5 > \dots$ and $A_2 < A_4 < A_6 < \dots$
(c) $A_1 < A_2 < A_3 < \dots$
(d) $A_1 < A_3 < A_5 < \dots$ and $A_2 > A_4 > A_6 > \dots$

50 Which of the following statements is correct?

- (a) $H_1 > H_2 > H_3 > \dots$
(b) $H_1 > H_3 > H_5 > \dots$ and $H_2 < H_4 < H_6 < \dots$
(c) $H_1 < H_2 < H_3 < \dots$
(d) $H_1 < H_3 < H_5 < \dots$ and $H_2 > H_4 > H_6 > \dots$

51 Let $a_1, a_2, \dots, a_{100}$ be an arithmetic progression such that

$$a_1 = 3 \text{ and } S_p = \sum_{i=1}^p a_i, \text{ where } 1 \leq p \leq 100.$$

For any integer n with $1 \leq n \leq 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not

depend on n , then a_2 is

- (a) 10 (b) 9
(c) 5 (d) -1

52 The harmonic mean (HM) of the roots of the equation $(5 + \sqrt{2})x^2 - (4 + \sqrt{5})x + 8 + 2\sqrt{5} = 0$ is

- (a) 2 (b) 4
(c) 6 (d) 8

53 Let $a_1, a_2, \dots, a_{10}$ be in A.P. and $h_1, h_2, \dots, h_{10}$ be in H.P. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then $a_4 h_7$ is

- (a) 2 (b) 3
(c) 5 (d) 6

54 For a positive integer n , let

$$a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{(2^n) - 1}.$$
 Then, which of the

following are correct?

- (i) $a(100) \leq 100$ (ii) $a(100) > 100$
(iii) $a(200) \leq 100$ (iv) $a(200) > 100$
(a) Only (iii) and (iv) (b) Only (i) and (iv)
(c) Except (i) and (ii) (d) (i), (ii) and (iii) only

55 Which one of the following options is true about the series given below?

$$S = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2^{32}}$$

- (a) $5 < S < 8\frac{1}{2}$ (b) $9 < S < 16\frac{1}{2}$
(c) $17 < S < 32\frac{1}{2}$ (d) $33 < S < 64\frac{1}{2}$

56 If $apq = 9b$ and a, b, p, q are positive real numbers, find the minimum value of $a^3 p + b^3 q$.

- (a) $6ab^2$ (b) $9a^2b$
(c) $6a^2b^2$ (d) $3\sqrt{3}ab^2$

57 Two friends Akhil and Manish decide to finish a 10 hour trip on the same bicycle. Akhil rides it for a hrs, then Manish enjoys on the pillion and when Manish rides it for b hrs, then Akhil enjoys on the pillion. If Akhil rides at the speed of $3a$ km/hr and Manish rides at the speed of b km/hr, what is the minimum possible distance (in km) they travel during this 10 hr trip, with no halts during the trip?

- (a) 75 (b) 42
(c) 96 (d) 84

- 58 The minimum value of the sum of the real numbers a^{-5} , a^{-4} , $3a^{-3}$, 1 , a^8 and a^{10} with $a > 0$ is

(a) 8 (b) 9
(c) 3 (d) none of these

- 59 If a , b , c are positive real number and

$$p = [(1+a)(1+b)(1+c)]^7, \text{ then}$$

(a) $p > 33^7 a^3 b^3 c^3$

$$(b) p > 3^7 a^{21} b^{14} c^7$$

$$(c) p > 77(1+a)^7(1+b)^7(1+c)^7$$

$$(d) p > 7^7 a^4 b^4 c^4$$

- 60 What's the minimum positive integer n for which

$$\sqrt{n} \sqrt{n+1} > \sqrt{n+1} \sqrt{n}$$

(a) 4 (b) 6
(c) 7 (d) 9

LEVEL 03 > Final Round

- 1 A bacteria gives birth to two new bacterias in each second and the life span of each bacteria is 5 seconds. The process of the reproduction is continuous untill the death of the bacteria. Initially there is one newly born bacteria at time $t = 0$, then find the total number of live bacterias just after 10 seconds :

(a) $\frac{3^{10}}{2}$ (b) $3^5(3^5 - 1)$
(c) $3^{10} - 2^{10}$ (d) $3^{10} - 2^5$

- 2 If S_n be defined as the sum of n terms of the series S . Such that $S \equiv 2^{66} - 2^{65} - 2^{64} \dots$ Find S_{20} :

(a) 400 (b) 2^{47}
(c) 2^{20} (d) none of these

- 3 If U_m be defined as the sum of m terms of the series V , where $V = S_m + S_{m+1} + S_{m+2} + \dots$ Find U_{34} :

(a) $2^{33} - 1$ (b) $2^{33} + 1$
(c) $2^{34} - 1$ (d) none of these

- 4 Let $a_1, a_2, a_3, \dots, a_n$ be an A.P. and S_1, S_2 and S_3 be the sum of first n , $2n$ and $3n$ terms respectively then $S_3 - S_2 - S_1$ is equal to, if a is first term and d is common difference :

(a) $3a - 2n - d$ (b) $a(n + 2d)$
(c) $3a + 2nd$ (d) $2n^2d$

- 5 Consider the sets

$$T_n = \{n, n+3, n+5, n+7, n+9\}$$

where $n = 1, 2, 3, \dots, 99$. How many of these sets contain 5 or any integral multiple there of (i.e. any one of the numbers 5, 10, 15, 20, 25, ...)?

(a) 81 (b) 79
(c) 80 (d) none of these

- 6 If $\{x\}$ is the least integer greater than or equal to x , then find the value of the following series :

$$\{\sqrt{1}\} + \{\sqrt{2}\} + \{\sqrt{3}\} + \{\sqrt{4}\} + \dots + \{\sqrt{99}\} + \{\sqrt{100}\}$$

(a) 715 (b) 55 (c) 157 (d) 835

- 7 If $[x]$ is the greatest integer less than or equal to x , then find the value of the following series

$$[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + [\sqrt{4}] + \dots + [\sqrt{323}]$$

(a) 3237 (b) 2373
(c) 3723 (d) none of these

Directions (for Q. Nos. 8 to 14) *King Dashratha of Ayodhya on his birthday decided to offer 100 coins of gold among his 4 sons and 3 wives. The denomination of each coin is ₹ 1. He put all the 100 coins in 7 bags in such a way that by taking a proper combination of various bags any integral sum (i.e. ₹ 1, 2, 3, 4, ..., 100) can be obtained and it is known that the only whole sum of any bag can be taken.*

- 8 What is the maximum amount received by any one of the recipient ?

(a) ₹ 79 (b) ₹ 96
(c) ₹ 37 (d) none of these

- 9 If all the coins, of these who have odd number of coins, are combined then minimum how many people are required to combine their coins to make the same amount having even number of coins :

(a) 2 (b) 3
(c) 4 (d) can't be determined

- 10 If the two least amounts are combined with highest one then minimum how many persons combined their coins to get the same amount :

(a) 2 (b) 3
(c) 4 (d) 5

- 11 If the king wanted to distribute the amount equally among them, then how many people would have received more amount (number of coins) than that of previously retaining two coins with himself :

(a) 6 (b) 3
(c) 4 (d) none

- 12 If the king also included Hanuman another 8th candidate, on this occasion and distributed 200 coins in similar fashion so that the combination of two or more person's coin could yield any integral sum from 1 to 200, then what was the maximum amount (in ₹) that anybody had received ?

(a) 128 (b) 73
(c) 37 (d) 64

- 13 In the previous problem (number 12), if the people who had obtained the coins equal to odd powers of 2 (i.e. $2^1, 2^3, 2^5, \dots$ etc.) can not combine their money with any other. Then maximum how many different sums can be made ?

(a) 5! (b) 28
(c) 31 (d) 158

- 14** In the problem number 12 if we assume Hanuman as a special guest who received the maximum amount but he distributed his sum in the same proportion as others had received previously leaving the person who had received initially the second highest amount. The number of coins that Hanuman was left with :

(a) 10 (b) 13
(c) 11 (d) 9

- 15** The sum of an infinite geometric progression (G.P.) is 2 and the sum of the G.P. made from the cubes of the terms of this infinite series is 24. The values a and r respectively (where a is the first term and r denotes common ratio of the series) :

(a) $-2, 3$ (b) $3, -\frac{1}{2}$
(c) $0, \frac{1}{2}$ (d) can't be determined

- 16** The sum of first ten terms of an A.P. is 155 and the sum of first two terms of a G.P. is 9. The first term of the A.P. is equal to the common ratio of the G.P. and the first term of the G.P. is equal to the common difference of the A.P. which can be the A.P. as per the given conditions ?

(a) 2, 4, 6, 8, 10, ... (b) $\frac{25}{2}, \frac{79}{6}, \frac{83}{6}, \dots$
(c) 2, 5, 8, 11, ... (d) both (b) and (c)

- 17** The sum of an infinite geometric series is 162 and the sum of its first n terms is 160. The inverse of its common ratio is an integer, then how many values of the common ratio (r) are possible ?

(a) 1 (b) 2
(c) 4 (d) 5

- 18** In the previous question how many values of n are possible ?

(a) 1 (b) 2
(c) 3 (d) 4

- 19** Let $a_1, a_2, a_3, \dots, a_{11}$ be the real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, 5, \dots, 11$.

If $a_1^2 + a_2^2 + \dots + a_{11}^2 = 90$, then the value of

$\frac{11}{a_1 + a_2 + a_3 + \dots + a_{11}}$ is equal to

(a) 0 (b) $3\sqrt{10}$
(c) 270 (d) 270/11

Directions (for Q 20-22) Answer the following three questions based on the information given below :

Let V_r denote the sum of the first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r - 1)$. Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for $r = 1, 2, 3, \dots$

- 20** The sum $V_1 + V_2 + \dots + V_n$ is

(a) $\frac{1}{12}n(n+1)(3n^2 - n + 1)$
(b) $\frac{1}{12}n(n+1)(3n^2 + n + 2)$
(c) $\frac{1}{2}n(2n^2 - n + 1)$
(d) $\frac{1}{3}(2n^2 - 2n + 3)$

- 21** T_r is always

(a) an odd number (b) an even number
(c) a prime number (d) a composite number

- 22** Which one of the following is a correct statement?

(a) $Q_1, Q_2, Q_3, \dots$ are in A.P. with common difference 5.
(b) $Q_1, Q_2, Q_3, \dots$ are in A.P. with common difference 6.
(c) $Q_1, Q_2, Q_3, \dots$ are in A.P. with common difference 11.
(d) $Q_1 = Q_2 = Q_3, \dots$

- 23** If $a_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \dots + (-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $b_n = 1 - a_n$,

then find the minimum natural number n_0 such that $b_n > a_n$; $\forall n > n_0$.

(a) 4 (b) 8
(c) 9 (d) 6

- 24** If total number of runs scored in n matches is $\left(\frac{n+1}{4}\right)(2^{n+1} - n - 2)$ when $n > 1$, and the runs scored in the k^{th} match are given by $k \cdot 2^{n+1-k}$, where $1 \leq k \leq n$. Find n .

(a) 9 (b) 6
(c) 7 (d) 3

- 25** Let $S_1, S_2, \dots$ be squares such that for each $n \geq 1$, the length of a side of S_n equals the length of a diagonal of S_{n+1} . If the length of a side of S_1 is 10 cm, then for which of the following value of n is the area of S_n less than 0.5 sq. cm?

(a) 7 (b) 8
(c) 9 (d) 10

Answers

✓ Introductory Exercise 18.1

1 (b)	2 (a)	3 (c)	4 (d)	5 (b)	6 (c)	7 (a)	8 (a)	9 (c)	10 (a)
11 (d)	12 (a)	13 (a)	14 (c)	15 (c)	16 (d)	17 (d)	18 (b)	19 (c)	20 (d)
21 (b)	22 (b)	23 (c)	24 (c)	25 (b)	26 (c)	27 (a)	28 (b)	29 (b)	30 (c)
31 (d)	32 (a)	33 (c)	34 (a)	35 (d)	36 (d)	37 (c)	38 (b)	39 (c)	40 (a)
41 (c)	42 (b)	43 (c)							

✓ Introductory Exercise 18.2

1 (c)	2 (c)	3 (b)	4 (a)	5 (b)	6 (c)	7 (b)	8 (a)	9 (b)	10 (c)
11 (a)	12 (d)	13 (c)	14 (a)	15 (a)	16 (a)	17 (b)	18 (c)	19 (b)	20 (a)
21 (d)	22 (c)	23 (b)	24 (b)	25 (b)	26 (b)	27 (d)	28 (c)	29 (b)	30 (b)
31 (c)	32 (d)	33 (b)	34 (c)	35 (c)	36 (d)	37 (c)	38 (c)	39 (d)	40 (c)

✓ Introductory Exercise 18.3

1 (c)	2 (b)	3 (d)

✓ Level 01 Basic Level Exercise

1 (c)	2 (b)	3 (a)	4 (b)	5 (d)	6 (b)	7 (d)	8 (b)	9 (c)	10 (c)
11 (c)	12 (b)	13 (d)	14 (c)	15 (c)	16 (a)	17 (b)	18 (c)	19 (c)	20 (d)
21 (a)	22 (b)	23 (c)	24 (b)	25 (c)	26 (c)	27 (b)	28 (c)	29 (c)	30 (b)
31 (c)	32 (c)	33 (c)	34 (b)	35 (d)	36 (c)	37 (b)	38 (b)	39 (c)	40 (b)
41 (d)	42 (b)	43 (d)	44 (c)	45 (a)	46 (b)	47 (b)	48 (d)	49 (d)	

✓ Level 02 Higher Level Exercise

1 (b)	2 (a)	3 (b)	4 (c)	5 (b)	6 (c)	7 (b)	8 (c)	9 (b)	10 (a)
11 (a)	12 (a)	13 (c)	14 (a)	15 (c)	16 (b)	17 (c)	18 (d)	19 (a)	20 (b)
21 (b)	22 (a)	23 (a)	24 (b)	25 (b)	26 (b)	27 (c)	28 (c)	29 (b)	30 (c)
31 (a)	32 (c)	33 (d)	34 (b)	35 (c)	36 (d)	37 (b)	38 (b)	39 (a)	40 (b)
41 (d)	42 (b)	43 (b)	44 (c)	45 (b)	46 (d)	47 (b)	48 (c)	49 (a)	50 (c)
51 (b)	52 (b)	53 (c)	54 (b)	55 (c)	56 (a)	57 (a)	58 (a)	59 (d)	60 (c)

✓ Level 03 Final Round

1 (b)	2 (b)	3 (a)	4 (d)	5 (b)	6 (a)	7 (c)	8 (c)	9 (b)	10 (a)
11 (c)	12 (b)	13 (c)	14 (c)	15 (b)	16 (d)	17 (d)	18 (c)	19 (a)	20 (b)
21 (d)	22 (b)	23 (d)	24 (c)	25 (d)					

Hints & Solutions

Introductory Exercise 18.1

1 $T_{41} = 3 + 40 \times 5 = 203$ ($\because T_n = a + (n-1)d$)

2 $T_{25} = 10 + 24(-4) = -86$ ($a = 10, d = -4$)

3 $T_n = l = a + (n-1)d \Rightarrow 2 + (n-1)3 = 56$
 $\Rightarrow 3(n-1) = 56 - 2 \Rightarrow n = 19$

4 Since there are only two consecutive terms so we can not find the pattern of the sequence that whether it is a non-progression sequence or A.P. or G.P. or H.P. etc. Hence (d) is correct.

5 $T_3 = a + 2d = 17$... (i)

$T_7 = a + 6d = 27$... (ii)

$\therefore$ Subtracting eq. (i) from eq. (ii), we get

$4d = 10 \Rightarrow d = 2.5$

$\therefore a + 2 \times 2.5 = 17$

$\Rightarrow a = 12$

6 $T_6 = a + 5d = 12$... (i)

$T_8 = a + 7d = 22$

$\Rightarrow 2d = 10$ or $d = 5$

$\therefore a + 25 = 12$ [from eq. (i)]

$\Rightarrow a = -13$

$\therefore T_n = -13 + (n-1)5$

$\Rightarrow T_n = 5n - 18$

Alternatively Check the options.

$5n - 18 \Rightarrow 5 \times 6 - 18 = 12$

and $5 \times 8 - 18 = 22$

Hence option (c) is correct.

7 $T_7 = a + 6d$

$T_{11} = a + 10d$

$\therefore 7 \cdot T_7 = 11 \cdot T_{11}$

$\Rightarrow 7(a + 6d) = 11(a + 10d)$

$\Rightarrow 4a = -68d$

$\Rightarrow a = -17d$

Now $T_{18} = a + 17d$

$\Rightarrow T_{18} = -17d + 17d \Rightarrow T_{18} = 0$

8 $T_p = a_1 + (p-1)d$

$T_q = a_1 + (q-1)d$

$T_r = a_1 + (r-1)d$

$\therefore$ L.H.S. $= a(q-r) + b(r-p) + c(p-q)$

$= \{a_1 + (p-1)d\}(q-r) + \{a_1 + (q-1)d\}(r-p)$
 $+ \{a_1 + (r-1)d\}(p-q)$

$= a_1 \cdot (q-r + r-p + p-q) + d[(p-1)(q-r)$
 $+ (q-1)(r-p) + (r-1)(p-q)]$

$= 0$

Alternatively Let us assume an A.P.

2, 5, 8, 11, 14, 17, 20, ...

Again assume $p = 2, q = 4, r = 5$

$\therefore T_p = T_2 = 5 = a$

$T_q = T_4 = 11 = b$

and $T_r = T_5 = 14 = c$

$\therefore a(q-r) + b(r-p) + c(p-q) = 5(-1) + 11(3) + 14(-2)$
 $= -5 + 33 - 28 = 0$

Hence (a) is the correct choice.

9 $T_p = a + (p-1)d = q$... (i)

and $T_q = a + (q-1)d = p$... (ii)

and $T_m = a + (m-1)d$... (iii)

From eqs. (i) and (ii), we get

$a = q - (p-1)d$

and $a = p - (q-1)d$

$\Rightarrow q - (p-1)d = p - (q-1)d$

$\Rightarrow (q-p) = -d$ $[(q-1) - (p-1)]$

$\Rightarrow d = -1$

Again, $a + (p-1)d = q$

$\Rightarrow a + (p-1) \times -1 = q$

$\Rightarrow a = p + q - 1$

$\therefore T_m = (p + q - 1) + (m-1) \times -1$

$\Rightarrow T_m = p + q - m$

10 $11 + 13 + 15 + \dots + 99$

Here number of terms $= 50 - 5 = 45$

or Number of terms $= \left(\frac{99-11}{2} \right) + 1 = 45$

NOTE When both the extremes of a sequence are

counted then number of terms $= \left(\frac{l-a}{d} \right) + 1$

Now the sum of the progression $= \left(\frac{a+l}{2} \right) n$
 $= \left(\frac{11+99}{2} \right) \times 45 = 2475$

Alternatively $11 + 13 + \dots + 99$

$= (1 + 3 + 5 + \dots + 99) - (1 + 3 + \dots + 9)$

$= (50)^2 - (5)^2 = 2475$

11 Number of terms $= \left(\frac{616-22}{6} \right) + 1 = \frac{594}{6} + 1 = 100$

Alternatively 22, 28, 34, ..., 616

$= (4, 10, 16, 22, 28, \dots, 616) - (4, 10, 16)$

$\Rightarrow (6, 12, 18, 24, 30, \dots, 618) - (6, 12, 18)$

$\Rightarrow (1, 2, 3, 4, 5, \dots, 103) - (1, 2, 3)$

$\Rightarrow 103 - 3 = 100$

☞ Alternatively 22, 28, 34, ..., 616

$$\Rightarrow 24, 30, 36, \dots, 618$$

$$\Rightarrow 4, 5, 6, \dots, 103$$

$$\Rightarrow 103 - 3 = 100$$

$$12 \text{ Number of terms} = \left(\frac{888 - 222}{2} \right) + 1 = 334$$

$$\begin{aligned} \therefore S_{334} &= \left(\frac{222 + 888}{2} \right) \times 334 \\ &= 555 \times 334 = 185370 \end{aligned}$$

☞ Alternatively 222 + 224 + ... + 888

$$= (2 + 4 + 6 + \dots + 888) - (2 + 4 + 6 + \dots + 220)$$

$$= (444 \times 445) - (110 \times 111)$$

$$= 111(4 \times 445 - 110) = 111(1780 - 110)$$

$$= 111 \times 1670 = 185370$$

$$13 \quad T_2 = a + d = 2 \quad \dots(i)$$

$$T_7 = a + 6d = 22 \quad \dots(ii)$$

$$\Rightarrow 5d = 20 \text{ (subtracting eq. (i) from eq. (ii))}$$

$$\Rightarrow d = 4$$

$$\therefore a = -2 \quad \text{(substituting 'd' in eq. (i))}$$

$$\therefore l = T_{35} = -2 + 34 \times 4 = 134$$

$$\therefore S_{35} = \left(\frac{-2 + 134}{2} \right) \times 35 = 2310$$

$$14 \quad T_{12} = a + 11d = -13 \quad \dots(i)$$

$$S_4 = 24 = \left(\frac{a + T_4}{2} \right) \times 4 \quad \left\{ \because S_n = \left(\frac{a + l}{2} \right) n \right\}$$

$$\Rightarrow a + T_4 = 12$$

$$\Rightarrow a + a + 3d = 12$$

$$\Rightarrow 2a + 3d = 12 \quad \dots(ii)$$

From eqs. (i) and (ii), we get

$$2a + 22d = -26$$

$$2a + 3d = 12$$

$$\begin{array}{r} - \quad - \quad - \\ 19d = -38 \end{array}$$

$$\Rightarrow d = -2$$

$$\therefore a = 9$$

$$\text{Hence } S_{10} = \left(\frac{a + T_{10}}{2} \right) 10$$

$$= \left[\frac{9 + (-9)}{2} \right] \times 10 = 0 \quad \{ \because T_{10} = -9 \}$$

$$15 \quad T_3 = a + 2d = \frac{1}{5} \quad \dots(i)$$

$$T_5 = a + 4d = \frac{1}{3} \quad \dots(ii)$$

$$\Rightarrow 2d = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$$

$$\Rightarrow d = \frac{1}{15} \therefore a = \frac{1}{15}$$

$$\therefore S_{15} = \left(\frac{a + T_{15}}{2} \right) 15$$

$$= \left(\frac{1/15 + 1}{2} \right) \times 15 \quad (\because T_{15} = 1)$$

$$= \left(\frac{16}{15} \right) \times 15 = 8$$

16 Let $n = 24$, then

$$T_{24} = 1 + 23 \times 3 = 70$$

$$\therefore S_{24} = \left(\frac{1 + 70}{2} \right) \times 24 = 71 \times 12 = 852$$

$$\therefore S_{24} = 852 < 925, \text{ hence wrong}$$

Thus it is obvious that n must be greater than 24, which gives us $n = 25$, as per the choices given.

$$\therefore S_{25} = \left(\frac{a + T_{25}}{2} \right) 25 \quad (\because T_{25} = T_{24} + d = 73)$$

$$= \left(\frac{1 + 73}{2} \right) \times 25 = 925$$

Hence (d) is correct.

$$\text{☞ Alternatively } S_n = 925 = \frac{n}{2} [2 \times 1 + (n-1)3]$$

$$\Rightarrow 1850 = 2n + 3n^2 - 3n$$

$$\Rightarrow 3n^2 - n - 1850 = 0$$

$$\Rightarrow 3n^2 - 75n + 74n - 1850 = 0$$

$$\Rightarrow 3n(n-25) + 74(n-25) = 0$$

$$\Rightarrow n = 25 \text{ or } n = -\frac{74}{3}$$

The only admissible value of $n = 25$

Since number of terms cannot be negative and as a fraction too.

Hence (d) is correct.

17 Best way is to go through options

$$20 + 16 + 12 = 48,$$

hence number of terms = 3

$$\text{Again } 20 + 16 + 12 + 8 + 4 + 0 + (-4) + (-8) = 48$$

hence number of terms = 8

Thus choice (d) is most appropriate answer.

☞ Alternatively

$$S_n = 48 = \frac{n}{2} [2 \times 20 + (n-1)(-4)]$$

$$\Rightarrow 96 = 40n - 4n^2 + 4n$$

$$\Rightarrow n^2 - 11n + 24 = 0$$

$$\Rightarrow (n-8)(n-3) = 0$$

$$\Rightarrow n = 8 \text{ or } n = 3$$

Hence (d) is the correct answer.

18 $p = (r - 2d), \quad q = (r - d), \quad r = r$

$$s = (r + d), \quad t = (r + 2d)$$

$$\therefore p + r + t = 3r = -12$$

$$\Rightarrow r = -4$$

Again $p \cdot q \cdot r = 8$
 $\Rightarrow p \cdot q = -2 \quad (\because r = -4)$
 $\Rightarrow (r - 2d)(r - d) = -2$
 $\Rightarrow (-4 - 2d)(-4 - d) = -2$
 $\Rightarrow (2 + d)(4 + d) = -1$
 $\Rightarrow d^2 + 6d + 9 = 0$
 $\Rightarrow (d + 3)^2 = 0$
 $\Rightarrow d = -3$
 $\therefore p = (r - 2d) = -4 - 2 \times (-3) = 2$

☞ Alternatively Go through options.

Let us consider choice (a)

If $p + r + t = -12$
 $\Rightarrow r = -4 \quad (\because r \text{ is the A.M. of } p, r, t)$

Now if $p = 3$, then

p	q	r	s	t
3	$-\frac{1}{2}$	-4	$-\frac{15}{2}$	-11

$\therefore p \cdot q \cdot r = 3 \times \left(-\frac{1}{2}\right) \times (-4) = 6$, which is wrong.

Again consider choice (b)

p	q	r	s	t
2	-1	-4	-7	-10

$\therefore p \cdot q \cdot r = 2 \times (-1) \times (-4) = 8$, which is correct.

Hence (b) is the right choice.

19 $l = a + (n - 1)d$

$l = 7 + (n - 1)3$

$\Rightarrow l = 3n + 4$

Let us assume option (d), then

$102 = 3n + 4$

$\Rightarrow n = \frac{98}{3}$, which is inadmissible

Since number of terms can never be a fraction.

Now, assume option (b)

$79 = 3n + 4 \Rightarrow n = 25$

$\therefore S_{25} = \left(\frac{7 + 79}{2}\right) \times 25 = 1075$

But since $S_{25} = 1075 < 1242$

Hence l must be greater than 79, therefore choice (c) is possible.

Let us consider choice (c).

$l = 85 = 3n + 4 \Rightarrow n = 27$

$\therefore S_{27} = \left(\frac{7 + 85}{2}\right) \times 27 = 1242$

Hence (c) is correct.

☞ Alternatively $S_n = \frac{n}{2}[2a + (n - 1)d]$

$1242 = \frac{n}{2}[14 + (n - 1)3]$

$\Rightarrow n(3n + 11) = 2484$

$\Rightarrow 3n^2 + 11n - 2484 = 0$

$\Rightarrow 3n^2 - 81n + 92n - 2484 = 0$

$\Rightarrow 3n(n - 27) + 92(n - 27) = 0$

$\Rightarrow n = 27 \text{ or } n = -\frac{92}{3}$

Hence $n = 27$ is the only admissible value.

$\therefore l = a + (n - 1)d = 7 + 26 \times 3 = 85$

20 The required numbers are 56, 63, 70, ..., 5551.

Number of terms = 786

$\therefore S_{786} = 56 + 63 + \dots + 5551$
 $= \left(\frac{56 + 5551}{2}\right) \times 786 = 2203551$

21 $T_1 = a = -14$

$T_5 = a + 4d = 2 \Rightarrow d = 4$

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$40 = \frac{n}{2}[-28 + (n - 1)4]$

$\Rightarrow n^2 - 8n - 20 = 0$

$\Rightarrow n = 10 \text{ or } n = -2$

Since number of terms can not be negative, hence $n = 10$.

☞ Alternatively Since S_n is very small, therefore we can solve it physically within seconds.

$T_1 = -14, T_5 = 2$

T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}
-14	-10	-6	-2	2	6	10	14	18	22
0									40

$\Rightarrow T_1 + T_2 + \dots + T_{10} = 0 + 40 = 40$

22 $S_n = \left(\frac{a + l}{2}\right)n$

$9153 = \left(\frac{-7 + 233}{2}\right)n$

$\Rightarrow 9153 = 113 \times n$

$\Rightarrow n = 81$

23 Let the three consecutive numbers in A.P. be p, q, r such that

$p = (q - d) \text{ and } r = (q + d)$

$\therefore p + q + r = 12$

$\Rightarrow q = 4$

and $p^3 + q^3 + r^3 = 408$

$\Rightarrow p^3 + r^3 = 344 \quad (\because q = 4)$

and $p + q + r = 12$

$\Rightarrow p + r = 8$

Now, let us assume $p = 1$, then $r = 7$

$\therefore p^3 + r^3 = 344$, which is true.

Hence $p = 1, q = 4$ and $r = 7$

$$\therefore p \cdot q \cdot r = 1 \times 4 \times 7 = 28$$

 **NOTE** Another possible value of $p = 7$ and $r = 1$.

Alternatively $p + q + r = 12$

$$\Rightarrow q = 4 \quad (\because q \text{ is A.M. of } p, q, r)$$

$$\Rightarrow p + r = 8$$

$$\text{and } p^3 + q^3 + r^3 = 408$$

$$\Rightarrow p^3 + r^3 = 344 \quad (\because q^3 = 64)$$

$$\text{Now, } (p + r)^3 = p^3 + r^3 + 3pr(p + r)$$

$$(8)^3 = 344 + 3pr(8)$$

$$\Rightarrow pr = 7$$

$$\therefore pqr = q \cdot pr = 4 \times 7 = 28$$

Alternatively $p = (q - d)$

$$q = q$$

$$r = (q + d)$$

$$\therefore (q - d) + q + (q + d) = 12$$

$$\Rightarrow q = 4$$

$$\text{and } (q - d)^3 + q^3 + (q + d)^3 = 408$$

$$\Rightarrow q^3 + 2qd^2 = 136$$

$$\Rightarrow 8d^2 = 72 \quad (\because q = 4)$$

$$\Rightarrow d = \pm 3$$

$$\therefore p = 1 \text{ and } r = 7 \quad \text{when } d = +3$$

$$\text{and } p = 7 \text{ and } r = 1 \quad \text{when } d = -3$$

$$\therefore p \cdot q \cdot r = 1 \times 4 \times 7 = 7 \times 4 \times 1 = 28$$

24 Go through options.

Let us consider choice (c)

$$\text{for } n = 1 \quad S_1 = (1)^3 + (1 - 1)^3 = 1$$

$$\text{for } n = 2 \quad S_2 = (2)^3 + (2 - 1)^3 = 9$$

$$\text{for } n = 3 \quad S_3 = (3)^3 + (3 - 1)^3 = 35$$

Hence choice (c) is correct.

$$\mathbf{25} \quad S_{365} = \frac{365 \times 366}{2} \quad \left(S_n = \frac{n(n+1)}{2} \right)$$

$$\text{or } S_{365} = 66795$$

$$\mathbf{26} \quad \frac{T_7}{T_3} = \frac{a + 6d}{a + 2d} = \frac{12}{5}$$

$$\Rightarrow 5(a + 6d) = 12(a + 2d)$$

$$\Rightarrow \frac{a}{d} = \frac{6}{7}$$

$$\text{Now, } \frac{T_{13}}{T_4} = \frac{a + 12d}{a + 3d} = \frac{\frac{a}{d} + 12}{\frac{a}{d} + 3} = \frac{\frac{6}{7} + 12}{\frac{6}{7} + 3} = \frac{10}{3}$$

27 The numbers which are even and divisible by 5 also are 10, 20, 30 ... 1000.

$$\therefore S_{100} = 10 + 20 + 30 + \dots + 1000$$

$$= 10(1 + 2 + 3 + \dots + 100)$$

$$= 10 \times \frac{100 \times 101}{2} = 50500$$

28

$$m \cdot T_m = n \cdot T_n$$

$$m[a + (m - 1)d] = n[a + (n - 1)d]$$

$$\Rightarrow m[a + md - d] = n[a + nd - d]$$

$$\Rightarrow ma + m^2d - md = na + n^2d - nd$$

$$\Rightarrow (m - n)a = (n^2 - m^2)d + (m - n)d$$

$$\Rightarrow (m - n)a = -(m + n)(m - n)d + (m - n)d$$

$$\Rightarrow (m - n)a = (m - n)d[1 - (m + n)]$$

$$\Rightarrow a = [1 - (m + n)]d$$

Now

$$T_{(m+n)} = a + [(m + n) - 1]d$$

$$= [1 - (m + n)]d + [(m + n) - 1]d = 0$$

29

$$S_{15} = \frac{15}{2}[2a + 14d] = 105$$

$$\Rightarrow 2a + 14d = 14 \quad \dots(i)$$

$$\text{and } S_{30} = \frac{30}{2}[2a + 29d] = 105 + 780 = 885$$

$$\Rightarrow 2a + 29d = 59 \quad \dots(ii)$$

$\therefore$ From eqs. (i) and (ii)

$$15d = 45 \Rightarrow d = 3$$

30

$$a \rightarrow 2$$

$$\text{and } S_5 = \frac{1}{4}(S_{10} - S_5)$$

$$\Rightarrow 5(S_5) = S_{10}$$

$$\Rightarrow 5\left[\frac{5}{2}(4 + 4d)\right] = \frac{10}{2}[4 + 9d] \Rightarrow d = -6$$

$$\therefore T_{20} = a + 19d = 2 + 19 \times (-6) = -112$$

31

$$S_6 = 42$$

$$\Rightarrow \frac{6}{2}[2a + 5d] = 42$$

$$\Rightarrow 2a + 5d = 14 \quad \dots(i)$$

$$\text{Now, } \frac{T_{10}}{T_{30}} = \frac{a + 9d}{a + 29d} = \frac{1}{3} \Rightarrow a = d$$

$\therefore$ from eq. (i)

$$a = d = 2$$

$$\therefore T_{40} = 2 + 39 \times 2 = 80$$

32

$$\frac{S_n}{S'_n} = \frac{7n + 1}{4n + 27}$$

$$\Rightarrow \frac{\frac{n}{2}[2a + (n - 1)d]}{\frac{n}{2}[2a' + (n - 1)d']} = \frac{7n + 1}{4n + 27}$$

$$\Rightarrow \frac{2a + (n - 1)d}{2a' + (n - 1)d'} = \frac{7n + 1}{4n + 27} \quad \dots(i)$$

Now,

$$\frac{T_{11}}{T'_{11}} = \frac{a + 10d}{a' + 10d'}$$

Substituting $n = 21$ in Eq. (i), we get

$$\frac{2a + 20d}{2a' + 20d'} = \frac{148}{111} \Rightarrow \frac{a + 10d}{a' + 10d'} = \frac{148}{111} = \frac{4}{3}$$

33

$$a + b + c = 15$$

or

$$(b - d) + b + (b + d) = 15$$

$d \rightarrow$ common difference

$\Rightarrow$

$$b = 5$$

Again

$$a^2 + b^2 + c^2 = 93$$

$\Rightarrow$

$$(b - d)^2 + b^2 + (b + d)^2 = 93$$

$\Rightarrow$

$$(b - d)^2 + (b + d)^2 = 68$$

$\Rightarrow$

$$(5 - d)^2 + (5 + d)^2 = 68$$

$\Rightarrow$

$$2(5^2 + d^2) = 68$$

$\Rightarrow$

$$25 + d^2 = 34$$

$\Rightarrow$

$$d = \pm 3$$

$\therefore$ If $d = 3$, then $a = 2$ and $c = 8$

If $d = -3$, then $a = 8$ and $c = 2$

Therefore the greatest value is 8.

☞ Alternatively Go through option.

Since middle term $b = 5$

and let $c = 10$, then $a = 0$

$\therefore a^2 + b^2 + c^2 = 125 \neq 93$, hence wrong

Again let $c = 8$, then $a = 2$

$\therefore a^2 + b^2 + c^2 = 4 + 25 + 64 = 93$ hence correct.

34

$$T_n = 4n - 1$$

$\therefore$

$$T_{30} = 119$$

$\therefore$

$$T_1 = 3, T_2 = 7, T_3 = 11, \dots, 8, T_{30} = 119$$

$\therefore$

$$S_{30} = \left(\frac{T_1 + T_{30}}{2} \right) \times 30 = \left(\frac{3 + 119}{2} \right) \times 30$$

or

$$S_{30} = 1830$$

35

$$S_n = 3n^2 + 5n$$

$$T_n = S_n - S_{(n-1)}$$

$$152 = [3n^2 + 5n] - [3(n-1)^2 + 5(n-1)]$$

$$152 = 6n + 2$$

$\Rightarrow$

$$n = 25$$

☞ Alternatively Go through options.

$$[3 \times (25)^2 + 5 \times 25] - [3 \times (24)^2 + 5 \times 24]$$

$$= (2000) - (1848) = 152$$

Hence (d) is correct.

36

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$0 = \frac{n}{2} [196 + (n-1) \times (-7)]$$

$\Rightarrow$

$$0 = n(203 - 7n)$$

$\Rightarrow$

$$7n = 203 \text{ or } n = 0$$

$\Rightarrow$

$$n = 29$$

Since $n = 0$ is not acceptable.

The A.P. is 98, 91, 84, ... 0, ..., -91, -98

(Try to understand it as it is a problem of common sense, there is no need to calculate with the help of formula.)

37 It is possible only when all the terms of A.P. are non-negative.

$\therefore$ Let T_n be the least possible non-negative term of the A.P. Then

$$T_n \geq 0$$

$\therefore$

$$a + (n-1)d \geq 0$$

$$17 + (n-1) \times (-3) \geq 0$$

$\Rightarrow$

$$20 - 3n \geq 0$$

$\Rightarrow$

$$n \leq \frac{20}{3}$$

$\Rightarrow$

$$n \leq 6 \quad (\text{only integral part is considerable})$$

$\therefore$

$$n = 6$$

$\therefore$

$$T_6 = 17 + 0.5 \times (-3)$$

$$T_6 = 2$$

$\therefore$

$$S_6 = \left(\frac{17 + 2}{2} \right) \times 6 = 57$$

(The required A.P. is 17, 14, 11, 8, 5, 2).

38 The least possible sum can be obtained only when all the terms of A.P. be non-positive.

Let the last term be T_n which must be non-positive.

$\therefore$

$$T_n \leq 0$$

$$a + (n-1)d \leq 0$$

$$-23 + (n-1)4 \leq 0$$

$$4n \leq 27$$

$$n \leq 6$$

$\therefore$

$$n = 6$$

$\therefore$

$$T_n = T_6 = -23 + 5 \times 4 = -3$$

$\therefore$

$$S_6 = \left[\frac{(-23) + (-3)}{2} \right] \times 6 = -78$$

(The required A.P. is -23, -19, -15, -11, -7, -3).

39 The required numbers are 1017, 1035, 1053, ..., 9999

$$\text{Total number of terms} = 500 \left[\because n = \left(\frac{9999 - 1017}{18} \right) + 1 \right]$$

$$\therefore \text{Sum of all the numbers} = \left(\frac{1017 + 9999}{2} \right) \times 500$$

$$= 2754000$$

40 Let an A.P. be 3, 7, 11, 15, 19, 23, 27, ...

and Let $p = 2, q = 3, r = 7$

then

$$T_p = a = 7$$

$$T_q = b = 11$$

$$T_r = c = 27$$

$\therefore$

$$p(b-c) + q(c-a) + r(a-b)$$

$$= 2(11 - 27) + 3(27 - 7) + 7(7 - 11)$$

$$= -32 + 60 - 28 = 0$$

Hence (a) is true.

41 Let an A.P. be 2, 5, 8, 11, 14, 17, 20, 23, ...

and let $p = 3, q = 5, r = 6$

then $S_p = a = (2 + 5 + 8) = 15$

and $S_q = b = (2 + 5 + 8 + 11 + 14) = 40$

and $S_r = c = (2 + 5 + \dots + 14 + 17) = 57$

$$\begin{aligned} \therefore \frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) \\ = \frac{15}{3}(5-6) + \frac{40}{5}(6-3) + \frac{57}{6}(3-5) \\ = -5 + 24 - 19 = 0 \end{aligned}$$

Hence (c) is correct.

42 Let p, q, r, s be the four consecutive terms in A.P., then

$$p = (m - 3d)$$

$$q = (m - d)$$

$$r = (m + d)$$

$$s = (m + 3d)$$

$$\therefore p + q + r + s = (m - 3d) + (m - d) + (m + d) + (m + 3d) = 20$$

$$\Rightarrow 4m = 20$$

$$\Rightarrow m = 5 \quad \dots(i)$$

$$\text{Again } (m - 3d)(m + 3d) : (m - d)(m + d) = 2 : 3$$

$$\Rightarrow \frac{m^2 - 9d^2}{m^2 - d^2} = \frac{2}{3}$$

$$\Rightarrow m = 5d$$

$$\therefore d = 1 \quad (\text{from eq. (i)})$$

$$\text{Hence } (m - 3d)(m + 3d) = (5 - 3)(5 + 3) = 16$$

☞ **Alternatively** Since p, q, r and s are in A.P.

$$\text{and } p + q + r + s = 20$$

$$\Rightarrow p + s = 10$$

$$\text{and } q + r = 10$$

Let $p = 1$, then $s = 9$

$$\therefore q = 1 + \frac{8}{3} = \frac{11}{3} \quad \left(\because d = \frac{8}{3} \right)$$

$$\text{and } r = \frac{11}{3} + \frac{8}{3} = \frac{19}{3}$$

$$\therefore \frac{p \cdot s}{q \cdot r} = \frac{1 \times 9}{\frac{11}{3} \times \frac{19}{3}} = \frac{9 \times 9}{11 \times 19} = \frac{81}{209} \neq \frac{2}{3}$$

Hence the values of p, q, r and s are wrong.

Again if $p = 2$, then $s = 8$

$$\therefore q = 2 + 2 = 4 \quad (\because d = 2)$$

$$\text{and } r = 4 + 2 = 6$$

$$\therefore \frac{p \cdot s}{q \cdot r} = \frac{2 \times 8}{4 \times 6} = \frac{2}{3}, \text{ hence true.}$$

Thus the values of p, q, r and s are correct.

$$\text{Hence } p \cdot s = 2 \times 8 = 16$$

$$43 \quad S_p = \frac{p}{2} [2a + (p-1)d] = q \quad \dots(i)$$

$$\text{and } S_q = \frac{q}{2} [2a + (q-1)d] = p \quad \dots(ii)$$

$$\text{From eq. (i)} \quad 2a + (p-1)d = \frac{2q}{p} \quad \dots(iii)$$

$$\text{and from eq. (ii)} \quad 2a + (q-1)d = \frac{2p}{q} \quad \dots(iv)$$

Subtracting eq. (iv) from Eq. (iii), we get

$$(p-q)d = \frac{-2(p+q)(p-q)}{pq}$$

$$\Rightarrow d = \frac{-2(p+q)}{pq}$$

Now, from eq. (iii)

$$2a + (p-1) \left(\frac{-2(p+q)}{pq} \right) = \frac{2p}{q}$$

$$\Rightarrow a = \frac{p^2 + q^2 + pq - (p+q)}{pq}$$

$$\begin{aligned} \therefore S_{p+q} &= \frac{(p+q)}{2} [2a + (p+q-1)d] \\ &= \frac{(p+q)}{2} \left[\frac{2(p^2 + q^2 + pq - (p+q))}{pq} - \frac{2(p+q-1)(p+q)}{pq} \right] \\ &= \frac{(p+q)}{2} \left[\frac{2}{pq} (p^2 + q^2 + pq - p - q - p^2 - q^2 - pq + p + q - pq) \right] \\ &= \frac{(p+q)}{pq} (-pq) \Rightarrow S_{(p+q)} = -(p+q) \end{aligned}$$

☑ Introductory Exercise 18.2

1 Every even term is equal to 2
common ratio of the G.P. is -1

$$2 \quad r = -2 \quad \left(\because r = \frac{\frac{1}{4}}{-\frac{1}{8}} = \frac{-1}{\frac{1}{4}} = -2 \right)$$

$$\therefore T_7 = \frac{-1}{8} \cdot (-2)^6 = -\frac{1}{8} \times 64 = -8$$

3

$$T_5 = a_1 \cdot r^4 = a$$

$$T_8 = a_1 \cdot r^7 = b$$

$$T_{11} = a_1 \cdot r^{10} = c$$

$$\therefore \text{Let } b^2 = ac$$

$$\text{then } (a_1 r^7)^2 = a_1 r^4 \cdot a_1 r^{10}$$

$$\Rightarrow a_1^2 r^{14} = a_1^2 r^{14}$$

Hence (b) is correct.

$$\begin{aligned}
 4 \quad T_5 &= ar^4 = 32 \\
 T_{12} &= ar^{11} = 4096 \\
 \therefore r^7 &= \frac{4096}{32} = 2^7 \Rightarrow r = 2 \\
 \therefore a &= 2 \quad (\because ar^4 = 32) \\
 \therefore T_n &= ar^{n-1} = 2 \cdot 2^{n-1} = 2^{1+n-1} = 2^n
 \end{aligned}$$

5 $S_n > 10^6$, for least possible $n \in I^+$

$$\begin{aligned}
 \text{then, } \frac{5(2^n - 1)}{(2 - 1)} &> 10^6 \\
 \Rightarrow 2^n - 1 &> 2 \times 10^5 \\
 \Rightarrow 2^n &> 200001
 \end{aligned}$$

$$\text{at } n = 17, 2^n = 2^{17} = 131072$$

$$\text{and at } n = 18, 2^n = 2^{18} = 262144$$

Hence the least possible value of $n = 18$

6 Let a, b be the two numbers then

$$\begin{aligned}
 \text{A.M.} &= \frac{a+b}{2} = 15 \\
 \Rightarrow a+b &= 30 \\
 \text{and G.M.} &= \sqrt{ab} = 12 \\
 \Rightarrow ab &= 144
 \end{aligned}$$

Now go through options

$$\text{let } a = 6, \text{ then } b = 24$$

$$\therefore a \cdot b = 6 \times 24$$

Hence choice (c) is correct.

$$\begin{aligned}
 \textcircled{D} \text{ Alternatively } (a-b)^2 &= (a+b)^2 - 4ab \\
 &= (30)^2 - 4 \times 144 \\
 &= 900 - 576 = 324
 \end{aligned}$$

$$\Rightarrow a - b = \pm 18$$

$$\therefore (a, b) = (6, 24) \text{ or } (24, 6)$$

7 Let a, b, c be the three numbers in G.P., then

$$\begin{aligned}
 a+b+c &= 38 \\
 \text{and } abc &= 1728 \\
 \Rightarrow b &= (abc)^{1/3} = (1728)^{1/3} = 12 \\
 \therefore a \times c &= 144 \\
 \text{and } a+c &= 26
 \end{aligned}$$

Now, go through options

$$\text{Let } c = 18, \text{ then } a = 8$$

$$\therefore a \cdot c = 8 \times 18 = 144$$

Hence choice (b) is correct

$\textcircled{D}$ Alternatively Let the three numbers in G.P. be

$$\begin{aligned}
 \frac{a}{r}, a, ar \\
 \therefore \frac{a}{r} + a + ar &= 38 \quad \dots(i)
 \end{aligned}$$

$$\text{and } \frac{a}{r} \cdot a \cdot ar = 1728 \quad \dots(ii)$$

$$\Rightarrow a = 12$$

$$\therefore \frac{12}{r} + 12 + 12r = 38$$

$$\text{or } \frac{6}{r} + 6 + 6r = 19$$

$$\Rightarrow 6r^2 - 13r + 6 = 0$$

$$\Rightarrow 6r^2 - 9r - 4r + 6 = 0$$

$$\Rightarrow (2r-3)(3r-2) = 0$$

$$\Rightarrow r = \frac{3}{2} \text{ or } r = \frac{2}{3}$$

$\therefore$ Three numbers in G.P. are 8, 12, 18 or 18, 12, 8.

Hence (b) is correct.

$$\begin{aligned}
 8 \quad \frac{S_3}{S_6} &= \frac{125}{152} = \frac{\frac{a(r^3-1)}{(r-1)}}{\frac{a(r^6-1)}{(r-1)}} = \frac{r^3-1}{r^6-1}
 \end{aligned}$$

$$\Rightarrow \frac{r^3-1}{(r^3)^2-1} = \frac{125}{152}$$

$$\Rightarrow \frac{(r^3-1)}{(r^3+1)(r^3-1)} = \frac{125}{152} \Rightarrow \frac{1}{r^3+1} = \frac{125}{152}$$

$$\Rightarrow r^3 + 1 = \frac{152}{125}$$

$$\Rightarrow r^3 = \frac{27}{125}$$

$$\Rightarrow r = \frac{3}{5}$$

$\textcircled{D}$ Alternatively Go through options.

$$9 \quad a+b+c = 14$$

$$\text{Let } abc = 125$$

$$\Rightarrow b = 5$$

$$\text{let } a = 1, \text{ then } c = 25$$

$$\therefore a+b+c = 31, \text{ which is wrong.}$$

$$\text{Again let } a \cdot b \cdot c = 64 \Rightarrow b = 4$$

$$\text{Let } a = 1 \text{ then } c = 16$$

$$\Rightarrow a+b+c = 21, \text{ which is wrong}$$

$$\text{again if } a = 2, \text{ then } c = 8$$

$$\therefore a+b+c = 2+4+8 = 14$$

Hence (b) may be correct

$$a = 2, \quad b = 4, \quad c = 8$$

$$a' = 3, \quad b' = 5, \quad c' = 7$$

$\therefore a', b' \text{ and } c' \text{ are in A.P.}$

Hence choice (b) is correct.

$$\textcircled{D} \text{ Alternatively } a+b+c = 14 \quad \dots(i)$$

since $(a+1), (b+1)$ and $(c-1)$ are in A.P.

$$\therefore (b+1) - (a+1) = (c-1) - (b+1)$$

$$\Rightarrow 2b+2 = a+c$$

Now, from Eq. (i), we get

$$\begin{aligned}
 &\Rightarrow 3b + 2 = 14 \\
 &\Rightarrow b = 4 \\
 \text{Again } &a = \frac{b}{r} \text{ and } c = br \\
 \therefore &a + b + c = 14 \\
 \Rightarrow &\frac{b}{r} + b + br = 14 \\
 \Rightarrow &\frac{4}{r} + 4 + 4r = 14 \\
 \Rightarrow &2r^2 - 5r + 2 = 0 \\
 \Rightarrow &r = 2 \text{ or } r = \frac{1}{2} \\
 \therefore &a \cdot b \cdot c = 2 \times 4 \times 8 = 8 \times 4 \times 2 = 64
 \end{aligned}$$

Shortcut: Since $b = 4$

$$\begin{aligned}
 \therefore abc &= b^3 \quad (\because a, b, c \text{ are in G.P.}) \\
 \therefore abc &= 64
 \end{aligned}$$

10 Let a, b, c, d, e be five consecutive terms in G.P., then

$$\begin{aligned}
 a &= \frac{c}{r^2}, \quad b = \frac{c}{r}, \quad d = cr, \quad e = cr^2 \\
 \therefore a \cdot b \cdot c \cdot d \cdot e &= \frac{c}{r^2} \cdot \frac{c}{r} \cdot c \cdot cr \cdot cr^2 = c^5 \\
 \therefore a \cdot b \cdot c \cdot d \cdot e &= 4^5 \quad (\because c = 4)
 \end{aligned}$$

11 Let a, b, c be in G.P., then

$$\begin{aligned}
 a + b + c &= 21 = 3(7) = 3(1 + 2 + 4) \\
 \text{and } a^2 + b^2 + c^2 &= 189 = 9(21) = 9(1 + 4 + 16)
 \end{aligned}$$

Hence common ratio = 2

☞ **Alternatively** $a + ar + ar^2 = 21$

$$\Rightarrow a(1 + r + r^2) = 21 \quad \dots(i)$$

$$\text{and } a^2 + (ar)^2 + (ar^2)^2 = 189$$

$$\therefore a^2(1 + r^2 + r^4) = 189$$

$$\Rightarrow a^2[(1 + r^2)^2 - r^2] = 189$$

$$\Rightarrow a^2[(1 + r^2 + r)(1 + r^2 - r)] = 189 \quad \dots(ii)$$

Dividing eq. (ii) by equation (i), we get

$$a(1 - r + r^2) = 9 \quad \dots(iii)$$

Dividing eq. (i) by equation (iii), we get

$$\frac{1 + r^2 + r}{1 + r^2 - r} = \frac{21}{9} = \frac{7}{3}$$

$$\Rightarrow \frac{(1 + r^2) + r}{(1 + r^2) - r} = \frac{7}{3} \Rightarrow \frac{1 + r^2}{r} = \frac{5}{2}$$

(By componendo and dividendo)

$$\Rightarrow 2r^2 - 5r + 2 = 0$$

$$\Rightarrow r = 2 \text{ or } r = \frac{1}{2}$$

$$\mathbf{12} \quad T_1 + T_3 = a + ar^2 = 15 \quad \dots(i)$$

$$\text{and } T_5 + T_7 = ar^4 + ar^6 = 240 \quad \dots(ii)$$

From eq. (i) $a(1 + r^2) = 15$

and eq. (ii) becomes $ar^4(1 + r^2) = 240$

$$\therefore r^4 = 16$$

$$\Rightarrow r = \pm 2$$

$$\therefore a = 3 \quad (\text{from eq. (i)})$$

$$\therefore T_9 = a \cdot r^8 = 3 \cdot (2)^8 = 768$$

$$\mathbf{13} \quad a + b + c = 42$$

$$abc = 512$$

$$\Rightarrow b = 8 \quad (\because (abc)^{1/3} = b = 8)$$

$$\therefore ac = 64$$

$$\text{and } a + c = 34$$

Now, go through options (or by hit and trial)

$\therefore$ Either $a = 2$, then $c = 32$

or $a = 32$, then $c = 2$

Hence, (c) is correct.

$$\mathbf{14} \quad a + b + c = 70$$

and $4a, 5b, 4c$ are in A.P.

$$\therefore 10b = 4(a + c)$$

$$\Rightarrow a + c = \frac{5b}{2}$$

$$\therefore a + b + c = \frac{7b}{2} = 70$$

$$\Rightarrow b = 20$$

$$\therefore a + c = 50$$

By hit and trial $a = 10$ and $c = 40$

$$\text{Now } 4a = 40, \quad 5b = 100$$

$$\text{and } 4c = 160$$

$$\text{Hence, } a \times b \times c = 10 \times 20 \times 40 = 8000$$

$$\text{☞ Alternatively } \frac{b}{r} + b + br = 70$$

$$\Rightarrow \frac{20}{r} + 20 + 20r = 70$$

$$\Rightarrow \frac{2}{r} + 2 + 2r = 7$$

$$\Rightarrow 2r^2 - 5r + 2 = 0$$

$$\Rightarrow r = 2 \text{ or } r = \frac{1}{2}$$

$$\therefore a, b, c = 10, 20, 40 \text{ or } 40, 20, 10$$

$$\therefore abc = 8000$$

$$\mathbf{15} \quad a + b + c + d = 312$$

$$a + ar + ar^2 + ar^3 = 312$$

$$\text{and } a + d = 252$$

$$\text{and } b + c = 312 - 252 = 60$$

$$\Rightarrow a + ar^3 = 252$$

$$\text{and } ar + ar^2 = 60$$

$$\Rightarrow a(1 + r^3) = 252$$

$$\begin{aligned}
 &\text{and} \quad a(r + r^2) = 60 \\
 \therefore \quad &\frac{a(1 + r^3)}{a(r + r^2)} = \frac{252}{60} = \frac{21}{5} \Rightarrow \frac{1 + r^3}{r + r^2} = \frac{21}{5} \\
 \Rightarrow \quad &\frac{(1 + r)(1 + r^2 - r)}{r(1 + r)} = \frac{21}{5} \\
 &(\because a^3 + b^3 = (a + b)(a^2 + b^2 - ab)) \\
 \Rightarrow \quad &\frac{1 + r^2 - r}{r} = \frac{21}{5} \\
 \Rightarrow \quad &5r^2 - 26r + 5 = 0 \\
 \Rightarrow \quad &r = 5 \quad \text{or} \quad r = \frac{1}{5} \\
 \therefore \quad &a = 2 \quad \text{and} \quad ar \times ar^2 = 500 \quad (\text{when } r = 5) \\
 \text{and} \quad &a = 250 \quad \text{and} \quad ar \times ar^2 = 500 \quad (\text{when } r = \frac{1}{5})
 \end{aligned}$$

☞ Alternatively Go through options.

- 16** Distance travelled before the first hit = 16 m

$$\begin{aligned}
 \text{Distance travelled before the second hit} &= 16 + \frac{1}{2} \times 16 \times 2 \\
 &= (16 + 16) \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Distance travelled before the third hit} \\
 &= 16 + 16 + \frac{1}{2} \times 8 \times 2 \\
 &= (16 + 16 + 8) \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Distance travelled before the fourth hit} \\
 &= 16 + 16 + 8 + \frac{1}{2} \times 4 \times 2 \\
 &= (16 + 16 + 8 + 4) \text{ m}
 \end{aligned}$$

and so on.

Hence the required distance

$$\begin{aligned}
 &= 16 + 16 + 8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \\
 &= 16 + \left(16 + 8 + 4 + \dots + \frac{1}{16} \right) \\
 &= 16 + \frac{16 \left(1 - \left(\frac{1}{2} \right)^9 \right)}{1 - \frac{1}{2}} = 16 + 16 \times \left(\frac{511}{512} \right) \times \frac{2}{1} \\
 &= 16 + \frac{511}{16} = 47 \frac{15}{16}
 \end{aligned}$$

- 17** $S_n = 7 + 77 + 777 + \dots$ to n terms

$$\begin{aligned}
 &= 7(1 + 11 + 111 + \dots \text{ to } n \text{ terms}) \\
 &= \frac{7}{9}(9 + 99 + 999 + \dots \text{ to } n \text{ terms}) \\
 &= \frac{7}{9}\{(10 - 1) + (100 - 1) + (1000 - 1) + \dots \text{ to } n \text{ terms}\} \\
 &= \frac{7}{9}\{(10 + 100 + 1000 + \dots \text{ to } n \text{ terms}) \\
 &\quad - (1 + 1 + 1 + \dots \text{ to } n \text{ terms})\}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{7}{9} \left\{ \frac{10 \cdot (10^n - 1)}{10 - 1} - n \right\} \\
 &= \frac{7}{9} \left\{ \frac{10}{9} (10^n - 1) - n \right\}
 \end{aligned}$$

☞ Alternatively Best way is to go through options.

Consider $n = 2$, then $S_2 = 7 + 77 = 84$

Now, put $n = 2$ in choice (b).

☞ NOTE This type of questions necessarily be solved by options and must take at least two values for $n = 2, 3$ etc. as a measure of safety.

- 18** It is very similar to the previous question.

- 19** Let a G.P. be 1, 3, 9, 27, 81, 243, 729, ...

Consider $p = 2, q = 5, r = 6$

$$\begin{aligned}
 \therefore \quad &T_p = 3 = a \\
 &T_q = 81 = b \\
 &T_r = 243 = c \\
 \therefore \quad &a^{q-r} \cdot b^{r-p} \cdot c^{p-q} = 3^{(5-6)} \cdot 81^{(6-2)} \cdot 243^{(2-5)} \\
 &= 3^{-1} \cdot 81^4 \cdot 243^{-3} \\
 &= 3^{-1} \cdot 3^{16} \cdot 3^{-15} \\
 &= 3^{-16} \cdot 3^{16} = 3^0 = 1
 \end{aligned}$$

- 20** Let the G.P. be 1, 2, 4, 8, 16, 32, 64, ...

Let $x = 3, y = 5$ and $z = 6$

$$\begin{aligned}
 \therefore \quad &T_x = 4 = a \\
 &T_y = 16 = b \\
 &T_z = 32 = c \\
 \therefore (y - z) \log a + (z - x) \log b + (x - y) \log c \\
 &= -1 \cdot \log 4 + 3 \log 16 + (-2) \log 32 \\
 &= \log 2^{-2} + \log 2^{12} + \log 2^{-10} \\
 &= \log 2^0 = \log 1 = 0
 \end{aligned}$$

- 21** Let a, b, c, d be four numbers such that a, b, c are in A.P. and b, c, d are in G.P.

$$\therefore a + c = 2b \quad \dots(i)$$

$$\text{and} \quad bd = c^2 \quad \dots(ii)$$

$$\text{Also} \quad a + c = 2 \quad \dots(iii)$$

$$\text{and} \quad b + d = 26 \quad \dots(iv)$$

$$\Rightarrow b = 1$$

from Eq. (iv), $d = 25$

$$\therefore \text{from Eq. (ii)} \quad c = \pm 5$$

$$\text{when } c = 5, \text{ then } a = -3$$

$$\text{when } c = -5, \text{ then } a = 7$$

Hence the four numbers are $-3, 1, 5, 25$ or $7, 1, -5, 25$

$$\mathbf{22.} \quad S_n = 1 + 3 + 7 + 15 + \dots + t_n \quad \dots(i)$$

$$\text{and} \quad S_n = 1 + 3 + 7 + \dots + t_{n-1} + t_n \quad \dots(ii)$$

subtracting, we get

$$0 = (1 + 2 + 4 + 8 + \dots + \text{to } n \text{ terms}) - t_n$$

$$\Rightarrow t_n = 1 + 2 + 4 + 8 + \dots \text{ to } n \text{ terms}$$

$$\Rightarrow t_n = \frac{1 \cdot (2^n - 1)}{(2 - 1)} = 2^n - 1$$

$$\begin{aligned} \text{again } S_n &= t_1 + t_2 + t_3 + \dots + t_n \\ &= (2 - 1) + (2^2 - 1) + (2^3 - 1) + \dots + (2^n - 1) \\ &= (2 + 2^2 + 2^3 + \dots + 2^n) - (n) \\ &= \frac{2(2^n - 1)}{(2 - 1)} - n = 2(2^n - 1) - n \end{aligned}$$

☞ **Alternatively** Go through options.

$$23 \quad S = \frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \frac{7}{2^4} + \dots + \frac{2n-3}{2^{n-1}} + \frac{2n-1}{2^n}$$

$$\text{and } \frac{1}{2}S = \frac{1}{2^2} + \frac{3}{2^3} + \frac{5}{2^4} + \dots + \frac{2n-3}{2^n} + \frac{2n-1}{2^{n+1}}$$

subtracting, we get

$$\begin{aligned} \frac{1}{2}S &= \frac{1}{2} + \frac{2}{2^2} + \frac{2}{2^3} + \dots + \frac{2}{2^n} - \frac{2n-1}{2^{n+1}} \\ &= \frac{1}{2} + 2 \left(\frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \dots + \frac{1}{2^n} \right) - \frac{2n-1}{2^{n+1}} \\ &= \frac{1}{2} + \frac{2 \cdot \frac{1}{2^2} \left\{ 1 - \left(\frac{1}{2} \right)^{n-1} \right\}}{1 - \frac{1}{2}} - \frac{2n-1}{2^{n+1}} \end{aligned}$$

$$\therefore S = 3 - \frac{2n+3}{2^n}$$

24 Go through options.

$$\begin{aligned} \text{☞ Alternatively } 11 + 102 + 1003 + \dots \\ &= (10 + 1) + (100 + 2) + (1000 + 3) \\ &= (10 + 100 + 1000 + \dots) + (1 + 2 + 3 + \dots) \end{aligned}$$

25 Go through options.

$$\text{☞ Alternatively } T_n = 1 + 3 + 9 + \dots$$

$$\text{or } T_n = \frac{1}{2} (3^n - 1)$$

$$\begin{aligned} \therefore S_n &= t_1 + t_2 + t_3 + \dots + t_n \\ S_n &= \frac{1}{2} [(3^1 + 3^2 + 3^3 + \dots + 3^n) - n] \\ &= \frac{1}{2} \left[\frac{3(3^n - 1)}{2} - n \right] \\ &= \frac{1}{4} [3(3^n - 1) - 2n] = \frac{3}{4} (3^n - 1) - \frac{n}{2} \end{aligned}$$

$$26 \quad S_n = 2 + 5 + 14 + 41 + \dots \text{ to } n \text{ terms} \quad \dots (i)$$

$$\text{and } S_n = 2 + 5 + 14 + 41 + \dots \text{ to } n \text{ terms} \quad \dots (ii)$$

subtracting, we get

$$0 = 2 + 3 + 9 + 27 + \dots - t_n$$

$$\Rightarrow t_n = 2 + 3 + 9 + 27 + \dots$$

$$\Rightarrow t_n = 2 + \{3 + 9 + 27 + \dots \text{ to } (n-1) \text{ terms}\}$$

$$\Rightarrow t_n = 2 + \frac{3(3^{n-1} - 1)}{2}$$

$$\therefore S_n = t_1 + t_2 + t_3 + \dots + t_n$$

$$\begin{aligned} S_n &= \Sigma t_n = t_1 + t_2 + t_3 + \dots + t_n \\ &= \Sigma \left\{ 2 + \frac{3(3^{n-1} - 1)}{2} \right\} \\ &= \Sigma \left\{ 2 + \frac{3^n}{2} - \frac{3}{2} \right\} = \Sigma \left\{ \frac{1}{2} + \frac{3^n}{2} \right\} \\ &= \left\{ \frac{n}{2} + \frac{1}{2} (3 + 3^2 + 3^3 + \dots + 3^n) \right\} \\ &= \left\{ \frac{n}{2} + \frac{1}{2} \left(\frac{3 \cdot (3^n - 1)}{2} \right) \right\} \\ &= \left\{ \frac{n}{2} + \frac{1}{4} 3 \cdot (3^n - 1) \right\} \\ &= \frac{3}{4} (3^n - 1) + \frac{n}{2} \end{aligned}$$

☞ **Alternatively** You should go through options.

Consider $n = 1, 2$ or 3 and verify the correct option.

As, when you put $n = 1$, in the correct formula you will get $S_1 = 2$

$$\text{and at } n = 2, S_2 = 7 \quad (2 + 5)$$

$$\text{and at } n = 3, S_3 = 21 \quad (2 + 5 + 14)$$

$$27 \quad S_n = 1 + 2x + 3x^2 + 4x^3 + \dots$$

$$x \cdot S_n = x + 2x^2 + 3x^3 + 4x^4 + \dots$$

subtracting, we get

$$(1 - x)S_n = (1 + x + x^2 + x^3 + \dots + x^{n-1}) - nx^n$$

$$(1 - x)S_n = \frac{1 - x^n}{(1 - x)} - nx^n$$

$$\Rightarrow S_n = \frac{1 - x^n}{(1 - x)^2} - \frac{nx^n}{(1 - x)}$$

☞ **Alternatively** Go through options.

28 Solve through options.

☞ **NOTE** This is very similar to the previous problem.

$$29 \quad \text{Let } S_n = 1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \dots + \frac{n}{3^{n-1}}$$

$$\therefore \frac{1}{3}S_n = \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \frac{4}{3^4} + \dots + \frac{n-1}{3^{n-1}} + \frac{n}{3^n}$$

subtracting, we get

$$\frac{2}{3}S_n = \left(1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \dots \right) - \frac{n}{3^n}$$

$$\Rightarrow \frac{2}{3}S_n = \left[\frac{1 \left\{ 1 - \left(\frac{1}{3} \right)^n \right\}}{1 - \frac{1}{3}} \right] - \frac{n}{3^n}$$

$$\Rightarrow \frac{2}{3}S_n = \frac{3}{2} \left\{ 1 - \left(\frac{1}{3} \right)^n \right\} - \frac{n}{3^n}$$

$$\Rightarrow S_n = \frac{9}{4} - \frac{3}{4} \left(\frac{3 + 2n}{3^n} \right)$$

- 30.** Let $S_n = 3 \cdot 2 + 5 \cdot 2^2 + 7 \cdot 2^3 + \dots + (2n-1) \cdot 2^n$
 $\therefore 2S_n = 3 \cdot 2^2 + 5 \cdot 2^3 + \dots + (2n-1) \cdot 2^n + (2n+1) \cdot 2^{n+1}$
 subtracting, we get
 $-S_n = 3 \cdot 2 + 2 \cdot 2^2 + 2 \cdot 2^3 + \dots + 2 \cdot 2^n - (2n+1) \cdot 2^{n+1}$
 $= 3 \cdot 2 + 2(2^2 + 2^3 + \dots + 2^n) - (2n+1) \cdot 2^{n+1} = \text{etc}$
 Now, simplify by yourself.

31 Go through options.

32
$$t_n = n(2n+1)^2$$

$$= 4n^3 + 4n^2 + n$$

 $\therefore S_n = \sum t_n = 4\sum n^3 + 4\sum n^2 + \sum n$
 then put $n = 20$

33
$$t_n = \frac{n^2(n+1)^2}{4n^2} = \frac{1}{4}(n^2 + 2n + 1)$$

 $\therefore S_n = \sum t_n = \frac{1}{4}[\sum n^2 + 2\sum n + n]$
 then put $n = 16$.

34 Let a, b, c and d be the required numbers, then a, b, c are in G.P. and b, c, d are in A.P.

a	b	c	d	
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
a	b	$b+6$	$b+12$	(common difference = 6)
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
$b+12$	b	$b+6$	$b+12$	($\because a = d$)

Now, since a, b, c are in G.P.

$$\therefore \frac{c}{b} = \frac{b}{a}$$

$$\Rightarrow \frac{b+6}{b} = \frac{b}{b+12}$$

$$\Rightarrow (b+6)(b+12) = b^2$$

$$\Rightarrow b = -4$$

$$\therefore c = b+6 = -4+6 = 2$$

☞ Alternatively Go through options.

35 Let a be the first term and r be the common ratio of the G.P. Then

$$S_\infty = a + ar + ar^2 + \dots = 16 = \frac{a}{1-r}, |r| < 1$$

$$\therefore \frac{a}{1-r} = 16 \quad \dots(i)$$

 and $a^2 + (ar)^2 + (ar^2)^2 + \dots = 153\frac{3}{5}$

$$\Rightarrow a^2 + a^2r^2 + a^2r^4 + \dots = \frac{768}{5}$$

$$\Rightarrow \frac{a^2}{1-r^2} = \frac{768}{5} \quad \dots(ii)$$

Dividing eq. (ii) by eq. (i), we get

$$\frac{\frac{a^2}{(1-r^2)}}{\frac{a}{(1-r)}} = \frac{\frac{768}{5}}{16}$$

$$\Rightarrow \frac{a}{1+r} = \frac{48}{5} \quad \dots(iii)$$

Dividing eq. (i) by eq. (iii)

$$\frac{\frac{a}{(1-r)}}{\frac{a}{(1+r)}} = \frac{16}{48} \Rightarrow \frac{1+r}{1-r} = \frac{5}{3}$$

$$\Rightarrow \frac{1}{r} = 4 \quad (\text{By componendo-dividendo rule})$$

$$\Rightarrow r = \frac{1}{4}$$

$$\therefore a = 16 \times \left(1 - \frac{1}{4}\right) \quad (\text{from Eq. (i)})$$

$$a = 12$$

$$\therefore T_4 = ar^3 = 12 \times \left(\frac{1}{4}\right)^3 = \frac{3}{16}$$

36 $x = 1 + a + a^2 + a^3 + \dots \infty$
 $\therefore x = \frac{1}{1-a} \quad \because |a| < 1$

Similarly, $y = \frac{1}{1-b}$

Let $S = 1 + ab + a^2b^2 + \dots \infty$
 $= 1 + ab + (ab)^2 + \dots \infty$
 $= \frac{1}{1-ab} \quad (\because |ab| < 1) \quad \dots(i)$

Now, go through option

$$\frac{xy}{x+y-1} = \frac{\frac{1}{1-a} \times \frac{1}{1-b}}{\frac{1}{1-a} + \frac{1}{1-b} - 1} = \frac{1}{1-ab}$$

Hence, $S = 1 + ab + a^2b^2$
 $= \frac{1}{1-ab} = \frac{xy}{x+y-1}$

37 His first payment = ₹ 100

His second payment = ₹ 90 = $\left(100 \times \frac{9}{10}\right)$

His third payment = ₹ 81 etc. = $\left(90 \times \frac{9}{10}\right)$

$\therefore$ The annual payments are 100, 90, 81, ... which are in G.P. with common ratio $\frac{9}{10}$ (< 1)

Therefore the sum to infinity of this G.P.
 $= 100 + 90 + 81 + \dots \infty$

$$= \frac{100}{1 - \frac{9}{10}} = \frac{100}{\frac{1}{10}} = 1000$$

Hence the person can receive maximum amount of ₹ 1000.

$$38 \quad S_{\infty} = \frac{1}{4} - \frac{3}{16} + \frac{9}{64} - \frac{27}{256} + \dots$$

$$S_{\infty} = \frac{\frac{1}{4}}{1 - \left(-\frac{3}{4}\right)} = \frac{\frac{1}{4}}{\frac{7}{4}} \quad \left(\because r = -\frac{3}{4} \right)$$

$$S_{\infty} = \frac{1}{7}$$

$$39 \quad a + ar = \frac{5}{3} \Rightarrow a(1 + r) = \frac{5}{3} \quad \dots(i)$$

$$\text{and} \quad \frac{a}{1 - r} = 3 \quad \dots(ii)$$

Dividing eq. (i) by eq. (ii), we get

$$(1 + r)(1 - r) = \frac{5}{9} \Rightarrow 1 - r^2 = \frac{5}{9}$$

$$\Rightarrow \quad r^2 = \frac{4}{9}$$

$$\Rightarrow \quad r = \pm \frac{2}{3}$$

If $r = \frac{2}{3}$, then $a = 1$ and if $r = -\frac{2}{3}$, then $a = 5$

- 40 Distance covered by ball just before it hits the ground for the first time = 96 ft.

In the first round (i. e. going up and returning to the earth)

$$\text{ball covers the distance} = \left(96 \times \frac{2}{3}\right) \times 2$$

In the third round distance covered by ball

$$= 96 \times \left(\frac{2}{3}\right)^2 \times 2$$

In the fourth round distance covered by ball

$$= 96 \times \left(\frac{2}{3}\right)^3 \times 2 \text{ etc.}$$

 **NOTE** In each round ball covers equal distance while going up and coming down to the earth).

Hence the required distance

$$= 96 + 2 \left[96 \times \frac{2}{3} + 96 \times \frac{4}{9} + 96 \times \frac{8}{27} + \dots \right]$$

$$= 96 + 96 \times 2 \left[\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \dots \right]$$

$$= 96 + 192 \left(\frac{\frac{2}{3}}{1 - \frac{2}{3}} \right) = 96 + 192 \left(\frac{\frac{2}{3}}{\frac{1}{3}} \right)$$

$$= 96 + 192 \times 2 = 480 \text{ ft}$$

✓ Introductory Exercise 18.3

- 1 Let A, G, H be the Arithmetic mean, Geometric mean and Harmonic mean of a and b respectively.

Then, we have

$$A = \frac{a+b}{2}, \quad G = \sqrt{ab}, \quad H = \frac{2ab}{a+b}$$

$$\therefore \quad A \cdot H = \frac{a+b}{2} \times \frac{2ab}{a+b} = ab = G^2$$

$$\begin{aligned} \text{Again } A - G &= \frac{a+b}{2} - \sqrt{ab} = \frac{a+b-2\sqrt{ab}}{2} \\ &= (\sqrt{a})^2 + (\sqrt{b})^2 - 2\sqrt{ab} = \frac{(\sqrt{a} - \sqrt{b})^2}{2} \geq 0 \end{aligned}$$

$$\therefore \quad A \geq G$$

$$\text{Again since } A \cdot H = G^2 = G \times G$$

$$\therefore \quad H \leq G \quad (\because A \geq G)$$

$$\text{i. e.} \quad G \geq H$$

$$\therefore \quad A \geq G \geq H$$

 **Alternatively** Let $a = 6$ and $b = 24$

$$\therefore \quad \text{A.M.} = \frac{6+24}{2} = 15$$

$$\text{and} \quad \text{G.M.} = \sqrt{6 \times 24} = 12$$

$$\text{and} \quad \text{H.M.} = \frac{2 \times 6 \times 24}{6+24} = 9.6$$

$$\therefore \quad \text{A.M.} > \text{G.M.} > \text{H.M.}$$

Hence (c) is the correct option.

 **NOTE** The equality depends upon values as $a = 2$ and $b = 2$

then $\text{A.M.} = 2$ and $\text{G.M.} = 2$ and $\text{H.M.} = 2$

then $\text{A.M.} = \text{G.M.} = \text{H.M.}$

- 2 Let a, b, c, d, e are taken such that a, b, c are in A.P. and b, c, d are in G.P. and c, d, e are in H.P. then

$$a, b, c, d, e$$

$$2, 4, 6, 9, 18$$

$$\therefore \quad a \cdot e = c^2$$

Hence choice (b) is correct

$$3 \quad \frac{a+b}{2} = 10 \Rightarrow a+b = 20$$

$$\sqrt{a \cdot b} = 8 \Rightarrow ab = 64$$

$$\therefore \quad a, b = 16, 4 \text{ or } 4, 16$$

$$\therefore \quad \text{H.M.} = \frac{2 \times 4 \times 16}{(4+16)} = 6.4$$

 **NOTE** $\text{G.M.} = \sqrt{\text{A.M.} \times \text{H.M.}}$

Hint Choices (a) and (b) are clearly wrong since $\text{H.M.} \leq \text{G.M.} \leq \text{A.M.}$

Now you are left with choices (c) and (d) only.

✓ Level 01 Basic Level Exercise

$$\begin{aligned}
 1 \quad S_n &= \frac{a(1-r^n)}{1-r}; \quad r < 1 \\
 S_n &= \frac{4095}{1024} = \frac{3(1-r^n)}{1-\frac{1}{4}} \\
 \Rightarrow \frac{4095}{1024} &= 4(1-r^n) \Rightarrow \frac{4095}{4096} = 1-r^n \\
 \Rightarrow r^n &= \frac{1}{4096} \Rightarrow r^n = \left(\frac{1}{4}\right)^6 \\
 \Rightarrow \left(\frac{1}{4}\right)^n &= \left(\frac{1}{4}\right)^6 \quad \left(\because r = \frac{1}{4}\right) \\
 \Rightarrow n &= 6
 \end{aligned}$$

2 Since $a > l$, therefore $r < 1$

Now, since $S_n = \frac{a-lr}{1-r}$ if $r < 1$

$$\begin{aligned}
 \therefore \frac{5187}{625} &= \frac{5 - \frac{32}{625}r}{1-r} \\
 (1-r) \frac{5187}{625} &= 5 - \frac{32r}{625} \Rightarrow r = \frac{2}{5}
 \end{aligned}$$

$$3 \quad r = \frac{2\sqrt{2}}{2\sqrt{3}} = \frac{\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$$

$$\begin{aligned}
 \therefore S_\infty &= \frac{a}{1-r}; |r| < 1 \\
 &= \frac{2\sqrt{3}}{1-\sqrt{\frac{2}{3}}} = \frac{2\sqrt{3} \times \sqrt{3}}{\sqrt{3}-\sqrt{2}} = \frac{6}{\sqrt{3}-\sqrt{2}} = 6(\sqrt{3} + \sqrt{2})
 \end{aligned}$$

4 (Sum of odd terms – sum of even terms) = 72 – 63

$$\begin{aligned}
 \Rightarrow \frac{\text{number of terms}}{2} \times \text{common difference} &= 9 \\
 \Rightarrow nd &= 18 \quad \dots(i)
 \end{aligned}$$

($n \rightarrow$ number of terms, $d \rightarrow$ common difference)

$$\begin{aligned}
 \text{Again } l - a &= 16.5 \\
 \Rightarrow (n-1)d &= 16.5 \quad \dots(ii)
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{From eqs. (i) and (ii), we get} \\
 d = 1.5 \quad \therefore n &= 12
 \end{aligned}$$

5 Go through option

$$\left\{ \begin{array}{l} \text{First Pocket} \\ 4, 7, 10 \end{array} \right\} \left\{ \begin{array}{l} \text{Second Pocket} \\ 5, 7, 9 \end{array} \right\}$$

$$\begin{aligned}
 6 \quad a \cdot b \cdot c &= a \cdot ar \cdot ar^2 = 216 \\
 \Rightarrow b = ar &= 6 \quad \dots(i) \\
 \text{and } ab + bc + ac &= a \cdot ar + ar \cdot ar^2 + a \cdot ar^2 = 126 \\
 \Rightarrow a^2r + a^2r^3 + a^2r^2 &= 126 \\
 \Rightarrow a^2r + a^2r^3 &= 90 \quad (\because ar = 6) \\
 \Rightarrow 6a + 36r &= 90 \\
 \Rightarrow a + 6r &= 15 \quad \dots(ii)
 \end{aligned}$$

From equation (i) and (ii), we get

$$\begin{aligned}
 \frac{6}{r} + 6r &= 15 \\
 \Rightarrow 6r^2 - 15r + 6 &= 0 \\
 \Rightarrow r &= 2 \quad \text{or} \quad \frac{1}{2}
 \end{aligned}$$

$$\therefore a, ar, ar^2 = 3, 6, 12 \text{ or } 12, 6, 3$$

$$\therefore a + b + c = 21$$

☞ **Alternatively** Go through options

$$7 \quad \text{Let } a, b, c, d \text{ be in A.P. then } \frac{ad}{bc} = \frac{9}{10}$$

$$\text{Also } a + b + c + d = 36$$

$$\Rightarrow (a + d) = (b + c) = 18$$

Now, go through options.

8 Let a, b, c, d be four integers in A.P.

$$\text{then } a + d = b + c = 12$$

Since $a \cdot b \cdot c \cdot d = 945$, it means one of the a, b, c and d has its unit digit 5.

$\therefore$ Therefore the obvious possibility is $b = 5$

So, if $b = 5$, then $c = 7$

$$\therefore a = 3 \quad \text{and} \quad d = 9$$

$$\therefore a \cdot b \cdot c \cdot d = 3 \times 5 \times 7 \times 9 = 945$$

Hence the values of a, b, c and d are correct.

$$\text{Therefore } a \times d = 3 \times 9 = 27$$

☞ **Alternatively** Option a is not suitable as

$$\begin{aligned}
 30 &= 1 \times 30 & \times \\
 30 &= 2 \times 15 & \times \\
 30 &= 3 \times 10 & \times \\
 30 &= 5 \times 6 & \times
 \end{aligned}$$

Since difference between a and d must be divisible by 3.

Option (c) is not suitable as

$$\begin{aligned}
 35 &= 1 \times 35 & \times \\
 35 &= 5 \times 7 & \times
 \end{aligned}$$

Since the difference between a and d is not divisible by 3.

Option (d) is also wrong as

$$\begin{aligned}
 39 &= 1 \times 39 & \times \\
 &= 3 \times 13 & \times
 \end{aligned}$$

Since the difference between a and d is not divisible by 3.

Choice (b) is correct as

$$\begin{aligned}
 27 &= 1 \times 27 & \times \\
 27 &= 3 \times 9 & \checkmark
 \end{aligned}$$

$$\text{If } a = 3 \quad \text{and} \quad d = 9$$

$$\text{then } b = 5 \quad \text{and} \quad c = 7$$

$$\therefore a \cdot b \cdot c \cdot d = 945$$

☞ **NOTE** Choices (a) and (d) are wrong since 30 and 39 are not the factors of 945.

☞ **Alternatively** Find the factors of 945 and then find the required values of b and c under the condition

$$a + b + c + d = 24$$

$$\begin{aligned}
 9 \quad T_{11} + T_{12} + T_{13} &= 114 \\
 \Rightarrow T_{12} &= \frac{114}{3} = 38 \\
 \Rightarrow a + 11d &= 38 \quad \dots(i) \\
 \text{and } T_{21} + T_{22} + T_{23} &= 204 \\
 \Rightarrow T_{22} &= 68 \\
 \Rightarrow a + 21d &= 68 \quad \dots(ii)
 \end{aligned}$$

From eq. (i) and (ii)

$$10d = 30 \Rightarrow d = 3$$

$$\therefore a = 5$$

$$\therefore T_1 + T_2 + T_3 = 5 + 8 + 11 = 24$$

$$\begin{aligned}
 \text{Alternatively } (T_1 + T_2 + T_3) + (T_{21} + T_{22} + T_{23}) \\
 = 2(T_{11} + T_{12} + T_{13})
 \end{aligned}$$

$$\Rightarrow (T_1 + T_2 + T_3) + 204 = 2 \times 114$$

$$\Rightarrow T_1 + T_2 + T_3 = 24$$

$$10 \quad a + ar + ar^2 + ar^3 + \dots \infty = 4$$

$$\Rightarrow \frac{a}{1-r} = 4$$

$$\text{or } \frac{a^3}{(1-r)^3} = 64 \quad \dots(i)$$

$$\begin{aligned}
 \text{and } a^3 + (ar)^3 + (ar^2)^3 + (ar^3)^3 + \dots \infty &= 192 \\
 = \frac{a^3}{1-r^3} &= 192 \quad \dots(ii)
 \end{aligned}$$

From eqs. (i) and (ii)

$$64(1-r)^3 = (1-r^3)192$$

$$\Rightarrow 2r^3 + 3r^2 - 3r - 2 = 0$$

$$r = -\frac{1}{2}, 1, -2$$

$$\text{But } |r| < 1$$

$$\therefore r = -\frac{1}{2} \text{ is the only admissible value}$$

$$\therefore \frac{a}{1-r} = 4 \Rightarrow \frac{a}{1-\left(-\frac{1}{2}\right)} = 4 \Rightarrow a = 6$$

$$11 \quad \frac{18500 - 12000}{500} = 13 \text{ years}$$

He has to work minimum 13 years to reach the highest scale of 18500.

$$12 \quad 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, \dots, 187$$

$$3, 7, 11, 15, 19, 23, 27, 31, 35, 39, \dots, 187$$

$$\text{The Common terms are } 7, 19, 31, 43, \dots, 187$$

$$\text{Therefore number of such terms} = \left(\frac{187-7}{12} \right) + 1 = 16$$

$$\begin{aligned}
 13 \quad 3^{1/3} \cdot 9^{1/18} \cdot 27^{1/81} \dots &= 3^{1/3} \cdot 3^{1/9} \cdot 3^{1/27} \dots \\
 &= 3^{1/3 + 1/9 + 1/27 + \dots} \\
 &= 3^{\left(\frac{1/3}{1-1/3} \right)} = 3^{1/2} = \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 14 \quad 1 + 4 + 9 + 16 + \dots + n^2 \\
 = 1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}
 \end{aligned}$$

15 Put $n = 2$ and 3 and then check for the correct choice

$$\text{Sum of 2 terms} = 11$$

$$\text{and Sum of 3 terms} = 46$$

$$\text{at } n = 2, \quad \frac{1}{6} \times 2(3)(11) = 11$$

$$\text{and at } n = 3, \quad \frac{1}{6} \times 3 \times 4 \times 23 = 46$$

Hence choice (c) is correct.

16 Let x, y, z be $1, 2, 4$ respectively

$$\therefore a^1 = b^2 = c^4$$

$$\Rightarrow 16^1 = 4^2 = 2^4$$

$$\Rightarrow a = 16, \quad b = 4 \quad \text{and} \quad c = 2$$

$$\therefore \log_4 16 = \log_2 4$$

$$\Rightarrow 2 = 2$$

Hence choice (a) is correct.

17 Since 913952088 is divisible by 7

$$\text{Alternatively } a = 119$$

$$a + d = 119 + 7 = 126$$

$$a + 2d = 119 + 14 = 133$$

$\therefore$ The numbers which are divisible by 7 are

$$119, 126, 133, \dots, 113113 \dots$$

$$\therefore \text{Number of terms} = \left(\frac{113113 - 119}{7} \right) + 1 = 16143$$

$$\begin{aligned}
 \therefore S_{16143} &= \left(\frac{119 + 113113}{2} \right) \times 16143 \\
 &= 913952088
 \end{aligned}$$

$$\text{Hint The unit digit will be 8 as } \left(\frac{9+3}{2} \right) \times 3$$

$$\Rightarrow 6 \times 3 \Rightarrow 8$$

Hence only choice (b) is appropriate.

18 Going through options

$$\begin{aligned}
 S_{21} - S_{20} &= T_{21} \\
 \Rightarrow [3(21)^2 + 5] - [3(20)^2 + 5] &= 3 \times 441 - 3 \times 400 \\
 &= 3(41) = 123
 \end{aligned}$$

Thus choice (c) is correct.

19 Let us consider choice (a)

$$\text{then } b - a = 3 - 4 = -1$$

$$c - b = 2 - 3 = -1$$

$$a = 4$$

Which are not in G.P., hence wrong.

Now, let us consider choice (c)

$$b - a = 1$$

$$c - b = 1$$

$$a = 1$$

Since $b - a, c - b, a$ (i.e. 1, 1, 1) are in G.P. hence correct.

$$\begin{aligned}
 20 \quad n=1, \quad S_n &= \frac{1}{2} \\
 n=2, \quad S_n &= \frac{5}{4} = \left(\frac{1}{2} + \frac{3}{4}\right) \\
 n=3, \quad S_n &= \frac{17}{8} = \left(\frac{1}{2} + \frac{3}{4} + \frac{7}{8}\right)
 \end{aligned}$$

Choice (a) is wrong

$$\text{Since at } n=2, \quad S_2 = 3 \neq \frac{5}{4}$$

Choice (b) is also wrong

$$\text{Since at } n=2, \quad S_2 = \frac{3}{4} \neq \frac{5}{4}$$

Choice (c) is also wrong

$$\text{Since at } n=2, \quad S_2 = 3 \neq \frac{5}{4}$$

Choice (d) is correct

$$\text{Since at } n=1, \quad S_1 = \frac{1}{2}$$

$$\text{at } n=2, \quad S_2 = \frac{5}{4}$$

$$\text{at } n=3, \quad S_3 = \frac{17}{8}$$

- 21 The required numbers are 1001, 1005, 1009, ..., 9997 and total number of terms in the above A.P. is 2250.

$$\begin{aligned}
 \therefore S_{2250} &= \frac{(1001 + 9997)}{2} \times 2250 = \frac{10998}{2} \times 2250 \\
 &= 5499 \times 2250 = 12372750
 \end{aligned}$$

Hint Choice (a) is the only suitable option since last two digits in the resultant value must be 50.

- 22 Let 3, 5, 7, 9, 11, 13, ... are in A.P. where 5, 7, 9 and 11 are 4 arithmetic means between 3 and 13.

Hence choice (b) is correct.

$$\begin{aligned}
 \text{Since } \frac{4}{2}(3 + 13) &= 5 + 7 + 9 + 11 \\
 32 &= 32
 \end{aligned}$$

- 23 Let 1, 2, 4, 8, 16 are in G.P. then 2, 4, 8 are 3 geometric means between 1 and 16. Hence choice (c) is correct.

$$\begin{aligned}
 \text{Since } (1 \times 16)^{3/2} &= 2 \times 4 \times 8 \\
 64 &= 64
 \end{aligned}$$

24. $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$
 $\Rightarrow a_1 + (a_1 + 4d) + (a_1 + 9d) + (a_1 + 14d)$
 $\quad + (a_1 + 19d) + (a_1 + 23d) = 225$
 $\Rightarrow 6a_1 + 69d = 225 \quad \dots(i)$
 Now, $a_1 + a_2 + a_3 + a_4 + \dots + a_{23} + a_{24} = 24a_1 + 276d$
 $= 4(6a_1 + 69d)$
 $= 4 \times 225 = 900$

25. $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 + \dots$
 $= (1-2)(1+2) + (3-4)(3+4) + (5-6)(5+6)$
 $\quad + (7-8)(7+8) + \dots$

$$\begin{aligned}
 &= -(1+2) - (3+4) - (5+6) \dots \\
 &= -[(1+2) + (3+4) + (5+6) + \dots] \\
 &= -[1+2+3+4+5+6+\dots] = -\left[\frac{n(n+1)}{2}\right]
 \end{aligned}$$

Alternatively Go through options.

26. If $n=1$, then $6+8=14$

$$\text{If } n=2, \text{ then } 66+88=154$$

$$\text{If } n=3, \text{ then } 666+888=1554$$

Now, go through options.

If you put $n=1, 2, 3$ etc. in choice (c) you will find satisfactory results.

27. If $n=1$, then sum of 1st group = 1

$$\text{If } n=2, \text{ then sum of 2nd group} = 35$$

$$\text{If } n=3, \text{ then sum of 3rd group} = 405$$

Now, going through options, we get choice (b) is correct

$$\text{since at } n=1, \quad Sg_1 = 1$$

$$\text{at } n=2, \quad Sg_2 = 35$$

$$\text{at } n=3, \quad Sg_3 = 405$$

28. Let the numbers be $a-d, a, a+d$

then $(a-d)^2, a^2, (a+d)^2$ are in G.P.

$$\therefore a^4 = (a-d)^2(a+d)^2$$

$$\Rightarrow d^4 - 2a^2d^2 = 0$$

$$\Rightarrow d = 0, \pm \sqrt{2}a$$

Hence d has three values.

29. Since we know that when $a \cdot b \cdot c = k$ (any constant value) then the minimum value of $a+b+c$ is obtained

$$\text{when } a = b = c$$

$$\therefore b^3 = 4 = 2^2$$

$$\Rightarrow b = 2^{2/3}$$

Alternatively Since a, b, c are in A.P.

$$\text{Therefore } a = b-d \text{ and } c = b+d$$

where d is the common difference.

$$\text{Now, } abc = 4 \Rightarrow (b-d) \cdot b \cdot (b+d) = 4$$

$$\Rightarrow b(b^2 - d^2) = 4$$

$$\text{But } b \cdot (b^2 - d^2) < b \cdot b^2 \quad (\because b^2 - d^2 < b^2)$$

$$\therefore b(b^2 - d^2) < b^3$$

$$4 < b^3$$

$$\Rightarrow 2^2 < b^3$$

$$\Rightarrow 2^{2/3} < b$$

Hence the minimum value of b is $2^{2/3}$.

30. Let the three numbers be $\frac{a}{r}, a, ar$. As the numbers form an increasing G.P.

Therefore $r > 1$

$$\text{Also } \frac{a}{r}, 2a, ar \text{ are in A.P.}$$

$$\therefore 2(2a) = \frac{a}{r} + ar$$

$$\begin{aligned} \Rightarrow r^2 - 4r + 1 &= 0 \\ \Rightarrow r &= 2 \pm \sqrt{3} \\ \Rightarrow r &= 2 + \sqrt{3} \quad (\because 2 - \sqrt{3} < 1) \end{aligned}$$

Hint If you go through options then options (a) and (d) are inadmissible.

$$\text{Since } 2 - \sqrt{3} < 1$$

$$\text{and } \sqrt{3} - 2 < 1$$

Now you have only two probable choices consider three terms in G.P. with $r = 2 + \sqrt{3}$ or $r = 3 + \sqrt{2}$ and then satisfy the condition.

- 31.** Best approach is to go through options. Choice (c) is correct.

Let $r = 4$, then a, b, c can be 1, 4, 16

$$\therefore 16 > 4 \times 4 - 3 \times 1$$

$$\Rightarrow 16 > 13$$

Let $r = -5$ then a, b, c can be 1, -5, 25

$$\therefore 25 > -5 \times 4 - 1 \times 3$$

$$\Rightarrow 25 > -23$$

Let $r = 2$ then a, b, c can be 1, 2, 4

$$\therefore 4 > 4 \times 2 - 3 \times 1$$

$$\Rightarrow 4 > 5, \quad \text{which is wrong.}$$

Hence choice (a) is wrong and choice (c) is correct also the range of choice (b) is the subset of the range of choice (c).

☞ Alternatively Let a, b, c be the first three consecutive terms of G.P. with common ratio r , then

$$b = ar \quad \text{and} \quad c = ar^2$$

Now,

$$c > 4b - 3a$$

$$\Rightarrow ar^2 > 4ar - 3a$$

$$\Rightarrow r^2 > 4r - 3$$

$$\Rightarrow r^2 - 4r + 3 > 0$$

$$\Rightarrow (r - 1)(r - 3) > 0$$

$$\Rightarrow r < 1 \quad \text{or} \quad r > 3$$

- 32.** Let d be the common difference of the A.P.

$$\begin{aligned} \text{Then } a_3 \cdot a_7 \cdot a_{11} &= (22 - 4d) \cdot 22 \cdot (22 + 4d) \\ &= 88(121 - 4d^2) \end{aligned}$$

Obviously, R.H.S. is greatest for $d = 0$

- 33** Given that A.M. = 2 G.M.

$$\Rightarrow \frac{a+b}{2} = 2\sqrt{ab}$$

$$\Rightarrow a + b = 4\sqrt{ab}$$

Now, let us consider option (c).

$$\therefore (2 + \sqrt{3}) + (2 - \sqrt{3}) = 4\sqrt{(2 + \sqrt{3})(2 - \sqrt{3})}$$

$$4 = 4 \cdot 1$$

$$4 = 4$$

Hence option (c) is correct.

$$\text{☞ Alternatively } a + b = 4\sqrt{ab}$$

$$\Rightarrow (a + b)^2 = 16ab$$

$$\Rightarrow a^2 + b^2 + 2ab = 16ab$$

$$\Rightarrow a^2 + b^2 = 14ab$$

$$\Rightarrow \frac{a}{b} + \frac{b}{a} = 14$$

$$\Rightarrow x + \frac{1}{x} = 14 \quad \left(\because \frac{a}{b} = x \right)$$

$$\Rightarrow x^2 - 14x + 1 = 0$$

$$\Rightarrow x = \frac{14 \pm \sqrt{192}}{2}$$

$$\Rightarrow x = 7 \pm 4\sqrt{3}$$

$$\Rightarrow \frac{a}{b} = 7 + 4\sqrt{3} = (2)^2 + (\sqrt{3})^2 + 4\sqrt{3}$$

$$\frac{a}{b} = (2 + \sqrt{3})^2 = \frac{2 + \sqrt{3}}{2 - \sqrt{3}}$$

$$\text{Now } x = 7 - 4\sqrt{3}$$

$$\Rightarrow x = \frac{2 - \sqrt{3}}{2 + \sqrt{3}} \neq \frac{a}{b}, \text{ since } a \text{ should be greater than } b.$$

$$\text{Hence, } x \neq 7 - 4\sqrt{3}$$

- 34** The two sequences are

$$17, 21, 25, 29, 33, 37, 41, \dots, 417$$

$$16, 21, 26, 31, 36, 41, \dots, 466$$

The common terms are

$$21, 41, 61, 81, \dots, 381, 401$$

$\therefore$ Number of terms (which are common) = 20

$$\begin{aligned} \text{35 } S_n &= \frac{1}{\sqrt{1} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{5}} + \dots + \frac{1}{\sqrt{2n-1} + \sqrt{2n+1}} \\ &= \frac{1}{2} [(\sqrt{3} - \sqrt{1}) + (\sqrt{5} - \sqrt{3}) + (\sqrt{7} - \sqrt{5}) + \dots \\ &\quad + \dots (\sqrt{2n+1} - \sqrt{2n-1})] \\ &= \frac{1}{2} (\sqrt{2n+1} - 1) \end{aligned}$$

☞ Alternatively Substitute $n = 1, 2, 3$ etc. and check the correct answer.

- 36** Let a be the first term and d be the common difference of the given A.P. then

$$T_m = \frac{1}{n} \Rightarrow a + (m-1)d = \frac{1}{n} \quad \dots(i)$$

$$T_n = \frac{1}{m} \Rightarrow a + (n-1)d = \frac{1}{m} \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we get

$$d = \frac{1}{mn} \quad \text{and} \quad a = \frac{1}{mn}$$

$$\begin{aligned} \therefore T_{mn} &= a + (mn-1)d \\ &= \frac{1}{mn} + (mn-1) \frac{1}{mn} = 1 \end{aligned}$$

☞ Alternatively Let $m = 2$ and $n = 4$, then

$$T_2 = \frac{1}{4} \quad \text{and} \quad T_4 = \frac{1}{2}$$

$$\therefore a + d = \frac{1}{4} \quad \dots(i)$$

$$\text{and } a + 3d = \frac{1}{2} \quad \dots(ii)$$

$$\Rightarrow 2d = \frac{1}{4} \Rightarrow d = \frac{1}{8}$$

$$\therefore a = \frac{1}{4} - \frac{1}{8} = \frac{1}{8} \quad [\text{from Eq. (i)}]$$

$$\begin{aligned} \therefore T_{mn} = T_8 &= a + 7d \\ &= \frac{1}{8} + 7 \times \frac{1}{8} = 1 \end{aligned}$$

Hence (c) is the correct.

37 Go through options

$$\text{Let } n = 2, \text{ then } S_n = 3 + 6 = 9$$

$$\therefore S_n = 2(3) + 2^2 - 1 = 9$$

$$\text{at } n = 3, S_n = 19$$

$$\therefore S_n = 3 \times 4 + 2^3 - 1 = 19$$

Hence choice (b) is correct.

☞ Alternatively $3 + 6 + 10 + 16 + \dots$

$$= (2 + 4 + 6 + 8 + \dots) + (1 + 2 + 4 + 8 + \dots)$$

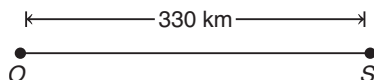
$$= n(n+1) + (2^n - 1)$$

38 Total number of cells $= 2 + (2^2 + 2^3 + 2^4 + \dots + 2^{12})$

$$= 2^1 + 2^2 + 2^3 + \dots + 2^{12}$$

$$= \frac{2(2^{12} - 1)}{2 - 1} = 8190$$

39



Div. + Su. $\rightarrow$ Total

1st day $30 + 20 \rightarrow 50$

2nd day $28 + 24 \rightarrow 52$

3rd day $26 + 28 \rightarrow 54$ and so on.

$$\therefore 50 + 52 + 54 + 56 + 58 + 60 = 330$$

Hence required number of days = 6

40 Best way is to go through options

$$\text{Let } n = 2, \text{ then } S_n = S_2 = 1 + (1 + 2) = 4$$

From option (b)

$$S_2 = \frac{2 \times 3 \times 4}{6} = 4$$

and for $n = 3$

$$S_n = S_3 = 1 + (1 + 2) + (1 + 2 + 3) = 10$$

$$\therefore \text{From choice (b), } S_3 = \frac{3 \times 4 \times 5}{6} = 10$$

Hence choice (b) is correct.

☞ Alternatively $T_n = \frac{n(n+1)}{2}$

$$\begin{aligned} \therefore S_n &= \Sigma \left[\frac{n(n+1)}{2} \right] = \Sigma \left[\frac{n^2 + n}{2} \right] = \frac{1}{2} [\Sigma n^2 + \Sigma n] \\ &= \frac{1}{2} \left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right] \\ &= \frac{n(n+1)}{4} \left[\frac{2n+1}{3} + 1 \right] \end{aligned}$$

$$= \frac{n(n+1)}{4} \left[\frac{2n+4}{3} \right] = \frac{n(n+1)(n+2)}{6}$$

$$\mathbf{41} \quad a = 2 \left(\begin{array}{l} \text{since } 3, 2, 1 \text{ are in A. P.} \\ b = 3 \left(\therefore 2, 3, 6 \text{ are in H. P.} \right) \text{ and } c = 6 \end{array} \right)$$

$$\therefore \left(\frac{1}{b} + \frac{1}{c} - \frac{1}{a} \right) \left(\frac{1}{c} + \frac{1}{a} - \frac{1}{b} \right) = \left(\frac{1}{3} + \frac{1}{6} - \frac{1}{2} \right) \left(\frac{1}{6} + \frac{1}{2} - \frac{1}{3} \right) = 0$$

Check the option (a)

$$\frac{3}{3^2} - \frac{2}{6} = 0, \text{ which is satisfied}$$

Again check the option (c)

$$\frac{2}{bc} - \frac{1}{b^2} = \frac{2}{18} - \frac{1}{9} = 0, \text{ which is also true.}$$

Hence (d) is the most appropriate answer.

42 Go through options

$$S_1 = 1 \cdot 3 \cdot 5 = 15$$

$$\text{and } S_2 = 1 \cdot 3 \cdot 5 + 3 \cdot 5 \cdot 7 = 120 \text{ etc.}$$

Now, put $n = 1$ in choice (b)

$$1(2 + 8 + 7 - 2) = 15$$

again put $n = 2$ in choice (b)

$$2(16 + 32 + 14 - 2) = 120$$

Hence choice (b) is correct.

☞ NOTE For the certainty of the result you can check for $n = 3$ also.

$$\begin{aligned} \mathbf{43} \quad \text{☞ Alternatively } T_n &= (2n-1)(2n+1)(2n+3) \\ &= 8n^3 + 12n^2 - 2n - 3 \end{aligned}$$

$$\begin{aligned} \therefore S_n &= 8 \Sigma n^3 + 12 \Sigma n^2 - 2 \Sigma n - 3n \\ &= 8 \left[\frac{n(n+1)}{2} \right]^2 + \frac{12(n)(n+1)(2n+1)}{6} \\ &\quad - \frac{2(n)(n+1)}{2} - 3n \\ &= n(2n^3 + 8n^2 + 7n - 2) \end{aligned}$$

$$\mathbf{43} \quad (1 - 2) + (3 - 4) + (5 - 6) + (7 - 8) + \dots + (99 - 100)$$

$$= (-1) + (-1) + (-1) + (-1) + \dots \text{ 50 times}$$

$$= -50$$

$$\mathbf{44} \quad \text{The sum of an infinite G.P.} = \frac{x}{1-r}$$

$$\therefore \frac{x}{1-r} = 5 \Rightarrow r = 1 - \frac{x}{5}$$

$$\therefore |r| < 1 \Rightarrow \left| 1 - \frac{x}{5} \right| < 1 \Rightarrow 0 < x < 10$$

Hence, choice (c) is the answer.

$$\mathbf{45} \quad 1 \times 1 + 0 = 1$$

$$2 \times 2 + 1 = 5$$

$$3 \times 3 + 5 = 14$$

$$4 \times 4 + 14 = 30$$

$$5 \times 5 + 30 = 55$$

$$6 \times 6 + 55 = 91$$

Hence, choice (a) is the answer.

46 Sum of the first 17 term = $17 \times 7 \frac{3}{17}$

Sum of the total 51 terms = $51 \times 21 \frac{9}{17}$

Sum of the last 34 terms = $51 \times 21 \frac{9}{17} - 17 \times 7 \frac{3}{17}$

Therefore, the required ratio = $\frac{17 \times 7 \frac{3}{17}}{51 \times 21 \frac{9}{17} - 17 \times 7 \frac{3}{17}}$

$$= \frac{17 \times \frac{122}{17}}{51 \times \frac{366}{17} - 17 \times \frac{122}{17}} = \frac{122}{3 \times 366 - 122}$$

$$= \frac{122}{122(3 \times 3 - 1)} = \frac{1}{8}$$

Hence, choice (b) is the correct one.

Hint Sum of n terms of an AP = $n \times$ mean of n terms

47 $n(n+1)(n+2)$

$$= n(n+1)(n+2) \frac{1}{4} [(n+3) - (n-1)]$$

$$= \frac{1}{4} [n(n+1)(n+2)(n+3) - (n-1)(n)(n+1)(n+2)]$$

Therefore, we have

$$3 \cdot 4 \cdot 5 = \frac{1}{4} [3 \cdot 4 \cdot 5 \cdot 6 - 2 \cdot 3 \cdot 4 \cdot 5]$$

$$4 \cdot 5 \cdot 6 = \frac{1}{4} [4 \cdot 5 \cdot 6 \cdot 7 - 3 \cdot 4 \cdot 5 \cdot 6]$$

$$\dots \dots \dots \dots \dots \dots \dots \dots$$

$$28 \cdot 29 \cdot 30 = \frac{1}{4} [28 \cdot 29 \cdot 30 \cdot 31 - 27 \cdot 28 \cdot 29 \cdot 30]$$

Adding up all the terms we get,

$$3 \cdot 4 \cdot 5 + 4 \cdot 5 \cdot 6 + 5 \cdot 6 \cdot 7 + \dots + 28 \cdot 29 \cdot 30$$

$$= \frac{1}{4} [28 \cdot 29 \cdot 30 \cdot 31 - 2 \cdot 3 \cdot 4 \cdot 5] = 188760$$

Hence choice (b) is the correct one.

48 $p^{21} + p^{22} + p^{23} + \dots + p^{399}$

$$= p^{21}(1 + p + p^2 + \dots + p^{378}) = p^{21} \left(\frac{p^{379} - 1}{p - 1} \right)$$

$$= (p^7)^3 \left(\frac{[(p^7)^{54} \times p] - 1}{p - 1} \right) = (1)^3 \left(\frac{(1 \times p) - 1}{p - 1} \right)$$

$$= 1 \times \left(\frac{p - 1}{p - 1} \right) = 1$$

Hence, choice (d) is the correct one.

- 49 Since $a > 0$ and $b \geq 9$. Therefore $a + b > 9$.
Similarly, since $a < 1$ and $b \leq 10$. Therefore $a + b < 11$.
Hence, choice (d) is the correct one.

Level 02 Higher Level Exercise

1. Let us consider $d = 1$, then the A.P. is

$$1, 2, 3, 4, 5, 6, 7, 8$$

$$\therefore 1 + 2 + 3 + 4 = 10$$

and $5 + 6 + 7 + 8 = 26$

Again $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$

$$\therefore 1 + 2 + 3 + 4 + 5 + 6 = 21$$

and $7 + 8 + 9 + 10 + 11 + 12 = 57$

$$\therefore \frac{10}{26} \neq \frac{21}{57}$$

hence $d \neq 1$

Now, consider $d = 2$, then the A.P. is

$$1, 3, 5, 7, 9, 11, 13, 15$$

$$\therefore 1 + 3 + 5 + 7 = 16$$

and $9 + 11 + 13 + 15 = 48$

Again $1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23$

$$\therefore 1 + 3 + 5 + 7 + 9 + 11 = 36$$

and $13 + 15 + 17 + 19 + 21 + 23 = 108$

$$\therefore \frac{16}{48} = \frac{36}{108} \Rightarrow \frac{1}{3} = \frac{1}{3}$$

Hence $d = 2$ is the correct answer.

Alternatively Let S_n denote the sum to n terms of the A.P. then,

$$\frac{S_n}{S_{2n} - S_n} = k \quad \text{for every } n \geq 1$$

$$\Rightarrow \frac{S_1}{S_2 - S_1} = \frac{S_2}{S_4 - S_2}$$

$$\Rightarrow S_1 S_4 = S_2^2$$

$$\Rightarrow a \cdot [2(2a + 3d)] = (2a + d)^2$$

$$\Rightarrow 2ad = d^2$$

$$\Rightarrow d = 2 \quad (\because a = 1)$$

- 2 Number of divisors of $187 = 4$

Number of divisors of $637 = 6$

Number of divisors of $1001 = 8$

Hence they are in A.P.

Hint To know more about how to find the number of divisors refer to chapter 1 "Fundamentals".

3 $S_1 = 1^2 = 1$

$$S_2 = 1^2 + (1^2 + 3^2) = 11$$

$$S_3 = 1^2 + (1^2 + 3^2) + (1^2 + 3^2 + 5^2) = 46$$

Now put $n = 1, 2, 3$ in choice (b) you will get

$$S_1 = \frac{1}{6} \times 1 \times 2 \times 3 = 1$$

$$S_2 = \frac{1}{6} \times 2(3)(11) = 11$$

$$S_3 = \frac{1}{6} \times 3(4)23 = 46$$

Hence choice (b) is correct.

☞ **Alternatively** $t_r = 1^2 + 3^2 + 5^2 + \dots + (2r-1)^2$

$$\begin{aligned} \therefore &= \sum_{k=1}^r (2k-1)^2 = \sum_{k=1}^r (4k^2 - 4k + 1) \\ &= 4 \left\{ \frac{1}{6} r(r+1)(2r+1) \right\} - \frac{4}{2} r(r+1) + r \\ &= \frac{1}{3} (4r^3 - r) \\ \therefore S_n &= \frac{4}{3} \sum_{r=1}^n r^3 - \frac{1}{3} \sum_{r=1}^n r \\ &= \frac{4}{3} \cdot \frac{1}{4} n^2(n+1)^2 - \frac{1}{3} \cdot \frac{1}{2} n(n+1) \\ &= \frac{1}{6} n(n+1)(2n^2 + 2n - 1) \end{aligned}$$

4 Since $|a| < 1$ and $|b| < 1$, therefore $|ab| = |a||b| < 1$

Also, $a(a+b) + a^2(a^2+b^2) + a^3(a^3+b^3) + \dots$

$$\begin{aligned} &= (a^2 + a^4 + a^6 + \dots \infty) + (ab + a^2b^2 + a^3b^3 + \dots \infty) \\ &= \frac{a^2}{1-a^2} + \frac{ab}{1-ab} \end{aligned}$$

5 Best way is to go through options by substituting the values of $n = 1, 2, 3, \dots$

$$\begin{aligned} \text{☞ Alternatively } &\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \dots \\ &= \left(1 - \frac{1}{2^2}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \dots + \left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right) \\ &= 1 - \frac{1}{(n+1)^2} = \frac{n^2 + 2n}{(n+1)^2} \end{aligned}$$

6 Best way is to go through options by substituting the values.

Sum of the terms of first row = 1

Sum of the terms of second row = $2 + 3 = 5$

Sum of the terms of third row = $4 + 5 + 6 = 15$ etc.

7 We have $\underbrace{1 \ 1 \ 1 \dots 1}_{119 \text{ times}} = 10^{118} + 10^{117} + \dots + 10^2 + 10^1 + 1$

$$\begin{aligned} &= \frac{(10^{119} - 1)}{10 - 1} = \left(\frac{10^{119} - 1}{10^7 - 1} \right) \left(\frac{10^7 - 1}{10 - 1} \right) \\ &= (10^{112} + 10^{105} + 10^{98} + \dots + 1) \\ &\quad (10^6 + 10^5 + \dots + 10 + 1) \end{aligned}$$

Thus $\underbrace{1 \ 1 \ 1 \dots 1}_{119 \text{ terms}}$ is not a prime number.

8 Since we know that the sum of the interior angles of a polygon of n sides is $(n-2)\pi$

$$\therefore \frac{n}{2} [2(120) + (n-1)5] = (n-2)180 \Rightarrow n = 9, 16$$

But if $n = 16$, then the greatest angle will be 195° and other interior angles are $190^\circ, 185^\circ, 180^\circ$. But no interior angle of a polygon can be equal to 180° . Hence $n = 9$.

9 If $n = 1$, then $1 + 10^4 = 10001 = 73 \times 137$

$\therefore 10^4 + 1$ is a composite number.

$$\begin{aligned} 10 \quad 0.123 &= \frac{1}{10} + \frac{23}{10^3} + \frac{23}{10^5} + \frac{23}{10^7} + \dots \\ &= \frac{1}{10} + \frac{\frac{23}{10^3}}{1 - \left(\frac{1}{10^2}\right)} \\ &= \frac{1}{10} + \frac{23}{1000 - 10} = \frac{122}{990} = \frac{61}{495} \end{aligned}$$

$$\begin{aligned} 11 \quad 11 - 2 &= 9 = 3^2 \\ 1111 - 22 &= 1089 = 33^2 \end{aligned}$$

$$111111 - 222 = 110889 = 333^2$$

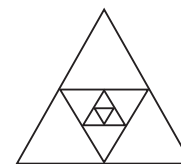
$$11111111 - 2222 = 11108889 = 3333^2 \text{ etc.}$$

12 Perimeter of the outer most triangle is 72 cm and perimeter of the second outer most triangle is 36 cm.

Similarly perimeter of the third outermost triangle is 18 cm.

$\therefore$ Sum of perimeters of all such triangles

$$\begin{aligned} &= 72 + 36 + 18 + 9 + 4.5 + \dots \infty \quad \left(\because S_\infty = \frac{a}{1-r} \right) \\ &= \frac{72}{1 - \frac{1}{2}} = \frac{72}{\frac{1}{2}} = 144 \text{ cm} \end{aligned}$$



13 Go through options.

$$\begin{aligned} 14 \quad \log a + \log \frac{a^3}{b} + \log \frac{a^5}{b^2} + \log \frac{a^7}{b^3} + \dots + \log \frac{a^{2n-1}}{b^{n-1}} \\ &= \log a \times \frac{a^3}{b} \times \frac{a^5}{b^2} \times \frac{a^7}{b^3} \times \dots \times \frac{a^{2n-1}}{b^{n-1}} \\ &= \log \frac{a^{(1+3+5+7+\dots+2n-1)}}{b^{(1+2+3+4+\dots+n-1)}} = \log \frac{a^{n^2}}{b^{(n-1)n/2}} \\ &= \log \left(\frac{a^{2n}}{b^{n-1}} \right)^{n/2} \end{aligned}$$

$$\begin{aligned} 15 \quad (0.2) \log_{\sqrt{5}} \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots \right) &= (0.2) \log_{\sqrt{5}} \left(\frac{1}{2} \right) \\ &= (5^{-1}) \log_{\sqrt{5}} \left(\frac{1}{2} \right) \end{aligned}$$

$$= 5 \log_{\sqrt{5}} 2 \quad (\because m \log n = \log (n)^m)$$

$$= (\sqrt{5})^2 \log_{\sqrt{5}} (2)$$

$$= (\sqrt{5}) \log_{\sqrt{5}} (4)$$

$$= 4$$

$$(\because {}_a \log_a b = b)$$

16 Since a, b, c are in H.P.

Therefore we can assume $a = 2, b = 3, c = 6$

$$\therefore \frac{b+a}{b-a} + \frac{b+c}{b-c} = \frac{5}{1} + \frac{9}{-3} = 2$$

17 Let $r = \frac{1}{2}$, then $(a, b, c) = (4, 2, 1)$

$\therefore (a, 2b, 3c) = (4, 4, 3)$ which are not in A.P.

Again if $r = \frac{1}{3}$ then

$$(a, b, c) = (9, 3, 1)$$

$\therefore (a, 2b, 3c) = (9, 6, 3)$ which are in A.P.

Hence choice (c) is correct.

18 Let $S = \frac{5}{13} + \frac{55}{(13)^2} + \frac{555}{(13)^3} + \frac{5555}{(13)^4} + \dots \infty$

$$\therefore \frac{S}{13} = \frac{5}{(13)^2} + \frac{55}{(13)^3} + \frac{555}{(13)^4} + \frac{5555}{(13)^5} + \dots \infty$$

$$\therefore \left(S - \frac{S}{13}\right) = \frac{5}{13} + \frac{50}{(13)^2} + \frac{500}{(13)^3} + \frac{5000}{(13)^4} + \dots \infty$$

$$S\left(1 - \frac{1}{13}\right) = \frac{5}{13} + \frac{5}{13}\left(\frac{10}{13}\right) + \frac{5}{13}\left(\frac{10}{13}\right)^2 + \frac{5}{13}\left(\frac{10}{13}\right)^3 + \dots \infty$$

$$\frac{12}{13}S = \frac{5/13}{3/13} \quad \left(\because S_{\infty} = \frac{a}{1-r}\right)$$

$$\frac{12}{13}S = \frac{5}{3} \Rightarrow S = \frac{65}{36}$$

19 Let $S = \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \dots \infty$

$$\Rightarrow S = \frac{1}{3} \left[\frac{3}{1 \cdot 4} + \frac{3}{4 \cdot 7} + \frac{3}{7 \cdot 10} + \dots \infty \right]$$

$$\Rightarrow S = \frac{1}{3} \left[\left(\frac{1}{1} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{10}\right) + \dots \infty \right]$$

$$\Rightarrow S = \frac{1}{3} [1] \therefore S = \frac{1}{3}$$

20. Let us add up one more number a_{n+1} to the given sequence

such that $|a_{n+1}| = |a_n + 1|$.

Now, squaring all the numbers, we get

$$a_1^2 = 0$$

$$a_2^2 = a_1^2 + 2a_1 + 1$$

$$a_3^2 = a_2^2 + 2a_2 + 1$$

$$a_4^2 = a_3^2 + 2a_3 + 1$$

$$\dots \dots \dots$$

$$\dots \dots \dots$$

$$a_n^2 = a_{n-1}^2 + 2a_{n-1} + 1$$

$$a_{n+1}^2 = a_n^2 + 2a_n + 1$$

$$\therefore a_1^2 + a_2^2 + a_3^2 + \dots + a_{n+1}^2 = a_1^2 + a_2^2 + \dots + a_n^2 + 2(a_1 + a_2 + \dots + a_n) + n$$

$$\Rightarrow 2(a_1 + a_2 + \dots + a_n) = -n + a_{n+1}^2 \geq -n$$

$$\Rightarrow \frac{a_1 + a_2 + \dots + a_n}{n} \geq -\frac{1}{2}$$

21 Let a, b, c, d are the first consecutive terms in G.P.

Let us consider choice (a)

$$\therefore \frac{b+c}{2} = 5 \Rightarrow b+c=10$$

Now if $(b, c) = (1, 9)$, then a and d can not be found so that $a+b+c+d=30$.

Similarly $(b, c) = (2, 8), (3, 7), (4, 6), (5, 5)$ are also not suitable.

Again consider choice (b)

$$\therefore \frac{b+c}{2} = 6 \Rightarrow b+c=12$$

Now if $(b, c) = (4, 8)$, then $a = 2$ and $d = 16$

$$\therefore a+b+c+d=30$$

Hence this can be the correct answer.

$$\text{Further } a^2 + b^2 + c^2 + d^2 = 4 + 16 + 64 + 256 = 340$$

Hence choice (b) is correct.

Alternatively Let a, ar, ar^2, ar^3 be the 4 consecutive terms of G.P. then

$$\therefore a + ar + ar^2 + ar^3 = 30 \quad \dots(i)$$

$$\text{and } a^2 + a^2r^2 + a^2r^4 + a^2r^6 = 340 \quad \dots(ii)$$

clearly $r \neq 1$

$$[\because (a + ar + ar^2 + ar^3)^2 \neq 4(a^2 + a^2r^2 + a^2r^4 + a^2r^6)]$$

$$\therefore \frac{(1+r+r^2+r^3)^2}{(1+r^2+r^4+r^6)} = \frac{(30)^2}{340} = \frac{45}{17}$$

$$\Rightarrow \frac{\left[\frac{(r^4-1)^2}{(r-1)^2}\right]}{\left(\frac{r^8-1}{r^2-1}\right)} = \frac{45}{17} \quad (\because r \neq 1)$$

$$\Rightarrow \frac{(r^4-1)^2}{(r-1)^2} \cdot \frac{(r^2-1)}{(r^8-1)} = \frac{45}{17}$$

$$\Rightarrow \frac{(r^4-1)(r^4-1)(r-1)(r+1)}{(r-1)(r-1)(r^4-1)(r^4+1)} = \frac{45}{17}$$

$$\Rightarrow \frac{(r^4-1)}{(r-1)} \cdot \frac{(r+1)}{(r^4+1)} = \frac{45}{17}$$

$$\Rightarrow (r^3 + r^2 + r - 1) \cdot \frac{(r+1)}{(r^4+1)} = \frac{45}{17}$$

$$\Rightarrow 17r^4 + 34r^3 + 34r^2 + 34r + 17 = 45r^4 + 45$$

$$\Rightarrow 14r^4 - 17r^3 - 17r^2 - 17r + 14 = 0$$

Dividing by r^2 , we get

$$14\left(r^2 + \frac{1}{r^2}\right) - 17\left(r + \frac{1}{r}\right) - (17) = 0$$

$$\Rightarrow 14\left[\left(r + \frac{1}{r}\right)^2 - 2\right] - 17\left(r + \frac{1}{r}\right) - 17 = 0$$

$$\Rightarrow 14y^2 - 17y - 45 = 0$$

$$\Rightarrow y = \frac{17 \pm \sqrt{17^2 + 4 \times 14 \times 45}}{28}$$

$$\Rightarrow y = \frac{17 \pm \sqrt{2809}}{28} = \frac{17 \pm 53}{28} = \frac{5}{2}, -\frac{9}{7}$$

$$\therefore \left(r + \frac{1}{r}\right) = \frac{5}{2} \Rightarrow r = 2, \frac{1}{2}$$

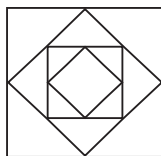
and $\left(r + \frac{1}{r}\right) = -\frac{9}{7}$ has no real roots.

$\therefore$ when $r = 2$, $a = 2$

and when $r = \frac{1}{2}$, $a = 16$

Thus the required progression is 2, 4, 8, 16, 32, 64, ... or 16, 8, 4, 2, 1, $\frac{1}{2}$, ...

- 22** Side of the first square is 4 cm,
side of the second square is $2\sqrt{2}$ cm
and side of the third square is
2 cm and so on. i.e., each side of first,
second, third square ...



$$= 4, 2\sqrt{2}, 2, \sqrt{2}, 1, \frac{1}{\sqrt{2}}, \dots$$

$\therefore$ Area of each of first, second, third square ...

$$= 16, 8, 4, 2, 1, \frac{1}{2}, \dots$$

$\therefore$ Sum of the area of first, second, third square ...

$$= 16 + 8 + 4 + 2 + \dots = \frac{16}{1 - \left(\frac{1}{2}\right)} = 32 \text{ cm}^2$$

- 23** Best way consider some suitable values to check it physically.

Let A.P. be 2, 9.5, 17, 24.5, 32

and G.P. be 2, 4, 8, 16, 32

$$\therefore A_s = 2 + 9.5 + 17 + 24.5 + 32 = 85$$

$$\text{and } G_s = 2 + 4 + 8 + 16 + 32 = 62$$

Since $A_s > G_s$

Hence option (a) is the correct.

NOTE For the fool proof testing you can consider some more values (i.e., Progressions)

- 24** Consider any A.P. and G.P. as per the given conditions ..., then

A.P. 2, 4, 6, 8, 10, 12, 14, ...

G.P. 2, 4, 8, 16, 32, 64, ...

Hence $a_i < g_i$ for $i > 2$ and $i \in N$

NOTE You can consider some more examples and then you will find that only choice (b) is correct.

$$\mathbf{25} \quad \frac{1^1 + 2^1 + 3^1 + 4^1}{1 + 2 + 3 + 4} = 1$$

$$\frac{1^3 + 2^3 + 3^3 + 4^3}{1 + 2 + 3 + 4} = \frac{100}{10} = 10 \text{ and so on.}$$

- 26** Let the A.P. be 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...

where $p = 1$, $q = 2$, $r = 4$, $s = 8$

$$\therefore p - q = -1 \text{ and } q - r = -2 \text{ and } r - s = -4$$

Hence $(p - q)$, $(q - r)$ and $(r - s)$ are in G.P. with common ratio 2 and first term -1.

$$\mathbf{27} \quad 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 = 1 + 2 + 4 + 8 + 16 + 32 = 63$$

$$\mathbf{28} \quad n = 5$$

The weight of each stone (in kg) is

1, 3, 9, 27, 81

$$1 \text{ kg} = 1 \text{ kg}$$

$$2 \text{ kg} = 3 - 1 = 2 \text{ kg}$$

$$3 \text{ kg} = 3 \text{ kg}$$

$$4 \text{ kg} = (3 + 1) = 4 \text{ kg}$$

$$5 \text{ kg} = 9 - (3 + 1) = 5 \text{ kg}$$

$$6 \text{ kg} = (9 - 3) = 6 \text{ kg}$$

$$7 \text{ kg} = (9 + 1) - 3 = 7 \text{ kg}$$

$$8 \text{ kg} = (9 - 1) = 8 \text{ kg}$$

$$9 \text{ kg} = 9 \text{ kg}$$

$$10 \text{ kg} = (9 + 1) = 10 \text{ kg and so on.}$$

Remember he is allowed to put the stones on either side of the balance.

$$\mathbf{29} \quad T_1 = 3^1 - 2$$

$$T_2 = 3^2 - 2$$

$$T_3 = 3^3 - 2$$

$$T_4 = 3^4 - 2$$

- 30** Try with the options.

$$\mathbf{31} \quad 2^1 - 1 \text{ souls become ghost in 1 second}$$

$$2^2 - 1 \text{ souls become ghost in 2 second}$$

$$2^3 - 1 \text{ souls become ghost in 3 second}$$

$$2^4 - 1 \text{ souls become ghost in 4 second}$$

$$\therefore (2^{43200} - 1) \text{ souls become ghost in 43200 seconds.}$$

- 32** Assume some values of n and k and then verify the correct answer.

Alternatively At a particular moment there are x souls left.

Just 1 second before there are

$$2^1 x + 1 = 2^1 x + 2^1 - 1 \text{ souls}$$

Just 2 seconds before there are

$$2^2 x + 2 + 1 = 2^2 x + 2^2 - 1 \text{ souls}$$

Just 3 seconds before there are

$$2^3x + 2^2 + 2 + 1 = 2^3x + 2^3 - 1 \text{ souls}$$

Hence n seconds before there are $2^n x + 2^n - 1$ souls left.

33 Savings = Income – Expenditure

Savings on 1st day = $3 - 3 = 0$

Savings on 2nd day = $6 - 5 = 1$

Savings on 3rd day = $11 - 7 = 4$

Savings on 4th day = $18 - 9 = 9$

$\therefore$ Total savings

$$= 1240 = 0 + 1 + 4 + 9 + 16 + 25 + \dots + 225$$

This will take 16 days.

34 Check through options.

Let us consider choice (a)

	E	M	L	T	
Lucknow (G.P.)	1	1	1	1	} 4
Kanpur (A.P.)	1	2	3	4	} 10

It is wrong since $M = E$

Again	E	M	L	T	
Lucknow (G.P.)	1	2	4	8	} 15
Kanpur (A.P.)	1	3	5	7	} 16

It is wrong since $15 \neq \frac{3}{2} \times 16$, also $2 \neq 3$

Again	E	M	L	T	
Lucknow (G.P.)	1	3	9	27	} 40
Kanpur (A.P.)	1	4	7	10	} 22

It is also wrong since $40 \neq \frac{3}{2} \times 22$, also $3 \neq 4$

Again	E	M	L	T	
Lucknow (G.P.)	2	4	8	16	} 30
Kanpur (A.P.)	2	4	6	8	} 20

It is correct since number of monkeys and number of elephants in both the places are equal respectively. Also total number of animals in zoological park is 50% more than that of Local zoo in Kanpur.

36 Go through options

Let $n = 4$, then

$$S_n = -1 + 1^2 - 2 + 2^2 = 2$$

from option (a) $\frac{4 \times 17}{3} = \frac{68}{3} \neq 2$

from option (b) $16 + 4 = 20 \neq 2$

from option (c) $\frac{4 \times 15}{3} = 20 \neq 2$

Hence choice (d) is correct.

37 Let $S = 1 + \frac{2}{3} + \frac{4}{9} + \frac{6}{27} + \frac{8}{81} + \frac{10}{243} + \dots \infty$

$$\therefore 3(S) = 3 + 2 + \frac{4}{3} + \frac{6}{9} + \frac{8}{27} + \frac{10}{81} + \dots \infty$$

$$\therefore 2(S) = 4 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81} + \dots \infty$$

$$\therefore 2(S) = 4 + 2 \left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots \infty \right)$$

$$\Rightarrow 2(S) = 4 + 2 \left[\frac{\frac{1}{3}}{1 - \left(\frac{1}{3}\right)} \right]$$

$$\Rightarrow 2(S) = 4 + 1$$

$$\Rightarrow 2(S) = 5 \therefore S = \frac{5}{2}$$

38 $\log a + \log \frac{a^2}{b} + \log \frac{a^3}{b^2} + \log \frac{a^4}{b^3} + \dots + \log \frac{a^n}{b^{n-1}}$

$$= \log \left(a \cdot \frac{a^2}{b} \cdot \frac{a^3}{b^2} \cdot \frac{a^4}{b^3} \dots \frac{a^n}{b^{n-1}} \right)$$

$$= \log \left[\frac{a^{(1+2+3+4+\dots+n)}}{b^{(1+2+3+4+\dots+(n-1))}} \right]$$

$$= \log \left[\frac{a^{\frac{n(n+1)}{2}}}{b^{\frac{(n+1)n}{2}}} \right] = \log \left[\frac{a^{(n+1)}}{b^{(n-1)}} \right]^{n/2}$$

39 Go through options

Let 1, 2, 3, 4, 5, ... be an A.P. then

$$\begin{aligned} & \frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \frac{1}{\sqrt{a_3} + \sqrt{a_4}} + \frac{1}{\sqrt{a_4} + \sqrt{a_5}} \\ &= \frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \frac{1}{\sqrt{4} + \sqrt{5}} \\ &= -(\sqrt{1} - \sqrt{2}) - (\sqrt{2} - \sqrt{3}) - (\sqrt{3} - \sqrt{4}) - (\sqrt{4} - \sqrt{5}) \\ &= \sqrt{5} - \sqrt{1} \\ &= \sqrt{5} - \sqrt{1} \times \frac{\sqrt{5} + \sqrt{1}}{\sqrt{5} + \sqrt{1}} = \frac{5 - 1}{\sqrt{5} + \sqrt{1}} \\ &= \frac{4}{\sqrt{1} + \sqrt{5}} = \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}} \end{aligned}$$

Alternatively

$$\begin{aligned} & \frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \frac{1}{\sqrt{a_3} + \sqrt{a_4}} \\ & \quad + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} \\ &= \frac{\sqrt{a_2} - \sqrt{a_1}}{(\sqrt{a_2} + \sqrt{a_1})(\sqrt{a_2} - \sqrt{a_1})} + \frac{\sqrt{a_3} - \sqrt{a_2}}{(\sqrt{a_3} + \sqrt{a_2})(\sqrt{a_3} - \sqrt{a_2})} + \\ & \quad \dots + \frac{\sqrt{a_n} - \sqrt{a_{n-1}}}{(\sqrt{a_n} + \sqrt{a_{n-1}})(\sqrt{a_n} - \sqrt{a_{n-1}})} \\ &= \frac{\sqrt{a_2} - \sqrt{a_1}}{a_2 - a_1} + \frac{\sqrt{a_3} - \sqrt{a_2}}{a_3 - a_2} + \dots + \frac{\sqrt{a_n} - \sqrt{a_{n-1}}}{a_n - a_{n-1}} \\ &= \frac{\sqrt{a_2} - \sqrt{a_1}}{d} + \frac{\sqrt{a_3} - \sqrt{a_2}}{d} + \dots + \frac{\sqrt{a_n} - \sqrt{a_{n-1}}}{d} \\ &= \frac{1}{d} [(\sqrt{a_2} - \sqrt{a_1}) + (\sqrt{a_3} - \sqrt{a_2}) + \dots + (\sqrt{a_n} - \sqrt{a_{n-1}})] \\ &= \frac{1}{d} [\sqrt{a_n} - \sqrt{a_1}] = \frac{a_n - a_1}{d(\sqrt{a_n} + \sqrt{a_1})} = \frac{n-1}{\sqrt{a_n} + \sqrt{a_1}} = \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}} \end{aligned}$$

- 40** Go through options. Remember if $\log a, \log b, \log c$ are in A.P., then a, b, c are in G.P.

$$\therefore 2, (2^x - 3) \text{ and } \left(\frac{17}{2} + 2^{x-1}\right) \text{ must be in G.P.}$$

Let $x = 3$, then the value of $2^x - 3 = 5$

$$\text{and } \frac{17}{2} + 2^{x-1} = \frac{25}{2}$$

Since $2, 5, \frac{25}{2}$ are in G.P.

Hence, the given terms of the sequence are in A.P.

- 41** Combining all of the three relations we get $x = y^{-1} = z^3$

$$\therefore 1, \log_y x, \log_z y, -15 \log_x z$$

$$\Rightarrow 1, -1, -3, -5, \text{ which are in A.P.}$$

Hence, choice (d) is most appropriate.

- 42** Let $x = 1, y = 2, z = 4$

$$\therefore a = 16, b = 4, c = 2 \quad (\because a^x = b^y = c^z)$$

$$\begin{aligned} \therefore \log_b a \cdot \log_b c &= \log_4 16 \cdot \log_4 2 \\ &= (2 \log_4 4) \cdot \left(\frac{\log 2}{2 \log 2}\right) \\ &= (2 \cdot 1) \cdot \left(\frac{1}{2}\right) = 1 \end{aligned}$$

- 43** Go through options.

Let us consider choice (b)

$$x = 1 - \log_2 5 = \log_2 2 - \log_2 5 = \log_2 \left(\frac{2}{5}\right)$$

$$\begin{aligned} \therefore \log_2 (5 \cdot 2^x + 1), \log_4 (2^{1-x} + 1), 1 \\ &= \log_2 (5 \cdot 2^{\log_2 2/5} + 1), \log_4 (2^{1 - \log_2 2/5} + 1), 1 \\ &= \log_2 \left(5 \times \frac{2}{5} + 1\right), \log_4 \left(\frac{2}{2^{\log_2 2/5}} + 1\right), 1 \\ &= \log_2 3, \log_4 \left(\frac{2}{2/5} + 1\right), 1 \\ &= \log_2 3, \log_4 6, 1 = \log_4 9, \log_4 6, \log_4 4 \\ \therefore 9, 6, 4 \text{ are in G.P.} \end{aligned}$$

Hence the assumed choice (b) is correct.

Hint If a, b, c are in G.P. then $\log a, \log b, \log c$ are in A.P.

- 44** Very similar to the previous problem.

- 45** Let $x = e, y = e^2$ and $z = e^3$, then

$$\begin{aligned} \frac{1}{1 + \ln e}, \frac{1}{1 + \ln e^2} + \frac{1}{1 + \ln e^3} &= \frac{1}{1 + 1}, \frac{1}{1 + 2}, \frac{1}{1 + 3} \\ &= \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \text{ which are in H.P.} \end{aligned}$$

- 46** Let d be the common difference of the A.P.

$$\text{then } a_{10} = 3$$

$$\Rightarrow a_1 + 9d = 3$$

$$\Rightarrow 2 + 9d = 3$$

$$\Rightarrow d = \frac{1}{9}$$

$$\therefore a_4 = a_1 + 3d = 2 + \frac{1}{3} = \frac{7}{3}$$

Let D be the common difference of

$$\frac{1}{h_1}, \frac{1}{h_2}, \dots, \frac{1}{h_{10}}$$

$$\text{Then, } \frac{1}{h_{10}} = \frac{1}{3}$$

$$\Rightarrow \frac{1}{h_1} + 9D = \frac{1}{3}$$

$$\Rightarrow \frac{1}{2} + 9D = \frac{1}{3}$$

$$\Rightarrow D = -\frac{1}{54}$$

$$\therefore \frac{1}{h_7} = \frac{1}{h_1} + 6D = \frac{1}{2} - \frac{1}{9} = \frac{7}{18}$$

$$\Rightarrow h_7 = \frac{18}{7}$$

$$\therefore a_4 h_7 = \frac{7}{3} \times \frac{18}{7} = 6$$

- 47**

$$S_4 < 1 \text{ cm}^2$$

Therefore the side of square also be less than 1 cm.

$$\therefore l_1 = 10, l_2 = \frac{10}{\sqrt{2}},$$

$$l_3 = \frac{10}{2}, l_4 = \frac{10}{2\sqrt{2}}$$

$$l_5 = \frac{10}{4}, l_6 = \frac{10}{4\sqrt{2}},$$

$$l_7 = \frac{10}{8}, l_8 = \frac{10}{8\sqrt{2}}$$

$$\text{Hence } l_8 = \frac{10}{8\sqrt{2}} < 1 \text{ cm}$$

$$\therefore \text{area of } S_8 < 1 \text{ cm}^2$$

where $l_1, l_2, \dots, l_{10}$ are the lengths of sides of different squares, from outwards to inwards.

$$\mathbf{48} \quad G_2 = \sqrt{A_1 H_1} = \sqrt{\frac{a+b}{2} \times \frac{2ab}{a+b}} = \sqrt{ab}$$

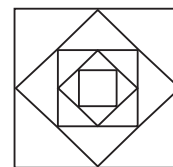
$$\begin{aligned} \text{Similarly, } G_3 &= \sqrt{A_2 H_2} \\ &= \sqrt{\frac{A_1 + H_1}{2} \times \frac{2A_1 H_1}{A_1 + H_1}} = \sqrt{A_1 H_1} = \sqrt{ab} \end{aligned}$$

$$\therefore G_1 = G_2 = G_3 = \dots$$

Hence, choice (c) is the correct one.

$$\mathbf{49} \quad A_1 = \frac{a+b}{2}, G_1 = \sqrt{ab} \text{ and } H_1 = \frac{2ab}{a+b}$$

$$A_n = \frac{A_{n-1} + H_{n-1}}{2};$$



$$G_n = \sqrt{A_{n-1}H_{n-1}}; H_n = \frac{2A_{n-1}H_{n-1}}{A_{n-1} + H_{n-1}}$$

Now, A_2 is A.M. of A_1 and H_1 , but $A_1 > H_1$

$$\Rightarrow A_1 > A_2 > H_1$$

Similarly, A_3 is A.M. of A_2 and H_2 , but $A_2 > H_2$

$$\Rightarrow A_2 > A_3 > H_2$$

$$\therefore A_1 > A_2 > A_3 > \dots$$

Hence, choice (a) is the correct one.

$$50 \quad H_2 = \frac{2A_1H_1}{A_1 + H_1}$$

$$\Rightarrow \frac{1}{A_1} + \frac{1}{H_1} = 2\left(\frac{1}{H_2}\right)$$

$$\Rightarrow \frac{1}{H_2} \text{ is the A.M. of } \frac{1}{A_1} \text{ and } \frac{1}{H_1}, \text{ but } \frac{1}{A_1} < \frac{1}{H_1}$$

$$\Rightarrow \frac{1}{A_1} < \frac{1}{H_2} < \frac{1}{H_1} \Rightarrow H_2 > H_1$$

$$\text{Similarly, } H_3 = \frac{2A_2H_2}{A_2 + H_2}$$

$$\Rightarrow \frac{1}{A_2} + \frac{1}{H_2} = 2\left(\frac{1}{H_3}\right)$$

$$\Rightarrow \frac{1}{H_3} \text{ is the A.M. of } \frac{1}{A_2} \text{ and } \frac{1}{H_2}, \text{ but } \frac{1}{A_2} < \frac{1}{H_2}$$

$$\Rightarrow \frac{1}{A_2} < \frac{1}{H_3} < \frac{1}{H_2} \Rightarrow H_3 > H_2$$

$$\therefore H_1 < H_2 < H_3 < \dots$$

Hence, choice (c) is the correct one.

$$51 \quad \frac{S_m}{S_n} = \frac{a_1 + a_2 + \dots + a_{5n}}{a_1 + a_2 + \dots + a_n} = \frac{\frac{5n}{2}[2 \times 3 + (5n-1)d]}{\frac{n}{2}[2 \times 3 + (n-1)d]}$$

$$\Rightarrow \frac{S_m}{S_n} = \frac{5(6 + 5nd - d)}{(6 + nd - d)}$$

$$\frac{S_m}{S_n} \text{ will be independent of } n \text{ when } d = 6.$$

$$\text{Therefore } a_2 = a_1 + d = 3 + 6 = 9$$

Hence, choice (b) is the answer.

52 Let α, β be the roots of the equation, then

$$\alpha + \beta = \frac{4 + \sqrt{5}}{5 + \sqrt{2}} \text{ and } \alpha \cdot \beta = \frac{8 + 2\sqrt{5}}{5 + \sqrt{2}}$$

$$\text{Therefore HM of } \alpha, \beta = \frac{2(\alpha \cdot \beta)}{\alpha + \beta} = 4$$

Hence, choice (b) is the answer.

$$53 \quad a_{10} = a_1 + 9d$$

$$\Rightarrow 3 = 2 + 9d$$

$$\Rightarrow d = \frac{1}{9}$$

$$\Rightarrow a_4 = a_1 + 3d = 2 + \frac{1}{3} = \frac{7}{3}$$

And since $h_1, h_2, \dots, h_{10}$ are in H.P., then $\frac{1}{h_1}, \frac{1}{h_2}, \dots, \frac{1}{h_{10}}$ are

in A.P.

$$\text{Therefore } \frac{1}{h_{10}} = \frac{1}{h_1} + 9D$$

$$\Rightarrow D = \frac{-1}{54}$$

$$\text{So, } \frac{1}{h_7} = \frac{1}{h_1} + 6D$$

$$\Rightarrow \frac{1}{h_7} = \frac{7}{18}$$

$$\text{Thus, } a_4 h_7 = \frac{7}{3} \times \frac{18}{7} = 6$$

Hence, choice (d) is the answer.

$$54 \quad a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{(2^n) - 1}$$

$$\Rightarrow a(n) = 1 + \left(\frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{4} + \dots + \frac{1}{7}\right)$$

$$+ \left(\frac{1}{8} + \dots + \frac{1}{15}\right) + \dots + \left(\frac{1}{2^{n-1}} + \dots + \frac{1}{2^n - 1}\right)$$

$$\Rightarrow a(n) < 1 + \left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{4} + \dots + \frac{1}{4}\right)$$

$$+ \left(\frac{1}{8} + \dots + \frac{1}{8}\right) + \dots + \left(\frac{1}{2^{n-1}} + \dots + \frac{1}{2^{n-1}}\right)$$

$$\Rightarrow a(n) < 1 + \left(\frac{2}{2}\right) + \left(\frac{4}{4}\right) + \left(\frac{8}{8}\right) + \dots + \left(\frac{2^{n-1}}{2^{n-1}}\right)$$

$$\Rightarrow a(n) < \underbrace{1 + 1 + 1 + \dots + 1}_{n \text{ times}}$$

$$\Rightarrow a(n) < n$$

$$\Rightarrow a(100) < 100$$

$$\text{Again, } a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{(2^{n-1}) - 1} + \frac{1}{(2^n) - 1}$$

$$\Rightarrow a(n) = 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \dots + \frac{1}{8}\right)$$

$$+ \dots + \left(\frac{1}{(2^{n-1}) + 1} + \dots + \frac{1}{2^n - 1} + \frac{1}{2^n}\right) - \frac{1}{2^n}$$

$$\Rightarrow a(n) > 1 + \frac{1}{2} + \left(\frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{8} + \dots + \frac{1}{8}\right)$$

$$+ \dots + \left(\frac{1}{2^n} + \dots + \frac{1}{2^n}\right) - \frac{1}{2^n}$$

$$\Rightarrow a(n) > 1 + \underbrace{\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \dots + \frac{1}{2}}_{n \text{ times}} - \frac{1}{2^n}$$

$$\Rightarrow a(n) > 1 + \frac{n}{2} - \frac{1}{2^n}$$

$$\Rightarrow a(200) > 1 + \frac{200}{2} - \frac{1}{2^{200}}$$

$$\Rightarrow a(200) > 100 + \left(1 - \frac{1}{2^{200}}\right) > 100$$

Hence, choice (b) is the answer.

$$\begin{aligned}
 55 \quad S &= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2^{32}} \\
 \Rightarrow S &= 1 + \left[\left(\frac{1}{2} \right) + \left(\frac{1}{3} + \frac{1}{4} \right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} \right) \right. \\
 &\quad \left. + \left(\frac{1}{9} + \dots + \frac{1}{16} \right) + \left(\frac{1}{17} + \frac{1}{18} + \dots + \frac{1}{32} \right) + 32 \text{ sets} \right] \\
 \Rightarrow S &> 1 + \left[\left(\frac{1}{2} \right) + \left(\frac{1}{4} + \frac{1}{4} \right) + \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \right) \right. \\
 &\quad \left. + \left(\frac{1}{16} + \dots + \frac{1}{16} \right) + \left(\frac{1}{32} + \frac{1}{32} + \dots + \frac{1}{32} \right) + \dots 32 \text{ sets} \right] \\
 \Rightarrow S &> 1 + \left[\left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) + \dots 32 \text{ sets} \right] \\
 \Rightarrow S &> 1 + \left(\frac{1}{2} \right) \times 32 \\
 \Rightarrow S &> 1 + (16) \Rightarrow S > 17
 \end{aligned}$$

Now, solve the same series for upper limit, as explained below.

$$\begin{aligned}
 S &= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2^{32}} \\
 \Rightarrow S &= \left[\left(\frac{1}{1} \right) + \left(\frac{1}{2} + \frac{1}{3} \right) + \left(\frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} \right) \right. \\
 &\quad \left. + \left(\frac{1}{8} + \frac{1}{9} + \dots + \frac{1}{15} \right) + \left(\frac{1}{16} + \frac{1}{17} + \dots + \frac{1}{31} \right) + 32 \dots \text{sets} \right] \\
 \Rightarrow S &< \left[\left(\frac{1}{1} \right) + \left(\frac{1}{2} + \frac{1}{2} \right) + \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \right) \right. \\
 &\quad \left. + \left(\frac{1}{8} + \frac{1}{8} + \dots + \frac{1}{8} \right) + \left(\frac{1}{16} + \frac{1}{16} + \dots + \frac{1}{16} \right) + \dots 32 \text{ sets} \right] \\
 \Rightarrow S &< [1 + (1) + (1) + (1) + (1) + \dots 32 \text{ sets}] + \frac{1}{2^{32}} \\
 \Rightarrow S &< (1) \times 32 + \frac{1}{2^{32}} \Rightarrow S < 32 + \frac{1}{2^{32}} \\
 \Rightarrow S &< 32 + \left(\frac{1}{2} \right) \quad \left(\because \frac{1}{2^{32}} < \frac{1}{2} \right) \\
 \Rightarrow S &< 32\frac{1}{2}
 \end{aligned}$$

Therefore, $17 < S < 32\frac{1}{2}$

Hence, choice (c) is correct.

☞ **Alternatively** $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} = \sum_{i=1}^n \frac{1}{i}$

$$\therefore \ln(n+1) \leq \sum_{i=1}^n \frac{1}{i} \leq \ln(n) + 1$$

$$\therefore \ln(2^{32} + 1) \leq \sum_{i=1}^{2^{32}} \frac{1}{i} \leq \ln(2^{32}) + 1;$$

$$(\ln 2^{32} = 32 \ln 2 = 32 \times 0.693 = 22.18)$$

$$\Rightarrow 22.18 \leq \sum_{i=1}^n \frac{1}{i} \leq 22.18 + 1$$

$$\Rightarrow 22.18 \leq \sum_{i=1}^n \frac{1}{i} \leq 23.18$$

$$\Rightarrow 22.18 < S < 23.18$$

Hence, choice (c) is correct.

☞ **NOTE** Whenever you see this problem in the exam, just find the value of natural log of that number.

56 As you know that $AM \geq GM$, so $\frac{a^3p + b^3q}{2} \geq \sqrt{(a^3p)(b^3q)}$

$$\Rightarrow a^3p + b^3q \geq 2[(a^2b^3)(apq)]^{1/2}$$

$$\Rightarrow a^3p + b^3q \geq 2[(a^2b^3)(9b)]^{1/2}$$

$$\Rightarrow a^3p + b^3q \geq 2[9(a^2b^4)]^{1/2}$$

$$\Rightarrow a^3p + b^3q \geq 6ab^2$$

Hence, choice (a) is the correct one.

57 $a + b = 10 \Rightarrow b = (10 - a)$

And the total distance completed by them $= 3a^2 + b^2$

$$\Rightarrow 3a^2 + (10 - a)^2$$

$$\Rightarrow 4a^2 - 20a + 100 \Rightarrow (2a - 5)^2 + 75$$

So, when $2a - 5 = 0$ or $a = \frac{5}{2}$, the above expression will have

the minimum value. Thus the minimum distance covered by them is 75 km.

Hence choice (a) is the correct one.

58 Using the property, A.M. $\geq$ G.M., you will have

$$\frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + 1 + a^8 + a^{10}}{8} \geq 1$$

$$\Rightarrow a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + 1 + a^8 + a^{10} \geq 8$$

Hence, choice (a) is the correct answer.

Hint $GM = \sqrt[8]{a^{-5} \times a^{-4} \times a^{-3} \times a^{-3} \times a^{-3} \times 1 \times a^8 \times a^{10}}$
 $= \sqrt[8]{a^0} = 1$

59 The best way is to substitute some numerals in place of variables (a, b, c) then test the validity against various given choices.

Let us consider $a = b = c = 1$, then $p = 8^7$. It shows that choices (a), (b) and (c) are wrong.

Hence, choice (d) is the answer.

☞ **Alternatively** $(1+a)(1+b)(1+c)$

$$= 1 + a + b + ab + bc + ac + abc$$

$$\Rightarrow (1+a)(1+b)(1+c) - 1$$

$$= a + b + c + ab + bc + ac + abc$$

Now, using the property $AM \geq GM$

$$\Rightarrow \frac{(1+a)(1+b)(1+c) - 1}{7} \geq (a \cdot b \cdot c \cdot ab \cdot bc \cdot ac \cdot abc)^{1/7}$$

$$\Rightarrow (1+a)(1+b)(1+c) - 1 \geq 7(a^4b^4c^4)^{1/7}$$

$$\Rightarrow (1+a)(1+b)(1+c) \geq 7(a^4b^4c^4)^{1/7}$$

$$\Rightarrow (1+a)(1+b)(1+c)^7 \geq 7^7(a^4b^4c^4)$$

60 Choice (c) is the correct one.

Level 03 Final Round

- 1 Total number of bacterias after 10 seconds

$$= 3^{10} - 3^5 = 3^5(3^5 - 1)$$

since just after 10 seconds all the bacterias (i.e. 3^5) are dead after living for 5-5 seconds.

2 $S_{20} = 2^{66} - 2^{65} - 2^{64} - 2^{63} - \dots - 2^{47}$

$$= 2^{65}(2 - 1) - 2^{64} - 2^{63} - \dots - 2^{47}$$

$$= 2^{65} - 2^{64} - 2^{63} - \dots - 2^{47}$$

$$= 2^{64}(2 - 1) - 2^{63} - \dots - 2^{47}$$

$$= 2^{64} - 2^{63} - 2^{62} - \dots - 2^{47}$$

$$= 2^{63}(2 - 1) - 2^{62} - \dots - 2^{47}$$

$$= 2^{63} - 2^{62} - 2^{61} - \dots - 2^{47}$$

$$\dots \dots \dots$$

$$\dots \dots \dots$$

$$= 2^{47}$$

3 Since $S_{67} = 2^0$ ($\because S_n = 2^{67-n}$)

$$S_{66} = 2^1$$

$$S_{65} = 2^2$$

$$S_{64} = 2^3$$

$$S_{63} = 2^4$$

$$\dots \dots \dots$$

$$\dots \dots \dots$$

$$S_{34} = 2^{33} \text{ etc.}$$

Now, $U_m = S_m + S_{m+1} + S_{m+2} + \dots + S_{m+(m-1)}$

$$\therefore U_{34} = S_{34} + S_{35} + S_{36} + \dots + S_{67}$$

$$\Rightarrow U_{34} = 2^{33} + 2^{32} + 2^{31} + \dots + 2^0$$

$$\Rightarrow U_{34} = 1 + 2^1 + 2^2 + 2^3 + \dots + 2^{32} + 2^{33}$$

$$\Rightarrow U_{34} = (2^{33} - 1)$$

- 4 Consider an A.P. then go through options

Let 1, 2, 3, 4, 5, 6 be an A.P. with 6 terms

i.e. $3n = 6, 2n = 4$ and $n = 2$

$$\therefore S_3 - S_2 - S_1$$

$$= (1 + 2 + \dots + 6) - (1 + 2 + 3 + 4) - (1 + 2)$$

$$= 21 - 10 - 3 = 8$$

Choice (a) gives

$$S_3 - S_2 - S_1 = 3a - 2n - d = 3 - 4 - 1$$

$$= -2, \text{ hence wrong}$$

Check the choice (d)

$$S_3 - S_2 - S_1 = 2n^2 d = 8$$

Hence it is correct.

☞ **Alternatively** Since $S_3 - S_2 - S_1$ is independent of a (i.e., first term).

Hence only choice (d) is correct.

5	$T_n = n$	$n + 3$	$n + 5$	$n + 7$	$n + 9$
	$T_1 = 1$	4	6	8	10
	$T_2 = 2$	5	7	9	11
	$T_3 = 3$	6	8	10	12
	$T_4 = 4$	7	9	11	13
	$T_5 = 5$	8	10	12	14
	$T_6 = 6$	9	11	13	15
	$T_7 = 7$	10	12	14	16
	$T_8 = 8$	11	13	15	17
	$T_n = 9$	12	14	16	18

In general when $n = 4, 9, 13, 17, \dots, 99$ does not contain 5 or its multiple.

i.e. $n = 5k - 1$ where $k = 1, 2, 3, \dots, 20$

Hence out of 99 sets 20 sets does not contain the 5 or its multiples.

Thus the required number of sets = $99 - 20 = 79$

6 Let $S = \{\sqrt{1}\} + \{\sqrt{2}\} + \{\sqrt{3}\} + \{\sqrt{4}\} + \dots + \{\sqrt{99}\} + \{\sqrt{100}\}$

$$\Rightarrow S = 1 + 2 + 2 + 2 + 3 + 3 + 3 + 3 + 3 + \dots + 10$$

$$\Rightarrow S = 1 + 3(2) + 5(3) + 7(4) + 9(5) + \dots + 19(10)$$

Since $T_n = (2n - 1)n = 2n^2 - n$

$$\therefore S_n = \Sigma(2n^2 - n) = 2\Sigma n^2 - \Sigma n$$

$$= \frac{2n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2} \left[\frac{2}{3}(2n+1) - 1 \right]$$

$$= \frac{n(n+1)}{2} \left[\frac{4n+2-3}{3} \right]$$

$$= \frac{n(n+1)(4n-1)}{6}$$

$$\therefore S_{10} = \frac{10 \times 11 \times 39}{6} = 715$$

7 Let $S = [\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \dots + [\sqrt{323}]$

$$\Rightarrow S = 1 + 1 + 1 + 2 + 2 + 2 + 2 + 2 + \dots + 17 + 17$$

$$\Rightarrow S = 1(3) + 2(5) + 3(7) + 4(9) + \dots + 17(35)$$

$$\therefore T_n = n(2n + 1)$$

$$\therefore S_n = \Sigma n(2n + 1) = 2\Sigma n^2 + \Sigma n$$

$$= \frac{2 \cdot n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2} \left[\frac{2}{3}(2n+1) + 1 \right]$$

$$= \frac{n(n+1)}{2} \left[\frac{4n+5}{3} \right] = \frac{n(n+1)(4n+5)}{6}$$

$$\therefore S = \frac{17 \times 18 \times 73}{6} = 3723$$

- 8** This can be done only by giving the number of coins as $2^0, 2^1, 2^2, 2^3, 2^4, \dots$ etc.

So, the amounts are 1, 2, 4, 8, 16, 32, 37.

Hence (c) is the correct answer.

- 9** $(1 + 37) = (32 + 4 + 2)$

Hence, 3 people are required.

- 10** $(1 + 2 + 37) = (32 + 8)$

Hence 2 people are required.

- 12** 1, 2, 4, 8, 16, 32, 64, 73

- 13** Since there are only 5 people left with their amount viz. 1, 4, 16, 64, 73, (excluding 2, 8, 32).

So total number of combinations are $2^5 - 1 = 31$

Hence option (c) is correct.

- 14** $1 + 2 + 4 + 8 + 16 + 32 = 63$

$$3(1 + 2 + 4 + 8 + 16 + 32) = 189$$

$$200 - 189 = 11$$

- 15** First of all choice (c) is ruled out since 'a' can not be zero.

Again choice (a) is ruled out because $|r| < 1$

Now let us check the option (b)

$$S_{\infty} = 3 + \left(-\frac{3}{2}\right) + \frac{9}{4} + \left(-\frac{27}{8}\right) + \dots \infty$$

$$\therefore S_{\infty} = \frac{3}{1 - \left(-\frac{1}{2}\right)} = \frac{3}{\frac{1}{2}} = 2$$

$$\text{Again, } 27 - \frac{27}{8} + \frac{729}{64} - \dots \infty$$

$$S_{\infty} = \frac{27}{1 - \left(-\frac{1}{8}\right)} = \frac{27}{\frac{7}{8}} = 24$$

Hence choice (b) is the appropriate choice.

Alternatively $S_1 = \frac{a}{1-r} = 2$

$$S_2 = \frac{a^3}{1-r^3} = 24$$

By solving these two equations, we get

$$a = 3, \quad r = -\frac{1}{2}$$

- 16** When you check option (a) it will be proved wrong. Again for convenience consider option (c) 2, 5, 8, 11, ...

then first term of that G.P. = 3

and the common ratio of G.P. = 2

Hence G.P. = 3, 6, 12, 24, ...

So $a_1 + a_2 = 9$, which is correct.

$$\text{and } S_n \text{ of A.P.} = \frac{10}{2} [2 \times 2 + 9 \times 3] = 155$$

which is also correct. Hence choice (c) is correct.

Similarly choice (b) is also correct.

Hence choice (d) is most appropriate.

Alternatively $a, a + d, a + 2d, \dots$ be an A.P.
and $A, Ar, Ar^2, Ar^3, \dots$ be a G.P.

$$\text{then, } \frac{n}{2} [2a + (n-1)d] = 155$$

$$5[2a + 9d] = 155$$

$$2a + 9d = 31 \quad \dots(i)$$

Again

$$A + Ar = 9$$

$$d + ad = 9 \quad \dots(ii)$$

Solving eqs. (i) and (ii) by substitution method,

$$a = 2, \frac{25}{2} \quad \text{and} \quad d = 3, \frac{2}{3}$$

Thus A.P. $\rightarrow 2, 5, 8, 11, \dots$

$$\text{G.P.} \rightarrow 3, 6, 12, 24, \dots$$

$$\text{or A.P.} \rightarrow \frac{25}{2}, \frac{79}{6}, \frac{83}{6}, \dots$$

$$\text{G.P.} \rightarrow \frac{2}{3}, \frac{25}{3}, \frac{625}{6}, \dots$$

Thus choice (d) is correct.

17 $\frac{a}{1-r} = 162, \quad \frac{a(1-r^n)}{1-r} = 160$

$$\therefore 1 - r^n = \frac{80}{81} \Rightarrow r^n = \frac{1}{81}$$

$$\text{Now, since } \frac{1}{r} \in \mathbb{Z}$$

$$\therefore \frac{1}{r^n} = 81 \Rightarrow \frac{1}{r} = 3^{4/n}$$

$$\Rightarrow \therefore n = 1, 2, 4$$

Here 1, 2, 4 are the factors of 4.

Hence	n	r	a
	1	$\frac{1}{81}$	160
	2	$\frac{1}{9}, -\frac{1}{9}$	144 and 180
	4	$\frac{1}{3}, -\frac{1}{3}$	108 and 216

Hence option (d) is correct.

19 $a_k = 2a_{k-1} - a_{k-2}$

$$\Rightarrow a_{k-2} + a_k = 2a_{k-1}$$

Therefore $a_1, a_2, a_3, \dots, a_{11}$ are in AP.

Let's assume that the common difference of this AP is d , then

$$a_1 = 15, a_2 = 15 + d, a_3 = 15 + 2d, \dots, a_{11} = 15 + 10d$$

$$\therefore \frac{a_1^2 + a_2^2 + \dots + a_{11}^2}{11} = 90$$

$$\Rightarrow \frac{(15)^2 + (15+d)^2 + \dots + (15+10d)^2}{11} = 90$$

$$[11(15)^2] + d^2[(1)^2 + (2)^2 + \dots + (10)^2] + 2 \times 15d$$

$$\Rightarrow \frac{(1 + 2 + \dots + 10)}{11} = 90$$

$$\Rightarrow 7d^2 + 30d + 27 = 0$$

$$\Rightarrow d = -3 \text{ or } d = -\frac{9}{7}$$

Since, $a_2 < \frac{27}{2}$, therefore $d = -3$

$$\therefore \frac{a_1 + a_2 + a_3 + \dots + a_{11}}{11} = \frac{15 + 12 + 9 + \dots + (-15)}{11} = 0$$

Hence, choice (a) is the answer.

$$20 \quad V_r = \frac{r}{2} [2r + (r-1)(2r-1)] = \frac{1}{2} (2r^3 - r^2 + r)$$

$$\therefore V_1 + V_2 + \dots + V_n = \sum_{r=1}^n \frac{1}{2} (2r^3 - r^2 + r)$$

$$\begin{aligned} &= \sum_{r=1}^n (r^3) - \frac{1}{2} \sum_{r=1}^n (r^2) + \frac{1}{2} \sum_{r=1}^n (r) \\ &= \left(\frac{n(n+1)}{2} \right)^2 - \frac{1}{2} \left(\frac{n(n+1)(2n+1)}{6} \right) + \frac{1}{2} \left(\frac{n(n+1)}{2} \right) \\ &= \frac{1}{12} n(n+1)(3n^2 + n + 2) \end{aligned}$$

Hence choice (b) is the answer.

$$21 \quad T_r = V_{r+1} - V_r - 2$$

$$\Rightarrow T_r = \left[\frac{1}{2} 2(r+1)^3 - (r+1)^2 + (r+1) - 2 - \frac{1}{2} (2r^3 - r^2 + r) - 2 \right]$$

$$\Rightarrow T_r = 3r^2 + 2r + 1$$

$$\Rightarrow T_r = (r+1)(3r-1)$$

It implies that T_r is a composite number.

Hence, choice (d) is the answer.

$$22 \quad T_r = 3r^2 + 2r + 1$$

$$\text{And } T_{r+1} = 3(r+1)^2 + 2(r+1) + 1 = 3r^2 + 8r + 4$$

$$\therefore Q_r = T_{r+1} - T_r$$

$$\Rightarrow Q_r = 6r + 5$$

$$\Rightarrow Q_{r+1} = 6(r+1) + 5$$

$$\Rightarrow Q_{r+1} - Q_r = 6$$

Therefore, common difference = 6

Hence, choice (b) is the answer.

$$23 \quad a_n = \frac{3}{4} - \left(\frac{3}{4} \right)^2 + \left(\frac{3}{4} \right)^3 + \dots + (-1)^{n-1} \left(\frac{3}{4} \right)^n$$

$$\Rightarrow a_n = \frac{\frac{3}{4} \left[1 - \left(-\frac{3}{4} \right)^n \right]}{1 + \frac{3}{4}} = \frac{3}{7} \left[1 - \left(-\frac{3}{4} \right)^n \right]$$

$$\text{Now, since } b_n = 1 - a_n \Rightarrow b_n + a_n = 1$$

$$\text{But } b_n > a_n \Rightarrow b_n > \frac{1}{2} \text{ and } a_n < \frac{1}{2}$$

$$\therefore \frac{3}{7} \left[1 - \left(-\frac{3}{4} \right)^n \right] < \frac{1}{2}$$

$$\Rightarrow -\frac{1}{6} < \left(-\frac{3}{4} \right)^n$$

It implies that the minimum natural number $n_0 = 6$

Hence, choice (d) is the answer.

24 Total runs = runs scored in (1st math match + 2nd match + ... + nth match)

$$\left(\frac{n+1}{4} \right) (2^{n+1} - n - 2) = \sum_{k=1}^n k \cdot 2^{n+1-k}$$

$$\Rightarrow \left(\frac{n+1}{4} \right) (2^{n+1} - n - 2) = 2^{n+1} \sum_{k=1}^n \frac{k}{2^k}$$

The best way to solve this problem is to go through the options.

$$\text{Choice (d) : } n = 3, \text{ then } \left(\frac{n+1}{4} \right) (2^{n+1} - n - 2) = 11$$

$$\text{And } 2^{3+1} \sum_{k=1}^3 \frac{k}{2^k} = 16 \left(\frac{1}{2} + \frac{2}{4} + \frac{3}{8} \right) = 22$$

Therefore, choice (d) is not valid.

$$\text{Choice (c) : } n = 7, \text{ then } \left(\frac{n+1}{4} \right) (2^{n+1} - n - 2) = 494$$

$$\text{And } 2^{7+1} \sum_{k=1}^7 \frac{k}{2^k}$$

$$= 256 \left(\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{5}{32} + \frac{6}{64} + \frac{7}{128} \right) = 494$$

Therefore, choice (c) is valid.

Hence, choice (c) is the correct answer.

$$\text{Alternatively } \left(\frac{n+1}{4} \right) (2^{n+1} - n - 2) = \sum_{k=1}^n k \cdot 2^{n+1-k}$$

$$\Rightarrow \left(\frac{n+1}{4} \right) (2^{n+1} - n - 2) = 2^{n+1} \sum_{k=1}^n \frac{k}{2^k}$$

$$\Rightarrow \left(\frac{n+1}{4} \right) (2^{n+1} - n - 2)$$

$$= 2^{n+1} \left(\frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + \dots + \frac{n}{2^n} \right)$$

$$\Rightarrow \left(\frac{n+1}{4} \right) (2^{n+1} - n - 2) = 2^{n+1} \left[2 \left(1 - \frac{1}{2^n} - \frac{n}{2^{n+1}} \right) \right]$$

$$\Rightarrow \left(\frac{n+1}{4} \right) (2^{n+1} - n - 2) = 2(2^{n+1} - n - 2)$$

$$\Rightarrow \left(\frac{n+1}{4} \right) = 2 \Rightarrow n = 7$$

Hint The expression $\left(\frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + \dots + \frac{n}{2^n} \right)$ is an

Arithmetico-Geometric Progression.

$$\text{Let us consider } P = \left(\frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + \dots + \frac{n-1}{2^{n-1}} + \frac{n}{2^n} \right)$$

$$\text{Therefore } 2P = \left(\frac{1}{2^0} + \frac{2}{2^1} + \frac{3}{2^2} + \dots + \frac{n-1}{2^{n-2}} + \frac{n}{2^{n-1}} \right)$$

$$\Rightarrow 2P - P = \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^{n-1}} - \frac{n}{2^n}$$

$$\Rightarrow P = \left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^{n-1}} \right) - \frac{n}{2^n}$$

$$\Rightarrow P = 2 \left(1 - \frac{1}{2^n} \right) - \frac{n}{2^n}$$

$$\Rightarrow P = 2 \left(1 - \frac{1}{2^n} - \frac{n}{2^{n+1}} \right)$$

- 25** Since the length of each side of S_n equals the length of the diagonal of S_{n+1} . So the each side of S_{n+1} is $\frac{1}{\sqrt{2}}$ times that

of S_n . Thus the area of S_{n+1} is half the area of S_n . So the answer can be obtained directly through finding the area of squares for $n = 1, 2, 3$, etc. Then, there is no need to calculate the side of different squares.

Choice	n	Side of S_n	Area of S_n
-	1	10	100
-	2	$10 / \sqrt{2}$	100/2
-	3	10/2	100/4
-	4	$10 / 2\sqrt{2}$	100/8
-	5	...	100/16
-	6	...	100/32
(a)	7	...	100/64
(b)	8	...	100/128
(c)	9	...	100/256
(d)	10	$10 / 16\sqrt{2}$	100/512

Thus, you can see that at $n = 10$, the area of S_n is less than 0.5.

Hence choice (d) is the answer.

CHAPTER 19

Permutations & Combinations

It is one of the most logical chapters that you are considering for your test preparation, so it's a really good thing for those who are pretty uncomfortable with the traditional Maths-essentially, the colossal equations, intricate graphs and confusing formulae. Well, if you feel that all these things are intimidating and discouraging, then here is a *nirvana* (or relief) from all such nasty things. Now, what you need to do is just stop relying upon the formulae and try getting the answers using logic (or call it common sense). And, the biggest advantage of using the logic is that you would be able to develop a knack for replicating and expanding the scope of logic in innumerable situations and settings; while there are huge limitations in memorizing, recalling and applying the formulae in the twisted and unfamiliar situations, especially under pressure. Usually, it is easy for our minds to apply the same logic in different situations, but it is truly painstaking in relating the same formula for different situations. For example, dividing the 10 identical chocolates among 3 kids is same as the number of non-negative integral solutions of $a + b + c = 10$.

My only advice to you is that please stay open minded, logical and creative enough. If you struggle to get the answer, break the problem down into pieces or take a similar but small example in order to understand the underlying idea and its logic. In P&C (Permutation & Combination) the understanding of problem is more important than knowing its solution, because the challenge is to decode the thought process, the calculation part is too simple to be bothered about. Even if you don't know the formula/property you can get the required answer just by becoming a bit imaginative, inventive and ingenious. As a business manager or a leader of a team your job is to arrange the events, things and select the best people and resources, so why don't you start it right here and right now.

Therefore, being a very logical chapter it is one of the favorite chapters for the paper setters of competitive exams. In CAT, you can expect approx. 1-3 problems from this chapter in almost every session/slot. Other exams like XAT and IIFT too ask approx. 2-4 sums from this chapter.

Chapter Checklist

- Counting
- Permutations
- Permutations of n Things Not All Different
- Circular Permutations
- Combinations
- Distribution/Division of Distinct things among Individuals/Groups
- Distribution/Division of Identical Objects Among Individuals/Groups
- Algebraic Properties
- Re-Arrangement or Derangement
- Number Properties
- Geometrical Properties
- CAT Test

19.1 Counting

Fundamental principle of counting

- (i) **Multiplication** : If one operation can be performed in ' m ' ways and corresponding to each way of performing the first operation a second operation can be performed in ' n ' ways, then the two operations together can be performed in $m \times n$ ways. If after two operations are performed in any one of the $m \times n$ ways a third operation can be performed in p ways, then the three operations together can be performed in $m \times n \times p$ ways and so on.

In general if there are n jobs (works/operations) $j_1, j_2, j_3, \dots, j_n$ such that job j_i can be performed independently in m_i ways. ($i=1, 2, 3, \dots, n$). Then the total number of ways in which all the jobs can be performed is $m_1 \times m_2 \times m_3 \times \dots \times m_n$.

Here the different jobs/operations are mutually inclusive. It implies that all the jobs are being done in succession. In this case we use the word '**and**' to complete the all stages of operation and the meaning of '**and**' is multiplication.

Exp. 1) A student has to select a letter from vowels and another letter from consonants, then in how many ways can he make this selection?

Solution Out of 5 vowels he can select one vowel (letter) in 5 ways and out of 21 consonants he can select one consonant (letter) in 21 ways. Thus a letter from the vowels and a letter from the consonants can be selected together in $5 \times 21 = 105$ ways.

- (ii) **Addition** : If there are two jobs such that they can be performed independently in m and n ways respectively, then either of the two jobs can be performed in $(m + n)$ ways.

In general if there are various jobs which are mutually exclusive, then they can be performed in $m_1 + m_2 + m_3 + \dots + m_n$ ways.

In this case we use the word '**or**' between various jobs and the meaning of '**or**' is addition.

Exp. 2) A student has to select a letter either from vowels 'or' from consonants, then in how many ways can he make this selection?

Solution Out of 5 vowels he can select one letter in 5 ways. Similarly he can select one letter from 26 consonants in 21 ways. Thus he can select one letter in $5 + 21 = 26$ ways.

Exp. 3) If a die is cast and then a coin is tossed, find the number of all possible outcomes.

Solution A die can fall in 6 different ways showing six different points 1, 2, 3, 4, 5, 6, ... and a coin can fall in 2 different ways showing head (H) or tail (T).

$\therefore$ The number of all possible outcomes from a die and a coin = $6 \times 2 = 12$

Hint (1, H), (1, T), (2, H), (2, T), (3, H), (3, T), (4, H), (4, T), (5, H), (5, T), (6, H), (6, T) i.e., the total number of all possible outcomes is 12. i.e., the total number of all possible outcomes is 12.

Exp. 4) There are 6 trains running between Indore and Bhopal. In how many ways can a man go from Indore to Bhopal and return by a different train?

Solution A man can go from Indore to Bhopal in 6 ways by any one of the 6 trains available. Then he can return from Bhopal to Indore in 5 ways by the remaining 5 trains, since he cannot return by the same train by which he goes to Bhopal from Indore.

Thus the required number of ways = $6 \times 5 = 30$

Exp. 5) In the previous question, if the person can return by any one of the 6 trains, then find the number of ways in which he can go from Indore to Bhopal and return from Bhopal to Indore.

Solution He can go from Indore to Bhopal in 6 ways and he can return from Bhopal to Indore in 6 ways also.

Hence the number of ways in which he can go from Indore to Bhopal and return from Bhopal to Indore = $6 \times 6 = 36$ ways.

Exp. 6) Four persons entered the lift cabin on the ground floor of a six floor office. Suppose each of them can leave the cabin independently at any floor beginning with the first. Find the total number of ways in which each of the four persons can leave the cabin at different floors.

Solution Suppose M_1, M_2, M_3 and M_4 are four persons. M_1 can leave at any of the 5 floors. So M_1 can leave the cabin in 5 ways. Now, M_2 can leave the cabin at any of remaining 4 floors. So M_2 can leave the cabin in 4 ways. Similarly M_3 and M_4 can leave the cabin in 3 ways and 2 ways respectively. Thus, the total number of ways in which each of the 4 persons can leave the cabin at different floors is $5 \times 4 \times 3 \times 2 = 120$.

Exp. 7) In the above question if the 4 persons can leave the cabin at any one of the 5 floors, then find the number of ways in which this can be done.

Solution Since each of the four person can leave the cabin at any one of the 5 floors. Hence each one of M_1, M_2, M_3 and M_4 can leave the cabin in 5 ways.

Hence total number of ways in which each of the four persons can leave the cabin at any of the 5 floors is $5 \times 5 \times 5 \times 5 = 625$

Exp. 8) In a MOCK CAT there are 3 sections viz, QA, English and D I containing 4, 6 and 11 questions respectively. In how many ways can a candidate select one question from each of the three sections?

Solution The candidate can select one question from Q A in 4 ways, one question from English in 6 ways and one

question from DI in 11 ways. Hence he can select one question from each of three sections in $4 \times 6 \times 11 = 264$ ways.

Exp. 9) A tricolour flag is to be formed having three adjacent strips of three different colours chosen from six different colours. How many different coloured flags can be formed with different design in which all the three strips are always in horizontal position?

Solution First strip can be coloured in 6 ways and second strip can be coloured in 5 ways and third strip can be coloured in 4 ways. Hence all the three strips can be coloured in $6 \times 5 \times 4 = 120$ ways.
Hence there can be 120 differently coloured flags.

Exp. 10) In how many ways can a chairman and vice-chairman be elected from a committee of 8 members who are equally eligible for the post?

Solution The chairman can be elected in 8 ways, and the vice chairman can be elected in 7 ways from the remaining 7 members.
Hence the required number of ways $= 8 \times 7 = 56$ ways.

Exp. 11) In how many ways can 5 prizes be given away to 7 boys when each boy is eligible for all the prizes?

Solution First prize can be given away to any one of 7 boys in 7 ways. Again second prize can be given away to any one of the 7 boys since each boy is eligible for all the prizes. Similarly third, fourth and fifth prize can also be given away in 7 ways.
Hence the required number of ways $= 7 \times 7 \times 7 \times 7 \times 7 = 7^5$

Exp. 12) A test paper consists of 10 questions and each question has 4 choices. If each question is necessarily attempted, then find the number of ways of answering the test paper.

Solution Since each question can be answered in 4 ways. So, the total number of ways of answering 10 questions $= 4 \times 4 \times 4 \times 4 \times 4 \dots 10 \text{ times} = 4^{10}$.

Exp. 13) A set of 6 questions contains true/false type questions. Maximum how many students can take the test if all the students answer differently from others and must attempt all the questions?

Solution A question can be answered in two ways i.e. either true or false. Hence all the 6 questions can be answered in $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = 64$ ways.
Thus there can be maximum 64 students.

Exp. 14) In a test paper first section contains 5 question each with 4 choices and second section contains 4 questions each with 3 choices. In how many different ways can the paper be answered if all the questions are attempted?

Solution First section can be answered in $4 \times 4 \times 4 \times 4 \times 4 = 4^5$ ways

Second section can be answered in $3 \times 3 \times 3 \times 3 = 3^4$ ways.
Hence the whole test paper can be answered in $4^5 \times 3^4$ ways.

Exp. 15) How many three letter words can be formed using only consonants but each only once?

Solution There are only 21 consonants in English alphabet. So, there are 21 ways of filling up the first place. Now, there are only 20 ways of filling up the second place and the third place can be filled up in 19 ways.
Hence, the required number of words $= 21 \times 20 \times 19 = 7980$

Exp. 16) How many four digit numbers can be formed with the digits 3, 5, 7 and 9 only?

4	4	4	4
Th	H	T	U

A four digit number has 4 places viz, thousands, hundreds, tens and unit place. All the four places can be filled up in $4 \times 4 \times 4 \times 4 = 256$ ways since repetition of digits is not restricted.

Exp. 17) How many 3 digit number can be formed whose unit digit is always zero and repetition of digits is not allowed?

Solution

9	8	1

 $\Rightarrow 9 \times 8 \times 1 = 72$

Since zero is fixed for the unit digit place. So only 9 digits are available for the hundreds place. Now only 8 digits are available for the tens place since out of 10 digits (i.e., 0, 1, 2, 3, ..., 9) two digits at two places have been used. Therefore required possible numbers $= 9 \times 8 \times 1 = 72$.

Exp. 18) How many numbers can be formed with digits 2, 4, 6, 8 without repetition?

Solution Case 1. There are 4 numbers of 1 digit.

Case 2.

4	3

 $\Rightarrow 4 \times 3 = 12$

There are 12 numbers of 2 digits.

Case 3.

4	3	2

 $\Rightarrow 4 \times 3 \times 2 = 24$

There are 24 numbers of 3 digits.

Case 4.

4	3	2	1

 $\Rightarrow 4 \times 3 \times 2 \times 1 = 24$

There are 24 numbers of 4 digits.

Hence total required numbers $= 4 + 12 + 24 + 24 = 64$

NOTE No any number of 5, 6, 7, ..., digits can be formed since we have only four digits and repetition of digits is not allowed.

Exp. 19) How many four digit numbers can be formed using 5 only once but 5 must be used in all such numbers?

Solution Case 1.

8	9	9	1

 $\Rightarrow 8 \times 9 \times 9 \times 1 = 648$

When 5 is fixed at unit place, then there are only 8 digits available for thousands place (i.e., except to 0 and 5).

Again hundreds and tens places can be filled by any one of digits from 0 to 9 except 5.

Case 2.

8	9	1	9

 $\Rightarrow 8 \times 9 \times 1 \times 9 = 648$

When 5 is fixed at tens, place then there are only 8 digits available for thousands place and 9 digits are available for each of the hundreds and unit places.

Case 3.

8	1	9	9

 $\Rightarrow 8 \times 1 \times 9 \times 9 = 648$

Here 5 is fixed at hundreds place.

Case 4.

1	9	9	9

 $\Rightarrow 9 \times 9 \times 9 = 729$

Here 5 is fixed at thousands place and each of the hundreds, tens and unit place can be filled up in 9 ways each.

Therefore, the total required numbers of ways
 $= 648 + 648 + 648 + 729 = 2673$

Exp. 20) How many four digit numbers can be formed with the digits 0, 2, 3, 5, 8, 9 if

- (i) repetition of digits is allowed?
- (ii) repetition of digits is not allowed?

Solution (i)

5	6	6	6

 $\Rightarrow 5 \times 6 \times 6 \times 6 = 1080$

Since at thousands place zero can not be placed.

(ii)

5	5	4	3

 $\Rightarrow 5 \times 5 \times 4 \times 3 = 300$

Since at thousands place zero cannot be placed so only 5 digits are available and there are only 5 remaining digits available for hundreds place and 4 digits for tens place and 3 digits for unit place.

19.2 Permutations

Out of the given set of finite number of things, you take all or some of them and arrange in distinct possible ways, then each such arrangement is called the permutation. In Permutation, order of the things is very important.

Permutation means selection and arrangement both, while Combination means only selection.

- A permutation is said to be a Linear Permutation if the objects are arranged in a line. A linear permutation is simply called as a permutation.
- A permutation is said to be a Circular Permutation if the objects are arranged in the form of a circle.
- The number of (linear) permutations that can be formed by taking r things at a time from a set of n distinct things is denoted by ${}^nP_r$ or $P(n, r)$, for every $1 \leq r \leq n$.

$${}^nP_r = n(n-1)(n-2)(n-3) \dots (n-r+1) = \frac{n!}{(n-r)!}$$

Thus the permutations of three letters (say) a, b, c taken two

at a time are

$$ab, ba, bc, cb, ac, ca$$

Therefore the number of permutations of three different things taken two at a time is

$${}^3P_2 = \frac{3!}{(3-2)!} = 6$$

Similarly, the permutations of the set of four letters S, W, A, G number of taken two at a time is

$$SW, WS, SA, AS, SG, GS, WA, AW, WG, GW, AG, GA$$

Therefore, the number of permutations of three different things taken two at a time is

$${}^4P_2 = \frac{4!}{(4-2)!} = 12$$

NOTE $n! = 1.2.3.4 \dots (n-1).n$

$$(n-r)! = 1.2.3.4 \dots (n-r-2)(n-r-1)(n-r)$$

$$0! = 1 = 1!$$

Permutations of n Different Things

1. Number of permutations of n different things taken all at a time $= {}^nP_n = n!$
2. Number of permutations of n different things taken r at a time $= {}^nP_r = \frac{n!}{(n-r)!}$
3. Number of permutations of n different things taken at most r at a time $= {}^nP_1 + {}^nP_2 + {}^nP_3 + \dots + {}^nP_r$
4. Number of permutations of n different things taken at least r at a time $= {}^nP_r + {}^nP_{r+1} + {}^nP_{r+2} + \dots + {}^nP_n$
5. Number of permutations of n different things taken r at a time, when one particular thing always occurs $= r \cdot ({}^{n-1}P_{r-1})$
6. Number of permutations of n different things taken r at a time, when one particular thing never occurs $= {}^{n-1}P_r$
7. Number of permutations of n different things taken r at a time, when k particular things, always occur $= ({}^rP_k)({}^{n-k}P_{r-k})$
8. Number of permutations of n different things taken r at a time, when k particular things never occur $= {}^{n-k}P_r$
9. Number of permutations of n different things, taken all at a time, when m specified things always come together $= m!(n-m+1)!$
10. Number of permutations of n different things, taken all at a time, when m specified things never come together $= n! - [m!(n-m+1)!]$

Introductory Exercise 19.1

- Find the value of ${}^9P_3$.
(a) 504 (b) 309 (c) 405 (d) 600
- Find the value of n , if ${}^nP_5 = 20 {}^nP_3$.
(a) 5 (b) 8 (c) 6 (d) 4
- If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$, find r .
(a) 14 (b) 20
(c) 41 (d) 21
- How many different numbers of 3 digits can be formed with the digits 1, 2, 4, 5, 7, 8 ; none of the digits being repeated in any of the numbers so formed?
(a) 120 (b) 1200
(c) 180 (d) 270
- How many even numbers less than 10,000 can be formed with the digits 3, 5, 7, 8, 9 without any repetition?
(a) 32 (b) 16 (c) 44 (d) 41
- How many numbers with different digits each greater than 4000 can be formed from the digits 0, 2, 5, 7, 8 ?
(a) 160 (b) 168 (c) 320 (d) 270
- How many numbers greater than 5000 can be formed with the digits, 3, 5, 7, 8, 9 no digit being repeated?
(a) 216 (b) 126 (c) 512 (d) 252
- How many 6-digit telephone numbers can be constructed with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 if each number starts with 35 and no digit appears more than once?
(a) 1600 (b) 1680 (c) 900000 (d) 9000
- How many 6-digit numbers can be formed with the digits 1, 2, 4, 5, 6, 7 (no digit being repeated) which are divisible by 5?
(a) 555 (b) 156
(c) 120 (d) none of these
- Find the sum of all the four digit numbers which are formed by the digits 1, 2, 5, 6.
(a) 933510 (b) 93324 (c) 65120 (d) 8400
- How many even numbers greater than 300 can be formed with the digits 1, 2, 3, 4, 5, such that no digit being repeated in any such number?
(a) 111 (b) 600
(c) 900 (d) none of these
- How many numbers, each lying between 100 and 1000, can be formed with the digits 0, 2, 3, 4, 5, such that no digit being repeated?
(a) 24 (b) 48 (c) 72 (d) 96

Directions (for Q. Nos. 13 to 22) : Answer these questions based on the following information.

After completing his MBA, Zafar shifted to a new flat where he bought a new bookshelf and a set of 7 novels. The names of these novels, in the chronological order, are A, B, C, D, E, F and G.

- Find the number of ways in which he can arrange all the 7 novels in his bookshelf.
(a) 5040 (b) 4050 (c) 1 (d) 7
- If he wants to keep exactly 4 novels in his bookshelf, find the number of ways in which he can arrange 4 novels.
(a) 420 (b) 24 (c) 28 (d) 840
- If he wants to keep at most 4 novels in his bookshelf, find the number of ways in which he can arrange the novels.
(a) 1100 (b) 1099 (c) 999 (d) 10
- If he wants to keep at least 4 novels in his bookshelf, find the number of ways in which he can arrange the novels.
(a) 13440 (b) 1440 (c) 12440 (d) 840
- If he always wants to keep the first novel of the series in his bookshelf, find the number of ways in which he can arrange total 4 novels in his bookshelf.
(a) 120 (b) 480 (c) 720 (d) 24
- If he never wants to keep the last novel of the series in his bookshelf, find the number of ways in which he can arrange total 4 novels in his bookshelf.
(a) 360 (b) 256 (c) 240 (d) 666
- If he always wants to keep the first two novels of the series (A, B) in his bookshelf, find the number of ways in which he can arrange total 4 novels.
(a) 345 (b) 210 (c) 240 (d) 360
- If he never wants to keep the last two novels of the series (F, G) in his bookshelf, find the number of ways in which he can arrange 4 novels.
(a) 360 (b) 240 (c) 256 (d) 120
- He wants to keep the novels in his bookshelf such that the first two novels of the series are always kept together, find the number of ways in which he can arrange all the novels.
(a) 120 (b) 2440
(c) 720 (d) 1440
- He wants to keep the novels in his bookshelf such that the last two novels of the series are never kept together, find the number of ways in which he can arrange all the novels.
(a) 3600 (b) 2500
(c) 1440 (d) 2880

Directions (for Q. Nos. 23 to 40) : *Answer these questions independently of each other.*

23. Find the number of ways in which the letters of the word RAINBOW can be arranged.
(a) 5040 (b) 4050 (c) 3040 (d) 8040
24. In the different arrangements of the word RAINBOW, how many words begin with *R*?
(a) 720 (b) 360 (c) 1440 (d) 480
25. In the different arrangements of the word RAINBOW, how many words begin with *R* and end with *W*?
(a) 120 (b) 240 (c) 180 (d) 360
26. In the different arrangements of the word RAINBOW, how many words are there in which *R* and *W* are at the end positions?
(a) 120 (b) 180 (c) 210 (d) 240
27. In the different arrangements of the word RAINBOW, how many words are there in which first and last letters are vowels?
(a) 360 (b) 720 (c) 1440 (d) 2880
28. In the different arrangements of the word RAINBOW, how many words are there in which *R* and *W* are together?
(a) 720 (b) 360 (c) 540 (d) 1440
29. In the different arrangements of the word RAINBOW, how many words are there in which *R* and *W* are never together?
(a) 3600 (b) 2400 (c) 1774 (d) 1440
30. In the different arrangements of the word RAINBOW, how many words are there in which vowels are never together?
(a) 720 (b) 1440 (c) 360 (d) 3660
31. In the different arrangements of the word RAINBOW, how many words are there in which *A* is always before *I* and *I* is always before *O*?
(a) 840 (b) 420 (c) 720 (d) 630
32. In the different arrangements of the word RAINBOW, how many words are there in which vowels are always before the consonants?
(a) 72 (b) 144 (c) 96 (d) 124
33. In the different arrangements of the word RAINBOW, how many words are there in which no two consonants are together?
(a) 441 (b) 420 (c) 360 (d) 144
34. In the different arrangements of the word RAINBOW, how many words are there in which vowels may occupy only even positions?
(a) 567 (b) 144 (c) 576 (d) 625
35. In the different arrangements of the word RAINBOW, how many words are there in which vowels occupy odd positions?
(a) 676 (b) 625 (c) 343 (d) 576

36. In the different arrangements of the word RAINBOW, how many words are there in which exactly two vowels always remain together?
(a) 2880 (b) 1440 (c) 3200 (d) 1600
37. If all the words formed by the letters of the word RAINBOW are arranged in a dictionary form, what is the position of the word RAINBOW in that dictionary?
(a) 3136 (b) 3631 (c) 3361 (d) 1363
38. In how many ways can the letters of the word STRANGE be arranged so that the vowels appear at the odd places?
(a) 720 (b) 1092 (c) 1440 (d) 1772
39. In how many ways can the letters of the word MOBILE be arranged so that the consonants always occupy the odd places?
(a) 18 (b) 36
(c) 54 (d) 72
40. In how many ways can the letters of the word MOBILE be arranged so that at least two consonants remain together?
(a) 2880 (b) 3200
(c) 576 (d) 1600

Directions (for Q. Nos. 41 to 47) : *Answer these questions independently of each other.*

41. 5 men and 4 women are to be seated for a dinner, in a row, such that no two persons of the same sex sit together. Find the number of ways in which this arrangement can be done.
(a) 2880 (b) 20
(c) 86400 (d) $9! - 2(5! \times 4!)$
42. 5 men and 4 women are to be seated for a dinner, in a row, such that no two men sit together. Find the number of ways in which this arrangement can be done.
(a) 1440 (b) 2880 (c) 1720 (d) 720
43. 5 men and 4 women are to be seated for a dinner, in a row, such that no two women sit together. Find the number of ways in which this arrangement can be done.
(a) 43200 (b) 2880
(c) 7200 (d) 21600
44. 5 men and 4 women are to be seated for a dinner, in a row, such that all the women do not sit together. Find the number of ways in which this arrangement can be done.
(a) 345600 (b) 348800 (c) 347200 (d) 216006
45. 5 men and 4 women are to be seated for a dinner, in a row, such that women occupy the even places. Find the number of ways in which this arrangement can be done.
(a) 2880 (b) 1440 (c) 720 (d) 2020

46. 6 men and 3 women are to be seated for a dinner, in a row, such that no two women sit together. Find the number of ways in which this arrangement can be done.
(a) 121500 (b) 151200 (c) 112500 (d) $6! \times 120$
47. 4 men and 4 women are to be seated for a dinner, in a row, such that men and women sit alternately. Find the number of ways in which this arrangement can be done.
(a) 1152 (b) 1252
(c) 576 (d) 40320

Directions (for Q. Nos. 48 to 52) : *Answer these questions independently of each other.*

48. In how many ways can 12 soldiers stand in a queue?
(a) 11! (b) 12!
(c) $12! \cdot 1$ (d) 1
49. In how many ways can 12 soldiers stand in two queues such that each queue has 6 soldiers?
(a) ${}^{12}P_6 \times {}^{12}P_6 \times 2!$ (b) ${}^{12}P_6 \times {}^6P_6$
(c) $6! \times 6! \times 2!$ (d) ${}^{12}P_6 \times 6! \times 2!$
50. In how many ways can 12 soldiers stand in three queues such that each queue has same number of soldiers?
(a) ${}^{12}P_4 \times {}^8P_4 \times {}^4P_4$ (b) ${}^{12}P_4 \times 3!$
(c) ${}^{12}P_4 \times {}^{12}P_4 \times {}^{12}P_4 \times 3!$ (d) ${}^{12}P_4 \times {}^8P_4 \times {}^4P_4 \times 3!$
51. In how many ways can 12 soldiers stand in three queues such that one queue has 3 soldiers, another queue has 4 soldiers and so the remaining row has 5 soldiers?
(a) ${}^{12}P_5 \times {}^7P_4 \times {}^3P_3$ (b) ${}^{12}P_3 \times {}^9P_4 \times {}^5P_5$
(c) ${}^{12}P_3 \times {}^9P_4 \times {}^5P_5 \times 3!$ (d) ${}^{12}P_3 \times {}^{12}P_4 \times {}^{12}P_5 \times 3!$
52. In how many ways can 12 soldiers stand in a queue such that no other soldier stands between three particular soldiers?
(a) 21772800 (b) 22778800
(c) 21278800 (d) 362880

Directions (for Q. Nos. 53 to 55) : *Answer these questions independently of each other.*

53. Find the number of ways in which 12 different books can be arranged on a shelf so that 2 particular books shall not be together.
(a) 399168000 (b) 299168000
(c) 199168000 (d) 369088000

54. Find the number of ways in which 12 different books can be arranged on a shelf so that all the 3 particular books shall not be together.
(a) $126 \times 10!$ (b) $1320 \times 9!$
(c) 387892 (d) 39916800
55. Find the number of ways in which 12 different books can be arranged on a shelf so that none of the 3 particular books shall be together.
(a) $126 \times 9!$ (b) $1320 \times 9!$
(c) $720 \times 10!$ (d) 261273600
56. There are 3 distinct books on Physics, 4 distinct books on Chemistry and 5 distinct books on Biology. In how many ways can these books be placed on a shelf if the books on the same subject are to be together?
(a) 103680 (b) 17280
(c) $\frac{12!}{3!}$ (d) 60

Directions (for Q. Nos. 57 to 60) : *Answer these questions independently of each other.*

57. How many different signals can be given using any number of flags from 6 flags of different colours?
(a) 1240 (b) 1956
(c) 3976 (d) 1976
58. How many different signals can be given using 3 coloured flag from 6 flags of different colours?
(a) 120 (b) 60 (c) 18 (d) 20
59. 10 students took a test and each one got the distinct score. In how many ways can three distinct prizes be given?
(a) 270 (b) 360
(c) 720 (d) 540
60. There are 10 stations on Konkan Railway Line that connects Mumbai and Mangalore at the two ends of it. How many different kinds of tickets of second class must be printed in order to make a passenger buy a ticket so that he can travel from one station to any other station?
(a) 90 (b) 45
(c) 135 (d) 100
61. In how many ways three different rings can be worn in four fingers such that at most one ring in any finger can be worn?
(a) 24 (b) 12
(c) 36 (d) 120

19.3 Permutations of n Things Not All Different

1. Number of permutations of n things taken all at a time when p of them are all alike and the rest are all different $= \frac{n!}{p!}$
2. Number of permutations of n things taken all at a time when p things are alike of one kind, q things are alike of other kind, r things are alike of another kind and rest all, if any, are different from p, q, r , then the number of

$$\text{permutations of } n \text{ things} = \frac{n!}{p!q!r!}$$

3. Number of permutations of n things taken all at a time when p_1 things are alike of one kind, p_2 things are alike of second kind, p_3 things are alike of third kind, ... p_r things are alike of r^{th} kind, and rest all, if any, are different from $p_1, p_2, p_3, \dots, p_r$, then the number of permutations of n things $= \frac{n!}{p_1!p_2!p_3!\dots p_r!}$

Introductory Exercise 19.2

1. In how many ways can the letters of the word REPEAT be arranged?
(a) 2240 (b) 232230 (c) 360 (d) 235760
2. In how many ways can the letters of the word TAMANNA be arranged?
(a) 120 (b) 420
(c) 840 (d) none of these
3. In how many ways can the letters of the word RECUPERATE be arranged?
(a) 234000 (b) 123000 (c) 232300 (d) 302400
4. In how many ways can the letters of the word ASSASSINATION be arranged?
(a) 181800 (b) 818100 (c) 108108 (d) 10810800
5. In the above question (number 4) find the number of arrangements of four different letters of the word in a row?
(a) 360 (b) 180
(c) 540 (d) none of these
6. How many arrangements can be made out of the letters of the word COMMITTEE, taken all at a time, such that the four vowels do not come together?
(a) 216 (b) 45360 (c) 1260 (d) 43200
7. How many arrangements can be made out of the letters of the word COMMITTEE, taken all at a time, such that none of the four vowels come together?
(a) 5400 (b) 360 (c) 2160 (d) 14400
8. In how many ways can the letters of the word DUMDUMDIGADIGA be arranged?
(a) 180 (b) 90
(c) 270 (d) none of these
9. How many different words can be formed with the letters of the word NAINITAL such that each of the word begin with L and end with T ?
(a) 90 (b) 80
(c) 88 (d) 82
10. How many words can be made from the word MATHEMATICS in which vowels are together?
(a) 12960 (b) 120960
(c) 15400 (d) none of (a), (b), (c)
11. How many words can be made from the word IMPORTANT in which both T do not come together?
(a) 141120 (b) 112244
(c) 113113 (d) 888222
12. How many words can be made from the word TING TING TRING in which vowels occupy even positions?
(a) 336000 (b) 85360
(c) 113600 (d) none of these
13. If the different permutations of the word PRODIGIOUS are listed as in a dictionary, then what is the rank of the word PRODIGIOUS?
(a) 200966 (b) 609902
(c) 12500 (d) 24800
14. If all the letters of the word SEQUESTERED be arranged as in a dictionary, what is 50th word?
(a) DEEEEURSTS (b) RUQDESTESEE
(c) ESSTREEUQDE (d) DQUESTREEES
15. How many 7 digit numbers can be formed using the digits 1, 2, 0, 2, 4, 2, 4?
(a) 120 (b) 360 (c) 240 (d) 424
16. How many 6 digit numbers can be formed out of the digits of the number 113113?
(a) 15 (b) 180 (c) 888 (d) 222
17. How many 6 digit numbers can be formed out of the number 567724, which are even?
(a) 360 (b) 480
(c) 180 (d) 220
18. How many numbers greater than a million can be formed with the digits 5, 5, 2, 2, 1, 7, 0?
(a) 980 (b) 1080
(c) 920 (d) 1200

19. How many different words can be formed with the letters of the word REGURGITATE so that the two T's are always together?
(a) 453600 (b) 543600 (c) 183600 (d) 5040
20. The number of ways in which the letters of the word SUMPTUOS can be arranged so that the two U's do not come together is :
(a) 5040 (b) 7560
(c) 38920 (d) none of (a), (b), (c)
21. In how many ways the letters of the word AFLATOON be arranged if the consonants and vowels must occupy alternate places?
(a) 144 (b) 143 (c) 288 (d) 248
22. In how many ways can the letters of the word SOOTHSAYER be arranged so that the two S's and two O's come together?
(a) 23400 (b) 302050 (c) 12340 (d) 40320
23. There are three copies of each of 4 different books. In how many ways can they be arranged on a shelf?
(a) 369600 (b) 366900 (c) 396600 (d) 306609
24. There are 3 red, 4 green and 5 pink marbles in a bag. They are drawn one by one and arranged in a row. Assuming that all the 12 marbles are drawn, determine the number of different arrangements.
(a) 22770 (b) 27720 (c) 22077 (d) 27270
25. There are 12 umbrellas placed in a row. Out of these, 3 identical umbrellas are of red colour, 4 identical umbrellas are of pink colour and 5 identical umbrellas are of white colour. In how many ways can these 12 umbrellas be arranged so that at least one umbrella is separated from the umbrellas of the same colour?
(a) 27714 (b) 27720 (c) $\frac{12!}{3!4!5!} \times 3!$ (d) 37216

Permutations Where Repetitions are Allowed

1. Number of permutations of n different things taken exactly r at a time, when each may be repeated any number of times in each arrangement $= n^r$
2. Number of permutations of n different things taken at most r at a time, when each may be repeated any number of times in each arrangement

$$= n + n^2 + n^3 + \dots + n^r = \frac{n(n^r - 1)}{n - 1}$$
3. Number of permutations of n different things taken at least r at a time, when each may be repeated any number of times in each arrangement

$$= n^r + n^{r+1} + n^{r+2} + \dots + n^n$$

$$= \frac{n^r (n^{(n-r+1)} - 1)}{n - 1}$$

Introductory Exercise 19.3

1. How many numbers of 5 digits can be formed with the digits 0, 2, 3, 4 and 5 if the digits may repeat?
(a) 2500 (b) 250 (c) 120 (d) 2400
2. How many numbers each lying between 9 and 1000 can be formed with the digits 0, 1, 2, 3, 7, 8 (numbers can be repeated) ?
(a) 30 (b) 120 (c) 210 (d) 90
3. How many five-figures numbers can be formed with the digits 1, 2, 3, 4, 5 given that an even digit may not occupy an even place if the digits may be repeated?
(a) 1125 (b) 5210 (c) 1152 (d) 2120
4. How many of the numbers from 1000 to 9999 (both inclusive) do not have four different digits?
(a) 4446 (b) 4664 (c) 4464 (d) 6444
5. Find the number of three digit numbers (repetitions allowed) such that atleast one of the digits is 9.
(a) 252 (b) 648
(c) 468 (d) 864
6. How many natural numbers not exceeding 4321 can be formed with the digits 1, 2, 3, 4 if repetition is allowed?
(a) 123 (b) 113 (c) 222 (d) 313
7. In how many ways 4 rings of different types can be worn in 3 fingers?
(a) 49 (b) 12 (c) 24 (d) 81
8. In how many ways can 5 prizes be given away to 4 boys, when each boy is eligible for all the prizes?
(a) 1024 (b) 20 (c) 625 (d) 540
9. In how many ways can n balls be randomly distributed in n cells?
(a) $n!$ (b) n^n (c) $n(n-1)$ (d) 2^n
10. In the above question, what will be the number of ways if it is specified that each cell is to be occupied?
(a) $n!$ (b) n^n (c) n^3 (d) 2^n
11. In how many ways can 5 letters be posted in 4 letter boxes?
(a) 512 (b) 1024 (c) 625 (d) 20

19.4 Circular Permutations

1. Number of circular permutations of n different things taken all at a time $= (n - 1)!$
2. Number of circular permutations of n different things taken all at a time in one direction (either clock-wise or anti-clock-wise) $= \frac{(n - 1)!}{2}$

Introductory Exercise 19.4

1. In how many ways can 6 boys form a ring?
(a) 120 (b) 720
(c) 119 (d) none of (a), (b), (c)
2. In how many ways can 6 boys be arranged at a round table so that 2 particular boys may be together?
(a) 24 (b) 48 (c) 360 (d) 60
3. In how many ways can 6 beads be strung into a necklace?
(a) 60 (b) 360 (c) 720 (d) 120
4. In how many ways can 5 men and 2 ladies be arranged at a round table if two ladies are never together?
(a) 5040 (b) 480 (c) 240 (d) 720
5. 9 persons were invited for a business meeting by Ambani, where the host be seated at a circular table. How many different arrangements are possible if two invitees viz., Sahara and Mahindra be seated on either side of the host Ambani?
(a) 10080 (b) 10800 (c) 9200 (d) 4600
6. In a G-20 meeting there were total 20 people representing their own country. All the representative sat around a circular table. Find the number of ways in which we can arrange them around a circular table so that there is exactly one person between two representatives namely Manmohan and Musharraf.
(a) $2 \times (17!)$ (b) $2 \times (18!)$
(c) $(3!) \times (18!)$ (d) $(17!)$
7. 6 men and 6 women are to be seated at a circular table such that no two women are adjacent, then the number of arrangements is :
(a) 86400 (b) 46800 (c) 12! (d) $\frac{1}{2}(12!)$
8. There are 3 boys and 4 girls, seated around a circular table so that no two boys are together. Find the number of ways in which this can be done.
(a) 36 (b) 120 (c) 144 (d) 132
9. 7 sisters dine at a round table. They dine together till each of them dine with different neighbours i.e., they do not like to dine with same neighbours in any two arrangements. In a year how many days they dine together?
(a) 180 (b) 504 (c) 720 (d) 360
10. Find the number of ways in which 10 different flowers can be strung to form a garland so that 3 particular flowers are always together.
(a) 30240 (b) 30420
(c) 23400 (d) None of (a), (b), (c)
11. 3 male and 3 female singers sat around a circular table. Alka is a female singer did not sat adjacent to a male singer and Sonu is a male singer did not sat adjacent to a female singer. In how many ways this could be done?
(a) 4 (b) 6 (c) 9 (d) 10
12. Three couples book a table for six people at a restaurant that has only circular tables. If no husband sits opposite his wife, in how many ways can three couples be arranged to sit around the table?
(a) 64 (b) 117 (c) 96 (d) 56
13. Twelve students are called for the group discussion of which no two students have the same rank. They sit in a circle for the group discussion in such a way that the students sitting just next to either side of any student should have either higher rank or lower rank. Find the number of ways in which they can be arranged.
(a) 14400 (b) 28560 (c) 86400 (d) 72000
14. In a single-occupancy co-ed hostel, consisting of 20 rooms, exactly 10 guys and 10 girls are allowed to stay in. Each one of them stays in a separate room. The hostel is designed in a circular form such that the door of a room opens opposite the door of another room. In how many ways can the rooms be allocated to them so that whenever a guy looks either side of his door there is always a girl next door?
(a) $2! \times (10!)^2$ (b) $20! - 19!$
(c) $9! \times 10!$ (d) $2! \times \frac{19!}{9! \times 10!}$
3. Number of circular permutations of n different things taken r at a time $= \frac{{}^nP_r}{r}$
4. Number of circular permutations of n different things taken r at a time in one direction (either clock-wise or anti-clock-wise) $= \frac{{}^nP_r}{2r}$

19.5 Combinations

A selection that can be formed by taking some or all of a finite set of things (or objects) is called a Combination.

Let A, B, C be three letters, then we can combine any two of them in the following ways:

$$AB, BC, CA$$

Similarly, if A, B, C, D are four letters, then we can combine any two of them in the following ways:

$$AB, AC, AD, BC, BD, CD$$

Similarly, we can combine any three of A, B, C and D as following: ABC, ABD, ACD, BCD

The number of combinations of n dissimilar things taken r at a time is denoted by ${}^nC_r$ or $C(n, r)$ or $\binom{n}{r}$

$${}^nC_r = \frac{n!}{r!(n-r)!}; \text{ Whereas } C \text{ denotes the combination and}$$

$$r \leq n$$

NOTE In the combination order of the letters (or things/people) does not matter. That means AB and BA are same. Similarly, $ABC, ACB, BAC, BCA, CAB, CBA$ are same, as order does not matter.

Essential Properties

- ${}^nC_r = \frac{{}^nP_r}{r!}$
- ${}^nC_0 = {}^nC_n = 1$
- ${}^nC_r = {}^nC_{n-r} \quad (0 \leq r \leq n)$
- ${}^nC_x = {}^nC_y \Leftrightarrow x + y = n \text{ or } x = y; (x, y \in W)$
- ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$
- If n is even, the greatest value of ${}^nC_r = {}^nC_m$ where $m = n/2$

$$7. \text{ If } n \text{ is odd, the greatest value of } {}^nC_r = {}^nC_m \text{ where } m = \frac{(n-1)}{2} \text{ or } m = \frac{(n+1)}{2}$$

$$8. {}^rC_r + {}^{r+1}C_r + {}^{r+2}C_r + \dots + {}^nC_r = {}^{n+1}C_{r+1}; r \leq n$$

$$9. {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n = 2^n$$

$$10. {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n = 2^n - 1$$

$$11. {}^nC_0 + {}^nC_2 + {}^nC_4 + \dots = 2^{n-1}$$

$$12. {}^nC_1 + {}^nC_3 + {}^nC_5 + \dots = 2^{n-1}$$

Combination of n Different Things

- Number of combination of n different things taken r at a time, when x particular things always occur $= {}^{n-x}C_{r-x}$
- Number of combination of n different things taken r at a time, when x particular things never occur $= {}^{n-x}C_r$
- Number of combination of n different things taken r at a time, when x particular things always occur and y particular things never occur $= {}^{n-x-y}C_{r-x}$
- Number of combination of n different things taken r at a time, when x particular things are not together in any selection $= {}^{n-x}C_{r-x}$
- Number of ways of selections of zero or more things from a group of n distinct things $= {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n = 2^n$
- Number of ways of selections of one or more things from a group of n distinct things $= {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n = 2^n - 1 \quad (\because {}^nC_0 = 1)$

Introductory Exercise 19.5

- Find the value of ${}^8C_3$.
(a) 56 (b) 8! (c) 65 (d) 3^8
- Find the value of ${}^{10}C_5$.
(a) 525 (b) 126 (c) 252 (d) 50
- If ${}^{13}C_x = {}^{13}C_y$ and $x \neq y$, what is the value of $x + y$?
(a) 6 (b) 7 (c) 13 (d) 26
- ${}^{17}C_r = {}^{17}C_{r+3}$. Find the value of r .
(a) 17 (b) 6 (c) 7 (d) 13
- ${}^7P_3 = n \cdot {}^7C_3$, find the value of n .
(a) 4 (b) 8 (c) 6 (d) 9
- If $C(2n, 3) : C(n, 2) = 12 : 1$, find n .
(a) 3 (b) 5 (c) 6 (d) 4
- What is the least possible value of n if :
 ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^nC_3$
(a) 6 (b) 8 (c) 7 (d) 12
- If ${}^nC_x = 56$ and ${}^nP_x = 336$, find n and x .
(a) 7, 3 (b) 8, 4 (c) 8, 3 (d) 9, 6
- What is the value of x when ${}^{11}C_x$ is maximum?
(a) 7 (b) 6 (c) 5 (d) both (b) and (c)

10. What is the maximum value of ${}^{12}C_x$ for any natural number x ?
(a) 6 (b) 7 (c) 8 (d) 4
11. For what value of x , ${}^{14}C_x$ is maximum?
(a) 5 (b) 6 (c) 7 (d) 8
12. How many different committees of 5 members may be formed from 6 gentlemen and 4 ladies?
(a) 181 (b) 357 (c) 603 (d) 252
13. In a test paper there are total 10 questions. In how many different ways can you choose 6 questions to answer?
(a) 210 (b) 540
(c) 336 (d) none of these
14. In the above question (no. 13) if the question number 1 is compulsory in how many ways can you choose to answer 6 questions in all?
(a) 53 (b) 63 (c) 126 (d) 210
15. Droupdi has 5 friends. In how many ways can she invite one or more of them to a dinner?
(a) 31 (b) 5^5 (c) 13 (d) 2^5
16. In how many ways can a committee of 6 members be formed from 7 men and 6 ladies consisting of 4 men and 2 ladies?
(a) 252 (b) 525 (c) 625 (d) 256
17. A committee of 5 persons is to be formed from a group of 6 gentlemen and 4 ladies. In how many ways can this be done if the committee is to be included atleast one lady?
(a) 123 (b) 113
(c) 246 (d) 945
18. A committee of 7 persons is to be chosen from 13 persons of whom 6 are Americans and 7 are Indians. In how many ways can the selection be made so as to retain a majority of Indians?
(a) 945 (b) 1057
(c) 923 (d) 1056
19. In how many ways can a committee of 4 men and 3 women be appointed from 6 men and 8 women?
(a) 480 (b) 408
(c) 420 (d) 840
20. In the previous question (no. 19) what will be the number of committees in which Miss A refuses to serve if Mr. B is a member?
(a) 210 (b) 420
(c) 630 (d) none of these
21. In how many ways 7 members forming a committee out of 11 be selected so that 3 particular members must be included?
(a) 60 (b) 130 (c) 80 (d) 70
22. In the previous question (no. 21) how many committees can be formed if 3 particular members must not be included?
(a) 6 (b) 8 (c) 10 (d) 61
23. A committee of 3 experts is to be selected out of a panel of 7 persons, three of them are engineers, three of them are managers and one is both engineer and manager. In how many ways can the committee be selected if it must have at least an engineer and a manager?
(a) 33 (b) 22 (c) 11 (d) 66
24. A committee of 5 persons is to be formed out of 6 gents and 4 ladies. In how many ways this can be done, when at most two ladies are included?
(a) 186 (b) 168 (c) 136 (d) 169
25. In an election, a voter may vote for any number of candidates not greater than the number to be chosen. There are 7 candidates and 4 members are to be chosen. In how many ways can a person vote?
(a) 89 (b) 98 (c) 79 (d) 101
26. At an election there are 5 candidates and 3 members are to be elected and a voter is entitled to vote for any number to be elected. In how many ways a voter can a vote?
(a) 25 (b) 5^3 (c) 3^5 (d) 15
27. A question has two parts, part A and part B, each containing 8 questions. If the students has to choose 6 from part A and 5 questions from part B, in how many ways can he choose the questions?
(a) 1268 (b) 1788 (c) 1024 (d) 1568
28. An examinee is required to answer six questions out of twelve questions which are divided into two groups each containing six questions and he is not permitted to answer more than four questions from any group. In how many ways can he answer six questions?
(a) 750 (b) 850 (c) 580 (d) 570
29. An examination paper consists of 12 questions divided into two parts, part A and part B. Part A contains 7 questions and part B contains 5 questions. A candidate is required to attempt 8 questions, selecting atleast 3 from each part. In how many ways can he select the questions?
(a) 240 (b) 60 (c) 420 (d) 480
30. In MOCK CAT Quantitative Aptitude Section was divided into 3 groups of 5, 5 and 4 questions respectively. A candidate is required to answer 6 questions taking at least two questions from each of the first two groups and atleast one question from the third group. In how many ways can he make up his choice?
(a) 1200 (b) 1400 (c) 1284 (d) 1560

31. Mr. Daruwala has 18 acquaintances of whom 13 are relatives. In how many ways he may invite 10 guests so that 8 of whom are relatives?
 (a) 12870 (b) 22022
 (c) 20222 (d) 12780
32. If there are 11 players in a cricket team, all of whom shake hands with each other, how many shake hands take place in the team?
 (a) 36 (b) 55
 (c) 66 (d) none of these
33. Amitabh has a list of 24 friends. He wishes to invite some of them in such a manner that he can enjoy maximum number of parties, but in each party the number of friends (*i.e.*, invitees) be same and each party must have different set of persons. Then how many parties can Amitabh enjoy?
 (a) 2704156 (b) 357600
 (c) 235763 (d) 270156
34. An ice-cream parlour offers only family packs of ice-creams with 11 different flavours. If each member of a family loves different flavours, then maximum how many such families can purchase the ice-cream if each family contains equal number of persons?
 (a) 246 (b) 462
 (c) 123 (d) ${}^{11}C_2$
35. In the previous question (no. 34) what is the maximum possible number of member in a family?
 (a) 4 (b) 5
 (c) 6 (d) 8
36. In a meeting everyone had shaken hands with everyone else, it was found that 66 handshakes were exchanged. How many members were present in the meeting?
 (a) 10 (b) 14 (c) 12 (d) 8
37. In how many ways can a cricket team of eleven players be chosen out of a batch of 16 players if a particular player is always chosen?
 (a) 2002 (b) 3003 (c) 1003 (d) 7603
38. In the previous question (no. 37) if a particular player is never chosen, then the number of ways in which a cricket team of eleven players can be chosen is :
 (a) 2365 (b) 2359 (c) 1365 (d) 1056
39. A cricket team of 11 players is to be formed from 16 players including 4 bowlers and 2 wicket keepers. In how many different ways can a team be formed so that the team has atleast 3 bowlers and atleast one wicket keeper?
 (a) 2472 (b) 2274 (c) 2427 (d) 1236
40. A cricket team of 11 players is to be formed from 20 players including 6 bowlers and 3 wicket keepers. In how many different ways can a team be formed so that the team contain exactly 2 wicket keepers and atleast 4 bowlers?
 (a) 22725 (b) 27225
 (c) 22275 (d) none of (a), (b), (c)
41. A team of 11 cricketers containing 1 wicket keeper, 2 bowlers, 3 all rounders and 5 batsmen, is to be formed from a group of 25 cricketers containing 2 wicket keepers, 8 bowlers, 5 all rounders and 10 batsmen?
 (a) 141120 (b) 111240
 (c) 101240 (d) none of (a), (b), (c)
42. Out of 3 books on Economics, 4 books on Corporate Strategy and 5 books on Philosophy, how many collection consists of exactly one book on each subject?
 (a) 40 (b) 36 (c) 60 (d) 120
43. In the previous question (no. 42) what is the number of different collections of books if atleast one book on each subject is taken?
 (a) 2365 (b) 3255 (c) 4224 (d) 1236
44. A box contains 7 red, 6 white and 4 blue balls. How many selection of three balls can be made so that all three are red balls?
 (a) 35 (b) 70 (c) 42 (d) 17
45. In the previous question (no. 44) if the red coloured ball is not taken, then the number of selections is :
 (a) 30 (b) 120
 (c) 60 (d) none of (a), (b), (c)
46. In the question (no. 44), if there is one ball of each colour, then the number of selections is :
 (a) 148 (b) 124 (c) 168 (d) 186
47. An urn contains 5 different red and 6 different green balls. In how many ways can 6 balls be selected so that there are atleast two balls of each colour?
 (a) 425 (b) 245 (c) 125 (d) 625
48. Find the number of different words that can be formed from 15 consonants and 5 vowels by taking 2 consonants and 4 vowels in each word.
 (a) 625 (b) 630 (c) 525 (d) 615
49. Find the number of ways of selecting 4 letters from the word EXAMINATION.
 (a) 136 (b) 126 (c) 252 (d) 525
50. How many words can be formed by using 4 letters at a time out of the letters of the word MATHEMATICS?
 (a) 2445 (b) 2454
 (c) 1243 (d) 1454

51. In how many ways can 3 ladies and 3 gentlemen be seated at a round table, so that any two and only two of ladies sit together?
(a) 72 (b) 36 (c) 24 (d) 144
52. Eighteen guests have to be seated half on each side of a long table. Four particular guests desire to sit on one particular side and three others on other side. Determine the number of ways in which the sitting, arrangements can be made.
(a) $18!$ (b) $462 \times (9!)^2$
(c) $(9!)^2$ (d) $7! \times (9!)^2$
53. Find the value of ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + \dots + {}^{10}C_{10}$.
(a) 100 (b) 10^2 (c) 2^{10} (d) $2^{10} - 1$
54. Find the value of ${}^{10}C_2 + {}^{10}C_3 + \dots + {}^{10}C_{10}$.
(a) 100 (b) $10^2 - 10$ (c) $2^{10} - 11$ (d) $2^{10} - 2$
55. Find the value of ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + \dots + {}^{10}C_9$.
(a) 100 (b) $10^2 - 10$ (c) $10^2 - 11$ (d) $2(2^9 - 1)$
56. Find the value of ${}^{10}C_0 + {}^{10}C_2 + {}^{10}C_4 + \dots$.
(a) ∞ (b) 2^9 (c) $2^{10} - 1$ (d) $4^9 - 1$
57. Find the value of $[({}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + \dots) \times ({}^{20}C_1 + {}^{20}C_3 + {}^{20}C_5 + \dots) \times ({}^{30}C_1 + {}^{30}C_3 + {}^{30}C_5 + \dots) \times \dots \times ({}^{100}C_1 + {}^{100}C_3 + {}^{100}C_5 + \dots)]$
(a) 540 (b) 55^2 (c) 2^{540} (d) $2^{55} - 1$
58. Larry has a long weekend off for 4 days that starts from Thursday and lasts till Sunday. She wants to attend some spiritual enrichment sessions that are held every day for four hours. In how many ways can she attend one or more sessions during this long weekend?
(a) 15 (b) 16 (c) 4 (d) $4 \times {}^4C_1$
59. Madhulika is planning to join International School for languages, which imparts the professional courses on numerous languages under various programs. Program 1 offers 1 language, program 2 offers 2 languages, program 3 offers 3 languages, and so on. No two programs offer any common language. One can choose any number of languages from a certain program, but all the languages must be chosen from any one program only. Currently, there are total 10 programs on offer with the program 10 offering exactly 10 languages. Madhulika is willing to enroll for exactly 4 languages, find the number of ways in which she can study any 4 languages.
(a) 462 (b) 210 (c) 40 (d) ${}^{10}C_1 \times {}^{49}C_4$
60. Paratha Inc. is a chain of food joints, which specializes in various kinds of parathas – the Indian dish, usually stuffed with various kinds of vegetables cooked and smashed. It has total 210 different

parathas in its menu. The menu is designed in such a way that n^{th} pack contains n parathas. Also, no two packs contain common parathas. However, you can choose any r parathas from any pack ($r \leq n$), but not from different packs, for any particular order.

The parathas can be ordered online only through home delivery service, and only one order in a day is accepted from a particular customer. Shivesh Raina — my foodie friend — wants to order 3 different parathas. Then, on a particular day, how many different sets of parathas can he choose from?
(a) 5985 (b) 1140 (c) 5040 (d) ${}^{20}C_1 \times {}^{206}C_3$

Directions (for Q. Nos. 61 and 62) : Answer these questions based on the following information.

To win the Mathematics Olympiad you can choose any one module from 40 different modules. The number of problems in each module is same as the number of module, i.e., n^{th} module has n problems. The module 1 has the toughest problem and so the module 40 has the easiest problems.

61. Nilekani, an aspirant, attempts only 2 problems from a particular module. The number of ways in which he can attempt 2 distinct problems is
(a) 10850 (b) 10660 (c) 50425 (d) ${}^{20}C_1 \times {}^{206}C_3$
62. Narainmurthi, another aspirant, attempts exactly 38 problems from a particular module. The number of ways in which he can attempt 38 distinct problems is
(a) 2460 (b) 1640 (c) 820 (d) 40^{38}

Directions (for Q. Nos. 63 to 66) : Answer these questions based on the following information.

This week Cinemax, a multi screen cinema, is showing 4 new movies. Shahid Chopra and Priyanka Kapoor, the movie freak couple, who never watch movies individually or separately, can watch as many movies as possible if they get good reviews, else they may not go for any movie if the movies are not entertaining enough.

63. Find the total number of ways in which they can watch any number of new movies this week, irrespective of the order of the movie?
(i) ${}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$
(ii) 2^4 (iii) 4^2 (iv) 4
(a) Only (i) (b) Only (ii) and (iii)
(c) Either (i) or (iv) (d) (i), (ii) and (iii)
64. If his girlfriend Priyanka insists on watching at least one new movie this week, irrespective of the order of the movie, then the number of ways in which they can watch the movies is
(i) ${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$ (ii) $2^4 - 1$
(iii) 3 (iv) $2^4 - {}^4C_0$
(a) Only (iii) (b) Only (iv)
(c) All, except (iii) (d) Only (i) and (ii)

65. If his ex-girlfriend Kareena Gandhi also joins the couple and now all of them want to watch at least 2 movies, irrespective of the order of the movie, then the number of ways in which they can watch the movies is

(i) ${}^4C_2 + {}^4C_3 + {}^4C_4$ (ii) $2^4 - 5$
 (iii) 2 (iv) $2^4 - {}^4C_0 - {}^4C_1$
 (a) Only (iii)
 (b) Only (iv)
 (c) All, except (iii)
 (d) Only (i) and (ii)

66. If Shahid and Priyanka realize, before they start watching any movie, that they have to do a long due shopping and so they decide not to watch more than 3 movies this week, then the number of ways, irrespective of the order of the movie, in which they can watch the movies is

(i) ${}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3$
 (ii) $2^4 - 1$
 (iii) $2^4 - {}^4C_4$
 (iv) 6
 (a) Only (iii) (b) Only (iv)
 (c) Except (i) only (d) Only (i), (ii) and (iii)

Directions (for Q. Nos. 67 to 70) : Answer these questions based on the following information.

Amirbhai Moviewala, regarded for his perfection, is a very fastidious actor as far selection of the scripts is concerned. This year he has been offered as many as 100 scripts to read before he confirms to act in any movie which would be based on these scripts only. Facing the problem of plenty and hectic shooting schedule this year, he may not go through some or all of the scripts. Also, he is not looking for any other assignment in whatsoever media or role-small screen, commercials, reality shows, cameo role etc.

67. Find the number of ways in which he can read these scripts, irrespective of their order.

(a) 100 (b) 101
 (c) 2^{100} (d) $2^{100} - 1$

68. If he decides not to read more than 50 scripts, then the number of ways in which he can read these scripts, irrespective of their order, is

(a) 2^{50} (b) $2^{50} - 1$
 (c) $2^{100} - 2^{50} - 1$ (d) $\frac{1}{2}(2^{100} + {}^{100}C_{50})$

69. If he decides to read at least 2 but not more than 98 scripts, then the number of ways in which he can read these scripts, irrespective of their order, is

(a) 97 (b) 2^{97}
 (c) $2^{100} - 202$ (d) $2^{100} - 2^{98} - 2$

70. If he decides to read at least 50 scripts this year, then the number of ways in which he can read these scripts, irrespective of their order, is

(a) $\frac{1}{2}(2^{100} - {}^{100}C_{50})$ (b) 2^{49}
 (c) $2^{100} - 2^{51}$ (d) $2^{99} + \frac{1}{2}({}^{100}C_{50})$

71. A coaching institute imparts training in 10 different subjects. It wants to develop various modules in such a way that each module has equal number of subjects and no two modules have all the same subjects. Find the number of subjects in each module when maximum number of modules can be developed.

(a) 5 (b) 4 (c) 8 (d) 6

72. A sweet shop sells 9 varieties of sweets. The shopkeeper wants to pick not more than one piece of any variety of sweets to make the assortment of sweets. He then packs equal number of pieces in each box from variety of sweets in order to have maximum number of distinct assortments. How many varieties of sweets should he pack in a box so that each box contains distinct assortment of sweets?

(a) 3 (b) 4
 (c) 5 (d) both (b) and (c)

Directions (for Q. Nos. 73 and 74) : Answer these questions based on the following information.

A team of 8 software engineers works on a project, for an American client, in Bengaluru, India. Every team member wants to go to the client's site in USA, as many times as it's possible for them. However, as per the company's policy every member will visit the site same number of times in a year. Each team member has to travel in groups of the same size each time the team goes to client's site. Not all members can be same in any two groups visiting the client's site in a year.

73. Maximum how many different groups can be formed?

(a) 4 (b) 16 (c) 70 (d) 32

74. Maximum how many times a team member can go to the client's site in a year?

(a) 32 (b) 35 (c) 49 (d) 7

Directions (for Q. Nos. 75 to 79) : Answer these questions based on the following information.

Shashi Huzoor – a prolific writer – is very vocal and straightforward in his views. He loves to opine through twitter, a website that allows anybody to tweet (or publish digitally) a message containing not more than 140 characters including the number of blank spaces.

But, it's not possible to tweet a blank message. Recently he got mired in a nasty tweet. Since then he has a superstition that if he tweets a post that uses odd number of alphanumeric characters, from the set of 26 alphabets and 10 numerals, it brings misfortune and a lot of trouble for him. So he prefers to use only even number of alphanumeric characters in all his tweets. That is to write a post on twitter he may select either 2 characters or 4 characters or 6 characters, and so on, from the set $\{a, b, c, \dots, z, 0, 1, 2, \dots, 9\}$ to write a tweet. Repetition of any character does not matter to him; that is total number of characters in a tweet can be either odd or even.

75. Find the number of ways in which he can select the characters to tweet for his ardent followers.
(a) 36 (b) 2^{36} (c) $4^{36} - 1$ (d) $2^{35} - 1$
76. Sunanda Khushker, his beautiful wife, loves his everything but his superstition with the usage of odd number of characters. This invokes her to insist him to write a tweet using nothing but an odd number of characters. Succumbed to his wife's love and insistence he decides to tweet a post using odd number of characters only. Find the number of ways in which he can select the characters for tweeting.
(a) $2^{35} + 1$ (b) 2^{36} (c) $4^{18} - 1$ (d) 2^{35}
77. If he selects even number of characters from alphabets and even number of characters from numerals, separately, then the number of ways in which he can select the characters for tweeting.
(a) $2^{35} + 1$ (b) $2^{34} - 1$
(c) $4^{36} - 1$ (d) $2^{35} - 1$
78. Seeing his dilemma, one of his fans tweeted back that Mr. Shashi should not be bogged down and further advises him that if he just selects an odd number of characters from alphabets and odd number of characters from numerals, separately, no trouble will occur to him. Then the number of ways in which he can select the characters for tweeting.
(a) $2^{35} + 1$ (b) 2^{34}
(c) $2^{35} - 2$ (d) $2^{35} - 1$
79. Mr. Shashi is also a globetrotter. Once on board he meets a numerologist, who tells him not to worry as even if he selects an odd number of characters from alphabets and an odd number of characters from numerals, separately, but if he selects overall an even number of characters from the set of 36 alphanumeric characters, all sorts of problems will be vanished. Then the number of ways in which he can select the characters for tweeting is
(a) $2^{35} + 1$ (b) $2^{35} - 1$
(c) $2^{35} - 2$ (d) 2^{34}

Combination of n Identical Things

- Number of ways of selections of r things from a group of n identical things = 1.
- Number of ways of selections of at most r things from a group of n identical things = $r + 1$
- Number of ways of selections of at least r things from a group of n identical things = $(n - r) + 1$
- Number of ways of selections of at least one thing from a group of n identical things = n
- Number of ways of selections of zero or more things from a group of n identical things = $n + 1$

Combination of n Things Not All Different

Number of ways of selection of some or all the things from a group of distinct subgroups:

- Number of ways of selection of zero or more things out of $(p + q + r + \dots)$ things, of which p are alike of one kind, q are alike of second kind, r are alike of third kind, and so on = $[(p + 1)(q + 1)(r + 1)\dots]$
- Number of ways of selection of at least one thing out of $(p + q + r + \dots)$ things, of which p are alike of one kind, q are alike of second kind, r are alike of third kind, and so on = $[(p + 1)(q + 1)(r + 1)\dots] - 1$
- Number of ways of selection of zero or more things from p identical things of one kind, q identical things of second kind, r identical things of third kind, and from remaining n things which are all different = $[(p + 1)(q + 1)(r + 1)2^n]$
- Number of ways of selection of at least one thing from p identical things of one kind, q identical things of second kind, r identical things of third kind, and from remaining n things which are all different = $\{[(p + 1)(q + 1)(r + 1)2^n] - 1\}$
- The number of ways of choosing k objects, from p objects of one kind, q objects of second kind, and so on, is the coefficient of x^k in the expansion $(1 + x + x^2 + \dots + x^p)(1 + x + x^2 + \dots + x^q)\dots$
- The number of ways of choosing k objects from p objects of one kind, q objects of second kind, and so on, such that one object of each kind must be included is the coefficient of x^k in the expansion $(x + x^2 + \dots + x^p)(x + x^2 + \dots + x^q)\dots$
- The number of ways in which r things can be selected from a group of n things of which p are identical is

$$\sum_{r=0}^n {}^{n-p}C_r; \text{ if } r \leq p$$

- The number of ways in which r things can be selected from a group of n things of which p are identical is

$$\sum_{r=p}^n {}^{n-p}C_r; \text{ if } r > p$$

Introductory Exercise 19.6

Directions (for Q. Nos. 1 and 2) : Answer these questions based on the following information.

A set contains 9 letters such that

$$S = \{A, A, B, B, B, C, C, C, C\}.$$

- Find the number of ways of selection of 4 letters.
(a) 7 (b) 11 (c) 15 (d) 18
- Find the number of ways of selection of 4 letters such that each of A, B, C must be there.
(a) 3 (b) 4 (c) 5 (d) 6

Directions (for Q. Nos. 3 to 10) : Answer these questions based on the following information.

At DLF mall Gurugram, there is a store that sells premium T-shirts. At this store there is a shelf that has exactly 10 red tees (or T-shirts) of same colour, size and style; and I am interested in buying tees from this shelf only.

- Find the number of ways of selecting 1 tee.
(a) 0 (b) 1 (c) 2 (d) 10
- Find the number of ways of selecting at least 1 tee.
(a) 0 (b) 1 (c) 2 (d) 10
- Find the number of ways of selecting at most 1 tee.
(a) 0 (b) 1 (c) 2 (d) 10
- Find the number of ways of selecting 4 tees.
(a) 0 (b) 1 (c) 2 (d) 10
- Find the number of ways of selecting at least 4 tees.
(a) 4 (b) 1 (c) 2 (d) none of these
- Find the number of ways of selecting at most 4 tees.
(a) 5 (b) 1 (c) 4 (d) 10
- Find the number of ways of selecting any number of tees.
(a) 0 (b) 1 (c) 2 (d) 11
- Find the number of ways of selecting 10 tees.
(a) 0 (b) 1 (c) 2 (d) 10

Directions (for Q. Nos. 11 to 25) : Answer these questions based on the following information.

There are 2 white candles, 3 red candles, 4 green candles and 3 yellow candles in a brand new packet. All the candles are uniform in shape and size, except their colour, as told above.

- Find the number of ways of selecting any number of candles.
(a) 2^{12} (b) 240
(c) 72 (d) 13
- Find the number of ways of selecting at least one candle.
(a) $2^{22} - 1$ (b) 239 (c) 72 (d) 12

- Find the number of ways of selecting one candle of each colour.
(a) 10 (b) 11 (c) 2 (d) 1
- Find the number of ways of selecting at least one candle of each colour.
(a) 0 (b) 72 (c) 2 (d) 10
- Find the number of ways of selecting 2 candles of each colour.
(a) 0 (b) 1 (c) 2 (d) 10
- Find the number of ways of selecting at least 2 candles of each colour.
(a) 12 (b) 2 (c) 10 (d) 11
- Find the number of candles of selecting at most 2 candles of each colour.
(a) 0 (b) 16 (c) 18 (d) 81
- Find the number of ways of selecting 2 candles.
(a) 0 (b) 1 (c) 2 (d) 10
- Find the number of ways of selecting 1 white, 2 red, 3 green and 2 yellow candles.
(a) 0 (b) 1 (c) 2 (d) 10
- Find the number of ways of selecting all the candles of each colour.
(a) 0 (b) 1 (c) 2 (d) 10
- Find the number of ways of selecting 4 candles.
(a) 0 (b) 29 (c) 2 (d) 10
- Find the number of ways of selecting 6 candles.
(a) 0 (b) 36 (c) 40 (d) 72
- Find the number of ways of selecting at least 6 candles.
(a) 0 (b) 140 (c) 72 (d) 120
- Find the number of ways of selecting at most 6 candles.
(a) 7 (b) 72 (c) 120 (d) 140
- Find the number of ways of selecting 6 candles, such that there must be at least one candle of each colour.
(a) 9 (b) 10 (c) 11 (d) 100

Directions (for Q. Nos. 26 to 34) : Answer these questions based on the following information.

An electronic goods store has 3 boxes of pendrives. The capacity of each pendrive is n GB, where $n \in N$.

The first box has 3 identical pendrives, each of 4 GB storage capacity; second box has 4 identical pen-drives, each of 2 GB storage capacity; and third box has 5 pendrives, each of varying or distinct storage capacities other than 2 GB and 4 GB. All the pendrives are of the same make, colour and design.

26. Find the number of ways of selecting any number of pen drives.
(a) 120 (b) 600 (c) 520 (d) 640
27. Find the number of ways of selecting at least one pendrive.
(a) 639 (b) 119 (c) 519 (d) 384
28. Find the number of ways of selecting at most 1 pendrive.
(a) 9 (b) 8 (c) 0 (d) 1
29. Find the number of ways of selecting 1 pendrive from each box.
(a) 5 (b) 3 (c) 60 (d) 50
30. Find the number of ways of selecting 3 pendrives.
(a) 49 (b) 36 (c) 64 (d) 220
31. Find the number of ways of selecting 3 distinct pendrives.
(a) 10 (b) 36 (c) 35 (d) 3
32. Find the number of ways of selecting 2 pen-drives of 4 GB, 3 pendrives of 2 GB and 4 pen-drives of different storage capacities other than 2 GB and 4 GB.
(a) 5 (b) 10
(c) 24 (d) 25
33. Find the number of ways of selecting at most 2 pendrives from the first box, at most 3 pendrives from the second box and at most 4 pendrives from the third box.
(a) 372 (b) 72
(c) 36 (d) 488

34. Find the number of ways of selecting any 2 identical pendrives.
(a) 0 (b) 1 (c) 2 (d) 4
35. Find the number of ways of selecting any 2 distinct pendrives.
(a) 19 (b) 20 (c) 21 (d) 66

Directions (for Q. Nos. 36 and 37) : *Answer these questions based on the following information.*

A shop has 10 mobile recharge coupons of Vodafone. Out of which 4 are identical of same value and remaining each coupon is of distinct value.

36. Find the number of ways of selecting 7 coupons.
(a) 3 (b) 21 (c) 42 (d) 22
37. Find the number of ways of selecting 3 coupons.
(a) 6 (b) 21 (c) 22 (d) 42

Combination of Contiguous Things

(A) Linear Combination

- I. Number of selections of k consecutive things out of n things in a row = $(n - k + 1)$
- II. Number of selection of k things out of n things in a row, such that no two of the selected objects are next to each other = ${}^{n-k+1}C_k$

(B) Circular Combination

Number of selections of k consecutive things out of n things in a circle = $\begin{cases} n, & \text{when } k < n \\ 1, & \text{when } k = n \end{cases}$

Introductory Exercise 19.7

Directions (for Q. Nos. 1 to 3) : *Answer these questions based on the following information.*

Last Diwali Chhota Bheem got 10 distinct coloured candles placed in a row. Then he asked his friend to lit 3 candles.

- Find the number of ways of selection of candles so that all the 3 candles are consecutive.
(a) 19 (b) 20 (c) 8 (d) 66
- Find the number of ways of selection of candles so that all the 3 candles are not consecutive.
(a) 112 (b) 120
(c) 132 (d) 66
- Find the number of ways of selection of candles so that at least 2 candles are consecutive.
(a) 66 (b) 33
(c) 17 (d) 64

Directions (for Q. Nos. 4 to 13) : *Answer these questions independently of each other.*

- In an amphitheater there are 10 distinguishable seats placed in a row. A family of five wants to sit together, while rest of the attendees can sit anywhere without any restriction. In how many ways can 5 seats be selected so that this family can sit together?
(a) 4 (b) 5 (c) 6 (d) ${}^{10}C_5$
- A rectangular field of 100×10 is plotted into 10 square plots of 10×10 . Happy Housing is a first customer who wants to buy 4 contiguous square plots to develop a society for women who are victims of domestic violence. In how many ways can he select 4 such plots that are all adjacent to each other?
(a) 7 (b) 6
(c) 5 (d) 40

6. Anna Hazare – a social activist who became world famous after his crusade against corruption – wants to sit on a fast for 4 days back-to-back in the next week. Sitting on fast is usually indicative of protest in a democratic system to let the government/authority know the differing, rising and unheard opinion of its stakeholders.

Find the number of selection of days in which he can sit on fast for 4 consecutive days.

- (a) 4 (b) 5 (c) 6 (d) 2

7. Divya, who just got married, is allowed to work from home. The only restriction is that in every calendar year she has to work in the office for exactly 4 months but she cannot work in the office for any two consecutive months. In a particular calendar year, in how many different ways can she choose to work from her office?

- (a) 369 (b) 156
(c) 126 (d) 123

8. At a very small railway station in Rajasthan, out of eight terrorists four terrorists enter into four distinct bogies of a passenger train that has total ten bogies, so that they could place the bombs in the train. However, no two terrorists enter in the contiguous bogies that are directly connected to each other. In how many ways can these terrorists board the train?

- (a) 58800 (b) 54200
(c) 14700 (d) 2450

9. Four teams – A, B, C and D, are attending a meet. They are made to sit in the 21 chairs placed beside one another in a row. The number of members in these teams are 2, 3, 5 and 6, respectively. However, no two members of different teams sit in the adjacent chairs and all the members of each team must sit together in the adjacent seats. In how many ways can they sit in these chairs?

- (a) 3036800 (b) 373248000
(c) 61036800 (d) 125646000

10. On the very auspicious day of Karva Chauth, six couples of my college decide to book the whole mini-theater, however they prefer to sit on the top row only, which has twenty seats. They sit in such a manner that the couple sits in any two adjacent chairs but there must be at least one seat vacant between any two couples. All the seats of top row are fixed in such a way that there is no gap between any two seats. Find the number of ways of sitting in the seats of top row.

- (a) 4456200 (b) 519300
(c) 697480 (d) 3870720

11. There are twelve security guards made to stand around a circular football stadium. Any six guards, who are adjacent to each other, need a replacement. In how many ways can these guards be selected for replacement?

- (a) 36 (b) 18 (c) 2 (d) 12

12. Five friends were dining at a plush restaurant sitting at a round dining table. During their dinner a wine glass fell off the table and wine was, of course, spilled on the two guys sitting next to each other. Find the number of ways in which any two chairs can be selected on which these two persons are sitting next to each other, provided each chair is distinguishable.

- (a) 2 (b) 10 (c) 4 (d) 5

13. A group of ten friends went to Lonavala in the late night. After reaching there all of them were almost nonplussed to realize that there is a nip in the air. Luckily there was a hut that had a bone-fire for sale. All of them paid money for tea and bone-fire and then sat around the fire. But it wasn't too late when four adjacent chairs caught the fire, which went unnoticed until one of the guys felt the burning sensation. Find the number of ways in which these four chairs can be selected, provided each chair is distinguishable.

- (a) 2 (b) 10 (c) 4 (d) 5

Directions (for Q. Nos. 14 and 15) : *Answer these questions based on the following information.*

All the triangles are formed from the 10-sided polygon by connecting all the vertices mutually.

14. Find the number of triangles so that at least one side of the triangle coincides with the side of the polygon.

- (a) 120 (b) 70 (c) 80 (d) 66

15. Find the number of triangles so that none of the sides of the triangle coincides with the side of the polygon.

- (a) 20 (b) 80 (c) 64 (d) 50

16. Sunburn is one of the biggest New Year carnivals of Goa. The party happens inside the restricted circular premises, which has exactly 10 gates. The event managers want 3 entrance gates and 7 exit gates for the heck of safety and security concerns. However, they want that none of the 3 entrances should be adjacent to each other. Each of the gates have a designated number, as in gate number 1, 2, ..., 10. In how many ways can 3 gates be selected from the 10 gates in order to have the entrance facility?

- (a) 150 (b) 50 (c) 60 (d) 160

17. Out of the following six statements how many statements are INCORRECT?

- Number of ways of arranging 6 different red chairs and 4 different blue chairs in a row such that the chairs of the same color are always together = $(6! \times 4!) \times 2!$
 - Number of ways of arranging 6 different red chairs and 4 different blue chairs in a row such that all the red chairs are together = $5! \times 6!$
 - Number of ways of arranging 6 different red chairs and 4 different blue chairs in a row such that no two blue chairs are together = ${}^7C_4 \times 4! \times 6!$
 - Number of ways of arranging 5 different red chairs and 5 different blue chairs in a row such that no two adjacent chairs are of the same color = $2 \times (5!)^2$
 - Number of ways of arranging 6 identical red chairs and 4 identical blue chairs in a row such that no two blue chairs are together = ${}^7C_4$
 - Number of ways of arranging 6 identical red chairs and 4 different blue chairs in a row such that no two blue chairs are together = $4! \times {}^7C_4$
- (a) 0 (b) 1 (c) 2 (d) 4

18. Suchirupa is a fitness conscious woman. She joined a club in her neighbourhood that offers seven fitness activities to choose from. However, on a particular day only one activity is supposed to be done by any of the members of this club. If Suchirupa does not do the same activity on any two consecutive days, in how many ways can she choose to work out the next week?

- 42×5^5
- $7 \times 6^3 \times 5^3$
- $7 \times (6)^6$
- 56

19. Deepika has two best friends, Yuvraj and Siddhartha. Everyday, she practices badminton with one of them, but she practices at least once a week with Siddhartha. If she practices badminton with Siddhartha more than once in a week she must practice with him on consecutive days only. She practices it only once in a day. In how many ways can she practice the badminton with her friends?

- 206
- 28
- 49
- 48

19.6 Distribution/Division of Distinct Things Among Individuals/Groups

1. Number of ways in which $(m+n)$ different things can be divided into two unequal groups containing m and n things, if the order of the group is not important

$$= {}^{m+n}C_m \times {}^nC_n = \frac{(m+n)!}{m!n!}$$

2. Number of ways in which $(m+n)$ different things can be divided into two unequal groups containing m and n things, if the order of the group is important

$$= \frac{(m+n)!}{m!n!} \times 2!$$

3. The number of ways in which $(m+n+p)$ different things can be divided into three different groups containing m , n and p things respectively, if the order of the group is not important

$$= {}^{m+n+p}C_m \times {}^{n+p}C_n \times {}^pC_p = \frac{(m+n+p)!}{m!n!p!}$$

4. The number of ways in which $(m+n+p)$ different things can be divided into three different groups containing m , n and p things respectively, if the order of the group is important = $\frac{(m+n+p)!}{m!n!p!} \times 3!$

5. The number of ways in which mn different items can be divided equally into m groups, each containing n objects when the order of the group is not important

$$= \left[\frac{(mn)!}{(n!)^m} \right] \frac{1}{m!}$$

6. The number of ways in which mn different items can be divided equally into m groups, each containing n objects when the order of the groups is important

$$= \left[\frac{(mn)!}{(n!)^m} \right]$$

7. The number of ways of dividing $2n$ different things into three groups of n each when the order of groups is not important = $\frac{(2n)!}{(n!)^2} \frac{1}{2!}$; the division by $2!$ indicates that

since 2 groups have equal number of things, so the two groups are indistinguishable.

8. The number of ways of dividing $3n$ different things into three groups of n each when the order of groups is important = $\left[\frac{(3n)!}{(n!)^3} \right]$

9. The number of ways of dividing $3n$ different things into three groups of n each when the order of groups is not important $= \frac{(3n)!}{(n!)^3 3!}$; the division by $3!$ indicates that

since 3 groups have equal number of things, so the three groups are indistinguishable.

10. The number of ways of dividing $3n$ different things into three groups of n each when the order of groups is important $= \left[\frac{(3n)!}{(n!)^3} \right]$

11. The number of ways in which n distinct things can be distributed to r different persons $= r^n$

NOTE Usually the phrase 'distribution' indicates that the things are given out (or doled out) to different people and so the order becomes important. As, normally, people are distinct so it becomes essential to distinguish that who receives what. The phrase 'division' usually suggests that the different things are just clubbed in different sets or groups. They may be kept in identical boxes or their receivers are identical, and therefore they are naturally indistinguishable. That's why no rearranging or ordering matters. However, authors/paper-setters normally use both the phrases interchangeably. So, it is your responsibility to figure it out whether order is important or not; and that can be done on the basis of the situation and language of the problem.

Exp. 1) Find the number of ways of dividing 4 distinct articles between 2 groups.

Solution Let a, b, c and d be the four distinct articles to be divided in two groups.

Then you have the following groups.

i	–	a, b, c, d
ii	a	b, c, d
iii	b	a, c, d
iv	c	a, b, d
v	d	a, b, c
vi	a, b	c, d
vii	a, c	b, d
viii	a, d	b, c

Therefore, total number of ways in which 4 distinct articles can be divided in 2 groups is 8.

Alternatively

Case I (4, 0) Number of ways $= {}^4C_4 \times {}^0C_0 = 1 \times 1 = 1$

Case II (3, 1) Number of ways $= {}^4C_3 \times {}^1C_1 = 4 \times 1 = 4$

Case III (2, 2) Number of ways

$$= {}^4C_2 \times {}^2C_2 \times \frac{1}{2} = 6 \times 1 \times \frac{1}{2} = 3$$

Therefore, total number of ways in which 4 distinct articles can be divided in 2 groups $= 1 + 4 + 3 = 8$.

Exp. 2) Find the number of ways of distributing the 4 distinct articles between 2 girls.

Solution Let a, b, c and d be the four distinct articles to be distributed in two girls.

Then you have the following distribution.

	Girl 1	Girl 2
i	–	a, b, c, d
ii	a	b, c, d
iii	b	a, c, d
iv	c	a, b, d
v	d	a, b, c
vi	a, b	c, d
vii	a, c	b, d
viii	a, d	b, c
ix	b, c	a, d
x	b, d	a, c
xi	c, d	a, b
xii	a, b, c	d
xiii	a, b, d	c
xiv	a, c, d	b
xv	b, c, d	a
xvi	a, b, c, d	–

Since each girl is distinct, so the number of ways in which the 2 groups can be arranged is $2!$ ways.

Therefore the number of ways of distribution of 4 distinct articles between 2 distinct girls $= 2! \times 8 = 16$

Alternatively

Case I (4, 0) Number of ways $= {}^4C_4 \times {}^0C_0 \times 2 = 1 \times 2 = 2$

Case II (3, 1) Number of ways $= {}^4C_3 \times {}^1C_1 = 4 \times 1 \times 2 = 8$

Case III (2, 2) Number of ways

$$= {}^4C_2 \times {}^2C_2 \times \frac{1}{2} \times 2 = 6 \times 1 \times \frac{1}{2} \times 2 = 6$$

Therefore, total number of ways in which 4 distinct articles can be divided in 2 groups $= 2 + 8 + 6 = 16$.

Exp. 3) Find the number of ways of distributing 4 distinct articles between 2 groups, such that one group has 1 article and another one has 3 articles.

Solution Let a, b, c and d be the four distinct articles to be divided in two groups, such that one group has 1 article and another one has 3 articles. Then you have the following groups.

i	a	b, c, d
ii	b	a, c, d
iii	c	a, b, d
iv	d	a, b, c

Therefore, total number of ways in which 4 distinct articles can be divided in 2 groups such that one group has 1 article and another one has 3 articles is 4.

☞ Alternatively Number of ways in which 4 distinct articles can be divided in 2 groups such that one group has 1 article and another one has 3 articles

$$= {}^4C_1 \times {}^3C_3 = \frac{4!}{1!3!} = 4$$

Exp. 4) Find the number of ways of dividing 4 distinct articles between 2 girls, such that one girl gets 1 article and another girl gets 3 articles.

Solution Let a, b, c and d be the four distinct articles to be distributed in two girls, such that one girl gets 1 article and another girl gets 3 articles.

Then you have the following distribution.

	Girl 1	Girl 2
i	a	b, c, d
ii	b	a, c, d
iii	c	a, b, d
iv	d	a, b, c
v	a, b, c	d
vi	a, b, d	c
vii	a, c, d	b
viii	b, c, d	a

Since each girl is distinct, so the number of ways in which the 2 groups can be arranged in $2!$ ways.

Therefore the number of ways of distribution of 4 distinct articles equally among 2 distinct girls $= 2! \times 4 = 8$

Alternatively: Number of ways in which 4 distinct articles can be divided in 2 groups such that one group has 1 article and another one has 3 articles

$$= {}^4C_1 \times {}^3C_3 = \frac{4!}{1!3!} = 4$$

Now these 2 groups of articles can be arranged among themselves in $2!$ ways.

Therefore the number of ways of distribution of 4 distinct articles among 2 distinct girls $= 2! \times 4 = 8$

Exp. 5) Find the number of ways of distributing 4 distinct articles between 2 groups, equally.

Solution Let a, b, c and d be the four distinct articles to be divided in two groups, equally.

Then you have the following groups.

i	$a, b,$	c, d
ii	a, c	b, d
iii	a, d	b, c

Therefore, total number of ways in which 4 distinct articles can be divided equally in 2 groups is 3.

☞ Alternatively Number of ways in which 4 distinct articles can be divided equally in 2 groups

$$= {}^4C_2 \times {}^2C_2 = \frac{4!}{2!2!} = 6$$

But since both the groups have equal number of articles, so

the two groups become identical. Therefore, the actual number of ways of division of articles

$$= ({}^4C_2 \times {}^2C_2) \times \frac{1}{2!} = \left[\frac{4!}{2!2!} \right] \times \frac{1}{2!} = 3$$

Exp. 6) Find the number of ways of dividing 4 distinct articles between 2 girls, equally.

Solution Let a, b, c and d be the four distinct articles to be distributed in two girls, equally.

Then you have the following distribution.

	Girl 1	Girl 2
i	a, b	c, d
ii	a, c	b, d
iii	a, d	b, c
iv	b, c	a, d
v	b, d	a, c
vi	c, d	a, b

Since each girl is distinct, so the number of ways in which the 2 groups can be arranged in $2!$ ways.

Therefore the number of ways of distribution of 4 distinct articles equally among 2 distinct girls $= 2! \times 3 = 6$

☞ Alternatively Number of ways in which 4 distinct articles can be divided equally in 2 groups

$$= {}^4C_2 \times {}^2C_2 = \frac{4!}{2!2!} = 6$$

But since both the 2 groups have equal number of articles, so the two groups become identical. Therefore, the actual number of ways of division of articles

$$= ({}^4C_2 \times {}^2C_2) \times \frac{1}{2!} = \left(\frac{4!}{2!2!} \right) \times \frac{1}{2!} = 3$$

However, the 2 girls are distinct so the required number of distribution of articles

$$= ({}^4C_2 \times {}^2C_2) \times \frac{1}{2!} \times 2! = \left(\frac{4!}{2!2!} \right) \times \frac{1}{2!} \times 2! = 6$$

19.7 Distribution/Division of Identical Objects Among Individuals/Groups

1. The number of ways of dividing n identical items among r persons (where, $0 \leq r \leq n$), each one of whom can receive any number (0, 1, 2,..., n) of items $= {}^{n+r-1}C_{r-1}$

Example : Divide 16 identical balls among 3 kids, in such a way that any kid can get any number of balls is ${}^{16+3-1}C_{3-1} = {}^{18}C_2$

2. The number of ways of dividing n identical items among r persons (where, $0 \leq r \leq n$), each one of whom must receive at least one item $= {}^{n-1}C_{r-1}$

Example : Divide 16 identical balls among 3 kids, in such a way that each kid must get at least 1 ball is

$${}^{16-1}C_{3-1} = {}^{15}C_2$$

3. The number of ways in which n identical items can be divided into r groups so that no group contains less than m items and more than k items (where $m < k$) is coefficient of x^n in the expansion of

$$(x^m + x^{m+1} + \dots + x^k)^r,$$

for every $m \leq x_i \leq k$

Example : Divide 16 identical balls among 3 kids, so that each kid gets at least 2 balls but not more than 7 balls is coefficient of x^{16} in the expansion of

$$(x^2 + x^3 + x^4 + x^5 + x^6 + x^7)^3$$

4. The number of ways in which n identical items can be divided into r groups such that the group x_i contains minimum a_i and maximum b_i items (where $a_i \leq b_i$) is coefficient of x^n in

$$(x^{a_1} + x^{a_1+1} + \dots + x^{b_1})(x^{a_2} + x^{a_2+1} + \dots + x^{b_2}) \dots (x^{a_r} + x^{a_r+1} + \dots + x^{b_r})$$

Example : Divide 16 identical balls among 3 kids, such that first kid cannot have less than 3 and more than 5 balls, second kid cannot have less than 2 and more than 8 balls, and the third kid cannot have less than 4 and more than 7 balls is coefficient of x^{16} in

$$(x^3 + x^4 + x^5)(x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8)(x^4 + x^5 + x^6 + x^7)$$

19.8 Algebraic Properties

A. For a natural number n , the given equation is

$$x_1 + x_2 + x_3 + \dots + x_r = n$$

- If $x_i \geq 0$, the number of integral solutions is ${}^{n+r-1}C_{r-1}$
- If $x_i \geq 1$, the number of integral solutions is ${}^{n-1}C_{r-1}$
- If $m \leq x_i \leq k$, the number integral solutions is coefficient of x^n in the expansion of $(x^m + x^{m+1} + \dots + x^k)^r$
- If $a_1 \leq x_1 \leq b_1, a_2 \leq x_2 \leq b_2, \dots, a_r \leq x_r \leq b_r$ the number of integral solutions is coefficient of x^n in $(x^{a_1} + x^{a_1+1} + \dots + x^{b_1})(x^{a_2} + x^{a_2+1} + \dots + x^{b_2}) \dots (x^{a_r} + x^{a_r+1} + \dots + x^{b_r})$

B. For a natural number n , if the given inequation is

$$x_1 + x_2 + x_3 + \dots + x_r \leq n.$$

Then, add a dummy variable x_{r+1} in the given equation, so that inequality can be converted into equality before finding the number of solutions.

5. If $x_i \geq 0$, the number of integral solutions to $x_1 + x_2 + x_3 + \dots + x_r \leq n$ is same as the number of integral solutions of $x_1 + x_2 + x_3 + \dots + x_r + x_{r+1} = n$. Thus the required number of integral solutions is ${}^{n+r}C_r$.

Example : If $x_i \geq 0$, the given inequation

$$x_1 + x_2 + x_3 + \dots \leq n \text{ can be expressed as } x_1 + x_2 + x_3 + x_4 = n$$

so the required number of integral solutions is ${}^{n+3}C_3$.

C. For a natural number n , number of solutions of $|x_1| + |x_2| + \dots + |x_m| = n$.

- The total number of integral solutions of $|a| + |b| = n$ is $4n$
- The total number of integral solutions of $|a| + |b| + |c| = n$ is $4n^2 + 2$
- The total number of integral solutions of $|a| + |b| + |c| + |d| = n$ is $\left(\frac{8n}{3}\right)(n^2 + 2)$

D. For a natural number n , number of terms of binomial and multinomial expressions.

- Total number of terms in $(x + y)^n = n + 1$
- Total number of terms in $(a_1 + a_2 + \dots + a_n)^m = {}^{m+n-1}C_{n-1}$
- Total number of terms in $(1 + x + x^2 + \dots + x^n)^m$ is $(m \cdot n) + 1$

NOTE $1 + x + x^2 + \dots + x^n = \frac{x^{n+1} - 1}{x - 1}$

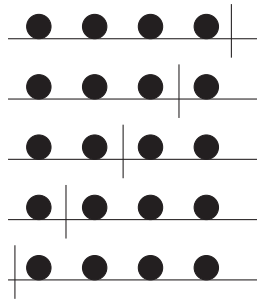
Exp. 1) Suppose you have 4 identical chocolates in your pocket. In how many ways can you distribute the four identical chocolates between the two kids (or children)?

Solution In the following you can see that there are total 5 ways of distributing the 4 chocolates between 2 kids.

	Number of Chocolates				
First Kid	4	3	2	1	0
Second Kid	0	1	2	3	4

NOTE Here chocolates are identical and kids are distinct. So, we focus on kids, as they are distinguishable, but not on the chocolates.

Alternatively Look at the other way of understanding the same problem. In this illustration there are 5 distinct possibilities of distributing the 4 chocolates between 2 kids. The number of chocolates on the left side of the bar (vertical line) is given to the first kid and the number of chocolates on the right side of the bar is given to the second kid. Thus, you can see that the number of ways of distributing the 4 chocolates is same as the number of ways of arranging 4 circles and 1 bar in a row.



And, you already know how to arrange the 5 objects in a row, which can be simply done in $5!$ ways. But since all the 4 circles are identical, so the actual number of ways in which 5 objects can be arranged is $5!/4! = 5$ ways.

NOTE As we need only one wall or partition to divide any physical object into two parts, so we need only one bar to divide any number of circles into 2 parts.

Alternatively Now look at the other way of understanding the same problem.

Find the number of non-negative integral solutions of

$$a + b = 4$$

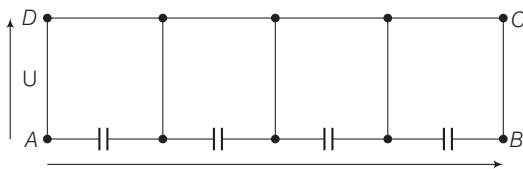
Therefore, we have (a, b)

$$= (4, 0), (3, 1), (2, 2), (1, 3), (0, 4).$$

Thus you can see that there are total 5 solutions to the above equation.

Alternatively Now look at the other problem, which works with the same logic as the original problem does.

Find the number of distinct shortest paths from A to C in the following diagram, which is a rectangular grid of 4 horizontal steps and 1 upward step.



In order to reach C from A you have to take any 4 horizontal steps and any 1 upward step in any of the 5 following orders.

HHHHU
HHHUH
HHUHH
HUHHH
UHHHH

Thus you can see that there are total 5 shortest ways of reaching C from A.

Mathematically, the answer is similar to arranging 5 objects (steps) in a row such that 4 of them are identical. That is

$$5!/4! = 5$$

So, essentially, what you are doing is nothing but distributing the total 5 steps in 2 different directions. Thus you can use the same logic in various kinds of problems.

Exp. 2) Find the number of ways in which you can distribute all the 12 identical chocolates among the 3 kids.

Solution This problem is like dividing 12 circles by 2 bars (vertical lines). Now, the number of ways of arranging

$$12 \text{ circles and } 2 \text{ bars} = \frac{(12 + 2)!}{12! \times 2!} = \frac{14!}{12! \times 2!} = 91$$

Therefore, the number of ways in which 12 identical chocolates can be divided among 3 kids is 91.

Hint The equivalent algebraic equation would be $a + b + c = 12$, such that $a, b, c \geq 0$.

NOTE You must be wondering that since the two kids are distinct, then why I have considered them identical. The reason is that the order of kids does not matter at all, as the kids are equivalent to the bars and the role of bars is just to separate the circles. So whenever the order is not important, things (or persons) may act as if they are identical.

Exp. 3) Find the number of ways in which you can distribute all the 12 identical chocolates among the 5 kids.

Solution As you have to distribute the chocolates among 5 kids that means you need to have 4 bars to make 5 divisions among 12 circles.

Thus the answer to this problem is same as finding the number of ways in which 16 objects (12 circles + 4 bars) can be arranged in a row, out of which 12 objects (circles) are identical and 4 other objects (bars) are also identical.

That is the number of ways of arrangements of 12 circles and 4 bars in a row = $\frac{(12 + 4)!}{12! \times 4!} = \frac{16!}{12! \times 4!} = 1820$

Therefore, the number of ways in which 12 identical chocolates can be divided among 5 kids is 1820.

Hint The equivalent algebraic equation would be

$$a + b + c + d + e = 12, \text{ such that } a, b, c, d, e \geq 0$$

Exp. 4) Find the number of ways in which you can distribute all the 12 identical chocolates among the 3 kids, such that each of them must get at least one chocolate.

Solution To simplify the solution what you need to do is distribute 1 chocolate to each of the 3 kids. Since chocolates are identical, so it does not matter who gets which chocolate.

Now you have just 9 chocolates left to distribute among

3 kids, in the second round. But the best part is that now you are totally free to give any number of chocolates, out of 9 chocolates, to any of the 3 kids. That means even if you give 0 chocolate to a particular kid, it won't violate the basic restriction, as each kid has already received 1 chocolate in the first round.

Then the modified problem is like distributing 9 identical chocolates among 3 kids, without any specific restriction.

And, this is equivalent to arranging the 9 circles and 2 bars, such that all the 9 circles are identical and the 2 bars are identical.

That is the number of ways of arrangements of 9 circles and 2 bars $= \frac{(9+2)!}{9! \times 2!} = \frac{11!}{9! \times 2!} = 55$

Therefore, the number of ways in which 12 identical chocolates can be divided among 3 kids, such that each kid must get at least 1 chocolate is 55.

Hint The equivalent algebraic equation would be $a + b + c = 12$, such that $a, b, c \geq 1$. or $a + b + c = 9$, such that $a, b, c \geq 0$

NOTE Whenever you see the lower limits imposed on the problem, you must give away the minimum required number of items to the deserving persons in the first round, then remaining number of items can be given away (or distributed) without any restriction. So, our objective is to deal with the restrictions, first. Then it becomes a very flexible problem to deal with.

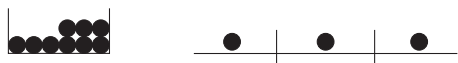
Alternatively The given problem can be stated as

$$a + b + c = 12; a, b, c \geq 1$$

Before the distribution of chocolates :



After distributing minimum 1 chocolate to each of the 3 kids :



Now, you can see that only 9 chocolates are remaining to be distributed among the 3 kids.

And, the best thing is that you are totally free to give any number of chocolates to anyone. As in, you can give all the 9 chocolates to one kid and no chocolates to the other 2 kids or you can give equal number of chocolates to each of them. And, that you can do in 55 ways, as explained below.

After ensuring that each kid must get 1 chocolate, the distribution problem transforms to

$$a + b + c = 9; a, b, c \geq 0$$

And, the number of solutions to this equation

$$= \frac{(9+2)!}{9! \times 2!} = \frac{11!}{9! \times 2!} = 55$$

NOTE Once you have ensured that each kid is getting the minimum requisite number of items, you are completely free to distribute the items using the very basic logic of distribution.

Exp. 5) Find the number of ways in which you can distribute all the 12 identical chocolates among the 3 kids, such that each of them must get at least two chocolates.

Solution To simplify the solution what you need to do is give 2 chocolates to each of the 3 kids. Since chocolates are identical, so it does not matter who gets which chocolates.

Now you have just 6 chocolates left to be distributed among 3 kids, in the second round.

But the best part is that now you are totally free to give any number of chocolates, out of 6 chocolates, to any of the 3 kids. That means even if you give 0 chocolate to a particular kid, it won't violate the basic restriction, as each kid has already received 2 chocolates, in the first round.

Then the modified problem is like distributing 6 identical chocolates among 3 kids, without any specific restriction.

And, this is equivalent to arranging the 6 circles and 2 bars, such that all the 6 circles are identical and the 2 bars are identical.

That is the number of ways of arrangements of 6 circles and 2 bars $= \frac{(6+2)!}{6! \times 2!} = \frac{8!}{6! \times 2!} = 28$

Therefore, the number of ways in which 12 identical chocolates can be divided among 3 kids, such that each kid must get at least 2 chocolates is 28.

Hint The equivalent algebraic equation would be $a + b + c = 12$ such that $a, b, c \geq 2$ or $a + b + c = 6$, such that $a, b, c \geq 0$

NOTE Whenever you see the lower limits imposed in the problem, you must give away the minimum required number of items to the deserving persons in the first round, then remaining number of items can be given away (or distributed) without any restriction. So, our objective is to do away with (or get rid of) the restrictions, first. Then it becomes a very flexible problem to deal with.

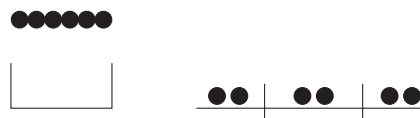
Alternatively The given problem can be stated as

$$a + b + c = 12; a, b, c \geq 2$$

Before the distribution of chocolates:



After distributing the minimum 2 chocolates to each of the 3 kids:



Now, you can see that only 6 chocolates are remaining to be distributed among the 3 kids. And, the best thing is that you are totally free to give any number of chocolates to anyone. As in, you can give all the 6 chocolates to one kid and no chocolates to the other 2 kids or you can give equal number of chocolates to each of them. And, that you can do in 28 ways, as explained on next page.

After ensuring that each kid must get 1 chocolate, the distribution problem transforms to

$$a + b + c = 6; a, b, c \geq 0$$

And, the number of solutions to this equation

$$= \frac{(6+2)!}{6! \times 2!} = \frac{8!}{6! \times 2!} = 28$$

NOTE Once you have ensured that each kid is getting the minimum requisite number of items, you are completely free to distribute the items using the very basic logic of distribution.

Exp. 6) Find the number of ways in which you can distribute all the 12 identical chocolates among the 3 kids, such that each of them must get at least three chocolates.

Solution Total number of identical chocolates = 12

In the first round total number of chocolates given to the 3 kids = $3 \times 3 = 9$

Number of chocolates remaining to be distributed without any restriction = $12 - 9 = 3$

Now this will be equivalent to arranging 5 objects (3 circles + 2 bars) in a row such that all the 3 circles are identical and all the 2 bars are also identical.

Thus the number of ways of arrangements of 3 circles and 2 bars = $\frac{(3+2)!}{3! \times 2!} = \frac{5!}{3! \times 2!} = 10$

Therefore, the number of ways in which 12 identical chocolates can be divided among 3 kids, such that each kid must get at least 3 chocolates is 10.

Hint The equivalent algebraic equation would be $a + b + c = 12$, such that $a, b, c \geq 3$ or $a + b + c = 3$, such that $a, b, c \geq 0$

NOTE Whenever you see the lower limits imposed in the problem, you must give away the minimum required number of items to the deserving persons in the first round, then remaining number of items can be given away (or distributed) without any restriction. So our objective is to do away with (or get rid of) the restrictions, first. Then, it becomes a very flexible problem to deal with.

Exp. 7) Find the number of ways in which you can distribute all the 12 identical chocolates among the 3 kids, such that each of them must get at least four chocolates.

Solution Total number of identical chocolates = 12

In the first round total number of chocolates given to the 3 kids = $3 \times 4 = 12$

Number of chocolates remaining to be distributed without any restriction = $12 - 12 = 0$

Now there are no any chocolate left to be distributed.

Thus, there is only 1 way to distribute the 12 identical chocolates among 3 kids such that each of them must get at least 4 chocolates. And that can be expressed as following.



Hint The equivalent algebraic equation would be $a + b + c = 12$, such that $a, b, c \geq 4$.

Exm. 8) Find the number of ways in which you can distribute all the 12 identical chocolates among the 3 kids, such that each of them must get at least five chocolates.

Solution Total number of identical chocolates = 12

In the first round total number of chocolates to be given to the 3 kids = $3 \times 5 = 15$

Since there are fewer chocolates available than the required number of chocolates, so there is no way to distribute the given chocolates.

Thus the required number of ways = 0.

Hint The equivalent algebraic equation would be $a + b + c = 12$, such that $a, b, c \geq 5$.

Exp. 9) Find the number of ways in which you can distribute 12 identical chocolates among the 3 kids, such that one kid gets at least 1 chocolate, another kid gets at least 2 chocolates and the third kid gets at least 3 chocolates.

Solution Total number of identical chocolates = 12

In the first round total number of chocolates given to the 3 kids = $1 + 2 + 3 = 6$

Number of chocolates remaining to be distributed without any restriction = $12 - 6 = 6$

Now, this will be equivalent to arranging 8 objects (6 circles + 2 bars) in a row such that all the 6 circles are identical and all the 2 bars are also identical.

That is the number of ways of arrangements of 6 circles and 2 bars = $\frac{8!}{6! \times 2!} = 28$.

Therefore, the number of ways in which 12 identical chocolates can be divided among 3 kids, such that one kid gets at least 1 chocolate, another kid gets at least 2 chocolates and the third kid gets at least 3 chocolates.

Hint The equivalent algebraic equation would be $a + b + c = 12$, such that $a \geq 1, b \geq 2, c \geq 3$ or $a + b + c = 6$, such that $a, b, c \geq 0$

NOTE Whenever you see the lower limits imposed in the problem, you must give away the minimum required number of items to the deserving persons in the first round, then remaining number of items can be given away (or distributed) without any restriction. So our objective is to do away with (or get rid of) the restrictions, first. Then it becomes a very flexible problem to deal with.

Exp. 10) Find the number of ways in which you can distribute 12 identical chocolates among 3 kids, such that there is one particular kid who gets the chocolates in the multiples of 3 only.

Solution The equivalent algebraic equation would be

$$3x + y + z = 12$$

For $x = 0$, we have $y + z = 12$ and the number of solutions

$$= \frac{13!}{12! \times 1!} = 13$$

For $x = 1$, we have $y + z = 9$ and the number of solutions

$$= \frac{10!}{9! \times 1!} = 10$$

For $x = 2$, we have $y + z = 6$ and the number of solutions

$$= \frac{7!}{6! \times 1!} = 7$$

For $x = 3$, we have $y + z = 3$ and the number of solutions

$$= \frac{4!}{3! \times 1!} = 4$$

For $x = 4$, we have $y + z = 0$ and the number of solutions

$$= \frac{1!}{0! \times 1!} = 1$$

Therefore, total number of solutions

$$= 1 + 4 + 7 + 10 + 13 = \frac{(1+13)}{2} \times 3 = 35$$

Exp. 11) Find the number of ways in which you can distribute 12 chocolates among 3 kids, such that there is one particular kid who gets the chocolates in the multiples of 3 only and no kid gets less than 1 chocolate.

Solution Since, $x, y, z \geq 1$ and x gets chocolates in multiple of 3, we must give 3 chocolates to x and 1 chocolate to each of y and z . Now, we are free to give the remaining 7 chocolates without any restriction.

The equivalent algebraic equation would be $3x + y + z = 7$

For $x = 0$, we have $y + z = 7$ and the number of solutions

$$= \frac{8!}{7! \times 1!} = 8$$

For $x = 1$, we have $y + z = 4$ and the number of solutions

$$= \frac{5!}{4! \times 1!} = 5$$

For $x = 2$, we have $y + z = 1$ and the number of solutions

$$= \frac{2!}{1! \times 1!} = 2$$

Therefore, total number of solutions $= 2 + 5 + 8 = 15$

Exp. 12) Find the number of ways in which you can distribute 12 chocolates among 3 kids, such that exactly two kids get the same number of chocolates.

Solution Let the three kids get x, y and z number of chocolates. Since two kids get same number of chocolates, so we can assume $x = y$. Then we have,

$$\Rightarrow x + y + z = 12$$

$$\Rightarrow 2x + z = 12$$

$$\Rightarrow x = \frac{12 - z}{2}$$

$$\Rightarrow \{y, z\} \equiv \{0, 12\}, \{1, 10\}, \{2, 8\}, \{3, 6\}, \{5, 2\}, \{6, 0\}$$

$$\Rightarrow \{x, y, z\} \equiv \{0, 0, 12\}, \{1, 1, 10\}, \{2, 2, 8\}, \{3, 3, 6\}, \{5, 5, 2\}, \{6, 6, 0\}$$

Thus there are 6 distinct pairs of $\{x, z\}$, such that $x = y \neq z$

Now, each such triplet $\{x, y, z\}$ can be arranged in 3 ways

$$\left(= \frac{3!}{2!} \right)$$

Therefore total number of non-negative integral solutions $= 6 \times 3 = 18$

$$\begin{aligned} &\{0, 0, 12\}, \{1, 1, 10\}, \{2, 2, 8\}, \{3, 3, 6\}, \{5, 5, 2\}, \{6, 6, 0\} \\ &\{0, 12, 0\}, \{1, 10, 1\}, \{2, 8, 2\}, \{3, 6, 3\}, \{5, 2, 5\}, \{6, 0, 6\} \\ &\{12, 0, 0\}, \{10, 1, 1\}, \{8, 2, 2\}, \{6, 3, 3\}, \{2, 5, 5\}, \{0, 6, 6\} \end{aligned}$$

So the number of ways in which 12 identical chocolates can be distributed among 3 kids such that exactly 2 kids get the same number of chocolates $= 18$.

Exp. 13) Find the number of ways in which you can distribute 12 chocolates among 3 kids, such that no two kids get the equal number of chocolates.

Solution Total number of ways in which 12 chocolates can be distributed among 3 kids, without any restriction

$$= \frac{(12 + 2)!}{12! \times 2!} = 91$$

But, this distribution involves those cases too, in which 2 or all the 3 kids get same number of chocolates.

Case I : When all 3 kids get the same number of chocolates.

$$x = y = z$$

$\Rightarrow$ 1 way only

Case II : When any 2 kids get same number of chocolates.

$$x + y + z = 12$$

$$\Rightarrow 2x + z = 12$$

$$\Rightarrow 18 \text{ ways}$$

Case III: When none of the kids get same number of chocolates.

$$\begin{aligned} x < y < z, y < z < x, z < x < y, \\ x < z < y, y < x < z, z < y < x \end{aligned}$$

Let α denotes the number of ways when each of the kids get the same number of chocolates, β denotes the number of ways when exactly two kids get the same number of chocolates and γ denotes the number of ways when none of the kids get same number of chocolates. Therefore,

$$\alpha + \beta + \gamma = 91$$

$$\Rightarrow 1 + 18 + 6(k) = 91$$

$$\Rightarrow 6k = 72 = \gamma$$

Therefore number of ways in which 12 identical chocolates can be distributed among 3 kids in such a way that no two kids get same number of chocolates $= 72$

Alternatively There are 4 possible cases when each of the three kids gets distinct number of chocolates.

Case I: $\{0, 1, 11\}, \{0, 2, 10\}, \{0, 3, 9\}, \{0, 4, 8\}, \{0, 5, 7\} \rightarrow 5$

Case II: $\{1, 2, 9\}, \{1, 3, 8\}, \{1, 4, 7\}, \{1, 5, 6\} \rightarrow 4$

Case III: $\{2, 3, 7\}, \{2, 4, 6\} \rightarrow 2$

Case IV: $\{3, 4, 5\} \rightarrow 1$

Total 12 $(=5+4+2+1)$ triplets. Each such triplet can be arranged in $3! = 6$ ways. Therefore there are total 72 ways to express $a + b + c = 12$, such that a, b and c are distinct.

Thus, the number of ways in which 12 identical chocolates can be distributed among 3 kids such that no two kids get the same number of chocolates $= 72$.

Exp. 14) Find the number of ways in which you can distribute 12 chocolates among 3 kids, such that all the three kids get distinct number of chocolates.

Solution It is the same problem as previous one, which is stated in different words.

Hence, the answer is 72.

Exp. 15) Find the number of ways in which you can distribute 12 chocolates among 3 kids. – Archimedes, Bernoulli and Carl Gauss, such that Archimedes gets more than Bernoulli and Bernoulli gets more than Carl Gauss.

Solution Total number of ways in which 12 chocolates can be distributed among 3 kids, without any restriction

$$= \frac{(12+2)!}{12! \times 2!} = 91$$

Now, we know that there are 13 different relations between the number of chocolates that 3 persons can receive.

Case I: When all 3 kids get the same number of chocolates.

$$a = b = c$$

$\Rightarrow$ 1 way only

Case II: When any 2 kids get same number of chocolates.

$$a + b + c = 12$$

$\Rightarrow 2a + c = 12$

$\Rightarrow$ 18 ways

Case III: When none of the kids get same number of chocolates.

$$a > b > c, b > c > a, c > a > b,$$

$$a > c > b, b > a > c, c > b > a$$

Let α denotes the number of ways when each of the kids get the same number of chocolates, β denotes the number of ways when exactly two kids get the same number of chocolates and γ denotes the number of ways when none of the kids get same number of chocolates. Therefore,

$$\alpha + \beta + \gamma = 91$$

$\Rightarrow 1 + 18 + 6(k) = 91$

$\Rightarrow 6k = 72 \Rightarrow k = 12$

So, out of 6 relations, as suggested in Case III, only 1 relation is in our favour ($a > b > c$).

Therefore, the required number of ways in which 12 identical chocolates can be distributed among A, B and C $= 72 \times \frac{1}{6} = 12$

☞ Alternatively If $a + b + c = n$; $a > b > c \geq 0$, the number of solutions $= \left\lfloor \frac{n^2 + 6}{12} \right\rfloor$

We have, $a + b + c = 12$; $a > b$

Therefore, the required number of ways

$$= \left\lfloor \frac{12^2 + 6}{12} \right\rfloor = \left\lfloor \frac{150}{12} \right\rfloor = \lfloor 12.5 \rfloor = 12$$

Exp. 16) Find the number of ways in which you can distribute 12 chocolates among 3 kids, such that each kid gets the odd number of chocolates.

Solution odd + odd + odd = odd

Since 12 is not an odd number, so we cannot distribute odd number of chocolates to each of the three kids.

The required number of ways = 0.

Exp. 17) Find the number of ways in which you can distribute 12 chocolates among 3 kids, such that each kid gets the even number of chocolates.

Solution Even + Even + Even = Even. Therefore it is possible to distribute even number of chocolates, as shown below.

$$\{0, 0, 12\}, \{0, 2, 10\}, \{0, 4, 8\}, \{0, 6, 6\} \rightarrow 3+6+6+3 = 18 \text{ ways}$$

$$\{2, 2, 8\}, \{2, 4, 6\} \rightarrow 3+6 = 9 \text{ ways}$$

$$\{4, 4, 4\} \rightarrow 1 \text{ way}$$

Total number of ways in which this can be done

$$= 18 + 9 + 1 = 28 \text{ ways}$$

Exp. 18) Find the number of ways in which you can distribute all the 12 chocolates among 3 kids, such that at least one of them must get more than 6 chocolates.

Solution In order to ensure that at least one of them must get more than 6 chocolates you have to find the number of ways in which one or more than one kid gets more than 6 chocolates using the inclusion-exclusion principle, as explained below.

(i) Finding the number of ways in which any one kid gets more than 6 chocolates.

So, first of all keep aside 7 chocolates and distribute the remaining 5 chocolates among 3 kids.

Now, the number of ways in which you can distribute 5 identical chocolates among 3 kids without restriction

$$= \frac{(5+2)!}{(5! \times 2!)} = \frac{7!}{5! \times 2!} = 21$$

Now, you can give away the 7 chocolates, which you had kept aside, to any one of the 3 kids in ${}^3C_1 = 3$ ways.

Therefore, the number of ways in which you can distribute 12 identical chocolates among 3 kids such that any one = of the 3 kids must get more than 6 chocolates $= 3 \times 21 = 63$

(ii) Finding the number of ways in which any two kids get more than 6 chocolates.

So, for that you have to keep aside 14 (=7+7) chocolates.

But, you have only 12 chocolates, so you cannot distribute 12 chocolates among 3 kids in such a way that at least 2 kids get more than 6 chocolates.

That is, the number of ways in which you can distribute 12 identical chocolates among 3 kids such that any 2 kids must get more than 6 chocolates = 0.

(iii) Finding the number of ways in which all three kids get more than 6 chocolates.

So, for that you have to keep aside 21 (=7+7+7) chocolates.

But, you have only 12 chocolates, so you cannot distribute 12 chocolates among 3 kids in such a way that all the 3 kids get more than 6 chocolates.

That is, the number of ways in which you can distribute 12 identical chocolates among 3 kids such that all the 3 kids must get more than 6 chocolates = 0.

Now using Inclusion-Exclusion principle, you can find the number of ways in which you can distribute 12 identical chocolates among 3 kids such that at least one of them must get more than 4 chocolates = $63 - 0 + 0 = 63$.

Hint ${}^3C_1 \left[\frac{(5+2)!}{5! \times 2!} \right] - {}^3C_2 (0) + {}^3C_3 (0) = 63$

Exp. 19) Find the number of ways in which you can distribute all the 12 chocolates among 3 kids, such that at least one of them must get more than 4 chocolates.

Solution In order to ensure that at least one of them must get more than 4 chocolates you have to find the number of ways in which one or more than one kid gets more than 4 chocolates using the inclusion-exclusion principle, as explained below.

- (i) Finding the number of ways in which any one kid gets more than 4 chocolates.

So, first of all keep aside 5 chocolates and distribute the remaining 7 chocolates among 3 kids.

Now, the number of ways in which you can distribute 7 identical chocolates among 3 kids without restriction

$$= \frac{(7+2)!}{7! \times 2!} = \frac{9!}{7! \times 2!} = 36$$

Now, you can give away the 5 chocolates, which you had kept aside, to any one of the 3 kids in ${}^3C_1 = 3$ ways.

Therefore, the number of ways in which you can distribute 12 identical chocolates among 3 kids such that any one of the 3 kids must get more than 4 chocolates = $3 \times 36 = 108$

- (ii) Finding the number of ways in which any two kids get more than 4 chocolates.

So, first of all keep aside 10 (=5+5) chocolates and distribute the remaining 2 chocolates among 3 kids.

Now, the number of ways in which you can distribute 2 identical chocolates among 3 kids without restriction

$$= \frac{(2+2)!}{2! \times 2!} = \frac{4!}{2! \times 2!} = 6$$

Now you have 2 identical packets of chocolates with you, such that each packet has 5 chocolates in it.

Now, you can give away the 2 identical packets of chocolates, which you had kept aside, to any 2 of the 3 kids in ${}^3C_2 = 3$ ways.

Therefore, the number of ways in which you can distribute 12 identical chocolates among 3 kids such that any two of the 3 kids must get more than 4 chocolates = $3 \times 6 = 18$.

- (iii) Finding the number of ways in which all three kids get more than 4 chocolates.

So, for that you have to keep aside 15 (=5+5+5) chocolates. But, you have only 12 chocolates, so you cannot distribute 12 chocolates among 3 kids in such a way that all the 3 kids get more than 4 chocolates. That is, the number of ways in which you can distribute 12 identical chocolates among 3 kids such that all the 3 kids must get more than 4 chocolates = 0.

Now using Inclusion-Exclusion principle, you can find the number of ways in which you can distribute 12 identical chocolates among 3 kids such that at least one of them must get more than 4 chocolates = $108 - 18 + 0 = 90$.

Hint ${}^3C_1 \left(\frac{(7+2)!}{7! \times 2!} \right) - {}^3C_2 \left(\frac{(2+2)!}{2! \times 2!} \right) + {}^3C_3 (0) = 90$

Alternatively Total number of ways in which you can distribute 12 identical chocolates among 3 kids without any

restriction = $\frac{(12+2)!}{12! \times 2!} = \frac{14!}{12! \times 2!} = 91$

And, the number of ways in which you can distribute not more than 4 chocolates to any of the 3 kids = 1

Therefore, the number of ways in which you can distribute 12 identical chocolates among 3 kids such that at least one of them must get more than 4 chocolates = $91 - 1 = 90$.

Hint There is only one way in which none of the 3 kids gets more than 4 chocolates, which is (4, 4, 4). In all other 90 combinations at least one kid must get more than 4 chocolates.

Exp. 20) Find the number of ways in which you can distribute all the 12 chocolates among 3 kids, such that at least one of them must get more than 2 chocolates.

Solution In order to ensure that at least one of them must get more than 2 chocolates you have to find the number of ways in which one or more than one kid gets more than 2 chocolates using the inclusion-exclusion principle, as explained below.

- (i) Finding the number of ways in which any one kid gets more than 2 chocolates.

So, first of all keep aside 3 chocolates and distribute the remaining 9 chocolates among 3 kids.

Now, the number of ways in which you can distribute 9 identical chocolates among 3 kids without restriction

$$= \frac{(9+2)!}{9! \times 2!} = \frac{11!}{9! \times 2!} = 55$$

Now, you can give away the 3 chocolates, which you had kept aside, to any one of the 3 kids in ${}^3C_1 = 3$ ways.

Therefore, the number of ways in which you can distribute 12 identical chocolates among 3 kids such that any one of the 3 kids must get more than 2 chocolates = $3 \times 55 = 165$.

- (ii) Finding the number of ways in which any two kids get more than 2 chocolates.

So, first of all keep aside 6 (=3+3) chocolates and distribute the remaining 6 chocolates among 3 kids.

Now, the number of ways in which you can distribute 6 identical chocolates among 3 kids without restriction

$$= \frac{(6+2)!}{6! \times 2!} = \frac{8!}{6! \times 2!} = 28$$

Now you have 2 identical packets of chocolates with you, such that each packet has 3 chocolates in it.

Now, you can give away the 2 packets of chocolates, which you had kept aside, to any 2 of the 3 kids in ${}^3C_2 = 3$ ways.

Therefore, the number of ways in which you can distribute 12 identical chocolates among 3 kids such that any two of the 3 kids must get more than 2 chocolates $= 3 \times 28 = 84$.

- (iii) Finding the number of ways in which all three kids get more than 2 chocolates.

So, for that you have to keep aside 9 ($= 3+3+3$) chocolates and distribute the remaining 3 chocolates among 3 kids.

Now, the number of ways in which you can distribute 3 identical chocolates among 3 kids without restriction $= \frac{(3+2)!}{3! \times 2!} = \frac{5!}{3! \times 2!} = 10$

Now you have 3 identical packets of chocolates with you, such that each packet has 3 chocolates in it.

Now, you can give away the 3 packets of chocolates, which you had kept aside, to the 3 kids in ${}^3C_3 = 1$ way.

Therefore, the number of ways in which you can distribute 12 identical chocolates among 3 kids such that at least two of the 3 kids must get more than 2 chocolates $= 1 \times 10 = 10$.

Now using Inclusion-Exclusion principle, you can find the number of ways in which you can distribute 12 identical chocolates among 3 kids such that at least one of them must get more than 2 chocolates $= 165 - 84 + 10 = 91$

Hint ${}^3C_1 \left(\frac{(9+2)!}{9! \times 2!} \right) - {}^3C_2 \left(\frac{(6+2)!}{6! \times 2!} \right) + {}^3C_3 \left(\frac{(3+2)!}{3! \times 2!} \right) = 91$

Alternatively Total number of ways in which you can distribute 12 identical chocolates among 3 kids without any restriction $= \frac{(12+2)!}{12! \times 2!} = \frac{14!}{12! \times 2!} = 91$

And, the number of ways in which you can distribute not more than 2 chocolates to any of the 3 kids $= 0$

Therefore, the number of ways in which you can distribute 12 identical chocolates among 3 kids such that at least one of them must get more than 4 chocolates $= 91 - 0 = 91$

Exp. 21) Find the number of ways in which you can distribute all the 12 chocolates among 3 kids, such that none of them gets more than 6 chocolates.

Solution In order to ensure that at least one kid must get more than 6 chocolates, keep aside 7 chocolates and later on give it to any of the 3 kids. Now, you have just 5 chocolates to be distributed among 3 kids, which you can distribute in any manner because when you give 7 chocolates to any one of the 3 kids he will certainly have more than 6 chocolates.

Therefore, the number of ways in which you can distribute 5 identical chocolates, without any restriction $= \frac{(5+2)!}{5! \times 2!} = \frac{7!}{5! \times 2!} = 21$

Now you can give away the remaining 7 chocolates, which you had kept aside, to any one of the 3 kids in 3 ways.

Therefore, the number of ways in which you can distribute 12 identical chocolates among 3 kids such that any one of the 3 kids must get more than 6 chocolates $= 12 \times 3 = 63$.

Number of ways in which you can distribute 12 identical chocolates among 3 kids such that any 2 kids get more than 6 chocolates $= 0$. Similarly, the number of ways in which you can distribute 12 identical chocolates among 3 kids such that all 3 kids get more than 6 chocolates $= 0$.

Now using Inclusion-Exclusion principle, you can find the number of ways in which you can distribute 12 identical chocolates among 3 kids such that at least one of them must get more than 4 chocolates $= 63 - 0 + 0 = 63$.

Thus, the number of ways in which you can distribute 12 identical chocolates among 3 kids such that none of the 3 kids get more than 6 chocolates $= 91 - 63 = 28$

Alternatively There are following 3 cases: when the highest value is 6, when the highest value is 5 and when the highest value is 4.

$$\{6, 6, 0\}, \{6, 5, 1\}, \{6, 4, 2\}, \{6, 3, 3\}$$

$$\rightarrow 3 + 6 + 6 + 3 = 18 \text{ ways}$$

$$\{5, 5, 2\}, \{5, 4, 3\} \rightarrow 3 + 6 = 9 \text{ ways } \{4, 4, 4\} \rightarrow 1 \text{ way}$$

Therefore, we can distribute 12 chocolates among 3 kids such that no kid gets more than 6 chocolates in 28 ways $(= 18 + 9 + 1)$.

Exp. 22) Find the number of ways in which you can distribute all the 12 chocolates among the 3 kids, such that none of them gets less than 1 chocolate and more than 6 chocolates.

Solution The problem suggests that each kid has to get at least 1, but at most 6 chocolates.

In order to ensure that no any kid gets more than 6 chocolates subtract the number of ways in which at least one kid must get more than 6 chocolates from the total number of ways.

How to calculate the total number of ways in which each kid gets at least 1 chocolate: In order to ensure that each kid must get at least 1 chocolate keep aside 3 chocolates and distribute the remaining 9 chocolates without any other constraint. The number of ways in which it can be done is $\frac{(9+2)!}{9! \times 2!} = \frac{11!}{9! \times 2!} = 55$

However, the remaining 3 chocolates can be given in such a way that each kid gets exactly 1 chocolate. This distribution can be done in 1 way. Therefore, the number of ways in which 12 chocolates can be distributed among 3 kids such that each kid must get at least 1 chocolate $= 55 \times 1 = 55$.

How to calculate the total number of ways in which at least one kid must get more than 6 chocolates and none of them gets less than 1 chocolate:

- (i) As you know that if you want to ensure that each kid must get at least 1 chocolate then you actually have only 9 chocolates to be distributed.

Now you have to ensure that any one kid must get more than 6 chocolates, so keep aside 6 chocolates. Then you have only 3 chocolates out of the 9 chocolates to be distributed among 3 kids.

Therefore, the number of ways in which 3 chocolates can be distributed among 3 kids = $\frac{(3+2)!}{3! \times 2!} = \frac{5!}{3! \times 2!} = 10$

Now, you can give away 3 chocolates, which you had kept aside, to any one of the 3 kids in ${}^3C_1 = 3$ ways.

Therefore, the number of ways in which you can distribute 12 identical chocolates among 3 kids such that any one of the 3 kids must get more than 6 chocolates and none of them gets less than 1 chocolate = $3 \times 10 = 30$

- (ii) Number of ways in which you can distribute 12 identical chocolates among 3 kids such that any 2 kids get more than 6 chocolates and none of them gets less than 1 chocolate = 0.
- (iii) Similarly, the number of ways in which you can distribute 12 identical chocolates among 3 kids such that all 3 kids get more than 6 chocolates and none of them gets less than 1 chocolate = 0.

Now using Inclusion-Exclusion principle, you can find the number of ways in which you can distribute 12 identical chocolates among 3 kids such that at least one of them must get more than 1 chocolate = $30 - 0 + 0 = 30$.

Thus, the number of ways in which you can distribute 12 identical chocolates among 3 kids such that none of them gets less than 1 chocolate and more than 6 chocolates = $55 - 30 = 25$

☞ **Alternatively** Let A, B and C be three kids. Then,

$$a + b + c = 12, \text{ such that } a, b, c \geq 1$$

$$\Rightarrow a + b + c = 9, \text{ such that } a, b, c \geq 0$$

The number of solutions to the above equation

$$= \frac{(9+2)!}{9! \times 2!} = \frac{11!}{9! \times 2!} = 55$$

Now, $a + b + c = 12$, such that $a, b, c \geq 1$

$$\Rightarrow a + b + c = 9, \text{ such that } a, b, c \geq 0$$

$$\Rightarrow a + b + c = 3, \text{ such that } a, b, c \geq 0 \text{ and any one of } a, b, c \geq 6$$

The number of solution to the above equation

$$= {}^3C_1 \left(\frac{(3+2)!}{3! \times 2!} \right) \\ = 3 \times \left(\frac{5!}{3! \times 2!} \right) = 30$$

Thus, the number of solutions of $a + b + c = 12$, such that $a, b, c \geq 1$ and one of $a, b, c \geq 6$ is $55 - 30 = 25$.

☞ **NOTE** You cannot give more than 6 chocolates to any 2 or 3 kids.

☞ **Alternatively** There are following 3 cases: when the highest value is 6, when the highest value is 5 and when the highest value is 4.

$$\{6, 5, 1\}, \{6, 4, 2\}, \{6, 3, 3\} \rightarrow 6 + 6 + 3 = 15 \text{ ways}$$

$$\{5, 5, 2\}, \{5, 4, 3\} \rightarrow 3 + 6 = 9 \text{ ways}$$

$$\{4, 4, 4\} \rightarrow 1 \text{ way}$$

Therefore, we can distribute 12 chocolates among 3 kids such that no kid gets less than 1 chocolate or more than 6 chocolates in 25 ways ($= 15 + 9 + 1$)

Exp. 23) Find the number of ways in which you can distribute at most 12 chocolates among 3 kids.

Solution First of all you must understand that the maximum number of chocolates that you can distribute is 12. It implies that out of the 12 chocolates it is not necessary that you distribute all the chocolates. If you distribute all the chocolates then the kids will receive the maximum number of 12 chocolates, but if you retain some chocolates with yourself, then the kids will receive less than 12 chocolates.

It implies that greater the number of chocolates you retain with yourself, lesser the number of chocolates you will distribute among the 3 kids. That means if you distribute X chocolates among 3 kids and retain Y chocolates with yourself, then

$$X + Y = 12$$

Now, let the number of chocolates received by first kid, second kid and third kid be a, b and c , the appropriate relation would be

$$a + b + c \leq 12 \text{ or } a + b + c + Y = 12$$

Now, the above equation is equivalent to the problem of distributing 12 identical chocolates among 4 kids. This, in turn, is equivalent to the problem of arranging the 15 objects (12 circles + 3 bars) in a row, out of which 12 circles are identical and 3 bars are also identical. That is the number of ways of arrangements of 12 circles and 3 bars = $\frac{15!}{12! \times 3!} = 455$

Therefore, the number of ways in which at most 12 identical chocolates can be divided among 3 kids is 455.

☞ **Alternatively** This solution is indeed not the smartest one, but it gives you a glimpse of what you can do to understand it how it works actually when you are not able to solve the problem smartly and quickly.

Case I: $a + b + c = 12$ The number of solutions

$$= \frac{14!}{12! \times 2!} = {}^{14}C_2$$

Case II: $a + b + c = 11$ The number of solutions

$$= \frac{13!}{11! \times 2!} = {}^{13}C_2$$

Case III: $a + b + c = 10$ The number of solutions

$$= \frac{12!}{10! \times 2!} = {}^{12}C_2$$

Case XII: $a + b + c = 10$ The number of solutions

$$= \frac{3!}{1! \times 2!} = {}^3C_2$$

Case XIII: $a + b + c = 0$ The number of solutions

$$= \frac{2!}{0! \times 2!} = {}^2C_2$$

Therefore, total number of solutions

$$= {}^2C_2 + {}^3C_2 + \dots + {}^{12}C_2 + {}^{13}C_2 + {}^{14}C_2$$

$$= {}^{14+1}C_{2+1} = {}^{15}C_3 = \frac{15!}{12! \times 3!} = 455$$

Hint ${}^rC_r + {}^{r+1}C_r + {}^{r+2}C_r + \dots + {}^nC_r = {}^{n+1}C_{r+1}$; $r \leq n$. Otherwise,

$$\frac{2!}{0! \times 2!} + \frac{3!}{1! \times 2!} + \frac{4!}{2! \times 2!} + \dots + \frac{14!}{12! \times 2!} = 1 + 3 + 6 + \dots + 91$$

$$= \frac{13 \times 14 \times 15}{6} = 455$$

Hint Since $1 + 3 + 6 + \dots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}$

Exp. 24) Find the number of ways in which you can distribute at least 6 and at most 12 chocolates among 3 kids.

Solution First of all you must understand that the number of chocolates that you can distribute is maximum 12 and minimum 6. That means if you distribute X chocolates among the 3 kids and retain Y chocolates with yourself, then

$$X + Y = 12$$

Now, let the number of chocolates received by first kid, second kid and third kid be a , b and c , then the appropriate relation is

$$a + b + c + Y = 12$$

Now, the above equation is equivalent to the problem of distributing 12 identical chocolates among 4 kids. This, in turn, is equivalent to the problem of arranging 15 objects (12 circles + 3 bars) in a row, out of which 12 circles are identical and 3 bars are also identical.

That is the number of ways of arrangements of 12 circles and 3 bars $= \frac{15!}{12! \times 3!} = 455$

Therefore, the number of ways in which at most 12 identical chocolates can be divided among 3 kids is 455.

Now, assume that instead of 12 chocolates you have just 5 chocolates. And, you can distribute at most 5 chocolates to the 3 kids. That means if you distribute P chocolates among the 3 kids and retain Q chocolates with yourself, then

$$P + Q = 5$$

Now, let the number of chocolates received by first kid, second kid and third kid be a , b and c , then the appropriate relation is, $a + b + c + Q = 5$

Now, the above equation is equivalent to the problem of distributing 5 identical chocolates among 4 kids.

This, in turn, is equivalent to the problem of arranging the 8 objects (5 identical circles + 3 identical bars) in a row.

That is the number of ways of arrangements of 5 circles and 3 bars $= \frac{8!}{5! \times 3!} = 56$

Therefore, the number of ways in which at most 5 identical chocolates can be distributed among 3 kids is 56.

Therefore, the number of ways in which you can distribute not less than 6 but not more than 12 identical chocolates among 3 kids = number of ways in which you can distribute at most 12 identical chocolates among 3 kids – number of ways in which you can distribute at most 5 identical chocolates among 3 kids $= 455 - 56 = 399$

Hint Number of non negative integral solutions of $6 \leq a + b + c \leq 12 =$ Number of non negative integral solutions of $a + b + c \leq 12 -$ Number of non negative integral solutions of $a + b + c \leq 5$

$$= ({}^2C_2 + {}^3C_2 + \dots + {}^{14}C_2) - ({}^2C_2 + {}^3C_2 + \dots + {}^7C_2)$$

$$= ({}^{15}C_3) - ({}^8C_3) = 455 - 56 = 399$$

Alternatively This solution is indeed not the smartest one, but it gives you a glimpse of what you can do in exam when you are not able to solve the problem smartly and quickly.

Case I : $a + b + c = 12$. The number of solutions

$$= \frac{14!}{12! \times 2!} = {}^{14}C_2$$

Case II : $a + b + c = 11$. The number of solutions

$$= \frac{13!}{11! \times 2!} = {}^{13}C_2$$

Case III : $a + b + c = 10$. The number of solutions

$$= \frac{12!}{10! \times 2!} = {}^{12}C_2$$

Case IV : $a + b + c = 9$. The number of solutions

$$= \frac{11!}{9! \times 2!} = {}^{11}C_2$$

Case V : $a + b + c = 8$. The number of solutions

$$= \frac{10!}{8! \times 2!} = {}^{10}C_2$$

Case VI : $a + b + c = 7$. The number of solutions

$$= \frac{9!}{7! \times 2!} = {}^9C_2$$

Case VII : $a + b + c = 6$. The number of solutions

$$= \frac{8!}{6! \times 2!} = {}^8C_2$$

Therefore, the required number of solutions

$$= ({}^8C_2 + {}^9C_2 + \dots + {}^{14}C_2)$$

$$= 28 + 36 + 45 + 55 + 66 + 78 + 91 = 399$$

NOTE Now, you must have observed that the previous solution is more logical and this one is more mechanical (or manual). But, this solution is helping you to visualize that what the actual logic behind the previous solution is. For smaller values alternative solution may appear to be easy one, but for greater values the previous solution is faster.

Exp. 25) You have 2 dice and you throw them simultaneously. Find the number of ways in which the total sum you get in any throw is not less than 5 and not more than 10.

Solution Even though the two dice are identical, still for the heck of our convenience we call them A and B

That is, $5 \leq a + b \leq 10$, such that, $(a, b) = 1, 2, 3, 4, 5, 6$

Then, $3 \leq a + b \leq 8$, such that, $(a, b) = 0, 1, 2, 3, 4, 5, 6$

Now, we can find the required number of ways as shown below.

$$a + b = 3, \text{ Number of ways} = \frac{4!}{3! \times 1!} = 4$$

$$a + b = 4, \text{ Number of ways} = \frac{5!}{4! \times 1!} = 5$$

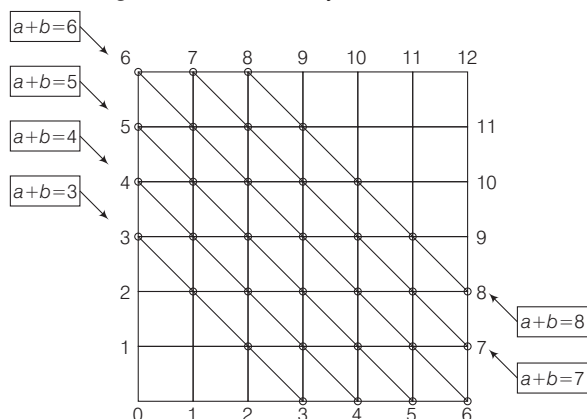
$$a + b = 5, \text{ Number of ways} = \frac{6!}{5! \times 1!} = 6$$

$$a + b = 6, \text{ Number of ways} = \frac{7!}{6! \times 1!} = 7$$

$$a + b = 7, \text{ Number of ways} = \frac{8!}{7! \times 1!} - 2 = 6$$

$$a + b = 8, \text{ Number of ways} = \frac{9!}{8! \times 1!} - 4 = 5$$

To get the above values you can use the following approach. Draw a 6×6 grid, as a die has only 6 numbers (1, 2, ..., 6) on it.



Now, you can find the number of ways by looking at the number of intersections on corresponding diagonals. As, $a + b = 3$ has 4 intersections, so it has 4 solutions. Similarly, $a + b = 6$ has 7 intersections, so it has 7 solutions. But, since 6 is the highest number that a die has, so after it reaches 6 it starts to decrease in the same manner as it increases before 6.

Thus the total number of ways = $4 + 5 + 6 + 7 + 6 + 5 = 33$

Exp. 26) You have 3 dice and you throw them simultaneously. Find the number of ways in which the total sum that you get in any throw is not less than 5 and not more than 15.

Solution Even though the three dice are identical, still for the heck of our convenience we call them A, B and C

Then, $5 \leq a + b + c \leq 15$, such that $(a, b, c) = 1, 2, 3, 4, 5, 6$

That is, $2 \leq a + b + c \leq 12$, such that $(a, b, c) = 0, 1, 2, 3, 4, 5, 6$

Now, we can find the required number of ways as shown below.

$$\text{Case I : } a + b + c = 2, \text{ Number of ways} = \frac{4!}{2! \times 2!} = 6$$

$$\text{Case II : } a + b + c = 3, \text{ Number of ways} = \frac{5!}{3! \times 2!} = 10$$

$$\text{Case III : } a + b + c = 4, \text{ Number of ways} = \frac{6!}{4! \times 2!} = 15$$

$$\text{Case IV : } a + b + c = 5, \text{ Number of ways} = \frac{7!}{5! \times 2!} = 21$$

$$\text{Case V : } a + b + c = 6, \text{ Number of ways} = \frac{8!}{6! \times 2!} = 28$$

$$\text{Case VI : } a + b + c = 7, \text{ Number of ways} = \frac{9!}{7! \times 2!} - 3 = 33$$

$$\text{Case VII : } a + b + c = 8, \text{ Number of ways} = \frac{10!}{8! \times 2!} - 9 = 36$$

$$\text{Case VIII : } a + b + c = 9, \text{ Number of ways} = \frac{11!}{9! \times 2!} - 18 = 37$$

$$\text{Case IX : } a + b + c = 10, \text{ Number of ways} = \frac{12!}{10! \times 2!} - 30 = 36$$

$$\text{Case X : } a + b + c = 11, \text{ Number of ways} = \frac{13!}{11! \times 2!} - 45 = 33$$

$$\text{Case XI : } a + b + c = 12, \text{ number of ways} = \frac{14!}{12! \times 2!} - 63 = 28$$

Thus the total number of ways

$$= 6 + 10 + 15 + 21 + 2(28 + 33 + 36) + 37 = 283$$

Hint From Case VI to Case XI, you see that I have subtracted some values. For example, in case VI, there are 3 ways/ arrangements (7, 0, 0), (0, 7, 0), (0, 0, 7) which do not satisfy the given criteria, as you cannot use any value greater than 6. That's why they have been discarded from the maximum possible number of ways/ solutions.

Similarly, in case VII, there are 9 solutions (8, 0, 0), (0, 8, 0), (0, 0, 8), (7, 1, 0), (7, 0, 1), (1, 0, 7), (1, 7, 0), (0, 1, 7), (0, 7, 1) which do not satisfy, as you cannot use any value greater than 6.

The same process is being followed for other cases too.

Exp. 27) Find the number of integral solutions of $|x| + |y| = 12$.

Solution It can be solved simply, by putting the minimum and maximum possible values.

x	y	Number of Solutions
0	± 12	2
± 1	± 11	4
± 2	± 10	4
.....		
± 10	± 2	4
± 11	± 1	4
± 12	0	2

Total number of solutions = $2 + 11(4) + 2 = 48$.

☞ **Alternatively** Since $|\pm k| = k$, it means each k accepts 2 non-zero values. But since 0 is a neutral number in the sense that it has no effect of positive or negative sign, so we have to consider it separately.

Case I : When none of x and y is 0.

$$x + y = 12; x, y \geq 1 \Rightarrow x + y = 10; x, y \geq 0$$

Therefore, the number of solutions of the equation

$$x + y = 10; x, y \geq 0 \text{ is } \frac{11!}{10! \times 1!} = 11$$

And, the number of solutions of the equation $|x| + |y| = 10$; $x, y \geq 0$ is $11 \times (2 \times 2) = 44$

Case II : When one of x and y is 0

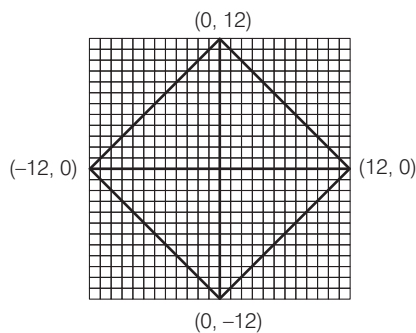
That is $(x, y) \equiv (0, 12), (0, -12), (12, 0), (-12, 0)$. So it has 4 solutions. Thus, the total number of required solutions of $|x| + |y| = 10$; $x, y \geq 0$ is $44 + 4 = 48$

☞ **NOTE** There are 44 solutions which have no zeros and 4 solutions have exactly one zero.

☞ **Alternatively** Look at the following coordinate graph.

The graph is drawn such that $-12 \leq x \leq 12$ and $-12 \leq y \leq 12$.

The number of solutions is equal to the number of intersections of the diagonals. As, there are 48 intersection points. So the number of solutions of $|x| + |y| = 12$ is 48.



Exp. 28) Find the integral solutions of $|x| + |y| \leq 12$.

Solution The number of integral solutions of $|x| + |y| \leq 12$ will be the sum of the number of integral solutions of $(|x| + |y| = 0) + (|x| + |y| = 1) + (|x| + |y| = 2) + \dots + (|x| + |y| = 12)$
 $= 1 + 4 + 8 + \dots + 48$
 $= 1 + 4(1 + 2 + 3 + \dots + 12) = 313$

Exp. 29) Find the integral solutions of $|x| + |y| + |z| = 12$.

Solution The total number of integral solutions of $|x| + |y| + |z| = n$ is $4n^2 + 2$. So the number of integral solutions of the given equation is $4(12^2) + 2 = 578$

☞ **Alternatively** The given problem can be solved in the following way.

Case I : When none of the x, y and z is zero, then the number of solutions of

$$x + y + z = 12 = {}^{11}C_2 = 55$$

But, since each of $|x|, |y|$ and $|z|$ takes 2 non zero values – one positive and one negative, so we will have 8 different ways $(2 \times 2 \times 2)$ to represent the 55 solutions.

Therefore, the number of solutions of $|x| + |y| + |z| = 12$ is $8 \times 55 = 440$

Case II : When exactly one of the x, y and z is zero, the number of solutions of

$$x + y + z = 12 = {}^{11}C_1 = 11$$

But, when $x = 0, |y| + |z|$ will have 4 different ways (2×2) to represent the 11 solutions

Similarly, when $y = 0, |x| + |z|$ will have 4 different ways to represent the 11 solutions

Similarly, when $z = 0, |x| + |y|$ will have 4 different ways to represent the 11 solutions

Therefore, the number of solutions of

$$|x| + |y| + |z| = 12 \text{ is } 12 \times 11 = 132$$

Case III : When exactly 2 of x, y and z are zero.

Then the number of solutions of $x + y + z = 12$ is 3.

But, since there are two ways to express a non-zero value $(+/-)$, so the total number of solutions of $|x| + |y| + |z| = 12$ is $2 \times 3 = 6$

Thus, by adding the number of solutions of case I, II and III, we get the total number of solutions, which is $440 + 132 + 6 = 578$.

Exp. 30) How many 3 digit numbers are there in which sum of all the three digits is 12?

Solution $a + b + c = 12; a \geq 1, b \geq 0, c \geq 0$

$$\Rightarrow a + b + c = 11; a \geq 1, b \geq 0, c \geq 0$$

Therefore the required number of solutions

$$= {}^{11+3-1}C_{3-1} = {}^{13}C_2 = 78$$

Alternatively: Total number of solutions of

$$a + b + c = 12;$$

$$(a \geq 0, b \geq 0, c \geq 0) \text{ is } {}^{14}C_2 = 91$$

Number of solutions of $a + b + c = 12$, when

$$a = 0, b \geq 0, c \geq 0 \text{ is } {}^{13}C_1 = 13$$

Therefore the required number of solutions

$$a + b + c = 12, \text{ when}$$

$$a \geq 1, b \geq 0, c \geq 0 \text{ is } 78 (= 91 - 13).$$

☞ **Alternatively** $a + b + c = 12$, such that $a \geq 1, b \geq 0, c \geq 0$

For $a = 1, b + c = 11 \Rightarrow$ The number of solutions $= {}^{12}C_1 = 12$

For $a = 2, b + c = 10 \Rightarrow$ The number of solutions $= {}^{11}C_1 = 11$

For $a=3, b+c=9 \Rightarrow$ The number of solutions $= {}^{10}C_1 = 10$

For $a=12, b+c=0 \Rightarrow$ The number of solutions $= {}^1C_1 = 1$

Therefore, the required answer is

$$78 (=12+11+10+\dots+3+2+1).$$

NOTE Since, ${}^rC_r + {}^{r+1}C_r + {}^{r+2}C_r + \dots + {}^nC_r = {}^{n+1}C_{r+1}; r \leq n$,
 Therefore ${}^1C_1 + {}^2C_1 + {}^3C_1 + \dots + {}^{12}C_1 = {}^{13}C_2 = 78$

Exp. 31) How many 3 digit number are there in which sum of all the three digits is not more than 12?

Solution $a+b+c \leq 12$, such that $a \geq 1, b \geq 0, c \geq 0$.

For $a+b+c=1$, the number of solutions is ${}^2C_2 = 1$

For $a+b+c=2$, the number of solutions is ${}^3C_2 = 3$

For $a+b+c=3$, the number of solutions is ${}^4C_2 = 6$

... ..

For $a+b+c=12$, the number of solutions is
 ${}^{13}C_2 = 78$

Therefore the required number

$$= 1 + 3 + 6 + \dots + 78 \\ = 364$$

Hint $1 + (1+2) + (1+2+3) + \dots + (1+2+3+\dots+n)$

$$= \left(\frac{n(n+1)(n+2)}{6} \right)$$

Exp. 32) Find the number of terms in $(x+y)^n$

Solution $(x+y)^n = {}^nC_0(x^n) + {}^nC_1(x^{n-1}y) + {}^nC_2(x^{n-2}y^2) \\ + \dots + C_{n-1}(xy^{n-1}) + {}^nC_n(y^n)$

(a) There are $n+1$ terms in the expansion of $(x+y)^n$

(b) In each term sum of the indices of x and y is equal to n .

Exp. 33) Find the total number of terms in the expansion of $(a+b+c)^2$.

Solution Look at the following expansions. These expansions show how the different terms are formed. However, the coefficients of these terms may not be correct as our emphasis is on finding the number of terms only.

$$(a+b)^2 = a^0b^2 + a^1b^1 + a^2b^0$$

$$(a+b)^3 = a^0b^3 + a^1b^2 + a^2b^1 + a^3b^0$$

$$(a+b)^4 = a^0b^4 + a^1b^3 + a^2b^2 + a^3b^1 + a^4b^0$$

$$(a+b+c)^2 = a^0b^0c^2 + a^0b^1c^1 + a^0b^2c^0 + a^1b^0c^1 + a^1b^1c^0 + a^2b^0c^0$$

So looking at the various terms of the expansion of above polynomials we can conclude that the number of terms in any polynomial expansion is like distributing n identical items among m distinct persons. And thus we get the following formula.

$$(a_1 + a_2 + \dots + a_n)^m = {}^{m+n-1}C_{n-1}$$

Exp. 34) Find the total number of terms in $(1+x+x^2)^3$.

Solution $(1+x)^2 = 1+x^2+2x$

$$(1+x)^3 = 1+3x+3x^2+x^3$$

$$(1+x)^4 = 1+4x+6x^2+4x^3+x^4$$

$$(1+x+x^2)^2 = 1+2x+3x^2+2x^3+x^4$$

$$(1+x+x^2)^3 = 1+3x+6x^2+7x^3+6x^4+3x^5+x^6$$

This shows that total number of terms in

$$(1+x+x^2+\dots+x^n)^m = (m.n) + 1$$

Exp. 35) What is the coefficient of $a^2b^3c^5$ in $(x+y+z)^{10}$?

Solution By Multinomial Theorem, it is $\frac{10!}{2!3!5!}$

Introductory Exercise 19.8

Directions (for Q. Nos. 1 to 12) : Answer these questions based on the following information.

A film library at FTII (Film and Television Institute of India) Pune has 12 distinct CDs on French cinema.

- Find the number of ways in which these CDs can be divided into two groups such that one group has 8 CDs and another one has 4 CDs.
 (a) 594 (b) 495 (c) 945 (d) 32
- Find the number of ways in which these CDs can be gifted to two French students who had recently visited FTII from France such that one student has 8 CDs and another one has 4 CDs.
 (a) 990 (b) 900 (c) 945 (d) 1188

- Find the number of ways in which these CDs can be divided into three groups of 6 CDs, 4 CDs and 2 CDs.
 (a) 12680 (b) 18360
 (c) 13860 (d) $\frac{12!}{6!+4!+2!}$
- Find the number of ways in which these CDs can be gifted to three French students who had recently visited FTII from France such that one student has 6 CDs, second student has 4 CDs and the third one has 2 CDs.
 (a) 31860 (b) 82160
 (c) 13860 (d) 83160

5. Find the number of ways in which these CDs can be divided equally into two groups.
(a) 1024 (b) 492 (c) 462 (d) 924
6. Find the number of ways in which these CDs can be distributed to two French students equally.
(a) 594 (b) 990 (c) 984 (d) 924
7. Find the number of ways in which these CDs can be divided equally into three groups.
(a) 34560 (b) 34650 (c) 7575 (d) 5775
8. Find the number of ways in which these CDs can be distributed among three French students equally.
(a) 34650 (b) 272100
(c) 43650 (d) 207900
9. Find the number of ways in which these CDs can be distributed to 6 French students equally.
(a) $\frac{12!}{(2!)^6}$ (b) $\frac{12!}{(6!)^2} \times 6!$
(c) $\frac{12!}{6! \times 2!} \times 6!$ (d) 32000084
10. Find the number of ways in which these CDs can be distributed to 6 French students such that anyone may have any number of CDs.
(a) 6^{12} (b) 12^6 (c) $\frac{12!}{6!}$ (d) $(6!)^{12}$
11. Find the number of ways in which these CDs can be distributed to 4 French students such that each one must have at least two CDs.
(a) 3450650 (b) 7050120
(c) 504090 (d) 6900520
12. Find the number of ways in which these CDs can be distributed to 4 French students such that no one has more than three CDs.
(a) $\frac{12!}{64}$ (b) $\frac{12!}{(3!)^4} \times 4!$
(c) $\frac{12!}{1296}$ (d) 360960

Directions (for Q. Nos. 13 to 20) : *Answer the questions based on the following information. Once Ruskin Bond meets 4 kids on his way to Dehradun. While chatting with them he decides to distribute all the 12 identical chocolates among these kids.*

13. Find the number of ways of distributing 12 chocolates among 4 kids such that each of them can receive any number of chocolates.
(a) 455 (b) 272 (c) 575 (d) 34650
 14. Find the number of ways of distributing 12 chocolates among 4 kids such that each of them must receive at least one chocolate.
(a) 365 (b) 1 (c) 8! (d) 165
 15. Find the number of ways of distributing 12 chocolates among 4 kids such that each of them must receive at least two chocolates.
(a) 35 (b) 1 (c) 8 (d) 65
 16. Find the number of ways of distributing 12 chocolates among 4 kids such that each of them must receive at least three chocolates.
(a) 36 (b) 1 (c) 3 (d) 16
 17. Find the number of ways of distributing 12 chocolates among 4 kids such that no kid receives less than 2 chocolates and more than 4 chocolates.
(a) 38 (b) 19 (c) 16 (d) 32
 18. Find the number of ways of distributing 12 chocolates among 4 kids such that these kids get 1, 2, 4, and 5 chocolates.
(a) 25 (b) 24 (c) 52 (d) 64
 19. Find the number of ways of distributing 12 chocolates among 4 kids such that taller the kid fewer the chocolates he gets. However, these kids get 1, 2, 4, and 5 chocolates as per their heights, assuming no two kids are of the same height.
(a) 25 (b) 24
(c) 52 (d) 1
 20. Find the number of ways of distributing 12 chocolates among 4 kids equally.
(a) 4 (b) 4^4
(c) $\frac{12!}{4!}$ (d) 1
- Directions** (for Q. No. 21 to 30) : *Answer the following questions independently of each other.*
21. Find the total number of non-negative integral solutions of $a + b + c = 30$.
(a) 306 (b) 406 (c) 396 (d) 496
 22. Find the total number of positive integral solutions of $a + b + c = 30$.
(a) 405 (b) 406 (c) 303 (d) 496
 23. Find the total number of even positive integral solutions of $a + b + c = 30$.
(a) 128 (b) 72 (c) 91 (d) 104
 24. Find the total number of odd positive integral solutions of $a + b + c = 30$.
(a) 30 (b) 42 (c) 1 (d) 0
 25. Find the total number of positive integral solutions of $a + b + c = 30$ such that a, b and c are distinct.
(a) 366 (b) 256 (c) 178 (d) 398
 26. Find the total number of non-negative integral solutions of $a + b + c = 30$ such that $a < b < c$.
(a) 63 (b) 85 (c) 105 (d) 75

27. Find the total number of non-negative integral solutions of $a + b + c = 30$ such that $a \leq b$.
(a) 231 (b) 225 (c) 256 (d) 132
28. Find the total number of positive integral solutions of $3a + 2b + c = 30$.
(a) 33 (b) 21 (c) 61 (d) 67
29. Find the total number of ways of selecting the 30 roses from the truckload of red, pink and yellow roses.
(a) 496 (b) 30! (c) 30^3 (d) 3^{30}
30. Find the total number of ways of selecting the 30 roses from truckload of red, pink and yellow roses, such that in each selection there are roses of all the three colours.
(a) 404 (b) 24 (c) 406 (d) 303

Directions (for Q. Nos. 31 to 38) : Answer the questions based on the following information. There are four numbers a, b, c, d such that $a + b + c + d = 12$.

31. Find the number of non-negative integral solutions of the equation.
(a) 25 (b) 244 (c) 255 (d) 425
32. Find the number of positive integral solutions of the equation.
(a) 125 (b) 165 (c) 143 (d) 156
33. Find the number of integral solutions of the equation, such that $2 \leq a, b, c, d \leq 4$.
(a) 19 (b) 24 (c) 52 (d) 34
34. Find the number of integral solutions of the equation, such that $a, b, c, d \geq 2$.
(a) 35 (b) 24 (c) 52 (d) 34
35. Find the number of integral solutions of the equation, such that $a > 0, b > 1, c > 2, d > 3$.
(a) 10 (b) 12 (c) 34 (d) 24
36. Find the number of integral solutions of the equation, such that $a > -4, b > -3, c > -2, d > -1$.
(a) 1330 (b) 1331 (c) 1690 (d) 340
37. Find the number of integral solutions of the equation, such that $1 \leq (a, b) \leq 4, c = 6, d = 0$.
(a) 18 (b) 9 (c) 5 (d) 4
38. Find the number of integral solutions of the equation, such that $-2 \leq (a, b, c, d) \leq 19$.
(a) 1991 (b) 1771 (c) 2552 (d) 434

Directions (for Q. No. 39 to 40) : Answer the questions based on the following information. There are four numbers a, b, c, d such that $a + b + c + d = -12$.

39. Find the number of integral solutions of the equation, such that $(a, b, c, d) < 0$.
(a) 195 (b) -165 (c) 92 (d) 165

40. Find the number of integral solutions of the equation, such that $(a, b, c, d) \leq 0$.
(a) 256 (b) -256 (c) 425 (d) -425

Directions (for Q. Nos. 41 to 44) : Answer the questions based on the following information. There are three integers a, b, c such that $a + b + c = p$.

41. If $p = 8$ and $0 \leq (a, b, c) \leq 4$, find the number of solutions of the given equation.
(a) 15 (b) 24 (c) 12 (d) 30
42. If $p = 8$ and $0 \leq (a, b, c) \leq 3$, find the number of solutions of the given equation.
(a) 6 (b) 12
(c) 3 (d) 21
43. If $p = 8$ and $0 \leq (a, b, c) \leq 2$, find the number of solutions of the given equation.
(a) 7 (b) 6
(c) 5 (d) none of these
44. If $p = 8$ and $0 \leq (a, b, c) \leq 1$, find the number of solutions of the given equation.
(a) 2 (b) 4
(c) 6 (d) 0

Directions (for Q. Nos. 45 to 47) : Answer the questions based on the following information. There are four integers a, b, c, d such that $a + b + c + d = p$

45. If $p = 30$ and $0 \leq (a, b, c) \leq 15$, find the number of solutions of the given equation.
(a) 2736 (b) 2456
(c) 5226 (d) 3436
46. If $p = 30$ and $0 \leq (a, b, c) \leq 10$, find the number of solutions of the given equation.
(a) 285 (b) 286
(c) 528 (d) 304
47. If $p = 30$ and $0 \leq (a, b, c) \leq 8$, find the number of solutions of the given equation.
(a) 25 (b) 10 (c) 0 (d) 30

Directions (for Q. Nos. 48 to 59) : Answer the questions based on the following information. Hirawala is a uber rich guy who lives in a swanky place, called Hiranandini, in Mumbai. He has 15 diamond rings and 5 daughters.

48. If the rings are distinct, find the number of ways of distributing these rings among his 5 daughters such that no any two daughters get equal number of rings but each daughter gets at least one ring and at most 5 rings.
(a) ${}^{15}C_5$ (b) $\frac{15!}{1!2!3!4!5!} \times 5$
(c) $\frac{15!}{1!2!3!4!5!}$ (d) $\frac{15!}{1!2!3!4!}$

49. If the rings are distinct, find the number of ways of distributing these rings among his 5 daughters such that elder the daughter fewer the rings she gets. Thus the youngest daughter gets 5 rings and the eldest daughter gets just 1 ring. No any two daughters are of the same age.

$$(a) \frac{15!}{1!2!3!4!5!} \times 5! \quad (b) \frac{15!}{1!2!3!4!5!}$$

$$(c) \frac{15!}{({}^{15}C_5)5!} \quad (d) \frac{15!}{1!2!3!4!5!} \times 5$$

50. If the rings are distinct, find the number of ways of distributing these rings among his 5 daughters such that elder the daughter fewer the rings she gets. The youngest daughter gets 5 rings and the eldest daughter gets just 1 ring. There are no twins among his daughters.

$$(a) \frac{15!}{3!5!} \times \frac{7}{144} \quad (b) \frac{15!}{1!2!3!4!5!} \times 3!$$

$$(c) 15^5 \quad (d) \frac{15!}{3!5!} \times \frac{8}{121}$$

51. If the rings are distinct, find the number of ways of distributing these rings among his 5 daughters such that none of the daughters gets less than 2 rings and more than 4 rings.

$$(a) \frac{15!}{2!2!3!4!4!} \quad (b) \frac{15!}{2!3!3!3!4!}$$

$$(c) ({}^{15}C_5) \times 5! \quad (d) \text{none of these}$$

52. If the rings are distinct, find the number of ways of distributing these rings among his 5 daughters equally.

$$(a) 1 \quad (b) \frac{15!}{(3!)^5} \times 5! \quad (c) \frac{15!}{(3!)^5} \quad (d) {}^{15}C_5$$

53. If the rings are distinct, find the number of ways of distributing these rings among his 5 daughters such that a daughter may get zero or all the rings.

$$(a) 75 \quad (b) 1$$

$$(c) 15^5 \quad (d) 5^{15}$$

54. If the rings are identical, find the number of ways of distributing these rings among his 5 daughters such that no daughters get equal number of rings but each daughter gets at least one ring and at most 5 rings.

$$(a) 120 \quad (b) 75 \quad (c) 12 \quad (d) 34$$

55. If the rings are identical, find the number of ways of distributing these rings among his 5 daughters such that elder the daughter fewer the rings she gets. Thus the youngest daughter gets 5 rings and the eldest daughter gets only 1 ring. No any two daughters are of the same age.

$$(a) 1 \quad (b) 5 \quad (c) 15 \quad (d) 3$$

56. If the rings are identical, find the number of ways of distributing these rings among his 5 daughters such that elder the daughter fewer the rings she gets. The youngest daughter gets 5 rings and the eldest daughter gets 1 ring only. There are no twins among his daughters.

$$(a) 3 \quad (b) 2 \quad (c) 1 \quad (d) 6$$

57. If the rings are identical, find the number of ways of distributing these rings among his 5 daughters such that none of the daughters gets less than 2 rings and more than 4 rings.

$$(a) 51 \quad (b) 15 \quad (c) 52 \quad (d) 25$$

58. If the rings are identical, find the number of ways of distributing these rings among his 5 daughters equally.

$$(a) 5 \quad (b) 1 \quad (c) 10 \quad (d) 25$$

59. If the rings are identical, find the number of ways of distributing these rings among his 5 daughters such that a daughter may get zero or all the rings.

$$(a) 38 \quad (b) 76 \quad (c) 3876 \quad (d) 34$$

Directions (for Q. Nos. 60 to 63) : Solve the following problems independently of each other.

Rollmall is a fast food outlet, which delivers the veg and non-veg rolls in its vicinity. Bawarchi is an employee, who is assigned to pack the rolls in the boxes for delivery. He has 5 rolls in hand to pack in the 3 boxes so that none of the boxes remains empty.

60. Find the number of ways of packing the rolls, if each role and each box both are distinct.

$$(a) 150 \quad (b) 50 \quad (c) 60 \quad (d) 90$$

61. Find the number of ways of packing the rolls, if each roll is identical but each box is distinct.

$$(a) 3 \quad (b) 5 \quad (c) 6 \quad (d) 4$$

62. Find the number of ways of packing the rolls, if each roll is distinct but all the boxes are identical.

$$(a) 120 \quad (b) 24 \quad (c) 25 \quad (d) 40$$

63. Find the number of ways of packing the rolls, if all the rolls and all the boxes are identical.

$$(a) 3 \quad (b) 0 \quad (c) 1 \quad (d) 2$$

Directions (for Q. Nos. 64 to 70) : Answer the following questions independently of each other.

64. In how many ways can 12 identical bouquets be kept in 3 identical crates?

$$(a) 20 \quad (b) 16 \quad (c) 19 \quad (d) 25$$

65. In how many ways can 12 identical bouquets be kept in 3 distinct crates?

$$(a) 19 \quad (b) 91 \quad (c) 81 \quad (d) 61$$

66. In how many ways can 12 identical bouquets be kept in 3 distinct crates so that no box remains empty?

$$(a) 36 \quad (b) 55 \quad (c) 66 \quad (d) 91$$

67. In how many ways can 12 identical bouquets be kept in 3 distinct crates so that at least 2 bouquets must be there in each bouquet?
(a) 28 (b) 24 (c) 42 (d) 36
68. In how many ways can 12 identical bouquets be kept in 3 distinct crates so that the smallest box has at least 2 bouquets, the largest box has at least 4 bouquets and the third one has at least 3 bouquets?
(a) 10 (b) 20 (c) 21 (d) 32
69. In how many ways can 12 identical bouquets be kept in 3 distinct crates so that no two boxes have the same number of bouquets?
(a) 36 (b) 32 (c) 72 (d) 64
70. In how many ways can 12 identical bouquets be kept in 3 distinct crates so that each box has the same number of bouquets?
(a) 36 (b) 1 (c) 2 (d) 12

Directions (for Q. No. 71-73): Answer the following questions independently of each other.

71. In how many ways can 4 balls be selected from the box that contains 10 balls of distinct colours?
(a) 360 (b) 320 (c) 90 (d) 210
72. In how many ways can 4 balls be selected from the box that contains balls of 10 distinct colours, in which there are thousands of balls of each colour?
(a) 124 (b) 210
(c) 715 (d) 640
73. In how many ways can 10 cookies be selected from a bakery that sells the cookies of 4 distinct types, such that there must be at least one cookie of each type?
(a) 12
(b) 21
(c) 84
(d) none of the above

19.9 Re-arrangement or Derangement

Rearrangement of objects such that none of the objects occupies its original place.

- (i) The number of ways in which exactly r letters can be placed in wrongly addressed envelopes when n letters are placed in n addressed envelopes

$$= {}^n P_r \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^r \frac{1}{r!} \right]$$

- (ii) The number of ways in which n different letters can be placed in their n addressed envelopes so that all the letters are in the wrong envelopes

$$= n! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!} \right]$$

Introductory Exercise 19.9

Directions (for Q. Nos. 1 to 4): Answer the following questions independently of each other.

- Find the number of ways in which all the 6 different letters are placed in 6 addressed envelopes so that all the letters are in the wrong envelopes.
(a) 265 (b) 365 (c) 252 (d) 356
- Find the number of ways of placing 6 letters in 6 envelopes which have different addresses in such a way that only two letters are correctly placed and 4 letters are not in the right envelopes.
(a) 235 (b) 135 (c) 152 (d) 325
- Find the number of ways of placing 6 letters in 6 envelopes, which have different addresses in such a way that at least two letters are correctly placed in the right envelopes.
(a) 191 (b) 171 (c) 362 (d) 121
- For every distinct natural number $a, b, c, d, e, f < 7$, a number $N = (a-1)(b-2)(c-3)(d-4)(e-5)(f-6)$ is an integer. Find the number of distinct sets $\{a, b, c, d, e, f\}$ for which N is a non-zero integer.
(a) 365 (b) 256
(c) 265 (d) none of these

19.10 Number Properties

If a natural number N can be expressed in the canonical form as $N = a^p b^q c^r \dots$, whereas $a, b, c, \dots$ are distinct prime numbers. Then

(i) Total number of factors of a composite number
 $= (p+1)(q+1)(r+1) \dots$

(ii) Sum of factors of a composite number

$$= \frac{(a^{p+1} - 1)}{(a - 1)} \times \frac{(b^{q+1} - 1)}{(b - 1)} \times \frac{(c^{r+1} - 1)}{(c - 1)} \times \dots$$

(iii) Product of factors of a composite number
 $= N^{1/2}$ (total number of factors of N)

(iv) Total number of odd factors of a composite number
 $= (p+1)(q+1)(r+1)$; where each of $p, q, r, \dots$ must be odd prime number only. That means if $N = 2^k a^p b^q c^r \dots$, then eliminate 2^k from the factors of N to obtain number of odd factors of N .

(v) Total number of even factors of a composite number
 $=$ Total number of factors of N – total number of odd factors.

(vi) Total number of co-primes to N , which are less than

$$N = N \left(1 - \frac{1}{a}\right) \left(1 - \frac{1}{b}\right) \left(1 - \frac{1}{c}\right) \dots$$

(vii) Sum of all the co-primes of N which are less than
 $N = \frac{N}{2} \times$ Number of co-primes to N which are less than N .

(viii) Total number of ways in which N can be written as a product of two co-primes $= 2^{n-1}$; where n is the number of distinct prime factors of N .

(ix) Total number of ways in which N can be expressed as a product of two factors

$$= \frac{\text{Total number of factors of } N}{2}$$

(a) If N has odd number of factors (i.e. when N is a perfect square), the number of ways in which N can be expressed as a product of two 'distinct' factors

$$= \frac{(\text{Total number of factors of } N) - 1}{2}$$

(b) If N has even number of factors (i.e. when N is not a perfect square), the number of ways in which N can be expressed as a product of two factors, including the 'similar' factors

$$= \frac{(\text{Total number of factors of } N) + 1}{2}$$

(x) Total number of ordered pairs (x, y) such that LCM of x and y is a composite number $= (2p+1)(2q+1)(2r+1) \dots$

Exp. 1) Find the number of factors 72.

Solution Let us consider $N = 72$, which can be expressed as $72 = 2^3 \times 3^2$. Now, there are total 12 factors of 72; namely 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72. Let's see how it happens.

$$\begin{aligned} 1 &= 2^0 3^0, 2 = 2^1 3^0, 3 = 2^0 3^1, \\ 4 &= 2^2 3^0, 6 = 2^1 3^1, 8 = 2^3 3^0, \\ 9 &= 2^0 3^2, 12 = 2^2 3^1, 18 = 2^1 3^2, \\ 24 &= 2^3 3^1, 36 = 2^2 3^2, 72 = 2^3 3^2 \end{aligned}$$

Thus you can see that all the factors of 72 are due to only two prime numbers namely 2 and 3. What's happening here is that we are taking all the possible combinations of 2^j and 3^k . Where $j = 0, 1, 2, 3$ and $k = 0, 1, 2$. In other words we can select prime number 2 in four ways (as $2^0, 2^1, 2^2, 2^3$) and prime number 3 in three ways (as $3^0, 3^1, 3^2$) to get the factors of 72.

It implies that $72 = 2^3 \times 3^2$ has $(3+1)(2+1)$ factors = 12 factors.

Thus it can be concluded that $N = a^p \times b^q \times c^r \times \dots$ has $(p+1)(q+1)(r+1) \dots$ factors.

NOTE When N is a perfect square total number of distinct factors would be odd and when N is a non perfect square total number of distinct factors would be even.

Exp. 2) Find the sum of the factors of 72.

Solution Let us consider $N = 72$, which can be expressed as $72 = 2^3 \times 3^2$. Now, there are total 12 factors of 72.

$$\begin{aligned} &(2^0 3^0), (2^0 3^1), (2^0 3^2), (2^1 3^0), (2^1 3^1), (2^1 3^2), \\ &(2^2 3^0), (2^2 3^1), (2^2 3^2), (2^3 3^0), (2^3 3^1), (2^3 3^2) \end{aligned}$$

Therefore sum of these factors

$$\begin{aligned} &= (2^0 3^0) + (2^0 3^1) + (2^0 3^2) + (2^1 3^0) + (2^1 3^1) + (2^1 3^2) \\ &\quad + (2^2 3^0) + (2^2 3^1) + (2^2 3^2) + (2^3 3^0) + (2^3 3^1) + (2^3 3^2) \\ &= (2^0 + 2^1 + 2^2 + 2^3)(3^0 + 3^1 + 3^2) \\ &= \frac{(2^{3+1} - 1)}{2 - 1} \times \frac{(3^{2+1} - 1)}{3 - 1} \end{aligned}$$

Thus we can conclude that sum of all the factors of

$$N = \frac{(a^{p+1} - 1)}{(a - 1)} \times \frac{(b^{q+1} - 1)}{(b - 1)} \times \frac{(c^{r+1} - 1)}{(c - 1)} \times \dots$$

Exp. 3) Find the product of all the factors of 72.

Solution Let us consider $N = 72$, which can be expressed as $72 = 2^3 \times 3^2$

Now, there are total 12 factors of 72; namely 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72.

We can write these factors in pairs like

$$72 = 1 \times 72, 72 = 2 \times 36,$$

$$72 = 3 \times 24, 72 = 4 \times 18,$$

$$72 = 6 \times 12, 72 = 8 \times 9$$

Therefore, product of all the factors

$$= 72 \times 72 \times 72 \times 72 \times 72 \times 72 = 72^6$$

It implies that product of all the factors of

$$N = (N)^{\text{half of the total number of factor}}$$

NOTE When N is a perfect square, the number of factors would be odd. In that case express N as $(\sqrt{N})^2$ so the product of all the factors of $N = (\sqrt{N})^{\text{total number of factors}}$

Exp. 4) Find the total number of odd factors of 72.

Solution Let us consider $N = 72$, which can be expressed as $72 = 2^3 \times 3^2$

Now, there are total 12 factors of 72.

$$(2^0 3^0), (2^0 3^1), (2^0 3^2), (2^1 3^0), (2^1 3^1), (2^1 3^2), \\ (2^2 3^0), (2^2 3^1), (2^2 3^2), (2^3 3^0), (2^3 3^1), (2^3 3^2)$$

But if we remove those factors which involve any positive power of 2, that is $2^1, 2^2, 2^3$, we will have those factors only which are odd. Thus there are 3 odd factors in 72 namely 1, 3 and 9.

So we can conclude that if $N = 2^k a^p b^q c^r \dots$; where $a, b, c, \dots$ are odd prime numbers, then the number of odd factors will be obtained by disregarding (or ignoring) the even prime number. Therefore in this case total number of odd factors of $N = (p+1)(q+1)(r+1) \dots$

Exp. 5) Find the total number of even factors of 72.

Solution Let us consider $N = 72$, which can be expressed as $72 = 2^3 \times 3^2$

Now, there are total 12 factors of 72; namely 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72. Simply if we subtract the odd factors from total factors, we will have only even factors.

Thus number of even factors = total number of factors – total odd factors

Thus you can see 72 has 3 odd factors and 9 even factors.

Exp. 6) Find the total number of co-primes to 72, which are less than 72.

Solution Let us consider $N = 72$, which can be expressed as $72 = 2^3 \times 3^2$

Now, there are total 12 factors of 72; namely 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72. You know that since 2 is a prime factor of 72, so all the multiples of 2 (i.e., 2, 4, 6, 8, ..., 72) will be factors of 72. Simply there are $72/2 = 36$ such numbers. Again since 3 is a factor of 72, so all the multiples of 3 (i.e., 3, 6, 9, ..., 72) will be factors of 72. Simply there are $72/3 = 24$ such numbers.

It is also very clear that if any number less than or equal to 72 is multiple of either 2 or 3 cannot be co-prime to 72. So in order to get co-prime numbers of 72 which are less than 72,

we will simply ignore those numbers which are multiple of 2 or 3 but less than or equal to 72.

Number of multiples of $2 = \frac{72}{2} = 36$, so the number of numbers which are not the multiple of

$$2 = 72 - \frac{72}{2} = 72 \left(1 - \frac{1}{2}\right)$$

Number of multiples of $3 = \frac{72}{3} = 24$, so the number of numbers which are not the multiple of

$$3 = 72 - \frac{72}{3} = 72 \left(1 - \frac{1}{3}\right)$$

Therefore, number of numbers which are neither the multiple of 2 nor of 3

$$= 72 \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right)$$

It implies that the number of co-primes of 72 which are less than 72 $= 72 \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) = 24$

Therefore number of co-primes to N , which is less than

$$N = N \left(1 - \frac{1}{a}\right) \left(1 - \frac{1}{b}\right) \left(1 - \frac{1}{c}\right) \dots$$

Exp. 7) Find the sum of all the co-primes of 72, which are less than 72.

Solution Let us consider $N = 24$, which can be expressed as $24 = 2^3 \times 3$

Therefore number of co-primes to 24, which are less than

$$24 = 24 \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) = 8$$

Thus there are total 8 co-prime factors of 24; namely 1, 5, 7, 11, 13, 17, 19, 23. Now you can see the conjugate pairs of co-prime factors which are (1, 23), (5, 19), (7, 17), (11, 13).

It shows that sum of both the terms in each parenthesis is 24.

Thus the sum of all the co-prime factors of 24, not exceeding $24 = 24 \times 4$

It implies that the sum of all the co-primes of N which are less than $N = \frac{N}{2} \times \text{Number of co-primes to } N \text{ which are less than } N$.

Exp. 8) Find the total number of ways in which 6, 30, 210 and 60 can be written as a product of two co-primes.

Solution Let us consider $N = 6$, which can be expressed as $6 = 2 \times 3$.

Now, there are total 4 factors of 6; namely 1, 2, 3, 6.

Then 6 can be expressed as product of two co-prime factors in two ways: 1×6 and 2×3 ,

Again consider $N = 30$, which can be expressed as $30 = 2 \times 3 \times 5$

Now, there are total 8 factors of 30; namely 1, 2, 3, 5, 6, 10, 15, 30.

Then 30 can be expressed as product of two co-prime factors in four ways: $1 \times 30, 2 \times 15, 3 \times 10$ and 5×6 .

Similarly, if we consider $N = 210$, which can be expressed as $210 = 2 \times 3 \times 5 \times 7$

Now, there are total 16 factors of 210; namely 1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105, 210.

Then 210 can be expressed as product of two co-prime factors in eight ways:

$$1 \times 210, 2 \times 105, 3 \times 70, 5 \times 42, \\ 6 \times 35, 7 \times 30, 10 \times 21 \text{ and } 14 \times 15.$$

Once again, if we consider $N = 60$, which can be expressed as $60 = 2^2 \times 3 \times 5$. Now, there are total 12 factors of 60; namely 1, 2, 3, 4, 5, 6, 7, 10, 12, 15, 20, 30, 60. Then there will be 6 pairs: $1 \times 60, 2 \times 30, 3 \times 20, 4 \times 15, 5 \times 12$ and 6×10 . But 2×30 and 6×10 are not valid.

Then 60 can be expressed as product of two co-prime factors in four ways: $1 \times 60, 3 \times 20, 4 \times 15$ and 5×12 .

Let $N = a^p \times b^q \times c^r \times \dots$ and $N = L \times R$, where L and R are co-prime (or relatively prime) numbers.

Then a^p can be placed in two ways either at the place of L or R . Similarly b^q can be placed in two ways either at the place of L or R . Similarly c^r can be placed in two ways either at the place of L or R . And so on...

Therefore if there are n factors of N , the number of ways in which N can be expressed as product of two co-prime factors = $\underbrace{2 \times 2 \times \dots \times 2}_{n \text{ times}} = 2^n$

But since L and R are interchangeable, so the number of ways in which N can be expressed as product of two factors = $\frac{2^n}{2} = 2^{n-1}$

Exp. 9) Find the total number of ways in which 72 and 36 can be expressed as a product of two factors.

Solution Let us consider $N = 72$, which can be expressed as $72 = 2^3 \times 3^2$.

Now, there are total 12 factors of 72; namely 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72. Now you see that

$$72 = (1 \times 72), 72 = (2 \times 36), \\ 72 = (3 \times 24), 72 = (4 \times 18), \\ 72 = (6 \times 12)$$

It implies that total number of ways in which N can be expressed as a product of two factor

$$= \frac{\text{Total number of factors of } N}{2}$$

Let us consider $N = 36$, which can be expressed as $36 = 2^2 \times 3^2$

Now, there are total 9 factors of 36; namely 1, 2, 3, 4, 6, 9, 12, 18, 36.

Now you see that

$$36 = (1 \times 36); 36 = (2 \times 18) \\ 36 = (3 \times 12); 36 = (4 \times 9) \\ 36 = (6 \times 6)$$

Thus 36 can be expressed as a product of two distinct factors = $\frac{(9-1)}{2} = 4$

And 36 can be expressed as a product of two factors including the product of similar factors = $\frac{(9+1)}{2} = 5$

Therefore it implies that, when N is a perfect square number

(a) The total number of ways in which N can be expressed as a product of two 'distinct' factors = $\frac{(\text{Total number of factors of } N) - 1}{2}$

(b) The total number of ways in which N can be expressed as a product of two factors, including the 'similar' factors factor = $\frac{(\text{Total number of factors of } N) + 1}{2}$

Exp. 10) Find the total number of ordered pairs (x, y) such that LCM of x and y is 72.

Solution Let us consider 72 is the LCM of two numbers x and y $72 = 2^3 \times 3^2$

Then the possible pairs can be determined as follows.

The various ways to distribute 2^3 to (x, y) are

$$(2^3, 1), (2^3, 2), (2^3, 2^2),$$

$(2^3, 2^3), (2^2, 2^3), (2, 2^3), (1, 2^3)$. Thus it can be distributed in $2 \times 3 + 1 = 7$ ways.

Similarly, 3^2 can be distributed to (x, y) in $2 \times 2 + 1 = 5$ ways.

Therefore the total number of ways = $7 \times 5 = 35$

These pairs are as follows:

$$(72, 1), (72, 3), (72, 9), (24, 9), (8, 9), \\ (72, 2), (72, 6), (72, 18), (24, 18), (8, 18) \\ (72, 4), (72, 12), (72, 36), (24, 36), (8, 36) \\ (72, 8), (72, 24), (72, 72), (24, 72), (8, 72) \\ (36, 8), (36, 24), (36, 72), (12, 72), (4, 72) \\ (18, 8), (18, 24), (18, 72), (6, 72), (2, 72) \\ (9, 8), (9, 24), (9, 72), (3, 72), (1, 72)$$

Thus the total number of ordered pairs (x, y) such that LCM of x and y is $72 = (2 \times 3 + 1)(2 \times 2 + 1) = 35$

Thus the total number of ordered pairs (x, y) such that LCM of x and y is a composite number = $(2p + 1)(2q + 1)(2r + 1)$.

19.11. Geometrical Properties

(i) In a plane if there are n points of which no three are collinear, then

- The number of straight lines that can be formed by joining them = ${}^nC_2$
- The number of triangles that can be formed by joining them = ${}^nC_3$
- The number of quadrilaterals that can be formed by joining them = ${}^nC_4$
- The number of polygons with k sides that can be formed by joining them = ${}^nC_k$

- (e) The number of diagonals in a polygon of n sides
 $= {}^nC_2 - n$

(ii) In a plane if there are n points out of which m points are collinear, then

- (a) The number of straight lines that can be formed by joining them $= {}^nC_2 - {}^mC_2 + 1$
 (b) The number of triangles that can be formed by joining them $= {}^nC_3 - {}^mC_3$
 (c) The number of polygons with k sides that can be formed by joining them $= {}^nC_k - {}^mC_k$

(iii) If n points are given on the circumference of the circle, then

- (a) The number of straight lines that can be formed by joining them $= {}^nC_2$
 (b) The number of triangles that can be formed by joining them $= {}^nC_3$
 (c) The number of quadrilaterals that can be formed by joining them $= {}^nC_4$
 (d) The number of polygons with k sides that can be formed by joining them $= {}^nC_k$
 (e) The number of diagonals in a polygon of n -sides $= {}^nC_2 - n$

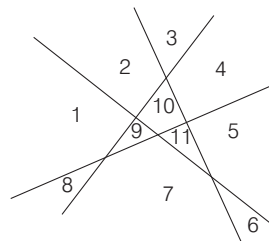
(iv) The number of regions in which a line/plane/space/sphere can be divided by n points/lines/circles/ellipses

Division by	Division of	Maximum Number of Parts / Regions
n Points	Line (1-Dimensional)	$\sum_{k=0}^1 {}^nC_k = {}^nC_0 + {}^nC_1$
n Lines	Plane (2-Dimensional)	$\sum_{k=0}^2 {}^nC_k = {}^nC_0 + {}^nC_1 + {}^nC_2$
n Planes	Solid (3-Dimensional)	$\sum_{k=0}^3 {}^nC_k = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3$
n Hyper Planes	Space (4-Dimensional)	$\sum_{k=0}^4 {}^nC_k = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4$

- (a) If n straight lines are drawn in the plane such that no two lines are parallel and no three lines are concurrent, then the maximum number of parts/regions into which these lines divide the plane is

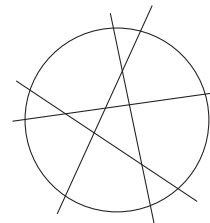
$${}^nC_0 + {}^nC_1 + {}^nC_2 = \frac{n(n+1)}{2} + 1$$

$$= \frac{n^2 + n + 2}{2}$$

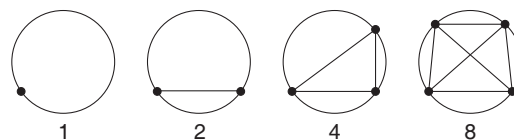


- (b) If n straight lines are intersecting a circle such that no two lines are parallel and no three lines are concurrent, the maximum number of parts/regions into which these lines divide the circle is

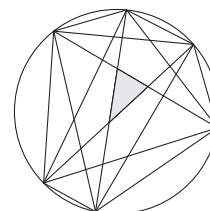
$${}^nC_0 + {}^nC_1 + {}^nC_2 = \frac{n(n+1)}{2} + 1 = \frac{n^2 + n + 2}{2}$$



- (c) If n points lie on the circumference of a circle and are connected by straight lines intersecting the circle such that no three lines are concurrent (that is no three lines ever pass through the same point), then the maximum number of parts/regions into which these lines divide the circle is ${}^nC_0 + {}^nC_2 + {}^nC_4$. The first few values are: 1, 2, 4, 8, 16, 31, 57, 99, 163, 256,



- (d) If n points are given on the circumference of a circle and the chords determined by them are drawn. If no three chords have a common point, number of triangles all of whose vertices lie inside the circle
 $= {}^nC_6 = \frac{n!}{6!(n-6)!}$



Every set of six points on the circumference of the circle can be paired in one and only one way such that the three lines joining pairs will form an admissible triangle.

Conversely, every admissible triangle has sides leading to six points on the circumference. Hence the number of admissible triangles is

$$= {}^nC_6 = \frac{n!}{6!(n-6)!}$$

- (e) If n circles are drawn in the plane such that no two of them are tangent, none of them lies entirely within or outside of another one and no three of them are concurrent, the maximum number of parts/regions into which these circles divide the plane is $n(n-1) + 2$

- (f) If n ellipses are drawn in the plane such that no two of them are tangent, none of them lies entirely within or outside of another one and no three of them are concurrent, the maximum number of parts/regions into which these ellipses divide the plane is $2n(n-1) + 2$

- (g) If there are n overlapping triangles, the maximum number of regions into which these triangles divide the plane is $3n^2 - 3n + 2$

- (h) If n planes are drawn in the space such that no four planes intersect at a single point and the intersection of any three planes are non-parallel lines and no two planes are parallel, the maximum number of spacial regions into which these planes divide the space is

$${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 = \frac{n^3 + 5n + 6}{6}$$

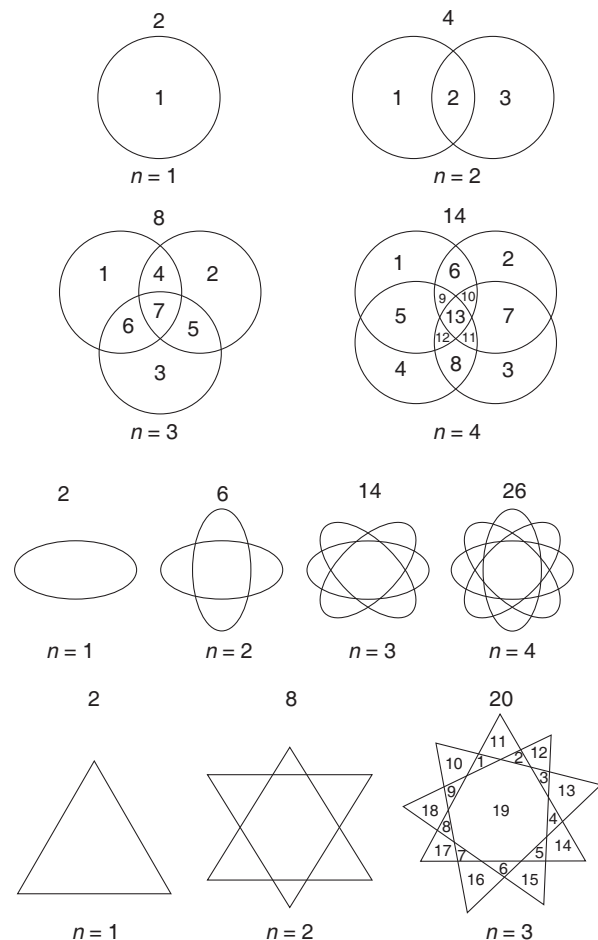
- (i) The maximum number of regions into which a 3-dimensional cube/sphere/cylinder can be partitioned by exactly n planes is

$${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 = \frac{n^3 + 5n + 6}{6}$$

- (j) A great circle is a circle drawn on a sphere is known as an 'equator'. That is, its center is also the centre of the sphere. There are n great circles on a sphere such that no three of which meet at any point, the number of regions into which these great circles divide the sphere is $n^2 - n + 2$

- (k) If n circles are intersecting each other, maximum number of points of intersection
- $$= 2 \times ({}^nC_2) = {}^nP_2 = n(n-1)$$

- (l) If n points in the plane are connected by straight lines in all the possible ways, and of these no two are coincident or parallel and no three of them are concurrent except at the given points, the number of points of intersection other than the given points of the lines so formed is
- $$\frac{1}{8} \times \frac{n!}{(n-4)!}$$



(v) Bounded and unbounded Regions

Since every new line added on the plane increases the number of unbounded regions by 2, so by using this logic we can determine the number of unbounded and bounded regions. As n th line creates n new regions and that of the n new regions introduced, $n-2$ of them must be new bounded regions. And the maximum number of bounded regions is the difference of total number of regions and unbounded regions.

Refer to the following table.

Number of lines n	Total regions	Total new regions added by n th line	Total unbounded regions	New unbounded regions added by n th line	Total bounded regions	Now bounded regions added by n th line
1	2	1	2	(1)	0	(0)
2	4	2	4	2	0	0
3	7	3	6	2	1	1
4	11	4	8	2	3	2
5	16	5	10	2	6	3
6	22	6	12	2	10	4
.....						
.....						
n	$\frac{n^2 + n + 2}{n}$	n	$2n$	2	$\frac{n^2 - 3n + 2}{2}$	$n - 2$

NOTE The figures, as shown in the table above, written in parentheses () indicate the exception to the pattern.

- Total number of regions may vary depending on the way each line is drawn. In order to have maximum number of regions no two lines should be parallel and no three lines should pass through any intersecting point.
- In the above table, the second column by default talks about MAXIMUM number of total regions.
- Total number of unbound regions varies as per NUMBER of lines drawn only, not on the WAY the lines are drawn on the plane.
- Total number of BOUNDED regions may vary depending on the way each line is drawn. In order to have maximum number of regions no two lines should be parallel and no three lines should pass through any intersecting point.
- In the above table, the sixth column by default talks about MAXIMUM number of total bounded regions.
- Each new line drawn on the plane increases minimum 1 new bounded region and maximum $n - 2$ bounded regions.
- Minimum number of bounded regions given by each n th line is $n - 2$.

(vi) Number of Squares/Rectangles/Quadrilaterals

- (a) Total number of squares in a square having n columns and n rows

$$= 1^2 + 2^2 + 3^2 + \dots + n^2 = \sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

- (b) Total number of rectangles in a square having n columns and n rows

$$= 1^3 + 2^3 + 3^3 + \dots + n^3 = \sum n^2 = \left(\frac{n(n+1)}{2} \right)^2$$

- (c) Total number of squares in a rectangle having m columns and n rows

$$= m \cdot n + (m-1)(n-1) + (m-2)(n-2) + \dots + 0$$

- (d) Total number of rectangles in a rectangle having m columns and n rows

$$= (1+2+3+\dots+m)(1+2+3+\dots+n)$$

- (e) Total number of quadrilaterals if m parallel lines intersect other n parallel lines $= {}^m C_2 \times {}^n C_2$

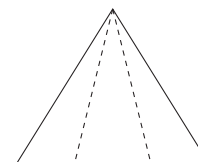
(vii) Number of Triangles in any Triangle

If you draw m lines from a vertex, these new lines will divide the side of the original triangle in $m + 1$ units (or parts).

- (a) When lines are drawn from only one vertex

If a side of a triangle is divided

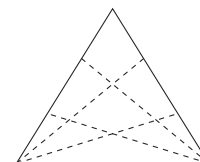
into n units, the total number of triangles $= \frac{n(n+1)}{2}$



NOTE In the above diagram, there are 6 triangles.

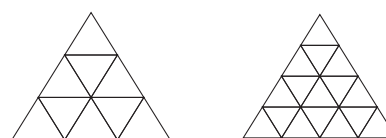
- (b) When lines are drawn from any two vertices

If each side of a triangle is divided into n units, the total number of triangles $= n^3$



NOTE In the above diagram, there are 27 triangles.

- (c) When the original triangle is an equilateral triangle and lines are drawn parallel to the sides of original triangle, as shown in the following figures.



$$= \frac{n(n+1)(n+2)}{6}$$
$$= \begin{cases} \frac{1}{24} (n) (n+2) (2n-1), & \text{when } n \text{ is even} \\ \frac{1}{24} (n^2-1) (2n+3), & \text{when } n \text{ is odd} \end{cases}$$

- In the above diagram, figure (i) has 13 triangles and figure (ii) has 27 triangles.
- In the above diagram, figure (i) has 10 upward pointing triangles and figure (ii) has 20 upward pointing triangles.
- In the above diagram, figure (i) has 3 downward pointing triangles and figure (ii) has 7 downward pointing triangles.

n	k										Total Triangles
	1	2	3	4	5	6	7	8	9	10	
1	1										1
2	3	1									4
3	6	3	1								10
4	10	6	3	1							20
5	15	10	6	3	1						35
6	21	15	10	6	3	1					56
7	28	21	15	10	6	3	1				84
8	36	28	21	15	10	6	3	1			120
9	45	36	28	21	15	10	6	3	1		165
10	55	45	36	28	21	15	10	6	3	1	220

[illegible]

(viii) Number of Triangles, Diagonals and Intersections in a Regular Polygon

A. Number of triangles formed by the vertices of a regular polygon of n sides .

- (a) Number of triangles formed by joining the 3 vertices of n - sided polygon

$$N = {}^nC_3 = \frac{n(n-1)(n-2)}{6}; \forall n \geq 3$$

- (b) Number of triangles having one side common with that of the polygon $N_1 = n(n-4); \forall n \geq 3$
 (c) Number of triangles having two sides common with that of the polygon $N_2 = n; \forall n \geq 3$
 (d) Number of triangles so that at least one side of the triangle coincides with the side of the polygon

$$= (N_1 + N_2) = n(n-3)$$

- (e) Number of triangles having no side common with that of the polygon

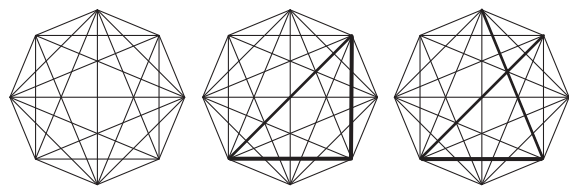
$$N_0 = N - (N_1 + N_2) = \frac{n(n-4)(n-5)}{6}; \forall n \geq 6$$

B. Number of triangles formed by intersecting diagonals of a regular polygon of n - sides.

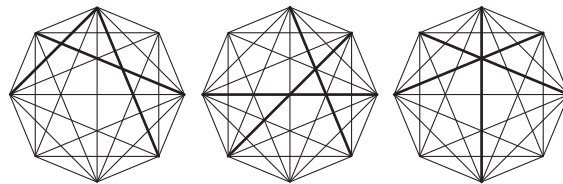
Let us assume $N(d_k)$ indicates the number of triangles with k number of diagonal endpoints

Number of sides in a polygon	Number of Triangles				
n	$N(d_3)$	$N(d_4)$	$N(d_5)$	$N(d_6)$	Total
3	1				1
4	4	4			8
5	10	20	5		35
6	20	60	30		110
7	35	140	105	7	287
8	56	280	280	16	632
9	84	504	630	84	1302
10	120	840	1260	180	2400

Let us consider an example of a regular octagon (8-sided polygon), as shown below. By connecting all the vertices it produces ${}^8C_2 = 28$ lines and 20 ($= 28 - 8$) diagonals.



A Regular Octagon 3 Diagonal Endpoints 4 Diagonal Endpoints



5 Diagonal Endpoints 6 Diagonal Endpoints 6 Diagonal Endpoints with False Triangle

Number of triangles with 3 Diagonal Endpoints $= {}^nC_3$

Number of triangles with 4 Diagonal Endpoints $= 4({}^nC_4)$

Number of triangles with 5 Diagonal Endpoints $= 5({}^nC_5)$

Number of triangles with 6 Diagonal Endpoints

$$= {}^nC_6 - [a_3(n)({}^3C_3) + a_4(n)({}^4C_3) + a_5(n)({}^5C_3) + a_6(n)({}^6C_3) + a_7(n)({}^7C_3) + \delta_2(n)({}^{n/2}C_3)]$$

Therefore, the number of triangles generated by intersecting diagonals of an n -sided regular polygon is

$${}^nC_3 + 4({}^nC_4) + 5({}^nC_5) + {}^nC_6 - [a_3(n)({}^3C_3) + a_4(n)({}^4C_3) + a_5(n)({}^5C_3) + a_6(n)({}^6C_3) + a_7(n)({}^7C_3) + \delta_2(n)({}^{n/2}C_3)]$$

For example, if we consider an 8-sided regular polygon, the total number of triangles

$$\begin{aligned} &= {}^8C_3 + 4({}^8C_4) + 5({}^8C_5) + {}^8C_6 - [8(1) + 0({}^4C_3) + 0({}^5C_3) + 0({}^6C_3) + 0({}^7C_3) + 1({}^4C_3)] \\ &= 56 + 280 + 280 + 28 - (8 + 4) \\ &= 56 + 280 + 280 + 16 = 632 \end{aligned}$$

Hint The expression $a_3(n)({}^3C_3) + a_4(n)({}^4C_3) + a_5(n)({}^5C_3) + a_6(n)({}^6C_3) + a_7(n)({}^7C_3) + \delta_2(n)({}^{n/2}C_3)$ indicates the number of false triangles. It happens when more than two diagonals intersect at any point inside the regular polygon.

- I. Here $a_m(n)$ denotes the number of intersection (interior) points other than the centre when m diagonals intersect each other.
- II. Here ${}^kC_3$ denotes the number of false triangles corresponding to a common point created by the intersection of k line segments for every $k > 2$.
- III. Here $\delta_2(n)$ denotes the contribution of the centre-point for even value of n . When n is odd, there is no centre point inside the regular polygon.

$$\text{Therefore, } \delta_2(n) = \begin{cases} 0, & \text{if } n \text{ is odd} \\ 1, & \text{otherwise} \end{cases}$$

NOTE By observation, the number of diagonals m is 2, 3, 4, 5, 6 or 7. In particular, it is impossible to have 8 or more diagonals of a regular n -gon meeting at a point other than the centre.

The maximum number of diagonals of a regular n -gon ($n > 4$) that meet at a point other than the centre is $m = 2$, if n is odd or n is 6

$$\begin{aligned} m &= 3, \text{ if } n \text{ is even but not divisible by 6,} \\ m &= 4, \text{ if } n \text{ is 12} \\ m &= 5, \text{ if } n \text{ is divisible by 6 but not 30, and,} \\ m &= 7, \text{ if } n \text{ is divisible by 30.} \end{aligned}$$

The number of intersection points formed inside a regular n -gon by its diagonals.

n	m	$a_m(n)$ [Intersection Points Other Than Centre]						Centre Point	Total Intersection Points	
		$a_2(n)$	$a_3(n)$	$a_4(n)$	$a_5(n)$	$a_6(n)$	$a_7(n)$		With-out Centre	With Centre
5	2	5							5	5
6	2	12						1	12	13
7	2	35							35	35
8	3	40	8					1	48	49
9	2	126							126	126
10	3	140	20					1	160	161
11	2	330							330	330
12	4	228	60	12				1	300	301
...	...	...							...	...
15	2	1365							1365	1365
...	...	...							...	...
18	5	1512	216	54	54			1	1836	1837
...	...	...							...	...
30	7	13800	2250	420	180	120	30	1	16800	16801

For a generic polygon, maximum number of intersection points formed inside a regular n -gon by its diagonals is ${}^nC_4$, because every four vertices would act as the end points of a unique pair of intersecting diagonals. But it can be less, because in a regular n -gon it may happen that three or more diagonals meet at an interior point, and then some of the ${}^nC_4$ intersection points will coincide.

Also, as we know that no three diagonals meet when n is odd, it implies that for odd n the number of intersection points

$$a_m(n) = {}^nC_4.$$

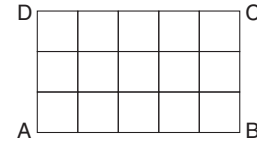
Here ${}^kC_3$ denotes the number of false triangles corresponding to a common point created by the intersection of k line segments for every $k > 2$.

Since, $k > 2$, it implies that this term $a_2(n)({}^nC_3)$ makes no sense, as you cannot select 3 things out of 2 things, and that's why it has been discarded from the formula.

Here $\delta_2(n)$ denotes the contribution of the centre-point for even value of n . Therefore, $\delta_2(n) = \begin{cases} 0, & \text{if } n \text{ is odd} \\ 1, & \text{otherwise} \end{cases}$.

Paths and Grids When m horizontal lines intersect the n vertical lines then a grid or network is formed.

In the following figure, a grid of 3×5 dimensions is shown, in which there are 4 horizontal lines and 6 vertical lines intersect each other. Using these lines as roads you can go from one corner to another one.



In mathematics, we are usually interested in determining the shortest paths and maximum number of paths available from one point to another one.

Suppose you have to go from A to C using the shortest paths, without backtracking, then how many shortest paths are there between A and C ?

If you look closely, then you will realize that while going from A to C you can move in only 2 directions – either rightward or upward. In other words, to reach from A to C , you have to use the combination of rightward and upward moves.

As you see in the diagram there are 5 RIGHT steps (between A and B) and 3 UP steps (between B and C). So it's obvious that to reach from A to C , you have to take total $8 (= 5 + 3)$ steps. Now, it's your choice how you choose these steps. Suppose if you want to go from A to C , then you can choose your paths as given below.

$RRRRRUUU$ or $RRRRURUU$ or $RRRRUURU$ or ... or $UUURRRRR$. It implies that you have to find out that in how many ways you can select 5 RIGHT steps and 3 UP steps out of total 8 step. And this can be done in

$${}^8C_5 \times {}^{8-5}C_3 = {}^8C_5 \times {}^3C_3 = \frac{8!}{5! \times 3!} \text{ ways}$$

In a sense, it shows that in how many ways you can arrange your 8 steps. Since out of 8 steps 5 steps are identical (RIGHTWARD) and 3 steps are identical (UPWARD).

$$\frac{8!}{5! \times 3!}$$

So all the 8 steps can be arranged in $8!(5! \times 3!)$ ways.

In general, if there is a grid of m horizontal lines and n vertical lines, there will be $(m-1)$ steps in the horizontal direction and $(n-1)$ steps in the vertical direction, thus the total number of steps will be $(m-1) + (n-1) = (m+n) - 2$.

Therefore, the total number of shortest paths that can be taken to reach from one corner to the opposite corner $= \frac{(m+n-2)!}{(m-1)!(n-1)!} = {}^{m+n-2}C_{m-1} \times {}^{n-1}C_{n-1}$.

Introductory Exercise 19.10

Directions (for Q. Nos. 1 and 2) : Answer these questions based on the following information.

There are four non-collinear points on a plane.

- Find the maximum number of straight lines formed by joining the four points.
(a) 3 (b) 2 (c) 1 (d) 6
- Find the maximum number of triangles formed by joining the four points.
(a) 4 (b) 2 (c) 1 (d) 6

Directions (for Q. Nos. 3 to 6) : Answer these questions based on the following information.

There are ten points on a plane, such that no three of them are collinear.

- Find the maximum number of straight lines formed by joining the ten points.
(a) 90 (b) 30 (c) 45 (d) 20
- Find the maximum number of triangles formed by joining the ten points.
(a) 100 (b) 120 (c) 90 (d) 60
- Find the maximum number of hexagons formed by joining the ten points.
(a) 210 (b) 120 (c) 240 (d) 66
- Find the maximum number of diagonals in a decagon formed by joining the ten points.
(a) 30 (b) 45 (c) 35 (d) 20

Directions (for Q. Nos. 7 to 10) : Answer these questions based on the following information.

There are fifteen points on the circumference of a circle.

- Find the maximum number of straight lines formed by joining the fifteen points.
(a) 105 (b) 210 (c) 15 (d) 60
- Find the maximum number of triangles formed by joining the fifteen points.
(a) 555 (b) 455 (c) 660 (d) 560
(e) 525
- Find the maximum number of quadrilaterals formed by joining the fifteen points.
(a) 365 (b) 1035
(c) 1155 (d) 1365
- Find the maximum number of octagons formed by joining the fifteen points.
(a) 6543 (b) 2345
(c) 7275 (d) 6435

Directions (for Q. Nos. 11 to 14) : Answer these questions based on the following information.

There are twelve points on a plane, out of which four are collinear points.

- Find the maximum number of straight lines formed by joining the twelve points.
(a) 33 (b) 60 (c) 62 (d) 61
- Find the maximum number of triangles formed by joining the twelve points.
(a) 215 (b) 216 (c) 220 (d) 196
- Find the maximum number of quadrilaterals formed by joining the twelve points.
(a) 495 (b) 491 (c) 494 (d) 400
- Find the maximum number of hexagons formed by joining the twelve points.
(a) 900 (b) 924 (c) 906 (d) 609

Directions (for Q. Nos. 15 to 18) : Answer these questions based on the following information.

There are n points on a plane, out of which m are collinear points, such that $m < n$

- Find the maximum number of straight lines formed by joining them.
(a) ${}^{n-m}C_2 + 1$ (b) ${}^nC_2 - {}^mC_2 - 1$
(c) ${}^{n-m}C_2 - 1$ (d) ${}^nC_2 - {}^mC_2 + 1$
- Find the maximum number of triangles formed by joining them.
(a) $n! - m!$ (b) ${}^mC_3 - {}^nC_3$
(c) ${}^{n-m}C_3 - {}^mC_3$ (d) ${}^nC_3 - {}^mC_3$
- Find the maximum number of quadrilaterals formed by joining them.
(a) ${}^mC_4 - {}^nC_4$ (b) ${}^nC_4 + {}^mC_4$
(c) ${}^nC_4 - {}^mC_4$ (d) ${}^nC_4 - {}^mC_4 + 1$
- Find the maximum number of hexagons formed by joining them.
(a) ${}^nC_6 - {}^mC_6$ (b) ${}^nC_6 - {}^mC_6 + 6$
(c) ${}^nC_6 + {}^mC_6$ (d) ${}^nC_6 + {}^mC_6 - 6$

Directions (for Q. Nos. 19 to 22) : Solve the following problems independently of each other.

- Find the number of diagonals in a heptagon.
(a) 14 (b) 21 (c) 12 (d) 16
- Find the number of diagonals in the n -sided polygon.
(a) ${}^nC_2 - n$ (b) ${}^nC_2$
(c) ${}^nC_2 + n$ (d) $\frac{n(n-3)}{2}$

21. A polygon has 54 diagonals. Find the number of its sides.
 (a) 18 (b) 12
 (c) 14 (d) 9
22. Find the value of m , if there are maximum 28 triangles formed by joining total 9 points on a plane, out of which m points are collinear.
 (a) 3 (b) 4
 (c) 1 (d) 8

Directions (for Q. Nos. 23 and 24) : Answer these questions based on the following information.

Two parallel lines each have a number of distinct points marked on them. On one line there are two points P and Q . On the other line there are eight points.

23. Find the maximum number of different triangles which could be formed having three of the ten points as their vertices.
 (a) 3 (b) 2 (c) 64 (d) 6
24. Find the maximum number of triangles which must have P as their vertex.
 (a) 3 (b) 2 (c) 36 (d) 6

Directions (for Q. Nos. 25 to 27) : Solve the following problems independently of each other.

25. If 20 straight lines be drawn in a plane such that no two of them being parallel and no three of them being concurrent, maximum how many points of intersection will there be?
 (a) 95 (b) 380
 (c) 400 (d) 190
26. If m parallel lines in a plane are intersected by a family of n parallel lines, find the maximum number of parallelograms thus formed.
 (a) m^n (b) $(m+1)(n+1)$
 (c) $\frac{(m-n)}{n!}$ (d) $\frac{mn(m-1)(n-1)}{4}$
27. In a plane there are 37 straight lines, of which 13 pass through the point A and 11 pass through the point B . Besides, no three lines pass through one point, no line passes through both the points A and B , and no two lines are parallel. Find the maximum number of points of intersection of the straight lines.
 (a) 535 (b) 525 (c) 235 (d) 355

Directions (for Q. Nos. 28 to 31) : Answer these questions independently of each other.

28. If 6 straight lines be drawn in a plane such that no two of them being parallel and no three of them being concurrent, the maximum number of regions into which this plane will be divided
 (a) 30 (b) 22 (c) 18 (d) 15

29. If a plane is divided into 22 distinct regions, find the minimum number of coplanar lines which divide this plane in such a way that neither two lines are parallel nor any three lines are concurrent.
 (a) 3 (b) 4 (c) 8 (d) 6
30. The maximum number of disjoint regions that can be formed in the plane by n simple closed curves which pair-wise meet in at most two points is
 (a) $n^3 - n + 2$ (b) $n^2 - n + 1$
 (c) $n^2 - n + 2$ (d) $n(n-1)$
31. If there are 6 points on the circumference of the circular sheet, find the maximum possible number of triangular regions that can be cut out of the circular sheet such that none of the vertices of these triangles are the points of the circumference.
 (a) 3 (b) 2 (c) 1 (d) 15

Directions (for Q. Nos. 32 to 35) : Answer these questions based on the following information.

Once, Amey and his friends decided to celebrate their success after they secured their admissions into various IIMs (Indian Institutes of Management). They ordered a thin crust cheese pizza from Pizza Hut and they told Pizza Hut to deliver it without cutting into pieces. The pizza was large enough to satiate their cravings. Once they receive the pizza, they were so excited that they didn't mind cutting the pizza into any shape and size. However, while cutting the pizza they cut it from edge to edge, that is they started from any point on the circumference and didn't lift the knife until it reaches another point on the circumference. It's like they cut it along various chords of a circle.

32. At most, how many pieces would they get if they made 8 cuts in this pizza?
 (a) 37 (b) 32 (c) 58 (d) 36
33. If they wanted to have 20 pieces of this pizza, minimum how many cuts would be required?
 (a) 4 (b) 5 (c) 10 (d) 6
34. Amey had just made 7 cuts making maximum possible number of pieces such that everyone present there could have had one piece each. But before anybody could have picked up a piece of it few more friends arrived at the party. Thus to serve the new guests at his party, he made one more cut keeping in mind that each new guest would get exactly one piece of pizza. What's the maximum number of new guests (friends) that showed up at the party?
 (a) 8 (b) 7 (c) 9 (d) 6
35. If Amey marked 6 points on the circumference of the pizza and then the pizza was cut into pieces such that all the cuts were straight and must passed through these points on the circumference, at most how many pieces they would have got out of this pizza?
 (a) 99 (b) 69 (c) 66 (d) 25

Directions (for Q. Nos. 36 to 38) : Answer these questions based on the following information.

Actually few of his friends love cheese so much so that they don't want to eat the pieces which have less cheese on it as they find it tasteless. For them, size of the piece does not matter as long as it has cheese uniformly spread on it. The problem is that there is no cheese spread on the edge (or circumference) of the pizza in order to avoid any untoward incident of the cheese falling on the clothes while eating. A piece of pizza that contains some part of circumference (or edge) has less or no cheese spread on it is called as Edger and another piece of pizza that is from the middle part of pizza which has proper cheese spread on it is called as Cheeser.

36. If there are 15 such guys and girls who don't eat the Edger and each such person must get a Cheeser one to eat, minimum how many cuts are required to get the desired pieces?
(a) 6 (b) 7 (c) 8 (d) 9
37. If there are 25 such guys and girls who don't want to eat Edger one and if only 8 cuttings are made to cut this pizza, minimum how many such persons have to compromise with an Edger piece of pizza?
(a) 0 (b) 1 (c) 2 (d) 4
38. Minimum how many cuts are required if they need more number of Cheeser than that of Edger pieces?
(a) 3 (b) 6 (c) 9 (d) 7

Directions (for Q. Nos. 39 and 40) : Answer these questions based on the following information.

A Venn diagram has two circles, which cuts the plane into 4 regions and another Venn diagram has 3 circles, which cuts the plane into 8 regions.

39. Find the maximum possible number of regions on a plane that can be created by a Venn diagram with 6 circles.
(a) 28 (b) 32 (c) 16 (d) 36
40. If you have ovals instead of circles, find the maximum possible number of regions on a plane that can be created by a Venn diagram with 5 ovals.
(a) 34 (b) 42 (c) 18 (d) 26

Directions (for Q. Nos. 41 and 42) : Answer these questions based on the following information.

In a class of origami, in Japan, my teacher Yamamotoyama gave me a circular paper, a knife and a scissors. She asked me to cut the paper in order to make triangles out of the circular sheet. She laid down some rules which were mandatory to follow for every origami student.

- (i) A sheet of paper has n points marked on its circumference.
(ii) One must cut through the marked points only.

- (iii) Any cut must last from one marked point to another marked point.
(iv) One has to cut the paper as many times as the number of chords are possible with n points on the circumference of a circle.
(v) One cannot displace the pieces until all the possible cuts are made.

41. Find the maximum possible number of triangles which do not have any of its vertices out of the n marked points.
(a) 0 (b) 3 (c) ${}^nC_3$ (d) ${}^nC_6$
42. If $n = 6$, find the maximum possible number of triangles that can be cut out of the circular sheet such that none of the vertices of these triangles are the points of the circumference.
(a) 3 (b) 6
(c) 1 (d) 2

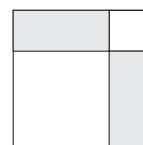
Directions (for Q. Nos. 43 and 44) : Answer these two questions based on the following information.

Khayamati is not just a foodie and extravagant but also a party animal. On her birthday her friends got an elephant size cake that was supposed to be distributed among all her friends. The cake was cylindrical in shape. To cut this cake they got a very large knife which was able to cross through the whole cake from any direction or angle. That is a single cut would be enough to make two pieces of the whole cake. By the way, they made n cuts to the cake from any arbitrary direction they could make. It was notable that until all the n cuts were done no one removed even a single piece.

43. If $n = 6$, find the maximum possible number of pieces that can be sliced out of that cake.
(a) 32 (b) 62 (c) 42 (d) 22
44. After cutting the cake if they got total 300 pieces, find the minimum possible number of cuts required.
(a) 13 (b) 17
(c) 11 (d) 12

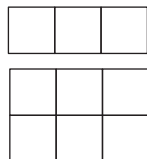
Directions (for Q. Nos. 45 to 60) : Solve these problems independently of each other.

45. Find the total number of squares and total number of rectangles, respectively, in the given figure. The main figure itself is a square.



- (a) 2 and 4 (b) 2 and 2
(c) 3 and 4 (d) 3 and 9

46. Find the total number of squares and total number of rectangles in the following figure. The main figure itself is a square.



- (a) 10 and 6 (b) 10 and 16
(c) 14 and 36 (d) 14 and 22
47. Find the total number of squares and total number of rectangles in the following figure.

The smallest unit (or cell) in this figure is a square.



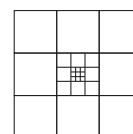
- (a) 10 and 20 (b) 10 and 16
(c) 14 and 45 (d) 14 and 35
48. Find the total number of squares in a square having 10 rows and 10 columns.
- (a) 385 (b) 360 (c) 100 (d) 275
49. Find the total number of rectangles in a square having 10 rows and 10 columns.
- (a) 3225 (b) 1625 (c) 1044 (d) 3025
50. Find the total number of squares in a rectangle having 8 rows and 15 columns.
- (a) 345 (b) 456 (c) 567 (d) 606
51. Find the total number of rectangles in a square having 8 rows and 15 columns.
- (a) 4321 (b) 6320 (c) 4320 (d) 3456
52. Find the total number of quadrilaterals in a square having 10 rows and 10 columns.
- (a) 3025 (b) 7744 (c) 2401 (d) 2601
53. Find the total number of quadrilaterals in a rectangle having 8 rows and 15 columns.
- (a) 4321 (b) 4320 (c) 1234 (d) 5440
54. Find the total number of quadrilaterals when 8 parallel lines intersect another set of 15 parallel lines.
- (a) 3636 (b) 2940 (c) 1990 (d) 3654

55. Find the total number of rectangles when 8 vertical lines intersect 15 horizontal lines such that all the vertical lines are parallel and equidistant to each other, and all the horizontal lines are also parallel and equidistant to each other.
- (a) 3996 (b) 6040 (c) 2940 (d) 6734

56. Find the total number of squares when 8 vertical lines intersect 15 horizontal lines such that all the vertical lines are parallel and equidistant to each other, and all the horizontal lines are also parallel and equidistant to each other.
- (a) 336 (b) 456 (c) 1096 (d) 376

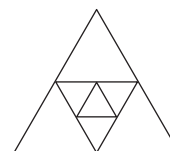
57. How many total squares are there in a standard chess board (or check board)?
- (a) 65 (b) 204 (c) 256 (d) 206
58. How many total rectangles are there in a standard chess board (or check board)?
- (a) 1296 (b) 512 (c) 0 (d) 2025

59. Find the total number of squares in the following diagram, where the main figure (i.e., the largest one) itself is a square.



- (a) 42 (b) 62 (c) 27 (d) 40

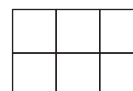
60. Find the total number of triangles in the given diagram.



- (a) 3 (b) 27 (c) 9 (d) 8

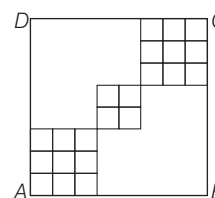
Directions (for Q. Nos. 61 to 66) : Solve the following problems independently of each other.

61. Find the total number of the shortest routes from one corner to its opposite corner in the given diagram.



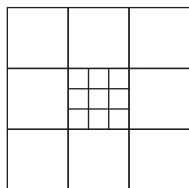
- (a) 10 (b) 15 (c) 6 (d) 9

62. Find the total number of the shortest routes from A to C in the given diagram.



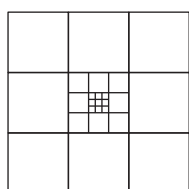
- (a) 4200 (b) 2402 (c) 2400 (d) 4800

63. Find the total number of the shortest routes from one corner to its opposite corner in the following diagram.



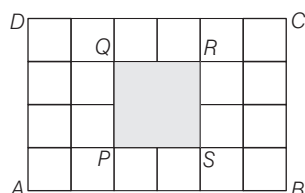
- (a) 42 (b) 84 (c) 92 (d) 40

64. Find the total number of the shortest routes from one corner to its opposite corner in the following diagram.



- (a) 800 (b) 400 (c) 388 (d) 380

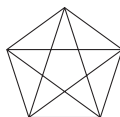
65. Find the total number of the shortest paths from A to C in the following diagram.



In this diagram, PQRS is a military area so one cannot enter inside the premises.

However, one may go along PQ, QR, PS and SR.

- (a) 100 (b) 110
(c) 168 (d) 204
66. Find the total number of triangles in a pentagon when all its vertices are joined with each other.



- (a) 10 (b) 35 (c) 25 (d) 20

Directions (for Q. Nos. 67 to 70): Solve the following problems based on the following information.

An industrial corridor is laid out along the Special Economic Zone (SEZ), which is in the shape of a regular polygon. Every vertex of the SEZ has exactly one manufacturing plant.

A particular road connects any two plants. No single road connects directly more than two plants. Engineers have laid down all the possible roads connecting all the plants with each other. Each road is designed to offer the shortest distance between any two plants.

Also, engineers have laid down all the possible rail-loops. Each loop connects any three plants. Every loop is designed along the shortest possible distance between any two plants connected through that loop.

Wherever any two or more than two roads intersect inside the SEZ (polygon), exactly one warehouse is to be built there.

Wherever any three or more than three roads intersect inside the SEZ (polygon), exactly one government office is to be built there.

Wherever any five or more than five roads intersect inside the SEZ (polygon), exactly one airport is to be built there.

67. Maximum how many rail-loops are there if the SEZ has 16 plants?
(a) 480 (b) 256
(c) 560 (d) none of these
68. Maximum how many warehouses are there if the SEZ has 25 plants?
(a) 7220 (b) 12650
(c) 14400 (d) none of these
69. Maximum how many government offices are there if the SEZ has 49 plants?
(a) 0
(b) 7
(c) 50
(d) none of the above
70. Maximum how many airports are there if the SEZ has 32 plants?
(a) 56 (b) 108
(c) 8648 (d) none of these

CAT-Test

Questions Helping you bell the CAT

LEVEL 01 > BASIC LEVEL EXERCISE

- 1 A letter lock consists of 4 rings, each ring contains 9 non-zero digits. This lock can be opened by setting a 4 digit code with the proper combination of each of the 4 rings. Maximum how many codes can be formed to open the lock?
 (a) 4^9 (b) ${}^9P_4$
 (c) 9^4 (d) none of these
- 2 In the previous question, if the lock opens in only one arrangement of 4 digits, how many unsuccessful attempts are possible in which lock cannot be opened?
 (a) $4^9 - 1$ (b) $9^4 - 1$
 (c) ${}^9C_4$ (d) none of these

Directions (for Q. Nos. 3 to 7) : Answer these questions based on the following information.

A group consists of four straight couples. That means each couple is having a male and a female.

- 3 In how many ways could they be arranged in a straight line such that the men and the women occupy alternate positions?
 (a) 1152 (b) 1278
 (c) 1296 (d) none of these
- 4 In how many ways can they be arranged in a straight line such that no two men were sitting together?
 (a) 1242 (b) 1440 (c) 3880 (d) 2880
- 5 In how many ways can they be seated such that each husband sits with his own wife?
 (a) 384 (b) 275
 (c) 184 (d) none of these
- 6 In how many ways can they be seated around a circular table such that the men and women occupy the alternate positions?
 (a) 288 (b) 144 (c) 72 (d) 720
- 7 In how many ways can they be seated on the single bench such that all the men sit together and all the women sit together?
 (a) 1512 (b) 1296
 (c) 1152 (d) none of these
- 8 How many different words can be made using the letters of the word 'HALLUCINATION' if all consonants are together?
 (a) 129780 (b) 1587600
 (c) 35600 (d) none of these
- 9 If all vowels occupy odd places, how many words can be formed from the letters of the word HALLUCINATION?
 (a) 129650 (b) 1587600
 (c) 78500 (d) none of these
- 10 In how many ways can the letters of the word SUCCESSFUL be arranged?
 (a) 1215700 (b) 1251600
 (c) 151200 (d) none of these
- 11 If all S's come together, then in how many ways the letters of the word SUCCESSFUL be arranged?
 (a) 10080 (b) 40080
 (c) 2378 (d) none of these
- 12 If all C's occur together and all U's also occur together, then how many arrangements are possible of the word SUCCESSFUL?
 (a) 5745 (b) 2760 (c) 6720 (d) 5432
- 13 What is the sum of all the 4 digit numbers which can be formed with the digits 1, 2, 3, 4 without repetition?
 (a) 15560 (b) 87660 (c) 45600 (d) 66660
- 14 What is the sum of all 5 digit numbers which can be formed with the digits 0, 1, 2, 3, 4 without repetition?
 (a) 2599980 (b) 235500 (c) 923580 (d) 765432
- 15 Which one of the following polygon has as many diagonals as the number of sides in it?
 (a) 4 (b) 6 (c) 5 (d) 7
- 16 In how many ways can the letters of the word PROPORTION be arranged by taking 4 letters at a time?
 (a) 123 (b) 758 (c) 658 (d) 578
- 17 In the previous question (no. 16) how many words can be formed without changing the relative positions of the vowels and consonants?
 (a) 217 (b) 720 (c) 920 (d) 1040
- 18 There are 16 executives including two brothers, Lehman and Mckinsey. In how many ways can they be arranged around the circular table if the two brothers cannot be seated together?
 (a) $(14!) \cdot 13$ (b) ${}^{14}P_3$
 (c) $\frac{14!}{3!}$ (d) none of these
- 19 In the previous question how many arrangements are possible if there is exactly one executive between these two brothers?
 (a) $(14!)^2$ (b) $(14!)$
 (c) $2 \cdot (14!)$ (d) none of these

- 20** In how many different ways can 6 different balls be distributed to 4 different boxes, when each box can hold any number of ball?
(a) 2048 (b) 1296
(c) $(24)^2$ (d) 4096
- 21** In how many different ways can 6 identical balls be distributed to 4 different boxes, when each box can have any number of balls?
(a) 12 (b) 18 (c) 84 (d) 88
- 22** If $a + b + c = 21$, what is the total number of non-negative integral solutions?
(a) 123 (b) 253 (c) 321 (d) 231
- 23** If $a + b + c = 21$ what is the total number of positive integral solutions?
(a) 109 (b) 190 (c) 901 (d) 910
- 24** What is the total number of ways of selecting atleast one object from 2 sets of 12 different objects, each set contains 6 objects?
(a) 4096 (b) 4095
(c) 2048 (d) none of these
- 25** What is the total number of ways of selecting atleast one object from 2 different sets, each set containing 6 identical objects?
(a) 24 (b) 48 (c) 76 (d) 96
- 26** What is the total number of ways of selecting atleast one item from each of the two sets containing 6 different items each?
(a) 2856 (b) 3969
(c) 480 (d) none of these
- 27** What is the total number of ways of selecting atleast one item from each of the two sets containing 6 identical items each?
(a) 16 (b) 36 (c) 32 (d) 18
- 28** A girl has to climb 12 steps. She climbs in either a single step or 2 steps simultaneously. In how many ways can she do it?
(a) 233 (b) 223 (c) 322 (d) 232
- 29** In how many ways can 8 identical apples be divided among 3 sisters?
(a) 25 (b) 65 (c) 45 (d) 24
- 30** In how many ways can 100 soldiers be divided into 4 squads of 10, 20, 30, 40 respectively?
(a) 1700 (b) 18!
(c) 190 (d) none of these
- 31** What is the total number of 4 digit numbers that can be formed using the digits 0, 1, 2, 3, 4, 5 without repetition, such that the number is divisible by 9?
(a) 36 (b) 28
(c) 15 (d) 18
- 32** How many 5 digit numbers contain exactly two 7's in them?
(a) 4268 (b) 6804 (c) 2340 (d) 1269
- 33** In how many ways can 15 billiard balls be arranged in a row if 3 are red, 4 are white and 8 are black?
(a) 12 (b) 18
(c) 96 (d) none of these
- 34** Seven delegates are to address a meeting. If a particular speaker is to speak before another particular speaker, find the number of ways in which this can be arranged.
(a) 1220 (b) 2520 (c) 3250 (d) 7826
- 35** How many 7 digit telephone numbers can be formed from the digits 0, 1, 2, ..., 9, where each telephone number begins with digit 2?
(a) 10^6 (b) 6^{10} (c) 10! (d) $^{10}P_6$
- 36** A man has 4 sons. There are 6 proposals of marriage for his sons. In how many ways can they select a proposal for their marriages, such that none of them marry with more than one girl?
(a) 180 (b) 270
(c) 360 (d) none of these
- 37** In how many ways can 4 books be selected out of 16 books on different subjects?
(a) 1208 (b) 1820
(c) 1296 (d) 1860
- 38** In how many ways can 4 books be arranged out of 16 books on different subjects?
(a) 34650 (b) 43680
(c) 43890 (d) none of these
- 39** In how many ways can 16 books on different subjects be divided equally into 4 groups?
(a) $\frac{16!}{(4!)}$ (b) $4! \times (16!)$
(c) $\frac{16!}{(4!)^5}$ (d) none of these
- 40** In a badminton tournament each player played one game with all the other players. How many players participated in the tournament if they played 105 games in all?
(a) 35 (b) 12
(c) 15 (d) none of these
- 41** There are 9 subjects in eighth standard but there are only 6 periods in a day. In how many ways can the time table be formed?
(a) 60480 (b) 23460
(c) 56780 (d) 61280
- 42** How many 6 digit numbers can be formed using the digit 2 two times and the digit 4 four times?
(a) 16 (b) 15
(c) 18 (d) 24
- 43** There are 4 different monitors and 6 different mother boards. How many different arrangements can be made to purchase a monitor and a motherboard?
(a) 12 (b) 24
(c) 18 (d) 6^4

- 44** The number of five digit numbers having atleast one of their digits repeated is :
 (a) 900 (b) 1000
 (c) 62784 (d) none of these
- 45** Four dice are rolled. The number of possible outcomes in which atleast one die shows 4 is :
 (a) 671 (b) 168
 (c) 176 (d) none of these
- 46** The number of signals that can be generated by using 5 differently coloured flags, when any number of them may be hoisted at a time is :
 (a) 235 (b) 253
 (c) 325 (d) none of these
- 47** If the letters of the word VERMA are arranged in all possible ways and these words are written out as in a dictionary, then the rank of the word VERMA is :
 (a) 108 (b) 117
 (c) 810 (d) 180
- 48** How many 5 digit numbers divisible by 3 can be formed using the digits 0, 1, 2, 3, 4 and 5, without repetition?
 (a) 108 (b) 216 (c) 810 (d) 180
- 49** In how many ways can a mixed double game can be arranged from amongst 8 married couples if no husband and wife play in the same game?
 (a) 840 (b) 240
 (c) 480 (d) none of these
- 50** If n objects are arranged in a row, then the number of ways of selecting three of these objects so that no two of them are next to each other is :
 (a) ${}^nC_3$ (b) $n-2C_3$
 (c) ${}^nC_2$ (d) none of these
- 51** The number of times the digit 8 will be written when listing the integers from 1 to 1000 is :
 (a) 100 (b) 200 (c) 300 (d) 400
- 52** Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls. The number of ways in which we can place the balls in the boxes so that no box remains empty is :
 (a) 30 (b) 150 (c) 600 (d) 900
- 53** The number of squares on a chessboard is :
 (a) 102 (b) 108 (c) 216 (d) 204
- 54** A train going from Lucknow to Mumbai stops at 7 intermediate stations. Five persons enter the train during the journey with five different tickets of the same class. How many different set of tickets they could have had ?
 (a) 98280 (b) 2898
 (c) 7325 (d) none of these
- 55** In how many ways can a committee of 4 women and 5 men be chosen from 9 women and 7 men, if Mr. A refuses to serve on the committee if Ms. B is a member?
 (a) 1608 (b) 1860
 (c) 1680 (d) 1806

Directions (for Q. Nos. 56 to 63) : There are 5 different caps c_1, c_2, c_3, c_4 and c_5 and 5 different boxes B_1, B_2, B_3, B_4 and B_5 . The capacity of each box is sufficient to accomodate all the 5 caps.

- 56** If any box can have any number of caps, in how many ways can all the caps be distributed?
 (a) 3125 (b) 1235 (c) 2358 (d) 1248
- 57** In how many arrangements does B_1 have cap C_1 ?
 (a) 5! (b) 5^4
 (c) ${}^5P_4$ (d) none of these
- 58** If all the caps are of different colours and each box can have only one cap, in how many ways can you arrange the caps among the 5 boxes?
 (a) 120 (b) 180
 (c) 360 (d) none of these
- 59** If atleast one cap has to be distributed and the caps have to be arranged such that any box can have a maximum of one cap only, in how many ways can you arrange the caps among 5 boxes?
 (a) 1 (b) 16
 (c) 1545 (d) none of these
- 60** If all the caps are identical, in how many ways can the caps be arranged in the different boxes such that no box is empty?
 (a) 1 (b) 2 (c) 6 (d) 8
- 61** If C_1 and C_5 are similar in all aspects, in how many ways can you arrange the caps in such a way that all the boxes have one cap?
 (a) 70 (b) 60 (c) 75 (d) 80
- 62** If B_3 can have only C_1 or C_5 , in how many ways can you arrange the caps such that all boxes have one cap?
 (a) 480 (b) 420 (c) 48 (d) 88
- 63** If B_1 and B_5 have the caps C_1 and C_5 among themselves, in how many ways can you arrange the caps among the 5 boxes ?
 (a) 400 (b) 485 (c) 500 (d) 365
- 64** Romeo and Juliet write love-letters to none but to each other. In a given period of time, Romeo writes 4 letters and Juliet writes 2 letters. During this period, at any given point of time Romeo writes greater than or equal to the number of letters written by Juliet. Find the number of ways of writing love-letters to each other.
 (a) 10 (b) 8 (c) 6 (d) 9
- 65** Malvika wants to colour a cubical box from outside. In how many ways can she colour it if she wants to colour each face of the box with either blue or pink colour?
 (a) 6 (b) 12 (c) 10 (d) 18
- 66** A gym opens twice a day – morning and evening. But, I don't work out twice a day. I attend it only 4 times a week. In how many ways can I attend the gym in a week?
 (a) 560 (b) 350
 (c) 280 (d) none of these

- 67** An n -digit number is a positive number with exactly n -digits. Nine hundred distinct n -digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is
(a) 6 (b) 7 (c) 8 (d) 9
- 68** Which of the following is not a possible number of regions into which three straight lines, of infinite extent, divide a plane?
(a) 4 (b) 5 (c) 6 (d) 7
- 69** If you travel between Mumbai and Pune, you have to go through a 10 km long tunnel. Lately there has been a surge in the incidents of loot and plunder and many passengers have been harassed and injured inside the tunnel. Considering the enormity of the law and order problem, highway police decided to establish 4 security posts in order to ensure the safety of life and luggage of commuters. The minimum distance between any two posts is 1 km. Each security post must be n km ($n = 0, 1, 2, 3, \dots$) apart from entry/exit of the tunnel and none of them will be outside the tunnel. Find the number of ways of selecting the spots for upcoming security posts.
(a) 330 (b) 110 (c) 66 (d) 120
- 70** A rectangle with sides $(2m - 1)$ and $(2n - 1)$ is divided into squares of unit length by drawing parallel lines as shown in the diagram, then the number of rectangles possible with odd side lengths is
(a) $(m + n + 1)^2$ (b) $4^{(m + n - 1)}$
(c) $m^2 n^2$ (d) $mn(m + 1)(n + 1)$
- 71** A black ant has to go from the origin to a point $(6, 4)$ on the Cartesian plane, using the coordinate axes. However, it avoids crossing the point $(4, 1)$, as a red ant is sitting there. Find the total number of shortest paths that it can go along.
(a) 150 (b) 160 (c) 190 (d) 240
- 72** The number of seven digit integers, with sum of digits equal to 10 and formed by using the digits 1, 2, and 3 only, is
(a) 55 (b) 66 (c) 77 (d) 88
- 73** Find the maximum possible number of regions created by 6 overlapping triangles.
(a) 36 (b) 63 (c) 92 (d) 96
- 74** Find the total number of three digit numbers, which have their digits in increasing order, as in, for a three digit number abc , there must be $a < b < c$.
(a) 84 (b) 67 (c) 97 (d) 79
- 75** There are three straight lines dividing the circle into maximum possible number of regions. If three additional lines are drawn, what is the maximum possible number of additional regions that can be formed in the circle?
(a) 6 (b) 9
(c) 14 (d) none of these
- 76** Spiritual Guru H. E. Dalai Lama is on a world tour to speak on peace and compassion. His itinerary mentions only four continents – Asia, Africa, Europe and America – for this visit. What is the minimum number of countries he should visit to ensure that at least 21 Asian or at least 16 African or at least 11 European or at least 6 American countries must be visited?
(a) 24 (b) 51 (c) 54 (d) 57
- 77** Arya, Bran, Cersei, Daenerys, Petyr and Tyrion are seated in six chairs placed in a row. Maximum how many seating arrangements are possible if Arya must sit next to Bran, but Petyr cannot sit next to Tyrion?
(a) 98 (b) 196 (c) 144 (d) 256
- 78** How many 6 digit telephone numbers can be formed if each number starts with 91 and sum of the digits is equal to 16?
(a) 210 (b) 336 (c) 168 (d) 84
- 79** Jwala Gutta and P. V. Sindhu play a one-day series. They stop playing matches as soon as one of them wins 3 matches and in this series no match is drawn. In how many ways can this series be won?
(a) 20 (b) 24 (c) 12 (d) 6
- 80** Let there be a random number n such that the product of its digits is 6. How many values of n are there, if n lies between 10^9 and 10^{10} ?
(a) 10 (b) 60 (c) 90 (d) 100
- 81** Sarvesh wants to prepare a bouquet with roses, lilies, orchids and carnations, so he walked into a large garden of flowers. In how many ways can he prepare a bouquet of 15 flowers such that he selects at least 1 rose, 2 lilies, 3 orchids and 4 carnations for his bouquet?
(a) 24 (b) 120 (c) 56 (d) 112

LEVEL 02 > HIGHER LEVEL EXERCISE

- In how many ways can the following prizes be given away to a class of 30 students, first and second in Mathematics, first and second in Physics, first in Chemistry and first in English?
 (a) $\frac{30!}{4!}$
 (b) $(30)^4 \times (29)^2$
 (c) $(30)^3 - 1$
 (d) $(30)^4 \times (29)^4$
- If P_r stands for ${}^rP_r$, then the value of $1 + 1 \cdot P_1 + 2 \cdot P_2 + 3 \cdot P_3 + \dots + n \cdot P_n$ is :
 (a) $\frac{(n-1)!}{2}$
 (b) $\frac{(n+1)!}{2}$
 (c) $2(n-1)!$
 (d) $(n+1)!$
- In an examination hall there are four rows of chairs. Each row has 6 chairs one behind the other. There are two classes sitting for the examination with 12 students in each class. It is desired that in each row, all students belong to the same class and that no two adjacent rows are allotted to the same class. In how many ways can these 24 students be seated?
 (a) $2(12!)^2$
 (b) $3! \times (12!)^2$
 (c) $(6!)^2$
 (d) none of these
- How many 3 digit even numbers are there such that if 3 is one of the digits then 9 is the next digit?
 (a) 365
 (b) 536
 (c) 210
 (d) 156
- A number plate of a vehicle has always a fixed code UP-32 for Lucknow city followed by the number of particular vehicle which is in two parts. First part is occupied by 2 English alphabets and second part is occupied by 4 digit numbers (0001, 0002, ... 9999). If the latest registration number of vehicle is $\left\{ \begin{array}{l} \text{UP - 32} \\ \text{SK - 0123} \end{array} \right\}$, find the number of vehicles registered before this vehicle number in Lucknow.
 (a) 2449744
 (b) 4779644
 (c) 4669235
 (d) 9235888222
- How many 4 digit numbers divisible by 4 can be formed without using the digits 0, 6, 7, 8, 9 if the repetition of digits is not allowed?
 (a) 67
 (b) 38
 (c) 24
 (d) 48
- In how many ways three girls and nine boys can be seated in two vans, each having numbered seats, 3 in the front and 4 at the back?
 (a) ${}^{14}C_{12}$
 (b) 9^4
 (c) ${}^{14}P_{12}$
 (d) none of these
- In the previous question, how many seating arrangements are possible if 3 girls sit together in a back row on adjacent seats?
 (a) $(3!)^2$
 (b) $6!$
 (c) $5!$
 (d) none of these
- In how many ways 5 plus (+) signs and 5 minus (-) signs be arranged in a row so that no two minus signs are together?
 (a) 6
 (b) 7
 (c) 8
 (d) 10
- In how many ways can 11 identical books on English and 9 identical books on Maths be placed in a row on a shelf so that two books on Maths may not be together?
 (a) 110
 (b) 220
 (c) 330
 (d) 440
- Find the total number of factors of 1680.
 (a) 40
 (b) 50
 (c) 60
 (d) 30
- $\sum_{k=m}^n {}^kC_r$ equals :
 (a) ${}^{n+1}C_{r-1}$
 (b) ${}^{n+1}C_{r+1} - {}^mC_r$
 (c) ${}^{n+1}C_{r+1} - {}^mC_{r+1}$
 (d) ${}^{n+1}C_{r+1} + {}^mC_{r+1}$
- In a certain test there are n questions. In this test 2^{n-k} students gave wrong answers to atleast k questions, where $k = 1, 2, 3, \dots, n$. If the total number of wrong answers given is 2047, then n is equal to :
 (a) 10
 (b) 11
 (c) 12
 (d) 13
- The number of triangles whose vertices are at the vertices of an octagon but none of the sides of such triangles are taken from the sides of the octagon.
 (a) 8
 (b) 15
 (c) 16
 (d) none of these
- The number of ways in which we can select 5 numbers from the set of numbers $\{1, 2, 3, \dots, 25\}$ such that none of the selections includes four consecutive numbers is :
 (a) 53109
 (b) 13350
 (c) 10035
 (d) none of these
- The number of integral solutions for the equation $a + b + c + d = 12$, where $(a, b, c, d) \geq -1$ is :
 (a) ${}^{19}C_3$
 (b) ${}^{18}C_4$
 (c) ${}^{20}C_4$
 (d) none of these
- If $n = {}^kC_2$, the value of ${}^nC_2$ is :
 (a) $2({}^{k+2}C_4)$
 (b) $(n-2)!$
 (c) $(k-2)!$
 (d) $3({}^{k+3}C_4)$
- The number of positive integral solutions of $abc = 42$ is :
 (a) 17
 (b) 27
 (c) 21
 (d) $3! \times 42$
- There are three piles of identical red, green and blue balls and each pile contains atleast 10 balls. The number of ways of selecting 10 balls if twice as many red balls as green balls are to be selected is :
 (a) 1
 (b) 2
 (c) 4
 (d) 6

- 20** Between Shirdee and Khandala, there are 10 intermediate stations. The number of ways in which a train can be made to stop at 4 of these stations so that no two of these halting stations are consecutive is :
 (a) 35 (b) 70 (c) 105 (d) 10
- 21** The number of ways in which an examiner can assign 30 marks to 8 questions, giving not less than 2 marks to any question is :
 (a) 11 (b) ${}^{21}C_7$ (c) 18 (d) 235
- 22** The exponent of 3 in $33!$ is :
 (a) 15 (b) 2
 (c) 11 (d) none of these
- 23** The total number of natural numbers of 6 digits that can be formed with digits 1, 2, 3, 4, if all the digits are to appear in the same number atleast once, is :
 (a) 740 (b) 1830
 (c) 1560 (d) none of these
- 24** The total number of seven digit numbers, the sum of whose digits is even are :
 (a) 250000 (b) 4500000
 (c) 35×10^5 (d) none of these
- 25** The number of ways in which 4 pictures can be hung from 6 picture nails on the wall is :
 (a) 4^6 (b) ${}^4P_6$ (c) ${}^6P_4$ (d) 6^4
- 26** The total number of ways of selecting 6 coins out of 10 one rupee coins, 6 fifty paise coins and 8 twenty paise coins is :
 (a) 28 (b) 14 (c) 13 (d) 19
- 27** Nargis has 8 children and she takes 3 at a time to children's park as often as she can without taking the same 3 children together more than once. The number of times she will go to the park is :
 (a) 56 (b) 14 (c) 28 (d) 76
- 28** In the previous problem (no. 27), the number of times each child will go to the park is:
 (a) 14 (b) 8
 (c) 21 (d) none of these
- 29** The sides AB , BC , CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. The total number of triangles that can be constructed by using these points as vertices is :
 (a) 105 (b) 307 (c) 420 (d) 205
- 30** The number of all the possible selections which a student can make for answering one or more questions out of 10 given questions in a paper, when each question has an alternative is :
 (a) 1345 (b) 23560 (c) 541340 (d) 59048
- 31** The number of ways in which a team of eleven players can be selected from 22 players including 2 of them and excluding 4 of them is :
 (a) ${}^{15}C_{10}$ (b) ${}^{16}C_{10}$
 (c) ${}^{16}C_9$ (d) none of these
- 32** There are 10 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated is :
 (a) 1022 (b) 1023
 (c) 1024 (d) none of these
- 33** How many 10 digits numbers can be formed by using the digits 2 and 3?
 (a) 2^{10} (b) 10^2
 (c) $10!$ (d) none of these
- 34** How many subsets containing at most n elements from the set of $(2n + 1)$ elements can be selected?
 (a) 2^n (b) 2^{n-1}
 (c) 2^{n+1} (d) 2^{2n}
- 35** The sum of the divisors of $2^3 \cdot 3^4 \cdot 5^2$ is :
 (a) 15625 (b) 1234
 (c) 56265 (d) 56789
- 36** The number of ways of selecting 10 balls out of an unlimited number of white, red, blue and green balls is :
 (a) 268 (b) 286 (c) 246 (d) 468
- 37** If 10 objects are arranged in a row, then the number of ways of selecting three of these objects so that no two of them are next to each other is :
 (a) 56 (b) 65 (c) 28 (d) 13
- 38** The number of non-negative integral solutions of $x_1 + x_2 + x_3 + x_4 \leq n$ (where n is a positive integer) is :
 (a) ${}^{n+3}C_3$ (b) ${}^{n+2}C_3$ (c) ${}^{n+4}C_4$ (d) ${}^{n+4}C_3$
- 39** The total number of permutations of $(n > 1)$ different things taken not more than r at a time, when each thing may be repeated any number of times is :
 (a) $\frac{n^r - 1}{(n - 1)}$ (b) $r^n - 1$
 (c) $\frac{n(n^r - 1)}{(n - 1)}$ (d) none of these
- 40** Number of rectangles on a chessboard is :
 (a) 1008 (b) 1296 (c) 1124 (d) 1600
- 41** Ten persons, amongst whom A , B and C are to speak at a function. The number of ways in which it can be done if A wants to speak before B and B wants to speak before C is :
 (a) $\frac{10!}{6}$ (b) $\frac{9!}{6}$
 (c) ${}^9C_6$ (d) none of these
- 42** The number of natural numbers which are smaller than $2 \cdot 10^8$ and which can be written by means of the digits 1 and 2 is :
 (a) 678 (b) 786 (c) 766 (d) 677
- 43** Twelve persons are arranged in a row. The number of ways of selecting four persons so that no two persons sitting next to each other are selected is :
 (a) 124 (b) 136
 (c) 126 (d) none of these

- 44** The number of n -bit (digits 0 and 1) strings having exactly k zeros with no two zeros consecutive is, where $2k < n$.
 (a) ${}^{n+1-k}C_k$ (b) ${}^{n+k}C_k$
 (c) ${}^{n-k}C_k$ (d) ${}^{n-1}C_k$
- 45** A question paper has two parts—Part A and Part B. Part A contains 5 questions and part B has 4. Each question in part A has an alternative. A student has to attempt atleast one question from each part. Find the number of ways in which the student can attempt the question paper.
 (a) 3360 (b) 1258 (c) 3850 (d) 3630
- 46** How many different vehicle licence plates can be made if the licences contain 2 letters of the English alphabet followed by a three digit number if repetitions are allowed?
 (a) 54320 (b) 67534 (c) 23456 (d) 675324
- 47** In the previous question (no. 46), how many different plates can be formed if the repetitions are not allowed?
 (a) 468000 (b) 13000
 (c) 15680 (d) none of these
- 48** $2m$ people are arranged along two sides of a long table with m chairs each side. r men wish to sit on one particular side and s on the other. In how many ways can they be seated? ($r, s \leq m$)
 (a) ${}^{48}C_r$ (b) $68m$
 (c) $\frac{(2m+r)!}{r!s!}$ (d) none of these
- 49** Management city has m parallel roads running East-West and n parallel roads running North-South. How many shortest possible routes are possible to go from one corner of the city to its diagonally opposite corner?
 (a) ${}^{m+n}C_{m-1}$ (b) ${}^{m+n}C_n$
 (c) ${}^{(m+n-2)}C_{(m-1)}$ (d) none of these
- 50** Priyanka has 11 different toys and Supriya has 8 different toys. Find the number of ways in which they can exchange their toys so that each keeps her initial number of toys.
 (a) ${}^{19}C_{11}$ (b) ${}^{18}C_{10}$ (c) ${}^{20}C_{11}$ (d) ${}^{19}C_{11} - 1$
- 51** The sum of the numbers of the n^{th} term of the series $(1) + (1 + 2) + (1 + 2 + 3) + (1 + 2 + 3 + 4) + \dots (1 + 2 + 3 + \dots n)$:
 (a) ${}^{n+1}C_3$ (b) ${}^{n+1}C_2$ (c) ${}^nC_2$ (d) ${}^{n+2}C_3$
- 52** Two packs of 52 playing cards are shuffled together. Find the number of ways in which a man can be dealt 26 cards so that he does not get two cards of the same suit and same denomination.
 (a) ${}^{52}C_{26} \times 2^{26}$ (b) $\frac{52!}{(2!)^{26}}$
 (c) 2^{26} (d) none of these
- 53** In how many ways we can select four cards of different suits and different values from a deck of 52 cards?
 (a) $10!$ (b) $\frac{13!}{10!}$
 (c) $\frac{13!}{9!}$ (d) none of these
- 54** How many 5 digit numbers can be formed having the digits 0 three times and 3 two times?
 (a) 4 (b) 6 (c) 8 (d) 10
- 55** How many different four digit numbers can be formed with the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 such that the digit 5 appears exactly once?
 (a) 1024 (b) 2048
 (c) 4096 (d) none of these
- 56** How many different four digit numbers can be formed using digits 1, 2, 3, 4, 5, 6, 7, 8, 9 without repetition such that the digit 5 appears exactly once?
 (a) 1243 (b) 1234 (c) 1344 (d) 1355
- 57** How many different eight digit numbers can be formed using only four digits 1, 2, 3, 4 such that the digit 2 occurs twice?
 (a) 20412 (b) 12042
 (c) 25065 (d) none of these
- 58** In how many ways can 8 distinct things be divided among three people such that any one can receive any number of things?
 (a) 1556 (b) 6561 (c) ${}^8C_2$ (d) ${}^7C_2$
- 59** There are 6 numbered chairs placed around a circular table. 3 boys and 3 girls want to sit on them such that neither of two boys nor two girls sit adjacent to each other. How many such arrangements are possible?
 (a) 36 (b) 58
 (c) 72 (d) none of these
- 60** In how many ways one black and one white rook can be placed on a chessboard so that they are never in an attacking position?
 (a) 1234 (b) 3136 (c) 9516 (d) 1024
- 61** There are 9 pairs of white shoes and 6 pairs of black shoes contained in a box. We are allowed to draw only one shoe at a time. Minimum how many shoes are required to be drawn out to get one pair of white shoes?
 (a) 11 (b) 22 (c) 33 (d) 15
- 62** In the previous question, minimum how many shoes must be drawn out to get at least 1 pair of either black or white shoes?
 (a) 15 (b) 18 (c) 16 (d) 22
- 63** There are four different coloured balls and four boxes of the same colour as that of each ball is. Find the number of ways in which exactly one ball can be put in a box so that the colour of the box and that of ball is distinct.
 (a) 16 (b) 10
 (c) ${}^4C_2$ (d) none of these

- 64** In the previous question (no. 63) find the number of ways in which only two balls can be put in the correct boxes i.e., the colour of box and the colour of contained ball be same.

(a) 3 (b) 6
(c) 9 (d) none of these

- 65** Find the number of digits required to write down the number of pages in a 300 pages book.

(a) 792 (b) 279
(c) 729 (d) none of these

- 66** Find the number of numbers that can be formed using all the digits 1, 2, 3, 4, 3, 2, 1 only once so that the odd digits occupy odd places only.

(a) 9 (b) 16 (c) 18 (d) 27

Directions (for Q. Nos. 67 to 74) : There are 5 different boxes and 7 different balls. All the 7 balls are to be distributed in the 5 boxes placed in a row so that any box can receive any number of balls.

- 67** In how many ways can these balls be distributed into these boxes?

(a) 5^7 (b) 7^5 (c) ${}^7C_5$ (d) ${}^7P_5$

- 68** In how many ways can these balls be distributed so that no box is empty?

(a) $7!$ (b) 16800
(c) 1775 (d) none of these

- 69** Suppose all the balls are identical, then in how many ways can all these balls be distributed into these boxes?

(a) 110 (b) 220 (c) 330 (d) 440

- 70** Suppose all the balls and all the boxes are also identical. Then in how many ways can all these balls be distributed into these boxes?

(a) 12 (b) 13 (c) 16 (d) 255

- 71** In the previous question (no. 70) in how many ways can all these balls be distributed into these boxes so that no box remains empty and no two consecutive boxes have the same number of balls?

(a) 1 (b) 4
(c) 12 (d) none of these

- 72** In how many ways can these balls be distributed so that box 2 and box 4 contain only 1 and 2 balls respectively?

(a) 5522 (b) 8505
(c) 2305 (d) none of these

- 73** In how many ways can these balls be distributed into these boxes if ball 2 can be put into either box 2 or box 4?

(a) 12360 (b) 31250 (c) 13490 (d) 31526

- 74** In how many ways can these balls be distributed such that no box is empty and ball 2 and ball 4 cannot be put in the same box?

(a) 1200 (b) 15000
(c) 3800 (d) none of these

Directions (for Q. Nos. 75 to 82) : There are 3 pots and 4 coins. All these coins are to be distributed into these pots where any pot can contain any number of coins.

- 75** In how many ways these coins can be distributed if all the coins and all the pots are different?

(a) 3^4 (b) 4^3 (c) ${}^3P_4$ (d) ${}^4P_3$

- 76** In how many ways these coins can be distributed if all the coins and all the pots are identical?

(a) 4 (b) 6 (c) 8 (d) 1

- 77** In how many ways all these coins can be distributed if all coins are identical but all pots are different?

(a) 15 (b) 16 (c) 17 (d) 81

- 78** In how many ways all these coins can be distributed if all coins are different but all pots are identical?

(a) 14 (b) 21
(c) 27 (d) none of these

- 79** In how many ways all these coins can be distributed such that no pot is empty if all coins are different but all pots are identical?

(a) 16 (b) 6 (c) 42 (d) 21

- 80** In how many ways all these coins can be distributed such that no pot is empty if all coins are identical but all pots are different?

(a) 6 (b) 3 (c) 9 (d) 27

- 81** In how many ways all these coins can be distributed if all coins are identical and two pots are also identical?

(a) 1 (b) 10 (c) 9 (d) 11

- 82** In how many ways all these coins can be distributed if out of 4 coins 2 coins are identical and all pots are different?

(a) 45 (b) 27
(c) 54 (d) none of these

- 83** Once Munnabhai gets admitted to a psychiatric hospital. In his room, there are two bulbs connected to two different switches, independently. One night, while accompanying him, his best buddy Circuit notices that there is, initially, no light in the room, but whenever a mosquito bites him he switches on the light and then immediately switches it off. Throughout the night, Munnabhai presses the switch (on/off) six times. Finally when Munnabhai stops playing around with the switches, Circuit notices that there is no light in the room. In how many ways Munnabhai ends up having no light in his room by pressing the given switches on or off exactly six times?

(a) 3 (b) 4 (c) 6 (d) 8

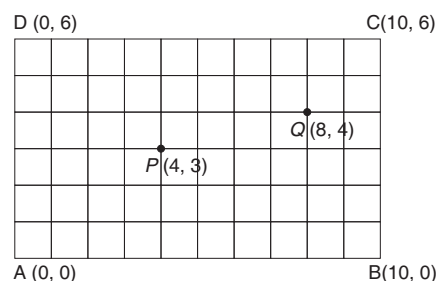
- 84** Mr. Been, who is getting older and senile, is standing on a crossroad totally confused about his directions. He can move in any of the four directions – North, South, East or West. He takes some steps and then comes back to the same point at the crossroad. In how many ways can he end up being at the same point where he is initially standing by taking total 8 steps?

(a) 4900 (b) 6400 (c) 360 (d) 238

- 85** Indian Democratic Alliance (IDA) is a coalition of three distinct regional parties. IDA decides to contest on 100 seats for the upcoming parliamentary elections. IDA wants to share these seats in such a way that no any ally gets the same number of seats to contest and no two allies contest for the same seat. Also each of the allies must get at least 1 seat to contest. Find the number of ways of allocating the number of seats for the three constituents of the IDA?
- (a) 4704 (b) 4851 (c) 4884 (d) 3667
- 86** Bill Gates personally announced that he would be building 49 toilets in 3 select Indian villages A, B and C, as part of the pilot project. In how many ways these toilets can be built in 3 villages such that village A gets more toilets than that of village B and village B gets more toilets than that of village C?
- (a) 98 (b) 196
(c) 188 (d) none of these
- 87** H2O is a chain of salons, employing one hair stylist at each salon. It has got centralized system in place to provide appointments through phone calls for all its 6 salons located in different areas in Pune. A family of four seeking the appointment for all its members for the hair cut. On a particular day, in how many ways the salons can give the appointments to them?
- (a) 3024 (b) 2524 (c) 3384 (d) 3084
- 88** A natural number N is such that $10^{10} < N < 10^{12}$ and the sum of its digits is 3. Find the total number of values of N .
- (a) 121 (b) 111 (c) 222 (d) 144
- 89** A spiritual organization, working for the world peace, has inducted 32 young volunteers none of whom knew each other before joining this organization. During the induction program they interacted with each other. Next day it was observed that most of the volunteers became friends on the Facebook, but one-fifth of the girls denied becoming Facebook friend with one-third of the boys. Considering the natural Facebook friendship among some of the volunteers, the head of the organization asked them to visit the various regions of the world and spread the message of the world peace. She told them that a region would have to be visited by exactly 2 volunteers who are the Facebook friends and no any such pair would have to visit more than one region. Which is the possible number of regions that these volunteers could have visited?
- (a) 525 (b) 496 (c) 421 (d) 487
- 90** Find the total number of non-negative integers less than 1000 for which the sum of the digits is 10.
- (a) 63 (b) 66 (c) 67 (d) 56

Directions (for Q. Nos. 91 and 92) : *Hirabhai after marrying Hiraben settled in Hiranandini, Mumbai. He has 15 diamond rings and 5 daughters. Among his 5 daughters, he has only one daughter who is not twins.*

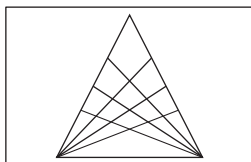
- 91** If the rings are distinct, find the number of ways of distributing these rings among his 5 daughters such that elder the daughter fewer the rings she gets. However none of them gets less than 1 and more than 5.
- (a) $\frac{15!}{3! \times (5!)^2}$ (b) $\frac{15!}{(2!)^5 \times (5!)^2}$
(c) $\frac{15!}{(2!)^2 \times (3!)^2 \times 5!}$ (d) none of these
- 92** If the rings are identical, find the number of ways of distributing these rings among his 5 daughters such that elder the daughter fewer the rings she gets. However none of them gets less than 1 and more than 5.
- (a) 180 (b) 150 (c) 75 (d) 6
- 93** Find the number of positive integral solutions of the equation $x_1 x_2 x_3 x_4 = 462$
- (a) 128 (b) 1024 (c) 256 (d) 64
- 94** Find the number of integral solutions of the equation $x_1 x_2 x_3 x_4 = 462$
- (a) 2048 (b) 256 (c) 24 (d) 1024
- 95** Rumour has it that Runnbeer, an alpha male, who easily gets to date the hometown hotties at the drop of his hat. Of late, he has been very promiscuous and ditched his two girls, who stay in the same city at two distinct locations Powai (P) and Qababchowk (Q).



Today, Runnbeer has got a new date and so he wants to take his date from her apartment at Arthur road (A) to a restaurant at Churchgate (C). However, he does not want to go through the places P and Q as his deserted girls, Priyankhan and Qatrinah, stay at these two places only. Find the total number of the shortest paths that he can follow to reach C from A, considering the roadmap given in the diagram.

- (a) 3298 (b) 6058 (c) 3880 (d) 3148
- 96** Sixty delegates from three different countries participated in a recent Disaster Management Conclave. Each such country is divided into four states and each such state is further divided into five cities. Each delegate was representing a different city. In that conclave any two delegates shook hands with each other exactly two times. However, if any two delegates are from the same country they shook hands five times and if any two delegates are from the same state they shook their hands nine times. What was the total number of handshakes amongst the delegates?
- (a) 1200 (b) 5730 (c) 7370 (d) 3540

- 97 DHL is a leading logistics company in India. In order to optimize its resources it has virtually divided the whole market region into four zones and each zone is again divided into five blocks. There are two dedicated trucks of DHL carrying the goods between any two blocks within each zone. Also there are three dedicated trucks carrying the goods between any two blocks of different zones. Exactly, how many trucks of DHL are engaged in the stated market?
(a) 72 (b) 660 (c) 530 (d) 484
- 98 There are 6 red and 4 green points on the circumference of a circle. How many convex polygons can be formed such that all the vertices of the polygon must be from these ten points only and having at least one green vertex?
(a) 1226 (b) 530 (c) 730 (d) 926
- 99 If m and n are relatively prime and $mn = 10!$, find the number of all rational numbers $\frac{m}{n}$ such that $0 < \frac{m}{n} < 1$.
(a) 8 (b) 15
(c) 30 (d) 21
- 100 How many distinct triangles are there in this figure?



- (a) 32 (b) 66
(c) 48 (d) 64
- 101 Sarvesh Verma has 3 mobile wallets – *PayPal*, *PayTM* and *PayU* – installed in his smartphone. Initially, he has only ₹ 100 in *PayPal* wallet and other two wallets have nothing. He can transfer his money from one wallet to any other wallet without any transaction charges or any incentives. One day, he performs seven online transactions, and in every transaction he empties his one wallet completely into another wallet. After seven transactions he has ₹ 100 in *PayPal* wallet. During this procedure neither he pays any amount to nor receives any amount from any third party by any means. In how many different ways can he make these transactions?
(a) 14 (b) 21 (c) 42 (d) 49
- 102 Kamlesh, the sulochan boy, has 100 identical matchsticks. How many distinct rectangles can he form using at most 100 matchsticks?
(a) 1225 (b) 625
(c) 25 (d) none of these
- 103 Two teams – Anadi and Khiladi – are playing a cricket match. Anadi requires 12 runs in the last 3 balls to win the match. Any team can score 0 to 6 runs, except 5 runs, from a particular ball. In how many different ways Team Anadi can score 12 runs without any no-ball and wide-ball in the last 3 balls?
(a) 11 (b) 12
(c) 13 (d) none of these
- 104 Each employee in our office at Lamamia must wear a shirt, a tie and a pair of pants to dress up oneself. Thus, any employee in our office can wear exactly 105 distinct combinations with the help of different colours. No two employees have equal number of quantities of all the three things individually. That is no two employees have x shirts, y ties and z pairs of pants. No two employees have the same colours of their clothes. Maximum how many employees are there in our office?
(a) 24 (b) 27 (c) 35 (d) 105
- 105 How many 8 digit numbers can be formed with the digits 1, 2, 3, 4, 5, 6, 7, 8 such that no digit repeats and none of the patterns 12, 34, 56, or 78 appears in any arrangement?
(a) 24096 (b) 24024
(c) 16296 (d) none of these

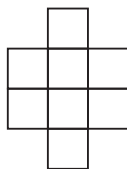
LEVEL 03 > Final Round

- 1 Maximum number of points of intersection of 6 circles, is
(a) 30 (b) 28
(c) 15 (d) none of these
- 2 Maximum number of points of intersection of 6 straight lines is :
(a) 30 (b) 15
(c) 28 (d) none of these
- 3 Maximum number of points into which 3 circles and 3 lines intersect is :
(a) 21 (b) 9 (c) 27 (d) 3!
- 4 Eight identical coins are arranged in a row. The total number of ways in which the number of heads is equal to the number of tails, is :
(a) 35 (b) 15 (c) 140 (d) 70
- 5 Two straight lines intersect at a point O . Points $A_1, A_2, A_3, A_4, A_5, \dots, A_m$ are taken on one line and points $B_1, B_2, B_3, \dots, B_n$ on the other. If the point O is not included, the number of triangles that can be drawn using these points as vertices, is :
(a) ${}^nC_2 + {}^mC_2$ (b) ${}^{2n}C_2$
(c) ${}^{m+n}C_2$ (d) none of these
- 6 How many different nine digit numbers can be formed from the number 22 33 55 888 by rearranging its digits so that the odd digits occupy even positions?
(a) 60 (b) 75
(c) 88 (d) 77

- 7 The straight lines l_1, l_2, l_3 are parallel and lie in the same plane. A total number of m points on l_1 , n points on l_2 , k points on l_3 are used to produce the triangles, the maximum number of triangles formed with vertices at these points are :
 (a) ${}^mC_3 \times {}^nC_3 \times {}^kC_3$
 (b) $({}^{m+n+k}C_3 - ({}^mC_3 + {}^nC_3 + {}^kC_3))$
 (c) $({}^{m+n+k}C_3)$
 (d) none of the above
- 8 Given that n is the odd, the number of ways in which three numbers in AP can be selected from 1, 2, 3, 4, ..., n is :
 (a) $\frac{(n-1)^2}{4}$ (b) n^2 (c) n^3 (d) $(n-2)^2$
- 9 Eight straight lines are drawn in the plane such that no two lines are parallel and no three lines are concurrent. The number of parts into which these lines divide the plane, is :
 (a) 73 (b) 37 (c) 17 (d) 72
- 10 Number of divisors of the form $4n + 2$ ($n \geq 0$) of the integer 240 is :
 (a) 6 (b) 4 (c) 3 (d) 12
- 11 The number of rectangles excluding squares from a rectangle of size of 12×8 is :
 (a) 1234 (b) 625
 (c) 2460 (d) 256
- 12 Two lines intersect at O . Points $A_1, A_2, \dots, A_n$ are taken on one of them and $B_1, B_2, \dots, B_n$ on the other the number of triangles that can be drawn with the help of these $(2n + 1)$ points is :
 (a) n (b) n^2 (c) n^3 (d) n^4
- 13 The number of ways in which 9 identical balls can be placed in three identical boxes is :
 (a) 12 (b) 6 (c) 9 (d) $\frac{9!}{3!}$
- 14 The number of ways in which 30 coins of one rupee each be given to six persons so that none of them receives less than 4 rupees is :
 (a) 246 (b) 462
 (c) $\frac{30!}{6!}$ (d) none of these
- 15 Find the number of circles that can be drawn out of 10 points, of which 7 points are collinear is :
 (a) 35 (b) 85
 (c) 70 (d) 124
- 16 The number of natural numbers of two or more than two digits in which digits from left to right are in increasing order is :
 (a) 205 (b) 520
 (c) 502 (d) none of these
- 17 In how many ways a cricketer can make a century with fours and sixes only?
 (a) 6 (b) 9
 (c) 8 (d) 10
- 18 The number of permutations of the letters of the word LUMINARY such that neither the pattern LURY nor MINA occurs is :
 (a) 46800 (b) 24600
 (c) 40086 (d) none of these
- 19 10 students are to be seated in two rows equally for the MOCK CAT in a room. There are two sets of papers, code A and code B. Each of the two rows can have only one set of paper but different that from the other row. In how many ways these students can be arranged?
 (a) 2775600 (b) 1200560
 (c) 125600 (d) 7257600
- 20 Aman Verma and Mini Mathur jointly host a TV programme in which one particular day n guests attend their show. In that show, each guest shakes hands with every other guest and each guest shakes hands with each of the hosts. If there happens to be total 65 handshakes, find the number of guests who attend the show.
 (a) 13 (b) 14
 (c) 10 (d) 9
- 21 The number of ways in which an examiner can assign 50 marks to 10 questions giving not less than 3 marks to any question is :
 (a) ${}^{29}C_9$ (b) ${}^{47}C_3$
 (c) ${}^{52}C_2$ (d) ${}^{40}C_{10}$
- 22 For $x \in R$, let $[x]$ denote the largest integer less than or equal to x , then the value of

$$E = \left[\frac{1}{3} \right] + \left[\frac{1}{3} + \frac{1}{100} \right] + \left[\frac{1}{3} + \frac{2}{100} \right] + \dots + \left[\frac{1}{3} + \frac{99}{100} \right]$$
 is :
 (a) 22 (b) 66
 (c) 33 (d) none of these
- 23 The number of positive integers from 1 to 10^6 (both inclusive) which are neither perfect squares, nor cubes, nor perfect fourth powers is :
 (a) 918990 (b) 998911
 (c) 998910 (d) none of these
- 24 Kumar's family consists of a grandfather, ' m ' sons and daughters and ' $2n$ ' grand children. They are to be seated in a row for dinner. The grandchildren wish to occupy the n seats at each end and the grandfather refuses to have a grandchild on either side of him. In how many ways can the Kumar's family be made to sit?
 (a) $(2n!) (m-1)!$
 (b) $(2n!) (m!) (m-1)$
 (c) $(2m!) (n!)$
 (d) none of the above

- 25** Six P s have to be placed in the squares of the diagram given below such that each column contains at least one P . In how many ways this can be done?



- (a) 26 (b) 13
(c) 8 (d) 24
- 26** A box contains two red, three green and four blue balls. In how many ways can three balls be drawn from the box if atleast one green ball is to be included in the draw?
(a) 23 (b) 64
(c) 46 (d) none of these
- 27** Ajay has 7 relatives, 3 men and 4 women. His wife Kajol also has 7 relatives, 3 women and 4 men. In how many ways can they invite 3 men and 3 women so that 3 of them are the Ajay's relatives and 3 his wife Kajol's?
(a) 485 (b) 458
(c) 365 (d) none of these
- 28** Serena and Venus were only two women participating in a chess tournament. Every participant played two games with every other participant. The number of games that men played between themselves proved to exceed by 66, compared to the number of games the men played with women. How many participants were there?
(a) 156 (b) 610 (c) 13 (d) 108
- 29** In the previous questions (no. 28) how many games were played?
(a) 208 (b) 156
(c) 316 (d) none of these
- 30** Six Burfis and six rasgullas are to be distributed among ten girls so that each girl receives at least one piece of sweets out of burfis and rasgullas. Find the number of ways in which the sweets can be distributed.
(a) 26250 (b) 22560
(c) 22330 (d) none of these
- 31** Mr. John has x children by his first wife and Ms. Bashu has $x + 1$ children by her first husband. They marry and have children of their own. The whole family has 10 children. Assuming that two children of the same parents do not fight, find the maximum number of fights that can take place among children.
(a) 33 (b) 22
(c) 111 (d) none of these
- 32** Find the number of natural numbers which are smaller than $2 \cdot 10^8$ and are divisible by 3 which can be written by means of the digits 0, 1 and 2.
(a) 7272 (b) 4373
(c) 3437 (d) none of these
- 33** Find the number of whole numbers formed on the screen of a calculator which can be recognised as numbers with (unique) correct digits when they are read inverted. The greatest number that can be formed on the screen of the calculator is 999999.
(a) 98970 (b) 89912 (c) 110050 (d) 100843

- 34** Find the number of ways of putting five distinct rings on four fingers of the left hand. (Ignore the difference of size of rings and the fingers)

(a) 1250 (b) 6720
(c) 5260 (d) none of these

- 35** If n distinct things are arranged in a circle, find the number of ways of selecting three of these things so that no two of them are next to each other.

(a) $n(n-2)(n-3)$ (b) $\frac{1}{6}n(n-4)(n-5)$

(c) $\frac{1}{3}(n^2 + 3n - 5)$ (d) none of these

- 36** The number of different selections of 5 letters from 1A, 2B's, 3C's, 4D's and 5E's is :

(a) 71 (b) 41
(c) 117 (d) none of these

- 37** In the entrance test of Wharton Business School, the maximum marks for each of three papers is n and that for the fourth paper is $2n$. Find the number of ways in which a candidate can get $3n$ marks.

(a) $\frac{1}{6}(n+1)(n+2)$ (b) $\frac{1}{6}(n+1)(5n^2 + 10n + 6)$

(c) $\frac{1}{2}(n^2 + 3n - 6)$ (d) none of these

- 38** An examination consists of 4 papers. Each paper has a maximum of ' n ' marks. Find the number of ways in which a students can get $2n$ marks in the examination.

(a) $\frac{1}{3}(n^2 - 5n + 4)$ (b) $\frac{1}{3}(n+1)(2n^2 + 4n + 3)$

(c) $\frac{1}{6}(n+1)(n+4)$ (d) none of these

- 39** Mark Shekhar Garg goes to a school to demonstrate how the kids can use the Facebook harmlessly and take the optimum advantage of socialization with fellow kids online at the early age of their social being. He tells the kids, after demonstrating the usage and safety features, that to make someone your facebook friend you have to send her/him a request and the other person has to accept it; as in out of two classmates A and B if first A sends request to B then B will accept it or if first B sends request to A then A will accept it and thus both A and B will become facebook friends.

As every kid responds to his instructions, within a minute Mark finds that every student in the class is now facebook friend of the other. In that class the number of female students is twice that of male students. He also notices that the number of friend-requests sent by the female kids is equal to the number of friend-requests sent by the male kids. Which of the following can be the number of friend-requests sent by female students to male students of that class?

(a) 42 (b) 99
(c) 105 (d) 120

- 40** A king calls 11 masseurs for his queen, as she wants the body massage from Monday to Friday, while on weekends she wants to go on the long trips with the king. She wants everyday a new masseur, so basically 5 masseurs are required. But a confidant of the king tells him that out of 11 masseurs 5 are familiar with the queen before her marriage and she used to get very close to them. But the king has no idea of these 5 people who are familiar with the queen. In spite of knowing this very odd fact he takes the risk of telling the masseurs that they can start giving massage to his lovely queen, as he doesn't want to make any fuss over it. However, he wants to identify the 5 masseurs who are close to the queen. Based on the queen's personal behaviour with him the wise king gets an idea that if in any week out of the 5 familiar masseurs any 3 masseurs give the massage to the beautiful queen he will certainly identify who all these 5 people familiar with the queen are. Minimum how many weeks are required to identify these 5 masseurs familiar with the queen?
- (a) 3 (b) 4
(c) 5 (d) 7
- 41** There is a set of 10 integers $\{0, 1, 2, 3, \dots, 9\}$. In how many ways can you permute this set of integers such that either 3 and 6 or 5 and 4 or 6 and 5 appear consecutively?
- (For example, we do not count (4, 5, 0, 6, 9, 2, 3, 7, 8, 1), as we want 5 and 4 to appear consecutively in that order. But we count (7, 2, 0, 1, 4, 3, 6, 5, 8, 9), both 3 and 6, and 6 and 5 appear consecutively in it.)
- (a) 962640 (b) 960049
(c) 368460 (d) 313200
- 42** How many integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 30$, such that $3 \leq x_1, x_2, x_3, x_4 \leq 10$.
- (a) 246 (b) 184
(c) 1084 (d) none of these
- 43** If ${}^{n-1}C_r = (k^2 - 3) {}^nC_{r+1}$, then $k \in R$
- (a) $(-\alpha - 2)$ (b) $[2, \alpha)$
(c) $[-\sqrt{3}, \sqrt{3}]$ (d) $(\sqrt{3}, 2]$
- 44** A shopping mall in Kashipur, a meditative and idyllic town near Nainital, has three food courts – Chinese, Thai and Italian, situated on its rooftop. Each food court is designed in such a way that all the restaurants are situated in a circle. Chinese, Thai and Italian food courts have 20, 15 and 10 restaurants, respectively. Having the change (coins and paper currency in smaller denomination) is usually a terrible job due to digital payments and rush in the food courts, so restaurants try to avoid giving the change to each other. Still it is found that each restaurant gives the change to any other restaurant at most three times a day and if the restaurants belong to the same food court then each restaurant gives the change to other restaurant at most two times a day. However, any two adjoining restaurants do not give change to each other at all. On a certain day at most how many times the change can be given to each other among all the restaurants on the rooftop?
- (a) 3970 (b) 5080 (c) 4370 (d) 6750
- 45** Find the total number of quadratic polynomial $ax^2 + bx + c$, if a, b, c are some three positive distinct integers less than 2000, such that $(x + 1)$ is the factor of the quadratic polynomial $ax^2 + bx + c$.
- (a) 19996002 (b) 1860004
(c) 19876004 (d) none of these
- 46** Find the number of isosceles triangles with integer sides, if no side exceeds 2014 unit.
- (a) 2426980 (b) 402800
(c) 2028098 (d) 3120600
- 47** A five-digit number is called a mountain number if the first three digits increase and the last three digits decrease. For example, 35763 is a mountain number, but 35663 is not. How many five-digit numbers greater than 70,000 are mountain numbers?
- (a) 69 (b) 36 (c) 63 (d) 96
- 48** If a triangle is inscribed in a regular decagon such that all the three vertices of the triangle are the vertices of the decagon, but none of the triangle's sides is the side of the decagon. How many such triangles can be formed?
- (a) 40 (b) 50
(c) 60 (d) 70
- 49** A wise old woman calls upon all her 7 sons at her eleventh hour and hands them over the keys of a box in which she keeps all her precious jewelry. She tells them that she has locked the box with multiple locks so that the box can be opened only when majority of you are present with the keys I am giving to you. After giving each of her sons the keys of certain locks she takes her last breath. How many locks she uses to secure her jewelry and how many keys does she hand over to each of the seven sons?
- (a) 35, 20 (b) 42, 21
(c) 28, 21 (d) 36, 18
- 50** An engineer in the urban planning department of IIM Kashipur, situated near Jim Corbett National Park in Uttarakhand, wants to connect all the 25 buildings (blocks) mutually. Each road connects only one pair of blocks. No single road connects more than 2 blocks. The roads are designed in such a way that one can reach from one block to any other block through a sequence of roads laid out at the campus. The number of roads (R) in the campus may satisfy the condition
- (a) $25 \leq R \leq 625$ (b) $24 \leq R \leq 288$
(c) $24 \leq R \leq 300$ (d) $25 \leq R \leq 225$

Answers

✓ Introductory Exercise 19.1

1. (a)	2. (b)	3. (c)	4. (a)	5. (d)	6. (b)	7. (a)	8. (b)	9. (c)	10. (b)
11. (a)	12. (b)	13. (a)	14. (d)	15. (b)	16. (a)	17. (b)	18. (a)	19. (c)	20. (d)
21. (d)	22. (a)	23. (a)	24. (a)	25. (a)	26. (d)	27. (b)	28. (d)	29. (a)	30. (b)
31. (a)	32. (b)	33. (d)	34. (b)	35. (d)	36. (a)	37. (b)	38. (c)	39. (b)	40. (c)
41. (a)	42. (b)	43. (a)	44. (a)	45. (a)	46. (b)	47. (a)	48. (b)	49. (d)	50. (d)
51. (c)	52. (a)	53. (a)	54. (a)	55. (d)	56. (a)	57. (b)	58. (a)	59. (c)	60. (a)
61. (a)									

✓ Introductory Exercise 19.2

1. (c)	2. (b)	3. (d)	4. (d)	5. (a)	6. (d)	7. (a)	8. (d)	9. (a)	10. (b)
11. (a)	12. (a)	13. (b)	14. (a)	15. (b)	16. (a)	17. (c)	18. (b)	19. (a)	20. (b)
21. (c)	22. (d)	23. (a)	24. (b)	25. (a)					

✓ Introductory Exercise 19.3

1. (a)	2. (c)	3. (a)	4. (c)	5. (a)	6. (d)	7. (d)	8. (a)	9. (b)	10. (a)
11. (b)									

✓ Introductory Exercise 19.4

1. (a)	2. (b)	3. (a)	4. (b)	5. (a)	6. (b)	7. (a)	8. (c)	9. (d)	10. (a)
11. (a)	12. (a)	13. (c)	14. (a)						

✓ Introductory Exercise 19.5

1. (a)	2. (c)	3. (c)	4. (c)	5. (c)	6. (b)	7. (b)	8. (c)	9. (d)	10. (a)
11. (c)	12. (d)	13. (a)	14. (c)	15. (a)	16. (b)	17. (c)	18. (b)	19. (d)	20. (c)
21. (d)	22. (b)	23. (a)	24. (a)	25. (b)	26. (a)	27. (d)	28. (b)	29. (c)	30. (b)
31. (a)	32. (b)	33. (a)	34. (b)	35. (c)	36. (c)	37. (b)	38. (c)	39. (a)	40. (b)
41. (a)	42. (c)	43. (b)	44. (a)	45. (b)	46. (c)	47. (a)	48. (c)	49. (a)	50. (b)
51. (a)	52. (b)	53. (d)	54. (c)	55. (d)	56. (b)	57. (c)	58. (a)	59. (a)	60. (a)
61. (b)	62. (c)	63. (d)	64. (a)	65. (c)	66. (d)	67. (c)	68. (d)	69. (c)	70. (d)
71. (a)	72. (d)	73. (a)	74. (b)	75. (d)	76. (d)	77. (b)	78. (b)	79. (d)	

✓ Introductory Exercise 19.6

1. (b)	2. (a)	3. (b)	4. (d)	5. (c)	6. (b)	7. (d)	8. (a)	9. (d)	10. (b)
11. (b)	12. (b)	13. (d)	14. (b)	15. (b)	16. (a)	17. (d)	18. (d)	19. (b)	20. (b)
21. (b)	22. (c)	23. (b)	24. (d)	25. (a)	26. (d)	27. (a)	28. (b)	29. (a)	30. (a)
31. (c)	32. (a)	33. (a)	34. (c)	35. (c)	36. (c)	37. (d)			

✓ Introductory Exercise 19.7

1. (c)	2. (a)	3. (d)	4. (c)	5. (a)	6. (a)	7. (c)	8. (a)	9. (b)	10. (d)
11. (d)	12. (d)	13. (b)	14. (b)	15. (d)	16. (b)	17. (a)	18. (c)	19. (b)	

✓ Introductory Exercise 19.8

1. (b)	2. (a)	3. (c)	4. (d)	5. (c)	6. (d)	7. (d)	8. (a)	9. (a)	10. (a)
11. (b)	12. (c)	13. (a)	14. (d)	15. (a)	16. (b)	17. (b)	18. (b)	19. (d)	20. (d)

21. (d)	22. (b)	23. (c)	24. (d)	25. (a)	26. (d)	27. (c)	28. (c)	29. (a)	30. (c)
31. (d)	32. (b)	33. (a)	34. (a)	35. (a)	36. (a)	37. (c)	38. (b)	39. (b)	40. (c)
41. (a)	42. (c)	43. (d)	44. (d)	45. (a)	46. (b)	47. (b)	48. (d)	49. (b)	50. (a)
51. (d)	52. (c)	53. (d)	54. (a)	55. (a)	56. (b)	57. (a)	58. (b)	59. (c)	60. (a)
61. (c)	62. (c)	63. (d)	64. (c)	65. (b)	66. (b)	67. (a)	68. (a)	69. (c)	70. (d)
71. (d)	72. (c)	73. (c)							

✓ **Introductory Exercise 19.9**

1 (a)	2. (b)	3. (a)	4. (c)

✓ **Introductory Exercise 19.10**

1 (d)	2. (a)	3. (c)	4. (b)	5. (a)	6. (c)	7. (a)	8. (b)	9. (d)	10. (d)
11. (d)	12. (b)	13. (c)	14. (b)	15. (d)	16. (d)	17. (c)	18. (a)	19. (a)	20. (a)
21. (b)	22. (d)	23. (c)	24. (c)	25. (d)	26. (d)	27. (a)	28. (b)	29. (d)	30. (c)
31. (c)	32. (a)	33. (b)	34. (a)	35. (a)	36. (b)	37. (d)	38. (d)	39. (b)	40. (b)
41. (d)	42. (c)	43. (c)	44. (a)	45. (d)	46. (c)	47. (c)	48. (a)	49. (d)	50. (b)
51. (c)	52. (a)	53. (b)	54. (b)	55. (c)	56. (a)	57. (b)	58. (a)	59. (d)	60. (c)
61. (a)	62. (b)	63. (c)	64. (d)	65. (b)	66. (b)	67. (c)	68. (b)	69. (a)	70. (d)

✓ **Level 01 Basic Level Exercise**

1 (c)	2. (b)	3. (a)	4. (d)	5. (a)	6. (b)	7. (c)	8. (b)	9. (b)	10. (c)
11. (a)	12. (c)	13. (d)	14. (a)	15. (c)	16. (b)	17. (b)	18. (a)	19. (c)	20. (d)
21. (c)	22. (b)	23. (b)	24. (b)	25. (b)	26. (b)	27. (b)	28. (a)	29. (c)	30. (d)
31. (a)	32. (b)	33. (d)	34. (b)	35. (a)	36. (c)	37. (b)	38. (b)	39. (c)	40. (c)
41. (a)	42. (b)	43. (b)	44. (c)	45. (a)	46. (c)	47. (c)	48. (b)	49. (a)	50. (b)
51. (c)	52. (b)	53. (d)	54. (a)	55. (d)	56. (a)	57. (b)	58. (a)	59. (c)	60. (a)
61. (b)	62. (c)	63. (c)	64. (d)	65. (c)	66. (a)	67. (b)	68. (b)	69. (a)	70. (c)
71. (b)	72. (c)	73. (c)	74. (a)	75. (d)	76. (b)	77. (c)	78. (d)	79. (a)	80. (d)
81. (c)									

✓ **Level 02 Higher Level Exercise**

1 (b)	2. (d)	3. (a)	4. (a)	5. (b)	6. (c)	7. (c)	8. (d)	9. (a)	10. (b)
11. (a)	12. (c)	13. (b)	14. (c)	15. (a)	16. (a)	17. (d)	18. (b)	19. (c)	20. (a)
21. (b)	22. (a)	23. (c)	24. (b)	25. (c)	26. (a)	27. (a)	28. (c)	29. (d)	30. (d)
31. (c)	32. (b)	33. (a)	34. (d)	35. (c)	36. (b)	37. (a)	38. (c)	39. (c)	40. (b)
41. (a)	42. (c)	43. (c)	44. (a)	45. (d)	46. (d)	47. (a)	48. (d)	49. (c)	50. (d)
51. (b)	52. (a)	53. (c)	54. (a)	55. (b)	56. (c)	57. (a)	58. (b)	59. (c)	60. (b)
61. (b)	62. (c)	63. (d)	64. (b)	65. (a)	66. (c)	67. (a)	68. (b)	69. (c)	70. (b)
71. (a)	72. (b)	73. (b)	74. (b)	75. (a)	76. (a)	77. (a)	78. (a)	79. (b)	80. (b)
81. (c)	82. (c)	83. (d)	84. (a)	85. (a)	86. (d)	87. (a)	88. (d)	89. (d)	90. (a)
91. (d)	92. (a)	93. (c)	94. (a)	95. (d)	96. (b)	97. (c)	98. (d)	99. (a)	100. (d)
101. (c)	102. (b)	103. (c)	104. (b)	105. (b)					

✓ **Level 03 Final Round**

1. (a)	2. (b)	3. (c)	4. (d)	5. (d)	6. (a)	7. (b)	8. (a)	9. (b)	10. (b)
11. (c)	12. (c)	13. (a)	14. (b)	15. (b)	16. (c)	17. (b)	18. (c)	19. (d)	20. (c)
21. (a)	22. (c)	23. (c)	24. (b)	25. (a)	26. (b)	27. (a)	28. (c)	29. (b)	30. (a)
31. (a)	32. (b)	33. (d)	34. (b)	35. (b)	36. (a)	37. (b)	38. (b)	39. (c)	40. (c)
41. (a)	42. (a)	43. (d)	44. (b)	45. (d)	46. (c)	47. (b)	48. (b)	49. (a)	50. (c)

Hints & Solutions

Introductory Exercise 19.1

$$1 \quad {}^9P_3 = \frac{9!}{6!} \quad \left(\because {}^nP_r = \frac{n!}{(n-r)!} \right)$$

$$= \frac{9 \times 8 \times 7 \times 6!}{6!} = 9 \times 8 \times 7 = 504$$

$$2 \quad {}^nP_5 = 20 \cdot {}^nP_3$$

$$\Rightarrow \frac{n!}{(n-5)!} = 20 \cdot \frac{n!}{(n-3)!} \Rightarrow \frac{(n-3)!}{(n-5)!} = 20$$

$$\Rightarrow \frac{(n-3)(n-4)(n-5)!}{(n-5)!} = 20$$

$$\Rightarrow (n-3)(n-4) = 20$$

$$\Rightarrow n = 8 \quad (n = -1 \text{ is inadmissible})$$

☞ Alternatively Go through options.

$$3 \quad \frac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}} = \frac{30800}{1}$$

$$\frac{56!}{(50-r)!} = \frac{30800}{1}$$

$$\frac{54!}{(51-r)!} = \frac{30800}{1}$$

$$\Rightarrow \frac{56 \times 55 \times (51-r)!}{(50-r)!} = \frac{30800}{1}$$

$$\Rightarrow \frac{(51-r)!}{(50-r)!} = \frac{10}{1}$$

$$\Rightarrow \frac{(51-r) \times (50-r)!}{(50-r)!} = \frac{10}{1} = 51-r=10 \Rightarrow r=41$$

☞ Alternatively Go through options

4 Since there are 6 digits available and out of 6 digits we have to take only 3 digits for the arrangement. Therefore, required number of 3 digits numbers = ${}^6P_3 = \frac{6!}{3!} = 120$

☞ Alternatively $\begin{array}{|c|c|c|} \hline 6 & 5 & 4 \\ \hline \end{array} \Rightarrow 6 \times 5 \times 4 = 120$

We have 6 digits to fill up hundreds place and only 5 digits for tens place and only 4 digits for unit place since repetition is not allowed.

5 **Case 1.** There is only one even number of one digit.

Case 2. $\begin{array}{|c|c|} \hline 4 & 1 \\ \hline \end{array} = 4 \times 1 = 4$

There are only 4 even numbers of two digit (Numbers are 38, 58, 78 and 98)

Case 3. $\begin{array}{|c|c|c|} \hline 4 & 3 & 1 \\ \hline \end{array} \Rightarrow 4 \times 3 \times 1 = 12$

There are only 12 even numbers of 3 digit.

Case 4. $\begin{array}{|c|c|c|c|} \hline 4 & 3 & 2 & 1 \\ \hline \end{array} \Rightarrow 4 \times 3 \times 2 \times 1 = 24$

There are 24 even numbers of 4 digits

Thus there are total $1 + 4 + 12 + 24 = 41$ even numbers.

🔔 **NOTE** In each case 8 is fixed at unit place, since if the unit digit is an even digit then the whole number is an even number.

6 $\begin{array}{|c|c|c|c|} \hline 3 & 4 & 3 & 2 \\ \hline \end{array} \Rightarrow 3 \times 4 \times 3 \times 2 = 72$

Thousands place can assume only 3 values viz., 5, 7, 8. Since required numbers are greater than 4000.

$\begin{array}{|c|c|c|c|c|} \hline 4 & 4 & 3 & 2 & 1 \\ \hline \end{array} \Rightarrow 4 \times 4 \times 3 \times 2 \times 1 = 96$

Ten thousands place can assume all the remaining non-zero digits and thousands place can assume zero also except the digit which has been filled up at thousands place.

Therefore total required numbers

$$= \text{number of 4 digit numbers} + \text{number of 5 digit numbers}$$

$$= 72 + 96 = 168$$

7 $\begin{array}{|c|c|c|c|} \hline 4 & 4 & 3 & 2 \\ \hline \end{array} \Rightarrow 4 \times 4 \times 3 \times 2 = 96$

Thousands place cannot assume 3 since required numbers are greater than 5000.

$\begin{array}{|c|c|c|c|c|} \hline 5 & 4 & 3 & 2 & 1 \\ \hline \end{array} \Rightarrow 5 \times 4 \times 3 \times 2 \times 1 = 120$

$$\text{Total required numbers} = 96 + 120 = 216$$

8 $\begin{array}{|c|c|c|c|} \hline 8 & 7 & 6 & 5 \\ \hline \end{array} \Rightarrow 8 \times 7 \times 6 \times 5 = 1680$

3 and 5 have been already used, so we have 8 digits for thousands place, then 7 digits for hundreds place, 6 digits for tens digit and remaining 5 digits for unit place.

☞ Alternatively ${}^8P_4 = \frac{8!}{4!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!}$

$$= 8 \times 7 \times 6 \times 5 = 1680$$

9 A number is divisible by 5 only if its unit digit is either 5 or 0. So we fix 5 as unit digit and then we fill up the remaining places.

$\begin{array}{|c|c|c|c|c|} \hline 5 & 4 & 3 & 2 & 1 \\ \hline \end{array} \Rightarrow 5 \times 4 \times 3 \times 2 \times 1 \times 1 = 120$

$$\text{Hence required number} = 120$$

10 $\begin{array}{|c|c|c|c|} \hline 4 & 3 & 2 & 1 \\ \hline \end{array} \Rightarrow 4 \times 3 \times 2 \times 1 = 24$

There are total 24 numbers of 4 digit.

Since we have only 4 digits it means we use each of the digit 6 $\left(= \frac{24}{4} \right)$ times in each of the unit, tens hundreds and thousands place.

Therefore the sum of the digits in the units place

$$= 6(1 + 2 + 5 + 6) = 84$$

Similarly, the sum of the digits in each of tens, hundreds, and thousands places = 84

Hence the required sum of all the numbers

$$\begin{aligned} &= 84 \times 1 + 84 \times 10 + 84 \times 100 + 84 \times 1000 \\ &= 84(1 + 10 + 100 + 1000) \\ &= 84 \times 1111 = 93324 \end{aligned}$$

- 11 Case 1.**

3	3	1

 $\Rightarrow 3 \times 3 \times 1 = 9$

When 2 is fixed at unit place we have 3, 4 and 5 i.e., 3 digits for hundreds place and remaining 3 digits (out of 5) for the tens place.

$$\begin{array}{|c|c|c|} \hline 2 & 3 & 1 \\ \hline \end{array} \Rightarrow 2 \times 3 \times 1 = 6$$

When 4 is fixed at unit place, we have 3 and 5 i.e., only two digits for hundreds place and remaining 3 digits for tens place.

Case 2.

4	3	2	2

 = 48

4 digit numbers = 48

Case 3.

4	3	2	1	2

 = 48

5 digit numbers = 48

Thus there are total $15 + 48 + 48 = 111$ even numbers greater than 300.

- 12**

4	4	3

 = 48

Hundred's place cannot assume 0.

- 13** The number of permutations in which he can arrange all his 7 novels together = $7! = 5040$

Hence choice (a) is the correct one.

- 14** The number of ways in which he can arrange any 4 novels
- $$= {}^7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = 840$$

Hence choice (d) is the correct one.

- 15** The number of ways in which he can arrange at most 4 novels

$$\begin{aligned} &= {}^7P_1 + {}^7P_2 + {}^7P_3 + {}^7P_4 \\ &= 7 + 42 + 210 + 840 = 1099 \end{aligned}$$

Hence choice (b) is the correct one.

Hint Either he can arrange 1 novel or 2 novels or 3 novels or 4 novels. But he cannot arrange more than 4 novels.

- 16** The number of ways in which he can arrange at least 4 novels

$$\begin{aligned} &= {}^7P_4 + {}^7P_5 + {}^7P_6 + {}^7P_7 \\ &= 840 + 2520 + 5040 + 5040 = 13440 \end{aligned}$$

Hence choice (a) is the correct one.

Hint Either he can arrange 4 novels or 5 novels or 6 novels or 7 novels. But he would not have to arrange less than 4 novels.

- 17** Number of permutations of 7 different novels taken 4 at a time, when the first novel is always there = $4 \cdot ({}^6P_3) = 480$
Hence choice (b) is the correct one.

Hint Since one novel is always there in the shelf, so he has to take any 3 novels from the remaining 6 novels to arrange them. But the first novel, which is already there in the bookshelf, can be arranged in 4 ways, as there are four places available to arrange the first novel. X, Y and Z denote the other 3 novels.

$$\_X\_Y\_Z\_$$

- 18** Number of permutations of 7 different novels taken 4 at a time, when the last novel is never kept in the shelf = ${}^6P_4 = 360$
Hence choice (a) is the correct one.

Hint Since the last novel is not to be considered for the arrangements, so there are only 6 novels left from which any 4 novels will be considered for the arrangement.

- 19** Number of permutations of 7 different things taken 4 at a time, when 2 particular novels always occur
- $$= ({}^4P_2) ({}^5P_2) = 240$$

Hence choice (c) is the correct one.

Hint Since 2 novels are already selected, so only 2 novels has to be selected from remaining 5 novels. Now arrange the 2 novels that you have just selected. The remaining 3rd novel can be arranged in 3 ways and the 4th novel can be arranged in 4 ways. Therefore total number of arrangements = $4 \times 3 \times ({}^5P_2)$. If you have any confusion, then solve this problem after studying the Combinations.

- 20** Number of permutations of n different novels taken 4 at a time, when 2 particular novels never occur = ${}^5P_4 = 120$

Hence choice (d) is the correct one.

- 21** Number of permutations of 7 different novels, taken all at a time, when 2 specified novels always come together is $2! \times (6!) = 1440$

Hence choice (d) is the correct one.

Hint Since 2 novels come together, then consider them as a single novel. Now there will be total 6 novels only, which can be arranged in $6!$ ways. But as the 2 novels can be mutually arranged in $2!$ ways, so the total number of arrangements = $2! \times 6!$.

- 22** Number of permutations of 7 different novels, taken all at a time, when 2 specified novels never come together is $7! - [2!(6!)] = 7 \times (6!) - 2 \times (6!) = 6! \times 5 = 3600$

Hence choice (a) is the correct one.

Hint Number of permutations when 2 novels never come together = Total number of permutations – Number of permutations when the 2 specified novels are always together.

- 23** There are 7 letters in the word RAINBOW and each letter is used only once. So all the 7 letters can be arranged in $7!$ ways.

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040.$$

- 24** If we fix R as the initial letter, then we have to arrange only 6 remaining letters.

$$\begin{aligned} \text{Hence required number of permutations} &= 6! \\ &= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720 \end{aligned}$$

25 If we fix R and W as the first and last letters then we have to arrange only 5 remaining letters which can be arranged in $5! = 120$ ways.

26 When R and W are the first and last letters of all the words then we can arrange them in $5!$ ways. Similarly when W and R are the first and last letters of the words then the remaining letters can be arranged in $5!$ ways.
Thus the total number of permutations $= 2 \times 5!$
 $= 2 \times 120 = 240$

☞ Alternatively Except R and W all the remaining 5 letters can be arranged in $5!$ ways and R and W can be arranged in $2!$ ways at the end positions.
Thus total number of permutations
 $= 2! \times 5! = 2 \times 120 = 240$

27 The first and last letters can be arranged in ${}^3P_2$ ways and the remaining letters can be arranged in $5!$ ways.
Hence required number of permutations
 $= {}^3P_2 \times 5! = 6 \times 120 = 720$

28 Assume R and W as a single letter, then we have only 6 letters for the arrangement.
That is A, B, I, N, O, RW , which can be arranged in $6!$ ways.
Now, R and W can also be mutually arranged in $2!$ ways.
Therefore, total number of arrangements $= 2! \times 6! = 1440$.

29 Total number of words $= 7! = 5040$
Number of words in which R and W are always together $= 1440$
Number of words in which R and W are never together $= 5040 - 1440 = 3600$.
Hence choice (a) is the correct one.

30 There are three vowels viz., A, I and O
First of all arrange all the four consonants which can be done in $4!$ ways. Now we have 5 places available for the three vowels to be filled up, which can be done in ${}^5P_3$ ways.
Thus the total number of arrangements $= 4! \times {}^5P_3 = 24 \times \frac{5!}{2!}$
 $= 24 \times 60 = 1440$

31 First of all we arrange all the 4 consonants in $4!$ ways.
Now we have 5 places for the 3 vowels as

1	C_1	2	C_2	3	C_3	4	C_4	5

1. When all the three vowels AIO are together then 5 places can be filled in 5 ways.
2. When A and I are together and O is separated from " AI " then the 5 places can be filled up in 10 ways.
3. When I and O are together and A is separated from " IO " then the 5 places can be filled up in 10 ways.
4. When A, I and O are separated from each other then the 5 places can be filled up in 10 ways.

Thus there are total $5 + 10 + 10 + 10 = 35$ ways in which 5 places can be filled up such that A is before I and I is before O .
Hence total number of ways in which all the 7 letters of

RAINBOW can be arranged in which A is before I and I is before $O = 4! \times 35 = 24 \times 35 = 840$.

☞ Alternatively All the 7 letters of the word RAINBOW can be arranged in $7!$ ways and 3 particular letters (A, I, O) can be arranged in $3!$ ways.
But the given condition is satisfied by only 1 out of 6 ways.
Hence required number of arrangements
 $= \frac{7!}{3!} = 7 \times 6 \times 5 \times 4 = 840$

32 Let all the vowels be in a single packet and all the consonants be in the other packet.
Now all the vowels can be arranged in $3!$ ways and all the consonants can be arranged in $4!$ ways.
Hence total number of arrangements
 $= 3! \times 4! = 6 \times 24 = 144$

33 First arrange the 3 vowels which can be done in $3!$ ways.
Now there are 4 places created by the 3 vowels which can be filled up by 4 consonants in ${}^4P_4$ ways.
Therefore total number of required permutations
 $= 3! \times {}^4P_4 = 6 \times 24 = 144$

34 $\underline{1} \quad \underline{2} \quad \underline{3} \quad \underline{4} \quad \underline{5} \quad \underline{6} \quad \underline{7}$
In order that vowels may occupy even places arrange all the 3 vowels, then arrange all the four consonants in four places.
Thus the required number of arrangements $= 3! \times 4! = 144$

35 $\underline{1} \quad \underline{2} \quad \underline{3} \quad \underline{4} \quad \underline{5} \quad \underline{6} \quad \underline{7}$
First of all arrange any 3 consonants at even places in ${}^4P_3$ ways. Now the newly created four odd places can be filled by the remaining letters which includes 3 vowels and 1 consonants, which can be done in ${}^4P_4$ ways. Hence the required number of permutation
 $= {}^4P_3 \times {}^4P_4 = 24 \times 24 = 576$

36 First of all arrange all the four consonants R, N, B, W in $4!$ ways.
Then there are 5 places to be filled up by the vowels. But any two vowels are always together then we assume that there are only two vowels which can be filled in 5 places in ${}^5P_2$ ways. But we have to take any two vowels together out of 3 vowels then this can be done in ${}^3P_2$ ways.
Hence the total number of permutation $= {}^5P_2 \times {}^3P_2 \times 4!$
 $= 20 \times 6 \times 24$
 $= 2880$

☞ Alternatively Since there can be only 3 cases
(i) When A, I and O are separate from each other
(ii) When A, I and O can always together
(iii) When any two vowels out of three vowels A, I and O are together.

Now we need to calculate the value of Case (iii).

$\therefore$ Required number of permutations

$= \text{Total number of permutations} - [\text{Case (i)} + \text{Case (ii)}]$

$\therefore 7! - (1440 + 720) = 5040 - (2160) = 2880$

- 37 The correct order of the letters is as follows :

A, B, I, N, O, R, W

Number of words begin with A = $6!$

Number of words begin with B = $6!$

Number of words begin with I = $6!$

Number of words begin with N = $6!$

Number of words begin with O = $6!$

Number of words begin with RAB = $4!$

Number of words begin with RAIB = $3!$

Now the next word is RAINBOW (it is the first word which begins with RAIN)

So the ranking of the word RAINBOW

$$= 5 \times 6! + 4! + 3! + 1 = 3631$$

- 38 There are 5 consonants and 2 vowels in the word STRANGE out of 7 places for the 7 letters, 4 places are odd and 3 places are even.

2 vowels can be arranged in 4 odd places in ${}^4P_2$ ways and then 5 consonants can be arranged in the remaining 5 places in ${}^5P_5$ ways.

Hence, the required number of ways = ${}^4P_2 \times {}^5P_5 = 1440$

- 39 $\frac{2}{1} \frac{4}{3} \frac{6}{5}$

First of all arrange 3 vowels at 3 even places, which can be done $3!$ ways. Again arrange the 3 consonants at 3 odd places which can be done in $3!$ ways.

Hence required number of permutations = $3! \times 3! = 36$ ways

☛ Alternatively

3	3	2	2	1	1

 = 36 ways

Assume vowels are even numbers say 2, 4, 6 and consonants are odd numbers say 1, 3, 5.

Now, if we have to form the numbers in which odd integers always occupy odd places then this can be done as above. (O, E, O, E, O, E)

- 40 At least two consonants remain together means

(i) Any two consonants are together

(ii) All the three consonants are together

which implies that all of the consonants are not separated from each other.

Hence required number of permutations

= (Total number of permutations

– number of permutations when all of the vowels are separated from each other)

$$= 6! - (3! \times {}^4P_3) = 720 - 144 = 576$$

- 41 Since neither two men nor two women sit together, it means they sit on the alternate positions.

M	W	M	W	M	W	M	W	M

Therefore first of all we arrange 4 women in $4!$ ways

then we arrange 5 men in newly created 5 places in

${}^5P_5 = 5!$ ways. Thus the total number of arrangements

$$= 4! \times 5! = 2880.$$

- 42 Since no two men can sit together, it means there must be 4 women separating all the 5 men. So, first of all let the 4 women sit in a row, then allow the 5 men to sit in such a way so that no two men sit together.

Essentially, this is the same problem as the previous one.

Therefore the required number of permutations

$$= 4! \times 5! = 2880.$$

- 43 Since no two women can sit together, it means there must be at least 3 men separating all the 4 women. But since men are more than 3, so let them sit before the women take their places. When men sit at 5 places (in $5!$ ways), there will be 6 places available for 4 women.

Therefore number of required permutations

$$= 5! \times {}^6P_4 = 43200.$$

- 44 Total number of persons = 9

So they can be arranged, without any restriction, in $9!$ ways.

But when all the four women sit together, consider them as one unit, then the number of ways in which they can be arranged = $(5 + 1)! \times 4! = 6! \times 4!$

Therefore the required number of ways in which all the four women do not sit together

$$= 9! - (6! \times 4!) = 9 \times 8 \times 7 \times (6!) - (6! \times 4!)$$

$$= 6!(7 \times 8 \times 9 - 4 \times 3 \times 2 \times 1)$$

$$= 345600.$$

- 45 There are total 9 places out of which 4 are even and rest 5 places are odd.

4 women can be arranged at 4 even places in $4!$ ways.

and 5 men can be placed in remaining 5 places in ${}^5P_5$ ways.

Hence, the required number of permutations

$$= 4! \times {}^5P_5 = 24 \times 120 = 2880$$

- 46 First of all the 6 men can be arranged in $6!$ ways then the 3 women can be arranged in 7 places in ${}^7P_3$ ways.

Therefore the required number of arrangements

$$= 6! \times {}^7P_3 = 720 \times 210 = 151200.$$

- 47 There are two cases : In the first case a man is sitting at the first chair and in the second case a woman is sitting at the first chair.

M	W	M	W	M	W	M	W
1	2	3	4	5	6	7	8
W	M	W	M	W	M	W	M

First case first chair can be occupied by any 1 of the 4 men, third chair can be occupied by 1 of the remaining 3 men, fifth chair can be occupied by the 1 of the remaining 2 men and the seventh chair can be occupied by the last man. Thus chair numbers 1, 3, 5 are 7 can be occupied by the 4 men in $4!$ ways. Similarly the remaining chairs can be occupied by the 4 women in $4!$ ways. Thus all the 8 people

can be seated in $4! \times 4!$ ways.

Second case in this case also, all the 8 people can be arranged in $4! \times 4!$ ways. Therefore total number of required ways = $4! \times 4! + 4! \times 4! = 2(4! \times 4!) = 1152$

48 12 soldiers can stand in a queue in $12!$ ways.

49 Any 6 soldiers can stand in one queue and the remaining 6 soldiers can stand in another queue. And then the two queues can be mutually arranged.

Therefore the required number of arrangements

$$= {}^{12}P_4 \times {}^6P_6 \times 2! = {}^{12}P_6 \times 6! \times 2!$$

50 The required number of arrangements

$$= {}^{12}P_4 \times {}^8P_4 \times {}^4P_4 \times 3!$$

51 The required number of ways = ${}^{12}P_3 \times {}^9P_4 \times {}^5P_5 \times 3!$

52 Assume that the 3 soldiers constitute only one soldier. So there are now $12 - 3 + 1 = 10$ soldiers. Thus 10 soldiers can be arranged in $10!$ ways. And the 3 soldiers can be mutually arranged in $3!$ ways. Therefore the required number of ways

$$= 10! \times 3! = 21772800.$$

53 Required number of ways = $12! - (11! \times 2!)$

$$= 11! \times (12 - 2)$$

$$= 11! \times 10 = 399168000$$

54 Total number of permutations = $12!$

Number of permutations when three particular books are together = $10! \times 3!$

$\therefore$ Number of permutations when three particular books are not together = $12! - 10! \times 3!$

$$= 10!(12 \times 11 - 6) = 10! \times 126 = 457228800$$



NOTE In this question out of the 3 particular books any 2 particular books can be together but all of the 3 particular books cannot be together.

55 First of all arrange the remaining 9 books in $9!$ ways. Then there will be 10 new places on the either side of these books where the 3 books can be arranged in ${}^{10}P_3 = 720$ ways.

Therefore the total number of ways in which none of the 3 particular books is together = $9! \times 720 = 261273600$.

56 All the 3 books on Physics can be mutually arranged in $3!$ ways.

Similarly, 4 books on Chemistry can be mutually arranged in $4!$ ways.

And 5 books on Biology can be arranged in $5!$ ways.

Besides, the three sets of Physics, Chemistry and Biology can be mutually arranged in $3!$ ways.

Therefore, required number of permutations

$$= 3! \times (3! \times 4! \times 5!) = 103680.$$

57 The signals can be made by using at a time one or two or three or four or five or six flags out of 6 different coloured flags.

$\therefore$ Required number of flags

$$= {}^6P_1 + {}^6P_2 + {}^6P_3 + {}^6P_4 + {}^6P_5 + {}^6P_6$$

$$= 6 + 30 + 120 + 360 + 720 + 720 = 1956$$

58 Required number of signals = ${}^6P_3 = 120$.

59 Required number of permutations = ${}^{10}P_3 = 720$

Alternatively First prize can be given to any of the 10 students and second prize can be given to any one of the remaining 9 students and the third prize can be given to any one of the remaining 8 students.

Therefore it can be done in

$${}^{10}P_1 \times {}^9P_1 \times {}^8P_1 = 10 \times 9 \times 8 = 720 \text{ ways.}$$

60 A passenger from any station may purchase ticket for anyone of the other 9 stations. Therefore, there must be 9 tickets in each station.

Therefore total number of different tickets = $9 \times 10 = 90$

61 Since we cannot wear more than one ring in any finger means repetition is not allowed.

Hence, the first ring can be worn in any of the available 4 fingers in 4 ways and second ring can be worn in any of the remaining 3 fingers in 3 ways and third ring can be worn in any of the remaining 2 fingers in 2 ways.

So the required number of ways in which all the 3 rings can be worn in 4 fingers = $4 \times 3 \times 2 = 24$.

Alternatively We have 4 places to be filled up by 3 different things (i.e., rings), which can be done in ${}^4P_3$ ways.

$${}^4P_3 = 24$$

Introductory Exercise 19.2

1 In the given word there are 6 letters of which E occurs 2 times.

$$\begin{aligned} \text{Hence the required number of ways} &= \frac{6!}{2!} \\ &= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2} = 360 \end{aligned}$$

2 There are total 7 letters of which A occurs 3 times and N occurs 2 times.

$$\text{Hence, the required number of ways} = \frac{7!}{3! \cdot 2!} = 420$$

3 There are total 10 letters of which R occurs 2 times, E occurs 3 times.

$$\text{Hence the required number of ways} = \frac{10!}{2! \cdot 3!} = 302400$$

4 Required number of permutations = $\frac{13!}{3! \cdot 4! \cdot 2! \cdot 2!}$

$$= 10810800$$

5 There are 6 different letters in the word which are A, S, I, N, T, O.

The number of arrangements of 6 different letters taken 4 at a time = ${}^6P_4 = 360$

- 6** There are total 9 letters in the word COMMITTEE in which there are 2M's, 2T's, 2E's.

∴ The number of ways in which 9 letters can be arranged

$$= \frac{9!}{2! \times 2! \times 2!} = 45360$$

There are 4 vowels O, I, E, E in the given word. If the four vowels always come together, taking them as one letter we have to arrange 5 + 1 = 6 letters which include 2Ms and 2Ts and this be done in $\frac{6!}{2! \times 2!} = 180$ ways.

In which of 180 ways, the 4 vowels O, I, E, E remaining together can be arranged in $\frac{4!}{2!} = 12$ ways.

∴ The number of ways in which the four vowels always come together = $180 \times 12 = 2160$.

Hence, the required number of ways in which the four vowels do not come together = $45360 - 2160 = 43200$

- 7** Consonants : C, M, M, T, T Vowels : O, I, E, E

Total number of consonants = 5

Number of ways of arranging 5 consonants = $\frac{5!}{2!2!} = 30$

Total number of places created for vowels = 6

Number of ways of arranging 4 consonants in 6 places = ${}^6P_4 = 360$

But since two vowels are same, so the 4 vowels can be arranged in $360 / 2 = 180$ ways

Therefore the required number of ways = $30 \times 180 = 5400$

- 8** $\frac{14!}{4! \times 2! \times 2! \times 2! \times 2! \times 2!} = 113513400$

- 9** When L and T are fixed as first and last letters of the word, then we have only 6 letters to be arranged.

Hence required number of permutations = $\frac{6!}{2! \times 2! \times 2!} = 90$

- 10** $\underbrace{M, M, T, T, H, C, S}_7 \underbrace{A, A, E, I}_4$

When all the vowels are together then $n = 7 + 1 = 8$

∴ Required number of permutations = $\frac{8! \times 4!}{2! \times 2! \times 2!} = 120960$

Hint (M, T and A occur 2 times and A A E I can be arranged mutually in $\frac{4!}{2!}$ ways)

- 11** I, M, P, O, R, T, A, N, T

Total number of permutations = $\frac{9!}{2!} = 181440$

Number of permutations when 2T's are together

$$= \frac{8! \times 2!}{2!} = 8! = 40320$$

∴ Number of permutations when 2T's are not together

$$= 181440 - 40320 = 141120$$

- 12** T, T, T, N, N, N, G, G, G, R, I, I, I

There are total 13 letters of which 3 are T's, 3 are N's, 3 are G's, 3 are I's.

Also there are 6 even places and 7 odd places. 3Is (i.e., vowels) can be arranged in the 6 even places in $\frac{{}^6P_3}{3!} = 20$ ways.

Now, we are left with 10 places in which 10 letters can be filled up in $\frac{10!}{3! \times 3! \times 3!} = 16800$

∴ The total number of permutations in which vowels occupy even places = $20 \times 16800 = 336000$

- 13** Correct order of the letters in D, G, I, I, O, O, P, R, S, U

Word begin with **Number of words**

D	$\frac{9!}{2!2!}$
G	$\frac{9!}{2!2!}$
I	$\frac{9!}{2!}$

Word begin with **Number of words**

O	$\frac{9!}{2!}$
PD	$\frac{8!}{2!2!}$
PG	$\frac{8!}{2!2!}$
PI	$\frac{8!}{2!}$
PO	$\frac{8!}{2!}$
PRD	$\frac{7!}{2!2!}$
PRG	$\frac{7!}{2!2!}$
PRI	$\frac{7!}{2!}$
PROD	$\frac{5!}{2!}$

Now the next two words are

PRODIGIOSU and PRODIGIOUS

Hence the rank of the word PRODIGIOUS

$$\begin{aligned}
 &= 2 \left(\frac{9!}{2!2!} \right) + 2 \left(\frac{9!}{2!} \right) + 2 \left(\frac{8!}{2!2!} \right) + 2 \left(\frac{8!}{2!} \right) \\
 &\quad + 2 \left(\frac{7!}{2!2!} \right) + \frac{7!}{2!} + \frac{5!}{2!} + 2 \\
 &= 3 \left(\frac{9!}{2!} \right) + 3 \left(\frac{8!}{2!} \right) + 7! + \frac{5!}{2!} + 2 \\
 &= 544320 + 60480 + 5040 + 60 + 2 \\
 &= 609902
 \end{aligned}$$

- 14 The correct order of letters is D, E, E, E, E, Q, R, S, S, T, U.

$$\text{Number of words begin with DEEEQR} = \frac{4!}{2!} = 12$$

$$\text{Number of words begin with DEEEQS} = 4! = 24$$

$$\text{Number of words begin with DEEEQT} = \frac{4!}{2!} = 12$$

Now, the next two words are

DEEEQURSST and DEEEQURSTS.

Hence the 50th word is DEEEQURSTS.

- 15 There are 7 digits 1, 2, 0, 2, 4, 2, 4 in which 2 occurs 3 times, 4 occurs 2 times.

$$\therefore \text{Number of 7 digit numbers} = \frac{7!}{3! \times 2!} = 420$$

But out of these 420 numbers there are some numbers which begin with '0' and they are not 7-digit numbers. The number of such numbers beginning with '0'.

$$= \frac{6!}{3! \times 2!} = 60$$

$$\text{Hence the required number of 7 digits numbers} = 420 - 60 = 360$$

☞ Alternatively

$$\begin{array}{|c|c|c|c|c|c|c|} \hline 6 & 6 & 5 & 4 & 3 & 2 & 1 \\ \hline \end{array} \Rightarrow 6 \cdot (6!)$$

Since the left most place can assume only non-zero digit hence it can consider only 6 digits. Now the remaining 6 places can be filled up in normal ways.

$$\text{But since 2 occurs 3 times, 4 occurs 2 times, therefore required number of 7 digit numbers} = \frac{6 \cdot (6!)}{3! \cdot 2!} = 360$$

- 16 Required number of 6 digit numbers = $\frac{6!}{4! \cdot 2!} = 15$

$$17 \begin{array}{|c|c|c|c|c|c|} \hline 5 & 4 & 3 & 2 & 1 & 3 \\ \hline \end{array} \Rightarrow 3 \cdot (5!)$$

Unit place can assume any of the three even digits viz., 2, 4, 6 and rest of the places can be filled up in 5! ways. But since digit 7 occurs 2 times.

$$\therefore \text{Required number of numbers} = \frac{3 \cdot (5!)}{2!} = 180$$

$$18 \begin{array}{|c|c|c|c|c|c|c|} \hline 6 & 6 & 5 & 4 & 3 & 2 & 1 \\ \hline \end{array}$$

Since left most place can assume only non-zero digit and rest can assume all the given digits.

Therefore the number of numbers = $6 \cdot (6!)$

But since 5 occurs 2 times and 2 occurs 2 times

$$\text{Hence the required number of numbers} = \frac{6 \cdot (6!)}{2! \cdot 2!} = 1080$$

- 19 REGUITA

REG T

There are total 11 letters, but since 2T's are together.

Therefore number of numbers when 2T's are together

$$= \frac{10!}{2! \times 2! \times 2! \times 2!} = 453600$$

$$\begin{array}{cccc} \uparrow & \uparrow & \uparrow & \uparrow \\ R & E & G & T \end{array}$$

Here R is 2 times, E is 2 times, G is 2 times and T is 2 times.

- 20 SUMPTO

S U

Number of ways in which 2U's are always together is

$$\frac{7!}{2!} \times \frac{2!}{2!} = 2520$$

$$\text{Total number of permutations} = \frac{8!}{2! \cdot 2!} = 10080$$

$$\therefore \text{Number of ways in which 2U's are not together} = 10080 - 2520 = 7560$$

- 21 AFLTON

A O

There are 4 consonants F, L, T and N and 4 vowels A, A, O, O

$$4 \text{ vowels can be arranged in } \frac{4!}{2! \cdot 2!} = 6 \text{ ways.}$$

$$\text{Now the 4 consonants can be arranged in 4 places in } 4! = 24 \text{ ways.}$$

$$\text{Therefore required number of permutations} = (6 \times 24) \times 2! = 288$$

Hint There are two cases :

(i) OCOCOCOC (ii) COCOCOCO
where O → vowel, C → consonant

- 22 SOTHAYER

S O

$$\text{Since two S's and two O's together therefore we can consider there are only 8 letters, which can be permuted in } \frac{8! \times 2! \times 2!}{2! \times 2!} = 8! = 40320 \text{ ways}$$

- 23 Total number of books = $3 \times 4 = 12$ in which each of 4 different books is repeated 3 times.

Hence the required number of permutations

$$= \frac{12!}{3! \times 3! \times 3! \times 3!} = \frac{12!}{(3!)^4} = 369600$$

- 24 There are total $3 + 4 + 5 = 12$ marbles of which 3 are red (alike), 4 are green (alike) and 3 are pink (alike).

$$\therefore \text{The required number of arrangements} = \frac{12!}{3! \times 4! \times 5!} = 27720$$

- 25 Total number of ways of arranging the 12 umbrellas

$$= \frac{12!}{3!4!5!}$$

Number of ways of arranging 12 umbrellas when the umbrellas of the same colour are together = 3!

Therefore the required number of ways

$$= \frac{12!}{3!4!5!} - 3! = 27714$$

Introductory Exercise 19.3

- 1 Ten thousands place can assume only non-zero digits hence ten thousands place can be filled up in 4 ways and thousands place can be filled up in 5 ways since repetition is allowed (and 0 can be filled up in this place). Similarly hundreds, tens and unit places can be filled up in 5 ways each.

$$\therefore \text{The required number of numbers} = 4 \times 5 \times 5 \times 5 \times 5 \\ = 2500$$

- 2 The numbers lying between 9 and 1000 consist of 2 or 3 digits in which repetitions are allowed.

Case 1. (For two digit numbers)

As the tens place can be filled by only non-zero digits and unit digit can be filled up in 6 ways.

$$\therefore \text{The number of 2 digit numbers} = 5 \times 6 = 30$$

Case 2. (For three digit numbers)

Hundreds place can be filled up by only non-zero digits hence it can be done in 5 ways and the rest of the places i.e., tens and unit places can be filled up in 6 ways each.

$$\therefore \text{The number of 3 digit numbers} = 5 \times 6 \times 6 = 180$$

$$\therefore \text{Total number of numbers} = 30 + 180 = 210$$

- 3 Since even digit may not occupy an even place therefore even places can be filled up in only 3 ways (with odd digits) and rest of the 3 odd places can be filled up in 5 ways with all the 5 digits.

Therefore the required number of numbers

$$= \begin{array}{|c|c|c|c|c|} \hline 5 & 3 & 5 & 3 & 5 \\ \hline \end{array}$$

$$= 5 \times 3 \times 5 \times 3 \times 5 = 1125$$

- 4 Number of numbers of 4 digit in which repetition allowed $= 9 \times 10 \times 10 \times 10 = 9000$

Number of numbers of 4 digit in which repetitions is not allowed $= 9 \times 9 \times 8 \times 7 = 4536$

Hence the required number of numbers of 4 digit

$$= 9000 - 4536 = 4464$$

- 5 Total number of 3 digit numbers $= 9 \times 10 \times 10 = 900$

Number of 3 digit numbers in which none of the digits is 9

$$= 8 \times 9 \times 9 = 648$$

$\therefore$ Number of 3 digit numbers in which there is atleast one digit is 9 $= 900 - 648 = 252$

- 6 Total one digit numbers $= 4$

Total 2 digit numbers $= 4 \times 4 = 16$

Total 3 digit numbers $= 4 \times 4 \times 4 = 64$

Number of 4 digit numbers beginning with

$$1 = 4 \times 4 \times 4 = 64$$

Number of 4 digit numbers beginning with

$$2 = 4 \times 4 \times 4 = 64$$

Number of 4 digit numbers beginning with

$$3 = 4 \times 4 \times 4 = 64$$

Number of 4 digit numbers beginning with 41 $= 4 \times 4 = 16$

Number of 4 digit numbers beginning with 42 $= 4 \times 4 = 16$

Number of 4 digit numbers beginning with 431 $= 4$

Number of 4 digit numbers beginning with 432 $= 1$

Total number of 4 digit numbers

$$= 64 + 64 + 64 + 16 + 16 + 4 + 1 = 229$$

Hence the total number of natural numbers not exceeding 4321 is $4 + 16 + 64 + 229 = 313$

- 7 First ring can be worn in any of the 3 fingers similarly second, third and fourth ring can be worn in any of the three fingers.

$$\therefore 4 \text{ rings can be worn in } 3 \times 3 \times 3 \times 3 = 81 \text{ ways.}$$

- 8 First prize can be given away to 4 boys in 4 ways.

Similarly second, third, fourth and fifth prizes can also be given away to four boys in 4 ways.

Hence the required number of ways in which all the 5 prizes can be given away to 4 boys

$$= 4 \times 4 \times 4 \times 4 \times 4 = 1024$$

- 9 The first ball can be placed in any one of the n cells in n ways. The second ball can also be placed in any one of the n cells in n ways.

$\therefore$ The first and second balls can be placed in n cells in $n \times n$ i.e., n^2 ways.

Similarly each of the rest balls can be placed in n ways.

Hence the required number of ways $= n \times n \times \dots \times n$ times $= n^n$

- 10 Since each cell is to be occupied, 1 ball is to be placed in one cell. First ball can be placed in any one of the n cells in n ways, 2nd ball can be placed in any one of the remaining $(n - 1)$ cells in $(n - 1)$ ways, similarly 3rd, 4th ... ball can be arranged in $(n - 2), (n - 3), \dots$ ways.

Hence the required number of ways

$$= n(n - 1)(n - 2) \dots 2 \cdot 1 = n!$$

- 11 First letter can be posted in 4 letter boxes in 4 ways.

Similarly second letter can be posted in 4 letter boxes in 4 ways and so on.

Hence all the 5 letters can be posted in

$$= 4 \times 4 \times 4 \times 4 \times 4 = 1024$$

Introductory Exercise 19.4

- 1 Required number of permutations = $5! = 120$
- 2 Assume there are only 5 boys since two boys always remain together and these two boys can be arranged mutually in 2 ways.
 $\therefore$ Required number of permutations = $4! \times 2 = 48$
- 3 Required number of permutations = $\frac{1}{2} \times 5! = 60$
 (Since there is no difference in clockwise and anticlockwise arrangements)
- 4 Total number of arrangements = $6! = 720$
 Number of arrangements in which two ladies are together = $2 \times 5! = 240$
 $\therefore$ Number of arrangements in which two ladies are never together = $720 - 240 = 480$
Alternatively First of all place the five men in $4!$ ways, then place the two ladies in any 5 spaces in ${}^5P_2$ ways.
 Hence, the required number of ways = $4! \times {}^5P_2 = 480$
- 5 Assuming Sahara, Ambani and Mahindra as a single personality, then there are total $(10 - 3) + 1 = 8$ person, which can be arranged in $7!$ ways.
 Also Sahara and Mahindra can be mutually permuted in 2 ways. Therefore required number of arrangements = $2 \times 7! = 10080$
- 6 A person can be chosen out of 18 people in 18 ways to be seated between Musharraf and Manmohan. Now consider Musharraf, Manmohan and the third person, sitting between them, as a single personality, we can arrange them in $17!$ ways but Musharraf and Manmohan can also be arranged in 2 ways.
 $\therefore$ Required number of permutations = $18 \times (17!) \times 2$
 $= 2 \times 18!$
- 7 6 Men can be arranged in $5!$ ways then 6 women can be arranged in 6 places in $6!$ ways.
 Hence the required number of ways = $5! \times 6!$
 $= 120 \times 720 = 86400$
- 8 First of all 4 girls can be arranged in $3!$ ways then 3 boys can be seated in 4 places in ${}^4P_3$ ways.
 $\therefore$ Required number of arrangements = $3! \times {}^4P_3$
 $= 6 \times 24 = 144$
- 9 Clearly 7 sisters can sit around a table in $6!$ ways. But in clockwise and anticlockwise arrangements each of the ladies have the same neighbours.
 So, the required number of ways = $\frac{1}{2} \times (6!) = 360$
 Hence they dine together only 360 days.
- 10 Since 3 particulars flowers are together
 Hence there can be total $(10 - 3) + 1 = 8$ flowers.

These 8 flowers can be arranged $7!$ ways but the 3 flowers which are together can be arranged mutually in $3!$ ways.
 Hence, the required number of ways = $3! \times 7! = 6 \times 5040$
 $= 30240$

- 11 Sonu can have only male singer as his neighbours which can be mutually arranged in $2!$ ways.
 Similarly Alka can have only female singers as her neighbours which can be arranged in $2!$ ways.
 Hence the required number of arrangements = $2! \times 2! = 4$
- 12 Let H_1, H_2, H_3 be the respective husbands of W_1, W_2, W_3 .
 As they sit in a circular arrangement, so the total number of ways in which they can sit around the table without any restriction = $5! = 120$

Now, we have to subtract all such cases in which the couple sits on the opposite chairs.

Case I When exactly one couple sits on opposite chairs.

Let us consider H_1 sits opposite W_1 . So, H_2 can sit on any of the 4 chairs in 4 ways and then W_2 can sit on any of the 2 chairs in 2 ways. After that H_3 can sit on any of the 2 chairs in 2 ways and W_3 can sit on the remaining chair in 1 way. Therefore when a couple $H_1 - W_1$ sits opposite all others can be arranged in $4 \times 2 \times 2 = 16$ ways. But, since there are three couples, so any one couple can be selected in 3 ways.

Therefore, the total number of ways in which a couple sits on opposite chairs = $3 \times 16 = 48$ ways.

Case II When 2 couples sit on opposite chairs.

When two couples sit on opposite chairs, the third couple will naturally be sitting on opposite chairs. So there are 0 ways in which 2 couples sit on opposite chairs.

Case III When all three couples sit on opposite chairs.

Arrange H_1 and W_1 opposite each other, which can be done in 1 way. Now H_2 can be arranged on any 4 chairs and W_2 can be arranged in 1 way as she has to sit opposite her husband. And then H_3 can sit in 2 ways and her wife W_3 can sit in 1 way. Therefore the number of ways in which all the three couples sit opposite = $1 \times 4 \times 1 \times 2 \times 1 = 8$ ways

Therefore the required number of ways in which no couple sits opposite each other = $120 - (48 + 0 + 8) = 64$

- 13 First of all arrange the six students with the higher ranks (1, 2, 3, 4, 5 and 6). Now, arrange the remaining students (of lower ranks 7, 8, 9, 10, 11 and 12) between them.
 The number of ways in which first 6 candidates can be arranged in $5!$ ways and the remaining 6 candidates can be arranged in $6!$ ways.
 Therefore the required number of ways = $5! \times 6! = 86400$
- 14 First of all 10 girls can be given 10 alternate rooms in $10!$ ways, as rooms are distinguishable. Now 10 guys can be given 10 alternate rooms in $10!$ ways. However, girls and guys can exchange their rooms in 2 ways.
 Therefore the required number of ways = $2! \times (10!)^2$

Introductory Exercise 19.5

$$1 \quad {}^8C_3 = \frac{8!}{3!5!} = \frac{8 \times 7 \times 6}{6} = 56$$

$$2 \quad {}^{10}C_5 = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5 \times 4 \times 3 \times 2 \times 1 \times (5!)} = 252$$

$$3 \quad \because {}^{13}C_x = {}^{13}C_y$$

$\Rightarrow$ either $x = y$ or $x + y = 13$
But since $x \neq y$, therefore $x + y = 13$

$$4 \quad \text{Since } {}^{17}C_r = {}^{17}C_{r+3}$$

$\Rightarrow$ either $r = r + 3$ or $r + r + 3 = 17$
But since $r \neq r + 3$
 $\therefore r + r + 3 = 17 \Rightarrow r = 7$

$$5 \quad {}^7P_3 = n \cdot {}^7C_3$$

$\Rightarrow 5 \times 6 \times 7 = n \times 5 \times 7 \Rightarrow n = 6$
Alternatively ${}^nP_r = r!({}^nC_r)$
 $\Rightarrow r! = 3! = 6 = n$

$$6 \quad {}^{2n}C_3 : {}^nC_2 = 12 : 1$$

$\Rightarrow \frac{2n(2n-1)(2n-2)}{3 \times 2 \times 1} : \frac{n(n-1)}{2 \times 1} = 12 : 1$
 $\Rightarrow (2n-1) = 9$
 $\Rightarrow n = 5$

Alternatively Go through options.

7 Go through options : The least possible value of n is 8. To verify the result put the value of n as given in the choices.
Let us consider choice (a) i.e., $n = 6$, then

$${}^5C_3 + {}^5C_4 > {}^6C_3$$

$10 + 5 > 20$, which is wrong.

Again consider choice (c) i.e., $n = 7$, then

$${}^6C_3 + {}^6C_4 > {}^7C_3$$

$20 + 15 > 35$, which is also wrong.

Now consider choice (b) i.e., $n = 8$, then

$${}^7C_3 + {}^7C_4 > {}^8C_3$$

$35 + 35 > 56$, which is correct.

Hence the least possible value of $n = 8$.

Now you need not to check the option (d).

$$8 \quad \frac{{}^nP_x}{{}^nC_x} = \frac{(n!)}{(n-x)!} = \frac{336}{56}$$

$$\Rightarrow \frac{{}^nP_x}{{}^nC_x} = x! = 6$$

$$\Rightarrow x! = 3!$$

$$\Rightarrow x = 3$$

Now since ${}^nP_3 = 336$

$$\Rightarrow n(n-1)(n-2) = 8 \times 7 \times 6$$

$$\Rightarrow n = 8$$

Alternatively Go through options.

9 If n is odd, the greatest value of ${}^nC_r = {}^nC_m$

$$\text{where } m = \frac{(n-1)}{2} \text{ or } m = \frac{(n+1)}{2}$$

$$\therefore x = \frac{(11-1)}{2} = 5 \text{ or } x = \frac{(11+1)}{2} = 6$$

Hence (d) is the correct choice.

10 If n is even, the greatest value of ${}^nC_r = {}^nC_m$

$$\text{where } m = \frac{n}{2} \Rightarrow x = \frac{12}{2} = 6$$

11 ${}^{14}C_x$ is maximum when $x = \frac{14}{2} = 7$

Alternatively Go through options.

12 Since there are total 10 persons and out of these 10 persons we have to select any 5 persons which can be done in ${}^{10}C_5$ ways.

$$\therefore {}^{10}C_5 = \frac{10!}{5! \times 5!} = 252$$

13 Out of 10 questions, 6 questions can be selected in ${}^{10}C_6$ ways

$$\therefore {}^{10}C_6 = 210$$

14 Since question number 1 is compulsory so we have to choose only 5 questions from rest of the 9 questions, which can be done in ${}^9C_5$ ways

$$\therefore {}^9C_5 = 126$$

15 She may invite one or more friends by selecting either 1 or 2 or 3 or 4 or 5 friends out of 5 friends.

$\therefore$ 1 friend can be selected out of 5 in ${}^5C_1$ ways

2 friends can be selected out of 5 in ${}^5C_2$ ways

3 friends can be selected out of 5 in ${}^5C_3$ ways

4 friends can be selected out of 5 in ${}^5C_4$ ways

5 friends can be selected out of 5 in ${}^5C_5$ ways

Hence the required number of ways

$$= {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$$

$$= 5 + 10 + 10 + 5 + 1 = 31$$

Alternatively ${}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$

$$= 2^5 - 1 = 31$$

Since, ${}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n = 2^n - 1$

16 4 men can be selected out of 7 men in ${}^7C_4$ ways

and 2 ladies can be selected out of 6 ladies in ${}^6C_2$ ways

Hence, the required number of ways $= {}^7C_4 \times {}^6C_2$

$$= 35 \times 15 = 525$$

- 17** A committee of 5 persons is to be formed from 6 gentlemen and 4 ladies by taking.

- (i) 1 lady out of 4 and 4 gentlemen out of 6.
- (ii) 2 ladies out of 4 and 3 gentlemen out of 6.
- (iii) 3 ladies out of 4 and 2 gentlemen out of 6.
- (iv) 4 ladies out of 4 and 1 gentleman out of 6.

In case I the number of ways

$$= {}^4C_1 \times {}^6C_4 = 4 \times 15 = 60$$

In case II the number of ways

$$= {}^4C_2 \times {}^6C_3 = 6 \times 20 = 120$$

In case III the number of ways

$$= {}^4C_3 \times {}^6C_2 = 4 \times 15 = 60$$

In case IV the number of ways

$$= {}^4C_4 \times {}^6C_1 = 1 \times 6 = 6$$

Hence, the required number of ways

$$= 60 + 120 + 60 + 6 = 246$$

- 18** A committee of 7 persons retaining a majority of Indians can be made from 6 Americans and 7 Indians by taking

- (i) 1 American out of 6 and 6 Indians out of 7.
- (ii) 2 Americans out of 6 and 5 Indians out of 7.
- (iii) 3 Americans out of 6 and 4 Indians out of 7.

In case I the number of ways = ${}^6C_1 \times {}^7C_6 = 6 \times 7 = 42$

In case II the number of ways = ${}^6C_2 \times {}^7C_5 = 15 \times 21 = 315$

In case III the number of ways

$$= {}^6C_3 \times {}^7C_4 = 20 \times 35 = 700$$

Hence, the required number of ways

$$= 700 + 315 + 42 = 1057$$

- 19** Required number of ways = ${}^6C_4 \times {}^8C_3 = 15 \times 56 = 840$

- 20** The number of committees in which both Miss. A and Mr. B are always members. We can select rest 3 men from the remaining 5 in ${}^5C_3$ ways and rest 2 women from the remaining 7 in ${}^7C_2$ ways.

$\therefore$ The required number of ways in which both Miss. A and Mr. B are always included

$$= {}^5C_3 \times {}^7C_2 = 10 \times 21 = 210$$

Hence the required number of ways in which Miss. A and Mr. B do not serve together = $840 - 210 = 630$

- 21** Since 3 particular members are already selected, then we are required to select only 4 members out of the remaining 8 members. It can be done in ${}^8C_4$ ways.

$$\therefore {}^8C_4 = 70$$

- 22** Since 3 particular members are not included hence we have to select 7 members out of remaining 8 members. It can be done in ${}^8C_7$ ways

$$\therefore {}^8C_7 = 8$$

- 23** 3 experts including atleast an engineer and a manager can be selected by taking.

(i) 2 managers out of 3 and 1 engineer out of 3.

(ii) 1 manager out of 3 and 2 engineer out of 3.

(iii) 2 persons out of 6 (3 managers and 3 engineers) and 1 person out of one who is both engineer and manager.

In case I, the number of ways = ${}^3C_2 \times {}^3C_1 = 9$

In case II, the number of ways = ${}^3C_1 \times {}^3C_2 = 9$

In case III, the number of ways = ${}^6C_2 \times {}^1C_1 = 15$

Hence, the required number of ways = $9 + 9 + 15 = 33$.

- 24** A committee of 5 persons, consisting of at most two ladies, can be formed in the following ways.

(i) selecting 5 gents only out of 6.

(ii) selecting 4 gents only out of 6 and one lady out of 4.

(iii) selecting 3 gents only out of 6 and two ladies out of 4.

In case I, the number of ways = ${}^6C_5$

In case II, the number of ways = ${}^6C_4 \times {}^4C_1$

In case III, the number of ways = ${}^6C_3 \times {}^4C_2$

$\therefore$ Required number of ways

$$= {}^6C_5 + {}^6C_4 \times {}^4C_1 + {}^6C_3 \times {}^4C_2$$

$$= 6 + 60 + 120 = 186$$

- 25** A voter may cast vote for either 1 candidate or 2 candidates or 3 candidates or 4 candidates out of 7.

The voter may cast vote for 1 candidate in ${}^7C_1$ ways

The voter may cast vote for 2 candidates in ${}^7C_2$ ways

The voter may cast vote for 3 candidates in ${}^7C_3$ ways

The voter may cast vote for 4 candidate in ${}^7C_4$ ways

Hence, the required number of ways

$$= {}^7C_1 + {}^7C_2 + {}^7C_3 + {}^7C_4$$

$$= 7 + 21 + 35 + 35 = 98$$

- 26** Required number of ways = ${}^5C_1 + {}^5C_2 + {}^5C_3$

$$= 5 + 10 + 10 = 25$$

- 27** Required number of ways = ${}^8C_6 \times {}^8C_5 = 28 \times 56 = 1568$

- 28** Required number of ways

$$= ({}^6C_4 \times {}^6C_2) + ({}^6C_3 \times {}^6C_3) + ({}^6C_2 \times {}^6C_4)$$

$$= (15 \times 15) + (20 \times 20) + (15 \times 15)$$

$$= 225 + 400 + 225 = 850$$

- 29** Required number of ways

$$= {}^7C_3 \times {}^5C_5 + {}^7C_4 \times {}^5C_4 + {}^7C_5 \times {}^5C_3$$

$$= 35 \times 1 + 35 \times 5 + 21 \times 10$$

$$= 35 + 175 + 210 = 420$$

- 30** Required number of ways

$$= ({}^5C_2 \times {}^5C_3 \times {}^4C_1) + ({}^5C_3 \times {}^5C_2 \times {}^4C_1) + ({}^5C_2 \times {}^5C_2 \times {}^4C_2)$$

$$= (10 \times 10 \times 4) + (10 \times 10 \times 4) + (10 \times 10 \times 6) = 1400$$

- 31** Required number of ways

$$= {}^{13}C_8 \times {}^5C_2 = 1287 \times 10 = 12870$$

- 32** One hand shake requires two different persons.
 $\therefore$ Number of hand shakes
 = number of ways of selecting two persons out of 11 persons
 $= {}^{11}C_2 = 55$

- 33** If n is even, ${}^nC_r$ is maximum when $r = \frac{n}{2}$
 $\therefore$ Number of invitees in a party = $\frac{24}{2} = 12$
 and maximum possible number of parties = ${}^{24}C_{12}$
 $= 2704156$

- 34** If n is odd, ${}^nC_r$ is maximum when

$$r = \frac{n-1}{2} \quad \text{or} \quad r = \frac{n+1}{2}$$

$$\therefore r = \frac{11-1}{2} = 5$$

$$\text{or} \quad r = \frac{11+1}{2} = 6$$

$$\therefore {}^{11}C_5 = {}^{11}C_6 = 462$$

- 35** Possible number of members in a family is either 5 or 6.
 So, the maximum possible number of family members = 6

- 36** Let there were n persons in the meeting, then number of handshakes = ${}^nC_2$

$$\therefore {}^nC_2 = 66$$

$$\Rightarrow \frac{n!}{2!(n-2)!} = 66$$

$$\Rightarrow \frac{n(n-1)}{2} = 66$$

$$\Rightarrow n(n-1) = 132$$

$$\Rightarrow n = 12$$

☞ **Alternatively** Go through options.

- 37** Since a particular player is always chosen therefore we have to choose only 10 players out of remaining 15 players.

$$\therefore \text{Required number of ways} = {}^{15}C_{10} = 3003$$

- 38** Since a player is never chosen, hence we have to choose 11 players out of remaining 15 players.

$$\therefore \text{Required number of ways} = {}^{15}C_{11} = 1365$$

- 39** Total number of bowlers = 4

Total number of wicket keepers = 2

Rest (normal) players = 10

Possible Combinations :

Bowlers	Wicket keepers	Normal players
3	1	7
3	2	6
4	1	6
4	2	5

$\therefore$ Required number of ways

$$= ({}^4C_3 \times {}^2C_1 \times {}^{10}C_7) + ({}^4C_3 \times {}^2C_2 \times {}^{10}C_6) \\ + ({}^4C_4 \times {}^2C_1 \times {}^{10}C_6) + ({}^4C_4 \times {}^2C_2 \times {}^{10}C_5) \\ = (3 \times 1 \times 7) + (3 \times 2 \times 6) + (4 \times 1 \times 6) + (4 \times 2 \times 5) \\ = 960 + 840 + 420 + 252 \\ = 2472$$

- 40** Total number of bowlers = 6

Total number of wicket keepers = 3

Total number of normal players = 11 [20 - (6 + 3)]

Possible combinations :

Bowlers	Wicket keepers	Normal players
4	2	5
5	2	4
6	2	3

$\therefore$ Required number of ways

$$= ({}^6C_4 \times {}^3C_2 \times {}^{11}C_5) + ({}^6C_5 \times {}^3C_2 \times {}^{11}C_4) \\ + ({}^6C_6 \times {}^3C_2 \times {}^{11}C_3) \\ = 20790 + 5940 + 495 = 27225$$

- 41** Total number of wicket keepers = 2

Total number of bowlers = 8

Total number of batsman = 10

Total number of all rounders = 5

1 Wicket keeper can be selected out of 2 in ${}^2C_1$ ways

2 Bowlers can be selected out of 8 in ${}^8C_2$ ways

5 Batsmen can be selected out of 10 in ${}^{10}C_5$ ways

3 All rounders can be selected out of 5 in ${}^5C_3$ ways

$\therefore$ Required number of ways = ${}^2C_1 \times {}^8C_2 \times {}^{10}C_5 \times {}^5C_3$

$$= 2 \times 28 \times 252 \times 10 = 141120$$

- 42** 1 book on Economics can be collected out of 3 in ${}^3C_1$ ways

1 book on Corporate strategy can be collected out of 4 in

${}^4C_1$ ways 1 book on Philosophy can be collected out of

5 in ${}^5C_1$ ways

Hence the required number of collections each of which consists of exactly one book on each subject

$$= {}^3C_1 \times {}^4C_1 \times {}^5C_1$$

$$= 3 \times 4 \times 5 = 60$$

- 43** At least one book on Economics can be collected out of 3 = ${}^3C_1 + {}^3C_2 + {}^3C_3 = 2^3 - 1 = 7$ ways.

Similarly one book on corporate strategy can be collected out of 4 = ${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 - 1 = 15$ ways.

Similarly one book on Philosophy can be collected out of 5 = ${}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 2^5 - 1 = 31$ ways.

Hence the required number of ways = $7 \times 15 \times 31 = 3255$

- 44 Since all the three balls are red which can be selected from 7 red balls only.

$$\therefore \text{Required number of selections} = {}^7C_3 = 35$$

- 45 Since red balls cannot be taken thus we can select 3 balls out of 6 white and 4 blue balls.

$$\therefore \text{Required number of selections} = {}^{10}C_3 = 120 \text{ ways}$$

- 46 The required number of selections = ${}^7C_1 \times {}^6C_1 \times {}^4C_1 = 168$

- 47 Total number of red balls = 5

$$\text{Total number of green balls} = 6$$

Possible combinations are :

Red	Green
2	4
3	3
4	2

$\therefore$ Required number of selections

$$= ({}^5C_2 \times {}^6C_4) + ({}^5C_3 \times {}^6C_3) + ({}^5C_4 \times {}^6C_2) = 425$$

- 48 Required number of words = ${}^{15}C_2 \times {}^5C_4$

$$= 105 \times 5 = 525$$

- 49 There are 11 letters in the given word of which 2 are A's, 2 are I's, 2 are N's and the remaining 5 letters are different. Thus we have 11 letters of 8 different kinds viz., (A, A), (I, I), (N, N), E, X, M, T, O.

A group of 4 letters can be classified as follows :

- (i) Two alike of one kind and two alike of another kind.
- (ii) Two alike and the other two different.
- (iii) All four different.

$$\text{In case I, the number of ways} = {}^3C_2 = 3$$

$$\text{In case II, the number of ways} = {}^3C_1 \times {}^7C_2 = 63$$

$$\text{In case III, the number of ways} = {}^8C_4 = 70$$

$$\text{Hence, the required number of ways} = 3 + 63 + 70 = 136$$

- 50 There are 11 letters in the given word where M occurs 2 times, A occurs 2 times and T occurs 2 times and rest letters occur one times each.

i.e., M, A, T, H, E, I, C, S

M, A, T

A group of 4 letters can be selected in the following manner.

- (i) Two alike of one kind and two alike of other kind.
- (ii) Two alike and the other two different.
- (iii) All four different.

$$\text{Case I. } {}^3C_2 \times \frac{4!}{2!2!} = 18$$

$$\text{Case II. } {}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 756$$

$$\text{Case III. } {}^8C_4 \times 4! = 1680$$

$$\therefore \text{Total required words of 4 letters} = 18 + 756 + 1680 = 2454$$

- 51 Let L_1, L_2, L_3 be 3 ladies and G_1, G_2, G_3 be 3 gents.

Any two of the ladies out of 3 can be selected in ${}^3C_2$ ways and can be arranged mutually in 2! ways.

Now we are left with 3 gents and 1 lady these 4 persons can be arranged at a round table in 3! ways.

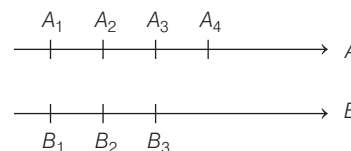
Now we have only two places among 4 persons to place the two selected ladies which can be done in ${}^2P_1$ ways.

Hence the required number of arrangements

$$= {}^3C_2 \times 2! \times 3! \times {}^2P_1$$

$$= 3 \times 2 \times 6 \times 2 = 72$$

- 52 Since 4 guests desire to sit on one side and the other 3 guests desire to sit on the other side.



Hence we are left with 11 guests only out of which we choose 5 guests for the side A in ${}^{11}C_5$ ways and remaining 6 guests can be selected for the side B in ${}^6C_6$ ways.

Further in each side 9 guests can be arranged in 9! ways.

Hence the required number of arrangements

$$= {}^{11}C_5 \times {}^6C_6 \times 9! \times 9!$$

$$= 462 \times (9!)^2$$

$$53 \quad {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + \dots + {}^{10}C_{10} = 2^{10} - 1$$

$$54 \quad \therefore {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + \dots + {}^{10}C_{10} = 2^{10} - 1$$

$$\therefore {}^{10}C_2 + {}^{10}C_3 + \dots + {}^{10}C_{10} = (2^{10} - 1) - 10 = 2^{10} - 11$$

$$55 \quad \therefore {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + \dots + {}^{10}C_9 + {}^{10}C_{10} = 2^{10} - 1$$

$$\therefore {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + \dots + {}^{10}C_9$$

$$= (2^{10} - 1) - 1 = 2^{10} - 2$$

$$= 2(2^9 - 1)$$

$$56 \quad {}^{10}C_0 + {}^{10}C_2 + {}^{10}C_4 + \dots + {}^{10}C_{10} = 2^{10-1} = 2^9$$

$$57 \quad [({}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + \dots) \times ({}^{20}C_1 + {}^{20}C_3$$

$$+ {}^{20}C_5 + \dots) \times ({}^{30}C_1 + {}^{30}C_3 + {}^{40}C_5 + \dots)$$

$$\times \dots \times ({}^{100}C_1 + {}^{100}C_3 + {}^{100}C_5 + \dots)]$$

$$= 2^9 \times 2^{19} \times 2^{29} \times \dots \times 2^{99}$$

$$= 2^{9+19+29+\dots+99} = 2^{540}$$

- 58 Larry can attend or miss the first session in 2 ways.

She can attend or miss the second session in 2 ways.

She can attend or miss the third session in 2 ways.

She can attend or miss the fourth session in 2 ways.

Thus she can attend the sessions in $2 \times 2 \times 2 \times 2 = 2^4$ ways.

But this includes the case when she does not attend even a single session. Therefore she can attend one or more sessions in $2^4 - 1 = 15$ ways.

Alternatively

She can attend the one session in ${}^4C_1$ ways

She can attend the two sessions in ${}^4C_2$ ways

She can attend the three sessions in ${}^4C_3$ ways

She can attend the four sessions in ${}^4C_4$ ways

Therefore she can attend 1 or 2 or 3 or 4 sessions in ${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 4 + 6 + 4 + 1 = 15$ ways.

- 59** The number of ways in which Madhulika can select exactly four languages $= {}^4C_4 + {}^5C_4 + {}^6C_4 + \dots + {}^{10}C_4$
 $= {}^{11}C_5 = 462$

NOTE ($\therefore {}^rC_r + {}^{r+1}C_r + {}^{r+2}C_r + \dots + {}^nC_r = {}^{n+1}C_{r+1}; r \leq n$)

Madhulika cannot select 4 languages from program 1, 2 or 3 as none of them has 4 languages.

- 60** $210 = 1 + 2 + 3 + \dots + 20$.

It means total 20 packs of parathas are there and each pack has distinct number of parathas. The number of ways in which Shivesh can select any 3 parathas

$$= {}^3C_3 + {}^4C_3 + {}^5C_3 + \dots + {}^{20}C_3$$

$$= {}^{21}C_4 = 5985$$

- 61** Since Nilekani attempts 2 problems so he can't attempt the module 1, as it has only 1 problem.

Therefore the number of ways in which he can attempt 2 distinct problems

$$= {}^2C_2 + {}^3C_2 + {}^4C_2 + \dots + {}^{40}C_2 = {}^{41}C_3 = 10660$$

- 62** Since Narainmurthi attempts 38 problems, so he has to attempt them from module 38 or module 39 or module 40 only, as other modules don't have sufficient number of problems to attempt from.

$$= {}^{38}C_{38} + {}^{39}C_{38} + {}^{40}C_{38} = {}^{41}C_{39} = 820$$

- 63** They can choose not to watch any movie or can go for 1 or 2 or 3 or all the four movies. Therefore the required number of ways in which they can watch the movies

$${}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 = 16 = 4^2$$

- 64** They can watch either 1 or more movies.

Therefore the required number of ways in which they can watch the movies

$$= {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 - {}^4C_0 = 2^4 - 1$$

- 65** They can watch 2 or more movies.

Therefore the required number of ways in which they can watch the movies ${}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 - {}^4C_0 - {}^4C_1 = 2^4 - 5$

- 66** They can watch at most 3 movies.

Therefore the required number of ways in which they can watch the movies

$$= {}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3$$

$$= 2^4 - {}^4C_4 = 2^4 - 1$$

- 67** He may choose to read none, one or any number of scripts offered.

Therefore the number of ways in which he can choose to read the scripts

$$= {}^{100}C_0 + {}^{100}C_1 + {}^{100}C_2 + \dots + {}^{100}C_{100} = 2^{100}$$

- 68** Since he reads at most 50 scripts.

Therefore the number of ways in which he can choose to read the scripts

$$= {}^{100}C_0 + {}^{100}C_1 + {}^{100}C_{50} = 2^{100} - ({}^{100}C_{51} + {}^{100}C_{52} + \dots + {}^{100}C_{100})$$

$$\Rightarrow ({}^{100}C_0 + {}^{100}C_1 + \dots + {}^{100}C_{49})$$

$$+ {}^{100}C_{50} = 2^{100} - ({}^{100}C_{51} + {}^{100}C_{52} + \dots + {}^{100}C_{100})$$

$$\Rightarrow P + {}^{100}C_{50} = 2^{100} - P$$

$$\Rightarrow P + {}^{100}C_{50} = 2^{100} - P - {}^{100}C_{50} + {}^{100}C_{50}$$

$$\Rightarrow P + {}^{100}C_{50} = 2^{100} - (P + {}^{100}C_{50}) + {}^{100}C_{50}$$

$$\Rightarrow 2(P + {}^{100}C_{50}) = 2^{100} + {}^{100}C_{50}$$

$$\Rightarrow P + {}^{100}C_{50} = \frac{1}{2}(2^{100} + {}^{100}C_{50})$$

$$\therefore ({}^{100}C_0 + {}^{100}C_1 + {}^{100}C_2 + \dots + {}^{100}C_{49})$$

$$+ {}^{100}C_{50} = \frac{1}{2}(2^{100} + {}^{100}C_{50})$$

$$\text{Hint } ({}^{100}C_0 + {}^{100}C_1 + \dots + {}^{100}C_{49})$$

$$= ({}^{100}C_{51} + {}^{100}C_{52} + \dots + {}^{100}C_{100})$$

$$\text{And } {}^{100}C_0 + {}^{100}C_1 + \dots + {}^{100}C_{100} = 2^{100}$$

- 69** The number of ways in which he can choose to read the scripts

$$= {}^{100}C_2 + {}^{100}C_3 + \dots + {}^{100}C_{98} = 2^{100} - ({}^{100}C_0$$

$$+ {}^{100}C_1 + {}^{100}C_{99} + {}^{100}C_{100})$$

$$\Rightarrow {}^{100}C_2 + {}^{100}C_3 + \dots + {}^{100}C_{98} = 2^{100} - (1 + 100 + 100 + 1)$$

$$\Rightarrow {}^{100}C_2 + {}^{100}C_3 + \dots + {}^{100}C_{98} = 2^{100} - 202$$

- 70** Since he reads at least 50 scripts.

Therefore the number of ways in which he can choose to read the scripts

$$= {}^{100}C_{50} + {}^{100}C_{51} + {}^{100}C_{100} = 2^{100} - ({}^{100}C_0$$

$$+ {}^{100}C_1 + \dots + {}^{100}C_{49})$$

$$\Rightarrow {}^{100}C_{50} + ({}^{100}C_{51} + {}^{100}C_{52} + \dots + {}^{100}C_{100})$$

$$= 2^{100} - ({}^{100}C_0 + {}^{100}C_1 + \dots + {}^{100}C_{49})$$

$$\begin{aligned}
\Rightarrow & {}^{100}C_{50} + P = 2^{100} - P \\
\Rightarrow & {}^{100}C_{50} + P = 2^{100} - P - {}^{100}C_{50} + {}^{100}C_{50} \\
\Rightarrow & {}^{100}C_{50} + P = 2^{100} - (P + {}^{100}C_{50}) + {}^{100}C_{50} \\
\Rightarrow & 2({}^{100}C_{50} + P) = 2^{100} + {}^{100}C_{50} \\
\Rightarrow & {}^{100}C_{50} + P = \frac{1}{2}(2^{100} + {}^{100}C_{50}) \\
\Rightarrow & {}^{100}C_{50} + P = 2^{99} + \frac{1}{2}({}^{100}C_{50}) \\
\therefore & ({}^{100}C_{50} + {}^{100}C_{51} + {}^{100}C_{100}) = 2^{99} + \frac{1}{2}({}^{100}C_{50})
\end{aligned}$$

Hint $({}^{100}C_{51} + {}^{100}C_{52} + \dots + {}^{100}C_{100})$
 $= ({}^{100}C_0 + {}^{100}C_1 + \dots + {}^{100}C_{49})$

- 71** If n is even, the greatest value of ${}^nC_r = {}^nC_m$; where $m = \frac{n}{2}$

It implies that the maximum number of modules can be developed when each module has $5\left(=\frac{10}{2}\right)$ subjects.

And the maximum number of modules would be ${}^{10}C_5 = 252$

- 72** If n is odd, the greatest value of ${}^nC_r = {}^nC_m$ where $m = \frac{(n-1)}{2}$ or $m = \frac{(n+1)}{2}$

It implies that the maximum number of distinct assortments can be obtained when each assortment has $\frac{(9-1)}{2} = 4$ or $\frac{(9+1)}{2} = 5$ varieties of sweets.

And the maximum number of distinct assortments $= {}^9C_4 = {}^9C_5 = 126$

- 73** If n is even, the greatest value of ${}^nC_r = {}^nC_m$; where $m = \frac{n}{2}$

It implies that the maximum number of groups can be formed when in each group there are $4\left(=\frac{8}{2}\right)$ members.

And the number of groups $= {}^8C_4 = 70$

- 74** For maximum number of trips/groups 4 members group will be travelling each time. So a particular member can go with any 3 members out of the remaining 7 members.

Therefore the maximum number of trips by a particular member $= {}^7C_3 = 35$

- 75** Mr. Shashi uses only even number of characters to write a tweet from the set of 36 characters.

$$\begin{aligned}
\therefore & {}^nC_0 + {}^nC_2 + {}^nC_4 + \dots + {}^nC_n = 2^{n-1} \\
\therefore & {}^nC_2 + {}^nC_4 + \dots + {}^nC_n = 2^{n-1} - {}^nC_0 \\
\Rightarrow & {}^nC_2 + {}^nC_4 + \dots + {}^nC_n = 2^{n-1} - 1 \\
\therefore & {}^{36}C_2 + {}^{36}C_4 + \dots + {}^{36}C_{36} = 2^{35} - 1
\end{aligned}$$

- 76** Mr. Shashi uses only odd number of characters to write a tweet from the set of 36 characters.

$$\begin{aligned}
\therefore & {}^nC_1 + {}^nC_3 + {}^nC_5 + \dots = 2^{n-1} \\
\therefore & {}^{36}C_1 + {}^{36}C_3 + {}^{36}C_5 + \dots + {}^{36}C_{35} = 2^{35}
\end{aligned}$$

- 77** Mr. Shashi uses only even number of characters from 26 alphabets and even number of characters from 10 numerals to write a tweet.

$$\begin{aligned}
\therefore & ({}^{26}C_0 + {}^{26}C_2 + {}^{26}C_4 + \dots + {}^{26}C_{26}) \\
& ({}^{10}C_0 + {}^{10}C_2 + {}^{10}C_4 + \dots + {}^{10}C_{10}) = 2^{25} \times 2^9 = 2^{34}
\end{aligned}$$

Thus we get the answer 2^{34} , which includes a case in which 0 characters are used. But, we know that he cannot tweet without writing anything in it, so 1 case has to be subtracted.

Therefore the number of ways in which he can select the required characters $= 2^{34} - 1$

- 78** Mr. Shashi uses only odd number of characters from 26 alphabets and odd number of characters from 10 numerals to write a tweet.

$$\begin{aligned}
\therefore & ({}^{26}C_1 + {}^{26}C_3 + {}^{26}C_5 + \dots + {}^{26}C_{25}) \\
& ({}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + \dots + {}^{10}C_9) = 2^{25} \times 2^9 = 2^{34}
\end{aligned}$$

- 79** When Mr. Shashi selects odd number of characters from 26 alphabets and odd number of characters from 10 numerals, he naturally he selects total even number of characters.

Thus the number of ways in which he can select the characters

$$\begin{aligned}
& = ({}^{26}C_1 + {}^{26}C_3 + {}^{26}C_5 + \dots + {}^{26}C_{25}) \\
& \quad ({}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + \dots + {}^{10}C_9) \\
& = 2^{25} \times 2^9 = 2^{34}
\end{aligned}$$

Introductory Exercise 19.6

- 1** The number of ways of choosing 4 letters from 2As, 3Bs and 4Cs = coefficient of x^4 in $(1+x+x^2)$
 $(1+x+x^2+x^3+x^4)$
 By multiplication we get $(1+\dots+11x^4+\dots+x^9)$.
 Thus we can see that the coefficient of x^4 is 11.

Therefore the required number of ways of choosing 4 letters from {A, A, B, B, B, C, C, C, C} is 11.

Hence choice (b) is the answer.

Alternatively

Case I : 0, 0 4 {CCCC}

Case II : 0, 1, 3 {ABBB}, {ACCC}, {BCCC}, {CBBB}

Case III : 0, 2, 2 {AABB}, {AACC}, {BBCC}

Case IV : 1, 1, 2 {AABC}, {BBCA}, {CCAB}

- 2 The number of ways of choosing 4 letters, such that at least 1 letter must be taken from each of 2As, 3Bs and 4Cs = coefficient of x^4 in

$$(x + x^2)(x + x^2 + x^3)(x + x^2 + x^3 + x^4)$$

By multiplication we get

$$(x^3 + \dots + 3x^4 + \dots + x^9).$$

Thus we can see that the coefficient of x^4 is 3.

Therefore the required number of ways of choosing 4 letters from {A, A, B, B, B, C, C, C, C} is 3.

Hence choice (a) is the answer.

☞ Alternatively

{AABC}, {ABBC}, {ABCC}

☞ Alternatively

First take out 1A, 1B, 1C.

Now you can select either 1A in 1 way or 1B in 1 way or 1C in 1 way.

So the total number of selections = $1 + 1 + 1 = 3$.

☞ Alternatively

{1A, 1B and 2C} or {1A, 2B, 1C} or {2A, 1B, 1C}

$$(1 \times 1 \times 1) + (1 \times 1 \times 1) + (1 \times 1 \times 1) = 3$$

- 3 Number of ways of selecting 1 tee = 1
Hence choice (b) is the answer.
- 4 Number of ways of selecting at least 1 tee
= selecting 1 tee or 2 tees or 3 tees or...or 10 tees
= $1 + 1 + 1 + \dots + 1 = 10$
Hence choice (d) is the answer.
- 5 Number of ways of selecting at most 1 tee = selecting no tee or 1 tee = $1 + 1 = 2$
Hence choice (c) is the answer.
- 6 Number of ways of selecting 4 tees = 1
Hence choice (b) is the answer.
- 7 Number of ways of selecting at least 4 tees
= selecting 4 tees or 5 tees or 6 tees or ... or 10 tees
= $1 + 1 + 1 + \dots + 1 = 7$
Hence choice (d) is the answer.
- 8 Number of ways of selecting at most 4 tees = selecting no tee or 1 tee or 2 tees or 3 tees or 4 tees
= $1 + 1 + 1 + 1 + 1 = 5$
Hence choice (a) is the answer.

- 9 Number of ways of selecting any number of tees
= selecting no tee or 1 tee or 2 tees or 3 tees or ... or 10 tees
= $1 + 1 + 1 + 1 + \dots + 1 = 11$

Hence choice (d) is the answer.

- 10 Number of ways of selecting 10 tees = 1

Hence choice (b) is the answer.

- 11 Number of ways of selection of zero or more things out of $(p + q + r + \dots)$ things, of which p are alike of one kind, q are alike of second kind, r are alike of third kind, and so on
= $[(p + 1)(q + 1)(r + 1)\dots]$

Therefore the number of ways of selection of any number (≥ 0) of candles out of $(2 + 3 + 4 + 3)$ candles, of which 2 are alike of white colour, 3 are alike of red colour, 4 are alike of green colour and 3 are alike of yellow colour

$$= (2 + 1)(3 + 1)(4 + 1)(3 + 1) = 240.$$

Hence choice (b) is the answer.



NOTE The above solution shows that out of 2 white candles you may select any number of candles (0 or 1 or 2); or out of 3 red candles you may select any number of candles (0 or 1 or 2 or 3); or out of 4 green candles you may select any number of candles (0 or 1 or 2 or 3 or 4); or out of 3 yellow candles you may select any number of candles (0 or 1 or 2 or 3).

Essentially, it manifests that you may select any number of candles (0 or 1 or 2 or ... or 12) from the whole lot of 12 candles.

- 12 Number of ways of selection of one or more things out of $(p + q + r + \dots)$ things, of which p are alike of one kind, q are alike of second kind, r are alike of third kind, and so on
= $[(p + 1)(q + 1)(r + 1)\dots] - 1$

Therefore number of ways of selection of at least one (≥ 1) candle out of $(2 + 3 + 4 + 3)$ candles, of which 2 are alike of white colour, 3 are alike of red colour, 4 are alike of green colour and 3 are alike of yellow colour
= $[(2 + 1)(3 + 1)(4 + 1)(3 + 1) - 1] = 239$.

Hence choice (b) is the answer.

Hint As it's clear from the previous question that there are 240 ways in which you may select 0 or 1 or 2 or ... or 12 candles. It obviously includes 1 way in which none of the candles can be selected. So if we subtract 1 way from 240 ways, we get the answer in which at least one candle must be selected.



NOTE Always keep in mind that the number of ways in which you can select nothing (i.e., number of ways in which you can select 0 things) is 1.

- 13 Number of ways in which you can select 1 candle of each colour = No. of ways of selecting 1 white candle $\times$ No. of ways of selecting 1 red candle $\times$ No. of ways of selecting 1 green candle $\times$ No. of ways of selecting 1 yellow candle
= $1 \times 1 \times 1 \times 1 = 1$. Hence choice (d) is the answer.

Hint Number of ways of selecting 1 article from n identical articles = 1.

- 14** Number of ways of selecting at least one candle of each colour = $2 \times 3 \times 4 \times 3 = 72$
Hence choice (b) is the answer.

Hint Number of ways of selecting r articles from n ($r \leq n$) identical articles = 1.

- 15** Number of ways in which you can select 2 candles of each colour = No. of ways of selecting 2 white candles $\times$ No. of ways of selecting 2 red candles $\times$ No. of ways of selecting 2 green candles $\times$ No. of ways of selecting 2 yellow candles
= $1 \times 1 \times 1 \times 1 = 1$.

Hence choice (b) is the answer.

Hint Number of ways of selecting r article from n ($r \leq n$) identical articles = 1.

- 16** Number of ways of selection of at least 2 white candles = 1
Number of ways of selection of at least 2 red candles = $1 + 1 = 2$
Number of ways of selection of at least 2 green candles = $1 + 1 + 1 = 3$. Number of ways of selection of at least 2 yellow candles = $1 + 1 = 2$
Therefore number of ways of selection of at least 2 candles of each colour = $1 \times 2 \times 3 \times 2 = 12$
Hence choice (a) is the answer.

- 17** Number of ways of selection of at most 2 white candles = $1 + 1 + 1 = 3$

Number of ways of selection of at most 2 red candles = $1 + 1 + 1 = 3$

Number of ways of selection of at most 2 green candles = $1 + 1 + 1 = 3$

Number of ways of selection of at most 2 yellow candles = $1 + 1 + 1 = 3$

Therefore number of ways of selection of at most 2 candles of each colour = $3 \times 3 \times 3 \times 3 = 81$

Hence choice (d) is the answer.

- 18** The number of ways of choosing r objects from p objects of one kind, q objects of second kind, and so on is the coefficient of x^r in the expansion $(1 + x + x^2 + \dots + x^p)(1 + x + x^2 + \dots + x^q) \dots$

Therefore number of ways of choosing 2 candles from 2 candles of white colour, 3 candles of red colour, 4 candles of green colour and 3 candles of yellow colour is the coefficient of x^2 in the expansion
 $(1 + x + x^2)(1 + x + x^2 + x^3)$

$$(1 + x + x^2 + x^3 + x^4)(1 + x + x^2 + x^3)$$

By multiplication we get $(1 + \dots + 10x^2 + \dots + x^{12})$.

Thus we can see that the coefficient of x^2 is 10.

Hence choice (d) is the answer.

Alternatively

Case I : 2, 0, 0, 0 {WW}, {RR}, {GG}, {YY}

Case II : 1, 1, 0, 0 {WR}, {WG}, {WY}, {RG}, {RY}, {GY}

Therefore there are total 10 ways of selection (or combination) of 2 candles. In case I both the candles have same colour and in case II each of the two candles has different colour.

No. of Combinations	Case I				Case II			
	White	Red	Green	Yellow	White	Red	Green	Yellow
1	2	0	0	0	1	1	0	0
2	0	2	0	0	1	0	1	0
3	0	0	2	0	1	0	0	1
4	0	0	0	2	0	1	1	0
5	–	–	–	–	0	1	0	1
6	–	–	–	–	0	0	1	1

- 19** Number of ways of selecting 1 white, 2 red, 3 green and 2 yellow candles = $1 \times 1 \times 1 \times 1 = 1$
Hence choice (b) is the answer.

Hint Number of ways of selecting r articles from n ($r \leq n$) identical articles = 1.

- 20** Number of ways of selecting 2 white, 3 red, 4 green and 2 yellow candles = $1 \times 1 \times 1 \times 1 = 1$

Hence choice (b) is the answer.

Hint Number of ways of selecting n articles from n identical articles = 1.

- 21** Number of ways of choosing 4 candles from 2 candles of white colour, 3 candles of red colour, 4 candles of green colour and 3 candles of yellow colour is the coefficient of x^4 in the expansion $(1 + x + x^2)(1 + x + x^2 + x^3)(1 + x + x^2 + x^3 + x^4)(1 + x + x^2 + x^3)$
By multiplication we get $(1 + 4x + 10x^2 + 19x^3 + 29x^4 + 37x^5 + 40x^6 + 37x^7 + 29x^8 + 19x^9 + 10x^{10} + 4x^{11} + x^{12})$.

Thus we can see that the coefficient of x^4 is 29.

Hence choice (b) is the answer.

Alternatively

Case I : 4, 0, 0, 0 $\rightarrow$ No. of selections = 1

Case II : 3, 1, 0, 0 $\rightarrow$ No. of selections = 9

Case III : 2, 2, 0, 0 $\rightarrow$ No. of selections = 6

Case IV : 2, 1, 1, 0 $\rightarrow$ No. of selections = 12

Case V : 1, 1, 1, 1 $\rightarrow$ No. of selections = 1

Therefore total number of required selections = $1 + 9 + 6 + 12 + 1 = 29$

- 22** Number of ways of choosing 6 candles from 2 candles of white colour, 3 candles of red colour, 4 candles of green colour and 3 candles of yellow colour is the coefficient of x^6 in the expansion $(1 + x + x^2)(1 + x + x^2 + x^3)(1 + x + x^2 + x^3 + x^4)(1 + x + x^2 + x^3)$

By multiplication we get

$$(1 + 4x + 10x^2 + 19x^3 + 29x^4 + 37x^5 + 40x^6 + 37x^7 + 29x^8 + 19x^9 + 10x^{10} + 4x^{11} + x^{12}).$$

Thus we can see that the coefficient of x^6 is 40.

Hence choice (c) is the answer.

☞ Alternatively

- Case I : 4, 2, 0, 0 → No. of selections = 3
 Case II : 4, 1, 1, 0 → No. of selections = 3
 Case III : 3, 3, 0, 0 → No. of selections = 3
 Case IV : 3, 2, 1, 0 → No. of selections = 18
 Case V : 3, 1, 1, 1 → No. of selections = 3
 Case VI : 2, 2, 2, 0 → No. of selections = 4
 Case VII : 2, 2, 1, 1 → No. of selections = 6
 Therefore total number of required selections
 = 3 + 3 + 3 + 18 + 3 + 4 + 6 = 40.

Hint Please keep in mind that you can select more than 2 candles from the set of red candles, not more than 3 candles from the set of red and yellow candles. There is only one set, of green candles, which have 4 candles. So it must be obvious to you that you cannot select more than 4 candles from any set of candles.

- 23** Number of ways of selecting at least 6 candles = Number of ways in which one can select either 6 candles or 7 candles or 8 candles or ... or 12 candles
 = 40 + 37 + 29 + 19 + 10 + 4 + 1 = 140

Hence choice (b) is the answer.

- 24** Number of ways of selecting at most 6 candles = Number of ways in which one can select either 6 candles or 5 candles or 4 candles or ... or 1 candle or no candle
 = 40 + 37 + 29 + 19 + 10 + 4 + 1 = 140

Hence choice (d) is the answer.

- 25** In order to select the 6 candles such that there must be 1 candle of each colour, first of all we can take out 4 candles 1 from each colour. Now 2 more candles have to be selected which can be done in the following ways:

- Case I: 2, 0, 0, 0 → No. of selections = 3
 Case II: 1, 1, 0, 0 → No. of selections = 6
 Therefore total number of required selections = 3 + 6 = 9
 Hence choice (a) is the answer.

☞ Alternatively

- Case I: 3, 1, 1, 1 → No. of selections = 3
 Case II: 2, 2, 1, 1 = No. of selections = 6
 Therefore total number of required selections = 3 + 6 = 9
 Hence choice (a) is the answer.

Hint Remember that you cannot select more than 2 candles of white colour, so in case-I, you will have only 3 ways of selections, instead of 4 ways.

☞ Alternatively

The number of ways of choosing k objects, from p objects of one kind, q objects of second kind, and so on, such that one

object of each kind must be included is the coefficient of x^k in the expansion $(x + x^2 + \dots + x^p)(x + x^2 + \dots + x^q) \dots$

Therefore required number of ways of selecting 6 candles, such that at least one candle of each colour must be there, is the coefficient of x^6 in the expansion of $(x + x^2)(x + x^2 + x^3)(x + x^2 + x^3 + x^4)(x + x^2 + x^3)$

By multiplication we get, $(x^4 + 4x^5 + 9x^6 + \dots + x^{12})$.

It shows that the coefficient of x^6 is 9. Therefore the required number of selections = 9.

- 26** Number of ways of selection of zero or more things from p identical things of one kind, q identical things of second kind, r identical things of third kind, and remaining n things are different = $[(p + 1)(q + 1)(r + 1)2^n]$

Therefore the number of ways of selection of zero or more pen-drives = $(3 + 1)(4 + 1)(2^5) = 640$

Hence choice (d) is the answer.

- 27** The number of ways of section of zero or more pen-drives = 640

Number of ways of selection of zero pen-drives = 1

Therefore number of ways of selection of one or more pen-drives = $640 - 1 = 639$.

Hence choice (a) is the answer.

- 28** The number of ways of selecting at most 1 pen-drive = number of ways of selecting no pen-drive

+ number of ways of selecting 1 pen drive.

Number of ways of not selecting any pen-drive = 1.

Number of ways of selecting 1 pen-drive is shown below:

	Number of pen-drives from		
	Box 1	Box 2	Box 3
Case I	1	0	0
Case II	0	1	0
Case III	0	0	1

Case I : $1 \times 1 \times 1 = 1$

Case II : $1 \times 1 \times 1 = 1$

Case III : $1 \times 1 \times 5 = 5$

Therefore total number of ways in which 1 pen-drive can be selected = $1 + 1 + 5 = 7$.

Thus the total number of ways in which at most 1 pen-drive can be selected = $1 + 7 = 8$.

Hence choice (b) is the answer.

- 29** The number of ways of selecting 1 pen-drive from each box = No. of ways of selecting 1 pen-drive from first box $\times$ No. of ways of selecting 1 pen-drive from second box $\times$ No. of ways of selecting 1 pen-drive from third box = $1 \times 1 \times 5 = 5$
 Hence choice (a) is the answer.

- 30** The number of ways of selecting 3 pen-drives is shown below:

Case	Number of pen-drives from			No. of ways of selection	Total
	Box 1	Box 2	Box 3		
I	3	0	0	$1 \times 1 \times 1 = 1$	12
	0	3	0	$1 \times 1 \times 1 = 1$	
	0	0	3	$1 \times 1 \times 10 = 10$	
II	2	1	0	$1 \times 1 \times 1 = 1$	32
	2	0	1	$1 \times 1 \times 5 = 5$	
	1	0	2	$1 \times 1 \times 10 = 10$	
	1	2	0	$1 \times 1 \times 1 = 1$	
	0	1	2	$1 \times 1 \times 10 = 10$	
III	0	2	1	$1 \times 1 \times 5 = 5$	5
	1	1	1	$1 \times 1 \times 5 = 5$	

Therefore total number of ways in which 3 pen-drives can be selected = $12 + 32 + 5 = 49$.

Hence choice (a) is the answer.

- 31** The number of ways of selecting 3 distinct pen-drives is shown below:

	Number of pen-drives from			No. of ways of selection
	Box 1	Box 2	Box 3	
Case I	0	0	3	$1 \times 1 \times 10 = 10$
Case II	0	1	2	$1 \times 1 \times 10 = 10$
	1	0	2	$1 \times 1 \times 10 = 10$
Case III	1	1	1	$1 \times 1 \times 5 = 5$

Therefore total number of ways in which 3 pen-drives can be selected $10 + 10 + 10 + 5 = 35$

Hence choice (c) is the answer.

- 32** The number of ways of selecting 2 pen-drives of 4 GB, 3 pen-drives of 2 GB and 4 pen-drives of different storage capacities other than 2 GB and 4 GB = $1 \times 1 \times 5 = 5$

Hence choice (a) is the answer.

- 33** Number of ways of selecting at most 2 pen-drives from the first box = 3
 Number of ways of selecting at most 3 pen-drives from the first box = 4
 Number of ways of selecting at most 4 pen drives from the

first box = 31

Therefore the required number of ways

$$= 3 \times 4 \times 31$$

$$= 372$$

Hence choice (a) is the answer.

- 34** The number of ways of selecting any 2 identical pen-drives = Number of ways of selecting 2 pen-drives either from box 1 or 2 pen-drives from box 2 = $1 + 1 = 2$

Hence choice (c) is the answer.

- 35** Number of ways of selecting any two distinct pen-drives is shown below:

	Number of pen-drives from			No. of ways of selection
	Box 1	Box 2	Box 3	
Case I	0	0	2	$1 \times 1 \times 10 = 10$
Case II	0	1	1	$1 \times 1 \times 5 = 5$
	1	0	1	$1 \times 1 \times 5 = 5$
Case III	1	1	0	$1 \times 1 \times 1 = 1$

Therefore total number of required ways

$$= 10 + 5 + 5 + 1 = 21.$$

Hence choice (c) is the answer.

- 36** We have to select 7 coupons from 4 identical coupons and 6 distinct coupons. Then the coupons can be selected in the following ways.

4 identical and 3 distinct coupons in $1 \times {}^6C_3$ ways

3 identical and 4 distinct coupons in $1 \times {}^6C_4$ ways

2 identical and 5 distinct coupons in $1 \times {}^6C_5$ ways

1 identical and 6 distinct coupons in $1 \times {}^6C_6$ ways

Hence the required number of ways

$$= {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$$

$$= 2^6 - ({}^6C_0 + {}^6C_1 + {}^6C_2) = 64 - 22 = 42$$

- 37** We have to select 3 coupons from 4 identical coupons and 6 distinct coupons. Then the coupons can be selected in the following ways.

3 identical and 0 distinct coupons in $1 \times {}^6C_0$ ways

2 identical and 1 distinct coupons in $1 \times {}^6C_1$ ways

1 identical and 2 distinct coupons in $1 \times {}^6C_2$ ways

0 identical and 3 distinct coupons in $1 \times {}^6C_3$ ways

Hence the required number of ways

$$= {}^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3$$

$$= 2^6 - ({}^6C_4 + {}^6C_5 + {}^6C_6) = 64 - 22 = 42$$

Introductory Exercise 19.7

- 1 The required number of ways = $10 - 3 + 1 = 8$
- 2 When there is no restriction, the number of ways of selection of 3 candles = ${}^{10}C_3 = 120$. The number of ways of selection of 3 consecutive candles = 8

Therefore the required number of ways = $120 - 8 = 112$

- 3 The number of ways of selection of 3 consecutive candles = $10 - 3 + 1 = 8$

The number of ways of selection of 2 consecutive candles = $10 - 2 + 1 = 9$

The number of ways of selection of 3 candles such that the third candle is not the adjacent to any of the two consecutive candles

$$= 7 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 7 = 56$$

Thus the number of selection of 3 candles such that at least 2 candles are consecutive = $8 + 56 = 64$

Hint Let $a, b, c, d, e, f, g, h, i, j$ be 10 consecutive candles in that order only. Then we have the following explanation.

2 consecutive candles	Third candle	Total number of cases
ab	Except c , any 7 candles	7
bc	Except a and d , any 6 candles	6
cd	Except b and e , any 6 candles	6
de	Except c and f , any 6 candles	6
ef	Except d and g , any 6 candles	6
fg	Except e and h , any 6 candles	6
gh	Except f and i , any 6 candles	6
hi	Except g and j , any 6 candles	6
ij	Except h , any 7 candles	7

- 4 Number of selections of k consecutive things out of n things in a row = $(n - k + 1)$. Therefore number of selection of 5 consecutive seats = $10 - 5 + 1 = 6$

Hence choice (c) is the answer.

Hint (1-5), (2-6), (3-7), (4-8), (5-9), (6-10)

- 5 Number of selections of k consecutive things out of n things in a row = $(n - k + 1)$

Therefore number of selection of 4 consecutive seats = $10 - 4 + 1 = 7$

Hence choice (a) is the answer.

- 6 Number of selections of k consecutive things out of n things in a row = $(n - k + 1)$

Therefore number of selection of 4 consecutive seats = $7 - 4 + 1 = 4$

Hence choice (a) is the answer.

- 7 The number of ways of selecting 4 months, out of 12 months, such that no two months are consecutive = ${}^{12-4+1}C_4 = {}^9C_4 = 126$

Hence choice (c) is the answer.

- 8 The number of ways of selecting 4 bogies out of 10 bogies, such that no two bogies are adjacent ones = ${}^{10-4+1}C_4 = 35$

Number of ways of selecting 4 terrorists out of 8 terrorists who board the train = ${}^8C_4 = 70$

And the number of ways of arranging 4 terrorists in the four bogies = $4! = 24$

Therefore the number of ways in which 4 terrorists board the train in the required manner = $35 \times 70 \times 24 = 58800$

- 9 Total number of teams = 4, Total number of members = 16

Total number of chairs = 21, Extra number of chairs = 5

Now for a moment, solve the problem considering the four teams as 4 individuals.

So, the number of seats available for 4 teams = $4 + 5 = 9$

Number of ways of selecting 4 seats out of 9 seats for the 4 teams, such that no teams sit in the adjacent chairs = ${}^{9-4+1}C_4 = 15$

Number of ways of mutually arranging the 4 teams = $4! = 24$

Number of ways of arranging the individuals within a team when they are sitting next to each other = $2! \times 3! \times 5! \times 6! = 1036800$

Number of ways of selecting 16 chairs out of 21 chairs so that no two members of different teams sit together and members of each team must sit together

$$= 15 \times 24 \times 1036800 = 373248000$$

- 10 Number of couples = 6, Number of seats required = 12

Number of seats available = 20, Number of extra seats = 8

For a moment consider each couple as an individual, then the number of seats available for the couples = $6 + 8 = 14$

Number of ways of selecting 6 seats out of 14 seats for the 6 couples, such that no two couples sit in the adjacent chairs = ${}^{14-6+1}C_6 = 84$.

Number of ways of mutually arranging the 6 couples = $6! = 720$

As each couple can be arranged in 2! ways so all the couples can be internally arranged in

$$2! \times 2! \times 2! \times 2! \times 2! \times 2! = 64 \text{ ways.}$$

Number of ways of selecting 12 seats out of 20 seats so that no two couples sit together and each couple must sit together = $84 \times 720 \times 64 = 3870720$

- 11** Number of selections of k consecutive things out of n things in a circle $= n$; when $k < n$.

Therefore number of selections of 6 consecutive lights out of 12 lights $= 12$.

Hence choice (d) is the answer.

- 12** Number of selections of k consecutive things out of n things in a circle $= n$; when $k < n$.

Therefore number of selections of 2 consecutive chairs out of 5 chairs $= 5$.

Hence choice (d) is the answer.

- 13** Number of selections of 4 consecutive chairs out of 10 chairs $= 10$.

Hence choice (b) is the answer.

- 14** Let $V_1, V_2, V_3, \dots, V_{10}$ be the 10 vertices of the 10-sided polygon, then there will be 10 consecutive pairs of vertices viz. $V_1V_2, V_2V_3, V_3V_4, \dots, V_{10}V_1$.

Now each pair can be connected with 7 other vertices. 1 vertex is not counted here to avoid the repetition.

For example, V_1V_2 can be connected to $V_3, V_4, V_5, V_6, V_7, V_8, V_9$.

Similarly, V_2V_3 can be connected to $V_4, V_5, V_6, V_7, V_8, V_9, V_{10}$.
...

Similarly, $V_{10}V_1$ can be connected to $V_2, V_3, V_4, V_5, V_6, V_7, V_8$.

So the total number of required triangles $= 10 \times 7 = 70$.

- 15** Total number of triangles obtained by connecting 10 vertices of the polygon $= {}^{10}C_3 = 120$

And the number of triangles with at least one coinciding side $= 70$

Therefore the number of triangles with no coinciding sides $= 120 - 70 = 50$.

- 16** The problem is equivalent to the selection of 3 points on the circumference of a circle so that none of the 3 points are adjacent to each other.

Let us consider the 10 points $P_1, P_2, P_3, \dots, P_{10}$ in that order on the circumference. Now let us select a point P_1 , then we cannot select the points P_2 and P_{10} as they are the adjacent points to P_1 .

Now we have 7 points $P_3, P_4, P_5, P_6, P_7, P_8, P_9$, from which we have to select 2 points such that they are not the adjacent to each other. This is like selecting 2 non-consecutive points from the 7 points in a row.

Number of ways of selection of 2 points out of 7 points, without any restriction $= {}^7C_2 = 21$

Number of ways of selection of 2 consecutive points out of 7 points in a row $= 7 - 2 + 1 = 6$

Therefore the number of ways of selection of 2 points which are non-consecutive $= 21 - 6 = 15$.

Since the first point (in this assumption it is P_1) can be selected in 10 ways, as there are total 10 points. So the total number of selections of 3 points such that none of them is adjacent to each other $= 10 \times 15 = 150$.

But, since in the above solution each combination occurs 3 times, so the required number of ways $= \frac{150}{3} = 50$.

NOTE When you select the first point P_1 , then the other two points will be (say) P_3 and P_5 . But when you select the first point P_3 , then the other two points will be (say) P_5 and P_1 . Similarly, when you select the first point P_5 , then the other two points will be (say) P_1 and P_3 . That's why I'm saying that each combination occurs 3 times.

- 17** (i) First of all keep all the chairs of same color together. Now the two groups can be arranged in $2!$ ways. Now the red chairs can be arranged in $6!$ ways and blue chairs can be arranged in $4!$ ways. Therefore the required number of arrangements $= 2! \times (6! \times 4!)$
- (ii) At first, consider all the 6 red chairs as a single bundle. Now, there are 4 blue chairs and 1 bundle of red chairs, so total 5 articles are to be arranged, which can be done in $5!$ ways. But just a while back we have assumed that 6 red chairs as a bundle, which can be arranged in $6!$ ways. Therefore the total number of arrangements $= 5! \times 6!$
- (iii) First of all arrange all the 6 red chairs, which can be done in $6!$ ways. Now after arranging 6 chairs there will be 7 places available for the 4 blue chairs to be arranged, which can be done in ${}^7C_4 \times 4! = 840$ ways. Therefore, the required number of ways $= {}^7C_4 \times 4! \times 6!$
- (iv) First of all arrange all the 5 red chairs in $5!$ ways and then arrange all the blue chairs in the right side of each red color chair in $5!$ ways. Similarly all the blue chairs can be arranged in the left side of the each red chair. Therefore the total number of arrangements $= 5! \times 5! + 5! \times 5! = 2(5! \times 5!)$
- (v) First of all arrange all the 6 identical red chairs in a row that can be done in 1 way. Now there will be 7 places for the 4 blue chairs to be arranged and that can be done in ${}^7C_4$ ways. Therefore, the required number of ways $= 1 \times {}^7C_4 = {}^7C_4$
- (vi) First of all arrange all the 6 identical red chairs in a row, which can be done in 1 way. Now there will be 7 places for the 4 blue chairs to be arranged and that can be done in ${}^7C_4 \times 4!$ ways. Therefore, the required number of ways $= 1 \times {}^7C_4 \times 4! = {}^7C_4 \times 4!$

Thus we see that none of the statements is wrong.

- 18** On the first day of the week she can choose any of the 7 activities, but on the remaining days she can choose only 6 activities as she cannot choose the activity that she chooses the previous day.

So the required number of ways

$$= 7 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 7 \times 6^6$$

Introductory Exercise 19.8

- 1** 8 CDs can be selected from 12 CDs in ${}^{12}C_8$ ways and 4 CDs can be selected from the remaining 4 CDs in ${}^4C_4$ ways.

Therefore total number of ways

$$= {}^{12}C_8 \times {}^4C_4 = \frac{12!}{8! \times 4!} \times \frac{4!}{4! \times 0!} = \frac{12!}{8! \times 4!} = 495$$

Hence choice (b) is the answer.

- 2** 8 CDs can be selected from 12 CDs in ${}^{12}C_8$ ways and 4 CDs can be selected from the remaining 4 CDs in ${}^4C_4$ ways. And since these CDs are given to 2 different students which can be exchanged mutually; that means if one student receives 8 CDs then another student will receive 4 CDs and vice-versa.

Therefore total number of ways in which 12 CDs can be given to 2 students

$$= ({}^{12}C_8 \times {}^4C_4) \times 2! = \left(\frac{12!}{8! \times 4!} \right) \times 2! = 990$$

Hence choice (a) is the answer.

- 3** 6 CDs can be selected from 12 CDs in ${}^{12}C_6$ ways and 4 CDs can be selected from 6 CDs in ${}^6C_4$ ways and the remaining 2 CDs can be selected from remaining 2 CDs in ${}^2C_2$ ways.

Therefore total number of ways

$$\begin{aligned} &= {}^{12}C_6 \times {}^6C_4 \times {}^2C_2 \\ &= \frac{12!}{6! \times 6!} \times \frac{6!}{4! \times 2!} \times \frac{2!}{2! \times 0!} \\ &= \frac{12!}{6! \times 4! \times 2!} = 13860 \end{aligned}$$

Hence choice (c) is the answer.

- 4** The number of ways in which these CDs can be divided in the groups of 6, 4 and 2 is $\frac{12!}{6! \times 4! \times 2!}$

But since these CDs can be exchanged among 3 students in 3! ways, so the number of ways in which these CDs can be given to three students = $\frac{12!}{6! \times 4! \times 2!} \times 3! = 13860 \times 6$

$$= 83160$$

Hence choice (d) is the answer.

- 19** If Siddhartha practices with Deepika only once in a week, it can be done in 7 ways. If Siddhartha practices with her twice a week, it can be done in 6 ways. Similarly, if Siddhartha practices with her for 3, 4, 5, 6 and 7 days, it can be done in 5, 4, 3, 2 and 1 ways.

Therefore the required number of ways

$$= 7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$$

- 5** 6 CDs can be selected from 12 CDs in ${}^{12}C_6$ ways and other 6 CDs can be selected from the remaining 6 CDs in ${}^6C_6$ ways.

Therefore total number of ways

$$\begin{aligned} &= {}^{12}C_6 \times {}^6C_6 \times \frac{1}{2!} = \frac{12!}{6! \times 6!} \times \frac{6!}{6! \times 0!} \times \frac{1}{2!} \\ &= \frac{12!}{6! \times 6!} \times \frac{1}{2!} = \frac{12!}{(6!)^2} \times \frac{1}{2!} = 462 \end{aligned}$$

The division by 2! indicates that since the number of CDs in each group is same so only half of the combinations will be there.

Hence choice (c) is the answer.

- 6** Number of ways in which 12 CDs can be distributed to two French students equally = $\left(\frac{12!}{(6!)^2} \times \frac{1}{2!} \right) \times 2! = 924$

The division by 2! indicates that the two groups are having equal number of CDs, so they are indistinguishable. And the multiplication by 2! indicates that since the two students are distinguishable so they can exchange the packets in 2! ways.

Hence choice (d) is the answer.

- 7** Number of ways in which 12 CDs can be divided equally into three groups = $\frac{12!}{4! \times 4! \times 4!} \times \frac{1}{3!} = \frac{12!}{(4!)^3} \times \frac{1}{3!} = 5775$

The division by 3! indicates that the three groups are having equal number of CDs, so they are indistinguishable.

Hence choice (d) is the answer.

- 8.** Number of ways in which 12 CDs can be distributed among three French students equally = $\frac{12!}{(4!)^3} \times \frac{1}{3!} \times 3! = 34650$

The division by 3! indicates that the three groups are having equal number of CDs, so they are indistinguishable. And the multiplication by 3! indicates that the three students are distinguishable so they can exchange the packets in 3! ways. Hence choice (a) is the answer.

- 9** Number of ways in which 12 CDs can be distributed among six French students equally = $\frac{12!}{(2!)^6} \times \frac{1}{6!} \times 6! = \frac{12!}{(2!)^6}$

Hence choice (a) is the answer.

- 10** Number of ways in which 12 CDs can be distributed to 6 French students
 = First CD can be given to any 6 students, similarly second CD, third CD....etc. can be given to any of the 6 students
 $= 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^{12}$
 Hence choice (a) is the answer.

- 11** First of all distribute 2 CDs to each of the four French students. Then distribute the remaining 4 CDs in all the possible ways. After distributing all the 12 CDs there will be five cases, as presented in the following table. Suppose A, B, C and D are four French students.

Case	A	B	C	D
i	2	2	2	6
ii	2	2	3	5
iii	2	2	4	4
iv	2	3	3	4
v	3	3	3	3

Case (i) $({}^{12}C_2 \times {}^{10}C_2 \times {}^8C_2 \times {}^6C_6) \times \frac{4!}{3!}$
 $= 83160 \times 4 = 110880$

Case (ii) $({}^{12}C_2 \times {}^{10}C_2 \times {}^8C_3 \times {}^5C_5) \times \frac{4!}{2!}$
 $= 166320 \times 12 = 1995840$

Case (iii) $({}^{12}C_2 \times {}^{10}C_2 \times {}^8C_4 \times {}^4C_4) \times \frac{4!}{2!2!}$
 $= 207900 \times 6 = 1247400$

Case (iv) $({}^{12}C_2 \times {}^{10}C_3 \times {}^7C_3 \times {}^4C_4) \times \frac{4!}{2!}$
 $= 277200 \times 12 = 3326400$

Case (v) $({}^{12}C_3 \times {}^9C_3 \times {}^6C_3 \times {}^3C_3) \times \frac{4!}{4!}$
 $= 369600 \times 1 = 369600$

Therefore total number of the required ways = 7050120

- 12** Since none of the 4 students can have more than 3 CDs, which implies that each one has to have exactly 3 CDs. This can be easily done using the following formula.

The number of ways in which 12 CDs can be distributed among 4 students equally

$$= \frac{12!}{(3!)^4} \times \frac{1}{4!} \times 4! = \frac{12!}{(3!)^4} = 369600$$

- 13** The number of ways of dividing n identical items among r persons (where $0 \leq r \leq n$), each one of whom can receive any number $(0, 1, 2, \dots, n)$ of items $= {}^{n+r-1}C_{r-1}$

Therefore the number of ways in which 12 identical chocolates can be distributed among 4 kids

$$= {}^{12+4-1}C_{4-1} = {}^{15}C_3 = 455$$

Hence choice (a) is the answer.

- 14** The number of ways of dividing n identical items among r persons (where $0 \leq r \leq n$), each one of whom must receive at least one item $= {}^{n-1}C_{r-1}$

Therefore the number of ways in which 12 identical chocolates can be distributed among 4 kids such that each of them must receive at least one chocolate

$$= {}^{12-1}C_{4-1} = {}^{11}C_3 = 165$$

Hence choice (d) is the answer.

Alternatively Since each kid has to receive at least one chocolate, so first of all give 1 chocolate to each of the 4 kids, which will be done in 1 way as the chocolates are identical and each one is getting equal number of chocolates. Now you have to distribute just 8 chocolates among 4 kids in such a way that anyone receives any number of chocolates. Since each kid has already received minimum 1 chocolate, so it won't matter anymore that any particular kid is not getting any chocolate from the remaining 8 chocolates.

Now, the number of ways in which 8 chocolates can be distributed among 4 kids such that any kid can get any number of chocolates $= {}^{8+4-1}C_{4-1} = 165$

Therefore the number of ways in which 12 identical chocolates can be distributed among 4 kids such that each of them must receive at least one chocolate $= 165 \times 1 = 165$

- 15** Since each kid has to receive at least two chocolates, so first of all give 2 chocolates to each of the 4 kids, which will be done in 1 way, as the chocolates are identical and each one is getting equal number of chocolates.

Now you have to distribute just 4 chocolates among 4 kids in such a way that anyone receives any number of chocolates. Since each kid has already received minimum 2 chocolates, so it won't matter anymore that any particular kid is not getting any chocolate from the remaining 4 chocolates. Now, the number of ways in which 4 chocolates can be distributed among 4 kids such that any kid can get any number of chocolates

$$= {}^{4+4-1}C_{4-1} = 35$$

Therefore the number of ways in which 12 identical chocolates can be distributed among 4 kids such that each of them must receive at least two chocolates $= 35 \times 1 = 35$.

- 16.** Since each of the 4 kids has to receive at least 3 chocolates, it means all the 12 chocolates will have to be distributed evenly (or equally). That means every one of them will get any 3 chocolates and this can be done in only 1 way as chocolates are identical and being distributed evenly.
- 17** Since each kid has to receive at least two chocolates, so first of all give 2 chocolates to each of the 4 kids, which will be done in 1 way as the chocolates are identical and each one is getting equal number of chocolates.

After distributing 8 chocolates, now you have to distribute remaining 4 chocolates among 4 kids.

- (i) Since each kid has to receive at least 2 chocolates, so now it's not necessary to give any chocolate to every kid, from the remaining 4 chocolates.
- (ii) Since each kid has to receive not more than 4 chocolates, so now it is obvious that no kid gets more than 2 chocolates, from the remaining 4 chocolates.

This can be done in the following ways:

Case I 0, 0, 2, 2 $\rightarrow$ No. of selections = 6

Case II 0, 1, 1, 2 $\rightarrow$ No. of selections = 12

Case III 1, 1, 1, 1 $\rightarrow$ No. of selections = 1

Therefore total number of required selection
 $= 6 + 12 + 1 = 19$

Hence choice (b) is the answer.

☞ **Alternatively**

Case I 2, 2, 4 $\rightarrow$ No. of selections = 6

Case II 2, 3, 3 $\rightarrow$ No. of selections = 12

Case III 3, 3, 3 $\rightarrow$ No. of selections = 1

Therefore total number of required selections
 $= 6 + 12 + 1 = 19$.

☞ **Alternatively** The number of ways in which n identical things can be divided into r groups so that no group contains less than m items and more than k items (where $m < k$) is coefficient of x^n in the expansion of $(x^m + x^{m+1} + \dots + x^k)^r$

Therefore the required number of ways of distribution of chocolates = coefficient of x^{12} in the expansion of $(x^2 + x^3 + x^4)^4$

$$\begin{aligned} \text{Now, } (x^2 + x^3 + x^4)^4 &= (x^2 + x^3 + x^4)^2 \times (x^2 + x^3 + x^4)^2 \\ \Rightarrow (x^2 + x^3 + x^4)^4 &= (x^4 + 2x^5 + 3x^6 + 2x^7 + x^8) \\ &\quad (x^4 + 2x^5 + 3x^6 + 2x^7 + x^8) \end{aligned}$$

$$\Rightarrow (x^2 + x^3 + x^4)^4 = (x^8 + \dots + 19x^{12} + \dots + x^{16})$$

Since the coefficient of x^{12} is 19, so the required number of ways = 19. Hence choice (b) is the answer.

- 18** Since the first kid can get any one lot out of 4 lots (1 or 2 or 4 or 5) of chocolates, then the second kid can get any one lot of remaining 3 lots of chocolates and so on.

Therefore the number of ways of distributing 12 chocolates = $4 \times 3 \times 2 \times 1 = 24$

Hence choice (b) is the answer.

- 19** Since it's already decided that who gets how many chocolates, then the number of ways in which 12 chocolates can be distributed is 1.

Hence choice (d) is the answer.

- 20** Since each kid gets 3 identical chocolates so the number of ways in which 12 chocolates can be distributed = 1. Hence choice (d) is the answer.

- 21** Total number of non-negative integral solutions of the given equation = ${}^{30+3-1}C_{3-1} = {}^{32}C_2 = 496$

- 22** Total number of positive integral solutions of the given equation = ${}^{30-1}C_{3-1} = {}^{29}C_2 = 406$

- 23** When $a = 2$, $b + c = 28$, and the number of even positive integral solutions is 13, namely (2, 26), (4, 24), (6, 22), ..., (26, 2)

Similarly we can find the number of solutions for every value of a , as shown in the following table.

a	$b + c$	Number of solutions
2	28	13
4	26	12
6	24	11
...	...	...
26	4	1

Therefore the total number of required solutions
 $= 1 + 2 + 3 + \dots + 13 = 91$

- 24** As you know, odd + odd + odd = odd, but since 30 is an even number.

So the required number of solutions = 0.

- 25** For $a = 1$, $b + c = 29$ and then it has 13 solutions such that $a < b < c$, namely $(b, c) = (2, 27), (3, 26), (4, 25), \dots, (14, 15)$

Similarly, we can find the number of solutions for $a = 2, 3, 4, \dots$ as shown in the table below.

a	$b + c$	Number of solutions
1	29	13
2	28	11
3	27	10
4	26	8
5	25	7
6	24	5
7	23	4
8	22	2
9	21	1

Therefore the number of solutions

$$= (1 + 4 + 7 + 10 + 13) + (2 + 5 + 8 + 11) = 61$$

Since a, b, c can be mutually arranged in $3!$ ways, so the required number of solutions

$$= 61 \times 3! = 366$$

- 26** For $a = 0$, $b + c = 30$ and then it has 14 solutions such that $a < b < c$, namely $(b, c) = (1, 29), (2, 28), (3, 27), (4, 26), \dots, (14, 16)$

Similarly, we can find the number of solutions for $a = 2, 3, 4, \dots$ as shown in the table below.

a	$b + c$	Number of solutions
0	30	14
1	29	13
2	28	11
3	27	10
4	26	8
5	25	7
6	24	5
7	23	4
8	22	2
9	21	1

Therefore the number of solutions

$$= (1 + 14) + (2 + 13) + (4 + 11) + (5 + 10) + (7 + 8) = 75$$

- 27** For $a = 0$, $b + c = 30$ and then it has 31 solutions such that $a \leq b$ and $0 \leq c \leq 30$, namely $(b, c) = (0, 30), (1, 29), (2, 28), (3, 27), (4, 26), \dots, (29, 1), (30, 0)$

Similarly, we can find the number of solutions for $a = 2, 3, 4, \dots$ as shown in the table below.

a	$b + c$	Number of solutions
0	30	31
1	29	29
2	28	27
3	27	25
4	26	23
...	...	...
...	...	...
15	15	1

Therefore the number of solutions

$$= (1 + 3 + 5 + \dots + 29 + 31) = 256$$

- 28** Consider the following table.

For example, when $a = 1$, $2b + c = 27$, it will have 13 positive solutions.

As in $(b, c) = (1, 25), (1, 23), (3, 21), \dots, (13, 1)$.

Similarly, you can find the number of solutions for other values of a .

a	$2b + c$	Number of solutions
1	27	13
2	24	11
3	21	10
4	18	8
5	15	7
6	12	5
7	9	4
8	6	2
9	3	1

Therefore total number of solutions

$$= 13 + 11 + 10 + 8 + 7 + 5 + 4 + 2 + 1 = 61$$

NOTE The above sum can be broken into two parts for the sake of convenience, as in $(13 + 10 + 7 + 4 + 1) + (11 + 8 + 5 + 2) = 61$. Clearly there are 2 arithmetic progressions.

- 29** Let the number of red, pink and yellow roses be r, p and y , respectively. Then, $r + p + y = 30$, such that $r, p, y \geq 0$.

Therefore the required number of ways = Number of ways of distributing 30 identical things among 3 persons such that each may get 0 or more things

$$= {}^{30+3-1}C_{3-1} = {}^{32}C_2 = 496$$

- 30.** Let the number of red, pink and yellow roses be r, p and y , respectively. Then, $r + p + y = 30$, such that $r, p, y \geq 1$.

Therefore the required number of ways = Number of ways of distributing 30 identical things among 3 persons such that each may get 1 or more things

$$= {}^{30-1}C_{3-1} = {}^{29}C_2 = 406$$

- 31** The number of non-negative integral solutions of the equation $x_1 + x_2 + x_3 + \dots + x_r = n$ is ${}^{n+r-1}C_{r-1}$

Therefore number of non-negative integral solutions of the equation $a + b + c + d = 12$ is ${}^{12+4-1}C_{4-1} = {}^{15}C_3 = 425$.

Hence choice (d) is the answer.

NOTE This problem is same as distribution of 12 identical chocolates among 4 different persons.

- 32** The number of positive integral solutions of the equation $x_1 + x_2 + x_3 + \dots + x_r = n$ is ${}^{n-1}C_{r-1}$.

Therefore number of positive integral solutions of the equation ${}^{12-1}C_{4-1} = {}^{11}C_3 = 165$

Hence choice (b) is the answer.

NOTE This problem is same as distribution of 12 identical chocolates among 4 different persons, in such a way that everyone gets at least 1 chocolate.

- 33** The number of solutions of $a + b + c + d = 12$ is the coefficient of x^{12} in the expansion of $(x^2 + x^3 + x^4)^4$

$$\begin{aligned} \text{Since, } (x^2 + x^3 + x^4)^4 &= (x^2 + x^3 + x^4)^2 \times (x^2 + x^3 + x^4)^2 \\ &\Rightarrow (x^2 + x^3 + x^4)^4 = (x^4 + 2x^5 + 3x^6 + 2x^7 + x^8) \\ &\quad (x^4 + 2x^5 + 3x^6 + 2x^7 + x^8) \end{aligned}$$

$$\Rightarrow (x^2 + x^3 + x^4)^4 = (x^8 + \dots + 19x^{12} + \dots + x^{16})$$

As the coefficient of x^{12} is 19, so the required number of solutions = 19. Hence choice (a) is the answer.

Alternatively Let us assume that there are 12 identical balls that you are supposed to distribute among four kids such that each of them receives at least 2 balls but not more than 4 balls.

So, first of all give 2 balls to each of the four kids, now you are left with four balls to distribute. That can be done in the following way.

Case I 0, 0, 2, 2 $\rightarrow$ No. of selections = ${}^4C_2 = 6$

Case II 0, 1, 1, 2 $\rightarrow$ No. of selections = ${}^4C_1 \times {}^3C_2 = 12$

Case III 1, 1, 1, 1 $\rightarrow$ No. of selections = 1

Therefore total number of required selections
= $6 + 12 + 1 = 19$

In case I, you can select any two kids out of four kids to dole out 2 balls to each one.

In case II, you can select 1 kid out of four kids to give her 2 balls, then you can select any 2 kids out of the remaining 3 kids to dole out 1 ball to each of them.

In case III, you can give 1 ball to each one of the four kids in 1 way.

Alternatively The number of integral solutions of the equation $a + b + c + d = 12$, such that $2 \leq a, b, c, d \leq 4$ is same as the number of non-negative integral solutions of $p + q + r + s = 4$, such that $0 \leq a, b, c, d \leq 2$ for $p = a - 2$, $q = b - 2$, $r = c - 2$, $s = d - 2$

Now the number of solutions of $p + q + r + s = 4$, such that $0 \leq a, b, c, d \leq 2$ = (number of solutions of $p + q + r + s = 4$, such that $0 \leq a, b, c, d \leq 4$) - (number of solutions of $p + q + r + s = 4$, such that $3 \leq a, b, c, d \leq 4$)

But number of solutions of $p + q + r + s = 4$, such that $0 \leq a, b, c, d \leq 4$ is ${}^{4+4-1}C_{4-1} = {}^7C_3 = 35$

And number of solutions of $p + q + r + s = 4$, such that $3 \leq a, b, c, d \leq 4$ is $4[{}^{(4-3)+4-1}C_{4-1}] = 4[{}^4C_3] = 16$

Therefore, number of solutions of $p + q + r + s = 4$, such that $0 \leq a, b, c, d \leq 2$ = $35 - 16 = 19$.

So the number of integral solutions of the equation $a + b + c + d = 12$, such that $2 \leq a, b, c, d \leq 4$ is 19.

- 34** The given equation $a + b + c + d = 12$ can be expressed as following.

$$(a - 2) + (b - 2) + (c - 2) + (d - 2) = 12 - (2 + 2 + 2 + 2)$$

$$\Rightarrow w + x + y + z = 4;$$

$$\text{(Where } w = a - 2, x = b - 2, y = c - 2, z = d - 2)$$

Now the number of non-negative integral solutions of $w + x + y + z = 4$ is ${}^{4+4-1}C_{4-1} = {}^7C_3 = 35$

Therefore the number of integral solutions of $a + b + c + d = 12$, when $a, b, c, d \geq 2$ is 35.

Hence choice (a) is the answer.

NOTE This problem is same as distribution of 12 identical chocolates among 4 different persons, in such a way that everyone gets at least 2 chocolates.

- 35** Finding the number of integral solutions of the equation, such that $a > 0, b > 1, c > 2, d > 3$ is same as finding the number of integral solutions of the equation, such that $a \geq 1, b \geq 2, c \geq 3, d \geq 4$

Now the given equation $a + b + c + d = 12$ can be expressed as following.

$$(a - 1) + (b - 2) + (c - 3) + (d - 4)$$

$$= 12 - (1 + 2 + 3 + 4)$$

$$\Rightarrow w + x + y + z = 2;$$

$$\text{(Where } w = a - 1, x = b - 2, y = c - 3, z = d - 4)$$

Now the number of non-negative integral solutions of $w + x + y + z = 2$ is

$${}^{2+4-1}C_{4-1} = {}^5C_3 = 10$$

Therefore the number of integral solutions of

$$a + b + c + d = 12, \text{ when } a > 0, b > 1, c > 2, d > 3 \text{ is } 10.$$

Hence choice (a) is the answer.

NOTE This problem is same as distribution of 12 identical chocolates among 4 different persons, in such a way that one particular person gets more than 1 chocolate, another particular person gets more than 2 chocolates, another particular person gets more than 3 chocolates and there is another person who gets more than 4 chocolates.

- 36** Finding the number of integral solutions of the equation, such that $a > -4, b > -3, c > -2, d > -1$ is same as finding the number of integral solutions of the equation, such that $a \geq -3, b \geq -2, c \geq -1, d \geq 0$.

The given equation $a + b + c + d = 12$ can be expressed as following.

$$(a + 3) + (b + 2) + (c + 1) + (d + 0) = 12 + (3 + 2 + 1 + 0)$$

$$\Rightarrow w + x + y + z = 18;$$

$$\text{(Where } w = a + 3, x = b + 2, y = c + 1, z = d + 0)$$

Now the number of non-negative integral solutions of $w + x + y + z = 18$ is

$${}^{18+4-1}C_{4-1} = {}^{21}C_3 = 1330$$

Therefore the number of integral solutions of

$$a + b + c + d = 12 \text{ when } a > -4, b > -3, c > -2, d > -1 \text{ is } 1330$$

Hence choice (a) is the answer.

- 37** The given equation $a + b + c + d = 12$ can be expressed as following $a + b + 6 + 0 = 12 \Rightarrow a + b = 6$

Now the number of non-negative integral solutions of the equation $a + b = 6$ such that $a \geq 1$ and $b \leq 4$.

a	b	$a + b$
2	4	6
3	3	6
4	2	6
5	1	6
6	0	6

Therefore the total number of solutions of the given equation will be 5. Hence choice (c) is the correct one.

- 38** The given equation $a + b + c + d = 12$ can be expressed as following.

$$(a + 2) + (b + 2) + (c + 2) + (d + 2) = 12 + (2 + 2 + 2 + 2)$$

$$\Rightarrow w + x + y + z = 20;$$

(where $w = a + 2$, $x = b + 2$, $y = c + 2$, $z = d + 2$)

Now, finding the number of integral solutions of the equation $a + b + c + d = 12$, such that $-2 \leq a, b, c, d \leq 19$ is same as finding the number of integral solutions of the equation $w + x + y + z = 20$, such that $0 \leq w, x, y, z \leq 21$. Since none of w, x, y or z can have any value greater than 20, so the upper limit (21) on w, x, y and z is irrelevant.

Thus the given problem is same as finding the number of non-negative integral solutions of $w + x + y + z = 20$.

Therefore the required number of solutions

$$= {}^{20+4-1}C_{4-1} = {}^{23}C_3 = 1771$$

- 39** The following two problems are just the same, the only difference lies in the sign.

There are four numbers a, b, c, d such that $a + b + c + d = -12$. Find the number of integral solutions of the equation, such that $a, b, c, d < 0$ Answer : 165	There are four numbers a, b, c, d such that $a + b + c + d = 12$. Find the number of integral solutions of the equation, such that $a, b, c, d > 0$ Answer : 165

- 40** The following two problems are just the same, the only difference lies in the sign.

There are four numbers a, b, c, d such that $a + b + c + d = -12$. Find the number of integral solutions of the equation, such that $a, b, c, d \leq 0$ Answer : 425	There are four numbers a, b, c, d such that $a + b + c + d = 12$. Find the number of integral solutions of the equation, such that $a, b, c, d \geq 0$ Answer : 425

Solutions (for Q. Nos. 41 to 44) $a + b + c = 8$

Case I	0	0	8 $\rightarrow$ 3 ways/solutions
Case II	0	1	7 $\rightarrow$ 6 ways/solutions
Case III	0	2	6 $\rightarrow$ 6 ways/solutions
Case IV	0	3	5 $\rightarrow$ 6 ways/solutions
Case V	0	4	4 $\rightarrow$ 3 ways/solutions
Case VI	1	1	6 $\rightarrow$ 3 ways/solutions
Case VII	1	2	5 $\rightarrow$ 6 ways/solutions
Case VIII	1	3	4 $\rightarrow$ 6 ways/solutions
Case IX	2	2	4 $\rightarrow$ 3 ways/solutions
Case X	2	2	33 $\rightarrow$ 3 ways/solutions

NOTE Please keep in mind that this is just a reference list to aid to your visualization of solutions. Actually, when the number of solutions are too big to determine by preparing the list we have to resort to more logical methods. So it is important to learn the following methods, otherwise whenever large figures (or numbers) will be involved you will be grouping in the dark with no way out to get the solutions.

- 41** The number of integral solutions of $a + b + c = 8$, if

$0 \leq a, b, c \leq 4$ (Number of integral solutions of $a + b + c = 8$, if $0 \leq a, b, c \leq 8$) – (number of integral solutions of $a + b + c = 8$, if any of a, b or c is such that $5 \leq a, b, c \leq 8$). Number of integral solutions of $a + b + c = 8$, if $0 \leq a, b, c \leq 8 = {}^{8+3-1}C_{3-1} = {}^{10}C_2 = 45$.

Number of integral solutions of $a + b + c = 8$, if any of a, b or c is such that $5 \leq a, b, c \leq 8$

$$= 3[{}^{(8-5)+3-1}C_{3-1}] = 3[{}^5C_2] = 30$$

Therefore number of solutions of $a + b + c = 8$, if

$$0 \leq a, b, c \leq 4 = 45 - 30 = 15.$$

Hence choice (a) is the answer.

Hint The above solution works on the basis of inclusion-exclusion principle. According to that principle, the set of integral solutions of $a + b + c = 8$, if $0 \leq (a, b, c) \leq 4$ do not include those solutions in which either of a, b or c is greater than 4.

However, the set of integral solutions of $a + b + c = 8$, if

$5 \leq (a, b, c) \leq 8$ includes only those solutions which have at least one variable out of a, b and c greater than or equal to 5; so it's obvious that few of the variables will be certainly greater than or equal to 5 and few of them must be less than 5. So in this case each of the variables may not be greater than or equal to 5. Why does it happen? It's because of the objective.

As our objective is to exclude all those solutions in which even if one variable is greater than or equal to 5 rather than to include all those solutions in which every variable is greater than or equal to 5. For better understanding have a look at the above list of solutions.

Alternatively The number of solutions of $a + b + c = 8$ is the coefficient of x^8 in the expansion of

$$(1 + x + x^2 + x^3 + x^4)^3$$

Since, $(1 + x + x^2 + x^3 + x^4)^3$

$$= (1 + x + x^2 + x^3 + x^4)^2 \times (1 + x + x^2 + x^3 + x^4)$$

$$\Rightarrow (1 + x + x^2 + x^3 + x^4)^3$$

$$= (1 + 2x + 3x^2 + 4x^3 + 5x^4 + 4x^5$$

$$+ 3x^6 + 2x^7 + x^8)(1 + x + x^2 + x^3 + x^4)$$

$$\Rightarrow (1 + x + x^2 + x^3 + x^4)^3$$

$$= (1 + 3x + \dots + 15x^8 + \dots + 3x^{11} + x^{12})$$

As the coefficient of x^8 is 15, so the required number of solutions = 15.

Alternatively

a	4	4	4	4	4	3	3	3	3	2	2	2	1	1	0
b	4	3	2	1	0	4	3	2	1	4	3	2	4	3	4
c	0	1	2	3	4	1	2	3	4	2	3	4	3	4	4

Thus there are total 15 solutions.

- 42** For the number of integral solutions of $a + b + c = 8$, for $0 \leq a, b, c \leq 3$, first of all find the total number of integral solutions of $a + b + c = 8$ for $0 \leq a, b, c \leq 8$ and then subtract the number of integral solutions of $a + b + c = 8$ if any of a, b or c is such that $4 \leq a, b, c \leq 8$.

The number of integral solutions of $a + b + c = 8$ for $0 \leq (a, b, c) \leq 8$ is ${}^{8+3-1}C_{3-1} = {}^{10}C_2 = 45$.

And the number of integral solutions of $a + b + c = 8$ if any of a, b or c is such that $4 \leq a, b, c \leq 8$ is $3[{}^{(8-4)+3-1}C_{3-1}] - 3[{}^{(8-2 \times 4)+3-1}C_{3-1}]$

$$= 3[{}^6C_2] - 3[{}^2C_2] = 42$$

Therefore number of integral solutions of $a + b + c = 8$, for $0 \leq a, b, c \leq 3$ is $45 - 42 = 3$.

Hence choice (c) is the answer.

☞ Alternatively The number of solutions of $a + b + c = 8$ is the coefficient of x^8 in the expansion of $(1 + x + x^2 + x^3)^3$ which is 3 as

$$(1 + x + x^2 + x^3)^3 = (1 + 3x + \dots + 3x^8 + x^9)$$

☞ Alternatively Given that $a + b + c = 8$ and $0 \leq a, b, c \leq 3$, then we have the following solutions.

a	b	c
3	3	2
3	2	3
2	3	3

Therefore the required number of total solutions = 3

- 43** To find the number of integral solutions of $a + b + c = 8$, for $0 \leq a, b, c \leq 2$, first of all find the total number of integral solutions of $a + b + c = 8$ for $0 \leq a, b, c \leq 8$ and then subtract the number of solutions of $a + b + c = 8$ if any of a, b or c is such that $3 \leq a, b, c \leq 8$. The number of solutions of $a + b + c = 8$ for $0 \leq a, b, c \leq 8$ is 45.

The equation $a + b + c = 8$ for $3 \leq a, b, c \leq 8$ suggests that we have to find the solutions of this equation in which at least one variable must be greater than or equal to 3. So, we see that there can be at most two variables that will be greater than or equal to 3. But we also know that the solutions which have only one variable greater than or equal to 3 also include those solutions which have two variables greater than or equal to 3.

Therefore number of solutions of $a + b + c = 8$ for $3 \leq a, b, c \leq 8$ is

$$3[{}^{(8-3)+3-1}C_{3-1}] - 3[{}^{(8-2 \times 3)+3-1}C_{3-1}] = 3[{}^7C_2] - 3[{}^4C_2] = 45$$

Thus the number of solutions of $a + b + c = 8$ for

$0 \leq a, b, c \leq 2$ is $45 - 45 = 0$.

It shows that there are no solutions to the equation $a + b + c = 8$ for $0 \leq a, b, c \leq 2$

Hence choice (d) is the answer.

☞ Alternatively The number of solutions of $a + b + c = 8$ is the coefficient of x^8 in the expansion of $(1 + x + x^2)^3$, which is 0, as the highest coefficient in this expansion is x^6 .

- 44** To find the number of integral solutions of $a + b + c = 8$, for $0 \leq (a, b, c) \leq 1$, first of all find the total number of solutions of $a + b + c = 8$ for $0 \leq (a, b, c) \leq 8$ and then subtract the number of solutions of $a + b + c = 8$ if any of a, b or c is such that $2 \leq (a, b, c) \leq 8$. The number of solutions of $a + b + c = 8$ for $0 \leq (a, b, c) \leq 8$ is 45.

And the number of solutions of $a + b + c = 18$ if any of a, b or c is such that $2 \leq (a, b, c) \leq 8$ is

$$3[{}^{(8-3)+3-1}C_{3-1}] - 3[{}^{(8-2 \times 2)+3-1}C_{3-1}] + [{}^{(8-2 \times 3)+3-1}C_{3-1}] = 3[{}^8C_2] - 3[{}^6C_2] + [{}^4C_2] = 45$$

Therefore number of integral solutions of $a + b + c = 8$, for $0 \leq (a, b, c) \leq 1$ is $45 - 45 = 0$

Hence choice (d) is the answer.

☞ Alternatively The number of solutions of $a + b + c = 8$ is the coefficient of x^8 in the expansion of $(1 + x + x^2)^3$, which is 0 as the highest coefficient in this expansion is x^6 .

☞ Alternatively Since the highest value of a, b and c is 2, so the total sum $a + b + c \leq 6$. Thus we cannot attain $a + b + c = 8$ even once.

Therefore we have no solution for the given equation.

- 45** The number of integral solutions of $a + b + c + d = 30$, if $0 \leq (a, b, c, d) \leq 15$ = (Number of integral solutions of $a + b + c + d = 30$, if $0 \leq (a, b, c, d) \leq 30$) - (number of integral solutions of $a + b + c + d = 30$, if any of a, b, c or d is such that $16 \leq (a, b, c, d) \leq 30$).

Number of integral solutions of $a + b + c + d = 30$, if $0 \leq (a, b, c, d) \leq 30 = {}^{30+4-1}C_{4-1} = {}^{33}C_3$

Number of integral solutions of $a + b + c + d = 30$, if any of a, b, c or d is such that $16 \leq (a, b, c, d) \leq 30$

$$= 4[{}^{(30-16)+4-1}C_{4-1}] = 4[{}^{17}C_3]$$

Therefore number of solutions of $a + b + c + d = 30$, if $0 \leq (a, b, c, d) \leq 15 = {}^{33}C_3 - 4[{}^{17}C_3] = 2736$.

Hence choice (a) is the answer.

☞ Alternatively The number of integral solutions of $a + b + c + d = 30$, if $0 \leq (a, b, c, d) \leq 15$ = Coefficient of x^{30} in the expansion of $(1 + x + x^2 + x^3 + \dots + x^{15})^4$

Now $(1 + x + x^2 + x^3 + \dots + x^{15})^4$

$$= (1 + 2x + 3x^2 + \dots + 14x^{13} + 15x^{14} + 16x^{15} + 15x^{16} + 14x^{17} + \dots + 2x^{29} + x^{30})^2$$

Therefore coefficient of

$$x^{30} = 2(1^2 + 2^2 + 3^2 + \dots + 15^2) + 16^2 = 2736$$

- 46** The number of integral solutions of $a + b + c + d = 30$, if $0 \leq (a, b, c, d) \leq 10 =$ (Number of integral solutions of $a + b + c + d = 30$, if $0 \leq (a, b, c, d) \leq 30$) - (Number of integral solutions of $a + b + c + d = 30$, if any of a, b, c or d is such that $11 \leq (a, b, c, d) \leq 30$).

Number of integral solutions of $a + b + c + d = 30$, if $0 \leq (a, b, c, d) \leq 30 = {}^{30+4-1}C_{4-1} = {}^{33}C_3$

Number of integral solutions of $a + b + c + d = 30$, if any a, b, c or d is such that $11 \leq (a, b, c, d) \leq 30$

$$= 4[{}^{(30+11)+4-1}C_{4-1}] - 6[{}^{(30-2 \times 11)+4-1}C_{4-1}]$$

$$= 4[{}^{22}C_3] - 6[{}^{11}C_3]$$

Therefore number of solutions of $a + b + c + d = 30$, if $0 < a, b, c, d \leq 6$

$$= {}^{33}C_3 - [4({}^{22}C_3) - 6({}^{11}C_3)] = 286.$$

Hence choice (b) is the answer.

Hint If any of a, b, c or d is such that $11 \leq a, b, c, d \leq 30$ then there are two cases.

(a) When one variable out of a, b, c is certainly at least 11, then one 11 is taken out from 30 and then remaining 19 is to be divided among a, b, c, d in ${}^{19+4-1}C_3 = {}^{22}C_3 = {}^{22}C_3$ ways. And this one 11 is later on arranged in ${}^4C_1 = 4$ ways.

(b) When any two variables out of a, b, c, d are certainly at least 11, then two 11s are taken out from 30 and then remaining 8 is to be divided among a, b, c, d in ${}^{8+4-1}C_3 = {}^{11}C_3$ ways. And these two 11s are later on arranged in ${}^4C_2 = 6$ ways.

But since case (a) includes case (b) also, so we have to subtract case (b) from case (a) as per inclusion-exclusion principle.

Alternatively The number of integral solutions of $a + b + c + d = 30$, if $0 \leq a, b, c, d \leq 10 =$ Coefficient of x^{30} in the expansion of $(1 + x + x^2 + x^3 + \dots + x^{10})^4$

$$\begin{aligned} \text{Now, } (1 + x + x^2 + x^3 + \dots + x^{10})^4 \\ = (1 + 2x + 3x^2 + \dots + 9x^8 + 10x^9 + 11x^{10} + 10x^{11} + 9x^{12} + \dots + 2x^{19} + x^{20})^2 \end{aligned}$$

Therefore coefficient of

$$\begin{aligned} x^{30} = 2(11 \times 1 + 10 \times 2 + 9 \times 3 + 8 \times 4 \\ + 7 \times 5) + (6 \times 6) = 286 \end{aligned}$$

- 47** The number of integral solutions of $a + b + c + d = 30$, if $0 \leq (a, b, c, d) \leq 8 =$ (Number of integral solutions of $a + b + c + d = 30$, if $0 \leq (a, b, c, d) \leq 30$) - (Number of integral solutions of $a + b + c + d = 30$, if any of a, b, c or d is such that $9 \leq (a, b, c, d) \leq 30$).

Number of integral solutions of $a + b + c + d = 30$, if

$$0 \leq (a, b, c, d) \leq 30 = {}^{30+4-1}C_{4-1} = {}^{33}C_3$$

Number of integral solutions of $a + b + c + d = 30$, if any of a, b, c or d is such that

$$\begin{aligned} 9 \leq (a, b, c, d) \leq 30 = 4[{}^{(30-9)+4-1}C_{4-1}] \\ - 6[{}^{(30-2 \times 9)+4-1}C_{4-1}] + 4[{}^{(30-3 \times 9)+4-1}C_{4-1}] \\ = 4[{}^{24}C_3] - 6[{}^{15}C_3] + 4[{}^6C_3] = 5446 \end{aligned}$$

Therefore number of solutions of $a + b + c + d = 30$, if $0 \leq (a, b, c, d) \leq 8 = {}^{33}C_3 - [4({}^{24}C_3) - 6({}^{15}C_3) + 4({}^6C_3)]$

$$= 5456 - 5446 = 10$$

Hence choice (b) is the answer.

Alternatively The number of integral solutions of $a + b + c + d = 30$, if $0 \leq (a, b, c, d) \leq 10 =$ Coefficient of x^{30} in the expansion of $(1 + x + x^2 + x^3 + \dots + x^8)^4$ which is 10.

- 48** The number of ways in which $(m + n + p + q + r)$ different things can be divided into five different groups containing m, n, p, q and r things respectively, if the order of the group is important = $\frac{(m + n + p + q + r)!}{m!n!p!q!r!} \times 5!$

Therefore number of ways of distributing 15 rings among his 5 daughters such that one daughter gets 1 ring, another daughter gets 2 rings, and so on = $\frac{15!}{1!2!3!4!5!} \times 5!$

Hence choice (d) is the answer.

NOTE Please take stock of the situation that the articles (rings) as well as receivers (girls) are different.

Since the articles are different so we will use the division formula $\frac{(m + n + p + q + r)!}{m!n!p!q!r!}$ for distinct things.

Further, since the receivers are distinct (as girls are distinguishable) so we can arrange the different sets of rings among different girls. That means the order is important, so we have to multiply the possible divisions of groups by 5! In order to bring out all the possibilities in which these 15 rings can be distributed among his 5 daughters.

- 49** Again rings as well as girls are distinct from each other. However, it is certain that who gets how many rings, so we cannot interchange the set of rings among these girls. That means we can only choose the different rings but not the number of rings for each girl. Since it is already decided that the eldest daughter gets 1 ring, second eldest daughter gets 2 rings, the middle one gets 3 rings, the second youngest daughter gets 4 rings and the youngest daughter gets 5 rings. Therefore number of ways of distributing 15 rings among his 5 daughters = $\frac{15!}{1!2!3!4!5!}$

Hence choice (b) is the answer.

- 50** Primarily, there are two different cases in which 15 different rings can be divided. As per the statement there will be only one girl who is eldest and only one girl

who is youngest. Since the problem states that there are no twins but it does not say that there can't be any triplet (i.e. three kids born at the same time). So the case II arises.

Case I. 1, 2, 3, 4, 5; Case II. 1, 3, 3, 3, 5

Case I. Number of ways of distributing 15 rings

$$= \frac{15!}{1!2!3!4!5!}$$

Here we are not multiplying by 5! because it is already known that which particular daughter gets the particular number of rings.

Case II. Number of ways of distributing 15 rings

$$= \frac{15!}{1!3!3!3!5!} \times \frac{1}{3!} \times 3!$$

Here multiplication by $1!/3!$ indicates that there are 3 groups containing the same number of rings. And the multiplication by $3!$ indicates that the 3 groups of equal number of rings can be arranged among 3 distinct girls. Note that the age of girls is same, but girls are distinguishable by their names, features and all sorts of attributes that may distinguish two or more individuals. Also, note that we are arranging it among the 3 girls only, because we already know that the youngest daughter gets 5 rings and the oldest daughter gets 1 ring, so these two girls cannot be considered for arrangement.

Therefore total number of ways of distributing 15 different rings

$$= \frac{15!}{1!2!3!4!5!} + \frac{15!}{1!3!3!3!5!} = \frac{15!}{3!5!} \times \left[\frac{1}{2! \times 4!} + \frac{1}{3! \times 3!} \right] = \frac{15!}{3!5!} \times \frac{7}{144}$$

Hence choice (a) is the answer.

- 51** Primarily, there are three different cases in which 15 different rings can be divided such that none of them gets less than 2 and more than 4 rings.

Case I: 2, 2, 3, 4, 4; Case II: 2, 3, 3, 3, 4;

Case III: 3, 3, 3, 3, 3

Case I Number of ways of distributing 15 rings

$$= \frac{15!}{2!2!3!4!4!} \times \frac{1}{2! \times 2!} \times 5!$$

Here multiplication by $\frac{1!}{2!2!}$ indicates that there are

2 groups containing 2 rings in each group and there are 2 groups containing 4 rings in each group. And the multiplication by 5! indicates that there 5 distinct girls among which all the ring are to be distributed.

Case II Number of ways of distributing 15 rings

$$= \frac{15!}{2!3!3!3!4!} \times \frac{1}{3!} \times 5!$$

Here multiplication by $1!/3!$ indicates that there are 3 groups containing the same number of rings. And the

multiplication by 5! indicates that there are 5 distinct girls among which all the rings are to be distributed.

Case III Number of ways of distributing 15 rings

$$= \frac{15!}{3!3!3!3!3!} \times \frac{1}{5!} \times 5!$$

Here multiplication by $1!/5!$ indicates that there are 5 groups containing the same number of rings. And the multiplication by 5! indicates that there are 5 distinct girls among which all the rings are to be distributed.

Therefore total number of ways of distributing 15 different rings

$$= 15! \times 5! \times \left[\frac{1}{(2!)^4 \times 3! \times (4!)^2} + \frac{1}{(2!) \times (3!)^4 \times 4!} + \frac{1}{(3!)^5 \times 5!} \right]$$

Hence choice (d) is the answer.

- 52** The number of ways of distributing 15 different rings among 5 girls equally $= \frac{15!}{3!3!3!3!3!} \times \frac{1}{5!} \times 5! = \frac{15!}{(3!)^5}$

Here multiplication by $1!/5!$ indicates that there are 5 groups containing the same number of rings. And the multiplication by 5! indicates that there are 5 distinct girls among which all the rings are to be distributed.

Hence choice (c) is the answer.

- 53** The number of ways of distributing 15 different rings among 5 girls such that a girl may get nothing or all the rings

$$= \underbrace{5 \times 5 \times 5 \times 5 \times 5 \times \dots \times 5}_{15 \text{ times}} = 5^{15}$$

Hence choice (d) is the answer.

- 54** The number of ways of selection of 1 ring, 2 rings, 3 rings, 4 rings and 5 rings $= 1 \times 1 \times 1 \times 1 \times 1 = 1$

Number of ways of distributing these 5 sets of rings containing 1, 2, 3, 4, 5 rings $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

Hence choice (a) is the answer.

Hint Number of ways of selecting 1 ring out of 15 rings = 1, selecting 2 rings out of remaining 14 rings = 1, and so on. Thus there will be 5 different sets of rings.

First set can be given to any 5 girls, second set can be given to any 4 girls, third set can be given to any three girls, fourth set can be given to any 2 girls and the last and fifth set can be given to the remaining girl.

- 55** The number of ways of selection of 1 ring, 2 rings, 3 rings, 4 rings and 5 rings $= 1 \times 1 \times 1 \times 1 \times 1 = 1$

Since it is already known that who gets how many rings, so the number of ways of distributing 5 different sets of rings = 1
 Hence choice (a) is the answer.

- 56** Primarily, there are two different cases in which 15 identical rings can be divided. As per the statement there will be only one girl who is the eldest and only one girl who is the youngest. Since the problem states that

there are no twins but it does not say that there can't be triplets. So the case II arises.

Case I 1, 2, 3, 4, 5; **Case II** 1, 3, 3, 3, 5

Case I Number of ways of distributing 15 rings = 1

Case II Number of ways of distributing 15 rings = 1

Therefore total number of ways of distributing 15 rings = 2

Hence choice (b) is the answer.

☞ **Alternatively** Let us consider a, b, c, d, e be his five daughters, then

$$a + b + c + d + e = 15 \quad 1 + 2 + 3 + 4 + 5 = 15$$

$$1 + 3 + 3 + 3 + 5 = 15$$

Thus there are only 2 ways of distribution as per the given conditions. Please note that the absence of twins in the family does not rule out the possibility of triplets.

- 57** The number of ways of dividing 15 identical rings among 5 girls such that none of them receives less than 2 or more than 4 = coefficient of x^{15} in the expansion of

$$(x^2 + x^3 + x^4)^5$$

$$\text{But } (x^2 + x^3 + x^4)^5 = (x^2 + x^3 + x^4)^4 (x^2 + x^3 + x^4)$$

$$\Rightarrow (x^2 + x^3 + x^4)^5 = (x^4 + 2x^5 + 3x^6 + 2x^7 + x^8)^2 (x^2 + x^3 + x^4)$$

$$\Rightarrow (x^2 + x^3 + x^4)^5 = (x^8 + 4x^9 + 10x^{10} + 16x^{11} + 19x^{12} + 16x^{13} + 10x^{14} + 4x^{15} + x^{16})(x^2 + x^3 + x^4)$$

$$\Rightarrow (x^2 + x^3 + x^4)^5 = (x^{10} + \dots + 51x^{15} + \dots + x^{20})$$

Since the coefficient of x^{15} in the expansion of $(x^2 + x^3 + x^4)^5$ is 51. Therefore the required number of ways = 51.

Hence choice (a) is the answer.

☞ **Alternatively** Primarily, there are three different cases in which 15 different rings can be divided such that none of them gets less than 2 and more than 4 rings.

$$\text{Case I: } 2, 2, 3, 4, 4; \quad \text{ways of distribution} = \frac{5!}{2! \times 2!} = 30$$

$$\text{Case II: } 2, 3, 3, 3, 4; \quad \text{ways of distribution} = \frac{5!}{3!} = 20$$

$$\text{Case III: } 3, 3, 3, 3, 3; \quad \text{ways of distribution} = \frac{5!}{5!} = 1$$

Therefore the number of ways in which 15 identical rings can be distributed among 5 girls such that none of them gets less than 2 or more than 4 is 51 (= 30 + 20 + 1).

☞ **Alternatively** This problem is similar to finding the number of solutions to the equation $a + b + c + d + e = 15$, such that $2 \leq a, b, c, d, e \leq 4$.

In turn, it is equivalent to finding the number of solutions to the equation $u + v + w + x + y = 5$, such that $0 \leq u, v, w, x, y \leq 2$

In turn number of solutions of the equation

$$u + v + w + x + y = 5, \text{ such that } 0 \leq u, v, w, x, y \leq 2 =$$

(number of solutions to the equation

$$u + v + w + x + y = 5, \text{ such that } 0 \leq u, v, w, x, y \leq 5) -$$

(number of solutions of the equation $u + v + w + x + y = 5$, such that at least one of the variables is $3 \leq u, v, w, x, y, \leq 5$)

Now the solution to the equation

$$u + v + w + x + y = 5, \text{ such that } 0 \leq u, v, w, x, y \leq 5 \text{ is } {}^{5+5-1}C_{5-1} = {}^9C_4 = 126.$$

And the solution to the equation $u + v + w + x + y = 5$, such that at least one of the variables is $3 \leq u, v, w, x, y \leq 5$ is $5 [{}^{(5-3)+5-1}C_{5-1}] = 5 [{}^6C_4] = 75$

Therefore number of solutions to the equation $u + v + w + x + y = 5$, such that $0 \leq u, v, w, x, y \leq 2$ is $= 126 - 75$

In turn, the number of solutions of the equation $a + b + c + d + e = 15$, such that $2 \leq a, b, c, d, e \leq 4$ is 51.

It implies that the number of ways in which 15 identical rings can be distributed among 5 girls such that none of them gets less than 2 or more than 4 is 51.

- 58** The number of ways of dividing 15 rings equally into 5 parts = 1

Now arranging the 5 sets of rings containing 5 rings each = 1

Therefore number of ways of distributing 15 identical rings equally among 5 girls = $1 \times 1 = 1$

Hence choice (b) is the answer.

☞ **Alternatively** This problem is similar to finding the number of solutions to the equation $a + b + c + d + e = 15$, such that $a = b = c = d = e$.

As $a + b + c + d + e = 15$, therefore $a = b = c = d = e = 3$.

Thus we have only 1 solution to this equation. So we can say that there is only 1 way of distribution of the 15 rings equally among his daughters.

- 59** The number of ways of dividing 15 identical rings among 5 girls such that these girls may get any number of rings $= {}^{15+5-1}C_{5-1} = {}^{19}C_4 = 3876$

Hence choice (c) is the answer.

- 60 Case I** 1, 1, 3

$$\text{Number of ways of selection of rolls} = {}^5C_1 \times {}^4C_1 \times {}^3C_3 = 20$$

Number of ways of arranging the 3 distinct boxes, where 2 boxes have the same number of rolls = $\frac{3!}{2!}$

$$\text{Number of ways of distribution of rolls} = {}^5C_1 \times {}^4C_1 \times {}^3C_3 \times \frac{3!}{2!} = 60$$

Case II: 1, 2, 2

$$\text{Number of ways of selection of rolls} = {}^5C_1 \times {}^4C_2 \times {}^2C_2$$

Number of ways of arranging the 3 distinct boxes, where 2 boxes have the same number of rolls = $\frac{3!}{2!}$

Number of ways of distribution of rolls

$$= {}^5C_1 \times {}^4C_2 \times {}^2C_2 \times \frac{3!}{2!} = 90$$

Therefore the required number of ways = $60 + 90 = 150$

☞ **Alternatively** Consider the following table.

Case	Box 1	Box 2	Box 3	Number of ways of distribution
i	1	1	3	$\frac{5!}{1!1!3!} = 20$
ii	1	3	1	$\frac{5!}{1!3!1!} = 20$
iii	3	1	1	$\frac{5!}{3!1!1!} = 20$
iv	1	2	2	$\frac{5!}{1!2!2!} = 30$
v	2	1	2	$\frac{5!}{2!1!2!} = 30$
vi	2	2	1	$\frac{5!}{2!2!1!} = 30$

Therefore the required number of ways

$$= 3 \times (20 + 30) = 150$$

61 Case I 1, 1, 3

Number of ways of selection of rolls = 1

Number of ways of arranging the 3 distinct boxes, where 2 boxes have the same number of rolls = $\frac{3!}{2!}$

Number of ways of distribution of rolls = $1 \times \frac{3!}{2!} = 3$

Case II 1, 2, 2

Number of ways of selection of rolls = 1

Number of ways of arranging the 3 distinct boxes, where 2 boxes have the same number of rolls = $\frac{3!}{2!}$

Number of ways of distribution of rolls = $1 \times \frac{3!}{2!} = 3$

Therefore the required number of ways = $3 + 3 = 6$

☞ Alternatively

Keeping 5 distinct rolls in 3 identical boxes so that none of the box remains empty implies that at least 1 roll must be kept in each box.

Therefore the required number of ways
 $= {}^{5-1}C_{3-1} = {}^4C_2 = 6$

62 Case I (1, 1, 3)

Number of ways of selection of rolls

$$= \frac{5!}{1!1!3!} \times \frac{1}{2!}$$

Number of ways of distribution of rolls

$$= 1 \times \frac{5!}{1!1!3!} \times \frac{1}{2!} = 10$$

Case II (1, 2, 2)

Number of ways of selection of rolls

$$= \frac{5!}{1!2!2!} \times \frac{1}{2!}$$

Number of ways of distribution of rolls

$$= 1 \times \frac{5!}{1!2!2!} \times \frac{1}{2!} = 15$$

Therefore the required number of ways = $10 + 15 = 25$

63 Case I : (1, 1, 3)

Number of ways of selection of rolls = $1 \times 1 \times 1 = 1$

Number of ways of distribution of rolls = 1

Case II : (1, 2, 2)

Number of ways of selection of rolls = $1 \times 1 \times 1 = 1$

Number of ways of distribution of rolls = 1

Therefore the required number of ways = $1 + 1 = 2$

64 Consider the following table. Assume that the crate

1 contains the least number of bouquets in every case.

Case	Crate 1	Crate 2	Crate 3	Total number of ways
i	0	0, 1, 2, 3, 4, 5, 6	12, 11, 10, 9, 8, 7, 6	7
ii	1	1, 2, 3, 4, 5	10, 9, 8, 7, 6	5
iii	2	2, 3, 4, 5	8, 7, 6, 5	4
iv	3	3, 4	6, 5	2
v	4	4	4	1

Therefore the required number of ways

$$= 7 + 5 + 4 + 2 + 1 = 19$$

☞ **Alternatively** Let us consider, for a moment, that the crates are distinct.

The number of ways of distribution of 12 identical bouquets into 3 distinct crates = ${}^{12+3-1}C_{3-1} = 91$

Case I When each crate has same number of bouquets.

(4, 4, 4)

Then it can be done in only 1 way.

Even though the crates are distinct, the number of ways of distribution will be 1 only, as each crate receives equal number of bouquets.

Case II When any 2 crates have same number of bouquets.

(0,0,12), (1,1,10), (2, 2, 8), (3,3,6), (5,5,2), (6,6,0)

Number of ways = 6

When we assume that the crates are distinct the number of

ways of distribution of bouquets = $\frac{3!}{2!} \times 6 = 18$

Case III When all the crates have distinct number of bouquets. Let us assume that the number of ways of distribution of 12 identical bouquets among 3 identical crates is k . Then the number of ways of distribution of 12 identical bouquets among 3 distinct crates $= 3! \times k = 6 \times k$. Therefore, we have $1 + 18 + 6k = 91$

Or $k = 12$

Thus the required number of ways $= 1 + 6 + 12 = 19$.

- 65** The number of ways in which 12 distinct bouquets can be kept in 3 distinct crates $= {}^{12+3-1}C_{3-1} = 91$

- 66** The number of ways in which 12 distinct bouquets can be kept in 3 distinct crates, so that no box remains empty $= {}^{12-1}C_{3-1} = 55$

- 67** In order to keep at least 2 bouquets in each of the crates you can take away 6 bouquets separately. Now you are left with just 6 bouquets to distribute among 3 distinct crates, which you can now distribute any number of bouquets among 3 crates. This is equivalent to the distribution of 6 identical bouquets among 3 distinct crates so that any number of bouquets can be kept in any crate $= {}^{6+3-1}C_{3-1} = 28$

- 68** Let us consider that a, b and c are three distinct crates, so that $a \geq 2, b \geq 3, c \geq 4$ and $a + b + c = 12$

Consider $a \geq 2 \Rightarrow a - 2 \geq 0 \Rightarrow x \geq 0$

Similarly, $b \geq 3 \Rightarrow b - 3 \geq 0 \Rightarrow y \geq 0$

And, $c \geq 4 \Rightarrow c - 4 \geq 0 \Rightarrow z \geq 0$

Therefore $a + b + c = 12$

$\Rightarrow x + y + z = 12 - (2 + 3 + 4)$

$\Rightarrow x + y + z = 3$

Thus the number of ways of distribution of 12 identical bouquets among 3 distinct crates so that one particular crate has at least 2 bouquets and another one has at least 3 bouquets and the third one has at least 4 bouquets is same as the number of distribution of 3 identical bouquets so that each of them can have any number of bouquets.

Therefore the required number of ways $= {}^{3+3-1}C_{3-1} = 10$

Alternatively First of all take away 2, 3 and 4 bouquets for crate 1, 2 and 3, assuming crate 1 is the smallest one and crate 3 is the largest one. Now you have to distribute the remaining bouquets, which can be done in the following ways.

Case I (0, 0, 3) Number of ways of distribution $= \frac{3!}{2!} = 3$

Case II (0, 1, 2) Number of ways of distribution $= 3! = 6$

Case III (1, 1, 1) Number of ways of distribution $= 1$

Therefore total number of required ways of distribution of 12 bouquets $= 3 + 6 + 1 = 10$

- 69** Let us consider that (a, b, c) is the notation for the number of bouquets distributed among three crates. It is imperative

to note that the number of selection in each case will be 1 as all the bouquets are identical. The only thing we need to calculate is the number of ways of distribution among 3 distinct crates.

Case I (0, 1, 11), (0, 2, 10), (0, 3, 9), (0, 4, 8), (0, 5, 7)

Case II (1, 2, 9), (1, 3, 8), (1, 4, 7), (1, 5, 6)

Case III (2, 3, 7), (2, 4, 6)

Case IV (3, 4, 5)

Total number of combinations $= 5 + 4 + 2 + 1 = 12$

And since crates are distinct and no two crates has same number of bouquets, so each combination further can be arranged in $3!$ ways.

Therefore the required number of ways of distributions $3! \times 12 = 72$

Alternatively

The number of ways of distribution of 12 identical bouquets among 3 distinct crates $= {}^{12+3-1}C_{3-1} = 91$

Case I When each crate has the same number of bouquets. (4, 4, 4)

Then it can be done in only 1 way.

Case II When any 2 crates have the same number of bouquets.

(0,0,12), (1,1,10), (2, 2, 8), (3,3,6), (5,5,2), (6,6,0)

Number of ways of selection of 12 bouquets $= 6$

Since there are 3 distinct crates out of which the 2 crates are having the same number of bouquets, therefore the number of arrangements of 3 crates $= \frac{3!}{2!}$.

Therefore the number of ways of distribution of 12 identical bouquets so that exactly 2 crates have the same number of bouquets $= \frac{3!}{2!} \times 6 = 18$

Case III When all the crates have distinct number of bouquets.

Then, let us assume that the number of ways of distribution of 12 identical bouquets among 3 distinct crates $= k$

Therefore, we have $1 + 18 + k = 91$ Or $k = 72$

Thus the required number of ways $= 72$

- 70** The number of selection of identical bouquets in three groups $= 1 \times 1 \times 1 = 1$

Further, since in each crate the number of bouquets are same, so the number of arrangements of crates $= 1$

Therefore the required number of ways of distribution $=$ number of ways of selection of bouquets

$\times$ number of ways of arrangements of crates $= 1$

It's like an equation $a + b + c = 12$, such that $a = b = c = 4$. This obviously has 1 solution as in $4 + 4 + 4 = 12$

- 71** Number of ways of selection of 4 balls from 10 balls of distinct colours $= {}^{10}C_4 = 210$

- 72** Number of ways of selection of 4 balls from a pool of unlimited balls of 10 distinct colours = number of non-negative integral solutions of

$$x_1 + x_2 + \dots + x_9 + x_{10} = 4,$$

such that $x_1, x_2, \dots, x_9, x_{10} \geq 0$

Therefore, the required number of selections
 $= {}^{4+10-1}C_{10-1} = {}^{13}C_9 = 715$

Introductory Exercise 19.9

- 1** Since the number of ways in which n different letters can be placed in their n addressed envelopes so that all the letters are in the wrong envelopes

$$= n! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!} \right]$$

Therefore the required number of ways

$$\begin{aligned} &= 6! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^6 \frac{1}{6!} \right] \\ &= 6! \left(1 - \frac{1}{1} + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} - \frac{1}{120} + \frac{1}{720} \right) \\ &= 720 \left(\frac{360 - 120 + 30 - 6 + 1}{720} \right) = 265 \end{aligned}$$

- 2** The number of ways in which exactly r letters can be placed in wrongly addressed envelopes when n letters are placed in n addressed envelopes

$$= {}^nP_r \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^r \frac{1}{r!} \right]$$

Therefore the required number of ways

$$\begin{aligned} &= {}^6P_4 \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \right] \\ &= 60 \left[1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} \right] \\ &= 360 \left[\frac{12 - 4 + 1}{24} \right] = 135 \end{aligned}$$

- 3** At least 2 letters are correctly placed
 = At most 4 letters are correctly placed
 = Either 4 letters are wrongly placed or 3 letters are wrongly placed or 2 letters are wrongly placed or 1 letter is wrongly placed or no letter is wrongly placed.

- 73** Number of ways of selection of 10 cookies from a pool of unlimited cookies of 4 distinct types = number of positive integral solutions of $x_1 + x_2 + x_3 + x_4 = 10$, such that $x_1, x_2, x_3, x_4 \geq 1$

Therefore, the required number of selections
 $= {}^{10-1}C_{4-1} = {}^9C_3 = 84$

Number of ways when 4 letters are wrongly placed

$$= {}^6P_4 \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \right] = 135$$

Number of ways when 3 letters are wrongly placed

$$= {}^6P_3 \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \right] = 40$$

Number of ways when 2 letters are wrongly placed

$$= {}^6P_2 \left[1 - \frac{1}{1!} + \frac{1}{2!} \right] = 15$$

Number of ways when 1 letter is wrongly placed

$$= {}^6P_1 \left[1 - \frac{1}{1!} \right] = 0$$

Number of ways when none of the letters is wrongly placed = 1

Therefore the required number of ways
 $= 135 + 40 + 15 + 0 + 1 = 191$

- 4** Given that a, b, c, d, e, f can take any value from the set $\{1, 2, 3, 4, 5, 6\}$.

N is a non-zero integer only when $a \neq 1, b \neq 2, c \neq 3, d \neq 4, e \neq 5, f \neq 6$. That is a can be anything but not 1, b can be anything but not 2, ..., f can be anything but not 6.

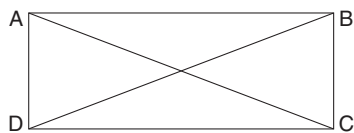
It means this problem is same as placing 6 letters in 6 envelopes such that no letter is placed in the right envelope.

Therefore the required number of ways

$$\begin{aligned} &= 6! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^6 \frac{1}{6!} \right] \\ &= 6! \left(1 - \frac{1}{1} + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} + \frac{1}{720} \right) \\ &= 720 \left(\frac{360 - 120 + 30 - 6 + 1}{720} \right) = 265 \end{aligned}$$

Introductory Exercise 19.10

Solutions (for Q. No. 1 and 2) : Consider the following diagram.



- 1 There are 4 points, A, B, C and D. So using these 4 points we have total 6 lines : AB, BC, CD, AD, AC and BD.

Since each line needs 2 points to be connected, therefore the total number of lines = ${}^4C_2 = 6$

- 2 Using any 3 of the 4 points we have total 4 triangles: ACD, ABC, DAB and BCD.

Since each triangle needs 3 points to be connected, therefore the total number of triangles = ${}^4C_3 = 4$

Solutions (for Q Nos. 3 to 6) : In these sums you have to consider those figures (lines, triangles, hexagons and decagons), whose vertices are the points already given in the plane.

- 3 The maximum number of straight lines that can be formed using 10 non-collinear points = ${}^{10}C_2 = 45$
- 4 The maximum number of triangles that can be formed using 10 non-collinear points = ${}^{10}C_3 = 120$
- 5 The maximum number of hexagons that can be formed using 10 non-collinear points = ${}^{10}C_6 = 210$
- 6 The maximum number of diagonals in a decagon that can be formed using 10 non collinear points

$$= {}^{10}C_2 - 10 = 45 - 10 = 35$$

Hint Number of diagonals = total number of lines
– number of sides

Solutions (for Q. Nos. 7 to 10) :

- 7 Since all the 15 points are on the circumference of a circle, it implies that no 3 lines are collinear. Therefore the number of straight lines formed by joining the 15 points = ${}^{15}C_2 = 105$
- 8 Since no three points are collinear, so the number of triangles that can be formed by joining 15 points = ${}^{15}C_3 = 455$
- 9 Since no three points are collinear, so the number of quadrilaterals that can be formed by joining 15 points = ${}^{15}C_4 = 1365$
- 10 Since no three points are collinear, so the number of octagons that can be formed by joining 15 points = ${}^{15}C_8 = 6435$

Solutions (for Q. Nos. 11 to 14) :

- 11 Total number of lines if 12 non-collinear points are connected = ${}^{12}C_2 = 66$

Number of lines if 4 non-collinear points are connected = ${}^4C_2 = 6$

Since 4 points are collinear, so there would be only 1 line instead of 6 lines.

Thus, the maximum number of lines = $(66 - 6) + 1 = 61$.

- 12 Since out of 12 points 4 points are collinear, so the maximum number of triangles = ${}^{12}C_3 - {}^4C_3 = 220 - 4 = 216$
- 13 Since out of 12 points 4 points are collinear, so the maximum number of quadrilaterals = ${}^{12}C_4 - {}^4C_4 = 495 - 1 = 494$
- 14 Since 4 points are not sufficient to form a hexagon, so it does not matter that the 4 points are collinear. Therefore the maximum number of hexagons = ${}^{12}C_6 = 924$

Solutions (for Q. Nos. 15 to 18) :

- 15 Maximum number of straight lines = ${}^nC_2 - {}^mC_2 + 1$
- 16 Maximum number of triangles = ${}^nC_3 - {}^mC_3$
- 17 Maximum number of quadrilaterals = ${}^nC_4 - {}^mC_4$
- 18 Maximum number of hexagons = ${}^nC_6 - {}^mC_6$

Solutions (for Q. Nos. 19 to 22) :

- 19 Number of sides in a heptagon = 7

Then the total number of lines that can be formed by connecting 7 points (vertices) of the heptagon = ${}^7C_2$

Therefore the total number of diagonals in the heptagon = ${}^7C_2 - 7 = 21 - 7 = 14$

- 20 Number of sides in a polygon = n

Then the total number of lines that can be formed by connecting n points of the polygon = ${}^nC_2$

Therefore the total number of diagonals in the polygon = ${}^nC_2 - n$

- 21 Number of diagonals in a polygon

$$= {}^nC_2 - n = \frac{n!}{(n-2)! \times 2!} - n$$

$$= \frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}$$

$$\therefore \frac{n(n-3)}{2} = 54$$

$$\Rightarrow n = 12$$

$$22 \quad \because {}^9C_3 - {}^mC_3 = 28 \Rightarrow {}^mC_3 = 56$$

$$\Rightarrow \frac{m(m-1)(m-2)}{6} = 56$$

$$\Rightarrow m(m-1)(m-2) = 8 \times 7 \times 6$$

$$\Rightarrow m = 8$$

Solutions (for Q. Nos. 23 and 24) :

$$23 \quad \begin{array}{c} \leftarrow \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \rightarrow \\ \quad \quad P \quad \quad \quad Q \end{array}$$

$$\begin{array}{c} \leftarrow \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \rightarrow \\ \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \end{array}$$

A triangle can be formed by selecting 1 point out of P and Q and selecting 2 points out of 8 points on the other parallel line.

or a triangle can be formed by selecting 2 points out of 2 points (P and Q) and selecting 1 point out of 8 points.

$$\begin{aligned} \therefore \text{Required number of triangles} &= ({}^2C_1 \times {}^8C_2) + ({}^2C_2 \times {}^8C_1) \\ &= (2 \times 28) + (1 \times 8) = 64 \end{aligned}$$

24 Number of triangles whose vertex is P = total number of triangles – number of triangles which do not contain P as a vertex.

$$\therefore \text{Number of triangles which do not contain } P \text{ as a vertex} = {}^1C_1 \times {}^8C_2 = 28$$

$$\therefore \text{Required number of triangles} = 64 - 28 = 36$$

Solutions (for Q. Nos. 25 - 27) :

25 An intersection point is formed by the intersection of two lines. Hence number of intersection points is equal to the number of ways of selecting 2 lines out of the given 20 non-parallel and non-concurrent lines.

$$\text{i.e., Required number of points} = {}^{20}C_2 = 190.$$

26 A parallelogram is formed by choosing two straight lines. From the set of m parallel lines and two straight lines from the set of n parallel lines.

Two straight lines from the set of m parallel lines can be chosen in ${}^mC_2$ ways and two straight lines from the set of n parallel lines can be chosen in ${}^nC_2$ ways. Hence the number of parallelograms formed

$$\begin{aligned} &= {}^mC_2 \times {}^nC_2 = \frac{m(m-1)}{2} \times \frac{n(n-1)}{2} \\ &= \frac{mn(m-1)(n-1)}{4} \end{aligned}$$

27 The number of points of intersection of 37 lines is ${}^{37}C_2$. But 13 straight lines out of the given 37 straight lines pass through the same point A . Therefore instead of getting ${}^{13}C_2$ points, we get only one point A . Similarly 11 straight lines out of the given 37 straight lines intersect at point B .

Therefore instead of getting ${}^{11}C_2$ points, we get only one point B . Hence the number of intersection points of the lines is ${}^{37}C_2 - {}^{13}C_2 - {}^{11}C_2 + 2 = 535$

Solutions (for Q. Nos. 28 to 31) :

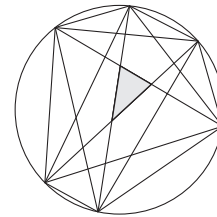
28 The number of regions into which this plane will be divided by 6 lines = ${}^6C_0 + {}^6C_1 + {}^6C_2 = 1 + 6 + 15 = 22$

$$29 \quad \frac{n^2 + n + 2}{2} = 22 \Rightarrow n = 6$$

$$\text{Hint } {}^nC_0 + {}^nC_1 + {}^nC_2 = \frac{n(n+1)}{2} + 1 = \frac{n^2 + n + 2}{2}$$

30 $n^2 - n + 2$ is the number of disjoint regions that can be formed in the plane by n simple closed curves which pair-wise meet in at most two points. It is same as you see with the Venn-diagrams.

31 If n points are given on the circumference of a circle and the chords determined by them are drawn. If no three chords have a common point, then the number of triangles all of whose vertices lie inside the circle = ${}^nC_6 = \frac{n!}{6!(n-6)!}$



$$\text{Therefore required number of triangles} = {}^6C_6 = 1$$

Solutions (for Q. Nos. 32 to 38) :

32 Maximum number of pieces of pizza

$$= {}^8C_0 + {}^8C_1 + {}^8C_2 = 1 + 8 + 28 = 37$$

$$33 \quad \because {}^nC_0 + {}^nC_1 + {}^nC_2 = \frac{n(n+1)}{2} + 1 = \frac{n^2 + n + 2}{2}$$

$$\therefore \frac{n^2 + n + 2}{2} = 20 \Rightarrow n(n+1) = 38 \Rightarrow n \geq 6$$

If $n = 5$, $n(n+1) < 38 \Rightarrow \frac{n^2 + n + 2}{2} < 20$, which is inadmissible.

34 Number of friends present, initially = maximum number of pieces obtained after making 7 cuts = $\frac{7^2 + 7 + 2}{2} = 29$

$$\begin{aligned} \text{Total number of friends including the ones who arrived later} &= \text{maximum number of pieces obtained after making 8 cuts} \\ &= \frac{8^2 + 8 + 2}{2} = 37 \end{aligned}$$

Therefore the maximum number of friends that arrived later at the party = $37 - 29 = 8$.

- 35** If n points lie on the circumference of a circle and are connected by straight lines intersecting the circle such that no three lines are concurrent (that is no three lines ever pass through the same point), then the maximum number of parts/regions into which these lines divide the circle is ${}^nC_0 + {}^nC_2 + {}^nC_4$.

Therefore the required number of pieces
 $= {}^8C_0 + {}^8C_2 + {}^8C_4 = 1 + 28 + 70 = 99$

- 36** When n lines intersect a circle, the maximum number of bounded regions created $= \frac{n^2 - 3n + 2}{2} = 15 \Rightarrow n = 7$

- 37** When n lines intersect a circle, the maximum number of bounded regions created

$$= \frac{n^2 - 3n + 2}{2} = \frac{8^2 - 3 \times 8 + 2}{2} = 21$$

Since there are only 21 Cheesers, so 4 ($= 25 - 21$) persons have to compromise with an Edger piece.

- 38** Let B and U denote the BOUNDED regions and UNBOUNDED regions, where bounded regions represent the Cheesers and unbounded regions represent the Edgers.

$$\text{Then, } B = \frac{n^2 - 3n + 2}{2} \Rightarrow U = 2n$$

$$\text{As, } B > U \text{ therefore } \frac{n^2 - 3n + 2}{2} > 2n \Rightarrow n^2 - 7n + 2 > 0$$

The above inequality is valid only when the minimum integral value of $n = 7$.

Solutions (for Q. Nos. 39 and 40) :

- 39** The maximum possible number of regions on a plane that can be created by a Venn diagram with n circles

$$= n(n-1) + 2 = n^2 - n + 2$$

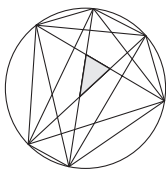
Therefore the required number of maximum regions
 $= 6^2 - 6 + 2 = 32$

- 40** If n ellipses are drawn in the plane such that no two of them are tangent, none of them lies entirely within or outside of another one and no three of them are concurrent, then the maximum number of parts/regions into which these ellipses divide the plane is $2n(n-1) + 2 = 2(n^2 - n + 1)$.

Therefore the required number of maximum regions
 $= 2(5^2 - 5 + 1) = 42$

Solutions (for Q. Nos. 41 and 42) :

- 41** If n points are given on the circumference of a circle and the chords determined by them are drawn.



If no three chords have a common point, number of triangles all of whose vertices lie inside the circle

$$= {}^nC_6 = \frac{n!}{6!(n-6)!}$$

- 42** The required number of triangles

$${}^nC_6 = \frac{n!}{6!(n-6)!} = \frac{6!}{6!(0)!} = 1$$

- 43** The maximum number of regions into which a 3-dimensional cube/sphere/cylinder can be partitioned by exactly n planes is

$${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 = \frac{n^3 + 5n + 6}{6}$$

Therefore the maximum number of required pieces

$$= \frac{6^3 + 5 \times 6 + 6}{6} = 42$$

- 44** Since, $\frac{n^3 + 5n + 6}{6} \geq 300$

$$\Rightarrow n^3 + 5n \geq 1794$$

$$\Rightarrow n > 12 \Rightarrow n = 13$$

Solutions (for Q. Nos. 45 to 60) :

- 45** Total number of squares = 3

(i) B

(ii) C

(iii) A + B + C + D

A	B
C	D

Total number of rectangles = 9

(i) A

(ii) B

(iii) C

(iv) D

(v) A + B

(vi) C + D

(vii) A + C

(viii) B + D

(ix) A + B + C + D

- 46** Total number of squares in a square having n columns and n rows $= 1^2 + 2^2 + 3^2 + \dots + n^2 = \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$

Since, $n = 3$, therefore total number of squares = 14.

Total number of rectangles in a square having n columns

$$\text{and } n \text{ rows} = 1^3 + 2^3 + 3^2 + \dots + n^3 = \Sigma n^3 = \left[\frac{n(n+1)}{2} \right]^2$$

Since, $n = 3$, therefore total number of rectangles = 36.

- 47** Total number of squares in a rectangle having m columns and n rows $= m \cdot n + (m-1)(n-1)$

$$+ (m-2)(n-2) + \dots + 0$$

Since, $m = 5$, $n = 2$, therefore total number of squares

$$= 10 + 4 = 14$$

Total number of rectangles in a rectangle having m columns and n rows

$$= (1 + 2 + 3 + \dots + m)(1 + 2 + 3 + \dots + n)$$

Therefore total number of rectangles $= (3) \times (15) = 45$

48 Total number of squares

$$= \sum 10^2 = 1^2 + 2^2 + \dots + 10^2 = 385$$

49 Total number of rectangles

$$= \sum n^3 = 1^3 + 2^3 + \dots + 3^3 = 3025$$

50 Total number of squares in a rectangle having m columns and n rows

$$= m \cdot n + (m-1)(n-1) + (m-2)(n-2) + \dots + 0$$

Since, $m = 8, n = 15$, therefore total number of squares

$$= 120 + 98 + 78 + 60 + 44 + 30 + 18 + 8$$

$= 456$

51 Total number of rectangles in a rectangle having m columns and n rows $= (1 + 2 + 3 + \dots + m)$

$$(1 + 2 + 3 + \dots + n)$$

Therefore total number of rectangles $= (36) \times (120) = 4320$

52 Total number of quadrilaterals in a square having n columns and n rows

$$= 1^3 + 2^3 + 3^3 + \dots + n^3 = \sum n^3 = \left[\frac{n(n+1)}{2} \right]^2$$

Since, $n = 10$, therefore total number of rectangles $= 3025$.

NOTE In this case quadrilaterals and rectangles are one and the same thing.

53 Total number of quadrilaterals in a rectangle having m columns and n rows

$$= (1 + 2 + 3 + \dots + m)(1 + 2 + 3 + \dots + n)$$

Therefore total number of quadrilaterals

$$= (36) \times (120) = 4320$$

NOTE In this case quadrilaterals and rectangles are one and the same thing.

54 Total number of quadrilaterals when a set of 8 parallel lines intersects another set of 15 parallel lines

$$= {}^8C_2 \times {}^{15}C_2 = 28 \times 105 = 2940$$

55 Total number of rectangles when 8 vertical lines intersect 15 horizontal lines such that all the vertical lines are parallel and equidistant to each other, and all the horizontal lines are also parallel and equidistant to each other $= {}^8C_2 \times {}^{15}C_2 = 28 \times 105 = 2940$

NOTE The only difference between a quadrilateral and a rectangle is that in a rectangle opposite sides are parallel and adjacent sides intersect each other perpendicularly, while it may not be the case in a normal quadrilateral.

56 The 8 vertical lines will create 7 columns and 15 horizontal lines will create 14 rows.

Now, the total number of squares in a rectangle having m columns and n rows

$$= m \cdot n + (m-1)(n-1) + (m-2)(n-2) + \dots + 0$$

Since, $m = 7, n = 14$, therefore total number of squares

$$= 98 + 78 + 60 + 44 + 30 + 18 + 8 = 336$$

57 In any standard chessboard there are total 8 rows and 8 columns, so the total number of squares

$$= 1^2 + 2^2 + 3^2 + \dots + n^2$$

$$= \sum n^2 = \frac{n(n+1)(2n+1)}{6} = \frac{8 \times 9 \times 17}{6} = 204$$

58 In any standard chessboard there are total 8 rows and 8 columns, so the total number of squares

$$= 1^3 + 2^3 + 3^3 + \dots + n^3 = \sum n^3$$

$$= \left[\frac{n(n+1)}{2} \right]^2 = \left(\frac{8 \times 9}{2} \right)^2 = 1296$$

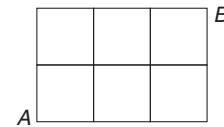
59 Total number of squares

$$= (9 + 4 + 1) + (9 + 4) + (9 + 4) = 40$$

Hint To get the answer easily, first you clear all the squares from the central square. Then you will have $9 + 4 + 1 = 13$ squares of distinct sizes. Then you can proceed like this for the central squares as well.

60 There are three different sizes of triangles. Total number of triangles $= 1 + 4 + 4 = 9$

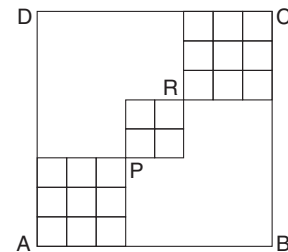
61 You can see in the following diagram that if you want to go from A to B, you have to take 3 steps towards the right side and 2 steps in the upward direction. Thus you have to take total 5 steps.



So the total number of ways

$$= {}^5C_3 \times ({}^{5-3}C_2) = {}^5C_3 \times {}^2C_2 = \frac{5!}{3!2!} = 10$$

62 Total number of paths from A to P $= {}^6C_3 \times {}^3C_3 = \frac{6!}{3!3!} = 20$



Total number of paths from P to

$$R = {}^4C_2 \times {}^2C_2 = \frac{4!}{2!2!} = 6$$

Total number of paths from R

$$\text{to } C = {}^6C_3 \times {}^3C_3 = \frac{6!}{3!3!} = 20.$$

Therefore the number of paths from A to B , via P and R
 $= 20 \times 6 \times 20 = 2400$

And the number of paths from A to C , via B and $D = 2$

Thus the required number of paths $= 2400 + 2 = 2402$

- 63** Let us consider you have to go from A to its opposite corner C .

$$\text{Total number of paths from } P \text{ to } R = {}^6C_3 \times {}^3C_3 = \frac{6!}{3!3!} = 20.$$

Total number of paths from A to C , via P to R , when there is a grid in the square $PQRS$ $=$ (number of paths from A to P) $\times$ (number of paths from P to R) $\times$ (number of paths from R to C)
 $= 2 \times 20 \times 2 = 80$

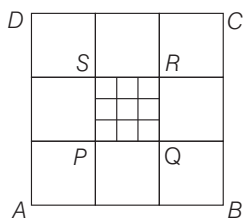
Total number of paths from A to C , when there is NO grid in the square $PQRS = {}^6C_3 \times {}^3C_3 = \frac{6!}{3!3!} = 20.$

Total number of paths from A to C , via P to R , when there is NO grid in the square $PQRS =$ (number of paths from A to P) $\times$ (number of paths from P to R) $\times$ (number of paths from R to C)

$$= 2 \times 2 \times 2 = 8$$

Therefore the required number of paths $=$ Total number of paths from A to C , when there is NO grid in the square $PQRS$ $+$ (Total number of paths from A to C , via P to R , when there is a grid in the square $PQRS$ $-$ Total number of paths from A to C , via P to R , when there is no grid in the square $PQRS$)

$$= 20 + (80 - 8) = 92$$



- 64** Using the same approach as in the previous sum, we can say that the total number of the shortest paths from P to $R = 92$

Total number of paths from A to C , via P to R , when there is a grid in the square $PQRS$

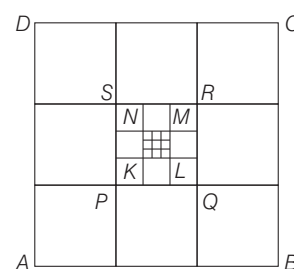
$$\begin{aligned} &= (\text{number of paths from } A \text{ to } P) \\ &\quad \times (\text{number of paths from } P \text{ to } R) \\ &\quad \times (\text{number of paths from } R \text{ to } C) \\ &= 2 \times 92 \times 2 = 368 \end{aligned}$$

Total number of paths from A to C , when there is NO grid in

$$\text{the square } PQRS = {}^6C_3 \times {}^3C_3 = \frac{6!}{3!3!} = 20$$

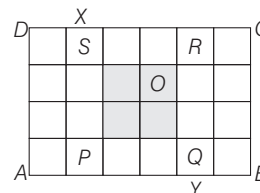
Total number of paths from A to C , via P to R , when there is NO grid in the square $PQRS =$ (number of paths from A to P) $\times$ (number of paths from P to R) $\times$ (number of paths from R to C) $= 2 \times 2 \times 2 = 8$

Therefore the required number of paths $=$ Total number of paths from A to C , when there is NO grid in the square $PQRS$ $+$ (Total number of paths from A to C , via P to R , when there is a grid in the square $PQRS$ $-$ Total number of paths from A to C , via P to R , when there is no grid in the square $PQRS$)
 $= 20 + (368 - 8) = 380$

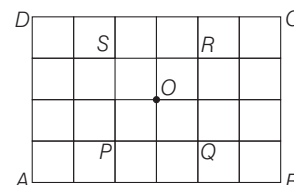


- 65** Total number of shortest paths from A to $C =$ (number of paths from A to Q) $\times$ (number of paths from Q to C) $+$ (number of paths from A to Y) $\times$ (number of paths from Y to C) $+$ (number of paths from A to S) $\times$ (number of paths from S to C) $+$ (number of paths from A to X) $\times$ (number of paths from X to C)

$$= 5 \times 10 + 1 \times 5 + 5 \times 10 + 1 \times 5 = 110$$



Alternatively This problem is like you can go through any point from A to C , but cannot pass through the point O .



The number of shortest paths from A to $O = 5! / 3!2! = 10$

The number of shortest paths from O to $C = 5! / 3!2! = 10$

Total number of shortest paths from A to C via O
 $= 10 \times 10 = 100$

Total number of shortest paths from A to $C = \frac{10!}{6!4!} = 210$

Therefore the required number of shortest paths

$=$ (Total number of paths from A to C) $-$ (Total number of paths from A to C , via O) $= 210 - 100 = 110$

- 66** Number of triangles with all three vertices taken from the pentagon = ${}^5C_3 = 10$

Number of triangles with two vertices taken from the two adjacent vertices of the outer pentagon and one vertex from the 3 vertices of inner pentagon = $5({}^2C_2 \times {}^3C_1) = 15$

Number of triangles with two vertices from the non-adjacent vertices of outer pentagon and 1 vertex from the inner pentagon = $({}^2C_2 \times {}^5C_1) = 5$

Number of triangles with one vertex from outer pentagon and two vertices from inner pentagon = ${}^5C_1 \times {}^2C_2 = 5$

Therefore total number of required triangles

$$= 10 + 15 + 5 + 5 = 35$$

- 67** A rail-loop can be formed by joining any 3 vertices of the regular polygon. It can be done in ${}^{16}C_3 = 560$ ways.

Therefore there are 560 rail-loops.

- 68** When n is odd, the maximum diagonals intersect at any point inside the polygon is 2. Now, the number of

maximum intersections when any two diagonals intersect each other inside a regular polygon with odd number of vertices is ${}^nC_4$. Therefore the maximum number of intersections = ${}^{25}C_4 = 12650$.

Thus there are maximum 12650 warehouses in the SEZ.

- 69** When n is odd, the maximum diagonals intersect at any point inside the polygon is 2. So, no 3 (or more than 3) diagonals intersect at the same point inside the polygon with 49 (odd number of) vertices. Therefore, there is no government office.

Hence choice (a) is correct.

- 70.** When n is even, but not divisible by 6, the maximum number of diagonals intersect at any point inside the regular polygon is 3. So there cannot be more than 3 diagonals intersecting at any common point inside the polygon. Since no 5 diagonals intersect at any common point, so the number of required intersection is zero. That is there is no airport in the given SEZ.

Hence choice (d) is correct.

✓ Level 01 Basic Level Exercise

1 $9 \times 9 \times 9 \times 9 = 9^4$.

- 2** $9^4 - 1$ (It is considered that the lock opens in the last attempt.)

3 Case I. MW MW MW MW

Case II. WM WM WM WM

Let us arrange 4 men in 4! ways, then we arrange 4 women in ${}^4P_4$ ways at 4 places either left of the men or right of the men.

Hence required number of arrangements

$$= 4! \times {}^4P_4 + 4! \times {}^4P_4$$

$$= 2 \times 576 = 1152$$

- 4** First of all we arrange 4 women in 4! ways then we arrange 4 men in 5 places in ${}^5P_4$ ways.

Hence, the total number of arrangements = $4! \times {}^5P_4 = 2880$

- 5** Let us arrange 4 persons in 4! ways then we place their respective wives adjacent to them in 2! ways i.e., either left or right sides of the husbands.

Hence, the required number of ways

$$= 4! \times 2! \times 2! \times 2! \times 2!$$

- 6** Let us first of all we arrange 4 men in 3! ways then 4 women can be arranged in 4 places in ${}^4P_4$ ways.

Hence the required number of ways = $3! \times {}^4P_4 = 144$.

- 7** All the 4 men can be arranged in 4! ways.

Similarly all the 4 women can be arranged in 4! ways but the two different groups of men and women can be mutually arranged in 2! ways.

Hence the required number of ways = $(4!)^2 \times 2! = 1152$

- 8 H L C N T A U I O
L N A I

There are total 13 letters out of which 7 are consonants and 6 are vowels. Also there are $2L$'s, $2N$'s, $2A$'s, and $2I$'s.

If all the consonants are together then the number of arrangements = $\frac{7!}{2! \cdot 2!}$, but the 7 consonants can be arranged themselves in $\frac{7!}{2! \cdot 2!}$ ways.

$$\text{Hence the required number of ways} = \frac{7!}{2! \times 2!} \times \frac{7!}{2! \times 2!} \\ = (1260)^2 = 1587600$$

- 9 There are 7 odd places and 6 vowels of which 2 vowels occur two times.

$$\text{Hence number of arrangements of vowels} = \frac{{}^7P_6}{2! \times 2!}$$

Now the remaining 7 consonants of which two consonants occur two times, can be arranged in 7 places in

$$\frac{7!}{2! \times 2!} \text{ ways.}$$

$$\text{Hence, the required number of ways} = \frac{{}^7P_6}{2! \times 2!} \times \frac{7!}{2! \times 2!} \\ = 1587600$$

- 10 S U C E F L

S U C

S

There are 10 letters in the word SUCCESSFUL and S occurs 3 times, U occurs 2 times and C occurs 2 times.

$\therefore$ The letters of the word SUCCESSFUL can be arranged in

$$= \frac{10!}{3! \times 2! \times 2!} = 151200 \text{ ways}$$

- 11 Since all three S's are together, hence there will be 8 letters of which C occurs 2 times and U occurs 2 times.

$$\text{Thus the required number of arrangements} = \frac{8!}{2! \times 2!} \\ = 10080$$

- 12 (C, C), (U, U), S, S, E, F, L

These 8 letters now can be arranged in $\frac{8!}{3!}$ ways.

- 13 Total number of 4 digit numbers without repetition of digits = $4! = 24$.

Each of the digits in each of the 4 places (thousands, hundreds and tens and unit) place will be $3!$ times i.e., 6 times.

Hence the sum of all the digits at each of the four places = $6(1 + 2 + 3 + 4) = 60$

$$\text{Hence the sum of all the required 4 digit numbers} \\ = 60(1000 + 100 + 10 + 1) \\ = 60 \times 1111 = 66660$$

- 14 Total number of 5 digit numbers (including which begins with zero) = $5! = 120$.

Number of 5 digit numbers which begin with zero = $4! = 24$.

Sum of all 5 digit numbers

$$= (0 + 1 + 2 + 3 + 4) \times 4! \times (11111) \\ = 240 \times 11111 = 2666640$$

Sum of all five digit numbers which begin with zero

$$= (1 + 2 + 3 + 4) \times 3! \times (1111) \\ = 66660$$

Hence the sum of the required numbers

$$= \text{sum of all 5 digit numbers including those numbers} \\ \text{which begin with zero} \\ - \text{sum of all 5 digit numbers which begin with zero} \\ = 2666640 - 66660 = 2599980$$

- 15 Number of polygons = ${}^nC_2$ - Number of sides (n)

$$\therefore {}^nC_2 - n = n$$

$$\Rightarrow {}^nC_2 = 2n$$

$$\Rightarrow \frac{n(n-1)}{2} = 2n$$

$$\Rightarrow n = 5$$

- 16 There are 10 letters in the given word PROPORTION and we have 10 letters of 6 different kinds viz., (O, O, O), (P, P), (R, R), T, I, N.

For a group of four letters, we have four cases :

- (i) Three alike and one different.
- (ii) Two alike and two other alike.
- (iii) Two alike and other two different.
- (iv) All four different.

In case I, the number of selections = ${}^1C_1 \times {}^5C_1 = 5$.

In case II, the number of selections = ${}^3C_2 = 3$.

[Since we can select two pairs out of 3 pairs (O, O), (P, P), (R, R)]

In case III, the number of selections = ${}^3C_1 \times {}^5C_2 = 30$.

($\therefore$ We can select one of 3 pairs and then two from the remaining 5 letters say, P, R, T, I, N.)

In case IV, the number of selections = ${}^6C_4 = 15$.

($\therefore$ We can select 4 different letters from 6 letters O, P, R, T, I, N.)

In case I, the number of arrangements = $5 \times \frac{4!}{3!} = 20$.

In case II, the number of arrangements = $3 \times \frac{4!}{2! \times 2!} = 18$.

In case III, the number of arrangements = $30 \times \frac{4!}{2!} = 360$.

In case IV, the number of arrangements = $15 \times 4! = 360$.

$$\text{Hence the required number of arrangements} \\ = 20 + 18 + 360 + 360 = 758$$

17 Vowels themselves can be permuted in $\frac{4!}{3!}$ ways

and consonants can be permuted in $\frac{6!}{2! \times 2!}$ ways

Therefore the required number of ways = $\frac{4!}{3!} \times \frac{6!}{2! \times 2!}$
 $= 4 \times 180 = 720$

18 Total number of ways of arranging 16 people = $15!$ ways

Number of ways in which two brothers are together = $14! \times 2$

$\therefore$ Number of ways in which two brothers are never together

$$= 15! - 14! \times 2$$

$$= 14!(15 - 2) = 14! \times 13$$

19 A person out of remaining 14 persons can be selected in ${}^{14}C_1 = 14$ ways.

Now consider Lehman, McKinsey and a third (selected person) as a single person, then we can arrange all of them in $13!$ ways. But the two brothers mutually can be arranged in $2!$ ways. Hence the required number of ways

$$= 14 \times 13! \times 2$$

$$= 14! \times 2$$

20 Every ball can be distributed in 4 ways.

Hence the required number of ways = $4 \times 4 \times 4 \times 4 \times 4 \times 4$
 $= 4^6 = 4096$

21 Number of ways of distribution of 'n' identical things taken r at a time = ${}^{n+r-1}C_{r-1}$

$\therefore$ Required number of ways = ${}^{6+4-1}C_{4-1} = {}^9C_3 = 84$

22 Number of non-negative integral solutions = ${}^{n+r-1}C_{r-1}$
 $= {}^{21+3-1}C_{3-1}$
 $= {}^{23}C_2 = 253$

23 Number of positive integral solutions = ${}^{n-1}C_{r-1}$
 $= {}^{21-1}C_{3-1} = {}^{20}C_2$
 $= 190$

24 Number of ways of selecting items from 6 different items = 2^6 .

Similarly from second set of 6 different objects we can select items in 2^6 ways.

Therefore, total number of selections = $2^6 \times 2^6$

but we have to select at least one item therefore we subtract one case in which none of the items is selected

$$= 2^6 \times 2^6 - 1 = 2^{12} - 1 = 4095$$

25 Number of ways of selecting the items from 6 identical items = $6 + 1 = 7$

Similarly we can select from other set of 6 identical items = 7

$\therefore$ Total number of ways in which at least one item can be selected from the two sets = $(7 \times 7) - 1 = 48$

Hint Since we can select either 0, 1, 2, 3, 4, 5, or 6 items which can be done in 7 ways.

26 We can select atleast one item from 6 different items = $(2^6 - 1)$

Similarly we can select atleast one item from other set of 6 different items in $(2^6 - 1)$ ways.

$$\therefore \text{ Required number of ways } = (2^6 - 1)(2^6 - 1)$$

$$= (2^6 - 1)^2 = 3969$$

27 We can select atleast one item from 6 identical items in 6 ways.

Similarly from other set also we can select atleast one item in 6 ways.

Hence, the required number of ways = $6 \times 6 = 36$.

28	Single step	Double step	Number of ways
	12	0	$\frac{12!}{12!} = 1$
	10	1	$\frac{11!}{10!} = 11$
	8	2	$\frac{10!}{(8! \times 2!)} = 45$
	6	3	$\frac{9!}{(6! \times 3!)} = 84$
	4	4	$\frac{8!}{(4! \times 4!)} = 70$
	2	5	$\frac{7!}{(2! \times 5!)} = 21$
	0	6	$\frac{6!}{6!} = 1$

$$\therefore \text{ Total number of ways } = 1 + 11 + 45 + 84 + 70 + 21 + 1$$

$$= 233$$

29 Using ${}^{n+r-1}C_{r-1}$, we get ${}^{8+3-1}C_{3-1} = {}^{10}C_2 = 45$

$$\mathbf{30} \quad {}^{100}C_{10} \times {}^{90}C_{20} \times {}^{70}C_{30} \times {}^{40}C_{40} = \frac{100!}{10! \times 20! \times 30! \times 40!}$$

31 There are two sets of numbers 0, 1, 3, 5 and 0, 2, 3, 4.

Therefore number of 4 digit numbers using digits 0, 1, 3, 5
 $= 3 \times 3 \times 2 \times 1 = 18$

Similarly number of 4 digit numbers using digits 0, 2, 3, 4
 $= 3 \times 3 \times 2 \times 1 = 18$

Hence the total required numbers = $18 + 18 = 36$.

32 When 7 is at ten thousands place →	[	1	1	9	9	9	→ 9^3
		1	9	1	9	9	→ 9^3
		1	9	9	1	9	→ 9^3
		1	9	9	9	1	→ 9^3
When 7 is not at the ten thousands place →	[	8	1	1	9	9	→ 8×9^2
		8	1	9	1	9	→ 8×9^2
		8	1	9	9	1	→ 8×9^2
		8	9	1	1	9	→ 8×9^2
		8	9	1	9	1	→ 8×9^2
		8	9	9	1	1	→ 8×9^2

Hence the required number of ways = $4 \times 9^3 + 6 \times 8 \times 9^2$
 $= 9^2 (36 + 48)$
 $= 81 \times 84 = 6804$

33 Required number of ways = $\frac{15!}{3! \times 4! \times 8!}$.

34 Total number of ways = $7!$

Let A has to speak before B.

Now since there are half of the total cases in which A speaks before B (Similarly in half of the total cases B speaks before A)

$\therefore$ Required number of ways = $\frac{1}{2} \times 7! = 2520$.

35

1	10	10	10	10	10	10

 $\Rightarrow 10^6$

Total number of 7 digit telephone numbers which begin with digit 2 = 10^6 .

36 ${}^6P_4 = 360$.

④ **Alternatively** First son can accept any of the 6 proposals and second son can accept any of the 5 proposals and so on.

Hence required number of ways = $6 \times 5 \times 4 \times 3 = 360$

37 ${}^{16}C_4 = 1820$

38 ${}^{16}P_4 = 43680$

39 ${}^{16}C_4 \times {}^{12}C_4 \times {}^8C_4 \times {}^4C_4 \times \frac{1}{4!} = \frac{16!}{(4!)^5}$.

40 ${}^nC_2 = 105$, where n is the number of players

$\Rightarrow \frac{n(n-1)}{2} = 105 \Rightarrow n = 15$

41 ${}^9P_6 = 60480$

42 $\frac{6!}{2! \times 4!} = 15$

43 $4 \times 6 = 24$

44 Total number of 5 digit numbers = $9 \times 10^4 = 90000$

Number of 5 digit numbers without any repetition

$= 9 \times 9 \times 8 \times 7 \times 6 = 27216$

$\therefore$ Required number of 5 digit numbers in which atleast one of the digit repeats = $90000 - 27216 = 62784$

45 Total number of possible outcomes = 6^4 .

The number of possible outcomes in which 4 does not appear on any die is 5^4 .

Therefore the number of possible outcomes in which atleast one die shows digit 4 = $6^4 - 5^4 = 671$

46 Required number of signals

$= {}^5P_1 + {}^5P_2 + {}^5P_3 + {}^5P_4 + {}^5P_5$

$= 5 + 20 + 60 + 120 + 120$

$= 325$

47 The number of words begin with A is $4!$.

The number of words begin with E is $4!$.

The number of words begin with M is $4!$.

The number of words begin with R is $4!$.

Number of words begin with VA is $3!$

Words begin with VE are VEAMR

VEARM

VEARM

VEMRA

VERAM

VERMA

$\therefore$ The rank of the word VERMA = $4 \times 4! + 3! + 6$

$= 96 + 6 + 6 = 108$

48 **Case I** 0, 1, 2, 4, 5

Case II 1, 2, 3, 4, 5

Since we know that a number is divisible by 3, if and only if the sum of its digits is divisible by 3.

Case I, number of 5 digit numbers = $4 \times 4 \times 3 \times 2 = 96$

Case II, number of 5 digit numbers = $5 \times 4 \times 3 \times 2 = 120$

$\therefore$ Total required numbers = 216

49 We can choose two men out of 8 in ${}^8C_2$ ways. Since no husband and wife are to play in the same game, two women out of the remaining 6 can be chosen in ${}^6C_2$ ways.

If M_1, M_2, W_1, W_2 are chosen, then a team may consist of M_1 and W_1 or M_1 and W_2 . Thus the number of ways of arranging the game is

$= {}^8C_2 \times {}^6C_2 \times 2 = 840$

50 Let us consider $n = 5$, then we have $\{a, b, c, d, e\}$ in a row.

Now we can select 3 of them so that no two are adjacent to each other is only one possible way that is $\{a, c, e\}$.

Now, let us consider $n = 6$, then we have $\{a, b, c, d, e, f\}$ in a row. Now we can select 3 of them so that no two are adjacent to each other is as follows.

$\{a, c, e\}, \{a, c, f\}, \{a, d, f\}, \{b, d, f\}$

That is total four ways.

Now, we know that if $n = 5$, required number of ways = 1 and if $n = 6$, required number of ways = 3.

This is true for ${}^{n-2}C_3$.

Basically, we know that the number of ways of selecting r things out of n things in a row so that no two of the selected things are adjacent to each other = ${}^{n+1-r}C_r$.

Hence choice (b) is correct.

- 51** Since 8 does not occur in 1000, we have to count the number of times 8 occurs when we list the integers from 1 to 999. Any number between 1 and 999 is of the form xyz , where $0 \leq x, y, z \leq 9$. Let us first count the numbers in which 8 occurs exactly once. Since 8 can occur at one place in ${}^3C_1$ ways. There are ${}^3C_1 (9 \times 9) = 3 \times 9^2$ such numbers. Next, 8 can occur in exactly two places in ${}^3C_2 (9) = 3 \times 9$ such numbers. Lastly, 8 can occur in all three digits in one number only. Hence, the number of times 8 occurs is
- $$1 \times (3 \times 9^2) + 2 \times (3 \times 9) + 3 \times 1 = 300$$

- 52 Case I**

2

2

1

Number of possible arrangements
 $= {}^5C_2 \times {}^3C_2 \times {}^1C_1 \times 3 = 90$

- Case II**

1

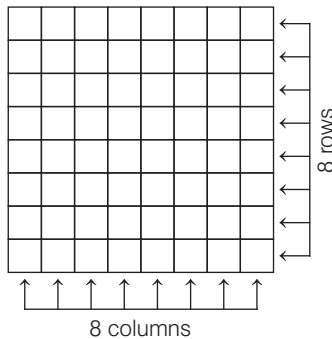
1

3

Number of possible arrangements
 $= {}^5C_1 \times {}^4C_1 \times {}^3C_3 \times 3 = 60$

Hence, the total number of ways is $90 + 60 = 150$.

- 53** In a chess board there are 8 columns and 8 rows.



Since number of columns = number of rows

Hence it is considered as a square.

$\therefore$ Number of square in a square of
 $n \times n = 1^2 + 2^2 + \dots + n^2$

Hence required number of squares

$$= 1^2 + 2^2 + 3^2 + \dots + 8^2$$

$$= \frac{8 \times 9 \times 17}{6} = 204 \quad \left[\because \sum n^2 = \frac{n(n+1)(2n+1)}{6} \right]$$

- 54**

Since these five people enter the train during journey it means they must have entered the train after Lucknow.

Hence if a person enters at S_1 he can have 7 different tickets for $S_2, S_3, \dots, S_7$ and Mumbai.

Similarly if a person enters at S_2 he can have 6 different tickets and if a person enters at S_3 he can have 5 different tickets and so on.

$\therefore$ Total number of possible tickets
 $= 7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$

Now, the five different tickets must be out of these 28 tickets. Therefore, the required number of ways

$$= {}^{28}C_5 = 98280$$

- 55** The number of ways of choosing the committee when Ms. B is a member (when Mr. A refuses to serve)

$$= {}^8C_3 \times {}^6C_5 = 56 \times 6 = 336$$

The number of ways of choosing the committee when Ms. B is not a member (when Mr. A can serve)

$$= {}^8C_4 \times {}^7C_5 = 70 \times 21 = 1470$$

Thus the required number of ways = $336 + 1470 = 1806$.

Alternatively Total number of ways of selecting 4 women and 5 men = ${}^9C_4 \times {}^7C_5 = 2646$.

Number of ways of selecting 4 women and 5 men in which Ms. B and Mr. A are not present together

$$= {}^8C_3 \times {}^6C_4 = 840$$

Hence the number of ways in which Ms. B and Mr. A do not serve together = $2646 - 840 = 1806$

- 56** Required number of ways = $5 \times 5 \times 5 \times 5 \times 5 = 3125$

- 57** Required number of arrangements = $5 \times 5 \times 5 \times 5 = 625$

Since we fix cap C_1 in box B_1 then we arrange the remaining 4 caps in any of the 5 boxes.

Alternatively Total number of arrangements = 5^5

Number of arrangements in which B_1 have cap $C_1 = \frac{5^5}{5} = 5^4$

- 58** Required number of arrangements = ${}^5P_5$

$$\therefore {}^5P_5 = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

- 59** 1 cap can be selected in ${}^5C_1$ ways.

Now this selected cap can be arranged in 5 boxes in ${}^5P_1$ ways. Again 2 caps can be selected in ${}^5C_2$ ways.

Now, these selected caps can be arranged in 5 boxes in ${}^5P_2$ ways and so on.

Hence the required number of ways

$$= {}^5C_1 \times {}^5P_1 + {}^5C_2 \times {}^5P_2 + {}^5C_3 \times {}^5P_3 + {}^5C_4 \times {}^5P_4 + {}^5C_5 \times {}^5P_5$$

$$= 25 + 200 + 600 + 600 + 120 = 1545$$

- 60** Since all the boxes are filled with exactly one cap and all the caps are identical.

Hence the required number of ways = 1.

- 61** Required number of ways = $\frac{5!}{2!} = 60$

(Since 2 caps are identical)

- 62 B_3 can be filled in $2!$ ways and rest can be filled in $4!$ ways.
Hence, required number of ways $= 2 \times 4! = 48$.
- 63 B_1 and B_5 can be filled up in $2 \times 2 = 4$ ways and B_2, B_3, B_4 can be filled up in $= 5 \times 5 \times 5 = 125$ ways.
Hence the required number of ways $= 4 \times 125 = 500$
- 64 Since Juliet never writes more number of letters than the number of letters Romeo writes, it implies that out of the total 6 letters first letter is written by Romeo. Now the remaining 5 letters can be written in the following order.

Case	Order of Letters in which they are written by Romeo and Juliet					
	1	2	3	4	5	6
I	R	R	R	R	J	J
II	R	R	R	J	R	J
III	R	R	R	J	J	R
IV	R	R	J	R	R	J
V	R	R	J	R	J	R
VI	R	R	J	J	R	R
VII	R	J	R	R	R	J
VIII	R	J	R	R	J	R
IX	R	J	R	J	R	R
X	R	J	J	R	R	R

Out of the above 10 cases, case X is invalid as in this case Juliet writes 2 letters by the time Romeo writes only 1 letter, which is contradictory to what is stated in the problem. Therefore total valid possibilities $= 10 - 1 = 9$.

☞ Alternatively

Since Juliet never writes more number of letters than that of Romeo, it implies that out of the total 6 letters Romeo writes first letter. Number of ways in which the remaining 5 letters can be arranged $= \frac{5!}{3!2!} = 10$. But if the second and

third both letter are written by Juliet then it would be contradictory and so unacceptable. That is the case when the order of writing the letters is RJRRR. Therefore the actual possible ways of writing the love-letters $= 10 - 1 = 9$

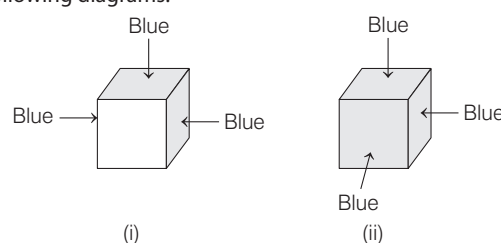
65

Case	Blue	Pink	Number of ways	Reason
I	6	0	1	Since all the faces are of the same color
II	5	1	1	Since there is a symmetric view
III	4	2	2	2 cases arise depending on whether the 2 pink faces are adjacent or opposite
IV	3	3	2	In one scenario all three blue faces are adjacent and in another scenario one blue face is adjacent to the other two blue faces but the other two blue faces are not adjacent to each other

Case	Blue	Pink	Number of ways	Reason
V	2	4	2	2 cases arise depending on whether 2 blue faces are adjacent or opposite
VI	1	5	1	Since there is a symmetric view
VII	0	6	1	Since all the faces are of the same color

Total number of ways $= 1 + 1 + 2 + 2 + 2 + 1 + 1 = 10$

NOTE In case IV there are two possibilities as depicted in the following diagrams.



In the figure (i) all three blue faces are not adjacent to each other. In figure (ii) all three faces are adjacent to each other.

- 66 The number of ways of selecting 4 days out of 7 days $= {}^7C_4 = 35$

The number of ways of selecting a shift from the 4 days $= 2^4 = 16$

Therefore total number of ways of attending the gym $= 35 \times 16 = 560$

- 67 If you do not repeat the use of digits you cannot form more than $3! = 6$ numbers. Since, you have to form 900 numbers, so you have to repeat the use of digits.

Since there are 3 digits namely, 2, 5, 7 and they have to be placed at n -places. So each place can be filled in 3 ways. Thus n places can be filled in 3^n ways.

It implies that $3^n \geq 900 \Rightarrow 3^{n-2} \geq 100 \Rightarrow n - 2 \geq 5 \Rightarrow n \geq 7$

So the least value of n is 7.

Hence choice (b) is the answer.

☞ Alternatively Consider the following table.

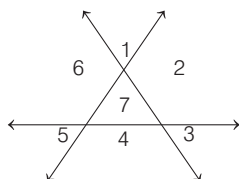
n	Number of n digit numbers	
1	3	3
2	3×3	9
3	$3 \times 3 \times 3$	27
4	$3 \times 3 \times 3 \times 3$	81
5	$3 \times 3 \times 3 \times 3 \times 3$	243
6	$3 \times 3 \times 3 \times 3 \times 3 \times 3$	729
7	$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$	2187

If n is less than 7, you cannot form 900 numbers. So n must be at least 7 to form 900 (or more) n -digit numbers.

- 68** Let us discuss the various possibilities of arranging the 3 lines in a plane.

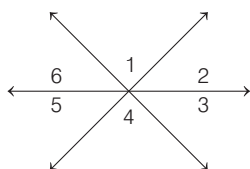
Case I No 2 lines are parallel and no 3 lines are concurrent.

Then the 3 lines divide the plane into 7 regions.



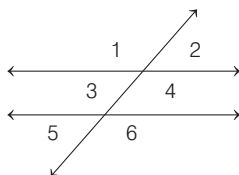
Case II No 2 lines are parallel but the 3 lines are concurrent.

Then the 3 lines divide the plane into 6 regions.



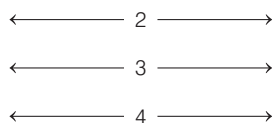
Case III Exactly 2 lines are parallel and the third line is a transversal.

Then the 3 lines divide the plane into 6 regions.



Case IV All the 3 lines are parallel.

Then the 3 lines divide the plane into 4 regions.



Thus it is evident that 3 lines can divide a plane into 4, 6 or 7 regions but not into 5 regions.

- 69** There will be total $10 + 1 = 11$ spots where 4 security posts can be established.

Therefore number of different ways of selecting 4 spots out of 11 spots $= {}^{11}C_4 = 330$

- 70** The rectangle is divided by $2m - 1$ vertical lines, so there are total $2m$ vertical lines.

Similarly the rectangle is divided by $2n - 1$ horizontal lines, so there are total $2n$ horizontal lines.

Since we are supposed to have the rectangles of odd dimensions (e.g., 1, 3, 5, ...). So we will select two vertical lines such that one is even numbered and another is odd numbered and similarly two horizontal lines.

Therefore the number of rectangles

$$= {}^mC_1 \times {}^mC_1 \times {}^nC_1 \times {}^nC_1 = m^2n^2$$

Hence choice (c) is the answer.

Alternatively

Number of possible rectangles

$$= [1 + 3 + 5 + \dots + (2m - 1)] \times [1 + 3 + 5 + \dots + (2n - 1)] \\ = m^2n^2$$

- 71** Total number of shortest paths from A to C

$${}^{10}C_6 \times {}^6C_6 = 210$$

Number of shortest paths from A to R

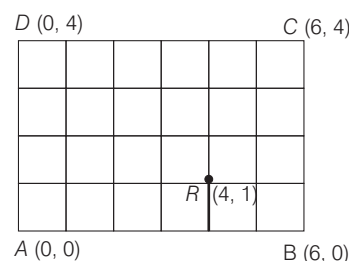
$$= {}^5C_4 \times {}^1C_1 = 5$$

Number of shortest paths from R to C

$$= {}^5C_2 \times {}^3C_3 = 10$$

Number of shortest paths from A to C, via R

$$= 5 \times 10 = 50$$



Therefore the required number of shortest paths from A to C, without going via R

$$= 210 - 50 = 160$$

- 72** The possible combinations are

(i) 1, 1, 1, 1, 1, 2, 3

(ii) 1, 1, 1, 1, 2, 2, 2

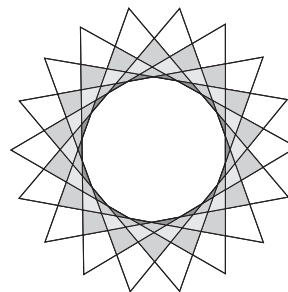
Hence total number of seven digits number

$$= \frac{7!}{5!} + \frac{7!}{4!3!} = 77$$

Hence choice (c) is the answer.

- 73** The maximum possible number of regions created by n triangles $= 3n^2 - 3n + 2$

Therefore the maximum possible number of regions created by 6 triangles $= 3(6)^2 - 3(6) + 2 = 92$



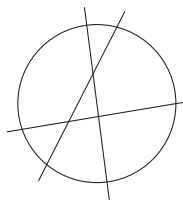
Six overlapping triangles, dividing the plane into 92 distinct regions

- 74** Here are few examples of the required numbers:
 123, 124, 125, ..., 129 and 134, 135, 136, ..., 139 and 145, 146, 147, ..., 149.
 For all such numbers please consider the following table.

a	b	c	Total values
1	2	3, 4, ..., 9	28
	3	4, 5, ..., 9	
	...		
	...		
	6	7, 8, 9	
	7	8, 9	
2	8	9	21
	3	4, 5, ..., 9	
	4	5, 6, ..., 9	
	...		
3	...		15
	...		
	8	9	
4	...	...	10
5	...	...	6
6	...	...	3
7	...	...	1
8	...	...	0
9	...	...	0

Therefore the total number of required three digit numbers
 $= 1 + 3 + 6 + 10 + 15 + 21 + 28 = 84$

- 75** When there is no line the number of regions = 1
 When 1st line is drawn, new regions added = 1
 When 2nd line is drawn, new regions added = 2
 When 3rd line is drawn, new regions added = 3
 When 4th line is drawn, new regions added = 4
 When 5th line is drawn, new regions added = 5
 When 6th line is drawn, new regions added = 6



Thus maximum number of regions when 3 lines are drawn
 $= 1 + 1 + 2 + 3 = 7$

Similarly, maximum number of regions when 6 lines are drawn
 $= 1 + 1 + 2 + 3 + 4 + 5 + 6 = 22$

Therefore maximum additional number of regions
 $= 22 - 7 = 15$

Alternatively If n straight lines are intersecting a circle such that no two lines are parallel and no three lines are concurrent, the maximum number of parts/regions into which these lines divide the circle is

$${}^nC_0 + {}^nC_1 + {}^nC_2 = \frac{n(n+1)}{2} + 1 = \frac{n^2 + n + 2}{2}$$

Then the maximum number of regions when there are

$$3 \text{ lines} = \frac{1}{2} (9 + 3 + 2) = 7$$

And, the maximum number of regions when there are

$$6 \text{ lines} = \frac{1}{2} (36 + 6 + 2) = 22$$

Therefore, maximum number of additional regions
 $= 22 - 7 = 15$

- 76** If the number of countries in the four continents are a, b, c and d , so he has to visit at least $(a-1) + (b-1) + (c-1) + (d-1)$ number of countries and then one more country from any continent.

	Asia	Africa	Europe	America	Total number of Countries
Number of countries to be visited	21	16	11	6	54
Minimum number of countries he must visit in each continent	20	15	10	5	50

First let him visit $20 + 15 + 10 + 5 = 50$ countries. Now, if he visits 1 more country in any of these four continents, then we can say that he visits at least 21 Asian or at least 16 African or at least 11 European or at least 6 American countries. Thus, he should visit minimum 51 countries to satisfy the given condition.

Hence choice (b) is the correct answer.

- 77** When A sits next to B, number of seating arrangements
 $= 5! \times 2! = 240$

When A sits next to B, and P sits next to T, number of arrangements
 $= 4! \times 2! \times 2! = 96$

Therefore, the required number of seating arrangements
 $= 240 - 96 = 144$

- 78** Given that $9 + 1 + a + b + c + d = 16$

$$\Rightarrow a + b + c + d = 6$$

$$\therefore \text{Number of integral solutions} = {}^9C_3 = 84$$

- 79** There can be total 3 main cases (Assuming Sindhu wins the series):

Case	Number of matches won by Jwala	Number of matches won by Sindhu	Total matches played to end the series	Number of ways to arrange the matches
I	0	3	3	$\frac{3!}{3!}$
II	1	3	4	$\frac{3!}{2!}$
III	2	3	5	$\frac{4!}{2! \times 2!}$

Case I Since all matches are won by only one person, so there is only 1 way Sindhu can win this series.

Case II Out of the 4 matches, Jwala cannot win the last match. If Jwala wins the last match, it means Sindhu has already won the first 3 matches, so there is no need to play the fourth match. Therefore, the last match has to be won by the winner of the series.

Valid arrangements, since the last winner is Sindhu – {JSSS, SJSS, SSJS}

Invalid arrangements, since Sindhu has already won the series before 4th match of the series – {SSSJ}

Since, we have observed that in this type of questions, it is the winner of the series who always plays the last winning match, so to solve this problem we should keep away the last winning match separately, and arrange the remaining matches.

In this case there are 4 matches (1 match is won by Jwala and other 3 are won by Sindhu), so we will keep the last match of Sindhu separately, and arrange the remaining matches (1 match of Jwala and 2 matches of Sindhu).

This can be done in $\frac{3!}{2!} = 3$ ways

Case III Since we know that the last match has to be definitely won by the winner of the series (Sindhu), so we have to arrange the remaining 2 matches in which Sindhu wins and the two matches in which Jwala wins.

This can be done in $\frac{4!}{2!2!} = 6$ ways

Valid arrangements, since the last winner is Sindhu – {JJSSS, JSJSS, JSSJS, SJSSS, SJSJS, SSJJS}

Invalid arrangements, since the Sindhu has already won the series before 4th/5th match of the series –

{SSSJJ, SSJSJ, SJSSJ, JSSSJ}

Thus, Sindhu wins the series in 10 (= 1 + 3 + 6) ways.

Similarly, there are 10 ways in which Jwala wins the series.

Therefore, total number of ways in which this series could be won = 20.

- 80** Given that $10^9 < n < 10^{10}$. It implies that n is a 10-digit number.

Case I $n = 6 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$

Number of possible numbers = 10

Case II $n = 3 \times 2 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$

Number of possible numbers = ${}^{10}P_2 = 90$

Total required numbers = 10 + 90 = 100

- 81** First of all he picks 1 rose, 2 lilies, 3 orchids and 4 carnations in order to fulfill the minimum requirement criterion. Thus 10 flowers have already been kept aside for the bouquet.

Now, the remaining 5 flowers can be chosen from any of the four types of flowers, such that $R + L + O + C = 5$ and $R, L, O, C \geq 0$.

Therefore, the required number of ways

$$= {}^{5+4-1}C_{4-1} = {}^8C_3 = 56$$

Level 02 Higher Level Exercise

- 1** First and second prizes can be given in Mathematics in (30×29) ways. First and second prizes can be given in Physics in (30×29) ways.

First prize can be given in Chemistry in 30 ways.

First prize can be given in English in 30 ways.

Hence, the number of ways to give prizes in all the four subjects

$$(30 \times 29) \times (30 \times 29) \times 30 \times 30 = (30)^4 \times (29)^2$$

- 2** $1 + 1 \cdot P_1 + 2 \cdot P_2 + 3 \cdot P_3 + 4 \cdot P_4 + \dots + n \cdot P_n$

$$= 1 + 1 + 2 \cdot 2! + 3 \cdot 3! + 4 \cdot 4! + \dots + n \cdot n!$$

$$= 1 + [(2! - 1!) + (3! - 2!) + (4! - 3!) + \dots$$

$$+ ((n+1)! - n!)]$$

$$= 1 + [(n+1)! - 1!] = (n+1)!$$

Alternatively Consider $n = 1, 2, 3, \dots$ etc. and then verify the result.

3

	Row-1	Row-2	Row-3	Row-4
Case 1.	Class 1	Class 2	Class 1	Class 2
Case 2.	Class 2	Class 1	Class 2	Class 1

Required seating arrangement can be done in two ways – case 1 and case 2. Total number of arrangements

= Number of arrangements in case 1

+ Number of arrangements in case 2

Now, 12 students of class 1 can be seated in 12 chairs in ${}^{12}P_{12}$ ways and 12 students of class 2 can be seated in 12 chairs in ${}^{12}P_{12}$ ways. Hence, the total number of arrangements

$$= (12! \times 12!) + (12! \times 12!) \quad (\because {}^{12}P_{12} = 12!)$$

$$= 2(12!)^2$$

- 4 There are 5 even digits viz., 0, 2, 4, 6 and 8. Digit 3 cannot be at the tens place, since if 3 is at tens place, then 9 must be at unit place which is impossible. Hence 3 can be only at hundred's place. Now there are two cases :

1. **When 3 is at hundred's place :** If 3 is at hundred's place, then 9 will be at ten's place. So unit place can be filled up in only 5 ways by using even digits.

So, the total number of even numbers = $1 \times 1 \times 5 = 5$.

2. **When 3 is not at hundred's place :** In this case hundred's place can be filled up in 8 ways (0 and 3 cannot be used). In tens place we can use any one of the 10 digits except 3. So tens place can be filled up in 9 ways and unit place can be filled up in 5 ways by using 0, 2, 4, 6 and 8.

So, the total number of even numbers = $8 \times 9 \times 5 = 360$

Hence the total number of required 3 digit even numbers
 $= 5 + 360 = 365$

- 5 Number of vehicles registered upto RZ 9999
 $= 18 \times 26 \times (9999)$

Number of vehicles registered between SA-0001

and SJ 9999 = $1 \times 10 \times 9999$

Therefore number of vehicles registered before SK-0123

$$= 18 \times 26 \times 9999 + 1 \times 10 \times 9999 + 122$$

$$= 9999 \times 478 + 122 = 4779644$$

- 6 Available digits are 1, 2, 3, 4 and 5. Now since we know that a number is divisible by 4 if and only if the number formed by last two digits is divisible by 4. So the following cases are possible.

Thousands	Hundreds	Tens	Unit
x	y	1	2
x	y	2	4
x	y	3	2
x	y	5	2

In each case thousands and hundreds places can be filled up in ${}^3P_2$ ways.

Hence the required number of ways = ${}^3P_2 \times 4 = 24$.

- 7 Total number of seats = $2(3 + 4) = 14$

Total number of persons = $3 + 9 = 12$

$\therefore$ Total number of required arrangements = ${}^{14}P_{12}$.

- 8 Three girls can be seated together in a back row on adjacent seats in the following ways :

1, 2, 3 or 2, 3, 4 of first van and 1, 2, 3 or 2, 3, 4 of second van.

In each of the above four cases all the three girls can be inter change among themselves in $3!$ ways.

So the total number of ways = $4 \times 3!$

Now, 9 boys can be seated in remaining 11 seats, which can be done in ${}^{11}P_9$ ways.

Hence, the required number of ways = ${}^{11}P_9 \times 4 \times 3!$

- 9 5 '+' (plus) signs can be arranged in 1 way, then we have 6 places to arrange 5 '-' (minus) signs in ${}^6C_5$ ways but since all the minus (-) signs are identical, therefore minus signs can be arranged in only 1 way in 5 chosen places out of 6.

Hence the required number of arrangements

$$= 1 \times {}^6C_5 \times 1 = 6$$

- 10 In order that no two books on Maths be together we first arrange 11 identical books on English which can be done in 1 way.

Now we can choose 9 places out of 12 (11 + 1) places for Maths books, which can be done in ${}^{12}C_9$ ways.

Again since all books on Maths are identical which can be mutually arranged in only 1 way.

Hence the required number of arrangements = $1 \times {}^{12}C_9 \times 1$
 $= 220$

11 $1680 = 2^4 \times 3 \times 5 \times 7$

Since if $N = a^p \cdot b^q \cdot c^r \cdot \dots$

we have number of factors of $N = (p + 1)(q + 1)(r + 1) \dots$

Hence the total number of factors of the given number

$$= (4 + 1)(1 + 1)(1 + 1)(1 + 1) = 40$$

12 $\sum_{k=m}^n {}^kC_r = {}^mC_r + {}^{m+1}C_r + {}^{m+2}C_r + \dots + {}^{(n-1)}C_r + {}^nC_r$

Now assume some values of m and n , then verify the correct result from the choices given in the question.

- 13 The number of students answering exactly k ($1 \leq k \leq n - 1$) questions wrongly is $2^{n-k} - 2^{n-k-1}$.

The number of students answering all n questions wrongly is 2^0 . Thus, the total number of wrong answers is :

$$1(2^{n-1} - 2^{n-2}) + 2(2^{n-2} - 2^{n-3}) + \dots$$

$$\dots + (n-1)(2^1 - 2^0) + n(2^0)$$

$$= 2^{n-1} + 2^{n-2} + 2^{n-3} + \dots + 2^0 = 2^n - 1$$

$$\Rightarrow 2^n - 1 = 2047 \Rightarrow 2^n = 2048 \Rightarrow n = 11$$

- 14 Required number of triangles = Total number of triangles

– Number of triangles having one side common with the octagon

– Number of triangles having two sides common with the octagon

$$= {}^8C_3 - ({}^8C_1 \times {}^4C_1) - 8 = 16$$

- 15 The number of ways of selecting any five numbers from the given set = ${}^{25}C_5$.

Now, the five consecutive numbers can be chosen in the following ways

(1, 2, 3, 4, 5); (2, 3, 4, 5, 6); (3, 4, 5, 6, 7); ... (21, 22, 23, 24, 25)

Thus the required number of ways = ${}^{25}C_5 - 21 = 53109$

- 16 $\therefore a \geq -1, b \geq -1, c \geq -1, d \geq -1$.

Let $u \geq 0, v \geq 0, w \geq 0, x \geq 0$.

$$\therefore a + b + c + d = 12$$

$$\begin{aligned} \Rightarrow (u-1) + (v-1) + (w-1) + (x-1) &= 12 \\ \Rightarrow u + v + w + x &= 16 \\ \therefore \text{Required number of solutions} &= {}^{16+4-1}C_{4-1} = {}^{19}C_3 \end{aligned}$$

- 17** Assume some value of k then find 'n' and hence ${}^nC_2$.
Now, verify the correct option.
- 18** $42 = 2 \times 3 \times 7$
Here each of a , b and c can take 3 values.
Hence the required number of solutions $= 3 \times 3 \times 3 = 27$.
- 19** Let (r, b, g) denotes the number of red, blue and green balls respectively.
Therefore the possibilities are $(0, 10, 0)$, $(2, 7, 1)$, $(4, 4, 2)$ and $(6, 1, 3)$.
Hence, the required number of ways $= 4$.
- 20** Let x_1 be the number of stations before the first halting station, x_2 between first and second, x_3 between second and third, x_4 between third and fourth and x_5 on the right of the fourth station, then
$$x_1 \geq 0, \quad x_2, x_3, x_4 \geq 1 \quad \text{and} \quad x_5 \geq 0$$
such that $x_1 + x_2 + x_3 + x_4 + x_5 = 6$.
Now, the total number of ways is the number of solutions of the above equation.
Let $y_2 \geq 0, \quad y_3 \geq 0, \quad y_4 \geq 0$.
$$\therefore x_1 + x_2 + x_3 + x_4 + x_5 = 6$$
$$\Rightarrow x_1 + (y_2 + 1) + (y_3 + 1) + (y_4 + 1) + x_5 = 6$$
$$\Rightarrow x_1 + y_2 + y_3 + y_4 + x_5 = 3$$
$$\therefore \text{The number of solutions} = {}^{3+5-1}C_{5-1} = {}^7C_4 = 35$$
- 21** Let x_i denote the marks assigned to the i^{th} question. Then
$$x_1 + x_2 + x_3 + \dots + x_7 + x_8 = 30$$
where $i = 1, 2, 3, \dots, 8$ and $x_i \geq 2$
and $y_1 + y_2 + \dots + y_7 + y_8 = 14$
where $y_i = x_i - 2, \quad i = 1, 2, 3, \dots$
 $\therefore$ The total number of solutions of this equation is
$${}^{14+8-1}C_{8-1} = {}^{21}C_7$$

- 22** $\therefore$ The exponent of 3 in 33! is 15.

$$\left. \begin{array}{r} 3 \overline{33} \\ 3 \overline{11} \leftarrow \\ 3 \overline{3} \leftarrow \\ 1 \leftarrow \end{array} \right\} 15$$

Hint For more information about exponents refer the first chapter of the book.

- 23** There are two distinct possibilities
(i) When any of the four digits may repeat thrice.
(ii) When any two of the four digits may repeat twice.
(i) $\frac{6!}{3!} \times {}^4C_1 = 480$

$$(ii) \frac{6!}{2!2!} \times {}^4C_2 = 1080$$

Therefore the required number $= 480 + 1080 = 1560$.

24

9	10	10	10	10	10	5

 $= 4500000$

Since if we fix first digit then unit digit will determine the odd/even nature of the numbers so formed.

But unit place can assume half even and half odd. Hence unit place can assume just 5 digits which makes the sum of previous six digits even.

- 25** Required number of ways $= {}^6P_4$.
- 26** Suppose x one rupee coins, y fifty paise coins and z twenty paise coins are selected.
Then $x + y + z = 6$
 $\therefore$ Total number of ways
$$= \text{Number of solutions of the above equation}$$
$$= {}^{6+3-1}C_{3-1} = {}^8C_2 = 28$$

- 27** Required number of times
 $=$ Number of selections of 3 children taking at a time
 $= {}^8C_3 = 56$
- 28** Each child will go as often as he or she can be accompanied by two others. Therefore the required number
$$= {}^7C_2 = 21$$
- 29** Total number of points $= 12$
Total number of triangles $= {}^{12}C_3$
But there are 3 cases which must be excluded for the required number of triangles.
Case 1. The number of triangles formed by 3 points on $AB = {}^3C_3 = 1$
Case 2. The number of triangles formed by 4 points on $BC = {}^4C_3 = 4$
Case 3. The number of triangles formed by 5 points on $CA = {}^5C_3 = 10$
Hence, the required number of triangles
$$= 220 - (10 + 4 + 1) = 205$$

- 30** Each question can be attempted in 3 ways i.e., either it can be omitted or one of the two parts can be attempted.
Hence required number of ways $= 3^{10} - 1 = 59048$.

- 31** Required number of ways $= {}^{16}C_9$.
- 32** Since each bulb has two choices, either switched on or off, therefore required number $= 2^{10} - 1 = 1023$.
- 33** Since each digit of a 10 digit number can be written as either 2 or 3, therefore required number of 10 digit number is 2^{10} .
- 34** Consider some value of n then verify the result.
- 35** Any divisor of $2^3 \cdot 3^4 \cdot 5^2$ is of the form $2^a \cdot 3^b \cdot 5^c$ where $0 \leq a \leq 3, 0 \leq b \leq 4$ and $0 \leq c \leq 2$. Thus, the sum of the

divisors of $2^3 \cdot 3^4 \cdot 5^2$ is
 $(1 + 2 + \dots + 2^3)(1 + 3 + \dots + 3^4)(1 + 5 + 5^2)$
 $= \frac{(2^4 - 1)(3^5 - 1)(5^3 - 1)}{(2 - 1)(3 - 1)(5 - 1)} = \frac{15 \times 242 \times 124}{1 \times 2 \times 4}$
 $= 56265$

36 Required number of ways $= {}^{10+4-1}C_{4-1} = {}^{13}C_3 = 286$

37 Since we know that out of n things arranged in a row then we can select r things out of n such that no two of them are adjacent to each other in ${}^{n+1-r}C_r$ ways.

Hence required number of ways $= {}^{10+1-3}C_3 = {}^8C_3 = 56$

☞ Alternatively To each selection of 3 objects we associated a binary sequence of the form 1001000100 where 1 at i^{th} place means the i^{th} object is selected and 0 at i^{th} place means the i^{th} object is not selected.

There exists one to one correspondence between the set of selections of 3 objects and set of binary sequences containing 7 zeros and 3 ones.

Actually we are concerned with the binary sequences in which 2 ones are consecutive we first arrange 7 zeros.

This can be done in just one way. Now, 3 ones can be arranged at any of the 3 places marked with a cross in the following arrangement.

$$\times 0 \times 0 \times 0 \times 0 \times 0 \times 0 \times 0 \times$$

We can arrange 3 ones at 8 places in ${}^8C_3$ ways $= 56$

38 Let x_5 be such that $x_1 + x_2 + x_3 + x_4 + x_5 = n$

Now we require the non-negative integral solutions of

$$x_1 + x_2 + x_3 + x_4 + x_5 = n$$

The number of required solutions $= {}^{n+5-1}C_{5-1} = {}^{n+4}C_4$

39 When we arrange things one at a time, the number of possible permutations is n .

When we arrange them two at a time, the number of possible permutations is $n \times n = n^2$ and so on.

Hence, the total number of required permutations is

$$n + n^2 + n^3 + \dots + n^r = \frac{n(n^r - 1)}{n - 1}$$

40 In a chess board there are 8 columns and 8 rows.

Hence the number of rectangles $= 1^3 + 2^3 + 3^3 + \dots + 8^3$

$$= \left(\frac{8 \times 9}{2} \right)^2 = 1296$$

☞ Alternatively A chessboard is a network of 9 vertical and 9 horizontal lines.

Since we know that number of quadrilaterals formed by m vertical and n horizontal lines is ${}^mC_2 \times {}^nC_2$.

Hence the required number of rectangles $= {}^9C_2 \times {}^9C_2 = 1296$

41 Order of speaking is equivalent to the order of placing the things.

Thus A , B and C can be placed in ${}^{10}C_3$ ways and the

remaining 7 persons can be placed in $7!$ ways.

Hence, the number of ways in which they can speak is

$${}^{10}C_3 \times 7! = \frac{10!}{3!}$$

☞ Alternatively All the 10 persons can speak in $10!$ ways in which A , B and C can speak in 6 different ways.

But we are required to find a single case out of 6 cases in which A speaks before B and B speaks before C .

Hence the required number of ways $= \frac{10!}{3!} = \frac{10!}{6}$

42 Total one digit number $= 2$

Total two digit numbers $= 2^2$

Total three digit numbers $= 2^3$

Total four digit numbers $= 2^4$

... ..

Total eight digit numbers $= 2^8$

There are total 9 digit numbers $= 2^9$

but out of these 2^9 numbers half of the numbers begin with digit 1 and rest half numbers begin with digit 2.

The greatest possible number less than $2 \cdot 10^8$ is 122222222, which in turn is the greatest possible number of 9 digit begin with 1. Thus the maximum possible number

of 9 digit numbers $= \frac{2^9}{2} = 2^8$

Hence, the required total numbers

$$= 2 + 2^2 + 2^3 + 2^4 + \dots + 2^8 + 2^8$$

$$= \frac{2(2^8 - 1)}{2 - 1} + 2^8 = 2^9 - 2 + 2^8 = 2^8(2 + 1) - 2 = 766$$

43 Required number of ways $= {}^{12+1-4}C_4 = {}^9C_4 = 126$.

44 Since $2k < n \Rightarrow k < \frac{n}{2}$

Hence, number of zeros is less than the number of ones.

Now, first of all we arrange $(n - k)$ ones as follows

$$\times 1 \times 1 \times 1 \times 1 \times 1 \times \dots \times 1 \times$$

The above arrangement leaves $(n - k + 1)$ places marked with $\times$.

Now we can arrange k zeros in the $(n - k + 1)$ places in ${}^{n-k+1}C_k$ ways.

45 Part A can be attempted in 3 ways

(i) The student do not solve the question.

(ii) The student attempts the first part of the question.

(iii) The student attempts the second (i.e., alternative) part of the question.

$\therefore$ Total number of choices in the first part $= 3^5$

Hence, the required number of ways in which a student must attempt at least one question $= 3^5 - 1 = 242$

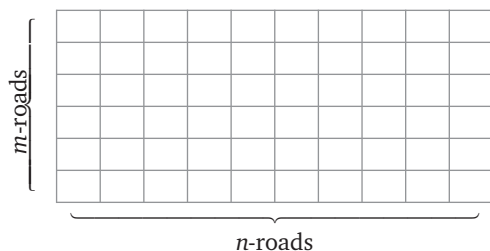
Similarly we can show that there are $2^4 - 1$

i.e. 15 choices for part B.

Hence the required number of ways $= 242 \times 15 = 3630$.

- 46** Required number of number plates of licences
 $= (26)^2 \times (999) = 675324$
- 47** Required number of plates $= {}^{26}P_2 \times {}^{10}P_3 = 468000$
- 48** We can arrange r persons on m chairs on a particular side in ${}^mP_r$ ways and s persons on m chairs on the other side in ${}^mP_s$ ways. Now, we can arrange $(2m - r - s)$ persons on the remaining $(2m - r - s)$ chairs in ${}^{(2m-r-s)}P_{(2m-r-s)}$ ways.
 Thus, the required number of ways of arranging the persons $= ({}^mP_r)({}^mP_s)({}^{(2m-r-s)}P_{(2m-r-s)})$

- 49** The diagram of roads can be as follows :



Suppose the distance between two successive parallel roads be one unit then a person can travel $(m - 1)$ steps in north-south direction and he can travel $(n - 1)$ steps in east-west direction.

Thus he has to travel $\{(m - 1) + (n - 1)\} = (m + n - 2)$ steps to reach from one corner to diagonally opposite corner.

Thus he can arrange his steps in

$${}^{(m+n-2)}C_{(m-1)} \times {}^{(n-1)}C_{(n-1)} \text{ ways}$$

i.e. ${}^{m+n-2}C_{m-1}$ ways.

- 50** The required number of ways is equal to the number of ways in which they can select the same number of toys from the collection of their combined number of toys – the number of ways in which each can select her original toys.
 Hence, required number of ways $= ({}^{19}C_{11} \times {}^8C_8) - 1$
 $= {}^{19}C_{11} - 1$
- 51** Go through options.

- 52** The number of ways of choosing 26 cards out of 52 cards is ${}^{52}C_{26}$. There are two ways in which each card can be dealt because a card can be either from a first pack or from the second pack.

Hence, the number of ways in which the card can be dealt is ${}^{52}C_{26} \times 2^{26}$

- 53** There are 4 suits and each suit contains 13 cards so we can select any one card from the first suit in 13 ways. Similarly we can select one card different from the first card out of second suit in 12 ways and so on.
 Hence, the required number of ways $= 13 \times 12 \times 11 \times 10$
 $= 13!/9!$
- 54** First digit (i.e. ten thousands place) can be filled up in $2!$ ways by the digit 3, but since 3 is being repeated twice, hence the actual number of ways $= \frac{2!}{2!} = 1$

Remaining 4 places can be filled up in $4!$ ways but since zero is being repeated thrice hence the actual number of ways $= \frac{4!}{3!} = 4$

$\therefore$ The required number of ways $= 1 \times 4 = 4$.

- 55** Digit 5 can be placed in any one of the 4 places in 4 ways. Now the remaining 3 places can be filled up with remaining 8 digits in 8^3 ways.

Hence, the required number of ways $= 4 \times 8^3 = 2048$.

- 56** Required number of ways $= 4 \times 8 \times 7 \times 6 = 1344$

- 57** Digit 2 can be arranged in two places out of 8 places in $\frac{{}^8P_2}{2!}$ ways.

Now, the remaining 6 places can be filled by the rest 3 digits in 3^6 ways.

Hence, the required number of ways $= \frac{{}^8P_2}{2!} \times 3^6$
 $= {}^8C_2 \times 3^6 = 20412$

- 58** Required number of ways $= 3^8 = 6561$.

- 59** Since chairs are numbered, hence these are distinguishable.

Therefore 3 boys can be arranged in $3!$ ways on alternate chairs and 3 girls can be arranged in remaining 3 chairs in $3!$ ways.

Also 3 girls can be arranged before the boys are arranged.

Hence the required number of ways $= 2 \times (3!)^2 = 72$

- 60** First rook can be selected in 64 ways ${}^{64}C_1$.

Second rook cannot be selected from the same row and same column from which the first rook has been selected hence there are only $64 - (8 + 7) = 49$ places for the second rook.

Hence, the second rook can be selected in ${}^{49}C_1$ ways.

$\therefore$ Required number of ways $= {}^{64}C_1 \times {}^{49}C_1 = 3136$.

- 61** Number of white shoes = 9 pairs

$\Rightarrow$ 9 left and 9 right shoes

Number of black shoes = 6 pairs

$\Rightarrow$ 6 left and 6 right

In such questions we always consider the worst situation.

Therefore we assume that initially we get $6 + 6 = 12$

black shoes and 9 white shoes either left or right.

Thus we have taken out $12 + 9 = 21$ shoes still we don't have required pair of white shoes.

Now we draw out just one white shoe which will fulfill our requirement of getting one pair of white shoes.

[One pair of white shoes means 1 left shoe and 1 right shoe.]

Thus we have to draw out $12 + 9 + 1 = 22$ shoes to get correct pair of white shoes.

- 62 The worst possible situations are :

White	Black
9 left	6 left
9 left	6 right
9 right	6 left
9 right	6 right

Thus before getting the required correct pair of shoes we have to draw out $9 + 6 = 15$ shoes.

Now when we will draw out one more shoe which will give the required combination of correct shoes either white or black.

Hence we have to draw out minimum $15 + 1 = 16$ shoes.

- 63 We cannot put only one incorrect ball since if we make a wrong choice it will simultaneously make another choice wrong. For example if we put red colour ball in either of green, blue or black box then we have to fill the red colour box with different colour ball, hence one colour combination can never be wrong.

Since atleast two combinations will be different.

- 64 Select any two balls out of 4 then interchange the combination of colours.

Hence the required number of ways $= {}^4C_2 = 6$.

☞ Alternatively

$\begin{Bmatrix} R & B & G & W \\ B & R & G & W' \end{Bmatrix}$	$\begin{Bmatrix} R & B & G & W \\ G & B & R & W \end{Bmatrix}$
$\begin{Bmatrix} R & B & G & W \\ W & B & G & R' \end{Bmatrix}$	$\begin{Bmatrix} R & B & G & W \\ R & G & B & W \end{Bmatrix}$
$\begin{Bmatrix} R & B & G & W \\ R & W & G & B' \end{Bmatrix}$	$\begin{Bmatrix} R & B & G & W \\ R & B & W & G \end{Bmatrix}$

- 65 For one digit number 1 to 9 $\rightarrow 1 \times 9 = 9$
 For two digit numbers 10 to 99 $\rightarrow 2 \times 90 = 180$
 For three digit numbers 100 to 300 $\rightarrow 3 \times 201 = 603$
 Hence, total number of required digits $= 9 + 180 + 603 = 792$

- 66 Four odd digits (1, 1, 3, 3) can be arranged in 4 odd places in $\frac{4!}{2!2!} = 6$ ways.

Now the remaining 3 even digits (2, 2, 4) can be arranged in 3 even place in $\frac{3!}{2!} = 3$ ways.

Hence the required number of ways $= 6 \times 3 = 18$.

- 67 Each of the 7 balls can be distributed into 5 ways.
 Hence the required number of ways $= 5^7$.

- 68 There can be two cases

- (1) 1, 1, 1, 1, 3
 (2) 1, 1, 1, 2, 2

Case 1. A box (in which 3 balls can be put) is selected in ${}^5C_1$ ways.

Now, 3 balls can be selected in ${}^7C_3$ ways and remaining 4 balls can be arranged in 4! ways.

Hence, total number of ways $= {}^5C_1 \times {}^7C_3 \times 4!$

Case 2. There are 2 boxes (each box contains 2 balls) is selected in ${}^5C_2$ ways.

Now, any two ball can be selected in ${}^7C_2$ ways and for the other selected box, 2 balls can be selected from the remaining 5 balls in ${}^5C_2$ ways and the remaining balls can be distributed in 3! ways.

Hence, the total number of ways $= {}^5C_2 \times ({}^7C_2 \times {}^5C_2) \times 3!$

$\therefore$ The required number of ways

$$= ({}^5C_1 \times {}^7C_3 \times 4!) + ({}^5C_2 \times {}^7C_2 \times {}^5C_2 \times 3!) = 16800$$

- 69 Required number of ways $= {}^{7+5-1}C_{5-1}$
 (using ${}^{n+r-1}C_{r-1} = {}^{11}C_4 = 330$)

- 70 Possible ways of distribution are as follows :

- (i) (7) (ii) (6 + 1)
 (iii) (5 + 2) (iv) (4 + 3)
 (v) (5 + 1 + 1) (vi) (4 + 2 + 1)
 (vii) (3 + 2 + 2) (viii) (3 + 3 + 1)
 (ix) (4 + 1 + 1 + 1) (x) (3 + 2 + 1 + 1)
 (xi) (2 + 2 + 2 + 1) (xii) (3 + 1 + 1 + 1 + 1)
 (xiii) (2 + 2 + 1 + 1 + 1)

Hence total number of ways of distribution $= 13$.

- 71 There is only one way, which is as follows :
 1, 2, 1, 2, 1.

- 72 One ball can be selected for the box 2 in ${}^7C_1$ ways and two balls can be selected out of remaining 6 balls for the box 4 in ${}^6C_2$ ways and the remaining 4 balls can be distributed in 3 boxes in 3^4 ways.

Hence, the required number of ways

$$= {}^7C_1 \times {}^6C_2 \times 3^4 = 8505$$

- 73 Ball 2 can be distributed between 2 boxes (viz., box 2 and box 4) in 2 ways.

Now, the remaining 6 balls can be distributed in any of the 5 boxes in 5^6 ways.

Hence, the required number of ways $= 2 \times 5^6$ ways
 $= 31250$ ways

- 74 Required number of distribution

= Total number of distribution

- Number of distribution in which ball 2

and ball 4 are together.

Here total number of distribution (as calculated in question number 68) $= 16800$.

Now, consider ball 2 and ball 4 are stuck together and this arrangement is assumed to be a single ball.

Thus we have 6 balls to be distributed into 5 boxes. Which can be done as 1, 1, 1, 1, 2 in ${}^5C_1 \times {}^6C_2 \times 4!$ ways $= 1800$.

${}^5C_1 \rightarrow$ Number of ways of selecting one box in which two balls are kept together.

${}^6C_2 \rightarrow$ Number of ways of selecting 2 balls out of 6 balls.

$4! \rightarrow$ Number of ways of distribution of remaining 4 balls in remaining 4 boxes.

Hence, the required number of ways

$$= 16800 - 1800 = 15000$$

75 Required number of ways = 3^4 .

76 Following 4 distributions are possible :

(4, 0, 0), (3, 1, 0), (2, 2, 0), (2, 1, 1)

77 Required number of ways = ${}^{4+3-1}C_{3-1}$

(using ${}^{n+r-1}C_{r-1}$) = ${}^6C_2 = 15$

78 Since pots are identical then there will be 4 cases (4, 0, 0), (3, 1, 0), (2, 2, 0) and (2, 1, 1) but since all coins are different hence selection of coins matters.

Therefore for the first case number of selections = ${}^4C_4 = 1$

For the second case number of selections = ${}^4C_3 \times {}^1C_1 = 4$

For the third case number of selections = $\frac{{}^4C_2 \times {}^2C_2}{2!} = 3$

For the fourth case number of selections

$$= \frac{{}^4C_2 \times {}^2C_1 \times {}^1C_1}{2!} = 6$$

Hence the total number of distributions

$$= 1 + 4 + 3 + 6 = 14.$$

79 Since no box is empty and all pots are identical so the possible case is (1, 1, 2).

But since all the coins are different, then the 2 balls can be selected in ${}^4C_2$ ways and rest can be put in $\frac{{}^2C_1 \times {}^1C_1}{2!}$ ways.

Hence, the required number of distribution

$$= \frac{{}^4C_2 \times {}^2C_1 \times {}^1C_1}{2!} = 6$$

80 Since no pot is empty and all coins are identical, the possible case is (1, 1, 2).

But since all three pots are different hence a pot (which contains 2 coins together) can be selected in ${}^3C_1$ ways.

Hence the required number of distribution = ${}^3C_1 \times 1 = 3$.

81 The possible arrangements are as follows :

(4, 0, 0) $\rightarrow$ can be done in 2 ways
i.e., 4 balls can be put either one of the two identical pots or can be put in different pot.

(3, 1, 0) $\rightarrow$ can be done in 3 ways
either distinct pot can be filled up with 3 balls, or 1 ball or remained empty.

(2, 2, 0) $\rightarrow$ can be done in 2 ways
either distinct pot can be filled by 2 balls or remained empty.

(2, 1, 1) $\rightarrow$ can be done in 2 ways
either distinct pot can be filled up with 1 ball or 2 ball.

Hence, the required number of ways = $2 + 3 + 2 + 2 = 9$.

82 The possible arrangements are as follows :

(4, 0, 0) $\rightarrow$ can be done in 3 ways
let the 4 coins be A, A, B and C then all these 4 coins as a single packet of coins can be arranged 3 different pots in 3 ways.

(3, 1, 0) $\rightarrow$ can be done in 18 ways
out of A, A, B and C we can select one coin in 3 ways i.e., either A or B or C.

Now we can arrange this selected coin in any 3 pots in 3 ways and the remaining 3 coins as a single packet of coins can be arranged in remaining 2 pots in 2 ways.

Hence, the required number of ways = $3 \times 3 \times 2 = 18$.

(2, 2, 0) $\rightarrow$ can be done in 12 ways

There are two possible cases :

(i) (A, A), (B, C) (ii) (A, B), (A, C)

In each of two cases we assume that there are two packets of coins which can be arranged in $3 \times 2 = 6$ ways.

Since there are two cases, hence the total number of required ways = $6 \times 2 = 12$.

(2, 1, 1) $\rightarrow$ can be done in 21 ways.

There are 4 possible cases :

(i) (A, A), (B), (C) (ii) (A, B), (A), (C)

(iii) (A, C), (A), (B) (iv) (B, C), (A), (A)

For the first 3 cases in each cases all the 3 packets of coins can be arranged in $3!$ ways.

Hence, the number of arrangements = $3 \times 3! = 18$.

Now in the fourth (iv) case two coins are identical so the third packet of coins can be arranged in 3 pots in 3 ways.

Hence, the total number of arrangements = $18 + 3 = 21$.

Thus the required number of ways

$$= 3 + 18 + 12 + 21 = 54 \text{ ways.}$$

83 Since he switches off the bulb immediately after switching it on, so he basically switches on 3 times (6 = 3 times on + 3 times off). So, for every time he wants to switch a bulb on he can choose any of the two bulbs, so for 3 different occasions he has 8 options = $(2 \times 2 \times 2)$.

Therefore the required number of ways = 8

84 In order to come back to the same point he has to take same number of steps in North and South directions or same number of steps in East and West directions. Then we have the following five cases:

Case	North	South	East	West	Number of ways
I	4	4	0	0	$\frac{8!}{4!4!} = 70$
II	3	3	1	1	$\frac{8!}{3!3!} = 1120$

Case	North	South	East	West	Number of ways
III	2	2	2	2	$\frac{8!}{2!2!2!2!} = 2520$
IV	1	1	3	3	$\frac{8!}{3!3!} = 1120$
V	0	0	4	4	$\frac{8!}{4!4!} = 70$

Therefore the total required number of ways
 $= 2(70) + 2(1120) + 2520 = 4900$

- 85** It is like dividing 100 identical things among 3 persons, such that each one must get at least 1 thing.

Therefore we have $^{100-1}C_{3-1} = 4851$

Since there are 3 parties and 100 seats, so it's obvious that all the 3 parties cannot get the same number of seats. However, there can be ways in which any 2 parties can get the same number of seats.

These 2 parties (or allies) can be selected in ${}^3C_2 = 3$ ways.

The 2 parties can get the same number of seats in the following ways.

(1, 1, 98), (2, 2, 96), (3, 3, 94), ..., (48, 48, 4), (49, 49, 2)

That means there are total 49 ways.

Thus the total number of ways in which any 2 parties can get the same number of seats $= 3 \times 49 = 147$

Number of ways in which all the three parties get the same number of seats = 0.

Therefore the required number of ways
 $= 4851 - 147 = 4704$.

- 86** If $a > b > c$, the number of solutions of $a + b + c = 49$, is $\lfloor \frac{n^2 + 6}{12} \rfloor$.

Therefore, the required number of solutions

$$= \left\lfloor \frac{49^2 + 6}{12} \right\rfloor = \left\lfloor \frac{2407}{12} \right\rfloor = \lfloor 200.5833 \rfloor = 200$$

Alternatively Consider the following explanation.

Number of toilets			Total Case
Village C	Village B	Village A	
0	1	48	24
...	...	...	
0	24	25	
1	2	46	22
...	...	...	
1	23	25	
2	3	44	21
...	...	...	
2	23	24	

3	4	42	19
...	...	...	
3	22	24	
...			3
14	15	20	
14	17	18	
15	16	18	1

Therefore total number of ways of building the toilets in 3 selected villages

$$\begin{aligned}
 &= 1 + (3 + 4) + (6 + 7) + (9 + 10) + (12 + 13) \\
 &\quad + (15 + 16) + (18 + 19) + (21 + 22) + 24 \\
 &= (1 + 7 + 13 + 19 + 25 + 31 + 37 + 43) + 24 \\
 &= \frac{(1 + 43)8}{2} + 24 = 200
 \end{aligned}$$

Hence, choice (d) is the answer.

Alternatively

Total number of ways of building 49 toilets in 3 villages.

$$= {}^{49+3-1}C_{3-1} = {}^{51}C_2 = \frac{51 \times 50}{2} = 1275$$

Now look out for all the possibilities when any two villages receive equal number of toilets.

Village 1	Village 2	Village 3
0	0	49
1	1	47
2	2	45
...	...	...
...	...	...
22	22	5
23	23	3
24	24	1

There are total 25 possibilities in which village 1 and village 2 will have equal number of toilets. Similarly there are 25 possibilities in which village 2 and village 3 will have equal number of toilets. And so there will be 25 possibilities when village 1 and village 3 will have equal number of toilets.

Thus, it shows that there are total 75 possibilities in which any two villages will have equal number of toilets.

Therefore the net possibilities in which all the three villages will have distinct number of toilets $= 1275 - 75 = 1200$

But there is only 1 case out of $3!$ cases in which village A will have more toilets than that of village B and village B will have more toilets than that of village C

Therefore the required number of ways $= \frac{1200}{3!} = 200$

Hint Please note that there is no any possibility in which all the three villages will have equal number of toilets, since 49 is not divisible by 3.

- 87** Let's say the numbers of persons of this family seeking the appointments at the 6 salons are a, b, c, d, e and f . Then we have $a + b + c + d + e + f = 4$.

The number of non-negative integral solutions of the above equation is ${}^{4+6-1}C_{6-1} = 126$

Since each person of the family is distinct (or distinguishable), so each of them can be mutually arranged in $4!$ ways.

Therefore, the required number of ways in which they can be given the appointments at these salons
 $= 4! \times 126 = 3024$.

☞ Alternatively

Case I When each person is given the appointment at distinct salons.

The number of ways of selection of 4 distinct salons ${}^6C_4 = 15$

The number of ways of arranging these people among 4 salons $= 4!$

The number of ways of getting the appointments at these 4 salons $= 4! \times 15 = 360$

Case II When 2 persons are given the appointment at one salon and the other 2 persons are given the appointment at 2 distinct salons.

The number of ways of selection of 3 distinct salons $= {}^6C_3 = 20$

The number of ways of arranging these people among 3 salons $= 3!$

The number of ways of selection of 2 persons who are given the appointment at the same salon $= {}^4C_2 = 6$

The number of ways of arranging the 2 persons who are given the appointment at the same salon $= 2!$

The number of ways of giving the appointments at the 3 salons $= 3! \times 20 \times 6 \times 2 = 1440$

Case III When 2 persons are given the appointment at one salon and the other 2 persons are given the appointment at the other salon.

The number of ways of selection of 2 distinct salons $= {}^6C_2 = 15$

The number of ways of arranging these people between 2 salons $= 2!$

The number of ways of selection of 2 people for each salon $= {}^4C_2 \times {}^2C_2 \times \frac{1}{2!} = 3$

The number of ways of arranging the 2 persons in each salon $= 2! \times 2! = 4$

The number of ways of giving the appointments at these 2 salons $= 2! \times 15 \times 3 \times 4 = 360$

Case IV When 3 persons are given the appointment at one salon and the 1 person takes the appointment at the other salon.

The number of ways of selection of 2 distinct salons $= {}^6C_2 = 15$

The number of ways of arranging these people between 2 salons $= 2!$

The number of ways of selecting 3 persons for the one salon $= {}^4C_3 = 4$

The number of ways of arranging 3 persons at one salon $= 3! = 6$

The number of ways of giving the appointments at these 2 salons $= 2! \times 15 \times 4 \times 6 = 720$

Case V When all the 4 persons are given the appointment at the same salon.

The number of ways of selection of 1 salon $= {}^6C_1 = 6$

The number of ways arranging the 4 persons at one salon $= 4!$

The number of ways of giving the appointments at a salon $= 4! \times 6 = 144$

Therefore the required number of ways
 $= 360 + 1440 + 360 + 720 + 144 = 3024$

- 88** There are two distinct cases:

Case I When there are 11 digits –

$$x_1 + x_2 + \dots + x_{11} = 3; \text{ for } x_1 \geq 1$$

$$\text{and } x_2, x_3, \dots, x_{11} \geq 0$$

$$\Rightarrow x_1 + x_2 + \dots + x_{11} = 2; \text{ for } x_1, x_2, x_3, \dots, x_{11} \geq 0$$

$$\text{Number of solutions} = {}^{2+11-1}C_{11-1} = {}^{12}C_{10} = 66$$

Case II When there are 12 digits –

$$x_1 + x_2 + \dots + x_{12} = 3; \text{ for } x_1 \geq 1 \text{ and } x_2, x_3, \dots, x_{12} \geq 0$$

$$\Rightarrow x_1 + x_2 + \dots + x_{12} = 2; \text{ for } x_1, x_2, x_3, \dots, x_{12} \geq 0$$

$$\text{Number of solutions} = {}^{2+12-1}C_{12-1} = {}^{13}C_{11} = 78$$

Therefore, the total required number of values of $N = 66 + 78 = 144$

☞ Alternatively As per the given information, N can be an 11 digit or a 12-digit number.

(A) **When N is an 11 digit number**

Case I Out of eleven digits three digits are 1 and eight digits are 0.

In this case a number will start with only one number that is 1. Then you have to arrange only 10 digits of which 2 digits will be same (i.e.) and other 8 digits will be same (i.e. 0).

$$\text{Therefore the total required numbers} = 1 \times \frac{10!}{8!2!} = 45$$

Case II Out of eleven digits one digit is 1, another digit is 2 and remaining nine digits are 0. In this case a number will start with two digits that is either 1 or 2. Then you have to arrange only 10 digits of which one digit is either 1 or 2, while rest of nine digits will be same. (i.e., 0).

Therefore the total required numbers = $2 \times \frac{10!}{9!} = 20$

Case III Out of the eleven digits one digit is 3 and the remaining ten digits are 0.

In this case a number will start with the digit 3 and the other 10 digits will be same (i.e., 0).

Therefore the total required numbers = $1 \times \frac{10!}{10!} = 1$

Thus the total 11-digit numbers = $45 + 20 + 1 = 66$.

(B) **When N is a 12 digit number**

Case I Out of twelve digits three digits are 1 and nine digits are 0.

In this case a number will start with only one number that is 1. Then you have to arrange only 11 digits of which 2 digits will be same (i.e., 1) and other 9 digits will be same (i.e., 0).

Therefore the total required numbers = $1 \times \frac{11!}{9! \times 2!} = 55$

Case II Out of the twelve digits one digit is 1, another digit is 2 and remaining ten digits are 0.

In this case a number will start with two digits that is either 1 or 2. Then you have to arrange only 11 digits of which one digit is either 1 or 2, while rest of the 10 digits will be same (i.e., 0).

Therefore the total required numbers = $2 \times \frac{11!}{10!} = 22$

Case III Out of the twelve digits one digit is 3 and the remaining eleven digits are 0.

In this case a number will start with the digit 3 and the other 11 digits will be same (i.e., 0).

Therefore the total required numbers = $1 \times \frac{11!}{11!} = 1$

Thus the total 11-digit numbers = $55 + 22 + 1 = 78$

Therefore, the total required number of values of $N = 66 + 78 = 144$

- 89** If out of 32 volunteers everybody befriended on Facebook, the maximum number of friendships = ${}^{32}C_2 = 496$

It means at most 496 pairs can be formed, which, in turn, implies that at most 496 regions can be visited.

However, since certain number of guys and girls are not the Facebook friends, so they are not allowed to visit the regions together. It implies that the required answer would be less than 496.

Let us consider that m guys and f girls don't befriend each other on the Facebook.

Therefore the total number of pairs who are Facebook

friends = $496 - m \times f$

Now we know that one-fifth of the girls deny becoming Facebook friend with one-third of the boys, so we must have $3m + 5f = 32$

Therefore the possible values of m and f are (4, 4) and (9, 1).

Now the possible number of pairs who are the Facebook friends = $496 - 4 \times 4 = 480$ and $496 - 9 \times 1 = 487$

Since 487 is given in the choices, so it could be the required possible value.

Thus the number of regions that these volunteers could have visited = 487.

- 90** Any number between 1 and 1000 must be less than 4 digits. Therefore it must be of the form $x_1 + x_2 + x_3 = 10$, where $0 \leq x_i \leq 9$ for every $i = 1, 2, 3$.

Therefore, the required number

$$\begin{aligned} &= \text{Coefficient of } x^{10} \text{ in } (1 + x + x^2 + \dots + x^9)^3 \\ &= \text{Coefficient of } x^{10} \text{ in } \left(\frac{1 - x^{10}}{1 - x} \right)^3 \\ &= \text{Coefficient of } x^{10} \text{ in } [(1 - x^{10})^3 (1 - x)^{-3}] \\ &= \text{Coefficient of } x^{10} \text{ in } [(1 - {}^3C_1 x^{10} \dots) (1 - x)^{-3}] \\ &= \text{Coefficient of } x^{10} \text{ in } (1 - x)^{-3} - {}^3C_1. \\ &\quad [\text{Coefficient of } x^{(10-10)} \text{ in } (1 - x)^{-3}] \\ &= {}^{3+10-1}C_{3-1} - 3 \cdot {}^{3+0-1}C_{3-1} \\ &= {}^{12}C_2 - 3 \cdot ({}^2C_2) = 66 - 3 = 63 \end{aligned}$$

Alternatively The number of non-negative integral solutions of $x_1 + x_2 + x_3 = 10$ will be ${}^{10+3-1}C_{3-1} = 66$

But out of these 66 solutions there are the following 3 solutions which need to be excluded.

$$\begin{aligned} 10 + 0 + 0 &= 10, & 0 + 10 + 0 &= 10, \\ 0 + 0 + 10 &= 10 \end{aligned}$$

Therefore the required number of solutions = $66 - 3 = 63$.

Alternatively Number of 1 digit number is zero.

Number of 2-digit numbers

$$\begin{aligned} &= \text{Number of solutions of } a + b = 10; (1 \leq a \leq 9, 0 \leq b \leq 9) \\ &= \text{Number of solutions of } a + b = 9; (0 \leq a \leq 9, 0 \leq b \leq 9) \\ &= {}^{10}C_1 = 10 \end{aligned}$$

But, this involves a solution in which $a = 10$. Which is unacceptable, so there are only 9 numbers of 3 digit numbers.

= Number of solutions of $a + b + c = 10$.

$$(1 \leq a \leq 9, 0 \leq b \leq 9, 0 \leq c \leq 9)$$

= Number of solutions of $a + b + c = 9$;

$$(0 \leq a \leq 9, 0 \leq b \leq 9, 0 \leq c \leq 9), {}^{11}C_2 = 55$$

But, this involves a solution in which $a = 10$, which is unacceptable, so there are only 54 numbers.

Thus, we have total 63 numbers (= 0 + 9 + 54).

- 91** Primarily, there are six different cases in which 15 different rings can be divided such that there are twins in the family and none of the daughters gets less than one and more than five rings.

Case I: 1, 1, 3, 5, 5; Case II: 1, 1, 4, 4, 5;
Case III: 1, 2, 2, 5, 5 Case IV: 1, 3, 3, 4, 4;
Case V: 2, 2, 3, 3, 5; Case VI: 2, 2, 3, 4, 4

Case I Number of ways of distributing 15 rings

$$= \frac{15!}{1!1!3!5!5!} \times \frac{1}{2! \times 2!} \times 2! \times 2!$$

Case II Number of ways of distributing 15 rings

$$= \frac{15!}{1!1!4!4!5!} \times \frac{1}{2! \times 2!} \times 2! \times 2!$$

Case III Number of ways of distributing 15 rings

$$= \frac{15!}{1!2!2!5!5!} \times \frac{1}{2! \times 2!} \times 2! \times 2!$$

Case IV Number of ways of distributing 15 rings

$$= \frac{15!}{1!3!3!4!4!} \times \frac{1}{2! \times 2!} \times 2! \times 2!$$

Case V Number of ways of distributing 15 rings

$$= \frac{15!}{2!2!3!3!5!} \times \frac{1}{2! \times 2!} \times 2! \times 2!$$

Case VI Number of ways of distributing 15 rings

$$= \frac{15!}{2!2!3!4!4!} \times \frac{1}{2! \times 2!} \times 2! \times 2!$$

Therefore total number of ways of distributing 15 different rings

$$= 15! \times \left[\frac{1}{3! \times (5!)^2} + \frac{1}{(4!)^2 \times 5!} + \frac{1}{(2!)^2 \times (5!)^2} + \frac{1}{(3!)^2 \times (4!)^2} + \frac{1}{(2!)^2 \times (3!)^2 \times 5!} + \frac{1}{(2!)^2 \times 3! \times (4!)^2} \right]$$

Hence choice (d) is the answer.

- 92** Primarily, there are six different cases in which 15 different rings can be divided such that there are twins in the family and none of the daughters gets less than one and more than five rings.

Case I : 1, 1, 3, 5, 5; Case II : 1, 1, 4, 4, 5;
Case III : 1, 2, 2, 5, 5 Case IV : 1, 3, 3, 4, 4;
Case V : 2, 2, 3, 3, 5; Case VI : 2, 2, 3, 4, 4

Case I Number of ways of distributing 15 rings = $\frac{5!}{2! \times 2!}$

Case II Number of ways of distributing 15 rings = $\frac{5!}{2! \times 2!}$

Case III Number of ways of distributing 15 rings = $\frac{5!}{2! \times 2!}$

Case IV Number of ways of distributing 15 rings = $\frac{5!}{2! \times 2!}$

Case V Number of ways of distributing 15 rings = $\frac{5!}{2! \times 2!}$

Case VI Number of ways of distributing 15 rings = $\frac{5!}{2! \times 2!}$

Therefore total number of ways of distributing 15 different rings = $6 \times \left(\frac{5!}{(2!)^2} \right) = \frac{6!}{(2!)^2} = 180$

Hence choice (a) is the answer.

- 93** $462 = 2 \times 3 \times 7 \times 11$

Since, there are exactly 4 prime factors, so each of the prime number can be assigned to each x_i in 4 ways. Therefore the total number of ways of assigning the prime factors to all the x_i is $4^4 = 256$

- 94** $462 = 2 \times 3 \times 7 \times 11$

Since, there are exactly 4 prime factors, so each of the four prime numbers can be assigned to x_i in 4 ways. Therefore the total number of assigning the prime factor to all the x_i is 4^4 .

The required solution will be positive only when either 0 or 2 or 4 prime factors are negative. This can be achieved in ${}^4C_0 + {}^4C_2 + {}^4C_4 = 1 + 6 + 1 = 8$ ways.

Thus the required number of solutions = $8 \times 4^4 = 2048$.

- 95** Let $L(A, C)$ denotes the total number shortest paths from A to C, without any restriction; $L(A, P, C)$ denotes the total number of shortest paths from A to C, via P; $L(A, Q, C)$ denotes the total number of shortest paths from A to C, via Q; $L(A, P, Q, C)$ denotes the total number of shortest paths from A to C, via P and Q Now,

$$L(A, C) = {}^{16}C_6 \times {}^6C_6 = 8008; L(A, P, C) = L(AP) \times L(PC)$$

$$= ({}^7C_4 \times {}^3C_3) \times ({}^9C_6 \times {}^3C_3) = 35 \times 84 = 2940$$

$$L(A, Q, C) = L(AQ) \times L(QC) = ({}^{12}C_8 \times {}^4C_4) \times ({}^4C_2 \times {}^2C_2) = 495 \times 6 = 2970$$

$$L(A, P, Q, C) = L(AP) \times L(PQ) \times L(QC)$$

$$= ({}^7C_4 \times {}^3C_3) \times ({}^5C_4 \times {}^1C_1) \times ({}^4C_2 \times {}^2C_2) = 1050$$

By the inclusion-exclusion principle, the total number of the shortest paths from A to C, such that Runnbeer doesn't go through any of the points P or Q

$$= L(A, C) - L(A, P, C) - L(A, Q, C) + L(A, P, Q, C)$$

$$= 8008 - 2940 - 2970 + 1050 = 3148$$

- 96** Total number of delegates in all 3 countries is 60, so the number of handshakes among the delegates of any two countries (only once) is ${}^{60}C_2$.

Total number of delegates in all 4 states of each country is 20, so the number of total number of handshakes among the delegates of any two states of each country (only once) is ${}^{20}C_2$.

Total number of delegates in all 5 cities of each state is 5, so the number of total number of handshakes among the delegates of any two cities of each state (only once) is ${}^5C_2$.

Out of 60 delegates any two delegates shake hands 2 times. Out of 20 delegates (in each country) any two delegates shake hands 5 times, but since some of their handshakes have been already counted, so we have to consider only 3 times, instead of 5 times.

Out of 5 delegates (in each state) any two delegates shake hands 9 times, but since some of their handshakes have been already counted, so we have to consider only 4 times, instead of 9 times.

Thus the total number of handshakes

$$\begin{aligned} &= [2({}^{60}C_2)] + 3[3({}^{20}C_2)] + 12[4({}^5C_2)] \\ &= 3540 + 1710 + 480 \\ &= 5730 \end{aligned}$$

97 Number of zones = 4

Number of blocks in each zone = 5

Total number of blocks = $4 \times 5 = 20$

Total number of connections between any two blocks across all the zones = ${}^{20}C_2 = 190$

Number of connections between any two blocks in a zone = ${}^5C_2 = 10$

Total number of trucks carrying goods between any two blocks across all the zones = 3

Number of trucks carrying goods between any two blocks in each zone = 2

Since the number of dedicated trucks between two cities of each block is less than the number of dedicated trucks between two cities across all the blocks of four zones, so we have to subtract the excess number of trucks.

Total number of trucks between any two blocks across all the zones = $3 \times 190 = 570$

Extra number of trucks in each zone = $1 \times 10 = 10$

Extra number of trucks across all the 4 zones = $4 \times 10 = 40$

Therefore the actual number of trucks engaged in the market

$$\begin{aligned} &= 3({}^{20}C_2) - 4[1 \times ({}^5C_2)] \\ &= 570 - 40 \\ &= 530 \end{aligned}$$

98 Consider the following table.

Case	Polygon	Value
I	Triangle	${}^{10}C_3 - {}^6C_3 = 100$
II	Quadrilateral	${}^{10}C_4 - {}^6C_4 = 195$
III	Pentagon	${}^{10}C_5 - {}^6C_5 = 246$
IV	Hexagon	${}^{10}C_6 - {}^6C_6 = 209$

Case	Polygon	Value
V	Heptagon	${}^{10}C_7 = 120$
VI	Octagon	${}^{10}C_8 = 45$
VII	Nonagon	${}^{10}C_9 = 10$
VIII	Decagon	${}^{10}C_{10} = 1$

Therefore the total required number of polygons such that at least one of the vertices is green and rest of the vertices are red = $100 + 195 + 246 + 209 + 120 + 45 + 10 + 1 = 926$

99 $10! = 2^a 3^b 5^c 7^d$

Since, there are four distinct prime factors so there will be $2^4 = 16$ subsets including the empty one, as in

$$\begin{aligned} &\{\}, \{2^a\}, \{3^b\}, \{5^c\}, \{7^d\}, \{2^a 3^b\}, \{2^a 5^c\}, \{2^a 7^d\}, \\ &\{3^b 5^c\}, \{3^b 7^d\}, \{5^c 7^d\}, \{2^a 3^b 5^c\}, \{2^a 3^b 7^d\}, \{2^a 5^c 7^d\}, \\ &\{3^b 5^c 7^d\}, \{2^a 3^b 5^c 7^d\}. \end{aligned}$$

In this kind of problems empty set should be replaced with a set containing the element 1.

Now we have the following combination for $m \times n$, as shown in the below.

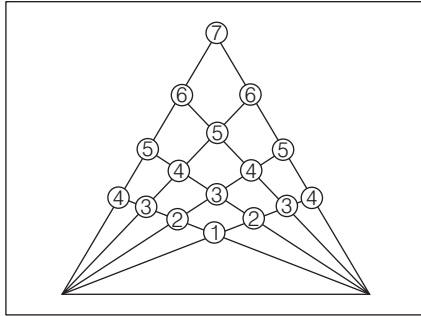
m	$\times$	n
1	$\times$	$2^a 3^b 5^c 7^d$
2^a	$\times$	$3^b 5^c 7^d$
3^b	$\times$	$2^a 5^c 7^d$
5^c	$\times$	$2^a 3^b 7^d$
7^d	$\times$	$2^a 3^b 5^c$
$2^a 3^b$	$\times$	$5^c 7^d$
$2^a 5^c$	$\times$	$3^b 7^d$
$2^a 7^d$	$\times$	$3^b 5^c$
$3^b 5^c$	$\times$	$2^a 7^d$
$3^b 7^d$	$\times$	$2^a 5^c$
$5^c 7^d$	$\times$	$2^a 3^b$
$2^a 3^b 5^c$	$\times$	7^d
$2^a 3^b 7^d$	$\times$	5^c
$2^a 5^c 7^d$	$\times$	3^b
$3^b 5^c 7^d$	$\times$	2^a
$2^a 3^b 5^c 7^d$	$\times$	1

In all such 16 cases there are half of the cases in which $m < n$.

Therefore, for $0 < \frac{m}{n} < 1$, the total number of required rational numbers

$$= \frac{1}{2} \times 2^4 = 8$$

- 100** Look at the following diagram and figure out all the unique triangles.



Since there are four slant lines emerging from each vertex at the base of the triangle.

Therefore the total number of triangles can be calculated as shown below.

$$1 \times 1 + 2 \times 2 + 3 \times 3 + 4 \times 4 + 3 \times 5 + 2 \times 6 + 1 \times 7 = 64$$

Alternatively Total number of triangles = the number of triangles with left bottom point as a vertex + the number of triangles with right bottom point as a vertex – the number of triangles with both bottom points as the vertices

$$= \frac{n \times n \times (n+1)}{2} + \frac{n \times n \times (n+1)}{2} - (n \times n) = n \times n \times n$$

Therefore the required number of triangles
 $= 4 \times 4 \times 4 = 64$

- 101** In the 1st transaction, PayTM or PayU can receive money from PayPal in 1 way.

In the 2nd transaction, PayPal can receive money either from PayTM or from PayU in 2 ways. But PayTM and PayU can receive money in 1 way from each other, as PayPal had no money after 1st transaction.

In the 3rd transaction, PayPal can receive money either from PayTM or PayU in 2 ways. But, each of PayTM and PayU can receive money in 3 ways.

Remember that we have to count the number of ways cumulatively. Now, follow the same principle and continue counting until the 7th transaction.

Transaction	1st	2nd	3rd	4th	5th	6th	7th
PayPal	–	2	2	6	10	22	42
PayTM	1	1	3	5	11	21	43
PayU	1	1	3	5	11	21	43

Thus we see that there are total 42 different ways in which PayPal wallet can have same amount after the 7th transaction that it had before the 1st transaction.

- 102** $2(l+b) \leq 100$ ($l, b \geq 0$)

$$\Rightarrow l+b \leq 50 \quad (l, b \geq 0)$$

$$\Rightarrow l+b \leq 48 \quad (l, b \geq 1)$$

$$\Rightarrow l+b+c = 48 \quad (l, b \geq 1)$$

$$\text{Number of solutions (N)} = {}^{48+3-1}C_{3-1} = {}^{50}C_2 = 1225$$

$$\Rightarrow N_{(l>b)} + N_{(l=b)} + N_{(l<b)} = 1225$$

Now, we know that when $l=b$, we have

$$\{l+b\} = \{1+1\}, \{2+2\}, \dots, \{25+25\}$$

$$\therefore N_{(l=b)} = 25$$

But, numerically, $N_{(l>b)} = N_{(l<b)}$

$$\therefore N_{(l>b)} = \frac{1225 - 25}{2} = 600$$

$$\begin{aligned} \text{Total number of rectangles} &= N_{(l>b)} + N_{(l=b)} \\ &= 600 + 25 = 625 \end{aligned}$$

Alternatively $2(l+b) = 100 \Rightarrow l+b = 50$. We will consider only combinations in which $l \geq b$, but we cannot consider $l < b$, as practically it violates the basic definition of rectangle.

$l+b$	Number of solutions	Solutions
2	1	{1, 1}
3	1	{2, 1}
4	2	{2, 2} {3, 1}
5	2	{3, 2}, {4, 1}
6	3	{3, 3}, {4, 2}, {5, 1}
7	3	{4, 3}, {5, 2}, {6, 1}
...	...	...
50	25	{25, 25}, {26, 24}, {27, 23}, ..., {49, 1}

Therefore total number of rectangles

$$= 2(1+2+3+\dots+24) + 25 = 625.$$

- 103** $a+b+c=12$; $\{0 \leq a, b, c \leq 6\} - \{a, b, c=5\}$

Number of solutions of $a+b+c=12$; $\{0 \leq a, b, c\}$ is ${}^{14}C_2 = 91$

If we allocate 7 to one of a, b, c we can make sure that the solution has $a, b, c > 6$.

Therefore, number of solutions of $a+b+c=12$ $\{0 \leq a, b, c \leq 6\}$ is equivalent to the number of solutions of $a+b+c=5$; $\{a, b, c \geq 0\}$ is ${}^7C_2 = 21$.

Since any of a, b, c can take value greater than 6, so it can be chosen in ${}^3C_1 = 3$ ways.

Now, we have following 3 sets in which one or more teams score 5 runs in a ball –

$$\{5, 6, 1\} \rightarrow 6$$

$$\{5, 5, 2\} \rightarrow 3$$

$$\{5, 4, 3\} \rightarrow 6$$

So, the number of solutions involving 5 is 15.

Therefore, required number of solutions

$$= 91 - 3(21) - 15 = 13$$

Alternatively There are certain triplets that can be arranged to find the valid possibilities.

$$\{6, 6, 0\} \rightarrow 3$$

$$\{6, 4, 2\} \rightarrow 6$$

$$\{6, 3, 3\} \rightarrow 3$$

$$\{4, 4, 4\} \rightarrow 1$$

Therefore, the possible number of ways

$$3 + 6 + 3 + 1 = 13$$

$$104 \quad 105 = N(\text{Shirts}) \times N(\text{Ties}) \times N(\text{Pants})$$

$$\Rightarrow 105 = 3 \times 5 \times 7$$

Since there are exactly three prime factors in 105, each of these 3 prime numbers can be allocated to three articles (shirts, ties and pants) in 3 ways. Therefore the required number of arrangements = $3^3 = 27$.

☞ **Alternatively** There are certain triplets that can be arranged to find the valid possibilities.

$$\{1 \times 1 \times 105\} \rightarrow 3$$

$$\{1 \times 3 \times 35\} \rightarrow 6$$

$$\{1 \times 5 \times 25\} \rightarrow 6$$

$$\{1 \times 7 \times 15\} \rightarrow 6$$

$$\{3 \times 5 \times 7\} \rightarrow 6$$

Therefore, the possible number of arrangements

$$= 3 + 6 + 6 + 6 + 6 = 27.$$

$$105 \quad \text{There are four pairs (12), (34), (56) and (78).}$$

Case I When only one pair exists, the number of arrangements = $7!$.

Since we can select any 1 pair out of 4 pairs in 4 ways, so the total number of arrangements = $4 \times (7!)$

Case II When exactly two pairs exist, the number of arrangements = $6!$

Since we can select any 2 pairs out of 4 pairs in 6 ways, so the total number of arrangements = $6 \times (6!)$

Case III When exactly three pairs exist, the number of arrangements = $5!$

Since we can select any 3 pairs out of 4 pairs in 4 ways, so the total number of arrangements = $3 \times (5!)$

Case IV When exactly two pairs exist, the number of arrangements = $4!$

Since we can select any 4 pairs out of 4 pairs in 1 way only, so the total number of arrangements = $1 \times (4!)$

By the Inclusion-Exclusion Principle, we have

$$\begin{aligned} |A_{12} \cup A_{34} \cup A_{56} \cup A_{78}| &= |A_{12}| + |A_{34}| + |A_{56}| + |A_{78}| \\ &- |A_{12} \cap A_{34}| - |A_{12} \cap A_{56}| - |A_{12} \cap A_{78}| - |A_{34} \cap A_{56}| \\ &- |A_{34} \cap A_{78}| - |A_{56} \cap A_{78}| + |A_{12} \cap A_{34} \cap A_{56}| \\ &+ |A_{12} \cap A_{34} \cap A_{78}| + |A_{12} \cap A_{56} \cap A_{78}| \\ &+ |A_{34} \cap A_{56} \cap A_{78}| - |A_{12} \cap A_{34} \cap A_{56} \cap A_{78}| \end{aligned}$$

$$\text{Therefore, } |A_{12} \cup A_{34} \cup A_{56} \cup A_{78}| = 4(7!) + 4(5!) - 4!$$

The total number of arrangements of numbers 1, 2, ... 8 is $8!$.

But, we need to find out the value of $8!$

$$- |A_{12} \cup A_{34} \cup A_{56} \cup A_{78}|.$$

Thus the required number of arrangements

$$= 8! - [4(7!) - 6(6!) + 4(5!) - 4!] = 24024$$

✓ Level 03 Final Round

$$1 \quad \text{Number of intersections} = 2 \times {}^6C_2 = {}^6P_2 = 30.$$

$$2 \quad {}^6C_2 = 15.$$

$$3 \quad 3 \text{ lines intersect each other in } {}^3C_2 = 3 \text{ points.}$$

$$3 \text{ circles intersect each other in } {}^3P_2 = 6 \text{ points.}$$

Every line cuts 3 circles into 6 points. Therefore 3 lines cuts 3 circles into 18 points. Therefore, the maximum number of points = $3 + 6 + 18 = 27$

$$4 \quad \frac{8!}{4!4!} = 70$$

$$5 \quad \text{Required number of triangles} = {}^{m+n}C_3 - {}^mC_3 - {}^nC_3.$$

$$6 \quad \text{Number of even places} = 4$$

$$\text{Number of even digits} = 5 (2, 2, 8, 8)$$

$$\text{Number of odd places} = 5$$

$$\text{Number of odd digits} = 4 (3, 3, 5, 5)$$

$$\text{Odd digits can be arranged in } \frac{4!}{2! \times 2!} \text{ ways} = 6 \text{ ways.}$$

$$\text{Even digits can be arranged in } \frac{5!}{2! \times 3!} = 10 \text{ ways.}$$

$$\text{Hence the required number of ways} = 6 \times 10 = 60 \text{ ways.}$$

$$7 \quad \text{Required number of triangles} \\ = {}^{m+n+k}C_3 - {}^mC_3 - {}^nC_3 - {}^kC_3$$

8 Let $n = 2m + 1$, for the three numbers are in AP we have the following patterns :

Common differences	Numbers	Number of ways
1	(1, 2, 3) (2, 3, 4) ... (n-2, n-1, n)	(n-2)
2	(1, 3, 5), (2, 4, 6) ... (n-4, n-2, n)	(n-4)
3	(1, 4, 7), (2, 5, 8) ... (n-6, n-3, n)	(n-6)
—		—
—		—
m	(1, m+1, 2m+1)	1

∴ Favourable number of ways

$$= (n-2) + (n-4) + (n-6) + \dots + 3 + 1 \text{ (total } m \text{ terms)}$$

$$= \frac{m}{2} (n-2+1) = \frac{(n-1)}{2} \cdot \frac{(n-1)}{2} = \frac{(n-1)^2}{4}$$

☞ **Alternatively** Consider some proper value of n and verify the result.

$$9 \quad \text{Required number of parts} = 1 + \sum_{r=1}^8 r = 1 + \frac{8 \times 9}{2} = 37$$

☞ **Alternatively** We can use the following formula too.

$${}^nC_0 + {}^nC_1 + {}^nC_2 = \frac{n \times (n+1)}{2} + 1 = \frac{n^2 + n + 2}{2}$$

$$\text{Therefore the required number of parts} = \frac{64 + 8 + 2}{2} = 37$$

$$10 \quad \frac{240}{4n+2} = k \in I$$

$$\therefore k = \frac{120}{2n+1} = \frac{2^3 \times 3 \times 5}{2n+1}$$

Since probable divisors are 1, 3, 5, 7, 9, 11, 13, ..., (2n + 1) but we have only 4 possible divisors 1, 3, 5, 15.

$$11 \quad \begin{aligned} \text{Total number of rectangles} \\ &= (1 + 2 + 3 + \dots + 12) \times (1 + 2 + \dots + 8) \\ &= \frac{12 \times 13}{2} \times \frac{8 \times 9}{2} = 2808 \end{aligned}$$

$$\begin{aligned} \text{Total number of squares} \\ &= (12 \times 8) + (11 \times 7) + (10 \times 6) + \dots + (5 \times 1) = 348 \\ \therefore \text{Required number of rectangles} &= 2808 - 348 = 2460 \end{aligned}$$

$$\text{Hint Number of squares} = \sum_{r=1}^8 (13-r)(9-r)$$

$$12 \quad \begin{aligned} \text{Required number of triangles} &= ({}^{2n}C_3 - {}^nC_3 - {}^nC_3) + (n \times n) \\ &= n^2(n-1) + n^2 = n^3 \end{aligned}$$

$$\text{Alternatively Required number of triangles} \\ = ({}^nC_2 \times {}^nC_1) + ({}^nC_1 \times {}^nC_2) + (n \times n) = n^3$$

$$13 \quad \begin{aligned} \text{There are 12 ways as follows :} \\ (9, 0, 0), (8, 1, 0), (7, 2, 0), (6, 3, 0), (5, 4, 0), (7, 1, 1), \\ (6, 2, 1), (5, 3, 1), (5, 2, 2), (4, 4, 1), (4, 3, 2), (3, 3, 3). \end{aligned}$$

$$14 \quad \begin{aligned} \text{First of all we give 4 coins to each of the six persons so that} \\ \text{each one must have at least 4 coins.} \\ \text{Now we are left with} = 30 - (6 \times 4) = 6 \text{ coins.} \\ \text{These remaining 6 coins can be distributed among} \\ \text{6 persons in such a way that any one can receive any} \\ \text{number of coins which can be done in} \\ {}^{6+6-1}C_{6-1} = {}^{11}C_5 \text{ ways} \end{aligned}$$

$$= 462 \text{ ways}$$

$$15 \quad \text{Required number of circles} = {}^{10}C_3 - {}^7C_3 = 85.$$

$$16 \quad \begin{aligned} 0 \text{ cannot be placed as the left most digit so we have only} \\ 9 \text{ digits to be placed.} \\ \text{Also we will not consider 1 digit numbers.} \\ \text{Now we can form a two digit number in } {}^9C_2 \text{ ways.} \\ \text{Now we can form a three digit number in } {}^9C_3 \text{ ways.} \\ \text{Now we can form a four digit number in } {}^9C_4 \text{ ways.} \end{aligned}$$

.....
.....

We can form 9 digit number in ${}^9C_9$ way.

$$\begin{aligned} \text{Hence the required numbers} &= {}^9C_2 + {}^9C_3 + {}^9C_4 + \dots + {}^9C_9 \\ &= 502 \end{aligned}$$

17 The possible ways are given below.

	I	II	III	IV	V	VI	VII	VIII	IX
Number of fours	25	22	19	16	13	10	7	4	1
Number of sixes	0	2	4	6	8	10	12	14	16

Therefore the required number of ways = 9.

$$18 \quad \begin{aligned} \text{Total number of permutations} &= 8! = 40320 \\ \text{Number of permutations when word LURY occurs} \\ &= (8 - 4 + 1)! = 5! = 120 \end{aligned}$$

$$\text{Number of permutations when word MINA occurs} = 5! = 120$$

$$\text{Number of permutation when words LURY and MINA both occurs} = 3! = 6$$

$$\begin{aligned} \text{Required number of permutations} \\ &= 40320 - (120 + 120) + 6 = 40086 \end{aligned}$$

$$19 \quad \begin{aligned} 5 \text{ students can be selected out of 10 students in } {}^{10}C_5 \text{ ways} \\ \text{remaining 5 students can be selected in } {}^5C_5 \text{ ways. These} \\ \text{students (in each row) can be arranged mutually in} \\ 5! \times 5! \text{ ways.} \end{aligned}$$

Besides it the two set of papers can be arranged in 2! ways between two rows.

$$\begin{aligned} \text{Hence, the required number of arrangements} \\ &= {}^{10}C_5 \times (5!)^2 \times 2 = 7257600 \end{aligned}$$

$$20 \quad \begin{aligned} \text{Number of handshakes among guests} &= {}^nC_2 \\ \text{Number of handshakes between the hosts and guests} &= 2n \\ \text{Therefore total number of handshakes} &= {}^nC_2 + 2n = 65 \end{aligned}$$

Now for your convenience go through the options and verify the correct one.

$$21 \quad \begin{aligned} \text{First of all deduce } 3 \times 10 = 30 \text{ marks to assign atleast} \\ 3 \text{ marks to each of the 10 students.} \\ \text{Now remaining 20 marks can be assigned to 10 students in} \\ {}^{20+10-1}C_{10-1} \text{ ways} = {}^{29}C_9 \text{ ways} \end{aligned}$$

$$22 \quad \begin{aligned} \left[\frac{1}{3} \right] &= \left[\frac{1}{3} + \frac{1}{100} \right] = \left[\frac{1}{3} + \frac{2}{100} \right] \\ &= \dots \left[\frac{1}{3} + \frac{65}{100} \right] = \left[\frac{1}{3} + \frac{66}{100} \right] = 0 \end{aligned}$$

$$\begin{aligned} \text{and } \left[\frac{1}{3} + \frac{67}{100} \right] &= \left[\frac{1}{3} + \frac{68}{100} \right] = \left[\frac{1}{3} + \frac{69}{100} \right] \\ &= \dots \left[\frac{1}{3} + \frac{98}{100} \right] = \left[\frac{1}{3} + \frac{99}{100} \right] = 1 \end{aligned}$$

$$\begin{aligned} \text{Hence, } E &= \left[\frac{1}{3} \right] + \left[\frac{1}{3} + \frac{1}{100} \right] + \dots + \left[\frac{1}{3} + \frac{99}{100} \right] \\ &= 33 \times 1 = 33 \end{aligned}$$

$$23 \quad \text{Total numbers} = 10^6 (1, 2, 3, 4, \dots, 10^6)$$

$$n^2 \Rightarrow \left\{ 1, 4, 9, 16, 25, \dots, 10^6 \right\} \rightarrow 10^3 = 1000$$

$$n^3 \Rightarrow \left\{ 1, 8, 27, 64, 125, \dots, 10^6 \right\} \rightarrow 10^2 = 100$$

$$n^4 \Rightarrow \left\{ 1, 16, 81, 256, \dots, 923521 \right\} \rightarrow 31$$

$$\text{Hint } \sqrt[4]{10^6} = 10^{6/4} = 10^{3/2} = \sqrt{1000} \Rightarrow 31$$

$$n^2 \cap n^3 \Rightarrow n^6 \left\{ 1, 64, 729, 4096, \dots, 10^6 \right\} \rightarrow 10$$

$$n^3 \cap n^4 \Rightarrow n^{12} \left\{ 1^{12}, 2^{12}, 3^{12} \right\} \rightarrow 3$$

$$\text{Hint } \sqrt[12]{10^6} = 10^{6/12} = 10^{1/2} = \sqrt{10} \Rightarrow 3$$

$$n^2 \cap n^4 \Rightarrow n^4 \left\{ 1, 16, 81, 256, \dots, (31)^4 \right\} \rightarrow 31$$

$$n^2 \cap n^3 \cap n^4 \Rightarrow n^{12} \left\{ 1^{12}, 2^{12}, 3^{12} \right\} \rightarrow 3$$

Hence the number of numbers which are either perfect square or perfect cube or perfect fourth powers or all of these

$$= n^2 + n^3 + n^4 - (n^2 \cap n^3 + n^3 \cap n^4 + n^2 \cap n^4) + n^2 \cap n^3 \cap n^4$$

$$= 1131 - 44 + 3 = 1090$$

Hence, the required number of ways

= Total numbers - Numbers which are perfect squares

or perfect cubes or perfect fourth powers

$$= 10^6 - 1090 = 998910$$

- 24** Total number of required seats = $1 + m + 2n$

The Grandchildren can occupy the n seats on either side of the table in $({}^{2n}P_{2n})$ ways. Remaining seats are $(1 + m)$.

Since grandfather can not occupy adjacent seats of the grandchildren hence the grandfather can access only $m + 1 - 2 = m - 1$ seats. Hence he can occupy a seat in $({}^{m-1}P_1)$ ways. Now the remaining seats can be occupied in ${}^mP_m$ ways by the ' m ' sons and daughters.

Hence the required number of ways

$$= {}^{2n}P_{2n} \times {}^mP_m \times {}^{m-1}P_1 = (2n!) (m!) (m - 1)$$

- 25** The 4 possible cases are as follows :

	C_1	C_2	C_3
$C_1 \rightarrow$ First column	2	3	1
$C_2 \rightarrow$ Second column	2	2	2
$C_3 \rightarrow$ Third column	1	4	1
	1	3	2

Hence, the required number of ways

$$\begin{aligned}
 &= {}^2C_2 \times {}^4C_3 \times {}^2C_1 + {}^2C_2 \times {}^4C_2 \times {}^2C_2 \\
 &\quad + {}^2C_1 \times {}^4C_4 \times {}^2C_1 + {}^2C_1 \times {}^2C_1 \times {}^4C_3 \times {}^2C_2 \\
 &= 1 \times 4 \times 2 + 1 \times 6 \times 1 + 2 \times 1 \times 2 + 2 \times 4 \times 1 \\
 &= 8 + 6 + 4 + 8 = 26
 \end{aligned}$$

- 26** The total number of balls in the box = $2 + 3 + 4 = 9$.

Total number of selections of 3 balls out of 9 balls = ${}^9C_3$

Number of selections in which no any green ball is selected = ${}^6C_3$

Hence the required number of selections = ${}^9C_3 - {}^6C_3 = 64$.

- 27** There are four possible cases.

	H	W
M	0	3
F	3	0

(i)

	H	W
M	1	2
F	2	1

(ii)

	H	W
M	2	1
F	1	2

(iii)

	H	W
M	3	0
F	0	3

(iv)

H $\rightarrow$ husband's relatives

W $\rightarrow$ wife's relatives

M $\rightarrow$ male

F $\rightarrow$ female

Hence the required number of ways

$$= ({}^4C_3 \times {}^4C_3) + ({}^3C_1 \times {}^4C_2)({}^4C_2 \times {}^3C_1) + ({}^3C_2 \times {}^4C_1)({}^4C_1 \times {}^3C_2) + ({}^3C_3 \times {}^3C_3)$$

$$= (4 \times 4) + (3 \times 6 \times 6 \times 3) + (3 \times 4 \times 4 \times 3) + (1 \times 1)$$

$$= 485$$

- 28** Let the number of men participating in the tournament be n . Since every participant played two games with every other participant.

Therefore the total number of games played among men is $2 \times {}^nC_2 = n(n - 1)$.

And the number of games played with each woman = $2n$.

but since there are two women, hence the total number of games men played with 2 women = $2 \times 2n = 4n$

$$\therefore \{n(n - 1)\} - 4n = 66$$

$$\Rightarrow n^2 - 5n - 66 = 0$$

$$\Rightarrow n = 11 \quad (\because n < 0, \text{ is not possible})$$

$$\therefore \text{Number of participants} = 11 \text{ men} + 2 \text{ women} = 13.$$

- 29** Number of games played by them is $2({}^{13}C_2) = 156$.

- 30** There are two possible cases in which 12 sweets can be distributed among 10 girls.

(i) Any 9 girls get one sweets each and the remaining 1 girl gets 3 sweets.

(ii) Any 8 girls get one sweets each and the remaining 2 girls get 2 pieces of sweets each.

Case I. 3 pieces of sweets can be given to a girl in the following four way :

Burfi	3	2	1	0
Rasgulla	0	1	2	3

After giving 3 pieces of sweets to a single girl. We can distribute the remaining 9 sweets to the 9 girls in the following ways :

$${}^9C_3 \times {}^6C_6 + {}^9C_4 \times {}^5C_5 + {}^9C_5 \times {}^4C_4 + {}^9C_6 \times {}^3C_3 \\ = 2({}^9C_3 + {}^9C_4)$$

One particular girl can be chosen in ${}^{10}C_1$ ways.

Therefore 3 sweets can be given to a single girl in

$${}^{10}C_1 \times 2 \times ({}^9C_3 + {}^9C_4) = 4200 \text{ ways.}$$

Case II. We can give two sweets to two girls (say A and B) in the following ways :

A	Burfi	2	1	0	2	1	0	2	1	0
	Rasgulla	0	1	2	0	1	2	0	1	2
B	Burfi	2	2	2	1	1	1	0	0	0
	Rasgulla	0	0	0	1	1	1	2	2	2

Now, remaining 8 pieces of sweets can be distributed among eight girls in the following ways

$$({}^8C_2 \times {}^6C_6) + ({}^8C_3 \times {}^5C_5) + ({}^8C_4 \times {}^4C_4) + ({}^8C_5 \times {}^3C_3) \\ + ({}^8C_4 \times {}^4C_4) + ({}^8C_5 \times {}^3C_3) \\ + ({}^8C_4 \times {}^4C_4) + ({}^8C_5 \times {}^3C_3) + ({}^8C_6 \times {}^2C_2) \\ = 2({}^8C_2) + 4({}^8C_3) + 3({}^8C_4) \\ = 26250$$

Further, two girls can be selected in ${}^{10}C_2$ ways.

Therefore two girls can get two sweets each in

$$({}^{10}C_2)[2({}^8C_2) + 4({}^8C_3) + 3({}^8C_4)] = 22050 \text{ ways}$$

Hence, the required number of ways = 4200 + 22050
= 26250

31 Number of common children of Mr. John and Ms. Bashu

$$= 10 - (x + x + 1) = 9 - 2x$$

Let N = The number of fights between children
of different parents

= (Total number of fights that can take place among
all the children)

– (The number of fights among the children
of the same parents)

$$= {}^{10}C_2 - [{}^xC_2 + {}^{x+1}C_2 + {}^{9-2x}C_2] \quad \dots(i)$$

$$= 45 - \left[\frac{x(x-1)}{2} + \frac{(x+1)(x)}{2} + \frac{(9-2x)(8-2x)}{2} \right]$$

$$= 45 - \frac{1}{2} [x^2 - x + x^2 + x + 72 - 34x + 4x^2]$$

$$= \frac{397}{12} - 3 \left(x - \frac{17}{6} \right)^2$$

For N to be maximum, x must be $\frac{17}{6}$. As x cannot be in

fraction, we take $x = 3 \left(\approx \frac{17}{6} \right)$.

Thus, maximum value of $N = 33$ which is attained at $x = 3$.

Alternatively After making the equation (i) go through options.

32 Let the form of the required numbers be $a_1, a_2 \dots a_9$, where $0 \leq a_1 \leq 1$ and $0 \leq a_i \leq 2$ for $i = 2, 3, \dots, 9$ and where all $a_1, a_2, \dots, a_9$ cannot be equal to zero.

Now, we can choose a_1 in two ways (0 or 1) and a_i for $i = 2, 3, \dots, 8$ in 3 ways (0, 1, 2).

After selecting $a_1, a_2, a_3, \dots, a_8$ we find the sum $s = a_1 + a_2 + \dots + a_8$ which is of the form $3m - 2, 3m - 1$ or $3m$. Now we can select a_9 in just one way.

Actually a_9 can be selected out of 2, 1 or 0 depending on whether $s = 3m - 2, 3m - 1$ or $3m$. Therefore, we can choose the numbers in $2 \times 3^7 \times 1 = 4374$ ways.

But this includes the case in which each of $a_i = 0$. Thus, the required number of numbers = $4374 - 1 = 4373$

33 The digits which can be recognised as digits on the screen of a calculator when they are read inverted i.e., upside down are 0, 1, 2, 5, 6, 8 and 9.

Since a number cannot begin with zero hence left most digit and right most digit can never be 0 as when an 'n' digit number read upside down it will become a number of less than n digit. Hence,

Number of digits	Total number of numbers
1	7
2	$6 \times 6 = 6^2$
3	$6 \times 7 \times 6 = 6^2 \times 7$
4	$6 \times 7 \times 7 \times 6 = 6^2 \times 7^2$
5	$6 \times 7 \times 7 \times 7 \times 6 = 6^2 \times 7^3$
6	$6 \times 7 \times 7 \times 7 \times 7 \times 6 = 6^2 \times 7^4$

Thus, the number of required numbers

$$= 7 + 6^2 + 6^2 \cdot 7 + \dots + 6^2 \cdot 7^4$$

$$= 7 + 6^2 \frac{(7^5 - 1)}{(7 - 1)} = 7 + 6(7^5 - 1)$$

$$= 6 \cdot 7^5 + 1 = 100843$$

34 Since rings are distinct, hence they can be named as

R_1, R_2, R_3, R_4 and R_5 .

The ring R_1 can be placed on any of the four fingers in 4 ways.

The ring R_2 can be placed on any of the four fingers in 5 ways since the finger in which R_1 is placed now has 2 choices, one above the R_1 and one below the ring R_1 .

Similarly R_3, R_4 and R_5 can be arranged in 6, 7 and 8 ways respectively.

Hence, the required number of ways

$$= 4 \times 5 \times 6 \times 7 \times 8 = 6720$$

35 We can select first object out of n objects in ${}^nC_1$ ways.

Now, number of ways of choosing two objects such that they are always together $(n - 4)$ ways.

Since we assume two objects as a single object. Further we can select three objects viz., the one object which has been already selected and two objects of one either side of the first object. Therefore the number of ways of choosing two objects such that they are not together $= {}^{(n-3)}C_2 - (n-4)$

$$= \frac{1}{2} (n-4)(n-5)$$

Since these two objects can be arranged in $2!$ ways, the number of ways of choosing three objects (in order of the first, second and third) is

$$n \times \frac{1}{2} (n-4)(n-5) \times 2 = n(n-4)(n-5)$$

But, since the order in which the objects are taken is immaterial, the number of ways of choosing the objects is $\frac{1}{6} n(n-4)(n-5)$.

36

Number of similar letters	Number of different letters	Number of selections
5	0	${}^1C_1 = 1$
4	1	${}^4C_1 \times {}^2C_1 = 8$
3	2	${}^3C_1 \times {}^4C_2 = 18$
3 of one type and 2 of another type	0	${}^3C_1 \times {}^3C_1 = 9$
2 of one type and 2 of another type	1	${}^4C_2 \times {}^3C_1 = 18$
2	3	${}^4C_1 \times {}^4C_3 = 16$
0	5	${}^5C_5 = 1$

Hence, the total number of selections

$$= 1 + 8 + 18 + 9 + 18 + 16 + 1 = 71$$

37

	Ist paper	2nd paper	3rd paper	4th paper
Max. marks	n	n	n	$2n$

Let us consider $n = 1$, then a candidate required 3 marks out of 5 marks, which can be done in the following ways :

Ist paper	2nd paper	3rd paper	4th paper	Total
0	0	1	2	3
0	1	0	2	3
1	0	0	2	3
1	1	1	0	3
1	1	0	1	3
1	0	1	1	3
0	1	1	1	3

Hence, there are total 7 ways.

Now, go through options.

Let us consider option (b).

Putting $n = 1$, we get

$$\frac{1}{6} (1+1)(5 \times 1^2 + 10 \times 1 + 6) = 7$$

Hence the choice (b) is correct.

38 Do this problem similarly as discussed in the previous problem.



NOTE The general solution of this type of problems involves higher mathematics. But we have given in question number 37 a very simple, lucid and a novel solution for the lay students.

39 Let the number of male students is x , the number of female students in that class will be $2x$.

The number of friendships among male students

$$= {}^xC_2 = \frac{x(x-1)}{2}$$

The number of friendships among female students

$$= {}^{2x}C_2 = \frac{2x(2x-1)}{2}$$

The number of friendships between male and female students $= x \times 2x = 2x^2$

Let's assume that the number of friend requests sent by female students to male students is y . Therefore, the number of friend requests sent by male students to female students $= 2x^2 - y$.

But you must know the following fact about intra group friend requests as in among boys only or among girls only.

The number of friendships = number of friend requests sent = number of friend requests accepted

So, the number of friend requests sent/accepted by male students among themselves $= \frac{x(x-1)}{2}$

Similarly, the number of friend requests sent/accepted by female students among themselves $= \frac{2x(2x-1)}{2}$

Further it is given that, The total number of friend requests sent by male students = Total number of friend requests sent by female students

$$\therefore \frac{x(x-1)}{2} + (2x^2 - y) = \frac{2x(2x-1)}{2} + y$$

$$\Rightarrow 4y = x(x+1)$$

Among the given choices, only choice (c) is valid.

- 40** Let $A, B, C, D, E, F, G, H, I, J, K$ be the 11 masseurs, then divide them in two groups say one group has 5 masseurs (A, B, C, D and E), while another group has 6 masseurs (F, G, H, I, J and K). In the first week, employ first group of 5 masseurs.

Case	Number of masseurs familiar with queen		Number of weeks required to identify	Reason
	First week	Second week		
I	5	0	1	All 5 are familiar in the first week itself
II	4	1	1	4 out of 5 are familiar in the first week itself
III	3	2	1	3 out of 5 are familiar in the first week itself
IV	2	3	>2	3 out of 6 are familiar so 3 must be strangers, thus, if all 3 strangers and any 2 familiar masseurs give the massage out of 5 days on the 2nd, 3rd or 4th week, king still can't identify them. So king needs 5th week to identify the masseurs familiar with the queen.
V	1	4	2	4 out of 6 are familiar and 2 are strangers, so even if all 2 strangers give the massage out of 5 days, 3 familiar masseurs will still be giving the massage in the second week, so king can identify them.
VI	0	5	2	5 out of 6 are familiar and 1 is stranger, so even if 1 stranger gives the massage out of 5 days, 4 familiar masseurs will still be giving the massage in the second week, so king can identify them.

Suppose in the first week A, B, C, D and E give her massage and the king could not identify them. It implies that there are maximum 2 familiar masseurs among A, B, C, D and E . In turn, it implies that there are minimum 3 familiar masseurs among the F, G, H, I, J, K . If there are 5 familiar masseurs in the group F, G, H, I, J and K , so whoever the 5 masseurs, among these 6 masseurs, are giving the massage king would certainly identify them.

Similarly, if there are 4 familiar masseurs in the group F, G, H, I, J and K , so whoever the 5 masseurs, among these 6 masseurs, are giving the massage king would certainly identify them.

But when there are only 3 familiar masseurs in the group F, G, H, I, J and K , then we will have the following possibilities assuming that I, J and K are the 3 familiar masseurs.

$\{F, G, H, I, J\}, \{F, G, H, I, K\}, \{F, G, H, J, K\}, \{F, G, I, J, K\},$
 $\{F, H, I, J, K\}, \{G, H, I, J, K\}$

Out of the 6 possibilities there are 3 cases in which king cannot identify the familiar masseurs.

It means in the worst case scenario king has to wait for 5 weeks to identify the 5 familiar masseurs as illustrated below.

Week 1: A, B, C, D, E	can't identify
Week 2: F, G, H, I, J	can't identify
Week 3: F, G, H, I, K	can't identify
Week 4: F, G, H, J, K	can't identify
Week 5: F, G, I, J, K	can identify
Week 6: F, H, I, J, K	can identify
Week 7: G, H, I, J, K	can identify

So, in the worst case scenario, king would require at least 5 weeks to be 100% sure if he wants to identify the masseurs familiar with the queen.

- 41** Let A be the set of permutations where 3 and 6 appear consecutively, B be the set of permutations, where 6 and 5 appear consecutively, and let C be the set of permutations, where 5 and 4 do. Then $|A| = |B| = |C| = 9!$

For example, permutations in A correspond to permutations of the alphabet $\{0, 1, 2, 36, 4, 5, 7, 8, 9\}$, where 36 is considered as a single symbol.

Similarly, $|A \cap B| = |B \cap C| = |C \cap A| = 8!$

For example, permutations in $A \cap B$ correspond to permutations of the alphabet with symbol 365 replacing 3, 5 and 6.

Similarly, $|A \cap B \cap C| = 7!$

$$\begin{aligned} \therefore |A \cup B \cup C| &= |A| + |B| + |C| - |A \cap B| - |B \cap C| \\ &\quad - |C \cap A| + |A \cap B \cap C| \\ \Rightarrow |A \cup B \cup C| &= 3(9!) - 3(8!) - 7! = 191(7!) = 962640 \end{aligned}$$

- 42** Number of solutions of $x_1 + x_2 + x_3 + x_4 = 30$ for $3 \leq x_1, x_2, x_3, x_4 \leq 10$ is same as the number of solutions of $y_1 + y_2 + y_3 + y_4 = 18$ for $0 \leq y_1, y_2, y_3, y_4 \leq 7$
 But the number of solutions of $y_1 + y_2 + y_3 + y_4 = 18$ for $0 \leq y_1, y_2, y_3, y_4 \leq 7$ = (number of solutions of $y_1 + y_2 + y_3 + y_4 = 18$ for $0 \leq y_1, y_2, y_3, y_4 \leq 18$) - (number of solutions of $y_1 + y_2 + y_3 + y_4 = 18$ for $8 \leq y_1, y_2, y_3, y_4 \leq 18$)
 Now, number of solutions of $y_1 + y_2 + y_3 + y_4 = 18$ for $0 \leq y_1, y_2, y_3, y_4 \leq 18$ is ${}^{18+4-1}C_{4-1} = {}^{21}C_3 = 1330$.

Now the number of solutions of $y_1 + y_2 + y_3 + y_4 = 18$ for $8 \leq y_1, y_2, y_3, y_4 \leq 18$ can be determined as follows.

Since, $y_i \geq 8$ so out of four values of y either one value of y or two values of y can be equal to or greater than 8 as $y_1 + y_2 + y_3 + y_4 = 18$.

Then number of solutions of $y_1 + y_2 + y_3 + y_4 = 18$ for $8 \leq y_1, y_2, y_3, y_4 \leq 18$ is

$$\begin{aligned} |A_1 \cup A_2 \cup A_3 \cup A_4| &= \sum_{1 \leq i \leq 4} |A_i| - \sum_{1 \leq i < j \leq 4} |A_i \cap A_j| \\ &= 4[(18-8) + 4-1]C_{4-1} - 6[(18-2 \times 8) + 4-1]C_{4-1} \\ &= 4(1^3C_3) - 6(5C_3) = 4 \times 286 - 6 \times 10 = 1084 \end{aligned}$$

Therefore the number of solutions of

$$y_1 + y_2 + y_3 + y_4 = 18 \text{ for } 0 \leq y_1, y_2, y_3, y_4 \leq 7 \text{ is } 1330 - 1084 = 246$$

So the number of solutions of $x_1 + x_2 + x_3 + x_4 = 30$ for $3 \leq x_1, x_2, x_3, x_4 \leq 10$ is 246.

Hint $|A_i|$ can be selected in 4 ways. For example, $|A_1|$ indicates that $z_1 + y_2 + y_3 + y_4 = 18 - 8$; where $z_1 = y_1 - 8$.

Similarly, $|A_i \cap A_j|$ can be selected in 6 ways. For example $|A_1 \cap A_2|$ indicates that $z_1 + z_2 + y_3 + y_4 = 18 - (8 + 8)$; where $z_1 = y_1 - 8$ and $z_2 = y_2 - 8$.

- 43** If $x \leq \sqrt{3}$, the given expression will be zero or negative, which is impossible. So choices (a) and (c) are wrong ones. Again if you consider k as a large number, the given relation won't satisfy.

Hence, choice (d) is the answer.

Alternatively ${}^{n-1}C_r = (k^2 - 3)({}^nC_{r+1})$

$$\begin{aligned} \Rightarrow \frac{(n-1)!}{(r)! \cdot (n-1-r)!} &= (k^2 - 3) \frac{(n)!}{(r+1)! \cdot (n-r-1)!} \\ \Rightarrow \frac{(n-1)!}{(r)! \cdot (n-1-r)!} &= (k^2 - 3) \frac{(n) \cdot [(n-1)]}{(r+1) \cdot [(r)!] \cdot (n-r-1)!} \\ \Rightarrow (k^2 - 3) &= \frac{r+1}{n} \end{aligned}$$

The minimum value of $\frac{r+1}{n}$ will be close to zero but positive when n is extremely high.

And the maximum value of $r+1/n$ will be 1 when $n = r+1$, since $r+1$ cannot exceed n .

$$\text{Therefore, } 0 < (k^2 - 3) \leq 1 \Rightarrow 3 < k^2 \leq 4 \Rightarrow \sqrt{3} < k \leq 2$$

Hence choice (d) is the answer.

- 44** Total number of restaurants = $20 + 15 + 10 = 45$

Number of transactions among 45 restaurants = ${}^{45}C_2$.

Number of transactions within each type of restaurants = ${}^{20}C_2 + {}^{15}C_2 + {}^{10}C_2$

Number of transactions among different categories of restaurants = ${}^{45}C_2 - ({}^{20}C_2 + {}^{15}C_2 + {}^{10}C_2)$

Since these restaurants can make up to 3 transactions a day, total number of transactions

$$= 3 \times \{ {}^{45}C_2 - ({}^{20}C_2 + {}^{15}C_2 + {}^{10}C_2) \}$$

Number of transactions among Chinese restaurants = ${}^{20}C_2$

But since no two neighboring restaurants make any transaction, so the actual number of transactions among

the Chinese restaurants = ${}^{20}C_2 - 20$

Similarly, number of transactions among Thai restaurants = ${}^{15}C_2 - 15$

And, the number of transactions among Italian restaurants = ${}^{10}C_2 - 10$

Total number of transactions

$$= ({}^{20}C_2 - 20) + ({}^{15}C_2 - 15) + ({}^{10}C_2 - 10)$$

But since these restaurants can make up to 2 transactions a day, total number of transactions

$$= 2 \times \{ ({}^{20}C_2 - 20) + ({}^{15}C_2 - 15) + ({}^{10}C_2 - 10) \}$$

Therefore, total number of transactions

$$\begin{aligned} &= 3 \times \{ {}^{45}C_2 - ({}^{20}C_2 + {}^{15}C_2 + {}^{10}C_2) \} \\ &\quad + 2 \times \{ ({}^{20}C_2 - 20) + ({}^{15}C_2 - 15) + ({}^{10}C_2 - 10) \} \end{aligned}$$

Now, as you know that when two people make a transaction so either of them can give or take change, so the total number of times the change can be given

$$\begin{aligned} &= 2 \times [3 \times \{ {}^{45}C_2 - ({}^{20}C_2 + {}^{15}C_2 + {}^{10}C_2) \} \\ &\quad + 2 \times \{ ({}^{20}C_2 - 20) + ({}^{15}C_2 - 15) + ({}^{10}C_2 - 10) \}] \\ &= 2 \times [3 \times ({}^{45}C_2) - \{ ({}^{20}C_2) + ({}^{15}C_2) + ({}^{10}C_2) \} - 2(20+15+10)] \\ &= 5080 \end{aligned}$$

NOTE I have used the term "transactions" which indicates, for my purpose, the flow of something in one direction, that's why I have multiplied by 2 in the last part to accommodate the two-way transactions.

- 45** Since $x+1$ is a factor of $ax^2 + bx + c$, therefore $x+1=0$ or $x=-1$.

Then we have $a-b+c=0 \Rightarrow a+c=b$. But, since $a \neq b \neq c$, so let us consider $a < c$, then we have the following possibilities for a, b and c .

a	c	b	Number of solutions
1	2, 3, ..., 1998	3, 4, ..., 1999	1997
2	3, 4, ..., 1997	5, 6, ..., 1999	1995
3	4, 5, ..., 1996	7, 8, ..., 1999	1993
...			
997	998, ..., 1002	1995, ..., 1999	5
998	999, 1000, 1001	1997, 1998, 1999	3
999	1000	1999	1

Total number of solutions

$$= 1 + 3 + 5 + \dots + 1993 + 1995 + 1997 = 999^2$$

Thus for $a < c$, we have total 999^2 number of solutions.

Similarly for $a > c$, we will have total 999^2 number of solutions.

Therefore the required number of solutions (or polynomials) $= 999^2 + 999^2 = 2(999)^2 = 1996002$

☞ **Alternatively** $a + c \leq 1999 \{a, c \geq 1\}$

$$\Rightarrow a + c \leq 1997 \quad \{a, c \geq 0\}$$

$$\Rightarrow a + c + d = 1997 \quad \{a, c, d \geq 0\}$$

Number of solutions of the above equation is

$${}^{1997+3-1}C_{3-1} = {}^{1999}C_2 = 999 \times 1999$$

When $a = c$, we have $2a + d = 1997$

$$\{a, c, d \geq 0\}$$

Then the number of solutions of $2a + d = 1997$ is 999, as $a = 0, 1, 2, \dots, 998$.

Therefore the number of required solutions when $a \neq c$, is

$$\begin{aligned} & 999 \times 1999 - 999 \\ &= 999(1998) = 999^2 \times 2 = 1996002 \end{aligned}$$

- 46** Let the three sides of isosceles triangle be a, a and b , then there will be a triangle if $2a > b$.

Case I If $1 \leq a \leq 1007$

a	b	Number of possibilities
1	1	1
2	1, 2, 3	3
3	1, 2, 3, 4, 5	5
...		
1007	1, 2, 3, ..., 2013	2013

Thus the number of triangles

$$= 1 + 3 + 5 + \dots + 2013 = (1007)^2$$

Case II If $1008 \leq a \leq 2014$

a	b	Number of possibilities
1008	1, 2, 3, ..., 2014	2014
1009	1, 2, 3, ..., 2014	2014
1010	1, 2, 3, ..., 2014	2014
...	...	...
2014	1, 2, 3, ..., 2014	2014

Thus the number of triangles

$$\begin{aligned} &= 2014 + 2014 + 2014 + \dots + 2014 \\ &= 1007 \times 2014 = 2(1007)^2 \end{aligned}$$

Therefore total number of isosceles triangles

$$= (1007)^2 + 2(1007)^2 = 3(1007)^2 = 2028098$$

- 47** The numbers in 2nd and 3rd position can only be 8 and 9 because they have to be strictly increasing. So the first 3 numbers have to be 789.

There are many possibilities for the numbers in position 4 and 5.

In position 4 we can have 8, 7, 6, 5, 4, 3, 2, 1

In position 5 what we have depends on what we have in previous position. If we have 8 in the previous position, we have 8 possibilities 7, 6, 5, 4, 3, 2, 1, 0. If the number is in the 4th position is 7, we have 7 possibilities - 6, 5, 4, 3, 2, 1, 0 and so on and finally if there is 1 in position 4, we have only 1 possibility 0.

So basically, the number of possibilities is

$$8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36$$

Hint We can use the formula for the sum of first n natural numbers, $\frac{n(n+1)}{2}$.

$$\text{Therefore, } 1 + 2 + 3 + \dots + 8 = \frac{8 \times 9}{2} = 36$$

- 48** Total number of triangles $= {}^{10}C_3 = 120$

Number of triangles, which share exactly one side of the decagon: first we can choose any 10 sides. Now, we cannot choose the two adjacent vertices of the chosen side, so we can choose the third vertex out of the remaining 6 ($= 10 - 4$) vertices. Therefore, the number of triangles that share only one side of the decagon $= 10 \times 6 = 60$.

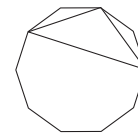
Number of triangles, which share exactly two sides of the decagon: first we can choose any two adjacent sides in 10 ways, out of the 10 sides of the decagon. The third side will be naturally formed by joining the two non-common vertices of the two chosen sides. Therefore, the number of triangles that share exactly two sides of the decagon $= 10$

Thus the required number of triangles

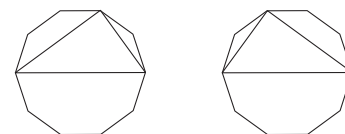
$$= 120 - (60 - 10) = 50$$

☞ **Alternatively** What we want is that any three vertices which are not adjacent to each other. So let us assume that A, B and C are the three vertices of the triangle taken clockwise, then have to select three vertices A, B, C such that at least one vertex of decagon lies between A and B and so between B and C . There are the various possibilities to choose from the remaining 7 ($= 10 - 3$) vertices between A, B and C .

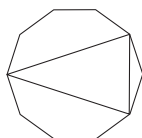
Case (i) (5, 1, 1)-Number of triangles $= 10$



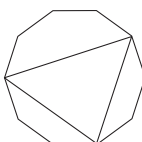
Case (ii) (4, 2, 1) - Number of triangles $= 20$



Case (iii) (3, 3, 1) - Number of triangles = 10



Case (iv) (3, 2, 2) - Number of triangles = 10



Thus the total number of require triangles

$$= 10 + 20 + 10 + 10 = 50$$

- 49** We can get a majority if we have 4 or more people together to unlock the box. If there are 3 or fewer people trying to open the box, they should not be able to open it.

So, in order to have the optimal solution to this problem, we will focus on the closest values. That is 3 and 4.

If we form groups of 3 people each, no such group should be able to open all the locks. But, when we form groups of 4 people each,

every such group must be able to open all the locks.

It shows that there are ${}^7C_3 = 35$ groups of 3 people each, and for each group there is a lock that the pertinent group cannot open it. But, any group of 4 people can open every lock. It implies that there must be at least 35 locks. If there are fewer than 35 locks, then all these locks can be opened without even majority being present, as there must be some group of 3 people having the keys of all the locks.

Thus the minimum number of locks = 35.

Now, associate each group of 3 people (G_i) with each lock (L_i) and give the keys of the corresponding lock to all other people not associated with a particular lock. Therefore, each person (in a group of 3) receives ${}^6C_3 = 20$ keys.

Thus the minimum number of keys = 20.

- 50** When a block is at the centre and connected with all other 24 blocks we have minimum number of required roads = 24

The maximum number of roads can be laid out when any blocks are directly connected with each other. It is

$${}^{25}C_2 = 300.$$

Therefore, we have $24 \leq R \leq 300$.

CHAPTER 20

Probability

Generally this chapter is considered as an extension of permutation and combination, since it is as logical in nature as permutation and combination. I hope that you must have been well-versed with the permutation and combination (previous chapter) which will enable you to solve the problems of this chapter quickly and easily.

This chapter is important for XAT, SNAP and other MBA entrance tests. Over the past years CAT has shown no great interest in this chapter, still you should learn it heartily due to uncertain syllabus and irregular pattern of CAT.

Remember that so far CAT has asked very simple and logical problems from this chapter which could be solved by applying just commonsense. The literal meaning of probability is the chance of occurrence of an event. For example, if a person is standing at the crossing of two roads which direct towards North, South, East and West.

Thus he has total four alternatives (*i.e.*, four different directions) to proceed. Now if he wish to go towards a particular direction then the probability of completing his wish is $1/4$ since he can choose only one direction at a time out of four directions.

Consider another example, A person has two different cars *viz.*, Scorpio and Safari, which he uses randomly, then it can be said that the probability of using Scorpio is $1/2$ since out of two cars he can use any one car at a time. Similarly the probability of using Safari is also $\frac{1}{2}$. Thus we can say that the probability of using any one car at

a time is $1/2$ *i.e.*, 50%. So there are chances that in 50% cases he can use Scorpio and in other 50% cases he can use Safari.

Hence from the above illustrations we can conclude that the probability of an event

$$= \frac{(\text{No. of ways in which favourable (or desired event occurs)})}{(\text{Total number of possible outcomes})}$$

Chapter Checklist

- Important Definitions
- Probability
- Important Values
- Important Addition Theorem
- Conditional Probability
- Multiplication Theorem
- Independent Events
- Law of Total Probability
- Baye's Rule
- CAT Test

20.1 Important Definitions

Experiment

An operation which can produce some well-defined outcomes, is known as an experiment.

The various experiments, when repeated under identical conditions, results. (*i.e.*, outcomes) in each case are same *e.g.*, standard scientific experiments.

But there are some other experiment, when repeated under identical conditions, results in each case are different *e.g.*, rolling of a fair die, tossing of a coin etc.

Random Experiment

If in each trial of an experiment conducted under identical conditions, the outcome is not unique, but may be any of the possible outcomes then such an experiment is known as a random experiment. *e.g.*, rolling of an unbiased die, tossing of a fair coin, drawing of a card from a well shuffled pack of cards.

Sample Space

The set of all possible outcomes in a random experiment is called a sample space and it is generally denoted by S .

If $E_1, E_2, E_3, \dots, E_n$ are the possible outcomes of a random experiment, then $S = \{E_1, E_2, \dots, E_n\}$. Also, each of the element of sample space ' S ' is called a **sample point**.

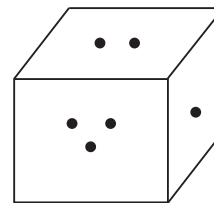
Exp. 1) In tossing of a fair coin, there are two possible outcomes, viz., head (H) and tail (T). So, the sample space in this random experiment is $S = \{H, T\}$

Exp. 2) When two fair coins are tossed together, the possible outcomes of the experiment are HH, HT, TH and TT . So the sample space is given by $S = \{HH, HT, TH, TT\}$

Exp. 3) When unbiased die is thrown, it gives 6 possible outcomes viz., 1, 2, 3, 4, 5 and 6. So, the sample space $S = \{1, 2, 3, 4, 5, 6\}$

NOTE A die is a cubical solid having 6 similar faces. In each of the six faces there are unique number of holes viz., 1 hole, 2 holes, 3 holes, 4 holes, 5 holes and 6 holes.

Remember that the sum of number of holes in any two opposite faces is always 7 viz., $1+6=2+5=3+4=7$.



- Die means one (single) die
- Dice means more than one die.
- In throwing a die, the outcome is the number of holes on the uppermost face

Exp. 4) When two unbiased dice are rolled (or tossed) simultaneously then there are total $6 \times 6 = 36$ possible outcomes. So, the sample space

$$S = \left\{ \begin{array}{l} (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) \\ (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) \\ (3, 1), \dots, \dots, \dots, (3, 6) \\ (4, 1), \dots, \dots, \dots, (4, 6) \\ (5, 1), \dots, \dots, \dots, (5, 6) \\ (6, 1), \dots, \dots, \dots, (6, 6) \end{array} \right\}$$

Exp. 5) When a die and a coin are tossed simultaneously, then there are total 12 possible outcomes. So the sample space

$$S = \left\{ \begin{array}{l} (1, H), (2, H), (3, H), (4, H), (5, H), (6, H) \\ (1, T), (2, T), (3, T), (4, T), (5, T), (6, T) \end{array} \right\}$$

Exp. 6) A coin is tossed twice. If the second throw results in a tail, then a die is thrown. So in this random experiment sample space.

$$S = \left\{ \begin{array}{l} HH, TH, HT1, HT2, HT3, HT4, HT5, HT6, \\ TT1, TT2, TT3, TT4, TT5, TT6, \end{array} \right\}$$

Exp. 7) From a bag containing 2 black and 3 white balls we draw two balls.

Let B_1, B_2 be the black balls and W_1, W_2 and W_3 be the white balls then the sample space

$$S = \left\{ \begin{array}{l} B_1W_1, B_1W_2, B_1W_3 \\ B_2W_1, B_2W_2, B_2W_3 \\ B_1B_2, W_1W_2, W_2W_3 \\ W_1W_3 \end{array} \right\}$$

Event

Any subset of a sample space is called an event.

Exp. 1) In a single throw of a die, the event of getting an even number is given by $E = \{2, 4, 6\}$

Clearly, here the sample space $S = \{1, 2, 3, 4, 5, 6\}$

Hence $E \subset S$ i.e., E is the subset of S .

Exp. 2) Consider a random experiment of tossing two dice at a time. The sample space $S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), \dots (6, 1), (6, 2), \dots (6, 6)\}$. Some different events associated with the above sample space are given below.

$$E_1 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$$

$$E_2 = \{(1, 8), (2, 7), (3, 6), (4, 5), (5, 4), (6, 3), (7, 2), (8, 1)\}$$

$$E_3 = \{(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6), (5, 2), (5, 4), (5, 6)\}$$

$$E_4 = \{(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5)\} \text{ etc.}$$

Clearly $E_1 \subset S, E_2 \subset S, E_3 \subset S, E_4 \subset S$

Where E_1 is the event of getting a doublet and E_2 is the event of getting 9 as the sum. E_3 is the event of getting odd number on the first die and an even number on the second die. E_4 is the event of getting only prime number on each of the two dice.

Exp. 3) Consider a random experiment of tossing three coins at a time. The sample space $S = \{HHH, HHT, HTH, THH, TTH, HTT, THT, TTT\}$

Following are the various events associated with the above sample space.

$$E_1 = \{HHH, HHT, HTH, THH\}$$

$$E_2 = \{HHT, HTH, THH, TTH, HTT, THT\}$$

$$E_3 = \{HHH, TTT\}$$

$$E_4 = \{THH, TTH, THT, TTT\}$$

Where E_1 is the event of getting at least two heads and E_2 is the event of getting at least one head and at most two heads.

E_3 is the event of getting all the three coins identical and E_4 is the event of getting tail on the first coin.

Elementary Events An event containing only a single sample point is called an elementary event, or simple event.

Exp. 1) In a simultaneous toss of two coins the sample space $S = \{HH, HT, TH, TT\}$

$$\text{then } E_1 = \{HH\}$$

$$E_2 = \{TT\} \text{ are the elementary events.}$$

Exp. 2) Consider a random experiment in which two dice rolled simultaneously, the sample space

$$S = \{(1, 1), (1, 2), \dots (6, 5), (6, 6)\}$$

$$\text{then } E_1 = \{(1, 1)\}$$

$$E_2 = \{(6, 6)\}$$

$$E_3 = \{(5, 5)\} \text{ are the elementary events.}$$

Here E_1 is the event of getting the sum of two.

E_2 is the event of getting the sum of twelve

E_3 is the event of getting the product of 25.

Compound Events Events which are not elementary are known as compound events or the events which contains more than one element are called compound or composite events.

Exp. 1) Consider a random experiment in which two dice are rolled simultaneously, the sample space

$$S = \{(1, 1), (1, 2), \dots (6, 5), (6, 6)\}.$$

$$\text{then } E_1 = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$$

$$E_2 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$$

$$E_3 = \{(1, 2), (2, 4), (3, 6)\}$$

are the compound events.

Here E_1 is the event of getting the sum of 6

E_2 is the event of getting identical results

E_3 is the event of getting the twice number by the second die than that by first die.

Occurrence of Events In a random experiment, let S be the sample space and E be the event such that $E \subseteq S$. Let w be an outcome of a trial such that $w \in A$, then we say that the event E has occurred. If $w \notin E$, we say that the event E has not occurred.

Exp. 1) Consider the random experiment of throwing an unbiased die. Let E be the event of getting an odd number, then $E = \{1, 3, 5\}$

Now, in a trial, let the outcome be 3, Since $3 \in E$, so in this trial, the event E has occurred. In another trial, let the outcome be 4.

Since $4 \notin E$ so in this trial, the event E has not occurred.

Exp. 2) Consider the random experiment of throwing a die. Let the outcome of the trial is 6. Then we can say that the following events have occurred.

- (i) Getting a number greater than or equal to 3, represented by the set $\{3, 4, 5, 6\}$
- (ii) Getting an even number represented by the set $\{2, 4, 6\}$

We can also say that the following events have not occurred

- (i) Getting an odd number represented by the set $\{1, 3, 5\}$.
- (ii) Getting a prime number represented by the set $\{2, 3, 5\}$

Impossible Events Let S be a sample space associated with a random experiment. Now, since $\phi \subseteq S$, so ϕ is an event, called an impossible event.

Sure (or certain) Event Let S be a sample space associated with a random experiment. Now, since $S \subseteq S$, so S is an event, called a sure or certain event.

Example Consider the random experiment of throwing an unbiased die. The sample space $S = \{1, 2, 3, 4, 5, 6\}$.

Let E_1 be the event of getting a number less than 1. and E_2 be the event of getting a number greater than 6. Here E_1 and E_2 both are impossible events.

Let E_3 be the event of getting a number less than or equal to 7. and E_4 be the event of getting a number multiple of 2 but less than 7. Here E_3 and E_4 are the certain events.

Equally Likely Events Events are said to be equally likely, if none of them is expected to occur in preference to the other.

Example If an unbiased die is rolled, then each outcome is equally likely to happen i.e., all elementary events are equally likely to happen. If however, the die is so formed that a particular face occurs most often, then the die is biased. So in this case, the outcomes are not equally likely to happen.

Favourable Events Let S be the sample space associated with a random experiment and let $E \subset S$. Then the elementary events belonging to E are known as the favourable events to A .

Exp. 1) Consider a random experiment of throwing a die. Let E be the event of getting an even number, then $E = \{2, 4, 6\}$. So there are three favourable events of event E viz. $\{2\}$, $\{4\}$ and $\{6\}$.

Exp. 2) Consider a random experiment of throwing a pair of dice. Let E be the event of getting the sum as 8, then $E = \{(1, 7), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (7, 1)\}$

So, there are 7 favourable events to event E .

Complementary Events In a random experiment, let S be the sample space and let E be an event. Then $E \subseteq S$. Clearly, $E^c \subseteq S$. So E^c is also an event, called the complementary of E . Sometimes E^c is denoted by $\bar{E}$ or E' . Where $\bar{E}$ is called as “not- E ”. Thus it is clear that $\bar{E}$ occurs only when E does not occur. Also in a trial one and only of E and $\bar{E}$ must occur.

Algebra of Events

In a random experiment, let the sample space be S . Let $E \subseteq S$, $F \subseteq S$ and $G \subseteq S$, then we can define the following relations in the set theory mode.

- (i) $\bar{E}$ is an event that occurs only when E does not occur.

- (ii) $(E \cup F)$ is an event that occurs only when E occurs or F occurs or both occurs.

- (iii) $(E \cap F)$ is an event that occurs only when each one of E and F occurs.

- (iv) $(E \cap \bar{F})$ is an event that occurs only when E occurs but not F .

- (v) $(\bar{E} \cap \bar{F}) = \overline{(E \cup F)}$ is an event that occurs when neither E nor F occurs.

- (vi) $(E \cup F \cup G)$ is an event that occurs when at least one of E , F or G occurs.

- (vii) $(E \cap F \cap G)$ is an event that occurs when all three E , F and G occurs.

- (viii) $(E \cap \bar{F}) \cup (\bar{E} \cap F)$ is an event that occurs when exactly one of E and F occurs.

- (ix) $(E \cap F \cap \bar{G}) \cup (E \cap \bar{F} \cap G) \cup (\bar{E} \cap F \cap G)$ is an event that occurs when exactly two of E , F and G occurs.

Mutually Exclusive Events

Let S be the sample space associated with a random experiment and let E_1 and E_2 be the two events. Then E_1 and E_2 are mutually exclusive events if $E_1 \cap E_2 = \phi$

Exp. 1) Consider a random experiment of throwing a die. Let E_1 , E_2 and E_3 be three events such that $E_1 = \{1, 3, 5\}$, the event of getting an odd number $E_2 = \{2, 4, 6\}$, the event of getting an even number $E_3 = \{2, 3, 5\}$, the event of getting a prime number.

Clearly $E_1 \cap E_2 = \phi$

and $E_1 \cap E_3 \neq \phi, E_2 \cap E_3 \neq \phi$

Hence E_1 and E_2 are mutually exclusive events but E_1 and E_3 , E_2 and E_3 are not mutually exclusive.

Mutually Exclusive & Exhaustive System of Events

Let S be the sample space associated with a random experiment.

Let $E_1, E_2, \dots, E_n$ be the subsets of S such that

- (i) $E_i \cap E_j = \phi$ for $i \neq j$ and

- (ii) $E_1 \cup E_2 \cup E_3 \cup \dots \cup E_n = S$

then the set of events $E_1, E_2, E_3, \dots, E_n$ is said to form a mutually exclusive and exhaustive system of events.

Exp. 1) In a sample space all the elementary events form a mutually exclusive and exhaustive system of events.

Exp. 2) Let $S = \{1, 2, 3, 4, 5, 6\}$ be the sample space when an unbiased die is rolled.

Let $E_1 = \{1, 3, 5\}$ and $E_2 = \{2, 4, 6\}$, then E_1 and E_2 form the mutually exclusive and exhaustive system of events.

Also, let $E_3 = \{1, 2\}$, $E_4 = \{4, 5\}$, then E_3 and E_4 do not form the mutually exclusive and exhaustive system of events since E_3 and E_4 do not include all the elementary events of S .

Again if $E_5 = \{3, 6\}$ and $E_6 = \{2, 4, 6\}$, then E_5 and E_6 do not form mutually exclusive and exhaustive system of events since $E_5 \cap E_6 \neq \emptyset$ and E_5, E_6 do not include all the elementary events of the sample space S . A pack of cards consists of 52 cards. There are four suits each containing 13 cards called as spades, clubs, hearts and diamonds.

All the spades and clubs are black faced cards while hearts and diamonds are red faced cards. The aces, kings, queens and jacks are known as face cards.

In each suit there is one ace, one king, one queen and one jack and rest 9 cards are numbered cards.

20.2 Probability

In a random experiment, let S be the sample space and let $E \subseteq S$, where E is an event.

The probability of occurrence of the event E is defined as

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} \\ = \frac{\text{number of elements in } E}{\text{number of elements in } S} = \frac{n(E)}{n(S)}$$

$$= \frac{\text{number of elementary events in } E}{\text{number of elementary events in } S}$$

From the above definitions it is clear that

$$(i) 0 \leq P(E) \leq 1$$

$$(ii) P(\emptyset) = 0$$

$$(iii) P(S) = 1$$

$$\text{Also, } P(\bar{E}) = \frac{\text{number of elementary events in } \bar{E}}{\text{number of elementary events in } S} \\ = \frac{n(S) - n(E)}{n(S)} \\ = 1 - \frac{n(E)}{n(S)} = 1 - P(E)$$

$$\Rightarrow P(\bar{E}) = 1 - P(E)$$

$$\therefore P(E) + P(\bar{E}) = 1$$

Odds in Favour of An Event and Odds Against An Event

In m be the number of ways in which an event occurs and n be the number of ways in which it does not occur, then

$$(i) \text{ odds in favour of the event} = \frac{m}{n} \text{ (or } m:n)$$

$$(ii) \text{ odds against the event} = \frac{n}{m} \text{ (or } n:m)$$

Introductory Exercise 20.1

- Find the probability of getting a head in a throw of a coin.
(a) $\frac{1}{2}$ (b) 1
(c) 2 (d) none of these

Directions (for Q. Nos. 2 to 5) *Two fair coins are tossed simultaneously. Find the probability of*

- Getting only one head.
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$
- Getting atleast one head.
(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{8}$
- Getting two heads.
(a) $\frac{2}{7}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{4}{5}$
- Getting atleast two heads :
(a) $\frac{3}{4}$ (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) 1

Directions (for Q. Nos. 6 to 12) *Three fair coins are tossed simultaneously. Find the probability of*

- Getting one head.
(a) 0 (b) $\frac{3}{4}$ (c) $\frac{5}{8}$ (d) $\frac{3}{8}$
- Getting one tail.
(a) 1 (b) $\frac{1}{4}$ (c) $\frac{5}{8}$ (d) $\frac{3}{8}$
- Getting atleast one head.
(a) $\frac{7}{8}$ (b) $\frac{1}{8}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$
- Getting two heads.
(a) $\frac{3}{5}$ (b) $\frac{3}{8}$ (c) $\frac{5}{8}$ (d) $\frac{2}{5}$
- Getting atleast two heads.
(a) $\frac{3}{8}$ (b) $\frac{7}{8}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$
- Getting atleast one head and one tail.
(a) $\frac{2}{8}$ (b) $\frac{1}{2}$ (c) $\frac{3}{10}$ (d) $\frac{3}{4}$

12. Getting more heads than the number of tails.
 (a) 2 (b) $\frac{7}{8}$ (c) $\frac{5}{8}$ (d) $\frac{1}{2}$

Directions (for Q. Nos. 13 to 16) *An unbiased die is rolled. Find the probability of*

13. Getting a number less than 7 but greater than zero.
 (a) 0 (b) $\frac{3}{4}$ (c) 1 (d) $\frac{7}{8}$
14. Getting a multiple of 3.
 (a) $\frac{1}{6}$ (b) $\frac{1}{3}$
 (c) $\frac{5}{6}$ (d) none of these
15. Getting a prime number.
 (a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) $\frac{5}{7}$ (d) $\frac{5}{8}$
16. Getting an even number.
 (a) $\frac{1}{2}$ (b) $\frac{4}{5}$ (c) $\frac{2}{8}$ (d) $\frac{3}{4}$

Directions (for Q. Nos. 17 and 18): *A coin is tossed successively three times. Find the probability of*

17. Getting exactly one head or two heads.
 (a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{8}$
18. Getting no heads.
 (a) 0 (b) 1 (c) $\frac{1}{8}$ (d) $\frac{7}{8}$

Directions (for Q. Nos. 19 to 27) *Two dice are rolled simultaneously. Find the probability of*

19. Getting a total of 9.
 (a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{8}{9}$ (d) $\frac{9}{10}$
20. Getting a sum greater than 9.
 (a) $\frac{10}{11}$ (b) $\frac{5}{6}$ (c) $\frac{1}{6}$ (d) $\frac{8}{9}$
21. Getting a total of 9 or 11.
 (a) $\frac{2}{99}$ (b) $\frac{20}{99}$ (c) $\frac{1}{6}$ (d) $\frac{1}{10}$
22. Getting a doublet.
 (a) $\frac{1}{12}$ (b) 0 (c) $\frac{5}{8}$ (d) $\frac{1}{6}$
23. Getting a doublet of even numbers.
 (a) $\frac{5}{8}$ (b) $\frac{1}{12}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$
24. Getting a multiple of 2 on one die and a multiple of 3 on the other.
 (a) $\frac{15}{36}$ (b) $\frac{25}{36}$ (c) $\frac{11}{36}$ (d) $\frac{5}{6}$
25. Getting the sum of numbers on the two faces divisible by 3 or 4.
 (a) $\frac{4}{9}$ (b) $\frac{1}{7}$ (c) $\frac{5}{9}$ (d) $\frac{7}{12}$

26. Getting the sum as a prime number.
 (a) $\frac{3}{5}$ (b) $\frac{5}{12}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

27. Getting atleast one '5'.
 (a) $\frac{3}{5}$ (b) $\frac{1}{5}$ (c) $\frac{5}{36}$ (d) $\frac{11}{36}$

Directions (for Q. Nos. 28 to 35) *One card is drawn from a pack of 52 cards, each of the 52 cards being equally likely to be drawn. Find the probability that*

28. The card drawn is black.
 (a) $\frac{1}{2}$ (b) $\frac{1}{4}$
 (c) $\frac{8}{13}$ (d) can't be determine
29. The card drawn is a queen.
 (a) $\frac{1}{12}$ (b) $\frac{1}{13}$
 (c) $\frac{1}{4}$ (d) $\frac{3}{4}$
30. The card drawn is black and a queen.
 (a) $\frac{1}{13}$ (b) $\frac{1}{52}$ (c) $\frac{1}{26}$ (d) $\frac{5}{6}$
31. The card drawn is either black or a queen.
 (a) $\frac{15}{26}$ (b) $\frac{13}{17}$ (c) $\frac{7}{13}$ (d) $\frac{15}{26}$
32. The card drawn is either king or a queen.
 (a) $\frac{5}{26}$ (b) $\frac{1}{13}$ (c) $\frac{2}{13}$ (d) $\frac{12}{13}$
33. The card drawn is either a heart, a queen or a king.
 (a) $\frac{17}{52}$ (b) $\frac{21}{52}$ (c) $\frac{19}{52}$ (d) $\frac{9}{26}$
34. The card drawn is neither a spade nor a king.
 (a) 0 (b) $\frac{9}{13}$ (c) $\frac{1}{2}$ (d) $\frac{4}{13}$
35. The card drawn is neither an ace nor a king.
 (a) $\frac{11}{13}$ (b) $\frac{1}{2}$ (c) $\frac{2}{13}$ (d) $\frac{11}{26}$
36. From a well shuffled pack of 52 cards, three cards are drawn at random. Find the probability of drawing an ace, a king and a jack.
 (a) $\frac{16}{5525}$ (b) $\frac{16}{625}$
 (c) $\frac{16}{3125}$ (d) none of these
37. Four cards are drawn at random from a pack of 52 cards. Find the probability of getting all the four cards of same number.
 (a) $\frac{17}{1665}$ (b) $\frac{1}{20825}$
 (c) $\frac{7}{25850}$ (d) none of these

38. From a well shuffled pack of 52 playing cards, four cards are accidentally dropped. Find the probability that one card is missing from each suit.

(a) $\frac{17}{20825}$ (b) $\frac{2197}{20825}$
 (c) $\frac{197}{1665}$ (d) none of these

39. Four cards are drawn at random from a pack of 52 cards. Find the probability of getting all the four cards of different numbers.

(a) $\frac{141}{4165}$ (b) $\frac{117}{833}$
 (c) $\frac{264}{4165}$ (d) none of these

Directions (for Q. Nos. 40 to 43) *Four dice are thrown simultaneously. Find the probability that*

40. All of them show the same face.

(a) $\frac{1}{216}$ (b) $\frac{15}{16}$ (c) $\frac{15}{36}$ (d) $\frac{1}{2}$

41. All of them show the different face.

(a) $\frac{3}{28}$ (b) $\frac{5}{18}$ (c) $\frac{15}{36}$ (d) $\frac{11}{36}$

42. Two of them show the same face and remaining two show the different faces.

(a) $\frac{4}{9}$ (b) $\frac{5}{9}$ (c) $\frac{11}{18}$ (d) $\frac{7}{9}$

43. Atleast two of them show the same face.

(a) $\frac{37}{72}$ (b) $\frac{11}{36}$ (c) $\frac{47}{72}$ (d) $\frac{25}{36}$

44. What is the probability that a number selected from the numbers 1, 2, 3, ..., 20, is a prime number when each of the given numbers is equally likely to be selected?

(a) $\frac{7}{10}$ (b) $\frac{2}{15}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$

45. Tickets are numbered from 1 to 18 are mixed up together and then 9 ticket is drawn at random. Find the probability that the ticket has a number, which is a multiple of 2 or 3.

(a) $\frac{1}{3}$ (b) $\frac{3}{5}$ (c) $\frac{2}{3}$ (d) $\frac{5}{6}$

46. In a lottery of 100 tickets numbered 1 to 100, two tickets are drawn simultaneously. Find the probability that both the tickets drawn have prime numbers.

(a) $\frac{2}{33}$ (b) $\frac{7}{50}$ (c) $\frac{7}{20}$ (d) $\frac{5}{66}$

47. In the previous question (number 46), find the probability that none of the tickets drawn has a prime number.

(a) $\frac{29}{66}$ (b) $\frac{17}{33}$ (c) $\frac{37}{66}$ (d) $\frac{17}{50}$

48. Find the probability that a leap year selected at random will contain 53 Sundays.

(a) $\frac{5}{7}$ (b) $\frac{3}{4}$ (c) $\frac{4}{7}$ (d) $\frac{2}{7}$

Directions (for Q. Nos. 49 to 53) *A bag contains 8 red and 4 green balls. Find the probability that*

49. The ball drawn is red when one ball is selected at random.

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{5}{6}$

50. All the 4 balls drawn are red when four balls are drawn at random.

(a) $\frac{17}{32}$ (b) $\frac{14}{99}$
 (c) $\frac{7}{12}$ (d) none of these

51. All the 4 balls drawn are green when four balls are drawn at random.

(a) $\frac{1}{495}$ (b) $\frac{7}{99}$ (c) $\frac{5}{12}$ (d) $\frac{2}{3}$

52. Two balls are red and one ball is green when three balls are drawn at random.

(a) $\frac{56}{99}$ (b) $\frac{112}{495}$
 (c) $\frac{78}{495}$ (d) none of these

53. Three balls are drawn and none of them is red.

(a) $\frac{68}{99}$ (b) $\frac{7}{99}$
 (c) $\frac{4}{495}$ (d) none of these

54. The odds in favour of an event are 2 : 7. Find the probability of occurrence of this event.

(a) $\frac{2}{9}$ (b) $\frac{5}{12}$ (c) $\frac{7}{12}$ (d) $\frac{2}{5}$

55. The odds against of an event are 5 : 7, find the probability of occurrence of this event.

(a) $\frac{3}{8}$ (b) $\frac{7}{12}$ (c) $\frac{2}{7}$ (d) $\frac{5}{12}$

56. If there are two children in a family, find the probability that there is atleast one girl in the family.

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
 (c) $\frac{3}{4}$ (d) none of these

57. From a group of 3 men and 2 women, two persons are selected at random. Find the probability that atleast one woman is selected.

(a) $\frac{1}{5}$ (b) $\frac{7}{10}$ (c) $\frac{2}{5}$ (d) $\frac{5}{6}$

58. A box contains 5 defective and 15 non-defective bulbs. Two bulbs are chosen at random. Find the probability that both the bulbs are non-defective.

- (a) $\frac{5}{19}$ (b) $\frac{3}{20}$
(c) $\frac{21}{38}$ (d) none of these

59. In the previous question (number 58), find the probability that atleast 3 bulbs are defective when 4 bulbs are selected at random.

- (a) $\frac{31}{969}$ (b) $\frac{7}{20}$
(c) $\frac{1}{20}$ (d) none of these

20.3 Important Value

Important Addition Theorems

$$1. P(E) \geq 0 \quad 2. P(\phi) = 0 \quad 3. P(S) = 1$$

Addition Theorem 1. If A and B are two events associated with a random experiment.

Then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Corollary If A and B are mutually exclusive events. then $P(A \cap B) = 0$, therefore $P(A \cup B) = P(A) + P(B)$

Addition Theorem 2. If A, B and C three events associated with a random experiment,

then $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Corollary If A, B and C are the three mutually exclusive events, then $P(A \cap B) = P(B \cap C) = P(A \cap C) = P(A \cap B \cap C) = 0$

$$= P(A \cap B \cap C) = 0$$

$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C)$

Theorem 3. If A and B be two events such that $A \subseteq B$, then $P(A) \leq P(B)$

Theorem 4. If E is an event associated with a random experiment, then $0 \leq P(E) \leq 1$

Theorem 5. For any two events A and B

$$P(A - B) = P(A) - P(A \cap B)$$

$$P(B - A) = P(B) - P(A \cap B)$$

$$P(\overline{A} \cap B) = P(B) - P(A \cap B)$$

$$P(A \cap \overline{B}) = P(A) - P(A \cap B)$$

Some Important Results

(A) If A, B and C are three events, then

(i) P [Exactly one of A, B, C occurs]

$$= P(A) + P(B) + P(C) - 2[(A \cap B) + (B \cap C) + (A \cap C)] + 3P(A \cap B \cap C)$$

(ii) P [Exactly two of A, B, C occur]

$$= P(A \cap B) + P(B \cap C) + P(A \cap C) - 3P(A \cap B \cap C)$$

(iii) P (Atleast two of A, B, C occur)

$$= P(A \cap B) + P(B \cap C) + P(A \cap C) - 2P(A \cap B \cap C)$$

(B) If A and B are two events, then

P (Exactly one of A, B occurs)

$$= P(A) + P(B) - 2P(A \cap B)$$

$$= P(A \cup B) - P(A \cap B)$$

Introductory Exercise 20.2

1. The probability of occurrence of two events A and B are $1/4$ and $1/2$ respectively. The probability of their simultaneous occurrence is $\frac{7}{50}$. Find the probability

that either A or B must occur.

- (a) $\frac{61}{100}$ (b) $\frac{29}{100}$
(c) $\frac{39}{100}$ (d) $\frac{56}{99}$

2. In the previous question, find the probability that neither A nor B occurs.

- (a) $\frac{25}{99}$ (b) $\frac{39}{100}$ (c) $\frac{61}{100}$ (d) $\frac{17}{100}$

3. If A and B be two events in a sample space such that $P(A) = \frac{3}{10}$ and $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{5}$.

Find $P(A \cup B)$.

- (a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

4. If A and B be two events in a sample space such that $P(A) = \frac{2}{5}$, $P(B) = \frac{1}{2}$ and $P(A \cup B) = \frac{3}{5}$, find $P(A \cap B)$.

- (a) $\frac{3}{10}$ (b) $\frac{7}{10}$
(c) $\frac{4}{7}$ (d) $\frac{4}{15}$

Directions (for Q. Nos. 5 to 10) If A and B be two mutually exclusive events in a sample space such that, $P(A) = \frac{2}{5}$ and

$P(B) = \frac{1}{2}$, then

5. Find $P(\bar{A})$:
 (a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{6}{7}$
6. Find $P(\bar{B})$:
 (a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{4}{5}$
7. Find $P(A \cup B)$:
 (a) $\frac{7}{16}$ (b) $\frac{9}{16}$ (c) $\frac{9}{10}$ (d) $\frac{1}{2}$
8. Find $P(\bar{A} \cap \bar{B})$:
 (a) $\frac{4}{5}$ (b) $\frac{1}{10}$ (c) $\frac{8}{9}$ (d) $\frac{13}{20}$
9. Find $P(\bar{A} \cap B)$:
 (a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) $\frac{4}{7}$ (d) $\frac{7}{15}$
10. Find $P(A \cap \bar{B})$:
 (a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{4}{15}$ (d) $\frac{3}{10}$
11. If $P(\bar{A}) = 0.65$, $P(A \cup B) = 0.65$, where A and B are two mutually exclusive events, then find $P(B)$.
 (a) 0.60 (b) 0.30
 (c) 0.70 (d) none of these
12. If A, B and C are three mutually exclusive and exhaustive events. Find $P(A)$, if $P(B) = \frac{3}{2}P(A)$ and $P(C) = \frac{1}{2}P(B)$.
 (a) $\frac{8}{13}$ (b) $\frac{5}{13}$ (c) $\frac{4}{13}$ (d) $\frac{9}{13}$
13. Two dice are tossed once. Find the probability of getting an even number on first die, or a total of 8.
 (a) $\frac{4}{9}$ (b) $\frac{2}{3}$ (c) $\frac{5}{9}$ (d) $\frac{1}{3}$
14. A die is thrown twice, what is the probability that atleast one of the two throws comes up with the number 5?
 (a) $\frac{11}{36}$ (b) $\frac{5}{6}$ (c) $\frac{15}{36}$ (d) none
15. In a single throw of two dice, find the probability that neither a doublet nor a total of 8 will appear.
 (a) $\frac{7}{15}$ (b) $\frac{5}{18}$ (c) $\frac{13}{18}$ (d) $\frac{3}{16}$
16. A die is thrown twice, what is the probability that atleast one of the two numbers is 6?
 (a) $\frac{11}{12}$ (b) $\frac{11}{36}$ (c) $\frac{1}{6}$ (d) $\frac{7}{36}$
17. A card is drawn at random from a well-shuffled deck of 52 cards. Find the probability of its being a heart or a king.
 (a) $\frac{4}{13}$ (b) $\frac{9}{13}$ (c) $\frac{8}{13}$ (d) $\frac{11}{26}$
18. Two cards are drawn at random from a well-shuffled pack of 52 cards. What is the probability that either both are red or both are queens?
 (a) $\frac{17}{112}$ (b) $\frac{55}{221}$ (c) $\frac{55}{121}$ (d) $\frac{33}{221}$
19. A card is drawn from a deck of 52 cards. Find the probability of getting a red card or a heart or a king.
 (a) $\frac{6}{13}$ (b) $\frac{7}{13}$ (c) $\frac{11}{26}$ (d) $\frac{15}{26}$
20. Four cards are drawn at a time from a pack of 52 playing cards. Find the probability of getting all the four cards of the same suit.
 (a) $\frac{5}{13}$ (b) $\frac{12}{65}$ (c) $\frac{44}{4165}$ (d) $\frac{44}{169}$
21. From a well-shuffled pack of 52 cards, a card is drawn at random. Find the probability that the drawn card is a king or a queen.
 (a) $\frac{2}{13}$ (b) $\frac{8}{13}$ (c) $\frac{11}{13}$ (d) none
22. Two cards are drawn at random from a pack of 52 cards. What is the probability that either both are black or both are jacks?
 (a) $\frac{65}{121}$ (b) $\frac{55}{221}$ (c) $\frac{17}{221}$ (d) none
23. A natural number is chosen at random from amongst the first 300. What is the probability that the number, so chosen is divisible by 3 or 5?
 (a) $\frac{48}{515}$ (b) $\frac{4}{150}$
 (c) $\frac{1}{2}$ (d) none of these
24. A natural number is chosen at random from the first 100 natural numbers. What is the probability that the number chosen is a multiple of 2 or 3 or 5?
 (a) $\frac{30}{100}$ (b) $\frac{1}{33}$
 (c) $\frac{74}{100}$ (d) $\frac{7}{10}$
25. A box contains 5 red balls, 8 green balls and 10 pink balls. A ball is drawn at random from the box. What is the probability that the ball drawn is either red or green?
 (a) $\frac{13}{23}$ (b) $\frac{10}{23}$ (c) $\frac{11}{23}$ (d) $\frac{13}{529}$

26. A basket contains 10 apples and 20 oranges out of which 3 apples and 5 oranges are defective. If we choose two fruits at random, what is the probability that either both are oranges or both are non defective?
- (a) $\frac{136}{345}$ (b) $\frac{17}{87}$ (c) $\frac{316}{435}$ (d) $\frac{158}{435}$
27. In a class 40% of the students offered Physics 20% offered Chemistry and 5% offered both. If a student is selected at random, find the probability that he has offered Physics or Chemistry only.
- (a) 45% (b) 55%
(c) 36% (d) none of these

20.4 Conditional Probability

Let A and B be two events associated with a random experiment. Then, the probability of occurrence of A under the condition that B has already occurred and $P(B) \neq 0$, is called the conditional probability and it is denoted by $P\left(\frac{A}{B}\right)$.

Thus, $P\left(\frac{A}{B}\right)$ = Probability of occurrence of A given that B has already occurred. Similarly, $P\left(\frac{B}{A}\right)$ = Probability of occurrence of B given that A has already occurred.

NOTE

- (i) Sometimes $P\left(\frac{A}{B}\right)$ is used to denote the probability of occurrence of A when B occurs.
- (ii) Similarly $P\left(\frac{B}{A}\right)$ is used to denote the probability of occurrence of B when A occurs. The above two cases happens due to the simultaneous occurrence of two events since the two events are the subsets of the same sample space.

Exp. 1) An urn contains 6 red and 9 blue balls. Two balls are drawn from the urn one after another without replacement. Find the probability of drawing a red ball when a blue ball has been drawn from the urn.

Solution Let A = drawing of a red ball in the second draw and B = drawing of a blue ball in the first draw

Now $P\left(\frac{A}{B}\right)$ = Probability of drawing a red ball in second draw when a blue ball has been drawn in the first draw. Now, since there are only 14 balls after drawing a blue ball in first draw and out of these 14 balls, 6 balls are red.

Therefore $P\left(\frac{A}{B}\right) = \frac{6}{14} = \frac{3}{7}$

Exp. 2) A pair of dice is thrown simultaneously, find the probability that the sum is obtained 9 when there is an odd number on the first die.

28. The probability that an MBA aspirant will join IIM is $\frac{2}{5}$ and that he will join XLRI is $\frac{1}{3}$. Find the probability that he will join IIM or XLRI.
- (a) $\frac{4}{15}$ (b) $\frac{7}{15}$ (c) $\frac{11}{15}$ (d) $\frac{8}{15}$
29. In a given race, the odds in favour of horses H_1, H_2, H_3 and H_4 are 1 : 2, 1 : 3, 1 : 4, 1 : 5 respectively. Find the probability that one of them wins the race.
- (a) $\frac{57}{60}$ (b) $\frac{1}{20}$ (c) $\frac{2}{7}$ (d) $\frac{7}{60}$

Solution Let A be the event of getting a sum of 9 and B be the event of getting an odd number on the first die.

$$\therefore A = \{(3, 6), (4, 5), (5, 4), (6, 3)\}$$

$$B = \{(1, 1), (1, 2), (1, 3) \dots (3, 1), (3, 2), (3, 3) \dots (5, 1), (5, 2) \dots (5, 6)\}$$

$$P(A) = \frac{4}{36} = \frac{1}{9} \quad \text{and} \quad P(B) = \frac{18}{36} = \frac{1}{2}$$

$$\therefore P\left(\frac{A}{B}\right) = \text{Probability of occurrence of } A \text{ when } B \text{ Occurs.}$$

$$\Rightarrow P\left(\frac{A}{B}\right) = \text{Probability of getting 9 as the sum when there is}$$

an odd number on first die.

$$\Rightarrow P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{2}{18} = \frac{1}{9} \quad [\text{Here}]$$

$$(A \cap B) = \{(3, 6), (6, 3)\}$$

20.5 Multiplication Theorem

Let A and B be two events associated with the same random experiment then

NOTE

$$P(A \cap B) = P(A) P\left(\frac{B}{A}\right), \text{ if } P(A) \neq 0 \quad \dots(i)$$

$$\text{or} \quad P(A \cap B) = P(B) P\left(\frac{A}{B}\right), \text{ if } P(B) \neq 0 \quad \dots(ii)$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} \text{ from (i) and } P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \text{ from (ii)}$$

In general, if $A_1, A_2, A_3 \dots A_n$ are events associated with the same random experiment, then $P(A_1 \cap A_2 \cap A_3 \cap \dots \cap A_n)$

$$= P(A_1) P\left(\frac{A_2}{A_1}\right) P\left(\frac{A_3}{A_1 \cap A_2}\right) \dots$$

$$P\left(\frac{A_n}{A_1 \cap A_2 \cap \dots \cap A_{n-1}}\right)$$

Exp. 1) Let A and B be the two events such that

$$P(A) = \frac{1}{2}, P(B) = \frac{1}{3} \text{ and } P(A \cap B) = \frac{1}{4}, \text{ find}$$

$$(i) P\left(\frac{A}{B}\right) \quad (ii) P\left(\frac{B}{A}\right) \quad (iii) P(A \cup B) \quad (iv) P\left(\frac{\bar{A}}{\bar{B}}\right)$$

Solution

$$\begin{aligned}
 \text{(i)} \quad P\left(\frac{A}{B}\right) &= \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/3} = \frac{3}{4} \\
 \text{(ii)} \quad P\left(\frac{B}{A}\right) &= \frac{P(A \cap B)}{P(A)} = \frac{1/4}{1/2} = \frac{1}{2} \\
 \text{(iii)} \quad P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
 &= \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{7}{12} \\
 \text{(iv)} \quad P\left(\frac{\bar{A}}{\bar{B}}\right) &= \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \frac{P(\overline{A \cup B})}{P(\bar{B})} \\
 &= \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{1 - \frac{7}{12}}{1 - \frac{1}{3}} = \frac{\frac{5}{12}}{\frac{2}{3}} = \frac{5}{8}
 \end{aligned}$$

Introductory Exercise 20.3

1. If A and B are two events such that $P(A) = 0.5$, $P(B) = 0.6$ and $P(A \cup B) = 0.8$. Find $P\left(\frac{A}{B}\right)$.

- (a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) none of these

2. If A and B are two events such that $P(A) = 0.4$, $P(B) = 0.8$ and $P\left(\frac{B}{A}\right) = 0.6$, find $P(A \cup B)$:

- (a) 0.24 (b) 0.96
(c) 0.04 (d) none of these

3. Three fair coins are tossed. Find the probability that they are all tails, if one of the coins shows a tail.

- (a) $\frac{2}{7}$ (b) $\frac{5}{14}$
(c) $\frac{1}{7}$ (d) none of these

4. A coin is tossed twice and the four possible outcomes are assumed to be equally likely. If E is the event, "Both head and tail have appeared and F be the event, at most one tail is observed; find $P\left(\frac{E}{F}\right)$ and $P\left(\frac{F}{E}\right)$.

- (a) $\frac{2}{3}, 1$ (b) $\frac{1}{3}, \frac{2}{3}$
(c) $1, \frac{2}{3}$ (d) none of these

5. A coin is tossed twice if the coin shows head it is tossed again but if it shows a tail then a die is tossed. If 8 possible outcomes are equally likely. Find the probability that the die shows a number greater than 4, if it is known that the first throw of the coin results in a tail.

- (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{2}{5}$ (d) $\frac{4}{15}$

Exp. 2) An urn contains 6 red balls and 9 green balls. Two balls are drawn in succession without replacement. What is the probability that first is red and second is green.

Solution Let A be the event of drawing a red ball in first draw and B be the event of getting a green ball in the second draw.

$$\therefore P(A) = \frac{{}^6C_1}{{}^{15}C_1} = \frac{6}{15} = \frac{2}{5}$$

$P\left(\frac{B}{A}\right)$ = Probability of getting a green ball in second draw

when a red ball has been selected in first draw = $\frac{{}^9C_1}{{}^{14}C_1} = \frac{9}{14}$

$$\therefore \text{Required probability} = P(A \cap B) = P(A) P\left(\frac{B}{A}\right) = \frac{2}{5} \times \frac{9}{14} = \frac{9}{35}$$

6. A die is rolled. If the outcome is an odd number, what is the probability that it is a prime number?

- (a) $\frac{3}{8}$ (b) $\frac{7}{9}$
(c) $\frac{2}{3}$ (d) none of these

7. A die is thrown twice and the sum of the numbers appearing is observed to be 9. What is the conditional probability that the number 4 has appeared at least once?

- (a) $\frac{1}{2}$ (b) $\frac{2}{3}$
(c) $\frac{3}{4}$ (d) none of these

8. Two dice are thrown. Find the probability that the sum is 8 or greater than 8, if 4 appears on the first die.

- (a) $\frac{3}{8}$ (b) $\frac{5}{8}$
(c) $\frac{1}{2}$ (d) none of these

9. A die is rolled. If the outcome is an odd number, what is the probability that it is a number greater than 1?

- (a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{3}{8}$ (d) $\frac{5}{6}$

10. In a class 45% students read English, 30% read French and 20% read both English and French. One student is selected at random. Find the probability that he reads English, if it is known that he reads French.

- (a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $\frac{5}{6}$ (d) none of these

11. In the previous question, find the probability that he reads French, if it is known that he reads English.
 (a) $\frac{4}{9}$ (b) $\frac{5}{9}$ (c) $\frac{2}{9}$ (d) $\frac{1}{9}$
12. A couple has two children. Find the probability that both are boys, if it is known that one of the children is a boy.
 (a) $\frac{1}{9}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{5}$
13. In the previous question find the probability that both are boys, if it is known that the older child is a boy.
 (a) $\frac{3}{8}$ (b) $\frac{1}{2}$ (c) $\frac{5}{8}$ (d) $\frac{3}{4}$

Directions (for Q. Nos. 14 to 17) *A bag contains 3 red and 4 black balls and another bag has 4 red and 2 black balls. One bag is selected at random and from the selected bag a ball is drawn. Let E be the event that the first bag is selected, F be the event that the second bag is selected, G be the event that ball drawn is red.*

14. Find $P(E)$.
 (a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{5}{8}$
15. Find $P(F)$.
 (a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$
16. Find $P\left(\frac{G}{E}\right)$.
 (a) $\frac{5}{6}$ (b) $\frac{5}{14}$ (c) $\frac{3}{7}$ (d) none
17. Find $P\left(\frac{G}{F}\right)$.
 (a) $\frac{2}{3}$ (b) $\frac{1}{9}$ (c) $\frac{5}{9}$ (d) $\frac{4}{5}$

20.6 Independent Events

Events are said to be independent, if the occurrence of one does not depend upon the occurrence of the other. Suppose an urn contains m red balls and n green balls. Two balls are drawn from the urn one after the other.

If the ball drawn in the first draw is not replaced back in the bag, then two events of drawing the ball are dependent because first draw of the ball determine the probability of drawing the second ball.

If the ball drawn in the first draw is replaced back in the bag, then two events are independent because first draw of a ball has no effect on the second draw.

Theorem 1. Two events A and B associated with the same sample space of a random experiment are independent if and only if $P(A \cap B) = P(A) \cdot P(B)$

18. Two balls are drawn from a bag containing 2 white, 3 red and 4 black balls one by one without replacement. What is the probability that atleast one ball is red?
 (a) $\frac{7}{12}$ (b) $\frac{5}{12}$
 (c) $\frac{3}{10}$ (d) none of these
19. A bag contains 6 red and 9 blue balls. Two successive drawing of four balls are made such that the balls are not replaced before the second draw. Find the probability that the first draw gives 4 red balls and second draw gives 4 blue balls.
 (a) $\frac{3}{715}$ (b) $\frac{7}{715}$
 (c) $\frac{15}{233}$ (d) none of these
20. An urn contains 4 white 6 black and 8 red balls. If 3 balls are drawn one by one without replacement, find the probability of getting all white balls.
 (a) $\frac{5}{204}$ (b) $\frac{1}{204}$
 (c) $13/204$ (d) none of these
21. Two numbers are selected at random from the integers 1 through 9. If the sum is even, find the probability that both numbers are odd.
 (a) $5/8$ (b) $3/8$
 (c) $3/10$ (d) none of these
22. A box contains 25 tickets, numbered 1, 2, 3, .. 25. A ticket is drawn and then another ticket is drawn without replacement. Find the probability that both tickets will show odd numbers.
 (a) $\frac{37}{50}$ (b) $\frac{13}{50}$
 (c) $13/25$ (d) none of these

Theorem 2. If $A_1, A_2, A_3 \dots A_n$ are independent events associated with a random experiment, then

$$P(A_1 \cap A_2 \cap A_3 \dots \cap A_n) = P(A_1) P(A_2) \dots P(A_n)$$

Theorem 3. If $A_1, A_2 \dots A_n$ are n independent events associated with a random experiment, then

$$P(A_1 \cup A_2 \cup \dots \cup A_n) = 1 - P(\bar{A}_1) P(\bar{A}_2) \dots P(\bar{A}_n)$$

Important Results

If A and B are independent events then the following events are also independent.

- (i) $A \cap \bar{B}$
- (ii) $\bar{A} \cap B$
- (iii) $\bar{A} \cap \bar{B}$

20.7 Law of Total Probability

Let $E_1, E_2, \dots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or ... or E_n , then

$$P(A) = P(E_1) P\left(\frac{A}{E_1}\right) + P(E_2) P\left(\frac{A}{E_2}\right) + \dots + P(E_n) P\left(\frac{A}{E_n}\right)$$

Baye's Rule

Let $E_1, E_2, \dots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or ... or E_n , then

$$P\left(\frac{E_i}{A}\right) = \frac{P(E_i) P\left(\frac{A}{E_i}\right)}{\sum_{i=1}^n P(E_i) P\left(\frac{A}{E_i}\right)}, i = 1, 2, \dots, n$$

Exp. 1) A coin is tossed twice and all 4 outcomes are equally likely. Let A be the event that first throw results in a head and B be the event that second throw results in a tail, then show that the events A and B are independent.

Solution $S = \{HH, HT, TH, TT\}$

$$A = \{HH, HT\}$$

$$B = \{HT, TT\}$$

$$A \cap B = \{HT\}$$

$$\therefore P(A) = \frac{2}{4} = \frac{1}{2}, P(B) = \frac{2}{4} = \frac{1}{2} \text{ and } P(A \cap B) = \frac{1}{4}$$

$$\text{Clearly } P(A \cap B) = \frac{1}{4} = P(A) \cdot P(B)$$

Hence A and B are independent events.

Exp. 2) A bag contains 10 red balls and 10 green balls. Two balls are drawn at random, one at a time, with replacement. Let A be the event that first ball is red, B be the event that second ball is green and C be the event that both balls are either red or green, then show that the events A, B and C are pair wise independent and A, B, C are mutually dependent.

Solution We have, $P(A) = \frac{1}{2}, P(B) = \frac{1}{2},$
 $P(C) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$

$P(A \cap B)$ = Probability that the first is red and second is green

$$\Rightarrow P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$P(B \cap C)$ = Probability that both the balls are green

$$\Rightarrow P(B) \cdot P(C) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

and $P(A \cap C)$ = Probability that both the balls are red

$$\Rightarrow P(A) P(C) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Hence A, B, C are pairwise independent

Now, $P(A \cap B \cap C)$ = Probability that the first ball is red and the second ball is green and the first and second are both red or green = 0

$$\text{and } P(A) P(B) P(C) = \frac{1}{8}$$

$$\therefore P(A \cap B \cap C) \neq P(A) P(B) P(C)$$

Thus, A, B, C are not mutually independent.

Exp. 3) (For explanation of law of total probability) There are two bags. The first bag contains 4 white and 5 black balls and the second bag contains 5 white and 4 black balls. Two balls are drawn at random from the first bag and are put into the second bag without noticing their colours. Then two balls are drawn from the second bag. Find the probability that the balls are white and black.

Solution A white and a black ball can be drawn from the second bag in the following mutually exclusive ways :

- By transferring 2 white balls from the first bag to the second bag and then drawing a white and a black ball from it.
- By transferring 2 black balls from the first bag to the second bag and then drawing a white and a black ball from it.
- By transferring 1 white and 1 black ball from first bag to the second bag and then drawing a white and a black ball from it.

Let A, B, C and D be the events as defined below :

A = Two white balls are drawn from the first bag

B = Two black balls are drawn from the first bag.

C = One white and one black ball is drawn from the first bag.

D = Two balls drawn from the second bag are white and black.

$$\text{We, have } P(A) = \frac{{}^4C_2}{{}^9C_2} = \frac{6}{36} = \frac{1}{6}$$

$$P(B) = \frac{{}^5C_2}{{}^9C_2} = \frac{10}{36} = \frac{5}{18}$$

$$P(C) = \frac{{}^4C_1 \times {}^5C_1}{{}^9C_2} = \frac{20}{36} = \frac{5}{9}$$

If A has already occurred, i.e. if two white balls have been transferred from the first bag to the second bag, then the second bag will contain 7 white and 4 black balls, therefore the probability of drawing a white and a black from the second bag is

$$P\left(\frac{D}{A}\right) = \frac{{}^7C_1 \times {}^4C_1}{{}^{11}C_2} = \frac{28}{55}$$

$$\text{Similarly, } P\left(\frac{D}{B}\right) = \frac{{}^5C_1 \times {}^6C_1}{{}^{11}C_2} = \frac{30}{55} = \frac{6}{11}$$

$$\text{and } P\left(\frac{D}{C}\right) = \frac{{}^6C_1 \times {}^5C_1}{{}^{11}C_2} = \frac{30}{55} = \frac{6}{11}$$

∴ By the law of total probability, we have

$$\begin{aligned} P(D) &= P(A) P\left(\frac{D}{A}\right) + P(B) P\left(\frac{D}{B}\right) + P(C) P\left(\frac{D}{C}\right) \\ &= \frac{1}{6} \times \frac{28}{55} + \frac{5}{18} \times \frac{6}{11} + \frac{5}{9} \times \frac{6}{11} \\ &= \frac{14}{165} + \frac{5}{33} + \frac{10}{33} = \frac{14 + 25 + 50}{165} = \frac{89}{165} \end{aligned}$$

Exp. 4) (For explanation of Baye's rule) Three boxes contain 6 red, 4 black, ; 5 red, 5 black and 4 red, 6 black balls respectively. One of the box is selected at random and a ball is drawn from it. If the ball drawn is red, find the probability that it is drawn from the first bag.

Solution Let A , B , C and D be the events defined as follows :

A = first box is chosen B = second box is chosen

C = third box is chosen D = ball drawn is red.

Since there are three boxes and one of the three boxes is chosen at random, therefore $P(A) = P(B) = P(C) = \frac{1}{3}$

If A has already occurred, then first box has been chosen which contains 6 red and 4 black balls. The probability of drawing a red ball from it is $\frac{6}{10}$. So, $P\left(\frac{D}{A}\right) = \frac{6}{10}$

Similarly, $P\left(\frac{D}{B}\right) = \frac{5}{10}$ and $P\left(\frac{D}{C}\right) = \frac{4}{10}$

We are required to find $P\left(\frac{A}{D}\right)$ i.e., given that the ball drawn

is red, what is the probability that it is drawn from the first box.

By Baye's rule.

$$\begin{aligned} P\left(\frac{A}{D}\right) &= \frac{P(A) \cdot P\left(\frac{D}{A}\right)}{P(A) \cdot P\left(\frac{D}{A}\right) + P(B) \cdot P\left(\frac{D}{B}\right) + P(C) \cdot P\left(\frac{D}{C}\right)} \\ &= \frac{\frac{1}{3} \times \frac{6}{10}}{\left(\frac{1}{3} \times \frac{6}{10}\right) + \left(\frac{1}{3} \times \frac{5}{10}\right) + \left(\frac{1}{3} \times \frac{4}{10}\right)} = \frac{2}{5} \end{aligned}$$

Introductory Exercise 20.4

Directions (for Q. Nos. 1 to 5) Let A and B be independent events such that $P(A) = 0.6$ and $P(B) = 0.4$

- Find $P(A \cap B)$.
(a) 0.24 (b) 0.76
(c) 0.56 (d) none of these
- Find $P(A \cup B)$.
(a) 0.24 (b) 0.76
(c) 0.36 (d) none of these
- Find $P(\bar{A} \cap B)$.
(a) 0.24 (b) 0.16
(c) 0.36 (d) none of these
- Find $P(A \cap \bar{B})$.
(a) 0.24 (b) 0.56
(c) 0.36 (d) 0.76
- Find $P(\bar{A} \cap \bar{B})$.
(a) 0.76 (b) 0.54
(c) 0.36 (d) 0.24
- If A and B are two independent events such that $P(\bar{A}) = 0.65$, $P(A \cup B) = 0.65$ and $P(B) = p$, find the value of p .
(a) $\frac{7}{13}$ (b) $\frac{6}{13}$
(c) $\frac{37}{65}$ (d) none of these

- An unbiased die is tossed twice. Find the probability of getting a 1, 2, 3 or 4 on the first toss and a 4, 5 or 6 on the second toss.
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{5}{6}$ (d) $\frac{1}{6}$
- Two persons A and B throw a die alternatively till one of them gets a three and wins the game. Find their respective probabilities of winning.
(a) $\frac{6}{11}, \frac{5}{11}$ (b) $\frac{5}{11}, \frac{8}{11}$
(c) $\frac{3}{11}, \frac{7}{11}$ (d) $\frac{8}{11}, \frac{3}{11}$
- Two persons A and B throw a coin alternatively till one of them gets head and wins the game. Find their respective probabilities of winning.
(a) $\frac{1}{3}, \frac{5}{6}$ (b) $\frac{3}{5}, \frac{4}{5}$
(c) $\frac{2}{3}, \frac{1}{3}$ (d) $\frac{1}{6}, \frac{5}{6}$
- Three persons A , B , C throw a die in succession till one gets a six and wins the game. Find their respective probabilities of winning.
(a) $\frac{36}{91}, \frac{30}{91}, \frac{25}{91}$ (b) $\frac{10}{71}, \frac{16}{91}, \frac{22}{81}$
(c) $\frac{13}{61}, \frac{15}{61}, \frac{17}{61}$ (d) none of these

11. A and B take turn in throwing two dice; the first to throw 9 being awarded. Find the ratio of probabilities of their winning if A has the first throw.
 (a) $\frac{7}{8}$ (b) $\frac{9}{8}$
 (c) $\frac{8}{7}$ (d) $\frac{9}{10}$
12. From a pack of 52 cards, two are drawn one by one without replacement. Find the probabilities that both of them are kings.
 (a) $\frac{11}{21}$ (b) $\frac{13}{121}$ (c) $\frac{1}{221}$ (d) $\frac{1}{121}$
13. Ashmit can solve 80% of the problem given in a book and Amisha can solve 70%. What is the probability that at least one of them will solve a problem, selected at random from the book?
 (a) 0.60 (b) 0.06
 (c) 0.94 (d) 0.56
14. The probability that A hits a target is $\frac{1}{3}$ and the probability that B hits it, is $\frac{2}{5}$. What is the probability that the target will be hit, if each one of A and B shoots the target?
 (a) $\frac{5}{6}$ (b) $\frac{3}{5}$ (c) $\frac{11}{15}$ (d) $\frac{1}{6}$
15. A problem is given to three students whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved?
 (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{7}{12}$
16. The probabilities of A, B, C solving a problem are $\frac{1}{3}$, $\frac{2}{7}$ and $\frac{3}{8}$ respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them will solve it.
 (a) $\frac{25}{52}$ (b) $\frac{25}{56}$
 (c) $\frac{13}{42}$ (d) none of these
17. A, B and C shoot to hit a target. If A hits the target 4 times in 5 trials, B hits it 3 times in 4 trials and C hits it 2 times in 3 trials. What is the probability that the target is hit by atleast 2 persons?
 (a) $\frac{5}{6}$ (b) $\frac{3}{4}$
 (c) $\frac{4}{5}$ (d) $\frac{1}{9}$
18. An airgun can take a maximum of 4 shots at a balloon at some distance. The probabilities of hitting the balloon at the first, second, third and fourth shot are 0.1, 0.2, 0.3 and 0.4 respectively. What is the probability that the balloon is hit?
 (a) 0.6976 (b) 0.6576
 (c) 0.786 (d) none of these

19. The probability that A can solve a problem is $\frac{2}{3}$ and the probability that B can solve the same problem is $\frac{3}{5}$. Find the probability that atleast one of A and B will be able to solve the problem.
 (a) $\frac{12}{15}$ (b) $\frac{13}{15}$
 (c) $\frac{19}{45}$ (d) none of these
20. In the previous question (number 19). Find the probability that none of the two will be able to solve the problem.
 (a) $\frac{13}{15}$ (b) $\frac{4}{15}$
 (c) $\frac{2}{15}$ (d) $\frac{23}{30}$

Directions (for Q. Nos. 21 to 24) *The probabilities that a student will receive an A, B, C or D grade are 0.4, 0.3, 0.2 and 0.1 respectively. Find the probability that a student will receive*

21. Not an A grade.
 (a) 0.4 (b) 0.6
 (c) 0.56 (d) none of these
22. At most a C grade.
 (a) 0.3 (b) 0.7
 (c) 0.36 (d) none of these
23. B or C grade.
 (a) 0.2 (b) 0.5 (c) 0.8 (d) 0.6
24. Atleast B grade.
 (a) 0.21 (b) 0.3
 (c) 0.7 (d) none of these

Directions (for Q. Nos. 25 to 28) *Ajay and his wife Kajol appear in an interview for two vacancies in the same post. The probability of Ajay's selection is $\frac{1}{7}$ and that of his wife Kajol's selection is $\frac{1}{5}$. What is the probability that*

25. Both of them will be selected?
 (a) $\frac{1}{12}$ (b) $\frac{1}{35}$
 (c) $\frac{13}{35}$ (d) $\frac{12}{35}$
26. Only one of them will be selected ?
 (a) $\frac{5}{7}$ (b) $\frac{1}{5}$ (c) $\frac{2}{7}$ (d) $\frac{2}{35}$
27. None of them will be selected?
 (a) $\frac{12}{35}$ (b) $\frac{6}{35}$
 (c) $\frac{24}{35}$ (d) none

28. At least one of them will be selected?

- (a) $\frac{11}{35}$ (b) $\frac{24}{35}$ (c) $\frac{2}{7}$ (d) $\frac{1}{35}$

29. A speaks truth in 60% and B in 80% of the cases. In what percentage of cases are they likely to contradict each other narrating the same incident?

- (a) 44% (b) 36% (c) 64% (d) 48%

Directions (for Q. Nos. 30 and 31) *The odds against a husband who is 50 years old, living till he is 70 are 7 : 5 and the odds against his wife who is now 40, living till she is 60 are 5 : 3. Find the probability that*

30. The couple will be alive 20 years hence.

- (a) $\frac{21}{32}$ (b) $\frac{5}{32}$
(c) $\frac{15}{32}$ (d) $\frac{12}{35}$

31. At least one of them will be alive 20 years hence.

- (a) $\frac{61}{96}$ (b) $\frac{31}{96}$
(c) $\frac{41}{70}$ (d) none of these

32. Three critics review a book. Odds in favour of the book are 5 : 2, 4 : 3 and 3 : 4 respectively for the critics. Find the probability that majority are in favour of the book.

- (a) $\frac{108}{343}$ (b) $\frac{209}{343}$
(c) $\frac{1}{7}$ (d) $\frac{1}{243}$

33. An article manufactured by a company consists of two parts X and Y. In the process of manufacture of the part X, 9 out of 100 parts may be defective. Similarly, 5 out of 100 are likely to be defective in the manufacture of the part Y. Calculate the probability that the assembled product will not be defective.

- (a) 0.6485 (b) 0.6565
(c) 0.8645 (d) none of these

34. A bag contains 4 red and 3 black balls. A second bag contains 2 red and 4 black balls. One bag is selected at random. From the selected bag, one ball is drawn. Find the probability that the ball drawn is red.

- (a) $\frac{23}{42}$ (b) $\frac{19}{42}$
(c) $\frac{7}{32}$ (d) $\frac{16}{39}$

35. In a toys making factory, machine A, B and C manufacture respectively 25%, 35% and 40% of the total toys. Of their output 5%, 4% and 2% respectively are defective toys. A toy is drawn at random from the product. What is the probability that the toy drawn is defective?

- (a) 0.225 (b) 0.345
(c) 0.235 (d) none of these

36. A box contains 20 bulbs. The probability that the box contains exactly 2 defective bulbs is 0.4 and the probability that the box contains exactly 3 defective bulbs is 0.6. Bulbs are drawn at random one by one without replacement and tested till the defective bulbs are found. What is the probability that the testing procedure ends at the twelfth testing?

- (a) 0 (b) 1
(c) can't be determined (d) none of these

37. In a toys making factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total toys of their output 5%, 4% and 2% respectively are defective toys. A toy is drawn at random from the product. If the toy drawn is found to be defective, what is the probability that it is manufactured by the machine B?

- (a) $\frac{17}{69}$ (b) $\frac{28}{69}$
(c) $\frac{35}{69}$ (d) none

38. An architecture company built 200 bridges 400 hospitals and 600 hotels. The probability of damage due to earthquake of a bridge, hospital and hotel is 0.01, 0.03 and 0.15 respectively. One of the construction gets damaged with earthquake. What is the probability that it is a bridge?

- (a) $\frac{1}{26}$ (b) $\frac{1}{52}$
(c) $\frac{7}{52}$ (d) none of these

39. There are 3 boxes each containing 3 red and 5 green balls. Also there are 2 boxes, each containing 4 red and 2 green balls. A green ball is selected at random. Find the probability that this green ball is from a box of the first group.

- (a) $\frac{54}{61}$ (b) $\frac{45}{61}$
(c) $\frac{8}{31}$ (d) none of these

40. A man speaks truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

- (a) $\frac{3}{8}$ (b) $\frac{5}{8}$
(c) $\frac{7}{8}$ (d) $\frac{1}{12}$

41. A card from a pack of 52 cards is lost. From the remaining cards of pack, two cards are drawn and are found to be diamonds. Find the probability of the missing card to be diamond.

- (a) $\frac{39}{50}$ (b) $\frac{11}{50}$
(c) $\frac{23}{25}$ (d) $\frac{3}{26}$

CAT-Test

Questions Helping you bell the CAT

LEVEL 01 > BASIC LEVEL EXERCISE

- 1 The digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are written in random order to form a nine digit number. Find the probability that this number is divisible by 4 :
 (a) $\frac{4}{9}$ (b) $\frac{2}{9}$
 (c) $\frac{17}{81}$ (d) none of these
- 2 A four digit number is formed with the digits 1, 3, 4, 5 without repetition. Find the chance that the number is divisible by 5 :
 (a) $\frac{3}{4}$ (b) $\frac{1}{4}$
 (c) $\frac{9}{16}$ (d) $\frac{1}{16}$
- 3 Five persons entered the lift cabin on the ground floor of an 8-floor house. Suppose each of them independently and with equal probability can leave the cabin at any floor beginning with the first. Find out the probability that all persons leaving at different floors :
 (a) $\frac{365}{2401}$ (b) $\frac{360}{2401}$
 (c) $\frac{35}{2410}$ (d) none of these
- 4 If from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is :
 (a) $\frac{13}{32}$ (b) $\frac{27}{32}$
 (c) $\frac{19}{32}$ (d) none of these
- 5 A fair coin is tossed repeatedly. If tail appears on first four tosses, then the probability of head appearing on fifth toss equals :
 (a) $\frac{5}{12}$ (b) $\frac{1}{2}$
 (c) $\frac{5}{6}$ (d) $\frac{1}{6}$
- 6 There are four calculators and it is known that exactly two of them are defective. They are tested one by one , in a random order till both the defective calculators are identified. Then the probability that only two tests are required is :
 (a) $\frac{5}{6}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$
- 7 20 girls, among whom are A and B sit down at a round table. The probability that there are 4 girls between A and B is :
 (a) $\frac{17}{19}$ (b) $\frac{2}{19}$ (c) $\frac{13}{19}$ (d) $\frac{6}{19}$
- 8 Two integers x and y are chosen with replacement out of the set {0, 1, 2, 3, ... 10}. Then the probability that $|x - y| > 5$ is :
 (a) $\frac{7}{11}$ (b) $\frac{40}{121}$ (c) $\frac{35}{121}$ (d) $\frac{30}{121}$
- 9 The probability that the birthdays of 4 different persons will fall in exactly two calendar months is :
 (a) $\frac{77}{1728}$ (b) $\frac{17}{87}$
 (c) $\frac{11}{144}$ (d) none of these
- 10 If 6 objects are distributed at random among 6 persons, the probability that atleast one of them will not get any thing is
 (a) $6 \cdot (6!)$ (b) $\frac{5^6}{6!}$
 (c) $\frac{6^6 - 6!}{6^6}$ (d) none of these
- 11 There is 4 volume encyclopaedia among 40 books arranged on a shelf in a random order. If the volumes are not necessarily kept side by side, the probability that they occur in increasing order from left to right is :
 (a) $\frac{1}{24}$ (b) $\frac{1}{12}$
 (c) $\frac{1}{10}$ (d) none of these

- 12** Four numbers are multiplied together. Then the probability that the product will be divisible by 5 or 10 is
 (a) $\frac{169}{625}$ (b) $\frac{369}{625}$
 (c) $\frac{169}{1626}$ (d) none of these
- 13** 8 couples (husband and wife) attend a dance show 'Nach Baliye' in a popular TV channel ; A lucky draw in which 4 persons picked up for a prize is held, then the probability that there is atleast one couple will be selected is :
 (a) $\frac{8}{39}$ (b) $\frac{15}{39}$
 (c) $\frac{12}{13}$ (d) none of these
- 14** Three persons A, B and C are to speak at a function along with 4 other persons. if they all speak in random order, the probability that A speaks before B and B speaks before C is :
 (a) $\frac{5}{6}$ (b) $\frac{1}{6}$
 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$
- 15** A bag contains 16 coins of which 2 coins are counterfeit with heads on the both sides. The rest are fair coins. One is selected at random from the bag and tossed. The probability of getting a head is :
 (a) $\frac{3}{16}$ (b) $\frac{13}{16}$
 (c) $\frac{9}{16}$ (d) $\frac{7}{16}$
- 16** A committee of five persons is to be chosen from a group of 9 people. The probability that a certain married couple will either serve together or not at all is :
 (a) $\frac{4}{9}$ (b) $\frac{5}{9}$
 (c) $\frac{13}{18}$ (d) none of these
- 17** A speaks truth in 60% cases and B speaks truth in 80% cases. The probability that they will say the same thing while describing a single event is :
 (a) 0.36 (b) 0.56
 (c) 0.48 (d) 0.20
- 18** Two squares are chosen at random on a chessboard, the probability that they have a side in common is :
 (a) $\frac{3}{32}$ (b) $\frac{1}{32}$
 (c) $\frac{1}{18}$ (d) none of these
- 19** An old person forgets the last two digits of a telephone number, remembering only that these are different dialled at random. The probability that the number is dialled correctly is :
 (a) $\frac{1}{90}$ (b) $\frac{81}{91}$
 (c) $\frac{2}{99}$ (d) none of these
- 20** Three squares of a chessboard are chosen at random, the probability that two are of one colour and one of another is
 (a) $\frac{67}{992}$ (b) $\frac{16}{21}$
 (c) $\frac{31}{32}$ (d) none of these
- 21** The probability that a leap year selected at random contains either 53 Sundays or 53 Mondays, is :
 (a) $\frac{17}{53}$ (b) $\frac{1}{53}$
 (c) $\frac{3}{7}$ (d) none of these
- 22** In order to get atleast once a head with probability $P \geq 0.9$, the number of times a coin needs to be tossed is :
 (a) 3 (b) 2
 (c) 5 (d) 4
- 23** Out of 13 applicants for a job, there are 5 women and 8 men. It is desired to select 2 persons for the job. The probability that atleast one of the selected persons will be a woman is :
 (a) $\frac{25}{39}$ (b) $\frac{31}{65}$ (c) $\frac{25}{69}$ (d) $\frac{5}{13}$
- 24** Nine squares are chosen at random on a chessboard. What is the probability that they form a square of size 3×3 ?
 (a) $\frac{9}{{}^{64}C_9}$ (b) $\frac{36}{{}^{64}C_9}$
 (c) $\frac{6}{{}^{64}C_9}$ (d) none of these
- 25** Seven digits from the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 are written in random order. The probability that this seven digit number is divisible by 9 is :
 (a) $\frac{7}{9}$ (b) $\frac{1}{9}$ (c) $\frac{2}{9!}$ (d) $\frac{4}{9}$
- 26** From the set of first ten natural numbers two distinct numbers are picked randomly. Find the probability that they are co-primes.
 (a) $\frac{31}{45}$ (b) $\frac{14}{45}$
 (c) $\frac{2}{3}$ (d) $\frac{28}{45}$

LEVEL 02 > HIGHER LEVEL EXERCISE

- 1 What is the probability that four S's come consecutively in the word MISSISSIPPI?
 - (a) $\frac{4}{165}$
 - (b) $\frac{4}{135}$
 - (c) $\frac{24}{165}$
 - (d) none
- 2 Each coefficient in the equation $ax^2 + bx + c = 0$ is determined by throwing ordinary six faced die. Find the probability that the equation will have real roots.
 - (a) $\frac{34}{161}$
 - (b) $\frac{43}{216}$
 - (c) $\frac{25}{36}$
 - (d) none of these
- 3 A and B throw alternately a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. Find their respective chance of winning, if A begins.
 - (a) $\frac{13}{16}, \frac{31}{16}$
 - (b) $\frac{30}{61}, \frac{31}{61}$
 - (c) $\frac{31}{61}, \frac{41}{61}$
 - (d) $\frac{38}{61}, \frac{23}{61}$
- 4 A consignment of 15 wristwatches contains 4 defectives. The wristwatches are selected at random, one by one and examined. The ones examined are not put back. What is the probability that ninth one examined is the last defective?
 - (a) $\frac{11}{195}$
 - (b) $\frac{17}{195}$
 - (c) $\frac{8}{195}$
 - (d) $\frac{16}{195}$
- 5 In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct, given that he copied it, is $\frac{1}{8}$. Find the probability that he knew the answer to the question, given that he correctly answered it.
 - (a) $\frac{17}{39}$
 - (b) $\frac{13}{29}$
 - (c) $\frac{24}{39}$
 - (d) $\frac{24}{39}$
- 6 Given that the sum of two non-negative quantities is 200, the probability that their product is not less than $\frac{3}{4}$ times their greatest product value is :
 - (a) $\frac{99}{200}$
 - (b) $\frac{101}{200}$
 - (c) $\frac{87}{100}$
 - (d) none of these
- 7 A pack of cards consists of 9 cards numbered 1 to 9. Three cards are drawn at random with replacement. Then the probability of getting 1 even and 2 odd numbered cards is :
 - (a) $\frac{3}{143}$
 - (b) $\frac{100}{243}$
 - (c) $\frac{50}{343}$
 - (d) $\frac{7}{72}$
- 8 Three numbers are to be selected at random without replacement from the set of numbers $\{1, 2, \dots, n\}$. The conditional probability that the third number lies between the first two, if the first number is known to be smaller than the second is :
 - (a) $\frac{1}{3}$
 - (b) $\frac{2}{3}$
 - (c) $\frac{5}{6}$
 - (d) $\frac{7}{12}$
- 9 Two numbers b and c are chosen at random with replacement from the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9. The probability that $x^2 + bx + c > 0$ for all $x \in R$ is :
 - (a) $\frac{23}{81}$
 - (b) $\frac{7}{9}$
 - (c) $\frac{32}{81}$
 - (d) $\frac{65}{729}$
- 10 The probabilities that a student passes in Mathematics, Physics and Chemistry are m , p and c , respectively. Of these subjects the student has 75% chance of passing in atleast one, a 50% chance of passing in atleast two and a 40% chance of passing in exactly, two, which of the following relations are true.
 - (a) $p + m + c = \frac{27}{20}$
 - (b) $p + m + c = \frac{13}{20}$
 - (c) $pmc = \frac{1}{10}$
 - (d) both (a) and (c)
- 11 A student appears for tests A, B and C. The student is successful if he passes either in tests A and B or tests A and C. The probabilities of the student passing in tests A, B, C are p , q and $\frac{1}{2}$ respectively. If the probability that the student is successful is $\frac{1}{2}$ then,
 - (a) $p = 1, q = 0$
 - (b) $p = 0, q = 1$
 - (c) $\frac{p}{q} = 1$
 - (d) infinitely many solutions
- 12 If $\frac{1+4p}{p}, \frac{1-p}{4}, \frac{1-2p}{2}$ are probabilities of three mutually exclusive events then
 - (a) $p = \frac{1}{2}$
 - (b) $p = \frac{3}{4}$
 - (c) $p = \frac{1}{3}$
 - (d) none of these
- 13 A letter is taken out at random from 'ASSISTANT' and another is taken out from 'STATISTICS'. The probability that they are the same letters is :
 - (a) $\frac{35}{96}$
 - (b) $\frac{19}{90}$
 - (c) $\frac{19}{96}$
 - (d) none of these

- 14** Two numbers a and b are chosen at random from the set of first 30 natural numbers. The probability that $a^2 - b^2$ is divisible by 3 is :
- (a) $\frac{37}{87}$ (b) $\frac{47}{87}$
(c) $\frac{17}{29}$ (d) none of these
- 15** A man takes a step forward with probability 0.4 and backward with probability 0.6. The probability that at the end of eleven steps he is one step away from the starting point is
- (a) ${}^{11}C_6(0.1)^{11}$ (b) ${}^{11}C_6(0.24)^5$
(c) ${}^{11}C_6(0.2)^{11}$ (d) none of these
- 16** Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with these three vertices is equilateral equals :
- (a) $\frac{1}{6}$ (b) $\frac{1}{3}$
(c) $\frac{1}{10}$ (d) none of these
- 17** A man can take a step forward, backward, left or right with equal probability. Find the probability that after nine steps he will be just one step away from his initial position.
- (a) $\frac{3696}{7^4}$ (b) $\frac{3969}{4^7}$
(c) $\frac{4^4}{10}$ (d) none of these
- 18** Urn A contains six red and four black balls and urn B has four red and six black balls. One ball is drawn at random from urn A and placed in urn B. Then one ball is transferred at random from urn B to urn A. If one ball is now drawn at random from urn A, find the probability that it is red.
- (a) $\frac{32}{65}$ (b) $\frac{32}{55}$
(c) $\frac{23}{55}$ (d) $\frac{56}{65}$
- 19** The digits 1, 2, 3, ..., 9 are written in random order to form a nine digit number. Find the probability that this number is divisible by 11.
- (a) $\frac{11}{63}$ (b) $\frac{11}{81}$
(c) $\frac{11}{126}$ (d) none of these
- 20** A terror outfit, works under the pseudonym of Mafia Wars, uses 12 satellite phones, during its operation to communicate with each other. Each of the 12 terrorists is armed with a satellite phone, an AK-47 and hand grenades among other explosives. Except the four terrorists, who are hidden in a tiny staff washroom of a prominent hotel, no two terrorists are positioned at the same location. The terrorists who are hidden in the washroom are codenamed as Alpha and the ones who are hidden at other surrounding locations are codenamed as Beta. Due to location differences Beta terrorists cannot communicate without the satellite phone, at all. In order to speak to any of the Alpha terrorist a Beta terrorist can call on any one of the phones available with Beta, similarly to speak to any Beta terrorist an Alpha can call from any one of the phones available with them. A conference call can be made between any three satellite phones. If three phones are chosen randomly from the 12 phones, then what is the probability that the terrorists can make a conference call?
- (a) 9/11 (b) 51/55
(c) 10/11 (d) 54/55
- 21** Zuckerberg, my facebook friend, recently returned from his honeymoon trip to three European cities – Paris, Milan and Zurich, where he had clicked some photos before uploading them on facebook in three different folders naming them on the cities he had visited. The folders Paris, Milan and Zurich have 3, 4, and 5 photos, respectively. He asks his six year old niece Olivia Bee that if she could download these photos and edit them using the Instagram and put them back but only one photo in each folder. Within no time she edits all the photos and uploads back quickly one photo in each folder. What's the probability that she uploads at least two photos of the same city and no folder has the photo uploaded back to its original folder, after editing?
- (a) 1/11 (b) 1/12
(c) 3/11 (d) 1/3
- 22** In a regular decagon there are diagonals of distinct sizes. If all the possible diagonals are drawn and you choose any one diagonal at random then what is the probability that it is neither the shortest one nor it is a longest one?
- (a) 4/7 (b) 3/7
(c) 6/7 (d) 5/7

Answers

✓ Introductory Exercise 20.1

1. (a)	2. (a)	3. (b)	4. (b)	5. (c)	6. (d)	7. (d)	8. (a)	9. (b)	10. (c)
11. (d)	12. (d)	13. (c)	14. (b)	15. (a)	16. (a)	17. (b)	18. (c)	19. (b)	20. (c)
21. (c)	22. (d)	23. (b)	24. (c)	25. (c)	26. (b)	27. (d)	28. (a)	29. (b)	30. (c)
31. (c)	32. (c)	33. (c)	34. (b)	35. (a)	36. (a)	37. (b)	38. (b)	39. (c)	40. (a)
41. (b)	42. (b)	43. (c)	44. (c)	45. (c)	46. (a)	47. (c)	48. (d)	49. (a)	50. (b)
51. (a)	52. (b)	53. (c)	54. (a)	55. (b)	56. (c)	57. (b)	58. (c)	59. (a)	

✓ Introductory Exercise 20.2

1. (a)	2. (b)	3. (c)	4. (a)	5. (b)	6. (c)	7. (c)	8. (b)	9. (a)	10. (b)
11. (b)	12. (c)	13. (c)	14. (a)	15. (b)	16. (b)	17. (a)	18. (b)	19. (b)	20. (c)
21. (a)	22. (b)	23. (c)	24. (c)	25. (a)	26. (c)	27. (b)	28. (c)	29. (a)	

✓ Introductory Exercise 20.3

1. (b)	2. (b)	3. (c)	4. (a)	5. (a)	6. (c)	7. (a)	8. (c)	9. (a)	10. (b)
11. (a)	12. (b)	13. (b)	14. (a)	15. (b)	16. (c)	17. (a)	18. (a)	19. (a)	20. (b)
21. (a)	22. (b)								

✓ Introductory Exercise 20.4

1. (a)	2. (b)	3. (b)	4. (c)	5. (d)	6. (b)	7. (a)	8. (a)	9. (c)	10. (a)
11. (b)	12. (c)	13. (c)	14. (b)	15. (c)	16. (b)	17. (a)	18. (a)	19. (b)	20. (c)
21. (b)	22. (a)	23. (b)	24. (c)	25. (b)	26. (c)	27. (c)	28. (a)	29. (a)	30. (b)
31. (a)	32. (b)	33. (c)	34. (b)	35. (b)	36. (d)	37. (b)	38. (b)	39. (b)	40. (a)
41. (b)									

✓ Level 01 Basic Level Exercise

1. (b)	2. (b)	3. (b)	4. (a)	5. (b)	6. (c)	7. (b)	8. (d)	9. (a)	10. (c)
11. (a)	12. (b)	13. (b)	14. (b)	15. (c)	16. (a)	17. (b)	18. (c)	19. (a)	20. (b)
21. (c)	22. (d)	23. (a)	24. (b)	25. (b)	26. (a)				

✓ Level 02 Higher Level Exercise

1. (a)	2. (b)	3. (b)	4. (c)	5. (c)	6. (b)	7. (b)	8. (a)	9. (c)	10. (d)
11. (d)	12. (a)	13. (b)	14. (b)	15. (b)	16. (c)	17. (b)	18. (b)	19. (c)	20. (d)
21. (a)	22. (a)								

Hints & Solutions

✓ Introductory Exercise 20.1

- 1** Here the sample space $S = \{H, T\} \Rightarrow n(S) = 2$
 Event of getting head = $\{H\} \Rightarrow n(E) = 1$
 $\therefore$ Probability of getting a head is given by $P(E) = \frac{1}{2}$

Solutions (for Q. Nos. 2 to 5)

- $S = \{HH, HT, TH, TT\} \Rightarrow n(S) = 4$
- 2** $E = \{HT, TH\} \Rightarrow n(E) = 2$
 $\therefore P(E) = \frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$
- 3** $E = \{HH, HT, TH\} \Rightarrow n(E) = 3$
 $\therefore P(E) = \frac{3}{4}$
- 4** $E = \{H, H\} \Rightarrow n(E) = 1$
 $\therefore P(E) = \frac{1}{4}$
- 5** $E = \{H, H\} \Rightarrow n(E) = 1$
 $\therefore P(E) = \frac{1}{4}$

Solutions (for Q. Nos. 6 to 12)

- $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$
 $\Rightarrow n(S) = 8$
- 6** $E = \{HTT, THT, TTH\} \Rightarrow n(E) = 3$
 $\therefore P(E) = \frac{3}{8}$
- 7** $E = \{HHT, HTH, THH\} \Rightarrow n(E) = 3$
 $\therefore P(E) = \frac{3}{8}$
- 8** $E = \{HHH, HHT, HTH, HTT, THH, THT, TTH\}$
 $\Rightarrow n(E) = 7$
 $\therefore P(E) = \frac{7}{8}$
- 9** $E = \{HHT, HTH, THH\} \Rightarrow n(E) = 3$
 $\therefore P(E) = \frac{3}{8}$
- 10** $E = \{HHH, HHT, HTH, THH\} \Rightarrow n(E) = 4$
 $\therefore P(E) = \frac{4}{8} = \frac{1}{2}$
- 11** $E = \{HHT, HTH, HTT, THH, THT, TTH\}$
 $\Rightarrow n(E) = 6$
 $\therefore P(E) = \frac{6}{8} = \frac{3}{4}$

- 12** $E = \{HHH, HHT, HTH, THH\} \Rightarrow n(E) = 4$
 $\therefore P(E) = \frac{4}{8} = \frac{1}{2}$

Solutions (for Q. Nos. 13 to 16)

- $S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$
- 13** $E = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(E) = 6$
 $\therefore P(E) = \frac{6}{6} = 1$
- 14** $E = \{3, 6\} \Rightarrow n(E) = 2$
 $\therefore P(E) = \frac{2}{6} = \frac{1}{3}$
- 15** $E = \{2, 3, 5\} \Rightarrow n(E) = 3$
 $\therefore P(E) = \frac{3}{6} = \frac{1}{2}$
- 16** $E = \{2, 4, 6\} \Rightarrow n(E) = 3$
 $\therefore P(E) = \frac{3}{6} = \frac{1}{2}$

Solutions (for Q. Nos. 17 and 18)

- $S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$
 $\Rightarrow n(S) = 8$
- 17** $E = \{HHT, HTH, THH, TTH, THT, HTT\}$
 $\Rightarrow n(E) = 6 \therefore P(E) = \frac{6}{8} = \frac{3}{4}$
- 18** $E = \{TTT\} \Rightarrow n(E) = 1$
 $\therefore P(E) = \frac{1}{8}$

Solutions (for Q. Nos. 19 to 27)

- $S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), \dots, (6, 5), (6, 6)\}$
 $\Rightarrow n(S) = 6 \times 6 = 36$
- 19** $E = \{(6, 3), (5, 4), (4, 5), (3, 6)\}$
 $n(E) = 4 \therefore P(E) = \frac{4}{36} = \frac{1}{9}$
- 20** $E = \{(6, 4), (5, 5), (4, 6), (6, 5), (5, 6), (6, 6)\}$
 $n(E) = 6$
 $\therefore P(E) = \frac{6}{36} = \frac{1}{6}$
- 21** $E = \{(6, 3), (5, 4), (4, 5), (3, 6), (6, 5), (5, 6)\}$
 $\Rightarrow n(E) = 6$
 $\therefore P(E) = \frac{6}{36} = \frac{1}{6}$

$$22 \quad E = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}; n(E) = 6$$

$$\therefore P(E) = \frac{6}{36} = \frac{1}{6}$$

$$23 \quad E = \{(2, 2), (4, 4), (6, 6)\} \Rightarrow n(E) = 3$$

$$\therefore P(E) = \frac{3}{36} = \frac{1}{12}$$

$$24. \quad E = \{(2, 3), (2, 6), (4, 3), (4, 6), (6, 3), (6, 6), (3, 2), (6, 2), (3, 4), (6, 4), (3, 6)\} \Rightarrow n(E) = 11$$

$$\therefore P(E) = \frac{11}{36}$$

$$25 \quad E = \{(1, 2), (1, 5), (2, 1), (2, 4), (3, 3), (3, 6), (4, 2), (4, 5), (5, 1), (5, 4), (6, 3), (6, 6), (1, 3), (2, 2), (2, 6), (3, 1), (3, 5), (4, 4), (5, 3), (6, 2)\} \Rightarrow n(E) = 20$$

$$\therefore P(E) = \frac{20}{36} = \frac{5}{9}$$

$$26 \quad E = \{(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)\}$$

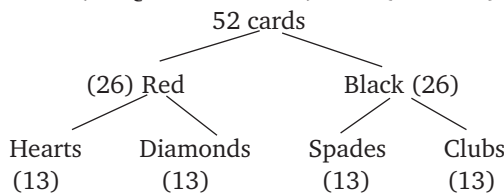
$$\Rightarrow n(E) = 15$$

$$\therefore P(E) = \frac{15}{36} = \frac{5}{12}$$

$$27 \quad E = \{(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6)\} \Rightarrow n(E) = 11$$

$$\therefore P(E) = \frac{11}{36}$$

Solutions (for Q. Nos. 28 to 35) : $S = \{52 \text{ cards}\}$



In each of the four suits there is one ace, one king, one queen and one jack (or knave) and rest 9 cards are numbered.

$$\therefore n(S) = 52$$

$$28 \quad n(E) = 26$$

$$\therefore P(E) = \frac{26}{52} = \frac{1}{2}$$

$$29 \quad n(E) = 4 \quad \therefore P(E) = \frac{4}{52} = \frac{1}{13}$$

30 Since drawn card must be black so there are only two queens.

$$\text{Hence } n(E) = 2$$

$$\therefore P(E) = \frac{2}{52} = \frac{1}{26}$$

31 There are 26 black cards (including two queens). Besides it there are two more queens (in red colours)

$$\text{Thus } n(E) = 26 + 2 = 28$$

$$\therefore P(E) = \frac{28}{52} = \frac{7}{13}$$

32 There are 4 kings and 4 queens

$$\therefore E = K \cup Q$$

$$\therefore n(E) = 4 + 4 = 8$$

$$\therefore P(E) = \frac{8}{52} = \frac{2}{13}$$

33 There are 13 hearts (including one queen and one king). Besides it there are 3 queens and 3 kings in remaining 3 suits each.

$$\text{Thus } n(E) = 13 + 3 + 3 = 19$$

$$\therefore P(E) = \frac{19}{52}$$

34 There are 13 spades (including one king). Besides there are 3 more kings in remaining 3 suits).

$$\text{Thus } n(E) = 13 + 3 = 16$$

$$\text{Hence } n(\bar{E}) = 52 - 16 = 36$$

$$\therefore P(\bar{E}) = \frac{36}{52} = \frac{9}{13}$$

35 There are 4 aces and 4 kings

$$\therefore n(E) = 4 + 4 = 8$$

$$\therefore n(\bar{E}) = 52 - 8 = 44$$

$$\therefore P(\bar{E}) = \frac{44}{52} = \frac{11}{13}$$

36 There are 4 aces, 4 kings and 4 jacks and their selection can be made in following ways.

$${}^{12}C_1 \times {}^8C_1 \times {}^4C_1 = 12 \times 8 \times 4$$

$$n(E) = 12 \times 8 \times 4$$

$$\text{Total selection can be made} = {}^{52}C_3 = 52 \times 51 \times 50$$

$$P(E) = \frac{12 \times 8 \times 4}{52 \times 51 \times 50} = \frac{16}{5525}$$

$$37 \quad E = \{(1, 1, 1, 1), (2, 2, 2, 2), \dots, (13, 13, 13, 13)\}$$

$$\therefore n(E) = 13$$

$$\text{and } n(S) = {}^{52}C_4 = 270725$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{13}{270725} = \frac{1}{20825}$$

$$38 \quad n(E) = {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = (13)^4$$

$$n(S) = {}^{52}C_4 = 270725$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{(13)^4}{270725} = \frac{2197}{20825}$$

$$39 \quad n(E) = {}^{13}C_1 \times {}^{12}C_1 \times {}^{11}C_1 \times {}^{10}C_1 = 13 \times 12 \times 11 \times 10$$

$$n(S) = {}^{52}C_4 = 270725$$

$$\therefore P(E) = \frac{13 \times 12 \times 11 \times 10}{270725} = \frac{264}{4165}$$

Solutions (for Q. Nos. 40 to 43) $n(S) = 6 \times 6 \times 6 \times 6 = 6^4$

$$40 \quad n(E) = 6$$

$$\therefore E = \{(1, 1, 1, 1), (2, 2, 2, 2), (3, 3, 3, 3), \dots, (6, 6, 6, 6)\}$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{6^4} = \frac{1}{6^3} = \frac{1}{216}$$

$$41 \quad n(E) = {}^6C_1 \times {}^5C_1 \times {}^4C_1 \times {}^3C_1 = 360 \quad \therefore P(E) = \frac{360}{6^4} = \frac{5}{18}$$

42 Select a number which occurs on two dice out of six numbers (1, 2, 3, 4, 5, 6). This can be done in ${}^6C_1$ ways.

Now select two distinct number out of remaining 5 numbers which can be done in ${}^5C_2$ ways. Thus these 4 numbers can be arranged in $\frac{4!}{2!}$ ways.

So, the number of ways in which two dice show the same face and the remaining two show different faces is

$${}^6C_1 \times {}^5C_2 \times \frac{4!}{2!} = 720$$

$$\therefore n(E) = 720 \quad \therefore P(E) = \frac{720}{6^4} = \frac{5}{9}$$

43 There are 3 possible cases-

(i) 2 similar faces + 2 different faces

(ii) 3 similar faces + 1 different face

(iii) all 4 faces are similar

$\therefore$ Required number of ways

$$= \left({}^6C_1 \times {}^5C_2 \times \frac{4!}{2!} \right) + \left({}^6C_1 \times {}^5C_1 \times \frac{4!}{3!} \right) + \left({}^6C_1 \times \frac{4!}{4!} \right)$$

$$= (6 \times 10 \times 12) + (6 \times 5 \times 4) + 6 = 846$$

$$\therefore n(E) = 846 \text{ and } n(S) = 6^4$$

$$\therefore P(E) = \frac{846}{6^4} = \frac{47}{72}$$

$$44 \quad E = \{2, 3, 5, 7, 11, 13, 17, 19\}$$

$$\Rightarrow n(E) = 8; S = \{1, 2, 3, 4, \dots, 20\} \Rightarrow n(S) = 20$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

$$45 \quad S = \{1, 2, 3, 4, \dots, 18\} \Rightarrow n(S) = 18$$

$$E_1 = \{2, 4, 6, 8, 10, 12, 14, 16, 18\} \Rightarrow n(E_1) = 9$$

$$E_2 = \{3, 6, 9, 12, 15, 18\} \Rightarrow n(E_2) = 6$$

$$(E_1 \cap E_2) = E_3 = \{6, 12, 18\} \Rightarrow n(E_3) = 3$$

$$\therefore E = E_1 \cup E_2 = E_1 + E_2 - E_3$$

$$n(E) = 9 + 6 - 3, n(E) = 12$$

where $E = \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{12}{18} = \frac{2}{3}$$

 **NOTE** For any problem refer set theory and number system.

$$46 \quad n(S) = {}^{100}C_2 = \frac{100 \times 99}{2} = 4950$$

$$n(E) = {}^{25}C_2 = 300 \quad \therefore P(E) = \frac{300}{4950} = \frac{2}{33}$$

Hint There are total 25 prime numbers upto 100.

$$47 \quad n(S) = {}^{100}C_2 = 4950$$

$$n(E) = {}^{75}C_2 = 2775 \quad (100 - 25 = 75)$$

$$\therefore P(E) = \frac{2775}{4950} = \frac{37}{66}$$

$$48 \quad 366 = 7 \times 52 + 2$$

It means a leap year has 52 full weeks and 2 more days. These 2 days can be :

(i) Sunday and Monday (ii) Monday and Tuesday

(iii) Tuesday and Wednesday (iv) Wednesday and Thursday

(v) Thursday and Friday (vi) Friday and Saturday

(vii) Saturday and Sunday

Clearly atleast there are 52 Sundays. Now, for having 53 Sundays in the year, one of the above 2 consecutive days must be Sunday. Thus, out of the above 7 possibilities, 2 possibilities are in favour [(i) and (vii)] of the event that one of the two days is a Sunday.

$$\therefore \text{Required probability} = \frac{2}{7}$$

$$49 \quad n(S) = 8 + 4 = 12; n(E) = 8$$

$$\therefore P(E) = \frac{8}{12} = \frac{2}{3}$$

$$50 \quad n(S) = {}^{12}C_4 = 495; n(E) = {}^8C_4 = 70$$

$$\therefore P(E) = \frac{70}{495} = \frac{14}{99}$$

$$51 \quad n(S) = {}^{12}C_4 = 495 \Rightarrow n(E) = {}^4C_4 = 1$$

$$\therefore P(E) = \frac{1}{495}$$

$$52 \quad n(S) = {}^{12}C_4 = 495 \Rightarrow n(E) = {}^8C_2 \times {}^4C_1 = 112$$

$$\therefore P(E) = \frac{112}{495}$$

$$53 \quad n(S) = 495 \Rightarrow n(E) = {}^4C_3 = 4$$

$$\therefore P(E) = \frac{4}{495}$$

$$54 \quad \text{Total number of outcomes} = 2 + 7 = 9$$

Favourable number of cases = 2

$$\therefore P(E) = \frac{2}{9}$$

$$55 \quad \text{Total number of outcomes} = 5 + 7 = 12$$

Number of cases against the occurrence of event = 5

$\therefore$ Number of cases in favour of event = 12 - 5 = 7

$$\therefore P(E) = \frac{7}{12}$$

$$56 \quad S = \{GG, GB, BG, BB\} \text{ and } E = \{GG, GB, BG\}$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$$

$$57 \quad n(S) = {}^5C_2 = 10; n(E) = ({}^2C_1 \times {}^3C_1) + ({}^2C_2) = 7$$

$$\therefore P(E) = \frac{7}{10}$$

$$58 \quad n(S) = {}^{20}C_2 = 190; n(E) = {}^{15}C_2 = 105$$

$$\therefore P(E) = \frac{105}{190} = \frac{21}{38}$$

$$59 \quad n(S) = {}^{20}C_4 = 4845; n(E) = ({}^5C_3 \times {}^{15}C_1) + ({}^5C_4) = 155$$

$$\therefore P(E) = \frac{155}{4845} = \frac{31}{969}$$

Introductory Exercise 20.2

- 1 $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{7}{50}$
 $\therefore P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $= \frac{1}{4} + \frac{1}{2} - \frac{7}{50} = \frac{61}{100}$
- 2 $P(\text{neither } A \text{ nor } B) = P(\bar{A} \text{ and } \bar{B})$
 $= P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - \frac{61}{100} = \frac{39}{100}$
- 3 $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{10} + \frac{1}{2} - \frac{1}{5} = \frac{6}{10} = \frac{3}{5}$
- 4 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $\frac{3}{5} = \frac{2}{5} + \frac{1}{2} - P(A \cap B) \Rightarrow P(A \cap B) = \frac{3}{10}$
- 5 $P(\bar{A}) = 1 - P(A) = 1 - \frac{2}{5} = \frac{3}{5}$
- 6 $P(\bar{B}) = 1 - P(B) = 1 - \frac{1}{2} = \frac{1}{2}$
- 7 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $= \frac{2}{5} + \frac{1}{2} - 0 = \frac{9}{10} \quad (\because P(A \cap B) = 0)$
- 8 $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - \frac{9}{10} = \frac{1}{10}$
- 9 $P(\bar{A} \cap B) = P(B) - P(A \cap B)$
 $= P(B) = \frac{1}{2} \quad (\because P(A \cap B) = 0)$
- 10 $P(A \cap \bar{B}) = P(A) - P(A \cap B) = 2/5$
- 11 $P(A \cup B) = P(A) + P(B)$
 $0.65 = 0.35 + P(B) \quad (\because P(A) = 1 - P(\bar{A}))$
 $\Rightarrow P(B) = 0.30$
- 12 Let $P(A) = x$, then $P(B) = \frac{3}{2}x$ and $P(C) = \frac{1}{2} \times \frac{3}{2}x = \frac{3}{4}x$
 $\therefore P(A) + P(B) + P(C) = P(A \cup B \cup C) = P(S) = 1$
 $\therefore x + \frac{3}{2}x + \frac{3}{4}x = 1 \Rightarrow x = \frac{4}{13}$
 $\therefore P(A) = \frac{4}{13}$
- 13 $n(S) = 6 \times 6 = 36$
Let A = Event of getting an even number on the first die
 $\therefore n(A) = 3 \times 6 = 18$
and B = Event of getting a total of 8
 $n(B) = 5$
 $A = \left\{ \begin{array}{l} (2, 1), (2, 2), (2, 3) \dots (2, 6) \\ (4, 1) \dots \dots \dots (4, 6) \\ (6, 1) \dots \dots \dots (6, 6) \end{array} \right\}$
 $B = \{(6, 2), (5, 3), (4, 4), (3, 5), (2, 6)\}$
 $\therefore n(A) = 3 \times 6 = 18$ and $n(B) = 5$

$$\begin{aligned} \therefore (A \cap B) &= \{(2, 6), (6, 2), (4, 4)\} \\ \therefore n(A \cap B) &= 3 \\ P(A) &= \frac{18}{36} = \frac{1}{2}, \quad P(B) = \frac{5}{36} \\ \text{and } P(A \cap B) &= \frac{3}{36} = \frac{1}{12} \\ \therefore P(\text{Even number on first die or a total of 8}) \\ &= P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ &= \frac{1}{2} + \frac{5}{36} - \frac{1}{12} = \frac{5}{9} \end{aligned}$$

14 $A = \{(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)\}$
 $B = \{(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5)\}$
 $A \cap B = \{(5, 5)\}$

Also $n(S) = 36$
 $\therefore P(A) = \frac{6}{36} = \frac{1}{6} \Rightarrow P(B) = \frac{6}{36} = \frac{1}{6}$
and $P(A \cap B) = \frac{1}{36}$
 $\therefore \text{Required probability} = P(A) + P(B) - P(A \cap B)$
 $= \frac{1}{6} + \frac{1}{6} - \frac{1}{36} = \frac{11}{36}$

15 $n(S) = 36$
 $A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$
 $B = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$
 $(A \cap B) = \{(4, 4)\}$
 $n(A) = 6, \quad n(B) = 5, \quad n(A \cap B) = 1$
 $\therefore \text{Required probability} = P(A \cup B)$
 $= P(A) + P(B) - P(A \cap B)$
 $= \frac{6}{36} + \frac{5}{36} - \frac{1}{36} = \frac{10}{36} = \frac{5}{18}$

16 See the solution of question number 14
 $P(\text{at least one die shows 6}) = \frac{11}{36}$

17 $n(S) = 52$
 $A \rightarrow$ The event of getting a heart
 $B \rightarrow$ The event of getting a king
then $A \cap B \rightarrow$ The event of getting a king of heart.
 $\therefore P(A) = \frac{13}{52} = \frac{1}{4}, \quad P(B) = \frac{4}{52} = \frac{1}{13}$ and $P(A \cap B) = \frac{1}{52}$
 $\therefore P(\text{a heart or a king}) = P(A \text{ or } B) = P(A \cup B)$
 $= P(A) + P(B) - P(A \cap B)$
 $= \frac{1}{4} + \frac{1}{13} - \frac{1}{52} = \frac{4}{13}$

18 $n(S) = {}^{52}C_2 = 1326$
Let A = event of getting both red cards
and B = event of getting both queens
then $A \cap B$ = event of getting two red queens

$$\begin{aligned}
 n(A) &= {}^{26}C_2 = 325, n(B) = {}^4C_2 = 6 \\
 n(A \cap B) &= {}^2C_2 = 1 \\
 \therefore P(A) &= \frac{325}{1326}, P(B) = \frac{6}{1326} = \frac{1}{221} \\
 P(A \cap B) &= \frac{1}{1326} \\
 \therefore P(\text{both red or both queens}) &= P(A \cup B) \\
 &= P(A) + P(B) - P(A \cap B) \\
 &= \frac{325}{1326} + \frac{1}{221} - \frac{1}{1326} = \frac{55}{221}
 \end{aligned}$$

19 $n(S) = 52$

Let A, B, C be the events of getting a red card, a heart and a king respectively.

then $n(A) = 26, n(B) = 13, n(C) = 4$

Clearly $n(A \cap B) = 13, n(B \cap C) = 1,$

$n(A \cap C) = 2, n(A \cap B \cap C) = 1$

$$\therefore P(A) = \frac{26}{52} = \frac{1}{2}, P(B) = \frac{13}{52} = \frac{1}{4}, P(C) = \frac{4}{52} = \frac{1}{13}$$

$$P(A \cap B) = \frac{13}{52} = \frac{1}{4}, P(B \cap C) = \frac{1}{52},$$

$$P(A \cap C) = \frac{2}{52} = \frac{1}{26} \Rightarrow P(A \cap B \cap C) = \frac{1}{52}$$

$\therefore P(\text{a red card, or a heart or a king}) = P(A \cup B \cup C)$

$$= P(A) + P(B) + P(C) - (P(A \cap B)$$

$$+ P(B \cap C) + P(A \cap C) + P(A \cap B \cap C))$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{13} - \left(\frac{1}{4} + \frac{1}{52} + \frac{1}{26} \right) + \frac{1}{52} = \frac{7}{13}$$

20 $n(S) = {}^{52}C_4$. Let E_1, E_2, E_3, E_4 be the events of getting all spades, all clubs, all hearts and all diamonds respectively.

Then $n(E_1) = {}^{13}C_4, n(E_2) = {}^{13}C_4$

$n(E_3) = {}^{13}C_4; n(E_4) = {}^{13}C_4$

$$\therefore P(E_1) = \frac{{}^{13}C_4}{{}^{52}C_4}, P(E_2) = \frac{{}^{13}C_4}{{}^{52}C_4},$$

$$P(E_3) = \frac{{}^{13}C_4}{{}^{52}C_4}, P(E_4) = \frac{{}^{13}C_4}{{}^{52}C_4}$$

Since E_1, E_2, E_3 and E_4 are mutually exclusive events.

$\therefore P(\text{getting all the 4 cards of the same suit})$

$$P(E_1 \text{ or } E_2 \text{ or } E_3 \text{ or } E_4) = P(E_1) + P(E_2) + P(E_3) + P(E_4)$$

$$= 4 \times \left(\frac{{}^{13}C_4}{{}^{52}C_4} \right) = \frac{44}{4165}$$

21 $n(S) = 52$

A = The event of getting a king

B = The event of getting a queen

$(A \cap B)$ = The event of getting a king and a queen both

$$\therefore n(A) = 4, n(B) = 4, n(A \cap B) = 0$$

$$P(A) = \frac{4}{52}, P(B) = \frac{4}{52}$$

$$\therefore P(A \cup B) = P(A) + P(B) = \frac{4}{52} + \frac{4}{52} = \frac{2}{13}$$

22 $n(S) = {}^{52}C_2 = 1326$

Let A be the event of getting two black cards

and B be the event of getting two jacks

and $(A \cap B)$ be the event of getting two black jacks.

$$\therefore n(A) = {}^{26}C_2, n(B) = {}^4C_2, n(A \cap B) = {}^2C_2$$

$$\therefore P(A) = \frac{{}^{26}C_2}{{}^{52}C_2}, P(B) = \frac{{}^4C_2}{{}^{52}C_2}, P(A \cap B) = \frac{{}^2C_2}{{}^{52}C_2}$$

$\therefore$ Required probability $= P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B) = \frac{{}^{26}C_2}{{}^{52}C_2} + \frac{{}^4C_2}{{}^{52}C_2} - \frac{{}^2C_2}{{}^{52}C_2}$$

$$= \frac{325}{1326} + \frac{1}{221} - \frac{1}{1326} = \frac{55}{221}$$

23 $n(S) = 300$

Let A be the event of getting a number divisible by 3 and

B be the event of getting a number divisible by 5 and

$(A \cap B)$ be event of getting a number divisible by 3 and 5 both

$$\therefore n(A) = 100, n(B) = 60, n(A \cap B) = 20$$

$$\therefore P(A) = \frac{100}{300} = \frac{1}{3}, P(B) = \frac{60}{300} = \frac{1}{5}, P(A \cap B) = \frac{20}{600} = \frac{1}{30}$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{3} + \frac{1}{5} - \frac{1}{30} = \frac{1}{2}$$

24 $n(S) = 100$

Let A be the event of getting a number divisible by 2 and

B be the event of getting a number divisible by 3 and

C be the event of getting of number divisible by 5.

$\therefore (A \cap B)$ be the event of getting a number divisible by both 2 and 3.

$(B \cap C)$ be the event of getting a number divisible by both 3 and 5.

$(A \cap C)$ be the event of getting a number divisible by both 2 and 5.

$(A \cap B \cap C)$ be the event of getting a number divisible by A, B and C .

Now, $n(A) = 50, n(B) = 33, n(C) = 20, n(A \cap B) = 16,$

$$n(B \cap C) = 6, n(A \cap C) = 10, n(A \cap B \cap C) = 3$$

$$\therefore P(A) = \frac{50}{100} = \frac{1}{2}, P(B) = \frac{33}{100},$$

$$P(C) = \frac{20}{100} = \frac{1}{5}, n(A \cap B) = \frac{16}{100}$$

$$n(B \cap C) = \frac{6}{100}, n(A \cap C) = \frac{10}{100},$$

$$n(A \cap B \cap C) = \frac{3}{100}$$

Required probability $= P(A \cup B \cup C)$

$$= P(A) + P(B) + P(C) - [P(A \cap B)$$

$$+ P(B \cap C) + P(A \cap C)] + P(A \cap B \cap C)$$

$$= \frac{50}{100} + \frac{33}{100} + \frac{20}{100} - \left(\frac{16}{100} + \frac{6}{100} + \frac{10}{100} \right) + \frac{3}{100}$$

$$= \frac{74}{100}$$

$$25 \quad n(S) = 5 + 8 + 10 = 23$$

$$n(A) = 5$$

$$n(B) = 8$$

$$[n(A \cap B) = 0]$$

$$\therefore P(A) = \frac{5}{23} \quad \text{and} \quad P(B) = \frac{8}{23} \quad P(A \cap B) = 0$$

$$\therefore P(A \cup B) = P(A) + P(B) = \frac{5}{23} + \frac{8}{23} = \frac{13}{23}$$

$$26 \quad n(S) = {}^{30}C_2$$

Let A be the event of getting two oranges and

B be the event of getting two non-defective fruits and

$(A \cap B)$ be the event of getting two non-defective oranges

$$\therefore P(A) = \frac{{}^{20}C_2}{{}^{30}C_2}, P(B) = \frac{{}^{22}C_2}{{}^{30}C_2} \quad \text{and} \quad P(A \cap B) = \frac{{}^{15}C_2}{{}^{30}C_2}$$

$$\begin{aligned} \therefore P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{{}^{20}C_2}{{}^{30}C_2} + \frac{{}^{22}C_2}{{}^{30}C_2} - \frac{{}^{15}C_2}{{}^{30}C_2} = \frac{316}{435} \end{aligned}$$

Hint There are 20 oranges, and $30 - (3 + 5) = 22$ non-defective fruits and $20 - 5 = 15$ non-defective oranges.

$$27 \quad n(S) = 100$$

$$n(A) = 40, \quad n(B) = 20, \quad n(A \cap B) = 5$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{40}{100} + \frac{20}{100} - \frac{5}{100}$$

$$P(A \cup B) = \frac{55}{100} = 55\%$$

$$28 \quad P(A) = \frac{2}{5}, \quad P(B) = \frac{1}{3}$$

$$\therefore P(A \cup B) = \frac{2}{5} + \frac{1}{3} = \frac{11}{15}$$

($\because A$ and B are mutually exclusive events)

$$29 \quad P(H_1) = \frac{1}{3}, P(H_2) = \frac{1}{4}, P(H_3) = \frac{1}{5}, P(H_4) = \frac{1}{6}$$

$$\begin{aligned} \therefore P(H_1 \cup H_2 \cup H_3 \cup H_4) &= P(H_1) + P(H_2) + P(H_3) + P(H_4) \\ &= \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{57}{60} \end{aligned}$$

Hint Here, H_1, H_2, H_3 and H_4 are the mutually exclusive events.

Introductory Exercise 20.3

$$1 \quad P\left(\frac{A}{B}\right) = \frac{1}{2} \quad (\text{see the example 1})$$

$$2 \quad P(A \cup B) = 0.96 \quad (\text{see the example 1})$$

$$3 \quad \text{Here } S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

Let A be the event that one of the coins shows a tail

$$\therefore A = \{HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

$$\therefore P(A) = \frac{7}{8}$$

Now, let B be the event that they are all tails

$$\therefore B = \{TTT\}$$

$$\therefore P(B) = \frac{1}{8}$$

$$\therefore (A \cap B) = \{TTT\} \Rightarrow P(A \cap B) = \frac{1}{8}$$

$$\therefore P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1/8}{7/8} = \frac{1}{7}$$

$$4 \quad \text{Here sample space } S = \{HH, HT, TH, TT\}$$

$$E = \{HT, TH\}$$

$$F = \{HH, HT, TH\}$$

$$\therefore E \cap F = \{HT, TH\}$$

$$\therefore P(E) = \frac{2}{4} = \frac{1}{2} \quad \text{and} \quad P(F) = \frac{3}{4}$$

$$\text{and} \quad P(E \cap F) = \frac{2}{4} = \frac{1}{2}$$

$$\therefore P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{1/2}{3/4} = \frac{2}{3}$$

$$P\left(\frac{F}{E}\right) = \frac{P(E \cap F)}{P(E)} = \frac{1/2}{1/2} = 1$$

$$5 \quad \text{Here sample space}$$

$$S = \{HH, HT, T1, T2, T3, T4, T5, T6\}$$

Let A be the event that the die shows a number greater than 4 and B be the event that the first throw of the coin results in a tail then,

$$A = \{T5, T6\}$$

$$B = \{T1, T2, T3, T4, T5, T6\}$$

$$\therefore \text{Required probability} = P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{2/8}{6/8} = \frac{1}{3}$$

$$6 \quad S = \{1, 2, 3, 4, 5, 6\}$$

Let A be the event of getting an odd number and

B be the event of getting a prime number

$$\therefore A = \{1, 3, 5\}, \quad B = \{2, 3, 5\}$$

$$\text{and} \quad (A \cap B) = \{3, 5\}$$

$$\therefore P(A) = \frac{3}{6} = \frac{1}{2}, \quad P(B) = \frac{3}{6} = \frac{1}{2}, \quad P(A \cap B) = \frac{2}{6} = \frac{1}{3}$$

$$\therefore \text{Required probability} = P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1/3}{1/2} = \frac{2}{3}$$

$$7 \quad \text{Let } A \text{ be the event of getting the sum 9 and } B \text{ be the event of getting atleast one 4.}$$

$$\text{Then } A = \{(3, 6), (4, 5), (5, 4), (6, 3)\}$$

$$B = \{(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4),$$

$$(4, 1), (4, 2), (4, 3), (4, 5), (4, 6)\}$$

then

$$A \cap B = \{(4, 5), (5, 4)\}$$

$$\therefore \text{Required probability} = P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{n(A \cap B)}{n(A)} = \frac{2}{4} = \frac{1}{2}$$

- 8** Let A = the event of getting 4 on the first die.
and B = the event of getting the sum 8 or greater
 $\therefore A = \{(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)\}$
 $B = \{(4, 4), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 2), (2, 6), (3, 5), (3, 6), (5, 3), (6, 3), (6, 4), (6, 5), (6, 6)\}$
 $\therefore A \cap B = \{(4, 4), (4, 5), (4, 6)\}$
 $\therefore$ Required probability $= P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$
 $= \frac{n(A \cap B)}{n(A)} = \frac{3}{6} = \frac{1}{2}$
- 9** Let A = event of getting an odd number
and B = the event of getting a number greater than 1.
 $\therefore A = \{1, 3, 5\}, B = \{3, 5\}, A \cap B = \{3, 5\}$
 $\therefore$ Required probability $= P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{n(A \cap B)}{n(A)} = \frac{2}{3}$
- 10** Let A be the event of reading English and
 B be the event of reading French.
Then $P(A) = \frac{45}{100} = \frac{9}{20}$,
 $P(B) = \frac{30}{100} = \frac{3}{10}$
and $P(A \cap B) = \frac{20}{100} = \frac{1}{5}$
 $\therefore P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{5}}{\frac{3}{10}} = \frac{2}{3}$
- 11** $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1/5}{9/20} = \frac{4}{9}$
- 12** $S = (B_1B_2, B_1G_2, G_1B_2, G_1G_2)$
Where $B \rightarrow$ Boy and $G \rightarrow$ Girl
Let A be the event that both are boys
and B be the event that one of two is a boy.
Then $A = \{B_1B_2\}$ and $B = \{B_1B_2, B_1G_2, G_1B_2\}$
So $A \cap B = \{B_1B_2\}$
 $\therefore$ Required probability $= P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{1}{3}$
- 13** A be the event that both children are boys
and B be the event that the other child is a boy.
then $A = \{B_1, B_2\}$ and $B = \{B_1B_2, B_1G_2\}$
So $(A \cap B) = \{B_1B_2\}$
 $\therefore$ Required probability $= P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{1}{2}$
- 14** $P(E) = \frac{1}{2}$ **15** $P(F) = \frac{1}{2}$
- 16** $P\left(\frac{G}{E}\right)$ = Probability of drawing a red ball when first bag is selected
= Probability of drawing a red ball from first bag $= \frac{3}{7}$
- 17** $P\left(\frac{G}{F}\right)$ = Probability of drawing a red ball from second bag
 $= \frac{4}{6} = \frac{2}{3}$
- 18** Let A be the event of not getting a red ball in first draw and
 B be the event of not getting a red ball in second draw.
Then Required probability
= Probability that atleast one ball is red
= $1 -$ Probability that none is red
 $= 1 - P(A \text{ and } B) = 1 - P(A \cap B)$
 $= 1 - P(A) \cdot P\left(\frac{B}{A}\right) = 1 - \left(\frac{2}{3} \times \frac{5}{8}\right)$
 $= \frac{7}{12}$
Here $P(A) = \frac{6}{9} = \frac{2}{3}$
and $P\left(\frac{B}{A}\right) = \frac{5}{8}$ [There are 5 balls (excluding 3 red balls)
after the selection of one non-red ball]
- 19** Let A be the event drawing 4 red ball in first draw and B be
the event of drawing 4 blue balls in the second draw.
Then $P(A) = \frac{{}^6C_4}{{}^{15}C_4} = \frac{15}{1365} = \frac{1}{91}$
 $P\left(\frac{B}{A}\right) = \frac{{}^9C_4}{{}^{11}C_4} = \frac{126}{330} = \frac{21}{55}$
Hence, the required probability $= P(A \cap B)$
 $= P(A)P\left(\frac{B}{A}\right)$
 $= \frac{1}{91} \times \frac{21}{55} = \frac{3}{715}$
- 20** Let A, B, C be the events of getting a white ball in first,
second and third draw respectively, then
Required probability $= P(A \cap B \cap C)$
 $= P(A)P\left(\frac{B}{A}\right)P\left(\frac{C}{A \cap B}\right)$
Now, $P(A)$ = Probability of drawing a white ball in first
draw $= \frac{4}{18} = \frac{2}{9}$
When a white ball is drawn in the first draw there are
17 balls left in the urn, out of which 3 are white.
 $\therefore P\left(\frac{B}{A}\right) = \frac{3}{17}$
Since the ball drawn is not replaced, therefore after
drawing a white ball in the second draw there are 16 balls
left in the urn, out of which 2 are white.
 $\therefore P\left(\frac{C}{A \cap B}\right) = \frac{2}{16} = \frac{1}{8}$
Hence the required probability
 $= \frac{2}{9} \times \frac{3}{17} \times \frac{1}{8} = \frac{1}{204}$

- 21** There are 4 even numbers and 5 odd numbers

Let A = the event of choosing odd numbers

B = the event of getting the sum an even number.

Then $A \cap B$ = The event of choosing odd numbers whose sum is even.

$$\therefore n(B) = {}^4C_2 + {}^5C_2 = 16$$

$$\text{and } n(A \cap B) = {}^5C_2 = 10$$

$$\begin{aligned} \therefore \text{Required probability} &= P\left(\frac{A}{B}\right) \\ &= \frac{n(A \cap B)}{n(B)} \\ &= \frac{10}{16} = \frac{5}{8} \end{aligned}$$

✓ Introductory Exercise 20.4

1 $P(A \cap B) = P(A) \cdot P(B) = 0.6 \times 0.4 = 0.24$

2 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $= 0.6 + 0.4 - 0.24 = 0.76$

3 $P(\bar{A} \cap B) = \{1 - P(A)\} P(B)$
 $= 0.4 \times 0.4 = 0.16$

4 $P(A \cap \bar{B}) = P(A) \{1 - P(B)\}$
 $= 0.6 \times 0.6 = 0.36$

5 $P(\bar{A} \cap \bar{B}) = \{1 - P(A)\} \{1 - P(B)\}$
 $= 0.4 \times 0.6 = 0.24$

6 $P(\bar{A}) = 0.65 \Rightarrow P(A) = 0.35$
 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $\Rightarrow P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$
 $\Rightarrow 0.65 = 0.35 + p - 0.35p$
 $\Rightarrow 0.65p = 0.30 \Rightarrow p = 6/13$

- 7** $S = \{1, 2, 3, 4, 5, 6\}$, for each case

Let A = event of getting a 1, 2, 3 or 4 on the first toss
 and B = event of getting a 4, 5 or 6 on the second toss

Then, clearly A and B are independent events.

$$\therefore P(A) = \frac{4}{6} = \frac{2}{3} \quad \text{and} \quad P(B) = \frac{3}{6} = \frac{1}{2}$$

So, required probability = $P(A \cap B)$

$$= P(A) \cdot P(B) = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

- 8** Let E = the event that A gets a three
 and F = the event that B gets a three

$$\begin{aligned} \text{Then, } P(E) &= \frac{1}{6}, & P(F) &= \frac{1}{6} \\ P(\bar{E}) &= \frac{5}{6}, & P(\bar{F}) &= \frac{5}{6} \end{aligned}$$

Suppose A wins then, he gets a three in 1st or 3rd or 5th ... throw etc.

- 22** Let A be the event of drawing an odd numbered ticket in the first draw and B be the event of drawing an odd numbered ticket in the second draw. Then

$$\text{Required probability} = P(A \cap B) = P(A) P\left(\frac{B}{A}\right)$$

$$P(A) = \frac{13}{25}, \text{ since there 13 odd numbers } 1, 3, 5, \dots, 25.$$

Since the ticket drawn in the first draw is not replaced, therefore second ticket drawn is from the remaining 24 tickets, out of which 12 are odd numbered.

$$\therefore P\left(\frac{B}{A}\right) = \frac{12}{24} = \frac{1}{2}$$

$$\text{Hence, Required probability} = \frac{13}{25} \times \frac{1}{2} = \frac{13}{50}$$

$$\begin{aligned} \therefore P(A \text{ wins}) &= P[E \text{ or } (\bar{E} \bar{F} E) \text{ or } (\bar{E} \bar{F} \bar{E} \bar{F} E) \text{ or } \dots \infty] \\ &= P[E \text{ or } (\bar{E} \text{ and } \bar{F} \text{ and } E) \text{ or } (\bar{E} \text{ and } \bar{F} \text{ and } \bar{E} \text{ and } \bar{F} \text{ and } E) \\ &\quad \text{or } \dots] \end{aligned}$$

$$= P(E) + P(\bar{E} \text{ and } \bar{F} \text{ and } E) + P(\bar{E} \text{ and } \bar{F} \text{ and } \bar{E} \text{ and } \bar{F} \text{ and } E) + \dots \infty$$

$$= P(E) + P(\bar{E}) P(\bar{F}) P(E) + P(\bar{E}) P(\bar{F})$$

$$\cdot P(\bar{E}) P(\bar{F}) \cdot P(E) + \dots \infty$$

$$= \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \dots \infty$$

$$= \frac{1}{6} + \frac{1}{6} \left(\frac{5}{6}\right)^2 + \frac{1}{6} \left(\frac{5}{6}\right)^4 + \dots \infty$$

$$= \frac{1}{6} \left[1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \dots \infty \right]$$

$$= \frac{1}{6} \cdot \frac{1}{1 - \left(\frac{5}{6}\right)^2} = \left(\frac{1}{6} \cdot \frac{36}{11}\right) = \frac{6}{11}$$

$$\text{Thus, } P(A \text{ wins}) = \frac{6}{11} \text{ and } P(B \text{ wins}) = \frac{5}{11}$$

- 9** We have, $P(H) = \frac{1}{2}$ and $P(T) = \frac{1}{2}$

Now, A wins if he throws a head in 1st, or 3rd or 5th or ... draw.

$$\therefore P(A \text{ wins}) = P[H \text{ or } (TTH) \text{ or } (TTTTH) \text{ or } (TTTTTTH) \text{ or } \dots]$$

$$= P(H) + P(TTH) + P(TTTTH) + \dots$$

$$= P(H) + P(T) P(T) P(H) + P(T) P(T) P(T) P(H) + \dots$$

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^2 \frac{1}{2} + \left(\frac{1}{2}\right)^4 \frac{1}{2} + \dots \infty$$

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \dots \infty$$

$$= \frac{1}{2} \left[1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \dots \infty \right]$$

$$= \frac{1}{2} \cdot \frac{1}{1 - \left(\frac{1}{2}\right)^2} = \left(\frac{1}{2} \times \frac{4}{3}\right) = \frac{2}{3}$$

Thus $P(A \text{ wins}) = \frac{2}{3}$ and $P(B \text{ wins}) = \left(1 - \frac{2}{3}\right) = \frac{1}{3}$

- 10** Let E be the event of 'getting a six'. Then $\bar{E}$ is the event of 'not getting a six'.

Then $P(E) = \frac{1}{6}$ and $P(\bar{E}) = \frac{5}{6}$

Now, A wins if he throws a six in 1st, 4th or 7th or ... draw.

$$\therefore P(A \text{ wins}) = P(E \text{ or } (\bar{E} \bar{E} \bar{E} E) \text{ or } (\bar{E} \bar{E} \bar{E} \bar{E} \bar{E} E) \text{ or } \dots)$$

$$= P(E) + P(\bar{E} \bar{E} \bar{E} E) + P(\bar{E} \bar{E} \bar{E} \bar{E} \bar{E} E) + \dots$$

$$= \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^6 \cdot \frac{1}{6} + \dots \infty$$

$$= \frac{1}{6} \left[1 + \left(\frac{5}{6}\right)^3 + \left(\frac{5}{6}\right)^6 + \dots \infty \right]$$

$$= \frac{1}{6} \cdot \frac{1}{1 - \left(\frac{5}{6}\right)^3} = \left(\frac{1}{6} \cdot \frac{216}{91}\right) = \frac{36}{91}$$

Now, B wins if he throws a six in 2nd or 5th or 8th or ... draw.

$$\therefore P(B \text{ wins}) = P[(\bar{E} E) \text{ or } (\bar{E} \bar{E} \bar{E} \bar{E} E) \text{ or } (\bar{E} \bar{E} \bar{E} \bar{E} \bar{E} \bar{E} E) \text{ or } \dots]$$

$$= P(\bar{E} E) + P(\bar{E} \bar{E} \bar{E} \bar{E} E) + P(\bar{E} \bar{E} \bar{E} \bar{E} \bar{E} \bar{E} E) + \dots$$

$$= \frac{5}{6} \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^7 \cdot \frac{1}{6} + \dots \infty$$

$$= \frac{5}{6} \left[\frac{1}{6} + \left(\frac{5}{6}\right)^3 + \left(\frac{5}{6}\right)^6 + \dots \infty \right]$$

$$= \frac{5}{36} \cdot \frac{1}{1 - \left(\frac{5}{6}\right)^3} = \left(\frac{5}{36} \cdot \frac{216}{91}\right) = \frac{30}{91}$$

Thus, $P(A \text{ wins}) = \frac{36}{91}$ and $P(B \text{ wins}) = \frac{30}{91}$

$$\therefore P(A \text{ wins or } B \text{ wins}) = P(A \text{ wins}) + P(B \text{ wins})$$

$$= \frac{36}{91} + \frac{30}{91} = \frac{66}{91}$$

and $P(C \text{ wins}) = 1 - P(A \text{ wins or } B \text{ wins})$

$$= 1 - \frac{66}{91} = \frac{25}{91}$$

- 11** Let E = Event of getting a sum 9 on two dice.

Then, $E = \{(3, 6), (4, 5), (5, 4), (6, 3)\}$

$$\therefore P(E) = \frac{4}{36} = \frac{1}{9}$$

and $P(\bar{E}) = \frac{8}{9}$

$$\therefore P(A \text{ wins}) = P[E_1 \text{ or } \bar{E}_1 \bar{E}_2 E_3 \text{ or } \bar{E}_1 \bar{E}_2 \bar{E}_3 \bar{E}_4 E_5 \text{ or } \dots]$$

$$= \left[\frac{1}{9} + \frac{8}{9} \cdot \frac{8}{9} \cdot \frac{1}{9} + \frac{8}{9} \cdot \frac{8}{9} \cdot \frac{8}{9} \cdot \frac{1}{9} + \dots \infty \right]$$

$$= \frac{9}{17}$$

$$\therefore P(B \text{ wins}) = \left(1 - \frac{9}{17}\right) = \frac{8}{17}$$

$$\therefore \text{Required ratio} = \frac{9}{8}$$

12 Required probability = $\frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$

- 13** Let A = the event that Ashmit solves the problem.

and B = the event that Amisha solves the problem.

Clearly, A and B are independent events.

Now, $P(A) = \frac{80}{100} = \frac{8}{10}$ and $P(B) = \frac{70}{100} = \frac{7}{10}$

$$\therefore P(A \cap B) = P(A) \cdot P(B) = \frac{8}{10} \times \frac{7}{10} = \frac{56}{100}$$

So $P(A \text{ or } B) = P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{8}{10} + \frac{7}{10} - \frac{56}{100} = \frac{94}{100} = 0.94$$

- 14** Let A = the event that A hits the target

and B = the event that B hits the target

As given, we have $P(A) = \frac{1}{3}$ and $P(B) = \frac{2}{5}$

Clearly, A and B are independent events

$$\therefore P(A \cap B) = P(A) \cdot P(B) = \frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$$

$$\therefore P(\text{target is hit}) = P(A \text{ hits or } B \text{ hits})$$

$$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{3} + \frac{2}{5} - \frac{2}{15} = \frac{3}{5}$$

- 15** Let A, B, C be the respective events of solving the problem and $\bar{A}, \bar{B}, \bar{C}$ be the respective events of not solving the problem. Then A, B, C are independent events

$\therefore \bar{A}, \bar{B}, \bar{C}$ are independent events

Now, $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(C) = \frac{1}{4}$

$$P(\bar{A}) = \frac{1}{2}, P(\bar{B}) = \frac{2}{3} \text{ and } P(\bar{C}) = \frac{3}{4}$$

$\therefore P(\text{none solves the problem})$

$$= P(\text{not } A) \text{ and } (\text{not } B) \text{ and } (\text{not } C)]$$

$$= P(\bar{A} \cap \bar{B} \cap \bar{C})$$

$$= P(\bar{A}) P(\bar{B}) P(\bar{C}) \quad (\because \bar{A}, \bar{B} \text{ and } \bar{C} \text{ are independent})$$

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$$

Hence, $P(\text{the problem will be solved})$

$$= 1 - P(\text{none solves the problem})$$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

- 16** Let A, B, C respectively. Then $\bar{A}, \bar{B}, \bar{C}$ are the respective events of not solving the problem by them

$$\text{Now, } P(A) = \frac{1}{3}, P(B) = \frac{2}{7} \text{ and } P(C) = \frac{3}{8}$$

$$\therefore P(\bar{A}) = \frac{2}{3}, P(\bar{B}) = \frac{5}{7} \text{ and } P(\bar{C}) = \frac{5}{8}$$

$\therefore$ The probability that exactly one of them will solve it
 $= P\{[A \cap (\text{not } B) \cap (\text{not } C)] \text{ or } [(\text{not } A) \cap B \cap (\text{not } C)]$

$$\text{or } [(\text{not } A) \cap (\text{not } B) \cap C]\}$$

$$= P\{(A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)\}$$

$$= P(A \cap \bar{B} \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{A} \cap \bar{B} \cap C)$$

$$[\because (A \cap \bar{B} \cap \bar{C}) \cap (\bar{A} \cap B \cap \bar{C}) \cap (\bar{A} \cap \bar{B} \cap C) = \phi]$$

$$= P(A)P(\bar{B})P(\bar{C}) + P(\bar{A}) \cdot P(B) \cdot P(\bar{C}) + P(\bar{A}) \cdot P(\bar{B}) \cdot P(C)$$

$$= \left(\frac{1}{3} \times \frac{5}{7} \times \frac{5}{8}\right) + \left(\frac{2}{3} \times \frac{2}{7} \times \frac{5}{8}\right) + \left(\frac{2}{3} \times \frac{5}{7} \times \frac{3}{8}\right)$$

$$= \frac{25}{168} + \frac{5}{42} + \frac{5}{28} = \frac{25}{56}$$

- 17** Let A, B, C be the events that A hits the target, B hits the target and C hits the target respectively.

$$\text{Then, } P(A) = \frac{4}{5}, P(B) = \frac{3}{4}, P(C) = \frac{2}{3}$$

$$P(\bar{A}) = \frac{1}{5}, P(\bar{B}) = \frac{1}{4}, P(\bar{C}) = \frac{1}{3}$$

Case I. $P(A, B, \text{ and } C, \text{ all hit the target}) = P(A \cap B \cap C)$

$$= P(A)P(B)P(C) = \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{2}{5}$$

Case II. $P(A \text{ and } B \text{ hit but not } C) = P[A \cap B \cap \bar{C}]$

$$= P(A)P(B)P(\bar{C})$$

$$= \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} = \frac{1}{5}$$

Case III. $P(A \text{ and } C \text{ hit but not } B) = P(A \cap C \cap \bar{B})$

$$= P(A)P(C)P(\bar{B})$$

$$= \frac{4}{5} \times \frac{2}{3} \times \frac{1}{4} = \frac{2}{15}$$

Case IV. $P(B \text{ and } C \text{ hit but not } A) = P(B \cap C \cap \bar{A})$

$$= P(B)P(C)P(\bar{A}) = \frac{3}{4} \times \frac{2}{3} \times \frac{1}{5} = \frac{1}{10}$$

All the above cases being mutually exclusive, we have the required probability $= \frac{2}{5} + \frac{1}{5} + \frac{2}{15} + \frac{1}{10} = \frac{5}{6}$

- 18** Let $P_1 = 0.1, P_2 = 0.2, P_3 = 0.3, P_4 = 0.4$

$\therefore P$ (The balloon is hit) $= P$ (the balloon is hit atleast once)

$$= 1 - P(\text{the balloon is hit in none of the shots})$$

$$= 1 - (1 - P_1)(1 - P_2)(1 - P_3)(1 - P_4)$$

$$= 1 - (0.9)(0.8)(0.7)(0.6) = 0.6976$$

- 19** $P(A) = \frac{2}{3}, P(B) = \frac{3}{5}$

Required probability $= P(A \text{ or } B) = P(A \cup B)$ be the events of solving the problem by A, B, C

$$= P(A) + P(B) - P(A)P(B) = \frac{2}{3} + \frac{3}{5} - \frac{2}{3} \cdot \frac{3}{5} = \frac{13}{15}$$

$$\text{20 } P(\text{none}) = 1 - P(\text{atleast one}) = 1 - P(A \cup B) = 1 - \frac{13}{15} = \frac{2}{15}$$

$$\text{21 } P(\text{not } A) = 1 - P(A) = 1 - 0.4 = 0.6$$

$$\text{22 } P(\text{at most } C) = P(D \text{ or } C) = P(D) + P(C) = (0.1) + (0.2) = 0.3$$

$$\text{23 } P(B \text{ or } C) = P(B) + P(C) \\ = (0.3) + (0.2) = 0.5$$

$$\text{24 } P(\text{atleast } B) = P(B \text{ or } A) = P(B) + P(A) = (0.3) + (0.4) = 0.7$$

Solutions (for Q. Nos. 25 to 28) Let E = the event that Ajay is selected and F = the event that Kajol is selected Clearly, E and F are independent events

$$\text{Now, } P(E) = \frac{1}{7} \text{ and } P(F) = \frac{1}{5}$$

$$\therefore P(\bar{E}) = \frac{6}{7} \text{ and } P(\bar{F}) = \frac{4}{5}$$

$$\text{25 } P(\text{Both of them will be selected}) = P(E \text{ and } F) = P(E \cap F) \\ = P(E)P(F) = \frac{1}{7} \times \frac{1}{5} = \frac{1}{35}$$

26 $P(\text{only one of them will be selected})$

$$= P[(E \text{ and not } F) \text{ or } (F \text{ and not } E)]$$

$$= P[(E \cap \bar{F}) \cup (F \cap \bar{E})]$$

$$= P(E \cap \bar{F}) + P(F \cap \bar{E})$$

$$= P(E)P(\bar{F}) + P(F)P(\bar{E}) = \frac{1}{7} \times \frac{4}{5} + \frac{1}{5} \times \frac{6}{7} = \frac{2}{7}$$

27 $P(\text{none of them will be selected})$

$$= P[(\text{not } E) \text{ and } (\text{not } F)]$$

$$= P(\bar{E} \cap \bar{F}) = P(\bar{E}) \times P(\bar{F}) = \frac{6}{7} \times \frac{4}{5} = \frac{24}{35}$$

28 $P(\text{atleast one of them will be selected})$

$$= 1 - P(\text{none will be selected})$$

$$= 1 - \frac{24}{35} = \frac{11}{35}$$

29 Let E = the event that A speaks the truth

and F = the event that B speaks the truth

then $\bar{E}$ = the event that A tells a lie

and $\bar{F}$ = the event that B tells a lie.

Clearly, E and F are independent events, So, E and $\bar{F}$ as well as $\bar{E}$ and F are independent.

$$\text{Now, } P(E) = \frac{60}{100} = \frac{3}{5}, P(F) = \frac{80}{100} = \frac{4}{5}$$

$$\therefore P(\bar{E}) = \frac{2}{5}, P(\bar{F}) = \frac{1}{5}$$

$\therefore P(A \text{ and } B \text{ contradict each other})$

$$= P(A \text{ speaks the truth and } B \text{ tells a lie})$$

or $(A \text{ tells a lie and } B \text{ speaks the truth})$

$$= P[(E \cap \bar{F}) \cup (\bar{E} \cap F)]$$

$$= P(E \cap \bar{F}) + P(\bar{E} \cap F)$$

$$= P(E)P(\bar{F}) + P(\bar{E})P(F)$$

$$= \frac{3}{5} \times \frac{1}{5} + \frac{2}{5} \times \frac{4}{5} = \frac{11}{25} \Rightarrow 44\%$$

So A and B contradict each other in 44% cases.

Solutions (for Q. Nos. 30 and 31) Let E = the event that the husband will be alive 20 years hence and F = the event that the wife will be alive 20 years hence.

$$\text{Then } P(E) = \frac{5}{12} \text{ and } P(F) = \frac{3}{8}$$

$$\therefore P(\bar{E}) = \frac{7}{12} \text{ and } P(\bar{F}) = \frac{5}{8}$$

Clearly, E and F are independent events.

30 $P(\text{Couple will be alive 20 years hence})$

$$\begin{aligned} &= P(E \text{ and } F) = P(E \cap F) \\ &= P(E) \cdot P(F) = \frac{5}{12} \times \frac{3}{8} = \frac{5}{32} \end{aligned}$$

31 $P(\text{atleast one of them will be alive 20 years hence})$

$$\begin{aligned} &= 1 - P(\text{none will be alive 20 years hence}) \\ &= 1 - P(\bar{E} \cap \bar{F}) \\ &= 1 - P(\bar{E})P(\bar{F}) \quad (\because \bar{E} \text{ and } \bar{F} \text{ are independent}) \\ &= 1 - \left(\frac{7}{12} \times \frac{5}{8}\right) = \frac{61}{96} \end{aligned}$$

32 Let A, B, C denote the events of favouring the book by the first, the second and the third critic respectively.

$$\text{Then } P(A) = 5/7, \quad P(B) = 4/7 \text{ and } P(C) = 3/7$$

$$\therefore P(\bar{A}) = 2/7, \quad P(\bar{B}) = 3/7 \text{ and } P(\bar{C}) = 4/7$$

$\therefore$ Required probability

$$\begin{aligned} &= P(\text{two favour the book or three favour the book}) \\ &= P(\text{two favour the book}) + P(\text{three favour the book}) \\ &= P[\{A \text{ and } B (\text{not } C)\} \cup \{A \text{ and } (\text{not } B) \text{ and } C\} \\ &\quad \cup \{(\text{not } A) \text{ and } B \text{ and } C\}] + P(A \text{ and } B \text{ and } C) \\ &= P[(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C)] \\ &\quad + P(A \cap B \cap C) \\ &= P(A)P(B)P(\bar{C}) + P(A)P(\bar{B})P(C) + P(\bar{A})P(B)P(C) \\ &\quad + P(A)P(B)P(C) \\ &= \left(\frac{5}{7} \times \frac{4}{7} \times \frac{4}{7}\right) + \left(\frac{5}{7} \times \frac{3}{7} \times \frac{3}{7}\right) + \left(\frac{2}{7} \times \frac{4}{7} \times \frac{3}{7}\right) + \left(\frac{5}{7} \times \frac{4}{7} \times \frac{3}{7}\right) \\ &= \frac{209}{343} \end{aligned}$$

33 Required probability

$$\begin{aligned} &= P(X \text{ not defective and } Y \text{ not defective}) \\ &= P(\bar{X})P(\bar{Y}) = \{1 - P(X)\} \{1 - P(Y)\} \\ &= \frac{91}{100} \times \frac{95}{100} = \frac{8645}{10000} = 0.8645 \end{aligned}$$

34 A red ball can be drawn in two mutually exclusive ways

(i) Selecting bag I and then drawing a red ball from it.

(ii) Selecting bag II and then drawing a red ball from it.

Let E_1, E_2 and A denote the events defined as follows :

E_1 = selecting bag I, E_2 = selecting bag II

A = drawing a red ball.

Since one of the two bags is selected randomly, therefore

$$P(E_1) = \frac{1}{2} \text{ and } P(E_2) = \frac{1}{2}$$

Now, $P\left(\frac{A}{E_1}\right)$ = probability of drawing a red ball when the first bag has been chosen = $4/7$

$P\left(\frac{A}{E_2}\right)$ = probability of drawing a red ball when the second bag has been selected = $2/6$

Using the law of total probability, we have

$$\begin{aligned} P(\text{red ball}) &= P(A) = P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) \\ &= \frac{1}{2} \times \frac{4}{7} + \frac{1}{2} \times \frac{2}{6} = \frac{19}{42} \end{aligned}$$

35 Let E_1, E_2, E_3 and A be the events defined as follows.

E_1 = the toy is manufactured by machine A

E_2 = the toy is manufactured by machine B

E_3 = the toy is manufactured by machine C

and A = the toy is defective then

$$P(E_1) = \frac{25}{100} = \frac{1}{4}$$

$$P(E_2) = \frac{35}{100} = \frac{7}{20}$$

$$P(E_3) = \frac{40}{100} = \frac{2}{5}$$

and $P\left(\frac{A}{E_1}\right)$ = Probability that the toy drawn is defective

given the condition that it is a manufactured by machine A = $5/100$

$$\text{Similarly, } P\left(\frac{A}{E_2}\right) = \frac{4}{100} \text{ and } P\left(\frac{A}{E_3}\right) = \frac{2}{100}$$

Using law of total probability, we have

$$\begin{aligned} P(A) &= P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right) \\ &= \frac{25}{100} \times \frac{5}{100} + \frac{35}{100} \times \frac{4}{100} + \frac{40}{100} \times \frac{2}{100} \\ &= 0.345 \end{aligned}$$

36 The testing procedure may terminate at the twelfth testing in two mutually exclusive ways

(i) When lot contain 2 defective bulbs.

(ii) When lot contains 3 defective bulbs .

Consider the following events :

A = testing procedure ends the twelfth testing

E_1 = lot contains 2 defective bulbs.

E_2 = lot contains 3 defective bulbs.

Required Probability = $P(A)$

$$= P(A \cap E_1) \cup P(A \cap E_2)$$

$$= P(A \cap E_1) + P(A \cap E_2)$$

$$= P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)$$

Now, $P\left(\frac{A}{E_1}\right)$ = Probability that first 11 draws contain 10 non defective and one defective and 12th draw contains a defective article.

$$= \frac{{}^{18}C_{10} \times {}^2C_1}{{}^{20}C_{11}} \times \frac{1}{9}$$

and $P\left(\frac{A}{E_2}\right)$ = Probability that first 11 draws contain

9 non-defective and 2 defective articles and twelfth draw contains a defective article.

$$= \frac{{}^{17}C_9 \times {}^3C_2}{{}^{20}C_{11}} \times \frac{1}{9}$$

Hence, Required probability

$$= 0.4 \times \frac{{}^{18}C_{10} \times {}^2C_1}{{}^{20}C_{11}} \times \frac{1}{9} + 0.6 \times \frac{{}^{17}C_9 \times {}^3C_2}{{}^{20}C_{11}} \times \frac{1}{9}$$

37 Let E_1, E_2, E_3 and A be the events defined as follows :

E_1 = the toy is manufactured by machine A

E_2 = the toy is manufactured by machine B

E_3 = the toy is manufactured by machine C

A = the toy is defective

Then $P(E_1)$ = probability that the toy drawn is manufactured by machine A = 25/100

Similarly,

$$P(E_2) = 35/100 \text{ and } P(E_3) = 40/100$$

$p(A/E_1)$ = probability that the toy drawn is defective given that it is manufactured by machine A = $\frac{5}{100}$

$$\text{Similarly } P\left(\frac{A}{E_2}\right) = \frac{4}{100}, \quad P\left(\frac{A}{E_3}\right) = \frac{2}{100}$$

Now, required probability

$$\begin{aligned} &= \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)} \\ &= \frac{\frac{35}{100} \times \frac{4}{100}}{\frac{25}{100} \times \frac{5}{100} + \frac{35}{100} \times \frac{4}{100} + \frac{40}{100} \times \frac{2}{100}} = \frac{28}{69} \end{aligned}$$

38 Let E_1, E_2, E_3 and A be the events defined as follows :

E_1 = construction chosen is a bridge.

E_2 = construction chosen is a hospital

E_3 = construction chosen is a hotel.

A = construction gets damaged

Since there are 1200 constructions, therefore

$$P(E_1) = \frac{200}{1200} = \frac{1}{6}, \quad P(E_2) = \frac{400}{1200} = \frac{1}{3}$$

$$\text{and } P(E_3) = \frac{600}{1200} = \frac{1}{2}$$

It is given that $P\left(\frac{A}{E_1}\right)$ = Probability that a construction gets damaged is a bridge = 0.01

$$\text{Similarly, } P\left(\frac{A}{E_2}\right) = 0.03 \text{ and } P\left(\frac{A}{E_3}\right) = 0.15$$

We are required to find $P\left(\frac{E_1}{A}\right)$ by Baye's rule.

$$\begin{aligned} P\left(\frac{E_1}{A}\right) &= \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)} \\ &= \frac{\frac{1}{6} \times 0.01}{\frac{1}{6} \times 0.01 + \frac{1}{3} \times 0.03 + \frac{1}{2} \times 0.15} = \frac{1}{1 + 6 + 45} = \frac{1}{52} \end{aligned}$$

39 Let E_1, E_2 and A be the events defined as follows :

E_1 = selecting a box from the first group

E_2 = selecting a box from the second group and

A = ball drawn is green

Since there are 5 boxes out of which 3 boxes belong to first group and 2 boxes belong to the second group. Therefore

$$P(E_1) = 3/5, \quad P(E_2) = 2/5$$

If E_1 has already occurred, then a box from the first group is chosen. The box chosen contains 5 green balls and 3 red balls. Therefore the probability of drawing a green ball from it is 5/8.

$$\text{So } P\left(\frac{A}{E_1}\right) = \frac{5}{8}$$

$$\text{Similarly, } P\left(\frac{A}{E_2}\right) = \frac{2}{6} = \frac{1}{3}$$

Now, we have to find $P\left(\frac{E_1}{A}\right)$

By Baye's rule, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)} = \frac{\frac{3}{5} \times \frac{5}{8}}{\frac{3}{5} \times \frac{5}{8} + \frac{2}{5} \times \frac{1}{3}} = \frac{45}{61}$$

40 Let E_1, E_2 and A be the events defined as follows :

E_1 = six occurs, E_2 = six does not occur

and A = the man reports that it is a six.

$$\text{We have, } P(E_1) = \frac{1}{6}, \quad P(E_2) = \frac{5}{6}$$

Now $P\left(\frac{A}{E_1}\right)$ = probability that the man reports that there is a six on the die given that six has occurred on the die.

= probability that the man speaks truth = 3/4

and $P\left(\frac{A}{E_2}\right)$ = probability that the man reports that there is six on the die given that six has not occurred on the die.

= probability that the man does not speak truth

$$= 1 - \frac{3}{4} = \frac{1}{4}$$

We have to find $P\left(\frac{E_1}{A}\right)$

By Baye's rule, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{1/6 \times 3/4}{1/6 \times 3/4 + 5/4 \times 1/4} = \frac{3}{8}$$

- 41 Let E_1, E_2, E_3, E_4 and A be the events as defined below :

E_1 = the missing card is diamond

E_2 = the missing card is heart

E_3 = the missing card is spade

E_4 = the missing card is club

A = drawing two diamonds cards from the remaining cards

Then $P(E_1) = \frac{13}{52} = \frac{1}{4}$, $P(E_2) = \frac{13}{52} = \frac{1}{4}$

$P(E_3) = \frac{13}{52} = \frac{1}{4}$ and $P(E_4) = \frac{13}{52} = \frac{1}{4}$

$P\left(\frac{A}{E_1}\right)$ = probability of drawing two diamonds cards given

that one diamond card is missing = $\frac{{}^{12}C_2}{{}^{51}C_2}$

$P\left(\frac{A}{E_2}\right)$ = probability of drawing two diamond cards given

that one heart card is missing = $\frac{{}^{13}C_2}{{}^{51}C_2}$

Similarly, $P\left(\frac{A}{E_3}\right) = \frac{{}^{13}C_2}{{}^{51}C_2}$ and $P\left(\frac{A}{E_4}\right) = \frac{{}^{13}C_2}{{}^{51}C_2}$

By, Baye's rule

Required probability = $P\left(\frac{E_1}{A}\right)$

$$= \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right) + P(E_4)P\left(\frac{A}{E_4}\right)}$$

$$= \frac{11}{50}$$

✓ Level 01 Basic Level Exercise

- 1 Total possible nine digit numbers = 9!

Out of these 9! numbers only those numbers are divisible by 4 which have their last digits as even natural number and the numbers formed by their last two digits are divisible by 4. The possible numbers of last two digits are 12, 32, 52, 72, 92, 24, 64, 84, 16, 36, 56, 76, 96, 28, 48, 68.

Thus there are 16 ways of choosing the last two digits. Corresponding to each of these ways the remaining 7 digits can be arranged in 7! ways. Therefore, the total number of 9 digits numbers divisible by 4 is $16 \times 7!$.

Hence, required probability = $16 \times 7! / 9! = 2/9$

- 2 Total possible number of 4 digits = $4! = 24$

The number is divisible by 5 if unit digit itself is 5. Therefore we fix 5 at unit place and then remaining 3 places can be filled up in 3! ways.

Hence, the required probability = $3! / 4! = 6/24 = 1/4$

- 3 Excluding ground floor there are 7 floors. A person can leave the cabin at any of the 7 floors. Therefore, the total number of ways in which each of the 5 persons can leave the cabin at any of the 7 floors = 7^5 .

$\therefore$ Exhaustive number of ways = 7^5

Five persons can leave the cabin at five different floors in ${}^7P_5$ ways.

$\therefore$ The favourable number of cases = ${}^7P_5$

Hence, the required probability = $\frac{{}^7P_5}{7^5} = \frac{360}{2401}$

- 4 Box 1 Box 2 Box 3

3W 2W 1W

1B 2B 3B

There can be three mutually exclusive cases of drawing 2 white balls and 1 black ball.

	Box 1	Box 2	Box 3
Case 1	1W	1W	1B
Case 2	1W	1B	1W
Case 3	1B	1W	1W

$\therefore$ Required probability

$$= P(W_1 \cap W_2 \cap B_3) \cup (W_1 \cap B_2 \cap W_3) \cup (B_1 \cap W_2 \cap W_3)$$

$$= P(W_1)P(W_2)P(B_3) + P(W_1)P(B_2)P(W_3)$$

$$+ P(B_1)P(W_2)P(W_3)$$

$$= \frac{3}{4} \times \frac{2}{4} \times \frac{3}{4} + \frac{3}{4} \times \frac{2}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{2}{4} \times \frac{1}{4} = \frac{26}{64} = \frac{13}{32}$$

- 5 Since coin is fair i.e., with equal probability a heads and a tail can be obtained. Also all the trials are independent so the probability that head appears on the fifth toss does not depend upon previous results of the tosses.

$\therefore$ Required probability = $1/2$

- 6 The total number of ways in which two calculators can be chosen out of four calculators is ${}^4C_2 = 6$. If only two tests are required to identify defective calculators, then in first two tests defective calculators are identified. This can be done in one way only.

$\therefore$ Required probability = $1/6$

- 7 20 girls can be seated around a round table in $19!$ ways.

So, exhaustive number of cases = $19!$

Excluding A and B , out of remaining 18 girls, 4 girls can be selected ${}^{18}C_4$ ways which can be arranged in $4!$ ways.

Remaining $20 - (4 - 2) = 14$ girls can be arranged in $14!$ ways. Also A and B mutually can be arranged in $2!$ ways.

$\therefore$ Required number of arrangements

$$= {}^{18}C_4 \times 4! \times 2! \times 14!$$

$$= 18! \times 2$$

$$\therefore \text{Required probability} = \frac{18! \times 2}{19!} = \frac{2}{19}$$

- 8 Since x and y can take values from 0 to 10. So, the total number of ways of selecting x and y is $11 \times 11 = 121$

Now, $|x - y| > 5 \Rightarrow x - y < -5$ or $x - y > 5$

There are 30 pairs of values of x and y satisfying these two inequalities, so favourable number of ways = 30

$$\text{Hence, required probability} = \frac{30}{121}$$

- 9 Since a person's birthday can fall in any of the 12 months.

So, total number of ways = 12^4 .

Now, any two months can be chosen in ${}^{12}C_2$ ways. The

4 persons birthday can fall in these two months in 2^4 ways.

Out of these 2^4 ways there are two ways when all of the four birthdays fall in one month.

$$\text{So, favourable number of ways} = \frac{{}^{12}C_2 \times (2^4 - 2)}{12^4} = \frac{77}{1728}$$

- 10 6 objects can be distributed among 6 persons in 6^6 ways.

$\therefore$ Total number of ways = 6^6

The number of ways of distribution in which each one gets only one thing is $6!$. So, the number of distribution in which atleast one of them does not get any thing is $6^6 - 6!$

$$\text{Hence, required probability} = \frac{6^6 - 6!}{6^6}$$

- 11 Total number of ways of arranging 40 books on a shelf = $40!$

Out of 40 places, 4 places for the four volumes can be chosen in ${}^{40}C_4$ ways. In the remaining 36 places the remaining 36 books can be arranged in $36!$ ways. In the 4 places four volumes of encyclopedia can be arranged in increasing order in one way only.

So, favourable number of ways = ${}^{40}C_4 \times 36!$

$$\text{Hence, required probability} = \frac{{}^{40}C_4 \times 36!}{40!} = \frac{1}{24}$$

- 12 The divisibility of the product of four numbers depends upon the value of the last digit of each number. The last digit of a number can be any of the 10 digits 0, 1, 2 ... 9. So, the total number of ways of selecting last digits of four numbers is $10 \times 10 \times 10 \times 10 = 10^4$.

If the product of the 4 numbers is not divisible by 5 or 10.

Then the number of choices for the last digit of each number is 8 (excluding 0 or 5).

So, favourable number of ways = $8 \times 8 \times 8 \times 8 = 8^4$

$\therefore$ The probability that the product is not divisible by 5 or 10

$$= \frac{8^4}{10^4} = \left(\frac{8}{10}\right)^4$$

$$\text{Hence, Required probability} = 1 - \left(\frac{8}{10}\right)^4 = \frac{369}{625}$$

- 13 $P(\text{selecting atleast one couple}) = 1 - P(\text{selecting none of the couples for the prize.})$

$$= 1 - \left(\frac{{}^{16}C_1 \times {}^{14}C_1 \times {}^{12}C_1 \times {}^{10}C_1}{{}^{16}C_4} \right) = \frac{15}{39}$$

- 14 Total number of ways in which 7 (= 4 + 3) persons can speak is $7!$.

The number of ways in which A, B, C speak in the given order is ${}^7C_3$ ways and remaining 4 persons can be arranged in $4!$ ways.

$\therefore$ Favourable number of ways = ${}^7C_3 \times 4!$

$$\therefore \text{Required probability} = \frac{{}^7C_3 \times 4!}{7!} = \frac{1}{6}$$

- 15 Let A be the event of selecting a counterfeit coin and B the event of getting head, then

$$\begin{aligned} \text{Required probability} &= P(A \cap B) \cup (\bar{A} \cap B) \\ &= P(A \cap B) + P(\bar{A} \cap B) \\ &= P(A)P\left(\frac{B}{A}\right) + P(\bar{A})P\left(\frac{B}{\bar{A}}\right) \\ &= \frac{2}{16} \times 1 + \frac{14}{16} \times \frac{1}{2} = \frac{9}{16} \end{aligned}$$

- 16 Total number of ways in which 5 people can be chosen out of 9 people = ${}^9C_5 = 126$

$$\begin{aligned} \text{Number of ways in which the couple serves the committee} \\ &= {}^7C_3 \times {}^2C_2 = 35 \end{aligned}$$

$$\begin{aligned} \text{Number of ways in which the couple does not serve the committee} \\ &= {}^7C_5 = 21 \end{aligned}$$

$\therefore$ Favourable number of cases = $35 + 21 = 56$

$$\text{Hence, the required probability} = \frac{56}{126} = \frac{4}{9}$$

- 17 E_1 = The event in which A speaks truth

E_2 = The event in which B speaks truth

$$\text{Then } P(E_1) = \frac{60}{100} = \frac{3}{5}, \quad P(E_2) = \frac{80}{100} = \frac{4}{5}$$

$$\text{and } P(\bar{E}_1) = \frac{2}{5}, \quad P(\bar{E}_2) = \frac{1}{5}$$

$\therefore$ Required probability = $P[(E_1 \cap E_2) \cup (\bar{E}_1 \cap \bar{E}_2)]$

$$= P(E_1 \cap E_2) + P(\bar{E}_1 \cap \bar{E}_2)$$

$$= P(E_1) \cdot P(E_2) + P(\bar{E}_1) \cdot P(\bar{E}_2)$$

$$= \left(\frac{3}{5} \times \frac{4}{5}\right) + \left(\frac{2}{5} \times \frac{1}{5}\right) = \frac{14}{25} = 0.56$$

- 18** Two different squares can be chosen in 64×63 ways. For each of the four corner squares, the favourable number of cases is 2. For each of the 24 non-corner squares on all the four sides of the chessboard, the favourable number of cases is 3. For each of the 36 remaining squares, the favourable number of cases is 4.

Thus, the total number of favourable cases

$$= 4 \times 2 + 24 \times 3 + 36 \times 4 = 224$$

$$\text{Hence, the required probability} = \frac{224}{64 \times 63} = \frac{1}{18}$$

- 19** The last two digits can be dialled in $^{10}P_2 = 90$ ways.

Out of these 90 cases only one case is favourable.

$$\text{Hence, the required probability} = \frac{1}{90}$$

- 20** 3 squares on a chessboard can be chosen in $^{64}C_3$ ways.

Two squares of one colour and third square of different colour can be chosen in two mutually exclusive way.

(i) 2 white and one black (ii) 2 black and one white

Thus the favourable number of cases

$$= {}^{32}C_2 \times {}^{32}C_1 + {}^{32}C_1 \times {}^{32}C_2 = 2({}^{32}C_2 \times {}^{32}C_1)$$

$$\text{Hence, the required probability} = \frac{2({}^{32}C_2 \times {}^{32}C_1)}{{}^{64}C_3} = \frac{16}{21}$$

- 21** A leap year contains 366 days comprising of 52 full weeks and 2 extra days. Thus there can be following 7 possibilities for 2 extra days.

(i) Sunday, Monday, (ii) Monday, Tuesday
(iii) Tuesday, Wednesday (iv) Wednesday, Thursday
(v) Thursday, Friday (vi) Friday, Saturday
(vii) Saturday, Sunday

Let A be the event that the leap year contains 53 Sundays.
and B be the event that leap year contains 53 Mondays.

$$\text{Then, we have } P(A) = \frac{2}{7}, \quad P(B) = \frac{2}{7}, \quad P(A \cap B) = \frac{1}{7}$$

So, required probability $= P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B) = \frac{2}{7} + \frac{2}{7} - \frac{1}{7} = \frac{3}{7}$$

- 22** Since, the probability of getting atleast one head in n times

$$= 1 - \left(\frac{1}{2}\right)^n$$

$$\text{Therefore, } 1 - \left(\frac{1}{2}\right)^n \geq 0.9 \Rightarrow \left(\frac{1}{2}\right)^n \leq 0.1$$

$$\Rightarrow 2^n \geq 10 \Rightarrow n \geq 4$$

Hence, the least value of n is 4.

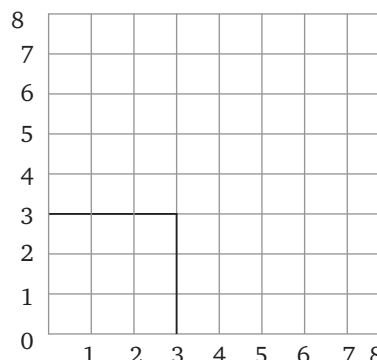
- 23** Total number of ways in which 2 persons can be selected out of 13 persons is $^{13}C_2$

$$\text{Now, favourable number of cases} = \frac{{}^5C_1 \times {}^8C_1 + {}^5C_2}{^{13}C_2} = \frac{25}{39}$$

- 24** We can choose 9 squares out of 64 squares in $^{64}C_9$ ways.

Hence, exhaustive number of cases $= {}^{64}C_9$

From the figure it is clear that the given square of size 3×3



can be formed by using four consecutive horizontal and 4 consecutive vertical lines, which can be done in

$${}^6C_1 \times {}^6C_1 = 36 \text{ ways}$$

Basically you can make 6 squares of size 3×3 in vertical direction and 6 squares of the size 3×3 in horizontal direction. Hence total $6 \times 6 = 36$ squares can be chosen.

$$\therefore \text{The required probability} = \frac{36}{{}^{64}C_9}$$

- 25** Total 7 digit numbers can be formed from the 9 digits $= {}^9P_7$.

There are four exclusive cases of selecting 7 digits out of 9 digits which can form 7 digit numbers which are divisible by 9.

$2, 3, 4, 5, 6, 7, 9 \}$ 36 removing 1 and 8

$1, 3, 4, 5, 6, 8, 9 \}$ 36 removing 2 and 7

$1, 2, 4, 5, 7, 8, 9 \}$ 36 removing 3 and 6

$1, 2, 3, 6, 7, 8, 9 \}$ 36 removing 4 and 5

All the 7 numbers of each of the 4 sets can be arranged in $7!$ ways. Hence the favourable number of numbers $= 4 \times 7!$

$$\therefore \text{Required probability} = \frac{4 \times 7!}{{}^9P_7} = \frac{1}{9}$$

- 26** Total number of ways of selecting 2 distinct numbers from the set of 10 natural numbers $= {}^{10}C_2 = 42$

Sets of co-primes are $(1, 2), (1, 3), (1, 4), \dots (1, 10); (2, 3), (2, 5), (2, 7), (2, 9); (3, 4), (3, 5), (3, 7), (3, 8), (3, 10); (4, 5), (4, 7), (4, 9); (5, 6), (5, 7), (5, 8), (5, 9); (6, 7); (7, 8), (7, 9), (7, 10); (8, 9); (9, 10)$

Thus total pairs of co-primes

$$= 9 + 4 + 5 + 3 + 4 + 1 + 3 + 1 + 1 = 31$$

Therefore the required probability $= 31/42$.

Hence choice (a) is the correct one.

Hint How to find the number of sets of two co-prime numbers?

Number of co-prime numbers of 10 which are below

$$10 = 10 \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{5}\right) = 4$$

Number of co-prime numbers of 9 which are below

$$9 = 9 \times \left(1 - \frac{1}{3}\right) = 6$$

Number of co-prime numbers of 8 which are below

$$8 = 8 \times \left(1 - \frac{1}{2}\right) = 4$$

Number of co-prime numbers of 7 which are below

$$7 = 7 \times \left(1 - \frac{1}{7}\right) = 6$$

Number of co-prime numbers of 6 which are below

$$6 = 6 \times \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) = 2$$

Number of co-prime numbers of 5 which are below

$$5 = 5 \times \left(1 - \frac{1}{5}\right) = 4$$

Number of co-prime numbers of 4 which are below

$$4 = 4 \times \left(1 - \frac{1}{2}\right) = 2$$

Number of co-prime numbers of 3 which are below

$$3 = 3 \times \left(1 - \frac{1}{3}\right) = 2$$

Number of co-prime numbers of 2 which are below

$$2 = 2 \times \left(1 - \frac{1}{2}\right) = 1$$

Therefore total number of sets of co-prime numbers (or pairs) = $4 + 6 + 4 + 6 + 2 + 4 + 2 + 2 + 1 = 31$.

✓ Level 02 Higher Level Exercise

- 1 Total number of words that can be formed from the letters of the word MISSISSIPPI is $\frac{11!}{4!4!2!}$

When all the S's are together then the number of words can be formed = $\frac{8!}{4!2!}$

$$\therefore \text{Required probability} = \frac{\frac{8!}{4!2!}}{\frac{11!}{4!4!2!}} = \frac{4}{165}$$

- 2 Since each of the coefficients a , b and c can take values from 1 to 6. Therefore the total number of equations = $6 \times 6 \times 6 = 216$

Hence the exhaustive number of cases = 216

Now, the roots of the equation $ax^2 + bx + c = 0$

will be real if $b^2 - 4ac \geq 0 \Rightarrow b^2 \geq 4ac$

Following are the number of favourable cases

a	c	ac	$4ac$	$b^2 (\geq 4ac)$	b	Number of cases
1	1	1	4	4, 9, 16, 25, 36	2, 3, 4, 5, 6	$1 \times 5 = 5$
1	2	2	8	9, 16, 25, 36	3, 4, 5, 6	$2 \times 4 = 8$
1	3	3	12	16, 25, 36	4, 5, 6	$2 \times 3 = 6$
1	4	4	16	16, 25, 36	4, 5, 6	$3 \times 3 = 9$
1	5	5	20	25, 36	5, 6	$2 \times 2 = 4$
1	6	6	24	25, 36	5, 6	$4 \times 2 = 8$
2	3	6	24	25, 36	5, 6	$4 \times 2 = 8$
2	4	8	32	36	6	$2 \times 1 = 2$
3	3	9	36	36	6	$1 \times 1 = 1$
Total						= 43

Note $\rightarrow ac = 7$ is not possible

Since $b^2_{(\max)} = 36$ and $4ac \leq b^2$ hence $ac = 10, 11, 12, \dots$ etc. is not possible.

Hence, total number of favourable cases = 43

So, the required probability = $\frac{43}{216}$.

- 3 6 can be thrown with a pair of dice in the following ways (1, 5), (5, 1), (2, 4), (4, 2), (3, 3)

So, probability of throwing a '6' = $\frac{5}{36}$

and probability of not throwing a '6' = $\frac{31}{36}$

And 7 can be thrown with a pair of dice in the following ways.

(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)

so probability of throwing a '7' = $\frac{6}{36} = \frac{1}{6}$

and probability of not throwing a '7' = $\frac{5}{6}$

Let E_1 be the event of the throwing a '6' in a single throw of a pair of dice and E_2 be the event of throwing a 7 in a single throw of a pair of dice.

Then $P(E_1) = \frac{5}{36}$, $P(E_2) = \frac{1}{6}$

and $P(\bar{E}_1) = \frac{31}{36}$, $P(\bar{E}_2) = \frac{5}{6}$

A wins if he throws '6' in first or third or fifth .. throws.

Probability of A throwing a 6 in first throw = $p(E_1) = \frac{5}{36}$

and probability of A throwing a 6 in third throw

$$= P(\bar{E}_1 \cap \bar{E}_2 \cap E_1) = P(\bar{E}_1) P(\bar{E}_2) P(E_1) = \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36}$$

Similarly, probability of A throwing a '6' in fifth throw

$$= P(\bar{E}_1) P(\bar{E}_2) P(\bar{E}_1) P(\bar{E}_2) P(E_1)$$

$$= \left(\frac{31}{36}\right)^2 \times \left(\frac{5}{6}\right)^2 \times \frac{5}{36}$$

Hence, probability of winning of A

$$\begin{aligned}
 &= P[E_1 \cup (\bar{E}_1 \cap \bar{E}_2 \cap E_1) \cup (\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_1 \cap \bar{E}_2 \cap E_1) \cup \dots] \\
 &= P(E_1) + P(\bar{E}_1 \cap \bar{E}_2 \cap E_1) + P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_1 \cap \bar{E}_2 \cap E_1) + \dots \\
 &= \frac{5}{36} + \left(\frac{31}{36} \times \frac{5}{6}\right) \times \frac{5}{36} + \left(\frac{31}{36} \times \frac{5}{6}\right)^2 \times \frac{5}{36} + \dots \\
 &= \frac{\frac{5}{36}}{1 - \left(\frac{31}{36}\right) \times \left(\frac{5}{6}\right)} = \frac{30}{61}
 \end{aligned}$$

Thus, probability of winning of $B = 1 - \frac{30}{61} = \frac{31}{61}$

- 4 Let A be the event of getting exactly 3 defectives in the examination of 8 wristwatches.
And B be the event of getting ninth wristwatch defective.
Then

$$\text{Required probability} = P(A \cap B) = P(A) P\left(\frac{B}{A}\right)$$

$$\text{Now, } P(A) = \frac{{}^4C_3 \times {}^{11}C_5}{{}^{15}C_8}$$

And $P\left(\frac{B}{A}\right)$ = Probability that the ninth examined wristwatch is defective given that there were 3 defectives in the first 8 pieces examined = $\frac{1}{7}$

$$\text{Hence, required probability} = \frac{{}^4C_3 \times {}^{11}C_5}{{}^{15}C_8} \times \frac{1}{7} = \frac{8}{195}$$

- 5 Let E_1, E_2, E_3 and A be the events defined as follows :

E_1 = the examinee guesses the answer

E_2 = the examinee copies the answer

E_3 = the examinee knows the answer and

A = the examinee answers correctly

$$\text{We have } P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{6}$$

Since E_1, E_2 and E_3 are mutually exclusive and exhaustive events therefore

$$\begin{aligned}
 P(E_1) + P(E_2) + P(E_3) &= 1 \\
 \Rightarrow P(E_3) &= \frac{1}{2}
 \end{aligned}$$

If E_1 has already occurred, then the examinee guesses. Since there are four choices out of which only one is correct, therefore the probability that he answers correctly given that he has made a guess is $\frac{1}{4}$ i.e., $P\left(\frac{A}{E_1}\right) = \frac{1}{4}$

It is a given that $P\left(\frac{A}{E_2}\right) = \frac{1}{8}$ and $P\left(\frac{A}{E_3}\right)$ is the probability that he answers correctly given that he knew the answer = 1
By Baye's rule,

$$\text{Required probability} = P\left(\frac{E_3}{A}\right)$$

$$\begin{aligned}
 &= \frac{P(E_3) P\left(\frac{A}{E_3}\right)}{P(E_1) P\left(\frac{A}{E_1}\right) + P(E_2) P\left(\frac{A}{E_2}\right) + P(E_3) P\left(\frac{A}{E_3}\right)} \\
 &= \frac{\frac{1}{2} \times 1}{\frac{1}{3} \times \frac{1}{4} + \frac{1}{6} \times \frac{1}{8} + \frac{1}{2} \times 1} = \frac{24}{29}
 \end{aligned}$$

- 6 Let x and y be the two non-negative integers

since $x + y = 200$

$$\therefore (xy)_{\max} = 100 \times 100 = 10000 \text{ (} xy_{\max} \text{ at } x = y \text{)}$$

$$\text{Now, } xy \leq \frac{3}{4} \times 10000 \Rightarrow xy \geq \frac{3}{4} \times 10000$$

$$\Rightarrow xy \geq 7500 \Rightarrow x(200 - x) \geq 7500$$

$$\Rightarrow x^2 - 200x + 7500 \leq 0$$

$$\Rightarrow 50 \leq x \leq 150$$

So favourable number of ways = $150 - 50 + 1 = 101$

Total number of ways = 200

$$\text{Hence, required probability} = \frac{101}{200}$$

- 7 Let $E_i (i = 1, 2, 3 \text{ etc.})$ denote the event of drawing an even numbered card in i^{th} draw and $F_i (i = 1, 2, 3)$ denote the event of drawing an odd numbered card in i^{th} draw, then required probability

$$= P[(E_1 \cap F_2 \cap F_3) \cup (F_1 \cap E_2 \cap F_3) \cup (F_1 \cap F_2 \cap E_3)]$$

$$= P(E_1) P(F_2) P(F_3) + P(F_1) P(E_2) P(F_3)$$

$$+ P(F_1) P(F_2) P(E_3)$$

$$= \frac{4}{9} \times \frac{5}{9} \times \frac{5}{9} + \frac{5}{9} \times \frac{4}{9} \times \frac{5}{9} + \frac{5}{9} \times \frac{5}{9} \times \frac{4}{9}$$

$$= 3 \times \frac{4 \times (5)^2}{(9)^3} = \frac{100}{243}$$

- 8 Consider the following events

A = The first number is less than the second number

B = The third number lies between the first and the second.

$$\text{Now, we have to find } P\left(\frac{B}{A}\right).$$

$$\text{Also, we have } P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

Any 3 numbers can be chosen out of n numbers in ${}^nC_3$ ways.

Let the selected numbers be x_1, x_2, x_3 . Then they satisfy exactly one of the following inequalities.

$$x_1 < x_2 < x_3, \quad x_1 < x_3 < x_2, \quad x_2 < x_1 < x_3,$$

$$x_2 < x_3 < x_1, \quad x_3 < x_1 < x_2, \quad x_3 < x_2 < x_1$$

The total number of ways of selecting three numbers and then arranging them = ${}^nC_3 \times 3! = {}^nP_3$

$$\therefore P(A) = \frac{{}^nC_3 \times 3}{{}^nP_3}$$

$$\text{and } P(A \cap B) = \frac{{}^nC_3}{{}^nC_3 \times 3!}$$

$$\text{Hence } P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1}{3}$$

- 9 Since b and c each can assume 9 values from 1 to 9.

So, total number of ways of choosing b and c is $9 \times 9 = 81$

Now, $x^2 + bx + c > 0$ for all $x \in R$

$$\Rightarrow D < 0$$

$$\Rightarrow b^2 - 4ac < 0 \Rightarrow b^2 - 4c < 0$$

$$\Rightarrow b^2 < 4c$$

☞ Alternatively $x^2 + bx + c > 0$

$$\Rightarrow \left(x + \frac{b}{2}\right)^2 + \frac{4c - b^2}{4} > 0$$

$$\Rightarrow 4c - b^2 > 0$$

$$\Rightarrow b^2 < 4c$$

Now, the following table shows the possible values of b and c for which $b^2 < 4c$

c	b	Total
1	1,	1
2	1, 2	2
3	1, 2, 3	3
4	1, 2, 3,	3
5	1, 2, 3, 4	4
6	1, 2, 3, 4	4
7	1, 2, 3, 4, 5	5
8	1, 2, 3, 4, 5	5
9	1, 2, 3, 4, 5	5
		32

So, favourable number of cases = 32

Hence required probability = $32/81$

- 10 We have, $P(A \cup B \cup C) = 3/4$

$$\text{i.e., } P(A) + P(B) + P(C) - P(A \cap B)$$

$$- P(B \cap C) - P(A \cap C) + P(A \cup B \cup C) = 3/4$$

$$\text{and } P(A \cap B) + P(B \cap C) + P(A \cap C) - 2P(A \cap B \cap C) = \frac{1}{2}$$

$$\text{and } P(A \cap B) + P(B \cap C) + P(A \cap C) - 3P(A \cap B \cap C) = \frac{2}{5}$$

Solving the above equations (last two), we get

$$P(A \cap B \cap C) = \frac{1}{2} - \frac{2}{5} = \frac{1}{10}$$

$$\Rightarrow P(A)P(B)P(C) = \frac{1}{10} \Rightarrow pmc = \frac{1}{10}$$

$$\text{Also, } P(A) + P(B) + P(C) - [P(A \cap B)$$

$$+ P(B \cap C) + P(A \cap C)] + P(A \cup B \cup C) = \frac{3}{4}$$

$$p + m + c - \left(\frac{1}{2} + \frac{2}{10}\right) + \frac{1}{10} = \frac{3}{4}$$

$$\Rightarrow p + m + c = 27/20$$

- 11 Let E, F, G be the events that the student is successful in tests A, B and C respectively. Then the probability that the students is successful is

$$\begin{aligned} &= P[(E \cap F \cap \bar{G}) \cup (E \cap \bar{F} \cap G) \cup (E \cap F \cap G)] \\ &= P(E \cap F \cap \bar{G}) + P(E \cap \bar{F} \cap G) + P(E \cap F \cap G) \\ &= P(E)P(F)P(\bar{G}) + P(E)P(\bar{F})P(G) + P(E)P(F)P(G) \\ &= pq\left(1 - \frac{1}{2}\right) + p(1 - q)\left(\frac{1}{2}\right) + pq\left(\frac{1}{2}\right) \\ &= \frac{pq + p - pq + pq}{2} = \frac{p(1 + q)}{2} \end{aligned}$$

But the probability that the student is successful = $1/2$

$$\therefore \frac{P(1 + q)}{2} = \frac{1}{2} \Rightarrow p(1 + q) = 1$$

This is satisfied by $p = 1, q = 0$

Also there are other values (infinite numbers) of p, q for which the above relation is satisfied.

Hence, (d) is the correct option.

- 12 Since $\frac{1 + 4p}{p}, \frac{1 - p}{4}, \frac{1 - 2p}{2}$ are the probabilities of

3 mutually exclusive events, therefore

$$0 \leq \frac{1 + 4p}{p} \leq 1, \quad 0 \leq \frac{1 - p}{4} \leq 1, \quad 0 \leq \frac{1 - 2p}{2} \leq 1$$

$$\text{and } 0 \leq \frac{1 + 4p}{p} + \frac{1 - p}{4} + \frac{1 - 2p}{2} \leq 1$$

$$\Rightarrow -\frac{1}{4} \leq p \leq \frac{3}{4}, \quad -1 \leq p \leq 1, \quad -\frac{1}{2} \leq p \leq \frac{1}{2} \text{ and } \frac{1}{2} \leq p \leq \frac{5}{2}$$

$$\Rightarrow \max\left\{-\frac{1}{4}, -1, -\frac{1}{2}, \frac{1}{2}\right\} \leq p \leq \min\left\{\frac{3}{4}, 1, \frac{1}{2}, \frac{5}{2}\right\}$$

$$\Rightarrow \frac{1}{2} \leq p \leq \frac{1}{2} \Rightarrow p = \frac{1}{2}$$

- 13 ASSISTANT $\rightarrow AA IN SSS TT$, STATISTICS $\rightarrow A I I C SSS TTT$
Here N and C are not common and same letters can be A, I, S, T . Therefore

$$\text{Probability of choosing } A = \frac{{}^2C_1}{{}^9C_1} \times \frac{{}^1C_1}{{}^{10}C_1} = \frac{1}{45}$$

$$\text{Probability of choosing } I = \frac{1}{{}^9C_1} \times \frac{{}^2C_1}{{}^{10}C_1} = \frac{1}{45}$$

$$\text{Probability of choosing } S = \frac{{}^3C_1}{{}^9C_1} \times \frac{{}^3C_1}{{}^{10}C_1} = \frac{1}{10}$$

$$\text{Probability of choosing } T = \frac{{}^2C_1}{{}^9C_1} \times \frac{{}^3C_1}{{}^{10}C_1} = \frac{1}{15}$$

$$\text{Hence, required probability} = \frac{1}{45} + \frac{1}{45} + \frac{1}{10} + \frac{1}{15} = \frac{19}{90}$$

- 14 Out of 30 numbers 2 numbers can be chosen in ${}^{30}C_2$ ways.

So, exhaustive number of cases = ${}^{30}C_2 = 435$

Since $a^2 - b^2$ is divisible by 3 iff either a and b are divisible by 3 or none of a and b is divisible by 3. Thus, the favourable numbers, of cases = ${}^{10}C_2 + {}^{20}C_2 = 235$

$$\text{Hence, required probability} = \frac{235}{435} = \frac{47}{87}$$

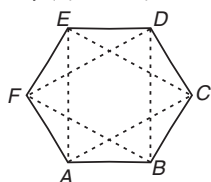
- 15** The man will be one step away from the starting point if (A) either he is one step ahead or (B) one step behind the starting point. Therefore, required probability = $P(A) + P(B)$
The man will be one step ahead at the end of eleven steps if he moves six steps forward and five steps backward. The probability of this event = ${}^{11}C_6(0.4)^6(0.6)^5$. The man will be one step behind at the end of the eleven steps if he moves six steps backward and five steps forward. The probability of this event = ${}^{11}C_6(0.6)^6(0.4)^5$.

Hence, the required probability

$$= {}^{11}C_6(0.4)^6(0.6)^5 + {}^{11}C_6(0.6)^6(0.4)^5$$

$$= {}^{11}C_6(0.4)^5(0.6)^5(0.4 + 0.6) = {}^{11}C_6(0.24)^5$$

- 16** There are 6 vertices in a hexagon. Using 3 vertices out of 6 vertices we can form ${}^6C_3$ triangles. But there can be only two triangles out of ${}^6C_3$ triangles which are equilateral (see the figure, (i) $\triangle ACE$, (ii) $\triangle BDF$)



Hence, the required probability = $2/{}^6C_3 = 2/20 = 1/10$

- 17** Let F, B, L and R denote the forward, backward, left and right steps (or movements) then the following mutually exclusive ways are possible.

F	B	L	R
0	0	4	5
1	1	3	4
2	2	2	3
3	3	1	2
4	4	0	1

F	B	L	R
4	5	0	0
3	4	1	1
2	3	2	2
1	2	3	3
0	1	4	4

In this case he cancels out his forward and backward movements by moving equal steps in forward and backward directions each and he creates a difference of 1 step by moving one step extra either in right or left direction.

The number of permutations of these five arrangements is

$$4 \left[\frac{9!}{5!4!} + \frac{9!}{1!1!3!4!} + \frac{9!}{2!2!2!3!} + \frac{9!}{3!3!1!1!} + \frac{9!}{4!4!1!1!} \right]$$

$$= 4(126 + 2520 + 7560 + 5040 + 630) = 4 \times 15876$$

But the total number of ways of arranging nine steps = 4^9 .

$$\therefore \text{The required probability} = \frac{4 \times 15876}{4^9} = \frac{3969}{4^7}$$

In this case he cancels out his left or right movement by moving equal number of steps in left and right directions each and he creates a difference of 1 step by moving one step extra either in forward or backward directions.

- 18** Let E_{rr} denote that a red colour ball is transferred from urn A to urn B then a red colour ball is transferred from urn B to urn

E_{rb} denote that a red colour ball is transferred from urn A to urn B then a black colour ball is transferred from urn B to urn A

E_{br} denote that a black colour ball is transferred from urn A to urn B then a red colour ball is transferred from urn B to urn A.

E_{bb} denote that a black colour ball is transferred from urn A to urn B then a black colour ball is transferred from urn B to urn A.

$$\text{Then } P(E_{rr}) = \left(\frac{6}{10}\right)\left(\frac{5}{11}\right) = \frac{3}{11}, P(E_{rb}) = \left(\frac{6}{10}\right)\left(\frac{6}{11}\right) = \frac{18}{55}$$

$$P(E_{br}) = \left(\frac{4}{10}\right)\left(\frac{4}{11}\right) = \frac{8}{55}, P(E_{bb}) = \left(\frac{4}{10}\right)\left(\frac{7}{11}\right) = \frac{14}{55}$$

Let A be the event of drawing a red colour ball after these

$$\text{transfers. Then } P\left(\frac{A}{E_{rr}}\right) = \frac{6}{10}, P\left(\frac{A}{E_{rb}}\right) = \frac{5}{10}$$

$$\Rightarrow P\left(\frac{A}{E_{br}}\right) = \frac{7}{10}, P\left(\frac{A}{E_{bb}}\right) = \frac{6}{10}$$

Therefore, the required probability is

$$P(A) = P(E_{rr})P\left(\frac{A}{E_{rr}}\right) + P(E_{rb})P\left(\frac{A}{E_{rb}}\right)$$

$$+ P(E_{br})P\left(\frac{A}{E_{br}}\right) + P(E_{bb})P\left(\frac{A}{E_{bb}}\right)$$

$$= \left(\frac{3}{11}\right)\left(\frac{6}{10}\right) + \left(\frac{18}{55}\right)\left(\frac{5}{10}\right) + \left(\frac{8}{55}\right)\left(\frac{7}{10}\right) + \left(\frac{14}{55}\right)\left(\frac{6}{10}\right)$$

$$= \frac{90 + 90 + 56 + 84}{550} = \frac{32}{55}$$

- 19** A number is divisible by 11 only if the difference of the sum of the digits at odd places and sum of the digits at even places is divisible by 11 i.e., 0, 11, 22, 33 ...

Here the sum of all the 9 digits (1, 2, 3, ..., 9) is 45.

We cannot create the difference of zero

since $x + y = 45$, which is odd hence cannot be broken into two equal parts in integers. Now, we will look for the possibilities of 11 which are as follows :

$$\{1, 2, 6, 8\}, \{1, 2, 5, 9\}, \{1, 3, 6, 7\}$$

$$\{1, 3, 5, 8\}, \{1, 3, 4, 9\}, \{1, 4, 5, 7\}$$

$$\{2, 3, 5, 7\}, \{2, 3, 4, 8\}, \{2, 4, 5, 6\} \text{ and } \{4, 7, 8, 9\}, \{5, 6, 8, 9\}$$

The above set of values either gives the sum of 17 or 28.

Since if the sum of 4 digits at even places be 17 or 28 then the sum of rest of the digits (i.e., digits at odd places) be 28 or 17 respectively and thus we can get the difference of 11.

Further we cannot get the difference of 22 or 33...

So there is only possible difference that can be created is 11 and there are only 11 set of values given above containing 4 digits which can be arranged in $4!$ ways and the remaining 5 digits can be arranged in $5!$ ways.

Thus the favourable number of numbers

$$= 11 \times 4! \times 5!$$

But the total number of ways of arranging a nine digit number is ${}^9P_9 = 9!$

$\therefore$ Exclusive number of cases = $9!$

$\therefore$ Required probability = $\frac{11 \times 4! \times 5!}{9!} = \frac{11}{126}$.

- 20** The number of ways of choosing any three phones out of total 12 phones = ${}^{12}C_3 = 220$

The number of conference calls that can be made from the 216 given phones = ${}^{12}C_3 - {}^4C_3 = 216$

Therefore the required probability = $\frac{216}{220} = \frac{54}{55}$

Hence choice (d) is the correct one.

- 21** Probability of uploading 3 photos, after editing, such that at least 2 photos are from the same folder
 $= 1 - \text{Probability of uploading 3 photos, after editing, from three different folders}$

$$= 1 - \frac{{}^3C_1 \times {}^4C_1 \times {}^5C_1}{{}^{12}C_3} = \frac{3}{11}$$

Probability that no folder has the photo uploaded back from its original folder, after editing

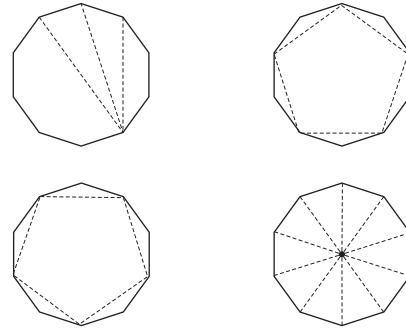
$$= 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} = \frac{1}{3}$$

Therefore, the required probability = $\frac{3}{11} \times \frac{1}{3} = \frac{1}{11}$

Hence choice (a) is the correct one.

- 22** First of all you must know that there are diagonals of four distinct lengths. If $d_1 < d_2 < d_3 < d_4$ be the diagonals in that of their increasing length, then d_1 is a diagonal which is formed by connecting any two vertices, leaving one vertex between them and d_2 is a diagonal which is formed by

connecting any two vertices, leaving two vertices between them and d_3 is a diagonal which is formed by connecting any two vertices, leaving three vertices between them and d_4 is a diagonal which is formed by connecting any two vertices, leaving four vertices between them.



Total number of diagonals = $\frac{10(7)}{2} = 35$

Number of shortest diagonals = $\frac{10(2)}{2} = 10$

Number of longest diagonals = $\frac{10}{2} = 5$

Number of diagonals which are neither the shortest nor the longest = $35 - (10 + 5) = 20$.

Therefore, the required probability = $\frac{20}{35} = \frac{4}{7}$

Hence choice (a) is the correct one.

Hint Total number of diagonals in a regular polygon of n side = $\frac{n(n-3)}{2}$

The number of the longest diagonals = number of pairs of opposite points in a decagon.

CHAPTER 21

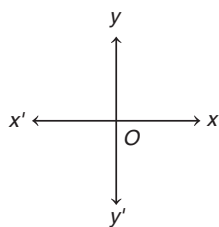
Co-ordinate Geometry

It is one of those chapters which contributes least number of problems in CAT. Perhaps this chapter is more or less algebraic instead of being logical, since most of the problems require formula to solve them. But some other MBA entrance exams ask the question from this chapter. It is not fool proof but this chapter may be optional particularly for CAT aspirants who feel difficulty in algebraic problems. But it does not mean that this chapter is difficult or can not be learned.

21.1 Cartesian Coordinate System

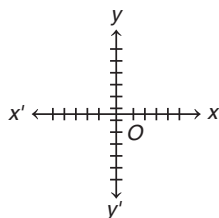
Rectangular Co-ordinate Axes

Let $X'OX$ and $Y'OY$ be the two mutually perpendicular lines through any point O in the plane of paper. The point O is called the origin. The line $X'OX$ is called the X -axis and the line $Y'OY$ is called the Y -axis. These two lines taken together are called the **co-ordinate axes**.



Cartesian Co-ordinate of a Point

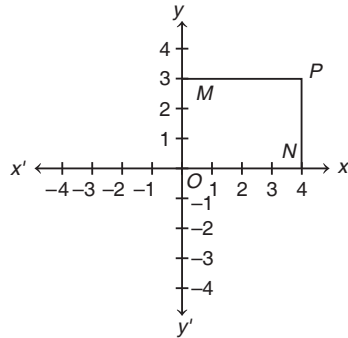
Each axes is calibrated as a scale into several points which are equidistant from each other.



Chapter Checklist

- Cartesian Coordinate System
- Some Important Formulae
- Condition of Collinearity of Three Points
- Important Results
- Important Points in a Triangle
- Shifting of Origin
- Straight Line
- Equations of Lines Parallel to the Co-ordinate Axes
- Different Standard Forms of the Equations of a Straight Line
- Reduction (or Transformation) of the General Equation
- Point of Intersection of Two Lines
- Condition of Concurrency of Three Lines
- Angle Between Two Straight Lines
- Condition for Parallelism and Perpendicularity of two lines
- Distance of a Point From a Given Line or Length of Perpendicular From the Point (x_1, y_1) to the Straight Line $ax+by+c=0$
- CAT Test

The point O is assumed to be zero.



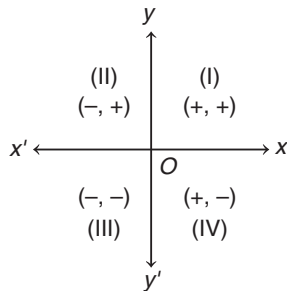
Let P be any point on the plane, then we draw a perpendicular from P on the X -axis and another on the Y -axis. Then the distance MP (which is parallel to the X -axis) is called the x -coordinate or abscissa of point P .

Similarly, the length PN (which is parallel to the Y -axis) is called the y -coordinate or ordinate of point P .

Then the position of the point P in the plane with respect to the coordinate axes is represented by the ordered pair (x, y) . The ordered pair (x, y) is called the coordinates of point P .

Quadrants

In the adjacent figure the two axes intersect each other perpendicular at point O . The regions XOY , $X'OY$, $X'OY'$ and $Y'OX$ are known as first, second, third and fourth quadrant respectively.



The nature of values of x and y in different quadrants is different which is shown in figure also given below

Quadrant	Sign of (x, y)
I	$(+, +)$
II	$(-, +)$
III	$(-, -)$
IV	$(+, -)$

For example, let P be a point in second quadrant, then it is denoted as $(-1, 2)$, $(-2, 3)$, $(-3, 5)$, $(-6, 8)$, $(-7, 10)$ etc.

The coordinates of point O (i.e., origin) are taken as $(0, 0)$. The coordinates of any point on X -axis are of the form $(x, 0)$ and the coordinates of any point on Y -axis are of the form $(0, y)$.

21.2 Some Important Formulae

Distance Formula

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Also, Distance of point (x_1, y_1) from origin $= \sqrt{x_1^2 + y_1^2}$

Section Formula

If $R(x, y)$ divides the line segment joined by the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio $m : n$ then

$$x = \frac{mx_2 + nx_1}{m + n}, y = \frac{my_2 + ny_1}{m + n}$$

If $R(x, y)$ divides the line segment joined by the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ externally in the ratio $m : n$, then

$$x = \frac{mx_2 - nx_1}{m - n}, y = \frac{my_2 - ny_1}{m - n}$$

Area of Plane

Area of a Triangle

The area of a triangle ABC whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is denoted by Δ .

$$\begin{aligned} \therefore \Delta &= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \\ &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \end{aligned}$$

Area of a Polygon

The area of the polygon whose vertices are

$$(x_1, y_1), (x_2, y_2), (x_3, y_3), \dots, (x_n, y_n) \text{ is}$$

$$\Delta = \frac{1}{2} [(x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2) + \dots + (x_n y_1 - x_1 y_n)]$$

21.3 Condition for Collinearity of Three Points

The three given points are collinear i.e., lie on the same straight line if

- Area of triangle ABC is zero.
- Slope of AB = Slope of BC = Slope of AC
- Distance between A and B + distance between B and C = Distance between A and C .
- Find the equation of the line passing through any two points, if third point satisfies the equation of the line then three points are collinear.

21.4 Important Results

1. If the vertices of a triangle have integral co-ordinates then the triangle can not be equilateral.
2. In order to prove that a given figure is a square, parallelogram, rectangle etc. We will prove the following points given in the table below corresponding the name of the figure.

S.N.	Name of the figure	Conditions
1.	Square	Four sides are equal and the diagonals are also equal
2.	Rhombus	Four sides are equal
3.	Rectangle	Opposite sides are equal and diagonals are also equal
4.	Parallelogram	Opposite sides are equal
5.	Parallelogram but not a rectangle	Opposite sides are equal but the diagonals are not equal.
6.	Rhombus but not a square	All sides are equal but the diagonals are not equal

21.5 Important Points in a Triangle

Centroid

If (x_1, y_1) , (x_2, y_2) and (x_3, y_3) are the vertices of a triangle, then the coordinates of its centroid are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

Incentre

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle ABC such that $BC = a$, $CA = b$ and $AB = c$, then the coordinates of its centre are

$$\left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c} \right)$$

NOTE If the equations of the sides of the triangle are given then we find the bisectors of internal angles and then their point of intersection to determine the incentre.

Circumcentre

If 'O' is the circumcentre of a triangle ABC , then $OA = OB = OC$ and OA is called the circumradius.

To find the circumcentre of $\triangle ABC$, we use the relation $OA = OB = OC$. This gives two simultaneous linear equation and their solution provides the coordinates of circumcentre.

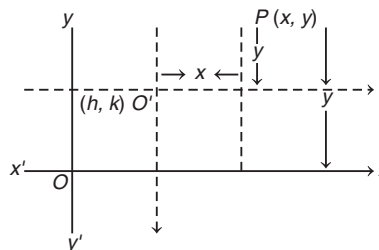
Orthocentre

To determine the orthocentre, first we find equations of lines passing through vertices and perpendicular to the opposite sides. Solving any two of these three equations we get the coordinates of orthocentre.

NOTE The circumcentre O , centroid G and orthocentre O' of a triangle ABC are collinear such that G divides $O'O$ in the ratio 2:1 i.e., $O'G : OG = 2:1$

21.6 Shifting of Origin

Let O be the origin and $X'OX$ and $Y'OY$ be the axis of x and y respectively. Let O' and P be two points in the plane having coordinates (h, k) and (x, y) respectively referred to $X'OX$ and $Y'OY$ as coordinate axes.



Let the origin be transferred to O' and let $X'O'X$ and $Y'O'Y$ be new rectangular axes. Let the co-ordinates of P referred to new axes as the co-ordinate axes be (X, Y)

Thus if (x, y) are coordinates of a point referred to old axis and (X, Y) are the coordinates of the same point referred to new axis then.

$$x = X + h$$

and

$$y = Y + k$$

If therefore the origin is shifted at a point (h, k) we must substitute $X + h$ and $Y + k$ for x and y respectively.

The transformation formula from new axes to old axes is

$$X = x - h, \quad Y = y - k$$

The coordinates of the old origin referred to the new axes are $(-h, -k)$

21.7 Straight Line

A straight line is a curve such that every point on the line segment joining any two points on it lies on it.

The general form of the equation of straight line

$$ax + by + c = 0$$

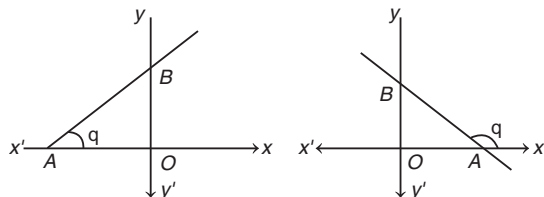
where a, b and c are real constants and x, y are two unknowns.

Slope (or Gradient) of a Line

$$m = \tan \theta = -\frac{a}{b}$$

$$\begin{aligned} \{ \because ax + by + c = 0 \Rightarrow by = -ax - c \Rightarrow y = -\frac{a}{b}x - \frac{c}{b} \\ \Rightarrow y = mx + c, \quad \text{where } m = -\frac{a}{b} \text{ and } c \text{ is a constant} \} \end{aligned}$$

Here m is called the slope or gradient of a line and c is the intercept on y -axis. The slope of a line is always measured in anticlockwise.

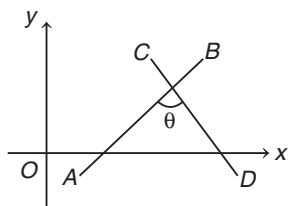


Slope of a line in terms of coordinates of any two points on it.

If (x_1, y_1) and (x_2, y_2) are coordinates of any two points on a line, then its slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Difference of ordinates}}{\text{Difference of abscissa}} = \frac{\text{rise}}{\text{run}}$$

Angle Between two Lines



$$\tan \theta = \pm \left(\frac{m_2 - m_1}{1 + m_1 m_2} \right)$$

Condition of Parallelism of Lines

If the slopes of two lines is m_1 and m_2 and if they are parallel then,

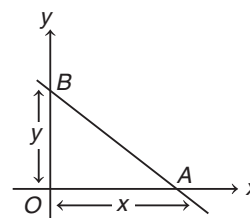
$$m_1 = m_2$$

Condition of Perpendicularity of Lines

If the slopes of two lines is m_1 and m_2 and if they are perpendicular to each other, then

$$m_1 \cdot m_2 = -1$$

Intercepts of a Line on The Axis



If a straight line cuts x -axis at A and the y -axis at B then OA and OB are known as the intercepts of the line on x -axis and y -axis respectively where O is the origin.

Introductory Exercise 21.1

- The point $(-2, 3)$ lies in the quadrant :
(a) First (b) Second
(c) Third (d) Fourth
- The point $(2, -3)$ lies in the quadrant :
(a) First (b) Second
(c) Third (d) Fourth
- Find the distance between the points $(-5, 3)$ and $(3, 1)$:
(a) $2\sqrt{7}$ (b) $3\sqrt{14}$ (c) $5\sqrt{17}$ (d) $2\sqrt{17}$
- Let the vertices of a triangle ABC be $(4, 3)$, $(7, -1)$, $(9, 3)$ then the triangle is :
(a) Scalene (b) Isosceles
(c) Equilateral (d) none of (a), (b), (c)
- Let the vertices of a triangle ABC be $(4, 4)$, $(3, 5)$, $(-1, -1)$, then the triangle is :
(a) scalene (b) equilateral
(c) right angled (d) none of (a), (b), (c)
- Let the vertices of a triangle ABC be $(7, 9)$, $(3, -7)$, and $(-3, 3)$ then the triangle is :
(a) right angled (b) equilateral
(c) isosceles (d) both (a) and (c)
- $A(-3, 2)$ and $B(5, 4)$ are the end points of a line segment, find the coordinates of the mid points of the line segment :
(a) $(1, 3)$ (b) $(2, 3)$ (c) $(3, 2)$ (d) $(4, 3)$
- In the previous question (no. 7) find the co-ordinates of the point which divides AB in the ratio $2 : 3$:
(a) $\left(\frac{1}{5}, \frac{14}{5}\right)$ (b) $(2, 3)$ (c) $\left(\frac{1}{5}, 3\right)$ (d) $\left(3, \frac{1}{2}\right)$
- Find the co-ordinates of the point which divides the join of the points $(2, 4)$ and $(6, 8)$ externally in the ratio $5 : 3$:
(a) $(5, 6)$ (b) $(12, 14)$ (c) $(3, 8)$ (d) $(2, 7)$
- The coordinates of the vertices of a triangle are $(3, 1)$, $(2, 3)$ and $(-2, 2)$. Find the coordinates of the centroid of the triangle ABC :
(a) $(1, 2)$ (b) $(2, 3)$ (c) $(4, 5)$ (d) $(5, 6)$
- Find the co-ordinates of the incentre of the triangle whose vertices are the points $(4, -2)$, $(5, 5)$ and $(-2, 4)$:
(a) $\left(\frac{7}{3}, \frac{2}{3}\right)$ (b) $\left(\frac{5}{2}, \frac{5}{2}\right)$
(c) $\left(\frac{6}{5}, 5\right)$ (d) none of these

12. Find the coordinates of the circum centre of the triangle whose vertices are (8, 6), (8, -2) and (2, -2):
 (a) (2, 3) (b) (5, 2)
 (c) (5, 3) (d) (7, 2)
13. If the coordinates of the mid points of the sides of a triangle are (1, 1), (2, -3) and (3, 4). Find its centroid:
 (a) $\left(3, \frac{2}{3}\right)$ (b) $\left(2, \frac{3}{4}\right)$
 (c) $\left(2, \frac{2}{3}\right)$ (d) none of these
14. If the previous problem (no. 13) find the incentre for the given triangle :
 (a) $\left(8, \frac{3}{2}\right)$ (b) $\left(\frac{7}{2}, 5\right)$
 (c) $(2\sqrt{3}, 5)$ (d) none of these
15. A(-2, -1), B(1, 0), C(4, 3) and D(1, 2) are the four points of a quadrilateral. The quadrilateral is a :
 (a) Square (b) Rhombus
 (c) Parallelogram (d) none of (a), (b), (c)
16. The three vertices of a parallelogram taken in a order are (-1, 0), (3, 1) and (2, 2) respectively. Find the coordinates of the fourth vertex :
 (a) (-1, 2) (b) (-2, 1)
 (c) (2, 3) (d) (3, -2)
17. If the coordinates of the mid points of the sides of a triangle are (1, 2), (0, -1) and (2, -1). Find the coordinates of its vertices :
 (a) (1, -4), (3, 2), (-1, 2) (b) (1, 2), (2, 3), (3, -4)
 (c) (3, 4), (5, 2), (1, 2) (d) none of these
18. Points A(4, -1), B(6, 0), C(7, 2) and D(5, 1) are the vertices of the following quadrilateral which is a :
 (a) Square (b) Rectangle
 (c) Rhombus (d) none of (a), (b), (c)
19. Find the area of quadrilateral formed by joining the points (-4, 2), (1, -1), (4, 1) and (2, 5):
 (a) 25.4 (b) 20.5
 (c) 24.5 (d) none of these
20. Find (x, y) if (3, 2), (6, 3), (x, y) and (6, 5) are the vertices of a parallelogram :
 (a) (5, 6) (b) (6, 5) (c) (9, 6) (d) (9, 5)
21. Determine the ratio in which $y - x + 2 = 0$ divides the line joining (3, -1) and (8, 9):
 (a) 3 : 5 (b) 4 : 3
 (c) 2 : 3 (d) none of these
22. If the area of the quadrilateral whose angular points taken in order are (1, 2), (-5, 6), (7, -4) and (h - 2) be zero, find the value of h :
 (a) 2 (b) 3 (c) 5 (d) 6
23. The co-ordinates of A, B, C are (6, 3), (-3, 5), (4, -2) respectively and P is a point (x, y), then find the value of $\frac{\Delta PBC}{\Delta ABC}$:
 (a) $\frac{x + y - 2}{7}$ (b) $\frac{x - y}{4}$
 (c) $\frac{x + y - 3}{5}$ (d) none of these
24. Find the slope of the line joining the points (7, 5) and (9, 7):
 (a) 1 (b) 2
 (c) $\frac{1}{2}$ (d) 3
25. If A(-2, 1), B(2, 3) and C(-2, -4) are three points, find the angle between BA and BC :
 (a) $\tan^{-1}\left(\frac{3}{5}\right)$ (b) $\tan^{-1}\left(\frac{2}{3}\right)$
 (c) $\tan^{-1}\left(\frac{3}{2}\right)$ (d) none of these
26. What is the slope of the line perpendicular to the line passing through the points (3, 5) and (-4, 2)?
 (a) $-\frac{3}{5}$ (b) $-\frac{2}{3}$
 (c) $-\frac{7}{3}$ (d) none of these
27. A line passes through the points A(2, -3) and B(6, 3). Find the slope of the line which is parallel to AB :
 (a) $\frac{2}{3}$ (b) $\frac{3}{2}$
 (c) $\frac{1}{2}$ (d) $\frac{3}{4}$
28. In the previous question (no. 27), find the slope of the line which is perpendicular to AB :
 (a) $-\frac{4}{3}$ (b) $\frac{5}{6}$
 (c) $-\frac{2}{3}$ (d) none of these
29. Points A(6, 6), B(2, 3) and C(4, 7) are the vertices of a triangle which is :
 (a) right angled
 (b) acute angled
 (c) obtuse angled
 (d) none of the above
30. Three points A(1, -2), B(3, 4) and C(4, 7) form :
 (a) a straight line
 (b) an equilateral triangle
 (c) a right angled triangle
 (d) none of (a), (b), (c)

21.8 Equations of Lines Parallel to the Co-ordinate Axes

Equation of a line parallel to x -axis and at a distance k from it is $y = k$. Equation of a line parallel to the y -axis and at a distance h from it is $x = h$.

Equation of the x -axis is $y = 0$

Equation of the y -axis is $x = 0$

21.9 Different Standard Forms of the Equations of a Straight Line

Slope Intercept Form

- (a) The equation of a line with slope m and making an intercept on y -axis is $y = mx + c$.
 (b) The equation of a line with slope m and passing through the origin (here $c = 0$) is $y = mx$.

Point Slope Form

The equation of a line passes through the point (x_1, y_1) and has the slope ' m ' is $y - y_1 = m(x - x_1)$

Two Point Form

The equation of a line passing through two points (x_1, y_1)

and (x_2, y_2) is $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1} \right) (x - x_1)$

Intercept Form

The equation of a line which cuts off intercepts a and b respectively from the x -axis and y -axis is $\frac{x}{a} + \frac{y}{b} = 1$.

Normal (or Perpendicular) Form

The equation of the straight line on which the length of the perpendicular from the origin $(0, 0)$ is p and this perpendicular makes an angle α with x -axis is $x \cos \alpha + y \sin \alpha = p$.

Distance (or Symmetrical) Form

The equation of the straight line passing through (x_1, y_1) and making an angle θ with positive direction of x -axis is

$$\frac{x - x_1}{\cos \theta} = \frac{y - y_1}{\sin \theta} = r$$

Where r is the distance of the point (x, y) on the line from the point (x_1, y_1)

- Since different values of r determine different points on the line, therefore the above form of the line is also called parametric form or symmetric form.

21.10 Reduction (or Transformation) of the General Equation

The general equation of a straight line is $Ax + By + C = 0$ which can be transformed into various standard forms.

- (i) General Equation ($Ax + By + C = 0$) $\rightarrow$ Slope Intercept form ($y = mx + c$)

$$Ax + By + C = 0 \Rightarrow By = -Ax - C$$

$$\Rightarrow y = \left(-\frac{A}{B} \right) x + \left(-\frac{C}{B} \right) \Rightarrow y = mx + c$$

$$\text{where } m = -\frac{A}{B} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } y}$$

$$\text{and intercept on } y\text{-axis} = -\frac{C}{B} = -\frac{\text{Constant term}}{\text{Coefficient of } y}$$

- (ii) General Equation ($Ax + By + C = 0$)

$$\rightarrow \text{Intercept form } \left(\frac{x}{a} + \frac{y}{b} = 1 \right)$$

$$Ax + By + C = 0$$

$$\Rightarrow \frac{Ax}{-C} + \frac{By}{-C} = 1$$

$$\Rightarrow \frac{x}{\left(-\frac{C}{A} \right)} + \frac{y}{\left(-\frac{C}{B} \right)} = 1$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \text{Where, intercept on } x\text{-axis} = -\frac{C}{A} = -\frac{\text{Constant term}}{\text{Coefficient of } x}$$

$$\text{Intercept on } y\text{-axis} = -\frac{C}{B} = -\frac{\text{Constant term}}{\text{Coefficient of } y}$$

- (iii) General Equation ($Ax + By + C = 0$)

$$\rightarrow \text{Normal Form } (x \cos \alpha + y \sin \alpha = p)$$

$$Ax + By + C = 0 \quad \dots(i)$$

$$x \cos \alpha + y \sin \alpha - p = 0 \quad \dots(ii)$$

Comparing eqs. (i) and (ii)

$$\frac{A}{\cos \alpha} = \frac{B}{\sin \alpha} = \frac{C}{-p}$$

$$\Rightarrow \cos \alpha = -\frac{Ap}{C} \quad \text{and} \quad \sin \alpha = -\frac{Bp}{C}$$

$$\Rightarrow 1 = \cos^2 \alpha + \sin^2 \alpha = \frac{p^2}{C^2} (A^2 + B^2)$$

or
$$p^2 = \frac{C^2}{A^2 + B^2} \Rightarrow p = \frac{C}{\sqrt{A^2 + B^2}}$$

$$\therefore \cos \alpha = \frac{-A}{\sqrt{A^2 + B^2}}, \sin \alpha = \frac{B}{\sqrt{A^2 + B^2}}$$

$\therefore$ The required equation is

$$-\frac{A}{\sqrt{A^2 + B^2}}x - \frac{B}{\sqrt{A^2 + B^2}}y = \frac{C}{\sqrt{A^2 + B^2}}$$

21.11. Point of Intersection of Two Lines

Let the equations of two lines be

$$a_1x + b_1y + c_1 = 0 \quad \dots(i)$$

and $a_2x + b_2y + c_2 = 0 \quad \dots(ii)$

Let (x_1, y_1) be the coordinates of the point of intersection of the two lines.

Then $a_1x_1 + b_1y_1 + c_1 = 0 \quad \dots(iii)$

and $a_2x_1 + b_2y_1 + c_2 = 0 \quad \dots(iv)$

$\therefore$ By the method of cross multiplication

$$\frac{x_1}{b_1c_2 - b_2c_1} = \frac{y_1}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\therefore x_1 = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, y_1 = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1},$$

where $a_1b_2 - a_2b_1 \neq 0$

Thus the coordinates of the point of intersection of the two lines are $\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

Remark To find the coordinates of the point of intersection of two non parallel lines, we solve the given equations simultaneously and the values of x and y so obtained determine the coordinates of the point of intersection.

21.12 Condition of Concurrency of Three Lines

Three lines are said to be concurrent if they pass through a common point *i.e.*, they meet at a single common point.

Thus if three lines are concurrent the point of intersection of any two lines lies on the third line.

Let $a_1x + b_1y + c_1 = 0 \quad \dots(i)$

$a_2x + b_2y + c_2 = 0 \quad \dots(ii)$

$a_3x + b_3y + c_3 = 0 \quad \dots(iii)$

be three concurrent lines then

$$a_3(b_1c_2 - b_2c_1) + b_3(c_1a_2 - c_2a_1) + c_3(a_1b_2 - a_2b_1) = 0$$

$$\Rightarrow \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Which is the required condition for concurrency of three lines.

- Find the point of intersection of any two lines by solving them simultaneously. If this point satisfies the third equation also, then the given lines are concurrent.

21.13 Angle Between Two Straight Lines

Let the two straight lines be

$$y = m_1x + c_1 \quad \dots(i)$$

and $y = m_2x + c_2 \quad \dots(ii)$

and θ be the angle between them,

then
$$\theta = \tan^{-1} \left| \frac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2} \right|$$

21.14 Conditions for Parallelism and Perpendicularity of Two Lines

- (i) If the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel,

$$\text{Then } m_1 = m_2 \Rightarrow -\frac{a_1}{b_1} = -\frac{a_2}{b_2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$$

- (ii) If the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular,

$$\text{then } m_1 \cdot m_2 = -1$$

$$\Rightarrow -\frac{a_1}{b_1} \times -\frac{a_2}{b_2} = -1 \Rightarrow a_1a_2 + b_1b_2 = 0$$

Thus two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

- (i) Coincident, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

- (ii) Parallel, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

- (iii) Intersecting, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

- (iv) Perpendicular, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

21.15 Distance of a Point From a Given Line or Length of Perpendicular From the Point (x_1, y_1) to the Straight Line $ax + by + c = 0$

Let the length of perpendicular be p ,

then
$$p = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Distance Between Parallel Lines

If two lines are parallel, then they have the same distance between these throughout.

Therefore to find the distance between two parallel lines find the coordinates of any point on one of the given lines, preferably putting $x = 0$ or $y = 0$

Then the perpendicular distance of this point from the other line is the required distance between the lines.

Alternatively Let the two parallel lines be $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$, then the perpendicular distance between the

lines is
$$\left| \frac{c_2 - c_1}{\sqrt{a^2 + b^2}} \right|$$

Equations of Straight Lines Passing Through (x_1, y_1) making angle θ with the given line $y = mx + c$

$$y - y_1 = \frac{m \pm \tan \theta}{1 \mp m \tan \theta} (x - x_1)$$

Introductory Exercise 21.2

- Find the equation of the line with slope $\frac{2}{3}$ and intercept on the y -axis is 5 :
 (a) $y = \frac{2}{3}x + 5$ (b) $y = \frac{3}{2}x + 5$
 (c) $y = 3x + 6$ (d) none of these
- Find the slope and the intercept on the y -axis of the line $\sqrt{3}x + 3y = 6$:
 (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{5}$
 (c) $\frac{2}{\sqrt{3}}$ (d) $-\frac{1}{\sqrt{3}}$
- Find the equation of the line passing through the point $(2, -3)$ and having its slope $\frac{5}{4}$:
 (a) $4x - 5y = 20$ (b) $3x - 2y = 5$
 (c) $5x - 4y = 22$ (d) none of these
- Find the equation of the line which cuts off intercepts 2 and 3 from the axes :
 (a) $9x - 7y = 6$ (b) $3x + 2y = 6$
 (c) $4x + 3y = 7$ (d) none of these
- Find the intercepts made by the line $3x + 4y - 12 = 0$ on the axes :
 (a) 2 and 3 (b) 4 and 3
 (c) 3 and 5 (d) none
- Find the equation of the line through the points $(-1, -2)$ and $(-5, 2)$:
 (a) $2x + y = 3$ (b) $3x - 2y + 7 = 0$
 (c) $x + y + 3 = 0$ (d) none of these
- Find the equation of the straight line passing through the point $(-1, 4)$ and having a gradient of 2.5 :
 (a) $2x - 5y + 13 = 0$ (b) $5x - 9y = 13$
 (c) $13x - 15y + 17 = 0$ (d) $5x - 2y + 13 = 0$

Equation of any Line Passing Through the Point of Intersection of Two Given Lines

Any line passing through the point of intersection of the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ can be represented by the equation $(a_1x + b_1y + c_1) + \lambda(a_2x + b_2y + c_2) = 0$

where λ is an arbitrary constant.

Position of a Point with Respect to a Line

Any line $ax + by + c = 0$ ($c \neq 0$) divides the whole plane into two parts (i) One part containing the origin called the origin side, (ii) the other part not containing the origin called non-origin side of the line. The length of the perpendicular

from the origin on the line $ax + by + c = 0$ is $\frac{c}{\sqrt{a^2 + b^2}}$; if $c > 0$

If $c > 0$, we say that the origin is on the positive side of the line and if $c < 0$, the origin is said to be on the negative side of the line. A point $P(x_1, y_1)$ is on the origin side or non-origin side of the line $ax + by + c = 0$ according as $ax_1 + by_1 + c$ and c are the same or opposite sign. If $c > 0$, then $P(x_1, y_1)$ is on the origin side or non-origin side of line $ax + by + c = 0$ according as $ax_1 + by_1 + c$ is positive or negative.

Equations of Bisectors of the Angles Between Two Given Lines

Let $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ be the equations of two given lines AB and AC respectively then the equations of the two bisectors are
$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

8. Find the equation of the straight line which makes equal intercepts on the axes passes through the point $(3, -5)$:
- (a) $x - y = 2$ (b) $x + y + 2 = 0$
 (c) $y - x + 2 = 0$ (d) none of these
9. Find the equation of the straight line making intercepts on the axes equal in magnitude but opposite in sign and passing through the point $(-5, -8)$:
- (a) $x - y = 7$ (b) $2x + y = 3$
 (c) $x - y = 3$ (d) none of these
10. Find the equation of the line passing through the point $(-4, -5)$ and perpendicular to the line joining the points $(1, 2)$ and $(5, 6)$:
- (a) $x + y + 17 = 0$ (b) $3x + 2y + 11 = 0$
 (c) $x + y + 9 = 0$ (d) $x - y + 20 = 0$
11. A straight line intersects the x-axis at A and the y-axis at B. AB is divided internally at C $(8, 10)$ in the ratio 5 : 4. Find the equation of AB :
- (a) $x + y = 18$ (b) $x + y + 2 = 0$
 (c) $x + y - 2 = 0$ (d) none of these
12. Find the equation of the straight line which passes through the point $(3, 4)$ and has intercepts on the axes such that their sum is 14 :
- (a) $4x + 3y = 24$
 (b) $x + y = 7$
 (c) $3x + 7y = 43$
 (d) both (a) and (b)
13. A straight line passes through the points $(a, 0)$ and $(0, b)$. The length of the line segment contained between the axis is 13 and the product of the intercepts on the axes is 60. Find the equation of the straight line :
- (a) $5x + 12y = 60$
 (b) $7x - 12y = 50$
 (c) $5x + 12y + 60 = 0$
 (d) both (a) and (c)
14. A firm produces 50 units of a good for ₹ 320 and 80 units for ₹ 380. Supposing that the cost curve is a straight line, estimate the cost of producing 110 units :
- (a) ₹ 330 (b) ₹ 1665
 (c) ₹ 440 (d) ₹ 365
15. Find the equation of the line on which length of the perpendicular from the origin is 5 and the angle which this perpendicular makes with the x-axis is 60° :
- (a) $x\sqrt{3} + 2y + 8 = 0$ (b) $x + \sqrt{2}y - 7 = 0$
 (c) $x + \sqrt{3}y = 10$ (d) none of (a), (b), (c)
16. Find the equation of the line which passes through the point $(3, -4)$ and makes an angle of 60° with the positive direction of x-axis :
- (a) $x\sqrt{2} + y\sqrt{3} = 0$
 (b) $\sqrt{3}x - y = 4 + 3\sqrt{3}$
 (c) $x\sqrt{3} + y = 3\sqrt{2} + 5$
 (d) none of (a), (b), (c)
17. Find the equation of the line joining the points of intersection of $2x + y = 4$ with $x - y + 1 = 0$ and $2x - y - 1 = 0$ with $x + y - 8 = 0$:
- (a) $2x + 3y + 6 = 0$ (b) $3x + 2y + 12 = 0$
 (c) $3x - 2y + 1 = 0$ (d) none of (a), (b), (c)
18. Find the equation of one of the two lines which pass through the point $(4, 5)$ which make an acute angle 45° with the line $2x - y + 7 = 0$:
- (a) $x - 2y = 0$ (b) $7x + 5y - 3 = 0$
 (c) $3x + y + 8 = 0$ (d) $x - 3y + 11 = 0$
19. Find the equation of the straight line which passes through the point $(5, -6)$ which is parallel to the line $8x + 7y + 5 = 0$:
- (a) $3x - 5y + 8 = 0$ (b) $7x + 8y + 5 = 0$
 (c) $7x - 8y + 2 = 0$ (d) $8x + 7y + 2 = 0$
20. Find the equation of the straight line which passes through the point of intersection of the straight lines $x + y = 8$ and $3x - 2y + 1 = 0$ and is parallel to the straight line joining the points $(3, 4)$ and $(5, 6)$:
- (a) $x - y + 2 = 0$ (b) $x + y - 2 = 0$
 (c) $3x - 4y + 8 = 0$ (d) none of these
21. Find the length of the perpendicular from the point $(3, -2)$ to the straight line $12x - 5y + 6 = 0$:
- (a) 5 (b) 4
 (c) 6 (d) 8
22. Find the distance between two parallel lines $5x + 12y - 30 = 0$ and $5x + 12y - 4 = 0$
- (a) 3 (b) 7
 (c) $\frac{5}{2}$ (d) 2
23. Find the equation of the line through the point of intersection of $2x - 3y + 1 = 0$ and $x + y - 2 = 0$ which is parallel to the y-axis.
- (a) $x = 1$ (b) $8x = 7$
 (c) $x + 3 = 0$ (d) $x = 6$
24. Find the equation of the line which passes through the point of intersection of the lines $x + 2y - 3 = 0$ and $4x - y + 7 = 0$ and is parallel to the line $y - x + 10 = 0$.
- (a) $2x + 2y + 5 = 0$ (b) $3x - 3y + 10 = 0$
 (c) $3x + 2y - 8 = 0$ (d) none of these

25. Find the equation of the line which passes through the point of intersection of the lines $2x - y + 5 = 0$ and $5x + 3y - 4 = 0$ and is perpendicular to the line $x - 3y + 21 = 0$
 (a) $2x + y + 8 = 0$ (b) $3x + 4y - 7 = 0$
 (c) $3x + y = 0$ (d) none of these
26. Find the equation of the line through the intersection of the lines $3x + 4y = 7$ and $x - y + 2 = 0$ having slope 3.
 (a) $4x - 3y + 7 = 0$ (b) $21x - 7y + 16 = 0$
 (c) $8x + y + 8 = 0$ (d) none of these
27. Find the equation of the straight line which passes through the point of intersection of the straight lines $3x - 4y + 1 = 0$ and $5x + y - 1 = 0$ and cuts off equal intercepts from the axis.
 (a) $32x + 32y + 11 = 0$ (b) $23x + 23y = 11$
 (c) $9x + 18y + 5 = 0$ (d) none of these
28. A straight line $\frac{x}{a} - \frac{y}{b} = 1$ passes through the point $(8, 6)$ and cuts off a triangle of area 12 units from the axes of coordinates. Find the equations of the straight line.
 (a) $3x - 2y = 12$ (b) $4x - 3y = 12$
 (c) $3x - 8y + 24 = 0$ (d) both (a) and (c)
29. Find the equations of the bisectors of the angle between the straight line $3x + 4y + 2 = 0$ and $5x - 12y - 6 = 0$.
 (a) $8x + y + 7 = 0$ (b) $16x - 12y - 1 = 0$
 (c) $x + 8y + 4 = 0$ (d) both (b) and (c)
30. Find the area of the triangle formed by the lines whose equations are $2y - x = 5$, $y + 2x = 7$ and $y - x = 1$.
 (a) $\frac{3}{10}$ (b) 10
 (c) 6 (d) $\frac{2}{5}$
31. Find the coordinates of the orthocentre of the triangle whose vertices are $(1, 2)$, $(2, 3)$ and $(4, 3)$.
 (a) $(2, 5)$ (b) $(3, 4)$
 (c) $(1, 6)$ (d) none of these
32. Two vertices of a triangle ABC are $B(5, -1)$ and $C(-2, 3)$. If the orthocentre of the triangle is the origin, find the third vertex.
 (a) $\left(\frac{7}{2}, \frac{13}{2}\right)$ (b) $\left(\frac{3}{2}, \frac{11}{2}\right)$
 (c) $(-4, -7)$ (d) none of these
33. The area of a triangle is 5. Two of its vertices are $(2, 1)$ and $(3, -2)$. The third vertex lies on $y = x + 3$. Find the third vertex :
 (a) $\left(\frac{2}{7}, \frac{13}{5}\right)$
 (b) $\left(\frac{7}{2}, \frac{13}{2}\right)$
 (c) $\left(\frac{9}{2}, \frac{13}{2}\right)$
 (d) $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(-\frac{3}{2}, \frac{3}{2}\right)$
34. A straight line L is perpendicular to the line $5x - y = 1$. The area of the triangle formed by the line L and coordinate axes is 5. Find the equation of the line :
 (a) $x + 5y = \pm 5\sqrt{2}$ (b) $x - 3y = 0$
 (c) $2x + y = 0$ (d) $x + 4y = 5\sqrt{2}$
35. $(1, 2)$ and $(3, 8)$ are a pair of opposite vertices of square. Find the diagonals of the square passing through $(1, 2)$:
 (a) $x - 2y = 1$ (b) $2x + 7y = 0$
 (c) $3x + 2y + 7 = 0$ (d) $3x - y = 1$

CAT-Test

Questions Helping you bell the CAT

LEVEL 01 > BASIC LEVEL EXERCISE

- The number of integral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices $(0, 0)$, $(0, 21)$ and $(21, 0)$ is
(a) 133 (b) 190
(c) 233 (d) 105
- Two squares have centers at the origin of a coordinate plane and sides parallel to the axes. The smaller square has an area of 27 sq. units and the larger square has an area of 91 sq. units. How many points with only integer coordinates are outside the smaller square region and inside the larger square region?
(a) 45 (b) 54
(c) 56 (d) 64
- The diagonals of a parallelogram $PQRS$ are along the lines $x + 3y = 4$ and $6x - 2y = 7$. Then $PQRS$ must be a
(a) Rectangle (b) Square
(c) Cyclic Quadrilateral (d) Rhombus
- What is the minimum value of $|c|$, if the lines $y = mx + 4$, $x = m + c$ and $y = 3$ are concurrent?
(a) 0 (b) 1
(c) 2 (d) 3
- Two points $(1, 1)$ and $(1, 5)$ lie on the circumference of a circle. Which of the following cannot be the area of that circle?
(a) $2\sqrt{2}\pi$ (b) 4π
(c) $3\sqrt{3}\pi$ (d) 6.67π
- Consider four circles $(x \pm 1)^2 + (y \pm 1)^2 = 1$. Find the equation of larger circle touching these four circles.
(a) $x^2 + y^2 = (\sqrt{2} + 1)^2$ (b) $x^2 + y^2 = (\sqrt{2} - 1)^2$
(c) $x^2 + y^2 = (2 - \sqrt{2})^2$ (d) none of these
- What's the area of a triangle whose coordinates are $(-7, 4)$, $(3, -2)$, $(-1, -5)$?
(a) 27 (b) 26
(c) 25 (d) 33

Answers

✓ Introductory Exercise 21.1

1 (b)	2 (d)	3 (d)	4 (b)	5 (c)	6 (d)	7 (a)	8 (a)	9 (b)	10 (a)
11 (b)	12 (b)	13 (c)	14 (d)	15 (c)	16 (b)	17 (a)	18 (c)	19 (c)	20 (c)
21 (c)	22 (b)	23 (a)	24 (a)	25 (b)	26 (c)	27 (b)	28 (c)	29 (a)	30 (d)

✓ Introductory Exercise 21.2

1 (a)	2 (d)	3 (c)	4 (b)	5 (b)	6 (c)	7 (d)	8 (b)	9 (c)	10 (c)
11 (a)	12 (d)	13 (d)	14 (c)	15 (c)	16 (b)	17 (c)	18 (d)	19 (d)	20 (a)
21 (b)	22 (d)	23 (a)	24 (b)	25 (c)	26 (b)	27 (b)	28 (d)	29 (d)	30 (a)
31 (c)	32 (c)	33 (d)	34 (a)	35 (d)					

✓ Level 01 Basic Level Exercise

1. (b)	2. (c)	3. (d)	4. (c)	5. (a)	6. (a)	7. (a)			

Hints & Solutions

Introductory Exercise 21.1

1 The point $(-2, 3)$ lies in the second quadrant.

2 The point $(2, -3)$ lies in the fourth quadrant.

3 Distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let $(x_1, y_1) \equiv (-5, 3)$ and $(x_2, y_2) \equiv (3, 1)$

$$\begin{aligned}\therefore \text{Required distance} &= \sqrt{(3+5)^2 + (1-3)^2} \\ &= \sqrt{64+4} = \sqrt{68} = 2\sqrt{17} \text{ unit}\end{aligned}$$

4 Let $A \equiv (4, 3)$, $B \equiv (7, -1)$, $C \equiv (9, 3)$

$$AB = \sqrt{(7-4)^2 + (-1-3)^2} = \sqrt{25} = 5$$

$$BC = \sqrt{(9-7)^2 + (3+1)^2} = \sqrt{20} = 2\sqrt{5}$$

$$CA = \sqrt{(4-9)^2 + (3-3)^2} = \sqrt{25} = 5$$

$$\therefore AB = CA = 5$$

Hence ABC is an isosceles triangle.

5 Let $A \equiv (4, 4)$, $B \equiv (3, 5)$, $C \equiv (-1, -1)$

$$\text{Then } AB = \sqrt{(3-4)^2 + (5-4)^2} = \sqrt{2}$$

$$BC = \sqrt{(-1-3)^2 + (-1-5)^2} = \sqrt{52}$$

$$AC = \sqrt{(-1-4)^2 + (-1-4)^2} = \sqrt{50}$$

$$\therefore AB^2 + AC^2 = BC^2$$

Hence by the Pythagoras Theorem, ABC is a right angled triangle.

6 Let $A \equiv (7, 9)$, $B \equiv (3, -7)$ and $C \equiv (-3, 3)$

$$\text{Then } AB = \sqrt{(3-7)^2 + (-7-9)^2} = \sqrt{272}$$

$$BC = \sqrt{(-3-3)^2 + (3+7)^2} = \sqrt{136}$$

$$AC = \sqrt{(7+3)^2 + (9-3)^2} = \sqrt{136}$$

$$\therefore AB^2 = BC^2 + AC^2$$

Hence by Pythagoras Theorem, triangle ABC is a right angled triangle.

Also, since $BC = CA$, hence ABC is an isosceles triangle.

Thus ABC is a right angled isosceles triangle.

7 The coordinates of the mid points of AB

$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-3+5}{2}, \frac{2+4}{2} \right) = (1, 3)$$

8 Here $(x_1, y_1) \equiv (-3, 2)$ and $(x_2, y_2) \equiv (5, 4)$

The coordinates of the point which divides AB in the ratio 2 : 3

$$= \left(\frac{2 \times 5 + 3 \times (-3)}{2+3}, \frac{2 \times 4 + 3 \times 2}{2+3} \right) = \left(\frac{1}{5}, \frac{14}{5} \right)$$

9 The required coordinates of the point which divides the join of $(2, 4)$ and $(6, 8)$ externally in the ratio 5 : 3 are $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n} \right)$

Here, $m : n = 5 : 3$, $(x_1, y_1) \equiv (2, 4)$, $(x_2, y_2) \equiv (6, 8)$

Hence the required co-ordinates $\equiv (12, 14)$.

10 Let the co-ordinate of the centroid of ΔABC be (x, y) , then

$$\begin{aligned}(x, y) &= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right) \\ &= \left(\frac{3+2-2}{3}, \frac{1+3+2}{3} \right) = (1, 2)\end{aligned}$$

11 Let $A \equiv (4, -2)$, $B \equiv (5, 5)$ and $C \equiv (-2, 4)$

$$\text{Then } a = BC = \sqrt{(-2-5)^2 + (4-5)^2} = 5\sqrt{2}$$

$$b = AC = \sqrt{(4+2)^2 + (-2-4)^2} = 6\sqrt{2}$$

$$c = AB = \sqrt{(5-4)^2 + (5+2)^2} = 5\sqrt{2}$$

and $(x_1, y_1) \equiv (4, -2)$, $(x_2, y_2) \equiv (5, 5)$,

$$(x_3, y_3) \equiv (-2, 4)$$

$\therefore$ The coordinates of the incentre of the ΔABC are

$$\begin{aligned}&\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right) \\ &= \left(\frac{5}{2}, \frac{5}{2} \right) \text{ \{Substitute the values of } (a, b, c)\end{aligned}$$

x_1, x_2, x_3, y_1, y_2 and y_3 }

12 Let $S(x, y)$ be the circumcentre, then $AS = BS = CS = R$, where R is the circumradius

Now $(AS)^2 = (BS)^2$ where $S \equiv (x, y)$,

$$A \equiv (8, 6), B \equiv (8, -2)$$

$$\therefore (x-8)^2 + (y-6)^2 = (x-8)^2 + (y+2)^2$$

$$\Rightarrow 36 - 12y = 4 + 4y$$

$$\Rightarrow y = 2$$

Again $(BS)^2 = (CS)^2$, where $S \equiv (x, y)$

$$B \equiv (8, -2), C \equiv (2, -2)$$

$$\therefore (x-8)^2 + (y+2)^2 = (x-2)^2 + (y+2)^2$$

$$\Rightarrow x^2 + 64 - 16x + y^2 + 4 + 4y$$

$$= x^2 + 4 - 4x + y^2 + 4 + 4y$$

$$\Rightarrow -12x + 60 = 0 \Rightarrow x = 5$$

$\therefore$ The circumcentre $S \equiv (x, y) \equiv (5, 2)$

- 13** Let $P(1, 1)$, $Q(2, -3)$, $R(3, 4)$ be the midpoints of sides AB , BC and CA respectively triangle ABC .

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of triangle ABC .

Then, P is the mid point of AB

$$\Rightarrow \frac{x_1 + x_2}{2} = 1, \frac{y_1 + y_2}{2} = 1$$

$$\Rightarrow x_1 + x_2 = 2 \text{ and } y_1 + y_2 = 2 \quad \dots(i)$$

Q is the mid point of BC

$$\Rightarrow \frac{x_2 + x_3}{2} = 2, \frac{y_2 + y_3}{2} = -3$$

$$\Rightarrow x_2 + x_3 = 4 \text{ and } y_2 + y_3 = -6 \quad \dots(ii)$$

R is the mid point of AC

$$\Rightarrow x_1 + x_3 = 6 \text{ and } y_1 + y_3 = 8 \quad \dots(iii)$$

From eqs (i), (ii) and (iii), we get

$$(x_1, y_1) \equiv (2, 8), (x_2, y_2) \equiv (0, -6)$$

$$\text{and } (x_3, y_3) \equiv (4, 0)$$

Then the coordinates of the centroid

$$\begin{aligned} &= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right) \\ &= \left(\frac{2 + 0 + 4}{3}, \frac{8 - 6 + 0}{3} \right) = \left(2, \frac{2}{3} \right) \end{aligned}$$

- 14** Since $(x_1, y_1) \equiv A(2, 8)$, $(x_2, y_2) \equiv B(0, -6)$
and $(x_3, y_3) \equiv C(4, 0)$
Now, $a = BC = 2\sqrt{3}$, $b = AC = 2\sqrt{17}$, $c = AB = 10\sqrt{2}$
The coordinates of the in-centre of the triangle ABC are

$$\left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c} \right)$$

$\therefore$ Required coordinates of incentre are

$$\left(\frac{2\sqrt{13} + 20\sqrt{2}}{\sqrt{13} + \sqrt{17} + 5\sqrt{2}}, \frac{2\sqrt{13} - 6\sqrt{17}}{\sqrt{13} + \sqrt{17} + 5\sqrt{2}} \right)$$

- 15** Let A, B, C and D be the vertices of the quadrilateral whose coordinates are $(-2, -1)$, $(1, 0)$, $(4, 3)$ and $(1, 2)$ respectively.
Now, $AB = \sqrt{10}$, $BC = \sqrt{18}$, $DC = \sqrt{10}$, $AD = \sqrt{18}$
 $\therefore AB = CD$ and $BC = AD$ i.e., the opposite sides are equal.
Hence $ABCD$ is a parallelogram.

- 16** Let $A(-1, 0)$, $B(3, 1)$, $C(2, 2)$ and $D(x, y)$ be the vertices of a parallelogram $ABCD$ taken in order. Since the diagonals of a parallelogram bisect each other.

$\therefore$ Coordinates of the mid point of AC

= Coordinates of the mid point of BD

$$\Rightarrow \left(\frac{-1 + 2}{2}, \frac{0 + 2}{2} \right) = \left(\frac{3 + x}{2}, \frac{1 + y}{2} \right)$$

$$\Rightarrow \left(\frac{1}{2}, 1 \right) = \left(\frac{3 + x}{2}, \frac{1 + y}{2} \right)$$

$$\Rightarrow \frac{3 + x}{2} = \frac{1}{2} \text{ and } \frac{1 + y}{2} = 1$$

$$\Rightarrow x = -2 \text{ and } y = 1$$

Hence the fourth, vertex of the parallelogram is $(-2, 1)$

- 17** Solve as the question no. 13 has been solved. The required vertices of the triangle are $(1, -4)$, $(3, 2)$, $(-1, 2)$.

- 18** Let the vertices be $A(4, -1)$, $B(6, 0)$, $C(7, 2)$, $D(5, 1)$, then the coordinates of the mid point of AC are

$$\left(\frac{4 + 7}{2}, \frac{-1 + 2}{2} \right) = \left(\frac{11}{2}, \frac{1}{2} \right)$$

Coordinates of the mid point of BD are

$$\left(\frac{6 + 5}{2}, \frac{0 + 1}{2} \right) = \left(\frac{11}{2}, \frac{1}{2} \right)$$

Thus AC and BD have the same mid point. Hence $ABCD$ is a parallelogram.

$$\text{Now, } AB = \sqrt{5}, BC = \sqrt{5}$$

$$\therefore AB = BC$$

So, $ABCD$ is a parallelogram whose adjacent sides are equal. Hence, $ABCD$ is a rhombus.

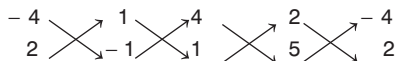
$$\text{Also, we have } AC = 3\sqrt{2}$$

$$\text{and } BD = \sqrt{2}$$

Clearly, $AC \neq BD$. So $ABCD$ is not a square.

- 19** Let A, B, C, D be the points $(-4, 2)$, $(1, -1)$, $(4, 1)$ and $(2, 5)$ respectively. Then the area of the quadrilateral $ABCD$

$$\begin{aligned} &= \frac{1}{2} \{-4 \times -1 - 2 \times 1 + 1 \times 1 - 4 \times -1 + 4 \times 5 - 2 \times 1 \\ &\quad + 2 \times 2 - 5 \times -4\} \\ &= \frac{1}{2} (4 - 2 + 1 + 4 + 20 - 2 + 4 + 20) \\ &= \frac{49}{2} = 24.5 \text{ square units.} \end{aligned}$$

Hint 

NOTE Area of quadrilateral $= \frac{1}{2} \{(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + (x_4y_1 - x_1y_4)\}$

- 20** Let A, B, C, D be the points $(3, 2)$, $(6, 3)$, (x, y) and $(6, 5)$ respectively.

Since $ABCD$ is a parallelogram, the diagonals AC and BD must bisect each other i.e., the mid points of AC and the mid point of BD must coincide and hence the coordinates of the two mid points are the same

$$\therefore \frac{3 + x}{2} = \frac{6 + 6}{2} \text{ and } \frac{2 + y}{2} = \frac{3 + 5}{2}$$

$$\text{or } 3 + x = 12 \text{ and } 2 + y = 8$$

$$\text{or } x = 9 \text{ and } y = 6$$

$$\text{Hence } (x, y) = (9, 6)$$

- 21** Let the line $y - x + 2 = 0$ divide the join of $(3, -1)$ and $(8, 9)$ at the point P in the ratio $k : 1$. Then the coordinates of P

$$\text{are } \left(\frac{k \cdot 8 + 1 \cdot 3}{k + 1}, \frac{k \cdot 9 + 1 \cdot (-1)}{k + 1} \right) = \left(\frac{8k + 3}{k + 1}, \frac{9k - 1}{k + 1} \right)$$

Since, this point lies on the line $y - x + 2 = 0$, we have

$$\frac{9k - 1}{k + 1} - \frac{8k + 3}{k + 1} + 2 = 0 \quad \text{or} \quad k = \frac{2}{3}$$

Hence the required ratio is $k : 1 = \frac{2}{3} : 1 = 2 : 3$

- 22** Let the points be $A(1, 2)$, $B(-5, 6)$, $C(7, -4)$, $D(h, -2)$

Given, area of the quadrilateral $ABCD = 0$

$$\text{or } \frac{1}{2} \{ (6 + 10) + (20 - 42) + (-14 + 4h) + (2h + 2) \} = 0$$

$$\text{or } h = 3$$

- 23** Area of $\Delta PBC = \frac{1}{2} \{ (5x + 3y) + (6 - 20) + (4y + 2x) \}$

$$= \frac{1}{2} (7x + 7y - 14) = \frac{7}{2} (x + y - 2)$$

and area of ΔABC

$$= \frac{1}{2} \{ (30 + 9) + (6 - 20) + (12 + 12) \}$$

$$= \frac{49}{2}$$

$$\text{Hence } \frac{\Delta PBC}{\Delta ABC} = \frac{x + y - 2}{7}$$

- 24** Slope of the line $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 5}{9 - 7} = 1$

Here $(x_1, y_1) \equiv (7, 5)$ and $(x_2, y_2) \equiv (9, 7)$

- 25** Let m_1 and m_2 be the slopes of BA and BC respectively.

$$\text{Then } m_1 = \frac{3 - 1}{2 - (-2)} = \frac{1}{2}$$

$$\text{and } m_2 = \frac{-4 - 3}{-2 - 2} = \frac{7}{4}$$

Let θ be the angle between BA and BC . Then

$$\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right| = \left| \frac{\frac{7}{4} - \frac{1}{2}}{1 + \frac{7}{4} \times \frac{1}{2}} \right| = \pm \frac{2}{3}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{2}{3} \right)$$

- 26** If m_1 be the slope of the line passing through the points $A(3, 5)$ and $B(-4, 2)$, then

$$m_1 = \frac{2 - 5}{-4 - 3} = \frac{3}{7}$$

If m_2 be the slope of the perpendicular line CD then

$$m_1 \cdot m_2 = -1$$

$$\text{or } \frac{3}{7} m_2 = -1$$

$$\text{or } m_2 = -\frac{7}{3}$$

- 27** Let m_1 be the slope of line AB

$$\therefore m_1 = \frac{3 - (-3)}{6 - 2} = \frac{3}{2}$$

If m_2 be the slope of a line parallel to AB , then

$$m_2 = m_1 = \frac{3}{2}$$

- 28** Let m_3 be the slope of line perpendicular to AB , then

$$m_1 \cdot m_3 = -1$$

$$\Rightarrow \frac{3}{2} \cdot m_3 = -1$$

$$\Rightarrow m_3 = -\frac{2}{3}$$

- 29** $m_1 = \text{Slope of } AB = \frac{3 - 6}{2 - 6} = \frac{3}{4}$

$$m_2 = \text{Slope of } BC = \frac{7 - 3}{4 - 2} = 2$$

$$\text{and } m_3 = \text{Slope of } AC = \frac{7 - 6}{4 - 6} = -\frac{1}{2}$$

$$\therefore m_2 \cdot m_3 = 2 \times -\frac{1}{2} = -1$$

This shows that BC is perpendicular to AC .

Hence, ABC is a right-angled triangle.

- 30** $m_1 = \text{Slope of } AB = \frac{4 - (-2)}{3 - 1} = 3$

$$\text{and } m_2 = \text{Slope of } BC = \frac{7 - 4}{4 - 3} = 3$$

$$\therefore m_1 = m_2$$

$\therefore AB$ is parallel to BC and B is common to both AB and BC .

Hence, the point $A(1, -2)$, $B(3, 4)$ and $C(4, 7)$ are collinear.

Introductory Exercise 21.2

- 1 The equation of the line with slope $2/3$ and intercept on the Y-axis 5 is $y = \frac{2}{3}x + 5$ ($\because y = mx + c$)

2 We have $\sqrt{3}x + 3y = 6$

or $3y = -\sqrt{3}x + 6$

or $y = -\frac{1}{\sqrt{3}}x + 2$

Comparing the above equation with $y = mx + c$ we get

$$m = -\frac{1}{\sqrt{3}} \quad \text{and} \quad c = 2$$

Hence slope is $\left(-\frac{1}{\sqrt{3}}\right)$ and intercept on the y-axis is 2.

3 We have $m = \frac{5}{4}$ and $(x_1, y_1) = (2, -3)$

$\therefore$ The equation of the line as point slope form is

$$y - y_1 = m(x - x_1)$$

or $y - (-3) = \frac{5}{4}(x - 2)$

or $y + 3 = \frac{5}{4}(x - 2)$

or $5x - 4y = 22$

4 Here $a = 2$ and $b = 3$

$\therefore$ The required equation of the line is $\frac{x}{2} + \frac{y}{3} = 1$

$$\Rightarrow 3x + 2y = 6$$

5 We have $3x + 4y - 12 = 0$

$$\Rightarrow 3x + 4y = 12$$

$$\Rightarrow \frac{3x}{12} + \frac{4y}{12} = 1 \Rightarrow \frac{x}{4} + \frac{y}{3} = 1$$

Which is of the form $\frac{x}{a} + \frac{y}{b} = 1$

Thus the required intercepts on the axes are 4 and 3.

6 The equation of the line through the points $(-1, -2)$ and

$$(-5, 2) \text{ is } (y - y_1) = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

where $(x_1, y_1) \equiv (-1, -2)$

and $(x_2, y_2) \equiv (-5, 2)$

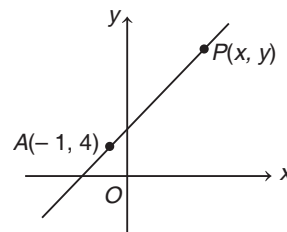
$\therefore$ Required equation is

$$y - (-2) = \frac{2 - (-2)}{-5 - (-1)} [x - (-1)]$$

or $y + 2 = \frac{4}{-4} (x + 1)$

or $x + y + 3 = 0$

- 7 Let $(-1, 4)$ be the point as shown in figure and let $P(x, y)$ be any point on the line. Then the gradient (or slope) of the line $\frac{y - 4}{x - (-1)} = 2.5$



$$\Rightarrow \frac{y - 4}{x + 1} = \frac{5}{2}$$

$$\Rightarrow 5x - 2y + 13 = 0$$

- 8 Let the equation of the straight line in the intercept form be

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \dots(i)$$

Since the intercepts are equal, therefore $a = b$

$\therefore$ From eq. (i)

$$x + y = a \quad \dots(ii)$$

Since this line passes through the point $(3, -5)$

$$\therefore 3 + (-5) = a \Rightarrow a = -2$$

$\therefore$ From eq. (ii), the required equation of the straight line is $x + y = -2$ or $x + y + 2 = 0$

- 9 Let the equation of the straight line be

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \dots(i)$$

Since intercepts a, b are equal in magnitude but opposite in sign.

$$\therefore b = -a$$

$$\therefore \text{From eq. (i)} \quad \frac{x}{a} + \frac{y}{(-a)} = 1$$

$$\text{or} \quad x - y = a \quad \dots(ii)$$

Since this line passes through the point $(-5, -8)$.

$$\therefore -5 - (-8) = a$$

$$\Rightarrow a = 3$$

Hence, from (ii) the required equation of the line is $x - y = 3$

- 10 Let m_1 = slope of the line 'joining' $(1, 2)$ and $(5, 6)$

$$\therefore m_1 = \frac{6 - 2}{5 - 1} = \frac{4}{4} = 1$$

If m_2 be the slope of the perpendicular line, then $m_1 m_2 = -1 \Rightarrow m_2 = -1$ ($\because m_1 = 1$)

∴ The required line has slope $m_2 = -1$ and passes through the point $(-4, -5)$

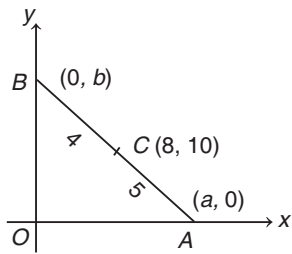
Hence, the required equation of the line in the point slope form is

$$(y - y_1) = m_2 (x_2 - x_1)$$

or $y - (-5) = -1 \{x - (-4)\}$

or $x + y + 9 = 0$

- 11** Let the equation of the line AB be $\frac{x}{a} + \frac{y}{b} = 1$... (i)



Then the coordinates of A and B are respectively $(a, 0)$ and $(0, b)$.

Since C(8, 10) divides AB in the ratio 5 : 4, we have

$$\frac{5 \times 0 + 4 \times a}{5 + 4} = 8$$

and $\frac{5 \times b + 4 \times 0}{5 + 4} = 10$

or $a = 18$ and $b = 18$

Hence from eq. (i), the required equation of the line AB is

$$\frac{x}{18} + \frac{y}{18} = 1 \text{ or } x + y = 18$$

- 12** Let the equation of the line in the intercept form be $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are intercepts on the axes.

Then $a + b = 14$ or $b = 14 - a$

Since the line $\frac{x}{a} + \frac{y}{b} = 1$ passes through the point $(3, 4)$;

∴ $\frac{3}{a} + \frac{4}{b} = 1$ or $\frac{3}{a} + \frac{4}{14 - a} = 1$

or $a^2 - 13a + 42 = 0$ or $(a - 6)(a - 7) = 0$

∴ $a = 6, 7$

If $a = 6$ then $b = 8$

If $a = 7$ then $b = 7$

Hence the required equation of the line are

$$\frac{x}{6} + \frac{y}{8} = 1 \text{ and } \frac{x}{7} + \frac{y}{7} = 1$$

or $4x + 3y = 24$ and $x + y = 7$

- 13** Since the line passes through $A(a, 0)$ and $B(0, b)$, it makes intercepts a and b on the axes of x and y . Let the equation of the line be

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \dots(i)$$

By the given conditions,

$$AB = 13 \Rightarrow a \cdot b = 60 \quad \dots(ii)$$

From eq. (i) $\sqrt{a^2 + b^2} = 13$

$\Rightarrow a^2 + b^2 = 169$

$\Rightarrow a + b = \pm 17$

Again $(a - b)^2 = (a + b)^2 - 4ab = 289 - 240 = 49$

∴ we get $a = 12, b = 5$ and $a = -12, b = -5$

∴ The required equations of the straight line are

$$\frac{x}{12} + \frac{y}{5} = 1 \text{ and } \frac{x}{-12} + \frac{y}{-5} = 1$$

i.e., $5x + 12y = 60$ and $5x + 12y + 60 = 0$

- 14** Let the equation of the cost curve as a straight line be

$$y = mx + c \quad \dots(i)$$

where x = number of units of a good produced

and y = cost of x units in rupees.

Given, when $x = 50, y = 320$ and when $x = 80, y = 380$

from eq. (i) $320 = 50m + c \quad \dots(ii)$

and $380 = 80m + c \quad \dots(iii)$

Subtracting eqs. (ii) from (iii), we get $m = 2$

Substituting $m = 2$ in eq. (ii), we get $c = 220$

∴ From eq. (i) $y = 2x + 220$

when $x = 110, y = 2 \times 110 + 220 = 440$

Hence, the required cost of producing 110 units is ₹ 440.

- 15** Here $p = 5$ and $\alpha = 60^\circ$

∴ The required equation of the line is

$$x \cos \alpha + y \sin \alpha = p$$

or $x \cos 60^\circ + y \sin 60^\circ = 5$

$\Rightarrow x + \sqrt{3}y = 10$

$$\left(\because \sin 60^\circ = \frac{\sqrt{3}}{2} \text{ and } \cos 60^\circ = \frac{1}{2} \right)$$

- 16** Here $(x_1, y_1) \equiv (3, -4)$ and $\theta = 60^\circ$

The required equation of the line in the symmetric form is

$$\frac{x - x_1}{\cos \theta} = \frac{y - y_1}{\sin \theta}$$

$\Rightarrow \frac{x - 3}{\cos 60^\circ} = \frac{y - (-4)}{\sin 60^\circ}$

$\Rightarrow \sqrt{3}x - y = 4 + 3\sqrt{3}$

- 17** We have $2x + y = 4 \quad \dots(i)$

and $x - y + 1 = 0 \quad \dots(ii)$

Solving eqs. (i) and (ii), we get

$$x = 1 \text{ and } y = 2$$

∴ The point of intersection of eqs. (i) and (ii) is $(1, 2)$.

Again $2x - y - 1 = 0 \quad \dots(iii)$

$x + y - 8 = 0 \quad \dots(iv)$

Solving eqs. (iii) and (iv), we get $(x, y) \equiv (3, 5)$

∴ The point of intersection of (3) and (4) is (3, 5)

∴ The required equation of the straight line joining the points of intersection (1, 2), (3, 5) is

$$y - 2 = \frac{(5-2)}{(3-1)} \cdot (x-1)$$

$$\Rightarrow 3x - 2y + 1 = 0$$

18 The equation of the line through the point (4, 5) is

$$y - 5 = m(x - 4) \quad \dots(i)$$

where m is the slope of the line.

Now the given line is $2x - y + 7 = 0$

$$\Rightarrow y = 2x + 7 \quad \dots(ii)$$

If m_1 be the slope of the line (ii), then $m_1 = 2$

If eq. (i) makes an angle of 45° with eq. (ii), then we have

$$\tan 45^\circ = \frac{m_1 - m}{1 + m \cdot m_1} = \frac{2 - m}{1 + 2m} \quad \text{i.e., } \frac{2 - m}{1 + 2m}$$

$$\therefore \text{either } 1 = \frac{m-2}{1+2m} \text{ or } 1 = \frac{2-m}{1+2m}$$

$$\text{If } \frac{m-2}{1+2m} = 1 \text{ then } m = -3$$

$$\text{If } \frac{2-m}{1+2m} = 1 \text{ then } m = \frac{1}{3}$$

Hence, from (i) the required equation of the two lines is

$$y - 5 = -3(x - 4) \text{ and } y - 5 = \frac{1}{3}(x - 4)$$

$$\Rightarrow 3x - y - 17 = 0 \text{ and } x - 3y + 11 = 0$$

19 The equation of any straight line parallel to the line $8x + 7y + 5 = 0$ is $8x + 7y + c = 0$... (i)

where c is an arbitrary constant.

If the line (i) passes through the point (5, -6), then

$$8 \times 5 + 7 \times (-6) + c = 0$$

$$\Rightarrow c = 2$$

Hence from (i), the required equation of the straight line is $8x + 7y + 2 = 0$

20 Solving $x + y = 8$ and $3x - 2y + 1 = 0$, we get the point of intersection.

∴ The point of intersection is (3, 5).

Now, the equation of the line joining the points

$$(3, 4) \text{ and } (5, 6) \text{ is } (y - 4) = \frac{(6-4)}{(5-3)} (x - 3)$$

$$\Rightarrow x - y + 1 = 0 \quad \dots(i)$$

∴ The equation of the line parallel to the line

$$x - y + 1 = 0 \text{ is}$$

$$x - y + c = 0 \quad \dots(ii)$$

Where c is an arbitrary constant. If the line passes through the point (3, 5) then

$$3 - 5 + c = 0 \text{ or } c = 2$$

Hence from (2), the required equation of the line is $x - y + 2 = 0$.

21 Length of the perpendicular from the point (3, -2) to the straight line $12x - 5y + 6 = 0$ is

$$\frac{12 \times 3 - 5 \times (-2) + 6}{\sqrt{(12)^2 + (-5)^2}} = \frac{36 + 10 + 6}{\sqrt{169}} = 4 \text{ units.}$$

22 Putting $y = 0$ in $5x + 12y - 30 = 0$, we get

$$5x - 30 = 0 \text{ or } x = 6$$

∴ (6, 0) is a point on the first line $5x + 12y - 30 = 0$

Required distance between the parallel lines = Perpendicular distance of the point (6, 0) from the second line $5x + 12y - 4 = 0$.

$$= \frac{5 \cdot 6 + 12 \cdot 0 - 4}{\sqrt{5^2 + 12^2}} = \frac{30 - 4}{13} = 2 \text{ units}$$

23 The equation of the line through the point of intersection of $2x - 3y + 1 = 0$ and $x + y - 2 = 0$, is

$$(2x - 3y + 1) + k(x + y - 2) = 0$$

$$\text{i.e., } (2 + k)x + (k - 3)y + (1 - 2k) = 0 \quad \dots(i)$$

If this line is parallel to the y -axis, then its equation must be of the form $x = h$, i.e., the coefficient of y in (i) must be zero.

$$\therefore k - 3 = 0 \text{ or } k = 3$$

Hence, from (i) the required equation of the line is $(2 + 3)x + 0 \cdot y + (1 - 2 \times 3) = 0$ [Putting $k = 3$]

$$\Rightarrow x = 1$$

24 The equation of any line passing through the point of intersection of the lines $x + 2y - 3 = 0$ and $4x - y + 7 = 0$ is

$$x + 2y - 3 + k(4x - y + 7) = 0 \quad \dots(i)$$

$$\text{or } (1 + 4k)x + (2 - k)y + (7k - 3) = 0 \quad \dots(ii)$$

$$m_1 = \text{Slope of the line (ii)} = \frac{4k + 1}{k - 2}$$

$$\text{and } m_2 = (\text{Slope of the line } y - x + 10 = 0) = 1$$

If the line (i) be parallel to the line $y - x + 10 = 0$

$$\text{then } \frac{4k + 1}{k - 2} = 1 \Rightarrow k = -1$$

Hence from (i), the required equation of the line is

$$(x + 2y - 3) - 1 \cdot (4x - y + 7) = 0$$

$$\Rightarrow 3x - 3y + 10 = 0$$

25 Solving $2x - y + 5 = 0$ and $5x + 3y - 4 = 0$, we get $x = -1$ and $y = 3$ i.e., the point of intersection of the given lines is $(-1, 3)$.

∴ The equation of any line perpendicular to the line

$$x - 3y + 21 = 0 \text{ is } 3x + y + k = 0 \quad \dots(i)$$

If this line (i) passes through the point $(-1, 3)$ then

$$3 \times (-1) + 3 + k = 0$$

$$\Rightarrow k = 0$$

∴ From (i), the required equation of the line is $3x + y = 0$.

- 26** The equation of any line passing through the intersection of the lines $3x + 4y - 7 = 0$ and $x - y + 2 = 0$ is

$$(3x + 4y - 7) + k(x - y + 2) = 0 \quad \dots(i)$$

$$\text{Slope of the line} = -\frac{3+k}{4-k} = 3$$

$$\Rightarrow k = \frac{15}{2}$$

Hence, from (i) the required equation of the line is

$$(3x + 4y - 7) + \frac{15}{2}(x - y + 2) = 0$$

$$\Rightarrow 21x - 7y + 16 = 0$$

- 27** The equation of any line passing through the point of intersection of the lines $3x - 4y + 1 = 0$ and $5x + y - 1 = 0$ is

$$(3x - 4y + 1) + k(5x + y - 1) = 0 \quad \dots(i)$$

for intercept of this line with the x -axis, $y = 0$

$$\therefore 3x + 1 + k(5x - 1) = 0$$

$$\Rightarrow x = \frac{k-1}{5k+3}$$

For intercept of the line (i) on the y -axis, $x = 0$

$$\therefore -4y + 1 + k(y - 1) = 0$$

$$\Rightarrow y = \frac{k-1}{k-4}$$

Since the intercepts on the axes are equal.

$$\therefore \frac{k-1}{5k+3} = \frac{k-1}{k-4} \Rightarrow k = 1, \text{ or } x = -\frac{7}{4}$$

But $k \neq 1$, because if $k = 1$, the line (i) becomes $8x - 3y = 0$ which passes through the origin and therefore cannot make non-zero intercepts on the axis.

$$\therefore k = -\frac{7}{4} \text{ and from (i), we get}$$

$$3x - 4y + 1 - \frac{7}{4}(5x + y - 1) = 0$$

$\Rightarrow 23x + 23y = 11$, which is the required equation of the line.

- 28** We have $\frac{x}{a} - \frac{y}{b} = 1 \quad \dots(i)$

Since (i) passes through the point $(8, 6)$

$$\therefore \frac{8}{a} - \frac{6}{b} = 1 \quad \dots(ii)$$

The line (i) meets the x -axis at the point given by $y = 0$ and from (i) $x = a$ i.e., the line (i) meets the x -axis at the point $A(a, 0)$.

Similarly, the line meets the y -axis ($x = 0$) at the point $B(0, -b)$.

By the given condition, area of $\Delta = 12$

$$\Rightarrow \frac{1}{2}ab = 12$$

$$\Rightarrow ab = 24 \Rightarrow b = \frac{24}{a}$$

Substituting $b = \frac{24}{a}$ in (ii), we get

$$\frac{8}{a} = \frac{6}{\frac{24}{a}} = 1 \Rightarrow a = 4 \text{ or } -8 \text{ and } b = 6 \text{ or } -3$$

Hence, from (i) the equation of the straight line are

$$\frac{x}{4} - \frac{y}{6} = 1 \quad \text{and} \quad \frac{x}{-8} - \frac{y}{-3} = 1$$

$$\Rightarrow 3x - 2y = 12 \quad \text{and} \quad 3x - 8y + 24 = 0$$

- 29** The equation of the lines may be written as $3x + 4y + 2 = 0$ and $-5x + 12y + 6 = 0$ in which the constant terms 2 and 6 are both positive.

The equation of the bisector of the angle in which the origin lies is $\frac{3x + 4y + 2}{\sqrt{3^2 + 4^2}} = \frac{-5x + 12y + 6}{\sqrt{(-5)^2 + (12)^2}}$

$$\Rightarrow 16x - 12y - 1 = 0$$

The equation of the other bisector is

$$\frac{3x + 4y + 2}{\sqrt{3^2 + 4^2}} = \frac{-(-5x + 12y + 6)}{\sqrt{(-5)^2 + (12)^2}}$$

$$\Rightarrow x + 8y + 4 = 0$$

- 30** Let the equations of the sides BC , CA and AB of the triangle ABC be represented by

$$2y - x = 5 \quad \dots(i)$$

$$y + 2x = 7 \quad \dots(ii)$$

$$y - x = 1 \quad \dots(iii)$$

Solving the above 3 eqs. (i), (ii) and (iii), we get

$$A(2, 3), B(3, 4) \text{ and } C\left(\frac{9}{5}, \frac{17}{5}\right)$$

$\therefore$ The area of ΔABC

$$= \frac{1}{2} \left\{ 2 \times 4 - 3 \times 3 + 3 \times \frac{17}{5} - 4 \times \frac{9}{5} + \frac{9}{5} \times 3 - \frac{17}{5} \times 2 \right\}$$

$$= \frac{1}{2} \left(8 - 9 + \frac{51}{5} - \frac{36}{5} + \frac{27}{5} - \frac{34}{5} \right) = \frac{3}{10} \text{ units.}$$

- 31** Let $A(1, 2)$, $B(2, 3)$ and $C(4, 3)$ be the vertices of ΔABC .

$$m_1 = \text{Slope of } BC = \frac{3-3}{4-2} = 0 \text{ i.e., } BC \text{ is parallel to the}$$

x -axis.

$\therefore$ The perpendicular from $A(1, 2)$ to BC is parallel to the y -axis and its equation is $x = h$, which passes through $A(1, 2)$

$\therefore h = 1$ i.e., the equation of the perpendicular from $A(1, 2)$ on BC is $x = 1$...(i)

$$m_2 = \text{Slope of } AC = \frac{3-2}{4-1} = \frac{1}{3}$$

If m'_2 be the slope of a line perpendicular to AC then
 $m_2 m'_2 = -1$ or $\frac{1}{3} \cdot m'_2 = -1$ or $m'_2 = -3$

$\therefore$ The equation of the perpendicular from $B(2, 3)$ on AC whose slope is -3 is

$$y - 3 = -3(x - 2)$$

$$\Rightarrow 3x + y = 9 \quad \dots(ii)$$

The orthocentre is the point of intersection of two lines (i) and (ii)

$\therefore$ From eqs. (i) and (iii), we get

$$3 \times 1 + y = 9$$

$$\Rightarrow y = 6$$

$\therefore$ The required coordinates of the orthocentre are $(1, 6)$

- 32** Let $A(x_1, y_1)$ be the third vertex. Let AD, BE, CF be the perpendiculars from the vertices on the opposite side BC, CA, AB respectively. Then the orthocentre $O(0, 0)$ is the point of intersection of AD, BE and CF .

Since AD i.e. OA is perpendicular to BC .

$\therefore$ Slope of $BA \times$ slope of $BC = -1$

$$\text{or } \frac{y_1 - 0}{x_1 - 0} \times \frac{3 - (-1)}{-2 - 5} = -1$$

$$\Rightarrow y_1 = \frac{7x_1}{4} \quad \dots(i)$$

Again, since OB is perpendicular to CA

$$\therefore \frac{-1 - 0}{5 - 0} \times \frac{y_1 - 3}{x_1 + 2} = -1$$

$$\Rightarrow 5x_1 + 10 = y_1 - 3$$

$$\Rightarrow x_1 = -4 \quad \left(\because y_1 = \frac{7x_1}{4} \right)$$

$$\text{From eq. (i), } y_1 = \frac{7x_1}{4} = \frac{7 \times (-4)}{4} = -7$$

Hence, the required coordinates of the third vertex A are $(x_1, y_1) = (-4, -7)$

- 33** Let (x_1, y_1) be the third vertex then

$$y_1 = x_1 + 3 \quad \dots(i)$$

The area of the triangle formed by the points $(2, 1), (3, -2)$ and (x_1, y_1)

$$= \frac{1}{2} \{-4 - 3 + 3y_1 + 2x_1 + x_1 - 2y_1\}$$

$$= \frac{1}{2} (3x_1 + y_1 - 7)$$

By the given condition,

$$\pm \frac{1}{2} (3x_1 + y_1 - 7) = 5$$

$$\Rightarrow 3x_1 + y_1 - 7 = \pm 10$$

$$\Rightarrow 3x_1 + y_1 = 17 \quad \dots(ii)$$

$$\text{and } 3x_1 + y_1 = -3 \quad \dots(iii)$$

Solving eqs. (i) and (ii) we get

$$x_1 = \frac{7}{2}, y_1 = \frac{13}{2}$$

Solving eqs. (i) and (iii) we get

$$x_1 = -\frac{3}{2} \text{ and } y_1 = \frac{3}{2}$$

Hence, the coordinates of the third vertex is either $\left(\frac{7}{2}, \frac{13}{2}\right)$

$$\text{or } \left(-\frac{3}{2}, \frac{3}{2}\right)$$

- 34** Equation of any line L perpendicular to

$$5x - y = 1 \text{ is } x + 5y = k \quad \dots(i)$$

Where k is an arbitrary constant.

If this line cuts x -axis at A and y -axis at B , then for $A, y = 0$ and from (i) $x = k$ i.e., A is the point $(k, 0)$ for $B, x = 0$ and

from (i) $y = \frac{k}{5}$ i.e., B is the point $\left(0, \frac{k}{5}\right)$

$$\therefore \text{Area of the given } \Delta OAB = \frac{1}{2} (x_1 y_2 - x_2 y_1)$$

$$= \frac{1}{2} \left(\frac{k^2}{5} - 0 \right) = \frac{k^2}{10}$$

$$\text{But the given condition } \frac{k^2}{10} = 5$$

$$\text{or } k^2 = 50$$

$$\therefore k = \pm 5\sqrt{2}$$

Hence, from (i), the required equation of the line is

$$x + 5y = 5\sqrt{2}$$

$$\text{or } x + 5y = -5\sqrt{2}$$

- 35** Let $ABCD$ be the square and let $(1, 2)$ and $(3, 8)$ be the coordinates of opposite vertices A and C respectively.

The equation of the diagonal AC is

$$y - 2 = \frac{8 - 2}{3 - 1} (x - 1)$$

$$\Rightarrow 3x - y = 1$$

✓ Level 01 Basic Level Exercise

1. Total integral points = $1 + 2 + 3 + \dots + 19 = \frac{19 \times 20}{2} = 190$.

Hence choice (b) is the correct one.

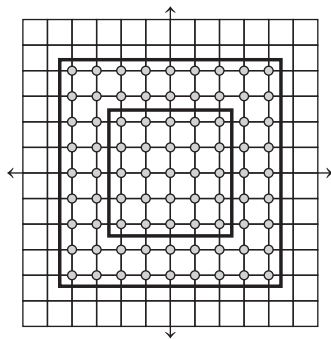
2. Area of the larger square is 91, so each side of this square will be little larger than 9 but less than 10.

From the following figure you can understand that each side of this square will have 9 integral points. So the total integral points this square will contain = $9 \times 9 = 81$.

And, area of the smaller square is 27, so each side of this square will be little larger than 5 but less than 6. From the following figure you can understand that each side of this square will have 5 integral points. So the total number of integral points contained by this square = $5 \times 5 = 25$

So the number of integer coordinates outside the smaller square region and inside the larger square region is

$$81 - 25 = 56$$



Hence choice (c) is the correct one.

3. Slope of $x + 3y = 4$ is $-\frac{1}{3}$. And slope of $6x - 2y = 7$ is 3.

Therefore, these two lines are perpendicular which shows that diagonals are perpendicular.

Hence PQRS must be a rhombus.

4. Since all three lines pass through a common point, therefore they will satisfy each other as follows.

$$3 = m(m + c) + 4$$

$$\Rightarrow m^2 + cm + 1 = 0$$

Since m is a real number, so the discriminant must be non-negative. That means, $c^2 - 4 \geq 0$

$$\Rightarrow c^2 \geq 4 \Rightarrow c \geq 2 \text{ or } -c \geq 2$$

$$\Rightarrow |c| \geq 2$$

Therefore, the least possible value of $|c|$ is 2.

Hence choice (c) is the correct one.

5. The distance between the given points

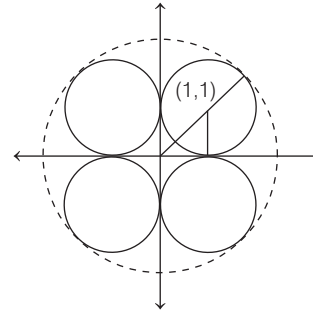
$$= \sqrt{(1-1)^2 + (1-5)^2} = 4$$

Since the given points lie on the circumference of the circle, so the diameter of the circle must be greater than or equal to 4. That means the radius of this circle must be greater than or equal to 2. That is $r \geq 2$

Therefore the area of the circle, $\pi r^2 \geq 4\pi$

Hence choice (a) is the correct one.

6. First we draw the four circles each with radius 1 unit and all of them would be symmetric as shown in the following diagram. So it is obvious that the centre of the larger circle would be origin (0, 0).



And the radius of the larger circle = distance between the origin and the centre of the smaller circle + radius of the smaller circle = $\sqrt{2} + 1$

Therefore the equation of the larger circle

$$= x^2 + y^2 = (\sqrt{2} + 1)^2$$

7. First of all draw a rectangle encapsulating the triangle.

Now, area of the desired triangle = Area of rectangle – Area of all the three triangles I, II and III.

Length of the rectangle = $3 - (-7) = 10$

Breadth of the rectangle = $4 - (-5) = 9$

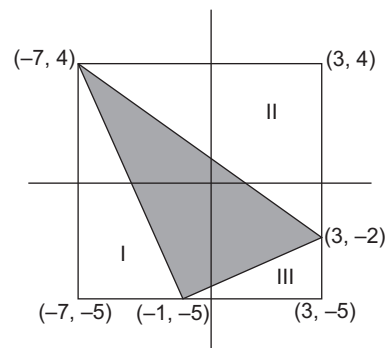
Therefore, area of the rectangle

$$= 10 \times 9 = 90$$

$$\text{Area of triangle I} = \frac{1}{2} (9 \times 6) = 27$$

$$\text{Area of triangle II} = \frac{1}{2} (10 \times 6) = 30$$

$$\text{Area of triangle III} = \frac{1}{2} (4 \times 3) = 6$$



Therefore area of the desired triangle

$$= 90 - (27 + 30 + 6) = 27.$$

Hence choice (a) is the correct one.